Off-track vibrations of head sliders in hard disk drives : simulation and a MEMs accelerometer for detection

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**NANYANG
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DISK DRIVES: SIMULATION AND A MEMS
ACCELEROMETER FOR DETECTION**

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YUAN YANHUI

2010

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2010**

OFF-TRACK VIBRATIONS OF HEAD SLIDERS IN HARD DISK DRIVES: SIMULATION AND A MEMS ACCELEROMETER FOR DETECTION

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Abstract

The areal density for hard disk drives has been increasing 100 percent per year for more than a decade since 1990's. To achieve the higher areal density, the head-disk spacing has to decrease accordingly. When the spacing is reduced to a few nanometers, intermittent contact between the head-slider and the disk is inevitable. The off-track vibration induced by intermittent head-disk contact may interrupt the reading and writing process, hence causing reading/writing errors. To further increase the areal density, the off-track vibrations must be well understood and controlled.

This dissertation deals with the off-track vibrations of head-sliders in hard disk drives. Off-track vibrations due to head-disk interactions were simulated. Based on the off-track acceleration range obtained from the simulations, a MEMS accelerometer was designed to detect the slider off-track motion. The micromachined accelerometer was characterized through vibration testing on a shaker.

In the first part of this dissertation, a five-DOF model including the transverse displacement of the actuator arm, the transverse displacement, vertical displacement, pitch angle and roll angle of the slider was developed to simulate the off-track vibrations induced by intermittent head-disk contact. Contact of the slider with a single bump as well as a large number of bumps on a rotating disk was simulated. Effects of the coefficient of friction, skew angle, bump spacing, and rotational speed of the disk on the slider off-track vibrations were analyzed. It was found that the off-track motion was dominated by the rotational mode of the actuator arm and the sway mode of the suspension.

Subsequently, a micro piezoresistive MEMS accelerometer ($1 \times 0.3 \times 0.3 \text{ mm}^3$) was designed for slider-level sensing of the off-track motion. Parametric analysis with geometric constraints was conducted to optimize the accelerometer structure for an optimal sensitivity with a specified natural frequency. The MEMS accelerometer was fabricated on a silicon-on-insulator (SOI) wafer. Vibration testing was carried out to characterize the accelerometer in comparison with a reference accelerometer. The background noise, sensitivity and nonlinearity of the accelerometer were obtained.

This study investigated the dynamics of the off-track vibration induced by head-disk contact and developed a MEMS accelerometer for detection of the off-track vibration. The simulation results obtained from the five-DOF model provided useful information for understanding various contributing factors to the off-track vibration. The fabricated and tested MEMS accelerometer offered a means for slider-level sensing of the slider off-track vibration. The acceleration signal generated by the accelerometer may be fed back to the servo controller to potentially prevent the off-track motion.

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List of Symbols

a	acceleration of the frame
b	width of the proof mass
B	width of the slider
c	damping coefficient
c_{arm}	damping coefficient of the actuator arm
c_d	damping coefficient of the disk
c_s	damping coefficient of the suspension
C	combined damping matrix
d	distance of z -axis of the center of mass to the rotational axis of the actuator
D	vertical distance from the origin of the disk coordinate system to that of the slider coordinate system
e	eccentricity
f_a	resonant frequency of the accelerometer
E	Young's modulus of single crystal silicon
F	friction force at the head-disk interface
F_i	inertial force of the proof mass due to the motion of the frame
F_l	longitudinal component of the friction force
F_n	normal contact force
$F_{n, ave}$	average contact force
$F_{n, max}$	maximum contact force
F_s	force exerted by the suspension
F_t	transverse component of the friction force

$\mathbf{F}$	vector of forces and moments induced by contact
h	distance of the frame to the bottom of the proof mass
h_b	bump height
h_f	air bearing film thickness
H	slider thickness
I_{arm}	mass moment of inertia of the actuator arm
I_{cm}	moment of inertia about z-axis of the center of mass of the actuator
I_θ	pitch moment of inertia
I_β	roll moment of inertia
J_{arm}	mass moment of inertia of the actuator arm about the rotational axis
k	effective spring constant of the flexure
k_{arm}	stiffness of the actuator arm
k_d	disk stiffness
k_e	equivalent spring constant of the central supporting beam
k_s	stiffness of the suspension
$\mathbf{k}_{diag}$	diagonal pico air bearing matrix
$\mathbf{k}_{nano}$	air bearing matrix for one particular nano slider
$\mathbf{k}_{pico}$	pico air bearing stiffness matrix
$\mathbf{k}^*$	normalized stiffness matrix
$\mathbf{k}_{nano}^*$	normalized nano air bearing stiffness matrix
$\mathbf{k}_{pico}^*$	normalized pico air bearing stiffness matrix
$\mathbf{K}$	suspension stiffness matrix
$\mathbf{K}_2$	stiffness matrix for the two-DOF model

$\mathbf{K}_c$	combined system stiffness matrix
l	length of the rear pad
l_c	length of the central supporting beam
l_s	length of the sensing beams
L	slider length
L_p	distance from the slider to the pivot
$\vec{L}$	vector from the center of the bump base to an arbitrary point on the bottom (air bearing) surface of the slider
m	slider mass
m_a	total mass of the actuator arm
m_{arm}	equivalent mass of the actuator arm
m_c	mass of the central supporting beam
m_p	proof mass
M	moment
M_θ	pitch moment
M_β	roll moment
$\mathbf{M}$	diagonal mass matrix
n	rotational speed of the disk
$\vec{n}$	inward unit normal vector of the bottom surface of the slider
p	thin-film pressure
p_a	ambient pressure.
P_{cr}	critical load
Q	flow factor to account for gaseous rarefaction effects

r	distance from the center of mass of the slider to the pivot axis of the arm
r_b	bump radius
R	original resistance of the piezoresistor
R_c	contact time ratio
$\mathbf{R}$	scaling matrix
S	sensitivity of the accelerometer
t_c	contact time of a single cycle
t_p	thickness of the proof mass
t_s	thickness of the sensing beams
T	period of the contact force
$\vec{T}$	vector from the center of the bump base to the tangential point
u	displacement of the frame
v_x	component of disk surface velocity in the x direction
v_y	component of disk surface velocity in the y direction
V	voltage
V_{input}	input voltage of the Wheatstone bridge
V_{output}	output voltage of the Wheatstone bridge
w_c	width of the central supporting beam
w_s	width of the sensing beams
x	transverse displacement of the slider
x_{arm}	transverse displacement of the actuator arm
x_b	x -coordinate of bump summit
x_c	x -coordinate of the contact point

x_g	x -coordinate of the slider's center of gravity
x_F	deflection caused by inertial force
x_M	deflection caused by moment
$x_{p, max}$	maximum x -coordinate of the rear pad corners
$x_{p, min}$	minimum x -coordinate of the rear pad corners
$x_{rotation}$	x -coordinate of the rotational axis of the actuator arm
$\dot{x}$	velocity of the slider in the transverse direction
$\dot{x}_{arm}$	velocity of the actuator arm in the transverse direction
$\ddot{x}$	acceleration of the slider in the transverse direction
$\ddot{x}_{arm}$	acceleration of the actuator arm in the transverse direction
$\mathbf{X}$	displacement vector
y_b	y -coordinate of the bump summit
y_c	y -coordinate of the contact point
y_g	y -coordinate of the slider's center of gravity,
z	vertical displacement of the center of mass of the slider
z_t	flying height
z_{t0}	flying height at the center of the trailing edge in the equilibrium state
β	roll angle of the slider
δ	proof mass displacement with respect to its frame
Δ_n	interference between the slider and bump
ΔR	resistance change of the piezoresistor
ε_1	axial strain of sensing beam 1
ε_2	axial strain of sensing beam 2

θ	pitch angle of the slider
θ_0	pitch angle in the equilibrium state
λ	flexibility matrix of the suspension
μ	coefficient of friction
ξ	shape factor of the contact force
π_l	longitudinal piezoresistance coefficient of single crystal silicon
π_{p-type}	p-type silicon piezoresistance coefficient for the <110> direction
π_t	transverse piezoresistance coefficient of single crystal silicon
ρ_{Si}	density of single crystal silicon
σ	stress
σ_l	longitudinal component of the stress in the piezoresistor
σ_t	transverse component of the stress in the piezoresistor
φ	coefficient of dynamic viscosity of air
ϕ	skew angle
ω	angular frequency
ω_a	angular natural frequency of the accelerometer

List of Abbreviations

BOE	Buffered Oxide Etchant
DOF	Degree of Freedom
DRIE	Deep-Reactive-Ion-Etching
FEA	Finite Element Analysis
FEM	Finite Element Methods
HAS	Head-Stack Assembly
HGA	Head-Gimbal Assembly
IC	Integrated Circuit
LDV	Laser Doppler Vibrometer
MEMS	Microelectromechanical Systems
ODE	Ordinary Differential Equation
PCB	Printed Circuit Board
PES	Position Error Signal
RIE	Reactive Ion Etching
SEM	Scanning Electron Microscopy
SOI	Silicon-on-Insulator
SNR	Signal-to-Noise-Ratio
TMR	Track Misregistration
VCM	Voice Coil Motor

Chapter 1 Introduction

1.1 Overview of Hard Disk Drives

The first hard disk drive IBM 350 was introduced by IBM in 1956 [1, 2]. Although it was as big as two refrigerators and had a capacity of only 5 Mbytes, it laid a foundation for modern hard disk drives. The main components in a hard disk drive on the current market as shown in Fig. 1.1 are almost the same as those in the historical IBM 350.

Information in the form of magnetic patterns is stored on rigid disks, also called platters. The platters are rotated at a high speed by a spindle motor. Special

Figure 1.1 A 3.5-inch Hitachi hard disk drive: Deskstar 7K400

electromagnetic read/write heads are mounted onto sliders to either write information onto the disks or read information from them. The sliders are suspended by stainless steel gimbal-suspension beams. As a whole, the head, slider and suspension are usually referred to as the head-gimbal assembly (HGA) [3]. The suspension, which is compliant in the direction perpendicular to the disk surface, keeps the slider at a constant height from the spinning disks with the aid of a self-pressurized air bearing. The spacing between the head on the slider and the disk surface is known as the flying height. Figure 1.2 illustrates the flying mechanism of the slider.

Figure 1.2 A slider flying on a rotating disk

The head gimbal assembly is connected to an actuator arm. The actuator arm driven by a voice coil motor (VCM) swings the HGA across the disk surface for the head to access a desired track. The HGA, actuator arm and voice coil motor are called the head-stack assembly (HAS) [4].

1.1.1 Areal Density and Position Error Signal

Based on the framework established by the IBM 350, the hard disk drive has seen tremendous advance in terms of smaller size, larger capacity, and higher access speed. Among all these achievements, the most remarkable progress is the astounding growth of the areal density for the data stored on the disk [5, 6].

The areal density is the product of the linear bit density in the circumferential direction and the track density in the radial direction. The first generation of the hard disk drive IBM 350 had an areal density of only 2 Kb/in² [2]. Forty years later, magnetic recording of 100 Gb/in² has been achieved by implementing the traditional longitudinal recording method [7]. Although for longitudinal recording further increase of the areal density over 100 Gb/in² appears to be limited by the issue of signal-to-noise ratio (SNR) [8], a newly introduced technology, perpendicular recording, has been predicted to reach 10 Tb/in² [9-11]. Using perpendicular recording, the areal density of current products is already approaching 400 Gb/in² [11]. Figure 1.3 shows the trend of the areal density. It can be seen that the areal density has been increasing 100 percent per year for more than a decade since 1990's.

Increasing the areal density can be attained by reducing the bit spot size and the track width. To ensure proper read/write, the head-media spacing has to decrease accordingly [12]. The decreasing trend of the head-media spacing with respect to the areal density for IBM hard disk drives is plotted in Fig. 1.4.

It is estimated that the flying height must be reduced to 6 nm for the areal density of 100 Gb/in² [13]. At such a small spacing, the flying height and the roughness of the

slider and the disk are comparable [14, 15]. As a result a near-contact (proximity) recording in which the slider makes contact with the disk surface intermittently is inevitable [16, 17].

Figure 1.3 Trend of areal density [18]

Head-disk interactions may cause off-track vibration of the suspension. As a result, the head will deviate from the desired track. By reading the information embedded on the disk, the difference between the actual and desired positions, the position error signal (PES), can be obtained [19]. The position error signal is then fed into the servo controller to control the track misregistration (TMR), which is a statistical distribution related to the off-track vibration of the read/write head. The feedback control system used to maintain the head on track is illustrated in Fig. 1.5 [5].

Figure 1.4 Areal density versus head-media spacing for IBM hard disk drives [6]

Figure 1.5 Block diagram of a typical head-positioning servo system

As it can be seen from above, the performance of the servo system in track following depends on the signal quality of PES. One prominent disadvantage of PES is its sampling rate is limited by the servo sectors available on the disk and the disk

spinning speed. This in turn limits the bandwidth of the servo controller and therefore its track following accuracy [20]. For a typical disk drive with a rotational speed of 7200 rpm and 128 servo bursts on the disk, the sampling rate is 15.36 kHz. As a result, in most servo controllers the closed-loop bandwidth is limited to 1-1.5 kHz, because in general the bandwidth of the servo controller has to be at least ten times lower than the sampling rate [21]. Another drawback about the current PES method is the servo sectors occupy precious disk space which otherwise can be used to store data.

1.1.2 Alternative Sensing Schemes to PES

Aimed at increasing the sampling rate as well as the bandwidth of the servo controller, additional sensors have been proposed to complement or replace PES.

Strain Sensing

Previous work largely concentrated on deploying strain gauges on the suspension [22, 23] or the actuator arm [24] to detect their vibrations. The strain signals, which are related to the suspension modes responsible for slider off-track motions, are fed into the control system to damp out the suspension resonances. Since the sampling rate of the strain signals is higher than PES, the bandwidth of the head-positioning system can be increased. Figure 1.6 is an illustration of using a strain gauge to detect suspension vibration.

There are some inherent drawbacks associated with the strain gauge schemes. First, even after the location of the strain gauges was optimized to some extent [25], the strain gauges still left out some off-track modes, while picked up some modes that

did not contribute to off-track vibration [26]. Therefore the signals from the strain gauges were not fully selective and inclusive for this specific application. As a result, off-track vibrations of the head slider could not be compensated effectively. Second, since a strain gauge functions by responding to elastic deformation of the suspension that leads to the displacement of the slider, its signal is only a measurement of the slider's off-track positions. In other words, they cannot be used to predict the slider's off-track motion. Similar to PES, a strain gauge signal can only trigger the servo to take a corrective action to bring back the slider on track.

Figure 1.6 A strain gauge attached to the suspension

If a signal can be generated at the time when the slider is still on track but starts to move off track, then an action can be taken upon this signal to stop the slider from deviating from its track. That means off-track errors can be theoretically prevented, not just corrected. Off-track motions are mostly induced by disturbances, such as intermittent head disk contact. During the disturbances, the slider usually experiences in-plane accelerations that come before the displacements. So an accelerometer is a better candidate than a strain gauge for a multi-sensing head-positioning system.

Acceleration Sensing

Previously, a number of researchers investigated the feasibility of using accelerometers to improve rejection of external disturbances [27-29]. Internal disturbances like head-disk contact were not addressed in their studies, because due to a much higher flying height head-disk interference in earlier lower-capacity hard disk drives was not as significant as in current hard disk drives. In their investigations, usually an accelerometer was mounted on the base casting of a hard disk drive to sense the vibrations of the drive, as illustrated in Fig. 1.7. The accelerometer in this configuration cannot detect the suspension vibrations directly and is totally insensitive to the slider vibrations induced by the head/disk contact.

Figure 1.7 External disturbance rejection using an accelerometer [29]

In an attempt to improve this accelerometer based disturbance rejection technique, Kobayashi et. al., [30] conducted a study in which an accelerometer was

put at the end of the actuator arm, as shown in Fig. 1.8.

Figure 1.8 An accelerometer located on the suspension base plate

Although this scheme achieved some improvement compared to the configurations in which accelerometers were mounted on the drive case, it also had some drawbacks. First, the accelerometer only detects out-of-plane vibrations, which are not among the major sources of slider off-track vibrations. Second, located on the actuator arm, the accelerometer does not respond to suspension vibrations, which are supposed to be the leading cause of slider off-track errors. As a result, only a slight reduction of TMR was reported.

1.2 Overview of Silicon Micromachining and MEMS Accelerometers

Microelectromechanical systems (MEMS) refer to devices or systems fabricated using micromachining technologies which originated from integrated circuit batch-processing technologies. These systems often consist of both electrical and

mechanical components with micrometer-level features [31-33].

Microfabrication technology, or more specifically silicon micromachining, has its roots in the well established integrated circuit (IC) fabrication technology [34-36]. There are two major fabrication technologies commonly used in silicon micromachining: bulk micromachining and surface micromachining [37].

Bulk micromachining is the technique used to shape the bulk of the silicon substrate to form desired three-dimensional microstructures. Etch masks and etch stop techniques are used in conjunction with silicon anisotropic etchants which selectively etch single crystal silicon along different crystal planes to create desired features.

Different from bulk micromachining, surface micromachining, developed more recently, builds up microstructures by successive deposition and etching of structural and sacrificial layers on the surface of a substrate wafer. Because surface micromachining does not depend on the silicon crystal orientation, it has more flexibility in fabricating different structures. Another advantage of surface micromachining over bulk micromachining is its compatibility with standard integrated circuit process. This means MEMS devices by surface micromachining can be integrated into integrated circuits more easily.

Accelerometers have been one of the most commercially successful MEMS products. Micromachined accelerometers are divided into two main categories in terms of sensing mechanism: piezoresistive and capacitive. Piezoresistive accelerometers generate a signal via resistance change in strain sensitive resistors, while capacitive accelerometers provide an output via capacitance change in a

capacitor formed by a proof mass and a fixed electrode. Early accelerometers were mostly of the piezoresistive type fabricated with bulk micromachining techniques. The first modern micromachined accelerometer was introduced in 1979 [38]. The piezoresistive accelerometer consisted of a cantilever with diffused piezoresistors on the root of the flexure. With the surface micromachining technology emerging in the early 1980s [39] and deep reactive ion etching in the early 90s [40], capacitive accelerometers had seen dramatic progress and have since become the most popular form for MEMS accelerometers. Compared with piezoresistive accelerometers, capacitive accelerometers have advantages of low temperature coefficients and easier fabrication. However, capacitive accelerometers have high output impedances and must be integrated with local amplifier circuits [41]. Monolithic integration was made possible by micromachining technology which is compatible with standard IC fabrication processes.

1.3 Objectives of the Thesis

Aimed to improve the tracking performance of the head slider in hard disk drives, this thesis had two objectives. The first objective was to simulate the intermittent head-disk contact to understand the slider off-track vibration. The second objective was to develop a MEMS accelerometer to detect the off-track vibration as studied in the first objective.

In order to gain understanding of the in-plane vibration of the head-slider under intermittent head-disk contact, main components of the head-stack assembly were

modeled by a five-DOF system, which consisted of transverse displacement of the actuator arm, and transverse displacement, vertical displacement, pitch angle and roll angle of the slider. Numerical simulations were conducted to examine the off-track behavior of the head-slider with a purpose to obtain the acceleration range for the subsequent accelerometer design.

A miniature micromachined piezoresistive accelerometer with a very high natural frequency and a desirable sensitivity was designed to be attached to the head-slider to detect its off-track accelerations. A simplified analytical model was presented for the analysis of the accelerometer. Parametric analysis was conducted to find an optimal design, in terms of sensitivity, frequency, and combinations of both. Design of masks, fabrication and characterization of the MEMS accelerometer were carried out.

Slider-Level Acceleration Sensing

In this project, a slider-level acceleration sensing scheme was proposed. A MEMS accelerometer is placed on the trailing side of the slider right above the read/write head. Figure 1.9 shows the proposed configuration. The sensing axis of the accelerometer is designed to be parallel to the slider trailing surface and is in the horizontal plane. In this direction, the slider off-track motion can be effectively detected.

Compared with previous schemes in which either a strain gauge or an accelerometer was deployed on the suspension [25, 30], this new scheme avoids the complexity and uncertainty in the effort of relating the suspension resonances to the

slider off-track motion. In cooperation with an actuated head that represents an emerging technology [42, 43], the in-plane accelerometer has the potential to greatly enhance the bandwidth and accuracy of the head-positioning system.

Figure 1.9 Proposed slider-level acceleration sensing scheme

1.4 Organization of the Chapters

Chapter 2 reviews previous work in the literature on dynamics of head-suspension assembly and MEMS accelerometers. Research status on off-track vibration induced by head-disk interactions is discussed. Several mainstream MEMS accelerometers are described with a focus on in-plane piezoresistive accelerometers.

Chapter 3 deals with modeling of slider off-track vibration due to head-disk interactions. The components of a head-disk assembly were studied, and the motions involved in the slider off-track vibration were identified. A one-DOF model on head-disk contact, a two-DOF model in the transverse direction and a five-DOF

model with one DOF from the actuator arm and four DOFs from the slider were established. A finite element model of the HSA was presented. The equivalent mass of the actuator arm was calculated, followed by the analysis of the sway mode of the suspension. A four-by-four stiffness matrix of the suspension was extracted from the finite element method (FEM) model. The scaling procedure for obtaining a pico air bearing matrix was described. A five-by-five stiffness matrix for the five-DOF model was obtained.

