Dual-actuator servo system design for fast data rate and high density hard disk drives

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DUAL-ACTUATOR SERVO SYSTEM DESIGN FOR FAST DATA RATE AND HIGH DENSITY HARD DISK DRIVES

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Summary

The dual-actuator structure has intrinsic property to meet fast data rate and high throughput requirements in hard disk drives (HDDs). In order to sustain the continuing increase of track density, the dual-actuator servo system needs to be developed for more precise head positioning accuracy in the presence of the disturbances and the interactions between the actuators. In this thesis, control structures and controller design methods are developed for the dual-actuator control system to compensate for the interactions as well as the other disturbance sources.

A linear coupled dual-input dual-output (DIDO) frequency response model to represent the dual-actuator servo plant is identified using the circle-fit modal testing method. In particular, the non-minimum phase (NMP) antiresonances in the interaction dynamics are explicitly identified such that both magnitude and phase are simultaneously matched between the identified and the measured model.

The dual-actuator tracking servo design specifications are described. In addition to the performance requirements in typical HDD servos, the interaction compensation is required for the dual-actuator servo system. Based on the use of information from the other actuator, the dual-actuator control structures are reviewed and classified into two categories: a fully coupled DIDO control structure and a

dual single-input single-output (SISO) control structure. Hence, the control design methods specific to the dual-actuator system can be seen clearly.

For the coupled DIDO controller design, the H_2 optimal control design provides systematic solution of the controller by formulating the dual-actuator tracking error specification as a standard H_2 control problem. Although the derived controller is impractically high-order, it gives the optimal tracking performance achievable in dual-actuator servo systems. Alternatively, the feedforward decoupling control (FDC) design contains a simple two-stage design process. The first stage is to design an optimal stable and causal feedforward decoupler that approximates an inverse-based perfect decoupler. The second stage is to design the feedback controller based on the decoupled dual-actuator plant. Simulation and experimental results of interaction compensation and performance robustness are presented.

To design the dual SISO controller, the control system is firstly analyzed via loop decomposition, which reveals that the interaction effects can be represented as both plant uncertainties and narrow-band disturbances acting on a SISO control loop. The stability condition is derived for the dual SISO control system. After that, the SISO controller design is separated into the feedback controller design and an add-on interaction filter design. The classical loop shaping methods are applied to design the controllers in both stages. Simulation and experimental results of interaction compensation and performance robustness are presented and compared with those obtained by coupled DIDO controller design.

Previous peak filter designs are generally applicable to reject narrow-band disturbances within a certain frequency range. Thus, a generalized optimal filter design to

reject narrow-band disturbances at all frequency ranges is developed. In addition to assign the filter poles for high gains, the filter zero is designed to minimally degrade the closed-loop system stability and obtain a smooth sensitivity curve around the disturbance frequency. A nonlinear optimization procedure is developed to select the filter parameters such that the statistical position error is minimized. Experimental results of a PZT-actuated head positioning control system demonstrate that the add-on filter can further reduce the mid-frequency PES NRRO by 8% and preserve the stability margin of the original feedback control system. The disturbance filter can be applied to reject the narrow-band RROs and NRROs in HDD servos.

The HDD tracking servo is conventionally performed by linear control because of its simplicity. However, the intrinsic limitation of linear control has constrained the achievable performance such as disturbance rejection. Thus, a nonlinear PID (NPID) controller for a class of linear plants with output feedback is studied. The controller gains are modulated by the control error and the rate of error change to yield a closed-loop system for improved disturbance rejection based on a linear PID control system. An optimization problem involving constraints written as linear matrix inequalities (LMIs) is solved to compute the controller parameters that can guarantee the stability of the NPID control system. The NPID control design is applied to the HDD track-following servo systems. Experimental results show that NPID control outperforms the linear PID control by 9% in disturbance rejection.

Nomenclature

The following abbreviations are frequently used throughout this thesis:

ARE	Algebraic Riccati Equation
BPI	Bits Per Inch
DIDO	Dual-Input Dual-Output
DSA	Dynamic Signal Analyzer
FDC	Feedforward Decoupling Control
FFT	Fast Fourier Transform
FPC	Flexible Printed Circuit
GM	Gain Margin
HDD	Hard Disk Drive
IVC	Initial Value Compensation
LDV	Laser Doppler Vibrometer
LHP	Left-Half Plane
LMI	Linear Matrix Inequality
LPID	Linear PID
MIMO	Multi-Input Multi-Output

NMP	Non-Minimum Phase
NPID	Nonlinear PID
NRRO	Non-Repeatable Runout
OLCF	Open Loop Crossover Frequency
PES	Position Error Signal
PID	Proportional-Integral-Derivative
PM	Phase Margin
PSD	Power Spectral Density
RAID	Redundant Arrays of Inexpensive Disks
RHP	Right-Half Plane
RRO	Repeatable Runout
SISO	Single-Input Single-Output
TMR	Track Mis-Registration
TPI	Tracks Per Inch
VCM	Voice Coil Motor

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Chapter 1

Introduction

Magnetic information storage technology has made astounding progress since its invention over 50 years ago. For the past several years, storage densities in hard disk drives (HDDs) have doubled every year. Smaller form factors enable the applications in various consumer electronics. Faster data transfer rate and higher areal density require novel mechanical structures and advanced servo technologies for more precise head positioning accuracy. This thesis focuses on the control design for a dual-actuator HDD to achieve this objective. In this chapter, an overview of the HDD mechanics and previous research on the HDD servo system design is firstly presented. Next, alternative high performance HDD structures are introduced. Finally, contributions and outline of the thesis are presented.

1.1 HDD System Overview

The HDDs never cease to strive for smaller form factors, faster data rate, denser areal density and cheaper price. The rapid evolution of HDDs in performance and cost

Figure 1.1: Components of a 2.5 inch HDD with top cover removed.

requires many cross-discipline technological innovations from the recording media, magnetic heads, disk technology, head-disk interface, spindle motor, signal processing, electronic circuit, mechanics and servo technologies. This section will give an overview of the servo mechanics technologies in HDDs.

1.1.1 HDD Components and Servomechanism

The HDD is a typical servomechanism with the specific task of controlling the position of the read/write heads. Figure 1.1 shows the components in a typical HDD. It consists of a stack of disks, sputtered with a thin magnetic layer or recording medium, which are written with data in concentric circles called tracks. The disks are stacked along a spindle and rotated by an electric motor inside the spindle, called the spindle motor. Electromagnetic read/write heads are used to either write data onto the disks or read information from them. The head is attached to a slider, an aerodynamically shaped block that allows the head to maintain a consistent flying height of only several nanometers on an air bearing above the disk surface. The

slider is connected to a suspension which is swaged onto the arm, or E-block, whose name comes from the “E” shape in the side view. The E-block is attached to a fixed pivot bearing and is driven by a voice coil motor (VCM) that swings between two permanent magnets. The VCM is a rotary actuator, which moves the head over the disk surface from the inner to the outer diameter by rotating the E-block about the pivot. The flexible printed circuit (FPC) attached to the side of the E-block conveys all electrical signals between the E-block and the stationary electronics block fixed on the baseplate.

In this thesis the HDD servo system refers to the VCM actuator servo system, or the head positioning servomechanism. A schematic drawing of the HDD servo system is shown in Figure 1.2. The traditional disk drive system is a noncollocated control system where the measured output occurs at the head which is not directly actuated, but the control effort is applied to the VCM. Hence, the mechanical resonance modes of the arm and suspension which are located between the VCM and the head would cause vibrations that affect the head position. The resonance modes

Figure 1.2: A schematic drawing of the HDD servo system.

in the total system must be taken into account during the control design.

The head position is not measured directly in HDDs. Instead, the servo controller must work on the position error signal (PES), which is the difference between the desired track center and the actual head position. The PES is demodulated from the servo sectors by a data channel. At the time of manufacturing, the HDD is placed in a servo track writer (STW) that embeds the PES bursts and track numbers directly on the disk at regular intervals between user data. The PES can be demodulated from the read back signal of the servo sectors as discrete-time signals. Thus, the HDD servomechanism is a sampled-data system in nature. The sampling rate of the PES can be computed by

$$f_s = N_{sector} \frac{V_{spindle}}{60}, \quad (1.1)$$

where f_s is the sampling rate in Hz, N_{sector} is the number of the servo sectors along one track, and $V_{spindle}$ is the spindle rotational speed in revolution per minute (RPM). Therefore, in the servo sector design, there is a conflict between the requirement for higher sampling rates for the servo and the fact that each sector reduces the available user data capacity.

The servo controller is programmed into a chip located on the printed circuit board. The servo controller is designed to minimize the PES by generating appropriate voltage command. The voltage demand is then fed through the power amplifier via a digital-to-analog converter (DAC). The output current of the power amplifier passes the coils of the VCM and produces torque to drive the head into proper position. The controller operates in a track-following mode when reading data on a track and in a track-seeking mode when moving to another track.

1.1.2 Design Specifications

The HDD performance specifications are designed differently according to the application environments. For example, in a server system faster data rate and higher reliability are the most important factors; while in mobile applications smaller form factor and anti-shock and vibration capability must be ensured. However, the performance trends in all applications are towards smaller form factors, faster data rate, larger storage capacity and lower cost. Table 1.1 lists the basic performance specifications of two recent Hitachi HDDs.

Table 1.1: Performance specifications of Hitachi HDDs (shipped in 2004).

Specifications	Ultrastar 15K147	Travelstar 5K100
Application	Enterprise server	Laptop
Form factor (inch)	3.5	2.5
Capacity (GBytes)	147	100
Disks/Heads	5/10	2/4
Areal density (Gbits/inch ²)	61.7	86
Rotational speed (RPM)	15,000	5,400
Interface data rate (MBytes/s)	320	100
Average seek time (ms)	3.7	12
Average latency (ms)	2	5.5
Operating shock (G)	15 (11 ms)	300 (2 ms)

Increase of the interface data rate is achieved by reducing the access time, which consists of the time required to get the head to a new track position and ready to

read data. The access time is commonly defined as

$$\begin{aligned} \text{Access time} = & \text{Command overhead time} + \text{Seek time} \\ & + \text{Settling time} + \text{Latency time.} \end{aligned} \quad (1.2)$$

Command overhead refers to the time that elapses from when a command is given to the HDD until something actually starts happening to fulfill the command. Seek time measures the amount of time required for the heads to move between tracks over the disk surfaces and it is commonly represented by an average seek time from one random track to any other. Settling time refers to the amount of time required, after the actuator has moved the head assembly during a seek, for the heads to stabilize sufficiently for the data to begin to be read. Latency refers to the time that the spindle takes to rotate the correct sector under the heads; thus latency is directly related to the spindle speed and is averagely defined as one half of a disk rotational time.

The seek time together with the settling time are the largest components of access time (about 80% but varies with drives). Hence, the objective of reducing access time has posed critical challenge on the servo system because the seek time and the settling time are the resulting performance of the head positioning servomechanism, which are referred to as the track-seeking servo and track-settling servo, respectively.

Increase of the HDD storage capacity is achieved by improving the areal density, which refers to the number of bits that can be stored in a unit of area on the disk surface. The areal density is usually expressed in bits per square inch (BPSI); and it is a two-dimensional measure computed as the product of two other one-dimensional density measures: bit density and track density. The bit density is a measure of how

many bits are packed within a length of track and is expressed in bits per inch (BPI). The track density is a measure of how many concentric tracks are packed along the radius on the disk surface and is expressed in tracks per inch (TPI). Therefore, increase of the areal density can be realized by the increase of the bit density or the track density.

The increase of the bit density is limited by the issues such as the thermal stability of high aspect ratio magnetic domains. Thus, it is relatively easy to increase the track density such that the areal density is increased and the squarer bits also bring benefits for reducing the magnetic noise and increasing the bit stability. However, the denser track density requires narrower track pitch and thus higher position accuracy of the head positioning servomechanism, which is referred to as the track-following servo. The head positioning accuracy is evaluated by track misregistration (TMR), which is defined by three times standard deviation of the PES, i.e.,

$$\text{TMR} = 3\sigma_{pes}. \quad (1.3)$$

The performance of TMR is required to be roughly less than 10% of the track pitch [126]. In turn, for a given HDD servo system the achievable track density can be estimated by the following equation

$$\text{Track density} = \frac{25.4 \times 10^3}{3\sigma_{pes}} \times 10\% = \frac{846.7}{\sigma_{pes}}, \quad (1.4)$$

where the unit of σ_{pes} is nanometer, and the unit of track density is kTPI.

The performance specifications in HDDs are not uncorrelated. In general, the improvement of one specification will accompany with the improvement or degradation of the other specifications. For example, increasing the spindle speed to improve the

data rate can simultaneously help to provide faster sampling rate according to (1.1). However, higher spindle speed would consume more power, generate more heat, and introduce more vibrations due to air turbulence [100], which severely affects the head positioning performance. Therefore, trade-offs have to be made during the design of HDDs.

1.1.3 HDD Servo System

The HDD servo system includes three main modes: the track-seeking mode, the track-settling mode and the track-following mode. For each servo mode, a lot of approaches have been proposed to improve the servo performance. A summary of the servo technologies for HDDs can be found in [5], [31], [85], [107] and [126]. Reviews of the servo algorithms for each servo mode are given as follows.

(1) Track-Seeking Mode

The control objective in track-seeking mode is to move the heads from one track to another in the minimum time. The proximate time optimal system (PTOS) has been popularly used for track-seeking ([31], [94] and [117]). The PTOS ensures an approximated time optimal solution as well as a smooth transition to the position servo. For long distance track-seeking, the PTOS is an excellent servo design because the saturation problem and jerk minimization have been solved. In the case that the power amplifier is not saturated during track-seeking, usually in short distance track-seeking, the two-degree-of-freedom (2DOF) control has already been a very popular servo method ([99] and [129]). The basic idea in 2DOF control is to design a stable feedforward compensator that produces an inverse dynamics of the plant.

Thus, many methods have been proposed for designing the feedforward compensator such as the zero-phase error tracking control (ZPETC) [105] and perfect tracking control (PTC) using multirate methods [33]. Moreover, other servo technologies have been proposed to deal with the issues on robustness, power, vibration and jerk. These methods include the adaptive control [112], the sliding-mode control [131], the model-following control [62] and the reference trajectory planning ([43], [51], [87], [89] and [102]).

(2) Track-Settling Mode

The control objective in track-settling mode is to provide fast and smooth settlement on a target track when the servo mode is switched from the track-seeking mode to the track-following mode. This servo mode is necessary in mode-switching control (MSC) where the servo mode is handed over from one controller to the other during a single servo action. In this case, the initial values of system state variables may have non-zero values that would cause undesirable output responses if not compensated. The technique called initial value compensation (IVC), which takes into account non-zero initial conditions during mode switching, has been proposed to improve the settling performance ([123] and [124]). The basic idea of the IVC design method is to cancel the undesirable poles of the z -transform of responses due to non-zero initial plant conditions. The issues for IVC include precise estimation of the initial values and keeping disturbance rejection capability. These problems are further solved using optimal control based on IVC, which can be found in [55], [80] and [92].

(3) Track-Following Mode

The control objective in track-following mode is to keep the head as close as possible to the track center. More specifically, the track-following servo is required to minimize the TMR represented by $3\sigma_{pes}$ under various disturbances and noises. Moreover, the servo system must be robust in the presence of environment changes, actuator parameters variations with aging and temperature change and so on. Basically, the track-following servo is a regulator system and is designed primarily based on linear control theory. The major obstacles to a high performance track-following servo include the limitations and trade-offs in all linear control systems such as the Bode integral theorem [37], the mechanical constraints due to the resonances and non-minimum phase (NMP) zeros in the actuator dynamics, the limited sampling rates and computation delay in electronics, and so on.

Since the track-following servo is to minimize the TMR, the contributions and sources to TMR have been investigated by [3], [4], [14], [23], [26], [50], [57], [125] and [127]. TMR can be divided into two main components: repeatable runout (RRO) and non-repeatable runout (NRRO). RRO is a repeatable periodic signal synchronized to spindle rotation. The spectral components of RRO appear at the spindle rotation frequency and its harmonics. Major sources of RRO include disk warping, disk slip, thermal expansion of the disks and the written-in TMR while servo writing. NRRO may be also like a periodic signal but is not synchronized to spindle rotation. The spectral components of NRRO contain broad-band spectra and narrow-band spectra with sharp peaks. Major sources of NRRO include spindle bearing runout, disk flutter, actuator windage due to air turbulence, external shock

and vibration, bias force due to pivot friction and FPC, and PES demodulation and quantization noise. The actual sources of NRRO are related to a variety of factors such as form factor, spindle speed, environment changes, mechanical and servo design.

To minimize the TMR caused by various disturbance sources, two approaches have been taken. One is to increase the servo bandwidth and the other is to suppress the specific disturbances. Increasing the servo bandwidth is an ideal way to suppress disturbances within a wider range of frequencies. Moreover, higher bandwidth typically means faster movements for track-seeking. However, the bandwidth has to be limited in order to keep robust stability [126]. Furthermore, too high a bandwidth will result in more noise in the true PES, which degrades the tracking performance. The servo and mechanical technologies developed for track-following mode can be roughly categorized as follows:

- **Loop shaping**

The loop shaping designs are based on the classical frequency response technique, which provides more intuition and a greater ability to tune designs to achieve desired performance than automated tools based on state space formulations. A new tool and a complex lead compensator are proposed in [86] for assisting the loop shaping servo design. The notch filters ([69] and [111]) have been widely used to compensate for the mechanical resonances with improved servo bandwidth but at the expense of phase margin loss. The peak filters ([103] and [118]) are typically used to increase the open-loop gain for low-frequency narrow-band disturbance rejection. The phase-stabilized design [71] is developed to suppress the vibrations caused by the high-

frequency mechanical resonances. Chapter 6 in this thesis proposes a generalized optimal filter design method to suppress the narrow-band disturbances with minimal degradation of system stability.

• Model-based control

A typical model-based control design is the disturbance observer (DOB), which uses the PES and a nominal plant model to create an estimate of the disturbance. This estimate is then used to compensate for the disturbance effects. The effectiveness of the DOB in rejecting shock and vibration disturbance below 100 Hz was demonstrated in [115]. The pivot friction compensation using an accelerometer and a DOB is proposed in [60]. The repetitive control [106] has been applied to the track-following mode for RRO compensation [18]. The internal-model-based repetitive controller with a loop shaping filter is further developed for RRO and NRRO compensation [41].

• Robust and optimal control

The linear-quadratic Gaussian (LQG) control design combined with loop transfer recovery (LTR) ([10] and [76]) is used to recover performance with reasonable stability margin under low sampling rate and calculation time delay ([14] and [113]). The bias effects are examined and compensated by choosing the optimal initial condition of the integrator using the linear quadratic regulator (LQR) method [25]. The H_2 optimal control used to evaluate the achievable highest TPI under the identified disturbance models can be found in [81] and also [23], which uses an LMI approach to solve the optimal control problem. The H_∞ control theory ([32], [133] and [134]) has been applied for robust stability ([49] and [128]). The μ -analysis tool is used

to design and evaluate the robust performance under the actuator uncertainty ([20] and [46]). The limitation of the robust and optimal control is that the controller is generally of high-order feature, which results in implementation complexity.

• Multirate control

In multirate control, the control input is changed at a higher sampling rate than that of the PES. The multirate feedback servo uses an observer to output an estimated PES a few (e.g., two or four) times during a sampling period. This approach can reduce phase shift and increase the servo bandwidth ([17], [39], [44] and [95]). Basically, if the PES sampling rate was relatively low, say less than eight times the open-loop crossover, then keeping everything else constant and raising the output sampling rate could result in a closed-loop bandwidth improvement of roughly 20%. If, on the other hand, the PES sampling rate was already at roughly 20 times the open-loop crossover, then the improvement was far smaller [5]. The multirate servo can improve the bandwidth, but its effect is not obvious from the viewpoint of positioning accuracy [126].

• Advanced control

Many advanced control theories have been applied to improve the positioning accuracy and performance. These methods include the repetitive learning control for bias force and RRO compensation [15]; the application of adaptive control for RRO compensation ([53] and [130]) and resonance mode compensation [119]; and the application of neural networks ([47] and [75]) and fuzzy control [54]. The nonlinear control schemes have been developed in [16], [77] and [120]. Chapter 7 in this thesis proposes a nonlinear PID of linear plants for improved disturbance rejection. The

nonlinear controllers were demonstrated to have improved performance compared to that of the linear controllers.

• Multisensing servo

The conventional HDD servo is constructed with only the PES feedback signal. Augmented servo loop with additional sensors is useful for active damping the mechanical resonances or rejecting the specific disturbance in an active way. The acceleration sensor [68] or a strain gauge ([56] and [79]) is attached to the head arm and the measured signal is fed back to the inner loop such that the gain of the dominant mechanical resonance is reduced and the servo bandwidth can thus be increased. Alternatively, sensors can be used to measure the disturbance and vibration. The sensor information is then fed into a feedforward controller to generate proper control signals to cancel the effects of the disturbance. The use of accelerometers has been widely applied to reject the rotational external disturbance and vibration ([2], [8], [65] and [114]). The disk-flutter-induced TMR is reduced by detecting the vertical vibration of the slider [40].

The major issue of the multisensing servo is the noise sensitivity of the sensor. It is not easy to apply to the products since the sensor is required to have a nanometers-level signal-to-noise ratio. However, since sensors are cheap and have enough reliability, this approach will be applied to the products, which combine with a microactuator to raise the bandwidth further [126].

• Integrated servo mechanical design

In order to increase the servo bandwidth, a lot of work on mechanics have been done by increasing the resonance frequency of the actuator ([73] and [84]) or lowering

the resonance peak value with a passive damper [64]. Moreover, several attempts have been made to integrate the servo and mechanical plant design simultaneously. For example, a unified design using the decoupling principle has been proposed in [61]. Recently, the in-phase actuator design method is developed in [6], where the vibration can be suppressed by the high-order in-phase resonance modes. This method makes the servo system easy to design while the resulting control system can achieve high robustness and positioning accuracy [11].

1.2 Alternative High Performance Disk Drive Structures

Since the first HDD was built in 1956 by IBM, the HDD technologies have gone through a series of dramatic transformations and innovations to meet the ever increasing requirement for high performance. For example, the actuator has been evolved from the early hydraulic actuator, the linear VCM actuator, to the current widely used rotary VCM actuator [5]. During the past few years, the dual-stage actuator has been quite extensively investigated and it is promising to be used in the near future products. Alternatively, there are some other disk drive structures that support specific high performance.

