Radiated emission models of power line communication systems

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**RADIATED EMISSION MODELS OF
POWER LINE COMMUNICATION SYSTEMS**

Luo Wu Qiong

SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

2007

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LINE COCOMMUNICATION SYSTEMS

Luo Wu Qiong

2007

Radiated Emission Models of Power Line Communication Systems

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University

in fulfillment of the requirement for the degree of
Doctor of Philosophy

2007

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me and has not been submitted for a higher degree to any other University or Institute.

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Date

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Luo Wuqiong

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Summary

This thesis contains the formulation of theoretical models and the results of experimental studies for the problems of radiated emissions from outdoor and indoor Power Line Communication (PLC) systems in the frequency range of 1-30 MHz.

For the outdoor power line system (both the overhead and underground lines), a radiated emission model is derived based on the full wave approach, i.e., the electric and magnetic Herz vectors subject to the boundary conditions of the dielectric-conductor and air-lossy ground surfaces. Appropriate approximations and assumptions according to PLC conditions (overhead and underground lines) are then applied to obtain a simplified transmission line model for a straight infinite-length PLC line. The effects of different types of ground and the geometry of the line on the characteristics and radiation of the PLC lines are investigated. In general, interaction between the PLC lines and the ground creates mutual impedance. This does not only affect the input impedance of the power lines, but also increases loss. The effect on the radiated emission of the PLC systems according to the electrical characteristics of the ground plane is studied. Experiments are made to check the validity and accuracy of our model.

Summary

Our results show that radiated emissions are approximately 20 dB smaller in average when the PLC lines are underground compared to the overhead PLC lines. Hence, more attention should be paid to the overhead lines when considering EMI to other communication systems.

The electromagnetic interference due to the PLC carriers to other power lines is also presented. Measurements were made to evaluate the accuracy of our model result. Our results show that the interference due to the PLC carrier to other power lines is unavoidable. The interference decreases greatly with the distance between the power lines especially when the lines are above good conducting ground.

The outdoor model mentioned above is useful for modeling the long distance PLC lines. However, for indoor PLC applications (short distance PLC systems), the PLC lines are not straight and the effects of the discontinuity along the lines, e.g., PLC lines bent with an arbitrary angle, and the terminations at both ends of the lines must be taken into account. A new radiated emission model, which is applicable to a finite-length PLC lines bent with an arbitrary angle above a lossy ground plane is proposed. The new model is derived based on the concept of distributed dipole antenna. In this approach, each power line is divided into small elements with the length of each element much smaller than the wavelength. Every small element is then treated as a dipole antenna. The tangential electric field and circumferential magnetic field at the surface of the power line is then determined. The current and voltage along

Summary

the PLC lines can then be calculated so that the radiated emission can be derived. Experiments were carried out to validate the model. To the best of the author's knowledge, no previous PLC radiation models have been formulated in this way before.

Finally, a computationally efficient distributed circuit model for finite-length PLC lines bent with an arbitrary angle above a lossy ground plane is presented. Comparisons between theoretical calculations with experimental data as well as with a full wave approach show the validity and applicability of our proposed model for such PLC applications.

Glossary

Abbreviation

PLC	Power Line Communication
LV	Low Voltage
MV	Middle Voltage
HV	High Voltage
NDGPS	Nationwide Differential Global Position System
LAN	Local Area Network
RegTP	German Regulatory Authority
p.u.l.	Per-unit-length
PVC	Polyvinyl Chloride
RSGB	Radio Society of Great Britain
EMC	Electromagnetic Compatibility
EDN	Electricity Distribution Network
PLT	Power Line Telecommunication
RAS	Radio Astronomy Service

Glossary

Symbols

Term	Symbol	Term	Symbol
Current	I	Current magnitude	I_0
Common mode current	I_c	Differential mode current	I_d
Voltage	V	Electric Hertz potential	Π_E
Magnetic Hertz potential	Π_H	Electric field strength	E
Magnetic field strength	H	Current density	J
Conductivity	σ	Magnetic constant	μ_0
Dielectric constant	ε_0	Relative permittivity	ε_r
Effective permittivity	ε_{eff}	Wave number in air	k_0
Propagation constant	γ	Wave number in wire	k_w
Wave number in ground	k_g	Ground impedance	Z_g
External impedance	Z_e	Series impedance	Z
Magnetic flux	Φ	Ground admittance	Y_g
External admittance	Y_e	Characteristic impedance	Z_c
Shunt admittance	Y	Coupled device admittance	Y_{cd}
Load admittance	Y_l	Frequency	f

Glossary

Angular frequency	ω	Loss tangent	$\tan \delta$
Modified Bessel function of first kind and zero order	I_0	Modified Bessel function of first kind and first order	I_1

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Chapter 1

Introduction

1.1 Background

With the Internet explosion and growth in multiple-PC homes and other Internet-enabled devices, the demand to send digital voice, video and Internet data to and between homes has been increasing continuously [1]. An in-home local area network can be either a wired or wireless network. A number of competing emerging access techniques ranging from copper enhancements (ADSL, SDSL, VDSL, HDSL) and fiber-to-the curb (FTTC) and fiber-to-the-home (FTTH) to wireless local loop (WLL) have been tested and evaluated [2].

The technologies that require new structured wiring provide a safe way to deploy new services in the home environment as they have been tested in the enterprise and business sectors. Since the most daunting cost of the home networking is the cabling installation, establishing new wires in an existing home is difficult and expensive.

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Moreover, most consumers are not willing to or unable to afford a large scale home rewiring. Great focus has been put on the “no-new-wires” solutions. These solutions are either using wireless technologies or exploiting existing home wires.

The wireless communication technologies aim to define a common air interface that supports both wireless voice and LAN data services in the home environment, ensuring interoperability among various wireless products being developed by PC, communications and consumer electronics vendors for the home market [3]. They are mainly used in broadcast video; video on demand, and broadband applications in the frequency range of 2-3 GHz and 28-38 GHz. Bluetooth is now used widely in the short distance communications. Its range is from 10 cm to 10 metres providing up to 1 Mbps in the unlicensed 2.4 GHz band. Wireless services appear almost ideal at first glance, but will face severe problems in practice, when several floors of a building have to be covered. PLC is not challenged by walls and other obstacles because the signal runs mainly along the wires and the coverage is complete even for large buildings.

Most houses world-wide are equipped with structured wiring, at least for electricity and phone distribution, and a large percentage of new buildings also have coaxial cabling for TV delivery. Reutilization of these networks for data distribution poses a good opportunity for in-home networking, as it minimizes installation costs.

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Power line carrier (PLC) is an important technique used extensively on electric power lines for communication and telemetry [4]. The carrier-frequency transmission of voice over high-voltage lines began in 1920 [5]. Important tasks to maintain the function of extensive high-voltage networks are operations management, monitoring, and troubleshooting. Coupling is normally implemented in parallel, i.e. within the building between neutral conductor and a phase. These systems have operated at comparatively low frequencies, typically below 450 kHz in the US and below 148.5 kHz in Europe [2]. Applications of PLC include protective relaying, telemetering, voice communications, and supervisory control [6].

In the 1980s, an innovative PLC communications system was developed for office use, which enabled computers and peripheral devices to communicate with each other within a building by using the normal ring main system [7]. The principle behind PLC technology appears simple: the low voltage end of an electric transformer in a power utility's low voltage network provides a telecommunication network gate (capable of serving 100-300 premises or up to distance of approximately 300 metres without repeater) so that it can distribute Internet access and telephony services to private households and business users [8]. The technology can use existing electric cables and the household wiring network to transmit data, with impressive throughput rates reaching several Mbps [9]. Reliable communications were achieved at 16,000 baud over considerable distances, in what is a particularly difficult and unpredictable environment [10].

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In 1995, it was demonstrated that Low Voltage (LV) power lines could be used to carry high frequency (>1 MHz) communication signals [7, 11-17]. In the vision of PLC, it is possible to build an in-house communication network or to access the Internet in a very cost-effective way as installing wires in an existing home is difficult and is not a solution amenable to the mass market [18]. This can be done with nearly no further expenditure on infrastructure installations, as the power line network already has an extensive coverage compared with any other networks[19]. Reutilization of these networks for data distribution poses a very good opportunity for in-home networking, as it minimizes installation costs [20]. In the Europe utility market, it is tempting to compete with classical telecom providers now over power lines for broadband internet access in mostly residential areas [21].

The usage of a frequency range up to more than 30 MHz offers excellent possibilities, for example, for various indoor applications, such as digital audio, video signal distribution, or even Internet access. Everybody can get online by simply plugging a PC into wall sockets. Power line will make worldwide communication, procurement of information, purchasing and trade as commonplace and omnipresent as drawing electric power from the wall plug is today. There is no doubt that this development is of enormous significance and consequence for the future of a highly industrialized society with global interconnections on all levels [7]. The overall system architecture and components of a PLC system for last mile access is shown in Fig. 1.1 [22]. The outdoor segment covers the public area, which

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is from transformer substation to the house access point (HAP), and the indoor segment is from the HAP to the electrical outlets inside a building. The outdoor master (OM) usually installed in the transformer substation controls the outdoor access point (OAP) at the customer premises. An outdoor repeater (OR) is available to cover remote buildings. In each served building, the independent indoor PLC system is governed by a so-called indoor controller (IC), which controls the indoor adapters (IA). The indoor and outdoor PLC are connected via an Ethernet link between the OAP and IC.

Fig.1.1 System architecture and components of a PLC system for last mile access [22]

In Europe, the outdoor PLC uses the lower frequency band, 1-10 MHz, while the indoor PLC uses the higher frequency band, 15-30 MHz. Each band can

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accommodate up to three carriers, each with a bandwidth of about 2 MHz. The selection of the utilized frequency band is optional. For example, in Singapore, it is from 1.6-30 MHz. In Fig. 1.2, a typical customer's connection example is shown [23].

Fig.1.2. Customer's connection example [23]

Recently, a few companies have been developing high data rate power line communications technology [22, 24-29]. Such systems would be deployed on distribution networks to provide telecommunication services, (Internet Access, Telephony etc.), or used within buildings to provide computer networking facilities without the need for traditional cabling to be installed. In order to accommodate the increased data bandwidth, often several Mbit/s, PLC developers have been forced to adopt carrier frequencies within the HF band, with systems typically operating between 1-30 MHz [30].

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At present, several companies have already put their products on the market. Yardena tested a new HomePlug networking product (85003-2004p024-2b) with a theoretical maximum speed of 14 megabits per second [26], and Phonex introduced Ethernet Bridge QX-202 and QX-201 with speed of 14 Mbps [27]. And such as Siemens SpeedStream 2501 (02, 21) Power Line USB Adapter [28], IOGEAR's Home-Plug Ethernet Bridge is also capable of a data speed up to 14 Mbps [29]. This technology which is already under evaluation in Brazil in several commercial trials, will raise all services from electric power distribution companies thanks to a data transmission that will allow services as a high speed Ethernet, digital telephony, videoconference and video surveillance, and residential automation, adding value to the electric grid infrastructure [25].

1.2 Motivation

In spite of the promising market for PLC technology, a lot of issues need to be considered. Since the power line is not established for the high frequency (1-30 MHz) communication, the signal channel attenuation is quite large compared with wires specially designed for high frequency signal transmission [31-34]. The propagation characteristics are unknown for most of the conditions. Path loss and local noise are the predominant factors for the outdoor range PLC systems. The path loss depends on the cable type, cable length, coupling mismatch and the topology of the power networks [35]. In component selection, it is said that the effects of parasitic capacitances are worse at high voltages while the effects of parasitic inductances are worse at high currents that should be considered in practical circuit design [36].

Unlike other wired communication mediums such as the optical cable, LV power lines present an extremely harsh environment for high frequency communication signals. Although PLC is intended to be wire-bound, a certain impact on wireless services seems inevitable, because power distribution networks are electromagnetically open. This could invoke different EMC problems, which have to be carefully analyzed and avoided by appropriate measures. There is a high probability that wide band emissions from power line telecommunication systems in

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the HF spectrum from connected domestic wiring would exceed the levels that allow radio and telecommunications apparatus to operate satisfactorily.

For typical in-building power lines, there are two types of external fields. The field of the guided wave, with a Poynting Vector parallel to the guiding wire, and the field of the radiated wave, with a Poynting Vector directed away from the wires. When the frequency is below 1 MHz, for example, 100 kHz, the free space wavelength of 3 km is large compared to the dimensions of the system. However when it is above 1 MHz region, e.g. 10 MHz with the free space wavelength of 3 metres, it is comparable to the dimensions of the system, and the power line will act as an effective antenna. In reality, the length of in-building power line is comparable to the wavelength within the frequency band of 1-30 MHz, and the lines will radiate emissions as antennas do.

The most desirable feature of a PLC system is the peaceful coexistence of established radio wireless technologies [37, 38]. It was stated that if Power Line Telecommunication (PLT) was permitted to operate without adequate protection of radio services, much of the HF spectrum would become unusable. The radiated emission from the PLC systems has invoked lots of attention. In 19th, February, 1998, Radio Society of Great Britain (RSGB) asked the Radio Astronomy (RA) Community to advise them on the significance of PLT emission (<http://www.rsgb.org.uk>) [39]. Radio Astronomy has frequency allocations in

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13.36-13.41 MHz and 25.55-25.67 MHz on a primary basis, worldwide. The powers from a single PLC system in the above radio astronomy bands are -33 dBW and -29.2 dBW, respectively. Therefore, the PLC systems seem to be a harmful interference source for the radio astronomical observation in the HF band. The Radio Society of Great Britain (RSGB) represents the interests of some 60,000 licensed radio amateurs in the UK and also a large number of short wave listeners who are potentially suspect to the PLT interference. In October, 2004, FCC adopted changes to Part 15 of its rules to encourage the development of Access Broadband over Power Line (Access BPL) systems while safeguarding existing licensed services against harmful interference [40].

1.3 Objectives

Specially, I aims to:

- a) Propose a basic PLC line channel model for high-speed data transmission.

The model should take into account the geometry of the line as well as the effect of a lossy ground plane. In the model, Hertz Potentials are obtained based on Maxwell's Equations subject to the boundary conditions at the interfaces of the air-ground and wire-dielectric, and Sommerfeld functions are applied to include the effect of the lossy ground plane.

- b) Propose a radiated emission model of straight PLC lines embedded in a lossy ground plane.

- c) Extend the above radiated emission models to include the effect of terminations at both ends of finite-length PLC lines and the effect of discontinuities along the PLC lines, i.e., a bend.

- d) Develop a channel model for the general PLC line bent with an arbitrary angle above a lossy ground plane. Emphasize the end effect of the finite-length straight PLC line and the bend effect of the PLC line.

- e) Carry out experiments to evaluate the validity and applicability of our proposed models.

1.4 Major Contributions of this Thesis

In this thesis, original contributions have been achieved and the relevant outcomes have led to publication and submission of several further publications. The major contributions are listed below.

- a) A model to investigate the effect of the lossy ground plane on radiated emission from overhead PLC lines has been proposed. The model was derived based on the full wave approach according to the boundary conditions at the interfaces of the wire-dielectric and the air-ground. In the model, the effect of the electric characteristics of the ground on the signal transmission has been studied, and explicit expressions for the effect of the lossy ground plane were presented. The effect of the height of the PLC line above the ground plane on the common mode current and the radiated emission have also been considered. Comparisons between the theoretical calculations with experimental data show good agreement.

- b) A radiated emission model for PLC lines embedded in the ground has been proposed, and the effects of the electrical properties of the ground on the common mode current and radiated emission discussed. Theoretical calculations were compared with our own measurements and experimental data reported in the literature.

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- c) A radiated emission model for finite-length PLC lines above a lossy ground plane has been derived. The model results were compared with our own measurement results.
- d) The finite-length model is extended to include the PLC lines bent with an arbitrary angle. Experiments were carried out to validate the model.
- e) A distributed circuit model is proposed to simulate the transmission characteristics of the high frequency PLC signals. The model was applied to the finite-length straight PLC lines and PLC lines bent with an arbitrary angle. With the circuit model, the characteristic impedance and propagation constant can be obtained easily. The model takes into account the end effects of the PLC lines, the lossy ground plane, and the discontinuity along the PLC line, i.e., the bend.

1.5 Organization of the Thesis

The thesis consists of seven chapters organized as follows:

Chapter 1 presents an introduction to the motivation, objectives and the contributions of the research work.

Chapter 2 gives a general introduction to the work done on the PLC channel characteristics and the radiated emissions. The introduction is divided into two parts. In the first part, research work on the transmission characteristics and radiated emission of the outdoor PLC line systems is introduced. In the second part, research work on the indoor power line system is surveyed.

Chapter 3 proposes a radiated emission model of an infinite-length PLC line based on the full wave approach according to the boundary conditions at the interfaces of the air-ground and wire-dielectric. The ground effect is enhanced to study the effect of the lossy ground plane. Formulations to derive the series impedance and shunt admittance are outlined. Current distributions along the wires are determined. The effect of the electric characteristics of the ground plane, i.e., the conductivity and permittivity of the ground to the transmission characteristics of the signal is discussed. The common mode current and radiated emission from the PLC lines is compared for different types of the ground plane. Finally, we detail our own

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experimental apparatus and procedures for common mode current and radiated emission measurements and the measurement sites where the data were collected. The experimental results of the common mode current and radiated emission are compared with the theoretical calculations for PLC lines above the concrete and conducting ground. The good agreement shows the accuracy and applicability of our model for such PLC applications.

In Chapter 4, the radiated emission model of a finite-length PLC line is proposed. The concept of a distributed dipole antenna is used to model the finite-length PLC line. By using transmission line theory and the chain matrices approach, the common mode current distribution and the radiated emission are calculated. Experimental results for PLC lines above the concrete and conducting ground are compared with the theoretical calculations.

In Chapter 5, the radiated emission model of finite-length PLC lines bent with an arbitrary angle is presented. The effect of the bend on the common mode current and the radiated emission is studied. Comparisons between the predicted and measured common mode current and the radiated emission of the bent PLC line above concrete and conducting ground are presented.

In Chapter 6, a distributed circuit model of a single-conductor straight PLC line above a lossy ground plane is proposed. The model is then extended to include

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multi-conductor PLC line. In this case, the mutual impedance is taken into account.

The effect of the terminations is studied with the circuit schematic. The method is then extended to include multi-conductor PLC line bent with an arbitrary angle.

