Micro-ring devices and circuits

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MICRO-RING DEVICES AND CIRCUITS

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“A man can do nothing better than to eat and drink and find satisfaction in his work. This too, I see, is from the hand of God,” (Ecclesiastes 2:24)

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ABSTRACT

A microring resonator is a realization of Fabry-Perot etalon in planar technology. The cavity mode of a microring resonator is normally excited by evanescent coupling from optical waveguides, which are analogous to semi-reflecting mirrors, and with resonance characteristics based on the ring circumference. Microring resonators have been extensively exploited in various active and passive functions, owing to their compact size and highly resonant behavior. Their planar nature allows integration with many resonators in a variety of array geometries, extending the functionalities even further to higher order filter and delay lines, which are important for optical interconnects. For the above reasons, the microring resonator is often thought as a good candidate in integrated optics.

This thesis is focused on the theoretical analysis, device design, and experimental demonstration of integrated optical devices derived from coupled microring resonators. The main idea of this research is to exploit inter-cavity interactions in certain multi-resonator configurations for application-specific purposes ranging from box-like filters to optical bistable switches. The proposed devices have been fabricated and experimentally verified using the CMOS-based silicon-on-insulator technology, under ePIXnet platform. Experimental results show excellent agreement with theoretical predictions.

The model used in this thesis is based on a simple ring resonator side-coupled to optical waveguides, which is then extended to two-cavity structures, and finally to multi-cavity arrays to overcome limitations that are intrinsic in one-cavity system. The finesse enhancement in two-cavity structure is investigated in this thesis, particularly in the rather strong coupling. In a one-cavity system, it is argued that the cavity finesse is more limited to practical concerns in realizing a very weak coupling and the fact that the roundtrip loss cannot be known before fabrication. As a result, there are difficulties in achieving near critically-coupled transmission in one-cavity systems, particularly when the roundtrip loss is so small that the required coupling would be unrealistically small.

On the contrary, by adjusting the relative sizes of both rings in two-cavity structure, we can have a very narrow resonance as a result of inter-pathway interference originating from both rings. It is theoretically shown that a two order of magnitude finesse enhancement can be obtained with realistic coupling coefficients. Experimentally, a factor of 10 to 20 finesse enhancement is observed and near critically-coupled transmission is demonstrated in $1\mu\text{m}$ tolerance in the two-ring-one-bus configuration. In addition, by adjusting resonance splitting and resonance broadening of the two-ring system, a buffer scheme is introduced and

compared with the other existing schemes. It is found that the proposed scheme exhibits superior buffering performance compared to other schemes reported in the literature, such as in immunity to higher order dispersions, delay-bandwidth-product, and device footprint. Compared to the conventional cascade of side-coupled resonators (APF) and coupled-resonator-optical waveguide structure (CROW), our proposed structure are 2x and 4x more compact for buffering the same number of bits, respectively.

In the nonlinear region, it is shown in this thesis that the shape of the resonance plays crucial role in bistability switching characteristics. In a one-cavity system, the symmetric nature of the resonance is shown responsible for fundamental tradeoff between switching threshold and modulation depth. Such limitation may be overcome in two-cavity system. By using asymmetric resonance shape, it is shown that high modulation depth and low switching threshold can be simultaneously obtained.

The transfer matrix formalism is used for the analysis of resonator arrays. By incorporating Bloch theorem, the band characteristics of three types of resonator arrays are investigated. It is shown that a box-like add-drop multiplexer can be obtained in two dimensional arrays of resonators which utilize the complementary bandgap properties in the rows and columns. Experimentally, a box-like response with a shape factor of 0.7-0.8 is demonstrated and certain non idealities are discussed. Similar to those in conventional photonic bandgap structures, the existence of defect modes in two types of periodic resonator arrays is also presented. By adjusting the size of the defect ring or inter-cavity spacing, the movements of donor and acceptor modes are theoretically shown and experimentally verified.

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LIST OF ACRONYMS

1R1B	One-Ring One-Bus system
1R2B	One-Ring Two-Bus system
2R1B	Two-Ring One-Bus system
2R2B	Two-Ring Two-Bus system
APF	All Pass Filter
AWG	Arrayed Waveguide Grating
CB	Conduction Band
CEA-LETI	Commissariat à l'Énergie Atomique — Laboratoire d'Electronique de Technologie de l'Information
CIFS	Coupling Induced resonance Frequency Shift
CMOS	Complementary Metal Oxide Semiconductor
CROW	Coupled Resonator Optical Waveguide
DBP	Delay Bandwidth Product
DBR	Distributed Bragg Reflector
DC	Directional Coupler
DVS-BCB	divinylsiloxane-benzocyclobutene
EA	Electro Absorption
EIT	Electromagnetically Induced Transparency
ePIXnet	The European FP6 Network of Excellence
ER	Extinction Ratio
FBG	Fiber Bragg Grating
FDTD	Finite Difference Time Domain
FP	Fabry P�rot
FSR	Free Spectral Range
FWHM	Full Width Half Maximum
IMEC	Interuniversity Microelectronics Center
LED	Light Emitting Diode
MD	Modulation Depth
MMI	Multimode Interferometer
MZI	Mach-Zehnder Interferometer
NRZ	Non-Return to Zero
OADM	Optical Add Drop Multiplexing
OLED	Organic Light Emitting Diode
QCSE	Quantum Confined Stark Effect
Q-factor	Quality Factor
QWI	Quantum Well Intermixing
REMZI	Resonator Enhanced Mach Zehnder Interferometer
RR	Ring Resonator
RZ	Return to Zero
Si	Silicon
SOI	Silicon on Insulator
VB	Valence Band
WDM	Wavelength Division Multiplexing

LIST OF FIGURES

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Chapter 1. An Overview of the Thesis: Background, Objectives, and Significant Contributions

1.1 Background / Motivation

Since the invention of laser in the 1960s and optical fiber in the 1980s, the field of photonics has grown exponentially over the years and generated emerging subfields of optical communications, optical signal processing, instrumentation, chemical/gas sensing, and biomedical optics. The much higher information capacity that can be transmitted by a single optical carrier has been used in optical communication which is the main backbone for internet worldwide. In increasingly complex optical networks and in the ever-increasing demand for high-speed data transmission, optical signal routing and switching become important particularly when signals need to be split and transmitted to other location at lower bit-rates.

Fig. 1.1. Illustration of add-drop multiplexing in optical communication, where channel λ_3 is dropped and channel λ_{N+1} is added.

For this reason, wavelength division multiplexing technology (WDM) has become the major step in the development of optical networking technology. The data transmission consists of densely packed optical carriers called channels, each of which contains a signal of certain bandwidth (e.g., 2.5Gb/s-10Gb/s), which dramatically increases the information capacity even in single fiber. The data is then switched and routed into different locations by optical add-drop multiplexing techniques (OADM), as illustrated in Fig. 1.1, by which a channel can either be dropped to specific location, or added to the main haul to be dropped somewhere else. The OADM is a simple yet elegant concept in complex optical networking.

The success of long haul optical fiber communication has given profound interest in the implementation of all optical signal processing on-chip. It has been suggested that optical

interconnection can solve the limitation of the conventional electrical interconnects on electronic chips. [1] The bit-rate capacity (or the RC time constant) of the electrical interconnect is not affected by the size scaling, while at the same time, the clock speed increases in the decreasing trend of CMOS gate length. This means the maximum speed of the electronic chips will be limited by the electrical interconnects, not by the clock speed of the CMOS devices. Furthermore, in increasing clock speed, electrical interconnects have other issues such as voltage isolation and timing accuracy. [2] For example, the resistance of the copper line is dependent on temperature and the frequency of the signal, which eventually also affects the effective delay. Thus, more complicated designs are needed to compensate the above issues. By contrast, optical interconnects work at optical frequencies which has the potential to carry much more bandwidth of information. In electrical interconnects, large data streams can be carried in many number of wires. With optical interconnects, however, one can easily add or drop wavelength channel in one optical waveguide to increase the signal bandwidth. The propagation loss and the effective delay are to first order independent of temperature and signal bandwidth, which makes the delay likely to be predictable, which can in turn solve the timing accuracy problem in electrical clock circuitry.

For the above applications, it is thus of great importance to realize optical devices in a small footprint in order to have high integration density and therefore low manufacturing cost. All-optical signal processing needs both passive and active devices including a simple waveguide, optical add-drop multiplexer, buffer/delay line, modulator, switch (thermo-optic, electro-optic, or nonlinear-optic), light source, and detectors. The simple optical waveguide is the basic fundamental building blocks for all integrated optics elements. This means that integration density in a given material platform is gauged by the critical parameters of optical waveguide: the propagation loss and minimum bending radius, which are index contrast (Δn) dependent. Different material systems have been proposed for integrated optics, most of which are based on III-V compound semiconductor materials, polymer, and silicon-on-insulator (SOI). Their strengths and weaknesses are briefly discussed in the following section.

Different material systems for photonic applications

III-V semiconductor compounds are very suitable for photonic active devices due to their direct bandgaps that are compatible with telecommunication wavelengths (1300nm and 1550nm). Light sources such as light emitting diodes (LED) and lasers have been widely fabricated with their operating wavelength tunable by means of quaternary combinations of

III-V compounds ($\text{In}_x\text{Ga}_y\text{As}_{1-x}\text{P}_{1-y}$). Bandgap wavelengths ranging from 1000nm to 1650nm are available for materials lattice matched to InP. Intrinsic nonlinear-optics (Kerr effect) and electro-optics coefficients (Pockels effect) as well as mechanisms of electro-absorption (EA) and quantum confined stark effect (QCSE) can be readily used to realize optical modulation and switching functions. Despite the active-friendly feature of III-V compounds, realizing passive functions becomes a challenge for the same operating wavelength because of the light-matter interaction that enhances the waveguide loss, e.g., the existence of two-photon absorption and free-carrier absorption. Quantum well intermixing technique (QWI) [3-6] is a simple yet effective method to separate active from passive functions, which locally adjusts the electronic bandgap in such a way that the light in active and passive devices is resonant and off-resonant with the material respectively. However, the demonstrated propagation loss based on submicron III-V waveguide reported to date is between 10dB/cm to 20dB/cm [7-11], which is still moderately high. The material cost for III-V compounds is relatively high, due to difficulty in pulling the crystal.

Polymer is best used for active functions, similar to III-V case. The successes in organic light emitting devices (OLED) has spurred interests in polymeric integrated optical devices. Active medium can be locally introduced via doping of optically active materials such as chromophores [12], organic laser dyes [13], and rare-earth light-amplifying complexes [14]. The soft feature of polymer permits a non-semiconductor approach in fabricating optical devices. Nano-imprinting method [15] has been suggested as a path to mass production of polymeric optical devices. The reported propagation losses of different polymeric waveguide range from 1dB/cm to 17dB/cm [16-18]. Although the propagation loss can be considered low, the bending radius is on the order of tens of microns due to the low index contrast (Δn) in polymers, which suggest lower integration density. Low material cost and flexibility for active and passive functions are the main features of polymer systems.

Silica glass has a wide transparency window (from visible to infrared), low intrinsic loss, and is also compatible with fiber-optic technology. The material loss is limited to Rayleigh scattering, thanks to the mature fiber optic technology that has minimized intrinsic loss around the infrared region due to water and OH contaminants [19]. Silica glass is markedly useful for realization of ultralong photon lifetime whispering gallery resonators. The fabrication of silica microsphere (or microtorus) generally employs surface tension mechanism, which forms atomically smooth surfaces and thus gives ultrahigh Q resonance. The index contrast between silica and air is considered small, in comparison to

semiconductor materials (InP, GaAs, Silicon, etc), thus the cavity size scales inversely. For the above reasons, fabrication of silica microsphere (or microtorus) is easier compared to the planar MR based on semiconductor materials. Typical microsphere has radius around 100 microns. In terms of nonlinear functionalities, foreign atoms can be introduced (e.g. implantation of Er and Yb atoms into silica) and nonlinear layers can be deposited on top of silica [20, 21]. Due to the exceptionally high-Q factor generated by this material, several parametric processes have been demonstrated at ultralow power, such as Raman lasing [22], erbium-doped lasing around $1.5\mu\text{m}$ [23], and green up-conversion lasing [24]. Microspheres fabricated by melting the tip of an optical fiber are generally difficult to integrate on planar chips. A better integrability is offered by silica-on-silicon fabrication technology where microtoroid and microdisk resonators are formed by heating the top silica part of short silicon posts acting as supports [25].

Silicon-on-Insulator (SOI) based integrated optics has gained enormous attention; particularly in the concern of implementing optical interconnects on-chip [26, 27]. This is because (1) silicon (Si) is the standard materials in electronic ICs, (2) the index-contrast (Δn) is probably the highest among available material systems, and (3) standard CMOS-based process can be used to fabricate SOI optical devices. Compared to other material systems, the SOI optical waveguide has the smallest dimensions and bending radius. The reported propagation loss to date is from 0.8dB/cm to about 3dB/cm [28-32]. Such a low loss is due to the fact that Si has native oxide (SiO_2), by which successive thermal oxidation can be used to reduce the sidewall roughness that is responsible for most propagation losses [30]. For active functions, free-carrier absorption/dispersion [33-36], two-photon absorption [37-39], and thermo-optic effects have been used in optical modulators and switches at GHz operational speed and sub-milliwatt switching power [40, 41]. There have been attempts for also making light sources and detectors by introducing silicon nanoclusters in silica host [42-44] and in Si/Ge semiconductor compound [45-47]. It has to be noted that in terms of active functions, silicon has more limited options than do the III-V compounds and polymers. However in terms of passive functions, SOI-based photonics is of ideal choice because of the CMOS-based processes that implies low manufacturing cost, highest integration density, and finally suitability for on-chip optical interconnects application [48, 49].

Although SOI technology offers a high-index contrast ($\sim 130\%$), the refractive index is not tunable. The relatively recent material systems, on the other hand, namely the silicon nitride (SiN) [50, 51], the silicon oxynitride (SiON) [52, 53], and the Hydrex® [54], have tunable

refractive indices of $\sim 30\%$ (for SiN and SiON) and 17% (for Hydrex®), and a demonstrated propagation loss in the range of $1.5\text{-}4\text{dB/cm}$ (SiN) [50, 51], $\sim 0.15\text{dB/cm}$ (SiON) [53], and a remarkably low for $\sim 0.06\text{dB/cm}$ [54] (Hydrex®). All the above key materials have their own advantages and disadvantages in terms of their linear and nonlinear functionalities, their ease of fabrication processing, and the range of refractive index contrasts that determine the allowable sizes of the resonator features. General properties of these materials are outlined in Table. 1.1

Table 1.1. Key properties of photonic material systems

Material system	Refractive index	Index contrast $(n_{\text{core}} - n_{\text{clad}})/n_{\text{clad}}$	Demonstrated loss (dB/cm)
Fused silica (SiO_2); silica-on-silicon.	1.44-1.47	~ 0.47	
Silicon-on-insulator (SOI)	3.47	$\sim 1.39\text{-}2.47$	0.8-3.4
Silicon Nitride (SiN)	1.44-1.99	$\sim 0.37\text{-}1$	$\sim 1.5\text{-}4$
Silicon Oxynitride (SiON)	1.44-1.99	< 0.045	~ 0.15
Hydrex®	1.44-1.7	~ 0.17	~ 0.06
Polymer	1.3-1.6	$\sim 0.2\text{-}0.6^*$	$\sim 3\text{-}17$
III-V compounds	3.17-3.37	$\sim 0.06(\text{vertical}),$ $< 2.37(\text{lateral})$	$\sim 10\text{-}20$

* The ~ 0.6 index contrast is obtained by vertical confinement through isotropic etching.

Heterogeneous integration for on-chip optical interconnects

Given the strengths and weaknesses of each material system, it is therefore apparent that the requirements of on-chip interconnects for light sources, detectors, optical waveguides, add-drop multiplexers, cannot be completely satisfied by one material platform. The material that is good for passive devices (e.g., silicon) usually is not good for light source and detector applications. Despite the growing interest in Si/Ge based photodetectors [47, 55] and in nanocrystal based light sources [43], it is apparent that integration between active and passive devices in silicon becomes challenging. There is however an alternative approach to vertically integrate SOI passive devices with III-V light sources and detectors [56-59]. This approach shows promises because (1) Si is a very good candidate in passive elements because Si is a standard material for electronic IC and the CMOS-based fabrication has been so matured to fabricate submicron devices, and (2) the III-V compound semiconductor materials have been widely used for light sources and detectors.

Fig. 1.2. Demonstration of silicon evanescent laser (SEL) [58], (a) the schematic and (b) the cross section SEM. The heterogeneous integration is assisted by O_2 plasma treatment (please refer to Ref. 59)

Fig. 1.3. (a) The schematic of die-to-wafer bonding technique developed by group from IMEC [60] (top) adhesive polymer bonding (bottom) molecular bonding; (b) Scanning electron microscopy (SEM) cross-section of a DVS-BCB bonding interface between InP/InGaAsP epitaxial layer stack with SOI waveguides.

There are two approaches for III-V/Silicon heterogeneous integration. In one approach, the bonding is assisted with plasma treatment. The surface of both III-V and SOI wafers are treated with O_2 plasma to form a very thin hydrophilic layer that facilitates the bonding of these two wafers [57-59]. This technique is used to demonstrate silicon evanescent laser (SEL), as shown in Fig. 1.2. The second approach [60] is based on either adhesive polymer DVS-BCB (known as adhesive polymer bonding) or SiO_2 (known as molecular bonding) as the bonding agent, as shown in the top and bottom panels of Fig. 1.3(a) respectively. Using this technique, III-V microdisk laser is integrated onto silicon plain wafer and lasing has been successfully demonstrated [61]. The success of these two approaches shows that the idea of on-chip optical interconnects is not so far from its realization.

Microring resonator as the basic building blocks in integrated optics

The optical filter is a key element in WDM systems. The ideal filter has box-shape transmission response with zero dispersion. The most common optical filters are based on Fabry-Perot cavities (FP), plane gratings, Mach-Zehnder Interferometers (MZI), Arrayed

Waveguide Gratings (AWG), Fiber Bragg Gratings (FBG), thin film filters, and ring resonators (RR) [62]. For on-chip WDM systems all photonics components need to be fabricated in planar form and the choices are thus limited to filters based on RR, FP, MZI, and AWG. Recently, much of the research is dedicated to microring resonators because of their versatility in realizing many optical functionalities [63] including add-drop filters [16, 32, 64-68], modulators [34, 69-72], switches [39, 41, 73, 74], and bio-chemical sensors [75-80].

Fig. 1.4 The simplest configuration of microring resonator coupled to one or two waveguide buses and their equivalence with Fabry-Perot cavity configuration. The 1R2B is equivalent with Fabry-Perot etalon with two semi-reflecting mirrors, while the 1R1B is equivalent with Girés-Turmois Interferometer, with one of the mirrors fully reflecting.

As schematically shown in Fig. 1.4, the simplest ring resonator configuration consists of a ring resonator side-coupled with one or two optical waveguides. Functionally, a microring resonator is identical to Fabry-Perot etalon in that the evanescent coupling between the straight waveguides and the ring is analogous to semi-reflecting mirrors in the FP cavity, and the ring circumference is analogous to the roundtrip cavity length. Realizing an FP cavity has been a challenge because the FP cavity (e.g., in the case of laser diode) needs high quality cleaving in the sidewalls, and the fact that the reflectivity, $R = [(n_2 - n_1) / (n_2 + n_1)]^2$, is dependent largely on the index contrast ($\Delta n = n_2 - n_1$). It is possible to tune the reflectivity in distributed Bragg reflector structure (DBR), however it still has the same challenge on high quality cleaving.

In contrast, such stringent requirements are not present with microring resonators since the mirrors have been replaced by evanescent coupling. The reflectivity can then be readily controlled via adjusting the gap separation between the ring and the waveguides or adjusting the coupler length in racetrack-shape rings. In high index contrast medium the waveguide can be bent in such a small radius, resulting in very compact ring resonators. Recently, rings with radius down to $1.5\mu\text{m}$ [81] have been fabricated in silicon-on-insulator. The applications of ring resonator however are not limited to single rings alone, as shown in Fig. 1.4. It has been shown that by coupling these resonators in certain ways, it is possible to extend the

functionalities to higher-order filters with box-like response [54-68], optical delay lines or buffers [48, 82]. Thus, the possibility for various optical functionalities in one-ring and multi-ring structures as well as their compact size makes the ring resonator a potential candidate in integrated optics building blocks.

1.2 Objectives

Based on the above background, the thesis focuses on the theoretical formulation, conceptualization and experimental demonstration of integrated optics devices based on coupled microring resonators. The objectives of this thesis are summarized as follows.

- Theoretically investigate the physics of different periodic arrays of microring resonators and devise their possible applications.
- Theoretically investigate the physics of the two-resonator system and its possible applications in active and passive media.
- Experimentally demonstrate device conceptualizations based on periodic microring resonator arrays and two-resonator systems.

1.3 Significant Contributions

The main work of this thesis can be classified into three parts: theoretical analysis of various multi-cavity configurations, device proposal and experimental demonstrations based on two-ring systems and periodic arrays of ring resonators. This research work has contributed 9 peer-reviewed journals and 7 conference proceedings. The original contributions of this work are outlined as follows.

Periodic arrays of microring resonators – It is widely known in the literature that the periodic arrangements of optical media can be constructed to form photonic crystal, in which there is a band of frequency within which the light propagation is forbidden inside the structure [83-85]. In this work, it is noted that the periodic configuration of ring resonators can be treated as to a hybrid resonator and photonic crystal structure that exhibit the properties of both. Three types of periodic configurations are considered. The first configuration (Type I) consists of evenly spaced ring resonators that are periodically side-coupled to common waveguide buses. The second configuration (Type II) consists of mutually (or directly) coupled ring resonators with the first and the last rings coupled to waveguide buses. The third configuration is the combination of the two former types, forming two-dimensional array of microring resonators. The properties of each configuration is derived from the photonic crystal perspective and based on transfer matrix formalism and

Bloch theorem. It is shown that the photonic bandgap properties of Type I and Type II always complement each other, provided the coupling strengths are identical. These non-overlapping photonic bandgaps properties then lead to the novel concept on building a near-perfect filter which has box-like transmission characteristics, where introduction of extra rows and columns can give significant suppression of out-of-band sidelobes and flattens the in-band ripples respectively. [86] The devices were fabricated in silicon-on-insulator and the experimental measurements are in excellent agreement with theory [87].

Next, the existence of defect modes is also investigated in Type I and Type II configurations when such a ‘defect’ is in the form of the absence of cavity, different inter-cavity spacing, or different cavity size. The concept of defect modes originated from photonic crystal, where a slight disorder in an otherwise periodic medium can induce a strong light localization in the vicinity of such defect [83, 84]. Using the same transfer matrix formalism, characteristic equations of defect modes for both Type I and Type II are derived [88]. The locations of the resonances associated with these defects are determined by the normalized defect size relative to the regular cavities. It is shown that these defect modes are analogous to donor and acceptor modes in doped semiconductor, where doping a donor (acceptor) atom is similar to a larger (smaller) defect rings. The experimental measurement for Type II defect modes agrees quite well with the theoretical prediction [89].

Two-Resonator systems and their applications – Due to the strong resemblance between atomic energy levels and resonant cavity decay modes, there have been attempts to optically mimic the atomic phenomena, in the literature, for example in the case of Rabi splitting, Fano resonance, and electromagnetically induced transparency (EIT) [90-92]. In this work, the two-ring system is theoretically analyzed by a simple extension from one-ring systems. By exciting the cavity modes in certain configurations, various devices such as optical buffer, highly resonant structure (high finesse), and bistable switch are proposed with better performances compared to those based on one-ring systems. Using the property of resonance splitting and resonance broadening, a concept of optical buffering is derived and shown to exhibit superior characteristics when compared to the existing buffering schemes based on resonators. Compared to one-ring systems, it is theoretically shown that the two-ring buffer has a better immunity towards inter-symbol interference, thereby improves the buffering capacity by about a factor of 2 [93].

The two-ring system, when coupled to two optical waveguides, can also be used to produce a very high finesse resonance by virtue of inter-pathway interference from two decaying modes from each resonator. By properly adjusting the relative size between

resonators, it is possible to realize a finesse enhancement scheme that is dependent on the intensity buildup factors between the two rings, since such enhancement is independent of the cavity size scaling [94]. In the one-ring system, high finesse resonance can only be achieved when both the coupling strength and the round trip loss are small, with the latter being smaller than the former. In the case of achieving finesse of more than 100 (>100), the coupling strength and the round trip loss has to be very weak. The power coupling strength required is below 2% and in practice it is challenging to have a precise control of the coupling coefficient because it is very sensitively dependent on the gap separation between the waveguide and the cavity. Furthermore, the lowest reported roundtrip loss to date is around 1%, thereby presenting a very stringent requirement should there be slight deviation in the fabrication.

On the other hand, in the two-ring system the finesse of > 100 can be achieved in the strong coupling scheme, due to the finesse enhancement scheme. The requirement to achieve high finesse resonance is therefore relaxed because now the roundtrip loss is not necessarily close to the coupling strength, unlike in the one-ring system. Finesse as high as 250 and a finesse enhancement up to 20 times have been experimentally demonstrated [95, 96]. When the same two-ring system is coupled to one optical waveguide (instead of 2 waveguides in the above case), the same finesse enhancement scheme can be used to ease the requirement of critical coupling condition that has been a challenge to achieve in one-ring system. Critical coupling occurs when the light is resonantly absorbed inside the system. The applications of critical coupling have varied from bio-chemical sensor to optical modulators [97]. In the one-ring system, critical coupling is achieved when the coupled light is exactly dissipated within one round trip in the ring. The problem arises when the ring has a very low cavity roundtrip loss which requires a very precise coupling strength to satisfy critical coupling situation. This is the same problem as achieving high finesse in one-ring system coupled to two optical waveguides. This is evident from various demonstrated attempts to match critical coupling in one-ring systems [98-100]. Using two-ring systems, however, the finesse enhancement mechanism enhances the effective loss which thereby relaxes the critical coupling condition [101]. Experimentally, it was demonstrated that the two-ring systems have high contrast resonance dip with broad range of coupling coefficients (corresponding to coupler length tolerance of $\sim 1\mu\text{m}$) [102].

Finally, the two-ring system is also theoretically investigated in the presence of Kerr nonlinearity (n_2). In the one-ring system, bistability occurs when there is a dynamic nonlinear

index change as a result of dynamic intensity buildup inside the resonators. The bistable switching characteristics of one-ring system is typically limited in that both low switching threshold ($n_2 I_{TH}$) and high modulation depth (the difference between ON and OFF state) cannot be simultaneously achieved. This is due to the symmetric nature of the resonance shape. On the other hand, using the two-ring system, the resonance is asymmetric and its asymmetry can be readily tuned by a relative size between two rings only. By using parametric approach, it is shown for the first time that achieving both low switching threshold and high modulation depth is possible in two-ring system [103].

1.4 Thesis organization

The thesis is divided into chapters that have their separate references, specific summaries and possible future directions. This thesis is outlined in the following. In Chapter 2, some general characteristics of whispering gallery modes are covered. In Chapter 3, the basic theory of two basic configurations of ring resonator is presented, and experimental setup as well as the near-vertical coupling scheme are briefly discussed. This chapter also presents experimental results of the fabricated ring resonators. Chapter 4 presents the analysis of two-ring system and its possible mode excitation. This chapter discusses about the finesse-enhancement scheme in two-ring systems and its experimental realization. Chapter 5 explores the theoretical possibilities to use two-ring system as an optical buffer. In this chapter, exhaustive comparison with existing buffering scheme is carried out and concluded. In Chapter 6, multi-ring structures are investigated from theoretical perspective to their experimental demonstrations. In this chapter, three types of periodic arrays are theoretically studied and experimentally demonstrated. Chapter 7 theoretically predicts and experimentally demonstrates existences of donor and acceptor modes in micro-resonator arrays, when a defect in the otherwise uniform arrays is introduced. Chapter 8 theoretically explores the possibility to use two-ring system as bistable switches and show how the switching performance can break the limitation of that in one-ring systems. Finally, Chapter 9 presents the conclusion and the outlook of this work. The author's publication and the necessary appendices are also listed.

Chapter 2. General characteristics of whispering gallery modes in the ring resonator

A ring resonator (Fig. 2.1) mainly consists of two basic building blocks: the coupling elements and the optical feedback. The coupling element consists of two optical waveguides separated by a certain distance, and the optical feedback is a ring-shaped optical waveguide. In many ways, ring resonator is similar to Fabry-Perot (FP) cavity with the evanescent coupling between the cavity and external waveguide functioning as a semi-reflecting mirror, and the ring circumference as the cavity length. The guided-mode of an optical waveguide evanescently transfers to the ring waveguide and circulates for certain numbers of round trips, which excites a steady-state field profile known as cavity-mode.

Fig. 2.1. Examples of fabricated ring resonators: (a) SOI ring resonator side-coupled to one optical waveguide bus (1R1B) and (b) the GaAs racetrack ring resonator side-coupled to two optical waveguide buses (1R2B). The inset shows the cross-section of the optical waveguide.

Unlike the FP cavity that supports standing waves, the ring resonator supports two degenerate modes which are traveling waves in clockwise and counterclockwise directions. The light is said to be on-resonance when the ring circumference is an integer multiple of the wavelength of light in the material (λ/n). In such situation, the coupled light interferes constructively with the circulating light, building up the intensity inside the resonator. For the sake of illustration, the dynamics of the fields in a resonator coupled to one optical waveguide [Fig. 2.1(a)] for a near-resonant light can be described by a phenomenological approach [1],

$$\begin{aligned} \frac{du_{\text{cav}}(t)}{dt} &= \left(-i\omega_0 - \frac{1}{\tau} \right) u_{\text{cav}}(t) + i\eta s_{\text{in}}(t), \\ s_{\text{out}}(t) &= s_{\text{in}}(t) + i\eta u_{\text{cav}}(t), \end{aligned} \quad (2.1)$$

for resonance frequency ω_0 , intra-cavity energy amplitude u_{cav} , and incoming (outgoing) field

amplitudes s_{in} (s_{out}). η is a mutual coupling coefficient and $1/\tau = 1/\tau_0 + 1/\tau_e$ is the total cavity decay rate of the resonator, consisting of external ($1/\tau_e$) and intrinsic ($1/\tau_0$) decay rate. In the continuous excitation $s_{\text{in}}(t) = s_{\text{in}} \exp(-i\omega t)$, the energy amplitude at the steady state can be expressed in the form of $u(t) = u_{\text{cav}} \exp(-i\omega t)$ and the buildup of intra-cavity power $|s_{\text{cav}}|^2 = v_g L_{\text{cav}}^{-1} |u_{\text{cav}}|^2$ with respect to the input power can be deduced as

$$\left| \frac{s_{\text{cav}}}{s_{\text{in}}} \right|^2 = \frac{\eta^2 v_g L_{\text{cav}}^{-1}}{(\omega - \omega_0)^2 + (1/\tau)^2} = \left(\frac{4}{\kappa^2} \right) \frac{(1/\tau_e)^2}{(\omega - \omega_0)^2 + (1/\tau)^2}, \quad (2.2)$$

where the η relates to the coupling coefficients “in-space” κ by $\eta^2 = \kappa^2 v_g L_{\text{cav}}^{-1}$ [1]. The governing equations for a ring resonator coupled to two optical waveguides [Fig. 2.1(b)] are quite similar to that of one optical waveguide, but with an additional $s_d(t) = i\eta u_{\text{cav}}(t)$ for coupling to the second waveguide and a modification of cavity decay-rate $1/\tau = 1/\tau_0 + 1/\tau_{e1} + 1/\tau_{e2}$, where $1/\tau_{e1,2}$ is the external decay rate to the two waveguides respectively. Eq. (2.2) shows that the intra-cavity intensity buildup at resonance ($\omega = \omega_0$) is of Lorentzian shape and inversely proportional to the coupling coefficient κ . Thus, a very large intra-cavity field can be contained inside the resonator when the coupling is very small, and this can be used to enhance various intensity-dependent and phase-sensitive mechanisms such as in nonlinear optics, optical switching, and bio-chemical sensing.

The above model describes the general working principle of a resonator, but it does not particularly describe the specific aspects such as the mode-coupling, geometry-dependent cavity mode profiles, and the intrinsic radiation loss which limits the cavity lifetime (Q factor). Furthermore, the validity of the above model only holds for weak-coupling regime and when the light is near-resonant. Thus, a more general approach is needed in order to analyze more complex structure, in particular when it involves many coupled resonators. In this chapter, various aspects of ring resonator, such as whispering gallery modes, mode-coupling, loss mechanisms in ring resonator will be discussed, and finally the linear frequency response of ring resonator is also be covered.

2.1 Whispering gallery modes

The whispering gallery mode is originated from “whispering gallery” in St Paul Cathedral and was first analyzed by Lord Rayleigh back in 19th century [2]. The same mechanism actually occurs in optical ring resonator in which the light is total-internally reflected along

the outer sidewalls of the ring. The existence of whispering gallery modes can be shown by rewriting Maxwell Equations in cylindrical coordinates and substituting all the fields with E_z and H_z , so that by neglecting the polarization coupling term, one can come at scalar Helmholtz equation,

$$\left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k_0^2 n^2(r) \right] E_z(r, \phi, z) = 0. \quad (2.3)$$

Generally, Eq. (2.3) is difficult to solve because of the radial dependence of the refractive index $n(r)$. Several methods have been used to solve (2.1), ranging from the use of Hankel functions [3], conformal mapping [4], Airy functions [5], and WKB method [6]. In one method involving Hankel functions, the field is assumed to be of the form $E_z(r, \phi) = E_z(r) \exp(im\phi)$, and the radial function $E_z(r)$ is expressed in terms of a combination of outward and inward cylindrical travelling waves, which are represented in Hankel function of the first and second kind respectively $H_m^{(1,2)}(r)$. The field profile then is obtained by matching the amplitude and the continuity of the fields along the interfaces. This method is rigorous however some fundamental characteristics such as the effective index¹ of the cavity-mode and the intrinsic radiation loss cannot be directly explained. One of the reasons is because the radial wave number (k_r) is already embedded inside the Hankel function, and so the azimuthal wave number $k_\phi = k_0 n_{\text{eff}} = \sqrt{k_0^2 n^2 - k_r^2}$ remains unknown. The asymptotical oscillatory trend of Hankel function² in $E_z(r)$ suggests the outwardly propagating light while most of the light propagates azimuthally, which implies the intrinsic radiation loss of whispering gallery modes.