1.2.1 Disk Array System

In server and network storage applications, storage systems with ultra large capacities, fast data rate and high reliability are essential. These tasks can not be

adequately fulfilled by a single HDD. Therefore, disk array systems have been used to meet the requirements. The fundamental principle of the disk array systems is the use of multiple HDDs mounted in an array that behaves like a single ultra large and fast HDD in most respects. Figure 1.3 shows an IBM DS6800 disk array system that comes with 16 internal HDDs offering up to 4.8 TB of storage capability.

The major issue in disk array systems is how to configure the HDDs to offer the excellent performance for a specific application environment. This can be accomplished by the technology called redundant arrays of inexpensive disks (RAID) levels [108]. Different RAID levels are defined to configure the disk array system in different ways such that the resulting storage system can offer large capacity, fast data rate or data security. Despite the increased cost and complexity, the disk array system offers many advantages over the use of a single HDD. In recent years, disk array systems are extending their applications on the exclusive high-end business to the personal desktop computers.

As the track density of HDDs increases, the tracking accuracy is becoming more sensitive to vibrations in the users' environments. This is more crucial in the disk array systems, because the drives are very frequently performing seek operations which cause a lot of mechanical vibrations. The servo technologies using acceleration

Figure 1.3: IBM DS6800 disk array system (adapted from [58]).

feedforward control ([8] and [65]) and a DOB [97] have been proposed to suppress these external vibrations.

1.2.2 Dual-Stage Actuator HDD

One of the best ways to increase the positioning accuracy is to use a microactuator as the second stage actuator which permits finer head positioning over closely-spaced data tracks. The open-loop bandwidth can thus be increased from current 1.0 – 1.5 kHz to over 3 kHz. Since an HDD has two actuators on one E-block, this method is called a dual-stage actuator servo. There are three categories of dual-stage actuator servos: a suspension-driven type [27], a slider-driven type [48] and a head element-driven type [59]. The piezoelectric (PZT), electrostatic and electromagnetic actuators have been used as the second stage actuator. The type of actuator is selected according to cost and reliability. Figure 1.4 shows a dual-stage actuator with a PZT-actuated suspension.

The dual-stage actuator servo is a multi-input single-output (MISO) system, which differs from the conventional single-input single-output (SISO) actuator servo. Thus, several servo algorithms have been proposed to achieve the high bandwidth

Figure 1.4: A picture of the dual-stage actuator with a PZT-actuated suspension.

and improved disturbance rejection. The alternative control structures proposed for the dual-stage actuator servo are summarized in [42]. More details of the methods can be found in [78] and the references therein. Since the servo bandwidth of the microactuator servo loop is higher than that of the main actuator servo loop, faster track-to-track seeking [70] and smoother settling [91] can be achieved by the dual-stage actuator servo than those by the single-stage actuator servo.

The dual-stage actuator servo can achieve highly precise positioning accuracy by improving the servo bandwidth significantly, but its cost and reliability have to be considered. In the future, it will be necessary to implement both microactuator and microsensor controls using MEMS technology to achieve nanometer positioning accuracy [126].

1.2.3 Multi-Actuator HDD

The multi-actuator HDD structure has been proposed to meet the high access performance and fast throughput requirement [35]. In the multi-actuator HDD, there are multiple VCM actuator assemblies mounted around the disks. For example, Figure 1.5 shows a top view of a six-actuator assemblies mounted in a 2.5 inch HDD form factor. The VCM actuator assemblies might work independently carrying their read/write heads to access the data at different radial tracks on the disk stack simultaneously.

The multi-actuator HDD performs quite similarly to the disk array system, but it integrates the VCM actuator assemblies into one disk drive enclosure and needs to replicate a portion of the electronics and data channels. The data rate can be

Figure 1.5: A schematic drawing of a multi-actuator HDD (adapted from [35]).

improved significantly with parallel data transfer, and the storage capacities can be enlarged with increased disks. However, there are several technical problems of the multi-actuator HDD. For example, having multiple E-blocks in a small form factor makes the assembly complex; the increased thermal expansion and contraction leads to issues related to magnetic media and disks; the logic is required to coordinate and optimize the operation of the actuators; and the mechanical interaction between the actuators affects the head positioning accuracy. Therefore, technological advances in many aspects needs to be developed to deal with these problems.

In this thesis, a dual-actuator HDD is adopted as the platform for investigation. Chapters 2, 3, 4 and 5 focus on the improvement of the servo system for the dual-actuator HDD. Especially, the mechanical interaction is identified and attenuated by the proposed control schemes. The developed control techniques are also applicable to the multi-actuator HDD and extensible to other applications such as the disk array systems and the servo tracking writing with multiple actuators.

1.3 Contributions of the Thesis

The contributions of this thesis are as follows:

- A linear coupled dual-input dual-output (DIDO) frequency response model to represent the dual-actuator plant is identified by using the circle-fit modal testing method. An approach to identify NMP antiresonances is presented. The approach is effective to match both the magnitude and the phase drop in the interaction modeling, which achieves an identification range up to 8 kHz, while only 2 kHz can be achieved using the usual least square identification method.
- The control structures for the dual-actuator track-following servo systems are reviewed based on the use of information from the other actuator, and classified into two categories: a fully coupled DIDO control structure and a dual single-input single-output (SISO) control structure. Hence, the control design methods specific to the dual-actuator system can be seen clearly.
- The H_2 optimal control design and the feedforward decoupling control (FDC) design are investigated for the coupled DIDO controller design. The dual-actuator H_2 control problem is formulated in terms of the tracking performance, and after that the H_2 control theory is applied to find the controller. The FDC provides a simple two-stage design method, which includes the first stage to design an optimal stable and causal feedforward decoupler that approximates an inverse-based perfect decoupler; and the second stage to design the feedback controller based on the decoupled dual-actuator plant. The con-

troller design methods in both stages are presented.

- The dual SISO control system is analyzed via loop decomposition revealing that the interaction effects can be represented as both plant uncertainties and narrow-band disturbances acting on a SISO control loop. The stability condition is derived for the dual SISO control system. After that, the SISO controller design is separated into the feedback controller design and an add-on interaction filter design. The classical loop shaping methods are applied to design the controllers. In particular, the interaction filter is designed as a phase-lead peak filter which preserves the system stability and effectively rejects the mid-frequency interaction disturbances.
- Previous peak filter designs are generally applicable to reject narrow-band disturbances within a certain frequency range. In this thesis, a generalized optimal filter design to reject narrow-band disturbances at all frequency ranges is developed. The filter zero is designed to minimally degrade the closed-loop system stability and obtain a smooth sensitivity curve around the disturbance frequency. A nonlinear optimization procedure is described to select the filter parameters such that the statistical position error is minimized. The disturbance filter can be applied to reject the narrow-band RROs and NRROs at any frequency range in HDD servos.
- The HDD tracking servo is conventionally performed by linear control because of its simplicity. However, the intrinsic limitation of linear control has constrained the achievable performance such as disturbance rejection. Thus,

a nonlinear PID (NPID) controller for a class of linear plants with output feedback is studied. The controller gains are modulated by the control error and the rate of error change to yield a closed-loop system for improved disturbance rejection based on a linear PID control system. An optimization problem formulated as linear matrix inequalities (LMIs) is solved to compute the controller parameters that can guarantee the stability of the NPID control system. Experimental results in a HDD tracking servo have demonstrated that NPID control outperforms the linear PID control by 9% in NRRO rejection.

1.4 Outline of the Thesis

The organization of this thesis is as follows:

Chapter 1 describes the HDD servomechanism system. Previous research of the HDD servo technologies has been reviewed and some alternative HDD structures for specific high performance are introduced.

Chapter 2 presents the identification of the dual-actuator system. The experimental setup of the control system is described. Frequency response modal testing method is presented to establish the linear coupled DIDO servo plant model for the dual-actuator system.

Chapter 3 lists the dual-actuator tracking servo design specifications, in which the interaction compensation is especially emphasized. After that, the dual-actuator control structures are classified into a fully coupled DIDO control structure and a dual single-input single-output (SISO) control structure. The features of both control structures are discussed and compared.

Chapter 4 investigates the coupled DIDO controller design methods, which include the H_2 optimal control and the feedforward decoupling control. The design methods are developed for the dual-actuator system. Simulation and experimental results of interaction compensation and performance robustness are presented.

Chapter 5 analyzes the dual SISO control system via loop decomposition revealing that the interaction effects can be represented as both plant uncertainties and narrow-band disturbances acting on a SISO control loop. The loop shaping method is used to design the feedback controller with an add-on interaction filter. Simulation and experimental results of interaction compensation and performance robustness are presented and compared with those obtained by coupled DIDO controller design.

Chapter 6 proposes a generalized optimal filter design method to reject narrow-band disturbances at all frequency ranges. The filter zero is designed to minimally degrade the system stability and obtain a smooth sensitivity curve around the disturbance frequency. The application to a PZT-actuated head positioning control system on the spinstand platform demonstrates that the filter improves the achievable track density to 300 kTPI by further reducing the NRROs.

Chapter 7 develops a NPID control scheme of linear plants for improved disturbance rejection. An optimization problem using LMIs is solved to compute the controller parameters that can guarantee the stability of the NPID control system. The application to a HDD servo shows that the NPID control improves the RROs and NRROs rejection based on the LPID control.

Chapter 8 concludes the major achievements of the thesis and summarizes the results obtained. Suggestions of future work are also discussed.

Chapter 2

System Identification of a Dual-Actuator HDD

The dual-actuator mechanical structure provides intrinsically fast data rate but is associated with interaction dynamics degrading the positioning accuracy. This chapter presents the identification of the dual-actuator HDD involving the interactions. The dual-actuator servo plant is analyzed and formulated as a linear coupled DIDO frequency response model. The experimental setup of the dual-actuator control system is described. Modal testing by experiments is investigated to extract the modal parameters of the DIDO plant model.

2.1 Dual-Actuator Mechanical Structure

Figure 2.1 shows a schematic diagram of the dual-actuator HDD mechanical structure. The size of the disks is 2.5 inch in diameter and the whole HDD is compacted into a standard 3.5 inch form factor. In comparison with the traditional single ac-

Figure 2.1: A dual-actuator HDD mechanical structure.

tuator HDD, the dual-actuator HDD contains two voice coil motor (VCM) actuator assemblies. The two VCM actuators are mounted in the diagonal corners of the disks. They can operate independently to allow parallel data transfer. The disks are carried by a rotational spindle motor. Each VCM actuator is composed of a VCM and a pivot bearing supported actuator arm. A flexible printed circuit (FPC) connects each VCM actuator with the digital signal processor (DSP) on which the servo control codes are loaded.

The dual-actuator HDD can significantly improve the data rate by parallel data transfer. Moreover, if the disk surface is divided into two separated regions along the radial direction, one for each actuator for data access, then the distance of track seek by each actuator is reduced with a decrease of the average seek time even with moderate performance and lower cost actuators. Furthermore, the dual-actuator structure reduces the rotational distance of the spindle motor to locate the correct sector to the nearer actuator for data access. Thus, the latency time is halved on average without having to boost the spindle speed. The allowable slower

spindle speed can in turn reduce the vibrations caused by turbulence. Therefore, the dual-actuator structure has the desired features for fast data rate and high storage capacity HDDs. Speculations about a 1 Tbits/in² HDD design are sketched out in [116], where a dual-actuator system is adopted to achieve a data rate of 375 MBytes/s and each actuator also with a high-bandwidth microactuator to support a track density of 360 kTPI translating to the required tracking accuracy of 7 nm $3\sigma_{pes}$ according to (1.4).

The major drawback of the dual-actuator system is that the two VCM actuators interact with each other in a mechanical way or in an electromagnetic way. In particular, the reaction forces from the VCM act on the baseplate, which in turn acts on the other actuator. This mechanical interaction transmitted through the baseplate would severely degrade the head positioning accuracy and decrease the achievable track density. Therefore, the interaction dynamics between the two actuators must be carefully investigated instead of being ignored. The following sections will be focused on the analysis of the dual-actuator servo plant model, especially the identification of the interaction dynamics.

2.2 Dual-Actuator Servo Plant Model

Figure 2.2 shows a complete model containing the major components of the dual-actuator servo plant. The dual-actuator servo plant consists of two VCM actuator servo models. Each VCM actuator is driven by a power amplifier. The actuator dynamics contain the resonance modes from the arm and suspension. Moreover, the two VCM actuators are coupled by the interaction dynamics due to the coupling

Figure 2.2: A complete model of the dual-actuator servo plant. The symbols denote control input (u), current of coil (i), force at the coil (f), head acceleration (a), head velocity (v), head position (y) and force constant (k_t).

resonance modes from the baseplate and the actuators. The position to be controlled is represented by y with subscript indicating the actuator index. The control input is u . Other state variables are shown in the figure.

The power amplifier for VCM is itself a closed-loop control system, which controls the VCM current i to follow the current command which is expressed in voltage u . By using the correct current sensing resistor, the output current is converted to a feedback voltage. The VCM voltage is disturbed by the back electro-motive-force (emf) voltage, which is proportional to the coil velocity v . In practice, the power amplifier will saturate if the voltage command u exceeds the saturation level, which is typically 3 V. During track-following operations, the power amplifier generally responds to the input command without saturation. Hence, the power amplifier can

be modeled as a first-order system as follows

$$\frac{I(s)}{U(s)} = \frac{k_a}{\tau_a s + 1}, \quad (2.1)$$

where k_a is the steady state gain, and τ_a is the time constant. Generally, τ_a can be tuned to provide a typical cutoff frequency 40 kHz. Therefore, during the track-following servo the power amplifier is often simplified as a constant gain k_a , which is true for an ideal current amplifier.

The pivot friction is a complex nonlinear phenomenon in principle. Various nonlinear models such as the viscous friction, coulomb friction and hysteresis have been used to identify the pivot nonlinearity [36]. During track-following operation when the signals are small, the pivot friction is generally linearized as a resonance mode with a second-order system. Due to the nonlinear effect, the linear resonance varies the natural frequency in the range from 0–100 Hz and the steady state gain in terms of the input amplitude [1]. The pivot friction decreases the plant gain at low frequencies, which makes the system more susceptible to disturbance. Since current actuators can achieve high servo bandwidth, the pivot friction is simply ignored during the control design. However, it is included in the plant model when evaluating the servo performance.

The actuator and interaction dynamics may contain a variety of resonance modes and nonlinearities. However, the number of resonances detectable at the head point and in the frequency of interest may be reduced to 7–10. Therefore, linearized simple models can be used to represent the actuator and interaction dynamics. In summary, the dual-actuator servo plant in Figure 2.2 can be simplified as a linear coupled dual-input dual-output (DIDO) frequency response model as shown in Figure 2.3,

Figure 2.3: A linear coupled DIDO frequency response model to simplify the dual-actuator servo plant model in Figure 2.2.

where P_{11} and P_{22} represent the VCM actuator models, P_{12} and P_{21} represent the interaction models between the two VCM actuators, respectively.

2.3 Modal Testing of the Dual-Actuator System

This section presents the modal testing method to identify the frequency response models in Figure 2.3 for the dual-actuator system.

2.3.1 Experimental Setup

The experimental setup for the dual-actuator control system is shown in Figure 2.4. It consists of a dual-actuator HDD prototype, two power amplifiers, two sets of laser Doppler vibrometer¹ (LDV) and a real-time DSP system dSPACE 1103², a dynamic signal analyser³ (DSA) and an oscilloscope⁴. The oscilloscope is used to collect the time traces of the signals. The dSPACE system is used to implement the controller

¹Polytec OFV 3001, Polytec, Waldbronn, Germany.

²dSPACE-DS1103, dSPACE GmbH, Paderborn, Germany.

³HP 35670A, Hewlett Packard Company, Washington.

⁴LeCroy 9304AM, LeCroy Corporation, USA.

(a) A picture of the experiment platform

(b) Block diagram of control system configuration

Figure 2.4: Experimental setup for the dual-actuator control system.

and connect the digital control platform with the actual dual-actuator plant. The two VCM actuator assemblies are of the same type taken from two IBM HDDs. They are mounted on the baseplate with the top cover firmly attached, and are placed diagonally around the disks rotated by a spindle motor. The baseplate is

clamped by the fixture placing on the pneumatic vibration isolation platform⁵. This experimental setup simulates the mounting mechanics of disk array systems, where the HDDs are mounted on a removable bracket as shown in Figure 1.3. We note that the dynamic interaction may vary with different mounting mechanism because the interactions are transmitted via the baseplate. This issue will be explored from the mechanical point of view in future work. Alternatively, dealing with the interaction uncertainties due to different mounting mechanism can be cast as a robustness problem in the control design, which is discussed in the latter chapters.

The head lateral positions $y_{1,2}$ of the two VCM actuators are measured by the LDVs, respectively. Strictly speaking, the feedback signals in real HDDs are the PES, which is the relative displacement between the head position and the sector servos on the disks. However, in our preliminary prototype, we can not access the sector servos. Thus, the disturbance samples $rro(k)$ and $nrro(k)$ that reflect the absolute position of the sector servos are artificially injected into the control system as shown in Figure 2.4(b). The $rro(k)$ and $nrro(k)$ are recovered through the measured PES samples in a real HDD. The resultant signals $pes_{1,2}$ are afterwards fed to the controller. This configuration is an approximate way to simulate the real HDD control system. In particular, the control system performance in terms of disturbance rejection can be effectively evaluated.

The dual-actuator model is identified from the measured frequency responses. The measurements are performed without feedback controllers and with independent single loop test method, in which one loop is excited while the other loop is kept

⁵Newport Corporation, CA, USA.

unexcited. The LDV gain is set as $2 \mu\text{m}/\text{V}$. The DSA is used to generate the swept sinusoidal excitation signals, which are applied to the VCM through the power amplifier. The excitation and response are fed to the DSA where the frequency response functions between the head lateral position $y_{1,2}$ and the input signal $u_{1,2}$ are extracted, respectively.

2.3.2 Frequency Response Model

The dual-actuator dynamics can be considered as a combination of a number of resonance modes. In Figure 2.3, the plant transfer function $P_{ij}(s)$, $i, j = 1, 2$, can be expressed by the following equation

$$P_{ij}(s) = k_{ij}[P_f^{ij}(s) + P_m^{ij}(s)], \quad (2.2)$$

$$P_f^{ij}(s) = \frac{A_0^{ij}}{s^2 + 2\zeta_0^{ij}\omega_0^{ij}s + (\omega_0^{ij})^2}, \quad (2.3)$$

$$P_m^{ij}(s) = \sum_{r=1}^{N_{ij}} \frac{B_r^{ij}s + A_r^{ij}}{s^2 + 2\zeta_r^{ij}\omega_r^{ij}s + (\omega_r^{ij})^2}, \quad (2.4)$$

where k_{ij} is a constant gain consisting of the power amplifier gain, the torque gain and the LDV gain. $P_f^{ij}(s)$ is the rigid body mode coupled by the linearized pivot friction model. $P_m^{ij}(s)$ is the summation of the resonance modes being considered in the frequency of interest. N_{ij} is the total number of modes. ζ_r^{ij} , ω_r^{ij} and A_r^{ij} are the damping ratio, the natural frequency and the modal constant of mode r ($r = 0 - N_{ij}$), respectively. B_r^{ij} is the resonance coupling parameter of mode r .

In the region between two adjacent resonances, an antiresonance occurs if there is a notch in the magnitude associated with a change in the phase angle [28]. Moreover, the antiresonance can be classified into two categories:

- (1) A minimum phase (MP) antiresonance if the phase angle is increasing;
- (2) A non-minimum phase (NMP) antiresonance if the phase angle is decreasing.

In practice, the antiresonance characteristics can be easily examined from the Bode plot of the measured frequency responses. Basically, a MP/NMP antiresonance results in two stable/unstable complex conjugated zeros that occur in the plant transfer function written in the form of a ratio of two polynomials. A MP antiresonance can be accurately identified by tuning A_r only; however, B_r is additionally required to be appropriately tuned to identify a NMP antiresonance.

The dashed lines in Figures 2.5 and 2.6 show the measured frequency responses of the VCM actuators. The actuator dynamics are dominated by the rigid body mode (R_0) in the low frequency range. The major resonance modes that contribute significantly to the head off-track motion include the butterfly mode (R_1) and the arm bending modes (R_2 and R_3). The butterfly mode caused by the flexibility of the pivot is the principle mode that limits the servo bandwidth. Therefore, R_0 – R_3 are considered to describe the VCM actuator dynamics. Moreover, there is a MP resonance between R_1 and R_2 and no NMP antiresonance is observed, thus B_r^{11} and B_r^{22} can be set as 0 in the actuator models $P_{11}(s)$ and $P_{22}(s)$, respectively.

The dashed lines in Figures 2.7 and 2.8 show the measured frequency responses of the interactions. The interactions are also dominated by the rigid body mode (R_0) in the low frequency range. However, several off-track resonance modes (R_1 – R_6), that can not be detected as shown in Figures 2.5 and 2.6, are excited in the interaction frequency responses. The actuator butterfly mode (R_7) is also excited. Therefore, R_0 – R_7 are considered to describe the interaction dynamics. Moreover, from the

Figure 2.5: Frequency responses of VCM actuator 1 dynamics: $P_{11}(s)$ (R_0 – R_3 : resonance modes).

Figure 2.6: Frequency responses of VCM actuator 2 dynamics: $P_{22}(s)$ (R_0 – R_3 : resonance modes).

Figure 2.7: Frequency responses of interaction dynamics: $P_{12}(s)$ (R_0 – R_7 : resonance modes; N_1 – N_4 : NMP antiresonance modes).

Figure 2.8: Frequency responses of interaction dynamics: $P_{21}(s)$ (R_0 – R_7 : resonance modes; N_1 – N_4 : NMP antiresonance modes).

phase plots, several NMP antiresonance modes (N_1 – N_4) are observed. Thus, identifying these NMP antiresonances needs to tune B_r^{12} and B_r^{21} as well as A_r^{12} and A_r^{21} in the interaction models $P_{12}(s)$ and $P_{21}(s)$, respectively. It should be emphasized that vibrations caused by the low-frequency modes R_1 – R_4 can be sufficiently suppressed by a typical HDD control system according to the sensitivity function shaping. However, vibrations caused by modes R_5 and R_6 , whose natural frequencies are close to the open-loop crossover frequency, must be further suppressed using advanced control analysis and design.

