Orthogonal and quasi-orthogonal space-time block codes

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Orthogonal and Quasi-Orthogonal Space-Time Block Codes

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2006

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Yuen Chau

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Contents

Acknowledgements.....	iii
Summary	iv
List of Abbreviations and Symbols	xi
List of Figures	xvi
List of Tables	xviii
CHAPTER 1 Introduction	1
1.1 Background Information	1
1.2 Motivations.....	6
1.3 Objectives.....	7
1.4 Contributions	8
1.5 Organizations.....	10
CHAPTER 2 Space-Time Block Code.....	11
2.1 Introduction	11
2.2 Signal Model of MIMO Channel	14
2.3 Signal Model of Space-Time Block Code.....	16
2.4 Design Criteria of Space-Time Block Code.....	19
2.4.1 <i>Decoding Performance Optimization Criteria</i>	19
2.4.2 <i>Power Distribution Criteria</i>	21
CHAPTER 3 Orthogonal STBC based on Amicable Complex Orthogonal Design...	
.....	23
3.1 Introduction	23
3.1.1 <i>Review of Construction of O-STBC From AOD</i>	23

3.1.2	<i>Motivations</i>	24
3.2	Amicable Complex Orthogonal Design	25
3.2.1	<i>Review of Orthogonal Designs</i>	25
3.2.2	<i>Existence of ACOD</i>	32
3.2.3	<i>Construction of ACOD</i>	35
3.3	Power-Balanced O-STBC	45
3.3.1	<i>Construction of Power-Balanced O-STBC From ACOD</i>	45
3.3.2	<i>Power Distribution of O-STBC</i>	53
3.4	Chapter Summary	56
CHAPTER 4	Quasi-Orthogonal STBC: Analysis and Optimization	58
4.1	Introduction	58
4.1.1	<i>Review of QO-STBC</i>	59
4.1.2	<i>Motivations</i>	63
4.2	Algebraic Structure of QO-STBC	64
4.2.1	<i>Decoding Complexity of a QO-STBC</i>	65
4.2.2	<i>Maximal Symbolwise Diversity of a QO-STBC</i>	67
4.3	Decoding of QO-STBC with Noise Whitening	68
4.3.1	<i>Noise Whitening Filter for QO-STBC</i>	69
4.3.2	<i>QO-STBC Decoder Schemes</i>	75
4.3.3	<i>Simulation Results</i>	77
4.3.4	<i>Section Summary</i>	81
4.4	Improving QO-STBC with GCLT	81
4.4.1	<i>Signal Model</i>	82
4.4.2	<i>Effect of CR on the Decoding Complexity of QO-STBC</i>	83
4.4.3	<i>Group-Constrained Linear Transformation</i>	88
4.4.4	<i>Performance Comparison</i>	98

4.4.5	Section Summary	102
4.5	Chapter Summary	103
CHAPTER 5 Quasi-Orthogonal STBC with Minimum Decoding Complexity ...		104
5.1	Introduction	104
5.1.1	Literature review	104
5.1.2	Motivations	106
5.2	Algebraic Structure of MDC-QOSTBC	106
5.3	Relationship between MDC-QOSTBC and AOD	108
5.3.1	Definition of Preferred AOD Pair	109
5.3.2	Relationship between MDC-QOSTBC and AOD through Preferred AOD Pair	110
5.3.3	Maximum Code Rate of MDC-QOSTBC	113
5.3.4	Examples of MDC-QOSTBC Constructed from Preferred AOD Pair ...	119
5.4	Systematic Construction of MDC-QOSTBC	123
5.4.1	Construction of MDC-QOSTBC from O-STBC	123
5.4.2	Performance optimization	126
5.4.3	MDC-QOSTBC for Odd Number of Transmit Antennas	132
5.4.4	Maximum Code Rate of MDC-QOSTBC from Construction 5.1	134
5.5	Performance Results	137
5.6	Chapter Summary	140
CHAPTER 6 Differential Detection of Quasi-Orthogonal STBC		142
6.1	Introduction	142
6.1.1	Motivations	142
6.1.2	DSTM Codeword Model	143
6.1.3	DSTM Design Criteria	143
6.2	Unitary DSTM	144

6.2.1	<i>Literature Review</i>	144
6.2.2	<i>Signal Model of Unitary DSTM Scheme</i>	145
6.2.3	<i>Proposed Double-Symbol-Decodable Unitary DSTM</i>	147
6.2.4	<i>Performance Comparison</i>	153
6.2.5	<i>Section Summary</i>	156
6.3	<i>Quasi-Unitary DSTM</i>	156
6.3.1	<i>Literature Review</i>	156
6.3.2	<i>Signal Model of Quasi-Unitary DSTM Scheme</i>	157
6.3.3	<i>Proposed Single-Symbol-Decodable Quasi-Unitary DSTM</i>	159
6.3.4	<i>Performance Comparison</i>	165
6.3.5	<i>Section Summary</i>	168
6.4	<i>Chapter Summary</i>	169
CHAPTER 7	<i>Conclusion And Recommendations</i>	170
7.1	<i>Conclusion</i>	170
7.2	<i>Recommendations</i>	172
	<i>Author's Publications</i>	174
	<i>Bibliography</i>	178
APPENDIX A	<i>Proof of Construction Method for ACOD</i>	186
APPENDIX B	<i>Derivation of ML Decision Metric in (5.15)</i>	190

SUMMARY

Future wireless communication systems will demand higher data rates to support a wide variety of data, Internet or multimedia applications. To support high transmission rates with sufficient robustness to radio channel impairments such as fading, *transmit diversity* has emerged as an attractive fading mitigation scheme. A simple and yet effective transmit diversity coding scheme is the *Space-Time Block Code* (STBC). *Orthogonal STBC* (O-STBC) is an important class of STBC, which can provide full transmit diversity with only linear maximum-likelihood (ML) decoding complexity. In this thesis, we first study the complex version of Amicable Orthogonal Design (AOD) called *Amicable Complex Orthogonal Design* (ACOD), and use it to construct O-STBCs with practical implementation advantages, such as balanced power distribution properties and rational-number code coefficients.

Despite its advantages, O-STBC suffers a reduced code rate when more than two transmit antennas and complex constellation are used. As a result, *Quasi-Orthogonal STBC* (QO-STBC) with *constellation rotation* (CR) is proposed to provide full transmit diversity with higher code rate than O-STBC. The ML decoding of QO-STBC can be implemented by jointly detecting a subset of the total transmitted data symbols. This is no longer the linear decoding complexity of O-STBC, but is nonetheless lower than other non-orthogonal STBC. In this thesis, we first study the decoding of QO-STBC with noise-whitening pre-filter, and compare the decoding performance of QO-STBC with and without CR under various symbol-by-symbol equalization schemes or joint-symbol detection schemes. It is found that the advantages of CR can only be reaped when joint-symbol detection schemes are used. Next, we derive the unifying algebraic structure of QO-STBC, which leads us to find out that CR increases the

decoding complexity of QO-STBC. Hence we propose *Group-Constrained Linear Transformation* (GCLT) as an alternative means to optimise the QO-STBC performance without increasing its decoding complexity. Specifically, the resultant QO-STBC with GCLT only needs to jointly detect half the number of symbols required by QO-STBC with CR. This leads to a significant reduction in the ML decoding search space with only a very slight tradeoff in decoding error probability performance. We also derive the optimum GCLT parameters that enable the QO-STBC to achieve full diversity and maximum coding gain.

GCLT has the limitation that it is only applicable to QO-STBC with regular square or rectangular QAM constellations. To provide full transmit diversity and higher code rate than O-STBC at the lowest possible decoding complexity among all QO-STBCs without limitation on the code constellation used, we propose a new class of QO-STBC called *QO-STBC with Minimum Decoding Complexity* (MDC-QOSTBC). The decoding complexity of MDC-QOSTBC is only next to O-STBC, as MDC-QOSTBC requires a joint detection of only two real symbols. In the thesis, we examine the relationship between the mathematical structures of MDC-QOSTBC and AOD, and found out that MDC-QOSTBC can be constructed from two AODs that form a *Preferred AOD Pair*, which is a new concept introduced in this thesis. We derive the theoretical maximum achievable code rate of MDC-QOSTBC and show that it has a code rate advantage of $1/4$ over the maximum achievable code rate of square O-STBC. We also propose a systematic way to construct higher-order MDC-QOSTBC from lower-order O-STBC, and derive the optimum constellation rotation angle required by MDC-QOSTBC to deliver full diversity and maximum coding gain. Besides requiring much lower decoding complexity, MDC-QOSTBC also has better flexibility in

supporting different numbers of transmit antennas and better power distribution properties than different existing QO-STBCs.

Finally, we also propose for the first time *differential space-time modulation* (DSTM) schemes based on QO-STBC and MDC-QOSTBC to provide blind transmit diversity. We employ *pair-wise modulation* and *customized constellation* to enable QO-STBC and MDC-QOSTBC to be differentially decoded without the knowledge of channel state information. The DSTM scheme based on QO-STBC is double-symbol decodable, while the DSTM scheme based on MDC-QOSTBC is single-symbol decodable, hence both have very low decoding complexity. They are able to achieve full transmit diversity, higher code rate than existing DSTM based on O-STBC, and much lower decoding complexity than existing DSTM schemes not based O-STBC.

LIST OF ABBREVIATIONS AND SYMBOLS

Abbreviations

ACF	amicable complex family
ACIOD	asymmetric co-ordinate interleaved orthogonal design
ACOD	amicable complex orthogonal design
AF	amicable family
AOD	amicable orthogonal design
A-PSK	amplitude-phase shift keying
AWGN	additive white Gaussian noise
BER	bit error rate
BLER	block error rate
bps/Hz	bits per sec per hertz
BPSK	binary phase shift keying
CF	complex family
CIOD	co-ordinate interleaved orthogonal design
COD	complex orthogonal design
CSI	channel state information
CR	constellation rotation
DFE	decision feedback equalizer
DSTM	differential space-time modulation
GCLT	group-constrained linear transformation
JD	joint detection
MDC-QOC	minimum-decoding-complexity quasi-orthogonality constraints

MDC-QOSTBC	minimum-decoding-complexity quasi-orthogonal space-time block code
MIMO	multiple input multiple output
MISO	multiple input single output
ML	maximum-likelihood
MLR	multi-dimensional lattice rotation
MMSE	minimum mean square error
MRC	maximal ratio combining
MSD	maximal symbolwise diversity
OD	orthogonal design
O-STBC	orthogonal space-time block code
PEP	pairwise error probability
PSK	phase-shift keying
QAM	quadrature amplitude modulation
QOC	quasi-orthogonality constraints
QPSK	quadrature phase shift keying
QO-STBC	quasi-orthogonal space-time block code
SD	sphere decoding
SIMO	single input multiple output
SNR	signal-to-noise ratio
STBC	space-time block code
STC	space-time code
STTC	space-time trellis code
ZF	zero-forcing

Symbols

$*$	Hadamard product
$\otimes$	Kronecker product
$\underline{\mathbf{A}}, \underline{\mathbf{B}}$	orthogonal design
$\underline{\mathbf{A}}, \underline{\mathbf{B}}$	matrices of an orthogonal design
$\underline{a}, \underline{b}$	variables of an orthogonal design
s, t	number of variables in an orthogonal design
u, v	types of an orthogonal design
$\rho(n)$	maximum number of variables of an orthogonal design
$\rho_l(n)$	maximum number of variables of an orthogonal design in an amicable orthogonal design
$\underline{\mathcal{C}}, \underline{\mathcal{D}}$	complex orthogonal design
$\underline{\mathbf{C}}, \underline{\mathbf{D}}$	matrices of a complex orthogonal design
$\underline{c}, \underline{d}$	variables of a complex orthogonal design
x, y	number of variables in an orthogonal design
f, g	types of a complex orthogonal design
$H(n)$	maximum number of variables of a complex family
$\underline{\mathbf{W}}$	weight or complex weighting matrix
$\mathbf{0}_n$	Zero matrix of size n -by- n
$\mathbf{A}, \mathbf{B}$	dispersion matrices of a STBC of size $P \times N_T$
$\mathbf{A}_{\text{CE}}$	codeword distance matrix
$\mathbf{B}_{\text{CE}}$	codeword difference matrix
b_p	source data bit at time index p
$\mathbf{C}$	STBC codeword of size $P \times N_T$

c_p	source data symbols at time index p
D	rank of codeword distance matrix
E_b	bit energy
E_s	total power transmitted across all transmit antennas
$\mathbf{H}$	equivalent channel matrix
$h_{i,k}$	channel fading coefficient from the i^{th} transmit to the k^{th} receive antenna
$\mathbf{I}_n$	identity matrix of size n -by- n
j	square root of -1
K	number of data symbols transmitted in a block of STBC
$\eta_p^{(i)}$	AWGN at the i^{th} receive antenna at time index p , superscript i is omitted if only one receive antenna
$\mathbf{N}_p$	noise matrix at time index p
N_o	one-sided noise density
N_R	number of receive antenna
N_T	number of transmit antenna
P	code length / delay of a STBC
θ	angle of rotation
ρ	signal-to-noise ratio
$r_p^{(i)}$	received symbol at the i^{th} receive antenna at time index p , superscript i is omitted if only one receive antenna
R	code rate of a STBC
$\mathbf{R}_p$	received signal matrix at time index p
$\mathbf{U}_p$	unitary or quasi-unitary code matrix in DSTM
$x_p^{(i)}$	transmitted symbol at the i^{th} receive antenna at time index p
$\mathbf{X}_p$	transmitted signal matrix at time index p

ζ	diversity product
$(.)^*$	complex conjugate
$(.)^H$	complex conjugate transpose / hermitian
$(.)^I$	imaginary part of a complex element, vector or matrix
$(.)^R$	real part of a complex element, vector or matrix
$(.)^T$	transpose
$ \cdot $	magnitude of complex element
$\ \cdot\ ^2$	Frobenius norm
$\lceil x \rceil$	smallest integer larger than x
$CW(n, w)$	complex weighting matrix of order n and weight w
$\text{Det}(\mathbf{M})$	determinant of a matrix $\mathbf{M}$
$E(\cdot)$	expectation operator
$\text{Max}(\cdot)$	maximization operator
$\text{Min}(\cdot)$	minimization operator
$\text{Rank}(\mathbf{M})$	rank of a matrix $\mathbf{M}$
$\text{Tr}(\mathbf{M})$	trace of a matrix $\mathbf{M}$
$W(n, w)$	weighting matrix of order n and weight w

LIST OF FIGURES

Figure 1.1 Feedforward transmit diversity with different diversity levels	4
Figure 2.1 Receive diversity with one transmit and two receive antennas.....	11
Figure 2.2 Transmit diversity with two transmit and one receive antenna.....	13
Figure 2.3 MIMO wireless communication model.....	15
Figure 4.1 Constellation rotation of BPSK constellation set.....	63
Figure 4.2 Model of QO-STBC Receiver with Noise Whitening.....	69
Figure 4.3 BLER Performance of J4 and J4_CR at 2 bits/s/Hz	79
Figure 4.4 BLER Performance of TBH8 and TBH8_CR at 2 bits/s/Hz.....	80
Figure 4.5 QAM-constellation and its minimum Euclidean distance d_{min}	93
Figure 4.6 Determinant values of J4_LT in (4.52) to (4.55) for various m, n values .	95
Figure 4.7 Grouping structure of dispersion matrices of J4 , J4_CR and J4_LT	96
Figure 4.8 Simulation results of QO-STBCs for four transmit antennas with spectral efficiency of 2 bits/sec/Hz	101
Figure 4.9 Simulation results of QO-STBCs for eight transmit antennas with spectral efficiency of 1.5 bits/sec/Hz	101
Figure 4.10 Simulation results of QO-STBCs for eight transmit antennas with spectral efficiency of 2 bits/sec/Hz	102
Figure 5.1 Grouping of dispersion matrices of an MDC-QOSTBC with $K=4$	107
Figure 5.2 Relationship of MDC-QOSTBC with AOD through Preferred AOD Pair	112
Figure 5.3 Construction of MDC-QOSTBC from O-STBC	124
Figure 5.4 Optimization of PSK rotation angle for MDC-QOSTBC	130
Figure 5.5 Simulation performance of STBCs with spectral efficiency of 2 bps/Hz	138

Figure 5.6 Simulation performance of STBCs with 4-QAM constellation for three and five transmit antennas	140
Figure 6.1 Optimization of constellation rotation angle $\theta = 2\pi m / M$ in (6.15).....	152
Figure 6.2 BLER of different DSTM schemes for four transmit and one receive antennas.....	155
Figure 6.3 Solution loci of the DSTM constellation design criteria (6.25)(i) and (ii).....	163
Figure 6.4 Optimization of coding gain.....	164
Figure 6.5 Optimized constellation sets for DSTM based on MDC-QOSTBC.....	164
Figure 6.6 BLER of DSTM schemes based on O-STBC and MDC-QOSTBC.....	166
Figure 6.7 BLER of DSTM schemes for four transmit and one receive antennas	167

LIST OF TABLES

Table 3.1 Values of δ for an AOD	28
Table 3.2 s and t values for $n = 4$	29
Table 3.3 s and t values for $n = 8$	30
Table 3.4 Power distribution characteristics of four-antenna O-STBCs with QPSK modulation	54
Table 3.5 Power distribution characteristics of eight-antenna O-STBCs with QPSK modulation	54
Table 4.1 Fulfillment of QOC for the dispersion matrices of J4	85
Table 4.2 Fulfillment of QOC for the dispersion matrices of J4_CR	87
Table 4.3 Number of real symbols required for joint detection for QO-STBC with square and rectangular QAM	88
Table 4.4 Comparison of 4QAM QO-STBCs with CR and GCLT	100
Table 5.1 Comparisons of STBCs for four transmit antennas	105
Table 5.2 s , t values for $n=4$	117
Table 5.3 s , t values for $n=8$	118
Table 5.4 Comparison of STBCs for four transmit antennas	131
Table 5.5 Comparison between square O-STBC and square QO-STBC	135
Table 5.6 Comparison of QO-STBCs for four transmit antennas	137
Table 5.7 Comparison of STBCs for three and five transmit antennas	139
Table 6.1 Comparison of coding gains and decoding complexity for DSTM schemes for four transmit antennas	154

CHAPTER 1 INTRODUCTION

1.1 BACKGROUND INFORMATION

In the past few years, consumer and manufacturer interest in wireless communication have exploded. This is due to the ability of wireless communication to provide the advantages of wide area coverage without wires, and most importantly allowing mobility while communicating. Beyond the success of the established technologies such as mobile telephony, a wide range of new wireless communications services are being developed. For example, there has been growing interest in providing broadband wireless Internet services with rich multimedia contents at near wire-line data rates.

However, the wireless channel suffers from random signal attenuation and phase distortion due to the destructive superposition of multiple received signals in a multipath propagation environment, a phenomenon commonly called *fading*. Fading makes it extremely difficult for the receiver to recover the transmitted signal unless the receiver is provided with some form of *diversity*, i.e. replicas of the same transmitted signal with uncorrelated attenuation. In fact, diversity combining technology has been one of the most important contributors to reliable wireless communications. Ways to achieve diversity include:

- *Temporal Diversity*: In this scheme, channel coding in conjunction with time interleaving is used. Thus replicas of the transmitted signal are provided to the receiver in the form of redundancy in the temporal domain. However, in slow fading channels, temporal diversity is not an option for delay-sensitive applications. Also, there is a penalty in time efficiency.

- *Frequency Diversity*: In this scheme, the fact that signals that transmitted on different frequencies tend to experience different fading effects is exploited. Thus replicas of the transmitted signal are provided to the receiver in the form of redundancy in the frequency domain. However, there is a penalty in frequency or bandwidth efficiency.
- *Spatial Diversity*: In this scheme, spatially separated antennas are used to provide diversity in the spatial domain. Diversity combining technique is then used to select or combine the signals that have been transmitted or received on different antennas.

Spatial diversity is attractive as diversity can be obtained with no penalty in time or bandwidth efficiency. It can be implemented by deploying multiple antennas at the transmitter and/or the receiver. Depending on the location of the antennas, we can classify wireless communication system employing spatial diversity into the following three configurations:

- *Single Input Multiple Output (SIMO)*: When there are single transmit antenna but multiple receive antennas, i.e. receive diversity.
- *Multiple Input Single Output (MISO)*: When there are multiple transmit antennas but one receive antenna, i.e. transmit diversity.
- *Multiple Input Multiple Output (MIMO)*: When there are multiple transmit antennas and multiple receive antennas, i.e. both transmit and receive diversity are possible.

Besides simply providing spatial diversity, it has been shown in [1,2] that the capacity of wireless channel grows linearly with the number of transmit and receive antennas, hence a MIMO system can be used to boost channel capacity too.

Considering the fact that common receivers are typically required to be small and cost efficient, it may not be practical to deploy receive diversity at the mobile station. This motivates many researchers to consider transmit diversity by deploying multiple antennas at the base station. Moreover, in economic terms, the cost of multiple transmit antennas at the base station can be amortized over numerous mobile users. Hence transmit diversity had been identified as one of the key contributing technologies to the downlinks of 3G wireless systems such as W-CDMA and CDMA2000 [3]. There are generally three categories of transmit diversity:

- *Feedback Scheme*: This involves the feedback of channel state information (CSI) from the receiver to the transmitter in order to adapt the transmitter to the channel during the next transmit epochs.
- *Feedforward Scheme*: This uses feedforward information (e.g. pilot symbols) for the receiver to estimate the channel, but no feedback information is sent back to the transmitter.
- *Blind Scheme*: This requires no feedback or feedforward CSI or pilots, and the receiver simply makes use of the received signal to attempt data recovery without the knowledge of CSI.

To demonstrate the benefit of transmit diversity under feedforward scheme, the bit error rate (BER) performance versus bit energy to noise ratio (E_b/N_o) of a typical transmit diversity scheme with various number of transmit antennas and one receive antenna at a spectral efficiency of 2 bits/sec/Hz (bps/Hz) is illustrated in Figure 1.1. These results are extracted from [4,5], and are achieved by *Space-Time Block Code* (STBC), a type of feedforward transmit diversity that will be discussed next. It can be seen that, when there is only one transmit antenna, more than 15dB increase in E_b/N_o is required over additive white Gaussian noise (AWGN) channel to achieve a BER of

10^{-3} , due to Rayleigh fading in the wireless channel. By employing multiple transmit antennas to provide transmit diversity, the decay rate of the BER curve can be significantly improved. So having more antennas results in better performance due to higher spatial diversity level. However, unlike receive diversity that can be achieved by simply performing the diversity combining at the receiver side, some form of processing, i.e. space time coding, on the transmitted signals is required in order to achieve transmit diversity.

Figure 1.1 Feedforward transmit diversity with different diversity levels

Space-Time Code (STC) is a technique that combines coding, modulation and signal processing to achieve transmit diversity. The first STC proposed in the literature is *Space-Time Trellis Code* (STTC) [6], which has a good decoding performance but its decoding complexity increases exponentially with the transmission rate.

In addressing the issue of decoding complexity of STTC, *Space-Time Block Code* (STBC) is proposed. Alamouti [4] discovered a remarkable STBC scheme for two transmit antennas. This scheme has a linear complexity for maximum-likelihood (ML) decoding, which is much less complex than the decoding of STTC. It can achieve the same diversity gain as a corresponding STTC for two transmit antennas, but there is generally a shortfall in coding gain compared to STTC. Despite this trade off, Alamouti's scheme is very appealing in terms of implementation simplicity; hence it motivates a search for similar schemes for more than two transmit antennas, to achieve diversity level higher than two. As a result, *Orthogonal Space-Time Block Code* (O-STBC) is introduced in [7]. It generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and retains the property of having linear maximum-likelihood decoding with full transmit diversity.

Although O-STBC can provide full diversity at low computational cost, [8] showed that it incurs a loss in capacity when 1) there are multiple receive antennas, 2) the code rate is less than one. As O-STBC design that provides full code rate is not possible for more than two transmit antennas [7], O-STBC with more than two transmit antennas will always suffer a capacity loss.

To address the issue of capacity loss, various non-orthogonal STBCs have been proposed in the literature. Among them, an interesting one is the *Quasi-Orthogonal STBC* (QO-STBC) [9,10,11], which has been designed to achieve a higher code rate than O-STBC by partially (rather than fully as in case of other non-orthogonal STBCs) relaxing the orthogonality of an O-STBC. For example, the ML decoding of the full-rate QO-STBC in [9] for four transmit antennas can be achieved by jointly detecting two complex symbols out of four complex symbols being transmitted in the same codeword. To achieve full diversity, QO-STBC is further optimized with the

constellation rotation (CR) [12,13,14] technique. Due to the full transmit diversity and low decoding complexity advantages of O-STBC and QO-STBC, we are motivated to investigate some of the unsolved issues in O-STBC and QO-STBC, and to seek their further improvements.

1.2 MOTIVATIONS

As discussed earlier, besides achieving a good diversity level and coding performance, the decoding and implementation complexity of the STC system is important as well. In this thesis, we focus on the feedforward and blind transmit diversity scheme with low decoding complexity, i.e. based on O-STBC and QO-STBC. Specifically, we will attempt to address the following research issues concerning these two classes of STBC:

- (i) O-STBC has been designed from *Amicable Orthogonal Design* (AOD) [15], but the existence of the complex version of AOD has been queried in [16]. On the other hand, existing O-STBCs do not have balanced power distribution. Although [17] and [18] attempted to design O-STBC with better power distribution properties, they fell short of developing a fully power balanced code.
- (ii) Many QO-STBCs proposed in the literature [9,10,11] have very similar properties such as decoding complexity, but there is no general framework to unify their algebraic structure and link it to their achievable decoding complexity.
- (iii) There is no comparative study on the decoding performance of QO-STBC under various optimal and sub-optimal decoding schemes. The effect of CR

on the decoding performance of QO-STBC under sub-optimal decoding scheme has not been reported too.

- (iv) We found that the well-known CR technique, which enables a QO-STBC to achieve full diversity, actually increases the joint detection search space (i.e. decoding complexity) of the QO-STBC. Whether an alternative technique exists to achieve the same performance benefit without the same complexity trade-off is an interesting research issue.
- (v) QO-STBC proposed in the literature has yet to reach its minimum achievable decoding complexity. How to construct QO-STBC with minimum decoding complexity in a systematic way for arbitrary code constellation has not been reported too.
- (vi) QO-STBC with coherent decoding, i.e. feedforward scheme, has been widely studied for its high code rate and reasonably low decoding complexity, but how to achieve such QO-STBC benefits in non-coherent or blind transmit diversity scheme has not been reported.

1.3 OBJECTIVES

The major objectives of this thesis are to design O-STBCs and QO-STBCs with low decoding complexity and good implementation properties, to achieve full transmit diversity and good decoding performance in both feedforward and blind schemes in quasi-static flat fading channel. Specifically, we aim to:

- (i) Prove the existence of the complex version of AOD and design its construction to achieve O-STBC with balanced power distributions.

- (ii) Unify the algebraic structure of generic QO-STBC.
- (iii) Study the decoding performance of QO-STBC with CR under representative optimal and sub-optimal decoding schemes including ML decoding, sphere decoding, zero forcing equalizer, minimum mean square error equalizer and decision feedback equalizer.
- (iv) Design a new constellation transformation technique to optimize the decoding performance of QO-STBC without increasing its decoding complexity.
- (v) Derive the algebraic structure of QO-STBC with minimum decoding complexity and find ways to construct it systematically. Derive its maximum achievable code rate and optimize its decoding performance.
- (vi) Propose differential QO-STBC schemes with higher code rate and/or lower decoding complexity than the existing differential transmit diversity schemes.

1.4 CONTRIBUTIONS

The major contributions of this thesis include the followings:

- (i) We have proven the existence of the complex version of AOD, namely *Amicable Complex Orthogonal Design* (ACOD), and proposed a systematic way to construct it. We have found the link between the ACOD parameters and the power distribution properties of O-STBC, and constructed new power-balanced O-STBC with ideal power distribution properties from ACOD.

- (ii) We have derived the algebraic structure of generic QO-STBC, and designed whitening filter for QO-STBC decoders. We have shown that CR can improve the decoding performance of QO-STBC only when ML or SD detection schemes are used. We have proposed *Group-Constrained Linear Transformation* (GCLT) as a means to achieve performance-optimized QO-STBC with lower decoding complexity than QO-STBC with CR.
- (iii) We have constructed single-symbol-decodable QO-STBC, called *Minimum-Decoding-Complexity QO-STBC* (MDC-QOSTBC) that has the lowest decoding complexity among all non-orthogonal STBCs. MDC-QOSTBC only needs the joint detection of two real symbols, so anything less complex (i.e. linear detection of only one real symbol) would be an O-STBC. We have proposed the concept of *Preferred AOD Pair* to show that MDC-QOSTBC can be formed by two AODs that form a Preferred AOD Pair. We have also derived the theoretical maximum code rate of an MDC-QOSTBC. In addition, we have designed a systematic way to construct higher-order MDC-QOSTBC from lower-order O-STBC, and derived its performance-optimized code parameters for QAM and PSK code constellations.
- (iv) We have developed differential QO-STBC and MDC-QOSTBC schemes to achieve double-symbol-decodable and single-symbol-decodable blind transmit diversity schemes respectively for the first time. The proposed differential QO-STBC and MDC-QOSTBC schemes achieve higher code rate than the existing differential scheme based on O-STBC, and much lower decoding complexity than the other existing differential schemes.

1.5 ORGANIZATIONS

The thesis is organized as follows. Chapter 2 provides an introduction of the technique of transmit diversity based on STBC. We introduce the signal models of MIMO system and linear STBC that will be used throughout the thesis, and we review the design criteria of STBC.

Chapter 3 discusses the construction of O-STBC from ACOD. We present ACOD and its construction. We show the relationship of the ACOD parameters with the power distribution properties of an O-STBC. We then design a fully power-balanced O-STBC based on the proposed ACOD's construction.

In Chapter 4, we derive the algebraic structure of a general QO-STBC, and the whitening filter for use with QO-STBC decoders, and then we study the impact of CR on the decoding performance of different decoding schemes. Finally, we discuss in detail how to apply GCLT to optimize the decoding performance of QO-STBC.

Chapter 5 discusses MDC-QOSTBC in detail and we link MDC-QOSTBC to AOD by proposing the concept of Preferred AOD Pair, and derive the maximum achievable code rate of MDC-QOSTBC. We also proposed a systematic way to construct MDC-QOSTBC, and compare its performance and attributes with existing QO-STBCs found in the literature.

In Chapter 6, we propose differential QO-STBC and MDC-QOSTBC schemes. We first derive the criteria for a QO-STBC and MDC-QOSTBC to be non-coherently decodable. From the criteria, we design the constellation sets that provide full transmit diversity.

Finally, we conclude the thesis and discuss possible future extensions of the current works in Chapter 7.

CHAPTER 2 SPACE-TIME BLOCK CODE

2.1 INTRODUCTION

Before we introduce the transmit diversity scheme based on Space-Time Block Code (STBC), we shall first review the traditional receive diversity scheme with *maximal ratio combining* (MRC) for one transmit antenna and two receive antennas, using Figure 2.1 as an example.

Figure 2.1 Receive diversity with one transmit and two receive antennas

Denote the transmitted signal at time p as c_p , and the received signals on the first and second receive antennas as $r_p^{(1)}$ and $r_p^{(2)}$ respectively, we obtain the following expressions:

$$\begin{aligned} r_p^{(1)} &= h_1 c_p + \eta_p^{(1)} \\ r_p^{(2)} &= h_2 c_p + \eta_p^{(2)} \end{aligned} \quad (2.1)$$

where h_1 and h_2 are the channel state information (CSI) from the transmit antenna to the first and second receive antennas respectively, and $\eta_p^{(1)}$ and $\eta_p^{(2)}$ are additive white Gaussian noise (AWGN) added to the respective received antennas at time instant p .

If only one receive antenna is available, the transmitted symbols can be estimated as follows:

$$\begin{aligned}\hat{c}_p &= h_1^* r_p^{(1)} \\ &= |h_1|^2 c_p + h_1^* \eta_p^{(1)}\end{aligned}\quad (2.2)$$

where $*$ denotes the complex conjugate and $|\cdot|$ denotes the magnitude of a complex element.

When multiple receive antennas are available, to retrieve the data symbols and utilizing the diversity provided by multiple receive antennas, we perform MRC as follow, assuming that the CSI are estimated accurately:

$$\begin{aligned}\hat{c}_p &= h_1^* r_p^{(1)} + h_2^* r_p^{(2)} \\ &= \left(|h_1|^2 + |h_2|^2\right) c_p + h_1^* \eta_p^{(1)} + h_2^* \eta_p^{(2)}\end{aligned}\quad (2.3)$$

The signal-to-noise ratio (SNR) distribution in (2.3) is better than that received signal in (2.2) because the chances that both h_1 and h_2 in (2.3) fade simultaneously is smaller than the chances that h_1 in (2.2) fades. Statistically, if h_1 and h_2 are Rayleigh-distributed and uncorrelated, $|h_1|^2$ will have a Chi-Square distribution with two degrees of freedom while $|h_1|^2 + |h_2|^2$ will have a Chi-Square distribution with four degree of freedom and lower probability of deep fade. This explains the diversity gain of a MRC receive diversity scheme over a non-diversity receiver scheme.

However, as mentioned in Chapter 1, the deployment of multiple receive antennas at the mobile station may not be feasible due to size and cost constraints, hence this motivates the research of transmit diversity, which relies on multiple transmit antennas at the base station. In [4], Alamouti proposed a simple transmit diversity scheme for two transmit antennas, which is shown later [7] to be the only Orthogonal STBC (O-STBC) that provides full diversity at full rate for any complex constellation, and that

achieves the same diversity gain as the receive MRC diversity with two receive antennas.

The Alamouti O-STBC is described as follows: considering a system with two transmit antennas and one receive antenna as shown in Figure 2.2, at a given symbol period, two signals are simultaneously transmitted from two antennas. At time $2p-1$, the signal c_{2p-1} is transmitted from the first antenna, while the signal c_{2p} is transmitted from the second antenna. During the next symbol period $2p$, signal $-c_{2p}^*$ is transmitted from the first antenna, and signal c_{2p-1}^* is transmitted from the second antenna. However in order to normalize the total transmission power to be the same as the receive diversity scheme in Figure 2.1, the transmission power from each transmit antenna is halved.

Figure 2.2 Transmit diversity with two transmit and one receive antenna

In this case, h_1 and h_2 are the CSI from the first and second transmit antenna to the receive antenna respectively, and they are assumed to be remained unchanged during two symbols period. The received signals at time $2p-1$ and $2p$ can be expressed respectively as:

$$\begin{aligned} r_{2p-1} &= \frac{1}{\sqrt{2}} (h_1 c_{2p-1} + h_2 c_{2p}) + \eta_{2p-1} \\ r_{2p} &= \frac{1}{\sqrt{2}} (h_2 c_{2p-1}^* - h_1 c_{2p}^*) + \eta_{2p} \end{aligned} \quad (2.4)$$

where the factor $1/\sqrt{2}$ accounts for the power normalization, η_{2p-1} and η_{2p} are AWGN added to the received signal at time $2p-1$ and $2p$ respectively.

Assuming that perfect CSI is known, the transmitted data symbols can be easily recovered by making use of linear combining as follows:

$$\begin{aligned}\hat{c}_{2p-1} &= h_1^* r_{2p-1} + h_2^* r_{2p} = \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2) c_{2p-1} + h_1^* \eta_{2p-1} + h_2^* \eta_{2p} \\ \hat{c}_{2p} &= h_2^* r_{2p-1} - h_1^* r_{2p} = \frac{1}{\sqrt{2}} (|h_1|^2 + |h_2|^2) c_{2p} + h_2^* \eta_{2p-1} - h_1^* \eta_{2p}\end{aligned}\quad (2.5)$$

Each of the resultant signals in (2.5) is similar to that obtained from receive diversity MRC in (2.3). Therefore, the diversity order of the above O-STBC system with simple linear receiver processing is the same as the corresponding receive diversity with MRC. This makes O-STBC attractive, as the multiple antennas can now be installed at the transmitter instead of the receiver. In order to further study the transmit diversity based on STBC, we will next introduce the signal models for MIMO channel and STBC transmission.

2.2 SIGNAL MODEL OF MIMO CHANNEL

We consider a MIMO wireless communication system with N_T transmit antennas at the base station and N_R receive antennas at the mobile as shown in Figure 2.3. At each time slot p , signal $x_p^{(i)}$, is transmitted from the i^{th} transmit antennas, where $i = 1, 2, \dots, N_T$. The channel is assumed to be a flat fading channel and the path gain from transmit antenna i to receive antenna k is denoted as $h_{i,k}$. It is assumed that $h_{i,k}$ and $h_{l,q}$ are independent for $1 \leq i, l \leq N_T$, $1 \leq k, q \leq N_R$ and for different i, k and l, q pairs. This condition is satisfied if the antennas are well separated by more than half of the wavelength of the transmitted wave, or by using antennas with different polarization. Since we focus on transmit diversity with just one receive antenna in our study, the

subscript k will be omitted for simplicity at a later part of the thesis, e.g. $h_{i,k}$ will be just written as h_i . The path gains are modeled as independent complex Gaussian random variables with variance 0.5 per dimension, i.e. $h_{i,k} = \alpha_{i,k} \exp(j\theta_{i,k})$, where $\alpha_{i,k}$ follows the Rayleigh distribution and $\theta_{i,k}$ is uniformly distributed. The channel fading is assumed to be quasi-static, so the path gains are constant over a frame of length F and only vary from frame to frame.

Figure 2.3 MIMO wireless communication model

We assume that the transmitted signal $x_p^{(i)}$ has unit power and E_s is the total energy transmitted from all antennas. Therefore, the energy transmitted from each transmit antenna is E_s / N_T . Signals arriving at different receive antennas undergo independent fading. The signal at each receive antenna is a noisy superposition of the faded versions of the N_T transmitted signals. At time p the signal $r_p^{(k)}$ received at the k^{th} antenna, where $k = 1, 2, \dots, N_R$, is given by

$$r_p^{(k)} = \sqrt{\frac{E_s}{N_T}} \sum_{i=1}^{N_T} (h_{i,k} x_p^{(i)} + \eta_p^{(k)}) \quad (2.6)$$

where the noise samples $\eta_p^{(k)}$ are independent samples of a zero-mean complex white Gaussian random variable with variance $N_o / 2$ per dimension. Since the total energy of the symbols transmitted from all transmit antennas is normalized to be E_s , the average power of the received signal at each receive antenna is E_s and the SNR is E_s / N_o per

receive antenna. Without loss of generality, we may assume E_s equal to one if it is not stated explicitly.

2.3 SIGNAL MODEL OF SPACE-TIME BLOCK CODE

Suppose that a generic STBC codeword is transmitted from N_T transmit antennas to N_R receive antennas over an interval of P symbol periods in which the propagation channel condition is time-invariant (i.e. $P \leq F$) and known to the receiver. The transmitted codeword can be written as a $P \times N_T$ matrix $\mathbf{C}$ that consists of K arbitrary complex constellation symbols. Its code length is P , and code rate is defined as $R = K / P$. Following the model in [19], $\mathbf{C}$ can be expressed as:

$$\mathbf{C} = \sum_{i=1}^K (c_i^R \mathbf{A}_i + jc_i^I \mathbf{B}_i) \quad (2.7)$$

where the info symbols are $c_i = c_i^R + jc_i^I$, and c_i^R and c_i^I are the real (I) and the imaginary (Q) components of c_i . Matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ are called the “*dispersion matrices*” and are of size $P \times N_T$.

To illustrate all the above definitions, we use Alamouti STBC in [4] as an example. Using the signal model in (2.7), we can write the Alamouti STBC codeword as follows:

$$\mathbf{C}_{\text{Alamouti}} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix} \quad (2.8)$$

where the corresponding dispersion matrices are:

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (2.9)$$

So one can easily see that for $N_T = 2$ transmit antennas, the Alamouti STBC takes $P = 2$ symbols period to transmit $K = 2$ complex symbols, and has a code rate of one.

For a given number of transmit antennas, the design of a STBC depends crucially on the choices of the parameters P , K , and the dispersion matrices $\{\mathbf{A}_i, \mathbf{B}_i\}$. The transmitted and received signals are related by [19]:

$$\tilde{\mathbf{r}} = \sqrt{\rho/N_T} \mathbf{H} \tilde{\mathbf{c}} + \tilde{\boldsymbol{\eta}} \quad (2.10)$$

$$\text{where } \tilde{\mathbf{r}} = \begin{bmatrix} \mathbf{r}_1^R \\ \mathbf{r}_1^I \\ \vdots \\ \mathbf{r}_{N_R}^R \\ \mathbf{r}_{N_R}^I \end{bmatrix}, \tilde{\mathbf{c}} = \begin{bmatrix} c_1^R \\ c_1^I \\ \vdots \\ c_K^R \\ c_K^I \end{bmatrix}, \tilde{\boldsymbol{\eta}} = \begin{bmatrix} \boldsymbol{\eta}_1^R \\ \boldsymbol{\eta}_1^I \\ \vdots \\ \boldsymbol{\eta}_{N_R}^R \\ \boldsymbol{\eta}_{N_R}^I \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} \mathcal{A}_1 \tilde{\mathbf{h}}_1 & \mathcal{B}_1 \tilde{\mathbf{h}}_1 & \dots & \mathcal{A}_K \tilde{\mathbf{h}}_1 & \mathcal{B}_K \tilde{\mathbf{h}}_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathcal{A}_1 \tilde{\mathbf{h}}_{N_R} & \mathcal{B}_1 \tilde{\mathbf{h}}_{N_R} & \dots & \mathcal{A}_K \tilde{\mathbf{h}}_{N_R} & \mathcal{B}_K \tilde{\mathbf{h}}_{N_R} \end{bmatrix},$$

$$\mathcal{A}_q = \begin{bmatrix} \mathbf{A}_q^R & -\mathbf{A}_q^I \\ \mathbf{A}_q^I & \mathbf{A}_q^R \end{bmatrix}, \mathcal{B}_q = \begin{bmatrix} -\mathbf{B}_q^I & -\mathbf{B}_q^R \\ \mathbf{B}_q^R & -\mathbf{B}_q^I \end{bmatrix}, \tilde{\mathbf{h}}_i = \begin{bmatrix} \mathbf{h}_i^R \\ \mathbf{h}_i^I \end{bmatrix}.$$

In the above equation, $\mathbf{r}_i$ and $\boldsymbol{\eta}_i$ ($1 \leq i \leq N_R$) are $P \times 1$ column vectors which contain the received signals and AWGN noises for the i^{th} receive antenna respectively, over P symbol periods. $\mathbf{H}$ is called the *equivalent channel matrix*, $\mathbf{h}_i$ is $N_T \times 1$ column vector that contains the fading coefficients of the spatial sub-channels between the N_T transmit antennas and i^{th} receive antenna. The normalization factor $\sqrt{\rho/N_T}$ in (2.10) ensures that $\rho = E_s / N_o$ is the signal to noise ratio (SNR) at each receive antenna, regardless of whatever N_T is.

To govern the distribution of transmission power across space and time, three Power Distribution Constraints were suggested in [19]:

$$\begin{aligned}
 \text{(i)} \quad & \sum_{i=1}^K \left[\text{Tr}(\mathbf{A}_i^H \mathbf{A}_i) + \text{Tr}(\mathbf{B}_i^H \mathbf{B}_i) \right] = 2PN_T \\
 \text{(ii)} \quad & \text{Tr}(\mathbf{A}_i^H \mathbf{A}_i) = \text{Tr}(\mathbf{B}_i^H \mathbf{B}_i) = \frac{PN_T}{K}, \quad 1 \leq i \leq K \\
 \text{(iii)} \quad & \mathbf{A}_i^H \mathbf{A}_i = \mathbf{B}_i^H \mathbf{B}_i = \frac{P}{K} \mathbf{I}_{N_T}, \quad 1 \leq i \leq K
 \end{aligned} \tag{2.11}$$

where $\text{Tr}(\mathbf{M})$ represents the trace of a matrix $\mathbf{M}$.

Constraint (i) – (iii) in (2.11) can be explained as follows: Constraint (i) is a constraint to ensure that the average total transmitted power is normalized to PN_T , i.e. $E[\text{Tr}(\mathbf{C}\mathbf{C}^H)] = PN_T$. Constraint (ii) is more restrictive and ensures that each of the I and Q signal (c_i^R and c_i^I) is transmitted with the same overall power over P symbol durations from all antennas. Constraint (iii) is the most stringent: it forces the symbols c_i^R and c_i^I to have equal energy in all spatial directions during *each* symbol duration. It is pointed out in [19] that codes satisfying the more stringer constraint in (2.11)(iii) generally give lower error rates in a feedforward / blind MIMO scheme. This is the same as the idea suggested by [20], which states that since the encoder does not have any information about the channel, it should transmit the code symbols uniformly in all space and time directions, assuming that all spatial channels be independence and identically distributed. It is further proposed in [73] that constraint (2.11)(iii) is the *maximal symbolwise diversity* (MSD) condition, which guarantees full diversity protection for one-symbol error events that have the smallest Euclidean distance.

