GPS-based control for wheeled mobile robots with wheel skidding and slipping

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2007

Low, C. B. (2007). GPS-based for wheeled mobile robots with wheel skidding and slipping.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/3492>

<https://doi.org/10.32657/10356/3492>

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GPS-based Control for Wheeled Mobile Robots with Wheel Skidding and Slipping

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School of Electrical and Electronic Engineering

A thesis submitted to Nanyang Technological University
in fulfillment of the requirements for the degree of
Doctor of Philosophy

2007

Dedicated to

My dear wife Liza and little daughter Gladys

Acknowledgements

I would like to express my deeply gratitude to my supervisor, Associate Professor Wang Dan Wei, for his invaluable and patient guidance throughout the study. His discussions on every issue were always helpful and put forward the way to the solutions. I am also greatly indebted to School of Electrical and Electronic Engineering, Division of Control and Instrumentations for the provision of budgets and equipments to the research project and of the scholarship for the author.

Many thanks are given to the colleagues: Dr. Pham Minh Tuan, as well as the technician in M&E Workshop: Mr. Tan Chai Soon. They were always being around for constructive discussions on theoretic issues and actual hands-on work on repairing and maintaining the vehicle and carrying out experiments.

Thank you all for being with me, always.

Summary

This thesis addresses the development and implementation of wheeled mobile robot controllers in the presence of wheel skidding and slipping using a Real-Time Kinematic Global Positioning System (RTK-GPS) and other aiding sensors. Critical issues are studied including theoretical development, simulation study and experimental implementation for the proposed controllers.

This thesis starts off with a comprehensive review of previous work related to wheeled mobile robot control in the presence of wheel skidding and slipping. The review is divided into three parts: (i) modeling of wheeled mobile robots with wheel skidding and slipping; (ii) applications of GPS for vehicles, in particular perturbation estimation and measurements; (iii) wheeled mobile robot controllers with wheel skidding and slipping. Subsequently, the thesis presents the following four main parts:

- **Modeling of wheeled mobile robots in the presence of wheel skidding and slipping:**

Kinematic models for four generic wheeled mobile robots are developed to give a general and unified presentation on modeling of wheeled mobile robots in the presence of wheel skidding and slipping from the perspective of control design. The skidding and slipping effects are explicitly described by physically appealing descriptions which lead to the classifications of the perturbations. These classifications reveal the difficulty levels of compensating for the perturbations while controlling the robots. Additionally, the explicit descriptions lead to practical control strategy to solve the mobile robot control problems. We also relate the mobile robot's maneuverability with the robot's controllability which provides a measure of the robot's ability to track a trajectory in the presence of wheel skidding and slipping. These insights lay a foundation for control problem formulation and the deployment of various control design techniques to overcome the addressed perturbations.

- **Posture, velocity and perturbation estimation for control:** Estimators are developed to provide high-update rate mobile robot posture, velocities, and perturbation estimates

for wheeled mobile robot control in the presence of wheel skidding and slipping. The estimators are based on Kalman Filters which combine the inertial sensors' measurements with centimeter accuracy RTK-GPS measurements to provide the robot's posture, velocities, and perturbation information. These estimators are applicable to the four mobile robots which are defined in Chapter 3 and are also able to provide reliable information at high-update rate during short occasional GPS outages.

The proposed estimation algorithms are implemented on a car-like wheeled mobile robot. A high-performance RTK-GPS and low-cost inertial sensors are mounted on the mobile robot for experiments. The experimental results suggest that with the careful modelling of mobile robots, the estimators are able to provide reliable and high-update rate information for mobile robot control applications in the presence of wheel skidding and slipping.

- **Path following control for wheeled mobile robots:** GPS-based path following control algorithms based on backstepping techniques are proposed for the four generic wheeled mobile robots with wheel skidding and slipping. The proposed control scheme exploits RTK-GPS and other aiding sensors to estimate/measure a mobile robot's posture, velocities, and perturbations in the presence of wheel skidding & slipping. This information is applied to compensate the path following errors based on the backstepping controllers. We address the local path following problem where initial path following errors are sufficiently small. Additionally, we consider the conditional global path following problem where initial errors are less restrictive compared with the local controller.

The proposed control laws are simulated using MATLAB SIMULINK. The reported simulation results confirm the effectiveness of the control scheme. Additionally, the control scheme is implemented on the car-like mobile robot. The experimental results show the effectiveness of the path following controllers.

- **Tracking control for wheeled mobile robots:** GPS-based tracking control algorithms based on backstepping techniques are proposed for the mobile robots to solve trajectory tracking problems in the presence of wheel skidding and slipping. Similarly, the control scheme uses RTK-GPS and other aiding sensors to estimate/measure a mobile robot's posture, velocities, and perturbations which are applied to compensate the tracking errors. We address the tracking problem where initial tracking errors are sufficiently small. The proposed tracking control laws are simulated using MATLAB SIMULINK. The sim-

ulation results validate the effectiveness of the control scheme. The tracking controller is also implemented on the car-like mobile robot and the experimental results validate the effectiveness of the GPS-based tracking controllers.

Finally, recommendations are given for future work on wheeled mobile robot control in the presence of wheel skidding and slipping.

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List of Symbols

General Notations

$\|\cdot\|$: Euclidean norm.

$|\cdot|$: absolute value of a real number.

$[g_1, g_2]$: Lie bracket of vector field g_1 and g_2 .

$\triangle$: distribution.

$\triangle_{\mathcal{C}}$: accessibility distribution.

I_2 : 2 by 2 identity matrix.

$L_g h(q)$: Lie derivative of function $h(q)$ along vector field g .

R^n : n- dimensional Euclidian space.

$R(\theta)$: rotation matrix.

Notations of Mobile Robot

θ : orientation angle of a mobile robot.

V : velocity of a mobile robot.

η : velocity of a mobile robot in body frame, a 2-dimensional vector of (V_l, V_y) .

V_l : linear velocity of a mobile robot.

V_y : lateral velocity of a mobile robot.

ψ : orientation of velocity V .

q : posture of a mobile robot, a 3-dimensional vector of (x, y, θ) .

ξ : position of a mobile robot in Cartesian coordinate, a 2-dimensional vector of (x, y) .

γ_i : the i -th independent orientation input of a mobile robot.

δ_i : the i -th perturbation a mobile robot.

d : longitudinal slipping perturbation of a mobile robot.

$\{\triangle^a, \triangle^b, \triangle^c, \triangle^d, \triangle^e\}$: generalized perturbations of a mobile robot.

$r_w \dot{\varphi}$: velocity control input of a mobile robot.

U : control input of a mobile robot.

r_w : free rolling radius of a wheel.

m : mobility of a mobile robot.

s : steerability of a mobile robot.

x : coordinate of a mobile robot.

y : coordinate of a mobile robot.

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$\{X_b, Y_b\}$: body axes of a mobile robot.

$\{X, Y\}$: axes of global coordinate frame.

Notations of Estimation and Measurement System

$u_{\bar{a}}$: acceleration of point P in robot's body coordinates.

$\ddot{\xi}$: acceleration of point P in global coordinates.

u_a : accelerometer measurements: a 2-dimensional vector of (a_x, a_y) .

a_x : accelerometer measurement along X_b axis.

a_y : accelerometer measurement along Y_b axis.

r : robot's turning rate.

r_m : gyroscope rate measurement.

Δt : discrete time sampling period.

w_a : accelerometer measurement noise: 2-dimensional vector.

w_r : gyroscope measurement noise.

$\{\mathbf{x}, \mathbf{X}\}$: system states

$\{\hat{\mathbf{x}}, \hat{\mathbf{X}}\}$: system state estimates

A : 2 by 2 Transformation matrix from body frame to global frame.

z_x : RTK-GPS positioning measurement in X coordinate.

z_y : RTK-GPS positioning measurement in Y coordinate.

$z_{\dot{x}}$: RTK-GPS velocity measurement along X axis.

$z_{\dot{y}}$: RTK-GPS velocity measurement along Y axis.

z_{θ} : absolute orientation measurement.

v_x : measurement noise of z_x .

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v_y : measurement noise of z_y .

v_θ : measurement noise of z_θ .

$v_{\dot{x}}$: measurement noise of $z_{\dot{x}}$.

$v_{\dot{y}}$: measurement noise of $z_{\dot{y}}$.

$\{\mathbf{z}, \mathbf{Z}\}$: measurement vectors.

σ_x^2 : variance of measurement noise of z_x .

σ_y^2 : variance of measurement noise of z_y .

$\sigma_{\dot{x}}^2$: variance of measurement noise $z_{\dot{x}}$.

$\sigma_{\dot{y}}^2$: variance of measurement noise $z_{\dot{y}}$.

σ_θ^2 : variance of measurement noise z_θ .

$\mathbf{f}$: discrete state transition vector.

$\{\mathbf{w}, \mathbf{W}\}$: process noise vectors.

$\{\mathbf{v}, \mathbf{V}\}$: measurement noise vectors.

$\mathbf{h}$: measurement vector equation.

$\mathbf{Q}$: process noise covariance matrix.

$\mathbf{R}$: measurement noise covariance matrix.

$\mathbf{P}$: error covariance matrix.

$\mathbf{K}$: Kalman filter gain matrix.

Φ : discrete-time transition matrix.

α : residual vector.

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$\nabla \mathbf{f}$: jacobian matrix of vector field $\mathbf{f}$.

$\nabla \mathbf{h}$: jacobian matrix of vector field $\mathbf{h}$.

$\mathbf{A}_i$: transition matrix of system i .

$\mathbf{B}_i$: input matrix of system i .

$\mathbf{C}_i$: observation matrix of system i .

$\mathbf{u}_i$: input vector of system i .

Chapter 1

Introduction

1.1 Background

Wheeled Mobile Robots (WMRs) have been an active area of research and development over the past decades and these robots are increasingly present in our modern society. The reasons for this long-term interest are the numerous practical applications that can be uniquely addressed by mobile robots due to their ability to work in large domains. In particular, wheeled mobile robots have been put into service for applications such as: i) transportation of materials, ii) mine excavation, iii) planetary exploration, and iv) military tasks. Based on the wide-range of applications described above, it is clear that wheeled mobile robots research is multidisciplinary and challenging by nature.

Some of the earliest autonomous wheeled mobile robots were the Stanford Cart built in 1979 and CMU rover built in 1983 [1]. Although these wheeled mobile robots are primitive, they place a golden landmark in the history of wheeled mobile robots. Wheeled mobile robots can be classified into several mobility configurations. The most common configuration for single-body wheeled mobile robots are differential drive and syncho-drive, tricycle or car-like, and omnidirectional wheeled mobile robots. In general, mobile robots can be divided into restricted and unrestricted robots based on the mobility of the robot. This measure suggests that restricted mobile robots (unicycle and car-like mobile robots) are more difficult to maneuver compared with the unrestricted robot (Omnidirectional mobile robot). A detailed study on these wheeled mobile robots is found in [2].

The multidisciplinary nature of wheeled mobile robots leads to several theoretical and practical developments that involve in addressing key issues of wheeled mobile robots such as nav-

igation, sensors and sensing technologies, and mobile robot controls [3–9]. These issues are closely-related to accomplish a robot’s designated task. Among these issues, the wheeled mobile robot controls can be regarded as one of the most challenging tasks. In general, wheeled mobile robot control problems can be classified into 3 categories as follows [4].

1. **Trajectory tracking:** Trajectory refers a geometric path with associated time law. The trajectory tracking problem is to control a wheeled mobile robot to reach and follow a trajectory in the Cartesian space from a given initial configuration.

Several researchers have examined the trajectory tracking control problem. One of the preliminary kinematic tracking controller is proposed in [10] based on approximate linearization. Approximate linearization technique was also applied along with linear quadratic regulator to solve the tracking problem in [11]. Although these results are local controllers, they show that the well-known Brockett condition is avoided in trajectory tracking problem. Not long after these works, control laws based on static and dynamic feedback linearization were reported in [12–16] to solve the tracking problem. Although this method achieves exponential convergence rate to the desired trajectory for both kinematic models and dynamics models, the dynamic feedback linearization control scheme is singular when at zero velocity. Recently, feedback linearization approach with stable internal dynamics is proposed to solve the full-state tracking problem [17]. In [18], backstepping technique is applied to achieve global tracking result for both kinematic and dynamics model. The method is extended to chained form in [19]. Neural network is also introduced to solve the tracking problem [20]. Recently, many researchers turn their attention to solve the tracking problem with parametric and nonparametric uncertainties associated with the mobile robot mechanical dynamics models. Control methods such as Adaptive control [21–23] and Robust control [24–26] are utilized to address the control problems.

2. **Path following:** Path here means a geometric path that is independent of time law. Path following problem is to control a wheeled mobile robot to reach and follow a path in the Cartesian space from a given initial configuration.

Without timing from the trajectory, the path following problem can be treated as a special case of trajectory tracking problem [18] in the sense that only lateral dynamics is relevant while the longitudinal dynamics can be neglected. In the path following problem, the wheeled mobile robot’s motion is modelled in term of path coordinates (arc length of path,

distance between the robot and the path, angle between the tangent to the path and the orientation of the robot). Clearly, the path following problem is less stringent compared with the trajectory tracking problem. Nevertheless, the solutions to the trajectory tracking and path following problems are practically appealing.

Many control methods were also studied to solve the path following problem. In [27,28], the problem is solved by Input-output feedback linearization technique. Linear control design via approximate linearization is applied to overcome the path following problem for small initial errors [3]. The small initial errors constraint is overcome by nonlinear control design [29]. Some stabilization control scheme [30] can also be used to solve the path following problem with practical implementation on a farm tractor [31]. Similarly, robust control of path following was investigated [32–35].

3. **Stabilization (Parking):** Stabilization problem is to control a wheeled mobile robot to reach a desired goal configuration from a given initial configuration.

Unlike the trajectory tracking and the path following problems, the stabilization problem is theoretically challenging; however, it is practically less meaningful compared with the preceding control problems. The stabilization problem is challenging because it cannot be solved by a smooth, time-invariant state feedback law due to the implications of Brockett's conditions [36]. In order to overcome this technical hurdle, sophisticated feedback control laws are applied to solve this problem. These control methods are continuous time-varying feedback laws, piecewise continuous control laws, and discontinuous control laws.

The application of continuous time-varying feedback was illustrated in [37], while [38] proposed an explicit time-varying control laws to stabilize a general class of driftless nonlinear system; however, these control laws have slow convergence rate. To enhance the transient performance, ρ -exponentially convergence control laws were proposed in [39,40] to stabilize classes of nonholonomic systems where wheeled mobile robots are a special case of these systems. In [41], a differentiable, time-varying controller was proposed to achieve global exponential stabilization for wheeled mobile robots. Discontinuous controllers [42–44] were developed to solve the stabilization problem with exponential convergence. Piecewise continuous time-varying method was also examined in [45]. Recently, controllers that simultaneously address the stabilization and trajectory

tracking problems were reported in [5, 46–49].

All above-mentioned works assumed the wheels of the mobile robots satisfy the *non-skidding* and *non-slipping* assumptions. These assumptions mean that the velocity of any robot's wheel has no component perpendicular to the wheel's plane, and the wheel's linear velocity is the product of wheel's free rolling radius and the wheel's angular velocity. In practice, these assumptions do not hold due to tire-deformation. Consequently, the performance and stability of these works are not guaranteed. In this thesis, the focus is to develop wheeled mobile robot controllers to solve the trajectory tracking and path following problems in the presence of wheel skidding and slipping. The following section conducts a literature review with focus on the mobile robots with wheel skidding and slipping. Related issues such as wheeled mobile robot modeling, perturbation estimation, and control in the presence of wheel skidding and slipping will be discussed.

1.2 Further Literature Survey

1.2.1 Modeling

One of the earliest kinematic model of mobile robot with wheel skidding effect was reported in [50] for a Type (2,0) unicycle wheeled mobile robot. The kinematic model was proposed for control design. In the model, perturbation due to wheel skidding is represented by general bounded term that possess no explicit physical meaning. Consequently, the lack of physical understanding resulted conservative robust control laws for the mobile robot to solve the stabilization problem. In [51], general kinematic and dynamic models were developed to study the effect of the wheel skidding and slipping on wheeled mobile robots using Singular perturbation analysis theory. The models were proposed to analyze the stability of the feedback linearization controller [15]. The analysis shows that the feedback control system remains stable for very mild wheel skidding and slipping. The dynamic model however suffers from some drawbacks. First, the model relies on accurate inertial parameters and a factor ϵ which are difficult to obtain in practice. Second, the models that describe the motion of the mobile robots are represented by general terms named pseudo-states. These representations are similar to the general perturbation representation used in [50] in the sense that they are derived from a pure mathematical approach which obscures useful explicit physical description. As a result, these models do not

provide useful insights from control perspective; therefore, these models are undesirable for control purposes.

Other dynamic models were also reported to study the motion of the unrestricted Omnidirectional mobile robot in the presence of wheel slip [52,53]. In [53], perturbations due to wheel longitudinal slippage are modelled using tire's adhesion coefficient and wheel slip information to enhance the robot's tracking performance. Another dynamic model was also proposed for the Omnidirectional mobile robot [52]. The model was not developed for real-time control purposes, but is to assist in understanding the wheel sliding dynamics. In contrast to the models proposed in [50,51], wheel skidding effect is not considered in these dynamic model. Additionally, these models were only for the unrestricted Omnidirectional mobile robot which is technically less challenging than other restricted mobile robots, e.g., Type (2,0), Type (1,1), Type (1,2), Type (2,1) configurations.

Recently, the path following problem with wheel skidding is studied based on a path following model of a farm tractor (a Type (1,1) mobile robot) [54,55]. The model is used for perturbation estimation and control design. Unfortunately, the path following model is restricted to path following control problem and it does not provide useful insights for control purposes. Moreover, wheel slippage is not considered in the problem. As a result, the perturbation estimator design based on the model is inefficient. Moreover, control designs that based on the path following model impose restrictive assumption on the skidding perturbations.

In this thesis, a general and unifying kinematic models were proposed for the four generic wheeled mobile robots with wheel skidding and slipping from the perspective of control design. These models relate the perturbations with physically meaningful descriptions to provide useful insights for control purposes. Most importantly, these models suggest that these perturbations can be measured using exteroceptive sensors, e.g., GPS. This finding reveals the possibility of using GPS to address the mobile robots control problems in the presence of wheel skidding and slipping.

1.2.2 GPS and INS integration

Global positioning system (GPS) provides positioning and velocity information at places on or near the surface of the earth where the GPS receiver maintains consistent lines of sight with GPS satellites. In the past decades, GPS has undergone intensive research and development to improve the system's reliability and accuracy. In fact, Real-time Kinematic GPS (RTK-GPS),

which is also known as Carrier-Phase Differential GPS has been shown to provide a positioning accuracy of about 2cm. Beside accurate positioning information, GPS can also provide velocity measurements with a typical accuracy of 0.05m/s. The ability to measure position and velocity with such high accuracy led to many advances in land, marine, and air navigation systems.

RTK-GPS has been exploited in outdoor automated vehicles to increase the efficiency and productivity in several applications. One example for such applications is the automated farming vehicle [31, 56, 57]. Unfortunately, GPS system suffers from several drawbacks. First, the system requires consistent lines of sight (LOS) between the GPS receiver and the GPS satellites to measure the receiver's position and velocity. Any obstacle that blocks these lines of sight will cause temporary loss of GPS satellite signals. Second, intermittent loss of communication can occur between the vehicle and the GPS base station if the radio communication is blocked by an obstacle. Third, GPS measurement has a typical low updating rate of 1-10Hz. In order for GPS-based control vehicles to gain major acceptance, the control system must not be halted during such outages. Additionally, the GPS-based positioning system of the vehicle must be able to provide sufficiently high-update rate information for high-speed maneuvers. A review of navigation literature shows that GPS and INS have complementary properties and they can be integrated to overcome these issues [56, 58–62]. Most importantly, these sensors provide measurements that are not corrupted by wheel skidding and slipping.

In [63], inertial sensors are utilized to decouple a car's skidding effect from the human driver. This paper not only demonstrated the importance of considering tire-slip angle in controlling a vehicle, but also illustrated the concept of using inertial sensors to address the skidding problem. Recently, much attention has been given to electric vehicle with four independently driven motors [64–67]. Similarly, inertial sensors are utilized to achieve stable vehicle side-slip control in the presence of wheel skidding. However, these works [63, 66, 67] require accurate knowledge of vehicle's mass and location of the vehicle's center of gravity. Fortunately, these issues can be avoided using GPS. In [68, 69], researchers utilize velocity measurements provided by a standalone GPS to measure the slip angles of an automobile for vehicle stability control application. Recently, this concept has been enhanced by the combination of GPS and INS to overcome the basic limitations of a GPS (e.g., GPS outages and low-update rate) [70, 71].

Absolute positioning information with respect to a reference initial navigation frame is not required in vehicle stability control applications; however, for wheeled mobile robot control applications with wheel skidding and slipping, we require high-update rate robot's positioning,

velocities, and perturbation information to design a non-conservative control laws. In [72], a navigation system based on laser scanner and inertial navigation system was proposed for an autonomous underground load, haul, and dump truck. In the work, the vehicle's wheel skidding perturbations are modelled as random white noise which may not be sufficiently accurate in representing the wheel skidding parameters.

RTK-GPS has also been used to compute the skidding perturbation of a farm tractor for path following control. Estimation algorithms were proposed based on the path following models developed in [54, 55]. Unfortunately, these estimators suffer from several deficiencies. First, they utilized the parameterized path following model for computing the perturbations. The computations require accurate instantaneous lateral deviation and curvilinear information. A mild error in the positioning measurement could possibly lead to an erroneous and noisy estimates. Moreover, these estimators are restricted to path following problem [54] and the update rates of these estimators are limited by the low-update rate GPS measurements. The perturbation estimator should be developed based on a wheeled mobile robot's kinematic model rather than the restrictive path following models since the skidding parameters are properties of the wheeled mobile robot and should be independent of any mobile robot control problem.

1.2.3 Wheeled Mobile Robot Control

In order to fully utilize a mobile robot, it is necessary to accurately control the robot. The ability for precise maneuver leads to advances in autonomous technology.

Several researchers have proposed controllers for the Type (2,0) mobile robot based on the kinematic model developed in [50] to address the wheel skidding problem. Under the assumption of the general and unknown perturbation being state-vanishing, an exponentially stable robust stabilizing controller was proposed [50]. Later the kinematic model was used in [26, 73] and uniformly boundedness solutions were proposed for stabilization and tracking problems without assuming the perturbation to be constant or state-vanishing. In [74], Dixon *et al.* extended their previous work [5] to address the wheel skidding by designing robust tracking and regulation controllers using the kinematic model. Recently, robust control design is also applied to path following and trajectory tracking problems of a farm-tractor [75, 76] based on the path following model developed in [54, 55]. These controllers offer uniformly boundedness solutions; however, if a high-precision control performance is desirable, these control laws would require high magnitude and fast switching control inputs. These methods can be constrained by

implementation and mechanical issues.

In [77, 78], controllers were proposed based on the dynamic model developed in [51] to address the tracking control problem with wheel skidding and slipping. A linear time varying controller was proposed to achieve uniform locally exponential stability for the Type (2,0) uni-cycle based on the dynamic model [77]. In [78], the model is applied to design a slow manifold controller to solve an output-tracking problem. Unfortunately, these dynamic models rely on the accurate inertial parameters and the factor ϵ which are not easily obtainable in practice.

RTK-GPS has also been widely used to address the path following problem of a farm tractor in the presence of wheel skidding effects [54, 55, 79, 80]. In these works, farm tractor's skidding perturbations are determined based on the path following model and RTK-GPS positioning measurements. The controllers compensate the path following errors based on these measurements. As we have fore-shadowed in the previous section, the approach of perturbation estimation suffers from several deficiencies. Additionally, these control laws assumed the skidding perturbations are time-invariant and the wheel longitudinal slippage is not considered.

1.3 Objectives and Contributions of This Thesis

In order for wheeled mobile robots to become widespread, they must be able to perform precise maneuver. Therefore, the objective of this thesis was to improve the capability of wheeled mobile robot controller using RTK-GPS in outdoor environments. These works encompass developments of new fundamental models and estimation techniques. Control algorithms that target to achieve precise trajectory tracking and path following were also proposed.

- **Modeling and Analysis of Wheeled Mobile Robots with skidding and slipping:** A general and unifying presentation on modeling of four generic wheel mobile robots with wheeled skidding and slipping was proposed from the control design perspective. We present kinematic models that explicitly relate the wheel skidding and slipping to vehicle perturbations. The perturbations are categorically classified as input additive, input multiplicative, and/or matched/unmatched perturbations. We also relate a mobile robot's maneuverability with the robot's controllability to provide a measure on the robot's ability to track a trajectory in the presence of wheel skidding and slipping. We found that a mobile robot with higher maneuverability (Type (1,2), Type (2,1)) has the ability to achieve better control performance than the lower maneuverability robots (Type (2,0),

Type (1,1)). Most importantly, these kinematic models reveal the potential of GPS in solving the mobile robots control problems in the presence of wheel skidding and slipping. These classifications and physical understanding of the perturbations are exploited in determining new higher limits on control performance. This work is presented in Chapter 3 based on [81] and [82].

- Integrated Estimation of Wheeled Mobile Robot's posture, velocity, and wheel skidding and slipping perturbations :** Unified kinematic estimators were proposed to provide reliable and high-update rate posture, velocities, and perturbations information for wheeled mobile robot control in the presence of wheel skidding and slipping effect. These estimators are applicable to the four generic wheeled mobile robots. Accurate measurements from a RTK-GPS receiver are blended with the high-update rate inertial sensors measurements based on the kinematic models and Kalman filtering. The estimators provide the ability to dead reckon using inertial measurements through short GPS outages. An experimental platform has been setup using a Type (1,1) car-like robot named Cycab. The platform carries a RTK-GPS and inertial sensors to study the estimation algorithms. The experimental outcomes show the effectiveness of the estimation scheme. This work is presented in Chapter 4 based on [83].
- GPS-based Path following control laws:** GPS-based path following control laws based on backstepping techniques were developed for the four generic wheeled mobile robots in the presence of wheel skidding and slipping. The proposed control scheme uses the robot's posture, velocities, and perturbations estimates/measurements determined by the sensors to compensate the path following errors. We first address the local path following problem where initial path following errors are sufficiently small. Additionally, we consider the path following problem where initial errors are less conservative. Sufficient condition is developed to achieve zero-converging lateral error and well-behaved orientation error for Type (2,0) and Type (1,1) mobile robots. On the other hand, sufficient condition for zero-converging lateral and orientation errors for Type (1,2) and Type (2,1) mobile robots is derived. These results illustrated mobile robots with higher maneuverability can achieve better control performance than the lower maneuverability mobile robots. The proposed control laws are simulated using MATLAB SIMULINK. The reported simulation results confirm the effectiveness of the controllers. Experiments were conducted based on the Type (1,1) experimental platform to evaluate the controllers. The

experimental results validate the control laws. This work is presented in Chapter 5 based on [83] and [84].

- **GPS-based Trajectory Tracking control laws:** GPS-based tracking controllers based on backstepping techniques were proposed for the mobile robots with wheel skidding and slipping. The developed control laws exploit the mobile robot's posture, velocities and perturbations estimates/measurements to achieve precision tracking. We address the tracking problem for the mobile robots where initial tracking errors are sufficiently small. Similarly, sufficient condition is developed to achieve zero-converging point tracking error and well-behaved orientation error for Type (2,0) and Type (1,1) mobile robots. Sufficient condition for Type (1,2) and Type (2,1) mobile robots is also derived for zero-converging point tracking and orientation errors. These results also demonstrated mobile robots with higher maneuverability achieve better tracking performance than the lower maneuverability mobile robots. These tracking control laws are simulated using MATLAB SIMULINK. Simulation results confirm the tracking control laws. The controller is also implemented on the experimental platform. The experimental results demonstrate the effectiveness of the GPS-based tracking controller. This work is presented in Chapter 6 based on [85] and [86].

1.4 Organisation of This Thesis

The rest of this thesis is organized as follows

- **Chapter 2:** Mathematical tools are briefly reviewed with the theories in controllability and stability of nonlinear systems.
- **Chapter 3:** Kinematic models of wheeled mobile robots with wheel skidding and slipping are developed for the four generic mobile robots. These kinematic perturbations are classified and several useful insights are gained from control perspectives. Additionally, we relate the maneuverability of each mobile robot with its controllability which provides a measure on the robot's ability to track a trajectory in the presence of wheel skidding and slipping.
- **Chapter 4:** Estimators are presented to estimate the mobile robot's posture, velocity and perturbations based on RTK-GPS and inertial sensors measurements. Experiments are

performed on a Type (1,1) car-like platform. Experimental results and discussions are included in the chapter.

- **Chapter 5:** Path following control problems are formulated based on the maneuverability of the four mobile robots. A set of GPS-based path following control laws are developed for the mobile robots. Local and conditionally global control laws are developed. Simulation results are conducted to validate the control laws. In addition, control laws are implemented on the Type (1,1) car-like platform. The experimental results are discussed in the Chapter.
- **Chapter 6:** Trajectory tracking problems are formulated based on the maneuverability of the mobile robots. A set of GPS-based tracking control laws are developed for the mobile robots. Simulation results are conducted to validate the control laws. Similarly, the tracking control laws are implemented on the experimental platform and experimental results are discussed in the chapter.
- **Chapter 7:** Conclusions are given and future research works are recommended.

Chapter 2

Mathematical Preliminaries

2.1 Introduction

In this chapter, we briefly summarize some useful concepts and results from controllability and stability theory of general nonlinear systems. For simplicity of the presentation, no proof is given. For details, one may refer to the references [4, 87–89].

2.2 Differential Geometry

For simplicity, we work with vectors $x \in R^n$, and denote the tangent space of R^n at x by $T_x(R^n)$. A smooth vector field $g : R^n \rightarrow T_x(R^n)$ is a smooth mapping assigning to each point $x \in R^n$ a tangent vector $g(x) \in T_x(R^n)$.

The *Lie derivative* of a scalar function $h : R^n \rightarrow R$ along g is defined as

$$L_g h(x) = \frac{\partial h}{\partial x} g(x). \quad (2.1)$$

Given two smooth vector fields g_1 and g_2 , we define *Lie bracket* as the new vector field $[g_1, g_2]$

$$[g_1, g_2] = \frac{\partial g_2}{\partial x} g_1(x) - \frac{\partial g_1}{\partial x} g_2(x). \quad (2.2)$$

A *Lie product* is nested of Lie brackets, e.g.,

$$[[g_1, g_2], [[g_1, [g_1, g_2]]]. \quad (2.3)$$

A smooth *distribution* Δ on R^n associated with m smooth vector field $\{g_1, \dots, g_m\}$ is the map that assigns to each point $x \in R^n$ a linear subspace of its tangent space, i.e.,

$$\Delta(x) = \text{span}\{g_1(x), \dots, g_m(x)\} \subset T_x(R^n). \quad (2.4)$$

Distribution Δ is said to be *nonsingular* if $\dim \Delta(x) = r$, constant for all x . In this case, r is called the *dimension* of the distribution.