Chapter 7 is the conclusion of the thesis and suggestions for future research.

Chapter 2

A Brief Survey of the Power Line Communication Systems

As an alternative last mile access technology, broadband power line communications (PLC) have been receiving tremendous interest in recent years. PLC access networks typically cover both the public area from the transformer substations to the customer premises (outdoor) to the private area within the customer buildings (indoor).

Much theoretical and experimental work has been reported in the area of the transmission characteristics and radiated emissions of PLC lines. Section 2.1 and 2.2 give a brief survey of the work done in the area of channel characteristics, common mode current, and the radiated emission for outdoor and indoor PLC systems.

2.1 Outdoor PLC Systems

Liu et al [41] described the network of the APLETT (Ascom Power Line Ethernet Technology Trial) containing two substations in Leichlingen. The nature of the PLC line was measured with unshielded underground cables. Figures for the measured attenuation and coupling loss of the PLC system were given. The authors found that the signal attenuation increased quickly with the operating frequency. If the tolerable attenuation is 40 dB, the transfer distance of the signal is less than 300 metres for frequencies above 2 MHz. Hooijen [42] studied the signal channel of the PLC line as a time-variant linear filter based on measurements taken in part of the RPLC (Residential Power Line Communication) experiment in Amsterdam. It was concluded that signal attenuation including coupling losses and line losses could be very high and this made it difficult to transmit over distances above 500 metres.

Fig.2.1 A Segment of power line network in Leichlingen [42]

Chapter 2 A Brief Survey of the Power Line Communication Systems

Zimmermann and Dostert [43] proposed an analytical model describing the transfer function $H(f)$ of a typical power line network with the parameters of the frequency responses based on physical signal propagation effects. Guo et al [44] studied the characteristics of a low voltage power line network through the measurement of S-parameters as well as the noise distribution in the frequency range of 3-500 kHz in China.

Aedreou et al [45] carried out a series of experiments to measure the distributed cable parameters and evaluate equations to calculate the parameters. Meng et al [46] simulated the PLC line with two transmission lines and obtained the characteristic impedance and propagation constant using transmission theory for a 3-core PLC line in a conduit. However, the model cannot be applied when the PLC line is above or embedded in a lossy ground plane. Olsen et al [47] reviewed the properties of current along a thin horizontal wire above a flat lossy ground.

Marthe et al [48] proposed a measurement technique for the analysis of PLC signals based on the scattering parameters representation of the various components (lines, cables, etc) of the network. Sutterlin [49] carried out research on PLC with the parameters obtained in advance. He suggested that capacitance is the dominator for PLC when terminated in a load impedance is much greater than the characteristic impedance of the wire. Besides the experimental research, Gonzalez et al [50] presented the software power line channel simulator adapted to the Galician (electrical

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experiment was always carried out) on a low voltage mains network in the 2-20 MHz band.

Fig. 2.2 Antenna array model [52]

With power line communications new broadband services will be operating in a network that generally consists of unshielded and heterogeneous cable types. Yazdani discussed the possibility of PLC in sub-sea communications with measurement and calculation results [51]. While transmitting signals at high frequencies over unshielded cables, such a wiring system may operate as an antenna and emit considerable radiation due to the lack of symmetry. The Radio Society of Great Britain (RSGB) [39] carried out research in 1998 to study the electromagnetic interference to amateur radio users. Measurements were carried out and it was said that the application system might cause interference to the amateur services. RSGB EMC Committee for the PLC Workshop in Brussels presented great interest in the compatibility between the PLC systems and the existing radio communication services. It was stated that there was a

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high probability that wide band emissions from PLC systems in the HF spectrum from connected domestic wiring would exceed levels which allow radio and telecommunications apparatus to operate as intended, contrary to UK Statutory Instrument 1992-No.2372, Part 1, Section 5, Clause 4. And, the effect of emissions from a large number of PLC sources could also cause interference to air born users of HF when flying over built-up areas. Ohishi et al [37] stated in their article that it was necessary to keep an adequate separation distance between a PLC system and the Radio Astronomy Service (RAS) to avoid harmful interference to the radio astronomy telescope. Calculations were made to ensure that interference level could be maintained. The conclusion was that it was difficult for the RAS to share frequencies with the widely deployed PLC. Lole [38] also stated the EMC impact of PLC systems on other wireless communication systems.

Esmailian [52] modeled the multi-conductor PLC line as a uniform linear antenna array. A comparison of experimental results and simulation was provided. Since the proposed equations of the antenna array related only for free space, the model could not take account of the effect of the real environment. Further more, this model could not be applied when the length of the PLC wire was longer or comparable to the wavelength. Gebhardt [53] presented the magnetic radiation measurement results for a residential building. The signal was injected within the house and the field was measured outside the building. He identified the typical property that the field regression depended on the distance. The high frequency signal was injected through a

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CW (continuous-wave) generator, directional coupler and a balun. The magnetic field was measured with a loop antenna connected to a receiver. A vertical antenna and a horizontal antenna were used to model the radiated emission from the residential building. Triantafyllidis et al [54] proposed a methodology using the Finite Element Method (FEM) for the calculation of the transmission line series impedances. This numerical method for overhead transmission line was used to solve the electromagnetic field equations in a region. From their model, the impedance was calculated with the total current density J by solving the two-dimensional electromagnetic diffusion problem. The FEM method used to solve the two-dimensional bounded electromagnetic field required large computation time.

Silva [55] discussed the impact of a PLC system (frequency range 40-490 kHz) on the Nationwide Differential Global Positioning System (NDGPS) considering the superposition of the frequency band from 283.5 kHz to 325 kHz. Measurements on the radiation from the overhead PLC system near the ground plane were carried out. It was reported that the PLC fields could have sufficient energy under or close to power lines to affect usage of the DGPS signals.

Issa et al [56] studied the influence of the inductive coupler in the MV (medium voltage) power line network on the signal transfer in the high frequency band. Signal attenuations along the MV power line network were measured with different states of inductive coupler. Dalichau [57] studied the coupling coefficient between two

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neighbouring PLC-LANs and its frequency dependence. The electromagnetic interference to a aeroplane flying over an area established with a typical PLC network was studied. It was proposed that the sum of the relative sky wave interference calculated using the equation $a_{\Sigma} = 20\log(E_{\Sigma} / E_0) = a_{SA} + a(r) + a_{Ionos} + a(M) + G_M$, where a_{SA} was the spherical angle of the part of the transmitted energy, $a(r)$ was the attenuation due to the average distance of the sky wave traveled, a_{Ionos} was the ionospheric loss, $a(M)$ was the energy of all PLC transmitters, and G_M was the average antenna gain of all PLC transmitters. Fenton [58] suggested the cumulative effects of the PLC systems on the radio reception, in the HF band. Signal propagation models were also studied with the free space propagation model and the Egli propagation model.

Brown [59] discussed the characterization of an electricity Distribution Network (EDN) with reference to the network parameters observed at the END terminal ports in HF. An experimental setup to measure the EMC from the EDN was proposed. Widmer [60] assessed the potential of global cumulative radio interference effects caused by over the horizon ionospheric propagation of unwanted emissions from broadband PLC systems operating in the HF band. In cities where PLC might be deployed with high density over a large area, summation of many weak far field disturbance components is expected. It was stated in his article that the vertical radiation could affect the HF radio services such as airborne HF radio service via ionospheric reflection. A large number of vertical radiations from the broadband PLC transmitter bounced back to earth by the

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ionosphere might increase the present sky wave noise that might impair HF radio reception. As shown in Fig. 2.3, radio waves with lower frequency (typical <5 MHz) are reflected by the E-layer, higher frequencies (typical >5 MHz) reflected by the F-layers, while the D-layer is attenuating. In his article, he also considered the street lamp as a radiation transmitter and a computer model was proposed.

Fig. 2.3 Ionospheric radio propagation [60]

Issa et al [61] characterized the outdoor radiation associated with the PLC signals in HF band in a common mode propagation approach. The model was established on the basis of outdoor underground cables. The Numerical Electromagnetic Code (NEC4) was used to simulate the radiation model. Chaffanjon and Issa [62] discussed the radiated emission from underground power line cables with signals in HF band. In

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their work, electric and magnetic fields were measured separately to find the relationship between the two. It was found that the difference between the E-field and H-field changed with the frequency. Measuring the H-field to calculate the E-field, a 20 dB overestimation at 4 MHz became a 5 dB underestimation at 17 MHz.

There is another factor that should be taken into account. Power lines usually have two or more conductors parallel to each other. The asymmetry of the power line will cause more radiated emission. Consider a two parallel conductors power line system, if the currents flowing along the two conductors are the same in magnitude while opposite in direction. The electrical fields at the far zone will be mostly cancelled if the two lines are close to each other. However for a power line above a lossy ground plane, there will be current flow on the surface of the ground. As a result, the currents flowing along the two conductors are no longer the same in magnitude [58, 63, 64]. In general, the current flow along a two-conductor power line can be divided into differential mode and common mode as shown in Fig. 2.4. In the figure, I_D and I_C are the differential and common mode current respectively. R is the distance from the power line to the observation point. The distance between the two conductors is d . Consider a small element $\Delta\ell$ of the power line, the electric field created by the differential and common mode current at the observation point can be expressed as:

$$E_{D_{\max}} = 1.316 \times 10^{-14} \frac{I_D f^2 \Delta\ell d}{R}$$

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$$E_{C_{\max}} = 1.257 \times 10^{-6} \frac{I_C f \Delta \ell}{R}$$

Fig. 2.4 Differential and common mode current

From the above two equations, we get,

$$\frac{E_{C_{\max}}}{E_{D_{\max}}} \approx \frac{10^8}{df} \cdot \frac{I_C}{I_D}$$

For a typical PLC system, the highest frequency utilized is 30 MHz and the distance d between the two conductors is in millimeter. That is, $E_{C_{\max}} \gg E_{D_{\max}}$ as long as $I_C \gg 10^{-4} I_D$. In the calculation of the radiated emission in the far field, the common mode current should be given more emphasis.

The effect of the common mode current in PLC systems was also studied in [65, 66].

Chaffanjon et al [65] studied the propagation modes of the power line carriers on the

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distribution networks in the frequency range from 3 to 148.5 kHz as well as the frequency range from 50 to 455 kHz. They found that common mode propagation caused crosstalk between neighbouring networks and the reinforcement of the electromagnetic fields radiated by the lines. The common mode field becomes dominant a few metres away from the line which was confirmed by the measurement result. In [66], Bridges studied the time domain of the current induced on an infinite bare or insulated cable buried in a lossy earth medium due to a transient plane wave.

2.2 Indoor PLC Systems

Liu et al [67] presented data characterizing the household AC power line in the frequency band 1-60 MHz. Tang [68] developed a channel model with the characteristics of power lines in Housing Development Board (HDB) flats to simulate their behavior for high speed data transmission of at least 1 Mbps. The transfer function was described by an echo model. From their measurement, the network impedance was found to be varying with frequency and in the range from 20 to 190 Ohms. A high value in impedance variation indicates a strong dependence on time. This was probably due to the presence of random electrical loads connected to the power line network, causing reflections, standing wave patterns, and complex reactance. Philipps [69] gave the measurements of the channel characteristics of power line channels in the frequency range of up to 30 MHz. The investigations dealt with power line inside buildings and were deduced for different types of buildings. The conclusions were that the main problems facing power line transmission was the high frequency dependent network impedance, which led to the impedance mismatch when coupling in and out. Tsuzuki et al [70] discussed the measurement of the transfer functions of an indoor VVF cable taking into account the impedance characteristics of household appliances and power line channels in the frequency band 70 kHz to 35 MHz. Sartenar and Delogne [71] studied the power line distribution cable as a multi-conductor transmission line with a surrounding screen around each cable. Corripio et al [72] contemplated the topology of the indoor power grid and the

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characteristics of the wires and loads to obtain the channel response and the noise incorporating statistics of time varying loads. Hooijen [73] discussed the relation between the RPC-network (residential power circuit network) topology and the signal attenuation. In his paper, different types of topology of the power network were considered, including radial network, ring network and meshed network. Measurement data obtained in an extensive measurement campaign executed in the city of Amsterdam, the Netherlands was provided. He found that signal attenuation levels ranging from 40 to 100 dB/km seemed to be reasonable according to the number of houses connected to the main line. If loads were evenly distributed over the main line, the relation between signal attenuation and distance could be considered to be approximately exponential. Anastasiadou and Antonakopoulos [10] proposed methods to calculate the load impedance characterization in the residential environment based on measurements. Papaleonidopoulos et al [74] gave a transfer model of indoor single-phase sheathed tricels with conductors of circular cross section in the 1-21 MHz frequency area. The model was based on the wave-propagation approach using distributed transmission-line analysis techniques. Tsuzuki et al [7] measured the impedance characteristics of household-appliances and power-line channels in the Japanese environment in the high frequency band, i.e., from 70 kHz to 35 MHz. It was found that the dispersion of the individual appliances' impedance tended to decrease gradually in the range from 500 kHz to 10 MHz. In the frequency range higher than 2 MHz, the impedance of the power line changed from an increasing function to a flat one, fluctuating around 40 Ohms.

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Neto et al [75] proposed an implementation of an I-V method to determine the impedance of power distribution circuits using a network analyzer and a circuit composed of a balun, a transformer and a high-pass band filter in the HF band. Marthe et al [76] presented experimental data measuring the indoor electromagnetic field to estimate the indoor emissions associated with PLC systems. The experiment was performed in two residential buildings as well as in the Power Systems Laboratory of the Swiss Federal Institute of Technology. The experiment setup consisted of a loop antenna/monopole antenna, a network analyzer, and a PC. The signal was injected into the indoor power line system through a directional coupler and a balun. They found that the field magnitude increased substantially with frequency (about 15 dB per decade) under MHz. Above a few MHz, the variation of the field magnitude became more irregular and some fall off was observed.

Minamitani et al [47] emphasized that the effect of the electric field strength leakage produced when a short wave band PLC was impressed on indoor power lines was mainly dependent on the balance of the indoor power line system. In their paper, different cables and different signal injection methods utilized in Japan were discussed and measurements carried out in a model house. It was confirmed that the balance of the power line changed greatly with the type of cable used and the power supply method. Tsuchiya et al [77] studied electromagnetic interference (EMI) problems on a PLC system. It was found that though the utilized frequency band was between 1-30

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MHz, the electromagnetic field caused by the PLC facility could be in the UHF band. In their experiment on measuring the emission, the background noise in the HF band increased about 30 dB when the PLC modem was on. In UHF band, some spurious emission around 327 MHz was found. Vick [64] showed the results of the magnetic field strength measured from indoor power line communication systems in the frequency band of 150 kHz to 30 MHz. It was recommended that there was a potential danger related to unregulated power-line communications.

2.3 Conclusion

An extensive review of the literature shows that most of the EMC problems of the PLC system were focused on the impact of the PLC system on other wireless communication systems. A lot of theoretical and measurement work has been reported but very few studies looked into the effects of the line geometry and environmental factors on the radiated emissions from the PLC system.

One of the main objectives of this thesis is to develop a general radiated emission model that takes into account the geometry of the PLC lines as well as the effect of the lossy ground plane.

Chapter 3

Radiated Emission Models for Straight Infinite-length PLC Systems

The radiated emission from a PLC system will change according to the electric characteristics of the ground plane. For outdoor power line systems, interaction with the ground creates mutual impedance. This not only affects the input impedance of the power line, but also increases loss. In this chapter, the radiated emission model proposed by Wait [78] is first introduced. Appropriate approximations and assumptions for PLC conditions are then applied to derive a simplified model for a straight infinite-length PLC line above lossy ground. Explicit formulae for the series impedance due to the lossy ground plane are derived. The effects of the ground on the transmission characteristics and radiated emission are emphasized. Finally, the experimental setup is described and the accuracy of the model verified by comparison with experimental data.

3.1 Introduction

The model proposed by Wait [78-80] was based on the exact formulation of Maxwell's equations. The propagation characteristics were calculated with the continuity of the tangential components of the electric field at the air-ground and air-wire interfaces as shown in Fig. 3.1. Based on the thin-wire approximation, Hertz vectors based on Maxwell's equation were employed [78].

Fig. 3.1 Geometrical configuration of single line above lossy ground

In Fig. 3.1, a thin wire with radius r_0 is arranged along the x -axis at a height of h from the ground plane. Let the current along the conductor $I(x)$ have the form $I_0 e^{-\gamma x} \cdot e^{j\omega t}$ where $\gamma = \alpha + j\beta$ is the propagation constant and ω is the angular frequency.

The components of the electromagnetic field scattered by the thin wire can be expressed as functions of the electric and magnetic Hertz Potentials, Π_E and Π_H respectively.