As mentioned earlier, the design of STBC depends on the number of info symbols transmitted within a codeword K and code length P . These two values are limited by the following constraints in (2.12) and (2.13), depending on the requirements of the final STBC.

$$\text{transmit diversity level} \leq \min(N_T, P) \tag{2.12}$$

$$K \leq P \min(N_T, N_R) \tag{2.13}$$

In order to make full use of the transmit diversity provided by the N_T transmit antennas, i.e. to achieve *full transmit diversity* of order N_T , we know from (2.12) that $P \geq N_T$. Hence a square design, i.e. $P = N_T$, gives the minimum possible code length for achieving full diversity, and STBC with such property is commonly called STBC with *minimum delay*. Furthermore, if only one receive antenna is used, i.e. $N_R = 1$, we get $K \leq P$ from (2.13). Hence the maximum achievable code rate $R = K / P$ of a transmit diversity scheme with only one receive antenna is one.

2.4 DESIGN CRITERIA OF SPACE-TIME BLOCK CODE

2.4.1 Decoding Performance Optimization Criteria

It has been pointed out in [21] that in evaluating the performance of a space-time code in slow flat fading channel, the critical parameter is the rank of the codeword difference matrix. In [6], Tarokh *et al.* further showed that the minimum rank of the codeword difference matrix quantifies the *diversity gain*, while the minimum product of the non-zero eigenvalues of the codeword distance matrix corresponding to the minimum rank quantifies the *coding gain*, of the space-time code. They will be elaborated in the following part.

We assume that all the codewords have equal transmission probability and consider the probability $P(\mathbf{C} \rightarrow \mathbf{E})$ that the codeword $\mathbf{C}$ was transmitted but the receiver decides erroneously in favor of codeword $\mathbf{E}$. This probability term is commonly called the *pairwise error probability* (PEP). With ideal CSI, PEP is well approximated as follows [6]:

$$P(\mathbf{C} \rightarrow \mathbf{E} | h_{i,k}, i = 1, 2, \dots, N_T; k = 1, 2, \dots, N_R) \leq \exp\left(-d^2(\mathbf{C}, \mathbf{E}) \frac{E_s}{4N_o}\right) \quad (2.14)$$

where $d^2(\mathbf{C}, \mathbf{E}) = \sum_{k=1}^{N_R} \sum_{p=1}^P \left| \sum_{i=1}^{N_T} h_{i,k} (c_{p,i} - e_{p,i}) \right|^2$, $c_{p,i}$ and $e_{p,i}$ are the entries in the p^{th} row and i^{th} column of $\mathbf{C}$ and $\mathbf{E}$.

Before simplifying the PEP expression, we first define two matrices, the first is the *codeword difference matrix* $\mathbf{B}_{\text{CE}}$ of size $P \times N_T$, which is defined as [6]:

$$\mathbf{B}_{\text{CE}} = \mathbf{C} - \mathbf{E} \quad (2.15)$$

and the second is the *codeword distance matrix* $\mathbf{A}_{\text{CE}}$ of size $N_T \times N_T$, which is defined as [22]:

$$\mathbf{A}_{\text{CE}} = \mathbf{B}_{\text{CE}}^H \mathbf{B}_{\text{CE}} \quad (2.16)$$

Furthermore, we also define $\lambda_1, \lambda_2, \dots, \lambda_D$ as the non-zero eigenvalues of $\mathbf{A}_{\text{CE}}$, where D denotes the rank of $\mathbf{A}_{\text{CE}}$, which is the same as the rank of $\mathbf{B}_{\text{CE}}$. At high SNR, the upper bound of the PEP in (2.14) can be simplified as [6]:

$$P(\mathbf{C} \rightarrow \mathbf{E}) \leq \left(\prod_{i=1}^D \lambda_i \right)^{-N_R} \left(\frac{E_s}{4N_o} \right)^{-DN_R} \quad (2.17)$$

From (2.17), we can define the following two quantities to account for the decoding performance of a STBC:

- *Diversity Gain*: It is defined as the minimum rank D of the matrix $\mathbf{A}_{\text{CE}}$ over all pairs of distinct codewords. It accounts for the slope of the bit error rate (BER) or block error rate (BLER) curve of the STBC. A STBC is said to achieve full diversity if $D = N_T$.
- *Diversity Product*: It is defined [14] as follows for a full diversity code,

$$\varsigma = \frac{1}{2\sqrt{N_T}} \text{Min} \left| \text{Det}(\mathbf{A}_{\text{CE}}) \right|^{1/(2P)} \quad (2.18)$$

where the factor $1/2\sqrt{N_T}$ guarantees that $0 \leq \zeta \leq 1$, and $\text{Det}(\mathbf{M})$ denotes the determinant of a matrix $\mathbf{M}$. It accounts for the *coding gain*, which determines the left or right shift of the BER or BLER curve of the STBC.

Based on the above, a set of code design criteria, commonly called the *Rank & Determinant Criteria*, can be stated as follows [6]:

- *Rank Criterion*: To maximize the diversity gain of the STBC, maximize the minimum rank D of the matrix $\mathbf{A}_{\mathbf{C}\mathbf{E}}$ over all distinct pairs of codewords. That is, in order to achieve the maximum diversity order of $N_T N_R$, the matrix $\mathbf{A}_{\mathbf{C}\mathbf{E}}$ has to be full rank for any codeword pair $\mathbf{C}$ and $\mathbf{E}$.
- *Determinant Criterion*: To maximize the coding gain of the STBC, maximize the minimum product of non-zero eigenvalues of the matrix $\mathbf{A}_{\mathbf{C}\mathbf{E}}$ with the minimum rank obtained in the above Rank Criterion over all distinct pairs of codewords.

Other performance design criteria for space-time codes have also been proposed in [5,22,23,24]. In this thesis, we employ the Rank and Determinant criteria, which has been commonly used in the literature for designing STBC.

2.4.2 Power Distribution Criteria

One of the targets of a good wireless front-end design is to minimize the linearity requirements of its power amplifier [17,18,32].

Assume that the average transmission power of the i^{th} transmit antenna at time period p is $\Psi_{i,p}$, and the MIMO system has N_T transmit antennas with a STBC of code length P , the definition of the above terms are defined as follows:

$$\text{Peak power: } \Psi_i^{\text{peak}} = \max_p (\Psi_{i,p}) \quad (2.19)$$

$$\text{Average power: } \Psi_i^{\text{ave}} = \left(\sum_{p=1}^P \Psi_{i,p} \right) / P \quad (2.20)$$

$$\text{Minimum power: } \Psi_i^{\text{min}} = \min_p (\Psi_{i,p}) \quad (2.21)$$

To achieve this, the transmitted signal should have the following power-distribution characteristics [17]:

- Low, ideally 1, peak-to-average (peak/ave) power ratio
- Low, ideally 1, average-to-minimum (ave/min) power ratio
- Low, ideally 0, probability P_o that an antenna transmits zero symbols

In general, space-time codes that have the above attributes are called “*power-balanced*” codes.

CHAPTER 3 ORTHOGONAL STBC BASED ON AMICABLE COMPLEX ORTHOGONAL DESIGN

3.1 INTRODUCTION

Orthogonal Space-Time Block Code (O-STBC) for two transmit antennas was first proposed in [4] and subsequently generalized in [7]. It can provide full transmit diversity with linear decoding complexity. Due to these advantages, O-STBC has drawn a lot of research attention, e.g. [15,18,25] focus on the construction of square O-STBC with minimal delay; [26,27,28] focus on the construction of rectangular O-STBC with higher code rate than square O-STBC; while [29,30] investigate the maximal achievable code rate of an O-STBC.

3.1.1 Review of Construction of O-STBC From AOD

An O-STBC can achieve full transmit diversity with linear decoding complexity [7], maximum SNR [15], and minimum PEP [31]. To design such a code, it has been proven in [7,15,31] that the dispersion matrices $\mathbf{A}_i$ and $\mathbf{B}_i$, for $1 \leq i \leq K$, of an O-STBC must satisfy the following constraints:

$$\begin{aligned}
 & \text{(i)} \quad \begin{aligned} \mathbf{A}_i^H \mathbf{A}_i &= \mathbf{I}_{N_T} \\ \mathbf{B}_i^H \mathbf{B}_i &= \mathbf{I}_{N_T} \end{aligned} & 1 \leq i \leq K \\
 & \text{(ii)} \quad \begin{aligned} \mathbf{A}_i^H \mathbf{A}_k &= -\mathbf{A}_k^H \mathbf{A}_i \\ \mathbf{B}_i^H \mathbf{B}_k &= -\mathbf{B}_k^H \mathbf{B}_i \end{aligned} & 1 \leq i \neq k \leq K \\
 & \text{(iii)} \quad \mathbf{A}_i^H \mathbf{B}_k = \mathbf{B}_k^H \mathbf{A}_i & 1 \leq i, k \leq K
 \end{aligned} \tag{3.1}$$

In addition, it is also desirable that the O-STBC has a high code rate and minimum code length, i.e. K as large as possible and P as small as possible. In [15], Amicable Orthogonal Design (AOD) is used to solve the O-STBC design constraints in (3.1) with minimum delay, i.e. a square design with $P = K$. [15] also shows that the maximum achievable code rate of a square O-STBC is $3/4$ for four transmit antennas and $1/2$ for eight transmit antennas.

3.1.2 Motivations

However, Ganesan and Stoica posted an open question regarding O-STBC based on AOD in [16]: whether the complex version or rectangular version of AOD exists and what is the maximum code rate of O-STBC achievable from such designs.

Furthermore, we note that some O-STBCs [15,25] require some of the antennas to transmit “zero” (i.e. antennas to be turned off) at regular intervals while other O-STBCs [7] have irrational numbers in the code coefficients. Regular transmission of “zeros” leads to undesirable low-frequency interference and high peak-to-average power ratio, which leads to challenges in front-end power amplifier design [17,18,32]. Irrational numbers in the code coefficients require floating-point multiplications in both the transmitter and receiver of the O-STBC system, which is less convenient to implement than having just ± 1 in the code-coefficients that requires simply additions/subtractions [33]. Hence, O-STBCs that do not suffer from the above-mentioned shortcomings are desirable.

In this chapter, we shall show that the complex version of AOD, i.e. *Amicable Complex Orthogonal Design* (ACOD), exists, and use it to construct new power-balanced square O-STBCs with minimum delay.

3.2 AMICABLE COMPLEX ORTHOGONAL DESIGN

A series of combinatorial matrices called *orthogonal design* (OD) [34], *amicable orthogonal design* (AOD) [35] and *complex orthogonal design* (COD) [36] have been reported in the literature. However, the existence of a complex version of AOD, herein called the *amicable complex orthogonal design* (ACOD), remains open according to [16]. In this section, we shall address this issue. We begin by defining ACOD and proving its existence. We will then derive its maximum achievable number of variables and propose a method to construct ACOD that achieves the maximum number of variables. These designs will then be used to construct new power-balanced O-STBC.

3.2.1 Review of Orthogonal Designs

We shall first review OD, AOD, COD and their related parameters. In this thesis, all the matrices related to an OD, AOD, COD or ACOD will be underlined, while the matrices related to a STBC will not.

Definition 3.1 [34]: A *weighting matrix* $\underline{\mathbf{W}}$ of weight w and order n is an $n \times n$ matrix with entries from $\{0, +1, -1\}$ satisfying

$$\underline{\mathbf{W}}^T \underline{\mathbf{W}} = w \mathbf{I}_n \quad (3.2)$$

We shall denote such a matrix by $W(n, w)$.

Definition 3.2 [34]: An *Orthogonal Design* (OD) $\underline{\mathbf{A}}$ of order n and of type $(u_1, \dots, u_s)$, u_i positive integers, is an $n \times n$ matrix, with entries from $\{0, \pm \underline{a}_1, \dots, \pm \underline{a}_s\}$ satisfying

$$\underline{\mathcal{A}}^T \underline{\mathcal{A}} = \left(\sum_{i=1}^s u_i \underline{a}_i^2 \right) \mathbf{I}_n. \quad (3.3)$$

$\underline{\mathcal{A}}$ can be expressed as

$$\underline{\mathcal{A}} = \underline{\mathbf{A}}_1 \underline{a}_1 + \underline{\mathbf{A}}_2 \underline{a}_2 + \cdots + \underline{\mathbf{A}}_s \underline{a}_s \quad (3.4)$$

where each $\underline{\mathbf{A}}_i$ is $W(n, u_i)$ satisfying:

$$\begin{aligned} (0) \quad & \underline{\mathbf{A}}_i * \underline{\mathbf{A}}_k = \mathbf{0} & 1 \leq i \neq k \leq s \\ (i) \quad & \underline{\mathbf{A}}_i^T \underline{\mathbf{A}}_i = u_i \mathbf{I}_n & 1 \leq i \leq s \\ (ii) \quad & \underline{\mathbf{A}}_i^T \underline{\mathbf{A}}_k + \underline{\mathbf{A}}_k^T \underline{\mathbf{A}}_i = \mathbf{0} & 1 \leq i \neq k \leq s \end{aligned} \quad (3.5)$$

and $*$ represents the Hadamard product.

The statement (3.5)(0) is called the “*disjointness statement*”. Member variables of $\underline{\mathbf{A}}_i$ satisfying this statement are said to be *pairwise disjoint*.

Definition 3.3 [37, Definition 1.1]: A *family* is a collection of matrices $\{\underline{\mathbf{A}}_1, \underline{\mathbf{A}}_2, \dots, \underline{\mathbf{A}}_s\}$ satisfying (3.5)(i) and (3.5)(ii) but not the “disjointness statement” (3.5)(0). This is usually called a *Hurwitz-Radon family* (H-R family).

Lemma 3.1 [37, Corollary 1.4]: The number of variables, s , in an orthogonal design of order $n = 2^a b$, b odd and $a = 4c + d$ where $0 \leq d < 4$, is upper bounded as follows:

$$s \leq \rho(n) \quad (3.6)$$

where

$$\rho(n) = 8c + 2^d \quad (3.7)$$

In the special cases of $n = 1, 2, 4$ and 8 , $\rho(n) = n$.

Definition 3.4 [35]: Let $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ be OD of the same order n where $\underline{\mathbf{A}}$ is of type $(u_1, \dots, u_s)$ on the variables $\{\underline{a}_1, \dots, \underline{a}_s\}$ and $\underline{\mathbf{B}}$ is of type $(v_1, \dots, v_t)$ on the variables $\{\underline{b}_1, \dots, \underline{b}_t\}$. Similar to $\underline{\mathbf{A}}$ in (3.4), $\underline{\mathbf{B}}$ can be expressed as

$$\underline{\mathbf{B}} = \underline{\mathbf{B}}_1 \underline{b}_1 + \underline{\mathbf{B}}_2 \underline{b}_2 + \dots + \underline{\mathbf{B}}_t \underline{b}_t \quad (3.8)$$

$\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ are said to be *Amicable Orthogonal Design* (AOD) if

$$\underline{\mathbf{A}}^T \underline{\mathbf{B}} = \underline{\mathbf{B}}^T \underline{\mathbf{A}} \quad (3.9)$$

A necessary and sufficient condition for the AOD as defined in *Definition 3.4* to exist is that there must exist a family of matrices $\{\underline{\mathbf{A}}_1, \dots, \underline{\mathbf{A}}_s; \underline{\mathbf{B}}_1, \dots, \underline{\mathbf{B}}_t\}$ of order n with entries from $\{0, 1, -1\}$ satisfying:

$$\begin{aligned} (0) \quad & \underline{\mathbf{A}}_i * \underline{\mathbf{A}}_p = \mathbf{0} & 1 \leq i \neq p \leq s \\ & \underline{\mathbf{B}}_k * \underline{\mathbf{B}}_q = \mathbf{0} & 1 \leq k \neq q \leq t \\ (i) \quad & \underline{\mathbf{A}}_i^T \underline{\mathbf{A}}_i = u_i \mathbf{I}_n & 1 \leq i \leq s \\ & \underline{\mathbf{B}}_k^T \underline{\mathbf{B}}_k = v_k \mathbf{I}_n & 1 \leq k \leq t \\ (ii) \quad & \underline{\mathbf{A}}_i^T \underline{\mathbf{A}}_p + \underline{\mathbf{A}}_p^T \underline{\mathbf{A}}_i = \mathbf{0} & 1 \leq i \neq p \leq s \\ & \underline{\mathbf{B}}_k^T \underline{\mathbf{B}}_q + \underline{\mathbf{B}}_q^T \underline{\mathbf{B}}_k = \mathbf{0} & 1 \leq k \neq q \leq t \\ (iii) \quad & \underline{\mathbf{A}}_i^T \underline{\mathbf{B}}_k - \underline{\mathbf{B}}_k^T \underline{\mathbf{A}}_i = \mathbf{0} & 1 \leq i \leq s, 1 \leq k \leq t \end{aligned} \quad (3.10)$$

Hence (3.10)(0)-(iii) act as the criteria to construct AOD matrices.

Definition 3.5 [37, Definition 5.7]: An *amicable family* (AF) of type $(u_1, \dots, u_s; v_1, \dots, v_t)$ of order n is a collection of matrices $\{\underline{\mathbf{A}}_1, \dots, \underline{\mathbf{A}}_s; \underline{\mathbf{B}}_1, \dots, \underline{\mathbf{B}}_t\}$ satisfying (3.10)(i), (ii), (iii), but not the “disjointness statement” (3.10)(0).

By comparing (3.10)(i)-(iii) with the O-STBC constraints in (3.1), it is clear that an AOD or AF can be used to construct an O-STBC [15]. Furthermore, from *Definition 3.4*, it can be noted that the number of variables in the OD $\underline{\mathbf{A}}$ of an AOD is denoted by s , while the number of variables in the OD $\underline{\mathbf{B}}$ of an AOD is denoted by t . The upper bounds of s and t have been derived in [37] and are summarized in *Lemma 3.2*.

Lemma 3.2 [37, Theorem 5.23]: For an AOD of order n , where $n=2^a b$, a and b are both integers and b is odd and $a = 4c + d$ where $0 \leq d < 4$,

$$\begin{aligned} t &\leq \rho(n) \\ s &\leq \rho_t(n) \end{aligned} \tag{3.11}$$

where $\rho(n)$ is as defined in *Lemma 3.1*, and $\rho_t(n)$ is defined as follows:

$$\rho_t(n) = 8c - t + \delta + 1 \tag{3.12}$$

where the values δ are given below in Table 3.1 [37, Theorem 5.23].

Table 3.1 Values of δ for an AOD

$d \backslash t(\text{mod}4)$	0	1	2	3
0	0	1	3	7
1	1	2	3	5
2	-1	3	4	5
3	-1	1	5	6

For $n = 4$ and 8, the corresponding values of s and t are listed in Table 3.2 and Table 3.3 respectively. For example, for $n = 4$, we know from (3.11) and (3.7) that:

- t must be less than or equal to $\rho(4) = 4$, so t may take values from 0 to 4 as shown in Table 3.2.
- s must be less than or equal to $\rho_t(4)$, so for $t = 3$ in Table 3.2, $s \leq \rho_3(4) = 3$ according to (3.12).

From Table 3.2 and Table 3.3, we note that $\rho_t(n)$ is a decreasing function of t .

Table 3.2 s and t values for $n = 4$

t (number of variables in $\underline{\mathcal{B}}$)	s (number of variables in $\underline{\mathcal{A}}$)	Max($s+t$) (max. number of variables in AOD)
4	$\leq \rho_4(4) = 0$	4
3	$\leq \rho_3(4) = 3$	6
2	$\leq \rho_2(4) = 3$	5
1	$\leq \rho_1(4) = 3$	4
0	$\leq \rho_0(4) = 4$	4

Table 3.3 s and t values for $n = 8$

t (number of variables in $\underline{\mathcal{B}}$)	s (number of variables in $\underline{\mathcal{A}}$)	Max($s+t$) (max. number of variables in AOD)
8	$\leq \rho_8(8) = 0$	8
7	$\leq \rho_7(8) = 0$	7
6	$\leq \rho_6(8) = 0$	6
5	$\leq \rho_5(8) = 1$	6
4	$\leq \rho_4(8) = 4$	8
3	$\leq \rho_3(8) = 4$	7
2	$\leq \rho_2(8) = 4$	6
1	$\leq \rho_1(8) = 5$	6
0	$\leq \rho_0(8) = 8$	8

Lemma 3.3 [37, Corollary 5.32]: For an AOD of order n , where $n=2^a b$, a and b are both integers and b is odd, the total number of variables in an AOD (i.e. $s + t$) is upper bounded by $2a + 2$, and that bound is achieved.

Definition 3.6: An AOD is called a “full design” if there is no zero entry in $\underline{\mathcal{A}}$ and $\underline{\mathcal{B}}$.

All the above discussions assume real numbers as the matrix entries. The following discussions will consider the inclusion of complex numbers as the matrix entries.

Definition 3.7 [36, Definition 1]: A complex weighting matrix $\underline{\mathbf{W}}$ of weight w and order n is an $n \times n$ matrix with entries from $\{0, \pm 1, \pm j\}$ satisfying

$$\underline{\mathbf{W}}^H \underline{\mathbf{W}} = w \mathbf{I}_n \quad (3.13)$$

We shall denote such a matrix by $CW(n, w)$.

Definition 3.8 [36, Proposition 3]: A *Complex Orthogonal Design* (COD) of order n and type $(f_1, \dots, f_x)$ (f_i positive integers, for $1 \leq i \leq x$) on the real commuting variables $\{\underline{c}_1, \dots, \underline{c}_x\}$ is an $n \times n$ matrix $\underline{\mathbf{C}}$ with entries from $\{0, \varepsilon_1 \underline{c}_1, \dots, \varepsilon_x \underline{c}_x\}$, where ε_i is a fourth root of 1} satisfying

$$\underline{\mathbf{C}}^H \underline{\mathbf{C}} = \left(\sum_{i=1}^x f_i \underline{c}_i^2 \right) \mathbf{I}_n \quad (3.14)$$

$\underline{\mathbf{C}}$ can be expressed as

$$\underline{\mathbf{C}} = \underline{\mathbf{C}}_1 \underline{c}_1 + \underline{\mathbf{C}}_2 \underline{c}_2 + \dots + \underline{\mathbf{C}}_x \underline{c}_x \quad (3.15)$$

where the $\underline{\mathbf{C}}_i$ are $CW(n, f_i)$ satisfying:

$$\begin{aligned} (0) \quad & \underline{\mathbf{C}}_i * \underline{\mathbf{C}}_k = \mathbf{0} & 1 \leq i \neq k \leq x \\ (i) \quad & \underline{\mathbf{C}}_i^H \underline{\mathbf{C}}_i = f_i \mathbf{I}_n & 1 \leq i \leq x \\ (ii) \quad & \underline{\mathbf{C}}_i^H \underline{\mathbf{C}}_k + \underline{\mathbf{C}}_k^H \underline{\mathbf{C}}_i = \mathbf{0} & 1 \leq i \neq k \leq x \end{aligned} \quad (3.16)$$

Definition 3.9: A *complex family* (CF) is a collection of matrices $\{\underline{\mathbf{C}}_1, \dots, \underline{\mathbf{C}}_x\}$ satisfying (3.16)(i) and (3.16)(ii), but not the “disjointness statement” (3.16)(0). This definition is analogues to the definitions of *family* and *amicable family* in Definition 3.3 and Definition 3.5 respectively.

Lemma 3.4 [36, Theorem 4]: Let $\tau(n)$ denote the maximum number of variables in a COD of order n , where $n=2^a b$, b odd. The value of $\tau(n)$ is upper bounded by $\tau(n) \leq H(n)$, and

$$H(n) = 2a + 2 \quad (3.17)$$

3.2.2 Existence of ACOD

Next we define an ACOD, using a similar approach as presented in [35] for defining an AOD from an OD.

Definition 3.10: Let the matrices $\underline{\mathbf{C}} = \underline{\mathbf{C}}_1 \underline{\mathbf{c}}_1 + \dots + \underline{\mathbf{C}}_x \underline{\mathbf{c}}_x$ and $\underline{\mathbf{D}} = \underline{\mathbf{D}}_1 d_1 + \dots + \underline{\mathbf{D}}_y d_y$ be two CODs of the same order n , where $\underline{\mathbf{C}}$ is of type $(f_1, \dots, f_x)$ on the variables $\{\underline{\mathbf{c}}_1, \dots, \underline{\mathbf{c}}_x\}$ and $\underline{\mathbf{D}}$ is of type $(g_1, \dots, g_y)$ on the variables $\{\underline{\mathbf{d}}_1, \dots, \underline{\mathbf{d}}_y\}$. $\underline{\mathbf{C}}$ and $\underline{\mathbf{D}}$ are said to be an *Amicable Complex Orthogonal Design* (ACOD) if

$$\underline{\mathbf{C}}^H \underline{\mathbf{D}} = \underline{\mathbf{D}}^H \underline{\mathbf{C}} \quad (3.18)$$

A necessary and sufficient condition for an ACOD of type $(f_1, \dots, f_x; g_1, \dots, g_y)$ as defined in *Definition 3.10* to exist is that there exists a family of matrices $\{\underline{\mathbf{C}}_1, \dots, \underline{\mathbf{C}}_x; \underline{\mathbf{D}}_1, \dots, \underline{\mathbf{D}}_y\}$ satisfying:

$$\begin{aligned}
 (0) \quad & \underline{\mathbf{C}}_i * \underline{\mathbf{C}}_p = \mathbf{0} & 1 \leq i \neq p \leq x \\
 & \underline{\mathbf{D}}_k * \underline{\mathbf{D}}_q = \mathbf{0} & 1 \leq k \neq q \leq y \\
 (i) \quad & \underline{\mathbf{C}}_i^H \underline{\mathbf{C}}_i = f_i \mathbf{I}_n & 1 \leq i \leq x \\
 & \underline{\mathbf{D}}_k^H \underline{\mathbf{D}}_k = g_k \mathbf{I}_n & 1 \leq k \leq y \\
 (ii) \quad & \underline{\mathbf{C}}_i^H \underline{\mathbf{C}}_p + \underline{\mathbf{C}}_p^H \underline{\mathbf{C}}_i = \mathbf{0} & 1 \leq i \neq p \leq x \\
 & \underline{\mathbf{D}}_k^H \underline{\mathbf{D}}_q + \underline{\mathbf{D}}_q^H \underline{\mathbf{D}}_k = \mathbf{0} & 1 \leq k \neq q \leq y \\
 (iii) \quad & \underline{\mathbf{C}}_i^H \underline{\mathbf{D}}_k - \underline{\mathbf{D}}_k^H \underline{\mathbf{C}}_i = \mathbf{0} & 1 \leq i \leq x, 1 \leq k \leq y
 \end{aligned} \tag{3.19}$$

where $\underline{\mathbf{C}}_i$ and $\underline{\mathbf{D}}_k$ are $CW(n, f_i)$ and $CW(n, g_k)$ respectively.

Definition 3.11: An *amicable complex family* (ACF) of type $(f_1, \dots, f_x; g_1, \dots, g_y)$ is a collection of matrices $\{\underline{\mathbf{C}}_1, \dots, \underline{\mathbf{C}}_x; \underline{\mathbf{D}}_1, \dots, \underline{\mathbf{D}}_y\}$ satisfying (3.19)(i-iii), but not the “disjointness statement” (3.19)(0). This definition is analogous to the definition of *amicable family* associated with AOD in *Definition 3.5*.

Comparing (3.19)(i)-(iii) with (3.1), it is clear that ACOD / ACF can be used to construct O-STBC, this is similar to the construction of O-STBC from AOD but allowing complex numbers as entries in the dispersion matrices. Next, we shall derive an upper bound on the number of variables that can exist in an ACOD, which is related to the maximum achievable code rate of an O-STBC constructed from ACOD.

Proposition 3.1: Assume that the ACODs $\underline{\mathbf{C}} = \underline{\mathbf{C}}_1 \underline{\mathbf{c}}_1 + \dots + \underline{\mathbf{C}}_x \underline{\mathbf{c}}_x$ and $\underline{\mathbf{D}} = \underline{\mathbf{D}}_1 d_1 + \dots + \underline{\mathbf{D}}_y d_y$ as defined in *Definition 3.10* exist. By letting:

$$\begin{aligned}\underline{\mathbf{E}}_i &= \underline{\mathbf{C}}_i & 1 \leq i \leq x \\ \underline{\mathbf{E}}_{x+k} &= j\underline{\mathbf{D}}_k & 1 \leq k \leq y\end{aligned}\tag{3.20}$$

a CF with $x + y$ variables of type $(f_1, \dots, f_x, g_1, \dots, g_y)$ can be formed by $\{\underline{\mathbf{E}}_1, \underline{\mathbf{E}}_2, \dots, \underline{\mathbf{E}}_{x+y}\}$.

Proof of Proposition 3.1: It can be shown that $\underline{\mathbf{E}}_z$ as defined in (3.20), for $1 \leq z \leq x + y$, satisfies all the constraints of a CF in (3.16)(i) and (3.16)(ii). For example, part of (3.16)(ii) can be verified as follows:

$$\begin{aligned}\underline{\mathbf{E}}_i^H \underline{\mathbf{E}}_p + \underline{\mathbf{E}}_p^H \underline{\mathbf{E}}_i & \quad 1 \leq i \leq x, x < p \leq x + y \\ &= \underline{\mathbf{C}}_i^H (j\underline{\mathbf{D}}_k) + (j\underline{\mathbf{D}}_k)^H \underline{\mathbf{C}}_i \quad 1 \leq k \leq y \\ &= j\underline{\mathbf{C}}_i^H \underline{\mathbf{D}}_k - j\underline{\mathbf{D}}_k^H \underline{\mathbf{C}}_i \\ &= \mathbf{0}\end{aligned}\tag{3.21}$$

Following (3.21), all the constraints of a CF in (3.16)(i) and (3.16)(ii) can be similarly verified for $\underline{\mathbf{E}}_z$ which follow all the ACODs constraints in (3.19). ■

Theorem 3.1: The total number of variables in an ACOD (i.e. $x + y$) of order n is bounded by $H(n)$ in (3.17), where $H(n) = 2a + 2$ for $n = 2^a b$, b odd.

Proof of Theorem 3.1: From *Proposition 3.1*, it is clear that whenever an ACOD with $x + y$ variables exists, a CF with $z = x + y$ variables will exist. Since *Lemma 3.4* states that $z \leq H(n)$, $x + y \leq H(n)$ follows. As a result, *Theorem 3.1* is proved. ■

Theorem 3.2: The maximum number of variables in an ACOD is the same as AOD, and the ACOD that achieves the maximum number of variables exists.

Proof of Theorem 3.2: We note from *Theorem 3.1* and *Lemma 3.3* that both ACOD and AOD have the same bound on maximum number of variables, even though an

ACOD permits complex numbers as its matrix entries while an AOD only permits real numbers. Since ACOD includes AOD as a special case, and an AOD that achieves its maximum of variables exists (*Lemma 3.3*), we can conclude that an ACOD that achieves the maximum number of variables also exists. ■

With the proof of *Theorem 3.2*, we have solved the question raised in [16] on whether the complex version of AOD exists. This result has been accepted for publication online at *SIAM Problems and Solutions*. Before proceeding, we would like to point out an interesting observation that although the maximum number of variables of a COD as stated in *Lemma 3.4* is greater than the maximum number of variables of an OD as stated in *Lemma 3.1*, the total number of variables of an ACOD is the same as the total number of variables of an AOD (*Theorem 3.2*), even though AOD is a subset of ACOD. Next, we will present two systematic ways to construct ACOD that achieves the maximum number of variables.

3.2.3 Construction of ACOD

We now propose two methods to construct higher-order ACOD from lower-order ACOD. In the first method, we construct an ACOD of order $4n$ from an ACOD of order n , while in the second method we construct an ACOD of order $2n$ from an ACOD of order n . We shall show that the higher-order ACOD constructed from both methods achieves the maximum number of variables if a lower-order ACOD that achieves the maximum number of variables is used to generate it. Furthermore, since ACOD is a generalized case of AOD, the code construction methods and associated discussions apply to AOD too.

Construction 3.1: If $\{ \underline{\mathbf{C}}_i, 1 \leq i \leq x ; \underline{\mathbf{D}}_k, 1 \leq k \leq y \}$ is an ACOD of order n with $x + y$ variables and type $(f_1, \dots, f_x ; g_1, \dots, g_y)$, then

$$\left\{ \begin{array}{l} \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_1, \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_2, \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_3, \underline{\mathbf{C}}_i \otimes \underline{\mathbf{I}}_4, 2 \leq i \leq x ; \\ \underline{\mathbf{C}}_1 \otimes \underline{\mathbf{N}}_1, \underline{\mathbf{C}}_1 \otimes \underline{\mathbf{N}}_2, \underline{\mathbf{C}}_1 \otimes \underline{\mathbf{N}}_3, \underline{\mathbf{D}}_k \otimes \underline{\mathbf{I}}_4, 2 \leq k \leq y \end{array} \right\}$$

is an ACOD of order $4n$ with $x + y + 4$ variables and type $(u_1g_1, u_2g_1, u_3g_1, f_2, \dots, f_x ; v_1f_1, v_2f_1, v_3f_1, g_2, \dots, g_y)$ where $\otimes$ denotes the Kronecker product, and the matrices $\underline{\mathbf{M}}_i$ and $\underline{\mathbf{N}}_i$, for $1 \leq i \leq 3$, satisfy the following conditions:

$$\begin{aligned} (0) \quad & \underline{\mathbf{M}}_i * \underline{\mathbf{M}}_k = \mathbf{0} & 1 \leq i \neq k \leq 3 \\ & \underline{\mathbf{N}}_i * \underline{\mathbf{N}}_k = \mathbf{0} \\ (i) \quad & \underline{\mathbf{M}}_i^H \underline{\mathbf{M}}_i = u_i \underline{\mathbf{I}}_4 & 1 \leq i \leq 3 \\ & \underline{\mathbf{N}}_i^H \underline{\mathbf{N}}_i = v_i \underline{\mathbf{I}}_4 \\ (ii) \quad & \underline{\mathbf{M}}_i^H \underline{\mathbf{M}}_k + \underline{\mathbf{M}}_k^H \underline{\mathbf{M}}_i = \mathbf{0} & 1 \leq i \neq k \leq 3 \\ & \underline{\mathbf{N}}_i^H \underline{\mathbf{N}}_k + \underline{\mathbf{N}}_k^H \underline{\mathbf{N}}_i = \mathbf{0} & (3.22) \\ (iii) \quad & \underline{\mathbf{M}}_i^H \underline{\mathbf{N}}_k - \underline{\mathbf{N}}_k^H \underline{\mathbf{M}}_i = \mathbf{0} & 1 \leq i, k \leq 3 \\ (iv) \quad & \underline{\mathbf{M}}_i * \underline{\mathbf{I}}_4 = \mathbf{0} & 1 \leq i \leq 3 \\ & \underline{\mathbf{N}}_i * \underline{\mathbf{I}}_4 = \mathbf{0} \\ (v) \quad & \underline{\mathbf{M}}_i^H + \underline{\mathbf{M}}_i = \mathbf{0} & 1 \leq i \leq 3 \\ & \underline{\mathbf{N}}_i^H + \underline{\mathbf{N}}_i = \mathbf{0} \end{aligned}$$

Proof of *Construction 3.1*: The proofs are given in the APPENDIX A . ■

An example of ACOD $\{\underline{\mathbf{M}}, \underline{\mathbf{N}}\}$ that satisfy the conditions in (3.22) are as follows:

$$\begin{aligned} \underline{\mathbf{M}}_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \underline{\mathbf{M}}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \underline{\mathbf{M}}_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}; \\ \underline{\mathbf{N}}_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \underline{\mathbf{N}}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \underline{\mathbf{N}}_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (3.23)$$

Based on *Construction 3.1*, we have the following remarks:

- An interesting observation is that since the conditions (3.22)(0-iii) are equivalent to the conditions (3.19)(0-iii), it implies that $\{\underline{\mathbf{M}}_1, \underline{\mathbf{M}}_2, \underline{\mathbf{M}}_3; \underline{\mathbf{N}}_1, \underline{\mathbf{N}}_2, \underline{\mathbf{N}}_3\}$ must themselves be an ACOD of order 4 with six variables and type $(u_1, u_2, u_3; v_1, v_2, v_3)$.
- If the target design is an ACF, then conditions (3.22)(0) and (3.22)(iv) in *Construction 3.1* can be neglected because they are related to the “disjointness statement”, as shown in the APPENDIX A. On the other hand, if the target design is an ACOD, then u_i and v_i must both be 1 for $1 \leq i \leq 3$. This is because in an ACOD of order 4, if u_i or v_i is greater than 1, the “disjointness statement” (3.22)(0) and (3.22)(iv) will be violated.
- If the lower-order design is an ACOD, the resultant higher-order design will also be an ACOD. However if the lower-order design is an ACF, the resultant higher-order design may be an ACOD or an ACF. For illustration, a ACOD of order 8 that is constructed from an ACF of order 2 is shown in *Example 3.3* later in this thesis.
- If both $\{\underline{\mathbf{C}}, \underline{\mathbf{D}}\}$ and $\{\underline{\mathbf{M}}, \underline{\mathbf{N}}\}$ only contain real numbers, i.e. both are AOD or AF, the resultant higher-order design would also contain real numbers only, i.e. an AOD or AF.

Theorem 3.3: The ACOD of order $4n$ constructed from *Construction 3.1* can achieve the maximum number of variables if the ACOD matrices $\{\underline{\mathbf{C}}, \underline{\mathbf{D}}\}$ of order n used for this construction achieve the maximum number of variables.

Proof of Theorem 3.3: From *Theorem 3.1*, it is known that the maximum number of variables for an ACOD of order n is bounded by $2a + 2$, where $n = 2^a b = 2^a$ (b is set to be one as it can be any odd number). This implies that the maximum number of variables for an ACOD of order $4n$ is bounded by $2(a+2)+2 = 2a+6$, as $4n = 4 \times 2^a = 2^{(a+2)}$. Therefore the difference between the maximum number of variables of an ACOD of order n and an ACOD of order $4n$ is at most 4. From *Construction 3.1*, the number of variables $(x + y + 4)$ of an ACOD of order $4n$ is also 4 more than the number of variables $(x + y)$ of an ACOD of order n that is used to generate it. Hence if an ACOD of order n that achieves the maximum number of variables is used, an ACOD of order $4n$ that achieves the maximum number of variables can be constructed. Hence *Theorem 3.3* is proved. ■

We will now construct some ACODs based on the proposed construction method, and they will be used later to construct new O-STBC.

Example 3.1: An ACOD of order 4 and type $(1, 1, 1; 1, 1, 1)$.

Consider an ACOD of order 1, i.e. $\underline{\mathbf{C}}_1 = [1]$; $\underline{\mathbf{D}}_1 = [1]$, and type $f_1 = 1$; $g_1 = 1$, or in short, ACOD $\{[1] ; [1]\}$ of order 1 and type $(1 ; 1)$. The following ACOD of order 4 and type $(1, 1, 1; 1, 1, 1)$, can be constructed by following *Construction 3.1* using $\{\underline{\mathbf{M}}, \underline{\mathbf{N}}\}$ in (3.23):

$$\begin{aligned}\underline{\mathcal{C}} &= \begin{bmatrix} 0 & \underline{c}_1 & \underline{c}_2 & \underline{c}_3 \\ -\underline{c}_1 & 0 & -\underline{c}_3 & \underline{c}_2 \\ -\underline{c}_2 & \underline{c}_3 & 0 & -\underline{c}_1 \\ -\underline{c}_3 & -\underline{c}_2 & \underline{c}_1 & 0 \end{bmatrix} \\ \underline{\mathcal{D}} &= \begin{bmatrix} 0 & \underline{d}_1 & \underline{d}_2 & \underline{d}_3 \\ -\underline{d}_1 & 0 & \underline{d}_3 & -\underline{d}_2 \\ -\underline{d}_2 & -\underline{d}_3 & 0 & \underline{d}_1 \\ -\underline{d}_3 & \underline{d}_2 & -\underline{d}_1 & 0 \end{bmatrix}\end{aligned}\quad (3.24)$$

Example 3.2: An ACOD of order 8 and type (1, 1, 1, 1; 1, 1, 1, 1).

Consider an ACOD $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -j & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ j & 0 \end{bmatrix} \right\}$ of order 2 and type (1, 1; 1, 1), which can be obtained by hand design due to its small size. The following ACOD of order 8 and type (1, 1, 1, 1; 1, 1, 1, 1), can be constructed by following *Construction 3.1* using $\{\underline{\mathbf{M}}, \underline{\mathbf{N}}\}$ in (3.23):

$$\begin{aligned}\underline{\mathcal{C}} &= \begin{bmatrix} 0 & \underline{c}_4 & \underline{c}_1 & 0 & \underline{c}_2 & 0 & \underline{c}_3 & 0 \\ -j\underline{c}_4 & 0 & 0 & -j\underline{c}_1 & 0 & -j\underline{c}_2 & 0 & -j\underline{c}_3 \\ -\underline{c}_1 & 0 & 0 & \underline{c}_4 & \underline{c}_3 & 0 & -\underline{c}_2 & 0 \\ 0 & j\underline{c}_1 & -j\underline{c}_4 & 0 & 0 & -j\underline{c}_3 & 0 & j\underline{c}_2 \\ -\underline{c}_2 & 0 & -\underline{c}_3 & 0 & 0 & \underline{c}_4 & \underline{c}_1 & 0 \\ 0 & j\underline{c}_2 & 0 & j\underline{c}_3 & -j\underline{c}_4 & 0 & 0 & -j\underline{c}_1 \\ -\underline{c}_3 & 0 & \underline{c}_2 & 0 & -\underline{c}_1 & 0 & 0 & \underline{c}_4 \\ 0 & j\underline{c}_3 & 0 & -j\underline{c}_2 & 0 & j\underline{c}_1 & -j\underline{c}_4 & 0 \end{bmatrix} \\ \underline{\mathcal{D}} &= \begin{bmatrix} 0 & \underline{d}_4 & \underline{d}_1 & 0 & \underline{d}_2 & 0 & \underline{d}_3 & 0 \\ j\underline{d}_4 & 0 & 0 & j\underline{d}_1 & 0 & j\underline{d}_2 & 0 & j\underline{d}_3 \\ -\underline{d}_1 & 0 & 0 & \underline{d}_4 & -\underline{d}_3 & 0 & \underline{d}_2 & 0 \\ 0 & -j\underline{d}_1 & j\underline{d}_4 & 0 & 0 & -j\underline{d}_3 & 0 & j\underline{d}_2 \\ -\underline{d}_2 & 0 & \underline{d}_3 & 0 & 0 & \underline{d}_4 & -\underline{d}_1 & 0 \\ 0 & -j\underline{d}_2 & 0 & j\underline{d}_3 & j\underline{d}_4 & 0 & 0 & -j\underline{d}_1 \\ -\underline{d}_3 & 0 & -\underline{d}_2 & 0 & \underline{d}_1 & 0 & 0 & \underline{d}_4 \\ 0 & -j\underline{d}_3 & 0 & -j\underline{d}_2 & 0 & j\underline{d}_1 & j\underline{d}_4 & 0 \end{bmatrix}\end{aligned}\quad (3.25)$$

Example 3.3: An ACOD of order 8 and type (2, 2, 2, 2; 2, 2, 2, 2).

Consider an AF $\left\{ \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \right\}$ of order 2 and type (2, 2; 2,

2), which is again obtained by hand design. The following ACOD of order 8 and type (2, 2, 2, 2; 2, 2, 2, 2), can be constructed by following *Construction 3.1* using $\{\underline{\mathbf{M}}, \underline{\mathbf{N}}\}$ in (3.23):

$$\underline{\mathbf{C}} = \begin{bmatrix} \underline{c}_4 & \underline{c}_4 & \underline{c}_1 & -\underline{c}_1 & \underline{c}_2 & -\underline{c}_2 & \underline{c}_3 & -\underline{c}_3 \\ \underline{c}_4 & -\underline{c}_4 & \underline{c}_1 & \underline{c}_1 & \underline{c}_2 & \underline{c}_2 & \underline{c}_3 & \underline{c}_3 \\ -\underline{c}_1 & \underline{c}_1 & \underline{c}_4 & \underline{c}_4 & \underline{c}_3 & -\underline{c}_3 & -\underline{c}_2 & \underline{c}_2 \\ -\underline{c}_1 & -\underline{c}_1 & \underline{c}_4 & -\underline{c}_4 & \underline{c}_3 & \underline{c}_3 & -\underline{c}_2 & -\underline{c}_2 \\ -\underline{c}_2 & \underline{c}_2 & -\underline{c}_3 & \underline{c}_3 & \underline{c}_4 & \underline{c}_4 & \underline{c}_1 & -\underline{c}_1 \\ -\underline{c}_2 & -\underline{c}_2 & -\underline{c}_3 & -\underline{c}_3 & \underline{c}_4 & -\underline{c}_4 & \underline{c}_1 & \underline{c}_1 \\ -\underline{c}_3 & \underline{c}_3 & \underline{c}_2 & -\underline{c}_2 & -\underline{c}_1 & \underline{c}_1 & \underline{c}_4 & \underline{c}_4 \\ -\underline{c}_3 & -\underline{c}_3 & \underline{c}_2 & \underline{c}_2 & -\underline{c}_1 & -\underline{c}_1 & \underline{c}_4 & -\underline{c}_4 \end{bmatrix}$$

$$\underline{\mathbf{D}} = \begin{bmatrix} \underline{d}_4 & \underline{d}_4 & -\underline{d}_1 & \underline{d}_1 & -\underline{d}_2 & \underline{d}_2 & -\underline{d}_3 & \underline{d}_3 \\ -\underline{d}_4 & \underline{d}_4 & \underline{d}_1 & \underline{d}_1 & \underline{d}_2 & \underline{d}_2 & \underline{d}_3 & \underline{d}_3 \\ \underline{d}_1 & -\underline{d}_1 & \underline{d}_4 & \underline{d}_4 & \underline{d}_3 & -\underline{d}_3 & -\underline{d}_2 & \underline{d}_2 \\ -\underline{d}_1 & -\underline{d}_1 & -\underline{d}_4 & \underline{d}_4 & -\underline{d}_3 & -\underline{d}_3 & \underline{d}_2 & \underline{d}_2 \\ \underline{d}_2 & -\underline{d}_2 & -\underline{d}_3 & \underline{d}_3 & \underline{d}_4 & \underline{d}_4 & \underline{d}_1 & -\underline{d}_1 \\ -\underline{d}_2 & -\underline{d}_2 & \underline{d}_3 & \underline{d}_3 & -\underline{d}_4 & \underline{d}_4 & -\underline{d}_1 & -\underline{d}_1 \\ \underline{d}_3 & -\underline{d}_3 & \underline{d}_2 & -\underline{d}_2 & -\underline{d}_1 & \underline{d}_1 & \underline{d}_4 & \underline{d}_4 \\ -\underline{d}_3 & -\underline{d}_3 & -\underline{d}_2 & -\underline{d}_2 & \underline{d}_1 & \underline{d}_1 & -\underline{d}_4 & \underline{d}_4 \end{bmatrix} \quad (3.26)$$

To our knowledge, (3.26) is the first ACOD / AOD of order 8 that is of “full design”, i.e. it contains no zero in its entries.