Chapter 4 presents simulation results of slider off-track motion. Intermittent contact of the slider with a single bump as well as an infinite number of bumps was simulated. Effects of the coefficient of friction, skew angle, rotational speed of the disk and the bump spacing on the transverse displacement were discussed. The phenomenon of transverse motion with zero skew was explained. Results on transverse motion from the two-DOF model and five-DOF model were compared. The acceleration range of off-track vibration under typical operational conditions was identified.

Chapter 5 is devoted to the design of the MEMS accelerometer. Parametric analysis with constraints such as axial deformation of the sensing beams, maximum dimensions, yield strength and critical load of buckling was conducted. The optimal design was found by considering both the natural frequency and sensitivity. Numerical results were presented together with finite element analysis.

Chapter 6 presents fabrication and characterization of the MEMS accelerometer. The fabrication process was described. Results on boron diffusion were obtained by a

four-point-probe and a semiconductor analyzer. Optical and SEM images of the fabricated accelerometer were presented. Post-processing including wire bonding and the amplifier circuit was also discussed. The experimental setup for characterization of the accelerometer was presented. The results on the sensor's noise, time response, sensitivity and linearity were analyzed.

Chapter 7 presents the conclusions and suggestions for future work.

Chapter 2 Literature Review

2.1 Dynamics of Head-Gimbal Assembly

The head-gimbal assembly plays a crucial role in track seeking (moving from one track to another) and track following. To ensure minimal seeking time and high tracking accuracy, the suspension is designed to be stiff in the lateral direction, while compliant in the vertical direction. The slider is attached to the suspension by a flexure (gimbal) and can rotate in two directions.

2.1.1 Suspension Dynamics

Compared to other components in the head-stack assembly, the suspension which holds the flying head slider fulfills more important and complicated functions. To meet the stringent requirements, the suspension must be carefully designed. An in-line suspension is shown in Fig. 2.1.

Figure 2.1 An in-line suspension assembly [44]

Since the head slider is flying on a spinning wavy disk, the suspension should be compliant out of plane to compensate for disk runout or waviness and at the same time, it should be stiff in plane so that the head slider follows the central line of a track despite off-track disturbances and accesses a new track at a very short time. The dynamics of the suspension plays a crucial role in improving the capacity and performance of hard disk drives, because tracking accuracy and the slider's flying ability are related to the suspension.

A lot of researchers made efforts to gain knowledge of the suspension dynamics and improve its design to enable higher areal densities. Mizoshita et al. [45] measured the slider's spacing fluctuation due to disturbances caused by the actuator arm vertical vibration and found many resonant peaks in the frequency region lower than the gas bearing resonant frequency. They calculated the modal shapes of the head assembly and clarified the lower resonant frequency peaks were caused by resonances of the suspension. Miu et al. [46] studied the dynamics of the suspension for the IBM 3370 head slider and differentiated vibration modes for compliance considerations from those for tracking considerations. They pointed out that even for linear actuators, the torsional and in-plane bending (sway) vibration modes of the suspension caused a substantial amount of transverse head motion and accounted for off-track errors. As rotary actuators took over linear actuators, the in-line suspension became very popular. In an in-line head mounting configuration, the torsional and sway modes tended to be easily excited to greater magnitudes than it would in a linear actuated configuration [47]. This new characteristic made off-track errors a more remarkable concern for

in-line suspensions. Yoneoka et al. [48] examined the slider responses under both vertical excitation and access excitation. Their results suggested that while the bending modes of the in-line suspension contribute to spacing variations, the torsional and sway modes contribute to transverse slider motions. Chiou and Miu [49] identified the sway mode which was a result of in-plane bending of the load beam as the primary source of off-track errors. They also reported that the initial deformed geometry of the suspension had direct effects on the in-plane stiffness of the suspension and therefore, on the sway mode frequency. Jeans [50] thoroughly investigated the sway mode with regard to its sensitivity to stiffness, thickness and shape of various parts of the load beam. They found both the sway and second torsion modes were strongly affected by the shape of the suspension near the beginning of the ribbed section but were not affected much by changes in the amount of preload and nor the specific shape of the bend region itself. Zeng and Bogy [51, 52] presented their research work on a Type 850AK which was widely used for 50% nano-sliders. They highlighted the effects of the head load, bend region geometry, rib height on the torsional and sway modes. Jen et al. [53] evaluated two types of suspensions for 30% pico-sliders focusing on the dynamic performance of the suspensions during shock and head slap. They noted a better shock resistance could be achieved with a smaller suspension equivalent mass and a lower nominal gram load.

In summary, it is believed that among the lower frequency resonances of the suspension, the sway mode, first torsional and second torsional modes are the main contributors to the off-track motions. These resonance modes also limit the bandwidth

of the head-positioning servo system. Controlling these resonance modes to increase the bandwidth of the servo and reduce track misregistration is a major research topic for hard disk drives.

Based on the established dynamics of the suspension, various efforts have been attempted to enhance its performance. The common target of these efforts was to improve both its in-plane tracking stiffness and out-of-plane compliance. Henze et al. [54] applied a constrained layer damper to the load beam to suppress the transverse vibrations of the suspension. Their results showed that while the damper was effective in reducing the amplitude of the torsional mode, the sway mode was only slightly affected. This was because a large amount of the suspension distortion which would contribute to shear strain in the damper occurred only over a very small region between the formed area and the load beam ribs. As a result, the tip region of the damper had a negligible contribution towards dissipation for the sway mode. Ohwe et al. [55] proposed a new suspension design for high-speed access in which a triangular, truss-bridge shaped load beam and a flexure supported at both ends were used to stiffen the load beam in the access direction and reduce the moment of the slider exerted by the access acceleration. The improvements saw the resonant frequency in the access direction twice and spacing fluctuation half that of the conventional in-line head assembly. Yumura et al. [56] found that a small change of the head load had a great influence on the gain of the first resonance peak. They related this phenomenon to the shear center shifting due to the small change of the load beam deformation under the head load. The shear center movement might have an influence on the

torsional moment which affected the vibration mode shapes. Aiming to eliminate the torsional moment, they optimized the geometry of the load beam rib and the bend region. Because the torsion mode was not excited and decoupled from the transverse bending mode, their proposed suspension had not only a high stiffness but also a stable flying height. To tackle the slider instability caused by the extreme friction force between the slider and the super-fine-finish disk surface, Arisaka et al. [57] developed a two-link mechanism which can set the center of motion of the slider on the sliding surface. The rotational moment of the frictional force could be removed during head/disk contact. The variation of friction coefficients was also suppressed by this mechanism. Ohwe et al. [58] presented a new integrated suspension for pico-sliders. Their suspension featured a resilient monolithic body including a load beam, a gimbal and a direct signal pattern to retain flying head stability. With a lighter equivalent mass, the 13 mm suspension was more compliant in the vertical direction and had a high transverse natural frequency. It could survive a high shock up to 600 g.

2.1.2 Slider Dynamics

The head-slider has been undergoing dramatic miniaturization which was motivated by lowering cost, decreasing head-media spacing to realize higher areal magnetic densities, improving flying stability and reducing access time. The first generation of flying sliders were designed for the ferrite ring heads used in the original Winchester hard disk drives, so they have been referred as the “full-size” Winchester ferrite sliders. Many generations of slider “form factors” have been

introduced since, as listed in Table 2.1 [59]. The percentage of each form factor is the linear scaling factor from the minislider.

Table 2.1 Head-slider form factors

Form factor	Dimension (mm)	Slider mass (mg)	Slider materials	Air-bearing features
Mini (100%)	$4 \times 3 \times 0.8$	55	Ferrite	Machined rails
Micro (70%)	$2.84 \times 2.23 \times 0.6$	16.2	$\text{Al}_2\text{O}_3\text{-TiC}$	Machined rails
Nano (50%)	$2.05 \times 1.6 \times 0.43$	5.9	$\text{Al}_2\text{O}_3\text{-TiC}$	Etched rails
Pico (30%)	$1.25 \times 1 \times 0.3$	1.6	$\text{Al}_2\text{O}_3\text{-TiC}$	Etched rails
Femto (20%)	$0.85 \times 0.7 \times 0.23$	0.6	Thin-film/composite	Head near or at contact

The miniaturization of slider dimensions and the evolution of its air-bearing surface are shown in Fig. 2.2.

Figure 2.2 Evolution of the slider and air-bearing surface [60]

The head-slider is attached to a cantilever suspension by a small flexure which permits two rotations, pitch and roll with the help of a gimbal to accommodate variations on the disk. The suspension flexure is flexible in the vertical direction, thus adding a third degree of freedom to the slider. The three degrees of freedom of the slider are shown in Fig. 2.3.

Pitch represents the angular rotation of the head-slider about the axis which is parallel to the plane of the disk and perpendicular to the recorded track, while roll is the rotation of the head-slider about the axis that is parallel to the tangent of the recorded track.

Figure 2.3 Degrees of freedom of the slider

The dynamics of the head-slider are governed by the equations of motion for the rigid slider body and the generalized Reynolds equation for the thin film air bearing. Simultaneously solving the coupled equations, the slider dynamic responses to a variety of excitations can be obtained.

Figure 2.4 shows a theoretical model of the head slider.

Figure 2.4 Physical model of a head-slider flying on a rigid disk surface

The dynamics of the rigid slider can be modeled as a three-degrees-of-freedom (DOF) system involving pitch, roll and vertical motions. The equations of motion are written as [61]

$$m \frac{d^2 z}{dt^2} = -F_s + \iint (p - p_a) dx dy \quad (2.1)$$

$$I_\theta \frac{d^2 \theta}{dt^2} = M_\theta + \iint (p - p_a)(x_g - x) dx dy \quad (2.2)$$

$$I_\beta \frac{d^2 \beta}{dt^2} = M_\beta + \iint (p - p_a)(y_g - y) dx dy \quad (2.3)$$

where z is the vertical displacement, θ and β are pitch and roll angles, m is the slider mass, I_θ and I_β are the slider's moments of inertia, x_g and y_g are the coordinates of the slider's center of gravity, F_s , M_θ and M_β are the force and moments exerted by the suspension, p is the thin-film pressure and p_a is the ambient pressure. Bushan [44] presented a more complicated model taking into account the stiffnesses and damping coefficients of the flexure, static moments and

fluid friction forces.

The thin-film pressure p between the slider and the disk is governed by the Reynolds equation as follows [62].

$$\frac{\partial}{\partial x} \left(ph_f^3 Q \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(ph_f^3 Q \frac{\partial p}{\partial y} \right) = 6\varphi v_x \frac{\partial(ph)}{\partial x} + 6\varphi v_y \frac{\partial(ph_f)}{\partial y} + 12\varphi \frac{\partial(ph_f)}{\partial t} \quad (2.4)$$

where h_f is the air bearing film thickness; φ is the coefficient of dynamic viscosity of air; Q is the flow factor to account for gaseous rarefaction effects; v_x and v_y are the components of disk surface velocity in the x and y directions.

The dynamic response of the slider can be simulated by solving the time-dependent lubrication equation in combination with the dynamic equations of motion of the slider. If the air bearing film thickness $h_f(x, y, t)$ is known, then Eq. (2.4) can be used to solve for the pressure distribution p of the air bearing. Subsequently, the pressure force exerted on the slider and its moment about any desired point can be calculated.

Usually, the solution of Eqs. (2.1), (2.2) and (2.3) is obtained by a Runge-Kutta scheme once the pressure p is known. The solution of Eq. (2.4), on the other hand, entails much more effort. Basically, different ways of solving Eq. (2.4) fall into two categories: the direct numerical integration method in the time domain with finite difference or finite element schemes [63-65] and the linearized analytical-numerical method using either the perturbation technique or the modal analysis in the frequency domain [66-68]. The direct numerical method can predict the nonlinear dynamic response of the slider, while the linearized analytic-numerical method in the frequency domain is very effective for obtaining general insights into the dynamic

characteristics of the slider, such as stiffnesses and coefficients. Comparisons of the two methods were presented by Hayashi [69].

Parallel to numerical simulations, experimental investigations as an alternative tool to analyze the slider dynamics have also continuously received attention. Earlier work to measure the air bearing separation by use of capacitive, inductive and optical interference techniques were summarized by Lin [70]. A laser Doppler vibrometer (LDV) with a single beam was first used to measure dynamic response of a Winchester-type slider by Miu et al. [71, 72]. Then, Ponnaganti et al. [73] used a dual beam approach to overcome the limitations of a single beam which could not measure the slider motion and the disk motion simultaneously. Later on, Riener et al. [74] further improved this technique by proposing a two channel laser Doppler vibrometer in which the spacing modulation between the slider and the disk could be measured without the need for time averaging. Since then laser Doppler vibrometer systems have been commonly used in the study of dynamic characteristics of head/disk interfaces.

Based on numerical simulations or experimental investigations as well as the combination of both, extensive research has been conducted on the slider dynamics under various disturbances. Miu and Bogy [65, 75] presented the experimental observation and numerical simulation of the head slider's transient responses to an artificially introduced crater-like surface imperfection and a sputtered rectangular step. They identified the roll mode and two coupled pitch and vertical modes. Excellent correlation was observed between theory and experiment. Ponnaganti et al. [76]

examined the dynamics of a slider subject to intermittent contact with an asperity on the disk. Benson et al. [61] also studied slider-disk contacts with an emphasis on the effects of coefficients of restitution and friction. Ruiz and Bogoy [77, 78] first noted that the suspension may have a significant impact on the air bearing dynamics and they developed a simulator which included the head/disk interface and main components of the suspension. The simulator was used to analyze the air bearing vibration due to a step bump, spacing variation caused by track accessing and actuator crash-stop. Birggs and Talke [79-81] investigated a series of sliders with different form factors using laser Doppler vibrometry. They compared vertical, pitch and roll frequencies of 100%, 70% and 50% sliders. Hu and Bogoy [82] studied the dynamic characteristics of three rail-shaped negative pressure sub-25 nm flying height sliders with various simulations including air bearing stiffness, impulse response, surface roughness induced spacing modulation, bump response and track seeking dynamics. More recent theoretical explorations and experimental investigations were attempted on the dynamics of pico or femto sliders with sub-5 nm flying height which is expected for 100 Gb/in² and later 1 Tb/in² [83, 84].

2.1.3 Off-Track Vibration Induced by Head-Disk Contact

The tracking accuracy during reading and writing required for high areal densities is in the range of a few nanometers, e.g., 1.5 nm (rms) for an areal density of 1 Tbit/in² [85]. Therefore, the slider off-track vibration induced by head-disk interactions may severely affect the tracking capability of the head. Off-track

vibration excited by the friction force can cause significant track misregistration. Takahashi and Bogy [86] reported a friction-induced peak transverse displacement of about 90 nm with the suspension at a skew angle of 8 degrees. Yoneoka et al. [48] analyzed the in-plane vibration of a slider resulting from the friction force by using a laser Doppler vibrometer. Wang et al. [87] found the track misregistration caused by contact induced off-track jitter was on the order of 20-70 nm. Although experimental work confirmed the effect of friction force on the off-track motion, its mechanism needs to be understood in detail. Yonemura et al. [88] identified the torsion and sway modes as the major contributors to the in-plane vibrations of the slider by experimental observation; however, they did not study quantitatively the characteristics of the two modes and the correlation between these modes and the off-track motion.

Although effects of disk surface defects, such as asperities and bumps, on the flying dynamics of the slider were studied extensively [61, 65, 75, 76], efforts were seldom made to examine the off-track motion. Miu and Bogy [65, 75] investigated the responses of the vertical displacement, pitch angle and roll angle of the slider due to a rectangular step on the disk surface. Ponnaganti et al. [76] modeled the slider motion with six degrees of freedom, namely, pitch, roll, yaw, in-plane radial, in-plane tangential, and out-of-plane vertical motions under the condition of a single large asperity passing along one of the slider rails. In their study, however, the suspension was essentially immune to in-plane radial disturbances, because it was driven by a linear actuator. Thus, the results were not relevant to the present-time suspensions that

are connected to rotary actuators. Benson et al. [61] conducted a more detailed study of slider dynamics during head-disk contacts by considering multiple asperities and a systematic numerical solution of the Reynolds equation and the equations of motion of the slider. Similar to the earlier work [65, 75], their interests were limited to the vertical displacement, pitch angle and roll angle.

More recently, disk surface profiles with waviness and roughness were introduced into the head-disk contact models [89-91]. Ono and Takahashi [89] used a single-degree-of-freedom model to study the vertical bouncing vibration of a slider in contact with a wavy disk surface with a harmonic profile. Similar analyses were performed in a more advanced way by using two-DOF models [90, 92]. Ono [91] further added random roughness to the disk waviness in the analysis of bouncing vibrations of the slider. Yu et al. [93] studied contact induced off-track vibrations using a complicated dynamic simulator comprising a finite element suspension model and an air bearing model based on the generalized Reynolds equation. Although theoretically a dynamic simulator can generate more accurate results for flying height analysis, it may not be necessary or efficient for off-track vibration simulation, since air bearing dynamics is not expected to affect off-track vibrations to a large extent. Therefore, a complete approach tends to include too many unimportant factors and may not be effective in differentiating the significant ones from the insignificant. The actuator arm rotation, which plays an important role in the slider off-track motion, was not considered in the foregoing analyses.

2.2 MEMS Accelerometers

Accelerometers have been one of the most commercially successful MEMS products. Due to their robustness, easy integration with microelectronics and low cost, MEMS accelerometers are used in large volume in passenger cars. Deployment of air bags has been the main application for MEMS accelerometers in vehicles. In addition, vehicle manufacturers have also been using MEMS accelerometers in vehicle suspension systems for better passenger comfort and drivability [94]. Furthermore, the application of MEMS accelerometers has been extended far beyond the automobile industry. More recently, accelerometers are starting to feature in consumer electronics, such as laptops and cell phones. They can be used for drop detection, or to expand the functionalities of portable electronic devices, like screen scrolling and gaming. The sales for MEMS accelerometers in the consumer electronics market is expected to reach \$212 million in 2009 [95].

A variety of MEMS accelerometers have been developed based on different sensing mechanisms [96]. Commercial MEMS accelerometers are mainly divided into three categories: pizoresistive, capacitive and piezoelectric. Applications of novel sensing techniques on MEMS accelerometers, such as tunneling, are also beginning to emerge.

2.2.1 Piezoresistive Accelerometers

The first micromachined piezoresistive accelerometer as shown in Fig. 2.5 was developed in 1979.

Figure 2.5 A cantilever piezoresistive accelerometer [38]

In piezoresistive accelerometers, piezoresistors are fabricated at the end of the suspension beam where the maximum stress occurs. A half or full Wheatstone bridge can be formed by incorporating two or four piezoresistors. Piezoresistors convert the stress into resistance change due to piezoresistivity. Resistance change can be converted to voltage change that is easy to be measured. Measuring acceleration is implemented by measuring the output electrical signal.

This design laid a foundation for today's piezoresistive accelerometers [97, 98].

The main advantage of piezoresistive accelerometers is the simplicity of their structure and fabrication process, as well as their readout circuitry. However, piezoresistive accelerometers suffer from large offset and temperature sensitivity. Their overall sensitivity is inferior to capacitive devices; hence a larger proof mass is preferred for them.

In-Plane Piezoresistive Accelerometers

While in most piezoresistive accelerometers the proof mass and cantilever deflects out-of-plane [38, 97, 98], under some circumstances it is more advantageous to have the sensing structure deflect in the same plane as the accelerometer frame. As shown in the proposed slider-level acceleration sensing scheme (see Fig. 1.9), since the accelerometer frame has to be attached to the trailing side of the slider, in this case an in-plane accelerometer might be the only choice. In an out-of-plane sensing structure, the piezoresistors are made by doping the top layer of the flexure near its end where the maximum stress appears. In an in-plane sensing structure, because the maximum stress is concentrated on the side walls, piezoresistor formation involves doping the side walls. An in-plane accelerometer fabricated with this technique is shown in Fig. 2.6. The fabrication process of this design was difficult to control and not well established. An innovative method is to use the central beam only to support and displace the proof mass, while use additional small flexures as the piezoresistors. A commercial accelerometer as shown in Fig. 2.7 was developed based on this method [99].

The small flexures on which piezoresistors are fabricated bridge the gap between

the proof mass and the frame. When the proof mass is displaced by inertial force, one beam will be in elongation and the other in compression. The tensile and compressive

Figure 2.6 An in-plane accelerometer with piezoresistors on its side walls [100]

Figure 2.7 An in-plane accelerometer with additional sensing beams [31]

stresses will cause resistances of the piezoresistors on the two beams to change in opposite directions.

Although this bulk micromachined product introduced a new type of in-plane accelerometers, it left much space for optimization and improvement. One of the issues to be resolved is that its sensing beams may not be in ideal compression and tension. Subject to bending, the thin sensing beams may easily break.

Targeted at achieving pure axial deformation in the sensing beams, a modified structure as shown in Fig. 2.8 was proposed [101]. The structure was three separate pieces bonded together. It is not feasible to implement this technique with high accuracy on structures with small dimensions.

Figure 2.8 An in-plane accelerometer with pure axially deformed sensing beams [101]

The performance of the accelerometer is likely to be compromised by errors in bonding alignment. With silicon-on-insulator (SOI) technology, the above structure

can be built without wafer bonding [102]. Although the condition for pure axial deformation condition was formulated, there is no solution for the chosen parameters when taking into consideration the geometric constraints on the dimensions of the accelerometer, such as those imposed by the slider geometry. Therefore the optimization scheme is not applicable to the accelerometer in this project. A different optimization scheme and a thorough numerical analysis are necessary in finding an optimal design.

