Dynamic analysis, modeling and control of a versatile micro air vehicle

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2010

Nguyen, H. Q. (2010). Dynamic analysis, modeling and control of a versatile micro air vehicle. Master's thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/35485>

<https://doi.org/10.32657/10356/35485>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**DYNAMIC ANALYSIS, MODELING AND CONTROL OF A
VERSATILE MICRO AIR VEHICLE**

**NGUYEN HONG QUAN
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING
2010**

DYNAMIC ANALYSIS, MODELING AND CONTROL OF A VERSATILE MICRO AIR VEHICLE

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Master of Engineering

2010

ACKNOWLEDGEMENTS

Much of my work for this research would not have been possible without the help and guidance from all the parties whom I wish to express my deep gratitude to:

Assistant Professor Yongki Go Tiaw Hiong, project supervisor, for his valuable advices, continuous guidance and support throughout this project

Suhartono Setiawan, Sheila Tobing and Adnan Maqsood, my group mates in the MAV research group, for working closely with me from the beginning of this project

Special thanks to the staffs of Main Aircraft Laboratory, CAD/CAM Laboratory and Service Workshop for their kind support and assistance.

Finally, my gratitude goes to the University for providing me with this great opportunity to work in this project.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	i
TABLE OF CONTENTS.....	ii
ABSTRACT.....	v
LIST OF FIGURES	vi
LIST OF TABLES	x
LIST OF SYMBOLS	xi
 Chapter 1: INTRODUCTION	 1
1. BACKGROUND	1
2. RESEARCH OBJECTIVES AND SCOPES	3
3. DOCUMENT ORGANIZATION.....	4
 Chapter 2: PREVIOUS WORK ON MAV DEVELOPMENT	 5
1. INTERESTING MAV DESIGNS	5
2. CONTROL SYSTEM DESIGNS	9
3. SYSTEM MODELING AND SIMULATION WITH MATLAB/SIMULINK	13
4. FINAL DESIGN OF THE MAV PROTOTYPE	16
 Chapter 3: SYSTEM MODELING AND SIMULATION BY MATLAB/SIMULINK	 18
1. INTRODUCTION	18
2. DATA PREPARATION FOR MODELING	18
2.1. Aerodynamic data	18
2.2. Trim conditions.....	22
3. SIMULATION MODEL.....	29
3.1. Manual control block.....	29
3.2. Environment model block	30
3.3. Aircraft model block	31
4. TESTING CONDITIONS	34
5. SIMULATION RESULTS	34
 Chapter 4: DYNAMIC ANALYSIS AND CONTROL SYNTHESIS OF CRUISING FLIGHT	 40

1. INTRODUCTION	40
2. EQUATIONS OF MOTION.....	40
3. AERODYNAMIC STABILITY AND CONTROL DERIVATIVES	41
3.1. Longitudinal plane	41
3.2. Longitudinal derivative transformation from stability to body axes systems.....	41
3.3. Lateral and directional plane.....	44
4. STATIC STABILITY ANALYSIS	46
5. DYNAMIC STABILITY AND RESPONSE ANALYSIS.....	47
5.1. Longitudinal case	47
5.2. Lateral – Directional case	47
6. DYNAMIC ANALYSIS OF THE MAV	48
6.1. Stability derivatives	48
6.2. Static stability discussions	49
6.3. Dynamic Response	49
7. AUTONOMOUS CONTROLLER DESIGN FOR MAV IN CRUISE	53
7.1. Pitch angle control system	54
7.2. Altitude control system	59
8. CONTROL SYSTEM IMPLEMENTATION TO THE NON-LINEAR MODEL	63
8.1. Altitude hold flight.....	63
8.2. Climbing flight.....	67
8.3. Turning flight	68
8.4. Combined flight profile.....	70
Chapter 5: PRELIMINARY HARDWARE IMPLEMENTATION AND TESTING	72
1. SENSOR SYSTEM	72
1.1 Introduction.....	72
1.2. Receiving and decoding raw data.....	73
1.3. Basics of Kalman filter	74
1.4. Model build-up	76
1.5. Experimental result	79
2. SERVO CONTROL SYSTEM	80
2.1. Introduction.....	80
2.2. Interface options	80
2.3. Stand alone servo board test	81

3. CONTROLLER TESTING	82
Chapter 6: CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK	86
1. CONCLUSIONS	86
2. RECOMMENDATIONS FOR FUTURE WORK	86
2.1. Design modifications	87
2.2. Wind-tunnel tests with the current prototype	87
2.3. Finalizing hover and transition control.....	87
2.4. Implementing the flight control system to the MAV	87
REFERENCES	88
Appendix A: AERODYNAMIC DATA FROM CFD SIMULATION	90
A1. WING – BODY COMBINATION	90
A2. VERTICAL AND HORIZONTAL TAILS.....	93
A3. STATORS.....	95
Appendix B: DETAILED DERIVATION OF LATERAL – DIRECTIONAL DERIVATIVES	97
B1. DERIVATIVES WITH RESPECT TO SIDESLIP ANGLE	97
B2. DERIVATIVES WITH RESPECT TO SIDESLIP ANGLE’S RATE OF CHANGE	98
B3. DERIVATIVES WITH RESPECT TO ROLL RATE AND YAW RATE.....	99
B4. DERIVATIVES WITH RESPECT TO VERTICAL TAIL DEFLECTION	109
B5. DERIVATIVES WITH RESPECT TO STATORS DEFLECTION	110
Appendix C: DETAILED CALCULATIONS OF STABILITY AND CONTROL DERIVATIVES.....	110
C1. LONGITUDINAL DERIVATIVES.....	110
C1.1. In stability axes system.....	110
C1.2. In body axes system	112
C2. LATERAL – DIRECTIONAL DERIVATIVES.....	113
Appendix D: : MAPLE CODE.....	115
D1. FINDING FORWARD FLIGHT’S STEADY-STATE CONDITIONS.....	115
D2. FINDING TURNING FLIGHT’S STEADY-STATE CONDITIONS.....	117

ABSTRACT

This research work is a part of a research project funded by Defense Science Organisation (DSO) to develop a vertical takeoff and landing (VTOL) micro air vehicle (MAV). Previous work has focused on conceptual design and numerical investigation of the aerodynamic properties of the MAV. This work focuses on the dynamic analysis and control system design of the current MAV design during non-hover flight. Both numerical simulation and analytical approaches were employed to obtain the results. For the simulation approach, a full 6 degree-of-freedom model was built using MATLAB Simulink and Aerospace Blockset.

The design of the flight control system was based on the linearised mathematical model of the MAV. Based on this linear model, appropriate control strategies were developed for various flight conditions such as cruise, climb, and turning flight. Finally, the controllers developed based on the linear approximation were applied to the non-linear system to examine their validity. It was proven that the designed controllers worked well for the non-linear aircraft model, even in the presence of disturbances.

Finally, preliminary implementation of the embedded control system, which consists of an Inertial Measurement Unit (IMU) and a servo control board, to the MAV prototype was developed.

LIST OF FIGURES

Figure 1.1: MQ1 – Predator UAV with Hellfire missiles (USAF)	1
Figure 1.2: Chinese UAV launched in June 2008 (4)	2
Figure 2.1: Black Widow MAV (7)	5
Figure 2.2: Initial design (left) and modified design of MicroStar MAV (5)	6
Figure 2.3: Wasp (left), Wasp II (middle) and Hornet (right) designs (8).....	6
Figure 2.4: Wasp III design (9).....	7
Figure 2.5: Allied Aerospace’s MAV – iSTAR (10).....	7
Figure 2.6: Honeywell’s MAV (12).....	8
Figure 2.7: The Israeli MAV – The Mosquito (13)	8
Figure 2.8: Control surfaces of Black Widow MAV (7)	9
Figure 2.9: MAV prototype of Blue Bear Systems Research Ltd. (left) and complete control board (right) (14).....	10
Figure 2.10: SLADe under fully automated flight control (left) and arrangement of control surfaces (right) (15).....	11
Figure 2.11: Fixed-wing prototype in its hovering orientation (16).....	12
Figure 2.12: A human pilot struggles to sustain a hover (top row) and the same MAV under autonomous hover control (bottom row) (16)	12
Figure 2.13: Horizon detection examples under different lighting conditions and video transmission noise degrees (18)	13
Figure 2.15: 3-DOF model of Wright brothers’ airplane (19).....	14
Figure 2.16: Lightweight four-seater monoplane (19).....	14
Figure 2.17: 3-DOF model of a lightweight airplane (19).....	15
Figure 2.18: Carolo MAV’s nonlinear model (20).....	15
Figure 2.19: Cruise to hover state model of a fixed-wing MAV (16).....	16
Figure 2.20: Final design of the MAV	17
Figure 2.21: Expected flight profile of the MAV.....	17
Figure 3.1: Effective angle of attack for horizontal tail.....	20
Figure 3.2: Effective angle of attack for vertical tail	21
Figure 3.3: Coordinate systems used in the theoretical approach	22
Figure 3.4: Forces acting on the MAV in steady state of cruise.....	25

Figure 3.5: Integration sequence to find trim thrust value.....	26
Figure 3.6: Geometry of steady-state, wing-level turn.....	27
Figure 3.7: Overall simulation model's blocks	29
Figure 3.8: Manual control block	30
Figure 3.9: Environment model	31
Figure 3.10: Aircraft model with 3 subsystem: Avionics system, sensors system and airframe	31
Figure 3.11: Avionics system block	32
Figure 3.12: Airframe block.....	33
Figure 3.13: Altitude, angle of attack and pitch attitude of the MAV's 3-DOF model in altitude-holding mode	35
Figure 3.14: Altitude, angle of attack and pitch attitude of the MAV's 3-DOF model in altitude-holding mode (with corrected thrust value)	36
Figure 3.15: Angle of attack and pitch attitude of the MAV's 3-DOF model due to $0.01rad$ change in horizontal tail's deflection	37
Figure 3.16: Estimation of phugoid mode's parameters by graphical method on pitch angle	37
Figure 3.17: Estimation of short-period mode's parameters by graphical method on angle of attack.....	38
Figure 3.18: Altitude, angle of attack and pitch attitude of the MAV's 3-DOF model in climb mode (with corrected thrust value)	39
Figure 4.1: Pitch response due to unit step input of horizontal tail	53
Figure 4.2: Typical displacement autopilot block diagram	54
Figure 4.3: Block diagram system with a displacement autopilot	55
Figure 4.4: Root locus of system with a displacement autopilot.....	56
Figure 4.5: System's response to unit step input (controller gain of 0.3).....	57
Figure 4.6: System's response to unit step input (controller gain of 1.5).....	57
Figure 4.7: System's response to unit step input (controller gain of 3).....	58
Figure 4.8: Pitch response with PID controller	59
Figure 4.9: Block diagram of an altitude control system	60
Figure 4.10: Root locus of the above altitude control system.....	60
Figure 4.11: System response to a unit step input with controller gain of 0.04.....	61
Figure 4.12: Response to a unit step input of system with a PD controller.....	62

Figure 4.13: Altitude, angle of attack and sideslip angle and bank angle of the MAV's 6-DOF model in altitude-hold mode (no wind)	63
Figure 4.14: Response of linear system and non-linear system to unit step input	64
Figure 4.15: Response of linear system and corrected non-linear system to unit step input	65
Figure 4.16: Wind model.....	65
Figure 4.17: Altitude, angle of attack and sideslip angle and bank angle of the MAV's 6-DOF model in altitude-hold mode (with wind disturbance).....	66
Figure 4.18: XY coordinate of the MAV's 6-DOF model in altitude-hold mode (with wind disturbance)	66
Figure 4.19: XY coordinate of the MAV's 6-DOF model in altitude-hold mode (with wind disturbance and heading angle control)	67
Figure 4.20: XY coordinate of the MAV's 6-DOF model in climbing-hold mode (with wind disturbance and heading angle control)	67
Figure 4.21: Altitude, angle of attack and sideslip angle and bank angle of the MAV's 6-DOF model in climbing mode (with wind disturbance)	68
Figure 4.22: XY coordinate of the MAV's 6-DOF model in turning mode (three rounds of turn)	69
Figure 4.23: Ground track during turning flight in wind	69
Figure 4.24: Angle of attack and sideslip angle, bank angle and altitude of the MAV's 6-DOF model in turning mode (with wind disturbance)	70
Figure 4.25: Ground track during combined mode.....	70
Figure 4.26: Angle of attack and sideslip angle, bank angle and altitude of the MAV's 6-DOF model in combined mode	71
Figure 5.1: LandMark 10 IMU (31)	72
Figure 5.2: Sensor system model	72
Figure 5.3: Algorithm to check for useful data package	74
Figure 5.4: Kalman filter's algorithm and explanation.....	78
Figure 5.5: Angle calculated by integrating gyroscope's data	79
Figure 5.6: Angle calculated with correction from Kalman filter	79
Figure 5.7: Angle sequence calculated without and with Kalman filter's correction	80
Figure 5.8: Pololu servo control board (33)	80
Figure 5.9: Block diagram for stand-alone servo board test.....	81
Figure 5.10: Simulink model for stand-alone servo board test.....	82

Figure 5.11: Block diagram for rapid control design.....	83
Figure 5.12: Attitude change feedback and horizontal tail's deflection of simulation model for a commanded sinusoidal attitude change of $0.5rad$	83
Figure 5.13: Attitude change feedback and horizontal tail's deflection of actual model for a commanded sinusoidal attitude change of $0.8rad$	84
Figure 5.14: Attitude change feedback and horizontal tail's deflection of simulation model for a commanded step attitude change of $0.1rad$	85
Figure 5.15: Attitude change feedback and horizontal tail's deflection of actual model for a commanded step attitude change of $0.5rad$	85
Figure A.1: Wing-body's drag coefficient vs. angle of attack.....	90
Figure A.2: Wing-body's lift coefficient vs. angle of attack.....	91
Figure A.3: Wing-body's pitching moment coefficient vs. angle of attack	91
Figure A.4: Wing – body's side force vs. side slip angle	92
Figure A.5: Wing – body's rolling moment vs. side slip angle	92
Figure A.6: Wing – body's yawing moment vs. side slip angle	93
Figure A.7: Vertical/horizontal tail's drag coefficient vs. angle of attack	94
Figure A.8: Vertical/horizontal tail's lift coefficient vs. angle of attack	94
Figure A.9: Vertical/horizontal tail's pitching moment coefficient vs. angle of attack.....	95
Figure A.10: Stators' drag coefficient vs. angle of attack.....	96
Figure A.11: Stators' rolling moment coefficient vs. angle of attack.....	96
Figure B.1: Side force at vertical tail in rolling flight (26).....	99
Figure B.2: Change in vertical tail's angle of attack due to yaw rate (26).....	100
Figure B.3: Change in wing's angle of attack in rolling flight (26)	101
Figure B.4: Change in wind speed due to yaw rate (26)	105

LIST OF TABLES

Table 3.1: Flight conditions for altitude-hold mode and climbing mode	34
Table 4.1: Longitudinal derivatives in stability axes system	41
Table 4.2: Longitudinal derivatives in body axes system	48
Table 4.3: Lateral - directional derivatives in body axes system	48
Table 4.4: Comparison of linear and non-linear models.....	50
Table 4.5: Longitudinal modes of some MAVs.....	51
Table 4.6: Lateral – Directional modes of some MAVs.....	52
Table 4.7: Summary of controller’s gains.....	62
Table A.1: Longitudinal aerodynamic data of wing - body combination	90
Table A.2: Lateral – directional aerodynamic data of wing – body combination	92
Table A.3: Aerodynamic data of vertical/horizontal tail.....	93
Table A.4: Aerodynamic data of stators.....	95

LIST OF SYMBOLS

Variable's notation	Description	Unit
A	Wing's area	m^2
b	Wingspan	m
c	Chord's length	m
C_D	Drag coefficient	
C_{D_α}	Variation of drag coefficient with respect to angle of attack	
C_l	Rolling moment coefficient	
C_{l_β}	Variation of rolling moment coefficient with respect to sideslip angle	
$C_{l_{\delta_S}}$	Variation of rolling moment coefficient with respect to stators' deflection	
C_L	Lift coefficient	
C_{L_α}	Variation of lift coefficient with respect to angle of attack	
C_m	Pitching moment coefficient	
C_{m_α}	Variation of pitching moment coefficient with respect to angle of attack	
$C_{m_{\delta_{HT}}}$	Variation of pitching moment coefficient with respect to horizontal tail's deflection	
C_n	Yawing moment	
C_{n_β}	Variation of yawing moment with respect to sideslip angle	
$C_{n_{\delta_{YT}}}$	Variation of yawing moment with respect to vertical tail's deflection	
D	Drag	N
dt	Sampling period of Kalman filter	s
$f_{A_x}, f_{A_y}, f_{A_z}$	Perturbed values of force components along XYZ	N
$F_{A_x}, F_{A_y}, F_{A_z}$	Aerodynamic force components along XYZ	N

l_A, m_A, n_A	Perturbed values of moment components about XYZ	Nm
L	Lift	N
L_A, M_A, N_A	Aerodynamic moment components about XYZ	Nm
m	Mass	kg
M	Mach number	
p, q, r	Perturbed values of angular velocity components about XYZ	rad / s
P, Q, R	Angular velocity components about XYZ	rad / s
$\bar{q}$	Dynamic pressure	N / m^2
T	Thrust	N
u, v, w	Perturbed values of airplane's velocity along XYZ	m / s
$\dot{u}, \dot{v}, \dot{w}$	Airplane's acceleration components along XYZ	m / s^2
U, V, W	Airplane's velocity components along XYZ	m / s
V_∞	Airplane's velocity (true airspeed)	m / s
Y	Side force	N

Variable's notation (Greek)	Description	Unit
α	Angle of attack	rad
$\dot{\alpha}$	Rate of change of angle of attack	rad / s
β	Sideslip angle	rad
$\dot{\beta}$	Rate of change of sideslip angle	rad / s
γ	Flight path angle	rad
δ	Control surface's deflection	rad
θ	Perturbed value of pitch angle	rad
Θ	Pitch angle	rad
ζ	Damping coefficient	
ρ	Air's density	kg / m^3
ϕ	Perturbed value of bank angle	rad

Φ	Bank angle	<i>rad</i>
ψ	Perturbed value of heading angle	<i>rad</i>
Ψ	Heading angle	<i>rad</i>
ω_d	Damped natural frequency	<i>rad / s</i>
ω_n	Natural frequency	<i>rad / s</i>

Subscript's notation	Description
<i>b</i>	Body axes system
<i>e</i>	Steady-state quantity
<i>HT</i>	Horizontal tail
<i>VT</i>	Vertical tail
<i>s</i>	Stability axes system
<i>S</i>	Stators
<i>WB</i>	Wing-body

Chapter 1: INTRODUCTION

1. BACKGROUND

After the Wright brothers invented the first powered airplane and successfully made the first heavier-than-air flight in 1903, people started developing unpowered aircrafts. This kind of aircrafts is usually called unmanned aerial vehicle (UAV). They can be remotely controlled or autonomously fly with an onboard autopilot which was preloaded with different control algorithms.

The first UAV was developed by A. M. Low in 1916 (1). It was intended to be used as an “aerial torpedo” during World War I. In the next few decades, UAVs were developed to be used only as “aerial torpedoes” and training target for anti-aircraft gunners until the 1960s. The US started using UAVs for reconnaissance missions to spy on Vietnam, China, and North Korea in the 1960s and early 1970s. With the advancement of technologies, reconnaissance UAVs were able to transmit live colored video to the ground control station. Many UAVs have been developed within the last 20 years, and were deployed by the US army in the Persian Gulf, Bosnia, Kosovo, Afghanistan and Iraq. Some of the UAVs could even carry weapons to attack targets on the ground (2). Figure 1.1 shows the MQ1 – Predator UAV, one of the most popular UAVs of the US Air Force carrying Hellfire missiles.

Figure 1.1: MQ1 – Predator UAV with Hellfire missiles (USAF)

Chapter 1: Introduction

These UAVs are not only proven useful in battlefield but also in other applications: It was reported that surveying the US border with Mexico by UAVs aided in the arrest of almost two thousands illegal immigrants and the seizure of four tons of drug in six months (3). They can also be used to search for survivors of disasters. Their applications can be any missions which are too “dull, dirty, or dangerous” for manned aircraft.

One of the latest UAVs was introduced in June 26, 2008 by Harbin Smart Special Aircraft Company. The Chinese company spent about 6 million Singapore dollars and 12 years developing its 1.2m-diameter prototype. It can hover at altitude of about 1000m, or fly at speed of 22m/s for up to 40 minutes (4).

Figure 1.2: Chinese UAV launched in June 2008 (4)

After achieving great success in development of UAVs, people started thinking about small-scale UAVs; however, developing small UAVs is a much more challenging task due to the limited size and weight. Those UAVs with largest dimension of 15 centimeters or less are classified as micro air vehicles (MAVs). This standard was defined by the Defense Advanced Research Projects Agency (DARPA) in 1997, and then, US\$35 million program was initiated to develop this kind of aircraft. However, after 4 years of running this program, DARPA ended this project in 2000, and the results were not as good as what they expected, proving

Chapter 1: Introduction

that a 15-centimeter UAV was too small to be useful or workable, at least at that time (5). Hence, today, a 500-gram and 50-centimeter UAV can be considered as a MAV (6).

