Carrier frequency offset estimation algorithms in orthogonal frequency division multiplexing systems

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**CARRIER FREQUENCY OFFSET ESTIMATION
ALGORITHMS IN ORTHOGONAL FREQUENCY
DIVISION MULTIPLEXING SYSTEMS**

Feng Yang

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctorate of Philosophy

2007

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me and has not been submitted for a higher degree to any other University or Institute.

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Date

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Feng Yang

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Abstract

Orthogonal frequency division multiplexing (OFDM) enables high data rate transmission over wireless communications. The OFDM scheme is robust to frequency-selective fading and avoids inter-symbol interference (ISI). Also, the bandwidth efficiency of OFDM schemes is high. Because of these excellent characteristics, OFDM has been proposed as a promising modulation scheme for the future wireless communication systems. However, OFDM systems are more sensitive to carrier frequency offset (CFO) than conventional single carrier systems. Carrier frequency offset is mainly caused by two sources. The first one is the mismatch of the carrier frequencies between the local oscillators in OFDM transceivers, which is usually considered as a stationary parameter. The other source is the time varying Doppler shift, which is caused by the variation of the relative speed between the transceivers in a wide outdoor environment. The focus of this thesis is on estimating the carrier frequency offset in OFDM systems.

A minimum output variance (MOV) estimator is proposed to estimate the fractional frequency offset in OFDM systems. The magnitude output is observed and

found that minimum output variance is achieved when carrier frequency offset is zero. This feature is exploited to estimate the carrier frequency offset. The theoretical probability density function (PDF) of the magnitude output is obtained, and the mean and variance of the magnitude output are derived based on the PDF. It validates the feature of minimum output variance. The accuracy of the proposed MOV estimator is then examined and numerical results support analytical results. The MOV estimator is effective over both additive white Gaussian noise (AWGN) and frequency-selective fading channels. The MOV estimator is reliable over frequency-selective fading channels even when the channel state information is not available at the estimator. An error floor is observed at high signal-to-noise ratio (SNR). To eliminate the error floor, the maximal ratio combining (MRC) technique is utilized.

To estimate the integer part of the frequency offset, a maximum-likelihood estimator (MLE) is investigated. After maximizing and simplifying the likelihood function, a simple MLE is derived based on only one OFDM block. To reduce the computational complexity and improve the bandwidth efficiency, a subset of pilot symbols is chosen from the OFDM block and a sub-optimum MLE (Sub MLE) is studied. Based on the hypothesis testing method, a threshold sub-optimum estimator (T-Sub MLE) is derived to further reduce the computational complexity. The PDF of the estimation metrics is examined and the mean and variance are obtained. The threshold in T-Sub MLE is set according to the mean value of the metrics and the theoretical error probability is obtained.

In MOV and MLE estimators, only the stationary frequency offset is considered. To track the variation of the frequency offset caused by the Doppler shift, a single-weight LMS-like estimator is proposed. The LMS-like estimator can be implemented by a simple adaptive process, which tracks the variation of the frequency offset based on pilot symbols. To improve bandwidth efficiency, a modified LMS-like (MLMS-like) estimator is proposed, which requires only one OFDM block as the pilot symbols. The MLMS-like estimator roughly estimates the frequency offset based on the pilot symbols and uses the estimated offset to compensate the consecutive OFDM block. The compensated block is exploited as the pilot symbols to track the variation of the frequency offset. To evaluate the accuracy of the proposed LMS-like estimator, the expression of the excess mean square error (EMSE) is obtained. The simulation results verify the theoretical results. The proposed LMS-like and MLMS-like estimators are effective over both AWGN and fading channels. Another superiority of the proposed estimators is that the LMS-like estimator can estimate the integer and fractional part of frequency offset simultaneously, which exhibits a wide estimation range and reduces the implementation complexity.

List of Abbreviations and Symbols

Abbreviations

A/D	analog-to-digital converter
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase-shift keying
BW	bandwidth
CDMA	code division multiple access
CFO	carrier frequency offset
CP	cyclic prefix
D/A	digital-to-analog converter
DAB	digital audio broadcasting
DFT	discrete Fourier transform
DOA	direction-of-arrival
DSP	digital signal processing
DVB	digital video broadcasting
EGC	equal gain combining

EM	expectation maximization
ETSI	European telecommunications standards institute
FFT	fast Fourier transform
ICI	inter-carrier interference
IDFT	inverse discrete Fourier transform
IFFT	inverse fast Fourier transform
ISI	inter-symbol interference
LMS	least mean square
MC-CDMA	multicarrier CDMA
MCM	multi-carrier modulation
MLE	maximum likelihood estimator
MOV	minimum output variance
MRC	maximal ratio combining
MSE	mean square error
OFDM	orthogonal frequency division multiplexing
PAPR	peak-to-average power ratio
PDF	probability density function
QAM	quadrature amplitude modulation
QPSK	quadrature phase-shift keying
RF	radio frequency
SAGE	space alternating generalized EM
SNR	signal to noise ratio

UWB	ultra wideband
WLAN	wireless local area network

Symbols

$E(\cdot)$	expectation operation
N	number of subcarriers
N_p	number of pilot symbols
$\mathbf{R}$	correlation matrix
T_s	symbol duration
$\text{var}(\cdot)$	variance operation
Δf	carrier frequency offset
f_c	carrier frequency
f_d	Doppler shift
v	normalized carrier frequency offset
P_{ICI}	ICI power
$ x $	absolute value of x
$\mathbf{G}$	channel gain matrix
$\text{tr}(\cdot)$	trace operation
ξ	mean square error
ζ	excess mean square error
P_e	error probability
$\hat{x}$	estimate of x

μ	step size
$\mathbf{W}$	IDFT matrix
$\mathbf{I}$	identity matrix
$(\cdot)^*$	transpose operation
$(\cdot)^H$	Hermitian operation
Φ	carrier frequency offset matrix
$\text{Im}(\cdot)$	imaginary part of the component
$\text{Re}(\cdot)$	real part of the component
T	threshold
$Q(\cdot)$	Q function

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Chapter 1

Introduction

1.1 Motivation

The introduction of high-bandwidth applications, such as streaming of video and high speed internet connection has created a huge demand for high data transmission to individual users, especially for wireless data transmissions. However, when the transmission rate increases, the symbol duration decreases. Over a wireless channel, the delay spread can be much longer than the symbol duration. Hence, inter-symbol interference (ISI) problem becomes severe. To avoid ISI, the complicated equalization is required at the receiver, which increases the implementation complexity.

To support high data rate transmission and avoid complicated equalization at the receiver, multi-carrier modulation (MCM) technique was proposed [1]. The principle of MCM is to split the high data rate stream into several lower data rate streams and transmit over parallel subcarriers. In this manner, the data rate is reduced at each subcarrier and ISI is not a serious problem to MCM schemes. Orthogonal frequency division multiplexing (OFDM) is a special form of MCM with

dense spaced subcarriers and overlapping spectra. Since the bandwidth resource is limited, the high bandwidth efficiency in OFDM schemes is an attractive advantage. After fast Fourier transform (FFT), inverse fast Fourier transform (IFFT) and modern digital signal processing (DSP) are introduced to the OFDM schemes, the OFDM schemes can easily be implemented and become popular. Because of the high bandwidth efficiency and robustness to multi-path fading, OFDM has been considered as a promising modulation for the next generation of wireless communications. Currently, OFDM has been adopted as the modulation scheme in many standards, including digital audio broadcasting (DAB) [2], terrestrial digital video broadcasting (DVB) [3], BRAN High-performance local area networks (HIPERLAN/2) [4], IEEE 802.11a [5], IEEE 802.16 [6], etc. Moreover, the basic OFDM system has been combined with various other systems to enhance the system performance. Multi-carrier CDMA is a good example of the combination of OFDM and CDMA [7–9]. Combined with multiple antennas at both the access point and mobile terminal to increase diversity gain and/or enhance system capacity on a time-varying multipath fading channel, a multiple-input multiple-output OFDM (MIMO-OFDM) system is proposed [10–12]. Combined OFDM modulation technique with a multiband-ing approach in ultra-wideband (UWB) communication system offers more flexible wideband applications [13–15].

Although OFDM has been found many applications in wireless communications, it has some drawbacks. Because of the overlapping spectra, the orthogonality among

subcarriers is exploited to identify the data transmitted over the overlapping subcarriers. Thus, the OFDM scheme is much more sensitive to carrier frequency offset (CFO) than the conventional schemes. A small mismatch of the carrier frequencies destroys the orthogonality among subcarriers. After the demodulation, the desired signal in subcarrier is attenuated and the interference comes from other subcarriers, resulting in inter-carrier interference (ICI). Thus, the transmitted data cannot be recovered properly and system performance degrades seriously. Thus, frequency synchronization is a very important task to OFDM schemes. This motivates us to find solutions to estimate and compensate CFO in OFDM systems and improve the system performance.

1.2 Objectives

In general, carrier frequency offset is mainly caused by two sources. The first one is the mismatch of the carrier frequencies between the local oscillators in OFDM transceivers. Because the OFDM scheme chooses very high radio frequency, the accuracy requirement of the oscillators is very high. Once the output frequency has an offset or drift, the mismatch incurs and carrier frequency offset is generated. The carrier frequency offset caused by oscillators is normally stationary. The other source, which causes the carrier frequency offset, is Doppler shift. In a wide outdoor environment, the relative speed between the transmitter and receiver is always changing from time to time. Thus, Doppler shift is another source of carrier frequency offset.

To avoid the system performance degradation, carrier frequency offset must be estimated and compensated. The main focus of this thesis is to provide solutions to accurately estimate the carrier frequency offset in OFDM systems and make the system degradation negligible after the CFO compensation. As the computational complexity is an important issue when implementing the algorithms in the practical systems, we also aim to provide lower complexity algorithms with affordable accuracy loss. If the lower complexity algorithms can meet the system requirement, they are attractive and can be adopted in practical systems.

1.3 Major Contribution of the Thesis

OFDM systems are more sensitive to carrier frequency offset than conventional single carrier systems. In this thesis, we focus on the algorithms to solve this problem. Three major algorithms are proposed to estimate the carrier frequency offset in this thesis.

The first algorithm is the minimum output variance (MOV) algorithm. In OFDM systems, carrier frequency offset attenuates the desired signal in each subcarrier and generates the inter-carrier interference (ICI). Observing one OFDM block, the whole signal power does not change. But more frequency offset causes more severe ICI. We investigate the magnitude of the output after the DFT demodulation. The probability density function (PDF) of the output magnitude is approximated as Gaussian

mixture. The expressions of the mean and variance of the magnitude output are obtained. It is observed that the mean and variance of the magnitude output are functions of the carrier frequency offset. Larger frequency offset causes smaller mean value and larger variance. In other words, the mean value reaches the maximum value and the variance reaches the minimum value when the frequency offset is zero. This feature is exploited to estimate the frequency offset. To evaluate the accuracy of the proposed MOV estimator, the performance analysis is conducted over AWGN channels. The proposed MOV estimator is robust to frequency-selective fading. The MOV estimator is still effective and accurate even when the fading channel state information is not available. But one drawback is that the mean square error (MSE) has an error floor due to the effect of fading. To mitigate the error floor, the maximal ratio combining (MRC) technique is used. With the channel gain based on the MRC criteria, the error floor is removed and the performance of the MOV estimator over frequency-selective fading channels is comparable to the performance over AWGN channels. The proposed MOV estimator is a blind estimator, which does not require pilot symbols or virtual carriers. Thus, the bandwidth efficiency is high. Because the MOV estimator can utilize the fast Fourier transform (FFT), the computational complexity is affordable. One drawback of the MOV estimator is that the estimation range is limited to $(-0.5, 0.5)$, which is a fine estimation of the frequency offset.

To achieve a coarse estimation, the second algorithm is proposed. The likelihood

function is simplified and maximized to derive the maximum-likelihood estimator (MLE). The MLE can be used to estimate the integer part of the frequency offset with the aid of pilot symbols. Compared with the previous coarse estimators, the MLE requires only one OFDM block as pilot symbols. To improve the bandwidth efficiency and reduce the computational complexity, a subset of the symbols among the subcarriers is chosen as the pilot symbols for the estimation of frequency offset. This is the sub-optimum MLE (Sub MLE). The mean and variance of the estimation metrics are obtained for the proposed estimators. A threshold sub-optimum estimator (T-Sub MLE) is subsequently proposed according to the hypothesis testing method. The T-Sub MLE can further reduce the computational complexity. The error probability of the proposed T-Sub MLE is obtained and validated.

In the first and second proposed estimators, the fractional and integer parts of the frequency offset are estimated individually. If the integer and fractional offsets are estimated simultaneously, the implementation complexity can be reduced. This motivates the proposal of the third algorithm in this thesis, called the adaptive least mean square like (LMS-like) algorithm. Following the LMS principle, a single-weight LMS-like estimator is proposed to estimate the integer and fractional offsets simultaneously. In the previously proposed MOV and MLE estimators, the carrier frequency offset is assumed to be a stationary parameter during the whole processing. This assumption is often adopted in the literature. But in a more practical environment, the frequency offset is time-varying due to the effect of Doppler

shift. The proposed adaptive LMS-like estimator can track the variation of the frequency offset caused by the Doppler shift. To improve the bandwidth efficiency, a modified LMS-like (MLMS-like) estimator is proposed. In MLMS-like estimator, only one OFDM block is used as the pilot symbols. After estimating the frequency offset based on the first pilot OFDM block, the frequency offset can be roughly estimated. And the estimated offset can be used to compensate the next OFDM block. The compensated block is utilized as the pilot symbols to track the variation of the frequency offset. In this manner, the variation of the frequency offset can be tracked block to block without the aid of more pilot symbols. The expression of the excess mean square error (EMSE) is obtained to assess the accuracy of the proposed LMS-like estimator and is validated by the simulation results. The adaptive LMS-like estimator can estimate the integer and fractional offsets simultaneously, which exhibits a large estimation range. Also, the estimation only requires simple adaptive process, resulting in low computational complexity.

1.4 Organization of the Thesis

The organization of the rest of the thesis is as follows. Chapter 2 provides an overview to OFDM systems. The history of the OFDM is first introduced and then followed by the OFDM fundamentals. The principle of OFDM and its scheme structure are exhibited. Based on the OFDM fundamentals, the advantages and disadvantages of OFDM are summarized. The major issue addressed in this thesis is the carrier frequency offset problem. Thus, the carrier frequency offset issue is

presented in greater detail. The sources of carrier frequency offset are explained and the effect of carrier frequency offset is illustrated. Also, the system degradation caused by carrier frequency offset is shown. Because carrier frequency offset is a serious problem in OFDM systems, many estimation algorithms have been proposed in the literature. At the end of Chapter 2, many existing carrier frequency offset estimation algorithms are classified and summarized.

In Chapter 3, a minimum output variance (MOV) estimator is proposed to estimate the fractional frequency offset. First, the OFDM system model is setup and then the probability density function (PDF) of the magnitude output is derived. The mean and variance of the magnitude output are obtained. Based on the minimum output variance feature, the minimum output variance estimator is proposed in this chapter. The performance analysis of the proposed MOV estimator is conducted over AWGN channels. The simulation results verify the analytical results and the accuracy of the MOV estimator is evaluated. The numerical results over frequency-selective fading channels are also provided in this chapter.

Chapter 4 presents a maximum likelihood estimator (MLE) to estimate the integer part of the frequency offset. After the simplification and maximization of the likelihood function, the MLE is derived. To reduce the computational complexity of the proposed MLE, a sub-optimum estimator (Sub MLE) is considered. A threshold estimator (T-Sub MLE) is proposed based on the hypothesis testing criteria. The

T-Sub MLE can further reduce the computational complexity. The mean and variance of the metrics are derived and validated. Based on the analytical mean and variance, the theoretical error probability of the T-Sub MLE is obtained.

The MOV estimator proposed in Chapter 3 can estimate the fractional offset only and the MLE estimators derived in Chapter 4 are limited to the integer offset only. Moreover, the frequency offset in both chapters is assumed to be a stationary parameter. In Chapter 5, we propose an adaptive LMS-like estimator to estimate the fractional and integer offsets simultaneously and we consider the time-varying part of the frequency offset caused by the Doppler shift. Based on the LMS criterion, a single-weight LMS-like estimator is proposed. The proposed LMS-like estimator also can track the variation of the frequency offset caused by the Doppler shift. To improve the bandwidth efficiency, a modified LMS-like (MLMS-like) is investigated. The accuracy of the proposed LMS-like estimator is analyzed and validated by numerical results.

Finally, the thesis is concluded in Chapter 6. The major contributions and conclusions are summarized in this chapter. Some suggestions are presented for the future research work.

Chapter 2

Overview of OFDM

Orthogonal frequency division multiplexing (OFDM) is a multi-carrier modulation (MCM) technique, which is proposed to support high data rate transmission. The demand of the high data rate transmission stimulates the development of OFDM in recent years. OFDM has been found many applications for high data rate transmission over frequency-selective fading channels. It has been considered as a promising modulation scheme for the next generation of wireless communication systems.

2.1 History of OFDM

The history of OFDM dates back to the mid 60's, when Chang published his paper on the synthesis of bandlimited signals for multichannel transmission [16]. The paper presented a principle for transmitting messages simultaneously through a linear bandlimited channel without interchannel interference and intersymbol interference (ISI). After the publication of Chang's paper, Saltzberg performed an analysis of the performance and drew an important conclusion: "The strategy of designing an efficient parallel system should concentrate more on reducing crosstalk between adjacent channels than on perfecting the individual channels themselves, since the

distortions due to crosstalk tend to dominate” [17]. This important conclusion was proven correct in the digital baseband processing a few years later. However, OFDM systems could not be realized efficiently since powerful semiconductor devices were not available at that time.

Weinstein and Ebert contributed significantly to OFDM by using the discrete Fourier transform (DFT) to perform baseband modulation and demodulation in 1971 [18]. Their work did not focus on “perfecting the individual channels”, but rather on introducing efficient processing to eliminate the banks of subcarrier oscillators. They used a guard space between the symbols and raised-cosine windowing in the time domain to combat the ISI and ICI. Although their system did not obtain perfect orthogonality among subcarriers over a dispersive channel, it was still a significant contribution to OFDM.

To solve the orthogonality problem, Peled and Ruiz introduced the cyclic prefix (CP) in 1980 [19]. They filled guard space with a cyclic extension of the OFDM symbol, instead of an empty guard space. This effectively simulates a channel performing cyclic convolution, which implies orthogonality over dispersive channels when the length of CP is longer than the impulse response of the channel. Many other papers were published and contributed to the early development of OFDM schemes [20–22].

Because of the excellent characteristics, OFDM is currently suggested and stan-

standardized for high-speed communications in Europe for digital audio broadcasting (DAB) [2] and terrestrial digital video broadcasting (DVB) [3]. Furthermore, OFDM is standardized for broadband wireless local area networks, e.g., European telecommunications standards institute (ETSI)-BRAN High-performance local area networks (HIPERLAN/2) [4], IEEE 802.11a [5], and for broadband wireless access, e.g., IEEE 802.16 [6]. OFDM plays a more and more important role in wireless communications.

2.2 Fundamentals of OFDM

2.2.1 OFDM Basic

The basic idea of OFDM is to split a high data rate stream into a number of lower data rate streams that are transmitted simultaneously over a number of parallel subcarriers [23, 24]. As the symbol duration is extended, the relative amount of dispersion in time caused by multipath delay spread is reduced. Then, intersymbol interference (ISI) is almost completely avoided by introducing a guard time between each OFDM block. In the guard time, the cyclic extension of the OFDM block is inserted to avoid inter-carrier interference (ICI). The OFDM signals are transmitted over narrowband subcarriers, which appear to be frequency-nonselective. Hence, OFDM systems are robust to frequency-selective fading. Also, the spectra of OFDM are overlapping and OFDM systems enjoy high bandwidth efficiency.

