Digital image enhancement algorithms for 2D ultrasound imaging systems

Zhang, Fan

2007

Zhang, F. (2007). Digital image enhancement algorithms for 2D ultrasound imaging systems. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/3478>

<https://doi.org/10.32657/10356/3478>

Nanyang Technological University

Downloaded on 26 Apr 2023 19:21:07 SGT

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**DIGITAL IMAGE ENHANCEMENT ALGORITHMS
FOR 2D ULTRASOUND IMAGING SYSTEMS**

ZHANG FAN

SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

2007

DIGITAL IMAGE ENHANCEMENT ALGORITHMS
FOR 2D ULTRASOUND IMAGING SYSTEMS

ZHANG FAN

2007

Digital Image Enhancement Algorithms For 2D Ultrasound Imaging Systems

Zhang Fan

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2007

Acknowledgements

First and foremost, I would like to thank my supervisor, Dr. Liang Mong Koh, for providing me the opportunity to conduct research in the field of medical image processing and for giving me broad guidance and a freethinking environment. His encouragement when my research progressed well and his patience when it did not were extremely helpful. Thanks are also due to him for helping me in the presentation of this thesis and making it more readable.

Many thanks are due to Dr. Yongmin Kim, Professor and Chair in Bioengineering at the University of Washington, Seattle, for inviting me to his Image Computing Systems Laboratory (ICSL). Thanks are also due to members of ICSL, in particular Yang Mo Yoo, Anup Agarwal, Fabio Schneider and Bo Zhuang, for critical comments and suggestions. Special thanks go to Yang Mo Yoo, for collecting phantom and real ultrasound data for my research.

The financial support from the Agency for Science, Technology, and Research (A*STAR) of the Republic of Singapore, for my one-month visit to Dr. Kim's ICSL at the University of Washington, Seattle, is gratefully acknowledged.

I am also thankful to other members in our home ultrasound research group, including Dr. Eddie Moon Chung Wong, Dr. Yvonne Ying Hung Lam, and Dr. Kei Tee Tiew, Ji Cao, Lichen Zhang, Changqing Gao, Vinay Sonu Sawant, Pengyu Song, and Jia Hao Cheong, for the many hours we spent in discussing our research and for their good suggestions.

Table of Contents

Summary	v
List of Figures	vii
List of Tables	ix
List of Notations	xi
Chapter 1 Introduction	1
1.1 Motivations	1
1.2 Objectives	4
1.3 Major Contribution of the Thesis	5
1.4 Organization of the Thesis	6
Chapter 2 Review of Speckle Reduction Techniques	7
2.1 Statistics of Speckle	7
2.1.1 Statistical Models of Backscattered Ultrasound Signals	7
2.1.2 Speckle After Logarithmic Compression in Real Ultrasound Machines	9
2.2 Compounding Methods	11
2.2.1 Spatial Compounding	12
2.2.2 Frequency Compounding	12
2.2.3 Temporal Compounding	13
2.2.4 Strain Compounding	13
2.2.5 Comments on Compounding Methods	13
2.3 Nonlinear Spatial Speckle Filters	14
2.3.1 Adaptive Weighted Median Filter	14
2.3.2 Directional Median Filter	15
2.3.3 Bamber, Lee, Kuan, and Frost Filters	16
2.3.4 Refined Lee Filter	19
2.3.5 Gamma MAP Filter	19
2.3.6 Segmentation or Region Growing Based Filters	21
2.3.7 Diffusion Based Spatial Filters	22

2.3.8	Comments on Nonlinear Spatial Speckle Filters	22
2.4	Wavelet Based Speckle Reduction Methods	23
2.4.1	Thresholding Techniques	24
2.4.2	Bayesian Estimation Techniques	26
2.4.3	Coefficients' Correlation Techniques	27
2.4.4	Singularity Detection Techniques	28
2.4.5	Comments on Wavelet Based Speckle Reduction Methods	29
2.5	Pyramid Based Speckle Reduction Methods	29
2.6	Conclusion and Discussion	30
Chapter 3	Pyramid Based Filtering Techniques	33
3.1	Introduction	33
3.2	Pyramid Decomposition and Reconstruction	33
3.3	Speckle Reduction in Pyramid Domain	38
3.4	Conclusion	42
Chapter 4	Nonlinear Diffusion in Laplacian Pyramid Domain for Speckle Reduction	43
4.1	Introduction	43
4.2	Noise Reduction by Nonlinear Diffusion	43
4.3	Laplacian Pyramid Based Nonlinear Diffusion	45
4.3.1	Basic Idea	45
4.3.2	Analysis of the Diffusion Process in Pyramid Domain	47
4.3.3	Automatic Determination of the Gradient Threshold	51
4.3.4	Discretization Scheme	52
4.3.5	Automatic Determination of the Diffusion Stopping Time	54
4.3.6	Computational Complexity of the LPND algorithm	59
4.4	Experimental Results and Comparisons	60
4.4.1	Results from Simulation Study	62
4.4.2	Results from Phantom Study	68
4.4.3	Results from <i>In Vivo</i> Study	71
4.5	Conclusion and Discussion	77

Chapter 5 Coupled Nonlinear Diffusion and Shock Filter in Laplacian Pyramid Domain for Ultrasound Image Enhancement	79
5.1 Introduction	79
5.2 Image Deblurring by Shock Filters	81
5.3 Laplacian Pyramid based Nonlinear Diffusion and Shock Filter	83
5.3.1 Basic Idea	83
5.3.2 Analysis of the Shock Filtering in Pyramid Domain	85
5.3.3 Discretization Scheme	87
5.3.4 Convergence of the Proposed LPNDSF	90
5.4 Experimental Results and Comparisons	91
5.4.1 Results from Simulation Study	93
5.4.2 Results from <i>In Vivo</i> Study	97
5.4.3 Temporal Consistency	104
5.5 Conclusion and Discussion	107
 Chapter 6 Conclusions and Recommendations	 111
6.1 Conclusions	111
6.2 Recommendations for Further Research	113
 Author's Publications	 117
 Bibliography	 119

Summary

Despite many obvious advantages, the usefulness of medical ultrasound imaging is at times limited by its low image quality, and speckle is one of the main reasons for this low quality. Speckle affects human interpretation of ultrasound images and the accuracy of computer-assisted diagnosis. In the past two decades, extensive research has been done on speckle suppression without degrading clinically useful information in ultrasound images. However, the state-of-the-art speckle reduction techniques have limited capability in preserving important image features and details. In this thesis, an effective speckle reduction algorithm, i.e., Laplacian pyramid based nonlinear diffusion (LPND), is proposed. The novelty of the proposed algorithm lies in extending the conventional single-scale nonlinear diffusion to a multiscale framework, i.e., Laplacian pyramid. A variation of median absolute deviation is used to automatically determine the gradient threshold, which greatly affects the performance of nonlinear diffusion. From simulation and phantom studies, an average gain of 1.55 dB and 1.34 dB in contrast-to-noise ratio (CNR) were obtained compared with recently-developed speckle reducing anisotropic diffusion (SRAD) and nonlinear coherent diffusion (NCD), respectively. The visual comparison of despeckled *in vivo* ultrasound images from liver and carotid artery also demonstrates that the proposed LPND method effectively suppresses speckles as well as preserves edges and detailed structures. To further enhance fuzzy boundaries in ultrasound images, Laplacian pyramid based nonlinear diffusion and shock filter (LPNDSF) is proposed. The proposed LPNDSF has demonstrated its ability to sharpen image edges and suppress speckle simultaneously. Apart from giving clinicians clearer and more refined ultrasound images, LPNDSF may be used to improve the performance of computer-assisted diagnosis such as automatic segmentation and boundary detection.

List of Figures

3.1	Block diagram showing the structure of Laplacian pyramid transform	35
3.2	The first four layers of the Gaussian and Laplacian pyramid of an image	36
3.3	Image filtered by Laplacian pyramid based Wiener filter and its four-layer Laplacian pyramid decomposition	41
4.1	Block diagram of the proposed LPND method, including pyramid decomposition, nonlinear diffusion, and pyramid reconstruction	46
4.2	The relationship between diffusion coefficients, flux functions, and image gradients	50
4.3	The convergence of MAE in the proposed LPND method	57
4.4	Flowchart of nonlinear diffusion in the proposed LPND method	58
4.5	Simulated B-mode image and its filtering results	63
4.6	Six regions of interests for calculating CNR in the simulation study	67
4.7	Phantom image and its filtering results	69
4.8	Real ultrasound liver image and its filtering results	72
4.9	Profiles along the highlight line in the liver image	73
4.10	Real ultrasound carotid artery cross-sectional image and its filtering results	75
4.11	Profiles along the highlight line in the carotid artery cross-sectional image	76
5.1	Block diagram of the proposed LPNDSF method, including pyramid decomposition, coupled nonlinear diffusion and shock filter process, and pyramid reconstruction	85
5.2	Flowchart of the coupled nonlinear diffusion and shock filter in the proposed LPND method	90
5.3	The convergence of MAE in the proposed LPNDSF method	91
5.4	Simulated B-mode image and its enhancing results	96
5.5	Real ultrasound liver image and its filtering results	100
5.6	Edge detection results on the images shown in Fig. 5.5	101

5.7	Real ultrasound kidney image and its filtering results	102
5.8	Edge detection results on the images shown in Fig. 5.7	103
5.9	Evolution of spatial correlation coefficients between consecutive frames over time	105

List of Tables

4.1	MSE, β , and SSIM values on 30 simulated images with the same speckle level	67
4.2	CNR values on 30 simulated images with the same speckle level ...	68
4.3	CNR values on a phantom image sequence containing 30 different frames	70
5.1	MSE, β , and SSIM values on 50 simulated images with the same speckle level	94
5.2	CNR values on 50 simulated images with the same speckle level ...	95
5.3	Spatial correlation coefficients between the LPND-denoised images and the LPNDSF-enhanced images.....	107

List of Notations

Symbols

W_0	:	Central weight in the window of AWMF
m	:	Mean
σ	:	Standard deviation
σ^2	:	Variance
$[x]$	:	Nearest integer to x if x is positive, otherwise zero
$\bar{I}_s$	:	Mean of I_s
$\hat{I}_s$	:	Estimation of I_s
K_s	:	Weighting function in the Lee, Kuan, and enhanced Lee filter
C_u	:	Coefficient of variance of speckle noise
C_s	:	Coefficient of variance of an image
$C_{\max}$	:	Pre-specified threshold
$\bar{u}$	:	Mean of speckle noise
$d_{s,p}$	:	Distance between the pixel s and the pixel p
i	:	X-coordinate of one pixel in an image
j	:	Y-coordinate of one pixel in an image
M	:	Number of columns in an image
N	:	Number of rows in an image
K	:	Damping factor in the Frost and enhanced Frost filter
L	:	Effective number of looks
$P(\)$	:	Probability distribution
T	:	BayesShrink threshold
∇	:	Gradient operator
Δ	:	Laplacian operator
div	:	Divergence operator
$ \dots $	:	Absolute value
c	:	Diffusion coefficient in nonlinear diffusion
ϕ	:	Flux function in nonlinear diffusion
k	:	Gradient threshold in nonlinear diffusion
t	:	Diffusion time
λ	:	Time step in nonlinear diffusion
n	:	Number of iterations in nonlinear diffusion
G_σ	:	Gaussian filter with standard deviation σ

∂	:	Partial derivative
W	:	Gaussian-like lowpass filter to generate Laplacian pyramid
l	:	Pyramid layer
Z	:	Total decomposition layer

Abbreviations

2D	:	Two dimensional
3D	:	Three dimensional
AOS	:	Additive operator splitting
AWGN	:	Additive white Gaussian noise
AWMF	:	Adaptive weighted median filter
BV	:	Background variance
CNR	:	Contrast to noise ratio
CT	:	Computed tomography
DSP	:	Digital signal processor
DV	:	Detail variance
DWT	:	Discrete wavelet transform
FFT	:	Fast Fourier transform
GP	:	Gaussian pyramid
IDWT	:	Inverse discrete wavelet transform
I/Q	:	In-phase and quadrature
iid	:	Independent, identical distributed
LP	:	Laplacian pyramid
LPND	:	Laplacian pyramid based nonlinear diffusion
LPNDSF	:	Laplacian pyramid based nonlinear diffusion and shock filter
LPW	:	Laplacian pyramid based Wiener filter
MAD	:	Median absolute deviation
MAP	:	Maximum <i>a posteriori</i>
ML	:	Maximum likelihood
MMSE	:	Minimum mean square error
MRI	:	Magnetic resonance imaging
MSE	:	Mean square error
NCD	:	Nonlinear coherent diffusion
ND	:	Nonlinear diffusion

NDSF	:	Nonlinear diffusion and shock filter
PDE	:	Partial differential equation
PDF	:	Probability density function
PET	:	Positron emission tomography
PSF	:	Point spread function
RF	:	Radio frequency
RMSE	:	Relative mean square error
ROI	:	Region of interest
SAR	:	Synthetic aperture radar
SNR	:	Signal to noise ratio
SRAD	:	Speckle reducing anisotropic diffusion
SSIM	:	Structure similarity
WTMS	:	Wavelet transform modulus sum

Chapter 1 Introduction

1.1 Motivations

Ultrasound imaging is based on interrogation of tissue with high frequency focused sound waves [55]. Compared with other medical imaging modalities, such as magnetic resonance imaging (MRI), computed tomography (CT), X-ray, and positron emission tomography (PET), ultrasound imaging has several favorable properties. It is non-invasive, non-ionizing, and safe for both patients and doctors. It is relatively inexpensive and portable. It provides soft tissue imaging, where conventional X-ray techniques have difficulties in showing details. Furthermore, because ultrasound images can be produced in real time, it allows imaging of dynamic structures such as the beating heart.

The main disadvantage of ultrasound imaging is its low image quality. The presence of speckle is one of the main reasons for this low image quality. Speckle is a granular pattern formed by constructive and destructive coherent interference of backscattered echoes from the scatterers that are typically much smaller than the spatial resolution in medical ultrasound systems (i.e., wavelength of an ultrasound wave) [11]. Speckle is a phenomenon common to many coherent imaging systems, such as synthetic aperture radar (SAR) imaging and laser holography. The speckle pattern generated in medical ultrasound imaging depends on the structure of the tissue and the frequency and geometry of the ultrasound transducer amongst others.

Because of its dependence on the microstructure of tissue parenchyma, for some clinical applications, speckle is used for diagnosis, e.g., the evaluation of diffuse liver diseases [37]. However, speckle is typically regarded as noise since it significantly reduces the image contrast and spatial resolution, resulting in the degradation in human interpretation of the images and the accuracy of computer-assisted diagnosis. For example, boundary detection and image segmentation are commonly applied to evaluate amniotic fluid volume in obstetrics, to quantify the volume of left ventricle in echocardiography, to detect prostate boundaries in prostate cancer treatment, and etc. Applying speckle reduction before boundary detection and image segmentation can improve the performance of these automatic image analysis techniques. Therefore, reducing speckle is an important preprocessing step for many ultrasound image analysis tasks, such as feature extraction, segmentation, classification, and quantitative measurements.

In the last two decades, various speckle reduction techniques have been reported in the literature, including compounding, nonlinear spatial filters, and multiscale (e.g., wavelet and pyramid) methods. Although some methods achieve encouraging results, each of them has its limitations when applied for ultrasound imaging. Present speckle reduction techniques have difficulty in preserving important image features and details while suppressing speckle in ultrasound images. Thus, a technique that can sufficiently remove the speckle and simultaneously maintain the clinically useful features is still in need.

In ultrasound imaging, another challenge is to obtain a sharper image from a blurred one. Apart from making it easier for clinicians to make diagnosis observing the image, a sharp image is also needed for subsequent computer-assisted diagnosis like boundary

detection and image segmentation. It may not be difficult for the human eyes to recognize and interpret the blurred edges, but it is unsuitable for computer analysis.

Image restoration methods have been investigated widely to recover clinically important details that appear blurred in ultrasound images. Much work has been done on deconvolution techniques, in which an ultrasound image is assumed to be linearly degraded by a convolution with a blurring kernel that is known *a priori*. A simple solution is the linear Wiener filter, which is optimal in the sense of mean square error (MSE). When the convolution kernel is unknown, blind deconvolution techniques are adopted. With this kind of techniques, a two dimensional (2D) radio frequency (RF) ultrasound image is considered to be the convolution of a 2D point spread function (PSF) of the ultrasound system with a 2D underlying echographic map that represents the overall reflection from the imaged tissue [60]. Given an acquired ultrasound image, deconvolution techniques perform either concurrent or consecutive estimation of the PSF of the ultrasound system and a sharpened image. The most important and difficult problem is the estimation of the PSF, and its accuracy will affect greatly the performance of these deconvolution techniques.

Another image deblurring strategy is to find a general sharpening operator that is not based on any assumption of the blurring kernel. Unsharp masking is a kind of classical techniques for image sharpening. With this kind of techniques, an input image is blurred first and its difference from the input image is multiplied by a certain weight before being added back to the input image. However, the unsharp masking methods may amplify noise, especially in ultrasound images where signal-dependent noise like speckle exists.

1.2 Objectives

The objective of this project is to research into existing speckle reduction and feature enhancement methods for medical ultrasound imaging and to develop an algorithm that reduces speckle and enhances key features at the same time to aid clinical diagnosis.

The limitation of existing speckle reduction and feature enhancement methods used in ultrasound imaging may be due to their single-scale representation of an image. A multiscale transform such as wavelet or pyramid represents an image at multiple scales, and has shown promising results for noise reduction and image enhancement. By representing an image using a certain multiscale transform, useful signal component and noise can be separated more easily, and they can be treated differently to get desired noise reduction and feature enhancement effects.

The research in this thesis is motivated by the promise of multiscale representations and the need to improve the image quality of ultrasound imaging systems. The major goal of this research is to develop reliable techniques for speckle reduction and edge enhancement in ultrasound imaging.

In this thesis, a novel speckle reduction method and a novel edge enhancement method for ultrasound imaging are proposed. The proposed speckle reduction method can improve the contrast-to-noise ratio (CNR) by suppressing speckle effectively while preserving important features and details in ultrasound images. The proposed edge enhancement method can achieve edge enhancement and speckle reduction simultaneously.

1.3 Major Contribution of the Thesis

The major contribution of this research is to the field of ultrasound imaging through the development of image enhancement methods. Particularly, the research in this thesis makes the following contributions.

A novel speckle reduction algorithm, which applies nonlinear diffusion filtering in Laplacian pyramid domain of an image, has been proposed. The conventional single-scale nonlinear diffusion is extended to a multiscale framework, i.e., Laplacian pyramid. A formula is given to automatically estimate the optimal value of the gradient threshold, which is one of the most important parameters to control the nonlinear diffusion and conventionally set by experience. Extensive experiments have shown that the proposed algorithm performs better than the state-of-the-art speckle reduction techniques on simulated, phantom, and several clinical ultrasound images, both quantitatively and qualitatively.

A novel simultaneous speckle reduction and edge enhancement algorithm has also been developed. This algorithm has the ability to tackle two challenging tasks: suppressing speckle effectively without damaging useful information and enhancing edges without amplifying speckle noise, which are nearly impossible to be achieved simultaneously by present speckle reduction and image enhancement algorithms.

The work presented in this thesis has resulted in two invention disclosures and two international journal papers, as listed in “Author’s Publications”.

1.4 Organization of the Thesis

After this introductory Chapter, Chapter 2 begins with a brief introduction to the statistics of ultrasound speckle, before presenting the detailed survey of existing speckle reduction methods. Their individual basic idea, advantages and disadvantages are introduced. By analyzing these methods, some characteristics of desired speckle reduction and edge enhancement algorithms are identified. In Chapter 3, pyramid transforms and the fundamental properties of filtering in the pyramid domain are introduced. They set the foundation for the proposed speckle reduction and edge enhancement methods, both of which are based on a particular pyramid transform, i.e., Laplacian pyramid. In Chapter 4, a novel speckle reduction method is proposed. Comparisons with other speckle reduction techniques are given to validate the superiority of the proposed method. Both quantitative and qualitative results from simulation, phantom, and *in vivo* studies are presented and discussed. Chapter 5 describes the proposed simultaneous speckle reduction and edge enhancement algorithm. In both Chapter 4 and Chapter 5, statistical results from hypothesis tests are given to compare the performance of the proposed methods with that of other speckle reduction methods, including the recently proposed speckle reducing anisotropic diffusion (SRAD) and nonlinear coherent diffusion (NCD). Chapter 6 concludes this thesis and gives some possible future directions to work on.

Chapter 2 Review of Speckle Reduction Techniques

2.1 Statistics of Speckle

2.1.1 Statistical Models of Backscattered Ultrasound Signals

The fine structure of ultrasound images has a mottled or granular appearance that has been called speckle because of its similarity to the equivalent optical phenomenon, laser speckle. The speckle results directly from the use of a coherent radiation for imaging [11]. It appears as brightness variations in an image and conceals fine details, affecting human interpretation of the images and the accuracy of computer-assisted diagnosis.

Backscattered ultrasound signals can be modeled as the sum of contributions from individual scatterers within one resolution cell. Thus, the statistical distribution of the backscattered ultrasound signal is related to the number of scatterers per resolution cell and their arrangement within the resolution cell. Analysis of this statistical distribution may be used for various applications, including speckle reduction.