A more intuitive picture can be obtained from conformal mapping approach, where the polar coordinate (r, ϕ) is transformed into the new coordinate (u, v) in such a way that Eq. (2.3) can be rewritten as

¹ Effective index (n_{eff}) is the eigenmode that satisfies the Maxwell equation that describes guiding of light in an optical waveguide by virtue of total internal reflection (TIR). Physically, it represents the average refractive index that the optical mode experiences as it propagates along the waveguide. The light is said to be in a *guided mode* when $n_{\text{clad}} < n_{\text{eff}} < n_{\text{core}}$, where n_{clad} and n_{core} are the refractive indices of the waveguide core and cladding respectively. When $n_{\text{eff}} < n_{\text{clad}}$, the light is said to be in a *leaky-mode* because the light propagates in the cladding instead of in the core. In this case, the light is leaking away to the cladding and therefore not guided. When $n_{\text{eff}} > n_{\text{core}}$, the lateral wave number becomes imaginary to satisfy $k_0^2 n^2 = k_x^2 + k_0^2 n_{\text{eff}}^2$ and the light is said to be in a *surface-mode*. In this case, the light propagates along the core-cladding interface with the maximum field at the interface and exponentially decays away from the interface.

² It can be shown that asymptotically Hankel function converges to $H_m^{(1,2)}(r) \sim \exp(\pm ikr)$, that are outward and inward cylindrical waves. Such an “oscillatory” function indicates the existence of an intrinsic radiation loss in whispering gallery modes.

$$\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + k_0^2 n_{eq}^2(u) \right) E_z(u, v, z) = 0, \quad (2.4)$$

where the conformal transformation is $u=R \ln(r/R)$, $v=R\phi$, R is the mean radius of the ring resonator and $n_{eq}(u)=n(u)\exp(u/R)$ is the equivalent index profile in (u,v) space. As illustrated in Fig. 2.2, in (u,v) space, the equivalent refractive index $n_{eq}(u)$ appears to be exponentially increasing with u due to the factor of $\exp(u/R)$, and this makes the optical mode asymmetric and more localized in the outward direction because light is naturally more concentrated at the higher index.

Fig. 2.2. The illustration of conformal transformation of step index curved waveguide

Comparison between optical mode in straight and curved waveguide is outlined in Fig. 2.3. In the straight waveguide [Fig. 2.3(a)], the guiding condition is met when $n_{clad} < n_{eff} < n_{core}$ (discrete red line), by which the light is bounded inside the waveguide core and evanescent in the cladding. In the curved waveguide [Fig. 2.3(b)], however, the light is only partially guided, due to the ever increasing nature of equivalent refractive index in (u,v) space. The guiding and radiating nature of whispering gallery modes are determined by the critical radius (r_c), that is the radius at which the effective index is equal to the equivalent refractive index $n_{eff} = n_{eq}(u_c)$. The light is in guided mode when $n_{eff} > n_{eq}(u_c)$ and in radiative mode when $n_{eff} < n_{eq}(u_c)$. The origin of radiation loss can be interpreted in the following [7]. As the light bends in the curved waveguide, the angular phase velocity should be constant for maintaining the modal shape. Thus, the light in the outer side of the ring must have higher tangential velocity $v_{ph} = \omega/k_\phi$ than those in the inner side.

Fig. 2.3. Comparison between straight and curved waveguides.

This means at certain radius outside the bend known as the *critical radius* (r_c), the phase velocity is equal to the maximum speed of light permitted in the given medium ($v_{ph}=c/n_{clad}$, $k_\phi=k_0n_{clad}$), and at radii beyond this, the tangential velocity exceeds that of light. In order to accommodate this, a radial wave number k_r must exist to compensate the lower k_ϕ ($k_\phi < k_0n_{clad}$) in order to increase the tangential phase velocity ($k_\phi < k_0n_{clad}$), thereby changing the mode from *evanescent* to *radiative*. The radiation (or bending) loss can be roughly determined by how much the light tunnels from the ring-core to the cladding, and the tunneling rate is determined by the gap between the outer ring and the critical radius in u -space ($\Delta u = u_c - u_{outer}$). The larger (smaller) the bending radius, the larger (smaller) also is the Δu , which results in lower (higher) radiation loss. The Δu is exponentially related with $n_{eq}(u) = n(u)\exp(u/R)$, and thus the radiation loss is also exponentially dependent on the bending radius.

2.2 Dispersion characteristics and radiation loss of whispering gallery modes

The scalar Helmholtz equation in Eq. (2.3) is sufficient in describing the fundamental properties of whispering gallery modes, but not really adequate in dealing with ring-resonator of sub-micron waveguide dimensions. The reason is because the scalar Helmholtz equation breaks down in small dimensions, and the polarization coupling terms in Maxwell Equation start to dominate. Therefore a numerical Full-Vectorial solution of the Maxwell equations is necessary for accurate calculation of the effective index and radiation loss of whispering gallery modes. Finite-difference time-domain method (FDTD) [8, 9] is chosen as a numerical tool because it solves Maxwell Equation without approximations and therefore gives a more accurate results compared to other existing techniques.

The waveguide geometries considered in this thesis is based on silicon-on-insulator (SOI) with buffer oxide of $2\mu\text{m}$, waveguide width of 450nm , and thickness of 220nm . The waveguide core is Si, while the upper cladding is SiO_2 . Fig. 2.4 shows the calculated effective index of various bent waveguide with radius varied from $1.1\mu\text{m}$ to $30\mu\text{m}$, with the dash line representing the effective index for a straight waveguide. The insets show the contour of the mode-fields for 3 different radii, from a very asymmetric one ($R=1.1\mu\text{m}$) to nearly symmetric one ($R=30\mu\text{m}$). In decreasing radius, the n_{eff} decreases a bit and then goes up significantly at the small radius ($<3\mu\text{m}$). This can be understood if one sees n_{eff} as an “averaging” of refractive index covered by the guided mode for a given mode-profile, where the averaging is intuitively weighted by the mode-profile itself.

Fig. 2.4. Effective index of the quasi-TE ring-mode as a function of bending radius. The insets show the contour plots of the ring-mode corresponding to (1) $1.1\mu\text{m}$, (2) $3\mu\text{m}$, and (3) $30\mu\text{m}$ bending radii.

When the light is tightly confined inside the waveguide core, the n_{eff} is very close to the core index, and when the light is extended to the cladding, the n_{eff} is closer to the cladding index. In curved-waveguide, the mode shifts in positive radial direction and the mode-tail of the inner side decreases whereas that of the outer-side increases. As a result the mode field inside the core increases due to the shrinking of inner tail and decreases due to the expanding outer-tail, and therefore affecting the average index covered by the cavity mode. At large radii, the mode is nearly symmetrical and the inner-tail is nearly unaffected while the outer-tail expands. This reduces n_{eff} of cavity modes. On the other hand, when the radius is smaller,

the inner-tail shrinks significantly and the field is so concentrated at the core (e.g., see the case of $R=1.1\mu\text{m}$), and so n_{eff} increases.

This is consistent with conformal mapping picture where the index contrast in u -space for the inner and outer walls is different. The available mode solutions $n_{\text{clad}} < n_{\text{eff}} < n_{\text{core}}$ now become different at the inner and outer cladding, and when the slope of $n_{\text{eq}}(u)$ increases the mode solutions are progressively restricted to that of outer-cladding only, since any sets of solutions pertaining to inner cladding will not be guided any further. In that sense, the n_{eff} increases exponentially at small radii. At large radius, the difference between the index contrast at the inner and outer cladding is not significant, however, the overall index contrast increases slightly. In the standard mode solving for single mode slab waveguide, this would mean a slight decrease in n_{eff} .³

Fig. 2.5. The radiation loss of 90 degree turns as a function of bending radius. The inset shows the contour plots of the guided and radiation field at the 90 degree bent when $R=1.1\mu\text{m}$.

Next, the radiation loss is calculated by measuring the transmission coefficient of a 90 degree bend for different bending radii. As shown in Fig. 2.5, the radiation loss decreases exponentially with increasing bending radii. Clearly there is a critical radius at which the radiation loss starts to increase very rapidly and this is related to the dispersion characteristics previously shown in Fig. 2.4. The overall loss of the 90 degree bend basically can be

³ The standard characteristic equation for a single mode slab waveguide is $S_{\text{clad}} = S_{\text{core}} \tan(S_{\text{core}} W / 2)$, with S_{clad} and S_{core} confined in a circle of $S_{\text{core}}^2 + S_{\text{clad}}^2 = k_0^2(n_{\text{core}}^2 - n_{\text{clad}}^2)$. $S_{\text{core}} = k_0(n_{\text{core}}^2 - n_{\text{eff}}^2)^{1/2}$ is the lateral wave number in the core while $S_{\text{clad}} = k_0(n_{\text{eff}}^2 - n_{\text{clad}}^2)^{1/2}$ is the evanescent wave number in the cladding. As index contrast Δn increases, the ‘radius’ of the circle also increases and this results in point of intersection at larger S_{core} . When the change of Δn is small, the n_{eff} is most likely to decrease. However, when the change is large, it is possible for n_{eff} to increase.

decomposed into pure radiation loss and waveguide loss. The waveguide loss is incorporated into the picture because there is an artificial roughness introduced from the meshing of the structure in the simulation. At large bending radii, there is no significant difference of index contrast at both inner and outer cladding and the mode-solution pertains more to a straight waveguide problem with slightly higher index contrast. In this situation, the light sees the structure more as a straight waveguide and the loss mechanism is therefore dominated by a waveguide loss. At decreasing bending radii, the solutions of the cavity modes become more restricted to the index range at the outer cladding, which shifts n_{eff} closer to the outer cladding index. In this situation, the light sees the outer cladding in u -space as a thin layer of lower index sandwiched by two higher indices and the probability of photon tunneling⁴ becomes higher, particularly when the “layer thickness” is smaller than the evanescent depth $\Delta u \sim k_0^{-1} [n_{\text{eff}}^2 - n_{\text{eq}}^2(u_{\text{outer}})]^{-1/2}$. Note that the radius at which the n_{eff} starts to increase rapidly after a slight decrease shown in Fig. 2.4 corresponds to the radius at which the radiation loss starts to dominate in Fig. 2.5. This clearly shows that the minimum in n_{eff} (Fig. 2.4) is actually a transition point from waveguide-loss dominated to radiation-loss dominated regions.

Fig. 2.6. The radiation loss for different sidewall roughness. The roughness is artificially incorporated by increasing the mesh size in the FDTD simulation.

In the FDTD simulations, the meshes introduce artificial roughness in the structure, especially in the round features. Thus, adjusting the mesh size can be used to illustrate qualitatively the effect of the actual sidewall roughness on the radiation loss. Although it

⁴ This is often known as *frustrated total internal reflection*.

should be noted that sidewall roughness essentially is a random process and cannot be emulated by simply adjusting the mesh size. The surface scattering loss is described by Tien's formula [10-13]

$$\alpha_{st} = 2\sigma^2(n_{\text{core}}^2 - n_{\text{clad}}^2)k_0^2(k_x / \beta)E_s^2, \quad (2.5)$$

for the mode propagation constant $\beta = k_0 n_{\text{eff}}$, standard deviation of surface roughness σ , transverse propagation constant k_x , and the amplitude of the modal fields at the surface E_s . The loss is thus mainly dependent on the field magnitude along the sidewall and the sidewall roughness σ . Since the mode extends to the outer cladding of the ring, the enhancement of scattering loss is to be expected. Fig. 2.6 illustrates the bending loss in different degree of sidewall roughness. Note that the radiation loss is not so much affected within the radiation-dominated region. However, in the roughness-dominated region, radiation loss in the presence of roughness increases by up to 6 times, which means that the effect of roughness cannot be neglected in designing high Q resonators.

2.3 Evanescent mode coupling

The design of coupler is important in controlling the cavity finesse and operating bandwidth of the resonator. The coupling strength between two closely spaced waveguides mainly depends on the gap separation between the waveguides (g) and the interaction length along which they are coupled together (L_C). In applications that require sharp resonance, such as in switching or bio-chemical sensor, one would need a weakly coupled resonator, and usually a point coupling scheme is used in this case [see Fig. 2.7(a)].

Fig. 2.7. Two types of ring resonator: (a) circular ring resonator and (b) racetrack ring resonator. (c) The schematic representation of evanescent coupling of modes. The L_π is defined as the coupler length at which the power is completely transferred to the other waveguide.

On the other hand, there are also applications that require a rather strong coupling. If point-coupler is to be used, the gap separation has to be tightly controlled down to nanometers scale, which is practically very difficult to achieve. Racetrack geometry can be used to solve this problem, where the straight section functions as coupler elements and the coupling strength can be conveniently controlled by adjusting the coupler length (L_C), while the radius of the circular section can be adjusted to control the cavity length to obtain the desired resonance wavelength [See Fig. 2.7(b)].

The working principle of coupler is based on overlapping of evanescent fields of the two waveguides. The mode of isolated waveguides WG_m ($m=1,2$) can be described by Helmholtz equation [14]

$$(\nabla_T^2 + k_0^2[\varepsilon_{BG} + \Delta\varepsilon_m(x, y)] - \beta_m^2)E_m(x, y, z) = 0,$$

$\nabla_T^2 = \partial_x^2 + \partial_y^2$ is the transverse Laplacian operator, $\partial_z = i\beta_m$ is the longitudinal dependency of a single mode waveguide, ε_{BG} is the background permittivity and $\Delta\varepsilon_m(x, y)$ denotes the index contrast profile of WG_m . In the presence of other waveguide, with assumption of adiabatic coupling and slowly varying amplitude along the coupling, the Helmholtz equation can be written in the following

$$(\nabla_T^2 + \partial_z^2 + k_0^2(\varepsilon_{BG} + \Delta\varepsilon_1 + \Delta\varepsilon_2))E = 0, \quad (2.6)$$

to which the solution is proposed as the linear combination of the isolated modes $E = C_1(z)E_1 \exp(i\beta_1 z) + C_2(z)E_2 \exp(i\beta_2 z)$. Multiplying the Eq. (2.6) with either E_1 or E_2 and perform integration over the cross section, the Eq. (2.6) can be decomposed into two coupled equations

$$\frac{d}{dz} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = i \begin{pmatrix} \beta_1 + k_{11} & k_{12} \\ k_{21} & \beta_2 + k_{22} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \quad (2.7)$$

where $k_{11} = \frac{k_0^2}{2\beta_1} \langle E_1 | \Delta\varepsilon_1 | E_1 \rangle$, $k_{12} = \frac{k_0^2}{2\beta_2} \langle E_1 | \Delta\varepsilon_1 | E_2 \rangle$, $k_{21} = \frac{k_0^2}{2\beta_1} \langle E_2 | \Delta\varepsilon_2 | E_1 \rangle$, $k_{22} = \frac{k_0^2}{2\beta_2} \langle E_2 | \Delta\varepsilon_2 | E_2 \rangle$ are the overlap integrals that define the self coupling of waveguide m (k_{mm}), and the cross-coupling from waveguide m to waveguide n (k_{mn}). The Eq. (2.7) is a standard eigen value problem which requires a solution of $[C_1 \ C_2]^T = \exp(i\beta z)[\tilde{C}_1 \ \tilde{C}_2]^T$, and the final solution of the fields in each waveguide is

$$\begin{aligned} E_1(x, y, z) &= A_1(0)E_1(x, y)\left(\cos(sz) + i(\Delta\tilde{\beta}/2s)\sin(sz)\right)\exp(i\bar{\beta}z), \\ E_2(x, y, z) &= A_1(0)E_2(x, y)\left(i(k_{12}/s)\sin(sz)\right)\exp(i\bar{\beta}z). \end{aligned} \quad (2.8)$$

Here $s^2 = (\Delta\tilde{\beta}/2)^2 + k_{12}k_{21}$ denotes the coupling strength, $\Delta\tilde{\beta} = \tilde{\beta}_1 - \tilde{\beta}_2$ is the phase mismatch between the isolated modes, $\bar{\beta} = (\tilde{\beta}_1 + \tilde{\beta}_2)/2$ is the effective propagation mode of the coupler, and $\tilde{\beta}_m = \beta_m + k_{mm}$ is the renormalization of the propagation mode of each waveguide due to the self-coupling term k_{mm} . The coupling of two closely spaced waveguides therefore can be described in the matrix form

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \cos(sz) + i(\Delta\tilde{\beta}/2s)\sin(sz) & i(k_{12}/s)\sin(sz) \\ i(k_{12}/s)\sin(sz) & \cos(sz) - i(\Delta\tilde{\beta}/2s)\sin(sz) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \quad (2.9)$$

If both waveguides are identical and when the self-coupling terms k_{mm} are neglected, then $k_{12} = k_{21} = \kappa$, $\Delta\tilde{\beta} = \Delta\beta = 0$, $s = \kappa$. Eq. (2.9) can be simplified to

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} \cos(\kappa z) & i\sin(\kappa z) \\ i\sin(\kappa z) & \cos(\kappa z) \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \equiv \begin{pmatrix} r & it \\ it & r \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}. \quad (2.10)$$

The matrix in Eq. (2.10) gives characteristics analogous to that of semi-reflecting mirrors with reflectivity r and transmittivity t . The coupling matrix in Eq. (2.10) will be used throughout this thesis as a description of phase-matched and lossless coupling in multi-resonator systems.

Some non-idealities, of course, are readily apparent from Eq. (2.9). First, the $\beta = k_0 n_{\text{eff}}$ of the cavity mode and the guided mode are generally different, suggesting that the mode coupling is inherently phase-mismatched. Second, the coupling is assumed to occur only along the straight section of the racetrack resonator. In reality, however, there is also field overlap along the curved section of the racetrack which also contributes some offset to the coupling. Third, the assumption that self-coupling term k_{mm} can be ignored is only valid for a rather weak coupling. In the case of stronger coupling, the k_{mm} can affect the resonance condition and shifts the resonant frequency.

There is also loss associated with mode coupling. Fig. 2.8 shows the scenario of coupling loss in ring and racetrack resonators. In the racetrack resonator, the n_{eff} for the straight and the curved section (*cf.* Fig. 2.4) of the racetrack resonator are different, and a Fresnel reflection (from curved to straight sections or vice versa) excites counter-propagating cavity mode which builds up as the light stays for many round trips in the resonator. The transmission

amplitude losses of straight-to-curved (τ_{sc}) and curved-to-straight (τ_{cs}) can be expressed in terms of Fresnel transmission and field-overlaps [15]

$$\tau_{sc} = 2\langle E_{cv} | E_{st} \rangle / (1 + \beta_{cv} / \beta_{st}), \tau_{cs} = 2\langle E_{st} | E_{cv} \rangle / (1 + \beta_{st} / \beta_{cv}), \quad (2.11)$$

where β_{cv}, β_{st} are respectively the propagation mode for the curved and the straight sections, and $\langle E_{cv} | E_{st} \rangle$ is the field overlap denoting the mode conversion efficiency between the two sections. The total coupling amplitude loss then is $\tau_c = \tau_{cs}^2 \tau_{sc}^2$. Note that this loss is generally very small, when the bending radius is beyond the radiation region, e.g., $R > 3\mu\text{m}$ in Fig. 2.5, that is when the n_{eff} for both sections are only slightly different. The other method to minimize this type of coupling loss is by lateral offset of the straight section so that the mode “center-of-mass” is aligned with the mode in the curved section in order to increase the field-overlap [15-17].

Fig. 2.8. The illustration of mode-conversion loss racetrack and ring resonators, the grey arrows represent the mode loss as a result from Fresnel reflection and field-overlap (for racetrack resonator) and phase-mismatch (ring resonator). The dash circle denotes azimuthal propagation direction of the coupled mode in the straight waveguide.

The second type of coupling loss is present in the point coupling scheme. In the coupling, when the intra-cavity field is reasonably strong, there is a tendency to extend the azimuthal propagation of the cavity mode to the waveguide (see the dash lines in Fig. 2.8). This means that the mode propagation angle of the coupled light is larger than that required for total internal reflection⁵ in the straight waveguide. As a result, the mode propagates in zigzag manner while giving off radiation for every reflection at the cladding. One method to reduce this loss is by decreasing the straight waveguide width, so as to limit the propagation angle of the coupled light inside the waveguide. This method was demonstrated in the fabrication of high Q ring resonator with $1.5\mu\text{m}$ radius [18].

⁵The mode propagation angle of single mode waveguide can be deduced as $\theta = \cos^{-1}(n_{\text{eff}}/n_{\text{core}})$.

2.4 Coupling induced resonance frequency shift (CIFS)

In this section, the shift of resonance frequency due to the self-coupling term (k_{mm}) is investigated. From coupled-mode theory, the mode splits into the orthogonal modes,

$$\beta_{e,o} \equiv \bar{\beta} \pm s = \bar{\beta} \pm \sqrt{(\Delta\tilde{\beta}/2)^2 + k_{12}k_{21}}, \quad (2.12)$$

which means that the coupling is actually an interference of these two co-propagating modes, where the $\pm s$ is responsible for lateral power transfer and $\bar{\beta}$ is the effective propagation mode along the coupling. Inserting $\beta_{e,o}$ to Eq. (2.7) gives the following characteristics matrix

$$\begin{pmatrix} \mp s & k_{12} \\ k_{21} & \mp s \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0,$$

which leads to eigen vectors

$$\begin{pmatrix} C_1^{(e)} \\ C_2^{(e)} \end{pmatrix} = \begin{pmatrix} k_{12} \\ s \end{pmatrix}, \quad \begin{pmatrix} C_1^{(o)} \\ C_2^{(o)} \end{pmatrix} = \begin{pmatrix} -k_{12} \\ s \end{pmatrix}, \quad (2.13)$$

corresponding to the even (β_e) and odd (β_o) field distributions in the two evanescently coupled waveguides. In the phase-matched coupling ($\beta_1 = \beta_2$), Eq. (2.13) reduces to $[1 \ 1]^T$ and $[-1 \ 1]^T$, signifying pure even and odd modes respectively. For simplicity, we can assume the uncoupled propagation mode of the cavity mode only differs slightly from that of the waveguide ($\beta_1 = \beta_2$), so that the ring-waveguide-ring term (k_{11}) and the waveguide-ring-waveguide term (k_{22}) is the same ($k_{11} = k_{22}$)⁶, and the propagation mode in the coupling section changes to $\beta_1' = \beta_1 + k_{11}$. The shift of the round trip phase then is

$$\Delta\phi = \Delta\omega(L_{\text{cav}} n_{\text{eff}} / c) = \beta_1(L_{\text{cav}} - L_C) + \beta_1' L_C - \beta_1 L_{\text{cav}} = k_{11} L_C, \quad (2.14)$$

and the coupling induced resonance frequency shift (CIFS) is $\Delta\omega = ck_{11} L_C / (n_{\text{eff}} L_{\text{cav}})$, that depends on the overlapping between the mode-field with the index perturbation $\Delta\epsilon(x)$ (controlled by gap separation g) and the coupling length (L_C). Note that the shift is indirectly proportional to the coupling strength ($\sim k_{12} L_C$). The effect of CIFS is very detrimental to the design of higher order optical filters based on multi-resonator configuration [19]. As illustrated in Fig. 2.9, the supposedly flat-top response will have many unwanted dips

⁶ When the two waveguides are identical and has high field confinement, the self coupling terms are the same $\langle E_1(x) | \Delta\epsilon(x - g - w) | E_1(x) \rangle = \langle E_1(x - g - w) | \Delta\epsilon(x) | E_1(x - w - g) \rangle$. The g and w are the gap separation and the waveguide width, respectively.

because the rings are no longer ‘identical’ to each other due to CIFS. This effect can be reduced when the rings are pre-distorted before fabrication so as to make the rings look “identical” after CIFS takes place.

Fig. 2.9. Illustration on coupling induced resonance frequency shift (CIFS) and detrimental effects of CIFS towards the flat-top filter response.

Chapter 3. The One-Cavity System: Theory and Experiments

The general aspects of propagation mode, coupling, mode-conversion and radiation loss of whispering gallery resonator have been discussed in the previous chapter. Although there are some non-idealities originated from mode-conversion loss in the mutual coupling and radiation loss in curved waveguide, they are not of particular interest in cases where only the amplitude and phase response of the device are concerned. Based on these reasons, the formulation of ring resonator can be simplified into simple algebraic relations of fields (or in matrix formalism for multi-cavity system), without the loss of generality. The assumptions used in this formalism are (1) that the mode coupling is phase-matched ($\beta_{\text{ring}} = \beta_{\text{wg}}$) and lossless ($r^2 + t^2 = 1$), (2) the coupling coefficients (r, t) are assumed to be frequency dependent and (3) that the effect of CIFS is omitted in the coupling. In the formalism, the round-trip cavity loss is the lump parameters of all the possible losses in the ring resonator, which is denoted as a constant a , and it consists of radiation, mode-conversion, and surface scattering loss.

3.1 Ring resonator coupled to one optical waveguide (1R1B)

The first ring resonator configuration is shown in Fig. 3.1, where the ring is coupled to one bus waveguide. The field amplitudes in 1R1B system (Fig. 3.1) can be related by the coupling matrix in Eq. (2.10)

$$\begin{pmatrix} c_2 \\ s_2 \end{pmatrix} = \begin{pmatrix} r & it \\ it & r \end{pmatrix} \begin{pmatrix} c_1 \\ s_1 \end{pmatrix},$$

and the continuity relation $c_1 = ac_2 \exp(-i\delta)$, where $\delta = k_0 n_{\text{eff}} L_{\text{cav}}$ is the normalized frequency and $a = a_{\text{wg}} a_{\text{bend}} a_{\text{coupling}}$ is the lumped round trip amplitude loss consisting of waveguide loss $a_{\text{wg}} = \exp(-\alpha_{\text{wg}} L_{\text{cav}} / 2)$, bending loss a_{bend} , and mode-conversion in the coupling loss a_{coupling} . Solving the sets of equations above, the transmittance $\tau = |\tau| \exp(-i\theta)$ is expressed as

$$\tau = \frac{s_2}{s_1} = \frac{r - a \exp(-i\delta)}{1 - ar \exp(-i\delta)}. \quad (3.1)$$

Here, the normalized frequency δ is used in the formalism for its simplicity in describing the system response in general without the need to over-specify the actual parameters such as the resonance wavelength, geometrically dependent waveguide propagation mode n_{eff} , and the

geometries of the resonator. The ring is said to be on *resonance* when the δ is in integer multiple of 2π , i.e., $\delta=2\pi m$, and on *anti-resonance* when δ is in odd multiple of π , i.e., $\delta=(2m+1)\pi$.

Fig. 3.1. (Left) The schematic of 1R1B configuration. (Right) The FDTD calculated field distribution of 1R1B configuration at resonance.

The 1R1B generally behaves as an all-pass phase-only filter (APF), which in ideal (lossless) case, exhibits a unity transmission and a sharp phase change around the resonance. In a more realistic case, however, the existence of loss is inevitable and the strong light localization inside the ring enhances the effect of loss which in turn gives a transmission dip. The Fig. 3.2 (left panel) shows the transmission $T = |\tau|^2$ and effective phase $\theta = -\arg(\tau)$ as a function of normalized detuning. The transmission spectrum is in the form on periodic dips, with $\Delta\delta=2\pi$ separation between adjacent peaks. The contrast of the dip is a function of coupling coefficients and roundtrip loss, and there is 2π phase swing across the resonance. The periodicity is defined as the resonator free spectral range (FSR) which describes the actual rotation frequency of the circulating light

$$\Delta f_{FSR} = \frac{c}{n_{\text{eff}} L_{\text{cav}}} \equiv \frac{1}{t_R}, \quad (3.2)$$

where t_R is the round-trip time. We can define the intensity buildup factor (B) as the ratio of the intensity of circulating light and the intensity of the input light

$$B = \left| \frac{c_2}{s_1} \right|^2 = \frac{1-r^2}{1+a^2 r^2 - 2ar \cos \delta}, \quad (3.3)$$

and it should be related to the cavity lifetime since the coherent buildup is a result from constructive interference between the circulating light and the incoming light for a periodic of time within which the light stays inside the cavity.

Fig. 3.2. (Left) The transmission and the phase of 1R1B as a function of normalized detuning. (Right) The resonant transmission and intensity buildup factor as a function of coupling coefficient r . The round trip loss is assumed to be $a=0.95$. The normalized detuning is the frequency detuning over free-spectral range, $(f - f_0) / \Delta f_{FSR} \equiv (\delta - \delta_0) / 2\pi$

In order to obtain the cavity lifetime, Eq. (3.3) can be rewritten in low-loss and weak-coupling assumption ($ar \sim 1$)

$$B = B_0 \frac{(\Delta\delta/2)^2}{\delta^2 + (\Delta\delta/2)^2} \equiv B_0 \eta, \quad (3.4)$$

where $\Delta\delta = 2(1-ar)/\sqrt{ar}$ is the cavity full-width half-maximum (FWHM) linewidth, $B_0 = (1-r^2)/(1-ar)^2$ is on-resonance intensity buildup factor, and the η is the Lorentzian lineshape function. The performance of a resonator is also characterized by cavity finesse F , which is the metric of the cavity lifetime with respect to cavity roundtrip time. Finesse is defined as the ratio of the free-spectral-range over the linewidth

$$F = \frac{2\pi}{\Delta\delta} = \frac{\pi\sqrt{ar}}{(1-ar)}. \quad (3.5)$$

It should be noted that the finesse is related to the decay rate in Eq. (2.2). By comparing the linewidth term of both equations, the cavity lifetime can thus be inferred as $t_{\text{cav}} = Ft_R / \pi$ and the effective number of roundtrips is $N = t_{\text{cav}}/t_R$, or

$$N \equiv \frac{t_{\text{cav}}}{t_R} = \frac{F}{\pi} = \frac{\sqrt{ar}}{1-ar}.$$

It is possible to re-express the intensity buildup factor in terms of effective amount of roundtrips. At resonance, the light fields continuously couples into the cavity, while the intra-

cavity field (E_{cav}) keeps on increasing for every completed roundtrip due to constructive interference until the net in-coupling is zero (see Fig. 3.1, right panel). Hence, the intra-cavity field is a coherent summation of the coupled field from all the roundtrips, $E_{\text{cav}} \sim N \times (1 - r^2)^{1/2} \times E_{\text{in}}$, and the intensity buildup is thus $|E_{\text{cav}}/E_{\text{in}}|^2 \cong B_0$, which is consistent with B_0 in Eq. (3.4).

Fig. 3.3. The transmission (T) and phase (θ) responses in under-coupled ($r > a$), critically-coupled ($r = a$), and over-coupled situations ($r < a$). The parameters used are $a = 0.98$, $r_{\text{undercoupled}} = 0.995$, and $r_{\text{overcoupled}} = 0.95$. The phase response of lossless ring is almost the same as that in over-coupled situation.

There are three operating regions of 1R1B: over-coupling ($r < a$), critical coupling ($r = a$), and under-coupling ($r > a$), as presented in Fig. 3.3. The dash line illustrates the unity transmission in the lossless case. In the *over-coupled* situation, the coupled power is larger than the cavity round trip loss, thereby giving a net field that will accumulate in the next round trip, and impart phase buildup across the resonance. The effective loss and thus the transmission contrast are also amplified. The contrast depends strongly on the cavity finesse. In the *critically-coupled* situation, the coupled power is exactly absorbed in one round trip. This corresponds to the maximum intensity buildup factors, as shown in Fig. 3.2 (right panel). In this situation, the light at the output port is under complete destructive interference and the phase information is therefore destroyed, rendering the discontinuity in the phase response at the resonance. Note that the intensity buildup factor at this condition serves as the prime limitation of how high the intra-cavity field can be enhanced inside the ring for a given roundtrip loss a , $B_0^{(\text{max})} = 1/(1 - a^2)$.

In the *under-coupled* situation, the coupled power is much lower than the absorption within one round trip and thus there is no phase buildup. The intensity buildup factor decrease rapidly in this condition. Note that, here the phase response resembles the real value of atomic susceptibility (χ'') in the vicinity of the material absorption resonance. It has been suggested that resonators often behave as dielectric atoms, because of the strong resemblance in its response. This has led to many interests in optical mimicking of atomic mechanisms, such as in resonance splitting in coupled resonators, photonic bandgap in periodic arrangement of cavities, and Fano resonance in specifically designed resonator system [1-3].

3.2 Ring resonator coupled to two optical waveguides (1R2B)

The second configuration is shown in Fig. 3.4, where the ring is coupled to two bus waveguides (1R2B). Here, the system functions as a channel dropping filter, where the light power is resonantly transferred from one waveguide to the other. Similar to 1R1B case, the fields are related by the same coupling matrix, with the following continuity relations $c_3 = c_2 \sqrt{a} \exp(-i\delta/2)$, $c_1 = c_4 \sqrt{a} \exp(-i\delta/2)$, $s_3 = 0$. It can be shown that the drop ($d=s_4/s_1$) and through ($\tau=s_2/s_1$) transmittance are

$$\tau = \frac{s_2}{s_1} = r \frac{1 - a \exp(-i\delta)}{1 - ar^2 \exp(-i\delta)}, \quad d = \frac{s_4}{s_1} = -\frac{(1 - r^2) \sqrt{a} \exp(-i\delta/2)}{1 - ar^2 \exp(-i\delta)}. \quad (3.6)$$

Fig. 3.5 (left panel) shows the transmissions ($D=|d|^2, T=|\tau|^2$) and the phase responses $\theta_{T,D}$ of 1R2B. When the light is off-resonant, the presence of the ring resonator is reduced to a waveguide splitter, each with amplitude splitting ratio of $d \sim (1-r^2)/(1+r^2)$ and $\tau \sim 2r/(1+r^2)$ in drop and through port respectively. The effective phase for the through port is relatively unperturbed, $\theta_r \sim 0$, while for the drop port is equivalent to a propagation of $L_{\text{cav}}/2$ path length which gives a linear phase $\theta_d \sim \delta/2$ around anti-resonance.