Example 3.4: A few ACODs/AODs of order 16 with different “type” parameters.

Consider the AOD of order 4 and type (1, 1, 2; 1, 1, 2) in [37, pp. 211]. ACODs / AODs of order 16 with ten variables and the following types can be constructed by

Construction 3.1 using $\{\underline{\mathbf{M}}, \underline{\mathbf{N}}\}$ in (3.23):

$$(1, 1, 2; 1, 1, 2) \rightarrow (1, 1, 1, 1, 2; 1, 1, 1, 1, 2)$$

$$(1, 1, 2; 2, 1, 1) \rightarrow (1, 1, 1, 1, 2; 2, 2, 2, 1, 1)$$

$$(2, 1, 1; 1, 1, 2) \rightarrow (2, 2, 2, 1, 1; 1, 1, 1, 1, 2)$$

$$(2, 1, 1; 2, 1, 1) \rightarrow (2, 2, 2, 1, 1; 2, 2, 2, 1, 1)$$

The corresponding ACODs / AODs are not shown here because there are too many of them, and the main purpose of this example to show how to generate ACODs / AODs of different types. From this example, we know that the “type” parameter can be treated as an explicit parameter for the construction of the higher order ACOD, since the sequences of $\{\underline{\mathbf{C}}, \underline{\mathbf{D}}\}$ matrices do not affect the algebraic structure of ACOD.

Construction 3.2: If $\{\underline{\mathbf{C}}_i, 1 \leq i \leq x; \underline{\mathbf{D}}_k, 1 \leq k \leq y\}$ is an ACOD of order n with $x + y$ variables of type $(f_1, \dots, f_x; g_1, \dots, g_y)$, then

$$\{\underline{\mathbf{D}}_1 \otimes \mathbf{N}_1, \underline{\mathbf{C}}_i \otimes \mathbf{I}_2, 1 \leq i \leq x; \underline{\mathbf{D}}_1 \otimes \mathbf{N}_2, \underline{\mathbf{D}}_1 \otimes \mathbf{N}_3, \underline{\mathbf{D}}_k \otimes \mathbf{I}_2, 2 \leq k \leq y\}$$

with

$$\mathbf{N}_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \mathbf{N}_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{N}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

is an ACOD of order $2n$ with $x + y + 2$ variables and type $(g_1, f_1, f_2, \dots, f_x; g_1, g_1, g_2, \dots, g_y)$.

Proof of Construction 3.2: The verifications are similar to the proof of *Construction 3.1*, hence they will not be repeated here. ■

Theorem 3.4: The ACOD of order $2n$ constructed by *Construction 3.2* can achieve the maximum number of variables if it is constructed using ACOD matrices $\{\underline{\mathbf{C}}; \underline{\mathbf{D}}\}$ of order n that achieve the maximum number of variables.

Proof of Theorem 3.4: From *Theorem 3.1*, it is known that the maximum number of variables for an ACOD of order n is bounded by $2a + 2$, where $n = 2^a$. This implies that the maximum number of variables for ACOD of order $2n$ is bounded by $2(a+1)+2 = 2a+4$, since $2n = 2 \times 2^a = 2^{(a+1)}$. Therefore the difference between the maximum number of variables of ACOD of order n and ACOD of order $2n$ is at most 2. From *Construction 3.2*, the number of variables of ACOD of order $2n$ is also 2 more than the number of variables of ACOD of order n used to generate it. Hence if ACOD of order n that achieves the maximum number of variables is used for the construction, ACOD of order $2n$ that achieves the maximum number of variables can be constructed. Hence *Theorem 3.4* is proved. ■

We will now construct some ACODs based on the proposed construction method, and they will be used later to construct new O-STBC.

Example 3.5: An ACOD of order 2 and type (1, 1; 1, 1).

Consider an ACOD $\{[1] ; [1]\}$ of order 1 and type (1 ; 1). The following ACOD of order 2 and type (1, 1; 1, 1), can be constructed from *Construction 3.2*:

$$\begin{aligned}\underline{\mathcal{C}} &= \begin{bmatrix} \underline{c}_2 & \underline{c}_1 \\ -\underline{c}_1 & \underline{c}_2 \end{bmatrix} \\ \underline{\mathcal{D}} &= \begin{bmatrix} \underline{d}_2 & \underline{d}_1 \\ \underline{d}_1 & -\underline{d}_2 \end{bmatrix}\end{aligned}\tag{3.27}$$

Example 3.6: An ACOD of order 4 and type (1, 1, 1; 1, 1, 1).

Consider an ACOD $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & j \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -j & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -j \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ j & 0 \end{bmatrix} \right\}$ of order 2 and type (1, 1; 1, 1). The following ACOD of order 4 and type (1, 1, 1; 1, 1, 1), can be constructed from *Construction 3.2*:

$$\begin{aligned}\underline{\mathcal{C}} &= \begin{bmatrix} \underline{c}_2 & \underline{c}_3 & \underline{c}_1 & 0 \\ -j\underline{c}_3 & j\underline{c}_2 & 0 & -j\underline{c}_1 \\ -\underline{c}_1 & 0 & \underline{c}_2 & \underline{c}_3 \\ 0 & j\underline{c}_1 & -j\underline{c}_3 & j\underline{c}_2 \end{bmatrix} \\ \underline{\mathcal{D}} &= \begin{bmatrix} \underline{d}_2 & \underline{d}_3 & \underline{d}_1 & 0 \\ j\underline{d}_3 & -j\underline{d}_2 & 0 & -j\underline{d}_1 \\ \underline{d}_1 & 0 & -\underline{d}_2 & \underline{d}_3 \\ 0 & -j\underline{d}_1 & j\underline{d}_3 & j\underline{d}_2 \end{bmatrix}\end{aligned}\tag{3.28}$$

Example 3.7: An ACOD of order 8 and type (1, 1, 1, 1; 1, 1, 1, 1).

Consider the ACOD of order 4 and type (1, 1, 1; 1, 1, 1) from *Example 3.6*. The following ACOD of order 8 and type (1, 1, 1, 1; 1, 1, 1, 1), can be constructed from *Construction 3.2*:

$$\begin{aligned}
 \underline{\mathcal{C}} &= \begin{bmatrix} \underline{c}_3 & \underline{c}_4 & \underline{c}_2 & 0 & 0 & 0 & \underline{c}_1 & 0 \\ -j\underline{c}_4 & j\underline{c}_3 & 0 & -j\underline{c}_2 & 0 & 0 & 0 & -j\underline{c}_1 \\ -\underline{c}_2 & 0 & \underline{c}_3 & \underline{c}_4 & \underline{c}_1 & 0 & 0 & 0 \\ 0 & j\underline{c}_2 & -j\underline{c}_4 & j\underline{c}_3 & 0 & -j\underline{c}_1 & 0 & 0 \\ 0 & 0 & -\underline{c}_1 & 0 & \underline{c}_3 & \underline{c}_4 & \underline{c}_2 & 0 \\ 0 & 0 & 0 & j\underline{c}_1 & -j\underline{c}_4 & j\underline{c}_3 & 0 & -j\underline{c}_2 \\ -\underline{c}_1 & 0 & 0 & 0 & -\underline{c}_2 & 0 & \underline{c}_3 & \underline{c}_4 \\ 0 & j\underline{c}_1 & 0 & 0 & 0 & j\underline{c}_2 & -j\underline{c}_4 & j\underline{c}_3 \end{bmatrix} \\
 \underline{\mathcal{D}} &= \begin{bmatrix} \underline{d}_3 & \underline{d}_4 & \underline{d}_2 & 0 & 0 & 0 & \underline{d}_1 & 0 \\ j\underline{d}_4 & -j\underline{d}_3 & 0 & -j\underline{d}_2 & 0 & 0 & 0 & -j\underline{d}_1 \\ \underline{d}_2 & 0 & -\underline{d}_3 & \underline{d}_4 & \underline{d}_1 & 0 & 0 & 0 \\ 0 & -j\underline{d}_2 & j\underline{d}_4 & j\underline{d}_3 & 0 & -j\underline{d}_1 & 0 & 0 \\ 0 & 0 & \underline{d}_1 & 0 & \underline{d}_3 & \underline{d}_4 & -\underline{d}_2 & 0 \\ 0 & 0 & 0 & -j\underline{d}_1 & j\underline{d}_4 & -j\underline{d}_3 & 0 & j\underline{d}_2 \\ \underline{d}_1 & 0 & 0 & 0 & -\underline{d}_2 & 0 & -\underline{d}_3 & \underline{d}_4 \\ 0 & -j\underline{d}_1 & 0 & 0 & 0 & j\underline{d}_2 & j\underline{d}_4 & j\underline{d}_3 \end{bmatrix} \quad (3.29)
 \end{aligned}$$

Example 3.8: An ACOD of order 4 and type (2, 2, 2; 2, 2, 2).

Consider an AF $\left\{ \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \right\}$ of order 2 and type (2, 2; 2,

2). The following AF of order 4 and type (2, 2, 2; 2, 2, 2), can be constructed from

Construction 3.2:

$$\begin{aligned}
 \underline{\mathcal{C}} &= \begin{bmatrix} -\underline{c}_2 + \underline{c}_3 & \underline{c}_2 + \underline{c}_3 & \underline{c}_1 & -\underline{c}_1 \\ \underline{c}_2 + \underline{c}_3 & \underline{c}_2 - \underline{c}_3 & \underline{c}_1 & \underline{c}_1 \\ -\underline{c}_1 & \underline{c}_1 & -\underline{c}_2 + \underline{c}_3 & \underline{c}_2 + \underline{c}_3 \\ -\underline{c}_1 & -\underline{c}_1 & \underline{c}_2 + \underline{c}_3 & \underline{c}_2 - \underline{c}_3 \end{bmatrix} \\
 \underline{\mathcal{D}} &= \begin{bmatrix} \underline{d}_2 + \underline{d}_3 & -\underline{d}_2 + \underline{d}_3 & \underline{d}_1 & -\underline{d}_1 \\ \underline{d}_2 - \underline{d}_3 & \underline{d}_2 + \underline{d}_3 & \underline{d}_1 & \underline{d}_1 \\ \underline{d}_1 & -\underline{d}_1 & -\underline{d}_2 + \underline{d}_3 & \underline{d}_2 + \underline{d}_3 \\ \underline{d}_1 & \underline{d}_1 & -\underline{d}_2 - \underline{d}_3 & -\underline{d}_2 + \underline{d}_3 \end{bmatrix} \quad (3.30)
 \end{aligned}$$

3.3 POWER-BALANCED O-STBC

We now construct new O-STBCs for four and eight transmit antennas based on the ACOD designs shown in the above examples, and highlight their practical advantages over existing O-STBC designs. We will also investigate the relationship between the “*type*” parameter of ACOD and the *power-distribution* and *code-coefficient* characteristics of O-STBC. The power distribution design criteria for a power-balanced code have been introduced in Section 2.4.2 of this thesis.

3.3.1 Construction of Power-Balanced O-STBC From ACOD

By comparing the constraints in (3.19) for the ACOD matrix with that in (3.1) for the O-STBC dispersion matrix, it can be seen that the $\underline{\mathbf{C}}_i$ and $\underline{\mathbf{D}}_i$ matrices defined in (3.19) can be used as the dispersion matrices of an O-STBC since they satisfy the design constraints in (3.1). The “*order*” parameter n of an ACOD corresponds to the number of transmit antennas N_T , while the code length P of the constructed O-STBC will be equal to N_T too, since ACOD is a square design. In addition, the “*type*” parameter of an ACOD, i.e. $(f_1, \dots, f_x; g_1, \dots, g_y)$, is related to the power and coefficient of info symbols of an O-STBC. This will be elaborated later in Section 3.3.2. Finally, the total number of variables, $x + y$, of the ACOD represents the number of real symbols carried by the O-STBC, hence the number of complex symbols transmitted is $K = (x + y) / 2$. Correspondingly, the code rate of an O-STBC constructed from ACOD is $K / P = (x + y) / 2n$.

Theorem 3.5: The maximum code rate of a square O-STBC constructed from an ACOD for $N_T = 2^a$ (a is an integer) transmit antennas is $(\log_2 N_T + 1) / N_T$.

Proof of *Theorem 3.5*: The upper bound of the maximum total number of variables of an ACOD of order n has been shown in *Theorem 3.1* to be $2a + 2$, if $n = 2^a b$, b odd. A square complex O-STBC for N_T transmit antennas constructed from ACOD has a code length $P = N_T = n$, and transmits $K = (x + y) / 2$ complex symbols. Hence the maximum code rate is $\max(K) / P = \max(x + y) / 2n = (2a + 2) / 2n = (a + 1) / n$ where $a = \log_2 n$, and $n = N_T$. As a result, *Theorem 3.5* is proved. ■

Theorem 3.5 further suggests that the maximum code rate of O-STBC constructed from ACOD is as high as the maximum code rate of O-STBC constructed from AOD [15] or Clifford Algebra [25], which is 3/4 for four transmit antennas and 1/2 for eight transmit antennas for square O-STBC. Two systematic ways to construct ACOD that reaches the maximum number of variables have been presented in Section 3.2.3, so O-STBC with maximum code rate can be constructed accordingly. We now construct such O-STBCs by using the ACOD examples presented in the previous section, and investigate the code properties in relation to the ACOD parameters.

A) Two Transmit Antennas

From the ACOD of order 2 in *Example 3.5* in (3.27), we have the followings:

$$\begin{aligned} \underline{\mathbf{C}} &= \begin{bmatrix} \underline{c}_2 & \underline{c}_1 \\ -\underline{c}_1 & \underline{c}_2 \end{bmatrix} = \underline{c}_1 \underline{\mathbf{C}}_1 + \underline{c}_2 \underline{\mathbf{C}}_2, \text{ where } \underline{\mathbf{C}}_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } \underline{\mathbf{C}}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \underline{\mathbf{D}} &= \begin{bmatrix} \underline{d}_2 & \underline{d}_1 \\ \underline{d}_1 & -\underline{d}_2 \end{bmatrix} = \underline{d}_1 \underline{\mathbf{D}}_1 + \underline{d}_2 \underline{\mathbf{D}}_2, \text{ where } \underline{\mathbf{D}}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } \underline{\mathbf{D}}_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{aligned} \quad (3.31)$$

then the following O-STBC can be constructed:

$$\begin{aligned}\mathbf{C2} &= \sum_{i=1}^x c_i^R \underline{\mathbf{C}}_i + \sum_{i=1}^y j c_i^I \underline{\mathbf{D}}_i \\ &= \begin{bmatrix} c_2 & c_1 \\ -c_1^* & c_2^* \end{bmatrix}\end{aligned}\quad (3.32)$$

Since c_1 and c_2 are arbitrary transmitted symbols, they can be interchanged and $\mathbf{C2}$ can be rewritten as:

$$\mathbf{C2} = \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix}\quad (3.33)$$

This turns out to be the famous Alamouti O-STBC proposed in [4].

B) Four Transmit Antennas

The O-STBC proposed in [7] by Tarokh, Jafarkhani and Calderbank and the O-STBC proposed in [15] by Ganesan and Stoica are rate-3/4 complex O-STBCs designed for four transmit antennas. They will be denoted as “**TJC4**” and “**GS4**” respectively, and used for comparison with our newly constructed O-STBCs.

$$\mathbf{TJC4} = \begin{bmatrix} c_1 & c_2 & \frac{c_3}{\sqrt{2}} & \frac{c_3}{\sqrt{2}} \\ -c_2^* & c_1^* & \frac{c_3}{\sqrt{2}} & -\frac{c_3}{\sqrt{2}} \\ \frac{c_3^*}{\sqrt{2}} & \frac{c_3^*}{\sqrt{2}} & \frac{-c_1 - c_1^* + c_2 - c_2^*}{2} & \frac{-c_2 - c_2^* + c_1 - c_1^*}{2} \\ \frac{c_3^*}{\sqrt{2}} & -\frac{c_3^*}{\sqrt{2}} & \frac{c_2 + c_2^* + c_1 - c_1^*}{2} & -\frac{(c_1 + c_1^* + c_2 - c_2^*)}{2} \end{bmatrix}\quad (3.34)$$

$$\mathbf{GS4} = \begin{bmatrix} c_1 & 0 & c_2 & -c_3 \\ 0 & c_1 & c_3^* & c_2^* \\ -c_2^* & -c_3 & c_1^* & 0 \\ c_3^* & -c_2 & 0 & c_1^* \end{bmatrix}\quad (3.35)$$

From *Example 3.6* in (3.28), *Example 3.8* in (3.30) and *Example 3.1* in (3.24), three new O-STBCs for four transmit antennas namely **C4a**, **C4b** and **C4c** can be constructed respectively.

$$\mathbf{C4a} = \begin{bmatrix} c_1 & c_2 & c_3 & 0 \\ -jc_2^* & jc_1^* & 0 & -jc_3 \\ -c_3^* & 0 & c_1^* & c_2 \\ 0 & jc_3^* & -jc_2^* & jc_1 \end{bmatrix} \quad (3.36)$$

$$\mathbf{C4b} = \begin{bmatrix} c_1 - c_2^* & c_1 + c_2^* & c_3 & -c_3 \\ c_1^* + c_2 & -c_1^* + c_2 & c_3 & c_3 \\ -c_3^* & c_3^* & c_1 - c_2 & c_1 + c_2 \\ -c_3^* & -c_3^* & c_1^* + c_2^* & -c_1^* + c_2^* \end{bmatrix} \quad (3.37)$$

$$\mathbf{C4c} = \begin{bmatrix} 0 & c_1 & c_2 & c_3 \\ -c_1 & 0 & -c_3^* & c_2^* \\ -c_2 & c_3^* & 0 & -c_1^* \\ -c_3 & -c_2^* & c_1^* & 0 \end{bmatrix} \quad (3.38)$$

It can be seen that the **TJC4** code contains the irrational number $1/\sqrt{2}$ in some of the codeword entries. This is because the **TJC4** code was formed by AOD of “*type*” (1, 1, 2; 1, 1, 2), hence one of the symbols has twice the power as the rest of the symbols. In order to normalize the power per symbol to be the same for all symbols, the scaling factor $1/\sqrt{2}$ is needed. Such multiplication operation (required at both the transmitter and receiver of the O-STBC system) may be less convenient to implement as compared to addition/subtraction operation associated with having ± 1 in the code coefficients of our proposed **C4b** code in (3.37), which is constructed from AOD of “*type*” (2, 2, 2; 2, 2, 2).

On the other hand, for the **GS** code, each transmit antennas is turned off once in every four symbol durations due to the presence of a zero coefficient in every codeword column. This is also true for **C4a** and **C4c**, and all of them are constructed

from AOD of “*type*” (1, 1, 1; 1, 1, 1). Switching on and off an antenna periodically will induce nonlinear effects in the power amplifier of the wireless MIMO system [32], this is undesirable for practical implementation. However, this shortcoming does not exist for the **C4b** code if its c_1 and c_2 symbols are taken from different modulation constellations, e.g. c_1 from quadrature phase shift keying (QPSK) and c_2 from rotated-QPSK.

Due to the above reasons, the proposed **C4b** code is more advantageous over the **TJC4** and **GS4** codes in terms of practical implementation.

C) Eight Transmit Antennas

Next, we consider the O-STBC designs for eight transmit antennas. Two types of rate-1/2 O-STBCs for eight transmit antennas have been proposed in the literature: non-square design [7] with code length 16, and square design from [25] with code length 8. Due to the advantages of square design, such as minimum decoding delay and applicability to differential/unitary space-time coding [38], we will only consider the square O-STBC here. The classical example is one from [25] by Tirkkonen and Hottinen, herein denoted as the “**TH8**” code and shown in (3.39). In [18], Tran, Seberry and *et. al.* construct two new O-STBC for eight transmit antennas that aim to reduce the unused time slots (i.e. number of zeros inside the codeword). One of them is constructed based on AOD of order 8 and “*type*” (1, 1, 1, 4 ; 1, 1, 1, 4). It is denoted herein as “**TS8**” and shown in (3.40).

$$\mathbf{TH8} = \begin{bmatrix} c_1 & c_2 & c_3 & 0 & c_4 & 0 & 0 & 0 \\ -c_2^* & c_1^* & 0 & -c_3 & 0 & -c_4 & 0 & 0 \\ -c_3^* & 0 & c_1^* & c_2 & 0 & 0 & -c_4 & 0 \\ 0 & c_3^* & -c_2^* & c_1 & 0 & 0 & 0 & c_4 \\ -c_4^* & 0 & 0 & 0 & c_1^* & c_2 & c_3 & 0 \\ 0 & c_4^* & 0 & 0 & -c_2^* & c_1 & 0 & -c_3 \\ 0 & 0 & c_4^* & 0 & -c_3^* & 0 & c_1 & c_2 \\ 0 & 0 & 0 & -c_4^* & 0 & c_3^* & -c_2^* & c_1^* \end{bmatrix} \quad (3.39)$$

$$\mathbf{TS8} = \begin{bmatrix} c_1 & 0 & c_3^R + jc_2^I & c_2^R + jc_3^I & \frac{c_4}{2} & \frac{c_4}{2} & \frac{c_4}{2} & \frac{c_4}{2} \\ 0 & c_1 & -c_2^R + jc_3^I & c_3^R - jc_2^I & \frac{c_4}{2} & -\frac{c_4}{2} & \frac{c_4}{2} & -\frac{c_4}{2} \\ -c_3^R + jc_2^I & c_2^R + jc_3^I & c_1^* & 0 & \frac{c_4}{2} & \frac{c_4}{2} & -\frac{c_4}{2} & -\frac{c_4}{2} \\ -c_2^R + jc_3^I & -c_3^R - jc_2^I & 0 & c_1^* & \frac{c_4}{2} & -\frac{c_4}{2} & -\frac{c_4}{2} & \frac{c_4}{2} \\ -\frac{c_4^*}{2} & -\frac{c_4^*}{2} & -\frac{c_4^*}{2} & -\frac{c_4^*}{2} & c_1^R - jc_3^I & c_2^* & c_3^R - jc_1^I & 0 \\ -\frac{c_4^*}{2} & \frac{c_4^*}{2} & -\frac{c_4^*}{2} & \frac{c_4^*}{2} & -c_2 & c_1^R + jc_3^I & 0 & c_3^R - jc_1^I \\ -\frac{c_4^*}{2} & -\frac{c_4^*}{2} & \frac{c_4^*}{2} & \frac{c_4^*}{2} & -c_3^R - jc_1^I & 0 & c_1^R + jc_3^I & -c_2^* \\ -\frac{c_4^*}{2} & \frac{c_4^*}{2} & \frac{c_4^*}{2} & -\frac{c_4^*}{2} & 0 & -c_3^R - jc_1^I & c_2 & c_1^R - jc_3^I \end{bmatrix} \quad (3.40)$$

Three new O-STBCs for eight transmit antennas constructed using ACOD / AOD from *Example 3.7* in (3.29), *Example 3.2* in (3.25), and *Example 3.3* in (3.26), are shown below as **C8a**, **C8b** and **C8c** respectively.

$$\mathbf{C8a} = \begin{bmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0 & c_4 & 0 \\ -jc_2^* & jc_1^* & 0 & -jc_3 & 0 & 0 & 0 & -jc_4 \\ -c_3^* & 0 & c_1^* & c_2 & c_4 & 0 & 0 & 0 \\ 0 & jc_3^* & -jc_2^* & jc_1 & 0 & -jc_4 & 0 & 0 \\ 0 & 0 & -c_4^* & 0 & c_1 & c_2 & c_3^* & 0 \\ 0 & 0 & 0 & jc_4^* & -jc_2^* & jc_1^* & 0 & -jc_3^* \\ -c_4^* & 0 & 0 & 0 & -c_3 & 0 & c_1^* & c_2 \\ 0 & jc_4^* & 0 & 0 & 0 & jc_3 & -jc_2^* & jc_1 \end{bmatrix} \quad (3.41)$$

$$\mathbf{C8b} = \begin{bmatrix} 0 & c_1 & c_2 & 0 & c_3 & 0 & c_4 & 0 \\ -jc_1^* & 0 & 0 & -jc_2^* & 0 & -jc_3^* & 0 & -jc_4^* \\ -c_2 & 0 & 0 & c_1 & -c_4^* & 0 & c_3^* & 0 \\ 0 & jc_2^* & -jc_1^* & 0 & 0 & -jc_4 & 0 & jc_3 \\ -c_3 & 0 & -c_4^* & 0 & 0 & c_1 & c_2^* & 0 \\ 0 & jc_3^* & 0 & jc_4 & -jc_1^* & 0 & 0 & -jc_2 \\ -c_4 & 0 & c_3^* & 0 & -c_2^* & 0 & 0 & c_1 \\ 0 & jc_4^* & 0 & -jc_3 & 0 & jc_2 & -jc_1^* & 0 \end{bmatrix} \quad (3.42)$$

$$\mathbf{C8c} = \begin{bmatrix} c_1 & c_1 & c_2^* & -c_2^* & c_3^* & -c_3^* & c_4^* & -c_4^* \\ c_1^* & -c_1^* & c_2 & c_2 & c_3 & c_3 & c_4 & c_4 \\ -c_2^* & c_2^* & c_1 & c_1 & c_4 & -c_4 & -c_3 & c_3 \\ -c_2 & -c_2 & c_1^* & -c_1^* & c_4^* & c_4^* & -c_3^* & -c_3^* \\ -c_3^* & c_3^* & -c_4 & c_4 & c_1 & c_1 & c_2 & -c_2 \\ -c_3 & -c_3 & -c_4^* & -c_4^* & c_1^* & -c_1^* & c_2^* & c_2^* \\ -c_4^* & c_4^* & c_3 & -c_3 & -c_2 & c_2 & c_1 & c_1 \\ -c_4 & -c_4 & c_3^* & c_3^* & -c_2^* & -c_2^* & c_1^* & -c_1^* \end{bmatrix} \quad (3.43)$$

It can be seen that **C8a** and **C8b** are similar to **TH8** in that half of their codeword entries are zero, hence four of the eight transmit antennas will be turned off at any one time for these codes. Note that they are constructed from AOD of “type” (1, 1, 1, 1; 1, 1, 1, 1). On the other hand, **TS8**, which is constructed from AOD of “type” (1, 1, 1, 4; 1, 1, 1, 4), has less zero entries than **TH8**, but still requires one transmit antenna to be turned off at any one time. In contrast, **C8c**, which is constructed from AOD of “type” (2, 2, 2, 2; 2, 2, 2, 2), does not require any transmit antenna to be turned off.

From the above observations, the following guidelines as listed in *Proposition 3.2* can be deduced for the design of a good O-STBC:

Proposition 3.2: To design an O-STBC without irrational number coefficients and zero entries, the following guidelines apply:

- To avoid irrational number coefficients inside an O-STBC codeword, ACOD having a constant “*type*” value for all variables, i.e. $f_i = g_i = \text{constant} \forall i$, is desired.
- To avoid zeros inside an O-STBC codeword, ACOD with the sum of “*type*” for all its variables equal to or greater than $2n$, i.e. $\Sigma(f_i + g_i) \geq 2n$, is desired.

Proof of *Proposition 3.2*: From all the above examples, we know that the “*type*” of a variable in an ACOD is related to the power of that transmitted symbols. If all the variables have equal “*type*”, all the transmitted symbols will have equal transmission power, this will eliminate the need for power normalization for individual symbols, and hence irrational number coefficients in the O-STBC codeword. Next, the “*type*” of a variable in an ACOD is also related to the number of occurrence of that variable in a row of the ACOD matrix. To eliminate zeros inside the codeword of an O-STBC, it is desired to have all the positions inside the codeword matrix at least filled by a complex symbol. So the sum of occurrence of every complex symbols must be equal to or greater than n , or the sum of “*type*” of all variables must be equal to or greater than $2n$ (since a complex symbol consists of two real symbols, and is represented by two variables in ACOD). However, this is only a necessary, but not sufficient, condition to eliminate the zeros inside an O-STBC codeword. ■

In the code construction examples shown above, it can be noted that **C4a**, **C4c**, **C8a** and **C8b** are similar to some existing O-STBC designs. This illustrates the generality of our code construction methods. In addition, we further showed that with careful selection of the “*type*” parameter according to the guidelines in *Proposition 3.2*, our proposed construction methods are able to construct improved O-STBC codes,

such as **C4b** and **C8c**. This is the major difference between our code construction methods and those surveyed in [27].

In the next section, we will show that besides having no irrational number and zero as code coefficient, O-STBC designed using the above guidelines also have more favorable power distribution properties (listed in Section 2.4.2) than the existing O-STBCs.

3.3.2 Power Distribution of O-STBC

The power distribution characteristics of our new rate-3/4 O-STBCs for four transmit antennas **C4b** and rate-1/2 O-STBCs for eight transmit antennas **C8c** are compared against existing O-STBCs in Table 3.4 and Table 3.5 respectively.

As shown in Table 3.4, our newly constructed **C4b** code achieves better peak/ave ratio and ave/min ratio than the **GS4** codes in [15], as well as the “power-balanced” version of **GS4** codes in [17]. Although the **TJC4** code has better power-distribution characteristics than **C4b**, it contains irrational-number codeword coefficients, which may lead to more complex practical implementation as discussed earlier. From Table 3.5, we can see that our proposed **C8c** code has much better power-distribution characteristics than the **TH8** and **TS8** codes. In fact, **C8c** can be considered to be optimally power-balanced as it has all the ideal power-distribution attributes, i.e. $\text{peak} / \text{ave} = 1$, $\text{ave} / \text{min} = 1$, and $P_o = 0$. Finally, it can be seen from both tables that codes that achieve $P_o = 0$ satisfy the second design guideline in *Proposition 3.2*, i.e. sum of “type” for all variables greater than or equal to $2n$.

Table 3.4 Power distribution characteristics of four-antenna O-STBCs with QPSK modulation

	peak/ave	ave/min	P_o	Σ “type”	Σ “type” ≥ 8 ?
TJC4 [7]	1.33	1.5^1	0	8	Yes
GS4 [15]	1.33	∞	25%	6	No
Power-balanced GS4 #1 [17]	3	3	0	NA	NA
Power-balanced GS4 #2 [17]	2.6	17.5	0	NA	NA
C4b ² (proposed)	2.28	2.56	0	12	Yes

Table 3.5 Power distribution characteristics of eight-antenna O-STBCs with QPSK modulation

	peak/ave	ave/min	P_o	Σ “type”	Σ “type” ≥ 16 ?
TH8 [25]	2	∞	50%	8	No
TS8 [18]	2	∞	12.5%	10	No
C8c (proposed)	1	1	0	16	Yes

All the O-STBCs constructed in this thesis have exactly the same coding and diversity gains as the existing O-STBCs with the same code rate. This can be proved by examining the codeword distance matrix of an O-STBC which has been defined in (2.16) (remember that the decoding performance of STBC is determined by the eigenvalues of its codeword distance matrix as discussed in Section 2.4.1.):

¹ This value has been corrected from the original value stated in [17].

² Assume that data symbol c_1 is QPSK, while c_2 is QPSK rotated 45 degree with respect to c_1 .

$$\begin{aligned}
 \mathbf{A}_{\text{CE}} &= \left(\sum_{i=1}^K (\Delta_i^R \mathbf{A}_i + j\Delta_i^I \mathbf{B}_i)^H \right) \left(\sum_{i=1}^K (\Delta_i^R \mathbf{A}_i + j\Delta_i^I \mathbf{B}_i) \right) \\
 &= \sum_{i=1}^K \left(|\Delta_i^R|^2 \mathbf{A}_i^H \mathbf{A}_i + |\Delta_i^I|^2 \mathbf{B}_i^H \mathbf{B}_i \right) + \\
 &\quad \sum_{i=1}^K \sum_{k=i+1}^K \left[\Delta_i^R \Delta_k^R (\mathbf{A}_i^H \mathbf{A}_k + \mathbf{A}_k^H \mathbf{A}_i) + \Delta_i^I \Delta_k^I (\mathbf{B}_i^H \mathbf{B}_k + \mathbf{B}_k^H \mathbf{B}_i) \right] + \\
 &\quad \sum_{i=1}^K \sum_{k=1}^K \left[j\Delta_i^R \Delta_k^I (\mathbf{A}_i^H \mathbf{B}_k - \mathbf{B}_k^H \mathbf{A}_i) \right]
 \end{aligned}$$

For an O-STBC that satisfies the constraints in (3.1), its codeword distance matrices can be simplified as follows:

$$\begin{aligned}
 \mathbf{A}_{\text{CE}} &= \sum_{i=1}^K \left(|\Delta_i^R|^2 \left(\underbrace{\mathbf{A}_i^H \mathbf{A}_i}_{=\mathbf{I}_{N_T} \text{ due to (3.1)(i)}} \right) + |\Delta_i^I|^2 \left(\underbrace{\mathbf{B}_i^H \mathbf{B}_i}_{=\mathbf{I}_{N_T} \text{ due to (3.1)(i)}} \right) \right) + \\
 &\quad \sum_{i=1}^K \sum_{k=i+1}^K \left[\Delta_i^R \Delta_k^R \left(\underbrace{\mathbf{A}_i^H \mathbf{A}_k + \mathbf{A}_k^H \mathbf{A}_i}_{=0 \text{ due to (3.1)(ii)}} \right) + \Delta_i^I \Delta_k^I \left(\underbrace{\mathbf{B}_i^H \mathbf{B}_k + \mathbf{B}_k^H \mathbf{B}_i}_{=0 \text{ due to (3.1)(ii)}} \right) \right] + \\
 &\quad \sum_{i=1}^K \sum_{k=1}^K \left[j\Delta_i^R \Delta_k^I \left(\underbrace{\mathbf{A}_i^H \mathbf{B}_k - \mathbf{B}_k^H \mathbf{A}_i}_{=0 \text{ due to (3.1)(iii)}} \right) \right] \\
 &= \sum_{i=1}^K \left(|\Delta_i^R|^2 + |\Delta_i^I|^2 \right) \mathbf{I}_{N_T}
 \end{aligned}$$

So an O-STBC that satisfy (3.1) always has full rank for its codeword distance matrix and the determinant of its codeword distance matrix will be

$$\left(\sum_{i=1}^K \left(|\Delta_i^R|^2 + |\Delta_i^I|^2 \right) \right)^{N_T}.$$

From the above, one may observe that the performance of O-STBC is only determined by the Euclidean distance of a single symbol rather than Euclidean distance across different symbols. Since all new O-STBCs constructed in the thesis are based on ACOD (which is a generalization of AOD) and they satisfy (3.19) which is a generalization of (3.1), they will have the same performance as the existing O-STBC that is based on AOD.

As for constellation rotation, the Euclidean distance between two QPSK constellation points or between two rotated QPSK constellation points are the same. It has also been shown above that the Euclidean distance across symbols does not contribute to the performance of O-STBC. Therefore the new code **C4b** also has the same performance as the existing O-STBC.

Since the simulations results will only show some overlapping lines, it is omitted from the thesis.

3.4 CHAPTER SUMMARY

In this chapter, ACOD (the complex version of AOD) is defined and its existence is proven. The maximum code rate of an O-STBC constructed from ACOD is shown to be as high as the existing O-STBCs constructed from AOD or Clifford Algebra. To facilitate the construction of O-STBC from ACOD, we have proposed two systematic ways to construct higher-order ACOD from lower-order ACOD. It is found that the “*type*” parameter of ACOD plays an important role in shaping the power distribution and code coefficient characteristics of the O-STBC. From these findings, we propose to use ACOD with constant “*type*” value for all variables, and sum of “*type*” for all variables equal to or greater than $2n$, to construct O-STBC with no irrational and zero code coefficient respectively, which are favorable implementation attributes.

Using the proposed ACOD construction methods and design guidelines, new square O-STBCs for four and eight transmit antennas that support complex constellations are constructed. Besides having no irrational-number and zero code coefficients, the newly constructed O-STBCs also have other advantageous attributes over existing O-STBCs because they possess better power distribution characteristics that ease the design of front-end power amplifiers. In particular, one of the newly

constructed O-STBCs for eight transmit antennas is found to be optimal in terms of power distribution properties.

Being square orthogonal designs, the new codes presented in this thesis also enjoy the benefits of achieving full diversity and minimum code length, and they can also be used as differential space-time codes for non-coherent detection. Furthermore, since ACOD is the generalized complex version of AOD, the code construction methods and the design guidelines proposed for the “*type*” parameter of ACOD are also applicable to the construction of O-STBC from AOD.

Finally, although not shown in this thesis, all the O-STBCs constructed in this thesis have exactly the same coding and diversity gains as the existing O-STBCs with the same code rate. So the above achievements are obtained without any loss in decoding performance in the proposed ACOD O-STBC.

CHAPTER 4 QUASI-ORTHOGONAL STBC: ANALYSIS AND OPTIMIZATION

4.1 INTRODUCTION

Orthogonal Space-Time Block Code (O-STBC) is an important class of space-time codes due to its ability to provide optimal transmit diversity by requiring only linear processing at the receiver. However, full-rate O-STBC for complex constellation exists only for two transmit antennas. For higher transmit diversity levels, O-STBCs exist only with code rate less than one, which implies a loss of bandwidth efficiency [7]. To achieve full-rate transmission while retaining much of the orthogonality benefits, Quasi-Orthogonal Space-Time Block Code (QO-STBC) has been proposed, such as the QO-STBC from [9], the ABBA code from [10], the transmit diversity scheme from [11], and others [39-42]. With its quasi-orthogonal code structure, the maximum-likelihood (ML) decoding of a QO-STBC can be performed by searching over (or joint detection of) only a subset of the total number of transmitted symbols, and the joint detection of different subsets can be performed in parallel. Hence the decoding complexity of *quasi-orthogonal* STBC is lower than the general *non-orthogonal* STBC. Although QO-STBC can achieve higher code rates than O-STBC, it generally does not provide full diversity. To address this limitation, constellation rotation of QO-STBC is proposed in [5,12-14,43] to achieve full diversity with ML decoding.

4.1.1 Review of QO-STBC

The QO-STBCs reported in the literature can be classified into the following three categories. We will quote one representative example for each category:

- Rate-1 QO-STBC for four transmit antennas

The four transmitted data symbols of this code can be grouped into two groups, and its ML decoding can be performed by jointly detecting two complex symbols (double-symbol decodable). An example of this code is the QO-STBC proposed by Jafarkhani in [9], which is denoted herein as **J4**:

$$\mathbf{J4} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & -c_4^* & c_1^* & c_2^* \\ c_4 & -c_3 & -c_2 & c_1 \end{bmatrix} \quad (4.1)$$

- Rate-3/4 QO-STBC for eight transmit antennas

The six transmitted data symbols of this code can be grouped into three groups, and its ML decoding can be performed by jointly detecting two complex symbols. An example of this code is the QO-STBC proposed by Jafarkhani in [9], which is denoted herein as **J8**:

$$\mathbf{J8} = \begin{bmatrix} c_1 & c_2 & c_3 & 0 & c_4 & c_5 & c_6 & 0 \\ -c_2^* & c_1^* & 0 & -c_3 & c_5^* & -c_4^* & 0 & c_6 \\ c_3^* & 0 & -c_1^* & -c_2 & -c_6^* & 0 & c_4^* & c_5 \\ 0 & -c_3^* & c_2^* & -c_1 & 0 & c_6^* & -c_5^* & c_4 \\ -c_4 & -c_5 & -c_6 & 0 & c_1 & c_2 & c_3 & 0 \\ -c_5^* & c_4^* & 0 & c_6 & -c_2^* & c_1^* & 0 & c_3 \\ c_6^* & 0 & -c_4^* & c_5 & c_3^* & 0 & -c_1^* & c_2 \\ 0 & c_6^* & -c_5^* & -c_4 & 0 & c_3^* & -c_2^* & -c_1 \end{bmatrix} \quad (4.2)$$

- Rate-1 QO-STBC for eight transmit antennas

The eight transmitted data symbols of this code can be grouped into two groups, and its ML decoding can be performed by jointly detecting four complex symbols (quad-symbol decodable). An example of this code is the QO-STBC proposed by Tirkkonen, Boariu and Hottinen in [10], which is denoted herein as **TBH8**:

$$\mathbf{TBH8} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 \\ -c_2^* & c_1^* & -c_4^* & c_3^* & -c_6^* & c_5^* & -c_8^* & c_7^* \\ c_3 & c_4 & c_1 & c_2 & c_7 & c_8 & c_5 & c_6 \\ -c_4^* & c_3^* & -c_2^* & c_1^* & -c_8^* & c_7^* & -c_6^* & c_5^* \\ c_5 & c_6 & c_7 & c_8 & c_1 & c_2 & c_3 & c_4 \\ -c_6^* & c_5^* & -c_8^* & c_7^* & -c_2^* & c_1^* & -c_4^* & c_3^* \\ c_7 & c_8 & c_5 & c_6 & c_3 & c_4 & c_1 & c_2 \\ -c_8^* & c_7^* & -c_6^* & c_5^* & -c_4^* & c_3^* & -c_2^* & c_1^* \end{bmatrix} \quad (4.3)$$

We will now use **J4** in (4.1) as an example to illustrate the simplified ML decoding of QO-STBC. The ML decoding metrics for **J4** have been derived in [9] and are reproduced in (4.4). It shows that the decoding decisions for symbols c_1 and c_4 are jointly obtained by minimizing the metric f_{14} , similarly the decoding decisions for symbols c_2 and c_3 are jointly obtained by minimizing the metric f_{23} . Since c_1 and c_4 (or c_2 and c_3) are each a complex symbol, their ML decoding requires the joint detection of two complex symbols (i.e. four real symbols) in total. Clearly, the joint detection of c_1 and c_4 can be performed independently from, and hence in parallel with, the joint detection of c_2 and c_3 .

$$\begin{aligned}
 f_{14}(c_1, c_4) &= \sum_{k=1}^{N_r} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) (|c_1|^2 + |c_4|^2) + \right. \\
 &\quad \left. 2 \operatorname{Re} \left\{ \begin{aligned} &c_1 (-h_{1,k} r_1^{(k)*} - h_{2,k}^* r_2^{(k)} - h_{3,k}^* r_3^{(k)} - h_{4,k} r_4^{(k)*}) + \\ &c_4 (-h_{4,k} r_1^{(k)*} + h_{3,k}^* r_2^{(k)} + h_{2,k}^* r_3^{(k)} - h_{1,k} r_4^{(k)*}) + \\ &c_1 c_4^* \times 2 \operatorname{Re}(h_{1,k} h_{4,k}^* - h_{2,k} h_{3,k}^*) \end{aligned} \right\} \right] \\
 f_{23}(c_2, c_3) &= \sum_{k=1}^{N_r} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) (|c_2|^2 + |c_3|^2) + \right. \\
 &\quad \left. 2 \operatorname{Re} \left\{ \begin{aligned} &c_2 (-h_{2,k} r_1^{(k)*} + h_{1,k}^* r_2^{(k)} - h_{4,k}^* r_3^{(k)} + h_{3,k} r_4^{(k)*}) + \\ &c_3 (-h_{3,k} r_1^{(k)*} - h_{4,k}^* r_2^{(k)} + h_{1,k}^* r_3^{(k)} + h_{2,k} r_4^{(k)*}) + \\ &c_2 c_3^* \times 2 \operatorname{Re}(-h_{1,k} h_{4,k}^* + h_{2,k} h_{3,k}^*) \end{aligned} \right\} \right] \quad (4.4)
 \end{aligned}$$

To examine the decoding error probability performance of **J4**, we look at its codeword distance matrix $\mathbf{A}_{\text{CE}}$ as defined in (2.16). The expression of $\mathbf{A}_{\text{CE}}(\mathbf{J4})$ has been reported in [5] and is re-produced below in (4.5):

$$\mathbf{A}_{\text{CE}}(\mathbf{J4}) = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \quad (4.5)$$

where

$$\begin{aligned}
 \alpha &= \sum_{i=1}^4 |\Delta_i|^2 \\
 \beta &= 2 \times \operatorname{Re}(\Delta_1 \times \Delta_4^* - \Delta_2 \times \Delta_3^*)
 \end{aligned} \quad (4.6)$$

and $\Delta_i = c_i - e_i$ represents the possible code symbols errors. The determinant of the codeword distance matrix of **J4** is

$$\begin{aligned}
 \operatorname{Det}(\mathbf{A}_{\text{CE}}(\mathbf{J4})) &= (\alpha + \beta)^2 \times (\alpha - \beta)^2 \\
 &= \left[\left(|\Delta_1 + \Delta_4|^2 + |\Delta_2 - \Delta_3|^2 \right) \times \right. \\
 &\quad \left. \left(|\Delta_1 - \Delta_4|^2 + |\Delta_2 + \Delta_3|^2 \right) \right] \quad (4.7)
 \end{aligned}$$

Due to the summation of two $|\cdot|^2$ terms in (4.7), the minimum value of (4.7) occurs when half of the symbols have error, i.e. either $\Delta_1 = \Delta_4 = 0$ or $\Delta_2 = \Delta_3 = 0$. Hence when considering the minimum determinant value, (4.7) can be simplified as [10]:

$$\text{Min}[\text{Det}(\mathbf{A}_{\text{CE}}(\mathbf{J4}))] = \left[(|\Delta_1 + \Delta_4|^2) \times (|\Delta_1 - \Delta_4|^2) \right]^2 \quad (4.8)$$

assuming that $\Delta_2 = \Delta_3 = 0$.