Figure 2.1: The OFDM transmitter model

Figure 2.2: The OFDM receiver model

The OFDM transmitter and receiver models are shown in Figs. 2.1 and 2.2, respectively. At the transmitter, the serial data stream is converted to parallel data streams by a serial-to-parallel converter. Then the OFDM signal is formulated by the N -point inverse discrete Fourier transform (IDFT) modulation and the k th output can be expressed as

$$d_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n e^{j2\pi kn/N} \quad (2.1)$$

Figure 2.3: The structure of cyclic prefix (CP)

where s_n is a quadrature amplitude modulation (QAM) or phase-shift-keying (PSK) symbol in the n th subcarrier and N is the number of subcarriers. A guard interval is inserted to avoid intersymbol interference. To maintain the orthogonality among subcarriers, a cyclic extension of the OFDM symbols is copied to be the guard interval. It is called cyclic prefix (CP) and the structure of CP is illustrated in Fig. 2.3. After the CP insertion, the signals go through the digital-to-analog (D/A) converter and are up-converted to radio frequency (RF).

The signals are transmitted over wireless channels. At the receiver, the received signal is down-converted to baseband and goes through the analog-to-digital (A/D) converter. The received signal can be expressed as

$$r_k = \sum_{l=1}^{L_p} h_{l,k-\tau} d_k + n_k \quad (2.2)$$

where $h_{l,k-\tau}$ denotes the channel response in l th path with delay τ , L_p is the number of multipath [25–27] and n_k denotes the additive white Gaussian noise (AWGN). After removing the CP, discrete Fourier transform (DFT) is performed to recover the signals

$$\begin{aligned} x_n &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_k e^{-j2\pi kn/N} \\ &= H_n s_n + N_n \end{aligned} \quad (2.3)$$

where H_n denotes the channel response in the frequency domain and N_n represents the noise after DFT demodulation. The IDFT modulation and DFT demodulation can be implemented by IFFT and FFT. With the high performance digital signal processing (DSP) chips, the modulation and demodulation can be realized efficiently.

2.2.2 Advantages of OFDM

High Bandwidth Efficiency

The OFDM signals are modulated to N orthogonal subcarriers by IFFT and the resultant signals can be recovered by FFT demodulation at the receiver. Because of the orthogonality among subcarriers, the overlapped spectra can be identified. The spectrum of individual subcarriers of OFDM systems is shown in Fig. 2.4. It is observed that OFDM signals are modulated to overlapped narrowband subcarriers, which makes bandwidth efficiency very high. As the bandwidth resources are limited, high bandwidth efficiency in OFDM systems is an attractive advantage.

Figure 2.4: Spectrum of individual subcarriers in OFDM systems

Immunity to Multi-path

For high data rate wireless transmission, multi-path is a serious problem. Because the transmitted symbol duration is short, the multi-path delay is longer than the symbol duration and intersymbol interference (ISI) occurs. To mitigate the ISI, complicated receivers are required, which increase system complexity and introduce processing delay. In an OFDM system, a high data rate stream is split into a number of lower data rate streams, which extends symbol duration and avoids ISI. Also, guard interval, which is chosen to be longer than the channel delay spread, is inserted between OFDM blocks. To maintain the orthogonality, cyclic extension of OFDM symbols is used to fill the guard space. Thus, the ISI problem can almost

be eliminated completely in OFDM systems.

Robustness to Frequency-Selective Fading

As mentioned previously, the high data rate stream is split into a number of lower data rate streams and transmitted over parallel subcarriers. In each subcarrier, the signal is narrowband and the bandwidth of the signal is less than the coherence bandwidth of the channel. The signal thus undergoes flat fading in each subcarrier. This feature avoids the complicated equalization in the frequency domain and the OFDM signals can be properly recovered at the receiver.

However, frequency selectivity still affects the system performance if the subcarriers are located in a region with deep fading. Thus, coded OFDM has been investigated [23, 28–30]. Channel coding and frequency interleaving have been introduced in OFDM to protect the system from transmission errors which frequently occurs in the absence of coding. In a block coding and hard-decision decoding approach, the code block length is equal to the DFT block length. With such a coding and decoding scheme, the OFDM systems are efficient if the code can correct the errors per block with a high probability. Another approach consists of convolutional coding with frequency-domain interleaving and soft-decision. The interleaver uniformly distributes the low-SNR samples over the channel bandwidth and the convolutional code has a large hamming distance. This approach improves the OFDM system performance.

2.2.3 Disadvantages of OFDM

Sensitivity to Phase Noise

The issue of phase noise in OFDM systems has been investigated in many studies [31, 32]. In [31], the power density spectrum of an oscillator signal with phase noise is modeled by a Lorentzian spectrum, which is equal to the squared magnitude of a first order lowpass filter transfer function. Phase noise basically has two effects. First, it introduces a random phase variation that is common to all subcarriers. Second, phase noise introduces inter-carrier interference (ICI) which degrades the performance seriously. The degradation D_{phase} caused by the phase noise is theoretically analyzed in [31, 33] and given by

$$D_{phase} \approx \frac{11}{6 \ln 10} 4\pi\beta T_s \frac{E_s}{N_0}. \quad (2.4)$$

Here, β is the -3 dB one-sided bandwidth of the power density spectrum of the carrier and E_s/N_0 is signal-to-noise ratio (SNR). The phase noise degradation is proportional to βT_s , which is the ratio of the linewidth and subcarrier spacing $1/T_s$. Figure 2.5 shows the SNR degradation in dB as a function of the normalized linewidth βT_s . The curves are shown for three different E_s/N_0 values, corresponding to the required values to obtain a bit error rate (BER) of 10^{-6} for uncoded QPSK, 16-QAM, and 64-QAM, respectively [33]. From the figure, it is observed that for a negligible SNR degradation of less than 0.1 dB, the -3 dB phase noise bandwidth has to be about 0.1 to 0.01 percent of the subcarrier spacing, depending on the modulation.

Figure 2.5: SNR degradation in dB versus the -3 dB bandwidth of the phase noise spectrum for (a) 64-QAM ($E_s/N_0 = 19$ dB), (b) 16-QAM ($E_s/N_0 = 14.5$ dB), (c) QPSK ($E_s/N_0 = 10.5$ dB)

Sensitivity to Frequency Offset

In an OFDM scheme, all subcarriers are orthogonal if they all have a different integer number of cycles within the FFT interval. If there is a frequency offset, then the number of cycles in the FFT interval is no longer an integer, and hence inter-carrier interference (ICI) occurs after the FFT. Then the FFT output for each subcarrier contains interfering terms from other subcarriers. The degradation is illustrated as follows [31]

$$D_{freq} \approx \frac{10}{3 \ln 10} (\pi \Delta f_s T_s)^2 \frac{E_s}{N_0} \quad (2.5)$$

Figure 2.6: SNR degradation in dB versus the normalized frequency offset for (a) 64-QAM ($E_s/N_0 = 19$ dB), (b) 16-QAM ($E_s/N_0 = 14.5$ dB), (c) QPSK ($E_s/N_0 = 10.5$ dB)

where Δf_s is the frequency spacing between subcarriers. This degradation is depicted in Fig. 2.6 as a function of the normalized frequency offset and for three different E_s/N_0 values [33]. Note that for a negligible degradation of about 0.1 dB, the maximum tolerable frequency offset is less than 1% of the subcarrier spacing.

The initial frequency error of a low-cost oscillator will not meet the requirement, so frequency offset occurs frequently. Thus frequency synchronization is needed before the FFT demodulation. This thesis concentrates on the frequency offset problem and the details will be presented in the subsequent chapters.

High Peak-to-Average Power Ratio (PAPR) Problem

An OFDM signal consists of a number of independently modulated subcarriers. When the signals are added up coherently, they produce a peak power [33,34]. When N signals are added with the same phase, they generate a peak power that is N times the average power. A large peak-to-average power ratio (PAPR) brings along many problems. It increases the complexity of the analog-to-digital and digital-to-analog converters. Also, it reduces the efficiency of the radio frequency (RF) power amplifier. To reduce the PAPR, many techniques have been proposed and basically can be divided into three categories. The first one is the signal distortion techniques, which reduce the peak amplitudes simply by nonlinearly distorting the OFDM signal at or around the peaks. Examples of distortion techniques are clipping, peak windowing [35] and peak cancelation [36]. The second category is coding techniques that use a special forward-error correcting code set that excludes OFDM symbols with a large PAPR [37]. The third techniques are scrambling techniques, which are based on scrambling each OFDM symbol with different scrambling sequences and selecting the sequence that gives the smallest PAPR [38,39].

2.3 Carrier Frequency Offset Issues

2.3.1 Sources of Carrier Frequency Offset

Multi-carrier modulation (MCM) schemes are more sensitive to carrier frequency offset than conventional single carrier schemes. There are two major sources causing

carrier frequency offset. The first source is the mismatch of the carrier frequencies between local oscillators in transmitter and receiver. OFDM applications usually use high radio frequency. IEEE 802.11 made the first WLAN standard for the 2.4-GHz industrial, scientific and medical band (ISM) [40]. The second IEEE 802.11 standard extension uses 5-GHz band [41]. In Europe, 5-GHz and 17-GHz bands are reserved for HIPERLAN, and Japan has allocated spectrum from 10- to 16-GHz to mobile broadband systems (MBS). For such a high radio frequency, the accuracy requirement of local oscillators is very high. A small frequency drift generated at the local oscillators or a small mismatch of carrier frequencies between the transmitter and receiver causes carrier frequency offset. Normally, the carrier frequency offset caused by the local oscillators is stable and can be considered as a stationary parameter.

Another source causing carrier frequency offset is Doppler shift. Because the relative speed between the transmitter and receiver changes from time to time in an outdoor wide area wireless environment, Doppler shift incurs and is given by [42]

$$\begin{aligned} f_d &= \frac{v_r}{\lambda} \cos \theta \\ &= \frac{v_r f_c}{c} \cos \theta \end{aligned} \tag{2.6}$$

where v_r is the relative velocity between the mobiles and base stations, λ is the wavelength, c denotes the light velocity, f_c is the carrier frequency and θ is the spatial angle between the direction of motion of the mobile and direction of arrival

of the wave. It can be seen from (2.6) that if the mobile is moving toward the direction of arrival of the wave, the Doppler shift is positive, and if the mobile is moving away from the direction of arrival of the wave, the Doppler shift is negative. The Doppler shift f_d is proportional to carrier frequency f_c . For OFDM applications, carrier frequency f_c is usually very high. Hence, the frequency offset caused by the Doppler shift is not negligible, especially for multi-carrier modulation schemes. Also, Doppler shift f_d is time varying due to the variation of the relative speed. This time varying feature makes the carrier frequency offset estimation more difficult to handle.

2.3.2 Effect of Carrier Frequency Offset

Carrier frequency offset is a serious drawback in OFDM systems. Carrier frequency offset destroys the orthogonality among subcarriers, results in inter-carrier interference and degrades system performance. If the carrier frequency offset cannot be corrected and compensated before the DFT (or FFT) demodulation, the transmitted data cannot be recovered properly at the receiver. Figure 2.7 shows the constellation of QPSK with the effect of CFO at an OFDM receiver. To highlight the effect of CFO, a high SNR of 30 dB is chosen. It is clear that the constellation becomes more blurred as CFO increases. When there is no carrier frequency offset, the constellation is clear after the demodulation. When there are 10% and 20% frequency offset, the constellations have a rotation and become blurred due to the ICI caused by the carrier frequency offset. When the frequency offset increases to 50%, the constellation is too blurred to be recovered.

Figure 2.7: Constellation of QPSK with the effect of CFO after demodulation in an OFDM system, $E_b/N_0 = 30$ dB

Figure 2.8 illustrates the effect of the carrier frequency offset in individual subcarriers. From the figure, it is seen that the desired signal in subcarrier i is attenuated when there is a frequency offset Δf . And the interferences from other subcarriers $i - 1$ and $i + 1$ affect the desired signal at subcarrier i . With the increase of frequency offset Δf , the desired signal at subcarrier i is attenuated more severely and the inter-carrier interference from other subcarriers is larger. Thus, the signal to interference ratio (SIR) is decreased due to the frequency offset, resulting in the system performance degradation.

Figure 2.8: Effect of carrier frequency offset: Attenuation and inter-carrier interference

In [31], the degradation caused by the carrier frequency offset is approximated by (2.5). An upper bound of the degradation is obtained as [43]

$$D_{\text{bound}} \leq 10 \log_{10} \left[\frac{1 + 0.5947(E_s/N_0) \sin(\pi \Delta f)^2}{\text{sinc}^2(\Delta f)} \right] \quad (2.7)$$

where

$$\text{sinc}(x) = \frac{\sin \pi x}{\pi x} \quad (2.8)$$

Figure 2.9: Upper bound of degradation caused by carrier frequency offset

and the factor 0.5947 is found from a lower bound of the summation of all interfering subcarriers. The upper bound of the degradation caused by the carrier frequency offset is plotted in Fig. 2.9. From (2.5) and (2.7), it is observed that the degradation increases as the frequency offset increases. In order to avoid severe degradation, the frequency offset should be less than 2% of subcarrier spacing [44]. In practice, this requirement is too strict to meet. Thus, carrier frequency offset estimation is needed to avoid system degradation.

2.3.3 Estimation of Carrier Frequency Offset

Carrier frequency offset causes a number of impairments, including the attenuation and phase rotation of each of the subcarriers and inter-carrier interference (ICI). Carrier frequency offset can be divided into an integer part, which is a multiple of the subcarrier spacing, and a fractional part, which is less than one half of the subcarrier spacing. Integer frequency offset causes the cyclic shift of the output after the demodulation. Fractional frequency offset attenuates the desired signal on each subcarrier and contains the interfering terms from other subcarrier, which is inter-carrier interference. With the carrier frequency offset, the system performance degrades severely. Thus, carrier frequency offset must be estimated and compensated before the DFT modulation.

In the literature, many estimation techniques have been proposed to estimate and correct the CFO before the demodulation. Based on the estimation range of the CFO, these estimation algorithms can be classified as fine estimation, which estimates the fractional part of the frequency offset, and coarse estimation, which estimates the integer part of the frequency offset. Based on the resources used for the estimation, the estimation algorithms can be classified as blind estimation, which exploits the characteristics of the signals to estimate the CFO without the aid of the pilot symbols, and pilot-based estimation, which utilizes prior knowledge of the pilot symbols to estimate the CFO. The classification of the carrier frequency offset estimation algorithms is illustrated in Fig. 2.10, where $|v|$ denotes the absolute value

Figure 2.10: Classification of carrier frequency offset estimation algorithms

of the normalized carrier frequency offset. Generally, blind estimation algorithms save the bandwidth but increase the computational complexity. Also, the accuracy of the blind estimation algorithms is in general lower than that of pilot-based estimation algorithms. For pilot-based estimation algorithms, pilot symbols are required, which add to the system overheads. The computational complexity of pilot-based estimations is normally lower than that of the blind estimation algorithms and the estimation accuracy is better. Thus, pilot-based algorithms are more suitable for the applications requiring fast and reliable synchronization. When choosing the estimation algorithms to tackle carrier frequency offset problem, the performance requirements and computational and implementation complexity should be jointly considered to optimize the overall system design.

Blind Carrier Frequency Offset Estimation Algorithms

In blind estimation, it always exploits the prior characteristics of the received signals. For instance, the redundancy between the OFDM block and cyclic prefix can be used to estimate the frequency offset and the orthogonality between the virtual carriers and information bearing carriers can be applied for the estimation of the frequency offset, etc. In general, blind estimation algorithms do not require extra overheads but the computational complexity increases and accuracy is affected.

Many blind estimation techniques have been proposed in the literature. Van de Beek *et al.* developed a maximum-likelihood (ML) estimator by exploiting the redundancy in the cyclic prefix (CP) of the OFDM symbol [45]. This method, however, suffers performance degradation when the delay spread of channel is comparable to the length of CP and generates the error floor effect in the presence of frequency-selective fading channels. A new likelihood function was derived to estimate carrier frequency offset by exploiting the properties of the cyclic prefix in [46]. This method can jointly estimate the timing errors. Luise *et al.* exploited the time-frequency-domain exchange inherent to the modulation scheme and proposed a blind algorithm for CFO recovery [47].

Tureli *et al.* utilized the shift invariant feature of the signal structure and extended ESPRIT algorithm [48] to estimate CFO [49]. Liu *et al.* exploited the inherent orthogonality between information bearing carriers and virtual carriers, and proposed

an algorithm to estimate the CFO [50, 51]. In this method, the inherent orthogonality is utilized, which is similar to the idea of MUSIC algorithm [52]. Because of the presence of the virtual carriers for the proposed algorithms in [49] and [51], the bandwidth efficiency is also reduced. In [53], a maximum likelihood estimator was proposed for carrier frequency offset in the presence of virtual carriers. The resulting algorithm was found to be identical to the algorithm in [51]. The equivalence between these two algorithms was explained using an intuitive projection argument.

A blind estimator was proposed in [54], which exploited the intrinsic phase shift between neighboring samples due to the carrier frequency offset. Perfect recovery of the CFO can be achieved in the noiseless case. In the presence of channel noise, the oversampling method leads to a simple root-finding problem that allows efficient computation. Relying on the maximization of the empirical cyclocorrelations, Ciblat *et al.* introduced an estimator for carrier frequency offset for noncircular constellation-based transmissions [55].

Cyclostationarity was exploited to jointly estimate frequency offset and symbol timing of a linearly modulated waveform transmitted through a frequency-flat fading channel for single-carrier systems [56]. This method exploited the received signal's second-order cyclostationarity. Bolcskei extended the cyclostationary approach to estimate the timing and carrier frequency offset in OFDM systems [57]. It relies on second-order statistics only.

The cyclic extension of OFDM blocks was used to synchronize the frame position and the carrier frequency at the same time [58]. In order to reduce the complexity, only the sign bits of the in-phase and the quadrature components of the received OFDM signal are used to estimate the frequency offset. The identifiability and performance of non-data-aided schemes for single-user OFDM with null subcarriers are investigated and used to estimate carrier frequency offset in [59]. A dual loop tracking structure which takes full advantage of the rotation property of OFDM data subcarriers in the presence of CFO was designed to estimate carrier frequency offset [60]. Relying on a kurtosis-type criterion, a low-complexity blind carrier frequency offset estimator was introduced in [61]. This blind CFO estimator depends on the channel's frequency selectivity and the input distribution. This approach can also be applied to blind CFO estimation in multi-input multi-output (MIMO) and multiuser OFDM systems.

Pilot-Based Carrier Frequency Offset Estimation Algorithms

Based on the bearing information in the pilot symbols, many pilot-based algorithms have been proposed to estimate carrier frequency offset. Moose proposed a scheme to estimate carrier frequency offset by repeating a data symbol and comparing the phase of each of the subcarriers between successive symbols [43]. Suppose that the carrier frequency offset is stationary between the two repeated symbols and the estimation can be accomplished by comparing the phases. This method is simple and the accuracy is high and it has been adopted in IEEE 802.11a standard [5].

However, the accuracy of the estimation is seriously affected if the carrier frequency offset varies between the two repeated symbols. Nogami and Nagashima presented algorithms to find the carrier frequency offset and sampling rate offset [62]. In this method, a null symbol is used to detect the beginning of the data frame and the carrier frequency offset is found in the frequency domain after applying a Hanning window and taking the FFT.