Fig. 3.4. (Left) The schematic of 1R2B configuration. (Right) The FDTD calculated field distribution of 1R2B configuration at resonance.

Fig. 3.5. (Left) The calculated transmissions and phase responses of Through and Drop ports of 1R2B. (Right) Resonant intensity buildup and Drop amplitude as a function of coupling coefficient (r).

Around the resonance, the light is under destructive and constructive interference at the through and drop port respectively. Ideally in lossless case ($a=1$), the input power is completely transferred to the drop port, and the phase information is completely lost, giving a phase discontinuity at the resonance, similar to the phase response of critically-coupled 1R1B configuration. In the presence of loss, the destructive interference is not complete and the phase information is not entirely lost, causing the phase response resembling that of under-coupled 1R1B. The phase response of the drop port resembles the phase response of over-coupled 1R1B and there is a π phase swing (instead of 2π in 1R1B) across the resonance, which is attributed to the fact that the light travels by $L_{\text{cav}}/2$ before entering the drop port. The intensity buildup is expressed as

$$B = \left| \frac{c_2}{s_1} \right|^2 = \frac{1-r^2}{1+a^2r^4-2ar^2\cos\delta}, \quad (3.7)$$

and at weak coupling regime (or high finesse cavity), the intensity buildup, drop, and through transmissions can be re-written in the form of Lorentzian lineshape η ,

$$B \cong B_0\eta, \quad D \cong D_0\eta, \quad T \cong T_0\eta + (1-\eta), \quad (3.8)$$

where $D_0 = a[(1-r^2)/(1-ar^2)]^2$, $T_0 = r^2[(1-a)/(1-ar^2)]^2$, $B_0 = (1-r^2)/(1-ar^2)^2$ are the on-resonance values of drop transmissions, through transmissions, intensity buildup factors,

respectively and $\Delta\delta = 2(1-ar^2)/r\sqrt{a}$ is the linewidth of the Lorentzian lineshape. The cavity finesse of 1R2B case is denoted as

$$F = \frac{\pi r \sqrt{a}}{1-ar^2}, \quad (3.9)$$

which intuitively should about half the finesse of 1R1B case since the light is coupled twice in 1R2B. This can be shown by comparing the finesse of 1R1B (F_1) with that of 1R2B (F_2), $F_1/F_2 = (1+r)/\sqrt{r}$, which is approximately 2 in the weak coupling ($r \sim 1$). Using the same approach as in the 1R1B case, the effective number of roundtrips is $N = F/\pi$ and the intra-cavity field is $E_{\text{cav}} \sim N \times (1-r^2)^{1/2} \times E_{\text{in}}$, which leads to intensity buildup factor of $\sim B_0$ in Eq. (3.8), as expected.

The plots of D_0 , T_0 , and B_0 as a function of coupling coefficients r are shown in Fig. 3.5 (right panel), where the 1% roundtrip loss ($a=0.99$) is assumed. In the over-coupling region ($r^2 < a$), the coupling dominates the interference and the intensity buildup increases in the increasing cavity finesse (higher r). In the critical-coupling situation, the coupled light is exactly dissipated inside the cavity ($r^2 = a$) and this corresponds to the highest achievable intensity buildup at a given round trip loss a , i.e., $B_0^{(\text{max})} = 1/[(1+a)(1-a^2)]$. In the under-coupling situation ($r^2 > a$), the loss dominates the interference process and the intensity buildup rapidly decreases in the increasing cavity finesse, however there is still a phase buildup and the drop transmission decreases at increasing cavity finesse, in contrast with 1R1B configuration.

3.3 Phase Sensitivity, Group Delay and Group Delay Dispersion of All Pass Filter

It has been shown that the coherent intensity buildup is the result of interference of N in-phase signals originated from previous roundtrips. However, it should be noted that the time taken to store and to release the light should be $2t_{\text{cav}}$. Thus, the light propagation inside the resonator is equivalent to a straight waveguide with effective path length of $L_{\text{eff}} = 2NL_{\text{cav}}$ that has a coherent delay of $T_g \sim 2Nt_R$. The N is dependent upon cavity finesse (F) which in turn is controllable by the coupling coefficients. Thus, the delay is tunable with coupling coefficients and providing a possibility for a ring resonator to functions as a compact yet effective tunable delay line and optical buffer. It is instructive to start the time-domain analysis from the phase response of 1R1B in the lossless case

$$\theta = \tan^{-1}\left(\frac{r \sin \delta}{1 - r \cos \delta}\right) - \tan^{-1}\left(\frac{\sin \delta}{r - \cos \delta}\right). \quad (3.10)$$

The phase response vs. δ for different coupling coefficients is presented in Fig. 3.6(a). It can be seen that near the resonance, the phase is significantly perturbed and the slope sensitively dependent on the coupling coefficient. Such a sensitivity is an indicative of a group delay of a pulse entering a resonator, normalized to roundtrip time t_R

$$\begin{aligned} T_g &= d\theta / d\delta \\ &= \left(\frac{r \cos \delta - r^2}{1 + r^2 - 2r \cos \delta} \right) - \left(\frac{r \cos \delta - 1}{1 + r^2 - 2r \cos \delta} \right) \\ &= \frac{1 - r^2}{1 + r^2 - 2r \cos \delta} \equiv B, \end{aligned} \quad (3.11)$$

which is in fact identical with the intensity buildup factor B . It is interesting to point out that when the coupling is very weak ($r \sim 1$), the buildup factor is approximately equal to $2N$,

$$T_g = B = \frac{1+r}{1-r} = \left(\frac{1+r}{\pi\sqrt{r}} \right) \left(\frac{\pi\sqrt{r}}{1-r} \right) \cong \frac{2}{\pi} F = 2N,$$

which is to be expected since the group delay converges to coherent delay when the light is nearly continuous wave.

Fig. 3.6. (a) Phase spectrum and (b) normalized delay for different coupling coefficients in 1R1B configuration.

The on-resonance group delay $T_g(0)$ for different coupling coefficients is given in Fig. 3.6(b), where the delay is shown to be exponentially related to the coupling coefficients and can go to thousands of roundtrip times in the very weak coupling ($r \sim 1$). Clearly, in the more

realistic case where the loss is present, one cannot have arbitrarily high delay in the same way as the case for intensity buildup factor discussed in the previous sections.

Fig. 3.7. The impulse response for resonant and anti-resonant light in 1R1B configuration.

Fig. 3.8. The time-domain response in over-, critical-, and under-coupled situations. The dash line represents the lossless case.

Since this is a linear problem, the time response can be calculated by multiplying the input spectrum with the resonator response in the frequency domain, and then perform inverse Fourier transform to have the output in time-domain. Fig. 3.7 presents the impulse response for resonant and anti-resonant light. Because the input pulse width is much smaller than the roundtrip time, the ring responds in pulse trains with decaying amplitudes and separated by exactly one roundtrip time. Two different cases are presented here. In the resonant case, the pulse trains are in the opposite sign which is an indicative of π phase shift at the resonance. The π phase shift also can be understood because each roundtrip involves two cross coupling, each of which impose $\pi/2$ phase shift. In anti-resonant case the pulse comes out with alternating signs. This is attributed to the fact that the light acquires a phase

of π for each roundtrip plus extra π phase shift from two cross couplings. This explains why the first pulse has positive sign and followed by a negative one, and so forth.

Fig. 3.8 presents the case for over-, critical, and under-coupled 1R1B for Gaussian input pulse with pulse width of cavity lifetime. In the over coupling case, the output pulse is broadened and delayed by approximately a cavity lifetime, with additional tail originated from cavity dispersion and distortion from the amplitude response. In general the group delay is not affected so much by the loss for over-coupling case. In the critical coupling case, we can see that the pulse “center-of-mass” is not delayed at all, and the pulse amplitude tends to decay rapidly in time. This is to be expected because of resonant absorption. In the under-coupling case, the pulse seems to travel faster than what is permissible in the given medium. This corresponds to the case of anomalous dispersion due to the change of sign in the phase response at under-coupling scheme. This phenomenon is often termed as fast light, as opposed to slow light in over-coupled situation. As clearly seen from the pulse propagation the light of different wavelengths experiences different delay, and the group delay dispersion (D_g) is defined as the gradient of the group delay,

$$D_g = \left(\frac{dT_g}{d\delta} \right) = -\frac{2\delta B_0}{(\Delta\delta/2)^2} \left[\frac{(\Delta\delta/2)^2}{\delta^2 + (\Delta\delta/2)^2} \right]^2 = -2B^2 \left(\frac{\delta}{\Delta\delta} \right), \quad (3.12)$$

where the $B_0 \cong 4/\Delta\delta$ for lossless case and the dispersion is normalized to t_R^2 .

Fig. 3.9. Time-domain characteristics of Gaussian input pulse of a fixed pulse width in (a) different cavity finesses and (b) in different number of cascade for a fixed cavity finesse.

The dispersion depends on a square of the buildup factor and the relative detuning over the cavity linewidth ($\delta/\Delta\delta$), which means that higher buildup imposes larger dispersion. From the detuning we can see that the dispersion changes sign from positive (pulse broadening) to

negative (pulse steepening), which means there is a pulse splitting after some time. Such a pulse splitting phenomenon corresponds to the third order dispersion in optical communication. Fig. 3.9(a) shows time domain characteristics of input Gaussian pulse in different coupling coefficients. Clearly, the pulse splitting is more apparent in higher cavity finesse, and the amplitude of the second pulse is becoming stronger as the cavity finesse is increased. This is characterized by the B^2 dependence in Eq. (3.12) that suggests very steep slope near the resonance in high finesse ($r > 0.95$). Since the delay is approximately $T_g \cong B$ when the loss is reasonably low, the dispersion can also be expressed in terms of the group delay itself $D_g = -2T_g^2(\delta / \Delta\delta)$. The second illustration is demonstrated in Fig. 3.9(b) where the 1R1B configurations are cascaded together and the same input pulse is launched onto the structure. Here the finesse of each cavity is fixed and the number of the cascade is increased. The delay and dispersion then are

$$T_g = \sum_m T_{g,m}, \quad D_g = \sum_m -2B_m^2 (\delta_m / \Delta\delta_m), \quad (3.13)$$

where B_m , δ_m , and $\Delta\delta_m$ are the buildup, detuning, and linewidth of the m^{th} cavity in the cascade respectively. As expected, the pulse starts to break down into multiple pulses as the number of cascade is increased. When the input consists of multiple pulses, the split pulses from each pulse interfere with the other input pulse, thus distorting the informational content in the input.

This is the reason why higher order dispersion is often termed as inter-symbol interference. It is interesting to note that at near resonance the dispersion is zero, but that does not mean the signal is free from distortion and broadening. This is attributed to the slope of dispersion which becomes the prime limitation of one-cavity system in optical delay line and optical buffer applications in which the information is to be preserved. One way to reduce higher order dispersion in the cascade structure is by adjusting the relative detuning of each cavity in the cascade so as to reduce the total sum of dispersion in broader bandwidth. This approach however is problematic in the practical sense since only few hundreds of nanometer difference in the cavity length can readily shift the resonance by more than the cavity linewidth.

Maximum dispersion can be easily found at $\delta_{\text{max}} = \pm \Delta\delta / 2\sqrt{3}$ and is related to the maximum nonlinear responsivity of a resonator ($dB/d\delta$). If the cavity is made of nonlinear medium with Kerr coefficients n_2 , the round trip phase becomes dependent on intra-cavity intensity $\delta = k_0(n_{\text{eff}} + \Delta n)L_{\text{cav}}$ with the nonlinear index change of $\Delta n = n_2 I_{\text{cav}}$. As the roundtrip

phase changes by $d\delta/dI_{\text{in}} \propto B$, the buildup factor changes by $dB/d\delta \propto B^2$ for nonzero detuning. It can be shown that the maximum nonlinear responsivity is $dB/dI_{\text{in}} = 9n_2B_0^2/32n_{\text{eff}}$ which is also proportional to the square of intensity buildup, similar to dispersion. The maximum dispersion is also related to what is known to be critical detuning in the nonlinear bistability switch, by which bistability only occurs when the detuning is larger than certain value, i.e., $|\omega - \omega_{\text{res}}| \geq \Delta\omega\sqrt{3}/2$. These aspects will be explained in the latter chapter.

3.4 Device fabrication

The devices of this work were fabricated by means of collaboration under ePIXnet platform [4] which uses silicon-on-insulator (SOI) technology and 248nm CMOS-compatible deep-UV (DUV) lithography system [5]. The key advantage of using SOI technology is the use of thermal oxidation for sidewall smoothening which is crucial for reducing the sidewall scattering loss. As shown in Fig. 3.10(a), the devices are fabricated with waveguide width and thickness of 450nm and 200nm, respectively. Both ends are integrated with grating couplers. The light from single mode fiber (SMF) is at off-normal incidence with the grating. The fiber-mode (β_{fiber}) and the waveguide mode (β_{wg}) are related by $\beta_{\text{wg}} = \beta_{\text{fiber}} \sin \theta_{\text{inc}} \pm mK$, where $K=2\pi/\Lambda$ is the grating momentum for a given grating pitch Λ , and m is an integer denoting the diffraction order. The coupling is optimum when the two propagation modes are phase-matched [see Fig. 3.10(b)]. The buffer oxide thickness (t_{ox}) is 2 μm , which is reasonable for suppressing the substrate leakage loss significantly. The SiO_2 upper cladding is deposited by plasma enhanced chemical vapor deposition (PECVD). The detailed geometries of the waveguide are presented in Fig. 3.10(c). It should be noted that the waveguide dimensions are chosen in this way so that only TE mode is supported and hence eliminates potential birefringence-related problems in the devices.

Fig. 3.10. (a) The schematic of near-vertical coupling in the waveguide measurement setup. (b) Illustration on the phase matching condition between the fiber and waveguide mode. The inset shows the actual fabricated gratings of ~ 10 microns length (c) The waveguide geometries.

3.5 Near-vertical coupling principle

The upper cladding deposited by PECVD technique is naturally amorphous, making it difficult to cleave the sample. Polishing is often necessary to ensure the waveguide edges smooth to improve the butt-coupling efficiency from the fiber to the waveguide. For the above reasons, we adopt the near-vertical coupling approach in our measurement setup, instead of the conventional butt-coupling technique. The working principle of grating coupler is well illustrated by equifrequency contour $k_x^2 + k_z^2 = \omega^2 n^2 / c^2$ of the two media representing free space light from the fiber (medium 1 of index n_1) and the optical waveguide (medium 2 with index n_2). The contour for isotropic medium for a given wavelength forms a circle with radius proportional to the refractive index of the medium and the angle of the refracted and reflected lights are obtained upon matching the k -vector parallel to the interface between the two media.

Two cases of mode-coupling are presented in Fig. 3.11. At normal-incidence (Fig. 3.11, upper left panel), the modified transverse k -vector is $\pm K$ and the vertical coupling occurs when $K = \beta$ (which is called resonant case), by which the light in-couples to two counter-propagating waveguide modes ($\pm \beta$) corresponding two first diffraction orders ($m = \pm 1$) and

downward radiating mode at $m=0$. In the out-coupling situation (Fig. 3.11, lower left panel), the waveguide mode ($+\beta$) is incident on the grating and there are three possible outputs which comprise of the transmitted waveguide mode ($+\beta$) at $m=0$, the upward and downward radiating modes at $m=-1$, and lastly the backward waveguide mode ($-\beta$) at $m=-2$. By conservation of energy, we can have $R+T+T_{UP}+T_{DOWN}=1$, where T_{UP} and T_{DOWN} are fractional power of the upward and downward radiations, and we can roughly estimate the in-coupling efficiency as $\sim T$ and the out-coupling efficiency as $\sim T_{UP}$. This means the excitation of backward waveguide mode is the prime limitation of the coupling efficiency at normal incidence.

Fig. 3.11. The equifrequency contour ($k_x^2+k_z^2=\omega^2n^2/c^2$) of normal incidence and off-normal incidence coupling. The black (gray) numbers denote the diffraction orders in the in-coupling (out-coupling) situation.

The excitation of backward waveguide mode can be avoided by slightly detuning the grating momentum from the waveguide mode ($K \neq \beta$) and the coupling occurs at oblique incidence. As shown in Fig. 3.11(upper right panel), the condition $(\beta - mK) \neq -\beta$ is always satisfied, which is indicative of the nonexistence of the backward modes. Hence, oblique incidence in-couples forward waveguide mode at $m=1$, downward leaky modes at $m = (-1, 0)$. In the out-coupling situation (Fig. 3.11, lower right panel), the forward mode (at $m=0$) will out-couple upward radiations and excites downward leaky modes at $m = (-1, -2)$. The conservation of energy then dictates $T+T_{UP}+T_{DOWN}=1$, which means the coupling efficiency for oblique scheme is inherently higher than the vertical scheme. Further improvements can be made, for example by adjusting the pitch in such a way that there is only one diffraction

order within the equifrequency of medium 1, or by breaking the vertical symmetry by adding reflective substrate at the bottom, adjusting the grating etch depth, and filling ratio (w_1/Λ).

3.6 Waveguide measurement setup

In this work, the design of the one-dimensional grating coupler was carried out by collaborators from university of Ghent, and had been shown to exhibit maximum coupling efficiency of $\sim 30\%$ [6]. The grating is designed so that the phase matching condition is satisfied when the fiber is butt-coupled 10° off vertical to the grating. The fiber-to-waveguide coupling efficiency has a Gaussian spectral profile of 30nm bandwidth. The device is excited with ASE broadband source with wavelength range from 1480nm to 1610nm. The output power is then passed through a 90:10 splitter, where 10% power goes to a fiber power meter for alignment purpose, while the rest goes to an optical spectrum analyzer (OSA) for normalization with the input spectrum. The measurement setup is schematically shown in Fig. 3.12.

Fig. 3.12. The schematic of the waveguide measurement setup

3.7 Characterization of waveguides and ring resonators

The fabricated waveguides are shown in Fig. 3.13(a). In order to investigate propagation and bending loss, the waveguides are fabricated in serpentine structure. The bending radius is

chosen to be $5\mu\text{m}$ so that bending loss is negligible for SOI waveguides [7] and the waveguide lengths are varied by introducing the elongation parameter L after individual 90° bends. The serpentine structure has 50 unit cells, each of which consists of four 90° bends and a length increment of $2L$. Thus, by varying L from $0\mu\text{m}$ to $15\mu\text{m}$, one can have waveguide length difference up to $1500\mu\text{m}$.

Fig. 3.13. (a) The fabricated waveguides with different lengths. The bending radius is $5\mu\text{m}$. (b) The fabricated 1R2B. The coupling length is varied from $0\mu\text{m}$ to $8\mu\text{m}$.

Fig. 3.14 shows the measured transmission for straight waveguide (T_{WG0}), bent waveguide but no elongation (T_{WG1}), and bent waveguide with $1500\mu\text{m}$ longer length (T_{WG2}). The decreasing trend of the transmission is caused by the bowing effect from the grating envelope whose center is around 1500nm . Thus, the propagation loss is

$$\alpha_{WG} = (T_{WG2} - T_{WG1}) / \Delta L_{21}, \quad (3.14)$$

and the measured result is $0.335 \pm 0.106 \text{ dB/mm}$, which agrees well with the demonstrated SOI waveguide of the same dimensions ($\sim 3 \text{ dB/cm}$). The pretty large noise is mainly caused by the TM mode that strongly leaks to the substrate. The bending loss (α_{BEND}) can then be obtained from the total loss of WG1 after deduction of the propagation loss,

$$\alpha_{BEND} = (T_{WG1} - T_{WG0} - \alpha_{WG} \Delta L_{10}) / (4N), \quad (3.15)$$

where $R = 5\mu\text{m}$ is the bending radius, $N = 50$ is the number of cells in the serpentine structure, and ΔL_{10} is the length difference between the two waveguides. The measured bending loss from the serpentine structure is $\alpha_{BEND} = 0.0241 \pm 0.0006 \text{ dB/turn}$. Since the serpentine

structures are fabricated with a fixed number of bends, the measured bending loss may not be accurate as there is an additional offset introduced from the gratings.

Fig. 3.14. The transmission for straight and serpentine waveguide structures. The length difference between WG1 and WG2 is 1500 μm .

A more accurate measurement can be done with the help of resonance effect in ring resonator coupled to two optical waveguides. Fig. 3.13(b) shows the fabricated racetrack resonator of 5 μm radius with coupling length varied from 0 μm to 8 μm . The gap separation between the waveguide and the ring is fixed at 200 nm. The Through transmissions are obtained by normalizing the output with the output from straight waveguide fabricated in the same sample. Thus, the parabolic envelope from the grating can be almost entirely removed, as evidenced by the offset Through transmissions of various 1R2Bs in Fig. 3.15. There is still residue of grating envelope in the spectra because grating efficiency may slightly differ in each device. Thus, for parameter extraction the grating factor G is introduced into the standard 1R2B theoretical model.

$$T = G \left| \frac{r(1 - a \exp(-i\delta))}{1 - ar^2 \exp(-i\delta)} \right|^2, D = \frac{aG(1 - r^2)^2}{|1 - ar^2 \exp(-i\delta)|^2}. \quad (3.16)$$

The curve-fitting procedure are explained in the following. First, the wavelength is normalized to the cavity length $L_{\text{cav}} = 2(\pi R + L_c)$ to give $\delta = 2\pi n_g L_{\text{cav}} / \lambda$ in such a way that only one resonance situated at every 2π . The initial guess of group index n_g is taken from free-spectral-range (FSR) $n_g = \lambda^2 / \Delta\lambda_{\text{FSR}} L_{\text{cav}}$, and then is fitted to obtain $\delta = 2\pi m$. This gives $n_g \sim 4.25$ which agrees very well with the group index calculated by mode-solver $n_g = n_{\text{eff}} - \lambda(dn_{\text{eff}}/d\lambda)$ and experimental measurement in Mach-Zehnder structure (MZI) as

well [8]. Second, the grating factor is adjusted from the measured Through transmission. When the coupling is weak, the Through transmission should reach almost unity at the anti-resonance. Thus, G can be obtained by comparing the ideal and the measured transmissions.

Fig. 3.15. The measured Through transmissions for various coupler lengths.

Third, the initial guess of the coupling coefficient r is obtained upon curve-fitting the Through transmission near the resonance. The amplitude of Through transmission is not sensitive to loss at the resonance for 1R2B. Finally, the obtained r is used in the Drop spectrum in order to extract the round trip loss a . The procedures are repeated to get the converged parameters, and the best fit parameters for 5 1R2B structures are shown in the left panel of Fig. 3.16(a). The evanescent coupling mechanism is based on the beating of even and odd modes, and should follow

$$r = \cos\left(\pi \frac{L_C + L_0}{2L_\pi}\right), \quad t = \sin\left(\pi \frac{L_C + L_0}{2L_\pi}\right). \quad (3.17)$$

Here, L_π denotes the beating length by which the odd and even modes are out-of-phase, which is related to κ in chapter 2 by $L_\pi = \pi/2\kappa$, and L_0 is the offset length that takes into account the coupling that occurs before the coupler section. The Eq. 3.17 can be transformed into linear function

$$\cos^{-1}(r) = \left(\frac{\pi}{2L_\pi}\right)L_C + \left(\frac{\pi L_0}{2L_\pi}\right), \quad (3.18)$$

where the slope and offset will determine L_π and L_0 respectively. The linear regression is

shown in the right panel of Fig. 3.16(b) and is performed for the wavelength around 1525nm. The linear trend clearly shows the agreement with the theoretical model in Eq. 3.17. It is found that $L_\pi \sim 19.25\mu\text{m}$ and $L_0 \sim 2.43\mu\text{m}$.

Fig. 3.16. (a) The fitting parameters for Drop transmission at a specific wavelength, (b) the plot of coupling coefficients vs. coupler length, and (c) the Losses in racetrack resonator.

The offset length L_0 can be explained in the perspective of the coupled-mode theory, where the effective coupling strength for a point-coupler is the summation of the coupling across half the cavity length $\int \kappa(\theta) R d\theta \equiv \kappa L_0$. Thus, larger radius will give longer effective length L_0 because of longer interaction between the ring and the waveguide. In this experiment however, we only consider the ring with $5\mu\text{m}$ radius and gap separation of 200nm . The measured group index for different racetrack lengths [inset of Fig. 3.16(b)] is relatively constant and only differs slightly with calculated values. The bending loss is extracted by subtracting the total round trip loss (a) with the propagation loss measured in the serpentine structure (α_{WG}) [see Fig. 3.16(c)]. The bending loss is shown to be $<0.01\text{dB/turn}$, which presents a more reasonable measurement of bending loss and in agreement with FDTD calculation performed in chapter 2 (*cf.* Fig. 2.6). This also shows the importance of grating factor G in the fitting. The bending loss seems to increase in the longer coupler length, which is possibly caused by the mode mismatch between the straight and the bend waveguide section. This fact is consistent with that of point coupler ($L_C=0$), where the cavity round trip loss is larger.

3.8 Summary

The one-ring system has been systematically studied from theoretical and experimental aspects. Meaningful parameters such as drop/through transmissions, cavity finesse, group delay, and higher order dispersions are derived and discussed in detail. The device fabrication and experimental setup is briefly outlined. Finally, experimental results show excellent agreement with theoretical prediction of transmission spectrum, coupling coefficients and round trip loss.

Chapter 4. Finesse enhancement in Two-Cavity Systems: Theory and Experiments⁷

High-Q optical resonance is known to have many interesting applications, such as slow light structures, bio-chemical sensing, filters, and ultra-low power all-optical switching. In the above applications, the realization of high Q factor in a small modal volume (V_{mod}) is the key to enhance the responsivity, which is mainly characterized by the resonance finesse ($F \sim Q/V_{\text{mod}}$). The best microring resonators have demonstrated round trip losses less than 1% in various material platforms [1-4], suggesting a best-case finesse of around 100 in a single-ring system. Although the finesse is fundamentally limited by the intrinsic cavity loss, practical aspects seem to be the prime limitation of achieving high finesse. In one-ring systems, the requirement to have a meaningful intensity buildup is that the total coupling should be larger than the cavity loss, or $r < a$ and $r^2 < a$ for 1R1B and 1R2B configurations respectively. Outside this criterion, the intensity buildup factor decreases with the slope inversely proportional to the roundtrip loss. This means in low-loss resonators ($a > 0.99$), such requirements becomes very stringent and a slight detuning of coupling coefficient could significantly decrease the intensity buildup. The controllability of coupling coefficients particularly in the weak coupling region ($r > 0.99$) is challenging due to the sensitive dependence on the gap separation between the ring and the waveguide, which is accompanied with the inevitable fabrication errors.

Fig. 4.1. (a) The two-resonator system and the possible ways of mode excitation: (b) Two-Ring One-Bus system (2R1B), and (c) Two-Ring Two-Buses System (2R2B). Two optical pathways are typically generated in these two configurations: one is resonating in the first ring (P1), while the other is in the second ring (P2).

⁷ The work of this chapter is a combination of several published papers: “Relaxation of critical coupling and characterization of coupling induced frequency shift in two-ring structures,” IEEE J. Sel. Top. Quant. Electron. (accepted for publication), “Finesse Enhancement in silicon-on-insulator two-ring resonator system,” Appl. Phys. Lett. **92**, 101122 (2008), “Experimental Verification of Finesse enhancement scheme in two-ring resonator system,” Proc. SPIE **6996**, 69960B (2008), “The Transmission Properties of one-bus two-ring devices,” IEICE Transactions on Electronics, **E91-C**, 167-172 (2008), and “Proposal of Ultranarrow Passband in two resonator configuration,” IEEE Photon. Technol. Lett. **19**, 1688 (2007).

There is a growing interest in the study of coupled cavity systems consisting only of two resonators for its strong resemblance to atomic systems [5-7]. The spectral characteristics of such system depend on the nature of interactions between the cavity modes and the mode excitation scheme. For example, when the two resonators only interact via two optical waveguides [7], the coherent interaction between two cavity modes is achieved upon adjusting the spacing between resonators and their relative size to each other, which gives the same spectrum as that in electromagnetically induced transparency (EIT) of the atomic systems. In this chapter, novel two-ring structures are explored with the intentionality to generate a resonance with finesse significantly larger than is achievable in one-ring system. The basic structure consists of two resonators mutually coupled to each other, and the cavity modes of these resonators are excited from the first ring, by either one or two waveguides (see Fig. 4.1). It is shown in this chapter that the finesse enhancement is achieved when the second ring is twice larger than the first ring and correlated with the intensity buildup inside the second ring.

4.1 Resonance properties of uncoupled two-ring system

In the bare two-ring structure shown in Fig. 4.1(a), as is well known in coupled cavity systems, the cavity modes evanescently couple to each other and give rise to resonance splitting. The fields inside two-ring systems are related by

$$\begin{pmatrix} E_{12} \\ E_{21} \end{pmatrix} = \begin{pmatrix} r & it \\ it & r \end{pmatrix} \begin{pmatrix} E_{11} \\ E_{22} \end{pmatrix},$$

where r and t are, analogous to semi-reflecting mirrors, the reflection and transmission coupling coefficients and $|r|^2 + |t|^2 = 1$. In the lossless and steady-state situation, the fields are continuous at the interface of the two rings, and based on previous discussions on 1R1B, the fields are related by

$$\tau_{nm} = \frac{E_{m2}}{E_{m1}} = \frac{r - \exp(-i\delta_n)}{1 - r \exp(-i\delta_n)}, \quad \frac{E_{m1}}{E_{m2}} = \exp(-i\delta_m). \quad (4.1)$$

The τ_{nm} is the effective transmittance that is induced by ring n on ring m , and $\delta_m = \omega n_{\text{eff}} L_m / c \equiv \omega T_m$ is the round-trip phase of ring m , where T_m and L_m are respectively the round-trip time and cavity length of ring m . By solving the set of equations in Eq. (4.1), we can derive the characteristic equation for arbitrary ring sizes:

$$\cos[\tfrac{1}{2}(\gamma+1)\delta_1] = r \cos[\tfrac{1}{2}(\gamma-1)\delta_1], \quad (4.2)$$

where $\gamma=\delta_2/\delta_1$ describes the ratio between the round-trip phase of Ring 2 relative to Ring 1. We are interested in the case where γ is varied from 1 to 2. When the rings are identical ($\gamma=1$), the resonances split symmetrically around the resonance of isolated ring 1 ($\omega_1/\Delta\omega_{\text{FSR1}}=\delta_1/2\pi=q$, where $q=\text{integer}$):

$$\delta_1^\pm = \pm \cos^{-1}(r) = \pm \sin^{-1}(t). \quad (4.3)$$

The stronger coupling between the rings (decreasing r), the wider is the splitting. On the other hand, when the Ring 2 is twice larger than Ring 1 ($\gamma=2$) there are three solutions to Eq. (4.2), two of which correspond to the resonance splitting

$$\delta_1^\pm = \pm \cos^{-1}[(1+r)/2]. \quad (4.4)$$

Note that the splitting for $\gamma=2$ is smaller than that for $\gamma=1$. The other solution of Eq. (4.2) corresponds to the narrow resonance located at the anti-resonance of the isolated ring 1 [$\omega_1/\Delta\omega_{\text{FSR1}}=(q+1)/2$]. These resonances are excited when the two-ring system is side-coupled in the configurations shown in Fig. 4.1(b) and Fig. 4.1(c), where different sets of optical pathways are generated. It should be noted that the modes of the optical pathways are also different in each configuration. In Fig. 4(b), where the two-ring system is side-coupled with one optical waveguide, two indirect pathways (i.e., P1 and P2) are generated. On the other hand, when only the lower ring is side-coupled with two optical waveguides [Fig. 4(c)], both direct (P1) and indirect (P2) pathways are generated. As will be shown later, the interference condition between these sets of optical pathways is the crucial factor that determines different resonance lineshapes.

4.2 Two-Ring One-Bus system (2R1B)

In the first two-ring configuration shown in Fig. 4.2, only the lower ring has access to the optical waveguide. Using the existing formulation for the one-ring system, the Through transmission (T) can be easily deduced by

$$T = \left| \frac{E_T}{E_{IN}} \right|^2 = \left| \frac{r_1 - a_1 \tau_{21} \exp(-i\delta_1)}{1 - a_1 r_1 \tau_{21} \exp(-i\delta_1)} \right|^2. \quad (4.5)$$

The $\tau_{21} = [r_2 - a_2 \exp(-i\delta_2)]/[1 - a_2 r_2 \exp(-i\delta_2)]$ is the loading factor imposed by Ring 2 on Ring 1, similar to the bare two-ring but with the presence of round trip loss

$a_m = \exp(-\alpha L_m / 2)$, where α is the total absorption coefficient. The r_1 and r_2 denote the coupling coefficient between ring 1 and the waveguide and between the rings respectively. By rewriting the loading factor of the form $|\tau_{21}| \exp(i\theta_{21})$, and defining the effective cavity loss in Ring 1 as $a = a_1 |\tau_{21}|$, the transmission can be reduced to the one-ring form

$$T = \left| \frac{r_1 - a \exp(-i\delta)}{1 - ar_1 \exp(-i\delta)} \right|^2, \quad (4.6)$$

where $\delta = \delta_1 - \theta_{21}$ is the phase difference between the two optical pathways illustrated in the right panel of Fig. 4.2,

$$\delta = \delta_1 - \left[\tan^{-1} \left(\frac{a_2 \sin \delta_2}{r_2 - a_2 \cos \delta_2} \right) - \tan^{-1} \left(\frac{a_2 r_2 \sin \delta_2}{1 - a_2 r_2 \cos \delta_2} \right) \right]. \quad (4.7)$$

The first term is the pathway that resonates in Ring 1 (P1) while the last two terms ($=\theta_{21}$) is the pathway that resonates in Ring 2 (P2). If P1 and P2 interfere destructively ($\delta/\pi = \text{odd}$), the light is not confined inside the two-ring system and thus gives maximum transmission. When P1 and P2 interfere constructively ($\delta/\pi = \text{even}$), the light is localized inside the two-ring system and gives minimum transmission. The location of such interferences depends on the relative size of both rings with respect to each other (γ).