2.2.2 Capacitive Accelerometers

In capacitive accelerometers, the proof mass displacement can be sensed by a capacitor comprising the proof mass and a fixed conductive electrode [103, 104]. Generally, differential capacitive sensing scheme is employed in order to linearize the output and compensate interferences in detection of the very small deflections. In this case, two variable capacitors are situated on opposite sides of the proof mass to form a differential capacitive bridge. Proof mass displacement results in capacitance increase in one capacitor, and capacitance decrease in the other. Figure 2.9 shows a capacitive accelerometer based on this principle.

High sensitivity, good DC response and noise performance, low drift, low temperature sensitivity and low power dissipation are among the main attractive characteristics of capacitive accelerometers. Because of susceptibility to parasitic capacitance, capacitive accelerometers need on-board electronic circuitry that requires more complicated fabrication process compared to piezoresistive accelerometers.

Figure 2.9 A capacitive accelerometer [31]

2.2.3 Piezoelectric Accelerometers

In a piezoelectric accelerometer, a piezoelectric film, e.g. PZT and ZnO, deposited on the suspension beam develops an internal dielectric dipole in response to the induced stress by beam deflection [105, 106]. The change of internal dipole displacement is usually detected in the form of a charge or voltage developed across a set of appropriately located electrodes surrounding the piezoelectric crystal. A surface micromachined piezoelectric accelerometer is illustrated in Figure 2.10.

The major advantage of micromachined piezoelectric accelerometers is that they can be fabricated in a simple process and easily incorporated in existing measuring systems.

However, the use of thin-film structures has some key limitations, including the need for careful stress control and the potential for large deflections with temperature

changes causing degradation of sensitivity and linearity. Piezoelectric accelerometers are not as commercially successful as their piezoresistive and capacitive counterparts.

Figure 2.10 Schematic of a piezoelectric accelerometer [106]

2.2.4 Tunneling Accelerometers

The high position sensitivity of electron tunneling can be utilized to make accelerometers [107-109]. As shown in Fig. 2.11, when the device is accelerated, the

Figure 2.11 A tunneling accelerometer [109]

proof mass begins to move relative to the tunneling tip. The feedback control circuit adjusts the electrostatic re-balance force on the proof mass to maintain a constant

tunneling current. The external acceleration is measured by the feedback deflection voltage that varies in direct proportion to the acceleration.

Thanks to the exponential relationship between the tunneling current and the tunneling gap, the sensitivity of the tunneling accelerometer is superior to that of other compact accelerometers, such as piezoresistive and capacitive accelerometers. Despite a growing research effort, many issues still remain to be solved, such as the fabrication processes necessary to produce appropriate electrode characteristics, the stability of tunneling transducers, and the packaging requirements necessary to preserve the sensor performance.

Chapter 3 Modeling of Slider Off-Track Vibrations due to Head-Disk Interactions

3.1 Introduction

Figure 3.1 shows a model of a head-stack assembly built after a 3.5-inch Fujitsu hard disk drive, of which the detailed specifications are listed in Table 3.1. The actuator arm has three arms which are able to hold a maximum of six head-gimbal

Table 3.1 Specifications of the 3.5-inch hard disk drive

Brand	Model No.	Dimensions (Width $\times$ Depth $\times$ Height)	Capacity	Number of Platters
Fujitsu	MPE3064AT	4 $\times$ 5.7 $\times$ 1 in ³	6.4 Gb	1

Figure 3.1 A head-stack assembly for a 3.5-inch hard disk drive

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assemblies. Since the hard disk drive in this study has only one platter, there is only one head-gimbal assembly attached to the actuator arm. The specifications of the head-gimbal assembly are listed in Table 3.2.

Table 3.2 Specifications of the head-gimbal assembly

Suspension length (mm)	Slider dimensions (mm ³)	Slider mass (mg)	Z-height (mil)	Gram load (g)	Slider flying height (nm)
12	$1.25 \times 1 \times 0.3$	1.58	28	2.5	10

Parameters of commercial hard disk drives can vary depending on the models and the manufacturers. However, for hard disk drives of the same form factor, a lot of components are standard, such as the disk, slider and suspension. To keep consistency of the parameters in this study, the parameters of the head-gimbal assembly and the operation conditions of the hard disk drive are determined or referenced from open literature based on the 3.5-inch hard disk drive and the pico slider.

Figure 3.2 shows the head-gimbal assembly (HGA). The load beam has ribs which stiffen the suspension in the lateral direction. It is connected to the base plate by the bent region without ribs. Therefore, the suspension is compliant in the vertical direction. As a result, the attached slider can move up and down to accommodate the variation on the disk surface. Suspended at the end of the load beam by a flexure, the slider is able to rotate about the dimple.

Figure 3.2 A head-gimbal assembly

3.2 Dynamic Friction at the Head-Disk Interface and Its Effects on Slider and Suspension Vibration

In order to reduce the magnetic distance between the read/write head on the slider and the magnetic layer of the disk to achieve a higher linear density, it is necessary to reduce the roughness of the disk surface. However, a smooth disk can bring about high friction force which will cause undesirable slider vibrations. Figure 3.3 is an illustration of the friction force F at the head-disk interface. F can be decomposed into two components F_l and F_t along the longitudinal and transverse directions. F_l and F_t are directly related to the skew angle ϕ . Because there exists a distance between the slider rotational center and where the friction force occurs, F_l and F_t can generate pitch and roll moments, which cause the slider to vibrate.

Figure 3.3 Friction at the head-disk interface

The actuator arm is the holding frame for the voice coil motor (VCM) and the suspension. It is thick and rigid. In the presence of a transverse disturbance, the actuator arm moves like a rigid body and contributes to the head-slider off-track motion. Figure 3.4 shows off-track motion caused by the actuator arm rotation.

Compared to the actuator arm, the suspension is thinner and more flexible. The bent region without ribs has a low stiffness. This allows the head slider to slide over a wavy disk with a stable flying height when subject to disturbances. Apart from the benefits, negative effects on the stability of the head slider are also brought about by the flexibility of the suspension. The elastic deformation of the suspension, mainly caused by the sway, first torsional and second torsional modes, displaces the head slider in a way different from the rigid actuator arm rotational mode. Figure 3.5 demonstrates track misregistration induced by the suspension sway mode.

Figure 3.4 Track misregistration induced by actuator arm rotational mode

Figure 3.5 Track misregistration induced by suspension sway mode

3.3 Two-DOF Model of the Head-Stack Assembly in the Transverse Direction

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From the analysis in the previous section, it is understood that when excited by the friction force, the rotational mode of the actuator arm and the sway mode of the suspension can contribute to the off-track motion of the slider. A two-DOF model is developed as follows to simulate the off-track motion (transverse motion) induced by the two modes.

Figure 3.6 shows the head-stack assembly and the two-DOF model. Since the distance from the slider to the pivot L_p is much greater than the transverse

Figure 3.6 Two-DOF model in the transverse direction

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displacement induced by the rotation of the actuator arm x_{arm} , the rotation of the actuator arm can be converted to linear motion in the transverse direction. Hence, the actuator arm is represented by an equivalent mass in the two-DOF model. The friction in the pivot is modeled by a spring-damper system [20]. Another spring-damper system is used to model the suspension. The equations of motion for the two-DOF model in the transverse direction are written as follows.

$$\begin{bmatrix} m_{arm} & \\ & m \end{bmatrix} \begin{bmatrix} \ddot{x}_{arm} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} c_{arm} & -c_s \\ -c_s & c_s \end{bmatrix} \begin{bmatrix} \dot{x}_{arm} \\ \dot{x} \end{bmatrix} + \begin{bmatrix} k_{arm} + k_s & -k_s \\ -k_s & k_s \end{bmatrix} \begin{bmatrix} x_{arm} \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix} \quad (3.1)$$

where m_{arm} , m , $\ddot{x}_{arm}$, $\ddot{x}$, c_{arm} , c_s , $\dot{x}_{arm}$, $\dot{x}$, k_{arm} , k_s , and F are the equivalent mass of the actuator arm for linear motion in the transverse direction, slider mass, acceleration of the actuator arm, acceleration of the slider, damping coefficient of the actuator arm, damping coefficient of the suspension, velocity of the actuator arm, velocity of the slider, stiffness of the actuator arm, stiffness of the suspension and friction force on the slider, respectively. Equation (3.1) can be solved for the time response of the slider's velocity and displacement in the transverse direction, if the initial velocities and displacements of the actuator arm and slider, as well as the friction force are known.

The two-DOF model has its limitations. It is not capable of simulation of the intermittent contact between the disk and slider. Since it only has two degrees of freedom, information on the flying height, pitch and roll cannot be obtained. Another inadequacy with the two-DOF model is its inability to calculate the friction force. Instead, it has to be fed from an external source. However the simple model also has its advantages. Since it only includes the two most important contributors to the

transverse motion of the slider, i.e. the actuator arm rotational mode and the suspension sway mode, it provides a means to estimate the transverse displacement of the slider with minimal computation cost.

3.4 Five-DOF Model of the Head-Stack Assembly with Air Bearing Effects

The two-DOF model was based on the understanding that the rotation of the actuator arm and the sway mode of the suspension are the two major contributors to the off-track vibration of the slider. Since the slider is attached to the suspension through a gimbal and dimple, their dynamics might be coupled. To simulate the off-track vibrations of the slider with a higher accuracy, the effects of the three degrees of freedom of the slider, namely vertical displacement, pitch and roll, on the off-track vibrations of the slider have to be considered. Therefore, in addition to the equivalent transverse displacement of the actuator arm and the transverse displacement of the slider in the two-DOF model, the vertical displacement, pitch and roll of the slider are added to form a five-DOF model. Since the stiffness of the air bearing is much higher than that of the suspension, it is assumed that changes in the air bearing stiffness have only a negligible effect on the suspension dynamics in the transverse direction. Furthermore, in this study the scenario for the slider transverse displacement is limited only to the track following state, where the slider is in the vicinity of its steady-state equilibrium position in the vertical direction. Therefore, the

nonlinear air bearing can be approximated by a linear stiffness-damping system for computational efficiency.

Mathematical Formulations

When the slider is in intermittent contact with the disk, the actuator arm will rotate and the suspension will deform in the transverse direction due to the transverse component of the friction force. At the same time, the normal contact force and the moments associated with the contact force will cause vibrations of the slider in the vertical position, pitch angle and roll angle. Assuming that the rotational angle of the head-stack assembly in a plane parallel to the disk is very small, or more precisely, that the transverse displacement of the slider is much smaller than the pivot-to-slider distance, the rotation of the actuator arm can be converted to equivalent linear motion, as shown in Fig 3.6.

Based on this conversion, a five-DOF model has been developed to analyze the off-track motion of the slider as well as its spacing, pitch angle and roll angle (Fig. 3.7). The coordinates z and x refer to the vertical direction and the transverse direction respectively. Considering the motion around the equilibrium position, the air bearing

Figure 3.7 Five-DOF model

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is simplified into a linear system described by a stiffness matrix $[k]$ and a damping matrix $[c]$.

When the slider and disk are in separation, the slider undergoes free vibration. In a matrix form, the equation of motion is

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}_c\mathbf{X} = \mathbf{0} \quad (3.2)$$

where $\mathbf{X} = [x_{arm}, x, z, \theta, \beta]^T$. x_{arm} is the transverse displacement of the actuator arm, x the transverse displacement of the slider, z the vertical displacement of the center of mass of the slider, θ the pitch angle, and β the roll angle. $\mathbf{M}$ is the diagonal mass matrix with the equivalent actuator arm mass m_{arm} , slider mass m , pitch moment of inertia I_θ , and roll moment of inertia I_β , on the major diagonal. m_{arm} can be calculated by using the formula, $m_{arm} = I_{arm}/r^2$, where I_{arm} is the mass moment of inertia of the arm, and r the distance from the center of mass of the slider to the pivot axis of the arm. $\mathbf{K}_c$ and $\mathbf{C}$ are the combined system stiffness matrix and damping matrix, respectively.

Assuming that a bump is present on the disk surface, contact occurs if an interference between the slider and the bump is detected with the x - and y -coordinates of the contact point being within a target domain, e.g., the rectangular area of a rear pad. For contact to occur, the following three conditions must be satisfied at the same time.

$$\Delta_n > 0, x_{p, min} < x_c < x_{p, max}, -L/2 < y_c < l-L/2 \quad (3.3)$$

where Δ_n , L , $x_{p, min}$, x_c , $x_{p, max}$, y_c , l are the interference between the slider and bump, slider length, minimum x -coordinate of the rear pad corners, x -coordinate of the

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contact point, maximum x -coordinate of the rear pad corners, y -coordinate of the contact point, and length of the rear pad, respectively. Δ_n depends on the bump position and geometry, and the slider thickness as well as its vertical displacement, pitch angle and roll angle. The origin of the xyz coordinate system is at the center of mass of the slider that is placed at its equilibrium position. Once the position of the slider is known, $x_{p, \min}$ and $x_{p, \max}$ can be calculated by geometry. The x - and y -coordinates of the contact point x_c and y_c are functions of the bump position and geometry as well as the pitch and roll of the slider.

When contact (interference) occurs, the slider experiences forced vibration due to the normal contact force and the friction force. The equation of motion is written as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}_c\mathbf{X} = \mathbf{F} \quad (3.4)$$

where $\mathbf{F}$ is the vector of forces and moments induced by the contact, $\mathbf{F} = [0, F_t, F_n, M_\theta, M_\beta]^T$. The normal contact force F_n , friction force component in the transverse direction F_t , pitch moment M_θ , and roll moment M_β are expressed as

$$F_n = k_d \Delta_n + c_d \dot{\Delta}_n \quad \text{for } \Delta_n > 0 \quad (3.5)$$

$$F_t = \mu F_n \sin \phi \quad (3.6)$$

$$M_\theta = F_n y_c + \frac{1}{2} H \mu F_n \cos \phi \quad (3.7)$$

$$M_\beta = -\frac{1}{2} H \mu F_n \sin \phi - (x_c - x) F_n \quad (3.8)$$

where H , k_d , c_d , μ and ϕ are the slider thickness, contact stiffness, contact damping coefficient, coefficient of friction and skew angle, respectively. The normal contact force F_n is calculated based on a contact model as shown in Fig. 3.8. Assuming point contact between the slider and the disk, the mass of the disk can be ignored. And

considering under a small contact load disturbance caused by an external excitation, the change of stiffness will be very small, the contacting asperities can be treated as a

Figure 3.8 Head-disk contact model: (a) Separation; (b) Contact

linear spring within this small contact load variation [110]. Hence, the disk is modeled as a massless spring-damper system, where the van der Waal's forces and adhesion forces are ignored [111]. The contact stiffness k_d and damping coefficient c_d can be calculated based on contact models, such as the Hertzian model [110]. k_d and c_d can also be measured experimentally based on the contact resonance method [112] or the ultrasonic technique [113]. In this study, values of k_d and c_d reported by Ono [89] will be used.

The slider-disk interaction is modeled as a three-dimensional slider in contact with a three-dimensional moving bump as shown in Fig. 3.9. The bump is defined as a circular paraboloid

$$\left(\frac{x-x_b}{r_b}\right)^2 + \left(\frac{y-y_b}{r_b}\right)^2 = \frac{h_b-z}{h_b} \quad (3.9)$$

where x_b , y_b , r_b and h_b are the x and y coordinates of the bump summit, bump radius and bump height, respectively.

Figure 3.9 Geometry of the bottom surface of the slider and a bump on the disk

The interference between the slider bottom and the bump Δ_n is expressed as follows

$$\Delta_n = \vec{n} \cdot (\vec{T} - \vec{L}) \quad (3.10)$$

where $\vec{n}$ is the inward unit normal vector of the bottom surface of the slider, $\vec{T}$ the vector from the center of the bump base to the tangential point, and $\vec{L}$ the vector from the center of the bump base to an arbitrary point on the bottom (air bearing) surface of the slider. If $\Delta_n > 0$, the slider and bump are in contact; otherwise, they are

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in separation. For small θ and β , the interference Δ_n between the slider and the bump as given in Eq. (3.10), can be approximated by

$$\Delta_n = h_b - \left[z + D - \frac{H}{2} + y_b \theta - (x_b - x) \beta \right] \quad (3.11)$$

where D is the vertical distance from the origin of the disk coordinate system to that of the slider coordinate system. It can be seen from Eq. (3.11) that, for small θ and β , the three-dimensional paraboloidal bump is equivalent to a sharp needle. Substituting Eq. (3.11) into Eq. (3.5), the normal contact force F_n is given by

$$F_n = k_d \left[h_b + \frac{1}{2} H - z - D - y_b \theta + (x_b - x) \beta \right] - c_d \left[\dot{z} + \dot{y}_b \theta + y_b \dot{\theta} - (\dot{x}_b - \dot{x}) \beta - (x_b - x) \dot{\beta} \right] \quad (3.12)$$

Equation (3.12) shows that the magnitude of the normal contact force is a function of the position and moving velocity of the bump, and the slider's pitch, roll, vertical and transverse displacements, and their velocities.

Under the same assumption of small θ and β , the x - and y -coordinates of the contact point are found to be

$$x_c = \frac{r_b^2 \beta}{2h_b} + x_b, \quad y_c = -\frac{r_b^2 \theta}{2h_b} + y_b \quad (3.13)$$

The fourth and fifth Runge-Kutta formulas in the software package MATLAB were used in solving Eqs. (3.2) and (3.4). In the computation, conditions specified by Inequalities (3.3) are monitored in each marching step. To locate a contact point precisely, the solution process uses a smaller marching step around the contact region. When contact is detected, integration of Eq. (3.2) will be terminated and the integration scheme will be switched to that for Eq. (3.4). The displacements, angular

displacements, velocities, and angular velocities at the end of the integration scheme for Eq. (3.2) are used as initial conditions for continuation of the solution process.

3.5 Stiffness Matrices for the Two-DOF Model and the Five-DOF Model

The equivalent mass for the actuator arm was calculated using a finite element model for the actuator arm. The stiffness matrix for the two-DOF model was formed by the stiffness of the actuator arm and that of the suspension in the transverse direction. The four-by-four suspension stiffness matrix was extracted from a finite element model. The air bearing stiffness matrix for a pico slider was obtained by scaling the air bearing stiffness matrix for a nano slider. By combining the suspension stiffness matrix, the air bearing stiffness matrix and the stiffness of the actuator arm in the transverse direction, the five-by-five stiffness matrix for the five-DOF model was constructed.

3.5.1 Finite Element Models

The equivalent mass of the actuator arm and the stiffness matrix of the suspension were extracted from their finite element models constructed with commercial finite element analysis (FEA) package Ansys 10.0. For the actuator arm, it was treated as a homogeneous rigid body made of stainless steel. The finite element model for the actuator arm with elements SOLID95 is shown in Fig. 3.10.

Figure 3.10 Finite element model of the actuator arm

Since the actuator arm (Fig. 3.10) is very rigid compared with the flexible head-gimbal assembly (Fig. 3.11), the deformation of the actuator arm can be neglected when the suspension is loaded. Therefore, the head-gimbal assembly is separated from the actuator arm for analysis. The structure of the head-gimbal assembly is much more complex than that of the actuator arm. It contains rigid parts, i.e. the slider and the dimple, as well as flexible sheets, i.e. the load beam and the gimbal. The element SOLID95 was used for the slider and dimple, while the element SHELL90 was used for the load beam and gimbal. The constituent parts of the head-suspension assembly are made of two materials, stainless steel for the load beam, gimbal and dimple, and $\text{Al}_2\text{O}_3\text{-TLC}$ for the slider. The suspension model is shown in Fig. 3.11. The material properties are summarized in Table 3.3.

Figure 3.11 Finite element model of the head-gimbal assembly

Table 3.3 Properties of the materials of the head-arm assembly

Material	Stainless steel	Al ₂ O ₃ -TLC
Young's modulus (Pa)	1.93×10^{11}	4.12×10^{12}
Density (kg/m ³)	8030	4250
Poisson's ratio	0.3	0.27

In the HGA model, the gimbal is attached to the top of the slider by coupling displacements at five points, respectively. Hence, there is no relative translation and rotation between the two parts. The gimbal is connected to the summit of the dimple geometrically, where the gimbal is coupled with the dimple by all three degrees of freedom of displacement. Since the node of the gimbal element (SHELL90) has three degrees of freedom of rotation, rotation is allowed between them. As the slider is

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attached to the gimbal, the slider can only rotate about the dimple with the three degrees of freedom of displacement fully coupled. As a result, separation and slip do not occur between the dimple and slider, with the exception of rotation.

Equivalent Mass of the Actuator Arm

The mass and moment of inertia of the actuator arm about its center of mass can be obtained from the finite element model (Fig. 3.10). They will be used to derive the equivalent mass of the actuator.

The following data are obtained from ANSYS.

Total mass of the actuator arm (m_a): 35.616 g.

Center of mass: (x_{cm}, y_{cm}, z_{cm}): 55.674 mm, 1.3914×10^{-4} mm, 4.5492 mm.

Moment of inertia about z-axis of the center of mass (I_{cm}): 5.9462×10^3 g·mm².

