Modified optical vortices : design and experiment

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Modified Optical Vortices: Design and Experiment

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SUMMARY

Optical vortices discussed in this thesis refer to the phase singularities arising from zeros of amplitude in scalar optical fields. As we all know, wavefront of a monochromatic laser beam is determined by the phase structure in the space. Therefore, we can expect an extraordinary wavefront for an optical vortex since the phase is undefined at the particular location of zero amplitude. Apart from usual spherical or plane wavefront, a laser beam carrying optical vortex does have a special helical wavefront centered in the phase singularity.

Normally optical vortices are generated by imposing the phase function $\exp(il\theta)$ to a TEM_{00} laser beam. Here θ is the polar angle and l is the topological charge that represents the strength of helical wavefront and takes an integer value. In this thesis, unconventional optical vortices with more general phase function in polar coordinates are studied. By manipulating the phase function of optical vortex in azimuthal direction, we are able to produce optical vortices with polygonal shapes in far-field and the collinear superposition of optical vortices with different topological charges. On the other hand, particular radial phase modulations are proved to suppress sidelobes of focused optical vortices.

Fundamental TEM_{00} mode emitted from a laser can be converted into various light beams embedded with unconventional optical vortices through diffractive optical elements. We choose to use phase-only diffractive optical elements in this study because of two major advantages brought by phase-only transmittance. One

is the high efficiency because a passive optical element will absorb a part of incoming power if amplitude modulation applies. The other is the ready implementation and fabrication of phase-only diffractive optical elements. However, the apparent trade-off in phase-only diffractive optical elements is the loss of information on the amplitude part of transmittance function. Hence, some optimizations must be applied to the design of phase-only diffractive optical elements in order to control this drawback, e.g. an iterative algorithm is developed in this thesis to generate superposition of multiple optical vortices by a phase-only element which is supposed to have a complex transmittance for 100% accuracy.

Applications of optical vortices are closely related to the well-known fact that the field of optical vortex carries orbital angular momentum. The quantized amount of orbital angular momentum is $l\hbar$ per photon for an optical vortex of topological charge equal to l . The orbital angular momentum can be transferred to microscopic objects trapped by optical tweezers and thus induces rotations. The combination of optical vortices and optical tweezers provides essential micro-opto-mechanical tools for the study of colloidal or optofluidic systems. We explore possible applications by incorporating two types of novel optical vortices, anisotropic optical vortices and concentric optical vortices, in an optical tweezers system. Another useful property of optical vortex is that the states of orbital angular momentum are orthogonal to each other for different topological charges. Collinear superposition of optical vortices generated by a dynamically modulated liquid-crystal spatial light modulator has shown the ability to realize multiplexing

of channels which are labeled by orthogonal states of orbital angular momentum in a free-space optical communication link. A possible multiple-signal-level model of orbital angular momentum multiplexing of optical channels is also discussed.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

A phase defect appears as a dark spot within a light beam where the amplitude vanishes, so the phase of this particular spot becomes undetermined [1]. Phase defects surrounded by helical wavefronts in optical fields are widely known as optical vortices [2]. The study of optical vortices has initiated a new branch in optics, i.e. singular optics. In 1992, Allen and *et al.* predicted that photons in optical vortices carry orbital angular momentum (OAM) [3]. The existence of OAM was demonstrated in 1995 by He and *et al.* in an experiment showing a rotation of micro-particle driven by the torque originated from OAM transferred from optical vortices [4].

A laser beam embedded with a single optical vortex is named as a helical beam due to the unique spiral wavefront. Helical beams have drawn a lot of interest in various research fields in the last two decades. For example, helical beams are used to extend the trapping ability and functionality of optical tweezers systems. Owing to the destructive interference arising from the spiral wavefront, a helical beam focuses to a ring instead of a point. This unusual intensity profile is employed to some optical tweezers systems [5-9] for optically trapping objects that are not suitable to be handled by focused Gaussian laser beams. The study on the phenomenon of OAM transfer to illuminated microscopic objects from helical

beams opens up an opportunity for developing novel optical tweezers that are capable of exerting torques on trapped objects [10-13]. Another important aspect of the applications of helical beam relies on the orthogonality of states of OAM [14]. Orthogonal OAM-states are exploited to create multi-dimensional quantum entanglements [15-19] and serve as information carriers in optical communication systems [20, 21].

Variations of helical beams expand the use of optical vortices to more research topics in addition to the above-mentioned applications. For example, the superposition of a helical beam and a Gaussian beam produces an oriented interference pattern used for manipulating asymmetric objects [22]. The other superposition of two helical beams of reverse topological charges has been implemented to form three-dimensional optical trapped structures [23] and optically selective trapping [24]. Given different Gouy phases, an optical bottle beam is generated by the superposition of two helical beams [25]. An optical bottle beam gives a dark focus surrounded by regions of high intensity and could be used to confine small particles like cold atom clouds repulsed from high-intensity regions. Furthermore, the combination of multiple optical vortices is exploited in optical communications [26]. Modulating the change of azimuthal phase function offers the irregular intensity profiles in the focus and thus shows the ability to control the rotating trajectory of microscopic objects [27]. Similarly the modulation on the radial phase also brings novel features to helical beams especially in the propagation direction. A well-know example is the high-order

Bessel beam labeled as “diffraction-free beam” within certain propagation distance [28, 29]. High-order Bessel beams can be generated by adding linear radial phase modulation to a normal helical beam [30, 31]. The high-order Bessel beam is found to be resistant to perturbations during propagations [32, 33] and the transfer of OAM to optical trapped particles are demonstrated [34, 35].

The consistently growing demand of incorporating optical vortices into new applications requires the usage of engineered optical vortex in each particular case. These variations of optical vortices can be generated by tailored diffractive optical elements. Some diffractive optical elements are even more flexible, e.g. electronic-addressed spatial light modulators whose transmittance functions are fed from computers. Mostly diffractive optical elements have phase-only or amplitude-only transmittances. However, sometimes the required transmittance is in between these two situations, i.e. a complex value. Thus, more efforts are needed to transform the complex transmittance to the simpler phase-only format. These challenges that arise under the interesting research topic of optical vortices and their applications have motivated the thesis study.

1.2 Objectives

The major purpose of this study is to develop various designs of diffractive optical elements for the generation of optical vortices and then investigate the characteristics and possible applications of these optical vortices. The research will include the following aims.

1. To identify the design problems in the generation of optical vortex for a specific application.
2. To devise methods of solving these design problems, this will involve the investigation of some algorithms and encoding techniques.
3. To study the characteristics of various optical vortices generated by the proposed methods.
4. To apply these novel optical vortices to specific applications and evaluate the performances.

More specifically, optical vortices created in this thesis are expected to deliver the desired optical fields according to specific requirements given by applications. At the same time, these optical fields should preserve the unique properties of optical vortices such as orbital angular momentum.

During the design process, the scalar diffraction theory may be taken into account in order to study the propagation of optical field. Because of the ready implementation and popular usage of phase-only diffractive optical elements, we focus mostly on designs that can be expressed as phase-only transmittance functions. In some cases, this requirement may not be fulfilled straightforwardly as complex transmittances emerge. Therefore, optimization approaches are facilitated to provide phase-only approximations to the complex transmittances.

1.3 Major contribution of the Thesis

This thesis makes three major contributions to the research field of optical vortices.

Generation of multi-helix beam

A phase-only diffractive optical element is designed to generate the collinear superposition of multiple optical vortices, i.e. a multi-helix beam. Generally this diffractive optical element is supposed to have a complex transmittance instead of phase-only one. However, an iterative algorithm is developed in this thesis to provide an approximation to the complex transmittance in the form of phase-only function. The algorithm is able to produce multi-helix beams with efficiencies higher than 80% and the errors maintained within acceptable values. Multi-helix beams have been proved to be useful in increasing the transmission capacity of free-space optical communication system. This is achieved by multiplexing the channels labeled as states of orbital angular momentum associated with optical vortices.

Anisotropic optical vortices

Conventional optical vortices have circularly symmetric amplitude profiles. However, anisotropic optical vortices with unsymmetrical intensity distributions can be created by modulating the azimuthal phase functions. A conventional optical vortex has the azimuthal phases increasing linearly in one direction and the azimuthal phase gradient is the same everywhere. In this thesis, we demonstrate a deterministic approach to modulating the azimuthal phase functions for generation of optical vortices that have polygonal intensity shapes in the far-field. And the value of phase gradient depends on the location in the azimuthal direction. The anisotropic optical vortex has shown to be able to circulate trapped micro-particles

in an unusual trajectory when it is coupled into optical tweezers system.

Radius-adjustable optical vortices with suppressed sidelobes

Like the rippling Airy disk, a focused optical vortex shows sidelobes on the outer side of the first bright ring. The magnitude of sidelobes increases with the topological charge of optical vortex. A segmented radial phase modulation is proposed to suppress the sidelobes. And it is noted the radial phase modulation term changes the radius of the primary ring. This technique is useful for constructing concentric rings system made up of optical vortices with different topological charges because the suppressed sidelobes reduce the interference between neighboring rings and the ability to adjust radius of primary ring enables the rearrangement of ring topology.

In summary, the first two variations of optical vortices come from the modulation on the azimuthal phase functions of optical fields. On the other hand, the last one is created by adding a radially modulated phase function to a conventional optical vortex.

1.4 Organization of the Thesis

The thesis is organized as follows after introducing motivations and objectives of this project in Chapter 1.

Chapter 2 presents the background knowledge of optical vortices and an overview of diffractive optical elements.

Chapter 3 describes an iterative algorithm developed to solve the practical problem encountered in the generation of collinear superposed optical vortices.

And this method is implemented to a free-space optical data link.

Chapter 4 presents a type of optical vortex with asymmetric shapes of intensity in the far-field. An approach to generating the anisotropic optical vortices is established. The anisotropic optical vortex is further incorporated into a typical optical tweezers system.

Chapter 5 provides a design of phase-only diffractive optical element which produces optical vortex with suppressed sidelobes in the far-field. The design can be used to generate concentric multiple rings in an optical tweezers system.

Chapter 6 summarizes these contributions and discusses the possible further work.

CHAPTER 2

BACKGROUND

In the first part of this Chapter, a literature review on the topic of optical vortices is provided. It starts with the historic origin of optical vortices and covers the associated orbital angular momentum of light. Then some generation techniques are presented and compared. At the end of this part, variations of optical vortices are introduced and various applications of optical vortices are mentioned.

Secondly, the basic concepts of diffractive optical element are introduced. We concentrate on the design and realization of the phase-only element. A reconfigurable element known as a spatial light modulator is described.

2.1 Optical vortices

2.1.1 Laguerre-Gaussian modes

In the field of laser physics, there is a well-known family of Laguerre-Gaussian (LG) modes possessing an azimuthally dependent term $\exp(i\ell\theta)$ that corresponds to a helical wavefront [36], where the integer ℓ termed topological charge and θ is the angular coordinate. LG modes are solutions to the paraxial wave equation in polar coordinates [37]

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial E}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 E}{\partial \varphi^2} + 2ik \frac{\partial E}{\partial z} = 0. \quad (2.1)$$

The complex amplitude of an LG mode is

$$\begin{aligned} E(LG_p^l) = & E_0 \frac{w_0}{w(z)} \left(\frac{\sqrt{2}\rho}{w(z)} \right)^{|l|} \exp \left(-\frac{\rho^2}{w(z)^2} \right) L_p^{|l|} \left(\frac{2\rho^2}{w(z)^2} \right) \\ & \times \exp \left\{ i \left[\frac{k\rho^2}{2R(z)} + l\theta + (2p + |l| + 1) \arctan \left(\frac{z}{z_R} \right) \right] \right\} \end{aligned} \quad (2.2)$$

where E_0 is the amplitude parameter, w_0 is the beam waist and $z_R = kw_0^2/2$

is the Rayleigh range. The transverse beam radius at different propagation distance

equals $w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2}$. The radius of the wavefront curvature is

$R(z) = z \left[1 + \left(\frac{z_R}{z} \right)^2 \right]$. There is an associated Laguerre polynomial involved in the

amplitude part with the definition as $L_p^l(x) = \frac{e^x x^{-l}}{p!} \frac{d^p}{dx^p} (x^{l+p} e^{-x})$, where l is the

azimuthal index and p is the radial index determining the number of nodes in the

radial direction. Fig. 2.1 demonstrates the transverse intensity profiles of LG

modes with various combinations of these two indices. The number of intensity

maxima in the radial direction is equal to $p + 1$ (including central maxima).

Figure 2.1 Intensity profiles of Laguerre-Gaussian transverse modes

LG mode with nonzero l has a vanishing amplitude at the center $\rho=0$ as indicated by Eq. 2.2. Therefore, the dark central point is considered as a phase singularity because the phase value is undefined when the amplitude equals zero. In order to avoid the complicated expression in Eq. 2.2, only the lowest radial mode with $p=0$ is studied in most cases. Thus, the complex amplitude of LG mode at the distance $z=0$ is simplified to

$$E(LG_0^l)|_{z=0} = E_0 \left(\frac{\sqrt{2}\rho}{w_0} \right)^{|l|} \exp\left(-\frac{\rho^2}{w_0^2}\right) \exp(il\theta) \quad (2.3)$$

as $R(0) \rightarrow \infty$ and $w(0) = w_0$. The unique phase term $\exp(il\theta)$ presents a helical

wavefront surrounding the phase singularity as illustrated in Fig. 2.2 and the amplitude part follows a Gaussian distribution modulated by a high-power term of ρ .

Figure 2.2 A helical wavefront [38]

Optical vortices are closely related to the phase singularities which occur at the zeros in light waves. They are characterized by the helical wavefronts appearing around the phase singularities. It was only possible to generate optical vortices after the invention of laser, a coherent light source. Thus, it is natural to find that the well-defined LG laser mode, which has both phase singularity and helical wavefront, is considered as a model to study optical vortices. However, it is difficult to produce a laser beam having exactly the amplitude expression addressed in Eq. 2.3. So a general expression as below for the complex amplitude of an optical vortex is commonly used.

$$E(\rho, \theta) = A(\rho) \exp(il\theta) \quad (2.4)$$

A question might be raised here as to whether this expression still represents an optical vortex. Suppose the function $A(\rho)$ is nonzero at the point of $\rho = 0$, as a result, there is no phase singularity in the center although the helical wavefront remains. In answering this question, a brief introduction to the scalar diffraction theory is given in the next section.

2.1.2 Scalar diffraction theory

Given the optical elements discussed in this thesis have large diffracting structure as compared with the wavelength of light and the sequential optical fields mostly propagate in free-space, a linear, isotropic, homogeneous and nondispersive medium, scalar diffraction theory is accurate in studying the light wave behavior [39].

Consider an optical field as described in Eq. 2.4 propagating a distance of z to the observation plane in cylindrical coordinates as illustrated in Fig. 2.2 and assume the propagation distance is much larger than the wavelength of light. Applying Huygens-Fresnel principle, the complex amplitude of a particular point (r, φ) in the observation plane is represented as the integral

$$E(r, \varphi) = \frac{z}{i\lambda} \iint_{\Sigma} E(\rho, \theta) \frac{\exp[ikR(\rho, \theta)]}{[R(\rho, \theta)]^2} \rho d\rho d\theta. \quad (2.5)$$

Here λ is the wavelength of light and wave number $k = 2\pi/\lambda$. $R(\rho, \theta)$ is the

distance between the points (ρ, θ) and (r, φ) while Σ is the aperture in the original plane. This integral becomes [40]

$$E(r, \varphi) = \frac{i}{\lambda z} \int_0^{2\pi} \int_0^\infty E(\rho, \theta) \exp \left\{ \frac{ik}{2z} [\rho^2 + r^2 - 2\rho r \cos(\theta - \varphi)] \right\} \rho d\rho d\theta \quad (2.6)$$

under the Fresnel approximation. The accuracy of this approximation is ensured if the propagation distance satisfies [39]

$$z^3 \gg \frac{\pi}{4\lambda} \max \left\{ [\rho^2 + r^2 - 2\rho r \cos(\theta - \varphi)]^2 \right\}. \quad (2.7)$$

Figure 2.3 Geometry for the propagation of wave in cylindrical coordinates

2.1.3 Helical wavefronts

Back to the question raised in section 2.1.1, after substituting Eq. 2.4 into Eq. 2.6

the evolving optical field becomes [40]

$$E(r, \varphi) = (-i)^{l-1} \frac{k}{z} \exp(ikz) \exp(il\varphi) \int_0^\infty A(\rho) \exp \left[\frac{ik}{2z} (\rho^2 + r^2) \right] J_l \left(\frac{k\rho r}{z} \right) \rho d\rho, \quad (2.8)$$

where $J_l(x)$ is the l th order Bessel function of the first kind defined as

$$J_l(x) = \frac{(-i)^l}{2\pi} \int_0^{2\pi} \exp[i(lt + x \cos t)] dt. \quad (2.9)$$

Now, the amplitude at the center point $r = 0$ is equal to zero because $J_l(0) = 0$ as long as $l \neq 0$. Hence, the phase singularity appears in the center of laser beam accompanied by the unchanged helical wavefront $\exp(il\varphi)$ no matter whether the original optical field has a phase singularity or not. The conclusion here indicates that helical wavefronts imply the presence of phase singularities in free-space. This makes the generations of optical vortices easier because only the phase term $\exp(il\theta)$ corresponding to the helical wavefront needs to be taken care of. It is noticed that Laguerre-Gaussian beams are often referred to a laser beam bearing helical wavefront or optical vortex in some literature although we know it is not exactly accurate because the radial complex amplitude part $A(\rho)$ might not be identical with the expression in Eq. 2.2 or Eq. 2.3. Alternatively, “helical beams” are used to describe such laser beams in this thesis.

It is worthy to note the phase singularity can exist without the presence of a helical wavefront. For example, if a line of zeros is perpendicular the beam axis, this singularity is called the edge dislocation.

An important parameter of these helical wavefronts is the topological charge. A topological charge or winding number of an optical vortex is defined by the line integral in Eq. 2.10 [36] as a measurement of the dislocation strength,

$$s = \frac{1}{2\pi} \oint_C \nabla \chi \cdot d\mathbf{r} \quad (2.10)$$

where χ is the phase function, $\mathbf{r}$ is the position vector, and C is some closed and simple curve enclosing the singular point. The parameter s describes the azimuthal gradient of the helical phase function. In the case of a helical wavefront $\exp(il\theta)$, the topological charge s equals l . The examples in Fig. 2.4 demonstrate that the topological charge controls the helicity and intensity distribution of an optical vortex. In general, the wavefront of an optical vortex with topological charge l has a $|l|$ -started helicoids and the sign of l determines the rotating direction. The topological charge has a positive sign if the screw rotates anticlockwise when looking along the propagation direction, negative if clockwise. The radius of doughnut transverse intensity distribution measured from the center to the radial maximum increases with the topological charge.