Fig. 4.2. The schematic of 2R1B.

Fig. 4.3(a) shows the transmission spectra for different values of γ . When the rings are identical, there is an even resonance splitting, as expected from Eq. (4.3), with equal intensity distribution in both rings. Light intensity distribution can be deduced using the relative intensity buildup B_{21} , defined as the light intensity in Ring 2 relative to light intensity in Ring 1 for the lossless case,

$$B_{21} = \frac{1 - r_2^2}{1 + r_2^2 - 2r_2 \cos \delta_2}. \quad (4.8)$$

By inserting $\delta_1^{(\pm)} = \pm \cos^{-1}(r_2)$ into Eq. (4.8), it can be shown that $B_{21}(\delta_1^{\pm}) = 1$. This is in agreement with the finite difference time domain calculation (FDTD) shown in the first two panels of Fig. 4.3(b), where the field amplitudes in both rings are the same. The symmetric splitting corresponds to the symmetric (S) and anti-symmetric (AS) fields with respect to the coupling point.

When γ is not integer, the resonances of both rings do not coincide and split in uneven manner. The narrower split-resonance corresponds to a situation where most of the light is localized in the ring that is not coupled to external waveguide (Ring 2). The situation is inverted in the broader split-resonance, where most of the light is localized in the ring that is coupled to the external waveguide (Ring1). As the resonances of the isolated rings are farther apart from each other, the transmission at the narrower split-resonance becomes progressively lower because the light circulates longer inside the system and in turn enhances the effective cavity loss. For convenience, these resonances may be addressed based on the optical pathway that dominates the system. In the narrow resonance, the path P2 dominates and thus can be addressed as P2-resonance. Similarly, the broader split-resonance where most light localization is in Ring 1, can be addressed as P1-resonance.

Fig. 4.3. (a) The transmission spectra for γ values from 1 to 2, the inset shows the broad and split-resonances near the resonance of the bare rings. (b) The field distribution for $\gamma = 1$ and $\gamma = 2$ as calculated using FDTD. Note that the two symmetric resonances have a symmetric (S) and an anti-symmetric (AS) transverse field profile at the coupling point. The NR stands for narrow resonance for $\gamma = 2$.

When $\gamma=2$, there is again an even resonance splitting, similar to $\gamma=1$ case, but with the circulating light in Ring 2 about two times smaller than that in the Ring 1, assuming weakly coupled rings. This can be easily verified by $B_{21}(\delta_1^\pm) = (1+r_2)^{-1} > 1/2$ and FDTD calculations in the second two panels of Fig. 4.3(b), showing the same symmetric (S) and anti-symmetric (AS) modes with respect to the coupling point. The symmetric and very narrow P2-resonance (NR) that is situated at the anti-resonance of Ring 1 ($\delta_1/\pi=\text{odd}$) is the third root of the characteristic equation for $\gamma=2$. Inserting $\delta_1=\pi$ ($\delta_2=2\pi$) to Eq. (4.8) gives $B_{21} = (1+r_2)/(1-r_2)$, which shows that almost all the light is confined inside Ring 2. This corresponds to the highest cavity finesse that can be obtained in the system. The effective cavity loss is also enhanced along with the finesse, and this could ease the critical coupling requirement for high contrast transmission that is generally difficult to obtain in the one-ring system.

Fig. 4.4. The round-trip paths in one-ring and two-ring systems. T_1 and T_2 are the roundtrip time for individual Ring 1 and Ring 2 respectively.

Fig.4.4 compares the round-trip paths for one-ring and two-ring systems. In the one-ring system, the amount of round-trips is equal to the intensity buildup factor $T_g = d\theta/d\delta \equiv B_1$ and the round-trip path is represented as a circular line. In the two-ring system, the round-trip path resembles 8-like shape where the amount of round-trips in Ring 2 is B_{21} times of that in Ring 1. Thus, it can be said that the light statistically returns to its original position after circulating $(1+B_{21})$ times, with one loop in Ring 1 and B_{21} loops in Ring 2. In total, by substituting $T_2=\gamma T_1$, where T_1 and T_2 are the roundtrip time for individual Ring 1 and Ring 2 respectively, the total round-trip time for the two-ring system is $(1+\gamma B_{21})T_1$ and the cavity finesse is enhanced by a factor of $(1+\gamma B_{21})$. The fraction of time spent inside Ring 2 can be easily deduced by $\eta=\gamma B_{21}/(1+\gamma B_{21})$. In $\gamma=1$ and $\gamma=2$, it can be shown that $\eta\sim 50\%$, meaning that there is an equal energy buildup in both cases. In the case of narrow resonance (NR),

$\eta \sim 100\%$, indicating that the light is almost isolated from external waveguide, and thus increasing the loss. These three cases agree well with the FDTD calculated fields shown in Fig. 4.3.

4.3 Finesse enhancement in 2R1B: relaxation of critical coupling condition

In one-ring system, critical coupling condition is satisfied when the light coupled into the cavity is exactly absorbed within one round trip ($r=a$) [8]. However both are strongly dependent of fabrication condition and generally the round trip loss can be only known after device fabrication. This becomes a problem in the low loss ring, particularly when the roundtrip loss is less than 1% ($a > 0.99$), as the coupling coefficient becomes very sensitive to the coupler length and the waveguide-ring gap separation. As illustrated in Fig. 4.5(a), the transmission is plotted as a function of coupling (r) and round-trip loss (a), with the region within -10dB (contrast of $>10\text{dB}$) denoting the range of near critical coupling condition. When the loss is moderate ($a < 0.99$), a contrast of 10dB is still possible with racetrack coupling. However, when round trip loss becomes low ($a > 0.99$), as in the fabricated rings demonstrated to date, the adjustment in gap separation is necessary and slight detuning from the requirement may incur large changes in the contrast.

Although near critically coupled rings have been experimentally demonstrated [9-11], in practice it is still difficult to match these two parameters, as is reflected in many attempts to achieve critically coupled ring in various ways, for example, in integrating semiconductor optical amplifier (SOA) into the ring [12], post-fabrication trimming in polymer ring resonator [13], and erbium doped fiber amplifier (EDFA) in fiber ring [14].

Fig. 4.5. Transmission contour of (a) one-ring and (b) two-ring systems.

Fig. 4.6. The critical coupling condition for different values of r_1 as a function of the inter-ring coupling coefficient r . The inset shows the field distribution of the narrow resonance

In 2R1B, these requirements can be made less stringent by resonant enhancement of cavity round trip loss in the P2-resonance of $\gamma=2$ case. In Fig. 4.5(b), we show the transmission for two low loss situations, i.e., $a_1=0.99$ ($\alpha L_1=10^{-2}$) and $a_1=0.998$ ($\alpha L_1=2 \times 10^{-3}$), as a function of waveguide-ring (r_1) and inter-resonator (r_2) coupling coefficients. It is evident that for both situations, contrast of $>10\text{dB}$ can be obtained in a broader range of coupling coefficients. The main advantage of the two-ring system is that critical coupling for low loss ring can be achieved in stronger coupling regime. Fig. 4.6 illustrates the slice of the contour in Fig. 4.5(b) for 4 different values of r_1 .

4.4 Two-Ring Two-Bus system (2R2B)

In the other two-ring system shown in Fig. 4.7, the lower ring is side-coupled to two external waveguides while the upper ring is only mutually coupled to the lower ring. Using the same one-ring analogy as in 2R1B, the Through (T) and Drop (D) transmission can be shown as

$$T = \left| \frac{E_T}{E_{IN}} \right|^2 = r_1^2 \left| \frac{1 - a_1 \tau_{21} \exp(-i\delta_1)}{1 - a_1 r_1^2 \tau_{21} \exp(-i\delta_1)} \right|^2, \quad (4.11)$$

$$D = \left| \frac{E_D}{E_{IN}} \right|^2 = \left| \frac{t_1^2 \sqrt{a_1} \tau_{21} \exp(-i\delta_1/2)}{1 - a_1 r_1^2 \tau_{21} \exp(-i\delta_1)} \right|^2.$$

Then, by following the same steps, Eq. (4.11) can be reduced to the one-ring form with the same definition of pathways phase difference δ and effective round trip loss a .

$$T = r_1^2 \left| \frac{1 - a \exp(-i\delta)}{1 - ar_1^2 \exp(-i\delta)} \right|^2, \quad D = a_1^{-1} \left| \frac{at_1^2}{1 - ar_1^2 \exp(-i\delta)} \right|^2. \quad (4.12)$$

The transmission properties of the 2R2B configuration for different γ values are shown in Fig. 4.8. In $\gamma=1$, there is an even resonance splitting with symmetric (S) and anti-symmetric (AS) field profiles shown in insets 3 and 4. When the resonance of both rings are detuned from each other ($\gamma \neq \text{integer}$), there is an uneven resonance splitting with the narrow and asymmetric resonance associated with light localization in the upper ring (P2-resonance), and the broader resonance associated with light localization in the lower ring (P1-resonance).

Fig. 4.7. The schematic of the 2R2B.

Fig. 4.8. (a) (Upper) The pathways phase difference δ for different values of γ with (Lower) their corresponding through spectra. The 2 by 2 panels show the corresponding FDTD calculations for the labeled resonances. (b) The Drop spectrum for $\gamma=2$, $r_{1,2}=0.95$ as a function of normalized detuning of Ring 1. The inset shows the corresponding field profile at the narrow resonance.

The light intensity distribution inside the two rings for each resonance follows the same relationship as in Eq. 4.8. When the resonances are more detuned from each other, the P2-resonance becomes narrower and more symmetrical and at $\gamma=2$ (or generally at even values), the P2-resonance is perfectly symmetrical and has the highest cavity finesse. In this situation, the light in the upper ring is completely trapped as both rings are anti-resonant with each other. This can be called narrow resonance (NR), which is similar in 2R1B case. This is shown in the Drop spectrum in Fig. 4.8(b), where the P2-resonance is situated at the anti-resonance of Ring 1.

The transmission pattern of 2R2B is almost similar to the 2R1B case. The only difference is that the P2-resonance in 2R2B case is generally asymmetric whereas the P2-resonance in 2R1B is symmetric. This is attributed to the fact that the transmission contrast in 2R1B case depends strongly on the critical coupling situation. In 2R1B, the cavity lifetime at the P2-resonance is generally longer, because the light needs to resonate in the lower ring before exiting the structure. This increases the effective cavity loss so as to enhance the transmission contrast. At the P1-resonance, however, the cavity lifetime is comparably short and the contrast is not that high. Therefore the envelope effect of P1-resonance is almost negligible on P2-resonance.

Fig. 4.9. (a) The envelope effect and asymmetry in 2R2B system. (b) Different asymmetricities are obtained in different resonance broadening of Ring 1.

In 2R2B system, there are two optical waveguides coupled to the lower ring. This gives transmission contrast almost independent of coupling strength. Thus, unlike in 2R1B, both the P1- and P2-resonances contribute equally in shaping the transmission spectrum and the envelope effect of one pathway resonance can be felt on the other. The envelope effect is provided by the P1-resonance and is determined by the broadening and the detuning from P2-

resonance, as illustrated in Fig. 4.9(a). Here, R1 and R2 denote the resonance of the isolated Ring 1 and Ring 2 respectively, while P1 and P2 denote the P1- and P2-resonances. The origin of asymmetry can be explained from the one-ring system resonantly “loaded” by Ring 2. At the R2-resonance, the light gains π phase shift as it couples from Ring 1 to Ring 2 and back to Ring 1, causing destructive pathway interference and a minimum in Drop spectrum. In the slight detuning from R2-resonance, the interference condition changes rapidly from destructive to constructive and produces maximum Drop transmission.

The sharpness of this transition depends on the relative intensity buildup factor of the two rings (B_{21}), and controlled by the coupling coefficients between the rings (r_2). As it moves away from R2-resonance, the loading effect of R2 becomes negligible and the system reduces to one-ring system with characteristics of Ring 1. In this situation, the transmission follows the transmission of Ring 1. The slight shift in the P1 and P2-resonance from the R1- and R2-resonances is because of the modification of the round-trip path from circular to 8-like. The asymmetry can be tuned accordingly, by adjusting the linewidth of the R1-resonance, as shown in Fig. 4.9(b). Such asymmetric lineshape could be used to overcome the limitation in bistable switching based on symmetric lineshape as in the case of one-ring system [15].

4.5 Finesse enhancement in 2R2B

The major performance characteristics are the finesse and the maximum drop efficiency (D_{MAX}). The finesse is independent of the ring size and depends only on the intrinsic and coupling losses. In the one-ring system (1R2B), high finesse and high D_{MAX} are achieved when both coupling strength and the round trip loss are small, with the former is several times larger than the former [4]. It has been pointed out that the cavity finesse is more limited to practical concerns rather than the fundamental limit of finesse itself. In the 2R2B case, it is possible to realize high finesse resonance in the strong coupling regime, which in principle is less affected by fabrication imperfections. Similar to the 2R1B case, the maximum obtainable finesse is achieved when $\gamma=2$ and the narrow and symmetric resonance (NR) is situated at the anti-resonances of the lower ring. The finesse enhancement in the lossless condition can be estimated by high-finesse approximation of the effective round-trip phase δ in Eq. (4.7) in the vicinity of NR

$$\delta \cong (1 + \gamma B_{21}) \delta_1, \quad (4.13)$$

where $B_{21} = (1 + r_2)/(1 - r_2)$ is the relative intensity buildup at $\delta_2=0$ ($\delta_1=\pi$). The $(1 + \gamma B_{21})$

signifies the scaling factor of round-trip time in the 8-like round-trip path that scales up the finesse by the same factor. Note that this is consistent with what has been intuitively deduced in Section 4.2, where statistically the light would go for B_{21} loops in Ring 2 before returning to Ring 1. Since the Ring 2 is γ times larger than Ring 1, the amount of time the light spends in Ring 2 before returning to Ring 1 is $\sim \gamma B_{21} T_1$, thereby adding up to $(1 + \gamma B_{21}) T_1$ as the total round-trip of the two-ring system. In the presence of loss, the finesse is calculated by numerical root finding $F = 2\pi / \Delta\delta_1(a_1, r_1, r_2)$, and the linewidth $\Delta\delta_1$ is a nonlinear function of coupling coefficients (r_1, r_2) and the round-trip losses (a_1, a_2) .

Fig. 4.10. The finesse- D_{MAX} contour plot of (a) 2R2B ($r_2=0.85$) and (b) 1R2B as a function of coupling coefficients and round trip loss.

The contour plot of the Finesse and the D_{MAX} as a function of round-trip loss and coupling coefficients for one-ring and two-ring systems are shown in Fig. 4.10(a) and Fig. 4.10(b) respectively. Note that there is generally a tradeoff between D_{MAX} and F [4]. The contour plots show the combination of a_1 and r_1 (while r_2 is fixed at 0.85) required to give a desired combination of D_{MAX} and F . The maximum r_1 is chosen to be the highest experimentally demonstrated in [9], while the feasible ranges of $(1 - a_1)$ based on real materials are indicated on the top bars. The lowest demonstrated round-trip loss in silicon-on-insulator (SOI) is 1.7-2.4dB/cm [16, 17], for which $a_1 \sim 0.999$ for 5 μ m ring. The finesse of $F \sim 500$ -2,000 and drop efficiency $D_{MAX} \sim 0.2$ -0.6 (the exact value depending on r_1) may be achieved

using the two-ring structure, whereas the corresponding values for the one-ring structure are $F \sim 20-600$ and $D_{MAX} \sim 0.4-1$.

In general, much larger finesse (albeit with a smaller D_{MAX}) can be achieved by the two-ring structure. To achieve a comparable finesse as the two-ring device, the one-ring device would need a much lower loss and much weaker coupling ($r_1 \sim 0.9999$) which is not practically feasible. In the lossless limit, the finesse of 2R2B is enhanced by $\gamma B_{21} = 2(1+r_2)/(1-r_2) = 24$ for $r_2 = 0.85$, and even more if r_2 is set higher (but D_{MAX} would be less, or else the loss would have to be much lower). In principle, D_{MAX} could be increased by using different coupling coefficients at the two waveguides to offset the loss-induced imbalance, but this is not easy to achieve in practice.

4.6 Fabricated two-ring devices

The 2R1B and 2R2B devices are shown in Fig. 4.11. Here, the coupling coefficients (r_1, r_2) are defined by the racetrack coupler with length L_{C1} and L_{C2} . The $\gamma = L_{cav2}/L_{cav1}$ is the relative size of cavity length of one ring relative to the other and extension parameter L_{C3} is used to match the desired values of γ . The fabricated two-ring devices are aimed for finesse-enhancement situation. In the 2R1B shown in Fig. 4.11 (a), more relaxed critical coupling condition in the moderate coupled system is to be demonstrated at $\gamma = 2$. The radius of Ring 1 is kept at $5\mu\text{m}$. Using the extracted parameters from 1R2B devices, the L_{C1} and L_{C2} are chosen so that (r_1, r_2) is situated within the range of critical coupling region. The measured transmissions are shown in Fig. 4.12. The L_{C1} is fixed at $5.5\mu\text{m}$ ($\sim 36\%$ power coupling), while the L_{C2} is varied from $2\mu\text{m}$ ($\sim 12\%$ power coupling) to $3\mu\text{m}$ ($\sim 18\%$ power coupling).

Fig.4.11. The fabricated (a) 2R1B and (b) 2R2B devices for finesse enhancement schemes.

Fig. 4.12. The measured transmission of the 2R1B structures. The inter-resonator coupler length is varied from $2\mu\text{m}$ to $3\mu\text{m}$.

Similar to theoretical calculation in Fig. 4.3, the high contrast narrow resonance is situated in between two low contrast split-resonances. The transmission contrast of more than 12dB is observed within $1\mu\text{m}$ difference in the coupling length. This shows a remarkable relaxation of the critical coupling condition for such a low loss resonators of $\sim 0.5\%$ ($a \sim 0.995$) round trip loss, which in one-ring system, would require 0.5% power coupling which is very difficult to achieve in practice. The total Q factor of these narrow resonances ranges from $\sim 18,000$ to $\sim 22,000$, while the intrinsic Q factor is measured to be $\sim 74,000$ from the fitted roundtrip loss and the resonance order. In terms of finesse, the narrow resonances have the finesse of ~ 186 with resonance order of ~ 118.5 (for $R=5\mu\text{m}$ and $L_{C1}=5.5\mu\text{m}$).

Fig. 4.13. The measured transmission of 1R1B with varying round-trip loss and coupling factors.

The finesse of the Ring 1 is ~ 13.6 , while the finesse of Ring 2 is ~ 40.8 (for $L_{C2}=2\mu\text{m}$) and ~ 28.1 (for $L_{C2}=3\mu\text{m}$). It should be noted that $F_{NR} > (F_{R1} + \gamma F_{R2})$, indicating that the finesse enhancement in the narrow resonance (NR) is larger than the total cavity finesse of the two individual rings $F_{R1} + \gamma F_{R2}$ (here γ is incorporated to scale the finesse of Ring 2 with respect to Ring 1). From interaction between cavity modes, the light gains additional finesse of ~ 90 (equivalent to ~ 30 roundtrips). To illustrate the stringent requirement in the one-ring system, 1R1B devices are fabricated with different coupling lengths and radius (Fig. 4.13). It can be seen that despite various round-trip losses (by varying the radius) or coupling factors, the 1R1B still has a relatively low contrast. This illustrates how difficult critical coupling condition is satisfied in low-loss one-ring systems.

Fig. 4.14. The measured Through transmissions of 2R2B, of $R_1=15\mu\text{m}$, $L_{C1}=5.3\mu\text{m}$, $L_{C2}=3\mu\text{m}$, and combinations of R_2 and L_{C3} to realize γ values of 1, 1.05, 1.5, and 2.

The fabricated 2R2B structures are shown in Fig. 4.11(b), where the round-trip loss (a_1 , a_2), coupling factors (r_1 , r_2) and γ are varied. There are four sets of 2R2B structures and each set comprises of 2R2B of different γ values from 1 to 2 and a fixed inter-resonator coupling at $L_{C2}=3\mu\text{m}$. In the first set (S1), the radius of Ring 1 is set to $R_1=15\mu\text{m}$, and the waveguide-Ring1 coupler is at $L_{C1}=5.3\mu\text{m}$. The Through transmissions are shown in Fig. 4.14. Theoretically for $\gamma=1$, there should be an even resonance splitting corresponding to equal light localization in both rings. However, measurement shows that the two resonances are not evenly split, suggesting that γ has deviated from designed value. Such deviation may be

attributed to fabrication variations and to the second order effects of coupling, i.e., coupling-induced resonance frequency shift (CIFS) [18]. The latter is more important because it is intrinsic mechanism that shifts the resonance frequency of the coupled cavity, and thus makes the two resonators non-identical.

Fig. 4.15. The curve-fitting of the Through and Drop transmission in the vicinity of resonance and anti-resonance of Ring 1. The slight asymmetry is caused by coupling induced resonance frequency shift (CIFS)

Simulation shows that a slight deviation of γ of less than 1% can shift one resonance from the other by much larger than FWHM linewidth, because of Vernier effect. The resonance shift becomes nonlinear particularly in large resonance orders (>300 for $15\mu\text{m}$ ring). In increasing γ , there are broad and narrow asymmetric resonances. The asymmetric feature depends on the overlap of the narrow resonance and the broader resonance. For example in the case of $\gamma=1.5$, one can see the highly asymmetric resonance around 1563nm and 1567nm . The $\gamma=2$ is shown in the last panel, where a very narrow resonance (NR) is seen roughly in the middle between two broad, split resonances for every resonance order. Note that all these measurements agree well with the theoretical results in Fig. 4.8, suggesting the accuracy of such a simple theoretical model described in this paper. Note that the NR does not necessarily occur at every even values of γ (as in $\gamma=2$ for the simplest case). The general condition for NR is that the Ring 1 is at anti-resonance and Ring 2 is at resonance, and this would suggest a more general condition of $\gamma=2p/(2q+1)$ ($p, q=\text{integer}$), even though such condition is not satisfied at every resonance order. This may explain the rather similar pattern near 1570nm in

$\gamma \sim 1.5$ case, where the $\gamma = 2p/(2q+1)$ is satisfied at a particular wavelength. This is possible because of slight deviation of γ that matches the general condition above.

The measured through (T) and drop (D) transmissions for $\gamma=2$ are fitted with theory (Fig. 4.15), where the general good fit is evident, and the best-fit parameters are given by $a_1 \sim 0.99$, $r_1 \sim 0.809$, $r_2 \sim 0.92$, and $\gamma \sim 2.0004$. The best fitted r_1 and r_2 are consistent with those of 1R2B, when the corresponding L_{C1} and L_{C2} are inserted into Eq. (3.17). The $\sim 1\%$ cavity round trip loss is verified independently by the drop transmission of a single ring of the same diameter coupled to two waveguide buses fabricated in the same sample, which shows no significant difference in the loss parameter. Finally, there is a $\sim 0.2\%$ deviation from $\gamma=2$ which results in a shift of the narrow resonance by about 5% of the free spectral range (FSR). Such sensitivity is due to the large resonance order of (>300) for Ring 1 of $15\mu\text{m}$ radius.

Fig. 4.16. Fitting of 2R2B of $R_1=5\mu\text{m}$, $L_{C1}=6\mu\text{m}$, $L_{C2}=3\mu\text{m}$, and $\gamma=2$. The slight asymmetry is caused by coupling induced resonance frequency shift (CIFS)

The fitting procedure is the same as in 1R2B case, the only difference is that the initial guess of the inter-resonator coupling is obtained from theoretical resonance splitting $\Omega = 2 \cos^{-1}[\frac{1}{2}(1+r_2)]$ for the case of $\gamma=2$. The measured finesse for the two-ring structure is ~ 94 , about 14 times larger than the single-ring value [19]. From the “drop” spectrum in Fig. 4.15, the resonance amplitude D_{MAX} is about 0.12. D_{MAX} and F can still be improved by decreasing the round trip loss. In the last three sets (S2, S3, and S4), we fabricated the 2R2B with radius of Ring 1 $R_1=5\mu\text{m}$. With the same fabrication condition, the loss should decrease

proportionally with the device size. In this sample, the racetrack length of Ring 1 L_{C1} is varied from $4\mu\text{m}$ to $8\mu\text{m}$, whereas the racetrack of Ring 2 is the same as the previous sample ($L_{C2}=3\mu\text{m}$). Fig. 4.16 shows the curve-fitting of the two-ring devices of $L_{C1}=6\mu\text{m}$ and $\gamma=2$, it is quite evident that the amplitude has increased from ~ 0.12 to ~ 0.2 (cf. Fig. 4.15), and the round trip loss improves from 1% to 0.6% ($a_1 \sim 0.994$) while the deviation $\Delta\gamma/\gamma$ increases from 0.02% to 0.05%, as expected from the same shift of resonance wavelength due to CIFS, or from the same fabrication variation.

The NR properties of 2R2B of L_{C1} of 4, 6, and $8\mu\text{m}$ are shown in Fig. 4.17. It is apparent that as the finesse is higher (the linewidth of NR is narrower), the contrast also increases. This is because of the less envelope-effect felt by the P2-resonance in the weaker coupling (smaller L_{C1}). However, such an increase in the contrast is at the expense of a decrease in resonance amplitude D_{MAX} , because the light in Ring 2 becomes more isolated. Thus, there is a tradeoff between the resonance amplitude and the contrast. The fitted round trip loss for the case of $L_{C1}=6\mu\text{m}$ and $L_{C1}=4\mu\text{m}$ are 0.994 and 0.995 respectively. Fig. 4.18 shows the calculated Finesse- D_{MAX} contours as a function of r_1 and a_1 for a fixed r_2 ($r_2=0.92$), with rectangular dots indicating 2R2B devices considered in this work.

Fig. 4.17. The illustration of the trade-off between the D_{MAX} and the contrast in the decreasing L_{C1} (increasing r_1). The fitting parameters for $L_{C1}=6\mu\text{m}$ and $L_{C1}=4\mu\text{m}$ are ($a_1 \sim 0.994$, $r_1 \sim 0.8$, $r_2 \sim 0.92$, $\Delta\gamma/\gamma = 0.06\%$) and ($a_1 \sim 0.995$, $r_1 \sim 0.88$, $r_2 \sim 0.92$, $\Delta\gamma/\gamma = 0.05\%$) respectively.

The measured finesse ($F = \lambda_{\text{FSR}}/\Delta\lambda$) is reasonably close to the predicted values ($F = 2\pi/\Delta\delta_1$), except for the device (3) whose finesse is between 160 (from measurement) and 300 (from calculation). Such a large difference is attributed to the fact that the NR resonance is near the

noise floor which has linewidth comparable with the FWHM of the NR , due to the resolution of the optical spectrum analyzer. It should be noted that because all the coupling coefficients and the round trip loss have been repeatedly verified, it is therefore believed that the limited resolution of in the equipment gives spurious broadening to the resonance, and it is possible that the finesse actually reaches 300. The corresponding finesse enhancement of Set 2 to Set 4 is around 20. For all devices, the Q factor can be calculated by multiplying the Finesse with the resonance orders. For device (1) the orders is ~ 300 , which gives $Q \sim 30,000$. For device (2)-(4) the resonance orders is around 100, which gives Q factors of $\sim 15,000$ [for S2 and S4] and $\sim 30,000$ [for S3].

Fig. 4.18. The measured devices performance shown on the $Finesse-D_{MAX}$ contour for the case $r_2 \sim 0.92$. (1) $R_1 = 15\mu m$, $r_1 \sim 0.809$, $a_1 \sim 0.99$, $\Delta\gamma/\gamma = 0.02\%$; (2) $R_1 = 5\mu m$, $r_1 \sim 0.82$, $a_1 \sim 0.993$, $\Delta\gamma/\gamma = 0.05\%$; (3) $R_1 = 5\mu m$, $r_1 \sim 0.88$, $a_1 \sim 0.995$, $\Delta\gamma/\gamma = 0.05\%$; (4) $R_1 = 5\mu m$, $r_1 \sim 0.8$, $a_1 \sim 0.994$, $\Delta\gamma/\gamma = 0.06\%$.

The fact of lossless finesse enhancement $(1 + \gamma B_{21}) = 49$ suggests that the present obtained finesses can still be improved by a factor of two. In our model, the round trip loss a_1 is a lumped parameter of bending loss, scattering loss, and coupling loss. In 2R2B devices, simple comparison between the round trip loss (a_1) of Ring 1 with radius $5\mu m$ (Device S3, $\sim 0.5\%$) and $15\mu m$ (Device S1, $\sim 1\%$) clearly shows that there is additional $\sim 0.2\%$ of loss, especially when the bending loss of $R \geq 5\mu m$ has been shown to be negligible, ($\leq 0.005\text{dB/turn}$) [20]. Therefore, it is possible that the mode-conversion loss is a more dominating factor and this is well supported from the data presented in the fabricated 1R2B in Fig. 3.16(c), where the bending loss seems to be higher in those of longer racetrack length and in point-coupler

($L_C=0$). The mode conversion loss is verified by using the propagation loss obtained from serpentine structures to device S3, giving round trip loss of $\sim 0.35\% \pm 0.11\%$. This suggests average mode conversion loss of $\sim 0.15\%$, in agreement with the earlier estimation. Optimization of such mode conversion can increase the present device performance to the Finesse and D_{MAX} of 400-600 and 0.2 to 0.4 respectively.

4.7 Summary

The simple theoretical framework for the two-ring structure is derived from a simple one-ring system, and is shown to agree well with experimental measurements. Experimentally, the finesse-enhancement have been demonstrated for both 2R1B and 2R2B structures. In 2R1B structure, it is manifested in the less stringent critical coupling condition in low loss resonators, by which a transmission contrast of nearly 10dB is achieved within $1\mu\text{m}$ tolerance of coupling length. This is caused by the enhanced effective loss inside the cavity system in the finesse enhancement situation. Difficulty in achieving near critical coupling situation is also experimentally demonstrated in one-ring system. In the 2R2B structure, finesse enhancements of 14 to 20 times have been measured. An average finesse of 150 can be obtained in two-ring system that has 33% external power coupling ($r_1 \sim 0.8$) and it is believed that finesse as high as 300 can be achieved by the two-ring system with 23% external power coupling ($r_1 \sim 0.88$). Optimization of the fabrication to minimize coupling loss will make the 2R2B useful for practical applications, where cavity finesse of 400-600 and D_{MAX} of 0.2-0.4 is possible.

Chapter 5. Optical buffering scheme based on two-ring resonator system⁸

Optical buffer temporarily holds and stores optical data stream without conversion to electrical format, thus plays an important role in all-optical information processing [1]. Conventionally, optical buffer is manifested by physically lengthen the propagation path by bent optical waveguides, where the amount of delay introduced is proportional with the length of the waveguide. Consequently, the delay of nanoseconds range can be achieved in a very long waveguides, which makes it not a suitable for on-chip integrated optical buffer with a small footprint. The group velocity then needs to be significantly slowed down in order to have much smaller footprint. Slow light schemes have been proposed by the use of optical [2-14] and electronic [15-20] resonances. Optically, the light propagation is delayed by circulating it through many round trips in a resonator, which in general can be of any forms, such as ring resonator, Fabry-Perot etalon, or defect modes in photonic crystals. The delay is extendable in cascade configuration, as in the array of mutually coupled resonators and in the array of side-coupled resonators.

Generally, optical scheme has more advantages compared to the electronic scheme in following aspects. First, the electronic scheme usually is realized in a very low temperature, whereas in the optical scheme, the optical resonance is nearly independent of temperature. Second, the large delay obtained in electronic schemes usually has a bandwidth on the order of MHz, which is very small compared to the required GHz bandwidth. In optical scheme, the bandwidth is inversely scalable with the device dimensions, thus in harmony with the requirement of small footprint. Third is the possibility to optically mimic the electronic schemes. For example, electronically the group velocity is greatly reduced by means of electromagnetically induced transparency (EIT), which is the result of destructive quantum interference between two coherently coupled atomic energy levels [14]. A very large delay, but associated with a very narrow bandwidth, has been demonstrated experimentally at very low temperatures [15]. Such a remarkable phenomenon causes a growing interest in optically mimicking EIT using two coupled resonators [4], [12], whose coupled resonances resemble the two energy levels in an atom, but without the limitation of low temperature which has been a major hurdle for electronic resonance. This method is interesting because it is a relatively simple configuration that can produce a large delay.