Based on a statistic in 2004, about 50 companies, academic institutions and government organizations in US have developed more than 150 UAV designs. 80% of these organizations were successful to have at least one working prototype built. 30% of them have already had their design in, or ready for, production. However, only 10% of the organizations were able to introduce a working MAV prototype (2).

2. RESEARCH OBJECTIVES AND SCOPES

This MAV research project was started in 2006 in Nanyang Technological University. It was funded by the Defense Science Organisation (DSO) to develop a MAV with largest dimension of 50 cm and endurance of 20 minutes. Currently, the MAV design was finalized, and a prototype has been fabricated. Aerodynamic properties of this design were also obtained by computational fluid dynamics simulation. Restricted tests were also conducted to examine the flying capability and controllability of this prototype.

This research work is intended to synthesize and develop the flight control system for the MAV, which can be used for autonomous cruising flight. It is divided in three main tasks:

A six degree-of-freedom model of the MAV will be developed with the available data. It will be used to analyze the MAV's flight characteristics during non-hover flight.

The flight control systems for the MAV during cruise need to be designed. The design procedure will be based on linearized mathematical model. The resulting controller will be implemented to the non-linear model and tested with different flight conditions.

Finally, preliminary hardware implementation needs to be carried out. This will include both implementation and testing of hardware to the MAV prototype. Various programs also need to be developed to eliminate error signal as well as to ensure proper communication between components.

3. DOCUMENT ORGANIZATION

This report includes six chapters which are listed as follows:

- **Chapter 1: Introduction**

This chapter starts with the background of how MAVs were developed, followed by the objectives and scopes of this research.

- **Chapter 2: Previous work on MAV development**

Recent MAV development activities around the world within the last ten years are covered in the chapter. At the end of this chapter, the investigated MAV is described to show some unique features of its design.

- **Chapter 3: System modeling and simulation by MATLAB/Simulink**

With the aerodynamic data obtained by computational fluid dynamics simulation, a 6-DOF non-linear model was built in MATLAB/Simulink. Testing conditions were selected as cruise and climb flights.

- **Chapter 4: Dynamic analysis and control synthesis of cruising flight**

This chapter gives the detailed analysis of dynamic stability of the current MAV design. Based on the analysis result, linearised model was developed. This model was used to design a control system, and then it was implemented to the non-linear model developed in chapter 3.

- **Chapter 5: Preliminary hardware implementation and testing**

Sensor system and servo control system were implemented to the prototype. A program was developed to get accurate data as well as eliminate drift effect from the sensor system. The program can also communicate with servo control board. Different control systems were tested by modifying the correspondent block in the Simulink model.

- **Chapter 6: Conclusions and recommendations for future work**

Conclusions of current research work and recommendations for future work are given in this chapter.

Chapter 2: PREVIOUS WORK ON MAV DEVELOPMENT

1. INTERESTING MAV DESIGNS

MAV development was initiated in mid of the 1990s by DARPA. They began a multi-year program to develop a flying vehicle with funding of US\$35 million. The MAV was required to be not larger than 15 centimeters, equipped with a camera and able to fly for about two hours, and all of this should be at a very low cost.

In 1998, Lockheed-Sanders and AeroVironment were awarded contracts by DARPA to develop their MAVs, and each of them has successfully developed their flying prototypes. AeroVironment's Black Widow is a 15-centimeter-wingspan MAV which uses an electric motor powered by a lithium battery; carries a color camera and weighs 80 grams (Figure 2.1) (7). Its flight in August 2000 was recorded with the following information: 30min endurance, 1.8km range and 235m altitude.

Figure 2.1: Black Widow MAV (7)

The Lockheed-Sanders' MicroStar MAV is bigger than the Black Widow; it has a 30-centimeter wingspan. The MicroStar's propulsion system is quite similar to the Black Widow's design; it also uses an electric motor powered by a lithium battery. Onboard GPS navigation system makes the MicroStar able to automatically follow a given flight path.

Chapter 2: Previous work on MAV development

Figure 2.2 shows the initial design of MicroStar with a single vertical tail and a pusher propeller at the back, and the later design with the winglets replacing the tail and a tractor propeller replacing the pusher propeller (5).

Figure 2.2: Initial design (left) and modified design of MicroStar MAV (5)

After successful development of the Black Widow MAV, AeroVironment continued their work to develop the next MAV which was named the Wasp and the Hornet. Although both of them are categorized as flying wing, their designs are slightly different which can be seen in Figure 2.3. The main difference is in the power source, the Wasp is powered by lithium batteries, while the Hornet is powered by fuel cells. In the Wasp's design, batteries are integrated into the wing structure; therefore, battery-capacity-to-MAV-size ratio is maximized (5). The Wasp has a wingspan of 33 centimeters (13 inches) and a weight of 210 grams (6 ounces). The Wasp can perform autonomous flight using its GPS navigation system or can be controlled by radio frequency signal. The Wasp II is the bigger version of the Wasp which was developed for operational use (8).

Figure 2.3: Wasp (left), Wasp II (middle) and Hornet (right) designs (8)

Chapter 2: Previous work on MAV development

The latest version of the Wasp family is the Wasp III; however, it is too large to be classified as a MAV (wingspan of 72 centimeters) (9).

Figure 2.4: Wasp III design (9)

iSTAR is a ducted fan MAV which was developed by Allied Aerospace in 2000. It is a high speed, vertical takeoff and landing (VTOL) MAV. Its size and weight are only 23-centimeter diameter and 1.8kg respectively (10).

Figure 2.5: Allied Aerospace's MAV – iSTAR (10)

Honeywell started flight testing an autonomous ducted-fan MAV at the beginning of 2005 (11). The ducted fan's VTOL capability allows it to operate in narrow area. Powered by a

Chapter 2: Previous work on MAV development

gasoline engine, this 33-centimeter MAV can go up to the impressive altitude of 3200 meters (12).

Figure 2.6: Honeywell's MAV (12)

Israel Aerospace Industries also introduced their own MAV, the Mosquito (Figure 2.7), in 2003. The design is quite similar to the Black Widow, with takeoff weight of 0.5kg and wingspan of 34 centimeters. This MAV can provide real-time image from a miniature video camera. The system can perform fully autonomous flights with its GPS navigation system, while flight path can be planned and monitored from the ground control station. Its performance is almost as same as the Black Widow: 60min endurance, 1.5km range and 90m altitude (13).

Figure 2.7: The Israeli MAV – The Mosquito (13)

Chapter 2: Previous work on MAV development

Companies, academic institutions and government organizations from other countries, such as France, Germany and China, also had their own MAV development programs. Although there are not many significant results announced yet, MAV development activities are very active all over the world now.

2. CONTROL SYSTEM DESIGNS

At the early stage, Black Widow, the MAV developed by AeroVironment which was mentioned in the previous part, was equipped with an electric motor and was manually controlled by elevons configuration. This configuration is the combination of elevator and ailerons, which can provide pitch control by symmetrical deflection and roll control by different deflection on each side. In the final design of this MAV, elevons configuration was replaced by an elevator in the middle of the wing's trailing edge and a rudder on the central fin (Figure 2.8). It can be seen that the rudder does not pass through the cg of the MAV; therefore, the rudder can also have some roll control power.

Figure 2.8: Control surfaces of Black Widow MAV (7)

In order to obtain some degree of autonomous control, a sensor system was implemented to the MAV. The sensor system consists of a two-axis magnetometer for compass heading sensing, a pitot-static tube and an absolute pressure sensor for altitude sensing, a differential pressure sensor for dynamic pressure sensing, and a gyroscope for turn rate sensing. Fully

Chapter 2: Previous work on MAV development

autonomous state has not been achieved yet; the MAV still needs to receive commands from the ground control station by a command uplink receiver and onboard computations are performed by two microprocessors (7).

Another flying wing MAV, developed by Blue Bear Systems Research Ltd (United Kingdom), is similar to Black Widow's design. It is two times bigger than the Black Widow, with total wingspan of 30cm (Figure 2.9). Control surface is also elevons configuration. However, due to the bigger size of the MAV, it can accommodate more components than the Black Widow. The complete control board is about 10cm long, consists of a tri-axis gyroscope, a tri-axis accelerometer, absolute and differential pressure sensors, on-board and environment temperature sensors, and a GPS module (14).

Figure 2.9: MAV prototype of Blue Bear Systems Research Ltd. (left) and complete control board (right) (14)

A team at Stellenbosch University (South Africa) built an autonomous ducted-fan UAV shown in Figure 2.10. It was named as SLADe (Surface Launched Aerial Decoy). Feedback signals are obtained by MEMS inertial sensors, a GPS receiver, a magnetometer and an ultrasonic altimeter. It is a counter-rotating ducted-fan aircraft (i.e. each motor at the top and bottom of the duct rotates in opposite direction); therefore, the motor torque can be minimized. Control of the MAV is governed by varying speeds of the two motors and deflection of eight flaps near the duct's exit. The flaps' arrangement is shown in Figure 2.10, too. To design the control system, mathematical model of the aircraft was developed and linearised about hover trim condition. Five proportional-integral (PI) control systems were implemented, allows the aircraft to navigate in three-dimensional space while maintaining

Chapter 2: Previous work on MAV development

an arbitrary heading angle, however, takeoff and landing still need to be manually performed (15).

Figure 2.10: SLADe under fully automated flight control (left) and arrangement of control surfaces (right) (15)

People at Drexel University (the USA) developed a very interesting fixed-wing MAV which can hover like a helicopter. The design is like a conventional aircraft with three basic types of control surface: Ailerons, elevator and rudder which are shown in Figure 2.11. Avionics system's components are quite standard with gyroscopes, accelerometers and a microprocessor.

Chapter 2: Previous work on MAV development

Figure 2.11: Fixed-wing prototype in its hovering orientation (16)

Although dashing and transition were not controlled autonomously, in hovering mode, the proportional-differential (PD) controller demonstrated its excellent performance. In the vertical orientation, the MAV was very unstable, and it required a skillful pilot to constantly manipulate the aircraft's yaw and pitch control surfaces to sustain the hover. But with the engagement of the hover control system, the aircraft could sustain a hover effortlessly in a more stable state (Figure 2.12). The authors also claimed that their work was the first to document autonomous hovering of a fixed-wing aircraft (17).

Figure 2.12: A human pilot struggles to sustain a hover (top row) and the same MAV under autonomous hover control (bottom row) (16)

Chapter 2: Previous work on MAV development

A team from University of Florida (the USA) and the National Aeronautics and Space Administration (NASA) built a very unique MAV. It is a flying wing with a vision-guided flight stability and autonomy system. It promoted the use of computer vision for MAV autonomy. The idea is to use the horizon as a reference to detect the MAV's attitudes. Given that most of the MAVs should be equipped with an on-board camera for their surveillance missions; vision-guided flight control system can help detecting body attitudes without the presence of a gyroscope. This leads to smaller payload size of MAVs. It was shown that when the captured images were clear enough, the algorithm was able to detect the horizon almost perfectly (over 99.9% of correct identification). Figure 2.13 shows some horizon detection examples different lighting conditions. Even when there is video transmission noise, the horizon was still detected correctly (18).

Figure 2.13: Horizon detection examples under different lighting conditions and video transmission noise degrees (18)

3. SYSTEM MODELING AND SIMULATION WITH MATLAB/SIMULINK

To demonstrate the application of MATLAB/Simulink in aerospace industry, the MathWorks included some complete models in the MATLAB package. There were two models which served as the basic for the simulation model developed in this project: *1903 Wright Flyer* and *Pilot with Scopes for Data Visualization* and *Lightweight Airplane Design* (19).The

Chapter 2: Previous work on MAV development

model of the Wright brothers' airplane is shown in Figure 2.14. It was modeled as a 3-DOF object with the only control surface is the elevator.

Figure 2.14: 3-DOF model of Wright brothers' airplane (19)

The lightweight four-seater monoplane in Figure 2.15 was also modeled as a 3-DOF object as shown in Figure 2.16. However, more details were added to this model compared to the previous one. Three control surfaces and the engine also presented in the model. Instruments' noise and environment's disturbance were simulated, too. Other lateral-directional parameters were included to make it a 6-DOF model. This really helps model builders to get a basic understanding of all the required components in a complete aircraft model.

Figure 2.15: Lightweight four-seater monoplane (19)

Chapter 2: Previous work on MAV development

Figure 2.16: 3-DOF model of a lightweight airplane (19)

MATLAB/Simulink is widely used by MAV developers. During the design phase of the Carolo MAV, developed by the Technical University of Braunschweig (Germany), a nonlinear aircraft model was built. The model contained the nonlinear vector differential equations, the calculations of forces and moments, the engine's gyroscopic effect, a standard environment model and other transformation blocks (Figure 2.17) (20).

Figure 2.17: Carolo MAV's nonlinear model (20)

In the early stage of development, the fixed-wing MAV that can hover like a helicopter was also modeled by MATLAB/Simulink. The model of aircraft's transition state from cruise to hover is shown in Figure 2.18 (16). After another year of work, this MAV was able to hover autonomously as described above (17).

Chapter 2: Previous work on MAV development

Figure 2.18: Cruise to hover state model of a fixed-wing MAV (16)

4. FINAL DESIGN OF THE MAV PROTOTYPE

The final design of the MAV prototype discussed here is a combination of both ducted-fan and fixed-wing designs as shown in Figure 2.19. It is able to perform vertical takeoff and landing. There are four control subsystems in the design:

- Main motor control system: To control the thrust
- Stators control system: To counterbalance the torque of the main motor as well as to control bank angle.
- Horizontal tail control system: To control pitch angle, therefore altitude can be maintained or adjusted.
- Vertical tail control system: To control heading angle, therefore flight direction can be controlled.

All of the control surfaces are arranged in such a way to minimize coupling effect between them. The aerodynamic centers of the horizontal tail and vertical tail are in the same plane with the cg, so there is no roll control power from the vertical tail. The stators also deflect in asymmetrical manner, so it is assured that only roll control is governed by the stators' deflection.

Chapter 2: Previous work on MAV development

Figure 2.19: Final design of the MAV

A normal flight profile of the prototype can be described as follows: The MAV starts with vertical takeoff, enters the transition stage then finally cruises like a conventional fixed-wing MAV. Thrust is highest during takeoff process; then it gradually decreases in transition phase and reaches minimum value in cruise phase (i.e. the aircraft stops accelerating). Research work presented in this document will focus on the cruise part of the flight profile.

Figure 2.20: Expected flight profile of the MAV

Chapter 3: SYSTEM MODELING AND SIMULATION BY MATLAB/SIMULINK

1. INTRODUCTION

MATLAB/Simulink is one of the most popular designing tools in engineering field. Its application can be found in many industries: Aerospace, biomedical, electrical/electronics, mechanical, etc. Working with MATLAB requires users to do a lot of programming work, while Simulink provides an interactive graphical environment and a customizable set of block libraries (21). For aerospace industry, there is one toolbox specially designed to build aerospace models quickly: Aerospace blockset. In this part, a detailed model of the investigated MAV which was built by this blockset is presented. Using this model, one can easily analyze common flight situations, as well as implement controllers to improve the system's response. In this model, most of the required hardware for the real aircraft is included; therefore, this model can also be used with hardware-in-the-loop test.

2. DATA PREPARATION FOR MODELING

2.1. Aerodynamic data

The investigated MAV was described at the end of chapter 2. Here are some highlights of its design: It is a combination of both fixed-wing and ducted-fan design. The duct in this case also contributes to the total lift of the aircraft during the cruise stage. The three control surfaces are arranged in such a way that the coupling's effect between them is minimal.

The aerodynamic data of the vehicle have been evaluated by the means of computational fluid dynamics in Reference (22), and summary of relevant data is given in appendix A. The simulation results were presented in the form of force and moment coefficients and normalized with reference data as:

- Wing area ($0.076m^2$)
- Wing span ($0.43m$)

Chapter 3: System modeling and simulation by MATLAB/Simulink

- Dynamic pressure at aircraft's forward speed of $15m/s$ with no atmospheric disturbance ($137.8125N / m^2$)

In the cruise's angle of attack range, these aerodynamic data can be approximated by first order (lift and moment terms) and second order (drag terms) polynomials. Based on simulation results with all the angles in radian, the aerodynamic force and moment coefficients of different components in the MAV are summarized as follows:

2.1.1. Longitudinal aerodynamic force and moment coefficients

- Wing and duct: There is a setting angle of 6 degrees between the wing and the duct; however, the effective angle of attack is measured by the duct's angle of attack only. The resultant aerodynamic force and moment coefficients due to wing and duct are:

$$C_{L_{WB}} = 2.3565\alpha + 0.0492 \quad (3.1)$$

$$C_{D_{WB}} = 2.3422\alpha^2 - 0.0787\alpha + 0.034 \quad (3.2)$$

$$C_{m_{WB}} = -0.1009\alpha - 0.0019 \quad (3.3)$$

- Horizontal tail:

$$C_{L_{HT}} = 0.2514\delta_{HT} \quad (3.4)$$

$$C_{D_{HT}} = 0.1629\delta_{HT}^2 + 0.0087 \quad (3.5)$$

$$C_{m_{HT}} = -0.1631\delta_{HT} \quad (3.6)$$

- Vertical tail: The vertical tail is identical to the horizontal tail in terms of shape and size. It only affects the total drag of the vehicle in the longitudinal plane, and its lift coefficient equation is used to determine the yawing moment in directional plane.

$$C_{D_{VT}} = 0.1629\delta_{VT}^2 + 0.0087 \quad (3.7)$$

- Stators: The stators are always configured in asymmetric setting; hence, in the longitudinal plane, only the total drag is affected by the stators.

$$C_{D_S} = 0.2557\delta_S^2 + 0.0371 \quad (3.8)$$

2.1.2. Lateral - Directional aerodynamic force and moment coefficients

- Wing and duct:

$$C_y = -1.5236\beta \quad (3.9)$$

$$C_l = -0.0083\beta \quad (3.10)$$

$$C_n = 0.0218\beta \text{ or } C_n = 0.2508\beta^3 + 0.0153\beta \quad (3.11)$$

- Stators: Only rolling moment is affected by stators' deflection.

$$C_l = -0.0425\delta_S \quad (3.12)$$

2.1.3. Dynamic pressure correction for stators and tails

Due to the geometry of the vehicle, the horizontal tail only exposes to the free air stream if the angle of attack is larger than the effective value which is shown in Figure 3.1 below:

Figure 3.1: Effective angle of attack for horizontal tail

This idea was first given by Suhartono (23), and the effective angle of attack will be calculated as: The duct's diameter is about 20cm, and the distance from duct's exit to the horizontal tail's trailing edge is about 18cm. Hence, the angle of attack at which the horizontal tail becomes exposed to the free air stream is:

$$\alpha_{effective} = \tan^{-1}\left(\frac{10}{18}\right) = 0.6rad (\approx 34^\circ) \quad (3.13)$$

Chapter 3: System modeling and simulation by MATLAB/Simulink

As this value is relatively high for cruising flight, it can be concluded that the horizontal tail is fully under the air stream exiting from the duct.

It can be seen from the above figure that the effective angle of attack at which the vertical tail becomes exposed to the free air stream is smaller than that angle for the case of horizontal tail. To make a rough estimation, it is assumed that if less than one quarter of the vertical tail area is exposed to the free air stream; the effective air speed will be the speed of the air stream exiting from the duct. This is illustrated in Figure 3.2. From experimental measurement, this angle is about 17 degrees, which can be considered relatively high for cruise; therefore, the air stream coming to the vertical tail will be taken as the one exiting from the duct.

Figure 3.2: Effective angle of attack for vertical tail

Velocity of the air stream at the duct's exit is calculated by this formula:

$$V_{exit} = V_{\infty} \cos(\alpha) + \sqrt{\frac{T}{\rho A_{duct}}} \approx V_{\infty} + \sqrt{\frac{T}{\rho A_{duct}}} \quad (3.14)$$

This velocity consists of two components: The free air stream's velocity (V_{∞}) and induced velocity by the propeller in a duct. The original formula for the latter term is given in Reference (24) as:

$$V_{induced} = \sqrt{\frac{L}{\rho A_R \sigma}} \quad (3.15)$$

Chapter 3: System modeling and simulation by MATLAB/Simulink

With ρ is density of air, A_R is the ducted-fan disk area, and $\sigma = \frac{A_e}{A_R}$ is ratio between the duct's exit area and disk area.

The correction factor for velocity is:

$$n = \frac{V_{exit}}{V_{\infty}} \quad (3.16)$$

For the aerodynamic data derived in the previous part, the only change is the velocity of the air stream; this change includes both magnitude and direction. Thus, the correction factor of magnitude for all the aerodynamic data of tails and stators is n^2 , and correction factor of direction is angle of α_e .

2.2. Trim conditions

2.2.1. Equations of motion

There are three coordinate systems used in this development which are shown in Figure 3.3:

- The body axes system, which is fixed to the aircraft, the equations of motion set is derived in this coordinate system.
- The stability axes system, of which x-axis is in line with the wind's velocity vector, it is used to derive the aerodynamic forces.
- Horizontal vertical axes system, of which orientation does not change with time, it serves as an inertial reference system.