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The electric and magnetic fields expressed in terms of $\bar{\Pi}_E$ and $\bar{\Pi}_H$ are given by:

$$\bar{E} = k_0^2 \bar{\Pi}_E + \nabla(\nabla \cdot \bar{\Pi}_E) - j\omega\mu \nabla \times \bar{\Pi}_H \quad (3.1a)$$

$$\bar{H} = k_0^2 \bar{\Pi}_H + \nabla(\nabla \cdot \bar{\Pi}_H) + (\sigma + j\omega\epsilon) \nabla \times \bar{\Pi}_E \quad (3.1b)$$

From Maxwell's equations,

$$\nabla \times \bar{H} = \bar{J} + (j\omega\epsilon + \sigma) \bar{E} \quad (3.2a)$$

$$\nabla \times \bar{E} = j\omega\mu \bar{H} \quad (3.2b)$$

We get the Laplace equations,

$$\nabla^2 \bar{\Pi}_E + k_0^2 \bar{\Pi}_E = -\bar{J} / (j\omega\epsilon + \sigma) \quad (3.3a)$$

$$\nabla^2 \bar{\Pi}_H + k_0^2 \bar{\Pi}_H = 0 \quad (3.3b)$$

Consider the electrical field outside the source region, that is $\bar{J} = 0$. Equation (3.3a) can be written as,

$$\nabla^2 \bar{\Pi}_E + k_0^2 \bar{\Pi}_E = 0. \quad (3.4)$$

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Π_E and Π_H can be solved by applying the boundary conditions for the tangential electric and magnetic fields at the air-ground interface ($z = 0$),

$$E_x(x, r_0, 0)|_{air} = E_x(x, r_0, 0)|_{ground} \quad (3.5a)$$

$$E_y(x, r_0, 0)|_{air} = E_y(x, r_0, 0)|_{ground} \quad (3.5b)$$

$$H_x(x, r_0, 0)|_{air} = H_x(x, r_0, 0)|_{ground} \quad (3.5c)$$

$$H_y(x, r_0, 0)|_{air} = H_y(x, r_0, 0)|_{ground} \quad (3.5d)$$

So we get [80],

$$\begin{aligned} \Pi_E(x, y, z) = \frac{I(x)}{4\pi j\epsilon_0\omega} \int_{-\infty}^{\infty} \left[\exp(-u_0|z-h|) + R_{E0}(\xi) \exp(-u_0(z+h)) \right] \\ \cdot \exp[-j\xi(y-r_0)] / u_0 d\xi \end{aligned} \quad (3.6a)$$

$$\begin{aligned} \Pi_H(x, y, z) = \frac{I(x)}{4\pi j\epsilon_0\omega} \int_{-\infty}^{\infty} R_{H0}(\xi) \exp[-u_0(z+h)] \cdot \exp[-j\xi(y-r_0)] / u_0 d\xi \end{aligned} \quad (3.6b)$$

In (3.6), $R_{E0}(\xi)$ and $R_{H0}(\xi)$ are expressed as:

$$R_{E0}(\xi) = \frac{k_0^2(u_0 - u_g) + \gamma^2(u_0 + u_g)}{(k_0^2 - \gamma^2)(u_0 + u_g)} - \frac{2k_0^2\gamma^2}{(k_0^2 - \gamma^2)(k_0^2 u_g + k_g^2 u_0)} \quad (3.7a)$$

$$R_{H0}(\xi) = \frac{2\xi\gamma u_0}{k_0^2 + \gamma^2} \cdot \frac{(\xi^2 - u_0 u_g)}{k_0^2 u_g + k_g^2 u_0} \cdot \frac{\omega\epsilon_0(k_0^2 - k_g^2)}{u_0(k_g^2 + \gamma^2) + u_g(k_0^2 + \gamma^2)} \quad (3.7b)$$

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where $u_0 = \sqrt{\xi^2 + \gamma^2 - k_0^2}$ and $u_g = \sqrt{\xi^2 + \gamma^2 - k_g^2}$. k_0 and k_g are the wave number in air and ground respectively, and ξ is the free parameter.

Substitute (3.6) into (3.1), the electric and magnetic fields can be expressed as,

$$E_x(x, y, z) = (-\gamma^2 + k_0^2) \bar{\Pi}_E \quad (3.8a)$$

$$E_y(x, y, z) = \frac{\partial^2 \bar{\Pi}_E}{\partial x \partial y} - j\omega\mu_0 \frac{\partial \bar{\Pi}_H}{\partial z} \quad (3.8b)$$

$$E_z(x, y, z) = \frac{\partial^2 \bar{\Pi}_E}{\partial x \partial z} + j\omega\mu_0 \frac{\partial \bar{\Pi}_H}{\partial y} \quad (3.8c)$$

$$H_x(x, y, z) = (-\gamma^2 + k_0^2) \bar{\Pi}_H \quad (3.8d)$$

$$H_y(x, y, z) = \frac{\partial^2 \bar{\Pi}_H}{\partial x \partial y} + j\omega\epsilon_0 \frac{\partial \bar{\Pi}_E}{\partial z} \quad (3.8e)$$

$$H_z(x, y, z) = \frac{\partial^2 \bar{\Pi}_H}{\partial x \partial z} - j\omega\epsilon_0 \frac{\partial \bar{\Pi}_E}{\partial y} \quad (3.8f)$$

The p.u.l. series impedance of the line in Fig. 3.1 is defined as:

$$Z = -\frac{d}{dx} \left[\frac{V(x)}{I(x)} \right] \quad (3.9a)$$

and the p.u.l. shunt admittance is:

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$$Y = -\frac{d}{dx} \left[\frac{I(x)}{V(x)} \right] \quad (3.9b)$$

where

$$V(x) = -\int_0^h E_z(x, r_0, z) dz \quad (3.10)$$

It should be noted that $V(x) \propto E_z(x, r_0, z) \propto I(x)$, so the series impedance Z and the shunt admittance Y are independent of current $I(x)$.

With Z and Y , the propagation constant can be determined:

$$\gamma = \sqrt{ZY}. \quad (3.11)$$

Fig. 3.2 Schematic configuration of single conductor PLC line

Consider a single-conductor PLC line with length ℓ as shown in Fig. 3.2. In the figure, I_{s0} is the source current, Y_{CD} and Y_L are the admittance connected at the source end and the load end respectively.

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In general, the current distribution along the wire can be rewritten as,

$$I(x) = \exp(-\gamma x)I_{01} + \exp(\gamma x)I_{02} \quad (3.12)$$

where I_{01} and I_{02} are the forward and backward currents determined by:

$$I_{01} = 0.5 \frac{Y_{in0} + Y_c}{Y_{in0} + Y_{CD}} I_{s0} \quad (3.13a)$$

$$I_{02} = 0.5 \frac{Y_{in0} - Y_c}{Y_{in0} + Y_{CD}} I_{s0} \quad (3.13b)$$

where the input admittance $Y_{in0} = \frac{\cosh(\gamma \ell)Y_L + Y\gamma^{-1} \sinh(\gamma \ell)}{\cosh(\gamma \ell) + \sinh(\gamma \ell)\gamma^{-1}ZY_L}$ and the characteristic admittance $Y_c = Y\gamma^{-1}$.

Finally, the electromagnetic field generated by the PLC wire can be obtained with,

$$\bar{E}(x, y, z) = I(x)\bar{\Gamma}(y, z) \quad (3.14a)$$

$$\bar{H}(x, y, z) = I(x)\bar{\Phi}(y, z) \quad (3.14b)$$

where the derivation of vectors $\bar{\Gamma}(y, z)$ and $\bar{\Phi}(y, z)$ are given in Appendix.

3.2 Overhead PLC Lines

Since the model proposed by Wait [78] is not specified for PLC application, tremendous computation time is required to evaluate the Sommerfeld integration [78] in calculating the Hertz Potentials. In order to reduce the computation cost, approximations have to be made according to power line communication conditions.

Fig. 3.3 Geometrical configuration of the overhead power distribution line

Consider the overhead PLC lines as shown in Fig. 3.3. The multi-conductor has n parallel wires with the same radius r_0 . The k^{th} conductor is at a height of h_k from the ground, and distant d_k from the origin. ϵ_g , σ_g and μ_g are the permittivity, conductivity and permeability of the ground, ϵ_0 and μ_0 are the permittivity and permeability of the air respectively.

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Assume the current propagation for the k^{th} conductor has the form $I_k \exp(-\gamma_k x + j\omega t)$ where γ_k is the propagation constant and ω is the angular frequency. The Hertz potentials at position (x, y, z) are the cumulation of the Hertz potentials from the source wires.

For the upper half medium where $z > 0$, the Hertz potentials are expressed as,

$$\begin{aligned} \Pi_{E0}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{I_k(x)}{4\pi j\epsilon_0 \omega} \int_{-\infty}^{\infty} \left[\exp(-u_{0k} |z - h_i|) + R_{E0k}(\xi) \exp(-u_{0k}(z + h_i)) \right] \\ \cdot \exp[-j\xi(y - d_i)] / u_{0k} d\xi \end{aligned} \quad (3.15a)$$

$$\begin{aligned} \Pi_{H0}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{I_k(x)}{4\pi j\epsilon_0 \omega} \int_{-\infty}^{\infty} R_{H0k}(\xi) \exp[-u_{0k}(z + h_i)] \\ \cdot \exp[-j\xi(y - d_i)] / u_{0k} d\xi \end{aligned} \quad (3.15b)$$

in which $I_k(x)$ is the current flowing in the k^{th} wire, and $u_{0k} = (\xi^2 + \gamma_k^2 - k_0^2)^{1/2}$ where k_0 is the wave number in air and ξ is a free parameter.

In (3.15a), the first term

$$\sum_{i=1}^n \sum_{k=1}^n \frac{I_k(x)}{4\pi j\epsilon_0 \omega} \int_{-\infty}^{\infty} \exp(-u_{0k} |z - h_i|) \cdot \exp[-j\xi(y - d_i)] / u_{0k} d\xi$$

is the primary field due to the currents flowing along the wires, and

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$$\sum_{i=1}^n \sum_{k=1}^n \frac{I_k(x)}{4\pi j\epsilon_0\omega} \int_{-\infty}^{\infty} R_{E0k}(\xi) \cdot \exp[-u_{0k}(z+h_i)] \cdot \exp[-j\xi(y-d_i)] / u_{0k} d\xi$$

is considered as the field reflected from the lossy ground.

For the lower half medium where $z < 0$, the Hertz potentials are,

$$\begin{aligned} \Pi_{Eg}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{I_k(x)}{4\pi j\epsilon_0\omega} \int_{-\infty}^{\infty} R_{Egk}(\xi) \cdot \exp(-u_{0k}h_i + u_{gk}z) \\ \cdot \exp[-j\xi(y-d_i)] / u_{0k} d\xi \end{aligned} \quad (3.16a)$$

$$\begin{aligned} \Pi_{Hg}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{I_k(x)}{4\pi j\epsilon_0\omega} \int_{-\infty}^{\infty} R_{Hgk}(\xi) \cdot \exp(-u_{0k}h_i + u_{gk}z) \\ \cdot \exp[-j\xi(y-d_i)] / u_{0k} d\xi \end{aligned} \quad (3.16b)$$

where, $u_{gk} = (\xi^2 + \gamma_k^2 - k_g^2)^{1/2}$ and k_g is the wave number of the ground.

The expressions for the reflection related functions $R_{E0k}(\xi)$, $R_{H0k}(\xi)$, $R_{Egk}(\xi)$

and $R_{Hgk}(\xi)$ in terms of k_0 , k_g , u_{0k} and u_{gk} are:

$$R_{E0}(\xi) = \frac{k_0^2(u_{0k} - u_{gk}) + \gamma_k^2(u_{0k} + u_{gk})}{(k_0^2 - \gamma_k^2)(u_{0k} + u_{gk})} - \frac{2k_0^2\gamma_k^2}{(k_0^2 - \gamma_k^2)(k_0^2 u_{gk} + k_g^2 u_{0k})} \quad (3.17a)$$

$$R_{Eg}(\xi) = \frac{2k_0^2 u_{0k}}{k_g^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \quad (3.17b)$$

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$$R_{H0}(\xi) = \frac{2\xi\gamma_k u_{0k}}{k_0^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \cdot \frac{\omega \varepsilon_0 (k_0^2 - k_g^2)}{u_{0k} (k_g^2 + \gamma_k^2) + u_{gk} (k_0^2 + \gamma_k^2)} \quad (3.17c)$$

$$R_{Hg}(\xi) = \frac{2\xi\gamma_k u_{0k}}{k_g^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \cdot \frac{\omega \varepsilon_g (k_g^2 - k_0^2)}{u_{gk} (k_0^2 + \gamma_k^2) + u_{0k} (k_g^2 + \gamma_k^2)}. \quad (3.17d)$$

The series impedance Z is obtained with (3.9a) in which the ground impedance is included. For the n -conductor PLC system, the impedance matrix is considered as

$$\begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix}. \text{ The element of the impedance matrix } Z_{ik} \text{ is given by,}$$

$$\begin{aligned} Z_{ik} = & \frac{j\omega\mu_0}{2\pi} \cdot \{K_0\{j(k_0^2 + \gamma_k^2)[(d_i - d_k)^2 + (h_i - h_k)^2]\} \\ & - K_0\{j(k_0^2 + \gamma_k^2)[(d_i - d_k)^2 + (h_i + h_k)^2]\} \\ & + \int_{-\infty}^{\infty} \frac{2k_0^2}{k_0^2 + \gamma_k^2} \cdot \frac{\xi^2 - u_{1k} u_{2k}}{k_0^2 u_g + k_g^2 u_0} \cdot \exp[u_{0k}(h_i + h_k) - j\xi(d_i - d_k)] d\xi \} \end{aligned} \quad (3.18)$$

where K_0 is the modified Bessel function of order zero.

The ground impedance Z_g can be obtained by applying the boundary conditions at the air-ground interface. They are,

$$E_{x1}(x, y, 0) = E_{x2}(x, y, 0) \quad (3.19a)$$

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$$E_{x2}(x, y, 0) = Z_g I(x) \quad (3.19b)$$

The element of the ground impedance matrix $Z_{g,ik}$ is,

$$Z_{g,ik} = \frac{j\omega\mu_0}{2\pi} \int_{-\infty}^{\infty} \frac{2k_0^2}{k_0^2 + \gamma_k^2} \cdot \frac{\xi^2 - u_{0k}u_{gk}}{k_0^2 u_g + k_g^2 u_0} \cdot \exp[u_{0k}(h_i - h_k) - j\xi(d_i - d_k)] d\xi \quad (3.20)$$

For a typical PLC system, k_0 has a value in the region 0.0210 to 0.6286 m^{-1} when the applied frequency changes from 1 MHz to 30 MHz, k_g is from 0.0702–0.0563 j to 1.2607–0.0939 j m^{-1} for poor ground, 0.1527–0.1293 j to 2.4466–0.2420 j m^{-1} for average ground, and 0.2909–0.2714 j to 3.2276–0.7339 j m^{-1} for good ground according to the conductivity and permittivity for the three types of ground. The condition $|k_1^2/k_2^2| \ll 1$ is satisfied, so $Z_{g,ik}$ can be rewritten as,

$$Z_{g,ik} = \frac{k_0^2}{4\pi j\omega\epsilon_0} \cdot \left(\ln \frac{\sqrt{(h_i + h_k)^2 + (d_i - d_k + rad)^2} - 2j/k_g}{\sqrt{(h_i + h_k)^2 + (d_i - d_k + rad)^2}} + \frac{4\pi j\omega\epsilon_0}{k_0^2 + k_g^2} \cdot \ln \frac{\sqrt{(h_i + h_k)^2 + (d_i - d_k + rad)^2} - jk_g/k_0^2}{\sqrt{(h_i + h_k)^2 + (d_i - d_k + rad)^2}} \right) \quad (3.21)$$

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For most overhead PLC wires, h_k is much larger than the distance between the wires ($h_k \gg d_{ik}$), and $\gamma_k \cong jk_0$. (3.21) can be approximated as,

$$Z_{ground,ik} = \frac{k_0^2}{4\pi j\omega\epsilon_0} \cdot \left(\ln \frac{(h_i + h_k) - 2j/k_g}{(h_i + h_k)} + \frac{4\pi j\omega\epsilon_0}{k_0^2 + k_g^2} \cdot \ln \frac{(h_i + h_k) - jk_g/k_0^2}{(h_i + h_k)} \right) \quad (3.22)$$

So the series impedance of the outdoor PLC system can be expressed as,

$$\begin{aligned} Z_{ik} = & \frac{j\omega\mu_0}{2\pi} \{ K_0 \{ j(k_0^2 + \gamma_k^2) [(d_i - d_k)^2 + (h_i - h_k)^2] \} \\ & - K_0 \{ j(k_0^2 + \gamma_k^2) [(d_i - d_k)^2 + (h_i + h_k)^2] \} + \frac{k_0^2}{4\pi j\omega\epsilon_0} \cdot (\ln \frac{(h_i + h_k) - 2j/k_g}{(h_i + h_k)} \\ & + \frac{4\pi j\omega\epsilon_0}{k_0^2 + k_g^2} \cdot \ln \frac{(h_i + h_k) - jk_g/k_0^2}{(h_i + h_k)}) \end{aligned} \quad (3.23)$$

Considering the wire-to-ground voltage expressed by the Hertz potentials, the shunt admittance can be obtained using (3.9b). The element in the admittance matrix Y_{ik} is expressed as,

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$$\begin{aligned}
 Y_{ik} = & j\omega\epsilon_0 2\pi \{ K_0 \{ j(k_0^2 + \gamma_k^2) [(d_i + d_k)^2 + (z - h_k)^2] \} \\
 & - K_0 \{ j(k_0^2 + \gamma_k^2) [(d_i - d_k)^2 + (z - h_k)^2] \} + \int_{-\infty}^{\infty} \frac{2k_0^2}{k_0^2 + \gamma_k^2} \cdot \frac{\gamma_k^2}{k_0^2 u_g + k_g^2 u_0} \\
 & \exp[u_{0k}(h_k + z) - j\xi(d_i - d_k)d\xi]_{z=0}^{z=h_i} \}^{-1}
 \end{aligned} \quad (3.24)$$

in which the ground admittance $Y_{g,ik}$ is included. After making the similar approximations similar to those used to obtain (3.21), the admittance can be written as,

$$Y_{g,ik} = \frac{j\pi k_g^2}{\omega\mu_1 \ln \frac{jk_g^2 + k_g^2/(h_i + h_k)}{jk_0^2 + k_g^2/(h_i - h_k)}} \quad (3.25)$$

The propagation constant and characteristic impedance can be derived,

$$\gamma = \sqrt{ZY} \ , \quad (3.26a)$$

$$Z_c = \sqrt{Z/Y} \ . \quad (3.26b)$$

Consider the circuit shown in Fig. 3.4. The injected signal is I_{s0} . Y_{CD} and Y_L are the admittances of the coupling device and the load respectively. The total length of

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the PLC line is ℓ . The relations between the terminal voltages and currents, at $x=0$ and $x=\ell$ can be expressed as,

$$I(0) = I_{s0} - Y_{CD}V(0) \quad (3.27a)$$

$$I(\ell) = Y_{CD}V(\ell) \quad (3.27b)$$

Y_{CD} and I_{s0} of the two-conductor line are given below:

$$Y_{CD} = \begin{bmatrix} Y_{cd} & Y_{cd} \\ Y_{cd} & Y_{cd} \end{bmatrix} \quad (3.28)$$

$$I_{s0} = [Y_{cd}V_{sd} \quad -Y_{cd}V_{sd}]^t \quad (3.29)$$

where the subscript t means transverse of the matrix.

Fig. 3.4 Schematic configuration of PLC

The line current can be calculated as,

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$$I(x) = NI_0(x) \quad (3.30)$$

where N is the modal transformation matrix for the line-currents, which diagonalizes the transpose of the propagation matrix defined by:

$$P = ZY \quad (3.31)$$

where Z and Y are the per-unit-length impedance and admittance matrices of the line.

