Adaptive control of robot manipulators with uncertain kinematics and actuator dynamics

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Adaptive Control of Robot Manipulators with Uncertain Kinematics and Actuator Dynamics

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirements for the degree of
Doctor of Philosophy

2006

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

.....

Date

.....

Liu Chao

To my family.

Acknowledgement

First and foremost, I would like to express my gratitude to my advisor, Prof. Cheah Chien Chern. He has been a constant source of inspiration, advice, and encouragement during all these years, both in my academic life and beyond. I benefited greatly from the unique perspective that he brought into the research leading to this work. I would like to thank him for sharing his profound insight in nonlinear systems and robot control theory, and for his endless words of encouragement. His enthusiasm has always been the driving force behind my research work.

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Summary

With the development of robot control theory and the requirement for robot to provide better performance in uncertain working environment, more and more research interests have been attracted to the two research topics of dealing with kinematic uncertainty existing in the robot system and considering actuator dynamics into the controller design during the past one decade. However, the studies in these two research areas have been developing separately in the literature.

This thesis is dedicated to solve the adaptive control problems of robot in the presence of all uncertainties in the kinematics, manipulator dynamics and actuator dynamics. The main contribution of the thesis is that a novel adaptive regressor design method is proposed which enables the adaptation of different kinds of uncertainties simultaneously during the control process. Based on the novel regressor design, adaptive control methods have been proposed for regulation and tracking control of robot with uncertain kinematics, manipulator and actuator dynamics. These approaches constitute the first results in the research area of robot control to deal with both kinematic and actuator dynamic uncertainties. Through Lyapunov analysis, the stability of the closed-loop systems are shown to be guaranteed and the sufficient conditions to be satisfied are explicitly presented.

The proposed control schemes have all been examined through simulation studies to confirm their effectiveness in dealing with the kinematic and dynamic uncertainties. Experiments were also conducted based on a vertical two-link robot driven by motors

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with current amplifiers.

Chapter 1

Introduction

Traditionally, robot manipulators are mainly used in industrial applications to increase speed of manufacturing or to reduce cost of production. The robot systems that are used in manufacturing applications perform mainly repetitive operations in a structured environment. With the rapid development in computing and sensor technologies, more sophisticated robots with high computing and sensing power can be designed. Robot systems are now used in applications where the environments are less structured than those inhabited by industrial robots, for example, space and undersea explorations, chemical and hazardous waste cleanup. Recently, there is also a growing market in using robots for service and entertainment purposes. Humanoid and toy robots are developed with skills like walking, catching and climbing.

All these new applications of robots present great challenge to previous robot control methods and more sophisticated and flexible control schemes are expected to meet the requirements of these new applications and provide high performance in the operations.

1.1 Literature Survey

The highly nonlinear and coupled nature of the dynamics and kinematics of a robot manipulator makes accurate robot motion control difficult. To solve the problem, many control schemes have been proposed in the literature.

The robot control schemes in the early era include the resolved motion rate control method as proposed by Whitney (1969) [94]. This method calculates the desired path of the tool endpoint of a manipulator in joint coordinates from a given desired path in Cartesian coordinates. Then by means of a velocity control servo-loop using the joint space desired path as the reference input signal, motion rate control can be realized. But this method gives rise to an accumulation of tracking errors in position level because positioning is established by taking an integral over a specified time interval.

Later on, an control scheme known as computed torque method was proposed independently by Bejczy (1974) [9], Takase (1976) [82], Vukobratovic (1982) [90], Hollerbach (1980) [40] and Luh et al (1980) [63]. This approach resort to nonlinear compensation to eliminate the interactions in robot dynamics. By computing the left hand-side of the dynamic equation for a given robot manipulator in real time for a predefined desired joint trajectory, the control torques are expected to be generated at each corresponding joint by actuators according to the online calculated command data. However, this method depends on exact kinematic and dynamic knowledge of the manipulator and hence there will be discrepancy between the computed torque and the actual torque in the presence of uncertainties in kinematics and dynamics. Moreover, even if the desired torque can be computed in real time with appropriate precision, it is still hard for the joint actuators to generate the required torque faithfully due to the uncertainties in actuator dynamics and the torque ripples. In addition, there will be a position error that can not be elimi-

nated if the actual initial state has some displacement from the measured state on which the desired input torque is computed. Control schemes were proposed later by Freund (1982) [35] based on linearization of Lagrange's equation by non-linear feedback, and then by Tarn et al. (1984) [86] based on a non-linear transformation proposed by Hunt (1983) [43]. These methods also require exact knowledge of the system kinematics and dynamics, and hence the problem of inaccurate calculation and generation of required torques arises in the presence of kinematic and dynamic uncertainties. The difficulty in obtaining the exact dynamic parameters also limits the application of the control method later proposed using differential geometry technique [27] [78].

In 1981, a new approach to the dynamic control of manipulator was developed from the viewpoint of mechanics by Takegaki and Arimoto [83]. Regardless of the complex dynamics of robotic motion, it was proved that a simple proportional and differential (PD) control with compensation for only the gravity term works well as far as setpoint control is concerned. Under this control method, the manipulator can be directly controlled in the task-oriented coordinate space and the stability of the system is easily assured. Moreover, this method is still applicable even when redundancy occurs in manipulator's coordinates and robust against uncertainty in physical parameters in the inertia and Coriolis and centrifugal terms except the gravity term. It was considered as an important landmark in robot control theory and also the first paper on application of Lyapunov's stability theory to robot control. In 1986, Arimoto and Miyazaki [5] furthered the PD control method by showing that the speed of convergence to the target position is of order of an exponentially decaying function and in case of robot arms with flexible joints, the globally asymptotically stable is still assured with this linear independent feedback law. The possibility of using desired gravity compensation instead of on-line gravity force compensation was also proposed without strict proof. The first rigorous proof came from Tomei (1991) [88] where the condition on the proportional gain matrix

was given explicitly in terms of the norm of the Hessian of the robot potential energy. While the PD control law is beautifully simple, it is far from perfect. If the gravity force is not compensated properly or unknown term of disturbances exist such as friction torques and evaluation errors of physical constants, steady offset appears. To reduce this kind of offset, Arimoto and Miyazaki (1983) [4] put forward a local PID feedback scheme for the position control of robot manipulators. In addition to the ordinary use of joint-angle position and velocity feedbacks at each joint actuator, the feedback of integration of position reference errors is introduced so as to reduce the steady-state offset. The PID feedback scheme can also be modified to suit the case when the target point is described in terms of the Cartesian coordinates or any other task-oriented coordinates. The PD, PID control methods having shown the power in regulation of robot manipulators, Kawamura, Miyazaki and Arimoto (1988) [47] showed that the two control methods were also efficient in robot motion tracking. It is proved that a robot can track a desired trajectory with sufficient accuracy, as long as a linear PD feedback loop is equipped at each joint of the robot and the velocity feedback gain can be set sufficiently large. In a similar way, a linear PID feedback controller that is capable of not only positioning but also trajectory tracking for robot motion is demonstrated.

In trying to find a local and independent PID-type servo-loop that would give rise to global asymptotic stability of setpoint control for non-linear mechanical systems, Arimoto [6] noted the importance of a class of artificial potentials that induce saturated vector fields. Then control methods SP-D, SP-ID, which use saturated-proportional error, were proposed [1] [2]. Using passivity analysis, systems under control of SP-D or SP-ID can be proved to be globally asymptotically stable. In the late 1980s, it was implicitly recognized among roboticists that one of the generic characteristics of robot dynamics must be “passivity” [3], which may enable simple control schemes such as the PD feedback control and the model based adaptive control to work well. The passivity approach tries to find the simplest output that makes

the system passive first, and then the Lyapunov function can be obtained as a kind of energy conservation law from a hyperstable feedback connection of the system with a linear positive real block. Later on, many results based on the “passivity” or “energy shaping” concepts are proposed for different control purposes.

In 1987, Slotine and Li [75] found a novel principle in adaptive control of mechanical systems which is a landmark for model-based adaptive control. The control algorithm proposed consists of estimated control input plus a PD feedback part, with the unknown manipulator and payload parameters being estimated online. The algorithm is computationally simple, because of an effective exploitation of the structure of manipulator dynamics. It requires neither feedback of joint accelerations nor inversion of the estimated inertia matrix. Conceptually different from the computed torque design techniques, the objective of this model-based adaptive control is not feedback linearization but preservation of the passivity properties of the rigid robot in the closed loop. Using the adaptive methodology, Tomei (1991) [88] proposed an adaptive PD controller. Different from Slotine and Li’s way, Tomei use regressor only for the gravity vector instead of the whole system. The proposed controller was designed to circumvent the problem of undesired steady-state errors in traditional PD control. The adaptive PD controller yields the global asymptotic stability of the whole system even if the inertia and gravity parameters are unknown, provided that upper and lower bounds of the inertia matrix are available. Following the same approach, an extension to the tracking problem is also presented [88]. Later, Kelly [48] presents an alternative analysis of the adaptive PD controller with desired gravity compensation for set-point control of robot manipulators developed in [88]. By using a suitable Lyapunov function, and without invoking LaSalle’s theorem, the closed-loop global asymptotic stability can be attained. By virtue of the adaptive design technique, many other tracking and regulation control schemes have been proposed in literature [23, 24, 33, 45, 84, 91, 93].

Noticing the facts that the majority of earlier robot controllers is based on complete feedback from the robot state variables and that the velocity measurements are often contaminated by noise, Nicosia and Tomei [66] proposed a robot control algorithm by using only joint position measurements. An asymptotic observer for estimating the joint speeds of robot manipulators was derived and then used in connection with existing state feedback controllers. The convergence was ensured provided that the observer gain constant satisfies suitable inequalities. Based on controller-observer combination method, a number of conceptually different methods for both regulation and tracking control of robots equipped with only position measurements had been developed. [10, 15, 34, 49].

After the experimental and theoretical studies carried out by Good et al. [38], Eppinger and Seering [31], Tarn et al. [85] showing the importance of inclusion of actuator dynamics in the control design, many approaches have been proposed regarding this research topic. Tarn et al. [85] applied nonlinear feedback linearization theory to this problem. Link acceleration measurement is needed for implementation. Taylor et al. [87] formulated the rigid-link electrically-driven (RLED) robot as a singular perturbation system. By using a time-scale separation, a controller was designed based on a “slow” reduced-order model of the original RLED system. Dawson et al. [26] derived a robust controller based on the general framework of [46] which achieves global uniform ultimate bounded stability. Other research works include robust control schemes [64] [60], adaptive schemes [12, 22, 36, 97], and hybrid schemes [39, 79, 80].

A very recent development in robot control theory is the study of manipulator control with uncertain kinematics. Several regulation control schemes have been proposed by Cheah et al. [16–18, 20], Dixon [29] and Huang et al. [42] respectively. In these works, the kinematics uncertainty is for the first time explicitly considered in the controller design and the proposed control schemes are shown to be capable of

assuring the system stability with the presence of uncertainty in robot kinematics.

1.2 Motivation of Research

For most robot applications, the desired position or path is specified in task space. To design controllers in joint space for such control tasks, inverse kinematics transformation should be carried out to get the desired joint-space position or trajectory. In order to avoid the problem of solving inverse kinematics, Takegaki and Arimoto [83] proposed a task-space controller for robot regulation in Cartesian space using a transposed Jacobian matrix and many other task-space control schemes are proposed later. Hence most research so far on robot control has assumed that the exact kinematics and Jacobian matrix of the manipulator from joint space to Cartesian space are known. This assumption implies that the exact lengths of the links, joint offsets and the object which the robot is holding must be known. Unfortunately, no physical parameters could be derived exactly. In addition, when the robot picks up objects or tools of different lengths, unknown orientations and gripping points, the overall kinematics is changing and therefore difficult to derive exactly. Hence if uncertainty exists in the robot kinematics, the controllers existed in the literature may give degraded performance or even incur instability.

Several research works [16–18, 20, 29, 42], as mentioned in the literature survey, have been proposed to study the kinematics uncertainty problem. However, these results are confined to the task-space regulation problem and no results for uncertain kinematics tracking control have been proposed. Moreover, in the proposed results the robot actuator dynamics is neglected in the controller design.

As pointed out by Good et al. [38], Eppinger and Seering [31], Tarn et al. [85], the actuator dynamics constitutes an important component of the complete robot dynamics and neglecting actuator dynamics may cause detrimental effects in the

performance of motion. In addition, the actuator dynamics are often found contain uncertainties due to the reasons that: the actuator dynamics parameters cannot be obtained exactly because of calibration error; some actuator parameters may change with the variation of working environment temperature and overheating of the motor.

It is observed that the studies in the area of uncertain robot kinematics and area of manipulator control including actuator dynamics and its uncertainty have developed separately in the literature. No results have been established that include the actuator dynamics in control design and can deal with the uncertainties in kinematics, manipulator dynamics and actuator dynamics. Also, for the research of another important actuator dynamics–joint flexibility, no results have been proposed involving the uncertain kinematics. These all present open problems in the robot control literature. An intuitive reasoning suggests that control schemes incorporating all these uncertainties will offer the robot more flexibility in operating in unknown working environment and better performance for applications with high speed and high accuracy requirements.

Robots are now finding their way into laboratories, research and exploration sites, hospitals and even outer space other than the traditional industrial plants. In these new applications, they are found facing totally different situations from that in industry. The environments can not be predicted in many cases and the system kinematics and/or dynamics may be uncertain or subject to change in operation. Traditional task space control schemes may no longer guarantee the control performance or even the system stability because of their design limits.

Both these new applications and the advancement of robot control theory itself request that more complicated control problems should be explored and high performance control schemes be developed which are able to deal with the kinematic, manipulator and actuator dynamic uncertainties and easy to apply for different

control tasks. This constitutes the main motivation of the research work in this thesis.

1.3 Major Contributions of the Thesis

The major contributions of this Ph.D thesis are summarized as follows:

- A novel adaptive regressor construction concept is proposed. The novelty of the adaptive regressor lies in that online updating information of the updated system parameters is utilized in the regressor instead of only using measurable system variables and parameters. It enables separate and simultaneous treatments of different kinds of uncertainties in system and also avoids the overparameterization problem often met in dealing with multiple uncertainties of the nonlinear systems. This regressor construction concept constitutes an important design technique for the results presented in this thesis.
- The first regulation results which can deal with the uncertainties in robot kinematics, manipulator dynamics and actuator dynamics all together are proposed. Three control methods are firstly proposed for robot with current-driven motors and uncertain kinematics. Then an adaptive control scheme is developed based on a backstepping procedure to solve the regulation problem of uncertain kinematics robot with voltage-driven motors.
- The first control approaches for tracking control of robot in the presence of uncertainties in kinematics, manipulator dynamics and actuator dynamics are proposed. The tracking results are developed based on nonlinear observers, and compared with other research works using the same observer design method no constraints are imposed on the initial system state variables.

- An adaptive control method is proposed as the first result for setpoint control of flexible (elastic) joint robot with uncertain kinematics.
- Rigorous proofs of the stability of closed-loop systems are provided through Lyapunov analysis method. Sufficient conditions to guarantee the system stability are explicitly presented.
- Simulation studies are provided for the control schemes proposed in this thesis and experiments based on a two-link robot were carried out for regulation and tracking control schemes of robots with current-driven motors and uncertain kinematics.

1.4 Organization of the Thesis

The remainder of the thesis is organized as follows:

In *Chapter 2* a brief introduction to the robot kinematics and dynamics is provided. The control task definition and kinematics properties of visual servoing are also briefly illustrated.

In *Chapter 3*, firstly, three task space controllers are proposed for the regulation problem of robot with uncertain kinematics and current-driven motors. A new adaptive control method is then proposed for the regulation of robot with voltage-driven motors and uncertain kinematics. Sufficient conditions for choosing the feedback gains and approximate Jacobian matrix are provided to guarantee system stability. Simulation results are presented to show the effectiveness of the proposed control scheme.

Chapter 4 addresses the tracking control problem of robot manipulator with uncertain kinematics and actuator dynamics. In the beginning, a new approximate Jacobian adaptive controller is proposed for trajectory tracking of robot with current-

driven motors. Then based on backstepping procedure, an approximate Jacobian control scheme is developed for the case of robot with voltage-driven motors. It is shown that the robot end effector is able to follow a desired trajectory with the uncertain kinematics and dynamics parameters being updated online by adaptive laws. Simulation results are presented to illustrate the performance of the controllers.

Chapter 5 addresses the uncertain kinematics regulation for robot manipulator with flexible joints. An adaptive regulation result is proposed to solve this problem. Simulation results are provided to verify the proposed control scheme.

Chapter 6 presents the practical implementations of control methods proposed in this thesis on a two-link direct drive robot. The setup of the experimental system is introduced. Experimental results show the practical feasibility of the control methods examined.

Chapter 7 gives conclusion of the thesis and suggests the topics of future research.

Chapter 2

Robot Kinematics and Dynamics

In all robot applications, completion of a generic task requires the execution of a specific motion prescribed to the manipulator's end-effector. The correct execution of the end-effector motion is entrusted to the control system which shall provide the joint actuators of the manipulator with the commands consistent with the desired motion trajectory. Control of end-effector motion demands an accurate analysis of the characteristics of the mechanical structure, actuators and sensors. The goal of such analysis is derivation of mathematical models of robot components.

In this chapter, the two main aspects of robot manipulator modelling: kinematics and dynamics (together with the actuator dynamics) are illustrated which constitute the basis of the control schemes synthesis in this thesis.

2.1 Robot Kinematics

Kinematic analysis of robot manipulator concerns the description of the manipulator motion with respect to a fixed reference coordinate frame by ignoring the forces and moments that cause motion of the manipulator.

2.1.1 Direct Kinematics and Jacobian Matrix

A robot manipulator consists of a series of rigid links connected by means of revolute or prismatic joints. The whole structure forms a kinematic chain. One end of the chain is constrained to a base and an end-effector or tool is connected to the other end. The mechanical structure of a manipulator is characterized by a number of degrees of freedom (DOF) which uniquely determine its configuration. Each degree of freedom is typically associated with a joint articulation and constitutes a joint variable. The space in which the vector of joint variables denoted by

$$q = [q_1 \ \cdots \ q_n]^T, \quad (2.1)$$

is defined is often referred to as *joint space*, where n denotes the number of manipulator joints and q_i is rotation angle for revolute joint and joint offset for prismatic joint. The space in which the manipulator task is specified is referred to as *task space* which could be Cartesian space or camera image space depending on different control task definitions.

2.1.1.1 Direct Kinematics

The direct kinematics equation of a manipulator allows the position and orientation of the end-effector frame to be expressed as a function of the joint variables with respect to the base frame or other task space frame. The direct kinematics equation can be written in the form: [3, 51, 52]

$$x = h(q), \quad (2.2)$$

where $x \in R^m$ represents the task space vector of the end-effector and $h(\cdot) \in R^n \rightarrow R^m$ represents a generally nonlinear function which computes the task space vari-

ables from the joint space variables.

2.1.1.2 Jacobian Matrix

In control applications, it's often necessary to calculate the analytical relationship between the joint motion and the end-effector motion in terms of velocities especially in task space control. If the end-effector position and orientation are specified in terms of a minimal number of parameters in the task spaces, the relationship can be obtained via differentiation of the direct kinematics function (2.2) with respect to the joint variables as: [72]

$$\dot{x} = J(q)\dot{q}, \quad (2.3)$$

where

$$J(q) = \frac{\partial h(q)}{\partial q} \quad (2.4)$$

is referred to as *analytical Jacobian matrix* or simply as *Jacobian matrix*. The Jacobian constitutes one of the most important tools for manipulator characterization. It's useful in finding singularity, analyzing redundancy, determining inverse kinematics, describing the mapping from forces applied to the end-effector to resulting torques at the joints and in designing task space control schemes.

A property of the kinematic equation described by equation (2.3) is stated as follows: [21]

Property 2.1 The right hand side of equation (2.3) is linear in a set of **kinematic parameters** $\theta_k = (\theta_{k1}, \dots, \theta_{kq})^T$, such as link lengths, joint offsets. Hence, equation (2.3) can be expressed as,

$$\dot{x} = J(q)\dot{q} = Y_k(q, \dot{q})\theta_k \quad (2.5)$$

where $Y_k(q, \dot{q}) \in R^{n \times q}$ is called the *kinematic regressor matrix*. ◇

2.1.1.3 A Simple Example of 3-Link Robot

To illustrate the formulation of direct kinematics equation and Jacobian of robot manipulator, a simple example of 3 link planar manipulator with all revolute joints as shown in Figure 2.1 is provided in the following.

Figure 2.1: 3 Link Planar Robot Manipulator

A Cartesian frame $[x \ y]$ with the base as the origin is chosen as the task space frame. Hence the task space position vector x can be derived in terms of the joint variables $q = [q_1 \ q_2 \ q_3]^T$ as:

$$x = \begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \end{bmatrix}, \quad (2.6)$$

where l_1, l_2, l_3 is the length of link 1, 2, 3 respectively and $c_1 = \cos(q_1)$, $c_{12} = \cos(q_1 + q_2)$, $c_{123} = \cos(q_1 + q_2 + q_3)$, $s_1 = \sin(q_1)$, $s_{12} = \sin(q_1 + q_2)$, $s_{123} = \sin(q_1 + q_2 + q_3)$.

Differentiate the direct kinematics equation with respect to time, one has

$$\begin{aligned}
 \dot{x} &= \begin{bmatrix} -l_1 s_1 \dot{q}_1 - l_2 s_{12}(\dot{q}_1 + \dot{q}_2) - l_3 s_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \\ l_1 c_1 \dot{q}_1 + l_2 c_{12}(\dot{q}_1 + \dot{q}_2) + l_3 c_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \end{bmatrix} \\
 &= \begin{bmatrix} -(l_1 s_1 + l_2 s_{12} + l_3 s_{123})\dot{q}_1 - (l_2 s_{12} + l_3 s_{123})\dot{q}_2 - l_3 s_{123}\dot{q}_3 \\ (l_1 c_1 + l_2 c_{12} + l_3 c_{123})\dot{q}_1 + (l_2 c_{12} + l_3 c_{123})\dot{q}_2 + l_3 c_{123}\dot{q}_3 \end{bmatrix} \\
 &= \begin{bmatrix} -(l_1 s_1 + l_2 s_{12} + l_3 s_{123}) & -(l_2 s_{12} + l_3 s_{123}) & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \\
 &= J(q)\dot{q}, \tag{2.7}
 \end{aligned}$$

where

$$J(q) = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12} + l_3 s_{123}) & -(l_2 s_{12} + l_3 s_{123}) & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \end{bmatrix} \tag{2.8}$$

is the Jacobian matrix for the 3 link manipulator.

In the presence of uncertainty in kinematic parameters, e.g., l_1, l_2, l_3 , the Jacobian matrix becomes uncertain and is expressed as

$$\hat{J}(q) = \begin{bmatrix} -(\hat{l}_1 s_1 + \hat{l}_2 s_{12} + \hat{l}_3 s_{123}) & -(\hat{l}_2 s_{12} + \hat{l}_3 s_{123}) & -\hat{l}_3 s_{123} \\ \hat{l}_1 c_1 + \hat{l}_2 c_{12} + \hat{l}_3 c_{123} & \hat{l}_2 c_{12} + \hat{l}_3 c_{123} & \hat{l}_3 c_{123} \end{bmatrix} \tag{2.9}$$

where $\hat{l}_1, \hat{l}_2, \hat{l}_3$ are the estimated values.

From equation (2.7), one has

$$\begin{aligned}
 \dot{x} &= J(q)\dot{q} \\
 &= \begin{bmatrix} -s_1\dot{q}_1 & -s_{12}(\dot{q}_1 + \dot{q}_2) & -s_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \\ c_1\dot{q}_1 & c_{12}(\dot{q}_1 + \dot{q}_2) & c_{123}(\dot{q}_1 + \dot{q}_2 + \dot{q}_3) \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \\
 &= Y_k(q, \dot{q})\theta_k
 \end{aligned} \tag{2.10}$$

where $Y_k(q, \dot{q})$ is the kinematic regressor matrix and θ_k is the kinematic parameter vector.

2.1.2 Robot Kinematics in Visual Servoing

The area of visual servoing has attracted great research interest in recent years [44]. The visual servo control system is characterized by using camera image information as the control feedback signal and hence the control goals are defined in camera image space directly. In this subsection the kinematics in visual servo system is presented as a special case.

Visual servo system typically adopts one of the two camera configurations [44, 53]: fixed camera, or camera-in-hand as illustrated by Figure 2.2 and Figure 2.3. In both configurations, the three-dimensional (3D) scene is captured by the camera and forms a two-dimensional (2D) array of brightness values. The projection of 3D object points in the scene space into the 2D image plane are referred to as *image feature points*. The captured 2D image can be processed using various techniques of image processing to enhance image quality and extract image feature points of interest. Fig 2.4 shows the simple model of a pinhole camera [55].

Let r represent the position of the end-effector in Cartesian coordinate frame and x represents the position vector of its image feature point in image coordinate frame

Figure 2.2: Robot System with Fixed Camera Configuration

[44]. The image velocity vector $\dot{x}$ is related to the joint velocity vector $\dot{q}$ as [32, 44, 69, 92],

$$\dot{x} = J_I(r)J_e(q)\dot{q} \quad (2.11)$$

where $J_I(r)$ is the image Jacobian matrix [44] and $J_e(q)$ is the manipulator Jacobian matrix of the mapping from joint space to Cartesian space. In the presence of uncertainties, the exact image Jacobian matrix and the manipulator Jacobian matrix cannot be obtained.

As an example, consider a 2-link planar robot with a fixed camera configuration and assume the camera is placed perpendicular to the plane where the robot evolves. The image Jacobian matrix is given by

$$J_I = \frac{f}{z - f} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}, \quad (2.12)$$

where β_1, β_2 denote the scaling factors in pixels/m, z is the perpendicular distance between the robot and the camera, f is the focal length of the camera. The Jacobian

Figure 2.3: Robot System with Camera-in-Hand Configuration

Figure 2.4: A Simple Camera Model

matrix $J_m(q)$ from joint space to Cartesian space for the 2-link robot is given by:

$$J_e(q) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}, \quad (2.13)$$

where l_1, l_2 are the link lengths, q_1 and q_2 are the joint angles, $c_1 = \cos q_1$, $s_1 = \sin q_1$, $c_{12} = \cos(q_1 + q_2)$, $s_{12} = \sin(q_1 + q_2)$. The constants $l_1, l_2, \beta_1, \beta_2, z$, and f are all unknown.

The image space velocity $\dot{x}$ can be derived as:

$$\begin{aligned} \dot{x} &= J_I J_e(q) \dot{q} \\ &= \frac{f}{z-f} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \\ &= \begin{bmatrix} -v_1 l_1 s_1 \dot{q}_1 - v_1 l_2 s_{12} (\dot{q}_1 + \dot{q}_2) \\ v_2 l_1 c_1 + v_2 l_2 c_{12} (\dot{q}_1 + \dot{q}_2) \end{bmatrix} \end{aligned} \quad (2.14)$$

where $v_1 = \frac{f\beta_1}{z-f}$, $v_2 = \frac{f\beta_2}{z-f}$.

Hence $\dot{x} = J_I J_e(q) \dot{q}$ can be written into the product of a known regressor matrix $Y_k(q, \dot{q})$ and an unknown constant vector θ_k where

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -s_1 \dot{q}_1 & -s_{12} (\dot{q}_1 + \dot{q}_2) & 0 & 0 \\ 0 & 0 & c_1 \dot{q}_1 & c_{12} (\dot{q}_1 + \dot{q}_2) \end{bmatrix} \begin{bmatrix} v_1 l_1 \\ v_1 l_2 \\ v_2 l_1 \\ v_2 l_2 \end{bmatrix} \\ &= Y_k(q, \dot{q}) \theta_k \end{aligned} \quad (2.15)$$

Note that for visual servoing applications, the image Jacobian matrix J_I and manipulator Jacobian matrix $J_e(q)$ can be written into one overall Jacobian matrix $J(q) = J_I J_e(q)$ of mapping from joint space to camera image space.

Remark 2.1: For 3D visual servoing, the non-adaptive Jacobian setpoint controllers still work as long as the Jacobian estimation error remains bounded. The adaptive tracking results are also effective in cases where the depth information is slowly time-varying. If the depth parameters are changing fast, then Neural Networks methods should be adopted to compensate the uncertainty. $\diamond$

2.2 Robot Dynamics

Kinematics of a manipulator represents the basis of a systematic and general derivation of manipulator dynamics, i.e., the equations of motion of the manipulator as a function of the forces and moments acting on it. Derivation of the dynamic model of robot manipulator and the according actuator system plays an important role for simulation of motion, analysis of system structures and design of control algorithms. In this section, the dynamics of a manipulator and some of its important properties are discussed first; then the actuator dynamics (electric motor and joint flexibility) are introduced to obtain a complete dynamical description of the arm plus actuators.

2.2.1 Dynamic Model of Robot Manipulator

The dynamic model of a manipulator provides a description of relationship between the joint actuator torques and the motion of the structure. Here Lagrange formulation is adopted for the derivation purpose. With Lagrange formulation, the equations of motion can be derived in a systematic way independently of the reference coordinate frame. The Lagrange's equation of motion for a conservative system is given by: [3]

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau, \quad (2.16)$$

where q is the n -vector of generalized coordinates q_i , τ is an n -vector of generalized forces τ_i , and the Lagrangian is the difference between the kinetic K and potential energy P

$$L = K - P. \quad (2.17)$$

In the derivation of robot manipulator dynamics, q will be the joint variable vector as previously defined and τ is a vector of torques corresponding to each joint. Then the Lagrangian of the manipulator system can be written as

$$\begin{aligned} L(q, \dot{q}) &= K(q, \dot{q}) - P(q) \\ &= \frac{1}{2} \dot{q}^T M(q) \dot{q} - P(q), \end{aligned} \quad (2.18)$$

where $M(q) \in R^{n \times n}$ is the manipulator inertial matrix.

Using the definition of $L(q, \dot{q})$, the dynamic equation of a robot manipulator with n degrees of freedom is given in joint space as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau, \quad (2.19)$$

where $C(q, \dot{q})\dot{q} = \dot{M}(q)\dot{q} - \frac{1}{2}\dot{q}^T \frac{\partial M(q)}{\partial q} \dot{q} \in R^n$ is the Coriolis/centripetal vector and $g(q) = \frac{\partial P(q)}{\partial q} \in R^n$ is the gravity force vector.

Some important properties of the dynamic equation (2.19) are summarized as follows which will be often used in the derivation of control schemes in this thesis: [3]

Property 2.2 The inertia matrix $M(q)$ is symmetric and positive definite. Moreover, it's bounded above and below, i.e.

$$\mu_m I \leq M(q) \leq \mu_M I, \quad (2.20)$$

where μ_m and μ_M are positive scalars denoting the lower and upper bound respectively. By saying that $\mu_m I \leq M(q)$ we mean matrix $(M(q) - \mu_m I)$ is positive semi-definite. $\diamond$

Property 2.3 The Coriolis/centripetal matrix $C(q, \dot{q})$ satisfies

$$C(q, x)y = C(q, y)x, \quad \forall x, y \in \mathbb{R}^n \quad (2.21)$$

$$\|C(q, x)\| \leq C_M \|x\|, \quad \forall q, x \in \mathbb{R}^n \quad (2.22)$$

where C_M is a positive scalar; in (2.20), (2.22) and throughout the rest of this thesis, the norm of a vector x is defined as

$$\|x\| = \sqrt{x^T x} \quad (2.23)$$

and the norm of a matrix A as

$$\|A\| = \sqrt{\lambda_{\max}(A^T A)} \quad (2.24)$$

with $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalue respectively. $\diamond$

Property 2.4 The matrix $S(q, \dot{q}) \equiv \frac{1}{2}\dot{M}(q) - C(q, \dot{q})$ is skew-symmetric and satisfies

$$y^T [\frac{1}{2}\dot{M}(q) - C(q, \dot{q})]y = 0 \quad \forall q, \dot{q}, y \in \mathbb{R}^n \quad (2.25)$$

$\diamond$

Property 2.5 The viscous friction matrix B_0 is positive definite. $\diamond$

Property 2.6 The gravity force vector $g(q)$ is bounded by a positive scalar:

$$\|g(q)\| \leq g_M \quad (2.26)$$

The gravity force vector $g(q)$ can be completely characterized by a set of physical parameters $\phi = (\phi_1, \dots, \phi_p)^T$ as [3, 48, 88]:

$$g(q) = Z(q)\phi = [z_1(q)\phi, \dots, z_n(q)\phi]^T, \quad (2.27)$$

where $Z(q) \in R^{n \times p}$ is the gravity regressor and $z_i(q) \in R^{1 \times p}$ ($i = 1, \dots, n$) is the i^{th} row of $Z(q)$, ϕ is the $p \times 1$ unknown parameter vector. $\diamond$

Property 2.7 The left side of equation (2.19) is linear in a set of physical parameters $\theta_d = (\theta_1, \dots, \theta_p)^T$ such as link masses, moments of inertia and damping factors and hence can be expressed as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Y_d(q, \dot{q}, \ddot{q})\theta_d, \quad (2.28)$$

where $Y_d(q, \dot{q}, \ddot{q})$ is called a dynamic regressor. $\diamond$

2.2.2 Actuator Dynamics

The dynamics of a rigid robot manipulator in joint space has been discussed in previous subsection. However, the robot needs actuators to generate the torques required to drive the manipulator links. Hence it's necessary to consider the actuator dynamics with manipulator dynamics to obtain a complete dynamic description of the manipulator plus actuators.