The rotational axis is situated at $x_{rotation} = 55$ mm. Mass moment of inertia about the rotational axis J_{arm} can be obtained from J_{cm} .

$$J_{arm} = J_{cm} + m_a \times d^2 \quad (3.14)$$

where $d = x_{cm} - x_{rotation}$ is the distance of z-axis of the center of mass to the rotational axis of the actuator arm. Substituting the parameters into Eq. (3.14), J_{arm} is obtained

$$\begin{aligned} J_{arm} &= J_{cm} + m_a \times d^2 \\ &= 5.9462 + 0.035616 \times (55.674 - 55)^2 \\ &= 5.962 \text{ kg} \cdot \text{mm}^2 \end{aligned} \quad (3.15)$$

Equivalent mass of the actuator arm at the end of the suspension m_{arm} is calculated as follows.

$$m_{arm} = \frac{J_{arm}}{r^2} \quad (3.16)$$

$$\begin{aligned}
 &= \frac{5.9624 \text{ kg} \cdot \text{mm}^2}{52.5^2 \text{ mm}^2} \\
 &= 2.163 \times 10^{-3} \text{ kg}
 \end{aligned}$$

where r represents the distance of the pivot axis to the end of the suspension.

3.5.2 Sway Mode of the Suspension

The suspension has a bent angle of 12.6 degrees, as shown in Fig. 3.12. Since the base plate of the suspension is attached to the actuator arm which is very rigid and fixed on the actuator axis in the vertical direction, the base plate is assumed to have no deformation and displacement. Hence, in the analysis the base plate is fully

Figure 3.12 Loaded suspension

constrained. When loaded onto a rotating disk, the slider is lifted by the air bearing force. Under the loaded state, the distance of the slider bottom plane to the base

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plate, called the Z-height, is approximately 28 mils [114]. To simulate the effect of the air bearing, non-zero displacement constraints are added on the corners at the bottom of the slider. The displacements are calculated according to the Z-height. For the two corners on the leading edge, the vertical displacement is 1.99 mm, and for the two corners on the trailing edge 2.26 mm. The reaction forces on the slider corners are shown in Fig. 3.13. The total reaction force 24.5 mN (2.5 gram) is in accordance with the gram load for pico sliders, which ranges from 2.5 gram to 3 gram.

Figure 3.13 Reaction forces on the slider

Under the loaded condition, modal analysis was then conducted. The first 10 extracted modes are listed in Table 3.4. The 5th mode of 9.4 kHz shown in Fig. 3.14 is identified as the second torsional mode. It is the main source for the sway motion of the suspension. Thus, it is also called the sway mode. Figure 3.15 shows the top view

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of the sway mode, which illustrates the transverse motion induced by torsion.

The resonant frequency of the sway mode may vary depending on the suspension design. For Hutchinson 830 TSA and Fujitsu Pico-Caps, the resonant frequencies of the sway mode are 12.8 kHz and 8.6 Hz, respectively [53]. Furthermore, the latter has an optimized sway mode of 10.2 kHz [115].

Table 3.4 The first ten modes of the suspension

Mode	1	2	3	4	5	6	7	8	9	10
Frequency (kHz)	1.6	4.4	5.2	8.5	9.4	10.9	11.6	14.7	17.9	21.1

Figure 3.14 Suspension sway mode

Figure 3.15 Top view of the sway mode

3.5.3 Suspension Stiffness Matrix

The suspension was analyzed to extract its stiffness matrix. A four-by-four flexibility matrix was first obtained and then inverted to a stiffness matrix. To calculate the terms in the flexibility matrix, a force or a moment was added on the slider. Resulted displacement and rotations were found and related flexibility terms can be calculated. A flexibility matrix could be assembled by filling in all the terms. Here the displacement constraints were converted to equivalent force loading by using the reaction forces as shown in Fig. 3.13.

The terms in the first column of the flexibility matrix $[\lambda]$ are defined as follows.

$$\lambda_{11} = \frac{U_x}{F_x}, \lambda_{21} = \frac{U_z}{F_x}, \lambda_{31} = \frac{\theta}{F_x}, \lambda_{41} = \frac{\beta}{F_x} \quad (3.17)$$

where U_x, U_z, θ, β are respectively the displacement in x direction and

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displacement in z direction at the center of gravity of the slider, pitch angle and roll angle induced by F_x at the center of gravity of the slider (Fig. 3.16).

Figure 3.16 Loaded slider subject to a transverse force F_x at the center of gravity

The terms in the second column of the flexibility matrix $[\lambda]$ are defined similarly as follows.

$$\lambda_{12} = \frac{U_x}{F_z}, \lambda_{22} = \frac{U_z}{F_z}, \lambda_{32} = \frac{\theta}{F_z}, \lambda_{42} = \frac{\beta}{F_z} \quad (3.18)$$

In this case, the force is changed to F_z in the z direction as shown in Fig. 3.17.

To obtain the third and fourth column of $[\lambda]$, a torque about the x -axis M_θ and the y -axis M_β are applied, as shown in Fig. 3.18 and Fig. 3.19, respectively. The third column is obtained by

$$\lambda_{13} = \frac{U_x}{M_\theta}, \lambda_{23} = \frac{U_z}{M_\theta}, \lambda_{33} = \frac{\theta}{M_\theta}, \lambda_{43} = \frac{\beta}{M_\theta} \quad (3.19)$$

Figure 3.17 Loaded slider subject to a vertical force F_z at the center of gravity

In a similar way, the fourth column can be found from

$$\lambda_{14} = \frac{U_x}{M_\beta}, \quad \lambda_{24} = \frac{U_z}{M_\beta}, \quad \lambda_{34} = \frac{\theta}{M_\beta}, \quad \lambda_{44} = \frac{\beta}{M_\beta} \quad (3.20)$$

Figure 3.18 Loaded slider subject to a pitch moment M_θ

Figure 3.19 Loaded slider subject to a roll moment M_β

The full flexibility matrix $[\lambda]$ was found to be

$$[\lambda] = \begin{bmatrix} 0.67 \times 10^{-3} & 0 & 0 & 2.74 \\ 0 & 8.81 \times 10^{-4} & -9.8 & 0 \\ 0 & -9.8 & 8.76 \times 10^3 & 0 \\ 2.74 & 0 & 0 & 1.62 \times 10^4 \end{bmatrix} \text{ (N, m, rad)} \quad (3.21)$$

Hence, the stiffness matrix was obtained by inverting $[\lambda]$ in Eq. (3.21)

$$[K] = \begin{bmatrix} 4.87 \times 10^3 & 0 & 0 & -0.82 \\ 0 & 12.99 & 1.46 \times 10^{-2} & 0 \\ 0 & 1.46 \times 10^{-2} & 1.31 \times 10^{-4} & 0 \\ -0.82 & 0 & 0 & 2 \times 10^{-4} \end{bmatrix} \text{ (N, m, rad)} \quad (3.22)$$

3.5.4 Air Bearing Stiffness Matrix for a Pico Slider

The air bearing stiffness and damping matrices can be identified using experimental modal analysis techniques [116, 117]. In open literature, there has not

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been any published full air bearing matrix for a pico slider. The air bearing matrix for the pico slider in this study was obtained from a diagonal air bearing matrix for a pico slider in conjunction with an air bearing matrix for a nano slider. The diagonal pico air bearing matrix is [118]

$$[k_{diag}] = \begin{bmatrix} 374.6 \times 10^3 & 0 & 0 \\ 0 & 133.1 \times 10^{-3} & 0 \\ 0 & 0 & 35.8 \times 10^{-3} \end{bmatrix} \quad (3.23)$$

The nano slider with dimensions of $2.05 \times 1.6 \times 0.43 \text{ mm}^3$ was the previous generation of the slider. An air bearing matrix for one particular nano slider is given below [119]

$$[k_{nano}] = \begin{bmatrix} 1.76 \times 10^6 & -8.55 \times 10^2 & -8.45 \\ -8.55 \times 10^2 & 0.967 & 1.04 \times 10^{-2} \\ -8.45 & 1.04 \times 10^{-2} & 0.383 \end{bmatrix} \text{ (N, m, rad)} \quad (3.24)$$

The normalization and scaling procedure used to derive the air bearing matrix for the pico slider is described as follows.

First, the nano air bearing matrix is normalized with respect to its dimensions.

The relationship among force, stiffness and displacement can be written as

$$\begin{bmatrix} F_z \\ M_\theta \\ M_\beta \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ \beta \end{bmatrix} \quad (3.25)$$

Expanding the first row of Eq. (3.25),

$$\begin{aligned} F_z &= k_{11}z + k_{12}\theta + k_{13}\beta \\ &= k_{11}z + \frac{2}{L}k_{12}z_\theta + \frac{2}{B}k_{13}z_\beta \end{aligned} \quad (3.26)$$

where z_θ and z_β are the equivalent displacements in z direction at $y = \frac{L}{2}$ and $x = \frac{B}{2}$, L and B are the length and width of the pico slider, respectively. Therefore, the

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stiffness terms corresponding to k_{12} and k_{13} for the normalized stiffness matrix $[\mathbf{k}^*]$ can be defined as

$$k_{11}^* = k_{11}, \quad k_{12}^* = \frac{2}{L} k_{12}, \quad k_{13}^* = \frac{2}{B} k_{13} \quad (3.27)$$

Expanding the second row of Eq. (3.25) in a similar way,

$$\begin{aligned} M_\theta &= k_{21}z + k_{22}\theta + k_{23}\beta \\ &= k_{21}z + \frac{2}{L} k_{22}z_\theta + \frac{2}{B} k_{23}z_\beta \end{aligned} \quad (3.28)$$

When substituting $M_\theta = F_\theta \times \frac{L}{2}$ into Eq. (3.28), where F_θ is the equivalent force,

Equation (3.28) becomes

$$F_\theta = \frac{2}{L} k_{21}z + \frac{4}{L^2} k_{22}z_\theta + \frac{4}{BL} k_{23}z_\beta \quad (3.29)$$

Hence,

$$k_{21}^* = \frac{2}{L} k_{21}, \quad k_{22}^* = \frac{4}{L^2} k_{22}, \quad k_{23}^* = \frac{4}{BL} k_{23} \quad (3.30)$$

$k_{31}^*, k_{32}^*, k_{33}^*$ can be obtained following the same procedure

$$k_{31}^* = \frac{2}{B} k_{31}, \quad k_{32}^* = \frac{4}{BL} k_{32}, \quad k_{33}^* = \frac{4}{B^2} k_{33} \quad (3.31)$$

Therefore, the normalized stiffness matrix is given by

$$[\mathbf{k}^*] = \begin{bmatrix} k_{11} & \frac{2}{L} k_{12} & \frac{2}{B} k_{13} \\ \frac{2}{L} k_{21} & \frac{4}{L^2} k_{22} & \frac{4}{BL} k_{23} \\ \frac{2}{B} k_{31} & \frac{4}{BL} k_{32} & \frac{4}{B^2} k_{33} \end{bmatrix} \quad (3.32)$$

Using the above principles, $[\mathbf{k}]$ can also be written in terms of elements of $[\mathbf{k}^*]$

$$[\mathbf{k}] = \begin{bmatrix} k_{11}^* & \frac{L}{2} k_{12}^* & \frac{B}{2} k_{13}^* \\ \frac{L}{2} k_{21}^* & \frac{L^2}{4} k_{22}^* & \frac{BL}{4} k_{23}^* \\ \frac{B}{2} k_{31}^* & \frac{BL}{4} k_{32}^* & \frac{B^2}{4} k_{33}^* \end{bmatrix} \quad (3.33)$$

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Substituting the numbers in $[k_{diag}]$ from Eq. (3.23) and $[k_{nano}]$ from Eq. (3.24) respectively, $[k_{diag}^*]$ and $[k_{nano}^*]$ can be calculated

$$[k_{diag}^*] = \begin{bmatrix} 3.75 \times 10^5 & 0 & 0 \\ 0 & 3.4 \times 10^5 & 0 \\ 0 & 0 & 1.43 \times 10^5 \end{bmatrix} \text{ (N, m)} \quad (3.34)$$

$$[k_{nano}^*] = \begin{bmatrix} 1.76 \times 10^6 & -8.34 \times 10^5 & -1.06 \times 10^4 \\ -8.34 \times 10^5 & 9.2 \times 10^5 & 1.27 \times 10^4 \\ -1.06 \times 10^4 & 1.27 \times 10^4 & 5.98 \times 10^5 \end{bmatrix} \text{ (N, m)} \quad (3.35)$$

The three ratios of the diagonal terms in $[k_{nano}^*]$ of Eq. (3.35) to those corresponding ones in $[k_{diag}^*]$ of Eq. (3.34) are found to be 4.7, 2.7 and 4.2. The off-diagonal ratios are obtained by averaging the neighboring diagonal ratios. Thus, the scaling matrix $\mathbf{R}$ is formed as

$$[\mathbf{R}] = \begin{bmatrix} 4.7 & 3.7 & 4.45 \\ 3.7 & 2.7 & 3.45 \\ 4.45 & 3.45 & 4.2 \end{bmatrix} \quad (3.36)$$

Dividing the elements of $[k_{nano}^*]$ by the elements of $[\mathbf{R}]$ respectively, a full normalized pico air bearing stiffness matrix $[k_{pico}^*]$ is obtained

$$[k_{pico}^*] = \begin{bmatrix} 3.75 \times 10^5 & -2.25 \times 10^5 & -2.38 \times 10^3 \\ -2.25 \times 10^5 & 3.4 \times 10^5 & 3.68 \times 10^3 \\ -2.38 \times 10^3 & 3.68 \times 10^3 & 1.43 \times 10^5 \end{bmatrix} \quad (3.37)$$

The normal full pico air bearing stiffness matrix $[k_{pico}]$ can be found by substituting the individual terms of $[k_{pico}^*]$ from Eq. (3.37) into Eq. (3.33)

$$[k_{pico}] = \begin{bmatrix} 3.75 \times 10^5 & -1.41 \times 10^2 & -1.19 \\ -1.41 \times 10^2 & 1.33 \times 10^{-1} & 1.15 \times 10^{-3} \\ -1.19 & 1.15 \times 10^{-3} & 3.58 \times 10^{-2} \end{bmatrix} \text{ (N, m, rad)} \quad (3.38)$$

Stiffness Matrix for the Two-DOF Model

As shown in Eq. (3.1), the stiffness matrix for the two-DOF model is written by

$$[\mathbf{K}_2] = \begin{bmatrix} k_{arm} + k_s & -k_s \\ -k_s & k_s \end{bmatrix} \quad (3.39)$$

where equivalent stiffness of the actuator arm k_{arm} is 216 N/m [20].

Stiffness Matrix for the Five-DOF Model

The system stiffness matrix $[\mathbf{K}_c]$ is formed by combining the equivalent stiffness of the actuator arm in the transverse direction, k_{arm} , the air bearing stiffness matrix $[\mathbf{k}_{pico}]$ and the suspension stiffness matrix $[\mathbf{K}]$, as follows.

$$[\mathbf{K}_c] = \begin{bmatrix} k_{arm} + K_{11} & -K_{11} & 0 & 0 & 0 \\ -K_{11} & K_{11} & 0 & 0 & K_{14} \\ 0 & 0 & K_{22} + k_{11} & K_{23} + k_{12} & k_{13} \\ 0 & 0 & K_{32} + k_{21} & K_{33} + k_{22} & k_{23} \\ 0 & K_{41} & k_{31} & k_{32} & K_{44} + k_{33} \end{bmatrix} \quad (3.40)$$

$[\mathbf{K}_c]$ can be constructed by substituting the values in Eqs. (3.22) and (3.38) into Eq. (3.40). Comparing $[\mathbf{k}_{pico}]$ and $[\mathbf{K}]$, it can be seen that the elements in the air bearing stiffness matrix is at least two orders greater than their counterparts in the suspension stiffness matrix. Therefore after adding $[\mathbf{k}_{pico}]$ to $[\mathbf{K}]$, the elements for z , θ and β in the combined matrix $[\mathbf{K}_c]$ will be nearly equal to the corresponding elements from the air bearing stiffness matrix. Substituting k_{arm} and the elements of $[\mathbf{k}_{pico}]$ and $[\mathbf{K}]$ into Eq. (3.40), the final system stiffness matrix $[\mathbf{K}_c]$ is obtained,

$$\mathbf{K}_c = \begin{bmatrix} 1.47 \times 10^4 & -1.45 \times 10^4 & 0 & 0 & 0 \\ -1.45 \times 10^4 & 1.45 \times 10^4 & 0 & 0 & -8.21 \times 10^{-1} \\ 0 & 0 & 3.75 \times 10^5 & -1.41 \times 10^2 & -1.19 \\ 0 & 0 & -1.41 \times 10^2 & 1.33 \times 10^{-1} & 1.15 \times 10^{-3} \\ 0 & -8.21 \times 10^{-1} & -1.19 & 1.15 \times 10^{-3} & 3.58 \times 10^{-2} \end{bmatrix}, \text{ (N, m, rad)} \quad (3.41)$$

3.6 Summary

A two-DOF model and a five-DOF model were developed to simulate the transverse vibration of the head-stack assembly.

The two-DOF model was based on the rotational mode of the actuator arm and the sway mode of the suspension. The actuator arm rotation was converted to linear motion of an equivalent mass in the transverse direction. The suspension was represented by a spring-damper system. When provided with the friction force, the model is capable of calculating the transverse displacement of the slider with minimal computation cost.

In addition to the transverse displacement of the actuator arm, the transverse displacement of the slider, the vertical displacement, pitch and roll were considered in the five-DOF model. It can simulate the dynamics behavior of the slider induced by intermittent contact between the slider and multiple bumps on a rotating disk. Normal contact force and friction force can be calculated. The model can be used to study the effects of the coefficient of friction, rotational speed of the disk, bump spacing, etc, on the transverse displacement of the slider.

The four-by-four suspension stiffness matrix was extracted from a finite element model. The air bearing stiffness matrix for a pico slider was obtained by scaling the air bearing stiffness matrix for a nano slider. By combining the suspension stiffness matrix, the air bearing stiffness matrix and the stiffness of the actuator arm in the transverse direction, the five-by-five stiffness matrix for the five-DOF model was constructed.

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Although the simulation models and the suspension stiffness matrix extraction method were developed based on a specific 3.5-inch hard disk drive, they can also be applied to other types of hard disk drives, such as 2.5-inch hard disk drives, due to their structural similarities. When studying the head-slider off-track vibrations for a hard disk drive with a different form factor, only the physical parameters, such as masses and stiffness matrices, need to be obtained and supplied to the simulation models.

Chapter 4 Modeling Results of Slider Off-Track Vibrations

The dynamic behavior of the slider due to the normal contact and friction forces has been investigated in this chapter. Effects of the coefficient of friction, skew angle, bump spacing, and rotational speed of the disk on the transverse displacement of the slider were analyzed.

4.1 Numerical Method for Solving the Governing Equations

The governing equations, Eq. (3.2) for separation and Eq. (3.4) for contact, are ordinary differential equations (ODEs). Runge-Kutta methods are the most commonly used numerical recipes for solving ODEs [120]. The ODE solver ODE45 in Matlab which is based on an explicit Runge-Kutta (4, 5) formula[121], was used in this work.

The solution scheme is shown in Fig. 4.1. In computation, conditions specified by

Figure 4.1 Computation scheme for solving the governing equations

Inequalities (3.3) are monitored in each marching step. The length of the marching step used is 1×10^{-7} s, when no event (contact or separation) occurs. If an event is detected, the marching step will be varied to find the optimal location of the event. When contact is detected, integration of Eq. (3.2) will be terminated and the integration scheme will be switched to that of Eq. (3.4). The displacements, angular displacements, velocities, and angular velocities at the end of the integration scheme for Eq. (3.2) are used as initial conditions for continuation of the solution process for Eq. (3.4).

4.2 Results and Discussion

4.2.1 Five-DOF Model

In the analysis, the stiffness k_d and damping coefficient c_d for the disk were set to 1.5×10^6 N/m and 0.3079 N-s/m, respectively [89]. As shown in Fig. 4.2, the flying height at the center of the trailing edge in the equilibrium state $z_{t0} = 10$ nm, and the actuator arm stiffness $k_{arm} = 216$ N/m. The bump height $h_b = 30$ nm. The pitch angle in the equilibrium state θ_0 was set to $190 \mu\text{rad}$, which is used as an initial condition for solving the ordinary differential equations in Eq. (3.2). The other initial conditions were given by $x_{arm} = 0$, $x = 0$, $z = 0$, and $\beta = 0$. The initial position of the bump was aligned with the leading edge of the slider.

Figure 4.2 Operation conditions for the head-gimbal assembly: (a) Slider flying state;

(b) Skew angle

A single Bump and a Small Number of Bumps

Compared to the period of the rotational mode of the actuator arm and that of the sway mode of the suspension, the contact time for a single bump or a small number of bumps is limited to a very short duration. Therefore, the excitation provided by the contact acts like an impulse and the responses of the slider are transient in nature. A reference case with a single bump was first studied, followed by cases of ten equally spaced bumps and an infinite number of bumps with varying rotational speeds of the disk. The effects of the coefficient of friction and the skew angle on the transverse displacement of the slider were also examined.

In the reference case, the major parameters were set as follows: the skew angle $\phi = 10^\circ$, rotational speed of the disk $n = 7200$ rpm, coefficient of friction $\mu = 0.5$ and the radius of the track where the slider was positioned was 40 mm. The results of F_n , x , θ , β and the flying height at the center of the trailing edge z_t are shown in Fig. 4.3.