The topological charge provides a parameter to describe the global phase structure of a helical wavefront; moreover, the total topological charge within a bounded area is conserved under continuous variation of the field [36] if no singularity crosses the boundary. This is the singularity conservation law which provides a fact that an optical vortex keeps the topological charge unchanged while propagating in free-space without disruptions.

Figure 2.4 Helical wavefronts with various values of topological charges. The top row shows a conventional Gaussian beam with plane wavefront. The rightmost column is the transverse phase distributions of these wavefronts. In the middle are the transverse intensity profiles. The propagation direction of wavefront is from left to right. [38]

2.1.4 Orbital angular momentum

One of the reasons why optical vortices attract so much interest is the connection between helical wavefronts and orbital angular momentum (OAM). There has been

a lot of literature about this since the seminal paper of Allen *et al* [3] shows that Laguerre-Gaussian modes of non-zero topological charges carry OAM.

As the OAM is associated with helical wavefronts, the counterpart for circular polarization is spin angular momentum (SAM). It is not surprising to find that SAM of light was verified and measured [41] much earlier than OAM because the generation of circular polarization does not require the coherent light source. OAM together with SAM compose the whole angular momentum of a vector light wave.

The linear momentum of light wave per unit volume (linear momentum density) is given by [42]

$$\mathbf{p} = \varepsilon_0 \mathbf{E} \times \mathbf{B}. \quad (2.11)$$

For a generalized helical beam with complex amplitude

$$u(r, \phi, z) = u(r, z) \exp(il\phi), \quad (2.12)$$

it has been shown in the paraxial approximation that the components of linear momentum density are [3, 43]

$$\begin{aligned} p_r &= \varepsilon_0 \frac{\omega k r z}{z_R^2 + z^2} |u|^2, \\ p_\phi &= \varepsilon_0 \left(\frac{\omega l}{r} |u|^2 - \frac{1}{2} \omega \sigma \frac{\partial |u|^2}{\partial r} \right), \\ p_z &= \varepsilon_0 \omega k |u|^2, \end{aligned} \quad (2.13)$$

where ω is the angular frequency of light, k is the wavenumber and z_R is the

Rayleigh range. The parameter σ describes the degree of polarization; $\sigma = +1/-1$ corresponds to left-handed and right-handed circular polarization respectively and $-1 < \sigma < 1$ for elliptically polarized light. These three orthogonal components are illustrated in Fig. 2.5.

Figure 2.5 The components of linear momentum density [43]

The angular momentum density of a light wave is given by [42]

$$\mathbf{j} = \mathbf{r} \times \mathbf{p}, \quad (2.13)$$

where $\mathbf{r}$ is the position vector. Using the result derived in Eq. 2.12, the angular momentum density in the direction of propagation becomes

$$j_z = \varepsilon_0 \left(\omega l |u|^2 - \frac{1}{2} \omega \sigma r \frac{\partial |u|^2}{\partial r} \right). \quad (2.14)$$

If the angular momentum density is integrated over the cross-section and then divided by the energy, the ratio is given by

$$\frac{J_z}{W} = \frac{l}{\omega} + \frac{\sigma}{\omega}. \quad (2.15)$$

This relation suggests that this laser beam carries OAM of $l\hbar$ per photon and SAM of $\sigma\hbar$ per photon [3].

Simplified from Eq. 2.14, the OAM density of a linearly polarized ($\sigma = 0$) helical beam is

$$j_z = \varepsilon_0 \omega l |u|^2 \quad (2.16)$$

For an arbitrary light field $u(r, \varphi)$, the calculation of the density of OAM is based on the decomposition into so-called helical modes [44, 45]

$$u(r, \varphi) = \sum_{l=-\infty}^{\infty} E_l(r) \exp(il\varphi) \quad (2.17)$$

where $E_l(r) = \frac{1}{2\pi} \int_{-\pi}^{\pi} u(r, \varphi) \exp(-il\varphi) d\varphi$. In so doing, each component is a simple

helical beam easy to calculate. Thus, the total OAM per unit volume is the superposition of contributions of all the component helical beams given by

$$j_z = \varepsilon_0 \omega \sum_{l=-\infty}^{\infty} l \times E_l(r) \quad (2.18)$$

The existence of OAM of light was proved in 1995 by transferring OAM to absorptive microparticles [4]. The torque inducing the orbital rotation of microparticle arises from the transfer of azimuthal momentum component p_ϕ in Eq. 2.13 to the object as illustrated in Fig. 2.6. Furthermore, the mechanical equivalence of a circularly polarized light with $\sigma = 1$ and a linearly polarized

helical beam with $l = 1$ was demonstrated in an optical tweezers system when the object is larger than the optical vortex in Fig. 2.7a [13]. On the contrary, if the size of the optical vortex is larger than the object as shown in Fig. 2.7b, the SAM transferred from a circular polarized helical beam makes the object spin and the OAM tends to rotate particles around the beam axis as indicated in Fig. 2.8a and 2.8b respectively [11].

Figure 2.6 Rotation of microparticle induced by the transfer of azimuthal momentum p_ϕ [38]

Figure 2.7 Configurations of optical vortices and objects (a) the object is larger than the optical vortex, (b) the object is smaller than the optical vortex.[38]

Figure 2.8 Rotations driven by optical vortices in optical tweezers (a) A birefringent calcite fragment ($\sim 3\mu\text{m} \times 1\mu\text{m}$) spins on its own axis due to the transfer of SAM from a circular polarized helical beam, (b) Driven by the torque exerted by the OAM transfer from the optical vortex, a high refractive index spherical particle (radius $\sim 1\mu\text{m}$) orbits along the bright ring of optical vortex with the topological charge $l = 8$. [38]

2.1.5 Generation of optical vortices

Recalled from section 2.1.3, the formation of optical vortices depends on helical wavefronts, i.e. the unique phase term $\exp(il\theta)$. As helical beams are originated from Laguerre-Gaussian laser cavity modes, it is natural to consider generating helical beams directly from a laser cavity [46, 47]. However, extra-cavity formations of optical vortices are widely used in laboratories because of the greater

flexibility and easier implementations. At present, there are three methods widely in use: mode converters, spiral phase plates and computer-generated holograms.

2.1.5.1 Mode converters

The usage of a mode converter in the helical beam generation was first proposed by Tamm and Weiss [48] and several researchers have worked on this method after that [49-52]. A mode converter is a combination of cylindrical lenses and other optical components where cylindrical lenses play an important role in the conversion from high-order Hermite-Gaussian (HG) modes to the corresponding high-order Laguerre-Gaussian (LG) modes.

HG modes form an orthogonal set in Cartesian coordinates and are expressed as [50]

$$u_{nm}^{HG}(x, y, z) = C_{nm}^{HG} \frac{1}{w(z)} \exp\left[-\frac{x^2 + y^2}{w^2(z)}\right] \exp\left[-\frac{ik(x^2 + y^2)}{2R(z)}\right] \times \exp[-i(n + m + 1)\Psi(z)] H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \quad (2.19)$$

while orthogonal LG modes are defined in Cylindrical coordinates as

$$u_{nm}^{LG}(r, \phi, z) = C_{nm}^{LG} \frac{1}{w(z)} \exp\left[-\frac{r^2}{w^2(z)}\right] \exp\left[-\frac{ikr^2}{2R(z)}\right] \times \exp[-i(n + m + 1)\Psi(z)] \exp[-i(n - m)\phi] \times (-1)^{\min(n,m)} \left[\frac{\sqrt{2}r}{w(z)}\right]^{|n-m|} L_{\min(n,m)}^{|n-m|}\left(\frac{2r^2}{w^2(z)}\right) \quad (2.20)$$

with

$$\begin{aligned} R(z) &= \frac{z_R^2 + z^2}{z}, \\ w(z) &= \frac{2(z_R^2 + z^2)}{kz_R}, \\ \Psi(z) &= \arctan\left(\frac{z}{z_R}\right), \end{aligned} \quad (2.21)$$

where $H_n(x)$ is the Hermite polynomial of order n and $L_p^l(x)$ is the associated Laguerre polynomial. Here, the normalized constants are given by

$$\begin{aligned} C_{nm}^{HG} &= 2^{-(n+m)/2} \sqrt{\frac{2}{\pi n!m!}}, \\ C_{nm}^{LG} &= \min(n, m)! \sqrt{\frac{2}{\pi n!m!}}. \end{aligned} \quad (2.22)$$

It is noted that the expression of LG modes differs from the previous one in Eq. 2.2 by a different index definition as the radial index $p = \min(n, m)$ and azimuthal index $l = n - m$.

Since both these two orthogonal bases are complete, they can be described by each other, e.g. [3, 49, 50]

$$u_{nm}^{LG}(x, y, z) = \sum_{k=0}^{n+m} i^k b(n, m, k) u_{n+m-k, k}^{HG}(x, y, z) \quad (2.23)$$

with

$$b(n, m, k) = \frac{1}{k!} \frac{d^k}{dt^k} \left[(1-t)^n (1+t)^m \right] \Big|_{t=0} \sqrt{\frac{(n+m-k)!k!}{2^{n+m} n!m!}}. \quad (2.24)$$

Amazingly, a HG mode with principle axis at the angle of 45° from the positive

part of x -axis can be decomposed into a very close form to Eq. 2.23, which is given by [3,49, 50]

$$u_{nm}^{HG} \left(\frac{x+y}{\sqrt{2}}, \frac{x-y}{\sqrt{2}}, z \right) = \sum_{k=0}^{n+m} b(n, m, k) u_{n+m-k, k}^{HG}(x, y, z) \quad (2.25)$$

with the same coefficients $b(n, m, k)$ defined in Eq. 2.24. The only difference is the additional phase factor i^k of Eq. 2.23. Some examples are given in Fig. 2.9. In order to produce $\pi/2$ phase difference between the neighboring components, different Gouy phases are introduced by a pair of parallel cylindrical lenses separated by a distance

$$d = f/\sqrt{2}, \quad (2.26)$$

where f is the focal length of cylindrical lens. The whole optical system used to generate LG modes from HG modes is shown in Fig. 2.10. The intensity profiles of high-order HG modes are obtained after lens f_1 and the consequent LG modes after mode conversion are collected at lens f_5 (see Fig. 2.11).

	(nm)	LG	
02			$= \frac{1}{2} \begin{matrix} \text{---} \\ \text{---} \end{matrix} + \frac{i}{\sqrt{2}} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix} - \frac{1}{2} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$
11			$= \frac{1}{\sqrt{2}} \begin{matrix} \text{---} \\ \text{---} \end{matrix} + \frac{1}{\sqrt{2}} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$
20			$= \frac{1}{2} \begin{matrix} \text{---} \\ \text{---} \end{matrix} - \frac{i}{\sqrt{2}} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix} - \frac{1}{2} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$
		HG	
02			$= \frac{1}{2} \begin{matrix} \text{---} \\ \text{---} \end{matrix} + \frac{1}{\sqrt{2}} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix} + \frac{1}{2} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$
11			$= \frac{1}{\sqrt{2}} \begin{matrix} \text{---} \\ \text{---} \end{matrix} - \frac{1}{\sqrt{2}} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$
20			$= \frac{1}{2} \begin{matrix} \text{---} \\ \text{---} \end{matrix} - \frac{1}{\sqrt{2}} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix} + \frac{1}{2} \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix}$

Figure 2.9 Examples showing the similarity between the decompositions of LG modes and tilted HG modes with the same indices. [50]

Figure 2.10 A sketch of a mode converter. Mirrors M_1 and M_2 form a resonant cavity with HeNe gain tube inside. Perpendicular metal wires W are placed in the cavity to select particular high-order HG modes. A beam splitter (BS) separates the output of laser into two directions. One of the outputs goes through a $\pi/2$ mode converter made up of two cylindrical lenses f_3 and f_4 . The distance between these two cylindrical lenses is controlled by Eq. 2.26. f_1 , f_2 and f_5 are convex lenses used for collimation or projection purposes. [50]

Figure 2.11 Experimental results showing the conversion of HG_{nm} laser modes to the corresponding LG_{nm} modes. [50]

As a theoretically lossless conversion process, the main advantage of mode converters is that the method can generate helical beams at very high power, which is useful in some applications like atom trapping and cooling [53-56], nevertheless this method requires critical alignments either in laser cavities or out of it and has difficulties in providing LG beams of high topological charge ($l > 10$) because the required high-order HG modes are not easy to generated from laser cavities.

2.1.5.2 Spiral phase plates

A more straightforward method of using spiral phase plates (SPPs) to produce optical vortices was proposed by Beijersbergen *et al* [57]. According to the conclusion in Section 2.1.3, a helical wavefront relevant to the phase term $\exp(il\theta)$ leads to an optical vortex in free-space. An SPP (see Fig. 2.12) generates a helical wavefront from an incoming plane wave by imprinting the spiral phase

term $\exp(il\theta)$ onto it. The thickness of phase plate gradually increases along the azimuthal direction until it reaches the height given by [57]

$$h = \frac{l\lambda}{\Delta n}, \quad (2.27)$$

where l is the desired topological charge of helical wavefront, λ is the wavelength of light and Δn is the refractive index difference between the phase plate and the surroundings. A typical setup as sketched in Fig. 2.13 converts a Gaussian beam into a helical beam by the use of SPP enclosed in a brass cell filled with index-matching fluid. The refractive index of the fluid is controlled by temperatures and thus the resultant topological charge l will be changed accordingly as shown in Fig. 2.14. When the surrounding refractive index is fixed, e.g. in vacuum, the height of SPP is proportional to the required topological charge. Sometimes it is difficult to fabricate such an SPP with that kind of height if the topological charge is large. Hence, a phase ramping method [paper IX] was proposed in a lithographic manufacture of SPPs in order to shrink the height (see Fig. 2.15). In that case, the maximum possible height is only $h = \lambda/\Delta n$. As we can see from these two cases, the feature size of this element is of the same order of wavelength. This demands an essential sub-wavelength tolerance control of fabrication.

Figure 2.12 The spiral phase plate. [57]

Figure 2.13 A sketch of the experimental setup using an SPP [57]

Figure 2.14 Far-field intensity distributions of helical beams produced by a SPP

immersed in an index-matching cell. The top row is based on numerical

calculations and the bottom one is measured in experiments. [57]

Figure 2.15 The ramped phase of SPP producing helical beams with topological charge of 4.

2.1.5.3 Computer-generated holograms

The technique of computer generated holograms (CGHs) introduced by Lohman *et al* [58, 59] is based on the principle of hologram: the interference pattern between an object beam with a reference beam is recorded and then the object beam can be restored from the recorded “hologram” by the same reference beam. Instead of

carrying out the interference in real experiments, CGH uses a numerical process to calculate the interference without real beams. So it can generate an object beam which never existed before. Therefore, it is convenient to produce optical vortices by CGHs [60, 61].

A hologram records the intensity of interference pattern between reference wave A and object wave a as [39]

$$\begin{aligned} I &= |A + a|^2 \\ &= |A|^2 + |a|^2 + A^*a + Aa^* \end{aligned} \quad (2.28)$$

Assuming a uniform intensity $|A|^2$ of reference wave, the amplitude transmittance of recording medium becomes [39]

$$t = t_0 + \beta(|a|^2 + A^*a + Aa^*) \quad (2.29)$$

where t_0 is a uniform transmittance caused by the reference wave and β is the responsibility of the recording medium. Illuminated by a read-out wave identical to the reference wave, the reconstructed field is given by [39]

$$u = t_0A + \beta A|a|^2 + \beta |A|^2 a + \beta A^2 a^* \quad (2.30)$$

Although the object wave a is recovered from the hologram as the third term in Eq. 2.30, it is overlapped with the other reconstructed waves. Evidently the Gabor hologram [62] has limitations in many applications because of the overlapping waves. An improvement made by Leith and Upatnieks [63] successfully solves this problem.

The major modification to the Gabor hologram is an offset reference wave given by

$$A' = A \exp(-i2\pi\alpha y) \quad (2.31)$$

with a constant α defined as the spatial frequency in the $\vec{y}$ direction. Now the reference wave is not parallel to the object wave any more. Instead, the angle between reference and object waves is determined by the spatial frequency as

$$\theta = \arcsin(\alpha\lambda). \quad (2.32)$$

As a result, the reconstructed field becomes

$$u = t_0 A + \beta A |a|^2 + \beta |A|^2 a \exp(i2\pi\alpha y) + \beta A^2 a^* \exp(-i2\pi\alpha y) \quad (2.33)$$

under an illumination of normally incident read-out wave A . The third term in Eq. 2.33 containing the original object wave a is deflected from the optical axis by an angle of θ and thus separated from the other background terms as illustrated in Fig. 2.16. It is noted that here we do not use a read-out wave exactly the same with the reference wave A' as the Gabor holograms.

Figure 2.16 Reconstructed field of a Leith-Upatnieks hologram [39]

In practice, the amplitude pattern in Eq. 2.29 is not often produced by the interference of real laser beams in many applications. Alternatively, the holograms are calculated in a computer and then reproduced onto recording media. The so-called computer-generated holograms (CGHs) are widely used in the field of optical vortices as the most convenient technique. To generate an optical vortex with the general complex amplitude $E(\rho, \theta) = a(\rho)\exp(il\theta)$, the interference pattern of Leith-Upatnieks hologram is given by

$$I = A^2 + a^2(\rho) + 2Aa(\rho)\cos(l\theta - 2\pi\alpha y) \quad (2.34)$$

with $A > 0$ is the uniform amplitude of reference wave and $a(\rho) > 0$ is the radial amplitude function of the optical vortex. Consider a simple example when the amplitude function of the optical vortex is uniformly distributed and equal to the magnitude of reference wave, i.e. $a(\rho) = A$, Eq. 2.34 becomes

$$I = \frac{1}{2}[1 + \cos(l\theta - 2\pi\alpha y)] \quad (2.35)$$

after neglecting the constant factor A^2 . The amplitude transmittance of hologram recording the interference pattern is given in a compact form

$$t = \frac{\beta}{2} [1 + \cos(l\theta - 2\pi\alpha y)], \quad (2.36)$$

where β is the responsibility of the recording medium. The hologram depicted in Fig. 2.17 can be viewed as a modification of regular sinusoidal amplitude gratings. The evident difference is the dislocation of gratings appearing in the middle of hologram. The strength of dislocation describes the degree of mismatch among the gratings and it is calculated by counting the different number of gratings between the upper and lower layers. For example, the strength of dislocation in Fig. 2.17 is three with the sign determined by the orientation of the gratings. It is noted that the strength of dislocation is just another name for the topological charge of optical vortex [36].