2.2.2.1 Dynamics of Robot with Current-Driven Motors

The actuators used to drive robot manipulator are generally either electric or hydraulic motors. In this thesis, the case of electric actuators is considered and the dynamic equation for hydraulic actuators can be described in a similar form.

If a permanent-magnet DC motor driven by an amplifier operating in *current mode* is used as an actuator at the i^{th} joint, then the differential equation of motion describing the rotational behavior of the motor is given by [3, 57]:

$$J_{mi}\ddot{q}_{mi} + B_{mi}\dot{q}_{mi} = K_{\tau i}I_{ai} - r_i\tau_i, \quad (2.29)$$

where q_{mi} denotes the angle of the motor rotor shaft, J_{mi} the inertia moment, B_{mi} the rotor damping coefficient, I_{ai} the motor armature current, $K_{\tau i}$ the motor torque constant. r_i is the transmission gear ratio defined as:

$$q_i = r_i q_{mi}. \quad (2.30)$$

From equations (2.19), (2.29) and (2.30), the dynamics of the robot with actuators can be given as:

$$(M_0 + M(q))\ddot{q} + (B_0 + B_c + C(q, \dot{q}))\dot{q} + g(q) = K_N I_a, \quad (2.31)$$

where B_0 denotes the viscous friction matrix, $M_0 = \text{diag}(J_{m1}/r_1^2, \dots, J_{mn}/r_n^2)$, $B_c = \text{diag}(B_{m1}/r_1^2, \dots, B_{mn}/r_n^2)$, $K_N = \text{diag}(K_{\tau 1}/r_1, \dots, K_{\tau n}/r_n)$, $I_a = (I_{a1}, \dots, I_{an})^T$.

For simplicity, the above dynamic equation can also be written as

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I, \quad (2.32)$$

where $M(q) \in R^{n \times n}$ represents the lumped positive definite inertia matrix, $B = B_0 + B_c$ denotes the damping matrix and $I \in R^n$ represents the armature current vector.

2.2.2.2 Dynamics of Robot with Voltage-Driven Motors

If DC motor driven by voltage amplifier is used as joint actuator and the armature inductance cannot be ignored, then combining motor equations with dynamic equation of robot manipulator leads to a set of third-order differential equations with armature voltage as control inputs [3, 22, 85]

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I, \quad (2.33)$$

$$L\dot{I} + RI + K_E \dot{q} = u. \quad (2.34)$$

where $I \in R^n$ is the vector of armature currents and $u \in R^n$ denotes the vector of armature voltages. $L \in R^{n \times n}$ represents the actuator inductance matrix; $R \in R^{n \times n}$ is the actuator resistance matrix, $K_E \in R^{n \times n}$ is the matrix characterizing the voltage constant of the actuator. L , R , K_E and K_N are positive definite constant diagonal matrices.

2.2.2.3 Dynamics of Rigid-Link Flexible-Joint Robot

Figure 2.5: Joint Flexibility

In most control scheme design, it's assumed that the coupling between the actuators and the robot links is provided through rigid gear trains. In some cases, the coupling suffers from gear train flexibility or elasticity. In this subsection, the dynamic equations of the manipulator with the flexible joints is briefly introduced.

The equations of motion for the rigid-link robot manipulator with flexible joints can be written as [30, 58, 77]

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_m(q_m - q), \quad (2.35)$$

$$J_m\ddot{q}_m + B_m\dot{q}_m + K_m(q_m - q) = u, \quad (2.36)$$

where $q_m = (q_{m1}, \dots, q_{mn})^T \in R^n$ with q_{mi} denoting the angle of motor rotor shaft, $K_m = \text{diag}\{k_{mi}\} \in R^{n \times n}$ denotes the positive definite stiffness constant matrix, $J_m = \text{diag}\{J_{mi}\}$ and $B_m = \text{diag}\{B_{mi}\}$ with J_{mi} , B_{mi} defined as in the previous subsection. u is the control torque input. Here for simplicity of expression and without loss of generality, it's assumed that the transmission gear ratio $r_i = 1$.

Remark 2.2: The research work presented in this thesis mainly deal with parametric uncertainties existing in robot kinematics and dynamics. However, it's noticeable that the three setpoint controllers proposed for robot with current-driven motors in Section 3.2 can also deal with structural uncertainty in kinematics. There a fixed approximate Jacobian matrix is used without adaptation in the kinematic parameters, and the sufficient stability conditions only concern the norm bound of estimation errors between the fixed approximate Jacobian and true Jacobian matrix. Hence the proposed controllers can tolerate both parametric and structural uncertainty in the Jacobian matrix. The details of the controllers development and stability proof are presented in Section 3.2. $\diamond$

Chapter 3

Adaptive Regulation of Robot with Uncertain Kinematics and Actuator Dynamics

3.1 Introduction

As discussed in Chapter 1, most of the control methods in the literature are designed in joint space and hence require to solve the inverse kinematics if the control task is defined in task space. Although transposed Jacobian methods are proposed to avoid the inverse kinematics problem, they are no longer valid when kinematic uncertainty exists and the Jacobian matrix becomes uncertain. Very recently, some results [17, 19, 29, 95] using approximate Jacobian matrix are proposed. However, these results neglected the actuator dynamics and hence can not guarantee high precision and control performance especially for high speed movements and highly varying loads.

This chapter is dedicated to the research of regulation problem of robots with un-

certain kinematics, manipulator dynamics and actuator dynamics. According to different motor types used to drive the robot manipulator and hence the different actuator dynamics, the problem is formulated based on robot with current-driven motors and robot with voltage-driven motors as discussed in Section 3.2 and Section 3.3 respectively. Several adaptive and non-adaptive control schemes are proposed for both systems to deal with the kinematic and dynamic uncertainties. The proposed schemes do not need accurate information about the actuator dynamics, manipulator dynamics and kinematics of the robot system. By using a novel adaptive regressor, the overparameterization problem which is often met in the adaptive control problem with uncertain motor torque transmission matrix is avoided. Hence compared with the existing results in literature, the dimension of the closed loop system is lower and the controller structure can be greatly simplified especially when degree of freedom of the robot increases. Sufficient conditions to guarantee system stability are provided and are illustrated in figure. Simulation results are presented in Section 3.4 to show the performance of the proposed controllers in the presence of uncertainties.

3.2 Adaptive Regulation of Robot with Current-Driven Motors

In this section, three setpoint control methods are first proposed for the regulation problem of robot with current-driven motors in the presence of uncertainties in kinematics, manipulator dynamics and motor torque transmission matrix. The result will be extended to robot with voltage-driven motors in the next section.

3.2.1 Adaptive Setpoint Control with Estimated Task-space Damping

As discussed in Section 2.2.2.1, the actuator system of robot with current-driven motors is characterized by a motor torque transmission matrix K_N and the dynamics of robot with n degrees of freedom can be expressed in joint space as:

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I, \quad (3.1)$$

In actual implementations of the robot controllers, calibration is necessary to identify the exact parameters of motor torque transmission matrix K_N in equation (3.1). However, the actuator model K_N may change as temperature varies due to overheating of motor or changes in ambient temperature. In addition, the robot kinematics and Jacobian matrix $J(q)$ may be uncertain in the presence of modeling error and when the robot picks up a tool of unknown length or gripping point. Hence, in the presence of modeling uncertainties or calibration errors of both actuator and robot kinematics, position error may be resulted.

According to the property of gravity force $g(q)$ as described in (2.27), the gravity force vector in the dynamic equation (3.1) can be completely characterized by a set of parameters $\phi = (\phi_1, \dots, \phi_p)^T$ as [3, 48, 88]:

$$g(q) = Z(q)\phi = [z_1(q)\phi, \dots, z_n(q)\phi]^T, \quad (3.2)$$

where $Z(q) \in R^{n \times p}$ is the gravity regressor and $z_i(q) \in R^{1 \times p}$ ($i = 1, \dots, n$) is the i^{th} row of $Z(q)$, ϕ is the $p \times 1$ unknown parameter vector. In the presence of manipulator dynamic uncertainty, ϕ is not available and hence $g(q)$ is estimated by

$$\hat{g}(q) = Z(q)\hat{\phi} = [z_1(q)\hat{\phi}, \dots, z_n(q)\hat{\phi}]^T, \quad (3.3)$$

where $\hat{\phi} \in R^p$ is an estimated parameter vector of ϕ .

Let $\hat{K}_N$ and $\hat{J}(q)$ be the motor torque transmission matrix and the approximate Jacobian matrix respectively chosen so that:

$$\|I - K_N \hat{K}_N^{-1}\| \leq \bar{\beta}, \quad (3.4)$$

$$\|J^T(q) - \hat{J}^T(q)\| \leq \bar{\gamma}, \quad (3.5)$$

where $\bar{\beta}$ and $\bar{\gamma}$ will be defined later.

Using the gravity regressor and approximate actuator model and Jacobian matrix, the control input is proposed as:

$$I = -\hat{K}_N^{-1}[\hat{J}^T(q)K_p s(e) + \hat{J}^T(q)K_v \hat{x} - Z(q)\hat{\phi} - Y(q, \hat{\phi})\hat{\varphi}], \quad (3.6)$$

$$\dot{\hat{\phi}} = -LZ^T(q)(\dot{q} + \alpha \hat{J}^+(q)s(e)), \quad (3.7)$$

$$\dot{\hat{\varphi}} = -L_a Y(q, \hat{\phi})(\dot{q} + \alpha \hat{J}^+(q)s(e)), \quad (3.8)$$

where $\hat{K}_N^{-1} \in R^{n \times n}$ is chosen to be positive definite and nonsingular such that its norm bound exists, $\hat{x} = \hat{J}(q)\dot{q}$ is the estimated task velocity vector, $e = x - x_d = (e_1, \dots, e_m)^T$ is a positional deviation from a desired position $x_d \in R^m$, x is measured by a sensor such as camera or laser tracking system, K_p , and K_v are positive definite diagonal feedback gains for the position and velocity respectively, L , L_a are positive definite diagonal matrices, α is a positive constant, $s_i(\cdot)$, $i = 1, \dots, n$ are saturated functions of e , $\hat{J}^+(q) = \hat{J}^T(q)(\hat{J}(q)\hat{J}^T(q))^{-1}$ is the pseudo-inverse of $\hat{J}(q)$ such that $\hat{J}(q)\hat{J}^T(q) \in R^{m \times m}$ is non-singular and $\hat{J}(q)\hat{J}^+(q) = I_E$, here I_E represents the identity matrix, and hence the proposed control method can apply to redundant robots.

The regressor $Y(q, \hat{\phi})$ is a diagonal matrix whose i^{th} diagonal element is the i^{th} entry

of estimated gravity force vector $\hat{g}(q) = [z_1(q)\hat{\phi}, \dots, z_n(q)\hat{\phi}]^T$ (see equation (3.3)):

$$Y(q, \hat{\phi}) = \begin{bmatrix} z_1(q)\hat{\phi} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & z_n(q)\hat{\phi} \end{bmatrix}, \quad (3.9)$$

and $\hat{\phi} \in R^n$ is the adaptive parameter vector whose updating law is given by equation (3.8). Note here the approximate actuator model $\hat{K}_N$ and the kinematic parameters used in $\hat{J}(q)$ are not updated by the control law.

Remark 3.1: In the controller, the gravity regressor $Z(q)$ and $\hat{\phi}$ are used to cope with the uncertainty in gravity force and the regressor $Y(q, \hat{\phi})$ and $\hat{\phi}$ are used to compensate the uncertainty caused by approximate actuator model. It is interesting to note that the novel regressor $Y(q, \hat{\phi})$ makes use of the updated information from $\hat{\phi}$ instead of fixed information. Using the adaptive regressor $Y(q, \hat{\phi})$, the overparameterization problem is avoided which is often met in adaptive control problem with uncertain actuator model. For a robot with n joints, if there are p unknown gravity dynamic parameters, the number of parameters updating for gravity compensation with uncertain actuator model is $n \times p$ in overparameterization method. Comparatively, using the control methods proposed in this thesis, the system dimension is same as the number of unknown parameters $n + p$ and hence complexity of the controller structure is greatly reduced especially when number of robot joints increases. $\diamond$

Let us define a scalar function $S_i(e)$ and its derivative $s_i(e)$ as shown in Fig. 3.1 and Fig. 3.2 and with the following properties [3]:

1. $S_i(e) > 0$ for $e \neq 0$ and $S_i(0) = 0$.
2. $S_i(e)$ is twice continuously differentiable, and the derivative $s_i(e) = \frac{dS_i(e)}{de}$ is strictly increasing in e for $|e| < \gamma_i$ with some γ_i and saturated for $|e| \geq \gamma_i$,

i.e. $s_i(e) = \pm s_i$ for $e \geq \pm \gamma_i$, and $e \leq -\gamma_i$ respectively where s_i is a positive constant.

3. There are constants $\hat{c}_i > 0$ such that for $e \neq 0$,

$$S_i(e) \geq \hat{c}_i s_i^2(e). \quad (3.10)$$

Figure 3.1: (a) Quasi-natural potential: $S(e)$

Figure 3.2: (b) Derivative of $S(e)$: $s(e)$

Some examples of the saturated function can be found in [3, 29, 50].

Remark 3.2: The saturation function $s(\cdot)$ was first introduced in joint space [3] to obtain global results for the theoretical proof of saturated PD and PID controllers. This is because the joint angles are usually revolute and hence, the joint error may not be bounded in practice. The objective of using $s(\cdot)$ in the approximate Jacobian controllers proposed in this thesis is similar. It is important to note that the positional error in task space is usually bounded since the robot workspace is usually finite. Hence, the saturation function may not be needed for most applications in task space. $\diamond$

Substituting equation (3.6) into equation (3.1), the closed-loop dynamic equation is obtained as

$$\begin{aligned} M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_v \hat{x} + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p s(e) + g(q) \\ - K_N \hat{K}_N^{-1} Z(q) \hat{\phi} - K_N \hat{K}_N^{-1} Y(q, \hat{\phi}) \hat{\phi} = 0. \end{aligned} \quad (3.11)$$

Since $(I - K_N \hat{K}_N^{-1})$ is a diagonal matrix, according to the definition of $Y(q, \hat{\phi})$ in equation (3.9), one has

$$\begin{aligned} (I - K_N \hat{K}_N^{-1}) Z(q) \hat{\phi} &= (I - K_N \hat{K}_N^{-1}) (z_1(q) \hat{\phi}, \dots, z_n(q) \hat{\phi})^T \\ &= \text{diag}[z_1(q) \hat{\phi}, \dots, z_n(q) \hat{\phi}] (1 - \frac{k_1}{\hat{k}_1}, \dots, 1 - \frac{k_n}{\hat{k}_n})^T \\ &= Y(q, \hat{\phi}) \phi_k, \end{aligned} \quad (3.12)$$

where $\phi_k = (1 - \frac{k_1}{\hat{k}_1}, \dots, 1 - \frac{k_n}{\hat{k}_n})^T$ is unknown since the exact actuator model is unknown, and k_i and $\hat{k}_i$ are the i^{th} diagonal elements of K_N and $\hat{K}_N$ respectively.

Substituting equation (3.12) into (3.11) and by using equation (3.2), the dynamic

equation can be written as:

$$\begin{aligned} M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_v \hat{x} + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p s(e) + Z(q)\phi \\ - Z(q)\hat{\phi} + Y(q, \hat{\phi})\phi_k - Y(q, \hat{\phi})K_N \hat{K}_N^{-1} \hat{\phi} = 0, \end{aligned} \quad (3.13)$$

note that $Y(q, \hat{\phi})$, $K_N \hat{K}_N^{-1}$ are diagonal matrices.

To study the stability of the closed-loop system, a Lyapunov function candidate V is proposed as

$$\begin{aligned} V = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \alpha \dot{q}^T M(q) \hat{J}^+(q) s(e) + \sum_{i=1}^m (\alpha k_{vi} + k_{pi}) S_i(e_i) + \frac{1}{2} (\phi - \hat{\phi})^T L^{-1} (\phi - \hat{\phi}) \\ + \frac{1}{2} (\hat{K}_N K_N^{-1} \phi_k - \hat{\phi})^T L_a^{-1} K_N \hat{K}_N^{-1} (\hat{K}_N K_N^{-1} \phi_k - \hat{\phi}), \end{aligned} \quad (3.14)$$

where $L_a^{-1} K_N \hat{K}_N^{-1}$ is a positive diagonal matrix. Since

$$\begin{aligned} \frac{1}{4} \dot{q}^T M(q) \dot{q} + \alpha \dot{q}^T M(q) \hat{J}^+(q) s(e) + \sum_{i=1}^m (\alpha k_{vi} + k_{pi}) S_i(e_i) \\ = \frac{1}{4} (\dot{q} + 2\alpha \hat{J}^+(q) s(e))^T M(q) (\dot{q} + 2\alpha \hat{J}^+(q) s(e)) \\ - \alpha^2 s^T(e) \hat{J}^{+T}(q) M(q) \hat{J}^+(q) s(e) + \sum_{i=1}^m (\alpha k_{vi} + k_{pi}) S_i(e_i) \\ \geq \sum_{i=1}^m (\alpha k_{vi} \hat{c}_i + k_{pi} \hat{c}_i - \alpha^2 \lambda_m) s_i^2(e_i), \end{aligned} \quad (3.15)$$

where α can be chosen small enough or k_{pi} and k_{vi} can be chosen large enough to satisfy the inequality,

$$\alpha k_{vi} \hat{c}_i + k_{pi} \hat{c}_i - \alpha^2 \lambda_m > 0, \quad (3.16)$$

where $\lambda_m = \lambda_{max}[\hat{J}^{+T}(q) M(q) \hat{J}^+(q)]$, $\lambda_{max}[A]$ denotes the maximum eigenvalue of a matrix A .

Substituting inequality (3.15) into equation (3.14), one has

$$\begin{aligned} V \geq & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \frac{1}{2}(\phi - \hat{\phi})^T L^{-1}(\phi - \hat{\phi}) + \sum_{i=1}^m (\alpha k_{vi}\bar{c}_{i1} + k_{pi}\bar{c}_{i2} - \alpha^2 \lambda_m) s_i^2(e_i) \\ & + \frac{1}{2}(\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi})^T L_a^{-1} K_N \hat{K}_N^{-1} (\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}) > 0. \end{aligned} \quad (3.17)$$

Hence, V is positive definite. Note that α must be chosen sufficiently small or K_v , K_p must be chosen sufficiently large to guarantee the positive definiteness of V .

Differentiating V with respect to time and substituting equations (3.7) and (3.8) into it, one has

$$\begin{aligned} \frac{d}{dt}V = & \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \alpha s^T(e)\hat{J}^{+T}\dot{M}(q)\dot{q} + \alpha s^T(e)\hat{J}^{+T}(q)M(q)\ddot{q} \\ & + \alpha \dot{q}^T M(q)\dot{\hat{J}}^+(q)s(e) + \alpha \dot{q}^T M(q)\hat{J}^+(q)\dot{s}(e) + \dot{x}^T K_p s(e) + \alpha \dot{x}^T K_v s(e) \\ & + (\dot{q} + \alpha \hat{J}^+(q)s(e))^T Z(q)(\phi - \hat{\phi}) + (\dot{q} + \alpha \hat{J}^+(q)s(e))^T Y(q, \hat{\phi})(\phi_k - K_N \hat{K}_N^{-1} \hat{\varphi}). \end{aligned} \quad (3.18)$$

From equation (3.1), one has

$$\begin{aligned} M(q)\ddot{q} = & -\{(B + C(q, \dot{q}))\dot{q} + K_N \hat{K}_N^{-1} \hat{J}^T(q)K_v \hat{x} + K_N \hat{K}_N^{-1} \hat{J}^T(q)K_p s(e) \\ & + Z(q)(\phi - \hat{\phi}) + Y(q, \hat{\phi})(\phi_k - K_N \hat{K}_N^{-1} \hat{\varphi})\}. \end{aligned} \quad (3.19)$$

Substituting equation (3.19) into equation (3.18), one has:

$$\frac{d}{dt}V = -W, \quad (3.20)$$

where

$$\begin{aligned}
W = & \dot{q}^T \{B + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_v \hat{J}(q)\} \dot{q} - \dot{q}^T \{J^T(q) K_p - K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p \\
& - \alpha \hat{J}^T(q) K_v \hat{J}(q) \hat{K}_N^{-1} K_N \hat{J}^+(q) + \alpha J^T(q) K_v\} s(e) + \alpha s^T(e) \hat{J}^{+T}(q) K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p s(e) \\
& + \alpha \{s^T(e) \hat{J}^{+T}(q) (B - \dot{M}(q) + C(q, \dot{q})) \dot{q} - \dot{s}^T(e) \hat{J}^{+T}(q) M(q) \dot{q} - s^T(e) \dot{\hat{J}}^{+T}(q) M(q) \dot{q}\}.
\end{aligned} \tag{3.21}$$

From the last term of equation (3.21), since $s(e)$ is bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that [3]:

$$\begin{aligned}
& \alpha |s^T(e) \hat{J}^{+T}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} - s^T(e) \dot{\hat{J}}^{+T}(q) M(q) \dot{q} \\
& - \dot{s}^T(e) \hat{J}^{+T}(q) M(q) \dot{q}| \geq -\alpha c_0 \|\dot{q}\|^2 - \alpha c_1 \|s(e)\|^2.
\end{aligned} \tag{3.22}$$

Substituting (3.22) into equation (3.21) and defining $\Delta_K = I - K_N \hat{K}_N^{-1}$, $\Delta_J = J^T(q) - \hat{J}^T(q)$, one has

$$\begin{aligned}
W \geq & \dot{q}^T [B + \hat{J}^T(q) K_v \hat{J}(q) - \Delta_K \hat{J}^T(q) K_v \hat{J}(q) - \alpha c_0 I] \dot{q} + \alpha s^T(e) [K_p \\
& - \hat{J}^{+T}(q) \Delta_K \hat{J}^T(q) K_p - \alpha c_1 I] s(e) - \dot{q}^T [\Delta_J K_p + \alpha \Delta_J K_v + \Delta_K \hat{J}^T(q) K_p \\
& + \alpha \hat{J}^T(q) K_v \hat{J}(q) \Delta_K \hat{J}^+(q)] s(e) \\
\geq & \{\lambda_{\min}[B + \hat{J}^T(q) K_v \hat{J}(q)] - c_2 \bar{\beta} \lambda_{\max}[K_v] - \alpha c_0\} \|\dot{q}\|^2 + \{\alpha \lambda_{\min}[K_p] \\
& - \alpha c_5 \bar{\beta} \lambda_{\max}[K_p] - \alpha c_1\} \|s(e)\|^2 - \{c_3 \bar{\beta} \lambda_{\max}[K_p] + \bar{\gamma} \lambda_{\max}[K_p] \\
& + \alpha c_4 \bar{\beta} \lambda_{\max}[K_v] + \alpha \bar{\gamma} \lambda_{\max}[K_v]\} \|\dot{q}\| \|s(e)\|,
\end{aligned} \tag{3.23}$$

Since,

$$-\|s(e)\| \cdot \|\dot{q}\| \geq -\frac{1}{2}(\|s(e)\|^2 + \|\dot{q}\|^2), \tag{3.24}$$

one has

$$\begin{aligned}
W \geq & \{\lambda_{\min}[B + \hat{J}^T(q)K_v\hat{J}(q)] - c_2\bar{\beta}\lambda_{\max}[K_v] - \frac{1}{2}c_3\bar{\beta}\lambda_{\max}[K_p] - \frac{1}{2}\bar{\gamma}\lambda_{\max}[K_p] \\
& - \frac{1}{2}\alpha c_4\bar{\beta}\lambda_{\max}[K_v] - \frac{1}{2}\alpha\bar{\gamma}\lambda_{\max}[K_v] - \alpha c_0\}\|\dot{q}\|^2 + \{\alpha\lambda_{\min}[K_p] - \alpha c_5\bar{\beta}\lambda_{\max}[K_p] \\
& - \frac{1}{2}c_3\bar{\beta}\lambda_{\max}[K_p] - \frac{1}{2}\bar{\gamma}\lambda_{\max}[K_p] - \frac{1}{2}\alpha c_4\bar{\beta}\lambda_{\max}[K_v] - \frac{1}{2}\alpha\bar{\gamma}\lambda_{\max}[K_v] - \alpha c_1\}\|s(e)\|^2,
\end{aligned} \tag{3.25}$$

where $c_2 = b_{jT}b_j$, $c_3 = b_{jT}$, $c_4 = b_{j+T}b_{jT}b_j$, $c_5 = b_{j+T}b_{jT}$, and b_{jT} , b_j , b_{j+} , b_{j+T} are the bounds for $\hat{J}^T(q)$, $\hat{J}(q)$, $\hat{J}^+(q)$, $\hat{J}^{+T}(q)$ respectively. Then one has:

$$W \geq (\lambda_{\max}[K_v]l_a - \alpha c_0)\|\dot{q}\|^2 + (\lambda_{\max}[K_v]l_b - \alpha c_1)\|s(e)\|^2, \tag{3.26}$$

where

$$\begin{aligned}
l_a &= \hat{\lambda}_1 - \bar{\beta}(c_2 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha), \\
l_b &= \alpha\hat{\lambda}_2a_1 - \bar{\beta}(\alpha c_5a_1 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha), \\
\hat{\lambda}_1 &= \frac{\lambda_{\min}[B + \hat{J}^T(q)K_v\hat{J}(q)]}{\lambda_{\max}[K_v]}, \quad \hat{\lambda}_2 = \frac{\lambda_{\min}[K_p]}{\lambda_{\max}[K_p]}, \quad a_1 = \frac{\lambda_{\max}[K_p]}{\lambda_{\max}[K_v]}
\end{aligned}$$

Hence, if the following conditions are satisfied:

$$\hat{\lambda}_1 - \bar{\beta}(c_2 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha) > 0, \tag{3.27}$$

$$\alpha\hat{\lambda}_2a_1 - \bar{\beta}(\alpha c_5a_1 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha) > 0. \tag{3.28}$$

That is

$$\min\left\{\frac{2\hat{\lambda}_1 - \bar{\gamma}(a_1 + \alpha)}{c_3a_1 + 2c_2 + \alpha c_4}, \frac{2\alpha\hat{\lambda}_2a_1 - \bar{\gamma}(a_1 + \alpha)}{2\alpha c_5a_1 + c_3a_1 + \alpha c_4}\right\} > \bar{\beta}, \tag{3.29}$$

$$\min\left\{\frac{2\hat{\lambda}_1 - \bar{\beta}(2c_2 + c_3a_1 + \alpha c_4)}{\alpha + a_1}, \frac{2\alpha\hat{\lambda}_2a_1 - \bar{\beta}(2\alpha c_5a_1 + c_3a_1 + \alpha c_4)}{\alpha + a_1}\right\} > \bar{\gamma}, \tag{3.30}$$

then $l_a > 0$ and $l_b > 0$ and hence K_v can be chosen large enough so that

$$l_a - \frac{\alpha c_0}{\lambda_{max}[K_v]} > 0, \quad l_b - \frac{\alpha c_1}{\lambda_{max}[K_v]} > 0, \quad (3.31)$$

and hence W is positive definite in $\dot{q}$ and $s(e)$.

From conditions (3.27) and (3.28), the tuning range for a_1 can be obtained with respect to uncertainty bounds:

$$\frac{\alpha c_4 \bar{\beta} + \alpha \bar{\gamma}}{2\alpha \hat{\lambda}_2 - (2\alpha c_5 + c_3)\bar{\beta} - \bar{\gamma}} < a_1 < \frac{\hat{\lambda}_1 - (2c_2 + \alpha c_4)\bar{\beta} - \alpha \bar{\gamma}}{c_3 \bar{\beta} + \bar{\gamma}} \quad (3.32)$$

As can be seen from the above inequality, a_1 can be chosen in a wider range when the estimation errors are smaller and a narrow range if the estimation errors are large.

Graphical illustrations of conditions (3.29) and (3.30) are shown in Figure 3.3, 3.4 and 3.2.3. Figure 3.3 shows the relation between $\bar{\beta}$ and a_1 with $\bar{\gamma} = 0$, i.e., with the actuator model uncertainty only, and Figure 3.4 shows the relation between $\bar{\gamma}$ and a_1 with $\bar{\beta} = 0$, i.e., with Jacobian matrix uncertainty only. The shaded area is the region where stability can be guaranteed. The figures show that with one uncertainty existing, a smaller a_1 allows more uncertainty in system. Figure 3.2.3 shows a 3-D illustration of the conditions when both uncertainties exist. Surface S_1 is described by condition (3.27) and surface S_2 is described by condition (3.28). The region between the two surfaces as indicated by the arrow is the region where stability can be guaranteed. From the figure, it's observed that the allowable bound $\bar{\beta}$ of actuator model uncertainty and bound $\bar{\gamma}$ of Jacobian uncertainty are inversely proportional to the ratio a_1 of positional feedback gain to velocity feedback gain. If the actuator model uncertainty $\bar{\beta}$ and/or the Jacobian uncertainty $\bar{\gamma}$ increase, a smaller a_1 is required. This relationship can also be seen from the condition (3.32). Hence, to allow more uncertainty in Jacobian matrix or actuator model, a_1 should

be kept smaller.