Figure 4.3 Response of the slider to a single bump: (a) normal contact force; (b) transverse displacement of the slider; (c) flying height; (d) pitch angle; (e) roll angle

The contact force with a duration of 4×10^{-6} s is just a narrow pulse compared to the period of the slider vibration in the x -direction as shown in Fig. 4.3(a, b). Thus, the motion of the slider in the x -direction is basically an impulse response which is reflected in Fig. 4.3(b) as well as Fig. 4.3(e) due to the coupling between x and β . The off-track motion of the slider caused by a single bump is only several nanometers as

shown in Fig. 4.3(b). However, this still affects the track misregistration appreciably for disk drives with extremely high areal densities. Comparing Fig. 4.3(d) and Fig. 4.3(e), only the pitch angle has slight oscillations while the roll mode is barely excited, with a very small amplitude ($< 0.2 \mu\text{rad}$). This is because the contact happens near the center of the trailing edge, creating only a small roll moment. The slider is elevated and rotates upon contact, with an increased flying height and a reduced pitch angle. After the bump leaves the contact, the slider vibrates freely and the vertical displacement and pitch angle with resonant frequencies of 77.5 kHz and 126.7 kHz respectively, die out quickly under damping (Fig. 4.3(c, d)). The damping ratios for the vertical displacement and pitch angle are 0.16 and 0.28 respectively, which are relatively high compared to the damping ratio of 0.016 for the transverse displacement. For the roll angle, the higher frequency component related to the air bearing also disappears quickly, while the lower frequency component coupled with the transverse displacement of the suspension lasts much longer (Fig. 4.3(e)).

Effects of the Coefficient of Friction and the Skew Angle

The effects of the coefficient of friction and the rotational speed of the disk on the maximum transverse displacement of the slider are shown in Fig. 4.4. The spacing for the ten equally spaced bumps was 50 μm . The bumps were situated on the track with a radius of 40 mm. The maximum transverse displacement increases nearly linearly with the coefficient of friction while it decreases with the rotational speed.

The shape of the contact force (Fig. 4.3(a)) is related to the rotational speed of the disk. For a higher rotational speed, the slider tends to disengage earlier from the slider

resulting in a shorter contact time, whereas the magnitude of the contact force becomes larger (Fig. 4.5). If the time that the slider and disk remain engaged is longer than the contact period, the contact force may complete more than one cycle as the case of $n = 7200$ rpm in Fig. 4.5. Therefore, the contact force may have more than one peak.

Figure 4.4 Maximum transverse displacement of the slider versus the coefficient of friction for different rotational speeds of the disk

Nevertheless, the effect of the reduced contact time outweighs that of the increased contact force, leading to a smaller impulse of the contact force. In Fig. 4.5, for $n = 7200$ rpm, the impulse is calculated to be 11.03 nN-s, while for $n = 10000$

rpm, it is 9.37 nN-s. A smaller impulse provides a smaller excitation, thus generating a smaller maximum displacement for $n = 10000$ rpm. Figure 4.4 also reveals that, for the same coefficient of friction and rotational speed of the disk, multiple bumps cause a greater transverse displacement due to an increased impulse. The effect of the skew angle ϕ on the maximum transverse displacement is very similar to that of the coefficient of friction μ , as shown in Fig. 4.6.

Figure 4.5 Normal contact force for a single bump versus time

This is because an increase in either ϕ or μ caused an increase in the excitation in the transverse direction according to Eq. (3.6). For a single bump as well as a small number of bumps, the maximum transverse displacement is almost zero, when the slider is flying at zero skew. The transverse displacement increases linearly with the

skew angle. Figure 4.6 also demonstrates the same effects of the rotational speed and multiple bumps on the maximum transverse displacement as shown in Fig. 4.4.

An Infinite Number of Bumps in a Sequence

For a rotating disk with equally spaced bumps on the circumference of a certain radius, it is equivalent to an infinite number of bumps in a sequence passing under the

Figure 4.6 Maximum transverse displacement of the slider versus the skew angle for different rotational speeds of the disk

slider. Since the slider is repeatedly excited by the contact interaction, its response will reach a steady state after a certain period of time. Therefore, in computation, only a sufficiently large finite number of bumps were needed for the slider to reach a steady state. The behavior of the slider for a large number of bumps is shown in Figs.

4.7 and 4.8. Unlike a single bump which produces only a single narrow pulse of the contact force as well as the friction force, multiple bumps provide repeated pulses. In the case of a bump spacing of 50 μm , for a rotational speed of 7200 rpm, the frequency of the pulses is about 603 kHz which is much higher than the natural frequency of the suspension sway mode of 10.3 kHz considered in this study.

Figure 4.7 Response of the slider to a large number of bumps: (a) normal contact force; (b) flying height; (c) pitch angle ($\theta_0 = 190 \mu\text{rad}$)

When the bumps come into contact one after another, the contact force forms a pulse train. As a result, the overall response behaves like that to a step function. Due to the lower natural frequency of the rotational mode of the actuator arm, the transverse displacement mainly follows the trend of the actuator arm after a certain

period of time. The vibration of the suspension with a relatively small amplitude is superimposed on the motion of the actuator arm in the x -direction, as seen in Fig. 4.8(a). The behavior of the roll angle exhibits a similar trend to the transverse displacement (Fig. 4.8(a, b)), this is because the two motions are strongly coupled. When the slider vibration reaches a steady state, the transverse displacement can continue to develop to a level of micrometers, indicating an undesirable situation of the magnetic head moving across a great number of data tracks.

Figure 4.8 Response of the slider to a large number of bumps: (a) transverse displacement; (b) roll angle

Figure 4.9 shows the transverse acceleration of the slider induced by a large number of bumps. It is composed of two frequencies, corresponding to those in the

transverse displacement as shown in Fig. 4.8(a). The maximum of the transverse acceleration is 22g, which the MEMS accelerometer should be able to detect. Therefore, in the design of the MEMS accelerometer as discussed in Chapter 5, its measurement range must be larger than 22g.

Figure 4.9 Transverse acceleration of the slider induced by a large number of bumps

Effects of the Bump Path Offset under a Zero Skew Condition

With the slider flying at a zero skew angle, transverse motion up to about 20 nm was experimentally observed [122]. Obviously, this phenomenon was not caused by the friction force component in the transverse direction since the latter was equal to zero for a zero skew angle. It may be explained by examining the effects of the bump path offset with the present model. When the skew angle is zero and the moving

bumps are passing along the center line of the slider (Fig. 4.10 (a)), the transverse motion is very small (< 3 nm) as indicated in Fig. 4.11.

Figure 4.10 Bump paths on a slider at a zero skew angle: (a) bumps moving along the center line; (b) bumps moving parallel to the center line

However, when the bump path is parallel to the center line but with an offset of $-80\text{ }\mu\text{m}$ (close to an edge of the rear pad in Fig. 4.10(b)), the transverse motion reaches a steady-state of 37 nm (Fig. 4.11).

Since the friction component in the transverse direction is zero in both cases, the transverse motion is not caused by friction. Instead, the strong coupling between the roll and transverse motions is the responsible factor. The roll motion is excited by a moment equal to the normal contact force multiplied by the offset distance from the center line. Figure 4.12 shows that the roll angle reaches a magnitude of about $9\text{ }\mu\text{rad}$, when the bump path has an offset of $-80\text{ }\mu\text{m}$, while the magnitude of the roll angle is only $0.6\text{ }\mu\text{rad}$ with the offset being zero.

Although the normal contact force and friction force cannot excite the roll angle when the bumps are passing through the center line with a zero skew, the roll angle is

Figure 4.11 Transverse displacement versus time with a zero skew angle

Figure 4.12 Roll angle versus time with a zero skew angle

not absolutely zero. The negligible roll angle in Fig. 4.12 is believed to be caused by the coupling between the pitch and roll.

Effects of the Bump Spacing

The level of the steady-state of the transverse displacement has a strong connection to the circumferential spacing of the bumps. This steady-state displacement level decreases with increasing bump spacing as shown in Fig. 4.13.

Figure 4.13 Transverse displacement versus time for different bump spacings

For a large number of bumps, the excitation from repeated contacts acts approximately like a step function. Therefore, the average contact force can be used as an indicator for the level of excitation, thus for the amplitude of the transverse displacement in the steady state. The average contact force can be expressed in terms

of the maximum contact force $F_{n, max}$, contact time ratio R_c , and a shape factor of the contact force ξ ,

$$F_{n, ave} = \xi R_c F_{n, max} \quad (4.1)$$

where R_c is defined as

$$R_c = \frac{t_c}{T} \quad (4.2)$$

where t_c is the contact time of a single cycle, and T is the period of the contact force, respectively. The shape factor ξ is defined as the ratio of the actual impulse to the maximum rectangular impulse of the contact force for a single cycle

$$\xi = \frac{\int_t^{t+t_c} F_n dt}{t_c F_{n, max}} \quad (4.3)$$

Plotted in Fig. 4.14 are $F_{n, max}$, R_c , ξ and $F_{n, ave}$. The contact time ratio tends to decrease with increasing bump spacing as shown in Fig. 4.14(b). Increasing the bump spacing also makes the average contact force deviate more from a rectangular pulse shape, thus leading to a decrease in ξ (Fig. 4.14(c)). The decreasing trend of the average contact force $F_{n, ave}$ as calculated from Eq. (4.1) and shown in Fig. 4.14(d), explains the transverse displacement that decreases with increasing bump spacing, as revealed in Fig. 4.13.

Effects of the Rotational Speed of the Disk

The steady-state transverse displacement of the slider changes slightly with the rotational speed of the disk (Fig. 4.15).

For the selected five speeds (1000, 2000, 5400, 7200 and 10000 rpm), the transverse displacement increases in the lower portion of the speed range. After it

Figure 4.14 Effects of the bump spacing on the contact force: (a) maximum contact force; (b) contact time ratio; (c) shape factor; (d) average contact force

Figure 4.15 Transverse displacement versus time for different disk rotational speeds

reaches a maximum at 5400 rpm, the transverse displacement starts to decrease. However, it can be concluded from Fig. 4.15 that the transverse displacement is fairly insensitive to changes in the rotational speed of the disk. The effect of contact force on the transverse displacement is determined by its maximum value, frequency, and shape under the steady state. Figure 4.16 shows two pulses of the contact force under steady state for rotational speeds of 2000 rpm, 5400 rpm and 7200 rpm. In order to compare, the starting point for all the pulses is assumed to be at the origin on the time axis. It can be seen from Fig. 4.17 that increasing the rotational speed has two opposite effects on the normal contact force. The magnitude of a single contact force impulse decreases, while the frequency of the normal contact force increases. Among

Figure 4.16 Two consecutive impulses of the contact force under steady state for different disk rotational speeds

the constituting aspects of the average contact force, $F_{n, max}$, R_c and ξ all change only slightly with the rotational speed as shown in Fig. 4.17. The maximum percentages of change for the maximum contact force $F_{n, max}$, contact time ratio R_c and the shape factor ξ are 22%, 33% and 16%, respectively. Their overall effects on the average contact force $F_{n, ave}$ are even smaller, with a maximum of change less than 3.8% for the latter (Fig. 4.17(d)). The small variation of the average contact force to the rotational speed explains the insensitivity of the transverse displacement to the rotational speed (Fig. 4.15). The low peak of the average contact force $F_{n, max}$ at 5400 rpm in Fig. 4.17 (d) also coincides with the maximum displacement at the same rotational speed in Fig. 4.15.

Figure 4.17 Effects of the disk rotational speed on the contact force: (a) maximum contact force; (b) contact time ratio; (c) shape factor; (d) average contact force

4.2.2 Two-DOF model

It can be found from Fig. 4.8 that the off-track motion is mainly a superposition of two components, the response of the actuator arm to a step function and the damped vibration of the slider due to the suspension sway mode at a higher frequency. To confirm the dominance of the relevant modes, a two-DOF model was also developed (Fig. 3.6). Applying the friction force extracted from the five-DOF model on the slider to the two-DOF model, the transverse displacement of the slider from the two-DOF model agrees to a large extent with that from the five-DOF model (Fig. 4.18). This indicates that the rotational mode of the actuator arm and the suspension sway mode (both simplified into equivalent modes in the x -direction) are the principal contributors to the transverse displacement of the slider. Figure 4.18 also shows that the steady-state transverse displacement from the two-DOF model is about 180 nm smaller than that from the five-DOF model (1658 nm) with the coupling elements between the roll angle and the transverse displacement (x - β) in the stiffness matrix $[K_c]$ considered. When setting the x - β coupling elements equal to zero, it was found that the steady-state transverse displacement from the two-DOF model is only 45 nm smaller than that from the five-DOF model (1523 nm).

This confirms that the roll mode, which is excited by the moments generated by the normal contact force and friction force, can induce the transverse displacement through the coupling effect between them. This coupling is represented specifically by the crossing elements, K_{14} and K_{41} , of the stiffness matrix of the suspension, which enter the combined stiffness matrix $[K_c]$ given by Eq. (3.40).

Figure 4.18 Slider transverse displacements from the 2-DOF and 5-DOF models

4.3 Summary

A five-DOF model was proposed to study the off-track motion of a slider subjected to intermittent contact with the disk. Contact situations with a single bump and multiple bumps were investigated. The main conclusions are given as follows.

1. The maximum transverse displacement of the slider increases with the coefficient of friction and the skew angle almost linearly.
2. For a single bump or a small group of bumps, increasing the rotational speed of the disk leads to a decrease of the maximum transverse displacement due to the reduced contact force impulse caused by a shorter contact time.
3. For multiple bumps in a sequence, a smaller bump spacing results in a larger contact time ratio R_c and a larger shape factor ξ , which means a longer contact time

and a more uniform contact force for each bump, respectively. Both lead to an increased average normal contact force, which in turn is responsible for a greater transverse displacement of the slider.

4. The transverse displacement is rather insensitive to changes in the rotational speed of the disk. This can be attributed to the fact that the maximum contact force, the contact time ratio and the shape factor of the contact force only change slightly with the rotational speed.

5. The roll angle can contribute significantly to the transverse displacement of the slider through the coupling between the transverse displacement and roll angle as represented by the stiffness matrix of the suspension. For a zero skew angle, when the contact occurs near an edge of the rear pad, the slider experiences a considerable transverse displacement. Although there is no transverse component of the friction force, the roll mode is excited by the moment due to the normal contact force, thus inducing motion in the transverse direction through coupling between the transverse displacement and roll angle.

6. The off-track motion of the slider is dominated by the rotational mode of the actuator arm and the suspension sway mode. This has been verified with a two-DOF model for the transverse motions of the actuator arm and slider subjected to the friction force obtained from the five-DOF model. This finding suggests that, once the contact force is obtained from some air bearing-based model, the off-track motion of the slider can, subsequently, be estimated easily by using a simple two-DOF model. If the interest is only in the steady-state off-track displacement, it can be obtained

simply by multiplying the flexibility matrix by the force vector, thus eliminating the need of solving the ODEs for the two-DOF model.

7. The maximum acceleration of slider in the transverse direction induced by multiple bumps is 22g. To detect the motion of the slider in the transverse direction, the measurement range of the MEMS accelerometer must be greater than 22g.

8. The stiffness matrices, coefficients of friction and other parameters for the simulation models were determined based on a 3.5-inch hard disk drive and its typical operational conditions. Deviations in the parameters affect the slider dynamics and the simulation results. Effects of parameters like the coefficient of friction, rotational speed of the disk and bump spacing on the slider off-track motion were investigated in this study. Similarly, the effects of stiffness matrices and other parameters can also be studied. The results presented in this study can serve as useful reference for hard disk drives with similar characteristics.

Chapter 5 Design of the MEMS Accelerometer

An in-plane piezoresistive accelerometer with a supporting beam and two sensing beams was designed in this chapter. In conventional cantilever-type accelerometers, the sensing elements (piezoresistors) are fabricated on the cantilever (supporting beam). In this study, the piezoresistors are located on two additional tiny beams where much greater stress is generated than that in the central supporting beam. Thus, a higher sensitivity can be achieved. The performance of the accelerometer was optimized for the resonant frequency and high sensitivity.

5.1 Operating Principles of Accelerometers

Generally, an accelerometer consists of a proof mass supported by an elastic cantilever. An accelerometer can be modeled by a mass-spring system as illustrated in Fig. 5.1.

Figure 5.1 Lumped model of an accelerometer

The equation of motion can be written as [123]

$$m_p \ddot{\delta} + c \dot{\delta} + k \delta = -m_p \ddot{u} \quad (5.1)$$

where m_p is the proof mass, δ the proof mass displacement with respect to its frame, c the damping constant, k the effective spring constant of the flexure and u the displacement of the frame. The motion of the frame is assumed to be harmonic and of the form

$$u(t) = U_0 \cos \omega t \quad (5.2)$$

where ω is the angular frequency and U_0 the amplitude of the motion, respectively.

The response of the proof mass is given by

$$\delta(t) = \delta_0 \cos(\omega t - \varphi) \quad (5.3)$$

where δ_0 is the measurement amplitude, and φ the phase. The steady-state solution to Eq. (5.1) is

$$\delta(t) = U_0 \left(\frac{\omega}{\omega_n} \right) \psi \cos(\omega t - \varphi) \quad (5.4)$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad (5.5)$$

$$\psi = \frac{1}{\sqrt{\left\{ \left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right]^2 + \left(2\zeta \frac{\omega}{\omega_n} \right)^2 \right\}}} \quad (5.6)$$

$$\varphi = \tan^{-1} \frac{2\zeta \frac{\omega}{\omega_n}}{\left(\frac{\omega}{\omega_n} \right)^2} \quad (5.7)$$

$$\zeta = \frac{c}{2\omega_n m} \quad (5.8)$$

where ω_n is the natural frequency of the accelerometer, ψ the amplitude of the

frequency response, ζ the damping coefficient. Equating Eq. (5.3) and Eq. (5.4) gives

$$\frac{\delta_0}{U_0} = \left(\frac{\omega}{\omega_n} \right)^2 \psi \quad (5.9)$$

For small values of the ratio ω/ω_n , ψ is nearly unity, hence the measurement amplitude δ_0 can be approximated by [124]

$$\delta_0 = \frac{1}{\omega_n^2} U_0 \omega^2 \quad (5.10)$$

Equation (5.10) indicates the measurement amplitude δ_0 is proportional to the acceleration of the frame $U_0 \omega^2$. The proportionality $1/\omega_n^2$ is referred to as the static sensitivity. Therefore, a trade-off exists between the resonant frequency and the sensitivity of an accelerometer. Increasing the spring constant and decreasing the proof mass lead to a higher resonance frequency and a lower static sensitivity. A good design is an optimal combination of the resonant frequency and sensitivity

In this study, a numerical analysis was carried out to optimize the accelerometer's parameters. Unlike in previous studies [101, 102] in which the only constraint came from the pure axial deformation of the sensing beams, in this work geometric constraints associated with the dimensions of the trailing side of the slider are also included in the optimization. The effects of the dimensions of the accelerometer's main components on its natural frequency, sensitivity and the combinations of both were investigated with numerical analysis.

5.2 Analytical Modeling

Since the accelerometer is to be installed on the trailing side of the pico slider and its motion is in the lateral direction to the slider, the thin substrate of the accelerometer has to be parallel to the trailing side. Therefore, it is not practical to use an out-of-plane sensing structure for the accelerometer. As discussed in Chapter 1, an in-plane sensing structure can be implemented by doping the side walls of the cantilever, which is not a well established technique, or using additional sensing beams on both sides of the cantilever. The latter scheme is easier to fabricate and its structure has the advantage of being able to achieve higher stress in the sensing beams. 5.2 is a schematic of the in-plane accelerometer with additional sensing beams.

Figure 5.2 The in-plane accelerometer under acceleration

Under the inertial force, the proof mass experiences lateral translation as well as rotation. The two motions lead to lateral displacements at the end of the sensing beams in opposite directions. Hence, the effect of the rotation on the horizontal displacement at the end of the sensing beams can be utilized to cancel out that of the lateral translation of the proof mass. Thus, the tiny sensing beams will be deformed in nearly pure tension or compression. Analysis of the two motions will be performed in the design of the structure to keep the sensing beams at zero lateral displacement.

The stress and strain generated in the sensing beams are much larger than those in the supporting beam. Thus, a large output voltage and a high sensitivity can be achieved. Under the condition of axial deformation in the sensing beams, the structure can be optimized in terms of both the sensitivity and the resonant frequency.

It can be seen from Fig. 5.2 that the displacement of the proof mass may depend on the supporting beam length, the position of the proof mass center, the shape and dimensions of the proof mass, and so on.

5.2.1 A Simplified Model

By assuming a square proof mass and a single thickness for the central supporting beam and the proof mass, the accelerometer turns into a simplified model as shown in Fig. 5.3.

Figure 5.3 The simplified model of the accelerometer

The major geometric parameters of the accelerometer are the length and width of the square proof mass b , the length of the central supporting beam l_c , the width of the central supporting beam w_c , the length of the sensing beams l_s , the distance of the fixed end of the central supporting beam to the bottom of the proof mass h , and the eccentricity e .

When acceleration is applied, the central supporting beam will deflect as a cantilever and the proof mass will move like a rigid body, as shown in Fig. 5.4. The center of the proof mass is denoted by C and C' before and after the deflection of the central supporting beam, respectively.

Figure 5.4 Deflection of the central supporting beam and movement of the proof mass

Figure 5.5 shows the force and moment on the proof mass and the central supporting beam. Since the sensing beams are very slim compared with the central supporting beam and the proof mass, they will not affect the rotation and translation

Figure 5.5 Free body diagrams of the proof mass and the central supporting beam

of the proof mass significantly. Therefore, they can be neglected in the mechanics analysis for the proof mass and the central supporting beam.

By beam deflection analysis, the displacement and the slope at the end of the central supporting beam can be obtained. Once point O' is known, other points like A' , B' and D' can be found by geometry, since the proof mass is regarded as a rigid body.

The inertial force of the proof mass due to the motion of the frame F_i is related to the acceleration by

$$F_i = -\rho_{Si} \times b^2 \times t_p \times a \quad (5.11)$$

where ρ_{Si} is the density of single crystal silicon, t_p thickness of the proof mass, and a acceleration of the frame.