Classen introduced a method to find both the symbol timing and carrier frequency offset [63]. However, the computational complexity is very high because it uses a trial-and-error method, where the carrier frequency is incremented in small steps over the entire acquisition range until the correct carrier frequency is found. It is impractical to do the exhaustive search and go through a large amount of computation at each possible carrier frequency offset. To reduce the computational complexity and extend the range for the acquisition, Schmidl and Cox modified the method in [63] and proposed an estimation algorithm for timing offset and carrier frequency offset estimation by using one unique symbol which has a repetition within half a symbol period [64]. An improved method was proposed to extend the estimation range with only one training symbol, at the price of increasing computational complexity [65]. In [66], one training sequence is utilized to estimate carrier frequency offset but the complexity is not increased. The training sequence has a differential coding structure in the frequency domain.

In [67], two consecutive OFDM blocks were observed. After simplifying and maximizing the likelihood function, the estimator was proposed based on maximum likelihood criterion. In [68], Choi *et al.* developed an ML estimator by assuming that the OFDM signal is complex Gaussian distributed, which is asymptotically true for circularly modulated OFDM symbols [69]. The proposed algorithm, however, requires perfect knowledge of the second-order statistics about the channel noise and the Rayleigh fading channel. Based on minimizing the Cramer-Rao lower bound (CRB) for frequency offset estimation with constraints on the peak and the total training-signal energies, and the training block length, the optimal training-signal structure was developed in [70]. The proof for the optimality and uniqueness of the training-signal structure was also presented. In order to handle a larger frequency offset, modified training-signal structures were discussed in [70].

A data-aided feedforward algorithm was proposed for carrier frequency offset estimation in M -ary phase-shift keying (PSK) transmissions over frequency-flat Rayleigh fading channels [71]. Its accuracy is very good but the estimation range may be limited under certain operating conditions. Also, its application requires knowledge of the Doppler bandwidth. To overcome these drawbacks, an extended method was proposed in [72] without sacrificing estimation accuracy and a simple technique was indicated to measure the Doppler bandwidth. This allows the algorithm to operate in an adaptive manner in a time-varying environment.

A robust symbol timing and carrier frequency synchronization method for OFDM systems was proposed in [73]. This method uses one specifically designed training symbol that has a steep rolloff timing metric trajectory. Three methods were presented based on the use of known training sequence for burst transmissions over frequency-selective channels [74]. An estimator was introduced to estimate the I/Q mismatch, dc offset, carrier frequency offset, and channel for receivers in frequency selective channels [75]. It was derived based on the maximum-likelihood (ML) criterion.

With known training signals and channel statistics, a carrier frequency offset estimator was presented in [76] based upon the ML criterion. This approach can be considered as an extension of [77]. In [78], a data-aided frequency offset estimator was developed based on least squares (LS) error criterion without the need for channel information. In [79], one training OFDM block with null subcarriers was used for CFO estimation. The null subcarriers are inserted in all odd subcarriers, which are utilized to estimate fractional offset and the almost-perfect autocorrelation sequences are allocated to the even null subcarriers, which are used to estimate the integer offset.

Chapter 3

Minimum Output Variance (MOV) Estimator

As introduced in Chapter 2, OFDM systems are much more sensitive to carrier frequency offset (CFO) than the conventional single-carrier systems. CFO destroys the orthogonality among subcarriers, resulting in intercarrier interference (ICI) and degrading bit error rate (BER) performance of OFDM systems. Thus, CFO must be accurately estimated and properly compensated before the FFT demodulation to avoid the degradation and guarantee system performance. In the subsequent chapters, we propose several algorithms to estimate the CFO in OFDM systems and evaluate the performance of the proposed algorithms.

3.1 Introduction

As explained in Chapter 2, CFO can be divided into an integer part, which is a multiple of the subcarrier spacing, and a fractional part, which is less than one half of the subcarrier spacing. The fractional offset attenuates the desired signal in subcarrier and generates ICI. Because of the existence of frequency offset, the

system performance degrades severely. In this chapter, a minimum output variance (MOV) estimator is proposed to estimate the fractional part of the frequency offset and we assume that the integer offset has been estimated and corrected by existing algorithms in the literature. In general, carrier frequency offset includes the stationary part, which is caused by the mismatch of carrier frequencies between local oscillators in the transceivers, and the time varying part of frequency offset, which is caused by the Doppler shift. In this chapter, we only consider the stationary part of the frequency offset and assume that CFO is a constant parameter during the whole processing. The time varying part of the frequency offset will be considered in Chapter 5.

The rest of this chapter is organized as follows. In Section 3.2, the OFDM system model is briefly introduced. In Section 3.3, the MOV estimator is described and its performance is analyzed over additive white Gaussian noise (AWGN) channels. The T&H estimator [51], proposed by Tureli and Hui, is also introduced for comparison. Numerical results are presented in both AWGN and frequency-selective fading channels in Section 3.4. Finally, conclusion is drawn in Section 3.5.

3.2 OFDM System Model

OFDM is a form of multicarrier modulation and consists of a number of narrowband orthogonal subcarriers transmitted in a synchronous manner. The m th OFDM data

block to be transmitted can be defined as

$$\mathbf{s}(m) = [s_0(m), s_1(m), \dots, s_{N-1}(m)]^T \quad (3.1)$$

where N is the number of subcarriers and $[\cdot]^T$ denotes the transpose. The OFDM signals can be formulated by inverse discrete Fourier transform (IDFT). Using the matrix representation, the m th block of the modulated signal is

$$\begin{aligned} \mathbf{x}(m) &= [x_0(m), x_1(m), \dots, x_{N-1}(m)]^T \\ &= \mathbf{W}\mathbf{s}(m) \end{aligned} \quad (3.2)$$

where $\mathbf{W}$ is the IDFT matrix given by

$$\mathbf{W} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{j\omega} & \dots & e^{j(N-1)\omega} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j(N-1)\omega} & \dots & e^{j(N-1) \times (N-1)\omega} \end{bmatrix}_{N \times N} \quad (3.3)$$

with $\omega = 2\pi/N$. After the IDFT modulation, a cyclic prefix (CP) is inserted and its length is assumed to be longer than the maximum delay spread of the channel to avoid intersymbol interference (ISI). The resultant baseband signal is up-converted to the radio frequency (RF) and transmitted over the multipath fading channel. At the receiver, the signal is down-converted and demodulated using discrete Fourier transform (DFT) to recover the desired signal. In the absence of CFO, the received

signal is given by

$$\mathbf{y}(m) = \mathbf{W}\mathbf{H}\mathbf{s}(m) + \mathbf{n}(m) \quad (3.4)$$

where

$$\mathbf{H} = \text{diag}(H_0(m), H_1(m), \dots, H_{N-1}(m)) \quad (3.5)$$

represents the channel response in the frequency domain and $\mathbf{n}(m)$ is the AWGN noise term. After the down-conversion, the carrier frequency offset Δf is incurred because of the mismatch of carrier frequencies between the transmitter and receiver. When the CP is removed, the received signal including the CFO can be expressed as

$$\mathbf{y}(m) = \mathbf{\Phi}\mathbf{W}\mathbf{H}\mathbf{s}(m) + \mathbf{n}(m) \quad (3.6)$$

where

$$\mathbf{\Phi} = \text{diag}(1, e^{j\phi_0}, \dots, e^{j(N-1)\phi_0}) \quad (3.7)$$

denotes the CFO matrix,

$$\phi_0 = 2\pi\Delta f T_s / N \quad (3.8)$$

and T_s is the OFDM block duration. Due to the existence of the CFO matrix $\mathbf{\Phi}$, the orthogonality among subcarriers is destroyed and the desired signals cannot be properly recovered after the DFT demodulation.

3.3 Estimation of Carrier Frequency Offset

Due to spectra overlapping, OFDM systems are sensitive to CFO. The CFO destroys the orthogonality among subcarriers, resulting in inter-carrier interference (ICI) and causes system performance degradation. To improve the system performance, the CFO must be estimated and compensated before the DFT demodulation. In this section, we propose a minimum output variance (MOV) estimator for carrier frequency offset estimation.

3.3.1 Minimum Output Variance (MOV) Estimator

In the presence of the CFO, the received signal, after the DFT demodulation, is given by

$$\begin{aligned}\mathbf{d}(m) &= \mathbf{W}^H \mathbf{y}(m) \\ &= \mathbf{W}^H \mathbf{\Phi} \mathbf{W} \mathbf{H} \mathbf{s}(m) + \mathbf{n}'(m)\end{aligned}\tag{3.9}$$

where

$$\mathbf{d}(m) = [d_0(m), d_1(m), \dots, d_{N-1}(m)]^T \tag{3.10}$$

is the recovered signal vector, $\mathbf{W}^H$ represents the DFT demodulation matrix and $\mathbf{n}'(m)$ is the noise term after the DFT operation. Because of the presence of the CFO, $\mathbf{W}^H \mathbf{\Phi} \mathbf{W}$ is no longer an identity matrix and the desired signal $\mathbf{s}(m)$ cannot be recovered properly. For the sake of simplicity, the block number m will not be included in the subsequent derivation. The recovered signal on subcarrier k is given

by

$$d_k = I_0 s_k + \sum_{l=0, l \neq k}^{N-1} I_{l-k} s_l \quad (3.11)$$

where

$$I_n = \frac{\sin(\pi \Delta f T_s)}{N \sin\left(\frac{\pi}{N}(\Delta f T_s + n)\right)} \cdot \exp\left(j \frac{\pi}{N}((N-1)\Delta f T_s - n)\right). \quad (3.12)$$

In (3.11), I_0 represents the attenuation and phase rotation of the desired signal and I_{l-k} represents the ICI coefficient from other subcarriers. From (3.11) and (3.12), it can be seen that the coefficient I_0 of s_k is independent of k . It implies that the desired signals on all subcarriers experience the same degree of attenuation and phase rotation. Note that the ICI term is the summation of many independent random variables and can be conveniently approximated as a complex Gaussian random variable with zero mean [26, 80]. Thus, the output of each subcarrier can be considered as a dominant signal embedded in AWGN. To illustrate the system performance, the BPSK modulation is employed in the OFDM scheme.

The power of the ICI is defined as

$$P_{ICI} \triangleq \mathbb{E} \left| \sum_{l=0, l \neq k}^{N-1} I_{l-k} s_l \right|^2 \quad (3.13)$$

and P_{ICI} can be obtained as [81, 82]

$$\begin{aligned} P_{ICI} &= \int_{-1}^1 (1 - |x|)(1 - e^{j2\pi\epsilon x}) dx \\ &= 1 - \text{sinc}^2(\epsilon) \end{aligned} \quad (3.14)$$

Figure 3.1: The theoretical and the simulation results of the ICI power

where

$$\epsilon = \Delta f T_s \in (-0.5, 0.5) \quad (3.15)$$

is the CFO normalized to the subcarrier spacing and

$$\text{sinc}(x) = \sin(\pi x)/(\pi x). \quad (3.16)$$

The theoretical and simulation results of the ICI power are presented in Fig. 3.1. It is observed that the theoretical curve based on (3.14) matches the simulation points well.

Since the ICI can be approximated as a Gaussian random variable with zero mean, its variance is equal to

$$\sigma^2 = P_{ICI} = 1 - \text{sinc}^2(\epsilon). \quad (3.17)$$

From (3.11) and (3.12), it is clear that the dominant signal on the k th subcarrier is $I_0 s_k$. Hence, the mean of the magnitude of the dominant signal on the k th subcarrier is

$$\begin{aligned} m &= |I_0 s_k| \\ &= \left| \frac{\sin \pi \epsilon}{\pi \epsilon} e^{j\pi \epsilon s_k} \right| \\ &= \text{sinc}(\epsilon). \end{aligned} \quad (3.18)$$

Based on the analysis above, the probability density function (PDF) of the output $|d_k|$ can be approximated as Gaussian mixture given by

$$p_{|d_k|}(x) = \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} + \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x+m)^2}{2\sigma^2}} \right] u(x) \quad (3.19)$$

where $u(x)$ is the unit step function. The approximated PDF of the output $|d_k|$ is validated by histogram of the PDF in Figs. 3.2, 3.3 and 3.4, where ϵ is equal to 0.1, 0.2 and 0.3, respectively. It is observed that the approximated PDF matches the histogram well, which supports that the approximation of the PDF is accurate.

Let us consider the expectation and variance of the output $|d_k|$. The expectation of

Figure 3.2: The approximated PDF of the output $|d_k|$ and the histogram of the PDF with $\epsilon = 0.1$

$|d_k|$ is

$$\begin{aligned}
 E(|d_k|) &= \int_{-\infty}^{\infty} x p_{|d_k|}(x) dx \\
 &= \sqrt{\frac{2(1 - \text{sinc}^2(\epsilon))}{\pi}} \exp\left\{-\frac{\text{sinc}^2(\epsilon)}{2(1 - \text{sinc}^2(\epsilon))}\right\} + \\
 &\quad \text{sinc}(\epsilon) - 2\text{sinc}(\epsilon) Q\left(\frac{\text{sinc}(\epsilon)}{\sqrt{1 - \text{sinc}^2(\epsilon)}}\right)
 \end{aligned} \tag{3.20}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt. \tag{3.21}$$

Figure 3.3: The approximated PDF of the output $|d_k|$ and the histogram of the PDF with $\epsilon = 0.2$

The second moment of $|d_k|$ is

$$\begin{aligned}
 E(|d_k|^2) &= \int_{-\infty}^{\infty} x^2 p_{|d_k|}(x) dx \\
 &= m^2 + \sigma^2 - (m^2 + \sigma^2 - 1) Q\left(\frac{m}{\sigma}\right) \\
 &= 1
 \end{aligned} \tag{3.22}$$

Based on $E(|d_k|)$ obtained in (3.20) and $E(|d_k|^2)$ obtained in (3.22), the variance of $|d_k|$ can be expressed as

$$\text{var}(|d_k|) = E(|d_k|^2) - E^2(|d_k|). \tag{3.23}$$

Figure 3.4: The approximated PDF of the output $|d_k|$ and the histogram of the PDF with $\epsilon = 0.3$

From (3.20), (3.22) and (3.23), it is observed that the expectation and variance of $|d_k|$ are functions of ϵ . Figures 3.5 (a), (b), (c) show the expectation of $|d_k|$, the variance of $|d_k|$ and the second moment of $|d_k|$ versus the normalized CFO, respectively. From these figures, it is observed that the simulation results match the theoretical results well. The figures also show that the variance of $|d_k|$ monotonically increases and the expectation of $|d_k|$ monotonically decreases when the normalized CFO $|\epsilon|$ increases. In other words, the minimum variance and the maximum mean are obtained when $\epsilon = 0$. Intuitively, this property can be explained as follows. The CFO

Figure 3.5: (a) Expectation of $|d_k|$ versus ϵ , (b) Variance of $|d_k|$ versus ϵ , (c) Second moment of $|d_k|$ versus ϵ

does not change the total signal power in one received symbol [83], but the CFO causes the uncertainty of the output. More frequency offset causes more uncertainties and thus, the variance of the output $|d_k|$ increases. This feature is exploited for the estimation of CFO in OFDM systems.

Based on this property, we propose a blind CFO estimator with minimum output variance (MOV) for OFDM systems. When N is sufficiently large, the expectation of $|d_k|$ is approximately equal to the average value of $|d_k|$ among N subcarriers. From the previous analysis, the average value of $|d_k|$ attains a maximum value if the

CFO is properly compensated. Defining

$$\mathbf{\Phi}_e = \text{diag}(1, e^{j\phi_e}, \dots, e^{j(N-1)\phi_e}), \quad (3.24)$$

we search for ϕ_e in the range of $(-\pi/N, \pi/N)$ such that

$$\Lambda(\phi_e) \triangleq \frac{1}{N} \sum_{i=1}^N |\mathbf{W}_i^H \mathbf{\Phi}_e^H \mathbf{\Phi} \mathbf{W} \mathbf{H} \mathbf{s}(m)| \quad (3.25)$$

achieves its maximum value for $\mathbf{\Phi}_e = \mathbf{\Phi}$, where $\mathbf{W}_i$ is the i th column of the matrix $\mathbf{W}$. If thermal noise is present, the accuracy of ϕ_e may be improved by searching for the maximal value in (3.25) with more blocks. Hence, the proposed MOV estimator is to search for one ϕ_e to maximize $\Lambda(\phi_e)$, i.e.,

$$\hat{\phi}_e = \arg \max_{\phi_e} \frac{1}{MN} \sum_{m=1}^M \sum_{i=1}^N |\mathbf{W}_i^H \mathbf{\Phi}_e^H \mathbf{y}(m)| \quad (3.26)$$

where $\mathbf{y}(m)$ is defined in (3.6) and M is the number of blocks used by the MOV estimator. Based on (3.26), the DFT operation $\mathbf{W}_i^H$ in the MOV estimator can be implemented efficiently by means of the fast Fourier transform (FFT) algorithm. Thus, the computational complexity can be further reduced. Although the above analysis is based on BPSK modulation, it is also applicable to higher-order modulation schemes such as QPSK, 8-QAM, 16-QAM and 64-QAM, etc. In Figs. 3.6, 3.7 and 3.8, the expectation, variance and second moment of $|d_k|$ for QPSK, 16QAM and 64QAM are presented, respectively. From the simulation results, it is observed

Figure 3.6: Expectation, variance and second moment of $|d_k|$ versus ϵ with QPSK modulation

that as long as the output variance is minimized, the estimator for carrier frequency offset is appropriate, even when higher-order modulation schemes are used.

3.3.2 Performance Analysis of the MOV Estimator

For the MOV estimator, we need to obtain the expected value of $|d_k|$ for each trial value of ϕ_e . The expectation of $|d_k|$ is approximated by the average value of $|d_k|$

Figure 3.7: Expectation, variance and second moment of $|d_k|$ versus ϵ with 16QAM modulation

among MN outputs, i.e.,

$$\begin{aligned}
 E(|d_k|) &= \mu_{\phi_e} \approx \hat{\mu}_{\phi_e} \\
 &= \frac{1}{MN} \sum_{m=1}^M \sum_{i=1}^N |\mathbf{w}_i^H \mathbf{\Phi}_e^H \mathbf{y}(m)|.
 \end{aligned} \tag{3.27}$$

When $|d_k|$ is embedded in AWGN, the accuracy of this approximation is affected and the difference between the expected value and the approximated average value is the major contribution to the error variance of the MOV estimator. It is known that $\hat{\mu}_{\phi_e}$ follows the Gaussian distribution with mean μ_{ϕ_e} and variance σ_w^2/MN [84,85], where

Figure 3.8: Expectation, variance and second moment of $|d_k|$ versus ϵ with 64QAM modulation

σ_w^2 represents the variance of AWGN noise. Thus, it has a probability that $\hat{\mu}_{\phi_e}$, ($\phi_e \neq \phi_0$), becomes the largest value and is greater than the theoretical maximum mean value $\hat{\mu}_{\phi_0}$. When this happens, the estimation accuracy is affected. For convenience, the analysis can be addressed equivalently as follows. Suppose that we have K trial values of ϕ_e among the estimation range and we obtain the average output for each trial value independently. For the k th trial value, the average output is

$$\hat{\mu}_k = \mu_k + n_k \quad (3.28)$$

where n_k denotes the noise term with zero mean and variance σ_w^2/MN . It is obvious that when the trial value is equal to ϕ_0 , the probability of this trial value becoming the largest is highest. Thus, if the PDF of $\hat{\mu}_k$ becomes the largest can be obtained, the variance represents the mean-square error (MSE) of the MOV estimator.