Figure 3.3: Coordinate systems used in the theoretical approach

Chapter 3: System modeling and simulation by MATLAB/Simulink

Chapter 1 of Reference (25) contains a detailed derivation of the equations of motion of a fixed-wing aircraft in a body-fixed axis system. The results are represented here as:

$$\sum F_x : m(\dot{U} - VR + WQ) = -mg \sin \Theta + F_{A_x} + F_{T_x} \quad (3.17)$$

$$\sum F_y : m(\dot{V} + UR - WP) = mg \sin \Phi \cos \Theta + F_{A_y} + F_{T_y} \quad (3.18)$$

$$\sum F_z : m(\dot{W} - UQ + VP) = mg \cos \Phi \cos \Theta + F_{A_z} + F_{T_z} \quad (3.19)$$

$$\sum L_x : I_{xx}\dot{P} + (I_{zz} - I_{yy})RQ - I_{xz}(\dot{R} + PQ) = L_A + L_T \quad (3.20)$$

$$\sum M_y : I_{yy}\dot{Q} + (I_{xx} - I_{zz})PR + I_{xz}(P^2 - R^2) = M_A + M_T \quad (3.21)$$

$$\sum N_z : I_{zz}\dot{R} + (I_{yy} - I_{xx})PQ - I_{xz}(\dot{P} - QR) = N_A + N_T \quad (3.22)$$

All the acceleration due to rotational effect is included in the left hand side of these equations. This is the most general set of equations of motion.

According to Reference (25), “The steady-state flight condition is defined as one for which all motion variables remain constant with time relative to the body-fixed axis system XYZ”. Hence, in Equations (3.17) to (3.22), all the acceleration variables are zero and other variables are equal to steady-state values.

$$\sum F_{x_e} : m(-V_e R_e + W_e Q_e) = -mg \sin \Theta_e + F_{A_{x_e}} + F_{T_{x_e}} \quad (3.23)$$

$$\sum F_{y_e} : m(U_e R_e - W_e P_e) = mg \sin \Phi_e \cos \Theta_e + F_{A_{y_e}} + F_{T_{y_e}} \quad (3.24)$$

$$\sum F_{z_e} : m(-U_e Q_e + V_e P_e) = mg \cos \Phi_e \cos \Theta_e + F_{A_{z_e}} + F_{T_{z_e}} \quad (3.25)$$

$$\sum L_{x_e} : (I_{zz} - I_{yy})R_e Q_e - I_{xz}P_e Q_e = L_{A_e} + L_{T_e} \quad (3.26)$$

$$\sum M_{y_e} : (I_{xx} - I_{zz})P_e R_e + I_{xz}(P_e^2 - R_e^2) = M_{A_e} + M_{T_e} \quad (3.27)$$

$$\sum N_{z_e} : (I_{yy} - I_{xx})P_e Q_e + I_{xz}Q_e R_e = N_{A_e} + N_{T_e} \quad (3.28)$$

Chapter 3: System modeling and simulation by MATLAB/Simulink

2.2.2. Steady-state cruise

Steady-state conditions of a straight, wing-level flight were selected as the trim conditions for the derivations. All of the acceleration terms in the steady-state equations of motion were removed. The free-body diagram of the MAV in cruise in Figure 3.4 indicates all the forces acting on the MAV. Aerodynamic forces are lift and drag from wing-duct and control surfaces. As this is steady-state flight in longitudinal plane, there is no force or moment equation in the lateral-directional planes, the cross-coupling between longitudinal and lateral-directional plane is neglected. Then the equations of motion can be simplified as:

$$\sum F_{x_e} : 0 = -mg \sin \Theta_e + F_{A_{x_e}} + F_{T_{x_e}} \quad (3.29)$$

$$\sum F_{z_e} : 0 = mg \cos \Theta_e + F_{A_{z_e}} + F_{T_{z_e}} \quad (3.30)$$

$$\sum M_{y_e} : 0 = M_{A_e} + M_{T_e} \quad (3.31)$$

The stability X axis is in line with the wind velocity, and the body X axis is in line with thrust vector. From the free-body diagram in Figure 3.4, the equations of motion of the MAV in stability axis system are expanded as:

$$T_e \cos \alpha_e - D_{WB_e} - (D_{S_e} + D_{VT_e} + D_{HT_e}) \cos \alpha_e - L_{HT_e} \sin \alpha_e = 0 \quad (3.32)$$

$$mg - T_e \sin \alpha_e - L_{WB_e} - L_{HT_e} \cos \alpha_e + (D_{S_e} + D_{VT_e} + D_{HT_e}) \sin \alpha_e = 0 \quad (3.33)$$

$$C_{m_{WB_e}} + C_{m_{HT_e}} = 0 \quad (3.34)$$

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.4: Forces acting on the MAV in steady state of cruise

Substituting the aerodynamic data in part 1 to Equations (3.20) to (3.22), applying small

angle assumptions ($\sin \varepsilon \approx \varepsilon, \cos \varepsilon \approx 1 - \frac{\varepsilon^2}{2}$) and rearranging these equations yield:

$$\begin{aligned} \frac{T_e}{10.4738} \left(1 - \frac{\alpha_e^2}{2}\right) - \left\{ \frac{0.2557(0.0888T_e)^2 + 0.0371}{+0.0087 + [0.1629\delta_{HT_e}^2 + 0.0087]} \right\} n^2 \left(1 - \frac{\alpha_e^2}{2}\right) \\ - (2.3422\alpha_e^2 - 0.0787\alpha_e + 0.034) - (0.2514\delta_{HT_e})n^2\alpha_e = 0 \end{aligned} \quad (3.35)$$

$$0.4296 + \left\{ \frac{0.2557(0.0888T_e)^2 + 0.0371}{+0.0087 + [0.1629\delta_{HT_e}^2 + 0.0087]} \right\} n^2\alpha_e - \frac{T_e\alpha_e}{10.4738} \quad (3.36)$$

$$- (2.3565\alpha_e + 0.0492) - (0.2514\delta_{HT_e})n^2 \left(1 - \frac{\alpha_e^2}{2}\right) = 0$$

$$(-0.1009\alpha_e - 0.0019) \times 4.5037 + (-0.1631\delta_{HT_e}) \times 4.5037 n^2 = 0 \quad (3.37)$$

Finding an analytical solution for these equations is difficult due to the non-linear relationship between the variables. A numerical method was proposed as follows:

- Step 1: Assign an initial value for thrust at trim point

Chapter 3: System modeling and simulation by MATLAB/Simulink

- Step 2: Get value of correction factor n
- Step 3: Solve Equations (3.24) and (3.25) for α_e and δ_{HT_e}
- Step 4: Substitute α_e and δ_{HT_e} to Equation (3.23) to solve for thrust value
- Step 5: Repeat from step 2 until thrust value converges to a final value

The whole above process can be summarized by this flow chart:

Figure 3.5: Integration sequence to find trim thrust value

All the above steps are done by a short Maple code. With the initial thrust value of $5N$, after about 15 iterations, it converged to the final value of $2.1941N$ with deviation of 5 decimal places. If the initial value was set at $1N$, the final value would also be $2.1941N$. Detailed code can be found in Appendix D1.

Hence, the trimmed values for thrust, angle of attack and horizontal tail deflection of the MAV during steady-state cruise were found as:

$$T_e = 2.1941N \quad (3.38)$$

$$\alpha_e = 0.1677 \quad (3.39)$$

$$\delta_{HT_e} = -0.052 \quad (3.40)$$

2.2.2. Steady-state turn

To initiate a turn, bank angle needs to be changed first. By changing the bank angle, component of lift in the vertical direction reduces, so we need to increase angle of attack to increase lift. However, because angle of attack increases, drag also increases; therefore, the aircraft's speed will slow down. So thrust needs to be increased, too. So the required action will be: Changing bank angle, increasing angle of attack and thrust.

The next parameters need to be considered are turn rate and turn radius. As shown in Figure 3.6, the equilibrium condition for centripetal force and acceleration must be satisfied:

$$F_c = m\dot{\psi}^2 R_t \quad (3.41)$$

with $\dot{\psi}$ is turn rate and R_t is turn radius.

The kinematic equation is also used to relate turn rate and turn radius:

$$V_\infty = R_t \dot{\psi} \quad (3.42)$$

Figure 3.6: Geometry of steady-state, wing-level turn

Chapter 3: System modeling and simulation by MATLAB/Simulink

Some assumptions to be noted here are:

- Only lift and thrust were considered, other forces were neglected.
- Turn rate and turn radius were estimated with values of angle of attack and thrust taken from steady-state cruise conditions.

The results calculated from these assumed values may be different from the real value; however, the difference should be insignificant. The force equations in vertical and horizontal direction are:

$$mg - (L + T_e \sin \alpha_e) \cos \phi_e = 0 \quad (3.43)$$

$$(L + T_e \sin \alpha_e) \sin \phi_e = m\dot{\psi}^2 R_t \quad (3.44)$$

Combining Equations (3.42), (3.43) and (3.44) gives us this relationship between bank angle and turn rate:

$$\dot{\psi} = \frac{g \tan \phi_e}{V_\infty} \quad (3.45)$$

The turn rate of 0.2 rad/s was selected, hence, the turn radius and bank angle were calculated as:

$$R_t = \frac{V_\infty}{\dot{\psi}} = 75 \text{ m} \quad (3.46)$$

$$\phi_e = \tan^{-1} \left(\frac{V_\infty \dot{\psi}}{g} \right) = 0.3 \text{ rad} \quad (3.47)$$

Initial control data need to be determined by solving for the steady-state parameters of turning flight. It is done by solving the six equations of motion for a general steady-state flight which were given above (Equations (3.23) to (3.28)). Detailed Maple code to solve these equations is given in Appendix D2. Final results are given as:

$$\begin{aligned} T_e &= 2.3946 \text{ N} & \alpha_e &= 0.1836 \text{ rad} & \beta_e &= -0.0055 \text{ rad} \\ \delta_{HT} &= -0.0544 \text{ rad} & \delta_{VT} &= -0.0002 \text{ rad} & \delta_S &= 0.0956 \text{ rad} \end{aligned} \quad (3.48)$$

Chapter 3: System modeling and simulation by MATLAB/Simulink

Comparing with results of steady-state wing-level flight conditions, it can be seen that both of angle of attack and thrust are higher, which complies with the expected conditions.

3. SIMULATION MODEL

The whole simulation model was made of 3 blocks: Manual control, environment model and aircraft model. The manual control block is to adjust the control surfaces to trim the aircraft to desired initial conditions. The environment model block is to generate effects of environment to the aircraft, such as wind and gravity. The aircraft model is the most important block of the whole simulation; it simulates the aircraft's responses under given control signals and environmental parameters.

Figure 3.7: Overall simulation model's blocks

3.1. Manual control block

For cruising flights, three scenarios are considered, which are altitude-holding, climbing and turning flights. In this block, varying altitude command will subsequently change the trimmed conditions of control surfaces. Climb rate was fixed at $1m/s$. Inputs are the initial control surfaces' deflection calculated in previous part to trim the aircraft at desired conditions.

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.8: Manual control block

3.2. Environment model block

The main functionality of this block is to simulate a wind model and transform gravity from earth to body axes system. Input signals are position and body angles of the aircraft. Air density is assumed to be constant within the given flight conditions. With the actual density variation of less than 2%, this assumption is valid. However, initial simulation does not include environmental disturbance as there is no control system implemented in the model yet. Disturbance will be introduced once the controller has been implemented.

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.9: Environment model

3.3. Aircraft model block

The aircraft model is made of 3 sub-models, which are avionics system, sensors and airframe. The sensors are considered as unity feedback block, and avionics system actually consists of 3 separate control loops to control altitude, heading angle and bank angle

Figure 3.10: Aircraft model with 3 subsystem: Avionics system, sensors system and airframe

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.11: Avionics system block

In the airframe block, aerodynamic forces and moments are calculated based on CFD simulation data which are given in appendix A. It also includes a propulsion block to simulate a motor with propeller (which generates both of thrust and torque). Resultant force and moment are fed to the 6DOF block, which consists of non-linear, 6-DOF equations of motion. Then the aircraft response is obtained by solving these equations.

Chapter 3: System modeling and simulation by MATLAB/Simulink

4. TESTING CONDITIONS

Without a flight control system implemented, the simulation model was used to simulate the aircraft in longitudinal plane only, which meant lateral-directional motion was not considered. As there were no forces in lateral – directional plane, it could be expected that there would be no lateral – directional motion at all. For cruising flight, two flight modes were analyzed, which were altitude-hold mode and climbing mode. Trimmed conditions were calculated previously in Part 2 of this chapter and they are presented in Table 3.1. During climbing state, thrust and pitch attitude would increase to accommodate the change in altitude.

	Altitude-hold mode	Climbing mode
Thrust	2.1941 <i>N</i>	2.5663 <i>N</i>
Horizontal tail's deflection	-0.052 <i>rad</i>	-0.0486 <i>rad</i>
Angle of attack	0.1677 <i>rad</i>	0.1653
Pitch angle	0.1677 <i>rad</i>	0.232
Stators' deflection	0.0904 <i>rad</i>	0.1001 <i>rad</i>

Table 3.1: Flight conditions for altitude-hold mode and climbing mode

5. SIMULATION RESULTS

The trim point conditions calculated previously were fed to the model as the initial control input, and were kept constant through the simulation. Figure 3.13 shows the system response under initial control signal.

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.13: Altitude, angle of attack and pitch attitude of the MAV's 3-DOF model in altitude-holding mode

It can be seen that the calculated thrust was not enough to hold the vehicle's altitude. After increasing the thrust to $2.46N$, the vehicle was able to hold its altitude without any control system implemented. New result is shown in Figure 3.14 below.

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.14: Altitude, angle of attack and pitch attitude of the MAV's 3-DOF model in altitude-holding mode (with corrected thrust value)

The next step was to introduce a control surface's deflection (i.e. horizontal tail's deflection) to observe how the aircraft responds, and then from the response's shape, oscillation mode could be estimated by graphical method.

Figure 3.15 shows the MAV's angle of attack and pitch attitude after an increase of 0.01 in horizontal tail deflection. Using the logarithmic decrement and damped natural frequency found in the pitch attitude response, the phugoid mode's natural frequency and damping ratio could be estimated (Figure 3.16).

Chapter 3: System modeling and simulation by MATLAB/Simulink

Figure 3.15: Angle of attack and pitch attitude of the MAV's 3-DOF model due to 0.01rad change in horizontal tail's deflection

Figure 3.16: Estimation of phugoid mode's parameters by graphical method on pitch angle

Logarithmic decrement:

$$\delta = \frac{1}{n} \ln \frac{x_0}{x_n} = 1.2862 \quad (3.49)$$

Chapter 3: System modeling and simulation by MATLAB/Simulink

Damping ratio:

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}} = 0.2 \quad (3.50)$$

Damped natural frequency:

$$\omega_d = \frac{2\pi}{T_d} = 0.7671 \text{ rad / s} \quad (3.51)$$

Natural frequency:

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} = 0.7829 \text{ rad / s} \quad (3.52)$$

Angle of attack plot was zoomed in when the disturbance started to find estimation for short-period mode as shown in Figure 3.17.

Figure 3.17: Estimation of short-period mode's parameters by graphical method on angle of attack

Due to the mixed response of the short-period mode and phugoid mode, what can be measured is the period of the damped oscillation of the short-period mode. And it was used to calculate the damped natural frequency as follows:

$$\omega_{d_{sp}} = \frac{2\pi}{0.75} = 8.38 \text{ rad / s} \quad (3.53)$$

This result will be verified when theoretical approach is employed in the next part.

Chapter 3: System modeling and simulation by MATLAB/Simulink

Simulation result for flight in climbing mode is shown in Figure 3.18. As is the case of the cruise mode, the calculated thrust was also in the low side, which made the MAV unable to perform at the desired climb rate. After increasing the thrust from $2.57N$ to $2.85N$, the simulation result showed that the MAV has no problem to achieve the desired performance.

Figure 3.18: Altitude, angle of attack and pitch attitude of the MAV's 3-DOF model in climb mode (with corrected thrust value)

In the next chapter, the control systems will be designed by analytical approach. A linear model will be developed to be used in the controller design procedure. After that, the control system will be implemented to the non-linear model, and then other more complicated flight conditions can be simulated and observed.

Chapter 4: DYNAMIC ANALYSIS AND CONTROL SYNTHESIS OF CRUISING FLIGHT

1. INTRODUCTION

The MAV's dynamic analysis starts with gathering all the aerodynamic data, aircraft design parameters and flight conditions. Then the trim point conditions need to be defined, so that all the derivatives can be derived about this trim point. Finally, the mathematical model can be obtained from those derivatives. Analyzing this model can give us some ideas about stability and controllability of the MAV.

2. EQUATIONS OF MOTION

Recalling the general equations of motion in Chapter 3 and substituting the perturbed variables to these equation, the perturbed state equations of motion can be obtained. Applying the small perturbed value assumption and selecting the steady-state conditions as wing-level, straight flight conditions, the perturbed state equations of motion can be simplified as:

$$\sum F_x : m(\dot{u} + W_e q) = -mg\theta \cos \Theta_e + f_{A_x} + f_{T_x} \quad (4.1)$$

$$\sum F_y : m(\dot{v} + W_e p) = -mg\phi \cos \Theta_e + f_{A_y} + f_{T_y} \quad (4.2)$$

$$\sum F_z : m(\dot{w} - U_e q) = -mg\theta \sin \Theta_e + f_{A_z} + f_{T_z} \quad (4.3)$$

$$\sum L_x : I_{xx} \dot{p} - I_{xz} \dot{r} = l_A + l_T \quad (4.4)$$

$$\sum M_y : I_{yy} \dot{q} = m_A + m_T \quad (4.5)$$

$$\sum N_z : I_{zz} \dot{r} - I_{xz} \dot{p} = n_A + n_T \quad (4.6)$$

All the derivatives derived below will be simplified with the corresponding inertial terms, i.e. force derivatives are divided by the mass and moment derivatives are divided by the moment of inertia, so that the equations of motion can be simplified further.

Chapter 4: Dynamic analysis and control synthesis of cruising flight

3. AERODYNAMIC STABILITY AND CONTROL DERIVATIVES

With the aid from Reference (25), (26), and (27), aerodynamic stability and control derivatives are given below. For the ease of derivation, all derivatives taken from the references were derived in stability axes system; they will be transformed back to body axes system to maintain compliance with other forces in the equations of motion.

3.1. Longitudinal plane

Variables	Axial force	Normal force	Pitching moment
u	$X_u = -\frac{2C_{D_e}\bar{q}_e S}{mU_e}$	$Z_u = -\frac{2C_{L_e}\bar{q}_e S}{mU_e}$	$M_u = 0$
α	$X_\alpha = -\frac{(C_{D_{\alpha_e}} - C_{L_e})\bar{q}_e S}{m}$	$Z_\alpha = -\frac{(C_{L_{\alpha_e}} + C_{D_e})\bar{q}_e S}{m}$	$M_\alpha = \frac{C_{m_{\alpha_e}}\bar{q}_e S b}{I_{yy}}$
q	$X_q = -\frac{C_{D_{a_{HT_e}}} l_{HT} \bar{q}_e S}{mU_e}$	$Z_q = -\frac{C_{L_{a_{HT_e}}} l_{HT} \bar{q}_e S}{mU_e}$	$M_q = -\frac{C_{L_{a_{HT_e}}} l_{HT}^2 \bar{q}_e S}{I_{yy} U_e}$
$\dot{\alpha}$	$X_{\dot{\alpha}} = 0$	$Z_{\dot{\alpha}} = 0$	$M_{\dot{\alpha}} = 0$
δ_{HT}	$X_{\delta_{HT}} = -\frac{C_{D_{\alpha_{HT_e}}} \bar{q}_e S}{m}$	$Z_{\delta_{HT}} = -\frac{C_{L_{\alpha_{HT_e}}} \bar{q}_e S}{m}$	$M_{\delta_{HT}} = \frac{C_{m_{\alpha_{HT}}} \bar{q}_e S b}{I_{yy}}$

Table 4.1: Longitudinal derivatives in stability axes system

3.2. Longitudinal derivative transformation from stability to body axes systems

All the forces and moment perturbation were derived in wind axes system. In order to apply these results to the equations of motion in part 1, they have to be transformed from wind axes system to body axes system, i.e. rotated about y axis through an angle of α_e . The rotational matrix for such transformation is:

$$\text{Stability axes system} \xrightarrow{\times \begin{bmatrix} \cos \alpha_e & 0 & -\sin \alpha_e \\ 0 & 1 & 0 \\ \sin \alpha_e & 0 & \cos \alpha_e \end{bmatrix}} \text{Body axes system}$$

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$\begin{bmatrix} (x)_b \\ (y)_b \\ (z)_b \end{bmatrix} = \begin{bmatrix} \cos \alpha_e & 0 & -\sin \alpha_e \\ 0 & 1 & 0 \\ \sin \alpha_e & 0 & \cos \alpha_e \end{bmatrix} \begin{bmatrix} (x)_s \\ (y)_s \\ (z)_s \end{bmatrix} \quad (4.7)$$

$$\text{or } \begin{bmatrix} (x)_s \\ (y)_s \\ (z)_s \end{bmatrix} = \begin{bmatrix} \cos \alpha_e & 0 & \sin \alpha_e \\ 0 & 1 & 0 \\ -\sin \alpha_e & 0 & \cos \alpha_e \end{bmatrix} \begin{bmatrix} (x)_b \\ (y)_b \\ (z)_b \end{bmatrix} \quad (4.8)$$