The moment M is determined by the inertial force F_i and the eccentricity e as

$$M = F_i \times e \quad (5.12)$$

The eccentricity e is the distance of the center of the proof mass to the free end of the central supporting beam

$$e = \overline{CO} = h + \frac{b}{2} - l_c \quad (5.13)$$

Substituting Eq. (5.13) into Eq. (5.12), one has

$$M = F_i \times \left(h + \frac{b}{2} - l_c \right) \quad (5.14)$$

Figure 5.5 shows that the central supporting beam is deflected by both F_i and M . The elastic curve of the central supporting beam with the point load F_i at its end is expressed by

$$x = \frac{F_i z^2}{2EI_c} \left(l_c - \frac{z}{3} \right) \quad 0 \leq z \leq l_c \quad (5.15)$$

where E is Young's modulus of single crystal silicon, I_c is the area moment of inertia of the central supporting beam cross section.

At the end of the central supporting beam, where $z = l_c$, the displacement is

$$\begin{aligned} x_F &= \frac{F_i l_c^2}{2EI_c} \times \left(l_c - \frac{l_c}{3} \right) \\ &= \frac{F_i l_c^3}{3EI_c} \end{aligned} \quad (5.16)$$

The deflection caused by M can be obtained by

$$x = \frac{Mz^2}{2EI_c} \quad (5.17)$$

Substituting $z = l_c$ into Eq. (5.17), deflection at the end of the central supporting beam caused by M is given by

$$x_M = \frac{Ml_c^2}{2EI_c} \quad (5.18)$$

Using the principle of superposition, the displacement at the end of the central supporting beam is

$$\begin{aligned} x_{O'} &= x_F + x_M \\ &= \frac{F_i l_c^3}{3EI_c} + \frac{Ml_c^2}{2EI_c} \end{aligned} \quad (5.19)$$

Upon the substitution of M from Eq. (5.14), Eq. (5.19) becomes

$$\begin{aligned} x_{O'} &= \frac{F_i l_c^3}{3EI_c} + \frac{Ml_c^2}{2EI_c} \\ &= \frac{F_i l_c^3}{3EI_c} + F_i \times \left(h + \frac{b}{2} - l_c \right) \times \frac{l_c^2}{2EI_c} \end{aligned}$$

$$= \frac{F_i}{EI_c} \left(-\frac{1}{6} l_c^3 + \frac{2h+b}{4} l_c^2 \right) \quad (5.20)$$

Similarly, the slope at the end of the central supporting beam can be derived. The slope caused by F_i is written as

$$\theta_F = \frac{F_i l_c^2}{2EI_c} \quad (5.21)$$

And the slope caused by M is given by

$$\theta_M = \frac{M l_c}{EI_c} \quad (5.22)$$

Combining θ_F and θ_M , the slope at the end of the central supporting beam is

$$\begin{aligned} \theta_{O'} &= \theta_F + \theta_M \\ &= \frac{F_i l_c^2}{2EI_c} + \frac{M l_c}{EI_c} \end{aligned} \quad (5.23)$$

Upon the substitution of M from Eq. (5.14), M can be removed from Eq. (5.23)

$$\begin{aligned} \theta_{O'} &= \frac{F_i l_c^2}{2EI_c} + \frac{M l_c}{EI_c} \\ &= \frac{F_i l_c^2}{2EI_c} + F_i \times \left(h + \frac{b}{2} - l_c \right) \frac{l_c}{EI_c} \\ &= \frac{F_i}{EI_c} \left(-\frac{1}{2} l_c^2 + \frac{2h+b}{2} l_c \right) \end{aligned} \quad (5.24)$$

Condition for Pure Axial Deformation of the Sensing Beams

It should be noted that only approximately pure axial deformation can be achieved in the two sensing beams, because it is impossible to attain zero horizontal displacement at points A' and D' simultaneously. The most desirable situation is to let A' and D' have equal horizontal displacements. Thus, neither of the two sensing beams will be subject to considerable bending. As a result, both can remain in

approximately pure axial deformation. To reach such a balanced status, the midpoint of the bottom edge of the proof mass in Fig. 5.4 should still lie on OB after deformation. In other words, the x coordinate of point B' has to be zero.

The x coordinate of point B' is related to that of point O' as

$$x_{B'} = x_{O'} - \overline{O'B'} \sin \theta_{O'} \quad (5.25)$$

Since $\sin \theta = \theta$ when θ is very small, Eq. (5.25) can be written in the form of

$$x_{B'} = x_{O'} - \overline{O'B'} \theta_{O'} \quad (5.26)$$

Considering the proof mass is a rigid body,

$$\overline{O'B'} = \overline{OB} = l_c - h \quad (5.27)$$

Substituting Eqs.(5.20), (5.24) and (5.27) into Eq.(5.26) gives

$$x_{B'} = \frac{F l_c^2}{EI_c} \left(\frac{1}{2}h + \frac{1}{4}b - \frac{1}{6}l_c \right) - (l_c - h) \frac{F l_c}{EI_c} \left(h + \frac{1}{2}b - \frac{1}{2}l_c \right) \quad (5.28)$$

Let $x_{B'} = 0$

$$\frac{F l_c^2}{EI_c} \left(\frac{1}{2}h + \frac{1}{4}b - \frac{1}{6}l_c \right) - (l_c - h) \frac{F l_c}{EI_c} \left(h + \frac{1}{2}b - \frac{1}{2}l_c \right) = 0 \quad (5.29)$$

Canceling out $\frac{F l_c}{EI_c}$ and collecting the remaining items,

$$4l_c^2 - (12h + 3b)l_c + 12h^2 + 6bh = 0 \quad (5.30)$$

The roots of the equation above can be written as

$$l_c = \frac{12h + 3b \pm \sqrt{(12h + 3b)^2 - 16(12h^2 + 6bh)}}{8} \quad (5.31)$$

From a practical point of view, l_c should be real and positive. Thus, the following constraint exists.

$$(12h + 3b)^2 - 16(12h^2 + 6bh) \geq 0 \quad (5.32)$$

Expanding and rearranging the terms on the left gives

$$-3(4h + 3b)(4h - b) \geq 0 \quad (5.33)$$

Because $4h + 3b$ is always positive, in order to satisfy the above condition, the following constraint must be imposed.

$$4h - b \leq 0, \quad h \leq \frac{1}{4}b \quad (5.34)$$

Therefore, in order to achieve the approximately axial deformation in the sensing beams, the length of the central supporting beam l_c , should be a function of the distance of the fixed end of the central supporting beam to the bottom of the proof mass h , and the width of the proof mass b , as stated in Eq. (5.31) with the constraint specified by Eq. (5.34).

5.2.2 Natural Frequency and Sensitivity

By converting the central supporting beam and the proof mass to an equivalent mass-spring system, the accelerometer natural frequency can be estimated.

The equivalent spring constant of the central supporting beam in Fig. 5.4 is given by

$$k_e = \frac{F_i}{x_F} \quad (5.35)$$

Substituting Eq. (5.16) into Eq. (5.35) yields

$$\begin{aligned} k_e &= \frac{F_i}{x_F} \\ &= \frac{3EI_c}{l_c^3} \end{aligned} \quad (5.36)$$

By the kinetic energy method, assuming a cubic shape for the beam deflection along

the beam span, the equivalent mass of the central supporting beam is known to be 0.2357 times its total mass [125]. Hence, the natural frequency of the accelerometer is expressed as

$$f_a = \frac{1}{2\pi} \sqrt{\frac{k_e}{m_p + 0.2357m_c}} \quad (5.37)$$

where m_p and m_c are the masses of the proof mass and the central supporting beam, respectively. Substituting Eq. (5.36), $m_p = \rho_{Si} b^2 t_p$ and $m_c = \rho_{Si} l_c w_c t_p$ into Eq. (5.37) yields

$$\begin{aligned} f_a &= \frac{1}{2\pi} \sqrt{\frac{k_e}{m_p + 0.2357m_c}} \\ &= \frac{1}{2\pi} \sqrt{\frac{\frac{3EI_c}{l_c^3}}{\rho_{Si} b^2 t_p + 0.2357 \rho_{Si} l_c w_c t_p}} \\ &= \frac{1}{2\pi} \sqrt{\frac{3EI_c}{\rho_{Si} b^2 t_p l_c^3 + 0.2357 \rho_{Si} l_c^4 w_c t_p}} \end{aligned} \quad (5.38)$$

The area moment of inertia of the central supporting beam cross section I_c is

$$I_c = \frac{t_p w_c^3}{12} \quad (5.39)$$

Upon the substitution of I_c from Eq. (5.39), Eq. (5.38) becomes

$$\begin{aligned} f_a &= \frac{1}{2\pi} \sqrt{\frac{3EI_c}{\rho_{Si} b^2 t_p l_c^3 + 0.2357 \rho_{Si} l_c^4 w_c t_p}} \\ &= \frac{1}{2\pi} \sqrt{\frac{3E \frac{t_p w_c^3}{12}}{\rho_{Si} b^2 t_p l_c^3 + 0.2357 \rho_{Si} l_c^4 w_c t_p}} \\ &= \frac{1}{4\pi} \sqrt{\frac{E w_c^3}{\rho_{Si} b^2 l_c^3 + 0.2357 \rho_{Si} l_c^4 w_c}} \end{aligned} \quad (5.40)$$

It is known from Eq. (5.40) that the natural frequency f_a is independent of the

thickness of the proof mass and central supporting beam t_p .

In general, the sensitivity of an accelerometer is the output voltage of the Wheatstone bridge per unit acceleration. The output voltage is related to the strain in the sensing elements. Since the sensing beams are under approximately pure axial deformation, their axial strain can be calculated after the z coordinates of points A' and D' are known. Due to the fact that the two sensing beams will undergo the same amount of strain with opposite signs, the following sensitivity analysis is based on only the strain analysis of beam 1.

As shown in Fig. 5.4, the z coordinate of A' is given by

$$z_{A'} = l_c - \overline{O'A'} \cos(\phi + \theta_{O'}) \quad (5.41)$$

where

$$\overline{O'A'} = \overline{OA} = \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \quad (5.42)$$

and

$$\tan \alpha = \frac{\frac{b}{2}}{l_c - h}, \quad \alpha = \text{atan}\left(\frac{b}{2(l_c - h)}\right) \quad (5.43)$$

Substituting Eqs. (5.24), (5.42) and (5.43) into Eq. (5.41), one obtains

$$\begin{aligned} z_{A'} &= l_c - \overline{O'A'} \cos(\alpha + \theta_{O'}) \\ &= l_c - \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \times \\ &\quad \cos\left(\frac{F_i}{EI_c} \left(-\frac{1}{2}l_c^2 + \frac{2h+b}{2}l_c\right) + \text{atan}\left(\frac{b}{2(l_c - h)}\right)\right) \end{aligned} \quad (5.44)$$

As shown in Fig. 5.4, the z coordinate of point A is simply

$$z_A = h \quad (5.45)$$

By the definition of strain, the axial strain of sensing beam 1 is written as

$$\varepsilon_1 = \frac{z_{A'} - z_A}{l_s} \quad (5.46)$$

Upon substitution of Eqs. (5.44) and (5.45), Eq. (5.46) becomes

$$\begin{aligned} \varepsilon_1 &= \frac{z_{A'} - z_A}{l_s} \\ &= \frac{1}{l_s} \left\{ l_c - \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \times \right. \\ &\quad \left. \cos\left(\frac{F}{EI_c} \left(-\frac{1}{2}l_c^2 + \frac{2h+b}{2}l_c\right) + \operatorname{atan}\left(\frac{b}{2(l_c - h)}\right)\right) - h \right\} \end{aligned} \quad (5.47)$$

In the general case where both transverse and longitudinal stresses are present, the resistance change ratio for a piezoresistor is

$$\Delta R / R = \pi_t \sigma_t + \pi_l \sigma_l \quad (5.48)$$

where ΔR and R are the resistance change and the original resistance of the piezoresistor; π_t and π_l are the transverse and longitudinal piezoresistance coefficients of single crystal silicon; σ_t and σ_l are the transverse and longitudinal components of the stress in the piezoresistor. Figure 5.6 shows the directional variation of the piezoresistive coefficients for p-type silicon, with maximum values of approximately $-70 \times 10^{-11} \text{ Pa}^{-1}$ and $+70 \times 10^{-11} \text{ Pa}^{-1}$, respectively for π_t and π_l along the $\langle 110 \rangle$ directions.

Figure 5.6 Piezoresistance coefficients π_l and π_t for the [100] plane of p-type single crystal silicon [126]

In this study, each sensing beam has an embedded p-type resistor along the $\langle 110 \rangle$ direction on a (100) wafer. Since the sensing beams are under approximately pure deformation, the resistance change ratio for sensing beam 1 is given by

$$\Delta R / R = \pi_{p-type} \sigma_l \quad (5.49)$$

where π_{p-type} is the p-type silicon piezoresistance coefficient for the $\langle 110 \rangle$ direction.

The stress and strain are related by Hooke's Law,

$$\sigma_l = E \varepsilon_l \quad (5.50)$$

Substituting Eq. (5.50) into Eq. (5.49), one has

$$\Delta R / R = \pi_{p-type} E \varepsilon_l \quad (5.51)$$

Upon the substitution of ε_l from Eq. (5.47), Eq. (5.51) becomes

$$\begin{aligned}
\Delta R / R &= \pi_{p\text{-type}} E \varepsilon_1 \\
&= \frac{\pi_{p\text{-type}} E}{l_s} \left\{ l_c - \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \times \right. \\
&\quad \left. \cos\left(\frac{F_i}{EI_c} \left(-\frac{1}{2} l_c^2 + \frac{2h+b}{2} l_c\right) + \operatorname{atan}\left(\frac{b}{2(l_c - h)}\right)\right) - h \right\} \quad (5.52)
\end{aligned}$$

A half Wheatstone bridge circuit used in this study is shown in Fig. 5.7.

Figure 5.7 Half Wheatstone bridge

The output voltage is expressed as

$$V_{output} = \frac{\Delta R}{2R} V_{input} \quad (5.53)$$

The sensitivity of the accelerometer S in terms of the output voltage and the frame acceleration a is given by

$$S = \frac{V_{output}}{a} \quad (5.54)$$

Substituting Eq. (5.53) into Eq. (5.54),

$$S = \frac{V_{output}}{a}$$

$$= \frac{\Delta R V_{input}}{2Ra} \quad (5.55)$$

Substituting Eq. (5.52) into the above equation gives

$$\begin{aligned} S &= \frac{\Delta R V_{input}}{2Ra} \\ &= \frac{V_{input} \pi_{p-type} E}{2a l_s} \left\{ l_c - \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \times \right. \\ &\quad \left. \cos\left(\frac{F_i}{EI_c} \left(-\frac{1}{2} l_c^2 + \frac{2h+b}{2} l_c\right) + \tan^{-1}\left(\frac{b}{2(l_c - h)}\right)\right) - h \right\} \end{aligned} \quad (5.56)$$

It is known that a trade-off exists between the natural frequency and the sensitivity for cantilever-type accelerometers. Hence, their products of suitable orders of power are used to systematically evaluate the performance of the accelerometer [127].

Combining Eq. (5.40) and Eq. (5.56) and replacing f_a with $\frac{\omega_a}{2\pi}$, the product of the angular resonant frequency and the sensitivity is obtained

$$\begin{aligned} \omega_a S &= \frac{V_{input} \pi_{p-type} E}{4a l_s} \sqrt{\frac{E w_c^3}{\rho_{Si} b^2 l_c^3 + 0.2357 \rho_{Si} l_c^4 w_c}} \times \left\{ l_c - \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \times \right. \\ &\quad \left. \cos\left(\frac{F_i}{EI_c} \left(-\frac{1}{2} l_c^2 + \frac{2h+b}{2} l_c\right) + \tan^{-1}\left(\frac{b}{2(l_c - h)}\right)\right) - h \right\} \end{aligned} \quad (5.57)$$

Likewise, the product of the angular natural frequency squared and the sensitivity is derived as

$$\begin{aligned} \omega_a^2 S &= \frac{V_{input} \pi_{p-type} E^2 w_c^3}{8a \rho l_s l_c^3 (b^2 + 0.2357 w_c l_c)} \left\{ l_c - \sqrt{\left(\frac{b}{2}\right)^2 + (l_c - h)^2} \times \right. \\ &\quad \left. \cos\left(\frac{F_i}{EI_c} \left(-\frac{1}{2} l_c^2 + \frac{2h+b}{2} l_c\right) + \tan^{-1}\left(\frac{b}{2(l_c - h)}\right)\right) - h \right\} \end{aligned} \quad (5.58)$$

5.2.3 Failure Analysis

For the accelerometer to work safely, maximum stresses in the central supporting beam and the sensing beams must not exceed the material strength of the single crystal silicon. Besides, as the sensing beams behave like long columns, the compressed one may buckle. Therefore, strength analysis for both the central supporting beam and the sensing beams as well as buckling analysis for the sensing beam under compression have to be considered in failure analysis.

Strength and Buckling Analysis for the Sensing Beams

To generate large strains in the sensing beams, the sensing beams have to be long and slender. As discussed in the previous sections, one of the two sensing beams is subject to axial compressive loading. Thus, buckling may occur to the compressed beam. And, in the worst situation the beam might be broken. Therefore, special attention must be given to the design of the sensing beams so that they can safely support their intended loadings without buckling.

The critical load for a column fixed at the base and free at the top is given by

$$P_{cr} = \frac{\pi^2 EI}{4L_{col}^2} \quad (5.59)$$

Where L_{col} is the length of the column, E Young's modulus and I area moment of inertia of the column cross section.

If the column is supported only at its ends and is free to buckle in any direction, then bending will occur about the principal centroidal axis having the smaller moment of inertia.

In this case, the smaller moment of inertia of the sensing beam cross section is

written as

$$I_s = \frac{w_s t_s^3}{12} \quad (5.60)$$

where w_s and t_s are the width and thickness of the sensing beams, respectively.

Substituting I_s in Eq. (5.60) for I in Eq. (5.59), the critical load for the sensing beam under compression can be obtained

$$P_{cr} = \frac{\pi^2 E w_s t_s^3}{48 l_s^2} \quad (5.61)$$

After the critical load is found, the corresponding critical stress can be calculated by dividing the load by the cross-sectional area. Hence, the critical stress for the sensing beam under compression is

$$\sigma_{cr} = \frac{P_{cr}}{w_s t_s} \quad (5.62)$$

Therefore, in order to avoid failure, the compressive stress in the sensing beam must not exceed the critical stress as well as the strength of single crystal silicon.

Strength Analysis for the Supporting Beam in Bending

For the safety of the supporting beam, the maximum stress that appears at its fixed end must be limited within the strength of single crystal silicon.

The moment of inertia for the cross-sectional area of the central supporting beam is given by

$$I_c = \frac{t_p w_c^3}{12} \quad (5.63)$$

where w_c is the width of the central supporting beam.

Generally, the normal stress at an arbitrary cross section of a beam under bending

is expressed as

$$\sigma = \frac{Mz}{I} \quad (5.64)$$

For the central supporting beam, the bending moment produced by the acceleration a , at the fixed end is written as

$$M = M_1 + M_2 \quad (5.65)$$

where M_1 is the bending moment produced by the resultant force at the fixed end of the supporting beam; while M_2 is the bending moment induced by the eccentricity e .

M_1 and M_2 are calculated as follows

$$M_1 = F \times l_c \quad (5.66)$$

$$M_2 = F \times e \quad (5.67)$$

Using Eqs. (5.65), (5.66) and (5.67), Eq. (5.64) is changed into the form of

$$\sigma = \frac{F(l_c + e)z}{I_c} \quad (5.68)$$

Considering a cross section has the maximum stress at $z = \frac{1}{2}w_c$ and substituting F from Eq. (5.11) into the above equation, the maximum stress in the central supporting beam is obtained

$$\begin{aligned} \sigma_{\max} &= \frac{F(l_c + e)z}{I_c} \\ &= - \frac{\rho_{Si} b^2 t_p a (l_c + e) \frac{1}{2} w_c}{I_c} \end{aligned} \quad (5.69)$$

Substituting Eq. (5.63) into Eq. (5.69) yields

$$\sigma_{\max} = - \frac{\rho_{Si} b^2 t_p a (l_c + e) \frac{1}{2} w_c}{I_c}$$

$$\begin{aligned}
&= -\frac{\rho_{Si} b^2 t_p a(l_c + e) \frac{1}{2} w_c}{\frac{t_p b_c^3}{12}} \\
&= -\frac{6\rho_{Si} b^2 a(l_c + e)}{w_c^2} \quad (5.70)
\end{aligned}$$

$\sigma_{\max}$ must always be smaller than the strength of single crystal silicon to allow safe operation of the accelerometer.

In the following section of numerical results, Figure 5.29 will show that the maximum stress in the central supporting beam $\sigma_{\max}$ is below the yield stress of silicon and much smaller than the stress in the sensing beams, which have a critical buckling stress smaller than the silicon strength. Therefore, the operation range will be determined by the resistance of the slender sensing beams against buckling.

5.3 Numerical Results

In this section, numerical results based on the analytic modeling discussed in preceding sections will be presented. These results show the effects of the principal dimensions on the characteristics of the accelerometer and provide guidelines for an optimal design. The accelerometer will be optimized to achieve a target natural frequency and a maximum sensitivity under the constraints of the slider dimensions.