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Buffering is not only characterized by the delay alone, but rather by the fact that the delay should be flat over a broad bandwidth with low insertion loss. The delay and the bandwidth are not independent of each other, but constrained by the constant delay-bandwidth product (DBP) as a consequence of causality principle. DBP is a measure of the number of bits that can be stored inside the buffer (N_{ST}). For buffers based on resonators the DBP is typically less than one. For example, in the simplest configuration of an all-pass filter (APF) which consists simply of one ring coupled to one waveguide bus, the delay-bandwidth product is given by

$$N_{ST}^{(APF)} = \tau \Delta f = (1+r)/\pi\sqrt{r} \cong 2/\pi < 1, \quad (5.1)$$

where Δf is the normalized FWHM of the resonance, τ is the maximum delay, and r is the coupling coefficient between the ring and the waveguide. The DBP of the cascade configuration is simply the DBP for single module multiplied by the number of modules N . Note that here FWHM is used only for convenience, and the usable bandwidth is actually smaller than that due to the presence of higher order dispersion reducing the N_{ST} . This is demonstrated in ref. 1, where the APF of 56 side-coupled resonators that should have buffered $\sim 56 \times 2/\pi = 35$ bits, is only able to buffer 10 bits without distortion. This is the evidence that the actual usable bandwidth is less than the FWHM due to the distortion introduced by the higher order dispersion [1].

Another distortion is also caused by the non-flat or rippling transmission spectrum. For example, the N_{ST} in CROW with N rings is shown as $N/2\pi$ [2]. However, recent experimental demonstration shows that a CROW with 100 rings is only able to buffer 1 bit without distortion [1]. This is because while the delay is increased by N -fold, the passband also has N ripples that effectively reduce the operating bandwidth by $1/N$. As a result the N_{ST} remains the same as that in the single ring. The ripple can be smoothened when the loss is high [3] or when the coupling coefficients are apodized [2]. The CROW with 12 rings is experimentally demonstrated to have a flat delay of ~ 110 ps over ~ 17 GHz bandwidth. The flat transmission is obtained because of high round trip loss of 17dB/cm, which corresponds to insertion loss of ~ 30 dB. The associated N_{ST} is 1.87, and it is close to the value of $(12/2\pi) = 1.9$ given by the theoretical estimate. On the other hand, with the apodization, the coupling between the adjacent rings is made weaker than the coupling between the waveguide and the ring. By this approach, the N_{ST} can be made close to the theoretical estimates with low insertion loss. However, the structure cannot be too compact since it takes about 6 resonators to buffer 1 bit.

In this chapter, we propose a novel scheme based on two-ring resonator structure, as shown in Fig. 5.1, that can exhibit theoretical DBP of $4/\pi$ (>1) with more immunity to higher order dispersion and low insertion loss at the same time. We show that one module of the two-ring system has comparable performance to 10-ring CROW, suggesting that our scheme is more compact. Using realistic loss parameters [21-23], one module of the two-ring system gives around 46ps delay with 15GHz bandwidth.

Fig. 5.1. The schematic of two-ring coupled to one waveguide bus. The excited optical pathways are shown in the right inset.

5.1 General transmission characteristics

The general spectral characteristics and its associated field distributions are already discussed in chapter 4. The reduced one-ring form of the two-ring system shown in Fig. 5.1 can be expressed as

$$T \equiv \left| \sqrt{T} \exp(i\theta) \right|^2 = \left| \frac{r_1 - a \exp(-i\delta)}{1 - ar_1 \exp(-i\delta)} \right|^2, \quad (5.2)$$

$$\theta = \tan^{-1} \left(\frac{ar_1 \sin \delta}{1 - ar_1 \cos \delta} \right) - \tan^{-1} \left(\frac{a \sin \delta}{r_1 - a \cos \delta} \right)$$

where T is the transmission, θ is the phase response of the two-ring system,

$$\delta = \delta_1 - \theta_{21} = \delta_1 - \left[\tan^{-1} \left(\frac{a_2 \sin \delta_2}{r_2 - a_2 \cos \delta_2} \right) - \tan^{-1} \left(\frac{a_2 r_2 \sin \delta_2}{1 - a_2 r_2 \cos \delta_2} \right) \right] \quad (5.3)$$

is the modified round trip phase, and $a = a_1 |\tau_{21}|$ is the effective loss. The group delay normalized to roundtrip of Ring 1 (T_1) is $T_g = \left(\frac{d\theta}{d\delta} \right) \left(\frac{d\delta}{d\delta_1} \right)$, and in the lossless case is given by

$$T_g = \left(\frac{d\theta}{d\delta} \right) \left(1 - \gamma \frac{d\theta_{21}}{d\delta_2} \right) = \left(\frac{1 - r_1^2}{1 + r_1^2 - 2r_1 \cos \delta} \right) \left(1 + \gamma \frac{1 - r_2^2}{1 + r_2^2 - 2r_2 \cos \delta_2} \right) \equiv B_1 (1 + B_{21}) \quad (5.4)$$

Note that the delay is proportional to the intensity buildup factor in Ring 1, B_1 , and the buildup factor in ring 2 relative to Ring 1, B_{21} , which are defined in Eq. (5.4).

5.2 The proposed Buffer scheme

Since the objective is to increase DBP, we propose to combine the two symmetrically split resonances to increase the DBP. The splitting and the broadening of each split resonance can be controlled so as to produce both flat transmission and delay spectra. As derived in the previous chapter, the splitting $\Omega_\gamma = \delta_{1,\gamma}^{(+)} - \delta_{1,\gamma}^{(-)}$ for $\gamma=1$ and $\gamma=2$ are

$$\Omega_1 = 2 \cos^{-1}(r_2), \quad \Omega_2 = 2 \cos^{-1}\left[\frac{1}{2}(1+r_2)\right], \quad (5.5)$$

which means that in any case the splitting is mainly controlled by the inter-resonator coupling. In the vicinity of the split resonance, the absorbance $A = 1 - T$, can be expressed in the form $A = (1 - a^2)B_1$, where

$$B_1 = \frac{[\delta_1^2 + (\Delta_2/2\gamma)^2]B_1(0)}{[\delta_1^2 + (\Delta_2/2\gamma)^2] + \frac{4}{\Gamma^2}[\delta_1^2 - (\Omega_\gamma/2)^2]^2} \quad (5.6)$$

is a split-Lorentzian function similar to that given in [13], which is derived using high finesse approximation and some trigonometric identities. The $B_1(0) = (1 - r_1^2)/(1 - a_1 r_1)^2$ is the maximum value of B_1 , $\Delta_m = 2(1 - a_m r_m)/\sqrt{a_m r_m}$ is the resonance linewidth of ring m , $\Gamma = \Delta_1 \sqrt{a_2 r_2}$ is the total decay rate from both resonators that signifies the linewidth of each split resonance, and $\Omega_\gamma = \delta_1^+ - \delta_1^-$ is the resonance splitting which depends on γ . When the finesse of ring 2 is high, we have $a_2 r_2 \sim 1$, which makes Γ mainly dependent on the waveguide-R1 coupling. Thus, the splitting and the broadening can be individually tuned so as to produce flat spectrum.

This is demonstrated in Fig. 5.2(a) which plots the evolution of the absorbance for different splitting-over-broadening ratio (Ω/Γ). When $\Omega > \Gamma$, the splitting dominates and there are two distinct resonances. When $\Omega < \Gamma$, the two resonances merge into one. When $\Omega/\Gamma \sim 0.6$, a broad resonance with minimum loss is obtained. This offers the optimal condition for the buffer and the criterion $\Omega/\Gamma = 0.6$ can be achieved by various combinations of (r_1, r_2) , as shown in Fig. 5.2(b). Note that in general the required value of r_2 is very high, and this makes

the difficulty in realizing this scheme for $\gamma=1$. However, in $\gamma=2$ such condition is relaxed because the splitting $\gamma=2$ case is naturally narrower compared to that in $\gamma=1$. Hence, it may be easier to implement the proposed scheme in $\gamma=2$. Using Eq. (5.4) and (5.6) the delay at the $\delta_1=0$ for the proposed scheme is shown to be

$$T_g \cong [4r_1/(1+r_1)^2](\Gamma/\Omega)^2 B_1(0), \quad (5.7)$$

where the fact that $r_2 \sim 1$ is used for both $\gamma=1$ and $\gamma=2$. Note that the delay is independent of γ . Using the criterion $\Omega/\Gamma \sim 0.6$, the delay for the two-ring system is almost $3B_1(0)$, or equivalent to 3 modules of APF with the same coupling parameters, for which $T_g = B_1(0)$. However, the FWHM bandwidth is somewhat smaller than that of the APF, giving rise to a net increase in DBP about twice. This is confirmed by the phase response.

Fig. 5.2. (a) The absorbance spectrum depends on the ratio Ω/Γ , displays a flat top shape when $\Omega/\Gamma \sim 0.6$. (b) The combinations of (r_1, r_2) required to achieve the criterion $\Omega/\Gamma \sim 0.6$. In all cases, $a_1=0.999$.

Fig. 5.3. (a) Different situations resulting from different values of r_1 or splitting-broadening ratio Ω/Γ . (b) Transparency and delay response for different loss parameters.

Fig. 5.3(a) shows the phase response which corresponds to three different situations according to Ω/Γ values. In the broadening-dominant situation ($r_1=0.65$), the phase converges to 2π swing similar to that in one-ring system. In the splitting-dominant situation ($r_1=0.95$), the phase has two 2π phase swings. And finally in the proposed scheme (dashed line), the phase has 4π swing. This suggests the delay bandwidth product is twice compared to the APF scheme.

$$N_{ST}^{(\text{TWO-RING})} \sim 4/\pi. \quad (5.8)$$

Fig. 5.3(b) shows the dependencies of the transparency and the delay with respect to different round trip loss. While the transparency is progressively lower for increasing round trip loss, the delay remains the same. The transparency is reduced to half with moderate loss of $a_1=0.99$, but much smaller loss of $a_1 \sim 0.999$ has been demonstrated [21-23], suggesting 90% transmission is possible. Finally, we note that the scheme can be extended to involve more rings. In the N rings, the situation is similar to apodized N -ring CROW coupled to one waveguide bus. However, the N cannot be too large as it will create photonic bandgap effect that concentrates high delay at the band edges, as happens in CROW scheme, which reduces the delay and therefore the DBP.

Fig. 5.4. The different signature in the transparency (T), phase response (θ), and delay (T_g) for (a) APF, (b) CROW, (c) EIT, and (d) the proposed scheme based on 2R1B configuration.

5.3 Discussion

In this section, we will compare the proposed scheme with other schemes. Fig. 5.4 summarizes APF, CROW, EIT, and proposed schemes, with their different signature in transparency (T), the phase response (θ), and the delay spectrum (τ). Here, we briefly deduce the DBP for each scheme. In the APF situation [Fig. 5.4(a)], the delay spectrum resembles the intensity buildup and has maximum value of $B_1(0)$. At resonance, the phase swings by 2π where the N_{ST} per module has been shown to be $2/\pi$ [see Eq. (5.1)] and therefore is twice smaller than our proposed scheme. In terms of delay bandwidth product, it appears that there is no difference between the proposed scheme and the cascade of 2 APFs, as the DBP will be the same. This is because the higher order dispersion has not been inserted in the comparison [14]. Fig. 5.5 shows a side-by-side comparison of the signal propagation between a single-ring and a two-ring cascade structures, with the same coupling parameters and for different number of rings in each case. In order to show the inter-symbol interference, we insert 8 return-to-zero (RZ) bits of logic 1 and plot the time domain results with the time normalized to the bit length. Note that the APF starts to show symbol ambiguity when $N > 40$ with ~ 6 buffered bits, whereas the proposed scheme starts to have inter-symbol interference only when there are more than 60 rings in the structure ($N > 30$) with ~ 14 buffered bits. Clearly this shows that the proposed scheme, due to its flatter delay spectrum, is more immune to high-order dispersion and can store twice the number of undistorted bits compared to the APF.

Another possible two-ring APF configuration is a cascade of two APFs with “detuned resonances” to double the 3-dB bandwidth while keeping the same delay. If the resultant spectrum is flat, then this configuration should be equally immune to inter-symbol interference while having the same N_{ST} as the proposed scheme. However, the main challenge for this configuration is the practical difficulty in achieving the desired detuning in a controlled manner. Simulations show that the resonance shift is not linear but rather random for very small size detunings of less than 1% and high resonance order of the order of 100 (corresponding to a ring radius of $> 5 \mu\text{m}$) because of the Vernier effect. In contrast, it is easier to achieve the $\Omega/T=0.6$ condition required for the proposed scheme, as illustrated by Fig. 5.2(b)

In the apodized CROW [see Fig. 5.4(b)], the transmission spectrum is flat at the center of the band which is at the resonance of the individual rings and the delay is also flat with high delay at the band edges. The phase response swings by $N \times K\Lambda$, where N is the number of

rings, K is the Bloch wavenumber, and Λ is the periodicity. The dispersion relation of CROW is [2]

$$\sin(\tfrac{1}{2}\delta) = \sqrt{1-r^2} \cos(K\Lambda), \quad (5.9)$$

where the δ is the roundtrip phase of individual ring and r is the coupling coefficient between the adjacent rings. The usable bandwidth (B) and the corresponding delay (T_g) are

$$B = \frac{1}{\pi} \sin^{-1} \sqrt{1-r^2}, \quad T_g \equiv -N \left. \frac{d(K\Lambda)}{d\delta} \right|_{\delta=0} = \frac{N}{2\sqrt{1-r^2}}. \quad (5.10)$$

Therefore by approximating $\sin^{-1} x \sim x$ in high finesse rings, one can obtain the DBP for CROW as

$$N_{ST}^{(CROW)} = N / 2\pi, \quad (5.11)$$

which is 4 times smaller than that of the proposed scheme. In the EIT scheme [see Fig. 5.4(c)], the full transparency is manifested as the two decaying modes from each ring cancels each other. This coherent interaction is obtained when the lower ring (R1) is under coupled while the upper ring (R2) is over-coupled [13] and the resonance splitting should be very narrow ($\Omega/\Gamma \ll 1$). This leads to condition $a_1 < r_1 < r_2 < a_2$, which suggests that the loss coefficient for each ring has to be different ($a_2 \neq a_1^*$).

Fig. 5.5. Side by side comparison of propagation of bits in different structure lengths. Note that APF suffer more inter-symbol interference as $N > 40$. The time domain responses are obtained from inverse Fourier transform of the output spectrum.

Alternatively, the EIT-like spectrum can be also realized using a different two-ring system [9,10], by side-coupling both rings with two waveguide buses. Using this

configuration, the rings have to be slightly non-identical in order to realize full transparency between the resonances of each ring. In any case, the phase response of both are the same, where at resonance the phase swings by less than π , and the spectrum resembles the combination of two under-coupled responses. Here the under-coupled response of each split resonance is actually similar to the transition between two energy states in atoms, which serves as a direct optical analogy of the electromagnetically induced transparency (EIT) in atomic medium. Note that the delay is negative near the two dips at which the full transparency is situated in between [see the delay spectrum in Fig. 5.4(c)]. Using the DBP of the APF as the baseline, we can deduce the DBP of EIT scheme as half of the APF scheme

$$N_{ST}^{(EIT)} \sim 1/\pi. \quad (5.12)$$

As a reality check, we also numerically calculate the delay-bandwidth product of another EIT configuration based on two rings side-coupled to two waveguide buses [10], which has a FWHM bandwidth of $\sim 0.13\text{nm}$ and a delay of 17.9ps . The N_{ST} is ~ 0.3 , which is close to $1/\pi$ as estimated in Eq. 5.12. Note that here we exclude the possibility for dynamic tuning, which is demonstrated in Ref. [11].

Fig. 5.6. Comparison of transparency and delay, between the proposed scheme with $r_1=0.95$ and $r_2=0.999$ (1), and the 10-ring CROW (2) with $r_{WG}=0.95$ and $r=0.999$, and the EIT schemes with (3) $r_1=0.9$, $r_2=0.999$, $a_1=0.88$ and (4) $r_1=0.96$, $r_2=0.999$, $a_1=0.95$. The second ring for the EIT scheme is assumed lossless, for other cases $a=0.999$. The parameters in (3) are extracted from ref. [12].

Fig. 5.6 shows the comparison between the proposed scheme (1) with the apodized CROW (2) and EIT schemes (3 and 4). It can be readily seen that one module of our simple proposed scheme can exhibit delay comparable with 10-ring CROW and EIT with about the same usable bandwidth (flat region) as the 10-ring CROW. The insertion loss of both

proposed scheme and the CROW are quite similar. The EIT appears to have the highest transparency (lowest insertion loss), but that is because the upper ring (R2) is assumed lossless. When the loss $a_2 = 0.999$ is inserted, the EIT scheme would have transparency around ~ 0.8 (for 3) and ~ 0.9 (for 4). This is not quite different with the insertion loss of the other two. For our proposed scheme, the group delay is $\sim 100T_1$ and the flat region of the delay has a bandwidth of $\Delta\delta_1/2\pi \sim 7 \times 10^{-3}$. For a ring with a radius of $5\mu\text{m}$ and a group effective index of 4.4, these values imply a group delay of $\sim 46\text{ps}$ and a bandwidth of 15GHz . This is a reflection of the fundamental delay-bandwidth product of each scheme.

Table. 5.1. The parameter comparison between CROW, EIT, and the proposed schemes

Scheme	Parameters	Insertion Loss (IL)	Normalized Delay (τ_D/T_1)	Normalized Bandwidth ($\Delta\delta_1$)	N_{ST}	N
CROW	$r_{WG} \sim 0.988$, $r_1 \sim 0.995$, $a \sim 0.93^*$	$> 30\text{dB}$	~ 60	$\sim 3.2 \times 10^{-2}$	~ 1.92	12
	$r_{WG} = 0.95$, $r_1 = 0.999$, $a = 0.999$	2.2dB	~ 270	$\sim 1.3 \times 10^{-2}$	~ 3.51	24
EIT	$r_1 = 0.9$, $r_2 = 0.999$, $a_1 = 0.88$, $a_2 = 0.999$	23dB	~ 2200	$\sim 2.9 \times 10^{-3}$	~ 6.3	24
	$r_1 = 0.96$, $r_2 = 0.999$, $a_1 = 0.95$, $a_2 = 0.999$	11dB	~ 930	$\sim 7.8 \times 10^{-3}$	~ 7.4	24
The proposed Scheme	$r_1 = 0.95$, $r_2 = 0.999$, $a_1 = 0.999$, $a_2 = a_1^2$	8dB	~ 830	$\sim 1.3 \times 10^{-2}$	~ 10.8	8
	$r_1 = 0.85$, $r_2 = 0.99$, $a_1 = 0.999$, $a_2 = a_1^2$	2.9dB	~ 330	$\sim 4.3 \times 10^{-2}$	~ 14	10

* The parameters are the adjustment of Ref. [2]: $|k_{WG}| = 0.15$, $|k_i| = 0.1$, loss = 17dB/cm, $R = 60\mu\text{m}$, and $\pi nR/c \sim 0.9182 \times 10^{-12}$

Except for the proposed scheme, the delay-bandwidth products for buffers involving resonators are typically less than one, thus a cascade configuration is necessary to buffer more than 1 bit. In Table 5.1, the relevant parameters and the performance of three different buffer schemes with N number of modules are summarized, where N is limited to around 10 so that the insertion loss is not excessive. In the first scheme, the apodized CROW configurations ($r_{WG} = 0.95$, $r = 0.999$) can have a flat spectrum with a small insertion loss of $\sim 2\text{dB}$ for 24 modules. The N_{ST} for this case is about 3.51, consistent with the value given by $N_{ST} = N/2\pi$ [2]. Although N_{ST} tends to be lower for CROW, the advantage of CROW is the low higher order dispersion, as long as the operating bandwidth is far from the edges of the CROW transmission band where higher order dispersion is dominant.

In the EIT scheme, a cascade of 4 modules is necessary to buffer 1 bit. It should also be noted that the second ring in reality is not lossless, thus there is insertion loss in the cascade EIT configuration. If $a_2=0.999$, the transparency per module would be ~ 0.8 for $a_1=0.88$ and $r_1=0.9$, which means a cascade of 24 modules would have an insertion loss (IL) of $10\log(0.8)^{24} \sim 23\text{dB}$. Similarly, the case of $a_1=0.95$ and $r_1=0.96$ would correspond to a transparency of ~ 0.9 and an insertion loss of $\sim 11\text{dB}$ for 24 modules. Finally, for our proposed scheme, two cases are presented. In the case with very weak inter-resonator coupling ($r_2=0.999$) we have an insertion loss of 8dB for an 8-module structure, and in the case with stronger inter-resonator coupling ($r_2=0.99$) we have IL = 2.9dB for a 10-module structure. The N_{ST} for both cases are 10.8 and 14. The insertion loss is reasonably low for such a large delay-bandwidth product. Note that the APF scheme is left out in Table 1 because, spectrally, the delay and 3dB bandwidth are of little difference from the 2R1B proposed scheme for the 2-APF-cascade. The only difference between the proposed scheme and the APF lies in the time-domain characteristics, where the proposed scheme exhibits more immunity towards higher-order dispersion. This has been previously discussed and shown in Fig 5.5.

Fig. 5.7. The comparison between the proposed scheme, APF, and CROW in a fixed 4(8) buffered RZ (NRZ) bits. The smaller the required number of modules, the larger is the delay-bandwidth product of the scheme

Finally, we compare the proposed scheme with APF and CROW in their ability to buffer a fixed number of bits. Here, the operating bandwidth is chosen to be smaller than the 3dB bandwidth to avoid inter-symbol interference and the coupling coefficients for each scheme are freely chosen to achieve the same undistorted buffering of 4 RZ bits. As a guideline we

may assume that the usable bandwidth is around half of the FWHM. The undistorted DBP for APF, CROW, EIT, and the proposed scheme then is

$$N_{ST}^{(APF)} \sim 1/\pi, N_{ST}^{(CROW)} = 1/(2\pi), N_{ST}^{(EIT)} \sim 1/(2\pi), N_{ST}^{(2R)} \sim 2/\pi. \quad (16)$$

The time domain results for the APF, CROW, and the proposed scheme are shown in Fig. 5.7. It is seen that in each scheme, different number of modules (N) is required to buffer the same 4 bits, and the ratio of modules, 18:43:78, is in agreement with the ratio of their delay-bandwidth products, 1:2.38:4.33. In other words, both frequency and time domain analyses show that our configuration is 2 and 4 times more compact than the APF and the CROW structures, respectively. Similarly, it should also be 4 times more compact compared with the EIT scheme.

Thus far comparisons are only drawn in theoretical aspects, which hold the general assumption that the devices are ideally fabricated. In reality this is not the case, since imperfections immediately come into play. Such imperfections include the non-uniformity of wafer thickness, dose variation, and ring radius, the proximity effects, and coupling induced non-idealities such as coupling induced resonance frequency shift (CIFS). The fabrication of cascade of coupled micro-ring resonator devices has been shown to be challenging due to these imperfection. Since all the schemes discussed in this chapter involve cascading and coupling of microrings, with the required precisions in the coupling coefficients as well as ring sizes (i.e., identical ring in CROW, $\gamma=2$ in proposed scheme and EIT), it is then apparent that fabrication imperfection equally affects their buffer performances.

For example, in CROW scheme, the rings should be identical with the coupling coefficients adiabatically changed from the strongest (at the edge of the device) to the weakest (at the center of the device). However, the presence of proximity effects and dose variation results in non-identical rings and this adds significant increase in the passband ripples which imparts distortion in the buffered signal. In APF scheme, the ring sizes become non-uniform due to the dose variation along the cascade structures, instead of proximity effects. The deviation of cavity length of $\sim 150\text{nm}$ (or $\sim 20\text{nm}$ in ring diameter) is sufficient to shift the resonance by more than half the free-spectral-range (FSR). In the structures employing two coupled resonators, i.e., EIT and the proposed scheme, the ring sizes and coupling coefficients are greatly affected by proximity effects and CIFS. A deviation in cavity size ratio (γ) of 0.1% is sufficient to distort the spectral characteristics. Such a strict requirement is also true for CROW structure. Thus, in the presence of fabrication imperfections, none of the schemes is more immune than the other. However, it should be

noted that such limitations are not inherent to the buffering schemes, but rather on the limitation of existing fabrication technologies.

5.4 Summary

A buffer scheme has been proposed based on indistinguishable split resonances in a two-ring structure. The delay bandwidth product, which is the fundamental parameter for optical buffer, is shown to be significantly higher than other schemes such as the cascaded side-coupled ring structure (APF), the coupled resonator optical waveguide (CROW), and the optical analog of EIT. Best case comparison for a given fixed number of buffered bits reveals that in terms of the number of cascade, the proposed scheme is 2x more compact than the APF scheme, while 4x more compact than the CROW and EIT schemes. The delay and transparency spectra are reasonably flat and the insertion loss for realistic parameters is quite low, which are all sought-after features in optical buffering. Time-domain simulations of signal propagation through long cascaded modular structures also show superior performance in buffering higher number of bits for the proposed scheme compared with APF and other schemes.

Chapter 6. Transfer matrix formalism and realization of box-like response based on 2D resonator arrays⁹

Ring resonators can be used as building blocks for high density optical circuits, especially for the synthesis of high-order optical filters [1-8]. High order optical filter based on ring resonators is of interest for potential applications in optical signal processing and routing in optical interconnection system. Filters with a box-like transmission spectrum, fast roll-offs, small ripples, and high in-band and out-of-band rejection ratio have been proposed and demonstrated in two types of array geometries. The first configuration [1-3], which we refer to Type 1, is an array of evenly spaced identical resonators indirectly coupled to two waveguide buses (Fig. 6.1). This configuration is identical to Bragg grating with each ring cavity functioning as frequency dependent mirrors. The light is resonantly dropped from one bus to the other and interferes with other dropped light from other rings. The flat top response is realized when the intercavity interference condition is optimized by setting the intercavity spacing to be half of the cavity length (ring circumference) ($2L_B=L_{CAV}$). The limitation of this approach is, while constructive intercavity interference produces flat-top spectrum around resonance, it also produces out-of-band sidelobes in the Drop (D) spectrum. In the Through (T) output, one has the inverse situation in which the response is of high out-of-band rejection ratio but with ripples within the band of interest.

The second configuration [4-8], which we refer to Type II, is shown in Fig. 6.1 (bottom left). This configuration is analogous to cascade of Fabry Perot mirrors. Here each ring interacts only with its adjacent neighbors, and the waveguide buses are only coupled to the first and last rings. The filter of this kind generally has high out-of-band rejection ratio at the expense of high in-band ripple, which is originated from mutual interaction between resonators. A flat-top response may be achieved by symmetric apodization of the coupling coefficients with the strongest coupling at the waveguide-ring section and the weakest at the center of the structure. However, a very tight fabrication control of the waveguide-ring and inter-ring gap separations is required [9, 10], and the ring radii need to be pre-distorted to compensate for the coupling induced resonance frequency shift (CIFS) from the inter-cavity coupling [11, 12]. It should be noted that Apodization has been proposed in Type I geometries as well, for suppressing the sidelobes, however this is obtained at the expense of

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reduced sharpness at the transition edges [2]. Filter shaping can be achieved by applying digital filter design theory [13] to synthesize any specified N th-order filter transfer function, which requires the N resonators to have N different specific values of coupling coefficients [14]. This level of control over the coupling coefficients is not always practical.

Fig. 6.1. Type I, Type II, and the proposed 2D array.

In this chapter, we consider the alternative approach of using a third configuration that is a geometric hybrid of the Type I and Type II configurations. This new configuration is a two-dimensional filter that can be constructed as a periodically coupled array with each column consisting of a higher-order coupled ring filter. The expectation is that by combining the complementary features of the other two, this configuration will give improved performance. We present an *analysis* of the 2-D filter to reveal its response functions, as opposed to the synthesis of a given filter function by calculating the coupling coefficients for each resonator. To our knowledge this is the first analytical study of a 2-D ring resonator array. Periodic structures of coupled resonators have been analyzed by the matrix method [7, 14-16] as well as in the time domain [2] and an analytical approach [3]. The transfer matrix method is most suitable for analyzing such periodic structures. It has been applied to 1D structures, but so far, not to a 2D structure. Here, we develop a uniform transfer matrix approach to analyze the 2D ring resonator array as a superposition of the two 1D arrays. An advantage of this approach is that it can be coupled easily with the Bloch theorem to derive the photonic bandgap (PBG) properties of these periodic structures. It will be shown that the PBG of the 2D array is the superposition of the PBG of the two one-dimensional arrays, which is important in explaining the resulting near-ideal filter characteristics of the two-dimensional array.

Note that in this work no attempt is made to optimize the parameters or analyze the tolerances of the filter. On the contrary, to focus on the transfer matrix analysis we will make

the simplifying assumptions that (i) all the rings are identical and lossless, and (ii) all the coupling coefficients between the bus waveguides and the resonators, and between the resonators themselves, are identical and lossless. These assumptions are not essential to the matrix approach, but are necessary if we are to analyze the system in terms of the photonic bandstructure. We will discuss the effect of loss at the end. The experimental verification of our proposal is also presented. The (3x4) and (3x8) arrays are fabricated in SOI platform and shape factor of 0.7 to 0.8 are measured with a usable bandwidth of 500 to 750GHz.

6.1 Analysis of Arrays of Identical Resonators

In this section we present an analysis of a general ($M \times N$) array of identical resonators between two parallel (input and output) waveguides. Such a lattice may be decomposed into a row sub-lattice of N uncoupled resonators and a column sub-lattice of M coupled resonators, as shown in Fig. 6.1. Likewise, it should be possible to understand the properties of the 2D filter in terms of the properties of the 1D filters. Therefore, these sub-lattices individually will be treated at some length, before the analysis and results of the 2-D array are presented.

Fig. 6.2. Ring resonator coupled to two waveguide buses.

To establish the notations, consider the simplest case of a single, lossless resonator coupled to two bus waveguides, as shown in Fig. 6.2. Assuming a single input a , the output in the backward direction is denoted as D and that in the forward direction is denoted T , where

$$T = \frac{r - r \exp(-i\delta)}{1 - r^2 \exp(-i\delta)}, \quad D = \frac{-(1 - r^2) \exp(-i\delta/2)}{1 - r^2 \exp(-i\delta)}. \quad (6.1)$$

A resonance in transmission occurs when $\delta = 2m\pi$, where m is an integer. The effective transmission phase shift across a resonance is π for the FP filter; varying the reflectivity r only changes the slope of the phase response. With both couplers being identical, the Fabry-Perot filter is symmetric and *reciprocal*; the amplitude and phase response from left to right is the same as from right to left. As a consequence, the transfer matrix which relates the fields in the left and the right planes of 1R2B will be *unimodular* (i.e., $\det(m) = 1$), even in the

presence of waveguide loss. The transfer matrix equation is given by

$$\begin{pmatrix} d \\ b \end{pmatrix} = \frac{1}{T} \begin{pmatrix} 1 & -D \\ D & T^2 - D^2 \end{pmatrix} \begin{pmatrix} c \\ a \end{pmatrix} \quad (6.2)$$

It follows that the transfer matrices for arrays of reciprocal resonators, to be considered below, will also be unimodular, even in the presence of loss. In the following, the resonators will at first be assumed lossless so as to focus on the frequency and geometric dependence embedded in δ . We will then discuss the effect of loss. Clearly, in the lossless case, $|D|^2 + |T|^2 = 1$, from which it can be shown the following corollary relations that will be useful later: (i) $|D+T|=1$; (ii) D/T is pure imaginary, and (iii) $(T^2 - D^2)/T = 1/T^*$.

6.2 Type I: Cascade of indirectly coupled resonators to side waveguides

We consider first a linear cascade of passive lossless resonators periodically coupled to a pair of bus waveguides with spacing L_B , as illustrated in Fig. 6.3. In general filter design, each of the ring resonators can be slightly different and the coupling can vary from one to another [13]. However, for simplicity all the resonators and all the couplers will be assumed identical. Although the individual resonators are not directly coupled to each other, they are still coupled indirectly through the input and output waveguides.

Fig. 6.3. The schematic of Type I array.

For the n^{th} ring cavity, the input and output field amplitudes are represented by (a_n, d_n) , and (b_n, c_n) , respectively. Obviously, by continuity, we have $a_{n+1} = b_n \exp(-ik_0 n_{\text{eff}} L_B)$, $d_n = c_{n+1} \exp(-ik_0 n_{\text{eff}} L_B)$, which may be written as

$$\begin{pmatrix} d_n \\ b_n \end{pmatrix} = \begin{pmatrix} \exp(-ik_0 n_{\text{eff}} L_B) & 0 \\ 0 & \exp(ik_0 n_{\text{eff}} L_B) \end{pmatrix} \begin{pmatrix} c_{n+1} \\ a_{n+1} \end{pmatrix}. \quad (6.3)$$

Eqs. (6.2) and (6.3) give the recursive transfer matrix relation:

$$\begin{pmatrix} c_{n+1} \\ a_{n+1} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} c_n \\ a_n \end{pmatrix} = \frac{1}{T} \begin{pmatrix} \exp(i\delta_B) & -D \exp(i\delta_B) \\ D \exp(-i\delta_B) & (T^2 - D^2) \exp(-i\delta_B) \end{pmatrix} \begin{pmatrix} c_n \\ a_n \end{pmatrix} \quad (6.4)$$

where $\delta_B = k_0 n_{\text{eff}} L_B$ is inter-cavity phase shift. Note that the matrix $[m]$ is unimodular and that $m_{22} = m_{11}^*$, and $m_{12} = m_{21}^*$. For N identical resonators, this relation may be iterated N times, i.e., $[M] = [m]^N$, and the Sylvester's theorem [17] can be applied, giving

$$\begin{pmatrix} c_{N+1} \\ a_{N+1} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ a_1 \end{pmatrix}, \text{ where}$$

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \frac{1}{\sin \theta} \begin{pmatrix} m_{11} \sin(N\theta) - \sin[(N-1)\theta] & m_{12} \sin(N\theta) \\ m_{21} \sin(N\theta) & m_{22} \sin(N\theta) - \sin[(N-1)\theta] \end{pmatrix}, \quad (6.5)$$

and $\cos \theta = \frac{1}{2}(m_{11} + m_{22}) = \frac{1}{2}(m_{11} + m_{11}^*)$ is real. Eq. (6.5) can also be expressed in terms of the Chebyshev polynomials of the second kind. Note that the matrix M is again unimodular, and $M_{22} = M_{11}^*$ and $M_{12} = M_{21}^*$. For the array of N resonators, the array *reflectivity*, or the drop transmittance D_I is the ratio c_1/a_1 , and can be found from the condition $d_N = c_{N+1} = 0$, giving

$$D_I \equiv \left| \frac{c_1}{a_1} \right| \exp(i\phi_R) = -\frac{M_{12}}{M_{11}} = \frac{T \exp(-i\delta_B) \sin(N\theta)}{\exp(i\delta_B) \sin(N\theta) - R \sin[(N-1)\theta]}. \quad (6.6)$$

Similarly, the *transmissivity*, or the through transmittance T_I of the resonator array is defined and given by

$$T_I \equiv \left| \frac{a_{N+1}}{a_1} \right| \exp(i\phi_T) = M_{22} - \frac{M_{21} M_{12}}{M_{11}} = \frac{1}{M_{11}}. \quad (6.7)$$

Eqs. (6.6) and (6.7) give both the amplitudes and phases of the output waves relative to the input wave. Note that $|D_I|^2 + |T_I|^2 = 1$ as expected by conservation of energy, so the transmission and reflection spectra are complementary. A semi-infinite periodic optical structure exhibits a bandstructure with photonic bandgaps (PBG) corresponding to frequencies in which light cannot propagate through the structure. Indeed, when the resonators are on resonance, light cannot propagate through but is reflected from the array, giving rise to *reflection* resonances that, in the limit of large N , correspond to photonic bandgaps. In addition, another type of reflection resonance and bandgap can appear that depends on the resonator spacing (L_B). This is because the double channels provide many

paths for the waves to feedback to earlier resonators, allowing Bragg resonances to develop when the *Bragg* condition, $\delta_B = j\pi$, or $\lambda_j = 2n_{\text{eff}}L_B/j$ ($j = 1, 2, 3, \dots$), is satisfied [16]. Hence, the device behaves as a hybrid between ring resonator and distributed-feedback grating [1].

