On the issues of localization and mapping in outdoor unstructured environments for mobile robot navigation

Perera Linthotage Dushantha Lochana

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On the Issues of Localization and Mapping in Outdoor Unstructured Environments for Mobile Robot Navigation

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School of Electrical and Electronic Engineering

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To my parents and wife

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Summary

The localization information is of paramount importance in several critical tasks such as path and mission planning and control, which are essential to achieve fully autonomous operation of mobile robots. Data association, use of multiple sensors, their fusion and effects of exteroceptive and proprioceptive sensor biases are identified as major issues that affect the performance of robot localization and mapping in outdoor unstructured environments. The importance of addressing these issues is justified using a comprehensive literature review and a discussion.

A generalized sliding window framework utilizing multiple measurement frames is established for the data association in robot navigation applications. It is argued that the feature extraction is a dimensionality reduction and a data compression technique for the data association framework. The issues pertaining to conditional measurement independence are then investigated in the context of the data association problem in Simultaneous Localization and Mapping (SLAM), Map Aided Localization (MAL) and Robotics Mapping (RM). Multi-frame sliding window algorithms for data association are synthesized for SLAM, MAL and RM for the first time using the conditional measurement independence assumption. However, when the vehicle uncertainty is large and the feature density is very high in the context of SLAM and MAL one cannot ignore the effects of strong spatial correlations on the measurements. Under such circumstances conditional measurement independence assumption does not hold. Thus, a new multiple frame temporal consistency criterion (MFTC) algorithm is synthesized for the multi-frame data association in SLAM and MAL relaxing the conditional measurement independence assumption. This method, which is an extension of batch data association methods over multiple temporal frames, incorporates both spatial and temporal correlations inherent to SLAM in the data association process. The search for the correct association is carried out using an interpretation tree search over multiple temporal frames. These multi-frame algorithms are shown to be robust to spurious measurements and clutter present in outdoor and dynamic environments. It has also been demonstrated that the multi-frame data association algorithms are more robust than the existing single

frame data association algorithms used for mobile robotics applications in outdoor unstructured environments. Several simulations and experiments were conducted to ascertain the performance of the multi-frame sliding window data association algorithms developed.

The use of multiple sensors in SLAM and approaches in sensor fusion and integration in outdoor environments are investigated. It is established that the group sensor fusion architecture is theoretically sound in multi sensor fusion in SLAM where strong correlations exist. A fully operational system, which is able to perform SLAM using the fusion of range-bearing and bearing only information, is demonstrated. A method of improving multiple sensor performance through information theoretic active sensor control strategy over a finite time window is formulated and verified using simulations.

The effects of exteroceptive and proprioceptive sensor biases, which adversely affect the data association problem, tracking accuracy, map convergence and localization accuracy and hence the performance of SLAM, MAL and RM in outdoor unstructured environments, are investigated. A rigorous theoretical discussion is presented on the estimation theoretic sensor bias correction problem in robot localization and mapping algorithms. The convergence properties of the bias estimates are derived and their lower bounds deduced. The observability properties of SLAM, MAL and RM problems with sensor bias correction are established. Remedies in the form of modified observation models are proposed for the partial observability problem. It was also verified using simulations and experiments that the inertial sensors can be utilized in SLAM and MAL with online bias correction effectively on the basis of the theoretical discussion.

Based on all the facts established it is argued that although the contribution by improving on a single issue affecting robot localization and mapping may not be very significant, the total contribution by improving on all the above issues pertaining to robot localization and mapping performance is substantial.

Chapter 1

Introduction

1.1. Motivations

In the last several decades, there has been a significant interest towards the deployment of autonomous systems described by many as mobile robots or autonomously guided vehicles (AGV) in several disciplines such as transportation, warfare, heavy industries, agriculture and aerospace. The applications of mobile robots extend from human habitats, industrial plants and underwater environments to outer space especially to hazardous and inaccessible areas to humans. A designer of a mobile robot navigation system faces three fundamental problems [1]. These are how to find out 1. Where the robot is located at, 2. Where it is heading for and 3. How it gets to the desired place. The first problem, known as the mobile robot localization [1], [2], is of paramount importance before one can address the remaining two problems, which are mostly linked with the local navigation, mission planning, path planning and control of the mobile robot.

Algorithms for mobile robot localization can be categorized into three. First one is Map Aided Localization (MAL) [2], where an a priori knowledge of the environment (known as a map) is used to determine the location of a mobile robot with respect to a global coordinate frame. In MAL, a mobile robot equipped with a sensor suite senses its surroundings and compares the information acquired with a stored map to determine where it is located in the world coordinate frame. With the advent of sophisticated sensor

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technologies and efficient sensor fusion algorithms, MAL has become mature and has been implemented in several military and commercial systems such as Cruise missile guidance [3], low altitude aircraft navigation, guidance and collision avoidance systems. However, MAL demands a bulk of prior work in preparing and compiling accurate maps and other information to suite various autonomous navigation tasks. In general, acquiring prior maps of large unexplored terrains having dynamically changing conditions is quite difficult and challenging.

Second approach to localization yielding promising accuracy is the use of active beacon based systems such as global positioning systems (GPS) [8]. In this approach signals transmitted by a constellation of low orbiting satellites are used to derive the global coordinates of a GPS receiver, which is mounted on the moving platform. Nevertheless, GPS systems fail intermittently due to multi-path effects of local terrain, periodic signal blockage by foliage, places having restricted views of the sky, hilly terrain, partial satellite occlusion and active RF jamming [1], making it inappropriate for stand-alone use in autonomous systems.

The Simultaneous Localization and Mapping (SLAM), or the Concurrent Mapping and Localization (CML) problem [4], [5] and [7] emerged as an answer to most of the above problems. In contrast to MAL and GPS based localization, the goal of a mobile robot performing SLAM is to build a map of its surroundings incrementally by using the uncertain information extracted from its sensors whilst simultaneously using the acquired map to localize itself with reference to a global coordinate frame and navigate in real time. Map Building stands for the correct identification and construction of robot's environment (essentially salient features or structure) in a useful manner. The independence of the localization solution obtained from the SLAM problem from any other external source has stimulated huge interest in the robotics community and has been widely regarded as the main key to achieve autonomous navigation capability for mobile

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robots [7]. A vehicle capable of performing SLAM using environmental features in its locality and sustaining hours and possibly days in a completely unknown environment would indeed make the system truly autonomous. Such a capability would necessarily eliminate the need for artificial active beacons or landmarks for autonomous vehicle navigation purposes. SLAM finds numerous applications in military reconnaissance, surveillance, aerospace, underwater and planetary exploration, mining, cargo handling, surveying and driver assistance systems. Further, the maps produced by SLAM algorithms can serve as a basis for a multitude of other tasks such as motion planning, exploration and control.

However, after over a decade and a half of hard work and having established a comprehensive framework of the problem researches still face several challenges in realizing fully deployable SLAM solutions especially in complex unstructured environments. This is because of the difficulties in perception, correspondence, modeling and sensor fusion related issues in SLAM. Had there been perfect sensors capable of providing perfectly accurate measurements and ideal models for vehicles and sensors, which can be used to exactly predict the vehicle motion and relative landmark positions there would not have been a localization and/or a map building problem.

Large inaccuracies accumulating over time in dead reckoning (proprioceptive) sensors (odometry and INS) due to integration of noisy information compel one to use complementing exteroceptive sensors such as lasers, RADARs or cameras together with reliable sensor fusion algorithms to achieve accurate positioning in robot navigation applications. The use of these sensors introduce the problem of correspondence or the data association, which is essential prior to any form of data fusion to determine what the robot observes in terms of the knowledge it currently possesses as a map. The problem of correspondence is one of the inadequately addressed issues in SLAM mainly because of

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its complexity. While modeling of random noise characteristics of sensors has received considerable attention in the literature, the effects of biases and drifts of sensors have not been adequately dealt with especially in the context of SLAM. If not appropriately compensated these errors exacerbate the data association problem.

Lack of proper treatment of the important issues of data association, sensor biases and related multi-sensor fusion characteristics are often reported to be the main causes of map inconsistency, filter divergence and over confident localization results. Hence, apart from effective fusion of sensory information from multiple sensors, it is absolutely necessary that the biases and drifts in the sensors be effectively compensated and the data association problem be effectively solved in order to realize full deployable MAL/SLAM algorithms in demanding outdoor environments.

Consequently, the ever demanding applications and prospects of mobile robotics overshadowed by the above challenges inspire and motivate us to investigate SLAM, MAL and Robotics Mapping problems concentrating on the issues of data association, sensor biases and multi-sensor fusion characteristics.

1.2 Objectives

The objective of this thesis is to study, investigate and analyze the important issues that affect the performance of mobile robot localization and mapping in outdoor unstructured environments. Data association problem, effects of exteroceptive and proprioceptive sensor biases and issues of multi-sensor fusion are addressed in detail to enhance the capabilities of presently available SLAM and MAL algorithms. As a result it is expected to achieve robust and consistent localization results and high integrity superior quality maps, which can be efficiently and effectively employed in autonomous vehicle navigation applications especially in outdoor unstructured environments.

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1.3 Major Contributions of the Thesis

- ♦ A generalized data association framework utilizing multiple temporal measurement frames in a sliding window is proposed for robot localization, robotics mapping and simultaneous localization and mapping problems.
- ♦ A multi-dimensional assignment based sliding window multiple frame data association algorithm utilizing discrete optimization is synthesized for SLAM assuming conditional measurement independence. Problems of violating the conditional measurement independence assumption are elaborated and a new Batch temporal Consistency Criterion (MFTC) based data association approach is introduced for SLAM and MAL relaxing the conditional independence assumption. The algorithms developed are tested with laser and vision sensors while addressing related issues involving track initiation and multi-sensor data fusion. Simulations and experiments are conducted to verify the performance gains of the proposed algorithms over conventional nearest neighbor and joint compatibility branch and bound algorithms.
- ♦ The use of multiple sensors in SLAM is investigated in the context of data fusion, correlations and measurement independence. It is established how group sensor fusion is more desirable in SLAM when information from multiple sensors are incorporated. An active sensing method is proposed to enhance the multiple sensor performance in SLAM. A generalized active sensing paradigm in a finite time window applicable to SLAM problem is formulated as a nonlinear optimization problem. The control of active bearing only sensor (eg. a monocular camera) to maximize the information gain, in the presence of multiple sensors is elaborated and illustrated in the context of the SLAM problem
- ♦ A rigorous and comprehensive analysis of the estimation theoretic sensor bias correction problem, which also affects the performance of SLAM and MAL in

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outdoor unstructured environments, is carried out. It is shown that the uncertainty of sensor bias terms cannot increase in a measurement update. It is also shown that the uncertainty of bias estimates reach a lower bound determined by the initial uncertainties of the vehicle and landmarks. The rates of convergence, observability and the stability of a class of linear models with respect to this problem are elucidated. The partial observability of the problem is shown to be the cause of these effects and methods of overcoming this problem using appropriately modified observation models are proposed. The developed concepts and proposed algorithms are verified experimentally using a low cost inertial navigation sensor suite.

1.4 Organization of the thesis

An overview of the work carried out in the course of the thesis and the structure of the thesis in concise form are outlined in this chapter. The rest of the thesis is organized in the following manner.

Chapter 2 describes the state of the Art in SLAM. Further, major work in data association, sensor and vehicle modeling, biases in the sensors and multi-sensor fusion as applied to SLAM are reviewed.

Chapter 3 describes how the data association problem of the MAL, mapping and SLAM problems can be cast as a discrete integer optimization problem employing a finite size sliding window of measurement frames. Further, the advantages, disadvantages and implications of the proposed framework for data association; in particular including the issues of measurement independence and the filtering of temporally correlated spurious measurements and dynamic objects are described. Chapter 3 synthesizes algorithms for data association in SLAM and MAL using conditional measurement independence assumption and also synthesizes a data association algorithm, which does not utilize the same assumption. The advantages and disadvantages of the proposed algorithms are

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investigated in detail and their performances compared against the existing Standard Nearest Neighbor (SNN) and Joint Compatibility Branch and Bound (JCBB) algorithms using extensive simulations and experiments.

Chapter 4 theoretically investigates the strong correlations that exist in the SLAM state in the context of multi-sensor fusion. A strategy of utilizing a laser range finder and two monocular cameras in an outdoor SLAM setting is detailed. Feature extraction, initialization, camera calibration and the experimental set up are detailed. A new performance metric quantifying the performance of multiple sensors in SLAM is proposed. A framework that improves multi-sensor performance in SLAM through active fusion concepts is developed.

Chapter 5 analyzes a significant and often neglected topic related to SLAM in general giving rise to data association failures and filter divergence; namely the effect of sensor biases. The estimation theoretic sensor bias correction is rigorously studied and analyzed in detail and several results of theoretical value encompassing diminishing uncertainty, lower uncertainty bounds, rates of convergence and observability are established. The partial observability characteristics are shown to be present in the problem and measures are proposed to overcome these with the use of modified observation models. It is further shown that the similar analysis is possible for the problem of estimation theoretic sensor bias correction in MAL and robotics mapping as well. The theoretical results are verified using extensive simulations. Experimental verification of the above sensor bias correction problem is elaborated using a low cost inertial navigation sensor suite.

Chapter 6 summarizes and concludes the work presented in the thesis and also provides recommendations for the future work

Chapter 2

Related Work

2.1. The Simultaneous Localization and Mapping Problem

The problem of Simultaneous Localization and Map Building (SLAM) has often been recognized as one of the key problems in building autonomous capabilities for mobile vehicles in the robotics literature. It is also widely accepted that the capability of simultaneous self-localization and map building without an a priori map would make a robot truly autonomous. The SLAM problem examines whether a vehicle in an unknown environment starting from an unknown location can build a map of its environment incrementally by using the uncertain information extracted from its sensors whilst simultaneously using that map to localize and navigate in real time.

The research in SLAM, robot localization using an a priori map (map aided localization) and robotics mapping given the exact robot path dates back to 1980s. The first solution to the SLAM problem was proposed in [4] by Smith, Self and Cheeseman. They emphasized the importance of map-vehicle correlations in the SLAM state vector comprising of the vehicle state and map-feature (landmark) locations and introduced the Extended Kalman Filter (EKF) based stochastic mapping framework for joint estimation of this augmented state vector. This work for the first time established that the localization and map building problems couldn't be solved independently from each other. This work was followed by those of Mourtartier and Chatila in [5], work on visual navigation by Ayache and Faugerras in [6] and several other alternative approaches including the more recent probabilistic and particle filter based methods in [9], [10] and [11].

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2.2. Probabilistic Solutions

2.2.1 A Bayesian Formulation

The SLAM problem can be formulated probabilistically in the following manner so that understanding the underlying concepts and reviewing of existing solutions to the problem in the literature are simplified. Let a vehicle navigating in an unknown environment be driven by a set of controls up to time $k-1$ denoted by $\mathbf{U}^{k-1} = [\mathbf{u}(0) \mathbf{u}(1) \mathbf{u}(2) \dots \mathbf{u}(k-1)]^T$ where $\mathbf{u}(k)$ is the control vector applied to the vehicle at time k . Also suppose that the set of measurements made by the sensors on board the vehicle up to time k be $\mathbf{Z}^k = [\mathbf{z}(1) \mathbf{z}(2) \dots \mathbf{z}(k)]^T$ where $\mathbf{z}(k)$ is the observation obtained by the sensors on board the vehicle at time k . Let the states of the vehicle and the map at time k be $\mathbf{x}_v(k)$ and $\mathbf{m}(k)$ respectively.

The recursive SLAM problem examines whether it is possible to calculate the joint probability of the vehicle state and the map state at time k given the states of the vehicle and the map at time $k-1$, the set of all past control inputs up to time $k-1$ and the set of all observations taken up to and including time k . In other words the SLAM problem is nothing but probabilistically calculating the joint posterior density function $P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^k, \mathbf{U}^{k-1})$ of $\mathbf{x}_v(k)$ and $\mathbf{m}(k)$ given $\mathbf{Z}^k$ and $\mathbf{U}^{k-1}$. Most approaches in the literature similar to the Kalman filter approach perform this calculation as a two step process consisting of a prediction step (known as the time update) and an updating step (known as the observation update).

In the prediction step a motion model is used to predict the vehicle and the map state vector at time k based on the previous map state and the vehicle state at time $k-1$ and the control inputs. Probabilistically, prediction step is equivalent to determining $P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1})$, which is the joint probability of obtaining $\mathbf{x}_v(k)$ and $\mathbf{m}(k)$ given $\mathbf{Z}^{k-1}$ and $\mathbf{U}^{k-1}$. Assuming a static map, the predicted joint posterior can be obtained

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as follows:

$$P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) = \int_{\mathbf{x}_v(k-1)} P(\mathbf{x}_v(k), \mathbf{m}(k), \mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) d\mathbf{x}_v(k-1) \quad (2.1)$$

Expanding $P(\mathbf{x}_v(k), \mathbf{m}(k), \mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1})$

$$\begin{aligned} &P(\mathbf{x}_v(k), \mathbf{m}(k), \mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) \\ &= P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{x}_v(k-1), \mathbf{m}(k-1), \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) P(\mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) \end{aligned} \quad (2.2)$$

Taking into account that the vehicle motion is independent of the measurements and the vehicle control inputs are independent of their previous values and substituting

$$P(\mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) = P(\mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-2}) \text{ in (2.1)}$$

$$\begin{aligned} &P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1}) \\ &= \int_{\mathbf{x}_v(k-1)} P(\mathbf{x}_v(k) | \mathbf{x}_v(k-1), \mathbf{u}(k-1)) P(\mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-2}) d\mathbf{x}_v(k-1) \end{aligned} \quad (2.3)$$

Equation (2.3) is the SLAM state prediction consisting of the motion model

$P(\mathbf{x}_v(k) | \mathbf{x}_v(k-1), \mathbf{u}(k-1))$ and the updated SLAM state at time $k-1$

$P(\mathbf{x}_v(k-1), \mathbf{m}(k-1) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-2})$. Using the Bayes theorem it is now possible to express

the updated vehicle and map state in the following manner.

$$P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^k, \mathbf{U}^{k-1}) = \frac{P(\mathbf{z}(k) | \mathbf{x}_v(k), \mathbf{m}(k)) P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1})}{P(\mathbf{z}(k) | \mathbf{Z}^{k-1}, \mathbf{U}^{k-1})} \quad (2.4)$$

The equation (2.4) provides a recursive update equation, which is the solution to the basic SLAM problem cast above. The entire process described can be illustrated in Figure 2.1.

This solution verifies the fact that the vehicle pose and the map are not conditionally independent [9] and also provides ground work for the process of reviewing the existing SLAM, MAL and RM algorithms discussed hereafter.

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Figure 2.1. The main elements of a recursive SLAM algorithm.

2.2.2 Estimation Theoretic Algorithms

The full Bayesian formulation presented in Section 2.2.1 is overshadowed by the huge complexity of representation of probability distributions and update operations. Therefore sub-optimal filters are in general sought in the full state estimation. These estimation theoretic algorithms either use the Extended Kalman Filter (EKF) [60], [61] and [62] or its variants as the main tool in several navigation and estimation problems including SLAM. The EKF estimator represents the state probability distributions by second order statistics, i.e the mean and the covariance. EKF based algorithms are adversely affected by their vulnerability to data association failures and inconsistent treatment of nonlinearities. However, they are widely used because of their uncomplicated treatment of vehicle and map uncertainties in the estimation process, simple recursion in the

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solution phase and availability of substantial set of techniques developed for tracking and aerospace applications, which are directly applicable in robotics. Most of these algorithms are feature based in the sense that they represent maps using sets of features, which are also known as landmarks. The EKF algorithm is still considered as the primary framework of most feature based SLAM algorithms and is also used as the basis of the material presented in this thesis.

The basic EKF SLAM framework of Smith, Self and Cheeseman [4] and Dissanayeke et al. [7] represents both the vehicle and landmark locations by absolute coordinates with reference to a global coordinate frame. A major attribute of this formulation is the map augmented SLAM state vector denoted by $\mathbf{X}(k)$. Let the vector consisting of concatenated landmark position vectors $\mathbf{m}(k)$ (known as the map), and the vehicle pose $\mathbf{x}_v(k)$ at time k be

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_v^T(k) & \mathbf{m}^T(k) \end{bmatrix}^T \quad (2.5)$$

where $\mathbf{m}(k) = [x_1(k) \ y_1(k) \ \dots \ x_n(k) \ y_n(k)]^T$ and $\mathbf{x}_v(k) = [x(k) \ y(k) \ \theta(k)]^T$ (assuming a 2D environmental model). Here $x_i(k)$ and $y_i(k)$ are x, y coordinates of the i^{th} feature location with reference to the global coordinate frame and $x(k)$, $y(k)$ and $\theta(k)$ denote the position coordinates and orientation of the vehicle (mid point of the rear axel) in the same coordinate frame. In general, the motion model ($P(\mathbf{x}_v(k) | \mathbf{x}_v(k-1), \mathbf{u}(k-1))$ in Section 2.2.1) of the vehicle is nonlinear and can be represented in closed form as

$$\mathbf{x}_v(k) = \mathbf{f}(\mathbf{x}_v(k-1), \mathbf{u}(k-1)) + \mathbf{v}(k) \quad (2.6)$$

where $\mathbf{u}(k-1) = [u(k-1) \ \gamma(k-1)]^T$ is the control input vector at time $k-1$ comprising of the speed input $u(k-1)$ and the steering angle input $\gamma(k-1)$ at time $k-1$ and $\mathbf{v}(k)$ is a

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zero mean temporally uncorrelated noise sequence with covariance matrix $\mathbf{Q}_v(k)$.

Assuming static landmarks, the process model of the map is:

$$\mathbf{m}(k) = \mathbf{m}(k-1) \quad (2.7)$$

The observation model $P(\mathbf{z}(k) | \mathbf{x}_v(k), \mathbf{m}(k))$ of Section 2.2 is represented by

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}(k)) + \mathbf{w}(k) \quad (2.8)$$

where $\mathbf{w}(k)$ is a zero mean temporally uncorrelated noise sequence with covariance matrix $\mathbf{R}(k)$. When the composite state covariance (including map and the vehicle pose) matrix is denoted by $\mathbf{P}(k)$ and the observation prediction is specified by $\hat{\mathbf{z}}(k)$, then the EKF predictor equations are as follows

$$\mathbf{X}(k | k-1) = \begin{bmatrix} \mathbf{f}(\mathbf{x}_v(k-1), \mathbf{u}(k-1)) \\ \mathbf{m}(k-1) \end{bmatrix} \quad (2.9)$$

$$\mathbf{P}(k | k-1) = \mathbf{F}\mathbf{P}(k-1 | k-1)\mathbf{F}^T + \mathbf{Q}(k) \quad (2.10)$$

$$\hat{\mathbf{z}}(k) = \mathbf{h}(\mathbf{x}_v(k | k-1), \mathbf{m}(k | k-1)) \quad (2.11)$$

where $\mathbf{F}$ is the Jacobian $(\partial \mathbf{X}(k | k-1) / \partial \mathbf{X})$ of the process model evaluated at time $k-1$ and $\mathbf{Q}(k)$ is the composite process noise covariance matrix. When observations $\mathbf{z}(k)$ are available at time k and after correct observation to feature associations are resolved using an appropriate data association algorithm, then the EKF update equations are as follows

$$\mathbf{e}(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k) \quad (2.12)$$

$$\mathbf{X}(k | k) = \mathbf{X}(k | k-1) + \mathbf{K}(k)\mathbf{e}(k) \quad (2.13)$$

$$\mathbf{P}(k | k) = \mathbf{P}(k | k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k) \quad (2.14)$$

where $\mathbf{e}(k)$, $\mathbf{S}(k)$, $\mathbf{H}$ and $\mathbf{K}(k) = \mathbf{P}(k | k-1)\mathbf{H}^T\mathbf{S}^{-1}(k)$ are the observation innovation, its covariance matrix, the Jacobian $(\partial \mathbf{h} / \partial \mathbf{X})$ and the Kalman gain with the usual notation.

Landmark track initiation, maintenance and deletion are carried out as in [7].

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2.2.3 Properties of the Estimation Theoretic SLAM Problem

Extended Kalman filter based estimation theoretic methods rely on number of assumptions, such as the accuracy of the state transition model, accuracy of sensor models, Gaussian noise and small size of time steps. Regardless of its popularity, feature based EKF formulation has several inherent shortcomings. Major shortcomings are its susceptibility to the data association errors, difficulty of describing highly skewed or multimodal vehicle and sensor models by the second order statistics, inconsistent treatment of nonlinearities and computational complexity (storage and time) due to the requirement of maintaining all the correlations (among map elements and the vehicle) explicitly in the state vector covariance matrix. To overcome these deficiencies in EKF formulation, several important solutions have been proposed over the last decade.

The general SLAM formulation described in equations (2.5) to (2.14) yields a time complexity of $O(N^3)$ and a storage complexity of $O(N^2)$, where N is the number of elements in the state vector. Regardless of the fact that the state vector contains N elements, only a small fraction of the state vector elements are observed at a time and thus it is possible to implement the above equations in several ways so as to achieve a computational complexity of $O(N^2)$ as in Knight [24] and Davison [25]. Even with $O(N^2)$ computational complexity it is almost impossible to perform SLAM for long duration missions in large unstructured outdoor areas consisting of possibly millions of features. To this end, there have been several solutions. For example, the suboptimal EKF based methods [24], [30] and [43], information filter based techniques [33] and [71], and sequential Monte-Carlo based methods [10] and [11] overcome the burden of complexity substantially.

The impact of the remaining challenges and other open issues and concerns of the localization and map building problem is discussed in the next sections of this chapter.

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Before investigating these problems and issues in detail, it is important to have the following fundamental measures of performance of the filtering algorithms in order to assess the goodness of SLAM algorithms.

- ♦ Conservativeness : An estimate is conservative if its true uncertainty is less than or equal to its estimated uncertainty.
- ♦ Consistency : An estimator of a nonrandom parameter is said to be consistent if the estimate (which is a random variable) converges to the true value in some stochastic sense (eg. in mean square sense)

2.3. Main Challenges

As described in the Introduction chapter and elaborated in the probabilistic formulation presented in the current chapter, the fundamental theoretical framework of the SLAM problem is well understood. As illustrated in Figure 2.1, the major elements of a typical SLAM algorithm can be identified as (1). Data Association or the correspondence, (2) Sensors and Perception, (3). Sensor and Vehicle Modeling and (4). Multi-sensor fusion.

Data association [7], [38], [39] and [40] is the process of establishing correspondence between the predictions of the observations based on the current knowledge and the true observations obtained in that particular instant. It is some times argued that the problems of data association mainly arise from the limitations of sensors. However, even if the sensors are nearly perfect, assigning a true measurement to a predicted measurement requires traversing a very large search space consisting of all the measurement and predicted measurement pairs. The problem gets even harder in the presence of sensor noise, environmental clutter, missed detections and false alarms. The combinatorial nature of the problem limits the development of statistically correct and computationally tractable data association algorithms. The impact of data association failures is adverse on the performance of SLAM and on many mobile robot navigation applications. In the

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context of SLAM, incorrect data association causes inconsistent localization results, poor quality maps and filter divergence.

Sensors [8], [67] and perception are responsible for providing the SLAM algorithms with sufficient information about the environment and the vehicle so as to update the knowledge of both the map and the vehicle. Vehicle modeling (Julier and Durrant-Whyte [26] and of Julier [27]) in general, is used to derive the belief of the vehicle and the environment at present instant given the knowledge of the previous state and the controls. Sensor modeling (Adams [51] and of Leonard and Durrant-Whyte [70]) is used to derive the measurement likelihoods given the belief of the vehicle and the map state. Although the random noise characteristics of sensors (including both exteroceptive and proprioceptive sensors) have received appropriate treatment in the context of SLAM [7], and [20], the effects of biases and drifts in the sensors and their compensation have largely been ignored. In most cases, either the effects of sensor biases and drifts have been ignored or compensated through an off-line correction method as in Kim and Sukkarieh [22] and Dissanayeke et al. [23]. However, the effect of sensor biases on SLAM performance is substantial [73] especially when using average sensors, which is typical of current SLAM applications. Sensor biases if not properly accounted for impact data association accuracy as well and results inconsistent localization estimates and map divergence.

Multi sensor fusion involves the combining of information from a number of sensors with the aim of obtaining an optimal or a better estimate of the state being observed. According to the information theory, the use of multiple sensors will always increase the information content available on the state being observed and thereby improving the estimator accuracy. Therefore it is crucial to have multiple sensors supporting SLAM in order to achieve robust and accurate localization and mapping results.

Presently there is a rich corpus of literature on SLAM. However, whatever the SLAM

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algorithm, the major issues of (1) Data association or the correspondence problem, (2) Treatment of biases and drifts sensors and (3) Related multi-sensor fusion issues hamper the task of deploying fully SLAM enabled platforms for extended periods of time in complex unstructured environments.

2.4. The Data Association Problem

Data association or the correspondence problem is often considered as one of the most difficult problems encountered in SLAM [4], [5] and [7]. Data association problem is difficult even in static environments and much more challenging in dynamic environments, which consists of moving objects. Almost every algorithm available today on state estimation has to deal with the correspondence problem in the form of either a maximum likelihood assignment or a correlation search to establish a match between elements of two data sets (in SLAM current features in the map and their current measurements). Large uncertainties in the vehicle pose, variable feature densities, dynamic objects, spurious measurements and clutter in the environment make the data association problem formidable. An efficient data association scheme must also aid feature or track initialization, maintenance, termination and management. Furthermore, in traversing large loops or cycles in the environment a mobile robot faces what is known as the “*Cycle Detection Problem*” or “*Loop Closing Problem*”, which is identifying its return to a previously mapped region. This problem introduces a significant overhead to the data association algorithm in the form of very large search space especially when mapping large areas. Therefore, the development of efficient and robust data association algorithms is a key area in robot localization and mapping, and will be addressed in detail in this thesis.

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2.4.1 Impact of Data Association on Estimation Theoretic Algorithms

All variants [7], [30], [31] and [43] of the estimation theoretic algorithms for SLAM require correct data association to be performed prior to state update operations. The most common choice of data association algorithms utilized in these approaches is the standard nearest neighbor (SNN) data association [7] scheme. The SNN uses a common statistical discriminator based on the normalized measurement innovation squared. This approach does not consider temporal and spatial correlations among features and observations in arriving at the data association decision. Data association inaccuracies give rise to large errors in the estimated states. The impact of data association errors is severe in the case of EKF/SLAM due to the propagating linearization errors and the misassociations invariably result in map inconsistency, and divergence [4], [5] and [7].

In the case of the relative filter of Csorba [28] and Neuman [29], the data association problem is so hard that the authors assume known data association in their filter implementations. In the absence of a globally known starting point, it is not possible to uniquely transform the relative map to an absolute one, which is essential for data association. Thus for actual implementations the method is impractical.

Other important estimation theoretic methods of interest are the extended information filters (EIF) [71] and sparse extended information filters (SEIF) [33]. The EIF SLAM represents posterior Gaussian in its information form [72], using the information matrix and the information form of the state vector. The data association in EIF is estimated as a joint posterior over augmented states and data association variables over an entire robot path. This means EIF SLAM processes an entire data set at a time to generate full SLAM posterior including the correspondence variables as well. The correspondence variables are in general found using a greedy search as in SNN (the same way it is carried out when using SNN in EKF/SLAM) at a particular calculation of the SLAM posterior. In general, it is relatively straightforward to undo past data associations in this framework. However,

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up to now there is no particular technique reported, which is able to determine whether a data association decision should be undone at a particular point in time. The main disadvantages of the EIF SLAM are that it is not an incremental algorithm and it is mostly suited to problems where one seeks a map from a data set of fixed size and the approach is fairly slow in implementation.

Thrun et al. [33] proposes an approximate constant time SLAM algorithm using sparsity of the information form of the Kalman filter known as SEIF. It uses the fact that even though the map covariance matrix is dense, information matrix or the inverse covariance matrix is sparse (only nearby features are linked through non zero elements) when the features in the map become more and more correlated. Despite the low time complexity, it has several problems. First the landmark states and vehicles states can be recovered only approximately in constant time and computing the exact values requires inverting the information matrix. Therefore, data association violates the constant time property of SEIF. Furthermore, the incremental greedy data association method, which is similar to SNN when used with SEIF, impairs data association failures due to spurious measurements.

2.4.2 Data Association in Sequential Monte Carlo Methods

Particle filters or sequential Monte Carlo methods [63], [64], [65] and [34] represent probability densities by random samples known as particles in such a manner that when the number of samples becomes very large the representation approaches the exact posterior probability density solution of the optimal Bayesian filter. Since the probability density functions contain all the information about the quantities that are being estimated, solution of the Bayesian filter is said to be complete and optimal. In the process of estimating dynamic states, optimal Bayesian filter tries to generate the posterior

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probability density function of the state based on the probabilistic motion models, measurement models and the prior probability distribution of the states. These methods have the advantage of proper treatment of nonlinearities and non-Gaussian noise characteristics in probabilistic models. In theory, particle filters are able to handle the full complexity of models as such they are able to solve the global localization problem (or Kidnapped Robot Problem) in MAL. Kalman filter based methods cannot recover from a large initial uncertainty even in map aided localization as the filter can diverge from the beginning. Whereas particle filters can represent the global uncertainty by a uniform prior probability density and has been experimentally verified to recover the robot pose [35].

However, robot localization is a low dimensional problem whereas SLAM is inherently a high dimensional problem in which the number of particles required to represent the state space increases almost exponentially with the dimension of the state space. In addition, the complexity of the data association problem for vast number of particles hampers the use of particle filters in real time SLAM. Recently Docet et al. [10] and Montemerlo et al. [11] used the important conditional independence properties that led to formulating the SLAM problem in an efficient manner. They exploited the fact that given the knowledge of the path of the robot, the individual feature locations become conditionally independent. This can be expressed in the following probabilistic form.

$$P(\mathbf{x}_v(k), \mathbf{m}(k) | \mathbf{Z}^k, \mathbf{U}^k) = P(\mathbf{x}_v(k) | \mathbf{Z}^k, \mathbf{U}^k) P(\mathbf{m}(k) | \mathbf{x}_v(k), \mathbf{Z}^k) \quad (2.15)$$

They represent the vehicle path state by a set of particles where on each particle a map is conditioned. Map states or the landmarks are represented by Kalman filters. The resulting approach known as the FastSLAM [11] is a form of Rao Blackwellized particle filters and has a complexity of $O(MN)$ where N is the number of features in the map and M is the number of particles used in the process. Finally the novelty in this method is the development of a tree based data structure that reduces the running time of FastSLAM to

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$O(M \log(N))$.

A significant development in FastSLAM is its data association function. In FastSLAM, data association is performed on a per particle basis by maximum likelihood method which is statistically correct unlike SNN data association in EKF SLAM due to the important conditional independence properties available to this problem. Per particle data associations per map in FastSLAM is conditioned on the entire vehicle path posterior. This means that each map attached with a particle can contain different data association hypotheses at the same time. Hence it is suggested that this is equivalent to maintaining multiple hypotheses over data associations at any particular instant and the particles with incorrect data association are expected to die away. Main drawbacks of this method are it is not trivial to integrate the different views of the map bourn by the particles into a single map and data association may be expensive as it must be performed for each particle independently.

The use of particle filters for the data association problem is also reported in the context of multi target tracking [74]. However the enormous complexity of representing the combinatorial hypothesis space by particles hampers the attempts of further development in this direction and the approach is yet to be proven in practice.

2.4.3 Treatment of Data Association in Other Algorithms

Other approaches of interest to the work presented in this thesis are, sum of Gaussians based SLAM [9] of Majumder, topological SLAM [36] and expectation maximization (EM) [35] based methods. Sum of Gaussians (SOG) approach represents map observations and vehicle state probability distributions by a sum of several Gaussian probability density functions. This approach is capable of representing complex unstructured environments in the sensor centric form and the data association is performed as a correlation search. The data association is achieved by reinforcement and

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inhibition of density functions [9]. However, the main problem in the data association of the SOG method is that data association decisions are hard and cannot be reversed at a latter stage.

The work of Choset and Nagatani [36] performs SLAM as a graph matching operation. The particular topological map of the environment is encoded in the form of a generalized Voronoi Graph. Thus the data association is performed as a topological graph matching in which nodes and adjacency relationships are matched together. However, it does not explicitly determine the robot's pose as in SLAM. Expectation maximization algorithms [35] and [37] are offline algorithms which consist of two steps known as the E Step (Expectation step) and the M Step (Maximization step). In the E Step, the posterior over all robot locations is calculated for a specified map and in the M Step the most likely map is determined based on those expected robot locations. Starting from an empty initial map, the EM algorithm generates an incrementally accurate map [35]. It has been demonstrated in Dellart [37] that data association can also be integrated into EM, however based on the conditional independence assumption discussed in Chapter 3. However, this approach is only suitable for offline SLAM implementations.

2.4.4 Categorizing Data Association Algorithms

Following the above discussion it is easy to see that data association problem in SLAM can be categorized into two based on the manner in which they arrive at data association decisions. Data association algorithms that use only the set of measurements (frame) received at the present time step are known as single frame methods or hard (see [39] and [40]) decision making methods. Data association algorithms that use several frames of measurement received at several time steps in the process of decision making are known as deferred decision making methods or soft decision making methods. In hard decision making methods decisions made with incomplete and ambiguous measurement

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information at an earlier time instant cannot be corrected when sufficient information is received at a later stage. Data association algorithms using deferred decision making methods on the other hand are capable of making soft data association decisions meaning that the decisions made at an earlier stage based on incomplete information can be revised at a later stage once more information is received. Furthermore, advantages of deferred decision making data association methods are that they can deal with spurious measurements and filter out measurement returns from unwanted dynamic objects and persistent clutter.

2.4.5 Hard Decision Making Methods

Most of the existing approaches to data association in robot localization applications such as in SLAM and MAL use only the current frame of measurements in the association process. Relevant examples in SLAM are standard nearest neighbor (SNN) [7], joint compatibility branch and bound (JCBB) [38] and combined constrained data association (CCDA) [20] methods. However, batch data association schemes such as Joint Compatibility Branch and Bound (JCBB) [38], Combined constrained data association (CCDA) [20] and Joint Assignment Matrix (JAM) [31] methods, which take into consideration groups of measurement to feature associations, are different from the SNN method. These methods consider the spatial correlations existing among the features of the SLAM state vector into consideration in decision making and therefore are much robust to increased vehicle uncertainty and measurement noise. Nevertheless, because of the single frame nature of all these data association algorithms they can't effectively filter out spurious measurements and returns due to dynamic objects.

Joint Probabilistic Data Association (JPDA) [39], [40] and [44] methods were proposed to provide a good solution to the clutter or spurious measurements and ambiguities of nearest neighbor method. JPDA associates all the measurements falling inside a suitably

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chosen validation region of a track to itself by a probabilistic weighting procedure and performs comparatively well when spurious measurements are relatively moderate. The downside is that it can be computationally prohibitive in terms of calculating weighting probabilities and the process may corrupt the feature recognition or discrimination information. Furthermore, JPDA is a single frame hard decision approach and in its standard form [39], does not explicitly provide a means of initiating tracks, which is vital for feature based map building applications.

2.4.6 Deferred Decision Making Methods

Use of multiple hypothesis tracking (MHT) method emerged as a solution to the problems encountered in the single frame data association algorithms. Multiple Hypotheses Tracking or MHT ([45], [40] and [46]) is the most structured and optimal approach employing deferred logic available for multi-target tracking and data association. Deferred logic schemes allow data association decisions to be delayed until a number of additional frames of measurements are received in successive scans so that incorrect associations made in the past can be corrected if necessary. For example, MHT defers association decisions in conflicting situations and forms a tree of all probable association hypotheses, which are then propagated through subsequent iterations in the belief that new information will most likely resolve conflicts if any. Therefore, MHT is capable of dealing with missed detections, spurious measurements and track initiations. However, the major drawback of MHT is that the hypothesis tree grows exponentially in time requiring exponential memory and computational resources making it impracticable for real-time implementation. A variant of MHT known as the “lazy data association” method [47] picks the most likely data association when a feature is observed as in a maximum likelihood data association method (MLDA) [7]. However, unlike in MLDA it stores and monitors the past and current data to detect whether different data associations (traversals

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of the hypothesis tree) can yield a map and path posterior of higher likelihood and hence revise the past data association decisions. However, use of all past correspondence variables in the method still makes it exponentially complex requiring heuristics and other ad-hoc strategies to prune the search space for real-time implementation. In [48], Nieto et al. proposes an MHT method for FastSLAM [11]. This method splits each particle representing a map in fastSLAM into further particles for keeping association hypotheses and the particles with wrong data associations are expected to die out in the re-sampling stage. The main disadvantage of this method is that the number of particles required for implementation of the algorithm can increase without bound causing computational problems.

2.4.7 Feature Initialization and Maintenance

Feature or track initialization and management is also a vital function associated with any data association algorithm suitable for SLAM. In any online SLAM algorithm, the map is first assumed empty. Once the new measurements are received they will be integrated into the map as features or new landmarks. However, since measurements are always uncertain and there may be outliers and spurious measurements among those received specialized feature initialization routines are required. The common practice [7], [20] is to initialize all the features into a tentative list using the inverse measurement model and then confirm the features to the permanent map after features are said to be stabilized (observed several times). The uncertainties of tentative features are also kept in a list. At the beginning of the SLAM algorithm the tentative list and the map are initialized to null. As and when features are observed they are associated with entries (if any) in the map using the data association algorithm. As such, any features observed but not associated with the features in the confirmed list are added to the tentative list of features. These features added to the tentative list are first checked for possible correspondence

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(association) with existing features in the tentative list using the data association algorithm. A record is made as to the number of times successful association is made with features observed for each feature in the tentative list. If this number exceeds a predetermined threshold, then the respective feature is considered sufficiently stable and thus is moved into the confirmed feature list. Thus a new feature is initiated. In the event of a successful association between an observed feature and a feature in the tentative list, the associated feature estimate and its covariance are updated using EKF. In the event of a successful association between an observed feature and a feature in the confirmed list, the observation is used to update the composite SLAM state vector (involving the feature vector and the vehicle pose).

However, this procedure of feature initiation has several shortcomings. If the tentative features are kept in the list independently from the map, the correlations that exist between the existing features in the map and the features in the tentative list through the vehicle location estimate are not accounted for. Thus the estimation of tentative features and their uncertainties are not properly reflected in this method and it is important that track initiation algorithms, which are statistically correct, be developed. It is possible to use the temporal correlations existing among measurements in track initiation and maintenance. This characteristic can be facilitated by the use of multiple frame data association schemes as the consideration of several measurement frames would enable them to consider temporal attributes of measurements received and to filter out the spurious measurements.

2.4.8 The Role of Feature Extraction in Data Association

Prior to any form of state estimation in robot localization and mapping, it is necessary to understand the environment in which the robot traverses. Various sensors are in general used to acquire information about the environment and about the vehicle in this context.

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In robot localization and mapping applications, representation of the environment can be done in two ways 1. Sensor centric approach, and 2. Feature extraction based approach. Raw sensor information is acquired and processed in its original form in the sensor centric approach as in scan matching [9], [15], [16], and [17]. The grid based method of Elfs [18] is a variant of the sensor centric approach where mapping is accomplished using a Bayesian updating method known as occupancy grids and localization is achieved using correlation of grid maps. It can be shown in both grid based and scan matching based methods that data association plays the major role.

In feature extraction based environmental representation [6] and [8], raw measurements are first processed by a feature extraction algorithm such as [12] in order to identify certain salient features and then used in the state estimation. The data association is then performed with the extracted features rather than with the raw measurements. Both the above architectures have certain advantages and disadvantages. The Sensor centric approaches do away with the world modeling and represent the world using raw sensor scans which is an advantage in representing highly unstructured complex environments having no proper parametric structure. Furthermore, it considers all the raw measurements in the state estimation. However, the inherent computational complexity of matching and estimating the state of each raw sensor point drastically limits the use of this approach and the balance is tilted in favor of the feature based environment representation and modeling, which enables a high degree of data compression and dimensionality reduction. For most indoor autonomous navigation applications, simplified geometric models such as lines, corners, circles [2] etc. are plenty and easy to model and detect. However, in an outdoor setting, automated perception is extremely difficult as simplified geometric objects are rare and there is a multitude of different objects such as trees, lamp posts, bushes, vegetation, rocks, buildings, etc., where modeling all of them is almost impractical. Therefore, the use of simplified clustering methods such as [57] and [58] is

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beneficial in outdoor feature extraction for SLAM applications. Under the general robot sensory perception, it is possible to interpret measurement vectors $\mathbf{Z}(k)$ in frame k as features or some form of data clusters out of a large data set of raw measurements perceived by robot's sensors.

Therefore the clustering or feature extraction process tries to reduce the dimension of $\mathbf{Z}(k)$ used in data association in an efficient and optimal manner. Had the raw measurements been used in matching process, a multiple frame data association algorithm should still have yielded accurate results but at the expense of increased computational complexity. However, if the feature extraction were not done optimally, then it would result in an increase in false alarms and missed detections in the data association process increasing its computational complexity.

Would it be possible to perform perfect feature extraction and would the features be sparsely distributed, the data association phase would become much simpler and accurate as would be seen by the less number of feature observations. However, this is not the case in real scenarios in complex unstructured environments. Clutter, occlusion, imperfect sensors and lack of perfect models of features complicate the feature extraction process and thereby the data association process. Conversely, it can be established that if the data association algorithm is robust we can develop robot localization and mapping algorithms with less reliance on efficient and effective feature extraction methods. It is argued in this work that paying less attention to feature extraction algorithms and using robust data association algorithms is much suited in autonomous navigation applications in complex and unstructured environments. Here we provide three examples from known estimation paradigms where correspondence plays the main role rather than the feature extraction (1. Cluster based feature extraction, 2. Iterative closest point algorithm, and 3. Occupancy grid based algorithms).

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Outdoor feature extraction is one of the main challenges in SLAM applications due to the complex and ever changing structure of the environment, which cannot be easily modeled. In contrast to the indoor environments, which are dense and resemble proper geometric primitives like curves and straight lines, outdoor environments are sparse in detail and lacks proper structure. Therefore, assuming that the data association algorithms may overcome the shortcomings of feature extraction simple clustering methods have been exploited successfully in outdoor robot navigation experiments in [7], [51] and [67]. These clustering methods consist of identifying the discontinuities in laser scans and grouping them based on a minimum distance criterion. The main rationale behind the working of this simple feature extraction strategy in SLAM is that only stable features sustain in the long run by the feature management and track maintenance function [7], of the data association algorithm used for SLAM. Cluster based feature extraction is robust to outliers and occlusions. However, it increases the data association complexity.

The iterative closest point (ICP) algorithm, first introduced in [66], is a clear example of the relationship between data association and feature extraction processes. Because of its simplicity, ICP algorithm has emerged as the dominant method for 2D and 3D point registration recently. The ICP algorithm, which only requires finding the closest point on a geometric entity to a given point, converges monotonically to the nearest local minimum of the selected distance metric. In simple terms ICP finds a relative translation and rotation to align two point sets with minimal disparity in an iterative manner by employing a reasonable initial guess of the translation and rotation terms. ICP based methods have been used by several researches [15], [16], [17] and [68] to recover locally consistent pose estimates in robot localization and mapping problems. The main attribute of the local scan matching approaches is that they do away with feature extraction. This can be equivalent to considering an entire laser scan at an instant as a composite feature and performing nearest neighbor data association point wise for each range-bearing

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measurement.

Grid based methods use a spatially distributed stochastic representation known as occupancy grids for robot perception and real world modeling. In particular, occupancy grid based methods use a multi-dimensional random field model that maintains estimates of the occupancy state of each cell in a spatial lattice [69]. Occupancy grid approaches generate dense world models, and therefore correlation based map matching approaches are common in robot localization using occupancy grids such as in [18] and [69]. These methods obviate any need for specific feature extraction method as such and use correlation techniques in recovering robot poses. A typical mapping algorithm encompasses a world map known as the global map and a local map, which consists of several local sensor views of the robot's sensor suit. These robot views taken from multiple points are integrated into the global map. The composition of these multiple views into a coherent global map requires an accurate estimate of the relative transformations between the global map and the local maps. The noise corrupted dead reckoning estimate is often inadequate to recover the relative transformation and therefore correlation based map matching approaches are applied to reduce motion uncertainty and ascertain precise map registration.

At the outset, it can be stated that the data association or the correspondence problem plays a prominent role in most feature extraction algorithms as well. However, just performing one extreme such as correspondence alone as in scan matching approaches and grid based methods is not quite advantageous in the context of SLAM or MAL because of the high computational complexity. On the other hand feature extraction based methods reduce the computational complexity of matching by reducing the dimensionality of the data before correspondence is established. Thus it can be concluded that the computational complexity of the data association and feature extraction problems in a robot localization and mapping algorithm can be reasonably managed by performing

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both functions in appropriate proportions, which has to be determined based on the available sensors and environmental characteristics.

2.4.9 Other Considerations

In traversing large loops, a robot is unable to recognize the loops in its path by standard SLAM algorithms due to large localization errors. This problem known as the Revisiting problem is one of the major difficulties a SLAM algorithm encounters in the long run. This is a result of accumulation of nonlinearities in the estimation process and the increase in uncertainty as a result. EKF/SLAM is adversely affected by this phenomenon because of the linearizations done in the process and observation models. The most successful method to get over with revisiting problem is the use of sub-maps instead of fully grown single map having all the feature states in it. Assuming independence of sub-maps, it is possible to confine the growth of nonlinearities into local regions where estimation is done independently from other sub-maps and thereby limiting the adverse effects in linearization. Decoupled stochastic mapping (DSM) [43] and Constrained local sub-map filter [42] are such attempts, which are proposed to overcome the linearization problems encountered in revisiting in SLAM.

In joint tracking and parameter estimation such as SLAM with sensor bias estimation, the effect of sensor bias errors can be significant as discussed in [41]. Errors due to bias estimation if not properly addressed impact both tracking accuracy and data association. Tractability is also a major concern in the data association. A truly tractable SLAM algorithm must in every respect be tractable for real time implementation. If tractability in state estimation provides significant overhead in data association and representational issues then it would not be feasible to use such algorithms in real world scenarios. Sparse extended information filter [33] and Relative Map Filter [29] are such examples where data association is not feasible.

2.5. Sensor Characteristics and Modeling Aspects

Using sensory information to locate robots in their environment is one of the most fundamental ways of providing them with autonomous capabilities. Sensors are in general used in mobile robotics to acquire a full or partial model of the world. Acquisition of a world model also supports other functions such as obstacle avoidance, threat assessment, surveillance, task planning, path planning and control.