The probability that $\hat{\mu}_k$ becomes the largest can be expressed as

$$\begin{aligned}\Pr(\hat{\mu}_k = \max(\hat{\mu}_1, \dots, \hat{\mu}_K)) &= \Pr(\hat{\mu}_k > \hat{\mu}_1, \hat{\mu}_k > \hat{\mu}_2, \dots, \hat{\mu}_k > \hat{\mu}_K) \\ &= \Pr(\mu_k + n_k > \mu_1 + n_1, \dots, \mu_k + n_k > \mu_K + n_K). \quad (3.29)\end{aligned}$$

Note that $n_1, n_2, \dots, n_K$ are independent noise. For a given n_k , the conditional probability can be expressed as

$$\begin{aligned}\Pr(\hat{\mu}_k = \max(\hat{\mu}_1, \dots, \hat{\mu}_K)|n_k) \\ &= \Pr(n_1 < \mu_k - \mu_1 + n_k, \dots, n_K < \mu_k - \mu_K + n_k) \\ &= \Pr(n_1 < \mu_k - \mu_1 + n_k) \times \dots \times \Pr(n_K < \mu_k - \mu_K + n_k) \\ &= \left(1 - Q\left(\frac{\mu_k - \mu_1 + n_k}{\sqrt{2\sigma_w^2/MN}}\right)\right) \times \dots \times \left(1 - Q\left(\frac{\mu_k - \mu_K + n_k}{\sqrt{2\sigma_w^2/MN}}\right)\right). \quad (3.30)\end{aligned}$$

Thus, the probability that $\hat{\mu}_k$ becomes the largest is obtained as

$$\Pr(\hat{\mu}_k = \max(\hat{\mu}_1, \dots, \hat{\mu}_K)) = \int_{-\infty}^{\infty} \Pr(\hat{\mu}_k|n_k)p_{n_k}(x)dx, \quad k = 1, 2, \dots, K \quad (3.31)$$

where $p_{n_k}(x)$ denotes the Gaussian PDF of n_k as

$$p_{n_k}(x) = \frac{1}{\sqrt{2\pi\sigma_w^2/MN}} \exp\left\{-\frac{x^2}{2\sigma_w^2/MN}\right\}. \quad (3.32)$$

When all the K probabilities in (3.31) are obtained, the PDF that $\hat{\mu}_k$ becomes the largest can be obtained and the mean and variance of the estimated offset can be obtained accordingly. The PDFs that $\hat{\mu}_k$ becomes the largest are numerically obtained and presented in Figs. 3.9, 3.10 and 3.11 with different E_b/N_0 and number of blocks M . Here the variance represents the MSE of the MOV estimator. The MSEs are validated in the next section.

3.3.3 T&H Estimator

In the next section, we mainly compare the performance of the proposed MOV estimator with that of the T&H estimator [51]. Thus, the T&H estimator is briefly introduced in this subsection.

The k th block of transmitted data is defined as

$$\mathbf{s}(k) = [s_1(k), s_2(k), \dots, s_P(k)]^T. \quad (3.33)$$

Here $P < N$ and $N - P$ is the number of virtual carriers. No signal is transmitted over the virtual carriers, which affects the bandwidth efficiency. After the IDFT

Figure 3.9: The PDF that $\hat{\mu}_k$ becomes the largest with $E_b/N_0 = 20$ dB and $M = 2$

operation, the resultant signals are given by

$$\begin{aligned} \mathbf{x}(k) &= [x_1(k), x_2(k), \dots, x_N(k)]^T \\ &= \mathbf{W}_P \mathbf{s}(k) \end{aligned} \quad (3.34)$$

where

$$\mathbf{W}_P = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{j\omega} & \dots & e^{j(P-1)\omega} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j(N-1)\omega} & \dots & e^{j(N-1) \times (P-1)\omega} \end{bmatrix}_{N \times P} \quad (3.35)$$

Figure 3.10: The PDF that $\hat{\mu}_k$ becomes the largest with $E_b/N_0 = 20$ dB and $M = 4$

with $\omega = 2\pi/N$. The resultant signal $\mathbf{x}(k)$ is transmitted over a wireless channel and received at the receiver. The received signal with CFO can be expressed as

$$\mathbf{y}(k) = \mathbf{\Phi} \mathbf{W}_P \mathbf{H} \mathbf{s}(k) + \mathbf{n}(k) \quad (3.36)$$

where

$$\mathbf{H} = \text{diag}(H(1), \dots, H(P)) \quad (3.37)$$

characterizes the channel effect, $\mathbf{n}(k)$ denotes the AWGN and the CFO matrix is given by

$$\mathbf{\Phi} = \text{diag}(1, e^{j\phi_0}, \dots, e^{j(N-1)\phi_0}) \quad (3.38)$$

Figure 3.11: The PDF that $\hat{\mu}_k$ becomes the largest with $E_b/N_0 = 30$ dB and $M = 4$

with

$$\phi_0 = 2\pi\Delta f T_s. \quad (3.39)$$

Since

$$\mathbf{W}_P^H \mathbf{\Phi} \mathbf{W}_P \neq \mathbf{I} \quad (3.40)$$

where $\mathbf{I}$ is the identity matrix, the matrix $\mathbf{\Phi}$ destroys the orthogonality among subcarriers. Note that $\mathbf{W}_P$ consists of a subset of the IDFT matrix $\mathbf{W}$ and its orthogonal complement

$$\mathbf{W}_P^\perp = [\mathbf{w}_{P+1}, \dots, \mathbf{w}_N] \quad (3.41)$$

is known *a priori*, where $\mathbf{w}_k$ represents the k th column of the matrix $\mathbf{W}$. Suppose that

$$\mathbf{Z} = \text{diag}(1, z, z^2, \dots, z^{N-1}) \quad (3.42)$$

it can be obtained that

$$\mathbf{w}_{P+i}^H \mathbf{Z}^H \mathbf{y}(k) = 0, \quad i = 1, \dots, N - P \quad (3.43)$$

when $\mathbf{Z} = \Phi$. This observation suggests the method to estimate the frequency offset.

Thus, the T&H estimator is proposed as [51]

$$\hat{z} = \arg \min_z \sum_{i=1}^L \sum_{k=1}^K \mathbf{w}_{P+i}^H \mathbf{Z}^H \mathbf{y}(k) \mathbf{y}^H(k) \mathbf{Z} \mathbf{w}_{P+i} \quad (3.44)$$

where $L = N - P$ is the number of the virtual carriers and K is the number of OFDM blocks used for the estimation. The performance of the T&H estimator improves as L or K increases, but meanwhile the computational complexity also increases. When L increases, less subcarriers can be used to transmit useful data. Thus, the bandwidth efficiency is reduced.

3.4 Numerical Results and Discussions

Numerical results are presented in this section to illustrate the performance of the proposed MOV estimator in OFDM systems. In particular, $N = 64$ subcarriers are used. The performance of the proposed MOV estimator is examined by the

normalized mean-square error (MSE) defined as

$$\begin{aligned} \text{MSE} &= \text{E} \left[\frac{|\hat{\phi}_e - \phi_0|^2}{(2\pi/N)^2} \right] \\ &\approx \frac{1}{P_{MC}} \sum_{p=1}^{P_{MC}} \frac{|\hat{\phi}_{e_p} - \phi_0|^2}{(2\pi/N)^2} \end{aligned} \quad (3.45)$$

where $\hat{\phi}_e$ is the estimate of ϕ_0 and P_{MC} is the number of Monte Carlo runs. Simulations are conducted in both AWGN and frequency-selective fading channels.

3.4.1 Performance Against AWGN

Figure 3.12 shows the normalized MSE of the proposed MOV estimator over an AWGN channel. From the simulation results, it is observed that the normalized MSE decreases as E_b/N_0 increases. By using more OFDM blocks, the accuracy of the proposed MOV estimator is generally improved. Note that the theoretical MSEs are plotted in corresponding solid lines in Fig 3.12. The simulation results match the theoretical results well at high E_b/N_0 , i.e., $E_b/N_0 > 10$ dB. The performance curve of the T&H estimator [51] is also included for comparison, where the number of subcarriers is $N = 64$, the number of virtual carriers is $L = 20$ and the number of blocks for estimation is $M = 4$. For the value of $\text{MSE} = 10^{-4}$, the proposed MOV algorithm outperforms the T&H algorithm by more than 8 dB in E_b/N_0 . The reason for the significant improvement is that the proposed MOV estimator takes the average of the magnitude output and is robust to noise, while the orthogonality property utilized by the T&H estimator is more sensitive to noise. Moreover, the

Figure 3.12: Normalized MSE of the proposed MOV estimator versus E_b/N_0 with various number of OFDM blocks over AWGN channels.

proposed MOV algorithm can be implemented by fast Fourier transform (FFT) with high efficiency. Thus, the computational complexity of the proposed MOV algorithm is generally lower than that of the T&H algorithm.

In Fig. 3.13, we plot the normalized MSE of the proposed MOV algorithm against the number of OFDM blocks for different values of E_b/N_0 . From the results, it is observed that the normalized MSE decreases with more OFDM blocks for estimation but using more blocks for estimation means an increase in estimator complexity. Also, the theoretical MSE results are validated by simulation results. Note that the

Figure 3.13: Normalized MSE of the proposed MOV estimator versus number of blocks over AWGN channels.

normalized MSE is less than 10^{-3} for moderate E_b/N_0 , which makes the degradation caused by the CFO negligible even when only one OFDM block is used for the estimation. The proposed MOV estimator is thus suitable for burst transmission.

3.4.2 Performance Against Frequency-Selective Fading

In our simulation of frequency-selective fading channels, a 6-path multipath delay profile is assumed. The amplitude of the k th path, A_k , follows the Rayleigh distribution and the power delay profile is $E[A_k^2] = e^{-k/3}$. The phase shift of each path is uniformly distributed over $[0, 2\pi)$. The length of cyclic prefix (CP) is 6 and

the maximal delay is not greater than the length of CP to avoid ISI. The MOV estimator uses the average of the output to search for the maximum mean output. Thus, it is also applicable over frequency-selective fading channels even when the channel information is not available. Figure 3.14 shows the normalized MSE results versus E_b/N_0 over frequency-selective fading channels. From the results, it is observed that the normalized MSE is less than 4×10^{-3} when $E_b/N_0 \geq 10$ dB. The error floor is caused by the channel dispersion. It shows that the algorithm achieves high accuracy with only a few OFDM blocks under moderately high E_b/N_0 . Thus, the proposed MOV algorithm is effective and reliable over frequency-selective fading channels.

In Fig 3.15, the normalized MSE of the proposed MOV algorithm is presented with different number of blocks over frequency-selective fading channels. It is observed that the accuracy is generally improved as the number of blocks increases but the improvement becomes insignificant for large M . Note that the complexity also increases as the number of blocks increases. Hence, there is a tradeoff between the estimation accuracy and implementation complexity. Considering performance requirements and system complexity, the number of estimation blocks M should be properly chosen.

Figure 3.14: Normalized MSE of the proposed MOV estimator versus E_b/N_0 with various number of OFDM blocks over frequency-selective fading channels

3.4.3 Performance Improvement with MRC

The MOV estimator takes the summation of the magnitude output among the N subcarriers. It is similar to combining techniques in multi-carrier CDMA (MC-CDMA) [7]. In MC-CDMA systems, many combining techniques have been applied to improve the system performance, e.g., equal-gain combining (EGC) and maximal-ratio combining (MRC), etc. With the MRC technique, the output SNR is maximized [26, 86]. It is interesting to see how the MRC technique improves the performance of the proposed MOV estimator over frequency-selective fading channels.

Figure 3.15: Normalized MSE of the proposed MOV estimator versus number of blocks over frequency-selective fading channels

We assume here that the channel estimation has been achieved. In addition, the channel attenuation and the phase shift on each subcarrier are known to the MOV estimator. Accordingly, the gain matrix for the MRC is given by [7, 8, 86]

$$\mathbf{G} = \text{diag}(H_0^*, \dots, H_{N-1}^*) \quad (3.46)$$

where H_n is the channel gain on the n th subcarrier in (3.5) and the superscript $(\cdot)^*$ denotes complex conjugation. Multiplying with the gain matrix $\mathbf{G}$, the stronger signals on subcarriers carry larger weights than weaker signals. Thus, the SNR of

the output is maximized and the performance of MOV estimator is improved.

Using the MRC approach, the MOV estimator is given by

$$\begin{aligned}\hat{\phi}_e &= \arg \max_{\phi_e} \Lambda(\phi_e) \\ &= \arg \max_{\phi_e} \frac{1}{MN} \sum_{m=1}^M \sum_{i=1}^N |\mathbf{W}_i^H \mathbf{\Phi}_e^H \mathbf{G} \mathbf{y}(m)|\end{aligned}\quad (3.47)$$

where $\mathbf{G}$ represents the gain matrix for MRC.

The normalized MSE results of the proposed MOV estimator versus E_b/N_0 with MRC over frequency-selective fading channels are presented in Fig. 3.16. From the simulation results, it is observed that the normalized MSE significantly decreases with MRC and the performance of the proposed MOV estimator with MRC is even comparable to that of the MOV estimator under the AWGN conditions. It is also clear that the error floor is eliminated. This supports the argument that the error floor in Fig. 3.15 is caused by channel dispersion. We also include the performance curve of the T&H estimator [51] for comparison. It is seen that the proposed MOV estimator still outperforms the T&H estimator over the same frequency-selective fading channel conditions.

Figure 3.16: Normalized MSE of the proposed MOV estimator versus E_b/N_0 with MRC technique over frequency-selective fading channels

3.4.4 Compensation for the CFO

After the CFO estimation is accomplished, frequency offset can be properly compensated and corrected to allow for data detection. Figure 3.17 shows the BER performance for three cases, namely, with 20% frequency offset, without frequency offset, and after offset correction over frequency-selective fading channels, respectively. It is observed that the BER performance degrades severely when there is a 20% frequency offset. The BER curve also has an error floor at high E_b/N_0 . The reason for the error floor is that the bit errors are mainly due to the ICI caused by the CFO, not due to the noise for high E_b/N_0 . The BER, after MOV estimation

Figure 3.17: BER versus E_b/N_0 over frequency-selective fading channels with carrier frequency offset correction

and correction, is very close to the BER without CFO, which validates that MOV estimator eliminates the effect of CFO effectively and the system performance is maintained.

3.5 Conclusion

In this chapter, we have proposed the minimum output variance (MOV) estimator to estimate carrier frequency offset (CFO) in OFDM systems. The probability density function (PDF) of the output magnitude has been approximated as Gaussian mixture, and the variance and expectation expressions of the output magnitude

have been derived. The minimum output variance feature was exploited for the carrier frequency offset estimation. Simulation results validated the derived PDF and also the mean and variance expressions. The performance of the proposed MOV estimator has been examined in this chapter. The theoretical MSE of the MOV estimator has been obtained and the theoretical MSE matches the simulation results well. From the simulation results, it is observed that the MOV algorithm has high accuracy in both AWGN and frequency-selective fading channels. Compared with the T&H algorithm, the MOV algorithm outperforms the T&H algorithm. The proposed MOV estimator can be implemented by FFT efficiently, which causes the computational complexity of the MOV estimator to be generally lower than that of the T&H estimator. Furthermore, the maximal-ratio combining (MRC) technique has been applied to eliminate the error floor and improve the estimation accuracy of the proposed MOV estimator over frequency-selective fading channels. Simulation results indicated that the MRC technique is effective and significantly improves the accuracy of the proposed MOV estimator over frequency-selective fading channels.

Chapter 4

Maximum Likelihood Estimator

4.1 Introduction

As explained in Chapter 2, carrier frequency offset (CFO) can be divided into an integer part, which is a multiple of the subcarrier spacing, and a fractional part, which is less than one half of the subcarrier spacing. The fractional offset attenuates desired signals and generates inter-carrier interference (ICI) at the output and the integer offset causes the cyclic shift of the output. In Chapter 3, we proposed a minimum output variance (MOV) estimator to estimate the fractional carrier frequency offset. In the literature, many estimation algorithms have been proposed for estimating the fractional part of carrier frequency offset [43, 45, 51, 64], which is called fine estimation. To complete the carrier frequency offset estimation, the integer frequency offset needs to be estimated, and the process is called coarse estimation. Some studies in the literature have been devoted to the estimation of the integer frequency offset [62, 64, 67, 87]. In [87], the authors jointly considered the timing offset and integer frequency offset using the concept of the coherence phase bandwidth. In [62], a null pilot was inserted to estimate the integer CFO,

but the method is not suitable for burst transmission. In [64] and [67], two consecutive OFDM blocks were examined to estimate the integer frequency offset. In these studies, it is assumed that the fractional frequency offset has been estimated and corrected by other estimation algorithms. In this chapter, we also adopt the same assumption and concentrate on the estimation of the integer frequency offset. To be specific, we derive a maximum-likelihood estimator (MLE) for coarse estimation of CFO, which requires only one OFDM block containing the pilot symbols. To reduce the computational complexity of the ML estimator and improve the bandwidth efficiency, we study a suboptimum estimator (Sub MLE). Based on the hypothesis testing method, a threshold Sub MLE (T-Sub MLE) is suggested to further reduce the computational complexity. The performance analysis of the T-Sub MLE is then conducted.

The rest of the chapter is organized as follows. OFDM signals are introduced in Section 4.2. The likelihood function is used to derive the MLE and two suboptimum estimators in Section 4.3. The performance of the proposed estimators in additive white Gaussian noise (AWGN) is also presented. Numerical results and discussions are given in Section 4.4. Finally, the conclusion is drawn in Section 4.5.

4.2 OFDM Signals

In an OFDM system, the OFDM signal is formulated by N -point inverse fast Fourier transform (IFFT) and can be expressed as

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c(n) e^{j2\pi kn/N} \quad (4.1)$$

where $c(n)$ is a quadrature amplitude modulation (QAM) or phase-shift-keying (PSK) symbol in the n th subcarrier. At the receiver, the CFO incurs due to the mismatch of the carrier frequencies between the oscillators in OFDM transmitter and receiver. The received signal with CFO is given by

$$x(k) = e^{j2\pi vk/N} s(k) + \eta(k) \quad (4.2)$$

where v is the frequency offset normalized to subcarrier spacing and $\eta(k)$ is modeled as the AWGN process with zero mean and variance

$$\sigma^2 = E[|\eta(k)|^2] \quad (4.3)$$

where $E[\cdot]$ denotes the expectation operation. The frequency offset can be divided into an integer part, m , and a fractional part, ϵ , i.e.,

$$v = m + \epsilon \quad (4.4)$$

where

$$m \triangleq \text{int}(v) \quad (4.5)$$

and $\epsilon \in [-0.5, 0.5]$. Before the integer part m is compensated, the fractional part ϵ can be estimated and compensated by many algorithms in the existing literature, including the repeated-training-symbols algorithms in [43, 64] and the minimum output variance (MOV) algorithm proposed in Chapter 3, etc. Here we follow the similar assumption presented in [64, 67] that the parameter ϵ has already been measured and compensated perfectly. In this chapter, we focus on the estimation of the integer part of the frequency offset, m .

4.3 Likelihood Function and CFO Estimator

4.3.1 Maximum Likelihood Estimator

The likelihood function with the unknown parameter m is given by

$$\Lambda_1(\tilde{m}) = \exp \left\{ -\frac{1}{2\sigma^2} \sum_{k=0}^{N-1} |z(k) - e^{j2\pi\tilde{m}k/N} s(k)|^2 \right\} \quad (4.6)$$

where $\tilde{m}$ denotes the trial value of m and

$$z(k) = x(k)e^{-j2\pi\epsilon k/N} \quad (4.7)$$

is the received sequence with the fractional carrier frequency offset ϵ compensated.