Figure 3.3: Variation of $\bar{\beta}$ with a_1 (with actuator uncertainty only)

Figure 3.4: Variation of $\bar{\gamma}$ with a_1 (with Jacobian uncertainty only)

We are now in a position to state the following Theorem:

Theorem 3.1 *The closed-loop system described by equation (3.7), (3.8) and (3.11) gives rise to the convergence of $x \rightarrow x_d$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$ if the feedback gains K_p and K_v are chosen to satisfy conditions (3.16), (3.29), (3.30), (3.31), $\hat{K}$ and $\hat{J}(q)$ are chosen to satisfy the condition (3.4) and (3.5) respectively.*

Figure 3.5: Variation of $\bar{\beta}$ and $\bar{\gamma}$ with a_1 (with both actuator uncertainty and Jacobian uncertainty)

Proof

Since W is positive definite in $s(e)$ and $\dot{q}$ and V is positive definite in $\dot{q}$, v , $s(e)$, $\phi - \hat{\phi}$ and $\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}$, V is bounded and $\dot{q}$, $s(e)$, $\hat{\phi}$, $\hat{\varphi}$ are all bounded. From inequality (3.26) and boundedness of V , one can conclude that $\dot{q}, s(e) \in L_2(0, \infty)$. From equation (3.11), it's easy to see that $\ddot{q}$ is bounded and hence $\dot{q}$ is uniformly continuous. Noting that $\dot{e} = J(q)\dot{q}$ and $\dot{q}$ is bounded, $s(e)$ is also uniformly continuous. It follows from [3] [28] that $\dot{q}$ and $s(e)$ converge to 0 as $t \rightarrow \infty$ which means $e = x - x_d$ also converge to 0 as $t \rightarrow \infty$. Hence the proof is complete. $\square$

Remark 3.3: The stability conditions (3.16), (3.29), (3.30), (3.31) presented in the Theorem are sufficient conditions to guarantee the stability of robot system in presence of uncertain actuator model and uncertain kinematics and dynamics. The conditions show the effects of control gains on system stability given a specific robot system and the approximate Jacobian and actuator model. Conditions (3.16), (3.31) mean that the feedback gains should be chosen sufficiently large. Conditions

(3.4), (3.5), (3.29) and (3.30) imply that the feedback gain K_p should be small as compared to K_v . If the system is unstable, $a_1 = \frac{\lambda_{max}[K_p]}{\lambda_{max}[K_v]}$ should be reduced so that the stable response could be achieved. (see Figure 3.3, 3.4 and Figure 3.2.3 also). As can be seen from condition (3.27), (3.28) and (3.31), when estimation errors $\bar{\beta}$ and $\bar{\gamma}$ are large, high control gains are required to stabilize the system. If $\bar{\beta}$ and $\bar{\gamma}$ are small, then small control gains may be sufficient to satisfy the stability conditions. However, if the estimation errors are too large, suitable control gains may not exist as can be seen from inequality (3.32). $\diamond$

Remark 3.4: A simpler but more conservative condition can also be derived on the estimation errors from condition (3.29) and (3.30) to guarantee the system stability. Let $\bar{\Lambda} = \max\{\bar{\beta}, \bar{\gamma}\}$, it can be got from inequality (3.27) and (3.28):

$$\min\left\{\frac{2\hat{\lambda}_1}{2c_2 + \alpha(c_4 + 1) + (c_3 + 1)a_1}, \frac{2\alpha a_1 \hat{\lambda}_2}{\alpha(c_4 + 1) + (2\alpha c_5 + c_3 + 1)a_1}\right\} > \bar{\Lambda}, \quad (3.33)$$

The graphical illustration of this condition is similar to Figure 3.3 or Figure 3.4 shown above. $\diamond$

3.2.2 Adaptive Setpoint Control without using Inverse Jacobian Matrix

In the previous subsection, a control method using pseudo-inverse $\hat{J}^+(q)$ in the updating laws is proposed. Therein the control signal may be very large when the manipulator operates near the singular points of the approximate Jacobian matrix. In this subsection, a control method without using the pseudo-inverse Jacobian matrix $\hat{J}^+(q)$ is proposed. Based on the approximate actuator model and Jacobian

matrix, the controller is proposed as

$$I = -\hat{K}_N^{-1}[\hat{J}^T(q)K_p s(e) + B_v \dot{q} - Z(q)\hat{\phi} - Y(q, \hat{\phi})\hat{\varphi}], \quad (3.34)$$

$$\dot{\hat{\phi}} = -LZ^T(q)(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (3.35)$$

$$\dot{\hat{\varphi}} = -L_a Y(q, \hat{\phi})(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (3.36)$$

where $B_v = b_v I_E$ is a positive definite diagonal matrix where I_E represents identity matrix. Hence the closed-loop dynamic equation of the system can be written as:

$$\begin{aligned} M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + K_N \hat{K}_N^{-1} \hat{J}^T(q)K_p s(e) + K_N \hat{K}_N^{-1} B_v \dot{q} \\ Z(\phi - \hat{\phi}) + Y(q, \phi)(\phi_k - K_N \hat{K}_N^{-1} \hat{\varphi}) = 0. \end{aligned} \quad (3.37)$$

Then a Lyapunov function candidate V' is defined with the form as follows:

$$\begin{aligned} V' = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \alpha \dot{q}^T M(q) \hat{J}^T(q) s(e) + \sum_{i=1}^m (\alpha b_v + k_{pi}) S_i(e_i) + \frac{1}{2} (\phi - \hat{\phi})^T L^{-1} (\phi - \hat{\phi}) \\ + \frac{1}{2} (\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi})^T L_a^{-1} K_N \hat{K}_N^{-1} (\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}), \end{aligned} \quad (3.38)$$

Since V' possesses a similar form with V , it's easy to verify that V' is positive definite provided that α , k_{pi} and b_v satisfy:

$$\alpha b_v \hat{c}_i + k_{pi} \hat{c}_i - \alpha^2 \lambda'_m > 0, \quad (3.39)$$

where $\lambda'_m = \lambda_{max}[\hat{J}(q)M(q)\hat{J}^T(q)]$. Differentiating V' with respect to time, one can get:

$$\frac{d}{dt} V' = -W', \quad (3.40)$$

where

$$\begin{aligned}
 W' = & \dot{q}^T (B + b_v K_N \hat{K}_N^{-1}) \dot{q} + \dot{q}^T [K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p + \alpha b_v K_N \hat{K}_N^{-1} \hat{J}^T(q) \\
 & - J^T(\alpha b_v I + K_p)] s(e) + \alpha s^T(e) \hat{J}(q) K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p s(e) \\
 & + \alpha \{ s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q} \}
 \end{aligned} \tag{3.41}$$

Applying the similar arguments as in previous subsection, one has:

$$\begin{aligned}
 W' & \geq (b_v + \lambda_{\min}[B] - b_v \bar{\beta} - \alpha c_0) \|\dot{q}\|^2 + (\alpha c_2 \lambda_{\min}[K_p] - \alpha c_2 \lambda_{\max}[K_p] \bar{\beta} - \alpha c_1) \|s(e)\|^2 \\
 & \quad - (\alpha b_v + \lambda_{\max}[K_p]) (\bar{\gamma} + \bar{\beta} c_3) \|\dot{q}\| \|s(e)\| \\
 & \geq (b_v l_c - \alpha c_0) \|\dot{q}\|^2 + (b_v l_d - \alpha c_1) \|s(e)\|^2
 \end{aligned} \tag{3.42}$$

where

$$\begin{aligned}
 l_c &= 1 + \hat{\lambda}_3 - (\tfrac{1}{2} \alpha c_3 + \tfrac{1}{2} a_2 c_3 + 1) \bar{\beta} - \tfrac{1}{2} (\alpha + a_2) \bar{\gamma}, \\
 l_d &= \alpha c_2 a_2 \hat{\lambda}_4 - (\tfrac{1}{2} \alpha c_3 + \tfrac{1}{2} a_2 c_3 + \alpha c_2 a_2) \bar{\beta} - \tfrac{1}{2} (\alpha + a_2) \bar{\gamma}, \\
 \hat{\lambda}_3 &= \frac{\lambda_{\min}[B]}{b_v}, \quad \hat{\lambda}_4 = \frac{\lambda_{\min}[K_p]}{\lambda_{\max}[K_p]}, \quad a_2 = \frac{\lambda_{\max}[K_p]}{b_v}
 \end{aligned} \tag{3.43}$$

and c_0, c_1, c_2, c_3 are defined in the same way as in last subsection.

Hence if the following conditions are satisfied:

$$\min \left\{ \frac{2 + 2\hat{\lambda}_3 - (\alpha + a_2) \bar{\gamma}}{\alpha c_3 + a_2 c_3 + 2}, \frac{2\alpha c_2 a_2 \hat{\lambda}_4 - (\alpha + a_2) \bar{\gamma}}{\alpha c_3 + a_2 c_3 + 2\alpha c_2 a_2} \right\} > \bar{\beta}, \tag{3.44}$$

$$\min \left\{ \frac{2 + 2\hat{\lambda}_3 - (\alpha c_3 + a_2 c_3 + 2) \bar{\beta}}{\alpha + a_2}, \frac{2\alpha c_2 a_2 \hat{\lambda}_4 - (2\alpha c_2 a_2 + \alpha c_3 + a_2 c_3) \bar{\beta}}{\alpha + a_2} \right\} > \bar{\gamma}, \tag{3.45}$$

then $l_a > 0$ and $l_b > 0$ and hence K_v can be chosen large enough so that

$$l_c - \frac{\alpha c_0}{b_v} > 0, \quad l_d - \frac{\alpha c_1}{b_v} > 0, \quad (3.46)$$

and hence W' is positive definite in $\dot{q}$ and $s(e)$.

Theorem 3.2 *The closed-loop system described by equation (3.35), (3.36) and (3.37) gives rise to the convergence of $x \rightarrow x_d$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$ if the feedback gains K_p and B_v are chosen to satisfy conditions (3.39), (3.44), (3.45), (3.46), $\hat{K}_N$ and $\hat{J}(q)$ are chosen to satisfy the condition (3.4) and (3.5) respectively.*

Proof

Since $W' = -\frac{d}{dt}V'$ is positive definite in $s(e)$ and $\dot{q}$ and V' is positive definite in $\dot{q}$, $v, s(e)$, $\phi - \hat{\phi}$ and $\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}$, V' is bounded and $\dot{q}$, $s(e)$, $\hat{\phi}$, $\hat{\varphi}$ are all bounded. From inequality (3.42), one can conclude that $\dot{q}, s(e) \in L_2(0, \infty)$. From equation (3.37), it's easy to see that $\ddot{q}$ is bounded and hence $\dot{q}$ is uniformly continuous. Noting that $\dot{e} = J(q)\dot{q}$ and $\dot{q}$ is bounded, $s(e)$ is also uniformly continuous. It follows from [3] [28] that $\dot{q}$ and $s(e)$ converge to 0 as $t \rightarrow \infty$ which means $e = x - x_d$ also converge to 0 as $t \rightarrow \infty$. Hence the proof is complete. $\square$

3.2.3 SPID Setpoint Control with Integration Compensation

The previous two subsections present the adaptive setpoint controllers possessing saturated proportional-derivative (SPD) structure with the gravity force compensated through the construction of gravity regressor $Z(q)$ and online adaptation of manipulator dynamic parameters. In this subsection, a SPID-formed controller is proposed which uses integration for gravity compensation and hence saves control design effort for the gravity regressor. Moreover, it's noticeable that in this control

method there's no adaptation for the actuator model uncertainty while the asymptotic stability of the system can still be guaranteed with certain conditions satisfied. Hence the controller proposed has a much simpler structure and consequently the online computation burden can be reduced.

Using the approximate actuator model $\hat{K}_N$ and Jacobian matrix $\hat{J}(q)$ which are chosen to satisfy

$$\|I - K_N \hat{K}_N^{-1}\| \leq \bar{\beta}, \quad (3.47)$$

$$\|J^T(q) - \hat{J}^T(q)\| \leq \bar{\gamma}, \quad (3.48)$$

the controller is proposed in the form:

$$I = -\hat{K}_N^{-1}(\hat{J}^T(q)K_p s(e) + \hat{J}^T(q)K_v \hat{x}) - \hat{K}_N^{-1}K_I \int_0^t y(\tau) d\tau, \quad (3.49)$$

where

$$y = \dot{q} + \alpha \hat{J}^+(q)s(e) \quad (3.50)$$

and x , $\hat{x}$, e , K_p , K_v , and $\hat{J}^+(q)$, $\hat{K}_N^{-1}$, $\hat{J}(q)$ are defined in the same way as in the previous subsections. K_I is a positive definite diagonal feedback gain.

Substituting the control input into equation (3.1), we can get the closed-loop system dynamics:

$$\begin{aligned} M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) - g(q_d) + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_v \hat{x} \\ + K_N \hat{K}_N^{-1} \hat{J}^T(q) K_p s(e) + K_N \hat{K}_N^{-1} K_I z = 0, \end{aligned} \quad (3.51)$$

and $K_I^{-1} \hat{K} K^{-1} g(q_d) + \int_0^t y(\tau) d\tau$, $K_N \hat{K}_N^{-1} K_I$ is a diagonal matrix since K_N , $\hat{K}_N^{-1}$ and K_I are all diagonal matrices. Note that q_d , $g(q_d)$ are introduced for the purpose of analysis and are not required in the controller as can be seen from equation (3.49).

A Lyapunov function candidate can then be defined as:

$$\begin{aligned} V'' = & \frac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^+(q)s(e) + \sum_{i=1}^m (k_{pi} + \alpha k_{vi})S_i(e_i) \\ & + P(q) - P(q_d) - \Delta q^T g(q_d) + \frac{1}{2}z^T K_N \hat{K}_N^{-1} K_I z, \end{aligned} \quad (3.52)$$

where $\Delta q = q - q_d$, k_{pi} , k_{vi} denote the i^{th} diagonal elements of K_p and K_v respectively.

$P(q)$ denotes the potential energy function of the manipulator.

Similarly as in previous two subsections, it can be obtained that

$$\begin{aligned} & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^+(q)s(e) + \sum_{i=1}^m \alpha k_{vi} S_i(e_i) \\ & \geq \sum_{i=1}^m \{\alpha(k_{vi}\bar{c}_i - \alpha\lambda_m)\} s_i^2(e_i) \end{aligned} \quad (3.53)$$

α and K_v are chosen to satisfy

$$k_{vi}\bar{c}_i - \alpha\lambda_m > 0 \quad (3.54)$$

where $\lambda_m = \lambda_{max}[\hat{J}^{+T}(q)M(q)\hat{J}^+(q)]$.

Since $P(q)$ is a trigonometric function of q , the following conditions with specified constant $k > 0$ can be satisfied simultaneously for any fixed q_d and any q :

$$P(q) - P(q_d) - \Delta q^T g(q_d) + \sum_{i=1}^n k_{pi} S_i(e_i) \geq k\|s(e)\|^2 \quad (3.55)$$

$$s(e)^T \hat{J}^{+T}(q)\{g(q) - g(q_d)\} + s(e)^T K_p s(e) \geq k\|s(e)\|^2 \quad (3.56)$$

Substituting equation (3.53) and (3.55) into equation (3.52), we have

$$\begin{aligned} V'' \geq & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^m \{\alpha(k_{vi}\bar{c}_i - \alpha\lambda_m)\} s_i^2(e_i) + k\|s(e)\|^2 \\ & + \frac{1}{2}z^T K_N \hat{K}_N^{-1} K_I z > 0. \end{aligned} \quad (3.57)$$

Differentiating V'' with respect to time and using equation (3.51), we have:

$$\frac{d}{dt}V'' = -W'', \quad (3.58)$$

where

$$\begin{aligned} W'' = & \dot{q}^T (B + \hat{J}^T(q)K_v\hat{J}(q) - \Delta_K\hat{J}^T(q)K_v\hat{J}(q))\dot{q} - \dot{q}^T\Delta_JK_ps(e) - \dot{q}^T\Delta_K\hat{J}(q)K_ps(e) \\ & - \alpha\dot{q}^T\Delta_JK_vs(e) - \alpha s^T(e)\hat{J}^{+T}(q)\Delta_K\hat{J}^T(q)K_v\hat{J}(q)\dot{q} - \alpha s^T(e)\hat{J}^{+T}(q)\Delta_K\hat{J}^T(q)K_ps(e) \\ & + \alpha s^T(e)\hat{J}^{+T}(q)[g(q) - g(q_d)] + \alpha s^T(e)K_ps(e) + \alpha\{s^T(e)\hat{J}^{+T}(q)[B - \frac{1}{2}\dot{M}(q) + S(q, \dot{q})]\dot{q} \\ & - \dot{q}^TM(q)\hat{J}^+(q)s(e) - \dot{q}^TM(q)\hat{J}^+(q)\dot{s}(e)\} \end{aligned} \quad (3.59)$$

where $\Delta_K = I - K_N\hat{K}_N^{-1}$, $\Delta_J = J^T(q) - \hat{J}^T(q)$.

Using inequality (3.56), we have

$$\begin{aligned} W'' \geq & \{\lambda_{\min}[B + \hat{J}^T(q)K_v\hat{J}(q)] - \bar{\beta}c_2\lambda_{\max}[K_v] - \frac{1}{2}\bar{\gamma}\lambda_{\max}[K_p] - \frac{1}{2}\bar{\beta}c_3\lambda_{\max}[K_p] \\ & - \frac{1}{2}\alpha\bar{\gamma}\lambda_{\max}[K_v] - \frac{1}{2}\alpha c_4\bar{\beta}\lambda_{\max}[K_v] - \alpha c_0\} \|\dot{q}\|^2 + \{\alpha k - \alpha c_5\bar{\beta}\lambda_{\max}[K_p] - \frac{1}{2}\bar{\gamma}\lambda_{\max}[K_p] \\ & - \frac{1}{2}\bar{\beta}c_3\lambda_{\max}[K_p] - \frac{1}{2}\alpha\bar{\gamma}\lambda_{\max}[K_v] - \frac{1}{2}\alpha c_4\bar{\beta}\lambda_{\max}[K_v] - \alpha c_1\} \|s(e)\|^2 \end{aligned} \quad (3.60)$$

where c_0, c_1, c_2, c_3 have the same definition as in previous section, $c_4 = b_{j_T}b_{j_T}b_{j_{+T}}$, $c_5 = b_{j_{+T}}b_{j_T}$.

Then we can have:

$$W'' \geq (\lambda_{\max}[K_v]l_e - \alpha c_0)\|\dot{q}\|^2 + (\lambda_{\max}[K_v]l_f - \alpha c_1)\|s(e)\|^2 \quad (3.61)$$

where

$$\begin{aligned} l_e &= \hat{\lambda}_5 - \bar{\beta}(c_2 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \bar{\gamma}(\frac{1}{2}a_1 + \frac{1}{2}\alpha), \\ l_f &= \alpha\hat{\lambda}_6a_1 - \bar{\beta}(\alpha c_5a_1 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \bar{\gamma}(\frac{1}{2}a_1 + \frac{1}{2}\alpha), \\ \hat{\lambda}_5 &= \frac{\lambda_{\min}[B + \hat{J}^TK_v\hat{J}]}{\lambda_{\max}[K_v]}, \hat{\lambda}_6 = \frac{k}{\lambda_{\max}[K_p]}, a_1 = \frac{\lambda_{\max}[K_p]}{\lambda_{\max}[K_v]}, \end{aligned} \quad (3.62)$$

Hence, if following conditions are satisfied:

$$\hat{\lambda}_5 - \bar{\beta}(c_2 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4) - \frac{1}{2}\bar{\gamma}(a_1 + \alpha) > 0 \quad (3.63)$$

$$\alpha\hat{\lambda}_6a_1 - \bar{\beta}[\alpha c_5a_1 + \frac{1}{2}c_3a_1 + \frac{1}{2}\alpha c_4] - \frac{1}{2}\bar{\gamma}(a_1 + \alpha) > 0 \quad (3.64)$$

That is

$$\min\left\{\frac{2\hat{\lambda}_5 - \bar{\gamma}(a_1 + \alpha)}{c_3a_1 + 2c_2 + \alpha c_4}, \frac{2\alpha\hat{\lambda}_6a_1 - \bar{\gamma}(a_1 + \alpha)}{(2\alpha c_5 + c_3)a_1 + \alpha c_4}\right\} > \bar{\beta} \quad (3.65)$$

$$\min\left\{\frac{2\hat{\lambda}_5 - (2c_2 + \alpha c_4 + c_3a_1)\bar{\beta}}{a_1 + \alpha}, \frac{2\alpha\hat{\lambda}_6a_1 - \bar{\beta}(2\alpha c_5a_1 + c_3a_1 + \alpha c_4)}{a_1 + \alpha}\right\} > \bar{\gamma} \quad (3.66)$$

then $l_e > 0$ and $l_f > 0$ and hence K_v can be chosen large enough so that

$$l_e - \frac{\alpha c_0}{\lambda_{\max}[K_v]} > 0, \quad l_f - \frac{\alpha c_1}{\lambda_{\max}[K_v]} > 0, \quad (3.67)$$

and hence W'' is positive definite in $s(e)$ and $\dot{q}$.

Theorem 3.3 *The closed loop system described by equation (3.51) gives rise to the asymptotic convergence of $x, \dot{q}$ to $x_d, 0$ as $t \rightarrow \infty$, if control gains α and K_p, K_v are chosen to satisfy the conditions (3.54), (3.55), (3.56), (3.65), (3.66), $\hat{K}_N$ and $\hat{J}(q)$ are chosen to satisfy the condition (3.47) and (3.48) respectively.*

Proof

Since $W'' = -\frac{d}{dt}V''$ is positive definite in $s(e)$ and $\dot{q}$ and V' is positive definite in $\dot{q}$, $s(e)$ and z , V'' is bounded and $\dot{q}$, $s(e)$, z are all bounded. From inequality (3.61), one can conclude that $\dot{q}, s(e) \in L_2(0, \infty)$. From equation (3.51), it's easy to see that $\ddot{q}$ is bounded and hence $\dot{q}$ is uniformly continuous. Noting that $\dot{e} = J(q)\dot{q}$ and $\dot{q}$ is bounded, $s(e)$ is also uniformly continuous. It follows from [3] [28] that $\dot{q}$ and $s(e)$ converge to 0 as $t \rightarrow \infty$ which means x converge to x_d as $t \rightarrow \infty$. Hence the proof is complete. $\square$

3.3 Adaptive Regulation of Robot with Voltage-Driven Motors

In this section, a task-space adaptive control scheme is proposed for the uncertain kinematics regulation problem of robots with voltage-driven motors. As discussed in Section 2.2.2.2, the dynamics of robot driven by voltage amplifier with n degrees of freedom can be expressed as:

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I, \quad (3.68)$$

$$L\dot{I} + RI + K_E \dot{q} = u, \quad (3.69)$$

where $I \in R^n$ is the vector of armature currents and $u \in R^n$ denotes the vector of armature voltages. In this thesis, it's assumed that the exact values of the dynamics parameters in $M(q)$, $C(q, \dot{q})$ and $g(q)$ and the actuator dynamic coefficient matrices L , R , K_E and K_N are not available; Jacobian matrix $J(q)$ is not known exactly due to uncertainties in robot kinematics.

Since the closed-loop system dynamics is third-order instead of second-order which complicates the controller design, a backstepping procedure is adopted to develop the control scheme. Firstly, based on the second-order manipulator subsystem dynamics

(3.68), a desired armature current signal I_d is designed to ensure that the task-space position errors converge with the presence of uncertainties in kinematics and motor torque constant matrix. Then based on the actuator subsystem dynamics (3.69), backstepping procedure is used to design a voltage control input u to guarantee that the actual armature current I tracks the desired current signal I_d in spite of the uncertain actuator coefficient matrices. Two Lyapunov functions are proposed for stability analysis of the control scheme proposed.

3.3.1 Desired Armature Current Design

Using the desired armature current I_d , the manipulator subsystem dynamics (3.68) can be rewritten as

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I_d + K_N \tilde{I} \quad (3.70)$$

where $\tilde{I} = I - I_d$ represents a current perturbation to the rigid-link robot dynamics. The revised subsystem (3.70) can be viewed as controlled by $K_N I_d$ with an input disturbance $K_N \tilde{I}$ which will be handled by the input control voltage designed in next subsection.

In this subsection, an adaptive Jacobian matrix $\hat{J}_A(q)$ is proposed according to property (2.5) as

$$\hat{J}_A^T(q)s(e) = Y_J(q, e)\hat{\theta}_J, \quad (3.71)$$

where $\hat{\theta}_J$ is the adaptive kinematics parameter vector to be updated by an updating law defined later.

Using the gravity regressor, approximate motor torque constant matrix and adaptive

Jacobian matrix, the desired armature current I_d is proposed as

$$I_d = -\hat{K}_N^{-1}(\hat{J}_A^T(q)K_p s(e) + K_v v - Z(q)\hat{\phi} - Y(q, e, \hat{\phi}, \hat{\theta}_J)\hat{\phi}),$$

$$\dot{v} = -\Lambda v + \dot{q} + \alpha \hat{J}^T(q)s(e), \quad (3.72)$$

$$\dot{\hat{\phi}} = -LZ^T(q)(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (3.73)$$

$$\dot{\hat{\phi}} = -L_a Y(q, e, \hat{\phi}, \hat{\theta}_J)(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (3.74)$$

$$\dot{\hat{\theta}}_J = L_J Y_J^T(q, e)(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (3.75)$$

where $K_p = k_p I_E$ and K_v are positive definite diagonal feedback gains; Λ , L_a , L_J , L are positive definite diagonal matrices and α is a positive constant. Note here a non-adaptive approximate Jacobian matrix $\hat{J}(q)$ is used in the updating laws (3.72)-(5.10) and it's chosen so that

$$\|J^T(q) - \hat{J}^T(q)\| \leq \bar{\gamma}, \quad (3.76)$$

where $\bar{\gamma}$ is a positive constant to be defined later.

Let $Z(q)\hat{\phi} - \hat{J}_A^T(q)K_p s(e) = [r_1, r_2, \dots, r_n]^T$, then the regressor $Y(q, e, \hat{\phi}, \hat{\theta}_J)$ is defined as $Y(q, e, \hat{\phi}, \hat{\theta}_J) = \text{diag}(r_1, \dots, r_n)$ such that

$$(I - K_N \hat{K}_N^{-1})(Z(q)\hat{\phi} - \hat{J}_A^T(q)K_p s(e)) = Y(q, e, \hat{\phi}, \hat{\theta}_J)\phi_k, \quad (3.77)$$

where $\phi_k = (1 - \frac{k_N^1}{\hat{k}_N^1}, \dots, 1 - \frac{k_N^n}{\hat{k}_N^n})^T$ is an unknown parameter vector with k_N^i and $\hat{k}_N^i$ being the i^{th} diagonal elements of K_N and $\hat{K}_N$ respectively.

Remark 3.5: In the desired current I_d design, an auxiliary variable v is proposed instead of using joint or task space damping $\dot{q}$ or $\dot{x}$ directly. Hence acceleration measurement is avoided in next step of the backstepping design as will be seen in subsection 3.3.2. $\diamond$

Substituting I_d as defined in (3.72) into the dynamic equation (3.70) and using equation (3.71), (5.12), one has

$$\begin{aligned} M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + K_N K_N^{-1} K_v v + k_p Y_J(q, e) \hat{\theta}_J + Z(q)(\phi - \hat{\phi}) \\ + Y(q, e, \hat{\phi}, \hat{\theta}_J)(\phi_k - K_N \hat{K}_N^{-1} \hat{\phi}) = K_N \tilde{I}, \end{aligned} \quad (3.78)$$

where equation (3.2) has been used and note that $Y(q, e, \hat{\phi}, \hat{\theta}_J)$, $K_N \hat{K}_N^{-1}$ are diagonal matrices.

Now the the stability of the closed loop system (3.78) with $\tilde{I} = 0$ can be studied and the current perturbation $\tilde{I}$ will be considered in the overall Lyapunov function construction in next subsection. A Lyapunov function candidate V_1 is proposed with the form:

$$\begin{aligned} V_1 = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \alpha \dot{q}^T M(q) \hat{J}^T(q) s(e) + \sum_{i=1}^m k_p S_i(e_i) + \frac{1}{2} v^T K_N \hat{K}_N^{-1} K_v v \\ + \frac{1}{2} (\phi - \hat{\phi})^T L^{-1} (\phi - \hat{\phi}) + \frac{1}{2} (\hat{K}_N K_N^{-1} \phi_k - \hat{\phi})^T K_N \hat{K}_N^{-1} L_a^{-1} (\hat{K}_N K_N^{-1} \phi_k - \hat{\phi}) \\ + \frac{1}{2} (\theta_J - \hat{\theta}_J)^T L_J^{-1} k_p (\theta_J - \hat{\theta}_J), \end{aligned} \quad (3.79)$$

Since

$$\begin{aligned} \frac{1}{4} \dot{q}^T M(q) \dot{q} + \alpha \dot{q}^T M(q) \hat{J}^T(q) s(e) + \sum_{i=1}^m k_p S_i(e_i) \\ = \frac{1}{4} (\dot{q} + 2\alpha \hat{J}^T(q) s(e))^T M(q) (\dot{q} + 2\alpha \hat{J}^T(q) s(e)) \\ - \alpha^2 s^T(e) \hat{J}(q) M(q) \hat{J}^T(q) s(e) + \sum_{i=1}^m k_p S_i(e_i) \\ \geq \sum_{i=1}^m (k_p \hat{c}_i - \alpha^2 \lambda_m) s_i^2(e_i), \end{aligned} \quad (3.80)$$

where α can be chosen small enough or k_p can be chosen large enough to satisfy the inequality,

$$k_p \hat{c}_i - \alpha^2 \lambda_m > 0, \quad (3.81)$$

where $\lambda_m = \lambda_{\max}[\hat{J}(q)M(q)\hat{J}^T(q)]$, $\lambda_{\max}[C]$ denotes the maximum eigenvalue of a matrix C .

Substituting inequality (3.80) into equation (3.79), one has

$$\begin{aligned} V_1 \geq & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^m (k_p \hat{c}_i - \alpha^2 \lambda_m) s_i^2(e_i) + \frac{1}{2}v^T K_N \hat{K}_N^{-1} K_v v \\ & + \frac{1}{2}(\phi - \hat{\phi})^T L^{-1}(\phi - \hat{\phi}) + \frac{1}{2}(\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi})^T L_a^{-1} K_N \hat{K}_N^{-1} (\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}) \\ & + \frac{1}{2}(\theta_J - \hat{\theta}_J)^T L_J^{-1} k_p (\theta_J - \hat{\theta}_J), > 0. \end{aligned} \quad (3.82)$$

Hence, V_1 is positive definite in $\dot{q}$, v , $s(e)$, $\phi - \hat{\phi}$, $\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}$ and $\theta_J - \hat{\theta}_J$ with condition (3.81) satisfied.

Differentiating equation (3.79) with respect to time and substituting equations (3.72)-(5.10) and (3.78) into it, one has:

$$\dot{V}_1 = -W_1 \quad (3.83)$$

where

$$\begin{aligned} W_1 = & \dot{q}^T B \dot{q} + v^T K_N \hat{K}_N^{-1} K_v \Lambda v + (\dot{q} + \alpha \hat{J}^T(q) s(e))^T k_p Y_J(q, e) \theta_J \\ & - k_p \dot{q}^T J^T(q) s(e) + \alpha \{s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} \\ & - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q}\}. \end{aligned} \quad (3.84)$$

From *Property 4*, since $Y_J(q, e) \theta_J = J^T(q) s(e)$ one can rewrite W_1 as

$$\begin{aligned} W_1 = & \dot{q}^T B \dot{q} + v^T K_N \hat{K}_N^{-1} K_v \Lambda v + \alpha k_p s^T(e) \hat{J}(q) J^T(q) s(e) \\ & + \alpha \{s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} \\ & - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q}\} \end{aligned} \quad (3.85)$$

Substituting $J^T(q) = \hat{J}^T(q) + (J^T(q) - \hat{J}^T(q))$ into (5.18), one has

$$\begin{aligned} W_1 = & \dot{q}^T B \dot{q} + v^T K_N \hat{K}_N^{-1} K_v \Lambda v + \alpha k_p s^T(e) \hat{J}(q) \hat{J}^T(q) s(e) \\ & + \alpha k_p s^T(e) \hat{J}(q) (J^T(q) - \hat{J}^T(q)) s(e) + \alpha \{s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} \\ & - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q}\} \end{aligned} \quad (3.86)$$

Since $s(e)$ is bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that [3]:

$$\begin{aligned} \alpha |s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} \\ - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q}| \geq -\alpha c_0 \|\dot{q}\|^2 - \alpha c_1 \|s(e)\|^2. \end{aligned} \quad (3.87)$$

Substituting inequality (3.87) into (3.86) and using inequality (3.76), one can get:

$$\begin{aligned} W_1 \geq & (\lambda_{\min}[B] - \alpha c_0) \|\dot{q}\|^2 + \lambda_{\min}[K_N \hat{K}_N^{-1} K_v \Lambda] \|v\|^2 \\ & + \alpha k_p (\lambda_{\min}[\hat{J}(q) \hat{J}^T(q)] - b_j \bar{\gamma} - \frac{c_1}{k_p}) \|s(e)\|^2. \end{aligned} \quad (3.88)$$

where b_j denotes the norm bound for $\hat{J}(q)$. Therefore, if

$$\bar{\gamma} < \frac{1}{b_j} (\lambda_{\min}[\hat{J}(q) \hat{J}^T(q)] - \frac{c_1}{k_p}), \quad (3.89)$$

then α can be chosen small enough so that

$$\lambda_{\min}[B] - \alpha c_0 > 0, \quad (3.90)$$

and hence W_1 is positive definite in $\dot{q}$, v and $s(e)$.

Lemma 3.1 *The closed-loop system described by equations (3.78) and (3.72)-(5.10) gives rise to the convergence of $x \rightarrow x_d$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$ if $\tilde{I} = 0$, the feedback gains K_p , α are chosen to satisfy conditions (3.81), (5.22) and $\hat{J}(q)$ are chosen to*

satisfy conditions (3.76) and (3.89).