The major functions and variables involved in the optimization problem are summarized in the following.

$$l_c = F_1(b, h) \quad (5.71)$$

$$e = F_2(b, h) \quad (5.72)$$

$$f_a = F_3(b, h, w_c) \quad (5.73)$$

$$S = F_4(b, h, w_c) \quad (5.74)$$

$$\omega_a S = F_5(b, h, w_c) \quad (5.75)$$

$$\omega_a^2 S = F_6(b, h, w_c) \quad (5.76)$$

where F_1, F_2 are functions of b and h ; F_3, F_4, F_5 , and F_6 are functions of b, h and w_c .

It is known that under the approximately pure axial deformation condition for the two sensing beams, the length of the central supporting beam l_c is dependent on the distance of the fixed end of the central supporting beam to the bottom of the proof mass h and the length of the proof mass b , as written in Eq. (5.71). Thus, l_c can be eliminated from the expressions of the eccentricity e , the natural frequency of the accelerometer f_a , the sensitivity of accelerometer S , the product of the angular natural frequency and the sensitivity $\omega_a S$, and the product of the angular natural frequency squared and the sensitivity $\omega_a^2 S$.

5.3.1 Optimization Objective

The objective and the constraint of the optimization problem are identified as

Objective

$$\text{Maximize } S = F_4(b, h, w_c)$$

Constraint

$$f_a \geq 300 \text{ kHz}$$

The accelerometer will be fabricated on a silicon-on-insulator (SOI) wafer with a thickness of 305 μm . Thus, the proof mass and the central supporting beam both have

a thickness of 305 μm . Since the thin sensing beams are not likely to affect the behavior of the central supporting beam, its dimensions are excluded from the analytic modeling and determined according to practical considerations. Table 5.1 lists the constants used in the numerical analysis.

Table 5.1 Constants used in the numerical analysis

Constant	Explanation	Value
t_p	Thickness of the proof mass and the central supporting beam	305 μm
l_s	Length of the sensing beams	20 μm
a	Frame acceleration	30 g
V_{input}	Input voltage for the half Wheatstone bridge	5 V
ρ_{Si}	Density of single crystal silicon	$2.3 \times 10^3 \text{ kg/m}^3$
E	Young's modulus of single crystal silicon	$190 \times 10^9 \text{ Pa}$
$\pi_{p\text{-type}}$	p-type silicon piezoresistance coefficient for the $\langle 110 \rangle$ direction	$70 \times 10^{-11} \text{ Pa}^{-1}$

5.3.2 Condition for Pure Axial Deformation of the Sensing Beams and Optimization of the Eccentricity e

It is understood in the previous discussion that the approximately pure axial deformation condition of the two sensing beams is satisfied when the constraints by Eq. (5.31) and Exp. (5.34) are met. There are three variables involved in this condition. By considering a simple model in which the sensing beams and the central supporting beam are rooted at the same level, h becomes a constant of 20 μm and there are only l_c and b left. Hence, l_c can be plotted as a function of b to examine the constraint exerted

by the approximately pure axial deformation condition. Figure 5.8 is the graphic description of this condition.

Figure 5.8 Representation of the approximately pure axial deformation condition with

$$h = 20 \mu\text{m}$$

According to Eq. (5.31) l_c may have two different roots or one single root depending on whether the determinant in Exp. (5.34) is greater than 0 or equal to 0. If l_c has a single root, then from Exp. (5.34), b and h are governed by

$$b = 4h \quad (5.77)$$

The single root in Eq. (5.31) can be written as

$$l_c = \frac{3}{8}b + \frac{3}{2}h \quad (5.78)$$

Substituting Eq. (5.77) into Eq. (5.78), l_c becomes

$$l_c = 3h \quad (5.79)$$

The eccentricity e , can be obtained by substituting Eq. (5.77) and Eq.(5.79) into Eq. (5.13)

$$\begin{aligned} e &= h + \frac{1}{2}b - l_c \\ &= h + 2h - 3h = 0 \end{aligned} \quad (5.80)$$

Therefore, the single root of l_c corresponds to zero eccentricity. When l_c has two positive real roots, then the bigger one of them is associated with a negative e , while the smaller one leads to a positive e . Equation (5.79) can be used as the criterion to decide the sign of the eccentricity e under a certain l_c . The three configurations of the central supporting beam are shown in Fig. 5.9.

Figure 5.9 Configurations of the central supporting beam: (a) $e > 0$, (b) $e = 0$, (c) $e < 0$

For a positive eccentricity e , l_c is obtained from Eq. (5.31) as

$$l_c = \frac{12h + 3b - \sqrt{(12h + 3b)^2 - 16(12h^2 + 6bh)}}{8} \quad (e > 0) \quad (5.81)$$

If b approaches infinity, l_c ($e > 0$) approximates to

$$l_c = \frac{12h + 3b - \sqrt{(12h + 3b)^2 - 16(12h^2 + 6bh)}}{8} \approx \frac{3}{2}h \quad (5.82)$$

Equation (5.82) is the asymptote for l_c ($e > 0$). That means for $e > 0$, l_c approaches the lower limit of $\frac{3}{2}h$, as b increases. This can be seen in Fig. 5.11.

Likewise, the upper limit for l_c ($e < 0$) can be obtained as

$$l_c = \frac{3}{4}b + \frac{3}{2}h \quad (5.83)$$

Figure 5.10 shows the eccentricities calculated from the qualified sets of b and l_c .

Figure 5.10 Eccentricity of the proof mass ($h = 20 \mu\text{m}$)

Substituting Eq. (5.82) and Eq. (5.83) into Eq. (5.13) respectively, one can calculate the asymptotes for the eccentricity e as

$$e = \frac{1}{2}b - \frac{1}{2}h \quad (5.84)$$

$$e = -\frac{1}{4}b - \frac{1}{2}h \quad (5.85)$$

Connecting with the proof mass below, at or above its center of mass, the central supporting beam has a different length for each case. To judge which design is the best, more information regarding the sensitivity, the natural frequency and their combinations of the accelerometer under these configurations is needed.

Figure 5.11 shows the sensitivity of the accelerometer with the central supporting beam width of 10, 15 and 20 μm , respectively. Since the highest aspect ratio of deep reactive ion etching (DRIE) is around 30 [37] and the central supporting beam is 305 μm thick, the lower limit for the central supporting beam width is set to 10 μm . With a certain proof mass, a negative eccentricity e yields a higher sensitivity. That's because

Figure 5.11 Sensitivity of the accelerometer ($h = 20 \mu\text{m}$)

a longer beam is more flexible and generates a larger displacement which means greater stresses in the two sensing beams. The width of the central supporting beam affects the sensitivity in a similar way. By contrast, as shown in Fig. 5.12, the opposite is true for the natural frequency. This is because of the known trade-off between the natural frequency and the sensitivity among cantilever accelerometers as discussed in previous sections.

A systematic approach to evaluate the performance of the accelerometer is to consider the products of the natural frequency and the sensitivity of suitable orders of power, e.g. $\omega_a S$ and $\omega_a^2 S$. Figure 5.13 and Fig. 5.14 show the results of $\omega_a S$ and $\omega_a^2 S$, respectively. Figure 5.13 and Fig. 5.14 both demonstrate that a positive e gives better performance in terms of greater $\omega_a S$ and $\omega_a^2 S$. Therefore, the free end of the central supporting beam should meet the proof mass below its center of mass. From

Figure 5.12 Natural frequency of the accelerometer ($h = 20 \mu\text{m}$)

Figure 5.13 Product of the natural frequency and the sensitivity ($h = 20 \mu\text{m}$)

Figure 5.14 Product of the natural frequency squared and the sensitivity ($h = 20 \mu\text{m}$)

Fig. 5.14, it can be seen that the product of the angular natural frequency squared and the sensitivity $\omega_a^2 S$ is independent of the width of central supporting beam w_c .

Due to the very high frequencies of the slider vibration, the accelerometer prefers a comparable natural frequency with an acceptable sensitivity. It can be seen in Fig. 5.12 that the natural frequency reaches the peak, when the proof mass has a length of about 90 μm .

However at that point, the sensitivity in Fig. 5.11 is very low and the output voltage of the accelerometer under the acceleration of 30 g may not be detectable, as shown in Fig. 5.15.

Figure 5.15 Output voltage with $h = 20 \mu\text{m}$ under a frame acceleration of 30g

5.3.3 Optimization of h and b

At $l_c = 51 \mu\text{m}$ and $b = 90 \mu\text{m}$ the natural frequencies are optimal, but the

sensitivity can not achieve a desirable level. This is because the distance of the fixed end of the central supporting beam to the bottom of the proof mass h is fixed at $20\text{ }\mu\text{m}$. By changing h while keeping the width of the supporting beam w_c a constant, the effect of changing h on the performance of the accelerometer can be seen. Since the eccentricity e is preferred to be positive for better performance as discovered in the previous discussions, only numerical results with positive eccentricities will be presented in the following analysis.

Figure 5.16 is the representation of the approximately pure axial deformation condition for the sensing beams with h being a variable and w_c fixed at $10\text{ }\mu\text{m}$.

Figure 5.16 Representation of the approximately pure axial deformation condition
with $w_c = 10\text{ }\mu\text{m}$

It is shown that with h increasing from $20\text{ }\mu\text{m}$ to $40\text{ }\mu\text{m}$, the curve moves to the

right. That is to say with a longer distance of the fixed end of the central supporting beam to the bottom of the proof mass h , the minimum central supporting beam length l_c and the minimum proof mass length b to satisfy the approximately pure axial deformation condition have to be larger, too. For a certain proof mass, a bigger h also requires a longer central supporting beam.

Figure 5.17 shows the curves of the eccentricity of the proof mass for a series of h . Combining Fig. 5.16, it is noticed that for a certain proof mass the central supporting beam has to be longer to suit a larger h , while the eccentricity decreases as a result. Each curve with a different h has an asymptote described by Eq. (5.84). If h equals 0, the asymptote becomes $e = b/2$, which is indicated in Fig. 5.17.

For a certain proof mass the central supporting beam becomes longer when h

Figure 5.17 Eccentricity of the proof mass ($w_c = 10 \mu\text{m}$)

increases. Hence, the sensitivity will increase and the natural frequency will decrease because of the trade-off effect. This can be seen from Fig. 5.18 and Fig. 5.19 that show the sensitivity and natural frequency, respectively.

Figure 5.18 Sensitivity of the accelerometer ($w_c = 10 \mu\text{m}$)

From Fig. 5.11 and Fig. 5.18, it is seen that the sensitivity can be increased by either decreasing the width of central supporting beam w_c or increasing the distance between the bottom of the proof mass and the fixed end of the central supporting beam h . From the perspective of device fabrication, a bigger w_c is preferred for easy processing. Therefore, increasing h is more desirable. However, when h increases, b also has to increase to guarantee the approximately pure axial deformation as shown in Fig. 5.16, as a result, h plus b might exceed the height of the pico slider that is 300

μm . For h of $40 \mu\text{m}$, the length of the proof mass for the optimal natural frequency is $176 \mu\text{m}$ as shown in Fig. 5.19. The length of the proof mass b plus the distance between the bottom of the proof mass and the fixed end of the central supporting beam h as shown in Fig. 5.3 is $216 \mu\text{m}$. Since space is needed for the frame, the maximum of h plus b is approximately $216 \mu\text{m}$. Hence, h is set to $40 \mu\text{m}$.

Figure 5.19 Natural frequency of the accelerometer ($w_c = 10 \mu\text{m}$)

Figure 5.19 shows that for $h = 40 \mu\text{m}$ and $w_c = 10 \mu\text{m}$, the maximum natural frequency is 123.3 kHz at $b = 176 \mu\text{m}$, which is below the target natural frequency. According to Fig. 5.12 the natural frequency can be increased by increasing the width of the central supporting beam w_c . Since the optimal natural frequency is independent of w_c , the natural frequency will be still optimal with the same h , b and l_c . Figure 5.20 and Fig. 5.21 show the products of $\omega_a S$ and $\omega_a^2 S$, respectively.

Figure 5.20 Product of the natural frequency and the sensitivity ($w_c = 10 \mu\text{m}$)

Figure 5.21 Product of the natural frequency squared and the sensitivity ($w_c = 10 \mu\text{m}$)

In Fig. 5.20, the optimal $\omega_a S$ curve corresponding to the optimal curve in Fig.

5.19 has an increasing trend. This means a bigger h leads to a better performance. In Fig. 5.21, it is seen that the optimal $\omega_a^2 S$ remains constant regardless of h . This is the same case with regard to w_c as shown in Fig. 5.14 where $\omega_a^2 S$ is found to be independent of w_c . Therefore, the optimization of $\omega_a S$ can be achieved by changing h without affecting $\omega_a^2 S$. Compared with Fig. 5.15, the output voltage shows improvement with h becoming larger, as shown in Fig. 5.22.

Figure 5.22 Output voltage with $w_c = 10 \mu\text{m}$ and an input voltage of 5 V

Figure 5.19 shows that for each h , the natural frequency reaches a maximum at a certain b . The corresponding output voltages for the optimal natural frequencies are plotted in Fig. 5.23. It is seen that the output voltage can be increased by increasing h . Due to the trade-off between the sensitivity and the natural frequency, h has an opposite effect on the natural frequency as shown in Fig. 5.19.

Figure 5.23 Output voltage for optimal natural frequencies with $w_c = 10 \mu\text{m}$ and an input voltage of 5 V

From the above analysis, for the optimal natural frequency and $\omega_a S$, the length of proof mass b , the distance between the bottom of the proof mass and the fixed end of the central supporting beam are determined to be $176 \mu\text{m}$ and $40 \mu\text{m}$, respectively. The length of central supporting beam l_c is calculated to be $103 \mu\text{m}$.

5.3.4 Determination of the Width of the Central Supporting Beam w_c

After b , h and l_c are determined, optimal natural frequencies can be found by choosing different widths for the central supporting beam. Figure 5.24 shows the optimal natural frequencies with varying widths of the central supporting beam w_c .

Figure 5.24 Optimal natural frequencies with $h = 40 \mu\text{m}$

For $f_a = 300 \text{ kHz}$, the width of the central supporting beam is calculated to be $18.1 \mu\text{m}$. Figure 5.25 shows the optimal natural frequency curve with h changing from $20 \mu\text{m}$ to $40 \mu\text{m}$ and w_c fixed at $18.1 \mu\text{m}$. Since the width of the central supporting beam w_c is left as the only variable, S and f_a can be plotted as a function of w_c in Fig. 5.26 and Fig. 5.27, respectively. Figure 5.26 indicates that the sensitivity is approximately 0.02 mV/g/V for a width of the central supporting beam of $18.1 \mu\text{m}$. The corresponding natural frequency as shown in Fig. 5.27 is 300 kHz . As shown in Fig. 5.28, the output voltage for a half Wheatstone bridge with the input voltage of 5 volts is 2.7 mV for a frame acceleration of 30 g .

Figure 5.25 Natural frequency of the accelerometer with $w_c = 18.1 \mu\text{m}$

Figure 5.26 Sensitivity as a function of w_c with b , l_c and h equal to 176, 103 and 40 μm , respectively

Figure 5.27 Natural frequency as a function of w_c with b , l_c and h equal to 176, 103 and 40 μm , respectively

Figure 5.28 Output voltage as a function w_c with b , l_c and h equal to 176, 103 and 40 μm , respectively

5.3.5 Failure Analysis of the Optimized Design

After the geometric parameters of the accelerometer are finalized, failure analysis is carried out to identify the operational acceleration range for the accelerometer. As discussed in the analytic modeling section, possible failures of the accelerometer include buckling or breaking of the sensing beams and breaking of the central supporting beam. Figure 5.29 shows the maximum stresses in the sensing beams and the central supporting beam in comparison with the yield stress of single crystal silicon and the critical stress of buckling for the sensing beam under compression. The length, width and thickness of the sensing beams are 20, 3 and 5 μm , respectively. It is seen from Fig. 5.29 that the stresses in the sensing beams are much larger than

Figure 5.29 Buckling and strength analysis for the sensing beams and central supporting beam

that in the central supporting beam, and when the acceleration increases to about 7700 g, the maximum stress in the compressed sensing beam first reaches the critical stress of buckling. This is the theoretical upper limit of the working range of the accelerometer. Adding a safety factor of 2, the measuring range of the accelerometer is 3850 g.

In summary, the analytically optimized geometric parameters and sensor specifications of the accelerometer are listed in Table 5.2. Compared with similar commercial products, the accelerometer of this design offers a much higher resonant frequency with a comparable sensitivity and is more than 10 times smaller in size and weight. The improvement can be attributed to the parametric optimization made to the structure specifically to enhance the natural frequency.

Table 5.2 Design specifications for the MEMS accelerometer

Dimensions of the proof mass (length×width×thickness), (μm)	176×176×305
Dimensions of the central supporting beam (length×width×thickness), (μm)	103×18×305
Dimensions of the sensing beams (length×width×thickness), (μm)	20×3×5
Sensitivity of the accelerometer (mV/g/V)	0.02
Natural frequency of the accelerometer (kHz)	300
Measuring Range (g)	3850

5.4 Finite Element Analysis

Since the end of the central supporting beam is located inside the proof mass, space between the proof mass and the central supporting beam must be allocated to allow deflection of the whole beam. The width of the trenches between the proof mass and the central supporting beam in the real model is 15 μm .

The element type chosen for the FEM model of the accelerometer is SOLID95. The material properties are shown in Table 5.3. The bottom of the accelerometer structure is fully constrained as shown in Fig. 5.30.

Table 5.3 Material properties for the accelerometer FEM model

Material	Single Crystal Silicon
Young's modulus (Pa)	190×10^9
Density (kg/m^3)	2.3×10^3
Poisson's ratio	0.24

When under an acceleration of 20g as shown in Fig. 5.31, the beam on the left is compressed, while the beam on the right elongated. The magnitude of their displacements in z -direction at the joint with the proof mass is the same, which is $0.113 \times 10^{-3} \mu\text{m}$. The two sensing beams also experience displacement in x -direction due to the constraint from the sensing beams themselves, which was ignored in the theoretical calculation. Since the deformation is symmetrical for the two sensing beams, the resistance change in the two piezoresistors will be equal and in opposite

Figure 5.30 The MEMS accelerometer structure with sensing beams

directions. Therefore, the half Wheatstone bridge configuration will not be affected. Substituting $0.113 \times 10^{-3} \mu\text{m}$ into Eq. (5.46), strain in z -direction in the two sensing beams can be calculated to be $0.113 \times 10^{-3} / 20 = 0.565 \times 10^{-5}$. After strain is known, the resistance change, output voltage and ultimately sensitivity of the accelerometer can be obtained. Substituting Eq. (5.51) into Eq.(5.55), the sensitivity S can be written as

$$S = \frac{V_{input}}{2a} \pi_{p-type} E \varepsilon_1 \quad (5.86)$$

The sensitivity S can be written independent of the input voltage V_{input} by dividing the right-hand side of Eq. (5.86) by V_{input} .

$$S = \frac{\pi_{p-type} E \varepsilon_1}{2a} \quad (5.87)$$

Figure 5.31 The accelerometer structure with sensing beams under an acceleration of 20g in x -direction

Substituting $a = 20g$, $\pi_{p-type} = 70 \times 10^{-11} \text{ Pa}^{-1}$, $E = 190 \times 10^9 \text{ Pa}$, and $\varepsilon_1 = 0.565 \times 10^{-5}$ into Eq. (5.87), the sensitivity S is found to be 0.0188 mV/g/V , slightly lower than the specification of 0.02 mV/g/V in the design.

The first mode of the accelerometer structure is shown in Fig. 5.32. Its natural frequency of 332 kHz is slightly higher than that targeted for in the design, which is 300 kHz as shown in Table 5.2. The reduced sensitivity and increased natural frequency can be attributed to a lighter proof mass in the FEM model because of the gap between the central supporting beam and the proof mass.

Figure 5.32 First modal shape of the accelerometer structure

5.5 Summary

An in-plane piezoresistive accelerometer was designed. The accelerometer has a proof mass supported by a central beam, with two sensing beams at each side of the central beam. The movement of the proof mass due to deflection of the central supporting beam induces greater stress in the sensing beams compared to that in the central beam. The accelerometer based on this “stress magnifying” structure can achieve a higher sensitivity with the same natural frequency compared to the conventional cantilever-type accelerometers. Its performance was optimized in terms of products of the natural frequency and sensitivity. Finite element analysis was conducted on the finalized design to verify the theoretical calculations.

Chapter 6 Fabrication and Testing of the MEMS Accelerometer

This chapter discusses fabrication and testing of the MEMS accelerometer. Characterization results of the diffused piezoresistors using theoretical calculation and the sheet resistance method are presented. The I-V characteristics of the p-type piezoresistors and a p-n-p path are compared. Major characteristics of the accelerometer, such as noise level, linearity and sensitivity are obtained through vibration testing.

6.1 Fabrication Process

The miniature sensor was built on a silicon-on-insulator (SOI) wafer. The sensing beams were carved out in the device layer, while the proof mass and central supporting beam utilized the full wafer thickness. The critical thickness of the sensing beams was controlled by using the buried silicon dioxide layer as an etch stop. The fabrication process involved diffusion, sputtering and deep-reactive-ion-etching (DRIE).

The fabrication sequences started with boron diffusion of resistors, followed by etching contact windows, deposition of pads and interconnections, cutting out the proof mass and beams using DRIE, and wet etching of the buried oxide layer to release the structure. The main processes are described below.