Figure 2.17 The CGH generates an optical vortex with topological charge 3. The white part is transparent and the black is opaque.

The amplitude hologram of Fig. 2.17 blocks a large part of the incoming light.

And like all the other gratings, this kind of CGH produces multiple diffraction orders and thus the energy of passed light spreads out among the orders. Hence, the theoretically calculated power efficiency of the sinusoidal amplitude hologram is as low as 6.25% for the first order [64]. The efficiency can be improved by converting the amplitude hologram into a phase-modulated one because the phase element will not affect the intensity of incoming light. Blazed phase structures are also used to further enhance the efficiency by diffracting the light into a particular diffraction order. A phase CGH was reported to achieve diffraction efficiency as more than 50% [65].

2.1.5.4 Summary

The advantages and limitations of the three major approaches discussed above are summarized as Table 2.1. Although mode converters made up of cylindrical lenses are capable of producing LG laser modes in high mode purity, this method is not suitable for the generation of various optical vortices with transverse profiles differed from LG laser modes. On the other hand, it is possible to design a spiral phase plate to produce a specific optical vortex. However, SPPs are still phase elements which cannot modulate the amplitude part of light waves. Computer-generated hologram (CGH) is the most promising technique for generating optical vortices because theoretically it can reconstruct any wavefront

and the implementation is simpler than the other two techniques.

Table 2.1 A comparison among three approaches to generating optical vortices

Generation methods	Versatility	Power efficiency	Implementation	Frequency of use
Mode converter	Poor	High	Difficult	Seldom
Spiral phase plate (SPP)	Medium	High	Medium	Often
Computer-generated hologram (CGH)	Good	Medium	Easy	Very often

2.1.6 Variations of helical beams

Various optical vortices share the same feature, i.e. the helical wavefront. Although sometimes it is not easy to identify the helical wavefront especially when two or more helical beams are superimposed [22-26]. By changing the radial amplitude profile while keeping the spiral azimuthal phase structure $\exp(il\theta)$, many variations of helical beam are created including Laguerre-Gaussian beams. Among them, it is worth mentioning the Bessel beam, also known as “non-diffracting” light. A n -th order Bessel beam is described as [28, 29]

$$E(\rho, \theta, z) = E_0 \exp(ik_z z) J_n(k_\rho \rho) \exp(in\theta), \quad (2.37)$$

where k_z and k_ρ are the longitudinal and radial wave numbers, respectively.

The two wave numbers satisfy the relation

$$k_z^2 + k_\rho^2 = k^2 \quad (2.38)$$

with $k = 2\pi/\lambda$ is the total wave number. The transverse intensity profiles of Bessel beams are shown in Fig. 2.18. As expected from the spiral phase term $\exp(in\theta)$, the Bessel beam of nonzero order has central darkness.

Figure 2.18 Transverse intensity distributions of Bessel beams. [31]

Although the ideal Bessel beam has an infinite transverse profile and thus appears impractical, an approximation to high-order Bessel beams ($n \neq 0$) can be generated by passing a helical beam with a uniform or Gaussian transverse amplitude profile through an axicon (see Fig. 2.19), an optical element producing extended focal line along the longitudinal direction [66]. It is noted the approximation maintains the “non-diffracting” property in a range of propagation distance. The maximum non-diffracting distance is given by [31]

$$Z_{\max} = w_0 \frac{k}{k_\rho}, \quad (2.39)$$

where w_0 is the waist of the incoming helical beam right in front of the axicon.

Within this range, the feature size of the transverse intensity profile of Bessel beam

hardly changes as illustrated in Fig. 2.20.

Figure 2.19 High-order Bessel beam generated by an axicon illuminated by a helical beam [31]

Figure 2.20 Numerically calculated transverse intensity profiles of 4th order Bessel beam in different propagation distances. [67]

The other interesting property of Bessel beams is the healing effect. A propagating Bessel beam obstructed by an on-axis amplitude or phase disturbance is able to restore the original optical field in some distance behind the obstacle [32,

33, 68]. Fig. 2.21 shows the simulated process of self-reconstruction of a high-order Bessel beam obstructed by a Gaussian absorbing disk.

Figure 2.21 Self-reconstruction of a perturbed Bessel beam with $n=2$. The propagation distance increases from left to right. The leftmost image shows the initial Bessel beam and the rightmost one is the reconstructed Bessel beam. [32]

Besides the Bessel beam, another kind of propagation-invariant optical vortex beam belongs to the family of Mathieu beam [69] in elliptic coordinates (ξ, η, h) [70] expressed as [71]

$$E(\xi, \eta, z; q) = [A_m(q)Je_m(\xi; q)ce_m(\eta; q) + iB_m(q)Jo_m(\xi; q)se_m(\eta; q)]\exp(ik_z z) \quad (2.40)$$

with the parameter $q = k_t^2 h^2 / 4$, $A_m(q)$ and $B_m(q)$ are coefficients related to the value of q . $Je_m(x)$ and $Jo_m(x)$ are the even and odd Mathieu functions of m th order, respectively. $ce_m(x)$ and $se_m(x)$ are m th order angular Mathieu functions [72]. High-order Mathieu beams demonstrate helical wavefronts and thus provide optical vortices as shown in Fig. 2.22.

Figure 2.22 Transverse intensity profiles of high-order Mathieu beams [71]

2.1.7 Applications of optical vortices

Applications of optical vortices mainly rely on two unique characteristics; one is helical wavefronts and the associated orbital angular momentum, the other is the propagating central darkness in the axial direction.

Optical vortices can exert a torque to illuminated microscopic objects and induce rotations [4, 5, 8]. This mechanical effect is explained by the transfer of OAM from helical beams to objects [11, 73, 74]. An optical vortex can be projected onto a miniaturized gear to drive a pump in a micro-fluidic environment. It has been shown an array of optical vortices forms a cooperative flow by rotating microparticles [75]. The superposition of a helical beam and a regular Gaussian beam produces spiral fringes which can be used to rotate asymmetrical objects [22]. The OAM-states labeled by the corresponding topological charges provides high dimensional quantum entanglements [15-19]. This is useful for the studies of

quantum computing, quantum information processing and quantum communications.

The central darkness along the axial direction of an optical vortex is caused by the destructive interference of the helical wavefront. This special intensity distribution is proved to be more suitable in optically trapping non-transparent and low-refractive-index objects [5-8, 10, 65, 76-78]. Optical vortices also create novel geometries for atom cooling and trapping [55, 56, 79, 80].

2.2 Optical vortices in optical tweezers

2.2.1 Introduction to optical tweezers

Single beam gradient force optical traps, also known as optical tweezers, pioneered by A. Ashkin in the mid-1980s [81], utilize a single tightly focused laser beam to create strong gradient force for trapping microscopic particles. Nowadays, optical tweezers have widely been used in various areas such as physics, chemistry and biology in particular.

A basic optical tweezers (see Fig. 2.23) usually consists of a laser and a focusing lens (objective lens). It is well-known that light carries momentum, so it is not surprising to observe an object in the specimen plane of the basic optical tweezers being pushing forward along the propagation direction of the laser beam. Indeed, this is what Ashkin tried to achieve in a series of preceding experiments in

studying the so-called *radiation pressure* for levitating and accelerating dielectric particles in air or water [83-88]. However, he also noticed an unexpected effect, i.e., the illuminated particles tend to be attracted to the laser axis with higher intensity, and even move with the laser beam. The mysterious transverse force was soon discovered as *gradient force*. A ray optics model [81] was proposed to explain the origin of the gradient force, and further investigation reveals another force known as *scattering force* in the laser beam propagation direction (see Fig. 2.24). In Fig. 2.24(b), a more intense light ray following the path $ABCD$ exerts a stronger force on a high refractive index dielectric particle than that in the other path $abcd$ because of the larger momentum change of photons. Therefore, the net force experienced by the particle is pointed towards the high intensity part of the laser beam. The net force can be decomposed into two orthogonal components, scattering force and gradient force, as shown in Fig. 2.24(a). For a focused laser beam, there is axial intensity gradient besides the transverse one and thus it also creates gradient force in the axial direction. In order to form a stable three-dimensional optical trap, the total gradient force must restore the position at the focus when the particle is displaced. Again, the ray optics model offers a good understanding on how a focused laser beam traps a spherical particle in three dimensions. As we can see from Fig. 2.25, the resultant net force always points to the focus no matter in which direction the displacement occurs. More rigorous calculation reveals that the equilibrium point is located slightly beyond the focus for a high NA lens. At the focus, the scattering force still exists due to the transfer

of photon momentum but the gradient force vanishes. In order to balance the scattering force, a stable optical trap must be displaced downstream from the focus. Here, we neglect the particle's weight and the buoyant force to simplify the discussion.

Although the ray optics model offers an intuitive picture of the mechanism of optical tweezers, it is invalid if the dimensions of particle are not much larger than the wavelength of light. Several models [91-102] are developed for rigorous calculations of the forces experienced by a spherical dielectric particle belonging to various radius ranges.

Figure 2.23 A sketch of basic optical tweezers. [82]

(a)

(b)

Figure 2.24 The ray optics model of optical trapping. (a) A high refractive index particle displaced from the laser beam axis [82], (b) The origin of total force experienced by the displaced particle [89].

Figure 2.25 Ray optics pictures of optical tweezers. (a) & (b) A high refractive index particle axially displaced from the focus, (c) transversely displaced [90].

For a particle much smaller than the wavelength i.e., the Rayleigh condition, the sphere is treated as a point dipole and the time-averaged scattering force is given by [96, 103]

$$F_{scatt} = \frac{I_0 \sigma n_m}{c} \quad (2.41)$$

with the scattering cross section

$$\sigma = \frac{128\pi^5 a^6}{3\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2} \right)^2. \quad (2.42)$$

On the other hand, the gradient force of a Rayleigh sphere in a focused laser beam is given by

$$F_{grad} = \frac{2\pi\alpha}{cn_m^2} \nabla I_0 \quad (2.43)$$

with the polarizability

$$\alpha = n_m^2 a^3 \frac{m^2 - 1}{m^2 + 2}, \quad (2.44)$$

where I_0 is the laser intensity, n_m is the refractive index of surrounding medium, c is the speed of light in vacuum, a is the radius of the sphere, λ is the wavelength of the laser, and m is the relative refractive index of the sphere defined as n_p / n_m (n_p is the refractive index the sphere).

The ray optical model holds for a spherical particle with the radius much larger than the wavelength of the laser used in the optical tweezers, i.e. Mie condition. The resultant scattering force and the gradient force are given by [89, 104]

$$F_{scatt} = \frac{n_m I_0}{c} \left\{ 1 + R \cos 2\theta - \frac{T^2 [\cos(2\theta - 2\phi) + R \cos 2\theta]}{1 + R^2 + 2R \cos 2\phi} \right\}, \quad (2.45)$$

and

$$F_{grad} = \frac{n_m I_0}{c} \left\{ R \sin 2\theta - \frac{T^2 [\sin(2\theta - 2\phi) + R \sin 2\theta]}{1 + R^2 + 2R \cos 2\phi} \right\}, \quad (2.46)$$

respectively, where θ is the incident angle of the light ray, ϕ is the refractive angle, R and T are reflectance and transmittance depending on the polarization of light, n_m is the relative refractive index of the medium. It is noted that the forces are independent of the particle size for a large particle in the ray optics model.

For an intermediate sphere with the radius comparable to the wavelength of the laser, the calculation of forces becomes much more complicated, which

involves more general electromagnetic theories. And these calculations are usually not well matched to the experimental data because it is difficult to characterize all the properties.

Most of optical tweezers share these common parts: a microscope with a high NA objective lens, a laser and some optics for alignment purpose. A typical optical tweezers system is illustrated in Fig. 2.26 where the lenses $L1$ and $L2$ forms an expanding telescope and the identical lens pair $L3$ and $L4$ is used to adjust the curvature of incoming wavefront by changing the distance in between and as such the axial location of focus changes. $DM1$ and $DM2$ are dichroic mirrors reflecting the laser beam while transmitting the illumination light.

Figure 2.26 A basic configuration of optical tweezers. [103]

2.2.2 Optical tweezers using optical vortices

As shown by Ashkin's calculation [104], in some cases, the laser beam of central intensity minimum (optical vortex) constructs a more stable optical trap than the normal Gaussian beam in term of trapping efficiencies. The reason is that the central part a focused laser beam contributes mostly to the scattering force trying to push the illuminated object away from the focus while the outer part attributes to the desired gradient force acting in the reverse way to pull the object back to the focus. Hence, in normal optical tweezers we need a highly focused laser beam to create a strong enough gradient of optical field. By eliminating the central part of a focused laser beam, we are expected to have the same trapping stiffness at the same laser power level. According to the theoretical calculation, an optical vortex increases the axial trapping force by around 25% while the drawback is that the transverse trapping force slightly drops around 10% [36, 104]. The improved axial trapping is observed by using an optical vortex as the trapping laser beam [105] and accounted for the reduction of the scattering force which pushes particles away from the traps. The trapping efficiency of optical tweezers is defined as the ratio of the trapping stiffness to a given laser power, which is usually essential for most of optical tweezers applications in biology because researchers try to minimize the undesired influences to the samples by lowering the power of the laser beam. The use of optical vortices improves the trapping efficiency and thus reduces the heat generated during the trapping.

The ray optics model of optical tweezers explains why high refractive index spheres are attracted to the intensity maxima. On the contrary, particles having

lower refractive index than the surrounding medium are repelled from the intensity maxima because the light ray inside the particles are refracted outwards instead. However, an optical vortex can trap low refractive index particles in the central intensity minimum transversely while these particles are confined by the container in the axial direction [6, 7, 10]. It has also been shown that reflective and absorptive particles are difficult to be trapped by the conventional optical tweezers for a similar reason. Nonetheless, optical vortices are successfully employed in manipulating these particles [4, 65, 77, 78].

Besides the central minimal intensity distribution, orbital angular momentum (OAM) is the other useful property of an optical vortex. There are many applications based on the mechanical effect associated with the OAM of optical vortices. The total amount of OAM carried by an optical vortex is given by [106]

$$\Gamma = \frac{P}{\omega} l, \quad (2.47)$$

where P is the laser power, ω is the angular frequency of the laser light and l is the topological charge of the optical vortex. The torque exerted on the illuminated particles is determined by the fraction of OAM transferred from the photons. Therefore, a spin induced by the torque is firstly observed in absorptive particles [4]. For transparent dielectric particles trapped in the bright intensity ring of an optical vortex, the torque drives the particles circling around the central axis (see Fig. 2.27).

Figure 2.27 The multi-exposed photo for a polystyrene sphere traveling around the center of an optical vortex with topological charge 40 [74].

2.3 Phase-only diffractive optical elements

2.3.1 Introduction to diffractive optical elements

Diffractive optical elements (DOEs) influence the electromagnetic field of light by diffraction instead of refraction. The reduced height of a surface relief DOE as shown in Fig. 2.28 is based on the simple principle, i.e. the optical path difference is equivalent even if an integer multiple of light wavelength is subtracted. DOEs are increasingly gaining research interests because of the compact sizes and novel features. Incorporated with modern lithographic fabrications, most DOEs are pixellated and the transmittance functions are quantized at each pixel. We will

focus on the design of DOEs especially the phase-only type for the generation of modified optical vortices in this thesis.

Generally speaking, for a comprehensive control over the electromagnetic field, a complex amplitude element is needed. Neglecting the thickness of a DOE and assuming that the incoming wavefront is $A_{in}(x, y)\exp[i\varphi_{in}(x, y)]$, the resulting field $A_{out}(x, y)\exp[i\varphi_{out}(x, y)]$ after going through the element with a complex transmission $A_E(x, y)\exp[i\varphi_E(x, y)]$ is given by [106]

$$\begin{cases} A_{out}(x, y) = A_{in}(x, y)A_E(x, y) \\ \varphi_{out}(x, y) = \varphi_{in}(x, y) + \varphi_E(x, y) \end{cases} . \quad (2.48)$$

However, a complex transmission is low in power efficiency because the amplitude modulation part $A_E(x, y)$ absorbs a portion of incident power. And the realization of a complex transmission function on DOEs is difficulty and costly. It is shown that sometimes the amplitude modulation part of the transmittance can be discarded without significantly degrading the reconstructed optical field after the DOE such as a kinoform, a phase-only hologram [107]. Hence, a lot of effort has been made to replace complex transmissions by phase-only elements while keeping the amplitude information. In practice, there are two methods widely in use: encoding techniques and optimization algorithms. They are discussed in the following sections.

Figure 2.28 An example showing the geometries of a refractive optical element and a diffractive optical element having the same phase function.

2.3.2 Encoding techniques

Urquhart *et al* [108] proposed a complex kinoform encoding technique in 1991. Each pixel of the phase element can be separated into two parts as illustrated in Fig. 2.29. The outer part is assigned the original phase information while the inner one is out of phase by π . The centered inner part has an area proportional to the amplitude strength. Hence the local destruction will produce the corresponding amplitude information. This method requires advanced fabrication techniques to control the size of inner area.

Figure 2.29 A single pixel of phase-only element using complex kinoform encoding technique.

High-carrier-frequency grating encoding technique was introduced by Kirk and Jones [109] and further developed by Noponen *et al* [110] and Arrizon [111]. The amplitude information is encoded as the local diffraction efficiencies are controlled by the grating groove-depth variations. Basically, this technique sacrifices the diffraction efficiency for including amplitude modulations. The wasted power is diffracted to the surrounding of the desired pattern as a noise. Since the noise and the signal are spatially separated, the method has rather high signal to noise ratio.

2.3.3 Optimization algorithms

Suppose a DOE is expected to perform a specific function and the ideal element only exists as a complex form, there is at least an optimal approximation in the

phase-only forms. The problem is how to develop an algorithm to find out the optimal solution. Usually these algorithms have iterative structures to increase the performances numerically.