2.3 Controllability

We now restrict our attention to control systems of the form

$$\dot{x} = f(x) + \sum_{i=1}^m g_i(x)u_i \quad x \in R^n, \quad u \in R^m \quad (2.5)$$

$f, g_1, \dots, g_m$ are smooth linearly independent vector fields on R^n . System (2.5) is affine in inputs. f is the *drift vector field* and $g_1, \dots, g_m$ are the *input vector fields*. We wish to determine conditions under which we can steer system from initial state $x_0 \in R^n$ to final state $x_f \in R^n$ by appropriate choice of piecewise continuous input $u_i(\cdot)$ for $i \in \{1, \dots, m\}$.

System (2.5) is *completely controllable*, if for any choice of $x_0, x_f \in R^n$, there exists a finite time $T > 0$ and input $u : [0, T] \rightarrow U$ such that system (2.5) satisfies $x(T, 0, x_0, u) = x_f$. This definition of controllability of nonlinear systems is difficult to proof using general conditions. To make it easily checkable, we express the system controllability in another way. We introduce $\mathcal{R}^\mathcal{V}(x_0, T)$ as set of states reachable at time T from x_0 with trajectories contained in the neighborhood $\mathcal{V}$ of x_0 for $t \geq T$. We also define

$$\mathcal{R}_T^\mathcal{V}(x_0) = \bigcup_{\tau \leq T} \mathcal{R}^\mathcal{V}(x_0, \tau) \quad (2.6)$$

which is the set of states reachable within time T from x_0 with trajectories contained in the neighborhood $\mathcal{V}$.

Then the control system (2.5) is called

- locally accessible* from x_0 if, for all neighborhoods $\mathcal{V}$ of x_0 and all T , $\mathcal{R}^\mathcal{V}(x_0)$ contains a non-empty open set.
- small-time locally controllable* from x_0 if, for all neighborhood $\mathcal{V}$ of x_0 and all T , $\mathcal{R}^\mathcal{V}(x_0)$ contains a non-empty neighborhood of x_0 .

The *accessibility algebra* $\mathcal{C}$ of the control system (2.5) is the smallest subalgebra of $\mathcal{V}(R^n)$ that contains $f, g_1, \dots, g_m$. Note that all the repeated Lie brackets of these vector fields also belong to $\mathcal{C}$. The accessibility distribution $\Delta_{\mathcal{C}}$ of system (2.5) is defined as

$$\Delta_{\mathcal{C}} = \text{span}\{v | v \in \mathcal{C}\} \quad (2.7)$$

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The computation of $\dim \Delta_C$ may be organized as an iterative procedure:

$$\Delta_C = \text{span}\{v | v \in \Delta_i, \forall i \geq 1\}, \quad (2.8)$$

with

$$\Delta_1 = \text{span}\{f, g_1, \dots, g_m\} \quad (2.9)$$

$$\Delta_i = \Delta_{i-1} + \text{span}\{[g, v] | g \in \Delta_1, v \in \Delta_{i-1}\}, i \geq 2. \quad (2.10)$$

The above procedure stops after κ steps, where κ is the smallest integer such that $\Delta_{\kappa+1} = \Delta_\kappa = \Delta_C$. Since $\dim \Delta_C \leq n$ necessarily, it follows that one stops after at most $n - m$ steps.

The following sufficient condition utilizes the accessibility distribution for local accessibility and controllability test which is stated as:

Theorem 2.3.1 [4] If the accessibility rank condition

$$\dim \Delta_C(x_0) = n \quad (2.11)$$

holds, then the control system (2.5) is locally accessible from x_0 . If the accessibility rank condition holds for all $x \in R^n$, the system is locally accessible. Conversely, if system (2.5) is locally accessible, then $\dim \Delta_C = n$ holds in an open and dense subset of R^n .

Moreover, if Theorem (2.3.1) is applied to a driftless control system, i.e. $f = 0$,

$$\dot{x} = \sum_{i=1}^m g_i(x)u_i \quad x \in R^n, \quad u \in R^m \quad (2.12)$$

it provides a sufficient condition for controllability.

Local controllability is not the same as complete controllability. However, for driftless system, the accessibility rank condition implies complete controllability by the following Theorem.

Theorem 2.3.2 [89] Suppose that $f = 0$ in (2.5), and let the set of vector fields

$$\mathcal{F} = \left\{ \sum_{i=1}^m g_i u_i : u = [u_1, \dots, u_m]^T \in U \right\}$$

be symmetric, i.e., $v \in \mathcal{F} \Rightarrow -v \in \mathcal{F}$ Then

1. If $\dim \Delta_C(x_0) = n$, then the system (2.5) is small-time locally controllable from x_0 .
2. If $\dim \Delta_C(x) = n$ for all $x \in R^n$, then the system (2.5) is completely controllable.

2.4 Lyapunov Stability

This section lists some results for stability of non-autonomous system. Consider the non-autonomous system

$$\dot{x} = f(t, x) \quad (2.13)$$

where $f : [0, \infty) \times D \rightarrow R^n$ is piecewise continuous in t and locally Lipschitz in x on $[0, \infty) \times D$, and $D \subset R^n$ is a domain that contains the origin $x = 0$.

Theorem 2.4.1 [88] Let $x = 0$ be an equilibrium point for (2.13) and $D \subset R^n$ be a domain containing $x = 0$. Let $V : [0, \infty) \times D \rightarrow R$ be a continuous differentiable function such that

$$k_1 \|x\|^a \leq V(t, X) \leq k_2 \|x\|^a \quad (2.14)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x) \leq -k_3 \|x\|^a \quad (2.15)$$

$\forall t \geq 0$ and $\forall x \in D$, where k_1, k_2, k_3 , and a are positive constants. Then $x = 0$ is *exponentially stable*. If $D = R^n$ and the assumptions hold globally, then $x = 0$ is *globally exponentially stable*.

The following result (*Barbalat's lemma*) is obtained from [88]. Readers are referred to the book for details.

Lemma 2.4.2 [88] Let $\phi : R \rightarrow R$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t \rightarrow \infty} \int_0^t \phi(\tau) d\tau$ exists and is finite. Then,

$$\phi(t) \rightarrow 0 \text{ as } t \rightarrow \infty \quad (2.16)$$

Note that the concept of *uniformly continuous function* is used by Barbalat's lemma. A function $f(t)$ is said to be uniformly continuous on $[0, \infty)$ if

$$\forall R > 0, \exists \eta(R), \forall t_1 \geq 0, t \geq 0, |t - t_1| < \eta \Rightarrow |g(t) - g(t_1)| < R \quad (2.17)$$

A sufficient condition for a function to be uniform continuous is that its derivative to bounded [87].

Another useful result that is frequently used for performance analysis is stated as follows.

Lemma 2.4.3 [87] If a real function $W(t)$ satisfies the inequality

$$\dot{W}(t) + \alpha W(t) \leq 0 \quad (2.18)$$

where α is a real number. Then

$$W(t) \leq W(0)e^{-\alpha t} \quad (2.19)$$

Chapter 3

Modeling and Analysis of Wheeled Mobile Robots with wheel skidding and slipping

3.1 Introduction

This chapter presents the development of kinematic models for four generic wheeled mobile robots: Type (2,0), Type (1,1), Type (1,2) and Type (2,1) robots. To facilitate the theoretical developments in the rest of the chapters, the kinematic models are studied from control perspectives. Firstly, the kinematic models of the wheeled mobile robots are derived using explicit description to represent the kinematic perturbations due to wheel skidding and slipping. Secondly, perturbations are classified into categories to study the impact of these perturbations to the mobile robot control problems. Finally, we relate a mobile robot's maneuverability with the robot's controllability which provide a measure on the robot's ability to track a trajectory in the presence of wheel skidding and slipping. These findings are useful in control problem formulations and designs in Chapters 5-6.

3.2 Wheel Modeling Under Skidding and Slipping

This section briefly describes wheel skidding and slipping physically and mathematically. In real running, tire deformation is unavoidable and is required to translate rotational torques into longitudinal traction force.

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3.2.1 Skidding

Figure 3.1: Tire deformation under a lateral force

Figure 3.1 illustrates a deformed tire in motion. When the wheel negotiates a turn, a lateral force, namely *cornering force*, is generated by the road-tire interaction [90]. The cornering force acts laterally on the wheel's contact patch, causing the wheel to transverse along a direction away from the wheel's plane. This happening is referred as *skidding*. The angle between the wheel's direction of travel and the wheel's plane is known as *slip angle* δ . When a wheel skids, the non-skidding assumption no longer holds. The lateral force F can be approximately described by $F = C_{tire}\delta$ for a small slip angle δ . C_{tire} denotes the tire-coefficient of the wheel's tire.

3.2.2 Slipping

Figure 3.2: Tire deformation under weight

Besides wheel skidding, tire deformation also causes wheel slippage (Figure 3.2). r_w denotes the wheel's free rolling radius with no weight, $\dot{\varphi}$ represents wheel's angular velocity, and V_l represents the wheel's linear velocity. Under pure-rolling assumption, the wheel's linear velocity is $V_l = r_w \dot{\varphi}$. However, this is not the case for a deformed wheel. The wheel slipping can

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be characterized by *slippage* $i = 1 - \frac{V_l}{r_w \dot{\phi}}$. $i = 0$ indicates no wheel slippage whereas $i = 1$ implies a complete slippage, i.e., the wheel is not moving linearly despite of its angular rotation. We refer the *longitudinal slip velocity* as $d = r_w \dot{\phi} - V_l$.

3.3 Kinematic Model with skidding and slipping

In this section, we use the tire deformation described in Section 3.2 to develop the kinematic perturbations due to wheel skidding and slipping in term of the slip angle and slip velocity. We develop kinematic models for four configurations which are generic nonholonomic wheeled mobile robots. These four configurations are the Type (2,0), (2,1), (1,1) and (1,2) robots. We also describe some associated properties in the perspective of control design. A wheeled mobile

Figure 3.3: A Wheeled Mobile Robot body frame

robot considered in this thesis has a body frame $\{X_b, Y_b\}$ attached to the reference point P with coordinates $\xi = (x, y)^T$ in global coordinate frame $\{X, Y\}$, as shown in Figure 3.3. θ denotes the orientation of the basis $\{X_b, Y_b\}$ with respect to the global frame. V denotes the velocity of the reference point P , V_y represents the velocity V projected along the direction of Y_b , and V_l denotes velocity V along the direction of X_b . These velocities (V_l, V_y) relate to the angle δ by the geometric relationships

$$\tan \delta = \frac{V_y}{V_l}, \quad \sin \delta = \frac{V_y}{V}. \quad (3.1)$$

The maneuverability index of a Type (m, s) mobile robot is defined as the sum of mobility index m and steerability index s [2]. In this thesis, all considered mobile robots have either maneuverability two (M2) or maneuverability three (M3). In the following development, the

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control input of a mobile robot is denoted by U , and the orthogonal rotation matrix is defined as

$$R(\theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}. \quad (3.2)$$

3.3.1 Type (2,0) robot

Figure 3.4: Type (2,0) robot: Ideal case.

Figure 3.5: Type (2,0) robot: In the presence of skidding and slipping

Figure 3.5 depicts the motion of a Type (2,0) wheeled mobile robot with M2 using a simplified knife model in the presence of skidding and slipping. In the ideal case of pure-rolling and non-skidding, the velocity V is constrained along X_b . However, when wheel skidding occurs, the velocity V deviates from the X_b axis by an angle δ_2 . $V_{\perp}$ denotes the direction orthogonal to the velocity V .

3.3. Kinematic Model with skidding and slipping

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To derive the kinematic model in the presence of skidding and slipping, we extend the equation formulation for ideal cases [2] to the situations with skidding and slipping. The vector $V_{\perp}$ is orthogonal to V and has zero length, i.e.,

$$[-\sin(\delta_2) \quad \cos(\delta_2)]R(\theta)\dot{\xi} = [0], \quad (3.3)$$

which has a solution

$$R(\theta)\dot{\xi} = \alpha_v \begin{bmatrix} \cos(\delta_2) \\ \sin(\delta_2) \end{bmatrix} \quad (3.4)$$

with $\alpha_v \neq 0$ being a scalar. In the global coordinate frame, the solution can be expressed as

$$\dot{\xi} = \alpha_v \begin{bmatrix} \cos(\theta) \cos(\delta_2) - \sin(\theta) \sin(\delta_2) \\ \sin(\theta) \cos(\delta_2) + \cos(\theta) \sin(\delta_2) \end{bmatrix}.$$

The scalar constant α_v can be interpreted as the magnitude of mobile robot's point velocity V . Using the geometric relation (3.1), the kinematic model of the wheeled mobile robot can be written as

$$\dot{\xi} = \begin{bmatrix} V_l \cos(\theta) - V_y \sin(\theta) \\ V_l \sin(\theta) + V_y \cos(\theta) \end{bmatrix}. \quad (3.5)$$

Comparing model (3.5) with the kinematic model in [50] shows that the general perturbation term used in [50] can be interpreted as the lateral skidding velocity V_y . The linear velocity of the robot V_l is related to the longitudinal slip velocity d and the velocity input $r_w \dot{\varphi}$ by $V_l = r_w \dot{\varphi} - d$.

The robot's yaw rate $\dot{\theta}$ can be easily shown as

$$\dot{\theta} = \gamma_1 + \delta_1 \quad (3.6)$$

where γ_1 is the controllable yaw rate input and δ_1 represents the yaw rate perturbation due to wheel's slippage. Equations (3.5) and (3.6) form the following kinematic model

$$\dot{x} = V_l \cos(\theta) - V_y \sin(\theta) \quad (3.7)$$

$$\dot{y} = V_l \sin(\theta) + V_y \cos(\theta) \quad (3.8)$$

$$\dot{\theta} = \gamma_1 + \delta_1 \quad (3.9)$$

For this Type (2,0) robot, the control input is $\mathbf{U} = (r_w \dot{\varphi}, \gamma_1)$. To gain insights to the kinematic perturbations from control perspective, the perturbations are defined and classified. We rearrange the kinematic model in the following form.

$$\dot{\xi} = (r_w \dot{\varphi} - \Delta^a) \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + \Delta^b \quad (3.10)$$

$$\dot{\theta} = \gamma_1 + \Delta^c \quad (3.11)$$

3.3. Kinematic Model with skidding and slipping

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where the perturbations $\{\Delta^a, \Delta^b, \Delta^c\}$ are defined as

$$\Delta^a = d, \quad \Delta^b = V_y \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}, \quad \Delta^c = \delta_1. \quad (3.12)$$

Property 3.3.1 The perturbations $\{\Delta^a, \Delta^b, \Delta^c\}$ are classified as: (i) Δ^a and Δ^c are input additive; and (ii) Δ^b is unmatched.

These classifications can be easily identified from equations (3.10) and (3.11). The perturbations Δ^a and Δ^c are input additive by inspection. On the other hand, the perturbation vector Δ^b is always orthogonal to input vector $(\cos(\theta), \sin(\theta))^T$ and thus is unmatched.

Property 3.3.2 Suppose that $\{\delta_1, \delta_2, d, V_l\}$ are bounded with $|\delta_1| < \rho_1$, $|\delta_2| < \rho_2 < \frac{\pi}{2}$, $|d| < \rho_3$, $|V_l| < \rho_4$ where ρ_i , $i = 1, 2, 3, 4$ are positive constants. The perturbations $\{\Delta^a, \Delta^b, \Delta^c\}$ are bounded as follows:

$$|\Delta^a| < \rho_3, \quad \|\Delta^b\| < \rho_4 \tan \rho_2, \quad |\Delta^c| < \rho_1 \quad (3.13)$$

Proof: Inequalities $|\Delta^a| < \rho_3$, $|\Delta^c| < \rho_1$ are implied from (3.12) with assumptions $|d| < \rho_3$ and $|\delta_1| < \rho_1$. The geometric relation (3.1) leads to $V_y = V_l \tan \delta_2$; and hence, $|V_y| < \rho_4 \tan(\rho_2)$. This implies that $\|\Delta^b\| = |V_y| \|\begin{bmatrix} \cos \theta & \sin \theta \end{bmatrix}^T\| \leq \rho_4 \tan(\rho_2)$ because of $\|\begin{bmatrix} \cos \theta & \sin \theta \end{bmatrix}^T\| = 1$.

□

The kinematic perturbations of this configuration are classified as *input-additive* and *unmatched*. The types of perturbations suggest the difficulty levels in compensating these perturbations from control perspective. For instance, input-additive perturbation is located in the same channel with a control input and can be eliminated without much difficulty if the instantaneous perturbation is known or measurable. On the other hand, *unmatched perturbation* does not fall into any channel with a control input. This unique characteristic suggests that any undesirable effects due to this unmatched perturbation cannot be compensated directly (without using other dynamical state) or in some cases cannot be compensated at all.

3.3.2 Type (1,1) robot

The configuration of Type (1,1) car-like robot is shown in Figures 3.6 and 3.7. This class of robot is equipped with front centered steerable wheels and fixed parallel rear wheels. γ_1 denotes the robot's front steering angle, a denotes the robot's wheelbase, and (δ_1, δ_2) represent the slip

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Figure 3.6: Type (1,1) robot: Ideal case.

Figure 3.7: Type (1,1) robot: In the presence of skidding and slipping

angles of the front and rear wheels. This type of wheeled mobile robot has maneuverability index two (M2).

The rear fixed wheel leads to the following constraint equation by projecting velocity V along $V_{\perp}$.

$$\begin{bmatrix} -\sin(\delta_2) & \cos(\delta_2) \end{bmatrix} R(\theta)\dot{\xi} = [0]. \quad (3.14)$$

For the front steerable wheel, we obtain

$$\begin{bmatrix} -\sin(\delta_1 + \gamma_1) & \cos(\delta_1 + \gamma_1) \end{bmatrix} R(\theta)\dot{\xi} + a \cos(\delta_1 + \gamma_1)\dot{\theta} = [0] \quad (3.15)$$

A solution to equations (3.14) and (3.15) is

$$\begin{bmatrix} R(\theta)\dot{\xi} \\ \dot{\theta} \end{bmatrix} = \alpha_v \begin{bmatrix} \cos(\delta_2) \\ \sin(\delta_2) \\ \frac{\sin(\delta_1 + \gamma_1) \cos(\delta_2) - \cos(\delta_1 + \gamma_1) \sin(\delta_2)}{a \cos(\delta_1 + \gamma_1)} \end{bmatrix}. \quad (3.16)$$

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The scalar α_v can be interpreted as the velocity magnitude V . Expressing the solution (3.16) in the global coordinate frame leads to the kinematic model

$$\dot{x} = V_l \cos(\theta) - V_y \sin(\theta) \quad (3.17)$$

$$\dot{y} = V_l \sin(\theta) + V_y \cos(\theta) \quad (3.18)$$

$$\dot{\theta} = \frac{V_l}{a} \tan(\gamma_1 + \delta_1) - \frac{V_y}{a} \quad (3.19)$$

This wheeled mobile robot has a linear velocity $V_l = r_w \dot{\varphi} - d$ and control input $\mathbf{U} = (r_w \dot{\varphi}, \gamma_1)$. To categorize the perturbations due to wheel skidding and slipping, the kinematic model (3.17)-(3.19) can be expressed as follows, using (3.16),

$$\dot{\xi} = (r_w \dot{\varphi} - \Delta^a) \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + \Delta^b \quad (3.20)$$

$$\dot{\theta} = (r_w \dot{\varphi} - \Delta^a) \frac{1}{a} \tan(\gamma_1 + \Delta^c) + \Delta^d \quad (3.21)$$

where the perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ are

$$\Delta^a = d, \Delta^b = V_y \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}, \Delta^c = \delta_1, \Delta^d = -\frac{V_y}{a}. \quad (3.22)$$

Property 3.3.3 The perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ are classified as follows: (i) Δ^a and Δ^c are input additive; (ii) Δ^b is unmatched, and (iii) Δ^d is matched.

By inspection, it is clear that Δ^a, Δ^c are input additive and Δ^d is matched. Similarly, by following the same arguments used in Type (2,0) case, we can show Δ^b being unmatched.

Property 3.3.4 Suppose that $\{\delta_1, \delta_2, d, V_l\}$ satisfy $|\delta_1| < \rho_1 < \frac{\pi}{2}$, $|\delta_2| < \rho_2 < \frac{\pi}{2}$, $|d| < \rho_3$, and $|V_l| < \rho_4$ where ρ_i , $i = 1, 2, 3, 4$ are positive constants. Then the perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ satisfy

$$|\Delta^a| < \rho_3, \quad \|\Delta^b\| < \rho_4 \tan \delta_2 \quad (3.23)$$

$$|\Delta^c| < \rho_1, \quad |\Delta^d| < \frac{1}{a} \rho_4 \tan \rho_2. \quad (3.24)$$

By following the same arguments used in the case of Type (2,0), we can show that the perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ satisfy the bounds since the wheelbase, a , of the robot is a positive finite constant.

Contrast to unmatched perturbation, a matched perturbation is located in a channel with a control input. However, when compared with input additive perturbation, matched perturbation

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requires more information for compensation. For instance, suppose that the matched perturbation Δ^d is known or measurable, we still require reliable information on $(r_w \dot{\phi}, \gamma_1, \Delta^a, \Delta^c)$ to compensate the undesirable effects from Δ^d using computed torque control technique. Nevertheless, this type of perturbation can still be compensated instantaneously from a control input.

3.3.3 Type (2,1) robot

Figure 3.8: Type (2,1) robot: Ideal case.

Figure 3.9: Type (2,1) robot: In the presence of skidding and slipping

The configuration of a Type (2,1) robot is shown in Figures 3.8 and 3.9. This class of robot is equipped with two directional control inputs. This robot has centered steerable wheels and rear off-centered steerable wheels. γ_2 denotes the front steering angle, α is the angle of the

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off-centered wheel, (δ_2, δ_1) are the front and rear wheels' slip angles, and (b, a) represent the respective lengths from the pivots to the wheels, respectively. This wheeled mobile robot has maneuverability index three (M3).

The centered steerable wheel leads to the following equation by projecting the velocity V along $V_\perp$.

$$\begin{bmatrix} -\sin(\gamma_2 + \delta_2) & \cos(\gamma_2 + \delta_2) \end{bmatrix} R(\theta)\dot{\xi} = [0]. \quad (3.25)$$

A solution for the constraint equation (3.25) is

$$R(\theta)\dot{\xi} = \alpha_v \begin{bmatrix} \cos(\gamma_2 + \delta_2) \\ \sin(\gamma_2 + \delta_2) \end{bmatrix}. \quad (3.26)$$

We choose $\alpha_v = V$ and the solution in global coordinates is

$$\dot{x} = V \cos(\theta + \gamma_2 + \delta_2) \quad (3.27)$$

$$\dot{y} = V \sin(\theta + \gamma_2 + \delta_2). \quad (3.28)$$

The robot's yaw rate can be derived by projecting the velocity vector V_1 of the off-centered wheels along $V_{1\perp}$, i.e.,

$$\begin{bmatrix} -\sin(\gamma_2 + \delta_2) & \cos(\gamma_2 + \delta_2) \end{bmatrix} R(\theta)\dot{\xi} - a\dot{\theta} \cos(\alpha + \delta_1) - (\gamma_1 + \dot{\theta})b \cos(\delta_1) = [0]. \quad (3.29)$$

Substituting (3.26) into (3.29) yields

$$\begin{aligned} & V \sin(\gamma_2 + \delta_2) \cos(\alpha + \delta_1) - V \cos(\gamma_2 + \delta_2) \sin(\alpha + \delta_1) \\ & - a\dot{\theta} \cos(\alpha + \delta_1) - (\gamma_1 + \dot{\theta})b \cos(\delta_1) = 0. \end{aligned} \quad (3.30)$$

Solving equation (3.30) leads to

$$\dot{\theta} = \frac{V \sin(\gamma_2 + \delta_2 - \alpha - \delta_1) - \gamma_1 b \cos(\delta_1)}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)}. \quad (3.31)$$

Equations (3.27), (3.28) and (3.31) form the kinematic model of this mobile robot. This robot's front steerable wheel has a linear velocity $\bar{V}_l = r_w \dot{\phi} - d$. By inspection, we can also relate the wheel linear velocity $\bar{V}_l$ with the mobile robot linear velocity via

$$V_l = \frac{\bar{V}_l \cos(\gamma_2 + \delta_2)}{\cos \delta_2}. \quad (3.32)$$

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The control inputs for this mobile robot is $\mathbf{U} = (r_w \dot{\varphi}, \gamma_1, \gamma_2)$. To sort the perturbations, we express the kinematic model as

$$\dot{\xi} = \{r_w \dot{\varphi} \Delta^a - \Delta^b\} \begin{bmatrix} \cos(\theta + \gamma_2 + \Delta^c) \\ \sin(\theta + \gamma_2 + \Delta^c) \end{bmatrix} \quad (3.33)$$

$$\dot{\theta} = \Delta^d + \Delta^e \gamma_1 \quad (3.34)$$

where the perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d, \Delta^e\}$ are defined as

$$\begin{aligned} \Delta^a &= \frac{1}{\cos \delta_2}, & \Delta^b &= \frac{d}{\cos \delta_2}, & \Delta^c &= \delta_2, \\ \Delta^d &= \frac{V \sin(\gamma_2 + \delta_2 - \alpha - \delta_1)}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)}, \\ \Delta^e &= \frac{b \cos \delta_1}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)}. \end{aligned}$$

Property 3.3.5 The perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d, \Delta^e\}$ are classified as follows: (i) Δ^b and Δ^c are input additive, (ii) Δ^a and Δ^e are input multiplicative, and (iii) Δ^d is matched.

Property 3.3.6 Suppose $|\alpha + \delta_1| < \frac{\pi}{2}$, $|\delta_1| < \rho_1 < \frac{\pi}{2}$, $|\delta_2| < \rho_2 < \frac{\pi}{2}$, $|d| < \rho_3$, $|V| < \rho_4$, $|\alpha| < \rho_5$ where $\rho_i, i = 1, 2, 3, 4, 5$ are positive constants, then the perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d, \Delta^e\}$ satisfy

$$\begin{aligned} 1 &\leq |\Delta^a| < \frac{1}{\cos \rho_2}, & |\Delta^b| &\leq \frac{\rho_3}{\cos \rho_2}, & |\Delta^c| &< \rho_2, \\ |\Delta^d| &< \frac{\rho_4}{\cos \rho_2} \left(\frac{1}{a \cos(\rho_1 + \rho_5) + b \cos \rho_1} \right), \\ 0 &< \frac{b \cos \rho_1}{a + b} \leq |\Delta^e| < \frac{b}{a \cos(\rho_1 + \rho_5) + b \cos \rho_1}. \end{aligned}$$

Proof: By definition, $\Delta^a = \frac{1}{\cos \delta_2}$. It is clear that Δ^a is lower bounded by unity and upper bounded by $\frac{1}{\cos \rho_2}$. Similarly, the upper and lower bounds of perturbation Δ^b can be determined since d satisfies $|d| \leq \rho_3$. As for perturbation Δ^c , its bound can be easily established by assumption $|\delta_2| < \rho_2$.

Next, before we show the bounds of perturbations Δ^d, Δ^e , we claim that $a \cos(\alpha + \delta_1) + b \cos \delta_1$ and $|V|$ satisfy

$$a \cos(\rho_1 + \rho_5) + b \cos \rho_1 \leq a \cos(\alpha + \delta_1) + b \cos \delta_1 \leq a + b \quad (3.35)$$

and

$$|V| \leq \frac{\rho_4}{\cos \rho_2}. \quad (3.36)$$

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Inequality (3.35) can be shown since $a \cos(\alpha + \delta_1)$ and $b \cos \delta_1$ satisfy respectively

$$a \cos(\rho_1 + \rho_5) \leq a \cos(\alpha + \delta_1) \leq a, \quad (3.37)$$

$$b \cos(\rho_1) \leq b \cos(\delta_1) \leq b. \quad (3.38)$$

Inequality (3.36) can be easily obtained by geometric relation $V \cos \delta_2 = V_l$.

From inequality (3.35), we have

$$\frac{1}{a+b} \leq \frac{1}{a \cos(\alpha + \delta_1) + b \cos \delta_1} \leq \frac{1}{a \cos(\rho_1 + \rho_5) + b \cos \rho_1}, \quad (3.39)$$

Furthermore,

$$|V \sin(\gamma_2 + \delta_2 - \alpha - \delta_1)| \leq |V| \leq \frac{\rho_4}{\cos \rho_2}. \quad (3.40)$$

Inequalities (3.39) and (3.40) lead to

$$|\Delta^d| < \frac{\rho_4}{\cos \rho_2} \left(\frac{1}{a \cos(\rho_1 + \rho_5) + b \cos \rho_1} \right). \quad (3.41)$$

Finally, the bounds of $|\Delta^e|$ can be established using inequalities (3.38) and (3.39).

□

Input multiplicative perturbation distorts the control input that commands the system state. This perturbation can completely disable the control input if the perturbation tends to zero. Fortunately, we have shown that the multiplicative perturbation Δ^e is lower bounded above zero for this type of mobile robot. Hence, perturbation Δ^e will not disable control input γ_1 .

3.3.4 Type (1,2) robot

The configuration of the Type (1,2) robot is shown in Figures 3.10 and 3.11. This class of wheeled mobile robot is identified by the centered front and rear steerable wheels. The robot is represented by a bicycle model equipped with a front and a rear virtual steering wheels on the center of the robot's front and rear axes. This mobile robot has maneuverability three (M3). (γ_1, γ_2) represent the front and rear steering angles. a denotes the robot's wheelbase and V denotes the robot's velocity, and (δ_1, δ_2) the front and rear wheels' slip angles.

By projecting velocity V along $V_\perp$, we obtain

$$\begin{bmatrix} -\sin(\delta_2 + \gamma_2) & \cos(\delta_2 + \gamma_2) \end{bmatrix} R(\theta) \dot{\xi} = [0]. \quad (3.42)$$

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Figure 3.10: Type (1,2) robot: Ideal case.