2.3.3 Modal Parameter Extraction Using Circle-Fit Method

The modal parameters in (2.2)–(2.4) can be extracted from the measured frequency responses data using the circle-fit method. The circle-fit method is a single-degree-of-freedom (SDOF) modal testing approach, which extracts the modal parameters of just one mode at a time. Thus, this method requires that the behavior of the plant near the resonance region be dominated by the mode under examination [28]. All the modes in the frequency range of interest are then analyzed one after the other. From Figures 2.5–2.8, these conditions are satisfied for the dual-actuator plant. Thus, the simple circle-fit method could be used to identify the modal parameters of each mode individually.

The circle-fit method [28] is based on the characteristic that a Nyquist plot of the frequency response data on a system includes sections of near-circular arcs (modal circles) corresponding to the regions near the natural frequencies. Figure 2.9 shows an illustration to extract the modal parameters of $P_{11}(s)$ from an individual modal

Figure 2.9: Nyquist plot illustrating the circle-fit method to extract the modal parameters of R_1 in $P_{11}(s)$ from the measured points in Figure 2.5.

circle corresponding to the mode R_1 in Figure 2.5. The procedure is as follows:

1. Fit a modal circle for the resonance mode by using the curve-fitting routine such as least-squares method [82], which gives a least-squares deviation from the measured points. Hence, the center and radius of the circle are specified.
2. The natural frequency ω_r is approximately equal to the frequency of the maximum response point.
3. Select the two half-power points ω_b and ω_a (points before and after ω_r , respectively) such that $\theta_a = \theta_b = 90^\circ$. Thus, the damping ratio can be obtained by

$$\zeta_r \approx \frac{\omega_a - \omega_b}{2\omega_r}. \quad (2.5)$$

4. The modal constant can be simply calculated by

$$|A_r| = 2D_r\omega_r^2\zeta_r, \quad (2.6)$$

where D_r is the diameter of the modal circle. The sign of A_r can be determined directly from the Nyquist plot. More specifically, A_r is negative if the circle is in the upper half of the plane; otherwise, positive.

5. In the interaction models, the parameter B_r is required to be tuned for identifying the NMP antiresonance. As aforementioned in Section 2.3.2, the NMP antiresonance between the resonance mode $r - 1$ and resonance mode r can be examined from the Bode plot of the measured frequency response, which directly provides the NMP antiresonance frequency ω_n , at which the lowest antiresonance notch locates.

Next, the guideline used here is to select a proper B_r such that the identified antiresonance is of negative damping ratio. More specifically, the transfer function in (2.4) only summed by the models of resonance mode $r - 1$ and r must have complex conjugated zeros with positive real parts. Thus, B_r can be tuned roughly from the initial value determined by the following equation

$$B_r \approx \frac{\omega_n^2 - 2\zeta_r\omega_r A_{r-1} - 2\zeta_{r-1}\omega_{r-1} A_r}{\omega_{r-1}^2}, \quad (2.7)$$

where ω_n is the aforementioned NMP antiresonance frequency.

Tables 2.1–2.4 list all the modal parameters extracted for $P_{11}(s)$, $P_{22}(s)$, $P_{12}(s)$ and $P_{21}(s)$, respectively, using the circle-fit method. The solid lines in Figures 2.5–2.8 show the simulated results of the identified models. The two VCM actuators and the interaction dynamics have similar frequency responses because the dual-actuator

prototype is designed by using the same type of VCM actuator assemblies and the symmetrical layout. The identified models show that the matched frequency range is obtained within 10 kHz in the actuator models and 8 kHz in the interaction models.

The identified frequency range is sufficient for a typical HDD control system. Thus, the dual-actuator models obtained here is used for the control analysis and design in the following chapters.

Table 2.1: Extracted Modal Parameters of $P_{11}(s)$ ($k_{11} = 2.6 \times 10^7$)

Mode (r)	ω_r^{11} (Hz)	ζ_r^{11}	A_r^{11}	B_r^{11}
R ₀	58	0.18	2.3	0
R ₁	5700	0.032	-1	0
R ₂	7300	0.005	-0.09	0
R ₃	9000	0.022	0.7	0

Table 2.2: Extracted Modal Parameters of $P_{22}(s)$ ($k_{22} = 2.5 \times 10^7$)

Mode (r)	ω_r^{22} (Hz)	ζ_r^{22}	A_r^{22}	B_r^{22}
R ₀	55	0.25	2.4	0
R ₁	5580	0.038	-1.8	0
R ₂	7550	0.004	-0.1	0
R ₃	8600	0.025	0.8	0

Table 2.3: Extracted Modal Parameters of $P_{12}(s)$ ($k_{12} = 1.0 \times 10^4$)

Mode (r)	ω_r^{12} (Hz)	ζ_r^{12}	A_r^{12}	B_r^{12}
R ₀	30	0.3	2.2	0
R ₁	110	0.08	-4.8	0
R ₂	370	0.025	2.0	0
R ₃	440	0.032	0.5	0.0008
R ₄	600	0.05	-2.5	0.0005
R ₅	1200	0.02	-2.9	0
R ₆	1600	0.013	-10	0.0001
R ₇	5700	0.05	-28	0.001

Table 2.4: Extracted Modal Parameters of $P_{21}(s)$ ($k_{21} = 1.2 \times 10^4$)

Mode (r)	ω_r^{21} (Hz)	ζ_r^{21}	A_r^{21}	B_r^{21}
R ₀	30	0.32	2.2	0
R ₁	110	0.09	-4.2	0
R ₂	370	0.025	2.0	0
R ₃	430	0.035	0.5	0.0007
R ₄	605	0.04	-1.0	0.0005
R ₅	1200	0.018	-2.5	0
R ₆	1600	0.016	-11	0.0001
R ₇	5700	0.08	-30	0.001

2.4 Summary

This chapter firstly presents the dual-actuator HDD mechanical structure, which provides intrinsically fast data rate but is associated with interaction dynamics degrading the positioning accuracy. Next, the dual-actuator servo plant is analyzed and formulated as a linear coupled DIDO frequency response model. The experiment platform is described with the control system configuration, where the feedback signals pes are generated by comparing the LDV position measurements with the disturbance samples collected from a real HDD. This configuration provides an effective way to evaluate the control system performance of disturbance rejection when the servo sectors are inaccessible.

The DIDO plant model is expressed by the summation of a number of parameterized second-order transfer functions, which respectively represent the resonance modes. From the measured frequency responses, the modal parameters are extracted by using the circle-fit identification method. In particular, a guideline of tuning the modal parameter B_r is given to identify the NMP antiresonance, which results in unstable complex conjugates zeros in the plant transfer function. The identification results show that the VCM actuator models are matched up to 10 kHz and the interaction models are matched up to 8 kHz by identifying the NMP antiresonances. The circle-fit method outperforms the usual least square identification method, which can only achieve 2 kHz identified frequency range for the dual-actuator plant.

The identified frequency range is sufficient for a typical HDD control system. Thus, the dual-actuator models obtained in this chapter is used for the control analysis and design in the following chapters.

Chapter 3

Dual-Actuator Track-Following Control Structures

This chapter gives the servo design specifications and analyzes the control structures for the dual-actuator track-following servo systems. Based on the use of information from the other actuator, the dual-actuator control structures can be classified into two categories: those using a fully coupled dual-input dual-output (DIDO) controller, and those using a dual single-input single-output (SISO) controller. The features of the controller structures are discussed.

3.1 Dual-Actuator Servo Design Specifications

The design specifications of the dual-actuator track-following servo systems include

1. Gain and phase margin: the minimal requirements are 5 dB gain margin and 30 deg phase margin in order to have sufficient stability robustness.

2. Bandwidth: the bandwidth in HDD servo systems is often characterized by the open-loop gain crossover frequency. For the dual-actuator which uses single-stage actuators, the servo bandwidth is required to be about 1 kHz.
3. Sensitivity function shape: the sensitivity function is usually required to have at least -35 dB at 100 Hz to reject the low-frequency disturbances effectively; meanwhile the maximum peak of the sensitivity function is required to be less than 6 dB to minimize the amplification of the disturbances by the servo system.
4. Interaction compensation: the track misregistration (TMR) induced by the interactions must be reduced to maintain a high positioning accuracy.
5. System integrity: system stability must be achieved while the positioning accuracy is maintained to some degree when either loop is open due to operating requirement or hardware failure.

Criteria 1, 2 and 3 apply to both single-actuator and dual-actuator servo systems, but Criteria 4 and 5 are particularly required for the dual-actuator servo systems because the dual-actuator plant is considered as a coupled DIDO system. Thus, they need to be taken into accounts when choosing the control structure and designing the controllers.

3.2 Coupled DIDO Controller Structure

For the coupled dual-actuator plant, the general controller structure is a fully cross-coupled DIDO controller as shown in Figure 3.1. In the structure, the controller

utilizes both the available feedback signals $pes_{1,2}$ from the two actuators together to make decisions on each control input $u_{1,2}$. From Figure 3.1, it gives that

$$u_1 = -C_{11}(s)pes_1 - C_{12}(s)pes_2, \quad (3.1)$$

$$u_2 = -C_{21}(s)pes_1 - C_{22}(s)pes_2. \quad (3.2)$$

The full controller structure is possible, in principle, to achieve optimal performance such as complete interaction compensation. A typical drawback of the controller structure is that a servo processor with large computational capability is required to implement the controller.

There are two controller design methodologies for the coupled DIDO control structure: design via modern optimal control theory such as H_2 and H_∞ , and design

Figure 3.1: Block diagram of a coupled DIDO controller structure for the dual-actuator track-following servo system. ($d_{1,2}$ denotes the lumped disturbance acting on the plant output. $u_{1,2}$ denotes the control input to the VCM actuator. $y_{1,2}$ denotes the actual head position of the VCM actuator. $pes_{1,2}$ denotes the position error signal (PES) fed back to the controller.)

based on decoupling control techniques. The modern control design based on state-space formulation is an automated design tool, which treats the dual-actuator and the interactions as a whole during the synthesis procedure. The decoupling control, which is relatively easy to understand, provides a two-stage design approach. The first stage is the decoupler design to compensate for the interactions. The second stage is the feedback controller design based on the assumption that the DIDO plant is decoupled into two independent SISO plants.

The coupled DIDO controller design using the H_2 optimal control theory and the feedforward decoupling control technique will be presented in Chapter 4.

3.3 Dual SISO Controller Structure

It is also essential to explore whether the dual-actuator servo performance can be achieved directly using the SISO design techniques because of its simplicity. This is the case when the dual-actuator plant has some specific property such as diagonal dominance.

A multi-input multi-output (MIMO) plant is said to be diagonal dominance if its $n \times n$ transfer function matrix $P(j\omega)$ has [38]

$$|P_{kk}(j\omega)| > \sum_{i=1; i \neq k}^n |P_{ik}(j\omega)| \quad \forall \omega \in \mathbb{R}. \quad (3.3)$$

The nice feature of a MIMO plant having the diagonal dominance property is that the multiple SISO controller structure can be generally applied to achieve acceptable servo performance.

According to the measured frequency responses as shown in Figures 2.5–2.8,

it is easy to verify that the dual-actuator plant is of diagonal dominance, which implies that the interaction dynamics have weaker effects on the system output than the actuator dynamics have. Therefore, this motivates us to study the dual-actuator servo systems using the dual SISO controller structure as shown in Figure 3.2. Such a structure when applied to the dual-actuator servo systems has the following advantages:

1. A variety of SISO control techniques can be employed directly to design the two independent controllers.
2. Compared to the fully coupled DIDO controller structure, the dual SISO controller structure requires a servo processor with less computational capability and thus is easier to implement.
3. System integrity is generally easier to achieve using the dual SISO controller

Figure 3.2: Block diagram of a dual SISO controller structure for the dual-actuator track-following servo systems.

structure because the control input in either loop is unrelated to the signals from the other loop.

4. The dual SISO control structure can be simply extended to a multiple SISO control structure for a multi-actuator servo system.

However, the structure constrained by a dual SISO controller will result in certain insufficient servo performance such as partial interaction compensation when compared to the servo system with the coupled DIDO controller structure.

The controller design methodologies for the dual SISO control structure is based on the assumption that the interactions are treated as a form of output disturbances. Consequently, the DIDO plant is approximated as two independent SISO plants, based on which the controller is then designed to achieve the desired performance and compensate for the interaction disturbances. However, in practice, the interaction disturbances involve the feedback information from the other loop. Thus, we need to analyze the overall control loop to investigate the interaction disturbance characteristics, which are then used to guide the controller design.

As mentioned above, the dual SISO controller can be designed using the sophisticated SISO control techniques such as the optimal control theory and the classical loop shaping design methods. The classical loop shaping methods can provide more intuition and a greater ability to tune designs to achieve performance than the methods using optimal control theory. Moreover, the loop shaping methods generally lead to a practically low-order controller. Hence, the dual SISO controller design using classical loop shaping methods is adopted and will be presented in Chapter 5.

3.4 Summary

In this chapter, the servo design specifications of the dual-actuator track-following servo systems are described. The interaction compensation and system integrity exclusively required in the dual-actuator servo systems are emphasized. The control structures for the dual-actuator system, which include the fully coupled DIDO controller structure and the dual SISO controller structure, are discussed. The coupled DIDO controller structure utilizes both the available feedback signals from the two actuators to generate the control inputs and thus in principle can achieve optimal performance. On the contrary, the dual SISO controller structure contains only two simple SISO controller blocks, which need to be well designed to achieve the acceptable performance.

The controller design based on these two control structures will be presented in Chapters 4 and 5.

Chapter 4

Coupled DIDO Controller Design

This chapter discusses the controller design for the dual-actuator track-following servo system based on the coupled DIDO controller structure as shown in Figure 3.1. Two control designs are presented and compared. The first design employs the H_2 optimal control design method. The dual-actuator control objective is formulated as a standard H_2 optimal control problem, the solutions of which are also given. The second design uses the feedforward decoupling control design method, which contains a simple two-stage design process. Simulation and experimental results using the two control designs are presented.

4.1 H_2 Optimal Control Design

The H_2 optimal control design [96] provides an automated design tool to synthesize the multivariable controller. Given a linear plant model and disturbance models in a state space description, the H_2 norm of a certain transfer function from an exogenous disturbance to a pertinent controlled output is minimized by an internally stabilizing

controller that can be obtained using the H_2 optimal control theory.

4.1.1 Dual-Actuator H_2 Optimal Control Problem

The dual-actuator control system can be represented as the standard block diagram of H_2 control system as shown in Figure 4.1, where

$w = [w_{i1} \ w_{o1} \ w_{n1} \ w_{i2} \ w_{o2} \ w_{n2}]^T$: is the disturbance vector in which all the signals are independent zero mean white noises with variance 1;

$G_{id}(s)$, $G_{od}(s)$ and $G_{mn}(s)$: are the disturbance models;

$y = [pesm_1 \ pesm_2]^T$: is the measured output vector, in which $pesm$ denotes the signals of the measured position error signal (PES);

$z = [pest_1 \ pest_2]^T$: is the controlled output vector, in which $pest$ denotes the signals of the true PES.

One of the control objectives in dual-actuator track-following servo systems is to minimize the track misregistration (TMR) of each actuator simultaneously subject to all the disturbance sources. For simplicity, we consider σ_{pest} as the performance index of TMR, where σ_{pest} is the standard deviation of $pest$ given by

$$\sigma_{pest} = \left[\frac{1}{n-1} \sum_{k=1}^n pest(k)^2 \right]^{1/2}, \quad (4.1)$$

where n represents a sufficiently large number of the true PES samples.

To associate σ_{pest} with the H_2 control design problem, in Figure 4.1, consider the H_2 norm of the system T_{zw} , which is the transfer function matrix from the

Figure 4.1: Block diagram of the dual-actuator control system represented as the standard H_2 control system.

disturbance input vector w to the controlled output vector z . It gives that

$$\begin{aligned}
 \|T_{zw}\|_2 &= \frac{\|z\|_2}{\|w\|_2} = \|z\|_2 \\
 &= \left\{ \frac{1}{n-1} \left[\sum_{k=1}^n pest_1(k)^2 + \sum_{k=1}^n pest_2(k)^2 \right] \right\}^{1/2} \\
 &\approx \sigma_{pest_1} + \sigma_{pest_2}.
 \end{aligned} \tag{4.2}$$

Note that $\|w\|_2 \equiv 1$ because the disturbance inputs are all considered as independent zero mean white noises with variance 1 [83]. Moreover, according to (4.2), the H_2 norm of T_{zw} is approximately equivalent to the sum of the σ_{pest_1} and σ_{pest_2} , which implies that the individual control objectives of the two actuators are undesirably correlated. However, since the dual-actuator plant has diagonally similar dynamics, the achievable servo performance of σ_{pest} of each actuator can be closely equivalent. Therefore, it can be approximated that minimizing the $\|T_{zw}\|_2$ can lead to a corresponding minimal σ_{pest_1} and σ_{pest_2} simultaneously.

Based on the above analysis, the dual-actuator control design problem to minimize σ_{pest_1} and σ_{pest_2} can be formulated as an H_2 optimal control problem to minimize $\|T_{zw}\|_2$. Hence, the controller can be found by using the H_2 optimal control theory.

4.1.2 Solutions of H_2 Optimization Problem

To obtain the coupled DIDO controller using the H_2 optimal control design, the system blocks of the standard H_2 control system in Figure 4.1 are generally required to be represented with state space descriptions. The realization of the transfer

function matrix G can be taken to be of the form

$$\Sigma_G : \begin{cases} \dot{x} = A x + B u + E w \\ y = C_1 x + D_{11}u + D_1 w \\ z = C_2 x + D_2 u + D_{22}w \end{cases} \quad (4.3)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^q$ is the disturbance input, $y \in \mathbb{R}^p$ is the measurement output, and $z \in \mathbb{R}^l$ is the controlled output. The following assumptions on Σ_G are made.

Assumption 4.1

1. (A, B) is stabilizable and (C_1, A) is detectable, which guarantee the stability of G by output feedback control.
2. D_{11} and D_{22} are both assumed to be zero without loss of generality [134]. ■

The H_2 optimal control design problem is to find an internally stabilizing proper controller C with the following state space description

$$\Sigma_C : \begin{cases} \dot{v} = A_c v + B_c y \\ u = C_c v + D_c y \end{cases} \quad (4.4)$$

such that the $\|T_{zw}\|_2$ is minimized.

The H_2 controller in (4.4) can be obtained by using the Riccati equations approach [96] or the linear matrix inequality (LMI) approach ([23] and [93]). In the following, we briefly describe the Riccati equations approach to obtain the H_2 controller under the regularity and singularity conditions.

Regular Case

The H_2 control system is called the regular case when the given system G in (4.3) satisfies the following specific regularity conditions [96]:

1. D_2 is of maximal column rank.
2. The subsystem characterized by the matrix quadruple (A, B, C_2, D_2) has no invariant zeros on the imaginary axis.
3. D_1 is of maximal row rank.
4. The subsystem characterized by the matrix quadruple (A, E, C_1, D_1) has no invariant zeros on the imaginary axis.

The H_2 optimal controller for the regular case is derived in [21], which can be constructed by

$$\Sigma_C : \begin{cases} \dot{v} = (A + BF + KC_1) v - K y \\ u = F v \end{cases} \quad (4.5)$$

where

$$F = -(D_2^T D_2)^{-1} (D_2^T C_2 + B^T P), \quad (4.6)$$

$$K = -(QC_1^T + ED_1^T)(D_1 D_1^T)^{-1}, \quad (4.7)$$

and where $P = P^T \geq 0$ and $Q = Q^T \geq 0$ are the solutions of the following algebraic Riccati equations (AREs), respectively,

$$A^T P + PA - (PB + C_2^T D_2)(D_2^T D_2)^{-1} (D_2^T C_2 + B^T P) + C_2^T C_2 = 0, \quad (4.8)$$

$$QA^T + AQ - (QC_1^T + ED_1^T)(D_1 D_1^T)^{-1} (D_1 E^T + C_1 Q) + EE^T = 0. \quad (4.9)$$

Furthermore, the achievable minimal H_2 norm of the closed-loop transfer function matrix T_{zw} can be computed as follows

$$\|T_{zw}\|_2 = \left\{ \text{trace}(E^T P E) + \text{trace} \left[(A^T P + PA + C_2^T C_2) Q \right] \right\}^{1/2}. \quad (4.10)$$

Singular Case

The H_2 control system is called the singular case when the conditions for the regular case are not satisfied. The H_2 controller for the singular case is generally suboptimal, and usually parameterized by a certain tuning parameter ε . As such, the suboptimal controller guarantees the internal stability of the closed-loop system, and the resulting closed-loop transfer function matrix T_{zw} , which is obviously a function of ε , has an H_2 norm arbitrarily close to the infimum as ε approaches to 0 [96].

The perturbation approach ([74] and [132]) can be applied to yield a suboptimal controller for the singular case. By adding some dummy variables to z , i.e., defining a new controlled output as follows

$$\tilde{z} = \begin{bmatrix} z \\ \varepsilon x \\ \varepsilon u \end{bmatrix} = \tilde{C}_2 x + \tilde{D}_2 u, \quad (4.11)$$

where

$$\tilde{C}_2 = [C_2^T \quad \varepsilon I \quad \phi]^T, \quad (4.12)$$

$$\tilde{D}_2 = [D_2^T \quad \phi \quad \varepsilon I]^T, \quad (4.13)$$

and defining another two new matrices

$$\tilde{E} = [E \quad \varepsilon I \quad \phi], \quad (4.14)$$

$$\tilde{D}_1 = [D_1 \quad \phi \quad \varepsilon I], \quad (4.15)$$

where I and ϕ in (4.12)–(4.15) are identity and zero matrix of appropriate dimensions, respectively. The newly constructed perturbed system $\tilde{G}$ can be described

by

$$\tilde{\Sigma}_G : \begin{cases} \dot{x} = A x + B u + \tilde{E} w \\ y = C_1 x + \tilde{D}_1 w \\ \tilde{z} = \tilde{C}_2 x + \tilde{D}_2 u \end{cases} \quad (4.16)$$

Obviously, $\tilde{D}_2$ is of maximal column rank and $(A, B, \tilde{C}_2, \tilde{D}_2)$ is free of invariant zeros for any $\varepsilon > 0$. Therefore, the perturb system $\tilde{G}$ satisfies the conditions of the regular case, and thus the previously stated solution to the regular case can be applied to find appropriate controllers. The suboptimal controller works for the original system for sufficiently small ε .