Proof

Clearly, V_1 is positive definite in $\dot{q}$, v , $s(e)$, $\phi - \hat{\phi}$, $\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}$ and $\theta_J - \hat{\theta}_J$. Since the time derivative of V_1 is non-positive, V_1 is bounded and hence $\dot{q}$, v , $s(e)$, $\hat{\phi}$, $\hat{\varphi}$, $\hat{\theta}_J$ are all bounded. From inequality (3.88), one can conclude also that $\dot{q}$, v , $s(e) \in L_2(0, \infty)$. From equations (3.72), (3.78), it's easy to see that $\ddot{q}$ and $\dot{v}$ are bounded and hence $\dot{q}$ and v are uniformly continuous. Noting that $\dot{e} = J(q)\dot{q}$ and $\dot{q}$ is bounded, $s(e)$ is also uniformly continuous. It follows from [3] [28] that $\dot{q}$, v and $s(e)$ converge to 0 as $t \rightarrow \infty$ which means $e = x - x_d$ also converge to 0 as $t \rightarrow \infty$. Hence the proof is complete. $\square$

Remark 3.6: The desired current design can deal with the parametric uncertainty in robot kinematics. To cope with the structural uncertainty in kinematics the non-adaptive approximate Jacobian matrix $\hat{J}(q)$ could be used in I_d instead of the adaptive Jacobian matrix $\hat{J}_A(q)$. In that case, the bound of allowable kinematics uncertainty will be more conservative. $\diamond$

Remark 3.7: Full adaptive version of this result without using the non-adaptive Jacobian matrix $\hat{J}(q)$ can be found in the very recent work by Dixon [29] without considering the actuator dynamics. The result in this thesis can also be extended by using projection algorithm in the adaptation of kinematic parameters to get a full adaptive version. $\diamond$

3.3.2 Input Control Voltage Design

The desired current input I_d as given by (3.72) in previous subsection is now used to design a voltage input u which will force armature current perturbation $\tilde{I}$ to zero. Since the actuator dynamic coefficient matrices L , R , K_E and K_N are unknown, approximate models $\hat{L}$, $\hat{R}$, $\hat{K}_E$ and $\hat{K}'_N$ are employed and the input voltage u is

proposed as:

$$u = \hat{L}\dot{I}_d + \hat{R}I_d + \hat{K}_E\dot{q} - N_N(y)\hat{\theta}_{K'_N} - K_I\tilde{I}, \quad (3.91)$$

$$\dot{\hat{\theta}}_L = -L_L N_L(\dot{I}_d)\tilde{I}, \quad (3.92)$$

$$\dot{\hat{\theta}}_R = -L_R N_R(I_d)\tilde{I}, \quad (3.93)$$

$$\dot{\hat{\theta}}_{K_E} = -L_E N_E(\dot{q})\tilde{I}, \quad (3.94)$$

$$\dot{\hat{\theta}}_{K'_N} = L_N N_N(y)\tilde{I}. \quad (3.95)$$

where $y = \dot{q} + \alpha \hat{J}^T(q)s(e)$, L_L, L_R, L_E, L_N, K_I are positive definite diagonal matrices. $N_L(\dot{I}_d) = \text{diag}(\dot{I}_{d1}, \dots, \dot{I}_{dn})$, $N_R(I_d) = \text{diag}(I_{d1}, \dots, I_{dn})$, $N_E(\dot{q}) = \text{diag}(\dot{q}_1, \dots, \dot{q}_n)$, $N_N(y) = \text{diag}(y_1, \dots, y_n)$ and hence $\hat{L}\dot{I}_d = N_L(\dot{I}_d)\hat{\theta}_L$, $\hat{R}I_d = N_R(I_d)\hat{\theta}_R$, $\hat{K}_E\dot{q} = N_E(\dot{q})\hat{\theta}_{K_E}$, $\hat{K}'_N y = N_N(y)\hat{\theta}_{K'_N}$. I is measured by sensor such as ammeter. The approximate models $\hat{L}$, $\hat{R}$, $\hat{K}_E$ and $\hat{K}'_N$ are updated online by the updating laws (3.92)-(3.95) during the regulation. Note that $\hat{K}'_N$ is updated online by the updating law (3.95) during the regulation process, comparatively $\hat{K}_N$ used in the desired armature current design is a constant matrix.

Substituting the input voltage (5.25) into the actuator dynamics (3.69), one has

$$\begin{aligned} L(\dot{I} - \dot{I}_d) + R(I - I_d) &= (\hat{L} - L)\dot{I}_d + (\hat{R} - R)I_d \\ &+ (\hat{K}_E - K_E)\dot{q} - N_N(y)\hat{\theta}_{K'_N} - K_I\tilde{I} \end{aligned} \quad (3.96)$$

which can be rewritten as

$$\begin{aligned} L\dot{\tilde{I}} + (R + K_I)\tilde{I} &= N_L(\dot{I}_d)(\theta_{\hat{L}} - \theta_L) + N_R(I_d)(\theta_{\hat{R}} - \theta_R) \\ &+ N_E(\dot{q})(\theta_{\hat{K}_E} - \theta_{K_E}) - N_N(y)\hat{\theta}_{K'_N} \end{aligned} \quad (3.97)$$

where θ_L , θ_R , θ_{K_E} , θ_{K_N} denote the unknown parameters in L , R , K_E , K_N respectively such that $L\dot{I}_d = N_L(\dot{I}_d)\theta_L$, $RI_d = N_R(I_d)\theta_R$, $K_E\dot{q} = N_E(\dot{q})\theta_{K_E}$, $K_N y =$

$N_N(y)\theta_{K_N}$. Another Lyapunov function candidate V can be proposed with the following form:

$$\begin{aligned} V = & V_1 + \frac{1}{2}\tilde{I}^T L \tilde{I} + \frac{1}{2}(\theta_L - \hat{\theta}_L)^T L_L^{-1}(\theta_L - \hat{\theta}_L) + \frac{1}{2}(\theta_R - \hat{\theta}_R)^T L_R^{-1}(\theta_R - \hat{\theta}_R) \\ & + \frac{1}{2}(\theta_{K_E} - \hat{\theta}_{K_E})^T L_E^{-1}(\theta_{K_E} - \hat{\theta}_{K_E}) + \frac{1}{2}(\theta_{K_N} - \hat{\theta}_{K'_N})^T L_N^{-1}(\theta_{K_N} - \hat{\theta}_{K'_N}) \end{aligned} \quad (3.98)$$

where V_1 is defined as in equation (3.79).

It's easy to see that V is non-negative if the condition (3.81) is satisfied. Differentiating V with respect to time and using equations (3.92)-(3.94),(3.97), one has:

$$\begin{aligned} \dot{V} = & -W_1 + y^T K_N \tilde{I} - \tilde{I}^T (R + K_I) \tilde{I} - \tilde{I}^T N_N(y) \hat{\theta}_{K_N} \\ & - \dot{\hat{\theta}}_{K'_N}^T L_N^{-1}(\theta_{K_N} - \hat{\theta}_{K'_N}) \end{aligned} \quad (3.99)$$

where W_1 is defined as in equation (3.86). Here it's noticeable that an additional term $y^T K_N \tilde{I}$ which is introduced by the differentiation of V_1 since the current perturbation $\tilde{I}$ is considered in this design step.

Substituting equation (3.95) into equation (3.99) and noting that $K_N y = N_N(y)\theta_{K_N}$, one can get:

$$\dot{V} = -W_1 - \tilde{I}^T (R + K_I) \tilde{I} \leq 0 \quad (3.100)$$

Hence, if the sufficient stability conditions (3.89) and (5.22) are satisfied $\dot{V}$ is non-positive.

We are now in a position to state the following Theorem:

Theorem 3.4 *If the control input voltage u given by (3.72)-(5.10) and (5.25)-(3.95) is applied on the rigid-link robot dynamics (3.68), (3.69), then the closed-loop system gives rise to the convergence of $x \rightarrow x_d$, $\dot{q} \rightarrow 0$ and $I \rightarrow I_d$ as $t \rightarrow \infty$, pro-*

vided that the feedback gains K_p , α are chosen to satisfy conditions (3.81), (5.22) and $\hat{J}(q)$ is chosen to satisfy conditions (3.76), (3.89).

Proof

Since V is positive definite in $\dot{q}$, v , $s(e)$, $\phi - \hat{\phi}$, $\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}$, $\theta_J - \hat{\theta}_J$, $\tilde{I}$, $\theta_L - \hat{\theta}_L$, $\theta_R - \hat{\theta}_R$, $\theta_{K_E} - \hat{\theta}_{K_E}$, $\theta_{K'_N} - \hat{\theta}_{K'_N}$ and its time derivative $\dot{V}$ is non-positive, V is bounded and hence $\dot{q}$, v , $s(e)$, $\tilde{I}$, $\hat{\theta}_J$, $\hat{\theta}_L$, $\hat{\theta}_R$, $\hat{\theta}_E$, $\hat{\theta}_{K'_N}$ are also bounded. From (3.88) and (3.100) it's easily verified that $\dot{q}, v, s(e), \tilde{I} \in L_2(0, \infty)$. From equations (3.72), (3.78), (3.97), one can see that $\dot{v}$, $\ddot{q}$ and $\ddot{\tilde{I}}$ are all bounded and hence $\dot{q}$, v and $\tilde{I}$ are uniformly continuous. Since $s(e)$ is differentiable and note that $\dot{e} = J(q)\dot{q}$ and $\dot{q}$ is bounded, $\dot{s}(e)$ is also bounded and hence $s(e)$ is uniformly continuous. It follows from [3] [28] that $\dot{q}$, v , $s(e)$ and $\tilde{I}$ converge to 0 as $t \rightarrow \infty$, which means $x \rightarrow x_d$, $\dot{q} \rightarrow 0$ and $I \rightarrow I_d$ as $t \rightarrow \infty$. $\square$

Remark 3.8: The setpoint controllers proposed in this chapter are applicable for both redundant and non-redundant joint robots. Based on several new Lyapunov functions, it has been proved through signal analysis method that the joint velocity $\dot{q}$ and task space endpoint position x will eventually converge to 0 and x_d respectively. The fact $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$ indicates that all the robot joints will be at rest at steady state and thus the so-called self-motion [7, 8, 73] is avoided eventually. It's understood that for redundant joint robot if only the convergence about $\dot{x} \rightarrow 0$ and $x \rightarrow x_d$ is concluded then it's not sure whether self-motion may arise or not, since there exists the possibility that the robot joints move without effecting the task space position of the endpoint. $\diamond$

3.4 Simulation Results

3.4.1 Adaptive Regulation for Robot with Current-Driven Motors

Figure 3.6: A three-link planar robot

In this subsection, simulation studies are conducted for the three setpoint control methods proposed in Section 3.2 and a simulation study using a simple task space PD control method is also provided for comparison with the three control methods proposed. The simulation study is based on a 3-link robot holding an object as shown in Figure 3.6. Using a fixed camera placed some distance away from the robot as the external sensor, the task space is defined in the vision coordinates described by $x = [x_s, y_s]^T$. The Jacobian matrix $J(q)$ of mapping from joint space

to visual space is given by the following equation [96].

$$J(q) = \frac{f_1}{z-f_1} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \times \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - (l_3 + l_0) s_{123} & -l_2 s_{12} - (l_3 + l_0) s_{123} & -(l_3 + l_0) s_{123} \\ l_1 c_1 + l_2 c_{12} + (l_3 + l_0) c_{123} & l_2 c_{12} + (l_3 + l_0) c_{123} & (l_3 + l_0) c_{123} \end{bmatrix} \quad (3.101)$$

where $s_1 = \sin(q_1)$, $s_{12} = \sin(q_1 + q_2)$, and $s_{123} = \sin(q_1 + q_2 + q_3)$. β_1 , β_2 denote scaling factors in pixels/m, δ represents the angle of rotation of the vision coordinates relative to Cartesian coordinates, the offset of the origins of the coordinates $d = (d_x, d_y)^T$ (see Figure 3.6) were set to 0 m, z is the perpendicular distance between the robot and the camera, f_1 is the focal length of the camera, and l_0 is the length of the object held, l_i is the length of link i .

In the simulations, the masses of the links m_1 , m_2 , m_3 were chosen as $1kg$, l_1 , l_2 and l_3 were set as $0.5m$, and the mass of the object chosen as $0.5kg$ with a length $0.3m$. f_1 was chosen as $16mm$, z was chosen as $1.5m$ and δ was chosen as 45° , $\beta_1 = \beta_2 = 78333$. The amplifiers were operating in current mode. The exact parameters of the actuator were set as $K_{\tau 1} = 18, K_{\tau 2} = 14, K_{\tau 3} = 16, r_1 = r_2 = r_3 = 1$. Hence, $K_N = \text{diag}(K_{\tau 1}/r_1, K_{\tau 2}/r_2, K_{\tau 3}/r_3) = \text{diag}(18, 14, 16)$. To show the effectiveness of the proposed control methods, the robot was required to move from the initial position of $[x(0), y(0)] = [480, 120]$ pixels to two different desired positions, $[x_d, y_d] = [975, 720]$ and $[x_d, y_d] = [750, 920]$ pixels respectively, in image space.

1. Simulation Results for the Control Scheme in Theorem 3.1

In this simulation, the control scheme proposed in *Theorem 3.1* is used to control the 3-link robot as illustrated in Figure 3.6. First, the camera parameters f_1 and z are estimated as $\hat{f}_1 = 12mm$, $\hat{z} = 1.2m$, the scaling factors are estimated as $\hat{\beta}_1 = \hat{\beta}_2 = 80000$, the lengths of the links and the object were estimated approximately as

$\hat{l}_1 = 0.55m, \hat{l}_2 = 0.45m, \hat{l}_3 = 0.7m, \hat{l}_0 = 0.4m$ respectively, the rotation angle δ was estimated as $\hat{\delta} = 80^\circ$ and the approximate Jacobian matrix $\hat{J}(q)$ can be obtained according to equation (3.101). The actuator model as $\hat{K}_N = \text{diag}(60, 35, 35)$. With $K_p = 0.0012I_E$, $K_v = 0.0001I_E$ and $\alpha = 1$, the robot performance to reach desired position $[x_d, y_d] = [975, 720]$ is shown in figure 3.7 and 3.8. With the same control settings, the robot performance to reach desired position $[x_d, y_d] = [750, 920]$ is shown in figure 3.9 and 3.10. Note that the entries of $\hat{J}(q)K_p$ and $\hat{J}(q)K_v$ are large since the scaling factors β_1, β_2 are very large.

Figure 3.7: Position error with the controller in *Theorem 3.1* ($\delta = 80^\circ$, $\hat{K}_N = \text{diag}(60, 35, 35)$, $L = L_a = 10I_E$)

Figure 3.8: The Control Inputs

Figure 3.9: Position error with the controller in *Theorem 3.1* ($\delta = 80^\circ$, $\hat{K}_N = \text{diag}(60, 35, 35)$, $L = L_a = 10I_E$)

Figure 3.10: The Control Inputs

Next, $\hat{K}_N$ and estimated rotation angle $\hat{\delta}$ were varied to examine the effects of the uncertainties in kinematics and actuator model on the robot's motion. System performance to reach desired position $[x_d, y_d] = [975, 720]$ with $\hat{\delta} = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $K_p = 0.0012I_E$, $K_v = 0.0001I_E$ and $\alpha = 1$ is shown in figure 3.11 and 3.12. Figure 3.13 and 3.14 show the system performance with the same control settings.

Figure 3.11: Position error with the controller in *Theorem 3.1* ($\delta = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $L = L_a = 10I_E$)

Figure 3.12: The Control Inputs

Figure 3.13: Position error with the controller in *Theorem 3.1* ($\delta = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $L = L_a = 10I_E$)

Figure 3.14: The Control Inputs

From the simulations results, it can be seen that in the presence of uncertainties in both kinematics and actuator model, the control method proposed can still guarantee the asymptotical stability. The simulation results also show that with a better estimated Jacobian matrix and actuator model the performance of the system was better. It is natural since more accurate information about the system was utilized in the control law.

2. Simulation Results for the Control Scheme in Theorem 3.2

In this simulation, the control scheme in *Theorem 3.2* is examined based on the same system configuration. The simulation studies in this simulation use the same settings as the simulations for control scheme in *Theorem 3.1*. Firstly, the camera parameters f_1 and z are estimated as $\hat{f}_1 = 12mm$, $\hat{z} = 1.2m$, the scaling factors are estimated as $\hat{\beta}_1 = \hat{\beta}_2 = 80000$, the lengths of the links and the object were estimated approximately as $\hat{l}_1 = 0.55m$, $\hat{l}_2 = 0.45m$, $\hat{l}_3 = 0.7m$, $\hat{l}_0 = 0.4m$ respectively, the rotation angle δ was estimated as $\hat{\delta} = 80^\circ$ and the approximate Jacobian matrix $\hat{J}(q)$ can be obtained according to equation (3.101). The actuator model as $\hat{K}_N = \text{diag}(60, 35, 35)$. With $K_p = 0.01I_E$, $B_v = 1200I_E$ and $\alpha = 0.0001$, the results for the adaptive regulation control method reaching desired position $[x_d, y_d] = [975, 720]$ are shown in figures 3.15 and 3.16. With the same control settings, the robot performance to reach desired position $[x_d, y_d] = [750, 920]$ is shown in figure 3.17 and 3.18. The control gains used here are different from previous simulations because of the different damping factors used in the controller design.

Figure 3.15: Position error with the controller in *Theorem 3.2* ($\delta = 80^\circ$, $\hat{K}_N = \text{diag}(60, 35, 35)$, $L = L_a = I_E$)

Figure 3.16: The Control Inputs

Figure 3.17: Position error with the controller in *Theorem 3.2* ($\delta = 80^\circ$, $\hat{K}_N = \text{diag}(60, 35, 35)$, $L = L_a = I_E$)

Figure 3.18: The Control Inputs

Next, $\hat{K}_N$ and estimated rotation angle $\hat{\delta}$ were varied to examine the effects of the uncertainties in kinematics and actuator model on the robot's motion. Simulation results with $\hat{\delta} = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $K_p = 0.01I_E$, $B_v = 1200I_E$ and $\alpha = 0.0001$ are shown in figures 3.19 and 3.20. With the same control settings, the robot performance to reach desired position $[x_d, y_d] = [750, 920]$ is shown in figure 3.21 and 3.22.

Figure 3.19: Position error with the controller in *Theorem 3.2* ($\delta = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $L = L_a = I_E$)

Figure 3.20: The Control Inputs

Figure 3.21: Position error with the controller in *Theorem 3.2* ($\delta = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $L = L_a = I_E$)

Figure 3.22: The Control Inputs

3. Simulation Results for the Control Scheme in Theorem 3.3

This simulation studies system performance with the control scheme proposed in *Theorem 3.3*. Firstly, the camera parameters f_1 and z are estimated as $\hat{f}_1 = 12mm$, $\hat{z} = 1.2m$, the scaling factors are estimated as $\hat{\beta}_1 = \hat{\beta}_2 = 80000$, the lengths of the links and the object were estimated approximately as $\hat{l}_1 = 0.55m$, $\hat{l}_2 = 0.45m$, $\hat{l}_3 = 0.7m$, $\hat{l}_0 = 0.4m$ respectively, the rotation angle δ was estimated as $\hat{\delta} = 80^\circ$ and the approximate Jacobian matrix $\hat{J}(q)$ can be obtained according to equation (3.101). The actuator model as $\hat{K}_N = diag(60, 35, 35)$. With $K_p = 0.008I_E$, $K_v = 0.002I_E$, $K_I = 10I_E$ and $\alpha = 6$, the results for the adaptive regulation control method reaching desired position $[x_d, y_d] = [975, 720]$ are shown in figures 3.23 and 3.24. With the same control settings, the robot performance to reach desired position $[x_d, y_d] = [750, 920]$ is shown in figure 3.25 and 3.26. The control gains used here are different from previous simulations because of the different damping factors used in the controller design.

Figure 3.23: Position error with the controller in *Theorem 3.3* ($\delta = 80^\circ$, $\hat{K}_N = \text{diag}(60, 35, 35)$)

Figure 3.24: The Control Inputs

Figure 3.25: Position error with the controller in *Theorem 3.3* ($\delta = 80^\circ$, $\hat{K}_N = \text{diag}(60, 35, 35)$)

Figure 3.26: The Control Inputs

Next, $\hat{K}_N$ and estimated rotation angle $\hat{\delta}$ were varied to examine the effects of the uncertainties in kinematics and actuator model on the robot's motion. Simulation results with $\hat{\delta} = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$, $K_p = 0.008I_E$, $K_v = 0.002I_E$, $K_I = 10I_E$ and $\alpha = 6$ are shown in figures 3.27 and 3.28. With the same control settings, the robot performance to reach desired position $[x_d, y_d] = [750, 920]$ is shown in figure 3.29 and 3.30.

Figure 3.27: Position error with the controller in *Theorem 3.3* ($\delta = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$)

Figure 3.28: The Control Inputs

Figure 3.29: Position error with the controller in *Theorem 3.3* ($\delta = 60^\circ$, $\hat{K}_N = \text{diag}(50, 50, 50)$)

Figure 3.30: The Control Inputs

From the simulations results, one can see that in the presence of uncertainties in both kinematics and actuator model, the control methods proposed can still guarantee the asymptotical stability. The simulation results also show that with a better estimated Jacobian matrix and actuator model the performance of the system was better. It is natural since more accurate information about the system was utilized in the control law.

4. Simulation Comparison by Using a Task Space PD Controller

In this simulation, a regular task space PD controller is used to compare with the three control methods proposed in this chapter in dealing with the kinematics and actuator model uncertainties. The PD controller used possesses the following form [54]:

$$I = -\hat{K}_N^{-1}[\hat{J}^T(q)K_p s(e) + \hat{J}^T(q)K_v \hat{x} - Z(q)\hat{\phi}], \quad (3.102)$$

Based on the same robot model, we set the kinematics and actuator model un-

certainties same as used in previous simulation studies: the camera parameters f_1 and z are estimated as $\hat{f}_1 = 12mm$, $\hat{z} = 1.2m$, the scaling factors are estimated as $\hat{\beta}_1 = \hat{\beta}_2 = 80000$, the lengths of the links and the object were estimated approximately as $\hat{l}_1 = 0.55m$, $\hat{l}_2 = 0.45m$, $\hat{l}_3 = 0.7m$, $\hat{l}_0 = 0.4m$ respectively, the rotation angle δ was estimated as $\hat{\delta} = 80^\circ$; the actuator model as $\hat{K}_N = diag(65, 30, 30)$. With control settings $K_p = 0.001I_E$, $K_v = 0.0001I_E$ and $\alpha = 1$, the system performance is illustrated in figures 3.31 and 3.32.

Figure 3.31: Position error with the PD controller ($\delta = 80^\circ$, $\hat{K}_N = diag(65, 30, 30)$)

Figure 3.32: The Control Inputs

As shown by the simulation results, regular task space PD controller (even with adaptive gravity compensation) can not achieve the regulation goal due to the lack of compensations to the effects caused by kinematics and actuator dynamics. Through the simulation comparison, the necessity and effectiveness of developing the control schemes in this chapter is justified.

3.4.2 Adaptive Regulation for Robot with Voltage-Driven Motors

In this subsection, the simulation results are presented to illustrate the performance of the proposed control scheme in Section 3.3. The simulation is based on a two-link robot holding an uncertain object as shown in Fig. 5.1. The manipulator endpoint is required to move from an initial position $x_0 = (0.8, 0.32)$ to different desired positions $x_d = (0.5, 0.55)$ and $x_d = (0.65, 0.5)$ respectively defined in Cartesian space.

Figure 3.33: A 2-Link Robot Holding an Object

In the simulation, the masses, lengths and gravity centers of link 1, 2 and the object are set as $m_1 = 17.4kg$, $m_2 = 4.8kg$, $m_o = 2kg$, $l_1 = 0.4318m$, $l_2 = 0.4318m$, $l_0 = 0.2m$, $l_{c1} = 0.068m$, $l_{c2} = 0.07m$, $l_{c0} = 0.1m$ respectively. The object grasping angle is set as $q_0 = 30^\circ$. The true actuator dynamic coefficient matrices are defined as $L = \text{diag}(1 \ 1)$, $R = \text{diag}(15 \ 15)$, $K_E = \text{diag}(10 \ 8)$, $K_N = \text{diag}(10 \ 8)$.

To test the ability of the proposed control schemes to deal with kinematics and actuator uncertainties, the kinematics parameters are estimated as $\hat{l}_1 = 0.5m$, $\hat{l}_2 = 0.5m$, $\hat{l}_0 = 0.25m$, $\hat{q}_0 = 45^\circ$ and the approximate actuator models as $\hat{L}(0) = \text{diag}(2.5 \ 2.5)$, $\hat{R}(0) = \text{diag}(10 \ 12)$, $\hat{K}_E(0) = \text{diag}(15 \ 10)$, $\hat{K}_N = \hat{K}'_N(0) = \text{diag}(15 \ 12)$ in the controller design. The system response with control gains $K_p = 400I_E$, $K_v = 400I_E$, $K_I = 100I_E$, $\alpha = 1.5$, $\Lambda = 5I_E$, $L = 40I_E$, $L_a = 0.35I_E$, $L_J = 2I_E$, $L_L = 0.5I_E$, $L_R = L_E = L_N = I_E$ and desired position $x_d = (0.5, 0.55)$ is illustrated by Fig. 3.34, 3.35, 3.36. The system response to the desired position $x_d = (0.65, 0.5)$ with the same system and control settings is shown in Fig. 3.37, 3.38 and 3.39. Next, the kinematics uncertainty is increased by setting $\hat{l}_1 = 0.6m$, $\hat{l}_2 = 0.6m$, $\hat{l}_0 = 0.3m$, $\hat{q}_0 = 60^\circ$ and keep the approximate actuator models unchanged. Figure 3.40, 3.41

and 3.42 show the system response to desired position $x_d = (0.5, 0.55)$ with the same control gains as used in the first simulation and Fig. 3.43, 3.44 and 3.45. As shown by the simulation results, the position errors and armature current tracking errors converge to 0 even in the presence of kinematics and dynamics uncertainties in the RLED robot system.

Figure 3.34: Position Errors

Figure 3.35: Currents Tracking Errors

Figure 3.36: Control Inputs

Figure 3.37: Position Errors

Figure 3.38: Currents Tracking Errors

Figure 3.39: Control Inputs

Figure 3.40: Position Errors

Figure 3.41: Currents Tracking Errors

Figure 3.42: Control Inputs

Figure 3.43: Position Errors

Figure 3.44: Currents Tracking Errors

Figure 3.45: Control Inputs

3.5 Conclusion

In this chapter, the regulation problems of robot manipulator with uncertain kinematics, manipulator dynamics and actuator dynamics are formulated and solved by using a novel regressor design concept. According to different types of actuator used in the robot system, several adaptive regulation methods are proposed. Asymptotic stability of the systems are proved through Lyapunov analysis and sufficient stability conditions are provided to guarantee the system stability. Simulation results based on robot holding an uncertain object are presented to illustrate the performance of the control methods proposed in this chapter.

Chapter 4

Adaptive Tracking Control of Robot with Uncertain Kinematics and Actuator Dynamics

4.1 Introduction

Although it has been almost two decades since the first globally tracking convergent adaptive controllers were derived for robot with dynamic uncertainties, the problem of adaptation to kinematic uncertainty has never been systematically solved. Also, as discussed in the Introduction, actuator dynamics constitutes an important part of the whole robot system dynamics and should be considered in the control scheme design to guarantee high-precision performance. The tracking control problem of robot with uncertainties existing in kinematics, manipulator dynamics and actuator dynamics has not been studied so far and remains an open problem in the robot control literature.

In this chapter, the trajectory tracking control of robot manipulators with uncer-

tain kinematics and actuator dynamics is presented. In *section 4.2*, the uncertain kinematics tracking control problem of robot with current-driven motors first addressed. Motor torque transmission matrix uncertainty characterizes the actuator dynamics uncertainty in current amplifier driven robots which may cause distorted control torque on robot manipulator joints. An adaptive Jacobian controller is proposed which does not require exact knowledge of either kinematics or manipulator dynamics and actuator dynamics. The trajectory tracking control problem in the presence uncertainties in the kinematics and manipulator and actuator dynamics is formulated and solved based on a Lyapunov-like analysis. By using sensory feedback of the robot end effector position, it is shown that the end effector is able to follow a desired trajectory with uncertainties in kinematics and dynamics. Novel adaptive laws, extending the capability of the standard adaptive algorithm [75] to deal with kinematics uncertainty, are proposed. A novel dynamics regressor using the estimated kinematics parameters is also proposed.

In *section 4.3*, the tracking control of robot with voltage-driven motors in the presence of kinematic and actuator dynamic uncertainties is addressed. In this case, the actuator dynamics leads to a third-order closed-loop system dynamics and hence presents a bigger challenge for the tracking control problem. A backstepping procedure is adopted to design the control voltage input and an observer-based adaptive control method is proposed to fulfill the control task. The control method guarantee the convergence of the robot tracking error with the presence of all uncertainties in kinematics, manipulator dynamics and actuator dynamics. Simulation results are presented to illustrate the performance of the proposed controllers.

4.2 Adaptive Jacobian Tracking Control of Robot with Current-Driven Motors

In this section, the tracking control problem of robot with current-driven motors in the presence of kinematic and actuator uncertainties is addressed. An adaptive Jacobian tracking controller is proposed to deal with all these uncertainties. The main idea of the derivation is to introduce an adaptive sliding vector based on estimated task-space velocity, so that kinematic, dynamic and actuator adaptation can be performed concurrently.

As discussed in *Chapter 2*, the dynamics of n -degree of freedom robot with current-driven motors can be expressed in joint space as:

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I, \quad (4.1)$$

where I is the vector of armature currents, $K_N \in R^{n \times n}$ is the motor torque transmission matrix characterizing the electromechanical conversion between current and torque. In practice, K_N may often be uncertain due to calibration error or temperature change of the working environment, hence it is assumed that K_N is not known exactly in this thesis.

In the presence of kinematic uncertainty, the parameters of the Jacobian matrix is uncertain and hence the estimated differential kinematics can be expressed as

$$\hat{\dot{x}} = \hat{J}(q, \hat{\theta}_k)\dot{q} = Y_k(q, \dot{q})\hat{\theta}_k \quad (4.2)$$

where $\hat{\dot{x}} \in R^n$ denotes an estimated task-space velocity, $\hat{J}(q, \hat{\theta}_k) \in R^{n \times n}$ is an approximate Jacobian matrix and $\hat{\theta}_k \in R^q$ denotes a set of estimated kinematic parameters.

Let $x_d(t) \in R^n$ be the desired trajectory in task space. The algorithm we shall now derive is composed of (i) a control law based on an approximate motor torque transmission matrix $\hat{K}_N$ as,

$$I = -\hat{K}_N^{-1} \hat{J}^T(q, \hat{\theta}_k)(K_v \Delta \dot{x} + K_p \Delta x) + \hat{K}_N^{-1} Y_d(q, \dot{q}, \ddot{q}_r) \hat{\theta}_d + \hat{K}_N^{-1} Y_a(\tau_o) \hat{\theta}_a, \quad (4.3)$$

where $\Delta x = x - x_d$, $\Delta \dot{x} = \dot{x} - \dot{x}_d$, and $\ddot{q}_r$ is defined based on the adaptive sliding vector as detailed later, $Y_d(q, \dot{q}, \ddot{q}_r)$ is a dynamic regressor defined later, (ii) a dynamic adaptation law

$$\dot{\hat{\theta}}_d = -L_d Y_d^T(q, \dot{q}, \ddot{q}_r) s \quad (4.4)$$

iii) a kinematic adaptation law

$$\dot{\hat{\theta}}_k = L_k Y_k^T(q, \dot{q})(K_v \Delta \dot{x} + K_p \Delta x) \quad (4.5)$$

and iv) an actuator adaptation law

$$\dot{\hat{\theta}}_a = -L_a Y_a(\tau_o) s \quad (4.6)$$

where $Y_a(\tau_o) = \text{diag}\{-\tau_{o1}, -\tau_{o2}, \dots, -\tau_{on}\}$ and τ_{oi} denotes the i^{th} element of the vector τ_o which is defined as

$$\tau_o = \hat{J}^T(q, \hat{\theta}_k)(K_v \Delta \dot{x} + K_p \Delta x) - Y_d(q, \dot{q}, \ddot{q}_r) \hat{\theta}_d \quad (4.7)$$

All gain matrices are symmetric positive definite. In the above controller, $\hat{K}_N^{-1}$ is used to transform the control torque to an approximate actuator input and $Y_a(\tau_o) \hat{\theta}_a$ is used to compensate for the uncertainty introduced by the estimated transmission matrix $\hat{K}_N$. The linear parameterization of the kinematic parameters is obtained from equation (2.5). The estimated kinematic parameters $\hat{\theta}_k$ of the approximate

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Jacobian matrix $\hat{J}(q, \hat{\theta}_k)$ is updated by the parameter update equation (4.5). The key novelties are that the algorithm is now augmented by a kinematic adaptation law (4.5), an actuator adaptation law (5.8) and that a specific choice of $\dot{q}_r$ is exploited throughout. In the proposed controller, x is measured from a position sensor. Many commercial sensors are available for measurement of x , such as vision systems, electromagnetic measurement systems, position sensitive detectors, or laser trackers.