It is easy to see from (4.8) that, when $\Delta_1 = \pm\Delta_4$, the determinant value of the codeword distance matrix of $\mathbf{J4}$ would become zero, which implies that the codeword distance matrix of $\mathbf{J4}$ would not have full rank and $\mathbf{J4}$ cannot achieve full diversity. Such a situation can easily happen due to the symmetry of conventional constellation sets, such as phase shift keying (PSK) and quadrature amplitude modulation (QAM). To achieve non-zero determinant value and hence full diversity, we need to have $\Delta_1 \neq \Delta_4$. This can be achieved by the constellation rotation (CR) technique.

For example, consider the binary phase shift keying (BPSK) constellation set $\{-1, 1\}$ for all symbols c_1 to c_4 in $\mathbf{J4}$, the possible values of Δ_i include $\{0, 2, -2\}$ for all values of i , and $|\Delta_1 \pm \Delta_4|$ can be easily be zero.

But if c_1 uses a non-rotated constellation set as shown in Figure 4.1(a) (hence Δ_1 takes values $\{0, 2, -2\}$), while c_4 uses a rotated constellation set as shown in Figure 4.1(b) (hence Δ_4 takes values $\{0, 2\exp(j\theta), -2\exp(j\theta)\}$), then a non-zero value for $|\Delta_1 \pm \Delta_4|$ can always be obtained (Δ_1 and Δ_4 cannot be both zero at the same time, otherwise there would not be any codeword difference). A similar BPSK CR can be applied to c_2 and c_3 .

Figure 4.1 Constellation rotation of BPSK constellation set

Besides facilitating full diversity, the CR angle gives us an extra degree of freedom to maximize the minimum determinant value in (4.8) to achieve maximum coding gain for the QO-STBC (according to the determinant criterion presented in Section 2.4.1 of this thesis). The **J4** code after appropriate CR, denoted as **J4_CR**, is able to achieve full diversity and is shown in (4.9):

$$\mathbf{J4_CR} = \begin{bmatrix} c_1 & c_2 & c_3 e^{j\pi/4} & c_4 e^{j\pi/4} \\ -c_2^* & c_1^* & -(c_4 e^{j\pi/4})^* & (c_3 e^{j\pi/4})^* \\ -(c_3 e^{j\pi/4})^* & -(c_4 e^{j\pi/4})^* & c_1^* & c_2^* \\ c_4 e^{j\pi/4} & -c_3 e^{j\pi/4} & -c_2 & c_1 \end{bmatrix} \quad (4.9)$$

where the factor $e^{j\pi/4}$ denotes the optimum CR angle for the QAM symbols c_3 and c_4 [5,14].

4.1.2 Motivations

To our knowledge, although different QO-STBCs proposed by different groups of researchers have similar properties such as code rate, decoding complexity and decoding performance, there is no general framework to unify all different designs of QO-STBCs. In this chapter, we will first derive the algebraic structure of QO-STBC to unify their code structure and decoding complexity.

Next, although the performance of QO-STBC with CR under ML decoding is well understood, the decoding performance of QO-STBC with CR under sub-optimal

decoding or equalization techniques has not been systematically studied. So we will study the decoding of QO-STBC with CR by utilizing noise-whitening filter and various equalization schemes.

Finally, we will show that by employing CR on QO-STBC with square or rectangular QAM constellation symbols, the decoding complexity of the code after CR is higher than that before CR. Hence we will propose a new constellation transformation technique, called the Group-Constrained Linear Transformation (GCLT), to optimize the decoding performance of QO-STBC such that the resultant code can achieve full diversity but will maintain the decoding complexity.

4.2 ALGEBRAIC STRUCTURE OF QO-STBC

The concept of quasi-orthogonality in QO-STBC means to divide the K transmitted symbols in a STBC codeword into G independent groups, such that symbols in any group are orthogonal to all symbols in the other groups after matched filtering, while strict orthogonality among the symbols within a group is not required. The ML decoding of QO-STBC can thus be performed by only jointly detecting symbols in the same group, while the detection of symbols in other groups can be performed separately and independently. In other words, the received symbols can be separated into G independent groups by linear matched filters, such that the ML decoding of different groups can be performed separately and in parallel, and the ML decoding of each group can be achieved by jointly detecting only K/G complex symbols within the group, instead of jointly detecting all the K symbols (which is clearly a very complex operation). In addition to achieving the above-mentioned reduction in ML decoding complexity, it is important to ensure that the QO-STBC can achieve full diversity too. Hence we will divide the design of QO-STBC into two parts, one governing the

grouping of code symbols, the other governing the optimization of decoding performance.

4.2.1 Decoding Complexity of a QO-STBC

Definition 4.1: A quasi-orthogonal STBC design $\mathbf{C}$ is such that the equivalent channel matrix, $\mathbf{H}$, as defined in (2.10) has the property that $\mathbf{H}^T \mathbf{H}$ is block-diagonal and consists of G smaller sub-matrices each with size $(2K/G) \times (2K/G)$. The symbols in different sub-matrices are said to be orthogonal, while the symbols within a sub-matrices are not orthogonal and said to be in a group.

It will be proven in *Theorem 4.2* in Section 4.3 that The ML decoding of QO-STBC $\mathbf{C}$ as defined in *Definition 4.1* can be performed by G independent decoders, and each of the decoder only needs to jointly detect K/G complex symbols.

Theorem 4.1: For any two symbols that belong to different groups (indexed using subscripts p and q) in a QO-STBC to be orthogonal as defined in *Definition 4.1*, their dispersion matrices $\{\mathbf{A}_p, \mathbf{B}_p\}$ and $\{\mathbf{A}_q, \mathbf{B}_q\}$ defined as per (2.7) must possess the following algebraic structure, herein referred to as *Quasi-Orthogonality Constraints* (QOC):

$$\begin{aligned} \text{(i)} \quad & \mathbf{A}_p^H \mathbf{A}_q = -\mathbf{A}_q^H \mathbf{A}_p \\ \text{(ii)} \quad & \mathbf{B}_p^H \mathbf{B}_q = -\mathbf{B}_q^H \mathbf{B}_p \quad 1 \leq q \notin \mathcal{G}(p) \leq K \\ \text{(iii)} \quad & \mathbf{A}_p^H \mathbf{B}_q = \mathbf{B}_q^H \mathbf{A}_p \end{aligned} \quad (4.10)$$

where $\mathcal{G}(p)$ represents a set of indexes of symbols in the same group as the symbol with index p , including p .

Proof of *Theorem 4.1*: To derive the QOC in (4.10), let us multiply a matched filter ($\mathbf{H}^T$) to the received signal $\tilde{\mathbf{r}}$ in (2.10) to obtain $\mathbf{H}^T \tilde{\mathbf{r}} = \sqrt{\rho/N_T} \mathbf{H}^T \mathbf{H} \tilde{\mathbf{c}} + \mathbf{H}^T \tilde{\mathbf{\eta}}$, and consider a segment of $\mathbf{H}^T \mathbf{H}$ as shown below:

$$\begin{aligned}
 & \mathbf{H}^T \mathbf{H} \\
 &= \begin{bmatrix} \vdots & \cdots & \vdots \\ \tilde{\mathbf{h}}_1^T \mathcal{A}_p^T & \cdots & \tilde{\mathbf{h}}_{N_R}^T \mathcal{A}_p^T \\ \tilde{\mathbf{h}}_1^T \mathcal{B}_p^T & \cdots & \tilde{\mathbf{h}}_{N_R}^T \mathcal{B}_p^T \\ \vdots & \cdots & \vdots \end{bmatrix} \begin{bmatrix} \cdots & \mathcal{A}_p \tilde{\mathbf{h}}_1 & \mathcal{B}_p \tilde{\mathbf{h}}_1 & \mathcal{A}_q \tilde{\mathbf{h}}_1 & \mathcal{B}_q \tilde{\mathbf{h}}_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \mathcal{A}_p \tilde{\mathbf{h}}_{N_R} & \mathcal{B}_p \tilde{\mathbf{h}}_{N_R} & \mathcal{A}_q \tilde{\mathbf{h}}_{N_R} & \mathcal{B}_q \tilde{\mathbf{h}}_{N_R} & \cdots \end{bmatrix}, \quad 1 \leq q \neq \mathcal{G}(p) \leq K \\
 &= \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_p^T \mathcal{A}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_p^T \mathcal{B}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_p^T \mathcal{A}_q) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_p^T \mathcal{B}_q) \tilde{\mathbf{h}}_i & \vdots \\ \vdots & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_p^T \mathcal{A}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_p^T \mathcal{B}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_p^T \mathcal{A}_q) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_p^T \mathcal{B}_q) \tilde{\mathbf{h}}_i & \vdots \\ \vdots & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_q^T \mathcal{A}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_q^T \mathcal{B}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_q^T \mathcal{A}_q) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{A}_q^T \mathcal{B}_q) \tilde{\mathbf{h}}_i & \vdots \\ \vdots & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_q^T \mathcal{A}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_q^T \mathcal{B}_p) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_q^T \mathcal{A}_q) \tilde{\mathbf{h}}_i & \sum_{i=1}^{N_R} \tilde{\mathbf{h}}_i^T (\mathcal{B}_q^T \mathcal{B}_q) \tilde{\mathbf{h}}_i & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix} \quad (4.11)
 \end{aligned}$$

In order to achieve orthogonality among two symbols in different groups, e.g. between symbols c_p and c_q , the summation terms included in the enclosed boxes in (4.11) are required to be zero. Note that these summation terms have the form of $\mathbf{v}^T \mathbf{M} \mathbf{v}$. For any vector $\mathbf{v}$ of size $v \times 1$, $\mathbf{v}^T \mathbf{M} \mathbf{v} = 0$ if and only if the matrix $\mathbf{M}$ of size $v \times v$ is skew-symmetric, i.e. $\mathbf{M}^T = -\mathbf{M}$. Hence $\mathcal{A}_p^T \mathcal{A}_q$, $\mathcal{A}_p^T \mathcal{B}_q$, $\mathcal{B}_p^T \mathcal{A}_q$, and $\mathcal{B}_p^T \mathcal{B}_q$ have to be skew-symmetric.

Next, from (4.10)(i), we get

$$\begin{aligned}
 \mathbf{A}_p^H \mathbf{A}_q &= -\mathbf{A}_q^H \mathbf{A}_p \\
 \Rightarrow (\mathbf{A}_p^R + j\mathbf{A}_p^I)^H (\mathbf{A}_q^R + j\mathbf{A}_q^I) &= -(\mathbf{A}_q^R + j\mathbf{A}_q^I)^H (\mathbf{A}_p^R + j\mathbf{A}_p^I) \\
 \Rightarrow \begin{cases} \text{real part : } (\mathbf{A}_p^R)^T \mathbf{A}_q^R + (\mathbf{A}_p^I)^T \mathbf{A}_q^I &= -(\mathbf{A}_q^R)^T \mathbf{A}_p^R - (\mathbf{A}_q^I)^T \mathbf{A}_p^I \\ \text{imag part : } (\mathbf{A}_p^R)^T \mathbf{A}_q^I - (\mathbf{A}_p^I)^T \mathbf{A}_q^R &= (\mathbf{A}_q^I)^T \mathbf{A}_p^R - (\mathbf{A}_q^R)^T \mathbf{A}_p^I \end{cases}
 \end{aligned} \tag{4.12}$$

Defining,

$$\begin{aligned}
 \mathbf{M} &\triangleq \text{Re}(\mathbf{A}_p^H \mathbf{A}_q) = (\mathbf{A}_p^R)^T \mathbf{A}_q^R + (\mathbf{A}_p^I)^T \mathbf{A}_q^I \\
 \mathbf{N} &\triangleq \text{Im}(\mathbf{A}_p^H \mathbf{A}_q) = (\mathbf{A}_p^R)^T \mathbf{A}_q^I - (\mathbf{A}_p^I)^T \mathbf{A}_q^R,
 \end{aligned} \tag{4.13}$$

then (4.12) shows that $\mathbf{M}$ is skew-symmetric (i.e. $\mathbf{M}^T = -\mathbf{M}$) and $\mathbf{N}$ is symmetric (i.e.

$\mathbf{N}^T = \mathbf{N}$). From the definition of $\mathcal{A}_q$ in (2.10), we get $\mathcal{A}_p^T \mathcal{A}_q = \begin{bmatrix} \mathbf{M} & -\mathbf{N} \\ \mathbf{N} & \mathbf{M} \end{bmatrix}$ and it is

skew-symmetric. Similar conclusions can be drawn on $\mathcal{A}_p^T \mathcal{B}_q$, $\mathcal{B}_p^T \mathcal{A}_q$, and $\mathcal{B}_p^T \mathcal{B}_q$.

In summary, by restricting the dispersion matrices $\{\mathbf{A}, \mathbf{B}\}$ of symbols in different groups in a QO-STBC to satisfy the QOC in (4.10), the $\mathcal{A}_p^T \mathcal{A}_q$, $\mathcal{A}_p^T \mathcal{B}_q$, $\mathcal{B}_p^T \mathcal{A}_q$, and $\mathcal{B}_p^T \mathcal{B}_q$ matrices of the QO-STBC are skew symmetric. This makes the $\mathbf{H}^T \mathbf{H}$ matrix of the QO-STBC block diagonal, and symbols in different groups (sub-matrices of $\mathbf{H}^T \mathbf{H}$) orthogonal to each other. Hence *Theorem 4.1* is proved. ■

4.2.2 Maximal Symbolwise Diversity of a QO-STBC

The QOC in *Theorem 4.1* only specifies the decoupling of symbols during the decoding process of a QO-STBC; it does not guarantee the good decoding performance of the QO-STBC. To ensure that the designed QO-STBC can achieve full diversity, we first employ the maximal symbolwise diversity (MSD) requirement [19,73] to add an additional constraint on the QO-STBC code structure:

$$\mathbf{A}_i^H \mathbf{A}_i = \mathbf{B}_i^H \mathbf{B}_i = \left(\frac{P}{K} \right) \mathbf{I}_{N_T} \quad 1 \leq i \leq K \quad (4.14)$$

where the factor P/K is to normalize the total transmission power. By (4.14), we guarantee full diversity protection for the most probable one-symbol error events, which has the smallest Euclidean distances [19,73]. To further protect against multiple-symbol error events so as to provide full diversity for all error events, constellation rotation can next be applied.

It can be shown that all the QO-STBCs proposed in the literature, such as those in [9,10,11], comply with the algebraic structure QOC specified in (4.10) and the MSD constraint specified in (4.14).

4.3 DECODING OF QO-STBC WITH NOISE WHITENING

In the literature, different groups of researchers have applied different type of decoding schemes on different QO-STBCs. The decoding schemes studied are usually specific to the codes, and cannot be generalized to other QO-STBCs. For example, in [9], the ML decision metric of the QO-STBC proposed therein was derived. In [10], least minimum mean square estimation was used for decoding of the proposed QO-STBC. In [11], the open-loop channel capacities rather than the decoding performance of their proposed QO-STBC with zero-forcing (ZF), minimum mean square error (MMSE) and ML decoding were analyzed. In [44], the diversity properties of ZF and MMSE linear receivers were studied for the QO-STBC proposed in [42]. In [45], sphere decoding (SD) was investigated for the QO-STBC proposed in [9]. However, to our knowledge, there is no study on the general decoding approach of QO-STBC. Furthermore, the benefit of CR on QO-STBC under sub-optimal decoding schemes instead of ML decoding is not reported. In this section, we propose to make a

generalized study on the various decoding options for QO-STBC with CR. To achieve this, we propose to first pre-filter the received signal after match filtering with a noise whitening filter, then investigate the decoding performance of QO-STBC using various optimal or sub-optimal, linear or non-linear, decoding schemes.

4.3.1 Noise Whitening Filter for QO-STBC

The received signal $\tilde{\mathbf{r}}$ of (2.10) after matched filtering by $\mathbf{H}^T$ is shown below:

$$\begin{aligned}\mathbf{H}^T \tilde{\mathbf{r}} &= \sqrt{\rho/N_T} \mathbf{H}^T \mathbf{H} \tilde{\mathbf{c}} + \mathbf{H}^T \tilde{\mathbf{\eta}} \\ \Rightarrow \tilde{\mathbf{r}}_c &= \mathbf{H}_c \tilde{\mathbf{c}} + \tilde{\mathbf{\eta}}_c\end{aligned}\quad (4.15)$$

where $\tilde{\mathbf{r}}_c = \mathbf{H}^T \tilde{\mathbf{r}}$, $\mathbf{H}_c = \sqrt{\rho/N_T} \mathbf{H}^T \mathbf{H}$, $\tilde{\mathbf{\eta}}_c = \mathbf{H}^T \tilde{\mathbf{\eta}}$.

Note that after the matched filtering in (4.15), $\tilde{\mathbf{\eta}}_c$ (with covariance matrix $E[\tilde{\mathbf{\eta}}_c \tilde{\mathbf{\eta}}_c^H] = N_o^2 \mathbf{H}^T \mathbf{H}$) is a colored noise. Hence traditional equalizer derived based on white noise assumption cannot be directly applied to decode the matched filter output $\tilde{\mathbf{r}}_c$, or else the optimal performance cannot be achieved. In this section, we propose to first whiten the noise in $\tilde{\mathbf{r}}_c$, so that standard ML decoding, sphere decoding, or equalization schemes can be applied to decode the whitened received signals. This alleviates the need to design customized colored-noise solutions on a case-by-case basis.

Figure 4.2 Model of QO-STBC Receiver with Noise Whitening

As shown in Figure 4.2, when the noise at the matched filter output is no longer uncorrelated, a whitening filter $\mathbf{H}_w$ can be employed to “whiten” the noise before the signal is processed by the decoder / equalizer $\mathbf{H}_E$. The resultant whitened noise $\tilde{\mathbf{n}}_w = \mathbf{H}_w \tilde{\mathbf{n}}_c = \mathbf{H}_w \mathbf{H}^T \tilde{\mathbf{n}}$ has covariance matrix $E[\tilde{\mathbf{n}}_w \tilde{\mathbf{n}}_w^H] = N_o^2 \mathbf{H}_w \mathbf{H}^T \mathbf{H} \mathbf{H}_w^T$. This implies that we should choose $\mathbf{H}_w$ to obtain $\mathbf{H}_w \mathbf{H}^T \mathbf{H} \mathbf{H}_w^T = \mathbf{I}$ so that $E[\tilde{\mathbf{n}}_w \tilde{\mathbf{n}}_w^H] = N_o^2 \mathbf{I}$. This gives a solution of $\mathbf{H}_w = (\mathbf{H}^T \mathbf{H})^{-1/2}$. Since $\mathbf{H}^T \mathbf{H}$ is symmetric, it can be decomposed according to Symmetric Schur Decomposition [46] as follows:

$$\mathbf{H}^T \mathbf{H} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^T = \sum_{i=1}^m \lambda_i \mathbf{q}_i \mathbf{q}_i^T \quad (4.16)$$

where $\mathbf{Q} \mathbf{Q}^T = \mathbf{I}$, $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_m)$, and m , λ_i and $\mathbf{q}_i$ ($1 \leq i \leq m$) are the rank, eigenvalues and eigenvectors of $\mathbf{H}^T \mathbf{H}$ respectively. So the whitening filter can be shown to be:

$$\begin{aligned} \mathbf{H}_w &= (\mathbf{H}^T \mathbf{H})^{-1/2} \\ &= \sum_{i=1}^m \lambda_i^{-1/2} \mathbf{q}_i \mathbf{q}_i^T \end{aligned} \quad (4.17)$$

and we can combine the equivalent channel matrix $\mathbf{H}$, the matched filter $\mathbf{H}^T$ and the whitening filter $\mathbf{H}_w$ into an equivalent filter $\mathbf{H}_e$:

$$\begin{aligned} \mathbf{H}_e &= \mathbf{H}_w \mathbf{H}^T \mathbf{H} \\ &= (\mathbf{H}^T \mathbf{H})^{-1/2} (\mathbf{H}^T \mathbf{H}) \\ &= (\mathbf{H}^T \mathbf{H})^{1/2} \\ &= \sum_{i=1}^m \lambda_i^{1/2} \mathbf{q}_i \mathbf{q}_i^T \end{aligned} \quad (4.18)$$

From (4.18), it can be noted that $\mathbf{H}_e$ has a similar structure as $\mathbf{H}^T \mathbf{H}$ as shown in (4.16), and the eigenvalues of $\mathbf{H}_e$ is the square root of the eigenvalues of $\mathbf{H}^T \mathbf{H}$. Finally, referring to Figure 4.2, the whitened received signal $\tilde{\mathbf{r}}_w$ can be modeled as:

$$\begin{aligned}
 \tilde{\mathbf{r}}_w &= \mathbf{H}_w \mathbf{H}^T \tilde{\mathbf{r}} \\
 &= \sqrt{\rho/N_T} \mathbf{H}_w \mathbf{H}^T \mathbf{H} \tilde{\mathbf{c}} + \mathbf{H}_w \mathbf{H}^T \tilde{\mathbf{\eta}} \\
 &= \mathbf{H}_e \tilde{\mathbf{c}} + \tilde{\mathbf{\eta}}_w
 \end{aligned} \tag{4.19}$$

Theorem 4.2: The equivalent filter $\mathbf{H}_e$ of a QO-STBC has the same block diagonal structure as $\mathbf{H}^T \mathbf{H}$. Hence the application of noise whitening filter will not alter the decoding complexity of a QO-STBC, i.e. the ML decoding of a QO-STBC defined in *Definition 4.1* can be performed by G independent sub-decoders, and each of the decoder only needs to jointly detect K/G complex symbols.

Proof of *Theorem 4.2:* One can assume the block diagonal matrix $\mathbf{H}^T \mathbf{H}$ of a QO-STBC as follows:

$$\begin{aligned}
 \mathbf{H}^T \mathbf{H} &= \begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}_G \end{bmatrix} \\
 &= \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^T \\
 &= \sum_i \lambda_i \mathbf{q}_i \mathbf{q}_i^T
 \end{aligned} \tag{4.20}$$

Since $\mathbf{H}^T \mathbf{H}$ is symmetry, $\mathbf{M}_i$ (for $1 \leq i \leq G$) are all symmetric matrices. So each of them can be expressed as:

$$\mathbf{M}_i^T \mathbf{M}_i = \mathbf{Q}_{M_i} \mathbf{\Lambda}_{M_i} \mathbf{Q}_{M_i}^T \tag{4.21}$$

It can be easily proved that:

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{M_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{M_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_{M_G} \end{bmatrix}$$

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_{M_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_{M_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Lambda}_{M_G} \end{bmatrix} \quad (4.22)$$

From (4.18) and (4.16)

$$\begin{aligned} \mathbf{H}_e &= (\mathbf{H}^T \mathbf{H})^{1/2} \\ &= \mathbf{Q} \sqrt{\mathbf{\Lambda}} \mathbf{Q}^T \\ &= \begin{bmatrix} \mathbf{Q}_{M_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{M_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_{M_G} \end{bmatrix} \begin{bmatrix} \sqrt{\mathbf{\Lambda}_{M_1}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sqrt{\mathbf{\Lambda}_{M_2}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \sqrt{\mathbf{\Lambda}_{M_G}} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_{M_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{M_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_{M_G} \end{bmatrix}^T \\ &= \begin{bmatrix} \mathbf{M}'_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}'_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{M}'_G \end{bmatrix} \end{aligned} \quad (4.23)$$

Since the equivalent channel $\mathbf{H}_e$ is also block diagonal with G sub-matrices $(\mathbf{M}'_1, \mathbf{M}'_2, \dots, \mathbf{M}'_G)$. The whitened received signal $\tilde{\mathbf{r}}_w$ in (4.19) can be separated into G smaller sub-blocks, and the ML decoding of each sub-block can be performed independently. This proves that the whitening filter will not alter the decoding complexity of a QO-STBC, and the ML decoding of a QO-STBC can be separately performed by G independent sub-decoders. ■

Take the QO-STBC codeword $\mathbf{J4}$ in (4.1) as an example. According to [9], $\mathbf{J4}$ has two quasi-orthogonal groups hence its $\tilde{\mathbf{r}}_{\mathbf{w}}$ in (4.24) can be separated into two independent parts as shown in (4.25).

$$\begin{aligned} \tilde{\mathbf{r}}_{\mathbf{w}} &= \mathbf{H}_{\mathbf{e}} \tilde{\mathbf{c}} + \tilde{\boldsymbol{\eta}}_{\mathbf{w}} \\ \begin{bmatrix} \tilde{\mathbf{r}}_{\mathbf{w}_1} \\ \tilde{\mathbf{r}}_{\mathbf{w}_2} \end{bmatrix} &= \begin{bmatrix} \mathbf{M}'_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{M}'_2 \end{bmatrix} \tilde{\mathbf{c}} + \begin{bmatrix} \tilde{\boldsymbol{\eta}}_{\mathbf{w}_1} \\ \tilde{\boldsymbol{\eta}}_{\mathbf{w}_2} \end{bmatrix} \end{aligned} \quad (4.24)$$

$$\begin{aligned} \tilde{\mathbf{r}}_{\mathbf{w}_1} &= \mathbf{M}'_1 \begin{bmatrix} c_1^R \\ c_1^I \\ c_4^R \\ c_4^I \end{bmatrix} + \tilde{\boldsymbol{\eta}}_{\mathbf{w}_1} = \mathbf{M}'_1 \mathbf{c}'_1 + \tilde{\boldsymbol{\eta}}_{\mathbf{w}_1} \\ \tilde{\mathbf{r}}_{\mathbf{w}_2} &= \mathbf{M}'_2 \begin{bmatrix} c_2^R \\ c_2^I \\ c_3^R \\ c_3^I \end{bmatrix} + \tilde{\boldsymbol{\eta}}_{\mathbf{w}_2} = \mathbf{M}'_2 \mathbf{c}'_2 + \tilde{\boldsymbol{\eta}}_{\mathbf{w}_2} \end{aligned} \quad (4.25)$$

where $\tilde{\mathbf{c}} = [c_1^R \ c_1^I \ c_4^R \ c_4^I \ c_2^R \ c_2^I \ c_3^R \ c_3^I]^T$ are the four complex symbols transmitted in $\mathbf{J4}$, $\mathbf{M}'_1$ and $\mathbf{M}'_2$ are the sub-matrices of the equivalent filter of $\mathbf{J4}$, and $\tilde{\boldsymbol{\eta}}_{\mathbf{w}_1}$ and $\tilde{\boldsymbol{\eta}}_{\mathbf{w}_2}$ are whitened noise vectors.

Each part in (4.25) has a smaller matrix size, hence this simplifies the decoding complexity and makes parallel processing possible. For simplicity, each of the decoupled equations in (4.25) can be expressed in a general form as shown below:

$$\mathbf{r}' = \mathbf{M}' \mathbf{c}' + \boldsymbol{\eta}' \quad (4.26)$$

where $\mathbf{r}' = \tilde{\mathbf{r}}_{\mathbf{w}_1}$ or $\tilde{\mathbf{r}}_{\mathbf{w}_2}$, $\mathbf{M}' = \mathbf{M}'_1$ or $\mathbf{M}'_2$, $\mathbf{c}' = \mathbf{c}'_1$ or $\mathbf{c}'_2$, and $\boldsymbol{\eta}' = \tilde{\boldsymbol{\eta}}_{\mathbf{w}_1}$ or $\tilde{\boldsymbol{\eta}}_{\mathbf{w}_2}$.

If constellation rotation is applied, we can express (4.25) as:

$$\tilde{\mathbf{r}}_{\mathbf{w}_1} = \mathbf{M}'_1 \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 & 0 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & \cos(\theta_4) & \sin(\theta_4) \\ 0 & 0 & -\sin(\theta_4) & \cos(\theta_4) \end{bmatrix} \begin{bmatrix} c_1^R \\ c_1^I \\ c_4^R \\ c_4^I \end{bmatrix} + \tilde{\boldsymbol{\eta}}_{\mathbf{w}_1} \quad (4.27)$$

where $\theta_1 = 0$ and $\theta_4 = \pi/4$ are the optimum CR angles shown in (4.9). Similar to (4.26), one can re-express (4.27) as:

$$\mathbf{r}' = \mathbf{M}'' \mathbf{c}' + \boldsymbol{\eta}' \quad (4.28)$$

where $\mathbf{M}'' = \mathbf{M}' \boldsymbol{\Theta}$,

$$\boldsymbol{\Theta} = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 & 0 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & \cos(\theta_4) & \sin(\theta_4) \\ 0 & 0 & -\sin(\theta_4) & \cos(\theta_4) \end{bmatrix} \quad (4.29)$$

and $\boldsymbol{\Theta}$ is a unitary matrix, i.e. $\boldsymbol{\Theta}^T \boldsymbol{\Theta} = \mathbf{I}$.

Since (4.28) is similar in form to (4.26), we will use the signal model (4.26) in the following discussions. The QO-STBC examples considered will be the double-symbol-decodable **J4** in (4.1) and quad-symbol-decodable **TBH8** in (4.3). The **J8** code in (4.2) will not be discussed, as it is also double-symbol-decodable, hence we expect it to have similar behavior as **J4**.

Remark

By comparing the STBC received signal in (2.10) and the signal of QO-STBC after matched filtering and noise whitening in (4.26), we can see that $\tilde{\mathbf{c}}$ in (2.10) is of dimension $2K \times 1$, whereas the $\mathbf{c}'$ in (4.26) is of dimension $2K/G \times 1$. Hence the matrices in (4.26) have smaller dimension than those in (2.10) because the QO-STBC symbols in different groups are orthogonal and hence separable. Such reduction in matrix dimension leads to simplification of the QO-STBC decoding operations, which will be discussed in the next section.

4.3.2 QO-STBC Decoder Schemes

In this section, two major types of decoders are considered. They are the symbol-by-symbol decoder and the joint-symbol decoder.

A) Symbol-by-Symbol Decoders

(1) Zero Forcing (ZF)

The ZF equalization matrix is $\mathbf{H}_{\text{E(ZF)}} = (\mathbf{M}')^{-1}$, where $\mathbf{M}'$ is defined in (4.26). It is well known that the ZF equalizer has noise amplification effect, hence it is expected to perform badly. Nonetheless, due to the quasi-orthogonal structure of **J4**, we only need to calculate the inverse of a 4×4 $\mathbf{M}'$ matrix (as **J4** has two groups with four real symbols per group), rather than the inverse of a 8×8 matrix (in the case of a fully non-orthogonal STBC). Similarly, the ZF equalization matrix of **TB8** only requires the inversion of an 8×8 $\mathbf{M}'$ matrix rather a 16×16 matrix.

(2) Minimum Mean Square Error (MMSE)

The MMSE equalization matrix is $\mathbf{H}_{\text{E(MMSE)}} = \left((\mathbf{M}')^T \mathbf{M}' + \frac{1}{\rho} \mathbf{I} \right)^{-1} (\mathbf{M}')^T$, where ρ is the signal-to-noise ratio at the receiver [47]. By taking ρ into consideration, the MMSE linear equalizer is able to reduce the noise amplification effect present in the ZF.

(3) Decision Feedback Equalizer (DFE)

DFE essentially performs successive interference cancellation, i.e. a symbol is first estimated based on a linear equalizer, then the estimated symbol is cancelled off from the other received symbols through a feedback filter [48]. This process is iterated until all transmitted symbols are adequately estimated. DFE is a non-linear equalization

approach due to its feedback cancellation mechanism. Although DFE generally achieves better performance than linear equalizers with a small increase in complexity, it suffers from the problem of error propagation, i.e. an initial estimation error will give rise to a burst of continuous errors. Depending on the type of linear equalizer used for the initial estimation, a DFE can be classified as either ZF-DFE or MMSE-DFE.

B) Joint-Symbol Decoder

The equalization methods discussed earlier, namely ZF, MMSE, ZF-DFE and MMSE-DFE, are not optimal. This is because different symbols in the received QO-STBC signal are cross-coupled, so intuitively it is better to estimate the entire block of symbols simultaneously, instead of estimating one symbol at a time or using the estimated symbols to estimate other symbols. In this way, we aim at minimizing the probability of choosing the wrong block of symbols. The optimal receiver structure to achieve this goal is commonly referred to as the maximum-likelihood (ML) receiver.

(1) Maximum-Likelihood (ML) Decoder

In a ML receiver, $\mathbf{c}'$ is chosen such that the following metric is minimized:

$$\text{Min}_{\mathbf{c}'} \|\mathbf{r}' - \mathbf{M}'\mathbf{c}'\|^2 \quad (4.30)$$

where $\mathbf{r}'$, $\mathbf{M}'$, $\mathbf{c}'$ are defined in (4.26).

With the noise-whitening filter approach, we can rely on (4.30) to perform ML decoding by jointly detecting the symbols in the same group only, this eliminate the need to analytically derive the dedicated ML decoding metric for a QO-STBC, such as the one shown in (4.4) for **J4** [9]. As the constellation size and antenna number of the QO-STBC increase, the derivation of dedicated ML metric can be expected to be very tedious.

(2) Sphere Decoding (SD)

Although the ML decoder for QO-STBC performs joint detection on only subsets of the transmitted symbols, the complexity of such joint detection could still grow exponentially when a large modulation constellation set is used. SD can be used to alleviate this problem [49].

SD attempts to minimize the same decision metric as ML, but the search is restricted to points of the lattice found within a sphere centered around the received signal $\mathbf{r}'$ in (4.26). Each time a valid lattice point is found, the search radius is reduced so that the newly discovered lattice point lies on the surface of the new sphere. This process is repeated until a solution is found. This greatly reduces the search space and hence computational complexity of the decode operation compared to full ML search. The readers are referred to [49] and the references therein for more detailed discussions on SD.

4.3.3 Simulation Results

In this section, four different QO-STBCs are simulated using various equalization and decoding methods described earlier and their performance will be compared. The QO-STBCs used in the simulations are **J4** (for four transmit antennas) and **TBH8** (for eight transmit antennas) without CR, as well as **J4** and **TBH8** with optimum CR, denoted as **J4_CR** and **TBH8_CR** respectively. In **J4_CR**, the code symbols c_3, c_4 are rotated by 45° with respect to c_1 and c_2 [14]; while in **TBH8_CR**, the code symbols c_2, c_7 are rotated by 45° ; c_3, c_4 are rotated by -22.5° and c_5, c_6 are rotate by 22.5° with respect to c_1 and c_8 [5]. Half-rate O-STBCs **G4C** and **G8C** from [7] are used for benchmarking the size-four and size-eight QO-STBCs respectively. For fair

comparison, we standardize the spectral efficiency of all codes to 2 bits/sec/Hz, hence 16QAM constellation is used for the half-rate O-STBC, while QPSK is used for the full-rate QO-STBCs.

Figure 4.3 shows the block error rate (BLER) performance of QO-STBC for four transmit antennas under various equalization schemes. As shown in Figure 4.3(a) for **J4** without CR, the DFE schemes (MMSE DFE and ZF DFE) always have an improvement of BLER over the linear equalization schemes (MMSE and ZF). In particular, the MMSE_DFE performs very close to the optimum ML performance. Next, the performance of SD is virtually identical to the optimum ML performance. This agrees with [49] that SD can match the decoding performance of ML if the initial radius for the lattice search is appropriately selected. Compared to QO-STBC, the half-rate O-STBC **G4C** performs worse in BLER because for the same spectral efficiency of 2 bits/sec/Hz, **G4C** uses 16QAM constellation while QO-STBC employs QPSK constellation that has a larger Euclidean distance.

After applying CR, Figure 4.3(b) shows that the decoding performance of **J4_CR** is greatly improved over the performance of **J4** for ML or SD decoders. By achieving the same asymptotic slope as the O-STBC **G4C**, the **J4_CR** is verified to have achieved full diversity gain with ML decoding or SD. However, the performance in Figure 4.3(b) **J4_CR** is the same as that of **J4** without CR in Figure 4.3(a) if linear equalizer or DFE is used. This means that QO-STBC can only achieve full diversity with CR when ML detection or SD is used.

Figure 4.3 BLER Performance of **J4** and **J4_CR** at 2 bits/s/Hz

Figure 4.4 shows the BLER decoding performance of QO-STBC for eight transmit antennas under various equalization schemes. In Figure 4.4(a), it can be observed that the MMSE_DFE's performance is not as close to the ML performance as in Figure 4.3(a). This is because there are now four complex symbols rather than two complex symbols to be jointly decoded in **TBH8**, and the error propagation problem of DFE becomes more significant. When optimal CR is used, the performance of **TBH8_CR** under ML decoding or SD is again significantly better than all the equalization techniques because the ML decoding is able to achieve full diversity but the linear equalizer or DFE is not able to.

Figure 4.4 BLER Performance of **TBH8** and **TBH8_CR** at 2 bits/s/Hz

The reason why QO-STBC with and without CR give the same performance in the symbol-by-symbol equalization schemes can be explained as follows. We use the ZF equalizer as an example due to its simplicity, and we re-express the received signal of QO-STBC after noise whitening in (4.26) (denoted as $\mathbf{r}'$ in (4.31)) and received signal of QO-STBC with CR after noise whitening in (4.28) (denoted as $\mathbf{r}'_{\text{CR}}$ in (4.32)) as follows:

$$\mathbf{r}' = \mathbf{M}'\mathbf{c}' + \boldsymbol{\eta}' \quad (4.31)$$

$$\mathbf{r}'_{\text{CR}} = \mathbf{M}''\mathbf{c}' + \boldsymbol{\eta}' \quad (4.32)$$

By applying the ZF filter $\mathbf{H}_{\text{E(ZF)}} = (\mathbf{M}')^{-1}$ or $(\mathbf{M}'')^{-1}$, we obtain:

$$(\mathbf{M}')^{-1}\mathbf{r}' = \mathbf{c}' + (\mathbf{M}')^{-1}\boldsymbol{\eta}' \quad (4.33)$$

$$(\mathbf{M}'')^{-1}\mathbf{r}'_{\text{CR}} = \mathbf{c}' + (\mathbf{M}'')^{-1}\boldsymbol{\eta}' \quad (4.34)$$

Since $\mathbf{M}'' = \mathbf{M}'\mathbf{\Theta}$, and $\mathbf{\Theta}$ is unitary, the decision statistics in (4.33) and (4.34) have exactly the same distribution. This explains why ZF gives the same performance for QO-STBC with and without CR. Similar explanations apply to MMSE and the other symbol-by-symbol detection schemes.

4.3.4 Section Summary

Noise whitening filter is proposed to facilitate the decoding of QO-STBC using standard linear equalizers, DFE, ML and SD. This simplifies the development of QO-STBC decoder as no dedicated decision metric need to be derived. Simulation results show that the decoding error probability of QO-STBC generally improves in the order of linear equalization, DFE, SD and ML decoding. Particularly, sphere decoding is always able to match the ML performance with reduced decoding complexity, while MMSE-DFE can achieve near-ML performance when the code is small and has no CR. The advantages of CR on QO-STBC can only be reaped by ML decoding or SD, but not linear equalizer and DFE schemes.

4.4 IMPROVING QO-STBC WITH GCLT

As explained earlier, QO-STBCs can achieve full transmit diversity with the help of CR [5,12-14,43]. To date, full-rate full-diversity QO-STBC for four transmit antennas requires joint detection of at least two complex symbols [12,13,14]. For eight transmit antennas, full-diversity QO-STBC requires joint detection of at least two complex symbols at a code rate of 3/4 [14], or four complex symbols at full rate [5].

In this section, we shall show that the number of symbols required for the joint detection of the existing full-diversity QO-STBCs with square or rectangular QAM constellations can actually be halved if, instead of CR, a novel “*Group-Constrained Linear Transformation* (GCLT)” is used to optimize the original QO-STBCs. To explain the principles of the proposed GCLT, we will examine the algebraic structure of existing QO-STBCs and study the impact of CR on their decoding complexity. We will also derive analytically the optimal *GCLT parameters* for the full-rate QO-STBC **J4** in (4.1) and rate-3/4 QO-STBC **J8** in (4.2) with square or rectangular QAM constellation, while the optimum GCLT parameters for **TBH8** will be obtained by computer search. The bit error rate (BER) performance of QO-STBC designed using CR and GCLT will then be compared.

4.4.1 Signal Model

Since a square or rectangular QAM symbol can be treated as two independent real PAM symbols, we can simplify the signal model of a STBC in (2.7) as follows:

$$\begin{aligned}
 \mathbf{C} &= \sum_{i=1}^K \left(c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i \right) \\
 &= \sum_{i=1}^K (d_i \mathbf{A}_i + d_{K+i} \mathbf{A}_{K+i}) \quad \text{where } \mathbf{A}_{k+p} = j \mathbf{B}_p \quad \forall 1 \leq p \leq K \quad (4.35) \\
 &= \sum_{k=1}^{2K} (d_k \mathbf{A}_k)
 \end{aligned}$$

where the complex data symbols are re-modeled as $c_i = d_i + j d_{K+i}$ for $1 \leq i \leq K$. Hence transmitting K regular square or rectangular QAM symbols is equivalent to transmitting $2K$ PAM symbols. With these modulation schemes, the K sets of $\{\mathbf{A}, \mathbf{B}\}$ dispersion matrices (each for a different complex symbol c_i) of a STBC can be mapped to $2K$ sets of $\{\mathbf{A}\}$ dispersion matrices (each for a different real symbol d_k) as shown in (4.35). By such modeling, the QOC in (4.10) can be simplified as follows:

Proposition 4.1: For the real symbol d_p defined in the simplified STBC model in (4.35) to be orthogonal to another real symbol d_q in the same STBC, their dispersion matrices $\mathbf{A}_p$ and $\mathbf{A}_q$ must follow the following version of *Quasi-Orthogonality Constraint* (QOC):

$$\mathbf{A}_p^H \mathbf{A}_q = -\mathbf{A}_q^H \mathbf{A}_p \quad \text{for } 1 \leq p, q \leq 2K \text{ and } q \notin \mathcal{G}(p) \quad (4.36)$$

Proof of *Proposition 4.1*: (4.36) can be readily obtained by setting $\mathbf{A}_{K+i} = j\mathbf{B}_i$ for $1 \leq i \leq K$ in (4.10). ■

In this section, we will adopt the simplified STBC signal model in (4.35) and simplified QOC in (4.36) in the discussions of GCLT.

4.4.2 Effect of CR on the Decoding Complexity of QO-STBC

We now examine the quasi-orthogonality relationship between the eight dispersion matrices of the code **J4** [9] (listed in (4.37) as matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_8$) based on the QOC in (4.36).

$$\begin{aligned}
 \mathbf{A}_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{A}_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} & \mathbf{A}_3 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \\
 \mathbf{A}_4 &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} & \mathbf{A}_5 &= \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & -j & 0 & 0 \\ 0 & 0 & -j & 0 \\ 0 & 0 & 0 & j \end{bmatrix} & \mathbf{A}_6 &= \begin{bmatrix} 0 & j & 0 & 0 \\ j & 0 & 0 & 0 \\ 0 & 0 & 0 & -j \\ 0 & 0 & -j & 0 \end{bmatrix} \\
 \mathbf{A}_7 &= \begin{bmatrix} 0 & 0 & j & 0 \\ 0 & 0 & 0 & -j \\ j & 0 & 0 & 0 \\ 0 & -j & 0 & 0 \end{bmatrix} & \mathbf{A}_8 &= \begin{bmatrix} 0 & 0 & 0 & j \\ 0 & 0 & j & 0 \\ 0 & j & 0 & 0 \\ j & 0 & 0 & 0 \end{bmatrix}
 \end{aligned} \tag{4.37}$$

The fulfillment of QO-Constraint of $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_8$ is shown in Table 4.1(a). For example, Table 4.1(a) shows that $\mathbf{A}_1$ is orthogonal to all other dispersion matrices except $\mathbf{A}_4$. Likewise, each of $\mathbf{A}_2, \dots, \mathbf{A}_8$ are orthogonal to all but one of the other dispersion matrices. By re-arranging the rows and columns of Table 4.1(a), we obtain Table 4.1(b), which shows that the dispersion matrices $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_8$ of the code **J4** can be grouped into four orthogonal groups: $\{\mathbf{A}_1, \mathbf{A}_4\}$, $\{\mathbf{A}_2, \mathbf{A}_3\}$, $\{\mathbf{A}_5, \mathbf{A}_8\}$, $\{\mathbf{A}_6, \mathbf{A}_7\}$. Since there are only two non-orthogonal dispersion matrices (modulating two real symbols) in each group, the ML decoding of **J4** can be achieved by joint detection of two real symbols, instead of two complex symbols as reported in [9].