2.5.1 Sensor Characteristics

Multiple sensors providing a combination of measurements are in general used in localization and map building applications as no single sensor modality alone can sufficiently capture all relevant details in a particular application. Most basic sensors used in SLAM and in many robot navigation algorithms are dead reckoning or proprioceptive sensors and these include odometers, encoders and inertial sensors. In a typical 2d ground based SLAM application, encoders are used to measure wheel rotation and steering angle, which are then used together with simple kinematic motion models of robots to estimate robot position and attitude with respect to a global coordinate frame. The main disadvantage of odometry based position estimation is the errors due to sensor misalignments, offsets and biases and uneven wheel radii and slippage of wheels. The errors in position estimates accumulate over time due to these errors as a result of integrating motion information over time. The method of dead reckoning is relatively straight forwards and can give a reasonable first estimate of the change in position. Some of the encoders [8] commonly available today are brush encoders, potentiometers, resolvers, optical encoders and magnetic encoders. Inertial sensors (eg. gyroscopes and accelerometers) are based on the principle that an object will remain in uniform motion unless disturbed by an external force. This force is measured and mathematically integrated by the inertial sensors to determine the change in velocity and position of the

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object with respect to an initial condition. Gyroscopes are used to measure angular rates, which are used as a basis for measuring heading. Today's optical gyroscopes (also known as laser gyros as in [1] and [8]) are lower in cost, accurate with little or no moving parts, and therefore require less maintenance. Accelerometers are also used as a basis for estimating velocities and positions. Biases, drifts noise in these sensors therefore can produce very large errors in the estimates over long periods of operation. Use of Inertial Navigation Systems (INS) [21], [22] and [23] and Inertial Measurement Units (IMU) as dead reckoning sensors is a mature area especially in aerospace applications. Nevertheless, the modeling of online bias and drift variations of inertial sensors are not addressed in the context of SLAM.

Exteroceptive sensors such as Laser Measurement Systems (LMS) [17], [20], millimeter wave RADAR (MMWR) [7], [19] and SONAR [2], [14] are used in SLAM applications as sensors (used for mapping), which provide external information about the environment.

These sensors represent the robot's environment either in the form of salient features or in the form of raw sensor scans. Extraction of external information of good quality (eg. range and resolution), and robustness to harsh environmental conditions are important metrics as far as the exteroceptive sensors are concerned. LMS is capable of providing high accuracy range and bearing sensing (5 cm, 0.25°) in the form of raw sensor scans and good salient features. MMWR is mostly used as a sensor providing raw sensor scans at the moment. High performance radars suitable for outdoor robotics is now available with high accuracies in the range of 12cm and narrow beams of the size 1° to 2° and for longer ranges [8]. MMWR is suitable under all weather conditions, which is not available with LMS. SONAR is rarely used in outdoor land based navigation applications. Vision is the most information rich, low cost and less power consuming passive sensor available for autonomous vehicle navigation applications today. However, extraction of visual features [12] in the form of landmarks, maps or object models is a difficult task because

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of the complexity in camera calibration and search for features which are often constituent parts of vision based algorithms. Camera calibration approaches are difficult in general because it is necessary to determine intrinsic camera parameters such as effective focal length, lens distortion parameters, image sensor size parameters and extrinsic camera parameters. There are also the extrinsic camera parameters that consist of three dimensional position and orientation of a camera coordinate system relative to the object or world coordinate system where the features are represented. Calibration process is also complicated because of the fact that calibration parameters are inter related and nonlinear.

2.5.2. Modeling Aspects

Sensor and vehicle modeling attempt to construct accurate probabilistic descriptions for measurements known as the sensor model and vehicle motion known as the vehicle model. The knowledge of the vehicle dynamics, kinematics and empirical findings are used to construct the vehicle model ([1] and [27]) in general. Similarly, developing an understanding of the sensed environment, the nature of the measurements provided by the sensor, sensor limitations and probabilistic understanding of the sensor in terms of measurement uncertainty and informativeness are used in sensor modeling [8]. Vehicle modeling also results in a significant impact to the navigation performance as reported in Julier and Durrant-Whyte [26] and Julier [27]. The work [26] and [27] also emphasize the disadvantages of over parameterization and exact modeling of vehicle process models. It also stresses the need to have a compromise between the exact modeling and observability of the model so as to achieve good localization results. In [8], sensor modeling is described as a major factor that determines the complexity in feature description, detection, and matching and positioning accuracy of robots. Although modeling of random sensor noise and its impacts on vehicle and sensor modeling are

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widely discussed, modeling of non-random sensor biases in the context of SLAM received less attention. The Chapter 5 of this thesis investigates the modeling aspects especially sensor biases on the performance of MAL and SLAM and their observability conditions.

2.6. Multi-Sensor Fusion Related Concerns

Combining several sensor measurements from multiple sensors in order to acquire information of greater quality and detail is known as sensor fusion. Even humans and animals perform a similar process in sensing and perception. Advantages of sensor fusion include reduced measurement uncertainty and improved observability of states. Techniques of data fusion have been drawn from diverse disciplines such as statistical estimation, signal processing and target tracking. Notable sensor fusion algorithms discussed in this thesis in the context of SLAM and navigation are EKF [4], [2], [7], [28], [29] and [31] evidence theory [18], [35] and [37] and Sequential Monte Carlo Methods [10] and [11].

Most approaches used in the SLAM literature utilize only one form of exteroceptive sensor (used in mapping) such as sonar [9] and [29], laser [7] and [20], vision [6] and [13] or RADAR [19]. However, several advantages can be achieved by using multi-sensors as discussed above. For example, in fusing vision with laser one can obtain features from vision (such as color, edges, corners etc.) even when the range features for laser are scarce such as in an open space. RADAR sensing can complement vision sensor's low visibility in bad weather. In this thesis, it is shown how monocular cameras can be used to complement laser range/bearing information in SLAM. Fusing information obtained by a monocular camera with laser in SLAM is challenging because a monocular camera can only provide bearing information. Feature initialization using bearing only measurements is hard as at least two readings obtained from measurements of the same feature must be

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obtained from two distinct robot locations are essential to successfully initialize the feature. This requires that proper association and track initiation algorithms be utilized in the process.

“What are the most suitable features for extraction?” and “What is the most efficient representation of the environment?” are two of the key questions raised in SLAM or any terrain aided navigation problem especially in outdoor unstructured environments. It is also arguable that one needs to keep track of only certain key features in a certain setting and there is no need in mapping and keeping track of every detail in the environment. Seeking only the salient features for observations is known as the Active Sensing Paradigm [13], [14] and [25]. There is still a huge potential in this area of exploration due to its advantages in providing good results by focusing on relatively few salient features (having high information content) rather than mapping all the observed features at the expense of high complexity in SLAM. The active control policies and fixating strategies still need development in this context.

In summary there are several open issues in SLAM related to multi sensor fusion, in particular to the fusion of vision and laser, which needs to be properly addressed in detail.

2.7. Conclusion

Although several algorithms have been reported of being able to successfully address the SLAM problem, most do not address every aspect of the problem in an effective manner. Some algorithms reduce the required computing resources for state estimation at the expense of data association and representation complexity. In particular the data association and related problems are not addressed in every possible perspective. For example, all the temporal and spatial correlations inherent to the SLAM problem are not very well taken care of. Treatment of nonlinearities and sensor biases are also not very well covered in most of the reported work in the context of SLAM. However, the

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theoretical discussions and reported work conjecture that these issues compromise SLAM performance considerably. Based on the discussion and examination of related work in SLAM the importance of the study of data association and related problems is well established beyond doubt. This thesis is mainly focused on the data association problem and the related problem; treatment of sensor biases in localization and mapping in outdoor environments.

Chapter 3

Sliding Window Data Association for SLAM Algorithms

3.1. Introduction

A typical data association algorithm resolves correspondence between the already known features with the observations received. This task is extremely complex as the measurements may consist of returns originating from true features as well as from false elements and new features or targets that are not known yet. Furthermore, some of the features or targets may not be visible after a short period and some may not generate measurements at a particular instance (missed detections) and again generate measurements after a few time steps. High measurement uncertainties and closely spaced features also add up to the ambiguities in the problem. Unwanted measurements (measurement clutter) known as false alarms may also be originated from unmodeled dynamic objects and from spurious measurements. As a result of the above and the combinatorial nature of the measurement to feature association, the problem of data association is complex in multi-sensor multi-target tracking [39], [40]. In the context of mobile robot navigation, the problem is exacerbated by the fact that the sensing platform is in motion. The use of deferred logic data association approaches hold promise in overcoming most of the deficiencies of the data association algorithms proposed for robot navigation applications. However, as reviewed in Chapter 2 most deferred logic data

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association approaches, which are nothing but variants of MHT are still computationally intensive. Therefore, it is more realistic to utilize a finite size window of measurement frames in the data association process instead of using all the measurement frames from the beginning. In this chapter, a finite size sliding window based multiple frame data association framework is introduced for robot localization and mapping.

The multiple frame data association framework utilizing a finite sliding window of measurement frames is shown to be superior in many aspects such as in respect of computational tractability, discriminating ability of spurious measurements and dynamic objects from true returns, and in support of feature initialization and management. The core attribute of this framework identified is the use of several measurement frames to determine the best associations for the current measurement frame. The optimal solution of the multi-frame data association methodology presented in this chapter is NP (Non-deterministically Polynomial) hard. In practice one may use sub-optimal solutions such as [40], [53], [55], [75] and [76].

This chapter details the formulations for multiple frame sliding window approach to the data association in SLAM. Sub-optimal solutions involving the linear programming, zero index auction algorithm and Lagrange Relaxation solutions are then presented. Feature initialization and maintenance are also described. The data association framework is extended for the Rao-Blackwellized particle filter based SLAM. The main disadvantage of the above algorithms is that they do not take the spatial correlations among the states in SLAM into account giving rise to problems in resolving dense feature measurement combinations. Towards this end a new batch data association algorithm, which is capable of addressing both spatial and temporal correlations in SLAM, is proposed. Data association in dynamic environments is also investigated.

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3.2 A Generalized Sliding Window Framework for Data Association

The bottlenecks of computational complexity and tractability of the deferred logic methods such as MHT and most of its variants [47] and [49] can be overcome by limiting the number of measurement frames to a finite size sliding window. A generalized sliding window based multiple measurement frame data association framework can thus be described as follows.

Figure 3.1. Multiple frame data association approach. The measurements (circles) obtained at each time interval in the window and feature states (ellipses) at the time intervals are shown.

3.2.1 Problem Formulation

Suppose N frames of measurement are obtained at time instances $k-N+1$, $k-N+2$, ..., $k-1$, and k over the period of time $[k-N+1, k]$. The sliding window defined at time k is spanned by the N measurement frames described above and is propagated forward one frame at each time step. Suppose that there are T states of interest, which are to be resolved with the observations received. In SLAM and robotics mapping these states are features of a map and in tracking they are referred to as targets or tracks. Hereafter, the term feature (in relation to feature based SLAM) is used to refer features, tracks or targets. The states of the autonomous vehicle is denoted by $\mathbf{x}_v$ (a vector consisting of the vehicle x , y coordinates and the heading) and the composite uncertainty of all the features

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and the vehicle are denoted by $\mathbf{P}$. The symbol $\mathbf{P}$ represents a notion of uncertainty and it may be a composite uncertainty matrix (as in SLAM or MAL) or may be a set of independent covariance matrices in the cases of robotics mapping and multi-target tracking.

Let the set of map features available and updated at time $k-N$ be $\Gamma = \{\tau_t \mid t = 1, 2, \dots, T\}$.

The problem of multiple frame data association is to assign appropriate measurement combinations of the N frames of measurement to the T features in an optimal and efficient manner subject to certain constraints. Let the set of measurements obtained in frame l be $\mathbf{Z}(l) = \{\mathbf{z}_{i_l} \mid i_l = 0, 1, 2, \dots, n_l\}$ where n_l is the actual number of measurements and $i_l = 0$ corresponds to a dummy or fictitious measurement used to accommodate any missed detections of features. Let Ω denote a specific but complete assignment of the T features to the measurements in N consecutive measurement frames. In other words, Ω is a partition of the product space $\Theta = \mathbf{Z}(k-N+1) \times \mathbf{Z}(k-N+2) \times \dots \times \mathbf{Z}(k) \times \Gamma$ of measurements and features. The solution to the multiple frame data association problem can now be formulated in general as a discrete optimization problem minimizing an appropriately chosen cost function C_Θ relating the measurements in N frames to T features subject to a set of constraints.

$$\eta = \arg \min_{\eta} C_\Theta(\Gamma, \mathbf{Z}(k-N+1), \dots, \mathbf{Z}(k-1), \mathbf{Z}(k), \mathbf{x}_v, \mathbf{P}, \Theta_\eta) \quad (3.1)$$

where Θ_η consists of binary variables $\eta(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k) \in \{0, 1\}$ such that $t \in \{1, 2, \dots, T\}$ and $i_{k-m} \in \{0, 1, 2, \dots, n_{k-m}\}$ for $m = N-1, N-2, \dots, 1, 0\}$. Here, $\eta = 1$ when the sequence of measurements $\mathbf{z}_{i_{k-N+1}}, \mathbf{z}_{i_{k-N+2}}, \dots, \mathbf{z}_{i_k}$ from the N successive frames are associated with the feature τ_t and $\eta = 0$ otherwise. The problem (3.1) is NP hard. Therefore, enumeration of all the possible combinations is required to obtain a solution. Thus, in general sub-optimal solutions are sought.

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The most striking feature of this sliding window formulation is its generality in explaining most available data association, sensor fusion and feature extraction algorithms. As an example, if the cost function used in this general framework is the distance between T features and measurements in each frame the result is a nonlinear optimization problem resembling N frame nearest neighbor data association. The Standard Nearest Neighbor data association scheme is a method, which uses a single frame of measurement and determines the feature to measurement association purely based on the minimum Euclidian distance between the features and the measurements. If the likelihood of assigning T features to measurements in N frames is used in the above formulation as the cost function, the data association problem reduces to a nonlinear 0-1 integer program resembling maximum likelihood assignment. When the number of features is known a priori, the above partitioning problem is similar to a supervised clustering problem. When the number of features is unknown, it is similar to an unsupervised clustering problem. If we use all the measurement frames available from the beginning up to the present time, the results of the algorithm will be equivalent to the multiple hypothesis tracking (MHT) algorithm [46]. However, because of the computational considerations, the advantages of the sliding window framework presented are more prominent than its competitor the MHT. Furthermore, the design issues of the cost function C_o and the inclusion of sensor origin uncertainty (the uncertainty of the moving platform) result in the approach presented for robot navigation applications complicated and clearly different from the sliding window approaches reported in the target tracking literature such as [40] and [49].

3.2.2 Constraining the Search Space

A straightforward implementation based on the sliding window framework presented in the last section is undoubtedly going to be computationally intensive as a result of huge dimensionality of the combinatorial search space and the presence of outliers and

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occlusions in the observed data. However, there are several sets of constraints, which can be imposed to limit the search space of the problem in general. Some of them are (1) Order Constraint [50] and [51], (2) Rigidity Constraint, (3) Uniqueness Constraint, (4) Visibility Constraint, (5) Proximity Constraint and (6) Epipolar Constraint in the case of computer vision. Order constraint imposes the order of the measurements received to constrain the search. Rigidity constraint assumes that the scene is static. The static environment assumption is utilized in most robot localization algorithms as in [2] and [7]. The uniqueness constraint assumes that a measurement can be associated with one feature and a feature can return only one measurement at most. Visibility constraint ensures that measurements are within the range of the sensor concerned. Proximity constraints on the other hand make sure that the measurements are in the close proximity [31] of the features. In addition, sparsity and the structure [52] of the problem are also used to further constrain the search space. The structure of the problem is also utilized in providing sub-optimal solutions such as greedy algorithms [53], simulated annealing [54], Lagrange relaxation [55], and randomized search. [56]. In order to synthesize a more tractable algorithm, one may now use the following uniqueness constraints.

1. Each measurement except dummy measurements can be assigned to only one or no features in the map. However, dummy measurements can be assigned to more than one feature in a frame as there can be several undetected features in a particular measurement frame. This is known as the single source constraint for measurements.

$$\sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_k) \leq 1, i_{k-N+1} = 1, 2, \dots, n_{k-N+1} \quad (3.2)$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-m-1}=0}^{n_{k-m-1}} \sum_{i_{k-m}=0}^{n_{k-m}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \leq 1 \quad (3.3)$$

$$i_{k-m} = 1, 2, \dots, n_{k-m}, m = N-2, N-3, \dots, 1$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-1}=0}^{n_{k-1}} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_k) \leq 1, i_k = 1, 2, \dots, n_k \quad (3.4)$$

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2. Each feature results in only one measurement in one measurement frame and hence the single return constraint:

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_k=0}^{n_k} \eta(t, i_{k-N+1}, \dots, i_k) = 1, \quad t = 1, 2, \dots, T \quad (3.5)$$

3. Maximum missed detections of features in a measurement frame cannot exceed the number of existing features. Therefore, the possibility of obtaining a missed detection for each feature in every measurement frame is considered. This yields the following constraints on the maximum number of dummy measurements:

$$\sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, 0, \dots, i_k) \leq T \quad (3.6)$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-m+1}=0}^{n_{k-m+1}} \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_{k-m+1}, 0, i_{k-m+1}, \dots, i_k) \leq T, \quad m = N-2, N-3, \dots, 1 \quad (3.7)$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-1}=0}^{n_{k-1}} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_{k-1}, 0) \leq T \quad (3.8)$$

4. Since η is an association variable, η must be either 0 or 1. Thus, the following 0-1 integer constraints are obtained.

$$\eta(t, i_{k-N+1}, \dots, i_k) = \{0, 1\}, \quad \forall i_{k-N+1}, \dots, i_k, \quad t = 1, 2, \dots, T \quad (3.9)$$

3.3. Measurement Independence and Correlations

The effects of including the vehicle location estimate in the measurement model make the vehicle uncertainty an integral part of the measurement uncertainty in robot navigation problems. The treatment of data association algorithms [38] is thus different in the contexts of Multi-target tracking, Robotics Mapping, Map aided robot localization and SLAM problems.

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3.3.1 Multi-Target Tracking

In multi-target tracking [40], the tracking sensor (or sensors) usually a range-bearing one, which is stationary, is used to measure the location and speed of a number of known targets moving according to known kinematic models. In the tracking literature the basic tracking problem considers that the location of the sensor is known with absolute certainty. Under these circumstances, the measurements of the moving targets observed by the sensor are assumed to be independent of each other. In other words, the measurement errors of the different targets can be assumed uncorrelated. This is an important assumption, which is often correct in multi-target tracking applications such as air traffic control and surveillance. However, this may not be the case if the targets move as a group or some form of sensor bias term causes the measurements to be correlated.

Let the moving targets and the sensor location in the case of multi-target tracking be denoted by $\Gamma = \tau_1, \tau_2, \dots, \tau_n$ and $\mathbf{x}_s$. The total likelihood $P(\mathbf{z} | \mathbf{x}_s, \Gamma)$ of a measurement $\mathbf{z}$ (or the probability of obtaining a measurement given the current state of all the targets and the sensor location) is then equal to the product of the individual likelihoods of a measurement given the state of the targets taken one at a time. In other words,

$$P(\mathbf{z} | \mathbf{x}_s, \Gamma) = \prod_{i=1}^n P(\mathbf{z} | \mathbf{x}_s, \tau_i) \quad (3.10)$$

This likelihood function can be used as a cost function in the sliding window data association framework presented in Section 3.2. Formulation of most data association approaches in multi-target tracking such as SNN [39], probabilistic data association (PDA) [39], joint probabilistic data association (JPDA) [40] and MHT [45] assume the above property of the likelihood function.

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3.3.2 Robotics Mapping

Robotics mapping [35] and [46] is defined as acquiring some form of a map of an environment by an autonomous vehicle moving on a predefined or known trajectory. Sensors used in robotics mapping are on board the moving vehicle. In this context, since the vehicle trajectory is known a priori the uncertainty of the sensor location can be assumed to be negligible. In feature based robotics mapping, the features extracted (or observed) at every instance are associated with the existing features and used for state update. Measurements that are not associated with any of the existing features are channeled through a feature initialization algorithm [46] to initialize new map features whose states are added to the map for estimation. Since the sensor location is always certain, the total likelihood of observing a map feature in robotics mapping is also equal to the product of individual likelihoods of observations given a map feature at a time.

Let $\mathbf{L}_i = [x_i \ y_i]^T$ for all $i = 1, 2, \dots, n$, $\mathbf{m}$ and $\mathbf{x}_v$ denote the existing features of a map, the map, and the vehicle location at a time instant then the following expression for total likelihood $P(\mathbf{z} | \mathbf{x}_v, \mathbf{m})$ holds as in (3.10).

$$P(\mathbf{z} | \mathbf{x}_v, \mathbf{m}) = \prod_{i=1}^n P(\mathbf{z} | \mathbf{x}_v, \mathbf{L}_i) \quad (3.11)$$

The implication of the expression is that any existing data association algorithm of multi-target tracking can also be applied to robotics mapping.

3.3.3 Map Aided Localization

The map aided localization (MAL) [2] and [3], problem is slightly different from the multi-target tracking and robotics mapping problems in respect of correlation of measurements and their uncertainties. In MAL, a robot platform is driven in an unknown trajectory. The problem in MAL is to determine the trajectory of the robot using measurements to known landmarks. Thus, the observations made by the sensors are not

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just subjected to the uncertainties of the sensors but also to the uncertainties of the moving platform. Hence the measurements made by the sensors on board a platform performing MAL at any particular instant are correlated.

Let the measurement equation of a sensor on board a vehicle performing MAL be

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}_s) + \mathbf{v}_i(k) \quad (3.12)$$

where $\mathbf{x}_v(k)$, $\mathbf{m}_s$ and $\mathbf{v}_i(k) \sim N(\mathbf{0}, \mathbf{R}) \forall i$ denote the state of the vehicle, the stored map and the temporally uncorrelated noise sequence respectively. Let the sensor now obtain two measurements $\mathbf{z}_1(k)$ and $\mathbf{z}_2(k)$ of the features $\mathbf{L}_1$ and $\mathbf{L}_2$ in the map at the same time instant. Suppose that the vehicle state uncertainty, individual innovations of the measurements and the composite measurement innovation are $\mathbf{P}_v(k)$, $\mathbf{e}_1(k)$, $\mathbf{e}_2(k)$ and $\mathbf{e}(k)$ then we have the following expressions for $\mathbf{e}(k)$ and the composite innovation covariance $\mathbf{P}_e(k)$.

$$\mathbf{e}(k) = \begin{bmatrix} \mathbf{z}_1(k) - \mathbf{h}(\mathbf{x}_v(k), \mathbf{L}_1) \\ \mathbf{z}_2(k) - \mathbf{h}(\mathbf{x}_v(k), \mathbf{L}_2) \end{bmatrix} \quad (3.13)$$

$$\mathbf{P}_e(k) = \begin{bmatrix} (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1} \mathbf{P}_v(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1}^T + \mathbf{R} & (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1} \mathbf{P}_v(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2}^T \\ (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2} \mathbf{P}_v(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1}^T & (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2} \mathbf{P}_v(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2}^T + \mathbf{R} \end{bmatrix} \quad (3.14)$$

where $(\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1}$ and $(\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2}$ are Jacobians of the vehicle state with respect to measurement function when observing features $\mathbf{L}_1$ and $\mathbf{L}_2$ respectively. From (3.14) it is clear that the two measurements are in fact correlated. As a result, the measurement independence assumptions (3.10) and (3.11) are not directly applicable in the case of MAL.

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3.3.4 The SLAM Problem

The SLAM problem [4], [5], [7], is characterized by the strong correlations that exist among map features and the vehicle. Further, the measurements are also characterized by the vehicle location uncertainty as in MAL. Hence, the measurements made by the sensors on board a vehicle are also correlated in the context of SLAM. Using the previous notation let the measurement equation of a sensor on board of a vehicle performing SLAM be given by

$$\mathbf{z}(k) = \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}(k)) + \mathbf{v}_i(k) \quad (3.15)$$

Suppose that the composite state vector given by $\mathbf{X}(k) = [\mathbf{x}_v^T(k) \mathbf{m}^T]^T$ has a composite covariance matrix $\mathbf{P}(k)$. When observing two features $\mathbf{L}_1(k)$ and $\mathbf{L}_2(k)$ (time varying with uncertainties) simultaneously, the composite innovation and its covariance are given by

$$\mathbf{e}(k) = \begin{bmatrix} \mathbf{z}_1(k) - \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}(k)) \\ \mathbf{z}_2(k) - \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}(k)) \end{bmatrix} \quad (3.16)$$

$$\mathbf{P}_e(k) = \begin{bmatrix} (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1} \mathbf{P}(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1}^T + \mathbf{R} & (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1} \mathbf{P}(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2}^T \\ (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2} \mathbf{P}(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_1}^T & (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2} \mathbf{P}(k) (\partial \mathbf{h} / \partial \mathbf{x}_v)_{\mathbf{L}_2}^T + \mathbf{R} \end{bmatrix} \quad (3.17)$$

where $(\partial \mathbf{h} / \partial \mathbf{X})_{\mathbf{L}_1}$ and $(\partial \mathbf{h} / \partial \mathbf{X})_{\mathbf{L}_2}$ are Jacobians of the composite state with respect to measurement function when observing features $\mathbf{L}_1(k)$ and $\mathbf{L}_2(k)$ respectively. From (3.17) it is clear that the two measurements are in fact correlated. Therefore, the measurement independence assumptions (3.10) and (3.11) in the strict sense are not applicable in the case of SLAM

3.3.5 Conditional Independence Assumption in Robotics and Its Implications

The notion of conditional independence [37] and [71] is central to the synthesis of

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probabilistic algorithms in robotics. Let A , B and C be random variables, then the following expression is known as the conditional independence relation

$$P(A, B | C) = P(A | C)P(B | C) \quad (3.18)$$

where $P(A, B | C)$, $P(A | C)$ and $P(B | C)$ denote the joint probability of events A and B given the event C , the probability of event A given the event C and the probability of event B given the event C respectively. In other words, given that the event C has occurred the occurrences of the two events A and B are independent. This does not necessarily imply that A and B independent in an absolute sense.

Many probabilistic robotics algorithms, in particular map building and localization algorithms [7], [15], [16], [37], [44], [47], [68], [77] and [78] assume conditional independence among features of a map. i.e.

$$P(\mathbf{z}(k) | \mathbf{x}_v(k), \mathbf{m}(k)) = \prod_{i=1}^n P(\mathbf{z}_i(k) | \mathbf{x}_v(k), \mathbf{m}(k)) \quad (3.19)$$

The above measurements are not independent in an absolute sense although the conditional independence of the feature measurements is assumed. The independence of measurements on vehicle state is useful for simplicity, mathematical tractability and computational efficiency. The theoretical justification of its applicability is as follows. Whether it is MAL or SLAM, it may be noted that the measurements are dependent on a common underlying vehicle state. In other words it is reasonable to assume that the underlying vehicle state is the only thing in common between the measurements sources. Therefore, it is reasonable to assume that the measurements conditioned on the common underlying vehicle state are independent. The abundance of literature on probabilistic robotics, including the seminal works in SLAM [7], [47] and [77], navigation [2], [15], [16] and [44], and structure from motion [37], [78], [79], and [80] exploit this assumption of conditional independence in the synthesis of algorithms. Practically, in the literature this state of affairs is verified and validated through proper simulations and actual

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implementations.

A brief review of the work in robotics, which utilizes conditional independence assumption, can be outlined as follows. In [7] pp. 241 Dissanayake and Durrant-Whyte et al. described a maximum likelihood data association and track maintenance algorithm for SLAM using the conditional measurement independence assumption and presented results obtained by radar observations. In [15] Section 3 Lu and Milios used independence assumption in aligning correlated range scan measurements obtained from uncertain robot locations in globally consistent range scan alignment. They have derived a simplified Mahalanobis distance based measure for minimization in order to calculate the set of all robot poses, which the robot has traversed in an optimal manner. In Folkesson and Christensen [77], an energy minimization based SLAM framework was developed and implementation described with acceptable results. In this approach Folkesson and Christensen used measurements obtained from a vehicle performing SLAM to be independent and derived energy terms for range bearing measurements and data association as detailed in Section IV pp. 384-386 of [39].

In [15], Dezert and Bar-Shalom developed a Joint Probabilistic Data Association (JPDA) method for autonomous vehicle navigation. It addressed the data association problem of robot localization given a priori map of known landmarks. In formulating the JPDA method, the work [15] assumed that measurements are conditionally independent (pp 1277-1281), although they are correlated and produced results verifying their framework. In [47] the MHT method termed as lazy data association assumed measurement independence. In calculating the likelihoods of individual hypotheses in the hypotheses tree, the work in [49] does not consider the dependencies among other feature to measurements. In [68], Sebastian Thrun implicitly assumes measurement independence for his formulation of aligning all the range scans over a time period to produce a globally consistent map. In [78] McLauchlan et al. and in [37] Dellart assume conditional

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independence of measurements obtained at several camera poses as independent in formulating several algorithms of structure and motion. Further, it is well known that over the decades photogrammetry community assumed the conditional independence of measurements in various structure from motion algorithms [78], [79] and [80].

As an aside it may be useful to look into recursive Bayesian estimation and data fusion. In recursive Bayesian estimation it is also necessary to make the conditional independence assumption without which it is not possible to update the propagated state estimate upon receipt of new measurements. Although strictly speaking, a measurement made from a moving platform is dependent on the state of the platform. It may be noted that EKF based SLAM is based on this notion (as described in Chapter 2). It assumes that the measurements are conditionally independent given the state of the process, although they are dependent over the common prediction. If this assumption is not valid the EKF SLAM based on Bayesian estimation is also not valid as measurements over several frames are correlated by the motion model of the vehicle. Furthermore, it is important to note that in sequential update of position estimates in particular when measurements are received asynchronously and are updated asynchronously as is customary in SLAM and sensor fusion algorithms the same assumption is made. i.e. although the measurements are dependent, the fact that they are inter-dependent through the underlying position of the vehicle state, the measurements are conditionally independent. In a similar vain, the conditional measurement independence is assumed to hold in the derivations that follow.

3.4. Algorithms Using Measurement Independence

3.4.1 An N Frame Likelihood Function

The conditional measurement independence assumption is exploited in this section to simplify the likelihood function of the SLAM measurement model (3.15) and to derive a simplified cost function for the generalized sliding window framework presented in (3.1).

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Using the similar notation utilized in Section 3.2, let $\mathbf{Z}(k-N+1)$, $\mathbf{Z}(k-N+2)$, ..., $\mathbf{Z}(k-1)$ and $\mathbf{Z}(k)$ be the sets of measurement obtained by a vehicle moving in an environment in N frames of measurement in the time interval $[k-N+1, k]$. Let $\mathbf{Z}(l) = \{z_{i_l} \mid i_l = 0, 1, 2, \dots, n_l\}$ and $l = k-N+1, \dots, k$ where n_l is the number of actual measurements obtained in frame l and $i_l = 0$ is the dummy or fictitious measurement used to accommodate any missed detections of features in frame l .

Let $\mathbf{\Omega}$ be a partition of assigning T existing features $\mathbf{\Gamma} = \{\tau_t \mid t = 1, 2, \dots, T\}$ to available measurements in N consecutive measurement frames. In other words, $\mathbf{\Omega}$ is a partition of the product space of measurements and features $\Theta = \mathbf{Z}(k-N+1) \times \mathbf{Z}(k-N+2) \times \dots \times \mathbf{Z}(k) \times \mathbf{\Gamma}$. The solution to the multiple frame data association problem can now be formulated in general as a discrete optimization problem minimizing a cost function $C_{\Theta}(\mathbf{\Omega})$ relating the measurements in N frames to T features subject to an appropriately chosen set of constraints. Let $\Lambda(\mathbf{\Omega})$ be the joint likelihood of the partition selected, then

$$\Lambda(\mathbf{\Omega}) = \Lambda_{true}(\mathbf{\Omega})\Lambda_{false}(\mathbf{\Omega}) \quad (3.20)$$

where $\Lambda_{true}(\mathbf{\Omega})$ is the joint likelihood of measurements associated with true features in $\mathbf{\Omega}$ and $\Lambda_{false}(\mathbf{\Omega})$ is the joint likelihood of measurements associating with spurious measurements and other unwanted objects in $\mathbf{\Omega}$. If it is assumed that the spurious measurements (or in general clutter) are uniformly distributed in the operating region having an area V_s (in 2D case) and that there are t_D^{k-N+1} , t_D^{k-N+2} , ..., t_D^{k-1} , t_D^k detected features in the measurement frames $k-N+1$, $k-N+2$, ..., $k-1$ and k respectively, then $\Lambda_{false}(\mathbf{\Omega})$ is

$$\Lambda_{false}(\mathbf{\Omega}) = (1/V_s)^{n_{k-N+1}-T_D^{k-N+1}} \dots (1/V_s)^{n_{k-1}-T_D^{k-1}} (1/V_s)^{n_k-T_D^k} \quad (3.21)$$

Given a feature τ_t the likelihood of a measurement $z_{i_{k-N+1}}$ in frame $k-N+1$ (the first frame

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in the sequence of N frames) is represented as $L(\mathbf{z}_{i_{k-N+1}} | \boldsymbol{\tau}_t)$. The likelihood of a measurement $\mathbf{z}_{i_{k-N+2}}$ in the frame $k-N+2$ given a feature $\boldsymbol{\tau}_t$ and a measurement $\mathbf{z}_{i_{k-N+1}}$ in the previous frame $k-N+1$ is then $L(\mathbf{z}_{i_{k-N+2}} | \mathbf{z}_{i_{k-N+1}}, \boldsymbol{\tau}_t)$. In a similar manner, we could express the likelihood of a measurement $\mathbf{z}_{i_{k-m}}$ in frame $k-m$ given the feature $\boldsymbol{\tau}_t$ and the measurements in all the frames from $k-N+1$ leading up to the $k-m$ th frame as $L(\mathbf{z}_{i_{k-m}} | \mathbf{z}_{i_{k-N+1}}, \dots, \mathbf{z}_{i_{k-m+1}}, \boldsymbol{\tau}_t)$. If the detection probability of a feature is assumed constant and equal to p_d , then the joint likelihood $\Lambda_{true}(\boldsymbol{\Omega})$ of measurements associating with features in $\boldsymbol{\Omega}$ can now be expressed as

$$\Lambda_{true}(\boldsymbol{\Omega}) = \prod_{\forall \boldsymbol{\tau}_t} (p_d L(\mathbf{z}_{i_{k-N+1}} | \boldsymbol{\tau}_t))^{e^{k-N+1}(i_{k-N+1})} (1-p_d)^{1-e^{k-N+1}(i_{k-N+1})} (p_d L(\mathbf{z}_{i_{k-N+2}} | \mathbf{z}_{i_{k-N+1}}, \boldsymbol{\tau}_t))^{e^{k-N+2}(i_{k-N+2})} \dots (1-p_d)^{1-e^{k-N+2}(i_{k-N+2})} (p_d L(\mathbf{z}_{i_k} | \mathbf{z}_{i_{k-N+1}}, \dots, \mathbf{z}_{i_{k-1}}, \boldsymbol{\tau}_t))^{e^k(i_k)} (1-p_d)^{1-e^k(i_k)} \quad (3.22)$$

where

$$e^{k-m}(i_m) = \begin{cases} 1 & \text{if } i_{k-m} \neq 0 \\ 0 & \text{if } i_{k-m} = 0 \end{cases} \quad \text{for all } m = N-1, N-2, \dots, 1, 0. \quad (3.23)$$

Substituting (3.21) and (3.22) in (3.20) yields $\Lambda(\boldsymbol{\Omega})$. A normalized likelihood of $\boldsymbol{\Omega}$ denoted by $\Lambda_N(\boldsymbol{\Omega})$ can now be defined as follows.

$$\Lambda_N(\boldsymbol{\Omega}) = \Lambda(\boldsymbol{\Omega}) / (V_s^{-n_k} V_s^{-n_{k+1}} \dots V_s^{-n_{k+N-1}}) \quad (3.24)$$

The data association problem can now be expressed as searching for $\boldsymbol{\Omega}$ that minimizes the cost function $C_{\ominus}$ equal to the minus log likelihood value of $\Lambda_N(\boldsymbol{\Omega})$

$$C_{\ominus}(\boldsymbol{\Omega}) = -\ln(\Lambda_N(\boldsymbol{\Omega})) \quad (3.25)$$

It is easy to see that searching for $\boldsymbol{\Omega}$ that minimizes $C_{\ominus}(\boldsymbol{\Omega})$ is equivalent to the assignment of one measurement from each of the N frames $k-N+1, k-N+2, \dots, k-1, k$ to each feature $\boldsymbol{\tau}_t$ ($\boldsymbol{\tau}_t, t=1, 2, \dots, T$) with an

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associated cost of $c(t, i_{k-N+1}, \dots, i_{k-1}, i_k)$ (The cost of associating feature τ_t with the measurement sequence $\mathbf{z}_{i_{k-N+1}}, \mathbf{z}_{i_{k-N+2}}, \dots, \mathbf{z}_{i_k}$ in the frames $k-N+1, k-N+2, \dots, \text{frame } k-1$, and frame k respectively), subject to an appropriate set of constraints. Thus, the problem is one of multi-dimensional assignment. This can be accomplished as follows. Let an association variable $\eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k)$ be defined in such a way that $\eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) = 1$ when the measurement sequence $\mathbf{z}_{i_{k-N+1}}, \mathbf{z}_{i_{k-N+2}}, \dots, \mathbf{z}_{i_k}$ in the frames $k-N+1, k-N+2, \dots, k-1$ and k respectively are associated with the feature τ_t and $\eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) = 0$ otherwise. Thus, the cost function can be expressed as follows.

$$C_{\Theta}(\mathbf{\Omega}) = \sum_{\mathbf{\Omega}} c(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \quad (3.26)$$

3.4.2 An Integer Programming Solution

This data association problem based on the conditional independence assumption can now be expressed as a search for integer variables $\eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k)$ for all $t, i_{k-N+1}, \dots, i_{k-1}$ and i_k . The search for $\mathbf{\Omega}$ that minimizes $C_{\Theta}(\mathbf{\Omega})$ can therefore be obtained by solving the following non-linear 0-1 integer programming problem.

$$\text{Minimize} \quad \sum_{i_{k-N+1}=0}^{n_{k-N+1}} \sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T c(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \quad (3.27)$$

Subject to the following uniqueness constraints

1). Single source constraints

$$\sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_k) \leq 1, i_{k-N+1} = 1, 2, \dots, n_{k-N+1} \quad (3.28)$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-m-1}=0}^{n_{k-m-1}} \sum_{i_{k-m+1}=0}^{n_{k-m+1}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \leq 1$$

$$i_{k-m} = 1, 2, \dots, n_{k-m}, m = N-2, N-3, \dots, 1 \quad (3.29)$$

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$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-1}=0}^{n_{k-1}} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_k) \leq 1, \quad i_k = 1, 2, \dots, n_k \quad (3.30)$$

2). Single return constraints

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_k=0}^{n_k} \eta(t, i_{k-N+1}, \dots, i_k) = 1, \quad t = 1, 2, \dots, T \quad (3.31)$$

3). Constraints on missed detections

$$\sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, 0, \dots, i_k) \leq T \quad (3.32)$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-m+1}=0}^{n_{k-m+1}} \sum_{i_k=0}^{n_k} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_{k-m+1}, 0, i_{k-m+1}, \dots, i_k) \leq T, \quad m = N-2, N-3, \dots, 1 \quad (3.33)$$

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-1}=0}^{n_{k-1}} \sum_{t=1}^T \eta(t, i_{k-N+1}, \dots, i_{k-1}, 0) \leq T \quad (3.34)$$

4). Integer constraints on association variables

$$\eta(t, i_{k-N+1}, \dots, i_k) = \{0, 1\} \quad \forall i_{k-N+1}, \dots, i_k, \quad t = 1, 2, \dots, T \quad (3.35)$$

However, the above integer programming problem is NP hard. An NP hard problem can only be solved by complete enumeration of all possible combinations. An optimal solution to an NP hard problem is intractable and requires time that is an exponential function of the problem size. When an NP hard problem is to be solved one approach is to use a polynomial time algorithm to approximate the solution and the answer thus obtained will not necessarily be optimal, but reasonably close to the optimal. Therefore, it is necessary that suitable sub-optimal algorithms be explored to solve the above multiple frame sliding window based data association problem.

3.4.3 Linear Programming Based Solutions

The above integer programming solution to the data association problem can be solved using linear programming techniques by relaxing the integer constraints (3.35) into non-negativity constraints as follows.

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$$\eta(t, i_{k-N+1}, \dots, i_k) \geq 0 \quad \forall i_{k-N+1}, \dots, i_k, \quad t = 1, 2, \dots, T \quad (3.36)$$

Thus, the data association problem is converted to the linear programming problem of minimizing (3.27) subject to the constraints (3.28) - (3.34) and (3.36).

However, the complexity of common linear programming methods such as the simplex method has an exponentially growing time complexity with the problem size. Methods that generate iterates lying in the interior of the feasible set (rather than on the boundary as simplex methods do) was due to Karmarkar [82] has proven to be several times faster. Karmarkar's method has made it possible to solve many large linear programming problems (e.g. a military aircraft optimal route selection problem with variables and constraints in the order of 10^5 and 10^4 respectively), which would have otherwise required a prohibitively large amount of computing resources when solved by the simplex algorithm. In this thesis, the linear program relaxation of the data association problem formulated in Section 3.4.3 is solved using Mehrotra's predictor corrector method [81], which is a primal dual interior point method. This algorithm is a faster variant of the kind of interior point algorithms available for solving large scale linear programming problems known as Primal-Dual Infeasible interior point methods. Unlike primal methods, which concentrate on solving the primal problem, primal-dual interior-point methods solve the Karush-Kuhn-Tucker (KKT) system of a linear program involving the primal, dual and slack variables. These methods solve the KKT system of equations using a damped Newton's method with a step size determined to satisfy all the non-negativity constraints.

3.4.3.1 A two frame formulation

The application of the above generalized N frame data association method to SLAM with a linear programming solution technique is formulated and verified for two consecutive frames of measurement as outlined below. Although an EKF based stochastic mapping approach described in Section 2.2.2 is employed in this formulation, the methodology can

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be easily extended to cater to the Rao-Blackwellised particle filter based methods [10] and [11] and numerous other methods [30] and [43]. The basic EKF SLAM framework presented in Chapter 2 and its notation is used in this formulation. Equations (2.5) to (2.14) describe all the models and estimation equations associated with this SLAM algorithm.

The two frame data association algorithm utilizes two frames of measurement made at times $k-1$ and k and features updated at time $k-2$ and assign measurements to features at time k based on the combined effect of measurement frames obtained at time $k-1$ and k . The data association problem in the form of multiple frame sliding window approach is to assign proper measurement combinations corresponding to existing features and new probable features in the two frames concerned in an efficient and optimal manner. Let $\mathbf{Z}(k-1) = \{\mathbf{z}_{i_{k-1}} \mid i_{k-1} = 0, 1, 2, \dots, n_{k-1}\}$ and $\mathbf{Z}(k) = \{\mathbf{z}_{i_k} \mid i_k = 0, 1, 2, \dots, n_k\}$ be the sets of measurement obtained in measurement frames $k-1$ and k respectively where n_{k-1} and n_k are the number of measurements obtained in frame $k-1$ and frame k respectively. Let the association variable $\eta(t, i_{k-1}, i_k)$ be defined in such a way that $\eta(t, i_{k-1}, i_k) = 1$ when measurement $\mathbf{z}_{i_{k-1}}$ in measurement frame $k-1$ and measurement $\mathbf{z}_{i_k}$ in measurement frame k are associated with the feature $\boldsymbol{\tau}_t$ and $\eta(t, i_{k-1}, i_k) = 0$ otherwise. Let $c(t, i_{k-1}, i_k)$ be the cost function of associating the same measurement sequence with feature $\boldsymbol{\tau}_t$, then the data association problem (based on (3.27)) is

$$\text{Minimize } \sum_{i_{k-1}=0}^{n_{k-1}} \sum_{i_k=0}^{n_k} \sum_{t=1}^T c(t, i_{k-1}, i_k) \eta(t, i_{k-1}, i_k) \quad (3.37)$$

Subject to the following constraints imposed on the association variables.

- 1) Single source constraint for measurements.

$$\text{In } k^{\text{th}} \text{ frame } \sum_{t=1}^T \sum_{i_{k-1}=0}^{n_{k-1}} \eta(t, i_{k-1}, i_k) \leq 1 \text{ for } i_k = 1, 2, \dots, n_k \quad (3.38)$$

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$$\text{In } k\text{-1}^{\text{th}} \text{ frame} \quad \sum_{t=1}^T \sum_{i_k=0}^{n_k} \eta(t, i_{k-1}, i_k) \leq 1 \quad \text{for } i_{k-1} = 1, 2, \dots, n_{k-1} \quad (3.39)$$

2) Single return constraint for features.

$$\sum_{i_{k-1}=0}^{n_{k-1}} \sum_{i_k=0}^{n_k} \eta(t, i_{k-1}, i_k) = 1 \quad \text{for } t = 1, 2, \dots, T \quad (3.40)$$

3) Constraint on maximum number of dummy measurements

$$\text{In } k^{\text{th}} \text{ frame} \quad \sum_{t=1}^T \sum_{i_{k-1}=1}^{n_{k-1}} \eta(t, i_{k-1}, 0) \leq T \quad (3.41)$$

$$\text{In } k\text{-1}^{\text{th}} \text{ frame} \quad \sum_{t=1}^T \sum_{i_k=1}^{n_k} \eta(t, 0, i_k) \leq T \quad (3.42)$$

4) Nonnegativity Constraints:

$$\eta(t, i_{k-1}, i_k) \geq 0 \quad \text{for } i_k = 0, 1, 2, \dots, n_k, \quad i_{k-1} = 0, 1, 2, \dots, n_{k-1}, \quad t = 1, 2, \dots, T \quad (3.43)$$

3.4.3.2 Reducing the number of association variables.

The number of variables associated with the linear program can be reduced by data preprocessing and gating of measurements. This step eliminates all the unlikely associations and thereby improves algorithm efficiency. Particularly in robotics mapping, SLAM and MAL, the number of features in the map can be very large (in the order of millions). Thus, it is essential that only likely observations be considered in the data association problem. The process of determining potential associations is known as gating. In gating, observations which are within a certain proximity of features are considered as feasible associations. Most frequently a distance measure such as Mahalanobis distance as in [39] and [40] is used to quantify the proximity. However, with the growth of the map size in SLAM the number of comparisons required for gating can increase without bound. This can be addressed by organizing the map features in efficient data structures such as kd trees and range segment trees [31] and [59], which enable

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efficient range queries be made.

As absolute feature locations and the measurement predictions are correlated over the vehicle state the relative measurements and their predictions are used in the sections that follow. Let the SLAM map state, vehicle state and SLAM composite state be $\mathbf{m}(k)$, $\mathbf{x}_v(k) = [x(k) \ y(k) \ \theta(k)]^T$ and $\mathbf{X}(k) = [\mathbf{x}_v^T(k) \ \mathbf{m}^T(k)]^T$ respectively where $x(k)$, $y(k)$ and $\theta(k)$ denote the position coordinates and heading of the vehicle. Let the absolute coordinate of a feature t in the state vector at time k and its location uncertainty be denoted by $\mathbf{x}_t(k) = [x_t(k) \ y_t(k)]^T$ and $\mathbf{P}_t(k)$. Assuming a range bearing measurement system such as SICK LMS 290, let the i_{k-1}^{th} relative range bearing measurement at frame $k-1$ be denoted by $\mathbf{z}_{i_{k-1}}$. Suppose that the prediction of this measurement is $\hat{\mathbf{z}}_{i_{k-1},t}$ and its covariance matrix is $\mathbf{P}_{i_{k-1},t}$, then

$$\hat{\mathbf{z}}_{i_{k-1},t} = \begin{bmatrix} \sqrt{(x(k-1|k-2) - x_t(k-1|k-2))^2 + (y(k-1|k-2) - y_t(k-1|k-2))^2} \\ \tan^{-1} \left(\frac{(y_t(k-1|k-2) - y(k-1|k-2))}{(x_t(k-1|k-2) - x(k-1|k-2))} \right) - \theta(k-1|k-2) \end{bmatrix} \quad (3.44)$$

$$= \mathbf{h}(\mathbf{x}_v(k-1|k-2), \mathbf{x}_t(k-1|k-2))$$

$$\mathbf{P}_{i_{k-1},t} = \mathbf{H}\mathbf{P}(k-1|k-2)\mathbf{H}^T \quad (3.45)$$

where $x(k-1|k-2)$, $y(k-1|k-2)$, $\theta(k-1|k-2)$, $\mathbf{P}(k-1|k-2)$, and $\mathbf{H}$ are the prediction of the vehicle pose, the prediction of the state covariance at time $k-1$, and the Jacobian $(\partial \hat{\mathbf{z}}_{i_{k-1},t} / \partial \mathbf{X})$ of $\hat{\mathbf{z}}_{i_{k-1},t}$ with respect to the composite state $\mathbf{X}$.

The innovation $\mathbf{e}_{i_{k-1},t}$ and its covariance $\mathbf{S}_{i_{k-1},t}$ are then calculated for all the combinations of measurements in frame $k-1$ and all the features t as follows.

$$\mathbf{e}_{i_{k-1},t} = \mathbf{z}_{i_{k-1}} - \hat{\mathbf{z}}_{i_{k-1},t} \quad (3.46)$$

$$\mathbf{S}_{i_{k-1},t} = \mathbf{P}_{i_{k-1},t} + \mathbf{R} \quad (3.47)$$

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The set of measurements $\mathbf{Z}^{k-1}$ satisfying the Chi-square statistical test given below is then selected for data association.

$$\mathbf{Z}^{k-1} = \{ \mathbf{z}_{i_{k-1}} \mid \mathbf{e}_{i_{k-1},t}^T \mathbf{S}_{i_{k-1},t}^{-1} \mathbf{e}_{i_{k-1},t} \leq g_1, \forall i_{k-1}, t \} \quad (3.48)$$

Here, g_1 is a threshold determined from Chi-square distribution of $\dim(\mathbf{z}_{i_{k-1}})$ degrees of freedom under a desired confidence level and i_{k-1} and t denote all the measurements received in frame $k-1$ and all available features. A validation matrix Ψ_{k-1} is then constructed in the following manner to represent the observations that fall inside the validation regions of features.

$$\Psi_{k-1} = \{ \lambda_{i_{k-1},t} \}_{(n_{k-1}+1)T}, i_{k-1}=0,1,2,\dots,n_{k-1}, t=1,2,\dots,T \quad (3.49)$$

$$\text{where } \lambda_{i_{k-1},t} = \begin{cases} 1 & \text{if } i_{k-1}=0, \text{ or } \mathbf{z}_{i_{k-1}} \in \mathbf{Z}^{k-1} \\ 0 & \text{otherwise} \end{cases}$$

Using the same procedure and notation described for frame $k-1$, all the set of observations $\mathbf{Z}^k$ in frame k satisfying the chi-square statistical test given below are selected.

$$\mathbf{Z}^k = \{ \mathbf{z}_{i_k} \mid \mathbf{e}_{i_k,t}^T \mathbf{S}_{i_k,t}^{-1} \mathbf{e}_{i_k,t} \leq g_2, \forall i_k, t \} \quad (3.50)$$

Here, g_2 is a threshold determined from chi-square distribution of $\dim(\mathbf{z}_{i_k})$ degrees of freedom under a desired confidence level, $\mathbf{e}_{i_k,t}$ denote the innovation of feature observation combinations at frame k , and $\mathbf{S}_{i_k,t}$ its covariance. Let i_k and t denote all the observations received in frame k and all the available features (T). A validation matrix $(\Psi_k = \{ \lambda_{i_k,t} \}_{(n_k+1)T}, i_k=0,1,2,\dots,n_k, t=1,2,\dots,T)$ is then constructed to represent the observations falling inside the validation regions of features. Here, $\lambda_{i_k,t} = 1$ if $i_k = 0$ or $\mathbf{z}_{i_k} \in \mathbf{Z}^k$ and $\lambda_{i_k,t} = 0$ otherwise.