Fig. 6.4. The bandstructure of Type I geometry in different inter-cavity spacing: (a) $p=0.5$, (b) $p=1.0$, and (c) $p=0.75$. The transmission spectra are shown side by side with the bandstructure.

The photonic band structure can be derived by implementing the Bloch theorem to the transfer matrix:

$$\begin{pmatrix} c_{n+1} \\ a_{n+1} \end{pmatrix} = \exp(ik\Lambda) \begin{pmatrix} c_n \\ a_n \end{pmatrix} \quad (6.8)$$

where k is the Bloch wavevector, and $\Lambda = L_B$ is the Bragg period. This relation strictly applies only for an infinite periodic array, but we will see that photonic bandgaps exist even for a finite array. Using the matrix relation in Eq.(6.4), Eq.(6.8) becomes an eigenvalue problem with the characteristic equation $\cos k\Lambda = \frac{1}{2}(m_{11} + m_{22}^*)$, where $m_{11} = m_{22}^* = (1/T)\exp(i\delta_B)$, or

$$\cos k\Lambda = \frac{1}{|T|} \cos(\delta_B - \theta_T) \quad (6.9)$$

where $T \equiv |T|\exp(i\theta_T)$ is given in Eq. (6.1). It is apparent that Eq. (6.9) is satisfied only when $-1 < \cos k\Lambda < 1$; those frequencies beyond this region are considered to form the PBG, in which k is imaginary. In Fig. 6.4, we show the dispersion (real value of k as a function of δ) and the reflection amplitude response for the lossless case. The transmission spectrum $|T|$ is calculated from Eq. (6.6) for $N = 20$ and $r = 0.95$.

Note that for this value of N , the transmission is uniformly zero within bands centered around the resonance wavelengths $\lambda_m = n_{\text{eff}} L_{\text{cav}} / m$ (or $\delta/2\pi = m$), which correspond to the photonic bandgaps (or *reflection* bands). The smaller satellite peaks shown in Fig. 6.4(a) are the Bragg resonances which occur at the Bragg wavelengths $\lambda_j = 2n_{\text{eff}} L_B / j$, or $(\delta/2\pi) = j/2p$, where $p = \delta_B / \delta$ [16]. These resonances are similar to the reflection of a passive Distributed Bragg Reflector (DBR) structure; the amplitude of these resonances is less than one, approaching unity only for very large number of rings. In general, these resonances do not overlap with the resonator resonances, but when $p = 0.5$ (or $2L_B = L_c$), they coincide because the resonator and Bragg resonance conditions are identical. The cases when resonator and Bragg gaps do not overlap are presented in Fig. 6.4(b) and 6.4(c). The bandgap is wider when both the two bandgaps overlap, thus the case of $p = 0.5$ is the most useful case and will be assumed henceforth.

The gap width is dependent on the coupling between the waveguide and the resonator. The stronger the coupling (i.e., the smaller the r), the stronger will be the feedback between the resonators, and hence the wider the bandgap. Outside the reflection bands are a number of *sidelobes* proportional to N (in the transmission spectrum these sidelobes become *ripples*). Here, it can be seen that the slope of the dispersion curve is quite constant, so the ring

structure behaves as a uniform slab with a constant group refractive index and a thickness proportional to N . Therefore, the ripples may be interpreted as the Fabry-Perot interference within the slab; the thicker the slab, the greater the number of ripples. Moreover, the wider the bandgap, the higher will be the group index (i.e., the smaller the slope of the dispersion curve), and hence the larger the ripple amplitude.

Fig. 6.5. The drop transmission $|R_I|^2$ and the phase response (ϕ_D) of lossless cascade of side-coupled array for $N = 1, 5$, and 10 . The coupling coefficient is fixed to $r=0.85$.

The *reflection* amplitude and phase responses for various values of N are shown in Fig. 6.5. The amplitude response $|D_I|$ changes from Lorentzian for a single ring to a broad square shape with only relatively small number of resonators. This is because each element in the array is a high-Q symmetric resonator with unity transmission at resonance and therefore has a significant contribution to the overall response [6]. Similarly, the phase response ϕ_R changes from a highly nonlinear shape for the single-ring case to become more linear with increasing N , but the effective phase shift across the reflection band remains roughly π just as for a single ring. Near the band edges, however, the phase becomes steeper and more nonlinear with increasing N , giving rise to higher group delay and more dispersion. Outside the band, at each zero crossing of the sidelobes there is a π -phase discontinuity which has no physical relevance, since the delay is undefined when the reflectance is zero. This phase ripple does not exist for a single ring and thus, like the reflection sidelobes, can be attributed to interference in the multi-ring structure. This behavior is similar to the Fiber Bragg Grating

(FBG) filter, and is related to the nature of a type of filter called *minimum phase* filter [18], which is important later on in understanding the behavior of 2D ring array.

6.2 Type II: Cascade of directly coupled resonators

The same analysis can be performed for a cascade of resonators in which the resonators are directly coupled to each other in a lattice architecture (Fig. 6.6). The cascade, in turn, is coupled to an input waveguide and an output waveguide. This structure is variously called multi-ring cavity [1], multiple-ring resonator filter [14,16], and coupled-resonator optical waveguide (CROW) [8, 16]. The M resonators in a CROW are analogous to a chain of coupled Fabry-Perot etalons or mirror stack. The input fields are denoted a_1 and d_{M+1} ($=0$) and the output fields are b_1 and c_{M+1} . The output wave is forward (backward) traveling if M is even (odd). Propagation through the CROW is allowed only when the resonators are on resonance. When the resonators are off resonance, the wave is reflected by the CROW. Hence, the transmission properties of the coupled resonator array are similar to a multi-layer stack, with narrow transmission bands and wide forbidden (reflection) regions. This feature is the reverse of the previous uncoupled case, which exhibits narrow reflection resonances (bandgaps) and broad transmissions bands.

Fig. 6.6. The schematic of Type II array.

Again all the couplers are assumed to be identical. Following the same steps, and noting that $a_{n+1} = c_n \exp(-i\delta/2)$, $d_n = b_{n+1} \exp(-i\delta/2)$, one can obtain the recursive transfer matrix relation:

$$\begin{aligned} \begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} &\equiv \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \\ &= \frac{1}{i\sqrt{1-r^2}} \begin{pmatrix} -\exp(-i\frac{\delta}{2}) & r \exp(-i\frac{\delta}{2}) \\ -r \exp(i\frac{\delta}{2}) & \exp(i\frac{\delta}{2}) \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \end{aligned} \quad (6.10)$$

Note that the matrix $[p]$ is unimodular and that $p_{22} = p_{11}^*$, and $p_{12} = p_{21}^*$. After iterating M times the $[P] = [p]^M$ is

$$\begin{aligned}
\begin{pmatrix} a_{M+1} \\ b_{M+1} \end{pmatrix} &\equiv \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} \\
&= \frac{1}{\sin \theta} \begin{pmatrix} p_{11} \sin(M\theta) - \sin[(M-1)\theta] & p_{12} \sin(M\theta) \\ p_{21} \sin(M\theta) & p_{22} \sin(M\theta) - \sin[(M-1)\theta] \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix},
\end{aligned} \tag{6.11}$$

where the matrix $[P]$ is again unimodular, and $P_{22} = P_{11}^*$ and $P_{12} = P_{21}^*$ can be found with the use of Eq. (6.11). Making use of the fact that $d_{M+1} = 0$, and $b_{M+1} = -ra_{M+1}$, the coupled array reflectivity $R_{II} = b_1/a_1$ is obtained as:

$$R_{II} = \left| \frac{b_1}{a_1} \right| \exp(i\phi_R) = -\frac{rP_{11} + P_{21}}{rP_{12} + P_{22}}. \tag{6.12}$$

Similarly, making use of the identity $c_{M+1} = ita_{M+1}$, the array transmissivity $T_{II} = c_{M+1}/a_1$ is given by:

$$T_{II} \equiv \left| \frac{c_{M+1}}{a_1} \right| \exp(i\phi_T) = i\sqrt{1-r^2} \left(\frac{P_{11}P_{22} - P_{12}P_{21}}{rP_{12} + P_{22}} \right) = \frac{i\sqrt{1-r^2}}{rP_{12} + P_{22}}. \tag{6.13}$$

It can be shown that $|R_{II}|^2 + |T_{II}|^2 = 1$ as expected, and that T_{II} is given by

$$T_{II} = \frac{-(1-r^2)\sin \theta}{\left[(1-r^2)\cos\left(\frac{\delta}{2}\right) + ir^2\sin\left(\frac{\delta}{2}\right) \right] \sin(M\theta) + i\sqrt{1-r^2}\cos(M\theta)}, \tag{6.14}$$

where $\cos \theta = (1/t)\sin(\frac{\delta}{2})$. Applying the Bloch theorem Eq. (6.8) to this case gives the characteristic equation

$$\cos k\Lambda = \left(\frac{1}{t} \right) \sin\left(\frac{\delta}{2} \right) \tag{6.15}$$

where $\Lambda = L_{\text{cav}}/2$ is the period of the unit cell as shown in Fig. 6.6, and t is the coupling factor of the coupler. Note that Eq. (6.15) is similar to Eq. (6.9) (when $p = 0.5$), as they both originate from Eq. (6.8) which suggests that the characteristic equation is determined by the “transmittivity” from one cell to the other multiplied by an oscillatory function. In the case of CROW, the transmittivity is t , but for the side-coupled (Fabry-Perot) case, it is given by T (see Fig. 6.2). The oscillatory function is sine in one case and cosine in the other, reflecting the fact that the photonic bandgaps are complementary (CROW exhibits band gaps when the resonators are off-resonance and the side-coupled case does when they are on resonance), as will be shown below.

The dispersion diagram, showing the real and imaginary parts of k as a function of δ , and the transmission amplitude and phase responses calculated for a CROW of 5 rings, are shown in Fig. 6.7. Note that the propagation of waves in CROW is arranged in narrow bands centered at the resonance frequencies at $\delta=m2\pi$, while the broad bandgaps (where k is imaginary) are centered at $\delta=(2m+1)\pi$, unlike the uncoupled case where the bandgaps are centered at $\delta=m2\pi$ (when the Bragg and ring resonances overlap). This means that the bandgaps are complementary and do not overlap, as mentioned earlier. The dispersion diagram is strictly defined only for an infinite CROW. Many interesting properties can be derived from the dispersion relation Eq. (6.15). For example, the group velocity, which is proportional to the slope of the dispersion curve, is given by:

$$v_g = \frac{d\omega}{dk} = \frac{2t\Delta\omega_{FSR}\Lambda \sin(k\Lambda)}{\cos\left(\frac{\delta}{2}\right)} \quad (6.16)$$

where $\Delta\omega_{FSR}$ is the free spectral range, and t is the inter-cavity coupling factor. From this and Fig. 6.7 it can be seen that the group velocity is maximum at the centre of the passband where $\sin(k\Lambda)=1$, and zero at the band edges where $\sin(k\Lambda)=0$. The group delay is related to the group velocity by $T_g = -d\phi_T/d\omega = M\Lambda/v_g$. This slowing of the wave at the edge is completely opposite to the single-resonator case where the group delay is a Lorentzian function with a maximum at resonance.

Fig. 6.7. Dispersion diagram showing the real (solid lines) and imaginary parts (dashed lines) of the Bloch eigenvalues, and the transmission spectra (amplitude squared and phase) for a waveguide-coupled CROW with 5 rings. The bandgaps are centered at $\delta/2\pi = m+1/2$.

Thus, for higher-order filters having large number of coupled resonators, the group delay and the dispersion are greatly enhanced and shifted to the band edges [8, 16]. Note that the group velocity is insensitive to geometry as the product $\Delta\omega_{FSR}\Lambda$ is approximately constant, but it can be increased or reduced by varying t . Changing t will also affect the transmission bandwidth, which can be found from the condition $\sin(\delta/2)/t = 1$ to be:

$$B = \left(\frac{2}{\pi}\right) \Delta\omega_{FSR} \sin^{-1}(t) \quad (6.17)$$

The larger the t , the greater the interaction between the rings, and hence the broader is the bandwidth. These properties are approximately satisfied also in a CROW of finite length. For a finite CROW of M rings, there are M -fold splitting in the transmission resonance, as shown in Fig. 6.7 for $M = 5$. The sharp peaks can be seen only if all the rings are identical and have equal coupling. As M increases, the band top becomes flatter, the edges become sharper and the width approaches B . Likewise, in the phase response, there are M -fold ripples. The effective phase shift across each ring is π , so the total phase shift is $M\pi$. As a result of these ripples, the derivative of the phase, or group delay, shows a multi-peak behavior, with the highest peaks at the band edges [19]. However, as M increases indefinitely, the envelope of these peaks approach the group delay given by the dispersion curve, showing the maxima at the band edges. The enhanced group delay at the edges is undesirable as it restricts the usable bandwidth of the filter.

6.3 Two-dimensional arrays of resonators

Both the 1D arrays discussed earlier fall short of the ideal filter because of various factors, such as the presence of sidelobes or ripples, and the nonlinear phase response at the bandedges. However, comparing Fig. 6.7 and Fig. 6.4, we notice that the sidelobes and ripples are absent in the bandgap regions. Since the bandgaps are completely complementary (i.e., the bandgap for the coupled case does not overlap with that for the uncoupled case as long as the Bragg and resonator resonances coincide), we infer that it is possible to achieve a bandpass filter without ripples and sidelobes everywhere by using a 2D array that combines the features of the coupled and uncoupled linear arrays. The generalized 2D array of N by M resonators, as shown in Fig. 6.1, can be analyzed column-wise first, then row-wise. The inputs are labeled as $a_{1,1}$ and $d_{M+1,N+1}$, and the outputs as $b_{1,N+1}$ at the “through” bus and $c_{M+1,1}$ at the “drop” bus. In order to form a bandgap structure, it is essential to have optical feedback among the columns of resonators. Hence, the number of row, M , must be an odd integer, so

that the wave in the output waveguide is backward traveling. Furthermore, to avoid the satellite Bragg resonances discussed earlier, we fix the spacing between columns, L_b , to be half the resonator cavity length (i.e., $p=0.5$). All the resonators are assumed lossless and identical.

Fig. 6.8. The schematic of two-dimensional arrays.

First, the column of coupled ring resonators is reduced to an equivalent resonator. To do so, we refer to Fig. 6.8 and rewrite Eq. (15) in the form:

$$\begin{pmatrix} a_{M+1,j} \\ b_{M+1,j} \end{pmatrix} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \begin{pmatrix} a_{1,j} \\ b_{1,j} \end{pmatrix}$$

where the subscript j has been added to denote the j^{th} column. The matrix $[a_{M+1,j} \ b_{M+1,j}]^T$ can be converted to $[c_{M+1,j} \ d_{M+1,j}]^T$, giving

$$\begin{pmatrix} c_{M+1,j} \\ d_{M+1,j} \end{pmatrix} = \frac{1}{it} \begin{pmatrix} -1 & r \\ -r & 1 \end{pmatrix} \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \begin{pmatrix} a_{1,1} \\ b_{1,1} \end{pmatrix} \equiv \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_{1,1} \\ b_{1,1} \end{pmatrix} \quad (6.18)$$

where

$$\begin{aligned} m_{11} &= -\frac{(P_{11} + rP_{21})}{it}, & m_{12} &= \frac{(rP_{11} + P_{21})}{it}, \\ m_{21} &= -\frac{(P_{12} + rP_{22})}{it}, & m_{22} &= \frac{(rP_{12} + P_{22})}{it} \end{aligned} \quad (6.19)$$

Since it comes from the multiplication of two unimodular matrices, $[m]$ is also unimodular. The next step is to make the 2D array appear like the cascade of uncoupled “equivalent

resonators”, as in Fig. 6.3. To do so, we need to first make a “rotation” from the column direction to the row direction, by transforming the matrix relation $[c \ d]^T = [m][a \ b]^T$ in Eq. (6.18) to the form $[d \ b]^T = [s][c \ a]^T$, where

$$[s] = \frac{1}{m_{12}} \begin{pmatrix} m_{22} & -1 \\ 1 & -m_{11} \end{pmatrix}$$

This gives

$$\begin{pmatrix} d_j \\ b_j \end{pmatrix} = -\frac{1}{rP_{22} + P_{12}} \begin{pmatrix} P_{22} + rP_{12} & -it \\ it & P_{11} + rP_{21} \end{pmatrix} \begin{pmatrix} c_j \\ a_j \end{pmatrix} \quad (6.20)$$

Using Eq. (6.3), the Eq. (6.20) can be written as a recursive equation:

$$\begin{aligned} \begin{pmatrix} c_{j+1} \\ a_{j+1} \end{pmatrix} &= \frac{1}{m_{12}} \begin{pmatrix} m_{22} \exp(i\delta_B) & -\exp(i\delta_B) \\ \exp(-i\delta_B) & -m_{11} \exp(-i\delta_B) \end{pmatrix} \begin{pmatrix} c_j \\ a_j \end{pmatrix} \\ &\equiv \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} \begin{pmatrix} c_j \\ a_j \end{pmatrix} \quad (1 \leq j \leq N) \end{aligned} \quad (6.21)$$

Using Eqs. (6.19), (6.10) and (6.5), it can be shown that the matrix $[q]$ is also unimodular. Therefore, for N columns, we have $[q]^N \equiv [Q]$, where again Sylvester theorem can be applied to find the matrix $[Q]$. Analogous to Eqs. (6.6) and (6.7), since $d_N = c_{N+1} = 0$, we again have

$$D_{2D} \equiv \left| \frac{c_1}{a_1} \right| \exp(i\phi_T) = -\frac{Q_{12}}{Q_{11}}, \quad T_{2D} \equiv \left| \frac{a_{N+1}}{a_1} \right| \exp(i\phi_R) = Q_{22} - \frac{Q_{12}Q_{21}}{Q_{11}}. \quad (6.22)$$

Finally, the bandstructure for the $M \times N$ array is given by $\cos k\Lambda = \frac{1}{2}(q_{11} + q_{22})$. Hence, the transfer function and the bandstructure of an $M \times N$ array are similar to those of a $1 \times N$ array. This is because the M coupled resonators in each column have been reduced to a single “equivalent” resonator. However, there are implicitly two *sinc* functions in the response, of the form $\sin(M\theta_y)/\sin(\theta_y)$ and $\sin(M\theta_x)/\sin(\theta_x)$. The first is from the M coupled resonators in each column and the second is from the array of N uncoupled equivalent resonators in the x -direction.

The bandstructure is 2D in character in the sense that it incorporates the bandgap of the periodic structure in the y -direction as well as that in the x -direction. Because the two bandgaps do not overlap, the two dimensional arrays are capable of suppressing the side-lobes outside as well as the ripples inside the filter passband. Fig. 6.9(a) shows, for example, a remarkably box-like and ripple-free *reflection (drop)* spectrum for a 7×20 array calculated by Eq. (6.22) (the transmission spectrum is complementary). The inset shows the 30-dB

sidelobe suppression while maintaining the maximally flat passband between the reflection spectra for 3x20 and 1x20 arrays. The strong suppression possible even with a depth of only three rings is because the additional periodic structure (CROW) provided in the y -direction opens up a bandgap in the frequency band of the sidelobes that effectively filters out the inter-resonator reflections in the x -direction, thereby removing the sidelobes, while leaving the resonance band unchanged as the CROWs are transparent in this region.

Fig. 6.9. (a) The Drop transmission spectrum at the “drop” port for a 7x20 2D array of resonators. The inset shows sidelobe suppression in 3x20 arrays, compared to that in 1x20 arrays. (b) Phase responses and the normalized group delay for the drop ports of 1x20 and 3x20 arrays. In all cases $r = 0.95$ is assumed.

Of equal importance to the filter performance is its phase response which determines the group velocity and the dispersion of a pulse passing through it. As shown in Fig. 6.9(b), the phase responses for the 1x20 and 3x20 arrays are approximately linear over 80% of the band. However, the slope, which corresponds to the group delay, exhibits a peak at the band edge similar to the 1D arrays discussed earlier [13,18]. The larger the arrays (larger M), the narrower is the flat, linear phase region. There is thus some tradeoff, as we increase the array size, between more square and ripple-free amplitude response on the one hand, and more dispersion and group delay at the bandedge on the other. However, as long as the signal frequency bandwidth is within 80% of the passband where the phase is linear, the dispersion will be minimal. The phase response is similar for transmission at the “through” port.

6.4 Design consideration of two-dimensional array filter

It has been theoretically shown that a general 2D resonator lattice may approximate an ideal bandpass filter with a square amplitude and linear phase response. In digital filter theory, this is characteristic of a class of filters called *minimum phase* filter (MPF) [18]. MP filters

are those in which the amplitude response uniquely determines the phase response and vice versa. This is the case for the single-ring resonator filter, the coupled FP filter (CROW), and the FBG type filter such as the side-coupled linear array considered earlier, for which the amplitude responses are related to the phase responses by the Hilbert transform. Some consequences of the Hilbert transform are: (i) a constant amplitude response implies linear phase; (ii) the phase is directly related to the change in the amplitude response, and hence will acquire higher order terms near the passband edges where the amplitude response changes radically. The result is that as the ideal amplitude characteristic is approached, the dispersion increases near the band edges, and the usable fraction of the passband is limited by the phase response (dispersion) rather than the amplitude response (flatness of the passband).

Fig. 6.10. (a) The Drop output spectrum of a $3 \times N$ array filter for several values of N , assuming $r=0.9$. (b) The Drop spectra, for a 3×10 array for different round trip losses.

Since the near-ideal filter characteristic is achieved only with a reasonably large 2D array of identical resonators, it may pose substantial difficulty in implementation. To reduce the complexity, an important parameter is thus the minimum array size required to achieve the optimum response. Typically, a column depth of $M=3$ gives sufficient sidelobe suppression. Fig. 6.10(a) shows the “drop” output spectrum of a $3 \times N$ array filter for several values of N , assuming $r=0.9$. Note that it takes about $N=10$ to filter out the ripples in the passband. Although this number may vary somewhat for different values of r , r is usually fixed by the required bandwidth, so it is safe to say that the smallest array size required to give a ripple-free (or ripple < 0.1 dB) transfer function is 3×10 , for a total of 30 ring resonators. If a larger ripple can be tolerated, then the array size may be reduced further. So far only lossless case is assumed in our theoretical analysis. In reality, any loss is detrimental. Fig. 6.10(b) shows the effect of propagation loss on a 3×10 array. It can be seen that even 5% ($a=0.95$) round-trip

loss is sufficient to significantly reduce the passband magnitude and degrades the transition edge. The rounding off at the shoulder is due to the larger increase in the imaginary part of k in the band-edge.

Low loss is particularly important for the coupled rings in the column, since the signal in the pass-band must pass through all of them. On the other hand, the number of rings in the row direction is larger, even though not all the light passes through all of them. Hence, these filter designs are suitable only if all the rings have very low loss. This requirement will severely restrict the implementation of these filters. Recently, very high order multi-ring filters with very high Q resonators have been realized using low-loss Hydex material [6] and silicon-on-insulator (SOI) [20], showing the technological possibility of realizing large and uniform resonator arrays. Although a full tolerance study is outside the scope of this chapter, simulations have shown that $\pm 5\%$ random variations in the ring radius and in the coupling coefficients on the 1D CROW is insignificant as they tend to cancel each other. It has also been shown for side-coupled rings that arrays are more robust to asymmetry in the coupling coefficients than a single ring [7].

6.5 Experimental Results

We fabricated 1x4, 1x8, 3x4, 3x8 configurations, using deep-UV (DUV) CMOS process [20] in silicon-on-insulator (SOI) technology, with the 1x8 and 3x8 arrays shown in Fig. 6.11. The dimensions of the waveguide and the propagation loss are the same as those in previous fabricated devices. The ring radius is $5\mu\text{m}$. The coupling strengths are defined via racetrack coupling of length $L_C=8\mu\text{m}$. Fig. 6.12 shows the measured Through (T) and Drop (D) spectra in 1x4 (right panel) and 3x4 (left panel) structures. The suppression of inter-column interaction in the 3x4 structure is clearly seen in the $\sim 10\text{dB}$ sidelobes reduction in the Drop spectrum, and in the band-flattening of the Through spectrum from $\sim 1.7\text{dB}$ in the 1x4 array to $< 0.2\text{dB}$ in the 3x4 array. The usable bandwidth is $\sim 4\text{nm}$, corresponding to $\Delta f = \Delta \lambda / \lambda_{\text{RES}}^2 = 500\text{GHz}$. The observable out-of-band contrast is $\sim 25\text{dB}$. The roll-off of the T spectrum has increased from $\sim 12.5\text{dB/nm}$ (1x4) to $\sim 21\text{dB/nm}$ (3x4). The shape factor (the ratio of the -1dB and the -20dB bandwidth) of the 3x4 structure is ~ 0.7 . The $8\mu\text{m}$ racetrack length corresponds to $\sim 50\%$ power coupling and is verified from the independent measurement of a fabricated one microring resonator coupled to two waveguide buses with the same racetrack length.

Fig. 6.11. The optical micrographs of fabricated 1D and 2D arrays

Fig. 6.13 shows the measurement results for the 1x8 and 3x8 configurations with the same coupling (racetrack length). The Through response of 1x8 exhibits high contrast ($\sim 25\text{dB}$) with the ripples ranging from $\sim 0.5\text{dB}$ to $\sim 3\text{dB}$. The roll-off increases by approximately twice to 27dB/nm compared with the 1x4 array, as expected from the fact that the structure has twice the number of rings. For the 3x8 configuration, the sidelobes in the spectrum cannot be observed due to the limitation of our equipment. However, the 10dB sidelobes suppression still can be seen. This is consistent with the comparison of 1x4 and 3x4. Due to the limitation of our equipment, the observable contrast is only 25dB . However our simulation shows that the out-of-band contrast can be as high as 50dB while the sidelobes is at -25dB . The usable bandwidth of 3x8 structure is $\sim 6\text{nm}$, corresponding to $\sim 755\text{GHz}$ bandwidth. The shape factor is ~ 0.8 and the measured roll-off is $\sim 43\text{dB/nm}$, again this is about twice that in the 3x4 structure.

Fig. 6.12. The measured spectra of 1x4 and 3x4 arrays.

Note that there are spiky features in the Drop spectrum of both 1xM and 3xM structures. In the 1xM structure, there is only one spike generally located around the center of the photonic bandgap. In the 3xM structure, there are two additional spikes. This can be attributed to two factors. First is the mismatch between the inter-coupling spacing and the cavity length. There are two kinds of bandgap in this structure. The first kind is the Bragg gap that arises from Bragg mechanism whose resonance wavelength depends on the ring spacing distance ($2n_{\text{eff}}L_B = m\lambda_B$). The second kind is the resonator gap that arises from the individual rings, located at every ring resonances ($n_{\text{eff}}L_{\text{CAV}} = m\lambda_{\text{RES}}$). These two bandgaps overlap only when $L_{\text{CAV}}=2L_B$, and this results in a more rapid forming of flat response. Simulations have confirmed that deviation from $L_{\text{CAV}}=2L_B$ separates the Bragg Gap from the Resonator Gap and forms narrow dip in the center of the bandgap.

Fig. 6.13. The measured spectra of (a) 1x8 array, and (b) 3x8 array.

The second factor, which may be the origin of two additional spikes in 3xM structure, is the coupling induced resonance frequency shift (CIFS). As a consequence of the self-

coupling between resonators, it has been argued that the shift of the mean resonance frequencies can be either positive or negative [11]. Two mutual interactions from three rows of resonator produce two different mean resonance frequency shifts and thus produce two distinct dips in the bandgap region. We also note that the presence of both fabrication mismatch and the CIFS only affect the Drop response while the Through response is relatively immune to these non-idealities. On-chip interconnect applications require that the flat-band should be about 15-20% of the free spectral range (FSR) [9]. So far, the bandwidth of the 3xM structures is about half the FSR. By decreasing the coupling factor, one can reduce the bandwidth in the Drop output while increasing the bandwidth in the Through output, and vice versa. The filter bandwidth can be further controlled by adjusting the ring radius to meet the specific bandwidth requirements. Thus, independent control of contrast and filter bandwidth is possible.

6.6 Summary

It has been shown in principle that a 2D lattice of lossless resonators can approximate a near-ideal bandpass filter, fundamentally because the 2D periodic structure exhibits non-overlapping (2D) photonic bandgaps. The minimum size of the array required to give such ideal characteristics is 3×10 . However, the loss must be kept to a minimum, as it limits ultimate size of the array. The fabricated 2D arrays in SOI platform have experimentally verified this theoretical prediction. Sidelobe reduction of at least 10 dB has been demonstrated for devices that have 50% power coupling factor. The filter exhibits passband of 500GHz to 755 GHz with the shape factor of 0.7 and 0.8 and edge sharpness of $\sim 21\text{dB/nm}$ and $\sim 43\text{dB/nm}$ for the 3×4 and 3×8 arrays, respectively. The presence of fabrication mismatch and CIFS produce spiky features in the Drop response. The Through response however remains relatively unaffected. In future implementations, pre-distorting the rings may help to reduce the effect of CIFS.

Chapter 7. Defect modes in micro-resonator arrays¹⁰

It is well known that a periodic arrangement of optical media exhibits a frequency band within which the propagation of light is forbidden. The photonic bandgap (PBG) mechanism [1, 2] bears a remarkable resemblance to the electronic bandgap in solid state physics. Similar to its electronic counterpart, introducing irregularities (or defects) inside the structure may excite a defect mode where light is localized near that defect. The concept of defect mode is not new and has been demonstrated in conventional photonic bandgap structures [3]. The usefulness of defect mode is in suppression of spontaneous emission [4], design of microcavity laser and filters.

Fig. 7.1. A single (point) defect embedded in a (a) Type I, and (b) Type II microring arrays. (c) The Fabry-Perot cavity formed by the defect and the two sub-arrays.

Defect mode was first demonstrated by McCall [5] in the microwave region. Guided and defect modes in the optical region have been predicted for a periodic dielectric waveguide consisting of dielectric rods or air holes [6,7], and experimentally demonstrated in a chain of coupled vertical-cavity microresonators [8]. However, the concept of defect modes in periodic arrays of microring resonators has not been extensively explored. This structure is somewhat different because of the hybrid properties of photonic bandgap and resonators. In this chapter, defect modes in two resonator arrays are studied with the aid of the transfer matrix already elaborated in the earlier chapter. Similar to defect modes in conventional photonic bandgap structure, a defect is introduced by removing one of the rings in Type I or by introducing larger (smaller) rings in Type II (see Fig. 7.1). Such an ‘addition’ or ‘removal’ of optical media consequently excites what is known as donor and acceptor modes in solid state electronics, as will be shown later.

¹⁰ Most work of this chapter has been published in: "Defect modes in micro-ring resonator arrays," *Opt. Express* **13**, 7800-7815 (2005), and , "Demonstration of defect modes in coupled microresonator arrays fabricated in silicon-on-insulator technology," *Opt. Lett.* **33**, 1939 (2008).