Table 4.1 Fulfillment of QOC for the dispersion matrices of **J4**

(a) QOC fulfillment for **J4**

	A₁	A₂	A₃	A₄	A₅	A₆	A₇	A₈
A₁	X			X				
A₂		X	X					
A₃		X	X					
A₄	X			X				
A₅					X			X
A₆						X	X	
A₇						X	X	
A₈					X			X

(b) Re-arrange rows and columns of (a)

	A₁	A₄	A₂	A₃	A₅	A₈	A₆	A₇
A₁	X	X						
A₄	X	X						
A₂			X	X				
A₃			X	X				
A₅					X	X		
A₈					X	X		
A₆							X	X
A₇							X	X

Legend:

X

 QO-Constraint is fulfilled

X

 QO-Constraint is not fulfilled

Table 4.2(a) examines the fulfillment of QOC for the dispersion matrices (listed in (4.38) as matrices $\mathbf{A}_{\text{CR}_1}, \mathbf{A}_{\text{CR}_2}, \dots, \mathbf{A}_{\text{CR}_8}$) of $\mathbf{J4_CR}$, the constellation-rotated $\mathbf{J4}$.

$$\begin{aligned}
 \mathbf{A}_{\text{CR}_1} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \mathbf{A}_{\text{CR}_2} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \\
 \mathbf{A}_{\text{CR}_3} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1+j & 0 \\ 0 & 0 & 0 & 1-j \\ -1+j & 0 & 0 & 0 \\ 0 & -1-j & 0 & 0 \end{bmatrix} & \mathbf{A}_{\text{CR}_4} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 1+j \\ 0 & 0 & -1+j & 0 \\ 0 & -1+j & 0 & 0 \\ 1+j & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{A}_{\text{CR}_5} &= \begin{bmatrix} j & 0 & 0 & 0 \\ 0 & -j & 0 & 0 \\ 0 & 0 & -j & 0 \\ 0 & 0 & 0 & j \end{bmatrix} & \mathbf{A}_{\text{CR}_6} &= \begin{bmatrix} 0 & j & 0 & 0 \\ j & 0 & 0 & 0 \\ 0 & 0 & 0 & -j \\ 0 & 0 & -j & 0 \end{bmatrix} \\
 \mathbf{A}_{\text{CR}_7} &= \frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1+j & 0 \\ 0 & 0 & 0 & -1+j \\ 1-j & 0 & 0 & 0 \\ 0 & -1-j & 0 & 0 \end{bmatrix} & \mathbf{A}_{\text{CR}_8} &= \frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 1+j \\ 0 & 0 & 1-j & 0 \\ 0 & 1-j & 0 & 0 \\ 1+j & 0 & 0 & 0 \end{bmatrix}
 \end{aligned}
 \tag{4.38}$$

By re-arranging the rows and columns of Table 4.2(a) to obtain Table 4.2(b), it is clear that the dispersion matrices of $\mathbf{J4_CR}$ can be grouped into only two orthogonal groups: $\{\mathbf{A}_{\text{CR}_1}, \mathbf{A}_{\text{CR}_4}, \mathbf{A}_{\text{CR}_5}, \mathbf{A}_{\text{CR}_8}\}$ and $\{\mathbf{A}_{\text{CR}_2}, \mathbf{A}_{\text{CR}_3}, \mathbf{A}_{\text{CR}_6}, \mathbf{A}_{\text{CR}_7}\}$. Since there are four non-orthogonal dispersion matrices (modulating four real symbols) in each group, the ML decoder for $\mathbf{J4_CR}$ needs to jointly decode four real symbols, an increase from the two real symbols required by $\mathbf{J4}$ before CR.

Table 4.2 Fulfillment of QOC for the dispersion matrices of **J4_CR**(a) QO-Constraint fulfillment for **J4_CR**

	$\mathbf{A}_{CR1}$	$\mathbf{A}_{CR2}$	$\mathbf{A}_{CR3}$	$\mathbf{A}_{CR4}$	$\mathbf{A}_{CR5}$	$\mathbf{A}_{CR6}$	$\mathbf{A}_{CR7}$	$\mathbf{A}_{CR8}$
$\mathbf{A}_{CR1}$	X			X				
$\mathbf{A}_{CR2}$		X	X					
$\mathbf{A}_{CR3}$		X	X			X	X	
$\mathbf{A}_{CR4}$	X			X	X			X
$\mathbf{A}_{CR5}$				X	X			X
$\mathbf{A}_{CR6}$			X			X	X	
$\mathbf{A}_{CR7}$			X			X	X	
$\mathbf{A}_{CR8}$				X	X			X

(b) Rearrange rows and columns of (a)

	$\mathbf{A}_{CR1}$	$\mathbf{A}_{CR4}$	$\mathbf{A}_{CR5}$	$\mathbf{A}_{CR8}$	$\mathbf{A}_{CR2}$	$\mathbf{A}_{CR3}$	$\mathbf{A}_{CR6}$	$\mathbf{A}_{CR7}$
$\mathbf{A}_{CR1}$	X	X						
$\mathbf{A}_{CR4}$	X	X	X	X				
$\mathbf{A}_{CR5}$		X	X	X				
$\mathbf{A}_{CR8}$		X	X	X				
$\mathbf{A}_{CR2}$					X	X		
$\mathbf{A}_{CR3}$					X	X	X	X
$\mathbf{A}_{CR6}$						X	X	X
$\mathbf{A}_{CR7}$						X	X	X

Legend: $\square$ QO-Constraint is fulfilled
 $\times$ QO-Constraint is not fulfilled

It can similarly be shown that for the other QO-STBCs such as **J8** and **TBH8**, the number of real symbols required for joint detection is doubled after CR, as summarized in Table 4.3 below.

Table 4.3 Number of real symbols required for joint detection for QO-STBC with square and rectangular QAM

Non full-diversity QO-STBC		Full-diversity QO-STBC with CR	
J4	2	J4_CR	4
J8	2	J8_CR	4
TBH8	4	TBH8_CR	8

4.4.3 Group-Constrained Linear Transformation

A) Definition of GCLT

In order to enable a QO-STBC to achieve full diversity and maximum coding gain while maintaining the original symbol groupings and hence decoding complexity, we propose the *Group-Constrained Linear Transformation* (GCLT) as defined in *Proposition 4.2*.

Proposition 4.2: By linearly combining the dispersion matrices $\mathbf{A}_i$ within the same group in accordance with (4.39) and (4.40), the resultant dispersion matrices, $\mathbf{A}_{\text{LT}_i}$, will satisfy the QOC stated in (4.36) with the same symbol grouping structure as the original matrices $\mathbf{A}_i$.

$$\tilde{\mathbf{A}}_{\text{LT}_i} = \sum_{k \in \mathcal{G}(i)} \alpha_{i,k} \mathbf{A}_k \quad \forall 1 \leq i \leq 2K \quad (4.39)$$

$$\mathbf{A}_{\text{LT}_i} = \phi_i \tilde{\mathbf{A}}_{\text{LT}_i} \quad \forall 1 \leq i \leq 2K \quad (4.40)$$

where $\alpha_{i,p}$ are the *GCLT parameters* and are real constants, and the scalar factor

$\phi_i = \sqrt{\frac{PN_T}{K} / \text{Tr}(\tilde{\mathbf{A}}_{\text{LT}_i}^H \tilde{\mathbf{A}}_{\text{LT}_i})}$ serves to ensure that every dispersion matrix of the QO-STBC after GCLT have equal transmission power, i.e. satisfying the power distribution constraint in (2.11)(ii).

Proof of Proposition 4.2: Applying (4.36) with $\mathbf{A}_p$ replaced by $\mathbf{A}_{\text{LT}_p}$ and $\mathbf{A}_q$ replaced by $\mathbf{A}_{\text{LT}_q}$ gives

$$\begin{aligned}
 & \mathbf{A}_{\text{LT}_p}^H \mathbf{A}_{\text{LT}_q} + \mathbf{A}_{\text{LT}_q}^H \mathbf{A}_{\text{LT}_p} \quad 1 \leq p, q \leq 2K \quad \text{and} \quad p \notin \mathcal{G}(q) \\
 &= \phi_p \left(\sum_{i \in \mathcal{G}(p)} (\alpha_{p,i} \mathbf{A}_i) \right)^H \phi_q \left(\sum_{k \in \mathcal{G}(q)} (\alpha_{q,k} \mathbf{A}_k) \right) + \\
 & \quad \phi_q \left(\sum_{k \in \mathcal{G}(q)} (\alpha_{q,k} \mathbf{A}_k) \right)^H \phi_p \left(\sum_{i \in \mathcal{G}(p)} (\alpha_{p,i} \mathbf{A}_i) \right) \quad (4.41) \\
 &= \phi_p \phi_q \left(\sum_{i \in \mathcal{G}(p)} \sum_{k \in \mathcal{G}(q)} (\alpha_{p,i} \alpha_{q,k} \mathbf{A}_i^H \mathbf{A}_k) \right) + \phi_q \phi_p \left(\sum_{k \in \mathcal{G}(q)} \sum_{i \in \mathcal{G}(p)} (\alpha_{q,k} \alpha_{p,i} \mathbf{A}_k^H \mathbf{A}_i) \right) \\
 &= \phi_p \phi_q \sum_{i \in \mathcal{G}(p)} \sum_{k \in \mathcal{G}(q)} \left(\alpha_{p,i} \alpha_{q,k} \underbrace{(\mathbf{A}_i^H \mathbf{A}_k + \mathbf{A}_k^H \mathbf{A}_i)}_{=0 \text{ as per QOC (4.36)}} \right) \\
 &= 0
 \end{aligned}$$

Since the above expression is equal to zero, the dispersion matrices $\{\mathbf{A}_{\text{LT}}\}$ satisfy the QOC (4.36) as long matrices $\{\mathbf{A}\}$ do, hence *Proposition 4.2* is proven. ■

We would like to mention that to obtain more general results, the power normalization operation in (4.40) could be relaxed to ensure that the total transmission power of the dispersion matrices satisfies (2.11)(i) instead of (2.11)(ii). This is a simple variation, so it will not be discussed in this thesis.

B) Optimization of GCLT parameters

The GCLT parameters α in (4.39) can be chosen such that $\{\mathbf{A}_{\text{LT}}\}$ satisfy certain chosen performance optimization criteria, such as the rank and determinant criteria discussed in Section 2.4. To provide a systematic way to optimize the GCLT parameters, the *Multi-dimensional Lattice Rotation* (MLR) technique in [50] can be employed. For simplicity, consider a QO-STBC with two real symbols per group (such as the code **J4**). Assume that the dispersion matrices $\mathbf{A}_p$ and $\mathbf{A}_q$ are in the same group, i.e. $\{p, q\} = \mathcal{G}(p) = \mathcal{G}(q)$, the GCLT of these dispersion matrices can be expressed as follows:

$$\begin{bmatrix} \tilde{\mathbf{A}}_{\text{LT}_p} \\ \tilde{\mathbf{A}}_{\text{LT}_q} \end{bmatrix} = \left(\mathbf{I}_{P \times P} \otimes \begin{bmatrix} \alpha_{p,p} & \alpha_{p,q} \\ \alpha_{q,p} & \alpha_{q,q} \end{bmatrix} \right) \begin{bmatrix} \mathbf{A}_p \\ \mathbf{A}_q \end{bmatrix} = (\mathbf{I}_{P \times P} \otimes \mathbf{L}_{\text{MLR}}) \begin{bmatrix} \mathbf{A}_p \\ \mathbf{A}_q \end{bmatrix} \quad (4.42)$$

where $\otimes$ represents the Kronecker product and $\mathbf{L}_{\text{MLR}}$ is an orthogonal matrix as specified in [50]. For example, for a two-dimensional case³, $\mathbf{L}_{\text{MLR}}$ maps four GCLT parameters into one variable θ using:

$$\mathbf{L}_{\text{MLR}} = \begin{bmatrix} \alpha_{p,p} & \alpha_{p,q} \\ \alpha_{q,p} & \alpha_{q,q} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \quad (4.43)$$

This streamlines the search or analysis of the optimum GCLT parameters because it reduces the number of optimization variables from four (i.e. $\alpha_{p,p}$, $\alpha_{p,q}$, $\alpha_{q,p}$, $\alpha_{q,q}$) to one (i.e. θ).

³ For extension to more than two dimensions, readers are referred to [50] for more details.

(1) GCLT of J4

Denoting the **J4** code after GCLT as **J4_LT**, the optimization of its GCLT parameters can be derived as follows. The determinant expression for the codeword distance matrix of **J4_LT** is:

$$\det = \left[\begin{array}{l} ((\tilde{\Delta}_1 + \tilde{\Delta}_4)^2 + (\tilde{\Delta}_2 - \tilde{\Delta}_3)^2 + (\tilde{\Delta}_5 + \tilde{\Delta}_8)^2 + (\tilde{\Delta}_6 - \tilde{\Delta}_7)^2) \times \\ ((\tilde{\Delta}_1 - \tilde{\Delta}_4)^2 + (\tilde{\Delta}_2 + \tilde{\Delta}_3)^2 + (\tilde{\Delta}_5 - \tilde{\Delta}_8)^2 + (\tilde{\Delta}_6 + \tilde{\Delta}_7)^2) \end{array} \right]^2 \quad (4.44)$$

where $\tilde{\Delta}_p = \Delta_p \cos \theta - \Delta_q \sin \theta$ and $\tilde{\Delta}_q = \Delta_p \sin \theta + \Delta_q \cos \theta$ for $\{(p,q)\} \in \{(1,4), (2,3), (5,8), (6,7)\}$, and Δ_p represents the possible error in the real PAM symbol d_p . Note that (p, q) denotes the symbol grouping in **J4** as indicated in Table 4.1.

Since the symbol grouping of **J4_LT** is such that d_1 and d_4 are in the same group, and they are independent of (i.e. orthogonal to) the other symbols in the ML decoding operation, the worst-case (i.e. minimum) determinant value in (4.44) can be simplified by assuming that only d_1 and d_4 have errors and the other symbols are error-free [12]:

$$\begin{aligned} \det &= \left[(\tilde{\Delta}_1 + \tilde{\Delta}_4)^2 \times (\tilde{\Delta}_1 - \tilde{\Delta}_4)^2 \right]^2 \text{ assuming that } \tilde{\Delta}_2 = \tilde{\Delta}_3 = \tilde{\Delta}_5 = \tilde{\Delta}_6 = \tilde{\Delta}_7 = \tilde{\Delta}_8 = 0 \\ &= \left[(\tilde{\Delta}_1)^2 - (\tilde{\Delta}_4)^2 \right]^4 \\ &= \left[(\Delta_1 \cos \theta - \Delta_4 \sin \theta)^2 - (\Delta_1 \sin \theta + \Delta_4 \cos \theta)^2 \right]^4 \\ &= \left[\Delta_1^2 \cos(2\theta) - 2\Delta_1\Delta_4 \sin(2\theta) - \Delta_4^2 \cos(2\theta) \right]^4 \end{aligned} \quad (4.45)$$

4-QAM Constellation

Consider first the optimization of GCLT parameters of **J4_LT** with 4-QAM constellation. The I and Q components of a 4-QAM symbol can be viewed as two independent 2-PAM symbols. Hence $\Delta_1, \Delta_4 \in \{0, \pm d_{\min}\}$ where $d_{\min}$ is the minimum Euclidian distance between two constellation points as shown in Figure 4.5, and Δ_1 and Δ_4 cannot be both zero in (4.45). To maximize the minimum determinant value in

(4.45) based on the Rank and Determinant Criteria in Section 2.4, the following four cases of (Δ_1, Δ_4) and their resultant determinant values in (4.45) are considered:

$$\text{Case 1: } (\Delta_1, \Delta_4) = \pm (0, d_{\min}) \rightarrow \det_1 = d_{\min}^8 [\cos(2\theta)]^4 \quad (4.46)$$

$$\text{Case 2: } (\Delta_1, \Delta_4) = \pm (d_{\min}, 0) \rightarrow \det_2 = d_{\min}^8 [\cos(2\theta)]^4 \quad (4.47)$$

$$\text{Case 3: } (\Delta_1, \Delta_4) = \pm (d_{\min}, d_{\min}) \rightarrow \det_3 = d_{\min}^8 [-2 \sin(2\theta)]^4 \quad (4.48)$$

$$\text{Case 4: } (\Delta_1, \Delta_4) = \pm (d_{\min}, -d_{\min}) \rightarrow \det_4 = d_{\min}^8 [2 \sin(2\theta)]^4 \quad (4.49)$$

Note that $\det_1 = \det_2$ and $\det_3 = \det_4$. In order to maximize the smaller value between $\det_1$ and $\det_3$, we equate $\det_1$ and $\det_3$ to get:

$$\begin{aligned} \cos(2\theta_{opt}) &= 2 \sin(2\theta_{opt}) \\ \Rightarrow \tan(2\theta_{opt}) &= 1/2 \\ \Rightarrow \theta_{opt} &= \frac{1}{2} \tan^{-1}\left(\frac{1}{2}\right) = 13.28^\circ \end{aligned} \quad (4.50)$$

So the optimum GCLT parameters for **J4_LT** are:

$$\begin{aligned} \alpha_{p,p} &= \alpha_{q,q} = \cos(\theta_{opt}) = \cos(13.28^\circ) \\ \alpha_{p,q} &= -\alpha_{q,p} = \sin(\theta_{opt}) = \sin(13.28^\circ) \end{aligned} \quad (4.51)$$

where $\{(p,q)\} \in \{(1,4), (2,3), (5,8), (6,7)\}$, and the minimum determinant value of the codeword distance matrix is $0.64d_{\min}^8$. Compared with **J4_CR**, which has a minimum determinant value $d_{\min}^8$ [5], **J4_LT** has a slightly smaller minimum determinant value (which will be shown later to give less than 0.5dB loss in coding gain), but the ML decoding of **J4_LT** requires the joint detection of only half the number of symbols required by **J4_CR**.

Figure 4.5 QAM-constellation and its minimum Euclidean distance d_{min}

M-ary Square QAM Constellation

We now derive the optimum GCLT parameters of **J4_LT** with larger QAM constellations. Consider the M -ary square-QAM constellation, where the I and Q components of a symbol can be viewed as two independent and identical $\sqrt{M}$ -ary PAM symbols. The following four cases of (Δ_1, Δ_4) and their resultant determinant values as per (4.45) are considered:

$$\text{Case 1: } (\Delta_1, \Delta_4) = \pm (0, nd_{min}) \rightarrow \det_5 = d_{min}^8 [n^2 \cos(2\theta)]^4 \quad (4.52)$$

$$\text{Case 2: } (\Delta_1, \Delta_4) = \pm (md_{min}, 0) \rightarrow \det_6 = d_{min}^8 [m^2 \cos(2\theta)]^4 \quad (4.53)$$

$$\text{Case 3: } (\Delta_1, \Delta_4) = \pm (md_{min}, nd_{min}) \rightarrow$$

$$\det_7 = d_{min}^8 [(m^2 - n^2) \cos(2\theta) - 2mn \sin(2\theta)]^4 \quad (4.54)$$

$$\text{Case 4: } (\Delta_1, \Delta_4) = \pm (md_{min}, -nd_{min}) \rightarrow$$

$$\det_8 = d_{min}^8 [(m^2 - n^2) \cos(2\theta) + 2mn \sin(2\theta)]^4 \quad (4.55)$$

where m and n are integers such that $1 \leq m, n \leq \sqrt{M} - 1$, and M is the dimensionality of the QAM constellation.

To maximize the smaller value of $\det_5$ to $\det_8$ for all values of m and n , consider first the smallest value of $m = n = 1$. For this case, $\det_5$ to $\det_8$ are identical to $\det_1$ to $\det_4$, hence the optimum θ value for (4.52) to (4.55) is the same as that derived in (4.50), i.e. $\theta_{\text{opt}} = \frac{1}{2} \tan^{-1}(\frac{1}{2})$, and the corresponding $\det_5$ to $\det_8$ values are identically $0.64d_{\min}^8$.

Next, consider $m, n > 1$. In this case, it can be shown that with $\theta = \frac{1}{2} \tan^{-1}(\frac{1}{2})$,

$$\begin{aligned} \det_5 &= n^8 (0.64d_{\min}^8) \\ \det_6 &= m^8 (0.64d_{\min}^8) \\ \det_7 &= [(m^2 - mn - n^2)]^4 (0.64d_{\min}^8) \\ \det_8 &= [(m^2 + mn - n^2)]^4 (0.64d_{\min}^8) \end{aligned} \tag{4.56}$$

which are all greater than or equal to $0.64d_{\min}^8$ for $m, n > 1$. Hence the worst-case (i.e. minimum) determinant value for **J4_LT** with M -ary square-QAM constellation occurs when $m = n = 1$, and it is optimized when $\theta = \theta_{\text{opt}} = \frac{1}{2} \tan^{-1}(\frac{1}{2})$.

M-ary Rectangular QAM Constellation

This is similar to the case of square-QAM, besides than the m and n may have different maximum value. However, it has been shown in the square-QAM case that the worst-case determinant value depends not on the maximum value of m and n , but on the minimum of m and n , i.e. $m = n = 1$. So the θ_{opt} in (4.50) and the optimum GCLT parameters in (4.51) apply to rectangular-QAM too. Therefore, we have shown that the optimum GCLT parameters derived in (4.51) applied to all square and rectangular QAM sizes.

To verify the above result graphically, the determinant values of **J4_LT** with square 16-QAM constellation is plotted as a function of θ in Figure 4.6. For square

16-QAM, the values of m and n can each have value 1, 2 or 3. A few combinations of m and n are considered in Figure 4.6 for illustration purpose. It shows that at θ_{opt} , the determinant values corresponding to all possible values of m and n are greater than or equal to the determinant value for $m = n = 1$.

Figure 4.6 Determinant values of $\mathbf{J4_LT}$ in (4.52) to (4.55) for various m, n values

The dispersion matrices of $\mathbf{J4_LT}$ obtained based on the optimum GCLT parameters derived in (4.51) are shown in (4.57) as $\mathbf{A}_{LT_1}, \mathbf{A}_{LT_2}, \dots, \mathbf{A}_{LT_8}$ and. In Figure 4.7, $\mathbf{J4_LT}$ is shown to have exactly the same number of symbol groups as $\mathbf{J4}$, hence using the proposed GCLT technique can improve the decoding performance of QO-STBC while maintaining its decoding complexity, which is not achieved by the CR technique.

$$\begin{aligned}
 \mathbf{A}_{\text{LT}_1} &= \begin{bmatrix} a & 0 & 0 & b \\ 0 & a & -b & 0 \\ 0 & -b & a & 0 \\ b & 0 & 0 & a \end{bmatrix} & \mathbf{A}_{\text{LT}_2} &= \begin{bmatrix} 0 & a & b & 0 \\ -a & 0 & 0 & b \\ -b & 0 & 0 & a \\ 0 & -b & -a & 0 \end{bmatrix} \\
 \mathbf{A}_{\text{LT}_3} &= \begin{bmatrix} 0 & -b & a & 0 \\ b & 0 & 0 & a \\ -a & 0 & 0 & -b \\ 0 & -a & b & 0 \end{bmatrix} & \mathbf{A}_{\text{LT}_4} &= \begin{bmatrix} -b & 0 & 0 & a \\ 0 & -b & -a & 0 \\ 0 & -a & -b & 0 \\ a & 0 & 0 & -b \end{bmatrix} \\
 \mathbf{A}_{\text{LT}_5} &= \begin{bmatrix} ja & 0 & 0 & jb \\ 0 & -ja & jb & 0 \\ 0 & jb & -ja & 0 \\ jb & 0 & 0 & ja \end{bmatrix} & \mathbf{A}_{\text{LT}_6} &= \begin{bmatrix} 0 & ja & jb & 0 \\ ja & 0 & 0 & -jb \\ jb & 0 & 0 & -ja \\ 0 & -jb & -ja & 0 \end{bmatrix} \\
 \mathbf{A}_{\text{LT}_7} &= \begin{bmatrix} 0 & -jb & ja & 0 \\ -jb & 0 & 0 & -ja \\ ja & 0 & 0 & jb \\ 0 & -ja & jb & 0 \end{bmatrix} & \mathbf{A}_{\text{LT}_8} &= \begin{bmatrix} -jb & 0 & 0 & ja \\ 0 & jb & ja & 0 \\ 0 & ja & jb & 0 \\ ja & 0 & 0 & -jb \end{bmatrix} \quad (4.57)
 \end{aligned}$$

where $a = \cos(13.28^\circ)$ and $b = \sin(13.28^\circ)$.

Figure 4.7 Grouping structure of dispersion matrices of **J4**, **J4_CR** and **J4_LT**

(2) GCLT of J8

Since **J8** is also double-symbol decodable as **J4**, it can similarly be shown that the optimum GCLT parameters for **J8** with M-ary QAM constellation are:

$$\begin{aligned}\alpha_{p,p} &= \alpha_{q,q} = \cos(13.28^\circ) \\ \alpha_{p,q} &= -\alpha_{q,p} = \sin(13.28^\circ)\end{aligned}\tag{4.58}$$

where $\{(p, q)\} \in \{(1, 10), (2, 11), (3, 12), (4, 7), (5, 8), (6, 9)\}$. The resultant code, denoted as **J8_LT**, has minimum determinant value of $0.4096(\sqrt{4/3}d_{\min})^{16}$, as compared with $(\sqrt{4/3}d_{\min})^{16}$ for **J8_CR**.

(3) GCLT of TBH8

THB8 requires a joint detection of four real symbols. Its corresponding $\mathbf{L}_{\text{MLR}}$ matrix in (4.42) is [50]:

$$\mathbf{L}_{\text{MLR}} = \prod_{1 \leq i \leq 3, i+1 \leq k \leq 4} \mathbf{G}(i, k, \theta_{ik})\tag{4.59}$$

where $\mathbf{G}(i, k, \theta_{ik})$ is a 4×4 matrix with entries at (i, i) and (k, k) equal to $\cos(\theta_{ik})$, entry at (i, k) equals to $\sin(\theta_{ik})$, and entry at (k, i) equals to $-\sin(\theta_{ik})$, one on the remaining diagonal positions and zero elsewhere. $\mathbf{G}(i, k, \theta_{ik})$ basically models a counter-clockwise rotation by θ degree with respect to the (i, k) plane. For example, for $i = 2, k = 3$, the $\mathbf{G}$ matrix becomes:

$$G(2, 3, \theta_{23}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_{23}) & \sin(\theta_{23}) & 0 \\ 0 & -\sin(\theta_{23}) & \cos(\theta_{23}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\tag{4.60}$$

The GCLT parameters are related to $\mathbf{L}_{\text{MCR}}$ by the following relationship:

$$\begin{bmatrix} \alpha_{p,p} & \alpha_{p,q} & \alpha_{p,m} & \alpha_{p,n} \\ \alpha_{q,p} & \alpha_{q,q} & \alpha_{q,m} & \alpha_{q,n} \\ \alpha_{m,p} & \alpha_{m,q} & \alpha_{m,m} & \alpha_{m,n} \\ \alpha_{n,p} & \alpha_{n,q} & \alpha_{n,m} & \alpha_{n,n} \end{bmatrix} = \mathbf{L}_{\text{MCR}}\tag{4.61}$$

where $\{(p, q, m, n)\} \in \{(1, 4, 6, 7), (2, 3, 5, 8), (9, 12, 14, 15), (10, 11, 13, 16)\}$ for **TBH8**. The mathematical derivation of optimum GCLT parameters for **TBH8** is difficult because of the large dimension involves. Hence we rely on computer search to find the optimization solution. The best solution found is: $\theta_{12} = -45.66^\circ$, $\theta_{23} = 9.43^\circ$, $\theta_{34} = -46.11^\circ$, $\theta_{14} = 37.78^\circ$, $\theta_{13} = 9.13^\circ$, $\theta_{24} = 44.24^\circ$.

4.4.4 Performance Comparison

A) ML Decoding Complexity

The ML decoding metrics of **J4_LT** are shown in (4.62). Note that they each rely on only two *real* symbols, in contrast to the decoding metric of **J4_CR** in (4.4) which relies on two *complex* symbols (note that **J4_CR** and **J4** have the same decision metric except that c_3 and c_4 in **J4_CR** are rotated). Hence GCLT can achieve a reduction of decoding complexity over CR. This complexity reduction will be increasingly significant with increasing QAM constellation size.

$$\begin{aligned}
 f_1(d_1, d_4) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) (|ad_1 - bd_4|^2 + |bd_1 + ad_4|^2) + \right. \\
 &\quad \left. 2 \operatorname{Re} \{ (ad_1 - bd_4)\alpha + (bd_1 + ad_4)\beta + (ad_1 - bd_4)(bd_1 + ad_4)\gamma \} \right] \\
 f_2(d_2, d_3) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) (|ad_2 - bd_3|^2 + |bd_2 + ad_3|^2) + \right. \\
 &\quad \left. 2 \operatorname{Re} \{ (ad_2 - bd_3)\chi + (bd_2 + ad_3)\delta + (ad_2 - bd_3)(bd_2 + ad_3)\phi \} \right] \\
 f_3(d_5, d_8) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) (|ad_5 - bd_8|^2 + |bd_5 + ad_8|^2) + \right. \\
 &\quad \left. 2 \operatorname{Re} \{ j(ad_5 - bd_8)\alpha + j(bd_5 + ad_8)\beta + (ad_5 - bd_8)(bd_5 + ad_8)\gamma \} \right] \\
 f_4(d_6, d_7) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) (|ad_6 - bd_7|^2 + |bd_6 + ad_7|^2) + \right. \\
 &\quad \left. 2 \operatorname{Re} \{ j(ad_6 - bd_7)\chi + j(bd_6 + ad_7)\delta + (ad_6 - bd_7)(bd_6 + ad_7)\phi \} \right]
 \end{aligned} \tag{4.62}$$

$$\begin{aligned}
\alpha &= -h_{1,r}r_1^* - h_{2,r}^*r_2 - h_{3,r}^*r_3 - h_{4,r}r_4^*, \\
\beta &= -h_{4,r}r_1^* + h_{3,r}^*r_2 + h_{2,r}^*r_3 - h_{1,r}r_4^*, \\
\gamma &= 2\text{Re}(h_{1,r}h_{4,r}^* - h_{2,r}h_{3,r}^*), \\
\text{where } \chi &= -h_{2,r}r_1^* + h_{1,r}^*r_2 - h_{4,r}^*r_3 + h_{3,r}r_4^*, \\
\delta &= -h_{3,r}r_1^* - h_{4,r}^*r_2 + h_{1,r}^*r_3 + h_{2,r}r_4^*, \\
\varphi &= 2\text{Re}(-h_{1,r}h_{4,r}^* + h_{2,r}h_{3,r}^*),
\end{aligned}$$

and $a = \cos(13.28^\circ)$ and $b = \sin(13.28^\circ)$.

The difference between the joint detection of two complex symbols (for the case of **J4_CR**) and two real symbols (for the case of **J4_LT**) is as follows:

	Decoding search space	
	J4_CR	J4_LT
QPSK	16	4
16QAM	256	16
Constellation with M points	M^2	M

The difference in the search space will become significant as the constellation size increases.

B) Decoding Performance

The diversity product defined in (2.18) and number of real symbols required for ML joint detection of **J4_CR**, **J4_LT**, **J8_CR**, **J8_LT**, **TBH8_CR** and **TBH8_LT** with 4-QAM are listed in Table 4.4. It shows that QO-STBCs with GCLT require only half of the symbols as QO-STBCs with CR for joint detection. The tradeoff is that **J4_LT**, **J8_LT** and **TBH8_LT** achieve a slightly lower diversity product (coding gain) than **J4_CR**, **J8_CR** and **TBH8_CR**, but such drops in diversity product will be shown later to give negligible performance loss in decoding BER.

Table 4.4 Comparison of 4QAM QO-STBCs with CR and GCLT

	No. of real symbols required for joint detection	Diversity product for 4-QAM
J4_CR [14]	4	0.3536
J4_LT	2	0.3344
J8_CR [14]	4	0.2887
J8_LT	2	0.2730
TBH8_CR [5]	8	0.2187
TBH8_LT	4	0.1531

The BER performance of the **J4**, **J4_CR** and **J4_LT** codes will next be compared, using the rate-1/2 O-STBC **G4C** from [7] as performance benchmark. Since **J4**, **J4_CR** and **J4_LT** are full-rate codes while the **G4C** is a half-rate code, 16-QAM constellation is used for **G4C** while 4-QAM constellation is used for the QO-STBCs to set a common efficiency of 2 bits/s/Hz for all codes. In Figure 4.8, it is observed that both **J4_CR** [14] and **J4_LT** achieve full transmit diversity as they have the same BER slope as the **G4C**. **J4_CR** and **J4_LT** also have lower BER than the **G4C** because the formers are full-rate codes with smaller QAM dimension and hence larger Euclidean distance. Although **J4_CR** has slightly better performance (corresponding to the larger diversity product shown in Table 4.4) than **J4_LT**, their performance difference is less than 0.3dB. **J4_LT**, however, needs joint detection of only two *real* symbols; hence it has a much lower decoding complexity than **J4_CR**, which requires joint detection of two *complex* symbols.

Figure 4.8 Simulation results of QO-STBCs for four transmit antennas with spectral efficiency of 2 bits/sec/Hz

Similar conclusions as **J4** can be drawn for **J8**, **J8_CR** and **J8_LT** as shown in Figure 4.9, as well as for **TBH8_CR** and **TBH8_LT** as shown in Figure 4.10, where their performances are benchmarked with **G8C**, a rate-1/2 O-STBC from [7].

Figure 4.9 Simulation results of QO-STBCs for eight transmit antennas with spectral efficiency of 1.5 bits/sec/Hz

Figure 4.10 Simulation results of QO-STBCs for eight transmit antennas with spectral efficiency of 2 bits/sec/Hz

4.4.5 Section Summary

In this section, we find that the CR technique, which is commonly used to optimize the decoding performance of a QO-STBC, actually increases the decoding complexity of the resultant QO-STBC when regular square and rectangular QAM constellations are used. Specifically, the number of symbols required for joint detection is doubled after CR is applied. Hence we propose GCLT as a means to optimize the decoding performance of a QO-STBC without increasing the joint detection search space. The optimum GCLT parameters for achieving maximum diversity and coding gains are derived analytically for square or rectangular QAM constellations. Simulation results show that QO-STBC with GCLT can achieve full diversity with negligible loss in decoding BER performance compared to QO-STBC with CR. Although only **J4**, **J8** and **TBH8** are used as examples in this thesis, the approach described in this section can be used to achieve the same reduction in decoding complexity (i.e. halving of the

number of symbols required for ML joint detection) for many other QO-STBCs reported in the literature [10,11,39-42].

4.5 CHAPTER SUMMARY

In this chapter, we first derive the generic algebraic structure of QO-STBC, called Quasi-Orthogonality Constraint (QOC), which can be shown to unify the structure of all existing QO-STBCs. We then propose to whiten the received QO-STBC signal after matched filtering, so that we can decode QO-STBC with standard decoders designed for white noise and avoid tedious derivation of dedicated decoding metric. We also investigate the impact of constellation rotation (CR) on the decoding performance QO-STBC under various decoding schemes. We find that the benefits of CR can only be obtained when optimal ML decoding or SD are used. When QO-STBC with CR are decoded with sub-optimal equalization schemes, it is not able to achieve full diversity. We also find that CR increases the decoding complexity of a QO-STBC as it doubles the number of symbols required for joint detection when square and rectangular QAM constellations are used. Hence we proposed to use Group-Constrained Linear Transformation (GCLT), as a new method to optimize QO-STBC. QO-STBC with GCLT can achieve full diversity, and only needs to jointly detect half of the number of symbols required by QO-STBC with CR. This lead to a great saving in decoding complexity over CR, especially when the QAM size is large, because the decoding search space of QO-STBC with CR is the square of the decoding search space of QO-STBC with LT.

CHAPTER 5 QUASI-ORTHOGONAL STBC

WITH MINIMUM DECODING

COMPLEXITY

5.1 INTRODUCTION

5.1.1 Literature review

Orthogonal Space-Time Block Code (O-STBC) can provide full transmit diversity with linear maximum-likelihood (ML) decoding complexity. O-STBC with phase shift keying (PSK) constellation requires real symbol-pair-wise decoding, O-STBC with square or rectangular quadrature amplitude modulation (QAM) constellation only requires real symbol-wise decoding. Despite these advantages, O-STBC is not bandwidth efficient because its maximum code rate will be less than one if more than two transmit antennas and complex constellation are used. As a result, researchers have proposed QO-STBC to mitigate this limitation.

Quasi-Orthogonal Space-Time Block Code (QO-STBC) achieves a higher code rate than O-STBC at the expense of slightly higher decoding complexity, and they can achieve full diversity by applying constellation rotation (CR). With CR, joint detection of two complex symbols (i.e. complex symbol-pair-wise decoding) is required for rate-1 QO-STBC for four transmit antennas and rate-3/4 QO-STBC for eight transmit antennas; while joint detection of four complex symbols is required for rate-1 QO-STBC for eight transmit antennas. In the last chapter, we have proposed a new

technique called Group-Constrained Linear Transformation (GCLT) to help QO-STBCs achieve full diversity while requiring only half the number of symbols required for joint detection as CR. However, such decoding complexity reduction by GCLT can only be achieved for square or rectangular QAM constellations.

On the other hand, the Co-ordinate Interleaved Orthogonal Design (CIOD) and Asymmetric CIOD (ACIOD) codes proposed in [51,52] are able to achieve real symbol-pair-wise ML decoding for any complex constellation. However, these codes require up to half of the transmit antennas to be turned off regularly, thus introducing low-frequency interference and high peak-to-average power ratio which is undesirable in terms of the power-distribution characteristic of the code [17,32,52].

A summary of the code rate and decoding complexity of the above-mentioned STBCs are shown in Table 5.1.

Table 5.1 Comparisons of STBCs for four transmit antennas

STBCs for four transmit antennas		No. of real symbols for joint detection in ML decoding	
	Code Rate	PSK Constellation	Square and Rectangular QAM Constellation
O-STBC	3/4	2	1
QO-STBC with CR	1	4	4
QO-STBC with GCLT	1	4	2
CIOD / ACIOD	1	2	2

5.1.2 Motivations

In this chapter, we propose a new class of QO-STBC whose ML decoding only requires the joint detection of two *real* symbols (i.e. real symbol-pair-wise). This is the lowest possible decoding complexity for any non-orthogonal STBCs. Hence we call it *Minimum-Decoding-Complexity QO-STBC* (MDC-QOSTBC). We shall derive its algebraic structure, propose systematic methods to construct it, and investigate its maximum achievable code rate. We will also compare its decoding performance, power distribution properties (which is related to the number of antennas to be turned off regularly) and antenna scalability (in supporting different number of transmit antennas) with the existing QO-STBCs, CIOD and ACIOD.

5.2 ALGEBRAIC STRUCTURE OF MDC-QOSTBC

In an MDC-QOSTBC, which carries K complex symbols per codeword, every complex symbol is orthogonal to all other complex symbols, but the I and Q components of the same complex symbol need not be orthogonal. As a result of such quasi-orthogonal relationship, the received symbols can be separated by simple linear matched filtering into K independent groups each containing two real symbols. ML decoding of different groups can then be performed separately, and in parallel. In each group, only two real symbols (i.e. the I and Q components of a complex symbol) need to be jointly detected. Hence MDC-QOSTBC is said to be single-symbol decodable.

To facilitate the discussion in the next section, we give an example in Figure 5.1 to illustrate the grouping concept of the dispersion matrices of an MDC-QOSTBC with $K = 4$. Figure 5.1 shows that the dispersion matrices of the MDC-QOSTBC are divided

into $K = 4$ groups, and every group consists of only two dispersion matrices, $\mathbf{A}_i$ and $\mathbf{B}_i$, of the complex symbol c_i .

Figure 5.1 Grouping of dispersion matrices of an MDC-QOSTBC with $K=4$

Definition 5.1: A *Minimum-Decoding-Complexity QO-STBC* (MDC-QOSTBC) is a QO-STBC such that its equivalent channel matrix $\mathbf{H}$ as defined in (2.10) has the property that $\mathbf{H}^T \mathbf{H}$ is block-diagonal with non-zero sub-matrices of size 2×2 .

It should be noted from *Definition 5.1* that MDC-QOSTBC is a special case of the QO-STBC defined in *Definition 4.1*, which states that the $\mathbf{H}^T \mathbf{H}$ of a QO-STBC which requires joint detection of m real symbols will be block-diagonal with $m \times m$ sub-block matrices. Also, MDC-QOSTBC has the minimum decoding complexity among all non-orthogonal STBC, because it only needs the joint detection of two real symbols, anything less complex (i.e. linear detection of only one real symbol) would be an O-STBC whose $\mathbf{H}^T \mathbf{H}$ is a diagonal matrix.

Theorem 5.1: For different complex symbols (indexed using subscripts i and k) in an MDC-STBC to be orthogonal to each other, the corresponding dispersion matrices $\{\mathbf{A}_i, \mathbf{B}_i\}$ and $\{\mathbf{A}_k, \mathbf{B}_k\}$ of the MDC-QOSTBC must possess the following algebraic structure, herein referred to as *Minimum Decoding Complexity Quasi-Orthogonality Constraints* (MDC-QOC):

$$\begin{aligned}
 & \text{(i) } \mathbf{A}_i^H \mathbf{A}_k = -\mathbf{A}_k^H \mathbf{A}_i \\
 & \text{(ii) } \mathbf{B}_i^H \mathbf{B}_k = -\mathbf{B}_k^H \mathbf{B}_i \\
 & \text{(iii) } \mathbf{A}_i^H \mathbf{B}_k = \mathbf{B}_k^H \mathbf{A}_i
 \end{aligned}
 \qquad 1 \leq i \neq k \leq K \qquad (5.1)$$

Proof of *Theorem 5.1*: This can be easily verified from *Theorem 4.1* by having only one complex symbols in a group, i.e. $\mathcal{G}(i) = \{i\}$. ■

Note that the difference between the properties of O-STBC in (3.1) and those of MDC-QOC in (5.1) is that (3.1)(iii) holds for all i and k , whereas (5.1)(iii) holds only when $i \neq k$. In addition, the condition (3.1)(i) is not required in (5.1) because it accounts for the maximal symbolwise diversity (MSD) and not the decoding complexity.

It can be verified that the CIOD and ACIOD codes from [51,52] comply with the algebraic structure stated in *Theorem 5.1*, although they were not designed from this perspective. This shows that our proposed MDC-QOC is generic and inclusive.

5.3 RELATIONSHIP BETWEEN MDC-QOSTBC AND AOD

In this section, we will investigate the algebraic structures of MDC-QOSTBC and AOD, and propose a new concept called Preferred Amicable-Orthogonal-Design Pair (Preferred AOD Pair)), to link MDC-QOSTBC with AOD. By doing so, we will be able to construct MDC-QOSTBC from AOD and derive the theoretical maximum achievable code rate of MDC-QOSTBC.

5.3.1 Definition of Preferred AOD Pair

Let $\underline{\mathbf{A}}_{\text{I}}$, $\underline{\mathbf{A}}_{\text{II}}$, $\underline{\mathbf{B}}_{\text{I}}$ and $\underline{\mathbf{B}}_{\text{II}}$ be OD (Orthogonal Design, as defined in *Definition 3.2*) of the same order n where $\underline{\mathbf{A}}_{\text{I}}$ is of type $(u_{\text{I}_1}, \dots, u_{\text{I}_{s_1}})$ on the variables $\{\underline{a}_{\text{I}_1}, \dots, \underline{a}_{\text{I}_{s_1}}\}$; $\underline{\mathbf{A}}_{\text{II}}$ is of type $(u_{\text{II}_1}, \dots, u_{\text{II}_{s_2}})$ on the variables $\{\underline{a}_{\text{II}_1}, \dots, \underline{a}_{\text{II}_{s_2}}\}$; $\underline{\mathbf{B}}_{\text{I}}$ is of type $(v_{\text{I}_1}, \dots, v_{\text{I}_{t_1}})$ on the variables $\{\underline{b}_{\text{I}_1}, \dots, \underline{b}_{\text{I}_{t_1}}\}$ and $\underline{\mathbf{B}}_{\text{II}}$ is of type $(v_{\text{II}_1}, \dots, v_{\text{II}_{t_2}})$ on the variables $\{\underline{b}_{\text{II}_1}, \dots, \underline{b}_{\text{II}_{t_2}}\}$. Let $\underline{\mathbf{A}}_{\text{I}}$ and $\underline{\mathbf{B}}_{\text{I}}$ form an AOD and $\underline{\mathbf{A}}_{\text{II}}$ and $\underline{\mathbf{B}}_{\text{II}}$ form another AOD according to *Definition 3.4*.