A basic design problem is to have a DOE generating the desired optical field in the observing plane. Depending on whether the reconstructed field is involved in calculating the transmittance of DOE in each iteration, these iterative algorithms can be categorized into two types, unidirectional or bidirectional (see Fig. 2.30).

Figure 2.30 Fundamental flowcharts of unidirectional and bidirectional iterative algorithms

Unidirectional iterative algorithms include direct binary search algorithm [112], simulated annealing (SA) algorithm [113-115] and genetic algorithm [116-118]. A typical flowchart of SA algorithm is illustrated in Fig. 2.31(a), in which the probability function [106]

$$P(\Delta E) = e^{-\frac{\Delta E}{T}} \quad (2.49)$$

with the difference of cost function $\Delta E = E_i - E_{i-1}$. The cost function indicates the degree of error between the current reconstructed optical field and the desired one. The temperature T is decrease in the iteration in order to approach a better approximation.

The Gerchberg-Saxton (GS) algorithm was first developed to solve the phase retrieval problems of electron microscopy imaging [119]. Then it becomes the foundation of a series of bidirectional iterative optimization algorithms such as input-output algorithm [120-123] and adaptive-additive (AA) algorithm [124-126]. The basic flowchart of GS algorithm is shown in Fig. 2.31(b). It begins with an initial guess of phase distribution which can be random. The initial phase element is mapped to the observation plane according to certain propagation rule. The function in the observation plane is imposed some constraints as required and then the modified function is reversely mapped to the DOE plane. Usually in this step the transmittance function of DOE is in a complex form. However, the amplitude information is abandoned then the phase-only function goes through the mapping again. This time the mapped field is evaluated by considering some criteria. If the change of performance is regarded as large, then the process will continue. Otherwise it will be terminated and the previous phase function is pulled out as the optimized phase function.

Figure 2.31 Flowcharts of iterative algorithms for the designs of phase-only DOEs.

(a) Stimulated annealing, (b) Gerchberg-Saxton.

AA algorithm imposes constraints in a different way. The resultant function in the reconstruction plane is a linear combination of the constrained and the mapped function. The combination is controlled by a relaxation parameter. Normally, AA algorithm reaches better convergences than GS algorithm. It is noted that GS algorithm is actually a special case of AA algorithm.

Usually, bidirectional algorithms converge at local minima in a few iterations. However, the local minima may not overlap with the desired global minima (see Fig. 2.32). On the other hand, unidirectional algorithms have better outputs since the deliberately induced errors help the process jump out the local minima at the cost of much slower converging speed. Fortunately, the results achieved by bidirectional algorithms are satisfying in most cases. Hence, bidirectional algorithms are more popular in DOE designs.

Figure 2.32 Illustration of a typical optimization problem.

2.3.4 Realization of phase-only diffractive optical element designs

Nowadays, various fabrication techniques such as direct printing [127, 128], diamond ruling [129] and lithography [130, 131] are available for the preparation of DOEs. However, reconfigurable elements are more desirable in some

applications such as adaptive optics when the transmittance of DOE need to change dynamically. A spatial light modulator (SLM) is such a device that can impose designated amplitude or phase information to an incoming light wave. There are several types of SLM adopting different modulation technologies including acousto-optical SLM, electro-optical SLM, photorefractive SLM, magneto-optical SLM, deformable mirror SLM and liquid crystal SLM [132, 133]. In this thesis, we focus on liquid crystal spatial light modulators (LC-SLMs) because these devices are able to deliver a phase shift up to 2π when working under the phase-only mode. This is due to the large birefringence of LC molecules within an electric field.

Normally a LC-SLM device has a two-dimensional pixellated structure with each pixel controlled electronically. In each pixel, LC molecules are sealed in a cell between two glass plates as illustrated in Fig. 2.33(a). For phase modulation, an untwisted nematic liquid crystal cell is used here as the orientations of alignment near two glass surfaces are parallel to each other. When an electric field is applied to the cell, these LC molecules in between two surfaces tend to align in the direction of the electric field. The tilting angle depends on the voltage applied (see Fig. 2.33(b)). The change of orientations increases the refractive index of the LC cell and thus induces a phase shift to the light passing through the cell.

(a)

(b)

Figure 2.33 An untwisted nematic liquid crystal cell. (a) a sketch of a liquid crystal cell controlled by electric field, (b) alignment changes due to electric fields. [134]

In this thesis, we choose LC-SLM_s fabricated by the LCoS (liquid crystal on silicon) technique [135] as major devices to realize designs of phase-only DOE. Currently this kind of LC-SLM can achieve a $9.5\ \mu\text{m}$ center to center pixel pitch and 256 phase-levels [136].

CHAPTER 3

MULTI-HELIX BEAMS

A multi-helix beam refers to the collinear superposition of multiple helical beams with various topological charges. Since the OAM-state of a helical beam is labeled by the topological charge, a multi-helix beam carries mixed OAM-states. A simple version of multi-helix beam is the superposition of two helical beams which has found a wide variety of applications [23-25].

3.1 Introduction

The superposition of two helical beams with opposite topological charges presents novel techniques for optical trapping [23, 24]. The superposition was achieved by using an interferometer [23] or modifying the wavefront of a TEM_{00} mode with an azimuthally alternating π -jump phase [24]. Zhang and Yuan [137] introduced radial spatial multiplexing of phase functions to produce a complete set of superposition of two helical modes. Khonina *et al* presented a design of phase elements that can generate up to 24 beams each matched to the superposition of two helical beams [138]. The superposition of more than two helical modes is expected to exhibit more unusual properties and find potential applications in both optical trapping and optical communication using OAM. For example, recently a

free-space optical communication system was proposed for the transfer of information carried by a single helical beam using a set of topological charges as coding alphabet [20]. It is natural to consider the possibility of transmitting more information through emitting multiple helical beams simultaneously.

Bouchal and Celechovsky [26] proposed a method for encoding information into superimposed OAM-states that are generated by a computer-generated hologram (CGH) experimentally. Although the CGH method is versatile and straightforward, it suffers from poor diffraction efficiency even if the CGH is converted into a phase hologram. Direct generation of mixed OAM-states by a phase-only element is thus more desirable because of the higher conversion efficiency and the readier implementation. In this Chapter, we develop an iterative algorithm for the design of the phase-only diffractive optical element (DOE) that can produce collinearly superposed helical beams.

3.2 Description of the design problem

We seek a phase-only transmittance that is imprinted on an incoming laser beam and then changes the laser beam into a superposition of multiple helical beams. Such a modified helical laser beam is referred to here as multi-helix beam and each helical beam as a component in a multi-helix beam is known as a helical mode. Usually a single helical mode is generated by an element with a phase-only transmission $\exp(il\theta)$ called circular harmonic [139] or angular harmonic [140].

As a single angular harmonic depends only on azimuthal angle θ , we assume that the synthesis of multiple angular harmonics depends only on the azimuthal angle too. For n angular harmonics, the mathematical description of the requirement for the phase-only transmittance is

$$B \exp[i\varphi(\theta)] = \sum_{m=1}^n A_{l_m} \exp(il_m \theta) \quad (3.1)$$

where B is a constant modulus, complex number $\{A_{l_m}\}$ comprises the weight coefficients, and $\{l_m\}$ denotes the n desired topological charges. Note that the phase $\varphi(\theta)$ is a function with real value. This formula implies [141]

$$BB^* = \sum_{m=1}^n \sum_{m'=1}^n A_{l_m} A_{l_{m'}} \exp[i(l_m - l_{m'})\theta] \quad (3.2)$$

In order to keep Eq. 3.2 valid for any value of θ within the range of $[0, 2\pi)$, the number of topological charges should be infinite except for a single topological charge [141], which means an azimuthally dependent phase-only element producing a multi-helix beam will consist of infinite angular harmonics. So there is no such phase-only element that can exactly transform the incoming laser beam into a desired multi-helix beam. However, we find that it is possible that a close enough phase-only approximation can be extracted from an iterative optimization algorithm. The resulting phase-only function can transfer 80%~90% or more of incoming power into the desired multi-helix beam.

3.3 Iterative algorithm

3.3.1 Main body of the iterative algorithm

We define the phase-only approximate function as $f(\theta) = \exp[i\phi(\theta)]$ by setting the constant amplitude modulus to unity. Expanding $f(\theta)$ in Fourier series, we obtain

$$f(\theta) = \sum_{m=-\infty}^{\infty} C_m \exp(im\theta), \quad (3.3)$$

where $C_m = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) \exp(-im\theta) d\theta$ is the complex coefficient, which can be represented in the phasor form as $C_m = |C_m| \exp(i\phi_m)$. According to Parseval's theorem,

$$\sum_{m=-\infty}^{\infty} |C_m|^2 = 1. \quad (3.4)$$

Because of the mathematical similarity between angular harmonics and the Fourier components, Eq. 3.4 means that the total power is distributed over the helical modes, and $|C_m|^2$ denotes the power of the specific angular harmonic $\exp(im\theta)$ corresponding to the helical mode with topological charge of m .

We intend to generate a multi-helix beam containing n helical modes $\{l_1, l_2, \dots, l_n\}$ and the respective weights are $\{A_{l_1}, A_{l_2}, \dots, A_{l_n}\}$ as illustrated in Fig.

3.1(a), and the total power is set to unity, i.e., $\sum_{m=1}^n |A_{l_m}|^2 = 1$. The objectives of

designing phase-only $f(\theta)$ are to ensure that the desired helical modes get most of the power and to maintain the ratio of weights. The ratio of weights is defined as $|A_{l_1}|^2 : |A_{l_2}|^2 : \cdots : |A_{l_n}|^2$. The mathematically similar problem exists in the design of optical-correlation filters. Schils and Sweeney [142] proposed an iterative method to determine such a phase-only function. We find that this method converges quickly and distributes most of the power to the desired helical modes. However, the optimized phase function does not fit the desired ratio, i.e. the particular distribution over helical modes well.

Figure 3.1 Power spectra of topological charges. (a) The power spectrum of ideal complex transmittance. (b) The power spectrum of initial phase-only transmittance $g(\theta)$. (c) The power spectrum of convergent transmittance $g(\theta)$. i and j ($i, j \neq l_m, m = 1, 2, \dots, n$) denote the additional topological charges introduced by the process.

We developed two different iterative processes for obtaining phase-only solutions that satisfy the requirements mentioned above. First we describe the core iterative part shared by both processes. We start from a phase-only transmittance $g(\theta) = \exp[i\psi(\theta)]$ with the phase function

$$\psi(\theta) = \text{Re} \left\{ -i \ln \left[\sum_{m=1}^n B_{l_m} \exp(il_m \theta) \right] \right\}, \quad (3.5)$$

where $\{B_{l_m}\}$ is given an initial value of $\{B_{l_m}^0\}$ and Re^1 denotes “real part of”. The choice of $\{B_{l_m}^0\}$ is discussed later. Discarding the imaginary part of the right-hand side of Eq. 3.5 is equivalent to setting the amplitude to unity, which ensures $g(\theta)$ is a phase-only transmittance. Decompose $g(\theta)$ into angular harmonics:

$$g(\theta) = \sum_{m=-\infty}^{\infty} C_m \exp(im\theta). \quad (3.6)$$

The corresponding power spectrum is shown in Fig. 3.1(b). Pick out the coefficients $\{C_{l_1}, C_{l_2}, \dots, C_{l_n}\}$ that concern us. According to the power difference between $\{|C_{l_1}|^2, |C_{l_2}|^2, \dots, |C_{l_n}|^2\}$ and the desired $\{|A_{l_1}|^2, |A_{l_2}|^2, \dots, |A_{l_n}|^2\}$, the coefficients $\{B_{l_1}, B_{l_2}, \dots, B_{l_n}\}$ are adjusted and substituted in Eq. 3.5 again. To evaluate the performance of $g(\theta)$, root-mean-square-error (*RMSE*) is defined relative to the total power $\sum_{m=1}^n |A_{l_m}|^2$, which is normalized to unity. *RMSE* serves as a measure of how close the actual power spectrum is to the desired power spectrum:

$$RMSE = \sqrt{\frac{\sum_{m=1}^n (|C_m|^2 - |A_m|^2)^2}{n}}. \quad (3.7)$$

If the parameter *RMSE* decreases, the improved $g(\theta)$ is saved to replace the previous one. The iterative process is terminated when the number of iterations

reaches a predefined limit N . As a rule of thumb, the limit is set to 1000 in the consideration of calculation time and $RMSE$. Of course, the $RMSE$ keeps dropping as long as the iteration runs. However, the improvement of outcome is tiny after the first hundred cycles as most of error-reduction algorithms. The optimized phase-only function $g(\theta)$ is derived when the iterative process is terminated. As shown in Fig. 3.1(c), the convergent $g(\theta)$ will obtain power spectrum similar to the ideal one in Fig. 3.1(a) but with a small portion of power leaking to unwanted helical modes. This imperfection is unavoidable because the phase-only transmittance having ideal power spectrum does not exist. The main body of the iterative algorithm is summarized as a flowchart in Fig. 3.2.

Figure 3.2 The main body of the iterative algorithms.

The design variable B_{l_m} ($m=1,2,\dots,n$) in the iteration has two degrees of freedom i.e. the modulus and the phase. We can choose to vary them sequentially or simultaneously, each of which corresponds to a different structure of algorithm.

3.3.2 Structure I—Sequential optimization

In the sequential structure, the initial value of $B_{l_m}^0$ is given as $|A_{l_m}| \exp(i\phi_{l_m})$, where the phase ϕ_{l_m} is first optimized by the iterative process illustrated in Fig. 3.3(a). In this method, the initial phase can be random, so we set it to zero for simplicity. This iteration optimizes the phase of the design variable B_{l_m}' . The new phase is equal to that of C_{l_m} so that

$$B_{l_m}' = \frac{|B_{l_m}'|}{|C_{l_m}|} \times C_{l_m}. \quad (3.8)$$

The resultant phase is determined when the current cycle is over. The optimized phase keeps unchanged in the second iterative process as shown in Fig. 3.3(b). The second iterative process varies the modulus of B_{l_m} in such a way that the increment is proportional to the difference between $|C_{l_m}|$ and $|A_{l_m}|$ as follows:

$$|B_{l_m}'| = |B_{l_m}| + \beta(|A_{l_m}| - |C_{l_m}|) \quad (3.9)$$

with the constant $\beta > 0$. To summarize, the sequential structure optimizes the phase and modulus of the design variable in two different iterative processes in sequence.

Figure 3.3 The sequential optimization structure. (a) The first iterative process optimizes the phase of B_{l_m} . (b) The other optimizes the modulus of B_{l_m} .

3.3.3 Structure II—Simultaneous optimization

It is noted that the two structures of the iterative processes in Fig. 3.3 are similar.

So it is possible to optimize both the phase and modulus of B_{l_m} in the same

iteration. The schematic diagram of the simultaneous optimization is shown in Fig. 3.4. Like the sequential optimization, the initial phase of B_{l_m} is also set to zero. The modulus and phase of B_{l_m} are now adjusted simultaneously in independent procedures each analogue to the optimizations (Eqs. 3.7 and 3.8) used in the sequential optimization.

Figure. 3.4 The simultaneous optimization structure.

3.4 Results and discussions

3.4.1 Comparison of the three algorithms

Besides the two algorithms described above, Schils and Sweeney's algorithm [142] can also be used in the synthesis of multiple helical modes. In this section, we present a design example using those three algorithms respectively.

The example is to design a phase-only transmittance that can generate a multi-helix beam containing four helical modes (topological charges 10, 20, 30, and 40) with equal weight. The total power is normalized to unity and each mode has one quarter of the total power. This requirement is illustrated in Fig. 3.5(a) as a power spectrum over topological charges of helical beams. The phase-only transmittances $g(\theta)$ calculated by the three different methods are decomposed into angular harmonics that represent relevant helical modes. The resulting power spectra are shown in Figs. 3.5(b), 3.5(c), and 3.5(d), respectively, and the numerical results on the performances are summarized in Table 3.1. Here the theoretical diffraction efficiency is defined as the ratio of the power falling into the desired modes to the total power, which has been normalized to unity. The experimental diffraction efficiency will be further limited by the specification of the available diffractive element. For this example, the three algorithms achieve similar diffraction efficiencies and there is no great difference among the diffraction efficiencies. However, Schils and Sweeney's algorithm optimizes only the phase of design variable B_{l_m} , so the RMSE is much worse than for the others.

Since the phase is optimized when the modulus is fixed in sequential optimization, the optimized phase generally will not coincide with the true optimum that is obtained when the modulus is also free to vary. So it is not surprising to expect better performance in simultaneous optimization than in sequential optimization. However, it seems confusing the spectrum in Fig. 3.5(c) has better uniformity than that in Fig. 3.5(d) although the latter has a better *RMSE*. It is because the parameter *RMSE* measures how far the power in each helical mode deviates from the desired level in average while the simultaneous optimization case achieves the higher diffraction efficiency, which makes the average power of each helical mode more close to the required value(0.25).

Figure 3.5 Power spectra over topological charges of helical modes. (a) The required power spectrum (b) Schils and Sweeney's algorithm (c) Sequential optimization (d) Simultaneous optimization

Table 3.1. Comparison of the three iterative algorithms

Algorithm	Optimization of Design Variables		Diffraction Efficiency [*]	RMSE
	Phase	Modulus		
Schils & Sweeney	Yes	No	90.5%	0.0665
Sequential Optimization	Yes	Yes	89.1%	0.0273
Simultaneous Optimization	Yes	Yes	90.6%	0.0240

*Here the diffraction efficiency is defined as the ratio of the power falling into the desired helical modes to the total power which has been normalized to unity.

3.4.2 Simulation and experiment results

It is well-known that a helical beam forms a doughnut-like diffraction pattern, and that the radius of ring increases with the absolute value of topological charge [43, 74]. So the multi-helix beam containing different helical beams should exhibit concentric bright rings. However, if the topological charges are close, the pattern of concentric bright rings will be disrupted significantly by interference. To illustrate this, we choose an example with widely spaced topological charges in order to differentiate them by direct observation. The weights are assigned in a manner such that the larger ring contains more power in order to make the brightness of each ring more consistent.