4.1.3 TMR Source Modeling

The disturbance models (TMR source models) as shown in Figure 4.1 are essential to find the H_2 controller for the dual-actuator servo system. In the HDD servo systems, TMR sources can be identified by direct measurement from experiments and/or analysis of the measured PES ([3], [4], [23], [26], [50], [57] and [127]). With the identified TMR source models, new servo algorithms can be evaluated; and servo performance of new HDD products can be predicted before the new HDD prototypes and firmware are made.

In what follows, we discuss a method of TMR source modeling, which is based primarily upon the spectral decomposition of the measured PES. Figure 4.2 shows a block diagram of the single-actuator HDD servo system with disturbance models that are to be identified.

The disturbances contributing to the servo error are classified into three major components: input disturbance (d_i), output disturbance (d_o) and measurement

noise (d_n), which are modeled as the outputs from the independent zero mean white noises with variance 1 (ω_i , ω_o and ω_n) passing through their corresponding disturbance models $G_{id}(s)$, $G_{od}(s)$ and $G_{mn}(s)$. The input disturbance d_i is caused by the external shock and vibration, bias torque due to the flex cable, pivot bearing friction, windage, amp offset, resolution of DAC. The output disturbance d_o contains various source of vibration due to spindle motor, bearing, disk and slider and RRO from STW. The measurement noise d_n is due to position sensing demodulation noise, resolution of ADC.

Due to the interest in the NRRO components, the *pesm* is assumed to have separated the RRO components by linear averaging method in time domain [127]. Moreover, it is assumed that d_n is white noise and d_i , d_o and d_n are completely uncorrelated with each other. In Figure 4.2, let the sensitivity function and the

Figure 4.2: Block diagram of a single-actuator HDD servo system (ω_i , ω_o and ω_n represent independent zero mean white noises with variance 1; y : actual head displacement; *pest*: true PES; *pesm*: measured PES; $C(s)$: controller; $P(s)$: actuator model; $G_{id}(s)$, $G_{od}(s)$ and $G_{mn}(s)$: disturbance models).

complementary sensitivity function be as follows

$$S(s) = \frac{1}{1 + P(s)C(s)}, \quad (4.17)$$

$$T(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}. \quad (4.18)$$

Therefore, the power spectral density (PSD) of the measured PES can be represented as follows

$$S_{pesm} = |S(s)P(s)|^2 S_{d_i} + |S(s)|^2 S_{d_o} + |S(s)|^2 S_{d_n}, \quad (4.19)$$

where S_a means the PSD of the signal a . Note that d_i , d_o and d_n are assumed to be generated by independent zero white noises with variance 1, it thus gives that

$$S_{d_i} = |P_{id}(s)|^2, \quad S_{d_o} = |P_{od}(s)|^2, \quad S_{d_n} = |P_{mn}(s)|^2. \quad (4.20)$$

Substituting (4.20) into (4.19) yields

$$S_{pesm} = |S(s)P(s)|^2 |P_{id}(s)|^2 + |S(s)|^2 |P_{od}(s)|^2 + |S(s)|^2 |P_{mn}(s)|^2. \quad (4.21)$$

In order to identify the disturbance models $G_{id}(s)$, $G_{od}(s)$ and $G_{mn}(s)$, the following steps [23] can be taken:

1. Collect the $pesm$ data with sufficient traces, e.g., 100 traces of 100 consecutive revolutions. Separate $pesm$ into RRO components and NRRO components by linear averaging method [127]. Calculate the PSD of the NRRO components, which exactly equals S_{pesm} according to our assumption.
2. Measure the transfer functions $S(s)$ and $P(s)$ according to the method in [26].
3. Find the baseline curve $S_b(j)$ of S_{pesm} ,

$$S_b(j) = \min_{i=1+(j-1)n}^{jn} S_{pesm}(i), \quad j = 1, 2, \dots, L/n, \quad (4.22)$$

where L is the length of S_{pesm} and n is as small as possible.

4. Compute $P_{id}(s)$ according to the following equation,

$$|P_{id}(s)|^2 = \frac{W_l S_b}{|S(s)P(s)|^2}, \quad (4.23)$$

where W_l is a low-pass weighting function used to select the low-frequency components in S_b , which are regarded as the contribution by w_i .

5. Compute $P_{mn}(s)$ according to the following equation,

$$|P_{mn}(s)|^2 = \frac{W_h S_b}{|S(s)|^2}, \quad (4.24)$$

where W_h is a high-pass weighting function used to select the high-frequency components in S_b , which are regarded as the contribution by w_n .

6. The residue of S_{pesm} is then regarded as the contribution by w_d . Hence, according to (4.21), $P_{od}(s)$ can be computed as

$$|P_{od}(s)|^2 = \frac{S_{pesm}}{|S(s)|^2} - \frac{|P(s)|^2 |P_{id}(s)|^2}{|S(s)|^2} - |P_{mn}(s)|^2. \quad (4.25)$$

Following the above steps, the approximate disturbance models of the NRRO sources are obtained from the measured PES in a commercial HDD. They are given directly by the following equations

$$G_{id}(s) = \frac{3620s^2 + 2.2 \times 10^7 s - 6.05 \times 10^{11}}{s^3 + 59000s^2 + 8 \times 10^8 s + 4.13 \times 10^9}, \quad (4.26)$$

$$G_{od}(s) = \frac{75000s^2 - 1.54 \times 10^9 s + 2.3 \times 10^{13}}{s^3 + 35400s^2 + 1.68 \times 10^9 s + 3.85 \times 10^{12}}, \quad (4.27)$$

$$G_{mn}(s) = \frac{9280s^2 - 4.08 \times 10^8 s + 6.89 \times 10^{12}}{s^3 + 6210s^2 + 4.06 \times 10^8 s + 1.21 \times 10^{12}}. \quad (4.28)$$

4.1.4 Controller Design Results

In order to find the H_2 controller, the augmented system G in Figure 4.1 combined with the dual-actuator model and disturbance models must be established in a state space formulation as Σ_G in (4.3). This can be accomplished by the following construction:

Let (A_p, B_p, C_p, D_p) , $(A_{id}, B_{id}, C_{id}, D_{id})$, $(A_{od}, B_{od}, C_{od}, D_{od})$ and $(A_{mn}, B_{mn}, C_{mn}, D_{mn})$ be respectively the state space models of the transfer function matrices

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \quad \begin{bmatrix} P_{id} & 0 \\ 0 & P_{id} \end{bmatrix}, \quad \begin{bmatrix} P_{od} & 0 \\ 0 & P_{od} \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} P_{mn} & 0 \\ 0 & P_{mn} \end{bmatrix}.$$

Define the state vector $x = (x_p^T \ x_{id}^T \ x_{od}^T \ x_{mn}^T)^T$, thus, the augmented system Σ_G can be established as

$$A = \begin{bmatrix} A_p & B_p C_{id} & \phi & \phi \\ \phi & A_{id} & \phi & \phi \\ \phi & \phi & A_{od} & \phi \\ \phi & \phi & \phi & A_{mn} \end{bmatrix}, \quad (4.29)$$

$$B = \begin{bmatrix} B_p & \phi & \phi & \phi \end{bmatrix}^T, \quad (4.30)$$

$$E = \begin{bmatrix} B_p D_{id} & \phi & \phi \\ B_{id} & \phi & \phi \\ \phi & B_{od} & \phi \\ \phi & \phi & B_{mn} \end{bmatrix}, \quad (4.31)$$

$$C_1 = \begin{bmatrix} C_p & D_p C_{id} & C_{od} & C_{mn} \end{bmatrix}, \quad (4.32)$$

$$D_1 = \begin{bmatrix} D_p D_{id} & D_{od} & D_{mn} \end{bmatrix}, \quad (4.33)$$

$$C_2 = \begin{bmatrix} C_p & D_p C_{id} & C_{od} & \phi \end{bmatrix}, \quad (4.34)$$

$$D_2 = D_p, \quad (4.35)$$

$$D_{11} = D_p, \quad (4.36)$$

$$D_{22} = \begin{bmatrix} D_p D_{id} & D_{od} & \phi \end{bmatrix}, \quad (4.37)$$

where ϕ in (4.29)–(4.37) denotes zero matrix of appropriate dimensions. Note that D_{11} and D_{22} are both equal to zero matrices because the dual-actuator plant and the disturbance models are strictly proper and thus the feedthrough terms D in their respective state space models are zero.

The system Σ_G is verified to be a regular case. Hence, the controller design presented in Section 4.1.2 is applied. During the control design, a reduced-order model of the dual-actuator plant is used to reduce the ill numerical condition. But the full-order model of the plant is used in simulating the final system response. The suboptimal H_2 controller is approached by decreasing the ε value until there is no solution to the AREs in (4.8) and (4.9). Finally, when $\varepsilon = 3 \times 10^{-7}$, a coupled DIDO H_2 controller is obtained and the Bode plots are shown in Figures 4.3 and 4.4. The frequency responses of the sensitivity function matrix of the closed-loop system is also shown in Figure 4.5.

Figures 4.3 and 4.4 show that the corresponding controller elements have similar dynamics. This is due to the fact that the dual-actuator DIDO model is of diagonally

Figure 4.3: Bode plots of H_2 controller elements (C_{11} : solid line; C_{12} : dashed line).

Figure 4.4: Bode plots of H_2 controller elements (C_{22} : solid line; C_{21} : dashed line).

Figure 4.5: Frequency responses of the sensitivity function matrix of the closed-loop system with a coupled DIDO H_2 controller.

similar dynamics and the disturbance models used in each loop are the same. In addition, the controllers C_{11} and C_{22} in the main loops are similar to the lead-lag controllers with some high-frequency notch filters. The cross-coupled controllers C_{12} and C_{21} exhibit additional notch effects around 1.6 kHz, which is expected to compensate for the interaction resonance dynamics. The sensitivity function is plotted in Figure 4.5, which reflects the design specifications in disturbance rejection are basically satisfied.

The designed H_2 controller contains four 23th-order controller elements, the order of which is too high to be implemented on our real-time DSP system (400 MHz) at 40 kHz sampling rate. Intuitively, the balanced model reduction method [134] is

performed to overcome this situation. However, we find that the notches around the interaction resonance (1.6 kHz) in S_{12} and S_{21} as shown in Figure 4.5 tend to vanish even when the controller order is reduced by two. This implies that the performance of interaction suppression is sensitive to the controller order reduction. The other robust model reduction methods will be further explored in future work. Since we are more concerned with the ideally optimal tracking performance obtainable for the dual-actuator system using the H_2 controller, we will present only the simulation results in the next section. The results can consequently be used as the benchmark to evaluate other controller designs.

4.1.5 Simulation Results

This section simulates the contributions of the TMR sources to PES using the designed H_2 controller. In particular, the interaction-induced TMR is studied. In what follows, only the $pest_1$ characteristics are shown for simplicity.

The simulations are carried out with the following steps:

1. Inject all the disturbance sources to the second actuator servo loop. Then, the torque disturbance, output disturbance, and measurement noise are alternately injected into the first servo loop. The $pest_1$ samples due to each disturbance source are subsequently collected one by one.
2. Remove all the disturbance sources from the first actuator servo loop and collect the $pest_1$ samples, which is induced only by the interaction dynamics.

Figure 4.6 shows the frequency spectrums of $pest_1$ due to the input disturbance, output disturbance, measurement noise and interaction-induced TMR, respectively.

(a) Input disturbance TMR.

(b) Output disturbance TMR.

(c) Measurement noise TMR.

(d) Interaction-induced TMR ($3\sigma_{pest1} = 1.5$ nm).

Figure 4.6: Frequency spectrums of $pest_1$ due to individual TMR sources (NRRO) using the H_2 controller.

Figure 4.7: Pie chart showing contributions of the TMR sources (NRRO) in dual-actuator H_2 control systems.

The spectrums of the interaction-induced TMR in Figure 4.6(d) indicate a spectrum area concentrating around 1.6 kHz, which corresponds to one of the interaction resonance frequency. Although the effect on TMR due to this resonance is not completely eliminated, it can be deduced that the effect is at least largely reduced because the sensitivity function S_{12} in Figure 4.5 shows a notch at this frequency. Thus, the cross-coupled controllers actively compensate for the interaction.

The $3\sigma_{pest_1}$ values due to the disturbance sources can be computed by the collected $pest_1$ samples, respectively. The $3\sigma_{pest_1}$ of the interaction-induced TMR is about 1.5 nm. The contributions of the disturbance sources to TMR including the interaction-induced TMR can be compared and shown in Figure 4.7. The interaction-induced TMR adds 2% to the overall TMR under the coupled DIDO H_2 controller. This is expected to be the optimal result, which can be used to evaluate the tracking performance obtained using other controller design methods.

4.2 Feedforward Decoupling Control Design

The feedforward decoupling control (FDC) design is based on a transformation of the coupled DIDO controller structure as shown in Figure 4.8. The coupled DIDO controller is reconfigured into two parts: feedforward decoupler and feedback controller. The advantage of the configuration is that the control design can be simply separated into two stages:

- (1) In the first stage, the feedforward decoupler is designed to compensate for the interaction dynamics. Once this objective is achieved, the dual-actuator plant is decoupled into two independent SISO plants.
- (2) In the second stage, the feedback controller is designed based on the decoupled SISO plants to satisfy the other servo performance, which is reduced to the conventional single-actuator servo design problem.

Figure 4.8: Feedforward decoupling control system of the dual-actuator plant.

4.2.1 Feedforward Decoupler Design

The feedforward decoupler design is an open-loop design process in order to compensate for the interaction dynamics. The design process is shown in Figure 4.9. Note that the decoupler is realized in a feedforward form, which leads to the advantages that the decoupler can be easily implemented as an add-on filter, and system integrity (system stability and performance) can be furthermore preserved even when the other loop is open [109].

According to Figure 4.9, it is easy to see that the decoupled dual-actuator plant can be achieved if the feedforward decoupler is designed as

$$D_{12} = \frac{P_{12}}{P_{11}}, \quad (4.38)$$

$$D_{21} = \frac{P_{21}}{P_{22}}. \quad (4.39)$$

Therefore, perfect decoupling, i.e., perfect interaction compensation is accomplished if the decouplers derived from (4.38) and (4.39) are invariably stable and causal [83],

Figure 4.9: Feedforward decoupler design process (Note that the decoupled dual-actuator plant in the right plot is seen from $[v_1 \ v_2]$ to $[y_1 \ y_2]$ in the left plot).

which is however difficult to satisfy because the decouplers are in fact inverse-based controllers.

For simplicity, in what follows we take D_{12} as a design example. Based on the identified dual-actuator model shown in Section 2.3.3, it gives that P_{11} contains a pair of complex non-minimum phase (NMP) zeros located at 7.7 kHz, which turns to be unstable poles in D_{12} according to (4.38). Thus, it is necessary to find a stable decoupler that can optimally approximate the original unstable decoupler. This problem can be solved using the methods such as the H_2 optimal control with internal model control (IMC) structure [88], the zero phase error tracking control (ZPETC) [105], and the model matching approach [22] using H_∞ optimization theory. Here, the model matching approach is applied and the corresponding optimization problem is stated as:

Find a stable $Q_0 \in \mathcal{S}$ such that

$$Q_0 = \arg \min_{Q \in \mathcal{S}} \|P_{12} - P_{11}Q\|_\infty, \quad (4.40)$$

where $\mathcal{S}$ denotes the set of all real rational stable transfer functions, and $P_{11}, P_{12} \in \mathcal{S}$. If (4.40) is solvable, then Q_0 is the stable controller that optimally approximates the unstable decoupler D_{12} .

By using the Nevanlinna's algorithm [22], the problem (4.40) can be solved. The obtained optimal Q_0 has a minimal model-matching error with $\|P_{12} - P_{11}Q_0\|_\infty = 7 \times 10^{-4}$. Figure 4.10 shows the pole locus of D_{12} and Q_0 , which indicates that the unstable poles of D_{12} are shifted from the right-half plane (RHP) to the left-half plane (LHP).

However, the problem of (4.40) does not guarantee a proper solution of Q_0 . In

Figure 4.10: Pole locus of the original unstable decoupler D_{12} and the approximate stable decoupler Q_0 .

our case, the Q_0 obtained is actually improper, in which the relative degree between the numerator and the denominator equals to 2. This problem is relatively easy to deal with by adding a low-pass filter of the order equivalent to the relative degree on Q_0 . Therefore, the resultant decoupler $\tilde{D}_{12}$ that is stable and causal for practical realization is of the following form:

$$\tilde{D}_{12} = Q_0 \frac{1}{(\tau s + 1)^k}, \quad (4.41)$$

where $\tau = \frac{1}{2\pi 20000}$, and $k = 2$.

The frequency responses in Figure 4.11 indicate that the designed decoupler $\tilde{D}_{12}$ has closely approximated the dynamics of the original unrealizable perfect decoupler D_{12} over the frequency range of interest. Thus, we can hopefully achieve the

Figure 4.11: Frequency responses of the original unrealizable perfect decoupler D_{12} (dashed line), and the approximate realizable decoupler $\tilde{D}_{12}$ (solid line).

decoupled dual-actuator plant and proceed to design the feedback controller based on the decoupled SISO plants using the SISO design methods.

4.2.2 Feedback Controller Design

The feedback controllers C_{11} and C_{22} are designed respectively based on P_{11} and P_{22} to meet the design specifications as listed in Section 3.1. Here, they are designed using the classical loop shaping methods and are given directly by the product of a lead-lag controller and a notch filter according to the following equation

$$C_{fb} = k_{fb} \cdot \frac{s + \omega_{g1}}{s + \omega_{g2}} \cdot \frac{T_{d1}s + 1}{T_{d2}s + 1} \cdot \frac{s^2 + 2\zeta_{d1}\omega_d s + \omega_d^2}{s^2 + 2\zeta_{d2}\omega_d s + \omega_d^2}. \quad (4.42)$$

Because P_{11} and P_{22} have closely similar dynamics, the parameters in (4.42) for C_{11} and C_{22} are selected as the same and given by

$$\begin{aligned}\omega_{g1} &= 2\pi 200, & \omega_{g2} &= 2\pi 35, \\ T_{d1} &= 1/(2\pi 200), & T_{d2} &= 1/(2\pi 2785), \\ \zeta_{d1} &= 0.02, & \zeta_{d2} &= 0.16, \\ \omega_d &= 2\pi 5700, & k_{fb} &= 0.2.\end{aligned}$$

The resulting two SISO closed-loop systems both achieve the servo performance as shown in Table 4.1.

Table 4.1: Servo performance of SISO feedback control system

Open-loop bandwidth	1.4 kHz
Phase margin	45 deg
Gain margin	12 dB
Sensitivity peak gain	5 dB
Sensitivity gain at 100 Hz	−40 dB

Finally, we evaluate the overall dual-actuator feedforward decoupling control system in Figure 4.8 with the designed decoupler (4.41) and the feedback controller (4.42). Figure 4.12 shows a Nyquist array of the open-loop transfer function matrix from $[v_1 \ v_2]$ to $[y_1 \ y_2]$, which is given by

$$L(s) = P(s)D(s)C(s) = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}, \quad (4.43)$$

where

$$P(s) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \quad C(s) = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix},$$

$$D(s) = \frac{1}{D_{12}D_{21} - 1} \begin{bmatrix} -1 & D_{12} \\ D_{21} & -1 \end{bmatrix}.$$

The Gershgorin bands in Figure 4.12 are obtained by superimposing a circle of the radius $|L_{12}(j\omega)|$ and $|L_{21}(j\omega)|$ on the loci of $L_{11}(j\omega)$ and $L_{22}(j\omega)$ at each frequency,

Figure 4.12: Nyquist array of the dual-actuator open-loop system $L(s)$ via feed-forward decoupling control design (L_{11} and L_{22} have similar unzoned or zoomed curves).

respectively. The figure shows that the Gershgorin bands exclude the critical point -1 , which infers the closed-loop stability [83]. In addition, the widths of the Gershgorin bands are less than 0.008 which are too narrow to be observed if not zoomed from the figure. This feature indicates that the system $L(s)$ is closely identical to two independent SISO systems. Therefore, it can be validated that the designed decoupler achieves adequate interaction compensation.

4.2.3 Simulation and Experimental Results

The designed controllers were implemented on the dual-actuator HDD. The suppression of interaction-induced TMR is demonstrated by experimental results. Simulations are carried out to evaluate the robustness. For simplicity, only the results of pes_1 are shown. The following configurations are carried out for comparison of the controller effectiveness:

without servo:	represents that the actuator 1 works in the open-loop servo without control input but actuator 2 works in the closed-loop servo;
with $C(s)$:	represents that both actuators work in the closed-loop servo but only with the feedback controller $C(s)$;
with $C(s) + D(s)$:	represents that both actuators work in the closed-loop servo with both the feedforward decouplers $D(s)$ and the feedback controller $C(s)$.

Suppression of Interaction-Induced TMR

The frequency spectrums of pes_1 are shown in Figure 4.13. The top plot shows the interaction-induced TMR caused by the effect of the actuator 2 loop. The spectrum of the interaction-induced TMR concentrates at low frequencies and the narrow-band frequency between 1.4–1.9 kHz. The middle plot shows that when the feedback controllers take effect, the low-frequency interaction-induced TMR is sufficiently suppressed, but the mid-frequency components are adversely amplified due to the waterbed effect. The bottom plot shows that the feedback controllers together with the add-on feedforward decouplers significantly compensate for the mid-frequency interaction-induced TMR while maintaining the spectrum components at other frequencies.

The time traces of pes_1 are given in Figure 4.14. The top plot shows that the interaction-induced TMR is dominated by a mid-frequency sinusoidal vibration superimposing on some low-frequency vibrations. The mid-frequency sinusoidal vibration has an explicit cycle of 0.6 ms, which corresponds to the interaction resonance mode at 1.6 kHz. The fast Fourier transform (FFT) of the time traces coincides with the frequency spectrums in Figure 4.13. The middle plot shows that the low-frequency vibrations are suppressed, but the mid-frequency sinusoidal vibration still remains and even has a higher amplitude than that in the top plot. The bottom plot shows that the feedforward decouplers further suppress the mid-frequency sinusoidal vibration. In summary, the $3\sigma_{pes1}$ value is reduced from 8.9 nm to 2.4 nm, which is a 73% reduction ratio. Compared to the H_2 optimal control design which achieved a 1.5 nm $3\sigma_{pes1}$ value of the interaction-induced TMR, the FDC design

still has difference with the optimal interaction compensation. This is because the designed decoupler is an approximation of the perfect decoupler and the low-order feedback controller is designed based on the loop shaping method instead of the optimal control method associated with the disturbance models.