Longitudinal force and moment perturbations in stability axes system are:

$$\begin{cases} (f_{A_x})_s = (X_u u)_s + (X_\alpha \alpha)_s + (X_q q)_s + (X_{\delta_{HT}} \delta_{HT})_s \\ (f_{A_z})_s = (Z_u u)_s + (Z_\alpha \alpha)_s + (Z_q q)_s + (Z_{\delta_{HT}} \delta_{HT})_s \\ (m_A)_s = (M_u u)_s + (M_\alpha \alpha)_s + (M_q q)_s + (M_{\delta_{HT}} \delta_{HT})_s \end{cases} \quad (4.9)$$

For the ease of transformation, angle of attack is replaced by normal velocity using the relationship:

$$\alpha \approx \tan \alpha \approx \frac{w}{V_e} \quad (4.10)$$

Hence, all the derivatives due to angle of attack are replaced by derivatives due to normal velocity:

$$X_w = \frac{X_\alpha}{V_e}; \quad Z_w = \frac{Z_\alpha}{V_e}; \quad M_w = \frac{M_\alpha}{V_e} \quad (4.11)$$

Then the longitudinal force and moment perturbation become:

$$\begin{cases} (f_{A_x})_s = (X_u u)_s + (X_w w)_s + (X_q q)_s + (X_{\delta_{HT}} \delta_{HT})_s \\ (f_{A_z})_s = (Z_u u)_s + (Z_w w)_s + (Z_q q)_s + (Z_{\delta_{HT}} \delta_{HT})_s \\ (m_A)_s = (M_u u)_s + (M_w w)_s + (M_q q)_s + (M_{\delta_{HT}} \delta_{HT})_s \end{cases} \quad (4.12)$$

Applying the relationship in Equation (4.7) to get the axial force perturbation in body axes system:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$\begin{aligned}
 (f_{A_x})_b &= (f_{A_x})_s \cos \alpha_e - (f_{A_z})_s \sin \alpha_e \\
 &= \left[(X_u u)_s + (X_w w)_s + (X_q q)_s + (X_{\delta_{HT}} \delta_{HT})_s \right] \cos \alpha_e \\
 &\quad - \left[(Z_u u)_s + (Z_w w)_s + (Z_q q)_s + (Z_{\delta_{HT}} \delta_{HT})_s \right] \sin \alpha_e
 \end{aligned} \tag{4.13}$$

Because q and δ_{HT} are the same in stability and body axes system, their derivatives in body axes system can be simply obtained as:

$$(X_q)_b = (X_q)_s \cos \alpha_e - (Z_q)_s \sin \alpha_e \tag{4.14}$$

$$(X_{\delta_{HT}})_b = (X_{\delta_{HT}})_s \cos \alpha_e - (Z_{\delta_{HT}})_s \sin \alpha_e \tag{4.15}$$

Velocity perturbations u and w in stability axes system can be resolved to body axes system as:

$$(u)_s = (u)_b \cos \alpha_e + (w)_b \sin \alpha_e \tag{4.16}$$

$$(w)_s = -(u)_b \sin \alpha_e + (w)_b \cos \alpha_e \tag{4.17}$$

Then the force perturbation due to velocity perturbations is:

$$\begin{aligned}
 &(X_u u + X_w w)_b \\
 &= \left[(X_u)_s \cos^2 \alpha_e - (X_w + Z_u)_s \sin \alpha_e \cos \alpha_e + (Z_w)_s \sin^2 \alpha_e \right] (u)_b \\
 &\quad + \left[(X_w)_s \cos^2 \alpha_e + (X_u - Z_w)_s \sin \alpha_e \cos \alpha_e - (Z_u)_s \sin^2 \alpha_e \right] (w)_b
 \end{aligned} \tag{4.18}$$

Hence, the stability derivatives due to velocity perturbation in body axes system are:

$$(X_u)_b = (X_u)_s \cos^2 \alpha_e - (X_w + Z_u)_s \sin \alpha_e \cos \alpha_e + (Z_w)_s \sin^2 \alpha_e \tag{4.19}$$

$$(X_w)_b = (X_w)_s \cos^2 \alpha_e + (X_u - Z_w)_s \sin \alpha_e \cos \alpha_e - (Z_u)_s \sin^2 \alpha_e \tag{4.20}$$

Following the same procedure as described above, all the longitudinal derivatives in stability axes system can be transformed to body axes system as follows:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$\begin{aligned}
 (X_u)_b &= (X_u)_s \cos^2 \alpha_e - (X_w + Z_u)_s \sin \alpha_e \cos \alpha_e + (Z_w)_s \sin^2 \alpha_e \\
 (X_w)_b &= (X_w)_s \cos^2 \alpha_e + (X_u - Z_w)_s \sin \alpha_e \cos \alpha_e - (Z_u)_s \sin^2 \alpha_e \\
 (X_q)_b &= (X_q)_s \cos \alpha_e - (Z_q)_s \sin \alpha_e \\
 (X_{\delta_{HT}})_b &= (X_{\delta_{HT}})_s \cos \alpha_e - (Z_{\delta_{HT}})_s \sin \alpha_e
 \end{aligned} \tag{4.21}$$

$$\begin{aligned}
 (Z_u)_b &= (Z_u)_s \cos^2 \alpha_e - (Z_w - X_u)_s \sin \alpha_e \cos \alpha_e - (X_w)_s \sin^2 \alpha_e \\
 (Z_w)_b &= (Z_w)_s \cos^2 \alpha_e + (Z_u + X_w)_s \sin \alpha_e \cos \alpha_e + (X_u)_s \sin^2 \alpha_e \\
 (Z_q)_b &= (Z_q)_s \cos \alpha_e + (X_q)_s \sin \alpha_e \\
 (Z_{\delta_{HT}})_b &= (Z_{\delta_{HT}})_s \cos \alpha_e + (X_{\delta_{HT}})_s \sin \alpha_e
 \end{aligned} \tag{4.22}$$

$$\begin{aligned}
 (M_u)_b &= (M_u)_s \cos \alpha_e - (M_w)_s \sin \alpha_e \\
 (M_w)_b &= (M_u)_s \sin \alpha_e + (M_w)_s \cos \alpha_e \\
 (M_q)_b &= (M_q)_s \\
 (M_{\delta_{HT}})_b &= (M_{\delta_{HT}})_s
 \end{aligned} \tag{4.23}$$

3.3. Lateral and directional plane

For the lateral-directional case, it is more complicated to derive a general derivative transformation method from stability to body axes system, therefore, each derivative will be derived in the stability axes system first, then transformed to body axes system individually. Detailed derivation can be found in Appendix B. Only the final results are presented here.

Side-force derivative due to sideslip angle:

$$Y_\beta = \frac{C_{y\beta} \bar{q}_e S}{m} \tag{4.24}$$

Rolling moment and yawing moment derivatives due to side slip angle in body axes system are:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$(L_\beta)_b = \frac{(C_{l_\beta} \bar{q}_e S b)_s \cos \alpha_e - (C_{n_\beta} \bar{q}_e S b)_s \sin \alpha_e}{I_{xx}} \quad (4.25)$$

$$(N_\beta)_b = \frac{(C_{l_\beta} \bar{q}_e S b)_s \sin \alpha_e + (C_{n_\beta} \bar{q}_e S b)_s \cos \alpha_e}{I_{zz}} \quad (4.26)$$

Derivatives due to rate of change of sideslip angle:

$$Y_{\dot{\beta}} = L_{\dot{\beta}} = N_{\dot{\beta}} = 0 \quad (4.27)$$

Side force derivative due to roll rate is zero:

$$(Y_p)_b = 0 \quad (4.28)$$

Side force derivative due to yaw rate is:

$$(Y_r)_b = C_{L_{\alpha_{VT}}} \frac{l_{VT}}{m(U_e)_b} \bar{q}_e S \quad (4.29)$$

The rolling moment derivatives with respect to roll rate and yaw rate in body axes system are:

$$(L_p)_b = -\frac{\rho(U_e)_s}{I_{xx}} \int_0^s c_y y^2 dy \left[\begin{aligned} &(C_{L_{\alpha_e}} + C_{D_e}) \cos^2 \alpha_e + 2C_{D_e} \sin^2 \alpha_e \\ &+ (C_{D_{\alpha_e}} + C_{L_e}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (4.30)$$

$$(L_r)_b = -\frac{\rho(U_e)_s}{I_{xx}} \int_0^s c_y y^2 dy \left[\begin{aligned} &-2C_{L_e} \cos^2 \alpha_e + (C_{D_{\alpha_e}} - C_{L_e}) \sin^2 \alpha_e \\ &+ (C_{L_{\alpha_e}} - C_{D_e}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (4.31)$$

And the yawing moment derivatives with respect to roll rate and yaw rate are:

$$(N_p)_b = \frac{\rho(U_e)_s}{I_{zz}} \int_0^s c_y y^2 dy \left[\begin{aligned} &(C_{D_{\alpha_e}} - C_{L_e}) \cos^2 \alpha_e - 2C_{L_e} \sin^2 \alpha_e \\ &+ (C_{D_e} - C_{L_{\alpha_e}}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (4.32)$$

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$(N_r)_b = \frac{\rho(U_e)_s}{I_{zz}} \int_0^s c_y y^2 dy \left[-2C_{D_e} \cos^2 \alpha_e - (C_{L_{\alpha_e}} + C_{D_e}) \sin^2 \alpha_e \right] + (C_{D_{\alpha_e}} + C_{L_e}) \sin \alpha_e \cos \alpha_e \quad (4.33)$$

$$-C_{L_{\alpha_{VT}}} \frac{l_{VT}^2}{I_{zz}(U_e)_b} \bar{q}_e S$$

The side force due to the vertical tail's deflection is:

$$Y_{\delta_{VT}} = \frac{C_{L_{\delta_{VT}}} \bar{q}_e S}{m} \quad (4.34)$$

The yawing moment derivative due to vertical tail's deflection is:

$$(N_{\delta_{VT}})_b = -\frac{C_{L_{\alpha_{VT}}} \bar{q}_e S l_{VT}}{I_{zz}} \quad (4.35)$$

The rolling moment derivative with respect to stators' deflection in body axes system is given by:

$$(L_{\delta_s})_b = \frac{(C_{l_{\delta_s}})_b \bar{q}_e S b}{I_{xx}} \quad (4.36)$$

4. STATIC STABILITY ANALYSIS

Static stability is related with the ability of the aircraft to self-recover from perturbed states. It can be analyzed separately in each plane:

- Longitudinal plane: An aircraft is statically stable in longitudinal plane if the curve slope of pitching moment coefficient with respect to angle of attack is negative, i.e. a negative change in angle of attack will result in positive change in pitching moment, and vice versa.
- Lateral plane: Similar to the longitudinal static stability criterion, a negative rolling moment derivative due to roll angle (bank angle) is required for lateral static stability.
- Directional plane: A negative yawing moment derivative due to yaw angle (heading angle) or positive yawing moment derivative due to side slip angle is required for directional static stability.

Chapter 4: Dynamic analysis and control synthesis of cruising flight

5. DYNAMIC STABILITY AND RESPONSE ANALYSIS

5.1. Longitudinal case

The derivatives derived in Part 3 are substituted to the equations of longitudinal motion in Part 2, and then the equations can be rewritten in matrix form as:

$$\begin{bmatrix} s - X_u & -X_w & g \cos \Theta_e - X_q s + W_e s \\ -Z_u & s - Z_w & g \sin \Theta_e - Z_q s - U_e s \\ -M_u & -M_w & s^2 - M_q s \end{bmatrix} \begin{bmatrix} \frac{u(s)}{\delta_{HT}(s)} \\ \frac{w(s)}{\delta_{HT}(s)} \\ \frac{\theta(s)}{\delta_{HT}(s)} \end{bmatrix} = \begin{bmatrix} X_{\delta_{HT}} \\ Z_{\delta_{HT}} \\ M_{\delta_{HT}} \end{bmatrix} \quad (4.37)$$

The system matrix is used to determine the stability characteristics of the vehicle by setting its determinant to be equal to zero. The resulting equation is called characteristic equation of the system, and its roots' location will affect the stability of the system.

The transfer function matrix can be determined by using Cramer's rule; then how a flying quantity respond to a control input can be determined.

5.2. Lateral – Directional case

The derivatives derived in Part 3 are substituted to the equations of lateral – directional motion in Part 2, and then the equations can be rewritten in matrix form as:

$$\begin{bmatrix} (U_e)_s s - Y_\beta & -(W_e s + Y_p s + g \cos \Theta_e) & (U_e - Y_r) s \\ -L_\beta & (s^2 - L_p s) & -\left(\frac{I_{xz}}{I_{xx}} s^2 + L_r s\right) \\ -N_\beta & -\left(\frac{I_{xz}}{I_{zz}} s^2 + N_p s\right) & (s^2 - N_r s) \end{bmatrix} \begin{bmatrix} \beta(s) \\ \phi(s) \\ \psi(s) \end{bmatrix} = \begin{bmatrix} Y_{\delta_{VT}} \delta_{VT}(s) \\ L_{\delta_S} \delta_S(s) \\ N_{\delta_{VT}} \delta_{VT}(s) \end{bmatrix} \quad (4.38)$$

Similar to the longitudinal case, the system matrix is used to determine the stability characteristics of the vehicle by setting its determinant to be equal to zero. The resulting equation is called characteristic equation of the system, and its roots' location will affect the stability of the system.

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Because there are two independent control inputs, response of a flying quantity to a particular control input can be determined by solving the equations while the other input is set to zero.

6. DYNAMIC ANALYSIS OF THE MAV

6.1. Stability derivatives

6.1.1. Longitudinal

The formulae derived in the previous part for calculating the aerodynamic stability and control derivatives in general cases only. The dimensionless results presented below have been accounted with both magnitude and direction correction factors. Detailed calculations can be found in Appendix C at the end of this report.

$X_u = -0.3765$	$X_w = 0.137$	$X_q = 0.0097$	$X_{\delta_{HT}} = 0.5622$
$Z_u = -0.6522$	$Z_w = -4.0564$	$Z_q = -0.1504$	$Z_{\delta_{HT}} = -8.7461$
$M_u = -0.0758$	$M_w = -6.7886$	$M_q = -4.043$	$M_{\delta_{HT}} = -254.3564$

Table 4.2: Longitudinal derivatives in body axes system

6.1.2. Lateral – Directional

From the results derived in Part 3, the aerodynamic stability and control derivatives in lateral – directional case were calculated and presented in the following table:

$Y_\beta = -34.7894$	$Y_p = 0$	$Y_r = 0.1465$	$Y_{\delta_{VT}} = 8.7461$
$L_\beta = -28.0786$	$L_p = -15.2959$	$L_r = 2.8508$	$L_{\delta_S} = -189.3693$
$N_\beta = 5.1798$	$N_p = -0.2313$	$N_r = -3.0209$	$N_{\delta_{VT}} = -175.7121$

Table 4.3: Lateral - directional derivatives in body axes system

Chapter 4: Dynamic analysis and control synthesis of cruising flight

6.2. Static stability discussions

Static stability of an aircraft requires these three conditions to be satisfied:

$$\frac{\partial C_m}{\partial \alpha} < 0 \text{ (longitudinal static stability)} \quad (4.39)$$

$$\frac{\partial C_l}{\partial \beta} < 0 \text{ (lateral static stability)} \quad (4.40)$$

$$\frac{\partial C_n}{\partial \beta} > 0 \text{ (directional static stability)} \quad (4.41)$$

From the simulation aerodynamic data, it can be seen that $C_{m\alpha}$ and $C_{l\beta}$ are negative and $C_{n\beta}$ is positive; hence, the MAV is statically stable in longitudinal and directional directions.

Rolling moment at 5 degree bank angle is -0.0042; hence, it can be inferred that rolling moment derivative due to bank angle is negative. Thus, the vehicle is also statically stable in lateral direction.

6.3. Dynamic Response**6.3.1. Longitudinal**

Substituting all the derivatives in Table 4.1 to Equation (4.37) and solving the characteristic equation of the system return the poles of the system as follows:

$$\begin{aligned} s_{1,2} &= -4.0648 \pm 9.9541i \\ s_{3,4} &= -0.1731 \pm 0.5276i \end{aligned} \quad (4.42)$$

Because all the poles are located on the left half plane, the system is said to be stable. Each pair of poles is associated with one mode of motion: The short period mode with higher natural frequency and damping coefficient; and the phugoid mode with lower natural frequency and damping coefficient. The poles can be expressed in terms of natural frequency and damping coefficient as:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$\begin{aligned}
s_{1,2} &= -\zeta_{1,2}\omega_{n_{1,2}} \pm i\omega_{n_{1,2}}\sqrt{1-\zeta_{1,2}^2} \\
s_{3,4} &= -\zeta_{3,4}\omega_{n_{3,4}} \pm i\omega_{n_{3,4}}\sqrt{1-\zeta_{3,4}^2}
\end{aligned} \tag{4.43}$$

Then the characteristics of each mode can be determined as:

$$\begin{aligned}
\zeta_{sp} &= 0.378 & \omega_{n_{sp}} &= 10.7521 \text{ rad / s} \\
\zeta_{ph} &= 0.3117 & \omega_{n_{ph}} &= 0.5553 \text{ rad / s}
\end{aligned} \tag{4.44}$$

Table 4.4 shows the results obtained from both of the non-linear and linear models. It can be seen that the linear model's phugoid mode has the same order of magnitude with the nonlinear model's result, which is lightly damped and low natural frequency. The short-period mode's results are quite close for both models. Possible reason for the different results is because the airspeed was assumed to be unchanged during perturbation of the linear model, while in the non-linear model, actual airspeed variation can be as large as 10% of the original value.

		Non-linear model	Linear model
Phugoid mode	ζ_{ph}	0.2	0.3117
	$\omega_{n_{ph}}$	0.78 rad / s	0.5553 rad / s
Short-period mode	ζ_{sp}	N.A.	0.378
	$\omega_{d_{sp}}$	8.38 rad / s	9.9544 rad / s

Table 4.4: Comparison of linear and non-linear models

Table 4.5 shows some comparisons between the MAV prototype and some other MAVs (28), (29). The frequencies of both modes are quite similar among the MAVs; however, the damping coefficients are quite different. In the current MAV prototype, the damping coefficient of the phugoid mode is higher because the tails is placed far from the cg; therefore, the control power is higher than other MAVs. However, all the damping values are within acceptable range for flyable aircrafts (30).

Chapter 4: Dynamic analysis and control synthesis of cruising flight

	MAV prototype	Zagi 400	StablEyes	Procerus	Dragonfly
$\omega_{sp} (rad / s)$	10.75	12.4	14.9	16.6	17.4
ζ_{sp}	0.38	0.68	0.99	0.30	0.41
$\omega_{ph} (rad / s)$	0.56	1.10	0.94	0.81	-
ζ_{ph}	0.31	0.05	0.16	0.05	-

Table 4.5: Longitudinal modes of some MAVs

The response transfer functions are obtained as follows:

$$\frac{u(s)}{\delta_{HT}(s)} = \frac{0.56(s + 1131.85)(s + 3.49 + 1.65i)(s + 3.49 - 1.65i)}{(s + 4.07 + 9.95i)(s + 4.07 - 9.95i)(s + 0.17 + 0.53i)(s + 0.17 - 0.53i)} \quad (4.45)$$

$$\frac{w(s)}{\delta_{HT}(s)} = \frac{-8.75(x + 429.64)(x + 0.19 + 0.59i)(x + 0.19 - 0.59i)}{(s + 4.07 + 9.95i)(s + 4.07 - 9.95i)(s + 0.17 + 0.53i)(s + 0.17 - 0.53i)} \quad (4.46)$$

$$\frac{\theta(s)}{\delta_{HT}(s)} = \frac{-254.36(s + 3.8)(s + 0.4)}{(s + 4.07 + 9.95i)(s + 4.07 - 9.95i)(s + 0.17 + 0.53i)(s + 0.17 - 0.53i)} \quad (4.47)$$

6.3.2. Lateral – Directional

Substituting all the derivatives in Table 4.2 to Equation (4.38) and solving the characteristic equation of the system return the poles of the system as follows:

$$\begin{aligned} s_{1,2} &= -2.7156 \pm 2.4513i \\ s_3 &= -0.225 \\ s_4 &= -15.0177 \\ s_5 &= 0 \end{aligned} \quad (4.48)$$

Because of the pole at origin, the system is expected to be neutrally stable. The complex pair of roots is associated with the Dutch roll mode, the real root closest to the origin is

Chapter 4: Dynamic analysis and control synthesis of cruising flight

associated with spiral mode, and the last non zero root is associated with the roll mode. The roots of Dutch roll mode can be expressed in terms of natural frequency and damping coefficient as:

$$s_{1,2} = -\zeta_{dr}\omega_{n_{dr}} \pm i\omega_{n_{dr}}\sqrt{1-\zeta_{dr}^2} \quad (4.49)$$

And the real roots can be expressed in terms of time constants as:

$$s_3 = -\frac{1}{T_s} \text{ (spiral mode)} \quad (4.50)$$

$$s_4 = -\frac{1}{T_r} \text{ (roll mode)} \quad (4.51)$$

Then the characteristics of each mode can be determined as:

$$\begin{aligned} \zeta_{dr} &= 0.7423 & \omega_{n_{dr}} &= 3.6583 \text{ rad / s} \\ T_s &= 4.44 \text{ s} \\ T_r &= 0.0666 \text{ s} \end{aligned} \quad (4.52)$$

Table 4.6 shows some comparisons between the MAV prototype and some other MAVs (28), (29). The frequencies of Dutch roll mode vary significantly among the MAVs; however, the damping coefficient of the investigated MAV prototype is the highest among those MAVs. It may be due to the control power from the vertical tail as well as higher moment of inertia about z axis.