A Preferred AOD Pair is then defined as follows:

Definition 5.2: $\{\underline{\mathbf{A}}_{\text{I}}, \underline{\mathbf{B}}_{\text{I}}\}$ and $\{\underline{\mathbf{A}}_{\text{II}}, \underline{\mathbf{B}}_{\text{II}}\}$ are said to be a *Preferred Amicable-Orthogonal-Design Pair* (Preferred AOD Pair) if:

- $s_1 = t_2$,
- $s_2 = t_1$, and
- by shifting any one variable $\underline{\mathbf{A}}_{\text{I}_p} \underline{a}_{\text{I}_p}$ ($1 \leq p \leq s_1$) from $\underline{\mathbf{A}}_{\text{I}}$ to $\underline{\mathbf{A}}_{\text{II}}$ (thus resulting in a new OD $\hat{\underline{\mathbf{A}}}_{\text{I}}$ with $s_1 - 1$ variables and another new OD $\hat{\underline{\mathbf{A}}}_{\text{II}}$ with $s_2 + 1$ variables as shown in (5.2)), and shifting another variable $\underline{\mathbf{B}}_{\text{II}_q} \underline{b}_{\text{II}_q}$ ($1 \leq q \leq t_2$) from $\underline{\mathbf{B}}_{\text{II}}$ to $\underline{\mathbf{B}}_{\text{I}}$ (thus resulting in a new OD $\hat{\underline{\mathbf{B}}}_{\text{II}}$ with $t_2 - 1$ variables and another new OD $\hat{\underline{\mathbf{B}}}_{\text{I}}$ with $t_1 + 1$ variables as shown in (5.2)), the new pair of ODs $\{\hat{\underline{\mathbf{A}}}_{\text{I}}, \hat{\underline{\mathbf{B}}}_{\text{I}}\}$ form an AOD, and so do $\{\hat{\underline{\mathbf{A}}}_{\text{II}}, \hat{\underline{\mathbf{B}}}_{\text{II}}\}$.

$$\begin{aligned}
 \hat{\underline{\mathbf{A}}}_I &= \sum_{i=1}^{p-1} \underline{\mathbf{A}}_{I_i} a_{I_i} + \sum_{i=p+1}^{s_1} \underline{\mathbf{A}}_{I_i} a_{I_i} && [s_1 - 1 \text{ variables}] \\
 \hat{\underline{\mathbf{B}}}_I &= \sum_{i=1}^{t_1} \underline{\mathbf{B}}_{I_i} b_{I_i} + \underline{\mathbf{B}}_{II_q} b_{II_q} && [t_1 + 1 \text{ variables}] \\
 \hat{\underline{\mathbf{A}}}_{II} &= \sum_{i=1}^{s_2} \underline{\mathbf{A}}_{II_i} a_{II_i} + \underline{\mathbf{A}}_{I_p} a_{I_p} && [s_2 + 1 \text{ variables}] \\
 \hat{\underline{\mathbf{B}}}_{II} &= \sum_{i=1}^{q-1} \underline{\mathbf{B}}_{II_i} b_{II_i} + \sum_{i=q+1}^{t_2} \underline{\mathbf{B}}_{II_i} b_{II_i} && [t_2 - 1 \text{ variables}]
 \end{aligned} \tag{5.2}$$

The above definition of Preferred AOD Pair includes the corresponding case of shifting the variables from $\underline{\mathbf{A}}_{II}$ to $\underline{\mathbf{A}}_I$ and from $\underline{\mathbf{B}}_I$ to $\underline{\mathbf{B}}_{II}$. The new AODs formed in this case are shown in (5.3).

$$\begin{aligned}
 \hat{\underline{\mathbf{A}}}_I &= \sum_{i=1}^{s_1} \underline{\mathbf{A}}_{I_i} a_{I_i} + \underline{\mathbf{A}}_{II_p} a_{II_p} && [s_1 + 1 \text{ variables}] \\
 \hat{\underline{\mathbf{B}}}_I &= \sum_{i=1}^{q-1} \underline{\mathbf{B}}_{I_i} b_{I_i} + \sum_{i=q+1}^{t_1} \underline{\mathbf{B}}_{I_i} b_{I_i} && [t_1 - 1 \text{ variables}] \\
 \hat{\underline{\mathbf{A}}}_{II} &= \sum_{i=1}^{p-1} \underline{\mathbf{A}}_{II_i} a_{II_i} + \sum_{i=p+1}^{s_2} \underline{\mathbf{A}}_{II_i} a_{II_i} && [s_2 - 1 \text{ variables}] \\
 \hat{\underline{\mathbf{B}}}_{II} &= \sum_{i=1}^{t_2} \underline{\mathbf{B}}_{II_i} b_{II_i} + \underline{\mathbf{B}}_{I_q} b_{I_q} && [t_2 + 1 \text{ variables}]
 \end{aligned} \tag{5.3}$$

5.3.2 Relationship between MDC-QOSTBC and AOD through Preferred AOD Pair

Comparing the MDC-QOC in (5.1) with the AOD Constraints in (3.10), some similarity between them can be observed.

1. The MDC-QOC (5.1)(i) and (ii) are exactly the same as the AOD Constraint (3.10) (ii). Hence they have the same solutions.
2. The MDC-QOC (5.1)(iii) is equivalent to the AOD Constraint (3.10)(iii) except that (3.10)(iii) applies for all i and k values, while (5.1)(iii) applies for $i \neq k$. Hence solutions to (5.1)(iii) will also satisfy (3.10)(iii), but not vice versa.

The above observations suggest that there exist some relationship between the MDC-QOSTBC dispersion matrices and the AOD matrices. This relationship is illustrated in Figure 5.2.

Figure 5.2(a) and (b) show the same eight dispersion matrices of the MDC-QOSTBC shown earlier in Figure 5.1, except that the $\mathbf{A}_i$ and $\mathbf{B}_i$ matrix positions in the third and fourth groups are vertically swapped in Figure 5.2(a), while the $\mathbf{A}_i$ and $\mathbf{B}_i$ matrix positions in the first, third and fourth groups are vertically swapped in Figure 5.2(b) (however this does not affect the group structure and decoding complexity of the MDC-QOSTBC).

First consider Figure 5.2(a): The top row of MDC-QOSTBC dispersion matrices $\{\mathbf{A}_1, \mathbf{A}_2; \mathbf{B}_3, \mathbf{B}_4\}$ satisfy (5.1). So they also satisfy (3.10) according to Observation (2) discussed above, and they form an AOD. Similarly, the bottom row of MDC-QOSTBC dispersion matrices $\{\mathbf{A}_3, \mathbf{A}_4; \mathbf{B}_1, \mathbf{B}_2\}$ form another AOD.

However, not *any* two AODs can form an MDC-QOSTBC because an MDC-QOSTBC requires $\{\mathbf{A}_i, \mathbf{B}_i\}$ and $\{\mathbf{A}_k, \mathbf{B}_k\}$ to satisfy (5.1) for all values of $1 \leq i \neq k \leq K$. For example, it requires $\mathbf{A}_1$ to not only satisfy (5.1) with $\mathbf{A}_2, \mathbf{B}_3$, and $\mathbf{B}_4$ in the top AOD, but also with $\mathbf{A}_3, \mathbf{A}_4$, and $\mathbf{B}_2$ in the bottom AOD in Figure 5.2(a). So if the $\mathbf{A}_1$ and $\mathbf{B}_1$ matrices in Figure 5.2(a) are swapped to obtain Figure 5.2(b), and the new rows of dispersion matrices $\{\mathbf{A}_2; \mathbf{B}_1, \mathbf{B}_3, \mathbf{B}_4\}$ and $\{\mathbf{A}_1, \mathbf{A}_3, \mathbf{A}_4; \mathbf{B}_2\}$ in Figure 5.2(b) again form two new AODs, this matches our proposed definition of Preferred AOD Pair. Therefore, we have shown that an MDC-QOSTBC can be constructed by using two AODs that form a Preferred AOD Pair.

Figure 5.2 Relationship of MDC-QOSTBC with AOD through Preferred AOD Pair

The above explanations imply that an MDC-QOSTBC for N_T transmit antenna can be constructed from AOD of order $n = N_T$ that formed a Preferred AOD Pair. Since AOD are square designs, the code length P of the square MDC-QOSTBC constructed will be $P = n = N_T$. The number of complex symbols K transmitted by the MDC-QOSTBC is equal to the number of its dispersion matrices pair $\{\mathbf{A}, \mathbf{B}\}$, which is in turn given by the total number of variables of the Preferred AOD Pair divided by two (since one variable in an AOD is related to one *real* symbol, while the code rate R of STBC is defined as the number of *complex* symbols transmitted per unit time). Therefore, for an MDC-QOSTBC constructed from Preferred AOD Pair, its K and R can be expressed as:

$$\begin{aligned} K &= \frac{s_1 + t_1 + s_2 + t_2}{2} \\ R &= \frac{K}{P} = \frac{s_1 + t_1 + s_2 + t_2}{2n} \end{aligned} \quad (5.4)$$

where s_1, s_2, t_1, t_2 and n are AOD variables defined in *Definition 5.2*.

5.3.3 Maximum Code Rate of MDC-QOSTBC

The code rate R expression in (5.4) indicates that the maximum code rate of an MDC-QOSTBC is related to the maximum number of variables of a Preferred AOD Pair. Hence, we now study the maximum number of variables that a Preferred AOD Pair can support.

Following the framework of deriving the upper bounds of s and t values for AOD presented in [37] (i.e. first fix the value of t , then find the maximum permissible value s), we propose below six propositions to derive an upper and lower bounds for the s_1, s_2, t_1 and t_2 values of a Preferred AOD Pair.

First consider the case that $\underline{\mathbf{A}}_{\text{I}}, \underline{\mathbf{A}}_{\text{II}}, \underline{\mathbf{B}}_{\text{I}}$ and $\underline{\mathbf{B}}_{\text{II}}$ all have non-zero number of variables.

Proposition 5.1: The maximum number of variables, t_1 and t_2 , that $\underline{\mathbf{B}}_{\text{I}}$ and $\underline{\mathbf{B}}_{\text{II}}$ as defined in *Definition 5.2* can have respectively is $\min\{\rho(n)-1, \hat{t}\}$, where $\hat{t}$ is a value such that $\rho_{\hat{t}-1}(n) \geq 1$.

Proof of *Proposition 5.1*: According to the definition of Preferred AOD Pair in *Definition 5.2*, the OD $\underline{\mathcal{B}}_I$ may be given one variable from $\underline{\mathcal{B}}_{II}$ to form a new OD $\hat{\underline{\mathcal{B}}}_I$ with t_1+1 variables as shown in (5.2). Since $\hat{\underline{\mathcal{B}}}_I$ is an OD whose maximum number of variables must be less than $\rho(n)$ as shown in (3.11), it implies that $t_1+1 \leq \rho(n)$, or $t_1 \leq \rho(n)-1$. In addition, referring to (5.3), $\underline{\mathcal{B}}_I$ is also possible to give a variable to $\underline{\mathcal{B}}_{II}$, and $\underline{\mathcal{A}}_I$ must accept one variable from $\underline{\mathcal{A}}_{II}$ in the mean time to form $\hat{\underline{\mathcal{A}}}_I$. In this case, $\hat{\underline{\mathcal{A}}}_I$ must have at least one variable, hence it is required that an $\hat{\underline{\mathcal{A}}}_I$ with at least one variable exists, i.e. $\rho_{t_1-1}(n) \geq 1$ by (3.11). Since $\rho(n)$ is a decreasing function of t , t_1 should be smaller than a value $\hat{t}$ where $\rho_{\hat{t}-1}(n) \geq 1$. Combining all of the above, the maximum value of t_1 should be the smaller value of $\rho(n)-1$ or $\hat{t}$ where $\rho_{\hat{t}-1}(n) \geq 1$. Similar argument applies to t_2 . Hence *Proposition 5.1* is proved. ■

Proposition 5.2: The maximum number of variables, s_1 , of $\underline{\mathcal{A}}_I$ is $\min\{\rho_{t_1+1}(n)+1, \rho_{t_1}(n), \rho_{t_1-1}(n)-1\}$. Similarly, the maximum number of variables, s_2 , of $\underline{\mathcal{A}}_{II}$ is $\min\{\rho_{t_2+1}(n)+1, \rho_{t_2}(n), \rho_{t_2-1}(n)-1\}$.

Proof of *Proposition 5.2*: Considering (5.2), if there are t_1+1 variables in $\hat{\underline{\mathcal{B}}}_I$, there will be s_1-1 variables in $\hat{\underline{\mathcal{A}}}_I$. Similarly, if there are t_1-1 variables in $\hat{\underline{\mathcal{B}}}_I$ in (5.3), there will be s_1+1 variables in $\hat{\underline{\mathcal{A}}}_I$. Hence for an AOD to exist, by (3.11) we have $s_1-1 \leq \rho_{t_1+1}(n) \Rightarrow s_1 \leq \rho_{t_1+1}(n)+1$; and $s_1+1 \leq \rho_{t_1-1}(n) \Rightarrow s_1 \leq \rho_{t_1-1}(n)-1$. In addition, since $\hat{\underline{\mathcal{A}}}_I$ and $\hat{\underline{\mathcal{B}}}_I$ are derived respectively from $\underline{\mathcal{A}}_I$ and $\underline{\mathcal{B}}_I$ that are themselves an AOD, (3.11) implies

that $s_1 \leq \rho_{t_1}(n)$. Combining all the above, s_1 must satisfy all the three constraints, i.e. $s_1 \leq \min\{\rho_{t_1+1}(n)+1, \rho_{t_1}(n), \rho_{t_1-1}(n)-1\}$. Similar proof applies to s_2 . ■

So far we have considered non-zero values for s_1 , s_2 , t_1 and t_2 . We now consider the remaining cases of $s_2=t_1=0$ or $s_1=t_2=0$. First consider the case of $s_2=t_1=0$, which corresponds to $\underline{\mathcal{A}}_{\text{I}}$ shifting a variable to $\underline{\mathcal{A}}_{\text{II}}$ and $\underline{\mathcal{B}}_{\text{II}}$ shifting a variable to $\underline{\mathcal{B}}_{\text{I}}$. In this case, the upper bound values for s_1 and t_2 can be obtained as follows:

Proposition 5.3: The maximum number of variables, t_2 , that $\underline{\mathcal{B}}_{\text{II}}$ can have is $\min\{\rho(n), \hat{t}\}$, where $\hat{t}$ is a value such that $\rho_{\hat{t}-1}(n) \geq 1$.

Proof of Proposition 5.3: Since $\underline{\mathcal{B}}_{\text{II}}$ is an OD, its maximum number of variable, t_2 , needs to satisfy $t_2 \leq \rho(n)$ according to (3.11). Furthermore, since $\underline{\mathcal{A}}_{\text{II}}$ will accept one variable from $\underline{\mathcal{A}}_{\text{I}}$ to form $\hat{\underline{\mathcal{A}}}_{\text{II}}$. Hence $\hat{\underline{\mathcal{A}}}_{\text{II}}$ must have at least one variable, which implies that it is required that $\hat{\underline{\mathcal{A}}}_{\text{II}}$ with at least one variable to be exist, i.e. $\rho_{t_2-1}(n) \geq 1$ by (3.11). Since $\rho(n)$ is a decreasing function of t , t_2 should be smaller than a value $\hat{t}$ where $\rho_{\hat{t}-1}(n) \geq 1$. Combining all of the above, the maximum value of t_2 should be the smaller value of $\rho(n)$ or $\hat{t}$ where $\rho_{\hat{t}-1}(n) \geq 1$. As a result Proposition 5.3 is proved. ■

Proposition 5.4: The maximum number of variables, s_1 , that $\underline{\mathcal{A}}_{\text{I}}$ can have is $\min\{\rho_1(n)+1, \rho_0(n)\}$.

Proof of Proposition 5.4: Since $\underline{\mathcal{B}}_{\text{I}}$ has $t_1=0$, s_1 must satisfy $s_1 \leq \rho_0(n)$ according to

(3.11). After shifting one variable from $\underline{\mathcal{A}}_{\text{I}}$ to $\underline{\mathcal{A}}_{\text{II}}$ and from $\underline{\mathcal{B}}_{\text{II}}$ to $\underline{\mathcal{B}}_{\text{I}}$, $\hat{\underline{\mathcal{A}}}_{\text{I}}$ will have $s_1 - 1$ variables and $\hat{\underline{\mathcal{B}}}_{\text{I}}$ will have one variable. In order for $\hat{\underline{\mathcal{A}}}_{\text{I}}$ and $\hat{\underline{\mathcal{B}}}_{\text{I}}$ to form an AOD, we must have $s_1 - 1 \leq \rho_1(n) \Rightarrow s_1 \leq \rho_1(n) + 1$. Hence *Proposition 5.4* is proved. ■

Finally, consider the case of $s_1 = t_2 = 0$, which corresponds to $\underline{\mathcal{A}}_{\text{II}}$ shifting a variable to $\underline{\mathcal{A}}_{\text{I}}$ and $\underline{\mathcal{B}}_{\text{I}}$ shifting a variable to $\underline{\mathcal{B}}_{\text{II}}$. In this case, the upper bound values for s_2 and t_1 can be obtained as follows:

Proposition 5.5: The maximum number of variables, t_1 , that $\underline{\mathcal{B}}_{\text{I}}$ can have is $\min\{\rho(n), \hat{t}\}$, where $\hat{t}$ is a value such that $\rho_{i-1}(n) \geq 1$.

Proof of *Proposition 5.5:* Similar to *Proposition 5.3*. ■

Proposition 5.6: The maximum number of variables, s_2 , that $\underline{\mathcal{A}}_{\text{II}}$ can have is $\min\{\rho_1(n) + 1, \rho_0(n)\}$.

Proof of *Proposition 5.6:* Similar to *Proposition 5.4*. ■

From the above six propositions, the upper bound values for s_1 , s_2 , t_1 and t_2 can be summarized as follows:

When $s_1, t_1, s_2, t_2 \neq 0$:

- $t_1, t_2 \leq \min\{\rho(n) - 1, \hat{t}\}$, where $\hat{t}$ is a value such that $\rho_{\hat{t}-1}(n) \geq 1$.
- $s_1 \leq \min\{\rho_{t_1+1}(n) + 1, \rho_{t_1}(n), \rho_{t_1-1}(n) - 1\}$ and
 $s_2 \leq \min\{\rho_{t_2+1}(n) + 1, \rho_{t_2}(n), \rho_{t_2-1}(n) - 1\}$.

When $s_2=t_1=0$:

- $t_2 \leq \min\{\rho(n), \hat{t}\}$, where $\hat{t}$ is a value such that $\rho_{\hat{t}-1}(n) \geq 1$.
- $s_1 \leq \min\{\rho_1(n) + 1, \rho_0(n)\}$.

When $s_1=t_2=0$:

- $t_1 \leq \min\{\rho(n), \hat{t}\}$, where $\hat{t}$ is a value such that $\rho_{\hat{t}-1}(n) \geq 1$.
- $s_2 \leq \min\{\rho_1(n) + 1, \rho_0(n)\}$

Since we are interested in MDC-QOSTBC with 4 and 8 transmit antennas in this study, we now use the results developed above to list the upper bound values of s_1 , s_2 , t_1 and t_2 $n = 4$ and 8 in Table 5.2 and Table 5.3 respectively.

Table 5.2 s, t values for $n=4$

t_1	$s_1 \leq$	t_2	$s_2 \leq$	Obtained from	$\text{Max}(s_1 + t_1 + s_2 + t_2)$
4	0	0	4	<i>Proposition 5.5</i> <i>Proposition 5.6</i>	8
3	1	1	3	<i>Proposition 5.1</i> <i>Proposition 5.2</i>	8
2	2	2	2		8
1	3	3	1		8
0	4	4	0	<i>Proposition 5.3</i> <i>Proposition 5.4</i>	8

Table 5.3 s, t values for $n=8$

t_1	$s_1 \leq$	t_2	$s_2 \leq$	Obtained from	$\text{Max}(s_1 + t_1 + s_2 + t_2)$
6	0	0	6	<i>Proposition 5.5</i> <i>Proposition 5.6</i>	12
5	1	1	5	<i>Proposition 5.1</i> <i>Proposition 5.2</i>	12
4	2	2	4		12
3	3	3	3		12
2	4	4	2		12
1	5	5	1		12
0	6	6	0	<i>Proposition 5.3</i> <i>Proposition 5.4</i>	12

Interestingly, it is noted that the $\text{max}(s_1+t_1+s_2+t_2)$ value for order 4 in Table 5.2 is always a constant value of 8, while the $\text{max}(s_1+t_1+s_2+t_2)$ value for order 8 in Table 5.3 is always a constant value of 12.

Theorem 5.2: The maximum achievable code rate of square MDC-QOSTBC is 1 for four transmit antennas and 3/4 for eight transmit antennas.

Proof of *Theorem 5.2:* Since the maximum total number of variables of a Preferred AOD Pair of order 4 is 8, it can be deduced from (5.4) that the maximum achievable code rate of an MDC-QOSTBC for four transmit antennas is $8/(2 \times 4)=1$. Likewise, for an MDC-QOSTBC for eight transmit antennas, which can be constructed by a Preferred AOD Pair of order 8 with maximum total number of variables of 12, the maximum achievable code rate is $12/(2 \times 8)=3/4$. ■

5.3.4 Examples of MDC-QOSTBC Constructed from Preferred AOD

Pair

An example of Preferred AOD Pair, with order 4 is listed in (5.5) and (5.6), while another with order 8 is listed in (5.9) and (5.10). The matrices $\{\underline{\mathbf{A}}_{\mathbf{I}_1}, \underline{\mathbf{A}}_{\mathbf{I}_2}; \underline{\mathbf{B}}_{\mathbf{I}_1}, \underline{\mathbf{B}}_{\mathbf{I}_2}\}$ in (5.5), form an AOD of order 4, similarly $\{\underline{\mathbf{A}}_{\mathbf{II}_1}, \underline{\mathbf{A}}_{\mathbf{II}_2}; \underline{\mathbf{B}}_{\mathbf{II}_1}, \underline{\mathbf{B}}_{\mathbf{II}_2}\}$ in (5.6) form another AOD of order 4. These two AODs form a Preferred AOD Pair with $s_1=t_1=s_2=t_2=2$, hence they achieve the upper bound of s and t values shown in the shaded row in Table 5.2. Consider shifting $\underline{\mathbf{A}}_{\mathbf{I}_1}$ from the first AOD to the second AOD, and $\underline{\mathbf{B}}_{\mathbf{II}_1}$ from the second AOD to the first AOD, two new AODs, $\{\underline{\mathbf{A}}_{\mathbf{I}_2}, \underline{\mathbf{B}}_{\mathbf{I}_1}, \underline{\mathbf{B}}_{\mathbf{I}_2}, \underline{\mathbf{B}}_{\mathbf{II}_1}\}$ and $\{\underline{\mathbf{A}}_{\mathbf{I}_1}, \underline{\mathbf{A}}_{\mathbf{II}_1}, \underline{\mathbf{A}}_{\mathbf{II}_2}; \underline{\mathbf{B}}_{\mathbf{II}_2}\}$, will be formed. These two new AODs can be shown to form a Preferred AOD Pair with $s_1=t_2=1$ and $t_1=s_2=3$, hence they achieve the upper bound of variables in the third row of Table 5.2. By continuing to shift the remaining pairs of $\underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ matrices, Preferred AOD Pairs that achieve all the other upper bound values of variables listed in Table 5.2 can be obtained. Likewise, the Preferred AOD Pair of order 8 with $s_1=t_1=s_2=t_2=3$ in (5.9) and (5.10) can be shown to achieve all the upper bound values of variables listed in Table 5.3 too. Hence we have shown that the proposed Preferred AOD Pair exists, and the upper bound on its number of variables can be reached.

$$\begin{aligned} \underline{\mathbf{A}}_{\mathbf{I}_1} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} & \underline{\mathbf{A}}_{\mathbf{I}_2} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \\ \underline{\mathbf{B}}_{\mathbf{I}_1} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \underline{\mathbf{B}}_{\mathbf{I}_2} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{aligned} \quad (5.5)$$

$$\begin{aligned}
 \underline{\mathbf{A}}_{\Pi_1} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & \underline{\mathbf{A}}_{\Pi_2} &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \\
 \underline{\mathbf{B}}_{\Pi_1} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} & \underline{\mathbf{B}}_{\Pi_2} &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}
 \end{aligned} \tag{5.6}$$

Using the Preferred AOD Pair of order 4 in (5.5) and (5.6), a full-rate MDC-QOSTBC for four transmit antenna as shown in (5.7) can be constructed using the dispersion matrix assignments in (5.8).

$$\begin{bmatrix} c_1^R + jc_3^I & c_2^R + jc_4^I & c_3^R + jc_1^I & c_4^R + jc_2^I \\ c_2^R - jc_4^I & -c_1^R + jc_3^I & c_4^R - jc_2^I & -c_3^R + jc_1^I \\ c_3^R + jc_1^I & -c_4^R - jc_2^I & -c_1^R - jc_3^I & c_2^R + jc_4^I \\ c_4^R - jc_2^I & c_3^R - jc_1^I & -c_2^R + jc_4^I & -c_1^R + jc_3^I \end{bmatrix} \tag{5.7}$$

$$\begin{aligned}
 \mathbf{A}_1 &= \underline{\mathbf{A}}_{\mathbf{I}_1} & \mathbf{A}_2 &= \underline{\mathbf{A}}_{\mathbf{I}_2} & \mathbf{A}_3 &= \underline{\mathbf{A}}_{\Pi_1} & \mathbf{A}_4 &= \underline{\mathbf{A}}_{\Pi_2} \\
 \mathbf{B}_1 &= \underline{\mathbf{B}}_{\Pi_1} & \mathbf{B}_2 &= \underline{\mathbf{B}}_{\Pi_2} & \mathbf{B}_3 &= \underline{\mathbf{B}}_{\mathbf{I}_1} & \mathbf{B}_4 &= \underline{\mathbf{B}}_{\mathbf{I}_2}
 \end{aligned} \tag{5.8}$$

Similarly, the rate-3/4 MDC-QOSTBC for eight transmit antenna in (5.11) can be constructed using the Preferred AOD Pair of order 8 in (5.9) and (5.10) using the dispersion matrix assignments in (5.12).

$$\begin{aligned}
 \underline{\mathbf{A}}_{\mathbf{I}_1} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} & \underline{\mathbf{A}}_{\mathbf{I}_2} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \\
 \underline{\mathbf{A}}_{\mathbf{I}_3} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} & \underline{\mathbf{B}}_{\mathbf{I}_1} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \\
 \underline{\mathbf{B}}_{\mathbf{I}_2} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \underline{\mathbf{B}}_{\mathbf{I}_3} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}
 \end{aligned}$$

(5.9)

$$\begin{aligned}
 \underline{\mathbf{A}}_{\Pi_1} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} & \underline{\mathbf{A}}_{\Pi_2} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \underline{\mathbf{A}}_{\Pi_3} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \underline{\mathbf{B}}_{\Pi_1} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \\
 \underline{\mathbf{B}}_{\Pi_2} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \underline{\mathbf{B}}_{\Pi_3} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.10)
 \end{aligned}$$

$$\begin{bmatrix} c_1^R + jc_4^I & c_2^R + jc_5^I & c_3^R + jc_6^I & 0 & c_4^R + jc_1^I & c_5^R + jc_2^I & c_6^R + jc_3^I & 0 \\ -c_2^R + jc_5^I & c_1^R - jc_4^I & 0 & -c_3^R - jc_6^I & c_5^R - jc_2^I & -c_4^R + jc_1^I & 0 & c_6^R + jc_3^I \\ c_3^R - jc_6^I & 0 & -c_1^R + jc_4^I & -c_2^R - jc_5^I & -c_6^R + jc_3^I & 0 & c_4^R - jc_1^I & c_5^R + jc_2^I \\ 0 & -c_3^R + jc_6^I & c_2^R - jc_5^I & -c_1^R - jc_4^I & 0 & c_6^R - jc_3^I & -c_5^R + jc_2^I & c_4^R + jc_1^I \\ -c_4^R - jc_1^I & -c_5^R - jc_2^I & -c_6^R - jc_3^I & 0 & c_1^R + jc_4^I & c_2^R + jc_5^I & c_3^R + jc_6^I & 0 \\ -c_5^R + jc_2^I & c_4^R - jc_1^I & 0 & c_6^R + jc_3^I & -c_2^R + jc_5^I & c_1^R - jc_4^I & 0 & c_3^R + jc_6^I \\ c_6^R - jc_3^I & 0 & -c_4^R + jc_1^I & c_5^R + jc_2^I & c_3^R - jc_6^I & 0 & -c_1^R + jc_4^I & c_2^R + jc_5^I \\ 0 & c_6^R - jc_3^I & -c_5^R + jc_2^I & -c_4^R - jc_1^I & 0 & c_3^R - jc_6^I & -c_2^R + jc_5^I & -c_1^R - jc_4^I \end{bmatrix}$$

(5.11)

$$\begin{aligned}
 \mathbf{A}_1 &= \underline{\mathbf{A}}_{\Pi_1} & \mathbf{A}_2 &= \underline{\mathbf{A}}_{\Pi_2} & \mathbf{A}_3 &= \underline{\mathbf{A}}_{\Pi_3} & \mathbf{A}_4 &= \underline{\mathbf{A}}_{\Pi_1} & \mathbf{A}_5 &= \underline{\mathbf{A}}_{\Pi_2} & \mathbf{A}_6 &= \underline{\mathbf{A}}_{\Pi_3} \\
 \mathbf{B}_1 &= \underline{\mathbf{B}}_{\Pi_1} & \mathbf{B}_2 &= \underline{\mathbf{B}}_{\Pi_2} & \mathbf{B}_3 &= \underline{\mathbf{B}}_{\Pi_3} & \mathbf{B}_4 &= \underline{\mathbf{B}}_{\Pi_1} & \mathbf{B}_5 &= \underline{\mathbf{B}}_{\Pi_2} & \mathbf{B}_6 &= \underline{\mathbf{B}}_{\Pi_3}
 \end{aligned} \quad (5.12)$$

5.4 SYSTEMATIC CONSTRUCTION OF MDC-QOSTBC

5.4.1 Construction of MDC-QOSTBC from O-STBC

Section 5.3 investigates the algebraic relationship between MDC-QOSTBC and AOD, and show that MDC-QOSTBC can be constructed by a Preferred AOD Pair. Although some examples are provided, there is no systematic way to construct Preferred AOD Pair. Preferred AOD Pair also can only give square MDC-QOSTBC. In this section, we propose a systematic method to construct MDC-QOSTBC from O-STBC. The proposed method consists of the four mapping rules listed in *Construction 5.1* to map the dispersion matrices of an O-STBC to the dispersion matrices of an MDC-QOSTBC.

Construction 5.1: Consider an O-STBC with code length P for N_T transmit antennas, which consists of K sets of dispersion matrices denoted as $\{\tilde{\mathbf{A}}_i, \tilde{\mathbf{B}}_i\}$, $1 \leq i \leq K$. An MDC-QOSTBC with code length $2P$ for $2N_T$ transmit antennas, which consists of $2K$ sets of dispersion matrices denoted as $\{\mathbf{A}_i, \mathbf{B}_i\}$, $1 \leq i \leq 2K$, can be constructed with the following four mapping rules:

$$\begin{aligned} \text{Rule 1: } \mathbf{A}_i &= \begin{bmatrix} \tilde{\mathbf{A}}_i & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}_i \end{bmatrix}; & \text{Rule 3: } \mathbf{A}_{K+i} &= \begin{bmatrix} j\tilde{\mathbf{B}}_i & \mathbf{0} \\ \mathbf{0} & j\tilde{\mathbf{B}}_i \end{bmatrix}; \\ & & & 1 \leq i \leq K \quad (5.13) \\ \text{Rule 2: } \mathbf{B}_i &= \begin{bmatrix} \mathbf{0} & j\tilde{\mathbf{A}}_i \\ j\tilde{\mathbf{A}}_i & \mathbf{0} \end{bmatrix}; & \text{Rule 4: } \mathbf{B}_{K+i} &= \begin{bmatrix} \mathbf{0} & \tilde{\mathbf{B}}_i \\ \tilde{\mathbf{B}}_i & \mathbf{0} \end{bmatrix}. \end{aligned}$$

Proof of *Construction 5.1*: Based on the structure of the O-STBC's dispersion matrices $\{\tilde{\mathbf{A}}_i, \tilde{\mathbf{B}}_i\}$ specified in (3.1), it can be proven that the mapping rules in (5.13) result in a

new set of dispersion matrices $\{\mathbf{A}_i, \mathbf{B}_i\}$ that satisfy the MDC-QOC in (5.1). For example, (5.1)(i) can be verified as follows:

$$\begin{aligned}
 & \mathbf{A}_i^H \mathbf{A}_k + \mathbf{A}_k^H \mathbf{A}_i \quad 1 \leq i \leq K, K < k \leq 2K \\
 &= \begin{bmatrix} \tilde{\mathbf{A}}_i & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}_i \end{bmatrix}^H \begin{bmatrix} j\tilde{\mathbf{B}}_{k-K} & \mathbf{0} \\ \mathbf{0} & j\tilde{\mathbf{B}}_{k-K} \end{bmatrix} + \begin{bmatrix} j\tilde{\mathbf{B}}_{k-K} & \mathbf{0} \\ \mathbf{0} & j\tilde{\mathbf{B}}_{k-K} \end{bmatrix}^H \begin{bmatrix} \tilde{\mathbf{A}}_i & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}_i \end{bmatrix} \\
 &= \begin{bmatrix} j\tilde{\mathbf{A}}_i^H \tilde{\mathbf{B}}_{k-K} & \mathbf{0} \\ \mathbf{0} & j\tilde{\mathbf{A}}_i^H \tilde{\mathbf{B}}_{k-K} \end{bmatrix} + \begin{bmatrix} -j\tilde{\mathbf{B}}_{k-K}^H \tilde{\mathbf{A}}_i & \mathbf{0} \\ \mathbf{0} & -j\tilde{\mathbf{B}}_{k-K}^H \tilde{\mathbf{A}}_i \end{bmatrix} \\
 &= j \begin{bmatrix} \tilde{\mathbf{A}}_i^H \tilde{\mathbf{B}}_{k-K} - \tilde{\mathbf{B}}_{k-K}^H \tilde{\mathbf{A}}_i & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}_i^H \tilde{\mathbf{B}}_{k-K} - \tilde{\mathbf{B}}_{k-K}^H \tilde{\mathbf{A}}_i \end{bmatrix} \\
 &= \mathbf{0} \quad \text{by (3.1)(iii)}
 \end{aligned}$$

The detailed proof is omitted, as the verifications of the other conditions in (5.1) are routine. ■

Figure 5.3 Construction of MDC-QOSTBC from O-STBC

A graphical illustration for the construction of an MDC-QOSTBC for four transmit antennas from the Alamouti O-STBC for two transmit antennas [4] is shown in Figure 5.3, where $\tilde{\mathbf{A}}_1, \tilde{\mathbf{B}}_1, \tilde{\mathbf{A}}_2, \tilde{\mathbf{B}}_2$ denote the dispersion matrices of the Alamouti O-STBC in (2.9) and $\mathbf{A}_1$ to $\mathbf{A}_4$, $\mathbf{B}_1$ to $\mathbf{B}_4$ denote the dispersion matrices of the constructed MDC-QOSTBC. The codeword $\mathbf{C}$ of the resultant MDC-QOSTBC is shown in (5.14).

$$\mathbf{C} = \begin{bmatrix} c_1^R + jc_3^R & c_2^R + jc_4^R & -c_1^I + jc_3^I & -c_2^I + jc_4^I \\ -c_2^R + jc_4^R & c_1^R - jc_3^R & c_2^I + jc_4^I & -c_1^I - jc_3^I \\ -c_1^I + jc_3^I & -c_2^I + jc_4^I & c_1^R + jc_3^R & c_2^R + jc_4^R \\ c_2^I + jc_4^I & -c_1^I - jc_3^I & -c_2^R + jc_4^R & c_1^R - jc_3^R \end{bmatrix} \quad (5.14)$$

As shown in APPENDIX B of this thesis, its ML decoding metric can be calculated as the sum $f_1 + f_2 + f_3 + f_4$, where the functions f_i to f_4 are given in (5.15). Since each f_i is just a function of a single complex symbol c_i the minimization of the ML metric of the MDC-QOSTBC codeword $\mathbf{C}$ in (5.14) is equivalent to minimizing the four f_i terms independently. This implies a lower decoding complexity than many existing QO-STBC, whose ML metrics typically rely on two complex symbols [9].

$$\begin{aligned} f_1(c_1) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) \left(|c_1^R|^2 + |c_1^I|^2 \right) + 2 \operatorname{Re} \{ c_1^R(\alpha) - c_1^I(\beta) - c_1^R c_1^I(\gamma) \} \right] \\ f_2(c_2) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) \left(|c_2^R|^2 + |c_2^I|^2 \right) + 2 \operatorname{Re} \{ c_2^R(\chi) - c_2^I(\delta) - c_2^R c_2^I(\varphi) \} \right] \\ f_3(c_3) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) \left(|c_3^R|^2 + |c_3^I|^2 \right) + 2 \operatorname{Re} \{ jc_3^R(\alpha) + jc_3^I(\beta) + c_3^R c_3^I(\gamma) \} \right] \\ f_4(c_4) &= \sum_{k=1}^{N_R} \left[\left(\sum_{i=1}^4 |h_{i,k}|^2 \right) \left(|c_4^R|^2 + |c_4^I|^2 \right) + 2 \operatorname{Re} \{ jc_4^R(\chi) + jc_4^I(\delta) + c_4^R c_4^I(\varphi) \} \right] \end{aligned} \quad (5.15)$$

where $\alpha = -h_{1,k}r_1^{(k)*} - h_{2,k}^*r_2^{(k)} - h_{3,k}r_3^{(k)*} - h_{4,k}^*r_4^{(k)}$,

$\beta = -h_{3,k}r_1^{(k)*} - h_{4,k}^*r_2^{(k)} - h_{1,k}r_3^{(k)*} - h_{2,k}^*r_4^{(k)}$, $\gamma = 2 \operatorname{Re}(h_{1,k}h_{3,k}^* + h_{2,k}h_{4,k}^*)$,

$\chi = -h_{2,k}r_1^{(k)*} + h_{1,k}^*r_2^{(k)} - h_{4,k}r_3^{(k)*} + h_{3,k}^*r_4^{(k)}$, $\delta = -h_{4,k}r_1^{(k)*} + h_{3,k}^*r_2^{(k)} - h_{2,k}r_3^{(k)*} + h_{1,k}^*r_4^{(k)}$,

$\varphi = 2 \operatorname{Re}(h_{1,k}h_{3,k}^* + h_{2,k}h_{4,k}^*)$,

$h_{i,k}$ are the CSI for the spatial channel between the i^{th} transmit and the k^{th} receive antennas, and $r_p^{(k)}$ are the received signals at the k^{th} receive antenna at time instant p .

5.4.2 Performance optimization

Similar to the QO-STBCs and CIOD/ACIOD proposed in the literature, MDC-QOSTBC constructed by *Construction 5.1* cannot achieve full transmit diversity yet. We therefore use the CR technique to help it attain full diversity, as well as to optimize its decoding performance.

A) Diversity Product of MDC-QOSTBC

We first derive the closed form expression of the diversity product (defined in (2.18)) of MDC-QOSTBC, which will subsequently be used to search for the optimum constellation rotation (CR) angle. Due to the construction of MDC-QOSTBC shown in (5.13), and the properties of the O-STBC dispersion matrices shown in (3.1), we can obtain the following expression for the codeword distance matrix $\mathbf{A}_{\text{CE}}$ (defined in (2.16)) of MDC-QOSTBC constructed from *Construction 5.1* in (5.13):

$$\begin{aligned}
 \mathbf{A}_{\mathbf{CE}} &= (\mathbf{C} - \mathbf{E})^H (\mathbf{C} - \mathbf{E}) \\
 &= \left[\sum_{i=1}^K (\Delta_i^R \mathbf{A}_i + j\Delta_i^I \mathbf{B}_i) \right]^H \left[\sum_{i=1}^K (\Delta_i^R \mathbf{A}_i + j\Delta_i^I \mathbf{B}_i) \right] \\
 &= \sum_{i=1}^K \left[(\Delta_i^R)^2 \mathbf{A}_i^H \mathbf{A}_i \right] + \sum_{i=1}^K \sum_{k=1, k \neq i}^K \left[\Delta_i^R \Delta_k^R \underbrace{(\mathbf{A}_i^H \mathbf{A}_k + \mathbf{A}_k^H \mathbf{A}_i)}_{=0 \text{ as MDC-QOC}} \right] + \\
 &\quad \sum_{i=1}^K \left[(\Delta_i^I)^2 \mathbf{B}_i^H \mathbf{B}_i \right] + \sum_{i=1}^K \sum_{k=1, k \neq i}^K \left[\Delta_i^I \Delta_k^I \underbrace{(\mathbf{B}_i^H \mathbf{B}_k + \mathbf{B}_k^H \mathbf{B}_i)}_{=0 \text{ as MDC-QOC}} \right] + \\
 &\quad \sum_{i=1}^K \left[j\Delta_i^R \Delta_i^I (\mathbf{A}_i^H \mathbf{B}_i - \mathbf{B}_i^H \mathbf{A}_i) \right] + \sum_{i=1}^K \sum_{k=1, k \neq i}^K \left[j\Delta_i^R \Delta_k^I \underbrace{(\mathbf{A}_i^H \mathbf{B}_k - \mathbf{B}_k^H \mathbf{A}_i)}_{=0 \text{ as MDC-QOC}} \right] \\
 &= \sum_{i=1}^K \left[(\Delta_i^R)^2 \mathbf{A}_i^H \mathbf{A}_i \right] + \sum_{i=1}^K \left[(\Delta_i^I)^2 \mathbf{B}_i^H \mathbf{B}_i \right] + \sum_{i=1}^K \left[j\Delta_i^R \Delta_i^I (\mathbf{A}_i^H \mathbf{B}_i - \mathbf{B}_i^H \mathbf{A}_i) \right] \\
 &= \sum_{i=1}^K \left[(\Delta_i^R)^2 \begin{bmatrix} \mathbf{I}_{N_T/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_T/2} \end{bmatrix} \right] + \sum_{i=1}^K \left[(\Delta_i^I)^2 \begin{bmatrix} \mathbf{I}_{N_T/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_T/2} \end{bmatrix} \right] \\
 &\quad + \sum_{i=1}^{K/2} \left[j\Delta_i^R \Delta_i^I \begin{bmatrix} \mathbf{0} & j2\mathbf{I}_{N_T/2} \\ j2\mathbf{I}_{N_T/2} & \mathbf{0} \end{bmatrix} \right] + \sum_{i=K/2+1}^K \left[j\Delta_i^R \Delta_i^I \begin{bmatrix} \mathbf{0} & -j2\mathbf{I}_{N_T/2} \\ -j2\mathbf{I}_{N_T/2} & \mathbf{0} \end{bmatrix} \right] \quad (5.16) \\
 &= \alpha \begin{bmatrix} \mathbf{I}_{N_T/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_T/2} \end{bmatrix} + \beta \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N_T/2} \\ \mathbf{I}_{N_T/2} & \mathbf{0} \end{bmatrix}
 \end{aligned}$$

where

$$\begin{aligned}
 \alpha &= \sum_{i=1}^K (\Delta_i^R)^2 + (\Delta_i^I)^2 \\
 \beta &= 2 \sum_{i=1}^{K/2} (-\Delta_i^R \Delta_i^I + \Delta_{K/2+i}^R \Delta_{K/2+i}^I)
 \end{aligned} \quad (5.17)$$

and $\mathbf{C}$ and $\mathbf{E}$ represent two different codewords, Δ_i represents the possible error of symbols c_i .

The determinant of the codeword distance matrix in (5.16) can be derived to be:

$$\det = \left[(\alpha + \beta)(\alpha - \beta) \right]^{\frac{N_T}{2}} \quad (5.18)$$

and its diversity product, ζ , is:

$$\begin{aligned}\zeta &= \frac{1}{2\sqrt{N_T}} \text{Min}_{\mathbf{C} \neq \mathbf{E}} [\det]^{1/2P} \\ &= \frac{1}{2\sqrt{N_T}} \text{Min}_{\mathbf{C} \neq \mathbf{E}} \left[\left[(\alpha + \beta)(\alpha - \beta) \right]^{N_T/2} \right]^{1/2P}\end{aligned}\quad (5.19)$$

In order to achieve full diversity, the diversity product has to be non-zero, and in order to achieve optimum coding gain, its value has to be maximized. Following [5,12-14], when considering the minimum value of the determinant in (5.18), we can assume that only one of the symbols (e.g. symbol c_i) makes an error and the rest of the symbols are error free, hence the minimum (worst-case) determinant value can be simplified to:

$$\begin{aligned}\det &= \left[\left((\Delta_i^R)^2 + (\Delta_i^I)^2 + 2\Delta_i^R \Delta_i^I \right) \left((\Delta_i^R)^2 + (\Delta_i^I)^2 - 2\Delta_i^R \Delta_i^I \right) \right]^{N_T/2} \\ &= \left[(\Delta_i^R)^2 - (\Delta_i^I)^2 \right]^{N_T}\end{aligned}\quad (5.20)$$

B) Optimum CR Angle for Square and Rectangular QAM

It can be easily seen from (5.20) that, without CR, the determinant in (5.20) and hence the diversity product in (5.19) may be zero due to the possible occurrence of $|\Delta^R| = |\Delta^I|$ in most standard constellations, such as QPSK and 16QAM. Hence CR technique must be used to avoid this situation. Assuming that c_i is now the rotated symbols, and Δ_i is the error in c_i , the determinant expression in (5.20) can now be expressed in (5.22) in terms of the error of an un-rotated constellation symbol, Λ_i , using the following relationship:

$$\begin{aligned}\Delta_i^R &= \Lambda_i^R \cos \theta - \Lambda_i^I \sin \theta \\ \Delta_i^I &= \Lambda_i^R \sin \theta + \Lambda_i^I \cos \theta\end{aligned}\quad (5.21)$$

$$\det = \left[(\Lambda_i^R)^2 \cos(2\theta) - 2\Lambda_i^R \Lambda_i^I \sin(2\theta) - (\Lambda_i^I)^2 \cos(2\theta) \right]^{N_T} \quad (5.22)$$

where θ represents the angle of CR. Note that (5.22) is common to all the symbols c_i , hence the optimum θ value will apply to all c_i symbols too.