For sequential optimization, we investigate the design of a multi-helix beam containing topological charges $\{10, -20, 30\}$, and two different schemes of power distribution are discussed: equal power distribution i.e., the power is evenly allocated to three helical modes and unequal power distribution where the proportions are $\{1/6, 1/3, 1/2\}$, respectively. By using the iterative process, both transmittances are calculated and their power spectra are depicted in Fig. 3.6(a) and 3.6(d), respectively. The actual power distributions of the equal power scheme and unequal power scheme are $\{0.275, 0.275, 0.275\}$ and $\{0.138, 0.265, 0.410\}$ respectively. Therefore the power ratios among the helical modes are 1:1:1 and 1:1.92:2.97 respectively, which indicates high uniformity in the desired ratios. The theoretical diffraction efficiency is 82.5% in the equal power case, and 81.3% in the unequal power case. Fig. 3.6(b) and 3.6(e) show the intensity patterns in the distance of 1m from the phase-only DOEs through numerical simulation. As a comparison, the intensity patterns generated by ideal complex DOEs are illustrated in Fig. 3.6(c) and 3.6(f). In all these simulations, the transverse amplitude function is assumed to have a Gaussian profile. According to [43] and [74], different helical modes correspond to different ring radii. Basically, if the absolute value of topological charge is larger, then the ring-like intensity pattern has a larger radius (see Fig. 2.4). Since we are generating a superposition of multiple optical vortices of different topological charges, we are supposed to observe concentric rings as a diffraction pattern. And we should be able to distinguish among the helical modes by the sizes of rings if they are

separated widely enough. However, a single optical vortex produces a primary ring along with subsidiary rings which will be discussed in details in Chapter 5. The spotty intensity discontinuity of each ring is due to the interference among the helical modes and these subsidiary rings. Nevertheless, we still are able to differentiate those primary rings representing helical modes as shown in Fig 3.6.

Figure 3.6 The numerical simulations of DOEs generating multi-helix beams. The upper row is in the equal power distribution scheme while the lower row is in the unequal power distribution scheme. (a) and (d) are the power spectra. (b) and (e) are the intensity patterns of reconstructed fields diffracted from the approximate phase-only DOEs. (c) and (f) are the reconstructed intensity patterns generated by ideal complex DOEs.

We tested the phase-only transmittance by using a liquid crystal spatial light modulator (LC-SLM). The phase-only transmittance shown in Fig. 3.7(a) or 3.7(c) was displayed on a reflective SLM (Holoeye, LCR 3000) with a 256-level grayscale and 1920×1200 pixels. A blazed grating phase was added to the transmittance to diffract the desired pattern into a certain direction. A TEM_{00} laser beam with the wavelength of 532.8nm coming from an Nd:YVO₄ laser (Coherent Verdi-V8) impinged on the SLM. The output beam was directed into a microscope (Carl-Zeiss Axiovert 25), and then the magnified images were captured by a CCD camera as shown in Fig. 3.7(b) and 3.7(d). Since the inner ring has the smallest radius and the width of the ring is almost the same as that of the others, the power distribution in the unequal power case is designed to have more weights on the larger rings in order to yield a uniform intensity profile. In Fig. 3.7, we observe three major concentric rings resulted from the three helical modes produced by the phase masks.

Figure 3.7. The observed intensity patterns at the laser power of 10mW. The upper row is in the equal power distribution scheme while the lower row is in the unequal power distribution scheme. The corresponding azimuthal modulated phase function is shown in (a) and (c) respectively.

For simultaneous optimization, we choose the multi-helix beam containing topological charges $\{5, -15, 30, -50\}$ with respective weights $\{0.15, 0.2, 0.25, 0.4\}$. The corresponding phase-only transmittance shown in Fig. 3.8(a) is calculated by

the simultaneous optimization algorithm described above. The transmittance has 81.1% theoretical diffraction efficiency and RMSE is 0.0263. The numerical reconstruction of the multi-helix beam with a Gaussian amplitude profile in the distance of 0.4m from the SLM is illustrated in Fig. 3.8(b). After adding a blazed grating phase at the spatial frequency of around 22mm^{-1} , the resulting transmittance was displayed on a reflective SLM (Boulder Nonlinear Systems P512 with 512×512 pixels and $15\ \mu\text{m}$ pixel pitch). A TEM_{00} laser beam with the wavelength of 632.8nm impinged on the SLM. The intensity pattern of the modulated beam at the distance 0.4m from the SLM is captured by a CCD beam profiler as shown in Fig. 3.8(c). The four rings corresponding to helical beams of topological charge 5, -15, 30 and -50 are separated as expected. The discontinuity of intensity is also due to the interferences among the helical modes.

Figure 3.8. The superposition of four collinear helical beams. (a) The phase-only transmittance (b) The numerical reconstruction of the multi-helix beam in the distance of 0.4m (c) The observed intensity patterns of multi-helix beam

3.5 Applications in free-space optical communication

Free-space optical communication provides a point to point data link without physical connections in between. Compared with the other wireless technology, radio frequency (RF), free-space optics (FSO) normally has faster speed (up to Gbit/s commercially), higher security (small divergent angle), and most of all, it is free from electromagnetic interference. The last two advantages are essential for military applications when the information security is one of the major concerns. By using the components well developed in optical fiber communication industry, the cost of FSO drops quickly and there is no license charge for these frequencies. Therefore, this technology is also now becoming competitive in commercial data communications. The largest challenge faced by FSO is the large attenuation caused by the atmosphere especially fog, which significantly reduces the propagation distance. However, this will not be a problem when FSO is used in inter-space craft or satellite communications.

3.5.1 Free-space optical communication using OAM-states

Ideally the information transmitted by an FSO system cannot be intercepted without positioning the detector directly in the path of propagation. However, there is still a portion of light scattered by aerosol, fog or rain drops if the laser beam is

transmitted in the atmosphere illustrated in Fig. 3.9. The scattered light could be picked up from a location that is far away from the path. Since most existing FSO systems rely on the modulation of intensity, frequency or polarization of light. The security of these systems is compromised by the fact that such modulated signals can be easily demodulated from the scattered light if the scattered power is large enough. Under this circumstance, the safety of information has to depend on the encryption of the software layer. Normally, the scattered parts of laser beams caused by small particles are of small angular extent. It has been proved that orbital angular momentum (OAM) states of light are difficult to restore from limited angular extent due to the inherent uncertainty of OAM [144]. The uncertainty principle refers to the expansion of topological charge spectrum when the angular aperture is restricted. In this case, the measurement of the original topological charge shows a lower accuracy than that under the condition of unobstructed angular aperture. The unique property of light would be useful in security when OAM-states are applied as the information carriers in an FSO system.

Figure 3.9 The scattering of propagating light in the atmosphere. [143]

3.5.1.1 System overview

Recently, a prototype of FSO system using OAM-states of light is demonstrated (see Fig. 3.10). The transmitter comprises a HeNe laser and a liquid crystal spatial light modulator (LC-SLM) controlled by a computer. The phase-only SLM converts the incoming TEM_{00} laser beam into a helical beam carrying a specific OAM-state. Together with a converging lens and a CCD camera, the receiver uses another LC-SLM to detect the topological charge of the transmitted OAM-state. The design of analyzing phase-only hologram displayed on the SLM is explained later. The optical link uses two telescopes to adjust the laser beam size. This system can work in the mode of a time-division multiplexing (TDM) because the SLM in the transmitter dynamically changes the transmittance selected from 8 holograms. The transmittance of the SLM at the receiving end

remains the same.

Figure 3.10 Free-space optical communication system using OAM-states [20]

3.5.1.2 Detection of topological charges

An important issue in the design of this FSO system is the detection of the OAM-state of transmitted helical beam because the information is encoded in the value and sign of topological charge characterizing an OAM-state. To measure the spiral phase structure of a propagating helical beam with the complex amplitude given by

$$E_v(\rho, \theta, z) = E_v^0 A(\rho) \exp \left\{ i \left[l\theta + \frac{k\rho^2}{2R_v(z)} + \Phi(z) \right] \right\}, \quad (3.10)$$

where $A(\rho)$ is the transverse amplitude profile, the second term in the phase

function $\frac{k\rho^2}{2R_v(z)}$ accounts for the wavefront curvature $R_v(z)$, and $\Phi(z)$ is the extra phase brought by Gouy phase shift, a common technique is to interference this helical beam with a coherent reference wave which can be an extended Gaussian beam or an ideal plane wave. The interference pattern is determined by the phase difference given by [36]

$$\Delta\Psi = l\theta - k\rho \sin \alpha \cos \theta + k\rho^2 \left(\frac{1}{2R_v(z)} - \frac{1}{2R_r(z)} \right) + \Delta\Phi(z) + \psi_0, \quad (3.11)$$

where α is the tilting angle of the reference wave with respect to the helical beam, $R_r(z)$ is the curvature of the reference wave, and ψ_0 is a relative phase difference. The bright fringes (constructive interference) of this pattern are described by the fringe equation [36]:

$$l\theta + k\rho \sin \alpha \cos \theta + k\rho^2 \frac{R_r - R_v}{2R_r R_v} = 2\pi n - \Delta\Phi(z) \quad (3.12)$$

with n is an integer number. In particular, the on-axis interference pattern is shown in Fig. 3.11 when the tilting angle α equals to zero. The value of topological charge can be determined by counting the number of fringes originated from the center. If the radii of two curvatures are not matched ($R_v \neq R_r$), the rotation of fringes appears. Although the rotating direction depends not only on the signs of topological charge but also the difference in the radii, it still provides useful information about the sign of topological charge when the sign of the difference between radii is fixed (see Figs. 3.11(b) and (e), Figs. 3.11(c) and (f)).

When the tilting angle α is large enough ($\alpha \gg \arcsin \sqrt{\frac{2}{kR_v(z)}}$) [36], the so-called off-axis interference exhibits a fork shape in the center (see Fig. 3.12). Similar to the computer-generated hologram introduced in Chapter 2, the difference in fringe number between upper and lower parts describes the value of topological charge. In this case, the direction of fork opening tells the sign of topological charge when the tilting angle is fixed, and the unmatched curvatures cause only the bending of fringes. In practice, the off-axis interference for the detection of topological charges is more desirable than the on-axis one because it offers higher resolution when multiple optical vortices are present in a transverse plane. In some rare cases, a coherent reference wave is difficult to produce. Therefore, an alternative way is to split the helical beam into two identical beams. The topological charge of one of them is then reversed by reflection of a mirror and interfered with the original copy of helical beam. The measured value of the topological charge doubles the original one [145, 146].

Figure 3.11 The fringe patterns of a helical beam interfered with a collinear reference wave. (a) $l = 3$, $R_v = R_r$, (b) $l = 3$, $R_r < R_v$, (c) $l = 3$, $R_r > R_v$, (d) $l = -3$, $R_v = R_r$, (e) $l = -3$, $R_r < R_v$, (f) $l = -3$, $R_r > R_v$

Figure 3.12 The off-axis fringe patterns of a helical beam interfered with a tilted reference wave. (a) $l=3$, $R_v = R_r$, $\alpha = \alpha_0$, (b) $l=3$, $R_v \neq R_r$, $\alpha = \alpha_0$, (c) $l=3$, $R_v \neq R_r$, $\alpha = -\alpha_0$, (d) $l=-3$, $R_v = R_r$, $\alpha = \alpha_0$, (e) $l=-3$, $R_v \neq R_r$, $\alpha = \alpha_0$, (f) $l=-3$, $R_v \neq R_r$, $\alpha = -\alpha_0$.

The other widely used technique for the measurement of topological charge is the conjugated phase matching method. Basically, these methods reverse the generation process of a helical beam $\exp(il\theta)$ by imposing a conjugated phase $\exp(-il\theta)$ to the helical beam. The resultant laser beam becomes a non-helical beam and thus carries no optical vortex. If the imposed phase function is not

matched, the output will be still a helical beam with central darkness. Similar to those optical vortex generation methods discussed in Chapter 2, we can also use mode converter [48, 147, 148], spiral phase plates [18, 19], and computer-generated hologram [16, 20] to measure the topological charges of helical beams.

3.5.1.3 Information security

The enhanced security of FSO system using OAM-states depends on the inherent uncertainty principle in the detection of topological charges [144, 149, 150]. Given the fact that the atmosphere scattering usually has a small transverse azimuthal aperture, the model applied in the study of the OAM-states of scattered light is considered as a mask with hard edge azimuthal aperture. The transmittance function of this mask is given by

$$T(\theta) = \begin{cases} 1 & -\frac{\Theta}{2} \leq \theta \leq \frac{\Theta}{2} \\ 0 & elsewhere \end{cases} \quad (3.13)$$

with the parameter Θ describing the opening of the aperture. A helical beam of topological charge l passing through this mask generates an optical field given by

$$E(\theta) = \exp(il\theta) \times T(\theta). \quad (3.14)$$

Expanding the mask transmittance in Fourier series, we get

$$T(\theta) = \frac{\Theta}{2\pi} \sum_{n=-\infty}^{+\infty} \sin c\left(\frac{n\Theta}{2}\right) \exp(in\theta). \quad (3.15)$$

Therefore, Eq. 3.14 becomes

$$E(\theta) = \frac{\Theta}{2\pi} \sum_{n=-\infty}^{+\infty} \sin c\left(\frac{n\Theta}{2}\right) \exp[i(n+l)\theta]. \quad (3.16)$$

Figure 3.13 The uncertainty relationship between the azimuthal opening $T(\phi)$ and the power spectrum over topological charges. In this example, the original topological charge is 1. [144]

And the power spectrum over topological charges measured from the scattered

light beam is [150]

$$P(m) = \frac{\Theta}{2\pi} \sin c \left[\frac{(m-l)\Theta}{2} \right]. \quad (3.17)$$

When the opening of the aperture, i.e. Θ increases, the power distribution over topological charge turns out to be more concentrated to those topological charges centered at the value of l (see Fig. 3.13). In reality, Θ usually is very small since the scattering traverse plane is tiny. Hence, the spread of power spectrum is great, which makes the accurate detection of topological charge difficult for the intentional eavesdropping from the scattered light.

It has been demonstrated the spread of power spectrum also happens when a helical beam is misaligned [151]. A tilted helical beam manifests a superposition of multiple OAM-states centered at the original topological charge as illustrated in Fig. 3.14. The broadening of the power spectrum becomes larger as the tilting angle increases. This effect can be used to prevent eavesdroppers from recovering information from reflected lights although it also makes the performance of the FSO system sensitive to the alignment.

Because of the unique characteristics of OAM-states discussed above, the FSO systems using helical beams significantly reduce the possibility of information leaking during propagation in the atmosphere.

Figure 3.14 The broadened power spectra over topological charges due to the misalignment of helical beams with $l=1$. Θ is the ratio of the tilting angle over the divergent angle of a helical beam. [144]

3.5.2 OAM multiplexing of optical signals

Combined with the iterative algorithm developed for producing collinear multiple OAM-states, we are able to set up a FSO system realizing the multiplexing of channels, each is labeled by a specific OAM-state. In the original FSO system proposed in [144], information is encoded in the value and sign of the topological

charge of single OAM-state chosen from a coding alphabet that comprises a set of topological charges. Apart from this system, our version uses topological charges as labels of channels in order to separate them in the detection process, and the binary information in each channel is represented by the “ON” and “OFF” statuses, which are defined respectively as whether the OAM-state exists in the multi-helix beam or not.

A phase-only DOE generating the multi-helix beam is designed by the iterative algorithm described above. Physically the phase element can be realized by a liquid crystal spatial light modulator (LC-SLM) which dynamically changes the phase of an incoming laser beam. Similar to the optical system illustrated in Fig. 3.10, we have a system made up with a transmitter and a receiver as shown in Fig. 3.15. In the transmitting part of the system, a computer-controlled LC-SLM (Boulder Nonlinear Systems P512 with 512×512 pixels and 15 μm pixel pitch) is employed to convert the TEM_{00} mode coming out of a He-Ne laser into helical beams. The OAM-states of laser beam are changed by feeding different data to the LC-SLM through computer. The iterative algorithm is incorporated to simultaneously generate multiple OAM-states embedded in one laser beam in order to realize the multiplexing of channels labeled by the topological charge of OAM-states within the system. After passing through an analyzing CGH in the receiving part, the diffracted pattern will be focused into a CCD beam profiler by a converging lens. The analyzing CGH produces two-dimensional far-field diffraction pattern in which each diffraction order is used to detect a value of

topological charge. Therefore, statuses of those channels labeled by topological charges are measured at the corresponding diffraction orders.

Fig. 3.15. Schematic of the free-space optical communication system using a multi-helix beam

The measurement of the topological charge of an OAM-state relies on the fact that a helical beam with $l \neq 0$ is focused into a central null ring-like intensity pattern rather than the maximum brightness in the center produced by a normal Gaussian laser mode with $l = 0$. Consider a helical beam with unknown topological charge l passing through a phase mask $\exp(il'\theta)$ with a predefined topological charge l' . The transmitted field on passing through the phase mask will have a helical wavefront with modified topological charge $l' + l$. When $l' = -l$ and the phase mask has a conjugated structure to the input waveform, the wavefront has zero resultant charge and its OAM-state may readily be detected from the centered bright spot in the diffraction pattern in the focal plane of a converging lens. Such a phase mask is suited only to the detection of a conjugated wavefront. In this thesis, we use a binary amplitude computer-generated hologram

(CGH) instead of a phase mask in the measurement of OAM-states. The CGH contains the conjugated phase functions of all the OAM-states to be used in the transmission and each conjugated state is projected into a specific direction determined by applying a relevant carrier spatial frequency. Then the diffracted field from the CGH is focused into a CCD beam profiler. By analyzing the intensity profile captured, one can detect the existence of multiple OAM-states simultaneously. The basic component of this CGH is a binary amplitude grating with a dislocation in the center as illustrated in Fig. 3.16(a). The CGH was first proposed to generate helical beams [152]. As shown in Fig. 3.16(b), the CGH gives mirror helical beams in the +1st and -1st diffraction orders which have reverse topological charges in the far field under the illumination of a plane wave. The beams have the characteristic ring patterns corresponding to the specific charge encoded in the CGH with the diameter of the ring of highest intensity proportional to l . If we multiply together transmittance functions of two binary CGHs with perpendicular grating orientations as in Fig. 3.16(c), the corresponding far-field pattern in Fig. 3.16(d) shows more diffraction orders and correspondingly more helical beams. There are four primary beams due to the two conjugate pairs arising from each CGH which provide the basis for analyzing four input OAM-states at the same time. However there are also additional diffraction orders and additional helical beams present (for example in Fig. 3.16(d) there are two visible further orders recorded symmetrically along the diagonal through the pattern center corresponding to $l = \pm 4$) due to two-dimensional

interferences. We have chosen a CGH design with only four beams of high intensity, but the option is open to employ a multi-level phase CGH modified by "anti-blazing" to enhance the power distributed to other diffraction orders thus widening the number of topological charges that can be analyzed simultaneously [153].