Figure 3.11: Type (1,2) robot: In the presence of skidding and slipping

As for the front steerable wheel, we have

$$\begin{bmatrix} -\sin(\delta_1 + \gamma_1) & \cos(\delta_1 + \gamma_1) \end{bmatrix} R(\theta)\dot{\xi} + a \cos(\delta_1 + \gamma_1)\dot{\theta} = [0] \quad (3.43)$$

A solution for the equations (3.42) and (3.43) is

$$\begin{bmatrix} R(\theta)\dot{\xi} \\ \dot{\theta} \end{bmatrix} = \alpha_v \begin{bmatrix} \cos(\gamma_2 + \delta_2) \\ \sin(\gamma_2 + \delta_2) \\ \frac{\sin(\delta_1 + \gamma_1) \cos(\gamma_2 + \delta_2) - \cos(\delta_1 + \gamma_1) \sin(\gamma_2 + \delta_2)}{a \cos(\delta_1 + \gamma_1)} \end{bmatrix} \quad (3.44)$$

where α_v can be interpreted as velocity V . Similarly, the kinematic model with skidding and

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slipping can be written as

$$\dot{x} = V \cos(\theta + \gamma_2 + \delta_2) \quad (3.45)$$

$$\dot{y} = V \sin(\theta + \gamma_2 + \delta_2) \quad (3.46)$$

$$\dot{\theta} = \frac{V}{a} \tan(\gamma_1 + \delta_1) \cos(\gamma_2 + \delta_2) - \frac{V}{a} \sin(\gamma_2 + \delta_2) \quad (3.47)$$

The longitudinal velocity of the rear steerable wheel is denoted by $\bar{V}_l$ and has the relationship with the wheel's slip velocity and control input $r_w \dot{\varphi}$ via $\bar{V}_l = r_w \dot{\varphi} - d$. Similarly, $\bar{V}_l$ is related the robot's linear velocity V_l via equation

$$V_l = \frac{\bar{V}_l \cos(\gamma_2 + \delta_2)}{\cos \delta_2}. \quad (3.48)$$

The control input of this mobile robot is $\mathbf{U} = (r_w \dot{\varphi}, \gamma_1, \gamma_2)$. The kinematic model can be expressed as

$$\dot{\xi} = (r_w \dot{\varphi} \Delta^a - \Delta^b) \begin{bmatrix} \cos(\theta + \gamma_2 + \Delta^c) \\ \sin(\theta + \gamma_2 + \Delta^c) \end{bmatrix} \quad (3.49)$$

$$\begin{aligned} \dot{\theta} &= \frac{1}{a} (r_w \dot{\varphi} \Delta^a - \Delta^b) \tan(\gamma_1 + \Delta^d) \cos(\gamma_2 + \Delta^c) \\ &+ \frac{1}{a} (r_w \dot{\varphi} \Delta^a - \Delta^b) \sin(\gamma_2 + \Delta^c) \end{aligned} \quad (3.50)$$

where the perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ are

$$\Delta^a = \frac{1}{\cos \delta_2}, \quad \Delta^b = \frac{d}{\cos \delta_2}, \quad \Delta^c = \delta_2, \quad \Delta^d = \delta_1. \quad (3.51)$$

Property 3.3.7 The perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ are classified as follows: (i) Δ^a is input multiplicative, (ii) Δ^b, Δ^c and Δ^d are input additive.

This statement can be easily seen from equations (3.49) and (3.50).

Property 3.3.8 Suppose that $|\delta_1| < \rho_1 < \frac{\pi}{2}$, $|\delta_2| < \rho_2 < \frac{\pi}{2}$, $|d| < \rho_3$ and $|V_l| < \rho_4$ where ρ_i , $i = 1, 2, 3, 4$ are positive constants. The perturbations $\{\Delta^a, \Delta^b, \Delta^c, \Delta^d\}$ satisfy

$$1 \leq |\Delta^a| \leq \frac{1}{\cos \rho_2}, \quad |\Delta^b| \leq \frac{\rho_3}{\cos \rho_2} \quad (3.52)$$

$$|\Delta^c| \leq \rho_2, \quad |\Delta^d| \leq \rho_1 \quad (3.53)$$

Property 3.3.8 can be shown similarly to Type (2,1) robot. The lower bound of the multiplicative perturbation $|\Delta^a| \geq 1$ implies that the velocity control input $r_w \dot{\varphi}$ will not be disabled by Δ^a . Other perturbations $\{\Delta^b, \Delta^c, \Delta^d\}$ influence the robot's control in a similar fashion as the preceding configurations.

3.4 Controllability

This section addresses the controllability of the mobile robots with wheel skidding and slipping. Without loss of generality, the kinematic model of a wheeled mobile robot can be written in the form of

$$\dot{\xi} = f_{m+s}^1(\theta, \mathbf{U}) \quad (3.54)$$

$$\dot{\theta} = f_{m,s}^2(\theta, \mathbf{U}) \quad (3.55)$$

Vectors $f_{m+s}^1 \in R^2$ and $f_{m,s}^2 \in R$ are given as follows depending on the robot's maneuverability index.

For mobile robots with M2, i.e., for each $(m, s) \in \{(2, 0), (1, 1)\}$, the vectors f_{m+s}^1 and $f_{m,s}^2$ are

$$f_2^1 = \begin{bmatrix} V_l \cos(\theta) - V_y \sin(\theta) \\ V_l \sin(\theta) + V_y \cos(\theta) \end{bmatrix} \quad (3.56)$$

$$f_{2,0}^2 = \gamma_1 + \delta_1 \quad (3.57)$$

$$f_{1,1}^2 = \frac{V_l}{a} \tan(\gamma_1 + \delta_1) - \frac{V_y}{a} \quad (3.58)$$

With the robot's control inputs $\mathbf{U} = (r_w \dot{\varphi}, \gamma_1)$.

For mobile robots with maneuverability three (M3), the vectors $f_3^1, f_{2,1}^2, f_{1,2}^2$ are

$$f_3^1 = \begin{bmatrix} V \cos(\theta + \gamma_2 + \delta_2) \\ V \sin(\theta + \gamma_2 + \delta_2) \end{bmatrix} \quad (3.59)$$

$$f_{2,1}^2 = \frac{V \sin(\gamma_2 + \delta_2 - \alpha - \delta_1) - \gamma_1 b \cos(\delta_1)}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)} \quad (3.60)$$

$$f_{1,2}^2 = \frac{V}{a} \tan(\gamma_1 + \delta_1) \cos(\gamma_2 + \delta_2) - \frac{V}{a} \sin(\gamma_2 + \delta_2) \quad (3.61)$$

The longitudinal wheel velocity $\bar{V}_l$, as shown in Figures (3.9) and (3.11), is related with the wheel's slip velocity and velocity control $r_w \dot{\varphi}$ via $\bar{V}_l = r_w \dot{\varphi} - d$. The control input of this mobile robots with M3 is $\mathbf{U} = (r_w \dot{\varphi}, \gamma_1, \gamma_2)$.

Assumption 3.4.1 Perturbations $\{\delta_1, \delta_2, d, \dot{\delta}_2, \dot{V}_y, \dot{V}_y\}$ are bounded and measurable with $|\delta_2| < \frac{\pi}{2}$.

Assumption 3.4.1 implies the perturbations are bounded and measurable. In the next chapter, we illustrate how these perturbations can be measured using RTK-GPS and other aiding sensors. For now, we proceed the analysis by assuming these perturbations are measurable. In

the ideal case where *non-slipping* and *non-skidding* assumptions are satisfied, the controllability of a mobile robot is referred as the ability to steer it from an initial posture to a final posture in a finite time, as stated as follows [4].

Definition 3.4.1 A wheeled mobile robot is said to be posture controllable if there exists a piecewise continuous input to steer the robot's configuration (x, y, θ) from an initial posture $\{x(t_0), y(t_0), \theta(t_0)\}$ to a final posture $\{x(t_f), y(t_f), \theta(t_f)\}$ in a finite-time interval.

In the presence of wheel skidding, robots with M2 do not have posture controllability. Nevertheless, these robots should be able to achieve point control by performing appropriate steering action to compensate the skidding and slipping perturbations.

Definition 3.4.2 A wheeled mobile robot is said to be point controllable if there exists a piecewise continuous input to steer the robot's reference point P from an initial point $\{x(t_0), y(t_0)\}$ to a final point $\{x(t_f), y(t_f)\}$ in a finite-time interval.

Now, we show that the kinematic model (3.54)-(3.55) of a robot with M2 can be transformed into a form similar to the ideal kinematic model of a Type (2,0) [10] without wheel skidding and slipping.

Lemma 3.4.3 Consider a wheeled mobile robot with M2. Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ satisfy Assumption 3.4.1, then there exists an invertible coordinate transformation and a corresponding invertible input change, respectively,

$$\bar{q} = \phi(q, \delta_2), \quad (3.62)$$

$$\mu = \beta_1(q, \delta_1, \delta_2, d, \mathbf{U}), \quad (3.63)$$

such that the transformed system becomes

$$\dot{\bar{q}} = G(\bar{q})\mu, \quad (3.64)$$

where

$$\bar{q} = \begin{bmatrix} x \\ y \\ \bar{\theta} \end{bmatrix}, \quad G(\bar{q}) = \begin{bmatrix} \cos \bar{\theta} & 0 \\ \sin \bar{\theta} & 0 \\ 0 & 1 \end{bmatrix}, \quad \mu = \begin{bmatrix} V \\ \omega \end{bmatrix}. \quad (3.65)$$

with $\bar{\theta} = \theta + \delta_2$.

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Proof: The proof is straightforward and constructive. Define $\bar{q}$ and μ as

$$\bar{q} = [x \ y \ \theta + \delta_2]^T \quad (3.66)$$

$$\mu = \left[\frac{r_w \dot{\varphi} - d}{\cos \delta_2} \ f_{m,s}^2 \right]^T \quad (3.67)$$

By inspection, the transformation equation (3.66) is invertible. We show that the input change (3.67) is also invertible. It is clear from (3.67) that the auxiliary V is invertible with respect to $r_w \dot{\varphi}$. As for ω , we need to show that $\omega = f_{m,s}^2$ is invertible for $\forall (m, s) \in \{(2, 0), (1, 1)\}$. For the Type (2,0) case, $\omega = \gamma_1 + \delta_1$ is invertible with respect to γ_1 since $f_{2,0}^2$ is a linear function. As for Type (1,1), ω is defined as

$$\omega = \frac{V}{a} \tan(\gamma_1 + \delta_1) \cos(\delta_2) - \frac{V}{a} \sin(\delta_2). \quad (3.68)$$

With some manipulations, we have

$$\gamma_1 = \tan^{-1} \left\{ \frac{\omega a}{V \cos \delta_2} + \tan \delta_2 \right\} - \delta_1. \quad (3.69)$$

For any nonzero V , ω is invertible; hence, the kinematic model of a mobile robot with M2 can be converted into (3.64).

□

With Lemma 3.4.3, we can state the following result.

Theorem 3.4.4 Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ of a wheeled mobile robot with M2 satisfy Assumption 3.4.1, then the robot is point controllable, but not posture controllable.

Proof: By Lemma 3.4.3, the kinematic model of a mobile robot with M2 can be transformed into (3.64) where input vector fields are

$$g_1 = \begin{bmatrix} \cos \bar{\theta} \\ \sin \bar{\theta} \\ 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (3.70)$$

Let Δ_C be the accessibility distribution generated by the vector fields $\{g_1, g_2\}$. The distribution has a rank of

$$\begin{aligned} \text{rank}\{\Delta_C\} &= \text{rank}\{\text{span}(g_1, g_2, [g_1, g_2])\} \\ &= 3 \end{aligned} \quad (3.71)$$

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for all $\bar{\theta}$. By applying Theorem 2.3.2, $\text{rank}\{\Delta_C\} = 3$ indicates that the system is controllable in coordinates $\bar{q}$. Since ξ is a subvector of $\bar{q}$, controllability in $\bar{q}$ implies controllability in $[x, y]^T$ coordinates.

To show that a robot with M2 is not *posture controllable* in the presence of skidding and slipping, we consider the point subsystem of a robot with M2,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = V \begin{bmatrix} \cos(\bar{u}) \\ \sin(\bar{u}) \end{bmatrix} \quad (3.72)$$

where we let $\bar{u} = \theta + \delta_2$ be a directional control input. If the reference point of the wheeled mobile robot ξ is to be steered from an initial point $\xi(0)$ to a final point $\xi(t_f)$, then the directional input control $\bar{u}$ is constrained along a feasible $\bar{u}_s(t)$ to achieve this goal. This implies that the orientation of the robot must satisfy $\theta(t) = \bar{u}_s(t) - \delta_s(t)$ to reach the desired point ξ_f . Since in general, the desired orientation $\theta_d(t)$ of a path that the robot follows is not $\bar{u}_s(t) - \delta_s(t)$, we conclude that the orientation of the robot cannot be equal to $\theta_d(t)$ in order to maintain point controllability. Hence, the robot is not posture controllable. □

In contrast to robots with M2, the additional orientation input γ_2 of a robot with M3 suggests that this class of wheeled mobile robot can handle the skidding perturbation more effectively. To confirm this intuition, we examine the controllability of the robots with M3.

Lemma 3.4.5 Consider a wheeled mobile robot with M3. Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ satisfy Assumption 3.4.1, then there exists an invertible input change,

$$\mu = \beta_2(q, \mathbf{U}) \quad (3.73)$$

such that the kinematic model of a wheeled mobile robot with M3 becomes

$$\dot{q} = G(q)\mu, \quad (3.74)$$

where

$$\dot{q} = \begin{bmatrix} \dot{\xi} \\ \dot{\theta} \end{bmatrix}, \quad G(q) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix}, \quad \mu = \begin{bmatrix} V \\ \omega \end{bmatrix}. \quad (3.75)$$

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Proof: By choosing $\gamma_2 = -\delta_2$ and auxiliary input

$$V = \frac{r_w \dot{\varphi} + d}{\cos \delta_2}, \quad (3.76)$$

the point subsystem of a robot with M3 becomes

$$\dot{x} = V \cos(\theta) \quad (3.77)$$

$$\dot{y} = V \sin(\theta) \quad (3.78)$$

Similarly, the auxiliary input (3.76) is invertible. Consider the orientation subsystem for a wheeled mobile robot with M3. For Type (1,2) robot, the auxiliary inputs are defined as

$$\omega = \frac{V}{a} \tan(\gamma_1 + \delta_1). \quad (3.79)$$

It is clear that auxiliary input (3.79) is invertible

$$\gamma_1 = \tan^{-1}\left(\frac{\omega a}{V}\right) - \delta_1 \quad (3.80)$$

for $V \neq 0$.

As for Type (2,1) robot, the auxiliary input for the robot's orientation subsystem is

$$\omega = \frac{-V \sin(\alpha + \delta_1) - \gamma_1 b \cos(\delta_1)}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)}. \quad (3.81)$$

We can see that the auxiliary input (3.81) is invertible since $|b \cos(\delta_1)| > 0$ by inequality (3.38) and the condition $|a \cos(\alpha + \delta_1) + b \cos(\delta_1)| > 0$ can be easily met in practice.

□

Lemmas 3.4.3 and 3.4.5 suggest the matched perturbations can be decoupled by direct compensation if the skidding and slipping parameters are measurable. These two lemmas lead the following result.

Theorem 3.4.6 Suppose that the perturbations $\{\delta_1, \delta_2, d\}$ of a wheeled mobile with M3 satisfy Assumption 3.4.1, then the robot is *posture controllable*.

Proof: By Lemma 3.4.5, the kinematic model of a mobile robot with M3 is input-equivalent to a nominal Type (2,0) kinematic model (3.75) which has an rank 3 accessibility distribution

$$\dim \Delta_C = 3 \quad (3.82)$$

for all θ ; hence, Theorem 2.3.2 implies the mobile robot is posture controllable.

□

Theorem 3.4.6 shows that in the presence of skidding and slipping, a wheeled mobile robot with M3 is more controllable compared with a robot with M2. Despite the high capability exhibited by the robots with M3, these robots could inherit a structural property of a mobile robot with M2. For instance, suppose the control input γ_2 of a robot with M3 is chosen as $\gamma_2 = -\delta_2$ but the maximum magnitude of the control input is lower than the maximum magnitude of the perturbation $\delta_2(t)$ for some time interval, i.e., $\gamma_{\max} < \sup_{t \geq 0} |\delta_2(t)|$, then the point system $\dot{\xi}$ of the robot with M3 becomes

$$\dot{\xi} = \begin{bmatrix} V \cos(\theta + \gamma_2 + \delta_2) \\ V \sin(\theta + \gamma_2 + \delta_2) \end{bmatrix} \quad (3.83)$$

with a nonzero $\gamma_2 + \delta_2$. By geometric relation (3.1), the point subsystem of a robot with M2 (3.56) can be expressed in the form of

$$\dot{\xi} = \begin{bmatrix} V \cos(\theta + \delta_2) \\ V \sin(\theta + \delta_2) \end{bmatrix}. \quad (3.84)$$

We can see that equation (3.83) has the same form as equation (3.84). Hence, a robot with M3 has the same point subsystem of a robot with M2 under condition $\gamma_{\max} < \sup_{t \geq 0} |\delta_2(t)|$. A wheeled mobile robot with M3 possesses an unmatched kinematic perturbation if the secondary control γ_2 cannot completely eliminate δ_2 due to the control input's maximum limit. Nevertheless, it is clear that a mobile robot with M3 in general has a better controllability than a mobile robot with M2. In the sequel, we highlight the implications of these results on the wheeled mobile robot tracking and path following problems.

3.5 Wheeled Mobile Robot Tracking Problem

In this section, we study the tracking and path following control problems under the influence of the kinematic perturbations due to skidding and slipping. Path following is a special case of tracking control problem in the sense that path following problem only considers lateral and orientation errors; whereas tracking control problem encompasses lateral, longitudinal and orientation errors. Hence, we focus on the tracking problem.

A tracking control problem is to maneuver the robot to follow a trajectory. In this paper, we

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consider the *posture tracking error*

$$\tilde{q} = \begin{bmatrix} \tilde{\xi} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \quad (3.85)$$

The reference trajectory (x_r, y_r, θ_r) represents the point coordinates and orientation of a reference trajectory which satisfies

$$\dot{x}_r = v_r \cos \theta_r \quad (3.86)$$

$$\dot{y}_r = v_r \sin \theta_r \quad (3.87)$$

$$\dot{\theta}_r = \omega_r \quad (3.88)$$

$\tilde{\xi}$ denotes the point tracking error and $\tilde{\theta}$ represents the orientation error. It can be shown that the dynamics of $\tilde{q}$ can be described by

$$\dot{\tilde{\xi}} = f_{m+s}^3 \quad (3.89)$$

$$\dot{\tilde{\theta}} = \omega_r - f_{m,s}^2 \quad (3.90)$$

where

$$f_2^3 = \begin{bmatrix} v_r \cos \theta - V_l + \tilde{y}\omega \\ -\tilde{x}\omega + v_r \sin \tilde{\theta} - V_y \end{bmatrix} \quad (3.91)$$

$$f_3^3 = \begin{bmatrix} v_r \cos \theta - V \cos(\gamma_2 + \delta_2) + \tilde{y}\omega \\ -\tilde{x}\omega + v_r \sin \tilde{\theta} - V \sin(\gamma_2 + \delta_2) \end{bmatrix} \quad (3.92)$$

The detailed derivations of these error dynamics are presented in Chapter 6. One assumption that is usually imposed on the moving reference trajectory in the tracking control problem is stated below.

Assumption 3.5.1 The yaw rate $\omega_r(t)$, velocity $v_r(t)$, and $\dot{v}_r(t)$ are *piecewise continuous* and *bounded functions*. Furthermore, $v_r(t)$ is positive and $\inf\{v_r(t); \forall t \geq t_0\} > 0$.

This assumption is assumed in the remainder of this chapter.

Wheeled mobile robot *point tracking problem* in the presence of skidding and slipping is said to be solvable if for a small initial tracking error $\tilde{q}(0)$, there exists a control input $\mathbf{U}(t)$ such that the point error $\tilde{\xi}$ converges to zero; and the *posture tracking problem* in the presence of skidding and slipping is said to be solvable if for a small initial tracking error $\tilde{q}(0)$, these

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exists a control input $\mathbf{U}(t)$ such that $\tilde{q}$ converges to zero. Many controllers have been proposed to solve the posture tracking problem for the Type (2,0) configuration ([5, 10, 18, 19]) under the non-skidding and non-slipping assumptions. We will show that there exists a control input such that a robot with M2 in the presence of skidding and slipping is able to track the trajectory with the point tracking error converges to zero. Similarly, we also show that there exists a control law such that the posture tracking error of a robot with M3 in the presence of skidding and slipping converges to zero. The following result summarizes this finding.

Theorem 3.5.1 Assume that the perturbations $\{\delta_1, \delta_2, d\}$ satisfy Assumption 3.4.1. Then in the presence of skidding and slipping,

1. a robot with M2 is point tracking solvable, but not posture tracking solvable.
2. a robot with M3 is posture tracking solvable.

Proof: We first consider a robot with M2. For the nominal case (in the absence of skidding and slipping), the tracking error dynamics $\dot{\tilde{q}}$ is as follows:

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (3.93)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - \omega \tilde{x} \quad (3.94)$$

$$\dot{\tilde{\theta}} = \omega_r - \omega \quad (3.95)$$

Integrator backstepping methodology [91] has been applied for tracking control problem in the absence of wheel skidding and slipping. Reader may refer to [18] [19] for further details. Here, we show that integrator backstepping can also be applied to solve the point tracking problem in the presence of skidding and slipping.

In the absence of skidding and slipping effects, it has been shown in [18] that by choosing $\tilde{\theta}$ as a virtual control input for point error subsystem (3.93),(3.94), there exists a velocity control $V_l = \alpha_1$, a continuously differentiable feedback control law

$$\tilde{\theta} = \alpha_2(\tilde{\xi}), \quad (3.96)$$

and a positive definite function $V_1(\tilde{\xi})$ such that its derivative satisfies

$$\dot{V}_1 \leq -W(\tilde{\xi}) \leq 0 \quad (3.97)$$

where $W(\tilde{\xi})$ is a positive definite function. Let $z = \tilde{\theta} - \alpha_2$. By integrator backstepping, there exists a positive definite function

$$V_2(\tilde{\xi}, z) = V_1 + \frac{1}{2}z^2 \quad (3.98)$$

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and a feedback control law ω which render $(\tilde{\xi}, z) \rightarrow 0$ as $t \rightarrow \infty$.

In the skidding and slipping case, we show that the following invertible auxiliary inputs

$$V_l = r_w \dot{\varphi} - d \quad (3.99)$$

$$\omega = f_{m,s}^2 \quad (3.100)$$

can eliminate the input-additive and matched perturbations inherent in the kinematic model of a wheeled mobile robot with M2. As a result, the tracking error dynamics of a robot with M2 can be expressed as

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (3.101)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - \omega \tilde{x} - V_y \quad (3.102)$$

$$\dot{\tilde{\theta}} = \omega_r - \omega \quad (3.103)$$

where V_l and ω can be converted to original control input $(r_w \dot{\varphi}, \gamma_1)$. Note that in the case of skidding and slipping, an additional unmatched term V_y appears in the lateral error dynamic $\dot{\tilde{y}}$. This observation suggests the choice of $V_l = \alpha_1$ and a differentiable feedback control law

$$\begin{aligned} \tilde{\theta} &= \sin^{-1} \left\{ \sin \alpha_2 + \frac{V_y}{v_r} \right\} \\ &= \alpha_3 \end{aligned} \quad (3.104)$$

renders the derivative of $\tilde{V}_1$ to satisfy condition (3.97). Similarly, by defining $z = \tilde{\theta} - \alpha_3$, we show that there exists a positive definite function

$$V_2(\tilde{\xi}, z) = V_1 + \frac{1}{2} z^2 \quad (3.105)$$

and a control law ω such that $(\tilde{\xi}, z) \rightarrow 0$ as $t \rightarrow \infty$ based on integrator backstepping. On the other hand, it is easy to show that the robot is not posture tracking solvable. Since Theorem 3.4.4 indicates that a robot with M2 cannot control its position without compromising the robot's orientation θ if δ_2 is nonzero; we conclude that the orientation of the mobile robot cannot approach θ_r when the robot's reference point converges to the desired reference point trajectory (x_r, y_r) .

As for the robots with M3, the proof is straightforward after applying Lemma 3.4.5. Since the robot is input equivalent to a nominal Type (2,0) robot, there exists tracking controllers that were designed based on non-skidding and non-slipping assumptions [5, 10, 18]; therefore, a mobile robot with M3 is posture tracking solvable and this completes the proof.

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□

Theorem 3.5.1 implies that there exists a control input for a robot with M2 such that the point error converges to zero if the unmatched perturbation V_y satisfies equality (3.104). In addition, the results also indicate the robot's orientation has to be "compromised" to achieve zero point tracking error if the skidding perturbation δ_2 is nonzero. Achieving good point tracking performance is desirable and in many practical cases is sufficient. In some cases, orientation error is as important as the point tracking error. The following results provide a measure on the orientation error while the point tracking error converges to zero.

Theorem 3.5.2 Suppose that there exists a continuously differentiable control input $\mathbf{U}(t)$ such that the point tracking error $\tilde{\xi}$ converges to zero. Then,

1. the orientation error $\tilde{\theta}$ of a wheeled mobile robot with M2 satisfies

$$\lim_{t \rightarrow \infty} (\tilde{\theta}(t) - \delta_2(t)) = 0. \quad (3.106)$$

2. the orientation error $\tilde{\theta}$ of a wheeled mobile robot with M3 satisfies

$$\lim_{t \rightarrow \infty} (\tilde{\theta}(t) - \gamma_2(t) - \delta_2(t)) = 0. \quad (3.107)$$

Proof: We first consider the case of a robot with M2. By assumption, the point tracking error $\tilde{\xi}$ approaches zero and $v_r, \dot{v}_r, \omega_r$ of the reference trajectory are bounded. The differentiability of the control input $\mathbf{U}$ guarantees the boundedness of $\ddot{\xi}$; hence $\dot{\xi}$ is uniformly continuous. By Barbalat lemma (see Lemma 2.4.2), we conclude that $\dot{\xi} \rightarrow 0$ as $t \rightarrow \infty$. One essential observation is that for any well-defined control input, the error dynamics of $\tilde{y}$ is governed by

$$\dot{\tilde{y}} = -\tilde{x}\omega + v_r \sin \tilde{\theta} - V_y, \quad (3.108)$$

and $(\tilde{\xi}, \dot{\tilde{\xi}}) \rightarrow 0$ implies

$$\lim_{t \rightarrow \infty} \left(\sin \tilde{\theta} - \frac{V_y}{v_r} \right) = 0. \quad (3.109)$$

Since $\dot{\tilde{\xi}} \rightarrow 0$ implies $V \rightarrow v_r$ as $t \rightarrow \infty$, geometric relation (3.1) leads equation (3.109) to

$$\lim_{t \rightarrow \infty} (\tilde{\theta}(t) - \delta_2(t)) = 0. \quad (3.110)$$

We can also show that $(\tilde{\xi}, \dot{\tilde{\xi}}) \rightarrow 0$ as $t \rightarrow \infty$ for a robot with M3. The error dynamic equation $\dot{\tilde{y}} = -\tilde{x}\omega + v_r \sin \tilde{\theta} - V \sin(\gamma_2 + \delta_2)$ of a M3 robot implies

$$\lim_{t \rightarrow \infty} \left(\sin \tilde{\theta} - \frac{V \sin(\gamma_2 + \delta_2)}{v_r} \right) = 0. \quad (3.111)$$

Similarly, $V \rightarrow v_r$ as $t \rightarrow \infty$. Hence, (3.111) leads to (3.107), and this completes the proof.

Theorem 3.5.2 indicates the orientation error of a robot with M2 tends to the slip angle δ_2 as $t \rightarrow \infty$. On the other hand, a robot with M3 does not have this limitation if the steerable wheel γ_2 is properly designed to compensate against perturbation δ_2 . Equality (3.107) suggests that the steady-state orientation error can be completely eliminated if the steering angle γ_2 is chosen such that the steady-state orientation error approaches zero. These findings show that it is impossible for a robot with M2 to solve the posture tracking problem if the slip angle δ_2 is nonzero. On the other hand, there exists a control input $\mathbf{U}(t)$ such that the posture tracking error of a robot with M3 converges to zero.

3.6 Concluding remarks

A set of kinematic models for four generic wheeled mobile robots has been developed to study the behavior of the robot in the presence of wheel skidding and slipping from a control perspective. By using explicit descriptions to describe the kinematic perturbations induced by wheel skidding and slipping, the kinematic models lead us to the classification of disturbance perturbations and associated properties. The developed models are unified for the four configurations of wheeled mobile robots. The perturbations are classified in the perspective of control designs; in particular, we have various types of the perturbations and we offer some comparisons as follows.

Among these perturbations, input-related (input-additive, input-multiplicative) and matched perturbations reduce the effectiveness of control inputs. But these types of perturbations can be easily compensated if the measurements of these uncertainties become available. On the other hand, an unmatched perturbation cannot be compensated by a control input directly. The existence of the unmatched perturbation in lower maneuverability mobile robots suggests that robust control approach is not advisable. On the other hand, even though there is no unmatched perturbation for the case of a robot with M3, the non-affine input structure of these mobile robots suggest that robust control approach is also not recommended.

Despite the difficulties resulted from the unmatched perturbation of a mobile robot with M2, Theorem 3.5.1 shows that the unmatched perturbation can still be compensated using the robot's orientation to control the robot's reference point if the measurement of the unmatched perturbation is available. This result suggests that *backstepping* control methodology is useful

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to design a controller to achieve point tracking maneuver if the information of the kinematics perturbations due to wheel skidding and slipping is available. Besides the possible control strategy that has been gained from these results, we also provide a measure on the orientation error for the wheeled mobile robot during point tracking maneuver.

From this development, we show that a mobile robot with higher maneuverability is more controllable than a mobile robot with lower maneuverability. These results have important implications on formulating control objectives for each class of mobile robots. Finally, the explicit descriptions of the kinematic perturbations shed light and motivation on the types of navigation sensors suitable for this control application.

Chapter 4

Integrated Estimation for Wheeled Mobile Robot posture, velocities, perturbations

4.1 Introduction

As we have fore-shadowed in Chapter 1 that it is critical to provide continuous control of a mobile robot during short GPS outages of GPS signals. The ability of an inexpensive dead-reckoning using low-cost inertial sensors to provide posture, velocities, and perturbations estimation is exploited during the short GPS outages. This chapter presents estimators to provide these information at high-update rate using Real-time Kinematic Global Positioning System (RTK-GPS) and inertial sensors for mobile robot control in the presence of wheel skidding and slipping. The estimators that based on Kalman Filtering combine the inertial sensors with centimeter accuracy RTK-GPS measurements to provide essential posture, velocities, and perturbation estimates. These estimators are applicable to the four wheeled mobile robots. Under some conditions, a simplified estimator is proposed using cascaded structure to estimate the information. The experimental results suggest that with careful modelling of mobile robots, the estimators are able to provide reliable and high-update rate information for wheeled mobile robots control applications in the presence of wheel skidding and slipping.