Theorem 5.3: The optimum CR angle of the MDC-QOSTBC with square and rectangular QAM is $\frac{1}{2} \tan^{-1}(1/2)$ for all code symbols, and the corresponding diversity product is:

$$\zeta = \frac{1}{2\sqrt{N_T}} \left(\frac{4}{5} \right)^{\frac{N_T}{4P}} d_{\min}^{N_T/P} \quad (5.23)$$

where $d_{\min}$ is the minimum Euclidean distance between two QAM constellation points.

Proof of Theorem 5.3: The worst-case determinant expression of MDC-QOSTBC in (5.22) is similar to that of QO-STBC with GCLT in (4.45), hence the optimum angle

of rotation is the same as in (4.50), i.e. $\theta_{opt} = \frac{1}{2} \tan^{-1}(\frac{1}{2}) = 13.28^\circ$. Note that this θ_{opt}

value is the same for all code symbols in the MDC-QOSTBC codeword. This may be surprising as CR angle usually refers to the rotation of one complex symbol with respect to another complex symbol. For MDC-QOSTBC, it turns out that it is the relative angle between the I and Q components of a code symbols (treated as two real symbols) that matters. After all, MDC-QOSTBC is real symbol-pair-wise decodable.

By substituting θ_{opt} and $\Lambda_i^R = \Lambda_i^I = d_{\min}$ into (5.22) we get:

$$\begin{aligned} \det_{\min} &= \left(\frac{4}{5} \right)^{\frac{N_T}{2}} d_{\min}^{2N_T} \\ \zeta &= \frac{1}{2\sqrt{N_T}} \left(\frac{4}{5} \right)^{\frac{N_T}{4P}} d_{\min}^{N_T/P} \end{aligned} \quad (5.24)$$

Hence *Theorem 5.3* is proved. ■

C) Optimum CR Angle for PSK

Unlike QAM, the analytical derivation of the optimum CR angle for MDC-QOSTBC with PSK constellation is unfortunately not as tractable, hence we rely on computer search to find it. As shown in Figure 5.4, the optimum CR angle is 31.7° for QPSK, and 4.9° for 8PSK. Likewise, these values are common for all PSK symbols in the codeword.

Figure 5.4 Optimization of PSK rotation angle for MDC-QOSTBC

Table 5.4 compares the diversity product and ML decoding complexity (in terms of number of real symbols required for joint detection (JD)) between O-STBC [15], QO-STBC with CR [14], QO-STBC with GCLT (Chapter 4 of this thesis), MDC-QOSTBC and CIOD [51] for 4-QAM and 8-PSK constellations. It shows that MDC-QOSTBC has a lower decoding complexity than the existing QO-STBC with CR. This decoding complexity reduction is traded-off by a slight reduction in the diversity

product, which is interestingly also observed for CIOD and QO-STBC with GCLT. This appears to be a fundamental price to pay to achieve a lower decoding complexity. Although QO-STBC with GCLT can achieve the same decoding complexity as MDC-QOSTBC, MDC-QOSTBC supports *any* complex constellation, while GCLT is only applicable for square or rectangular QAM constellations. For PSK constellations, MDC-QOSTBC has the same ML decoding complexity as O-STBC (since both require two real symbols for joint detection), but MDC-QOSTBC achieves a higher spectral efficiency of 3 bps/Hz because it has full code rate, while O-STBC has a lower spectral efficiency of 2.25 bps/Hz due to a lower code rate of 3/4.

Table 5.4 Comparison of STBCs for four transmit antennas

	4-QAM			8-PSK		
STBC	Bps/Hz	Diver. Pdt.	No. of <i>real</i> syms for JD	Bps/Hz	Diver. Pdt.	No. of <i>real</i> syms for JD
Rate-3/4 O-STBC	1.5	0.4082	1	2.25	0.2209	2
QO-STBC CR [14]	2	0.3536	4	3	0.1674	4
MDC- QOSTBC	2	0.3344	2	3	0.1453	2
QO-STBC GCLT	2	0.3344	2	N.A. for 8-PSK		
CIOD [51]	2	0.3344	2	Not reported		

5.4.3 MDC-QOSTBC for Odd Number of Transmit Antennas

The construction method in *Construction 5.1* specifies how to construct MDC-QOSTBC for even number of transmit antennas. Other numbers of antennas may be necessary under practical considerations. In this section, we will prove that by removing *any* number of columns from a full-diversity MDC-QOSTBC codeword, the resultant code is still a valid full-diversity MDC-QOSTBC that supports a smaller number of transmit antennas at the same code rate.

Lemma 5.1 [53, Theorem 4.3.15]: Let $\mathbf{M}$ be a Hermitian matrix with size m , and d be an integer with $1 \leq d \leq m$, and let $\mathbf{M}_d$ denote any d -by- d principal submatrix of $\mathbf{M}$ (obtained by deleting $m-d$ rows and the corresponding columns from $\mathbf{M}$). For each integer k in the range of $1 \leq k \leq d$, we have

$$\lambda_k(\mathbf{M}) \leq \lambda_k(\mathbf{M}_d) \leq \lambda_{k+m-d}(\mathbf{M}) \quad (5.25)$$

where $\lambda_k(\mathbf{M})$ represents the k^{th} eigenvalue of matrix $\mathbf{M}$, ordered in increasing eigenvalues magnitude.

Theorem 5.4: By removing any column in the codeword of an MDC-QOSTBC that achieves full diversity, the resultant codeword is still an MDC-QOSTBC that achieves full diversity.

Proof of Theorem 5.4: The removal of any column from a STBC can be represented as follows:

$$\bar{\mathbf{C}} = \mathbf{C}\mathbf{T} \quad (5.26)$$

where $\bar{\mathbf{C}}$ of size $P \times \bar{N}_T$ is the new STBC after column removal, and $\mathbf{T}$ is a truncated version of an identity matrix after column removal at the same column positions as $\bar{\mathbf{C}}$ (hence $\mathbf{T}$ is an rectangular matrix of size $N_T \times \bar{N}_T$). For example, to remove the last column of $\mathbf{C}$ so as to reduce the number of transmit antennas from $N_T = 4$ to $\bar{N}_T = 3$, the following $\mathbf{T}$ should be used:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad (5.27)$$

In such cases, the dispersion matrices of the new STBC $\bar{\mathbf{C}}$ would be:

$$\begin{aligned} \bar{\mathbf{A}}_i &= \mathbf{A}_i \mathbf{T} \\ \bar{\mathbf{B}}_i &= \mathbf{B}_i \mathbf{T} \end{aligned} \quad (5.28)$$

It can be easily verified that as long as the dispersion matrices $\{\mathbf{A}_i, \mathbf{B}_i\}$ satisfy the MDC-QOC in (5.1), $\{\bar{\mathbf{A}}_i, \bar{\mathbf{B}}_i\}$ will satisfy the MDC-QOC too. Furthermore, the new STBC will have the following codeword distance matrix:

$$\begin{aligned} \bar{\mathbf{A}}_{\text{CE}} &= \mathbf{T}^H (\mathbf{C} - \mathbf{E})^H (\mathbf{C} - \mathbf{E}) \mathbf{T} \\ &= \mathbf{T}^H \mathbf{A}_{\text{CE}} \mathbf{T} \\ &= \bar{N}_T\text{-by-}\bar{N}_T \text{ submatrix of } \mathbf{A}_{\text{CE}} \end{aligned} \quad (5.29)$$

Since $\bar{\mathbf{A}}_{\text{CE}}$ and $\mathbf{A}_{\text{CE}}$ are both Hermitian matrixes, *Lemma 5.1* applies. As a result, if $\mathbf{A}_{\text{ce}}$ does not have any zero eigenvalue (since $\mathbf{C}$ is assumed to achieve full diversity), $\bar{\mathbf{A}}_{\text{CE}}$ will not have any zero eigenvalue too (as all k^{th} eigenvalue of $\bar{\mathbf{A}}_{\text{CE}}$ will be at least equal to the k^{th} eigenvalue of $\mathbf{A}_{\text{CE}}$), and hence $\bar{\mathbf{C}}$ can also achieve full diversity. Therefore, *Theorem 5.4* is proved. ■

5.4.4 Maximum Code Rate of MDC-QOSTBC from *Construction 5.1*

Construction 5.1 implies that the maximum achievable code rate of an MDC-QOSTBC is related to the maximum achievable code rate of an O-STBC. To be more specific, an MDC-QOSTBC for $2N_T$ transmit antennas will consist of $2K$ dispersion matrices, each of duration $2P$. Hence its code rate is $2K / 2P = K/P$, which is the same as the code rate of the lower-order O-STBC used to generate it. Based on the maximum achievable code rate of O-STBC, the maximum achievable code rate of an MDC-QOSTBC constructed from *Construction 5.1* can be derived as follows:

Theorem 5.5: The maximum achievable code rate of a square MDC-QOSTBC constructed from *Construction 5.1* is:

$$R_{\text{MDC-QOSTBC}} = \frac{2 \log_2 k}{k} \quad \text{where } k = 2^{\lceil \log_2 N_T \rceil} \quad (5.30)$$

Proof of Theorem 5.5: In [15,25], it has been shown that a square O-STBC with a code length of $\tilde{N}_T$ has a maximum code rate of

$$R_{\text{O-STBC}} = \frac{\log_2 i + 1}{i} \quad \text{where } i = 2^{\lceil \log_2 \tilde{N}_T \rceil} \quad (5.31)$$

Since an MDC-QOSTBC constructed from (5.13) has the same code rate as the half-size O-STBC used to construct it, we get the following maximum achievable code rate of square MDC-QOSTBC by substituting $N_T = 2\tilde{N}_T$ into (5.31):

$$\begin{aligned} R_{\text{MDC-QOSTBC}} &= \frac{\log_2 2^{\lceil \log_2 (N_T/2) \rceil} + 1}{2^{\lceil \log_2 (N_T/2) \rceil}} \\ &= \frac{2 \log_2 2^{\lceil \log_2 (N_T/2) \rceil}}{2^{\lceil \log_2 (N_T/2) \rceil}} \\ &= \frac{2 \log_2 k}{k} \quad \text{where } k = 2^{\lceil \log_2 (N_T) \rceil} \end{aligned} \quad (5.32)$$

Hence *Theorem 5.5* is proved. ■

Table 5.5 Comparison between square O-STBC and square QO-STBC

Tx. Antennas	O-STBC			QO-STBC			
	Ref.	P	R	Reference	P	R	Complexity
2	[4]	2	1	N.A.			
3-4	[15] [25]	4	3/4	QO-STBC with CR [14]	4	1	4
				CIOD/ACIOD [51,52]	4	1	2
				MDC-QOSTBC	4	1	2
5-8	[15] [25]	8	1/2	QO-STBC with CR [14]	8	3/4	4
				CIOD/ACIOD [51,52]	8	3/4	2
				MDC-QOSTBC	8	3/4	2

In Table 5.5, we give a comparison of the code length P , maximum achievable code rate R and decoding complexity (i.e. the number of real symbols required for joint ML detection) of square full-diversity MDC-QOSTBC versus the square O-STBC, full-diversity QO-STBC and full-diversity CIOD/ACIOD for various number of transmit antennas. The comparison shows that our proposed MDC-QOSTBC achieves:

- higher code rate than O-STBC with the same diversity level (number of transmit antennas);
- lower decoding complexity than many existing full-diversity QO-STBC with CR at the same code rate;

- same code rate and decoding complexity as full-diversity CIOD/ACIOD with CR.

The advantageous attributes of full-diversity MDC-QOSTBC over full-diversity CIOD/ACIOD will be discussed in later section.

From Table 5.5, we also notice that the maximum achievable code rate of the square proposed MDC-QOSTBC for four and eight transmit antennas are the same as the one derived in *Theorem 5.2*, which was derived using the property of Preferred AOD Pair.

It has been shown in [28] that a non-square O-STBC has a higher achievable code rate than square O-STBC for more than 4 transmit antennas. We will next derive the maximum achievable code rate of a non-square MDC-QOSTBC constructed from non-square O-STBC using *Construction 5.1*.

Theorem 5.6: The maximum achievable code rate of a non-square MDC-QOSTBC constructed using *Construction 5.1* is:

$$R_{\text{MDC-QOSTBC}} = \frac{k+1}{2k} \quad \text{where } k = \left\lceil \frac{N_T}{4} \right\rceil \quad (5.33)$$

Proof of Theorem 5.6: It has been shown in [28] that a non-square O-STBC for $\tilde{N}_T$ transmit antennas has the maximum code rate of:

$$R_{\text{O-STBC}} = \frac{i+1}{2i} \quad \text{where } i = \left\lceil \frac{\tilde{N}_T}{2} \right\rceil \quad (5.34)$$

Following the proof for *Theorem 5.5*, *Theorem 5.6* can be easily proved by substituting $N_T = 2\tilde{N}_T$ into (5.34). ■

It should be noted that, although the non-square MDC-QOSTBC constructed from non-square O-STBC has a higher code rate, it has a higher code length, which will lead to longer decoding latency. For example, for 16 transmit antennas, the *non-square* MDC-QOSTBC constructed from non-square O-STBC will have a code rate of 5/8 and a code length of 112, whereas the *square* MDC-QOSTBC constructed from square O-STBC will have a slightly lower code rate of 4/8 but a much shorter code length of just 16.

5.5 PERFORMANCE RESULTS

The optimum CR angle, minimum determinant (coding gain) and P_o (probability that an antenna transmits the “zero” symbol as defined in Section 2.4.2) values of QO-STBC, CIOD and MDC-QOSTBC with 4QAM constellation for four transmit antennas are compared in Table 5.6, while their bit error rate (BER) are compared in Figure 5.5.

Table 5.6 Comparison of QO-STBCs for four transmit antennas

	Optimum Constellation Angle	No. of Real Symbols for ML Joint Detection	Minimum Determinant	P_o
QO-STBC with CR	45^0	4	0.3536	0
CIOD	31.72^0	2	0.3344	50%
MDC-QOSTBC	13.29^0	2	0.3344	0

These results show that our proposed MDC-QOSTBC suffers reduced diversity product and a slight 0.5 dB loss at BLER of 10^{-4} compared to the existing QO-STBCs (which have higher decoding complexity). Interestingly, the same performance loss is also observed for CIOD. Hence it appears to be a fundamental price to pay in order to achieve a lower decoding complexity. Next, comparing MDC-QOSTBC against CIOD, we observe that although they have almost identical decoding performance, our proposed MDC-QOSTBC does not transmit zero symbols (hence achieving the ideal value of $P_o = 0$), while CIOD requires half of the transmit antennas to transmit zero symbols at any one time (hence $P_o = 50\%$). So our MDC-QOSTBC has an advantage over CIOD in terms of practical implementation (as discussed in Section 2.4.2). Also shown in Figure 5.5 that the performance gap remains around 0.5 dB between MDC-QOSTBC and QO-STBC with constellation rotation for four or eight transmit antennas.

Figure 5.5 Simulation performance of STBCs with spectral efficiency of 2 bps/Hz

For the cases of three and five transmit antennas, corresponding comparisons between MDC-QOSTBC, CIOD and ACIOD with 4QAM constellation are presented in Table 5.7 and Figure 5.6. CIOD and MDC-QOSTBC for three transmit antennas are obtained by removing the last column from their counterparts for four transmit antennas, while CIOD and MDC-QOSTBC for five transmit antennas are obtained by removing the first and last two columns from their counterparts for eight transmit antennas based on the guideline given in [51]. The results in Table 5.7 and Figure 5.6 show that our proposed MDC-QOSTBC can achieve a higher diversity product, and hence lower BLER than CIOD. Furthermore, our code performs comparably with ACIOD but does not require any transmit antennas to transmit zero symbol (to be turned off), while ACIOD for three transmit antennas requires 1/3 of the transmit antennas to be turned off at any one time. Combined with earlier observations from Table 5.6 and Figure 5.5, we can summarize that our proposed MDC-QOSTBC is more versatile in supporting both odd and even number of transmit antennas, while CIOD only performs well for even number of transmit antennas and ACIOD only supports odd number of transmit antennas.

Table 5.7 Comparison of STBCs for three and five transmit antennas

		3 Tx Antennas		5 Tx Antennas	
	Optimum Constellation Angle	Diversit y Product	P_o	Diversit y Product	P_o
CIOD [51]	31.72^0	0.2504	50%	0.2489	50%
ACIOD [52]	31.72^0	0.3564	33%	0.2946	20%
MDC-QOSTBC	13.29^0	0.3641	0	0.2996	0

Figure 5.6 Simulation performance of STBCs with 4-QAM constellation for three and five transmit antennas

5.6 CHAPTER SUMMARY

We have derived the algebraic structure of MDC-QOSTBC, which has the lowest possible decoding complexity for any QO-STBC, because its ML decoding only requires a joint detection of two real symbols. When PSK constellations are used, MDC-QOSTBC can achieve the same low decoding complexity as O-STBC, but supports a higher code rate than O-STBC. By investigating the algebraic relationship between MDC-QOSTBC and AOD, we propose the concept of Preferred AOD Pair and show that a square MDC-QOSTBC can be constructed by two AODs that form a

Preferred AOD Pair. The maximum achievable code rate of MDC-QOSTBC constructed from AOD is derived to be 1 for four transmit antennas and $3/4$ for eight transmit antennas. Next, a set of dispersion matrix mapping rules is proposed to systematically construct MDC-QOSTBC for an even number of transmit antennas from O-STBCs. The optimum CR angle for the modulation to be used by such MDC-QOSTBC to achieve full diversity and optimum coding gain is found to be 13.29° for square or rectangular QAM, 31.7° for QPSK, and 4.9° for 8PSK. In order to support odd number of transmit antennas, columns of an MDC-QOSTBC codeword can be truncated without loss of diversity gain or increase in decoding complexity. Compared with QO-STBC with CR, MDC-QOSTBC can always achieve a lower ML decoding complexity with less than 0.4 dB loss in coding gain. Compared with QO-STBC with GCLT, MDC-QOSTBC has the same decoding performance and decoding complexity, but it can support any complex constellation while QO-STBC with GCLT can only support square and rectangular QAM constellation. Compared with the CIOD and ACIOD, MDC-QOSTBC has better power distribution property as it does not require any transmit antenna to be turned off and it is more versatile in supporting different number of transmit antennas. In addition, MDC-QOSTBC has better decoding performance than CIOD for odd number of transmit antennas.

CHAPTER 6 DIFFERENTIAL DETECTION OF QUASI-ORTHOGONAL STBC

6.1 INTRODUCTION

The QO-STBC transmit diversity schemes discussed so far are designed for coherent detection, with estimates of spatial path gains and phases assumed available at the receiver. However, the complexity and cost of MIMO channel estimation grow with the number of transmit and receive antennas [54]. As a result, *Differential Space-Time Modulation* (DSTM) is proposed to achieve transmit diversity without requiring channel estimation. It is equivalent to the blind detection scheme mentioned in Section 1.1.

6.1.1 Motivations

Similar to the coherent transmit diversity scheme, DSTM with high code rate and low decoding complexity is always desired. Existing DSTM schemes based on Orthogonal Space-Time Block Code (O-STBC) [38] have the lowest decoding complexity but non-full code rate for more than two transmit antennas, while other DSTM schemes not based on O-STBC, such as those reported in [54, 57-63], have high decoding complexity. Hence a DSTM scheme that can achieve higher code rate

than the DSTM based on O-STBC, but without much increase in decoding complexity, is desired.

6.1.2 DSTM Codeword Model

In order to be consistent with the signal model used in the literature for differential / non-coherent / blind communication [38,55,56], in this chapter we employ a slightly different signal model from the one used in Chapter 2, which is for coherent / feed-forward communication.

Most differential space-time modulations are designed based on a set of unitary or quasi-unitary matrices [38,54-58]. For the DSTM schemes described in [38,55,56], the set of unitary or quasi-unitary matrices $\{\mathbf{U}_i\}$ are constructed from a STBC, whose codeword $\mathbf{C}$ is represented as follows:

$$\mathbf{C} = \frac{1}{\sqrt{K}} \sum_{i=1}^K (c_i^R \mathbf{A}_i + jc_i^I \mathbf{B}_i) \quad (6.1)$$

Note that (6.1) and (2.7) are similar except the factor $1/\sqrt{K}$ in (6.1), which is used to render the STBC codeword unitary or quasi-unitary matrices. Also, the STBC codeword $\mathbf{C}$ in (6.1) and the dispersion matrices $\mathbf{A}$ and $\mathbf{B}$ are of size $N_T \times P$ rather than $P \times N_T$ as in (2.7), where N_T and P are the number of transmit antennas and code length respectively.

6.1.3 DSTM Design Criteria

The decoding error probability performance of DSTM scheme with optimal differential decoder has been analyzed in [54]. Specifically, the diversity gain that can be achieved by DSTM is defined as:

$$\text{Min}_{k \neq l} [\text{Rank}(\mathbf{U}_k - \mathbf{U}_l)] \quad (6.2)$$

To achieve full transmit diversity, the minimum rank in (6.2) must be equal to N_T . For a full-rank DSTM code, its *coding gain* is defined in [38,54] as

$$\text{Min}_{k \neq l} \left[N_T \times \text{Det} \left((\mathbf{U}_k - \mathbf{U}_l)(\mathbf{U}_k - \mathbf{U}_l)^H \right)^{1/N_T} \right] \quad (6.3)$$

To achieve optimum decoding error probability, the coding gain in (6.3) must be maximized. Hence the diversity gain in (6.2) is the same as that defined in Section 2.4, while the coding gain in (6.3) is analogous to the diversity product defined in (2.18).

6.2 UNITARY DSTM

6.2.1 Literature Review

Several unitary DSTM schemes have been designed. Hochwald and Marzetta proposed the use of unitary space-time codes, in which the signals transmitted by different antennas are mutually orthogonal [59,60]. Optimal receivers, error bounds, and design criteria for unitary codes were derived in [59], and some specific code constructions were given in [60]. Hughes designed a DSTM based on group codes [54,61]. A similar idea was also given in [58]. Jing and Hassibi [57] proposed a DSTM scheme based on $\text{Sp}(2)$, which can make use of sphere decoding (SD) to simplify the decoding algorithm. Tarokh and Jafarkhani proposed a differential transmit diversity scheme by using the O-STBC [62,63]. Ganesan and Stoica [38] developed a differential scheme based similarly on O-STBC, but with a much simpler decoder. Among these DSTM schemes, the scheme in [38] is single-symbol decodable, hence it has the lowest decoding complexity. However, its maximum achievable code rate is

limited to $3/4$ for four antennas and $1/2$ for eight antennas as it is based on O-STBC. On the other hand, all the DSTM schemes besides [38] have high decoding complexity as their decoders need to search over fairly large search space.

To achieve differential transmit diversity with higher code rate than the DSTM scheme based on O-STBC and lower decoding complexity than the other DSTM schemes mentioned above, we propose to construct DSTM scheme using unitary matrices derived from double-symbol-decodable Quasi-Orthogonal Space-Time Block Code (QO-STBC) such as those in [9,10,11], or using quasi-unitary matrices derived from single-symbol-decodable QO-STBC such as the MDC-QOSTBC proposed in Chapter 5. We will derive the design criteria of the proposed unitary DSTM schemes, and examine their decoding complexity and decoding performance in this section.

6.2.2 Signal Model of Unitary DSTM Scheme

Consider a MIMO communication system, with N_T transmit antennas and N_R receive antennas. Let $\mathbf{H}_p$ be the $N_R \times N_T$ fading channel gain matrix at a time p . The $(i, k)^{\text{th}}$ element of $\mathbf{H}_p$ is the channel coefficient for the signal path from the k^{th} transmit antenna to the i^{th} receive antenna. Let $\mathbf{X}_p$ be the $N_T \times P$ DSTM codeword transmitted at time p . Then, the received $N_R \times P$ signal matrix $\mathbf{R}_p$ can be written as

$$\mathbf{R}_p = \mathbf{H}_p \mathbf{X}_p + \mathbf{N}_p \quad (6.4)$$

where $\mathbf{N}_p$ is the additive white Gaussian channel noise. In this thesis, the code length P is set equal to N_T as in [38,54], so that the transmitted codeword $\mathbf{X}_p$ is a square matrix.

At the start of transmission, we transmit a *known* codeword $\mathbf{X}_0$, which is a unitary matrix of size $N_T \times N_T$. The codeword $\mathbf{X}_p$ transmitted at a time p is differentially encoded by

$$\mathbf{X}_p = \mathbf{X}_{p-1} \mathbf{U}_p \quad (6.5)$$

where $\mathbf{U}_p$ is a unitary matrix (such that $\mathbf{U}_p \mathbf{U}_p^H = \mathbf{I}$), called the *code matrix*, that contains information of the transmitted data. Since $\mathbf{X}_0$ and $\mathbf{U}_p$ are both unitary, it follows that $\mathbf{X}_p$ is unitary at all time. Hence the requirement for the code matrix $\mathbf{U}_p$ to be unitary ensures that all the transmitted codewords have constant power at all times. If we assume that the channel remains unchanged during two consecutive code periods, i.e. $\mathbf{H}_p = \mathbf{H}_{p-1}$, the received signal $\mathbf{R}_p$ at a time p can be expressed [38] as

$$\begin{aligned} \mathbf{R}_p &= \mathbf{H}_p \mathbf{X}_{p-1} \mathbf{U}_p + \mathbf{N}_p \\ &= (\mathbf{R}_{p-1} - \mathbf{N}_{p-1}) \mathbf{U}_p + \mathbf{N}_p \\ &= \mathbf{R}_{p-1} \mathbf{U}_p + \tilde{\mathbf{N}}_p \end{aligned} \quad (6.6)$$

Based on (6.6), the received signal $\mathbf{R}_p$ can be differentially decoded as it depends only on the previous received signal block $\mathbf{R}_{p-1}$, the code matrix $\mathbf{U}_p$ and an equivalent additive white Gaussian noise $\tilde{\mathbf{N}}_p = -\mathbf{N}_{p-1} \mathbf{U}_p + \mathbf{N}_p$. Since $\mathbf{N}_p$ and $\mathbf{N}_{p-1}$ are white and $\mathbf{U}_p$ is unitary, it is shown in [57] that the equivalent noise $\tilde{\mathbf{N}}_p$ is white. Hence the decision metric for (6.6) is

$$\begin{aligned} \hat{\mathbf{U}}_p &= \arg \min_{\mathbf{U}_p \in \mathcal{U}} \text{Tr} \left(\left\{ \mathbf{R}_p - \mathbf{R}_{p-1} \mathbf{U}_p \right\}^H \left\{ \mathbf{R}_p - \mathbf{R}_{p-1} \mathbf{U}_p \right\} \right) \\ &= \arg \max_{\mathbf{U}_p \in \mathcal{U}} \text{Re} \left\{ \text{Tr} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{U}_p \right) \right\} \end{aligned} \quad (6.7)$$

where $\text{Tr}(\mathbf{M})$ represents the trace of a matrix $\mathbf{M}$ and $\mathcal{U}$ denotes the set of all possible code matrices $\mathbf{U}_p$.

6.2.3 Proposed Double-Symbol-Decodable Unitary DSTM

A) Unitary DSTM Based on Double-Symbol-Decodable QO-STBC

In this section, we shall develop a new unitary DSTM scheme based on square double-symbol-decodable QO-STBC, such as those in [9,10,11]. Let us use the QO-STBC **J4** for four transmit antennas in (4.1) as an example. It has the following property:

$$(\mathbf{J4})(\mathbf{J4})^H = \frac{1}{4} \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix} \quad (6.8)$$

where

$$\begin{aligned} \alpha &= \sum_{i=1}^4 |c_i|^2 \\ \beta &= 2 \operatorname{Re}(c_1 \times c_4^* - c_2 \times c_3^*) \end{aligned} \quad (6.9)$$

and c_i are the code symbols in **J4**.

The determinant of the codeword distance matrix of **J4** has been shown in [5] to be:

$$\det = \frac{1}{4} \left[\left(|\Delta_1 + \Delta_4|^2 + |\Delta_2 - \Delta_3|^2 \right) \times \left(|\Delta_1 - \Delta_4|^2 + |\Delta_2 + \Delta_3|^2 \right) \right]^2 \quad (6.10)$$

where Δ_i , $1 \leq i \leq 4$, represents the possible error in c_i , that is, $\Delta_i = c_i - e_i$ if the receiver decides erroneously in favor of e_i if c_i is transmitted. As explained in Section 4.1.1, when considering the minimum determinant value of (6.10), we can assume half of the codeword errors to be zero (e.g. $\Delta_2 = \Delta_3 = 0$), hence the minimum value of (6.10) can be simplified as:

$$\det = \frac{1}{4} \left[\left(|\Delta_1 + \Delta_4|^2 \right) \times \left(|\Delta_1 - \Delta_4|^2 \right) \right]^2 \quad \text{assuming } \Delta_2 = \Delta_3 = 0 \quad (6.11)$$

In order to achieve full transmit diversity and optimum coding gain, the value in (6.11) has to be non-zero and maximized.

To use **J4** as the unitary code matrix of a DSTM scheme, its α and β values in (6.8) must be equal to 4 and 0 respectively. However it is clear from (6.9) that these cannot be achieved if c_1 to c_4 are memoryless PSK or QAM symbols.

To have $\beta = 0$, (6.9) shows that $\text{Re}(c_1 c_4^*)$ must be equal to $\text{Re}(c_2 c_3^*)$. This suggests that c_1 and c_4 should be jointly mapped to a symbol-pair $\{a_i, b_i\}$, while c_2 and c_3 should be mapped to another symbol-pair $\{a_k, b_k\}$, such that $\text{Re}(a_i b_i^*) = \text{Re}(a_k b_k^*)$ for all i and k . To further achieve $\alpha = 4$, (6.9) requires that these symbol-pairs must further satisfy $|a_i|^2 + |b_i|^2 + |a_k|^2 + |b_k|^2 = 4$, or $|a_i|^2 + |b_i|^2 = |a_k|^2 + |b_k|^2 = 2$ if all symbol-pairs are assumed to have equal power. The collection of all symbol-pairs $\{a_i, b_i\}$, denoted as the joint constellation set $\mathcal{M}$, must hence satisfy the constraints $\text{Re}(a_i b_i^*) = v$ where v is any constant value, and $|a_i|^2 + |b_i|^2 = 2$. To maximize the coding gain of the DSTM scheme, $\mathcal{M}$ should further be designed to achieve highest possible value of the worst-case determinant value in (6.11).

Summarizing the above, the design requirements on the joint constellation set $\mathcal{M}$, which consists of L sets of complex-valued symbol-pair $\{a_i, b_i\}$, for $1 \leq i \leq L$, are:

- (i) Unitary Criterion: $\text{Re}(a_i b_i^*) = v$
- (ii) Power Criterion: $|a_i|^2 + |b_i|^2 = 2$ (6.12)
- (iii) Performance Criterion: Maximize $\text{Min} \left\{ \left[|\Delta a_{ik} + \Delta b_{ik}|^2 \times |\Delta a_{ik} - \Delta b_{ik}|^2 \right]^2 \right\}$

where v can be any constant value, and $\Delta a_{ik} = a_i - a_k$, $\Delta b_{ik} = b_i - b_k$ for all $1 \leq i \neq k \leq L$. The spectral efficiency, R , of the resultant DSTM scheme based on full-rate QO-STBC is $R = 2(\log_2 L)/N_T$ bps/Hz.

As an example, consider a system with four transmit antennas ($N_T = 4$) and a desired spectral efficiency of $R = 2$ bps/Hz. From the equation of R , the required constellation size is $L = 16 = 2^4$. Hence, in the *encoding process*, we will map 4 data bits to one of the 16 symbol-pairs $\{a_i, b_i\}$ in $\mathcal{M}$ to form the code symbols $\{c_1, c_4\}$ in $\mathbf{J4}$, then map another 4 data bits to the symbol-pair $\{a_k, b_k\}$ in $\mathcal{M}$ to form the code symbols $\{c_2, c_3\}$ in $\mathbf{J4}$. The resultant $\mathbf{J4}$ with c_1 to c_4 assigned as above will be unitary, and it can now be used as the DSTM code matrix $\mathbf{U}_p$ in (6.5).

In the *decoding process*, with $\mathbf{U}_p$ in (6.5) set to $\mathbf{J4}$, the decision metrics in (6.7) can be simplified to:

$$\begin{aligned}\hat{\mathbf{U}}_p &= \arg \max_{\mathbf{U}_p \in \mathcal{U}} \text{Re} \left\{ \text{Tr} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{U}_p \right) \right\} \\ &= \arg \max_{c_1, \dots, c_4 \in \mathcal{M}} \text{Re} \left\{ \text{Tr} \left[\mathbf{R}_p^H \mathbf{R}_{p-1} \left(\sum_{i=1}^4 (c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i) \right) \right] \right\} \\ &= \arg \max_{c_1, \dots, c_4 \in \mathcal{M}} \sum_{i=1}^4 \text{Re} \left\{ \text{Tr} \left[\mathbf{R}_p^H \mathbf{R}_{p-1} (c_i^R \mathbf{A}_i + j c_i^I \mathbf{B}_i) \right] \right\}\end{aligned} \quad (6.13)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the dispersion matrices of $\mathbf{J4}$.

Since c_1 and c_4 are modulated independently from c_2 and c_3 , (6.13) can be separated into two as follows:

$$\begin{aligned}\{\hat{c}_1, \hat{c}_4\} &= \arg \max_{\{c_1, c_4\} \in \mathcal{M}} \left[\sum_{i=1,4} \left[\text{Re} \left\{ \text{Tr} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{A}_i \right) \right\} c_i^R + \text{Re} \left\{ \text{Tr} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} j \mathbf{B}_i \right) \right\} c_i^I \right] \right] \\ \{\hat{c}_2, \hat{c}_3\} &= \arg \max_{\{c_2, c_3\} \in \mathcal{M}} \left[\sum_{i=2,3} \left[\text{Re} \left\{ \text{Tr} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{A}_i \right) \right\} c_i^R + \text{Re} \left\{ \text{Tr} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} j \mathbf{B}_i \right) \right\} c_i^I \right] \right]\end{aligned} \quad (6.14)$$

As shown in (6.14), the proposed differential QO-STBC modulation scheme can be decoded by the joint detection of two complex symbols, and the two decision metrics can be computed in parallel. So the proposed DSTM scheme does **not** increase the decoding complexity as compared to the original coherent $\mathbf{J4}$ code. Furthermore, it

has a lower decoding complexity than all the unitary DSTM schemes reported in the literature besides [38], as they generally require a larger joint detection search space dimension for the same spectral efficiency and antenna number. This will be elaborated in Section 6.2.4.

B) Design of Constellation Set

In this section, we propose a constellation set $\mathcal{M}$ with good scalability in spectral efficiency that satisfies all three requirements in (6.12) with $\nu = 0$. The proposed constellation set $\mathcal{M}$ is:

$$\begin{aligned} \{a_i, b_i\} &= \begin{cases} a_i = \sqrt{2} \exp[j(2k\pi/M)] \\ b_i = 0 \end{cases} & \text{for } 1 \leq i \leq L/2 \\ \{a_i, b_i\} &= \begin{cases} a_i = 0 \\ b_i = \sqrt{2} \exp[j(2(k-L/2)\pi/M + \theta)] \end{cases} & \text{for } L/2 < i \leq L \end{aligned} \quad (6.15)$$

where $M = L/2$ is an integer, and θ lies between 0 and $2\pi/M$.

Note that in the pair-wise constellation set $\mathcal{M}$ proposed in (6.15), a_i belongs to a M -ary PSK constellation for $1 \leq i \leq L/2$, while b_i belongs to a rotated M -ary PSK constellation for $L/2 < i \leq L$. The parameter M is related to the spectral efficiency R by $R = 2(\log_2 2M)/N_T$ (since $L = 2M$), hence a DSTM scheme with a wide range of spectral efficiency can be systematically designed from (6.15) by adjusting M . The constellation rotation (CR) angle θ for a quarter of the symbols in $\mathcal{M}$ provides an extra degree of freedom to maximize the diversity and coding gain of the resultant DSTM scheme.

Theorem 6.1: For the DSTM constellation set defined in (6.15), the optimum value of the CR angle θ to meet the Performance Optimization Criterion (6.12)(iii) is π/M when M is even, and $\pi/2M$ or $3\pi/2M$ when M is odd.

Proof of Theorem 6.1: The proof is given in four cases, considering different values of i and k .

Case 1: $1 \leq i, k \leq L/2$, and $i \neq k$.

Since b_i and b_k are always zero in this case, Δb_{ik} is always zero, but not Δa_{ik} (since $i \neq k$). Hence the determinant value $\left[|\Delta a_{ik} + \Delta b_{ik}|^2 \times |\Delta a_{ik} - \Delta b_{ik}|^2 \right]^2$ in (6.12)(iii) can never be zero, and its value is independent of θ . This implies that full diversity is always achieved in this case, and the coding gain optimization does not depend on θ .

Case 2: $L/2 < i, k \leq L$ and $i \neq k$.

The proof is similar to Case 1 with the roles of Δa_{ik} and Δb_{ik} interchanged.

Case 3: $1 \leq i \leq L/2$ and $L/2 < k \leq L$.

For this case, the determinant value $\left[|\Delta a_{ik} + \Delta b_{ik}|^2 \times |\Delta a_{ik} - \Delta b_{ik}|^2 \right]^2$ in (6.12)(iii) can be simplified to:

$$\begin{aligned} \det &= \frac{1}{16} \left[\left(\left| \exp[j(2i\pi/M)] + \exp[j(2(k-L/2)\pi/M + \theta)] \right|^2 \right) \times \left(\left| \exp[j(2i\pi/M)] - \exp[j(2(k-L/2)\pi/M + \theta)] \right|^2 \right) \right]^2 \\ &= \left(\sin^2 \left(\frac{2\pi i}{M} - \frac{2\pi q}{M} - \theta \right) \right)^2 \quad 1 \leq i, q \triangleq k - \frac{L}{2} \leq M \\ &= \sin^4 \left(\frac{2(l-m)\pi}{M} \right) \quad 0 \leq l \triangleq i - q \leq M - 1 \end{aligned} \quad (6.16)$$

where i, k, l, q are integers, $M \triangleq \frac{L}{2}$, and $m \triangleq \frac{M\theta}{2\pi}$ is a real number between 0 and 1 inclusively.

To maximize the minimum determinant value in (6.16), we first consider *even* values of M . As shown in Figure 6.1(a), the function $\sin[2l\pi/M]$ (solid line) is zero if $l = 0$ or $M/2$, as shown by the *triangular* markers. This results in a zero determinant value in (6.16) and the resultant DSTM will not deliver full diversity. In order to achieve full diversity and maximum coding gain, a right shift m can be introduced to obtain the function $\sin[2(l-m)\pi/M]$ (dashed line) such that its minimum *absolute* values at integer values of l are non-zero and maximized, as shown by the *circular* markers in Figure 6.1(a). Clearly, this optimum point is reached when $m \triangleq \frac{M\theta}{2\pi} = 0.5$, which corresponds to $\theta = \pi/M$. Similarly, as indicated by the circular markers in Figure 6.1(b), the optimum m value for *odd* values of M is 0.25 or 0.75, which corresponds to $\theta = \pi/2M$ or $3\pi/2M$.

Figure 6.1 Optimization of constellation rotation angle $\theta = 2\pi m / M$ in (6.15)

Case 4: $1 \leq k \leq L/2$ and $L/2 < i \leq L$.

The proof is similar to Case 3.

As the determinant value in (6.12)(iii) does not depend on θ for Cases 1 and 2, the optimum θ value is determined by Cases 3 and 4 to be π/M for M even and $\pi/2M$ or $3\pi/2M$ for M odd, hence *Theorem 6.1* is proved. ■

6.2.4 Performance Comparison

In Table 6.1, our proposed double-symbol-decodable DSTM with optimized joint constellation set as specified in (6.15) and *Theorem 6.1* is compared with DSTM schemes based on O-STBC [38] and group codes [54] in terms of coding gain and decoding complexity. It shows that our proposed DSTM provides the highest coding gain at both spectral efficiency values of 1.5bps/Hz and 2 bps/Hz. Our proposed DSTM also has lower decoding complexity, with a decoding search space dimension of 8 at 1.5bps/Hz and 16 at 2 bps/Hz, than the group-code DSTM in [54].

Although the DSTM based on rate-3/4 O-STBC [38] has a lower decoding complexity than ours at a spectral efficiency of 1.5bps/Hz, it suffers a lower coding gain. At a spectral efficiency of 2 bps/Hz, our proposed DSTM has higher coding gain and equal decoding search space dimension as the DSTM based on rate-1/2 O-STBC with 16PSK [38].

Table 6.1 Comparison of coding gains and decoding complexity for DSTM schemes
for four transmit antennas

Spectral Efficiency	DSTM Scheme	Constellation	Coding Gain	Decoding Search Space Dimension
1.5	Group Code [54]	64PSK	1.85	64
1.5	Proposed Double-Symbol-Decodable DSTM	(6.15) with $M = 4, \theta = \pi/4$	2.83	8
1.5	Rate-3/4 O-STBC [38]	QPSK	2.70	4
2	Group Code [54]	256PSK	0.78	256
2	Proposed Double-Symbol-Decodable DSTM	(6.15) with $M = 8, \theta = \pi/8$	1.17	16
2	Rate-1/2 O-STBC [38]	16PSK	0.31	16

In Figure 6.2, we compare the simulated block error rate (BLER) performances of our proposed DSTM scheme and the DSTM schemes reported in [38,57,64], assuming four transmit and one receive antennas. Note that the Cayley's and $\text{Sp}(2)$ DSTM schemes cannot support 2 bps/Hz spectral efficiency exactly.

Figure 6.2 BLER of different DSTM schemes for four transmit and one receive antennas

Compared with Cayley's DSTM scheme of spectral efficiency 1.75 bps/Hz (obtained from [64] with $Q=7$, $r=2$), our DSTM scheme provides a higher spectral efficiency of 2 bps/Hz at comparable BLER performance. Compared with the Sp(2) DSTM scheme of spectral efficiency 1.94 bps/Hz and decoding search space dimension of 225 (obtained from [57] with $M=5$, $N=3$), our proposed DSTM scheme performs 1dB worse in BLER but has a much smaller decoding search space dimension of 16 and a slightly higher spectral efficiency. Finally, compared with the rate-1/2 O-STBC [38] DSTM scheme of spectral efficiency of 2 bps/Hz, our proposed DSTM has a 5 db gain at $\text{BLER} = 10^{-3}$ with the same spectral efficiency and decoding search space dimension. This is mainly because our DSTM scheme has a higher code

rate, hence it can use a constellation with larger minimum Euclidean distance to support a given spectral efficiency.

6.2.5 Section Summary

We have shown that a new unitary double-symbol-decodable DSTM scheme can be constructed from double-symbol-decodable QO-STBC. The main idea is to force the QO-STBC codeword to be a unitary matrix by using pair-wise symbol modulation with a specially designed constellation set. The design criteria for the pair-wise constellation set are derived. We also present a constellation set that meets the design criteria and supports a wide range of spectral efficiency, and derive its optimum parameter. Our proposed DSTM scheme has the merits of QO-STBC, hence it achieves higher code rate and better decoding performance than DSTM scheme based on O-STBC, and lower decoding complexity (smaller decoding search space dimension) than other DSTM schemes not based on O-STBC with comparable decoding performance.

6.3 QUASI-UNITARY DSTM

6.3.1 Literature Review

As discussed in Section 6.2, first-generation DSTM schemes were designed based on unitary matrices. Due to the unitary requirement on the underlying space-time code, unitary DSTMs mostly employ phase-shift keying (PSK) constellation. It is well known that PSK constellation becomes less power and spectrally efficient as the constellation size increases. As a result, DSTMs based on quasi-unitary matrices have been proposed. In [65], a DSTM based on O-STBC with Amplitude-PSK (A-PSK)

constellation is first proposed. In subsequent publications [55,56], DSTMs based on O-STBC with general QAM constellation are designed. They are shown to have better performance than DSTM schemes with PSK constellations. Likewise, a DSTM based on QO-STBC with non-constant matrix norm is proposed in [66].

In this section, we propose a new *quasi-unitary* and *single-symbol decodable* DSTM scheme that is based on the Minimum-Decoding-Complexity Quasi-Orthogonal Space-Time Block Code (MDC-QOSTBC) proposed in Chapter 5. Compared with the unitary double-symbol-decodable DSTM proposed earlier by us in Section 6.2, this new quasi-unitary single-symbol-decodable DSTM has even smaller decoding search space dimension, hence lower decoding complexity.⁴ We will derive its code design criteria, design its constellation set, and compare its error probability performance and decoding complexity with the existing DSTM schemes.