As the association variable $\eta(t, i_{k-1}, i_k)$ indicates that the measurement i_{k-1} in frame $k-1$ and measurement i_k in frame k are inside the validation regions of a feature t , it is

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possible to select the set of variables ($\mathbf{E}_R$) required to perform the data association function in the following manner.

$$\mathbf{E}_R = \{\eta(t, i_{k-1}, i_k) \mid \lambda_{i_{k-1}, t} \lambda_{i_k, t} = 1, \forall i_{k-1} = 0, 1, \dots, n_{k-1}, i_k = 0, 1, \dots, n_k, t = 1, 2, \dots, T\} \quad (3.51)$$

3.4.3.3 Likelihood calculation

Using the same notation used in Section 3.4.1, the following expressions can be obtained for the 2 frame data association approach. From (3.22)

$$\Lambda_{true}(\mathbf{\Omega}) = \prod_{t=1}^T \{ (p_d L(\mathbf{z}_{i_{k-1}} \mid \boldsymbol{\tau}_t))^{\varepsilon^{k-1}(i_{k-1})} (1-p_d)^{1-\varepsilon^{k-1}(i_{k-1})} (p_d L(\mathbf{z}_{i_k} \mid \mathbf{z}_{i_{k-1}}, \boldsymbol{\tau}_t))^{\varepsilon^k(i_k)} (1-p_d)^{1-\varepsilon^k(i_k)} \} \quad (3.52)$$

From (3.20)

$$\Lambda(\mathbf{\Omega}) = \Lambda_{true}(\mathbf{\Omega}) V_s^{-n_{k-1} + T_D^{k-1}} V_s^{-n_k + T_D^k} \quad (3.53)$$

$$\Lambda_N(\mathbf{\Omega}) = \Lambda(\mathbf{\Omega}) / (V_s^{-n_{k-1}} V_s^{-n_k}) \quad (3.54)$$

From (3.25) and (3.26)

$$c(t, i, j) = -2\ln(1-p_d) - (\varepsilon^{k-1}(i_{k-1}) + \varepsilon^k(i_k))\ln(p_d V_s / (1-p_d)) - \varepsilon^{k-1}(i_{k-1})\ln(L(\mathbf{z}_{i_{k-1}} \mid \boldsymbol{\tau}_t)) - \varepsilon^k(i_k)\ln(L(\mathbf{z}_{i_k} \mid \mathbf{z}_{i_{k-1}}, \boldsymbol{\tau}_t)) \quad (3.55)$$

Costs $c(t, i_{k-1}, i_k)$ in (3.55) were calculated by incorporating the vehicle location uncertainty and the second order statistics of the likelihoods in the measurement model.

Equation (3.55) reflects four different cases depending on the value of i_{k-1} and i_k .

1) When a feature $\boldsymbol{\tau}_t$ is not detected in both frames: $i_{k-1} = 0$ & $i_k = 0$ i.e.

$$\varepsilon^{k-1}(i_{k-1}) = \varepsilon^k(i_k) = 0$$

$$c(t, 0, 0) = -2\ln(1-p_d) \quad (3.56)$$

2) When the feature $\boldsymbol{\tau}_t$ is detected in frame $k-1$ but not detected in frame

$$k.: i_{k-1} \neq 0 \text{ \& } i_k = 0, \text{ i.e. } \varepsilon^{k-1}(i_{k-1}) = 1 \text{ and } \varepsilon^k(i_k) = 0.$$

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$$c(t, i_{k-1}, 0) = -\ln(p_d(1-p_d)V_s L(\mathbf{z}_{i_{k-1}} | \boldsymbol{\tau}_t)) \quad (3.57)$$

$$L(\mathbf{z}_{i_{k-1}} | \boldsymbol{\tau}_t) = \exp(-0.5 \mathbf{e}_{i_{k-1},t}^T \mathbf{S}_{i_{k-1},t}^{-1} \mathbf{e}_{i_{k-1},t}) / \sqrt{(2\pi)^{\dim(\mathbf{z}_{i_{k-1}})} \det(\mathbf{S}_{i_{k-1},t})} \quad (3.58)$$

where $\mathbf{e}_{i_{k-1},t}$ and $\mathbf{S}_{i_{k-1},t}$ are obtained from (3.46) and (3.47).

3) When the feature $\boldsymbol{\tau}_t$ is detected in frame k but not detected in frame

$k-1$: $i_{k-1} = 0$ & $i_k \neq 0$, i.e. $\varepsilon^{k-1}(i_{k-1}) = 0$ and $\varepsilon^k(i_k) = 1$.

$$c(t, 0, i_k) = -\ln(p_d(1-p_d)V_s L(\mathbf{z}_{i_k} | \mathbf{z}_0, \boldsymbol{\tau}_t)) \quad (3.59)$$

$$L(\mathbf{z}_{i_k} | \mathbf{z}_0, \boldsymbol{\tau}_t) = \exp(-0.5 \mathbf{e}_{i_k,t}^T \mathbf{S}_{i_k,t}^{-1} \mathbf{e}_{i_k,t}) / \sqrt{(2\pi)^{\dim(\mathbf{z}_{i_k})} \det(\mathbf{S}_{i_k,t})} \quad (3.60)$$

where $\mathbf{e}_{i_k,t}$ and $\mathbf{S}_{i_k,t}$ are the observation innovation associated with the measurement $\mathbf{z}_{i_k}(k)$ in frame k and feature $\boldsymbol{\tau}_t$ in the same frame and its innovation covariance respectively.

4) When a feature $\boldsymbol{\tau}_t$ is detected in both frames $k-1$ and k : $i_{k-1} \neq 0$ & $i_k \neq 0$, i.e.

$\varepsilon^{k-1}(i_{k-1}) = 1$ and $\varepsilon^k(i_k) = 1$.

$$c(t, i_{k-1}, i_k) = -\ln((p_d V_s)^2 L(\mathbf{z}_{i_{k-1}} | \boldsymbol{\tau}_t) L(\mathbf{z}_{i_k} | \mathbf{z}_{i_{k-1}}, \boldsymbol{\tau}_t)) \quad (3.61)$$

Let a feature $\boldsymbol{\tau}_t$ at time $k-2$ be denoted by $\mathbf{x}_t(k-2)$ and its covariance matrix be denoted by $\mathbf{P}_t(k-2)$. Given the relative measurement vector $\mathbf{z}_{i_{k-1}}$, corresponding measurement prediction $\hat{\mathbf{z}}_{i_{k-1},t}$ and its uncertainty $\mathbf{P}_{i_{k-1},t}$, the innovation $\mathbf{e}_{i_{k-1},t}$ of feature measurement combination and its covariance matrix $\mathbf{S}_{i_{k-1},t}$ can be obtained from (3.44), (3.45), (3.46) and (3.47). Extending the same notation, the Kalman gains $\mathbf{K}_t$ and $\mathbf{K}_v$ of the SLAM update corresponding to the update of the feature t and vehicle pose with the measurement $\mathbf{z}_{i_{k-1}}$ are given by

$$\mathbf{K}_t = (\mathbf{P}_v(k-1|k-2)\mathbf{H}_v^T + \mathbf{P}_t(k-1|k-2)\mathbf{H}_t^T) \mathbf{S}_{i_{k-1},t}^{-1} \quad (3.62)$$

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$$\mathbf{K}_v = \left(\mathbf{P}_{vv}(k-1|k-2)\mathbf{H}_v^T + \mathbf{P}_{tv}^T(k-1|k-2)\mathbf{H}_t^T \right) \mathbf{S}_{i_{k-1},t}^{-1} \quad (3.63)$$

where $\mathbf{P}_{vv}(k-1|k-2)$, $\mathbf{H}_v$ and $\mathbf{H}_t$ denote the covariance prediction of feature (t) and vehicle (v), the Jacobian $\left(\partial \hat{\mathbf{z}}_{i_{k-1},t} / \partial \mathbf{x}_v \right)$ of $\hat{\mathbf{z}}_{i_{k-1},t}$ with respect to the vehicle state $\mathbf{x}_v$ and the Jacobian $\left(\partial \hat{\mathbf{z}}_{i_{k-1},t} / \partial \mathbf{x}_t \right)$ of $\hat{\mathbf{z}}_{i_{k-1},t}$ with respect to feature t . The updated values of the feature t $\mathbf{x}_t(k-1|k-1)$, its uncertainty $\mathbf{P}_{tt}(k-1|k-1)$, vehicle pose $\mathbf{x}_v(k-1|k-1)$ and its uncertainty $\mathbf{P}_{vv}(k-1|k-1)$ can then be calculated using the standard Kalman filter update equations. Similarly update of $\mathbf{P}_{tv}(k-1|k-2)$ is

$$\mathbf{P}_{tv}(k-1|k-1) = \mathbf{P}_{tv}(k-1|k-2) - \mathbf{K}_t \mathbf{S}_{i_{k-1},t} \mathbf{K}_v^T \quad (3.64)$$

The predictions of $\mathbf{x}_v(k-1|k-1)$, $\mathbf{P}_{vv}(k-1|k-1)$, and $\mathbf{P}_{tv}(k-1|k-1)$ at time k can then be computed as

$$\mathbf{x}_v(k|k-1) = \mathbf{f}(\mathbf{x}_v(k-1|k-1), \mathbf{u}(k-1)) \quad (3.65)$$

$$\mathbf{P}_{vv}(k|k-1) = \mathbf{F}_v \mathbf{P}_{vv}(k-1|k-1) \mathbf{F}_v^T + \mathbf{Q}_v(k) \quad (3.66)$$

$$\mathbf{P}_{tv}(k|k-1) = \mathbf{P}_{tv}(k-1|k-1) \mathbf{F}_v^T \quad (3.67)$$

The value of $\mathbf{P}_{tt}(k-1|k-1)$ remains the same, $\mathbf{u}(k-1)$ is the control input and $\mathbf{F}_v = \partial \mathbf{f} / \partial \mathbf{x}_v$. The measurement prediction $\hat{\mathbf{z}}_{i_k,t}^*$ and its covariance $\mathbf{P}_{i_k,t}^*$ of the updated feature t , the innovation $\mathbf{e}_{i_k,t}^*$ with measurement $\mathbf{z}_{i_k}$, and its covariance $\mathbf{S}_{i_k,t}^*$ are

$$\hat{\mathbf{z}}_{i_k,t}^* = \mathbf{h}(\mathbf{x}_v(k|k-1), \mathbf{x}_t(k|k-1)) \quad (3.68)$$

$$\mathbf{P}_{i_k,t}^* = \mathbf{H}_v \mathbf{P}_{vv}(k|k-1) \mathbf{H}_v^T + \mathbf{H}_t \mathbf{P}_{tt}(k|k-1) \mathbf{H}_t^T + \mathbf{H}_v \mathbf{P}_{tv}(k|k-1) \mathbf{H}_t^T + \mathbf{H}_t \mathbf{P}_{tv}(k|k-1) \mathbf{H}_v^T \quad (3.69)$$

$$\mathbf{e}_{i_k,t}^* = \mathbf{z}_{i_k} - \hat{\mathbf{z}}_{i_k,t}^* \quad (3.70)$$

$$\mathbf{S}_{i_k,t}^* = \mathbf{P}_{i_k,t}^* + \mathbf{R} \quad (3.71)$$

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where $\mathbf{H}_v = \partial \mathbf{h} / \partial \mathbf{x}_v$ and $\mathbf{H}_t = \partial \mathbf{h} / \partial \mathbf{x}_t$ are the Jacobians of $\mathbf{h}$ with respect to $\mathbf{x}_v$ and $\mathbf{x}_t$.

The likelihood of $\mathbf{z}_{i_k}$ given $\mathbf{z}_{i_{k-1}}$ and feature t is

$$L(\mathbf{z}_{i_k} | \mathbf{z}_{i_{k-1}}, \mathbf{t}) = \exp\left(-0.5(\mathbf{e}_{i_k,t}^*)^T (\mathbf{S}_{i_k,t}^*)^{-1} (\mathbf{e}_{i_k,t}^*)\right) / \sqrt{(2\pi)^{\dim(\mathbf{z}_{i_k})} \det(\mathbf{S}_{i_k,t}^*)} \quad (3.72)$$

The data association problem in SLAM can now be solved in the form of a linear programming problem. An efficient and faster interior point algorithm known as Mehrotra's predictor corrector method [81] having a polynomial complexity is used as the solution technique.

3.4.3.4 Fractional solutions

As the non-linear 0-1 integer programming problem is solved through the relaxation of the integer constraint on the association variables in the linear programming solution techniques, the data association solution may be occasionally sub-optimal. This means although very rarely, there can be occasions where the association variables may evaluate to fractional values necessitating appropriate handling. One approach to cope the problem is to discard all the associations giving rise to fractional solutions. Another alternative is to interpret the resulting fractional solutions for the association variables as association probabilities specifying the correspondence between features and the measurements.

Let $w_{i_k,t} = \sum_{i_{k-1}=0}^{n_{k-1}} \eta(t, i_{k-1}, i_k), \forall i_k$, then using, $\sum_{i_k=0}^{n_k} \sum_{i_{k-1}=0}^{n_{k-1}} \eta(t, i_{k-1}, i_k) = 1, \forall t = 1, 2, \dots, T$ from

$$(3.40) \quad \sum_{i_k=0}^{n_k} w_{i_k,t} = 1, \forall t = 1, 2, \dots, T. \text{ Hence } w_{i_k,t} \text{ for all } i_k \text{ and } t \text{ can be interpreted as}$$

association probabilities. During such instances, a JPDA type of update is utilized as done in [40] in the state update process by modifying the EKF estimation equations where necessary as

$$\mathbf{e}_{i_k}(k) = \mathbf{z}_{i_k} - \hat{\mathbf{z}}_{i_k} \quad (3.73)$$

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$$\bar{\mathbf{e}}(k) = \sum_{i_k=0}^{n_k} w_{i_k} \mathbf{e}_{i_k}(k) \quad (3.74)$$

$$\mathbf{X}(k | k) = \mathbf{X}(k | k-1) + \mathbf{K}(k) \bar{\mathbf{v}}(k) \quad (3.75)$$

$$\mathbf{P}(k | k) = w_{0_t} \mathbf{P}(k | k-1) + (1 - w_{0_t}) \mathbf{P}^*(k) + \mathbf{K}(k) \mathbf{W}^*(k) \mathbf{K}^T(k) \quad (3.76)$$

$$\mathbf{P}^*(k) = (\mathbf{I} - \mathbf{K}(k) \mathbf{H}(k)) \mathbf{P}(k | k-1) \quad (3.77)$$

$$\mathbf{W}^*(k) = \sum_{i_k=0}^{n_k} \left(w_{i_k} \bar{\mathbf{e}}(k) \mathbf{e}_{i_k}^T(k) - \bar{\mathbf{e}}(k) \mathbf{e}_{i_k}^T(k) \right) \quad (3.78)$$

where $\mathbf{K}(k)$ is the Kalman gain and $\hat{\mathbf{z}}_{i_k}$ is the predicted value of the observation $\mathbf{z}_{i_k}$.

3.4.3.5 Feature initialization and management

Feature initiation plays a vital role in all SLAM algorithms. In the case of SLAM, the feature map is built incrementally. That is one has to have a strategy for initializing features in to the map before any data association can be carried out. It is shown here, how the multiple frame data association methodology proposed can be utilized effectively for this task as well. When observations are received they are first associated with the landmarks in the SLAM state vector using the multiple frame data association algorithm. If an association is found, that observation is used to update the SLAM state vector. If an association is not found, the observation is considered a potential new feature i.e. a tentative feature and added into the state vector. The rationale being that the multiple frame data association algorithm can effectively filter out spurious measurements and returns due to dynamic objects more effectively than the single frame schemes. Thus, it is logical to add every new (tentative) feature measurement straightaway in to the SLAM state vector as these have a higher possibility of being genuine (confirmed) features. The new (tentative) feature $\mathbf{x}_{new}$ is thus appended to the full SLAM state vector $\mathbf{X}_{aug}(k)$ and the covariance $\mathbf{P}_{aug}(k)$ is updated as follows

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$$\mathbf{g}(k) = \mathbf{x}_{new}(k) = \begin{bmatrix} x(k|k) + r_{new} \cos(\theta_{new} + \theta(k|k)) \\ y(k|k) + r_{new} \sin(\theta_{new} + \theta(k|k)) \end{bmatrix} \quad (3.79)$$

$$\mathbf{X}_{aug}(k) = \begin{bmatrix} \mathbf{X}(k|k) \\ \mathbf{x}_{new}(k) \end{bmatrix} \quad (3.80)$$

$$\mathbf{P}_{aug}(k) = \begin{bmatrix} \mathbf{P}(k|k) & \mathbf{P}^T(k|k)(\nabla_{\mathbf{x}}\mathbf{g})^T \\ (\nabla_{\mathbf{x}}\mathbf{g})\mathbf{P}(k|k) & (\nabla_{\mathbf{v}}\mathbf{g})\mathbf{P}_{vv}(k|k)(\nabla_{\mathbf{v}}\mathbf{g})^T + (\nabla_{\mathbf{z}}\mathbf{g})\mathbf{R}(\nabla_{\mathbf{z}}\mathbf{g})^T \end{bmatrix} \quad (3.81)$$

where $\mathbf{x}_{new}(k)$, r_{new} , θ_{new} , $\mathbf{g}(k)$, $\mathbf{X}_{aug}(k)$, $\mathbf{P}_{aug}(k)$, $\mathbf{P}_{vv}(k|k)$, $\nabla_{\mathbf{x}}\mathbf{g}$, $\nabla_{\mathbf{v}}\mathbf{g}$, $\nabla_{\mathbf{z}}\mathbf{g}$ denote the absolute coordinates of the new feature, range of the new feature, bearing of the new feature, inverse observation function, new augmented SLAM state vector, its covariance matrix, vehicle covariance matrix and Jacobians of $\mathbf{g}(k)$ with respect to $\mathbf{X}(k|k)$, vehicle state $\mathbf{x}_v$ and measurement $[r_{new} \ \theta_{new}]^T$ respectively.

However, it could still happen that the new features (tentative) added could be spurious measurements and their frequency of occurrences dependent on the number of frames used in data association. Thus, as a part of the feature maintenance strategy we monitor the ‘quality’ of these tentative or newly added features over a number of consecutive frames of measurement. The quality measure we use is the number of successive frames (or scans) over which the newly added feature (tentative feature) is successfully associated with measurements using the multiple frame data association method. If this predetermined number is exceeded the tentative feature in the SLAM state vector is declared confirmed or else it is deleted from the SLAM state vector. Only measurements associated with the confirmed map features are used to update the SLAM state vector.

The properties of the linear programming algorithm solution can also be used to determine the initialization of features. When converting constraint inequalities given by (3.38) into equations in linear programming solution algorithms, slack variables s_{i_k} are used. Therefore, n_k slack variables are required to convert the constraint inequality (3.38)

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into equation form. If $s_{i_k} = 1$ for any i_k , then that observation $\mathbf{z}_{i_k}$ in frame k cannot be associated with any existing feature and therefore most probably originated from a new feature. Such features can be added into tentative features for further confirmation.

3.4.4 Performance Evaluation

The performance of the multiple frame data association (MFDA) algorithm for two frames of measurement in EKF SLAM was initially evaluated in a simulated environment. The feature measurements are assumed to be from a scanning range measurement system such as a SICK LMS 290 (exteroceptive sensor). The control input to the vehicle kinematic model is assumed to be from a set of wheel encoders.

3.4.4.1 Scenario 1 – Illustration of MFDA in SLAM

As the first step to illustrate the operation of MFDA in EKF SLAM a simple scenario is considered. The vehicle is assumed to be traveling in a 20m radius circular path in an environment consisting of 10 randomly generated (located at randomly generated coordinates) features. The spurious measurements are assumed uniformly distributed in the environment and their returns are assumed Poisson distributed [39]. In other words, it is assumed that the number of spurious measurements in the operating environment at a particular instant of time is given by the Poisson probability distribution. Thus, the spurious measurement density is the average of the Poisson probability distribution. Hence it was assumed that the exteroceptive sensor receives returns from a number of spurious measurements, which are given by the Poisson probability distribution according to a pre-specified density. The locations of these given number of spurious measurements were taken as uniform. That is the probability of having a spurious measurement at any particular location in the operating environment is assumed constant.

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Figure 3.2. Feature to measurement association hypotheses. The figure shows possible association hypothesis in one instant. Measurement combinations shown by squares indicate the measurement indices in two consecutive frames of measurements.

$$\Psi_{k-1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \quad \Psi_k = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix}$$

Figure 3.3. Validation matrices for the association hypotheses shown in Figure 3.2. The columns in validation matrices consist of available features and rows correspond to the measurements in respective frames.

The simulation specific parameters are; speed input error 0.5 m/s, steering angle encoder error 0.05 radians, range measurement noise 0.1 m, bearing measurement error 0.5 degrees and spurious measurement or clutter density 0.002 m^{-2} . Figure 3.2 shows an association scenario at a particular instant ($k=15$) with five features in the state vector. The number of spurious measurements in the preceding frame $k-1^{\text{th}}$ (14^{th}) and current k^{th}

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frame (15th) are 1 and 4 and the number of true measurements in both frames is 5. For this state of affairs, the validation matrices of the two consecutive frames are as shown in Figure 3.3. At this instant there are in all 22 hypotheses corresponding to the different combinations of the measurements of the two frames and the features in the state vector. It is seen from Figure 3.2 that the association variables evaluate to 1 for correct measurement to feature associations and to negligibly small values for incorrect assignments.

Table 3.1 Slack variables

Value of Slack variable in single source constraint	Constraint No.	Possible New Tentative Tracks
1.2698e-011	1	×
9.478e-011	2	×
1.2556e-011	3	×
1.2834e-011	4	×
1.263e-011	5	×
1	6	Measurement 6 is a possibility
1.2825e-011	7	×
8.7541e-011	8	×
1.2483e-011	9	×
1.3339e-011	10	×
1.2705e-011	11	×
1	12	Measurement 12 is a possibility
1	13	Measurement 13 is a possibility
1	14	Measurement 14 is a possibility
1	15	Measurement 15 is a possibility
Parameters used in the illustration	No. of association hypothesis	27
	No. of returns in frame k from spurious measurements or new features	4
	No. of true measurements in frame k	5
	No. of true measurements in frame k-1	5
	No. of returns in frame k-1 from spurious measurements or new features	1

Table 3.1 shows the values of slack variables utilized in the constraints. As discussed in Section 3.4.3.5, the values of the slack variables indicate whether there is a possibility of obtaining a measurement from a new feature. This could have either originated from a new feature or a spurious measurement. Table 3.1 suggests that there are 4 measurements (measurements 12, 13, 14 and 15), which are most probably from new features and can be resolved by the track initialization algorithm.

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3.4.4.2 Scenario 2: - High spurious measurement conditions

This simulation demonstrates the MFDA algorithm's performance in SLAM when there is a significant proportion of spurious measurements (that may be due to the nature of environment or poor sensor performance or both) and high feature density. This scenario is quite demanding for any data association scheme. Here, the vehicle is assumed to be traveling in a arbitrary path (Figure 3.4) in a $100 \times 100 \text{m}^2$ area environment consisting of 100 randomly generated features. The spurious measurement spread and the simulation parameters are as in the Scenario 1. The MFDA data association filter's consistency is clearly seen from the plot of normalized position innovation squared versus time (Figure 3.5). It is apparent from the results that the performance of SLAM with MFDA data association is good even under high spurious measurement and feature densities.

Figure 3.4. The simulation environment.

Figure 3.5. Normalized (x, y) position innovation squared plot.

Localization accuracy of Standard Nearest Neighbor (SNN) data association, Multiple Frame Data Association (MFDA) and joint Compatibility Branch and Bound (JCBB) were compared with respect to the localization accuracy. Localization error plots (Figures. 3.6 (a) (b) (c) and Figures 3.7 (a), (b) and (c)) were drawn to the same scale in the simulations. These error plots (Figure. 3.6 and Figure. 3.7) verify that the localization accuracy is high when using MFDA instead of SNN and JCBB data association algorithms when the spurious measurements are high.

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Figure 3.6. Lateral position error in SLAM. Figure 3.6 (a). With MFDA. Figure 3.6 (b). With JCBB. Figure 3.6 (c). With SNN. Dotted lines show 95% confidence limits of the estimated error and the thin continuous line represents the estimated error.

Figure 3.7. Heading error in SLAM. Figure 3.7 (a). With MFDA. Figure 3.7 (b). With JCBB. Figure 3.7 (c). With SNN. Dotted lines show 95% confidence limits of the estimated error and the thin continuous line represents the estimated error.

The MFDA data association approach in EKF-SLAM is compared with the standard nearest neighbor data association filter (SNN) and joint compatibility branch and bound (JCBB) by performing several Monte-Carlo runs using the same conditions and the same set of simulation parameters used in Scenario 1. A good measure of performance of data association in this context is the percentage missed associations or percentage track loss measure [83], which is the ratio of missed associations to total observed instances of a feature as a percentage. The reduction in track loss or the percentage missed associations is used by several researchers in the tracking community such as Poor [84], Bar-Shalom [83], and Blackman [40] to quantify the reduction in tracking accuracy. The reduction in track loss amounts to the reduction in missed associations and there by reduction in inclusion of less accurate information in the measurement update equation of the SLAM state vector. Since all the estimated map and vehicle errors are bound by the strong correlations [4], [7] and [38] in the state vector, low accuracy in one measurement update causes the propagation of errors throughout the state vector not only at the particular time

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step but also during rest of the time. Thus, we can conclude that the reduction in track loss leads to reasonable degree of improvement in the localization accuracy. This can be quantified in SLAM by plotting the number of percentage missed associations versus the localization accuracy in a SLAM run. The evaluation was done in the same simulation environment described in Section 3.4.4.1. The same set of simulation parameters used in Section 3.4.4.1 was used. The data associations were assumed known a priori in the simulation. The number of percentage missed associations was simulated so that in each set of hundred 100 ms time steps in which a feature was observed, there were a specified number of misassociations for the particular feature. The average standard deviation of the localization error is calculated for 50 Monte-Carlo runs in order to get a plot of percentage missed associations versus average standard deviation of the localization error in SLAM. The average standard deviation of localization error is an indicator as in [26] and [27], which quantifies the variation of the confidence limits of the robot localization error. Thus, the higher is the average standard deviation of localization error the smaller will be the localization accuracy. The results shown in figure 3.8 confirm that when the number of percentage missed associations (track loss) increase, so does the average standard deviation of the localization error. It is also observed that the reduction of localization accuracy is significant with the increase of the percentage missed associations. Thus, it can be concluded that the reduction in track loss or the number of percentage missed associations leads to a reasonable degree of improvement in localization accuracy.

Table 3.2 depicts track loss of the various methods with varying densities of spurious measurement (.01 at 99% Gate and 0.5 at 99% Gate). The term “0.01 at 99% Gate” means that the average number of spurious measurements, which can be expected probabilistically (given by the Poisson probability distribution) in the area of the gate spanned by the 99% confidence limit of a SLAM feature is 0.01.

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Figure 3.8. Localization accuracy (heading) versus percentage missed associations. Figure 3.8(a). Lateral position accuracy. Figure 3.8(b). Heading accuracy

Table 3.2 Comparison of data association algorithms in SLAM

Data Association Algorithm	% Missed Associations (Track Loss)			
	Density of spurious measurements			
	0.01 at 99% Gate		0.5 at 99% Gate	
	Feature 1	Feature 2	Feature 1	Feature 2
SNN	2.7	2.8	36.5	36.2
MFDA (2 Frame)	1.0	1.1	9.4	9.2
JCBB	1.0	1.0	11.2	11.1

Table 3.2 shows that the performance of data association in SLAM is significantly improved with MFDA with two frames compared to the standard nearest neighbor data association especially in high densities of spurious measurement. The MFDA algorithm's performance is comparable to JCBB at lower densities of spurious measurement. However, with increasing densities of spurious measurement there is a marked improvement in performance of MFDA over JCBB.

Table 3.3 shows that map feature estimation accuracy is very high when the MFDA (Multi-Frame Data Association) data association algorithm is used in SLAM instead of SNN (Standard Nearest Neighbor) or JCBB (Joint Compatibility Branch and Bound) data association algorithms, which are single frame methods. The indicator used to establish map accuracy is the average standard deviation ([26] and [27]) of the lateral and longitudinal errors of a map feature. This indicator shows the variation of the confident

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limits of estimated accuracy of a map feature under a given set of conditions. If the average standard deviation [26] of an estimated quantity is small, the particular quantity can be estimated with high confidence. Thus, a small average standard deviation signifies that the accuracy of the quantity is high.

Table 3.3 Comparison of map accuracy in SLAM-simulations

Data Association Algorithm	Average Standard Deviation [m]			
	Longitudinal Error		Lateral Error	
	Feature 1	Feature 2	Feature 1	Feature 2
SNN	0.41	0.38	0.48	0.48
MFDA (2 Frame)	0.14	0.13	0.20	0.22
JCBB	0.25	0.23	0.29	0.31

3.4.4.3 Complexity and computational efficiency

The complexity of the linear program can be reduced significantly by the preprocessing steps as described in Section 3.4.3.2. It can be shown as in Potra and S. J. Wright [110] that the complexity of primal dual infeasible interior point method has an iteration complexity of $O(\sqrt{n}L)$ and a computational complexity of $O(n^3L)$ where n is the number of association variables and L is the bit length of the input data. In this work several studies were conducted to compare the actual computation time required by MFDA, SNN, and JCBB in SLAM. The CPU time requirement for the preprocessing and data association functions are determined using a Pentium 4, 2.4 GHz, 512 MB RAM PC and is given in Table 3.4

Of the three, the computation time for JCBB is the highest. Although the computation time for MFDA is much higher than SNN, it is still feasible for real-time implementation in EKF-SLAM. The computation time of MFDA can be appreciably reduced with pre-processing of features before association and thereby reducing the number of association variables

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Table 3.4 Comparison of average computational load

Data Association Algorithm	CPU Time (s)	
	<i>Number of Association Hypotheses</i>	
	50	100
Nearest Neighbor	0.018	0.042
Two frame assignment	0.098	0.214
JCBB	0.154	0.452

3.4.4.4 Experimental verification

The experimental verification of the MFDA data association in EKF-SLAM was done using the in-house built autonomous vehicle Generic Outdoor Mobile Explorer (GenOME), a car like mobile robot (Appendix A). The sensors onboard the vehicle amongst others include a SICK LMS 290 laser measurement system, a fiber optic gyroscope and rear and front wheel encoders. The SICK LMS 290 provides range measurements to objects ahead at 1 degree intervals over a span of 180 degrees in one scan. SICK laser scanners extensively used in mobile robotics provide range and bearing information to points, which reflect the laser beam. It is also possible to cluster the points (raw measurements) obtained by these laser scanners into some known complex landmark forms such as buildings, trees, lamp posts etc. Nevertheless, propagating the uncertainty measures and the location information of such complex landmark forms over longer periods of time is computationally infeasible in SLAM. Moreover, data association schemes utilizing such complex landmark forms and measurements are not well developed. Subsequently, the estimation theoretic Simultaneous Localization and Mapping (SLAM) and Map Aided Localization (MAL) formulations used in the thesis require that landmarks be modeled as point landmarks as described in the seminal work of Dissanayeke et al. [7]. Consequently we model the landmark data association and measurement updating in SLAM, MAL and Robotics Mapping as point tracking problems (eg. work of Leonard and Feder [2]) similar to the ones in radar target tracking (eg [39]

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and [40]). Thus, the scanned data arriving at 0.1s intervals from this range measurement system is segmented and clustered as described in [20] to extract features.

The outputs of the wheel and steering encoders are used as inputs to the vehicle kinematic model for vehicle pose prediction. The output of the fiber optic gyroscope is also used as a vehicle orientation measurement. Features extracted from the laser and the measurements from encoders are then used to implement the EKF-SLAM with the particular data association scheme.

The vehicle is driven at an average speed of 3m/s along an approximately 170m stretch of road in a 60x60m² area car park in the campus as shown in the Figure 3.9 in the first experiment. In this experiment it is observed that the features extracted per scan varies from about zero to 35 with an average of 18. The mapped feature (lamp posts and tree trunks) locations (as point landmarks) and the vehicle path (171m length) estimates obtained using EKF SLAM with MFDA data association are shown in Figure 3.9. The plot of range innovation metric is used to measure the data association filter consistency. As shown in Figure 3.10, the range innovation plot is well within the 95% confidence limits demonstrating the MFDA filter consistency.

The experimental map accuracy is determined by measuring average standard deviation of a map feature as done in the simulation (Table 3.3). The results are shown in Table 3.5. The same performance indicator (average standard deviation) was used to establish map accuracy when using SNN, MFDA and JCBB data association algorithms in experiments. The experimental results also affirm the map accuracy results obtained in simulations (Table 3.3). In other words, map accuracy of SLAM when using a multiple frame data association algorithm is high as compared to using single frame data association algorithms such as SNN and JCBB. The MFDA, JCBB and SNN algorithms were further evaluated using the measured ground truth of selected landmarks with reference to the vehicle starting position, which is chosen as the origin of the global reference coordinate

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frame. These landmarks include lampposts on the roadside and artificially placed fiberglass posts. The locations of these are measured with respect to the origin of the global reference frame (starting position of vehicle). The EKF-SLAM algorithm was run with MFDA, JCBB and SNN data association algorithms for the case in Figure 3.9. Figure 3.11. and Figure 3.12 show the true errors associated with two landmarks (Landmarks one and two used for ground truth evaluation are shown in Figure 3.9). It can be observed that the errors are bounded by 95% confidence limits of the estimated landmark uncertainties and thus demonstrate the accuracy of map estimation (and hence the vehicle path estimate), filter consistency and the good performance of the MFDA with two frames. It can be noted in Figure 3.11 and Figure 3.12 that the standard deviations of the estimation errors of all the map features vary in the ascending order of data association algorithms MFDA, JCBB and SNN. Hence, it can be concluded that the multiple frame algorithms result in better map accuracies than single frame ones as predicted.

The loop closing error in a real SLAM experiment can be measured by starting the vehicle at a marked location and stopping at the same place after traversing a loop. For this experimental scenario the loop closing error obtained is within 0.6m (Figure 3.9). These good performance results demonstrate not just the feasibility but also the effectiveness of the MFDA data association in EKF-SLAM. It is also observed that the loop closing error in MFDA is less than that of NN by 8.3% (X error), 8.1% (Y error) for the experiment in Figure 3.9. Figure 3.13 shows the computation times for MFDA, JCBB and NN data association algorithms for the experiment conducted in the car park (Figure 3.9).

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Figure 3.9. Estimated vehicle path (thick line) and feature locations (circles) (SLAM experiment in a campus car park).

Figure 3.10. Filter performance: Range innovation (dotted line) and its 95% confidence limits (thick line).

It may be noted that the computation time for NN is the least. The computation time for MFDA is higher than NN and but consistent with time. The computation time of JCBB is always higher, varies significantly and becomes very significant especially with increasing spurious measurements and features (Figure 3.14). As illustrated in the plot of Figure 3.14, the increased computational requirements of JCBB is due to the variable but large number of traversals of the hypothesis tree required at each time step by the joint compatibility algorithm, even with branch and bound pruning.

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Table 3.5 Comparison of map accuracy in SLAM-experiments

Data Association Algorithm	Average Standard Deviation [m]			
	Longitudinal Error		Lateral Error	
	Feature 1	Feature 2	Feature 1	Feature 2
SNN	0.51	0.48	0.55	0.61
MFDA (2 Frame)	0.15	0.18	0.18	0.24
JCBB	0.31	0.33	0.32	0.34

Figure 3.11. Estimated location errors for landmark 1. Figure 3.11(a). Error in X Figure 3.11(b). Error in Y. The unmarked thin continuous line, the dotted line and thick dashed line represent landmark error in SLAM when utilizing MFDA, JCBB and SNN data association algorithms respectively.

Figure 3.12. Estimated location errors for landmark 2. Figure 3.12(a). Error in X Figure 3.12(b). Error in Y. The unmarked thin continuous line, the dotted line and thick dashed line represent landmark error in SLAM when utilizing MFDA, JCBB and SNN data association algorithms respectively.

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Figure 3.13. Comparison of the time complexity of data association algorithms. Time is shown for data sampled at 0.1s in the actual experiment..

Figure 3.14. Number of association hypotheses traversed by JCBB method in each time step and the number of features extracted.(shown as measurements)

Table 3.6 depicts track loss of the JCBB, MFDA and SNN data association algorithms with varying densities of spurious measurement in experimental setting (experiment described in Section 3.4.4.4). A low clutter (static) and high clutter (when there are several moving people and vehicles) scenarios were selected for the experiments. Table 3.6 shows that the performance of data association in SLAM is significantly improved with MFDA with two-frames compared to the standard nearest neighbor data association especially in high densities of spurious measurement affirming the simulation results (Table 3.2). The MFDA algorithm's performance is comparable to JCBB at lower densities of spurious measurement. However, with increasing densities of spurious measurement there is a marked improvement in performance of MFDA over JCBB.

Table 3.6 Comparison of data association algorithms in SLAM experiments

Data Association Algorithm	% Missed Associations (Track Loss)			
	Density of spurious measurements			
	<i>Low</i>		<i>High</i>	
	<i>Feature 1</i>	<i>Feature 2</i>	<i>Feature 1</i>	<i>Feature 2</i>
SNN	3.6	3.8	21.4	22.3
MFDA (2 Frame)	1.5	1.6	6.9	7.1
JCBB	1.6	1.8	8.5	8.9

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Figure 3.15. A Large scale SLAM experiment in a university campus environment. The estimated vehicle path is shown by the thick line and features are shown by crosses

Another set of experiments was conducted in a large outdoor environment. The vehicle was driven at an average speed of about 4m/s along an approximately 1 km stretch of road in the campus. A digital map (scale 1:5000) of the area of interest is shown in Figure 3.15. It was observed in this experiment that the number of features extracted varies from zero to 55 with an average of 27. The results of the estimated vehicle path ($\approx 1\text{km}$) and the feature locations are shown superimposed on the digital map in Figure 3.15. Results verify that MFDA can sustain large scale outdoor SLAM exercises. Figure 3.16 illustrates the performance of SLAM algorithm with the two frame data association algorithm at selected time instances.

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Figure (a)

Figure (b)

Figure (c)

Figure (d)

Figure (e)

Figure (f)

Figure 3.16. Large scale SLAM experiment with 2 frame MFDA algorithm. Features are shown by crosses and 95% confidence limits are shown by the ellipses surrounding the features. The landmark number in the SLAM state vector and the appropriate associated feature measurement number are also shown.

The loop closing error [17], [20], [38], [42] and [43] is the error accumulated over a long period when performing simultaneous localization and mapping over large loops. When the loop closing error is large it is difficult to recognize the loops in vehicle path. Localization accuracy of the experimental results was compared by measuring accumulated loop closing error when performing different data association algorithms

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(SNN, MFDA and JCBB) as done in [38] and [137]. The vehicle is driven along the path shown in Figure 3.15 and stopped at the starting position after making a loop. Table 3.7 includes the measured loop closing errors observed when traversing the loop while using SNN, MFDA and JCBB as data association algorithms in SLAM. It is observed that the loop closing errors when using JCBB and SNN in SLAM are respectively 15% and 28% more with respect to using MFDA. These results show that MFDA is more robust and accurate than both JCBB and SNN in large outdoor areas.

Table 3.7 Comparison of localization accuracy via loop closing error

Data Association Algorithm	Loop Closing Error
MFDA	12.5
JCBB	15.2
SNN	16.0

The above experimental results were verified further by a similar SLAM run conducted in a crowded area (Figure 3.17). Figures 3.18 and 3.19 show a plot of possible hypotheses with the number of measurements received in MFDA and JCBB methods. Figures 3.20 (a) and 3.20 (b) depict a comparison of computational load. The comparison demonstrates that MFDA can perform in real time under several challenging scenarios whereas JCBB is not feasible under the same conditions because of the increasing computational requirements as a result of the combinatorial explosion in the number of hypothesis traversed.

The above results show that implementing JCBB can be quite challenging from a computational perspective with large number of spurious measurements and feature densities, which is typical of many outdoor environments. In contrast, MFDA performs relatively well under all conditions discussed.

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Figure 3.17. SLAM experiment in a car park. Thick line shows the estimated robot trajectory. Crosses show the landmark estimates and curves surrounding them are uncertainty ellipses.

Figure 3.18. Features extracted and hypotheses traversed by JCBM.

Figure 3.19. Features extracted and hypotheses traversed by MFDA.

Figure (a)

Figure (b)

Figure 3.20. Complexity of data association algorithms. Logarithms of time (Figure 3.20 (a)) and ratios of MFDA with JCBM with SNN (Figure 3.20 (b)) are shown.

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3.4.5 Lagrange Relaxation Approach

The solution of MFDA based data association involving more than two frames is computationally tractable with a multi-stage Lagrangian Relaxation (LR) approach [84], [85] and [76]. This approach can also be extended to support track initialization, maintenance and termination.

It is worthwhile to consider the following feature-measurement sequences (shown as a sequence of index variables) with the same notation to extend the formulation in Section 3.4.2. A false alarm at m^{th} frame out of N frames can be denoted by the index sequence $(0, 0, \dots, 0, i_m, 0, \dots, 0)$. A new feature initialized at m^{th} frame can be denoted by the index sequence $(0, 0, \dots, 0, i_m, i_{m+1}, \dots, i_N)$. The index for the existing feature (first index) in this case is also zero as the particular feature is not yet initialized. A feature termination at m^{th} frame can be denoted by the index sequence $(t, i_1, i_2, \dots, i_{m-1}, 0, 0, \dots, 0)$. A missed detection at m^{th} frame is given by the index sequence $(t, i_1, i_2, \dots, i_{m-1}, 0, i_{m+1}, \dots, i_N)$. All the above sequences can be accommodated in the formulation given in Section 3.4.2 by removing the single source constraints of measurements (3.28), (3.29) and (3.30), constraints on missed detections (3.32), (3.33) and (3.34), and relaxing the nonzero condition for the index representing existing features. That is the index t is allowed to vary from zero to T (number of existing features) instead of from 1 to T . Thus, the data association problem can be expressed as follows.

$$\text{Minimize } \sum_{t=0}^T \sum_{i_{k-N+1}=0}^{n_{k-N+1}} \sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} c(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \quad (3.82)$$

Subject to the constraints

$$\sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_k=0}^{n_k} \eta(t, i_{k-N+1}, \dots, i_k) = 1, t = 1, 2, \dots, T \quad (3.83)$$

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$$\sum_{t=0}^T \sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} \eta(t, i_{k-N+1}, \dots, i_k) = 1, i_{k-N+1} = 1, 2, \dots, n_{k-N+1} \quad (3.84)$$

$$\sum_{t=0}^T \sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-m-1}=0}^{n_{k-m-1}} \sum_{i_{k-m+1}=0}^{n_{k-m+1}} \dots \sum_{i_k=0}^{n_k} \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) = 1, i_{k-m} = 1, 2, \dots, n_{k-m} \quad (3.85)$$

$, m = N-2, N-3, \dots, 1$

$$\sum_{t=0}^T \sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-1}=0}^{n_{k-1}} \eta(t, i_{k-N+1}, \dots, i_k) = 1, i_k = 1, 2, \dots, n_k \quad (3.86)$$

$$\eta(t, i_{k-N+1}, \dots, i_k) = \{0, 1\}, \forall i_{k-N+1}, \dots, i_k, t \quad (3.87)$$

This problem can be solved in the standard Lagrange relaxation procedure [84] and [85], as follows.

3.4.5.1 Lagrange relaxation solution

Suppose the problem (a) is defined such that,

$$\text{Problem (a)} = (3.82) \text{ subject to } (3.83), (3.84), (3.85), (3.86) \text{ and } (3.87) \quad (3.88)$$

Let the set of constraints given by (3.83) be appended to the objective function (3.82) as outlined below.

$$\begin{aligned} \text{Minimize } & \sum_{t=0}^T \sum_{i_{k-N+1}=0}^{n_{k-N+1}} \sum_{i_{k-N+2}=0}^{n_{k-N+2}} \dots \sum_{i_k=0}^{n_k} c(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \\ & + \sum_{i_k=0}^{n_k} \lambda_{i_k} \left(\left(\sum_{t=0}^T \sum_{i_{k-N+1}=0}^{n_{k-N+1}} \dots \sum_{i_{k-1}=0}^{n_{k-1}} \eta(t, i_{k-N+1}, \dots, i_k) \right) - 1 \right) \end{aligned} \quad (3.89)$$

where λ_{i_k} are Lagrange multipliers. The problem (b) and the associated N dimensional assignment problem (c) are then formed as follows.

$$\text{Problem (b)} = (3.89) \text{ subject to } (3.83), (3.84), (3.85) \text{ and } (3.86) \quad (3.90)$$

The problem (c) is formed by the following substitutions to problem (3.90).

$$j_k = \arg \min \left\{ c(t, i_{k-N+1}, \dots, i_{k-1}, i_k) + \lambda_{i_k} \mid i_k = 0, 1, \dots, n_k \right\} \quad (3.91)$$

$$\bar{c}(t, i_{k-N+1}, \dots, i_{k-1}, i_k) = c(t, i_{k-N+1}, \dots, i_{k-1}, j_k) + \lambda_{j_k}, \forall t, i_{k-N+1}, \dots, i_{k-1} \quad (3.92)$$

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$$\bar{c}(0, \dots, 0, 0) = \sum_{i_k=0}^{n_k} \text{Minimum} \{0, c(0, \dots, 0, i_k) + \lambda_{i_k}\} \quad (3.93)$$

$$\bar{\eta}(t, i_{k-N+1}, \dots, i_{k-1}) = \sum_{i_k=0}^{n_k} \eta(t, i_{k-N+1}, \dots, i_{k-1}, i_k) \quad (3.94)$$

Problem (c) is solved optimally using the zero index auction algorithm [86] when $N = 3$ and solved sub-optimally using a similar LR algorithm when $N > 3$. This approach is recursively carried out for successive stages of problem (c). A solution to problem (b) is then constructed based on the solution to problem (c) as in [76] of Deb et al.. The recovery procedure of determining a solution to problem (a) based on problem (b), which results in a 2D recovery assignment problem is then solved by the zero index auction algorithm. The above recursive procedure is repeated until the termination criteria of the optimization problem “Minimize $C_\Theta(\eta, \lambda)$ ” are reached. This problem is one of non-differentiable optimizations and therefore solved by iterative non-smooth optimization techniques such as [87] and [88]. Non-smooth optimization techniques are used to maximize the Lagrangian multipliers used in appending relaxed constraints to the objective function (Lagrangian dual).

3.4.5.2 Likelihood calculation

Using the likelihood function (3.22) and including feature initiation, maintenance and termination as discussed the following cost function can be obtained for LR problem (a) defined in Section 3.4.5.1.

$$c(t, i_{k-N+1}, \dots, i_{k-1}, i_k) = - \sum_{m=1}^{N+1} \log \left((P_1^m)^{\alpha_{i_m}} \left(\left(\frac{(1-P_x^m)P_d P_{z_{i_m}}}{P_{false}^m} \right)^{\delta_{i_m}} \left(\frac{P_{new}^m}{P_{false}^m} \right)^{n_{i_m}} \right)^{(1-\alpha_{i_m})} \right) \quad (3.95)$$

where

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$$P_1^m = \begin{cases} P_x^m & \text{if the sequence } (t, i_{k-N+1}, \dots, i_{k-1}, i_k) \text{ terminates at scan } m \\ (1 - P_x^m)(1 - P_d) & \text{if the sequence has a missed detection at scan } m \\ 1 & \text{otherwise} \end{cases} \quad \text{and,}$$

$\alpha_{i_m} = 1$ if $i_m = 0$ and $\alpha_{i_m} = 0$ otherwise. $\delta_{i_m} = 1$ if $\mathbf{z}_{i_{k-m}}$ belongs to an existing feature and $\delta_{i_m} = 0$ otherwise. $n_{i_m} = 1$ if $\mathbf{z}_{i_{k-m}}$ belongs to a new feature. P_d , P_f , P_n and P_x represent detection probability of features, probability of detecting a false alarm, probability of detecting a new feature and probability of feature termination in $k-m^{\text{th}}$ frame respectively. The probability $P_{\mathbf{z}_{i_m}}$ denotes the probability of obtaining $\mathbf{z}_{i_{k-m}}$ at $k-m^{\text{th}}$ frame given the measurement sequence from $k-N+1^{\text{th}}$ frame up to $k-m-1^{\text{th}}$ frame. As an example, the probability of $\mathbf{z}_{i_{k-m}}$ being a false alarm is zero. The likelihood calculations are carried out for every measurement combination considering robot position uncertainty as well. Further, $c(0, 0, \dots, 0, 0) = 0$ and the cost of a false alarm $c(0, 0, \dots, 0, i_m, 0, \dots, 0, 0) = 0$ is assumed.

3.4.5.3 Algorithm performance

The multi-frame data association algorithm solved using the LR approach described above was extensively tested in a simulated environment consisting of several point features. In the simulation it is assumed that a vehicle is performing feature based SLAM with the help of a range bearing sensor and encoders. A uniform clutter model [39] with Poisson distributed clutter returns was also utilized.

The multiple frame data association scheme was compared using several Monte-Carlo runs and using SNN algorithm (2D hard), 2D Auction algorithm (2D soft), 3 frame and 4 frame assignment based data association using linear programming and LR methods. Table 3.8 shows the comparison and it clearly verifies the superior performance of data association algorithms when the number of frames used in the algorithm increases. This is

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more evident when the clutter densities increase as shown in Table 3.8.