This chapter firstly describes the development of the estimation model based on the kinematic models presented in Chapter 3. Secondly, Extended Kalman filter is applied to estimate a robot's posture, velocities and perturbations. Thirdly, a simplified estimation algorithm is presented. Finally, experimental results and discussions will end the chapter.

4.2 Sensors

In this section, we provide a brief description about inertial measurement unit (IMU) and GPS that are used as the key sensors for the estimators. For further details on these sensors, the reader may refer to [60, 92].

4.2.1 Inertial measurement unit

An IMU unit consists of linear accelerometers and gyroscopes for general measurements of linear acceleration and angular rate in dynamic environments. An IMU provides high-update rate measurements which are not affected by wheel skidding and slipping. IMU has its limitations. The major sources of errors are the sensor bias which drifts with time, and the inaccurate readings caused by misalignment of the unit. Another factor that contributes to erroneous measurement is the sensitivity error of the IMU. Many different error models have been proposed to model the errors of inertial sensors [58, 59]. Since the focus of this chapter is not on error modeling of IMU, we assume the bias is known and constant during operations.

4.2.2 Global positioning system

GPS is an external sensor that provides absolute positioning information with bounded errors. A GPS receiver relies on either the code phase or the carrier phase measurements received from several satellites that are not geostationary. Knowing the georeference (absolute satellite position) from the satellite signals, the antenna's position of the GPS receiver can be calculated based on the range information between the receiver and the satellites. Four or more satellite signals are required to compute the position of the receiver.

GPS measurement suffers from erroneous delays along the paths from the satellites and these delays lead to measurement noise in measurement readings. There are two forms of GPS differential technologies to overcome these errors: standard code phase differential GPS (DGPS) and Carrier phase differential GPS (RTK-GPS). In these differential modes, one GPS receiver is used as a base station, and another is used as a rover station where its location is to be determined by the user. Common errors due to ionospheric and tropospheric effects are cancelled using information transmitted from the base station to achieve a better absolute position measurement with respect to the base station. These corrections are transmitted via a radio modem from the base station to the rover receiver. Even when a GPS system operates in these

differential modes, the measurements still possess high-frequency faults when the GPS signals are distorted by multipath. This multipath error occurs when the GPS signals are reflected off from one or more surfaces before it reaches the receiver's antenna. In comparison with DGPS and RTK-GPS, RTK-GPS has a better accuracy than DGPS since the distance information is computed based on the carrier phase of the satellites carrier signals instead of the code phase modulated on the carrier signals. The typical accuracy of DGPS is 1.5-5 m whereas RTK-GPS has a typical accuracy of 0.01-0.05 m (depending on the quality of the GPS receiver).

Besides positioning information, a GPS receiver which operates with carrier-phase measurement is also capable of providing velocity measurements. The receiver determines velocity of the receiver based on the doppler shift of the GPS carrier wave. This velocity information includes the velocity orientation where its error variance is inversely proportional to the magnitude of the velocity [69]. The typical accuracy of the velocity measurements is between 0.01-0.05m/sec for each axis. In general, the accuracy of the GPS measurements (position or velocity) varies with satellite geometry and the receiver multi-path error.

4.3 Objectives

In brief, the estimation problem considered in this paper is to utilize the high-update rate inertial sensors and the low-update rate RTK-GPS measurements to estimate a robot's

$$\{x, y, \theta, V_l, V_y, d, \delta_1, \delta_2\} \quad (4.1)$$

at an update rate that the inertial sensors can provide.

4.4 Kinematic modeling for estimation

In this section, we derive a state-space model for Kalman filtering. Figure 4.1 shows a mobile robot moving on a 2-D surface. For convenience, we choose the inertial navigation frame to be the global coordinate frame $\{X, Y\}$. Without loss of generality, the accelerometer is placed at point P to measure the acceleration of point P . The sensor's sensitivity axes are aligned with the robot's body axes $\{X_b, Y_b\}$. Similarly, RTK-GPS receiver is also placed at point P to measure the position $\xi = (x, y)$ and velocity $\dot{\xi} = (\dot{x}, \dot{y})$ of the point with respect to the global coordinate frame by letting the Y be North and X be East.

Figure 4.1: Motion of a mobile robot on a 2-D surface

The velocity of a mobile robot can be expressed as

$$\dot{\xi} = A(\theta)\eta \quad (4.2)$$

where

$$A(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \quad \eta = \begin{bmatrix} V_l \\ V_y \end{bmatrix}. \quad (4.3)$$

To relate $\dot{\eta}$ with acceleration $\ddot{\xi}$, we differentiate equation (4.2) and we have

$$\begin{aligned} \ddot{\xi} &= \dot{A}\eta + A\dot{\eta} \\ &= \frac{\partial A}{\partial \theta}\dot{\theta}\eta + A\dot{\eta}, \end{aligned}$$

where

$$\frac{\partial A}{\partial \theta} = \begin{bmatrix} -\sin \theta & -\cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix}.$$

Since matrix A is an orthogonal matrix, we have

$$\begin{aligned} \dot{\eta} &= A^T \ddot{\xi} - A^T \frac{\partial A}{\partial \theta} \dot{\theta} \eta \\ &= u_{\bar{a}} + \begin{bmatrix} V_y \\ -V_l \end{bmatrix} \dot{\theta} \end{aligned} \quad (4.4)$$

$$= u_{\bar{a}} + \bar{P}\eta \quad (4.5)$$

where

$$\bar{P} = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}. \quad (4.6)$$

4.4. Kinematic modeling for estimation

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$u_{\bar{a}} = A^T \ddot{\xi}$ is the acceleration of point P expressed in the body frame and r denotes the robot's turning rate.

In this work, we assume that the slow varying biases of the accelerometers and gyroscope are constant and known. Hence without loss of generality, equation (4.5) can be written as

$$\dot{\eta} = \bar{P}\eta + u_a + w_a \quad (4.7)$$

$u_a = [a_x \ a_y]^T$ is the acceleration measurements of point P which is expressed in the robot's body frame. w_a denotes the accelerometers measurements noise which are assumed to be Gaussian white with a sampled variance of $\{\sigma_{a,x}^2, \sigma_{a,y}^2\}$. Similarly, we can relate the gyroscope rate measurement with the robot's orientation via

$$\dot{\theta} = r_m + w_r. \quad (4.8)$$

r_m represents the gyroscope rate measurement and w_r denotes gyroscope's measurement noise which is assumed to be zero-mean Gaussian white noise with sampled variance of σ_r^2 . Equations (4.2),(4.7),(4.8) can be written in the following state-space form.

$$\dot{\xi} = A(\theta)\eta \quad (4.9)$$

$$\dot{\eta} = \bar{P}\eta + u_a + w_a \quad (4.10)$$

$$\dot{\theta} = r_m + w_r \quad (4.11)$$

When GPS measurements (both position and velocities) are available, the observation equations that relate the measurements with the states (ξ, η) can be described by

$$z_x = x + v_x \quad (4.12)$$

$$z_y = y + v_y \quad (4.13)$$

$$z_{\dot{x}} = \cos \theta V_l - \sin \theta V_y + v_{\dot{x}} \quad (4.14)$$

$$z_{\dot{y}} = \sin \theta V_l + \cos \theta V_y + v_{\dot{y}} \quad (4.15)$$

where measurement noise $\{v_x, v_y, v_{\dot{x}}, v_{\dot{y}}\}$ are assumed to be zero-mean Gaussian white with sampled variance $\{\sigma_x^2, \sigma_y^2, \sigma_{\dot{x}}^2, \sigma_{\dot{y}}^2\}$. Similarly, the observation equation that relates an absolute orientation measurement with the state θ is simply

$$z_{\theta} = \theta + v_{\theta} \quad (4.16)$$

where v_{θ} is also zero-mean Gaussian white noise with sampled variance σ_{θ}^2 . Sensors that provide absolute orientation measurement are: magnetic compass, gyrocompass, and a two-antenna

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GPS system. Equations (4.9)-(4.11) and observation equations (4.12)-(4.16) lead to a continuous state-space model.

In the ideal case where there is no wheel skidding and slipping, the velocity orientation $\psi(t)$ is

$$\psi(t) = \begin{cases} \theta(t) & \text{for robot with M2} \\ \theta(t) + \gamma_2(t) & \text{for robot with M3} \end{cases} \quad (4.17)$$

This can be observed from equations (3.56) and (3.59) when the skidding parameters $\{V_y, \delta_2\}$ are zero. Note that γ_2 is a control input which is known or measurable. Equation (4.17) suggests that we can apply the velocity's orientation measurement provided by a GPS receiver to calibrate the orientation estimation error accumulated by integrating the mobile robot's turning rate. However, in the presence of wheel skidding where the slip angle δ_2 is nonzero, the velocity orientation is no longer equal to (4.17) and becomes

$$\psi(t) = \begin{cases} \theta(t) + \delta_2(t) & \text{for robot with M2} \\ \theta(t) + \gamma_2(t) + \delta_2(t) & \text{for robot with M3} \end{cases} \quad (4.18)$$

This relation suggests that δ_2 is not observable using $\psi(t)$ measurement alone. An additional orientation sensor is required to determine the slip angle $\delta_2(t)$. Therefore, in general, we cannot utilize ψ to calibrate the estimation orientation error accumulated by dead-reckoning. Nevertheless during any time interval where the robot is in a linear trajectory (straight motion) or is travelling at low speed, we could equate $\psi = \theta$ to reset the dead-reckoning error accumulated by integrating the robot's turning rate since wheel skidding is minimal in these maneuvers.

4.5 Wheeled Mobile Robot posture, velocities, and perturbation estimation system

Kalman Filter is a recursive estimator that produces optimum estimate in least square sense. There are two basic models in Kalman filtering: the *process model* or *state prediction model* which is governed by the system's dynamic equations, and the *observation model* which relates sensors measurements with the system states by measurement equations. In brief, Kalman filter combines both estimates from each model by choosing a suitable gain, namely Kalman gain based on the strengths of the process and measurement noise. This information leads the Kalman gain to weight more on observation information if the acquired measurements are

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accurate, or weights the state prediction model more if it is accurate. Readers may refer to [93] for details. The estimation scheme and the filter derivation are presented in the following subsections.

4.5.1 Estimation system

Figure 4.2: Structure of the Estimation Scheme

Figure 4.2 shows the architecture of the estimation system based on Kalman filtering. At time k , the states $\{x, y, \theta, V_l, V_y\}$ are estimated by the Extended Kalman Filter. This is achieved by using the inertial, RTK-GPS, and absolute orientation measurements.

At any time instant when there is no absolute observation, the state prediction predicts the states by integrating the kinematic model (4.9)-(4.11) using the high bandwidth inertial measurements. In this manner, the prediction estimates have a maximum update rate that the inertial sensors can offer. The rate of error growth depends on the quality of the inertial measurements. A high quality IMU can provide estimates using integration without any online error calibration for a longer duration. At any time instant when the low-update rate absolute GPS and orientation measurements are available, the integration errors accumulated by the integration are reset by the measurement update based on the respective absolute measurements and observation equations. Once the states $\{x, y, \theta, V_l, V_y\}$ are estimated, the scheme computes the skidding and slipping perturbations $\{\delta_1, \delta_2, d\}$ using control input $U(t)$ and the robot's kinematic model. The

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control inputs are usually known or measurable using high bandwidth sensors, e.g., incremental encoders and absolute encoder. In this way, the estimation system is able to provide posture, velocities, and perturbation estimates at an update rate that the inertial sensors can offer.

4.5.2 Discrete kinematic model

To implement the Extended Kalman Filter (EKF), we discretize the continuous model (4.9)-(4.11), where the absolute sensors observation equations are sampled at a regular interval. The discrete state-space model of equations (4.9)-(4.11) can be established using first order *Euler* integration and is written in the form (4.19). A higher order integration may be used to enhance the accuracy of the discrete model. k denotes the discrete time index.

The discretized state transition vector equation is

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, k) + \mathbf{w}_k \quad (4.19)$$

where

$$\mathbf{f}(\mathbf{x}_k, k) = \begin{bmatrix} x_k + \Delta t V_{l,k} \cos(\theta_k) - \Delta t V_{y,k} \sin(\theta_k) \\ y_k + \Delta t V_{l,k} \sin(\theta_k) + \Delta t V_{y,k} \cos(\theta_k) \\ V_{l,k} + \Delta t V_{y,k} r_k + \Delta t a_{x,k} \\ V_{y,k} - \Delta t V_{l,k} r_k + \Delta t a_{y,k} \\ \theta_k + \Delta t r_{m,k} \end{bmatrix}, \quad (4.20)$$

$\mathbf{x}_k = [x_k \ y_k \ V_{l,k} \ V_{y,k} \ \theta_k]^T$, and the process noise vector $\mathbf{w}_k = [0 \ 0 \ \Delta t w_{a,x} \ \Delta t w_{a,y} \ \Delta t w_r]^T$.

The time-varying parameters $\{a_{x,k}, a_{y,k}, r_{m,k}\}$ at time k are provided by the accelerometer and gyroscope. Δt denotes the sample time of the discrete system. We assume the instantaneous yaw rate r_k is measurable by a low-noise gyroscope; hence, we let $r_k = r_m$.

The observation vector $\mathbf{z}_k = [z_{x,k} \ z_{y,k} \ z_{\dot{x},k} \ z_{\dot{y},k} \ z_{\theta,k}]^T$ consists of absolute position, velocity and orientation readings. The absolute measurements vector $\mathbf{z}_k$ is related with the states $\mathbf{x}_k$ via the observation equations (4.12)-(4.16) sampled at time k which can be written as

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \quad (4.21)$$

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where

$$\mathbf{h}(\mathbf{x}_k) = \begin{bmatrix} x_k \\ y_k \\ \cos \theta_k V_{l,k} - \sin \theta_k V_{y,k} \\ \sin \theta_k V_{l,k} + \cos \theta_k V_{y,k} \\ \theta_k \end{bmatrix}, \quad (4.22)$$

and observation noise $\mathbf{v}_k = [v_{x,k} \ v_{y,k} \ v_{\dot{x},k} \ v_{\dot{y},k} \ v_{\theta,k}]^T$.

4.5.3 State Prediction

The state prediction $\hat{\mathbf{x}}_{k+1}^-$ based on information up to time k is given by

$$\hat{\mathbf{x}}_{k+1}^- = \mathbf{f}(\hat{\mathbf{x}}_k, k). \quad (4.23)$$

Note that $\hat{\mathbf{x}}_k$ is the best estimate at time k . The error covariance between the true state and the predicted state $\hat{\mathbf{x}}_{k+1}^-$ is given by

$$\mathbf{P}_{k+1}^- = \nabla \mathbf{f}(\hat{\mathbf{x}}_k) \mathbf{P}_k \nabla \mathbf{f}^T(\hat{\mathbf{x}}_k) + \mathbf{Q}_k. \quad (4.24)$$

$\nabla \mathbf{f}(\cdot)$ denotes the Jacobian of $\mathbf{f}(\cdot)$ at time k , and $\mathbf{Q}_k$ is the covariance matrix of the discretized noise vector $\mathbf{w}_k$. $\nabla \mathbf{f}(\mathbf{x}_k)$ and $\nabla \mathbf{h}(\mathbf{x}_k)$ are given by equations

$$\nabla \mathbf{f}(\mathbf{x}_k) = \begin{bmatrix} 1 & 0 & \Delta t \cos \theta_k & -\Delta t \sin \theta_k & -\Delta t V_{l,k} \sin \theta_k - \Delta t V_{y,k} \cos \theta_k \\ 0 & 1 & \Delta t \sin \theta_k & \Delta t \cos \theta_k & \Delta t V_{l,k} \cos \theta_k - \Delta t V_{y,k} \sin \theta_k \\ 0 & 0 & 1 & \Delta t r_k & 0 \\ 0 & 0 & -\Delta t r_k & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.25)$$

$$\nabla \mathbf{h}(\mathbf{x}_k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cos \theta_k & -\sin \theta_k & -\sin \theta_k V_{l,k} - \cos \theta_k V_{y,k} \\ 0 & 0 & \sin \theta_k & \cos \theta_k & \cos \theta_k V_{l,k} - \sin \theta_k V_{y,k} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.26)$$

where $\mathbf{Q}_k = \mathbf{diag}\{0, 0, \Delta t^2 \sigma_{a,x}^2, \Delta t^2 \sigma_{a,y}^2, \Delta t^2 \sigma_r^2\}$. Note that the best estimate $\hat{\mathbf{x}}_k$ and $\mathbf{P}_k$ are chosen as $\hat{\mathbf{x}}_k^+$ and $\mathbf{P}_k^+$ if there are absolute measurements $\mathbf{z}_k$ available at time k . On the other hand, $\hat{\mathbf{x}}_k$ and $\mathbf{P}_k$ are chosen as $\hat{\mathbf{x}}_k^-$ and $\mathbf{P}_k^-$ if there is no measurement $\mathbf{z}_k$ available at time k .

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4.5.4 Measurement Update

Assuming that there is a predicted state $\hat{\mathbf{x}}_k^-$, we have the predicted observation

$$\hat{\mathbf{z}}_k = \mathbf{h}(\hat{\mathbf{x}}_k^-). \quad (4.27)$$

Suppose the absolute measurements $\mathbf{z}_k$ at time k are available, the prediction observation error which is also known as residual is defined as $\alpha_k = \mathbf{z}_k - \hat{\mathbf{z}}_k$ with the covariance

$$\mathbf{S}_k = \nabla \mathbf{h}(\hat{\mathbf{x}}_k^-) \mathbf{P}_k^- \nabla \mathbf{h}^T(\hat{\mathbf{x}}_k^-) + \mathbf{R}_k. \quad (4.28)$$

The Jacobian $\nabla \mathbf{h}(\cdot)$ is given by equation (4.26), where the observation covariance is

$\mathbf{R}_k = \text{diag}\{\sigma_x^2, \sigma_y^2, \sigma_{\dot{x}}^2, \sigma_{\dot{y}}^2, \sigma_\theta^2\}$. The state estimate and covariance update equations of a EKF is as follows:

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k(\alpha_k) \quad (4.29)$$

where $\mathbf{K}_k$ is the Kalman gain given by

$$\mathbf{K}_k = \mathbf{P}_k^- \nabla \mathbf{h}^T(\hat{\mathbf{x}}_k^-) \mathbf{S}_k^{-1} \quad (4.30)$$

and the state estimate covariance matrix is

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \nabla \mathbf{h}(\hat{\mathbf{x}}_k^-)) \mathbf{P}_k^-. \quad (4.31)$$

To improve the reliability of the estimation scheme, a validation mechanism under the framework of Kalman filter can be applied to inspect the absolute measurements $\mathbf{z}_k$ so that occasional bad measurements will not affect the estimates [60]. This approach has been implemented to improve the reliability of the estimation system [59]. Here, we may also implement this validation mechanism to increase the integrity of the estimator. To implement the validation gate, a threshold $\lambda_{i,k}$ which to be selected by the designer such that

$$\frac{\alpha_{i,k}^2}{\sigma_{i,k}^2} > \lambda_{i,k} \quad (4.32)$$

where $\alpha_{i,k}$ is the i th element of α_k and $\sigma_{i,k}^2$ is the estimate of the variance of the $\alpha_{i,k}$. Note that value of $\frac{1}{\sigma_{i,k}^2}$ is the i th diagonal element of $\mathbf{S}_k^{-1}$ [94].

If condition (4.32) is satisfied, the corresponding measurement will be declared as faulty measurement. Note that the parameter $\lambda_{i,k}$ should be tuned by experimental trials. In the case where the measurements $\mathbf{z}_k$ are not available or faulty, the estimator simply applies state prediction to estimate the states using inertial measurements until the next absolute measurement $\mathbf{z}_k$ arrives.

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4.5.5 Perturbation computation

With the knowledge of $\hat{\mathbf{x}}_k$, whether is by state prediction or by measurement update, we can make use of the estimate and the kinematic model of each robot to compute the kinematic perturbations $\{d, \delta_1, \delta_2\}$ at time k using the following equations.

Type (2,0) robot

$$\hat{\delta}_2 = \tan^{-1} \frac{\hat{V}_y}{\hat{V}_l} \quad (4.33)$$

$$\hat{d} = r_w \dot{\varphi} - \hat{V}_l \quad (4.34)$$

$$\hat{\delta}_1 = r_m - \gamma_1 \quad (4.35)$$

Type (1,1) robot

$$\hat{d} = r_w \dot{\varphi} - \hat{V}_l \quad (4.36)$$

$$\hat{\delta}_1 = \tan^{-1} \left(\frac{r_m a + \hat{V}_y}{\hat{V}_l} \right) - \gamma_1 \quad (4.37)$$

$$\hat{\delta}_2 = \tan^{-1} \frac{\hat{V}_y}{\hat{V}_l} \quad (4.38)$$

Type (2,1) robot

$$\hat{\delta}_2 = \tan^{-1} \frac{\hat{V}_y}{\hat{V}_l} - \gamma_1 \quad (4.39)$$

$$\hat{d} = r_w \dot{\varphi} - \hat{V} \cos \hat{\delta}_2 \quad (4.40)$$

$$\begin{aligned} \hat{\delta}_1 = \tan^{-1} \left\{ \frac{\hat{V} \sin(\gamma_2 + \delta_2 - \alpha)}{\hat{V} \cos(\gamma_2 + \delta_2 - \alpha) + a r_m \sin \alpha} \right. \\ \left. + \frac{-a r_m \cos \alpha - (\gamma_1 + r_m) b}{\hat{V} \cos(\gamma_2 + \delta_2 - \alpha) + a r_m \sin \alpha} \right\} \end{aligned} \quad (4.41)$$

Type (1,2) robot

$$\hat{\delta}_2 = \tan^{-1} \frac{\hat{V}_y}{\hat{V}_l} - \gamma_1 \quad (4.42)$$

$$\hat{d} = r_w \dot{\varphi} - \hat{V} \cos \hat{\delta}_2 \quad (4.43)$$

$$\hat{\delta}_1 = \tan^{-1} \left(\frac{r_m a + \hat{V}_y}{\hat{V}_l} \right) - \gamma_1 \quad (4.44)$$

$\hat{V}$ is simply determined by taking the magnitude of its components $\{\hat{V}_l, \hat{V}_y\}$. These angular terms $\{r_w \dot{\varphi}, \gamma_1, \gamma_2, \alpha\}$ of these mobile robots are assumed to be known or measurable. They can be easily measured by high bandwidth sensors, e.g., absolute encoder and incremental encoder.

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4.5.6 Estimation using absolute GPS position only

In the preceding section, we assume that the RTK-GPS receiver is able to provide both position and velocity measurements. Here, we consider the case where the GPS receiver only provides absolute positioning information. This may be due to insufficient interfacing I/O ports either on the receiver or PC, or it may be due to an interface malfunction where only positioning information is available. In this case, we may still estimate the velocity based on reduced observation equations (4.12), (4.13) and (4.15). However, if the robot is maneuvering at low speed and the sample time Δt is small, then the velocity estimate based on position measurements is noisy. This is because the velocity estimate is based on numerical differentiation although this operation is implicitly performed by the filter. As a result, the signal to noise ratio of the velocity estimate is low.

One ad hoc solution to mitigate this problem is to differentiate the position fixes explicitly using a time interval larger than Δt and use the differentiated velocity as fictitious velocity measurements to calibrate the integration error due to the state prediction. The following analysis shows the noise characteristic of the approximated velocity. Without loss of generality, we only consider the X axis since the analysis applies to Y axis as well. The approximated velocity computed by backward differencing based on Δt_1 interval samples is

$$\hat{x}_{k_1} = \frac{1}{\Delta t_1} \{ \tilde{x}_{k_1} - \tilde{x}_{k_1-1} \} \quad (4.45)$$

where $\Delta t_1 > \Delta t$. $\tilde{x}_{k_1}$ is the measured position at time k_1 . In the presence of the positioning white measurement noise v_{x,k_1} , with a variance σ_x^2 , equation (4.45) becomes

$$\begin{aligned} \hat{x}_{k_1} &= \frac{1}{\Delta t_1} \{ x_{k_1} + v_{x,k_1} - \tilde{x}_{k_1-1} + v_{x,k_1-1} \} \\ &= \frac{1}{\Delta t_1} \{ \Delta x_{k_1} \} + \bar{v}_{x,k_1} \end{aligned} \quad (4.46)$$

where $\Delta x_{k_1} = x_{k_1} - x_{k_1-1}$ and the approximation error due to the measurement noise is $\bar{v}_{x,k_1} = \{v_{x,k_1} + v_{x,k_1-1}\}/\Delta t_1$. Since v_{x,k_1} and v_{x,k_1-1} have the same variance, zero mean, and uncorrelated, $\bar{v}_{x,k_1}$ is also white with a variance of

$$\sigma_v^2 = \frac{4}{\Delta t_1^2} \sigma_x^2. \quad (4.47)$$

Equation (4.47) shows that the velocity approximation error due to positioning measurement noise has a $\frac{4}{\Delta t_1^2}$ variance gain. The equation also suggests that by choosing a suitable Δt_1 , we

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can reduce the amplification of noise and let the approximated velocity $\hat{x}_{k_1}$ be the fictitious absolute velocity measurement with variance (4.47). On the other hand, the interval time Δt_1 must not be large, because $\dot{x}_{k_1}$ may not be well approximated by $\Delta x_{k_1}/\Delta t_1$. In general, Δt_1 should be chosen based on the speed of vehicle so that the approximated velocity has an acceptable signal to noise ratio for control purposes.

Figure 4.3: Approximated orientation error by position fixes

Similarly, we can also approximate the velocity orientation ψ when the robot is in motion based on two position fixes. Figure 4.3 shows a worst case error analysis of the approximated orientation based on the two position fixes. The circle represents the possible positioning reading from the GPS receiver at time k_1 . The length of the circle's radius represents the accuracy of a point measurement. l_1 denotes the distance travelled between the two chosen samples which is approximated by $l \approx v_{k_1} \Delta t_1$. Without loss of generality, we choose $\psi = 0$; hence we have

$$\begin{aligned}
 \hat{\psi}_{k_1} &= \tan^{-1} \left\{ \frac{\tilde{y}_{k_1} - \tilde{y}_{k_1-1}}{\tilde{x}_{k_1} - \tilde{x}_{k_1-1}} \right\} \\
 \Rightarrow \psi_e &= \tan^{-1} \left\{ \frac{0 + v_{x,k_1} - 0 + v_{x,k_1-1}}{l_1 + v_{y,k_1} + 0 + v_{y,k_1-1}} \right\} \\
 &\approx \frac{v_{x,k_1} + v_{x,k_1-1}}{l_1 + v_{y,k_1} + v_{y,k_1-1}} \\
 &\approx \frac{v_{x,k_1} + v_{x,k_1-1}}{l_1}
 \end{aligned} \tag{4.48}$$

$$\tag{4.49}$$

Equation (4.49) shows the approximated orientation error of $\hat{\psi}_{k_1}$. It is clear that the longer the distance l_1 , the smaller the error ψ_e . The error also has a white noise characteristics with a variance of $\frac{4}{l^2} \sigma_x^2$. Similarly, Δt_1 must also not be large in order to achieve a good approximate of the absolute orientation ψ .

To apply these fictitious velocity and orientation approximations to the proposed estimation scheme, we simply let these approximated values be the absolute measurements $\{z_{\dot{x},k_1}, z_{\dot{y},k_1}, z_{\theta,k_1}\}$

and apply the measurement update using the respective measurement equations to reset the accumulating integration error due to the state prediction at every Δt_1 interval when the approximations are performed.

Naturally, estimation using both position and velocity measurements is expected to have a better performance than using positioning measurement only; nevertheless, the approach provides an ad hoc solution to estimate the vehicle posture, velocities, and perturbations.

4.6 A simplified estimation system

Figure 4.4: Simplified estimation scheme

In Section 4.5, a suboptimal EKF is applied to the nonlinear kinematic model to estimate the states $\{x, y, \theta, V_l, V_y\}$. By inspection, we see that if the instantaneous robot's orientation θ is known, then states equations (4.9) and (4.10) form a time-varying linear system. This motivates us to decouple the orientation subsystem from the position and velocity subsystem and estimate the states using two linear Kalman filters. In this way, we can perform the filtering with a lower computation load and also optimal estimation can be achieved. Figure 4.4 shows the simplified estimation scheme. Assuming that the orientation estimate is accurate, the kinematic model (4.9)-(4.11) are decoupled into two state-space models for estimating the states $\{x, y, \theta, V_l, V_y\}$. The first model, which is denoted by *System 1*, consists of equation (4.11) and measurement equation (4.16). The second model named *System 2*, consists of equations (4.9)-(4.10) and observation equations (4.12)-(4.15). At every time instant k , Kalman filter (KF1) estimates the

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robot's orientation $\hat{\theta}$. With this information, another linear Kalman filter (KF2) is applied to System 2 assuming that $\theta = \hat{\theta}$. Once the states are estimated by the filters, the perturbations can be computed using respective equations in section 4.5.5. The equations of the Kalman filters are presented in the following sections.

4.6.1 Discrete kinematic model

We rewrite the discretized state transition function (4.19) into two discretized state functions for System 1 and System 2.

System 1 is described by

$$\mathbf{X}_{1,k+1} = \mathbf{A}_{1,k}\mathbf{X}_{1,k} + \mathbf{B}_1\mathbf{u}_{1,k} + \mathbf{W}_{1,k} \quad (4.50)$$

where $\mathbf{X}_{1,k} = \theta_k$, $\mathbf{A}_{1,k} = 1$, $\mathbf{B}_1 = \Delta t$, $\mathbf{u}_{1,k} = r_{m,k}$, and $\mathbf{W}_{1,k} = \Delta tw_r$. The orientation measurement is related with the θ by

$$\mathbf{Z}_{1,k} = \mathbf{C}_1\mathbf{X}_{1,k} + \mathbf{V}_{1,k} \quad (4.51)$$

where the observation matrix is $\mathbf{C}_1 = 1$ and $\mathbf{V}_{1,k} = v_\theta$.