Figure 3.16. Some examples of binary amplitude CGH used to measure OAM-states. For (a) and (c), the white color and black color correspond to transmission of “1” and “0”. (a) the pattern of CGH generating two helical beams of topological charge +7 and -7, (b) the far-field intensity profile of pattern (a) illuminated by a plane wave, (c) the pattern of CGH generating four helical beams of topological charge +3, -3, +7 and -7, (d) the far-field intensity profile of pattern (c) illuminated by a plane wave.

We choose four OAM-states of $l = \pm 3, \pm 7$ as the working channels. Each channel has two states, “1” or “0” for “ON” or “OFF”, respectively. For example, if two OAM-states $l = -3, -7$ are transmitted, the status of the four working channels labeled by topological charges $(+3, -3, +7, -7)$ will be $(0, 1, 0, 1)$ respectively. The analyzing CGH at the receiving end is designed to have four major diffraction orders which correspond to the measurement of four OAM-states. We used a printed transparency illustrated in Fig. 3.16(c) as the binary amplitude CGH to generate the 2×2 grid for OAM-states detection in the far-field as shown in Fig. 3.17(a). The locations of helical beams are different from those in Fig. 3.16(d) because of the orientation of CGH. Taking x, y as the horizontal direction and vertical direction respectively, we use (x, y) to mark locations of orders. The origin is set to be the bright zero-order in the center of the diffracted field. Now the orders of $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$ are used to detect the conjugated OAM-states of $+7, +3, -7, -3$ respectively. In Fig. 3.17(b), there is only one central bright spot in the location of $(1, 0)$, which indicates only the status of the channel represented by the OAM-state of topological charge $+7$ is “1” and the others are “0” because the OAM-state carried by the laser beam is canceled by the original OAM-state -7 appearing in the order $(1, 0)$ as shown in Fig. 3.17(a). On the other hand, the results shown in Fig. 3.18 demonstrate the simultaneous detection of multiple OAM-states. For example, the iterative algorithm is used to calculate the phase element which produces a laser beam embedded with three multiplexed OAM-states representing the channel status $(1, 0, 1, 1)$. In the receiving end, we

observe three central intensity peaks at diffraction orders of (1,0), (0,1) and (-1,0) of Fig. 3.18(b) in agreement with three major components within the incoming laser beam's power spectrum over topological charges illustrated in Fig. 3.18(a). Another example depicted in Fig. 3.18(c) and 3.18(d) shows the measured far-field intensity profile when all the channels are transmitting data "1". It is noted that in the calculated power spectrum of Fig. 3.18(a), the ratio of signal power level over the noise appearing in the channel of topological charge -3 is more than 8 times. The unwanted channels other than the four channels will not contribute to the measurement because they are not the conjugated OAM-states of analyzing CGH ,and thus they will add up to the central intensities within rings. We test transmission of data through the four channels. A CCD camera is connected to a computer running a measurement program which detects the intensities at those interested locations and compare them with the threshold. The refreshing rate of the SLM is set to be 1Hz and the distance between transmitter and receiver is 5m. We have gone through all the 16 combinations of channel status and successfully differentiate them in real time. The intensity experienced by each diffraction order may be different due to either slightly tilting or different diffraction efficiencies. Therefore, we use different threshold value for each channel. The threshold values are set to be proportional to the maximum intensity values detected in the specific locations. The crosstalk problem among different channels is not serious because of the binary nature of transmitted signals and the wide separation among the values of topological charges used in the experiment.

Figure 3.17 The intensity patterns captured by the CCD beam profiler. (a) under the illumination of a TEM_{00} laser mode, the numbers indicate topological charges appearing in corresponding orders when the channel state is $(0,0,0,0)$ (b) single OAM-state of topological charge +7 detection, i.e. the status of each channel is $(0,0,1,0)$ in the order of channels $(+3, -3, +7, -7)$.

Figure 3.18 Multiplexed OAM-states measurement (a) power spectrum of status (1, 0, 1, 1), (b) measured far-field intensity profile of status (1, 0, 1, 1), (c) power spectrum of status (1, 1, 1, 1), (d) measured far-field intensity profile of status (1, 1, 1, 1).

Apart from the equal power distribution scheme chosen in the examples shown in Fig. 3.18, we calculate a phase element which generates the collinear superposition of four OAM-states with a desired unequal power spectrum shown in Fig. 3.19(a). The ratio of power among topological charge -7, -3, +3, +7 is 4:3:2:1.

The optimized phase element (see Fig. 3.19(b)) has a power spectrum depicted in Fig. 3.19(c). The actual ratio among these four OAM-states measured from the corresponding intensity peaks in the simulated intensity pattern in Fig. 3.19(d) is 3.98:2.93:2.12:1, which is very close to the desired ratio. However, we failed to observe this ideal result in the experiment carried out on the FSO system. We believe it is due to the imperfect quality of the amplitude analyzing CGH which is printed on a transparency which curls naturally. This affects the power distribution over diffraction orders and thus the detection of multi-level signals sensitive to power fluctuations. Nonetheless, this multi-level signal could be useful for optical interconnection if a more accurate analyzer is provided. In such a system, each OAM-state serves as an independent channel and the power level of each channel may be more than just high and low. Hence, it could provide larger transmission capacity than the FSO system using binary signals.

Figure 3.19 Numerical simulations of OAM state measurement. (a) the desired spectrum over topological charges (b) the calculated phase distribution (c) the actual power spectrum generated by the phase element (d) the intensity pattern captured in the Fourier plane after passing through the analyzing CGH.

3.6 Conclusions

A so-called multi-helix beam is introduced in this Chapter. A high-efficiency method to generate such a beam by a phase-only DOE is proposed and

implemented experimentally. The phase-only transmittance of the DOE is calculated by an iterative algorithm. It is noted that the designed DOEs only depend on the azimuthal angle, so the radial variation of an incoming laser will not affect the performance. This advantage enables the application of multi-helix beams to the widely used fundamental Gaussian beam whose intensity profile varies in the radial direction. Two different structures of iterative algorithm are developed and compared. Either one provides better performance than the algorithm proposed by Schils and Sweeney [142]. Out of the two optimization structure, the simultaneous optimization structure demonstrates slightly higher performance.

The multi-helix beam can be exploited in free-space optical communication systems as discussed by Gibson, *et al* [20] to replace the conventional helical beam with single topological charge. By introducing OAM-states multiplexing, we can significantly increase the transmission capacity of existing OAM communication systems without additional optical components. For example, if a system operating on single channel has a transmission speed at T bit/sec, the modified speed is $N \times T$ bit/sec after adopting the multiplexing of N channels. It is noted that the number of channels has a practical upper bound due to the unwanted noise channels generated by the iterative algorithm itself. An estimated maximum number of working channels would be around 10 in this system. The speed of the multiplexed system is mostly limited by the refreshing rate of SLM, which is usually less than 1 kHz. Therefore, the maximum transmission rate will be around

10 kHz. However, it is possible to fabricate fixed phase masks each corresponding to an independent light source, e.g. a laser diode. By switching these laser diodes on and off, the transmitter can operate at a very high speed. The azimuthal modulated phase-only element can be also used to replace the complex mask in the application of vortex beams in optical fiber communications reported recently by Bouchal, *et al* [21]. Of course the technique also has a potential to form more complicated three-dimensional optical trapping structures similar to [23].

CHAPTER 4

ANISOTROPIC OPTICAL VORTICES

A conventional optical vortex demonstrates an isotropic, circularly symmetric intensity profile in the transverse plane during propagation. In contrast, an anisotropic optical vortex presents a more complicated transverse intensity distribution than conventional optical vortex. The anisotropic intensity distribution is closely related to the phase function of an optical vortex because the original amplitude function is usually symmetric. In this Chapter, the study of anisotropic optical vortices mainly focuses on the relationship between the shape of intensity distribution and the phase function.

4.1 Introduction

An anisotropic optical vortex and an isotropic one with topological charge equal to unity are depicted in the contour maps of Fig. 4.1. A global parameter such as a topological charge is well suited for an anisotropic optical vortex because it cannot describe the local structure of the anisotropic optical vortex. Therefore, a few parameterization methods are proposed to characterize an anisotropic optical vortex [154-157]. For example, the phase function of the elliptical optical vortex illustrated in Fig. 4.1(b) can be expressed as

$$\Phi(\theta) = \arctan[\tan \varepsilon \tan(\theta - \varphi)] + \gamma, \quad (4.1)$$

where the geometrical parameter ε describes the ellipticity, φ is the rotation angle measured from the x -axis, and γ is the initial phase shift. For a more complicated anisotropic optical vortex, the description of phase and amplitude would include superposition of isotropic optical vortices [154].

Figure 4.1 Intensity (dotted line) and phase (solid line) contours of optical vortices. (a) an isotropic optical vortex, (b) an anisotropic optical vortex. The phase values of adjacent phase contour lines differ from each other by $\pi/6$. [154]

As we can see from Fig. 4.1(b), the dark core of optical vortex extends further away the center where the phase contour lines are more concentrated (high azimuthal phase gradient). Using the relationship between phase and intensity distributions of an optical vortex propagating in free-space, modified helical beams have been proposed and studied as novel beams for generation of anisotropic

optical vortices in certain shapes rather than the conventional symmetrical doughnut shape [27, 158, 159]. Curtis and Grier demonstrated an approach to generating an anisotropic optical vortex with a Lissajous pattern in the focal plane of a converging lens by adding a sinusoidal modulation part to the phase function of a normal helical beam[27]:

$$\varphi(\theta) = l[\theta + \alpha \sin(m\theta + \beta)], \quad (4.2)$$

where the parameters α , β and m control the geometry of a Lissajous pattern as illustrated in Fig. 4.2. The modified phase function of a helical beam is supposed to produce a m -fold symmetrical Lissajous pattern in far-field with the depth controlled by α and the orientation controlled by β .

Figure 4.2 Lissajous anisotropic optical vortices. The patterns in the top row have the same parameter $\alpha = 0.1$, and those in the bottom row share the same parameter $m = 4$. The parameter β is chosen in a way that the orientations of these patterns are identical. [27]

It is noted that this work was implemented in an implicit way that the anisotropic optical vortex only forms a Lissajous pattern in far-field and the intensity patterns of the modified helical beams were only changed restrictedly by varying the constants of the Lissajous pattern. However, a direct and deterministic technique for generation of helical beams with various specific intensity patterns explicitly will be more useful in practice. It has been shown that by using a spatial light modulator (SLM) for a phase-only hologram a modified helical beam embedded with a dark core of anisotropic optical vortex in a desired shape can be reconstructed within certain propagation distance when the Fresnel diffraction is taken into account [158]. Furthermore, a modified helical beam carrying an anisotropic elliptical optical vortex was generated by a micro-fabricated spiral phase plate and it was then employed for optical trapping recently [159]. However, the pattern of intensity distribution of an anisotropic optical vortex will change with the propagation distance not only in the scale but also in the shape. In general, an anisotropic optical vortex is the superposition of some isotropic optical vortices [154]. And these optical vortices usually have different Gouy phase shift during propagation because of the different topological charges. For example, the third phase term $(2p + |l| + 1) \arctan\left(\frac{z}{z_R}\right)$ in a propagating Laguerre-Gaussian mode defined in Eq. 2.2 accounts for the relevant Gouy phase shift and it depends on the topological charge l . Hence, the relative phase difference between any two constituent isotropic optical vortices normally changes during propagation and as such the whole intensity pattern of the anisotropic optical vortex will change. It is

noted that in many applications it is required to focus a helical beam. Hence, the method used in [158] and [159] to generate anisotropic optical vortices within the near-field range could encounter some problems in terms of the shape of the desired dark core of optical vortices in far-field. Therefore, this Chapter introduces a deterministic method to show how to obtain an anisotropic optical vortex with specified dark core shape as a far-field pattern.

4.2 Designs of anisotropic optical vortex

A conventional helical beam can be approximated by the phase function $\exp(il\theta)$ while the phase function of a modified helical beam carrying an anisotropic optical vortex can be written in a generalized form of $f(\theta) = \exp[i\varphi(\theta)]$, where $\varphi(\theta)$ represents the phase values and $\varphi(\theta) = l\theta$ in the case of a conventional helical beam. According to [27], the radius of the maximum intensity of the optical vortex at certain azimuth angle in the focal plane after the modified helical beam passes through a focusing lens is given by

$$R_l(\theta) = a_l \frac{\lambda}{NA} \left[1 + \frac{1}{b_l} \frac{d\varphi(\theta)}{d\theta} \right], \quad (4.3)$$

where a_l and b_l are constants depending on the helical beam's radial amplitude profile, λ is the wavelength of light and NA is the numerical aperture of the focusing lens. Obviously, $R_l(\theta)$ forms a closed loop about the origin. Compared to the circumference of the dark core of an optical vortex, the maximum intensity

loop $R_l(\theta)$ is not easy to be visualized in terms of an intensity pattern. So the main purpose of this design method is to realize an optical vortex with a desired dark core shape. But this method is also well suited to design the high-intensity loop $R_l(\theta)$ by adjusting the value of certain parameter.

Generally, the perimeter of the dark core will not locate at exactly the high-intensity loop $R_l(\theta)$. For the sake of brevity, the perimeter of the dark core is referred to as the perimeter below. Since the experimental result in [27] indicated that the perimeter almost kept the same shape as the maximum intensity loop, it is reasonable to suggest that the perimeter of the dark core $R(\theta)$ is also dependent on the azimuthal phase gradient $\frac{d\varphi(\theta)}{d\theta}$ linearly:

$$R(\theta) = a \frac{\lambda}{NA} \left[1 + \frac{1}{b} \frac{d\varphi(\theta)}{d\theta} \right] = \frac{a}{b} \frac{\lambda}{NA} \left[b + \frac{d\varphi(\theta)}{d\theta} \right] \quad (4.4)$$

where a and b are constants. For a conventional helical beam $\exp(il\theta)$, since the phase gradient along azimuthal direction is a constant, i.e. the topological charge l , the perimeter is formed in a circular shape because the radius is constant in this case. For modified helical beams, since the phase values can be modified to increase by the azimuth angle in a nonlinear fashion, it results in different local radii at different angles. The term $\frac{a\lambda}{b \times NA}$ in Eq. 4.4 can be neglected during the calculation of the specific phase function because it does not affect the shape of the perimeter but only the size, whereas the size can be altered by choosing different focusing lens later. Therefore, Rewriting Eq. 4.4, we have

$$R(\theta) \propto b + \frac{d\varphi(\theta)}{d\theta}. \quad (4.5)$$

Given phase function $\varphi(\theta)$ in a range between 0 and $l \times 2\pi$, where l is the overall topological charge of the anisotropic optical vortex, and in order to keep the overall topological charge as an integer l , we assume

$$R(\theta) = \frac{\int_0^{2\pi} R(t) dt}{2\pi(l+b)} \left[b + \frac{d\varphi(\theta)}{d\theta} \right]. \quad (4.6)$$

Now, if a certain shape of the perimeter $R(\theta)$ is determined in Eq. 4.6, then the phase function $\varphi(\theta)$ can be expressed as

$$\varphi(\theta) = 2\pi(l+b) \frac{\int_0^\theta R(t) dt}{\int_0^{2\pi} R(t) dt} - b\theta, \quad (4.7)$$

where we assume the starting phase $\varphi(0) = 0$.

Without losing generality, only a polygonal perimeter is discussed because polygons approximate most useful shapes. Consider a generalized polygon with n vertices $(P_1, P_2, \dots, P_{n-1}, P_n)$ as illustrated in Fig. 4.3. For the sake of simplicity, vertex P_1 is rotated onto the axis $\theta = 0$.

Figure 4.3 The desired polygonal perimeter $R(\theta)$ of the anisotropic optical vortex

The polygonal perimeter can be expressed as follows:

$$R(\theta) = \begin{cases} \frac{-r_1 r_2 \sin \theta_2}{r_2 \sin(\theta - \theta_2) - r_1 \sin \theta} & \text{for } 0 \leq \theta < \theta_2 \\ \frac{r_2 r_3 \sin(\theta_2 - \theta_3)}{r_3 \sin(\theta - \theta_3) - r_2 \sin(\theta - \theta_2)} & \text{for } \theta_2 \leq \theta < \theta_3 \\ \vdots & \\ \frac{r_i r_{i+1} \sin(\theta_i - \theta_{i+1})}{r_{i+1} \sin(\theta - \theta_{i+1}) - r_i \sin(\theta - \theta_i)} & \text{for } \theta_i \leq \theta < \theta_{i+1}, 2 \leq i \leq n-1. \\ \vdots & \\ \frac{r_{n-1} r_n \sin(\theta_{n-1} - \theta_n)}{r_n \sin(\theta - \theta_n) - r_{n-1} \sin(\theta - \theta_{n-1})} & \text{for } \theta_{n-1} \leq \theta < \theta_n \\ \frac{r_n r_1 \sin \theta_n}{r_1 \sin \theta - r_n \sin(\theta - \theta_n)} & \text{for } \theta_n \leq \theta < 2\pi \end{cases} \quad (4.8)$$

Combined with Eq. 4.8, the phase function becomes

$$\varphi(\theta) = 2\pi(l + b) \frac{I(\theta)}{I_{total}} - b\theta. \quad (4.9)$$

where

$$\begin{aligned}
 I(\theta) = & \left[\begin{aligned}
 & D_{1,2} \left[\ln \left| \tan\left(\frac{\theta - \alpha}{2}\right) \right| - \ln \left| \tan\left(\frac{\alpha}{2}\right) \right| \right] \\
 & \text{for } 0 \leq \theta < \theta_2 \\
 & D_{2,3} \ln \left| \tan\left(\frac{\theta - \alpha}{2}\right) \right| - D_{1,2} \ln \left| \tan\left(\frac{\alpha}{2}\right) \right| \\
 & + (D_{1,2} - D_{2,3}) \ln \left| \tan\left(\frac{\theta_2 - \alpha}{2}\right) \right| \\
 & \text{for } \theta_2 \leq \theta < \theta_3 \\
 & \vdots \\
 & D_{i,i+1} \ln \left| \tan\left(\frac{\theta - \alpha}{2}\right) \right| - D_{1,2} \ln \left| \tan\left(\frac{\alpha}{2}\right) \right| \\
 & + \sum_{m=1}^{i-1} (D_{m,m+1} - D_{m+1,m+2}) \ln \left| \tan\left(\frac{\theta_{m+1} - \alpha}{2}\right) \right| \\
 & \text{for } \theta_i \leq \theta < \theta_{i+1} \\
 & \vdots \\
 & D_{n-1,n} \ln \left| \tan\left(\frac{\theta - \alpha}{2}\right) \right| - D_{1,2} \ln \left| \tan\left(\frac{\alpha}{2}\right) \right| \\
 & + \sum_{m=1}^{n-2} (D_{m,m+1} - D_{m+1,m+2}) \ln \left| \tan\left(\frac{\theta_{m+1} - \alpha}{2}\right) \right| \\
 & \text{for } \theta_{n-1} \leq \theta < \theta_n \\
 & D_{n,1} \ln \left| \tan\left(\frac{\theta - \alpha}{2}\right) \right| - D_{1,2} \ln \left| \tan\left(\frac{\alpha}{2}\right) \right| \\
 & + (D_{n-1,n} - D_{n,1}) \ln \left| \tan\left(\frac{\theta_n - \alpha}{2}\right) \right| \\
 & + \sum_{m=1}^{n-2} (D_{m,m+1} - D_{m+1,m+2}) \ln \left| \tan\left(\frac{\theta_{m+1} - \alpha}{2}\right) \right| \\
 & \text{for } \theta_n \leq \theta < 2\pi
 \end{aligned} \right]
 \end{aligned}$$

with

$$\alpha = \arctan\left(\frac{r_2 \sin \theta_2 - r_1 \sin \theta_1}{r_2 \cos \theta_2 - r_1 \cos \theta_1}\right),$$

$$D_{p,q} = \frac{r_p r_q \sin(\theta_p - \theta_q)}{\left[(r_q \cos \theta_q - r_p \cos \theta_p)^2 + (r_q \sin \theta_q - r_p \sin \theta_p)^2\right]^{1/2}},$$

$$I_{total} = (D_{n,1} - D_{1,2}) \ln \left| \tan\left(\frac{\alpha}{2}\right) \right| + (D_{n-1,n} - D_{n,1}) \ln \left| \tan\left(\frac{\theta_n - \alpha}{2}\right) \right| \\ + \sum_{m=1}^{n-2} (D_{m,m+1} - D_{m+1,m+2}) \ln \left| \tan\left(\frac{\theta_{m+1} - \alpha}{2}\right) \right|,$$

and $2 \leq i \leq n-1$.