Figure 4.15 shows the transient responses of pes_1 under the step output disturbances $d_1 = d_2 = 0.5 \mu\text{m}$. The top plot shows that the interaction-induced TMR causes some oscillations on the step responses of pes_1 , which lengthen the settling time. The bottom plot shows that the oscillations are significantly suppressed by the add-on feedforward decouplers. Thus, the settling time is reduced from 15 ms to 6 ms.

Evaluation of Performance Robustness

The performance robustness is evaluated by simulation. Since the feedforward decouplers are designed based on the plant models, the tracking performance is tested under the variations of the resonance frequencies in the actuator and the interaction plant models, which are possibly caused by the different HDD mounting mechanism.

Figure 4.16 shows the test results of $3\sigma_{pes1}$ under $\pm 5\%$ variations of the P_{11} resonance frequencies, respectively. The results indicate that the performance of $3\sigma_{pes1}$ is retained quite closely to the nominal performance. The experimental results in previous section have shown that the major frequency spectrums of interaction-induced TMR are concentrated below 2 kHz, but the P_{11} resonance frequencies are far away from 2 kHz, hence, the tracking performance under FDC is expected to have robustness against the variations of the resonance frequencies in actuator dynamics.

For the $3\sigma_{pes1}$ under $\pm 5\%$ variations of the P_{12} resonance frequencies, the test results are given in Figure 4.17, which indicates that the performance of $3\sigma_{pes1}$ is quite sensitive to the resonance frequency at 1600 Hz but robust against the other resonance frequencies. This is because the interaction-induced TMR is dominated by this 1600 Hz resonance while the feedforward decouplers are designed based on P_{12} . The more detailed test results on this dominant mode are summarized in Figure 4.18. It is observed that a robust performance range within only $\pm 2\%$ variations is not sufficient because the uncertainties in the interaction dynamics occurs within $\pm 5\%$ according to the measurement.

From the test results, we conclude that the FDC is able to achieve desired nominal tracking performance but robust performance is inadequate. In order to improve the robustness, the feedforward decouplers using adaptive algorithm may be employed. Alternatively, the dual SISO controller design introduced in the next chapter in fact can improve the robustness.

4.3 Summary

The coupled DIDO controller design methods are presented in this chapter for the dual-actuator servo systems. The H_2 optimal control design provides direct solution for the controller when the dual-actuator control objective, i.e., σ_{pest} , is formulated as a standard H_2 control problem. Due to the involvement of disturbance models in the controller solution, the TMR source modeling method for HDD servo systems is also discussed. Although the derived controller is impractically high-order, simulations show that it gives an optimal $3\sigma_{pest1}$ value (1.5 nm) of the interaction-induced TMR

in the dual-actuator servo systems. The result can be used to evaluate the interaction compensation using other controller design methods.

The feedforward decoupling control design method contains a simple two-stage design process. In the first stage, an optimal stable and causal feedforward decoupler is designed to approximate an inverse-based controller, which can achieve perfect interaction compensation. In the second stage, the feedback controller can be designed using the SISO design methods based on the assumption that the dual-actuator plant is perfectly decoupled. The stability of the overall DIDO control system and the interaction compensation performance are finally validated from simulations. Experimental results show that the $3\sigma_{pest_1}$ value of the interaction-induced TMR can be reduced to 2.4 nm. However, simulation results indicate that the interaction compensation performance is sensitive to the interaction uncertainties with a tolerance within $\pm 2\%$ variation only.

Figure 4.13: Frequency spectrums of pes_1 (top: without servo; middle: with $C(s)$ only; bottom: with $C(s) + D(s)$).

Figure 4.14: Time traces of pes_1 (top: without servo, $3\sigma_{pes1} = 8.4$ nm; middle: with $C(s)$ only; $3\sigma_{pes1} = 8.9$ nm; bottom: with $C(s) + D(s)$, $3\sigma_{pes1} = 2.4$ nm).

Figure 4.15: Transient responses of pes_1 to step output disturbance $d_1 = d_2 = 0.5$ μm (top: with $C(s)$ only, settling time=15 ms, bottom: with $C(s) + D(s)$, settling time=6 ms).

Figure 4.16: Performance robustness against P_{11} resonance frequency variations (dashed-dot line: nominal performance with $C(s)$ only; dashed line: nominal performance with $C(s) + D(s)$).

Figure 4.17: Performance robustness against P_{12} resonance frequency variations (dashed-dot line: nominal performance with $C(s)$ only; dashed line: nominal performance with $C(s) + D(s)$).

Figure 4.18: Performance robustness against ω_6 variations of P_{12} (acceptable range: $-3\% \sim 2\%$).

Chapter 5

Dual SISO Controller Design

This chapter presents the controller design for the dual-actuator track-following servo system based on the dual SISO controller structure as shown in Figure 3.2. The analysis of the servo system by loop decomposition reveals that the interaction dynamics can be represented as plant uncertainties and narrow-band disturbances acting on a SISO control loop. The stability condition of the dual SISO control system is given. After that, a two-stage design method including the feedback controller design and the add-on interaction filter design is developed to establish the dual SISO controller. The design method is applied to the dual-actuator plant. Simulation and experimental results are presented to validate the effectiveness of the control design.

5.1 Analysis of Dual SISO Control System

This section analyzes the dual SISO control system of the dual-actuator by loop decomposition. The interaction effects on system stability and tracking performance

are investigated. The results are subsequently used to guide the controller design in the next section.

5.1.1 Loop Decomposition

Recall the block diagram in Figure 3.2 showing the dual SISO control structure for the dual-actuator servo system. The disturbance inputs to each control output involve both d_1 and d_2 if the interaction coupling is considered. Thus, it is easy to derive that the pes_1 of the first loop is given by

$$pes_1 = \frac{1}{1 + C_{11}G_{11}}d_1 - \frac{D_{12}}{1 + C_{11}G_{11}}d_2, \quad (5.1)$$

$$D_{12} = \frac{P_{12}C_{22}}{1 + P_{22}C_{22}}, \quad (5.2)$$

$$G_{11} = P_{11} + \Delta_{11}, \quad (5.3)$$

$$\Delta_{11} = \frac{-P_{12}P_{21}C_{22}}{1 + P_{22}C_{22}}. \quad (5.4)$$

Similarly, the pes_2 of the second loop can be derived as follows

$$pes_2 = \frac{1}{1 + C_{22}G_{22}}d_2 - \frac{D_{21}}{1 + C_{22}G_{22}}d_1, \quad (5.5)$$

$$D_{21} = \frac{P_{21}C_{11}}{1 + P_{11}C_{11}}, \quad (5.6)$$

$$G_{22} = P_{22} + \Delta_{22}, \quad (5.7)$$

$$\Delta_{22} = \frac{-P_{21}P_{12}C_{11}}{1 + P_{11}C_{11}}. \quad (5.8)$$

Equations (5.1) and (5.5) indicate that the PES is subject to not only the disturbance from its main loop, but also the disturbance from the other loop via the interaction path. Moreover, (5.2) – (5.4) indicate that the interaction effect from the second loop to the first loop can be represented as the plant uncertainties Δ_{11}

Figure 5.1: Loop decomposition of the dual SISO control system.

and the disturbance model D_{12} . Similar results hold for the second loop. Therefore, the dual SISO control system coupled by the interactions can be equivalently decomposed into two separated control loops as shown in Figure 5.1.

The decomposed control system can provide a block diagram to investigate the interaction effects on the system stability and tracking performance in a simple and explicit way. Moreover, the controller design can be simplified into a SISO loop design based on such a decomposed structure.

5.1.2 Interaction Effects on Stability

The following terms are defined with reference to Figure 5.1.

$$S_1 = \frac{1}{1 + P_{11}C_{11}}, \quad (5.9)$$

$$S_2 = \frac{1}{1 + P_{22}C_{22}}, \quad (5.10)$$

$$S'_1 = \frac{1}{1 + G_{11}C_{11}}, \quad (5.11)$$

$$S'_2 = \frac{1}{1 + G_{22}C_{22}}. \quad (5.12)$$

The stability condition for the dual SISO control system of the dual-actuator in Figure 3.2 can be stated in terms of Figure 5.1 as follows

Theorem 5.1 *Consider the stable dual-actuator plant with the stable feedback controllers C_{11} and C_{22} , as shown in Figure 3.2, assume that S_1 and S_2 are stable. Then the closed-loop system in Figure 3.2 is stable if and only if S'_1 or S'_2 is stable. ■*

Proof The return difference operator H for the closed-loop system in Figure 3.2 can be obtained as

$$H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \quad (5.13)$$

$$= \begin{bmatrix} 1 + P_{11}C_{11} & P_{12}C_{22} \\ P_{21}C_{11} & 1 + P_{22}C_{22} \end{bmatrix}. \quad (5.14)$$

It follows from Theorem 2.8 in [83] that the closed-loop system is stable if and only if the determinant of H has no zeros in the closed right-half plane (RHP), i.e.,

$$\begin{aligned} \det(H) &= (1 + P_{11}C_{11})(1 + P_{22}C_{22}) - P_{12}P_{21}C_{11}C_{22} \\ &\neq 0 \text{ in the closed RHP.} \end{aligned} \quad (5.15)$$

Substitute (5.3) and (5.4) into (5.11), then it can be claimed that the return difference operator H'_1 of S'_1 in (5.11) has no zeros in the closed RHP because S'_1 is stable, i.e.,

$$\begin{aligned} H'_1 &= 1 + (P_{11} + \Delta_{11})C_{11} \\ &= [(1 + P_{11}C_{11})(1 + P_{22}C_{22}) - P_{12}P_{21}C_{11}C_{22}] \times S_2 \\ &\neq 0 \text{ in the closed RHP.} \end{aligned} \quad (5.16)$$

Therefore, (5.15) follows immediately because of the stability of S_2 .

Identically, the return difference operator H'_2 of S'_2 in (5.12) has no zeros in the closed RHP because S'_2 is stable, i.e.,

$$\begin{aligned} H'_2 &= [(1 + P_{11}C_{11})(1 + P_{22}C_{22}) - P_{12}P_{21}C_{11}C_{22}] \times S_1 \\ &\neq 0 \text{ in the closed RHP,} \end{aligned} \tag{5.17}$$

which also implies (5.15) because of the stability of S_1 . ■

Remark 5.1 The stability condition provides a guideline for the controller design. That is, the controllers C_{11} and C_{22} can be designed independently based on the plants P_{11} and P_{22} with the closed-loop robust stability to tolerate the uncertainties Δ_{11} and Δ_{22} , respectively. If the robust stabilities of the separated control loops are satisfied simultaneously, the overall stability of the dual SISO control system coupled by the interactions can be guaranteed.

Remark 5.2 The magnitudes of Δ_{11} and Δ_{22} in the dual-actuator servo system can be evaluated by substituting a typical HDD controller such as (4.42) into (5.4) and (5.8). The results show that Δ_{11} and Δ_{22} have less than 0.003 dB magnitudes relative to those of P_{11} and P_{22} over the whole frequency range of interest. Therefore, the uncertainties Δ_{11} and Δ_{22} due to the interaction effects can be easily tolerated to guarantee stability, when the separated SISO control loop achieves the closed-loop robust stability in term of the design specifications as listed in Section 3.1, i.e. with minimal gain margin of 5 dB and phase margin of 30 deg. ■

5.1.3 Interaction Disturbances Analysis

The interaction effects on the tracking performance is regarded as disturbances acting on the system outputs as shown in Figure 5.1. The interaction disturbances can be further reflected by the dynamics D_{12} and D_{21} .

Consider equation (5.2), we see that D_{12} is the product between the interaction model and the sensitivity function multiplied by the controller. The sensitivity function in HDD servo systems is generally designed to have low gains at low frequencies but at the cost of high gains at mid-high frequencies due to the waterbed effect. Thus, the sensitivity function can adequately compensate for the interaction resonances at low frequencies, but amplify those at mid-high frequencies, which lead to the D_{12} dynamics and exhibit lightly damped resonances.

The above analysis of the interaction disturbance models is validated from the

Figure 5.2: Magnitude plot of the interaction disturbance model D_{12} .

simulation of D_{12} , where C_{22} is considered as a typical HDD controller such as (4.42). The magnitude of the frequency responses of D_{12} is plotted in Figure 5.2. It indicates a lightly damped resonance mode around 1.6 kHz, and the magnitudes at other frequency ranges are relatively flat. It is easy to infer that the resonance around 1.6 kHz corresponds to the resonance R_6 in the interaction P_{12} .

Remark 5.3 The lightly damped resonance in the interaction disturbance models will amplify the disturbance source and behave as mid-frequency narrow-band disturbances acting on the PES. Thus, the resonance mode must be actively damped. An effective way is to design an interaction filter added onto the original controller such that the resultant sensitivity function is shaped to have sufficiently low gains around the interaction resonance. ■

5.2 Dual SISO Controller Design

Based on the analysis in previous section, the SISO controller can be constructed by two individual parts as shown in Figure 5.3, which can be designed by the following two stages:

- (1) In the first stage, the feedback controller $C_{fb}(s)$ is designed based on P_{11} to provide basic robust stability and servo performance.
- (2) In the second stage, the add-on interaction filter $F_{if}(s)$ is designed based on the baseline control system in the first stage to reject the mid-frequency narrow-band interaction disturbances.

Figure 5.3: Block diagram of the SISO controller structure.

A variety of SISO design methods can be employed to design the controllers in both stages. The classical loop shaping method is used here, because it is easy to understand and leads to low-order controllers. For simplicity, only controller design for the first loop is illustrated in the following sections, but the same design procedure can also be applied to the second loop.

5.2.1 Feedback Controller Design

The feedback controller C_{fb} is designed by using a lead-lag controller and a notch filter. The controller equation and parameters are the same as those in (4.42). The Bode plot of the open-loop control system is shown in Figure 5.4. The sensitivity function plot can refer to Figure 5.8. The resultant closed-loop servo performance specifications are the same as those in Table 4.1.

It can be validated from the results obtained that the stability margin is sufficient to provide robustness against the uncertainties Δ_{11} due to the interaction effects. Therefore, the stability of the overall coupled control system is guaranteed according to Theorem 5.1. The feedback control system obtained here provides the baseline control system to design the interaction filter in the next section.

Figure 5.4: Bode plot of the open-loop control system with C_{fb} only.

5.2.2 Interaction Filter Design

The interaction filter is designed based on the above baseline control system to reject the narrow-band interaction disturbances around 1.6 kHz. This can be achieved by the add-on of a proper high-gain filter such that the sensitivity function is further shaped to have low gains around 1.6 kHz. Moreover, the stability margin should be retained because the frequencies around 1.6 kHz are close to the open-loop crossover frequency (OLCF), where if a pure high-gain filter is embedded, then the phase margin must be badly degraded. For example, the conventional peak filter [103] has a sharp peak for narrow-band disturbance rejection but at the cost of phase loss scaled with the peak gain at the center frequency. The phase loss might not cause stability problem if the filter center frequency is at very low frequency, but it negatively reduces the phase margin significantly when the filter center frequency is

around the OLCF.

A phase-lead peak filter (PLPF) which provides pure phase-lead feature is developed as the interaction filter to reject the interaction disturbances as well as maintain the stability margin. The interaction filter F_{if} can be represented as the following transfer function

$$F_{if}(s) = \frac{\alpha_f s^2}{s^2 + 2\zeta_f \omega_f s + \omega_f^2}, \quad (5.18)$$

$$\alpha_f = \frac{2\Delta\omega_f - \Delta^2}{(\omega_f - \Delta)^2}, \quad (5.19)$$

$$\zeta_f = \frac{\alpha_f}{2\sqrt{10^{M/10} - 1}}, \quad (5.20)$$

where ω_f is the disturbance frequency, Δ is the disturbance bandwidth and M (dB) is the desired gain attenuation of the disturbance. The parameters have an approximate correspondence to the performance specifications indicated on the sensitivity function plots (see Figure 5.8). In our case, the parameters in (5.18)–(5.20) are set to be

$$\omega_f = 2\pi 1600, \quad \Delta = 2\pi 300, \quad M = 10 \text{ dB}. \quad (5.21)$$

The Bode plots of the designed interaction filter F_{if} and the overall SISO controller $C_{11} = C_{fb}(1 + F_{if})$ are shown in Figure 5.5. Figure 5.6 shows the open-loop servo system plot with C_{fb} and F_{if} . The key advantage of the add-on F_{if} is that it increases the phase of the open-loop function around $\omega_f = 1.6$ kHz rather than decreases. This can relax the limitation of the maximum M for disturbance rejection while maintaining acceptable PM. However, the open-loop gain is greater than 0 dB at ω_f , which results in three open-loop crossover frequencies. Thus, there are three candidate PM values corresponding to ω_{c1} , ω_{c2} , and ω_{c3} , respectively, as clearly

shown in Figure 5.7. Since the phase margin requires the least phase perturbation to drive the system to instability, we must choose the minimum of the three possible PMs [30], i.e. the PM at ω_{c3} . Therefore, the PM needs to be reevaluated, especially when ω_f is close to the -180 -deg crossover frequency.

Figure 5.8 shows the magnitude plots of the sensitivity function. It indicates that a desired gain attenuation around ω_f is achieved by adding the interaction filter F_{if} whereas an undesired gain amplification occurs around ω_f with the original baseline servo only.

Table 5.1 compares the servo performance obtained under the baseline servo and those with the add-on interaction filter F_{if} . The robust stability with the add-on interaction filter is only a bit lower. The great improvement is that the sensitivity gain at 1.6 kHz is reduced significantly.

Table 5.1: Comparison of servo performance of SISO control system

Specifications	Baseline servo	with add-on interaction filter (F_{if})
Open-loop bandwidth	1.4 kHz	1.0 kHz
Phase margin	45 deg	39 deg
Gain margin	12 dB	8 dB
Sensitivity peak gain	5 dB	6 dB
Sensitivity gain at 100 Hz	-40 dB	-40 dB
Sensitivity gain at 1.6 kHz	3.5 dB	-10 dB

Figure 5.5: Bode plots of F_{if} (solid line) and $C_{11} = C_{fb}(1 + F_{if})$ (dashed line).

Figure 5.6: Bode plots of the open-loop servo system with $C_{fb} + F_{if}$.

Figure 5.7: Nyquist plots showing phase margin (PM) candidates (with F_{if} : there are 3 PM candidates; without F_{if} : only 1 PM).

Figure 5.8: Magnitude plots of sensitivity function $S_1 = (1 + P_{11}C_{11})^{-1}$.

5.3 Simulation and Experimental Results

The designed controllers were implemented on the dual-actuator HDD. The injected disturbance sources are the same as those used in previous chapter. The suppression of interaction-induced TMR is demonstrated by experimental results. Simulations are carried out to evaluate the robustness. For simplicity, only the results of pes_1 are shown. The following configurations are carried out for comparison of the controller effectiveness:

without servo:	represents that the actuator 1 works in the open-loop servo without control input but actuator 2 works in the closed-loop servo with the feedback controller;
with C_{fb} only:	represents that both actuators work in the closed-loop servo but only with the feedback controller C_{fb} ;
with $C_{fb} + F_{if}$:	represents that both actuators work in the closed-loop servo with the feedback controller C_{fb} along with the interaction filter F_{if} switched on.

Suppression of Interaction-Induced TMR

The frequency spectrums of pes_1 are shown in Figure 5.9. The top plot shows the interaction-induced TMR caused by the effect of the actuator 2 loop. The spectrum of the interaction-induced TMR concentrates at low frequencies and the narrow-band frequency around 1.6 kHz. The middle plot shows that when the feedback controllers take effect, the low-frequency interaction-induced TMR is sufficiently suppressed, but the mid-frequency components are adversely amplified due to the

waterbed effect. The bottom plot shows that the C_{fb} together with the add-on F_{if} effectively compensate for the narrow-band interaction-induced TMR around 1.6 kHz while the frequency spectrums at other frequency bands are not obviously amplified.

The time traces of pes_1 are given in Figure 5.10. The results indicate that when F_{if} is added onto the control system with C_{fb} , the $3\sigma_{pes1}$ value due to the interaction-induced TMR is reduced from 9.4 nm to 3.7 nm, which is a 61% reduction ratio. Figure 5.11 shows the transient responses of pes_1 to a 0.5 μm step reference input. The results indicate that the add-on F_{if} results in 17% more overshoot and 25% longer settling time but smaller steady-state error compared to those without F_{if} . This is due to the high-gain effect of F_{if} acting on the closed-loop system. However, the noise rejection at high frequencies are maintained, and the maximum control input is nearly the same. Therefore, these trade-offs may well be acceptable.

Evaluation of Performance Robustness

The performance robustness is evaluated by simulation. Since F_{if} is designed according to the characteristic of the resonance mode R_6 in the interaction dynamics P_{12} , the tests are carried out by varying the resonance frequency ω_6 and damping ratio ζ_6 , respectively. After that, the corresponding $3\sigma_{pes1}$ is calculated. The results obtained are summarized in Figure 5.12. The tracking performance can tolerate variations of ζ_6 within $\pm 30\%$ because the PES 3σ values are almost maintained near the nominal value. The acceptable PES 3σ is maintained when the ω_6 varies within the range of $\pm 20\%$.

Figure 5.9: Frequency spectrums of pes_1 (top: without servo; middle: with C_{fb} only; bottom: with $C_{fb} + F_{if}$).

Figure 5.10: Time traces of pes_1 (F_{if} disabled: $3\sigma_{pes1} = 9.4$ nm; F_{if} enabled: pes $3\sigma_{pes1} = 3.7$ nm).

Figure 5.11: Transient responses of pes_1 to a step reference input in the decomposed SISO servo system.

Figure 5.12: Performance robustness of F_{if} against interaction P_{12} modal parameter uncertainties (the pes 3σ is nearly maintained within $\pm 20\%$ of ω_6 variations, and almost insensitive to ζ_6 variations).

5.4 Summary

This chapter presents the dual SISO controller design for the dual-actuator servo system. The dual SISO control system coupled by interactions is analyzed via loop decomposition, which reveals that the interaction effects can be represented as both plant uncertainties and narrow-band disturbances acting on a SISO control loop. The stability condition of the dual SISO control system is given. After that, a two-stage design method including the feedback controller design and the add-on interaction filter design is developed to establish the dual SISO controller. The design method is applied to the dual-actuator plant. A phase-lead peak filter is presented to reject the mid-frequency interaction disturbances with preserved system

stability margin.