Using multi-conductor transmission line theory, the voltage and current are related by the chain parameter matrix [84],

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} \Phi_{11}(x) & \Phi_{12}(x) \\ \Phi_{21}(x) & \Phi_{22}(x) \end{bmatrix} \cdot \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} \quad (3.32)$$

For an n -conductor system, the voltages and current in (3.32) are both $n \times 1$ matrices and $\Phi_{11}(x)$, $\Phi_{12}(x)$, $\Phi_{21}(x)$, $\Phi_{22}(x)$ in the chain parameter matrix are all $n \times n$ matrices,

$$\Phi_{11}(x) = T \cosh(\gamma x) T^{-1} \quad (3.33a)$$

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$$\Phi_{12}(x) = -T \sinh(\gamma x) \gamma^{-1} T^{-1} \quad (3.33b)$$

$$\Phi_{21}(x) = -Y T \gamma^{-1} \sinh(\gamma x) T^{-1} \quad (3.33c)$$

$$\Phi_{22}(x) = Z^{-1} T \cosh(\gamma x) T^{-1} Z \quad (3.33d)$$

T is the $n \times n$ similarity transformation matrix which can be solved with the equation:

$$\sqrt{YZ} = T \gamma T^{-1} \quad (3.34)$$

and,

$$\cosh(\gamma \ell) = \text{diag}\{\cosh(\gamma_k \ell)\}, \quad k = 1, \dots, n \quad (3.35a)$$

$$\sinh(\gamma \ell) = \text{diag}\{\sinh(\gamma_k \ell)\}, \quad k = 1, \dots, n \quad (3.35b)$$

The vector of the modal currents $I_0(x)$ at generic abscissa x can be expressed as the superposition of the current waves traveling in the forward and backward directions:

$$I_0(x) = \exp(-\gamma x) I_{01} + \exp(\gamma x) I_{02} \quad (3.36a)$$

where I_{01} and I_{02} are determined by the terminal conditions in (3.27),

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$$I_{01} = 0.5N^{-1}(Y_{in0} + Y_c)(Y_{in0} + Y_{CD})^{-1}I_{s0} \quad (3.36b)$$

$$I_{02} = 0.5N^{-1}(Y_{in0} - Y_c)(Y_{in0} + Y_{CD})^{-1}I_{s0} \quad (3.36c)$$

where Y_c and Y_{in0} are the characteristic and input admittance matrices of the line respectively, given by:

$$Y_c = YM\gamma^{-1}M^{-1} \quad (3.37a)$$

$$Y_{in0} = [\Phi_{22}Y_L - \Phi_{21}][\Phi_{11} - \Phi_{12}Y_L]^{-1} \quad (3.37b)$$

In (3.37), Y_L is the load matrix of the system, in our case $Y_L = Y_{CD}$, and M is the voltage modal transformation matrix given by:

$$M = N_t^{-1}$$

The common mode current is defined as:

$$I_{com}(x) = \sum_{k=1}^n I_k(x) \quad (3.38)$$

The radiated emission in air ($z > 0$) can then be calculated with the current distribution by the following equations:

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$$E_x(x, y, z) = -\frac{j\omega\mu_0}{\pi}\Gamma_x(\gamma, y, z)I(x) \quad (3.39a)$$

$$E_y(x, y, z) = \frac{\gamma}{j\omega\pi\epsilon_0}\Gamma_y(\gamma, y, z)I(x) \quad (3.39b)$$

$$E_z(x, y, z) = \frac{\gamma}{j\omega\pi\epsilon_0}\Gamma_z(\gamma, y, z)I(x) \quad (3.39c)$$

$$H_x(x, y, z) = \frac{j\gamma}{\pi}\Theta_x(\gamma, y, z)I(x) \quad (3.39d)$$

$$H_y(x, y, z) = \frac{1}{\pi}\Theta_y(\gamma, y, z)I(x) \quad (3.39e)$$

$$H_z(x, y, z) = -\frac{1}{\pi}\Theta_z(\gamma, y, z)I(x) \quad (3.39f)$$

where the expressions of $\Gamma_x(\gamma, y, z)$, $\Gamma_y(\gamma, y, z)$, $\Gamma_z(\gamma, y, z)$, $\Theta_x(\gamma, y, z)$, $\Theta_y(\gamma, y, z)$ and $\Theta_z(\gamma, y, z)$ are given in Appendix.

It should be noted that our proposed model does not require evaluation of the Sommerfeld integrations, thus the computation time is greatly reduced compared with the original infinite-length model. The explicit formula for ground impedance also makes it easy to study the effect of the ground plane.

3.3 Underground PLC Lines

Consider the underground PLC lines as shown in Fig. 3.5. This figure is obtained by changing the relative positions of the two media of Fig. 3.3, i.e., the upper half medium ($z > 0$) is now ground, and the lower half medium ($z < 0$) is air. The n PLC wires are parallel to the x - y plane embedded at a depth of $h_1, h_2, \dots, h_n$ from the air-ground interface.

Fig. 3.5 Geometry of the underground power distribution line

The electric and magnetic fields can be expressed in terms of Hertz Potentials as given below:

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$$\bar{E} = (k_g^2 + \nabla \nabla \bullet) \bar{\Pi}_E - j\mu_g \omega \nabla \times \bar{\Pi}_H \quad (3.40a)$$

$$\bar{H} = (\sigma_g + j\epsilon_g \omega) \nabla \times \bar{\Pi}_E + (-k_g^2 + \nabla \nabla \bullet) \bar{\Pi}_H \quad (3.40b)$$

where $jk_g = jk_0 \sqrt{\epsilon_{rg} - j \frac{\sigma_g}{\omega \epsilon_0}}$ is the propagation constant in the ground, and Π_E

and Π_H are the electric and magnetic Hertz potentials in the ground respectively.

Applying the same approach as described in Section 3.2 for the overhead PLC line, the Hertz potentials can be written as:

$$\begin{aligned} \Pi_{E0}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{j\omega\mu_g I_k(x)}{4\pi k_g^2} \cdot \int_{-\infty}^{\infty} \{ R_{E0k}(\xi) \cdot \exp(-u_{gk} h_i + u_{0k} z) \\ \cdot \exp[-j\lambda(y - d_i)] / u_{gk} \} d\xi \end{aligned} \quad (3.41a)$$

$$\begin{aligned} \Pi_{H0}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{j\omega\mu_g I_k(x)}{4\pi k_g^2} \cdot \int_{-\infty}^{\infty} \{ R_{H0k}(\xi) \cdot \exp(-u_{gk} h_i + u_{0k} z) \\ \cdot \exp[-j\lambda(y - d_i)] / u_{gk} \} d\xi \end{aligned} \quad (3.41b)$$

$$\begin{aligned} \Pi_{Eg}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{j\omega\mu_g I_k(x)}{4\pi k_g^2} \cdot \int_{-\infty}^{\infty} \left[\exp(-u_{gk} |z - h_i|) + R_{E2k}(\xi) \cdot \exp(-u_{gk} (z + h_i)) \right] \\ \cdot \exp[-j\lambda(y - d_i)] / u_{gk} d\xi \end{aligned} \quad (3.41c)$$

$$\begin{aligned} \Pi_{Hg}(x, y, z) = \sum_{i=1}^n \sum_{k=1}^n \frac{j\omega\mu_g I_k(x)}{4\pi k_g^2} \cdot \int_{-\infty}^{\infty} \{ R_{Hgk}(\xi) \cdot \exp[-u_{gk} (z + h_i)] \\ \cdot \exp[-j\lambda(y - d_i)] / u_{gk} \} d\xi \end{aligned} \quad (3.41d)$$

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where the new form of reflection related functions $R_{E0k}(\xi)$, $R_{H0k}(\xi)$, $R_{Egk}(\xi)$

and $R_{Hgk}(\xi)$ are given by,

$$R_{E0k}(\xi) = \frac{2k_g^2 u_{gk}}{k_0^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \quad (3.42a)$$

$$R_{Egk}(\xi) = -1 + \frac{2k_g^2 u_{gk}}{k_g^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \quad (3.42b)$$

$$R_{H0k}(\xi) = \frac{2\xi \gamma_k u_{gk}}{k_0^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \cdot \frac{\omega \varepsilon_0 (k_0^2 - k_g^2)}{u_{0k} (k_g^2 + \gamma_k^2) + u_{gk} (k_0^2 + \gamma_k^2)} \quad (3.42c)$$

$$R_{Hgk}(\xi) = \frac{2\xi \gamma_k u_{gk}}{k_g^2 + \gamma_k^2} \cdot \frac{(\xi^2 - u_{0k} u_{gk})}{k_0^2 u_{gk} + k_g^2 u_{0k}} \cdot \frac{\omega \varepsilon_0 (k_0^2 - k_g^2)}{u_{0k} (k_g^2 + \gamma_k^2) + u_{gk} (k_0^2 + \gamma_k^2)} \quad (3.42d)$$

Substitute (3.41) to (3.40), which is modified for the underground condition

according to Fig. 3.5, the elements of the series impedance matrix Z_{ik} are obtained

with (3.9a),

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$$\begin{aligned}
 Z_{ik} = & \frac{j\omega\mu_g}{2\pi} \cdot \{K_0\{j(k_g^2 + \gamma_k^2)[(d_i - d_k)^2 + (h_i - h_k)^2]\} \\
 & - K_0\{j(k_g^2 + \gamma_k^2)[(d_i - d_k)^2 + (h_i + h_k)^2]\} + \int_{-\infty}^{\infty} \frac{2k_g^2}{k_g^2 + \gamma_k^2} \cdot \frac{\xi^2 - u_{0k}u_{gk}}{k_0^2u_2 + k_g^2u_1} \\
 & \cdot \exp[u_{gk}(h_i - h_k) - j\lambda(d_i - d_k)]d\xi\}
 \end{aligned} \tag{3.43}$$

It is worth noting that in the case of underground PLC lines, $k_g^2 + \gamma_k^2 \cong 0$ is no longer satisfied. However, $|k_0^2|$ is still much smaller than $|k_g^2|$, thus, (3.43) can be further simplified to give,

$$\begin{aligned}
 Z_{ik} = & \frac{j\omega\mu_g}{4\pi} \cdot \ln \left[\frac{(h_i + h_k)^2 + (d_i + rad - d_k)^2}{(h_i - h_k)^2 + (d_i + rad - d_k)^2} \right] \\
 & + \frac{1}{4\pi j\omega\mu_g} \cdot \left[\ln \frac{\sqrt{(h_i + h_k)^2 + rad^2} \sqrt{k_g^2 + \gamma_k^2} - 2}{\sqrt{(h_i + h_k)^2 + rad^2} \sqrt{k_g^2 + \gamma_k^2}} \right. \\
 & \left. + 4\pi j\omega\mu_g \ln \frac{k_g^2 \sqrt{k_g^2 + \gamma_k^2} \sqrt{(h_i - h_k)^2 + (d_i + rad - d_k)^2} - k_g^2}{k_g^2 \sqrt{k_g^2 + \gamma_k^2} \sqrt{(h_i - h_k)^2 + (d_i + rad - d_k)^2}} \right]
 \end{aligned} \tag{3.44}$$

Similarly, the element of the shunt admittance Y_{ik} can be expressed as,

$$\begin{aligned}
 Y_{ik} = & 2\pi j\omega(\varepsilon_g + j\frac{\sigma_g}{\omega}) \cdot \{K_0[j(k_g^2 + \gamma_k^2)((d_i + rad - d_k)^2 + (y + h_k)^2)] \\
 & - K_0[j(k_g^2 + \gamma_k^2)((d_i + rad - d_k)^2 + (y + h_k)^2)] \\
 & + \int_{-\infty}^{\infty} \frac{2k_g^2}{k_g^2 + \gamma_k^2} \cdot \frac{\gamma_k^2}{k_0^2u_g + k_g^2u_0} \cdot \exp[u_{gk}(y + h_k) - j\xi((d_i + rad - d_k))]d\xi \Big|_{y=0}^{y=h_k} \}^{-1}
 \end{aligned} \tag{3.45}$$

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With Z and Y , the characteristic impedance Z_c and propagation constant γ can be calculated with (3.26). The current distribution along the wires and the radiated emission are calculated following the method described in Section 3.2 for the overhead PLC lines.

3.4 Experiment Description

To evaluate the accuracy and applicability of our PLC models, we have performed a series of experiments to measure the common mode current and radiated emission under “control” environments. The experiments were conducted on the floor of the School of Electrical and Electronic Engineering Wing, Block S2.2 where the PLC lines were placed above a concrete ground, as well as on the South Spine, SS2 Roof where the lines were placed above a conducting ground.

A clamp-on RF current probe (ESH2 - Z1) connected to a spectrum analyzer was used to measure the common mode current. Fig. 3.6 shows the common mode current measurement setup when the 2-core power line cable was on the concrete ground. The common mode current was measured with the clamp-on RF current probe every 1 metre along the wires.

Fig. 3.6 Apparatus for common mode current measurements

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Fig. 3.7 Apparatus for radiated emission measurements

Fig. 3.8 Cross section of 2-core and 3-core (BS6500) power lines

The loop antenna with diameter of 12 inches (30 cm) was used to measure the radiated emission as shown in Fig. 3.7. The loop antenna was connected to a hand-held spectrum analyzer, placed 10 metres away from the power line.

A 26 metres 2-core PVC coated power line (BS6500) with radius of 0.69 mm as

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well as a 30 metres 3-core power line was used in the experiment. The cross sections of the 2-core and 3-core power lines utilized in the experiment are shown in Fig. 3.8. The signal generated by a signal generator was injected into the live and neutral wires of the mains cable. A load of 50 Ohms was connected between the two wires. A schematic diagram of the experiment setup is shown as Fig. 3.9.

Fig. 3.9 Schematic diagram of apparatus for common mode current and radiated emission measurements

3.5 Results and Discussion

In our theoretical calculations, all conductors are assumed to be copper which is often used in in-building power line systems. The relative permittivity and the loss tangent of the PVC [17] are assumed to be,

$$\varepsilon_r = -\frac{0.9}{\log 20} \log f + 7.9506 \quad (3.46)$$

$$\tan \delta = -\frac{0.004}{19 \times 10^6} f + 0.0521 \quad (3.47)$$

3.5.1 PLC Lines above the Ground Plane

The series impedance Z and shunt admittance Y are calculated with the physical parameters of the 3-core power line BS6500. The ground impedance Z_g in equation (3.21) is used to investigate the effect of the ground plane on the PLC wires.

The calculated frequency-spectra of the real parts of Z and Y of the PLC line positioned at 0.1 metre above the ground plane are shown in Fig. 3.10 and Fig. 3.11.

Three ground classifications were made according to their electrical characteristics.

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They are: good, average, and poor ground. The relative permittivity of each kind of ground is 25, 15 and 4 for good, average and poor ground respectively. The conductivity is 0.02 S/m, 0.005 S/m and 0.001 S/m accordingly [81]. Comparing the three types of ground, the relative permittivity of the poor ground is much smaller than the relative permittivities of the good and the average ground. On the other hand, the conductivity of the good ground is much greater than those of the average and poor ground, but the difference between the per-unit-length impedance of the average and good ground is small. It is noted that the series impedance has closer relationship to the permittivity than to the conductivity.

Fig. 3.10 Per-unit-length series impedance of PLC wire at 0.1m above the lossy ground

From Fig. 3.10 and Fig. 3.11, it can be seen that the ground impedance is the dominating part of the series impedance and the self-impedance may be neglected. From these two figures, it is seen that at frequencies higher than 10 MHz, the difference between the series impedance for the three types of ground planes is greater than that at frequencies below 10 MHz.

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Fig. 3.11 Per-unit-length ground impedance of PLC wire at 0.1m above the lossy ground

Since the radiated emission in the far field due to the common mode current is much larger than that due to the differential mode current, the common mode current must be studied carefully. The common mode current of the PLC lines is determined by the asymmetry of the system, e.g., the arrangement of the PLC wires with respect to the ground plane.

Fig. 3.12(a) Side-by-side arrangement of the 2-core power line

Fig. 3.12(b) Up-down arrangement of the 2-core power line

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In order to study the effect of the arrangement of the PLC lines to the common mode current, the current distribution of both the side-by-side and up-down arrangements (as shown in Fig. 3.12(a) and Fig. 3.12(b)) were investigated.

For the side-by-side arrangement, both our theoretical and experimental results show that the common mode current is negligible. This is due to the symmetry of the two conductors with respect to the ground plane. The leakage currents from the two conductors cancel each other.

Consider the up-down arrangement as shown in Fig. 3.12(b) in which the two conductors are positioned at different heights from the ground, the asymmetric arrangement of the conductors with respect to the ground plane will lead to much larger common mode current. Fig. 3.13 shows the predicted differential and common mode currents of the up-down arrangement at different positions along the power line with the injected source amplitude 0.7 V. In our calculation, the length of the PLC line ℓ , the distance between the conductors d , and the height of the line h are assumed to be 2 metres, 2.85 mm and 0.1 metre respectively. Our results show that the height of the power line has less effect on the differential mode current compared with the common mode current. This is because the differential mode current is along the wire-wire loop while the common mode current is along the wire-ground loop. On the other hand, due to the asymmetry, the common mode current in this case is much greater than that of the side-by-side arrangement.

Fig. 3.13 Predicted common mode and differential mode current of up-down arrangement

In the case of radiated emission, the far field radiation is more important than the near field radiation because of the radiation cumulation in far field. The dominant contribution of the radiated emissions in the far field for PLC wires is due to the common mode current. This is because the radiated emissions in the far field created by differential mode current subtract from each other while the fields created by common mode current add.

Due to the asymmetry condition, the common mode currents of the 2-core power line and 3-core power line are different. We have carried out a series of experiments to measure the common mode current with 2-core and 3-core power lines both on the concrete ground and conducting ground. In our experiment, the length of the

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2-core PLC lines is 26 metres and the 3-core is 30 metres. A 50 Ohms load was connected to the end. Fig. 3.14 shows a comparison of the predicted and measured common mode current between the 2-core and 3-core power lines. In our experiment, the 2-core power line is arranged up-down above the ground plane (see Fig. 3.12(b)). From Fig. 3.14, both theoretical calculations and experimental data show that the common mode current of the 2-core power line is about 15 dB larger than the 3-core power line. The good agreement with these measurements indicates that our model works well for such PLC lines with different ground geometry factors. Our results above confirmed that the common mode current is greatly affected by the arrangement of these conductors with respect to ground.