System 2 is described by

$$\mathbf{X}_{2,k+1} = \mathbf{A}_{2,k}\mathbf{X}_{2,k} + \mathbf{B}_2\mathbf{u}_{2,k} + \mathbf{W}_{2,k} \quad (4.52)$$

where $\mathbf{A}_{2,k}$ and $\mathbf{B}_2$ are given as

$$\mathbf{A}_{2,k} = \begin{bmatrix} 1 & 0 & \Delta t \cos \theta_k & -\Delta t \sin \theta_k \\ 0 & 1 & \Delta t \sin \theta_k & \Delta t \cos \theta_k \\ 0 & 0 & 1 & \Delta tr_k \\ 0 & 0 & -\Delta tr_k & 1 \end{bmatrix} \quad (4.53)$$

$$\mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (4.54)$$

We define state vector $\mathbf{X}_{2,k} = [x_k \ y_k \ V_{l,k} \ V_{y,k}]^T$, $\mathbf{u}_{2,k} = [a_{x,k} \ a_{y,k}]^T$, and the process noise vector $\mathbf{W}_{2,k} = [0 \ 0 \ \Delta tw_{a,x} \ \Delta tw_{a,y}]^T$. Similarly we assume that a low-noise gyroscope is used;

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hence, we let r_k of matrix $\mathbf{A}_{2,k}$ be $r_{m,k}$. The absolute GPS position and velocity measurements $\mathbf{Z}_{2,k} = [z_{x,k} \ z_{y,k} \ z_{\dot{x},k} \ z_{\dot{y},k}]^T$ are related with state $\mathbf{X}_{2,k}$ by

$$\mathbf{Z}_{2,k} = \mathbf{C}_2 \mathbf{X}_{2,k} + \mathbf{V}_{2,k} \quad (4.55)$$

where the observation matrix of this system is

$$\mathbf{C}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \theta_k & -\sin \theta_k \\ 0 & 0 & \sin \theta_k & \cos \theta_k \end{bmatrix} \quad (4.56)$$

and $\mathbf{V}_{2,k} = [v_{x,k} \ v_{y,k} \ v_{\dot{x},k} \ v_{\dot{y},k}]^T$. Note that in the following development, we let θ_k of matrix $\mathbf{A}_{2,k}$ be $\hat{\theta}_k$ computed by KF1.

4.6.2 State prediction

For $i = 1, 2$, the state prediction for the filters are described by

$$\hat{\mathbf{X}}_{i,k+1}^- = \mathbf{A}_{i,k} \hat{\mathbf{X}}_{i,k} + \mathbf{B}_i \mathbf{u}_{i,k} \quad (4.57)$$

with an error covariance of

$$\mathbf{P}_{i,k+1}^- = \mathbf{A}_i \mathbf{P}_{i,k} \mathbf{A}_i^T + \mathbf{Q}_i \quad (4.58)$$

where $\mathbf{Q}_1 = \triangle t^2 \sigma_r^2$ and $\mathbf{Q}_2 = \mathbf{diag}\{0, 0, \triangle t^2 \sigma_{a,x}^2, \triangle t^2 \sigma_{a,y}^2\}$. In the same way, the best estimate $\hat{\mathbf{x}}_{i,k}$ and $\mathbf{P}_{i,k}$ are chosen as $\hat{\mathbf{x}}_{i,k}^+$ and $\mathbf{P}_{i,k}^+$ if measurements $\mathbf{Z}_{i,k}$ are available at time k ; and $\hat{\mathbf{x}}_{i,k}$ and $\mathbf{P}_{i,k}$ are chosen as $\hat{\mathbf{x}}_{i,k}^-$ and $\mathbf{P}_{i,k}^-$ if there is no measurement $\mathbf{Z}_{i,k}$ available at time k .

4.6.3 Measurement Update

For $i = 1, 2$, we have the predicted observation

$$\hat{\mathbf{Z}}_{i,k} = \mathbf{C}_i \hat{\mathbf{X}}_{i,k}^- \quad (4.59)$$

With measurements $\mathbf{Z}_{i,k}$ at time k , the residual is $\alpha_{i,k} = \mathbf{Z}_{i,k} - \hat{\mathbf{Z}}_{i,k}$ with the covariance

$$\mathbf{S}_{i,k} = \mathbf{C}_i \mathbf{P}_{i,k}^- \mathbf{C}_i^T + \mathbf{R}_i \quad (4.60)$$

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where $\mathbf{R}_1 = \sigma_\theta^2$ and $\mathbf{R}_2 = \text{diag}\{\sigma_x^2, \sigma_y^2, \sigma_{\dot{x}}^2, \sigma_{\dot{y}}^2\}$. The state estimate and covariance update equations of the linear Kalman filters are

$$\mathbf{K}_{i,k} = \mathbf{P}_{i,k}^- \mathbf{C}_i^T \mathbf{S}_{i,k}^{-1} \quad (4.61)$$

$$\hat{\mathbf{X}}_{i,k}^+ = \hat{\mathbf{X}}_{i,k}^- + \mathbf{K}_{i,k} \boldsymbol{\alpha}_{i,k} \quad (4.62)$$

$$\mathbf{P}_{i,k}^+ = (\mathbf{I} - \mathbf{K}_{i,k} \mathbf{C}_i^T) \mathbf{P}_{i,k}^- \quad (4.63)$$

The measurement validation condition (4.32) can also be used to monitor occasional bad measurements $\mathbf{Z}_{i,k}$ for each filter. If the GPS receiver only provides positioning measurement, then the approximated velocity (4.45) and the approximated velocity's orientation (4.48) can be applied to this simplified Kalman filters during every time interval Δt_1 .

We can clearly see that by decoupling the state-space model (4.9)-(4.11) into two lower dimension linear models, we can apply Kalman filters with a lower computation load compared with the EKF. If the orientation estimate $\hat{\theta}$ computed by KF1 is closed to θ , then KF2 has a better performance than the suboptimal EKF since there is no linearization involves in the computation. This low error orientation estimation can be realized using accurate absolute orientation measurement sensors, e.g., a two-antenna GPS system provides absolute orientation measurement with a low error of ± 0.2 deg [71].

Besides the computational advantages, this simplification allows us to see that the state $\mathbf{X}_{2,k}$ is observable assuming that we only have orientation and position measurements $\{z_{\theta,k}, z_{x,k}, z_{y,k}\}$. To show this, we consider System 2 with the instantaneous orientation being known. The observation matrix without velocity measurement is

$$\bar{\mathbf{C}}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (4.64)$$

The state transition matrix of the discretized System 2 can be expressed as

$$\mathbf{A}_{2,k} = \begin{bmatrix} I_2 & \Delta t A(\theta) \\ 0 & I_2 + \Delta t P \end{bmatrix}. \quad (4.65)$$

I_2 denotes a 2×2 identity matrix. Let $\bar{\mathbf{Z}}_k = [z_{x,k} \ z_{y,k}]^T$ and observation equations of two consecutive measurements can be expressed as

$$\bar{\mathbf{Z}}_0 = \bar{\mathbf{C}}_2 \mathbf{X}_{2,0} \quad (4.66)$$

$$\bar{\mathbf{Z}}_1 = \bar{\mathbf{C}}_2 \mathbf{X}_{2,1} = \bar{\mathbf{C}}_2 \mathbf{A}_{2,0} \mathbf{X}_{2,0} \quad (4.67)$$

where

$$\begin{aligned}\bar{\mathbf{C}}_2 \mathbf{A}_{2,0} &= \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} I & \Delta t A(\theta) \\ 0 & I + \Delta t P \end{bmatrix} \\ &= \begin{bmatrix} I & \Delta t A(\theta) \end{bmatrix}.\end{aligned}\quad (4.68)$$

Let

$$\mathbf{O} = \begin{bmatrix} \bar{\mathbf{C}}_2 \\ \bar{\mathbf{C}}_2 \mathbf{A}_{2,0} \end{bmatrix} = \begin{bmatrix} I & 0 \\ I & \Delta t A(\theta) \end{bmatrix} \quad (4.69)$$

where its determinant $\det \mathbf{O} = \det(I) \det(\Delta t A(\theta_k))$ indicates that $\mathbf{O}$ is nonsingular since $\det(A(\theta)) \neq 0$ for $\forall \theta$. Therefore we conclude that the state $\mathbf{X}_{2,k}$ of system 2 is observable using orientation and position measurements. On the other hand, it is clear that the state is not observable if we only have velocity measurements since it is impossible to determine an initial position $\{x_0, y_0\}$ from velocity measurements. This short analysis highlights the importance of RTK-GPS positioning measurement in this estimation application.

This simplification comes with limitations. Since the approach requires accurate orientation measurement, it will increase the overall cost of the estimation system. If the orientation estimate is inaccurate due to low quality absolute orientation sensor, then the estimates computed by KF2 are no longer meaningful due to the discrepancy of the linear model (4.9) and (4.10). By comparing this simplified approach with the EKF, the states estimates and the approximated covariances computed by the EKF are meaningful even when $\hat{\theta}$ is not closed to θ .

4.7 Experiments

4.7.1 Experimental setup

Experiments were conducted in an open and flat area at our school. The open area was selected so that the effects of inertial sensors' bias due to gravitational effect were minimized and also to maintain consistent lines of sight between the receiver and the GPS satellites during the trial. The proposed algorithm was tested on a car-like mobile robot platform that is equipped with a high-performance RTK-GPS, a Fibre Optic Gyroscope (FOG), and a tri-axial accelerometer (see Appendix A). Additionally, an absolute encoder and incremental encoders are installed on the robot to measure the robot's steering angle and the wheels angular velocities. The GPS receiver is programmed to provide measurements at an update rate of 10Hz where the higher

Figure 4.5: NTU Campus

update rate sensors were sampled at 20Hz. The mobile robot is not equipped with an absolute orientation sensor that measures the robot's orientation θ . Since the robot is to maneuver at a low speed about $V_l = 1.0 \text{ ms}^{-1}$ where the rear slip angle δ_2 of the robot is negligible; hence, we let the velocity orientation ψ measurement provided by the GPS receiver be the robot's absolute orientation measurement z_θ for the estimators.

To apply the estimators effectively, the variance of the sensor noises are to be estimated. Since the inertial measurements are sampled using a 12 bits DAQ system. Additional measurement noise is incurred at the acquisition process. Hence, we examine the inertial data and the GPS measurements collected using the DAQ system when the robot is stationary. Figures 4.6 and 4.7 show the collected measurements from the sensors mounted on the vehicle. The error variance estimated from the data provides a guideline in choosing the covariance matrices for the estimation. Further fine tuning of the covariance matrices $\{\mathbf{Q}_k, \mathbf{R}_k\}$ may be required to compensate for unmodelled errors, for example the mild gravitational effects due to uneven ground.

4.7.2 Experimental results I

During the trial run, the robot maneuvered in a circular path while the sensors data was collected for the computations. Both velocity and position measurements are provided by the RTK receiver. Figure 4.8 shows the position and velocity estimates computed by the EKF. Figure 4.9

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Figure 4.6: GPS measurements at stationary

Figure 4.7: Inertial measurements at stationary

depicts the kinematic perturbation estimates computed by equations (4.36) and (4.37). These estimates have an update rate of 20Hz. The spikes occurred in $\hat{\delta}_1$ estimate during $0 \leq t \leq 3$ sec were due to the zero velocity when the robot was stationary. Note that we cannot measure orientation using GPS when the receiver is stationary. We can simply let $\hat{\delta}_1 = 0$ when the robot is stationary since there should have no wheel skidding when the robot is motionless. The sharp spikes of the $\hat{d}$ estimates were due to the high frequency incremental encoder measurement noise which can be eliminated by implementing a low-pass filter.

Figure 4.10 depicts the approximated variances of the estimates computed by the filter. The standard deviations of the position and velocity estimates were approximately

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Figure 4.8: Posture, velocities, and Perturbation estimates

Figure 4.9: Perturbation $\{d, \delta_1\}$ estimates

$\{\sigma_x = 0.0173 \text{ m}, \sigma_y = 0.0174 \text{ m}, \sigma_\theta = 0.02 \text{ rad}, \sigma_{V_l} = 0.048 \text{ ms}^{-1}, \sigma_{V_y} = 0.052 \text{ ms}^{-1}\}$. The bounds of these error variances indicate that the EKF is well-behaved and stable. Finally, Figure 4.11 presents the measurement residuals of the estimator. The whiteness and zero bias of these residuals indicate the consistency of the filter. The large spikes occurred during interval $0 \leq t \leq 3$ sec in the orientation measurement residual is mainly due to the initial noisy measurement z_θ when the robot is stationary. The sudden increments of the residuals during the run were caused by the GPS multipath errors when the robot crossed some of the palm trees along the path. A

Figure 4.10: Variances of the estimates

threshold level may be defined for the measurement validation condition to reject a faulty GPS measurements whenever the residual exceeds the pre-defined threshold level. The validation implementation will enhance the reliability of the estimation system, especially in areas where multipath error is dominant.

The experimental results demonstrate that the combination of inertial sensors and RTK-GPS increases the update rate of the estimates. Additionally, they show that the dominant perturbations in the trial run is the robot's front slip angle δ_1 compared with V_y . The strength of the longitudinal slippage d is also less significant compared with the front slip angle.

4.7.3 Experimental results II

This section presents the estimates using the same data collected in the run and we assumed the GPS receiver only provides positioning measurement. In the computation, we choose $\Delta t_1 = 0.5$ sec to provide the approximated velocities and orientation for the EKF using equations (4.47) and (4.48) (see Figure 4.12). Similarly, the large orientation estimation error during $0 \leq t \leq 3$ sec is because the robot was stationary.

Absolute information is used to calibrate the error drift accumulated by the state prediction.

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Figure 4.11: Measurement residuals

Figure 4.12: Approximated velocity and orientation ψ

The absolute position measurement is updated every 10Hz whereas the approximated orientation and velocity are updated at every 2Hz. Figures 4.13 and 4.14 depict the 20Hz estimates

Figure 4.13: Posture, velocities, and perturbation estimates

Figure 4.14: Perturbation $\{d, \delta_1\}$ estimates

computed by the EKF. In general, the estimates are close with the estimates computed in the previous case where doppler velocities are available for the user. The prominent saw-tooth characteristic of the velocity estimates (See Figure 4.13) is due to the larger integration error that grow between the 2Hz approximated velocity updates. From these results, we can see that estimation using both position and velocity measurements (section 4.7.2) performs better compared with estimation using only positioning measurement. Nevertheless, the results in this section

show that we can still estimate the posture, velocities, and perturbation information based on inertial and FIX-RTK position measurements.

4.7.4 Experimental results III: Estimation using simplified Kalman filters

Figure 4.15: Posture, velocities, and Perturbation estimates

Figure 4.16: Perturbation $\{d, \delta_1\}$ estimates

Figures 4.15-4.18 show that the fused data computed by the simplified Kalman filters presented in Section 4.6. Figures 4.15 and 4.16 depict the posture, velocities, and perturbation estimates. The computed 20Hz estimates are closely resemblance with the estimates computed using EKF in Section 4.7.2. Similarly, we may apply the measurement validation condition

Figure 4.17: Variances of the estimates

to each filter to reject any faulty absolute measurement to enhance the estimation scheme's reliability.

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Figure 4.18: Measurement residuals

4.8 Conclusions

This chapter has developed reliable and high-update rate estimators to provide critical posture, velocities, and kinematic perturbation estimates for control in the presence of wheel skidding and slipping for the mobile robots.

The kinematic estimators combine the low-update rate RTK-GPS measurements with high-update rate inertial measurements based on Kalman Filtering. In this way, the update rate of the estimates is now limited by the bandwidth of the inertial sensors instead of the low-update rate absolute measurements. The estimators are general and applicable to the four wheeled mobile robots to estimate the vehicle posture, velocities, and perturbations information for mobile robot control purposes. One advantage of these kinematic estimators is that the inertial parameters of the robot, which are usually unknown, are not required to estimate the kinematic perturbations. Another advantage is that the system can provide estimates using inertial sensors for short durations where the GPS signals are unavailable or corrupted by high frequency multipath error. With these high-update rate estimates, the mobile robot control laws that are designed to compensate the kinematic perturbations due to wheel skidding and slipping can be applied more effectively. The proposed estimation scheme may also be used in GPS-unfriendly environment as long as there exists an absolute position sensor which is able to provide sufficiently accurate positioning and velocity measurements. One example of such sensor is the NAV200 [95].

Chapter 5

GPS-based Path following Control

5.1 Introduction

In the last chapter, it has been demonstrated that a robot's posture, velocities, and perturbations can be determined using RTK-GPS and inertial sensors. In this chapter, these information are exploited to control the mobile robots in the presence of wheel skidding and slipping. A GPS-based path control scheme based on backstepping technique is proposed for the four generic wheeled mobile robots. The proposed control scheme uses information provided by the sensors to compensate the path following errors based on backstepping controllers. This chapter begins with the development of path following models and problem formulations based on the maneuverability of a mobile robot. Controllers are developed based on the path following models and the information provided by the RTK-GPS and inertial sensors. Firstly, path following controllers for maneuverability two mobile robots are developed. Secondly, the local path following problem is solved where the initial path following errors are sufficiently small. We also address the conditional global path following problem where initial errors are larger compared with the initial errors assumed in the local path following problem. Thirdly, these control laws are applied to higher maneuverability mobile robots to solve the path following problems. Simulation results and discussions are presented to verify the controllers. Additionally, the control laws are implemented on the car-like experimental platform. The experimental results and discussion will end the chapter.

Figure 5.1: Path following problem

5.2 Path following Control Formulation

Figure 5.1 depicts a mobile robot moving on a horizontal plane. The path following problem is to find a steering controller such that the mobile robot's reference point P follows a differentiable geometric curve C . M is a point on C which is closest to the robot's reference point P , s_p is the curvilinear coordinate of point M along C , $c(s_p)$ denotes the curvature of the path at the point, θ_d denotes the orientation of the tangent to C at point M , $\vec{n}_t$ denotes the tangent vector to the path at point M , $\vec{n}_n$ is the unitary vector normal to C at that point, l is the lateral deviation which is defined as $\overrightarrow{MP} = l\vec{n}_n$, and the orientation error is defined as $\tilde{\theta} = \theta - \theta_d$.

5.2.1 Path following problem for wheeled mobile robots with M2

This section presents the path following models for the mobile robots with M2, i.e., for each $(m, s) \in \{(2, 0), (1, 1)\}$. From Figure 5.1, it follows that

$$\dot{\theta}_d = \frac{\dot{s}_p}{\frac{1}{c(s_p)}} = \frac{V \cos(\tilde{\theta} + \delta_2)}{\frac{1}{c(s_p)} - l}. \quad (5.1)$$

5.2. Path following Control Formulation

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Based on equation (5.1), we can deduce

$$\dot{s}_p = \frac{V \cos(\tilde{\theta} + \delta_2)}{1 - c(s_p)l} \quad (5.2)$$

$$\dot{\theta}_d = \frac{V c(s_p) \cos(\tilde{\theta} + \delta_2)}{1 - c(s_p)l} \quad (5.3)$$

It is also clear from Figure 5.1 that

$$\dot{l} = V \sin(\tilde{\theta} + \delta_2). \quad (5.4)$$

As for $\dot{\tilde{\theta}}$, by utilizing equations (3.9), (3.19) and (5.3), we establish

$$\dot{\tilde{\theta}} = \dot{\theta} - \dot{\theta}_d = \omega_{m,s} - \frac{V_l c(s_p) \cos \tilde{\theta}}{1 - c(s_p)l} + \frac{V_y c(s_p) \sin \tilde{\theta}}{1 - c(s_p)l} \quad (5.5)$$

where

$$\omega_{2,0} = \gamma_1 + \delta_1 \quad (5.6)$$

$$\omega_{1,1} = \frac{V_l}{a} \tan(\gamma_1 + \delta_1) - \frac{V_y}{a}. \quad (5.7)$$

Equation (5.4) suggests that the orientation error does not converge to zero if the lateral deviation error $l \rightarrow 0$ with any nonzero δ_2 . This implies that we should not formulate a path following problem such that both errors $(l, \tilde{\theta})$ converge to zero with a nonzero δ_2 . Nevertheless, equations (5.4) and (5.5) suggest that we can achieve $l \rightarrow 0$ with a well-behaved orientation error $\tilde{\theta}$. Additionally, it is natural to demand the steady-state path following errors $(l, \tilde{\theta}) \rightarrow 0$ when $\delta_2 \rightarrow 0$. With these observations in mind, we state the control objectives as follows.

Definition 5.2.1 *Local path following control problem for wheeled mobile robots with M2:*

Given a desired curve C , the path following control problem is to find control law for γ_1 such that, for small initial errors $\{l(t_0), \tilde{\theta}(t_0)\}$,

1. l and $\tilde{\theta}$ are uniformly bounded.
2. l converges to zero as $t \rightarrow \infty$.
3. $\tilde{\theta}$ converges to a neighborhood containing $\tilde{\theta} = 0$ as $t \rightarrow \infty$. Moreover, the steady-state orientation error $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0$ if $\lim_{t \rightarrow \infty} \delta_2(t) = 0$.

Definition 5.2.2 *Conditional global path following control problem for wheeled mobile robots*

with M2: Given a desired curve C , the path following control problem is to find control law for γ_1 such that for large initial errors $\{l(t_0), \tilde{\theta}(t_0)\}$,

5.2. Path following Control Formulation

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1. l and $\tilde{\theta}$ are uniformly bounded.
2. l converges to zero as $t \rightarrow \infty$.
3. $\tilde{\theta}$ converges to a neighborhood containing $\tilde{\theta} = 0$ as $t \rightarrow \infty$. Moreover, the steady-state orientation error $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0$ if $\lim_{t \rightarrow \infty} \delta_2(t) = 0$.

5.2.2 Path following problem for wheeled mobile robots with M3

Similarly, the path following models for the mobile robots with M3, i.e., for each $(m, s) \in \{(2, 1), (1, 2)\}$, are derived as follows.

$$\dot{s}_p = \frac{V \cos(\tilde{\theta} + \gamma_2 + \delta_2)}{1 - c(s_p)l} \quad (5.8)$$

$$\dot{l} = V \sin(\tilde{\theta} + \gamma_2 + \delta_2) \quad (5.9)$$

$$\dot{\tilde{\theta}} = \omega_{m,s} - \frac{V_l c(s_p) \cos \tilde{\theta}}{1 - c(s_p)l} + \frac{V_y c(s_p) \sin \tilde{\theta}}{1 - c(s_p)l} \quad (5.10)$$

where

$$\omega_{2,1} = \frac{V \sin(\gamma_2 + \delta_2 - \alpha - \delta_1) - \gamma_1 b \cos(\delta_1)}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)} \quad (5.11)$$

$$\begin{aligned} \omega_{1,2} &= \frac{V}{a} \tan(\gamma_1 + \delta_1) \cos(\gamma_2 + \delta_2) \\ &\quad - \frac{V}{a} \sin(\gamma_2 + \delta_2). \end{aligned} \quad (5.12)$$

Unlike the robots with M2, equation (5.9) reveals that the additional control input γ_2 of the mobile robots with M3 allows the path following errors $(l, \tilde{\theta})$ to achieve zero convergence for nonzero slip angle δ_2 . This observation leads to the following control objectives.

Definition 5.2.3 *Local path following control problem for wheeled mobile robots with M3:*

Given a desired curve C , the path following control problem is to find control laws for γ_1 and γ_2 such that, for small initial errors $\{l(t_0), \tilde{\theta}(t_0)\}$,

1. l and $\tilde{\theta}$ are uniformly bounded.
2. l and $\tilde{\theta}$ converge to zero as $t \rightarrow \infty$.

Definition 5.2.4 *Conditional global path following control problem for wheeled mobile robots with M3:* Given a desired curve C , the path following control problem is to find control laws for γ_1 and γ_2 such that for large initial errors $\{l(t_0), \tilde{\theta}(t_0)\}$,

1. l and $\tilde{\theta}$ are uniformly bounded.
2. l and $\tilde{\theta}$ converge to zero as $t \rightarrow \infty$.

The *path following control problems for wheeled mobile robots with M3* demand the path following errors $(l, \tilde{\theta})$ converges to zero, whereas the *path following problems for wheeled mobile robots with M2* aim to achieve zero convergence for the lateral deviation error with a well-behaved orientation error. We formulate stricter control problems for mobile robots with M3 because they are equipped with higher number of control inputs compared with the M2 case. Nevertheless, these control problems are physically meaningful and the solutions to these problems are useful in real practices.

In the remainder of this thesis, we assume the following conditions.

Assumption 5.2.1 $\{\delta_1, \delta_2, d, V_l\}$ are uniformly bounded with $|\delta_1| < \rho_1$, $|\delta_2| < \rho_2 < \frac{\pi}{4}$, $|d| < \rho_3$, and $0 < |V_l| < \rho_4$ where $\rho_i, i=1,2,3,4$ are positive constants.

Assumption 5.2.2 The derivatives $\{\dot{V}_y, \dot{V}_l\}$ are uniformly bounded.

Note that the uniformly boundedness of δ_2 and V_l imply the lateral velocity V_y is also uniformly bounded since $V_y = V_l \tan \delta_2$. These assumptions on the wheels' slip angles, longitudinal slip velocity, linear velocity and their derivatives are always true in practice. They lay a base for the following control development.

5.3 GPS-based Path following Control

5.3.1 GPS-based Control scheme

In this chapter, the objective is to propose a practical control scheme to solve the path following problem without imposing restrictive assumptions. To achieve this objective, we utilize exteroceptive RTK-GPS and other aiding sensors to determine the skidding and slipping perturbations for compensation. As we have discussed in Chapter 4 that a robot's posture, velocities, and perturbations can be estimated/measured using the sensors. This information is fed to the control laws to perform efficient and precise control in the presence of wheel skidding and slipping perturbations. Figure 5.2 summarizes the control scheme's structure.

Figure 5.2: GPS-based Control scheme

5.3.2 Control design for mobile robots with M2

This section presents the control designs for the robots with M2. The additive nature of $\{\delta_1, d\}$ suggests that they can be compensated by providing suitable amount of additional control inputs. On the other hand, the unmatched characteristic of V_y implies the perturbation cannot be compensated directly from the control inputs. Fortunately, it has been shown by Theorem 3.5.1 that V_y can be compensated through suitable orientation adjustment. This corrective maneuver can be translated into control laws using *integrator backstepping* methodology.

The control objective is to achieve uniformly bounded and exponentially-converging path following errors using the robot's posture, velocities and perturbations information. Since we have this information, we can define an auxiliary input

$$u = \omega_{m,s} - \frac{V_l c(s_p) \cos \tilde{\theta}}{1 - c(s_p)l} + \frac{V_y c(s_p) \sin \tilde{\theta}}{1 - c(s_p)l} \quad (5.13)$$

for each $(m, s) \in \{(2, 0), (1, 1)\}$. The input change is invertible in the sense we can convert from u back to original control input γ_1 .

Using equations (3.1) and (5.13), we express the path following model (5.4)-(5.5) as

$$\dot{l} = V_l \sin \tilde{\theta} + V_y \cos \tilde{\theta} \quad (5.14)$$

$$\dot{\tilde{\theta}} = u. \quad (5.15)$$

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5.3.2.1 Local path following controller

We introduce

$$V_1 = \frac{1}{2}l^2, \quad (5.16)$$

and $\alpha_1 = \sin \tilde{\theta}$ as the intermediate control variable. The derivative of V_1 becomes

$$\dot{V}_1 = l(V_l \alpha_1 + V_y \cos \tilde{\theta}). \quad (5.17)$$

Equation (5.17) suggests the choice

$$\alpha_1 = \frac{-V_y \cos \tilde{\theta} - k_1 l}{V_l} \quad (5.18)$$

renders $\dot{V}_1$ into a negative definite function. By defining new coordinates (l, z) where $z = \sin \tilde{\theta} - \alpha_1$, equations (5.14)-(5.15) become

$$\dot{l} = V_l(z + \alpha_1) + V_y \cos \tilde{\theta} \quad (5.19)$$

$$\begin{aligned} \dot{z} &= \cos \tilde{\theta} \dot{\tilde{\theta}} - \left\{ \frac{\partial \alpha_1}{\partial \tilde{\theta}} \dot{\tilde{\theta}} + \frac{\partial \alpha_1}{\partial V_y} \dot{V}_y + \frac{\partial \alpha_1}{\partial l} \dot{l} + \frac{\partial \alpha_1}{\partial V_l} \dot{V}_l \right\} \\ &= \beta_2 u + \beta_1 \end{aligned} \quad (5.20)$$

where

$$\begin{aligned} \beta_1 &= -\frac{\partial \alpha_1}{\partial V_y} \dot{V}_y - \frac{\partial \alpha_1}{\partial l} \dot{l} - \frac{\partial \alpha_1}{\partial V_l} \dot{V}_l \\ \beta_2 &= \cos \tilde{\theta} - \frac{\partial \alpha_1}{\partial \tilde{\theta}}, \\ \frac{\partial \alpha_1}{\partial \tilde{\theta}} &= \frac{V_y \sin \tilde{\theta}}{V_l}, \quad \frac{\partial \alpha_1}{\partial V_y} = \frac{-\cos \tilde{\theta}}{V_l}, \quad \frac{\partial \alpha_1}{\partial l} = -\frac{k_1}{V_l} \\ \frac{\partial \alpha_1}{\partial V_l} &= \frac{V_y \cos \tilde{\theta} + k_1 l}{V_l^2}. \end{aligned}$$

For notational simplicity, let $\chi = [l \ z]^T$. Now we consider the Lyapunov function

$$V_2(\chi) = \frac{1}{2}l^2 + \frac{1}{2}z^2, \quad (5.21)$$

with its derivative

$$\dot{V}_2(\chi) = l\{V_l(z + \alpha_1) + V_y \cos \tilde{\theta}\} + z\{u\beta_2 + \beta_1\}. \quad (5.22)$$

By choosing the control input u as

$$u = \frac{-k_2 z - V_l l - \beta_1}{\beta_2}, \quad (5.23)$$

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$\dot{V}_2$ becomes

$$\dot{V}_2(\chi) = -k_1 l^2 - k_2 z^2 < 0. \quad (5.24)$$

Note that equation (4.7) allows us to determine $\{\dot{V}_l, \dot{V}_y\}$ based on inertial sensors measurements and $\{V_l, V_y\}$ estimates.

The performance of the control system is stated as follows.