7.1 The mapping of defect modes

Any configurations can be represented as a defect layer of thickness L_D sandwiched by two sub-arrays functioning as frequency dependent mirrors [see Fig. 7.1(c)], where $R_{1,2}$ and $T_{1,2}$ are the reflections and transmission coefficients of the respective sub-arrays. The system transmission coefficient (T_D) is found by totaling the transmitted waves:

$$\begin{aligned} T_D &= T_1 T_2 \exp(-i\delta_D) \sum_{m=0}^{m=\infty} [R_1 R_2 \exp(-i2\delta_D)]^m \\ &= \frac{T_1 T_2 \exp(-i\delta_D)}{1 - R_1 R_2 \exp(-i2\delta_D)} \end{aligned} \quad (7.1)$$

where $\delta_D = k_0 n_{\text{eff}} L_D$ is the phase change accumulated a defect. The transmission is on resonance when the condition $\phi_{R1} + \phi_{R2} - 2\delta_D = m2\pi$ is satisfied. This resonance condition determines the defect mode frequencies. Normally the defect mode exists for $m = 0$. For a symmetric structure ($R_1 = R_2 = R$) the defect modes are then determined by the condition

$$\phi_R = \delta_D \quad (7.2)$$

where ϕ_R is the phase response of reflection from any type of finite sub-arrays. If the sub-arrays are semi-infinite (i.e., ideal case), the reflection of both types then can be deduced from Bloch theorem

$$R_I = -\frac{m_{12}}{m_{11} - \exp(ik\Lambda)}, \quad R_{II} = -\frac{p_{12}}{p_{11} - \exp(ik\Lambda)}, \quad (7.3)$$

where $[m]$ and $[p]$ are, respectively, the matrix formalisms defined in the previous chapter for Type I [Eq. (6.6)] and Type II [Eq. (6.12)].

The Eq. (7.2) is solved for Type I configuration and the associated defect resonance frequency is mapped with the normalized defect size (L_D/L_{CAV}) in Fig. 7.2(a). As inter-cavity spacing increases from $0.5L_{\text{CAV}}$ to $0.6L_{\text{CAV}}$, there are more defect modes excited in the higher frequency, similar to guided modes in optical waveguides. Furthermore, the mapped defect resonance moves from higher frequency (known as photonic conduction band) to lower frequency (photonic valence band). This resembles donor modes in solid state physics, because $2L_D > L_{\text{CAV}}$ (the roundtrip in the defect) and the structure treats the defect as a ‘donor’. The case of acceptor modes is when $2L_D < L_{\text{CAV}}$ and the defect modes moves from lower frequency to higher frequency (not shown here). When $L_D = 0.5L_{\text{CAV}}$, there is no defect mode. This can be explained in two ways. In geometrical perspective, $L_D = 0.5L_{\text{CAV}}$ means that the system consists of $2N+1$ identical resonators. Because there is no defect of any sorts then

defect modes do not exist. In the perspective of Eq. (7.2), $L_D=0.5L_{CAV}$ means the defect resonance coincides with the cavity resonance and the reflection from sub-arrays (even in finite case) is unity. Thus, excitation of defect modes is impossible. Fig. 7.2(b) shows the defect transmission $|T_D|$ at $L_D = 0.52$ where the defect is sandwiched between two sub-arrays of 6 resonators each and $r = 0.85$. The agreement between the narrow peak locations and the mapping is excellent.

Fig. 7.2. (a) The frequencies of the defect modes vs. the defect length for infinite Type I periodic structure. (b) shows the cross sectional view for $L_D = 0.52L_{CAV}$.

The case for type II is presented in Fig. 7.3(a). The L_D in this case represents half the actual defect length. Here, the defect size is progressively increased from $1.0L_{CAV}$ to $1.2L_{CAV}$. Again the defect modes correspond to donor modes in this case ($L_D > L_{CAV}$). The same trend holds for type II by which multiple defect modes are excited as the defect size increases. Fig. 7.3(b) illustrates the finite-case transmission when $L_D=1.05L_{CAV}$ for a defect ring is sandwiched by sub-arrays of 3 cavities of coupling coefficient $r=0.65$. One can see that the defect resonance is located near the center of the bandgap (i.e., mid-gap) of Type II and have a very narrow resonance linewidth. When the defect resonance exactly coincides with the center-gap, excitation of defect modes becomes impossible since the reflection is unity (at infinite case), which is the same situation as that in type I case when $L_D=0.5L_{CAV}$.

Analogous to Fabry-Perot, using Eq. (7.2) the resonance FWHM can be approximately given by $\Delta\delta_{FWHM} = (1 - R_1^4)^{1/2} / R_1^2$. Indeed, the Q factor (of cavity finesse) of defect modes should depend on its detuning from the mid-gap as well as the size of the sub-array because it gives higher reflection. This then raises a practical concern on exciting the defect modes. For an infinite array exhibiting true photonic bandgap in which $|R| = 1$, the defect mode will have an infinite Q , but is also impossible to excite or extract from the cavity. The field distribution

of the defect mode in both types is calculated based on the eigenvector $\tilde{x}$, $A_D(n) = \tilde{x} \exp(\mp n \kappa \Lambda)$ with n signifies the n^{th} resonator from the defect. The field decays discretely as a function of n , which implies the constant amplitude (in the lossless system) until it proceeds to next resonator.

Fig. 7.3. (a) The resonant frequency of defect modes vs. normalized defect length L_D/L_{CAV} . (b) Defect transmission $|T_D|^2$ for $M = 3$ and $r = 0.65$. The defect is a racetrack resonator whose cavity length is $L_D/L_{CAV} = 1.05$.

Fig. 7.4. Absolute amplitude of the field distribution calculated by FDTD for (a) Type I ($L_D/L_C = 0.75$) and (b) Type II ($L_D/L_C = 1.1$).

In order to excite the defect modes in simulations as well as in experiments, we allow the input sub-array to have a finite transmission (T_1) by choosing the appropriate coupling parameters and array size. In the FDTD simulations, the Gaussian pulse is inserted to the structure as the light excitation with detectors recording the fields along the waveguides. The defect mode corresponds to a frequency that has the highest amplitude spectrum. This frequency then is used as the input excitation to extract the field distribution. The finite-difference time-domain (FDTD) calculations are shown in Fig. 7.4 for both types I and II. As expected from Bloch theorem, for Type I configuration, the field amplitude along the waveguide channel is piecewise constant (the Bloch theorem for type I describes the light propagation along the channel waveguides). The field distribution in the resonators are

skewed due to the fact that $r \sim 0.85$ correspond to moderate coupling, the distribution can be less skewed in weaker coupling (*cf.* field distribution of 1R2B at resonance). Similarly for the type II defect, the field distribution is piecewise constant in each resonator and begins to decay for resonators away from the defect.

In terms of FDTD simulation it is very hard to excite high Q defect modes externally in a large amount of arrays, which limits the minimum resonance linewidth. In the Type I, the r is set to be fairly large (~ 0.85) for finite arrays where the Gaussian pulse is inserted to excite the defect mode externally, as illustrated in Fig. 6.4(a). However in Type II, with $r \sim 0.85$, it is very difficult to externally excite the modes since the light may not even ‘penetrate’ the first sub-arrays due to the strong reflection. Thus from simulation point of view, the defect mode can be seen when it is internally excited, as illustrated in Fig. 6.4(b). Therefore to optimize the excitation, one can set fairly weak coupling for Type I and moderately strong coupling for Type II.

7.2 Experimental demonstration of defect modes in Type II configuration

In order to experimentally demonstrate the existence and dispersion of these defect modes, we designed a set of mutually coupled microring arrays (Type II), each with 4 identical rings and one defect ring at the center (see Fig. 7.5). The number of rings is kept low in order to limit the effect of loss in the measurement. The devices were fabricated using deep-UV (DUV) CMOS process in silicon-on-insulator (SOI) technology [9]. The waveguide width is 450nm and the silicon core thickness is 200nm. The ring radius is 8 μ m. The defect size is normalized to the cavity length $k_D = L_D / L_{CAV}$. The couplings between the rings and between the input waveguide and the ring are introduced by racetrack coupling of length $L_C = 6\mu$ m, which is designed to give a fairly large coupling in order to make it easier to excite the defect mode. Finally, to demonstrate the existence of donor and acceptor modes, the defect ring size is varied such that the defect parameter ranges from $k_D = 0.7$ to $k_D = 2$. The defect size is varied by adjusting the extension length (L) and defect radius (R_D) in the defect ring, as shown in Fig. 7.5.

First, we show the traces of donor-like modes in Fig. 7.6, which plots the measured Drop transmissions for increasing defect size. The defect modes are the resonances located within the bandgap region. For clarity the transmissions for different defect modes are offset by -22dB. As the insertion loss is still high, to make the results clearer, the underlying noises are filtered out by setting a lower transmission threshold of -15dB. In the theoretical fitting, a vertical offset up to 2dB is used to compensate the additional insertion loss in each

measurement, and a fixed group index is used to adjust the location of the defect mode resonance.

Fig. 7.5. The fabricated devices showing the defect ring in the center of the 4-ring array coupled to two bus waveguides. The defect parameter k_D is controlled by varying L_2 and R_D .

Fig. 7.6. Traces of how donor modes shift in wavelength with increasing defect size. The red lines indicate the theoretical fitting of each device.

The theoretical fitting for each device is indicated by the red curve. Due to the dispersive properties of the devices, i.e., the group index is only fixed for a narrow range of wavelength, a slight horizontal offset between theory and experiment is to be expected. Nevertheless, there is good agreement between theory and experiment. In the fitting, the power coupling factor is found to be $\sim 43\%$, and the cavity loss is about $\sim 3\text{dB/cm}$. These parameters are confirmed with independent measurements from a one-ring two-bus (1R2B) structure with

the same racetrack length fabricated in the same sample. The photonic bandgap is identified from the low and flat transmission band as indicated in the shaded gray region. The continuous transmission region with shorter wavelength is the conduction band (CB) and that with the longer wavelength is the valence band (VB). When the defect ring is only slightly different from the other rings ($k_D \sim 1$), one can observe a defect mode (labeled 1) very close to the conduction band edge. When k_D is progressively increased, mode 1 begins to move from CB to VB. At $k_D \sim 1.7$, a second donor mode (mode 2) emerges near the CB edge, and it moves in the same direction along with mode 1. Finally, in the last graph where $k_D \sim 2$ we see yet another mode (mode 3) being excited and emerging near the CB.

Fig. 7.7. Traces of acceptor (A) and donor (D) modes with varying k_D . The red lines indicate the theoretical fitting of each device.

Finally, we show the traces of both acceptor-like modes and donor-like modes together in Fig. 7.7, which plots the Drop transmissions from smaller ($k_D < 1$) to larger defect ring size ($k_D > 1$). In solid state physics, an acceptor mode moves from VB to CB when the acceptor level is increased. By analogy, in our photonic structure, the acceptor mode should move to the shorter wavelength with decreasing defect size ($k_D < 1$). Note that this is to be distinguished from the movement of the two donor modes in Fig. 7.6 that occurs for $k_D > 1$. When $k_D \sim 1$, there exists two modes at the CB and VB edges, and we address them as acceptor mode (A) and donor mode (D), respectively. For $k_D < 1$ (the smaller defect rings), mode (A) shifts to

shorter wavelength while mode (D) is no longer observed. The opposite occurs for $k_D > 1$ (the larger defect rings).

Fig. 7.8. The measured Q -factor of Type II defect modes situated at different normalized detunings.

The measured Q -factor for different defect modes are plotted as a function of their location (λ_{DM}) relative to the midband wavelength ($\lambda_{midband}$) normalized to the free-spectral-range (λ_{FSR}), $(\lambda_{DM} - \lambda_{midband})/\lambda_{FSR}$, as shown in Fig. 7.8. Here, the donor and acceptor modes are denoted by triangular and rectangular marks respectively. The FSR is measured by taking the wavelength difference between identical patterns in at $k_D \sim 1$, while the midband wavelength is measured by taking the average of two bandedge wavelengths (see Fig. 7.6). As expected, as the defect mode is more closely situated to the midband, the Q -factor monotonically increases. The highest measured Q -factor for donor modes (with $\sim 43\%$ power coupling) is $Q_{DM} \sim 22,000$. From theoretical fitting, the resonance order is found to be ~ 172 (for $\sim 60.2\mu\text{m}$ cavity length), suggesting resonance finesse of $F_{DM} \sim 127$, which is higher by $\sim 23x$ compared to that of the individual ring, i.e., $F_R = \pi r \sqrt{a} / (1 - ar^2) \sim 5.3$. Compared to the total of finesse of isolated rings in the Type II structure ($4F_{Ring} + F_{DefectRing} \sim 32$), the finesse of the defect mode is $4x$ higher. The Q -factor of the acceptor mode is generally lower than the donor modes, because the ‘acceptor ring’ is smaller than the regular ring, scaling down the Q -factor accordingly.

7.3 Summary

The existence of defect mode in linear periodic arrays of resonator has been theoretically studied. It has been shown that high finesse resonance can be obtained when a defect resonance is tuned closer to the center of the photonic bandgap. Due to the strong reflection

near the cavity resonance, the photonic bandgaps develop faster in periodic resonator arrays and the existence of defect modes can be observed for sub-arrays consisting only of few resonators. High- Q defect modes in general are difficult to excite because of very strong reflection from the sub-arrays. Thus, certain schemes need to be implemented theoretically and experimentally demonstrating defect modes such as weak coupling in Type I structure and rather strong coupling in Type II structure.

The experimental demonstration was carried out in Type II structure. The unique ability of the hybrid resonator structure in generating the photonic bandgap with very few unit cells has been verified. The movements of donor and acceptor modes are consistent with theoretical prediction. It is observed that the Q of the defect resonance is the highest when it is approaching the center of the photonic bandgap, but the amplitude is also smallest because of the accentuated effect of loss. The Q -factor obtainable in 43% power coupled system is as high as 22,000, corresponding to finesse of 127, which is 23x higher than that of individual resonator. In general, the Q can be significantly enhanced by slightly increasing the number of unit cells and reducing all the coupling coefficients.

So far the experimental validation of Type II Defect Mode structures is successfully demonstrated because the fabrication non-idealities do not significantly affect the spectral properties around the bandgap region. This can be explained from the fact that the photonic bandgap formed in Type II structures occurs at the ring anti-resonance which is devoid of inter-cavity interference. In Type I structures, on the other hand, the photonic bandgap is formed by inter-cavity interferences such as the resonator and Bragg gaps (see Chapter 6). The presence of fabrication imperfections consequently results in spurious resonances within the bandgap, belonging to the additional pathlength induced by proximity effects and CIFS. Therefore, care must be taken in the geometric design of Type I structures so as to minimize these non-idealities. Therefore experimental validation of Type I Defect Mode structures will be included to future work.

Chapter 8. Asymmetric Fano resonance and bistability for high extinction ratio, large modulation depth, and low power switching¹¹

After discussions of linear properties of single, double, and multiple cavity systems in the previous chapters, it is important to study some of nonlinear characteristics such as bistability switching in these configurations. The term bistability originates from the fact that two possible output states are produced as a result of dynamic intensity buildup in a resonator with Kerr nonlinearities. Bistability (or in general multi-stability) have been demonstrated in various configurations, such as in the ring resonator coupled to one or two bus waveguides [1,2], the ring resonator arrays [3], Bragg gratings [4], and photonic bandgap cavities [5]. A good optical switch is characterized by low input threshold power, high extinction ratio (i.e. >10 dB), high modulation depth, and reasonable optical bandwidth. The extinction ratio (ER) is defined as the ratio between the logic “1” and “0”, the modulation (MD) is defined as the difference between the logical “1” and “0” normalized by the incident power, and the optical bandwidth is defined as the signal wavelength tolerance for a given ER. While low threshold has been achieved using cavity-enhanced nonlinear response, the demonstrated extinction ratio and modulation depth are usually very low (as an example, for the first Silicon ring resonator switch using thermal nonlinearity [6], MD ~0.3 and ER = 3 dB, according to our definitions). There is thus a need to search for the optimal switch configurations with significantly better performance.

In this chapter we show that the extinction ratio and modulation depth are determined by the shape of the resonance while the switching power is dependent on its sharpness. It is difficult to achieve both high ER and large MD for a single-ring with a symmetric resonance. On the other hand, they can both be significantly enhanced by using an asymmetric Fano resonance [7]. Such a Fano resonance can be generated by the conventional ring-enhanced MZI (REMZI) where the resonator is coupled to a Mach-Zehnder interferometer [8]. Alternatively, it can be generated, with more flexibility, by using a new and novel two-ring configuration proposed in this chapter. The present two-ring configuration is different from other reported two-ring structures [9,10], as it is designed to harness the narrow and asymmetric lineshape of the Fano resonance to achieve optimal switching performance. It is

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shown that both configurations can achieve optical switching with very low threshold ($n_2 I_{\text{IN}} \sim 10^{-6}$), high extinction ratio ($> 30\text{dB}$) and large modulation depth (~ 1). We also consider their sensitivity to wavelength and show that a larger optical bandwidth can be obtained but at the expense of switching threshold.

This chapter is outlined in the following. In section 8.1 we discuss bistability in the conventional bus-coupled one-ring structures using the parametric approach. In section 8.2 we introduce the Fano resonance through the ring-enhanced MZI and consider its switching performance. This work complements and improves on that reported in [8]. Finally, in section 8.3 we discuss the new two-ring switching configuration in great detail, culminating in a comparison between the one- and two-ring structures.

8.1 Bistability in a one-ring system

A single ring can be coupled with one or two bus waveguides. Although the operation is different between these two cases, the result as an optical switch is similar and for this reason, only the one-bus case is considered below. A lossless single waveguide ring resonator coupled with a bus waveguide, as shown in Fig. 8.1 is an all-pass filter, but in the presence of loss exhibits a resonant transmission profile given by

$$T = \left| \frac{E_{\text{OUT}}}{E_{\text{IN}}} \right|^2 = \frac{r^2 + a^2 - 2ar \cos \delta}{1 + a^2 r^2 - 2ar \cos \delta} \quad (1)$$

where δ is the round trip phase, $a = \exp(-\frac{1}{2}\alpha L_{\text{cav}})$ is the round-trip amplitude for a cavity with length L_{cav} and linear loss coefficient α . The transmission is symmetric and minimum on resonance when $\delta = 2m\pi$ (m is an integer).

Fig. 8.1. Schematic of a ring resonator coupled to one bus waveguide.

On resonance, the transmission is zero when $r=a$, a condition known as critical coupling. In the case where the ring is a Kerr medium with nonlinear coefficient n_2 , the round-trip phase is the sum of a linear part and a nonlinear part:

$$\delta \equiv \delta_L + \delta_{NL} = \int_0^{L_{\text{cav}}} k_0 [n_{\text{eff}} + n_2 I_0^2 \exp(-\alpha s)] ds = k_0 (n_{\text{eff}} + n_2 I_{\text{cav}} \eta) L_{\text{cav}} \quad (8.2)$$

where $k_0 = 2\pi/\lambda$, n_{eff} is the waveguide effective index, $\eta = [1 - \exp(-\alpha L_{\text{cav}})]/(\alpha L_{\text{cav}})$ is the length reduction factor which accounts for the power loss in the ring, and I_{cav} is the maximum intensity in the ring. The intensity buildup then depends on the intensity of the circulating light, which in the high-finesse approximations can be written as

$$B = \frac{I_{\text{cav}}}{I_{\text{IN}}} = B_0 \frac{(\Delta\delta/2)^2}{\delta_L^2 [1 + (n_2/n_{\text{eff}})\eta I_{\text{cav}}]^2 + (\Delta\delta/2)^2}. \quad (8.3)$$

It should be noted that the nonlinear absorption such as two photon absorption (TPA) is assumed to be negligible, as is usually done [11]. Eq. (8.3) describe a cubic relation between I_{cav} and I_{IN} , and thus for a suitable value of I_{IN} there can exist three real solutions.

Fig. 8.2. The transmission as a function of incident power for a critically coupled ring for (a) $r=a=0.95$ for various near-resonant wavelengths, and (b) at a fixed wavelength, $\lambda = 1567$ nm, for various losses under the critical coupling condition.

Some examples are shown in Fig. 8.2(a) for several wavelengths below the resonance (the resonance is at $\lambda = 1568$ nm), all under the critical coupling condition ($r=a$). These examples are for the case where n_2 is negative, as is the case for polymers [12]. The radius has radius and effective index of $R=15\mu\text{m}$ and $n_{\text{eff}}=1.6$ respectively, as taken from Ref. [2]. The operating wavelengths are all below the resonance wavelength. Note that the transmission shows bistability behavior only at some wavelengths, but not all (e.g., no bistability exists at 1567.8nm). This can be explained in the following. Mathematically, the intra-cavity fields becomes unstable when $dI_{\text{cav}}/dI_{\text{IN}} = \infty$, which gives the following quadratic equation:

$3\delta^2 + 2\delta_0\delta + (\Delta\delta/2)^2 = 0$ for initial detuning $\delta_0 = (\omega - \omega_0)n_{\text{eff}}L_{\text{cav}}/c$. Clearly, the roots of this equation correspond to bistability,

$$\delta^{(\uparrow\downarrow)} = -\frac{1}{3}\delta_0 \pm \frac{1}{3}\sqrt{\delta_0^2 - \left(\Delta\delta\frac{\sqrt{3}}{2}\right)^2}, \quad (8.4)$$

where the $(\uparrow\downarrow)$ are upward (turn-on) and downward (turn-off) transitions at bistable point. One can see that Eq. (8.4) is only valid when $\delta_0 \geq \Delta\delta\sqrt{3}/2$ because the complex frequency is physically meaningless in this case. Hence, bistability occurs only when the initial detuning is larger than the critical detuning, i.e., $|\omega - \omega_0| \geq \Delta\omega_{\text{FWHM}}\sqrt{3}/2$, and this was pointed out by Miller for the first time in Ref. [11]. If the incident light is detuned exactly at the critical detuning, then we have bistability occurring at $\delta = \delta_0/3 = \Delta\delta/2\sqrt{3}$, which corresponds maximum nonlinear responsivity (slope of intensity buildup B).

The bistability, therefore, can be illustrated in the following way. As the input light charges up the cavity, the nonlinear phase shift increases and pulls the resonant wavelength towards the incident wavelength, which in turn increases the intensity buildup, leading to unstable situation. This positive feedback increases the slope of the leading edge of the transmission spectrum relative to the linear case [13]. This positive feedback continuous until the resonance wavelength is pulled to the other side of the incident wavelength, where the slope becomes negative and therefore providing negative feedback to stop the nonlinear shift. This is indicated as the downward transition in Fig. 8.2(a). The same happens when the input intensity is decreased afterward, where one can have the denoted upward transition in Fig. 8.2(a). The critical detuning originates from the fact that the positive feedback cannot be formed when the nonlinear responsivity (or the slope of intensity buildup factor) decreases with the increasing intra-cavity fields. This means bistability occurs only when $\delta_L + \delta_{\text{NL}} \geq \Delta\delta/2\sqrt{3}$, that is before the nonlinear responsivity (or the slope of B) reaches maximum. By substituting $\delta = \Delta\delta/2\sqrt{3}$ into Eq. (8.2) and (8.4), we can derive the switching threshold for 1R1B configuration under critical coupling condition (the complete derivation is in Appendix A)

$$n_2 I_{\text{TH}}^{(1\text{R1B})} = 4 / \left(3\sqrt{3}k_0 B_0^2 \eta L_{\text{cav}} \right). \quad (8.5)$$

So the required normalized switching power $n_2 I_{\text{TH}}$ is inversely proportional to the square of

the intensity buildup and the cavity size and the effect of loss is embedded in the reduction factor η , which approximates to unity in lossless case. In the case of single ring coupled to two waveguides (1R2B), the switching threshold can be similarly deduced, but with modification of effective cavity length, i.e., $\eta L_{\text{cav}} \rightarrow \frac{1}{2}(1+ar^2)\eta_{1/2}L_{\text{cav}}$ (see Appendix A), where $\eta_{1/2} = [1 - \exp(-\frac{1}{2}\alpha L_{\text{cav}})]/(\frac{1}{2}\alpha L_{\text{cav}})$ is the length reduction factor over half cavity length. Thus, the deduced switching threshold for 1R2B is

$$n_2 I_{\text{TH}}^{(1\text{R}2\text{B})} = 8 / \left[3\sqrt{3}k_0 B_0^2 (1+ar^2)\eta_{1/2}L_{\text{cav}} \right], \quad (8.6)$$

and it can be shown that for the switching threshold for 1R1B case is about 8x smaller compared to the 1R2B case, for lossless case (see Appendix A).

Note that the minimum transmission is zero only under critical coupling. The ON/OFF ratio at the turn-off point is generally smaller than that at turn-on, and determines the extinction ratio (ER) of the switch if the power is held here in the OFF state. Hence, to maximize the ER and modulation depth, it is important to minimize the off-state transmission at the turn-off point, while maximizing the on-state transmission at the initial detuning. To align the minimum transmission point closer to the turn-off point, one way is to operate very close to the critical detuning, but the problem with this is that the ON transmission amplitude is also substantially reduced (hence the modulation depth is small), and the difference between the turn-on and turn-off powers becomes very small. Similar effect may be achieved by increasing the round-trip loss, as shown in Fig. 8.2(b). This is because increasing the loss broadens the resonance and increases the critical detuning, which has the same effect as having the operating wavelength closer to critical detuning. However, the broadened linewidth increases the switching threshold, as expected because of the smaller build-up factor. To maximize the build-up factor at the given loss, one must further satisfy the critical coupling condition $r = a$. Under this condition the achievable input threshold is in the order $n_2 I_{\text{IN}} \sim 10^{-4}$.

8.2 Ring-enhanced Mach-Zehnder Interferometer (REMZI)

It has been shown that in the simple one-ring case, it is not possible to achieve simultaneously high extinction ratio and large modulation depth. This is fundamentally because of the symmetric shape of the resonance which gives rise to the general behavior seen in Fig. 8.2. To achieve a high ER one has to operate very close to critical detuning and inevitably the linear transmission amplitude will drop leading to limited modulation depth.

To maximize the ER and MD simultaneously, one would need to engineer the minimum turning point of the transmission curve to be directly below the turn-off point without depressing the ON transmission level. Such a situation can be achieved by using an *asymmetric* resonance such as the Fano resonance. Fano resonance is a result of interference between two pathways [7]. One way to generate a Fano resonance is by the use of a ring resonator coupled to one arm of a Mach-Zehnder interferometer (MZI), with a static bias in the other arm, as shown in Fig. 8.3.

Fig. 8.3 Schematic of a REMZI with phase bias $\Delta\phi$.

The outputs of such a ring-enhanced MZI (REMZI) depend on the interference between the frequency-sensitive phase response of the ring resonator and the static phase bias. Interference between the direct and indirect paths only takes place when the ring is near resonance. To maximize the interference the power division in the upper and lower arms is kept equal and the ring is designed with low loss and far from the critical coupling ($r=a$). Under this condition, the outputs at the two arms are given approximately by

$$T_{BAR} = \sin^2 \frac{1}{2}(\theta - \Delta\phi), \quad T_{CROSS} = \cos^2 \frac{1}{2}(\theta - \Delta\phi), \quad (8.7)$$

where θ is the phase response of 1R1B and $\Delta\phi$ is the static phase bias on the lower MZI arm. The phase bias can be used to tune the asymmetry of the resonance, as seen in Fig. 8.4 which shows the cross port transmission, T_{CROSS} , near resonance.

The resonance is very sharp as we have assumed $r=0.95$. A large r is desirable as a sharp resonance minimizes the switching threshold. When the phase bias is zero, the output is a symmetric function of δ . But as $\Delta\phi$ is increased the asymmetry also increases and the Fano resonance shifts slightly to the left. In the lossless case ($a=1$), by expanding the phase response of a ring for a given detuning δ in the first order: $\theta \cong \pi + B_0\delta$, it can be shown that the shift of the minimum transmission point (where $T_{CROSS} = 0$) is approximately given by $\delta_{MIN} = -\Delta\phi(1-r)/(1+r) = -\Delta\phi/B_0$, i.e., the shift is *reduced*, relative to the phase bias, by the

maximum build-up factor in the ring. Far away from resonance, the transmission approaches asymptotically the value given by $\cos^2 \frac{1}{2} \Delta\phi$.

Fig. 8.4. The Fano resonances for various values of $\Delta\phi$.

Fig. 8.5. The Cross output transmission as a function of incident power for various phase biases. The inset shows a blow-up of the curves at the switch-off points.

Since the Fano resonance curves in Fig. 8.4 are plotted as a function of total phase $\delta = \delta_L + \delta_{NL}$, where δ_{NL} depends on the intensity, they indirectly trace the transmission as a function of intensity for a particular initial detuning (i.e., operating wavelength). For a material with negative n_2 , the intensity increases from right to left. The intra-cavity intensity is related to the incident intensity by the same build-up factor given in Eq. (8.3). Therefore, following similar approach as for the one-ring configuration, one can plot the transmission as a function of the input intensity. Some examples are shown in Fig. 8.5, where the set of colored curves correspond to the set of $\Delta\phi$ values in Fig. 8.4. Note that by operating very close to the resonance, the initial transmission at low intensity is high. At the same time,

$\Delta\phi$ can be used to tune the minimum transmission to align with the turn-off point to achieve the maximum ER. The maximum ER can be as high as 30dB, which is much larger compared with that achievable with a symmetric resonance ($\Delta\phi=0$). On the other hand, varying $\Delta\phi$ does not change the switching power (as shown in the inset) as I_{IN} does not depend on $\Delta\phi$ [see Eq. (8.3)]. The switching threshold is of the order of 10^{-5} because of the extremely narrow Fano resonance.

Fig. 8.6. The achievable bandwidth can be up to 20GHz for relatively high ER. The highest ER can be as much as 40dB, with phase bias $\Delta\phi = 0.2\pi$ and $r = 0.9$.

Because of the narrow resonance, the ER is also very sensitive to wavelength, thereby limiting the device optical bandwidth. As a tradeoff between ER and bandwidth, it may be necessary to compromise the linewidth of the Fano resonance. Fig. 8.6 shows that an ER higher than 10dB can be achieved within a 20GHz ($\Delta\lambda=0.2\text{nm}$) optical bandwidth.

8.3 The Two-Ring Two-Bus (2R2B) configuration

A narrow asymmetric Fano resonance can also be generated in the two-resonator configuration shown in Fig. 8.7. In this case, one path is formed by the lower ring (Ring 1) while the other involves the light propagating through the upper ring (Ring 2). Fano resonance arises from the interference between the resonant modes in the two ring cavities. Note that the two rings can have different radii, and hence different resonance frequencies. Consequently, the position of the Fano resonance can be tuned by adjusting the relative position of one ring resonance to the other. The detunability of the Fano resonance distinguishes it from that of REMZI. Similarly, the coupling between the bus waveguide and the ring, denoted r_1 , and that between the two rings, r_2 , may be different, and this variability can be used to optimize the asymmetry of the Fano resonance to achieve maximum ER,

just as $\Delta\phi$ does in the case of REMZI. As a four-port device, there are two possible output ports, denoted as “drop” (D) and “through” (T), respectively, and the switch can function in a pump and probe configuration [14,15].

Fig. 8.7. The two ring resonator configuration with “through” (T) and “drop” (D) outputs. The fields at various points in the structure are indicated.

Fig. 8.8. The drop spectrum for different values of γ . The rings are assumed to be lossless and the coupling is $r_{1,2}=0.95$. The ring parameters follow the polymer ring in Ref. [2].

The linear transmissions of 2R2B configurations have been discussed in the previous chapters. Here, we recall the characteristics of 2R2B linear transmission, whereby, the spectral shape of the Drop and Through transmission mainly depends on the size detuning of both rings (γ), and coupling coefficients between the waveguides and the Ring 1 (r_1). The evolution of the Drop spectrum for different values of γ is shown in Fig. 8.8, while the calculated field distributions are shown in Fig. 8.9. For identical rings ($\gamma=1$), the resonances split symmetrically, and corresponds to equal light localization in both rings. For non-identical rings ($\gamma \neq 1$), the resonances split into broad resonance, dominated by light localization in Ring 1, and a narrow asymmetric resonance, dominated by light localization in

Ring 2 and often termed as Fano resonance. The more the resonances of the two rings detuned from each other, the sharper and more symmetric is the Fano resonance. As discussed in Chapter 4, the field distributions at both resonances in any situations are symmetric and anti-symmetric fields relative to the coupling point, where the lower (higher) frequency corresponds to the symmetric (anti-symmetric) profiles.

Fig. 8.9. The analytically calculated “Through” spectra of a linear two-ring structure for different γ values ($r_{1,2}=0.95$). The right panel shows the associated field distribution for each resonance calculated by FDTD.

Fig. 8.10. Fano resonances in the “Drop” spectra with different asymmetries can be obtained by adjusting r_1 , with $\gamma=1.05$ and $r_2 = 0.99$.

Fig. 8.10 shows the drop transmission for different values of r_1 . Note that while the right edges of the Fano resonances follow the envelope of the main resonance, the steepness on the

left edges is relatively stable due to the proximity of the minimum in D , which location is independent of r_1 . The amplitude at the dip changes somewhat for different r_1 but remains small. By operating at a wavelength just before the “dip”, it is possible to achieve switching with high extinction ratio, large modulation depth and low threshold power, as shown below.

The nonlinear transmission is calculated using the parametric approach [2]. In this approach, the fields in various parts of the structure are expressed in terms of a single parametric field in the upper ring, which in this case is chosen as E_9 . The set of equations required to solve the drop and through amplitudes are

$$\begin{aligned}
 E_D &= it_1 \tau_{21} \sqrt{a_1} \exp(i\gamma_{NL}^{14} |E_1|^2) \exp(-i\delta_1/2) E_1 \\
 E_T &= \left[1 - a_1 \tau_{21} \exp(-i\delta_1) \exp(i\gamma_{NL}^{16} |E_1|^2) \right] (r_1/it_1) E_1 \\
 E_{IN} &= \left[1 - a_1 r_1^2 \tau_{21} \exp(-i\delta_1) \exp(i\gamma_{NL}^{16} |E_1|^2) \right] (1/it_1) E_1 \\
 E_3 &= \left[r_2 - a_2 \exp(-i\delta_2) \exp(i\gamma_{NL}^{98} |E_9|^2) \right] (1/it_2) E_9 \\
 E_2 &= \left[1 - a_2 r_2 \exp(-i\delta_2) \exp(i\gamma_{NL}^{98} |E_9|^2) \right] (1/it_2) E_9 \\
 \tau_{21} &= E_3/E_2, |E_2|^2 = \sqrt{a_1} |E_1|^2, a_{1,2} = \exp(-\frac{1}{2} \alpha L_{cav}^{(1,2)})
 \end{aligned} \tag{8.8}$$

where $\gamma_{NL}^{pq} = k_0 n_{\text{eff}} n_2 c \varepsilon_o [1 - \exp(-\alpha L_{pq})] / (2\alpha)$ and $\gamma_{NL}^{pq} |E_i|^2$ is the cumulative nonlinear phase over the path L_{pq} from point p to point q . In the linear case, γ_{NL} is set to zero. It can be seen that by fixing $|E_9|^2$, we can obtain E_2 and E_3 , and thus τ_{21} , which contains the phase perturbation due to the upper ring. From $|E_2|^2$ we get $|E_1|^2$, which leads to the Through ($T = |E_T/E_{IN}|^2$) and the Drop ($D = |E_D/E_{IN}|^2$) transmission. To calculate the nonlinear response, the wavelength is fixed while E_9 is varied from zero to infinity. The nonlinear response is sensitive to wavelength, and bistability exists only for certain wavelengths which depend on the other design parameters (γ , r_1 and r_2).