Minimizing the rms error is very important if the localization and mapping of very large areas over longer distances and spanning very large loops are to be carried out using autonomous vehicles for extended periods of time. It is known in SLAM research community that if the accumulated errors are increased over a time the loop closing error [17], [20], [38], [42] and [43] increases making it very difficult to recognize the loops in vehicle path. The rms error is an indicator, which signifies of the size of error accumulated at a particular instant. Hence the rms error over a period of time is an important measure, which stands for the long-term accuracy of a SLAM algorithm. Thus, the RMS error is monitored during 50 Monte-Carlo runs in SLAM with SNN, 2D auction, 2 frame (LR) and 3 frame (LR) based data association algorithms. It is observed that the RMS error in both lateral and heading of the vehicle decreases when the number of measurement frames in the data association algorithm increases (Table 3.9). However, increase from frames 2 to 3 has not resulted in a significant improvement as this depends on the density of clutter and persistent nature of clutter in successive frames. If the amount of persistent clutter in the environment were high then this improvement would have been much significant. The performance of data association algorithms and thus the localization accuracy of an unmanned vehicle are much improved by using multiple frame data association algorithms when the density of spurious measurements is high. The performance of data association algorithms is very clear if rms lateral errors are plotted for varying densities of spurious measurement. In fact, the rms error when utilizing multiple measurement frames decreases by a large factor as the density of spurious measurements is increased. The 2D Auction based algorithm is superior to SNN algorithm (Figure 3.21). The 2D Auction method removes the associations when there is a true measurement and spurious measurements are associated with a particular feature, whereas the SNN selects the nearest measurement to the feature in such a scenario.

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Table 3.8. Track loss criterion.

Data Association Algorithm	% Track Loss			
	False Alarm Density 0.01 at 99% Gate		False Alarm Density 0.5 at 99% Gate	
	Feature 1	Feature 2	Feature 1	Feature 2
SNN	2.7	2.8	36.5	36.2
2D Auction	2.5	2.7	31.8	32.2
LR-3D	1.0	1.1	9.4	9.2
LR-4D	0.8	0.8	8.6	8.1

Table 3.9. Comparison of RMS error

Data Association Algorithm	RMS Error	
	<u>Lateral [m]</u>	<u>Heading [rad]</u>
Nearest Neighbor	0.4042	0.0324
2D Auction based single frame	0.3850	0.0295
Two frame (LR)	0.2861	0.0187
Three frame (LR)	0.2530	0.0171

Figure 3.21. Comparison of RMS lateral error

The performance of the above data association algorithm in SLAM was experimentally verified by implementing the algorithm with data obtained using the vehicle and the sensor set up described in Section 3.4.4.4.

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Figure 3.22. Estimated vehicle path (thick line) and feature locations (crosses) of a SLAM experiment in a campus car park.

Figure 3.23. Filter performance: Range innovation (thick line) and its 95% confidence limits (dotted line).

As in the previous experiments, feature extraction is carried out using the simple clustering of the range bearing measurements obtained as done in Bailly [20]. The estimated path and the feature locations of the SLAM experiment performed in a campus car park are shown in Figure 3.22. The range innovation plot bounded by the 95% confidence limits shown in Figure 3.23 verifies that the performance of SLAM is satisfactory with the MFDA approach utilizing LR techniques.

The effects of data association algorithms on the loop closing error of SLAM was evaluated using a data set (consisting of odometry and 2D laser scanners) collected in a campus car park where moving vehicles and people are present during the experiment. The vehicle was driven at an average speed of 3 m/s in the car park along a loop of approximately 350m. The mapped feature (lamp posts and tree trunks) locations and estimated vehicle trajectories using EKF SLAM with the data set for SNN, 2D Auction based, 2 frame (LR) and 3 frame (LR) based data association algorithms are shown in Figure 3.24. Loop closing errors of 1.4m for 3 frame (LR), 1.7m for 2 frame (LR), 3.2m for 2D auction and 3.4m for SNN based data association algorithms were observed in this experiment. The results justify that the loop closing errors can be reduced significantly if the multiple frame data association algorithms are utilized instead of single frame ones.

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Figure 3.24. Comparison of data association algorithms using the loop closing error

3.5. Conditional Independence and Rao-Blackwellised Methods

As discussed in Section 2.4.2 data association in Rao-Blackwellized particle filter based methods take a different form from the algorithms presented for EKF/SLAM. These methods utilize the inherent conditional independence properties that are present in the SLAM framework for efficient estimation of the SLAM posterior and the correspondence variables.

An important relationship utilized in Rao-Blackwellized particle filter based methods is

$$P(\mathbf{x}_v^k, \mathbf{m}(k) | \mathbf{Z}^k, \mathbf{U}^k) = P(\mathbf{x}_v^k | \mathbf{Z}^k, \mathbf{U}^k) \prod_{i=1}^n P(\mathbf{x}_i(k) | \mathbf{x}_v^k, \mathbf{Z}^k, \mathbf{U}^k) \quad (3.96)$$

where $\mathbf{x}_v^k$, $\mathbf{Z}^k$, $\mathbf{U}^k$, $\mathbf{x}_i(k)$ and $\mathbf{m}(k)$ are the entire vehicle path history, observation history, control history, a feature estimate and n feature map estimate. The expression (3.96) is derived as in [10] and [11] on the basis that the knowledge of the robot's true

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path renders estimates of landmark positions to be conditionally independent. The above factorization in (3.96) establishes that the full SLAM posterior can be decomposed into a product of $n+1$ recursive estimators: one estimator over robot paths and n independent estimators of landmark positions or maps each conditioned on the path estimate.

Figure 3.25. Experiment in a small car park (170m), with FastSLAM and two frame data association

Figure 3.26. Experiment in a large area (1.1 km), with FastSLAM and two frame data association

Thus, the most important result with respect to Rao-Blackwellized methods and sliding window data association approaches is their important conditional independence properties present in the maps conditioned on path estimates. This causes the data association problem in Rao-Blackwellized methods equal to that in the robotics mapping. Hence, the sliding window approach to data association performs relatively better in this approach. Figures 3.25 and 3.26 illustrate the performance of two frame data association approach (described in Section 3.4.3.1) in FastSALM 1.0 [89]. The experimental setup was similar to the one described in Section 3.4.4.4. The ability of these methods to retain multiple data association hypotheses over several maps conditioned on path particles also enhances the performance of the data association. Nevertheless, the number of particles required to consistently approximate the map and the vehicle pose grow with map size and also increases data association overhead.

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3.6. Batch Data Association Extensions

The previous sections of this chapter described several algorithms, which can be exploited to find efficient solutions for the proposed multiple frame data association paradigm in robot navigation applications based on conditional measurement independence assumption in SLAM and MAL. These algorithms were derived based on the assumption that the measurements conditioned on the common underlying vehicle state are independent. Although this assumption is mostly valid and is used by many (please see discussion in Section 3.3), in extreme cases it has its limitations. More specifically, in SLAM and MAL problems this assumption gives rise to difficulties in the data association process when the vehicle uncertainty is quite large and the feature density is very high. Because, in the context of SLAM and MAL, when the vehicle uncertainty is large and the feature density is very high one cannot ignore the effects of strong spatial correlations on the measurements.

Joint Compatibility Branch and Bound (JCBB) [38] and Combined Constrained Data Association (CCDA) [20] are well known batch data association schemes, which take spatial correlation into account implicitly by considering batches of target-measurement pairs in the association process. The major disadvantage of such schemes however, are as reviewed earlier (Section 3.3.5), their inability to make soft decisions or in other words, their inability to permit reversal or corrections of decisions at a later stage in the presence of new evidence. Furthermore, JCBB and CCDA only take into account the measurements in the current time frame and ignore temporal attributes of the collective measurements in decision making. Hence, JCBB and CCDA suffer from missed associations in dealing with spurious measurements or dynamic objects. In the next section it is shown how the proposed multiple frame multi-dimensional data association framework can be extended to include the strong spatial and temporal correlations, whilst permitting soft decision making.

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3.6.1 The Multiple Frame Temporal Consistency Criterion

An effective strategy towards overcoming the limitations of the conditional measurement independence assumption and being able to make soft data association decisions is to combine a batch scheme with the sliding window approach. However, extending batch data association methods over multiple frames for MAL and SLAM problems is NP hard. Thus, to reduce the search space, we propose a sub-optimal method (in the sense we don't traverse the entire search space and instead select most probable hypotheses from a temporal consistency criterion). In this sub-optimal method, measurement-feature pairs consistent over a multiple frame sliding window are selected as probable associations for SLAM using a coarse gating technique. Measurement-target associations over multiple frames are then placed in an extended interpretation tree [90] consisting of all possible measurement feature combinations. If there are n_k measurements in the current frame the interpretation tree is constructed to accommodate n_k levels. If measurement i_k in frame k consists of f_k feature-measurement combinations in the window of multiple frames (allowing for missed detections, spurious measurements and the map feature repetitions in the tree) we have a search space of $\prod_{k=1}^{n_k} f_k$ hypotheses for measurement to feature associations in the selected sliding window of measurement frames. However, if the single return and single source constraints ((3.28)-(3.31)) are imposed we have a reduced number of hypotheses. Traversing the hypothesis tree can now be done as in [20] or [90]. The method [90] is used here to determine the hypothesis with the largest number of jointly consistent pairings as the most probable (measurement, feature) association set. We term the association hypothesis satisfying the above largest number of jointly consistent pairings as satisfying the multiple frame temporal consistency (MFTC) criterion.

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3.6.1.1 Selection of feature and measurement combinations

The advantage of the proposed batch sliding window data association scheme employing multiple frames of measurement is that it resolves ambiguities arising from persistent clutter and moving objects and permits the reversal of the previously taken data association decisions as in MHT. On the one hand, this batch method can be further extended to the MHT form by considering all the measurement combinations having an infinite window, however, with insurmountable computational complexity. On the other hand if the sliding window is limited to one frame of measurement the scheme essentially reduces to the batch association method. Thus, one could deploy the algorithm proposed with a reasonable number of measurement frames to permit soft decision making and to correctly account for the spatial and temporal correlations of the measurements.

Let $\mathbf{X}(k-N+1|k-N)$ and $\mathbf{P}(k-N+1|k-N)$ be the prediction of composite state vector and its covariance at time $k-N+1$ from those at time $k-N$. As in Anderson and More [91], in each subsequent prediction of the state vector and its covariance we define the modified state vector and its covariance by adding the current vehicle trajectory state to the last position of the state vector. As an example, the predicted state vector and its covariance at time $k-N+2$ are

$$\mathbf{X}(k-N+2|k-N+1) = \begin{bmatrix} \mathbf{f}(\mathbf{x}_v(k-N+1|k-N), \mathbf{u}_{k-N+1}) \\ \mathbf{m}(k-N+1|k-N) \\ \mathbf{x}_v(k-N+1|k-N) \end{bmatrix} \quad (3.97)$$

$$\mathbf{P}(k-N+2|k-N+1) = \begin{bmatrix} \mathbf{F}_v \mathbf{P}_{\mathbf{x}_v \mathbf{x}_v}(k-N+1|k-N) \mathbf{F}_v^T + \mathbf{Q}_v & \mathbf{F}_v \mathbf{P}_{\mathbf{x}_v \mathbf{m}}(k-N+1|k-N) & \mathbf{F}_v \mathbf{P}_{\mathbf{x}_v \mathbf{x}_v}(k-N+1|k-N) \\ \mathbf{P}_{\mathbf{m} \mathbf{x}_v}(k-N+1|k-N) \mathbf{F}_v^T & \mathbf{P}_{\mathbf{m} \mathbf{m}}(k-N+1|k-N) & \mathbf{P}_{\mathbf{m} \mathbf{x}_v}(k-N+1|k-N) \\ \mathbf{P}_{\mathbf{x}_v \mathbf{x}_v}(k-N+1|k-N) \mathbf{F}_v^T & \mathbf{P}_{\mathbf{x}_v \mathbf{m}}(k-N+1|k-N) & \mathbf{P}_{\mathbf{x}_v \mathbf{x}_v}(k-N+1|k-N) \end{bmatrix} \quad (3.98)$$

where $\mathbf{F}_v$ is the Jacobian $(\partial \mathbf{f} / \partial \mathbf{x}_v)$ of the vehicle motion model evaluated at time $k-N+1$. The symbols $\mathbf{m}$ and $\mathbf{x}_v$ denote the entire map state and the vehicle state

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respectively. On subsequent predictions in the window of frames, this procedure is repeated by augmenting the state vector from the appropriate trajectory states and predicting until the state vector and the covariance matrix at time k is obtained. Calculation of the measurement Jacobian $\mathbf{H}$ of the measurement model (equation (2.11)) now consists of three parts Jacobian $\mathbf{H}_v$ corresponding to the vehicle state, Jacobian $\mathbf{H}_m$ corresponding to the map state and Jacobian $\mathbf{H}_{traj}$ corresponding to the trajectory states added to the state vector.

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_v & \mathbf{H}_m & \mathbf{H}_{traj} \end{bmatrix} \quad (3.99)$$

Let $\mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)$ and $\mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)$ where $t = 1, 2, \dots, T$, $i_k = 1, 2, \dots, n_k$ and $i_{k-m} = 0, 1, 2, \dots, n_{k-m}$, $\forall m > 0$ denote the Jacobian and the measurement innovation representing a feature τ_t associated with measurement sequence $\mathbf{z}_{i_{k-N+1}}, \mathbf{z}_{i_{k-N+2}}, \dots, \mathbf{z}_{i_{k-1}}$, and $\mathbf{z}_{i_k}$ in N frames ($k-N+1, k-N+2, \dots, k-1$ and k) of measurement. Here, $i_{k-m} = 0$ denotes that there is no matching between the feature and the i_{k-m}^{th} measurement at frame $k-m$. If this happens, the composite innovation does not contain a corresponding entry for this particular measurement frame. The non-matching is however allowed at the intermediate time frames only and it is not allowed at frame k . $\mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)$ and $\mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)$ are then obtained by concatenating innovation sequences and appropriate Jacobians in the form of a composite innovation vector and a composite Jacobian matrix.

$$\mathbf{e}(t, i_{k-N+1}, \dots, i_k) = \left[(\mathbf{z}_{i_{k-N+1}} - \hat{\mathbf{z}}_{i_{k-N+1}})^T \quad (\mathbf{z}_{i_{k-N+2}} - \hat{\mathbf{z}}_{i_{k-N+2}})^T \quad \dots \quad (\mathbf{z}_{i_k} - \hat{\mathbf{z}}_{i_k})^T \right]^T \quad (3.100)$$

$$\mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k) = \begin{bmatrix} \mathbf{H}_{i_{k-N+1}}^T & \mathbf{H}_{i_{k-N+2}}^T & \dots & \mathbf{H}_{i_k}^T \end{bmatrix}^T \quad (3.101)$$

where $\mathbf{z}_{i_{k-m}}$ and $\hat{\mathbf{z}}_{i_{k-m}}$ for all $m = N-1, N-2, \dots, 1, 0$ consist of the true measurement of the feature t at time $k-m$ and its prediction at $k-m$. $\mathbf{H}_{i_{k-m}}$ is the Jacobian corresponding to

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the innovation vector $(\mathbf{z}_{i_{k-m}} - \hat{\mathbf{z}}_{i_{k-m}})$. Omitting the index term, $(t, i_{k-N+1}, \dots, i_k)$, the covariance $\mathbf{S}$ of the association is

$$\mathbf{S} = \mathbf{H}\mathbf{P}\mathbf{H}^T + \mathbf{R}_{aug} \quad (3.102)$$

where $\mathbf{R}_{aug}$ is the covariance of the composite measurement noise vector whose diagonal terms consist of $\mathbf{R}$. The Mahalanobis distance $D^2 = \mathbf{H}^T \mathbf{S}^{-1} \mathbf{H}$ is then calculated and the association hypothesis is accepted only if the value of D^2 is less than the value of Chi-squared distribution of $\dim(\mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k))$ degrees of freedom at a selected confidence level. Once the measurement target combinations are accepted they are used to construct the interpretation tree. Traversing of the interpretation tree is done as in Grimson [90] and the hypothesis having the maximum number of consistent pairs is taken as the correct assignment.

For example, suppose that there are 5 features $\{f_i \mid i=1,2,3,4,5\}$ in the map constructed by a SLAM algorithm. Let 4 measurements $\{m_i^{k-1} \mid i=1,2,3,4\}$ in frame $k-1$ and 2 measurements $\{m_i^k \mid i=1,2\}$ in frame k fall inside the gates of these features as shown below in Figure 3.27 where the gates are shown by dashed ellipses. Figures 3.28 and 3.29 summarise the possible feature to measurement associations. Arrows in the figures indicate a measurement falling inside the 95% confidence region of a feature. Measurements m_1^k and m_2^k have 4 and 8 possible combinations of association including the possibility of non-association. Thus, the hypothesis space of 2 frame MFTC consists of $4 \times 8 = 32$ hypotheses. The interpretation tree can now be constructed to accommodate all the 32 hypotheses. The association ambiguities are then resolved using the MFTC criterion.

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Figure 3.27. Feature-measurement association. The 95% confidence limits of features are shown by ellipses (dashed lines). Features are shown by the rectangles and the measurements are shown by the circles.

Figure 3.28. Possible associations for measurement m_1^k

Figure 3.29. Possible associations for measurement m_2^k

3.6.1.2 Construction of the interpretation tree

An example interpretation tree is illustrated in Figure 3.30. Assume that there are n features ($\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_n$) and m measurements ($\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_m$) in a particular association scenario. The symbol $\mathbf{m}_0$ is used to denote the missed detection.

Further, the possible association pairings for i^{th} measurement is assumed to be n_i where $n_i \leq n \forall i$. The tree defining all the partial interpretations among features and measurements can be represented by the interpretation tree structure [90]. Each node of

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the tree defines a partial interpretation where each ancestor defines a particular measurement (Figure 3.30) and a branch leading to each node defines the corresponding feature. The hypothesis space of this interpretation tree is $\prod_{i=1}^m (n_i + 1)$ allowing repetitions and spurious measurements. This type of an interpretation tree is constructed to represent possible associations prior to any MFTC calculation as depicted in the previous paragraph with association combinations in the sliding window as features and the measurements in the current frame as measurements of the interpretation tree.

Figure 3.30. A typical interpretation tree.

3.6.1.3 Evaluation of the MFTC criterion

Suppose that there is an l node branch in the interpretation tree, which needs to be evaluated. Let the feature measurement association combinations in this branch be denoted by $\eta(t, i_{k-N+1}, \dots, i_k)_i$ where $i = 1, 2, \dots, l$, $i_k > 0$, $t > 0$ and $i_{k-m} \geq 0$, $\forall m > 0$. using the previous notation. $\mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_i$ and $\mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_i$ can now be calculated for all $i = 1, 2, \dots, l$. The algorithm of finding the maximum consistent pairs is as follows.

Step (a) $i = l$

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Step (b) Calculate:

$$\mathbf{e}_{comp} = \begin{bmatrix} \mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_1 \\ \mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_2 \\ \dots \\ \mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_i \end{bmatrix}, \quad \mathbf{H}_{comp} = \begin{bmatrix} \mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_1 \\ \mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_2 \\ \dots \\ \mathbf{H}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_i \end{bmatrix} \quad \text{and}$$

$\mathbf{R}_{comp} = \text{diag}(\mathbf{R}_{aug,1}, \mathbf{R}_{aug,2}, \dots, \mathbf{R}_{aug,i})$, where $\mathbf{R}_{aug,i}$ is the covariance matrix of the composite measurement noise vector corresponding to $\mathbf{e}(t, i_{k-N+1}, i_{k-N+2}, \dots, i_k)_i$.

Step (c) Calculate:

$$\mathbf{S}_{comp} = \mathbf{H}_{comp} \mathbf{P} \mathbf{H}_{comp}^T + \mathbf{R}_{comp} \quad \text{and} \quad D^2 = \mathbf{H}_{comp}^T \mathbf{S}_{comp}^{-1} \mathbf{H}_{comp}$$

Step (d) Find if D^2 is less than or equal to the 95% confidence value determined from the Chi-squared distribution of $\dim(\mathbf{e}_{comp})$ degrees of freedom is satisfied.

Step (e) If the condition in Step (d) is satisfied, the maximum number of pairs is equal to i and the search is completed. Otherwise decrease i by 1 and go to step (b).

The data association satisfying the MFTC criterion is now the set of measurement-feature combinations as determined by the maximum number of consistent pairs.

3.6.2 Algorithm Performance

Data association algorithm performance in SLAM is tested with Multiple Frame Temporal Consistency (MFTC) using two temporal frames of measurement in a batch. The feature measurements are assumed to be from a scanning range measurement system such as a SICK LMS 290. The control input to the vehicle kinematic model is assumed to be from wheel encoders. The vehicle is assumed to be traveling in a manually generated path (Figure 3.4) in a $100 \times 100 \text{m}^2$ area environment consisting of 100 randomly generated

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features.

Figure 3.31 (a) Lateral position error

Figure 3.31 (b) Heading error

Figure 3.31. Error performance of SLAM using MFTC criterion in data association using two measurement frames. Thick lines are used to show the errors in position and heading and the dashed line is used to show 95% confidence limits of the estimated error.

The spurious measurements (that may be due to the nature of environment or poor sensor performance or both) are assumed uniformly distributed in the environment and their returns are assumed Poisson distributed [39]. The simulation specific parameters are speed input error 0.5 m/s, steering angle encoder error 0.05 radians, range measurement noise 0.1 m, bearing measurement error 0.5 degrees and spurious measurement or clutter density 0.001 m^{-2} . Figure 3.31 establish that the error bounds of position (only lateral position error is shown) and orientation of MFTC algorithm are bounded by the 95% confidence limits demonstrating the effectiveness of MFTC data association algorithm in EKF SLAM.

The MFDA & MFTC data association schemes in EKF-SLAM are compared with the SNN data association filter and JCBB by performing several Monte-Carlo runs under the same conditions and using the same set of simulation parameters.

A good measure of performance of data association in this case is the percentage missed associations or percentage track loss measure [40], which is the ratio of missed associations to total observed instances of a feature as a percentage. Table 3.10 depicts track loss of the various methods with varying densities of spurious measurement. It also

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shows that the performance of data association in SLAM is significantly improved with MFDA with two-frames and MFTC with two frames compared to the standard nearest neighbor data association especially in high densities of spurious measurement.

Table 3.10 Comparison of algorithms with spurious measurements

Data Association Algorithm	Average % Track Loss per Feature	
	False Alarm Density Per Unit Area <u>0.001</u>	<u>0.01</u>
SNN	2.7	15.1
JCBB	1.5	11.4
MFDA	1.3	9.3
MFTC	1.1	8.8

The MFDA & MFTC algorithms' performances are comparable to JCBB at lower densities of spurious measurement. However, with increasing densities of spurious measurement there is a marked improvement in performance of MFDA & MFTC over JCBB. This is because MFDA & MFTC both take temporal attributes of features into account filtering out spurious measurement. MFTC is also superior in performance to MFDA in high densities of spurious measurement (Table 3.10) as it takes the spatial correlations of the measurements into consideration.

3.6.2.1 Computational complexity

The preprocessing and gating can reduce the complexity of MFDA & MFTC significantly. Complexity of the MFTC data association can be reduced significantly by traversing the interpretation tree branches having the lowest D^2 first enabling to reach branches of the interpretation tree having higher degree of consistency prior to others [90]. When traversing branches of the interpretation tree, a heuristic criteria to minimize traversal excessively is doing consistency tests first on the current frame of measurements and testing the largest hypothesis in the current frame with previously resolved hypotheses for past frames. If this test gives the largest number of consistent pairs there is

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no need to search the entire exponential search space.

Table 3.11 Comparison of average computational load

Data Association Algorithm	CPU Time (s)	
	Number of Association Hypotheses	
	<u>50</u>	<u>100</u>
SNN	0.018	0.042
MFDA	0.098	0.214
JCBB	0.154	0.452
MFTC	0.173	0.514

In this work several studies were conducted to compare the actual computation time required by MFDA, MFTC, SNN and JCBB in SLAM. The CPU time requirement (Table 3.11) for the preprocessing and data association functions are determined using several Monte-Carlo runs in MATLAB environment by using a Pentium 4, 2.4 GHz, 512 MB RAM PC. Of the four algorithms compared, the computation time for MFTC is the highest. Although the computation time for MFDA is much higher than SNN, it is still feasible for real-time implementation in EKF-SLAM. This concludes that the most optimal MFTC is computationally demanding. However, it is the most robust approach to data association of the four methods compared, with abilities to filter out both temporal and spatial correlations arising from spurious measurements and dynamic objects.

3.6.2.2 Experiments

The experimental verification of the MFTC data association algorithm in EKF-SLAM was done using the experimental setup as described in Section 3.4.4.4. The estimated path and the feature locations of the SLAM experiment performed with 2 frame data association (MFTC) in this environment is shown in Figure 3.32. The car park is a dynamic environment with people, cars and bicycles moving around. The well-bounded innovations shown in Figure 3.33 verify that the performance of SLAM is satisfactory with the data association algorithm. Figure 3.34 shows the estimated lateral and

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longitudinal uncertainty of the SLAM algorithm, which indicates that estimated errors are reduced when MFTC algorithm is used to filter out the dynamic objects.

The MFTC algorithm is further evaluated using the measured ground truth of landmarks with reference to the vehicle starting position, which is chosen as the origin of the global reference coordinate frame. The EKF-SLAM algorithm with two-frame MFTC data association is run for the case in Figure 3.35 (A case with approximately 170m stretch of road in a $60 \times 60 \text{m}^2$ area car park in the campus). Figure 3.36 shows the true errors associated with the measured landmark position. Landmark one used for ground truth evaluation is shown in Figure 3.35.

Figure 3.32. Estimated vehicle path (thick line) and feature locations (circles) (SLAM experiment in a campus car park).

The errors bounded by 2σ limits demonstrate the accuracy of map estimation (and hence vehicle path estimate), filter consistency and the good performance of the MFTC criterion in the data association in SLAM.

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Figure 3.33 Filter performance: Range innovation (thick line) and its 95 % confidence limits (dotted line).

Figure 3.34. Variation of lateral (thick line) and longitudinal (dashed line) uncertainty

Figure 3.35. Estimated vehicle path (thick line) and feature locations (crosses) and landmark uncertainty ellipses are shown around landmark estimates. (SLAM experiment in a campus car park).

Figure 3.36. Difference between the actual and estimated location for landmark 1. The 95% confidence limit is shown by dotted lines for SLAM with MFTC.

Figures 3.37 and Figure 3.38 show the variation of the total number of hypotheses traversed and the maximum number of compatible pairs detected in an iteration of the

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MFTC algorithm in the above experiment (The experimental run shown in Figure 3.35).

The number of features extracted is also plotted in both the figures for comparison.

Figure 3.37. Total number of the hypotheses traversed and the features extracted over the time

Figure 3.38. Maximum number of compatible pairs detected and the number of features extracted over the time

These results demonstrate the effectiveness of MFTC in a real world setting and its ability to sustain the available computing resources without exponentially increasing the number of hypotheses traversed in an iteration.

3.7. Data Association in Dynamic Environments

Presence of dynamic objects in the environment violates the static map assumption in the standard SLAM problem. In addition, the presence of dynamic objects (eg pedestrians, animals, cars, buses and bicycles etc.) with varying sizes and moving at varying speeds make the feature extraction process complex and error prone. There is also a high probability of misinterpreting temporarily static objects such as vehicles stopped at traffic lights as static objects. Thus, if we are to deploy autonomous platforms running SLAM algorithms in dynamic environments such as urban crowded environments and industrial premises there should be proper means of filtering out moving objects from the static objects. Further, the data association problem becomes difficult due to occlusion and clutter resulting from dynamic objects. Experiments in crowded environments show significant increase in false alarm rates and a rise of false matches therein.

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A dynamic or moving object's motion is modelled using a Brownian motion model (where the velocity of the object $\mathbf{v}_k$ at time k is given by $\mathbf{v}_{k+1} = \mathbf{v}_k + \mathbf{v}_n(k)$ where $\mathbf{v}_n(k) \sim N(0, Q_n)$) in the simulations. The Brownian model was used by Montermerlo, Thurn and Whittakar 2002 in [136] and [89] to simulate the dynamic objects successfully in mobile robotics. This model assumes nothing about the speed and direction of the dynamic object in particular. i.e. a dynamic object can travel at any direction at any time. i.e. the velocity of the moving object is subject to random perturbations. The number of dynamic objects at a given instance is obtained from a Poisson distribution whose density is varied from 0.001 /m² to 0.01/ m². A uniform probability density function (PDF) is used to initialize the position and a Gaussian PDF to initialize the speed of an object. A simulation environment (figure 3.39) is used to model the performance of SLAM with different data association algorithms in SLAM. Parameters of the simulation were as used in Section 3.6.2. Table 3.12 shows the vehicle's X-position rms error measure with MFDA, MFTC, JCBB and NN data association methods in SLAM.

Figure 3.39. Performance of data association with dynamic objects. Rectangles, circles and crosses represent dynamic object true locations, true landmarks and estimated landmarks. Dashed line show the true robot path and the thick line show the estimated trajectory.

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Table 3.12 Error performance with dynamic objects

Data Association Algorithm	RMS Error in Localization in X Direction	
	<i>Dynamic Object Density Per Unit Area</i>	
	<i>0.001</i>	<i>0.01</i>
SNN	0.4523	0.6820
JCBB	0.3522	0.4260
MFDA	0.2891	0.3720
MFTC	0.2574	0.3251

Table 3.13 Track loss measure with dynamic objects

Data Association Algorithm	Average % Track Loss per Feature	
	<i>Dynamic Object Density Per Unit Area</i>	
	<i>0.001</i>	<i>0.01</i>
SNN	3.2	18.4
JCBB	2.5	12.0
MFDA	1.7	9.8
MFTC	1.4	8.9

Table 3.13 depicts the variation of track loss measure. It is observed that MFDA & MFTC clearly outperform both JCBB and SNN. Although JCBB tests consistency of groups of measurements in the current time frame thereby dealing with the spatial correlations in the present time frame, it lacks the ability to filter out temporal correlations over several time frames. In contrast, MFDA & MFTC filter out spurious measurements or returns due to dynamic objects by checking for consistency of measurement to feature associations over several consecutive frames. Therefore, MFDA & MFTC are capable of addressing the problems due to dynamic objects and temporarily persistent clutter. Moreover, a dynamic object is most probably not located in the same place in the validation gate (as viewed by the data association algorithm) of the estimated landmark in more than one consecutive measurement frame. In contrast, a static object is always located in the same gate of a landmark as viewed from the data association algorithm in more than one consecutive measurement frame. Thus, multiple frame data association algorithms can easily distinguish dynamic objects from static objects. Therefore, if we are using a multiple frame data association algorithm we don't have to classify the dynamic objects from the static objects.

3.8. Conclusion

A generalized sliding window framework employing multiple frames of measurement was proposed to address the data association problem in robot navigation applications. Most data association algorithms reported in the tracking and robotics literature were shown to be within the scope of this framework. Based on this generalized data association framework, a sliding window based multiple frame data association algorithm known as MFDA was formulated for SLAM and MAL applications assuming that the feature measurements are conditionally independent. A sub-optimal solution based on linear programming was proposed to overcome the NP complexity of the optimal formulation. The MFDA algorithm was shown to be a practical and effective alternative to the theoretically optimal MHT. Compared to single measurement frame methods, the MFDA was shown to resolve association incompatibilities and ambiguities more effectively and to yield consistent maps and vehicle states especially with increased spurious measurements (clutter), feature densities and in the presence of moving objects. In particular, it was shown that even though the complexity of the two-frame MFDA is high it significantly outperforms SNN data association.

It was also shown that MFDA outperforms JCBB (which takes full account of the spatial correlations between vehicle and feature measurements) at relatively higher clutter and in the presence of moving objects. In addition, MFDA was shown to be more efficient computationally and amenable to real time implementation unlike JCBB (despite JCBB's use of branch and bound pruning of its exponential search space). This was observed mainly in dynamic environments with high spurious measurement densities. When clutter is high and features are sparse, the compatibility information of features of a single measurement frame (as in SNN and JCBB) is not sufficient to make effective data association decisions thus compromising performance of single frame based methods. However, in a multiple measurement frame approach the availability of more than one

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frame of measurement provides for more effective data association decisions as consistency of measurements are looked at in several frames of measurement. Thus, the performance of MFDA has shown a marked improvement over other data association algorithms such as SNN and JCBB, in high clutter and in the presence of dynamic objects. Moreover, it was shown that the MFDA algorithm could be readily deployed using standard PC computing resources in real time without any custom processing hardware. This was observed primarily due to the polynomial time complexity of the primal-dual infeasible-interior point approach which was used to obtain a sub-optimal solution to the 0-1 integer-programming problem of multi-dimensional assignment.

The conditional independence assumption used to formulate MFDA data association in SLAM was then relaxed and an extended data association algorithm employing both spatial and temporal attributes of the measurement to feature correlations (MFTC) was developed. The effectiveness of MFTC over the common data association algorithms (SNN, JCBB, and MFDA) was illustrated in the context of performance in higher densities of spurious measurement, moving objects and high vehicle uncertainty. Development of MFTC therefore results in more robust and statistically correct avenues to be opened towards multiple-frame paradigm of data association in SLAM. Moreover, robust data association algorithms such as MFTC would definitely contribute in developing localization algorithms with less reliance on efficient and effective feature extraction methods.

Chapter 4

Use of Multiple Sensors in SLAM

4.1. Introduction

As discussed in the previous chapters, data association plays a vital role in robot localization and mapping in highly cluttered outdoor environments. However, if autonomous vehicles are to be deployed successfully in the outdoor environments as in [1], [3] and [8] to perform mission critical tasks, one must also take into consideration other important issues such as multi-sensor integration and fusion and sensor modeling aspects. Because the sensors currently available for robot localization are too inaccurate and unreliable, it is always important to provide complementary sensing mechanisms to achieve enhanced performance and acquire greater detail of the environment where the mobile robot traverses. Humans and animals often rely on several different sensors to gather information and to make decisions. Towards this end, multi-sensor fusion is used in mobile robot localization to combine inputs from many independent sources (sensors) of limited accuracy and reliability to acquire information of good quality and greater detail. Advantages of general multi-sensor data fusion include improved state estimation accuracy, improved observability by the relative placement of sensors, increased reliability, wide coverage and adaptability to changing weather and environment conditions. In particular, sensor fusion provides SLAM and MAL with several advantages in outdoor unstructured environments. Use of multiple sensors provides a good chance of

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observing many features and thereby improving the accuracy of the map and the robot's position in sparse outdoor environments. Features not seen by one sensor can be visible to another if the vantage points of two sensors are different providing complementary information for the state estimation. In addition, use of multiple sensors improves the performance of SLAM algorithms in all weather conditions. For example in rainy and gloomy conditions use of RADAR [19] can complement laser or vision [6].

Although use of many exteroceptive sensors in SLAM is seldom reported, several types of individual sensor modalities [1], [7], [9], [12], [13], [20], [24], [30] and [77] are utilized in SLAM as exteroceptive sensors. Millimeter Wave Radar (MMWR) provides all weather all environment performance in sensing often not present in laser and vision, while providing the directivity and accuracy. However, use of MMWR in feature based SLAM such as in [7] and [19] is shown to be extremely difficult due to difficulties in feature extraction and interpretation. Specular reflections and large noise present in MMWR prevent accurate feature extraction as in the case of laser. Laser range finder [8], [77], [30] and [20], which provides accurate range-bearing information is one of the widely used sensors in SLAM. Laser is a very reliable sensor, which gives good performance in both indoor and outdoor environments in feature based and sensor centric environmental representations. Use of vision as an exteroceptive sensor in SLAM [24], [13], [12] and [100], is also reported in several indoor applications. Vision is desirable as an exteroceptive sensor because of the low cost, low power consumption, compact space requirement and rich information content it delivers. However, vision as in [94] and [95] alone is not robust in several environmental conditions where changing illumination, shading and low visibility conditions prevail. Further, use of a monocular camera cannot provide range information to a feature directly as it provides only the bearing [100] of the feature at a particular instant. Possibility of incorporating more than two cameras in stereo configuration in a single base line or in multiple base lines as in [96] and [97] can be used

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if direct range information to features is expected. Nevertheless, the calibration problems and the processing power requirements limit the use of stereo in SLAM especially in outdoor setting.

The use of multiple exteroceptive sensors received little attention of the SLAM research community in particular in outdoor environments. In addition, the theoretical issues in SLAM related to sensor fusion architectures are not well addressed. The work of Castellanos et al. [93] involves a laser range finder and a monocular camera in SLAM for the indoor environments. However, this work ([93]) fuses data obtained from multiple sensors sequentially (in Section C, page 911 of [93]) and therefore does not take into consideration that features and measurements in the composite SLAM state vector can be fully correlated [7]. Further, the use of one narrow angle monocular camera as in [93] is quite limiting, especially in an outdoor setting where features are sparse and distributed in a large area. A configuration of multiple sensors used in mobile robot localization and world modeling is described in Cid et al. [135]. This work ([135]) uses both a color monocular camera and a stereovision system to recognize and classify landmarks. However, as to how this information is used to localize the robot is not described in detail. It appears as though a separate DGPS system is used in the localization of the mobile platform.

Thus, a promising strategy of using multiple sensors in outdoor SLAM would be to use a laser range finder and a monocular camera together. Vision provides a large field of view. However, with a monocular camera only bearing information of features will be available. Therefore, to facilitate initialization of these features a laser range measurement can be used. This strategy of using vision and laser together can provide for robust and reliable navigation over long periods of operation over large outdoor spaces. Further, if necessary the rich information content of vision (such as color or texture) can be used as additional attributes to aid feature initialization and data association.

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This chapter is organized in the following manner. Theoretical issues pertaining to the correlations that exist among map feature states and the vehicle state in relation to the multi-sensor fusion in SLAM are investigated initially. A strategy of utilizing a laser range finder and two monocular cameras in SLAM in an outdoor setting is then elaborated. Effects on feature extraction, initialization, camera calibration and the experimental set up are detailed. A new performance metric quantifying the use of multiple sensors in SLAM is proposed. A theoretical framework and justification of improving multi-sensor performance in SLAM through active fusion concepts are also developed.

4.2. Sensor Fusion in SLAM

There are two main sensor fusion methods used to fuse data from multiple sensors. They can be categorized as (1). Sequential sensor fusion method and (2). Group sensor fusion method. In sequential sensor fusion method, observations received at the same instance of time are used sequentially to update the estimated state. As an example, let the estimated SLAM state be $\mathbf{X}(k)$ and its covariance matrix be $\mathbf{P}(k)$. Also suppose that two measurements $\mathbf{z}_1(k)$ and $\mathbf{z}_2(k)$ from two sensors are received at time k . The sequential sensor fusion method first updates $\mathbf{X}(k)$ and $\mathbf{P}(k)$ using $\mathbf{z}_1(k)$ and then updates the resulting updated state of $\mathbf{X}(k)$ and its covariance matrix $\mathbf{P}(k)$ by $\mathbf{z}_2(k)$. The sequential sensor fusion method therefore assumes that the two measurements are independent from each other. However, in the context of SLAM and MAL the measurements are dependent on the vehicle state and therefore the measurements are strictly not independent. Thus, the sequential sensor fusion method can only be justified if it is assumed that both the measurements conditioned on the vehicle state are independent. In any case, in the case of asynchronous measurements, sequential sensor fusion method is the only choice.

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In the group sensor fusion method, the measurements obtained at a particular instant of time are grouped into a composite measurement vector and the corresponding update operation is carried out in one step. This method of sensor fusion can in theory support simultaneous measurements from any number of sensors and the prediction phase is invariant to the number of observations. Let there be n sensors, which receive measurements $\mathbf{z}_i(k)$; $i=1,2,\dots,n$ at time k . Each measurement $\mathbf{z}_i(k)$ of sensor i consists of measurements of map features, false alarms and outliers. After correct association with the map features in the state vector we have the following measurement vectors from each sensor $i=1,2,\dots,n$.

$$\mathbf{z}_i(k) = \begin{bmatrix} \mathbf{z}(i,1) \\ \mathbf{z}(i,2) \\ \dots \\ \mathbf{z}(i,n_i) \end{bmatrix} \quad (4.1)$$

where $\mathbf{z}(i, j)$ is a measurement j obtained by sensor i at time k associated with a feature in the map. Let the measurement predictions of $\mathbf{z}_i(k)$ be $\hat{\mathbf{z}}_i(k)$ for $i=1,2,\dots,n$. Thus, the sensor fusion using EKF can be done using by the following equations.

$$\mathbf{e}(k) = \begin{bmatrix} (\mathbf{z}_1(k) - \hat{\mathbf{z}}_1(k))^T & (\mathbf{z}_2(k) - \hat{\mathbf{z}}_2(k))^T & \dots & \dots & (\mathbf{z}_n(k) - \hat{\mathbf{z}}_n(k))^T \end{bmatrix}^T \quad (4.2)$$

$$\mathbf{H}(k) = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \dots & \dots & \mathbf{H}_n^T \end{bmatrix}^T \quad (4.3)$$

$$\mathbf{S}(k) = \mathbf{H}(k)\mathbf{P}(k | k-1)\mathbf{H}^T(k) + \mathbf{R}(k) \quad (4.4)$$

where $\mathbf{R}(k)$ is the block diagonal matrix representing the measurement noises of individual sensors along the diagonal and $\mathbf{H}_i$ denote the Jacobian of $\hat{\mathbf{z}}_i(k)$ with respect to $\mathbf{X}(k)$. The EKF state update equations are as follows.

$$\mathbf{X}(k | k) = \mathbf{X}(k | k-1) + \mathbf{K}(k)\mathbf{e}(k) \quad (4.5)$$

$$\mathbf{P}(k | k) = \mathbf{P}(k | k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k) \quad (4.6)$$

where $\mathbf{e}(k)$, $\mathbf{S}(k)$, and $\mathbf{K}(k) = \mathbf{P}(k | k-1)\mathbf{H}^T\mathbf{S}^{-1}(k)$ are the observation innovation, its covariance matrix, and the Kalman gain with the usual notation. The main point of significance in group sensor fusion method is that it takes into consideration the correlations that exist among observations taken at any particular instance in the update

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operation through the calculation of $S(k)$ and the Kalman gain. Thus, measurements from any number of sensors can be combined using the group sensor fusion method at the same time taking into consideration the strong correlations [7] that exist in the SLAM state vector. Hence the group sensor fusion approach is the appropriate choice when measurements received simultaneously from several exteroceptive sensors are to be fused in the context of SLAM or MAL. In this work the concepts of fusing multiple sensors in SLAM were demonstrated using two monocular cameras and an LMS.

4.3. Combining Laser and Monocular Vision in SLAM

4.3.1 Incorporating Bearing Only Information from Monocular Cameras in SLAM

4.3.1.1 Background

In general, monocular cameras provide bearing information only of the features in an environment. Hence, it is quite important to investigate the insight of incorporating bearing information in SLAM prior to attempting any implementation. Before one looks into using bearing information from monocular cameras in SLAM (Bearing only SLAM), it may be useful to consider a problem in vision known as Structure from Motion (SFM) [37], [78], [101], [102] and [103], which is very akin to SLAM. In computer vision and photogrammetry SFM involves the use of monocular camera image sequences to reconstruct the scene and the camera motion simultaneously. Had the structure of the scene (the map in the context of SLAM) been known a priori (as in MAL), it would have been straightforward to recover the camera trajectory (localization information in MAL) or the camera motion. In the same manner, had the camera trajectory (motion) been known a priori (as in RM) the reconstruction of the scene structure (map) would have been trivial. SFM can therefore provide the basis for incorporating bearing information from cameras in the related problem of SLAM. However, most of these SFM techniques implicitly exploit the conditional independence assumption [37].

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The most noteworthy distinction between SLAM and SFM using bearing information from a camera is that the former aims to carry out its operation in a sequential manner, whereas the latter performs the estimation offline in batch mode. Batch estimation techniques treat all data received from the beginning to the end with equal importance and find optimal parameter estimates. These batch techniques are thus rendered useless in real-time SLAM implementations. Therefore, SFM techniques are not directly suitable to the bearing only SLAM problem.

The following examples illustrate the common SFM techniques and their implications when directly applied to SLAM. Bundle adjustment, when applied to SFM as in [78] uses nonlinear optimization techniques to update estimates of all the structure and motion parameters using all the available observations. Since the bundle adjustment when applied to SLAM applications optimizes all motion and structure parameters at every step, the state vector of SLAM will consist of the entire history of robot pose and the map, which can become very large. Hence the bundle adjustment when applied to SLAM is not scalable. The other major disadvantage is that most feature based recursive SFM algorithms such as [37], [102] and [103] assume that the data association (correspondence among image features in several images) is known a priori, which is not the case in real world applications. Especially, in robotics data association has to be solved on the fly using a statistical method in order to perform autonomous localization and mapping.

Factorization techniques such as [101] can be extended to bearing only SLAM formulations. However, there is no method currently known, which can incorporate odometry information in the bearing only SLAM problem. Moreover, factorization is a batch estimation technique. Disadvantages of factorization techniques are (1).each iteration becomes slower with time, (2). it can take several iterations to converge and (3). solutions are not always guaranteed to be physically meaningful. If motion information is not utilized in SFM, camera motions and scene geometry can be scaled arbitrarily as in

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[25] without any inconsistency with the image measurements. In bearing only SLAM, this scale ambiguity can be resolved by using odometry and there is no known way of estimating the states consistently without using odometry.

4.3.1.2 Feature extraction

Feature extraction using monocular image sequences is vital for incorporating bearing only information in SLAM. In particular in outdoor navigation, the process of feature extraction is difficult due to varying illumination conditions [104], shading and the ambiguous nature of measurements. Prior to extraction of features it is important to segment the image in to regions of interest. Features of significance in intensity images are such points or areas having high discontinuity of intensity levels. Edges are points having such high intensity gradients in one direction only, whereas corners are defined as points having steep changes of intensity in more than one direction. Vertical edges [105], straight lines [106], corners [105] and color features are useful in semi structured outdoor environments such as urban and suburban environments, theme parks etc. consisting of occasional buildings, lampposts, fixed structures etc. In outdoor environments, corners are of little use as good features because they are huge in number and ambiguous to changes in lighting and shading. However, region based segmentation, which segments homogeneous regions from grey level and color images is a method of choice when extracting features from images in outdoor environments. Region based segmentation uses some form of clustering based on image properties. This method is utilized for feature extraction in this work and will be described in Section 4.3.6.

4.3.1.3 Feature initialization

Feature initialization is one of the most important issues that come into play in SLAM when incorporating bearing only information. This is because of the fact that a single

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bearing measurement of a feature in 2D space from only one robot location does not provide adequate information to initialize [100] feature's 2D location. In other words, at least two bearing measurements of the same feature taken from distinct vantage points are required to initialize a feature. The problem is exacerbated due to ill conditioned measurements resulting from vehicle pose uncertainties and bearing measurement uncertainties. Nevertheless, there are several possible ways to perform the feature initialization. Use of a priori information such as feature location constraints is one way to work around to feature initialization. However, this method is not well suited in SLAM due to the lack of any a priori knowledge of the environment. Use of particle filters as in [63] and [107] is another method used for feature initialization. Nevertheless, the use of particle filters in feature initialization requires certain heuristics in conjecturing the measurements associated with a specific feature. The problem is exacerbated when the features are dense. Perceptual grouping with RANSAC (Random Sample Consensus) [108] is also utilized in feature initialization. The use of RANSAC is statistically sound but not very robust in high clutter outdoor environments. One way to initialize features when there are two sensors consisting of a bearing only sensor and a range bearing sensor is to use the range-bearing sensor to initialize features and to use the bearing only sensor to incorporate measurements of the initialized features. This method is utilized in the experiment that follows due to its robustness in incorporating bearing information when the same entities are observed by both the vision and the laser measurement system (LMS).

4.3.2 Experimental Setup

The implementation of laser and monocular vision aided SLAM was carried out in an outdoor neighborhood environment. A SICK LMS 290 laser measurement system (LMS) was used as a range-bearing sensor and two fixed MCL-1500 monocular color cameras

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were used as the bearing only sensors in this implementation. The sensors were fixed to the front of the Genome test vehicle on a plane perpendicular to the vehicle base as shown in Figure 4.1. The LMS was mounted facing in the direction of travel and to provide range information on a semi circular region (180°) with 1° angle steps. The cameras were mounted in such a way that the optical axes were parallel to the ground plane. Further, the right camera axis was aligned at an angle of 30° to the right of the vehicle heading and the left camera axis was aligned at an angle of 30° to the left of the vehicle heading. This configuration involving two monocular cameras is essential to overcome the narrow field of view of each of the cameras. All the details of the LMS and the cameras on Genome test bed are listed in Appendix A.

Figure 4.1. Sensor configurations. Figure 4.1 (a) depicts the front view of the sensors and Figure 4.1 (b) depicts a top view of the same.

The LMS can deliver range and bearing values of the environment within its range in a direct manner under its intrinsic calibration. However, the monocular cameras need to be calibrated both extrinsically and intrinsically prior to acquiring any useful information of bearings of features.

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4.3.3 Calibration of Monocular Cameras

Camera calibration and distortion correction is used to find the extrinsic and intrinsic parameters associated with the transformation from a world coordinate frame to the camera image pixel coordinate frame. A comprehensive survey of distortion correction methods can be found in Clarke and Frye [138]. If wide-angle cameras are used, a distortion correction model such as Brown's Plumb-Bob model [139], which utilizes 6th order radial distortion model can be used. This model uses a tangential distortion correction due to imperfect centering of the lens components and other manufacturing defects in a lens. In this vision and LMS aided SLAM experiment, we have used very narrow angle cameras (maximum angle is about 48^0). Thus, the distortion is not a serious issue as reported by the researchers [25], [100] and [140] in using bearing only monocular vision sensor measurements in mobile robotics. Further, the inclusion of lens distortion in the bearing only SLAM models introduces unnecessary complexity into the equations resulting in highly nonlinear observation models, which can some times lead to observability problems as well. Thus, in this work we have not included lens distortion into the sensor modeling equations ((4.7)-(4.15)). For calibration and modeling of cameras, the pinhole model [105] of the camera with perspective projection is used. The intrinsic parameters are invariant to camera pose and its orientation in space whereas extrinsic parameters define the pose of the camera with respect to a reference world coordinate frame. There are numerous intrinsic and extrinsic camera calibration methods such as [111]-[115] in the literature. The method [114] and [115] of Zhang was utilized in this implementation together with the library of software functions of the Intel open source [116]. This approach requires a camera to observe a planar pattern shown at a few (at least two) different orientations. Either the camera or the fixed pattern can be freely moved while taking observations. The procedure consists of a closed form solution followed by a nonlinear refining based on the maximum likelihood criterion. Both

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cameras were calibrated using the same technique. The intrinsic camera parameters calculated were based on the transformation of a world point $\mathbf{P} \equiv [X_c \ Y_c \ Z_c]^T$ (Figure 4.2) in camera coordinate frame (with the focal point C as the origin and camera axis as the Z axis) to the pixel frame. The principal point (image centre) C_i is the origin of the image plane and $P_i = [x \ y]^T$ is the image of the point $\mathbf{P}$ in the image plane as shown in Figure 4.2. $[u_0 \ v_0]^T$ is the coordinates of the image center or the principal point on the pixel frame.