Experimental results show that the $3\sigma_{\text{pest}_1}$ value of the interaction-induced TMR can be reduced to 3.7 nm with the dual SISO controller. Compared to the result 2.4 nm obtained by FDC in Section 4.2.3, the dual SISO control design has less reduction of interaction disturbances. However, simulation results show that the dual SISO control design can tolerate interaction frequency variations within $\pm 20\%$, which is a significant improvement compared to the range of $\pm 2\%$ achieved by FDC. Furthermore, the dual SISO controller is of only 6th-order, which is quite easy to implement.

Motivated by the phase-lead peak filter design used for mid-frequency narrow-band disturbance rejection, the next chapter proposes a generalized filter design method. The filter is efficient in rejecting narrow-band disturbances at all frequency range with minimally degradation of the system stability.

Chapter 6

Generalized Optimal Narrow-Band Disturbance Filter Design

Previous peak filter designs are only effective to reject the narrow-band disturbances in a limited frequency range. This chapter presents a generalized optimal filter design to reject narrow-band disturbances at all frequency ranges. In addition to assign the filter poles for high gains, the filter zero is designed to minimally degrade the system stability and obtain a smooth sensitivity curve around the disturbance frequency. A nonlinear optimization procedure is developed to select the filter parameters such that the statistical position error is minimized. Experimental results of a PZT-actuated head positioning control system on spinstand demonstrate that the add-on filter can further reduce the PES NRRO by 8% and preserve the stability margin of the original feedback control system. The disturbance filter is also applicable to the single-actuator or the dual-actuator servo systems to reject the narrow-band RROs and NRROs.

6.1 Introduction

The narrow-band disturbances with spectral energies concentrating at narrow frequency bands commonly exist in a practical servomechanism, e.g., the hard disk drive (HDD) servo system. In HDDs, the track misregistration (TMR) is composed of many factors such as the repeatable runout (RRO) and the nonrepeatable runout (NRRO). Typically, a large portion of the NRRO is contained within narrow frequency bands [26]. In order to meet the requirement for a high track density HDD, many control techniques have been applied to reject the RROs and the narrow-band NRROs [126]. These techniques include the classical loop shaping methods and the modern control theories such as H_2 and H_∞ optimal control. The modern control design based on state-space formulations is an automated design tool, which however often results in an impractically high-order controller. Recently, a peak filter using adaptive control technique [67] was proposed to reduce the NRROs, in which the filter's center frequency is adaptively searched for based on the position error signal (PES) in order to identify a dominant spectral peak. The classical loop shaping methods can provide more intuition and a greater ability to tune designs to achieve performance than the automated tools [86]. The narrow-band disturbance filter proposed in this chapter is also based on the classical loop shaping technique.

The frequency range can be divided into three regions in terms of the servo system at hand. The low-frequency range denotes those frequencies substantially lower than the open-loop crossover frequency (OLCF); the mid-frequency range denotes those frequencies around the OLCF, and the high-frequency range denotes those frequencies substantially higher than the OLCF. Hence, the narrow-band disturbance

can be further characterized according to its disturbance frequency. For example, the mid-frequency narrow-band disturbance denotes the narrow-band disturbance with its disturbance frequency located in the mid-frequency range.

In the HDD servo, the peak filter was effectively employed to reject the low-frequency (100–600 Hz) narrow-band disturbance caused by disk shift, disk warp and spindle vibration ([103] and [118]). However, the peak filter is hardly applied to reject the mid-frequency disturbances because of its intrinsic phase loss that negatively impacts the phase margin and distorts the sensitivity gain around the disturbance frequency. Thus, a phase-lead peak filter in Section 5.2.2 was proposed to reject the mid-frequency (1.6 kHz) narrow-band disturbances. The filter is improved by adding a differentiator to provide additional $\pi/2$ phase lead such that the phase margin is preserved and the sensitivity curve is smoothly shaped. For the high-frequency (4–10 kHz) narrow-band disturbance rejection, a phase-stabilized servo controller [71] was developed to suppress the windage disturbance caused by suspension vibrations. The controller should be designed to keep the phase of the open-loop system at the disturbance frequency within $-2\pi \pm \pi/2$ and ensure a second phase margin (> 40 deg) to maintain the robust stability.

Previous peak filters are only effective to reject the narrow-band disturbances in a limited frequency range. This chapter generalizes the filter design to minimally degrade the closed-loop system stability and effectively reduce the disturbances in an unlimited frequency range by assigning the filter zero. The filter was applied to a PZT-actuated head positioning servo system on a spinstand platform. Experimental results demonstrated that the filter can further reduce the NRRO by 8%.

6.2 Optimal Narrow-Band Disturbance Filter Realization and Design

This section presents an optimal narrow-band disturbance filter with parallel realization added on a baseline servo system. The design process of the filter parameters is developed such that the resultant servo system can achieve optimal tracking accuracy by rejecting the narrow-band disturbance.

6.2.1 Disturbance Filter Structure with Parallel Realization

The disturbance filter structure with parallel realization added on to a baseline servo system is shown in Figure 6.1. The baseline servo system is assumed to have basic stability and performance. The filter is connected to the baseline controller in a parallel form such that the filter can be easily enabled or disabled in the tracking mode or seeking mode. Moreover, the parallel realization has better numerical resolution than cascade realization in the case of fixed-point implementation. The

Figure 6.1: Block diagram of a disturbance filter structure with parallel realization added on a baseline servo system (y : controlled output; y_m : measured output, d : output disturbance, n : noise).

most important advantage for this kind of structure is that the control design can be decoupled into two stages [41]. This can be illustrated by the transfer function from the disturbance d to the controlled output y in Figure 6.1, which is given by

$$\begin{aligned} S(s) &= \frac{1}{1 + PC(1 + F)} \\ &= \frac{1}{1 + PC} \frac{1 + PC}{1 + PC + PCF} \\ &= S_0 S_F, \end{aligned} \tag{6.1}$$

where

$$S_0(s) = \frac{1}{1 + PC}, \tag{6.2}$$

$$S_F(s) = \frac{1}{1 + T_0 F}, \tag{6.3}$$

$$T_0(s) = \frac{PC}{1 + PC}. \tag{6.4}$$

Note that S_0 and T_0 are the sensitivity function and complementary sensitivity function of the baseline servo system, respectively. The equation (6.1) shows that the overall sensitivity function of the closed-loop system is the multiplication of two subsystem S_0 and S_F , which implies that the controllers can be designed by a two-stage approach:

- (1) In the first stage, we can design the baseline controller $C(s)$ for basic closed-loop stability and disturbance rejection performance indicated by S_0 .
- (2) In the second stage, we can design the filter $F(s)$ based on the pseudo-plant T_0 as shown in (6.3) such that S_F is shaped to a desired curve for rejecting disturbances in some frequency ranges.

Since we aim at rejecting the narrow-band disturbances, the disturbance filter of the following form can be adopted

$$F(s) = K \frac{s[\omega_0 \cos(\varphi) - \sin(\varphi)s]}{s^2 + 2\zeta\omega_0 s + \omega_0^2}, \quad (6.5)$$

where

ω_0 : is the disturbance frequency, at which high disturbance rejection is required;

ζ : is the damping ratio with $\zeta \in (0, 1)$;

φ : is the phase angle determined by

$$\varphi = \arg[T_0(j\omega_0)] \in [-\pi, \pi]; \quad (6.6)$$

K : is the positive filter gain. Moreover, the closed-loop system will be guaranteed to be stable if

$$0 < K < \gamma, \quad (6.7)$$

where γ is the minimal positive real solution of K in the following two equations

$$\begin{cases} \operatorname{Re}[Q(\omega, K)] = 0 \\ \operatorname{Im}[Q(\omega, K)] = 0 \end{cases} \quad (6.8)$$

where

$$Q(\omega, K) = 1 + T_0(j\omega)F(j\omega). \quad (6.9)$$

Here, $\operatorname{Re}(\cdot)$ and $\operatorname{Im}(\cdot)$ denote the real and imaginary part of a complex number, respectively. Note that if (6.8) has no solution except $K = \omega = 0$, then $\gamma = +\infty$.

The disturbance filter in (6.5) is quite a general high-gain controller structure to reject narrow-band disturbances in a wide frequency range because the filter zero location can be automatically shifted according to the disturbance frequency associated with the baseline servo system. The next two sections will discuss the servo properties of stability and sensitivity gain shaping due to the zero location and the filter gain.

6.2.2 Stability Properties Using the Disturbance Filter

The disturbance filter has two complex poles at

$$p_{1,2} = \omega_0 e^{\pm j\theta}, \quad (6.10)$$

$$\theta = \arctan\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}\right). \quad (6.11)$$

The poles can provide the high loop gain at the disturbance frequency. The filter contains two real zeros at

$$z_1 = 0, \quad (6.12)$$

$$z_2 = \omega_0 \tan(\varphi). \quad (6.13)$$

z_1 is specified at the origin in order to maintain the DC gain substantially below the disturbance frequency as that of the baseline servo system such that the corresponding sensitivity gains are not affected. The other zero z_2 is specified in order to achieve phase stabilization, more specifically, the zero will make the departure angles of the filter poles approach to π in the root locus of $T_0(s)F(s)$. This is the correct choice since the poles move in the most stable direction, which also provides a phase margin of $\pm \frac{\pi}{2}$ [101]. This property can be verified by Figure 6.2. Applying

the rule for departure angles from the root locus design method [30], the departure angle from the pole p_1 is given by

$$\phi_{\text{dep}} = \theta + \alpha + \beta - \frac{\pi}{2} + \pi i, \quad (6.14)$$

$$i = \begin{cases} 1, & \text{for } \varphi \in [-\pi, 0] \\ 0, & \text{for } \varphi \in [0, \pi] \end{cases} \quad (6.15)$$

where

$$\alpha = \arg[T_0(p_1)] \quad (6.16)$$

denotes the sum of the angles from the zeros of $T_0(s)$ to p_1 minus the sum of the angles from the poles of $T_0(s)$ to p_1 . In practice, the filter damping ratio ζ is chosen

Figure 6.2: Root locus for the system $T_0(s)F(s)$. The departure angles from the filter poles $p_{1,2}$ approach to π by assigning the location of the filter zero z_2 .

as a small value (< 0.1) to provide the high gain at the disturbance frequency. Thus, the filter poles will be very close to the imaginary axis, and we can make the following approximations

$$\theta \approx \frac{\pi}{2}, \quad (6.17)$$

$$\alpha \approx \arg[T_0(j\omega_0)] = \varphi, \quad (6.18)$$

$$\beta \approx \begin{cases} -\varphi, & \text{for } \varphi \in [-\pi, 0] \\ \pi - \varphi, & \text{for } \varphi \in [0, \pi] \end{cases} \quad (6.19)$$

Substituting (6.17)–(6.19) into (6.14), it gives that

$$\phi_{\text{dep}} \approx \pi. \quad (6.20)$$

Therefore, the root locus at the filter poles will move towards the left-half plane (LHP) when the loop gain K increases from 0. The point where the locus crosses the imaginary axis can be computed by (6.8); and the corresponding K value equals to γ , below which the poles of the closed-loop system are all in the LHP, which guarantees the system stable.

6.2.3 Sensitivity Shape Properties Using the Disturbance Filter

The zero placement of the disturbance filter can furthermore lead to another important advantage of minimizing the sensitivity gain at the disturbance frequency without obviously distorting the gains at other frequency bands. This can be confirmed by the sensitivity function of (6.3), from which it gives that

$$|S_F(j\omega_0)| = \frac{1}{|1 + T_0(j\omega_0)F(j\omega_0)|}$$

$$\geq \frac{1}{1 + |T_0(j\omega_0)| |F(j\omega_0)|}. \quad (6.21)$$

By computing the phase angle of $F(s)$ at the disturbance frequency $s = j\omega_0$, it is easy to derive that

$$\arg[T_0(j\omega_0)] + \arg[F(j\omega_0)] = 0. \quad (6.22)$$

Therefore, the equal mark in (6.21) holds, which implies that $|S_F(j\omega_0)|$ approaches the minimum value that can be derived as follows

$$|S_F(j\omega_0)|_{\min} = \frac{1}{1 + \frac{K}{2c}|T_0(j\omega_0)|}. \quad (6.23)$$

Figure 6.3: Nyquist plot. The filter zero provides the exact phase lead such that T_0F at ω_0 is located on the positive real axis. Moreover, the overall curve of T_0F is moved away from the -1 circle, implying reduced sensitivity gains around ω_0 and no obvious distortion of the gains at other frequencies. The “ -1 circle” represents the unit circle whose center is at -1 point.

In fact, the equation (6.22) indicates that the filter zero provides the exact phase lead amounting to the phase lag of $T(s)$ at ω_0 such that the point of $T_0(j\omega_0)F(\omega_0)$ is assured to locate on the positive real axis, where $|S_F(j\omega_0)|$ reaches the minimum. This property can be clearly illustrated from the Nyquist plot in Figure 6.3. Another feature observed from the figure is that the phase lead provided by the filter zero tends to move the overall curve of T_0F away from the -1 circle. Thus, the sensitivity gains around ω_0 are reduced, and in the meantime the sensitivity gains at other frequencies are not obviously increased because the curve segments at these frequencies are closed to the neighbourhood of -1 circle.

Figure 6.4 further shows the comparison of the sensitivity gains obtained by different filter zero placements with the disturbance frequency located at low, mid, and high frequency range. The compared filters are all based on the same baseline servo system and the reduction ratio at the disturbance frequency is equivalently normalized by tuning the filter gain K . It is obvious that the filter with optimal zero design achieves a smooth sensitivity curve around the disturbance frequency, around which the filters with different zeros exhibit obvious peaks.

6.2.4 Optimal Rejection of the Narrow-Band Disturbance

According to the waterbed effect in linear systems [22], the push-down sensitivity gains around the narrow-band disturbance frequency is generally accompanied with the pop-up sensitivity gains somewhere else. This phenomenon can be observed from Figure 6.4, where the curve achieved by the proposed filter has no sharp peak but still has some region about 0 dB. Therefore, it is necessary to determine the peak value

(a) Low-frequency

(b) Mid-frequency

(c) High-frequency

Figure 6.4: Frequency responses of the sensitivity gain of $S_F(s)$ with different filter zero placements and disturbance frequency located at different frequency range. The curve achieved with optimal zero is smooth without exhibiting obvious peaks. In addition, the top plot shows that the sensitivity gain is adversely increased instead of attenuated when the zero is placed at an opposite direction.

and bandwidth of the filter associated with K and ζ such that the disturbances along the entire frequency band is statistically minimized while maintaining the other performance such as the stability margin and servo bandwidth.

The selection for the disturbance filter parameters K and ζ can be determined by an optimization process based on a MATLAB/Simulink¹ model with the block diagram in Figure 6.1. It is assumed that the plant model $P(s)$, the baseline controller $C(s)$ and the disturbance source d are known and the measurement noise n is white noise. In the case that the disturbance is unmeasurable, the method proposed by [72] can be employed to estimate the equivalent disturbance. The method can be simply described as: collect the time traces of the measured output under the baseline servo, say $y_m(k)$, $k = 1, \dots, N$, where N represents a sufficiently large number of measurement points (e.g., $N = 20000$); then the disturbance time traces can be estimated from the following equation

$$d(k) = Z^{-1}\{S_0(z)^{-1}y_m(z)\} - n(k), \quad (6.24)$$

where $Z^{-1}\{\bullet\}$ denotes the inverse z transform of a signal; and $n(k)$ has the same PSD (power spectral density) magnitude as the baseline PSD magnitude of $y_m(k)$.

When the disturbance time traces are available, it is easy to obtain the corresponding frequency spectrum with a plot similar to Figure 6.5. Hence, from the plot the disturbance frequency ω_0 of the dominant narrow-band disturbance can be read out directly where the magnitude is relatively high. The disturbance is then injected into the Simulink model and the controlled output $y(k)$ are regenerated for different filter parameters. Finally, the simulated results are evaluated according to the

¹MATLAB and Simulink are registered trademarks of The Mathworks, Inc.

performance criteria such that the optimal filter parameters are determined. Our objective is to minimize the standard deviation of $y(k)$ under the disturbance $d(k)$. Thus, the optimization problem of the filter parameters is formulated as follows

$$\min_{x=(K,\zeta)} \sigma[y(x)] \quad (6.25)$$

subject to the constraints :

$$\text{PM}(x) \geq \text{PM}_0,$$

$$\text{GM}(x) \geq \text{GM}_0,$$

$$f_c(x) \geq f_{c0},$$

$$(0, 0) \leq x \leq (\gamma, 1),$$

where PM_0 , GM_0 and f_{c0} are the minimal requirements of phase margin (PM), gain margin (GM) and open-loop crossover frequency, respectively; and γ can be

Figure 6.5: A typical frequency spectrum of narrow-band disturbance (ω_0 : disturbance frequency; Δ : disturbance bandwidth).

determined by (6.8) for each fixed ζ .

The optimization problem can be solved by using the constrained minimization function *fmincon* in the MATLAB Optimization Toolbox [19]. The following remarks are in order:

- (1) The disturbance filter generally causes multiple open-loop crossover frequencies, which leads to multiple candidate PM values. Since the phase margin requires the least phase perturbation to drive the system to instability, we must choose the minimum of all the possible PMs [30]. Therefore, the PM along with the open-loop crossover frequency need to be reevaluated.
- (2) A tighten constraint and bound of the variables can help to achieve global minimization. Moreover, a good starting guesses of the variables can improve the execution efficiency and help to locate the global minimum instead of a local minimum. An initial estimate of the filter parameters can be set as

$$\zeta = \frac{\Delta(\omega_0 + 0.5\Delta)}{4\omega_0^2}, \quad (6.26)$$

$$K = (10^{M/20} - 1) \frac{2\zeta}{|T_0(j\omega_0)|}, \quad (6.27)$$

where Δ is the disturbance bandwidth as shown in Figure 6.5, which is defined as the frequency difference between the two points whose magnitudes are $1/\sqrt{2}$ times of the peak value, and M (unit: dB) is the desired reduction ratio of the narrow-band disturbance.

- (3) The solution might be trivial when the narrow-band disturbance is not the dominant disturbance factor on the controlled output.

6.3 Applications to a PZT-Actuated Head Positioning Control System

In this section, the disturbance filter design method is applied to a PZT-actuated head positioning control system on a spinstand platform for improved tracking accuracy by rejecting the mid-frequency NRROs.

In disk drive industry, the spinstands are used for the testing and evaluation of magnetic media and heads before the components are assembled into the disk drive during production. With the increasing requirement of a high areal density HDD, it is urgent to increase the track density. Therefore, high precision and efficient servomechanism is needed to position the head on the desired track to support the increased demand for high track density under the spinstand platform. Moreover, precise positioning and noise rejection capability are required for disk media and head testing, track profile and track interference analysis.

We have upgraded the capability of a Guzik spinstand (1701A) with the new design of a PZT head cartridge base [45], which has a displacement range of 2 μm and resolution of 0.02 nm as the positioning device for quick and precise tracking purpose. A picture of the platform setup is shown in Figure 6.6. The disk is rotated by the spindle at 4000 RPM. The head position servo patterns are prewritten on the disk using a multi-frequency servo encoding and decoding scheme [110]. A dual-frequency PES demodulator implemented within a digitizer board with on-board FPGA (Acqiris AC240) is used to generate the PES based on the readback signals of the servo patterns on real-time with the sampling frequency of 40 kHz. The PES

Figure 6.6: A PZT-actuated head positioning device on spindstand.

is then fed back to the controller for reader servo control. Various controllers ([24] and [29]) have been tested with an external PC-based servo system to reduce the RROs from servo track writing (STW) and the NRROs of disk and spindle vibration.

6.3.1 System Description

The plant model, i.e., the controlled object on the spindstand platform consists of the PZT translator chip, the head cartridge base and a suspension carrying the read/write head. The control input is applied to the PZT translator via a PZT amplifier. The control variable PES is the relative error between the head position

Figure 6.7: Frequency responses from PZT amplifier input to PES.

and the servo sectors prewritten on the disk surface. Here, the plant model $P(s)$ is identified using the following equation with the mechanical system model $P_m(s)$ and the equivalent time delay model $P_d(s)$

$$P(s) = P_m(s)P_d(s), \quad (6.28)$$

$$P_m(s) = k \sum_{i=1}^3 \frac{r_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}, \quad (6.29)$$

$$P_d(s) = \frac{1}{\frac{T}{2}s + 1} e^{-\lambda s}. \quad (6.30)$$

The modal parameters of the plant model are listed in Table 6.1. $P_d(s)$ includes an approximate transfer function of the zero-order hold [30] and a time delay term due to process and computation delay. The frequency responses of the identified and measured plant model are shown in Figure 6.7. Note that $P_d(s)$ causes phase lag which would degrade the phase margin and limit the servo bandwidth.

Table 6.1: Modal Parameters of $P(s)$

$r_1=1$	$\zeta_1=0.015$	$\omega_1 = 2\pi 11500$
$r_2=-0.4$	$\zeta_2=0.03$	$\omega_2 = 2\pi 15500$
$r_3=-0.3$	$\zeta_3=0.01$	$\omega_3 = 2\pi 18500$
$k = 2 \times 10^{11}$	$T = 25 \mu s$	$\lambda = 20 \mu s$

6.3.2 Baseline Servo Design

The baseline controller $C(s)$ is designed such that the servo system ensures the basic stability margin and disturbance and noise rejection performance. The designed $C(s)$ is of the following equation

$$C(s) = C_{PI}(s) \cdot F_{nf}(s) \cdot [1 + F_{pf}(s)], \quad (6.31)$$

$$C_{PI}(s) = 0.003 \frac{s + 2\pi 13000}{s}, \quad (6.32)$$

$$F_{nf}(s) = \frac{s^2 + 2890s + (2\pi 11500)^2}{s^2 + 130100s + (2\pi 11500)^2}, \quad (6.33)$$

$$F_{pf}(s) = \frac{5429s}{s^2 + 603.2s + (2\pi 600)^2}, \quad (6.34)$$

where C_{PI} is a PI controller; F_{nf} is a notch filter to gain-stabilize the first resonance mode; and F_{pf} is a peak filter used to suppress the low-frequency disturbances caused by disk flutter and spindle vibrations. The baseline servo system achieves an open-loop crossover frequency $f_c = 1400$ Hz, GM = 10 dB, PM = 45 deg (see Figure 6.9) and desired disturbance rejection at low frequencies (see Figure 6.10). The frequency spectrum of the PES NRRO with baseline servo is shown in Figure 6.8.

Figure 6.8: Frequency spectrum of the PES NRRO with baseline servo.