Theorem 5.3.1 Consider a wheeled mobile robot with M2. For small initial conditions $\{l(t_0), \tilde{\theta}(t_0)\}$ which satisfy

$$\|\chi(t_0)\| < \frac{1}{\sup c(s_p)}, \quad (5.25)$$

there exist positive constants $\{k, k_3, \rho_5\}$ such that the path following errors $\{l(t), \tilde{\theta}(t)\}$ of the closed-loop system (5.14), (5.15), and (5.23) are governed by the following statements

1. the path following errors $\{l(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2. the lateral error l exponentially converges to zero and satisfies the inequality

$$|l(t)| \leq \|\chi(t_0)\| e^{-k_3(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (5.26)$$

3. the orientation error $\tilde{\theta}$ exponentially converges to a neighborhood containing $\tilde{\theta} = 0$ and satisfies the inequality

$$|\sin \tilde{\theta}(t)| \leq k \|\chi(t_0)\| e^{-k_3(t-t_0)} + \rho_5 \quad \forall t \geq t_0 \geq 0. \quad (5.27)$$

4. the steady-state orientation error satisfies

$$\lim_{t \rightarrow \infty} (\tilde{\theta}(t) + \delta_2(t)) = 0. \quad (5.28)$$

Proof: We first show the point $(l, z) = (0, 0)$ is an isolated equilibrium point of the closed-loop system. By some algebraic manipulations, the closed-loop system can be expressed as

$$\begin{aligned} \dot{l} &= V_l z - k_1 l \\ \dot{z} &= -k_2 z - V_l l. \end{aligned} \quad (5.29)$$

The closed-loop system (5.29) can be put into a matrix form

$$\dot{\chi} = \bar{A}\chi \quad (5.30)$$

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where

$$\bar{A} = \begin{bmatrix} -k_1 & V_l \\ -V_l & -k_2 \end{bmatrix}. \quad (5.31)$$

Since k_1, k_2 are positive constants, the $\det \bar{A}$ is nonzero; hence, $(l, z) = (0, 0)$ is an isolated equilibrium point of the closed-loop system.

The quadratic nature of (5.21) and (5.24) indicate that there exist positive constants k_3 such that

$$V_2(\chi) = \frac{1}{2} \|\chi\|^2 \quad (5.32)$$

$$\dot{V}_2(\chi) \leq -k_3 \|\chi\|^2. \quad (5.33)$$

where $k_3 = \min(k_1, k_2)$. By Theorem 2.4.1, χ satisfies

$$\|\chi(t)\| \leq \|\chi(t_0)\| e^{-k_3(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (5.34)$$

Inequality (5.34) implies $(l, \tilde{\theta})$ are uniformly bounded since V_y is assumed to be bounded. The lateral deviation is a component of vector χ , hence we have

$$|l(t)| \leq \|\chi(t)\| \leq \|\chi(t_0)\| e^{-k_3(t-t_0)} \quad (5.35)$$

and inequality (5.26) is established. Inequalities (5.25) and (5.35) ensure the input change (5.13) is always well-defined.

Next, we show that the intermediate control law α_1 satisfies

$$\begin{aligned} |\alpha_1| &= \left| \frac{-k_1 l}{V_l} - \frac{V_y}{V_l} \cos \tilde{\theta} \right| \\ &\leq \left| \frac{k_1 l}{V_l} \right| + \left| \frac{V_y}{V_l} \cos \tilde{\theta} \right| \\ &\leq \frac{k_1}{\rho_4} |l| + |\tan \delta_2|. \end{aligned} \quad (5.36)$$

Since there exists a positive constant $\rho_5 = \tan \rho_2$ such that $|\tan \delta_2| \leq \rho_5 < 1$, inequality (5.36) implies

$$|\alpha_1| \leq \frac{k_1}{\rho_4} |l| + \rho_5. \quad (5.37)$$

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Furthermore, definition $z = \sin \tilde{\theta} - \alpha_1$ leads us to (5.27) as follows.

$$\begin{aligned}
 |z| &= |\sin \tilde{\theta} - \alpha_1| \\
 &\geq |\sin \tilde{\theta}| - |\alpha_1| \\
 |\sin \tilde{\theta}| &\leq |z| + |\alpha_1| \\
 &\leq |z| + \frac{k_1}{\rho_4} |l| + \rho_5 \\
 &\leq \left(1 + \frac{k_1}{\rho_4}\right) \|\chi(t_0)\| e^{-k_3(t-t_0)} + \rho_5 \\
 &= k \|\chi(t_0)\| e^{-\lambda(t-t_0)} + \rho_5
 \end{aligned} \tag{5.38}$$

where $k = (1 + \frac{1}{\rho_4})$.

To show (5.28), $z \rightarrow 0$ implies

$$\begin{aligned}
 \lim_{t \rightarrow \infty} \left\{ \sin \tilde{\theta} - \alpha_1 \right\} &= 0 \\
 \lim_{t \rightarrow \infty} \left\{ \sin \tilde{\theta} + \frac{V_y}{V_l} \cos \tilde{\theta} \right\} &= 0 \\
 \lim_{t \rightarrow \infty} \left\{ \tan \tilde{\theta} \cos \tilde{\theta} + \tan \delta_2 \cos \tilde{\theta} \right\} &= 0 \\
 \lim_{t \rightarrow \infty} \left\{ \left(\tan \tilde{\theta} + \tan \delta_2 \right) \cos \tilde{\theta} \right\} &= 0
 \end{aligned} \tag{5.39}$$

From (5.27), we know that there exists a positive constant c such that $|\cos \tilde{\theta}| \geq c$ for $\forall t \geq t_0$; hence equality (5.39) shows $\lim_{t \rightarrow \infty} (\tan \tilde{\theta} + \tan \delta_2) = 0$ and this establishes

$$\lim_{t \rightarrow \infty} \left(\tilde{\theta}(t) + \delta_2(t) \right) = 0. \tag{5.40}$$

for $\forall t \geq t_0$ and this completes the proof.

□

Theorem 5.3.1 solves the path following problem based on the information determined by the sensors. The control results provide estimates on the effect of control gains $\{k_1, k_2\}$. For instance, the convergence rate and the transient behaviors (maximum overshoot) of the path following errors can be estimated using inequalities (5.26)-(5.27).

We note that β_2 appears as a denominator of the control law (5.23). If the initial orientation error is sufficiently small (e.g., $|\tilde{\theta}(t_0)| < \frac{\pi}{4}$), then β_2 is always greater than zero which implies the control law (5.23) is well-defined.

The proposed controller achieves local stability performance instead of global stability performance since $z = 0$ implies more than one isolated equilibrium points in the original

state-space $(l, \tilde{\theta})$. Nevertheless, it would be sufficient to consider the neighborhood about $(l, \tilde{\theta}) = (0, 0)$ with $|\tilde{\theta}| < \frac{\pi}{2}$.

In [54, 55], RTK-GPS has been used to enhance path following control of a farm-tractor to compensate the kinematic perturbations due to wheel skidding. Besides the differences in the perturbation computation approaches, the compensator proposed in these works are mainly for farm guidance operations (straight line following, constant curvature following) where the vehicle's wheel slip angles are assumed to be time-invariant. The control scheme proposed in this thesis avoids this restrictive assumption where the perturbations can be time-varying.

Equation (5.28) implies $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0$ if $\lim_{t \rightarrow \infty} \delta_2(t) = 0$. Due to the low maneuverability of the robot, $\tilde{\theta}$ does not converge to zero when $l \rightarrow 0$ and δ_2 is nonzero; nevertheless in many practical cases, a zero-converging lateral deviation error and a well-behaved orientation error are sufficient.

5.3.2.2 Conditional global path following controller

This section solves the conditional global path following problem for the wheeled mobile robots with M2 where the initial conditions are less restrictive compared with the initial conditions considered in the local control problem. Similarly, after applying the input change (5.13), the path following model (5.4)-(5.5) becomes

$$\dot{l} = V \sin(\tilde{\theta} + \delta_2) \quad (5.41)$$

$$\dot{\tilde{\theta}} = u. \quad (5.42)$$

We introduce

$$V_1 = \frac{1}{2}l^2 \quad (5.43)$$

and $\alpha_2 = \tilde{\theta}$ as the intermediate control variable. The derivative of V_1 becomes

$$\dot{V}_1 = l \{V \sin(\alpha_2 + \delta_2)\}. \quad (5.44)$$

By choosing

$$\alpha_2 = \arctan(-c_1 l) - \delta_2 \quad (5.45)$$

where c_1 is a positive constant, $\dot{V}_1$ becomes

$$\dot{V}_1 = Vl \sin(\arctan(-c_1 l)) \quad (5.46)$$

$$= -V \frac{c_1 l^2}{\sqrt{1 + l^2}}. \quad (5.47)$$

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Since $V > 0$, $\dot{V}_1$ is a negative definite function. By defining $z = \tilde{\theta} - \alpha_2$, we express equations (5.41) and (5.42) as

$$\dot{l} = V \sin(z + \alpha_2 + \delta_2) \quad (5.48)$$

$$\begin{aligned} \dot{z} &= \dot{\tilde{\theta}} - \dot{\alpha}_2 \\ &= u - \frac{\partial \alpha_2}{\partial l} \dot{l} - \frac{\partial \alpha_2}{\partial \delta} \dot{\delta}. \end{aligned} \quad (5.49)$$

where

$$\frac{\partial \alpha_2}{\partial l} = -\frac{1}{1 + c_1^2 l^2}, \quad \frac{\partial \alpha_2}{\partial \delta} = -1. \quad (5.50)$$

Similarly, we define $\chi = [l \ z]^T$ and a positive definite Lyapunov function

$$V_2(\chi) = \frac{1}{2} l^2 + \frac{1}{2} z^2. \quad (5.51)$$

Clearly, Lyapunov function (5.51) is positive definite and is radially unbounded. The derivative of V_2 is

$$\dot{V}_2(\chi) = lV \sin(z + \alpha_2 + \delta_2) + z\dot{z}. \quad (5.52)$$

We can segregate the unwanted cross term z within the $\sin(\cdot)$ function by choosing

$$\sin(z + \alpha_2 + \delta_2) = \sin(\alpha_2 + \delta_2) + z\bar{\eta} \quad (5.53)$$

where

$$\bar{\eta} = \frac{\sin(z + \alpha_2 + \delta_2) - \sin(\alpha_2 + \delta_2)}{z}. \quad (5.54)$$

$\bar{\eta}$ is a well-defined function for $\forall z \neq 0$ and approaches $\cos(\alpha_2 + \delta_2)$ as $z \rightarrow 0$. Using (5.53), equation (5.52) can be written as

$$\dot{V}_2(\chi) = lV \sin(\alpha_2 + \delta_2) + lV z\bar{\eta} + z\dot{z}. \quad (5.55)$$

Equation (5.55) suggests that by choosing the control law u to be

$$u = -l\bar{\eta}V - c_2 z + \left\{ \frac{\partial \alpha_2}{\partial l} \dot{l} + \frac{\partial \alpha_2}{\partial \delta} \dot{\delta} \right\} \quad (5.56)$$

where c_2 is a positive constant, equation (5.55) becomes

$$\dot{V}_2 = -V \frac{c_1 l^2}{\sqrt{1 + l^2}} - c_2 z^2. \quad (5.57)$$

Note that $\dot{\delta}_2(t)$ can be determined by

$$\frac{d\delta_2(t)}{dt} = \frac{\frac{\dot{V}_y}{V_l} - \frac{V_y \dot{V}_l}{V_l^2}}{1 + \left(\frac{V_y}{V_l} \right)^2} \quad (5.58)$$

based on the $\{\dot{V}_l, \dot{V}_y\}$ and $\{V_y, V_l\}$ estimates.

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Theorem 5.3.2 Consider a wheeled mobile robot with M2. For any initial conditions $\{l(t_0), \tilde{\theta}(t_0)\}$ that satisfy

$$\|\chi(t_0)\| < \frac{1}{\sup c(s_p)}, \quad (5.59)$$

the path following errors $\{l(t), \tilde{\theta}(t)\}$ of the closed-loop system (5.41), (5.42), and (5.56) are governed by the following statements

1. the path following errors $\{l(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2. the lateral error l converges to zero, i.e.,

$$\lim_{t \rightarrow \infty} l(t) = 0. \quad (5.60)$$

3. the orientation error $\tilde{\theta}$ converges to a neighborhood containing $\tilde{\theta} = 0$ with the steady-state orientation error satisfies

$$\lim_{t \rightarrow \infty} (\tilde{\theta}(t) + \delta_2(t)) = 0. \quad (5.61)$$

Proof: Since $\dot{V}_2$ is a negative definite function, the states $\{l, z\}$ are uniformly bounded and this implies the boundedness of the path following errors $\{l, \tilde{\theta}\}$. The derivatives of $\{\delta_2, V\}$ are uniformly bounded since $\{\dot{V}_y, \dot{V}_l\}$ are assumed to be uniformly bounded; hence, $\dot{V}_2$ is uniformly continuous. By Barbalat's lemma, it follows that $\dot{V}_2$ converges to zero as t goes to ∞ and this implies $\{l, z\}$ converges to zero as t goes to ∞ . Similarly, to show (5.61), the definition $z = \tilde{\theta} - \alpha$ and the zero convergence of $\{l, z\}$ implies $(\tilde{\theta} + \delta_2) \rightarrow 0$ as t goes ∞ . Finally, we show that input change (5.13) is well-defined, i.e., $l \neq \frac{1}{c(s)}$ for $\forall t \geq t_0$ when inequality (5.59) is satisfied. Note that the negative definiteness of $\dot{V}_2$ and inequality (5.59) imply

$$\frac{1}{2} \|\chi\|^2 \leq \frac{1}{2} \|\chi(t_0)\|^2 < \frac{1}{2} \frac{1}{(\sup c(s))^2} < \frac{1}{2} \frac{1}{c(s)^2} \quad (5.62)$$

Since l is a subvector of χ , $|l| \leq \|\chi\|$ leads (5.62) to $l < \frac{1}{c(s)}$ for $\forall t \geq t_0$ and this completes the proof. □

Theorem 5.3.2 shows the lateral deviation error converges to zero where the uniformly bounded orientation error converges to a neighborhood containing zero for any initial states $\{l(t_0), \tilde{\theta}(t_0)\}$ that satisfy inequality (5.59). The result has yet to show its transient performance. Next, we show that the convergence rate of the conditional global closed-loop system is locally exponential.

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Corollary 5.3.3 Under the conditions of Theorem 5.3.2, for small initial errors $\{l(t_0), \tilde{\theta}(t_0)\}$, $\chi(t)$ exponentially converges to zero.

Proof: For a neighborhood that contains $(l, z) = (0, 0)$, there exists a positive constant c_3 such that

$$\dot{V}_2 \leq -c_3 \|\chi\|^2 \quad (5.63)$$

Since, V_2 is a quadratic Lyapunov function, there exists a positive constant c_4 such that

$$\dot{V}_2 \leq -2c_3 V_2. \quad (5.64)$$

By invoking comparison lemma 2.4.3, we have

$$V_2(t) \leq V_2(t_0) e^{-c_3(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (5.65)$$

With some manipulations, inequalities (5.38) implies

$$\|\chi(t)\| \leq \|\chi(t_0)\| e^{-c_3(t-t_0)} \quad (5.66)$$

and this completes the proof.

By following the same argument used in Theorem 5.3.1, we show that the path following errors $(l(t), \tilde{\theta}(t))$ satisfy (5.26)-(5.27) for some positive constants $\{k, k_3, \rho_5\}$. Although the analysis shows the conditional global controller guarantees a local exponential convergence in a neighborhood about the origin $(l, \tilde{\theta}) = (0, 0)$; the controller drives the path following errors towards the origin from a larger set of initial conditions compared with the local control law. Does Corollary 5.3.3 means the local controller which has developed previously is redundant? By comparing between the controllers, we observe that the local controller has some advantages over the conditional global controller. First, the local controller possesses a faster convergence in the lateral deviation error compared with the conditional global controller for a given control gain $k_1 = c_1, k_2 = c_2$. This characteristic can be noticed since the $-k_1 l^2$ term of (5.24) is always smaller than $-k_1 l^2 / \sqrt{1 + l^2}$ of (5.57) for any nonzero l . This difference becomes significant for large l . The exponential convergence rate of the conditional closed-loop system diminishes as $l \rightarrow \infty$. Second, the local control development is more mathematically tractable compared with the conditional global version. For instance, the transient performance of the local controller can be explicitly estimated by the analytical bounds (5.26)-(5.27).

From the development, it is clear that the longitudinal slippage d does not pose any difficulty in the path following problem as long as the robot is in motion, i.e., $V_l > 0$. However, in

applications where we want the robot to achieve a desired linear velocity, i.e., $V_l(t) = V_{ld}(t)$, a simple feedforward compensation

$$r_w \dot{\varphi}(t) = V_{ld}(t) + d(t) \quad (5.67)$$

can be applied to achieve the objective based on d measurement.

5.3.3 Control design for wheeled mobile robots with M3

This section illustrates how a wheeled mobile with M3 achieves path following using the GPS-based control scheme. The path following models we consider are (5.9) and (5.10). We define an invertible auxiliary control input:

$$u = \omega_{m,s} - \frac{V c(s_p) \cos(\tilde{\theta} + \delta_2 + \gamma_2)}{1 - c(s_p)l}, \quad (5.68)$$

for each $(m, s) \in \{(2, 1), (1, 2)\}$. Similarly, the auxiliary u can be converted back to original control input γ_1 . Equation (5.68) leads to

$$\dot{l} = V \sin(\tilde{\theta} + \delta_2 + \gamma_2), \quad (5.69)$$

$$\dot{\tilde{\theta}} = u. \quad (5.70)$$

Equation (5.69) suggests that by choosing

$$\gamma_2(t) = -\delta_2(t), \quad (5.71)$$

the path following model becomes

$$\dot{l} = V \sin(\tilde{\theta}) \quad (5.72)$$

$$\dot{\tilde{\theta}} = u. \quad (5.73)$$

Equations (5.72)-(5.73) suggest that nominal path following control laws [3, 29, 31] that based on non-skidding and non-slipping assumptions can be applied to solve the path following problem along with the perturbation decoupling inputs (5.68) and (5.71) under the GPS-based control scheme. This strategy assumes that (5.71) completely eliminate $\delta_2(t)$ out of equation (5.69), i.e., $\gamma_2(t) + \delta_2(t) = 0$.

Unfortunately, if $\gamma_2(t) + \delta_2(t)$ is nonzero due to the limited range of control input $\gamma_2(t)$, then the above-mentioned approach fails to solve the path following problems, i.e., the robot will not follow the geometric path with high-precision. To exemplify this point, consider control input

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$\gamma_2(t)$ has a maximum limit $\gamma_{\max}$ such that $|\gamma_2(t)| \leq \gamma_{\max}$, then under the choice of (5.71), we can model $\gamma_2(t)$ as

$$\gamma_2 = \begin{cases} -\delta_2 & \text{if } |\delta_2| \leq \gamma_{\max} \\ -\text{sgn}(\delta_2)\gamma_{\max} & \text{if } |\delta_2| > \gamma_{\max} \end{cases} \quad (5.74)$$

To analyze the saturation effect, without loss of generality we define $\delta_s = \delta_2 + \gamma_2$ where equation (5.69) can be expressed as

$$\dot{l} = V \sin(\tilde{\theta} + \delta_s). \quad (5.75)$$

If the control input γ_2 is saturated while compensating δ_2 , i.e., $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$, then δ_s is nonzero and it would be undesirable to apply the nominal path following control laws e.g., [3, 29], with (5.68) and (5.71) to solve the path following problems.

On the other hand, the combination of (5.68), (5.71) and the backstepping control laws developed in this chapter results a zero converging lateral error and a well-behaved orientation error even if $\gamma_2(t)$ saturates at its physical output limit. This positive feature of the backstepping control laws becomes explicit in the following statements.

Theorem 5.3.4 Consider a wheeled mobile robot with M3 with the control input γ_2 satisfies

$$|\gamma_2(t)| \leq \gamma_{\max} \quad (5.76)$$

where $\gamma_{\max}$ is the maximum of the control input γ_2 . For small initial conditions $\{l(t_0), \tilde{\theta}(t_0)\}$ which satisfy

$$\|\chi(t_0)\| < \frac{1}{\sup c(s_p)}, \quad (5.77)$$

there exist positive constants $\{k, k_3, \rho_5\}$ such that the path following errors $\{l(t), \tilde{\theta}(t)\}$ of the closed-loop system (5.23), (5.69), (5.70) and (5.71) are governed by the following statements

1. the path following errors $\{l(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2. the lateral error l exponentially converges to zero and satisfies the inequality

$$|l(t)| \leq \|\chi(t_0)\| e^{-k_3(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (5.78)$$

3. the orientation error $\tilde{\theta}$ satisfies the inequality

$$|\sin \tilde{\theta}(t)| \leq k \|\chi(t_0)\| e^{-k_3(t-t_0)} + \rho_5 \quad \forall t \geq t_0 \geq 0. \quad (5.79)$$

where

$$\rho_5 = \begin{cases} 0 & \text{if } \sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max} \\ \tan(\rho_2 - \gamma_{\max}) & \text{otherwise} \end{cases} \quad (5.80)$$

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4. the steady-state orientation error satisfies

$$\begin{aligned} \lim_{t \rightarrow \infty} \tilde{\theta}(t) &= 0 && \text{if } \sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max} \\ \lim_{t \rightarrow \infty} (\tilde{\theta}(t) + \delta_2(t) + \gamma_2(t)) &= 0 && \text{otherwise} \end{aligned} \quad (5.81)$$

Proof: The proof of this result is straightforward. By following the same argument used in Theorem 5.3.1, condition (5.77) ensures input change (5.68) is well-defined for $\forall t \geq t_0$. We let $\delta_s(t) = \gamma_2(t) + \delta_2(t)$ and the path following model (5.69)-(5.70) becomes

$$\dot{l} = V \sin(\tilde{\theta} + \delta_s) \quad (5.82)$$

$$\dot{\tilde{\theta}} = u. \quad (5.83)$$

$\delta_s(t)$ is analogous to ' $\delta_2(t)$ ' of the M2 robots. Similarly, equation (5.82) can be expressed as

$$\dot{l} = V_l \sin(\tilde{\theta}) + V_y \cos(\tilde{\theta}) \quad (5.84)$$

where $V_y = V \sin \delta_s$ and $V_l = V \cos \delta_s$. Equations (5.83) and (5.84) match with the path following model (5.14)-(5.15) of a robot with M2. Hence, by applying control law (5.23), the path following errors $(l, \tilde{\theta})$ satisfy

$$\begin{aligned} |l(t)| &\leq \|\chi(t_0)\| e^{-k_3(t-t_0)} \\ |\sin \tilde{\theta}(t)| &\leq k \|\chi(t_0)\| e^{-k_3(t-t_0)} + \rho_5 \quad \forall t \geq t_0 \geq 0 \end{aligned}$$

where $k_3 = \min(k_1, k_2)$ and the bound ρ_5 depends on the magnitude of $\delta_s(t)$. Note that $\{k_1, k_2\}$ are the control gains of control law (5.23). If $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$, then the choice (5.71) implies $\delta_s(t) = 0$ for $\forall t \geq t_0$ and this leads to $\rho_5 = 0$. On the other hand, if $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$, then control law (5.71) results a nonzero $\delta_s(t)$ such that $|\delta_s(t)| \leq \rho_2 - \gamma_{\max}$ since γ_2 saturates at $\gamma_2 = -\text{sgn}(\delta_2)\gamma_{\max}$. This implies $\rho_5 = \tan(\rho_2 - \gamma_{\max})$. Finally, since $(\tilde{\theta}(t) + \delta_s(t))$ converges to zero as $t \rightarrow \infty$ and $\delta_s(t) = 0$ if $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$, we conclude (5.81) and this completes the proof. □

Theorem 3 shows that the combination of perturbation decoupling control law (5.68), (5.71) and the backstepping control law (5.23) forces the path following errors $\{l(t), \tilde{\theta}(t)\}$ to converge exponentially to origin.

The control result demonstrates that the additionally orientation input $\gamma_2(t)$ possesses by a mobile robot with M3 allows the path following errors $\{l(t), \tilde{\theta}(t)\}$ converge to zero for nonzero

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δ_2 . This control performance is superior to the M2 case (assuming there is no saturation occurs at the control input γ_2 , i.e., $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$).

In the case where the robot's max steering $\gamma_{\max}$ is smaller than $|\delta_2(t)|$ for some t interval, then the combination with the backstepping control laws can at least ensure the lateral error l exponentially converges to zero with a well-behaved orientation error.

Theorem 5.3.4 solves the *local path following control problem for the wheeled mobile robots with M3*. Using the same approach, we can also combine the conditional global control law (5.56) with control laws (5.68) and (5.71) to solve the *conditional global path following control problem for wheeled mobile robots with M3*. This result is stated as follows.

Theorem 5.3.5 Consider a wheeled mobile robot with M3 with the control input γ_2 satisfies

$$|\gamma_2(t)| \leq \gamma_{\max} \quad (5.85)$$

where $\gamma_{\max}$ is the maximum of the control input γ_2 . For any initial conditions $\{l(t_0), \tilde{\theta}(t_0)\}$ that satisfy

$$\|\chi(t_0)\| < \frac{1}{\sup c(s_p)^2}, \quad (5.86)$$

the path following errors of the closed-loop system (5.56), (5.69), (5.70) and (5.71) are governed by the following statements

1. the path following errors $\{l(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2. the lateral error l converges to zero, i.e.,

$$\lim_{t \rightarrow \infty} l(t) = 0. \quad (5.87)$$

3. the steady-state orientation error $\tilde{\theta}$ satisfies

$$\begin{aligned} \lim_{t \rightarrow \infty} \tilde{\theta}(t) &= 0 && \text{if } \sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max} \\ \lim_{t \rightarrow \infty} (\tilde{\theta}(t) + \delta_2(t) + \gamma_2(t)) &= 0 && \text{otherwise} \end{aligned} \quad (5.88)$$

Proof: Similarly, input change (5.68) is well-defined for $\forall t \geq t_0$ since the initial conditions satisfy inequality (5.86). Definition $\delta_s(t) = \delta_2(t) + \gamma_2(t)$ leads to

$$\dot{l} = V \sin(\tilde{\theta} + \delta_s) \quad (5.89)$$

$$\dot{\tilde{\theta}} = u \quad (5.90)$$

which match with equations (5.41)-(5.42). By applying the conditional global control law (5.56), we conclude that the path following errors are uniformly bounded with the lateral deviation error l converges to zero as $t \rightarrow \infty$ and $\lim_{t \rightarrow \infty}(\tilde{\theta}(t) + \delta_s(t)) = 0$. By using the same argument applied in Theorem 5.3.4, we establish (5.88) and this completes the proof.

□

We can also exploit the $d(t)$ measurements to achieve a desired robot's longitudinal velocity $V_{ld}(t)$, i.e., $V_l(t) = V_{ld}(t)$ using a simple compensation

$$r_w \dot{\varphi}(t) = \frac{V_{ld}(t) \cos \delta_2(t)}{\cos(\gamma_2(t) + \delta_2(t))} + d(t). \quad (5.91)$$

based on equations (3.32) and (3.48).

5.4 Simulation results

The control algorithms developed in this paper were simulated in MATLAB SIMULINK. The designated geometric path is a circle with a radius of 30 m centered at $(x, y) = (0, 0)$, and the desired longitudinal velocity is chosen as $V_{ld} = 1 \text{ ms}^{-1}$. The control gains used in this simulation are selected as

$$k_1 = 1, \quad k_2 = 1, \quad c_1 = 1, \quad c_2 = 1. \quad (5.92)$$

5.4.1 Path following control for wheeled mobile robots with M2: Type (2,0) robot

In this section, the local and conditional global control laws are simulated on a Type (2,0) robot. The kinematic perturbations $\{\delta_1, \delta_2, d, V_y\}$ are chosen as piecewise continuous signals depicted in Figure 5.3. Figures 5.4 and 5.5 show the path following errors and the corresponding control inputs for small initial errors $(l(0), \tilde{\theta}(0)) = (-1.5 \text{ m}, 0.1 \text{ rad})$. The responses of the local control law (5.23) are represented by dash-dot lines where the responses of the conditional global control law are denoted by solid lines. From the results, we observe that the lateral deviation errors exponentially converge to zero and the orientation errors exponentially converge to a neighborhood about zero. Additionally, the orientation errors of the control laws tend to $-\delta_2(t)$ as $t \rightarrow \infty$. The control inputs of these control laws are smooth. Because of implementation considerations, the smooth control inputs suggest that these control laws are preferable

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Figure 5.3: Perturbations

compared with discontinuous control inputs. We can also notice the effects of longitudinal slip velocity d are eliminated by (5.67) with the linear velocity $V_l(t) = V_{ld}(t)$.

These simulation results not only illustrate the effectiveness of the path following controllers but also providing insights on the difference between the usefulness of the local and conditional global control laws. From Figure 5.4, we can observe that the local control law (5.23) drives the lateral deviation error to zero slightly faster than the conditional global control law (5.56). We can quantify this difference using the error measure $E = \int_0^T l^2(\tau) d\tau$ over the time period $[0, T]$. The values are determined as

$$E_{\text{local}} = 1.6 \quad E_{\text{Cond-global}} = 1.9. \quad (5.93)$$

On the other hand, the conditional global controller can be used for large initial errors; however the convergence rate will be slow. We demonstrate this point by performing a simulation on the conditional global control law with large initial errors of $(l(0), \tilde{\theta}(0)) = (-15 \text{ m}, 0.1 \text{ rad})$. The results are shown in Figures 5.6 and 5.7. Figure 5.6 shows that the lateral error converges to zero in a linearly fashion unlike the case for small initial errors where the lateral error converges to zero exponentially. In brief, the conditional global control law allows large initial errors; but

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Figure 5.4: Path following errors $\{l, \tilde{\theta}\}$: path following with small initial errors $(l(0), \tilde{\theta}(0)) = (-1.5 \text{ m}, 0.1 \text{ rad})$

Figure 5.5: Control inputs $\{\gamma_1, r_w \dot{\varphi}\}$: path following with small initial errors $(l(0), \tilde{\theta}(0)) = (-1.5 \text{ m}, 0.1 \text{ rad})$

Figure 5.6: Path following errors $\{l, \tilde{\theta}\}$: Conditional global path following with initial error $(l(0), \tilde{\theta}(0)) = (-15 \text{ m}, 4.4 \text{ deg})$

for small initial errors, the local control law is preferred. These simulation results show the consistencies of Theorem 5.3.1 and Theorem 5.3.2.

5.4.2 Path following controllers for wheeled mobile robots with M3: Type (1,2) robot

This section validates the controllers for M3 robots (Theorem 5.3.4 and Theorem 5.3.5). In this simulation, the results are simulated on a Type (1,2) robot with a wheelbase of $a = 1.2 \text{ m}$. We consider two cases: (i) control input γ_2 satisfies condition $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$, i.e., no saturation for the control input; (ii) $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$ where the control input γ_2 saturates for some time intervals. In this simulation, the path to be followed and the control gains remain unchanged. The maximum of γ_2 is chosen as $\gamma_{\max} = 0.1 \text{ rad}$. For illustration purposes,

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Figure 5.7: Control inputs $\{\gamma_1, r_w \dot{\varphi}\}$: Conditional global path following

we only consider small initial errors of $(l(0), \tilde{\theta}(0)) = (-1.5 \text{ m}, 0.1 \text{ rad})$. The perturbations chosen for this simulation are shown in Figure 5.8 with a $\sup\{|\delta_2(t)|; \forall t \geq t_0\} = 0.15 \text{ rad}$.