Let $[u \ v]^T$ be the pixel coordinates of the point $\mathbf{P}$ in pixel coordinate frame (with u and v correspond to x and y in image coordinate frame) and S_x (mm/pixel) and S_y (mm/pixel) be the effective sizes of a pixel in the horizontal and vertical directions respectively. The following expression can then be derived for the homogeneous (projective) coordinates of the scene point and the corresponding pixel point.

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f/S_x & 0 & u_0 & 0 \\ 0 & f/S_y & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} \quad (4.7)$$

where $f_x = f/S_x$ and $f_y = f/S_y$ with both the quantities having the unit of pixels.

Thus, the calculated camera parameters are depicted in Table 4.1. The extrinsic parameters were calculated by placing the calibration frame in front of the cameras in the world coordinate frame (this was a coordinate frame with respect to the vehicle in this case to determine the location of camera projection centre with respect to the vehicle body) with known orientation and known distances away from the vehicle body and taking images by the two cameras. Since the cameras were properly aligned to the vehicle structure, the rotation component was not considered. The location of the camera projection center with reference to the vehicle was thus determined with the use of

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intrinsic parameters calculated.

Figure 4.2. Image and camera coordinate frames. Camera and image coordinate systems consist of 3 and 2 perpendicular axes respectively

Table 4.1 Camera calibration results

Parameter	Left Camera	Right Camera
f_x (pixels)	804.75191	815.79319
Error in f_x (3σ in pixels)	4.7558	8.64978
f_y (pixels)	805.43259	815.75338
Error in f_y (3σ in pixels)	4.80202	8.58461
u_0 (pixels)	264.12533	257.64556
Error in u_0 (3σ in pixels)	9.65675	12.06982
v_0 (pixels)	211.27448	219.22707
Error in v_0 (3σ in pixels)	8.96137	11.0557

Once the extrinsic parameters of the two cameras with respect to the vehicle body are known, the locations of the LMS and the two cameras on the vehicle body are also known. It was assumed that the plane spanned by the scan line of the LMS sensor was parallel to the plane spanned by the vehicle axels and there was no rotation of the LMS with respect to the vehicle's body frame. The optical centre of the LMS sensor was calculated by placing a distinguishable object such as a wooden pole before the LMS sensor at a known location in front of the vehicle. By measuring the distance to the pole

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in the LMS, it is straightforward to estimate the optical centre of the LMS with respect to the vehicle body frame. Thus, the transformation that relates the LMS and the two cameras were determined.

4.3.4 Modeling of Monocular Cameras

One of the important steps in multiple sensor fusion implementations is the accurate modeling of sensors and their measurements. Modeling the vision sensors (cameras) consist of the construction of measurement models, associated error models and sensor noise characteristics. In the case of cameras, the pixel values of the features have to be taken as measurements. The bearing of the landmarks can be determined from these measurement models. A comprehensive treatment of noise in vision sensors can be found in the work [141], of Boie and Cox. When using vision sensors, they perform well under high signal-to-noise conditions when the scene is bright, contrasts are high, the lens is good and the focus is sharp. Under such conditions (these conditions normally exist in outdoor setting) camera noise is relatively low. A well-designed CCD camera normally includes a front-end amplifier that is designed for low noise and provides sufficient signal gain such that all subsequent noise sources in the signal processing chain become very low and even. Thus, it is a common practice to model the noise of the vision sensors as Gaussian [141], [142]. Hence in the following formulation Gaussian noise assumption as required by the EKF is utilized.

Figure 4.3 depicts a scenario in which the left camera is observing a landmark P. The same notation is used in Figures 4.2 and 4.3. This situation is further illustrated by Figure 4.4 where α is the angle of the landmark with the camera coordinate axis Z_c , β is the anti clockwise angle of the left camera with the vehicle heading, θ is the vehicle heading angle, θ_i is the bearing of the observed landmark with respect to the world coordinate

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frame X axis, a_c is the distance to the projection centre of the camera from the vehicle rear axis and b_c is the displacement of the left camera from the vehicle front mid point. The point C is the projection centre of the camera and X_c , Y_c and Z_c are coordinate axes of the camera coordinate frame.

Figure 4.3. Observing a landmark. The bearing observed by the landmark is shown by α .

Figure 4.4. Configuration of the left camera.

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Let $[u \ v]^T$ be the pixel coordinates of the landmark P with the same notation used in the camera calibration section. The following expressions can be obtained.

$$\tan \alpha = Y_c / Z_c = (v - v_0) / f_y \quad (4.8)$$

Assuming the $Y_c C Z_c$ plane is parallel to the world coordinate XY plane

$$\alpha = \beta + \theta - \theta_i \quad (4.9)$$

Let the landmark i has the world coordinates (x_i, y_i) and the coordinates of the camera projection centre in world frame be (x_s, y_s) then $\theta_i = \tan^{-1}((x_i - x_s) / (y_i - y_s))$. Thus, the observation model of the left camera can be expressed as

$$v = f_y \tan(\beta + \theta - \tan^{-1}((x_i - x_s) / (y_i - y_s))) + v_0 \quad (4.10)$$

The error models of v , and the sensor location are

$$\delta v = (\partial v / \partial (x_s, y_s, \theta_s)) [\delta x_s \ \delta y_s \ \delta \theta_s]^T + (\partial v / \partial (x_i, y_i)) [\delta x_i \ \delta y_i]^T + (\partial v / \partial \beta) \delta \beta + (\partial v / \partial v_0) \delta v_0 + (\partial v / \partial f_y) \delta f_y \quad (4.11)$$

$$\begin{bmatrix} x_s \\ y_s \\ \theta_s \end{bmatrix} = \begin{bmatrix} x + a_c \cos \theta - b_c \sin \theta \\ y + a_c \sin \theta + b_c \cos \theta \\ \theta \end{bmatrix} \quad (4.12)$$

$$[\delta x_s \ \delta y_s \ \delta \theta_s]^T = (\partial v / \partial (x, y, \theta)) [\delta x \ \delta y \ \delta \theta]^T \quad (4.13)$$

where x , y and θ are the coordinates of the mid point of the rear axel of the vehicle and its heading respectively. Similarly, for the right camera (Figure 4.5), $\alpha = \theta - \beta - \theta_i$ and $y_s = y + a_c \sin \theta - b_c \cos \theta$. All the other equations involving measurement prediction and update using EKF are standard as usual.

The bearing observation for landmark initialization and its variance σ_{θ_i} are

$$\theta_i = \beta + \theta - \tan^{-1}((v - v_0) / f_y) \quad (4.14)$$

$$\sigma_{\theta_i}^2 = \sigma_{\theta}^2 + (f_y / r)^2 (\sigma_v^2 + \sigma_{v_0}^2) + ((v - v_0) / r)^2 \sigma_{f_y}^2 \quad (4.15)$$

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where σ_θ , σ_v , σ_{v_0} and σ_{f_y} are the standard deviations of θ , v , v_0 and f_y respectively and $r = f_y^2 + (v - v_0)^2$. It is assumed that the errors of v , v_0 and f_y are uncorrelated in deriving (4.15).

Figure 4.5. Configuration of the right camera.

4.3.5 Data Association with Bearing and Range-Bearing Measurements

The data association in multi-sensor fusion in SLAM is done in a manner similar to the single sensor case described in Chapter 3. However, the measurements obtained simultaneously from two sensors (bearing only information obtained from the vision sensor and range-bearing information obtained from the laser scanner) can be processed sequentially (data association of measurements from one sensor first and the other after that) assuming these measurements are conditionally independent. Although the measurements obtained from bearing sensor is practically independent from the measurements obtained from the range-bearing sensor, their errors are correlated over the common error term of the vehicle pose. Thus, measurements of both sensors must be processed jointly to achieve good results. In the following sections, two methods of addressing the data association problem of bearing measurement (vision sensor) and the range bearing measurement (laser sensor) are discussed.

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4.3.5.1 Data association assuming measurement independence

The data association of both sensors can be done using the linear program based multiple frame (MFDA) method discussed in Section 3.4.3 assuming that the measurements are conditionally independent. This is to some extent justifiable as the only thing common to the sensors are the underlying vehicle state. Thus, calculating the likelihoods as required by the equations (3.52)-(3.72) of MFDA only requires that the measurement model of the range-bearing measurement be adequately represented by any given landmark state coordinates.

Let the landmark i has the world coordinates (x_i, y_i) and the coordinates of the LMS in world coordinate frame be (x_s, y_s) . The location of LMS is then given by (4.12) with $b_c = 0$. Thus, when observing a range r_i and bearing θ_i of the i^{th} landmark, the true measurement $\mathbf{z}_i$, measurement prediction $\hat{\mathbf{z}}_i$ and measurement innovation $\mathbf{e}_i$ are given by

$$\mathbf{z}_i = [r_i \quad \theta_i]^T \quad (4.16)$$

$$\hat{\mathbf{z}}_i = \begin{bmatrix} \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2} \\ \tan^{-1}((y_i - y_s)/(x_i - x_s)) - \theta_s \end{bmatrix} \quad (4.17)$$

$$\mathbf{e}_i = \mathbf{z}_i - \hat{\mathbf{z}}_i \quad (4.18)$$

Thus, the innovation $\mathbf{e}_i$ and its uncertainty are functions of known quantities (i.e the uncertainty of the landmark i and the vehicle pose). Hence the likelihoods of any combination of predictions of the landmark i and the true range-bearing measurements can be calculated using (3.52)-(3.60).

In the same manner a measurement obtained from the bearing only vision sensor constitutes the bearing of a landmark in the map and thus the measurement predictions and the innovations together with the innovation covariance can be calculated assuming conditional independence. Thus the association likelihood of a bearing measurement with

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a landmark can be easily deduced. Suppose the landmark i is observed using the left camera and the true measurement $\mathbf{z}_i$, received is v pixels. The measurement prediction $\hat{\mathbf{z}}_i$, and the measurement innovation $\mathbf{e}_i$ of the bearing measurement are then given by

$$\mathbf{z}_i = v \quad (4.19)$$

$$\hat{\mathbf{z}}_i = f_y \tan\left(\beta + \theta - \tan^{-1}\left((x_i - x_s)/(y_i - y_s)\right)\right) + v_0 \quad (4.20)$$

$$\mathbf{e}_i = \mathbf{z}_i - \hat{\mathbf{z}}_i \quad (4.21)$$

Thus, the innovation $\mathbf{e}_i$ and its uncertainty are functions of the uncertainty of the landmark i and the vehicle pose, which are known quantities. Hence, the likelihoods of any combination of predictions of the landmark i and the true bearing measurement can be calculated using (3.52)-(3.60).

However, the calculation of $c(t, i_{k-1}, i_k)$ in equation (3.61) needs special attention when measurement in frame $k-1$ is a bearing only measurement and measurement in frame k is a range-bearing measurement or vice-versa. Let us consider the case when the measurement in frame $k-1$ is a bearing only measurement and that of the frame k is a range-bearing measurement. The calculations of (3.44)-(3.47), (3.62)-(3.64), updated values of the feature $\mathbf{x}_i(k-1|k-1)$, its uncertainty $\mathbf{P}_i(k-1|k-1)$, vehicle pose $\mathbf{x}_v(k-1|k-1)$ and its uncertainty $\mathbf{P}_v(k-1|k-1)$ as specified in Chapter 3 should be done using (4.19) to (4.21) of the bearing only measurement. The rest of the likelihood calculations (3.65) to (3.72) have to be then done using the range-bearing measurement obtained in frame k . Once the likelihoods are calculated, the rest of the MFDA formulation is straightforward as described in Chapter 3 by (3.37)-(3.43).

4.3.5.2 Data association using MFTC criterion

As outlined in Section 3.6, the batch data association extensions using MFTC criterion is also applicable to multi-sensor fusion in SLAM. The method described in Section 3.6 is

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applicable in a straightforward manner to both the bearing sensor measurements and the range-bearing sensor measurements as a single batch. However, update and Mahalanobis distance calculations as well as Jacobian calculations have to be modified to incorporate the bearing only measurement models where appropriate. For example, when incorporating a bearing only measurement in (3.100), the term $(\mathbf{z}_{i_{k-m}} - \hat{\mathbf{z}}_{i_{k-m}})$ is calculated as

$$(\mathbf{z}_{i_{k-m}} - \hat{\mathbf{z}}_{i_{k-m}}) = v - f_y \tan\left(\beta + \theta - \tan^{-1}\left((x_i - x_s)/(y_i - y_s)\right)\right) \quad (4.22)$$

When incorporating a range-bearing measurement in (3.100), the term $(\mathbf{z}_{i_{k-m}} - \hat{\mathbf{z}}_{i_{k-m}})$ is calculated as

$$(\mathbf{z}_{i_{k-m}} - \hat{\mathbf{z}}_{i_{k-m}}) = \begin{bmatrix} r_i & \theta_i \end{bmatrix}^T - \begin{bmatrix} \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2} \\ \tan^{-1}((y_i - y_s)/(x_i - x_s)) - \theta_s \end{bmatrix} \quad (4.23)$$

Hence, the Jacobians of (3.101) have to be modified appropriately taking into consideration of the measurement type whether it is range-bearing or bearing only.

4.3.6 The Outdoor SLAM Experiment

The experimental setup consisted of an LMS and two monocular cameras mounted on the front of the test platform as shown in Figure 4.1. The output of the sensors were synchronized with a common clock signal and used in the SLAM algorithms. The LMS data was sampled at 10 Hz and the image data from both the cameras was recorded at 5 Hz. Since landmark bearing based information was used in the sense of estimation theoretic EKF formulation of SLAM, it was not possible to initialize the landmarks consistently in a noisy out door setting. Although several researchers have suggested different bearing only landmark initialization methods [100], [63], [107] and [108], they are not statistically correct in the strict sense giving rise to several occlusions and false

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matches. Thus, in this experimental evaluation of sensor fusion in SLAM, range and bearing information of laser scanners were used to initialize landmarks. Vision sensors were used to provide measurements only if cameras observe the bearings of already initialized landmarks (i.e landmarks initialized by the laser scanner). In this context and for simplicity of implementation a set of artificial beacons (constructed by fiber glass) were utilized. The set of beacons were laid out along the vehicle path at random locations on both sides of the path. Since the beacons were of the same color, feature extraction using monocular camera images was simplified. Hence the feature extraction using monocular images was carried out using a simple color segmentation strategy with the prior knowledge of the beacon color distribution. The LMS features were extracted using the algorithm described in [20]. Therefore, LMS was also able to detect the artificial beacons as distinguishable features.

Figure 4.6. Feature extraction by color histogram thresholding. The beacon is the orange color pole in Figure 4.6 (a). In Figure 4.6 (b), the vertical line represents the observed v coordinate of the beacon in pixel coordinate frame.

The color pixels having the distribution of the beacon color were clustered using histogram thresholding with a suitable threshold value. Figure 4.6 (a) shows a beacon image recorded and Figure 4.6 (b) shows how the image pixel corresponding to horizontal direction is marked as a thin vertical line.

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Figure 4.7 Laser and vision aided SLAM in an outdoor neighborhood environment. The thick line is the estimated vehicle path and the dotted lines show the matched landmarks at the current time step

Figure 4.8 Outdoor SLAM with two monocular cameras and LMS. The images of two cameras are shown on the map. Thick line represents the estimated robot path. Landmarks are shown as crosses.

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Figure 4.9. The estimated vehicle path (thick line) and landmarks (crosses). Figure 4.9 (a). The estimated path. Figure 4.9 (b). An enlarged view of the vehicle path. Landmarks and 95% confidence limits of the landmark error before sensor fusion (dotted ellipsi) and after sensor fusion (continuous ellipsi).

The thin vertical line represents the observation of the pixel value v as defined in (4.10). The feature initialization for bearing only sensors is performed by using LMS. Data association is performed using two consecutive frames of measurement using MDA algorithm. Figure 4.7 shows the laser and vision fusion in outdoor SLAM. The estimated vehicle path and the landmarks are drawn on a digital map of scale 1:1300. Figure 4.8 illustrates an instance where the color based beacons are extracted in images and at the same time laser based features are also utilized in data fusion. The dotted lines mark the matched features used in the fusion operation at the instant. Figure 4.9 depicts the same path and landmark estimates drawn on a graph for the SLAM experiment shown in Figure 4.7, and an enlarged view of the landmark estimates with 2σ error ellipses. It can be observed that the error ellipses have been reduced (Fig 4.9) after fusing both vision and laser information. Figure 4.10 illustrates the error performance of localization achieved by the fusion of laser and vision in SLAM. Figure 4.11 depicts the error performance of the same experiment conducted with the laser only SLAM. By closely examining the two figures it is evident that the uncertainty is high when only the laser is used. To quantify the variation of uncertainty rigorously, the mean value of the variance of a state is defined as a performance measure. This value reflects the variation of uncertainty of a particular state throughout the experiment.

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Figure 4.10 Error performance in laser and vision aided SLAM. STD=Standard deviation.

Figure 4.11 Error performance in laser only SLAM.

Table 4.2. Mean standard deviation measure in SLAM

Quantity	Laser only SLAM	Fusion of vision and laser in SLAM	Percentage decrease from Fusion
X error mean STD [m]	0.2971	0.2734	7.98
Y error mean STD [m]	0.3052	0.2804	8.13
Heading error mean STD [rad]	0.0133	0.0127	4.51

Table 4.2 shows the variation of the mean value of standard deviation in vehicle pose states when using laser only and when laser is fused with vision for SLAM implementation respectively. The results in Table 4.2 verify that there are advantages of sensor fusion as the fusion of vision and laser slightly improves the localization errors in SLAM. Figures 4.12 (a), (b) and (c) show the variations of number of feature measurements received in each scan, number of association hypotheses for consideration after preprocessing and the number of matches in the two frame MFDA using linear

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programming technique. The results show that the number of hypotheses is manageable even in outdoor setting with multiple sensor based algorithms.

Figure 4.12 Data association performance. Variation of measurements, number of association hypotheses and number of matches are given by Figures (a), (b) and (c) respectively.

Even though the performance of vision and laser is demonstrated with success in outdoors it has few failure modes as well. Region based segmentation of feature extraction in vision fails due to changes in illumination conditions and due to shading. This happens as the contrast and texture in an image are functions of the illumination changes. It is also questionable if the features viewed from the fixed camera locations are the most informative ones in the visible range. Therefore, it is important to investigate whether the same performance of sensor fusion algorithms can be achieved by selectively focusing on the most informative features in the environment.

4.4 Active Sensing Paradigm

Active sensing is the act of controlling a sensor so as to achieve certain performance criteria. It is one of the ways in which multiple sensors can be exploited to maximize the sensor resources through increasing the sensing range and measurement quality. Instead of using several monocular cameras, a single narrow angle monocular camera can be used in active control mode to increase its visibility region. Here it is demonstrated by using simulations how new active sensing strategies can be developed in SLAM in the bearing only context to increase the visibility region of sensors. The resulting active sensing

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algorithm is not implemented here with real sensor platforms because of the hardware limitations (required hardware is not available). The work in active sensing reported in the literature especially on robot localization does not explicitly concentrate on multi-sensor fusion aspects. The work of Feder et al. [14] and Bourgault et al. [109] describe the SLAM implementations, which try to achieve active sensing by trajectory control of the robot so as to maximize the information content. However, in SLAM this is not very realistic as one doesn't control his trajectory just because he has to increase the information content only but try to follow a trajectory specified by the mission he has to complete.

In the work [13], Davison describes an active control of a stereo rig for an indoor SLAM implementation. Davison's method can be briefly outlined as follows. Initially all the landmarks of the state vector in the visibility region of the stereo cameras are determined. The covariance matrices of the measurement innovations of all the landmarks in the visibility region are then determined. Of all the features, it is conjectured that the feature with the largest error ellipse is the hardest to predict in the visibility region. The stereo head is then driven to fixate that particular feature. As stipulated in [13], there is no proof however, that the above active sensing policy is the optimal choice or criterion from an information theoretic point of view. However, it is argued that fixating on the feature, which is the hardest to predict would provide the maximum information gain possible. Further, active control of monocular cameras compensates the angle limitations of the most monocular cameras (they will become a virtual wide angle camera) in use. Therefore, in the next section an information theoretic active sensing strategy is proposed to control an active monocular head providing bearing only information, which can aid an associated range-bearing sensor.

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4.4.1 Control of an Active Monocular Camera Aiding SLAM

An active sensing problem in controlling a monocular active camera is investigated in this section. The generalized active sensing problem is formulated for a finite time window and its implementation in a single time step is elaborated.

4.4.1.1 Generalized active sensing in a finite time window

The generalized active sensing paradigm in a finite time window $[k, k + N - 1]$ can be formulated in the following form where k is the current time step and N is the finite time window size in time steps. Suppose that the robot and its active sensors are controlled by a set of actions at each time step denoted by $\mathbf{a} = \{\mathbf{a}_k, \mathbf{a}_{k+1}, \dots, \mathbf{a}_{k+N-1}\}$ where $\mathbf{a}_t$ for $t = k, k + 1, \dots, k + N - 1$ is the vector of actions specifying the robot control and the sensor placement commands at time t . Let $\mathbf{U}_t$ and $\mathbf{C}_t$ for all $t = k, k + 1, \dots, k + N - 1$ denote the sets of uncertainty terms and sets of cost terms at time step t , which are affected by the set of actions in the given time window as follows.

$$\mathbf{U}_t = \{\mathbf{u}_{it}(\mathbf{a}_k, \mathbf{a}_{k+1}, \dots, \mathbf{a}_{k+N-1}) \mid i = 1, 2, \dots, n_t\} \quad (4.24)$$

$$\mathbf{C}_t = \{\mathbf{c}_{it}(\mathbf{a}_k, \mathbf{a}_{k+1}, \dots, \mathbf{a}_{k+N-1}) \mid i = 1, 2, \dots, m_t\} \quad (4.25)$$

where n_t and m_t denote the number of uncertainty terms and the number of cost terms at time step t . The terms $\mathbf{u}_{it}$ and $\mathbf{c}_{it}$ denote the i^{th} uncertainty term and i^{th} cost term at the time step t . The active sensing problem can now be cast in the following form.

$$\mathbf{a} = \arg \min_{\mathbf{a}} (\lambda U(\mathbf{U}_k, \mathbf{U}_{k+1}, \dots, \mathbf{U}_{k+N-1}) + \mu C(\mathbf{C}_k, \mathbf{C}_{k+1}, \dots, \mathbf{C}_{k+N-1})) \quad (4.26)$$

Subject to the constraints

$$\mathbf{g}(\mathbf{X}(k), \mathbf{X}(k+1), \dots, \mathbf{X}(k+N-1), \mathbf{a}_k, \mathbf{a}_{k+1}, \dots, \mathbf{a}_{k+N-1}) \leq \mathbf{g}_{th} \quad (4.27)$$

where $U(\cdot)$ is the composite scalar function representing uncertainty, $C(\cdot)$ is the complex scalar function representing the costs, λ and μ are two weighting coefficients of

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uncertainties and the costs, $\mathbf{X}$ is the composite SLAM map augmented state vector, $\mathbf{g}(\cdot)$ is the nonlinear constraint vector function and $\mathbf{g}_{th}$ is a vector consisting of threshold values of the constraints respectively. The constraints can be used to provide conditions for maximum and minimum of the control inputs, distance to the obstacles and the allowed sensor movements etc. However, this problem is exponentially complex even for a small horizon size. Thus, a simplified version of the problem is demonstrated in the next section.

4.4.1.2 Active sensing for the control of a bearing only sensor

Figure 4.13 Active sensing using a bearing only sensor. The active bearing sensor and fixed sensor are mounted at the front of the vehicle with active sensor on top of fixed one. S denotes the active sensor location.

The control of an active bearing only sensor in the presence of a fixed range-bearing sensor is considered in this context. The active bearing sensor (a monocular active camera) and the fixed range-bearing sensor (eg. SICK LMS 290) are located on the vehicle as depicted in Figure 4.13. The fixed range-bearing sensor is looking forward with 180° scanning range and the bearing only sensor is having a span angle of α_{max} to the both sides from the direction of travel (ie from the forward direction of the robot) the active bearing sensor is fixated at an angle α from the robot heading as shown in Figure 4.13.

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The objective function to determine the single step active sensor fixation angle is taken as the total area of the uncertainty ellipses of the robot pose error and the individual landmark errors. The rationale behind the selection of this objective function is that maximizing information about a state estimate is tantamount to minimizing the error covariances.

$$\Phi_A = \min. \left(\pi \sqrt{\det(\mathbf{P}_{vv}(k|k))} + \pi \sum_{i=1}^n \sqrt{\det(\mathbf{P}_{L_i L_i}(k|k))} \right) \quad (4.28)$$

The observation model of the range-bearing sensor is given by (A.5) of the Appendix A. The observation model of the bearing only sensor when observing a landmark $[x_i(k) \ y_i(k)]^T$ is

$$z_1(k) = \tan^{-1}((y_i(k) - y_s(k))/(x_i(k) - x_s(k))) - \theta(k) - \alpha(k) + (\pi/2) + w_1(k) \quad (4.29)$$

where $x_s(k) = x(k) + a_2 \cos(\theta(k))$ and $y_s(k) = y(k) + a_2 \sin(\theta(k))$. The cost function in

(4.26) is denoted by $C = \pi \sqrt{\det(\mathbf{P}_{vv}(k|k))} + \pi \sum_{i=1}^n \sqrt{\det(\mathbf{P}_{L_i L_i}(k|k))}$. The solution technique

is as follows. Discretize the angle α into small angles (for example in this case from -60° to 60° in 5° steps.). At each α , determine all the landmarks (by using the SLAM state) in the visibility region of the active sensor after updating the full SLAM state by the measurements obtained from the range-bearing sensor. For each of these landmark observations, calculate the Jacobian $\mathbf{H}_i$, the composite Jacobian $\mathbf{H}_{comp}$ of the state vector and composite measurement innovation covariance matrix $\mathbf{S}_{comp}$ for particular α as follows.

$$\mathbf{H}_{comp} = [\mathbf{H}_1^T \ \mathbf{H}_2^T \ \dots \ \mathbf{H}_n^T]^T \quad (4.30)$$

$$\mathbf{S}_{comp} = \mathbf{H}_{comp} P(k|k) \mathbf{H}_{comp}^T + \mathbf{R}_{comp} \quad (4.31)$$

where $\mathbf{R}_{comp}$ is the composite diagonal measurement covariance matrix corresponding to observing the set of landmarks in the visibility region corresponding to α . The Jacobian

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$\mathbf{H}_{comp}$ is a nonlinear function of the present landmark and vehicle estimates. Thus the updated covariance can be calculated from, the standard EKF update equations (2.13) and (2.14). Thus for each α the value of C can be calculated.

Figure 4.14 SLAM with a range-bearing sensor and an active bearing sensor. The camera is shown by the thick lines. The dotted lines show the features observed by the active monocular camera.

Hence, by the set of discrete values of α it is possible to obtain a set of values C for the cost function. Thus, the value of α corresponding to minimum of C can be always found. This is the angle to which the active sensor is to be fixated to achieve maximum information gain. Thus, according to the definition, the value of α satisfying (4.28) results in the smallest state covariance matrix possible for SLAM at that particular time instant.

Figure 4.14 shows the performance of an active sensor in such configuration. The simulation parameters are speed input error 0.5 m/s, steering angle encoder error 0.05

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radians, range measurement noise 0.1 m, bearing measurement error 0.5 degrees, maximum angle sensed by the monocular camera is 50° and $\alpha_{\max} = 60^\circ$.

Figure 4.15 Localization error in active sensor aiding in SLAM. Thick line shows the error and dashed lines show 3σ error bounds

Figure 4.15 shows the error performance of the active sensing in SLAM. The errors are bounded by 3σ limits. Figure 4.16 shows the multi-sensor performance with active head. The active sensor always observes fewer features than the range-bearing one as its angle of view is small. Figures 4.17 and 4.18 depict the cost and active camera fixation angle, which gives the minimum entropy.

Figure 4.16. Observations received by sensors. Sensor 1 is the range-bearing one and sensor 2 is the active one.

Doing several Monte Carlo runs (50 runs) establish that the use of active camera head decreases the rms position error by 12% and the heading error by 5.5% in the above simulation scenario. When the active head is fixed the decrease in error reduces to 4.3% and 2.1% in position error and the heading error respectively. Thus use of multiple sensors enhances the performance of SLAM and the amount of information gain. Further,

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when using a narrow angle bearing sensor the performance can be further enhanced by making it an active one.

Figure 4.17. Variation of cost function

Figure 4.18. Active head angle which results in the minimum entropy

4.5 Conclusion

The importance of paying due consideration to the issues of multi-sensor integration and fusion to achieve enhanced localization and mapping performance are established. The theoretical issues pertaining to multi-sensor data fusion in the context of SLAM such as the effect of the sensor fusion architecture, conditional measurement independence and active sensing are shown to be vital and elaborated in detail. It was established that out of the group sensor fusion architecture and sequential sensor fusion architecture only group sensor fusion architecture is theoretically sound in the strict statistical sense in SLAM for multiple sensor fusion. It is also shown that contrary to the common notion of using sequential sensor fusion in SLAM, the sequential sensor fusion architecture in SLAM is usable only under the conditional measurement independence assumption.

An effective strategy of using multiple monocular cameras providing bearing measurement to features aided by a laser range measurement system for SLAM was proposed and demonstrated. Effects on feature extraction, initialization, camera calibration and the experimental set up were outlined. A new performance metric

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quantifying the performance of multiple sensors in SLAM was also proposed.

A method of enhancing multiple sensor performance through information theoretic active control strategy over a finite time window was formulated. In this context, a generalized active sensing paradigm in a finite time window applicable to SLAM problem was formulated as a nonlinear optimization problem. The control of active bearing only sensor (eg. a monocular camera) to maximize the information gain, in the presence of multiple sensors was elaborated in the context of the SLAM problem.

Chapter 5

Effects of Sensor Biases

5.1. Introduction

In most mobile robot navigation applications, sensor biases and drifts due to unmodeled kinematics and sensing cause adverse effects especially in outdoor complex unstructured environments. In particular, accumulated bias errors cause significant errors in robot pose estimates in MAL and SLAM. In SLAM the errors due to biases in the sensors also give rise to inconsistent maps. Further, the effects of sensor biases often degrade data association and tracking accuracy [41] and causes further inaccuracies and inconsistencies in the pose and map estimates. Presence of sensor biases often cause SLAM algorithms to perform in extremely poor manner especially in large unexplored terrains including cycles. As more and more features are added to the map, the existence of map vehicle correlations result in the spread of bias errors throughout the map causing divergence and instability in feature based EKF-SLAM [73]. Nonlinear transformations in EKF also cause the Gaussian assumptions in the measurement and process models invalid. It has been demonstrated in small-scale SLAM implementations that non-linearity in state and measurement models can be offset [117] by adding more stabilizing process and measurement noise. However, this does not ensure consistent results in the presence of large exteroceptive and proprioceptive sensor biases inevitably present in practical situations in large, complex and unstructured environments. Throughout most part of the SLAM, MAL and mapping literature, the modeling aspects of sensors are not quite

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comprehensively addressed and investigated. Mostly, the random noise components in sensors are modeled and compensated for in most filtering algorithms. However, the nonrandom variations are not treated adequately.

Although offline calibration of the biased sensors can reduce the ill effects to some extent, the calibration process adds up to longer setup and processing times. Moreover, during operation the sensors' calibration may often be subject to changes or drifts requiring regular resetting and initialization. A convenient, appropriate and an effective approach to overcome problems associated with biases in sensors has been to explicitly model and estimate the bias parameters concurrently with the vehicle state online using an augmented state space approach. Thus, in this work we use the augmented state vector method to compensate for sensor biases because of its simplicity and its similarity to the structure of the map augmented state vector representation of the SLAM problem.

5.2. Estimation Theoretic Sensor Bias Correction in SLAM

There are several bias estimation methods such as [132], [119], [120] and [121] in the literature. The work of Krishanan [132] uses an augmented state vector method in the case of offline testing and calibration of inertial sensors. Work by Krishanan [132] and Fang and Wan [133] are notable early attempts to estimate vehicle parameters and input biases in the context of inertial sensors. However, most of these techniques deal with the offline calibration and error modeling. Separate bias estimation methods [119], [120] and [121] are advantageous when the number of bias terms in the estimation process is comparatively high compared to the bias free state vector, which is not true in the high dimensional state vector of SLAM.

In [122], Bar Shalom uses several radar systems mounted on moving platforms to track targets. The platform location (sensor location) estimates are obtained from GPS receivers, which are subjected to slowly varying biases. Since there are no fixed known

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targets he uses targets of opportunity (which are nothing but fixed targets at unknown locations) to estimate the biases in the GPS receivers. It is noted that under certain observability conditions these biases can be estimated. This work is further extended by the work of Okello and Challa [123] for radar sensor's bias estimation, joint registration and track-to-track fusion based on measurements generated by geographically separated radar trackers. Inspired by the works [122] and [123], our approach uses unknown fixed landmark location measurements obtained by biased sensors (whose locations are also uncertain) to estimate the sensors' intrinsic bias parameters and the vehicle control input bias parameters.

5.2.1 Concurrent Bias Estimation with SLAM

In this thesis it is shown, for the first time in a mathematically rigorous manner, how any slowly varying bias in proprioceptive and exteroceptive sensor(s) can be estimated and compensated for in the context of the SLAM problem. For simplicity, the derivation is given for a single exteroceptive sensor with range and bearing biases together with two biased proprioceptive sensors without losing the generality of including many sensors.

5.2.1.1 Single degree of freedom SLAM problem

To better appreciate concurrent estimation of sensor biases in SLAM, consider the problem of single degree-of-freedom SLAM having n landmark map. In this context, a vehicle traversing on a straight line path (along the x-axis, with vehicle pose $x_v(k)$) is assumed. The vehicle localizes on the line and acquires a map $\mathbf{m}(k)$ of concatenated one dimensional landmarks (locations) situated on the same line based on the correct association of landmarks observed on the line with those stored in the estimated map. When the vehicle's proprioceptive and exteroceptive sensors' biases are $u_b(k)$ and $s_b(k)$ at time k respectively, the process model (incorporating the biases) and the measurement

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model of the problem when observing n landmarks are

$$\mathbf{X}(k) = \begin{bmatrix} 1 & 1 & 0 & \mathbf{0}_{1 \times n} \\ 0 & 1 & 0 & \mathbf{0}_{1 \times n} \\ 0 & 0 & 1 & \mathbf{0}_{1 \times n} \\ \mathbf{0}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{I}_{n \times n} \end{bmatrix} \begin{bmatrix} x_v(k-1) \\ u_b(k-1) \\ s_b(k-1) \\ \mathbf{m}(k-1) \end{bmatrix} + \begin{bmatrix} u(k-1) \\ \mathbf{0}_{(n+2) \times 1} \end{bmatrix} + \begin{bmatrix} v(k-1) \\ \mathbf{0}_{(n+2) \times 1} \end{bmatrix} \quad (5.1)$$

$$\mathbf{z}(k) = \begin{bmatrix} -\mathbf{1}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{1}_{n \times 1} & \mathbf{I}_{n \times n} \end{bmatrix} \begin{bmatrix} x_v(k) & u_b(k) & s_b(k) & \mathbf{m}^T(k) \end{bmatrix}^T + \mathbf{w}(k) \quad (5.2)$$

where $v(k) \sim N(0, q^2)$, $\mathbf{w}(k) \sim N(\mathbf{0}_{n \times 1}, \text{diag}(r^2, \dots, r^2))$, q^2 and r^2 denote the proprioceptive sensor input noise, the measurement noise, the variances of $v(k)$ and exteroceptive measurement noise when observing one landmark. The quantities $\mathbf{I}_{n \times n}$ and when a is any integer $\mathbf{a}_{n_1 \times n_2}$ denote the identity matrix of n diagonal elements and an $n_1 \times n_2$ matrix with all elements equal to a where n_1 and n_2 are positive integers.

5.2.1.2 Two dimensional SLAM problem on a flat terrain

The concurrent exteroceptive and proprioceptive sensor bias estimation problem in more realistic 2D EKF SLAM can be formulated as follows by extending expressions given in Section 5.2.1.1. Again for simplicity we consider only a single exteroceptive sensor for making observations of landmarks, although the derivation that follows can be extended for many. Let the lumped proprioceptive sensor biases, i.e. biases in the input vehicle speed $u(k)$ and input steering angle $\gamma(k)$ be $u_b(k)$ and $\gamma_b(k)$ respectively. Suppose that the exteroceptive sensor's biases in the range and bearing be $r_b(k)$ and $\alpha_b(k)$ respectively. The vector of biases $\mathbf{x}_b(k)$ incorporating all the biases of both proprioceptive and exteroceptive sensors can be formed as follows:

$$\mathbf{x}_b(k) = \begin{bmatrix} u_b(k) & \gamma_b(k) & r_b(k) & \alpha_b(k) \end{bmatrix}^T \quad (5.3)$$

As was illustrated in the ID case for online concurrent bias estimation and filtering with SLAM, a new composite state vector $\mathbf{X}$ is formed by concatenating the vehicle state

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vector, map state vector and the vector of biases as follows.

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_v^T(k) & \mathbf{x}_b^T(k) & \mathbf{m}^T(k) \end{bmatrix}^T \quad (5.4)$$

The process models of the vehicle model (see Appendix A), takes the following form.

$$\begin{aligned} \mathbf{x}_v(k) &= \begin{bmatrix} x(k-1) + \Delta t(u_a(k-1))\cos(\theta(k-1)) \\ y(k-1) + \Delta t(u_a(k-1))\sin(\theta(k-1)) \\ \theta(k-1) + \Delta t u_a(k-1) \tan(\gamma_a(k-1))/a_1 \end{bmatrix} + \mathbf{v}(k-1) \\ &= \mathbf{f}_v(\mathbf{x}_v(k-1), u_a(k-1), \gamma_a(k-1)) + \mathbf{v}(k-1) \end{aligned} \quad (5.5)$$

$$u_a(k-1) = u_a(k-1) + u_b(k-1) + u_n(k-1) \quad (5.6)$$

$$\gamma_a(k-1) = \gamma(k-1) + \gamma_b(k-1) + \gamma_n(k-1) \quad (5.7)$$

where $\mathbf{x}_v(k) = [x(k) \ y(k) \ \theta(k)]^T$, $x(k)$, $y(k)$ and $\theta(k)$ denote the vehicle pose vector, vehicle X and Y coordinates and the heading. $u_n(k) \sim N(0, \sigma_{u_n}^2)$ and $\gamma_n(k) \sim N(0, \sigma_{\gamma_n}^2)$ denote noise properties of u and γ , which are assumed Gaussian. a_1 is the vehicle wheel base and Δt is the sampling time. $\mathbf{v}(k) \sim N(\mathbf{0}, \mathbf{Q}_a(k))$ is a temporally uncorrelated noise sequence representing the modeling uncertainties having the covariance matrix $\mathbf{Q}_a(k)$.

We assume that the biases of proprioceptive sensors exhibit a random walk behavior and those of exteroceptive sensors constant. Thus, we have

$$u_b(k) = u_b(k-1) + u_{bn}(k-1) \quad (5.8)$$

$$\gamma_b(k) = \gamma_b(k-1) + \gamma_{bn}(k-1) \quad (5.9)$$

$$r_b(k) = r_b(k-1) \quad (5.10)$$

$$\alpha_b(k) = \alpha_b(k-1) \quad (5.11)$$

where $u_{bn} \sim N(0, \sigma_{ub}^2)$ and $\gamma_{bn} \sim N(0, \sigma_{\gamma_b}^2)$. The concatenated bias state can now be expressed as

$$\mathbf{x}_b(k) = \mathbf{x}_b(k-1) + \mathbf{v}_b(k-1) \quad (5.12)$$

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where $\mathbf{v}_b(k) \sim N(\mathbf{0}, \text{diag}(\sigma_{ub}^2, \sigma_{\gamma b}^2, 0, 0))$ with σ_{ub}^2 and $\sigma_{\gamma b}^2$ are assumed to be the known variances of $u_{bn}(k)$ and $\gamma_{bn}(k)$ respectively. Should the bias parameters be time varying according to a different model, it is straightforward to incorporate their time varying characteristics in the formulation by appropriate modeling. The process model of the map is expressed as follows assuming static landmarks.

$$\mathbf{m}(k) = \mathbf{m}(k-1) \quad (5.13)$$

The observation model when modified to incorporate the biases of exteroceptive sensors becomes:

$$\begin{aligned} \mathbf{z}(k) &= \begin{bmatrix} \sqrt{(x_i(k) - x_s(k))^2 + (y_i(k) - y_s(k))^2} \\ \tan^{-1}((y_i(k) - y_s(k)) / (x_i(k) - x_s(k))) + \pi/2 - \theta(k) \end{bmatrix} + \begin{bmatrix} r_b(k) \\ \alpha_b(k) \end{bmatrix} + \mathbf{w}(k) \\ &= \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}(k), r_b(k), \alpha_b(k)) + \mathbf{w}(k) \end{aligned} \quad (5.14)$$

where the sensor location (x_s, y_s) is given by $x_s(k) = x(k) + a_2 \cos(\theta)$, $y_s(k) = y(k) + a_2 \sin(\theta)$ and a_2 is the absolute position of the exteroceptive sensor from the vehicle's rear axel center (see Appendix A). $\mathbf{w}(k)$ is the vector of observation noise in range and bearing assumed Gaussian with zero mean and covariance $\mathbf{R}(k)$.

When state covariance matrix of $\mathbf{X}(k-1)$ is $\mathbf{P}(k-1|k-1)$ at time $k-1$ then the EKF prediction equations are as follows:

$$\mathbf{X}(k|k-1) = \mathbf{f}(\mathbf{x}_v(k-1), \mathbf{u}(k-1), \mathbf{x}_b(k-1), \mathbf{m}(k-1)) \quad (5.15)$$

$$\mathbf{P}(k|k-1) = \mathbf{F}\mathbf{P}(k-1|k-1)\mathbf{F}^T + \mathbf{Q}(k-1) \quad (5.16)$$

$$\hat{\mathbf{z}}(k) = \mathbf{h}(\mathbf{x}_v(k), \mathbf{m}(k), r_b(k), \alpha_b(k)) \quad (5.17)$$

where $\mathbf{f}(\cdot) = [(\mathbf{f}_v(\cdot))^T \mathbf{x}_b^T(k-1) \mathbf{m}^T(k-1)]^T$, $\mathbf{F}$ is the Jacobian $(\partial \mathbf{f} / \partial \mathbf{X})$ of the process model evaluated at time $k-1$ and $\mathbf{Q}(k)$ represents the covariance matrix of the uncertainties of proprioceptive sensor noise and modeling uncertainties. When the true observation $\mathbf{z}(k)$ is available at time k and after correct observation to map

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feature associations are resolved using an appropriate data association algorithm, the EKF update equations are as follows:

$$\mathbf{e}(k) = \mathbf{z}(k) - \hat{\mathbf{z}}(k) \quad (5.18)$$

$$\mathbf{X}(k | k) = \mathbf{X}(k | k-1) + \mathbf{K}(k)\mathbf{e}(k) \quad (5.19)$$

$$\mathbf{P}(k | k) = \mathbf{P}(k | k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k) \quad (5.20)$$

where $\mathbf{e}(k)$, $\mathbf{S}(k)$, $\mathbf{H}$ and $\mathbf{K}(k) = \mathbf{P}(k | k-1)\mathbf{H}^T\mathbf{S}^{-1}(k)$ denote the observation innovation, its covariance matrix, the Jacobian $(\partial\mathbf{h}/\partial\mathbf{X})$ and the Kalman gain with the usual notation.

It may be noted that the system of equations (5.3)-(5.20) describing concurrent bias estimation in the context of SLAM is non linear. And hence to gain insight and to start the theoretical analysis (next section), the system of equations are first linearized as given below.

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_v(k) \\ \mathbf{x}_b(k) \\ \mathbf{m}(k) \end{bmatrix} = \begin{bmatrix} \partial\mathbf{f}/\partial\mathbf{x}_v & \partial\mathbf{f}/\partial\mathbf{x}_b & \mathbf{0}_{3 \times 2n} \\ \mathbf{0}_{4 \times 3} & \mathbf{I}_{4 \times 4} & \mathbf{0}_{4 \times 2n} \\ \mathbf{0}_{2n \times 3} & \mathbf{0}_{2n \times 4} & \mathbf{I}_{2n \times 2n} \end{bmatrix} \begin{bmatrix} \mathbf{x}_v(k-1) \\ \mathbf{x}_b(k-1) \\ \mathbf{m}(k-1) \end{bmatrix} + \begin{bmatrix} \partial\mathbf{f}/\partial\mathbf{u} \\ \mathbf{0}_{(2n+4) \times 2} \end{bmatrix} \mathbf{u}_a(k-1) + \begin{bmatrix} \mathbf{v}^T(k-1) & \mathbf{v}_b^T(k-1) & \mathbf{0}_{1 \times 2n} \end{bmatrix}^T + \mathbf{d}_f(k) \quad (5.21)$$

$$\mathbf{z}(k) = \begin{bmatrix} \partial\mathbf{h}/\partial\mathbf{x}_v & \partial\mathbf{h}/\partial\mathbf{x}_b & \partial\mathbf{h}/\partial\mathbf{m} \end{bmatrix} \begin{bmatrix} \mathbf{x}_v^T(k) & \mathbf{x}_b^T(k) & \mathbf{m}^T(k) \end{bmatrix}^T + \mathbf{w}(k) + \mathbf{d}_h(k) \quad (5.22)$$

where $\mathbf{d}_f(k)$ and $\mathbf{d}_h(k)$ can be determined from the following equations.

$$\mathbf{d}_f(k) = \mathbf{f}(\hat{\mathbf{x}}_v(k-1), \hat{\mathbf{x}}_b(k-1), \mathbf{u}(k-1), \hat{\mathbf{m}}(k-1)) - \begin{bmatrix} \partial\mathbf{f}/\partial\mathbf{x}_v & \partial\mathbf{f}/\partial\mathbf{x}_b & \mathbf{0}_{3 \times 2n} \\ \mathbf{0}_{4 \times 3} & \mathbf{I}_{4 \times 4} & \mathbf{0}_{4 \times 2n} \\ \mathbf{0}_{2n \times 3} & \mathbf{0}_{2n \times 4} & \mathbf{I}_{2n \times 2n} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_v(k-1) \\ \hat{\mathbf{x}}_b(k-1) \\ \hat{\mathbf{m}}(k-1) \end{bmatrix} - \begin{bmatrix} (\partial\mathbf{f}/\partial\mathbf{u})^T & \mathbf{0}_{2 \times (2n+4)} \end{bmatrix}^T \mathbf{u}(k-1) \quad (5.23)$$

$$\mathbf{d}_h(k) = \mathbf{h}(\hat{\mathbf{x}}_v(k), \hat{\mathbf{m}}(k), r_b(k), \alpha_b(k)) - \begin{bmatrix} \partial\mathbf{h}/\partial\mathbf{x}_v & \partial\mathbf{h}/\partial\mathbf{x}_b & \partial\mathbf{h}/\partial\mathbf{m} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_v^T(k) & \hat{\mathbf{x}}_b^T(k) & \hat{\mathbf{m}}^T(k) \end{bmatrix}^T \quad (5.24)$$

where the quantities $\hat{\mathbf{x}}_v(k-1)$, $\hat{\mathbf{x}}_b(k-1)$, $\hat{\mathbf{u}}_b(k-1)$, $\hat{\gamma}_b(k-1)$, $\mathbf{m}(k-1)$, $\hat{\mathbf{x}}_v(k)$, $\hat{\mathbf{x}}_b(k)$,

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$\hat{r}_b(k)$, $\hat{\alpha}_b(k)$ and $\hat{\mathbf{m}}(k)$ denote the estimates of the states $\mathbf{x}_v$, $\mathbf{x}_b$, u_b , γ_b and $\mathbf{m}$ at time $k-1$ and the predictions of the states $\mathbf{x}_v$, $\mathbf{x}_b$, r_b and α_b at time k given the states at time $k-1$. It may be noted that these linearized equations have the same general structure as the simple linear bias estimation algorithm for the linear SLAM case.

5.2.2 Analysis of the Problem

In this section, observability, convergence and bounds of uncertainty of the estimated bias parameters are investigated in the context of SLAM.

5.2.2.1 Diminishing uncertainties of bias estimates

In the case of the idealized linear case ((5.1) and (5.2)) or in the more realistic 2D nonlinear case ((5.21)-(5.24)), the SLAM with joint bias estimation assuming constant biases (where $\sigma_{ub}^2 = \sigma_{\gamma b}^2 = 0$) can be summarized as follows. The bias vector of (5.3) can be concisely expressed as

$$\mathbf{x}_b(k) = \begin{bmatrix} \mathbf{u}_b^T(k) & \mathbf{s}_b^T(k) \end{bmatrix}^T = \mathbf{x}_b(k-1) \quad (5.25)$$

where $\mathbf{u}_b(k)$ and $\mathbf{s}_b(k)$ are proprioceptive and exteroceptive sensor bias vectors respectively.

$$\mathbf{X}(k) = \mathbf{F}\mathbf{X}(k-1) + \begin{bmatrix} \mathbf{B}_v(\mathbf{u}(k-1)) \\ \mathbf{0}_{n_1 \times 1} \end{bmatrix} + \begin{bmatrix} \mathbf{v}(k) \\ \mathbf{0}_{n_1 \times 1} \end{bmatrix} \quad (5.26)$$

where $\mathbf{B}_v$ is the transition matrix of the proprioceptive sensor measurements, $\mathbf{v}(k) \sim N(\mathbf{0}, \mathbf{Q}_v(k))$ and $\mathbf{X}(k)$ is given by (5.4). n_1 is the total dimension of bias states and map states. The matrix $\mathbf{F}$ is

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_b & \mathbf{0}_{bm} \\ \mathbf{0}_{bm}^T & \mathbf{I}_{mm} \end{bmatrix} \text{ and } \mathbf{F}_b = \begin{bmatrix} \mathbf{F}_v & \mathbf{B}_v & \mathbf{0}_{vs} \\ \mathbf{0}_{vu}^T & \mathbf{I}_{uu} & \mathbf{0}_{us} \\ \mathbf{0}_{vs}^T & \mathbf{0}_{us}^T & \mathbf{I}_{ss} \end{bmatrix} \quad (5.27)$$

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where subscripts m , v , u and s denote the entire map, the vehicle, the proprioceptive sensor bias vector and the exteroceptive sensor bias vector respectively. Matrices $\mathbf{I}_{ss}$, $\mathbf{I}_{uu}$ and $\mathbf{I}_{mm}$ are identity matrices with dimensions $(\dim(\mathbf{s}_b))^2$, $(\dim(\mathbf{u}_b))^2$ and $(\dim(\mathbf{m}))^2$ respectively. $\mathbf{0}_{vs}$, $\mathbf{0}_{vu}$ and $\mathbf{0}_{us}$ are null matrices having appropriate dimensions. The observation model corresponding to (5.2) and (5.22) is

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{w}(k) \quad (5.28)$$

where $\mathbf{H}(k) = [-\mathbf{H}_v(k) \quad \mathbf{0}_{uu} \quad \mathbf{I}_{ss} \quad \mathbf{H}_m(k)]$. $-\mathbf{H}_v(k)$, $\mathbf{H}_m(k)$ and $\mathbf{0}_{uu}$ denote the parts of $\mathbf{H}(k)$ corresponding to vehicle states and map states and a null matrix having dimension $(\dim(\mathbf{u}_b))^2$. The zero mean observation noise vector is $\mathbf{w}(k) \sim N(\mathbf{0}, \mathbf{R}(k))$ and $\mathbf{R}(k)$ is the covariance matrix of $\mathbf{w}(k)$. The rest of the prediction and update equations then take the form of (5.15) to (5.20) with the process and observation models ((5.25) and (5.28)) replacing those given in the general formulation outlined in Section 5.2.1.2. By investigating the above models the following important result can be arrived at.