6.3.3 Optimal Filter Design for NRRO Rejection

Figure 6.8 indicates that a narrow-band mid-frequency NRRO occurs at the center frequency 1300 Hz, at which the baseline servo amplifies the disturbances. It is expected that the rejection of the disturbance at this band by the add-on disturbance filter can further reduce the NRRO. Hence, the disturbance filter in (6.5) has the following directly determined parameters

$$\omega_0 = 2\pi 1300, \quad \varphi = -71 \text{ deg.} \quad (6.35)$$

After carrying out the optimization procedure in Sec. 6.2.4 with the constraints chosen as $\text{PM}_0 = 40 \text{ deg}$, $\text{GM}_0 = 6 \text{ dB}$, $f_{c0} = 1300 \text{ Hz}$, the remaining filter parameters are obtained as

$$K = 0.26, \quad \zeta = 0.11. \quad (6.36)$$

Thus, the disturbance filter can be constructed as

$$F(s) = 0.26 \frac{s(0.943s + 2708)}{s^2 + 1873s + (2\pi 1300)^2}. \quad (6.37)$$

The resulting servo system with the add-on filter has the servo characteristics according to Figure 6.9:

$$f_c = 1800 \text{ Hz}, \text{GM} = 7 \text{ dB}, \text{PM} = 50 \text{ deg}. \quad (6.38)$$

The measured frequency responses in Figure 6.10 show that the add-on filter achieves gain attenuations around 1300 Hz without obviously distorting the shape at other frequencies on the sensitivity curve, while the phase lead feature has preserved the stability margin as shown in Figure 6.9.

6.3.4 Experimental Results

The frequency spectrum of the PES NRRO with the add-on filter is shown in Figure 6.11, which indicates that the mid-frequency NRROs around 1300 Hz are significantly attenuated while the NRROs at other frequency bands are not obviously amplified. The PES NRRO 3σ value is further reduced from 0.121 μinch with the baseline servo to 0.111 μinch with the add-on filter, which is a 8% reduction ratio. The improvement can be translated into an increase of the achievable track density from 275 kTPI to 300 kTPI as shown in Figure 6.12.

Figure 6.9: Measured frequency responses of the open-loop system.

Figure 6.10: Measured frequency responses of the sensitivity function.

Figure 6.11: Frequency spectrum of the PES NRRO with the add-on filter, the NRRO components around 1300 Hz are attenuated compared to those in Figure 6.8.

Figure 6.12: Achievable track density (Track density=10%/PES NRRO 3σ).

6.4 Summary

This chapter presents a generalized optimal disturbance filter design method to reject the narrow-band disturbances at all frequency ranges. In addition to assign the filter poles for high gains, the filter zero is assigned to minimally degrade the closed-loop system stability and obtain a smooth sensitivity curve around the disturbance frequency. An optimization procedure is also presented to determine the filter parameters such that the standard deviation of the tracking error is minimized. The disturbance filter was applied to a PZT-actuated head positioning servo system on the spinstand platform. Experimental results showed that the add-on filter further reduced the overall 3σ value of PES NRRO by 8% and preserved the stability margin of the original baseline servo system.

The disturbance filter can be simply plugged into a HDD control system to reject the RROs and NRROs that contribute to the TMR.

Chapter 7

Nonlinear PID Control for Improved Disturbance Rejection

The HDD tracking servo is conventionally performed by linear control because of its simplicity. However, the intrinsic limitation of linear control has constrained the achievable performance such as disturbance rejection. This chapter studies a nonlinear PID (NPID) controller for a class of linear plants with output feedback. The controller gains are modulated by the control error and the rate of error change to yield a closed-loop system for improved disturbance rejection based on a linear PID control system. An optimization problem involving constraints written as linear matrix inequalities (LMIs) is solved to compute the controller parameters that can guarantee the stability of the NPID control system. The NPID control design is applied to the HDD track-following servo systems. Simulation and experimental results are presented.

7.1 Overview of NPID Control Methods

The conventional linear proportional-integral-derivative (LPID) feedback control is to date the most common structure to deal with the servo problems due to its simplicity. A general LPID control structure is of the form

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} + k_i \int_0^t e(\eta) d\eta, \quad (7.1)$$

where k_p , k_d and k_i are real constant parameters, and $e(t)$ is the control error with $e(t) = r(t) - y(t)$, where $r(t)$ and $y(t)$ represents the reference input and the system output, respectively. In order to make the LPID practically applicable under different situations, a variety of modifications of the LPID algorithms have also been proposed and summarized in [9]. However, the performance of LPID control is constrained by the limitations inherent in linear control systems [37]. Therefore, NPID control have been developed to achieve higher performance than that of LPID control.

Previous studies and applications have employed the NPID control on linear plants to improve the specific performance. It has been demonstrated that NPID control has the capability to

- improve tracking accuracy ([63], [98] and [122]);
- reduce overshoot ([12] and [120]);
- decrease rise time [16];
- compensate for friction [7].

From the perspective of control laws, NPID control can be categorized into two broad classes: magnitude-based and phase-based. The magnitude-based NPID controllers are those with the controller gains modulated by the magnitude of the control error. The phase-based NPID controllers are those with the controller gains modulated by not only the control error but also the rate of error change explicitly.

The magnitude-based NPID control design can be found in [12], [16], [98] and [120]. For example, Shahruz [98] has proposed a magnitude-based NPI controller as follows

$$\dot{\xi}(t) = \frac{e(t)}{1 + \mu^2 e^2(t)}, \quad \xi(0) = 0, \quad (7.2)$$

$$u(t) = k_i \xi(t) + [k_p + g_p \exp(\lambda |e(t)|)] e(t), \quad (7.3)$$

where k_p , k_i , g_p , λ and μ are real constant parameters to be tuned. Here, the selection of μ in the integral term (7.2) can make the system dynamics softer for large errors and the selection of g_p and λ in the proportional term (7.3) can make the system dynamics stiffer. Thus, the transient behavior of NPI control system is superior to that of a linear PI control system. In addition, Shahruz has provided an optimization technique to obtain the optimal parameters.

The phase-based NPID design was firstly proposed by Jacobsen [63] for the MIT/Utah hand; and has been further developed by Xu [122] and Armstrong [7]. A phase-based NPD controller proposed by Xu [122] is of the form

$$u(t) = k_p(\cdot) e(t) + k_d(\cdot) \dot{e}(t), \quad (7.4)$$

$$k_p(\cdot) = \frac{k_{p1}}{1 + \beta \exp[\alpha \text{sign}(\dot{e}(t))e(t)]} + k_{p0}, \quad (7.5)$$

$$k_d(\cdot) = \frac{k_{d1}}{1 + \beta \exp[\alpha \text{sign}(\dot{e}(t))e(t)]} + k_{d0}, \quad (7.6)$$

where $k_p(\cdot)$ and $k_d(\cdot)$ are the nonlinear proportional and derivative gains, k_{p0} and k_{d0} are the minimum values of the gains and $k_{p0} + k_{p1}$ and $k_{d0} + k_{d1}$ are the maximum values of the gains, respectively. The constant β determines whether the value of the function is closer to its maximum (smaller β) or minimum (larger β). The constant α determines the width of the transient region from low to high gain. The most important feature of NPD control is that the nonlinear gains can create an improved damping field according to the moving direction of the system output and thus achieve higher disturbance rejection and robustness to time delay. In [121], Xu has proved the stability for a second order plant under NPD control but for a high order plant, only simulation results are given but not proved.

This chapter is concerned with the phase-based NPID control of a class of linear plants with output feedback. For simplicity, the term phase-based NPID control is abbreviated as NPID control in what follows. A new NPID controller structure is developed for improved disturbance rejection. The NPID controller is formulated in state space and the controller parameters can be designed using an LMIs approach. The developed NPID control design will guarantee the stability of a high order plant under NPID control and obtain improved disturbance rejection.

7.2 NPID Control Scheme

This section presents a new NPID controller structure based on the control law proposed in [122]. The NPID controller is formulated in state space and the controller parameters can be designed using an LMIs approach, which will be presented in next section.

7.2.1 System Model in State Space

Consider a linear time-invariant (LTI) and strictly proper plant with its state space representation as follows

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ y(t) = C_1x(t) + D_1w(t) \\ z(t) = C_2x(t) + D_2w(t) \end{cases} \quad (7.7)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the control input, $w(t) \in \mathbb{R}^m$ is the disturbance vector, $y(t) \in \mathbb{R}$ is the measurement output, and $z(t) \in \mathbb{R}$ is the system output, respectively. A , B , G , C_1 , D_1 , C_2 and D_2 are known real constant matrices of appropriate dimensions. The following assumptions on the plant (7.7) are made.

Assumption 7.1

1. The plant is stabilizable with a LPID controller of the form in (7.1).
2. Output feedback is used, hence, only $y(t)$ is measurable.
3. Regulation problem is considered here, i.e., $r(t) \equiv 0$. ■

The LPID controller can be designed using linear control approaches. However, it is well known that the pure differentiator in (7.1) is not physically implementable because differentiation of the control error containing high frequency noise components will generate large control signal. The problem can be alleviated by implementing the differentiator as follows

$$\hat{e}(t) = \frac{s}{\tau_d s + 1} e(t), \quad (7.8)$$

where τ_d is the time constant with $\tau_d = \frac{0.2}{\omega_0}$, and ω_0 is the desired closed-loop bandwidth [30]. The modified PID controller is then represented in state space as follows

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c e(t) \\ y_c(t) = C_c x_c(t) + D_c e(t) \\ u(t) = K y_c(t) \end{cases} \quad (7.9)$$

where $x_c \in \mathbb{R}^2$ is the controller state vector and $y_c \in \mathbb{R}^3$ is the measurable controller output, which are functions of the control error as follows

$$x_c = \begin{bmatrix} \tau_d \hat{e}(t) - e(t) \\ \int_0^t e(\eta) d\eta \end{bmatrix}, \quad y_c = \begin{bmatrix} e(t) \\ \hat{e}(t) \\ \int_0^t e(\eta) d\eta \end{bmatrix}. \quad (7.10)$$

The matrices in (7.9) are of the form

$$A_c = \begin{bmatrix} -\frac{1}{\tau_d} & 0 \\ 0 & 0 \end{bmatrix}, B_c = \begin{bmatrix} -\frac{1}{\tau_d} \\ 1 \end{bmatrix}, C_c = \begin{bmatrix} 0 & 0 \\ \frac{1}{\tau_d} & 0 \\ 0 & 1 \end{bmatrix}, D_c = \begin{bmatrix} 1 \\ \frac{1}{\tau_d} \\ 0 \end{bmatrix},$$

$$K = \begin{bmatrix} k_p & k_d & k_i \end{bmatrix}. \quad (7.11)$$

Folding the controller (7.9) into the plant (7.7), then the closed-loop system is given by

$$\begin{cases} \dot{x}_a(t) = A_a x_a(t) + G_a w(t) \\ z(t) = C_{2a} x_a(t) + D_2 w(t) \end{cases} \quad (7.12)$$

where $x_a \in \mathbb{R}^{n+2}$ is the augmented state vector with

$$x_a = \begin{bmatrix} x_c \\ x \end{bmatrix}; \quad (7.13)$$

and

$$A_a = \left[\begin{array}{c|c} A_c & -B_c C_1 \\ \hline B K C_c & A - B K D_c C_1 \end{array} \right] \quad (7.14)$$

is the augmented closed-loop system matrix;

$$G_a = \left[\begin{array}{c} -B_c D_1 \\ \hline G - B K D_c D_1 \end{array} \right] \quad (7.15)$$

is the augmented disturbance input vector; and

$$C_{2a} = [\phi, C_2] \quad (7.16)$$

is the augmented system output vector, $\phi \in \mathbb{R}^2$ is the zero vector.

Here, the closed-loop system matrix A_a is a function of the controller gain vector K ; it is however fixed once K is determined. Hence, the system performance is unchangeable online. In the next section, a NPID controller with a time-varying $K(\cdot)$ is presented to yield a closed-loop system for improved disturbance rejection.

7.2.2 NPID Control Law

Our objective is to design a NPID control law that maintains the system output closest to zero under the influence of disturbances. This can be accomplished by shifting the constant controller gain vector K in (7.11) into a time-varying controller gain vector

$$K(\cdot) = \left[\begin{array}{ccc} k_p(\cdot) & k_d(\cdot) & k_i \end{array} \right], \quad (7.17)$$

such that the closed-loop system dynamics is instantly changed for favorable disturbance rejection. The NPID control law studied here is motivated by that in

[122]. More specifically, $K(\cdot)$ is modulated according to whether the system output is moving away from or towards the desired target:

1. Movement away from the target: $k_d(\cdot)$ decreases while $k_p(\cdot)$ increases to provide high stiffness.
2. Movement towards the target: $k_p(\cdot)$ decreases while $k_d(\cdot)$ increases to provide high damping.

The NPID control law is illustrated in Figure 7.1. The improvement of the NPID control on disturbance rejection can be qualitatively explained as: at the shaded areas the higher control gain acts as a stiff spring to constrain the system output moving away from the target; while at the non-shaded areas the lower gain and higher damping act as an energy absorber to remove the energy from the overall system. Thus, the NPID control benefits from the nonlinear controller gains and

Figure 7.1: Illustration of NPID control law (adapted from [122]).

achieves higher disturbance rejection than that under LPID control.

7.2.3 NPID Controller Structure

A nonlinear gain vector function $K(\cdot)$ satisfying the NPID control law above can be constructed as follows

$$k_p(\cdot) = k_p [1 + \nu \alpha \rho(e, \hat{e})], \quad (7.18)$$

$$k_d(\cdot) = k_d [1 - (1 - \nu) \alpha \rho(e, \hat{e})], \quad (7.19)$$

$$\rho(e, \hat{e}) = \begin{cases} 1 & \text{if } e(t)\hat{e}(t) > 0 \\ 0 & \text{if } e(t)\hat{e}(t) = 0 \\ -1 & \text{if } e(t)\hat{e}(t) < 0 \end{cases} \quad (7.20)$$

where $\nu \in [0, 1]$ is the weighting factor indicating the extent to which the nonlinear gain takes effect on k_p and k_d , respectively; and $\alpha \in [0, \min(\frac{1}{\nu}, \frac{1}{1-\nu})]$ is the tuning parameter which determines the range of the nonlinear gain and assures the gain invariably positive. It is desired that α be maximized for best performance improvement. This will be illustrated later with an example shown in Figure 7.2.

Combining (7.18)–(7.20) into (7.12) gives a nonlinear closed-loop control system of the form

$$\begin{cases} \dot{x}_a(t) = A_N(\rho)x_a(t) + G_N(\rho)w(t) \\ z(t) = C_{2a}x_a(t) + D_2w(t) \end{cases} \quad (7.21)$$

where

$$A_N(\rho) = A_a + \alpha A_\Delta(\rho), \quad (7.22)$$

$$A_\Delta(\rho) = \left[\begin{array}{c|c} \phi & \phi \\ \hline BK_\Delta(\rho)C_c & -BK_\Delta(\rho)D_cC_1 \end{array} \right], \quad (7.23)$$

$$K_{\Delta}(\rho) = \begin{bmatrix} k_p \nu \rho, & -k_d(1 - \nu)\rho, & 0 \end{bmatrix}. \quad (7.24)$$

7.3 NPID Controller Parameters Synthesis Using an LMIs Approach

For the derived NPID control system (7.21), suppose a linear gain vector K has been designed to assure A_a stable and ν is known, then a question naturally arises: how to select the maximum α that assures the nonlinear closed-loop system asymptotically stable. The problem can be solved analytically for a low order system as shown in the following example.

Example 7.1 Consider a second-order plant of the form

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \quad (7.25)$$

Assume both states of the plant are measurable and the NPD control in (7.18)–(7.20) is applied. Then the nonlinear closed-loop system is derived by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = [a_1 - k_p(\cdot)]x_1 + [a_2 - k_d(\cdot)]x_2. \end{cases} \quad (7.26)$$

To study the stability condition of the system, a natural Lyapunov function candidate may be taken as the energy-like function

$$V(x) = \int_0^{x_1} [k_p(\cdot) - a_1]y dy + \frac{1}{2}x_2^2. \quad (7.27)$$

Let $k_p(\cdot) > a_1$, then $V(x)$ is positive definite and

$$\begin{aligned} \dot{V}(x) &= [k_p(\cdot) - a_1]x_1x_2 + x_2\dot{x}_2 \\ &= -[k_d(\cdot) - a_2]x_2^2. \end{aligned} \quad (7.28)$$

Figure 7.2: Simulation results of NPID control in Example 7.1: disturbance rejection improved when α increases.

Given that $k_d(\cdot) > a_2$, $\dot{V}(x)$ is negative semidefinite. Hence, the maximum α satisfying the stability condition is simply derived as

$$\alpha = \min\left\{\frac{1}{\nu}, \frac{1}{1-\nu}, \frac{k_p - a_1}{\nu k_p}, \frac{k_d - a_2}{k_d - \nu k_d}\right\}. \quad (7.29)$$

A numerical simulation of the example was carried out. The parameters are selected as: $a_1 = -10^5$, $a_2 = -180$, $k_p = 10^7$, $k_d = 10^3$, $\nu = 0.7$ and output disturbance source $\omega(t) = 2\sin(2\pi 300t)$. Thus, from (7.29), it follows that

$$\alpha = \min\{1.43, 3.33, 1.44, 3.93\} = 1.43. \quad (7.30)$$

The simulation results for the NPID control system are shown in Figure 7.2. The result obtained demonstrates that disturbance rejection is gradually improved when α increases within the region that satisfies the stability conditions. ■

For a high order system the stability problem becomes extremely complicated and no systematic methods are available to find appropriate Lyapunov functions that satisfy the stability conditions. However, we note that the stability condition of NPID control system can be formulated as an optimization problem involving constraints written as linear matrix inequalities (LMIs), which is then solved to compute the controller parameters.

7.3.1 Overview of Linear Matrix Inequalities

A wide variety of problems ranging from control theory to system identification and structural design can be reduced to a few standard convex or quasiconvex optimization problems that involve LMIs. The resulting optimization problems can be solved efficiently in practice. Therefore, in this section, an overview of LMIs is introduced. More details of the theories and proofs of LMIs can refer to [13], [34], [52] and [90].

Linear Matrix Inequalities

A linear matrix inequality is a matrix inequality of the form

$$F(x) = F_0 + \sum_{i=1}^m x_i F_i > 0, \quad (7.31)$$

where $x \in \mathbb{R}^m$ is the variable and $F_i \in \mathbb{R}^{n \times n}$, $i = 0, \dots, m$, are given symmetric matrices, i.e., $F_i = F_i^T$. The inequality symbol “ > 0 ” means that $F(x)$ is positive definite. In the case where the matrices F_i are diagonal, the LMI $F(x) > 0$ only constitutes a set of n linear inequalities.

The LMI (7.31) is a convex constraint on the variable space x , i.e., the set $\{x | F(x) > 0\}$ is convex. The LMI (7.31) can represent a wide variety of convex constraints on x . In particular, linear inequalities, convex quadratic inequalities,

matrix norm inequalities, and constraints that arise in control theory, such as Lyapunov and convex quadratic matrix inequalities, can all be cast in the form of an LMI.

Multiple LMIs, $F_1(x) > 0, F_2(x) > 0, \dots, F_k(x) > 0$ can be combined as a single block diagonal LMI of the form

$$\begin{bmatrix} F_1(x) & & & \\ & F_2(x) & & \\ & & \ddots & \\ & & & F_k(x) \end{bmatrix} > 0, \quad (7.32)$$

where the diagonal blocks are the individual LMIs at hand. Accordingly, there is no distinction between a set of LMIs and a single LMI.

Nonlinear convex matrix inequalities can easily be converted to LMI form using Schur complements. The basic idea is as follows: the set of nonlinear inequalities

$$R(x) > 0, \quad Q(x) - S(x)R(x)^{-1}S(x)^T > 0, \quad (7.33)$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ depends affinely on x , are equivalent to the LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0. \quad (7.34)$$

More often, we will deal with problems in which the variables are matrices in the LMIs, e.g., the Lyapunov inequality

$$A^T P + P A < 0, \quad (7.35)$$

where $A \in \mathbb{R}^{n \times n}$ is given and $P = P^T$ is the variable. In this case, the LMI will not be written explicitly in the form $F(x) > 0$, but instead clarify which matrices are

variables. Undoubtedly, the inequality (7.35) can be readily put in the form (7.31), as follows. Let $F_0 = 0$ and $F_i = -A^T P_i - P_i A$, where $P_1, \dots, P_m$ are a basis for symmetric $n \times n$ matrices and $m = \frac{n(n+1)}{2}$. Writing LMIs in a compact form such as (7.35), in addition to saving notation, may lead to more efficient computation [13].

Standard LMI Problems

LMIs can be used to represent a wide variety of convex constraints in optimization problems. Most of the problems can be identified as either LMI problem (LMIP), eigenvalue problem (EVP) or generalized eigenvalue problem (GEVP).

The LMIP is to find x^{feas} such that the given LMI $F(x^{\text{feas}}) > 0$ or determine that the LMI is infeasible. This is called a convex feasibility problem. Many control problems involving stability conditions can be formulated as such LMIPs, e.g., giving a system described by the linear differential inclusion (LDI) $\dot{x} = A_i x$, where $A_i \in \mathbb{R}^{n \times n}$, $i = 1, \dots, k$, thus, a necessary and sufficient condition for quadratic stability of the system is

$$P = P^T > 0, \quad A_i^T P + P A_i < 0 \quad \text{for all } i = 1, \dots, k. \quad (7.36)$$

Therefore, determining the quadratic stability for the system is an LMIP in the variable P .

The EVP is to minimize the maximum eigenvalue of a matrix which depends affinely on a variable, subject to an LMI constraint, i.e.,

$$\begin{aligned} & \text{minimize} \quad \lambda \\ & \text{subject to} \quad \lambda I - A(x) > 0, \text{ and } B(x) > 0, \end{aligned} \quad (7.37)$$

where λ is a scalar, A and B are symmetric matrices that depend affinely on the optimization variable x .

The GEVP is to minimize the maximum generalized eigenvalue of a pair of matrices which depend affinely on a variable, subject to an LMI constraint. The general form of a GEVP is:

$$\begin{aligned} & \text{minimize} \quad \lambda \\ & \text{subject to} \quad \lambda B(x) - A(x) > 0, \quad B(x) > 0 \quad \text{and} \quad C(x) > 0, \end{aligned} \quad (7.38)$$

where λ is a scalar, A , B and C are symmetric matrices which depend affinely on the optimization variable x .