Figure 5.8: Perturbations

Figure 5.9: Path following errors $\{l, \tilde{\theta}\}$

5.4.2.1 Path following control with no γ_2 saturation

Figures 5.9 and 5.10 show the path following responses of (i) closed-loop system $\{(5.23), (5.69), (5.70), (5.71)\}$ for the local controller; and (ii) closed-loop system $\{(5.56), (5.69), (5.70), (5.71)\}$ for the conditional global version with the condition $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$. The results show that both lateral and orientation errors exponentially converge to zero even when the perturbation δ_2 is nonzero. Unlike the robots with M2, the additional control input γ_2 of a M3 robot eliminates the perturbation δ_2 , resulting zero convergence for both lateral and orientation errors.

5.4.2.2 Path following control with γ_2 saturation

Figures 5.11 and 5.12 depict the path following responses of the closed-loop systems with $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$. Figure 5.12 depicts the control input saturates at -0.1 rad after $t = 5$ sec. In this case, the control input γ_2 cannot completely eliminate the undesirable effects of $\delta_2(t)$. Nevertheless, the lateral errors still exponentially converge to zero with well-behaved orientation errors converge to $-\delta_2(t) - \gamma_2(t)$. Similarly, we see that the longitudinal

Figure 5.10: Control inputs $\{\gamma_1, \gamma_2, r_w \dot{\phi}\}$

slip perturbation d is eliminated by (5.91) and $V_l = V_{ld}$ is attained. These simulation results confirm Theorem 5.3.4 and Theorem 5.3.5.

5.5 Experimental results

5.5.1 Experimental setup

The experimental validation of the GPS-based path following control scheme was again performed on the Cycab mobile robot in Appendix A. The control algorithms are implemented in the high-level Visual C++ on the host PC. For convenience, we let $\{X, Y\}$ be the East and North axes, and the origin be the location of the GPS base station. The robot is not equipped with an absolute orientation sensor; therefore, the orientation of the vehicle is determined by integrating the measured yaw rate $\dot{\theta}$ using trapezoidal integration from a known initial orientation $\theta(0)$. This approach is feasible since the time duration of the trial run is short due to the limited open space at our school campus. The experimental site is an open area (see Figure 5.13) which allows consistent GPS signals during the run. In the trial runs, vehicle is moving

Figure 5.11: Path following errors $\{l, \tilde{\theta}\}$

at low speed and the low-level control system is running at 10Hz. This low speed maneuver suggests that estimators proposed in Chapter 4 is not required in this particular implementation. Instead, we illustrate the concept of the GPS-based control scheme is demonstrated based on the observation equations (4.12)-(4.15) and the sensors measurements to estimate $\{x, y, V_l, V_y\}$.

The desired velocity V_{ld} chosen in the experiments is 0.8ms^{-1} . Figure 5.14 shows the circular geometric path which the vehicle is to follow. The circle has a radius of 11 m, i.e., $c(s_p) = 1/11$. We demonstrate the path following performances of the local control law (5.23) and the conditional global control law (5.56).

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Figure 5.12: Control inputs $\{\gamma_1, \gamma_2, r_w \dot{\phi}\}$

Figure 5.13: Experimental site

Figure 5.14: Geometric path

5.5.2 Experimental results: Local controller

Figure 5.15: Measured perturbations

5.5.2.1 Experiments without perturbations compensation

In the experiments, the robot was first controlled by the backstepping control law without any compensation action, i.e., $(\delta_1 = 0, V_y = 0, d = 0)$. The velocity control input is set to $r_w \dot{\phi} = 0.8 \text{ ms}^{-1}$.

The control gains chosen in the experiments are

$$k_1 = 0.1, \quad k_2 = 0.3. \quad (5.94)$$

The robot was initially driven to a starting point

$$[x(0), y(0), \theta(0)] = [-60.08 \text{ m}, 9.6 \text{ m}, 326.1 \text{ deg}] \quad (5.95)$$

where the corresponding initial error is

$$l(0) = -0.05 \text{ m} \quad \tilde{\theta}(0) = 9 \text{ deg}. \quad (5.96)$$

Figure 5.16: Path following errors

The path following results are shown in Figure 5.16 (solid lines). We see that the path following errors $(l, \tilde{\theta})$ converge to a neighborhood about point $(l, \tilde{\theta}) = (0, 0)$ with a steady-state error of $l = 0.07\text{m}$ and $\tilde{\theta} = 4\text{ deg}$. These steady-state errors motivated us to introduce corrective actions into the control laws to compensate the bias errors.

5.5.2.2 Experiments with perturbations compensation

Wheel skidding compensation was then activated by including the perturbation terms in the control laws. Similarly, the vehicle began from the same starting point with the same control gains $\{k_1, k_2\}$ and velocity input $r_w \dot{\phi}$. Figure 5.15 depicts the perturbations computed using the GPS measurements. Figure 5.17 shows the robot's linear velocity V_l and the smooth steering input during the experiment. Since $V_l \approx 0.8\text{m/s}^{-1}$ and $r_w \dot{\phi} = 0.8\text{ ms}^{-1}$, the longitudinal slipping velocity d is insignificant and additional velocity compensation is not required. The measured V_y shown in Figure 5.15 has a zero mean. In contrast, the front slip angle δ_1 is dominant compared with V_y and d . The path following errors are represented by the dashed-lines in Figure 5.16. We can see that introducing perturbations compensation reduces the steady-

Figure 5.17: Control inputs

state lateral deviation error l to zero and the steady-state orientation error to $\tilde{\theta} = 3$ deg. The mild nonzero steady-state orientation error is due to the accumulated integration error of the orientation estimate. The reduction of the path following errors, specifically the lateral error, is mainly due to the elimination of the front wheel slip angle δ_1 during the run.

5.5.3 Experimental results: Conditional global controller

5.5.3.1 Conditional global path following control without perturbations compensation

In the same manner as in the local case, the wheeled mobile robot was initially driven to the starting point for control without any compensation action. The initial errors must not be large since it will saturate the physical limit of the control input (γ_1). On the other hand, the choice of small control gains and large initial errors avoids this initial saturation at the control input; however, the convergence rate of the path following errors will be slow and the experimental site is not sufficiently long for the maneuver. Therefore, the robot was driven to the starting point corresponds to small initial errors of $(l(0), \tilde{\theta}(0)) = (0, -2 \text{ deg})$ and the control gains are

Figure 5.18: Measured perturbations

chosen as

$$c_1 = 0.1, \quad c_2 = 0.1. \quad (5.97)$$

Figures 5.18-5.20 depict the experimental results of the conditional global control law (5.56). The path following results are shown in Figures 5.19 (Solid lines). We can see that the path following errors maintain in a neighborhood about the origin $(l, \tilde{\theta}) = (0, 0)$. The results reveal the steady-state lateral error has a positive bias of about 0.05 m while the orientation error is closed to zero. The positive bias of the lateral error motivated us to introduce corrective actions in the control law to improve the control performance.

5.5.3.2 Conditional global path following control with perturbations compensation

Perturbations compensation was then activated by including the perturbation terms in the control law (5.56). The vehicle began from the same starting point with the same control gains $\{c_1, c_2\}$ and velocity input $r_w \dot{\phi}$. Figure 5.18 depicts the perturbations computed using the GPS measurements. Figure 5.20 shows the robot's linear velocity V_l and the smooth steering input during the experiment. The measured V_y shown in Figure 5.18 has a zero mean. The path fol-

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Figure 5.19: Path following errors

lowing errors with compensation are represented by dash-dot lines in Figure 5.19. We observe that introducing perturbation compensation reduces the steady-state bias of the lateral deviation error close to zero while the steady-state orientation error maintains about zero.

Figure 5.20: Control inputs

5.6 Conclusions

In this chapter, path following control problems are formulated and solved using a GPS-based control scheme for the four generic wheeled mobile robots with wheel skidding and slipping. This scheme relies on high-accuracy RTK-GPS and other aiding sensors to measure the skidding and slipping perturbations for compensations. Both local and conditional global path following problems have been solved for the robots using the backstepping control laws. It has been demonstrated that robots with higher maneuverability has the ability to achieve better control performance compared with robots with lower maneuverability. By Lyapunov analysis, we show that the developed control laws achieve precise maneuvers without imposing restrictive assumptions on the skidding and slipping perturbations. The control scheme is also validated by means of simulation and experiments. Although the experimental setting is not designed to validate all theoretical aspects of the control scheme, the experimental results do reveal some positive features. We show that the GPS-guided system is able to compensate skidding perturbations based on RTK-GPS and other aiding sensors measurements. Additionally, the results suggest that the proposed control laws possess a certain degree of robustness, in particular to the sensors' noise.

Chapter 6

GPS-based Tracking Control

6.1 Introduction

This chapter aims to solve the tracking problem for the four mobile robots with wheel skidding and slipping. A GPS-based tracking control scheme based on backstepping technique is proposed for the mobile robots. Similarly, the proposed control scheme exploits the robot's posture, velocities, and perturbations information determined by the sensors to compensate the tracking errors. This chapter firstly presents the development of tracking error models which are to be used for tracking control designs. Secondly, tracking problems are formulated based on the maneuverability of a mobile robot. Thirdly, the tracking controller for maneuverability two mobile robots is developed. The problem considers the case where initial tracking errors are sufficiently small. These control laws are extended to higher maneuverability mobile robots to solve the tracking problems. Simulation results are presented to verify the controllers. Additionally, the control laws are implemented on the car-like experimental platform. The experimental results and discussion will end the chapter.

6.2 Tracking Control Formulation

This section presents the tracking control problem in the presence of wheel skidding and slipping (see Figure 6.1). In brief, the tracking problem is to find control laws $\mathbf{U}$ such that a mobile robot tracks a *reference trajectory* $q_r = (x_r, y_r, \theta_r)^T$ which is described by

Figure 6.1: Tracking Control problem

$$\begin{aligned}
 \dot{x}_r &= v_r \cos \theta_r \\
 \dot{y}_r &= v_r \sin \theta_r \\
 \dot{\theta}_r &= \omega_r.
 \end{aligned} \tag{6.1}$$

Throughout the development of this chapter, we assume that $v_r(t)$, $\omega_r(t)$, and $\dot{v}_r(t)$ are both *piecewise continuous and bounded functions*. Additionally, we assume that the reference velocity v_r is positive and $\inf\{v_r(t); \forall t \geq t_0\} > 0$. In this tracking problem, we choose the *posture tracking error* as

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \tag{6.2}$$

6.2.1 Tracking problem for wheeled mobile robots with M2

This section presents derivation of the tracking models for the mobile robots with M2, i.e., for each $(m, s) \in \{(2, 0), (1, 1)\}$. By differentiating the posture error (6.2) and using equality

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$\dot{x}_r \sin \theta_r = \dot{y}_r \cos \theta_r$, the error dynamics for the robots are:

$$\begin{aligned}\dot{\tilde{x}} &= (\dot{x}_r - \dot{x}) \cos \theta + (\dot{y}_r - \dot{y}) \sin \theta \\ &\quad - (x_r - x) \dot{\theta} \sin \theta + (y_r - y) \dot{\theta} \cos \theta \\ &= \dot{x}_r \cos \theta - \dot{x} \cos \theta + \dot{y}_r \sin \theta - \dot{y} \sin \theta + \omega \tilde{y} \\ &= v_r \cos \tilde{\theta} - V_l + \omega \tilde{y}\end{aligned}\tag{6.3}$$

$$\begin{aligned}\dot{\tilde{y}} &= -\dot{x}_r \sin \theta + \dot{x} \sin \theta + \dot{y}_r \cos \theta - \dot{y} \cos \theta \\ &\quad - (x_r - x) \dot{\theta} \cos \theta - (y_r - y) \dot{\theta} \sin \theta \\ &= v_r \sin \tilde{\theta} + \dot{x} \sin \theta - \dot{y} \cos \theta - \tilde{x} \omega \\ &= v_r \sin \tilde{\theta} - V_y - \tilde{x} \omega\end{aligned}\tag{6.4}$$

$$\dot{\tilde{\theta}} = \omega_r - \omega_{m,s}\tag{6.5}$$

where

$$\omega_{2,0} = \gamma_1 + \delta_1\tag{6.6}$$

$$\omega_{1,1} = \frac{V_l}{a} \tan(\gamma_1 + \delta_1) - \frac{V_y}{a}.\tag{6.7}$$

As it has been highlighted in Chapter 3 that $\tilde{\xi} \rightarrow 0$ results a nonzero steady-state orientation error. Nevertheless, equations (6.3) and (6.4) suggest that we can achieve $\tilde{\xi} \rightarrow 0$ with a well-behaved orientation error $\tilde{\theta}$. Additionally, it is natural to demand the steady-state tracking errors $(\tilde{\xi}, \tilde{\theta}) \rightarrow 0$ when $\delta_2 \rightarrow 0$. This characteristic motivates us to formulate the following tracking problem with wheel skidding and slipping.

Definition 6.2.1 *Trajectory tracking problem for wheeled mobile robots with M2:* Find control laws for $r_w \dot{\varphi}$ and γ_1 such that for small initial errors $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$,

1. the tracking errors $\tilde{\xi}$ and $\tilde{\theta}$ are uniformly bounded.
2. the point tracking error $\tilde{\xi}$ converges to zero as $t \rightarrow \infty$.
3. $\tilde{\theta}$ converge to a neighborhood about $\tilde{\theta} = 0$ as $t \rightarrow \infty$. Moreover, the steady-state orientation error $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0$ if $\lim_{t \rightarrow \infty} \delta_2(t) \rightarrow 0$.

6.2.2 Tracking problem for wheeled mobile robots with M3

Similarly, the tracking error dynamics for mobile robots with M3, i.e., for each $(m, s) \in \{(2, 1), (1, 2)\}$ are derived as follows.

$$\begin{aligned}
 \dot{\tilde{x}} &= (\dot{x}_r - \dot{x}) \cos \theta + (\dot{y}_r - \dot{y}) \sin \theta \\
 &\quad - (x_r - x) \dot{\theta} \sin \theta + (y_r - y) \dot{\theta} \cos \theta \\
 &= \dot{x}_r \cos \theta - \dot{x} \cos \theta + \dot{y}_r \sin \theta - \dot{y} \sin \theta + \omega \tilde{y} \\
 &= \dot{x}_r \cos \theta + \dot{y}_r \sin \theta - \{\dot{x} \cos \theta + \dot{y} \sin \theta\} + \omega \tilde{y} \\
 &= v_r \cos \tilde{\theta} - V \cos(\gamma_2 + \delta_2) + \omega \tilde{y}
 \end{aligned} \tag{6.8}$$

$$\begin{aligned}
 \dot{\tilde{y}} &= -\dot{x}_r \sin \theta + \dot{x} \sin \theta + \dot{y}_r \cos \theta - \dot{y} \cos \theta \\
 &\quad - (x_r - x) \dot{\theta} \cos \theta - (y_r - y) \dot{\theta} \sin \theta \\
 &= v_r \sin \tilde{\theta} + \{\dot{x} \sin \theta - \dot{y} \cos \theta\} - \tilde{x} \omega \\
 &= v_r \sin \tilde{\theta} - \{V \sin(\gamma_2 + \delta_2)\} - \tilde{x} \omega
 \end{aligned} \tag{6.9}$$

$$\dot{\tilde{\theta}} = \omega_r - \omega_{m,s} \tag{6.10}$$

where

$$\omega_{2,1} = \frac{V \sin(\gamma_2 + \delta_2 - \alpha - \delta_1) - \gamma_1 b \cos(\delta_1)}{a \cos(\alpha + \delta_1) + b \cos(\delta_1)} \tag{6.11}$$

$$\begin{aligned}
 \omega_{1,2} &= \frac{V}{a} \tan(\gamma_1 + \delta_1) \cos(\gamma_2 + \delta_2) \\
 &\quad - \frac{V}{a} \sin(\gamma_2 + \delta_2).
 \end{aligned} \tag{6.12}$$

Unlike the robots with M2, equations (6.8)-(6.9) reveal that the additional control input γ_2 of the robots with M3 allows the tracking errors $(\tilde{\xi}, \tilde{\theta})$ to achieve zero convergence for nonzero slip angle δ_2 . This observation leads to the following tracking control objectives.

Definition 6.2.2 *Trajectory tracking problem for wheeled mobile robots with M3:* Find control laws for $\{r_w \dot{\varphi}, \gamma_1, \gamma_2\}$ such that for small initial errors $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$, the tracking errors $\tilde{\xi}$ and $\tilde{\theta}$ are uniformly bounded and converges to zero.

The *tracking control problem for wheeled mobile robots with M3* demands the tracking errors $(\tilde{\xi}, \tilde{\theta})$ converge to zero, whereas the *tracking control problem for wheeled mobile robots with M2* aims to achieve zero-converging point tracking error $\tilde{\xi}$ and a well-behaved orientation error. Similarly, we formulate a stricter control problem for robots with M3 due to the higher

number of control inputs compared with the M2 case. Nevertheless, these formulated tracking problems are physically meaningful and the solutions to these problems are useful in real practices. The following tracking control developments also assume the perturbations satisfy Assumptions 5.2.1 and 5.2.2.

6.3 GPS-based Tracking Control

6.3.1 GPS-based Tracking Control scheme

The objective of this chapter is to propose a practical tracking control scheme to solve the formulated tracking problems without imposing restrictive assumptions. To achieve this objective, we utilize exteroceptive RTK-GPS and other aiding sensors to estimate/measure the necessary information for compensation. The key attribute of these sensors is that the measurements provided by these devices are not affected by wheel skidding and slipping. With this information, the control module is able to maneuver a robot with precision in the presence of wheel skidding and slipping perturbations. Figure 6.2 summarizes the tracking control scheme's structure.

Figure 6.2: GPS-based Tracking Control scheme

6.3.2 Control design for wheeled mobile robots with M2

By following the same argument used in the Chapter 5, backstepping technique is applied to solve the tracking problem. The controller aims to achieve uniformly bounded and exponential-

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converging tracking errors using the robot's posture, velocities and perturbations information. Since, we have these information, we define auxiliary inputs

$$V_l = r_w \dot{\phi} - d \quad (6.13)$$

$$u = \omega_r - \omega_{m,s} \quad (6.14)$$

for each $(m, s) \in \{(2, 0), (1, 1)\}$. The auxiliary inputs are invertible in the sense that we can convert from $\{V_l, u\}$ back to original control input $\{r_w \dot{\phi}, \gamma_1\}$. With these input transformations, the error dynamics becomes

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (6.15)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - V_y - \omega \tilde{x} \quad (6.16)$$

$$\dot{\tilde{\theta}} = u \quad (6.17)$$

6.3.2.1 Local tracking controller

We let

$$V_1 = \frac{1}{2} \tilde{x}^2 + \frac{1}{2} \tilde{y}^2. \quad (6.18)$$

Considering $\alpha_1 = \sin \tilde{\theta}$ as the intermediate control variable, the derivative of V_1 becomes

$$\dot{V}_1 = \tilde{x} \{v_r \cos \tilde{\theta} - V_l + \omega \tilde{y}\} + \tilde{y} \{v_r \alpha_1 - V_y - \tilde{x} \omega\}. \quad (6.19)$$

By choosing

$$V_l = v_r \cos \tilde{\theta} + k_1 \tilde{x}, \quad (6.20)$$

$$\alpha_1 = \frac{V_y - k_2 \tilde{y}}{v_r}, \quad (6.21)$$

equation (6.19) becomes

$$\dot{V}_1 = -k_1 \tilde{x}^2 - k_2 \tilde{y}^2. \quad (6.22)$$

Let $\chi = [\tilde{x} \ \tilde{y} \ z]^T$ where $z = \sin \tilde{\theta} - \alpha_1$. The error dynamics (6.15)-(6.17) can be expressed as

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (6.23)$$

$$\dot{\tilde{y}} = v_r (z + \alpha_1) - V_y - \tilde{x} \omega \quad (6.24)$$

$$\dot{z} = \beta_2 u + \beta_1, \quad (6.25)$$

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where

$$\beta_1 = -\frac{\partial \alpha_1}{\partial \tilde{y}} \left\{ v_r \sin \tilde{\theta} - V_y - \tilde{x} \omega_r \right\} \quad (6.26)$$

$$-\frac{\partial \alpha_1}{\partial V_y} \dot{V}_y - \frac{\partial \alpha_1}{\partial v_r} \dot{v}_r \quad (6.27)$$

$$\begin{aligned} \beta_2 &= \cos \tilde{\theta} - \frac{\partial \alpha_1}{\partial \tilde{y}} \tilde{x} \\ \frac{\partial \alpha_1}{\partial \tilde{y}} &= -\frac{k_2}{v_r}, \quad \frac{\partial \alpha_1}{\partial V_y} = \frac{1}{v_r}, \quad \frac{\partial \alpha_1}{\partial v_r} = \frac{k_2 \tilde{y} - V_y}{v_r^2}. \end{aligned} \quad (6.28)$$

Now we consider the Lyapunov function

$$V_2(\chi) = \frac{1}{2} \tilde{x}^2 + \frac{1}{2} \tilde{y}^2 + \frac{1}{2} z^2 \quad (6.29)$$

with its derivative

$$\dot{V}_2 = -k_1 \tilde{x}^2 + \tilde{y} \{ v_r (z + \alpha_1) - V_y \} + z \{ u \beta_2 + \beta_1 \}. \quad (6.30)$$

Equation (6.30) suggests that by choosing u as

$$u = \frac{-\beta_1 - k_3 z - v_r \tilde{y}}{\beta_2}, \quad (6.31)$$

$\dot{V}_2$ becomes

$$\dot{V}_2 = -k_1 \tilde{x}^2 - k_2 \tilde{y}^2 - k_3 z^2. \quad (6.32)$$

Similarly, equation (4.7) allows us to determine $\{\dot{V}_l, \dot{V}_y\}$ based on inertial sensors measurements and $\{V_l, V_y\}$ estimates.

The performance of the control system is stated as follows.

Theorem 6.3.1 Consider a wheeled mobile robot with M2. For small initial conditions $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$, there exist positive constants k, k_4, c_1 such that the tracking errors of the closed-loop system (6.15)-(6.17), (6.20), (6.31) are governed by the following statements

1. the tracking error $\{\tilde{\xi}(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2. the point tracking error $\tilde{\xi}$ exponentially converges to zero and satisfies the inequality

$$\|\tilde{\xi}(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (6.33)$$

3. the orientation error $\tilde{\theta}$ exponentially converges to a neighborhood containing $\tilde{\theta} = 0$ and satisfies the inequality

$$|\sin \tilde{\theta}(t)| \leq k \|\chi(t_0)\| e^{-k_4(t-t_0)} + c_1 \quad \forall t \geq t_0 \geq 0. \quad (6.34)$$

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4. the steady-state orientation error satisfies

$$\lim_{t \rightarrow \infty} (\tilde{\theta}(t) - \delta_2(t)) = 0. \quad (6.35)$$

Proof: The quadratic nature of functions (6.29) and (6.32) implies there exists a positive constant k_4 such that the functions satisfy

$$V_2(\chi) = \frac{1}{2} \|\chi\|^2 \quad (6.36)$$

$$\dot{V}_2(\chi) \leq -k_4 \|\chi\|^2. \quad (6.37)$$

where $k_4 = \min\{k_1, k_2, k_3\}$. Consequently, we have

$$\dot{V}_2 \leq -2k_4 V_2. \quad (6.38)$$

By Comparison Lemma 2.4.3, inequality (6.38) leads to

$$V_2 \leq V_2(t_0) e^{-2k_4(t-t_0)} \quad (6.39)$$

which implies

$$\|\chi(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (6.40)$$

Inequality (6.40) implies the tracking errors $(\tilde{\xi}, \tilde{\theta})$ are uniformly bounded and $\chi \rightarrow 0$. Since $\tilde{\xi}$ is a subvector of χ , inequality (6.40) implies

$$\|\xi(t)\| \leq \|\chi(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \quad \forall t \geq t_0 \geq 0 \quad (6.41)$$

and this establishes condition (6.35). The intermediate control law α_1 satisfies

$$\begin{aligned} |\alpha_1| &\leq \left| \frac{V_y}{v_r} \right| + \left| \frac{k_2}{v_r} \right| |\tilde{y}| \\ &\leq c_1 + c_2 |\tilde{y}| \end{aligned} \quad (6.42)$$

where $\{c_1, c_2\}$ are positive constants such that $|V_y/v_r| < c_1$ and $|k_2/v_r| < c_2$. Furthermore, definition $z = \sin \tilde{\theta} - \alpha_1$ leads to (6.34) as follows.

$$\begin{aligned} |z| &= |\sin \tilde{\theta} - \alpha_1| \\ &\geq |\sin \tilde{\theta}| - |\alpha_1| \\ |\sin \tilde{\theta}| &\leq |z| + |\alpha_1| \\ &\leq |z| + c_1 + c_2 |\tilde{y}| \\ &\leq k \|\chi(t_0)\| e^{-k_4(t-t_0)} + c_1 \end{aligned}$$

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where $k = k(1 + c_2)$.

To show (6.35), we first establish that $V \rightarrow v_r$ as $t \rightarrow \infty$. Inequality (6.41) indicates the point tracking error $\tilde{\xi}$ is uniformly bounded and approaches zero. Since $v_r, \dot{v}_r, \omega_r, \dot{V}_y$ are uniformly bounded by assumption, the convergence of $\tilde{\xi}$ and differentiability of the control laws imply the boundedness of $\ddot{\tilde{\xi}}$; hence $\dot{\tilde{\xi}}$ is uniformly continuous. Since $\tilde{\xi}$ is finite, by Barbalat lemma, we conclude that $\dot{\tilde{\xi}} \rightarrow 0$ as $t \rightarrow \infty$ and this means $V \rightarrow v_r$ as $t \rightarrow \infty$. The convergence of z implies that

$$\begin{aligned} \lim_{t \rightarrow \infty} \{\sin \tilde{\theta} - \alpha_1\} &= 0 \\ \lim_{t \rightarrow \infty} \{\sin \tilde{\theta} - \frac{V_y}{v_r}\} &= 0 \\ \lim_{t \rightarrow \infty} \{\sin \tilde{\theta} - \frac{V_y}{V}\} &= 0 \end{aligned} \quad (6.43)$$

Geometric relation (3.1) and equality (6.43) implies $\lim_{t \rightarrow \infty} (\tilde{\theta}(t) - \delta_2(t)) = 0$ and this completes the proof.

□

Theorem 6.3.1 solves the tracking problem for wheeled mobile robots with M2 based on backstepping control laws. The result provides estimates on the effect of control gains $\{k_1, k_2, k_3\}$. For instance, the convergence rate and the maximum overshoot of the tracking errors can be estimated using inequalities (6.33)-(6.34).

We can see that β_2 appears as the denominator of the control law (6.31). If the initial condition is sufficiently small (e.g., $|\tilde{\theta}| < \frac{\pi}{4}$ rad and $|\tilde{x}| < 0.5$ m), then β_2 is always greater than zero which implies the control law is well-defined.

The proposed controller achieves local stability performance instead of global stability performance since $z = 0$ implies more than one isolated equilibrium points in the original state-space $(\tilde{\xi}, \tilde{\theta})$. Nevertheless, it would be sufficient to consider the neighborhood about $(\tilde{\xi}, \tilde{\theta}) = (0, 0)$ with $|\tilde{\theta}| < \frac{\pi}{2}$.

Equation (6.35) implies $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0$ if $\lim_{t \rightarrow \infty} \delta_2(t) = 0$. Due to the low maneuverability of the robot, $\tilde{\theta}$ does not converge to zero if δ_2 is nonzero; nevertheless in many practical cases, a zero-converging point tracking error and a well-behaved orientation error are sufficient.

Besides $(\tilde{\xi}, \tilde{\theta}) = (0, \delta_2)$, the point $(\tilde{\xi}, z) = (0, 0)$ also corresponds to $(\tilde{\xi}, \tilde{\theta}) = (0, \pi + \delta_2)$ in the original tracking error coordinates. If the initial tracking errors $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$ are large, then the control laws will drive the tracking errors towards $(\tilde{\xi}, \tilde{\theta}) = (0, \pi + \delta_2)$. The

physical interpretation of this convergence is that the robot tries to track the reference trajectory $\{x_r, y_r, \theta_r\}$ in reverse direction, i.e., $\tilde{\xi} \rightarrow 0$ with $V_l < 0$. Therefore, the small initial condition is required to apply the tracking control laws.

6.3.3 Control design for wheeled mobile robots with M3

This section presents the control design for the wheeled mobile robots with M3 to solve the tracking problems. Consider the tracking error dynamics

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (6.44)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - V_y - \tilde{x} \omega \quad (6.45)$$

$$\dot{\tilde{\theta}} = \omega_r - \omega_{m,s} \quad (6.46)$$

where

$$V_l = V \cos(\gamma_2 + \delta_2), \quad V_y = V \sin(\gamma_2 + \delta_2). \quad (6.47)$$

We define invertible auxiliary inputs

$$V_l = \frac{r_w \dot{\varphi} - d}{\cos \delta_2} \cos(\gamma_2 + \delta_2) \quad (6.48)$$

$$u = \omega_r - \omega_{m,s} \quad (6.49)$$

for each $(m, s) \in \{(2, 1), (1, 2)\}$. Similarly, the auxiliary inputs $\{V_l, u\}$ can be converted back to original control inputs $\{r_w \dot{\varphi}, \gamma_1\}$. Equations (6.44)-(6.46) become

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (6.50)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - V \sin(\gamma_2 + \delta_2) - \tilde{x} \omega \quad (6.51)$$

$$\dot{\tilde{\theta}} = u. \quad (6.52)$$

Equation (6.47) suggests that choosing

$$\gamma_2(t) = -\delta_2(t) \quad (6.53)$$

eliminates V_y and leads the tracking model into

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (6.54)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - \tilde{x} \omega \quad (6.55)$$

$$\dot{\tilde{\theta}} = u. \quad (6.56)$$

Equations (6.54)-(6.56) indicate that nominal tracking controllers, e.g., [10, 18, 19] that were designed based on non-skidding and non-slipping assumptions can be applied along with (6.48), (6.49), (6.53) to solve the tracking problems for the robots with M3. This strategy is one direct way of exploiting the sensors measurements to achieve precise maneuver for the robots with M3. However, this approach assumes δ_2 is eliminated from equation (6.45), i.e., $\gamma_2(t) + \delta_2(t) = 0$.