- (1) Four resistors were fabricated in the top silicon layer by light boron doping

(Fig. 6.1(b)). Silicon dioxide was used as a diffusion mask. The process comprised two steps, predeposition and drive-in. During predeposition the device layer was placed facing a source wafer (BN-975) for 20 minutes at 900 °C. The wafer then was dipped into buffered oxide etchant (BOE) for two minutes to remove the remaining boron oxide on the surface. A 20-minute dry oxidation at a low temperature of 750 °C followed to oxidize the boron alloy skin. After that, a second BOE dipping was used to get rid of the oxidized boron skin. To push the boron atoms deeper down the wafer to form a p-n junction with a desired depth, the wafer went through the drive-in process for 90 minutes at 1100 °C in oxygen ambience. The silicon dioxide of about 150 nm grown during the process was used for insulation, on which the pads and interconnections would be deposited.

(2) Contact vias were cut out in the silicon dioxide layer to enable the pads and interconnections to have contact with the resistors (Fig. 6.1(c)). Since the dimensions of the resistors were only 3 μm by 20 μm , significant undercutting occurred. Therefore, instead of wet etching, reactive ion etching (RIE) was used in this case.

(3) Titanium/Gold was sputtered onto the wafer to make pads and interconnections (Fig. 6.1(d)). The thickness was 50 nm for Titanium and 500 nm for Gold, respectively.

(4) The SOI wafer underwent front-side deep reactive ion etching (DRIE) as the first step to create the sensing beams, the central supporting beam and the proof mass (Fig. 6.1(e)).

(5) The central supporting beam and the proof mass were cut down from the

back-side using DRIE etching to make them about 50 μm thinner than the frame (Fig. 6.1(f)). This is to allow the proof mass and the central supporting beam to move freely when the accelerometer is bonded onto a substrate.

(6) The central supporting beam and the proof mass were created by etching through the SOI wafer from the back side down to the buried silicon dioxide layer (Fig. 6.1(g)).

(7) The buried silicon dioxide layer was removed by BOE (Fig. 6.1(h)), cutting out all the devices from the wafer.

The fabrication process of the accelerometer is illustrated in Fig. 6.1.

Figure 6.1 Flow chart of the fabrication process

The six masks for boron diffusion, contact vias, pads and interconnections, front-side structure, back-side cavity, and back-side structure respectively, are shown in appendix A.

6.2 Fabrication Results

During boron diffusion, a patterned dummy wafer went through the same processes as the device wafer. The sheet resistance of the dummy wafer was measured in different phases. Process parameters and corresponding sheet resistance values are listed in Table 6.1.

Table 6.1 Process parameters and sheet resistances during boron diffusion

Process	Parameters	Sheet resistance (ohm/square)
Start		15
Predeposition	900 °C, 15 minutes	198
Low-temperature dry oxidation	750 °C, 20 minutes	235
Drive-in (dry oxidation)	1100 °C, 90 minutes of O ₂ , 40 minutes of N ₂	318

The number of squares of the resistor ($5\text{ }\mu\text{m} \times 20\text{ }\mu\text{m}$) as shown in Fig. 6.2 is calculated to be 2.78. According to the sheet resistance after drive-in, the final resistance value of the resistor is $2.78 \times 318 = 884\Omega$.

To form a proper p-n junction, the junction depth has to be smaller than the thickness of the device layer of $5\text{ }\mu\text{m}$. The junction depths following the predeposition

and drive-in steps are verified using the diffusion theory as discussed in [128]. The concentration of boron N during predisposition is governed by a complementary error function (erfc)

Figure 6.2 Geometry of the piezoresistor

$$N(x,t) = N_0 \operatorname{erfc}(x / 2\sqrt{Dt}) \quad (6.1)$$

where x is the depth from the wafer surface, t time, N_0 the boron concentration at the wafer surface ($x = 0$), D the diffusion coefficient. N_0 is determined by the solid-solubility limit of boron at a given temperature. At 900 °C, N_0 is approximately 1.1×10^{20} atoms/cm³. The diffusion coefficient depends exponentially on temperature

$$D = D_0 \exp\left(-\frac{E_A}{kT}\right) \quad (6.2)$$

where D_0 is the maximum diffusion coefficient, k the Boltzmann constant, E_A the activation energy for diffusion, T the temperature. For boron, $D_0 = 10.5$ cm²/sec, $E_A = 3.69$ eV. Substituting D_0 , E_A , $k = 8.617 \times 10^{-5}$ eV/K and the predeposition temperature $T_p = 1173$ K into Eq. (6.2), the diffusion coefficient for the predeposition

step D_p is found to be

$$\begin{aligned} D_p &= D_0 \exp\left(-\frac{E_A}{kT_p}\right) \\ &= 10.5 \exp\left(-\frac{3.69}{8.617 \times 10^{-5} \times 1173}\right) \\ &= 1.45 \times 10^{-15} \text{ cm}^2 / \text{sec} \end{aligned} \quad (6.3)$$

Likewise, the diffusion coefficient for the drive-in step D_d is calculated to be $2.96 \times 10^{-13} \text{ cm}^2 / \text{sec}$.

The junction is formed at the point at which the concentration of boron N is equal to the background concentration of phosphorous N_B . The doping concentration for a 0.45 ohm-cm n-type wafer is $1.17 \times 10^{16} \text{ atoms/cm}^3$. Therefore, the junction depth following the predeposition step x_d can be obtained by equating N from Eq. (6.1) to N_B

$$1.1 \times 10^{20} \operatorname{erfc}\left(\frac{x_p}{2\sqrt{D_p t_p}}\right) = 1.17 \times 10^{16} \quad (6.4)$$

Substituting D_p and the predeposition time $t_p = 900 \text{ sec}$ into Eq. (6.4), x_p is can be found

$$\begin{aligned} x_p &= 2\sqrt{D_p t_p} \operatorname{erfc}^{-1}\left(\frac{1.17 \times 10^{16}}{1.1 \times 10^{20}}\right) \\ &= 2\sqrt{1.45 \times 10^{-15} \times 900} \times 2.7405 \text{ cm} \\ &= 0.063 \mu\text{m} \end{aligned} \quad (6.5)$$

The dose Q deposited into the wafer during the predeposition step is measured by the boron atoms per unit area and given by

$$Q = 2N_0\sqrt{Dt/\pi} \quad (6.6)$$

Inserting N_0 , D_p and t_p into Eq. (6.6), Q is obtained as

$$\begin{aligned}
Q &= 2N_0\sqrt{D_p t_p / \pi} \\
&= 2 \times 1.1 \times 10^{20} \sqrt{1.45 \times 10^{-15} \times 900 / \pi} \\
&= 1.43 \times 10^{14} \text{ atoms/cm}^2
\end{aligned} \tag{6.7}$$

The boron atoms that entered the wafer during the predeposition step provide a limited source for the drive-in step. The resulting profile is described by a Gaussian function

$$N(x, t) = (Q / \sqrt{\pi t}) \exp \left[- \left(\frac{x}{2\sqrt{Dt}} \right)^2 \right] \tag{6.8}$$

Plugging in Q , D_d for D and the drive-in time $t_d = 5400$ sec for t in Eq. (6.8) and setting it equal to N_B , the junction depth following the drive-in step x_d is found to be 1.83 μm . The surface concentration after drive-in $N(0, t_d)$ is

$$\begin{aligned}
N(0, t_d) &= Q / \sqrt{\pi t_d} \\
&= 1.43 \times 10^{14} / \sqrt{\pi \times 5400} \\
&= 2 \times 10^{18} \text{ atoms/cm}^3
\end{aligned} \tag{6.9}$$

The sheet resistance value can be estimated via the Irvin's Curve as shown in Appendix B. The sheet resistance-junction depth product is approximately 600 ohm-cm. Dividing by a junction depth of 1.83 μm yields a sheet resistance of 328 ohm/square, which is close to the experimental value 318 ohm/square.

The I-V characteristics of the dummy resistor were measured with a semiconductor parameter analyzer HP4156B. A voltage sweep was performed through two probes as shown in Fig. 6.3 (a) and the current flowing in the resistor was sampled. To measure the current leakage in the substrate, a voltage sweep was performed on a p-n-p path as shown in Fig 6.3 (b).

Figure 6.4 shows the resistor has good linearity. The slope of the p-p curve is found to be 0.0011. Therefore, the resistance of the resistor is 909.1 Ω , which is close

to the calculated value from the sheet resistance method ($884\ \Omega$). At 5 volts, the ratio

Figure 6.3 Probe arrangement for I-V characteristics measurement: (a) p-p; (b) p-n-p

Figure 6.4 I-V characteristics of piezoresistor and p-n-p path

of the current through the resistor to that through the p-n-p path is found to be 255.

This indicates that the insulation between p-type areas is largely acceptable.

Figure 6.5 shows a SEM picture of the fabricated accelerometer temporarily bonded to a substrate by photoresist. A close-up view of a sensing beam is shown in Fig. 6.6. It has a width of 3 microns and is connected to the proof mass via the device layer. The bottom side of the accelerometer is shown in Fig. 6.7. The proof mass and the supporting beam are about 50 microns lower than the frame. This ensures that the proof mass can move freely, when the bottom side is bonded to a substrate.

Figure 6.5 Top side of the fabricated accelerometer

Figure 6.6 Close-up of the sensing beam

Figure 6.7 Bottom side of the fabricated accelerometer

Postprocessing

Because of the small dimensions of the MEMS accelerometer ($1 \times 0.3 \times 0.3 \text{ mm}^3$) and its small pads ($200 \times 120 \text{ }\mu\text{m}^2$), it is difficult to handle and connect it to external circuits. In this case, the accelerometer was first bonded to a silicon substrate with bigger gold pads ($2 \times 2 \text{ mm}^2$). The pads on the accelerometer were then connected to the bigger pads using wire bonding as shown in Fig. 6.8.

Figure 6.8 The MEMS accelerometer connected to external pads via wire bonding

The close-up view of the wire bonds sticking to the pads of the accelerometer is shown in Fig. 6.9. After wire bonding, regular electrical wires were soldered to the external pads (Fig. 6.10). The accelerometer is circled in Fig. 6.10. Finally, the accelerometer was connected to an amplifier circuit (Fig. 6.11). The chosen amplifier

Figure 6.9 Wires bonded to the pads of the accelerometer

Figure 6.10 Wires soldered to the external pads

Figure 6.11 PCB board with the accelerometer and an amplifier circuit

was an AD620. Its gain adjustment resistor was $216.8 \, \Omega$, giving an amplification of 228.9 [129]. The metal holder housing the printed circuit board (PCB) board can be connected to the shaker by the screw as shown in Fig. 6.11.

6.3 Testing Results

The MEMS accelerometer was characterized using a commercial accelerometer as a reference. The measured acceleration from the reference accelerometer was treated as the true acceleration of the shaker. With the output voltage from the MEMS accelerometer and the acceleration obtained by the reference accelerometer, the performance of the MEMS accelerometer can be evaluated. The experimental methods are described below, followed by analyses of the results.

6.3.1 Experimental Setup

The MEMS accelerometer was tested on a shaker (B&K 4810). Acceleration measured by a commercial accelerometer (B&K 4374) was used as the reference (true acceleration of the shaker). The complete setup is shown in Fig. 6.12. The location of the B&K accelerometer (type 4374) can be seen in the close-up view in Fig. 6.13.

Figure 6.12 Experimental setup for characterization of the MEMS accelerometer

Figure 6.13 Commercial accelerometer B&K 4374 with the MEMS accelerometer

6.3.2 Time Response

The MEMS accelerometer was tested with multiple excitation frequencies and amplitudes to evaluate its time domain performance. The background noise of the MEMS accelerometer was obtained by recording the output voltage when there was no excitation provided to the shaker. Figure 6.14 shows the noise of the MEMS accelerometer compared with that of the commercial accelerometer. The noise peak value is 0.08 mV for the amplifier output of the MEMS sensor, which is about ten times higher than that of the commercial accelerometer. Assuming the noise in the amplifier circuit is negligible, the noise from the bridge output of the MEMS accelerometer is $0.08 \text{ mV} / 228.9 = 0.349 \text{ } \mu\text{V}$.

Figure 6.14 Output voltage without excitation: (a) B&K 4374; (b) MEMS

accelerometer

The sources of the noise may include electrical noise and mechanical noise [130]. The peak noise value of the commercial accelerometer is about one-tenth of that of the MEMS accelerometer. Amplifier noise, Johnson noise associated with piezoresistors and an unencapsulated proof-mass-cantilever structure are the possible factors that make the MEMS sensor prone to noise. However, the absolute peak value of the noise is 0.08 mV for the MEMS sensor, which is insignificant compared with the bridge output in the operational range of the MEMS sensor. Figure 6.15 shows the responses of the two accelerometers to acceleration at 500 Hz. From the output voltage of the commercial accelerometer B&K 4374, the amplitude of the acceleration was found to be 20g. The amplitude of the output voltage for the MEMS accelerometer is 420 mV.

Figure 6.15 Time response to a 20g sinusoidal acceleration of 500 Hz: (a) the commercial accelerometer (B&K 4374); (b) the MEMS accelerometer

The wave forms of the output voltage for the two accelerometers agree well in terms of the dominant component, which was caused by the periodic excitation of the shaker. The difference in the wave forms is mainly caused by the high-frequency components that come from different noise sources for the two accelerometers. And the different locations to which the two accelerometers were attached can also bring different vibration modes of the holding structure into their output wave forms.

The FFT analysis of the amplifier output of the MEMS accelerometer in Fig. 6.15(b) is shown in Fig. 6.16. The peak of the power spectrum is at 500 Hz, which

Figure 6.16 Power spectrum of the MEMS accelerometer's response to a 20g sinusoidal acceleration of 500 Hz

shows that the dominant part of the output signal is associated with the excitation acceleration of 500 Hz. It can be seen from Fig. 6.16 that the noise roughly consists of

two categories, $1/f$ noise and white noise [131]. Below 10 kHz, the power of the noise is inversely proportional to the frequency. Thus, it is mainly $1/f$ noise. Above 10 kHz, the power spectrum of the noise becomes independent of the frequency (flat), which is characteristic of white noise. White noise includes thermal noise and shot noise. Thermal noise is related to the piezoresistors of the Wheatstone bridge of the accelerometer, while shot noise is generated by the current flow passing the Schottky junction, which is formed beneath the metal pads of the accelerometer, i.e., the interface between metal and silicon.

6.3.3 Linearity and Sensitivity

Figure 6.17 shows the bridge output of the MEMS accelerometer at 500 Hz with a fitted straight line. The acceleration was changed by varying the amplitude of the excitation voltage. The weight of the aluminum holder limited the maximum acceleration of the shaker (B&K 4810) to approximately 20g. Therefore, the accelerometer was only tested under acceleration up to 20g. For the acceleration range of the accelerometer, a theoretical estimate was discussed in Chapter 5. Using the fitted straight line as the ideal output, the nonlinearity of the MEMS accelerometer can be calculated by the following formula [131]

$$\text{Nonlinearity} = \frac{\text{maximum output deviation}}{\text{full scale output}} \quad (6.10)$$

The nonlinearity was found to be 6.2% FS, which is relatively high compared to some commercial accelerometers [132].

Figure 6.18 shows the sensitivity of the accelerometer. The average sensitivity is

Figure 6.17 Output of Wheatstone bridge versus acceleration at 500 Hz

Figure 6.18 Sensitivity of the MEMS Accelerometer versus acceleration

89.8 $\mu\text{V/g/5V}$ (0.018 mV/g/V), which is close to the simulated sensitivity in the FEM analysis (0.0188 mV/g/V). The characteristics of the accelerometer are listed in Table 6.2.

Table 6.2 Characteristics of the MEMS accelerometer

Size (mm^3)	Noise (μV)		Sensitivity (mV/g/V)		Nonlinearity (% FS)
	With amplifier	Without amplifier	With amplifier	Without amplifier	
1×0.3×0.3	80	0.349	4.12	0.018	6.2

6.4 Summary

A MEMS accelerometer was fabricated on a SOI wafer. The four doped p-type resistors were characterized and found to be well insulated from the rest of the substrate through junction insulation. The miniature MEMS accelerometer was wire-bonded to gold pads and interfaced with an external amplifier circuit on a PCB board, which was mounted on a shaker for characterization of the accelerometer. The accelerometer's noise level, sensitivity, and nonlinearity were obtained via vibration testing. The characterized average sensitivity was 0.018 mV/g/V , which is close to the design specification of 0.02 mV/g/V (Table 5.2). Due to the limited acceleration range of the shaker, the accelerometer was only tested under acceleration up to 20g, which was close to the maximum slider transverse acceleration induced by multiple bumps (22g). The measurement range of the accelerometer was estimated to be 3850g by analytical calculation in Chapter 5.

Chapter 7 Conclusions and Future Work

7.1 Conclusions

This thesis presents a study on the slider off-track vibrations induced by head-disk interactions. There were two main objectives for the study. The first objective was to model the off-track vibrations of the slider induced by intermittent head-disk contact. The second objective was to design, fabricate and characterize a MEMS accelerometer for detection of the slider off-track vibration. The transverse acceleration of the slider was obtained from the modeling results of a slider under typical operation conditions. This determines the resolution and measurement range required for the subsequently developed MEMS accelerometer.

Two physical models were developed to simulate the slider off-track vibrations induced by head-disk contact. The first was based on a simplified two degrees of freedom model, in which the actuator arm and the head-gimbal assembly were represented by two mass-spring-damper systems. The second model was based on a five-DOF system. In addition to the transverse displacements of the actuator arm and the slider in the two-DOF model, three additional degrees of freedom, namely vertical displacement, pitch and roll of the slider were also considered. The slider air bearing was incorporated into the model through a stiffness matrix and a damping matrix. The five-DOF model has the capability to simulate intermittent head-disk contact induced by multiple bumps on a rotating disk. Compared with other simulation models in the literature, the five-DOF model developed in this thesis has two major advantages.

First, the contribution of the actuator arm rotation to the transverse displacement of the slider was modeled in the proposed five-DOF model, which other models failed to take into account. Results showed that the contribution of the actuator arm rotation to the transverse displacement of the slider can be dominant in the scenario of repeated excitation by multiple bumps. Second, compared with previous models which combine FEM models of the structural components with Reynolds equations of the slider air bearing, the five-DOF model was based on a simplified mass-spring-damper system and much smaller stiffness and damping matrices to model the behavior of the structural components as well as the air bearing. Therefore it requires much less computation resources and as a result is much more efficient.

Contact situations with a single bump and multiple bumps were investigated using the models developed. Effects of the coefficient of friction, skew angle, rotational speed of the disk and bump spacing on the slider off-track vibration were studied. The off-track motion of the slider was found to be dominated by the rotational mode of the actuator arm and the suspension sway mode. The maximum acceleration of the slider in the transverse direction induced by multiple bumps was found to be 22g.

An in-plane piezoresistive accelerometer for detection of the slider off-track motion was subsequently designed, fabricated and characterized. Its structure was optimized to achieve the desired natural frequency and an optimal sensitivity. The wire-bonded accelerometer with an external amplifier circuit was placed on a PCB board for vibration test on a shaker. Main characteristics of the MEMS accelerometer were obtained. The sensor was found to give an average sensitivity of 0.018 mV/g/V

and a moderate noise level of $\pm 80 \mu\text{V}$. The nonlinearity of the accelerometer was found to be 6.2% FS. The small size and light weight of the accelerometer make it possible to be mounted on the trailing side of a pico slider for detection of the slider off-track motion.

7.2 Future Work

Possible future work includes expansion of the five-DOF model, further characterization of the MEMS accelerometer and experimental investigation of the slider transverse vibration using the MEMS accelerometer.

1. The yaw motion of the slider and the down-track motion of the suspension may also play an important role in the transverse displacement of the slider. The five-DOF model may be extended to a seven-DOF model to include these effects. To analyze the head-disk contact more accurately, a non-linear Hertzian contact model can substitute for the linear spring-damper contact model. Adhesion on the head-disk interface, such as van der Waal's bonding forces can be added to the contact model. Furthermore, instead of using bumps, a more advanced surface with fractal roughness can be used to better model the disk surface.

2. In addition to the characterized noise level, resolution, sensitivity and linearity of the MEMS accelerometer, the frequency characteristics also need to be understood. Since the MEMS accelerometer was designed with a natural frequency of 300 kHz, a shaker with a frequency capability exceeding 300 kHz must be used to obtain the frequency response of the accelerometer. Alternatively, an impulse excitation might be

provided by a force hammer for characterization of the frequency response.

3. The MEMS accelerometer can be mounted on the slider to study its off-track vibration under different circumstances. Transverse acceleration induced by track seeking and head-disk contact can be detected and compared with simulation results and those obtained by other experimental tools, such as a LDV.

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Appendix A

Masks for fabrication of the MEMS accelerometer

Mask 1 – Boron diffusion

Mask 2 – Vias (for one resistor)

Mask 3 – Metallization

Mask 4 – Front-side structure

Mask 5 – Back-side cavity

Mask 6 – Back-side structure

Appendix B

Surface impurity concentration versus the sheet resistance-junction depth product for different silicon background concentration at 300 K [128]

Appendix C

List of Publications

Ao, H. R., Wang, S., Yuan, Y. H., Liu, Y., Ng, Q. Y., Gan, S., and Chai, M. C. (2006) Dynamic Simulation of the Suspension Lift-Tab on a Ramp during the Unloading Process of a Hard Disk Drive. *Proceedings of the Asia-Pacific Magnetic Recording Conference (APMRC2006)*, pp. DR-07-01 to DR-07-02.

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Yuan, Y. H., Du, H. J., and Wang, S. (2009) A Miniature In-Plane Piezoresistive MEMS Accelerometer for Detection of Slider Off-Track Motion in Hard Disk Drives. *Microsystem Technologies*, in press.