	MAV prototype	Zagi 400	StablEyes	Procerus	Dragonfly
$\omega_{dr} \text{ (rad / s)}$	3.66	5.7	9.0	10.3	121
ζ_{dr}	0.74	0.15	0.16	0.10	0.02
$T_s \text{ (s)}$	4.44	-0.15	-0.79	-0.11	0.93
$T_r \text{ (s)}$	0.07	0.10	0.06	0.09	0.09

Table 4.6: Lateral – Directional modes of some MAVs

By setting $\delta_S(s) = 0$, the response due to vertical tail deflection transfer functions is obtained as follows:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

$$\frac{\beta(s)}{\delta_{VT}(s)} = \frac{0.58(s + 300.54)(s + 14.84)(s - 0.12)}{(s + 15.02)(s + 0.22)(s + 2.72 + 2.45i)(s + 2.72 - 2.45i)} \quad (4.53)$$

$$\frac{\psi(s)}{\delta_{VT}(s)} = \frac{-175.83(s + 15.03)(s + 1.95)(s + 0.62)}{(s + 15.02)(s + 0.22)(s + 2.72 + 2.45i)(s + 2.72 - 2.45i)s} \quad (4.54)$$

And by setting $\delta_{VT}(s) = 0$, the bank-angle-to-stators-deflection transfer function is:

$$\frac{\phi(s)}{\delta_S(s)} = \frac{-187.09(s + 2.67 + 2.22i)(s + 2.67 - 2.22i)}{(s + 15.02)(s + 0.22)(s + 2.72 + 2.45i)(s + 2.72 - 2.45i)} \quad (4.55)$$

It can be seen that, due to the zero pole in the heading-angle-to-vertical-tail-deflection transfer function, there is a linear term in heading angle response, thus, it becomes uncontrollable.

7. AUTONOMOUS CONTROLLER DESIGN FOR MAV IN CRUISE

From Equation (4.47), open-loop system response is plotted as shown in Figure 4.1 below:

Figure 4.1: Pitch response due to unit step input of horizontal tail

Chapter 4: Dynamic analysis and control synthesis of cruising flight

The open loop system's response was not good because percentage of overshoot and steady-state error were high. Therefore, a controller should be added to the system to help stabilizing the system faster, as well as improve the system's response.

Detailed procedure for designing the pitch angle and altitude will be presented here. Pitch angle control system can be used to maintain the MAV's attitude during hover. It is also used in hardware test which will be presented in the next part. Altitude control system is a modified version of the original pitch attitude controller, and design procedure of this controller will also be presented here.

7.1. Pitch angle control system

The pitch response of the vehicle due to a unit step input from horizontal tail is shown in Figure 4.1. Although the system is stable, the response is not desirable because it takes about $15s$ to reach steady-state value with maximum overshoot of about 70%.

One of the possible simplest controllers for this purpose is the displacement autopilot, shown in Figure 4.2. The system dynamics is the pitch-angle-to-horizontal-tail-deflection transfer function:

$$\frac{\theta(s)}{\delta_{HT}(s)} = \frac{-254.3564s^2 - 1066.3888s - 387.4048}{s^4 + 8.4689s^3 + 118.7992s^2 + 42.7402s + 36.1987} \quad (4.56)$$

Figure 4.2: Typical displacement autopilot block diagram

The aircraft is initially trimmed to straight and level flight, and pitch attitude is aligned with the reference value. If there is a disturbance to the system or change in reference pitch angle, the resultant pitch attitude will deviate from the reference value. The comparator will generate an error signal, which will be amplified and input to the servo at horizontal

Chapter 4: Dynamic analysis and control synthesis of cruising flight

tail. The horizontal tail will try to correct the pitch attitude error, i.e. bring the pitch attitude as close to reference value as possible. If the response cannot be corrected perfectly, there will be steady-state error. The next step is applying this controller to our vehicle model derived previously to improve the response of the system.

A typical servo can be modeled as:

$$\frac{\delta_{HT}(s)}{v(s)} = \frac{1}{\tau s + 1} \quad (4.57)$$

τ is time constant, and it is about 0.05 to 0.25 for typical servo motors. In this case, it is selected to be 0.1, therefore, the amplifier and horizontal tail servo can be combined together as a single block with transfer function of:

$$G_c(s) = \frac{K_a}{s + 10} \quad (4.58)$$

K_a is a positive number, and the minus sign in the aircraft dynamics' transfer function indicates that pitch attitude and horizontal tail deflection are always inversely proportional to each other.

The gyroscope's gain is simply assumed as 1, so that the pitch attitude response will be fed directly to the comparator. The resultant block diagram for this configuration is shown in Figure 4.3:

Figure 4.3: Block diagram system with a displacement autopilot

Chapter 4: Dynamic analysis and control synthesis of cruising flight

The root locus for the MAV with this controller is shown in Figure 4.4:

Figure 4.4: Root locus of system with a displacement autopilot

When K_a is equal to zero, all the poles are located at the open-loop pole positions as shown in Figure 4.4 above. When K_a is increased starting from zero, the phugoid mode poles will move away from the imaginary axis while the short period mode poles move closer to the imaginary axis. When K_a is about 2.5, phugoid poles are on the real axis, the short period mode becomes more significant. Although the system is still stable, all poles are in the left hand side half plane, this should be avoided. When K_a is about 6.5, the short period poles are located in the right hand side half plane, and the system is unstable. System responses of three different values of K_a are presented below, with input signal of a unit step function:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.5: System's response to unit step input (controller gain of 0.3)

Figure 4.6: System's response to unit step input (controller gain of 1.5)

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.7: System's response to unit step input (controller gain of 3)

It can be seen that when the short period mode becomes more significant, the response is not desirable. Hence, to get better system performance, the controller gain should not exceed 2.5.

To help reducing the oscillation and steady-state error in the response, derivative and integral terms can be added to the controller to make it a PID controller. After fine tuning, the integral term is chosen as 0.4 and the derivative term is 0.1. The response with this PID controller is shown in Figure 4.8 below:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.8: Pitch response with PID controller

7.2. Altitude control system

Altitude is controlled by commanding a zero reference climb rate, i.e. $\dot{h}_{ref} = 0$. The climb rate is controlled by the elevator, or for the current MAV prototype, it is controlled by the horizontal tail. Altitude-to-horizontal-tail-deflection transfer function can be derived by pitch-angle-to-horizontal-tail-deflection and angle-of-attack-to-horizontal-tail-deflection transfer functions:

$$\dot{h} = (U_e)_s \sin \gamma \approx (U_e)_s \gamma = (U_e)_s (\theta - \alpha) \quad (4.59)$$

Applying Laplace transform to Equation (4.59) leads to:

$$sh(s) = (U_e)_s [\theta(s) - \alpha(s)] \quad (4.60)$$

$$\text{or } \frac{h(s)}{\delta_{HT}(s)} = \frac{(U_e)_s}{s} \left[\frac{\theta(s)}{\delta_{HT}(s)} - \frac{\alpha(s)}{\delta_{HT}(s)} \right] \quad (4.61)$$

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Substituting $\frac{\theta(s)}{\delta_{HT}(s)}$ and $\frac{\alpha(s)}{\delta_{HT}(s)}$ from previous part to Equation (4.61), the altitude transfer function can be obtained:

$$\frac{h(s)}{\delta_{HT}(s)} = \frac{8.7465s^3 - 52.752s^2 - 14610.1215s - 4355.1375}{s^5 + 8.4759s^4 + 118.7294s^3 + 42.5313s^2 + 35.6487s} \quad (4.62)$$

The block diagram for an altitude control system and its root locus are shown in Figure 4.9 and 4.10 below:

Figure 4.9: Block diagram of an altitude control system

Figure 4.10: Root locus of the above altitude control system

Chapter 4: Dynamic analysis and control synthesis of cruising flight

It can be seen from Figure 4.10 that, the phugoid mode becomes unstable as the control gain is larger than 0.062. The system's time response to a unit step input with control gain of 0.04 (K_a) is given in Figure 4.11.

Figure 4.11: System response to a unit step input with controller gain of 0.04

Following the same procedure as pitch control system design, a PID controller was added to the system with derivative term of 0.05. There is no integral term in the controller as it has little effect on the overall result. Response of this control system is given in Figure 4.12.

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.12: Response to a unit step input of system with a PD controller

With the damper in the controller, system response has been improved significantly. There is little oscillation in the signal, and the settling time has been reduced to about 10 seconds. As suggested in Reference (25), a lead-lag compensator also helps to stabilize the overall system.

Following the same procedure to design the controllers for lateral – directional cases, the final controller gains used in the model can be summarized as:

	Proportional term	Integral term	Derivative term
Altitude controller	-0.004	-	-0.005
Heading angle controller	-0.05	-0.25	-0.05
Bank angle controller	-0.1	-0.05	-0.01

Table 4.7: Summary of controller's gains

The gains are negative because the control surface's deflections are opposite with the aircraft's response.

8. CONTROL SYSTEM IMPLEMENTATION TO THE NON-LINEAR MODEL

8.1. Altitude hold flight

In the first scenario, the MAV was set to operate in the altitude-hold mode. There were three control surfaces used in the model: Altitude controller (by horizontal tail), heading angle controller (by vertical tail) and bank angle controller (by ailerons). All of the controllers designed in the previous part were implemented, but different control systems would be activated for different flight conditions, i.e. during altitude hold mode, MAV's performance was only affected by the altitude control system. Selected gain values were given in Table 4.7. There was no wind in this scenario, and the aircraft had no problem to stabilize itself as shown in Figure 4.13.

Figure 4.13: Altitude, angle of attack and sideslip angle and bank angle of the MAV's 6-DOF model in altitude-hold mode (no wind)

The controller used in this case was designed by using the linear model. There were some differences in the response of linear and non-linear models as shown in Figure 4.14. The differences were mainly due to the fact that in linear model, flying parameters were assumed to be unchanged during perturbed state. In this case, a commanded change of altitude caused a change in pitch angle and angle of attack, thus, drag was increased while

Chapter 4: Dynamic analysis and control synthesis of cruising flight

thrust remained unchanged. Therefore, the vehicle slowed down during this transient state which reduced the dynamic pressure. All of these resulted in slower response of the non-linear model; it took twice the time of the linear model to reach the desired steady-state value. However, those differences did not have much effect on the overall system's performance, response of both of the systems had the same shape and the results were acceptable. The non-linear system's response can be corrected to match the linear system's response by increasing the proportional gain of the controller as shown in Figure 4.15, but this is not really necessary because the overall system's response was still acceptable.

Figure 4.14: Response of linear system and non-linear system to unit step input

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.15: Response of linear system and corrected non-linear system to unit step input

Moving to the next situation, a wind model as shown in Figure 4.16 was introduced to the system. The wind velocity components are 2 m/s , 3.5 m/s , and 2.75 m/s along the X, Y, Z direction respectively, so the wind speed is approximately 5 m/s . The MAV model was able to maintain the altitude without any problem as shown in Figure 4.17; however, due to the lack of a heading angle control system, the MAV could not follow the desired heading direction as shown in Figure 4.18.

Figure 4.16: Wind model

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.17: Altitude, angle of attack and sideslip angle and bank angle of the MAV's 6-DOF model in altitude-hold mode (with wind disturbance)

Figure 4.18: XY coordinate of the MAV's 6-DOF model in altitude-hold mode (with wind disturbance)

When the heading angle control system was activated, the deviation in Y axis was within 10m for a flying range of 1500m and wind speed of 5m/s as shown in Figure 4.19 below:

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.19: XY coordinate of the MAV's 6-DOF model in altitude-hold mode (with wind disturbance and heading angle control)

8.2. Climbing flight

In this scenario, the MAV was commanded to change its altitude from $100m$ to $150m$. To command a climb, all the control parameters were changed to climb-mode values which were shown in Table 3.1. Figure 4.20 and 4.21 show the MAV's response during a climbing flight, with the presence of wind disturbance.

Figure 4.20: XY coordinate of the MAV's 6-DOF model in climbing-hold mode (with wind disturbance and heading angle control)

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.21: Altitude, angle of attack and sideslip angle and bank angle of the MAV's 6-DOF model in climbing mode (with wind disturbance)

In this mode, due to the change in thrust, bank angle also varied due to the induced rolling moment from the main motor. Thus, the bank angle control system was activated to maintain the zero-bank angle.

8.3. Turning flight

To initiate a turning action, the required commands are: Changing bank angle, increasing angle of attack and thrust. By changing the control parameters accordingly with the calculated values in Chapter 3 for a turning flight, the MAV started performing a turning flight with the expected turn rate of 0.2 rad/s and expected turn radius of 75 m . Figure 4.22 shows the XY coordinate of the MAV during a turning flight. It can be seen that the result was very close to the calculated values: The turn radius was about 75 m and turn rate was about 0.185 rad/s (the MAV took 34 s to finish a complete circle). During this simulation, the MAV performed three rounds of turn, and these circles are almost identical in Figure 4.22.

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.22: XY coordinate of the MAV's 6-DOF model in turning mode (three rounds of turn)

When there was presence of wind disturbance, the MAV model was still able to maintain its bank angle, therefore, the observed ground track in Figure 4.23 still shows some kind of circular pattern that the MAV created when performing a turn.

Figure 4.23: Ground track during turning flight in wind

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.24: Angle of attack and sideslip angle, bank angle and altitude of the MAV's 6-DOF model in turning mode (with wind disturbance)

8.4. Combined flight profile

In this final situation, the MAV model was set to finish a complete flight profile in which it started with climbing mode, followed by cruise mode, then made a half circular turn, and finally, headed back to the original position. The control inputs were changed accordingly with different flight modes. Figure 4.25 shows the actual ground track of the MAV, and Figure 4.26 shows the flight's parameters during this simulation.

Figure 4.25: Ground track during combined mode

Chapter 4: Dynamic analysis and control synthesis of cruising flight

Figure 4.26: Angle of attack and sideslip angle, bank angle and altitude of the MAV's 6-DOF model in combined mode

Chapter 5: PRELIMINARY HARDWARE IMPLEMENTATION AND TESTING

1. SENSOR SYSTEM

1.1 Introduction

The MAV prototype is equipped with an inertial measurement unit (IMU) to record angular rates and translational acceleration data. The IMU model is LandMark 10 which is produced by Gladiator Technologies. It is powered by 3 AA batteries, and connection to PC is established through RS-232 interface (31).

Figure 5.1: LandMark 10 IMU (31)

Once having been powered up, the IMU outputs signal automatically at the rate of 100Hz (i.e. at every 0.01s). This signal is captured and processed by a Simulink model as described in Figure 5.2 below. The Kalman filter is required because it helps reduce uncertainty from noisy measurements. This practice is commonly applied when dealing with IMU.

Figure 5.2: Sensor system model

Chapter 5: Preliminary hardware implementation and testing

1.2. Receiving and decoding raw data

Signal from the IMU is sent in 18-byte package, with checksum capability to prevent losing data, however, Simulink model is not able to receive the whole package data; it can only receive one byte of data at a time. To ensure that at least a complete package is received, the Simulink model will wait until 35 bytes are received, then the decoding work can start. Due to this limitation, the sensor system model cannot operate at any rate higher than 50Hz.

Decoding process starts with finding the complete 18-byte package in the received 35 bytes. The first byte of the 18-byte package is 62; this was defined by the manufacturer of the IMU. However, not any byte equivalent to 62 is the first byte. Detailed algorithm to find the first byte and deal with the exceptional case is shown in Figure 5.3. The procedure can be described as follows: The first step is to find the byte equal to 62. This byte is the starter of the 18-byte package if and only if sum of all bytes in the package can be divided by 256 without any remainder. The exceptional case is when the 1st byte is 62, the 18th byte is 62 and the 19th byte is 63, then it can be concluded that the complete data package starts at the 18th byte. The 1st byte, which is equal to 62, is actually the check sum of the previous package. After the complete message is found, the useful data can be extracted which includes data of 3 angular rate values and 3 translational acceleration values.

Chapter 5: Preliminary hardware implementation and testing

Figure 5.3: Algorithm to check for useful data package

1.3. Basics of Kalman filter

Kalman filter is an iterative filter that requires 2 inputs: The first input is used to predict the desired output, and the second input is the estimation of the desired output. After each round of iteration, the Kalman filter will try to get the desired output from the 2 given inputs. In this case, the only 2 inputs are accelerometer and gyroscope data from the IMU (32).

Theoretically, the attitude of the vehicle can be obtained by simply integrating the gyroscope data. However, in reality, there is an effect that makes the integration drift away from the true value over time. Therefore, we need the 2nd input from the accelerometers to overcome this effect. The accelerometers give us the acceleration components in body axes system, this acceleration always includes gravity. By assuming that acceleration of the

Chapter 5: Preliminary hardware implementation and testing

vehicle is relatively small compared to the gravity, we can calculate the Euler's angles of the vehicle based on the rotation matrix:

$$\begin{bmatrix} accX \\ accY \\ accZ \end{bmatrix}_{body} = D \times \begin{bmatrix} accX \\ accY \\ accZ \end{bmatrix}_{earth} \quad (5.1)$$

For the current MAV and IMU configuration, the direction cosine matrix is given by:

$$D = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi & \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi & \sin \phi \cos \theta \\ \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi & \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi & \cos \phi \cos \theta \end{bmatrix} \quad (5.2)$$

And the acceleration in earth axes system is:

$$\begin{bmatrix} accX \\ accY \\ accZ \end{bmatrix}_{earth} = \begin{bmatrix} 0 \\ 0 \\ 9.81 \end{bmatrix} \quad (5.3)$$

Therefore, the acceleration in body axes system can be simplified as followed:

$$\begin{bmatrix} accX \\ accY \\ accZ \end{bmatrix}_{body} = D \times \begin{bmatrix} 0 \\ 0 \\ 9.81 \end{bmatrix} = \begin{bmatrix} -9.81 \sin \theta \\ 9.81 \sin \phi \cos \theta \\ 9.81 \cos \phi \cos \theta \end{bmatrix} \quad (5.4)$$

The acceleration in body axes system can be obtained from the IMU, then the angles θ and ϕ can be estimated. However, this estimation will not work if the pitch attitude is 90 degree (i.e. the MAV is in hover position) because at that moment, the above equation is undefined. Therefore, in this experiment, only pitch attitude estimation is used. Another point to be noted is this estimation will be invalid if the acceleration in body X direction is larger than $9.81m/s^2$. However, this case is very unlikely to happen; the maximum acceleration in X direction during cruise is $5m/s^2$ (maximum thrust during cruise is about $2.5N$ and mass of the MAV is about $0.5kg$), the only possibility is the case of vertical accelerating when pitch angle is 90 degree. In order to cover this case, a 90 degree pitch angle is assumed when the acceleration in body X direction is larger than $9.81m/s^2$.

Chapter 5: Preliminary hardware implementation and testing

With the acceleration in body axes system is the IMU's data and the acceleration in earth axes system is gravity, Euler's angles can be obtained. These angles are used to eliminate the drift effect of integration of gyroscope data.

1.4. Model build-up

In order to use Kalman filter, the model must be described by a linear system. A linear system is a process that can be described by the following equations:

$$\text{State equation: } x_{k+1} = Ax_k + Bu_k$$

$$\text{Output equation: } y_k = Cx_k$$

In the above equations, A, B and C are matrices, k is time index, x is the state of the system, u is the known input and y is the measured output. For this IMU model, these parameters are selected as follows:

$$A = \begin{bmatrix} 1 & -dt \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} dt \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x_k = \begin{bmatrix} \text{angle} \\ \text{error} \end{bmatrix}_k \quad u_k = k^{\text{th}} \text{ gyroscope's data}$$

dt is the sampling period. The 2nd element of state vector (which is the error) is less interested because we do not have any data for verifying this.

To complete the Kalman filter model, there are three more parameters:

S_z : The measurement process noise covariance, which is calculated by:

$$S_z = E(z_k \cdot z_k^T)$$

z_k is the noise from the process of measuring y. In our model, this is the noise from accelerometer's data.

S_w : The process noise covariance matrix, which is calculated by:

$$S_w = E(w_k \cdot w_k^T)$$

Chapter 5: Preliminary hardware implementation and testing

w_k is the noise from the process of calculating x . In our model, this is the noise from gyroscope's data.

P : Prediction error covariance, which is related to the process noise covariance matrix S_w . Initially, P is equal to S_w . After each round of iteration, the prediction error covariance is recalculated from the previous value.

To determine S_z and S_w , experimental data need to be collected and analyzed. We use the IMU to measure some standard angles (0, 45, 90 degrees), then the standard deviation of the measurement can be calculated and the noise matrices can be found as:

$$S_z = 0.012$$

$$S_w = \begin{bmatrix} 0.00005 & 0.00089 \\ 0.00089 & 0.01788 \end{bmatrix}$$

The whole algorithm with explanation is summarized in the flow chart below:

Chapter 5: Preliminary hardware implementation and testing

Figure 5.4: Kalman filter's algorithm and explanation

Chapter 5: Preliminary hardware implementation and testing

1.5. Experimental result

The IMU was left untouched in the table, therefore, all the gyroscope's readings should be zero, and the angles should be constants. However, if we apply direct integration to the gyroscope's data, the angles keep increasing as shown in Figure 5.5, which is due to the effect of drift.