Let us now detail the proof. First, define a vector $\dot{x}_r \in R^n$ as

$$\dot{x}_r = \dot{x}_d - \alpha(x - x_d) \quad (4.8)$$

Differentiating equation (4.8) with respect to time, one has

$$\ddot{x}_r = \ddot{x}_d - \alpha(\dot{x} - \dot{x}_d) \quad (4.9)$$

where $\ddot{x}_d = \frac{d\dot{x}_d}{dt} \in R^n$ is the desired acceleration in task space.

Next, define an **adaptive task-space sliding vector** using equation (4.2) as,

$$\hat{s}_x = \hat{\dot{x}} - \dot{x}_r = \hat{J}(q, \hat{\theta}_k)\dot{q} - \dot{x}_r \quad (4.10)$$

where $\hat{J}(q, \hat{\theta}_k)\dot{q} = Y_k(q, \dot{q})\hat{\theta}_k$ as indicated in equation (4.2). The above vector is adaptive in the sense that the parameters of the approximate Jacobian matrix is updated by a parameter update law (4.5). Differentiating equation (4.10) with respect to time, one has,

$$\dot{\hat{s}}_x = \hat{\ddot{x}} - \ddot{x}_r = \dot{\hat{J}}(q, \hat{\theta}_k)\dot{q} + \hat{J}(q, \hat{\theta}_k)\ddot{q} - \ddot{x}_r \quad (4.11)$$

where $\hat{\ddot{x}}$ denotes the derivative of $\hat{\dot{x}}$. Next, let

$$\dot{q}_r = \hat{J}^{-1}(q, \hat{\theta}_k)\dot{x}_r \quad (4.12)$$

In this thesis, it's assumed that the robot is operating in a finite task space such that the approximate Jacobian matrix is of full rank. From equation (4.12), one has

$$\ddot{q}_r = \hat{J}^{-1}(q, \hat{\theta}_k)(\ddot{x}_r - \dot{\hat{J}}(q, \hat{\theta}_k)\dot{q}_r) \quad (4.13)$$

where the definition $\dot{\hat{J}}^{-1}(q, \hat{\theta}_k) = -\hat{J}^{-1}(q, \hat{\theta}_k)\dot{\hat{J}}(q, \hat{\theta}_k)\hat{J}^{-1}(q, \hat{\theta}_k)$ has been used.

Hence, one has an adaptive sliding vector in joint space as,

$$s = \dot{q} - \dot{q}_r = \dot{q} - \hat{J}^{-1}(q, \hat{\theta}_k)\dot{x}_r \quad (4.14)$$

and

$$\dot{s} = \ddot{q} - \ddot{q}_r \quad (4.15)$$

Next, multiplying both side of equation (4.14) by $\hat{J}(q, \hat{\theta}_k)$ and using equation (4.10), one can get

$$\hat{J}(q, \hat{\theta}_k)s = \hat{J}(q, \hat{\theta}_k)\dot{q} - \dot{x}_r = \hat{s}_x \quad (4.16)$$

Remark 4.1 For the case of redundant robot being used, since $\hat{J}(q, \hat{\theta}_k)$ is not a square matrix, the inverse Jacobian $\hat{J}^{-1}(q, \hat{\theta}_k)$ cannot be calculated directly. Then a generalized inverse of the approximate Jacobian matrix

$$\hat{J}^+(q, \hat{\theta}_k) = \hat{J}^T(q, \hat{\theta}_k)(\hat{J}(q, \hat{\theta}_k)\hat{J}^T(q, \hat{\theta}_k))^{-1}$$

can be used instead of $\hat{J}^{-1}(q, \hat{\theta}_k)$ in $\dot{q}_r$ as:

$$\dot{q}_r = \hat{J}^+(q, \hat{\theta}_k)\dot{x}_r + (I_E - \hat{J}^+(q, \hat{\theta}_k)\hat{J}(q, \hat{\theta}_k))\psi \quad (4.17)$$

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where $\psi \in R^n$ is minus the gradient of the convex function to be optimized [65, 68].

Then from equation (4.17), one has

$$\ddot{q}_r = \hat{J}^+(q, \hat{\theta}_k)(\ddot{x}_r - \dot{\hat{J}}(q, \hat{\theta}_k)\dot{q}_r) + (I_E - \hat{J}^+(q, \hat{\theta}_k)\hat{J}(q, \hat{\theta}_k))[\dot{\psi} + \dot{\hat{J}}^T(q, \hat{\theta}_k)\hat{J}^{+T}(q, \hat{\theta}_k)(\dot{q}_r - \psi)] \quad (4.18)$$

Hence the adaptive sliding vector s can be written as

$$s = \dot{q} - \dot{q}_r = \dot{q} - \hat{J}^+(q, \hat{\theta}_k)\dot{x}_r - (I_E - \hat{J}^+(q, \hat{\theta}_k)\hat{J}(q, \hat{\theta}_k))\psi \quad (4.19)$$

and from equations (4.19) and (4.10), one has

$$\hat{J}(q, \hat{\theta}_k)s = \hat{J}(q, \hat{\theta}_k)\dot{q} - \dot{x}_r = \hat{s}_x \quad (4.20)$$

since $\hat{J}(q, \hat{\theta}_k)\hat{J}^+(q, \hat{\theta}_k) = I_E$ and $\hat{J}(q, \hat{\theta}_k)(I_E - \hat{J}^+(q, \hat{\theta}_k)\hat{J}(q, \hat{\theta}_k)) = 0$. Then in equation (4.20) the same relationship between s and $\hat{s}_x$ is obtained as defined in (4.16) and hence the result can be extended to the case of redundant robots. $\diamond$

Substituting equations (4.14) and (4.15) into equation (4.1), the equations of motion can be expressed as,

$$M(q)\dot{s} + (B + C(q, \dot{q}))s + M(q)\ddot{q}_r + (B + C(q, \dot{q}))\dot{q}_r + g(q) = K_N I \quad (4.21)$$

Note that the last four terms of equation (4.21) are linear in a set of **dynamics parameters** θ_d and hence can be expressed as,

$$M(q)\ddot{q}_r + (B + C(q, \dot{q}))\dot{q}_r + g(q) = Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\theta_d$$

so the dynamic equation (4.21) can be written as,

$$M(q)\dot{s} + (B + C(q, \dot{q}))s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\theta_d = K_N I \quad (4.22)$$

Consider now the adaptive control law (4.3), where $K_v \in R^{n \times n}$ and $K_p \in R^{n \times n}$ are symmetric positive definite matrices. The first term is an approximate Jacobian transpose feedback law of the task-space velocity and position errors, and the last term is an estimated dynamic compensation term based on equation (4.22). Update the estimated dynamic parameters $\hat{\theta}_d$ using (4.4), and the estimated kinematic parameters using (4.5), where $L_k \in R^{q \times q}$, $L_d \in R^{p \times p}$ are symmetric positive definite matrices and $L_a \in R^{n \times n}$ is a positive definite and diagonal matrix. The closed-loop dynamics is obtained by substituting (4.3) into equation (4.22) to give

$$\begin{aligned} M(q)\dot{s} + (B + C(q, \dot{q}))s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Delta\theta_d + \hat{J}^T(q, \hat{\theta}_k)(K_v\Delta\dot{x} + K_p\Delta x) \\ + (K_N\hat{K}_N^{-1} - I)\tau_o - K_N\hat{K}_N^{-1}Y_a(\tau_o)\hat{\theta}_a = 0 \end{aligned} \quad (4.23)$$

where τ_o is defined in equation (4.7).

The estimated kinematic parameters $\hat{\theta}_k$ of the approximate Jacobian matrix $\hat{J}(q, \hat{\theta}_k)$ is updated by the parameter update equation (4.5). The estimated parameters $\hat{\theta}_k$ is then used in the inverse approximate Jacobian matrix $\hat{J}^{-1}(q, \hat{\theta}_k)$ and hence $\dot{q}_r$ and $\ddot{q}_r$ in the dynamic regressor matrix. Note that $\hat{\theta}_k$ (like q and $\dot{q}$) is just part of the states of the adaptive control system and hence can be used in the control variables even if it is nonlinear in the variables (provided that a linear parameterization can be found else where in the system model i.e. equation (2.5)). Since $\hat{J}(q, \hat{\theta}_k)$ and its inverse $\hat{J}^{-1}(q, \hat{\theta}_k)$, are updated by q and $\hat{\theta}_k$, $\dot{\hat{J}}(q, \hat{\theta}_k)$ is a function of q , $\dot{q}$, $\hat{\theta}_k$, Δx and $\Delta\dot{x}$ because $\dot{\hat{\theta}}_k$ is described by equation (4.5).

Since K_N , $\hat{K}_N$ and $Y_a(\tau_o)$ are diagonal matrices, the last two terms of equation (4.23) can be expressed as

$$(K_N\hat{K}_N^{-1} - I)\tau_o - K_N\hat{K}_N^{-1}Y_a(\tau_o)\hat{\theta}_a = Y_a(\tau_o)(\theta_a - K_N\hat{K}_N^{-1}\hat{\theta}_a), \quad (4.24)$$

where $\theta_a = [1 - \frac{k_N^1}{k_N^1} \cdots 1 - \frac{k_N^n}{k_N^n}]^T$ with k_N^i and $\hat{k}_N^i$ being the i^{th} diagonal element of

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K_N and $\hat{K}_N$ respectively.

Substituting equation (4.24) into (4.23), one has

$$\begin{aligned} M(q)\dot{s} + (B + C(q, \dot{q}))s + Y_d(q, \dot{q}, y_r, \dot{y}_r)\Delta\theta_d \\ + \hat{J}^T(q, \hat{\theta}_k)(K_v\Delta\dot{x} + K_p\Delta x) + Y_a(\tau_o)\Delta\theta_a = 0, \end{aligned} \quad (4.25)$$

where $\Delta\theta_a = \theta_a - K_N\hat{K}_N^{-1}\hat{\theta}_a$ and hence $\Delta\dot{\theta}_a = -K_N\hat{K}_N^{-1}\dot{\hat{\theta}}_a$.

Let us define a Lyapunov-like function candidate as

$$\begin{aligned} V = \frac{1}{2}s^T M(q)s + \frac{1}{2}\Delta\theta_d^T L_d^{-1}\Delta\theta_d + \frac{1}{2}\Delta\theta_k^T L_k^{-1}\Delta\theta_k + \frac{1}{2}\Delta\theta_a^T L_a^{-1}\hat{K}_N K_N^{-1}\Delta\theta_a \\ + \frac{1}{2}\Delta x^T (K_p + \alpha K_v)\Delta x \end{aligned} \quad (4.26)$$

where $\Delta\theta_k = \theta_k - \hat{\theta}_k$. Differentiating with respect to time, one has

$$\begin{aligned} \dot{V} = s^T M(q)\dot{s} + \frac{1}{2}s^T \dot{M}(q)s - \Delta\theta_d^T L_d^{-1}\dot{\Delta\theta}_d - \Delta\theta_k^T L_k^{-1}\dot{\Delta\theta}_k - \Delta\theta_a^T L_a^{-1}\dot{\Delta\theta}_a \\ + \Delta x^T (K_p + \alpha K_v)\Delta\dot{x} \end{aligned}$$

Substituting $M(q)\dot{s}$ from equation (4.25), $\dot{\hat{\theta}}_k$ from equation (4.5), $\dot{\hat{\theta}}_d$ from equation (4.4) and $\dot{\hat{\theta}}_a$ from equation (5.8) into the above equation, using equation (2.25), equation (4.14) and equation (4.16) yields,

$$\begin{aligned} \dot{V} = & -s^T B s - \hat{s}_x^T (K_v\Delta\dot{x} + K_p\Delta x) + \Delta x^T (K_p + \alpha K_v)\Delta\dot{x} \\ & - \Delta\theta_k^T Y_k^T(q, \dot{q})(K_v\Delta\dot{x} + K_p\Delta x) \end{aligned} \quad (4.27)$$

From equations (4.10), (2.5) and (4.8), one has

$$\hat{s}_x = \Delta\dot{x} + \alpha\Delta x - Y_k(q, \dot{q})\Delta\theta_k \quad (4.28)$$

where

$$Y_k(q, \dot{q})\Delta\theta_k = J(q)\dot{q} - \hat{J}(q, \hat{\theta}_k)\dot{q} = \dot{x} - \hat{\dot{x}} \quad (4.29)$$

Substituting equation (4.28) into equation (4.27) yields,

$$\dot{V} = -s^T B s - \Delta\dot{x}^T K_v \Delta\dot{x} - \alpha \Delta x^T K_p \Delta x \leq 0 \quad (4.30)$$

We are now in a position to state the following Theorem:

Theorem 4.1 *The adaptive Jacobian tracking control law (4.3) and the parameter update laws (4.5), (4.4) and (5.8) for the robot system (4.1) result in the convergence of position and velocity tracking errors. That is, $x - x_d \rightarrow 0$ and $\dot{x} - \dot{x}_d \rightarrow 0$, as $t \rightarrow \infty$.*

Proof:

Since $M(q)$ is uniformly positive definite, V in equation (4.26) is positive definite in s , Δx , $\Delta\theta_k$, $\Delta\theta_d$ and $\Delta\theta_a$. Since $\dot{V} \leq 0$, V is also bounded, and therefore s , Δx , $\Delta\theta_k$, $\Delta\theta_d$ and $\Delta\theta_a$ are bounded vectors. This implies that $\hat{\theta}_k$, $\hat{\theta}_d$, $\hat{\theta}_a$ are bounded, x is bounded if x_d is bounded, and $\hat{s}_x = \hat{J}(q, \hat{\theta}_k)s$ is bounded as seen from equation (4.16). Since Δx is bounded, $\dot{x}_r$ in equation (4.8) is also bounded if $\dot{x}_d$ is bounded. Therefore, $\ddot{q}_r$ in equation (4.12) is also bounded if the approximate Jacobian matrix is non singular. From equations (4.14), $\dot{q}$ is bounded and the boundedness of $\dot{q}$ means that $\dot{x}$ is bounded since the Jacobian matrix is bounded. Hence, $\Delta\dot{x}$ is bounded and $\ddot{x}_r$ in equation (4.9) is also bounded if $\ddot{x}_d$ is bounded. From equation (4.5), $\dot{\hat{\theta}}_k$ is therefore bounded since Δx , $\Delta\dot{x}$, $\dot{q}$ are bounded and $Y_k(\cdot)$ is a trigonometric function of q . Therefore, $\ddot{q}_r$ in equation (4.13) is bounded. From the closed-loop equation (4.25), one can conclude that $\dot{s}$ is bounded. The boundedness of $\dot{s}$ imply the boundedness of $\ddot{q}$ as seen from equation (4.15). From equation (4.11), $\dot{\hat{s}}_x$ is therefore bounded. Finally, differentiating equation (4.28) with respect to time and

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re-arranging yields,

$$\Delta\ddot{x} + \alpha\Delta\dot{x} = \dot{\hat{s}}_x + \dot{Y}_k(q, \dot{q}, \ddot{q})\Delta\theta_k - Y_k(q, \dot{q})\dot{\hat{\theta}}_k$$

which means that $\Delta\ddot{x} = \ddot{x} - \ddot{x}_d$ is also bounded.

To apply Barbalat's lemma, let us check the uniform continuity of $\dot{V}$. Differentiating equation (4.30) with respect to time gives,

$$\ddot{V} = -2s^T B\dot{s} - 2\Delta\dot{x}^T K_v \Delta\ddot{x} - 2\alpha\Delta x^T K_p \Delta\dot{x}$$

This shows that $\ddot{V}$ is bounded since Δx , $\Delta\dot{x}$, $\Delta\ddot{x}$ are all bounded. Hence, $\dot{V}$ is uniformly continuous. Using Barbalat's lemma, one has $\Delta x = x - x_d \rightarrow 0$, $\Delta\dot{x} = \dot{x} - \dot{x}_d \rightarrow 0$ and $s \rightarrow 0$ as $t \rightarrow \infty$. $\square$

Remark 4.2 To avoid the need for measuring task-space velocity, a known signal y based on filtered differentiation of the measured position x can be introduced as,

$$\dot{y} + \lambda y = \lambda \dot{x} \quad (4.31)$$

The signal y can be computed by measuring x alone. With p the Laplace variable, y can be written from equations (2.5) and (4.31) as

$$y = \frac{\lambda p}{p + \lambda} x = W_k(t)\theta_k \quad (4.32)$$

where

$$W_k(t) = \frac{\lambda}{p + \lambda} Y_k(q, \dot{q})$$

with $y(0) = 0$ and $W_k(0) = 0$. Then the kinematic adaptation law (4.5) is modified to

$$\dot{\hat{\theta}}_k = -L_k W_k^T(t) K_v (W_k(t)\hat{\theta}_k - y) + L_k Y_k^T(q, \dot{q}) (K_p + \alpha K_v) \Delta x \quad (4.33)$$

and accordingly the control input is modified as

$$I = -\hat{K}_N^{-1} \hat{J}^T(q, \hat{\theta}_k)(K_v \Delta \dot{x} + K_p \Delta x) + \hat{K}_N^{-1} Y_d(q, \dot{q}, \ddot{q}_r) \hat{\theta}_d + \hat{K}_N^{-1} Y_a(\tau_o) \hat{\theta}_a, \quad (4.34)$$

where $\Delta \dot{x} = \dot{x} - \dot{x}_d$. It can be verified that with the modified adaptation law the system stability is still retained. $\diamond$

Remark 4.3 If the kinematic parameter update equation (4.5) is modified as,

$$\dot{\hat{\theta}}_k = -P Y_k^T(q, \dot{q}) P(\dot{x} - \dot{x}_d) + L_k Y_k^T(q, \dot{q})(K_v \Delta \dot{x} + K_p \Delta x)$$

where P is a symmetric positive definite matrix. This add to $\dot{V}$ minus the P square norm of $Y_k(q, \dot{q}) \Delta \theta_k$. Hence, if the persistent excitation condition is satisfied, the convergence of $\hat{\theta}_k$ can be achieved. $\diamond$

Remark 4.4 From equation (4.28), the adaptive sliding vector can be expressed as:

$$\hat{s}_x = \Delta \dot{x} + \alpha \Delta x + Y_k(q, \dot{q}) \hat{\theta}_k - Y_k(q, \dot{q}) \theta_k \quad (4.35)$$

Hence, the sign of the parameter update laws in equations (4.5) and (4.4) are different because the last term in equation (4.3) is positive while the last term in equation (4.35) is negative. $\diamond$

Remark 4.5 In this thesis, it's assumed that the robot is operating in a finite task space such that the approximate Jacobian matrix is of full rank. Note from equation (4.12) that $\hat{J}^{-1}(q, \hat{\theta}_k)$ is used only in the definition of control variable $\dot{q}_r$. Therefore, one should be able to control this by bounding the variable or using a singularity-robust inverse of the approximate Jacobian matrix [65]. Often, singularities only depend on q , not $\hat{\theta}_k$. It may also be possible to avoid boundary singularities by originally overestimating the size of the robot. $\diamond$

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Remark 4.6 As in [68], the definition (4.12) and its derivative (4.13) can also be implemented as filtered signals as,

$$\ddot{q}_r + \lambda \dot{q}_r = \hat{J}^{-1}(q, \hat{\theta}_k)(\ddot{x}_r + \lambda \dot{x}_r - \dot{\hat{J}}(q, \hat{\theta}_k)\dot{q}_r)$$

with $\lambda > 0$. This implies that,

$$\frac{d}{dt}(\hat{J}(q, \hat{\theta}_k)\dot{q}_r) + \lambda \hat{J}(q, \hat{\theta}_k)\dot{q}_r = \ddot{x}_r + \lambda \dot{x}_r$$

so that $\hat{J}(q, \hat{\theta}_k)\dot{q}_r$ and its derivative tend to $\dot{x}_r$ and its derivative. $\diamond$

4.3 Adaptive Jacobian Tracking Control of Robot with Voltage-Driven Motors

In this subsection, the tracking control problem of robot with voltage-driven motors is addressed. Different from current driven robot, the closed-loop system dynamics is third-order because of the inductance existed in the actuator system. Hence a backstepping procedure similar as in *Chapter 3* is adopted to solve the problem. A desired intermediary signal I_d is defined for the armature current I , then based on the actuator dynamics a voltage control input u is designed to force the armature current to track desired current signal I_d . Two adaptive observers are proposed to avoid accelerations in the control voltage design.

As discussed in *Subsection 2.2.2.2*, the dynamics of robot driven by voltage amplifier with n degrees of freedom can be expressed as:

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_N I, \quad (4.36)$$

$$L\dot{I} + RI + K_E \dot{q} = u, \quad (4.37)$$

where $I \in R^n$ is the vector of armature currents and $u \in R^n$ denotes the vector of armature voltages. $L \in R^{n \times n}$ represents the actuator inductance matrix; $R \in R^{n \times n}$ is the armature resistance matrix, $K_E \in R^{n \times n}$ is the matrix characterizing the voltage constant of the actuator. L , R , K_E and K_N are positive definite constant diagonal matrices. In this thesis, it's assumed that the exact values of the dynamics parameters in $M(q)$, $C(q, \dot{q})$ and $g(q)$ and the actuator dynamic coefficient matrices L , R , K_E and K_N are not available; Jacobian matrix $J(q)$ is not known exactly due to uncertainties in robot kinematics.

In the presence of kinematic uncertainty, the parameters of the Jacobian matrix is uncertain and hence the estimated differential kinematics can be expressed as

$$\hat{x} = \hat{J}(q, \hat{\theta}_k) \dot{q} = Y_k(q, \dot{q}) \hat{\theta}_k \quad (4.38)$$

where $\hat{x} \in R^n$ denotes an estimated task-space velocity, $\hat{J}(q, \hat{\theta}_k) \in R^{n \times n}$ is an approximate Jacobian matrix and $\hat{\theta}_k \in R^q$ denotes a set of estimated kinematic parameters.

Using the desired armature current I_d , the manipulator subsystem dynamics (4.36) can be rewritten as

$$M(q) \ddot{q} + (B + C(q, \dot{q})) \dot{q} + g(q) = K_N I_d + K_N \tilde{I}, \quad (4.39)$$

where $\tilde{I} = I - I_d$ represents a current perturbation to the robot manipulator dynamics which will be handled by the voltage control input u defined later.

Let $x_d(t) \in R^n$ be the desired task space trajectory which is differentiable up to third order. Using an approximate motor torque transmission matrix $\hat{K}_N$, the desired armature current signal can be proposed as

$$I_d = \hat{K}_N^{-1} [-\hat{J}^T(q, \hat{\theta}_k) K_p \Delta x - K_d(\hat{\dot{q}}_o - \dot{q}_r) + Y_d(q, \hat{\dot{q}}_o, \dot{q}_r, \hat{\ddot{q}}_r) \hat{\theta}_d + Y_a(I_o) \hat{\theta}_a], \quad (4.40)$$

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where $\Delta x = x - x_d$, x is measured by a sensor, e.g., camera or laser tracking system; $K_p \in R^{n \times n}$, $K_d \in R^{n \times n}$ are positive definite diagonal matrices.

The regressor $Y_a(I_o) = \text{diag}\{I_{o1}, I_{o2}, \dots, I_{on}\}$ where I_{oi} denotes the i^{th} element of the vector I_o which is defined as

$$I_o = -\hat{J}^T(q, \hat{\theta}_k)K_p\Delta x - K_d(\hat{q}_o - \dot{q}_r) + Y_d(q, \hat{q}_o, \dot{q}_r, \hat{q}_r)\hat{\theta}_d. \quad (4.41)$$

$Y_a(I_o)\hat{\theta}_a$ is used to compensate the uncertainty brought by the approximate motor torque transmission matrix $\hat{K}_N$ and $\hat{\theta}_a \in R^n$ is updated through the following adaptation law

$$\dot{\hat{\theta}}_a = -L_a Y_a(I_o)s, \quad (4.42)$$

where $L_a \in R^{n \times n}$ is positive definite diagonal matrix. $\hat{\theta}_k$ in $\hat{J}(q, \hat{\theta}_k)$ is updated using the adaptation law as

$$\dot{\hat{\theta}}_k = L_k Y_k^T(q, \dot{q})(K_p\Delta x + K_x\tilde{x}), \quad (4.43)$$

where $K_x \in R^{n \times n}$ is a positive definite diagonal matrix, $\tilde{x} = x - \hat{x}_o$ and $\hat{x}_o \in R^n$ is the observed signal for x which is updated through the following observer

$$\dot{\hat{x}}_o = \hat{J}(q, \hat{\theta}_k)\hat{q}_o + \Lambda(x - \hat{x}_o), \quad (4.44)$$

where $\dot{\hat{x}}_o$ is the time derivative of $\hat{x}_o$; $\Lambda \in R^{n \times n}$ is a positive definite diagonal matrix; $\hat{q}_o$ is the observed signal of $\dot{q}$ and is updated through the observer:

$$\begin{aligned} \dot{\hat{q}}_o = & \hat{M}^{-1}(q)\{\hat{K}_N I - [\hat{B} + \hat{C}(q, \dot{q})]\hat{q}_o - \hat{g}(q) + \hat{J}^T(q, \hat{\theta}_k)K_x\tilde{x} + K_q\tilde{q} \\ & + [K_d + Z_r^T(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{J}^{-1}(q, \hat{\theta}_k)\hat{M}(q) - \hat{C}^T(q, \dot{q}_r)]s - \hat{K}_N Y_a(I)\hat{\theta}_a'\}, \end{aligned} \quad (4.45)$$

where $\hat{\dot{q}}_o$ is the time derivative of $\hat{q}_o$; $\tilde{q} = \dot{q} - \hat{\dot{q}}_o$; $\hat{M}(q)$, $\hat{B}$, $\hat{C}(q, \dot{q}_r)$ and $\hat{g}(q)$ are the approximate dynamic matrices and vector respectively using the estimated dynamic parameters in $\hat{\theta}_d$. The definition and role of the known matrix $Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})$ will be made clear later.

The observed signals $\hat{q}_o$ and $\hat{x}_o$ are introduced to avoid task space and joint space accelerations in the design of control voltage input u which will utilize the time derivative of I_d . $Y_a(I)\hat{\theta}'_a$ in $\hat{\dot{q}}_o$ plays the same role as $Y_a(I_o)\hat{\theta}_a$ in the desired armature current design and $\hat{\theta}'_a \in R^n$ is updated through the adaptation law

$$\dot{\hat{\theta}}'_a = -L'_a Y_a(I) \tilde{q}, \quad (4.46)$$

where $L'_a \in R^{n \times n}$ is a positive definite diagonal matrix. The observer for $\dot{q}$ as defined by (4.45) contains the online information of I and it will be shown that the combination of controller and observer retains the passivity of the system.

Here $\dot{q}_r \in R^n$ is an auxiliary signal defined using the observed signal $\hat{x}_o$ as

$$\dot{q}_r = \hat{J}^{-1}(q, \hat{\theta}_k) [\dot{x}_d - \alpha(\hat{x}_o - x_d)]. \quad (4.47)$$

and the time derivative of $\dot{q}_r$ is given as follows

$$\ddot{q}_r = \dot{\hat{J}}^{-1}(q, \hat{\theta}_k) [\dot{x}_d - \alpha(\hat{x}_o - x_d)] + \hat{J}^{-1}(q, \hat{\theta}_k) [\ddot{x}_d - \alpha(\dot{\hat{x}}_o - \dot{x}_d)]. \quad (4.48)$$

By using the definition $\dot{\hat{J}}^{-1}(q, \hat{\theta}_k) = -\hat{J}^{-1}(q, \hat{\theta}_k) \dot{\hat{J}}(q, \hat{\theta}_k) \hat{J}^{-1}(q, \hat{\theta}_k)$ and equations (4.43) and (4.47), $\ddot{q}_r$ can be written into the form

$$\begin{aligned} \ddot{q}_r &= -\hat{J}^{-1}(q, \hat{\theta}_k) \dot{\hat{J}}(q, \hat{\theta}_k) \hat{J}^{-1}(q, \hat{\theta}_k) [\dot{x}_d - \alpha(\hat{x}_o - x_d)] + \hat{J}^{-1}(q, \hat{\theta}_k) [\ddot{x}_d - \alpha \dot{\tilde{x}}] \\ &= -\hat{J}^{-1}(q, \hat{\theta}_k) \dot{\hat{J}}(q, \hat{\theta}_k) \dot{q}_r + \hat{J}^{-1}(q, \hat{\theta}_k) [\ddot{x}_d - \alpha \dot{\tilde{x}}] \\ &= -\hat{J}^{-1}(q, \hat{\theta}_k) Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x}) \dot{q} + \hat{J}^{-1}(q, \hat{\theta}_k) [\ddot{x}_d - \alpha \dot{\tilde{x}}] \end{aligned} \quad (4.49)$$

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where $\dot{J}(q, \hat{\theta}_k)$ is a function of $\dot{q}$ and $\dot{\hat{\theta}}_k$; $\dot{q}_r$ is a function of $\hat{\theta}_k$, $\hat{x}_o$, x_d and $\dot{x}_d$.

As can be seen from equation (4.49), since $\dot{q}$ appears in $\ddot{q}_r$, if $\ddot{q}_r$ is directly used in the desired armature current design I_d (see equation (4.40)) then the joint acceleration $\ddot{q}$ will appear in $\dot{I}_d$ which will be used in the control voltage u design in next step. The purpose of introducing $Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})$ is to extract $\dot{q}$ out from $\dot{J}(q, \hat{\theta}_k)$ and replace it by the observed signal $\hat{q}_o$ to form a modified variable for $\ddot{q}_r$. The modified variable $\hat{\ddot{q}}_r$ using the observed velocity $\hat{q}_o$ is defined as

$$\begin{aligned}\hat{\ddot{q}}_r &= -\hat{J}^{-1}(q, \hat{\theta}_k)Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{q}_o + \hat{J}^{-1}(q, \hat{\theta}_k)[\ddot{x}_d - \alpha(\dot{\hat{x}}_o - \dot{x}_d)] \\ &= \ddot{q}_r + \hat{J}^{-1}(q, \hat{\theta}_k)Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})\tilde{q}.\end{aligned}\quad (4.50)$$

where $\tilde{q} = \dot{q} - \hat{q}_o$.

The dynamic regressor matrix $Y_d(q, \hat{q}_o, \dot{q}_r, \hat{\ddot{q}}_r)$ in I_d is defined as

$$Y_d(q, \hat{q}_o, \dot{q}_r, \hat{\ddot{q}}_r)\hat{\theta}_d = \hat{M}(q)\hat{\ddot{q}}_r + [\hat{B} + \hat{C}(q, \hat{q}_o)]\dot{q}_r + \hat{g}(q), \quad (4.51)$$

Substituting equation (4.50) into (4.51) and using *Property 2.1*, one has

$$\begin{aligned}Y_d(q, \hat{q}_o, \dot{q}_r, \hat{\ddot{q}}_r)\hat{\theta}_d &= \hat{M}(q)\ddot{q}_r + [\hat{B} + \hat{C}(q, \dot{q})]\dot{q}_r + \hat{g}(q) \\ &\quad + \hat{M}(q)\hat{J}^{-1}(q, \hat{\theta}_k)Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})\tilde{q} - \hat{C}(q, \dot{q}_r)\tilde{q} \\ &= Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\theta}_d + \hat{M}(q)\hat{J}^{-1}(q, \hat{\theta}_k)Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})\tilde{q} - \hat{C}(q, \dot{q}_r)\tilde{q},\end{aligned}\quad (4.52)$$

where

$$Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\theta}_d = \hat{M}(q)\ddot{q}_r + [\hat{B} + \hat{C}(q, \dot{q})]\dot{q}_r + \hat{g}(q). \quad (4.53)$$

The estimated dynamic parameters vector $\hat{\theta}_d$ is updated by the adaptation law

$$\dot{\hat{\theta}}_d = -L_d[Y_d^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)s + Y_d^T(q, \dot{q}, \hat{q}_o, \hat{q}_o)\tilde{q}], \quad (4.54)$$

where $L_d \in R^{p \times p}$ is a positive definite diagonal matrix.