The RF signal or the envelope of the backscattered echo signal has been studied extensively for their statistical properties [25, 26, 69, 70, 76]. The B-mode medical ultrasound imaging systems use the envelope signal to display, because most information such as the location and amplitude of backscatter is present in the envelope signal and the phase is a uniform random function under normal conditions of scattering. In the envelope signal, the nature of speckle pattern can be categorized into one of three classes according to the number of scatterers in one resolution cell (i.e., the smallest detectable structure), their spatial distributions and the characteristics of the

imaging system. The three classes are described briefly as follows. Their details can be found in [25].

The first class of speckle noise is fully developed (or formed) speckle, which occurs when there is a large number of randomly located scatterers within small scatter spacing compared to the wavelength of ultrasound and thus the scatterer density is above 10 scatterers per resolution cell. In this case, the amplitude of the backscattered signal can be modeled by Rayleigh distribution with a constant signal-to-noise-ratio (SNR) of 1.92. This SNR level can be used to detect fully developed speckle. Blood cells are typical examples of this type of scatterers. The Rayleigh distribution model requires a high scatterer density (i.e., larger than 10 scatterers per resolution cell) and a large number of spatially uniformly distributed scatterers, which are seldom satisfied in real situations. Normally, the scatterer density is smaller or there is a coherent component present in the envelope signal.

In the second class, an additional non-random coherent structure is present in the envelope signal, which may arise because of unresolved periodically located scatterers or strong specular scattering, the probability density function (PDF) of the backscattered signal becomes close to Rician distribution. In this situation, the SNR is above 1.92 and increases linearly with the coherent component of the signal for large coherent component level. Therefore in the case of high scatterer density, SNR is a good indicator for estimating the coherent component level in the envelope signal. Examples of this type of scatterers include organ surfaces and blood vessels.

The above two speckle patterns assume that each resolution cell contains a large number of scatterers and that the scatterers' positions and scattering strengths are

independent of each other. However, these assumptions are seldom met exactly. In general, the scatterer density may not be that large or the scatterers may be correlated, so the effective number of scatterers is finite and the scatterer density is normally below 10 scatterers per resolution cell. A more general distribution known as the K distribution has been proposed to model this situation. In this case, speckle is partially developed, and the SNR becomes smaller than 1.91 and decreases with the scatterer density.

Other distribution models have also been proposed, such as the homodyned K distribution [26], the generalized K distribution [69], and the Nakagami distribution (together with phase analysis) [70]. These distribution models encompass two or all the three distributions described above, i.e., Rayleigh, Rician, and K distributions, and approximate them under various limiting cases.

2.1.2 Speckle After Logarithmic Compression in Real Ultrasound Machines

In a real medical ultrasound system, the envelope signal is subject to several transformations that change its statistics. The most important transformation is the logarithmic compression, which is executed to reduce the large dynamic range of the envelope signal to match the lower dynamic range of the display device. For example, the envelope signal may have a dynamic range of 50-70 dB, while the dynamic range of a typical display device would be 20-30 dB. This log-compression and some other signal processing stages inside the scanner such as lowpass filtering and interpolation modify the statistics of the envelope signal. Thus Rayleigh, Rician, and K distributions described above cannot be used directly to model the signal after log-compression.

One way to overcome this problem is to decompress the signal if the compression parameters are known. Some medical ultrasound machines provide information of the compression parameters. If the dynamic range of the input and that of the output are known, the compression parameter can be estimated, and thus the log-compressed signal can be decompressed to get back the envelope signal. A better solution would be to derive statistical models of the compressed signal. Such statistical models can be used to develop speckle reduction filters that work on the log-compressed signal and can be used directly on final displayed images. The distribution of log-compressed fully developed speckle can be modeled by Fish-Tippet density function [38]. For log-compressed partial developed speckle, it is difficult to derive a closed form expression. But in general, the statistics of log-compressed speckle can be approximated by the combination of a Gaussian and a long-tailed outlier producing distribution [84].

In the envelope signal, which is the output of the beamformer and prior to the log-compression in an ultrasound imaging system, speckle noise is generally modeled [36] by the following equation

$$f(x, y) = g(x, y)\eta_m(x, y) + \eta_a(x, y) \quad (2.1)$$

where $g(x, y)$ is a noise-free original image which is to be recovered, $f(x, y)$ is a noisy observation of $g(x, y)$, $\eta_m(x, y)$ and $\eta_a(x, y)$ are multiplicative and additive noise respectively, x and y are the spatial locations, and $(x, y) \in R^2$.

Normally, the effect of additive noise (such as sensor noise) is considerably smaller than the multiplicative noise, i.e., $\|\eta_a(x, y)\|^2 \ll \|g(x, y)\eta_m(x, y)\|^2$. Thus (2.1) can be approximated by

$$f(x, y) = g(x, y)\eta_m(x, y) \quad (2.2)$$

A log-transform on both sides will separate the noise term $\eta_m(x, y)$ from the signal $g(x, y)$:

$$\log(f(x, y)) = \log(g(x, y)) + \log(\eta_m(x, y)) \quad (2.3)$$

or

$$f^{\log}(x, y) = g^{\log}(x, y) + \eta_m^{\log}(x, y) \quad (2.4)$$

Using uniform sampling, the discrete version of (2.4) is obtained

$$f^{\log}(i, j) = g^{\log}(i, j) + \eta_m^{\log}(i, j) \quad (2.5)$$

In this way, the multiplicative noise is transformed to additive noise, and thus conventional noise reduction techniques that are originally proposed for getting rid of additive noise can be applied to suppress speckle. For example, the Wiener filter, which is developed for additive noise, is used in such a homomorphic manner in [36] to remove speckle.

Various techniques have been employed to reduce speckle noise in ultrasound imaging in the last two decades. Those speckle reduction techniques can be classified into three groups: compounding methods, nonlinear spatial filtering, and multiscale (e.g., wavelet or pyramid) methods. In the remaining part of this Chapter, all these three groups of techniques will be reviewed.

2.2 Compounding Methods

Compounding techniques [23, 50, 57, 73] have been widely investigated and are currently used in several commercial medical ultrasound systems. All compounding methods try to limit the formation of speckle before an image is formed, and thus can

be called pre-acquisition methods. While the other two kinds of techniques, nonlinear spatial filtering and multiscale methods are post-acquisition methods.

2.2.1 Spatial Compounding

Partially correlated measurements of a speckle-generating object can be obtained by several methods. For example, a one-dimensional transducer array can be divided into several sub-arrays, and each sub-array is used to image the same object. Decorrelation between these measurements is introduced because of the statistical independence of speckle viewed from different directions. This method is known as spatial compounding [23]. However, because each image is acquired using only a portion of the entire array, contrast resolution is improved at the cost of lateral resolution. An alternative is to use the entire array to acquire several images by manually or mechanically moving the transducer in the lateral direction. Although the entire physical array is used, the lateral resolution is still not optimal because a narrower acoustic beam can be synthesized if image data from different directions are coherently combined.

2.2.2 Frequency Compounding

Frequency compounding techniques are based on the assumption that speckle produced at one imaging frequency is not correlated with that produced at other frequencies. This method uses bandpass filters to partition a wideband received echo signal into separate imaging channels. In each of these channels the RF signal is full-wave rectified and then lowpass filtered, to produce the demodulated signal. After this demodulation process, an analog sum of the separate channels is formed to produce the output [73]. Because of the reduced pulse bandwidth, speckle fluctuations are reduced at the cost of axial resolution.

2.2.3 Temporal Compounding

Temporal compounding is an alternative approach. The method requires a time sequence of images as input. Pixels from the same imaging point are averaged over a sequence of images. In the case where no motion is present, this method is straightforward to apply. An output frame is produced from each input frame by taking, at each pixel, the average of the current pixel and pixels at the same position in previous (or subsequent) frames. However, because speckle is largely uncorrelated with time while image structures are correlated, applying this straightforward approach in the presence of motion results in motion blur and loss of resolution. A more sophisticated approach is to use temporal compounding with motion compensation [57].

2.2.4 Strain Compounding

Recently, a new compounding technique has been proposed [50]. This approach is based on decorrelation of images under different strain states. The external force deforms the image object and produces three-dimensional (3D) tissue motion. The in-plane motion can be estimated by speckle tracking and corrected such that images under different strain conditions are spatially matched. The out-of-plane motion, on the other hand, is not correlated. And hence, the images under different strain states are partially correlated. Thus, speckle brightness variations can be reduced by incoherently averaging these partially correlated images.

2.2.5 Comments on Compounding Methods

While these compounding methods can reduce speckle noise and improve image quality, they suffer from degraded spatial resolution and increased system complexity.

On the positive side, post-acquisition image processing techniques do not require significant hardware modifications.

2.3 Nonlinear Spatial Speckle Filters

There is a diverse range of nonlinear spatial filters available for speckle reduction. The use of each filter depends on the specification for a particular application. Each of these filters has a unique speckle reduction approach that performs spatial filtering in a square-moving window known as kernel. The filtering is based on the statistical relationship between the central pixel and its surrounding pixels.

A spatial filter's performance depends heavily on the choice of the local window size and orientation. A larger filter window means that a larger area of the image can be used for calculation and generally requires more computation time. If the size of filter window is too large, the important details will be lost due to over-smoothing. On the other hand, if the size of the filter window is too small, speckle may not be reduced thoroughly.

2.3.1 Adaptive Weighted Median Filter

Loupas *et al.* [54] proposed the adaptive weighted median filter (AWMF). The weighted median of a window is defined as the pure median of an extended sequence formed by replicating each pixel in the window for W_{ij} times, where W_{ij} is the corresponding weight coefficient. The weight coefficient of one pixel is calculated using the following equation:

$$W_{ij} = \left\lceil W_0 - cd \frac{\sigma^2}{m} \right\rceil \quad (2.6)$$

where W_0 is pre-specified weight assigned to the central pixel in the filter window, c is a scaling constant, d is the distance between the pixel (i, j) and the central pixel of the window, and m and σ^2 are the respective local mean and variance of pixels in the filter window.

The use of pixel replication in the AWMF eliminates the requirement of the pure median filter that speckle be smaller than half the filter window size. It is an improvement over the pure median filter, since it selectively applies greater smoothing only in locations where the image has no significant features. However, its performance is quite sensitive to the values of its empirically selected parameters such as W_0 and c in Eq. (2.6), especially when the filter window is small.

2.3.2 Directional Median Filter

Czerwinski *et al.* [21] proposed to filter an image using short line segments (called “sticks”) in different angular orientations as templates and selecting the orientation at each point that is most likely to represent a line in the image. They defined a set of short lines passing through the central pixel of a squared filter window. At every pixel of the original image, they computed a set of median values, each corresponding to a stick of different angular orientation and calculated by taking the median of pixel gray values along that stick. The filtered image is obtained by selecting the maximum median value at each pixel. This stick filtering method can be represented by

$$\hat{I}_s(i, j) = \max_{\theta} \{ \text{median}_{\xi, \eta} [I_s(i - \xi, j - \eta) s_{\theta}(\xi, \eta)] \} \quad (2.7)$$

where I_s is the original image, $\hat{I}_s$ is the filtered image, ξ and η are used to represent the relative coordinates of pixels on a stick with respect to the filtered pixel $\hat{I}_s(i, j)$, and s_{θ} is a stick of angular orientation θ .

This directional median filter can be related to the AWMF in the sense that each orientation filter can be described as a block from which some elements are given unity weights while others are given zero weights in the median operator.

The directional median filter requires a tradeoff between effective edge enhancement and good speckle reduction.

2.3.3 Bamber, Lee, Kuan, and Frost Filters

The Bamber [6, 7], Lee [48], Kuan [45], Frost [29], and enhanced Lee and Frost [53] filters share a similar basic idea for speckle reduction. The Bamber filter was proposed for medical ultrasound imaging, while the other five filters come from SAR community.

The Bamber filter [6, 7] can be expressed as

$$\hat{I}_s = \bar{I}_s + K_s(I_s - \bar{I}_s) \quad (2.8)$$

where $\hat{I}_s$ is the filtered gray value of a pixel to be calculated from the old value I_s , $\bar{I}_s$ is the mean value of all pixels' gray values within the filter window, and K_s is the weighting function to control the filtering strength. The parameter K_s is determined by a measure of similarity and can be calculated by

$$K_s = 1 - \frac{C_u}{C_s} \quad (2.9)$$

where

$$C_u = \frac{\sigma_u^2}{\bar{u}} \quad (2.10)$$

and

$$C_s = \frac{\sigma_s^2}{\bar{I}_s} \quad (2.11)$$

The terms C_u and C_s are the respective coefficients of variance of speckle noise and that of the image. σ_s^2 and $\bar{I}_s$ are the variance and mean of gray values of all pixels in the filter window respectively; σ_u^2 and $\bar{u}$ are the variance and mean of gray values in a region consisting entirely of fully developed speckle.

The local statistic C_s plays an essential role in controlling the filter: if $C_s \rightarrow C_u$, then $K_s \rightarrow 0$; if $C_s \rightarrow \infty$, then $K_s \rightarrow 1$. In general, the values of K_s approaches zero in homogeneous areas, leading to the same result as that of the mean filter. On the other hand, the value of K_s approaches unity at edges, resulting in little modification to the pixel values near edges.

The Lee filter [48] uses similar equations but adopts the ratio of local standard deviation to local mean (i.e., $C_u = \frac{\sigma_u^2}{\bar{u}^2}$ and $C_s = \frac{\sigma_s^2}{\bar{I}_s^2}$) to control the filtering process.

The Kuan filter [45] has the same form as the Lee filter, but with a different weighting function, which is given as

$$K_s = \frac{1 - C_u^2 / C_s^2}{1 + C_u^2} \quad (2.12)$$

where C_u and C_s have the same expressions as those in the Lee filter.

The Frost filter [29] uses an exponentially damped convolution kernel. The weighting factor decreases as the variance within the filter window reduces. The filtering output is determined by

$$\hat{I}_s = \sum_{p \in \eta} m_p I_p / \sum_{p \in \eta} m_p \quad (2.13)$$

where
$$m_p = \exp(-K \cdot C_s^2 \cdot d_{s,p}) \quad (2.14)$$

$$d_{s,p} = \sqrt{(i_s - i_p)^2 + (j_s - j_p)^2} \quad (2.15)$$

K is the damping factor, C_s is the same as that used in the Lee and Kuan filters, $d_{s,p}$ is the distance between the pixel s and the central pixel p , and η denotes the set of all pixel values in the filter window. The factor K is chosen such that when in a homogeneous region $K \cdot C_s^2$ approaches zero, becoming a mean filter; at an edge location $K \cdot C_s^2$ becomes so large that filtering is inhibited completely.

Lopes *et al.* [53] proposed to divide an image into areas of three classes. The first class corresponds to the homogeneous areas in which speckle may be eliminated simply by applying a mean filter. The second class corresponds to the heterogeneous areas in which speckle is to be reduced while preserving textures. In this class, the enhanced filters have the same forms as the original ones. The third class is areas containing isolated point targets, where the filter should preserve the observed value. Compared to the original ones, the enhanced Lee and enhanced Frost filters preserve edges and other texture features of an image better.

All the above six filters are based on the multiplicative noise model. However, log-compression is often executed in a medical ultrasound system to reduce the large dynamic range of the envelope detected echo signals. This log-compression and some other signal processing stages inside the scanner modify the statistics of the original signal. Dutt *et al.* [27] investigated the effect of log-compression on ultrasonic speckle

statistics and suggested an unsharp masking filter, which is similar to Bamber's filter but uses the statistics of log-compressed images to adjust the smoothing level.

These seven filters [6, 7, 27, 29, 45, 48, 53] share the same basic idea and have the same limitation. Their main disadvantage is their inefficiency in removing speckle noise in the areas near or on image edges and lines.

2.3.4 Refined Lee Filter

To improve the performance in regions around edges, Lee [49] proposed a refinement to the original Lee filter, in which the neighborhood used in high variance areas for the calculation of the local statistics takes into account the orientation of a possible edge. For each pixel with local variance exceeding a set threshold, oriented gradients are computed and used to select a subset of the neighborhood pixels that are on one side of the edge and most like the central pixel. Local variance estimated from this subset will in general be lower than that from the whole neighborhood pixels, allowing more accurate filtering of noise. However, the involved edge detection is not optimized for speckle corrupted images in which local variance are related not only to the edges but also to the underlying average intensity level. The performance of this refined Lee filter is highly dependent on its edge detection process.

2.3.5 Gamma MAP Filter

The application of maximum *a posteriori* (MAP) approaches to speckle reduction was first proposed by Kuan *et al.* [46]. The MAP estimation of I is obtained by maximizing the *a priori* PDF (the Bayes criterion) with respect to I :

$$P(I | g) = \frac{P(g | I)P(I)}{P(g)} \quad (2.16)$$

where I is the ideal image without speckle noise, g is the observed image, and they follow the multiplicative model $g = uI$, where u is a signal-independent noise with unit mean. $P(g | I)$ denotes the PDF of the speckle. Because the logarithm function is a monotonically increasing function, we can take the logarithm on both sides of the above function and then differentiate it with respect to I . Thus the MAP equation is obtained as follows:

$$\left. \frac{\partial \ln P(g|I)}{\partial I} + \frac{\partial \ln P(I)}{\partial I} \right|_{I=I(MAP)} = 0 \quad (2.17)$$

Here it is assumed that $P(g)$ is constant with respect to I . The first term in the above equation is the maximum likelihood (ML) term, and the second term is *a priori* term of the incoherent image. The output of a MAP filter can be achieved by taking the real and positive root, whose value is between the average and original one, of the MAP equation.

By assuming different *a priori* models, different roots from the MAP equation can be obtained. Kuan *et al.* [46] assumed a Gaussian distribution. Lopes *et al.* [52] modified it by assuming a Gamma distribution and classifying an image into three classes, similar to enhanced Lee and enhanced Frost filters. The Gamma MAP filter is given as below:

$$\hat{I}_s = \begin{cases} \bar{I}_s, & C_s \leq C_u \\ \frac{(\alpha - L - 1)\bar{I}_s + \sqrt{\bar{I}_s^2 (\alpha - L - 1)^2 + 4\alpha L \bar{I}_s I_s}}{2\alpha}, & C_u < C_s < C_{\max} \\ I_s, & C_{\max} \leq C_u \end{cases} \quad (2.18)$$

where L is the effective number of looks and can be computed by $L = \frac{1}{C_u^2}$,

$$C_{\max} = \sqrt{2}C_u \quad (2.19)$$

and

$$\alpha = \frac{1 + C_u^2}{C_s^2 - C_u^2} \quad (2.20)$$

The MAP filter requires a large window for accurate estimation of the PDF and the filter parameters.

2.3.6 Segmentation or Region Growing Based Filters

Some researchers use segmentation based filtering methods [41]. A signal is said to be stationary if all average values of interest are time (space) independent, and is nonstationary if one or more of the average values of interest vary in time (space). Speckle filtering can be effective only in locally stationary regions, and regions where the signals are nonstationary have to be filtered separately. The underlying idea of the segmentation based filtering methods is that, with the aid of a suitable segmentation algorithm, a statistically nonstationary image can be divided into approximately stationary regions that can, in turn, be processed by filters designed on the basis of the corresponding statistics.

A group of similar methods is region growing based filtering methods [17, 35, 40, 42]. The fundamental step in both kinds of methods is defining the extent of regions in the image. Both kinds of methods group pixels that are similar in terms of both gray level and connectivity. These pixels are usually contextually related and are likely to belong to the same object or region. After such a grouping, filtering is performed based on the local statistics of adaptive regions whose size and shape are based on the information content of the image, rather than of arbitrarily formed rectangular regions.

Rangayyan *et al.* [67] argued that region growing based methods are superior to segmentation based methods because that of these two classes of regions, namely non-overlapping regions obtained using segmentation techniques, and overlapping regions obtained from region growing techniques, overlapping regions are preferred, as processing disjoint segments of an image may result in noticeable edge artifacts and an inferior restored image. The main difficulty of these methods is in designing appropriate similarity criteria for region growing.

2.3.7 Diffusion Based Spatial Filters

Alternatively, diffusion based spatial filtering methods have been proposed [1, 83]. Diffusion based methods remove noise from an image by modifying the image via a partial differential equation (PDE). More details about this kind of spatial filters will be introduced in Chapter 4. Yu *et al.* [83] introduced an edge sensitive diffusion method (i.e., SRAD) to suppress speckle while preserving edge information. Abd-Elmoniem *et al.* [1] presented a tensor-based anisotropic diffusion method for speckle reduction and coherence enhancement (i.e., NCD). Both of the two diffusion methods can preserve or even enhance prominent edges when removing speckle noise, but they have limitation in retaining subtle features such as small cysts and lesions in ultrasound images.

2.3.8 Comments on Nonlinear Spatial Speckle Filters

Most of these nonlinear spatial filters utilize variations of the statistical properties of the pixels, observed through a fixed-size moving window, to obtain the output of the filter. The difficulties in implementing such techniques are related to determination of the proper weighting coefficients for the filter and design of an appropriately sized moving window.