Figure 5.5: Angle calculated by integrating gyroscope's data

After introducing the Kalman filter, the angles' values become stable as shown in Figure 5.6:

Figure 5.6: Angle calculated with correction from Kalman filter

Figure 5.7 shows some readings when the IMU was in motion. The actual sequence introduced is $90^{\circ} - 45^{\circ} - 120^{\circ} - 90^{\circ}$. It can be seen that the IMU was able to detect the actual

Chapter 5: Preliminary hardware implementation and testing

motion quite closely. The drift effect can also be seen quite clearly in the left graph, which has been corrected in the right graph.

Figure 5.7: Angle sequence calculated without and with Kalman filter's correction

2. SERVO CONTROL SYSTEM

2.1. Introduction

The servo control board used in the MAV prototype is the Micro serial servo controller made by Pololu Robotics and Electronics. One Pololu board, as shown in Figure 5.8, can control 8 servos which is sufficient for the MAV (33).

Figure 5.8: Pololu servo control board (33)

2.2. Interface options

There are two protocols to communicate with the servo board. The first protocol is called Pololu mode, which is enabled when the blue protocol selection jumper is removed from

Chapter 5: Preliminary hardware implementation and testing

the board, and the second protocol is called Mini SSC II mode, which is enabled when the protocol selection pins on the board are shortened by a jumper. The Pololu mode offers access to all special features of the servo controller; therefore, it needs more data to be transferred at one time. Comparing with the Mini SSC II mode, the required data to be transferred can be 2 to 2.5 times more, depends on the instruction given to the servo. For every command to be sent in Pololu mode, 6 to 7 bytes are required, and there is only one 3-byte command can be sent in Mini SSC II mode. The maximum data transmission rate of the first mode is 40000 baud, and that of the second mode is 9600 baud. However, operating the servo board at higher speed will result in more energy consumption, hence, the Mini SSC II mode was selected due to its smaller data package and lower operating speed. The experimental result will verify if this mode is sufficient for controlling the MAV. The other communication protocol needs to be employed in case the simpler mode fails to meet our requirements.

2.3. Stand alone servo board test

Figure 5.9: Block diagram for stand-alone servo board test

The controller used in the servo board test is a normal gaming joystick with one control stick, one scroll button and several push buttons. Due to the capability of changing output value in a wide range, only the control stick and the scroll button are used to simulate the control signal sent to the servo board. The Simulink control model is shown in Figure 5.10.

Chapter 5: Preliminary hardware implementation and testing

Figure 5.10: Simulink model for stand-alone servo board test

3. CONTROLLER TESTING

Until now, controllers can be implemented to the system by modifying the yellow block in Figure 5.11. However, the MAV prototype needs to be connected with the computer, therefore, available tests are very limited. It is highly recommended that PC block should be replaced by an embedded system, so that more tests can be carried out.

Chapter 5: Preliminary hardware implementation and testing

Figure 5.11: Block diagram for rapid control design

For the current system, the PID controller brought from previous part was used to do some simple tests. The pitch angle was set at 90° , i.e. the prototype was left untouched on the table. Then the control system was activated. All the control surfaces would move accordingly with the MAV's orientations in order to bring the MAV to the original state.

To compare the simulation model and the actual model, the pitch angle control derived in Chapter 4 was implemented to both of the simulation and actual models. The simulation model was commanded to quickly increase the pitch angle by 0.5rad , then horizontal tail's deflection was recorded as about 0.02rad and actual pitch angle change was about 0.3rad . Results are shown in Figure 5.12:

Figure 5.12: Attitude change feedback and horizontal tail's deflection of simulation model for a commanded sinusoidal attitude change of 0.5rad

Chapter 5: Preliminary hardware implementation and testing

For the actual model, the commanded attitude change is the yellow curve shown in the left part of Figure 5.13, which is about $0.8rad$. Then the actual model was moved manually, trying to generate the same pattern with the simulation model, which is the purple curve in the left part of Figure 5.13. It can be seen in the horizontal tail's deflection that the controller trying to move the tail toward negative direction, thus positive pitching moment can be generated to pitch the aircraft up. To return the pitch attitude to the initial value, horizontal tail's deflection should also revert back to the initial value, but due to the imperfect manual movement, it can only return to a state close to initial one.

Figure 5.13: Attitude change feedback and horizontal tail's deflection of actual model for a commanded sinusoidal attitude change of $0.8rad$

Another trial is the step change in pitch attitude. The horizontal tail's deflection is expected to follow the same pattern with the simulation result, which is shown in Figure 5.14 below:

Chapter 5: Preliminary hardware implementation and testing

Figure 5.14: Attitude change feedback and horizontal tail's deflection of simulation model for a commanded step attitude change of 0.1rad

And the recorded horizontal tail's deflection of the actual model is shown in Figure 5.15. It can be seen that the same pattern of the simulation model's response is captured in the actual model's response. The spikes in the result are due to the noise from the measurement equipments.

Figure 5.15: Attitude change feedback and horizontal tail's deflection of actual model for a commanded step attitude change of 0.5rad

Chapter 6: CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

1. CONCLUSIONS

This research has successfully developed a general 6-DOF non-linear model for the current MAV design in cruising flight conditions with the aerodynamic data obtained from CFD simulation. Conclusions drawn from this work can be divided into two parts:

On the software part: The modeling work started with the linear mathematical model of the MAV. It was shown that the investigated MAV was stable in longitudinal plane, but it needed to be aided with a control system to be stable in lateral – directional plane. Therefore, autonomous control systems were designed based on this linear model, after that, they were implemented to the non-linear model. Results showed that the control systems designed with linear model can improve the non-linear model's performance significantly. The final model was able to perform altitude hold mode, climbing and turning flight without any problem, even in the presence of wind disturbance.

On the hardware part: After all the theoretical analysis had been done, hardware was eventually implemented to the system, the Simulink model was also modified to interact with the hardware. Up to now, the sensor system and servo control board were implemented in the prototype. The interfacing software was developed so that the MATLAB/Simulink model data can collect feedback data and send control signal to the servos. Filtering algorithm (Kalman filter) was implemented to eliminate drift effect in the sensor which leads to inaccurate feedback data. The computer serves as the flight control system of the MAV now; therefore, the whole system becomes a base tool for rapid implementation of different control schemes. When embedded system is employed, more autonomous features can be developed in future.

2. RECOMMENDATIONS FOR FUTURE WORK

The final objective of this project is to develop a fully autonomous MAV. In order to achieve this, the future work should focus on the following topics:

2.1. Design modifications

This electric motor does not provide enough thrust for the MAV during hover test. It turns out that the actual weight of the MAV is more than what was calculated. There is also collision between the vertical and horizontal tail planes. This makes the movable range of both control planes limited.

2.2. Wind-tunnel tests with the current prototype

As the current MAV is not able to reach the cruise flight conditions, wind-tunnel test should be carried out to verify the simulation model of the MAV. During this test, a constant flow of air blowing to the MAV model is as same as the MAV flying in no wind conditions. Moreover, wind-tunnel test can also be used to validate the numerical aerodynamic investigation which was carried out previously.

2.3. Finalizing hover and transition control

In order to switch to cruise conditions, the current MAV needs to be able to fly in hover and transition mode. Therefore, hover and transition control needs to be finalized before cruise control can be implemented.

2.4. Implementing the flight control system to the MAV

Once the design modifications have been performed, the next step should be implementing the control system to the MAV by using an embedded system. With the vehicle attached to the rotary rig, although the degree of freedom is limited, a controlled environment is created; therefore, any unexpected response of the vehicle can be captured and corrected. Once the limited flight test phase has been completed, free flight tests can be performed.

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Appendix A: AERODYNAMIC DATA FROM CFD SIMULATION

A1. WING – BODY COMBINATION

$\alpha(rad)$	0	0.174533	0.349066
C_D	0.033998	0.091604	0.291904
C_L	0.014699	0.529414	0.837266
C_m	-0.000033	-0.002386	-0.00245

Table A.1: Longitudinal aerodynamic data of wing - body combination

Figure A.1: Wing-body's drag coefficient vs. angle of attack

Figure A.2: Wing-body's lift coefficient vs. angle of attack

Figure A.3: Wing-body's pitching moment coefficient vs. angle of attack

At trim point condition ($\alpha = 0.1677$), both second order approximation and linear approximation give similar results for pitching moment curve slope. Therefore, linear approximation result is used to make the analysis simple but still accurate.

$\beta(rad)$	-0.174533	-0.087266	0	0.087266	0.174533
C_y	0.264032	0.137062	0	-0.137296	-0.2636
C_l	0.001483	0.000729	0	-0.000719	-0.001435

C_n	-0.0040	-0.0015	0	0.0015	0.0040
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Table A.2: Lateral – directional aerodynamic data of wing – body combination

Figure A.4: Wing – body's side force vs. side slip angle

Figure A.5: Wing – body's rolling moment vs. side slip angle

Figure A.6: Wing – body's yawing moment vs. side slip angle

As shown in Figure A6, both linear approximation and third order approximation give a close result of C_n , to get a more accurate value of $C_{n\beta}$ around 0 degree point, third order approximation should be used in lateral dynamic analysis.

A2. VERTICAL AND HORIZONTAL TAILS

$\alpha(rad)$	0	0.174532778	0.349065556
C_D	0.009116813	0.013425287	0.028643809
C_L	0.000004369	0.044569239	0.08740477
C_m	0.000005405	-0.026782949	-0.057762215

Table A.3: Aerodynamic data of vertical/horizontal tail

Figure A.7: Vertical/horizontal tail's drag coefficient vs. angle of attack

Figure A.8: Vertical/horizontal tail's lift coefficient vs. angle of attack

Figure A.9: Vertical/horizontal tail's pitching moment coefficient vs. angle of attack

A3. STATORS

$\alpha(rad)$	0	0.087266389	0.174532778
C_D	0.03711	0.039	0.04487
C_l	0.000026	N.A.	-0.007423834

Table A.4: Aerodynamic data of statoms

Figure A.10: Stators' drag coefficient vs. angle of attack

Figure A.11: Stators' rolling moment coefficient vs. angle of attack

Appendix B: DETAILED DERIVATION OF LATERAL – DIRECTIONAL DERIVATIVES

For the lateral-directional case, it is more complicated to derive a general derivative transformation method from stability to body axes system, therefore, each derivative will be derived in the stability axes system first, then transformed to body axes system individually. In the stability axes system, the lateral-directional force and moments are expressed in terms of non-dimensional coefficients as:

$$F_{A_y} = C_y \bar{q} S \quad (\text{B.1})$$

$$L_A = C_l \bar{q} S b \quad (\text{B.2})$$

$$N_A = C_n \bar{q} S b \quad (\text{B.3})$$

B1. DERIVATIVES WITH RESPECT TO SIDESLIP ANGLE

Partially differentiate Equations (B.1) with respect to β and normalize with the mass of the MAV, we have:

$$Y_\beta = \frac{C_{y_\beta} \bar{q}_e S}{m} \quad (\text{B.4})$$

Because body y-axis and stability y-axis are the same, side force derivative does not change when transformed from stability to body axes system.

Partially differentiate Equations (B.2) and (B.3) with respect to β :

$$\left. \frac{\partial L_A}{\partial \beta} \right|_e = \left. \frac{\partial C_l}{\partial \beta} \bar{q} S b \right|_e = C_{l_\beta} \bar{q}_e S b \quad (\text{B.5})$$

$$\left. \frac{\partial N_A}{\partial \beta} \right|_e = \left. \frac{\partial C_n}{\partial \beta} \bar{q} S b \right|_e = C_{n_\beta} \bar{q}_e S b \quad (\text{B.6})$$

The perturbed rolling moment and yawing moment due to side slip angle are:

$$(l_A)_s = (C_{l_\beta} \bar{q}_e S b)_s \beta \quad (\text{B.7})$$

$$(n_A)_s = (C_{n_\beta} \bar{q}_e S b)_s \beta \quad (\text{B.8})$$

l_A and n_A are along stability X and Z axis, respectively. Applying the relationship in Equation (4.7) to transform l_A and n_A to body axes system leads to:

$$\begin{aligned} (l_A)_b &= (l_A)_s \cos \alpha_e - (n_A)_s \sin \alpha_e \\ &= (C_{l_\beta} \bar{q}_e S b)_s \beta \cos \alpha_e - (C_{n_\beta} \bar{q}_e S b)_s \beta \sin \alpha_e \end{aligned} \quad (\text{B.9})$$

$$\begin{aligned} (n_A)_b &= (l_A)_s \sin \alpha_e + (n_A)_s \cos \alpha_e \\ &= (C_{l_\beta} \bar{q}_e S b)_s \beta \sin \alpha_e + (C_{n_\beta} \bar{q}_e S b)_s \beta \cos \alpha_e \end{aligned} \quad (\text{B.10})$$

Then, rolling moment and yawing moment derivatives due to side slip angle in body axes system can be obtained as:

$$(L_\beta)_b = \frac{(C_{l_\beta} \bar{q}_e S b)_s \cos \alpha_e - (C_{n_\beta} \bar{q}_e S b)_s \sin \alpha_e}{I_{xx}} \quad (\text{B.11})$$

$$(N_\beta)_b = \frac{(C_{l_\beta} \bar{q}_e S b)_s \sin \alpha_e + (C_{n_\beta} \bar{q}_e S b)_s \cos \alpha_e}{I_{zz}} \quad (\text{B.12})$$

B2. DERIVATIVES WITH RESPECT TO SIDESLIP ANGLE'S RATE OF CHANGE

It is assumed that effect of sideslip angle's rate of change on the aircraft performance can be neglected. Hence, all the derivatives with respect to $\frac{\dot{\beta} b}{2U_e}$ are equal to zero, and no

transformation from stability to body axes system is necessary.

$$\left. \frac{\partial F_{A_y}}{\partial \left(\frac{\dot{\beta} b}{2U_e} \right)} \right|_e = \left. \frac{\partial L_A}{\partial \left(\frac{\dot{\beta} b}{2U_e} \right)} \right|_e = \left. \frac{\partial N_A}{\partial \left(\frac{\dot{\beta} b}{2U_e} \right)} \right|_e = 0 \quad (\text{B.13})$$

$$\text{or } Y_{\dot{\beta}} = L_{\dot{\beta}} = N_{\dot{\beta}} = 0 \quad (\text{B.14})$$

B3. DERIVATIVES WITH RESPECT TO ROLL RATE AND YAW RATE

Side force derivatives can be derived directly in body axes system, but other derivatives need to be derived in stability axes system first, then transformed to body axes system because the drag and lift data are involved in their derivation.

It is assumed that the side force due to roll rate perturbation and yaw rate perturbation is contributed by the vertical tail only. If the vehicle experiences a roll rate perturbation p in body axes system, let's consider a chord-wise strip element of the vertical tail, with thickness of ∂h at coordinate h measured from the body x axis as shown in Figure B.1 below:

Figure B.1: Side force at vertical tail in rolling flight (26)

When there is a roll rate disturbance p applying to the aircraft, the strip element will have a lateral velocity component ph . The resultant velocity V is at angle of attack α' relative to the vertical tail:

$$\alpha' \approx \tan \alpha' = \frac{ph}{U_e} \quad (B.15)$$

The change in angle of attack causes a change in vertical tail lift, which is resolved into a lateral force increment on this strip element. Sum of all these force increments is the resultant side force due to roll rate generated by the vertical tail. However, for the current MAV design, we can always find a pair of strip elements of the vertical tail which are symmetric about the body X axis. The force increments generated by these two elements are equal in magnitude but opposite in direction; hence, they will cancel each other out. As a result, the total side force due to a roll rate perturbation is zero, i.e. side force derivative due to roll rate is zero:

$$(Y_p)_b = 0 \quad (B.16)$$

In a yaw rate perturbation, the vertical tail's angle of attack in body axes system is:

$$\alpha' \approx \tan \alpha' = \frac{r l_{VT}}{(U_e)_b} \quad (B.17)$$

This change in vertical tail's angle of attack generates a lift and drag at the vertical tail as shown in Figure B.2. It can also be seen that a positive yaw rate perturbation will generate a positive side force.

Figure B.2: Change in vertical tail's angle of attack due to yaw rate (26)

The resultant lift and drag generated by the vertical tail are resolved into body Y axis to give a rise of Y_{VT} to side force:

$$\begin{aligned}
 Y_{VT} &= L_{VT} \cos \alpha' + D_{VT} \sin \alpha' \\
 &= C_{L_{\alpha_{VT}}} \alpha' \bar{q}_e S \cos \alpha' + C_{D_{\alpha_{VT}}} \alpha' \bar{q}_e S \sin \alpha' \approx C_{L_{\alpha_{VT}}} \frac{r l_{VT}}{(U_e)_b} \bar{q}_e S \quad (B.18)
 \end{aligned}$$

Hence, the side force derivative due to yaw rate is:

$$(Y_r)_b = C_{L_{\alpha_{VT}}} \frac{l_{VT}}{m (U_e)_b} \bar{q}_e S \quad (B.19)$$

The following part is to derive the relationship of wing – body rolling moment and yawing moment due to roll rate perturbation p and yaw rate perturbation r in stability axes system because the moment contribution from the wing is easier to derive in stability axes system. Then the results will be transformed to body axes system. For the contribution from the vertical and horizontal tail, it will be derived directly in body axes system, then all the results will be combined together to produce the final derivatives in body axes system.

Referring to Figure B.3 below, let's consider a pair of symmetric chord-wise strip elements of the wing. Each pair of elements has the following characteristics:

- Located at a distance y from the center line
- Length of c_y (chord at y), width of ∂y
- Lift curve slope of $C_{L_{\alpha_y}}$ and drag curve slope of $C_{D_{\alpha_y}}$

Figure B.3: Change in wing's angle of attack in rolling flight (26)

When the aircraft experiences a positive roll rate perturbation p , the right wing strip element will have an increment in angle of attack of:

$$\alpha \approx \tan \alpha = \frac{py}{U_e} \quad (\text{B.20})$$

Lift and drag in steady state are the same for both right wing and left wing strip elements:

$$L_e = \frac{1}{2} \rho U_e^2 c_y dy C_{L_{\alpha_y}} \alpha_e \quad (\text{B.21})$$

$$D_e = \frac{1}{2} \rho U_e^2 c_y dy C_{D_{\alpha_y}} \alpha_e \quad (\text{B.22})$$

And in perturbed state, lift and drag from right wing strip element become:

$$(L_1)_r = \frac{1}{2} \rho U_e^2 c_y dy C_{L_{\alpha_y}} (\alpha_e + \alpha) \quad (\text{B.23})$$

$$(D_1)_r = \frac{1}{2} \rho U_e^2 c_y dy C_{D_{\alpha_y}} (\alpha_e + \alpha) \quad (\text{B.24})$$

For the opposite strip element in the left wing, there is a decrement of α in angle of attack, hence, the perturbed state lift and drag from the left wing strip element are:

$$(L_1)_l = \frac{1}{2} \rho U_e^2 c_y dy C_{L_{\alpha_y}} (\alpha_e - \alpha) \quad (\text{B.25})$$

$$(D_1)_l = \frac{1}{2} \rho U_e^2 c_y dy C_{D_{\alpha_y}} (\alpha_e - \alpha) \quad (\text{B.26})$$

Changes in normal force and axial force due to the right wing strip element are:

$$\begin{aligned} (\delta Z_1)_r &= -[(L_1)_r \cos \alpha + (D_1)_r \sin \alpha - L_e] \\ &\approx -\frac{1}{2} \rho U_e^2 c_y dy [C_{L_{\alpha_y}} (\alpha_e + \alpha) + C_{D_{\alpha_y}} (\alpha_e + \alpha) \alpha - C_{L_{\alpha_y}} \alpha_e] \\ &\approx -\frac{1}{2} \rho U_e^2 c_y dy [C_{L_{\alpha_y}} \alpha + C_{D_{\alpha_y}} (\alpha_e + \alpha) \alpha] \\ &\approx -\frac{1}{2} \rho U_e^2 c_y dy (C_{L_{\alpha_y}} \alpha + C_{D_{\alpha_y}} \alpha_e \alpha) \end{aligned} \quad (\text{B.27})$$

$$\begin{aligned}
 (\delta X_1)_r &= -[(D_1)_r \cos \alpha - (L_1)_r \sin \alpha - D_e] \\
 &\approx -\frac{1}{2} \rho U_e^2 c_y dy [C_{D_{\alpha_y}} (\alpha_e + \alpha) - C_{L_{\alpha_y}} (\alpha_e + \alpha) \alpha - C_{D_{\alpha_y}} \alpha_e] \\
 &\approx -\frac{1}{2} \rho U_e^2 c_y dy [C_{D_{\alpha_y}} \alpha - C_{L_{\alpha_y}} (\alpha_e + \alpha) \alpha] \\
 &\approx -\frac{1}{2} \rho U_e^2 c_y dy (C_{D_{\alpha_y}} \alpha - C_{L_{\alpha_y}} \alpha_e \alpha)
 \end{aligned}
 \tag{B.28}$$