Fig. 8.11 shows several examples of through transmission as a function of $n_2 I_{IN}$ for the asymmetric case ($\gamma=1.05$). In (a) we show the dependence on wavelength, with $r_{1,2}=0.85$ and $a_{1,2}=0.99$. Bistability exists for the two curves on the right. It can be seen that the ER is greater than 10 dB even if the wavelength changes by 0.1 nm. With careful wavelength tuning, an ER of 20 dB can be achieved. In (b), we show the dependence on asymmetry. We make r_1 and r_2 different so that the Fano resonance becomes more asymmetric (other parameters are $\lambda=1555$ nm and $a=0.999$). It is evident that the more asymmetric Fano resonance ($r_1 \neq r_2$) gives a much larger extinction ratio compared with the case where $r_1=r_2$. In

fact, the latter is similar to the one-ring case shown in Fig. 8.2(b). We further note that the extinction ratio generally decreases when r approaches a . In fact, the dotted curve shows that under the critical coupling condition (when $r_2=a$), bistability disappears as power is entirely absorbed in the upper ring resonance and the Fano interaction between the two rings is quenched. Hence, critical coupling is undesirable for the two-ring configuration, unlike in the single-ring case.

Fig. 8.11. Through transmission vs. $n_2 I_{IN}$ curves: (a) For various wavelengths, with $r_{1,2}=0.85$, and $a_{1,2}=0.99$. (b) For different asymmetricities, as defined by the different (r_1, r_2) combinations. The dotted curve shows the loss of bistability when the upper ring is under the critical coupling condition.

Since 2R2B is an extension from 1R2B, it is then important to compare the bistability characteristics of both configurations, as shown in Fig. 8.12. In this case, the roundtrip loss is chosen as 0.999 (for 2R2B, $a_1=0.999$), and the coupling coefficients are $r=0.95$ for 1R2B and $r_1=0.85, r_2=0.95$ for 2R2B, while γ is fixed at 1.05. At particular wavelength near the critical detuning [Fig. 8.12(a)], the switching threshold of 2R2B is lower and the MD is higher than those of 1R2B. As demonstrated in Fig. 8.12(b), the modulation depth is still high (with $ER > 10$ dB) and the threshold power does not change so much within 10 GHz (0.1 nm) range. This is not the case in 1R2B case, where closer detuning decreases both threshold power and modulation depth. The threshold of 2R2B is still lower than that of 1R2B with twice the size. This signifies the finesse enhancement in 2R2B that is independent from the cavity size scaling.

Fig. 8.12. The nonlinear Through transmission of one-ring and two-ring systems: (a) at particular incident wavelength and (b) within 10GHz.

The maximum extinction ratio achievable with an asymmetric Fano resonance is limited primarily by loss in the lower ring approximately as $\sim 1/(1-a)$. This can be shown by taking the maximum and minimum values of through and drop transmissions. In the simplest case where the loss in the upper ring is negligible (i.e., $|\tau_{21}|=1$), this gives:

$$ER_{\max}^{(T)} = \frac{T_{\max}}{T_{\min}} = \left(\frac{1+a_1}{1-a_1} \right)^2 \left(\frac{1-a_1 r_1^2}{1+a_1 r_1^2} \right)^2, \quad ER_{\max}^{(D)} = \frac{D_{\max}}{D_{\min}} = \left(\frac{1+a_1 r_1^2}{1-a_1 r_1^2} \right)^2 \quad (8.9)$$

The calculated ER for both T and D , without assuming $|\tau_{21}|=1$, are plotted as a function of a_1 in Fig. 8.13 for several values of r_1 (while r_2 is fixed). Note that the ER for T decreases, while that for D increases, with increasing r_1 . The Drop port (D) generally has a higher ER, but is

exceeded by the Through port (T) when the loss is sufficiently small.

Fig. 8.13. Extinction ratio as a function of a_1 , for both T and D outputs, and for various values of r_1 .

For $a=0.999$, the ER can be as high as 40dB. Such a low loss is possible, as demonstrated by some recent reports. First, a very high order multi-ring filter with very high Q resonators has been realized using low-loss Hydrex material [16]. For polymer microring devices a thermal-reflow technique can be used to greatly reduce scattering loss [17]. Finally, an ultrahigh quality silicon-on-insulator (SOI) microring resonator with a Q of 139,000 has been reported [18]. All these suggest that it may be feasible to realize a high-ER switch using the two-ring device. The tradeoff, however, is that both extinction ratio and threshold power will be highly sensitive to wavelength, another consequence of the narrowness of the Fano resonance. Therefore, to increase the optical bandwidth one has to compromise the resonance sharpness and the switching threshold somewhat.

Finally, we consider the dependence on the size of the ring and compare the 1R1B and 2R2B configurations. The two-ring device performance varies with the ring size in a way subtly different from the one-ring structure. The switching threshold in a one-cavity system is inversely proportional to the cavity size, in accordance with the inverse relation between the resonance linewidth and the cavity size. This rule applies also to REMZI. In the two-ring structure, the resonance broadening has little effect on the slope of the Fano resonance (the effect is similar to that of varying r_1 in Fig. 8.10), which means that the slope is relatively unchanged with cavity size. There are also more parameters (such as r_1 , r_2 , and γ) that can be used to optimize the performance. A comparison between the two cases, for various ring sizes,

is shown in Fig. 8.14. Note that a two-ring structure with smaller rings still has lower threshold than a one-ring configuration with a larger ring. This clearly shows that the two-ring devices generally have much better performance than the one-ring configuration, and are not constrained by the conventional size dependence. The switching threshold can be one order of magnitude smaller, while the ON transmission is higher giving a better modulation depth. This winning edge is due to a tunable, narrow and highly asymmetric Fano resonance made possible by the field dynamics in a two-cavity structure.

Fig. 8.14. The switching characteristics of the two-ring (solid) and one-ring (dashed) configurations with different cavity sizes. The cavity length is varied from 0.5 to 2 times the original length in both configurations, and the λ in each case is adjusted to achieve the lowest threshold. The coupling coefficient in both configurations is fixed, $r=r_2=0.95$. The (γ, r_1) for each of the two-ring cases is different, and from 0.5x to 2x, are given by (1.005, 0.8), (1.05, 0.85), and (1.05, 0.75), respectively.

8.4 Summary

The switching performance of three different ring configurations has been considered. It is shown that high extinction ratio (> 10 dB), large modulation depth (~ 1), and low switching threshold ($n_2 I_{IN} \sim 10^{-5}$) over a signal bandwidth of 0.1 nm (10 GHz) can be simultaneously achieved by both the ring-enhanced MZI and the simple two-ring configuration. In contrast, for the one-ring case, both one-bus and two-bus, although they can achieve a similar ER, they inevitably fare poorer in terms of modulation depth, switching threshold and signal bandwidth. In both cases, the switching mechanism is fundamentally based on a tailorable asymmetric Fano resonance which can be significantly sharper than the symmetric ring resonance. The Fano resonance arises from the interference between two pathways.

In the two-ring case, the interference causes a splitting of the resonance; the amount of splitting and the shapes of the resonances can be tuned by varying the physical parameters of the rings and their coupling. In the case of REMZI, the Fano resonance arises from the interference between the resonant phase of the ring and the static phase bias in the MZI. The sharpness and the shape asymmetry of the Fano resonance contribute, respectively, to the low switching threshold, the high extinction ratio, and high modulation depth of the optical switches. The ER may be maximized by minimizing the ring loss, and by optimizing the coupling between the ring and the bus waveguides. In general, the switching threshold for the two-ring device and the REMZI may be one to two orders of magnitude smaller than the one-ring configurations. In practical terms, assuming a polymer material such as the PTS whose Kerr index is $n_2=5 \times 10^{-12} \text{ cm}^2/\text{W}$ [19], and a waveguide with a cross section area of $1 \text{ } \mu\text{m}^2$, $n_2 I_{\text{IN}} \sim 10^{-6}$ implies a threshold power of 2mW. This theoretical value is 30x smaller compared with the theoretical result of Dumeige which is based on the dual-bus one-ring structure [2]. In reality, one must also consider two-photon absorption and free carrier refraction, which will further reduce the switching threshold [19].

Chapter 9. Conclusions and Future Outlook

9.1 Conclusions

In this thesis, theoretical analysis, device design, as well as experimental demonstration of multi-resonator configurations are presented. Starting from a rather rigorous analysis of whispering gallery resonators, some key parameters such as radiation loss, coupling strength, coupling loss, and propagation mode of the cavity mode are discussed. It is shown from finite-difference time-domain method that the cavity radiation loss is negligible when the radius is larger than $5\mu\text{m}$, and dominated more to a waveguide-like loss characterized by cavity surface roughness. Coupled-mode theory suggests that evanescent coupling intrinsically imparts additional phase, thereby introduces a shift in the cavity resonance wavelength that could be detrimental in filter-design. From these key parameters, simple matrix formalism is then developed and used as a fundamental theoretical framework throughout this thesis. The analysis begins with the one-cavity system from which some basic parameters such as intensity buildup factor, cavity finesse (Q factor), group delay and dispersion are derived.

The formulation is then extended to the two-cavity system, in which the concept of inter-pathway interference is introduced. Although the structure is just a simple geometric extension of one-cavity system, the spectral characteristics and the underlying physics are very different. By adjusting the frequencies of the inter-pathways interference, the two-cavity system may offer some advantages in various applications such as optical buffer, bistable switching, and high finesse structure. The optical buffer based on 2R1B has been shown to offer certain advantages compared to other schemes. It is well known that the key problem in optical buffer is the third order dispersion which distorts the buffered signal. Our proposed scheme is based on two independently controlled parameters, namely the resonance broadening and the resonance splitting. Adjusting the ratio between these two parameters around 0.6 (i.e., analogous to Rayleigh criterion in diffraction), a flat and high delay spectrum can be realized. The delay spectrum based on this scheme is not concentrated at the band-edges, as is the case in coupled resonator optical waveguide (CROW) and the cause of the low delay-bandwidth-product in CROW.

Comprehensive comparison both in frequency response and time-domain shows that the proposed scheme has higher delay-bandwidth-product and more immunity to higher order dispersion compared to the existing schemes. In terms of the number of cascade, the proposed

scheme is 2x more compact than the APF scheme, while 4x more compact than the CROW and the EIT schemes. In terms of immunity to inter-symbol interference (higher order dispersion), the proposed scheme has $\sim 2x$ the number of buffered bits before significant distortion takes place, in comparison to APF scheme with equal number of rings.

In the nonlinear bistable switch, it is shown that the bistability characteristic of one-cavity system is intrinsically limited by the symmetrical shape of the resonance. This means, a high extinction ratio (or low switching threshold) is always obtained at the expense of low modulation depth, due to the critical detuning condition. Thus, we propose asymmetric resonance based on inter-pathway interference that can simultaneously achieve high modulation depth and low switching threshold (or high extinction ratio). The finesse enhancement in the two-ring structure is independent from the cavity size scaling, and may reduce the switching threshold by one or two orders of magnitude assuming low cavity roundtrip loss.

The highest achievable resonance finesse is obtained when the upper ring is on anti-resonance with the lower ring, and it is satisfied when the upper ring is twice larger than the lower ring. The enhancement exponentially depends on the intensity buildup factor of upper ring relative to the lower ring, and can be readily tuned by adjusting the racetrack length or gap separation between the rings. This finesse enhancement scheme overcomes the problems of achieving high-finesse in 1R2B and critical coupling in 1R1B that requires unrealistically small coupling strength. In 2R1B structure, a transmission contrast of nearly 10dB is achieved within $1\mu\text{m}$ tolerance of coupling length, in contrast to the low-contrast transmission of the fabricated 1R1B structures that have different ring radius and coupling coefficients. This experimentally demonstrates the difficulty of achieving critical coupling in low-loss resonators, such as those based on silicon-on-insulator (SOI). The critical coupling is relaxed in 2R1B, because the much increased effective roundtrip loss in the finesse-enhancing situation. In 2R2B structures, the finesse enhancements of up to 20 times have been measured. An average finesse of 150 can be obtained in two-ring system that has 33% external power coupling ($r_1 \sim 0.8$) and it is believed that finesse as high as 300 can be achieved by the two-ring system with 23% external power coupling ($r_1 \sim 0.88$). Optimization of the fabrication to minimize the coupling loss will make the 2R2B useful for practical applications, where cavity finesse of 400-600 and D_{MAX} of 0.2-0.4 is possible.

The analysis extends further to multi-cavity systems that constitute a hybrid resonator photonic crystal structures. There are three types of array considered in this thesis. The first

type termed as Type I, is a configuration of which each resonator is evenly spaced with each other and side-coupled via two common waveguide buses. This structure mimics the Bragg gratings with each resonator functions as a frequency dependent mirror, and the photonic bandgap is formed when the light is on resonance with the cavity and also when the Bragg condition is satisfied. The photonic bandgap therefore consists of resonator gap and Bragg gap, which can be made overlap when the inter-resonator spacing is half the cavity length. The second type termed as Type II, is a configuration of which each resonator is mutually coupled with each other, with two waveguides directly coupled to the first and last resonator. This configuration is often termed as coupled resonator optical waveguide (CROW) and exhibits photonic bandgap at cavity anti-resonances.

The third type is a two-dimensional array of resonators, with Type I in the x -direction and Type II in y -direction. It is shown that near-ideal filter characteristics of dispersion-free and box-like output response can be obtained using this array, provided that the coupling coefficients are identical and the rows are odd in numbers. The main idea is on harnessing the non-complementary photonic bandgap properties of the two previous arrays. Introduction of extra rows provides a significant suppression of out-of-band sidelobes, while introduction of extra columns flattens the in-band transmissions ripple as a consequence of a Bragg reflection. A box-like response is experimentally demonstrated in 3×4 and 3×8 arrays of $8\mu\text{m}$ racetrack length. The respective shape-factor is 0.7 and 0.8 and the passband is 500GHz and 750GHz. More than 10dB sidelobe suppression has been measured. The measured spectra are in excellent agreement with the matrix formalism. The presence of fabrication mismatch and CIFS produce spiky features in the Drop response. The Through response however remains relatively unaffected. Pre-distorting the rings may help to reduce the effect of CIFS in the next fabrication batch.

As the concept of defect modes is well known in photonic crystal community, it is more complete to also study the existence of defect modes in Type I and Type II arrays. The defect in this context is in the form of the absence of cavity, different inter-cavity spacing, or different cavity size. Using matrix formalism, the resonance properties of the defect modes are derived. It is shown that these defect modes are analogous to donor and acceptor modes in doped semiconductor, where doping a donor (acceptor) atom is similar to a larger (smaller) defect rings. The movements of donor and acceptor modes are consistent with theoretical prediction. It is observed that the Q of the defect resonance is highest when it is located near the center of the photonic bandgap (in our case it is $\sim 22,000$, with finesse ~ 127), but the

amplitude is also smallest because of the accentuated effect of loss. In general, the Q can be significantly enhanced by slightly increasing the number of unit cells and reducing all the coupling coefficients.

9.2 Future work

Despite many of our proposed designs have been experimentally verified, there are still many improvements to be done. The existing resonator loss thus far is not low enough to unleash the potentiality of our devices, e.g., the two-ring resonator configuration. In the progressive betterment of fabrication technology, it is hoped that our future designs exhibit a much better performance. Theoretically the radiation loss is negligible for radius down to $3\mu\text{m}$. This means our present designs can be realized in smaller footprint. In the case of two-dimensional resonator arrays, the effect of CIFS should be minimized if we have the required fabrication control in pre-distorting resonator size. We should note that this is also important in designing optical buffer in the future. The presence of CIFS significantly skews the resonance splitting properties of two-ring structure, thereby distorting the delay spectrum required for optical buffer. The improvements can come from the designs themselves. Theoretically, our designs should be applicable to any type of cavity including those based on photonic crystals (PhC). Thus, we believe it is worthwhile to realize our designs in PhC platform, not only because of much smaller footprint, but also because PhC cavity has be largest nonlinearity enhancements due to its very large Q/V_{mod} .

Thus far, most of our experimental works are limited to our present equipments where only linear characteristics can be investigated. However, there are still many aspects that have not been investigated such as nonlinear characteristics and time-domain response. It is evident that silicon may not be an ideal material for the study of nonlinear interaction. Thus, it would be interesting to see the nonlinear characteristics of our devices under the material of high Kerr nonlinearity, such as in III-V compounds (AlGaAs, InP, etc) or polymers. Moreover, incorporating artificial atoms like quantum dots, silicon nanocrystals, or implantation of external atom sources may be a good research direction concerning light-matter interaction. Besides for pure time-domain characteristics, it is also needed to construct a pump-probe station for all-optical switching in our coupled cavity systems. It is also possible to normally incident a pulse laser onto our devices for the purpose of dynamic tuning. It has been shown that dynamic tuning gives a lot of physical insights even in a simple resonator, for example in overcoming the delay-bandwidth limitation in slow light

structure and in nonlinear frequency conversion of light. If for example the incident optical pump is replaced by ultraviolet source (e.g., excimer laser), permanent change of refractive index may be introduced, which is useful for post-fabrication trimming.

Author's publications

Peer-reviewed journals

Thesis related work

1. **Landobasa Y. M. Tobing**, S. Darmawan, D. R. Lim, T. Mei, "Relaxation of critical coupling and characterization of coupling induced frequency shift in two-ring structures," IEEE J. Sel. Top. Quant. Electron. (accepted for publication).
2. **Landobasa Y. M. Tobing**, P. Dumon, R. Baets, M. K. Chin, "Boxlike filter response based on complementary photonic bandgap in two-dimensional microresonator arrays," Opt. Lett. **33**, 2512 (2008).
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16. **Landobasa YM Tobing**, P. Dumon, R. Baets, and MK Chin, "Box-like filter response of two-dimensional array of microring resonator fabricated in silicon-on-insulator technology," Group IV Photonics 2008. (Oral Presentation)
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Book Chapter

23. **Landobasa Y. M. Tobing** and Pieter Dumon, "Fundamental principles of operation and notes on fabrication of photonic microresonator ," in *Photonic Microresonator Research and Applications*, Ioannis Chremmos, Otto Schwelb, and Nikolaos Uzunoglu, eds. (Springer, 2009) (invited, in press).

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Chapter 1

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Appendix A. Parametric fields approach for one-ring systems

A.1 Maxwell equation in Kerr nonlinear medium

We start from the Helmholtz equation describing light propagation inside ring resonator

$$\partial_s^2 E + (\omega/c)^2 (\varepsilon + \varepsilon_2 |E|^2) E = 0, \quad (\text{A.1})$$

where s is the curvilinear distance in the resonator and ε_2 is the effective Kerr permittivity. The ring resonator is assumed to support counter-propagating modes, which makes each propagating mode does not experience the same nonlinearity (provided their amplitude is not the same) as the counter-propagating one. This is because the two counter propagating waves form a standing wave pattern which, because of its intensity dependence, forms a grating that couples the two counter-propagating modes. By assuming the solution is of the form $E = E_F(s) \exp(i\phi_F) \exp(-i\beta_{\text{real}}s) + E_B(s) \exp(i\phi_B) \exp(i\beta_{\text{real}}s)$ and that the envelope is slowly varying, we can separate the equations in (A.1) to

$$\frac{\partial \phi_F}{\partial s} = -\frac{1}{2} i \beta_{\text{real}} c N_2 \varepsilon_0 (|E_F|^2 + 2|E_B|^2), \quad (\text{A.2})$$

$$\frac{\partial \phi_B}{\partial s} = \frac{1}{2} i \beta_{\text{real}} c N_2 \varepsilon_0 (|E_B|^2 + 2|E_F|^2), \quad (\text{A.3})$$

$$\frac{\partial E_F}{\partial s} = -\frac{\alpha}{2} E_F, \quad (\text{A.4})$$

$$\frac{\partial E_B}{\partial s} = \frac{\alpha}{2} E_B, \quad (\text{A.5})$$

The Eq. (A.2) and (A.3) signify the nonlinear phase which is dependent on the corresponding field amplitude. Eqs. (A.4) and (A.5) clearly means that the field amplitudes do not depend on the nonlinear phase. This is because the third harmonics as well as the nonlinear loss have been neglected. The Eqs. (A.4) and (A.5) indicate that the linear absorption is needed to excite the nonlinearity, which is consistent with the fact that nonlinearity is a light-matter interaction, and the light needs to be absorbed in the material (at least linearly) before experiencing nonlinear interaction. The propagation constant β is defined as $\beta \equiv \beta_{\text{real}} + \beta_{\text{imag}} = k_0 n_{\text{eff}} - i \frac{1}{2} \alpha$. Since the field varies transversally, the effective nonlinear index is defined as $N_2 = \int n_2(x) |E|^4 dx / \int |E|^2 dx$. It is important to note that the coupling between the forward and backward field only occurs when both fields are present, if there is only one propagating field then the Eq. (A.2)-(A.5) is rewritten as

$$\begin{aligned}\frac{\partial \phi_F}{\partial s} &= -\frac{1}{2} i \beta_{\text{real}} c N_2 \varepsilon_0 |E_F|^2, \\ \frac{\partial E_F}{\partial s} &= -\frac{\alpha}{2} E_F.\end{aligned}\tag{A.6}$$

Solving (A.6) gives

$$\begin{aligned}E_F &= E_{F0} \exp(-\tfrac{1}{2} \alpha s), \\ \phi_F &= -\tfrac{1}{2} i \beta_{\text{real}} c N_2 \varepsilon_0 \int_0^L E_{F0}^2 \exp(-\alpha s) ds \\ &= -i \beta_{\text{real}} c N_2 \varepsilon_0 E_{F0}^2 [1 - \exp(-\alpha L)] / (2\alpha).\end{aligned}\tag{A.7}$$

Fig. A.1. The schematic of 1R1B and 1R2B configurations

A.2 One Ring One Bus Configuration (1R1B)

The fields of 1R1B can be parametrically formulated according to E_1 :

$$\begin{aligned}|E_{\text{IN}}|^2 &= \frac{1}{t^2} [1 + a^2 r^2 - 2ar \cos(\delta - \gamma_{NL}^{12} |E_1|^2)] |E_1|^2, \\ |E_{\text{OUT}}|^2 &= |E_{\text{IN}}|^2 - |E_1|^2 (1 - a^2),\end{aligned}\tag{A.8}$$

where the loss is $a = \exp(-\alpha L/2)$, the $\gamma_{NL}^{pq} = k_0 n_{\text{eff}} N_2 \varepsilon_0 c \bar{L}_{pq} / 2$ is a nonlinear phase factor from field p to q and the characteristic length $\bar{L}_{pq}$ is defined as:

$$\bar{L}_{p,q} = [1 - \exp(-\alpha L_{pq})] / \alpha.\tag{A.9}$$

A.3 One Ring Two Bus Configuration (1R2B)

The fields of 1R2B can be parametrically formulated in the following.

$$\begin{aligned}|E_{\text{T}}|^2 &= \frac{r^2}{t^2} [1 - 2a \cos(\delta - \gamma_{NL}^{14} |E_1|^2) + a^2] |E_1|^2, \\ |E_{\text{IN}}|^2 &= \frac{1}{t^2} [1 - 2ar^2 \cos(\delta - \gamma_{NL}^{14} |E_1|^2) + a^2 r^4] |E_1|^2, \\ |E_{\text{D}}|^2 &= at^2 |E_1|^2.\end{aligned}\tag{A.10}$$

The total roundtrip phase for 1R2B is calculated as

$$\begin{aligned}
 \int_0^{L_{\text{cav}}} E_1^2 \exp(-\alpha s) ds &= \int_{L_1}^{L_2} E_1^2 \exp(-\alpha s) ds + \int_{L_3}^{L_4} E_3^2 \exp(-\alpha s) ds \\
 &= \eta_{12} \bar{L}_{12} |E_1|^2 + \eta_{34} \bar{L}_{34} |E_3|^2 \\
 &= |E_1|^2 (1 + ar^2) (\frac{1}{2} \eta_{1/2} L_{\text{cav}}),
 \end{aligned} \tag{A.11}$$

where $E_3 = r\sqrt{a}E_1$, $\eta_{12} = \eta_{34} = \eta_{1/2}$ and $L_{12} = L_{34} = \frac{1}{2}L_{\text{cav}}$. Hence, the effective cavity length is $\bar{L}_{14} = \bar{L}_{12} + ar^2 \bar{L}_{34}$.

Figure A. 2. (a) The graphical representation of intensity buildup B and I_R/I_{IN} . (b) The situations for the upward transition. The switching threshold can be found by finding the gradient at the intersected tangential point. (c) The relation of input intensity I_{IN} with I_R , here the I_{IN} is represented by A/B from (b).

A.4 Bistable switching Threshold

The mechanism of bistability can be graphically illustrated by relating intensity buildup B with $|E_1/E_{IN}|^2$ in either Eq. (A.8) or (A.10), as shown in Fig. A.2. The buildup factor is the usual Airy (or Lorentzian) lineshape, while the $|E_1/E_{IN}|^2 = I_R/I_{IN}$ is a linear line whose slope is inversely dependent upon the I_{IN} . It should be noted that the horizontal offset of B is determined by the initial frequency detuning δ_0 , as evident from Eq. (A.8) and Eq. (A.10). Upward and downward bistable transitions occur when the two curves tangentially intersect, as represented in yellow circles in Fig. A.2(a). The more offset is the buildup factor from the origin (0,0), the higher is the required input intensity, because the slope is lower. This suggests that adjusting the initial detuning is of crucial importance in attaining low threshold switching power.

The ON-transition corresponds to the flatter line, for which the input intensity is higher (lower slope), while the OFF-transition corresponds to the intersection with the steeper line. Fig. A.2(b) shows the upward transition in which three regions can be defined along with the increment of input intensity. In region I, increasing I_{IN} pulls the resonance frequency slightly

closer to the operating light frequency. In this situation, the buildup factor is still relatively low. This process continues to the point where the increase of buildup reaches instability (Region II). In this situation, there is a positive feedback between the buildup intensity and the resonance frequency shifting, where the resonance shift enhances the intra-cavity intensity which shifts the cavity resonance frequency that in turn enhance the intra-cavity intensity and so on. This process stabilizes at Region III, where the frequency pulling has passed the peak of the buildup factor, which further detunes the cavity resonance frequency away from the operating frequency away, and thus reduces the intensity buildup and slows down the resonance pulling effect. The intersections can be found by

$$dB/dI_R = 1/I_{IN}, \quad B(I_R) = I_R/I_{IN}, \quad (\text{A.12})$$

where it should be noted that the condition in (A.12) is essentially the same as the condition $dI_R/I_{IN} = \infty$ in Chapter 8.

The B used in the Fig. A.2 is the build-up factor for any configuration (1R1B or 1R2B) shown in Fig. A.1. Adopting the high finesse approximation, the I_{IN} may be written as (in this case, single ring one bus configuration is assumed)

$$\gamma_{NL}I_{IN} = \frac{\gamma_{NL}I_1}{B_1} \left[\frac{\phi^2 + (\Delta\delta/2)^2}{(\Delta\delta/2)^2} \right] \quad (\text{A.13})$$

where $\Delta\delta = 2(1-ar)/\sqrt{ar}$ is the full width half maximum FWHM. The total round trip phase is defined by $\phi = \gamma_{NL}I_1 - \delta_0$. The extreme point of I_{IN} from Eq. (A.13) is obtained by $dI_{IN}/dI_1 = [(\gamma_{NL}ar)/(1-r^2)][\phi^2 + (\Delta\delta/2)^2 + (\phi + \delta)2\phi] = 0$, yielding a quadratic equation

$$3\phi^2 + 2\delta\phi + (\Delta\delta/2)^2 = 0 \quad (\text{A.14})$$

By defining the minimum detuning for bistability as $\Delta\bar{\delta} = \Delta\delta\sqrt{3}/2$, the roots of the Eq. (A.14) are

$$\begin{aligned} \phi^{(\uparrow\downarrow)} &= -\frac{1}{3}\delta \pm \frac{1}{3}\sqrt{\delta^2 - \Delta\bar{\delta}^2} \\ \bar{\gamma}_{NL}I_1^{(\uparrow\downarrow)} &= \frac{2}{3}\delta \pm \frac{1}{3}\sqrt{\delta^2 - \Delta\bar{\delta}^2}. \end{aligned} \quad (\text{A.15})$$

Clearly, the two possible roots show two possible transitions. They are the upward and downward transition as discussed earlier as noted by the $(\uparrow\downarrow)$ arrow in (11). Moreover, the existence of bistability is determined by the initial detuning δ . The bistability only occurs

at $\delta > \Delta\bar{\delta}$ so that the roots are real number, which verifies the critical detuning condition for bistability, that is $|\omega - \omega_{\text{res}}| \geq \Delta\omega\sqrt{3}/2$. The threshold can be obtained by inserting Eq. (A.15) to Eq. (A.13). Note that, it is difficult to derive a simple analytical expression for the threshold at arbitrary detuning. However, since at $\delta \sim \Delta\bar{\delta}$, the switching threshold is at minimum, then it is more important to formulate the minimum switching threshold rather than finding it at arbitrary detuning. Hence by using $\delta \cong \Delta\bar{\delta}$ in Eq. (A.13) and Eq. (A.15), the threshold in (A.13) is expressed as

$$\gamma_{NL} I_{TH} = \frac{4}{3\sqrt{3}} \left(\frac{\Delta\delta}{B_1} \right) \quad (\text{A.16})$$

Substituting the $\gamma_{NL} = k_0 n_2 \bar{L}$ to Eq. (A.16), we have

$$n_2 I_{TH}^{(1R1B)} = \left(\frac{4\Delta\delta}{3\sqrt{3}B_1} \right) \left(\frac{\ln(a)}{\pi(a^2 - 1)} \right) \left(\frac{\lambda}{L_{\text{cav}}} \right) \quad (\text{A.17})$$

where the characteristic length expressed in terms of a : $\bar{L} = (a^2 - 1)L / 2\ln(a)$, from $a = \exp(-\alpha L / 2)$. For the one ring coupled to two waveguide buses, we need to replace $\Delta\delta / 2 = (1 - ar^2) / r\sqrt{a}$ and $\gamma_{NL} = k_0 n_2 \bar{L}_{1/2} (1 + ar^2)$ to the previous definitions, where the $\bar{L}_{1/2}$ is the characteristic length for a ring half circumference, $\bar{L}_{1/2} = (a - 1)L / 2\ln(a)$. Therefore,

$$n_2 I_{TH}^{(1R2B)} = \left(\frac{4\Delta\delta}{3\sqrt{3}(1 + ar^2)B_2} \right) \left(\frac{\ln(a)}{\pi(a - 1)} \right) \left(\frac{\lambda}{L_{\text{cav}}} \right) \quad (\text{A.18})$$

It is evident from Eq. (A.17) and (A.18) that the switching threshold is dependent on loss a and inversely proportional with the normalized cavity length L_{cav}/λ . Implicitly, It is also inversely related with the cavity build up factor B , we can re-express the (14) and (15) according to its respective operating condition

$$\begin{aligned} n_2 I_{TH}^{(1)} &\sim \frac{4}{3\pi\sqrt{3}B_1^2} \left(\frac{\lambda}{L_{\text{cav}}} \right), & \text{for } r = a \\ n_2 I_{TH}^{(2)} &\sim \frac{8}{3\pi\sqrt{3}(1 + r^2)B_2^2} \left(\frac{\lambda}{L_{\text{cav}}} \right), & \text{for } a \sim 1 \end{aligned} \quad (\text{A.19})$$

where the maximum build-up factor for the single bus case is $B_1 = 1/(1 - r^2)$ (for $r=a$) and for double bus case is $B_2 = 1/(1 - r^2)$. The switching threshold for both configurations is shown

in Fig. A.3, where the normalized intensity $n_2 I_{TH}$ is represented in dB. Intuitively, the switching threshold for 1R1B should be lower than that of 1R2B, in general, due to the fact that the intensity buildup is intrinsically higher in 1R1B. The ratio between the thresholds is

$$\frac{I_{TH}^{(2)}}{I_{TH}^{(1)}} = \left(\frac{2}{1 + ar^2} \right) \left(\frac{\Delta\delta_2}{\Delta\delta_1} \right) \left(\frac{B_1}{B_2} \right). \quad (\text{A.20})$$

The contour plot of Eq. (A.20) is shown in Fig. A.4. The dashed line represents the critical coupling condition for the single bus case. It is evident that $I_{TH}^{(1)} < I_{TH}^{(2)}$ for all (r, a) . In the high finesse limit ($r \sim 1$), one can show that the ratio in (A.20) approximates to 8.

Figure. A.3. The contour plot of switching threshold for: (a) single bus and (b) double bus configurations. The dashed black line in (a) is the critical coupling condition.

Figure. A.4. The contour plot of the threshold ratio between the single and double bus configuration.