Then an adaptive sliding vector s can be defined using $\dot{q}_r$ as

$$s = \dot{q} - \dot{q}_r, \quad (4.55)$$

such that

$$\dot{s} = \ddot{q} - \ddot{q}_r, \quad (4.56)$$

and

$$\begin{aligned} \hat{J}(q, \hat{\theta}_k)s &= \hat{J}(q, \hat{\theta}_k)\dot{q} - \dot{x}_d + \alpha(\hat{x}_o - x_d) \\ &= \dot{x} - \dot{x}_d + \hat{J}(q, \hat{\theta}_k)\dot{q} - \dot{x} + \alpha(x - x_d) - \alpha(x - \hat{x}_o) \\ &= \Delta\dot{x} + \alpha\Delta x - Y_k(q, \dot{q})\Delta\theta_k - \alpha\tilde{x} \end{aligned} \quad (4.57)$$

where $\tilde{x} = x - \hat{x}_o$ and equation (4.38) have been used.

Substituting equation (4.40) into (4.39), one can get the closed-loop manipulator dynamic equation

$$\begin{aligned} M(q)\ddot{q} + [B + C(q, \dot{q})]\dot{q} + g(q) + \hat{J}^T(q, \hat{\theta}_k)K_p\Delta x + K_d(\hat{q}_o - \dot{q}_r) - Y_d(q, \hat{q}_o, \dot{q}_r, \hat{q}_r)\hat{\theta}_d \\ + (I - K_N\hat{K}_N^{-1})I_o - K_N\hat{K}_N^{-1}Y_a(I_o)\hat{\theta}_a = K_N\tilde{I} \end{aligned} \quad (4.58)$$

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Using equation (4.52) and substituting (4.55) and (4.56) into (4.58), one has

$$\begin{aligned} M(q)\dot{s} + [B + C(q, \dot{q})]s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\Delta\theta_d - \hat{M}(q)\hat{J}^{-1}(q, \hat{\theta}_k)Z_r(q, \hat{\theta}_k, \Delta x, \tilde{x})\tilde{q} \\ + \hat{C}(q, \dot{q}_r)\tilde{q} + \hat{J}^T(q, \hat{\theta}_k)K_p\Delta x + K_d(\hat{q}_o - \dot{q}_r) \\ + Y_a(I_o)(\theta_a - K_N\hat{K}_N^{-1}\hat{\theta}_a) + C(q, \dot{q}_r)s = K_N\tilde{I} \end{aligned} \quad (4.59)$$

where *Property 2.7* has been used; $\theta_a = [1 - \frac{k_N^1}{\hat{k}_N^1} \cdots 1 - \frac{k_N^n}{\hat{k}_N^n}]^T$ with k_N^i and $\hat{k}_N^i$ being the i^{th} diagonal element of K_N and $\hat{K}_N$ respectively.

Next, the closed-loop observer dynamics is to be developed. From equation (4.45), note that

$$\begin{aligned} \hat{K}_NI - \hat{M}(q)\hat{q}_o - [\hat{B} + \hat{C}(q, \dot{q})]\hat{q}_o - \hat{g}(q) + \hat{J}^T(q, \hat{\theta}_k)K_x\tilde{x} + K_q\tilde{q} \\ + [K_d + Z_r^T(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{J}^{-1}(q, \hat{\theta}_k)\hat{M}(q) - \hat{C}^T(q, \dot{q}_r)]s - \hat{K}_NY'_a(I)\hat{\theta}'_a = 0, \end{aligned} \quad (4.60)$$

which can be further written as

$$\begin{aligned} K_NI - \hat{M}(q)\hat{q}_o - [\hat{B} + \hat{C}(q, \dot{q})]\hat{q}_o - \hat{g}(q) + \hat{J}^T(q, \hat{\theta}_k)K_x\tilde{x} \\ + [K_d + Z_r^T(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{J}^{-1}(q, \hat{\theta}_k)\hat{M}(q) - \hat{C}^T(q, \dot{q}_r)]s + K_q\tilde{q} \\ + \hat{K}_N(I - \hat{K}_N^{-1}K_N)I - \hat{K}_NY'_a(I)\hat{\theta}'_a = 0, \\ K_NI - \hat{M}(q)\hat{q}_o - [\hat{B} + \hat{C}(q, \dot{q})]\hat{q}_o - \hat{g}(q) + \hat{J}^T(q, \hat{\theta}_k)K_x\tilde{x} \\ + [K_d + Z_r^T(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{J}^{-1}(q, \hat{\theta}_k)\hat{M}(q) - \hat{C}^T(q, \dot{q}_r)]s + K_q\tilde{q} \\ + \hat{K}_NY'_a(I)\theta_a - \hat{K}_NY'_a(I)\hat{\theta}'_a = 0, \\ K_NI - Y_d(q, \dot{q}, \hat{q}_o, \hat{q}_o)\hat{\theta}_d + \hat{J}^T(q, \hat{\theta}_k)K_x\tilde{x} + K_q\tilde{q} \\ + [K_d + Z_r^T(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{J}^{-1}(q, \hat{\theta}_k)\hat{M}(q) - \hat{C}^T(q, \dot{q}_r)]s \\ + \hat{K}_NY'_a(I)(\theta_a - \hat{\theta}'_a) = 0, \end{aligned} \quad (4.61)$$

where it can be seen that $Y_a(I)$ possesses the same structure as $Y_a(I_o)$ but uses I

instead of I_o on the diagonal; $Y_d(q, \dot{q}, \hat{q}_o, \hat{q}_o)$ is defined similarly as $Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$:

$$Y_d(q, \dot{q}, \hat{q}_o, \hat{q}_o)\hat{\theta}_d = \hat{M}(q)\hat{q}_o + [\hat{B} + \hat{C}(q, \dot{q})]\hat{q}_o + \hat{g}(q), \quad (4.62)$$

Substituting equation (4.36) into (4.61), the closed-loop observer dynamic equation is obtained as

$$\begin{aligned} M(q)\ddot{\tilde{q}} + [B + K_q + C(q, \dot{q})]\dot{\tilde{q}} + Y_d(q, \dot{q}, \hat{q}_o, \hat{q}_o)\Delta\theta_d + \hat{J}^T(q, \hat{\theta}_k)K_x\tilde{x} \\ + [K_d + Z_r^T(q, \hat{\theta}_k, \Delta x, \tilde{x})\hat{J}^{-T}(q, \hat{\theta}_k)\hat{M}(q) - \hat{C}^T(q, \dot{q}_r)]s \\ + \hat{K}_N Y'_a(I)(\theta_a - \hat{\theta}'_a) = 0 \end{aligned} \quad (4.63)$$

where $\tilde{q} = \ddot{q} - \hat{q}_o$.

Now that the closed-loop dynamics of the manipulator and observer have been obtained, one can proceed to analyze the stability of closed-loop system with the desired armature current input proposed.

To analyze the system stability, a Lyapunov function candidate V is proposed as

$$\begin{aligned} V = \frac{1}{2}s^T M(q)s + \frac{1}{2}\Delta x^T K_p \Delta x + \frac{1}{2}\tilde{q}^T M(q)\tilde{q} + \frac{1}{2}\tilde{x}^T K_x \tilde{x} + \frac{1}{2}\Delta\theta_d^T L_d^{-1} \Delta\theta_d + \frac{1}{2}\Delta\theta_k^T L_k^{-1} \Delta\theta_k \\ + \frac{1}{2}(\hat{K}_N K_N^{-1} \theta_a - \hat{\theta}_a)^T L_a^{-1} K_N \hat{K}_N^{-1} (\hat{K}_N K_N^{-1} \theta_a - \hat{\theta}_a) + \frac{1}{2}(\theta_a - \hat{\theta}'_a)^T \hat{K}_N L_a'^{-1} (\theta_a - \hat{\theta}'_a) \end{aligned} \quad (4.64)$$

It's obvious that V is positive definite in all the signals contained. Differentiating V with respect to time, one can get

$$\begin{aligned} \dot{V} = s^T M(q)\dot{s} + \frac{1}{2}s^T \dot{M}(q)s + \Delta x^T K_p \Delta \dot{x} + \dot{\tilde{q}}^T M(q)\tilde{q} + \frac{1}{2}\dot{\tilde{q}}^T \dot{M}(q)\tilde{q} \\ + \tilde{x}^T K_x \dot{\tilde{x}} - (\theta_d - \hat{\theta}_d)^T L_d^{-1} \dot{\hat{\theta}}_d - (\theta_k - \hat{\theta}_k)^T L_k^{-1} \dot{\hat{\theta}}_k \\ - (\hat{K}_N K_N^{-1} \theta_a - \hat{\theta}_a)^T L_a^{-1} K_N \dot{\hat{K}}_N^{-1} \hat{\theta}_a - (\theta_a - \hat{\theta}'_a)^T \hat{K}_N L_a'^{-1} \dot{\hat{\theta}}'_a \end{aligned} \quad (4.65)$$

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where $\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}_o$.

Substituting equations (4.59) and (4.63) into equation (4.65), one can get

$$\begin{aligned}
\dot{V} = & -s^T(B + K_d)s - s^T \hat{J}^T(q, \hat{\theta}_k) K_p \Delta x - s^T Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \Delta \theta_d \\
& - s^T Y_a(I_o)(\theta_a - K_N \hat{K}_N^{-1} \hat{\theta}_a) + \Delta x^T K_p \Delta \dot{x} - \tilde{q}^T (B + K_q) \tilde{q} \\
& - \tilde{q}^T Y_d(q, \dot{q}, \hat{q}_o, \ddot{q}_o) \Delta \theta_d - \tilde{q}^T \hat{J}^T(q, \hat{\theta}_k) K_x \tilde{x} - \tilde{q}^T Y_a(I) \hat{K}_N (\theta_a - \hat{\theta}'_a) \\
& + \tilde{x}^T K_x \dot{\tilde{x}} - (\theta_d - \hat{\theta}_d)^T L_d^{-1} \dot{\hat{\theta}}_d - (\theta_k - \hat{\theta}_k)^T L_k^{-1} \dot{\hat{\theta}}_k \\
& - (\theta_a - K_N \hat{K}_N^{-1} \hat{\theta}_a)^T L_a^{-1} \dot{\hat{\theta}}_a - (\theta_a - \hat{\theta}'_a)^T \hat{K}_N L_a'^{-1} \dot{\hat{\theta}}'_a + s^T K_N \tilde{I}
\end{aligned} \quad (4.66)$$

Note from equations (4.38) and (4.44) that

$$\hat{J}(q, \hat{\theta}_k) \tilde{q} = \hat{J}(q, \hat{\theta}_k) \dot{q} - \hat{J}(q, \hat{\theta}_k) \hat{q}_o = \dot{\tilde{x}} + \Lambda \tilde{x} - Y_k(q, \dot{q}) \Delta \theta_k. \quad (4.67)$$

Hence substituting equations (4.57) and (4.67) into equation (4.66) leads to

$$\begin{aligned}
\dot{V} = & -s^T(B + K_d)s - \tilde{q}^T (B + K_q) \tilde{q} - \alpha \Delta x^T K_p \Delta x - \tilde{x}^T \Lambda K_x \tilde{x} \\
& + \alpha \tilde{x}^T K_p \Delta x + \Delta \theta_k^T (Y_k^T(q, \dot{q}) (K_p \Delta x + K_x \tilde{x}) - L_k^{-1} \dot{\hat{\theta}}_k) \\
& - \Delta \theta_d^T (Y_d^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) s + Y_d^T(q, \dot{q}, \hat{q}_o, \ddot{q}_o) \tilde{q} + L_d^{-1} \dot{\hat{\theta}}_d) \\
& - (\theta_a - K_N \hat{K}_N^{-1} \hat{\theta}_a)^T (Y_a^T(I_o) s + L_a^{-1} \dot{\hat{\theta}}_a) \\
& - (\theta_a - \hat{\theta}'_a)^T \hat{K}_N (Y_a^T(I) \tilde{q} + L_a'^{-1} \dot{\hat{\theta}}'_a) + s^T K_N \tilde{I}.
\end{aligned} \quad (4.68)$$

Bring the adaptation laws (4.42), (4.43), (4.46), (4.54) into (4.68), $\dot{V}$ can be simpli-

fed into

$$\begin{aligned}
\dot{V} &= -s^T(B + K_d)s - \tilde{q}^T(B + K_q)\tilde{q} - \alpha\Delta x^T K_p \Delta x - \tilde{x}^T \Lambda K_x \tilde{x} \\
&\quad + \alpha\tilde{x}^T K_p \Delta x + s^T K_N \tilde{I} \\
&\leq -\lambda_{\min}\{B + K_d\}\|s\|^2 - \lambda_{\min}\{B + K_q\}\|\tilde{q}\|^2 - \alpha\lambda_{\min}\{K_p\}\|\Delta x\|^2 \\
&\quad - \lambda_{\min}\{\Lambda K_x\}\|\tilde{x}\|^2 + \alpha\lambda_{\max}\{K_p\}\|\tilde{x}\|\|\Delta x\| + s^T K_N \tilde{I} \\
&\leq -\lambda_{\min}\{B + K_d\}\|s\|^2 - \lambda_{\min}\{B + K_q\}\|\tilde{q}\|^2 \\
&\quad - [\|\Delta x\| \|\tilde{x}\|] P [\|\Delta x\| \|\tilde{x}\|]^T + s^T K_N \tilde{I}
\end{aligned} \tag{4.69}$$

where P is a matrix of the form

$$P = \begin{bmatrix} \alpha\lambda_{\min}\{K_p\} & -\frac{1}{2}\alpha\lambda_{\max}\{K_p\} \\ -\frac{1}{2}\alpha\lambda_{\max}\{K_p\} & \lambda_{\min}\{\Lambda K_x\} \end{bmatrix} \tag{4.70}$$

Note that the matrix P is positive definite if the following condition is satisfied:

$$4\lambda_{\min}\{K_p\}\lambda_{\min}\{\Lambda K_x\} > \alpha\lambda_{\max}^2\{K_p\} \tag{4.71}$$

then

$$\begin{aligned}
\dot{V} &\leq -\lambda_{\min}\{B + K_d\}\|s\|^2 - \lambda_{\min}\{B + K_q\}\|\tilde{q}\|^2 - \lambda_{\min}\{P\}\|\Delta x\|^2 \\
&\quad - \lambda_{\min}\{P\}\|\tilde{x}\|^2 + s^T K_N \tilde{I}
\end{aligned} \tag{4.72}$$

Hence $\dot{V}$ is negative definite in s , $\tilde{q}$, Δx and $\tilde{x}$ when the armature current perturbation $\tilde{I} = 0$.

Lemma 4.1 *In the absence of armature current perturbation, i.e., $\tilde{I} = 0$ and with the condition (4.71) satisfied, the desired current design (4.40), the parameter adaptation laws (4.42), (4.43), (4.46), (4.54) together with the observers (4.44) and*

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(4.45) guarantee the convergence of $x - x_d \rightarrow 0$ and $\dot{x} - \dot{x}_d \rightarrow 0$ as $t \rightarrow \infty$. Moreover, $x - \hat{x}_o \rightarrow 0$ and $\dot{q} - \hat{\dot{q}}_o \rightarrow 0$ as $t \rightarrow \infty$.

Proof:

From equation (4.64) it can be seen that V is positive definite in $s, \hat{q}, \Delta x, \tilde{x}, \Delta\theta_d, \Delta\theta_k, \hat{K}_N K_N^{-1} \theta_a - \hat{\theta}_a$ and $\theta_a - \hat{\theta}'_a$. Since $\dot{V}$ is negative definite in $s, \tilde{q}, \Delta x$ and $\tilde{x}$, V is bounded as well the variables included in V . This implies that $\hat{\theta}_k, \hat{\theta}_d, \hat{\theta}_a, \hat{\theta}'_a$ are bounded, x and $\hat{x}_o$ are also bounded if x_d is bounded. Hence $\dot{q}_r$ in equation (4.47) is bounded if the approximate Jacobian matrix is non-singular and $\dot{x}_d$ is bounded. From the boundedness of s , it can be seen from equation (4.55) that $\dot{q}$ is also bounded which means $\dot{x} = J(q)\dot{q}$ is bounded. Since $\Delta\dot{x} = \dot{x} - \dot{x}_d$, $\Delta\dot{x}$ is bounded if $\dot{x}_d$ is bounded. Then boundedness of $\hat{\dot{q}}_o$ can be concluded because $\tilde{\dot{q}} = \dot{q} - \hat{\dot{q}}_o$ is bounded. From equation (4.44) it can be seen now that $\hat{\dot{x}}_o$ is bounded and hence $\tilde{\dot{x}} = \dot{x} - \hat{\dot{x}}_o$ is also bounded. From equation (4.43), $\hat{\dot{\theta}}_k$ is bounded since $\dot{q}, \Delta x, \tilde{x}$ are bounded. Then from equation (4.49), $\ddot{q}_r$ is bounded. Hence it can be seen from equation (4.40) that I_d is bounded since all signals contained in it are bounded. In the absence of armature current perturbation, $\tilde{I} = 0$ or $I = I_d$ and hence from equation (4.45) it has $\hat{\dot{q}}_o$ is bounded. The assumption $\tilde{I} = 0$ will be removed in next design step and it will be shown that the actual armature current I is still bounded and converge to I_d with the proposed voltage control input. Then from equation (4.39) it can be seen that $\ddot{q}$ is bounded, hence $\tilde{\ddot{q}} = \ddot{q} - \hat{\ddot{q}}_o$ and $\dot{s} = \ddot{q} - \ddot{q}_r$ are also bounded. Now it has been shown that $\dot{s}, \tilde{\dot{q}}, \Delta\dot{x}$ and $\tilde{\dot{x}}$ are all bounded which means $s, \tilde{q}, \Delta x$ and $\tilde{x}$ are uniformly continuous. Since V is bounded and $\dot{V}$ is negative definite in $s, \tilde{q}, \Delta x$ and $\tilde{x}$ when the armature current perturbation $\tilde{I} = 0$, it's obvious that $s, \tilde{q}, \Delta x, \tilde{x} \in L_2(0, \infty)$. Then it follows from [3] [28] that $s, \tilde{q}, \Delta x, \tilde{x} \rightarrow 0$ as $t \rightarrow \infty$. Since $\ddot{q}$ is bounded, $\ddot{x} = \dot{J}(q)\dot{q} + J(q)\ddot{q}$ is also bounded and hence $\Delta\ddot{x} = \ddot{x} - \ddot{x}_d$ is bounded if $\ddot{x}_d$ is bounded. The boundedness of $\Delta\ddot{x}$ means that $\Delta\dot{x}$ is uniformly continuous, also Δx has been shown to be bounded, then according to Babalat's Lemma [76], $\Delta\dot{x} \rightarrow 0$ as $t \rightarrow \infty$ and hence the proof is complete. $\square$

Now that it has been shown that the desired armature current design can guarantee the convergence of the tracking errors in the absence of armature current perturbation $\tilde{I}$, it's necessary to propose a voltage control input u such that $\tilde{I}$ converges to 0.

Since the actuator dynamic coefficient matrices L , R , K_E and K_N are unknown, the approximate models $\hat{L}$, $\hat{R}$, $\hat{K}_E$ and $\hat{K}'_N$ are employed and the input voltage u is proposed as:

$$u = \hat{L}\dot{I}_d + \hat{R}I_d + \hat{K}_E\dot{q} - N_N(s)\hat{\theta}_{K'_N} - K_I\tilde{I}, \quad (4.73)$$

$$\dot{\hat{\theta}}_L = -L_L N_L(\dot{I}_d)\tilde{I}, \quad (4.74)$$

$$\dot{\hat{\theta}}_R = -L_R N_R(I_d)\tilde{I}, \quad (4.75)$$

$$\dot{\hat{\theta}}_{K_E} = -L_E N_E(\dot{q})\tilde{I}, \quad (4.76)$$

$$\dot{\hat{\theta}}_{K'_N} = L_N N_N(s)\tilde{I}. \quad (4.77)$$

where L_L, L_R, L_E, L_N, K_I are positive definite diagonal matrices; $\tilde{I} = I - I_d$ and I_d is defined in (4.40). $N_L(\dot{I}_d) = \text{diag}(\dot{I}_{d1}, \dots, \dot{I}_{dn})$, $N_R(I_d) = \text{diag}(I_{d1}, \dots, I_{dn})$, $N_E(\dot{q}) = \text{diag}(\dot{q}_1, \dots, \dot{q}_n)$, $N_N(s) = \text{diag}(s_1, \dots, s_n)$ and hence $\hat{L}\dot{I}_d = N_L(\dot{I}_d)\hat{\theta}_L$, $\hat{R}I_d = N_R(I_d)\hat{\theta}_R$, $\hat{K}_E\dot{q} = N_E(\dot{q})\hat{\theta}_{K_E}$, $\hat{K}'_N y = N_N(y)\hat{\theta}_{K'_N}$. I is measured by sensor such as ammeter. The approximate models $\hat{L}$, $\hat{R}$, $\hat{K}_E$ and $\hat{K}'_N$ are updated online by the updating laws (4.74)-(4.77) during the control process. Note that $\hat{K}'_N$ is updated online by the updating law (4.77) during the control process, comparatively $\hat{K}_N$ used in the desired armature current design is a constant matrix.

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Substituting the input voltage (4.73) into the actuator dynamics (4.37), one has

$$\begin{aligned} L(\dot{I} - \dot{I}_d) + R(I - I_d) &= (\hat{L} - L)\dot{I}_d + (\hat{R} - R)I_d \\ &+ (\hat{K}_E - K_E)\dot{q} - N_N(s)\hat{\theta}_{K'_N} - K_I\tilde{I} \end{aligned} \quad (4.78)$$

which can be rewritten as

$$\begin{aligned} L\dot{\tilde{I}} + (R + K_I)\tilde{I} &= N_L(\dot{I}_d)(\theta_{\hat{L}} - \theta_L) + N_R(I_d)(\theta_{\hat{R}} - \theta_R) \\ &+ N_E(\dot{q})(\theta_{\hat{K}_E} - \theta_{K_E}) - N_N(s)\hat{\theta}_{K'_N} \end{aligned} \quad (4.79)$$

where θ_L , θ_R , θ_{K_E} , θ_{K_N} denote the unknown parameters in L , R , K_E , K_N respectively such that $L\dot{I}_d = N_L(\dot{I}_d)\theta_L$, $RI_d = N_R(I_d)\theta_R$, $K_E\dot{q} = N_E(\dot{q})\theta_{K_E}$, $K_N s = N_N(s)\theta_{K_N}$. Now another Lyapunov function candidate V is proposed with the following form:

$$\begin{aligned} V' = V &+ \frac{1}{2}\tilde{I}^T L \tilde{I} + \frac{1}{2}(\theta_L - \hat{\theta}_L)^T L_L^{-1}(\theta_L - \hat{\theta}_L) + \frac{1}{2}(\theta_R - \hat{\theta}_R)^T L_R^{-1}(\theta_R - \hat{\theta}_R) \\ &+ \frac{1}{2}(\theta_{K_E} - \hat{\theta}_{K_E})^T L_E^{-1}(\theta_{K_E} - \hat{\theta}_{K_E}) + \frac{1}{2}(\theta_{K_N} - \hat{\theta}_{K'_N})^T L_N^{-1}(\theta_{K_N} - \hat{\theta}_{K'_N}) \end{aligned} \quad (4.80)$$

where V is defined as in equation (4.64).

Differentiating V with respect to time and using equations (4.74)-(4.76),(4.79), one has:

$$\dot{V}' = \dot{V} - \tilde{I}^T(R + K_I)\tilde{I} - \tilde{I}^T N_N(s)\hat{\theta}_{K'_N} - \dot{\hat{\theta}}_{K'_N}^T L_N^{-1}(\theta_{K_N} - \hat{\theta}_{K'_N}). \quad (4.81)$$

Substituting equations (4.69) and (4.77) into equation (4.81) and using the inequal-

ity (4.72), one can get:

$$\begin{aligned}\dot{V}' \leq & -\lambda_{\min}\{B + K_d\}\|s\|^2 - \lambda_{\min}\{B + K_q\}\|\tilde{q}\|^2 - \lambda_{\min}\{P\}\|\Delta x\|^2 \\ & - \lambda_{\min}\{P\}\|\tilde{x}\|^2 - \lambda_{\min}\{R + K_I\}\|\tilde{I}\|^2 \leq 0\end{aligned}\quad (4.82)$$

Note that the term $s^T K_N \tilde{I}$ which is contained in $\dot{V}$ has been cancelled out by the adaptive parameter vector $\hat{\theta}_{K'_N}$ and its adaptation law (4.77). Hence, if the sufficient stability condition (4.71) are satisfied, $\dot{V}'$ is negative definite in $s, \tilde{q}, \Delta x, \tilde{x}$ and $\tilde{I}$.

We are now in a position to state the following Theorem:

Theorem 4.2 *If the control input voltage u given by (4.40), (4.42)–(4.46), (4.54) and (4.73)–(4.77) is applied on the rigid-link robot dynamics (4.36), (4.37), then the closed-loop system gives rise to the convergence of $x \rightarrow x_d, \dot{x} \rightarrow \dot{x}_d$ and $I \rightarrow I_d$ as $t \rightarrow \infty$, if the feedback gains are chosen as in condition (4.71).*

Proof

Since V' is non-negative and $\dot{V}'$ is negative definite in $s, \tilde{q}, \Delta x, \tilde{x}$ and $\tilde{I}$, V' is bounded which means variables $\hat{\theta}_k, \hat{\theta}_d, \hat{\theta}_a, \hat{\theta}'_a, \hat{\theta}_l, \hat{\theta}_R, \hat{\theta}_{KE}, \hat{\theta}_{K'_N}, s, \tilde{x}, \Delta x, \tilde{q}$ are bounded. It's noticeable that the current perturbation $\tilde{I}$ is also bounded since $\tilde{I}$ is included in V' . Similarly as in the proof for Lemma 4.1, it's easy to verify that variables $x, \hat{x}_o, \dot{q}_r, \dot{q}, \hat{q}_o, \dot{\hat{x}}_o$ and $\Delta\dot{x}, \dot{\tilde{x}}$ are all bounded, hence $\ddot{q}_r$ is bounded from equation (4.49). From equation (4.40) it can be seen that I_d is bounded. Since $\tilde{I}$ is also bounded, the armature current I is also bounded. Then from equations (4.36) and (4.45) it's easily verified that $\ddot{q}$ and $\hat{\hat{q}}_o$ are bounded. From the boundedness of $\ddot{q}, \ddot{q}_r$ and $\hat{\hat{q}}_o, \tilde{\tilde{q}}$ and $\dot{s}$ can be concluded to be bounded. Now we have shown the boundedness of $\dot{s}, \tilde{\tilde{q}}, \Delta\dot{x}, \dot{\tilde{x}}$ and $\dot{\tilde{I}}$. Since V' is bounded and $\dot{V}'$ is negative definite in $s, \tilde{q}, \Delta x, \tilde{x}$ and $\tilde{I}$, it's clear that $s, \tilde{q}, \Delta x, \tilde{x}, \tilde{I} \in L_2(0, \infty)$. Then it follows from [3] [28] that $s, \tilde{q}, \Delta x, \tilde{x}, \tilde{I} \rightarrow 0$ as $t \rightarrow \infty$. Since $\ddot{q}$ is bounded, $\ddot{x} = \dot{J}(q)\dot{q} + J(q)\ddot{q}$ is also bounded and hence $\Delta\ddot{x} = \ddot{x} - \ddot{x}_d$ is bounded if $\ddot{x}_d$ is

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bounded. The boundedness of $\Delta\ddot{x}$ means that $\Delta\dot{x}$ is uniformly continuous, also Δx has been shown to be bounded, then according to Babalat's Lemma [76], $\Delta\dot{x} \rightarrow 0$ as $t \rightarrow \infty$. Hence the proof is complete. $\square$

Remark 4.7 To guarantee $\hat{M}^{-1}(q)$ used in $\hat{q}_o$ is non-singular, a projection algorithm can be used for the adaptation of $\hat{\theta}_d$ so that $\hat{\theta}_d$ is kept in a range $\theta_{dmin} \leq \hat{\theta}_d \leq \theta_{dmax}$ where θ_{dmax} , θ_{dmin} are the upper and lower estimation bounds for $\hat{\theta}_d$ [71]. $\diamond$

Remark 4.8 As discussed in *Remark 4.1*, the result presented in this section can also be extended to redundant robots by using the pseudo inverse Jacobian $\hat{J}^+(q, \hat{\theta}_k)$ instead of $\hat{J}^{-1}(q, \hat{\theta}_k)$ in the controller development. $\diamond$

Remark 4.9 In this thesis, the use of observer is to avoid acceleration in the voltage control input design. The observer design for signal $\hat{q}_o$ as described in equation (4.45) is based on the passivity observer design concept proposed by H. Berghuis and H. Nijmeijer [11]. The research works using this design approach in dealing with actuator dynamics [74] assume that the kinematics is known and normally require the system initial states to satisfy certain conditions to stabilize the observer-controller system. The constraint on system initial states problem is dealt with by using a robust observer [75], but at the cost of tracking errors. The proposed control method in this thesis does not require the exact kinematics parameters and imposes no assumption on system initial states. $\diamond$

Remark 4.10 By defining a fixed point as the desired trajectory, the tracking control methods proposed in this chapter can solve the setpoint problems as studied in Chapter 3. However, tracking control is a more sophisticated control task, the tracking control methods often need to use full robot manipulator dynamics information and hence result in complicated controller design. Comparatively, setpoint control methods depend much less on the manipulator dynamics, in most cases only the gravity force, due to the simplicity nature of the regulation task and hence can be synthesized in a much simpler and more instinctive way. Thus for practical

regulation applications, the results proposed in Chapter 3 can save remarkable control design effort as compared with the tracking results proposed in this chapter. However, it's important to notice that, for fast trajectory tracking applications, the setpoint controllers are not capable to achieve the control goal and tracking control methods should be used instead. $\diamond$

Remark 4.11 For adaptive Jacobian tracking control of robot with current-driven motors as studied in *Section 4.2*, the recursive calculation algorithm as suggested by G. Niemeyer and J. J. E. Slotine [68] can be implemented to improve the computation efficiency of the proposed tracking result since the dynamic regressor $Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ is defined in a similar way and the reference velocity $\dot{q}_r$, reference acceleration $\ddot{q}_r$ are explicitly given as in equation (4.12) and (4.13). However, the implementation of the algorithm in the tracking results for robot with voltage-driven motors remains a research issue because of the nonlinear observer involved and the complexity of the backstepping design procedures. $\diamond$

4.4 Simulation Results

In this section, simulation results are presented for the two control methods proposed in this chapter. The simulations are based on the model of a 2-link robot holding an unknown object as shown in Figure 4.1.

In the simulation, the masses, lengths and gravity centers of link 1, 2 and the object are set as $m_1 = 17.4kg$, $m_2 = 4.8kg$, $m_o = 2kg$, $l_1 = 0.4318m$, $l_2 = 0.4318m$, $l_0 = 0.2m$, $l_{c1} = 0.068m$, $l_{c2} = 0.07m$, $l_{c0} = 0.1m$ respectively. The object grasping angle is set as $q_0 = 30^\circ$.

Figure 4.1: A 2-Link Robot Holding an Object

4.4.1 Tracking Control for Robot with Current-Driven Motors

In this simulation, the controller presented in *Theorem 4.1* is implemented. The manipulator endpoint is required to follow a circular trajectory in Cartesian space with a radius of 0.2m and center at (0.35, 0.35). The desired trajectory is specified as:

$$\begin{aligned} X_d &= 0.35 + 0.2\sin(3t) \\ Y_d &= 0.35 + 0.2\cos(3t) \end{aligned} \quad (4.83)$$

The exact motor torque transmission matrix K_N is set as $K_N = \text{diag}(18 \ 14)$. To test the ability of the proposed control method to deal with kinematic and actuator uncertainties, the initial estimated kinematic parameters are set as $\hat{l}_1(0) = 0.5m$, $\hat{l}_2(0) = 0.5m$, $\hat{l}_0(0) = 0.3m$ and $\hat{q}_0 = 45^\circ$; the estimated motor torque transmission matrix $\hat{K}_N$ is set as $\hat{K}_N = \text{diag}(13 \ 10)$. The estimated manipulator link masses are

$\hat{m}_1(0) = 15kg$, $m_2(0) = 6kg$ and the object mass as $m_o(0) = 1.5kg$.