Expanding the likelihood function in (4.6), we have

$$\begin{aligned}
 \Lambda_1(\tilde{m}) &= \exp \left\{ -\frac{1}{2\sigma^2} \sum_{k=0}^{N-1} |z(k) - e^{j2\pi\tilde{m}k/N} s(k)|^2 \right\} \\
 &= \exp \left\{ -\frac{1}{2\sigma^2} \sum_{k=0}^{N-1} [z(k) - e^{j2\pi\tilde{m}k/N} s(k)] [z(k) - e^{j2\pi\tilde{m}k/N} s(k)]^* \right\} \\
 &= \exp \left\{ -\frac{1}{2\sigma^2} \sum_{k=0}^{N-1} [|z(k)|^2 + |s(k)|^2 - z(k)e^{-j2\pi\tilde{m}k/N} s^*(k) \right. \\
 &\quad \left. - e^{j2\pi\tilde{m}k/N} s(k)z^*(k)] \right\} \tag{4.8}
 \end{aligned}$$

where the superscript $*$ represents the complex conjugation. It is observed that

$$\begin{aligned}
 \sum_{k=0}^{N-1} |s(k)|^2 &= \sum_{k=0}^{N-1} \left| \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c(n) e^{j2\pi kn/N} \right|^2 \\
 &= \frac{1}{N} \sum_{k=0}^{N-1} \left[\left(\sum_{n=0}^{N-1} c(n) e^{j2\pi kn/N} \right) \left(\sum_{n=0}^{N-1} c(n) e^{j2\pi kn/N} \right)^* \right] \\
 &= \frac{1}{N} \sum_{k=0}^{N-1} \left[\left(c(0) + \dots + c(N-1) e^{j2\pi k(N-1)/N} \right) \right. \\
 &\quad \left. \times \left(c(0) + \dots + c(N-1) e^{j2\pi k(N-1)/N} \right)^* \right] \\
 &= N \tag{4.9}
 \end{aligned}$$

where

$$|c(n)|^2 = 1 \tag{4.10}$$

is assumed. Note that $|z(k)|^2$ is uncorrelated to the unknown parameters. Dropping the terms $|z(k)|^2$ and $|s(k)|^2$ in (4.8) yields

$$\begin{aligned}\Lambda_2(\tilde{m}) &= \exp\left\{-\frac{1}{2\sigma^2} \sum_{k=0}^{N-1} [-z(k)e^{-j2\pi\tilde{m}k/N} s^*(k) - e^{j2\pi\tilde{m}k/N} s(k)z^*(k)]\right\} \\ &= \exp\left\{\frac{1}{\sigma^2} \sum_{k=0}^{N-1} \text{Re}[z(k)e^{-j2\pi\tilde{m}k/N} s^*(k)]\right\}\end{aligned}\quad (4.11)$$

where $\text{Re}[\cdot]$ denotes the real part of the component. Substituting $s(k)$ into (4.11), the objective function is further simplified to

$$\begin{aligned}\Lambda_2(\tilde{m}) &= \exp\left\{\frac{1}{\sigma^2} \sum_{k=0}^{N-1} \text{Re}\left[z(k)e^{-j2\pi\tilde{m}k/N} \left(\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c^*(n)e^{-j2\pi nk/N}\right)\right]\right\} \\ &= \exp\left\{\frac{1}{\sigma^2} \sum_{n=0}^{N-1} \text{Re}\left[c^*(n) \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} z(k)e^{-j2\pi\tilde{m}k/N} e^{-j2\pi nk/N}\right]\right\} \\ &= \exp\left\{\frac{1}{\sigma^2} \sum_{n=0}^{N-1} \text{Re}[c^*(n)Z(n + \tilde{m})]\right\}\end{aligned}\quad (4.12)$$

where $Z(n)$ is defined as

$$Z(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} z(k)e^{-j2\pi nk/N} \quad (4.13)$$

and represents the n th FFT output. Here the pilot symbols $c(n)$'s form a pseudonoise sequence and satisfy the following relation

$$\text{E}[c(l)c^*(i)] = \begin{cases} 1, & l = i \\ 0, & l \neq i \end{cases} \quad (4.14)$$

Maximizing the likelihood function $\Lambda_2(\tilde{m})$ is equivalent to maximizing

$$\Lambda_3(\tilde{m}) = \text{Re} \left\{ \sum_{n=0}^{N-1} [c^*(n)Z(n + \tilde{m})] \right\}. \quad (4.15)$$

Thus, the estimation of the m is obtained by maximizing the metric in (4.15), i.e.,

$$\hat{m} = \arg \max_{|\tilde{m}| \leq M} \Lambda_3(\tilde{m}) \quad (4.16)$$

where M denotes the largest possible value of $|m|$, $\tilde{m}$ is the integer trial value of m and this proposed estimator is denoted as the maximum likelihood estimator (MLE).

4.3.2 Suboptimum Maximum Likelihood Estimator

To reduce the complexity of the MLE, we investigate another suboptimum estimator.

We may select a subset of the pilots $c(n)$ for the estimation [67], i.e., a total of $N_p (\ll N)$ pilot symbols are selected at known locations in an OFDM block. Hence, the estimation metric in (4.15) can be expressed as

$$\Lambda_3(\tilde{m}) = \text{Re} \left\{ \sum_{n \in \Omega} c^*(n)Z(n + \tilde{m}) + \sum_{n \notin \Omega} c^*(n)Z(n + \tilde{m}) \right\} \quad (4.17)$$

where Ω is the set of indices corresponding to the N_p pilot carriers. Note that the transmitted signals $c(n)$'s, where n does not belong to the subset Ω , are unknown

to the estimator. Accordingly,

$$\sum_{n \notin \Omega} c^*(n)Z(n + \tilde{m}) \approx 0 \quad (4.18)$$

because $c(n)$ and $Z(n + \tilde{m}), n \notin \Omega$, can be considered as two uncorrelated sequences with zero mean. Thus, the estimation metric becomes

$$\Lambda(\tilde{m}) = \text{Re} \left\{ \sum_{n \in \Omega} c^*(n)Z(n + \tilde{m}) \right\} \quad (4.19)$$

and the suboptimum estimator (Sub MLE) is

$$\hat{m} = \arg \max_{|\tilde{m}| \leq M} \Lambda(\tilde{m}). \quad (4.20)$$

Since the number of pilot symbols $N_p \ll N$, other subcarriers can be used to transmit useful symbols and the bandwidth efficiency is improved.

4.3.3 Threshold Suboptimum Maximum Likelihood Estimator

The MLE and Sub MLE are based on the exhaustive search for all possible values of $|m| \leq M$, which requires high computational complexity. To further reduce the computational complexity of the MLE and Sub MLE, we next suggest a threshold Sub MLE (T-Sub MLE).

The mean and variance of the metric of $\Lambda(\tilde{m})$ in (4.19) can be obtained by using the method similar to that in [64] and [66]. In AWGN channels, the FFT outputs are given by

$$Z(n) = c(n - m) + W(n) \quad (4.21)$$

where $W(n)$ is the noise term after the FFT operation. Let us consider the distribution of $\Lambda(\tilde{m})$ when $\tilde{m} = m$. It is assumed that the metric $\Lambda(\tilde{m})$ can be approximated as a Gaussian random variable [64, 66] and written as

$$\Lambda(\tilde{m}) = \text{Re} \left\{ \sum_{n \in \Omega} c^*(n) [c(n) + W(n + \tilde{m})] \right\}. \quad (4.22)$$

Defining

$$\text{E}[|c(n)|^2] = S \quad (4.23)$$

and

$$\text{E}[|W(n)|^2] = \sigma_W^2, \quad (4.24)$$

the mean and variance of the metric are, respectively,

$$\mu_\Lambda = N_p S \quad (4.25)$$

and

$$\sigma_\Lambda^2 = N_p S \sigma_W^2. \quad (4.26)$$

Note that the above analysis of the mean and variance is also valid for the metric of the MLE when $N_p = N$.

The T-Sub MLE is based on a hypothesis testing method. The hypothesis testing is selected between a null hypothesis $\mathcal{H}_0 : \hat{m} \neq m$ and an alternative hypothesis $\mathcal{H}_1 : \hat{m} = m$. If the estimation metric is larger than a threshold T , $\mathcal{H}_1$ is accepted; otherwise, $\mathcal{H}_1$ is rejected. We set a threshold T based on the mean of the estimation metric and declare the offset $\hat{m} = \tilde{m}$ when the estimation metric is larger than this threshold. Thus, we have the following hypothesis testing:

$$\begin{aligned}\mathcal{H}_0 : \hat{m} &\neq m & (\Lambda(\tilde{m}) \leq T) \\ \mathcal{H}_1 : \hat{m} &= m & (\Lambda(\tilde{m}) > T)\end{aligned}\tag{4.27}$$

According to the hypothesis testing, the probability of error, P_e , can be expressed as [66, 88]

$$P_e = \Pr\{\Lambda(\tilde{m}) < T | \mathcal{H}_1\} \approx Q\left(\frac{\mu_\Lambda - T}{\sigma_\Lambda}\right)\tag{4.28}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt.\tag{4.29}$$

Substituting the mean and variance derived in (4.25) and (4.26), the error probability of the T-Sub MLE can be obtained. The analytical error probability will be validated by simulations in the next section.

4.4 Numerical Results and Discussions

Computer simulations are conducted to illustrate the performance of the proposed estimators in this section. The OFDM symbols are QPSK modulated. We also include the results of the estimators proposed in [64] and [67] for comparison, where pilot symbols are inserted into two consecutive OFDM blocks, $c_0(n)$ and $c_1(n)$, and $d(n) = c_0(n)c_1^*(n)$, $(n \in \Omega)$ forms the pilot sequence. The Schmidl and Cox estimator (SCE) proposed in [64] is given by

$$\hat{m} = \arg \max_{|\tilde{m}| \leq M} \left\{ \left| \sum_{n \in \Omega} c_0(n)c_1^*(n)Z_0^*(n + \tilde{m})Z_1(n + \tilde{m}) \right|^2 \right\} \quad (4.30)$$

and the Morelli, Andrea and Mengali estimator (MAME) proposed in [67] is given by

$$\hat{m} = \arg \max_{|\tilde{m}| \leq M} \left\{ 2\text{Re} \left[\sum_{n \in \Omega} c_0(n)c_1^*(n)Z_0^*(n + \tilde{m})Z_1(n + \tilde{m}) \right] + \sum_{n=0}^{N_u-1} [|Z_0(n + \tilde{m})|^2 + |Z_1(n + \tilde{m})|^2] \right\} \quad (4.31)$$

where $c_0(n)$ and $c_1(n)$ are the two consecutive pilot symbols, $Z_0(n)$ and $Z_1(n)$ are the two consecutive received symbols, and $N_u (\leq N)$ is the number of the modulated subcarriers.

In Fig. 4.1, the error probability of the MLE, P_e , versus SNR over an AWGN channel is presented, where the number of subcarriers $N = 16$ and the largest possible

Figure 4.1: The error probability of the MLE, P_e , versus SNR over an AWGN channel, $N = 16$

value of $|m|$ is set to $M = 4$. From the simulation results, it is observed that the error probability decreases as SNR increases. The results obtained from SCE and MAME are also included for comparison. Since we assume $M = 4$, the maximum number of modulated subcarriers in MAME is $N_u = 12$. It is observed that the MLE outperforms SCE and MAME. Note that the MLE is also applicable to larger values of N and the particular choice $N = 16$ used in Fig. 4.1 is for illustration.

In Fig. 4.2, the performance of the Sub MLE is presented, where the number of subcarriers is $N = 256$. Similar to [67], the number of pilot symbols, N_p , is set to be 10 and 12, respectively. The parameter M is limited to 10. From the simulation

Figure 4.2: The error probability of the Sub MLE, P_e , versus SNR over an AWGN channel, $N = 256$

results, it is observed that the error probability decreases as the SNR or the number of pilot symbols N_p increases. Also, the Sub MLE outperforms the MAME and SCE. The reason why the MLE and Sub MLE outperforms MAME is explained below.

We apply the same method presented in Section 4.3.3 to analyze the mean and variance of the existing MAME. Because the second term in the metric (4.31) has no significant contribution to the estimation compared to the first term, we only analyze the first term for simplicity. Then the estimation metric can be rewritten

as

$$\hat{m} = \arg \max_{|\tilde{m}| \leq M} \left\{ \text{Re} \left[\sum_{n \in \Omega} c_0(n) c_1^*(n) Z_0^*(n + \tilde{m}) Z_1(n + \tilde{m}) \right] \right\}. \quad (4.32)$$

The metric in (4.32) can be approximated as a Gaussian random variable and written

as

$$\Lambda_{MAME}(\tilde{m}) = \text{Re} \left\{ \sum_{n \in \Omega} c_0(n) c_1^*(n) [c_0^*(n) + W_0^*(n + \tilde{m})] [c_1(n) + W_1(n + \tilde{m})] \right\}. \quad (4.33)$$

The mean of the metric $\Lambda_{MAME}(\tilde{m})$ is

$$\mu_{\Lambda_{MAME}} = N_p S^2 \quad (4.34)$$

and the variance is

$$\sigma_{\Lambda_{MAME}}^2 = N_p (2S^3 \sigma_W^2 + S^2 \sigma_W^4). \quad (4.35)$$

The means and variances of the metrics $\Lambda(\tilde{m})$ in (4.19) and $\Lambda_{MAME}(\tilde{m})$ in (4.33) versus SNR are shown in Figs. 4.3 and 4.4, respectively. It is clear that the simulation results match well with the theoretical results. It implies that the Gaussian assumption for the PDFs of the metrics is acceptable. From the comparison, it is evident that the variance of the proposed metric is smaller than that of the MAME's metric. In other words, the proposed estimator is more robust to the noise. This partially explains the reason why the MLE and Sub MLE outperform the MAME.

Figure 4.3: The mean of the metrics versus SNR over an AWGN channel with $N = 256$ and $N_p = 32$

Figure 4.5 shows the error probability of the T-Sub MLE, where the number of subcarriers is $N = 256$, the number of pilot symbols is $N_p = 32$ and the threshold is $T = 0.75\mu_\Lambda$. It is observed that the analytical results in (4.28) match the simulation results well. It also supports that the Gaussian assumption for the PDFs is acceptable.

The MLE is derived under the AWGN conditions, but it is applicable to frequency-selective fading channels. Incorporating the effect of fading, the estimation metric

Figure 4.4: The variance of the metrics versus SNR over an AWGN channel with $N = 256$ and $N_p = 32$

in (4.19) can be rewritten as

$$\Lambda(\tilde{m}) = \text{Re} \left\{ \sum_{n \in \Omega} c^*(n) H(n + \tilde{m}) Z(n + \tilde{m}) \right\} \quad (4.36)$$

where $H(\cdot)$ denotes the fading characteristics in subcarrier. In OFDM systems, the high data rate streams are transmitted over parallel subcarriers and it is assumed that the transmitted signals undergo flat fading over each subcarrier. Although the correlation value between the pilot symbols and received symbols are affected due to the attenuation of the received signals, the location of the maximal correlation value is not affected seriously. The correlation curves over an

Figure 4.5: The error probability of the T-Sub MLE, P_e , versus SNR over an AWGN channel with $N = 256$ and $N_p = 32$

AWGN and a frequency-selective fading channel are illustrated in Fig. 4.6, where $N = 256$, $N_p = 32$, $E_b/N_0 = 10$ dB and the true frequency offset $m = 1$. From the correlation curves, it is observed that the maximum correlation value over frequency-selective fading channels is smaller than the maximum correlation value over AWGN channels. But both of the correlation curves achieve the maximum value at the location of the true frequency offset. Thus, the proposed estimators are still applicable to estimate frequency offset over frequency-selective fading channels. When the transmitted signals undergo deep fading, the location of the maximal correlation value may be affected. It makes the frequency offset estimation fail. This is the

reason why the error probability obtained over fading channels is greater than the error probability obtained over AWGN channels. In that case, we have two methods to handle deep fading. The first one is to increase the number of pilot N_p . Because the probability of all the pilot symbols undergoing deep fading is low, increasing the number of pilot symbols decreases the error probability effectively. The other method is to equalize the received signals before the frequency offset estimation. When the channel condition is too poor, both channel estimation and equalization are required. After the equalization, the frequency offset estimation can be accomplished properly. Figure 4.7 shows the results obtained under frequency-selective fading channels where the channel state information is unknown to the estimator. Here we assume that the frequency-selective fading channel has 15 independent paths. The amplitude of the k th path, A_k , follows the Rayleigh distribution and the power delay profile is $E[A_k^2] = e^{-k/3}$. The phase shift of each path is uniformly distributed over $[0, 2\pi)$ [67]. From the simulation results, it is obvious that higher SNR causes lower error probability P_e . The corresponding results of SCE and MAME are also included for comparison under the same fading channel conditions. It is observed that the proposed estimator still outperforms the SCE and MAME over frequency-selective fading channels.

4.5 Conclusion

In this chapter, we have investigated a maximum-likelihood estimator (MLE) for coarse carrier frequency offset (CFO) estimation in OFDM systems. The proposed

Figure 4.6: Correlation curves over AWGN and frequency-selective fading channels with $N = 256$, $N_p = 32$, $E_b/N_0 = 10$ dB, true frequency offset $m = 1$

estimator needs only one OFDM block for estimation of CFO. To reduce the computational complexity and improve the bandwidth efficiency of the MLE, a sub optimum estimator (Sub MLE) has been studied. Using the hypothesis testing method, a threshold Sub MLE (T-Sub MLE) has been considered to further reduce the computational complexity. The performance analysis of the T-Sub MLE has been conducted to evaluate the accuracy. From the simulation and analytical results, it is concluded that the proposed estimators are effective and reliable in both AWGN and frequency-selective fading channels. Also, the proposed estimators outperform the estimators proposed in [64] and [67].

Figure 4.7: The error probability of the Sub MLE, P_e , versus SNR over a frequency-selective fading channel, $N = 256$

Chapter 5

Adaptive LMS-like Estimator

5.1 Introduction

In Chapter 2, we claimed that CFO can be divided into an integer part, which is a multiple of the subcarrier spacing, and a fractional part, which is less than one half of the subcarrier spacing. The integer frequency offset causes the cyclic shift of the output, and the fractional frequency offset attenuates the desired signal in each subcarrier and contains inter-carrier interference (ICI) from other subcarriers. We proposed a minimum output variance (MOV) estimator in Chapter 3 to estimate the fractional frequency offset. In Chapter 4, a maximum likelihood estimator (MLE) and two sub-optimum estimators were proposed to estimate the integer frequency offset. In the literature, most of algorithms estimate either the fractional offset or the integer offset. The two-level implementation increases the system complexity. Another problem is that the residual integer offset affects the accuracy of the following fractional offset estimation or vice versa. This motivates us to develop an algorithm to estimate the integer and fractional frequency offset simultaneously.