Most of these filters fail to achieve spatial adaptation because they only employ a local window with fixed size and shape. An adjustable window is expected to have superior despeckling performance. The refined Lee filter [49] is a good example of using an adaptive window, and possesses a good balance between noise reduction and feature preservation. The success of the refined Lee filter is dependent on its edge detection process, which is not optimized for images corrupted by speckle. Region growing based methods [17, 35, 40, 42] adjust the size of filter window according to the formed adaptive regions, but it is difficult to derive an appropriate similarity criteria for region growing based on the local statistics. Diffusion methods have achieved promising results for additive noise reduction. However, they still cannot provide the desired reduction in speckle because of its heavy tailed property. They also introduce artifacts (e.g., jagged boundaries) when applied to ultrasound images. Two diffusion methods developed recently specifically for speckle reduction [1, 83] cannot preserve detailed features in the image well.

2.4 Wavelet Based Speckle Reduction Methods

Wavelet multiresolution analysis has the very useful property of space and scale localization, so it provides great promise for image feature detection at different scales. Wavelets have found successful applications in a variety of signal processing problems, including image coding and denoising. The wavelet based denoising techniques consist of three procedures: discrete wavelet transform (DWT), manipulation on wavelet coefficients, and then inverse discrete wavelet transform (IDWT). The second step is most crucial, and distinguishes one wavelet based denoising method from another. The manipulation on the wavelet coefficients is mostly based on a classification. With a certain criterion, the coefficients are divided into two groups. The first group contains important, regular coefficients that are contributed mainly by useful signal; while the

second group consists of irregular coefficients that are caused by noise. Then these two groups are processed in different ways, with the noisy coefficients often set to be zero. According to the different criteria used to classify the two groups of coefficients, wavelet based denoising methods can be roughly classified into the following four groups: thresholding, Bayesian estimator, coefficients' correlation, and singularity detection.

Speckle is often characterized as multiplicative random noise, whereas most wavelet based denoising algorithms were developed for additive white Gaussian noise (AWGN), because AWGN is most common in imaging systems. To take advantage of the available noise-related wavelet models in the publications, the common practice is to apply a logarithmic transform to convert the multiplicative speckle model to an additive model. After performing the above mentioned three steps of wavelet based denoising on the logarithmic transformed image, an exponential operation is applied in order to transform the image back to a non-logarithmic format.

2.4.1 Thresholding Techniques

The basic idea of this kind of technique is to use DWT to transform the data into a different domain, where large coefficients correspond to the noise-free signal, while small ones represent mostly noise.

A thresholding function determines how the thresholds are applied to the coefficients. Two most often used thresholding functions are soft and hard thresholding. For each wavelet coefficient, if its amplitude is smaller than a predefined threshold, it will be set to zero; otherwise it will be kept unchanged (hard thresholding) or cut in the absolute value by an amount of the threshold (soft thresholding).

The key decision in the thresholding techniques is the selection of an appropriate threshold. If this value is too small, the processed image will remain noisy. On the other hand, if the value is too large, important image details will be smoothed out. Donoho [24] proposed the universal thresholding rule, which is simple and efficient but not spatially adaptive because only a single threshold is used globally. The universal threshold can be expressed by $T = \sigma_n \sqrt{2 \log N}$, where σ_n is the noise standard deviation and N is the sample size. It has been shown that with a very large sample size, the above universal threshold tends to smear out image details.

Following this, some researchers have focused on developing spatially adaptive thresholding methods instead of using a global uniform threshold. One good example is Chang *et al.*'s work [14], in which the image is first segmented into three kinds of regions: edges, textured areas, and homogeneous areas. Then three different thresholds are adapted to the three different regions. The limitation of this method is that the three different thresholds are selected in an *ad hoc* way.

Chang *et al.* [15] also proposed an approximate minimum mean-square error (MMSE) solution to the soft-thresholding. The so-called BayesShrink threshold is calculated as

$$T = \frac{\sigma_\eta^2}{\sigma_g} \quad (2.21)$$

where σ_η^2 and σ_g are the noise variance and the image standard deviation, respectively. This threshold is designed to adapt to each individual subband at each resolution level.

In [28], a comprehensive evaluation of various thresholding methods was carried out, and BayesShrink was found to be the best thresholding rule considering the simplicity of implementation along with the denoising performance.

Other thresholding techniques for speckle reduction have been proposed in the literature [33, 86]. Current state-of-the-art wavelet based denoising and image enhancement techniques employ a combination of soft and hard thresholding together with a generalized adaptive gain for feature emphasis. For example, Zong *et al.* [86] applied soft thresholding at fine scales and hard thresholding within middle levels to eliminate noise, followed by nonlinear processing of feature energy to enhance contrast. The five parameters that determine the empirical generalized adaptive gain function in [86] are tuned experimentally to achieve appropriate nonlinear stretching of wavelet coefficients that accomplishes the desired contrast enhancement. Hao *et al.* [33] separated an image into two parts by using the adaptive weighted median filter of Loupas *et al.* [54]. After that, both parts were processed by a wavelet soft thresholding method modified from the Donoho's approach [24] and then recombined.

The main drawback of thresholding methods is the determination of an appropriate threshold, which is the most important parameter for this kind of methods.

2.4.2 Bayesian Estimation Techniques

In recent years, some researchers [2, 81] showed that Bayesian estimation is preferable to thresholding methods in many image processing applications, e.g., medical image and remote sensing image processing. In general, the Bayesian estimation will end up with a continuous shrinking function imposed on noisy observations, in contrast with the thresholding methods, which usually involves a binary thresholding action. To

develop a continuous shrinking function, the probability of one wavelet coefficient to be noisy or clean needs to be calculated, according to a certain criterion. To calculate this probability, an *a priori* PDF for regular coefficients is needed. Two estimators, MAP and MMSE, have been used. Under the assumptions that the noise-free signal is independent, identical distributed (i.i.d.) Gaussian noise with zero mean and variance σ_x^2 and that the noise is AWGN and the signal and noises are uncorrelated, both estimators provide the same solution:

$$\hat{g}_i = \frac{\sigma_g^2}{\sigma_g^2 + \sigma_\eta^2} f_i \quad (2.22)$$

where $\hat{g}_i$ is the estimation of the noise-free signal g_i , f_i is the observed noisy signal, σ_g^2 and σ_η^2 are the variance of the noise-free signal g and the noise η respectively.

However, the above assumptions are usually not satisfied for natural images. To solve these disagreements, Achim *et al.* [2] suggested a generalization of Gaussian distribution, i.e., the alpha-stable distribution, to model the noise-free signal, and Xie *et al.* [81] proposed a mixture of two Gaussian distributions to model natural images.

Bayesian estimation methods can perform better than thresholding methods given a reasonable assumed distribution model. However, appropriate distribution models of the noise-free signal and those of the noise are crucial for the success of this kind of techniques.

2.4.3 Coefficients' Correlation Techniques

In [65], an undecimated (or overcomplete) wavelet domain denoising method was proposed to utilize the correlation of regular wavelet coefficients across different

scales. The preliminary classification step in [65] is related to the one in [82] but avoids its iterative procedure. The result of this preliminary coefficient classification is used to empirically estimate the statistical distributions of the noise-free signal and those of the noise. After that, a soft modification of the wavelet coefficients is performed, instead of adopting a binary decision to select the regular coefficients as in [82].

This method does not rely on the exact prior knowledge of the noise distribution so that it is more flexible and robust compared to other wavelet based methods. However, experiments conducted showed that this method could not suppress enough speckle noise in ultrasound images.

2.4.4 Singularity Detection Techniques

This kind of methods utilizes the evolution of wavelet coefficients across different scales to separate regular coefficients from irregular ones [56]. Generally speaking, the singularity of useful signal has a long-term range and the corresponding regular coefficients have large values at coarse scales, whereas the singularity of noise has larger coefficients at finer scales.

The wavelet transform modulus sum (WTMS) algorithm proposed by Hsung *et al.* [34] has good edge preservation ability based on multiscale singularity detection in wavelet domain. The proposed WTMS method classifies the wavelet coefficients at each scale into two groups, one corresponding to irregular coefficients mainly caused by noise, and the other corresponding to edge-related and regular coefficients mainly contributed by useful signal. By discarding those irregular coefficients and preserving the regular coefficients, noise can be reduced and useful signal is retained.

However, the simple binary processing of irregular and regular coefficients limits the performance of the WTMS method. It degrades the visibility of small structures when applied to medical imaging [85]. This can be explained by the fact that, irregular coefficients contain some signal information and those regular coefficients also contain some noise especially for the lowest decomposition layer. Zhong *et al.* [85] improved the WTMS method by processing the irregular coefficients and also the regular coefficients at the lowest decomposition layer with a minimum mean square error estimation method, instead of using a binary decision.

2.4.5 Comments on Wavelet Based Speckle Reduction Methods

It is well known that the critically sampled orthogonal discrete wavelet transform is not translation invariant. One approach to solve this problem is to perform wavelet denoising to circularly shifted versions of the noisy image and averages the results after shifting them back. This kind of approach is known as cycle spinning. An alternative is to use the undecimated wavelets. It has been proved that, selecting the right wavelets, an undecimated wavelet is equivalent to the cycle spinning scheme [18]. In fact, critically subsampled wavelets are preferred for redundancy suppression, e.g., signal or image compression. In other applications such as image denoising, undecimated wavelets introduce a redundancy that will benefit processing.

2.5 Pyramid Based Speckle Reduction Methods

Besides the wavelets, another multiscale representation is pyramid transform [12, 22, 32, 58, 59, 71, 74]. In the pyramid decomposition scheme, a signal is successively decomposed into a decimated lowpass version and a signal containing residual information. This residual signal is computed as a difference between the signal on a finer scale and the interpolated signal from a coarser scale. More details about pyramid

transform will be given in Chapter 3. Most pyramid denoising methods share the same idea with the wavelet thresholding methods. Pyramid based filters have also been used for reducing speckle [4, 47, 68].

Sattar *et al.* [68] adopted the Feaveau's pyramid transform. In this method, edge detection on interpolated detailed images is used to determine which pixels of the detailed images are to be included in the reconstruction stage. However, none of the three proposed edge detection approaches in [68] are sophisticated enough to perform well on ultrasound images. Considering the multiplicative nature of speckle noise, Aiazzi *et al.* [4] introduced a ratio Laplacian pyramid. In this method, the conventional Kuan filter [47] is extended to multiscale domain by processing the interscale layers of the ratio Laplacian pyramid, but it suffers from the need to estimate the noise variance in each interscale layer. In [47], nonlinear adaptive filtering of the bandpass layers of the pyramid was adopted to reduce noise in medical X-ray and MRI images. The adaptivity is controlled by image gradients calculated from the coarser layer. However, this method cannot achieve satisfactory results on ultrasound images, mainly due to the simply designed adaptive filtering.

2.6 Conclusion and Discussion

As described above, the compounding methods suppress speckle at the expense of reduced spatial resolution and increased system complexity. On the other hand, nonlinear spatial filters do not need many hardware modifications, but their performances and computational complexity are highly dependent on the selected window size, orientation, and the algorithm employed. Conventional diffusion methods may generate saw-smoothed edges when applied to ultrasound images, and recently developed diffusion filtering techniques (i.e., SRAD and NCD) cannot preserve subtle

features and details in ultrasound images. Meanwhile, most multiscale denoising methods suffer from the selection of an appropriate threshold and a proper distribution model of the noise-free image. Overall, all these techniques have difficulty in preserving important image features and details while suppressing speckle in ultrasound images. A technique that can sufficiently remove the speckle and maintain the clinically useful image edges is still absent. Image details and object visibility are very important to medical diagnosis, and the human eyes are very sensitive to image edges. Therefore, a desired speckle reduction algorithm should have the following properties.

First, it should be adaptive. It means that the algorithm adapts automatically to the nature of the pixels to be processed, reducing noise while preserving diagnostic information. In homogeneous (i.e., no texture or structure) regions, smoothing is applied equally in all directions to maximally suppressing noise. In the vicinity of an edge, smoothing is applied along the edge direction, and edge enhancement is performed perpendicularly to the edge direction. At the same time, the denoising algorithm should not generate any visual artifacts.

Second, it should be a multiscale method. The reason why a multiscale method is desired is that human vision being superior to any computerized image processing method sees objects in a multiscale manner. Two common multiscale representations are wavelet and pyramid [51]. Wavelet and pyramid based methods have recently received more and more attention and achieved promising results for image denoising. Two multiscale post-processing speckle reduction techniques implemented in commercial ultrasound machines are Philips' XRES (<http://www.medical.philips.com>) and ContextVision's GOP (<http://www.contextvision.com>). This means that they adopt a certain kind of wavelet, pyramid or similar representation. In multiscale processing,

an input image is separated into several scales, and the denoised image is the combined denoising results from all the different scales.

Last but not least, the smoothing level of the speckle reduction algorithm should be adjustable by users, i.e., clinicians or sonographers. Almost any noise reduction algorithm is an information-reducing process. The more noise it suppresses, the more likely it is to degrade clinically useful information. In real applications, clinicians are often worried that clinically important details might disappear as well when performing speckle reduction. Therefore, to practically use a speckle reduction algorithm as a diagnostic aid, it is a requirement that users have the option to adjust the level of smoothing applied to the images.

The pyramid is one common multiscale representation. It will be shown in Chapter 3 that certain properties of pyramid transform make it more suitable for noise reduction, compared to wavelet transform. Speckle is a kind of signal-dependent noise and thus not distributed uniformly in different scales. The signal decomposition via pyramid transform allows for frequency-specific filtering, and thus benefits speckle reduction. The nonlinear diffusion performs adaptive filtering according to local image gradients, and has shown promising results for additive noise reduction. The smoothing level of the nonlinear diffusion can be adjusted by controlling the diffusion time, which is reflected as the number of iterations. Performing nonlinear diffusion in a pyramid domain satisfy the three properties of a desired speckle reduction algorithm: it is adaptive, multiscale, and allows adjustment on the level of smoothing. In the next chapters, pyramid transform will be analyzed and nonlinear diffusion will be utilized in a pyramid domain to achieve speckle reduction and feature preservation.

Chapter 3 Pyramid Based Filtering Techniques

3.1 Introduction

An inherent property of real-world objects is that they only exist as meaningful entities over certain ranges of scales. The noise can easily be distinguished from signal in scale space [51]. Although it is not clear how to define scale space, it is generally agreed that scale is an indication of how far an object is being viewed. During the last few decades, a number of multiscale representations have been developed, including wavelet and its variations, and several pyramid representations [12, 22, 32, 58, 59, 71, 74]. Wavelet representations are complete and orthogonal, and thus have advantages in image compression. The pyramid representations, on the other hand, are more redundant and allow for efficient filtering algorithms.

3.2 Pyramid Decomposition and Reconstruction

The general structure of pyramid transforms consists of decomposition and reconstruction stages and can be described by smoothing and interpolating filters. In the decomposition stage, a signal is successively decomposed into a decimated smoothed signal and a signal containing residual information. This residual signal is computed as the difference between the signal on a finer scale and the interpolated signal from a coarser scale. A finer scale corresponds to a lower pyramid layer. The lowest pyramid layer has the same size as the original image. A pyramid is specified by its decimation factor and the smoothing and interpolating filters.

The Laplacian pyramid (LP) introduced by Burt and Adelson [12] is one of the earliest and most popular pyramid transforms. Because the lowpass filter for smoothing is a

Gaussian-like lowpass filter, the resultant decimated images form a Gaussian pyramid (GP). Whereas the difference between two images is similar to the Laplacian operator commonly used in image enhancement, thus the decomposition is called LP decomposition scheme.

To introduce LP, two operators REDUCE and EXPAND are defined. The REDUCE operator performs a 2D lowpass filtering followed by a sub-sampling by a factor of two in both directions. With a $2k+1$ -by- $2k+1$ Gaussian-like lowpass filter W , the REDUCE operator can be expressed as:

$$G_l(i, j) = \sum_{m=-k}^k \sum_{n=-k}^k W(m, n) G_{l-1}(2i + m, 2j + n) \quad (3.1)$$

where (i, j) is the position of an image pixel, and l is the pyramid layer. The original image is at layer 0.

The EXPAND operator enlarges an image to twice the size in both directions by up-sampling (insertion of zeros) and lowpass filtering followed by a multiplication by a factor of four, which is necessary to maintain the average intensity being reduced by the insertion of zeros. The EXPAND operator can be expressed as:

$$G_{l,0} = G_l \quad G_{l,n}(i, j) = 4 \sum_{m=-k}^k \sum_{n=-k}^k W(m, n) G_{l,n-1}\left(\frac{i-m}{2}, \frac{j-n}{2}\right) \quad (3.2)$$

By representing the original image by I , GP at layer l by G_l , and LP at layer l by L_l , where $l = 0, 1 \dots Z-1$, and Z is the total decomposition layer, the GP and LP at layer l are defined as follows

$$G_0 = I \quad (3.3)$$

$$G_l = REDUCE[G_{l-1}] \quad (3.4)$$

$$L_l = G_l - \text{EXPAND}[G_{l+1}] \quad (3.5)$$

Figure 3.1 shows the structure of LP transforms, for the case of four decomposition layers. And Fig. 3.2 shows a four-layer LP decomposition of an ultrasound phantom image. The envelope of the acquired data was log-compressed and then decomposed into its GP and LP domain. As can be seen in Figs. 3.1 and 3.2, GP contains a set of lowpass filtered copies of the original image at different sizes, while LP decomposes the original image into a set of bandpass images and a final lowpass image. The image at the highest layer of GP is same as that at the highest layer of LP.

Fig. 3.1. Block diagram showing the structure of Laplacian pyramid transform

Fig. 3.2. The first four layers of the Gaussian and Laplacian pyramid of an image.

(a) Original image I ; (b) Gaussian pyramid, from left to right: G_0 , G_1 , G_2 , and G_3 ;

(c) Laplacian pyramid, from left to right: L_0 , L_1 , L_2 , and L_3 .

The scale of the Laplacian operator doubles from layer to layer of the pyramid, while the center frequency of the passband is reduced by an octave [12]. This is of interest because there is psychophysical evidence that the mechanisms in the human visual systems tuned to different spatial frequencies are approximately an octave apart. Ideally the lowpass filter cuts the spectrum of the image at half of the Nyquist frequency to enable sub-sampling by a factor of two. The finest subband L_0 represents a highpass band containing frequencies down to half of the Nyquist frequency of the original image. Consequently, each subband L_l contains frequencies from $\frac{1}{2^{l+1}}$ to $\frac{1}{2^l}$ times the Nyquist frequency of the original image. This decomposition scheme results from the sub-sampling procedure, which reduces the Nyquist frequency of the current scale to half of that of the proceeding scale [66]. Reconstruction of the image from the LP is achieved by simply reversing the decomposition steps. A perfect reconstruction can be obtained if no processing takes place in the individual LP layers.

The LP is an oversampling scheme in the sense that the total number of transform coefficients is $\frac{4}{3}$ the number of pixels in the original image. Therefore, in compression applications, it is often replaced by subband coding or wavelet transforms, which are critically sampled schemes and often orthogonal decompositions. Though the oversampling of LP transform is a problem for image compression, it has little or no impact in denoising applications. The LP has the advantage over the critically sampled wavelet schemes that each pyramid layer generates only one bandpass signal, even for multidimensional cases. This property makes it easy to apply many multiscale algorithms using a coarse to fine strategy to LP. Furthermore, the resultant bandpass signals of LP do not suffer from the “scrambled” frequencies as in the critical sampling

scheme. This frequency scramble happens in the wavelet filter bank when a highpass channel after downsampling is folded back into the low frequencies, and thus its spectrum is reflected. In LP, this effect can be avoided by downsampling only the lowpass signals [22].

Because of the signal-dependent property of speckle noise in ultrasound images, noise is not uniformly distributed in different scales. The LP separates the major signal component and noise into different layers, and thus helps to achieve a good balance between speckle reduction and feature preservation of ultrasound images. Applying adaptive filtering to each bandpass image, except the lowpass image at the coarsest scale, allows reducing noise independently on slowly varying signal components and thus preserving fine details of the image.

3.3 Speckle Reduction in Pyramid Domain

Techniques of pyramid based denoising are similar to those wavelet based methods. They are consisted of three procedures: pyramid decomposition, manipulation on pyramid coefficients, and finally pyramid reconstruction. The manipulation of the pyramid coefficients is mostly based on a classification. With a certain criterion, the pyramid coefficients are divided into two groups. The first group contains important, regular coefficients that are contributed by useful signal, while the second group consists of coefficients that are considered as irregular and caused mainly by noise. Then these two groups are processed in different ways, with the noisy coefficients often replaced by zero.

Roughly speaking, there are two groups of pyramid based denoising methods, i.e., thresholding [58] and filtering of pyramid coefficients [4, 47, 59]. Two most often used

thresholding functions are soft and hard thresholding. For each pyramid coefficient, if its amplitude is smaller than a predefined threshold, it will be set to zero; otherwise it will be kept unchanged (hard thresholding), or cut in the absolute value by an amount of the threshold (soft thresholding). The hard thresholding scheme preserves feature better while the soft thresholding scheme provides smoothness. The pyramid based thresholding methods share the similar idea as the wavelet based thresholding methods. The difficulty in the thresholding techniques is the selection of an appropriate threshold, which is the most important design parameter for this kind of methods. On the other hand, filtering of transform coefficients have not been successfully applied to wavelet based methods, due to the critically sampling property of wavelet. This property means that connected spatial structures are split into different subbands that are uncorrelated with each other, and their spatial connectivity is lost. Therefore performing filtering to the wavelet coefficients will damage the image textures after inverse wavelet transform of the filtered image. On the contrary, LP is redundant and represents connected image structures on multiple scales, and thus can avoid the impairments of image textures [4].