Fig. 3.14 Comparison of predicted and measured common mode current of 2-core and 3-core power line above concrete ground

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Fig. 3.15 Predicted and measured common mode current of 2-core power line above conducting and concrete ground

Fig. 3.15 shows the predicted and measured common mode current of a power line above a conducting and concrete ground plane. It can be seen that the common mode current of power line above the conducting ground is about 12 dB larger than that of power line above the concrete ground. This is because more current leaks into the conducting ground than into the concrete ground. These results show that the common mode current is greatly affected by the electrical characteristics of the ground.

The predicted common mode current at 30 MHz of the 2-core PLC lines with different height to the different types of the ground plane is shown in Fig. 3.16. It can be seen that the common mode current of the PLC line above the good ground

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plane is more sensitive to the change in height than the other two types of the ground plane. This is because good ground with large conductivity σ_g and permittivity ε_g has more contribution to the wire-ground loop than the other two types of ground. It can be seen in the figure that when the height is 5 metres, the common mode current of the PLC line above good conducting ground is larger than that of the PLC lines above the other two types of the ground plane.

Fig. 3.16 Comparison of predicted common mode current for PLC wires at different height above different types of ground plane (good ground ($\varepsilon_r = 25$, $\delta = 0.02S/m$), average ground ($\varepsilon_r = 15$, $\delta = 0.005S/m$) and poor ground ($\varepsilon_r = 4$, $\delta = 0.001S/m$))

The common mode currents along the conductors of the 3-core power line on the concrete and conducting ground planes with different heights were measured. The comparisons of the predicted and measured common mode current are shown in Fig.

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3.17 and Fig. 3.18 for concrete and conducting ground respectively. The relative permittivity and conductivity of an average ground were assumed, i.e., $\epsilon_r = 15$ and $\sigma_g = 0.005 \text{ S/m}$. In the experiment, the heights of the power line from the ground were set to 0.0 metre, 0.4 metre and 0.8 metre.

When the height of the power line from the ground changed from the 0.0 metre to 0.4 metre, the model predicted the change of current with an error which is less than 3 dB out of a change of 45 dB. From Fig. 3.17 and Fig. 3.18, it can be noted that the effect from the ground becomes insignificant when the PLC line is 0.4 metre or more above the ground. The common mode current of the power line then becomes less sensitive to changes in height.

Fig. 3.17 Comparison of predicted and measured common mode current for PLC wires above a concrete ground

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Fig. 3.18 Comparison of predicted and measured common mode current for PLC wires above a conducting ground

So far we mainly focused on the electromagnetic impact on the wireless communication systems in the far field. However, the radiated emission will also interfere with communication channels in the near field where there are other communication cables operating in the same frequency range. Experiments were carried out to study the leakage current from the PLC line (Line 1) to a nearby no-sourced power line (Line 2) as shown in Fig. 3.19. In these experiments, two 2-core power lines was arranged parallel to each other positioned at the same height h from the ground plane. The distance between the two lines d was changed from 0.02 metre to 0.05 metre. The experiments were conducted both on the concrete and conducting ground.

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Fig. 3.19 Experiment setup to measure leakage current from Line 1 to Line 2

Fig. 3.20 Comparison of predicted and measured common mode current along Line 2

A comparison of the measured and calculated common mode currents along the no-source line (Line 2) is shown in Fig. 3.20 when the PLC lines were on the concrete and the conducting ground plane. It can be seen that when the PLC lines were on the concrete ground, more common mode current was induced into Line 2.

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From our model and the experimental results, it can be seen that when d is smaller than about 10 cm, the change of d does not have much effect on the induced current along the no-sourced line when the PLC lines are on the concrete ground. When the PLC lines were on the conducting ground, the common mode current was more sensitive to d . Our theoretical results also show the same scenario. When the PLC lines were on the conducting ground, more current was induced into the ground that has almost the same conductivity of the PLC lines so less current flew along Line 2. When Line 2 is far away from Line 1, less current leaked into Line 2. Our simulation results show that the induced current in Line 2 will become larger if $h \gg d$. For example, in the case of out-door PLC lines that are much higher from the ground than the lines in our experiment, the power lines near the PLC carrier will be greatly interfered.

Fig. 3.21 Magnetic field of PLC wire at 0.1m above the ground for three types of ground

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Fig. 3.21 shows our theoretical results of the radiated emissions when the PLC lines were above the three types of ground plane. It is worth noting that the radiated emission is large for good ground while the common mode current does not show the same behavior in Fig. 3.16. This can be explained with (3.42), where R_{H0k} increases when ϵ_g and σ_g increase. So when the magnitude of the common mode current is almost the same, the electromagnetic field is determined by the reflection related functions, R_{0k} and R_{H0k} in (3.42).

Fig. 3.22 Predicted and measured electric field strength for PLC wire with different ground conductivities at $x=1\text{m}$

Fig. 3.23 Predicted and measured electric field strength for PLC wire with different concrete permittivity at $x=1\text{m}$

In order to study the effect of the characteristics of the ground plane on the radiated emission, model results of radiated emission from PLC lines with different ground conductivity and permittivity are shown in Fig. 3.23 and Fig. 3.24. Because of the lack of experimental results, our theoretical calculation results were compared with the measurement data proposed by Esmailian [52]. The parameters used in our model were set to be the same as in [52]. The length of the power line was 2 metres. A sinusoidal signal in the frequency range of 1-30 MHz was transmitted. Fig. 3.22 shows the predicted and measured electric field strength for PLC wire with different ground conductivity. It can be seen that the spectrum of the radiated emission

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changes with the ground conductivity. In Fig. 3.22, there are dips of the spectrum of the radiated emission of the PLC lines at the frequency of 17 MHz. When the applied frequency changed from 20 MHz to 30 MHz, k_g changed from 1.1587-0.6815j to 1.5906-0.7446j, while when the frequency changes from 1 MHz to 15 MHz, k_g changes from 0.2015-0.1959j to 0.9426-0.6282j m⁻¹. The big change of k_g leads to the obvious difference of the radiated emissions at the frequencies below 15 MHz.

Fig.3.24 Comparison of predicted and measured magnetic field of PLC wires above a concrete ground

The radiated emissions for different frequencies from 15 MHz to 30 MHz from the power line at the distance of 10 metres were also measured. The results of the experiment and the model are shown in Fig. 3.24 and Fig. 3.25. In these figures, the difference between the experiment and the model is about 6%. The good agreement

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between our theoretical results and experimental results show the applicability and accuracy of our model for such PLC applications.

Fig. 3.25 Comparison of predicted and measured magnetic field of PLC wires above a conducting ground

3.5.2 PLC Lines Embedded in the Ground Plane

In order to emphasize the effect of the ground and compare with the overhead PLC lines described in Section 3.5.1, the PLC wires are embedded at a depth of 0.1 metre from the ground-air interface.

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The frequency-spectra of the real part of the series impedance for three different types of ground are shown in Fig. 3.26.

Fig. 3.26 Per-unit-length series impedance of PLC wire embedded at 0.1m under the ground

The relative permittivity and conductivity for good, average and poor ground are (25, 0.02 S/m) , (15, 0.005 S/m) and (4, 0.01 S/m) respectively. The series impedance Z of the under ground line is much sensitive to the change in the ground condition. In general, the higher the conductivity of the ground is, the higher the attenuation of the voltage and current wave, which leads to higher series impedance as shown in Fig. 3.26.

Fig. 3.27 shows the radiated emission from the PLC line embedded under the ground plane. It can be seen that the radiated emissions from the embedded PLC line

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are about 20 dB smaller compared with the radiated emission from the overhead lines. Hence more attention should be paid to overhead lines when considering electromagnetic interference from PLC systems to other communication systems in the far field. The comparison between the experiment and model results shows that our model can provide good EMI predictions for power line communications.

Fig. 3.27 Magnetic field of PLC wires at 0.1m above and under the ground for three types of ground

From the model result, it is obvious that the ground characteristics have more effect on the PLC impedance when the PLC wires are under the ground than when they were above the ground. The effect of the ground affects the transmission characteristics.

3.6 Conclusions

In this chapter, the radiated emission model of a straight infinite-length PLC line when it was placed above different types of ground plane was provided. The model was derived based on the full wave approach. Appropriate approximations were made according to the PLC conditions to simplify formulations. Explicit formulae were obtained to calculate the transmission characteristics of the PLC lines. The effects of the different types of ground on the characteristics of the PLC wires were studied.

The same process was applied to the condition when the PLC line was embedded in the ground. The difference between the overhead lines and under ground lines was high lighted.

In order to predict the effect of the ground plane on the carrier channels of the PLC, the series impedance of the PLC wires above different types of ground planes was investigated. Common mode current was compared with the measurement data when the PLC lines were above concrete ground and conducting ground with different heights. The effects of ground conductivity and permittivity to the radiated emission were investigated. The model results were compared with the experimental results when the PLC lines were above concrete and conducting ground.

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Considering different arrangement of the PLC lines, our results show that the common mode current is mainly generated by the asymmetry of the PLC system. Predicted and measured radiated emissions for both side-by-side arrangement and up-and-down arrangement show that the radiated emission for the side-by-side arrangement is about 12 dB less than that of the up-down one.

The electromagnetic interference of the PLC carries to other power line is also studied. Measurements were carried out to evaluate the accuracy of our model. Our results show that interference between the PLC carriers to other power line is un-avoidable. The interference decreased greatly with the distance between the two lines when they are above good conducting ground.

Chapter 4

Radiated Emission Models for Finite-length PLC Systems

4.1 Introduction

In Chapter 3, we presented a comprehensive radiated emission model for an infinite length power line communication system. The proposed model is useful for long distance outdoor PLC system. However, for indoor PLC applications, the effects at both ends of the line have to be taken into account. It is necessary to propose a more flexible model to deal with such finite-length PLC lines.

The purpose of this chapter is to establish a basic radiated emission model for a straight PLC line of finite length above a lossy ground plane. The model is derived based on the full wave approach. In the model, the characteristic impedance and the propagation constant of the power line are derived based on the Maxwell Equations

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using the concept of the distributed dipole antenna [82]. The currents along the power line conductors can then be calculated so that the radiated emission can be derived. Finally, experiments were carried out to validate the model.

4.2 Theory and Formulation

Fig. 4.1 Geometry of the finite-length PLC line

The geometry of a finite-length PLC line is shown in Fig. 4.1. In the figure, the two wires (Line 1 and Line 2) with length ℓ have the same radius r_0 . ϵ_0 and μ_0 are the permittivity and permeability of the air, and ϵ_g , μ_g and σ_g are the permittivity, permeability and conductivity of the ground plane respectively. The wires are parallel to the infinite lossy ground plane with heights h_1 and h_2 as shown in Fig. 4.1.

To determine the current distribution along the wires, each wire is divided into k small elements of length $\Delta\ell$. If $\Delta\ell$ is small enough, the current along each element can be treated as constant $I(x)$ where x is the coordinate at the middle of each element. Every small element is then considered as a dipole antenna.

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In the frequency domain, the current along the wire excited by the external source can be expressed in the form,

$$I(x) = I_0 \exp(-\gamma x) \quad (4.1)$$

where γ is the propagation constant.

The propagation constant of the electromagnetic wave in the air jk_0 can be expressed in terms of ϵ_0 and μ_0 as,

$$k_0 = \omega \sqrt{\epsilon_0 \mu_0} . \quad (4.2)$$

The propagation constant in the ground jk_g is,

$$k_g = k_0 \sqrt{\frac{\epsilon_g}{\epsilon_0} - \frac{j\sigma_g}{\omega\epsilon_0}} . \quad (4.3)$$

Hertz potentials of the dipole antenna ($\Delta\ell$ in Fig. 4.1) can be expressed as follows:

For the upper half medium ($z > 0$),

$$\Pi_x = \frac{I_0 \Delta\ell}{4\pi j\omega\epsilon_0} \int_0^\infty \frac{\xi}{u_0} \left[e^{-u_0(z-h)} + R(\xi)e^{-u_0(z+h)} \right] J_0(\xi\rho) d\xi \quad (4.4)$$

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$$\Pi_z = \frac{I_0 \Delta \ell}{4\pi j\omega \varepsilon_0} \frac{\partial}{\partial x} \int_0^\infty S(\xi) e^{-u_0(z+h)} J_0(\xi \rho) d\xi \quad (4.5)$$

where $\rho = \sqrt{(x-x')^2 + (y-r_0)^2}$.

For the lower half medium ($z < 0$),

$$\Pi_x = \frac{I_0 \Delta \ell}{4\pi j\omega \varepsilon_0} \int_0^\infty T(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (4.6)$$

$$\Pi_z = \frac{I_0 \Delta \ell}{4\pi j\omega \varepsilon_0} \frac{\partial}{\partial x} \int_0^\infty Q(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (4.7)$$

where the reflection related functions $R(\xi)$, $S(\xi)$, $T(\xi)$ and $Q(\xi)$ are:

$$R(\xi) = \frac{u_0 - u_1}{u_0 + u_1} \quad (4.8)$$

$$S(\xi) = \frac{2\lambda(u_1 - u_0)}{\gamma_0^2(N^2 u_0 + u_1)} \quad (4.9)$$

$$T(\xi) = \frac{2\xi}{(u_0 + u_1)N^2} e^{-u_0 h} \quad (4.10)$$

$$Q(\xi) = \frac{2\xi(u_1 - u_0)}{\gamma_1^2(N^2 u_0 + u_1)} e^{-u_0 h} \quad (4.11)$$

Using the relation $\int_0^\infty \frac{\xi}{u_0} e^{-u_0|z-h|} \cdot J_0(\xi \rho) d\xi = \frac{e^{-\gamma_0 R_0}}{R_0}$, Π_x in (4.4) can be rewritten as:

$$\Pi_x(x, y, z) = \frac{I_0 \Delta \ell}{4\pi j\varepsilon_0 \omega} \left(\frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_1}}{R_1} + M \right), \quad (z > 0) \quad (4.12)$$

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where

$$M = \int_0^\infty \frac{\xi}{u_0} \cdot \frac{2u_0}{u_0 + u_1} e^{-u_0(z+h)} d\xi,$$

$$R_0 = \sqrt{(x - x')^2 + r_0^2},$$

$$R_1 = \sqrt{(x - x')^2 + (2h + r_0)^2},$$

and $\rho = |x - x'|$.

Let d is the distance between the two wires, that is, $d = h_1 - h_2$. Assuming $d \ll \Delta\ell$,

M can be approximated in a form without integration:

$$M \cong \frac{2e^{-\gamma_0 R_1}}{R_1^2} + \frac{4e^{-\gamma_0 R_1}}{(k_0 + k_g)R_1^2} \left(1 + \frac{\rho^2}{R_1^2}\right) \quad (4.13)$$

So we get Π_x :

$$\Pi_x(x, y, z) \cong \frac{I_0 \Delta\ell}{4\pi j\epsilon_0 \omega} \left(\frac{e^{-\gamma_0 R_0}}{R_0} - \frac{e^{-\gamma_0 R_1}}{R_1} + \frac{2e^{-\gamma_0 R_1}}{R_1^2} + \frac{4e^{-\gamma_0 R_1}}{(k_0 + k_g)R_1^2} \left(1 + \frac{\rho^2}{R_1^2}\right) \right) \quad (4.14)$$

To simplify Π_z in (4.5), we let $f(\xi) = \frac{2\xi(u_1 - u_0)}{\gamma_1^2 u_0 + \gamma_0^2 u_1} (-u_0) e^{-u_0(z+h)}$ and

$g(\xi) = \frac{2\xi(u_1 - u_0)}{\gamma_1^2 u_0 + \gamma_0^2 u_1} e^{-u_0(z+h)}$, and write $f(\xi)$ and $g(\xi)$ using the Taylor expansion

that [83],

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$$f(\xi) = \sum_{k=0}^{\infty} \frac{f^{(N)}(0)}{k!} \xi^k \quad (4.15)$$

$$g(\xi) = \sum_{k=0}^{\infty} \frac{g^{(N)}(0)}{k!} \xi^k \quad (4.16)$$

Using (4.15) and (4.16), Π_z can be simplified as,

$$\Pi_z = \frac{I_0 \Delta \ell}{4\pi j\omega \epsilon_0} \frac{\partial}{\partial x} \sum_{k=0}^{\infty} 2^k \frac{f^{(N)}(0)}{k!} \rho^{-k-1} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)}, \quad (z > 0) \quad (4.17)$$

With (4.14) and (4.17) electromagnetic fields due to $\Delta \ell$ in the upper half medium ($z > 0$) can be written as,

$$E_x = -\gamma_0^2 \Pi_x + \frac{\partial^2 \Pi_x}{\partial x^2} + \frac{I_0 \Delta \ell}{4\pi j\omega \epsilon_0} \sum_{k=0}^{\infty} 2^k \frac{f^{(N)}(0)}{k!} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)} \frac{\partial^2 \rho^{-k-1}}{\partial x^2} \quad (4.18)$$

$$E_y = \frac{\partial^2 \Pi_x}{\partial x \partial y} + \frac{I_0 \Delta \ell}{4\pi j\omega \epsilon_0} \sum_{k=0}^{\infty} 2^k \frac{f^{(N)}(0)}{k!} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)} \frac{\partial^2 \rho^{-k-1}}{\partial x \partial y} \quad (4.19)$$

$$E_z = -\gamma_0^2 \Pi_z + \frac{\partial^2 \Pi_x}{\partial x \partial z} + \frac{I_0 \Delta \ell}{4\pi j\omega \epsilon_0} \sum_{k=0}^{\infty} 2^k \frac{g^{(N)}(0)}{k!} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)} \frac{\partial^2 \rho^{-k-1}}{\partial z^2} \quad (4.20)$$

$$H_x = -j\omega \epsilon_0 \frac{\partial \Pi_z}{\partial y} \quad (4.21)$$

$$H_y = -j\omega \epsilon_0 \left(\frac{\partial \Pi_z}{\partial x} - \frac{\partial \Pi_x}{\partial z} \right) \quad (4.22)$$

$$H_z = -j\omega \epsilon_0 \frac{\partial \Pi_x}{\partial y} \quad (4.23)$$

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For PLC applications in the frequency range of 1-30 MHz, the wavelength λ is between 10-300 metres and r_0 is in the order of millimetres. To simplify the above expressions, $\Delta\ell$ is chosen to be 1 metre so that $\Delta\ell \ll \lambda$ and $\Delta\ell \gg r_0$.