Result : *The uncertainties in the estimates of the proprioceptive and exteroceptive sensor bias parameters diminish monotonically in an update of the SLAM algorithm.*

Proof :

The covariance matrix $\mathbf{P}(k|k)$ of the composite state vector $\mathbf{X}(k)$, can be partitioned as follows:

$$\mathbf{P}(k|k) = \begin{bmatrix} \mathbf{P}_{vv}(k|k) & \mathbf{P}_{vu}(k|k) & \mathbf{P}_{vs}(k|k) & \mathbf{P}_{vm}(k|k) \\ \mathbf{P}_{vu}^T(k|k) & \mathbf{P}_{uu}(k|k) & \mathbf{P}_{us}(k|k) & \mathbf{P}_{um}(k|k) \\ \mathbf{P}_{vs}^T(k|k) & \mathbf{P}_{us}^T(k|k) & \mathbf{P}_{ss}(k|k) & \mathbf{P}_{sm}(k|k) \\ \mathbf{P}_{vm}^T(k|k) & \mathbf{P}_{um}^T(k|k) & \mathbf{P}_{sm}^T(k|k) & \mathbf{P}_{mm}(k|k) \end{bmatrix} \quad (5.29)$$

Let the partitions of $\mathbf{P}(k|k)$ be defined as follows:

$$\mathbf{P}_{bb}(k|k) = \begin{bmatrix} \mathbf{P}_{vv}(k|k) & \mathbf{P}_{vu}(k|k) & \mathbf{P}_{vs}(k|k) \\ \mathbf{P}_{vu}^T(k|k) & \mathbf{P}_{uu}(k|k) & \mathbf{P}_{us}(k|k) \\ \mathbf{P}_{vs}^T(k|k) & \mathbf{P}_{us}^T(k|k) & \mathbf{P}_{ss}(k|k) \end{bmatrix} \quad (5.30)$$

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$$\mathbf{P}_{bm}(k|k) = \begin{bmatrix} \mathbf{P}_{vm}^T(k|k) & \mathbf{P}_{um}^T(k|k) & \mathbf{P}_{sm}^T(k|k) \end{bmatrix}^T \quad (5.31)$$

From (5.16) the prediction of the composite state covariance matrix is determined as follows:

$$\mathbf{P}(k|k-1) = \begin{bmatrix} \mathbf{F}_b \mathbf{P}_{bb}(k-1|k-1) \mathbf{F}_b^T + \mathbf{Q}(k-1) & \mathbf{F}_b \mathbf{P}_{bm}(k-1|k-1) \\ \mathbf{P}_{bm}^T(k-1|k-1) \mathbf{F}_b^T & \mathbf{P}_{mm}(k-1|k-1) \end{bmatrix} \quad (5.32)$$

where $\mathbf{Q}(k-1)$ represents noise injections from the vehicle motion and bias terms. Let

$$\mathbf{F}_b \mathbf{P}_{bb}(k-1|k-1) \mathbf{F}_b^T + \mathbf{Q}(k-1) = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_4 & \mathbf{P}_5 \\ \mathbf{P}_4^T & \mathbf{P}_2 & \mathbf{P}_6 \\ \mathbf{P}_5^T & \mathbf{P}_6^T & \mathbf{P}_3 \end{bmatrix} \quad (5.33)$$

where

$$\begin{aligned} \mathbf{P}_1 = & \mathbf{F}_v \mathbf{P}_{vv}(k-1|k-1) \mathbf{F}_v^T + \mathbf{F}_v \mathbf{P}_{vu}(k-1|k-1) \mathbf{B}_v^T + \mathbf{B}_v \mathbf{P}_{uv}(k-1|k-1) \\ & + \mathbf{B}_v \mathbf{P}_{vu}(k-1|k-1) \mathbf{B}_v^T + \mathbf{F}_v \mathbf{P}_{vu}(k-1|k-1) + \mathbf{Q}_v(k) \end{aligned} \quad (5.34)$$

$$\mathbf{P}_2 = \mathbf{P}_{uu}(k-1|k-1) \quad (5.35)$$

$$\mathbf{P}_3 = \mathbf{P}_{ss}(k-1|k-1) \quad (5.36)$$

$$\mathbf{P}_4 = \mathbf{F}_v \mathbf{P}_{vu}(k-1|k-1) + \mathbf{B}_v \mathbf{P}_{uu}(k-1|k-1) \quad (5.37)$$

$$\mathbf{P}_5 = \mathbf{F}_v \mathbf{P}_{vs}(k-1|k-1) + \mathbf{B}_v \mathbf{P}_{us}(k-1|k-1) \quad (5.38)$$

$$\mathbf{P}_6 = \mathbf{P}_{us}(k-1|k-1) \quad (5.39)$$

As $\mathbf{P}(0|0)$, $\mathbf{Q}(k)$ and $\mathbf{R}(k)$ are positive semi definite (PSD) matrices, by the properties (see appendix B) of PSD matrices [124], $\mathbf{S}(k)$ and $\mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k)$ are also PSD matrices. Hence from (6.20)

$$\det(\mathbf{P}(k|k)) = \det(\mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k)) \quad (5.40)$$

$$\det(\mathbf{P}(k|k)) \leq \det(\mathbf{P}(k|k-1)) \quad (5.41)$$

Since any principal sub-matrix of a PSD matrix is also PSD

$$\det(\mathbf{P}_{uu}(k|k)) \leq \det(\mathbf{P}_{uu}(k|k-1)) \quad (5.42)$$

$$\det(\mathbf{P}_{ss}(k|k)) \leq \det(\mathbf{P}_{ss}(k|k-1)) \quad (5.43)$$

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But from (5.35) and (5.36) $\mathbf{P}_{uu}(k|k-1) = \mathbf{P}_{uu}(k-1|k-1)$ and $\mathbf{P}_{ss}(k|k-1) = \mathbf{P}_{ss}(k-1|k-1)$. Therefore, from (5.42) and (5.43)

$$\det(\mathbf{P}_{uu}(k|k)) \leq \det(\mathbf{P}_{uu}(k-1|k-1)) \quad (5.44)$$

$$\det(\mathbf{P}_{ss}(k|k)) \leq \det(\mathbf{P}_{ss}(k-1|k-1)) \quad (5.45)$$

Since the determinant of a matrix is proportional to its volume, determinant of covariance matrices indicate the volume or size of their uncertainty ellipses and therefore it can be concluded that the errors in the estimates of the bias parameters diminish in an update of the SLAM algorithm. Since the estimation is assumed to be synchronous, this decrease in uncertainty is monotonic.

5.2.2.2 Lower bounds on bias uncertainty

In this section we examine rigorously if the decrease in bias estimation errors over successive observations as shown in Section 5.2.2.1 will eventually become zero (biases completely determined with certainty) or approach a non-zero but finite lower limit. It may be noted that the lowest covariance estimate is obtained when the proprioceptive sensor noise and the modeling uncertainty (process noise) are at their minimum (i.e when $\mathbf{Q}$ is the lowest) and the observation noise is small. Such a scenario is encountered when the vehicle is stationary ($\mathbf{Q} = \mathbf{0}$) whilst observing an unknown landmark. However, under the circumstances it is not possible to obtain an estimate of the proprioceptive sensor bias terms as they are not observable when the vehicle is not in motion. Hence we calculate lower bounds for the covariance of exteroceptive sensor bias vector $\mathbf{s}_b$ when the vehicle is stationary. The composite SLAM state vector in this situation is

$$\mathbf{X}(k) = [\mathbf{x}_v^T(k) \quad \mathbf{L}^T(k) \quad \mathbf{s}_b^T(k)]^T \quad (5.46)$$

where $\mathbf{L}(k)$ is the unknown landmark location vector, which has to be estimated. The

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observation model $\mathbf{H}(k) = [-\mathbf{H}_v(k) \quad \mathbf{H}_L(k) \quad \mathbf{I}_{ss}]$ takes the form (5.28) where $-\mathbf{H}_v(k)$ and $\mathbf{H}_L(k)$ denote the parts of the observation matrix corresponding to vehicle state and the landmark state respectively. Since the vehicle is stationary, the predicted covariance matrix $\mathbf{P}(k | k-1)$ for all k is

$$\mathbf{P}(k | k-1) = \mathbf{P}(k-1 | k-1) \quad (5.47)$$

Suppose the initial covariance matrix of $\mathbf{X}(k)$ be $\mathbf{P}(0 | 0) = \text{diag}(\mathbf{P}_v, \mathbf{P}_L, \mathbf{P}_b)$ where $\mathbf{P}_v$, $\mathbf{P}_L$ and $\mathbf{P}_b$ denote the initial covariance matrices of the vehicle, landmark and the sensor biases respectively. Using the inverse covariance form of the Kalman filter [117]

$$\mathbf{P}^{-1}(k | k) = \mathbf{P}^{-1}(k | k-1) + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \quad (5.48)$$

Hence when taking k observations

$$\begin{aligned} \mathbf{P}^{-1}(k | k) &= \begin{bmatrix} \mathbf{P}_v^{-1} & 0 & 0 \\ 0 & \mathbf{P}_L^{-1} & 0 \\ 0 & 0 & \mathbf{P}_b^{-1} \end{bmatrix} + k \begin{bmatrix} -\mathbf{H}_v^T \\ \mathbf{H}_L^T \\ \mathbf{I}_{ss} \end{bmatrix} \mathbf{R}^{-1} [-\mathbf{H}_v \quad \mathbf{H}_L \quad \mathbf{I}_{ss}] \\ \mathbf{P}^{-1}(k | k) &= \left[\begin{array}{cc|c} \mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v & -k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_L & -k\mathbf{H}_v^T \mathbf{R}^{-1} \\ -k\mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_v & \mathbf{P}_L^{-1} + \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L & k\mathbf{H}_L^T \mathbf{R}^{-1} \\ \hline -k\mathbf{R}^{-1} \mathbf{H}_v & k\mathbf{R}^{-1} \mathbf{H}_L & \mathbf{P}_b^{-1} + k\mathbf{R}^{-1} \end{array} \right] = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \end{aligned} \quad (5.49)$$

$$\text{where } \mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v & -k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_L \\ k\mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_v & \mathbf{P}_L^{-1} + \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -k\mathbf{H}_v^T \mathbf{R}^{-1} \\ k\mathbf{H}_L^T \mathbf{R}^{-1} \end{bmatrix}, \mathbf{C} = \mathbf{B}^T$$

and $\mathbf{D} = \mathbf{P}_b^{-1} + k\mathbf{R}^{-1}$. Let $\mathbf{A}^{-1} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{bmatrix}$, then using the matrix inversion Lemma (see

Appendix B) $\mathbf{a}_{11} = (\mathbf{A}_{11} - \mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{A}_{21})^{-1}$ and similar identities

$$\mathbf{a}_{11} = (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v - (k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_L (\mathbf{P}_L^{-1} + \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L)^{-1} k\mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_v))^{-1} \quad (5.50)$$

$$\begin{aligned} \mathbf{a}_{12} &= (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} (-k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_L) \{ (k\mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_v) \\ &\quad (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} (k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_L) - (\mathbf{P}_L^{-1} + \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L) \}^{-1} \end{aligned} \quad (5.51)$$

$$\begin{aligned} \mathbf{a}_{21} &= \{ (k\mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_v) (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} (k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_L) - \\ &\quad (\mathbf{P}_L^{-1} + \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L) \}^{-1} (-k\mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_v) (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} \end{aligned} \quad (5.52)$$

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$$\mathbf{a}_{22} = ((\mathbf{P}_L^{-1} + \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L) - k^2 \mathbf{H}_L^T \mathbf{R}^{-1} \mathbf{H}_L (\mathbf{P}_v^{-1} + k \mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} \mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} \quad (5.53)$$

Let $\mathbf{P}(k|k) = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix}$ where the covariance matrix of the exteroceptive

sensor biases is $\mathbf{P}_{22}$. Using the matrix inversion lemma

$$\mathbf{P}_{22} = \{(\mathbf{P}_b^{-1} + k \mathbf{R}^{-1}) + k \mathbf{R}^{-1} \mathbf{H}_v (-\mathbf{a}_{11} \mathbf{H}_v^T \mathbf{R}^{-1} + \mathbf{a}_{12} k \mathbf{H}_L^T \mathbf{R}^{-1}) - k \mathbf{R}^{-1} \mathbf{H}_L (-\mathbf{a}_{21} \mathbf{H}_v^T \mathbf{R}^{-1} + \mathbf{a}_{22} k \mathbf{H}_L^T \mathbf{R}^{-1})\}^{-1} \quad (5.54)$$

Since the value of $\mathbf{P}_{22}$ is obtained by assuming zero vehicle and proprioceptive sensor uncertainties, its inverse represents the maximum information gain conceivable or realizable for the algorithm. In the limit when a large number of successive observations are made using the on board biased sensors while the vehicle is stationary, the upper limit of this information gain will be reached if at all a limit exists. Conversely, if this finite limit exists the covariance of the exteroceptive sensor bias term $\mathbf{P}_{22}$ will be lower bounded. Therefore, this limit if exists is a lower bound $\mathbf{P}_{22}^*$ for the covariance $\mathbf{P}_b(k|k)$ of the exteroceptive sensor bias term under the given circumstances. i.e.

$$\mathbf{P}_{22}^* = \lim_{k \rightarrow \infty} (\mathbf{P}_{22}) \quad (5.55)$$

The above lower bound of the covariance of the sensor biases can be obtained in closed form for the following special cases.

Case 1: When the initial uncertainties of the exteroceptive sensor biases are infinite. i.e. completely unknown.

From (5.45) to (5.51)

$$\mathbf{P}_b(k|k) = \lim_{\mathbf{P}_b \rightarrow \infty} (\mathbf{P}_{22}^*) = \mathbf{H}_L \mathbf{P}_L \mathbf{H}_L^T + \mathbf{H}_v \mathbf{P}_v \mathbf{H}_v^T \quad (5.56)$$

Therefore, the lower bound of the uncertainty of exteroceptive sensor biases when the bias terms are initially completely unknown is $\mathbf{H}_L \mathbf{P}_L \mathbf{H}_L^T + \mathbf{H}_v \mathbf{P}_v \mathbf{H}_v^T$. Hence, the lower bound of the sensor bias terms in this case is determined by the initial covariance of the

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vehicle ($\mathbf{P}_v$) and the landmark ($\mathbf{P}_L$) estimates. Otherwise $\mathbf{P}_b(k|k)$ should reach zero or a value independent of initial conditions $\mathbf{P}_v$ and $\mathbf{P}_L$ depending on the observability conditions. The cause of this phenomenon can be attributed to the partial observability of the problem and will be discussed in detail in a latter section.

Case 2 : When the initial landmark position is very large.

$\mathbf{P}_L \rightarrow \infty$. i.e $\mathbf{P}_L^{-1} = 0$. After careful rearrangement of terms and simplification of (5.50) to (5.55)

$$\mathbf{P}_{22} = \mathbf{P}_b \quad (5.57)$$

Hence $\mathbf{P}_b(k|k) = \mathbf{P}_b$ for all k . Therefore, the uncertainties of exteroceptive sensor biases cannot be reduced by observing initially completely unknown landmark from a stationary vehicle. These observations can be reiterated for a more idealistic situation involving a single degree of freedom SLAM problem [125]. Consider the simplified process and observation models derived from (5.1) and (5.2) removing the proprioceptive bias term in the state vector for 1 landmark map.

$$\mathbf{X}(k) = [x_v(k) \quad L(k) \quad s_b(k)]^T \quad (5.58)$$

$$z(k) = [-1 \quad 1 \quad 1][x_v(k) \quad L(k) \quad s_b(k)]^T \quad (5.59)$$

By comparing with (6.28), this is equivalent to $\mathbf{H} = [-1 \quad 1 \quad 1]$, $\mathbf{H}_v = 1$ and $\mathbf{H}_L = 1$.

Using the scalar versions of uncertainties for initial covariance matrix of $\mathbf{X}(k)$ given by

$\mathbf{P}(0|0) = \text{diag}(P_v, P_L, P_b)$ and using the standard inverse covariance form of the Kalman filter as in (6.48)

$$\mathbf{P}(k|k) = \frac{1}{R + kP_b + kP_L + kP_v} \begin{bmatrix} (R + kP_b + kP_L)P_v & kP_vP_L & kP_vP_b \\ kP_vP_L & (R + kP_b + kP_v)P_L & -kP_LP_b \\ kP_vP_b & -kP_LP_b & (R + kP_L + kP_v)P_b \end{bmatrix} \quad (5.60)$$

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$$\mathbf{P}(\infty) = \lim_{k \rightarrow \infty} \mathbf{P}(k|k) = \frac{1}{P_b + P_L + P_v} \begin{bmatrix} (P_b + P_L)P_v & P_v P_L & P_v P_b \\ P_v P_L & (P_b + P_v)P_L & -P_L P_b \\ P_v P_b & -P_L P_b & (P_L + P_v)P_b \end{bmatrix} \quad (5.61)$$

where $\mathbf{P}(\infty)$ is the error covariance matrix of $\mathbf{X}(k)$ given a large number of observations.

Decrease in exteroceptive sensor bias variance $\delta P_b(\infty)$ is given by

$$\delta P_b(\infty) = P_b - \frac{(P_L + P_v)P_b}{P_b + P_L + P_v} = \frac{P_b^2}{P_b + P_L + P_v} > 0 \quad (5.62)$$

suggesting that the exteroceptive sensor bias variance in the limit is strictly monotonically decreasing and bounded by (5.61).

5.2.2.3 Observability conditions

It is always important to know the observability properties of process and observation models in designing the bias estimation algorithms so that all the states are estimated consistently. Because of the growing correlations among the constituent states and as a result of partial observability [127] in the standard SLAM problem, the full reconstruction of the map state vector is not possible with typical measurement models. Thus it is interesting to investigate whether the SLAM with bias estimation problem is also subjected to the same limitations as the standard SLAM problem. We analyze the observability conditions of SLAM with joint bias correction problem for several scenarios to arrive at a conclusion on the estimator performance and to find the means of rectifying problems if any. As the first case we analyze the observability of 1D SLAM with n landmark locations in the state vector when observing the same n landmarks utilizing biased proprioceptive and exteroceptive sensors. The process and observation equations after writing in terms of linearized error states are as follows with the usual notation. The input noise vector $\begin{bmatrix} v(k-1) & \mathbf{0}_{1 \times (n+2)} & [w_1 \ w_2 \ \dots \ w_n] \end{bmatrix}$ is written in terms of concatenated process and measurement noise terms.

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$$\tilde{\mathbf{X}}(k) = \mathbf{F}\tilde{\mathbf{X}}(k-1) + \mathbf{B} \begin{bmatrix} v(k-1) & \mathbf{0}_{1 \times (n+2)} & w_1 & w_2 & \dots & w_n \end{bmatrix}^T \quad (5.63)$$

$$\tilde{\mathbf{z}}(k) = \mathbf{H}\tilde{\mathbf{X}}(k) + \mathbf{H}_w \begin{bmatrix} v(k-1) & \mathbf{0}_{1 \times (n+2)} & w_1 & w_2 & \dots & w_n \end{bmatrix}^T \quad (5.64)$$

where $\mathbf{H} = \begin{bmatrix} -\mathbf{1}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{1}_{n \times 1} & \mathbf{I}_{n \times n} \end{bmatrix}$, $\mathbf{H}_w = \begin{bmatrix} \mathbf{0}_{n \times (n+3)} & \mathbf{I}_{n \times n} \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} 1 & 1 & \mathbf{0}_{1 \times (n+1)} \\ 0 & 1 & \mathbf{0}_{1 \times (n+1)} \\ \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times 1} & \mathbf{I}_{(n+1) \times (n+1)} \end{bmatrix}$,

$$\mathbf{B} = \begin{bmatrix} 1 & \mathbf{0}_{1 \times (2n+2)} \\ \mathbf{0}_{(n+2) \times 1} & \mathbf{0}_{(n+2) \times (2n+2)} \end{bmatrix}, \quad \tilde{\mathbf{X}}(k) = \mathbf{X}(k) - \mathbf{X}(k | k-1), \quad \tilde{\mathbf{X}}(k-1) = \mathbf{X}(k-1) - \mathbf{X}(k-1 | k-1) \quad \text{and}$$

$\tilde{\mathbf{Z}}(k) = \mathbf{Z}(k) - \mathbf{Z}(k | k-1)$. The quantities $\mathbf{X}(k)$, $\mathbf{X}(k-1)$, $\mathbf{Z}(k)$ and $\mathbf{Z}(k | k-1)$ denote the true values of the composite state at time k and $k-1$, true value of the measurement at time k and the measurement prediction at time k based on the estimated state at time $k-1$.

It is now possible to compute observability and controllability Grammians $\mathbf{G}_o$ and $\mathbf{G}_c$.

For linear time invariant systems such as (5.63) and (5.64) $\mathbf{G}_o$ and $\mathbf{G}_c$ can be determined as follows:

$$\mathbf{G}_o = \left[(\mathbf{H}\mathbf{F})^T \quad (\mathbf{H}\mathbf{F}^2)^T \quad \dots \quad (\mathbf{H}\mathbf{F}^i)^T \right]^T \quad (5.65)$$

$$\mathbf{G}_c = \left[\mathbf{B} \quad \mathbf{F}\mathbf{B} \quad \mathbf{F}^2\mathbf{B} \quad \dots \quad \mathbf{F}^{i-1}\mathbf{B} \right] \quad (5.66)$$

where i is the dimension of the state vector $\tilde{\mathbf{X}}(k)$. The system is said to be controllable if $\mathbf{G}_c$ has a column rank of i and observable if $\mathbf{G}_o$ has a row rank of i .

$$\mathbf{G}_o^T = \begin{bmatrix} -\mathbf{1}_{1 \times n} & -\mathbf{1}_{1 \times n} & \dots & -\mathbf{1}_{1 \times n} \\ -\mathbf{1}_{1 \times n} & -\mathbf{2}_{1 \times n} & \dots & -\mathbf{n}_{1 \times n} \\ \mathbf{1}_{1 \times n} & \mathbf{1}_{1 \times n} & \dots & \mathbf{1}_{1 \times n} \\ \mathbf{I}_{n \times n} & \mathbf{I}_{n \times n} & \dots & \mathbf{I}_{n \times n} \end{bmatrix} \quad (5.67)$$

$$\text{Image}(\mathbf{G}_c) = \begin{bmatrix} 1 \\ \mathbf{0}_{(n+3) \times 1} \end{bmatrix} \quad (5.68)$$

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$$\text{Image}(\mathbf{G}_O^T) = \begin{bmatrix} \mathbf{1}_{1 \times n} & 0 \\ \mathbf{0}_{1 \times n} & 1 \\ -\mathbf{1}_{1 \times n} & 0 \\ -\mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} \end{bmatrix} \quad (5.69)$$

where $\text{Image}(\mathbf{X})$ denotes a basis of the image of the vector space specified by $\mathbf{X}$. Hence from (5.65) and (5.66), it is clear that the rank of the observability Grammian is $n+1$ and therefore it is rank deficient by 2. Thus the sensor bias correction problem in SLAM is partially observable. Under such partial observability conditions, the angle ψ between the controllable and observable subspaces is an indication of the closeness of observable states from fully revising the process state and therefore this angle is a measure of partial observability of the problem.

Result : *The angle ψ between the controllable and observable subspaces in single degree of freedom SLAM problem with constant bias estimation is a function of the number of landmarks in the map only and is given by*

$$\psi = \cos^{-1} \left(\sqrt{\frac{n}{2n+1}} \right) \quad (5.70)$$

Proof :

As given by the equations (5.68) and (5.69), $\text{Image}(\mathbf{G}_O^T)$ and $\text{Image}(\mathbf{G}_C)$ consists of $n+1$ and 1 basis vectors respectively. Let $\underline{\mathbf{r}}_i$ for $i=1,2,\dots,n+1$ be the basis vectors of $\text{Image}(\mathbf{G}_O^T)$ and $\underline{\mathbf{q}}$ be the basis vector of $\text{Image}(\mathbf{G}_C)$. Suppose $\underline{\mathbf{p}}$ be the projection of the basis of $\text{Image}(\mathbf{G}_C)$ into the basis spanned by $\text{Image}(\mathbf{G}_O^T)$. Thus using the algebra of vector transformations and projections;

$$\underline{\mathbf{p}} = \sum_{i=1}^{n+1} \frac{\underline{\mathbf{q}}^T \underline{\mathbf{r}}_i}{\underline{\mathbf{r}}_i^T \underline{\mathbf{r}}_i} \underline{\mathbf{r}}_i = \sum_{i=1}^n \frac{\underline{\mathbf{q}}^T \underline{\mathbf{r}}_i}{\underline{\mathbf{r}}_i^T \underline{\mathbf{r}}_i} \underline{\mathbf{r}}_i + \frac{\underline{\mathbf{q}}^T \underline{\mathbf{r}}_{n+1}}{\underline{\mathbf{r}}_{n+1}^T \underline{\mathbf{r}}_{n+1}} \underline{\mathbf{r}}_{n+1}, \quad \text{Since } \underline{\mathbf{q}}^T \underline{\mathbf{r}}_{n+1} = 0, \quad \underline{\mathbf{r}}_i^T \underline{\mathbf{r}}_i = 3 \quad \text{for all } i \quad \text{and}$$

$$\underline{\mathbf{p}} = \frac{1}{3} \sum_{i=1}^n \underline{\mathbf{r}}_i. \quad \text{Thus, } \|\underline{\mathbf{p}}\| = \sqrt{2(n/3)^2 + n(1/3)^2} = \sqrt{(2n^2 + n)/9} \quad \text{and } \underline{\mathbf{p}}^T \underline{\mathbf{q}} = n/3.$$

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Thus, the angle between the controllable and observable subspaces is

$$\psi = \cos^{-1} \left(\frac{(\underline{\mathbf{p}}^T \underline{\mathbf{q}}) / (\|\underline{\mathbf{p}}\| \|\underline{\mathbf{q}}\|)}{\left((n/3) / \left(\sqrt{2n^2 + n/3} \right) \right)} \right) = \cos^{-1} \left(\sqrt{\frac{n}{2n+1}} \right)$$

Since ψ is a monotonically decreasing function of n (Figure 5.1), as the number of landmarks increases the observable subspace gets closer to the controllable part of the state space. However, ψ cannot be reduced below $\pi/4$. Therefore, it is important to propose methods of providing full observability for the bias estimation problem in SLAM.

Figure 5.1. Angle between controllable and observability subspaces. Figure 5.1(a) show the variation of the function specifying the cosine of the angle and Figure 5.2(b) show the variation of the angle.

An important 2nd case arises when the proprioceptive sensor bias varies with a random walk. In this case $\mathbf{X}(k)$, $\mathbf{F}$, $\mathbf{H}$ and $\mathbf{z}(k)$ take the same forms as if they are in the previous case. The model (5.63) is changed as follows:

$$\tilde{\mathbf{X}}(k) = \mathbf{F}\tilde{\mathbf{X}}(k-1) + \mathbf{B} \begin{bmatrix} v(k) & v_b(k) & \mathbf{0}_{1 \times (n+1)} & w_1 & w_2 & \dots & w_n \end{bmatrix}^T \quad (5.71)$$

and $\mathbf{B} = \begin{bmatrix} 1 & 0 & \mathbf{0}_{1 \times (2n+1)} \\ 0 & 1 & \mathbf{0}_{1 \times (2n+1)} \\ \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times (2n+1)} \end{bmatrix}$ where $v_b(k)$ is a Gaussian random noise sequence of

the proprioceptive sensor. The bases of the images of observability and controllability Grammians are

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$$\text{Image}(\mathbf{G}_O^T) = \begin{bmatrix} \mathbf{1}_{1 \times n} & 0 \\ \mathbf{0}_{1 \times n} & 1 \\ -\mathbf{1}_{1 \times n} & 0 \\ -\mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} \end{bmatrix} \quad (5.72)$$

$$\text{Image}(\mathbf{G}_C) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times 1} \end{bmatrix} \quad (5.73)$$

In this case also the rank of $\mathbf{G}_O^T$ is $n+1$. i.e. it is rank deficient. However, it is difficult to obtain an analytical expression for the angle between controllable and observable subspaces in this case. Nevertheless, the angle can be calculated using the following definition.

Definition :

Let columns of matrices $\mathbf{Q}_F \in \mathbb{R}^{n \times p}$ and $\mathbf{Q}_G \in \mathbb{R}^{n \times q}$ form orthonormal bases for the subspaces $\mathbf{F}$ and $\mathbf{G}$ respectively. The reduced singular value decomposition is $\mathbf{U}^T \mathbf{Q}_F^T \mathbf{Q}_G \mathbf{V} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_q)$ and $1 \geq \sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_q \geq 0$ where $\mathbf{U} \in \mathbb{R}^{p \times q}$ and $\mathbf{V} \in \mathbb{R}^{q \times q}$ both have orthonormal columns. The principal angle can then be computed as, $\theta_k = \cos^{-1}(\sigma_k)$, $k = 1, 2, \dots, q$ where $0 \leq \sigma_1 \leq \sigma_2 \leq \dots \leq \sigma_q \leq \frac{\pi}{2}$. The angle between the two subspaces $\mathbf{F}$ and $\mathbf{G}$ is given by the smallest singular value ($\min(\sigma_k)$).

Using the above definition, it can be verified that ψ closely follows (as shown in Figure 5.2) the expression (5.70).

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Figure 5.2. Angle between controllable and observable subspaces for the 2nd case.

The above partial observability situation can be rectified by incorporating a known landmark observation and absolute position information in the form of an absolute position sensor (such as GPS) in the measurement vector in the 1D case. Thus, the observation matrix $\mathbf{H}$ of the fully observable 1D SLAM algorithm when observing n unknown landmarks with sensor bias estimation will become

$$\mathbf{H} = \begin{bmatrix} -\mathbf{1}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{1}_{n \times 1} & \mathbf{I}_{n \times n} \\ -1 & 0 & 1 & \mathbf{0}_{1 \times n} \\ 1 & 0 & 0 & \mathbf{0}_{1 \times n} \end{bmatrix} \quad (5.74)$$

$$\mathbf{G}_O^T = \begin{bmatrix} -\mathbf{1}_{1 \times n} & -1 & 1 & -\mathbf{1}_{1 \times n} & -1 & 1 & .. & -\mathbf{1}_{1 \times n} & -1 & 1 \\ -\mathbf{1}_{1 \times n} & -1 & 1 & -\mathbf{2}_{1 \times n} & -2 & 2 & .. & -\mathbf{n}_{1 \times n} & -n & n \\ \mathbf{1}_{1 \times n} & 1 & 0 & \mathbf{1}_{1 \times n} & 1 & 0 & .. & \mathbf{1}_{1 \times n} & 1 & 0 \\ \mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} & \mathbf{0}_{n \times 1} & .. & \mathbf{I}_{n \times n} & \mathbf{0}_{n \times 1} & \mathbf{0}_{n \times 1} \end{bmatrix} \quad (5.75)$$

The $\text{rank}(\mathbf{G}_O^T) = n + 3$, which is equal to the dimension of the state vector. Hence the system is now fully observable. In this case, $\text{Image}(\mathbf{G}_C) = \begin{bmatrix} 1 & \mathbf{0}_{1 \times (n+3)} \end{bmatrix}^T$. Thus, the angle between the controllable and observable subspaces is now zero.

In general, there are no specific rules governing the observability of nonlinear stochastic systems and therefore it is difficult to have prior judgments on the viability of EKF in 2D SLAM. However, an observability test similar to that is suitable for linear time invariant systems may be carried out for nonlinear systems as in [128] and [134]. Here, the

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observation and process models are linearized around a state using Taylor series and the rank of the observability matrix constructed from the Jacobians of the observation and process models are examined assuming the linearized models are piecewise constant. Although this rank analysis is not always sufficient to ensure observability, it provides necessary conditions [128].

The observability analysis of the planer (2D) nonlinear SLAM is carried out in a similar manner by the symbolic manipulation of the models (5.3) to (5.20) with a single landmark. The analysis shows that the observability matrix in this case is rank deficient by 4 (Dimension of the state vector is 9, and rank of $\mathbf{G}_o^T$ is 5). The empirical study (using simulations) of the observability matrix with increasing number of landmarks in the state vector further establishes that the rank deficiency remains 4 no matter of the increase in the number of landmarks.

It is further established that the matrix $\mathbf{G}_o$ becomes full rank when observing more than one known landmark at the same time. Thus, for any number of landmarks in the state vector it is necessary that at least two of the landmarks are known and observed simultaneously for the observability of the 2D non-linear SLAM with bias correction.

5.2.2.4 Rates of convergence of uncertainties of sensor biases

From a practical implementation perspective it is important to know the rate of convergence of the bias estimation algorithm. The phenomenon of convergence can be elucidated by studying the 1D SLAM problem introduced in Section 5.2.1.1 by (5.1) and (5.2) together with the modified observability matrix (5.74) that ensures full observability. The map state is assumed to consist of only one landmark ($n = 1$).

Closed form solutions for the uncertainty terms of the bias estimates and the vehicle location estimate can be derived using continuous time equivalents of (5.1) and (5.74) for one landmark map. The factors that affect the rates of convergence of these uncertainties

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are clearly visible from the closed form solution.

The continuous time process model of the discrete time model (5.1) is

$$\begin{bmatrix} \dot{x}_v(t) \\ \dot{u}_b(t) \\ \dot{s}_b(t) \\ \dot{L}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_v(t) \\ u_b(t) \\ s_b(t) \\ L(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [u(t) + v(t)] \quad (5.76)$$

where $x_v(t)$, $u_b(t)$, $s_b(t)$, $L(t)$ and $u(t)$ denote the position of the robot, bias of the proprioceptive sensor, bias of the exteroceptive sensor, landmark location (map) and proprioceptive sensor measurement in the continuous space at time t respectively. $\mathbf{v}(t) \sim N(0, q^2)$ represents the proprioceptive sensor noise and modeling uncertainties if any. The analogous continuous time state vector of the problem is given by $\mathbf{X}(t) = [x_v(t) \ u_b(t) \ s_b(t) \ L(t)]^T$. The observation model incorporating an external absolute sensor measurement of the vehicle position, a known landmark and an unknown landmark is given by:

$$\mathbf{z}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{w}(t) \quad (5.77)$$

where $\mathbf{w}(t) \sim N(\mathbf{0}_{3 \times 1}, \mathbf{R})$, $\mathbf{R} = \text{diag}(r^2, r^2, r_e^2)$, r^2 and r_e^2 denote the measurement noise, covariance matrix of $\mathbf{w}(t)$, variances of exteroceptive sensor measurement noise and absolute sensor measurement noise. $\mathbf{H}$ is given by (5.74) when $n=1$.

Let $\mathbf{G} = [1 \ 0 \ 0 \ 0]^T$, $\mathbf{F} = \begin{bmatrix} 0 & 1 & \mathbf{0}_{1 \times 2} \\ \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 1} & \mathbf{0}_{3 \times 2} \end{bmatrix}$, and $\mathbf{P}(0) = \text{diag}(0, \sigma_{bu}^2, \sigma_{bs}^2, \sigma_l^2)$ where $\mathbf{P}(0)$ is the

initial covariance matrix of model (5.76) and $\mathbf{I}$ denote the identity matrix of size 4x4. The quantities σ_{bu}^2 , σ_{bs}^2 and σ_l^2 denote the initial variances of proprioceptive and exteroceptive sensor biases and the unknown landmark respectively. The initial uncertainty of the vehicle is assumed zero. $\mathbf{P}(t)$ is governed by the continuous time Riccati equation

$$\dot{\mathbf{P}}(t) = \mathbf{F}\mathbf{P}(t) + \mathbf{P}(t)\mathbf{F}^T + \mathbf{G}\mathbf{Q}\mathbf{G}^T - \mathbf{P}(t)\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{P}(t) \quad (5.78)$$

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The solution to the Riccati equation [126] is of the form $\mathbf{P}(t) = \mathbf{M}(t)\mathbf{N}^{-1}(t)$ where $\mathbf{M}(t)$ and $\mathbf{N}(t)$ are given by the following equations.

$$\begin{bmatrix} \dot{\mathbf{M}}(t) \\ \dot{\mathbf{N}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{F} & \mathbf{G}\mathbf{Q}\mathbf{G}^T \\ \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} & -\mathbf{F}^T \end{bmatrix} \begin{bmatrix} \mathbf{M}(t) \\ \mathbf{N}(t) \end{bmatrix} \quad (5.79)$$

$$\begin{bmatrix} \mathbf{M}(0) \\ \mathbf{N}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{P}(0) \\ \mathbf{I} \end{bmatrix} \quad (5.80)$$

The characteristic equation $\mathbf{C}(t)$ of the system and the m^{th} row n^{th} column element $\mathbf{P}(m,n)$ of $\mathbf{P}(t)$ for all m and n can then be obtained by solving (5.79) and (5.80) as follows:

$$\mathbf{C}(t) = (c_1 e^{-2\alpha t} + c_2)t^3 + (c_3 e^{-2\alpha t} + c_4)t^2 + (c_5 e^{-2\alpha t} + c_6 e^{-\alpha t} + c_7)t + c_8 e^{-2\alpha t} + c_9 e^{-\alpha t} + c_{10} \quad (5.81)$$

$$\begin{aligned} \mathbf{P}(1,1) = (1/\mathbf{C}(t)) & ((c_{11} e^{-2\alpha t} + c_{12})t^3 + (c_{13} e^{-2\alpha t} + c_{14} e^{-\alpha t} + c_{15})t^2 \\ & + (c_{16} e^{-2\alpha t} + c_{17} e^{-\alpha t} + c_{18})t + c_{19} e^{-2\alpha t} + c_{20} e^{-\alpha t} + c_{21}) \end{aligned} \quad (5.82)$$

$$\mathbf{P}(1,2) = (1/\mathbf{C}(t)) ((c_{22} e^{-2\alpha t} + c_{23})t^2 + (c_{24} e^{-2\alpha t} + c_{25} e^{-\alpha t} + c_{26})t + c_{27} e^{-2\alpha t} + c_{28} e^{-\alpha t} + c_{29}) \quad (5.83)$$

$$\mathbf{P}(1,3) = (1/\mathbf{C}(t)) c_{30} (1 - e^{-\tau t})^2 (c_{31} t^2 + c_{32} t + c_{33}) \quad (5.84)$$

$$\mathbf{P}(1,4) = (1/\mathbf{C}(t)) c_{34} (1 - e^{-\tau t})^2 (c_{35} t^2 + c_{36} t + c_{37}) \quad (5.85)$$

$$\mathbf{P}(2,2) = (1/\mathbf{C}(t)) ((c_{38} e^{-2\alpha t} + c_{39})t^2 + (c_{40} e^{-2\alpha t} + c_{41})t + c_{42} e^{-2\alpha t} + c_{43}) \quad (5.86)$$

$$\mathbf{P}(2,3) = (1/\mathbf{C}(t)) c_{44} (1 - e^{-\tau t})^2 (c_{45} t^2 + c_{46} t + c_{47}) \quad (5.87)$$

$$\mathbf{P}(2,4) = (1/\mathbf{C}(t)) c_{48} (1 - e^{-\tau t})^2 (c_{49} t^2 + c_{50} t + c_{51}) \quad (5.88)$$

$$\mathbf{P}(3,3) = (1/\mathbf{C}(t)) ((c_{52} e^{-2\alpha t} + c_{53})t^2 + (c_{54} e^{-2\alpha t} + c_{55})t + c_{56} e^{-2\alpha t} + c_{57} e^{-\alpha t} + c_{58}) \quad (5.89)$$

$$\mathbf{P}(3,4) = (1/\mathbf{C}(t)) ((c_{59} e^{-2\alpha t} + c_{60})t^2 + (c_{61} e^{-2\alpha t} + c_{62})t + c_{63} e^{-2\alpha t} + c_{64} e^{-\alpha t} + c_{65}) \quad (5.90)$$

$$\mathbf{P}(4,4) = (1/\mathbf{C}(t)) ((c_{66} e^{-2\alpha t} + c_{67})t^2 + (c_{68} e^{-2\alpha t} + c_{69})t + c_{70} e^{-2\alpha t} + c_{71} e^{-\alpha t} + c_{72}) \quad (5.91)$$

$$\alpha = (q/(rr_e)) (2r_e^2 + r^2)^{0.5} \quad (5.92)$$

where c_1 to c_{72} are functions of either one or more variables out of q , r , r_e , σ_l , σ_{bu} and

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σ_{bs} . Here, $1/\alpha$ is the time constant of the decay of the covariance terms. All the other terms of the covariance matrix can be derived from (5.81) to (5.92) using the symmetry of $\mathbf{P}(t)$. The equations ((5.81)-(5.92)) show that the error covariance terms of $\mathbf{X}(t)$ decay exponentially initially at a time constant of $1/\alpha$ and then converge asymptotically to the steady state covariance according to the same expressions. This asymptotic convergence of the error covariance terms can be a disadvantage in actual practice. It may be interesting to note that this convergence rate is not affected by the biases or their initial uncertainties and depends only on the proprioceptive sensor noise term, exteroceptive sensor noise term, absolute sensor noise term and the number of unknown landmarks in the map. When the variance of the absolute sensor noise r_e^2 is very large compared to r^2 (i.e landmark based information dominates the absolute sensor based information) the time constant approaches $\sqrt{r^2/(2q^2)}$. In other words, the smaller is the ratio between the exteroceptive sensor noise and the process noise (proprioceptive sensor noise and the modeling uncertainty) or the larger is the number of landmarks observed simultaneously, the faster will be the convergence of the state variances to their steady state values. This is an important result worthy of consideration when selecting sensors to achieve good performance. The steady state covariance $\mathbf{P}(\infty)$ of $\mathbf{X}(t)$ can be deduced from (5.81) to (5.92) in the limit as $t \rightarrow \infty$. Thus, it can be noted that all the state covariance terms except that of the vehicle position approaches to zero in the limit. The vehicle position variance approaches to q^2/α in the limit. Thus, we can conclude that the rates of convergence of state uncertainties do not depend on the bias parameters or their initial uncertainties. But they depend only on the amount of proprioceptive sensor noise, modelling uncertainty and the exteroceptive sensor noise. These theoretically established properties provide important insights into the problem and therefore useful in practical applications.

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5.2.3. Simulation of SLAM with Bias Estimation

In this section we consider important simulation scenarios to verify the theoretical results established on the observability, convergence and lower bounds and to assess the importance of estimating biases on line in the context of SLAM. Simulations were done for 2D SLAM (models (5.3) to (5.20)) with biased sensors having range bias 0.5m, bearing bias 2° , speed sensor input bias 0.25m/s and steering input angle sensor bias 1° and the random noise sequences with variances 0, 0, 0.0004 m^2/s^4 and $4 \times 10^{-6} rad^2/s^2$ respectively. It is specifically important to note that when there are sensor biases and no bias estimation is utilized it is almost impossible to estimate the vehicle position consistently. The effects of the sensor biases on the performance of 2D SLAM was first examined by deliberately adding the above set of biases and drift parameters to a real data set obtained in a large loop in the NTU university campus with SICK LMS 290 laser measurement system (LMS), steering and speed encoders in an approximately 1.1 km stretch. The sensors were mounted on an in-house built car like mobile robot (Appendix A). The feature extraction for SLAM was carried out using a simple clustering (Chapter 2) of adjacent laser range bearing readings. Figure 5.3 (a) shows the estimated path and the approximate true path of the vehicle when biased data set is used without the bias estimation algorithm in standard SLAM. Figure. 5.3 (b) is obtained by using the biased data set running a SLAM algorithm with bias estimation algorithm incorporated. Landmarks and the map are drawn in a digital map of the environment (scale 5000:1) in Figure 5.3 (a) and (b).

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Figure 5.3. Simulation of SLAM in 2D terrain with bias estimation: Figure (a) shows the map divergence in the case of large scale SLAM in loop closing experiments. Dashed line marks the true path robot should travel and the thick line is the estimated robot trajectory without bias compensation. Figure (b) represents the same experiment in loop closing with on line bias compensation. Figures (c), (d) and (e) show the 2D SLAM, localization error in x, y and heading with online bias estimation. Figures (f)-(i) show sensor bias estimates. Dashed lines in Figures (c), (d) and (e) show 95% confidence limits and the thick line in the other figures show the average bias estimated. Figures (j) and (k) show the variation of standard deviation of sensor biases.

The results shown in Figure 5.3 (a) illustrate the map divergence and inconsistent localization results in the performance of 2D SLAM in the presence of sensor biases. It is also evident that the inconsistent estimation is corrected and the loop closing error

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reduced when using online bias estimation with imposing fully observable conditions (Figure 5.3 (b)). This step is carried out by always providing at least 2 known feature observations throughout the simulation.

The SLAM with online bias estimation and compensation and its performance are further evaluated in a simulated environment consisting of 25 point landmarks. An autonomous vehicle is assumed to be navigating in an approximately circular path of 20m radius with a 2D scanning range bearing sensor (eg. SICK LMS 290 laser measurement system) and wheel encoders. Figures 6.3 (c)-(k) were obtained for SLAM with online bias estimation and correction algorithm with fully observable measurement models (i.e always incorporating two known landmark observations). The results show that the SLAM works well with sensor bias estimation and with ever diminishing uncertainties of bias estimates highlighted in the theoretical discussion in Section 5.2.2. Figure 5.3 (j) and (k) also show that the uncertainties of bias estimates reach a lower bound. These results (i.e. the convergence of estimated bias parameters to the actual values) further corroborate the theoretical results given in Section 5.2.2.3.

5.3. Estimation Theoretic Sensor Bias Correction in MAL

Even though the MAL [2], [98], [99], [129] and [130] per se has received considerable interest in localization, the effects and implications of exteroceptive and proprioceptive sensor biases on MAL are not adequately addressed and understood. In MAL, the propagation of sensor uncertainties due to systematic and nonsystematic errors often results in inconsistent and inaccurate localization results. Nonlinear transformations in the EKF also cause the Gaussian assumptions in the measurement and process models to be invalid. Persistent and slowly varying biases due to modeling errors, sensor imperfections and imperfect calibrations also contribute to poor performance. Thus, bias and modeling offsets and their cumulative effects cause significant problems in large-scale outdoor

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MAL implementations. The work described here analyses the augmented state vector method in the online estimation of bias parameters to overcome the above problems in the context of EKF/MAL.

5.3.1 Bias Estimation

In this thesis, it is shown how any slowly varying biases in a mobile robot's proprioceptive and exteroceptive sensors can be estimated and compensated for in the context of the MAL problem. For simplicity, the derivation is given for a single exteroceptive sensor with range and bearing biases together with proprioceptive sensor biases without losing the generality of including many sensors.

5.3.1.1 Single degree of freedom MAL problem

Consider the problem of single degree-of-freedom MAL with concurrent sensor bias estimation. Here, a vehicle traversing a straight line path (along the x-axis) is assumed to localize itself on the line based on the correct association of landmarks observed on the line with those stored in an a priori map $\mathbf{m}$. When the vehicle's proprioceptive and exteroceptive sensors' biases are $u_b(k)$ and $s_b(k)$ at time k respectively, the process model (incorporating the biases) and the measurement models of the problem when observing a single landmark are

$$\mathbf{X}(k) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v(k-1) \\ u_b(k-1) \\ s_b(k-1) \end{bmatrix} + \begin{bmatrix} u(k-1) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} v(k-1) \\ 0 \\ 0 \end{bmatrix} \quad (5.93)$$

$$z(k) = [-1 \quad 0 \quad 1] \begin{bmatrix} x_v(k) & u_b(k) & s_b(k) \end{bmatrix}^T + L + w(k) \quad (5.94)$$

where $\mathbf{X}(k) = [x_v(k) \quad u_b(k) \quad s_b(k)]^T$, $v(k) \sim N(0, q^2)$ is the proprioceptive sensor input noise, L is the coordinate of a stored landmark, $w(k) \sim N(0, r^2)$ is the exteroceptive

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sensor noise, q^2 and r^2 are the variances of $v(k)$ and $w(k)$

5.3.1.2 Two dimensional MAL problem on a flat terrain

Using the same notation and conventions used in Section 5.2.1.2 and appendix A, the vector of biases $\mathbf{x}_b(k)$ is given by (5.3), bias model is given by (5.8)-(5.12) and the vehicle model is given by (5.5)-(5.7). The composite state vector $\mathbf{X}(k)$ is formed by concatenating the vehicle state vector and the vector of biases as follows:

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_v^T(k) & \mathbf{x}_b^T(k) \end{bmatrix}^T \quad (5.95)$$

For simplicity, the landmarks in the environment are represented as point features. Thus, the stored map is a vector $\mathbf{m}$ of n landmark points $\mathbf{L}_i = \begin{bmatrix} m_i^x & m_i^y \end{bmatrix}^T$, $i = 1..n$ where (m_i^x, m_i^y) represents the (x, y) coordinate pair of the i^{th} landmark. If an exteroceptive sensor such as a laser range measurement system (e.g. SICK LMS 290) is used to observe the i^{th} landmark's range and bearing, we now obtain the following observation model:

$$\begin{aligned} \mathbf{z}(k) &= \begin{bmatrix} \sqrt{(m_i^x - x_s(k))^2 + (m_i^y - y_s(k))^2} \\ \tan^{-1} \left((m_i^y - y_s(k)) / (m_i^x - x_s(k)) \right) + \pi/2 - \theta(k) \end{bmatrix} + \begin{bmatrix} r_b(k) \\ \alpha_b(k) \end{bmatrix} + \mathbf{w}(k) \\ &= \mathbf{h}(\mathbf{x}_v(k), \mathbf{L}_i, r_b(k), \alpha_b(k)) + \mathbf{w}(k) \end{aligned} \quad (5.96)$$

where $\mathbf{w}(k)$ is the vector of observation noise in range and bearing assumed Gaussian with zero mean and covariance $\mathbf{R}(k)$ and $(x_s(k), y_s(k))$ is the absolute position of the exteroceptive sensor mounted on the vehicle (Appendix A). $r_b(k)$ and $\alpha_b(k)$ are range and bearing biases of the exteroceptive range-bearing sensor. The EKF prediction equations of MAL algorithm are as follows:

$$\mathbf{X}(k | k-1) = \mathbf{f}(\mathbf{x}_v(k-1), \mathbf{u}(k-1), \mathbf{x}_b(k-1)) \quad (5.97)$$

$$\mathbf{P}(k | k-1) = \mathbf{F}\mathbf{P}(k-1 | k-1)\mathbf{F}^T + \mathbf{Q}(k-1) \quad (5.98)$$

$$\hat{\mathbf{z}}(k) = \mathbf{h}(\mathbf{x}_v(k), \mathbf{L}_i, r_b(k), \alpha_b(k)) \quad (5.99)$$

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where $\mathbf{f}(\cdot) = [(\mathbf{f}_v(\cdot))^T \ \mathbf{x}_b^T(k-1)]^T$ and $\mathbf{F}$ is the Jacobian $(\partial \mathbf{f} / \partial \mathbf{X})$ of the process model evaluated at time $k-1$ and $\mathbf{Q}(k)$ represents the covariance matrix of the uncertainties of proprioceptive sensor noise and modeling uncertainties. When the true observation $\mathbf{z}(k)$ is available at time k and after correct observation to map feature associations are resolved using an appropriate data association algorithm, the EKF update equations are as (5.18)–(5.20). It may be noted that the system of equations (5.95)–(5.99) describing concurrent bias estimation in the context of MAL is non linear. And hence to gain an insight and for the theoretical analysis (next section) the system of equations are first linearized.