6.3.2 Signal Model of Quasi-Unitary DSTM Scheme

Consider a MIMO channel with N_T transmit antennas and N_R receive antennas. Let $\mathbf{H}_p$ be the $N_R \times N_T$ fading channel gain matrix at a time p . The $(i, k)^{\text{th}}$ element of $\mathbf{H}_p$ is the channel coefficient for the signal path from the k^{th} transmit antenna to the i^{th} receive antenna. Let $\mathbf{X}_p$ be the $N_T \times N_T$ DSTM square codeword transmitted at a time p . Then, the received $N_R \times N_T$ signal matrix $\mathbf{R}_p$ can be written as

$$\mathbf{R}_p = \mathbf{H}_p \mathbf{X}_p + \mathbf{N}_p \quad (6.17)$$

where $\mathbf{N}_p$ is the additive white Gaussian channel noise.

⁴ MDC-QOSTBC cannot be used in the previous unitary DSTM scheme because only a solution that provides a spectral efficiency of 1 bps/Hz can be provided. At 1 bps/Hz, a good solution can be easily provided by DSTM based on rate-1 O-STBC with real constellation, hence a unitary DSTM with MDC-QOSTBC at 1 bps/Hz provides no decoding performance and complexity advantages.

At the start of the transmission, we transmit a known unitary codeword $\mathbf{X}_0$. The codeword $\mathbf{X}_p$ transmitted at a time p is then differentially encoded by

$$\mathbf{X}_p = a_{p-1}^{-1} \mathbf{X}_{p-1} \mathbf{U}_p \quad (6.18)$$

where $\mathbf{U}_p$ is a quasi-unitary matrix (i.e. $\mathbf{U}_p \mathbf{U}_p^H = a_p^2 \mathbf{I}$) called the *code matrix* that contains information of the transmitted data and a_{p-1} is the square root of the diagonal element of $\mathbf{U}_{p-1} \mathbf{U}_{p-1}^H$. To ensure that the average transmission power is maintained constant, it is required that

$$\mathbb{E}(\mathbf{X}_p \mathbf{X}_p^H) = \mathbb{E}(\mathbf{U}_p \mathbf{U}_p^H) = \mathbf{I}, \text{ or } \mathbb{E}(a_p^2) = 1. \quad (6.19)$$

If we assume that the channel remains unchanged during two consecutive code periods, i.e. $\mathbf{H}_p = \mathbf{H}_{p-1}$, then the received signal $\mathbf{R}_p$ at a time p can be expressed as

$$\mathbf{R}_p = a_{p-1}^{-1} \mathbf{R}_{p-1} \mathbf{U}_p + \tilde{\mathbf{N}}_p \quad (6.20)$$

where $\tilde{\mathbf{N}}_p = -a_{p-1}^{-1} \mathbf{N}_{p-1} \mathbf{U}_p + \mathbf{N}_p$. Since $\tilde{\mathbf{N}}_p$ is not independent of a_{p-1} and $\mathbf{U}_p$, the optimal DSTM decoder is a frame-based decoder which has high computational complexity, hence a near-optimal block-based differential decoder has been proposed in [55] to estimate $\mathbf{U}_p$ as follows:

$$\begin{aligned} \hat{\mathbf{U}}_p &= \arg \min_{\mathbf{U}_p \in \mathcal{U}} \left\| \mathbf{R}_p - a_{p-1}^{-1} \mathbf{R}_{p-1} \mathbf{U}_p \right\|^2 \\ &= \arg \min_{\mathbf{U}_p \in \mathcal{U}} \text{Tr} \left(a_{p-1}^{-2} \mathbf{U}_p^H \mathbf{R}_{p-1}^H \mathbf{R}_{p-1} \mathbf{U}_p - a_{p-1}^{-1} \mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{U}_p - a_{p-1}^{-1} \mathbf{U}_p^H \mathbf{R}_{p-1}^H \mathbf{R}_p \right) \quad (6.21) \\ &= \arg \min_{\mathbf{U}_p \in \mathcal{U}} \text{Tr} \left[a_{p-1}^{-2} \mathbf{R}_{p-1}^H \mathbf{R}_{p-1} \mathbf{U}_p \mathbf{U}_p^H - 2a_{p-1}^{-1} \text{Re}(\mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{U}_p) \right] \end{aligned}$$

where $\mathcal{U}$ denotes the set of all possible code matrices, and a_{p-1} can be estimated from the previous decision $\hat{\mathbf{U}}_{p-1}$ ⁵. It has been shown in [55] that the performance loss of the near-optimal differential decoder in (6.21) compared to the optimal differential

⁵ This may lead to error propagation, but it has been shown in [55,56,66] that the effect of error propagation is small. This will also be confirmed by our simulation results in Figure 6.6 later in this thesis.

decoder is small, but the near-optimal differential decoder could lead to a significant reduction in decoding complexity. Hence in this section, we adopt the near-optimal decoder for our proposed DSTM scheme.

Furthermore, since the performance of the near-optimal differential decoder is similar to that of the optimal decoder [55], we will adopt the design criteria in Section 6.1.3 for designing our quasi-unitary DSTM scheme in this section.

6.3.3 Proposed Single-Symbol-Decodable Quasi-Unitary DSTM

A) Quasi-Unitary DSTM Based on Single-Symbol-Decodable MDC-QOSTBC

To use the single-symbol-decodable MDC-QOSTBC proposed in Chapter 5 as the code matrix $\mathbf{U}_p$ of a quasi-unitary DSTM scheme, the MDC-QOSTBC codeword $\mathbf{C}$ must have the property that $\mathbf{C}\mathbf{C}^H = \gamma\mathbf{I}$ where γ is a constant value. For the MDC-QOSTBC constructed from (5.13):

$$\mathbf{C}\mathbf{C}^H = \frac{\alpha}{K} \begin{bmatrix} \mathbf{I}_{N_T/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_T/2} \end{bmatrix} + \frac{\beta}{K} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N_T/2} \\ \mathbf{I}_{N_T/2} & \mathbf{0} \end{bmatrix} \quad (6.22)$$

where

$$\begin{aligned} \alpha &= \sum_{i=1}^K |c_i|^2 \\ \beta &= 2 \sum_{i=1}^{K/2} \left(-c_i^R c_i^I + c_{(K/2)+i}^R c_{(K/2)+i}^I \right) \end{aligned} \quad (6.23)$$

and its minimum determinant value in (5.20) is:

$$\det = \frac{1}{K} \left[\left(\Delta_i^R \right)^2 - \left(\Delta_i^I \right)^2 \right]^{N_T} \quad (6.24)$$

where Δ_i , $1 \leq i \leq K$, represents the possible error in c_i , that is, $\Delta_i = c_i - e_i$ if the receiver decides erroneously in favor of e_i if c_i is transmitted.

For $\mathbf{C}$ to be quasi-unitary in (6.22), it requires that $\beta = 0$ and $E(\alpha) = K$ in (6.23), where $E(\cdot)$ is the expectation operator⁶. However, if the code symbols c_i in $\mathbf{C}$ are memoryless PSK or QAM symbols, then β in (6.23) may not be zero and $\mathbf{C}\mathbf{C}^H \neq \gamma\mathbf{I}$. Hence $\mathbf{C}$ with memoryless PSK or QAM constellation is not quasi-unitary and cannot be used as a DSTM code matrix.

To achieve $\beta = 0$, we know from (6.23) that $c_i^R c_i^I$ must be equal to $c_{(K/2)+i}^R c_{(K/2)+i}^I$. For example, if $K = 4$, we must have $c_1^R c_1^I = c_3^R c_3^I$ and $c_2^R c_2^I = c_4^R c_4^I$. This can be achieved by mapping c_i to a complex constellation symbol $z_k = x_k + jy_k$ such that $x_k y_k = v \forall k$ where v is a constant. Next, to obtain $E(\alpha) = K$, equation (6.23) suggests that the mapped constellation symbol $z_k = x_k + jy_k$ must satisfy the additional constraint of $E(x_k^2 + y_k^2) = 1$, assuming that all transmitted symbols have the same power. Finally, to maximize the coding gain, the constellation symbols z_k should also be designed to achieve maximum possible value of the minimum determinant value in (6.24). In summary, the collection of constellation symbols $z_k = x_k + jy_k$, $1 \leq k \leq M$, denoted as constellation set $\mathcal{M}$, for use in a quasi-unitary DSTM based on MDC-QOSTBC, should be designed with the following requirements:

- (i) Quasi-Unitary Criterion: $x_k y_k = v$
- (ii) Power Criterion: $E(x_k^2 + y_k^2) = 1$ (6.25)
- (iii) Performance Criterion: maximize $\text{Min} \left\{ \left[(\Delta x_{ik})^2 - (\Delta y_{ik})^2 \right]^{N_T} \right\}$

where v can be any constant value, and $\Delta x_{ik} = x_i - x_k$, $\Delta y_{ik} = y_i - y_k$ for $1 \leq i \neq k \leq M$.

⁶ The presence of this expectation operator is the main difference between the quasi-unitary and unitary schemes.

The spectral efficiency of the resultant DSTM scheme constructed using an MDC-QOSTBC with code rate R will be $(R \log_2 M)$ bps/Hz. Its decoding decision metric in (6.21) can be reduced to:

$$\begin{aligned}
 \hat{\mathbf{U}}_p &= \arg \min_{\mathbf{U}_p \in \mathcal{U}} \text{Tr} \left[a_{p-1}^{-2} \mathbf{R}_{p-1}^H \mathbf{R}_{p-1} \mathbf{U}_p \mathbf{U}_p^H - 2a_{p-1}^{-1} \text{Re}(\mathbf{R}_p^H \mathbf{R}_{p-1} \mathbf{U}_p) \right] \\
 &= \arg \min_{c_1, \dots, c_K \in \mathcal{M}} \text{Tr} \left[a_{p-1}^{-2} \mathbf{R}_{p-1}^H \mathbf{R}_{p-1} \left(\sum_{i=1}^K \frac{|c_i|^2}{K} \mathbf{I}_{N_T} \right) - \right. \\
 &\quad \left. 2a_{p-1}^{-1} \text{Re} \left(\mathbf{R}_p^H \mathbf{R}_{p-1} \left(\frac{1}{\sqrt{K}} \sum_{i=1}^K (c_i^R \mathbf{A}_i + jc_i^I \mathbf{B}_i) \right) \right) \right] \quad (6.26) \\
 &= \arg \min_{c_1, \dots, c_K \in \mathcal{M}} \sum_{i=1}^K \text{Tr} \left[a_{p-1}^{-2} \left(\frac{|c_i|^2}{K} \right) \mathbf{R}_{p-1}^H \mathbf{R}_{p-1} - \right. \\
 &\quad \left. \frac{2a_{p-1}^{-1}}{\sqrt{K}} \text{Re}(\mathbf{R}_p^H \mathbf{R}_{p-1} (c_i^R \mathbf{A}_i + jc_i^I \mathbf{B}_i)) \right]
 \end{aligned}$$

where the $\mathbf{A}_i$ and $\mathbf{B}_i$ are the dispersion matrices defined in (5.13). Since every trace term in (6.26) is dependent on only a single c_i symbol, minimizing the sum of all terms is equivalent to minimizing each term first. Hence the individual symbols c_i in $\hat{\mathbf{U}}_p$ in (6.26) can be separately detected using the simplified expression below:

$$\hat{c}_i = \arg \min_{c_i \in \mathcal{M}} \text{Tr} \left[a_{p-1}^{-2} \mathbf{R}_{p-1}^H \mathbf{R}_{p-1} \left(\frac{|c_i|^2}{K} \mathbf{I}_{N_T} \right) - 2a_{p-1}^{-1} \text{Re}(\mathbf{R}_p^H \mathbf{R}_{p-1} (c_i^R \mathbf{A}_i + jc_i^I \mathbf{B}_i)) \right] \quad (6.27)$$

As shown in (6.27), the proposed MDC-QOSTBC DSTM scheme is single-symbol decodable as its decoding can be achieved by the joint detection of two real symbols (c_i^R and c_i^I), or one complex symbol equivalently. This results in a lower decoding complexity than all the existing DSTMs not based on O-STBC, as well as the double-symbol-decodable DSTM scheme proposed earlier in Section 6.2.

B) Constellation Design

The solution of the three constellation design criteria in (6.25) will be discussed in this section. As shown in Figure 6.3, the solutions of (x_k, y_k) to (6.25)(ii) can be viewed as points on L concentric circles with radius $r_1, r_2, \dots, r_L$, where r_i are subject to the constraint $\sum_{i=1}^L r_i^2 = L$ so as to achieve $E(x_k^2 + y_k^2) = E(r_k^2) = 1$. Next, the solution loci to (6.25)(i) can be represented as a hyperbola. Hence the intersection points of the hyperbola with the concentric circles represent constellation points that satisfy both (6.25)(i) and (ii). Every circle has at most four intersection points with the hyperbola, illustrated as A, B, C and D in Figure 6.3. However, due to their geometrical symmetry, the pair of constellation points A and B, or C and D, will lead to a zero value for determinant value in (6.24) and the resultant DSTM will not have full diversity. Hence, only the pair of constellation points A and C, or B and D, are permissible on each circle, as indicated by the different markers (x and o) in Figure 6.3. Finally, to further comply with (6.25)(iii) in order to maximize the Euclidean distance between the constellation points, we propose to use the (A,C) constellation pairs and the (B,D) constellation pairs alternately on the adjacent circles.

Figure 6.3 Solution loci of the DSTM constellation design criteria (6.25)(i) and (ii)

The above approach enables us to reduce the variables to be optimized in (6.25) (iii) from M pairs of (x_k, y_k) to v and $r_1, r_2, \dots, r_L$, where $L = M/2$, since every circle eventually gives two constellation points as the solutions to (6.25)(i)-(iii). Through computer optimization, the optimum coding gain of the proposed DSTM scheme with $M = 4$ and 8 constellation points are shown in Figure 6.4 for different values of v . The figure shows that the coding gain increases as v decreases, and $v = 0$ gives the best coding gain for both values of M .

It should be pointed out that (6.25)(iii) is a max-min optimization problem, whose numerical solutions depend on the initial values and may give only local maxima. For this reason, we have used several different initial values in our optimization to ensure that the solutions converge. Hence we believe that the coding gain values shown in Figure 6.4 are close to the global maxima. In Figure 6.5, we show two optimized

constellation sets $\mathcal{M}_1$ and $\mathcal{M}_2$ with $M = 4$ and 8 constellation points respectively, both of which were optimized based on $\nu = 0$.

Figure 6.4 Optimization of coding gain

Figure 6.5 Optimized constellation sets for DSTM based on MDC-QOSTBC

6.3.4 Performance Comparison

In our simulations, the channel is assumed to be flat fading and quasi-static. To facilitate result comparison with [66], every frame from each antenna is assumed to have 132 symbols.

In Figure 6.6, we compare the BLER performances of our proposed single-symbol-decodable DSTM scheme and the DSTM scheme based on O-STBC [55,56] under common spectral efficiency values. For the case of eight transmit antennas and spectral efficiency of 1.5 bps/Hz, our proposed DSTM scheme based on rate-3/4 MDC-QOSTBC and four-point constellation set $\mathcal{M}_1$ outperforms the DSTM scheme based on rate-1/2 O-STBC and 8QAM constellation, with a 1.5 dB gain at BLER = 10^{-4} . For the case of four transmit antennas and spectral efficiency of 2 bps/Hz, our proposed DSTM scheme based on rate-1 MDC-QOSTBC and four-point constellation set $\mathcal{M}_1$ again outperforms the DSTM scheme based on rate-1/2 O-STBC and 16QAM constellation, with a 3 dB gain at BLER = 10^{-4} . Finally, for four transmit antennas and spectral efficiency of 3 bps/Hz, our proposed DSTM scheme with rate-1 MDC-QOSTBC and eight-point constellation set $\mathcal{M}_2$ performs comparably with the DSTM scheme based on rate-3/4 O-STBC and 16QAM constellation. These results suggest that our proposed DSTM scheme can offer good performance gain over DSTM schemes based on rate-1/2 O-STBC. This is mainly because our proposed DSTM scheme has a higher code rate, hence it can use a constellation with larger Euclidean distance for the same spectral efficiency.

Figure 6.6 BLER of DSTM schemes based on O-STBC and MDC-QOSTBC

In addition, we have also included in Figure 6.6 the “Genie” BLER results, which are obtained by assuming that the exact values of a_{p-1} are available to the DSTM decoding process in (6.27). These “Genie” results show that the effects of error propagation due to the estimation of a_{p-1} in (6.27) are negligible, hence we will use the practical (non-Genie) DSTM decoder in the subsequent study. Finally, Figure 6.6 shows that our proposed DSTM scheme has the same BLER slop as the DSTM schemes based on O-STBC. This proves that our proposed DSTM schemes achieve full diversity.

In Figure 6.7, we compare the BLER performance and *decoding complexity* of our proposed single-symbol-decodable DSTM scheme for four transmit and one receive antennas with the DSTM schemes based on rate-1 double-symbol-decodable QO-

STBC described in Section 6.2 and in [66], and $\text{Sp}(2)$ [57]. First, we compare the BLER performance of our unitary double-symbol-decodable DSTM scheme from Section 6.2 with this quasi-unitary single-symbol-decodable DSTM scheme. Under a spectral efficiency of 3 bps/Hz, our single-symbol-decodable DSTM scheme based on MDC-QOSTBC performs comparably as our double-symbol-decodable DSTM scheme, but the decoding search space of the latter (64 constellation points) is the square of the formers (8 constellation points). Likewise can be said for the DSTM scheme from [66] with spectral efficiency of 2 bps/Hz.

Figure 6.7 BLER of DSTM schemes for four transmit and one receive antennas

Next, when compared against the $\text{Sp}(2)$ DSTM scheme from [57], our single-symbol-decodable DSTM scheme with spectral efficiency of 3 bps/Hz has a 0.5dB

performance advantage at $\text{BLER}=10^{-4}$ over the $\text{Sp}(2)$ DSTM scheme with spectral efficiency of 3.13 bps/Hz, but our DSTM scheme is single-symbol decodable with a decoding search space of 8 while the $\text{Sp}(2)$ DSTM scheme has a much larger decoding search space of 5929. At a lower spectral efficiency, the $\text{Sp}(2)$ DSTM scheme with 1.95 bps/Hz and a decoding search space of 225 outperforms our single-symbol-decodable DSTM scheme, but our scheme has a slightly higher spectral efficiency of 2 bps/Hz and significantly smaller decoding search space of 4. These results suggest that our single-symbol-decodable quasi-unitary DSTM scheme is able to offer significant reduction in decoding complexity with comparable decoding error probability over the DSTM schemes in [66,57] at similar spectral efficiency.

6.3.5 Section Summary

We have shown that a single-symbol-decodable DSTM scheme with full diversity can be constructed using the MDC-QOSTBC. The main idea is to render the MDC-QOSTBC codeword quasi-unitary by customizing its symbol constellation. We have derived the constellation design criteria and obtained optimized solutions for the corresponding constellation set. Our proposed DSTM scheme achieves higher code rate (rate 1 for four transmit and rate 3/4 for eight transmit antennas) than a DSTM scheme based on O-STBC, and much lower decoding complexity than any DSTM schemes not based on O-STBC. The code rate advantage leads to better error probability performance than DSTM based on rate-1/2 O-STBC, while the decoding complexity advantage is achieved with no or little trade-off in decoding error probability performance than DSTM schemes based on $\text{Sp}(2)$ or double-symbol-decodable QO-STBC.

6.4 CHAPTER SUMMARY

We have shown that double-symbol-decodable QO-STBC and the single-symbol-decodable MDC-QOSTBC can be used to construct new DSTM schemes with full code rate and very low decoding complexity. Existing double-symbol-decodable QO-STBC can be used to construct a new double-symbol-decodable DSTM, while the single-symbol-decodable MDC-QOSTBC proposed in Chapter 5 can be used to construct a new single-symbol-decodable DSTM. The proposed double-symbol-decodable DSTM requires pair-wise symbols modulation, while the proposed single-symbol-decodable DSTM requires only symbol-by-symbol modulation. Both of them require customized constellation sets, whose design criteria are derived in this chapter. The new DSTM schemes can achieve a higher code rate than the DSTM schemes based on O-STBC and a lower decoding complexity than the rest of the DSTM schemes at a slight trade-off in decoding performance.

CHAPTER 7 CONCLUSION AND RECOMMENDATIONS

7.1 CONCLUSION

In this thesis, we have mainly focused on the design, analysis, construction and performance optimization of transmit diversity schemes based on Orthogonal Space-Time Block Code (O-STBC) and Quasi-Orthogonal Space-Time Block Code (QO-STBC). We have first proven the existence of Amicable Complex Orthogonal Design (ACOD), which is the complex version of the existing Amicable Orthogonal Design. We have also proposed new construction methods to construct ACOD. We have then related the power distribution properties of O-STBC to the “type” parameters of ACOD, and used this insight to construct better power-balanced O-STBCs for four and eight transmit antennas. In particular, we have constructed the first O-STBC for eight transmit antenna that achieving *ideal* power distribution properties. Due to their better power distribution properties, the new O-STBCs do not transmit zero symbols (hence do not suffer high peak-to-average power ratio problem) and do not have irrational number coefficients in the codeword (hence leading to simpler hardware implementation) than existing O-STBCs.

Next, we have derived the generic algebraic structure of QO-STBC to unify the structural requirements of all QO-STBC. We have also derived the whitening filter that permits the generalized decoding of QO-STBC using standard linear and decision feedback equalization schemes, sphere decoding (SD) or maximum-likelihood (ML) decoding. We have studied the impact of various kinds of joint-symbol and symbol-by-symbol detection schemes on constellation rotation (CR), and find that the full-

diversity advantages of CR can only be reaped if joint-symbol detections, like ML decoder or SD, are used. In addition, with our derived algebraic structure of QO-STBC, we have found that although CR helps QO-STBC to attain full diversity, it actually increases the decoding complexity of the QO-STBC when regular square or rectangular quadrature amplitude modulation (QAM) constellation are used, hence we have proposed a new constellation transformation technique called Group-Constrained Linear Transformation (GCLT) for optimizing the decoding performance of QO-STBC without increasing its decoding complexity. We have also derived the optimum GCLT parameters for square and rectangular QAM to achieve full diversity and maximum coding gain. The proposed QO-STBC with GCLT only needs to jointly detect half of the number of symbols required by QO-STBC with CR and only with a very slight tradeoff in decoding performance.

As GCLT can only be applied for square and rectangular QAM constellation, we have further proposed QO-STBC with minimum decoding complexity (MDC-QOSTBC), which is applicable for any constellation, and derived its algebraic structure requirements. MDC-QOSTBC is single-symbol decodable (i.e. its ML decoding only need to jointly detect two real symbols), hence it has the lowest decoding complexity among all QO-STBC, and a decoding complexity only next to O-STBC. We have proposed the new concept of Preferred AOD Pair to link MDC-QOSTBC to AOD (i.e. an MDC-QOSTBC can be constructed from two AODs that form a Preferred AOD Pair) and used it to derive the theoretical maximum achievable code rate of MDC-QOSTBC, which turns out to be 1 for four transmit antennas and $3/4$ for eight transmit antennas. We have also designed systematic construction rules to construct MDC-QOSTBC from O-STBC, and derived the optimum constellation rotation angle for such MDC-QOSTBC construction. For square and rectangular

QAM, the optimum constellation rotation angle is found to be 13.3^0 for every symbol. The proposed MDC-QOSTBC is shown to achieve full diversity, higher code rate than O-STBC, and better power distribution properties as well as better flexibility to support different number of transmit antennas than two competing same decoding complexity codes in the literature: Coordinate Interleaved Orthogonal Design (CIOD) and Asymmetry CIOD (ACIOD).

Finally, in an effort to develop new blind transmit diversity schemes with high code rate and low decoding complexity, we have designed new Differential Space-Time Modulation (DSTM) scheme based on double-symbol-decodable QO-STBC and single-symbol-decodable MDC-QOSTBC. We have proposed the use of joint modulation and specially designed constellation set such that a set of unitary / quasi-unitary matrices can be constructed from QO-STBC / MDC-QOSTBC. DSTM schemes based on these unitary / quasi-unitary matrices have the advantage of a very low decoding complexity because they can achieve ML decoding by jointly decode two complex symbols (double-symbol-decodable) / two real symbols (single-symbol-decodable) respectively. These are the lowest ML decoding complexity ever reported for DSTM schemes not based on O-STBC. Compared with DSTM scheme based on O-STBC, the proposed DSTMs schemes can achieve higher code rate and decoding performance. Compared with other DSTM schemes such as $\text{Sp}(2)$, the proposed DSTMs have a much smaller decoding search space with only a slight trade-off in decoding performance.

7.2 RECOMMENDATIONS

The works reported in this thesis may be extended into the following areas:

- Super Orthogonal STTC (Space-Time Trellis Code) and Super Quasi-Orthogonal STTC have been proposed in the literature [74,75]. These STTCs use the codewords of O-STBC and QO-STBC as the codewords of a STTC in order to achieve higher coding gain than STBC and lower decoding complexity than traditional STTC. The extension of this idea to MDC-QOSTBC can be investigated as it promises to achieve even lower decoding complexity than Super Quasi-Orthogonal STTC.
- In this thesis, we focus on the feedforward and blind schemes of transmit diversity of QO-STBC. However, it is well understood that, when channel state information is available to the transmitter, the decoding performance can be further improve. Although the feedback scheme of QO-STBC has been proposed in [67,68], a feedback scheme based on MDC-QOSTBC has yet to be explored.
- The design of MDC-QOSTBC in this thesis is targeted for frequency flat fading channel. The extension of QO-STBC to frequency-selective fading channel have been studied in [69,70], similar extension for the MDC-QOSTBC will be of interest too.
- O-STBC and MDC-QOSTBC with one receive antenna have been proved to have a maximum code rate of one. When more than two receive antennas are available, the STBC code rate can exceed one. Although a rate-5/4 QO-STBC has been obtained by computer search in [71], QO-STBC with code rate greater than one is still worth a further study.
- Existing DSTM schemes mostly have a maximum code rate of one. Design of DSTM with code rate higher than one remains an interesting, though challenging, topic.

AUTHOR'S PUBLICATIONS

Journal Publications

1. C. Yuen; Y. L. Guan; T. T. Tjhung, "Quasi-Orthogonal STBC with Minimum Decoding Complexity", *IEEE Trans. Wireless Comms.*, vol. 4, Sept. 2005, Pages: 2089 – 2094.
2. C. Yuen; Y. L. Guan; T. T. Tjhung, "Partial Answer to a Question Concerning Generalized Amicable Orthogonal Designs", *SIAM Problems and Solutions*, available at: <http://www.siam.org/journals/problems/01-006.htm>, March 2004.
3. C. Yuen; Y. L. Guan; T. T. Tjhung, "On The Search for High-rate Quasi-Orthogonal Space-time Block Code", *invited to submit to International Journal of Wireless Information Networks*.
4. C. Yuen; Y. L. Guan; T. T. Tjhung, "Single-Symbol-Decodable Differential Space-Time Modulation Based on QO-STBC", *accepted to be published in IEEE Trans. Wireless Comms.*
5. C. Yuen; Y. L. Guan; T. T. Tjhung, "Algebraic Relationship between Quasi-Orthogonal STBC with Minimum Decoding Complexity and Amicable Orthogonal Design", *submitted to IEEE Trans. Communications*, on Dec 2004.
6. C. Yuen; Y. L. Guan; T. T. Tjhung, "Unitary Differential Transmit Diversity Based On Quasi-Orthogonal Space-Time Block Code", *submitted to IEEE Trans. Vehicular Technology*, on May 2005.
7. C. Yuen; Y. L. Guan; T. T. Tjhung, "Amicable Complex Orthogonal Design", *submitted to Applied Mathematics Letters*, June 2004.

8. C. Yuen; Y. L. Guan; T. T. Tjhung, "Optimizing Quasi-Orthogonal STBC Through Group-Constrained Linear Transformation", *submitted to IEEE Trans. Vehicular Technology*, on June 2004.
9. C. Yuen; Y. L. Guan; T. T. Tjhung, "Power-Balanced Orthogonal Space-Time Block Code from Amicable Complex Orthogonal Design", *submitted to IEEE Trans. Information Theory*, on Jan 2004.

Conference Publications

1. C. Yuen; Y. L. Guan; T. T. Tjhung, "Algebraic Relationship between Quasi-Orthogonal STBC with Minimum Decoding Complexity and Amicable Orthogonal Design", *accepted by IEEE International Conference on Communications (ICC) 2006*, Istanbul, Turkey.
2. C. Yuen; Y. L. Guan; T. T. Tjhung, "New High-Rate STBC with Good Dispersion Property", *accepted for IEEE Proceedings on Personal, Indoor And Mobile Radio Communications (PIMRC) 2005*, Berlin, Germany.
3. C. Yuen; Y. L. Guan; T. T. Tjhung, "A Class of Four-Group Quasi-Orthogonal Space-Time Block Code Achieving Full Rate and Full Diversity for Any Number of Antennas", *accepted for IEEE Proceedings on Personal, Indoor And Mobile Radio Communications (PIMRC) 2005*, Berlin, Germany.
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5. C. Yuen; Y. L. Guan; T. T. Tjhung, "Single-Symbol Decodable Differential Space-Time Modulation Based on QO-STBC", *International Conference on Acoustics, Speech, and Signall Processing (ICASSP)*, Philadelphia, PA USA, March 2005.
6. C. Yuen; Y. L. Guan; T. T. Tjhung, "Quasi-Orthogonal STBC with Minimum Decoding Complexity: Further Results", *IEEE Wireless Communications and Networking Conference (WCNC)*, New Orleans, LA USA, March 2005.
7. C. Yuen; Y. L. Guan; T. T. Tjhung, "Improved Quasi-Orthogonal STBC with Group-Constrained Linear Transformation", *IEEE Global Telecommunications Conference (Globecom)*, Dallas, Texas USA, Nov 2004.
8. C. Yuen; Y. L. Guan; T. T. Tjhung, "Differential Transmit Diversity Based on Quasi-Orthogonal Space-Time Block Code", *IEEE Global Telecommunications Conference (Globecom)*, Dallas, Texas USA, Nov 2004.
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APPENDIX A PROOF OF CONSTRUCTION

METHOD FOR ACOD

In this appendix, we shall prove that the new ACOD matrices constructed from *Construction 3.1* fulfill the ACOD constraints in (3.19), if the $\underline{\mathbf{M}}$ and $\underline{\mathbf{N}}$ matrices used in *Construction 3.1* fulfill the conditions in (3.22). Firstly, some properties of the Kronecker product, $\otimes$, are listed below. They will be used in the proof subsequently.

$$\begin{aligned}
 & \text{(i) } (\mathbf{X} \otimes \mathbf{Y})(\mathbf{W} \otimes \mathbf{Z}) = (\mathbf{XW}) \otimes (\mathbf{YZ}) \\
 & \text{(ii) } (\mathbf{X} \otimes \mathbf{Y})^H = (\mathbf{X}^H) \otimes (\mathbf{Y}^H) \\
 & \text{(iii) } (\mathbf{X} \otimes \mathbf{Y}) + (\mathbf{X} \otimes \mathbf{Y})^H = 0 \\
 & \quad \text{if } \mathbf{X} = \mathbf{X}^H \text{ \& } \mathbf{Y} = -\mathbf{Y}^H \\
 & \quad \text{or if } \mathbf{X} = -\mathbf{X}^H \text{ \& } \mathbf{Y} = \mathbf{Y}^H \\
 & \text{(iv) } (\mathbf{X} \otimes \mathbf{Y}) - (\mathbf{X} \otimes \mathbf{Y})^H = 0 \\
 & \quad \text{if } \mathbf{X} = \mathbf{X}^H \text{ \& } \mathbf{Y} = \mathbf{Y}^H \\
 & \quad \text{or if } \mathbf{X} = -\mathbf{X}^H \Rightarrow \mathbf{Y} = -\mathbf{Y}^H
 \end{aligned} \tag{A.1}$$

Properties (A.1)(i) and (ii) can be found in [72], while (A.1)(iii) and (iv) can be easily verified.

For simplicity in notation, we shall denote the new ACOD matrices proposed in *Construction 3.1* as $\{\tilde{\mathbf{C}}_1, \tilde{\mathbf{C}}_2, \dots, \tilde{\mathbf{C}}_{x+2}; \tilde{\mathbf{D}}_1, \tilde{\mathbf{D}}_2, \dots, \tilde{\mathbf{D}}_{y+2}\}$, using the following definitions:

$$\left\{ \begin{array}{l} \tilde{\mathbf{C}}_1 \triangleq \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_1, \tilde{\mathbf{C}}_2 \triangleq \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_2, \tilde{\mathbf{C}}_3 \triangleq \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_3, \tilde{\mathbf{C}}_{2+i} \triangleq \underline{\mathbf{C}}_i \otimes \mathbf{I}_4, 2 \leq i \leq x ; \\ \tilde{\mathbf{D}}_1 \triangleq \underline{\mathbf{C}}_1 \otimes \underline{\mathbf{N}}_1, \tilde{\mathbf{D}}_2 \triangleq \underline{\mathbf{C}}_1 \otimes \underline{\mathbf{N}}_2, \tilde{\mathbf{D}}_3 \triangleq \underline{\mathbf{C}}_1 \otimes \underline{\mathbf{N}}_3, \tilde{\mathbf{D}}_{2+k} \triangleq \underline{\mathbf{D}}_k \otimes \mathbf{I}_4, 2 \leq k \leq y \end{array} \right\}$$

Verification of property in (3.19)(0)

$$\begin{aligned}
 & \tilde{\mathbf{C}}_i * \tilde{\mathbf{C}}_k & 1 \leq i \neq k \leq 3 \\
 & = (\underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_i) * (\underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_k) \\
 & = \underline{\mathbf{D}}_1 \otimes (\underline{\mathbf{M}}_i * \underline{\mathbf{M}}_k) \\
 & = \mathbf{0}
 \end{aligned}$$

because (3.22)(0) specifies that $\underline{\mathbf{M}}_i * \underline{\mathbf{M}}_k = \mathbf{0}$, where $*$ represents Hadamard product.

Next,

$$\begin{aligned} \tilde{\underline{\mathbf{C}}}_i * \tilde{\underline{\mathbf{C}}}_k & \quad 1 \leq i \leq 3, 4 \leq k \leq x+2 \\ & = (\underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_i) * (\underline{\mathbf{C}}_p \otimes \underline{\mathbf{I}}_4) \quad 2 \leq p \leq x \\ & = \mathbf{0} \end{aligned}$$

because (3.22)(iv) specifies that $\underline{\mathbf{M}}_i * \underline{\mathbf{I}}_4 = \mathbf{0}$.

Next,

$$\begin{aligned} \tilde{\underline{\mathbf{C}}}_i * \tilde{\underline{\mathbf{C}}}_k & \quad 4 \leq i \neq k \leq x+2 \\ & = (\underline{\mathbf{C}}_p \otimes \underline{\mathbf{I}}_4) * (\underline{\mathbf{C}}_q \otimes \underline{\mathbf{I}}_4) \quad 2 \leq p \neq q \leq x \\ & = \mathbf{0} \end{aligned}$$

because $\underline{\mathbf{C}}_p * \underline{\mathbf{C}}_q = \mathbf{0}$ as specified in (3.19)(0).

Following similar approach as above, all the $\tilde{\underline{\mathbf{D}}}$ matrices can be shown to satisfy property (3.19)(0) too. Hence all the matrices constructed in *Construction 3.1* satisfy the ACOD constraint (3.19)(0).

Verification of property in (3.19)(i)

$$\begin{aligned} \tilde{\underline{\mathbf{C}}}_i^H \tilde{\underline{\mathbf{C}}}_i & \quad 1 \leq i \leq 3 \\ & = (\underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_i)^H (\underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_i) \\ & = \underline{\mathbf{D}}_1^H \underline{\mathbf{D}}_1 \otimes \underline{\mathbf{M}}_i^H \underline{\mathbf{M}}_i \\ & = \underline{\mathbf{g}}_1 \underline{\mathbf{I}}_n \otimes \underline{u}_i \underline{\mathbf{I}}_4 \\ & = \underline{u}_i \underline{\mathbf{g}}_1 \underline{\mathbf{I}}_{4n} \end{aligned}$$

because $\underline{\mathbf{D}}_1^H \underline{\mathbf{D}}_1 = \underline{\mathbf{g}}_1 \underline{\mathbf{I}}_n$ by definition (3.19)(i) and $\underline{\mathbf{M}}_i^H \underline{\mathbf{M}}_i = \underline{u}_i \underline{\mathbf{I}}_4$ by (3.22)(i).

Following similar approach as above, all the $\tilde{\underline{\mathbf{C}}}$ and $\tilde{\underline{\mathbf{D}}}$ matrices constructed in *Construction 3.1* can be shown to satisfy property (3.19)(i).

Verification of property in (3.19)(ii)

$$\begin{aligned}
 & \tilde{\mathbf{C}}_i^H \tilde{\mathbf{C}}_k + \tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_i & 1 \leq i \neq k \leq 3 \\
 & = (\mathbf{D}_1 \otimes \mathbf{M}_i)^H (\mathbf{D}_1 \otimes \mathbf{M}_k) + (\mathbf{D}_1 \otimes \mathbf{M}_k)^H (\mathbf{D}_1 \otimes \mathbf{M}_i) \\
 & = (\mathbf{D}_1^H \mathbf{D}_1 \otimes \mathbf{M}_i^H \mathbf{M}_k) + (\mathbf{D}_1^H \mathbf{D}_1 \otimes \mathbf{M}_k^H \mathbf{M}_i) \\
 & = (\mathbf{D}_1^H \mathbf{D}_1 \otimes \mathbf{M}_i^H \mathbf{M}_k) + (\mathbf{D}_1^H \mathbf{D}_1 \otimes \mathbf{M}_i^H \mathbf{M}_k)^H \\
 & = \mathbf{0}
 \end{aligned}$$

because (A.1)(iii) and $\mathbf{M}_i^H \mathbf{M}_k + \mathbf{M}_k^H \mathbf{M}_i = \mathbf{0}$ by (3.22)(ii).

Next,

$$\begin{aligned}
 & \tilde{\mathbf{C}}_i^H \tilde{\mathbf{C}}_k + \tilde{\mathbf{C}}_k^H \tilde{\mathbf{C}}_i & 1 \leq i \leq 3, 4 \leq k \leq x+2 \\
 & = (\mathbf{D}_1 \otimes \mathbf{M}_i)^H (\mathbf{C}_p \otimes \mathbf{I}_4) + (\mathbf{C}_p \otimes \mathbf{I}_4)^H (\mathbf{D}_1 \otimes \mathbf{M}_i) & 2 \leq p \leq x \\
 & = (\mathbf{D}_1^H \mathbf{C}_p \otimes \mathbf{M}_i^H) + (\mathbf{C}_p^H \mathbf{D}_1 \otimes \mathbf{M}_i) \\
 & = (\mathbf{D}_1^H \mathbf{C}_p \otimes \mathbf{M}_i^H) + (\mathbf{D}_1^H \mathbf{C}_p \otimes \mathbf{M}_i^H)^H \\
 & = \mathbf{0}
 \end{aligned}$$

because (A.1)(iii), $\mathbf{D}_1^H \mathbf{C}_p = \mathbf{C}_p^H \mathbf{D}_1$ by (3.19)(iii), and $\mathbf{M}_i^H + \mathbf{M}_i = \mathbf{0}$ by (3.22)(v).

Following similar approach as above, all the $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ matrices constructed in

Construction 3.1 can be shown to satisfy property (3.19)(ii).

Verification of property in (3.19)(iii)

$$\begin{aligned}
 & \tilde{\mathbf{C}}_i^H \tilde{\mathbf{D}}_k + \tilde{\mathbf{D}}_k^H \tilde{\mathbf{C}}_i & 1 \leq i, k \leq 3 \\
 & = (\mathbf{D}_1 \otimes \mathbf{M}_i)^H (\mathbf{C}_1 \otimes \mathbf{N}_k) - (\mathbf{C}_1 \otimes \mathbf{N}_k)^H (\mathbf{D}_1 \otimes \mathbf{M}_i) \\
 & = (\mathbf{D}_1^H \mathbf{C}_1 \otimes \mathbf{M}_i^H \mathbf{N}_k) - (\mathbf{C}_1^H \mathbf{D}_1 \otimes \mathbf{N}_k^H \mathbf{M}_i) \\
 & = (\mathbf{D}_1^H \mathbf{C}_1 \otimes \mathbf{M}_i^H \mathbf{N}_k) - (\mathbf{D}_1^H \mathbf{C}_1 \otimes \mathbf{M}_i^H \mathbf{N}_k)^H \\
 & = \mathbf{0}
 \end{aligned}$$

because (A.1)(iv), $\mathbf{D}_1^H \mathbf{C}_1 = \mathbf{C}_1^H \mathbf{D}_1$ by (3.19)(iii), and $\mathbf{M}_i^H \mathbf{N}_k - \mathbf{N}_k^H \mathbf{M}_i = \mathbf{0}$ by (3.22)

(iii).

Next,

$$\begin{aligned}
 & \tilde{\mathbf{C}}_i^H \tilde{\mathbf{D}}_k + \tilde{\mathbf{D}}_k^H \tilde{\mathbf{C}}_i & 1 \leq i \leq 3, 4 \leq k \leq y+2 \\
 & = (\mathbf{D}_1 \otimes \mathbf{M}_i)^H (\mathbf{D}_p \otimes \mathbf{I}_4) - (\mathbf{D}_p \otimes \mathbf{I}_4)^H (\mathbf{D}_1 \otimes \mathbf{M}_i) & 2 \leq p \leq y \\
 & = (\mathbf{D}_1^H \mathbf{D}_p \otimes \mathbf{M}_i^H) - (\mathbf{D}_p^H \mathbf{D}_1 \otimes \mathbf{M}_i) \\
 & = (\mathbf{D}_1^H \mathbf{D}_p \otimes \mathbf{M}_i^H) - (\mathbf{D}_1^H \mathbf{D}_p \otimes \mathbf{M}_i^H)^H \\
 & = \mathbf{0}
 \end{aligned}$$

because (A.1)(iv), $\mathbf{D}_1^H \mathbf{D}_p = -\mathbf{D}_p^H \mathbf{D}_1$ by (3.19)(ii), and $\mathbf{M}_i^H + \mathbf{M}_i = \mathbf{0}$ by (3.22)(v).

Following similar approach as above, all the $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ matrices constructed in *Construction 3.1* can be shown to satisfy property (3.19)(iii).

In summary, as long as the matrices $\{\mathbf{M}, \mathbf{N}\}$ used in *Construction 3.1* follow the constraints in (3.22), a higher order ACOD that follows (3.19) can be obtained. Hence *Construction 3.1* is proved. ■

APPENDIX B DERIVATION OF ML DECISION METRIC IN (5.15)

The received signal model in (2.6) is reproduced below:

$$r_p^{(k)} = \sqrt{\frac{E_s}{N_T}} \sum_{i=1}^{N_T} (h_{i,k} x_p^{(i)} + \eta_p^{(k)}) \quad (\text{B.1})$$

Assuming that perfect channel state information (CSI) is available, the receiver computes the decision metric as follows [9]:

$$\sum_{k=1}^{N_R} \sum_{p=1}^P \left| r_p^{(k)} - \sum_{i=1}^{N_T} h_{i,k} x_p^{(i)} \right|^2 \quad (\text{B.2})$$

Considering the Minimum-Decoding-Complexity Quasi-Orthogonal Space-Time Block Code (MDC-QOSTBC) in (5.14),

$$\mathbf{C} = \begin{bmatrix} c_1^R + jc_3^R & c_2^R + jc_4^R & -c_1^I + jc_3^I & -c_2^I + jc_4^I \\ -c_2^R + jc_4^R & c_1^R - jc_3^R & c_2^I + jc_4^I & -c_1^I - jc_3^I \\ -c_1^I + jc_3^I & -c_2^I + jc_4^I & c_1^R + jc_3^R & c_2^R + jc_4^R \\ c_2^I + jc_4^I & -c_1^I - jc_3^I & -c_2^R + jc_4^R & c_1^R - jc_3^R \end{bmatrix} \quad (\text{B.3})$$

where $x_p^{(i)}$ in (B.2) is the transmitted symbol at i^{th} transmit antenna at p^{th} symbol period, so is the p^{th} row and i^{th} column entry of the STBC codeword $\mathbf{C}$ in (B.3).

The decision metric in (B.2) becomes:

$$\sum_{k=1}^{N_R} \left\{ \begin{aligned} & \left| r_1^{(k)} - \left[h_{1,1}(c_1^R + jc_3^R) + h_{2,1}(c_2^R + jc_4^R) + h_{3,1}(-c_1^I + jc_3^I) + h_{4,1}(-c_2^I + jc_4^I) \right] \right|^2 + \\ & \left| r_2^{(k)} - \left[h_{1,2}(-c_2^R + jc_4^R) + h_{2,2}(c_1^R - jc_3^R) + h_{3,2}(c_2^I + jc_4^I) + h_{4,2}(-c_1^I - jc_3^I) \right] \right|^2 + \\ & \left| r_3^{(k)} - \left[h_{1,3}(-c_1^I + jc_3^I) + h_{2,3}(-c_2^I + jc_4^I) + h_{3,3}(c_1^R + jc_3^R) + h_{4,3}(c_2^R + jc_4^R) \right] \right|^2 + \\ & \left| r_4^{(k)} - \left[h_{1,4}(c_2^I + jc_4^I) + h_{2,4}(-c_1^I - jc_3^I) + h_{3,4}(-c_2^R + jc_4^R) + h_{4,4}(c_1^R - jc_3^R) \right] \right|^2 \end{aligned} \right\} \quad (\text{B.4})$$

By expanding (B.4) followed by some simplifications, we will be able to obtained the decision metric shown in (5.15).