As mentioned in Section 2.1.2, a log-compression will transform the multiplicative speckle noise to additive noise. Because LP transform is a linear transformation, the obtained pyramid coefficients after performing LP transform to both sides of Eq. (2.5) can be represented as

$$LP[f^{\log}(i, j)] = LP[g^{\log}(i, j)] + LP[\eta_m^{\log}(i, j)] \quad (3.6)$$

where

$$LP[f^{\log}(i, j)] = \{ (L_l[f^{\log}(i, j)])_{0 \leq l \leq Z-2}, L_{Z-1}[f^{\log}(i, j)] \} \quad (3.7a)$$

$$LP[g^{\log}(i, j)] = \{ (L_l[g^{\log}(i, j)])_{0 \leq l \leq Z-2}, L_{Z-1}[g^{\log}(i, j)] \} \quad (3.7b)$$

$$LP[\eta_m^{\log}(i, j)] = \{ (L_l[\eta_m^{\log}(i, j)])_{0 \leq l \leq Z-2}, L_{Z-1}[\eta_m^{\log}(i, j)] \} \quad (3.7c)$$

The definitions of symbols are same as before, i.e., $L_l(i, j)$ represents LP at layer l , $l = 0, 1 \dots Z-1$, and Z is the total decomposition layer. The terms $(L_l[f^{\log}(i, j)])_{0 \leq l \leq Z-2}$, $(L_l[g^{\log}(i, j)])_{0 \leq l \leq Z-2}$, and $(L_l[\eta_m^{\log}(i, j)])_{0 \leq l \leq Z-2}$ are the bandpass images after the LP transform of the log-compressed noisy image $f^{\log}(i, j)$, noise-free image $g^{\log}(i, j)$, and noise $\eta_m^{\log}(i, j)$, respectively. And the terms $L_{Z-1}[f^{\log}(i, j)]$, $L_{Z-1}[g^{\log}(i, j)]$, and $L_{Z-1}[\eta_m^{\log}(i, j)]$ are the lowpass images at the coarsest scale.

It is known that speckle noise exists at high frequency, thus $L_{Z-1}[\eta_m^{\log}(i, j)] \rightarrow 0$. To get $LP[g^{\log}(i, j)]$, which is the LP transform of the log-transformed noise-free image $g^{\log}(i, j)$, an appropriate filter can be applied to the bandpass images $(L_l[f^{\log}(i, j)])_{0 \leq l \leq Z-2}$ to remove the noise component $LP[\eta_m^{\log}(i, j)]$. A certain degree of feature enhancement besides noise reduction can be achieved by enhancing the feature energy of bandpass images $(L_l[g^{\log}(i, j)])_{0 \leq l \leq Z-2}$. After performing filtering of the bandpass images, the pyramid is reconstructed to get the image $g^{\log}(i, j)$. And finally, an exponential transformation is performed to recover the noise-free image $g(i, j)$.

To demonstrate the effectiveness of LP based filtering for speckle reduction, a LP based Wiener filter was applied to the envelope-detected image of the image in Fig. 3.2(a). The envelope-detected image was first log-compressed, followed by LP decomposition. Then the Wiener filter was performed on each bandpass image in the LP, and the filtered pyramid was reconstructed. Finally, an exponential function was applied to the reconstructed image to get the despeckled image, as shown in Fig. 3.3(a).

The four-layer LP decomposition of the despeckled image is depicted in Fig. 3.3(b). It can be seen from Figs. 3.3(a) and (b) that much speckle noise was removed and the image edges were preserved. However, a simple filtering method such as Wiener filter cannot suppress speckle thoroughly.

(a)

(b)

Fig. 3.3. Image filtered by Laplacian pyramid based Wiener filter and its four-layer Laplacian pyramid decomposition. (a) Filtered image; (b) The first four layers of the Laplacian pyramid decomposition of the filtered image.

3.4 Conclusion

The LP is a useful multiscale representation approach for image denoising. It enables frequency-selective noise reduction by separating major signal component and noise into different scales. Such a property makes it suitable for speckle reduction, since speckle is not distributed uniformly across the frequency spectrum. Speckle noise can be reduced by suppressing those irregular pyramid coefficients. Enhancement of some image features can be achieved by selectively amplifying those pyramid coefficients that represent the features of interests. In this way, both noise reduction and image enhancement can be achieved by performing adaptive filtering in the pyramid domain. Whereas, a simple approach like LP based Wiener filter cannot suppress enough speckle noise, and consequently a superior method is needed to perform filtering in the LP domain.

Chapter 4 Proposed Nonlinear Diffusion in Laplacian Pyramid Domain for Speckle Reduction

4.1 Introduction

As discussed in Chapter 2, present speckle reduction methods have difficulty in preserving important image features and details while suppressing speckle in ultrasound images. In this Chapter, a novel speckle reduction method, i.e., Laplacian pyramid based nonlinear diffusion (LPND), is proposed to effectively suppress speckle noise while preserving clinically useful edges and detailed features for medical ultrasound imaging. In this method, speckle is removed by nonlinear diffusion filtering of the bandpass ultrasound images in Laplacian pyramid domain. For nonlinear diffusion in each pyramid layer, a gradient threshold is automatically determined by a variation of median absolute deviation (MAD) estimator. The performance of the proposed LPND method is compared with that of other speckle reduction methods, including the recently proposed speckle reducing anisotropic diffusion (SRAD) and nonlinear coherent diffusion (NCD). Quantitative results from simulation and phantom studies, and visual comparison of despeckled clinical ultrasound images are given to evaluate the proposed speckle reduction algorithm.

4.2 Noise Reduction by Nonlinear Diffusion

Diffusion filtering removes noise from an image by modifying the image via a PDE.

The linear diffusion equation is given by

$$\frac{\partial I}{\partial t} = \text{div}[c \cdot \nabla I], \quad I(t=0) = I_0 \quad (4.1)$$

where $\text{div}[\cdot]$ is the divergence operator, ∇I is the image gradient, c is the diffusion constant, I_0 is the original image, and t is the diffusion time. Processing an image using this linear diffusion equation is equivalent to filtering the image with a Gaussian lowpass filter whose variance is $\sigma^2 = 2t$.

Perona and Malik [64] proposed the nonlinear diffusion as described by the following equation

$$\frac{\partial I}{\partial t} = \text{div}[c(|\nabla I|) \cdot \nabla I], \quad I(t=0) = I_0 \quad (4.2)$$

Where $|\cdot|$ is the magnitude operator, $c(|\nabla I|)$ is the diffusion coefficient or diffusivity function. If the function $c(|\nabla I|)$ keeps constant for all the image pixels, the diffusion process becomes linear. For nonlinear diffusion, the diffusion coefficient $c(|\nabla I|)$ is a monotonically decreasing function of the image gradient. Thus, diffusion is discouraged across regions containing large image gradients and encouraged within regions containing small image gradients. This leads to improved structure preservation and image denoising properties. Perona and Malik [64] suggested two diffusion coefficients which are scalar-valued functions varying with locations:

$$c_1(|\nabla I|) = \frac{1}{1 + (|\nabla I|/k)^2} \quad (4.3)$$

$$c_2(|\nabla I|) = \exp[-(|\nabla I|/k)^2] \quad (4.4)$$

where k is a constant called the gradient threshold. It plays an important role in determining the degree of smoothing and enhancing in the diffusion process. Nonlinear diffusion described in (4.2) is called “anisotropic diffusion” by Perona and Malik [64] and some other researchers. However, in this thesis, “anisotropic diffusion” is reserved

for the case when the diffusion coefficient $c(|\nabla I|)$ is a tensor-valued function varying with both location and edge orientation.

4.3 Laplacian Pyramid Based Nonlinear Diffusion

4.3.1 Basic Idea

Similar to wavelet based denoising methods, pyramid based nonlinear diffusion methods consist of three steps: transformation of an image into its pyramid domain, manipulation of pyramidal coefficients by nonlinear diffusion, and then reconstruction of the pyramid. Note that other pyramid transform may also be adopted. In this study, the LP proposed by Burt and Adelson [12] is arbitrarily chosen, mainly because of its simplicity. The block diagram of the proposed LPND method is shown in Fig. 4.1.

First, an input image I is decomposed into GP structure (i.e., G_0, G_1, G_2 and G_3 in Fig. 4.1) and LP structure (i.e., L_0, L_1, L_2 and L_3 in Fig. 4.1). After decomposing an image into its pyramid structure of decreasing frequencies, the main noise (i.e., speckle) and useful signal components of the image exist in different layers because they have different frequency characteristics. Therefore, performing spatial adaptive filtering to each bandpass layer can more effectively suppress speckle noise without degrading slow varying signal too much. Then, each detailed image in the LP (i.e., L_0, L_1 and L_2 in Fig. 4.1) is filtered by a nonlinear diffusion process, which will be discussed in details later. This nonlinear diffusion process differentiates the proposed speckle reduction method from other LP-based denoising methods like those proposed in [4, 47,

58, 59, 68]. After getting the nonlinear diffusion filtered images (i.e., D_0, D_1 and D_2 in Fig. 4.1), with the image in the highest layer of the LP kept untouched (i.e., $R_3 = D_3 = L_3$ in Fig. 4.1), the LP is reconstructed to get the denoised image R_0 . Another GP (i.e., R_0, R_1, R_2 and R_3 in Fig. 4.1) is formed in the reconstruction process.

Fig. 4.1. Block diagram of the proposed LPND method, including pyramid decomposition, nonlinear diffusion, and pyramid reconstruction.

The developed speckle reduction method can be applied to the envelope signal after receive beamforming, to the signal after log-compression, or to the scan-converted

image. When applied to the envelope signal, the speckle reduction method is used in a homomorphic manner, i.e., log-compress the envelope signal, then apply the speckle reduction method and finally perform exponential transform. The log-compression operation changes the multiplicative noise contained in the envelope signal to additive noise which is suitable for the developed speckle reduction technique. If the speckle reduction method is performed after log-compression or scan conversion in an ultrasound imaging system, it does not require these extra steps.

4.3.2 Analysis of the Diffusion Process in Pyramid Domain

The diffusion coefficients described in (4.3) and (4.4) may create problems such as failure in convergence and high sensitivity to noise [13]. Very close images may produce divergent solutions. This is caused by the fact that the diffusion coefficients in (4.3) and (4.4) may act locally as an inverse heat equation which is ill posed and could develop singularities of any order in arbitrarily small time. When applied to images containing noise, this possible inverse diffusion represents a severe drawback. This problem can be handled by applying regularizing finite difference discretization, and the diffusion is usually observed to be stable. (The only observable instability is a “staircasing” effect, in which a single ramp edge may be broken into many discrete steps. Fortunately, it has been shown in [80] that this staircasing tends to occur only on wide smooth edges, as in a blurred disk image. For a wide range of images that have only narrow boundary regions, this phenomenon is not evident.)

It is desirable to have the regularization that does not depend on discretization effects.

In the proposed LPND, a Gaussian regularization strategy [13], which estimates the gradient ∇I on a Gaussian lowpass filtered version of the image, is adopted. Based on this strategy, Eq. (4.2) becomes

$$\frac{\partial I}{\partial t} = \text{div}[c(|\nabla(G_\sigma * I)|) \cdot \nabla I] \quad (4.5)$$

where G_σ is a Gaussian filter with standard deviation σ . This parameter σ is a controller for the roughness of the smoothed image. The parameter σ should be large enough to suppress the emphasis of speckle noise in ultrasound images, and also not too large to blur edges and details of the image.

The diffusion coefficient can be computed using Eq. (4.3) or Eq. (4.4). It has been found that a greater decay rate of the diffusion coefficient will create sharper edges that persist over longer time intervals in a narrower range of edge slopes, and that a more gradual decay rate will sharpen edges over a wider range of edge slopes. The choice of the gradient threshold k gives a lower limit on the gradients of edges that are enhanced in the diffusion process. To analyze the relationship between the threshold k and the image gradient ∇I , a flux function (i.e., $\phi(\nabla I)$) can be defined as $\phi(\nabla I) = c(|\nabla I|) \cdot \nabla I$ [80]. Then, Eq. (4.2) becomes $\frac{\partial I}{\partial t} = \phi'' \nabla' I^2 + \phi' \nabla'' I$. For a local maximum of gradients, $\nabla' I = 0$ and $\nabla'' I < 0$. If $\phi' < 0$, then $\frac{\partial I}{\partial t} > 0$, meaning that these edges will become steeper over time. By taking a derivative of $\phi_1(\nabla I) = c_1(|\nabla I|) \cdot \nabla I$ over ∇I and letting it equal zero, $\nabla I = k$ is obtained. Thus, the maximum of the flux $\phi_1(\nabla I)$ happens in regions where the gradients equal k . The condition $\phi' < 0$ is satisfied when $\nabla I > k$, so the image edges having gradients

above k will be enhanced. If $|\nabla I| < k$, then $\phi' > 0$, and consequently $\frac{\partial I}{\partial t} < 0$. Thus, the edges having gradients under k will be blurred away. The flux of edges with much higher gradients than k will be close to zero, resulting in stopping of diffusion across these edges. A similar analysis finds that the turning point of the flux function $\phi_2(|\nabla I|) = c_2(|\nabla I|) \cdot |\nabla I|$ is at $|\nabla I| = k/\sqrt{2}$.

Figure 4.2 shows the curves of the two diffusion coefficients and their flux functions versus gradient, under a special case of $k = 1$. As shown in Fig. 4.2, below k , the flux reduces to zero, representing the case of homogeneous regions. Above k , the flux also reduces to zero, stopping diffusion across high gradients. As noted by Perona and Malik [64], these two diffusion coefficients generate different diffusions: $c_1(|\nabla I|)$ favors wide regions over smaller ones, while $c_2(|\nabla I|)$ favors high contrast edges over low contrast ones.

(a)

(b)

Fig. 4.2. The relationship between diffusion coefficients, flux functions, and image gradients. (a) Diffusion coefficients $c(|\nabla I|)$ versus gradient ∇I . Curve 1 represents $c_1(|\nabla I|)$ in (4.3), and curve 2 represents $c_2(|\nabla I|)$ in (4.4). ($k=1$) (b) Flux functions $\phi(\nabla I)$ versus ∇I . Curve 1 represents $\phi_1(\nabla I) = c_1(|\nabla I|) \cdot \nabla I$, and curve 2 represents $\phi_2(\nabla I) = c_2(|\nabla I|) \cdot \nabla I$. ($k=1$)

The diffusion coefficient $c_2(|\nabla I|)$ can be modified to be

$$c_2(|\nabla I|) = \exp\left[-(|\nabla I|^2 / (2k^2))\right] \quad (4.6)$$

Thus Eqs. (4.3) and (4.6) will provide similar results for the same k value. Adopting a Gaussian regularization scheme, the above equation becomes

$$c_2(|\nabla(G(\sigma) * I)|) = \exp\left[-(|\nabla(G(\sigma) * I)|^2 / (2k^2))\right] \quad (4.7)$$

4.3.3 Automatic Determination of the Gradient Threshold

The choice of the gradient threshold k plays an important role in determining the parts of an image that will be blurred or enhanced in the diffusion process. If its value is set too high, the diffusion will act as a smoothing filter, diffusing across the edge boundaries. While if it is too low, small diffusion will result in a large number of iterations. At some small k values, extra edges may be introduced between the regions of high and low intensities.

In conventional nonlinear diffusion methods, the gradient threshold is mainly adjusted and determined depending on the noise level and edge strength. When the noise level is unknown, this parameter needs to be selected by trial-and-error to get an optimal result. This heuristic procedure is inappropriate for clinical applications where multiple frames are dynamically utilized. Perona and Malik [64] used Canny's noise estimator to select the gradient threshold automatically. They set the parameter k equal to 90 percent of the cumulative histogram of the absolute gradients throughout the whole image. However, their automatic threshold selection mechanism requires a bi-peak histogram [64], which is uncommon in medical ultrasound images. Currently, the gradient threshold k is estimated using the robust MAD estimator [8]

$$k = \frac{MAD(|\nabla I|)}{0.6745} = \frac{\text{median}(|\nabla I| - \text{median}(|\nabla I|))}{0.6745} \quad (4.8)$$

where the constant 0.6745 is derived from the fact that the MAD of a zero mean normal distribution with unit variance is 0.6745.

The experiments show that the gradient threshold estimated by (4.8) is close to its optimal value for conventional nonlinear diffusion in most cases. No significance difference has been observed when using the gradient threshold estimated by (4.8) or using a manually selected optimal gradient threshold. However, this value needs to be adjusted in each pyramid layer for the proposed LPND. The new decision rule on the gradient threshold for the LPND can be represented by

$$k(l) = \frac{MAD(|\nabla I(l)|)}{0.6745} \sqrt{2 \log((l+1)/l)} \quad (4.9)$$

where l is the pyramidal layer. The image gradient ∇I is computed using the pyramidal coefficients in the corresponding pyramid layer. In this new gradient threshold decision rule, a relatively large value is used in the lowest pyramid layer where speckle noise dominates, in order to remove it more thoroughly. On the other hand, to enhance larger structure boundaries, small gradient thresholds are applied in higher layers. In this way, the new gradient threshold decision rule given in Eq. (4.9) can help the LPND suppress noise thoroughly and preserve important image features.

After applying regularized nonlinear diffusion in each pyramid layer except the highest one where the prominent structure information is mainly remained, the despeckled image is generated by the reconstruction of the pyramid.

4.3.4 Discretization Scheme

Many numerical schemes can be used for the solution of the diffusion equation, such as fast Fourier transform (FFT), wavelet transform, finite element scheme, neural network,

and multigrid method [72]. In most cases, a finite difference scheme is preferred because of its relatively easy implementation for 2D digital images. In the finite difference scheme, continuous derivatives are approximated by discrete differences. Using the finite difference scheme and central differencing in spatial domain, the 2D discrete nonlinear diffusion equation may be expressed as

$$I(i, j, t+1) = I(i, j, t) + \lambda \cdot [c_N \cdot \nabla I_N(i, j, t) + c_S \cdot \nabla I_S(i, j, t) + c_W \cdot \nabla I_W(i, j, t) + c_E \cdot \nabla I_E(i, j, t)] \quad (4.10)$$

where λ is the time step and it controls the diffusion speed. The local gradients are approximated by directional differences:

$$\begin{aligned} \nabla I_N(i, j) &= I(i-1, j) - I(i, j) \\ \nabla I_S(i, j) &= I(i+1, j) - I(i, j) \\ \nabla I_W(i, j) &= I(i, j-1) - I(i, j) \\ \nabla I_E(i, j) &= I(i, j+1) - I(i, j) \end{aligned} \quad (4.11)$$

Allowing $|\nabla I|^2$ to be discretized as an average of the four squared directional differences, the gradient magnitude $|\nabla I|$ can be computed by

$$|\nabla I| = 0.5 \cdot \sqrt{|\nabla I_N|^2 + |\nabla I_S|^2 + |\nabla I_W|^2 + |\nabla I_E|^2} \quad (4.12)$$

To solve the PDE, a boundary condition needs to be imposed to fit the solutions to actual problems. Since zero-padding and periodic boundary conditions may lead to undesirable false discontinuities, the Neumann boundary condition that assumes values beyond the border are equivalent to the border values is utilized.

The limitation of this explicit discretization scheme given in (4.10) is the requirement of stability which limits the time step to be no larger than $1/2D$, where D is the number

of dimensions (i.e., $D = 2$ in this study) along which the gradients are calculated. As discussed in [62], iteration and multiscale processing have a similar effect as they both perform local operations to produce global effects. A multiscale iterative filter combines the advantages of both multiscale processing and iterative methods so that it needs much smaller number of iterations than that from single scale processing. The proposed LPND can be classified as a multiscale iterative method. Thus, the time step limitation (i.e., $\lambda \leq 0.25$) is not prohibitive in the LPND. When the image contains a large amount of speckle noise, the additive operator splitting (AOS) scheme [79] can be adopted to use a large time step. When the noise level is moderate, the explicit discretization scheme is preferred because a small diffusion time is needed in this case and a large time step allowed by the AOS scheme may affect the accuracy of solution.

4.3.5 Automatic Determination of the Diffusion Stopping Time

A diffusion filter is iterative. One challenge in diffusion filtering is to decide when to stop the diffusion process. In the described discrete algorithm, the diffusion coefficients $c_1(|\nabla I|)$ and $c_2(|\nabla I|)$ would never drop to zero because the gradients as computed by finite differences are bounded by the difference between minimum and maximum intensity values in the original image. So given enough time, the process as computed by finite differences will converge to a single value. To get a nontrivial result from a diffusion filtering process, a single finite time which is equal to the product of time step and the number of iterations needs to be selected to stop the diffusion process. A small diffusion time gives more weights to the original image and keeps more noise and also image details untouched, while a large diffusion time will make the resultant image

close to the piecewise constant model that is inherent in the diffusion process.