In the simulation, the control gains are set as $K_p = 350I_E$, $K_v = 50I_E$ and $\alpha = 12$. The parameters adaptation gains are set as $L_d = 50I_E$, $L_k = 0.1I_E$ and $L_a = 0.01I_E$. The initial position of the manipulator endpoint is specified as $(X(0), Y(0)) = (0.43, 0.5)$. Figure 4.2 shows the first round tracking trajectory and Figure 4.3 shows the last round tracking trajectory. In Figure 4.4, the position and velocity tracking errors are illustrated. As shown in the simulation results, both position and velocity tracking errors converge despite the presence of kinematic and dynamic uncertainties.

Figure 4.2: First Round Trajectory

Figure 4.3: Last Round Trajectory

Figure 4.4: Tracking Errors

4.4.2 Tracking Control for Robot with Voltage-Driven Motors

In this subsection, simulation results are presented for the tracking control of robot with voltage-driven motors based on the same manipulator model as used in the previous simulation. The desired trajectory is defined as a circular trajectory in Cartesian space with a radius of 0.2m and center at (0.35, 0.35):

$$\begin{aligned} X_d &= 0.35 + 0.2\sin(t) \\ Y_d &= 0.35 + 0.2\cos(t) \end{aligned} \quad (4.84)$$

The exact actuator dynamic models are set as $L = \text{diag}(4 \ 4)$, $R = \text{diag}(15 \ 15)$, $K_E = \text{diag}(10 \ 8)$, $K_N = \text{diag}(10 \ 8)$. The initial estimated actuator models are set as $\hat{L}(0) = \text{diag}(5 \ 5)$, $\hat{R}(0) = \text{diag}(10 \ 12)$, $\hat{K}_E(0) = \text{diag}(15 \ 10)$, $\hat{K}_N = \hat{K}'_N(0) = \text{diag}(15 \ 12)$. The robot kinematic parameters are estimated as $\hat{l}_1(0) = 0.5m$, $\hat{l}_2(0) = 0.5m$, $\hat{l}_0(0) = 0.25m$, $\hat{q}_0(0) = 45^\circ$ and the masses of the links and object are estimated as $\hat{m}_1(0) = 16kg$, $\hat{m}_2(0) = 5.5kg$ and $\hat{m}_o = 1.5kg$.

In the simulation, the control gains are set as $K_p = 450I_E$, $K_v = K_x = K_q = K_I = 250I_E$, $\alpha = 15$, $\Lambda = 40I_E$. The parameters adaptation gains are set as $L_d = 1.5I_E$, $L_k = 0.35I_E$, $L_a = L'_a = 0.05I_E$, $L_L = L_R = L_E = L_N = 0.04I_E$. Figure 4.5 shows the first round tracking trajectory and Figure 4.6 shows the last round tracking trajectory. In Figure 4.7 the position and velocity tracking errors are illustrated. And Figure 4.8 shows the armature current tracking errors with the proposed control voltage input. As illustrated through the simulation results, despite all the uncertainties in robot kinematics, manipulator dynamics and actuator dynamics, the both manipulator tracking error and armature tracking error converge to 0.

Figure 4.5: First Round Trajectory

Figure 4.6: Last Round Trajectory

Figure 4.7: Tracking Errors

Figure 4.8: Current Tracking Errors

4.5 Conclusion

This chapter addresses the tracking control problems of robot with kinematic and actuator dynamic uncertainties. First, the tracking control problem of robot with current-driven motors is formulated and solved. An adaptive controller is proposed and the closed-loop system is shown to be asymptotic stable through Lyapunov analysis. Then the tracking control of robot with voltage-driven motors is addressed. The controller synthesis is based on a backstepping procedure and two adaptive observers are proposed to avoid task and joint acceleration in the control voltage design without the exact knowledge of the kinematics. Compared with other research works in dealing with uncertain actuator dynamics using the same nonlinear observer design concept, the proposed control method imposes no constraint on the initial system states. Both position and armature current tracking errors converge to zero and the overall system is shown to be stable. Simulation results illustrate the performance of both control methods in the presence of kinematic and actuator dynamic uncertainties.

Chapter 5

Adaptive Regulation of Rigid-Link Flexible-Joint Robot with Uncertain Kinematics

5.1 Introduction

For robot motion control, it has been a usual procedure to model the manipulator by chains of rigid links with rigid transmission joints to simplify dynamic analysis and the controller design. Many controllers for robots with on rigid joints have been proposed and successfully implemented in industrial robot manipulator applications. However, with the increasing demands being placed on robot manipulators to perform high speed, high precision tasks, the problem of dealing with joint flexibility has become important. Joint flexibility usually comes from gear elasticity, shaft wind-up and the use of harmonic drives. As the experimental investigations by Sweet and Good [81] show, the effects of joint flexibility can limit the robustness and performance of a given robot controller.

As a benchmark for control designers, Spong [77] proposed a rigid-link flexible-joint (RLFJ) robot model which is globally feedback linearizable and reduces to the standard rigid-link robot model as the joint stiffness tends to infinity. As a result, this model has been widely used by many control researchers in the development of their controllers. Ghorbel et al. [37] proposed using an adaptive controller designed for rigid robots in conjunction with a fast controller to damp out oscillation due to joint flexibility; Khorasani et al. [56] illustrated how standard adaptive control schemes for rigid robots may be generalized for RLFJ robots under a certain set of assumptions. Dawson et al. [25] designed a robust tracking controller that achieved global uniform ultimate bounded tracking despite the presence of bounded disturbances and parameter uncertainty. In [61], Lozano et al. proposed an asymptotic link position tracking adaptive controller for RLFJ robots without a priori knowledge of joint flexibilities. Nicosia and Tomei [67] used a reduced-order RLFJ robot model to design an adaptive controller which guaranteed global convergence of unknown parameters. In [70], Qu proposed a robust control scheme, developed for a special class of nonlinear uncertain systems, for the control of RLFJ robots. Bridges et al. designed robust controllers for RLFJ robots equipped with redundant actuators [13] or driven by harmonic drives [14]. In [89], Tomei proposed a robust tracking controller which can compensate for unknown disturbances. Based on exact knowledge of actuator dynamic models and availability of full state measures, De Luca et al [62] introduced an inversion algorithm for the synthesis of dynamic feedback control law that gives input-output decoupling and full state linearization.

For the research works cited above, one main disadvantage is that they are based in joint-space and therefore no longer implementable in the presence of kinematic uncertainty if the control task is defined in task space. There have also been a few Cartesian-space controllers [59] for flexible-joint robot, however, these results assume exact kinematic knowledge and hence suffer the same drawback as the joint-space controllers.

In this chapter, the first study on task-space regulation problem of RLFJ robot with uncertain kinematics is provided and a new adaptive control scheme is proposed to solve this problem. The control scheme proposed can deal with kinematic uncertainty as well as uncertainty in actuator dynamic parameters, e.g., stiffness constants, actuator inertia and actuator viscous friction coefficients. The adapting of kinematic and actuator dynamic estimations separately is realized based on the novel adaptive regressor used in *Chapter 3* and *Chapter 4*. Compared with the recent backstepping results [30] [58] for flexible joint robots, the proposed method in this chapter utilizes less control development levels and less auxiliary variables, and hence is conceptually simpler. Stability of the whole system is shown to be guaranteed by choosing the control parameters to satisfy certain conditions. Simulation results are provided to confirm the effectiveness of the proposed setpoint controller in the presence of both kinematic and dynamic uncertainties.

5.2 Adaptive Regulation of RLFJ Robot with Uncertain Kinematics

In this section, an adaptive controller for regulation of RLFJ robot with uncertain kinematics is developed. The control task is formulated as to move the robot manipulator endpoint to a specified task-space position denoted by x_d in the presence of uncertainty in both kinematics and actuator dynamics.

As introduced in Section 2.2.2.3, the equations of motion for the robot manipulator with flexible joints can be written in joint space as

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_m(q_m - q), \quad (5.1)$$

$$J_m\ddot{q}_m + B_m\dot{q}_m + K_m(q_m - q) = u, \quad (5.2)$$

where $q_m = (q_{m1}, \dots, q_{mn})^T \in R^n$ with q_{mi} denoting the angle of motor rotor shaft, $K_m = \text{diag}\{k_{mi}\} \in R^{n \times n}$ denotes the positive definite stiffness constant matrix, $J_m = \text{diag}\{J_{mi}\}$ and $B_m = \text{diag}\{B_{mi}\}$ denote the rotor inertia matrix and damping matrix respectively. u is the control torque input. In this chapter, besides the kinematic uncertainty, the uncertainties in the actuator dynamic matrices K_m , J_m , B_m are also considered.

Since the whole flexible-joint robot dynamics, similar with that of robot with voltage driven motors, constitutes of two subsystems as described by equation (5.1) and (5.2), a backstepping procedure may be used to synthesize the control torque input u as in Chapter 3 and 4. However, note that robot with flexible joints is a fourth order system, hence compared with the third order system for robot with voltage-driven motors the controller design presents a bigger challenge. To synthesize the control torque input u , the control design is separated into two steps, similar to backstepping procedure: first, a desired actuator position trajectory denoted by q_{md} is formulated based on the link subsystem (5.1) which should guarantee the manipulator move to the desired task-space position x_d ; then a control torque input u is designed to eliminate the effect of actuator position perturbation $\tilde{q}_m$ and hence guarantee the asymptotic stability of the whole closed-loop robot system.

5.2.1 Desired Actuator Position Trajectory Design

With the desired actuator position trajectory q_{md} , the link subsystem dynamics (5.1) can be rewritten as:

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_m(q_{md} - q) + K_m\tilde{q}_m, \quad (5.3)$$

where $\tilde{q}_m = q_m - q_{md}$. The rewritten subsystem (5.3) can be deemed as controlled by $K_m(q_{md} - q)$ with an actuator rotor perturbation $\tilde{q}_m$ which will be dealt with by

the control torque input u in next design step.

By defining q_{md} using an auxiliary variable q'_{md} as

$$q_{md} = q'_{md} + q, \quad (5.4)$$

we can cancel off q from $K_m(q_{md} - q)$ and get $K_m(q_{md} - q) = K_m q'_{md}$.

Hence equation (5.3) can then be further written as

$$M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + g(q) = K_m q'_{md} + K_m \tilde{q}_m, \quad (5.5)$$

Now that a clearer expression of the link subsystem (5.5) has been obtained for analysis, we should next design a proper q'_{md} so that it will force the manipulator move to the desired position in the presence of all uncertainties without considering the presence of actuator position perturbation $\tilde{q}_m$. It's noticeable that because the joint stiffness matrix K_m is uncertain, q'_{md} can not just adopt the existing regulation algorithms devoted to standard rigid-link robots to fulfill the task.

An auxiliary signal q'_{md} is defined with a form similar with the result in Section 3.3 as in following:

$$q'_{md} = -\hat{K}_m^{-1}(\hat{J}_A^T(q)K_p s(e) + K_v v - Z(q)\hat{\phi} - Y(q, e, \hat{\phi}, \hat{\theta}_J)\hat{\varphi}), \quad (5.6)$$

$$\dot{v} = -\Lambda_1 v + \dot{q} + \alpha \hat{J}^T(q)s(e), \quad (5.7)$$

$$\dot{\hat{\phi}} = -LZ^T(q)(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (5.8)$$

$$\dot{\hat{\varphi}} = -L_a Y(q, e, \hat{\phi}, \hat{\theta}_J)(\dot{q} + \alpha \hat{J}^T(q)s(e)), \quad (5.9)$$

where $\hat{K}_m$ is the estimated constant joint stiffness matrix, $K_p = k_p I$ and K_v are positive definite diagonal feedback gains. v , $\hat{\phi}$, $\hat{\varphi}$ are updated through adaptation laws (5.7), (5.8) and (5.9) respectively. The adaptive kinematic parameter vector

$\hat{\theta}_J$ used in $\hat{J}_A(q)$ is updated through

$$\dot{\hat{\theta}}_J = L_J Y_J^T(q, e)(\dot{q} + \alpha \hat{J}^T(q)s(e)). \quad (5.10)$$

Λ_1 , L_a , L_J , L are all positive definite diagonal matrices and α is a positive constant.

Note here a *non-adaptive* approximate Jacobian matrix $\hat{J}(q)$ is used in the updating laws (5.7)-(5.10) and it's chosen so that

$$\|J^T(q) - \hat{J}^T(q)\| \leq \bar{\gamma}. \quad (5.11)$$

where $\bar{\gamma}$ is a positive constant estimation error bound to be defined later.

The regressor $Y(q, e, \hat{\phi}, \hat{\theta}_J)$ is defined as $Y(q, e, \hat{\phi}, \hat{\theta}_J) = \text{diag}\{Z(q)\hat{\phi} - \hat{J}_A^T(q)K_p s(e)\}$ such that

$$(I - K_m \hat{K}_m^{-1})(Z(q)\hat{\phi} - \hat{J}_A^T(q)K_p s(e)) = Y(q, e, \hat{\phi}, \hat{\theta}_J)\phi_k, \quad (5.12)$$

where $\text{diag}\{\cdot\}$ denotes a diagonal matrix with $\cdot$ as the diagonal vector; $\phi_k = (1 - \frac{k_m^1}{k_m^1}, \dots, 1 - \frac{k_m^n}{k_m^n})^T$ is an unknown parameter vector with k_m^i and $\hat{k}_m^i$ being the i^{th} diagonal elements of K_m and $\hat{K}_m$ respectively.

Substituting q'_{md} defined in (5.6) into (5.5) and using equation (5.12), the closed-loop link subsystem dynamics can be obtained as

$$\begin{aligned} M(q)\ddot{q} + (B + C(q, \dot{q}))\dot{q} + K_m \hat{K}_m^{-1} K_v v + k_p Y_J(q, e)\hat{\theta}_J + Z(q)(\phi - \hat{\phi}) \\ + Y(q, e, \hat{\phi}, \hat{\theta}_J)(\phi_k - K_m \hat{K}_m^{-1} \hat{\phi}) = K_m \tilde{q}_m, \end{aligned} \quad (5.13)$$

note that $Y(q, e, \hat{\phi}, \hat{\theta}_J)$, $K_m \hat{K}_m^{-1}$ are diagonal matrices.

To analyze the stability of the closed-loop link subsystem with the desired actuator position trajectory $q_{md} = q'_{md} + q$, a Lyapunov function candidate V_1 is proposed

with the following form:

$$\begin{aligned}
V_1 = & \frac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^T(q)s(e) + \sum_{i=1}^m k_p S_i(e) + \frac{1}{2}v^T K_m \hat{K}_m^{-1} K_v v \\
& + \frac{1}{2}(\phi - \hat{\phi})^T L^{-1}(\phi - \hat{\phi}) + \frac{1}{2}(\hat{K}_m K_m^{-1} \phi_k - \hat{\varphi})^T K_m \hat{K}_m^{-1} L_a^{-1}(\hat{K}_m K_m^{-1} \phi_k - \hat{\varphi}) \\
& + \frac{1}{2}(\theta_J - \hat{\theta}_J)^T L_J^{-1} k_p (\theta_J - \hat{\theta}_J).
\end{aligned} \tag{5.14}$$

Since

$$\begin{aligned}
& \frac{1}{4}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^T(q)s(e) + \sum_{i=1}^m k_p S_i(e) \\
& = \frac{1}{4}(\dot{q} + 2\alpha\hat{J}^T(q)s(e))^T M(q)(\dot{q} + 2\alpha\hat{J}^T(q)s(e)) \\
& \quad - \alpha^2 s^T(e)\hat{J}(q)M(q)\hat{J}^T(q)s(e) + \sum_{i=1}^m k_p S_i(e) \\
& \geq \sum_{i=1}^m (k_p \hat{c}_i - \alpha^2 \lambda_{JM}) s_i^2(e),
\end{aligned} \tag{5.15}$$

where $\lambda_{JM} = \lambda_{\max}[\hat{J}(q)M(q)\hat{J}^T(q)]$, $\lambda_{\max}[\cdot]$ denotes the maximum eigenvalue of a matrix. It's easy to see that α can be chosen small enough or k_p can be chosen large enough so that the following inequality stands:

$$k_p \hat{c}_i - \alpha^2 \lambda_{JM} > 0, \tag{5.16}$$

By substituting inequality (5.15) into equation (5.14), we can have

$$\begin{aligned}
V_1 \geq & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^m (k_p \hat{c}_i - \alpha^2 \lambda_{JM}) s_i^2(e) + \frac{1}{2}v^T K_N \hat{K}_N^{-1} K_v v + \frac{1}{2}(\phi - \hat{\phi})^T L^{-1}(\phi - \hat{\phi}) \\
& + \frac{1}{2}(\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi})^T L_a^{-1} K_N \hat{K}_N^{-1}(\hat{K}_N K_N^{-1} \phi_k - \hat{\varphi}) + \frac{1}{2}(\theta_J - \hat{\theta}_J)^T L_J^{-1} k_p (\theta_J - \hat{\theta}_J) > 0.
\end{aligned} \tag{5.17}$$

Hence V_1 is positive definite with the condition (5.16) satisfied.

Differentiating V_1 with respect to time and substituting equations (5.7)-(5.10), (5.13)

into it, we have

$$\begin{aligned}\dot{V}_1 = & -\dot{q}^T B \dot{q} - v^T K_N \hat{K}_N^{-1} K_v \Lambda_1 v - \alpha k_p s^T(e) \hat{J}(q) J^T(q) s(e) \\ & - \alpha \{s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q}\}\end{aligned}\quad (5.18)$$

Note that the effect of the actuator position perturbation $\tilde{q}_m$ in equation (5.13) is not considered in this step since we are considering the case when the actuator rotor tracks the desired trajectory q_{md} exactly. The position perturbation $\tilde{q}_m$ will be included in the analysis of overall closed-loop system in next step and it will be shown that its effect to the system stability will be dealt with by the proposed control torque input u .

Since $s(e)$ is bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that [3]:

$$\begin{aligned}\alpha |s^T(e) \hat{J}(q) [B - \dot{M}(q) + C(q, \dot{q})] \dot{q} - \dot{s}^T(e) \hat{J}(q) M(q) \dot{q} - s^T(e) \dot{\hat{J}}(q) M(q) \dot{q}| \\ \geq -\alpha c_0 \|\dot{q}\|^2 - \alpha c_1 \|s(e)\|^2.\end{aligned}\quad (5.19)$$

Substituting inequality (5.19) into (5.18) and using inequality (5.11), we can get:

$$\dot{V}_1 \leq -\lambda_1 \|\dot{q}\|^2 - \lambda_v \|v\|^2 - \alpha k_p \lambda_2 \|s(e)\|^2. \quad (5.20)$$

where $\lambda_1, \lambda_v, \lambda_2$ are positive and

$$\lambda_1 = \lambda_{\min}[B] - \alpha c_0, \quad \lambda_v = \lambda_{\min}[K_N \hat{K}_N^{-1} K_v \Lambda_1], \quad \lambda_2 = \lambda_{\min}[\hat{J}(q) \hat{J}^T(q)] - b_j \bar{\gamma} - \frac{c_1}{k_p}, \quad (5.21)$$

and b_j denotes the norm bound for $\hat{J}(q)$.

Therefore, if

$$\frac{1}{b_j}(\lambda_{\min}[\hat{J}(q)\hat{J}^T(q)] - \frac{c_1}{k_p}) > \bar{\gamma}, \quad (5.22)$$

and α is chosen small enough so that

$$\lambda_{\min}[B] - \alpha c_0 > 0, \quad (5.23)$$

then λ_1, λ_2 are positive and hence $\dot{V}_1$ is negative definite in $\dot{q}, v$ and $s(e)$.

Lemma 5.1 *The closed-loop system described by equations (5.13) and (5.7)-(5.10) gives rise to the convergence of $x \rightarrow x_d$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$, if $\tilde{q}_m = 0$, the feedback gains K_p, α are chosen to satisfy conditions (5.16) (5.23) and $\hat{J}(q)$ are chosen to satisfy conditions (5.11) and (5.22).*

Proof:

If condition (5.16) is satisfied, V_1 is positive definite in $\dot{q}, v, s(e), \phi - \hat{\phi}, \hat{K}_m K_m^{-1} \phi_k - \hat{\varphi}$ and $\theta_J - \hat{\theta}_J$. Since the time derivative of V_1 is non-positive with the conditions (5.22) (5.23) satisfied, V_1 is bounded and hence $\dot{q}, v, s(e), \hat{\phi}, \hat{\varphi}, \hat{\theta}_J$ are all bounded. From inequality (5.20), we can conclude also that $\dot{q}, v, s(e) \in L_2(0, \infty)$. From equations (5.7), (5.13), it's easy to see that $\dot{v}$ and $\ddot{q}$ are bounded and hence v and $\dot{q}$ are uniformly continuous. Noting that $s(e)$ is differentiable and $\dot{e} = J(q)\dot{q}$ is bounded, hence $s(e)$ is also uniformly continuous. It follows from [3] [28] that $\dot{q}, v$ and $s(e)$ converge to 0 as $t \rightarrow \infty$ which means $e = x - x_d$ also converge to 0 as $t \rightarrow \infty$. The proof is complete. $\square$

5.2.2 Control Torque Input Design

It has been shown that the regulation task can be achieved with the desired actuator position trajectory q_{md} despite of the kinematics and dynamics uncertainties. In this step of design, we propose a control torque input u which will deal with the actuator position perturbation $\tilde{q}_m$ and guarantee the convergence of manipulator task-space

position error.

According to the actuator dynamics defined by equation (5.2), we can directly come up with a control torque design in the form of combination of adaptive feedforward terms and a PD feedback part as follows

$$u = \hat{J}_m \ddot{q}_{md} + \hat{B}_m \dot{q}_{md} + \hat{K}'_m (q_{md} - q) - K_{mp} \tilde{q}_m - K_{mv} \dot{\tilde{q}}_m, \quad (5.24)$$

where $\hat{J}_m$, $\hat{B}_m$, $\hat{K}'_m$ are the approximate models of J_m , B_m , K_m respectively, K_{mp} and K_{mv} are two positive definite control gain matrices. However, from the definition of q_{md} and q'_{md} as in equation (5.4) and (5.6), it can be seen that the control torque design (5.24) requires acceleration information since $\ddot{q}_{md}$ is directly employed.

In order to avoid accelerations, we replace q_{md} in (5.24) with an observed signal $\hat{q}_{md}$ and propose the control torque input as

$$u = \hat{J}_m \ddot{\hat{q}}_{md} + \hat{B}_m \dot{\hat{q}}_{md} + \hat{K}'_m (\hat{q}_{md} - q) - K_{mp} \Delta q_m - K_{mv} \Delta \dot{q}_m, \quad (5.25)$$

where $\Delta q_m = q_m - \hat{q}_{md}$, $\Delta \dot{q}_m = \dot{q}_m - \dot{\hat{q}}_{md}$. The approximate models $\hat{J}_m$, $\hat{B}_m$, $\hat{K}'_m$ are updated by the adaptation laws

$$\dot{\hat{\theta}}_{J_m} = -L_{J_m} N_{J_m}(\ddot{\hat{q}}_{md})(\Delta \dot{q}_m + \beta \Delta q_m), \quad (5.26)$$

$$\dot{\hat{\theta}}_{B_m} = -L_{B_m} N_{B_m}(\dot{\hat{q}}_{md})(\Delta \dot{q}_m + \beta \Delta q_m), \quad (5.27)$$

$$\dot{\hat{\theta}}_{K'_m} = -L_{K_m} N_{K_m}(\hat{q}_{md} - q)(\Delta \dot{q}_m + \beta \Delta q_m), \quad (5.28)$$

where $\hat{\theta}_{J_m}$, $\hat{\theta}_{B_m}$, $\hat{\theta}_{K'_m}$ are estimated actuator dynamic parameter vectors, $N_{J_m}(\ddot{\hat{q}}_{md})$, $N_{B_m}(\dot{\hat{q}}_{md})$, $N_{K_m}(\hat{q}_{md} - q) \in R^{n \times n}$ are regressors defined as $N_{J_m}(\ddot{\hat{q}}_{md}) = \text{diag}\{\ddot{\hat{q}}_{md}\}$, $N_{B_m}(\dot{\hat{q}}_{md}) = \text{diag}\{\dot{\hat{q}}_{md}\}$, $N_{K_m}(\hat{q}_{md} - q) = \text{diag}\{\hat{q}_{md} - q\}$. β is a positive control gain, L_{J_m} , L_{B_m} , $L_{K_m} \in R^n$ are positive definite diagonal adaptation gain matrices.

Substituting (5.25) into (5.2), one can get the closed-loop dynamic equation for the

actuator subsystem as

$$\begin{aligned}
& J_m \Delta \ddot{q}_m + (B_m + K_{mv}) \Delta \dot{q}_m + (K_m + K_{mp}) \Delta q_m \\
&= (\hat{J}_m - J_m) \ddot{q}_{md} + (\hat{B}_m - B_m) \dot{q}_{md} + (\hat{K}'_m - K_m)(\hat{q}_{md} - q), \\
&= -N_{J_m}(\ddot{q}_{md}) \Delta \theta_{J_m} - N_{B_m}(\dot{q}_{md}) \Delta \theta_{B_m} - N_{K_m}(\hat{q}_{md} - q) \Delta \theta_{K_m}, \quad (5.29)
\end{aligned}$$

where $\Delta \theta_{J_m} = \theta_{J_m} - \hat{\theta}_{J_m}$, $\Delta \theta_{B_m} = \theta_{B_m} - \hat{\theta}_{B_m}$, $\Delta \theta_{K_m} = \theta_{K_m} - \hat{\theta}_{K'_m}$.

The observer dynamics is proposed in the following form

$$\dot{\hat{q}}_{md} = \eta + \Lambda_2 e_m, \quad (5.30)$$

$$\dot{\hat{\eta}} = \ddot{\hat{q}}_{md} - \Lambda_2 \dot{e}_m = \hat{J}_{mo}^{-1}(u - \hat{B}_{mo} \eta - \hat{K}_{mo}(q_m + \hat{q}_{md} - q - q_{md}) + K_s z) \quad (5.31)$$

where

$$z = \dot{q}_m - \eta = \Delta \dot{q}_m + \Lambda_2 e_m \quad (5.32)$$

and η is an auxiliary variable, Λ_2 is a positive definite diagonal constant matrix, $e_m = q_{md} - \hat{q}_{md}$. $\hat{J}_{mo}$, $\hat{B}_{mo}$, $\hat{K}_{mo}$ are the approximate models used in the observer dynamics and $\hat{\theta}_{J_{mo}}$, $\hat{\theta}_{B_{mo}}$, $\hat{\theta}_{K_{mo}}$ are the vectors of estimated parameters updated through adaptation laws defined in the following:

$$\dot{\hat{\theta}}_{J_{mo}} = -L_{J_{mo}} N_{J_{mo}}(\dot{\eta}) z, \quad (5.33)$$

$$\dot{\hat{\theta}}_{B_{mo}} = -L_{B_{mo}} N_{B_{mo}}(\eta) z, \quad (5.34)$$

$$\dot{\hat{\theta}}_{K_{mo}} = -L_{K_{mo}} N_{K_{mo}}(q_m + \hat{q}_{md} - q - q_{md}) z, \quad (5.35)$$

where $L_{J_{mo}}, L_{B_{mo}}, L_{K_{mo}} \in R^{n \times n}$ are positive definite constant matrices, $N_{J_{mo}}(\dot{\eta}) = \text{diag}\{\dot{\eta}\}$, $N_{B_{mo}}(\eta) = \text{diag}\{\eta\}$, $N_{K_{mo}}(q_m + \hat{q}_{md} - q - q_{md}) = \text{diag}\{q_m + \hat{q}_{md} - q - q_{md}\}$.

Using equations (5.2) and (5.31), we can get the closed-loop observer dynamics as

$$\begin{aligned}
& J_m \dot{z} + (B_m + K_s)z + K_m e_m \\
&= (\hat{J}_{mo} - J_m)\dot{\eta} + (\hat{B}_{mo} - B_m)\eta + (\hat{K}_{mo} - K_m)(q_m + \hat{q}_{md} - q - q_{md}) \\
&= -N_{J_{mo}}(\dot{\eta})\Delta\theta_{J_{mo}} - N_{B_{mo}}(\eta)\Delta\theta_{B_{mo}} - N_{K_{mo}}(q_m + \hat{q}_{md} - q - q_{md})\Delta\theta_{K_{mo}} \quad (5.36)
\end{aligned}$$

where $\Delta\theta_{J_{mo}} = \theta_{J_m} - \hat{\theta}_{J_{mo}}$, $\Delta\theta_{B_{mo}} = \theta_{B_m} - \hat{\theta}_{B_{mo}}$, $\Delta\theta_{K_{mo}} = \theta_{K_m} - \hat{\theta}_{K_{mo}}$.

A Lyapunov function candidate V_2 then can be proposed for the actuator-observer integrated closed-loop system as

$$\begin{aligned}
V_2 = & \frac{1}{2}(\Delta\dot{q}_m + \beta\Delta q_m)^T J_m(\Delta\dot{q}_m + \beta\Delta q_m) + \frac{1}{2}\Delta q_m^T(\beta B_m + \beta K_{mv} + K_m + K_{mp} - \beta^2 J_m)\Delta q_m \\
& + \frac{1}{2}z^T J_m z + \frac{1}{2}\Delta\theta_{J_m}^T L_{J_m}^{-1} \Delta\theta_{J_m} + \frac{1}{2}\Delta\theta_{B_m}^T L_{B_m}^{-1} \Delta\theta_{B_m} + \frac{1}{2}\Delta\theta_{K_m}^T L_{K_m}^{-1} \Delta\theta_{K_m} + \frac{1}{2}\Delta\theta_{J_{mo}}^T L_{J_{mo}}^{-1} \Delta\theta_{J_{mo}} \\
& + \frac{1}{2}\Delta\theta_{B_{mo}}^T L_{B_{mo}}^{-1} \Delta\theta_{B_{mo}} + \frac{1}{2}\Delta\theta_{K_{mo}}^T L_{K_{mo}}^{-1} \Delta\theta_{K_{mo}} \quad (5.37)
\end{aligned}$$

where the control gain K_{mv} should be chosen large enough so that

$$\lambda_{\min}[B_m + K_{mv}] > \beta\lambda_{\max}[J_m] \quad (5.38)$$

and hence V_2 is positive definite.

Differentiating V_2 with respect to time and using equations (5.26)-(5.29) and (5.33)-(5.36) one has

$$\begin{aligned}
\dot{V}_2 = & -\Delta\dot{q}_m^T(B_m + K_{mv} - \beta J_m)\Delta\dot{q}_m - \beta\Delta q_m^T(K_m + K_{mp})\Delta q_m \\
& - z^T(B_m + K_s)z - \Delta\dot{q}_m^T K_m e_m - e_m^T \Lambda_2 K_m e_m \\
\leq & -(\lambda_{\min}[B_m + K_{mv}] - \beta\lambda_{\max}[J_m])\|\Delta\dot{q}_m\|^2 - \beta(\lambda_{\min}[K_m] + \lambda_{\min}[K_{mp}])\|\Delta q_m\|^2 \\
& - \lambda_{\min}[B_m + K_s]\|z\|^2 - \lambda_{\min}[\Lambda_2 K_m]\|e_m\|^2 + \lambda_{\max}[K_m]\|\Delta\dot{q}_m\|\|e_m\| \quad (5.39)
\end{aligned}$$

Now an overall Lyapunov function candidate V can be proposed for the whole closed-loop robot system as

$$V = V_1 + V_2, \quad (5.40)$$

where V_1 is defined in equation (5.14) and it's easy to see that V is positive definite with conditions (5.16) and (5.38) satisfied.