For outdoor PLC applications, $h \gg r_0$, $Z(x)$ obtained with (3.9a) can be approximated as,

$$Z(x) = \int_0^\ell \frac{1}{I(x)} \left\{ \frac{I(x')\Delta\ell}{4\pi j\omega\epsilon_0} \left[-\gamma_0^2 \frac{e^{-\gamma_0|x'-x|}}{|x'-x|} + \frac{(x'-x)^3 \gamma_0^2 e^{-\gamma_0|x'-x|} + 2(x'-x)e^{-\gamma_0|x'-x|} [\gamma_0(x'-x) + 1]}{(x'-x)^4} \right] \right. \\ \left. + \frac{I(x')\Delta\ell}{4\pi j\omega\epsilon_0} \sum_{k=0}^{\infty} 2^k (k+1)(k+2) \frac{f^{(N)}(0)}{k!} |x'-x|^{-(k+3)} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)} \right\} dx' \quad (4.24)$$

Since $\Delta\ell \ll \lambda$, $I(x)$ is assumed as constant along $\Delta\ell$. $Z(x)$ can be rewritten as the summation of each element $\Delta\ell$,

$$Z(x) = \sum_{n=1}^N \left\{ \frac{\sin(\gamma_0 n \Delta\ell) \Delta\ell}{\sin(\gamma_0 x) 4\pi j\omega\epsilon_0} \left[-\gamma_0^2 \frac{e^{-a\gamma_0}}{|n\Delta\ell - x|} \right. \right. \\ \left. \left. + \frac{(n\Delta\ell - x)^3 \gamma_0^2 e^{-\gamma_0|n\Delta\ell - x|} + 2(n\Delta\ell - x)e^{-\gamma_0|n\Delta\ell - x|} [\gamma_0(n\Delta\ell - x) + 1]}{(n\Delta\ell - x)^4} \right] \right. \\ \left. + \frac{\sin(\gamma_0 n \Delta\ell) \Delta\ell}{\sin(\gamma_0 x) 4\pi j\omega\epsilon_0} \sum_{k=0}^{\infty} 2^k (k+1)(k+2) \frac{f^{(N)}(0)}{k!} |n\Delta\ell - x|^{-(k+3)} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)} \right\}$$

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(4.25)

Using a similar approach, the shunt admittance $Y(x)$ in (3.9b) can be written as,

$$Y(x) = \sum_{n=1}^N \frac{-j\omega\epsilon_0 \cos(\gamma_0 n \Delta\ell) \Delta\ell}{\cos(\gamma_0 x) 4\pi j\omega\epsilon_0} \sum_{k=0}^{\infty} 2^k (k+1)(k+3) \frac{g^{(N)}(0)}{k!} \cdot (n\Delta\ell - x) r_0 \sqrt{(n\Delta\ell - x)^2 + r_0^2}^{-(k+5)} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}k)}{\Gamma(\frac{1}{2} - \frac{1}{2}k)} \quad (4.26)$$

With the series impedance $Z(x)$ and the shunt admittance $Y(x)$, the characteristic impedance $Z_c(x)$ and propagation constant $\gamma(x)$ of the power line can be calculated with

$$Z_c(x) = \sqrt{\frac{Z(x)}{Y(x)}} \quad (4.27)$$

$$\gamma(x) = \sqrt{Z(x)Y(x)} \quad (4.28)$$

Consider the signal carrier channel of a PLC system described in Fig. 4.2. The carrier signal V_{in} is injected by the sending device (SD) with impedance Z_0 , and the load Z_l is connected to the load end.

Fig. 4.2 Schematic configuration of signal channel on distribution power line

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The relations between the vectors of the terminal voltages and currents at $x=0$ and $x=\ell$ are,

$$V(0) = V_{in} - Z_0 I(0) \quad (4.29)$$

$$V(\ell) = Z_\ell I(\ell) \quad (4.30)$$

where Z_0 and Z_ℓ are both 2 by 2 matrices given below,

$$Z_0(s) = \begin{bmatrix} Z_0 & -Z_0 \\ -Z_0 & Z_0 \end{bmatrix}$$

$$\text{and } Z_\ell(s) = \begin{bmatrix} Z_\ell & -Z_\ell \\ -Z_\ell & Z_\ell \end{bmatrix}.$$

Since the ground plane shown in Fig. 4.1 is considered as the reference plane, V_{in} , $V(0)$, $V(\ell)$, $I(0)$ and $I(\ell)$ are all vectors of 2 elements.

The voltage and current at the cascade section of the power line denoted $\Delta\ell_j$ (j is from 2 to k) can be expressed by the voltage and current at the section denoted $\Delta\ell_{j-1}$ by the chain parameter matrix $\Phi_{j-1}(\Delta\ell)$,

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$$\begin{bmatrix} V(\Delta\ell_j) \\ I(\Delta\ell_j) \end{bmatrix} = \Phi_{j-1}(\Delta\ell) \begin{bmatrix} V(\Delta\ell_{j-1}) \\ I(\Delta\ell_{j-1}) \end{bmatrix} \quad (4.31)$$

In (4.31),

$$\Phi_{j-1}(\Delta\ell_{j-1}) = \begin{bmatrix} T \cosh(\gamma\Delta\ell_{j-1})T^{-1} & -T \sinh(\gamma\Delta\ell_{j-1})\gamma^{-1}T^{-1}Z \\ -YT\gamma^{-1} \sinh(\gamma\Delta\ell_{j-1})T^{-1} & Z^{-1}T\gamma \cosh(\gamma\Delta\ell_{j-1})\gamma^{-1}T^{-1} \end{bmatrix} \quad (4.32)$$

and,

$$\cosh(\gamma\Delta\ell_{j-1}) = \text{diag}\{\cosh(\gamma_N\Delta\ell_{j-1})\}, \quad N = 1, 2 \quad (4.33)$$

$$\sinh(\gamma\Delta\ell_{j-1}) = \text{diag}\{\sinh(\gamma_N\Delta\ell_{j-1})\}, \quad N = 1, 2 \quad (4.34)$$

The k elements can be considered as k cascaded connected networks as shown in Fig. 4.3.

Therefore, the relation for voltage and current from the input end of the power line to the end of the line ℓ is,

$$\begin{bmatrix} V(\Delta\ell) \\ I(\Delta\ell) \end{bmatrix} = \Phi_1(\Delta\ell) \times \Phi_2(\Delta\ell) \times \cdots \times \Phi_k(\Delta\ell) \begin{bmatrix} V(0) \\ I(0) \end{bmatrix} \quad (4.35)$$

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Fig. 4.3 Representation of a line as a cascade of uniform sections

Assume the multi-conductor PLC line is excited by an external source, and $I(x)$ is the currents propagating along the conductors, i.e. $I(x) \propto e^{-\gamma x}$. The vector of the line currents is expressed as a function of the modal current vector $I_m(x)$ by means of the following linear transformation,

$$I(x) = TI_m(x) \quad (4.36)$$

Here T is the 2×2 transformation matrix [84]. In the frequency domain, the N^{th} (N is the number of the wire, in our case, $N = 2$) coefficient of the modal current vector is expressed in the form,

$$I_{mN}(x) = I_{mN0} e^{(-\gamma_N x)} \quad (4.37)$$

In which I_{mN0} is the maximum amplitude and γ_N is the propagation constant of the N^{th} mode.

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The common mode current then can be calculated with the known current $I(x)$ using the equation,

$$I_{com}(x) = I_1(x) + I_2(x) \quad (4.38)$$

Finally, the radiated emission can be calculated based on the Hertz potential mentioned in (4.4) and (4.5) with the known current distribution. The electric and magnetic fields of the dipole antenna are described in the Appendix.

Let us assume the electric field from the source positioned at x' is $E(x, x')$ at the element positioned at x . The electric and magnetic fields on the surfaces of wire 1 and wire 2 are the sum of the electromagnetic fields due to the current along the two conductors as,

$$E(x) = \int_0^\ell E_1(x, x') dx' + \int_0^\ell E_2(x, x') dx' \quad (4.39)$$

$$H(x) = \int_0^\ell H_1(x, x') dx' + \int_0^\ell H_2(x, x') dx' \quad (4.40)$$

where x is the local coordinate for the field point on the surface of the conductors and x' is the source point on the axes.

4.3 Results and Discussion

Experiments were carried out to measure the common mode current and the radiated emission of power lines above the concrete ground as well as on the conducting ground. The measurement was carried out as described in Chapter 3.

Fig. 4.4 and Fig. 4.5 show the common mode current when the PLC line was on the concrete ground and conducting ground respectively. The results show that the common mode current of the PLC line on the conducting ground is about 8 dB larger than that of the PLC line on the concrete ground.

Fig. 4.4 Predicted and measured common mode current of PLC above concrete ground

Comparing Fig. 4.4 and Fig. 4.5 with Fig. 3.17 and Fig. 3.18, it can be seen that the finite-length PLC model is more accuracy than the infinite-length PLC model.

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Fig.4.5 Predicted and measured common mode current of PLC above conducting ground

Fig.4.6 Predicted and measured magnetic field of PLC wire above concrete ground

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Fig.4.7 Predicted and measured magnetic field of PLC wire above conducting ground

Fig. 4.6 and Fig. 4.7 show the frequency spectra of the radiated emission when the PLC line was on the concrete ground plane and on the conducting ground respectively. From these two figures, it can be concluded that the finite-length PLC model has clear advantage in modeling the radiated emission compared with the infinite-length PLC model described in Chapter 3.

4.4 Conclusions

In this chapter, the radiated emission model of a finite-length multi-conductor PLC line is presented. The theoretical results are compared with the measurement data and the results calculated by the infinite-length PLC line model. From these comparisons, we can consider the finite-length PLC model as an improved version of the infinite-length PLC model.

Chapter 5

Radiated Emission Models for Finite-length PLC Lines Bent with an Arbitrary Angle above a Lossy Ground Plane

5.1 Introduction

The purpose of this chapter is to extend the straight finite-length length model described in Chapter 4 to include the effect of discontinuities along a power line. The effects of the discontinuity, i.e., bends and terminations at both ends of the line as well as the effect of the lossy ground plane will be studied carefully.

5.2 Theory and Formulation

The geometry of the PLC line is shown in Fig. 5.1. Theoretically, the formulation is applicable to a N-core PLC line with different bend angles and for different lengths of the line. For simplicity, we will restrict ourselves to a 2-core PLC line with the same bend angle as shown in Fig. 5.1. The wires are parallel to the infinite lossy ground plane (x-y plane) and θ is the bend angle. For indoor power line wires, θ is usually 90 degrees. The total length of each PLC line is $L_1 + L_2$ and the radius of the two wires is r_0 . In Fig. 5.1, arm P_1 (P_{11} and P_{12}) has the length of L_1 is arranged along the x-axis, and arm P_2 (P_{21} and P_{22}) is the other arm with length of L_2 directed at an angle θ to the x-axis in the x-y plane. The heights of the two wires from the ground plane are h_1 and h_2 respectively.

The permittivity, permeability and conductivity of the wires and the ground plane are $(\epsilon_w, \mu_w, \sigma_w)$ and $(\epsilon_g, \mu_g, \sigma_g)$ respectively. Each arm is divided into k small elements of length $\Delta\ell$ so that the length of each element is much smaller than the wavelength. If $\Delta\ell$ is small enough, the current along each element can be treated as constant $I(s)$ in which s is the coordinate at the middle of each element. Each of the small elements is then treated as a dipole antenna.

Sommerfeld integrals are employed to take into account the effect of the lossy ground plane as described in Chapter 4.

Fig.5.1 Geometrical configuration of power line with a bend

Consider arm P_i , the components of Hertz Potentials in air ($z > 0$) can be written as,

$$\Pi_{P_x}(s) = \frac{I(s)}{4\pi i \epsilon_0 \omega} \int_0^\infty \frac{\xi}{u_0} \left[e^{-u_0|z-h|} + R_m(\lambda) e^{-u_0(z+h)} \right] J_0(\xi \rho) d\xi \quad (5.1)$$

$$\Pi_{P_z} = \frac{I(s)}{4\pi i \omega \epsilon_0} \frac{\partial}{\partial x} \int_0^\infty S(\xi) e^{-u_0(z+h)} J_0(\xi \rho) d\xi \quad (5.2)$$

In the ground medium ($z < 0$), we have,

$$\Pi_{P_x} = \frac{I(s)}{4\pi i \omega \epsilon_0} \int_0^\infty T_m(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (5.3)$$

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$$\Pi_{P_z} = \frac{I(s)}{4\pi i \omega \epsilon_0} \frac{\partial}{\partial x} \int_0^\infty Q(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (5.4)$$

Similarly, the components of Hertz Potentials in air due to the arm P_2 can be written as

$$\Pi_{P_x}(s) = \frac{I(s)}{4\pi i \epsilon_0 \omega} \sin \theta \cdot \int_0^\infty \frac{\xi}{u_0} \left[e^{-u_0 |z-h|} + R_m(\xi) e^{-u_0(z+h)} \right] J_0(\xi \rho) d\xi \quad (5.5)$$

$$\Pi_{P_y}(s) = \frac{I(s)}{4\pi i \epsilon_0 \omega} \cos \theta \cdot \int_0^\infty \frac{\xi}{u_0} \left[e^{-u_0 |z-h|} + R_m(\xi) e^{-u_0(z+h)} \right] J_0(\xi \rho) d\xi \quad (5.6)$$

$$\Pi_{P_z}(s) = \frac{I(s)}{4\pi i \omega \epsilon_0} \frac{\partial}{\partial x} \int_0^\infty S(\xi) e^{-u_0(z+h)} J_0(\xi \rho) d\xi \quad (5.7)$$

and in the ground ($z < 0$), we have

$$\Pi_{P_x} = \frac{I(s)}{4\pi i \omega \epsilon_0} \sin \theta \cdot \int_0^\infty T_m(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (5.8)$$

$$\Pi_{P_y} = \frac{I(s)}{4\pi i \omega \epsilon_0} \cos \theta \cdot \int_0^\infty T_m(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (5.9)$$

$$\Pi_{P_z} = \frac{I(s)}{4\pi i \omega \epsilon_0} \frac{\partial}{\partial x} \int_0^\infty Q(\xi) e^{u_1 z} J_0(\xi \rho) d\xi \quad (5.10)$$

where $\rho = \sqrt{(s' - s \cos \theta)^2 + (s \sin \theta)^2 + r_0^2}$ and $R_m(\xi)$, $T_m(\xi)$, $S(\xi)$ and $Q(\xi)$

have the same expression as (4.8)-(4.11).

The electric field at position s' along the wire from the source positioned at s is,

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$$E_x(s, s', z) = -\gamma^2 \Pi_x((s' - s), z) + \frac{\partial}{\partial x} [\text{div} \bar{\Pi}((s' - s), z)] \quad (5.11)$$

The tangential magnetic field H_Φ consists of H_z and H_y . Since H_y is much smaller than H_z , H_Φ is approximated as,

$$H_\Phi(s, s', z) \cong -(\sigma + j\omega\epsilon_0) \left(\frac{\partial \Pi_z(s, s', z)}{\partial (s \cdot \cos \theta)} - \frac{\partial \Pi_x(s, s', z)}{\partial z} \right) \quad (5.12)$$

The electric and magnetic fields on the surfaces of conductor 1 and conductor 2 are the sum of the electromagnetic fields due to the current along the two conductors as,

$$\bar{E}(s, h) = \int_{-L_1}^0 \bar{E}_{P_{11}}(s, s', h) ds' + \int_0^{L_2} \bar{E}_{P_{21}}(s, s', h) ds' + \int_{-L_1}^0 \bar{E}_{P_{12}}(s, s', h) ds' + \int_0^{L_2} \bar{E}_{P_{22}}(s, s', h) ds' \quad (5.13)$$

$$\bar{H}(s, h) = \int_{-L_1}^0 \bar{H}_{P_{11}}(s, s', h) ds' + \int_0^{L_2} \bar{H}_{P_{21}}(s, s', h) ds' + \int_{-L_1}^0 \bar{H}_{P_{12}}(s, s', h) ds' + \int_0^{L_2} \bar{H}_{P_{22}}(s, s', h) ds' \quad (5.14)$$

where s is the local coordinate for the field point on the surface of the conductors and s' is the source point on the axis.

The series impedance $Z(s)$ of each element $\Delta\ell$ can be expressed as,

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$$Z(s) = -\lim_{\rho \rightarrow r_0} \frac{E_{\tan}(s)}{I(s)} \quad (5.15)$$

where $I(s)$ is the current of the element $\Delta\ell$ and $E_{\tan}(s)$ is the tangential electric field along the conductors at point s .

The shunt admittance $Y(s)$ of each element is,

$$Y(s) = -2\lim_{\rho \rightarrow r_0} \frac{H_{\text{cir}}(s)}{V(s)} \quad (5.16)$$

where $V(s)$ is the voltage of the element $\Delta\ell$ with respect to ground. $H_{\text{cir}}(s)$ is the circumferential magnetic field along the conductors at position s .

The tangential component of the electric field at arm P_1 is,

$$E_{\tan P_1}(s) = \int_{-L_1}^0 \left(\bar{E}_{P_{11}}(s, s') + \bar{E}_{P_{12}}(s, s') \right) \cdot \bar{s} \cdot ds' + \int_0^{L_2} \left(\bar{E}_{P_{21}}(s, s') + \bar{E}_{P_{22}}(s, s') \right) \cdot \bar{s} \cdot ds' \quad (5.17)$$

and at arm P_2 is,

$$E_{\tan P_2}(s) = \int_{-L_1}^0 \left(\bar{E}_{P_{11}}(s, s') + \bar{E}_{P_{12}}(s, s') \right) \cdot \bar{s} \cdot ds' + \int_0^{L_2} \left(\bar{E}_{P_{21}}(s, s') + \bar{E}_{P_{22}}(s, s') \right) \cdot \bar{s} \cdot ds'.$$

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(5.18)

The circumferential component of the magnetic field at arm P_1 is,

$$H_{\text{cir}P_1}(s) = \int_{-L_1}^0 \left(\bar{H}_{P_{11}}(s, s') + \bar{H}_{P_{12}}(s, s') \right) \cdot \bar{z} \cdot ds' + \int_0^{L_2} \left(\bar{H}_{P_{21}}(s, s') + \bar{H}_{P_{22}}(s, s') \right) \cdot \bar{z} \cdot ds' \quad (5.19)$$

and at arm P_2 is,

$$H_{\text{cir}P_2}(s) = \int_{-L_1}^0 \left(\bar{H}_{P_{11}}(s, s') + \bar{H}_{P_{12}}(s, s') \right) \cdot \bar{z} \cdot ds' + \int_0^{L_2} \left(\bar{H}_{P_{21}}(s, s') + \bar{H}_{P_{22}}(s, s') \right) \cdot \bar{z} \cdot ds'. \quad (5.20)$$

The series impedance at point s along the conductor is expressed as $Z(s)$ and shunt admittance $Y(s)$. The characteristic impedance and propagation constant of the power line can be calculated using the formulas in (3.9).