A diffusion process can be stopped manually by setting a fixed number of iterations. However, in real applications, different images may need different numbers of diffusion iterations. Thus, a mechanism to stop the diffusion automatically is preferred. Several criteria for estimating the optimal stopping time have been proposed. Weickert's relative variance [78] and Mrázek's decorrelation [59] criteria have been implemented and tested in this study. Although working well for conventional diffusion methods, they are inappropriate to be used for the LPND.

Weickert's criterion is based on the fact that the relative variance $r(I(t)) = \frac{\text{var}(I(t))}{\text{var}(I_0)}$, i.e., the ratio between the variance of diffused image at time t and the variance of original image, should decrease monotonically from 1 to 0. Prescribing a threshold for $r(I(t))$ will stop the diffusion automatically. Since different pyramid layers in the LPND have different noise levels, they need different thresholds. In practice, without any *prior* knowledge of the speckle noise variance, it is a challenging task to determine the different thresholds based on experience and trial-and-error.

Mrázek's criterion is based on the correlation between the filtered image at time t and the noise, which is assumed to be uncorrelated to the useful signal and can be estimated as the difference between the original image and the filtered image at time t :

$$\text{corr}(I_0 - I(t), I(t)) = \frac{\text{cov}(I_0 - I(t), I(t))}{\sqrt{\text{var}(I_0 - I(t)) \cdot \text{var}(I(t))}} \quad (4.13)$$

This correlation should decrease first, that is to say, the noise is reduced, and increase later, implying the useful signal is to be affected. Thus, the diffusion should stop when the correlation reaches its minimum. However, this correlation does not have a unique minimum for the LPND since the lowest pyramid layer contains very little signal and is close to the degenerate case. Therefore, the decorrelation criterion is not suitable for determining the stopping time of the LPND.

In the LPND, the stopping time is determined based on the mean absolute error (MAE) between two adjacent diffusion steps:

$$\text{MAE}(I(t)) = \frac{1}{M \times N} \sum_{(i,j)=1}^{M,N} |I(i, j, t) - I(i, j, t-1)| \quad (4.14)$$

where $I(i, j, t)$ and $I(i, j, t-1)$ are the filtered value of the pixel (i, j) at time t and $t-1$, and M and N are the number of columns and rows in the image, respectively.

As discussed in [3] and the references therein, the convergence condition of a nonlinear diffusion process is dependent on the image size, the diffusion coefficient, and the numerical scheme. Some diffusion implementations will diverge whereas a convergent nonlinear diffusion process converges exponentially. To study the convergence of the proposed LPND, the MAE values from a four-layer LPND process (i.e., nonlinear diffusion is applied in three bandpass layers) with the above scheme on a real ultrasound liver image are plotted in Fig. 4.3. The time step is 0.2, and the number of

iterations is 100. This number of iterations far exceeds the necessary number of iterations to get a smooth image, and thus is considered enough for examining the convergence of the LPND. As shown in Fig. 4, the MAE value decreases exponentially with the number of iterations, and the diffusion iteration can be stopped by setting a threshold for the MAE value.

Fig. 4.3. The convergence of MAE in the proposed LPND method.

When the change between two adjacent diffusion steps is smaller than a prescribed threshold value, the diffusion filtering will stop automatically. Alternatively, this threshold for stopping the diffusion filtering can be adjusted by clinicians according to their preferences and different purposes for speckle reduction. For example, a user can adjust the smoothing level by changing the MAE value described in Eq. (4.14) to optimize the speckle reduction process for a specific clinical application. If speckle reduction is applied to improve the visual lesion detectability, a short diffusion time is suitable to remove noise without blurring fine structures and boundaries. If the speckle reduction technique is used as a preprocessing step before some image analysis tasks,

such as segmentation and classification, it can create more homogeneous regions and clear boundaries using a longer diffusion time. In clinical applications, clinicians are often afraid that important diagnostic details might disappear as well when performing speckle reduction. Therefore, to practically apply a speckle reduction technique in ultrasound imaging's routine clinical use, it is desirable that clinicians be given the flexibility to adjust its smoothing level. The threshold for MAE provides clinicians such a flexibility to adjust the smoothing level of the LPND.

Fig. 4.4. Flowchart of nonlinear diffusion in the proposed LPND method.

The flowchart of the nonlinear diffusion filtering in each LP layer is shown in Fig. 4.4. First the gradient threshold is estimated from the input image, which is a detailed image in the LP domain, using (4.9), (4.11) and (4.12). Then within each iteration of the nonlinear diffusion, a Gaussian lowpass filtering is performed on the image to be filtered. Following that, the image gradients are computed on the Gaussian lowpass filtered image using (4.11), and then the diffusion coefficients are computed by (4.7). Finally, the image is updated by (4.10). After evolving the image by a certain number of iterations, the diffusion is automatically stopped according to the MAE in (4.14).

4.3.6 Computational Complexity of the LPND algorithm

The computational complexity of the proposed LPND can be analyzed by dividing it into two parts. For a 2-D image of $M \times N$ pixels, the computational complexity of Laplacian pyramid decomposition and reconstruction is $O(M \times N)$ (i.e., the order of $M \times N$). For the same image, the computational complexity of nonlinear diffusion is $i_{dif} \cdot O(M \times N)$, where i_{dif} is the number of iterations. Using a four-layer decomposition as shown in Fig. 4.1, the computational complexity of the proposed LPND can be written as the sum of two main parts: one is Laplacian pyramid decomposition and reconstruction of an image of $M \times N$ pixels, and the other is three nonlinear diffusions on three images of different sizes (i.e., $M \times N$, $\frac{M \times N}{4}$, and $\frac{M \times N}{16}$). Therefore, the overall computational complexity of the LPND is $\left(1 + i_0 + \frac{i_1}{4} + \frac{i_2}{16}\right) \cdot O(M \times N)$, where i_0 , i_1 , and i_2 are the numbers of iterations for

nonlinear diffusion in the first, second, and third layers of Laplacian pyramid (i.e., L_0 , L_1 , and L_2 in Fig. 4.1), respectively.

In our current implementation using the explicit discretization scheme and time step $\lambda = 0.2$, and stopping diffusion automatically by the MAE criterion where the threshold for MAE is 0.005, it takes around 6 s (15, 27, and 45 iterations in the first, second, and third layers of Laplacian pyramid, respectively) for the LPND to process a 266×512 unfiltered envelope-detected ultrasound image in Matlab[®] (Mathworks Inc., Natick, MA, USA) on a Pentium 4 (2.4 GHz) PC. By adopting the AOS scheme and time step $\lambda = 2$, the number of iterations for nonlinear diffusion in each layer of the Laplacian pyramid can be reduced to two. This number of iterations is comparable to the one discussed in [1], where a real-time implementation is achieved using the AOS scheme and three iterations of their diffusion process.

4.4 Experimental Results and Comparisons

To evaluate the performance of the LPND, simulation, phantom and *in vivo* studies were carried out. The simulation and phantom studies give quantitative performance analysis, and the *in vivo* study shows the usefulness of the proposed method in real applications.

In each study, the performance of the LPND is compared with those of LP-based Wiener filter (LPW), Gaussian regularized nonlinear diffusion (ND), speckle reducing anisotropic diffusion (SRAD) [83], and the nonlinear coherent diffusion (NCD) [1].

LPW uses Wiener filters at all passbands of an image pyramid. The Wiener filter was implemented using the built-in function "wiener2" in Matlab[®] (Mathworks Inc., Natick, MA, USA), and its filter window was set to be 7×7 , 5×5 , and 3×3 from the lowest pyramid layer to the second highest one. For both LPND and LPW, four decomposition layers and 9×9 binomial filters for both REDUCE and EXPAND operators were used. Using more layers did not guarantee improved performance. A smaller binomial filter window would blur image edges too much, whereas a larger binomial filter window would increase the computational complexity and did not improve performance much. In the ND, Eq. (4.5) was utilized for speckle reduction. Its gradient threshold was determined by (4.8). The diffusion coefficients for the LPND and ND were computed using (4.7), the standard deviation of Gaussian filter and time step were 1.0 and 0.2, respectively. The stopping time for LPND, ND, and NCD were all determined by the MAE given in Eq. (4.14), and the thresholds for the MAE were all 0.005. The SRAD needed a relatively small time step λ , which was set to 0.02. Its stopping time was also determined automatically by the MAE criterion, and the threshold for the MAE was 0.05, 0.1, and 0.2 to give optimal results for simulation, phantom, and *in vivo* studies, respectively. The coherence stopping level in NCD was set as $s = 0.07$ to achieve optimal coherence enhancement without introducing artifacts. Other parameters of SRAD and NCD were same as those used in [83] and [1] respectively. The LPND, LPW, ND, and NCD were implemented in homomorphic manners on the envelope-detected images, which were formed by computing $B = \sqrt{R_I^2 + R_Q^2}$, where R_I and R_Q are the in-phase and quadrature (I/Q) echo signals respectively. After taking the logarithm of the envelope-detected images, they were processed using the

LPND, LPW, ND, or NCD method, and finally an exponential transform of the processed data was performed. The SRAD was applied directly on the envelope-detected images, as proposed in [83].

4.4.1 Results from Simulation Study

The five different speckle reduction methods (i.e., LPND, LPW, ND, SRAD, and NCD) were applied on simulated B-mode images, which were generated by considering the RF image to be the convolution of the 2D PSF of the ultrasound system with a 2D underlying echographic map, as adopted in [83]. Simulated images with different parameters were tested, and similar results were obtained in all these cases. Thus, only the results obtained with the following settings are presented: the center frequency was 5 MHz, the pulse width was 1.2, the lateral beam width was 1.5, and the variance of the underlying complex Gaussian random field was 1.0.

The echographic map of 256×256 pixels was presumably simulated as follows. A ventricular cavity, an artery interior and a vascular wall, a rectangular target, a bright circular target, a dark circular target, an elliptical target, three small cysts, and four point targets, whose mean values of the ultrasound cross sections are 2, 2.5, 20, 25, 18, 3.5, 3, 1.5, and 40 respectively, were placed at different locations in the image. The mean value of background was set to 10. Figures 4.5(a) and 4.5(b) show the underlying echographic map and a simulated envelope detected image respectively. Both images are log-compressed and then normalized with the same function for display. The underlying echographic map was used as the reference image for calculating

quantitative measures.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

Fig. 4.5. Simulated B-mode image and its filtering results. (a) Underlying echographic map. (b) Simulated speckled image. (c)-(g) Filtered images with LPND, LPW, ND, SRAD, and NCD.

To quantify the achieved performance improvements, three metrics were computed based on the reference and the filtered images. One measure is MSE defined as

$$MSE = \frac{1}{M \times N} \sum_{(i,j)=1}^{M,N} (\hat{S}(i,j) - S(i,j))^2 \quad (4.15)$$

where S and $\hat{S}$ are the reference and filtered images respectively, and M and N are the number of columns and rows in the image respectively.

To evaluate the performance of edge preservation, the second parameter β as used in [2, 33, 68] is computed by

$$\beta = \frac{\Gamma(\Delta S - \overline{\Delta S}, \Delta \hat{S} - \overline{\Delta \hat{S}})}{\sqrt{\Gamma(\Delta S - \overline{\Delta S}, \Delta S - \overline{\Delta S}) \cdot \Gamma(\Delta \hat{S} - \overline{\Delta \hat{S}}, \Delta \hat{S} - \overline{\Delta \hat{S}})}} \quad (4.16)$$

where ΔS and $\Delta \hat{S}$ are the highpass filtered versions of S and $\hat{S}$, respectively,

obtained with a 3×3 standard approximation of the Laplacian operator, $\overline{\Delta S}$ and $\overline{\hat{\Delta S}}$ are mean values of pixels in the region of ΔS and $\hat{\Delta S}$ respectively, and

$$\Gamma(S_1, S_2) = \sum_{(i,j)=1}^N (S_1(i, j) \cdot S_2(i, j)) \quad (4.17)$$

The range of β is between $[0, 1]$, with unity for ideal edge preservation.

The third measure, the structure similarity (SSIM) [77], is calculated to evaluate the overall processing quality.

$$SSIM = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (4.18)$$

where μ_i is the mean of image intensity, σ_i is the intensity standard deviation of pixels in a local window, and C_i is the constant to avoid instability. The SSIM is calculated between two images x and y . All involved parameters were set as suggested in [77]. The range of SSIM value is also $[0, 1]$, with unity for ideal processing quality, i.e., when the filtered image is equal to the reference image.

The CNR, which is sometimes referred as lesion signal-to-noise ratio [16, 75], was also computed by

$$CNR = \frac{|\mu_1 - \mu_2|}{\sqrt{\sigma_1^2 + \sigma_2^2}} \quad (4.19)$$

where μ_1 and σ_1^2 are the mean and variance of the region of interest (ROI), and μ_2 and σ_2^2 are the mean and variance of a background region that has the same size as the ROI to be compared with. Both the ROI and the background region were considered circular regions and manually selected.

The obtained values of MSE, β and SSIM from all the methods are summarized in Table 4.1. To reduce the sample bias in the random process of noise generation, the experiment was repeated 30 times under the same settings except the noise was simulated using different random seeds. Table 4.1 shows the mean and standard deviation values of the 30 samples. The results in Table 4.1 show that the proposed LPND outperforms all the other four methods in terms of MSE and SSIM, while ND, SRAD, and NCD achieve comparable β values to LPND. The two-sided paired t-tests were conducted under the null hypothesis that there is no statistically significant difference between the β values achieved by LPND, ND, SRAD, and NCD. A value of $p < 0.05$ was considered statistically significant. The β values achieved by LPND are significantly different from those achieved by ND ($p < 10^{-5}$). However, there are no significant difference between the β values achieved by LPND and those by SRAD ($p = 0.1612$), and no significant difference between the β values achieved by LPND and those by NCD ($p = 0.8555$).

The mean and standard deviation of CNR values from each method on the 30 simulated images are summarized in Table 4.2. The six ROIs in the reference image (i.e., Fig. 4.5(a)) are marked in Fig. 4.6. The two-sided paired t-tests were conducted under the null hypothesis that there is no statistically significant difference between the CNR values obtained from the LPND and those from other four methods for each ROI. For ROI 1, 2, 3, 4, and 6, the hypothesis was rejected with $p < 0.001$. For ROI 5, LPND was also significantly different from LPW, ND and NCD ($p < 0.0011$). On the other

hand, no significant difference between LPND and SRAD was found for ROI 5 ($p = 0.5401$).

TABLE 4.1

MSE, β , and SSIM values on 30 simulated images with the same speckle level

	MSE	β	SSIM
Noisy	260.46 ± 7.15	0.0285 ± 0.0057	0.3922 ± 0.0038
LPND	76.88 ± 3.11	0.1009 ± 0.0119	0.8107 ± 0.0038
LPW	93.05 ± 3.53	0.0317 ± 0.0053	0.7535 ± 0.0049
ND	86.38 ± 3.52	0.0805 ± 0.0189	0.7934 ± 0.0043
SRAD	144.06 ± 4.68	0.0955 ± 0.0169	0.7290 ± 0.0039
NCD	82.49 ± 2.96	0.1003 ± 0.0126	0.7992 ± 0.0039

Fig. 4.6. Six regions of interests (ROIs) for calculating CNR in the simulation study.

Figures 4.5(c)-(g) gives a visual comparison of their performances on the simulated

image in Fig. 4.5(b). All these filtered images are log-compressed and then normalized with the same function for display, so that they have the same dynamic range. Compared to the other approaches, LPND shows the most noise reduction while preserving small structures like the three small cysts and the four point targets. LPW does not give satisfactory speckle suppression. ND enhances edges but it does not keep correct edge locations. SRAD dilates bright regions and erodes dark regions. With the SRAD, the boundaries of bright regions are broadened and those of dark regions are shrunk. NCD enhances the edge coherence but cannot suppress enough noise. And the three small cysts and the four point targets are blurred as shown in Fig. 4.5 (g).

TABLE 4.2

CNR values on 30 simulated images with the same speckle level

CNR	ROI 1	ROI 2	ROI 3	ROI 4	ROI 5	ROI 6
Original	1.37 ± 0.06	1.18 ± 0.06	0.71 ± 0.08	1.06 ± 0.07	1.04 ± 0.09	1.15 ± 0.05
LPND	4.73 ± 0.34	4.14 ± 0.33	2.29 ± 0.36	3.58 ± 0.49	2.68 ± 0.39	3.17 ± 0.27
LPW	3.07 ± 0.18	2.69 ± 0.21	1.51 ± 0.20	2.32 ± 0.29	2.00 ± 0.22	2.38 ± 0.19
ND	3.44 ± 0.25	3.03 ± 0.23	1.77 ± 0.25	2.64 ± 0.35	2.29 ± 0.29	2.43 ± 0.27
SRAD	3.34 ± 0.25	3.02 ± 0.25	1.97 ± 0.27	2.60 ± 0.31	2.62 ± 0.36	2.15 ± 0.22
NCD	3.65 ± 0.27	3.16 ± 0.24	1.85 ± 0.26	2.74 ± 0.35	2.38 ± 0.28	2.67 ± 0.28

4.4.2 Results from Phantom Study

The five speckle reduction methods including the proposed LPND were applied on a

commercial gray scale phantom image (i.e., 403GS LE, Gammex RMI, USA). A sequence of I/Q data containing 30 frames was acquired using a commercial ultrasound machine (i.e., EUB-6000, Hitachi Medical Corp., Japan), equipped with a special programmable board for data acquisition. The programmable board was connected with a PC, and the acquired I/Q data were transferred to the PC for processing. The envelopes of the acquired data were used for testing all the five methods.

Fig. 4.7. Phantom image and its filtering results. (a) Original noisy image. (b)-(f) Filtered images with LPND, LPW, ND, SRAD, and NCD.

Figure 4.7 shows their results. For quantitative analysis, the CNR was computed on the filtered image. The results on the 30 frames are summarized in Table 4.3. ROIs 1-4 are the four contrast regions from left to right in the reference image as shown in Fig. 4.7(a). They are a high scatter region relative to the background (ROI 1), a gray scale target of +6 dB (ROI 2), a gray scale target of -6 dB (ROI 3), and an anechoic cyst (ROI 4), respectively.

TABLE 4.3

CNR values on a phantom image sequence containing 30 different frames

CNR	ROI 1	ROI 2	ROI 3	ROI 4
Original	1.80 ± 0.10	0.51 ± 0.08	1.52 ± 0.07	3.81 ± 0.13
LPND	6.34 ± 0.81	2.17 ± 0.45	6.19 ± 0.57	6.86 ± 0.34
LPW	4.45 ± 0.40	1.35 ± 0.26	4.28 ± 0.29	6.61 ± 0.33
ND	5.57 ± 0.55	1.72 ± 0.33	5.06 ± 0.31	8.61 ± 0.56
SRAD	5.93 ± 0.50	1.77 ± 0.32	5.11 ± 0.38	7.33 ± 0.74
NCD	5.59 ± 0.58	1.76 ± 0.33	5.12 ± 0.31	8.07 ± 0.57

For each ROI, the two-sided paired t-tests were conducted under the null hypothesis that there is no statistically significant difference between the CNR values achieved by LPND and the other four methods. For ROIs 1-3, the hypothesis between the CNR values from LPND and those from SRAD in ROI 1 was rejected by $p < 0.023$, and all

the other hypotheses were rejected by $p < 10^{-3}$. However, for ROI 4, which is an anechoic cyst, pyramid-based methods gave lower CNR values compared to diffusion-based methods.

4.4.3 Results from *In Vivo* Study

Two sequences of I/Q data were acquired from liver and carotid artery of a volunteer using 3.5-MHz linear and 7.5-MHz convex array transducers. The results from different frames are similar therefore only one of the frames from each set after scan conversion is used here for illustration. Fig. 4.8 shows the result on a liver image obtained using a 3.5-MHz convex array transducer. To show the differences between them more clearly, a line presenting the pixel values on row 175 is highlighted, and the pixel values on this line from the original and filtered images are shown in Fig. 4.9. Fig. 4.10 shows that on a common carotid artery cross-sectional image obtained using a 7.5-MHz linear array transducer operating at 10 MHz. The pixel values on column 180 from the original and filtered images are compared in Fig. 4.11. All the five methods were applied on the envelope-detected data.

As can be seen from the Figs. 4.8 - 4.11, LPND preserves image edges and retains small structures while suppressing effectively the speckle noise in background. Due to its optimal noise reduction, LPND outperforms the other four methods, resulting in increased contrast and improved detectability of small structures in each image. All the other four methods have limited speckle reduction performance. ND shows sharper but jagged edges, and gives a relatively low contrast. SRAD has smeared out small

structures and edges before reaching a satisfactory noise reduction level. Although NCD enhances the coherence of organ surfaces, it also causes blurring of small structures, and the contrast of its output image is not as good as that of LPND.