An equivalent formulation of the GEVP is

$$\begin{aligned} & \text{minimize} \quad \lambda \\ & \text{subject to} \quad A(x, \lambda) > 0, \end{aligned} \quad (7.39)$$

where $A(x, \lambda)$ is affine in x for fixed λ and affine in λ for fixed x , and $A(x, \lambda)$ is monotonic in λ , i.e., $\lambda > \mu \implies A(x, \lambda) \geq A(x, \mu)$. An example of GEVP includes measuring quadratic stability margin of a system, e.g., giving a system $\dot{x} = (A_0 + A(t))x$, where $A(t) \in \alpha \mathbf{Co}\{A_1, \dots, A_k\}$, and the largest nonnegative α is defined as the quadratic stability margin, for which the system is quadratically stable. Thus, α can be computed by solving the GEVP in P and α as follows

$$\begin{aligned} & \text{maximize} \quad \alpha \\ & \text{subject to} \quad P > 0, \quad \alpha \geq 0, \end{aligned} \quad (7.40)$$

$$A_0^T P + P A_0 + \alpha(A_i^T P + P A_i) < 0, \quad i = 1, \dots, k.$$

Solution Methods

The standard LMI problems may be solved in polynomial-time, and thus are tractable at least in a theoretical sense. The solution methods include the simple ellipsoid algorithm and the powerful and efficient interior-point methods. In this section, only a general overview of the methods will be outlined.

The ellipsoid algorithm assumes that the problem being solved has at least one optimal point, i.e., the constraints are feasible for at least one point in the variable space. Suppose there exists an ellipsoid ϵ_0 that is guaranteed to contain an optimal point. A cutting plane through the center x_0 of ϵ_0 can then be computed, which guarantees that an optimal point lies in a half-space. Thus, an optimal point must lie in the half-ellipsoid, i.e., the intersection of the ellipsoid ϵ_0 and the half-space containing the optimal point. Next, compute the ellipsoid ϵ_1 of minimum volume that contains the half-ellipsoid of ϵ_0 . ϵ_1 is then guaranteed to contain an optimal point. The process is repeated until the center point of some ellipsoid ϵ satisfies the LMI constraints and exceeds the global minimum by less than some prespecified accuracy, i.e., is optimal. The sequence of ellipsoids generated are guaranteed to contain an optimal point, additionally, their volume decreases geometrically implying polynomial time convergence [13].

The interior-point methods can exploit the problem structure to increase efficiency. Most of the computational effort is devoted to computing the Newton direction of a barrier or similar function. It turns out that this Newton direction can be expressed as the solution of a weighted least-squares problem of the same size as the original problem. Using conjugate-gradient and other related methods to solve these least-squares systems gives two advantages. First, by exploiting problem structure in the conjugate-gradient iterations, the computational effort required to solve the least-squares problems is much smaller than by standard direct methods such as QR or Cholesky factorization. Second, it is possible to terminate the conjugate-gradient iterations before convergence, and still obtain an approximation

of the Newton direction suitable for interior-point methods.

The remarkable advantage for LMIs formulation is that efficient software packages such as the LMI Control Toolbox for MATLAB [34] are developed to solve the standard problems involving LMIs.

7.3.2 Formulation of Parameters Synthesis Using LMIs

The proposed NPID control system (7.21) is essentially switched among three linear subsystems as follows

$$A_N(\rho) = \begin{cases} A_a + \alpha A_{\Delta 1} & \text{if } \rho = 1 \\ A_a + \alpha A_{\Delta 2} & \text{if } \rho = 0 \\ A_a + \alpha A_{\Delta 3} & \text{if } \rho = -1 \end{cases} \quad (7.41)$$

Therefore, a sufficient stability condition presented in [66] can be applied to the system (7.21) directly and restated as follows:

Proposition 7.1 *The NPID control system (7.21) is exponentially stable if there exists a common matrix $P = P^T > 0$ for all the subsystems in (7.41) such that*

$$(A_a + \alpha A_{\Delta i})^T P + P(A_a + \alpha A_{\Delta i}) < 0, \quad \text{for } i = 1, 2, 3. \quad (7.42)$$

■

Proof The above proposition can be proved using a globally quadratic Lyapunov function $V(x) = x^T P x > 0$ and thus $\dot{V}(x) = x^T [(A_a + \alpha A_{\Delta i})^T P + P(A_a + \alpha A_{\Delta i})] x < 0$ for all i . Then the system (7.21) is exponentially stable for every trajectory in the state space. ■

Consequently, finding the maximum α that assures the stability condition in Proposition 7.1 can be formulated as an optimization problem involving constraints written as LMIs, i.e., an GEVP similar to (7.39). The solution of the LMI problem is then equivalent to solving a convex optimization problem. The remarkable advantage here is that efficient software [34] is already available to provide reliable numerical solutions. Therefore, the following LMI problem gives the conditions for the existence of the optimal α .

LMI Problem: Giving the known matrices $A_{\Delta i}$ and stable A_a , the maximum α that assures the NPID control system (7.21) exponentially stable is the solution of the following optimization problem in the symmetric matrix P and scalar α :

$$\text{maximize } \alpha \tag{7.43}$$

subject to

$$\begin{aligned} P &> 0, \quad 0 \leq \alpha \leq \min\left(\frac{1}{\nu}, \frac{1}{1-\nu}\right), \\ A_a^T P + P A_a + \alpha(A_{\Delta i}^T P + P A_{\Delta i}) &< 0, \\ i &= 1, 2, 3. \end{aligned} \tag{7.44}$$

■

Remark 7.1 To speed up the computation, $\alpha = \min(\frac{1}{\nu}, \frac{1}{1-\nu})$ can be set as the initial value. If there exists a P satisfying (7.44), then no more iterations are required. The initial value is exactly the desired maximal α . The undesirable case happened when $\alpha = 0$ because in this case the NPID control reduces to LPID control. Thus, there always exists a P satisfying (7.44) because A_a is assumed to be stabilized under a LPID controller. ■

7.3.3 NPID Controller Design Procedure

From the results obtained, the design procedure for the NPID controller (7.17) can be summarized as the following steps:

1. Design a LPID controller (7.9) for the plant (7.7) using an appropriate method such that the resulting closed-loop system (7.12) has basic stability margin and sensitivity magnitude for disturbance rejection.
2. Replace the LPID controller gains in (7.12) with the NPID controller gains (7.18)–(7.20) such that (7.21) is obtained. Rewrite (7.22) as (7.41).
3. Select ν according to the disturbance characteristics, e.g., if the disturbance spectrum is more intensive below the open-loop crossover, then $\nu > 0.5$ is suggested because k_p improves the disturbance rejection within this range more effectively than k_d . Substitute ν into $A_{\Delta i}$ and thus the matrices A_a and $A_{\Delta i}$ are known. Find the maximum α by solving the LMI problem (7.44). Thus, the NPID controller (7.17) is found with the nonlinear gains given in (7.18)–(7.20).

To smooth the transition among the nonlinear controller gains and thus reduce chattering, the switching function (7.20) can be implemented with an approximating function as follows

$$\rho(e, \hat{e}) = \frac{\exp[\beta e \operatorname{sgn}(\hat{e})] - 1}{\exp[\beta e \operatorname{sgn}(\hat{e})] + 1}, \quad (7.45)$$

where the constant β indicates the switching rate, and it is determined by the order of magnitude of e such that $\rho(\cdot)$ has a fast switch to ensure the improvement of performance. The approximating function of ρ is sketched in Figure 7.3 according

Figure 7.3: A sketch of the approximating function of ρ in (7.45) with different selection of β .

to a sinusoid signal of e with different selections of β . It can be seen that when β increases, the switching rate get faster, which indicates a stronger effect of the non-linear gains. However, the fast switching rate will result in chattering of the control signal. Therefore, the selection of β requires a trade-off between the performance improvement and implementation feasibility.

7.4 Applications to HDD Track-Following Servo Systems

In order to increase the track density, the HDD track-following servo system is required to efficiently compensate for the disturbances caused by disk vibration, spindle run-out, windage and external vibrations. The task is conventionally fulfilled

by a LPID-like compensator [69]. However, such linear controllers are inherently linear systems and thus have to compromise performance among good disturbance rejection, insensitive to noise, robustness, and so on [37]. In this section, the NPID control is applied to the HDD track-following servo system. Experimental results are presented to demonstrate the effectiveness of NPID control for higher disturbance rejection compared to that of LPID control while the stability robustness is maintained.

7.4.1 State Space Model of the VCM Actuator

The VCM actuator assembly in the dual-actuator prototype is employed as the controlled object. The control input u is the voltage to the power amplifier. The control variable is the position error signal (PES), which is the relative error between the actual head position y and the servo sectors prewritten on the disk surface. The feedback loop is closed with LDV in the laboratory setup. The complete plant model has been identified with the parameters listed in Table 2.1. The complete plant model is used in simulations. For the control design, the plant is simplified by modeling the rigid bode mode along with the first resonance mode. A state space representation of the simplified plant model is as follows

$$\left\{ \begin{array}{l} \dot{x} = \begin{bmatrix} 0 & k & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & w_n \\ 0 & 0 & -w_n & -2\zeta w_n \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_n \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x \end{array} \right. \quad (7.46)$$

where $k = 2 \times 10^6$ is the loop gain, $\omega_n = 2\pi 5700$ is the natural frequency and $\zeta = 0.12$ is the damping ratio of the first resonance mode, respectively.

7.4.2 Controller Design Results

The NPID controller is designed based on the plant model in (7.46) following the design procedure described in previous section.

Firstly, using the loop shaping techniques, the LPID controller (7.9) is obtained with the following controller parameters

$$k_p = 15, \quad k_d = 0.003, \quad k_i = 1500, \quad \tau_d = 3 \times 10^{-5}.$$

The frequency responses of the resulting open-loop system under LPID control is shown in Figure 7.4. The system specifications are derived as: phase margin 45 deg, gain margin 10 dB, and the open-loop crossover frequency 1.2 kHz.

Figure 7.4: Frequency responses of the open-loop system under LPID control.

Secondly, construct the corresponding NPID control system (7.21) for the VCM actuator servo system.

Finally, select $\nu = 0.7$ since disturbances with low and midrange frequencies are more concerned in our study. Solving the corresponding LMI problem (7.44) gives $\alpha = 1.2$. A positive definite solution P that guarantees the system stability is obtained as follows

$$P = \begin{bmatrix} 2.0 \times 10^{-4} & -2.8 \times 10^{-3} & -1.4 \times 10^{-4} & 2.7 \times 10^{-2} & -2.5 \times 10^{-7} & 9.9 \times 10^{-7} \\ -2.8 \times 10^{-3} & 31.1 & -3.4 \times 10^{-3} & -1.3 & -1.7 \times 10^{-5} & -3.6 \times 10^{-5} \\ -1.4 \times 10^{-4} & -3.4 \times 10^{-3} & 1.7 \times 10^{-4} & -1.2 \times 10^{-2} & 4.2 \times 10^{-7} & -5.9 \times 10^{-7} \\ 2.7 \times 10^{-2} & -1.3 & -1.2 \times 10^{-2} & 7.7 & 3.2 \times 10^{-5} & 2.4 \times 10^{-4} \\ -2.5 \times 10^{-7} & -1.7 \times 10^{-5} & 4.2 \times 10^{-7} & 3.2 \times 10^{-5} & 5.2 \times 10^{-9} & 1.2 \times 10^{-9} \\ 9.9 \times 10^{-7} & -3.6 \times 10^{-5} & -5.9 \times 10^{-7} & 2.4 \times 10^{-4} & 1.2 \times 10^{-9} & 1.2 \times 10^{-8} \end{bmatrix}.$$

Hence, the NPID controller is established as follows

$$k_p(\cdot) = 15[1 + 0.84\rho(e, \hat{e})],$$

$$k_d(\cdot) = 0.003[1 - 0.36\rho(e, \hat{e})].$$

Moreover, the switching function $\rho(e, \hat{e})$ is implemented with (7.45). In the HDD servo, the order of PES magnitude is nanometer, thus, a typical $\beta = 3 \times 10^8$.

7.4.3 Experimental Results

The NPID was implemented on the actual VCM actuator assembly and compared with the LPID by setting $\alpha = 0$. Firstly, repeatable runouts (RROs) compensation is tested. The RROs correspond to the harmonics generated by a 7200 RPM spindle motor, which contains a fundamental frequency of 120 Hz. In the laboratory test

Figure 7.5: Time traces of PES responses to a RRO harmonic with magnitude = 1 μm and frequency = 720 Hz.

stand, the LDV output is used as a simulated PES and RRO and NRRO disturbances are artificially added. To check the effectiveness of the NPID at different frequencies, experiments are conducted individually by introducing one harmonic of RRO disturbance with a magnitude of 1 μm . The PES responses to a 720 Hz RRO are shown in Figure 7.5, which indicates that a lower PES magnitude is obtained by the NPID and the NPID control input magnitude is nearly not increased compared to that of the LPID. The test results of all RRO harmonics are summarized in Figure 7.6, which shows that the NPID achieves higher attenuation for the midrange frequency RROs. This is because the weighting factor ν was selected to have more effect on the midrange disturbance rejection. Figure 7.6 also shows that the control signal RMS (Root Mean Square) values under NPID control are nearly the same as those under LPID control. Thus, the NPID has the benefit of improved disturbance rejection without increasing the control inputs significantly.

(a) PES magnitude

(b) Control signal RMS

Figure 7.6: Summary of RRO compensation tests with RRO magnitude = $1 \mu\text{m}$ and frequency = 120–3600 Hz.

Next, tests are carried out to compensate for the non-repeatable runout (NRRO) dominated by disk flutter modes. The disk flutter vibrations have the frequency spectrums located at the midrange frequencies. The midrange frequency disturbances are generally inadequately suppressed by LPID control due to the low open-loop gains within this range. However, Figure 7.7 shows the results of the PES spectrums, which indicates a 9% reduction of PES 3σ under NPID control is obtained based on the LPID control.

Furthermore, the transient responses of PES to $0.4\ \mu\text{m}$ step output disturbance are given in Figure 7.8. The results show that NPID control reduces the overshoot and consequently decreases settling time from 0.5 ms to 0.4 ms, which is a 20% reduction ratio. Therefore, the NPID control has a resultant contribution to the transient performance by improving the disturbance rejection.

Finally, simulations are performed to evaluate the stability robustness under NPID control. The plant uncertainties are assumed to be a $\pm 5\%$ variation of the first resonance frequency in the VCM actuator. Figure 7.9 shows that the responses of PES to impulse input disturbance. The results indicate that the PES still decay to zero fast under the resonance frequency variations.

(a) LPID control

(b) NPID control

Figure 7.7: Compensation for NRRO: 9% reduction of PES 3σ under NPID control compared with LPID control.

Figure 7.8: Step ($0.4 \mu\text{m}$) output disturbance response: 20% reduction of settling time under NPID control compared with LPID control.

Figure 7.9: Simulated impulse input disturbance responses to $\pm 5\%$ variations of the VCM actuator first resonance frequency ω_n under NPID control.

7.5 Summary

In this chapter, a NPID controller is presented for a class of linear plants with output feedback. The controller gains are modulated by the control error and the rate of error change to yield a closed-loop system for improved disturbance rejection based on the LPID control system. An optimization problem involving constraints written as linear matrix inequalities (LMIs) is solved to compute the controller parameters that can guarantee the stability of the NPID control system. The NPID control design is applied to the HDD track-following servo systems. Experimental results show that the NPID control outperforms the LPID control by 9% in disturbance rejection along with a consequent reduction by 20% in settling time performance. Simulation results show that the stability robustness of the NPID control is maintained with $\pm 5\%$ variations of the first resonance frequency in the VCM actuator.

In this chapter, the NPID control design is applied and tested on the single-actuator HDD. In principle, it can be employed in the dual-actuator HDD to design the feedback controller as shown in Sections 4.2.2 and 5.2.1.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

The dual-actuator system is developed for fast data rate and large storage capacity in hard disk drives (HDDs). In order to sustain the continuing increase in track density, the dual-actuator servo system needs to be developed for more precise head positioning accuracy in the presence of internal and external disturbances especially including the interactions between the actuators. In this thesis, control structures and controller design methods were developed for the dual-actuator control system to compensate for the interactions as well as the other disturbance sources.

A linear coupled dual-input dual-output (DIDO) frequency response model was established to represent the dual-actuator servo plant. After that, the plant model was identified by modal testing using the circle-fit modal parameter identification method. An approach to identify NMP antiresonances was also presented. This approach is effective to match both the magnitude and the phase drop in the inter-

action modeling, which achieves an identification range up to 8 kHz, while only 2 kHz can be achieved using the usual least square identification method. The identified dual-actuator plant is sufficiently accurate for the control analysis and design.

The dual-actuator tracking servo design specifications were described, where the requirement for interaction compensation was emphasized. Based on the use of information from the other actuator, the dual-actuator control structures were classified into two categories: a fully coupled DIDO control structure and a dual single-input single-output (SISO) control structure. The coupled DIDO control structure can achieve optimal performance in principle but has a complex fully coupled controller. On the contrary, the dual SISO control structure has a simple controller but may not be able to achieve the optimal performance.

For the coupled DIDO controller design, the H_2 optimal control design provides systematic solution for the controller by formulating the dual-actuator tracking error specification as a standard H_2 control problem. Although the derived controller is impractically high-order, it gives the optimal tracking performance achievable in dual-actuator servo systems. The feedforward decoupling control (FDC) design contains a two-stage design process. The first stage is to design an optimal stable and causal feedforward decoupler that approximates an inverse-based perfect decoupler. The second stage is to design the feedback controller based on the decoupled dual-actuator plant. Simulation results showed that the interaction-induced TMR could be optimally reduced to 1.5 nm by the H_2 control. The FDC achieved a 2.4 nm interaction-induced TMR according to the experimental results.

The dual SISO control system was analyzed via loop decomposition, which re-

vealed that the interaction effects could be represented as both plant uncertainties and narrow-band disturbances acting on a SISO control loop. The stability condition is derived for the dual SISO control system. After that, the SISO controller design is separated into the feedback controller design and an add-on interaction filter design. The classical loop shaping methods are applied to design the controllers in both stages. In particular, the interaction filter is designed as a phase-lead peak filter which can preserve the system stability and effectively reject the mid-frequency interaction disturbances. Experimental results showed that interaction-induced TMR is reduced to 3.7 nm with the dual SISO controller. Compared to the coupled DIDO control, it had less reduction of interaction disturbances. However, simulation results indicated that the dual SISO controller could tolerate interaction frequency variations within $\pm 20\%$, which was a significant improvement compared to the range of $\pm 2\%$ achieved by FDC. Furthermore, the designed SISO controller was of only 6th-order, which was quite easy to implement.

A generalized optimal filter design to reject narrow-band disturbances at all frequency ranges was developed. The filter zero was designed to minimally degrade the closed-loop system stability and obtain a smooth sensitivity curve around the disturbance frequency. A nonlinear optimization procedure was developed to select the filter parameters such that the statistical position error is minimized. Experimental results of a PZT-actuated head positioning control system on spinstand platform demonstrated that the add-on filter could further reduce the mid-frequency PES NRRO by 8% and preserve the stability margin of the original feedback control system. As a result, the track density can be improved from the original 275 kTPI to

300 kTPI.

The nonlinear PID (NPID) control of linear plants was proposed to achieve improved disturbance rejection. The controller gains are modulated by the control error and error rate to yield a closed-loop system for improved disturbance rejection. An optimization problem involving constraints written as linear matrix inequalities (LMIs) is solved to compute the controller parameters that can guarantee the stability of the NPID control system. The NPID control design was applied to the HDD track-following servo systems. Experimental results showed that the NPID control outperformed the linear PID control by 9% in disturbance rejection along with a consequent reduction by 20% in settling time performance. Simulation results showed that the stability robustness of the NPID control was maintained with $\pm 5\%$ variations of the first resonance frequency in the VCM actuator.

8.2 Future Work

There are several interesting topics related to the research in this thesis remained to be explored. The suggestions on future work are listed as follows:

- **Integrated Servo-Mechanical Design of Dual-Actuator System**

The head positioning accuracy in the dual-actuator system can be improved by mechanical optimization. In particular, the mounting system transmits the mechanical interaction between the actuators. An evaluation system for calculating the residual vibration caused by a HDD mounting system is presented in [104]. Hence, different types of supporting mechanisms for mounting the dual-actuator system can be investigated to reduce the interaction vibration.

In addition, the integrated servo-mechanical design of the dual-actuator system deserved to be explored. The key issue is how to formulate the plant as a parameterized model, which is then folded into the servo loop for performance optimization. The synthesized results can thus be used to design the actuator mechanics.

• Adaptive Control Design Based on the FDC Structure

The FDC developed in Chapter 4 is able to eliminate the interaction theoretically, but it is not robust against the parameter variations. The robustness may be improved by the adaptive feedforward decouplers as shown in Figure 8.1. As such, the decouplers are considered to automatically redesign themselves as the plant changes. By matching the uncertainty dynamics with proper adaptation algorithms, the outputs of the feedforward decouplers are able to eliminate the interaction-induced vibrations. The primary difficulty of adaptive control system is the large amounts of computation that need to be considered in the HDD.

Figure 8.1: Adaptive feedforward decoupling control of the dual-actuator system.

- **Track-Seeking Control of Dual-Actuator System**

This thesis is focused on the track-following servo system of the dual-actuator HDD. The developed control structures might be directly applied to the short distance track-seeking where saturation does not occur. However, for long distance track-seeking, the saturation generally occurs in order to minimize the seeking time. In this case, the interactions are severely generated due to the large reaction force of the actuators and the residual interaction-induced vibrations would degrade the settling performance. Therefore, track-seeking control must take the saturation into account while dealing with the interaction-induced vibration during track-seeking.

- **Multisensing Servo of Dual-Actuator System**

The multisensing servo for HDDs has been well developed to improve the head positioning accuracy. Thus, it is expected to apply to the dual-actuator system for reducing the interactions and external disturbances. The key issues include the selection of the type of sensor in terms of cost and reliability and their optimal locations, such that the resulting servomechanism achieves the highest positioning performance and robustness against the disturbances.

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Curriculum Vitae

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1. J. Zheng, G. Guo and Y. Wang, "Feedforward decoupling control design for dual-actuator system in hard disk drives," *IEEE Trans. Magn.*, vol. 40, no. 4, pp. 2080-2082, July 2004.
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