Now the only undetermined parameter in Eq. 4.9 is the constant b . Consider the simplest polygon case, i.e. a triangular anisotropic optical vortex with $l=10$, and the amplitude of the modified helical beam is given a Gaussian profile. We have the complex amplitude of the modified helical beam given by

$$E(\rho, \theta) = \exp\left(\frac{\rho^2}{w_0^2}\right) \exp[i\varphi_t(\theta)], \quad (4.10)$$

where w_0 is the beam waist and $\varphi_t(\theta)$ is the phase function derived from Eq. 4.9. The corresponding far-field complex amplitude in the focal plane of a converging lens is calculated by the scalar diffraction integral:

$$E_f(r, \varphi) = \frac{i}{\lambda f} \exp(ikf) \exp\left(i \frac{k}{2f} r^2\right) \\ \times \int_0^{2\pi} \int_0^\infty E(\rho, \theta) \exp\left[\frac{ik}{f} \rho r \cos(\theta - \varphi)\right] \rho d\rho d\theta \quad (4.11)$$

where f is the focal length of the lens. The numerically reconstructed far-field intensity distribution is shown in Fig. 4.4 in which we find that the value of the

constant b has a significant influence on the shape of the roundness of the reconstructed perimeter.

Figure 4.4 Far-field intensity distributions of triangular anisotropic optical vortices with different values of b

It is noted that as the value of b decreases the shape of the perimeter tends to be a circle. In Fig. 4.4, obviously an optimized value of $b = -3$ exists. Furthermore, an empirical formula $b = -0.3l$ fitting well for $l \geq 10$ can be found by altering the value of topological charge in numerical simulations. It is because Eq. 4.5 is only applied to helical beams with topological charges much greater than unity. Now the phase function becomes

$$\varphi(\theta) = 1.4\pi l \frac{I(\theta)}{I_{total}} - 0.3\theta. \quad (4.12)$$

According to Eq. 4.12, we explored and implemented a series of phase-only computer-generated holograms for reconstruction of anisotropic optical vortices with arbitrary dark core shapes using an SLM. A TEM_{00} laser beam shining onto a reflective type SLM (Boulder Nonlinear Systems P512 with 512×512 pixels) was imposed with the designated phase function $\varphi(\theta)$. Then the modified helical beam passed through a focusing lens to generate the far-field intensity distribution

as shown in Fig. 4.5 captured by a CCD beam profiler in the focal plane. A constant azimuthal phase can be added to cancel out the Gouy phase shift effect in order to keep the patterns the same orientation with that of the designs. The phase structures of these anisotropic optical vortices are shown in Fig. 4.6. The far-field distribution of phase shows similarity to the phase in the element plane.

Figure 4.5 Far-field intensity distributions of the modified helical beams with designated shapes of perimeter such as triangle, Greek cross, star and three letters (N, T, and U). The uppermost row shows the desired shapes of perimeter. The row in the middle shows the far-field distribution of corresponding modified helical beams through simulation. The bottommost is the experimentally observed far-field patterns of the anisotropic optical vortices. All these patterns are designed with overall topological charge $l = 60$.

Figure 4.6 Phase distributions of anisotropic optical vortices in the far-field. (a) triangle (b) Greek cross (c) star (d) the letter N (e) the letter T (f) the letter U.

4.3 Optical tweezers incorporated with anisotropic optical vortices

One of the most common applications of optical vortices involves optical tweezers technology. The conventional optical tweezers apply a highly focused Gaussian beam on microscopic dielectric particles. Due to the forces originated from the transfer of photon momentum, the particles are attracted to the focus of the laser beam. Therefore, the trapped particles will follow the trajectory of the focus when the laser beam moves. It has been proven to be a successful non-contact technique for manipulating small objects ranging from 10nm to 100mm in diameter.

By replacing the Gaussian laser beam with various helical beams, the optical vortices formed in the focal plane bring new aspects of applications because of the unique orbital angular momentum and intensity rings with central minima.

Anisotropic optical vortices proposed in this Chapter are also employed for optical trapping in an optical tweezers system. As shown in Fig. 4. 7, a modified helical beam generated by the SLM passes through a telescope system and enters into an inverted microscope. The far-field pattern is formed in the focal plane of a high-NA objective lens mounted in the microscope. As a result of optical trapping demonstrated in Fig. 4.8, an optical vortex with a triangle shape of perimeter is surrounded by six $3\mu\text{m}$ diameter polystyrene spheres dispersed in water.

Similar to a conventional optical vortex, the anisotropic optical vortex exerts a torque on the trapped particles. But the anisotropic optical vortex rotates the trapped particles in an irregular trajectory rather than the usual circle. All these

particles circulated along the bright rim of the anisotropic optical vortex in the same direction that is determined by the sign of topological charge, which indicates the existence of orbital angular momentum. The snapshots in Fig. 4.9 captured from a video clip show that the orbital angular momentum induced rotation of polystyrene spheres trapped in the bright rim of triangle shape optical vortex.

Figure 4.7 A sketch of the optical tweezers system.

Figure 4.8 Polystyrene spheres are trapped by an anisotropic optical vortex with triangle perimeter.

Fig. 4.9. Polystyrene spheres are trapped by an anisotropic optical vortex with triangle perimeter. Eight polystyrene spheres move along perimeter of triangle. The spheres spent around 17 sec completing a circle by using 532nm laser at 800mW. One of the spheres is indicated by dot and arrow.

4.4 Conclusions

In this Chapter, a deterministic method for generation of anisotropic optical vortices with desired far-field patterns (polygons) is demonstrated. The semi-analytical expression of the radial phase distribution corresponding to a polygonally shaped dark core of optical vortex is derived by solving the governing differential equation. Using this technique, we are able to modify the intensity

distribution of optical vortex in the far-field to match the targeted particle in optical tweezers and assemble particle in a specific way with a single diffractive optical element. Accompanied with the engineered intensity pattern, the torque originated from the transferred OAM drives microscopic particles along a particular two-dimensional trajectory controlled by the design of anisotropic optical vortex.

CHAPTER 5

RADIUS-ADJUSTABLE OPTICAL VORTICES WITH SUPPRESSED SIDELOBES

5.1 Focused optical vortex

The study of far-field distribution of a helical beam is important for many applications, especially in optical tweezers since a helical beam need to be focused to a small area to increase the intensity. Usually a helical beam is produced by passing a plane wave through an azimuthally modulated phase element with a transmission function $\exp(il\theta)$, where the integer l is the topological charge of the optical vortex. Curtis and Grier [74] found that the intensity profile of such an optical vortex in the focal plane has a primary intensity ring accompanied by concentric outer rings of diminishing intensity (see Fig. 5.1). These outer rings, referred to as sidelobes below, become prominent when the topological charge increases. They also found that the radius of the primary intensity ring, measured from the center to the first intensity maximum in the radial direction, scales approximately linearly with the topological charge l (see Fig. 5.2) instead of the square root of it for Laguerre-Gaussian beams that are described by Padgett and Allen [43]. The discrepancy is explained by investigating the optical field of a focused optical vortex which is a Fourier transform of the complex amplitude of

incoming helical beam [39]. An impression is given in [74] that the optical vortex created in the far field has radius dependence on l because of the truncated Fourier transform caused by the limited aperture of focusing lens.

Figure 5.1 A focused optical vortex of topological charge 40. The central bright dot is due to a collinear Gaussian beam. [74]

Figure 5.2 The relation between the radius of an optical vortex and the corresponding topological charge. [74]

However, we find that, besides the topological charge, the amplitude profile of the helical beam is another major factor in determining the radius. The conclusion becomes clear when comparing the relations between radii and topological charges

obtained from two helical beams with different amplitude distributions. Fig. 5.3(a) and (b) show the relationships between the radius and the topological charge for a Gaussian amplitude distribution

$$\exp\left(-\frac{r^2}{w^2}\right) \quad (5.1)$$

and a Laguerre-Gaussian amplitude distribution ($p = 0$, see Eq. 2.2)

$$\left(\frac{r}{w}\right)^l \exp\left(-\frac{r^2}{w^2}\right), \quad (5.2)$$

respectively, where w is the beam waist. Notice that these two beams have the same helical phase if the topological charges are identical. It is evident that the former vortex scales with l and the latter scales with $\sqrt{l}$. So the amplitude profile of an optical vortex has a pronounced effect on radius of optical vortex that is actually neglected in [74], and the different amplitude profiles presented in [43] and [74] are indeed the source of the discrepancy between these two radius dependencies.

Figure 5.3 Dependence of the radius of a focused optical vortex on the topological charge. (a) for Gaussian amplitude profile, (b) for Laguerre-Gaussian amplitude profile.

The complex amplitude of a focused optical vortex with Gaussian amplitude profile is expressed by the Fraunhofer diffraction integral [39]

$$E(\rho, \theta) = \frac{ik}{2\pi f} \exp(ikf) \exp\left(i \frac{k}{2f} \rho^2\right) \times \int_0^\infty \int_0^{2\pi} u(r, \varphi) \exp\left[-\frac{ik}{f} r \rho \cos(\varphi - \theta)\right] r dr d\varphi, \quad (5.3)$$

where $u(r, \varphi)$ is the incoming optical field, k is the wave-number and f is the focal length. For a helical beam with a Gaussian amplitude profile defined as in Eq. 5.1, the far-field amplitude is given by [163, 164]

$$A(x) = \frac{\pi^{3/2} w^2}{4} x \exp\left(-\frac{x^2}{8}\right) \left[I_{l-1/2}\left(\frac{x^2}{8}\right) - I_{l+1/2}\left(\frac{x^2}{8}\right) \right], \quad (5.4)$$

where $x = \frac{\pi w \rho}{f \lambda}$, and I_n is the n -th order modified Bessel function of the first kind. By using the asymptotic expansion [164, 165]:

$$I_\nu(v\xi) - I_{\nu+1}(v\xi) \approx \frac{\exp(v\eta)}{\sqrt{2\pi\nu}} \frac{1 + \xi - (1 + \xi^2)^{1/2}}{\xi(1 + \xi^2)^{1/4}} \quad (5.5)$$

with $\eta = \sqrt{1 + \xi^2} + \ln\left(\frac{\xi}{1 + \sqrt{1 + \xi^2}}\right)$ for $\nu \gg 1$, Eq. 5.4 becomes [164]

$$A(x) \approx \frac{\pi w^2}{4} \frac{x}{\sqrt{2\nu}} \exp[\nu(\eta - \xi)] \frac{1 + \xi - (1 + \xi^2)^{1/2}}{\xi(1 + \xi^2)^{1/4}} \quad (5.6)$$

with $\nu = \frac{l-1}{2}$ and $\xi = \frac{x^2}{4(l-1)}$. The position of the primary maximum of this amplitude profile is shown to scale with the topological charge. [164]

For a helical beam with uniform amplitude profile limited by a circular opening:

$$T(r) = \begin{cases} 1 & r \leq R \\ 0 & r > R \end{cases} \quad (5.7)$$

, the amplitude profile in the far field is given by [166]

$$A(y) = \frac{1}{(l+2)l!} \frac{2\pi R^2}{f\lambda} y^l \left| {}_1F_2 \left[\frac{l+2}{2}, \frac{l+4}{2}, l+1; -y^2 \right] \right| \quad (5.8)$$

with $y = \frac{\pi R \rho}{f\lambda}$, where ${}_1F_2(a, b, c; x)$ is a hypergeometric function [165].

Although there is not an explicit formula for the location of primary maximum, numerical calculation results indicate it still scales with the topological charge.

5.2 Suppression of sidelobes

Guo *et al* [161, 162] showed that these secondary intensity rings in the far-field intensity distribution of an optical vortex can be removed by blocking a large central area of the phase element (see Fig. 5.4). However, the intensity of primary ring will be reduced significantly if the width of annulus is too narrow. Fig. 5.5 illustrates the influence of the width of annular aperture to the far-field intensity profile of an optical vortex. The annular aperture used in Fig. 5.5(c) has better performance than the other three apertures in terms of power efficiency and contrast. The optimal value of width of the annular aperture is found to correlate

with the topological charge. Optimal width for each particular topological charge is numerically calculated and these optimal values are fitted by a curve given by [162]

$$\Delta R_{opt}(l) = 1.4043 R_0 l^{-0.5363}. \quad (5.9)$$

However, the remaining primary intensity ring merely accounts for a portion of the incoming energy, e.g., around 40% in the case of an optical vortex of topological charge 40. The power efficiency of this method will be worse if the incident laser beam has a Gaussian profile with higher intensity at the center. Hence, we propose a novel method using phase-only radial modulation to suppress these secondary intensity rings without block the aperture.

Figure 5.4 Phase-only elements and the corresponding optical vortices in the far-field, (a) a conventional focused optical vortex generated by a phase element with full aperture, (b) a sidelobes-suppressed optical vortex generated by an annular phase element. The central bright spots are due to the coaxial Gaussian beams. [161]

Figure 5.5 Radial intensity profiles of focused optical vortices generated by phase elements with various annular apertures. The width of annular aperture decreases from (a) to (d) starting from an unobstructed circular aperture in (a). [161]

5.3 Radially modulated spiral phase

In this Chapter, a radial modulation function is added to the conventional phase element to concentrate most of the incoming power into a single intensity ring and hence suppress the secondary intensity rings. The radius of the intensity ring no longer scales with the topological charge and is determined by the radial modulation part and the topological charge together. This high-efficiency technique could be used to enhance the trapping stiffness in optical trapping systems and generate a multi-ring system without interference with each other.

A helical beam of topological charge l in the plane of phase mask $\exp(il\theta)$ can be expressed as

$$E(r, \theta) = A(r) \exp(il\theta), \quad (5.10)$$

where $A(r)$ is the amplitude profile of the incoming laser beam. For the sake of simplicity, we consider the uniform illumination, i.e., $A(r) = 1$. After focusing by a lens of focal length f , the field in the focal plane is given by the Fourier transform in polar coordinates:

$$E_f(\rho, \phi) = \frac{k}{f} \exp\left(i \frac{k}{2f} \rho^2\right) \exp(il\phi) \int_0^R J_l\left(\frac{kr\rho}{f}\right) r dr, \quad (5.11)$$

where k is the wave-number and $J_l(x)$ is the l -th order Bessel function of the first kind. Those constant phase factors are neglected in Eq. 5.11. On introducing a variable $u = kr\rho/f$, the rotationally symmetric amplitude part of $E_f(\rho, \phi)$ is given by

$$A_f(\rho) = \frac{f}{k\rho^2} \int_0^{\frac{k\rho R}{f}} J_l(u) u du. \quad (5.12)$$

The first maximum of $A_f(\rho)$ occurs approximately at

$$kR\rho/f = x_1 \quad (5.13)$$

(see Appendix), where x_1 is the first root of $J_l(x)$. It is also noted that the second maximum corresponds to the third root x_3 of $J_l(x)$, as illustrated in Fig. 5.6. If we consider an annular zone of the phase mask, the amplitude at the focal point contributed by the annular zone becomes

$$A_f(\rho) = \frac{f}{k\rho^2} \left[\int_0^{\frac{k\rho R_2}{f}} J_l(u) u du - \int_0^{\frac{k\rho R_1}{f}} J_l(u) u du \right], \quad (5.14)$$

where R_2 and R_1 denote the inner and the outer radii of the annular zone, respectively.

Figure 5.6 The 40-th Bessel function of the first kind (dash curve) and amplitude profile (solid curve) created by focusing helical beam of $l = 40$. The horizontal axis denotes a dimensionless variable $kR\rho/f$. The vertical dotted lines indicate the positions of first two odd roots of the Bessel function.

Following the above discussion, the second maximum of the first integral in Eq. 5.13 appears at $\rho = fx_3/(kR_1)$, while the first maximum of the other integral in Eq.

5.13 occurs at $\rho = fx_1/(kR_2)$. If these two maximum coincide with each other, which indicates

$$R_2 = \frac{x_1}{x_3} R_1, \quad (5.15)$$

the sidelobe of the field intensity distribution $I_f = |A_f|^2$ in the focal plane can be suppressed as depicted in Fig. 5.7. Above 90% of the power coming from the annular zone falls into the intensity peak. Our purpose in the above investigation is to find a simple way to divide the circular aperture into annular zones. The result in Eq. 5.15 is different with Guo's annular aperture in Eq. 5.9 [161] calculated by a trial-and-error method. However, the difference in the resultant intensity distributions is small and will not affect the discussion below.