As can be seen from Figs. 4.8 - 4.11, LPND preserves image edges and small structures (e.g., the regions numbered 4, 5, and 6 in Figs. 4.9 and 4.11) while effectively suppressing speckle in background (e.g., the regions numbered 1, 2, and 3 in Figs. 4.9 and 4.11). Due to its optimal noise reduction, LPND outperforms the other four methods, resulting in increased contrast and improved visibility of small structures in each image. All the other four methods have limited noise reduction performance. ND shows sharper but jagged edges and gives a relatively low contrast. SRAD broadens the boundaries of bright regions (the regions numbered 4, 5, and 6 in Fig. 4.11) and shrinks those of dark regions (the regions numbered 4 and 5 in Fig. 4.9). Although NCD enhances the coherence of organ surfaces, it also causes blurring of small structures, and the contrast of its output image is not as good as that of LPND.

(a)

(b)

(c)

(d)

(e)

(f)

Fig. 4.8. Real ultrasound liver image and its filtering results. (a) Original noisy image. (b)-(f) Filtered images with LPND, LPW, ND, SRAD, and NCD.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

Fig. 4.9. Profiles along the highlight line in the liver image. (a) Original noisy image showing the highlight line. (b)-(g) Profiles along the highlight line in the images filtered by LPND, LPW, ND, SRAD, and NCD.

Fig. 4.10. Real ultrasound carotid artery cross-sectional image and its filtered results.

(a) Original noisy image. (b)-(f) Filtered images with LPND, LPW, ND, SRAD, and NCD.

(a)

(b)

(c)

(d)

(e)

Fig. 4.11. Profiles along the highlight line in the carotid artery cross-sectional image.

(a) Original noisy image showing the highlight line. (b)-(g) Profiles along the highlight line in the images filtered by LPND, LPW, ND, SRAD, and NCD.

4.5 Conclusion and Discussion

A new nonlinear diffusion method in LP domain for ultrasonic speckle reduction has been investigated. It consists of the following three steps. First, the input image is decomposed into its LP domain. Then, a regularized nonlinear diffusion process is performed in each pyramid layer except the highest one to remove the speckle. Finally, the diffused LP is reconstructed to get the despeckled image. An automatic estimation of the gradient threshold by MAD is also integrated in the LPND to make the algorithm practical. The MAE criterion that is used to stop the diffusion automatically offers the LPND the flexibility in adjusting its smoothing strength.

The proposed LPND preserves edges and small structures while thoroughly removing speckles. From simulation and phantom studies, an average gain of 1.55 dB and 1.34

dB in CNR were obtained compared with SRAD and NCD, respectively. (The CNR gain in dB were calculated by $20\log_{10}\frac{CNR(LPND)}{CNR(SRAD \text{ or } NCD)}$) The visual comparison of despeckled *in vivo* ultrasound images from liver and carotid artery shows that the proposed LPND method effectively suppresses speckles as well as preserves edges and detailed structures. These preliminary results indicate that the proposed speckle reduction method could improve image quality of medical ultrasound imaging through the improvement in CNR and also the detectability of small structures and fine details. Thus, it can be used for improving the diagnostic capability of current ultrasound imaging and enhancing the performance of computer-assisted diagnosis as well.

Chapter 5 Coupled Nonlinear Diffusion and Shock Filter in Laplacian Pyramid Domain for Ultrasound Image Enhancement

5.1 Introduction

In Chapter 4, a novel image processing method, LPND, has been introduced for speckle reduction. Extensive experiments have shown that applying nonlinear diffusion in Laplacian pyramid domain can reduce speckle noise effectively while preserving subtle features and details in ultrasound images. However, the nonlinear diffusion mainly acts as an edge-preserving process and cannot achieve satisfactory edge enhancement effects. Consequently, some important diagnostic features in the image may still be blurred by LPND.

Conventional image enhancement algorithms only work well on images with well defined edges. When they are applied on ultrasound images that contain heavy speckle noise, conventional edge enhancement methods tend to amplify noise. One possible scheme to limit noise amplification is to apply image enhancement on the image filtered by a certain speckle reduction technique. Since the proposed LPND can reduce the speckle noise effectively with minimally degradation of useful features in the image, it may be a suitable candidate for this purpose. A long diffusion time may be set for the LPND to generate a smooth image, prior to performing image enhancement on the filtered image. However, there are two main limitations for such a scheme. First, some noise will be amplified if no additional measures are adopted to prevent it, because the

LPND (or any other speckle reduction techniques) cannot reduce all speckle noise. Second, if some faint edges in the image are blurred too much or even eliminated by the speckle reduction technique, it is almost impossible to restore them or enhance them to a satisfactory level. It is therefore desirable to perform denoising and edge enhancement simultaneously. Combining them into one process helps to solve the problems of amplifying noise and losing faint edges with low gradients.

In this Chapter, such a combined speckle reduction and edge enhancement algorithm, i.e., Laplacian pyramid based nonlinear diffusion and shock filter (LPNDSF) will be introduced. The proposed LPNDSF uses a coupled nonlinear diffusion and shock filter process in the LP domain of an image. Speckle is suppressed by nonlinear diffusion while edges are enhanced by shock filter, of bandpass images in the LP domain. The nonlinear diffusion and shock filter is combined into one PDE, and applied as one coupled process. The ratio between shock filter and nonlinear diffusion is adaptive according to the local gradients. More shock filtering is allowed at locations where the local gradients are high, to achieve more enhancement on image edges. On the other hand, low local gradients represent smooth regions, where less shock filtering is performed to avoid enhancement of these regions and amplification of speckle noise. The superiority of LPNDSF will be evaluated by comparing it with LPND, NCD, and single-scale coupled nonlinear diffusion and shock filter (NDSF) on simulated and *in vivo* images.

5.2 Image Deblurring by Shock Filter

In the PDE community, the nonlinear diffusion process has some sharpening effects, but it mainly acts as an edge-preserving noise reduction process and cannot achieve satisfactory edge enhancement effects. Another PDE-based method, shock filter, is a kind of well-known deblurring algorithm that behaves similarly to deconvolution [60]. The shock filter was first proposed by Kramer and Bruckner in a fully discrete manner [44]. The basic idea is to use a dilation operation around maxima and an erosion operation around minima. Whether a pixel belongs to the influence zone of a maximum or minimum is determined by the sign of its Laplacian. A negative Laplacian means that the pixel is in the influence zone of a maximum and vice versa. Iterating this process produces a segmentation-like result with piecewise constant segments that are separated by discontinuities, i.e., shocks.

The term shock filter and the first continuous version based on PDEs were developed by Osher and Rudin [63]. The equation describing the shock filter is

$$\frac{\partial I}{\partial t} = -\text{sign}(\Delta I)|\nabla I|, \quad I(t=0) = I_0 \quad (5.1)$$

where $|\nabla I|$ is the gradient of the image I , and $\Delta I = I_{xx} + I_{yy}$ is the Laplacian operator.

When a pixel is in the influence zone of a local maximum, its Laplacian is negative, so the above equation becomes $\frac{\partial I}{\partial t} = |\nabla I|$. Evolution with this equation for time t is equal to a dilation process with a disk-shaped structuring element of radius t . At the influence zone of a local minimum, the Laplacian is positive, so the equation becomes $\frac{\partial I}{\partial t} = -|\nabla I|$,

which is equal to an erosion process with a disk-shaped structuring element of radius t [10].

It is shown in [63] that the image's second derivative along the gradient direction $I_{\eta\eta}$ with η representing the gradient direction is a better edge detector than the Laplacian operator ΔI . However, this process is extremely sensitive to noise and thus has little practical use. In order to increase its robustness, a common idea is to replace the edge detector $I_{\eta\eta}$ by a lowpass filtered version [5]. Adopting a Gaussian filter for this purpose, the shock filter becomes

$$\frac{\partial I}{\partial t} = -\text{sign}(G_\sigma * I_{\eta\eta})|\nabla I| \quad (5.2)$$

where G_σ is a Gaussian with standard deviation σ , and $*$ denotes convolution.

Introducing a Gaussian filter will decrease the magnitudes of the inflection points caused by noise, but cannot eliminate them. Thus there will still be change of sign at these points, which will lead the flow to produce shocks [31]. Therefore, a better solution is to add a denoising part when using the shock filter for image deblurring. The common idea is to use a diffusion process together with the shock filter, with a certain weight between them [5]:

$$\frac{\partial I}{\partial t} = -\text{sign}(G_\sigma * I_{\eta\eta})|\nabla I| + cI_{\xi\xi} \quad (5.3)$$

where c is a positive constant, and ξ represents the direction perpendicular to the gradient direction.

5.3 Laplacian Pyramid Nonlinear Diffusion and Shock Filter

5.3.1 Basic Idea

In the proposed speckle reduction and edge enhancement algorithm, both nonlinear diffusion and shock filter process are performed in the LP domain. The procedure of the proposed LPNDSF is summarized in Fig. 5.1. First, the image is decomposed into its GP (Gaussian pyramid) structure (i.e., G_0, G_1, G_2 and G_3 in Fig. 5.1) and LP structure (i.e., L_0, L_1, L_2 and L_3 in Fig. 5.1). Second, each bandpass image in the pyramid (i.e., L_0, L_1 and L_2 in Fig. 5.1) is evolved by a PDE that combines nonlinear diffusion and shock filter to reduce speckle noise and enhance image edges. And finally, the pyramid is reconstructed to get the enhanced image R_0 . From Fig. 4.1 and Fig. 5.1, we can find the following main differences between the LPND and the LPNDSF. The LPND uses a nonlinear diffusion process in the LP domain, whereas the LPNDSF uses a coupled nonlinear diffusion and shock filter process in the LP domain. While both of them estimate the gradient threshold from details images in the LP domain, the LPNDSF also performs edge map estimation using images in the GP domain.

The coupled nonlinear diffusion and shock filter in the second step can be represented by the following equation:

$$\frac{\partial I}{\partial t} = \text{div}(c(|\nabla I|)\nabla I) - r \cdot (1 - c(|\nabla I|)) \cdot \text{sign}(G_\sigma * I_{\eta\eta})|\nabla I| \quad (5.4)$$

where $c(|\nabla I|) = \exp\left[-|G_\sigma * \nabla I|^2 / (2k^2)\right]$, and r is the ratio between the nonlinear diffusion and the shock filter in the coupled process. When $r \rightarrow 0$, the above equation becomes a P-M nonlinear diffusion; when $r \rightarrow \infty$, it acts as a conventional shock filter

without smoothing. Both the term $(1 - c(|\nabla I|))$ and the parameter r are used to adjust the edge enhancement of the coupled process. They will be discussed in details later.

The process in Eq. (5.4) is different from existing variations of combined diffusion and shock filter, such as those introduced in [5, 19, 43]. The method in [5], as described by Eq. (5.3), performs diffusion only in the direction perpendicular to the gradient. However, noise also exists along the gradient direction, especially in homogeneous regions. A certain amount of denoising is also needed in the gradient direction, and such a process was proposed in [43], where the diffusion is an anisotropic process, i.e., decomposed into the direction perpendicular to the gradient and the gradient direction. The nonlinear diffusion is preferred to the anisotropic diffusion, because the nonlinear diffusion performs better in homogeneous regions and at points of high curvature. Compared to the scheme in [19] which was originally proposed for image classification, the scheme in Eq. (5.4) has a higher flexibility and can result in more edge enhancement.

As in the LPND method, the LPNDSF can be applied on the envelope signal after receive beamforming, on the signal after log-compression, or on the scan-converted image. When applied on the envelope signal, the LPNDSF is used in a homomorphic manner. If the LPNDSF is performed after log-compression or scan conversion in an ultrasound imaging system, it can be used directly with the three steps described above.

Fig. 5.1. Block diagram of the proposed LPNDSF method, including pyramid decomposition, coupled nonlinear diffusion and shock filter process, and pyramid reconstruction.

5.3.2 Analysis of the Shock Filter in Pyramid Domain

The most important step in the shock filter is to get correct locations of image edges. The accuracy of edge detection will greatly affect the performance of the shock filter. Taking noise mistakenly as edges will amplify noise, while missing some edges will prevent these edges from being enhanced. Replacing the edge detector $I_{\eta\eta}$ by its Gaussian lowpass filtered version may help to suppress noise amplification to some extent, but has a limited capability to solve this problem. Adopting a small filtering window for the

Gaussian convolution cannot reduce noise sufficiently, while a big filtering window will blur or even eliminate many image details and blurry edges.

It is known that the bandpass images in the LP contain high and middle frequency components, a large part of which are contributed by noise and edges. To separate edges from noise in the LP is not a trivial task. As a matter of fact, calculating $I_{\eta\eta}$ and its lowpass filtered version on the bandpass images in the LP is not an effective approach to detect edges, and an alternative way is needed.

A Gaussian pyramid (GP) is formed when decomposing an image into its LP domain. The GP contains a set of lowpass filtered versions of the original image at different sizes, and each layer of the GP has a same-sized counterpart in the LP. Since the GP contains less noise than the LP, edges can be detected by calculating $I_{\eta\eta}$ on each lowpass image in the GP domain, and the detected edge map can be used to control the shock filter on the corresponding bandpass image in the LP domain.

Though the lowpass images in the GP contain less noise than the bandpass images in the LP, these lowpass images are not noise-free. Therefore, detecting edges directly from the images in the GP will take some noise mistakenly as edges. To limit noise amplification, the images in the GP need to be smoothed before calculating its $I_{\eta\eta}$. At the same time, there is also a concern that some blurry edges may be eliminated during the smoothing process and consequently they will not be detected and enhanced. Taking these into consideration, nonlinear diffusion is performed on the images in the GP and edges are

detected on the diffused images to control the shock filter in the LP. The nonlinear diffusion in the GP domain is performed together with the coupled nonlinear diffusion and shock filter process in the LP domain, and the detected edge map is updated iteratively from the GP. Detecting edges from the diffused images in the GP guarantees that most edges especially those with low gradients are enhanced while noise amplification is limited.

The term $(1 - c(|\nabla I|))$ is utilized to control the edge enhancement strength of the shock filter in Eq. (5.4). With such a monotonically increasing function of $|\nabla I|$, more edge enhancement is encouraged in regions containing large image gradients and discouraged across regions containing small image gradients. The parameter r given in Eq. (5.4) is set by experience. The following guidelines are given to help tune this parameter. To avoid noise amplification, a small value of r should be used for the lowest layer (i.e., the finest scale) of the LP, since most noise exists in this layer. To enhance blurry edges, a relatively large value of r is desired in the second layer of the LP. To prevent smooth regions from being enhanced, relatively small values of r should be used in higher layers of the LP.

5.3.3 Discretization Scheme

The proposed process as shown in (5.4) is a PDE-based method, and needs to be discretized for implementation. As mentioned in [19], the discretization of a coupled nonlinear diffusion and shock filter process is not straightforward. If the image gradient in the shock filter is discretized by centered directional differences, the stability of such

a process is not guaranteed. Therefore, the nonlinear diffusion and shock filter in (5.4) are discretized by different schemes. For the nonlinear diffusion part, the same discretization scheme as presented in Section 4.3.4 is followed and thus omitted here; for the shock filter part, the non-centered scheme proposed by Osher and Rudin [63] is adopted as described below.

The image gradient is defined by

$$\nabla I(i, j) = \sqrt{DI(i, j)_i^2 + DI(i, j)_j^2} \quad (5.5a)$$

$$DI(i, j)_i = m(\Delta_+^i I(i, j), \Delta_-^i I(i, j)) \quad (5.5b)$$

$$DI(i, j)_j = m(\Delta_+^j I(i, j), \Delta_-^j I(i, j)) \quad (5.5c)$$

where

$$\Delta_{\pm}^i I(i, j) = \pm(I(i \pm 1, j) - I(i, j)) \quad (5.6a)$$

$$\Delta_{\pm}^j I(i, j) = \pm(I(i, j \pm 1) - I(i, j)) \quad (5.6b)$$

and $m(x, y)$ is the minmod function

$$m(x, y) = \begin{cases} \text{sign}(x) \cdot \min(|x|, |y|) & \text{if } xy > 0 \\ 0 & \text{if } xy \leq 0 \end{cases} \quad (5.7)$$

The second derivative along the gradient direction is given by

$$I_{\eta\eta} = I_{xx}(I_x)^2 + 2I_{xy}I_xI_y + I_{yy}(I_y)^2 \quad (5.8)$$

where the first order derivatives I_x and I_y , and the second order derivatives I_{xx} , I_{xy} ,

and I_{yy} are approximated by standard finite directional differences:

$$I_x = \frac{1}{2}(I(i, j+1) - I(i, j-1)) = \frac{1}{2}(\nabla I_E(i, j) - \nabla I_W(i, j)) \quad (5.9)$$

$$I_y = \frac{1}{2}(I(i+1, j) - I(i-1, j)) = \frac{1}{2}(\nabla I_S(i, j) - \nabla I_N(i, j)) \quad (5.10)$$

$$I_{xx} = I(i, j+1) + I(i, j-1) - 2I(i, j) = \nabla I_w(i, j) + \nabla I_e(i, j) \quad (5.11)$$

$$I_{yy} = I(i+1, j) + I(i-1, j) - 2I(i, j) = \nabla I_n(i, j) + \nabla I_s(i, j) \quad (5.12)$$

$$I_{xy} = \frac{1}{2}(I_x(i+1, j) - I_x(i-1, j)) \quad (5.13)$$

For both nonlinear diffusion and shock filter, the Neumann boundary condition is utilized. The time step is limited to be no larger than 0.25.

The flowchart of the coupled nonlinear diffusion and shock filter used in the proposed LPNDSF is summarized in Fig. 5.2. Within each iteration of the coupled process, a lowpass image in the GP is first filtered by a nonlinear diffusion process represented by (4.7) and (4.9)-(4.12). Then the filtered lowpass image's second derivative along the gradient direction $I_{\eta\eta}$ is calculated using (5.8)-(5.13); and the term $(1 - c(|\nabla I|))$ is also calculated on the filtered lowpass image, using (4.12) to get the gradient of the filtered lowpass image, (4.9) to get the gradient threshold, and (4.7) to compute $c(|\nabla I|)$. At the same time, (5.5)-(5.7) are computed on the corresponding bandpass image in the LP which has the same size as the lowpass image in the GP, and the image gradients obtained are utilized to calculate the term $\text{sign}(G_\sigma * I_{\eta\eta})|\nabla I|$ in (5.4). The nonlinear diffusion process represented (4.7) and (4.9)-(4.12) is also computed on the bandpass image in the LP domain. And at last, the bandpass image in the LP domain is updated using (5.4). The coupled process will stop automatically after a certain number of iterations, according to the MAE setting in (4.14).

Fig. 5.2. Flowchart of the coupled nonlinear diffusion and shock filter in the proposed LPNDSF method.

5.3.4 Convergence of The Proposed LPNDSF

To study the convergence of the proposed LPNDSF, the MAE values from a four-layer LPNDSF process (i.e., coupled nonlinear diffusion and shock filter process was applied on three bandpass layers) with the above scheme on a real ultrasound liver image are plotted in Fig. 5.3. The time step was 0.2 and the number of iterations was 100. This number of iterations far exceeds the necessary number of iterations to get a smooth image, and thus is considered enough for examining the convergence of the LPND. As

shown in Fig. 5.3, the MAE value decreases exponentially with the number of iterations.

Therefore, the iteration can be stopped by setting a threshold for the MAE value.

Fig. 5.3. The convergence of MAE in the proposed LPND method.

5.4 Experimental Results and Comparisons

To evaluate the performance of the LPNDSF, simulation study using computer-generated images and *in vivo* study using clinical images were carried out. The simulation study gives quantitative performance analysis, and the *in vivo* study shows the usefulness of the proposed method in real applications.

In each study, the performance of LPNDSF was compared with that of three recently-developed diffusion-based methods, i.e., speckle reducing anisotropic diffusion (SRAD) [83], nonlinear coherent diffusion (NCD) [1], and Laplacian pyramid based nonlinear diffusion (LPND). The involved parameters were adjusted to give

optimal results for each method. For LPNDSF and LPND, four decomposition layers and 9×9 binomial filters for REDUCE and EXPAND operators were used. The Gaussian kernel to smooth second derivative along the gradient direction in the shock filter part of the LPNDSF was 1.0. The weights between shock filter and nonlinear diffusion in LPNDSF were 0.1, 1, and 0.1 for the first, second and third layer, respectively. The diffusion coefficient for LPNDSF and LPND was computed by (4.7) where the standard deviation of Gaussian filter was 1.0. The coherence stopping level in NCD was set as $s = 0.07$ to achieve optimal coherence enhancement without introducing artifacts. The time step for LPNDSF, NCD, and LPND was 0.2. The stopping time for the three methods was determined by the MAE criterion given in (4.14), and the threshold for the MAE was 0.0025 for LPNDSF, and 0.005 for NCD and LPND. The SRAD needed a relatively small time step, which was set to 0.02. Its stopping time was also determined by the MAE criterion, and the threshold for the MAE was 0.05 and 0.2 for simulation and *in vivo* studies respectively. Other parameters of SRAD and NCD were same as those used in [83] and [1].

All the four methods were implemented in homomorphic manners on the envelope detected images. We took the logarithm of the envelope-detected images, processed it using the LPNDSF, LPND or NCD method, and finally performed exponential transform of the processed data. The logarithm operation was used to change the speckle in the envelope-detected images from multiplicative noise to additive noise. The SRAD was applied directly on the envelope-detected images as proposed in [83].