Similarly, changes in normal force and axial force due to the left wing strip element are:

$$\begin{aligned}
 (\delta Z_1)_l &= -[(L_1)_l \cos \alpha - (D_1)_l \sin \alpha - L_e] \\
 &\approx -\frac{1}{2} \rho U_e^2 c_y dy [C_{L_{\alpha_y}} (\alpha_e - \alpha) - C_{D_{\alpha_y}} (\alpha_e - \alpha) \alpha - C_{L_{\alpha_y}} \alpha_e] \\
 &\approx \frac{1}{2} \rho U_e^2 c_y dy (C_{L_{\alpha_y}} \alpha + C_{D_{\alpha_y}} \alpha_e \alpha)
 \end{aligned}
 \tag{B.29}$$

$$\begin{aligned}
 (\delta X_1)_l &= -[(D_1)_l \cos \alpha + (L_1)_l \sin \alpha - D_e] \\
 &\approx -\frac{1}{2} \rho U_e^2 c_y dy [C_{D_{\alpha_y}} (\alpha_e - \alpha) + C_{L_{\alpha_y}} (\alpha_e - \alpha) \alpha - C_{D_{\alpha_y}} \alpha_e] \\
 &\approx -\frac{1}{2} \rho U_e^2 c_y dy (-C_{D_{\alpha_y}} \alpha + C_{L_{\alpha_y}} \alpha_e \alpha)
 \end{aligned}
 \tag{B.30}$$

Change in normal force will change rolling moment, and similarly, change in axial force will change yawing moment. Rolling moment increment is given by:

$$\begin{aligned}
 \delta l_{A_1} &= (\delta Z_1)_r y_r + (\delta Z_1)_l y_l = (\delta Z_1)_r y_r - (\delta Z_1)_l y_r \\
 &= [(\delta Z_1)_r - (\delta Z_1)_l] y = -\rho U_e^2 c_y dy (C_{L_{\alpha_y}} + C_{D_{\alpha_y}} \alpha_e) \alpha y \\
 &= -\rho U_e^2 c_y dy (C_{L_{\alpha_y}} + C_{D_{\alpha_y}} \alpha_e) \frac{py}{U_e} y \\
 &= -\rho U_e p (C_{L_{\alpha_y}} + C_{D_{\alpha_y}} \alpha_e) y^2 c_y dy
 \end{aligned}
 \tag{B.31}$$

Thus, the total rolling moment in stability axes system due to roll rate perturbation is:

$$\begin{aligned}
 (l_{A_1})_s &= \int \delta l_{A_1} = -\rho U_e p \int_0^s (C_{L_{\alpha_y}} + C_{D_{\alpha_y}} \alpha_e) y^2 c_y dy \\
 &\approx -\rho U_e p (C_{L_{\alpha_e}} + C_{D_e}) \int_0^s y^2 c_y dy
 \end{aligned}
 \tag{B.32}$$

Yawing moment increment due to change in axial force from a pair of chord-wise strip elements is:

$$\begin{aligned}
 \delta n_{A_1} &= -(\delta X_1)_r y_r - (\delta X_1)_l y_l = -(\delta X_1)_r y_r + (\delta X_1)_l y_r \\
 &= -[(\delta X_1)_r - (\delta X_1)_l] y = \rho U_e^2 c_y dy (C_{D_{\alpha_y}} - C_{L_{\alpha_y}} \alpha_e) \alpha y \\
 &= \rho U_e^2 c_y dy (C_{D_{\alpha_y}} - C_{L_{\alpha_y}} \alpha_e) \frac{py}{U_e} y \\
 &= \rho U_e p (C_{D_{\alpha_y}} - C_{L_{\alpha_y}} \alpha_e) y^2 c_y dy
 \end{aligned}
 \tag{B.33}$$

Then the total yawing moment due to roll rate is given by:

$$\begin{aligned}
 (n_{A_1})_s &= \int \delta n_{A_1} = \rho U_e p \int_0^s (C_{D_{\alpha_y}} - C_{L_{\alpha_y}} \alpha_e) y^2 c_y dy \\
 &\approx \rho U_e p (C_{D_{\alpha_e}} - C_{L_e}) \int_0^s y^2 c_y dy
 \end{aligned}
 \tag{B.34}$$

Referring to Figure B.4 below, let's consider a pair of symmetric chord-wise strip elements of the wing. Each pair of elements has the following characteristics:

- Located at a distance y from the center line
- Length of c_y (chord at y), width of ∂y
- Lift coefficient of C_{L_y} and drag coefficient of C_{D_y}

Figure B.4: Change in wind speed due to yaw rate (26)

When the aircraft experiences a positive yaw rate perturbation r , the right wing strip element will have a decrement in wind speed of ry , and the resultant wind speed at that element is:

$$(V)_r = U_e - ry \quad (\text{B.35})$$

Lift and drag in steady state for each wing strip element were calculated in Equations (B.21) and (B.22). And in perturbed state, lift and drag from right wing strip element become:

$$(L_2)_r = \frac{1}{2} \rho (V)_r^2 c_y dy C_{L_y} \approx \frac{1}{2} \rho (U_e^2 - 2U_e ry) c_y dy C_{L_y} \quad (\text{B.36})$$

$$(D_2)_r = \frac{1}{2} \rho (V)_r^2 c_y dy C_{D_y} \approx \frac{1}{2} \rho (U_e^2 - 2U_e ry) c_y dy C_{D_y} \quad (\text{B.37})$$

For the opposite strip element in the left wing, there is an increment of ry in wind speed, hence, the perturbed state lift and drag from the left wing strip element are:

$$(L_2)_l = \frac{1}{2} \rho (V)_l^2 c_y dy C_{L_y} \approx \frac{1}{2} \rho (U_e^2 + 2U_e ry) c_y dy C_{L_y} \quad (\text{B.38})$$

$$(D_2)_l = \frac{1}{2} \rho (V)_l^2 c_y dy C_{D_y} \approx \frac{1}{2} \rho (U_e^2 + 2U_e ry) c_y dy C_{D_y} \quad (\text{B.39})$$

Changes in normal for and axial force at these chord-wise strip elements are:

$$\begin{aligned} (\delta Z_2)_r &= -[(L_2)_r - L_e] \\ &= -\frac{1}{2} \rho (U_e^2 - 2U_e ry) c_y dy C_{L_y} + \frac{1}{2} \rho U_e^2 c_y dy C_{L_y} \\ &= \rho U_e r C_{L_y} c_y y dy \end{aligned} \quad (\text{B.40})$$

$$\begin{aligned}
 (\delta X_2)_r &= -[(D_2)_r - D_e] \\
 &= -\frac{1}{2}\rho(U_e^2 - 2U_e r y)c_y dy C_{D_y} + \frac{1}{2}\rho U_e^2 c_y dy C_{D_y} \\
 &= \rho U_e r C_{D_y} c_y y dy
 \end{aligned} \tag{B.41}$$

$$\begin{aligned}
 (\delta Z_2)_l &= -[(L_2)_l - L_e] \\
 &= -\frac{1}{2}\rho(U_e^2 + 2U_e r y)c_y dy C_{L_y} + \frac{1}{2}\rho U_e^2 c_y dy C_{L_y} \\
 &= -\rho U_e r C_{L_y} c_y y dy
 \end{aligned} \tag{B.42}$$

$$\begin{aligned}
 (\delta X_2)_l &= -[(D_2)_l - D_e] \\
 &= -\frac{1}{2}\rho(U_e^2 + 2U_e r y)c_y dy C_{D_y} + \frac{1}{2}\rho U_e^2 c_y dy C_{D_y} \\
 &= -\rho U_e r C_{D_y} c_y y dy
 \end{aligned} \tag{B.43}$$

Rolling moment increment is caused by change of normal force, and it is given by:

$$\begin{aligned}
 \delta l_{A_2} &= (\delta Z_2)_r y_r + (\delta Z_2)_l y_l = (\delta Z_2)_r y_r - (\delta Z_2)_l y_r \\
 &= [(\delta Z_2)_r - (\delta Z_2)_l] y = 2\rho U_e r C_{L_y} c_y y^2 dy
 \end{aligned} \tag{B.44}$$

Thus, resultant rolling moment due to yaw rate is:

$$(l_{A_2})_s = \int \delta l_{A_2} = 2\rho U_e r \int_0^s C_{L_y} c_y y^2 dy \approx 2\rho U_e C_{L_e} r \int_0^s c_y y^2 dy \tag{B.45}$$

Yawing moment increment is caused by change in axial force, and it is given by:

$$\begin{aligned}
 \delta n_{A_2} &= -(\delta X_2)_r y_r - (\delta X_2)_l y_l = -(\delta X_2)_r y_r + (\delta X_2)_l y_r \\
 &= -[(\delta X_2)_r - (\delta X_2)_l] y = -2\rho U_e r C_{D_y} c_y y^2 dy
 \end{aligned} \tag{B.46}$$

Then the resultant yawing moment due to yaw rate is:

$$(n_{A_2})_s = \int \delta n_{A_2} = -2\rho U_e r \int_0^s C_{D_y} c_y y^2 dy \approx -2\rho U_e C_{D_e} r \int_0^s c_y y^2 dy \tag{B.47}$$

Total rolling moment due to roll rate and yaw rate is:

$$\begin{aligned} (l_A)_s &= (l_{A_1})_s + (l_{A_2})_s \\ &= -\rho(U_e)_s \int_0^s c_y y^2 dy \left[(C_{L_{\alpha_e}} + C_{D_e})(p)_s - 2C_{L_e}(r)_s \right] \end{aligned} \quad (\text{B.48})$$

And total yawing moment due to roll rate and yaw rate is:

$$\begin{aligned} (n_A)_s &= (n_{A_1})_s + (n_{A_2})_s \\ &= \rho(U_e)_s \int_0^s c_y y^2 dy \left[(C_{D_{\alpha_e}} - C_{L_e})(p)_s - 2C_{D_e}(r)_s \right] \end{aligned} \quad (\text{B.49})$$

Rolling and yawing moments contributed by the wing – body have been derived in stability axes system. The next step is to transform them to body axes. Rolling and yawing moments in stability axes system are transformed to body axes system by applying the relationship in Equation (4.7):

$$\begin{aligned} (l_A)_b &= (l_A)_s \cos \alpha_e - (n_A)_s \sin \alpha_e \\ &= -\rho(U_e)_s \int_0^s c_y y^2 dy \left\{ \left[(C_{L_{\alpha_e}} + C_{D_e})(p)_s - 2C_{L_e}(r)_s \right] \cos \alpha_e \right. \\ &\quad \left. + \left[(C_{D_{\alpha_e}} - C_{L_e})(p)_s - 2C_{D_e}(r)_s \right] \sin \alpha_e \right\} \end{aligned} \quad (\text{B.50})$$

$$\begin{aligned} (n_A)_b &= (l_A)_s \sin \alpha_e + (n_A)_s \cos \alpha_e \\ &= \rho(U_e)_s \int_0^s c_y y^2 dy \left\{ \left[(C_{D_{\alpha_e}} - C_{L_e})(p)_s - 2C_{D_e}(r)_s \right] \cos \alpha_e \right. \\ &\quad \left. - \left[(C_{L_{\alpha_e}} + C_{D_e})(p)_s - 2C_{L_e}(r)_s \right] \sin \alpha_e \right\} \end{aligned} \quad (\text{B.51})$$

Applying the Equation (4.7), roll rate and yaw rate in stability axes system are resolved into body axes system as:

$$(p)_s = (p)_b \cos \alpha_e + (r)_b \sin \alpha_e \quad (\text{B.52})$$

$$(r)_s = -(p)_b \sin \alpha_e + (r)_b \cos \alpha_e \quad (\text{B.53})$$

Substituting Equations (B.52) and (B.53) to Equation (B.50) and (B.51) and rearranging them lead to:

$$(l_A)_b = -\rho(U_e)_s \int_0^s c_y y^2 dy \times \left\{ \left[(C_{L_{\alpha_e}} + C_{D_e}) c^2 \alpha_e + (C_{D_{\alpha_e}} + C_{L_e}) s \alpha_e c \alpha_e + 2C_{D_e} s^2 \alpha_e \right] (p)_b \right. \\ \left. + \left[-2C_{L_e} c^2 \alpha_e + (C_{L_{\alpha_e}} - C_{D_e}) s \alpha_e c \alpha_e + (C_{D_{\alpha_e}} - C_{L_e}) s^2 \alpha_e \right] (r)_b \right\} \quad (B.54)$$

$$(n_A)_b = \rho(U_e)_s \int_0^s c_y y^2 dy \times \left\{ \left[(C_{D_{\alpha_e}} - C_{L_e}) c^2 \alpha_e + (C_{D_e} - C_{L_{\alpha_e}}) s \alpha_e c \alpha_e - 2C_{L_e} s^2 \alpha_e \right] (p)_b \right. \\ \left. + \left[-2C_{D_e} c^2 \alpha_e + (C_{D_{\alpha_e}} + C_{L_e}) s \alpha_e c \alpha_e - (C_{L_{\alpha_e}} + C_{D_e}) s^2 \alpha_e \right] (r)_b \right\} \quad (B.55)$$

The notation $c \alpha_e$ and $s \alpha_e$ stand for $\cos \alpha_e$ and $\sin \alpha_e$ respectively. It should be reminded that these moments are only contributed by the wing – body. The contribution of the tails to the rolling moment due to roll rate perturbation is insignificant because their span is relatively small. However, their contribution to yawing moment due to yaw rate perturbation needs to be considered, and it can be seen in Figure B.2 that most of this contribution is from the vertical tail due to the side force generated in yawing motion. Yawing moment from vertical tail due to yaw rate perturbation is:

$$n_{A_{VT}} = -Y_{VT} l_{VT} = -C_{L_{\alpha_{VT}}} \frac{r l_{VT}^2}{(U_e)_b} \bar{q}_e S \quad (B.56)$$

Then the total yawing moment is now changed to:

$$(n_A)_b = -C_{L_{\alpha_{VT}}} \frac{l_{VT}^2}{(U_e)_b} \bar{q}_e S (r)_b + \rho(U_e)_s \int_0^s c_y y^2 dy \times \left\{ \left[(C_{D_{\alpha_e}} - C_{L_e}) c^2 \alpha_e + (C_{D_e} - C_{L_{\alpha_e}}) s \alpha_e c \alpha_e - 2C_{L_e} s^2 \alpha_e \right] (p)_b \right. \\ \left. + \left[-2C_{D_e} c^2 \alpha_e + (C_{D_{\alpha_e}} + C_{L_e}) s \alpha_e c \alpha_e - (C_{L_{\alpha_e}} + C_{D_e}) s^2 \alpha_e \right] (r)_b \right\} \quad (B.57)$$

The rolling moment derivatives with respect to roll rate and yaw rate in body axes system can be simply obtained as:

$$(L_p)_b = -\frac{\rho(U_e)_s}{I_{xx}} \int_0^s c_y y^2 dy \left[\begin{aligned} &(C_{L_{\alpha_e}} + C_{D_e}) \cos^2 \alpha_e + 2C_{D_e} \sin^2 \alpha_e \\ &+ (C_{D_{\alpha_e}} + C_{L_e}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (\text{B.58})$$

$$(L_r)_b = -\frac{\rho(U_e)_s}{I_{xx}} \int_0^s c_y y^2 dy \left[\begin{aligned} &-2C_{L_e} \cos^2 \alpha_e + (C_{D_{\alpha_e}} - C_{L_e}) \sin^2 \alpha_e \\ &+ (C_{L_{\alpha_e}} - C_{D_e}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (\text{B.59})$$

And the yawing moment derivatives with respect to roll rate and yaw rate are:

$$(N_p)_b = \frac{\rho(U_e)_s}{I_{zz}} \int_0^s c_y y^2 dy \left[\begin{aligned} &(C_{D_{\alpha_e}} - C_{L_e}) \cos^2 \alpha_e - 2C_{L_e} \sin^2 \alpha_e \\ &+ (C_{D_e} - C_{L_{\alpha_e}}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (\text{B.60})$$

$$(N_r)_b = \frac{\rho(U_e)_s}{I_{zz}} \int_0^s c_y y^2 dy \left[\begin{aligned} &-2C_{D_e} \cos^2 \alpha_e - (C_{L_{\alpha_e}} + C_{D_e}) \sin^2 \alpha_e \\ &+ (C_{D_{\alpha_e}} + C_{L_e}) \sin \alpha_e \cos \alpha_e \end{aligned} \right] \quad (\text{B.61})$$

$$-C_{L_{\alpha_{VT}}} \frac{l_{VT}^2}{I_{zz}(U_e)_b} \bar{q}_e S$$

B4. DERIVATIVES WITH RESPECT TO VERTICAL TAIL DEFLECTION

The side force due to the vertical tail deflection is actually the lift generated by the vertical tail, thus, the side force derivative due to vertical tail deflection is:

$$Y_{\delta_{VT}} = \frac{C_{L_{\delta_{VT}}} \bar{q}_e S}{m} \quad (\text{B.62})$$

This derivative is already derived in body axes systems because the air stream coming to the vertical tail is in the body X axis' direction.

The yawing moment derivative due to vertical tail's deflection is simply the side force derivative due to vertical tail deflection multiplied by the distance from cg to vertical tail's aerodynamic center, and it is also in body axes system:

$$\left(N_{\delta_{VT}}\right)_b = -\frac{C_{L_{\alpha_{VT}}} \bar{q}_e S l_{VT}}{I_{zz}} \quad (\text{B.63})$$

The minus sign means, a positive side force from the tail will generate a negative yawing moment, and vice versa. Obviously, the vertical tail deflection does not have any effect on the rolling moment; hence, there is no rolling moment derivative due to vertical tail's deflection.

B5. DERIVATIVES WITH RESPECT TO STATORS DEFLECTION

In body axes system, the statoms are arranged in asymmetric configuration; hence, change in statoms deflection only affects rolling moment. The rolling moment derivative with respect to statoms deflection in body axes system is given by:

$$\left(L_{\delta_s}\right)_b = \frac{\left(C_{l_{\delta_s}}\right)_b \bar{q}_e S b}{I_{xx}} \quad (\text{B.64})$$

Due to the asymmetric configuration of the statoms, it is obvious that statoms deflection does not have any effect on the yawing moment and side force.

Appendix C: DETAILED CALCULATIONS OF STABILITY AND CONTROL DERIVATIVES

C1. LONGITUDINAL DERIVATIVES

C1.1. In stability axes system

$$\begin{aligned} \left(X_u\right)_s &= -\frac{2C_{D_e} \bar{q}_e S}{mU_e} \\ &= -\frac{2 \left[C_{D_{WB_e}} + C_{L_{HT_e}} n^2 \sin \alpha_e \right. \\ &\quad \left. + \left(C_{D_{S_e}} + C_{D_{VT_e}} + C_{D_{HT_e}} \right) n^2 \cos \alpha_e \right] \bar{q}_e S}{mU_e} = -0.5638 \end{aligned}$$

$$(Z_u)_s = -\frac{2C_{L_e}\bar{q}_e S}{mU_e}$$

$$= -\frac{2\left[C_{L_{WB_e}} + C_{L_{HT_e}} n^2 \cos \alpha_e - (C_{D_{HT_e}} + C_{D_{VT_e}} + C_{D_{S_e}}) \sin \alpha_e\right] \bar{q}_e S}{mU_e} = -1.2435$$

$$(X_w)_s = \frac{(X_\alpha)_s}{U_e} = -\frac{(C_{D_{\alpha_e}} - C_{L_e}) \bar{q}_e S}{mU_e}$$

$$= -\frac{\left\{C_{D_{\alpha_{WB_e}}} - \left[C_{L_{WB_e}} + C_{L_{HT_e}} n^2 \cos \alpha_e - (C_{D_{HT_e}} + C_{D_{VT_e}} + C_{D_{S_e}}) \sin \alpha_e\right]\right\} \bar{q}_e S}{mU_e} = -0.4543$$

$$(Z_w)_s = \frac{(Z_\alpha)_s}{U_e} = -\frac{(C_{L_{\alpha_e}} + C_{D_e}) \bar{q}_e S}{mU_e}$$

$$= -\frac{\left\{C_{L_{\alpha_{WB_e}}} + \left[C_{D_{WB_e}} + C_{L_{HT_e}} n^2 \sin \alpha_e + (C_{D_{S_e}} + C_{D_{VT_e}} + C_{D_{HT_e}}) n^2 \cos \alpha_e\right]\right\} \bar{q}_e S}{mU_e} = -3.8691$$

$$(M_w)_s = \frac{(M_\alpha)_s}{U_e} = \frac{C_{m_{\alpha_e}} \bar{q}_e S b}{I_{yy} U_e} = \frac{C_{m_{\alpha_{WB_e}}} \bar{q}_e S b}{I_{yy} U_e} = -6.8852$$

$$(X_q)_s = -\frac{(C_{D_{\alpha_{HT_e}}} \cos \alpha_e + C_{L_{\alpha_{HT_e}}} \sin \alpha_e) n^2 l_{HT} \bar{q}_e S}{mU_e} = -0.0156$$

$$(Z_q)_s = -\frac{(C_{L_{\alpha_{HT_e}}} \cos \alpha_e - C_{D_{\alpha_{HT_e}}} \sin \alpha_e) n^2 l_{HT} \bar{q}_e S}{mU_e} = -0.1499$$

$$(M_q)_s = -\frac{C_{L_{\alpha_{HT_e}}} l_{HT}^2 \bar{q}_e S}{I_{yy} U_e} = -4.043$$

$$\left(X_{\delta_{HT}}\right)_s = -\frac{\left(C_{D_{\alpha_{HT_e}}} \cos \alpha_e + C_{L_{\alpha_{HT_e}}} \sin \alpha_e\right) n^2 \bar{q}_e S}{m} = -0.9056$$

$$\left(Z_{\delta_{HT}}\right)_s = -\frac{\left(C_{L_{\alpha_{HT_e}}} \cos \alpha_e - C_{D_{\alpha_{HT_e}}} \sin \alpha_e\right) n^2 \bar{q}_e S}{m} = -8.7172$$

$$\left(M_{\delta_{HT}}\right)_s = \frac{C_{m_{\alpha_{HT}}} n^2 \bar{q}_e S b}{I_{yy}} = -254.3564$$