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_v(k) \\ \mathbf{x}_b(k) \end{bmatrix} = \begin{bmatrix} \partial \mathbf{f} / \partial \mathbf{x}_v & \partial \mathbf{f} / \partial \mathbf{x}_b \\ \mathbf{0}_{4 \times 3} & \mathbf{I}_{4 \times 4} \end{bmatrix} \begin{bmatrix} \mathbf{x}_v(k-1) \\ \mathbf{x}_b(k-1) \end{bmatrix} + \begin{bmatrix} \partial \mathbf{f} / \partial \mathbf{u} \\ \mathbf{0}_{4 \times 2} \end{bmatrix} \mathbf{u}_a(k-1) + \begin{bmatrix} \mathbf{v}^T(k-1) & \mathbf{v}_b^T(k-1) \end{bmatrix}^T + \mathbf{d}_f(k) \quad (5.100)$$

$$\mathbf{z}(k) = \begin{bmatrix} \partial \mathbf{h} / \partial \mathbf{x}_v & \partial \mathbf{h} / \partial \mathbf{x}_b \end{bmatrix} \begin{bmatrix} \mathbf{x}_v^T(k) & \mathbf{x}_b^T(k) \end{bmatrix}^T + \mathbf{w}(k) + \mathbf{d}_h(k) \quad (5.101)$$

where $\mathbf{u}_a(k-1) = [u_a(k-1) \ \gamma_a(k-1)]^T$, and $\mathbf{d}_f(k)$ and $\mathbf{d}_h(k)$ can be determined from the following equations:

$$\mathbf{d}_f(k) = \mathbf{f}(\hat{\mathbf{x}}_v(k-1), \hat{\mathbf{x}}_b(k-1), \mathbf{u}(k-1)) - \begin{bmatrix} \partial \mathbf{f} / \partial \mathbf{x}_v & \partial \mathbf{f} / \partial \mathbf{x}_b \\ \mathbf{0}_{4 \times 3} & \mathbf{I}_{4 \times 4} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_v(k-1) \\ \hat{\mathbf{x}}_b(k-1) \end{bmatrix} - \begin{bmatrix} (\partial \mathbf{f} / \partial \mathbf{u})^T & \mathbf{0}_{2 \times 4} \end{bmatrix}^T \mathbf{u}(k-1) \quad (5.102)$$

$$\mathbf{d}_h(k) = \mathbf{h}(\hat{\mathbf{x}}_v(k), \mathbf{L}_i, r_b(k), \alpha_b(k)) - \begin{bmatrix} \partial \mathbf{h} / \partial \mathbf{x}_v & \partial \mathbf{h} / \partial \mathbf{x}_b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_v^T(k) & \hat{\mathbf{x}}_b^T(k) \end{bmatrix}^T \quad (5.103)$$

where the quantities $\hat{\mathbf{x}}_v(k-1)$, $\hat{\mathbf{x}}_b(k-1)$, $\hat{\mathbf{u}}_b(k-1)$, $\hat{\gamma}_b(k-1)$, $\hat{\mathbf{x}}_v(k)$, $\hat{\mathbf{x}}_b(k)$, $\hat{r}_b(k)$ and $\hat{\alpha}_b(k)$ stand for the same quantities given in Section 5.2.1.2.

5.3.2 Analysis of the Problem

In this section we investigate the observability, convergence and bounds of uncertainty of

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the estimated parameters in the context of MAL with joint sensor bias estimation, as they are vital for the viability and robustness of the algorithm.

5.3.2.1 Diminishing uncertainties of the bias estimates in MAL

In accordance with the formulation of MAL with biases either for the ideal linear case ((5.93) and (5.94)) or for the more realistic 2D non-linear case (linearized models given by (5.100)-(5.103)) we can summarize the MAL with joint bias estimation assuming constant biases ($\sigma_{ub}^2 = \sigma_{\gamma b}^2 = 0$) as follows. The bias vector is expressed as (5.25).

$$\mathbf{X}(k) = \mathbf{F}\mathbf{X}(k-1) + \mathbf{B}_v\mathbf{u}(k-1) + [\mathbf{v}^T(k) \ \mathbf{0}]^T \quad (5.104)$$

where $\mathbf{B}_v$ is the transition matrix of the proprioceptive sensor measurements, $\mathbf{v}(k) \sim N(\mathbf{0}, \mathbf{Q}_v(k))$ and $\mathbf{X}(k)$ is given by (5.95). The matrix $\mathbf{F}$ is

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_v & \mathbf{B}_v & \mathbf{0}_{vs} \\ \mathbf{0}_{vu}^T & \mathbf{I}_{uu} & \mathbf{0}_{us} \\ \mathbf{0}_{vs}^T & \mathbf{0}_{us}^T & \mathbf{I}_{ss} \end{bmatrix} \quad (5.105)$$

where $\mathbf{I}_{ss}$ and $\mathbf{I}_{uu}$ are identity matrices with dimensions $(\dim(\mathbf{s}_b))^2$ and $(\dim(\mathbf{u}_b))^2$ respectively. $\mathbf{0}_{vs}$, $\mathbf{0}_{vu}$ and $\mathbf{0}_{us}$ are null matrices having appropriate dimensions. The observation model corresponding to (5.94) and (5.101) is

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{X}(k) + \mathbf{L} + \mathbf{w}(k) \quad (5.106)$$

where $\mathbf{H}(k) = [-\mathbf{H}_v(k) \ \mathbf{0}_{uu} \ \mathbf{I}_{ss}]$, $\mathbf{L}$ denotes a constant vector corresponding to the observed landmark in the stored map and $\mathbf{w}(k) \sim N(\mathbf{0}, \mathbf{R}(k))$ denotes the observation noise. $\mathbf{0}_{uu}$ is a null matrix having dimension $(\dim(\mathbf{u}_b))^2$.

The rest of the prediction and update equations then take the form of (5.97)-(5.99) and (5.18)-(5.20) with the process and observation models ((5.104) and (5.106)) replacing those given in the general formulation outlined in the previous section. By investigating

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the above models the following important result is arrived at.

Result : *The uncertainties in the estimates of the proprioceptive and exteroceptive sensor bias parameters diminish monotonically in an update of the MAL algorithm.*

Proof :

Let $\mathbf{P}(k|k)$ be the covariance matrix of the estimate of $\mathbf{X}(k)$ given by $\mathbf{X}(k|k)$. In MAL, $\mathbf{P}(k|k)$ can be expressed as

$$\mathbf{P}(k|k) = \begin{bmatrix} \mathbf{P}_{vv}(k|k) & \mathbf{P}_{vu}(k|k) & \mathbf{P}_{vs}(k|k) \\ \mathbf{P}_{vu}^T(k|k) & \mathbf{P}_{uu}(k|k) & \mathbf{P}_{us}(k|k) \\ \mathbf{P}_{vs}^T(k|k) & \mathbf{P}_{us}^T(k|k) & \mathbf{P}_{ss}(k|k) \end{bmatrix} \quad (5.107)$$

where the subscripts v, u, and s denote the vehicle state, proprioceptive sensor and

exteroceptive sensor bias vectors respectively. Let $\mathbf{P}(k|k-1) = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_4 & \mathbf{P}_5 \\ \mathbf{P}_4^T & \mathbf{P}_2 & \mathbf{P}_6 \\ \mathbf{P}_5^T & \mathbf{P}_6^T & \mathbf{P}_3 \end{bmatrix}$. Using the

standard Kalman prediction equations, the values of $\mathbf{P}_1$ to $\mathbf{P}_6$ can now be determined from (5.34) to (5.39) as described in Section 5.2.2.1. As $\mathbf{P}(0|0)$, $\mathbf{Q}(k)$ and $\mathbf{R}(k)$ are positive semi definite (PSD) matrices in the case of MAL problem, by the properties (see appendix B) of PSD matrices [124], $\mathbf{S}(k)$ and $\mathbf{K}(k)\mathbf{S}(k)\mathbf{K}^T(k)$ are also PSD matrices. Hence from standard EKF covariance update equation (5.20) and using the same reasoning as in (5.40) to (5.41) the following inequalities result.

$$\det(\mathbf{P}_{uu}(k|k)) \leq \det(\mathbf{P}_{uu}(k-1|k-1)) \quad (5.108)$$

$$\det(\mathbf{P}_{ss}(k|k)) \leq \det(\mathbf{P}_{ss}(k-1|k-1)) \quad (5.109)$$

Since the determinant of a matrix is proportional to its volume, determinant of covariance matrices indicate the volume or size of their uncertainty ellipses and therefore it can be concluded that the errors in the estimates of the bias parameters involving both proprioceptive and exteroceptive sensors diminish in each successive update of the MAL algorithm.

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5.3.2.2 Lower bounds on bias uncertainty

An investigation is carried out in this section to ascertain if the decrease in bias estimation errors over successive observations will eventually become zero (biases are determined with complete certainty) or approach a lower limit. It may be noted that the lowest covariance estimate is obtained when the proprioceptive sensor noise and modeling uncertainty is at its minimum (i.e when $\mathbf{Q}$ is lowest) and the observation noise is small. Such a scenario corresponds to the vehicle is stationary ($\mathbf{Q}=\mathbf{0}$) whilst observing a landmark in the stored map. However, under such circumstances, it is not possible to obtain an estimate of the proprioceptive sensors' bias terms as they are not observable. Hence, only the lower bounds of the uncertainty of exteroceptive sensor bias vector $\mathbf{s}_b$ is determined. Under the above conditions, the composite state vector is

$$\mathbf{X}(k) = [\mathbf{x}_v^T(k) \quad \mathbf{s}_b^T(k)]^T \quad (5.110)$$

The observation model takes the form (5.106) with $\mathbf{H}(k) = [-\mathbf{H}_v(k) \quad \mathbf{I}_{ss}]$. Since the vehicle is stationary, the predicted covariance matrix $\mathbf{P}(k | k-1)$ for all k is

$$\mathbf{P}(k | k-1) = \mathbf{P}(k-1 | k-1) \quad (5.111)$$

Suppose the initial value of $\mathbf{P}(k | k)$ is $\mathbf{P}(0 | 0) = \text{diag}(\mathbf{P}_v, \mathbf{P}_b)$ where $\mathbf{P}_v$ and $\mathbf{P}_b$ denote the initial covariance matrices of the vehicle and the exteroceptive sensor biases respectively.

Using the inverse covariance form of the Kalman filter (5.48)

$$\mathbf{P}^{-1}(k | k) = \begin{bmatrix} \mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v & -k\mathbf{H}_v^T \mathbf{R}^{-1} \\ -k\mathbf{R}^{-1} \mathbf{H}_v & \mathbf{P}_b^{-1} + k\mathbf{R}^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \quad (5.112)$$

Let $\mathbf{P}(k | k) = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{bmatrix}$ where the covariance term of the exteroceptive

sensor biases is $\mathbf{P}_{22}$. Thus, using $\mathbf{P}_{11} = (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}$ and similar expressions

$$\mathbf{P}_{11} = (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v - k\mathbf{H}_v^T \mathbf{R}^{-1} (\mathbf{P}_b^{-1} + k\mathbf{R}^{-1})^{-1} k\mathbf{R}^{-1} \mathbf{H}_v)^{-1} \quad (5.113)$$

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$$\mathbf{P}_{12} = (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} (-k\mathbf{H}_v^T \mathbf{R}^{-1}) \{ (k\mathbf{R}^{-1} \mathbf{H}_v) (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} (k\mathbf{H}_v^T \mathbf{R}^{-1}) - (\mathbf{P}_b^{-1} + k\mathbf{R}^{-1}) \}^{-1} \quad (5.114)$$

$$\mathbf{P}_{21} = \mathbf{P}_{12}^T \quad (5.115)$$

$$\mathbf{P}_{22} = (\mathbf{P}_b^{-1} + k\mathbf{R}^{-1} - k^2 \mathbf{R}^{-1} \mathbf{H}_v (\mathbf{P}_v^{-1} + k\mathbf{H}_v^T \mathbf{R}^{-1} \mathbf{H}_v)^{-1} \mathbf{H}_v^T \mathbf{R}^{-1})^{-1} \quad (5.116)$$

Since the value of $\mathbf{P}_{22}$ is obtained by assuming zero vehicle and proprioceptive sensor uncertainties, its inverse represents the maximum information gain conceivable or realizable for the algorithm. In the limit when a large number of successive observations are made using on board sensors while the vehicle is stationary, the upper limit of this information gain will be reached if at all a limit exists. Conversely, if this finite limit exists the covariance of the exteroceptive sensor bias term $\mathbf{P}_{22}$ will reach its lower bound. Therefore, this limit if exists is a lower bound $\mathbf{P}_{22}^*$ for the covariance $\mathbf{P}_b(k|k)$ of the exteroceptive sensor bias term under the given circumstances. i.e.

$$\mathbf{P}_{22}^* = \lim_{k \rightarrow \infty} (\mathbf{P}_{22}) \quad (5.117)$$

The above covariance lower bound of the sensor biases can be obtained in closed form for the following special cases.

Case 1 : When the initial uncertainties of exteroceptive sensor biases are infinite. i.e. completely unknown.

From (5.113) to (5.117)

$$\mathbf{P}_b(k|k) = \lim_{\mathbf{P}_b \rightarrow \infty} (\mathbf{P}_{22}^*) = \mathbf{H}_v \mathbf{P}_v \mathbf{H}_v^T \quad (5.118)$$

Therefore, the lower bound of the uncertainty of exteroceptive sensor biases when the bias terms are initially completely unknown is $\mathbf{H}_v \mathbf{P}_v \mathbf{H}_v^T$. Hence the lower bound of the uncertainty of the bias terms in this case is determined by the initial covariance of the vehicle pose or in other words $\mathbf{P}_v$. Otherwise $\mathbf{P}_b(k|k)$ should reach to zero or to a value independent of initial conditions depending on the observability conditions.

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Case 2 : When the initial vehicle position is completely unknown. (i.e resembling the Kidnapped Robot Problem)

$\mathbf{P}_v \rightarrow \infty$. i.e $\mathbf{P}_v^{-1} = 0$. After careful rearrangement of terms and simplification from (5.113) to (5.117)

$$\mathbf{P}_{22} = \mathbf{P}_b \quad (5.119)$$

Hence $\mathbf{P}_b(k | k) = \mathbf{P}_b$ for all k . Therefore, the uncertainty of sensor biases can't be reduced by observing a known landmark from a stationary position unknown vehicle.

These observations can be reiterated for a more realistic situation involving a single degree of freedom MAL problem. Consider the simplified process and observation models derived in (5.93) and (5.94) and the removal of the proprioceptive bias term in the state vector.

$$\mathbf{X}(k) = [x_v(k) \quad s_b(k)]^T \quad (5.120)$$

$$z(k) = [-1 \quad 1][x_v(k) \quad s_b(k)]^T + L \quad (5.121)$$

By comparing with (5.106), this is equivalent to $\mathbf{H} = [-1 \quad 1]$, $\mathbf{H}_v = 1$. Using the scalar versions of uncertainties for initial covariance matrix of $\mathbf{X}(k)$ given by $\mathbf{P}(0 | 0) = \text{diag}(P_v, P_b)$ and using (5.48)

$$\mathbf{P}(k | k) = \frac{1}{R + kP_b + kP_v} \begin{bmatrix} (R + kP_b)P_v & kP_vP_b \\ kP_vP_b & (R + kP_v)P_b \end{bmatrix} \quad (5.122)$$

$$P_b(\infty) = \lim_{k \rightarrow \infty} \frac{(R + kP_v)P_b}{R + kP_b + kP_v} = \frac{P_vP_b}{P_b + P_v} \quad (5.123)$$

where $P_b(\infty)$ is the steady state error covariance of the exteroceptive sensor bias term for the single degree of freedom MAL, as for the previous cases discussed above. Decrease in bias variance $\delta P_b(\infty)$ is given by

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$$\delta P_b(\infty) = \lim_{k \rightarrow \infty} \left(P_b - \frac{(R + kP_v)P_b}{R + kP_b + kP_v} \right) = \frac{P_b^2}{P_b + P_v} > 0 \quad (5.124)$$

suggesting that the exteroceptive sensor bias variance in the limit is strictly monotonically decreasing and bounded by (5.124).

5.3.2.3 Observability conditions

Understanding the observability properties of process and observation models is vital in designing the observation model for the MAL with joint bias estimation. We analyze the observability conditions for several scenarios to arrive at a conclusion on the estimator performance and to find the means of rectifying problems if any. As the first case we analyze the observability of 1D MAL when observing n landmarks with biased proprioceptive and exteroceptive sensors. The process and observation equations after writing in terms of linearized error states are as follows with the usual notation. The input noise vector $[v(k-1) \quad \mathbf{0}_{1 \times 2} \quad [w_1 \ w_2 \ \dots \ w_n]]$ is written in terms of concatenated process and measurement noise terms.

$$\tilde{\mathbf{X}}(k) = \mathbf{F}\tilde{\mathbf{X}}(k-1) + \mathbf{B}[v(k-1) \quad \mathbf{0}_{1 \times 2} \quad [w_1 \ w_2 \ \dots \ w_n]]^T \quad (5.125)$$

$$\tilde{\mathbf{z}}(k) = \mathbf{H}\tilde{\mathbf{X}}(k) + \mathbf{H}_w[v(k-1) \quad \mathbf{0}_{1 \times 2} \quad [w_1 \ w_2 \ \dots \ w_n]]^T \quad (5.126)$$

where $\mathbf{H} = [-\mathbf{1}_{n \times 1} \quad \mathbf{0}_{n \times 1} \quad \mathbf{1}_{n \times 1}]$, $\mathbf{H}_w = [\mathbf{0}_{n \times 3} \quad \mathbf{I}_{n \times n}]$, $\mathbf{F} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & \mathbf{0}_{1 \times (n+2)} \\ \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times (n+2)} \end{bmatrix}$.

It is now possible to compute observability and controllability Grammians $\mathbf{G}_o$ and $\mathbf{G}_c$ from (5.65) and (5.66). Hence for (5.125) and (5.126) it is clear that the rank of the $\mathbf{G}_o$ is 2 and thus it is rank deficient by 1. Therefore, the sensor bias estimation problem in the 1D MAL is partially observable. 2nd Case arises when the bias in vehicle speed proprioceptive measurement has a random walk behavior as modeled by (5.8). In this case

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$\mathbf{X}(k)$, $\mathbf{F}$, $\mathbf{H}$ and $\mathbf{z}(k)$ take the same form as in the previous system ((5.125) and (5.126)) with changes in

$$\tilde{\mathbf{X}}(k) = \mathbf{F}\tilde{\mathbf{X}}(k-1) + \mathbf{B} \begin{bmatrix} v(k) & v_b(k) & \mathbf{0}_{1 \times (n+1)} \end{bmatrix}^T \quad (5.127)$$

$$\text{and } \mathbf{B} = \begin{bmatrix} 1 & 0 & \mathbf{0}_{1 \times (n+1)} \\ 0 & 1 & \mathbf{0}_{1 \times (n+1)} \\ \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times (n+1)} \end{bmatrix}$$

In this case, the rank of $\mathbf{G}_o$ is 2. i.e. rank deficient by one. However, the rank of $\mathbf{G}_c$ in this case is two, which is higher than the rank of $\mathbf{G}_c$ in the previous case with constant bias term. The results also show that angle between the controllable and observable subspaces for the first case is $\pi/2$ and for this case is $\pi/4$. This is an indication suggesting the exteroceptive sensor measurement noise driven corrections to the state estimate could reconstruct the vehicle state much closer to the actual state in the variable bias case than in the constant bias case. The above partial observability situation can be rectified by incorporating absolute position information in the measurement vector in the form of an external sensor. Thus the observation matrix $\mathbf{H}$ of the fully observable ID MAL algorithm when observing n known landmarks in a map with sensor bias estimation will become

$$\mathbf{H} = \begin{bmatrix} -\mathbf{1}_{n \times 1} & \mathbf{0}_{n \times 1} & \mathbf{1}_{n \times 1} \\ 1 & 0 & 0 \end{bmatrix} \quad (5.128)$$

In general there are no specific rules governing the observability of nonlinear stochastic systems and therefore it is difficult to have prior judgments on the viability of EKF based MAL with bias estimation in two dimensions (2D). However, an observability test similar to that is suitable for linear time invariant systems may be carried out as done in Section 5.2.2.3.

The observability analysis of the planer (2D) nonlinear MAL is carried out by the symbolic manipulation of the models (5.95) to (5.99) with a single known landmark

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observation. The analysis shows that the observability matrix in this case is rank deficient by 2 (Dimension of the state vector is 7 and rank of $\mathbf{G}_o$ is 5). However, it is established that the matrix $\mathbf{G}_o$ becomes full rank when observing more than one known landmark at the same time because observing two or more landmarks provides the adequate number of independent rows to the observability matrix. Thus, it is a pre-requisite that at least two landmarks are to be observed simultaneously for observability in 2D non-linear MAL with joint bias correction.

It is important to note that there can be certain conditions which can still prevent the system from achieving full observability depending on the robot trajectory as well as the relative configuration of the observed landmarks in the stored map. This can be explained using the following example. Let the robot pose be denoted by $\mathbf{X}(k) = [x(k) \ y(k) \ \theta(k)]^T$ according to the usual notation. Assume that the exteroceptive sensor on the robot (at location $\mathbf{V} \equiv (x, y)$) is observing two landmarks denoted by $\mathbf{LM}_1 \equiv (x_1, y_1)$ and $\mathbf{LM}_2 \equiv (x_2, y_2)$ where x_1, y_1 and x_2, y_2 are the x and y coordinates of the observed landmarks in the stored map. Assuming the vehicle location and the exteroceptive sensor coordinates coincide, the measurement Jacobian $\mathbf{H}$ when observing both the landmarks simultaneously can be expressed as

$$\mathbf{H} = \begin{bmatrix} \Delta x_1 / \Delta r_1 & \Delta y_1 / \Delta r_1 & 0 & 0 & 0 & 1 & 0 \\ -\Delta y_1 / (\Delta r_1)^2 & -\Delta x_1 / (\Delta r_1)^2 & -1 & 0 & 0 & 0 & 1 \\ \Delta x_2 / \Delta r_2 & \Delta y_2 / \Delta r_2 & 0 & 0 & 0 & 1 & 0 \\ -\Delta y_2 / (\Delta r_2)^2 & -\Delta x_2 / (\Delta r_2)^2 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5.129)$$

where $\Delta x_1 = x(k | k-1) - x_1$, $\Delta y_1 = y(k | k-1) - y_1$, $\Delta x_2 = x(k | k-1) - x_2$ and $\Delta y_2 = y(k | k-1) - y_2$. Thus, it can be established that when the first and the third rows or the second and the fourth rows of $\mathbf{H}$ are equivalent, the observability matrix of the system for this particular linearization is rank deficient. The conditions for the above situation can be derived from (5.129) as

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$$\Delta x_1 / \Delta x_2 = \Delta y_1 / \Delta y_2 = \Delta r_1 / \Delta r_2 \quad (5.130)$$

$$\Delta x_1 / \Delta x_2 = \Delta y_1 / \Delta y_2 = (\Delta r_1)^2 / (\Delta r_2)^2 \quad (5.131)$$

Thus, it can be established that the system is not fully observable when (5.130) or (5.131) is satisfied. This demonstrates that even though a minimum of two landmarks is simultaneously observed, under the conditions (5.130) or (5.131) the observability matrix is still rank deficient. Thus, it is always desirable to observe as many landmarks as possible simultaneously to retain the full observability.

5.3.2.4 Rates of convergence of uncertainties of sensor biases

It is important to know the rate of convergence of the bias estimation algorithm as for SLAM as done in Section 5.2.2.4. The phenomenon of convergence can be elucidated by studying the 1D MAL problem introduced by (5.93) and the modified observability matrix introduced by (5.128) that ensure full observability.

Closed form solutions for the uncertainty terms of the bias estimates and the vehicle location estimate can be derived using continuous time equivalents of (5.93) and (5.128). The factors that affect the rates of convergence of these uncertainties are clearly visible from this closed form solution.

The continuous time process model of the discrete time model (5.93) is

$$\begin{bmatrix} \dot{x}_v(t) \\ \dot{u}_b(t) \\ \dot{s}_b(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_v(t) \\ u_b(t) \\ s_b(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [u(t) + v(t)] \quad (5.132)$$

where $x_v(t)$ is the position of the robot, $u_b(t)$ is the bias of the proprioceptive sensor, $s_b(t)$ is the bias of the exteroceptive sensor, $u(t)$ is proprioceptive sensor measurement in the continuous space at time t . $v(t) \sim N(0, q^2)$ represents the proprioceptive sensor noise and modeling uncertainties if any. The analogous continuous time state vector of

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the problem is given by $\mathbf{X}(t) = [x_v(t) \ u_b(t) \ s_b(t)]^T$. The observation model incorporating an external absolute sensor measurement of the vehicle position and n known landmarks is given by

$$\mathbf{z}(t) = \mathbf{H}\mathbf{X}(t) + [\mathbf{L}^T \ 0]^T + \mathbf{w}(t) \quad (5.133)$$

where $\mathbf{w}(t) \sim N(\mathbf{0}_{(n+1) \times 1}, \mathbf{R})$ is the measurement noise, $\mathbf{R} = \text{diag}(r^2, r^2, \dots, r^2, r_e^2)$ is the covariance matrix of $\mathbf{w}(t)$, r^2 is the variance of $\mathbf{w}(t)$, r_e^2 is the variance of the absolute sensor measurement noise, $\mathbf{L}$ is the vector of n known 1D landmarks stored in the map and $\mathbf{H}$ is given by (5.128). Let $\mathbf{G} = [1 \ 0 \ 0]^T$ and $\mathbf{F} = \begin{bmatrix} [0 \ 1 \ 0]^T & \mathbf{0}_{3 \times 2} \end{bmatrix}^T$. Let $\mathbf{P}(0) = \text{diag}(0, \sigma_{bu}^2, \sigma_{bs}^2)$ be the initial error covariance matrix of $\mathbf{P}(t)$ (model (5.132)) and $\mathbf{I}$ denotes the identity matrix of size 3x3. The quantities σ_{bu}^2 and σ_{bs}^2 denote the initial variances of the proprioceptive and exteroceptive sensor biases. The initial uncertainty of the vehicle is assumed zero. $\mathbf{P}(t)$ is governed by the continuous time Riccati equation (5.78). The characteristic equation $\mathbf{C}(t)$ of the system and the i^{th} row j^{th} column element $\mathbf{P}(i, j)$ of $\mathbf{P}(t)$ for all i and j can be obtained by solving (5.79) and (5.80) as follows:

$$\begin{aligned} \mathbf{C}(t) = & (nr_e^2 + r^2)(nq^2r^2\sigma_{bu}^2\sigma_{bs}^2(1-e^{-\alpha t})^2t^2 + ((\sigma_{bu}^2(nr_e^2 + r^2) + nq^2\sigma_{bs}^2)(1+e^{-2\alpha t})q^2r^2 + \\ & nr^2r_e^2\sigma_{bu}^2\sigma_{bs}^2\alpha(1-e^{-2\alpha t}))t) + q^4r^2(nr_e^2 + r^2)(1+e^{-2\alpha t}) - 2n^2r_e^4r^2\sigma_{bu}^2\sigma_{bs}^2(1-e^{-\alpha t})^2 \\ & + r^2r_e^2\alpha(n^2r_e^2q^2\sigma_{bs}^2 - r^2\sigma_{bu}^2(nr_e^2 + r^2))(1-e^{-2\alpha t})) \end{aligned} \quad (5.134)$$

$$\mathbf{P}(1,1) = (1/\mathbf{C}(t))(2n^2r_e^4\sigma_{bs}^2(1-e^{-\alpha t})^2 + r^2r_e^2\alpha(nr_e^2 + r^2 + n\sigma_{bs}^2t)(1-e^{-2\alpha t}))(q^2 + \sigma_{bu}^2t)q^2r^2 \quad (5.135)$$

$$\mathbf{P}(1,2) = (1/\mathbf{C}(t))(2n^2r_e^4\sigma_{bs}^2(1-e^{-\alpha t})^2 + r^2r_e^2\alpha(nr_e^2 + r^2 + n\sigma_{bs}^2t)(1-e^{-2\alpha t}))\sigma_{bu}^2q^2r^2 \quad (5.136)$$

$$\mathbf{P}(1,3) = (1/\mathbf{C}(t))(1-e^{-\alpha t})^2r^2r_e^2q^2(nr_e^2 + r^2)(q^2 + \sigma_{bu}^2t)n\sigma_{bs}^2 \quad (5.137)$$

$$\mathbf{P}(2,2) = (1/\mathbf{C}(t))(r^2q^2(nr_e^2 + r^2)(1+e^{-2\alpha t}))(nr_e^2 + r^2 + n\sigma_{bs}^2t) + n^2r_e^4r^2\alpha(1-e^{-2\alpha t})q^2\sigma_{bu}^2 \quad (5.138)$$

$$\mathbf{P}(2,3) = (1/\mathbf{C}(t))\sigma_{bu}^2\sigma_{bs}^2q^2r^2n(nr_e^2 + r^2)(1-e^{-\alpha t})^2 \quad (5.139)$$

$$\mathbf{P}(3,3) = (1/\mathbf{C}(t))(q^2(nr_e^2 + r^2)(1+e^{-2\alpha t}))(q^2 + \sigma_{bu}^2t) - r^2r_e^2\alpha\sigma_{bu}^2(1-e^{-2\alpha t})\sigma_{bs}^2(nr_e^2 + r^2)r^2 \quad (5.140)$$

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$$\alpha = (q/(rr_e))(nr_e^2 + r^2)^{0.5} \quad (5.141)$$

Here, $1/\alpha$ is the time constant of the decay of the covariance terms. All the other terms of the covariance matrix can be derived from (5.134) to (5.141) using the symmetry of $\mathbf{P}(t)$. The equations (5.134)-(5.141) show that the error covariance terms of $\mathbf{X}(t)$ decay exponentially initially at a time constant of $1/\alpha$ and then converge asymptotically to the steady state covariance according to the same expressions. This asymptotic convergence of the error covariance terms can be a disadvantage in actual practice. It may be interesting to note that the convergence rate is not affected by the biases or their initial uncertainties and depends only on the proprioceptive sensor noise term, exteroceptive sensor noise term, external sensor noise term and the number of known landmarks in the stored map. When the variance of the external sensor noise r_e^2 is very large compared to r^2 (i.e landmark based information dominates the external sensor based information) the time constant approaches $\sqrt{r^2/(nq^2)}$. In other words, the smaller is the ratio between exteroceptive sensor noise and the process noise (proprioceptive sensor noise and the modeling uncertainty) or the larger is the number of landmarks observed simultaneously, the faster will be the convergence of the state variances to their steady state values. This is an important result when selecting sensors to achieve good performance is concerned. The steady state covariance of $\mathbf{X}(t)$, $\mathbf{P}(\infty)$ can be obtained from (5.134) to (5.141) by taking the limit as $t \rightarrow \infty$. Thus it can be seen that all the state covariance terms except that of the vehicle position approach zero in the limit. The vehicle position variance approaches q^2/α in the limit. Thus, we can conclude that the rates of convergence of state uncertainties do not depend on the bias parameters or their initial uncertainties. But they depend only on the amount of proprioceptive sensor noise, modelling uncertainty and the exteroceptive sensor noise. These theoretically established properties give an

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important insight to the problem and are therefore useful in practical applications.

5.3.3 Simulation of MAL with bias estimation

In this section we consider simulation scenarios to verify the theoretical results established on the observability, convergence and lower bounds and to assess the importance of estimating biases on line in the context of MAL.

MAL with online bias estimation, compensation and its performance were evaluated in a simulated environment consisting of 25 point landmarks. An autonomous vehicle is assumed to be navigating in an approximately circular path of 20m radius with a 2D scanning range bearing sensor (eg. SICK LMS 290 laser measurement system) and wheel encoders as described in Section 5.2.3. Simulations were conducted for 2D MAL (models (5.95) to (5.99)) with biased sensors having LMS range bias 0.5m, angle bias 2° , speed sensor input bias 0.25m/s and steering input angle sensor bias 1° and the random noise sequences with variances 0, 0, $0.0004 \text{ m}^2/\text{s}^4$ and $4 \times 10^{-6} \text{ rad}^2/\text{s}^2$ respectively.

It is important to note that when there are sensor biases and no bias estimation is utilized it is almost impossible to estimate the vehicle position consistently.

Figures 5.4 (a)-(f) and Figure 5.5 (a) and (b) were obtained for the MAL with online bias estimation and correction algorithm ((5.95) to (5.99)). The results highlight that the MAL works well with joint sensor bias estimation and with ever diminishing uncertainties of bias estimates as shown in the theoretical discussion. Figure 6.5 (a) and (b) also show that the uncertainties of bias estimates reach a lower bound. These results (i.e. the convergence of estimated bias parameters to the actual values) further substantiate the theoretical results given in Section 5.3.2.3 that observing two or more known landmarks makes the 2D MAL with joint sensor bias estimation problem observable without the need for any absolute sensor such as GPS.

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Figure 5.4. 2D MAL with bias estimation Figures 5.4 (a) and (b) show the 2D MAL, localization error in x and heading with online bias estimation. Figures 5.4 (c), (d), (e) and (f) show sensor bias estimates and their true value variation.

Figure 5.5. Standard deviation (STD) of sensor biases. Figure 5.5 (a) shows the STDs of LMS range bias and the speed biases. Figure 5.5 (b) shows the STDs of LMS bearing bias and the steering angle bias.

5.4. Estimation Theoretic Sensor Bias Correction in Robotics Mapping

A brief description of robotics mapping (RM) [35] and [46] from a known robot path is given in Section 3.3.2. In this problem, a robot moving on a known path tries to estimate the map of the environment in some form. The standard feature based estimation theoretic

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RM algorithms use point landmark based map representations and the EKF based estimation algorithms. An investigation is carried out here on the effects of exteroceptive sensor biases to the RM problem.

If exteroceptive sensors free of biases are used in the RM problem, the standard practice is to consider the map estimation as a set of individual landmark estimates, which are independent of each other. Let the inverse measurement model of landmark initialization be

$$\mathbf{L}_i = \mathbf{g}(\mathbf{s}_b(k), \mathbf{z}_i(k), \mathbf{x}_v(k)) \quad (5.142)$$

where $\mathbf{L}_i$, $\mathbf{s}_b(k)$, $\mathbf{z}_i(k)$ and $\mathbf{x}_v(k)$ denote the i^{th} initialized landmark location vector, exteroceptive bias vector, the measurement and the true vehicle location vector respectively. Thus, the error in the initialized landmark can be expressed as

$$\delta \mathbf{L}_i = (\partial \mathbf{g} / \partial \mathbf{s}_b) \delta \mathbf{s}_b + (\partial \mathbf{g} / \partial \mathbf{z}_i) \delta \mathbf{z}_i \quad (5.143)$$

where the variables prefixed by δ denote the corresponding error quantities. It follows from (5.143) that if two or more such landmark initializations are considered their errors are correlated over the errors in the exteroceptive sensor bias estimates. Thus, it is necessary to consider all these correlations in the problem of RM with bias estimation. Therefore, the correct way to deal with this estimation problem is to consider a concatenated state vector consisting of all the landmark location vectors and exteroceptive bias states as a composite state.

In the same manner the measurement model of a landmark also consists of the common terms due to $\mathbf{s}_b(k)$. Thus, if we take two or more measurements from a biased sensor, their errors are correlated causing the measurements to be dependent. This results in a significant impact on the performance of data association algorithms as the measurements are not independent (Note-most data associations in the tracking literature consider measurement independence as the fundamental assumption. See Chapter 3 of this thesis).

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5.4.1 Single Degree of Freedom RM Problem

In the single degree of freedom RM problem with joint bias estimation, a vehicle is assumed to traverse on a straight line path (with the vehicle pose $x_v(k)$) tries to estimate the map of the landmarks located on the same line based on the linear range measurements originated from the same set of landmarks. Also assume that the exteroceptive sensor onboard the vehicle has a constant bias $s_b(k)$ at time k and is used to obtain measurements of the 1D landmarks. Using the notation used in Section 5.2.1.1, the process model and the measurement models of the problem are

$$\mathbf{X}(k) = \mathbf{I}_{(n+1) \times (n+1)} \mathbf{X}(k-1) + \begin{bmatrix} \mathbf{0}_{n \times 1} \\ v_b(k-1) \end{bmatrix} \quad (5.144)$$

$$z(k) = [\mathbf{h}_L \quad 1] \mathbf{X}(k) + w(k) \quad (5.145)$$

where $\mathbf{X}(k) = [L_1(k) \dots L_n(k) \ s_b(k)]^T$, $L_i(k) \ \forall i$ and $\mathbf{h}_L$ denote the composite map and bias state vector, i^{th} landmark location and a row vector having all zeros except at the position corresponding to the observed landmark in the state vector, which is equal to one.

5.4.2 RM Problem on a Flat 2D Terrain

Using the notation used in this chapter the process and observation models of the RM problem on a flat 2D terrain is as follows. The observation model assumes the use of a range-bearing sensor.

$$\mathbf{X}(k) = \mathbf{I}_{(2n+2) \times (2n+2)} \mathbf{X}(k-1) + \begin{bmatrix} \mathbf{0}_{2n \times 1} \\ \mathbf{v}_b(k-1) \end{bmatrix} \quad (5.146)$$

$$\begin{aligned} \mathbf{z}(k) &= \begin{bmatrix} \sqrt{(x_i(k) - x_s(k))^2 + (y_i(k) - y_s(k))^2} \\ \tan^{-1}((y_i(k) - y_s(k))/(x_i(k) - x_s(k))) + \pi/2 - \theta(k) \end{bmatrix} + \begin{bmatrix} r_b(k) \\ \alpha_b(k) \end{bmatrix} + \mathbf{w}(k) \\ &= \mathbf{h}(\mathbf{x}_v(k), \mathbf{L}_i, r_b(k), \alpha_b(k)) + \mathbf{w}(k) \end{aligned} \quad (5.147)$$

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where $\mathbf{L}_i = [x_i(k) \ y_i(k)]^T \ \forall i$ with $x_i(k)$ and $y_i(k)$ as the X and Y coordinates is the i^{th} landmark observed by the range-bearing sensor onboard the vehicle. All the other symbols carry the defined meaning in this chapter and $\mathbf{X}(k) = [\mathbf{L}_1^T(k) \ \dots \ \mathbf{L}_n^T(k) \ \mathbf{s}_b(k)]^T$.

5.4.3 Convergence of 1D RM

As the first step towards understanding the problem, the simple 1D RM problem with sensor bias estimation is studied with 2 initialized landmarks. The continuous time process and observation models are thus $\dot{\mathbf{X}}(t) = \mathbf{0}_{3 \times 3} \mathbf{X}(t)$, $z(t) = \mathbf{H}\mathbf{X}(t) + \mathbf{w}(t)$ with the

usual notation where $\mathbf{X}(t) = [L_1(t) \ L_2(t) \ s_b(t)]^T$, $\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$,

$\mathbf{w}(t) \sim N(\mathbf{0}_{2 \times 1}, \text{diag}(r^2, r^2))$ and r^2 is the variance of the exteroceptive sensor noise.

Thus, the characteristic equation $C(t)$ of the system and the m^{th} row n^{th} column element $\mathbf{P}(m, n)$ of $\mathbf{P}(t)$ (the covariance matrix of the state) for all m and n can be obtained by solving the Riccati equation with the initial conditions as $\mathbf{P}(0) = \text{diag}(p_1, p_2, p_3)$. Here, the initial states of the landmarks and bias are taken as uncorrelated for simplicity although in real scenarios they are correlated.

$$C(t) = (p_1 p_2 + p_2 p_3 + p_3 p_1) t^2 + r(p_1 + p_2 + 2p_3) t + r^2 \quad (5.148)$$

$$P(1, 1) = (1/C(t)) \left(p_1 (p_2 p_3 t^2 + r(p_2 + 2p_3) t + r^2) \right) \quad (5.149)$$

$$P(2, 2) = (1/C(t)) \left(p_2 (p_1 p_3 t^2 + r(p_1 + 2p_3) t + r^2) \right) \quad (5.150)$$

$$P(3, 3) = (1/C(t)) \left(p_3 (r + t p_1) (r + t p_2) \right) \quad (5.151)$$

Therefore, in the steady state all the states are fully correlated and all state variances approach to $p_1 p_2 p_3 / (p_1 p_2 + p_2 p_3 + p_3 p_1)$. The fact that all the steady state variances are dependent on initial variances is an indication of the partial observability of the problem.

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5.4.4 Observability Conditions

Firstly, the observability conditions are analyzed for n landmark 1D RM problem (5.144) and (5.145). After writing (5.144) and (5.145) in error terms;

$$\tilde{\mathbf{X}}(k) = \mathbf{F}\tilde{\mathbf{X}}(k-1) + \mathbf{B}[\mathbf{0}_{1 \times n} \ v_b(k-1) \ w_1 \ w_2 \ \dots \ w_n]^T \quad (5.152)$$

$$\tilde{\mathbf{z}}(k) = \mathbf{H}\tilde{\mathbf{X}}(k) + \mathbf{H}_w[\mathbf{0}_{1 \times n} \ v_b(k-1) \ w_1 \ w_2 \ \dots \ w_n]^T \quad (5.153)$$

where $\mathbf{F} = \mathbf{I}_{(n+1) \times (n+1)}$, $\mathbf{B} = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{0}_{n \times 1} & \mathbf{0}_{n \times n} \\ \mathbf{0}_{1 \times n} & 1 & \mathbf{0}_{1 \times n} \end{bmatrix}$, $\mathbf{H} = [\mathbf{I}_{n \times n} \ \mathbf{1}_{n \times 1}]$ and $\mathbf{H}_w = [\mathbf{0}_{n \times n} \ 0 \ \mathbf{I}_{n \times n}]$.

It is now possible to compute observability and controllability Grammians $\mathbf{G}_o$ and $\mathbf{G}_c$.

Thus, the rank of $\mathbf{G}_o$ in this case is n as $\text{rank}(\mathbf{G}_o) = \text{rank}(\mathbf{H})$ for all n . Therefore, the n landmark RM with bias observation is partially observable. However, it is possible to show that the problem of partial observability of this problem can be overcome by always

observing a known landmark. In such a scenario $\mathbf{H} = \begin{bmatrix} \mathbf{I}_{n \times n} & \mathbf{1}_{n \times 1} \\ \mathbf{0}_{1 \times n} & 1 \end{bmatrix}$. If a known landmark is observed at all times, then the rank of the observability matrix can be shown to be $n+1$, i.e. ($\text{rank}(\mathbf{G}_o) = \text{rank}(\mathbf{H})$).

The dependence of observability of the 1D MAL with joint sensor bias estimation on the number of landmarks can be stated as follows.

Result : *The angle ψ between the controllable and observable subspaces in single degree of freedom RM problem with constant bias estimation is a function of the number of landmarks in the map only and is given by*

$$\psi = \cos^{-1} \left(\sqrt{\frac{n}{n+1}} \right) \quad (5.154)$$

Proof : Using the models (5.152) and (5.153):

$$\text{Image}(\mathbf{G}_c) = [\mathbf{0}_{1 \times n} \ 1]^T \quad (5.155)$$

$$\text{Image}(\mathbf{G}_o^T) = \text{Image}(\mathbf{H}^T) = [\mathbf{I}_{n \times n} \ \mathbf{1}_{n \times 1}]^T \quad (5.156)$$

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Let $\underline{r}_i$ for $i=1,2,\dots,n$ be the basis vectors of $\text{Image}(\mathbf{G}_o^T)$ and $\underline{q}$ be the basis vector of $\text{Image}(\mathbf{G}_c)$. Suppose $\underline{p}$ be the projection of the basis of $\text{Image}(\mathbf{G}_c)$ into the vector space spanned by the basis of $\text{Image}(\mathbf{G}_o^T)$. Using the algebra of vector transformations and projections:

$$\underline{p} = \sum_{i=1}^n \frac{\underline{q}^T \underline{r}_i}{\underline{r}_i^T \underline{r}_i} \underline{r}_i, \quad \underline{r}_i^T \underline{r}_i = 2, \quad \forall i \quad \text{and} \quad \underline{p} = \frac{1}{2} \sum_{i=1}^n \underline{r}_i.$$

Thus, it follows that $\|\underline{p}\| = \sqrt{(n/2)^2 + n(1/2)^2} = \sqrt{(n^2 + n)/4}$ and $\underline{p}^T \underline{q} = n/2$. Therefore, the angle between the controllable and observable subspaces is

$$\psi = \cos^{-1} \left(\frac{(\underline{p}^T \underline{q})}{(\|\underline{p}\| \|\underline{q}\|)} \right) = \cos^{-1} \left(\frac{(n/2)}{\left(\sqrt{(n^2 + n)/4} / 2 \right)} \right) = \cos^{-1} \left(\sqrt{\frac{n}{n+1}} \right)$$

Since ψ is a monotonically decreasing function of n , as the number of landmarks increases the observable subspace gets closer to the controllable part of the state space. (ψ can be reduced to zero by observing a very large number of landmarks). Therefore, it is important to provide full observability to the models in general.

For the realistic 2D mapping problem (5.146) and (5.147), the observability conditions are difficult to be ascertained due to the nonlinear process and observation models. However, using simulations it can be verified (by observing all the landmarks of the map at the same time) that the observability Grammian is always rank deficient by 2. (the number of exteroceptive sensor bias terms). Further it is also verified that the above effect can be corrected easily by providing the observations from a known 2D landmark all the time. The observability Grammian then becomes full rank ensuring the full observability of the 2D RM problem with sensor bias estimation.

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5.5. Use of Inertial Sensors in Robot Localization

The most important application area of online bias estimation problem in SLAM and MAL arises when using inertial sensors. With the advent of low cost inertial sensor technology, the use of inertial sensor suits has become viable for autonomous and other land navigation systems. These sensor suits are essential in field robotics where large uneven terrains have to be explored and mapped. However, the major disadvantage of inertial sensor suits is their inherent sensor biases, drifts and scale factors. Random walks present in angular rate and acceleration sensors [21] cause the offline calibration methods less effective. In this work, it is demonstrated how the above theoretical discussion and the principles can be extended for the case of online bias estimation problem in SLAM and MAL with a low cost inertial sensor suite.

5.5.1 Inertial Sensors and Coordinate Transformations

Inertial navigation systems (INS) use accelerometers and gyroscopes to measure accelerations and angular velocities with reference to an inertial coordinate frame. An inertial frame is a non-accelerating and non-rotating reference frame. When the area of travel is not very large, it is customary to use an earth fixed local tangent frame as the navigation frame, the coordinate frame respect to which the vehicle position is estimated. This frame has its origin located at a local fixed point with the x-axis, y-axis and z axis usually pointing North, East and Down respectively. However, it is important to note that this frame is non-inertial because it is both accelerating and rotating. Nevertheless, for navigation on a small region it is safe to assume that the effects due to rotation of the earth and the changing gravity are negligible as shown in [23] and [131].

Inertial sensors can be classified into two categories based on their implementation. In the Gimbaled arrangement, sensors are attached to a stable system mounted on gimbals so that the sensors are always in fixed alignment with the navigation frame. In the strapdown

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arrangement, the inertial sensors are attached to the vehicle body and remain fixed with respect to the body of the vehicle. Although low in cost, it is required to transform the accelerations and rotation rates to and from body frame to navigation frame when using the strapdown version. In this work, a low cost strapdown inertial measurement unit is used to experimentally verify the on line bias correction algorithms and their properties discussed in the theoretical studies.

5.5.2 Vehicle Motion on a Flat Terrain

The fundamental assumption of a flat terrain model is utilized in INS aided localization using MAL and SLAM. That is the vehicle is assumed to move on a flat surface in a static environment. For mathematical simplicity it is assumed that the landmarks are static and their positions are represented by their (x,y) coordinate pairs. The inertial sensor assembly is mounted at the centre of gravity (COG) of the vehicle. The vehicle motion is referenced to a global coordinate frame, which is the earth reference frame or the navigation frame $\{\mathbf{n}\}$ with orthogonal set of axes North, East and Down (Figure 5.6). The vehicle body reference frame $\{\mathbf{v}\}$ with origin at COG of vehicle coincides with the inertial measurement unit's coordinate frame. The vehicle frame $\{\mathbf{v}\}$'s X-axis is pointing along the heading direction and the Y-axis and Z-axis are shown in the Figure 5.6. The orientation of the vehicle body frame $\{\mathbf{v}\}$ is represented by the three Euler angles ϕ (rotation about vehicle X axis or roll), θ (rotation about vehicle Y axis or pitch) and ψ (rotation about vehicle Z axis or yaw) as shown in Figure 5.6. Since it is assumed that the vehicle traverses a flat surface, the vehicle state vector $\mathbf{x}_v(k)$ at time k is:

$$\mathbf{x}_v(k) = \begin{bmatrix} x_v^n & y_v^n & v_x^n & v_y^n & v_z^n & \phi & \theta & \psi & b_{x_a}^v & b_{y_a}^v & b_{z_a}^v & b_{x_{\omega}}^v & b_{y_{\omega}}^v & b_{z_{\omega}}^v \end{bmatrix} \quad (5.157)$$

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Figure 5.6. Reference coordinate frames

where x_v^n , y_v^n , v_x^n , v_y^n and v_z^n are X, Y coordinates of the vehicle and velocities of the vehicle along X axis, Y axis and Z axis all with reference to the fixed navigation frame (**n**). The concatenated bias vector is $\mathbf{b}^v(k)$ at time k is given by

$$\mathbf{b}^v(k) = [b_{x_a}^v(k) \quad b_{y_a}^v(k) \quad b_{z_a}^v(k) \quad b_{x_\omega}^v(k) \quad b_{y_\omega}^v(k) \quad b_{z_\omega}^v(k)]^T \quad (5.158)$$

where $b_{x_a}^v$, $b_{y_a}^v$, $b_{z_a}^v$, $b_{x_\omega}^v$, $b_{y_\omega}^v$ and $b_{z_\omega}^v$ are the biases in the IMU acceleration measurements along vehicle X, Y and Z axes and biases in the IMU angular velocity measurements about vehicle X, Y and Z axes respectively.