As introduced in Chapter 2, CFO might be caused by two sources. The first one is the mismatch of carrier frequencies between local oscillators in transmitter and receiver, which is considered as a stationary part. The other one is caused by the Doppler shift, which is due to the variation of relative speed between the transmitter and receiver and may change from time to time [43]. In the literature, most research papers assume CFO as a stationary parameter and do not consider the variation of the frequency offset caused by the Doppler shift. In Chapters 3 and 4, we also followed this assumption. But for a more practical environment, the frequency offset includes the stationary part and the time-varying part. As a matter of fact, the accuracy of many existing algorithms is affected when frequency offset changes from time to time. In [43], two repeated training blocks are transmitted and the phase of each of the subcarriers between the training blocks is compared for the estimation of CFO. However, when the CFO changes from block to block, the estimation accuracy is affected. In this chapter, we propose an adaptive LMS-like algorithm to estimate the CFO, which can track the variation of the frequency offset caused by the Doppler shift. To improve the bandwidth efficiency, we also propose a modified LMS-like (MLMS-like) algorithm, which requires only one OFDM block as training symbols, to track the variation of the frequency offset. In the literature, most of the existing algorithms estimate either the fractional part [43], which is called fine estimation, or the integer part [67, 87], which is called coarse estimation. Some papers estimate the fractional and integer parts by two separate methods [64, 66]. A blind algorithm has been proposed to estimate both the integer and fractional parts of

the frequency offset, which exploited the inherent orthogonality between information bearing carriers and virtual carriers [51]. However, it is based on the exhaustive search in a particular offset range and requires very high computational complexity. The proposed LMS-like and MLMS-like algorithms can estimate both the fractional and integer parts of the frequency offset simultaneously with the simple adaptive process and the estimation range is not limited to a particular range.

This chapter is organized as follows. Section 5.2 briefly introduces the OFDM systems and describes the frequency offset issue. In Section 5.3, the LMS-like and MLMS-like algorithms are proposed to estimate the CFO. Performance analysis of the proposed algorithms is presented in Section 5.4. Numerical results and discussions are presented in Section 5.5 and finally, conclusion is drawn in Section 5.6.

5.2 OFDM Systems

In an OFDM system, the OFDM signal is formulated by the N -point inverse fast Fourier transform (IFFT) modulation and can be expressed as

$$d_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} s_n e^{j2\pi kn/N} \quad (5.1)$$

where s_n is a quadrature amplitude modulation (QAM) or phase-shift-keying (PSK) symbol in the n th subcarrier. After the modulation, cyclic prefix (CP) is inserted to avoid intersymbol interference (ISI). It is assumed that the length of the CP is longer

than the delay of the channel spread. The formulated signals are up-converted to the radio frequency (RF) and transmitted over wireless channels. At the receiver, the received signals are down-converted and the CP is removed. The received signal with CFO can be expressed as

$$x_k = e^{j2\pi vk/N} d_k + \eta_k \quad (5.2)$$

where

$$v = \epsilon_o + \epsilon_d \quad (5.3)$$

is the frequency offset normalized to subcarrier spacing, which includes the frequency offset caused by the mismatch of oscillators, ϵ_o , and the normalized Doppler shift, ϵ_d , and η_k is the complex additive white Gaussian noise (AWGN) with zero mean and variance σ^2 . Note that the frequency offset ϵ_o is assumed to be stationary and the Doppler shift ϵ_d can be assumed to be stationary within one OFDM block. However, ϵ_d may change from block to block because of the relative motion between OFDM transceivers in an outdoor wide area wireless environment [89]. The Doppler shift in frequency is given by [42]

$$f_d = \frac{v_r}{\lambda} \cos \theta \quad (5.4)$$

where v_r is the relative velocity, λ is the wavelength and θ is the spatial angle between the direction of motion of the mobile and direction of arrival of the wave. It can be seen from (5.4) that if the mobile is moving toward the direction of arrival of the waveform, the Doppler shift is positive, and if the mobile is moving away from

the direction of arrival of the signal, the Doppler shift is negative. The normalized Doppler shift is given by

$$\epsilon_d = f_d T_s \quad (5.5)$$

where T_s is the OFDM block duration.

Because of the presence of the frequency offset, the orthogonality among the sub-carriers is destroyed. Moreover, inter-carrier interference (ICI) is incurred after the FFT demodulation, which severely affects the system performance. Thus, CFO must be estimated and properly compensated before the FFT demodulation.

5.3 Estimation of Carrier Frequency Offset

5.3.1 Principle of LMS

The principle of least mean square (LMS) was presented in [90–92]. Suppose that we have the M tap inputs $x_k, x_{k-1}, x_{k-2}, \dots, x_{k-M+1}$ and the corresponding tap weights $\mathbf{w} = [w_{0,k}, w_{1,k}, w_{2,k}, \dots, w_{M-1,k}]^T$. The vector of tap inputs at time k is denoted by $\mathbf{x}_k = [x_k, x_{k-1}, x_{k-2}, \dots, x_{k-M+1}]^T$ and the desired response at time k is denoted by d_k . Comparing the corresponding estimate of the desired response with d_k , the estimation error is given by

$$\epsilon_k = d_k - \mathbf{w}^T \mathbf{x}_k \quad (5.6)$$

and the mean square error is defined as

$$\xi_k = \text{E}[\epsilon_k^2]. \quad (5.7)$$

To develop the LMS algorithm, we take ϵ_k^2 itself as an estimate of ξ_k . Then, at each iteration in the adaptive process, the gradient is estimated as

$$\nabla_k = -2\epsilon_k \mathbf{x}_k. \quad (5.8)$$

With this simple estimate of the gradient, we can specify a steepest descent type of adaptive algorithm as

$$\begin{aligned} \mathbf{w}_{k+1} &= \mathbf{w}_k - \mu \nabla_k \\ &= \mathbf{w}_k + 2\mu \epsilon_k \mathbf{x}_k \end{aligned} \quad (5.9)$$

where μ is the step size. The block diagram of the LMS is shown in Fig. 5.1. The step size μ regulates the speed and stability of the adaption. Since the weight changes at each iteration are based on imperfect gradient estimates, the adaptive process is noisy. It is noticed that the LMS algorithm can be implemented in a practical system without squaring, averaging or differentiation and is elegant in its simplicity and efficiency. Because of these characteristics, the LMS algorithm has been investigated, analyzed and applied to many applications [93–99].

Figure 5.1: The adaptive linear combiner

5.3.2 Adaptive LMS-like Estimator

We propose here a single-weight LMS-like estimator for the estimation of the CFO in OFDM systems, where the weight is defined as follows. By comparing the estimation with the training symbols d_k , we define the error as

$$e_k = d_k - e^{j2\pi k w_k / N} x_k \quad (5.10)$$

where w_k is a real-valued weight and x_k is the received signal defined in (5.2). The mean-square error is defined as

$$\begin{aligned} \xi_k &= E[|e_k|^2] \\ &= E[|d_k|^2] - E[e^{j2\pi k w_k / N} x_k d_k^*] - E[e^{-j2\pi k w_k / N} x_k^* d_k] + E[|x_k|^2] \end{aligned} \quad (5.11)$$

Figure 5.2: Block diagram of the adaptive LMS-like estimator

where $(\cdot)^*$ denotes the complex conjugation. We take $|e_k|^2$ itself as an estimate of mean-square error ξ_k [91]. Then, at each iteration of the adaptive process, we have a gradient estimate of the form

$$\begin{aligned} \nabla_k &= \frac{\partial |e_k|^2}{\partial w_k} \\ &= \frac{4\pi k}{N} \text{Im}(e^{j2\pi k w_k / N} x_k d_k^*) \end{aligned} \quad (5.12)$$

where $\text{Im}(\cdot)$ represents the imaginary part of the component. It is observed that $\nabla_k = 0$ is satisfied when $w_k = -v$ under the noise free condition. Thus, we exploit this feature to estimate the CFO.

With the simple estimate of the gradient in (5.12), we specify a type of steepest

descent adaptive algorithm as follows,

$$\begin{aligned} w_{k+1} &= w_k - \mu \nabla_k \\ &= w_k - \mu \frac{4\pi k}{N} \text{Im}(e^{j2\pi k w_k / N} x_k d_k^*) \end{aligned} \quad (5.13)$$

where μ is the step size. In general, a larger step size gives a faster convergence rate but larger misadjustment. Thus, the selection of the step size is a tradeoff between the convergence rate and misadjustment. After K iterations, we estimate the CFO by $\hat{v} = -w_K$, where the number of iterations is equal to the number of pilot symbols. The block diagram of the proposed adaptive LMS-like estimator is illustrated in Fig. 5.2.

We summarize the steps of the proposed LMS-like algorithm in the followings:

1. *Initialization:* $w_0 = 0$.
2. *Weight updating:*

$$w_{k+1} = w_k - \mu \frac{4\pi k}{N} \text{Im}(e^{j2\pi k w_k / N} x_k d_k^*) \quad (5.14)$$

The updating process stops at the K th iteration.

3. *Frequency offset estimation:* $\hat{v} = -w_K$.

In the LMS-like algorithm, the weight updating is based on the desired training

symbols d_k . To further improve the bandwidth efficiency, we propose a modified LMS-like (MLMS-like) algorithm. In the MLMS-like algorithm, only one OFDM block is required as the training symbols. After the updating based on the first OFDM training block, we can roughly estimate the frequency offset. According to this frequency offset estimation, we compensate the CFO for the consecutive OFDM block and estimate the received signals $\hat{d}_k$. Then, the estimated signals $\hat{d}_k$ are used as the pilot symbols for the subsequent updating and CFO estimation. In this manner, the frequency offset estimation can be accomplished block to block without the aid of more pilot symbols. The proposed MLMS-like algorithm is summarized as follows:

1. *Initialization:* $w_0 = 0$.
2. *Weight updating for the first OFDM training block:*

$$w_{k+1} = w_k - \mu \frac{4\pi k}{N} \text{Im}(e^{j2\pi k w_k / N} x_k d_k^*) \quad (5.15)$$

The updating process stops at the N th iteration.

3. *Frequency offset estimation:* $\hat{v}_1 = -w_N$.
4. *Frequency offset compensation for the consecutive OFDM block based on the previous estimation of the CFO and estimate the signals $\hat{d}_k$ as the pilot symbols.*
5. *Weight updating based on the estimated symbols $\hat{d}_k$:*

$$w_{k+1} = w_k - \mu \frac{4\pi k}{N} \text{Im}(e^{j2\pi k w_k / N} x_k \hat{d}_k^*) \quad (5.16)$$

6. *Frequency offset estimation after K iterations: $\hat{v} = -w_K$.*
7. *Repeat steps 4–6 based on the estimation of $\hat{v}$ and CFO can be estimated block to block without the aid of more pilot symbols.*

As explained in Section 5.2, the frequency offset changes from block to block due to the variation of the Doppler shift in OFDM systems. In the proposed MLMS-like algorithm, the frequency offset in the consecutive OFDM block can be compensated based on the previous frequency offset estimation and the compensated OFDM block is used as the pilot symbols to track the variation of the frequency offset. Based on this criterion, the proposed MLMS-like algorithm can track the variation of the frequency offset block to block without the aid of more pilot symbols.

5.3.3 Moose's Estimator

In the numerical results section, the proposed LMS-like estimator is compared with Moose's estimator [43] to illustrate the capability of the proposed LMS-like estimator to track the variation of the frequency offset. Hence, Moose's estimator is briefly introduced in this subsection.

An OFDM transmission symbol is given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}. \quad (5.17)$$

At the receiver, the received symbols with CFO can be expressed as

$$y_n = \frac{1}{\sqrt{N}} \left[\sum_{k=0}^{N-1} X_k e^{j2\pi n(k+v)/N} \right] + w_n \quad (5.18)$$

where v is the normalized frequency offset and w_n is the complex envelope of AWGN noise.

For the Moose's estimator, two repeated OFDM blocks are transmitted. Hence, the $2N$ point sequence, in the absence of noise, can be expressed as

$$r_n = \frac{1}{\sqrt{N}} \left[\sum_{k=0}^{N-1} X_k e^{j2\pi n(k+v)/N} \right] \quad n = 0, 1, \dots, 2N - 1. \quad (5.19)$$

The k th element of the N point DFT of the first N points is given by

$$R_{1k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r_n e^{-j2\pi nk/N} \quad k = 0, 1, \dots, N - 1 \quad (5.20)$$

and the k th element of the second N points is

$$\begin{aligned} R_{2k} &= \frac{1}{\sqrt{N}} \sum_{n=N}^{2N-1} r_n e^{-j2\pi nk/N} \\ &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} r_{n+N} e^{-j2\pi nk/N} \quad k = 0, 1, \dots, N - 1 \end{aligned} \quad (5.21)$$

From (5.19), it is observed that

$$r_{n+N} = r_n e^{j2\pi v} \quad (5.22)$$

and it is obtained that

$$R_{2k} = R_{1k}e^{j2\pi v}. \quad (5.23)$$

Including the effect of AWGN noise, it is obtained that

$$Y_{1k} = R_{1k} + W_{1k}, \quad k = 0, 1, \dots, N-1 \quad (5.24)$$

and

$$Y_{2k} = R_{1k}e^{j2\pi v} + W_{2k}, \quad k = 0, 1, \dots, N-1. \quad (5.25)$$

Assuming that the frequency offset v is constant between the two repeated OFDM blocks, the frequency offset can be estimated by

$$\hat{v} = \frac{1}{2\pi} \tan^{-1} \left\{ \left(\sum_{k=0}^{N-1} \text{Im}[Y_{2k} Y_{1k}^*] \right) / \left(\sum_{k=0}^{N-1} \text{Re}[Y_{2k} Y_{1k}^*] \right) \right\}. \quad (5.26)$$

Note that Moose's estimator compares the phase of each of the subcarriers between the two repeated OFDM symbols to estimate the frequency offset. If the frequency offset between the two blocks is varying, the accuracy of Moose's estimator will be seriously affected.

5.4 Performance Analysis of the LMS-like Algorithm

In this section, we investigate the performance of the proposed LMS-like algorithm. Here we provide the steady state analysis. An important performance measure for an adaptive filter is its steady state MSE, which is defined as [100]

$$\begin{aligned}
 \text{MSE} &= \lim_{k \rightarrow \infty} \text{E}[|e_k|^2] \\
 &= \lim_{k \rightarrow \infty} \text{E}[|d_k - x_k W_k|^2] \\
 &= \lim_{k \rightarrow \infty} \text{E}[|\eta_k + d_k \tilde{w}_k|^2]
 \end{aligned} \tag{5.27}$$

where

$$\tilde{w}_k = W_k - W^o \tag{5.28}$$

denotes the weight error. Here W_k denotes the weight and W^o is the optimum weight to be obtained. It is assumed that the noise sequence η_k is i.i.d. (independent, identically distributed) and statistically independent of d_k . The MSE is equivalently expressed as

$$\begin{aligned}
 \text{MSE} &= \sigma^2 + \lim_{k \rightarrow \infty} \text{E}[|d_k \tilde{w}_k|^2] \\
 &= \sigma^2 + \lim_{k \rightarrow \infty} \text{E}[|\tilde{w}_k|^2]
 \end{aligned} \tag{5.29}$$

where we assume

$$\mathbb{E}[|d_k|^2] = 1 \quad (5.30)$$

and d_k is independent of $\tilde{w}_k$.

The steady state excess mean square error (EMSE) is defined as [100]

$$\zeta = \lim_{k \rightarrow \infty} \mathbb{E}[|\tilde{w}_k|^2]. \quad (5.31)$$

According to the steady state analysis in [100, 101], the EMSE ζ is obtained as

$$\zeta = \frac{\mu}{2} \sigma^2 \text{tr}(\mathbf{R}) \quad (5.32)$$

where

$$\mathbf{R} = \mathbb{E}[\mathbf{x}_k \mathbf{x}_k^H] \quad (5.33)$$

is the correlation matrix of the data sequence $\mathbf{x}_k$ and $\text{tr}(\cdot)$ represents the trace of the matrix. The element of $\mathbf{x}_k$ has been defined in (5.2). The trace of the correlation matrix $\mathbf{R}$ can be easily obtained as

$$\text{tr}(\mathbf{R}) = (1 + \sigma^2)/N \quad (5.34)$$

where the trace is normalized to N subcarriers. Thus, the EMSE is obtained as

$$\zeta = \frac{\mu}{2N} \sigma^2 (1 + \sigma^2). \quad (5.35)$$

The EMSE reflects the variance of the weight error and can be used to evaluate the accuracy of the proposed LMS-like estimator. The theoretical results will be validated by simulations in the next section.

5.5 Numerical Results and Discussions

In this section, the numerical results of the proposed LMS-like and MLMS-like algorithms are presented. To evaluate the accuracy of the proposed estimators, we use the normalized mean-square-error (MSE) defined as

$$\text{MSE} = E(|\hat{v} - v|^2) \quad (5.36)$$

where $\hat{v}$ is the estimated frequency offset and v is the true frequency offset. Note that the MSE describes the accuracy between the estimated offset and the true frequency offset. The number of subcarriers $N = 256$ is used for the following simulations.

Figure 5.3 presents the MSE versus E_b/N_0 for various step size μ in OFDM systems. It is observed that the MSE results in general decrease as the step size μ decreases or E_b/N_0 increases. It is because a smaller step size gives smaller misadjustment error after the convergence. The theoretical MSE results can be approximated by (5.35) and are also plotted accordingly in Fig. 5.3 with the solid lines. It is observed

Figure 5.3: Mean-square-error (MSE) of the proposed LMS-like estimator versus E_b/N_0 with $K = 512$ and various step size μ

that the simulation and theoretical results are matched very well. It verifies that the theoretical approximation obtained in (5.35) is reasonable and accurate.

The learning curves of the LMS-like estimator with different step sizes are presented in Figs. 5.4, 5.5 and 5.6, respectively. Figure 5.4 shows the learning curves of the LMS-like estimator with step size $\mu = 0.05$. It is observed that the misadjustment after the convergence is small, which gives an accurate estimation of carrier frequency offset. The weights converge at around 380 iterations and the convergence rate is relatively slow. On the other hand, the misadjustment is larger for the step

size of $\mu = 0.1$ as shown in Fig. 5.5 and $\mu = 0.3$ as shown in Fig. 5.6, which affects the estimation accuracy. For a larger step size, the convergence rate is faster. It is seen that the weights converge at around 300 iterations for step size of $\mu = 0.1$ and about 220 iterations for step size of $\mu = 0.3$. If the adaptive process converges within a limited number of iterations, a smaller step size should be chosen to achieve better performance. When the true frequency offset is large, it takes more iterations for a smaller step size to converge to the true value. If the updating weights cannot converge within the training symbols, the accuracy of the frequency offset estimation is seriously affected. As the learning curve shown in Fig. 5.7, the weights cannot converge within two OFDM training blocks for a small step size of $\mu = 0.1$. Note that the true frequency offset is $\nu = 1.4$. In this case, a larger step size must be chosen to guarantee the convergence. As an illustration, the weights converge within the two training blocks for a larger step size of $\mu = 0.5$. Thus, the selection of the step size is a tradeoff between the convergence rate and misadjustment. It should be properly selected according to the system performance requirement.

Figure 5.8 shows the capability of the proposed LMS-like estimator to track the variation of the frequency offset. Here two OFDM blocks are used as the training blocks. Note that the Doppler shift ϵ_d can be assumed to be stationary within one OFDM block, but it may change in adjacent blocks due to the variation of the vehicle speed [89]. The variation of the Doppler shift ϵ_d between the two blocks is defined as $\Delta\epsilon_d$. From the simulation results, it is observed that the performance

Figure 5.4: Learning curve of the proposed LMS-like estimator with $\mu = 0.05$, $E_b/N_0 = 10$ dB, exact frequency offset $\nu = 0.2$ and $K = 512$

of the proposed LMS-like algorithm is more or less the same for different values of $\Delta\epsilon_d$. It supports that the variation of the frequency offset does not affect the accuracy of the proposed LMS-like estimator. In other words, it verifies that the proposed LMS-like estimator can track the variation of the frequency offset, $\Delta\epsilon_d$, well. Here we select the step size $\mu = 0.05$. We also include the performance curve using Moose's algorithm [43] for comparison, where two repeated training blocks are used. It is observed that the proposed LMS-like algorithm has the comparable performance to Moose's algorithm when $\Delta\epsilon_d = 0$. However, the performance of Moose's algorithm degrades seriously when $\Delta\epsilon_d \neq 0$. It shows that the frequency offset variation between the OFDM blocks affects the estimation accuracy of Moose's

Figure 5.5: Learning curve of the proposed LMS-like estimator with $\mu = 0.1$, $E_b/N_0 = 10$ dB, exact frequency offset $\nu = 0.2$ and $K = 512$

algorithm seriously and it also supports that the proposed LMS-like algorithm has the capability to trace the variation of the frequency offset caused by the Doppler shift.