5.4.1 Results from Simulation Study

The four different ultrasound image enhancement methods (i.e., LPNDSF, LPND, SRAD, and NCD) were applied on simulated B-mode images, as adopted in Section 4.4.1. Images with different simulating parameters were tested, and similar results were obtained in all these cases. Thus, we only present the result obtained with the following parameters: the center frequency was 5 MHz, the pulse width was 1.2, the lateral beam width was 1.5, and the variance of the underlying complex Gaussian random field was 1.0. The simulated echogeneity map of 256×256 pixels was the same as adopted in Section 4.4.1. Figures 5.3 (a) and (b) show the underlying echographic map and a simulated envelope detected image respectively. Both images are log-compressed and then normalized with the same function for better visualization. The echogeneity map was used as the reference image for calculating quantitative measures.

To quantify the achieved performance improvements, four metrics MSE, β , SSIM, and CNR as used in Section 4.4.1 were computed on the reference and filtered images. The obtained values of MSE, β and SSIM from all the methods are summarized in Table 5.1. To reduce the sample bias in the random process of noise generation, the experiment was repeated 50 times under the same parameter settings except that the noise was simulated using different random seeds. Table 5.1 shows the mean and standard deviation values obtained from the 50 samples. The results in Table 5.1 show that the proposed LPNDSF outperformed the other three methods in terms of all the three metrics. The two-sided paired t-tests were conducted under the null hypotheses

that there are no statistically significant differences between the MSE, β and SSIM values achieved by LPNDSF, LPND, SRAD, and NCD. A value of $p < 0.05$ was considered statistically significant. All the hypotheses were rejected at very high level ($p < 10^{-7}$).

TABLE 5.1

MSE, β , and SSIM values on 50 simulated images with the same speckle level

	MSE	β	SSIM
Noisy	257.76 ± 5.92	0.0267 ± 0.0051	0.3926 ± 0.0034
LPNDSF	72.90 ± 2.61	0.1249 ± 0.0119	0.8212 ± 0.0035
LPND	75.96 ± 2.65	0.0998 ± 0.0109	0.8113 ± 0.0035
SRAD	142.63 ± 3.91	0.0912 ± 0.0203	0.7299 ± 0.0034
NCD	81.22 ± 2.90	0.1005 ± 0.0138	0.8005 ± 0.0039

The mean and standard deviation of CNR values from each method on the 50 simulated images are summarized in Table 5.2. The six ROIs in the reference image (i.e., Fig. 5.4(a)) are marked in Fig. 4.6. The two-sided paired t-tests were conducted under the null hypothesis that there is no statistically significant difference between the CNR values obtained from LPNDSF and those from the other three methods for each ROI. A value of $p < 0.05$ was considered statistically significant. The results from LPNDSF were significantly different from those from SRAD and NCD for all the six ROIs ($p < 10^{-3}$). Compared to LPND, LPNDSF gave significantly higher CNR values for ROI 1-5 ($p < 10^{-4}$ for ROI 1-3 and $p < 0.007$ for ROI 4-5) and gave comparable

CNR values for ROI 6 ($p = 0.61$).

TABLE 5.2

CNR values on 50 simulated images with the same speckle level

CNR	ROI 1	ROI 2	ROI 3	ROI 4	ROI 5	ROI 6
Original	1.37 ± 0.06	1.19 ± 0.05	0.75 ± 0.07	1.05 ± 0.08	1.03 ± 0.08	1.16 ± 0.06
LPNDSF	4.99 ± 0.41	4.48 ± 0.42	2.80 ± 0.41	3.90 ± 0.58	2.95 ± 0.43	3.19 ± 0.42
LPND	4.64 ± 0.34	4.09 ± 0.34	2.46 ± 0.33	3.61 ± 0.48	2.69 ± 0.35	3.23 ± 0.32
SRAD	3.30 ± 0.25	2.99 ± 0.24	2.17 ± 0.29	2.65 ± 0.32	2.63 ± 0.38	2.15 ± 0.22
NCD	3.62 ± 0.26	3.14 ± 0.25	2.04 ± 0.27	2.80 ± 0.37	2.44 ± 0.30	2.69 ± 0.24

Figures 5.4(c)-5.4(f) give a visual comparison of their performance on the simulated image in Fig. 5.4(b). All these filtered images are log-compressed and then normalized with the same functions, so that they have the same dynamic range. Compared to the other methods, LPNDSF reduced more noise in homogeneous areas and also preserved better small structures such as the three small cysts and the four point targets. In addition, LPNDSF showed clearer boundaries of the objects. The LPND suppressed more speckle noise than SRAD and NCD, and preserved small structures better than these two methods, but the LPND did not enhance the blurry boundaries in the image. The SRAD dilated bright regions and eroded dark regions. The NCD blurred the three small cysts and the four point targets and only enhanced sharp edges such as the synthetic artery walls.

Fig. 5.4. Simulated B-mode image and its filtering results. (a) Underlying echographic map. (b) Simulated speckled image. (c)-(f) Filtered images with LPNDSF, LPND, SRAD, and NCD.

5.4.2 Results from *In Vivo* Study

Five sequences of I/Q data were acquired from liver, carotid artery and kidney of a volunteer using 3.5-MHz linear and 7.5-MHz convex array transducers. The results are similar for all these cases. Here, one sequence of liver images and one sequence of kidney images that contain fuzzy edges are selected, since these images are good candidates for demonstrating the edge enhancement performance of a filter. Furthermore, Canny edge detector [36] was applied on filtered images to show their corresponding edge maps. It may be valuable to evaluate the performance of LPNDSF on edge enhancement using more sophisticated edge detection techniques like deformable models. However, most of these techniques are also based on simple edge information such as gradients, as in Canny edge detection. Therefore, using Canny edge detection for evaluation is meaningful.

All the four methods were applied on the envelope-detected images. To achieve better edge enhancement results, the weights between shock filter and nonlinear diffusion were set to be 0.1, 5, and 0.1 for the first, second, and third layer respectively. All the other parameters of each method used the same values as in the simulation study. The Canny edge detector was implemented using the built-in function “edge” in Matlab and the same parameters were used on the images filtered using the four methods. The results of applying the four methods on a liver image obtained by a 3.5-MHz convex array are given in Fig. 5.5, and their corresponding detected edges are shown in Fig. 5.6. The results on a kidney image obtained by a 7.5-MHz linear array operating at 10 MHz

and their detected edges are shown in Fig. 5.7 and Fig. 5.8 respectively.

Observing the filtered images in Figs. 5.5 and 5.7, it can be found that LPNDSF could enhance the image edges without resulting in noticeable noise amplification effect. It revealed blurry edges and generated smooth boundaries. In addition, the overshoot artifacts were not observed in the images filtered by LPNDSF. Both LPNDSF and LPND can suppress speckle noise thoroughly. LPND blurred fuzzy edges with low gradients, while LPNDSF gave sharper edges. Both NCD and NDSF did not reduce speckle noise to a satisfactory level. As shown in Figs. 5.5(d)(e) and Figs. 5.7(d)(e), their filtered images contained some blocks, indicating that much speckle noise existed in the filtered images. NCD only enhanced the coherence of some boundaries. Compared to the images filtered by LPND, there was no obvious edge enhancement effect in the images filtered by NCD. Though NDSF gave sharper edges than LPND and NCD, it generated jagged boundaries like those produced by the conventional ND. This is predictable, since NDSF is based on ND and both are single-scale methods that cannot perform effectively on ultrasound images containing heavy speckle noise. Comparing the filtered images and the original images, the blurry edges that are unclear in the original images are easier to be observed in the images filtered by LPNDSF. Due to the heavy speckle noise inherent in ultrasound images, some weak edges are hardly recognizable in the original images. Of the four methods, only LPNDSF can make those weak edges visually clearer in the filtered images.

Though there is no ideal edge map for these *in vivo* images, comparison of their Canny

edge detected images as shown in Figs. 5.6 and 5.8 can be used to evaluate the performance of the four filtering methods. The original images are so noisy that it is hard to recognize any clear boundaries after applying Canny edge detection on them. All the filtering methods reduce noise and thus make the boundaries easier to be recognized by a computer. Among the four edge maps detected from the filtered images, the boundaries obtained from LPNDSF were the most continuous and complete. All the other three methods produced broken and incomplete boundaries. Since LPND reduced speckle well, its errors caused by classification of noise as edges were relatively smaller than the other methods. But some boundaries in its filtered images were not clear, and thus the detected edge maps were not complete in Fig. 5.6(c) and Fig. 5.8(c). Both NCD and NDSF left much speckle in the images, and introduced more classification errors in their edge-detected images. This comparison of their edge detection results validates the superiority of LPNDSF over the other three methods.

The LPNDSF is computationally more complex than the other methods in the experiments, due to the fact that it combines two iterative PDE-based methods and needs both Laplacian and Gaussian pyramids to control filtering process. In the current implementation using the explicit discretization scheme and time step $\lambda = 0.2$ and stopping the process automatically by the MAE criterion where the threshold for MAE is 0.005, it takes around 16 s for LPNDSF to process a 266×512 unfiltered envelope detected image in Matlab on a Pentium 4 (3.2GHz) PC, while it takes about 6 s for LPND, 18 s for SRAD and NCD. Adopting a larger time step and parallel computing of filtering in Laplacian and Gaussian pyramids will speed up the calculations.

(a)

(b)

(c)

(d)

(e)

Fig. 5.5. Real ultrasound liver image and its filtering results. (a) Original noisy image.

(b)-(e) Filtered images with LPNDSF, LPND, SRAD, and NCD.

(a)

(b)

(c)

(d)

(e)

Fig. 5.6. Edge detection results on the images shown in Fig. 5.5.

(a)

(b)

(c)

(d)

(e)

Fig. 5.7. Real ultrasound kidney image and its filtering results. (a) Original noisy

image; (b)-(e) Filtered images with LPNDSF, LPND, SRAD, and NCD.

(a)

(b)

(c)

(d)

(e)

Fig. 5.8. Edge detection results on the images shown in Fig. 5.7.

5.4.3 Temporal Consistency

Because the LPNDSF does not use any temporal information over frames, the temporal consistency of its enhancement effect should be studied to ensure that no temporal artifacts are introduced. Two experiments were conducted as suggested in [9].

First, the spatial correlation curves for the image sequences denoised by LPND and those for the corresponding sequences enhanced by LPNDSF were compared. The correlation coefficient was computed between consecutive frames. As mentioned in [9], if the enhancement is consistent and no temporal artifacts are introduced, the evolution of the spatial correlation curves over time should be same. The correlation curves for a sequence of 90 liver images, those for a sequence of 70 kidney images, and those for a sequence of 40 carotid artery cross-sectional images are shown in Figs. 5.9(a), (b), and (c) respectively. To study the temporal consistency of the enhancement effect more fully, the spatial correlation for the sequences enhanced by LPNDSF were calculated under four different enhancement levels, with the following four sets of weights between shock filter and nonlinear diffusion for the first, second, and third layer: (0.1, 1, 0.1), (0.1, 5, 0.1), (1, 1, 1), and (1, 5, 1). Fig. 5.9 illustrates that the curves for LPND and LPNDSF evolve similarly over time, meaning that the temporal correlations of the LPND-denoised images are well preserved by LPNDSF. In Fig. 5.9, deep spikes appear when there are relatively big differences (i.e., tissue movements) between two consecutive frames.

(a)

(b)

(c)

Fig. 5.9. Evolution of spatial correlation coefficients between consecutive frames over time. (a) Sequence 1, 90 liver images; (b) Sequence 2, 70 kidney images; (c) Sequence 3, 40 carotid artery cross-sectional images.

Note: “LPNDSF, (0.1, 1, 0.1); 0.9429” (as in Fig. 5.9(a)) represents the curve obtained when the weights between shock filter and nonlinear diffusion were 0.1, 1, and 0.1 from the first to the third layer respectively, and its correlation coefficient with the LPND-denoised curve is 0.9429.

In the second experiment, the spatial correlations between the LPND-denoised images and the LPNDSF-enhanced images were computed for each frame. If no temporal artifacts are introduced by LPNDSF, the spatial correlations between the denoised images and the enhanced images should be almost constant for all the frames. Table 5.3

gives the mean and standard deviation of this measure on the three sequences, with sequence 1 consisting of 90 liver images, sequence 2 consisting of 70 kidney images and sequence 3 consisting of 40 carotid artery cross-sectional images, respectively. The small values of the standard deviations indicate that the enhancement effect of LPNDSF is temporally consistent.

TABLE 5.3

Spatial correlation between the denoised images and the enhanced images

(r_0, r_1, r_2)	(0.1, 1, 0.1)	(0.1, 5, 0.1)	(1, 1, 1)	(1, 5, 1)
Sequence 1	0.9824 ± 0.0031	0.9766 ± 0.0037	0.9629 ± 0.0055	0.9579 ± 0.0059
Sequence 2	0.9747 ± 0.0068	0.9669 ± 0.0086	0.9570 ± 0.0082	0.9511 ± 0.0090
Sequence 3	0.9806 ± 0.0015	0.9664 ± 0.0017	0.9589 ± 0.0022	0.9455 ± 0.0023

5.5 Conclusion and Discussion

In this Chapter, a new speckle reduction and edge enhancement method has been introduced. The proposed LPNDSF consists of the following three steps. First, the input image is decomposed into its GP and LP domain. Then, a coupled nonlinear diffusion and shock filtering process is performed in each LP layer except the highest one. The nonlinear diffusion is mainly used for speckle reduction, and the shock filter is for edge enhancement. The threshold in the nonlinear diffusion is estimated automatically from the images in the LP domain, and the GP is utilized to obtain an edge map to control the shock filter in the LP. Finally, the processed LP is reconstructed to give the output.

The parameters for the LPNDSF can be divided into three groups: the parameters for the nonlinear diffusion, those for the shock filter, and the ratio between shock filter and nonlinear diffusion. The first two groups can be fixed as constants for images acquired under similar conditions. On the other hand, the ratios between the two processes need to be adjusted to achieve optimal results for different images. Generally, small details and edges having high gradients exist in fine scales, while fuzzy boundaries representing the general structure of the image exist in coarse scales. For those images where diagnostic interests are mainly small details, more emphases on shock filter in fine scales can improve the detectability of these small details and give sharper edges. Usually, fuzzy boundaries need to be enhanced, rather than the sharp edges that are already easy to be detected. In this situation, a relatively large weight on shock filtering is desired in coarse scales, which contain the fuzzy boundaries of interests. In the conducted experiments, it usually gives good results by allowing more shock filter in the second finest layer of the LP, where fuzzy boundaries mainly exist. However, the optimal value for the ratio between shock filter and nonlinear diffusion in this layer varies for different images.

The superior performance of the LPNDSF indicates that it has the potential of helping clinicians in making diagnosis visually. With the speckle reduced and edge enhanced images, clinicians may find additional diagnostic information that is hidden in the original images. Because it can enhance image edges while reducing speckle noise thoroughly, the proposed LPNDSF can also serve as a preprocessing step in computer

assisted image analysis tasks, such as segmentation, classification and quantitative measurements. A better result can be achieved by performing these image analysis tasks on the LPNDSF filtered images than conducting them directly on the original images.

Chapter 6 Conclusions and Recommendations

6.1 Conclusions

This thesis has concentrated on utilizing digital image processing techniques to reduce speckle noise and improve the image quality of ultrasound imaging systems. Two partial differential equation (PDE) based algorithms within the Laplacian pyramid framework have been proposed. These are Laplacian pyramid based nonlinear diffusion (LPND) for speckle removal, and Laplacian pyramid based nonlinear diffusion and shock filter (LPNDSF) for simultaneous speckle removal and edge enhancement.

The LPND algorithm first decomposes an image into its Laplacian pyramid domain, before applying nonlinear diffusion filtering to each detail image in the Laplacian pyramid domain. After decomposing an image into its pyramid structure of decreasing frequencies, the main noise and useful signal components of the image exist in different layers because their frequency characteristics are different. Therefore, performing spatial adaptive filtering to each detailed pyramid layer can suppress noise without degrading slow varying signal too much. For nonlinear diffusion in each detailed pyramid layer, a formula has been developed to automatically estimate the optimal gradient threshold from the image, by a variation of median absolute deviation estimator. The proposed algorithm has been validated using simulated images, phantom images, and clinical ultrasound images. It can suppress speckle effectively while preserving important features and details in ultrasound images. Compared with other speckle reduction schemes including the recently proposed speckle reducing anisotropic diffusion (SRAD) [83] and nonlinear coherent diffusion (NCD) [1], LPND performs better in terms of noise reduction and overall image quality improvement.

While in terms of edge preservation, LPND is comparable to NCD and performs better than the conventional nonlinear diffusion and SRAD. The proposed LPND is also capable of providing further improvement on the contrast-to-noise ratio than the conventional nonlinear diffusion, SRAD, and NCD.

Although LPND has demonstrated its superior performance in reducing speckle noise and preserving important image features such as edges and small structures, it has a limited capability in enhancing image edges. Conventional image enhancement algorithms can only work well on images with well defined edges. When they are applied to ultrasound images that contain heavy speckle noise, they tend to amplify the noise. To improve the image quality further, LPNDSF has been developed by utilizing another PDE based method, the shock filter, which is a classical technique for image deblurring. The proposed LPNDSF uses a coupled nonlinear diffusion and shock filtering process in the Laplacian pyramid domain. The nonlinear diffusion part in the coupled process is mainly used for speckle reduction, same as in LPND. The shock filter part is added to enhance edges, and is controlled by an edge map derived from the Gaussian pyramid. The nonlinear diffusion and shock filter is combined into a PDE and applied simultaneously to the image, and the ratio between them is dynamically adjusted according to the local gradients. Experimental results from simulated and clinical ultrasound images have shown that LPNDSF has a better edge enhancement capability than LPND while both of them are effective in suppressing speckle noise. Quantitatively, LPNDSF results in more accurate edge localization and steeper edge slopes than LPND. Qualitative studies through application of Canny edge detector on the filtered clinical images demonstrate that LPNDSF can reveal unclear edges and present more complete and smooth boundaries. No similar methods have been found in the literature to achieve speckle reduction and edge enhancement simultaneously on

ultrasound images. The NCD algorithm [1] can enhance the coherence of prominent edges when removing speckle, but its speckle reduction effect is not as thorough as the proposed LPNDSF and its edge enhancement effect is not as effective as LPNDSF. Besides, both LPND and LPNDSF achieve better performance than NCD in retaining subtle features such as small cysts and lesions in ultrasound images.

In conclusion, the proposed speckle reduction and edge enhancement methods have the potential to improve the diagnostic capability of ultrasound imaging. They are promising to help clinicians in visual diagnoses and to enhance the performance of computer-assisted tasks such as image segmentation, classification and quantitative measurements as well.

6.2 Recommendations for Further Research

More work is needed to extend the research presented here to useful real applications. Three areas of further research have been identified.

6.2.1 Further Evaluation on Clinical Ultrasound Images

The proposed algorithms have only been tested on the following limited sets of images: a series of simulated images, two image sequences from a multipurpose phantom, and six sequences of *in vivo* images from human liver, kidney, and carotid artery. More tests should be carried out with a variety of clinical ultrasound images and comparisons of the obtained results should be made with other speckle reduction techniques to validate the superiorities and also to find out the limitations, if any, of the proposed algorithms. Collaboration with clinicians is important in the evaluation process. Only with the feedback from clinicians, the usefulness of the proposed algorithms can be confirmed.

6.2.2 Extending to 3D Domain

Three dimensional (3D) medical ultrasound imaging has drawn more and more attention in recent years. Conventional 2D ultrasound imaging systems have limitations in quantifying the volume of structures of interests in human body. The 3D ultrasound imaging systems can tackle this problem. However, 3D ultrasound imaging also suffers from speckle noise, which increases the difficulty for subsequent image segmentation and volume quantification. A simple way to achieve speckle reduction and edge enhancement for a 3D image is to perform speckle reduction and edge enhancement on each 2D slice in the 3D image. But block artifacts may arise in the direction perpendicular to the slices, since no information between adjacent slices is utilized in this simple scheme. A more effective approach is to develop a 3D speckle reduction and edge enhancement method which works directly on volume data instead of slices. In this way, information between slices can be explored, and artifacts can be reduced.

A 3D speckle reduction and edge enhancement method can be developed based on the proposed 2D algorithms introduced in this thesis. As mentioned in [30], the Laplacian pyramid technique for 2D images can be extended easily to 3D volume data, by replacing 2D Gaussian lowpass filters with 3D Gaussian lowpass filters for REDUCE and EXPAND operators. Both nonlinear diffusion and shock filter are based on the well studied areas of PDE and numerical analysis, and thus not difficult to be extended to 3D domain. However, extensive experiments on clinical 3D ultrasound images need to be conducted, to validate the proposed automatic determination of the gradient threshold using a variation of MAD estimator in the 3D domain.

6.2.3 Optimization for Real-time Implementation

Since the ultimate goal is to implement the speckle reduction and edge enhancement algorithms into a real medical ultrasound system, the proposed algorithms need to be optimized for dedicated mediaprocessors or digital signal processors (DSPs) implementation at 20-60 frame/s image rates. Two schemes for mapping algorithms to mediaprocessors are introduced in [39]. The same approaches can be adopted for DSP implementations. If the proposed algorithms do not require the full processing power of an underlying mediaprocessor or DSP, they can be implemented in generic C and optimized by typical C optimizations. On the other hand, if generic C's performance is deficient for performance-critical functions, C intrinsics can be adopted instead of generic C. If the estimated ideal performance is not good enough for real-time implementation, some components of the proposed algorithms will have to be modified.