Unfortunately, if $\gamma_2(t) + \delta_2(t)$ is nonzero due to the range limitation of control input γ_2 , then the above-mentioned approach fails to solve the tracking problem, i.e., the wheeled mobile robot will not track the reference trajectory with precision. To illustrate this point, we consider the control input $\gamma_2(t)$ has a maximum limit $\gamma_{\max}$ such that $|\gamma_2(t)| \leq \gamma_{\max}$, then under the choice of (6.49), we can model $\gamma_2(t)$ as

$$\gamma_2 = \begin{cases} -\delta_2 & \text{if } |\delta_2| \leq \gamma_{\max} \\ -\text{sgn}(\delta_2)\gamma_{\max} & \text{if } |\delta_2| > \gamma_{\max} \end{cases} \quad (6.57)$$

To analyze the saturation effects, we let $\delta_s = \delta_2 + \gamma_2$ and equation (6.51) can be expressed as

$$\dot{y} = v_r \sin \tilde{\theta} - V_y - \tilde{x}\omega \quad (6.58)$$

where $V_y = V \sin \delta_s$.

If the control input γ_2 is saturated while compensating $\delta_2(t)$, i.e., $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$, then δ_s is nonzero and it is undesirable to apply the nominal tracking control laws e.g., [10, 18, 19], with (6.48), (6.49), and (6.53) to solve the tracking problem. In contrast, the combination of $\{(6.48), (6.49), (6.53)\}$ with the backstepping control laws $\{(6.20), (6.31)\}$ can at least achieve zero-converging point tracking error even when $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$. This positive feature becomes explicit in the following statements.

Theorem 6.3.2 Consider a wheeled mobile robot with M3 with control input satisfies

$$|\gamma_2(t)| \leq \gamma_{\max} \quad (6.59)$$

where $\gamma_{\max}$ is the maximum of the control input γ_2 . For small initial conditions $\{\tilde{\xi}(t_0), \tilde{\theta}(t_0)\}$, there exist positive constants k, k_4 such that the tracking errors of the closed-loop system (6.20), (6.31), (6.50)-(6.52), and (6.53) are governed by the following statements

1. the tracking errors $\{\tilde{\xi}(t), \tilde{\theta}(t)\}$ are uniformly bounded.
2. the point tracking error $\tilde{\xi}$ exponentially converges to zero and satisfies the inequality

$$\|\tilde{\xi}(t)\| \leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \quad \forall t \geq t_0 \geq 0. \quad (6.60)$$

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3. the orientation error $\tilde{\theta}$ exponentially converges to a neighborhood containing $\tilde{\theta} = 0$ and satisfies the inequality

$$|\sin \tilde{\theta}(t)| \leq k \|\chi(t_0)\| e^{-k_4(t-t_0)} + c_1 \quad \forall t \geq t_0 \geq 0 \quad (6.61)$$

where $c_1 = 0$ if $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$ and $c_1 > 0$ if $\sup\{|\delta_2(t)|; \forall t \geq t_0\} < \gamma_{\max}$.

4. the steady-state orientation error satisfies

$$\begin{aligned} \lim_{t \rightarrow \infty} \tilde{\theta}(t) &= 0 && \text{if } \sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max} \\ \lim_{t \rightarrow \infty} \left(\tilde{\theta}(t) - \delta_2(t) - \gamma_2(t) \right) &= 0 && \text{otherwise} \end{aligned} \quad (6.62)$$

Proof: The proof of this result is straightforward. Without loss of generality, we express the tracking error model (6.50)-(6.52) as

$$\dot{\tilde{x}} = v_r \cos \tilde{\theta} - V_l + \omega \tilde{y} \quad (6.63)$$

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - V \sin(\delta_s) - \tilde{x} \omega \quad (6.64)$$

$$\dot{\tilde{\theta}} = u. \quad (6.65)$$

$\delta_s(t)$ is analogous to the slip angle ' $\delta_2(t)$ ' of the M2 case. Similarly, equation (6.64) can be expressed as

$$\dot{\tilde{y}} = v_r \sin \tilde{\theta} - V_y - \tilde{x} \omega \quad (6.66)$$

where $V_y = V \sin \delta_s$. Equations (6.63)-(6.65) match with the tracking error model (6.15)-(6.17) of a robot with M2. Hence, by applying control laws (6.20) and (6.31), the tracking errors $\{\tilde{\xi}, \tilde{\theta}\}$ satisfy

$$\begin{aligned} \|\tilde{\xi}(t)\| &\leq \|\chi(t_0)\| e^{-k_4(t-t_0)} \\ |\sin \tilde{\theta}(t)| &\leq k \|\chi(t_0)\| e^{-k_4(t-t_0)} + c_1 \quad \forall t \geq t \geq 0 \end{aligned}$$

where $k_4 = \min\{k_1, k_2, k_3\}$ and $c_1 \geq 0$ such that $|V_y/v_r| < c_1$ for $\forall t \geq t \geq 0$. Note that $\{k_1, k_2, k_3\}$ are the control gains of control laws (6.20) and (6.31). If $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$, then the choice (6.53) implies $\delta_s = 0$, i.e., $V_y = 0$ for all t and this leads to $c_1 = 0$. If $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$, then δ_s is nonzero for some time intervals such that $|\delta_s| \leq \rho_2 - \gamma_{\max}$ since γ_2 saturates at $\gamma_2 = -\text{sgn}(\delta_2)\gamma_{\max}$. Consequently, V_y is nonzero and hence $c_1 > 0$. By following the same argument used in Theorem 6.3.1, we know that $\tilde{\theta} \rightarrow \delta_s$ as $t \rightarrow \infty$. Since $\delta_s = 0$ if $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$, we conclude (3.63) and this completes the proof.

□

Theorem 3 shows that the combination of perturbation decoupling control laws (6.48), (6.49), (6.53) and the backstepping control laws (6.20), (6.31) converge the tracking errors $(\tilde{\xi}(t), \tilde{\theta}(t))$ exponentially to origin.

The control result demonstrates that the additional orientation input $\gamma_2(t)$ possesses by a mobile robot with M3 allows zero-converging tracking errors $(\tilde{\xi}(t), \tilde{\theta}(t))$ for nonzero δ_2 . This control performance is superior to the M2 case (assuming there is no saturation occurs at the control input γ_2 , i.e., $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$).

In the case where the robot's max steering $\gamma_{\max}$ is smaller than $|\delta_2(t)|$ for some t intervals, then the combination of the control laws $\{(6.20), (6.31), (6.48), (6.49), (6.53)\}$ can at least ensure the point tracking error $\tilde{\xi}$ exponentially converges to zero with a well-behaved orientation error for small initial errors.

6.4 Simulation results

The control algorithms developed in this paper were simulated using MATLAB SIMULINK. The reference velocity and reference angular input are selected as $\{v_r, \omega_r\} = \{1 \text{ ms}^{-1}, 0.1 \text{ rad/sec}\}$. The control gains used in this simulation are selected as

$$k_1 = 1, \quad k_2 = 1, \quad k_3 = 1. \quad (6.67)$$

6.4.1 Tracking control for wheeled mobile robots with M2: Type (2,0) robot

In this section, the tracking control laws are simulated on a Type (2,0) wheeled mobile robot. The kinematic perturbations $\{\delta_1, \delta_2, d, V_y\}$ chosen in this simulation are depicted in Figure 6.3. Figures 6.4 and 6.5 depict the simulation results for small initial condition

$$(\tilde{x}, \tilde{y}, \tilde{\theta}) = (-0.4\text{m}, 0.4 \text{ m}, 5.73 \text{ deg}).$$

The responses of the tracking control laws $\{(6.20), (6.31)\}$ are shown in the figures. The point tracking error $\tilde{\xi} = (\tilde{x}, \tilde{y})$ converges to zero exponentially, whereas the orientation error $\tilde{\theta}(t)$ tends to $\delta_2(t)$ as $t \rightarrow \infty$. Additionally, the simulation results show the smooth continuity of the control inputs. The smooth control inputs suggest that the tracking control laws are preferable

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Figure 6.3: Perturbations

Figure 6.4: Tracking errors $\{\tilde{x}, \tilde{y}, \tilde{\theta}\}$

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Figure 6.5: Control inputs $\{r_w\dot{\varphi}, \gamma_1\}$

for implementation compared with discontinuous or fast-switching control laws. The effects of longitudinal slip d are eliminated by injecting more velocity via input transformations (6.13)-(6.14) to the robot. These simulation results show the consistencies of Theorem 6.3.1.

6.4.2 Tracking control for wheeled mobile robots with M3: Type (1,2) robot

This section validates the control laws for wheeled mobile robots with M3 (Theorem 6.3.2). In this simulation, the results are simulated on a Type (1,2) robot with a wheelbase of $a = 1.2$ m. We consider two cases: (i) control input γ_2 satisfies condition $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$, i.e., no saturation for the control input; (ii) $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$ where the control input γ_2 saturates at some time intervals. The designated reference trajectory $\{x_r(t), y_r(t), \theta_r(t)\}$ to be followed and the control gains remain unchanged. The maximum magnitude of γ_2 is chosen as $\gamma_{\max} = 0.1$ rad. Similarly, we consider initial errors of $(\tilde{x}, \tilde{y}, \tilde{\theta}) = (-0.4 \text{ m}, 0.4 \text{ m}, 5.73 \text{ deg})$. The perturbations chosen for this simulation are shown in Figure 6.6 with a $\sup\{|\delta_2(t)|; \forall t \geq t_0\} = 0.15$ rad.

Figure 6.6: Perturbations

6.4.2.1 Tracking control with no γ_2 saturation

Figures 6.7 and 6.8 show the tracking performances of the closed-loop system $\{(6.20), (6.31), (6.50) - (6.52), (6.53)\}$ with condition $\sup\{|\delta_2(t)|; \forall t \geq t_0\} \leq \gamma_{\max}$. The point and orientation errors $\{\tilde{\xi}, \tilde{\theta}\}$ exponentially converge to zero even when the perturbation δ_2 is nonzero. Unlike the robots with M2, the additional control input γ_2 eliminates the perturbation δ_2 , resulting zero convergence for both point tracking and orientation errors.

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Figure 6.7: Tracking errors $\{\tilde{x}, \tilde{y}, \tilde{\theta}\}$ Figure 6.8: Control inputs $\{r_w \dot{\varphi}, \gamma_1, \gamma_2\}$

Figure 6.9: Tracking errors $\{\tilde{x}, \tilde{y}, \tilde{\theta}\}$

6.4.2.2 Tracking control with γ_2 saturation

Figures 6.9-6.10 depict the tracking errors of the closed-loop systems with $\sup\{|\delta_2(t)|; \forall t \geq t_0\} > \gamma_{\max}$. Figure 6.10 depicts the control input saturates at -0.1 rad for $\forall t \geq 5$ sec. In this case, the control input γ_2 cannot completely eliminate the undesirable effects of $\delta_2(t)$. Nevertheless, the point tracking error $\tilde{\xi}$ exponentially converges to zero and the orientation error converges to $\delta_2(t) + \gamma_2(t)$. Similarly, we see that the effects of longitudinal slip perturbation d is eliminated by (6.48). These simulation results confirm Theorem 6.3.2.

Figure 6.10: Control inputs $\{r_w \dot{\varphi}, \gamma_1, \gamma_2\}$

6.5 Experimental results

6.5.1 Experimental setup

Figure 6.11: Experimental site

The experimental validation of the GPS-based tracking control scheme was again performed on the Cycab mobile robot (see Appendix A). Similarly, we let $\{X, Y\}$ be the East and North axes, and the origin be the location of the GPS base station. The orientation of the vehicle is determined by integrating the measured yaw rate $\dot{\theta}$ using trapezoidal integration. The trial run is conducted at the same open space at our school campus as shown in 6.11. Similarly, the observation equations (4.12)-(4.15) and the sensors measurements are used to determine $\{x, y, V_l, V_y\}$ for control.

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6.5.2 Experimental results

Figure 6.12: Desired Path

Figure 6.13: Measured Perturbations

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Figure 6.14: Tracking performances

Figure 6.15: Control inputs

6.5.2.1 Experiments without perturbations compensation

To validate the GPS-based tracking control scheme, the wheeled mobile robot was first controlled by the backstepping control laws without any compensation action, i.e., $(\delta_1 = 0, V_y = 0, d = 0)$. The control gains chosen in the experiments are

$$k_1 = 0.1, \quad k_2 = 0.1, \quad k_3 = 0.3. \quad (6.68)$$

The mobile robot was initially driven to a starting point such that the initial tracking errors are $(\tilde{\xi}(0), \tilde{\theta}(0)) = (0, 0)$. The velocity and angular rate inputs of the reference trajectory are chosen as $(v_r, \omega_r) = (0.8 \text{ ms}^{-1}, -0.08 \text{ rad s}^{-1})$ such that the corresponding geometric path is a curve as shown in Figure 6.12.

The control scheme was activated to perform the tracking control task. The tracking errors are shown in Figure 6.14 (dashed-dot lines). We see that the tracking errors $(\tilde{\xi}, \tilde{\theta})$ remain in a neighborhood about point $(\tilde{\xi}, \tilde{\theta}) = (0, 0)$ with $\tilde{x}$ converges to a steady-state value of 0.05 m, $\tilde{y}$ converges to a nonzero bias of -0.06 m, and orientation error $\tilde{\theta}$ converges to a low amplitude close to zero. These steady-state errors in particular the nonzero bias of tracking error $\tilde{y}$ motivates us to introduce perturbations corrective actions into the tracking control laws to enhance the tracking performance.

6.5.2.2 Experiments with perturbations compensation

Wheel skidding compensation was then activated by including the perturbation terms in the control laws. Similarly, the vehicle began from the same starting point with the same control gains $\{k_1, k_2, k_3\}$. Figure 6.13 depicts the perturbations computed using the GPS measurements. Figure 6.15 shows the robot's linear velocity V_l and the smooth steering input γ_1 recorded during the experiment. Since $V_l \approx r_w \dot{\varphi}$, the longitudinal slipping velocity d is insignificant and additional velocity compensation is not required. The estimated V_y shown in Figure 6.13 also has a zero mean. On the other hand, the front slip angle δ_1 is dominant compared with V_y and d . The tracking performance are shown by the solid-lines in Figure 6.14. We can see that introducing perturbation compensation reduces the steady-state bias of $\tilde{y}$. This reduction of the bias is due to the compensation of perturbation δ_1 by the control laws during the run. The mild nonzero steady-state orientation error could due to the accumulated integration error of the orientation estimate. Although the experimental setting is not designed to validate all theoretical aspects of the tracking control scheme, the experimental results show that the GPS-based control scheme

is able to compensate skidding perturbations based on RTK-GPS and other aiding sensors measurements.

6.6 Conclusions

In this chapter, tracking problems are formulated and solved using a GPS-based control scheme for the four generic wheeled mobile robots with wheel skidding and slipping. This scheme relies on high-accuracy RTK-GPS and other aiding sensors measurements for precise tracking. Similarly, we demonstrate that a robot with higher maneuverability has the ability to achieve better tracking performance than robots with lower maneuverability. We show that the developed control laws achieve uniform boundedness with exponential convergence without imposing restrictive assumptions on the skidding and slipping perturbations. The control scheme is validated by means of simulation and experiments. The implementation is conducted on the Type (1,1) experimental platform. The experimental results show that the idea of exploiting RTK-GPS and other aiding sensors measurements to address the tracking control problem with wheel skidding and slipping is positive and effective.

Chapter 7

Conclusions and Future Research

7.1 Conclusions

We have discussed and contributed the wheeled mobile robot control problems in the presence of wheel skidding and slipping whose motions are described by kinematic models. The overall goal has been to improve maneuvering precision using RTK-GPS and nonlinear control theory. The achievements include the development of kinematic models for the wheeled mobile robots; the development of integrated estimation systems based on RTK-GPS and other aiding sensors to estimate the information required for precise control; and the development of control laws to solve the practical path following and trajectory tracking problems.

The first achievement is the development of kinematic models for four general mobile robots with wheel skidding and slipping from control perspective. The kinematic models facilitate efficient and practical control designs by describing the kinematic perturbations due to wheel skidding and slipping using physically appealing geometric descriptions. These explicit descriptions lead us to useful insights and control strategy to address the problem. The kinematic models show the limitations of a mobile robot when it skids. This understanding allows suitable control problem formulations for each robot based on its capability. Additionally, perturbations are classified to measure the difficulties they impose to the control problems. This classification leads to suitable control design technique to address the control problems. Most importantly, these kinematic models show that exteroceptive sensors, in particular RTK-GPS, is useful in measuring the perturbations for compensations. These findings shed light on strategies for the development of efficient control scheme to achieve precise mobile robot control in the presence of wheel skidding and slipping.

The second achievement is the development of estimation schemes to provide continuous estimates on a robot's posture, velocities, and perturbations for precise control. The estimation schemes rely on RTK-GPS, inertial sensors and an absolute orientation sensor for estimation. Firstly, kinematic estimation model is derived based on the mobile robot's kinematic model. Measurement equations are established to measure these information using the absolute sensors. Extended Kalman Filter is applied based on the model and measurements to estimate the information. Under the assumption of accurate orientation sensor, a simplified cascaded estimator is developed based on the kinematic estimation model and linear Kalman Filters. This cascaded estimator reduces the real-time computation load. We also consider a special case where the RTK-GPS only provides positioning measurements. Through this integration, the estimators update rate are now limited by the high-bandwidth of the inertial sensors. Additionally, they also provide estimates during short GPS-outages. These algorithms are implemented on a Type (1,1) experimental platform and the results show the effectiveness of the estimators. With these systems, the GPS-based control system will no longer limited by the low-update rate and intermitted loss of GPS signals.

The third achievement is the development of GPS-based path following control scheme to address the path following problem in the presence of wheel skidding and slipping. The control scheme applies to the four mobile robots. Path following models are developed for each mobile robot. Path following control problems are formulated based on the controllability of each mobile robot. The GPS-based control scheme relies on the robot's posture, velocities, and perturbations information provided by the RTK-GPS, inertial sensors, and an absolute orientation sensor for control. Backstepping control laws are developed to attain precise path following control. Sufficient condition for Type (2,0) and Type (1,1) robots are determined to ensure the lateral error converges to zero with a well-behaved orientation error. For Type (2,1) and Type (1,2) robots, sufficient condition is also established to achieve zero-converging lateral and orientation errors. These conditions assume the initial conditions are small. We also address conditional global path following problem where initial errors can be large. The control scheme is implemented on the experimental platform to verify the theories. The results show the feasibility of exploiting the RTK-GPS and other aiding sensors to enhance the robot's control performance in the presence of wheel skidding and slipping. With this control scheme, the mobile robots are able to follow a geometric path with precision in outdoor environments.

The last but definitely not least achievement is the development of GPS-based tracking con-

trol scheme to attain precise trajectory tracking with wheel skidding and slipping for the mobile robots. Firstly, tracking error models are derived based on the robot kinematic models. Tracking control problems are formulated for each mobile robot. Similarly, the GPS-based tracking control scheme exploits the information determined by the sensors for control. Tracking control laws are developed for the robots based on backstepping control design to attain precise tracking control. Sufficient condition for Type (2,0) and Type (1,1) robots are determined to ensure the point tracking error converges to zero with a well-behaved orientation error. For Type (2,1) and Type (1,2) robots, sufficient condition is also established to achieve zero-converging point and orientation errors. These conditions assume the initial tracking errors are small. The tracking scheme is tested on the Type (1,1) experimental platform and the experimental results reveal the positive features of the control scheme.

Above all, the thesis has dealt with the wheeled mobile robot control problems with wheel skidding and slipping from the methodology proposal and theoretic development to simulation and experimental implementation of the estimators and controllers. Relevant issues regarding solving the path following and tracking control problems based on kinematic models have been discussed in detail. In conclusion, the proposed GPS-based path following and tracking control schemes are ready to be applied in actual applications.

7.2 Further Research

Based on the works in this thesis, some further research topics are recommended as follows.

- The mobile robot models utilized in this thesis are kinematic models. Kinematic solutions are proposed to the mobile robot control problems. Next, we may consider the mobile robot control problem from the mechanical dynamic level where control input are torques or forces. To develop effective controllers using dynamic model, the skidding and slipping disturbances in the dynamic equation of motion has to be considered. An explicit dynamic model of a mobile robot is required to provide useful information for efficient and practical control designs. Mobile robot control design at dynamic level is always more challenging compared with control design at kinematic level. This is due to the larger number of states and the inertial uncertainties at the dynamic level. These considerations pose tough problems in designing efficient and applicable controllers to address the control issues with wheel skidding and slipping.

- In this thesis, single-body wheeled mobile robots are considered. In some applications, multi-body wheeled mobile robot, e.g., a car with a single trailer [96], is found useful in automated farming application. Multi-bodies mobile robot control problems are clearly more challenging than the single-body wheeled mobile robots. This motivates the development of control systems for these robots that are capable of handling wheel skidding and slipping. Similarly, issues like robot modeling, estimation and control designs are to be addressed in order to achieve precise maneuvering of these robots.
- The GPS-based backstepping controllers proposed in this thesis are based on stability in Lyapunov notions. These controllers are the first set of controllers proposed in the GPS-based framework to satisfy stability conditions in the Lyapunov sense. In the future, we will look for other better control laws that are based on other practical stability notions.
- As the control schemes proposed in this thesis rely on GPS, it is clear that the GPS-based control system is restricted to outdoor environments where GPS signals are consistent. If the mobile robot is to maneuver with precision in environment where GPS signals are frequently obstructed for long duration, the estimation systems proposed in this thesis may be insufficient. In this case, an alternate absolute positioning sensor that is able to work in these GPS-unfriendly environments is required to replace the role of the RTK-GPS sensor to provide the critical absolute information for precision control. Recently, a laser-based 360⁰ scanner named NAV200 is developed by SICK INC [95]. This sensor is capable of providing positioning and velocity information based on triangulation principle. The sensor relies on the time-of-flight taken for the transmitted laser to some reflective beacons to compute the location and velocity of the laser scanner. Additionally, the system can cover up to a range of 50m per zone with centimeter accuracy. Most importantly, NAV200 is exteroceptive in nature and therefore its measurements are not affected by wheel skidding and slipping. A preliminary work has been proposed to provide continuous positioning information for real-time control [97] using NAV200 and RTK-GPS sensors. This work suggests that NAV200 can also be integrated with the inertial sensors based on the estimation systems proposed in this thesis to enhance the coverage of precision control in environments where GPS signals are obstructed for long duration.

Appendix A

Experimental Hardware

All the experiments discussed in this thesis were implemented on the Type (1,1) robot namely Cycab (see Figure A.1).

Figure A.1: Type (1,1) car-like robot

A.1 Low-level Control System of Cycab

The mobile robot has four wheels which are driven by four independent DC motors. An incremental encoder is fitted on each DC motor to measure the wheel's angular velocity. An absolute encoder is installed on the robot's steering system to measure the steering angle γ_1 . A Pentium PC with Window operating system hosts the vehicle control software. The software is written

using Visual C++ to communicate with the Cycab's onboard computer for steering and driving actions. The software is also interfaced with a 12-bits DAQ card to acquire the signals from the INS sensors and the encoders.

A.1.1 Hardware configuration

Figure A.2: Hardware configuration of Cycab

Figure A.2 shows the structure of the Cycab. The low-level control system is a computer controlled system using a Motorola MC68LC040-based Embedded Controller Motion control board to control the four motor controllers and one steering controller.

For each DC-motor, there is one PID feedback control loop to drive the motor to a desired velocity. The PID controller is implemented on the Motion control board with adjustable PID gains by software. The PID controller receives the motor speed measurement from the incremental encoder mounted with the motor as shown in Figure A.3. The output of the PID controller is converted to DC voltage which is treated as reference voltage to motor controller. The motor controller functions as an amplifier which generates a high-current PWM voltage proportional to the reference voltage to drive the motor.

The steering system is also a closed-loop system. The steerable wheels are mechanically linked together and an electro-hydraulic system is used to transform a reference voltage into a force acting on the mechanical links to steer them. A PID controller oversees the whole process and takes feedback of angular position from an absolute encoder. In Figure A.4, the absolute encoder is hidden under the flat surface and firmly linked to the pistol of the hydraulic cylinder.

Figure A.3: Cycab's driving system

The translation of the pistol is created under the pressure of the oil inside the cylinder, which in turn is pumped by a set of a DC motor and a hydraulic pump. Similarly to the driving system, the motion of the DC motor of the steering system is also generated by a current-modulated PWM voltage from a motor controller (or amplifier). This PWM voltage is generated proportionally based on a $\pm 10V$ reference voltage from the PID controller.

Figure A.4: Cycab's steering system

A.1.2 Software configuration

The onboard computer-based embedded controller runs on a real-time operating system, namely *ALBATROSTM*, specially designed for multi-axis devices real-time control. The features of *ALBATROSTM* include a real-time kernel, all I/O drivers, generalized PID regulators, trajectory generators, sensor read modules and a command interpreter. The real-time kernel is to generate the real-time clocks for the entire system to synchronize operations. The closed-loop PID regulators and the trajectory generators are two periodic tasks that base on the real-time clocks. A trajectory generator is to create a reference trajectory (position with respect to time)

Table A.1: Frequently used commands of Blue Cycab

Command	Description
serv on t=bv d=1	Turn on PID control loops for velocities
serv of d=1	Turn off control loops for driving
serv on t=bp d=2	Turn on PID control loop for steering angle
serv of d=2	Turn off control loop for steering
move v ac=<fl>,<fr>,<rl>,<rr>	Set desired velocity for each driving motor with resolution of $0.1m/s$ <fl> : front-left desired velocity <fr> : front-right desired velocity <rl> : rear-left desired velocity <rr> : rear-right desired velocity
move v rc=<fl>,<fr>,<rl>,<rr> p=<n>	Set relative velocity with a preset time to complete → to control accelerations <n> : number of control periods to reach to the command
move p ac=<fs>,0 d=2	Set desired steering angle with resolution of 0.1° <fs> : desired steering angle

whenever a new desired position/velocity is set. A PID regulator drives the motor to follow this reference trajectory. In general, the sampling period of the trajectory generator is a multiple of that of the PID regulator. By default, they are 10ms for PID controllers and 40ms for trajectory generators. A wide set of text-based commands is also provided through which users can use to take virtually all the controls to any part, even the kernel, of the system. Some examples of the commands are listed in Table A.1.

A.2 Sensing System

The mobile robot is equipped with a MS-750 RTK-GPS receiver which is able to provide both position and velocities measurements simultaneously. Throughout the experiments discussed in this thesis, we let the $\{X, Y\}$ axes of the global coordinate system be the East and North axes. Additionally, a KVH RA1100 Fibre Optic gyroscope and a CXL01LF3 tri-axial accelerometer are attached on the robot's reference point P to provide high-update rate inertial information.

The following subsections present the technical specifications of the sensors.

A.2.1 Sensors Specifications

A.2.1.1 IMU

Performance	
Input Range (g)	$\pm g$
Zero g Drift (mV)	± 30
Sensitivity (V/g)	2
Alignment Error (deg)	± 2
Noise (mg rms)	0.5
Bandwidth (Hz)	50
Electrical	
Supply Voltage (Volts)	$+5 \pm 0.25$
Zero g output (Volts)	$+2.5 \pm 0.15$
Physical	
Dimensions	$1.9 \times 4.76 \times 2.54$ cm
Weight	46 gm

Table A.2: Specification of CXL01LF3 Accelerometer

The IMU installed on the Cycab encompasses a low-cost solid state tri-axis accelerometer and a single axis fibre optic gyroscope to provide high-update rate navigation information.

The CXL01F3 accelerometer is a strapdown inertial sensor which is designed for general measurement of linear acceleration in dynamic environments. The sensor is bonded to a high quality ceramic substrate where it is coupled to signal conditioning electronics. The device offers analog voltage linearly proportional to the acceleration of each axis. These voltages can be easily interfaced with a PC using a standard data acquisition system. The accelerometer module's offset and sensitivity are factory calibrated and tested. The technical specification of the accelerometer is shown in Table A.2.

The KVH RA1100 Fibre Optic gyroscope (FOG) is a single axis rate sensor. Similarly, the rate sensor employs an analog electronics signal processor and all-fiber optical system. The device provides analog voltage output linearly proportional to its rotational rate which can also

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be sampled by a standard data acquisition system. The technical specification of the FOG is shown in Table A.3.

Performance	
Input Rotation rate (deg/sec)	± 100
Resolution Rate (deg/sec)	0.014
Scale factor(mv/deg/sec)	20
Bias stability (deg/sec p-p)	0.12
Angle Random Walk (deg/rt-Hz)	0.08
Bandwidth (Hz)	100
Start up time	1 second
Electrical	
Supply Voltage (V)	+9 to +18 VDC
Analog output	+2.5VDC $\pm$ 2.0 VDC
Physical	
Dimensions	11.2 $\times$ 10.8 $\times$ 4.3 cm
Weight	340gm

Table A.3: Specification of KVH-RA1100 Gyroscope

A.2.1.2 MS750 RTK GPS

MS750 RTK (Real-time kinematic) system is a high-grade GPS measurement system that provides accurate positioning and velocity information for surveying or dynamic positioning. The RTK GPS system includes two MS750 receivers. One receiver functions as a base station and another receiver measures the position and velocity of another receiver. Each MS750 receiver can track up to nine satellites and it determines the position of the receiver either using GPS code phase or carrier phase information. This system operates using dual frequencies (L1/L2). The base station provides the RTK differential correction via an UHF radio data link. The RTK integer ambiguities are solved using on the fly (OTF) method and centimeter accuracy position fixes is attained once the integer ambiguities are solved. The receivers are equipped with two RS-232 series ports which allow the receiver to output both position and velocity information simultaneously through two different NMEA output formats.

Performance	
Max update rate	20Hz
start up	< 90 secs
RTK Initialization	Typically < 1 mins
Accuracy (RTK)	2cm+2ppm (Horizatonal)
Range	up to +18 VDC
Output formats	NMEA-0183: GPK, GGA, ZDA VTG, GST, PJT and PKJ
Physical	
Dimensions	14.5cm W × 5.1cmH × 23.9cmD
Weight	1.0 kg
Power	12VDC/24DC,9 Watts

Table A.4: Specification of MS750

A.3 Communication and control of the Cycab

Figure A.5 depicts the overall block diagram of the control system for controlling the vehicle. In brief, there is a host computer which performs the control algorithm computation and estimations using measurements acquired from the sensors. At every iteration, positioning and velocity measurements from GPS receiver are interfaced with the Visual C++ program with RS-232 communication. The analog measurements of the inertial sensors are interfaced with the program using a 12-Bit DAQ card. The absolute encoder and the incremental encoders that attached to the wheels are also interfaced with the software via the DAQ card. Once the control laws are computed at each iteration, the host computer accesses the low-level onboard computer to perform the necessary control actions. The host computer and the onboard computer are connected via RS-232 communication.

Figure A.5: Overall Block Diagram of control system

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- [2] Chang Boon Low, Danwei Wang, Robust Path following of car-like WMR in the presence of skidding effects. In *Proc. IEEE International Conference on Mechatronics*, Taipei, July 2005.
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