5.3 Results and Discussion

A series of experiments were carried out to measure the common mode current of a bent PLC line. The length of the 2-core power line (BS6500) used in the experiment is 26 metres. A signal of 30 MHz with a voltage of 1 V rms was injected into the Line 1 (the live line) and Line 2 (the neutral wire) of the 2-core power line. A load of 50 Ohms was connected between Line 1 and Line 2. To obtain the worst-case condition in the radiation configuration, the two conductors are arranged as up and down (see Fig. 3.12(b)). In our model, the relative permittivity and conductivity of the ground were chosen to be 20 and 0.005 S/m respectively.

Fig. 5.2 Predicted and measured common mode current of the PLC line bent with an angle

The common mode currents along the conductors of the multi-conductor power line system both above the concrete and conducting ground were measured. Fig. 5.2 shows the common mode current of the 2-core power line when it was above the

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conducting ground. The common mode currents are compared when the power line was straight and had a bend of 90 degrees 16 metres away from the source. Comparing Fig. 5.2 and Fig. 4.5, it can be seen that when the power line was on a good conducting ground plane, the bend has not much effect on the common mode current.

In Fig. 5.3, the common mode current distribution along the PLC line bent with an angle of 90 degrees at a position 15 meters away from the source end is shown. The solid line is the model result calculated using the transmission line theory and the line with stars is the measured data. From Fig. 5.3, it is shown that the solid line and the line with stars are not well matched with each other, especially at the positions near the bend and the ends of the line. At these positions, the difference between our model results and experimental data is up to 10 dB. These discrepancies arise because the current clamp measured the common mode current by assuming that the magnetic field around the wire is circumferential. This assumption is only valid for long wire away from the discontinuity. Near the discontinuity e.g. near the bend or at the ends of the line, the magnetic fields around the wire are not circumferential. It should be noted that distributed circuit parameters are derived based on the tangential fields at the surface of the PLC wires but the diameter of the current clamp (3 cm) used in the experiment is much larger than that of the PLC wires. To have a fair comparison with experiment, we have recomputed the common current distribution by using the tangential fields at the same positions of the current clamp

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and the results are plotted as dotted line in Fig. 5.3. The good general agreement shows the validity and accuracy of our proposed model.

Fig. 5.4 is the common mode current distribution of 20 MHz bent at a position 20 meters away from the source end. In this figure, the dotted line calculated with the field theory with the radius of the current clamp fits the experimental data well.

Fig. 5.3 Predicted and measured common mode current of the power line above concrete ground plane at 30 MHz

Fig. 5.4 Predicted and measured common mode current of the power line above concrete ground plane at 20MHz

The radiated emission from the PLC system was measured with the loop antenna and the spectrum analyzer. Because of the high background noise in the frequency band of 1 MHz to 19 MHz, the radiated emissions for the frequency band from 20 MHz to 30 MHz from the power line at the distance of 10 metres were measured. For the PLC lines with a bend, the observation position is 10 metres away from both arms. The results of the experiment and the model are shown in Fig 5.5. In this figure, we can find the difference of the radiated emission levels between the two conditions is not significant because roughly both have the same common mode

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current. Other measurements with the 3-core power line also showed the same scenario.

Comparison between the predicted and the experimental results shows that our model can provide good predictions of the radiated emission from power line communication.

Fig.5.5 Radiated emission from the power line above the concrete ground

5.4 Conclusion

In this chapter, a radiated emission model was provided to predict the radiated emission from the carrier channels of power line communications for finite-length PLC lines bent at an arbitrary angle. The proposed model allows the calculation of the radiated EM field components as well as the common mode current along the conductors by considering the losses in the ground plane. A comparison between the experimental and theoretical results shows that the model can provide good EMI predictions for power line communications.

Chapter 6

Distributed Circuit Models for PLC Systems

6.1 Introduction

The radiated emission models described in the previous chapters are applicable to a finite-length PLC line and PLC line bent with an arbitrary angle above and embedded in the lossy ground plane. These models are derived based on Maxwell's Equations and required the evaluation of the Sommerfeld integrals. The accuracy of these full wave models is achieved at the expense of longer computation time. The purpose of this chapter is to establish computationally efficient distributed circuit models for infinite or finite-length straight or bent PLC line above a lossy ground plane.

6.2 Straight Finite-length Power Lines

Fig. 6.1 shows the geometry of a straight PLC line system, where r_0 is the radius of the conductor, ϵ_0 and μ_0 are the permittivity and permeability of air respectively. ϵ_g , μ_g and σ_g are the permittivity, permeability and conductivity of the ground plane respectively. The wire of length ℓ is parallel to the ground plane at a height h from the ground plane.

Fig.6.1 Geometry of a straight finite-length power line

Considering the physical characteristics of the power line, the self-impedance of the wire itself can be written as [78],

$$Z_i = \frac{(j\mu_w\omega)^{1/2}}{2\pi(\sigma_w + j\epsilon_w\omega)^{1/2}} \cdot \frac{I_0(jk_w r_0)}{I_1(jk_w r_0)} \quad (6.1)$$

where ϵ_w , μ_w , and σ_w are the permittivity, permeability and conductivity of the

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conductor respectively. k_w is the propagation constant of the electromagnetic field in the conductor. I_0 and I_1 are the first kind modified Bessel function of order zero and order one.

For PLC applications (in the frequency range of 1-30 MHz), it can be shown that

$|k_w| \gg |\gamma|$. Z_i can be approximated as,

$$Z_i = 1/(2\pi\sigma_w\delta) + j/(2\pi\sigma_w\delta) \quad (6.2)$$

As described in previous chapters, the wire is divided into k small elements of length $\Delta\ell$ so that the length of each element is much smaller than the operating wavelength. According to the applied frequency band (from 1 MHz to 30 MHz), $\Delta\ell$ is chosen to be 1 metre so that it is much smaller than the operating wavelength. Every small element is then considered as a dipole antenna.

Using the antenna theory, the series impedance can be calculated as follow:

$$Z(x) = -\lim_{\rho \rightarrow a} \frac{E_x(x, r_0, h)}{2\pi\rho \cdot H_\phi(x, r_0, h)} \quad (6.3)$$

where $E_x(x, r_0, h)$ and $H_\phi(x, r_0, h)$ are the summation of the elements along the wire.

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The tangential components of the electromagnetic field in (5.11) and (5.12) can be expressed as,

$$E_x(x, r_0, h) = A(x, r_0, h) \cdot (j\omega) + B(x, r_0, h) + C(x, r_0, h) \cdot (j\omega)^{-1} + D(x, r_0, h) \cdot (j\omega)^{-2} \quad (6.4)$$

$$H_\phi(x, r_0, h) = a(x, r_0, h) \cdot (j\omega) + b(x, r_0, h) + c(x, r_0, h) \cdot (j\omega)^{-1} \quad (6.5)$$

Since $\Pi_z(x, y, z)$ is much smaller than $\Pi_x(x, y, z)$ in (5.11) and (5.12), $A(x, r_0, h)$, $B(x, r_0, h)$, $C(x, r_0, h)$, $D(x, r_0, h)$, $a(x, r_0, h)$, $b(x, r_0, h)$ and $c(x, r_0, h)$ are approximated as following,

$$\begin{aligned} A(x, r_0, h) = & \frac{1}{4\pi\epsilon_0} \int_0^L I(x') \cdot \left[\left(K_0^2 \cdot \left(\frac{EX_0 \cdot (x-x')^2}{R_0^3} - \frac{EX_1 \cdot (x-x')^2}{R_1^3} - \frac{EX_0}{R_0} + \frac{EX_1}{R_1} \right) \right) \right. \\ & + \left(\frac{2}{K_0^2 - K_1^2} \left(K_0^4 \cdot \left(\frac{EX_0}{R_0} - \frac{EX_1 \cdot (x-x')^2}{R_1^3} \right) \right. \right. \\ & \left. \left. - K_1^2 \frac{EX_1}{R_1^2} \left(K_1^2 (x-x')^2 (1-\rho^2) - K_0^2 (1-(x-x')^2) \right) \right) \right] dx' \end{aligned} \quad (6.6a)$$

$$\begin{aligned} B(x, r_0, h) = & \frac{1}{4\pi\epsilon_0} \int_0^L I(x') \cdot K_0 \cdot \left[EX_0 \cdot \left(\frac{-1}{R_0^2} + \frac{(x-x')^2}{R_0^4} \right) \right. \\ & \left. + \frac{2K_1 EX_1}{K_0^2 - K_1^2} \left(K_1^2 \left(\frac{-10(x-x')^2 \rho^2}{R_1^5} + \frac{10(x-x')^2}{R_1^3} - \frac{1}{R_1} \right) + K_0^2 \left(\frac{2}{R_1} - \frac{3(x-x')^2}{R_1^3} \right) \right) \right] dx' \end{aligned} \quad (6.6b)$$

$$\begin{aligned} C(x, r_0, h) = & \frac{1}{4\pi\epsilon_0} \int_0^L I(x') \cdot (EX_0 - EX_1) \left(\frac{-1}{R_0^3} + \frac{3(x-x')^2}{R_0^5} \right) \\ & + \frac{2K_1^2 EX_1}{K_0^2 - K_1^2} \left(\frac{-42(x-x')^2 \rho^2}{R_1^7} + \frac{43(x-x')^2}{R_1^5} - \frac{5}{R_1^3} \right) dx' \end{aligned} \quad (6.6c)$$

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$$D(x, r_0, h) = \frac{1}{4\pi\epsilon_0} \int_0^L I(x') \cdot \frac{2K_1 EX_1}{K_0^2 - K_1^2} \left(\frac{-72(x-x')^2 \rho^2}{R_1^8} + \frac{76(x-x')^2}{R_1^6} - \frac{10}{R_1^4} \right) dx' \quad (6.6d)$$

and,

$$a(x, r_0, h) = (2\pi\rho K_0) \int_0^L I(x') \cdot \left(\frac{-(z-h)EX_0}{R_0^2} + \frac{(z+h)EX_1}{R_1^2} + \frac{2(z+h)EX_1 K_0^2}{K_0^2 - K_1^2} \cdot \left(\frac{(x-x')^2}{R_1^5} - \frac{1}{R_1^3} \right) \right) dx' \quad (6.7a)$$

$$b(x, r_0, h) = (2\pi\rho K_0) \int_0^L I(x') \cdot \left(\frac{-(z-h)EX_0}{R_0^3} + \frac{(z+h)EX_1}{R_1^3} + \frac{2(z+h)EX_1 K_0^2}{K_0^2 - K_1^2} \cdot \left(\frac{4(x-x')^2}{R_1^5} - \frac{3}{R_1^3} \right) \right) dx' \quad (6.7b)$$

$$c(x, r_0, h) = \frac{(4\pi\rho K_0)\Delta\ell}{K_0^2 - K_1^2} \int_0^L I(x') \cdot (z+h)EX_1 \cdot \left(\frac{6(x-x')^2}{R_1^6} - \frac{4}{R_1^4} \right) dx' \quad (6.7c)$$

where $EX_0 = e^{-jk_0 R_0}$, $EX_1 = e^{-jk_0 R_1}$, $\rho = \sqrt{(z-h)^2 + (y-r_0)^2}$. In order to abstract the term $(j\omega)$, in equations (6.6) and (6.7), $K_0 = k_0/(j\omega)$ and $K_1 = k_g/(j\omega)$.

$R_0 = \sqrt{(x-x')^2 + 2r_0^2}$ and $R_1 = \sqrt{(x-x')^2 + (2h)^2 + r_0^2}$. The above equations show that it is possible to eliminate the Sommerfield integrals so that the computation time will be decreased greatly.

$Z_g(x)$ is expressed as,

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$$Z_g(x) = \frac{A \cdot (j\omega) + B + C \cdot (j\omega)^{-1} + D \cdot (j\omega)^{-2}}{a \cdot (j\omega) + b + c \cdot (j\omega)^{-1}} \quad (6.8)$$

Equation (6.8) can be written in a ladder network form:

$$Z_g(x) = R_{g1} + \frac{1}{j\omega C_{g2} + G_{g2} + \frac{1}{j\omega L_{g3} + R_{g3} + \frac{1}{j\omega C_{g4} + G_{g4}}}} \quad (6.9)$$

In (6.9),

$$R_1 = A/a \quad (6.10a)$$

$$C_2 = a/E \quad (6.10b)$$

$$G_2 = \left(b - \frac{aF}{E} \right) / E \quad (6.10c)$$

$$L_3(x) = E/e \quad (6.10d)$$

$$R_3(x) = (F - f \cdot E/e) / e \quad (6.10e)$$

$$C_4(x) = e/m \quad (6.10f)$$

$$G_4(x) = f/m \quad (6.10g)$$

where,

$$E = B - Ab/a$$

$$F = C - Ac/a$$

$$e = c - aD/E - F/E \cdot (b - aF/E)$$

$$f = D \cdot (aF/E - b) / E$$

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$$m = D - f \cdot (F - f \cdot E / e) / e .$$

Assuming the ground plane as a reference plane, the model can be viewed as Fig. 6.2(a), which is equal to Fig. 6.2(b).

Fig. 6.2(a) Distributed circuit model of straight finite-length power line above a lossy ground plane

Fig. 6.2(b) Distributed circuit model of a straight finite-length power line above a lossy ground plane

By symetry, it can be showed that $G_{g2} = G_{g4} = G_g$ and $C_{g2} = C_{g4} = C_g$.

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Fig. 6.3 Distributed circuit model for long PLC line

The infinite-length line consists of cascading an infinite number of small elements each with length $\Delta \ell$ as shown in Fig. 6.3. After the circuit model is established, the series impedance and shunt admittance of the power line can be calculated.

In order to compare our model results with the method suggested by D'Amore [80], the ground impedance and admittance are calculated. Fig. 6.4 showed the capacitance and inductance of the line due to the effect of the ground plane that was compared with the infinite length method suggested by D'Amore.

In Fig. 6.4, the per-unit-length capacitance and conductance along the line is shown. The applied frequency is 30 MHz. The ground relative permittivity and conductivity are 5 and 0.005 S/m respectively. Fig. 6.4 shows that at the both ends of the line, the capacitance and conductance have a fluctuation due to the end effect. R_g and G_g also show the same scenarios as L_g and C_g .

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Fig. 6.4 Comparison of per-unit-length capacitance and inductance of finite-length and infinite-length methods

6.3 Finite-length Multi-conductor Power Lines

Fig.6.5 Geometrical configuration of the multi-conductor power line

For an indoor power line communication system, the installed power line has two cores (for example, in China), three cores (for example, in Singapore) or four cores (in most European countries). For simplicity, the two-line system is considered as shown in Fig. 6.5; systems with 3 or more lines can be derived using the approach described below.

As described in Chapter 6.2, the electromagnetic field of the multi-conductor power line can be calculated. Compared with the single-conductor power line, the circuit model for the wire of the multi-conductor power line system should include the mutual

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impedance due to the adjacent wire.

The expressions of the series impedance and shunt admittance are complex due to the complex expression of the electromagnetic field in accounting the ground effect.

According to the following assumptions:

- $d \ll h_1$ and h_2
- $d \ll \lambda$

Using the concept of distributed dipole antenna, the major effect of the mutual coupling between the two wires is capacitive because they are within the near field range, that is, $Y_m(x) \approx j\omega C_m$.

Fig. 6.6 Distributed circuit model for multi-conductor power line above a lossy ground plane

In our calculation, we assume that the 2-core power line (BS6500) radius $r_0 = 0.69$ mm and the PLC lines are at 10 metres height ($h = 10$ metres) from the lossy ground plane, and the ground conductivity and permittivity are $\sigma_g = 0.005$ S/m and $\epsilon_g = 5\epsilon_0$ respectively.

Fig. 6.7(a) Frequency spectral of the real part of the ground impedance compared with D'Amore's model [85]

Fig. 6.7(b) Frequency spectral of the imaginary part of the ground impedance compared with D'Amore's model [85]

The series impedance due to the lossy ground Z_g is calculated using (6.9). The frequency spectrum of the real and imaginary part of Z_g for a PLC line with length $\ell = 100$ metres is compared with the result calculated based on the model proposed by D'Amore [85].

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Fig. 6.7 shows that our model can give reasonable results in calculating the characteristics of the power line communication wires in the frequency band from 1 to 30 MHz. It shows that in the specified frequency band, the smaller the frequency, and the more accurate the model is. As shown in these two figures, the fluctuation of the ground impedance of the finite length power line is because of the resonance of the capacitor and inductor in the circuit model. Since $k_g = k_0 \sqrt{\frac{\epsilon_g}{\epsilon_0} - j \frac{\sigma_g}{\omega \epsilon_0}}$ and $k_g / k_0 = 3.4399 - 2.6140j$, the effective wavelength in the ground is 19.1 metres at 6 MHz. Because the electric fields of the wire and the equivalent image are enhanced by each other, there's a peak near 6 MHz in Fig. 6.7(a). Consider Fig. 6.7(b), the resonance from the per-unit ground impedance has a peak at 6 MHz. The peak is determined by the height from the ground and $\sqrt{\epsilon_g / \epsilon_0 - j \sigma_g / (\omega \epsilon_0)}$. When the height is increased, the frequency of the peak will move to the left.

Fig. 6.8(a) Frequency spectral of the attenuation constant

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Fig. 6.8(b) Frequency spectrum of wave number

Fig. 6.8 shows a comparison of the frequency spectrum of the propagation constant of the finite length and the infinite length PLC line. From Fig. 6.8(a) and Fig. 6.8(b), it shows that when the length of the power lines is larger than several wavelengths, the frequency spectral of the PLC characteristics of the finite-length is approximately the same as the infinite length power line. Since the two wires are near to each other ($d \ll \lambda$), the propagation constant is nearly the same, the figure showed γ of Line 1 as shown in Fig. 6.6.