Author's Publications

Invention Disclosures:

1. Fan Zhang, Yang Mo Yoo, Liang Mong Koh, and Yongmin Kim, *Speckle reduction based on multiscale nonlinear diffusion*, ITTO ref. TD/064/05, Nanyang Technological University, Singapore.
2. Fan Zhang, Yang Mo Yoo, Liang Mong Koh, and Yongmin Kim, *Multiscale nonlinear diffusion and shock filter for simultaneous speckle reduction and edge enhancement*, ITTO ref. TD/078/05, Nanyang Technological University, Singapore.

Journal Papers:

1. Fan Zhang, Yang Mo Yoo, Liang Mong Koh, and Yongmin Kim, "Nonlinear diffusion in Laplacian pyramid domain for ultrasonic speckle reduction", *IEEE Transactions on Medical Imaging*, vol. 26, no. 2, pp. 200-211, Feb 2007.
2. Fan Zhang, Lichen Zhang, and Liang Mong Koh, "Multiscale nonlinear diffusion and shock filter for simultaneous speckle reduction and edge enhancement in ultrasound images", to be submitted to *IEEE Transactions on Image Processing*.

Conference Papers:

1. Fan Zhang and Liang Mong Koh, "A novel multiscale nonlinear diffusion method for ultrasonic speckle reduction," *Proc. of the 4th IEEE International Symposium on Signal Processing and Information Technology*, pages 34-37, Rome, Italy, Dec. 2004.
2. Fan Zhang, Yang Mo Yoo, Lichen Zhang, Liang Mong Koh, and Yongmin Kim, "Multiscale nonlinear diffusion and shock filter for ultrasound image enhancement," *Proc. of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 1972-1977, New York, Jun., 2006.

Bibliography

- [1] K. Z. Abd-Elmoniem, A. M. Youssef, and Y. M. Kadah, "Real-time speckle reduction and coherence enhancement in ultrasound imaging via nonlinear anisotropic diffusion," *IEEE Trans. Biomed. Eng.*, vol. 49, no. 9, pp. 997-1014, Sep. 2002.
- [2] A. Achim, A. Bezerianos, and P. Tsakalides, "Novel Bayesian multiscale method for speckle removal in medical ultrasound images," *IEEE Trans. Med. Imaging*, vol. 20, no. 8, pp. 772-783, Aug. 2001.
- [3] S. T. Acton, "Multigrid anisotropic diffusion," *IEEE Trans. Image Processing*, vol. 7, no. 3, pp. 280-291, Mar. 1998.
- [4] B. Aiazzi, L. Alparone, and S. Baronti, "Multiresolution local-statistics speckle filtering based on a ratio Laplacian pyramid," *IEEE Trans. Geoscience and Remote Sensing*, vol. 36, no. 5, pp. 1466-1476, Sep. 1998.
- [5] L. Alvarez and L. Mazorra, "Signal and image restoration using shock filter and anisotropic diffusion," *SIAM J. Numer. Anal.*, vol. 31, no. 2, pp. 590-605, Apr. 1994.
- [6] J. C. Bamber and C. Daft, "Adaptive filtering for reduction of speckle in ultrasound pulse-echo images," *Ultrasonics*, vol. 24, no. 1, pp. 41-44, Jan. 1986.
- [7] J. C. Bamber and J. V. Phelps, "Real-time implementation of coherent speckle suppression in B-scan images," *Ultrasonics*, vol. 29, no. 3, pp. 218-224, May 1991.
- [8] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger, "Robust anisotropic diffusion," *IEEE Trans. Image Processing*, vol. 7, no. 3, pp. 421-432, Mar. 1998.
- [9] D. Boukerroui, J. A. Noble, M. C. Robini, and M. Brady, "Enhancement of contrast regions in suboptimal ultrasound images with application to echocardiography," *Ultrasound Med. Biol.*, vol. 27, no. 12, pp. 1583-1594, Dec. 2001.

- [10] R. W. Brockett and P. Maragos, "Evolution equations for continuous-scale morphology," *Proc. IEEE ICASSP*, vol. 3, pp. 125-128, San Francisco, CA, Mar. 1992.
- [11] C. B. Burckhardt, "Speckle in ultrasound B-mode scans," *IEEE Trans. Sonics Ultrason.*, vol. 25, no. 1, pp. 1-6, Jan. 1978.
- [12] P. J. Burt and E. H. Adelson, "The Laplacian pyramid as a compact image code," *IEEE Trans. Commun.*, vol. 31, no. 4, pp. 532-540, Apr. 1983.
- [13] F. Catté, P.-L. Lions, J.-M. Morel, and T. Coll, "Image selective smoothing and edge detection by nonlinear diffusion," *SIAM J. Numer. Anal.*, vol. 29, no. 1, pp. 182-193, Feb. 1992.
- [14] S. G. Chang, B. Yu, and M. Vetterli, "Spatially adaptive wavelet thresholding with context modeling for image denoising," *IEEE Trans. Image Processing*, vol. 9, no. 9, pp. 1522-1531, Sep. 2000.
- [15] ----, "Adaptive wavelet thresholding for image denoising and compression," *IEEE Trans. Image Processing*, vol. 9, no. 9, pp. 1532-1546, Sep. 2000.
- [16] Y. Chen, S. Broschat, and P. Flynn, "Phase insensitive homomorphic image processing for speckle reduction," *Ultrasonic Imaging*, vol. 18, no. 2, pp. 122-139, Apr. 1996.
- [17] Y. Chen, R. Yin, P. Flynn, and S. Broschat, "Aggressive region growing for speckle reduction in ultrasound images," *Pattern Recognition Letters*, vol. 24, no. 4-5, pp. 677-691, Feb. 2003.
- [18] R. R. Coifman and D. L. Donoho, "Translation-invariant denoising", *Wavelets and Statistics*, A. Antoniadis, G. Oppenheim (Eds.), Lecture Notes, Springer, Berlin, Germany, pp. 125-150, 1995.

- [19] O. Coulon and S. R. Arridge, "Dual echo MR image processing using multi-spectral probabilistic diffusion coupled with shock filters," *Proc. MIUA*, London, UK, 2000.
- [20] T. Crimmins, "The geometric filter for speckle reduction," *Applied Optics*, vol. 24, no. 10, pp. 1438-1443, May 1985.
- [21] R. N. Czerwinski, D. L. Jones, and W. D. O'Brien, "Detection of lines and boundaries in speckle images—Application to medical ultrasound," *IEEE Trans. Med. Imaging*, vol. 18, no. 2, pp. 126-136, Feb. 1999.
- [22] M. N. Do and M. Vetterli, "Framing pyramids," *IEEE Trans. Signal Processing*, vol. 51, no. 9, pp. 2329-2342, Sep. 2003.
- [23] M. O'Donnell and S. D. Silverstein, "Optimum displacement for compound image generation in medical ultrasound," *IEEE Trans. Ultrason. Ferroelect. Freq. Constr.*, vol. 35, no. 4, pp. 470-476, July 1988.
- [24] D. L. Donoho, "De-noising by soft thresholding," *IEEE Trans. Inform. Theory*, vol. 41, no. 3, pp. 613-627, May 1995.
- [25] V. Dutt, *Statistical Analysis of Ultrasound Echo Envelope*, Ph.D. Dissertation, Mayo Graduate School, Rochester, MN, 1995.
- [26] V. Dutt and J. F. Greenleaf, "Ultrasound echo envelope analysis using a homodyned K distribution signal model," *Ultrasonic Imaging*, vol. 16, no. 4, pp. 265-287, Oct. 1994.
- [27] ----, "Adaptive speckle reduction filter for log-compressed B-scan images," *IEEE Trans. Med. Imaging*, vol. 15, no. 6, pp. 802-813, Dec. 1996.
- [28] I. K. Fodor and C. Kamath, "Denoising through wavelet shrinkage: an empirical study," *Journal of Electronic Imaging*, vol. 12, no. 1, pp. 151-160, Jan. 2003.

- [29] V. S. Frost, J. A. Stiles, K. S. Shanmugan, and J. C. Holtzman, "A model for radar image and its application to adaptive digital filtering of multiplicative noise," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 4, no. 2, pp. 157-166, Mar. 1982.
- [30] M. H. Ghavamnia and X. D. Yang, "Direct rendering of Laplacian pyramid compressed volume data", *Proc. IEEE Visualization Conference (VIS'95)*, pp. 192-199, Atlanta, Georgia, Oct. 29 - Nov. 03, 1995.
- [31] G. Gilboa, *Super-resolution Algorithms Based on Inverse Diffusion-type Processes*, Ph.D. Thesis, Technion - Israel Institute of Technology, Mar. 2004.
- [32] J. Goutsias and H. Heijmans, "Nonlinear multiresolution signal decomposition schemes — Part I: Morphological pyramids," *IEEE Trans. Image Processing*, vol. 9, no. 11, pp. 1862-1876, Nov. 2000.
- [33] X. Hao, S. Gao, and X. Gao, "A novel multiscale nonlinear thresholding method for ultrasonic speckle suppressing," *IEEE Trans. Med. Imaging*, vol. 18, no. 9, pp. 787-794, Sep. 1999.
- [34] T.-C. Hsung, D.P.-K. Lun, and W.-C. Siu, "Denoising by singularity detection," *IEEE Trans. Signal Processing*, vol. 47, no. 11, pp. 3139-3144, Nov. 1999.
- [35] H-C Huang, J-Y Chen, S-D Wang, and C-M Chen, "Adaptive ultrasonic speckle reduction based on the slope-facet model," *Ultrasound Med. Biol.*, vol. 29, no. 8, pp. 1161-1175, Aug. 2003.
- [36] A. K. Jain, *Fundamentals of Digital Image Processing*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1989.
- [37] Y. M. Kadah, A. A. Farag, J. M. Zurada, A. M. Badawi, and A-B. M. Youssef, "Classification algorithms for quantitative tissue characterization of diffuse liver disease from ultrasound images," *IEEE Trans. Med. Imaging*, vol. 15, no. 4, pp. 466-478, Aug. 1996.

- [38] D. Kaplan and Q. Ma, "On the statistical characteristics of log-compressed Rayleigh signals: theoretical formulation and experimental results," *J. Acoust. Soc. Am.*, vol. 95, no. 3, pp. 1396-1400, 1994.
- [39] K. Karadayi, V. Markandey, J. Golston, R. J. Gove, and Y. Kim, "Strategies for mapping algorithms to mediaprocessors for high performance," *IEEE Micro.*, vol. 23, no. 4, pp. 58-70, Jul./Aug. 2003.
- [40] M. Karaman, M. A. Kutay, and G. Bozdagi, "An adaptive speckle suppression filter for medical ultrasonic imaging," *IEEE Trans. Med. Imaging*, vol. 14, no. 2, pp. 283-292, Jun. 1995.
- [41] E. Kofidis, S. Theodoridis, C. Kotropoulos, and I. Pitas, "Nonlinear adaptive filters for speckle suppression in ultrasonic image analysis," *Signal Processing*, vol. 52, no. 3, pp. 357-372. 1996
- [42] J. I. Koo and S. B. Park, "Speckle reduction with edge preservation in medical ultrasonic images using a homogeneous region growing mean filter (HRGMF)," *Ultrasonic Imaging*, vol. 13, no. 3, pp. 211-237, July 1991.
- [43] P. Kornprobst, R. Deriche, and G. Aubert, "Image coupling, restoration and enhancement via PDE's," *Proc. IEEE ICIP*, pp. 458-461, Santa Barbara, CA, 1997.
- [44] H. P. Kramer and J. B. Bruckner, "Iterations of a non-linear transformation for enhancement of digital images," *Pattern Recognition*, vol. 7, no. 1-2, pp. 53-58, Jun. 1975.
- [45] D. T. Kuan, A. A. Sawchuk, T. C. Strand, and P. Chavel, "Adaptive noise smoothing filter for images with signal dependent noise," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 7, no. 2, pp. 165-177, Mar. 1985.
- [46] ----, "Adaptive restoration of images with speckle," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol.35, no. 3, pp. 373-383, Mar. 1987.

- [47] D. Kunz, K. Eck, H. Fillbrandt, and T. Aach, "A nonlinear multi-resolution gradient adaptive filter for medical images," *Proc. SPIE Medical Imaging*, vol. 5032, pp. 732-742, Feb. 2003.
- [48] J. S. Lee, "Digital image enhancement and noise filtering by use of local statistics," *IEEE Trans. Pattern Anal. Machine Intell.*, vol.2, no. 2, pp. 165-168, Mar. 1980.
- [49] ----, "Refined filtering of image noise using local statistics," *Comput. Graph. Image Process.*, vol. 15, no.2, pp. 380-389, Apr. 1981.
- [50] P. -C. Li and M. -J. Chen, "Strain compounding: A new approach for speckle reduction," *IEEE Trans. Ultrason. Ferroelect. Freq. Constr.*, vol. 49, no. 1, pp. 39-46, Jan. 2002.
- [51] Lindeberg, *Scale-Space Theory in Computer Vision*, Kluwer Academic Publishers, Dordrecht, Netherlands, 1994.
- [52] A. Lopes, E. Nezry, R. Touzi, and H. Laur, "Maximum a posteriori speckle filtering and first order texture models in SAR image," *International Geoscience and Remote Sensing Symposium (IGARSS' 1990)*, pp. 2409-2412, College Park, Maryland, May 20-24, 1990.
- [53] A. Lopes, R. Touzi, and E. Nezry, "Adaptive speckle filters and scene heterogeneity," *IEEE Trans. Geosci. Remote Sensing*, vol.28, no. 6, pp. 992-1000, Nov. 1990.
- [54] T. Loupas, W. N. McDicken, and P. L. Allan, "An adaptive weighted median filter for speckle suppression in medical ultrasonic images", *IEEE Trans. Circuits Syst.*, vol.36, no. 1, pp. 129-135, Jan. 1989.
- [55] A. Macovski, *Medical Imaging Systems*, Prentice-Hall, New Jersey, 1983.
- [56] S. Mallat and W. L. Hwang, "Singularity detection and processing with wavelets," *IEEE Trans. Inform. Theory*, vol. 38, no. 2, pp. 617-643, Mar. 1992.

- [57] L. Markosien, *A Motion-compensated Filter for Ultrasound Image Sequences*, Tech. Report CS-96-14, May 1996.
- [58] V. Melnik, I. Shmulevich, K. Egiazarian, and J. Astola, "Block-median pyramidal transform: Analysis and denoising applications," *IEEE Trans. Signal Processing*, vol. 49, no. 2, pp. 364-372, Feb. 2001.
- [59] ---, "Image denoising based on locally-adaptive filtering in the median pyramidal transform domain," *IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing (NSIP-01)*, Baltimore, Maryland, Jun. 2001.
- [60] O. V. Michailovich and D. Adam, "A novel approach to the 2-D blind deconvolution problem in medical ultrasound," *IEEE Trans. Med. Imaging*, vol. 24, no. 1, pp. 86-104, Jan. 2005.
- [61] P. Mrázek and M. Navara, "Selection of optimal stopping time for nonlinear diffusion filtering," *Int. J. Comput. Vision*, vol. 52, no. 2, pp. 189-203, May 2003.
- [62] P. Mrázek, J. Weickert, G. Steidl, and M. Welk, "On iterations and scales of nonlinear filters," *Computer Vision Winter Workshop 2003*, O. Drbohlav (ed.), Valtice, Czech Republic, pp. 61-66, Feb. 2003.
- [63] S. Osher and L. I. Rudin, "Feature-oriented image enhancement using shock filters," *SIAM J. Numer. Anal.*, vol. 27, no. 4, pp. 919-940, Aug. 1990.
- [64] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 12, no. 7, pp. 629-639, Jul. 1990.
- [65] A. Pizurica, W. Philips, I. Lemahieu, and M. Acheroy, "A versatile wavelet domain noise filtration technique for medical imaging," *IEEE Trans. Med. Imaging*, vol. 22, no. 3, pp. 323-331, Mar. 2003.
- [66] S. Ranganath, "Image filtering using multiresolution representations," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 13, no. 5, pp. 426-440, May 1991.

- [67] R. M. Rangayyan and A. Das, "Filtering multiplicative noise in images using adaptive region-based statistics," *Journal of Electronic Imaging*, vol. 7, no. 1, pp. 222-230, Jan. 1998.
- [68] F. Sattar, L. Floreby, G. Salomonsson, and B. Lövdström, "Image enhancement based on a nonlinear multiscale method," *IEEE Trans. Image Processing*, vol. 6, no. 6, pp. 888-895, Jun. 1997.
- [69] P. M. Shankar, "A model for ultrasonic scattering from tissues based on the K distribution," *Phys. Med. Biol.* vol. 40, no. 10, pp. 1633-1649, Oct. 1995.
- [70] ----, "A general statistical model for ultrasonic backscattering from tissues," *IEEE Trans. Ultrason. Ferroelect. Freq. Constr.*, vol. 47, no. 3, pp. 727-736, May 2000.
- [71] E. P. Simoncelli and W. T. Freeman, "The steerable pyramid: a flexible architecture for multi-scale derivative computation," *Proc. ICIP*, pp. 444-447, Oct. 1995.
- [72] A. Spira, N. Sochen, and R. Kimmel, "Geometric filters, diffusion flows, and kernels in image processing," *Handbook of Computational Geometry for Pattern Recognition, Computer Vision, Neurocomputing and Robotics*, Eduardo Bayro-Corrochano (ed.), Springer Verlag, pp. 203-230, Feb. 2005.
- [73] G. E. Trahey, J. W. Allison, S. W. Smith, and O. T. von Ramm, "A quantitative approach to speckle reduction via frequency compounding," *Ultrasonic Imaging*, vol. 8, no. 3, pp. 151-164, Jul. 1986.
- [74] M. Unser, A. Aldroubi, and M. Eden, "The L_2 polynomial spline pyramid," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 15, no. 4, pp. 364-379, Apr. 1993.
- [75] J. T. M. Verhoeven, J. M. Thijssen, and A. G. M. Theeuwes, "Improvement of lesion detection by echographic image processing: Signal-to-noise-ratio imaging," *Ultrasonic Imaging*, vol. 13, no. 3, pp. 238-251, Jul. 1991.

- [76] R. F. Wagner, S. W. Smith, J. M. Sandrik, and H. Lopez, "Statistics of speckle in ultrasound B-scans," *IEEE Trans. Sonics Ultrason.*, vol. 30, no. 3, pp. 156-163, May 1983.
- [77] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, "Image quality assessment: From error visibility to structural similarity," *IEEE Trans. Image Processing*, vol. 13, no. 4, pp. 600-612, Apr. 2004.
Available: <http://www.cns.nyu.edu/~zwang/files/research/ssim/index.html>
- [78] J. Weickert, "Coherence-enhancing diffusion of colour images," *Image and Vision Computing*, vol. 17, no. 3, pp. 201-212, Mar. 1999.
- [79] J. Weickert, B. M. ter Haar Romeny, and M. Viergever, "Efficient and reliable schemes for nonlinear diffusion filtering," *IEEE Trans. Image Processing*, vol. 7, no. 3, pp. 398-410, Mar. 1998.
- [80] R. Whitaker and S.M. Pizer, "A multi-scale approach to nonuniform diffusion," *CVGIP: Image Understanding*, vol. 57, no. 1, pp. 99-110, Jan. 1993.
- [81] H. Xie, L. Pierce, and F. T. Ulaby, "SAR speckle reduction using wavelet denoising and Markov random field modeling," *IEEE Trans. Geosci. Remote Sensing*, vol. 40, no. 10, pp. 2196-2211, Oct. 2002.
- [82] Y. Xu, J. B. Weaver, D. M. Healy, and J. Lu, "Wavelet transform domain filters: a spatially selective noise filtration technique," *IEEE Trans. Image Processing*, vol. 3, no. 6, pp. 747-758, Nov. 1994.
- [83] Y. J. Yu and S. T. Acton, "Speckle reducing anisotropic diffusion," *IEEE Trans. Image Processing*, vol. 11, no. 11, pp. 1260-1270, Nov. 2002.
- [84] Y. Yue, M. M. Croitoru, A. Bidani, J. B. Zwischenberger, and J. W. Clark, Jr., "Ultrasonic speckle suppression using robust nonlinear wavelet diffusion for LV volume quantification," *Proc. IEEE EMBS*, pp. 1609-1612, Sep. 2004.

- [85] J. Zhong, R. Ning, and D. Conover, "Image denoising based on multiscale singularity detection for cone beam CT breast imaging," *IEEE Trans. Med. Imaging*, vol. 23, no. 6, pp. 696-703, Jun. 2004.
- [86] X. Zong, A. F. Laine, and E. A. Geiser, "Speckle reduction and contrast enhancement of echocardiograms via multiscale nonlinear processing," *IEEE Trans. Med. Imaging*, vol. 17, no. 4, pp. 532-540, Aug. 1998.