Suppose that the IMU angular rate measurements along the X, Y and Z axes of the vehicle reference frame $\{\mathbf{v}\}$ be ω_x^v , ω_y^v and ω_z^v and acceleration measurements along the axes X, Y and Z with reference to $\{\mathbf{v}\}$ be a_x^v , a_y^v and a_z^v all measured at time $k-1$. The temporally uncorrelated noise terms of the measurements a_x^v , a_y^v , a_z^v , ω_x^v , ω_y^v and ω_z^v at time $k-1$ are assumed as $\eta_{x_a}^v$, $\eta_{y_a}^v$, $\eta_{z_a}^v$, $\eta_{x_\omega}^v$, $\eta_{y_\omega}^v$ and $\eta_{z_\omega}^v$ respectively. Let η_x^n and η_y^n represent the modeling uncertainties of the x, y coordinates of the vehicle with reference to the navigation frame $\{\mathbf{n}\}$. The vehicle states and the bias states then evolve as follows:

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$$\begin{bmatrix} x_v^n(k) \\ y_v^n(k) \end{bmatrix} = \begin{bmatrix} x_v^n(k-1) + \Delta t v_x^n(k-1) \\ y_v^n(k-1) + \Delta t v_y^n(k-1) \end{bmatrix} + \begin{bmatrix} \eta_x^n \\ \eta_y^n \end{bmatrix} \quad (5.159)$$

$$\begin{bmatrix} v_x^n(k) \\ v_y^n(k) \\ v_z^n(k) \end{bmatrix} = \begin{bmatrix} v_x^n(k-1) \\ v_y^n(k-1) \\ v_z^n(k-1) \end{bmatrix} + \left(\mathbf{C}_{vn} \begin{bmatrix} a_x^v + b_{x_a}^v(k-1) + \eta_{x_a}^v \\ a_y^v + b_{y_a}^v(k-1) + \eta_{y_a}^v \\ a_z^v + b_{z_a}^v(k-1) + \eta_{z_a}^v \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \right) \Delta t \quad (5.160)$$

$$\begin{aligned} \phi(k) = \phi(k-1) + ((\omega_x^v + b_{x_\omega}^v(k-1) + \eta_{x_\omega}^v) + ((\omega_y^v + b_{y_\omega}^v(k-1) + \eta_{y_\omega}^v)(\sin \phi(k-1)) + \\ (\omega_z^v + b_{z_\omega}^v(k-1) + \eta_{z_\omega}^v) \cos(\phi(k-1))) \tan(\theta(k-1))) \Delta t \end{aligned} \quad (5.161)$$

$$\begin{aligned} \theta(k) = \theta(k-1) + ((\omega_y^v + b_{y_\omega}^v(k-1) + \eta_{y_\omega}^v) \cos(\phi(k-1)) - \\ (\omega_z^v + b_{z_\omega}^v(k-1) + \eta_{z_\omega}^v) \sin(\phi(k-1))) \Delta t \end{aligned} \quad (5.162)$$

$$\begin{aligned} \psi(k) = \psi(k-1) + ((\omega_y^v + b_{y_\omega}^v(k-1) + \eta_{y_\omega}^v) \sin(\phi(k-1)) \sec(\theta(k-1))) \Delta t \\ + ((\omega_z^v + b_{z_\omega}^v(k-1) + \eta_{z_\omega}^v) \cos(\phi(k-1)) \sec(\theta(k-1))) \Delta t \end{aligned} \quad (5.163)$$

$$\mathbf{b}^v(k) = \mathbf{A}_b \mathbf{b}^v(k-1) + \mathbf{c}_b^v + \boldsymbol{\eta}_b^v \quad (5.164)$$

$$\mathbf{C}_v^n(k) = \begin{bmatrix} C_\theta C_\psi & -C_\phi S_\psi + S_\phi S_\theta C_\psi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & -S_\phi C_\psi + C_\phi S_\theta S_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} \quad (5.165)$$

where Δt is the sampling time of the inertial sensor, g is the gravitational constant, $\mathbf{C}_{vn}$ is the rotational matrix describing the orientation of the vehicle frame $\{\mathbf{v}\}$ with respect to the navigation frame $\{\mathbf{n}\}$ and $C_x = \cos(x)$ and $S_x = \sin(x)$ for $x \in \{\phi, \theta, \psi\}$. The matrix $\mathbf{A}_b$ is a 6×6 diagonal matrix, the vector $\mathbf{c}_b^v$ is a 6×1 constant vector and $\boldsymbol{\eta}_b^v$ is a 6×1 temporally uncorrelated noise vector having the diagonal covariance matrix $\mathbf{Q}_b$. The values of $\mathbf{A}_b$, $\mathbf{c}_b^v$ and $\mathbf{Q}_b$ can be experimentally determined. The offsets and drifts in bias phenomena of inertial sensors are modeled as given by (5.164). The equation (5.164) is capable of accommodating exponential variation [118] of biases or a constant but unknown component and a random walk component [21]. The error models of the gyroscopes and accelerometers are as follows:

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$$\boldsymbol{\omega}_{i/p} = \mathbf{M}_{gyro} (\boldsymbol{\omega}_{o/p} + \boldsymbol{\omega}_{bias}) + \boldsymbol{\eta}_{\omega} \quad (5.166)$$

$$\mathbf{a}_{i/p} = \mathbf{M}_{acc} (\mathbf{a}_{o/p} + \mathbf{a}_{bias}) + \boldsymbol{\eta}_a \quad (5.167)$$

where $\boldsymbol{\omega}_{i/p}$ is the actual input rotational rate vector, $\boldsymbol{\omega}_{o/p}$ is the measured rotational rate vector $[\omega_x^v, \omega_y^v, \omega_z^v]^T$, $\boldsymbol{\omega}_{bias}$ is the three axis gyro bias vector $[b_{x_{\omega}}^v, b_{y_{\omega}}^v, b_{z_{\omega}}^v]^T$, $\boldsymbol{\eta}_{\omega}$ is the gyroscope measurement noise, $\mathbf{a}_{i/p}$ is the actual input acceleration vector, $\mathbf{a}_{o/p}$ is the measured accelerations vector $[a_x^v, a_y^v, a_z^v]^T$, $\mathbf{a}_{bias}$ is the , three axis acceleration bias vector $[b_{x_a}^v, b_{y_a}^v, b_{z_a}^v]^T$, and $\boldsymbol{\eta}_a$ is the accelerometer measurement noise all along the three axes in $\{\mathbf{v}\}$. $\mathbf{M}_{gyro}$ and $\mathbf{M}_{acc}$ are 3x3 matrices used for scale factor and misalignment compensation of gyroscopes and the accelerometers respectively. Scale factor matrices, in this formulation are assumed to be identity matrices. The process model of $\mathbf{x}_v(k)$ can now be expressed as

$$\mathbf{x}_v(k) = \mathbf{f}_v(\mathbf{x}_v(k-1), \mathbf{u}^v(k-1), \mathbf{b}^v(k-1), \boldsymbol{\eta}(k-1)) \quad (5.168)$$

where $\mathbf{u}^v(k-1) = [a_x^v, a_y^v, a_z^v, \omega_x^v, \omega_y^v, \omega_z^v]$, $\mathbf{f}_v(\cdot)$ is given by concatenated right hand sides of the equations (5.159) to (5.164) and $\boldsymbol{\eta}(k-1)$ is a zero mean process noise sequence. The covariance $\mathbf{Q}_v(k)$ representing the entire noise characteristics of IMU measurements and biases is given by

$$\mathbf{Q}_v(k) = \left(\frac{\partial \mathbf{f}_v}{\partial \mathbf{u}^v} \right) \mathbf{Q}_u \left(\frac{\partial \mathbf{f}_v}{\partial \mathbf{u}^v} \right)^T + \begin{bmatrix} \mathbf{Q}_{xy} & \mathbf{0}_{2 \times 6} & \mathbf{0}_{2 \times 6} \\ \mathbf{0}_{6 \times 2} & \mathbf{0}_{6 \times 6} & \mathbf{0}_{6 \times 6} \\ \mathbf{0}_{6 \times 2} & \mathbf{0}_{6 \times 6} & \mathbf{Q}_b \end{bmatrix} \quad (5.169)$$

where $\mathbf{Q}_u$ and $\mathbf{Q}_{xy}$ are zero mean diagonal noise covariance matrices of the concatenated IMU measurement noise vector $[\eta_{x_a}^v, \eta_{y_a}^v, \eta_{z_a}^v, \eta_{y_{\omega}}^v, \eta_{y_{\omega}}^v, \eta_{y_{\omega}}^v]^T$ and the noise vector $[\eta_x^a, \eta_y^a]^T$. The observation model is derived assuming that the vehicle traverses on a flat

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horizontal plane. It is assumed that the landmarks are observed using an onboard exteroceptive sensor such as a 2D range-bearing sensor. It is important to note that given a 3D sensor such as 3D laser measurement system or stereo vision, it would be straightforward to incorporate the Z dimension of the vehicle location coordinates and the landmark coordinates to the estimation problem and to relax the flat surface assumption. As shown in Figure 5.6, the exteroceptive sensor frame $\{s\}$ is assumed to be aligned but not coincident with the vehicle frame $\{v\}$. Let $\mathbf{x}_s^v$ represents the position of the exteroceptive sensor in the vehicle frame $\{v\}$, $\mathbf{x}_s^n$ be the position of the exteroceptive sensor in the navigation frame $\{n\}$ and $\mathbf{x}_v^n$ be the vehicle's coordinates in the navigation frame $\{n\}$. Let $\mathbf{x}_L^n$ be the coordinates of a point landmark in $\{n\}$, and $\mathbf{x}_L^s = [L_x \ L_y \ L_z]^T$ be its coordinates in the exteroceptive sensor's frame $\{s\}$. Since the vehicle moves on a flat horizontal plane whose elevation is known, the z coordinates of the position vectors $\mathbf{x}_s^v$, $\mathbf{x}_s^n$, $\mathbf{x}_v^n$, and $\mathbf{x}_L^s$ are known a priori. Since $\mathbf{C}_{vn}$ is the relative orientation between $\{v\}$ and $\{n\}$, and assuming there is no relative orientation between $\{v\}$ and $\{s\}$ we have

$$\mathbf{x}_s^n = \mathbf{x}_v^n + \mathbf{C}_{vn} \mathbf{x}_s^v, \quad \mathbf{x}_L^s = (\mathbf{C}_{vn})^T (\mathbf{x}_L^n - \mathbf{x}_s^n) \quad (5.170)$$

$$\mathbf{z}(k) = \begin{bmatrix} \sqrt{L_x^2 + L_y^2} \\ \tan^{-1}(L_y/L_x) + \pi/2 \end{bmatrix} \quad (5.171)$$

where it is assumed that there is no relative rotation between the coordinate frames $\{v\}$ and $\{s\}$.

Since the vehicle motion is constrained by the nonholonomic constraints, (i.e vehicle velocities along the Y-axis v_y^v and Z-axis v_z^v of the vehicle reference frame $\{v\}$ are zero) we use these constraints [23] as virtual observations also in the online bias estimation. Let

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the vehicle velocity on vehicle frame X-axis be v_x^v , then

$$\begin{bmatrix} v_x^v & v_y^v & v_z^v \end{bmatrix}^T = (\mathbf{C}_{\mathbf{v}^n})^T \begin{bmatrix} v_x^n & v_y^n & v_z^n \end{bmatrix}^T \quad (5.172)$$

$$v_y^v = 0 + \eta_x \quad (5.173)$$

$$v_z^v = 0 + \eta_z \quad (5.174)$$

where η_x and η_z are Gaussian noise sequences, which represent the amount of constraint violation. The INS based SLAM with bias estimation problem has a composite state vector $\mathbf{X}(k)$ as

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}_v^T(k) & \mathbf{m}^T(k) \end{bmatrix} \quad (5.175)$$

where the model for $\mathbf{x}_v(k)$ is given by (5.168) and the model of the map $\mathbf{m}(k)$ of concatenated 2D landmark locations is given by (5.13). Observed landmarks in (5.171) are in general part of the estimated map state $\mathbf{m}(k)$. EKF prediction equations take the form (5.15)-(5.17) where $\mathbf{f}(\cdot) = \begin{bmatrix} (\mathbf{f}_v(\cdot))^T & \mathbf{m}^T(k-1) \end{bmatrix}^T$ and EKF update equations are given by (5.18)-(5.20).

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5.5.3 SLAM with Bias Estimation

5.5.3.1 Simulations

As discussed before, it necessary that the observability of the system in the presence of biases be investigated before any attempt is made to estimate these parameters on-line. Symbolic manipulation of process and observation models of standard INS aided SLAM with one unknown landmark without on line bias estimation yields the following results. When the bias is not estimated jointly with the SLAM state vector, there are total of 10 states. This includes the 8 vehicle states from (5.159) to (5.163) and the two states arising from the unknown landmark. In this case if nonholonomic constraints, which serve as virtual observations are ignored, the observability Grammian yields rank 7. Therefore the Grammian is rank deficient by 3. When the nonholonomic constraints are used as virtual observations, the rank of the observability matrix becomes 8. It still has a rank deficiency of 2 in the observability Grammian. However, if the measurement model is modified to include a known 2D landmark measurement (as in priori map based robot localization) or an absolute sensor measurement such as GPS, then the system becomes full rank (i.e the observability matrix has a rank of 10).

We now investigate the observability of the INS aided SLAM with online bias estimation (algorithm (5.159)-(5.175)) by symbolic manipulation. The problem is analyzed for the case of only one 2D landmark in the SLAM state vector. The results are further verified using empirical SLAM runs. In this case, SLAM state vector has 16 states including 8 vehicle states, 6 bias states and 2 landmark states. When the online bias estimation is carried out without imposing nonholonomic constraints, the rank of the observation matrix is 9. When the nonholonomic constraints are used as virtual observations, the rank of the observation matrix increased to 12. It is also observed that INS aided SLAM with one unknown landmark becomes fully observable when the nonholonomic constraints are imposed and if it is assumed that two known landmarks are observed simultaneously.

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Simulations verify that this result can be generalized to any number of unknown landmarks in the SLAM state vector. That is the problem of INS aided SLAM with joint on-line estimation of biases involving any number of unknown 2D landmarks is fully observable if the non-holonomic constraints are imposed and at least two known 2D landmarks are observed simultaneously.

Figure 5.7. IMU aided SLAM simulation. Estimated path is shown by the dark color dashed line and the true path is shown by the thick light color line. True landmarks are shown as circles while the estimated landmarks are marked by crosses.

The performance of the online bias estimation problem is studied by conducting simulations in an environment having several point landmarks (Figure 5.7). An autonomous vehicle performing SLAM is assumed to travel in an approximately elliptic path making several loops. The vehicle is assumed to be equipped with a range/bearing sensor and an IMU (having three angular rate sensors and three accelerometers) mounted as discussed in Section 5.5.2. The simulation parameters consist of angular rate biases of $1.5^\circ s^{-1}$ constant bias component and a random walk component in all 3 axes and acceleration biases of $12mg$ constant component and a random walk component. The random walk components due to the measurement noise of the angular rate sensors and

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those of the accelerometers were taken as $4.5^0/\sqrt{hr}$ and $0.1\text{ ms}^{-1}/\sqrt{hr}$ respectively.

Figure 5.8. IMU aided SLAM. Localization errors of X, Y coordinates and velocities of the vehicle in X, Y, Z axes in the navigation frame, roll, pitch and yaw.

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Figure 5.9. IMU aided SLAM. Estimation of accelerometer and rotational rate sensor biases and their uncertainties

Figures 5.8 (a)-(h) show the localization error distributions of the IMU aided SLAM with online bias estimation. It may be seen from Figures 5.9 (a)-(h) that the errors are bounded by 95% confidence limits. It is also observed that in general, the uncertainties (variances) of the proprioceptive INS sensor biases settle to a lower bound as discussed in Section 5.3.3.2.

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Figure 5.10. Location estimation errors in navigation frame in SLAM without bias estimation and with offline inertial sensor calibration

Figures 5.10 (a)-(d) show the results obtained assuming that the constant bias terms are known a priori. That is it is assumed that the biases are calibrated offline. Since the biases vary and cause the accumulated errors, inconsistent localization results are produced as shown in Figure 5.10. The errors in Figures 5.10 (a) and (b) (velocity estimation errors) and Figures 5.10 (c) and (d) (location estimation errors) diverge after a small duration after initialization. Moreover, the results also verify that offline calibration methods are successful only for smaller periods of time between resetting (initialization) for smaller variations of biases.

5.5.3.2 Experiments

Experimental evaluation of the online bias estimation algorithm in INS aided SLAM was carried out using the in-house built autonomous vehicle platform (Appendix A). The vehicle was equipped with an uncalibrated IMU (Crossbow DMU-AHRS), which was fixed to the vehicle frame as discussed in Section 5.5.2.

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Figure 5.11. IMU aided SLAM with online bias estimation - Estimated path and landmarks.

Figure 5.12. IMU Aided SLAM landmark estimates and their uncertainty

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A SICK LMS 290 range/bearing measurement sensor was used as the exteroceptive sensor. The feature extraction is done using a simple clustering strategy based on the distance between contiguous and adjacent range points of laser scans (Section 2.4.8).

Figure 5.13. Estimated accelerometer biases.

Figure 5.14. Estimated rotational rate sensor biases

Figure 5.15. Estimated 2σ uncertainty bound of accelerometer biases.

Figure 5.16. Estimated 2σ uncertainty bound of rotational rate sensor biases

(a)

(b)

Figure 5.17. Estimated location error of landmark 1, The 95% confidence limit is shown by dotted lines. Figure 5.17 (a). Error in X. Figure 5.17 (b). Error in Y.

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Figure 5.18. Estimated location error of landmark 2. The 95% confidence limit is shown by dotted lines. Figure 5.18 (a) shows error in X. Figure 5.18 (b) shows error in Y

The experiment was performed in a car park where the surface was relatively flat and horizontal. Figure 5.11 illustrates the estimated vehicle path. Figure 5.12 shows the estimated landmarks and their 2σ uncertainty ellipse. The results verify that the landmark estimated errors are bounded by their 2σ uncertainty ellipses. Figures 5.13 and 5.14 show the variation of biases and Figures 5.15 and 5.16 show the variation of 95% confidence bound of the uncertainty of the bias estimation. Figures 5.17 and 5.18 evaluate the ground truth of the static landmarks. The landmark locations are hand measured and their true errors plotted against time with their 95% confidence bounds of the uncertainty. The 95% confidence limit bounded uncertainties in the location estimates suggest that the algorithm gives consistent results. Therefore, as shown and discussed in the simulations the results of the experiment also verify that given sufficient number of known landmark observations the biases of the accelerometer and rotational rate sensor (proprioceptive sensors) can be estimated online indicating the full observability of the system.

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5.5.4 MAL with Bias Estimation

5.5.4.1 Simulations

In the case of INS based MAL, all the equations (5.157)-(5.174) used in SLAM with online bias estimation remain intact except the composite state vector $\mathbf{X}(k)$ which is given by (5.168) is equal to $\mathbf{x}_v(k)$. Furthermore, observed landmarks in (5.171) are always part of a known map as described in Section 5.3. EKF prediction equations take the form (5.97)-(5.99) where $\mathbf{f}(\cdot) = \mathbf{f}_v(\cdot)$ and EKF update equations are given by (5.18)-(5.20).

The observability of the MAL with online bias estimation is also investigated by symbolic manipulation. We consider maps with varying number of 2D known landmarks. The results are further verified using empirical MAL runs with bias estimation. For the case under consideration, the MAL state vector has 14 states (i.e. 8 vehicle states and 6 bias states). When online bias estimation is incorporated without imposing nonholonomic constraints and observing one known landmark, the rank of the observability Grammian is 9. When two landmarks are observed, the rank is increased to 11. Increasing the number of known landmarks observed to three and beyond only increases the rank of the Grammian to 12. However, the observability Grammian is still rank deficient by 2. The full rank can only be achieved by using the non-holonomic constraints, as there is no sensor, which gives information about the z dimension. These results verify that to achieve the necessary observability in IMU aided MAL, it is necessary to observe at least two landmarks and apply nonholonomic constraints (virtual observations) at the same time. Further, by analyzing the observability Grammian of the system when observing two landmarks, it is inferred that there is a little chance that the equations such as (5.130) or (5.131) are satisfied at the same time, which will make the system unobservable. This is because the number of conditions that must be satisfied at the same time for the rows of measurement Jacobian are dependent is larger than in the case of measurement Jacobian

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(5.129) for standard 2D MAL with bias estimation.

Figure 5.19. IMU aided MAL. Localization errors of X, Y coordinates and velocities of the vehicle in navigation frame, roll, pitch and yaw.

The performance of the online bias correction problem in IMU aided MAL was studied by conducting simulations as described in Section 5.5.3.1 with the same set of simulation parameters.

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Figure 5.20. IMU aided MAL, Bias Estimation. Estimation of accelerometer and rotational rate sensor biases and their uncertainties

Figures 5.19 (a)-(h) show the localization error distributions of the IMU aided MAL with online bias estimation. The errors are bounded by 95% confidence limits as illustrated. Figures 5.20 (a)-(l) depict the estimation of sensor biases and their uncertainties. The results verify that all the errors are bounded by the 95% confidence limits. However, non-availability of the observations from time to time and the fact that there exist random walk behavior in the biases cause the uncertainties of bias terms to increase at the instances depicted in Figures 5.20 (a), (c), (g), (i) and (k). For example, the increases in bias uncertainties at the time instances 40s, 100s and 180s are due to the lack of adequate

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measurement updates in all the bias terms except that of $b_{z_a}^y(k)$. The estimate of $b_{z_a}^y(k)$ gains information in the form of nonholonomic constraints.

Figure 5.21. Location estimation errors in navigation frame in MAL without bias estimation and with offline inertial sensor calibration

Figures 5.21 (a)-(d) show the results obtained assuming that the constant bias terms are known a priori. In practice this is equivalent to offline calibration and initialization of the biased sensors. In the simulations, the parameters used in Section 5.5.3.1 to model the random walk behavior of biases are used. It is noted that the estimation errors in the velocity (Figure 5.21 (a) and (b)) and location (Figure 5.21 (c) and (d)) diverge shortly after initialization. Further, the results also indicate that such offline calibration methods are successful only for smaller periods of time between resetting (initialization) and for smaller variations of biases.

5.5.4.1 Experiments

Experimental evaluation of the online bias estimation algorithm in MAL was carried out using the same system and in the same outdoor setting as described in Section 5.5.3.2.

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The experiment was performed in a car park where the surface is relatively flat and horizontal. The stored map is created by hand measuring the locations of a set of features (landmarks) such as lampposts and trees in the vicinity with respect to the predefined earth fixed navigation frame. The vehicle starting location is also established with reference to this coordinate system. Figure 5.22 illustrates the estimated vehicle path when performing IMU aided MAL. The small circles in Figure 5.22 show the locations of hand-measured features, which are shown as true landmarks.

Figure 5.22. IMU aided MAL with online bias estimation. Estimated vehicle path and the true landmark locations used as the stored map.

As in the simulations, the results of the experiment also verify that given a sufficient number of landmark observations, the biases of the accelerometer and rotational rate sensor (proprioceptive sensors) can be estimated online. Figures 5.23 (a) and (b) show the variation of biases in MAL and Figures 5.24 (a) and (b) show the variation of 2σ uncertainty bound of the bias estimation.

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Figure 5.23. Estimation of accelerometer and gyroscope biases. Figure 5.23 (a) Accelerometer biases. Figure 5.23 (b) Rotational rate sensor biases

Figure 5.24. Estimation of 2σ uncertainty bound of accelerometer and gyroscope biases. Figure 5.23 (a) Uncertainties of accelerometer biases. Figure 5.23 (b) Uncertainties of rotational rate sensor biases

5.6 Conclusion

The effects of the exteroceptive and proprioceptive sensor biases on the problems of simultaneous localization and mapping, map aided localization and robotics mapping in unstructured environments were investigated in detail. In particular, with regards to SLAM with online bias estimation the following were established: (1). Uncertainties in both the proprioceptive and exteroceptive sensor biases diminish in each successive update and approach a certain lower bound. (2). In 1D problem, the observability is ensured by observing one known landmark and incorporating absolute position information of the vehicle. (3). In the standard 2D problem for full observability, it is

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necessary that at least two known landmarks be observed simultaneously in addition to observing the landmarks in the map, and (4). In 1D problem, the rate of convergence is not affected by the biases or their initial uncertainties and depends only on the proprioceptive sensor noise term, exteroceptive sensor noise term, absolute sensor noise term and the total number of unknown landmarks in the estimated map and the number of known landmarks, which are being observed. Moreover, the smaller is the ratio between the exteroceptive sensor noise and the process noise (proprioceptive sensor noise and the modeling uncertainty) or the larger is the number of landmarks observed simultaneously (either known landmarks or the estimated landmarks or the both), the faster will be the convergence of the state variances to their steady state values.

It was shown analytically that if sensor biases are estimated jointly with the vehicle pose in a MAL framework: 1) The uncertainties of the estimated errors in the bias parameters of both proprioceptive and exteroceptive sensors diminish in each update. 2) A derived lower bound is reached in exteroceptive sensor bias estimate. 3) The rate of convergence to this lower bound is also derived. 4) In order to guarantee observability in MAL with bias estimation, it is necessary to observe at least two distinct landmarks.

In the RM problem with joint exteroceptive sensor bias estimation, it was shown that 1). In the steady state all the map states are correlated and 2). The RM problem in 1D or 2D is observable when one known 1D or 2D landmark (depending on the problem) is observed.

In conclusion, the theoretical findings and results substantiated by simulations and experiments provide greater insight into the problem of estimation theoretic bias correction in SLAM, MAL and RM. The investigations pave the way for the improved design and judicious choice of process and observation models and the selection of appropriate sensors for the enhanced performance of localization algorithms depending on the requirements of the mobile robot application.

Chapter 6

Conclusions and Recommendations

6.1. Conclusions

In the recent years, deploying autonomous vehicles in outdoor unstructured environments has received considerable interest throughout the military, homeland security, surveying, mapping, automotive and aerospace industries and several other domains. In this context, localization of a mobile platform with respect to an a priori map or simultaneous localization with respect to an incrementally built map is of paramount importance. More specifically, localization is a prerequisite to most critical functions associated with autonomous operation such as path planning, control, obstacle avoidance and task execution. Great deal of research has been done in the area of autonomous localization in indoor environments, which are usually structured or semi-structured. However, in outdoor environments, which are essentially unstructured, the issues of data association, use of multiple sensors, their fusion and effects of their biases have not been adequately addressed. The importance of these issues was comprehensively dealt with in Chapters 1 and 2. It was shown through appropriate synthesis, analysis, simulations and experimental verifications of algorithms in each of these areas that significant gains are realizable in robustness, reliability and accuracy of localization and mapping.

In Chapter 3, a generalized discrete optimization based framework was synthesized employing multiple frames of measurement for data association in robot navigation applications. Within this framework, it was shown how different data association

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algorithms including the most common single measurement frame based nearest neighbor method and the multiple measurement frames based optimal MHT methods and their variants can be synthesized by an appropriate choice of the cost function. The framework proposed and developed was also shown to provide insight into the correspondence, relationship and interplay of data association and feature extraction. In particular, it is shown that correlation based matching that completely avoids feature extraction (or data compression) as in scan matching and grid based approaches is one end of the spectrum of data association where feature extraction function is completely avoided. Such approaches were shown to be not quite advantageous in the context of SLAM or MAL. It was argued that the computational complexity of the data association and feature extraction problems in a robot localization and mapping algorithm could be reasonably managed by performing both functions, i.e. feature extraction and data association in appropriate proportions. The amount of the mix between data association and feature extraction has to be determined based on the available sensors and environmental conditions.

Furthermore, data association algorithms utilizing a sliding window of multiple measurement frames (known as MFDA) in SLAM, MAL and RM were proposed for the first time in Chapter 3. The issues of correlation of measurements and conditional measurement independence were investigated in the context of SLAM, MAL and RM. MFDA based data association using two frames of measurement was rigorously formulated for concurrent robot localization and mapping by incorporating the sensor location uncertainty. The MFDA based algorithm was shown to be a practical and effective alternative to the theoretically optimal MHT. Compared to single measurement frame methods, the MFDA resolves association incompatibilities and ambiguities more effectively and yields consistent maps especially with increased spurious measurements (clutter), feature densities and moving objects. In particular, this work established that

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even though the complexity of two-frame data association is relatively high, it significantly outperforms standard nearest neighbor data association. As compared to JCBB, which takes full account of the spatial correlations between vehicle and features in a single frame, MFDA does better in settings with very high spurious measurement densities, and in the presence of moving objects. In addition, MFDA is more efficient computationally and amenable to real time implementation unlike JCBB despite JCBB's use of branch and bound pruning of its exponential search space. This is especially so in dynamic environments with high spurious measurement densities. A new multiple frame temporal consistency criterion (MFTC) was then introduced for the multiple frame data association in SLAM and MAL relaxing the conditional measurement independence assumption. The MFTC method takes into account the strong correlations present in SLAM and at the same time takes temporal attributes of measurement feature combinations into consideration however, at an increased computational complexity. It was established that the multiple frame data association algorithms were much more robust than the existing data association algorithms in outdoor unstructured environments using several experiments conducted under varying terrain conditions.

In Chapter 4, issues pertaining to the use of multiple sensors and approaches in sensor fusion and integration in outdoor setting were investigated in the context of monocular vision and laser sensors. It is established that of the group sensor fusion architecture and the sequential sensor fusion architecture, only the former is theoretically sound in the strict statistical sense in SLAM for multiple sensor fusion. It is also shown that contrary to the common belief, using sequential sensor fusion in SLAM is justifiable only under the conditional measurement independence assumption.

A strategy of utilizing a laser range finder and two monocular vision sensors in outdoor SLAM setting was demonstrated. A new performance metric quantifying the performance of multiple sensors in SLAM was proposed based on the mean value of the estimated

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uncertainty bound. A method of enhancing multiple sensor performance through information theoretic active control strategy over a finite time window was formulated. A generalized active sensing paradigm in a finite time window applicable to SLAM problem was formulated as a nonlinear optimization problem. The control of active bearing only sensor (eg. a monocular camera) to maximize the information gain, in the presence of multiple sensors was elaborated in the context of the SLAM problem.

The effect of sensor biases on the performance of SLAM, MAL and RM is an important issue, which has drawn little attention of the research community. In outdoor unstructured environments, the problem affects most localization algorithms and causes adverse effects to the data association function, tracking accuracy, map convergence and localization accuracy. A rigorous investigation was carried out in Chapter 5 on the estimation theoretic sensor bias correction problem to analyze these effects on robot localization and mapping algorithms. The convergence properties of the bias estimates and their lower bounds were derived. The work also theoretically established the partial observability properties of SLAM, MAL and RM problems with joint sensor bias correction and proposed remedies in the form of appropriately modified observation models to ensure full observability. On the basis of the theoretical discussion, it was shown how the inertial sensors could be utilized for the SLAM and MAL with online sensor bias correction effectively. Consequently, the theoretical findings and results substantiated by simulations and experiments was shown to provide greater insight into the problem of estimation theoretic sensor bias correction in SLAM, MAL and RM. The analysis and theoretical results pave the way for the improved design and judicious choice of process and observation models and the selection of appropriate sensors for the enhanced performance of localization algorithms.

In conclusion the work of this thesis addressed data association, effects of sensor biases and efficient integration of multiple sensors, which are challenging issues in outdoor

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unstructured environments and proposed efficient solutions to these problems. It is argued that the combined effect of improving on the above major challenges is substantial for robot localization and mapping algorithms in outdoor unstructured environments and will be a significant leap towards achieving the final goal of truly autonomous systems.

6.2. Recommendations for Future Work

Although, the work presented in this thesis elucidates and elaborates on the structure and provides insight to some of the important problems encountered in localization and mapping in outdoor unstructured environments and proposed efficient and effective solutions, it opened up several issues that need further investigation and study at the same time. One of them is the complexity of the combinatorial search in the data association algorithms. It was suggested that the introduction of efficient data structures, use of graph theoretic approaches and utilizing of new search algorithms would contribute towards reducing the complexity of interpretation tree search in the data association using MFTC criterion and also providing efficient techniques for the solution of multi-dimensional assignment problems.

Development of feature extraction algorithms having a high compression ratio and at the same time retaining most of the information, which is essential to the robot localization, is also an important open challenge. The present feature extraction algorithms rely on specific kinds of environments and predefined feature types only. On the other hand, the next generation feature extraction algorithms are expected to extract features of multiple shapes, size and structure. It is also possible to couple the data association and feature extraction as discussed in this thesis (Chapter 2) and thereby enhance the robustness of the entire robot navigation process. Thus, it is recommended that more work be done towards the development of robust feature extraction and data association algorithms, which is appropriate to outdoor unstructured environments.

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Another challenging problem, which comes into light in the course of this work, is the use of online sensor calibration and bias correction. This is shown to be vital and solutions are provided for the case of the sensor bias correction. The work can be further extended by including both sensor calibration and bias parameters in robot navigation and estimation algorithms as done in the sensor bias correction. This type of work will enhance the performance of multiple sensors and improve the sustainability in harsh and rough environmental conditions. As an example, the use of vision sensors can be made more robust by including online camera calibration in the estimation problem. However, it is also important to study the observability, controllability and stability of these models and convergence properties of the estimators in order to achieve good performance.

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Appendix A

The Mobile Robot Platform

A.1. Generic Outdoor Mobile Explorer

Generic Outdoor Mobile Explorer (GenOME), a car like mobile robot (Figure A.1), was used as the experimental test bed in implementing mobile robot navigation applications.

Figure A.1. Generic outdoor mobile explorer test bed

The hardware architecture of the navigation system is shown in Figure A.2. The local area network on the vehicle consists of three single board computers (PIII, 650 Mhz), designated as “Path planner”, “Navigator” and “Controller” providing the required computing resources. Path planner acts as the host computer and is able to communicate with a remote terminal via a wireless Ethernet within 150m and via a radio modem for distances exceeding 150m. A remote terminal can be used to monitor and control the platform from a distance. The path planner is also employed in control and data acquisition of the cameras. Laser scanners, differential global positioning system (DGPS),

Appendix A: The Mobile Robot Platform

eleven infra-red (IR) sensors and a sonar ring is connected to the Navigator for data acquisition. Controller oversees all the low-level controlling tasks, such as controlling the drive system actuator, steering system actuator and braking system actuator. Furthermore, it processes the data from three axis accelerometers and three rate sensors of the Crossbow Altitude and Heading Reference System (DMU-AHRS), five encoders, one gyroscope and one compass.

Figure A.2. The hardware architecture of GenOME. The symbols and acronyms IR, LMI 400, PCL 833, D I/O, A I/O, LAN, RS 232 and IEEE 1394 denote the infra red, laser measurement interface, PCL 833 3 axis quadrature encoder and counter, digital input output, analogue input output, local area network, RS 232 serial interface and IEEE 1394 interface respectively.

A.2. Sensor Specifications

A.2.1. SICK LMS 290 Laser Measurement System

Figure A.3. SICK LMS 290 laser measurement system

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Table A.1 Specifications of LMS 290 S-05

Model	LMS 290
Range	Maximum. 80 m
Angular resolution	0.25°/0.5°/1° (selectable)
Response time	53 ms/26 ms/13 ms
Measurement resolution	10mm
System error	Typ. ± 60 mm, range 1,...,4m Typ. ± 35 mm, range 4,...,20m Environmental conditions: good visibility, Ta=23° C, reflectivity 10%...10,000%
Statistical error	Typ. ± 10 mm (at range 1...20 m/ $\geq 10\%$ reflectivity/ ≤ 5 klux)
Data interface	RS 232/ RS 422 (configurable)
Transfer rate	9.6/19.2/38.4/500 kbaud
Switching outputs, standard variants	3xPNP; typ. 24VDC OUT A, OUT B max. 250mA, OUT C max. 100mA
Switching outputs, relay variants	OUT A, OUT B (relay) max. switching voltage 48VDC/ 26 VAC (protected low voltage, safe isolation from mains) max. switching current 0.7A; max. switching power 30W OUT C/ weak (PNP) typ. 24VDC, max. 100 mA.
Supply voltage	24 VDC $\pm 15\%$ (max. 500 mV ripple); current requirement max. 1.8 A (incl. Output load)
Power uptake	Approx. 20W (without output load)
Electrical protection class	Safety insulated, protection class 2
Laser protection class	1 (eye-safe)
Interference resistance	Acc. To IEC 801, part 2-4; EN 50081-1/50082-2
Operating ambient temperature	-30...+50° C
Storage temperature	-30...+70° C
Weight	Approx. 9kg
Vibration fatigue limit	Acc. To IEC 68 part 2-6, table 2c, frequency range 10...150Hz, amplitude 0.35mm or 5g single impact IEC 68 part 2-27, table 2, 15 g/11ms, shock absorbers are recommended for heavy vibration and impact demands (e.g. AGV applications)

A.2.2. Fiber Optical Gyroscope (HOFG 1(A))

Figure A.4. Fiber optical gyroscope

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Table A.2 Specifications of HOFG-1(A)

Parameter	Serial output	Analog output ****
Model	HOFG-1(A)	
Rotation rate	-60^0 to $+60^0$ /s	
Angle measurement range	-360^0 to $+360^0$ /s	-180^0 to $+180^0$ /s
Random walk	$\leq 0.1^0 / \sqrt{h}$	$\leq 0.15^0 / \sqrt{h}$
Zero drift (rate integration)	$\leq 1^0 / h$ *	$\leq 1^0 / h$ *
Nonlinearity of scale factor	Within $\pm 1.0^0$ **	Within $\pm 1.5^0$ **
Response time	Typ. 20ms ***	
Data output interval	Min. 15ms	Typ. 10ms
Time constant	Typ. 20ms	
Warm-up time	Typ. 5.0 s	
Supply voltage	-0.3 VDC to 18 VDC	
Input voltage (RS232)	-15 V to 15 V	
Operating temperature	-10^0 C to 60^0 C	
Operating humidity	20-80% RH (Non condensing)	
Data interface	RS232C	
Transfer rate	9,600 baud	

* Clipping level $n = 6 = 0.0157^0 / s$

** For $|\Omega| \geq 2^0 / s$, where Ω is the rotation rate; in the case of $|\Omega| < 2^0 / s$, nonlinearity of scale factor is mainly determined by random walk

*** Between command acquisition and response completion

**** Measured using a voltmeter with an error of less than $\pm 0.1\%$

A.2.3. Encoder (IE-58)

Table A.3 Specifications of IE-58

Model	IE-58
Supply voltage	11-27VDC
Power dissipation (No load)	<4 W
Output	Push-pull, max. current 30mA; Incremental signal A, A neg., B, B neg.; Channel A leads channel B when rotating in clockwise, Marker pulse Z, Z neg., 1 pulse per revolution, output frequency <1000 PPR = 30kHz, >1000 PPR = 160kHz; Rise time of edge <500 ns
Max. revolutions per minute	(Output frequency [Hz]/PPR)x 60 s
Number of pulses per revolution	1-10,000
Electromagnetic compatibility	EN 61000-4-2 (IEC-801-2)/ EN 61000-4-4 (IEC-801-4)
Operating temperature	0^0 to 70^0 C
Relative humidity	98% (non-condensing)
Protection class	IP 65 (DIN 40 050)
Maximum speed	6000 RPM
Maximum load on shaft	40N axial, 60N radial (at end of shaft)
Weight	0.3 kg
Max. angular acceleration	$\leq 10^4 \text{ rad} / s^2$
Vibration (50-2000 Hz sinusoidal) DIN IEC 68-2-6	$\leq 100m / s^2$
Shock (11 ms) IEC 68-2-27	$\leq 1000m / s^2$
Standard connector	PG 9 axial cable (pigtail)

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Figure A.5. Incremental encoder assembly. Figure (a) shows the IE-58 encoder and figure (b) shows its mounting.

A.2.4. Crossbow DMU-AHRS IMU

DMU-AHRS is a nine axis inertial measurement system that gives acceleration, angular rates and magnetic orientation as output. The unit consists of silicon MEMS accelerometers, Coriolis effect angular rate sensors and magnetometers.

Table A.4 Specifications of DMU-AHRS

Performance		Application Dependent Typical
Roll, pitch and heading angle :		
Dynamic accuracy	$\pm 1^\circ$	
Repeatability	$\pm 0.5^\circ$	
Full scale span (analog outputs)	± 4.096 VDC	
Roll, pitch angle range	$\pm 90^\circ$	
Heading angle range	$\pm 180^\circ$	
Bandwidth	10 Hz	
Linearity	<1%	
Power		
Input supply voltage	14.5 V-30 V	(1 ms half sine wave)
Input supply current	275 mA	
Environmental		
Operating Temperature Range	-40 to 85 ⁰ C	
Storage Temperature Range	-55 to 85 ⁰ C	
Package	Aluminum housing	
Weight	1.25 lb.	
Mechanical Shock	1000 G	
Vibration	10 G RMS	
Digital Data Output Rate		
Voltage Mode	166 Hz	Minimum
Scaled Sensor Mode	156 Hz	
Angle Mode	100 Hz	
Analog Data Update Rate	200 Hz	

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Figure A.6. Crossbow DMU-AHRS (Altitude and Heading Reference System).

A.2.5. MCL-1500 Color Camera

MCL 1500 is a CCD color camera with built in 10-time motorized zoom lens and is suited for surveillance applications. It also comes with Windows NT set up software for camera and lens control via RS 232C interface.

Table A.5 Specifications of MCL-1500 color camera

Scanning System	625 lines, 25 frames/sec.
CCD Sensor	Color 1/3" Hyper HAD IT CCD
Sensing Area	752 (h)x582(v)
Resolution (horizontal)	450 TV line
Sensitivity	0.5 Lux, F1.4
S/N ratio	>48 dB (AGC OFF)
Video Output	Composite VBS signal 1.0 Vpp at 75 Ohm
Video Output Y/C	Y/C signal 1.0 Vpp at 75 Ohm
TV Standard	NTSC/PAL
Zoom Lens	10 times, 5.8 mm to 58mm
Lens Focal Length	5.8 to 58mm
Field of View	46° to 4.8°
Lens Iris	Auto
Focus Function	Auto focus/Manual
Focus Range Wide	0.3 m to ∞
Focus Range Tele	1 m to ∞
Gamma	0.45-1.0
Gain Functions	BLC On/AGC On/BLC & AGC Off
AGC Range	0 to +15 dB
BLC Pattern	Potmeter adjustment: Center/all/peripheral
Level Adjust	Potmeter for AGC, CCD iris, Auto iris
White Balance Manual	2600k 9000k
White Balance Fixed	4600k
White Balance Auto	2600k 9000k
Shutter Function	CCD iris On, Off, Manual
Shutter Range	1 field to 1/30000 second
Serial Control	RS 232C 9600 bps 8 bit data no parity
Synchronization	Internal X-tal or external composite sync.
Operating Temperature	-5 to 45° C
Humidity	20-80% non-condensing
Power	12 V DC ±10% 8W
Dimensions	60×55×95 mm (H×W×D)
Weight	340g

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Figure A.7. MCL 1500 camera

A.3. Modeling the Platform

Figure A.8. Kinematic model of the mobile test platform

The kinematic model of the mobile robot can be described by Figure A.8. The mid point of the rear axel of the vehicle (denoted by point $V \equiv (x(k), y(k))$ where $x(k)$ and $y(k)$ are the x and y absolute coordinates on a reference world coordinate frame) is used as the vehicle coordinates. The vehicle heading angle is θ . The exteroceptive sensor S is located at the front middle point is a laser scanner such as SICK LMS 290 laser measurement system (LMS), which gives range and bearing measurements. Figure A.8 illustrates when the LMS is observing a landmark (considered as a point feature), $P \equiv (x_i(k), y_i(k))$ where $x_i(k)$ and $y_i(k)$ denote the absolute x and y coordinates of the landmark with reference to the same world coordinate frame. The symbols γ , α , $\theta(k)$, θ_i , a_1 and a_2 denote the steering angle input angle, the measurement bearing of the landmark as recorded by the LMS, vehicle heading, the angle of the landmark observed by the LMS with reference to

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the global x-axis, the vehicle wheel base and the distance along the vehicle horizontal axis from point V to point S.

The process model of the vehicle can be expressed as

$$\mathbf{x}_v(k) = \begin{bmatrix} x(k-1) + \Delta t(u_a(k-1))\cos(\theta(k-1)) \\ y(k-1) + \Delta t(u_a(k-1))\sin(\theta(k-1)) \\ \theta(k-1) + \Delta t(u_a(k-1))\tan(\gamma_a(k-1))/a_1 \end{bmatrix} + \mathbf{v}(k-1) \quad (\text{A.1})$$

$$= \mathbf{f}_v(\mathbf{x}_v(k-1), u_a(k-1), \gamma_a(k-1)) + \mathbf{v}(k-1)$$

$$u_a(k-1) = u(k-1) + u_n(k-1) \quad (\text{A.2})$$

$$\gamma_a(k-1) = \gamma(k-1) + \gamma_n(k-1) \quad (\text{A.3})$$

where $u(k-1)$ and $\gamma(k-1)$ denote the speed and the steering angle inputs at time k-1 respectively as measured by the proprioceptive sensors (eg. odometers, gyroscopes etc.). The lumped proprioceptive sensor input can be expressed as $\mathbf{u}(k-1) = [u(k-1) \ \gamma(k-1)]^T$ and the vehicle pose vector is given by $\mathbf{x}_v(k) = [x(k) \ y(k) \ \theta(k)]^T$. $u_n(k) \sim N(0, \sigma_{u_n}^2)$ and $\gamma_n(k) \sim N(0, \sigma_{\gamma_n}^2)$, denote noise properties of $u(k)$ and $\gamma(k)$, which are assumed Gaussian and Δt is the sampling time. $\mathbf{v}(k) \sim N(\mathbf{0}, \mathbf{Q}_a(k))$ is a temporally uncorrelated noise sequence representing the modeling uncertainties having the covariance matrix $\mathbf{Q}_a(k)$. The covariance matrix $\mathbf{Q}_v(k)$ of the vehicle location prediction can now be expressed as

$$\mathbf{Q}_v(k) = \left(\partial \mathbf{f} / \partial (u, \gamma) \right) \mathbf{Q}_u(k) \left(\partial \mathbf{f} / \partial (u, \gamma) \right)^T + \mathbf{Q}_a(k) \quad (\text{A.4})$$

where $\mathbf{Q}_u(k) = \text{diag}(\sigma_{u_n}^2, \sigma_{\gamma_n}^2)$. The observation model of the exteroceptive sensor can be shown as

$$\mathbf{z}(k) = \begin{bmatrix} \sqrt{(x_i(k) - x_s(k))^2 + (y_i(k) - y_s(k))^2} \\ \tan^{-1}((y_i(k) - y_s(k))/(x_i(k) - x_s(k))) + \pi/2 - \theta(k) \end{bmatrix} + \mathbf{w}(k) \quad (\text{A.5})$$

$$= \mathbf{h}(\mathbf{x}_v(k), x_i(k), y_i(k)) + \mathbf{w}(k)$$

where the sensor coordinate S is given by $x_s(k) = x(k) + a_2 \cos(\theta(k))$ and $y_s(k) = y(k) + a_2 \sin(\theta(k))$. $\mathbf{w}(k)$ is a zero mean random vector representing the exteroceptive sensor measurement uncertainties.

The inverse measurement model representing landmark initialization can be expressed as

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$$\begin{aligned}\mathbf{x}_L(k) &= \begin{bmatrix} x(k) + a_2 \cos(\theta(k)) + r_i \cos(\theta_i + \theta(k) - \pi/2) \\ y(k) + a_2 \sin(\theta(k)) + r \sin(\theta_i + \theta(k) - \pi/2) \end{bmatrix} \\ &= \mathbf{g}(\mathbf{x}_v(k), r_i, \theta_i)\end{aligned}\tag{A.6}$$

where $\mathbf{x}_L(k)$, r_i and θ_i denote the x, y absolute coordinates of the initialized landmark in the world frame in concatenated form, range reading and the bearing measurements of the LMS respectively

Appendix B

Matrix Identities

B.1. Properties of Positive Semi-definite Matrices

A matrix $\mathbf{A}_{n \times n}$ is positive semi definite (PSD) if $\underline{\mathbf{x}}^T \mathbf{A} \underline{\mathbf{x}} \geq 0$ for all non zero $\underline{\mathbf{x}}_{n \times 1}$ such that $\underline{\mathbf{x}} \in \mathbf{R}^n$.

If $\mathbf{A}_{n \times n}$ and $\mathbf{B}_{n \times n}$ are PSD matrices, then

1. $\mathbf{A} + \mathbf{B}$ is PSD
2. $\mathbf{AB}$ is PSD
3. Diagonal entries of $\mathbf{A}$ are nonnegative
4. $\det(\mathbf{A} + \mathbf{B}) \geq \det(\mathbf{A}) + \det(\mathbf{B})$
5. Any principal sub matrix of $\mathbf{A}$ is PSD

B.2. Matrix Inversion Lemma

If $\mathbf{A}_{m \times m}$ is a positive definite (PD) matrix, which has the form $\mathbf{A} = \mathbf{B}^{-1} + \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^H$ where $\mathbf{B}$ is a PD matrix of size $m \times m$ and $\mathbf{D}$ is a PD matrix of size $n \times n$, then the inverse of $\mathbf{A}$ is given by

$$\mathbf{A}^{-1} = \mathbf{B} - \mathbf{B}\mathbf{C}(\mathbf{D} + \mathbf{C}^H\mathbf{B}\mathbf{C})^{-1}\mathbf{C}^H\mathbf{B} \quad (\text{B.1})$$

where $\mathbf{A}^H$ is the Hermitian of $\mathbf{A}$.

B.3. Inversion of a Partitioned Matrix

Let $(m+n) \times (m+n)$ matrix $\mathbf{E}$ be partitioned as $\mathbf{E} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$ where both $m \times m$ matrix

$\mathbf{A}$ and $n \times n$ matrix $\mathbf{D}$ are invertible the inversion of the matrix can be expressed as

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} (\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & -\mathbf{A}^{-1}\mathbf{B}(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & (\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1} \end{bmatrix} \quad (\text{B.2})$$