Suppose a phase element $\exp(il\theta)$ with a circular aperture of radius R is divided with a series of concentric circles in the radial direction. The radii $\{R_i\}$ of these circles satisfy

$$R_i = \left(\frac{x_1}{x_3}\right)^i R, \quad i = 1, 2, \dots. \quad (5.16)$$

According to Eq. 5.15, each annular zone enclosed with two adjacent circles will produce a similar intensity peak with only scale and location differences, because the shape of the intensity distribution of an annular zone is determined only by the ratio between the inner radius and the outer radius. Once the ratio is fixed, as in our design, the value of the inner or the outer radius affect only the scale and the location of the intensity distribution. The annular zone farther from the center

contributes to the intensity ring with a smaller radius and a higher intensity peak. Normally only a few intensity peaks generated by the outermost few annular zones will be considered, since these already occupy most the area of the phase mask. We find that the intensity peak of the ring shifts away from the beam axis if a blazed phase grating $\exp(i\tau r)$ is added to an annular zone of the phase mask as illustrated in Fig. 5.8 and the shift distance increases with the parameter τ . The intensity of the shifted peak will drop because of the expanded area of its modified intensity distribution. It is possible to shift intensity peaks

Figure 5.7 The focal plane intensity distribution of a helical beam of $l = 40$ with a circular aperture of radius R (dash curve) and the intensity distribution contributed by an annular zone with inner radius $(x_1/x_3)R$ and outer radius R (solid curve).

generated from different zones into the same region if a different τ is applied to each annular zones. To achieve a desired location for the primary peak, a radial modulation term, $\exp(i\tau r)$, is added to the outermost annulus. The shifted distance from the center is found to increase with the value of τ monotonically. Therefore, it is possible to find a particular value corresponding to the desired shift distance. If the shifted intensity peak turns out to be farther from the centre than the desired location, τ is decreased to move the peak toward the center, and vice versa. Next, the second intensity peak generated by the neighboring annular zone is shifted to the desired location to combine with the previous one by the same method of trial and error. The process continues until most of incoming power is projected in the desired location.

Figure 5.8 The shift of intensity peak due to the radial modulation added to the annular zone of the phase element.

Figure 5.9 The phase element divided into annular zones by dash circles.

For example, a phase mask is divided into five annular zones and a central circular zone as illustrated in Fig. 5.9. The intensity distribution of a focused flat-top helical beam in Fig. 5.10(a) can be decomposed into five primary intensity peaks and the remnant as shown in Fig. 5.10 (b). The parameters used in the following calculation of are $\lambda = 632.8nm$, $f = 30cm$, $R = 2.5mm$, and $l = 40$. These five intensity peaks $\{I_i\}$ are attributed to the annular zones with $\{R_{i-1}\}$ as inner radii and $\{R_i\}$ as outer radii respectively, where the radii fulfill Eq. 5.16 and $i = 1, 2, 3, 4, 5$. I_R represents the intensity distribution generated by the residual region of the phase mask, which receives only 14% of the incoming power. Consider a phase mask $g(r)\exp(il\theta)$ with the radial phase modulation given in Eq. 5.17 calculated by the above trial-and-error method,

$$g(r) = \begin{cases} \exp(i40.9wr), & qR < r \leq R \\ \exp(i38.9wr), & q^2R < r \leq qR \\ \exp(i36wr), & q^3R < r \leq q^2R \\ \exp(i32.3wr), & q^4R < r \leq q^3R \\ \exp(i23.8wr), & q^5R < r \leq q^4R \\ 1, & 0 \leq r \leq q^5R \end{cases} \quad (5.17)$$

where $w = \pi/R$ and $q = x_1/x_3$. Fig. 5.11(a) shows the corresponding intensity distribution in the focal plane. The five intensity peaks are shifted to the location of intensity peak of I_R . Around 80% of the total incoming power falls into the central sharp peak with suppressed sidelobes. Although this phase element is designed under the condition of uniform illumination, the intensity distribution as shown in Fig. 5.11(b) remains almost the same as in Fig. 5.11(a) when the incoming beam has a Gaussian amplitude profile. In Fig. 5.12(a), the intensity profile generated by an optical vortex with $l=120$ has a primary intensity peak located in the same place with the shifted one in Fig. 5.11(a). However, the intensity distribution in Fig. 5.11(a) is far more concentrated than in Fig. 5.12(a). In Fig. 5.12(b), we can even produce an intensity peak higher than the original primary intensity peak by combining the first two intensity peaks following Eq. 5.18. The other parts of intensity are removed by blocking the central region of the phase mask. This cannot be achieved by simply applying an annular aperture without apply the radial modulation.

$$g(r) = \begin{cases} \exp(i10.2wr), & qR < r \leq R \\ 1, & q^2R < r \leq qR \\ 0, & 0 \leq r \leq q^2R \end{cases} \quad (5.18)$$

Figure 5.10 Radial intensity distributions of conventional optical vortices in the focal plane. (a) produced by phase element $\exp(i40\theta)$, (b) five primary peaks of $I_1 \sim I_5$ generated by five annular zones and the remained intensity generated by the central region of the phase element.

Figure 5.11 Radial intensity distributions of sidelobe-suppressed optical vortices in the focal plane. The optical vortices are produced by the phase element $g(r)\exp(i40\theta)$. (a) uniformly illuminated, (b) illuminated by a Gaussian beam whose intensity drops to 10% of the central maximum at the aperture edge.

Figure 5.12 Radial intensity distributions of optical vortices in the focal plane. (a) produced by the conventional phase element $\exp(i120\theta)$, (b) combination of first two intensity peaks of the optical vortex.

We tested the phase element $g(r)\exp(il\theta)$ of Eq. 5.17 by employing a liquid-crystal spatial light modulator (Boulder Nonlinear Systems P512 with 512×512 pixels and 15 μm pixel pitch). Fig. 5.13(a) and (b) illustrate a conventional phase mask of $l = 40$ within a circular aperture of radius 2.5mm and a modified one with radial modulation respectively. An additional grating phase was added to the phase function before it was loaded into the SLM in order to diffract the desired pattern into a specific direction. The additional grating phase is added only within the central circular area, so only the light illuminating in that region will be diffracted into the desired direction. A collimated laser beam coming from a He-Ne laser at the wavelength of 632.8nm illuminated on the SLM and the diffracted beam was focused by a lens. The intensity distribution in the focal plane was captured by a CCD camera. Fig. 5.14(a) shows an optical vortex of topological charge 40 generated by the conventional phase mask focused by a lens of focal length 30cm. In Fig. 5.14(b), an optical vortex of the same topological charge generated by the proposed phase mask with radial modulation given in Eq. 5.17 was focused by the same lens. The modified optical vortex exhibits a clear single intensity ring of linewidth comparable with the primary intensity ring of the original one. In order to compare these two optical vortices on similar scales, we changed the focal length of lens to 10cm and then recaptured the modified optical vortex in Fig. 5.14(c). According to our design, the single ring in Fig 5.14(c) occupies 85% of incoming laser power in comparison to 40% in the primary ring

of Fig. 5.14(a) (neglecting diffraction and conversion losses). Meanwhile, the radius of ring in Fig. 5.14(c) is reduced to one third of that in Fig. 5.14(a) because of the shorter focal length. So we expect that the intensity of the ring in Fig. 5.14(c) is roughly 6 times that in Fig. 5.14(a).

Figure 5.13 The designs of spiral phase elements (a) without radial modulation, (b) with radial modulation.

Figure 5.14 The experimental intensity patterns in the focal plane. (a) the conventional optical vortex focused by a lens of $f=30\text{cm}$, (b) the modified optical vortex focused by a lens of $f=30\text{cm}$, (c) the modified optical vortex focused by a lens of $f=10\text{cm}$.

5.4 Concentric optical vortices

Focusing two coaxial optical vortices with different topological charges usually results in a grainy intensity distribution instead of two separate rings because of the

interference between each other (see Fig. 5.15). Because the part within the primary ring of a single optical vortex has little power (see Fig. 5.10(a)), the interference is mainly due to the extended sidelobes beyond the primary intensity ring. When the difference between the values of these two topological charges is large, which means the two relative primary rings are spatially separated, the optical vortex with smaller topological charge is hardly disrupted and the larger one is severely discontinued due to the interference with the sidelobes from the inner ring (see Fig. 5.15(a), (b) and (c)).

Figure 5.15 The far-field interference patterns of two optical vortices. The inner ring is the primary intensity distribution of the optical vortex with smaller topological charge. (a) $l_1 = 20$, $l_2 = -1$, (b) $l_1 = 20$, $l_2 = -3$, (c) $l_1 = 20$, $l_2 = -8$, (d) $l_1 = 20$, $l_2 = -12$, (e) $l_1 = 20$, $l_2 = -15$, (f) $l_1 = 20$, $l_2 = -20$.

However, it is essential to maintain smooth intensity rings for some applications. For example, when two coaxial optical vortices were coupled into an

optical tweezers system, we found that the broken outer optical vortex cannot induce rotation of particles because the local “hot spot”, intensity maxima formed by constructive interferences, tend to immobilize the particles through gradient forces. For the intact inner ring, the rotation of particles is similar to a single optical vortex. Recently, Ladavac and Grier [167] used the annular aperture method proposed in [161, 162] to suppress the sidelobes and thus reduce the interference between two coaxial optical vortices (see Fig. 5.16). The two-ring structure was employed in an optical tweezers system and demonstrated smooth rotation of particles on both rings.

Figure 5.16 Concentric optical vortices. (a) The phase element with two annulus corresponding to the optical vortices of topological charge -30 and 80, respectively. (b) The focused concentric optical vortices generated by the phase element. (c) The rotation of polystyrene particles with the diameter $0.99\ \mu\text{m}$. The rotation direction is controlled by changing the sign of topological charge. [167]

The limitations for using annular apertures in creating concentric optical vortices are obvious. First, the power efficiency is low especially for an incoming

Gaussian beam with a central intensity minimum because a large central portion of phase element is wasted. For example, the design in Fig. 5.16 has only about 50% power efficiency when the incoming laser beam has a Gaussian profile with the beam waist at the edge of phase element. In contrast, our design offers 100% usage of incoming laser power because the whole area of element is illuminated. In this comparison, we neglect other losses such as diffraction and reflection. Second, the radius of each individual ring is fixed by the inherent nature of optical vortex. So it is difficult to change the structure of concentric ring system once the topological charges and the incoming laser beam are determined. Nevertheless, we demonstrated a more flexible concentric optical vortices system can be produced by the radially modulated phase element. The circular aperture of a phase element is divided into two regions: the inner circular region for an optical vortex with topological charge -5 and the outer annular one for an optical vortex with topological charge 20. It is noted that an optical vortex with small topological charge value has negligible sidelobes, so we did not add radial modulation to the optical vortex of topological charge -5. For the outer region, the radial phase modulation function is calculated by the process introduced above. The resultant phase element and the generated coaxial optical vortices in far-field are shown in Fig. 5.17. The resultant two-ring system has a smoother outer ring compared to those in Fig. 5.15. However, we still observe the uneven intensity distribution on the outer ring caused by the slight sidelobes of the inner ring. These two coaxial optical vortices are directed into an optical tweezers system and demonstrate the

ability to induce counter-directional rotations in each ring (see Fig. 5.18).

Figure 5.17 Modified coaxial optical vortices. (a) Radially modulated phase element. (b) Focused coaxial optical vortices generated by the phase element. The inner ring and outer ring correspond to topological charges -5 and 20, respectively.

Figure 5.18 Optical trapping by using coaxial optical vortices. Two $0.99\ \mu\text{m}$ polystyrene particles are trapped in the inner ring and rotating anticlockwise. Some particles are trapped in the outer ring and rotating clockwise.

5.5 Conclusions

A high order optical vortex in the far-field is usually accompanied by sidelobes outside the primary ring. The forming mechanism of sidelobes is studied and we have demonstrated that by adding a radially modulated phase function to the conventional phase mask $\exp(il\theta)$, the subsidiary intensity rings are suppressed and the radius of optical vortex increases. Therefore, a conventional optical vortex generated by phase mask $\exp(il_1\theta)$ could be replaced with an optical vortex of lower topological charge generated by phase mask $g(r)\exp(il_2\theta)$ in the applications that require sharp intensity distribution in the focal plane. Usually we will not block the aperture of the phase mask, so the power efficiency is higher than the method based on applying an annular aperture. This method is also suitable for designing multiple concentric optical vortices trapping structures in which the interference among optical vortices caused by sidelobes seriously limits applications. These undesired sidelobes can be suppressed by the additional radial phase function imposed. As a result, the interference is significantly reduced. An optical trap consisting of two optical vortices with inverted signs of topological charges is shown to be able to drive particles rotating in reverse directions. Compared to the other approach of generating concentric optical vortices from different annular zones of a phase mask, this method offers higher power efficiency and a possible control on the radius of each optical vortex.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

In this thesis, the physics of optical vortices is studied and the techniques for designing diffractive optical elements are investigated. Several novel designs regarding modified optical vortices are explored and implemented.

First, an iterative algorithm is proposed for encoding multiple helical modes with a single phase-only element. Two different structures of the proposed algorithm are investigated. Furthermore, we compare the algorithm with an existing algorithm solving a mathematically similar problem. The new algorithm demonstrates a much lower relative error as compared to the present algorithm. A superposition of multiple helical modes is demonstrated by using a spatial light modulator experimentally. Our method is proved to be useful for free-space optical communications. Now we can realize the channel multiplexing by labeling each channel with a unique OAM-state. A prototype of such a multiplexed free-space optical communication system is demonstrated.

Second, a deterministic method is presented to generate modified helical beams which create optical vortices with desired dark core intensity patterns in the far-field. An analytical expression gives the modified azimuthal phase function of

a high order helical beam which will produce an anisotropic optical vortex with a polygonal dark core in the far-field. The experiments are implemented and verified by a spatial light modulator, which imprints a phase function onto the incident wavefront of a TEM_{00} laser mode to transform it into a modified helical beam. The phase function can be calculated once a specific dark core shape of an optical vortex is required. The modified helical beam is exploited in optical manipulation with verification of its orbital angular momentum experimentally.

Last, we propose a design of a phase mask for generating an optical vortex with suppressed side lobes in the focal plane and the radius of the intensity ring is variable. A radial modulation added to conventional azimuthally modulated phase mask $\exp(il\theta)$ projects the light diffracted from different annular zones into a single intensity ring in the focal plane. The technique is employed in the design of a two-ring optical trapping structure in which two groups of micro-particles trapped in separate rings rotate in reverse directions.

6.2 Recommendations for further research

In this thesis, designs of phase-only diffractive optical elements focus on the generation of modified optical vortices. Some novel applications and characteristics of these optical vortices remain undiscovered. For example, the multi-helix beam has been used in a free-space optical communication system. However, the application of such a beam in optical fiber communications will also

attract both scientific and commercial interests. A further study of optical vortices propagation in optical fibers and corresponding experiments are needed. Furthermore, the anisotropic optical vortices will be exploited to generate an array of modified optical vortices which may have the potential to sort particles by shape. In addition, the OAM distribution of a modified optical vortex will be studied quantitatively.

It is noted that a series of applications of conventional optical vortices in the field of image processing have been reported recently [168-173]. It is natural to consider the performance of modified optical vortices to optimize the performances. Following the investigations made in this thesis, much work can be done on the novel design of modified optical vortices.

Another promising field of research on optical vortices will be the study of associated orbital angular momentum and its applications in the emerging area of quantum informatics including quantum communications, quantum computing, quantum cryptography, etc. Polarization states are commonly used as the encoding media for information in these applications. However, polarization only provides two orthogonal states, which significantly restricts the performance. By using theoretically unlimited orthogonal OAM-states in quantum informatics, we can expect a major advance in this field and new areas to be discovered.

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APPENDIX

THE LOCATIONS OF MAXIMA OF THE FAR-FIELD INTENSITY PROFILE OF A FLAT-TOP HELICAL BEAM

The locations (in term of the normalized dimensionless variable $y = kR\rho/f$) of the maxima of the intensity distribution are not exactly coincident with the roots of Bessel function. But the difference is negligible, which can be explained as below.

The intensity distribution is expressed as

$$I_f(\rho) = |A_f(\rho)|^2 = \left| \frac{f}{k\rho^2} \int_0^{\frac{k\rho R}{f}} J_1(u) u du \right|^2 \quad (\text{A.1})$$

The locations of maxima are calculated by the derivative of Eq. A.1:

$$\frac{dI_f(\rho)}{d\rho} = 2A_f(\rho) \frac{dA_f(\rho)}{d\rho} = 0. \quad (\text{A.2})$$

Apparently Eq. A.2 has already implied an assumption, which is $A_f(\rho) \geq 0$. If a further assumption $A_f(\rho) \neq 0$ is given, we have

$$\frac{dA_f(\rho)}{d\rho} = 0. \quad (\text{A.3})$$

Now, if the variable ρ is replaced by the dimensionless variable $y = kR\rho/f$, Eq. A.3 becomes

$$\frac{d \left[\frac{1}{y^2} \int_0^y J_l(u) u du \right]}{dy} = 0. \quad (\text{A.4})$$

According to numerical calculations, the first two maxima appear at $y = 45.74$ and 56.44 while the first two odd roots of Bessel function are $x = 46.648$ and 56.658 .

The conclusion that the maxima of intensity coincide with the odd roots of Bessel function only happens when Eq. A.4 is replaced by neglecting the term of $(1/y^2)$:

$$\frac{d \left[\int_0^y J_l(u) u du \right]}{dy} = J_l(u) u = 0 \Rightarrow J_l(u) = 0. \quad (\text{A.5})$$

This is a good approximation because Bessel function varies much faster than $1/y^2$. The rigorous mathematical derivation of the zeros in Eq. A.4 is difficult which is shown below:

$$\frac{d \left[\frac{1}{y^2} \int_0^y J_l(u) u du \right]}{dy} = \frac{y^2 J_l(y) - 2 \int_0^y J_l(u) u du}{y^3} = 0. \quad (\text{A.6})$$

Further simplify Eq. A.6, we have

$$y^2 J_l(y) - 2 \int_0^y J_l(u) u du = 0. \quad (\text{A.7})$$

The solutions to Eq. A.7 are not explicit.