C1.2. In body axes system

To transform the derivatives from stability axes system to body axes system, relationship in Equations (4.21) to (4.23) was recalled, and then the derivatives in body axes system are calculated as:

$$\left(X_u\right)_b = \left(X_u\right)_s \cos^2 \alpha_e - \left(X_w + Z_u\right)_s \sin \alpha_e \cos \alpha_e + \left(Z_w\right)_s \sin^2 \alpha_e = -0.3765$$

$$\left(X_w\right)_b = \left(X_w\right)_s \cos^2 \alpha_e + \left(X_u - Z_w\right)_s \sin \alpha_e \cos \alpha_e - \left(Z_u\right)_s \sin^2 \alpha_e = 0.137$$

$$\left(X_q\right)_b = \left(X_q\right)_s \cos \alpha_e - \left(Z_q\right)_s \sin \alpha_e = 0.0097$$

$$\left(X_{\delta_{HT}}\right)_b = \left(X_{\delta_{HT}}\right)_s \cos \alpha_e - \left(Z_{\delta_{HT}}\right)_s \sin \alpha_e = 0.5622$$

$$\left(Z_u\right)_b = \left(Z_u\right)_s \cos^2 \alpha_e - \left(Z_w - X_u\right)_s \sin \alpha_e \cos \alpha_e - \left(X_w\right)_s \sin^2 \alpha_e = -0.6522$$

$$\left(Z_w\right)_b = \left(Z_w\right)_s \cos^2 \alpha_e + \left(Z_u + X_w\right)_s \sin \alpha_e \cos \alpha_e + \left(X_u\right)_s \sin^2 \alpha_e = -4.0564$$

$$\left(Z_q\right)_b = \left(Z_q\right)_s \cos \alpha_e + \left(X_q\right)_s \sin \alpha_e = -0.1504$$

$$\left(Z_{\delta_{HT}}\right)_b = \left(Z_{\delta_{HT}}\right)_s \cos \alpha_e + \left(X_{\delta_{HT}}\right)_s \sin \alpha_e = -8.7461$$

$$\left(M_u\right)_b = \left(M_u\right)_s \cos \alpha_e - \left(M_w\right)_s \sin \alpha_e = -0.0758$$

$$\left(M_w\right)_b = \left(M_u\right)_s \sin \alpha_e + \left(M_w\right)_s \cos \alpha_e = -6.7886$$

$$\left(M_q\right)_q = \left(M_q\right)_s = -4.043$$

$$\left(M_{\delta_{HT}}\right)_q = \left(M_{\delta_{HT}}\right)_s = -254.3564$$

C2. LATERAL – DIRECTIONAL DERIVATIVES

$$(Y_\beta)_b = \frac{C_{y_\beta} \bar{q}_e S}{m} = -34.7894$$

$$(L_\beta)_b = \frac{(C_{l_\beta} \bar{q}_e S b)_s \cos \alpha_e - (C_{n_\beta} \bar{q}_e S b)_s \sin \alpha_e}{I_{xx}} = -28.0786$$

$$(N_\beta)_b = \frac{(C_{l_\beta} \bar{q}_e S b)_s \sin \alpha_e + (C_{n_\beta} \bar{q}_e S b)_s \cos \alpha_e}{I_{zz}} = 5.1798$$

$$(L_p)_b = -\frac{\rho(U_e)_s}{I_{xx}} \int_0^s c_y y^2 dy \left[(C_{L_{\alpha_e}} + C_{D_e}) \cos^2 \alpha_e + 2C_{D_e} \sin^2 \alpha_e \right. \\ \left. + (C_{D_{\alpha_e}} + C_{L_e}) \sin \alpha_e \cos \alpha_e \right] \\ = -15.2959$$

$$(N_p)_b = \frac{\rho(U_e)_s}{I_{zz}} \int_0^s c_y y^2 dy \left[(C_{D_{\alpha_e}} - C_{L_e}) \cos^2 \alpha_e - 2C_{L_e} \sin^2 \alpha_e \right. \\ \left. + (C_{D_e} - C_{L_{\alpha_e}}) \sin \alpha_e \cos \alpha_e \right] \\ = -0.2313$$

$$(Y_r)_b = C_{L_{\alpha_{VT}}} \frac{l_{VT}}{m(U_e)_b} n^2 \bar{q}_e S = 0.1465$$

$$(L_r)_b = -\frac{\rho(U_e)_s}{I_{xx}} \int_0^s c_y y^2 dy \left[-2C_{L_e} \cos^2 \alpha_e + (C_{D_{\alpha_e}} - C_{L_e}) \sin^2 \alpha_e \right. \\ \left. + (C_{L_{\alpha_e}} - C_{D_e}) \sin \alpha_e \cos \alpha_e \right] \\ = 2.8508$$

$$(N_r)_b = \frac{\rho(U_e)_s}{I_{zz}} \int_0^s c_y y^2 dy \left[-2C_{D_e} \cos^2 \alpha_e - (C_{L_{\alpha_e}} + C_{D_e}) \sin^2 \alpha_e \right. \\ \left. + (C_{D_{\alpha_e}} + C_{L_e}) \sin \alpha_e \cos \alpha_e \right] \\ - C_{L_{\alpha_{VT}}} \frac{l_{VT}^2}{I_{zz}(U_e)_b} \bar{q}_e S \\ = -3.0209$$

$$\left(Y_{\delta_{VT}}\right)_b = \frac{C_{L_{\alpha_{VT}}} n^2 \bar{q}_e S}{m} = 8.7461$$

$$\left(N_{\delta_{VT}}\right)_b = -\frac{C_{L_{\alpha_{VT}}} n^2 \bar{q}_e S l_{VT}}{I_{zz}} = -175.7121$$

$$\left(L_{\delta_S}\right)_b = \frac{\left(C_{l_{\delta_S}}\right)_b n^2 \bar{q}_e S b}{I_{xx}} = -189.3693$$

Appendix D: : MAPLE CODE

D1. FINDING FORWARD FLIGHT'S STEADY-STATE CONDITIONS

$T1 := 5;$

5

$Ve := 15;$

15

$vtail := evalf(Ve \cdot \cos(0) + 5.0975 \cdot \sqrt{T1});$

26.3983565

$T2 := 0;$

0

$T3 := 0;$

0

$qS := 0.5 \cdot 1.225 \cdot Ve^2 \cdot 0.076$

10.4737500

$qSb := qS \cdot 0.43$

4.503712500

while $|T2 - T1| > 0.00001$ **do**:

$$n := \frac{v_{tail}}{V_e};$$

$$delt1 := -\frac{0.1009 \cdot alph1 + 0.0019}{0.1631 \cdot n^2};$$

$$eq1 := expand \left(T1 \cdot alph1 + (2.3565 \cdot alph1 + 0.0492) \cdot qS \right. \\ \left. + (0.2514 \cdot delt1) \cdot qS \cdot n^2 \cdot \left(1 - \frac{alph1^2}{2} \right) - (0.1629 \cdot delt1^2 \right. \\ \left. + 0.0087) \cdot qS \cdot n^2 \cdot alph1 - (0.0087 \cdot qS \cdot n^2 \cdot alph1) - (0.2557 \right. \\ \left. \cdot (0.0888 \cdot T1)^2 + 0.0371) \cdot qS \cdot n^2 \cdot alph1 - 4.5 = 0 \right);$$

$solu := solve(eq1);$

$alph2 := solu[1];$

$delt2 := subs(alph1 = alph2, delt1);$

$$T3 := \frac{1}{\cos(alph2)} \left(((2.3422 \cdot alph2^2 - 0.0787 \cdot alph2 + 0.034) \cdot qS \right. \\ \left. + (0.2514 \cdot delt2) \cdot qS \cdot n^2 \cdot \sin(alph2) + (0.1629 \cdot delt2^2 \right. \\ \left. + 0.0087) \cdot qS \cdot n^2 \cdot \cos(alph2) + (0.0087 \cdot qS \cdot n^2 \cdot \cos(alph2)) \right. \\ \left. + (0.2557 \cdot (0.0888 \cdot T1)^2 + 0.0371) \cdot qS \cdot n^2 \cdot \cos(alph2) \right);$$

$T2 := T1;$

$T1 := T3;$

$v_{tail} := evalf(V_e \cdot \cos(alph2) + 5.0975 \cdot \sqrt{T1});$

end do:

$alph2; delt2$

0.167676965

-0.0495942922

$deltS := 0.0888 \cdot T2;$

0.223643947

$T1; T2; T3;$

2.51850740

2.51851292

2.51850740

n

1.52528462

D2. FINDING TURNING FLIGHT'S STEADY-STATE CONDITIONS

$$m := \frac{4.5}{9.81};$$

$$0.458715596;$$

$$Pe := -0.2 \cdot (alph);$$

$$-0.2 \, alph$$

$$Qe := 0.2 \cdot (ph);$$

$$0.2 \, ph$$

$$Ree := 0.2;$$

$$0.2$$

$$Ue := 15;$$

$$15$$

$$Ve := 15 \cdot (bet);$$

$$15 \, bet$$

$$We := 15 \cdot (alph);$$

$$15 \, alph$$

$$Vinf := 15;$$

$$15$$

$$vtail := evalf(Vinf \cdot \cos(0.1836258348 + 5.0975 \cdot \sqrt{2.4622}));$$

$$22.7465120;$$

$$qS := 0.5 \cdot 1.225 \cdot Vinf^2 \cdot 0.076$$

$$10.4737500$$

$$qSb := qS \cdot 0.43$$

$$4.503712500$$

$$n := \frac{vtail}{Vinf};$$

1.51643413:

$$\begin{aligned}
 eq1 := m \cdot (-Ve \cdot Ree + We \cdot Qe) + 4.5 \cdot (alph) = Te + (2.3565 \cdot alph \\
 + 0.0492) \cdot qS \cdot (alph) - (2.3422 \cdot alph^2 - 0.0787 \cdot alph + 0.034) \\
 \cdot qS - (0.1629 \cdot deltHT^2 + 0.0087) \cdot qS \cdot n^2 - (0.1629 \cdot deltVT^2 \\
 + 0.0087) \cdot qS \cdot n^2 - (0.2557 \cdot deltS^2 + 0.0371) \cdot qS \cdot n^2 \\
 - 1.376146789bet + 1.376146789alph \ ph + 4.5 \ alph = Te \\
 + 10.4737500(2.3565 \ alph + 0.0492) \ alph - 24.53161725alph^2 \\
 + 0.8242841250alph - 1.668748030 - 3.923470500deltHT^2 \\
 - 3.923470500deltVT^2 - 6.158572172deltS^2
 \end{aligned}$$

$$\begin{aligned}
 eq2 := m \cdot (Ue \cdot Ree - We \cdot Pe) - 4.5 \cdot (ph) = (-1.5236 \cdot bet) \cdot qS \\
 + (0.2514 \cdot deltVT) \cdot qS \cdot n^2 \\
 1.376146789 + 1.376146789alph^2 - 4.5 \ ph = -15.95780550bet \\
 + 6.055006039deltVT
 \end{aligned}$$

$$\begin{aligned}
 eq3 := m \cdot (-Ue \cdot Qe + Ve \cdot Pe) - 4.5 = -(2.3565 \cdot alph + 0.0492) \cdot qS \\
 - (2.3422 \cdot alph^2 - 0.0787 \cdot alph + 0.034) \cdot qS \cdot (alph) - (0.2514 \\
 \cdot deltHT) \cdot qS \cdot n^2 \\
 - 1.376146789ph - 1.376146789bet \ alph - 4.5 = -24.68139188alph \\
 - 0.5153085000 - 10.4737500(2.3422 \ alph^2 - 0.0787 \ alph \\
 + 0.034) \ alph - 6.055006039deltHT
 \end{aligned}$$

$$\begin{aligned}
 eq4 := (0.00566 - 0.00439) \cdot Ree \cdot Qe + 0.00007 \cdot Pe \cdot Qe = -0.0083 \\
 \cdot bet \cdot qSb - 0.0425 \cdot deltS \cdot qSb \cdot n^2 + Te \cdot 0.0175 \\
 0.0000508ph - 0.0000028alph \ ph = -0.03738081375bet \\
 - 0.4401560674deltS + 0.0175 \ Te
 \end{aligned}$$

$$\begin{aligned}
 eq5 := (0.00154 - 0.00566) \cdot Pe \cdot Ree - 0.00007 \cdot (Pe^2 - Ree^2) = \\
 - (0.1009 \cdot alph + 0.0019) \cdot qSb - 0.1631 \cdot deltHT \cdot qSb \cdot n^2 \\
 0.0001648alph - 0.0000028alph^2 + 0.0000028 = -0.4544245912alph \\
 - 0.008557053750 - 1.689163637deltHT
 \end{aligned}$$

$$\begin{aligned}
 eq6 := (0.00439 - 0.00154) \cdot Pe \cdot Qe - 0.00007 \cdot Qe \cdot Ree = (0.2508 \\
 \cdot bet^3 + 0.0153 \cdot bet) \cdot qSb - 0.2514 \cdot deltVT \cdot qS \cdot n^2 \cdot 0.2479
 \end{aligned}$$

$$-0.0001140\alpha ph - 0.0000028ph = 1.129531095bet^3 \\ + 0.06890680125bet - 1.501035997deltVT$$

$$eq7 := subs(ph = 0.2968, eq1);$$

$$-1.376146789bet + 4.908440367\alpha ph = Te \\ + 10.4737500(2.3565\alpha ph + 0.0492)\alpha ph - 24.53161725\alpha ph^2 \\ + 0.8242841250\alpha ph - 1.668748030 - 3.923470500deltHT^2 \\ - 3.923470500deltVT^2 - 6.158572172deltS^2$$

$$eq8 := subs(ph = 0.2968, eq2);$$

$$0.040546789 + 1.376146789\alpha ph^2 = -15.95780550bet \\ + 6.055006039deltVT$$

$$eq9 := subs(ph = 0.2968, eq3);$$

$$-4.908440367 - 1.376146789bet\alpha ph = -24.68139188\alpha ph \\ - 0.5153085000 - 10.4737500(2.3422\alpha ph^2 - 0.0787\alpha ph \\ + 0.034)\alpha ph - 6.055006039deltHT$$

$$eq10 := subs(ph = 0.2968, eq4);$$

$$0.00001507744 - 8.3104 \cdot 10^{-7}\alpha ph = -0.03738081375bet \\ - 0.4401560674deltS + 0.0175Te$$

$$eq11 := subs(ph = 0.2968, eq5);$$

$$0.0001648\alpha ph - 0.0000028\alpha ph^2 + 0.0000028 = -0.4544245912\alpha ph \\ - 0.008557053750 - 1.689163637deltHT$$

$$eq12 := subs(ph = 0.2968, eq6);$$

$$-0.00003383520\alpha ph - 8.3104 \cdot 10^{-7} = 1.129531095bet^3 \\ + 0.06890680125bet - 1.501035997deltVT$$

$\text{solve}(\{eq7, eq8, eq9, eq10, eq11, eq12\}, [Te, alph, bet, deltHT, deltVT, deltS])$

$$\begin{aligned} & [[Te = 47.40398810 + 88.42673996i, alph = 0.1669160365bet \\ & = -1.852548960deltHT = -0.04998807297deltVT \\ & = -4.869314453deltS = 2.042013520 + 3.515725589i], [Te \\ & = 47.40398810 - 88.42673996i, alph = 0.1669160365bet \\ & = -1.852548960deltHT = -0.04998807297deltVT \\ & = -4.869314453deltS = 2.042013520 - 3.515725589i], [Te \\ & = 2.394646264alph = 0.1836262495bet = -0.005543479637 \\ & deltHT = -0.05448513235deltVT = -0.0002499152568deltS \\ & = 0.09564472168], [Te = 100.3039075alph = 0.1836262495bet \\ & = -0.005543479637deltHT = -0.05448513235deltVT \\ & = -0.0002499152568deltS = 3.988382319], [Te = 55.33026822 \\ & + 81.65655406i, alph = 0.2038422376bet = 1.858176381deltHT \\ & = -0.05992566610deltVT = 4.913313884deltS = 2.042013520 \\ & + 3.246552307i], [Te = 55.33026822 - 81.65655406i, alph \\ & = 0.2038422376bet = 1.858176381deltHT = -0.05992566610 \\ & deltVT = 4.913313884deltS = 2.042013520 - 3.246552307i], [Te \\ & = 50.30618277 - 96.27463822i, alph = -0.06661219740 \\ & + 1.038513578i, bet = -1.899338194 - 0.005665503796i, deltHT \\ & = 0.01285745006 - 0.2794860322i, deltVT = -5.243067465 \\ & - 0.04637588580i, deltS = 2.161373984 - 3.827264121i], [Te \\ & = 44.30192764 + 96.25033602i, alph = -0.06661219740 \\ & + 1.038513578i, bet = -1.899338194 - 0.005665503796i, deltHT \\ & = 0.01285745006 - 0.2794860322i, deltVT = -5.243067465 \\ & - 0.04637588580i, deltS = 1.922653056 + 3.827264121i], [Te \\ & = 102.0304910 - 3.494148968i, alph = -0.07509361990 \\ & + 0.9856958412i, bet = 0.08234116057 + 0.01307791683i, \\ & deltHT = 0.01514015810 - 0.2652716866i, deltVT \\ & = 0.004167140418 + 0.0008210634838i, deltS = 4.049564700 \\ & - 0.1400313560i], [Te = 1.043538729 + 3.549925428i, alph = \\ & -0.07509361990 + 0.9856958412i, bet = 0.08234116057 \\ & + 0.01307791683i, deltHT = 0.01514015810 - 0.2652716866i, \\ & deltVT = 0.004167140418 + 0.0008210634838i, deltS \\ & = 0.03446234064 + 0.1400313560i], [Te = 56.18595874 \\ & - 73.16565880i, alph = -0.08508572613 + 0.9379067579i, bet \\ & = 1.816955063 - 0.007463327149i, deltHT = 0.01782939866 \\ & - 0.2524106603i, deltVT = 4.596951851 - 0.05594348995i, deltS \\ & = 2.079534826 - 2.908330384i], [Te = 54.29850383 \\ & + 73.13368569i, alph = -0.08508572613 + 0.9379067579i, bet \\ & = 1.816955063 - 0.007463327149i, deltHT = 0.01782939866 \\ & - 0.2524106603i, deltVT = 4.596951851 - 0.05594348995i, deltS \end{aligned}$$

$$\begin{aligned}
 &= 2.004492214 + 2.908330384i, [Te = 56.18595874 \\
 &+ 73.16565880i, alph = -0.08508572613 - 0.9379067579i, bet \\
 &= 1.816955063 + 0.007463327149i, deltHT = 0.01782939866 \\
 &+ 0.2524106603i, deltVT = 4.596951851 + 0.05594348995i, deltS \\
 &= 2.079534826 + 2.908330384i], [Te = 54.29850383 \\
 &- 73.13368569i, alph = -0.08508572613 - 0.9379067579i, bet \\
 &= 1.816955063 + 0.007463327149i, deltHT = 0.01782939866 \\
 &+ 0.2524106603i, deltVT = 4.596951851 + 0.05594348995i, deltS \\
 &= 2.004492214 - 2.908330384i], [Te = 102.0304910 \\
 &+ 3.494148968i, alph = -0.07509361990 - 0.9856958412i, bet \\
 &= 0.08234116057 - 0.01307791683i, deltHT = 0.01514015810 \\
 &+ 0.2652716866i, deltVT = 0.004167140418 \\
 &- 0.0008210634838i, deltS = 4.049564700 + 0.1400313560i], [Te \\
 &= 1.043538729 - 3.549925428i, alph = -0.07509361990 \\
 &- 0.9856958412i, bet = 0.08234116057 - 0.01307791683i, \\
 &deltHT = 0.01514015810 + 0.2652716866i, deltVT \\
 &= 0.004167140418 - 0.0008210634838i, deltS = 0.03446234064 \\
 &- 0.1400313560i], [Te = 50.30618277 + 96.27463822i, alph = \\
 &- 0.06661219740 - 1.038513578i, bet = -1.899338194 \\
 &+ 0.005665503796i, deltHT = 0.01285745006 + 0.2794860322i, \\
 &deltVT = -5.243067465 + 0.04637588580i, deltS = 2.161373984 \\
 &+ 3.827264121i], [Te = 44.30192764 - 96.25033602i, alph = \\
 &- 0.06661219740 - 1.038513578i, bet = -1.899338194 \\
 &+ 0.005665503796i, deltHT = 0.01285745006 + 0.2794860322i, \\
 &deltVT = -5.243067465 + 0.04637588580i, deltS = 1.922653056 \\
 &- 3.827264121i]]
 \end{aligned}$$