In Fig. 5.9, the performance of the proposed MLMS-like algorithm is evaluated. We observe two OFDM blocks: the first OFDM block is the training block and the second block is the transmitted data. We compensate the frequency offset in the second block based on the frequency offset estimation from the first training block and use the estimated data as the pilot symbols to estimate the frequency offset for the second OFDM block. It is observed that the proposed MLMS-like algorithm

Figure 5.6: Learning curve of the proposed LMS-like estimator with $\mu = 0.3$, $E_b/N_0 = 10$ dB, exact frequency offset $\nu = 0.2$ and $K = 512$

is able to track the variation of the frequency offset. When $E_b/N_0 > 10$ dB, the estimation accuracy is almost the same as that of the LMS-like algorithm, which uses two OFDM blocks as the training symbols. Figure 5.10 shows the frequency offset tracking capability of the proposed MLMS-like estimator. Here we observe four OFDM blocks, only the first OFDM block includes the training symbols and the remaining three OFDM blocks contain the transmitted symbols. Each block is assumed to have different frequency offset due to the variation of the Doppler shift. In Fig. 5.10, the solid curve is the estimated frequency offset and the dashed line is the exact frequency offset variation. It is observed that the proposed MLMS-like

Figure 5.7: Learning curve of the proposed LMS-like estimator with $\mu = 0.1$ and $\mu = 0.5$, respectively, $E_b/N_0 = 10$ dB, exact frequency offset $v = 1.4$ and $K = 512$

estimator can track the variation of the frequency offset block to block closely.

Previously, it was assumed that frequency offset is stationary within one OFDM block but it varies from block to block. Using the LMS-like algorithm, this assumption can be further relaxed. In the following, we assume that the varying frequency offset v_k is expressed as

$$\begin{aligned}
 v_k &= \epsilon_o + \epsilon_d \cos(\pi k/N) \\
 &= -0.1 + 0.02 \cos(\pi k/N).
 \end{aligned} \tag{5.37}$$

Figure 5.8: Mean-square-error (MSE) results of the proposed LMS-like estimator versus E_b/N_0 with $\mu = 0.05$, $K = 512$, $v = 0.1$ and various $\Delta\epsilon_d$

Note that the frequency offset is changing within one OFDM block. The tracking capability of the proposed LMS-like estimator is presented in Fig. 5.11. It is observed that the proposed LMS-like estimator can track the variation of the frequency offset even when the frequency offset varies within one OFDM block.

The proposed LMS-like estimator can also be applied to estimate the frequency offset over multi-path fading channels. The numerical results are presented in Fig. 5.12. Here we assume that the frequency-selective fading channel has 6 independent paths. The amplitude of the k th path, A_k , follows the Rayleigh distribution and the power

Figure 5.9: Mean-square-error (MSE) of the proposed MLMS-like estimator versus E_b/N_0 with $\mu = 0.05$, $K = 512$ and various $\Delta\epsilon_d$

delay profile is $E[A_k^2] = e^{-k/3}$. The phase shift of each path is uniformly distributed over $[0, 2\pi)$. It shows that the proposed LMS-like estimator is still effective over frequency-selective fading channels, although the estimation accuracy is affected by the attenuation of the receiving signals. We observe from Fig. 5.12 that a smaller step size gives better performance, which is consistent to the results over AWGN channels.

To evaluate the performance of the proposed LMS-like estimator over a more practical environment, we use 3GPP channel models [102] for the following simulations.

Figure 5.10: Frequency offset tracking capability of the proposed MLMS-like estimator with $\mu = 0.05$ and $E_b/N_0 = 10$ dB

The carrier frequency is $f_c = 2.5$ GHz and the number of subcarriers is $N = 1024$. We verify the LMS-like estimator with three different channel models, namely, “*urban micro*”, “*urban macro*”, and “*suburban macro*”, respectively. The channel parameters are listed in Table 5.1. The simulation results are presented in Fig. 5.13, Fig. 5.14, and Fig. 5.15, respectively. It is observed from the simulations that a smaller step size gives smaller MSE, which is still consistent with the AWGN scenario. From the simulation results verified with 3GPP channel models, it is concluded that the proposed LMS-like estimator is effective over a practical environment.

Figure 5.11: Learning curve of the proposed LMS-like estimator with varying frequency offset, $\mu = 0.05$, and $E_b/N_0 = 20$ dB

Table 5.1: 3GPP channel parameters

	Cell Radius (km)	BS-MS Dis- tance (m)	Vehicle Speed (km/h)	Tap Delay(μ s)
Urban micro	0.5	100	100	[0, 0.20, 0.28, 0.66, 0.81, 0.92]
Urban macro	1.5	1000	100	[0, 0.25, 0.36, 1.04, 2.73, 4.60]
Suburban macro	1.5	1000	250	[0, 0.06, 0.14, 0.40, 1.38, 2.83]

It is well known that recursive least-squares (RLS) algorithm has a faster convergence rate than the LMS algorithm. Typically, the convergence rate of the RLS

Figure 5.12: MSE of the proposed LMS-like estimator versus E_b/N_0 over frequency-selective fading channels, $K = 512$

algorithm is an order of magnitude faster than that of the LMS algorithm. If the RLS algorithm is utilized for CFO estimation, better performance can be expected. The RLS algorithm exploits a relation in matrix algebra known as the matrix inversion lemma, which is presented as follows. Let $\mathbf{A}$ and $\mathbf{B}$ be two positive-definite M -by- M matrices related by

$$\mathbf{A} = \mathbf{B}^{-1} + \mathbf{C}\mathbf{D}^{-1}\mathbf{C}^H, \quad (5.38)$$

where $\mathbf{D}$ is a positive-definite N -by- M matrix and $\mathbf{C}$ is an M -by- N matrix. According to the matrix inversion lemma, the inverse of the matrix $\mathbf{A}$ is obtained

Figure 5.13: MSE of the proposed LMS-like estimator versus E_b/N_0 over urban micro channels, $N = 1024$, $K = 2048$

from

$$\mathbf{A}^{-1} = \mathbf{B} - \mathbf{B}\mathbf{C}(\mathbf{D} + \mathbf{C}^H\mathbf{B}\mathbf{C})^{-1}\mathbf{C}^H\mathbf{B} \quad (5.39)$$

Assume the following identifications:

$$\mathbf{A} = \Phi(n) \quad (5.40)$$

$$\mathbf{B}^{-1} = \lambda\Phi(n-1) \quad (5.41)$$

$$\mathbf{C} = \mathbf{u}(n) \quad (5.42)$$

Figure 5.14: MSE of the proposed LMS-like estimator versus E_b/N_0 over urban macro channels, $N = 1024$, $K = 2048$

and

$$\mathbf{D} = 1, \quad (5.43)$$

where $\mathbf{u}(n)$ is the input data, $\Phi(n)$ is the correlation matrix of input data, which is assumed to be nonsingular and therefore invertible, and λ is a positive constant.

The matrix inversion lemma is applied to obtain a recursive equation for the tap-weight updating in the RLS algorithm, which is summarized in [92].

The RLS algorithm *whitens* the input data by using the inverse correlation matrix of the data and thus, gives faster convergence rate. However, this improvement

Figure 5.15: MSE of the proposed LMS-like estimator versus E_b/N_0 over suburban macro channels, $N = 1024$, $K = 2048$

in performance is achieved at the expense of an increase in computational complexity. Compared with the LMS algorithm, the RLS algorithm exploits the inverse correlation matrix instead of the gradient descent [92]. This is the fundamental difference between both algorithms. However, the gradient estimate is utilized for CFO estimation in the proposed LMS-like algorithm. The real-valued weight in the LMS-like algorithm has a close relation with the frequency offset. When the gradient estimate ∇_k is equal to zero, the updated weight can be used to estimate the frequency offset. Thus, RLS algorithm cannot be directly used for CFO estimation, although it has a faster convergence rate.

5.6 Conclusion

In this chapter, we have proposed an adaptive LMS-like algorithm to estimate the integer and fractional parts of the carrier frequency offset (CFO) simultaneously in OFDM systems. The LMS-like estimator achieves comparable performance and a wider estimation range compared to Moose's estimator [43]. The time-varying nature of the Doppler shift causes the frequency offset to change from time to time, which affects the estimation accuracy of many existing estimators. The proposed LMS-like estimator can track the variation of the Doppler shift closely and the estimation accuracy is not sensitive to frequency offset over a wide range. This superiority makes the proposed LMS-like estimator more reliable in time-varying environments. To further improve the bandwidth efficiency, we have proposed a modified LMS-like (MLMS-like) algorithm, which requires only one OFDM block as the training symbols. The MLMS-like algorithm can track the variation of the frequency offset from block to block without the aid of more pilot symbols and the estimation accuracy of the MLMS-like algorithm is almost the same as that of the LMS-like algorithm. The analytical weight error expressions have been obtained to evaluate the performance of the proposed LMS-like algorithm and the theoretical results are in good agreement with the simulation results. The numerical results verify that the proposed estimators are effective in OFDM systems over both AWGN and frequency-selective fading channels.

Chapter 6

Conclusion and Recommendations

6.1 Conclusion

In this thesis, we have addressed the carrier frequency offset (CFO) issue. We focus on the estimation algorithms of carrier frequency offset in orthogonal frequency division multiplexing (OFDM) systems and the performance analysis of the proposed algorithms have been conducted.

We proposed a minimum output variance (MOV) estimator to estimate the fractional part of the frequency offset. The probability density function (PDF) of the magnitude output could be approximated as Gaussian mixture. Based on the PDF, the expressions of the mean and variance of the magnitude output have been obtained. The mean value monotonically increases and the variance monotonically decreases as the absolute value of carrier frequency offset decreases. It is concluded that the maximum mean value and minimum variance are achieved when carrier frequency offset is zero. This feature is utilized to estimate the carrier frequency offset. The performance analysis of the proposed MOV estimator has been conducted

over additive white Gaussian noise (AWGN) channels. Numerical results were presented to validate the accuracy of the MOV estimator and verify the theoretical analysis. The accuracy of the MOV estimator is generally improved as the number of blocks increases or signal-to-noise ratio (SNR) increases. The T&H estimator proposed in [51] has been briefly introduced and compared with the MOV estimator. The MOV estimator outperforms the T&H estimator. Also, the proposed MOV estimator could be implemented efficiently by fast Fourier transform (FFT). Thus, the computational complexity is generally lower than that of the T&H estimator. The orthogonality among the information bearing carriers and virtual carrier might be used to estimate carrier frequency offset in the T&H estimator. Because of the existence of the virtual carriers, the bandwidth efficiency would be reduced. In a MOV estimator, all of the subcarriers could be used to transmit useful data, which exhibits high bandwidth efficiency. The MOV estimator is effective over frequency-selective fading channels even when the channel state information is not available to the MOV estimator. However, an error floor is generated because of the channel dispersion. To mitigate the error floor, MRC has been applied. With the aid of the MRC technique, the accuracy of the MOV estimator is significantly improved and the error floor is eliminated.

A maximum likelihood estimator (MLE) has been proposed to estimate the integer part of the frequency offset. After simplifying and maximizing the likelihood function, the MLE has been derived based on only one OFDM block as pilot symbols.

To reduce the computational complexity of the proposed MLE, a sub-optimum estimator (Sub MLE) has been investigated. A subset of symbols in one OFDM block could be chosen to be the pilot symbols. The proposed Sub MLE not only reduces the computational complexity but also improves the bandwidth efficiency. Based on the hypothesis testing, a threshold sub-optimum estimator (T-Sub MLE) was studied. The proposed T-Sub MLE could further reduce the computational complexity. After examining the mean and variance of the estimation metrics, the analytical error probability of the T-Sub MLE has been achieved. The theoretical results were also validated by simulations, which verify that the proposed estimators are effective and reliable in OFDM systems over both AWGN and frequency-selective fading channels. Moreover, the proposed estimators outperform the estimators proposed in [64] and [67].

To estimate the integer and fractional part of frequency offset simultaneously, an adaptive LMS-like estimator has been suggested. The LMS-like estimator is a single-weight LMS estimator, which is proposed according to the principle of the LMS. The carrier frequency offset includes not only the stationary part, which is caused by the mismatch of the carrier frequencies between the local oscillators in transceivers, but also the time varying part, which is caused by the Doppler shift. The time varying frequency offset affects the accuracy of many existing estimators. Because of the adaptive nature of the proposed LMS-like estimator, the LMS-like estimator can track the variation of the frequency offset caused by the Doppler shift. This

superiority makes the proposed LMS-like estimator more reliable and flexible in a practical time-varying environment. To further improve the bandwidth efficiency, a modified LMS-like (MLMS-like) estimator has been proposed, which requires only one OFDM block as the training symbols. Using the training symbols, the carrier frequency offset could roughly be estimated. The estimated frequency offset would be used to compensate the next OFDM block. The compensated block could be used to track the variation of the frequency offset. Thus, the MLMS-like estimator is able to track the variation of the frequency offset from block to block without the aid of more pilot symbols and the estimation accuracy of the MLMS-like algorithm is almost the same as that of the LMS-like algorithm. The analytical weight error expressions have been obtained to evaluate the performance of the proposed LMS-like algorithm and the theoretical results are in good agreement with the simulation results. The numerical results support that the proposed estimators are effective in OFDM systems over both AWGN and fading channels. Since the proposed estimators can estimate the integer and fractional part of frequency offset simultaneously, a wide estimation range is exhibited.

6.2 Recommendations for Future Research

In this thesis, we focus on the carrier frequency offset estimation algorithms in OFDM systems. Three major algorithms have been proposed and the performance analysis has been conducted. The scope of current work can be further extended.

The expectation maximization (EM) algorithm is a well-known iterative method for locating modes of a likelihood function and is characterized by simple implementation and stability. The space alternating generalized EM (SAGE) has the potential to speed up convergence and preserves the advantage of simple implementation. The EM and SAGE algorithms have been proposed for direction-of-arrival (DOA) estimation [103] and channel estimation [104]. It is possible to utilize the EM and/or SAGE to estimate the carrier frequency offset in OFDM systems.

In the literature, most of algorithms for coarse carrier frequency offset estimation are based on pilot symbols. If a blind estimation algorithm is used for coarse carrier frequency offset, the bandwidth efficiency can be further improved. In [105], the authors provide the theory of offset DFTs. It is interesting to utilize the eigenvectors and eigenvalues of the offset DFTs to estimate the carrier frequency offset in OFDM systems. Utilizing the characteristics of offset DFTs, the coarse carrier frequency offset can be estimated blindly.

In this thesis, we concentrate on carrier frequency offset estimation. As a mat-

ter of fact, timing offset also affects the system performance. It is interesting to jointly estimate the timing offset and carrier frequency offset. Then the estimation is more feasible for practical wireless environments. Also, many channel estimation algorithms do not consider the carrier frequency offset. If the carrier frequency offset is not estimated and eliminated before the channel estimation, the accuracy of channel estimation is seriously affected. Thus, it is beneficial to develop an algorithm to estimate the carrier frequency offset and channel state jointly. In this way, the residual errors of carrier frequency offset or channel estimation do not degrade the quality of the subsequent processing. Recently, the in-phase and quadrature-phase (IQ) imbalance problem has been investigated in OFDM systems [106, 107]. It is interesting to consider the carrier frequency offset problem with the effect of IQ imbalance.

The combination of OFDM with other techniques enables many application in future wireless communications. Multi-carrier CDMA (MC-CDMA) is the combination of OFDM with CDMA. It provides a solution for multi-carrier modulation (MCM) schemes to support multi-users. In a MC-CDMA system, the carrier frequency offset is also a serious problem. It is beneficial to develop an algorithm to estimate carrier frequency offset in MC-CDMA systems. In MC-CDMA systems, carrier frequency offset causes not only inter-carrier interference (ICI) but also multi-user interference (MUI). To eliminate the ICI and MUI is an important task in MC-CDMA systems. However, the estimation of carrier frequency offset is more difficult and

complicated in MC-CDMA schemes. The multiple-input multiple-output OFDM (MIMO-OFDM) system and multi-band OFDM UWB system also encounter carrier frequency offset problem. The carrier frequency in these systems is higher than conventional OFDM systems. Thus, it is more important to design proper algorithms to solve the frequency offset problem for these systems.

Author's Publications

1. F. Yang, K. H. Li, and K. C. Teh, "A carrier frequency offset estimator with minimum output variance for OFDM systems," *IEEE Communications Letters*, vol. 8, no. 11, pp. 677–679, Nov. 2004.
2. F. Yang, K. H. Li, and K. C. Teh, "Performance analysis of the blind minimum output variance estimator for carrier frequency offset in OFDM systems," *EURASIP Journal on Applied Signal Processing* **2006** (2006), Article ID 49257, 8 pages.
3. F. Yang, K. H. Li, and K. C. Teh, "Adaptive LMS-like algorithm for carrier frequency offset estimation in OFDM systems," submitted to *IEE Proceedings on Communications*.
4. F. Yang, K. H. Li, and K. C. Teh, "Maximum likelihood estimator for coarse carrier frequency offset estimation in OFDM systems," submitted to *IEEE Transactions on Vehicular Technology*.

5. F. Yang, K. H. Li, and K. C. Teh, “Performance of carrier frequency offset estimation in MC-CDMA systems over frequency selective fading channels,” in *Proc. of 4th International Conference on Information, Communications & Signal Processing*, pp. 673–677, Dec. 2003, Singapore.
6. F. Yang, K. H. Li, and K. C. Teh, “Adaptive LMS-like algorithm for carrier frequency offset estimation in OFDM systems,” in *Proc. of 8th International Symposium on Signal Processing and Its Applications*, pp. 131–134, Aug. 2005, Australia.

Bibliography

- [1] J. A. C. Bingham, “Multicarrier modulation for data transmission: An idea whose time has come,” *IEEE Commun. Mag.*, vol. 28, no. 5, pp. 5–14, May 1990.
- [2] E. T. Standard, “Radio Broadcast Systems: Digital Audio Broadcasting (DAB) to Mobile, Portable, and Fixed Receivers,” *preETS 300 401, Tech. Rep.*, Mar. 1994.
- [3] U. Reimers, “DVB-T: The COFDM based system for terrestrial television,” *Electron. Commun. Eng. J.*, vol. 9, pp. 28–32, Feb. 1995.
- [4] J. Khun-Jush, P. Schramm, U. Wachsmann, and F. Wenger, “Structure and performance of the HIPERLAN/2 physical layer,” in *Proc. IEEE Veh. Technol. Conf.*, 1999, vol. 5, pp. 2667–2671.
- [5] IEEE Std. 802.11a, “Wireless LAN medium access control (MAC) and physical layer (PHY) specifications: High-speed physical layer extension in the 5-GHz band,” *IEEE*, 1999.