Then the time derivative of V can be obtained according to inequalities (5.20) and (5.39) as

$$\begin{aligned} \dot{V} \leq & -\lambda_1 \|\dot{q}\|^2 - \lambda_v \|v\|^2 - \alpha k_p \lambda_2 \|s(e)\|^2 + (\dot{q} + \alpha \hat{J}^T(q)s(e))^T K_m \tilde{q}_m \\ & - (\lambda_{\min}[B_m + K_{mv}] - \beta \lambda_{\max}[J_m]) \|\Delta \dot{q}_m\|^2 - \beta (\lambda_{\min}[K_m] + \lambda_{\min}[K_{mp}]) \|\Delta q_m\|^2 \\ & - \lambda_{\min}[B_m + K_s] \|z\|^2 - \lambda_{\min}[\Lambda_2 K_m] \|e_m\|^2 + \lambda_{\max}[K_m] \|\Delta \dot{q}_m\| \|e_m\| \end{aligned} \quad (5.41)$$

Note that the term $(\dot{q} + \alpha \hat{J}^T(q)s(e))^T K_m \tilde{q}_m$ in the first line of the above inequality is introduced by the perturbation term $\tilde{q}_m$ in equation (5.13). This term appears here because the stability analysis in this step is based on the whole closed-loop robot system instead of only the link subsystem.

Since $\tilde{q}_m = (q_m - \hat{q}_{md}) - (q_{md} - \hat{q}_{md}) = \Delta q_m - e_m$, inequality (5.41) can be written

as

$$\begin{aligned}
\dot{V} &\leq -\lambda_1 \|\dot{q}\|^2 - \lambda_v \|v\|^2 - \alpha k_p \lambda_2 \|s(e)\|^2 + \lambda_{\max}[K_m] \|\dot{q}\| (\|\Delta q_m\| + \|e_m\|) \\
&+ \alpha \lambda_{\max}[K_m] b_j \|s(e)\| (\|\Delta q_m\| + \|e_m\|) - (\lambda_{\min}[B_m + K_{mv}] - \beta \lambda_{\max}[J_m]) \|\Delta \dot{q}_m\|^2 \\
&- \beta (\lambda_{\min}[K_m] + \lambda_{\min}[K_{mp}]) \|\Delta q_m\|^2 - \lambda_{\min}[B_m + K_s] \|z\|^2 - \lambda_{\min}[\Lambda_2 K_m] \|e_m\|^2 \\
&\quad + \lambda_{\max}[K_m] \|\Delta \dot{q}_m\| \|e_m\| \\
&= -\lambda_1 \|\dot{q}\|^2 - \lambda_v \|v\|^2 - \alpha k_p \lambda_2 \|s(e)\|^2 - \lambda_{\min}[B_m + K_s] \|z\|^2 - \lambda_{\min}[\Lambda_2 K_m] \|e_m\|^2 \\
&\quad - \beta (\lambda_{\min}[K_m] + \lambda_{\min}[K_{mp}]) \|\Delta q_m\|^2 - (\lambda_{\min}[B_m + K_{mv}] - \beta \lambda_{\max}[J_m]) \|\Delta \dot{q}_m\|^2 \\
&\quad + \lambda_m \|\dot{q}\| \|\Delta q_m\| + \lambda_m \|\dot{q}\| \|e_m\| + \alpha \lambda_m b_j \|s(e)\| \|\Delta q_m\| + \alpha \lambda_m b_j \|s(e)\| \|e_m\| + \lambda_m \|\Delta \dot{q}_m\| \|e_m\| \\
&= - \begin{bmatrix} \|\dot{q}\| & \|s(e)\| & \|\Delta q_m\| & \|\Delta \dot{q}_m\| & \|e_m\| \end{bmatrix} A \begin{bmatrix} \|\dot{q}\| & \|s(e)\| & \|\Delta q_m\| & \|\Delta \dot{q}_m\| & \|e_m\| \end{bmatrix}^T \\
&\quad - \lambda_v \|v\|^2 - \lambda_{\min}[B_m + K_s] \|z\|^2 \tag{5.42}
\end{aligned}$$

where

$$A = \begin{bmatrix} \lambda_1 & 0 & \frac{1}{2}\lambda_m & 0 & \frac{1}{2}\lambda_m \\ 0 & \alpha k_p \lambda_2 & \frac{1}{2}\alpha \lambda_m b_j & 0 & \frac{1}{2}\alpha \lambda_m b_j \\ \frac{1}{2}\lambda_m & \frac{1}{2}\alpha \lambda_m b_j & \beta \psi_1 & 0 & 0 \\ 0 & 0 & 0 & \psi_2 & \frac{1}{2}\lambda_m \\ \frac{1}{2}\lambda_m & \frac{1}{2}\alpha \lambda_m b_j & 0 & \frac{1}{2}\lambda_m & \lambda_{\min}[\Lambda_2 K_m] \end{bmatrix} \tag{5.43}$$

with $\lambda_m = \lambda_{\max}[K_m]$, $\psi_1 = \lambda_{\min}[K_m] + \lambda_{\min}[K_{mp}]$ and $\psi_2 = \lambda_{\min}[B_m + K_{mv}] - \beta \lambda_{\max}[J_m]$.

Then if matrix A is positive definite, then inequality (5.41) can be written as

$$\begin{aligned}
\dot{V} &\leq -\lambda_v \|v\|^2 - \lambda_{\min}[B_m + K_s] \|z\|^2 - \lambda_{\min}[A] (\|\dot{q}\|^2 + \|s(e)\|^2 + \|\Delta q_m\|^2 \\
&\quad + \|\Delta \dot{q}_m\|^2 + \|e_m\|^2) \tag{5.44}
\end{aligned}$$

and $\dot{V}$ is negative definite in v , z , $\dot{q}$, $s(e)$, Δq_m , $\Delta \dot{q}_m$ and e_m .

According to Sylvester's Theorem [41], the symmetric matrix A is positive definite iff all its principal minors A_i ($i = 1, 2, \dots, 5$) be strictly positive, where principle minor is defined as $A_1 = a_{11}$, $A_2 = a_{11}a_{22} - a_{12}a_{21}$, $\dots$, $A_5 = \det(A)$.

Since λ_1, λ_2 are positive with conditions (5.22), (5.23) satisfied, it's easy to see that A_1 and A_2 are positive. A_3 can be calculated as

$$A_3 = \frac{\alpha}{4}[4\beta k_p \lambda_1 \lambda_2 \psi_1 - \alpha \lambda_m^2 b_j^2 \lambda_1 - \lambda_m^2 k_p \lambda_2]. \quad (5.45)$$

By choosing α small and k_p, ψ_1 large (i.e., K_p, K_{mp} large) so that

$$4\beta \lambda_1 \lambda_2 \psi_1 - \frac{\alpha}{k_p} \lambda_m^2 b_j^2 \lambda_1 - \lambda_m^2 \lambda_2 > 0, \quad (5.46)$$

the positiveness of A_3 can be achieved easily. From the structure of A , it can be seen that A_4 is simply $A_4 = A_3 \cdot \psi_2$. From (5.38), we have

$$\psi_2 = \lambda_{\min}[B_m + K_{mv}] - \beta \lambda_{\max}[J_m] > 0. \quad (5.47)$$

Hence, A_4 is positive.

The determinant $\det(A)$ or A_5 can be calculated as

$$A_5 = \frac{\alpha}{4} k_p (\psi_2 \lambda_{\min}[\Lambda_2 K_m] + \beta \psi_1 \psi_2 - \frac{1}{4} \lambda_m^2) \{ 4\lambda_1 \lambda_2 \beta \psi_1 (1 - \frac{\beta \psi_1 \psi_2}{\psi_2 \lambda_{\min}[\Lambda_2 K_m] + \beta \psi_1 \psi_2 - \frac{1}{4} \lambda_m^2}) - \lambda_m^2 (\frac{\alpha}{k_p} b_j + \lambda_2) \}, \quad (5.48)$$

where Λ_2 can be chosen large enough so that

$$1 - \frac{\beta \psi_1 \psi_2}{\psi_2 \lambda_{\min}[\Lambda_2 K_m] + \beta \psi_1 \psi_2 - \frac{1}{4} \lambda_m^2} > 0, \quad (5.49)$$

then α can be chosen small and k_p , ψ_1 can be chosen large enough to satisfy

$$4\lambda_1\lambda_2\beta\psi_1\left(1 - \frac{\beta\psi_1\psi_2}{\psi_2\lambda_{\min}[\Lambda_2 K_m] + \beta\psi_1\psi_2 - \frac{1}{4}\lambda_m^2}\right) - \lambda_m^2\left(\frac{\alpha}{k_p}b_j + \lambda_2\right) > 0 \quad (5.50)$$

By using sufficiently large Λ_2 and ψ_1 , it can be easily achieved that

$$\psi_2\lambda_{\min}[\Lambda_2 K_m] + \beta\psi_1\psi_2 - \frac{1}{4}\lambda_m^2 > 0, \quad (5.51)$$

Hence

$$A_5 = \frac{\alpha}{4}k_p(\psi_2\lambda_{\min}[\Lambda_2 K_m] + \beta\psi_1\psi_2 - \frac{1}{4}\lambda_m^2)\left\{4\lambda_1\lambda_2\beta\psi_1\left(1 - \frac{\beta\psi_1\psi_2}{\psi_2\lambda_{\min}[\Lambda_2 K_m] + \beta\psi_1\psi_2 - \frac{1}{4}\lambda_m^2}\right) - \lambda_m^2\left(\frac{\alpha}{k_p}b_j + \lambda_2\right)\right\} > 0. \quad (5.52)$$

We can now state the following Theorem:

Theorem 5.1 *The rigid-link flexible-joint robot system described by equations (5.13), (5.29) together with the parameter adaptation laws (5.7)-(5.10), (5.26)-(5.28) and (5.33)-(5.35) gives rise to the convergences of $x \rightarrow x_d$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$ if the feedback gains α , K_p , K_{mp} , K_{mv} and Λ_2 are chosen to satisfy conditions (5.16), (5.23), (5.38), (5.46), (5.52), $\hat{J}(q)$ is chosen to satisfy the condition (5.11) and (5.22).*

Proof:

Since V is positive definite in $\dot{q}$, $s(e)$, v , $\phi - \hat{\phi}$, $\hat{K}_m K_m^{-1} \phi_k - \hat{\phi}$, $\theta_J - \hat{\theta}_J$, $\Delta \dot{q}_m + \beta \Delta q_m$, Δq_m , z , $\Delta \theta_{J_m}$, $\Delta \theta_{B_m}$, $\Delta \theta_{K_m}$, $\Delta \theta_{J_{mo}}$, $\Delta \theta_{B_{mo}}$, $\Delta \theta_{K_{mo}}$ with conditions (5.16) (5.38) satisfied, and $\dot{V}$ is negative definite in v , z , $\dot{q}$, $s(e)$, Δq_m , $\Delta \dot{q}_m$, e_m with conditions (5.22) (5.23) (5.46) (5.52) satisfied, the variables included in V are all bounded. Since $\tilde{q}_m = \Delta q_m - e_m$, $\tilde{q}_m$ is also bounded. From equation (5.13), it can be seen that $\ddot{q}$ is also bounded. Since $s(e)$ is differentiable and note that $\dot{e} = \dot{x} = J(q)\dot{q}$ is bounded, the time derivative $\dot{s}(e)$ is also bounded. Now that $\ddot{q}$ and $\dot{s}(e)$ are bounded, $s(e)$ and $\dot{q}$ are uniformly continuous. From inequality (5.44), it's easy to verify that

$s(e)$, $\dot{q} \in L_2(0, \infty)$. Then it can be concluded from [3] [28] that $s(e) \rightarrow 0$ and $\dot{q} \rightarrow 0$ as $t \rightarrow \infty$. Since $s(e) = 0$ means $x = x_d$, the proof is complete. $\square$

Remark 5.1: From inequalities (5.46)-(5.51), it can be seen that the positive definiteness of A can be guaranteed by simply choosing α small and K_p , K_{mp} , K_{mv} , Λ_2 large enough. $\diamond$

5.3 Simulation Results

In this section, simulation results are provided to show the feasibility of the control scheme proposed in *Theorem 5.1*. The simulation is based on the two-link robot holding an object as illustrated by Fig. 5.1. In the simulation, the masses, lengths and gravity centers of link 1, 2 and the object were set as $m_1 = 17.4kg$, $m_2 = 4.8kg$, $m_o = 2kg$, $l_1 = 0.43m$, $l_2 = 0.43m$, $l_0 = 0.2m$, $l_{c1} = 0.068m$, $l_{c2} = 0.07m$, $l_{co} = 0.1m$ respectively. The object grasping angle was set as $q_0 = 30^\circ$. The exact actuator models used are $J_m = \text{diag}\{5, 4\}$, $B_m = \text{diag}\{5, 5\}$ and $K_m = \text{diag}\{600, 600\}$. The control task is to move the manipulator endpoint from an initial position $x_0 = (0.82, 0.32)$ to a desired position $x_d = (0.5, 0.55)$ in Cartesian space.

To test the ability of the proposed control schemes to deal with kinematics and actuator uncertainties, the kinematics parameters were set as $\hat{l}_1 = 0.5m$, $\hat{l}_2 = 0.5m$, $\hat{l}_0 = 0.25m$, $\hat{q}_0 = 45^\circ$ and the approximate actuator models were set as $\hat{J}_m(0) = \hat{J}_{mo}(0) = \text{diag}\{8, 6\}$, $\hat{B}_m(0) = \hat{B}_{mo}(0) = \text{diag}\{8, 8\}$, $\hat{K}_m = \hat{K}'_m(0) = \hat{K}_{mo}(0) = \text{diag}\{550, 550\}$ in the controller design. The system response with control gains $K_p = 500I$, $K_v = 500I$, $K_{mp} = 800I$, $K_{mv} = 800I$, $K_s = I$, $\alpha = 3$, $\beta = 3$, $\Lambda_1 = 50I$, $\Lambda_2 = I$, $L = 15I$, $L_a = 0.001I$, $L_J = 0.01I$, $L_{J_m} = L_{J_{mo}} = 0.01I$, $L_{B_m} = L_{B_{mo}} = 0.01I$, $L_{K_m} = L_{K_{mo}} = I$ is illustrated by Fig. 5.2.

Next, the kinematics parameters were set as $\hat{l}_1 = 0.6m$, $\hat{l}_2 = 0.6m$, $\hat{l}_0 = 0.3m$,

Figure 5.1: 2-Link Robot Holding Uncertain Object

$\hat{q}_0 = 60^\circ$ and the approximate actuator models were changed to $\hat{J}_m(0) = \hat{J}_{mo}(0) = \text{diag}\{10, 8\}$, $\hat{B}_m(0) = \hat{B}_{mo}(0) = \text{diag}\{10, 10\}$, $\hat{K}_m = \hat{K}'_m(0) = \hat{K}_{mo}(0) = \text{diag}\{700, 700\}$.

Fig. 5.3 shows the system response with the same control gains as used in the first simulation. As shown by the figures, with the proposed controller the position errors converge to zero despite the uncertainties existing in the kinematics, link dynamics and actuator dynamics.

5.4 Conclusion

In this chapter, the adaptive regulation problem of rigid-link flexible-joint robot with uncertain kinematics is addressed. A task-space regulation method with uncertain kinematics and actuator dynamics is presented. Compared with the recent

Figure 5.2: Simulation Study I

Figure 5.3: Simulation Study II

backstepping control methods for RLFJ robots [30] [58], only two control levels and one intermediate virtual control input are utilized in this method and hence the controller development is simplified. The proposed control scheme is conceptually simple and consistent with the design methodologies proposed in the previous chapters of this thesis. It's verified through simulation that the regulation scheme proposed can fulfill the control task with the presence of uncertainties in robot kinematics, manipulator dynamics and actuator dynamics.

Chapter 6

Experimental Results

In this chapter, experimental results are provided to show the practical feasibility of control methods proposed in this thesis. The regulation control methods proposed for robot with current-driven motors proposed in Section 3.2 and the tracking control scheme for robot with current-driven motors described in Section 4.2 were implemented in the experiments and the experimental results show that the control methods proposed are effective in dealing with the kinematic and dynamic uncertainties in practical applications. Due to hardware limitations as will be explained in Section 6.1, not all control schemes are examined.

6.1 Experiment System Setup

Figure 6.1: Experiment System Setup

The experiments are conducted based a vertical two-link robot as shown in Figure 6.1. The robot links are driven direct drive motors. The controller is running on a Pentium II processor PC with Microsoft Windows NT and VenturCom's RTX (a hard real-time extension). The sampling period is set as 3 ms. The PC is mounted with a servo-to-go I/O card which consists of a digital to analog (DA) converter, analog to digital (AD) converter and a pulse counter. The control signals are fed through the DA converters of the servo I/O card to the two current amplifiers that drive the two motors attached to the two links of the robot. Joint velocities are

obtained from differentiation of the joint angles. A PSD camera (position sensitive detector) manufactured by Hamamatsu is used to measure the position of the robot end effector and the camera is placed parallel to the robot.

The masses of the first and second link are approximately equal to $1.6kg$ and $1kg$ respectively, and the masses of the first and second motors are approximately equal to $9.5kg$ and $3kg$ respectively. The inertias of joint 1 and joint 2 are unknown. The lengths of the first and second links are approximately measured to be $l_1 = 0.3m$ and $l_2 = 0.34m$ respectively. At the end of the second link, an object is attached through screw as can be seen in Figure 6.1. The length and posture angle of the object can be adjusted to obtain different kinematic descriptions of the robot system. And by adjusting the length and angle of the object different kinematic uncertainties can be introduced. The mass of the object is approximately $0.12kg$. The motor torque transmission matrix gained through calibration is $K_N = diag(2.63, 1.03)$.

Since the robot links are driven by motors with current amplifier directly in this experiment system and the robot joints and motors are linked rigidly, the control schemes proposed for robot with voltage-driven motors and robot with flexible joints were not implemented. In this chapter, the regulation and tracking results for robot with current-driven motors as discussed in Section 3.2 and 4.2 respectively were implemented and the experimental results are shown in the next two sections.

6.2 Experimental Results for Regulation with Uncertain Kinematics and Motor Torque Transmission Matrix

In this section the regulation control methods proposed in Section 3.2 were implemented. The robot's end point is required to move to a desired position $(0.3, 0.5)$

specified in Cartesian space from a initial position. In this experiment, the robot holds an unknown object at the end of the second link. The measured length and grasping angle of the object are $l_o = 0.12m$ and $q_o = 30^\circ$ respectively.

Accordingly to the conditions developed in Section 3.2, systematic tuning procedures for the control parameters can be established. First of all, a large K_p is used (which is common in regulation control design) so that condition (3.16) and (3.39) may be satisfied. A very large K_v is not desirable at the beginning to avoid introducing large measurement noise. A very small α may also be avoided since α contribute much to the steady state errors of the regulation. Since K_p is large, condition (3.16) and (3.39) can still be guaranteed even though α is not very small and K_v or B_v is not very large. If the system is not stable, which means conditions (3.29)-(3.31) or conditions (3.44)-(3.46) for the second controller are not met, then K_v or B_v should be increased ($a_1 = \frac{\lambda_{max}[K_p]}{\lambda_{max}[K_v]}$ and $a_2 = \frac{\lambda_{max}[K_p]}{b_v}$ be decreased) such that these conditions could be satisfied (see Figure 3.3, Figure 3.4 and Figure 3.2.3).

First the controller in *Theorem 3.1* is implemented on the robot. To examine the effects of kinematics uncertainty, the link lengths were estimated as $\hat{l}_1 = 0.28m, \hat{l}_2 = 0.28m, \hat{l}_o = 0.06m$ and the grasping angle was estimated as $\hat{q}_o = 60^\circ$. The estimated actuator model is $\hat{K}_N = diag(3.57, 1.33)$. Experimental result with $K_p = diag(120, 80), K_v = diag(6, 4), L_1 = 10I_E, L_2 = I_E, \alpha = 10$ is presented in Figure 6.2. As can be seen from the result, the position errors converged with the control scheme using approximate actuator model and Jacobian matrix.

6.2 Experimental Results for Regulation with Uncertain Kinematics and Motor Torque Transmission Matrix

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Figure 6.2: Stable Response of the Result in Theorem 3.1

In the next experiment, the controller in *Theorem 3.2* was implemented on the robot. The kinematics, manipulator and actuator dynamics parameters used in the controller are the same as in the previous experiment. The control gains are set as $K_p = \text{diag}(180, 80)$, $B_v = 1.2I$, $L_1 = 10I$, $L_2 = I$, $\alpha = 10$. The experimental result in Figure 6.3 shows that system stability was guaranteed with the presence of uncertainties in both actuator model and Jacobian matrix.

Figure 6.3: Stable Response of the Result in Theorem 3.2

6.3 Experimental Results for Tracking Control with Uncertain Kinematics and Motor Torque Transmission Matrix

In this experiment, the tracking control method proposed in Section 4.2 was implemented. The robot is required to hold an object with uncertain length and grasping angle and follow a circular trajectory specified in Cartesian space as:

$$\begin{aligned} X_d &= 0.33 + 0.1\sin(0.54 + 3t) \\ Y_d &= 0.41 + 0.1\cos(0.54 + 3t). \end{aligned} \tag{6.1}$$

6.3 Experimental Results for Tracking Control with Uncertain Kinematics and Motor Torque Transmission Matrix

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The relationship between the velocities in task space and the velocities of the joints is given by

$$\dot{x} = J(q)\dot{q} = \begin{bmatrix} -s_1\dot{q}_1 & -s_{12}(\dot{q}_1 + \dot{q}_2) & -c_{12}(\dot{q}_1 + \dot{q}_2) \\ c_1\dot{q}_1 & c_{12}(\dot{q}_1 + \dot{q}_2) & -s_{12}(\dot{q}_1 + \dot{q}_2) \end{bmatrix}, \quad (6.2)$$

where l_1, l_2 are the link lengths, l_o and q_o are the length and grasping angle of the object respectively, $c_1 = \cos(q_1)$, $s_1 = (\sin q_1)$, $c_{12} = \cos(q_1 + q_2)$, $s_{12} = \sin(q_1 + q_2)$, $c_{12o} = \cos(q_1 + q_2 + q_o)$, $s_{12o} = \sin(q_1 + q_2 + q_o)$, $c_o = \cos q_o$, $s_o = \sin q_o$. The proposed controller in the Theorem 4.1 was implemented on the robot holding an object with uncertain length and grasping angle. The length of the object was approximately set as $0.06m$ and the grasping angle was approximately set as 45° . The object length and grasping angle were estimated as $\hat{l}_o(0) = 0.12m$ and $\hat{q}_o(0) = 20^\circ$ respectively and the link lengths were set as $\hat{l}_1(0) = 0.25m$, $\hat{l}_2(0) = 0.27m$. The initial position of the robot end effector was specified as $(X(0), Y(0)) = (0.28, 0.52)$. The actuator model is estimated as $\hat{K}_N = \text{diag}(2.45, 0.95)$. Experimental results with $L_a = \text{diag}(0.15, 0.10)$, $L_k = \text{diag}(0.13, 0.15, 0.015)$, $L_d = \text{diag}(0.01, 0.002, 0.002, 0.002, 0.015, 0.01, 0.01)$, $K_v = \text{diag}(2, 2)$, $K_p = \text{diag}(420, 380)$, $\alpha = 1.2$ are presented in Figure 6.4 and Figure 6.5. As seen from the results, the tracking errors converge with updating of the estimated actuator, kinematic and dynamic parameters.

Figure 6.4: Tracking Trajectory

Figure 6.5: Tracking Errors

6.3 Experimental Results for Tracking Control with Uncertain Kinematics and Motor Torque Transmission Matrix

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Next, the actual length and grasping angle of the object were adjusted to $0.1m$ and 60° respectively. The controller using the same estimated kinematic and dynamic information and same control parameters was implemented to control the manipulator endpoint track the same desired trajectory again. As shown by the experimental results in Figure 6.6 and Figure 6.7, the control method proposed can still guarantee a good performance in the presence of different kinematics uncertainty.

Figure 6.6: Tracking Trajectory

Figure 6.7: Tracking Errors

Chapter 7

Conclusion and Recommendations

7.1 Conclusion

In this thesis, the regulation and tracking control problems of robot manipulator with uncertain kinematics and actuator dynamics are addressed. For control tasks defined in task space, traditional solutions in literature resort to inverse kinematics or transposed Jacobian methods to develop the control schemes. However, when uncertainty exists in robot kinematics, these control methods may no longer be able to guarantee the system stability or may lead to regulation or tracking errors in the performance since they depend on exact kinematic knowledge. Recently, some researchers proposed several approaches to deal with the uncertain kinematics problem, but their results are confined to the task-space regulation problem and no results for uncertain kinematics tracking control have been proposed. Moreover, the proposed results, as well as many existed control methods in the literature, omit the robot actuator dynamics in the controller design. As has been verified through experimental studies, the actuator dynamics constitutes an important part of whole robot dynamics and contributes much to the robot performance especially for high speed and high load variance applications.

This thesis is dedicated to develop control schemes for robot manipulator which can deal with the uncertainties in kinematics, manipulator dynamics and actuator dynamics. Although several results dealing with kinematic uncertainty and results that consider actuator dynamics in the controller design have both been proposed in literature, no results have been established to cope with the two uncertainties together. The control schemes proposed for this purpose will be more flexible to the uncertainties existing in the real working environment and hence will be capable for many new and more challenging control tasks where stable and high precision performance is required. Toward this objective, a systematic study is conducted in this thesis and the adaptive control schemes proposed for regulation and tracking control problems are presented. Some conclusions of this thesis are summarized as follows:

- The regulation problem of robot manipulator with uncertain kinematics and driven by electric motors is formulated and solved. Three controllers are first presented for robot with current-driven motors with uncertain kinematics based on a novel adaptive regressor. The novelty of the adaptive regressor design concept compared with traditional regressors is that it utilizes the online adaptation information of other uncertain parameters instead of only using system feedback signals in the regressor construction. The use of this adaptive regressor enables the adaptation of more than one kind of uncertainties simultaneously during the control process and hence also avoids the overparameterization problem often met in dealing with multiple uncertainties. An adaptive controller is then proposed for robot with voltage-driven motors. Since the overall system is third order, a backstepping procedure is adopted to design the actual voltage control input. The four regulation schemes constitute the first results in literature that can deal with uncertainties in both robot kinematics and actuator dynamics.

- The trajectory tracking control problems for robot with uncertain kinematics, manipulator dynamics and actuator dynamics are formulated and solved by incorporating the actuator dynamics into the control design. The result for robot with current-driven motors is first developed based on the novel adaptive regressor. Then the control scheme for robot with voltage-driven motors is proposed based on a backstepping procedure. In this design, two adaptive observers are used. The purpose of introducing the observers are to avoid task-space and joint-space accelerations in the control voltage design. Compared with other control methods using the same observer design approach in dealing with the uncertain actuator dynamics, this result does not require the exact kinematic knowledge and imposes no assumption on initial system states.
- The uncertain kinematics regulation problem of robot with another kind of actuator dynamic uncertainty-joint flexibility is addressed. The control task is more challenging since the overall system is fourth order. To synthesize the control torque input u , the control design is separated into two steps, similar to backstepping procedure: first, a desired actuator position trajectory denoted by q_{md} is formulated based on the link subsystem which should guarantee the manipulator move to the desired task-space position x_d ; then a control torque input u is designed to eliminate the effect of actuator position perturbation $\tilde{q}_m$. By adjusting the control gains to satisfy certain sufficient conditions the stability of the closed-loop system can be guaranteed. This control method presents the first result towards solving the regulation problem of rigid-link flexible-joint robot with uncertain kinematics in the literature.
- Experimental results for several adaptive control methods are presented in this thesis. Although simulation results are provided in the end of each previous chapter, experimental results are still important to verify the practical feasibility of the proposed methods. Due to the reason that the robot system

our experiments based is driven by motors with current amplifiers, only the results for robot with current-driven motors are implemented. The experimental results confirmed that the control schemes proposed are effective in dealing with all the uncertainties in kinematics, manipulator dynamics and actuator dynamics.

7.2 Recommendations for Future Research

In this section, we present some future research works and the possible extensions of this work to other problems.

- Since time derivative of the desired armature current design is used in the control voltage input, the regulation result for robot with voltage-driven motors in Chapter 3 and the tracking control methods presented in Chapter 4 require task-space velocity. The velocity is often obtained from differentiation of the position which is contaminated by noises. Observed signals could be introduced to replace the task-space velocities to overcome this problem. An alternative possible solution is to use the estimated task-space velocities based on the approximate Jacobian matrix.
- Since in Chapter 5 only the regulation control problem for rigid-link flexible-joint robot is addressed, it's of research interest to extend the study to the tracking control problem of rigid-link flexible-joint robot to develop control schemes for the tracking control tasks with the presence of uncertainties in the kinematics and manipulator joint flexibility.
- Chapter 5 considers the kinematic uncertainty and the actuator dynamic uncertainty in joint flexibility. The dynamics of the electric motor is not considered. Future research work would be devoted to include the motor dynamics

into consideration also and get a more complete result to counteract the negative effects brought by the motor dynamic uncertainty.

- The area of visual servoing has attracted more and more research interest in recent years. In this thesis, the extension of the proposed results to visual servoing applications is briefly discussed, future research work would include more specific study of visual servoing control problems of robot taking the actuator dynamics into consideration.
- The link flexibility of robot manipulator presents another source of uncertainty in both kinematics and dynamics. It is often introduced by a long reach and slender/lightweight construction of the manipulator. To consider the link flexibility together with other kinematic/dynamic uncertainties discussed in this thesis in the control design presents another interesting and challenging research topic.

Author's Publications

International Journal Papers

1. C. Liu and C. C. Cheah, "Task-space Adaptive Setpoint Control for Robots with Uncertain Kinematics and Actuator Model", *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1854–1860, Nov. 2005.
2. C. C. Cheah, C. Liu and J. J. E. Slotine, "Adaptive Tracking Control for Robots with Unknown Kinematic and Dynamic Properties", *International Journal of Robotics Research*, vol. 25, no. 3, pp. 283–296, Mar. 2006
3. C. Liu, C. C. Cheah and J. J. E. Slotine, "Adaptive Jacobian Tracking Control of Rigid-Link Electrically Driven Robots based on Visual Task-Space Information", *Automatica* (accepted as **regular paper**).
4. C. C. Cheah, C. Liu and J. J. E. Slotine, "Adaptive Jacobian Tracking Control of Redundant Robots with Uncertainties in Kinematic, Dynamic and Actuator Models", *IEEE Transactions on Automatic Control* (accepted).
5. C. Liu, C. C. Cheah and J. J. E. Slotine, "Adaptive Task-Space Regulation of Rigid-Link Flexible-Joint Robots with Uncertain Kinematics", submitted to *IEEE Transactions on Automatic Control*.

International Conference Papers

1. C. Liu and C.C. Cheah, "Task-space Adaptive Setpoint Control for Robots with Uncertain Kinematics and Actuator Model" *Proceeding of the American Control Conference*, Boston, Massachusetts, pp. 5268-5273, June 30 - July 2 2004.
2. C. Liu, C.C. Cheah and H.C. Liaw, "Adaptive Regulation of Robots using Approximate Models" (**Invited Paper**) *Proceedings of the IEEE International*

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- Symposium on Intelligent Control*, Taipei, Taiwan, pp. 180-185, September 2004.
3. C.C. Cheah, C. Liu and J.J.E. Slotine, "Approximate Jacobian Adaptive Control for Robot Manipulators" *Proceeding of the International Conference on Robotics and Automation*, New Orleans, LA, pp.3075-3080, April 2004.
 4. C.C. Cheah, C. Liu and J.J.E. Slotine, "Experiments on Adaptive Control of Robots with Uncertain Kinematics and Dynamics" *Proceeding of the 9th International Symposium on Experimental Robotics*, Singapore, June 2004.
 5. C.C. Cheah, C. Liu and H.C.Liaw, "Stability of Inverse Jacobian Control for Robot Manipulator" *Proceedings of the IEEE International Conference on Control Applications*, Taipei, Taiwan, pp. 321-326, September 2004.
 6. C. Liu and C.C. Cheah, "Adaptive Regulation of Rigid-Link Electrically Driven Robots with Uncertain Kinematics" *Proceedings of IEEE International Conference on Robotics and Automation*, Barcelona, Spain, pp. 3273-3278, April 2005.
 7. C.C. Cheah, C. Liu and J.J.E. Slotine, "Adaptive Jacobian Tracking Control of Robots based on Visual Task-space Information" *Proceedings of IEEE International Conference on Robotics and Automation*, Barcelona, Spain, pp. 3509-3514, April 2005.
 8. C. Liu, C.C. Cheah and J.J.E. Slotine, "Adaptive Task-Space Regulation of Rigid-Link Flexible-Joint Robots with Uncertain Kinematics" *Proceedings of IEEE International Conference on Robotics and Automation*, Orlando, USA, May 2006. (to appear)

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