6.4 Multi-conductor PLC Line Bent with an Arbitrary Angle

The geometry of the PLC line is shown in Fig. 6.9. Theoretically, the formulation is applicable to a N-core PLC line with different bend angles and for different lengths of line. For simplicity, we will restrict ourselves to a 2-core PLC line ($M = 2$) with the same bend angles θ as shown in Fig. 6.9. The total length of each PLC line is $L_1 + L_2$ and the radius of the two wires is set to be r_0 . The wires (or power lines) are assumed to be parallel to the infinite lossy ground plane (x-y plane), and θ is the bent angle. The permittivity, permeability and conductivity of the wires and the ground plane are $(\epsilon_w, \mu_w, \sigma_w)$ and $(\epsilon_g, \mu_g, \sigma_g)$ respectively.

Fig. 6.9 Geometry configuration of power line with a bend

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In Fig. 6.9, arm P_1 (P_{11} and P_{12}) with length of L_1 is arranged along the x-axis, and P_2 (P_{21} and P_{22}) is the other arm with length L_2 directed at an angle θ to the x-axis in the x-y plane. P_{ij} denotes arm i and line j . The heights of the two wires from the ground plane are h_1 and h_2 respectively.

In Fig. 6.9, s denotes the axis along the wires. The Hertz Potentials of the elements with length $\Delta\ell$ along arm P_1 are $\Pi_x(s, z)$ and $\Pi_z(s, z)$ which are referred to as $\Pi_s(s, z)$ and $\Pi_z(s, z)$. Similarly, the Hertz Potentials of the elements with length $\Delta\ell$ along arm P_2 has the components of $\Pi_x(x, y, z)$, $\Pi_y(x, y, z)$ and $\Pi_z(x, y, z)$ which are considered as $\Pi_s(s, z)$ and $\Pi_z(s, z)$.

Assuming the electric field from the source point at s' on the surface of the conductor positioned at s is $E(s, s')$ and magnetic field is $H(s, s')$. The electric and magnetic fields on the surfaces of Line 1 and Line 2 are the summation of the electromagnetic fields due to the current along the two conductors.

Fig.6.10 Equivalent per-unit-length 2-cone transmission line circuit

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Assuming the ground plane is the reference plane, the equivalent circuit model of the two-conductor transmission line is shown as Fig. 6.10.

In Fig. 6.10, Z_s is the self-impedance, Z_m is the mutual impedance and Z_g is the impedance due to the lossy ground. Y_m and Y_g are the mutual admittance and the admittance due to the ground plane respectively.

The tangential component of the electric field expressed with the Hertz potentials of arm P_1 is as following,

$$E_{s,P_1}(s, z) = \int_{-L_1}^0 \left\{ -\gamma_0^2 \Pi_{s,1}(s', z) + \frac{\partial}{\partial s} [\text{div} \Pi_1(s', z)] \right\} dx' + \cos \theta \cdot \int_0^{L_2} \left\{ -\gamma_0^2 \Pi_{s,2}(s', z) + \frac{\partial}{\partial s} [\text{div} \Pi_2(s', z)] \right\} ds' \quad (6.11)$$

$$E_{s,P_2}(s, z) = \int_{-L_1}^0 \left\{ -\gamma^2 \Pi_{s,2}(s', z) + \frac{\partial}{\partial s} [\text{div} \Pi_2(s', z)] \right\} dx' - \cos \theta \cdot \int_0^{L_2} \left\{ -\gamma^2 \Pi_{s,1}(s', z) + \frac{\partial}{\partial s} [\text{div} \Pi_1(s', z)] \right\} ds' \quad (6.12)$$

where s' is the source position along the wire and s is the observation position along the wire. The Hertz potentials of any element on arm P_1 includes $\Pi_{s,1}(s, z)$ and $\Pi_{z,1}(s, z)$. The Hertz potential of any element on arm P_2 includes $\Pi_{s,2}(s, z)$ and $\Pi_{z,2}(s, z)$.

After neglecting $\Pi_z(s, z)$ of the wires, the magnetic fields are expressed as follows,

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$$H_{\Phi P_1}(s, z) = (\sigma + j\omega\epsilon_0) \int_{-L_1}^0 \frac{\partial \Pi_{s,1}}{\partial z} ds' + \cos \theta \cdot \int_0^{L_2} \frac{\partial \Pi_{s,2}}{\partial z} ds' \quad (6.13)$$

$$H_{\Phi P_2}(s, z) = (\sigma + j\omega\epsilon_0) \int_{-L_1}^0 \frac{\partial \Pi_{s,2}}{\partial z} ds' - \cos \theta \cdot \int_0^{L_2} \frac{\partial \Pi_{s,1}}{\partial z} ds' \quad (6.14)$$

Assume that $E_s(s, z)$ and $H_\Phi(s, z)$ have the forms the same as (5.11) and (5.12), terms $A(x, r_0, h)$, $B(x, r_0, h)$, $C(x, r_0, h)$, $D(x, r_0, h)$, $a(x, r_0, h)$, $b(x, r_0, h)$ and $c(x, r_0, h)$ are expressed as,

$$A(s, h) = A_{P_{11}}(s, h) + A_{P_{12}}(s, h) + A_{P_{21}}(s, h) + A_{P_{22}}(s, h) \quad (6.15a)$$

$$B(s, h) = B_{P_{11}}(s, h) + B_{P_{12}}(s, h) + B_{P_{21}}(s, h) + B_{P_{22}}(s, h) \quad (6.15b)$$

$$C(s, h) = C_{P_{11}}(s, h) + C_{P_{12}}(s, h) + C_{P_{21}}(s, h) + C_{P_{22}}(s, h) \quad (6.15c)$$

$$D(s, h) = D_{P_{11}}(s, h) + D_{P_{12}}(s, h) + D_{P_{21}}(s, h) + D_{P_{22}}(s, h) \quad (6.15d)$$

$$a(s, h) = a_{P_{11}}(s, h) + a_{P_{12}}(s, h) + a_{P_{21}}(s, h) + a_{P_{22}}(s, h) \quad (6.15e)$$

$$b(s, h) = b_{P_{11}}(s, h) + b_{P_{12}}(s, h) + b_{P_{21}}(s, h) + b_{P_{22}}(s, h) \quad (6.15f)$$

$$c(s, h) = c_{P_{11}}(s, h) + c_{P_{12}}(s, h) + c_{P_{21}}(s, h) + c_{P_{22}}(s, h) \quad (6.15g)$$

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Fig. 6.11 Capacitance and inductance of the element of the bent power line above lossy ground

Fig. 6.12 Real part of the impedance and shunt admittance of a bent power line above lossy ground

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The calculated results of the inductance and capacitance are shown in Fig. 6.11 with the series impedance and shunt admittance of Line 1 are shown in Fig. 6.12. Here θ is assumed to be 90 degrees. The radius r_0 of the 2-core power line (BS6500) is 0.69 mm. The distance between the two wires is assumed to be 2.85 mm. Comparing Fig. 6.12 and Fig. 6.4, it is obvious that the effect of the bend on the series impedance and shunt admittance is large. With the bend, the input impedance will be changed so that the current along the wires will also be changed as shown in Fig5.3 and Fig. 5.4.

6.5 Conclusion

In this chapter, distributed circuit models for broadband power line systems are presented. The model can be used to predict the propagation characteristics of the power line communications for finite-length PLC lines bent with an angle. The model is derived based on the concept of distributed dipole antenna that takes into account the effect of the lossy ground plane. Without the need to evaluate the Sommerfeld integrations, the computation time is greatly reduced by more than fifteen times relative to the full wave approach. The simulation result of our model is compared with the infinite-length model result. The results show that the distributed circuit model is reasonable both for straight and bent PLC wires.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

In this thesis, we have completed the formulation of a series of radiated emission models and a series of measurements for Power Line Communication systems in the frequency band of 1 to 30 MHz for both outdoor and indoor applications. The models have been developed based on the full wave approach, i.e., the electric and magnetic Hertz vectors subjected to the boundary conditions at the interfaces of the air-ground and dielectric-conductor. Sommerfeld integral is used to take into account the effect of the lossy ground plane.

The models are applied to a sequence of increasingly complex PLC systems to achieve improved versatility, flexibility and accuracy.

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This sequence progresses in the following order: from (a), a straight infinite-length PLC line system to (b), a straight finite-length PLC system to (c), a multi-conductor finite-length PLC lines bent with an arbitrary angle and at last to (d), a distributed circuit model for multi-conductor finite-length PLC lines bent with an arbitrary angle above a lossy ground plane.

At each stage of the development of our models, each model is applied to a specific PLC system for comparison with our own measurement data and experimental data reported in the literature. The good agreement with these measurements indicates that our models work well for such PLC applications.

The important features of the results applied to various PLC systems may be summarized as follows:

1. A radiated emission model to predict the EM field from the straight infinite-length PLC line above a lossy ground plane is established. With the assumptions and approximations made according to the PLC condition, the computation time is greatly reduced. Explicit formulae to describe the effect of the lossy ground are also presented. The effects of the ground permittivity and conductivity were discussed. Our theoretical and experimental results show that the electrical properties of the ground plane have a large effect both on the transmission characteristics and the radiated emissions.

2. The study of the electromagnetic interference from PLC line to the adjacent electric line shows that the interference from the PLC carrier to other power lines was quite large when the two lines were nearby. From our model, it is obvious that the interference between the PLC carrier and other power line is un-avoidable. The interference decreases greatly with the distance between the two lines when they are above good conducting ground. When two lines were near to the ground plane, the electrical properties of the ground would also affect the common mode current along the no-sourced line. The common mode current along the no-sourced line was smaller when the two lines were above conducting ground plane than that when they were above concrete ground. Our results also show that the common mode current is greatly affected by the electrical characteristics of the ground.

3. The radiated emissions for both side-by-side and up down arrangements were investigated. The common mode current and radiated emission of the up – down PLC wires is much larger than that of the side – by – side one. The reason for the large common mode current for the up down arrangement has been identified, i.e., due to the asymmetry of the wires with respect to the lossy ground plane.

4. A radiated model of the PLC line when it was embedded in the ground plane was proposed to predict the radiated emission from the under-ground PLC system. The transmission characteristics of the PLC line were calculated and results were shown. The radiated emission in the air was shown and compared with that of the PLC line when it was above ground. Our model results show that the radiated emission in air of the underground PLC is about 20 dB less than that of the above ground PLC.

5. Radiated emission models were proposed to predict the electromagnetic radiated emission from the carrier channels of the power line communications both for finite-length straight PLC line and PLC line bent with an arbitrary angle. The proposed model allows the calculation of the radiated EM field components as well as the common mode current along the conductors by considering the losses in the ground plane. The comparison between the experiment and model results shows that the model can provide good EMI predictions for power line communications. The end effect of the straight power line and the bend effect of the bent power line could be taken account into the common mode current with our model. When the power line was on concrete ground, the effect of the bend on the common mode current is larger than 5 dB.

Chapter 7 Conclusion and Future Work

6. A distributed circuit model for broadband power line systems was described.

With this model, the PLC line is described with the cascading of ladder networks. With our model, explicit forms of R , L , C and G of the PLC lines for finite-length PLC line and PLC line with a bend are obtained. This makes it easier to study the geometry effect of the line and the current distribution along the wires. Computation time is decreased greatly without the need to evaluate the Sommerfeld integrations.

7.2 Suggestions for Future Research

Our models may be extended or modified to include the effects of non-homogeneity of wall and ground media. Further improvement in the use of our radiated emission models would involve techniques to extract the electrical features for real building.

There is also a great need for more published measurements with well-documented parameters of the line geometry and the electrical properties of the ground surfaces to enable comparisons of theory with experiment. Measurements are also needed to characterize the body walls and ground surfaces for broadband PLC systems.

Author's Publications

Journal Papers:

1. Wu Qiong Luo, Soon Yim Tan and Boontiong Tan, "Effects of the Ground on Power-Line Communications", published in IEEE Transactions on Microwave Theory and Techniques, Vol. 53, No.10, pp 3191-3198, Oct., 2005
2. Wu Qiong Luo, Soon Yim Tan and Boontiong Tan, "A Radiated Emission Model for Power Line Communications", published in IEEE Transaction on Power Delivery, Vol. 21, No. 3, pp 1245-1249, July, 2006
3. Wu Qiong Luo and Soon Yim Tan, "A Distributed Circuit Model for Power-Line Communications", accepted by IEEE Transactions on Power Delivery.

Conference Papers

4. "Radiated Emission Model for Broadband Power Line Communications", presented at Asia-Pacific Microwave Conference (APMC04), New Delhi, India, 2004.
5. "Modeling and Measurements of Radiated Emissions Associated with Power Line Communication", presented at The Seventeenth Asia-Pacific Microwave Conference (APMC-2005), Suzhou, China, December 4-7, 2005.

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Appendix

The coefficients Γ_x , Γ_y , Γ_z , Θ_x , Θ_y and Θ_z in Chapter 3 are expressed as follows,

$$\Gamma_x(\gamma, y, z) = \frac{k_0^2 - \gamma^2}{2k_0^2} \Lambda_1(\gamma, y, z) + S_1(\gamma, y, z) - \frac{\gamma^2}{k_0^2} S_2(\gamma, y, z) \quad (\text{A1a})$$

$$\Gamma_y(\gamma, y, z) = 0.5(k_0^2 - \gamma^2)^{1/2} \Lambda_2(\gamma, y, z) + k_0^2 S_3(\gamma, y, z) - \gamma^2 S_4(\gamma, y, z) - S_6(\gamma, y, z) \quad (\text{A1b})$$

$$\Gamma_z(\gamma, y, z) = 0.5(k_0^2 - \gamma^2)^{1/2} \Lambda_3(\gamma, y, z) + S_8(\gamma, y, z) \quad (\text{A1c})$$

$$\Theta_x(\gamma, y, z) = S_9(\gamma, y, z) - S_{10}(\gamma, y, z) \quad (\text{A2a})$$

$$\Theta_y(\gamma, y, z) = 0.5j(k_0^2 - \gamma^2)^{1/2} \Lambda_3(\gamma, y, z) + S_7(\gamma, y, z) \quad (\text{A2b})$$

$$\Theta_z(\gamma, y, z) = 0.5j(k_0^2 - \gamma^2)^{1/2} \Lambda_2(\gamma, y, z) - k_0^2 S_3(\gamma, y, z) + \gamma^2 S_4(\gamma, y, z) + S_5(\gamma, y, z) \quad (\text{A3c})$$

where $\Lambda_1(\gamma, y, z)$, $\Lambda_2(\gamma, y, z)$, $\Lambda_3(\gamma, y, z)$ and $S_1(\gamma, y, z)$ to $S_{10}(\gamma, y, z)$ are as follows,

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$$\Lambda_1(\gamma, y, z) = K_0 \left\{ j[(y-h)^2 + z^2]^{1/2} (k_0^2 - \gamma^2)^{1/2} \right\} - K_0 \left\{ j[(y+h)^2 + z^2]^{1/2} (k_0^2 - \gamma^2)^{1/2} \right\} \quad (\text{A4a})$$

$$\Lambda_2(\gamma, y, z) = (y+h)[(y+h)^2 + z^2]^{-1/2} K_1 \left\{ j[(y+h)^2 + z^2]^{1/2} (k_0^2 - \gamma^2)^{1/2} \right\} \\ - (y-h)[(y-h)^2 + z^2]^{-1/2} K_1 \left\{ j[(y-h)^2 + z^2]^{1/2} (k_0^2 - \gamma^2)^{1/2} \right\} \quad (\text{A4b})$$

$$\Lambda_3(\gamma, y, z) = z[(y+h)^2 + z^2]^{-1/2} K_1 \left\{ j[(y+h)^2 + z^2]^{1/2} (k_0^2 - \gamma^2)^{1/2} \right\} \\ - z[(y-h)^2 + z^2]^{-1/2} K_1 \left\{ j[(y-h)^2 + z^2]^{1/2} (k_0^2 - \gamma^2)^{1/2} \right\} \quad (\text{A4c})$$

$$S_1(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\exp[-u_0(y+h)]}{u_0 + u_g} \exp(-jz\lambda) d\lambda \quad (\text{A5a})$$

$$S_2(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\exp[-u_0(y+h)]}{k_g^2 k_0^{-2} u_0 + u_g} \exp(-jz\lambda) d\lambda \quad (\text{A5b})$$

$$S_3(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\exp[-u_0(y+h)]}{(u_0 + u_g)u_0} \exp(-jz\lambda) d\lambda \quad (\text{A5c})$$

$$S_4(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\exp[-u_0(y+h)]}{(k_g^2 k_0^{-2} u_0 + u_g)u_0} \exp(-jz\lambda) d\lambda \quad (\text{A5d})$$

$$S_5(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\exp[-u_0(y+h)]}{\lambda^2 (u_0 + u_g)u_0} \exp(-jz\lambda) d\lambda \quad (\text{A5e})$$

$$S_6(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\exp[-u_0(y+h)]}{(k_g^2 k_0^{-2} u_0 + u_g)u_0 \lambda^2} \exp(-jz\lambda) d\lambda \quad (\text{A5f})$$

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$$S_7(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\lambda \exp[-u_0(y+h)]}{(u_0 + u_g)} \exp(-jz\lambda) d\lambda \quad (\text{A5g})$$

$$S_8(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\lambda \exp[-u_0(y+h)]}{(k_g^2 k_0^{-2} u_0 + u_g)} \exp(-jz\lambda) d\lambda \quad (\text{A5h})$$

$$S_9(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\lambda \exp[-u_0(y+h)]}{(u_0 + u_g)u_0} \exp(-jz\lambda) d\lambda \quad (\text{A5i})$$

$$S_{10}(\gamma, y, z) = \frac{1}{2} \int_{-\infty}^{+\infty} \frac{\lambda \exp[-u_0(y+h)]}{(k_g^2 k_0^{-2} u_0 + u_g)u_0} \exp(-jz\lambda) d\lambda . \quad (\text{A5j})$$

Here K_0 is the modified Bessel Function of second kind, and of order zero and K_1 is the modified Bessel Function of second kind, of order 1.