- [6] *Air Interface for Fixed Broad-Band Wireless Access Systems. Part A: Systems Between 2-11 GHz*, IEEE 802.16ab-01/01 Std., Rev. 1, July 2001.
- [7] N. Yee, J. P. Linnartz, and G. Fettweis, “Multi-carrier CDMA in indoor wireless radio networks,” in *Proc. IEEE PIMRC*, 1993, pp. 109–113.
- [8] S. Hara and R. Prasad, “Design and performance of Multicarrier CDMA system in frequency-selective Rayleigh fading channels,” *IEEE Trans. Veh. Technol.*, vol. 48, no. 5, pp. 1584–1595, Sept. 1999.
- [9] K. Fazel, “Performance of CDMA/OFDM for mobile communication system,” in *Proc. Int. Conf. on Universal Personal Communications*, Oct. 12–15, 1993, pp. 975–979.
- [10] H. Yang, “A road to future broadband wireless access: MIMO-OFDM-based air interface,” *IEEE Commun. Mag.*, vol. 43, no. 1, pp. 53–60, Jan. 2005.
- [11] G. L. Stuber, J. R. Barry, S. W. McLaughlin, Y. (Geoffrey) Li, M. A. Ingram, and T. G. Pratt, “Broadband MIMO-OFDM wireless communications,” *IEEE Proceedings*, vol. 92, no. 2, pp. 271–294, Feb. 2004.
- [12] H. Sampath, S. Talwar, J. Tellado, V. Erceg, and A. Paulraj, “A fourth-generation MIMO-OFDM broadband wireless system: design, performance, and field trial results,” *IEEE Commun. Mag.*, vol. 40, no. 9, pp. 143–149, Sept. 2002.
- [13] A. Batra, J. Balakrishnan, G. R. Aiello, G. R. Foerster, and A. Dabak, “Design of a multiband OFDM system for realistic UWB channel environments,”

- IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 9, pp. 2123–2138, Sept. 2004.
- [14] L. Yang and G. B. Giannakis, “Ultra wideband communications: an idea whose time has come,” *IEEE Signal Processing Mag.*, vol. 21, no. 6, pp. 26–54, Nov. 2004.
- [15] W. C. Chung, N. J. August, and D. S. Ha, “Signaling and multiple access techniques for ultra wideband 4G wireless communication systems,” *IEEE Wireless Commun.*, vol. 12, no. 2, pp. 46–55, Apr. 2005.
- [16] R. W. Chang, “Synthesis of band-limited orthogonal signals for multichannel data transmission,” *Bell System Tech. J.*, vol. 45, pp. 1775–1796, 1966.
- [17] B. R. Saltzberg, “Performance of an efficient parallel data transmission system,” *IEEE Trans. Commun.*, vol. 15, no. 6, pp. 805–811, Dec. 1967.
- [18] S. B. Weinstein and P. M. Ebert, “Data transmission by frequency division multiplexing using the discrete Fourier transform,” *IEEE Trans. Commun.*, vol. 19, no. 5, pp. 628–634, Oct. 1971.
- [19] A. Peled and A. Ruiz, “Frequency domain data transmission using reduced computational complexity algorithms,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Procssing*, Denver, Dec. 1980, pp. 964–967.
- [20] B. Hirosaki, “An orthogonally multiplexed QAM system using the discrete Fourier transform,” *IEEE Trans. Commun.*, vol. 29, no. 7, pp. 982–989, July 1981.

- [21] R. W. Chang and R. A. Gibby, "A theoretical study of performance of an orthogonal multiplexing data transmission scheme," *IEEE Trans. Commun. Technol.*, vol. 16, no. 4, pp. 529–540, Aug. 1968.
- [22] L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 33, no. 7, pp. 665–675, July 1985.
- [23] W. L. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. Broadcasting*, vol. 41, no. 1, pp. 1–8, Mar. 1995.
- [24] O. Edfors, M. Sandell, J. J. van de Beek, D. Landstrom, and F. Sjoberg, "An introduction to orthogonal frequency division multiplexing," *Tech. Rep., Lulea University of Technology, Sweden*, Sept. 1996.
- [25] H. Hashemi, "The indoor radio propagation channel," *IEEE Proceedings*, vol. 81, no. 7, pp. 943–968, July 1993.
- [26] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1978. new ed. 1994.
- [27] W. C. Y. Lee, *Mobile Communications Engineering: Theory and Applications*, McGraw-Hill, 2nd ed., 1997.
- [28] H. Sari, G. Karam, and I. Jeanclaude, "Transmission techniques for digital terrestrial TV broadcasting," *IEEE Commun. Mag.*, vol. 33, no. 2, pp. 100–109, Feb. 1995.

- [29] Y. Gong and K. B. Letaief, “An efficient space-frequency coded OFDM system for broadband wireless communications,” *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 2019–2029, Nov. 2003.
- [30] Z. Wang and G. B. Giannakis, “Complex-field coding for OFDM over fading wireless channels,” *IEEE Trans. Inf. Theory*, vol. 49, no. 3, pp. 707–720, Mar. 2003.
- [31] T. Pollet, M. Van Bladel, and M. Moeneclaey, “BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise,” *IEEE Trans. Commun.*, vol. 43, no. 2, pp. 191–193, Feb. 1995.
- [32] L. Tomba, “On the effects of Wiener phase noise in OFDM systems,” *IEEE Trans. Commun.*, vol. 46, no. 5, pp. 580–583, May 1998.
- [33] R. V. Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Boston: Artech House, 2000.
- [34] S. H. Han and J. H. Lee, “An overview of peak-to-average power ratio reduction techniques for multicarrier transmission,” *IEEE Wireless Commun.*, vol. 12, no. 2, pp. 56–65, Apr. 2005.
- [35] M. Pauli and H. P. Kuchenbecker, “Minimization of the intermodulation distortion of a nonlinearly amplified OFDM signal,” *Wireless Personal Communications*, vol. 4, no. 1, pp. 93–101, Jan. 1997.

- [36] T. May and H. Rohling, “Reducing the peak-to-average power ratio in OFDM radio transmission systems,” in *Proc. IEEE Veh. Technol. Conf.*, Canada, May 18–21, 1998, pp. 2774–2778.
- [37] R. V. Nee, “OFDM codes for peak-to-average power reduction and error correction,” in *Proc. IEEE Global Telecom. Conf.*, London, Nov. 18–22, 1996, pp. 740–744.
- [38] S. H. Muller, W. Bauml, R. F. H. Fischer, and J. B. Huber, “OFDM with reduced peak-to-average power ratio by multiple signal representation,” *Annals of Telecommunications*, vol. 52, no. 1–2, pp. 58–67, Feb. 1997.
- [39] S. H. Muller and J. B. Huber, “OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences,” *IEE Electronics Lett.*, vol. 33, no. 5, pp. 368–369, Feb. 1997.
- [40] IEEE, “Supplement to standard for telecommunications and information exchange between systems – LAN/MAN specific requirements – Part II: Wireless MAC and PHY specifications: Higher speed physical layer extension in the 2.4-GHz band,” *P802.11b/D7.0*, July 1999.
- [41] IEEE, “Supplement to standard for telecommunications and information exchange between systems – LAN/MAN specific requirements – Part II: Wireless MAC and PHY specifications: Higher speed physical layer in the 5-GHz band,” *P802.11b/D7.0*, July 1999.

- [42] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, 2nd ed., 2002.
- [43] P. H. Moose, “A technique for orthogonal frequency division multiplexing frequency offset correction,” *IEEE Trans. Commun.*, vol. 42, no. 10, pp. 2908–2914, Oct. 1994.
- [44] L. Wei and C. Schlegel, “Synchronization requirements for multi-user OFDM on satellite mobile and two-path Rayleigh fading channels,” *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 887–895, Feb/Mar/Apr 1995.
- [45] J. J. van de Beek, M. Sandell, and P. O. Borjesson, “ML estimation of time and frequency offset in OFDM systems,” *IEEE Trans. Signal Processing*, vol. 45, no. 7, pp. 1800–1805, July 1997.
- [46] N. Lashkarian and S. Kiaei, “Class of cyclic-based estimators for frequency offset estimation of OFDM systems,” *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 2139–2149, Dec. 2000.
- [47] M. Luise, M. Marselli, and R. Reggiannini, “Low-complexity blind carrier frequency recovery for OFDM signals over frequency-selective radio channels,” *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1182–1188, July 2002.
- [48] R. Roy, A. Paulraj, and T. Kailath, “ESPRIT-A subspace rotation approach to estimation of parameters of cisoids in noise,” *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. ASSP-34, no. 5, pp. 1340–1342, Oct. 1986.

- [49] U. Tureli, H. Liu, and D. Zoltowski, "OFDM blind carrier offset estimation: ESPRIT," *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1459–1461, Sept. 2000.
- [50] H. Liu and U. Tureli, "A high-efficiency carrier estimator for OFDM communications," *IEEE Commun. Lett.*, vol. 2, no. 4, pp. 104–106, Apr. 1998.
- [51] U. Tureli, D. Kivanc, and H. Liu, "Experimental and analytical studies on a high-resolution OFDM carrier frequency offset estimator," *IEEE Trans. Veh. Technol.*, vol. 50, no. 2, pp. 629–643, Mar. 2002.
- [52] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagat.*, vol. AP-34, no. 3, pp. 276–280, Mar. 1986.
- [53] B. Chen, "Maximum likelihood estimation of OFDM carrier frequency offset," *IEEE Signal Processing Lett.*, vol. 9, no. 4, pp. 123–126, Apr. 2002.
- [54] B. Chen and H. Wang, "Blind estimation of OFDM carrier frequency offset via oversampling," *IEEE Trans. Signal Processing*, vol. 52, no. 7, pp. 2047–2057, July 2004.
- [55] P. Ciblat and L. Vandendorpe, "Blind carrier frequency offset estimation for noncircular constellation-based transmissions," *IEEE Trans. Signal Processing*, vol. 51, no. 5, pp. 1378–1389, May 2003.
- [56] F. Gini and G. B. Giannakis, "Frequency offset and symbol timing recovery in flat-fading channels: A cyclostationary approach," *IEEE Trans. Commun.*, vol. 46, no. 3, pp. 400–411, Mar. 1998.

- [57] H. Bolcskei, “Blind estimation of symbol timing and carrier frequency offset in wireless OFDM systems,” *IEEE Trans. Commun.*, vol. 49, no. 6, pp. 988–999, June 2001.
- [58] M. Hsieh and C. Wei, “A low complexity frame synchronization and frequency offset compensation scheme for OFDM systems over fading channels,” *IEEE Trans. Veh. Technol.*, vol. 48, no. 5, pp. 1596–1609, Sept. 1999.
- [59] X. Ma, C. Tepedelenlioglu, G. B. Giannakis, and S. Barbarossa, “Non-data-aided carrier offset estimators for OFDM with null subcarriers: Identifiability, algorithm and performance,” *IEEE J. Select. Areas Commun.*, vol. 19, no. 12, pp. 2504–2515, Dec. 2001.
- [60] L. Kuang, Z. Ni, J. Lu, and J. Zheng, “A time-frequency decision-feedback loop for carrier frequency offset tracking in OFDM systems,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 367–373, Mar. 2005.
- [61] Y. Yao and G. B. Giannakis, “Blind carrier frequency offset estimation in SISO, MIMO, and multiuser OFDM systems,” *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 173–183, Jan. 2005.
- [62] H. Nogami and T. Nagashima, “A frequency and timing period acquisition technique for OFDM systems,” in *Proc. IEEE PIMRC '95*, 1995, pp. 1010–1015.

- [63] F. Classen and H. Meyr, "Synchronization algorithms for an OFDM system for mobile communication," in *Proc. ITG-Fachtagung*, Oct. 26–28, 1994, pp. 105–113.
- [64] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for OFDM," *IEEE Trans. Commun.*, vol. 45, no. 12, pp. 1613–1621, Dec. 1997.
- [65] M. Morelli and V. Mengali, "An improved frequency offset estimator for OFDM applications," *IEEE Commun. Lett.*, vol. 3, no. 3, pp. 75–77, Mar. 1999.
- [66] Y. H. Kim, I. Song, S. Yoon, and S. R. Park, "An efficient frequency offset estimator for OFDM systems and its performance characteristics," *IEEE Trans. Veh. Technol.*, vol. 50, no. 5, pp. 1307–1312, Sept. 2001.
- [67] M. Morelli, A. N. D'Andrea, and U. Mengali, "Frequency ambiguity resolution in OFDM systems," *IEEE Commun. Lett.*, vol. 4, no. 4, pp. 134–136, Apr. 2000.
- [68] Y. Choi, P. J. Voltz, and F. A. Cassara, "ML estimation of carrier frequency offset for multicarrier signals in Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 2, pp. 644–655, Mar. 2001.
- [69] S. Wei, D. L. Goeckel, and P. E. Kelly, "A modern extreme value theory approach to calculating the distribution of the peak-to-average power ratio in

- OFDM systems,” in *Proc. IEEE Int. Conf. Commun.*, New York, Dec. 2002, pp. 156–159.
- [70] H. Minn and S. Xing, “An optimal training signal structure for frequency offset estimation,” *IEEE Trans. Commun.*, vol. 53, no. 2, pp. 343–355, Feb. 2005.
- [71] W. Kuo and M. P. Fitz, “Frequency offset compensation of pilot symbol assisted modulation in frequency flat fading,” *IEEE Trans. Commun.*, vol. 45, no. 11, pp. 1412–1416, Nov. 1997.
- [72] M. Morelli, U. Mengali, and G. M. Vitetta, “Further results in carrier frequency estimation for transmissions over flat fading channels,” *IEEE Commun. Lett.*, vol. 2, no. 12, pp. 327–330, Dec. 1998.
- [73] H. Minn, V. K. Bhargava, and K. B. Letaief, “A robust timing and frequency synchronization for OFDM systems,” *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 822–839, July 2003.
- [74] M. Morelli and U. Mengali, “Carrier-frequency estimation for transmissions over selective channels,” *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1580–1589, Sept. 2000.
- [75] G. Gil, I. Sohn, J. Park, and Y. H. Lee, “Joint ML estimation of carrier frequency, channel, I/Q mismatch, and DC offset in communication receivers,” *IEEE Trans. Veh. Technol.*, vol. 54, no. 1, pp. 338–349, Jan. 2005.

- [76] M. G. Hebley and D. P. Taylor, "The effect of diversity on a burst-mode carrier-frequency estimator in the frequency-selective multipath channel," *IEEE Trans. Commun.*, vol. 46, no. 4, pp. 553–560, Apr. 1998.
- [77] M. Luise and R. Reggiannini, "Carrier frequency recovery in all-digital modems for burst-mode transmissions," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp. 1169–1178, Feb./Mar./Apr. 1995.
- [78] E.-R. Jeong, S.-K. Jo, and Y. H. Lee, "Least squares frequency estimation in frequency-selective channels and its application to transmissions with antenna diversity," *IEEE J. Select. Areas Commun.*, vol. 19, no. 12, pp. 2369–2380, Dec. 2001.
- [79] D. Huang and K. B. Letaief, "Carrier frequency offset estimation for OFDM systems using null subcarriers," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 813–823, May 2006.
- [80] J. P. Linnartz, "Performance analysis of synchronous MC-CDMA in mobile Rayleigh channel with both delay and doppler spreads," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1375–1387, Nov. 2001.
- [81] M. Russell and G. L. Stuber, "Interchannel interference analysis of OFDM in a mobile environment," in *Proc. IEEE Veh. Technol. Conf.*, 1995, pp. 820–824.
- [82] Y. (Geoffrey) Li and L. J. Cimini, "Bounds on the interchannel interference of OFDM in time-varying impairments," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 401–404, Mar. 2001.

- [83] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, no. 3, pp. 365–369, Mar. 1999.
- [84] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, 4th ed., 2002.
- [85] S. M. Kay, *Fundamentals of Statistical Signal Processing Volume I: Estimation Theory*, Prentice Hall, 1993.
- [86] J. G. Proakis, *Digital Communications*, McGraw-Hill, 4th ed., 2001.
- [87] K. Bang, N. Cho, J. Cho, H. Jun, K. Kim, H. Park, and D. Hong, "A coarse frequency offset estimation in an OFDM system using the concept of the coherence phase bandwidth," *IEEE Trans. Commun.*, vol. 49, no. 8, Aug. 2001.
- [88] S. M. Kay, *Fundamentals of Statistical Signal Processing Volume II: Detection Theory*, Prentice Hall, 1998.
- [89] Y. Zhao and S.-G. Haggman, "Sensitivity to doppler shift and carrier frequency errors in OFDM systems - the consequences and solutions," in *Proc. IEEE Veh. Technol. Conf.*, 1996, vol. 3, pp. 1564–1568.
- [90] B. Widrow and M. E. Hoff, "Adaptive switching circuits," in *Proc. IRE WESCON Conf. Rec.*, 1960, pp. 96–104.
- [91] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Prentice Hall, 1985.

- [92] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, 1995.
- [93] J. F. Galdino, E. L. Pinto, and M. S. de Alencar, “Analytical performance of the LMS algorithm on the estimation of wide sense stationary channels,” *IEEE Trans. Commun.*, vol. 52, no. 6, pp. 982–991, June 2004.
- [94] M. Godavarti and A. O. Hero, “Partial update LMS algorithms,” *IEEE Trans. Signal Processing*, vol. 53, no. 7, pp. 2382–2399, July 2005.
- [95] Y. Gong and C. F. N. Cowan, “An LMS style variable tap-length algorithm for structure adaptation,” *IEEE Trans. Signal Processing*, vol. 53, no. 7, pp. 2400–2407, July 2005.
- [96] G. G. Yin and V. Krishnamurthy, “LMS algorithms for tracking slow Markov chains with applications to hidden Markov estimation and adaptive multiuser detection,” *IEEE Trans. Inf. Theory*, vol. 51, no. 7, pp. 2475–2490, July 2005.
- [97] L. Lindbom, M. Sternad, and A. Ahlen, “Tracking of time-varying mobile radio channels—Part I: The Wiener LMS algorithm,” *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2207–2217, Dec. 2001.
- [98] R. H. Kwong and E. W. Johnston, “A variable step size LMS algorithm,” *IEEE Trans. Signal Processing*, vol. 40, no. 7, pp. 1633–1642, July 1992.
- [99] J. C. M. Bermudez and N. J. Bershad, “Transient and tracking performance analysis of the quantized LMS algorithm for time-varying system identification,” *IEEE Trans. Signal Processing*, vol. 44, no. 8, pp. 1990–1997, Aug. 1996.

- [100] N. R. Yousef and A. H. Sayed, “A unified approach to the steady state and tracking analyses of adaptive filters,” *IEEE Trans. Signal Processing*, vol. 49, no. 2, pp. 314–324, Feb. 2001.
- [101] S. K. Jones, R. K. Cavin, and W. M. Reed, “Analysis of error-gradient adaptive linear estimators for a class of stationary dependent processes,” *IEEE Trans. Inf. Theory*, vol. IT-28, pp. 318–329, Mar. 1982.
- [102] 3GPP TR 25.996, *Spacial channel model for Multiple Input Multiple Output (MIMO) simulations*.
- [103] P. J. Chung and J. F. Bohme, “Comparative convergence analysis of EM and SAGE algorithms in DOA estimation,” *IEEE Trans. Signal Processing*, vol. 49, no. 12, pp. 2940–2949, Dec. 2001.
- [104] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. I. Pedersen, “Channel parameter estimation in mobile radio environments using the SAGE algorithm,” *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 434–450, Mar. 1999.
- [105] S. Pei and J. Ding, “Generalized eigenvectors and fractionalization of offset DFTs and DCTs,” *IEEE Trans. Signal Processing*, vol. 52, no. 7, pp. 2032–2046, July 2004.
- [106] A. Tarighat, R. Bagheri, and A. H. Sayed, “Compensation schemes and performance analysis of IQ imbalances in OFDM receivers,” *IEEE Trans. Signal Processing*, vol. 53, no. 8, pp. 3257–3268, Aug. 2005.

- [107] G. Xing, M. Shen, and H. Liu, “Frequency offset and I/Q imbalance compensation for direct-conversion receivers,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 673–680, Mar. 2005.