Inventory models for reverse logistics

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**NANYANG
TECHNOLOGICAL
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**INVENTORY MODELS FOR REVERSE
LOGISTICS**

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NANYANG BUSINESS SCHOOL

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ABSTRACT

In this thesis, we analyze and develop solution methodologies for several inventory models that arise in reverse logistics.

The first model we consider is a product recovery system with deterministic demand and return. It is assumed that recovered items (recoverable items that are returned and remanufactured) are as good as new and satisfy the same demand as new items. The demand rate and return fraction rate per unit time are constant and known in advance. The relevant costs are setup cost for remanufacturing recoverable items, setup cost for manufacturing, holding cost for recoverable (or returned) items in stock, and holding cost for serviceable (i.e. newly manufactured or remanufactured/recovered) items in stock. The objective is to minimize the long run average total costs.

Prior literature has proposed integer-ratio policy based heuristics where either one remanufacturing cycle is followed by an integer number of identical manufacturing cycles, or vice versa. The manufacturing (remanufacturing) lot sizes are identical across cycles. However these heuristics do not provide any apriori performance guarantee.

We first analyze the apriori performance guarantee of heuristics for the problem. We first develop a lower bound for the problem. We consider the above mentioned integer ratio policy. We show that the optimal integer ratio policy is within 1.5% of the lower bound, and the integer ratio policy proposed in the previous literature is within 6% of the lower bound.

We then study inventory models with product recovery with stochastic demand and return rates. As in the deterministic model, all returned items can be

remanufactured and recovered, and the recovered items are as good as new and satisfy the same demands as new items. Demands that are not satisfied immediately are assumed to be backlogged. In addition to setup costs for manufacturing and remanufacturing, and holding cost for the new (or serviceable) items and recoverable item, in the stochastic model there is also the penalty cost for backlogging of demand.

Past work has suggested classes of policies that has been broadly labeled as Push and Pull policies. Under a Push policy, remanufacturing of a lot is commenced as soon as there is sufficient inventory of the recoverable item, irrespective of the inventory position of the new products. Manufacturing of a new lot is scheduled if the inventory position of new (or serviceable) items is low and there are not enough recoverable item to remanufacture. In Pull policies, manufacturing or remanufacturing of a lot is commenced only when the inventory position of new/recovered products is sufficiently low. Several classes of policies under this umbrella of Push and Pull policies have been suggested in the literature. In general, Pull policies outperform Push policies. However all these policies suggested in the literature involve evaluating the steady state probabilities of a Markov chain with two states. Therefore even for a single set of numerical values for the policy parameters, the total cost can be evaluated only numerically. No closed form optimal solution or a systematic search algorithm for determining the optimal solution is available. Therefore optimal policies can be evaluated only by a completely extensive numerical search.

In this thesis, we propose two new policies, both under the class of Pull policies. Using the recent research on cost function behavior of (r, Q) policies, we are able to develop optimal algorithms for these particular classes of policies.

First, we derive a basic (r, Q) policy, assuming the same reorder point and ordering quantity for both manufacturing and remanufacturing orders. The policy

works as follows: as soon as the inventory position of the serviceable item drops to the level r , we place an order for a lot of size Q . If there is sufficient inventory of the recoverable item, we place a remanufacturing order; if not, we place an order for manufacturing new products. We analyze the cost function for this policy and develop an efficient optimal algorithm for this policy.

Then, we consider an (r_p, Q_p, r_r, Q_r) policy, which has different reorder point and order quantity for manufacturing and remanufacturing orders. The policy works as follows: as soon as the inventory position of the serviceable item drops to r_r , a remanufacturing lot of size Q_r is setup if there is sufficient inventory of the recoverable item. If a remanufacturing lot is ordered, this raises the inventory position of the serviceable item to $r_r + Q_r$. Otherwise, we let the inventory position of the serviceable item decrease further. As soon as it drops to the level r_p ($r_p \leq r_r$), a manufacturing order of size Q_p is placed. We also compare the proposed policy with this policy.

For the deterministic model, we also propose a heuristic for a more general class of policies, where there are R remanufacturing cycles for every P manufacturing cycles. While P and R are integers, their ratio needs not be an integer value. We implement this heuristic through an innovative interleaving of the lot schedule. If holding cost for the recoverable item is not included, our policy clearly outperforms the existing policies. When the holding cost for the recoverable item is included and the manufacturing (remanufacturing) lot sizes are kept stationary, the proposed interleaving based heuristic does not outperform the existing methods. However, when variable lot sizes are used across different cycles, the proposed heuristic generates solution with lower cost than existing methods for many problem

instances.

Chapter 1. Introduction

In the past decade, reverse logistics has been receiving increasing attention both from the industrial practitioners as well as from scientific researchers. Rogers and Tibben-Lembke (1999) define reverse logistics as the process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal.

Reverse logistics is practiced in many industries, including those producing steel, commercial aircraft, computers, automobiles, chemicals, appliances and medical items. Companies that have practiced reverse logistics include BMW, Delphi, General Motors and Hewlett-Packard. The effective use of reverse logistics can help a company to compete in its industry, especially when confronting intense competition and low profit margins (Thierry, Salomon, Van Nunen, et al. 1995).

In 1994, paper recycling in Europe amounted to 27.7 million tons with an annual growth rate of about 7%, signifying a recovery rate (in percentage of total paper consumption) of about 43%. European glass recycling grew by almost 10% (in tons collected) in 1994 to more than 7 million tons, which is a recycling rate (in percentage of total glass consumption) of roughly 60% (EUROSTAT 1997). According to Dowlatshahi (2005), remanufacturing is a \$53 billion industry in the US alone. Costs derived from reverse logistics activities in the US exceed \$35 billion per year. The customer returns rate may be as high as 15% of sales in the coming years, and in sectors such as catalogues sales and e-commerce it could reach as much as 35%.

Obviously, reuse of products and materials, such as metal scrap, waste paper,

softdrink bottles are common and economically more attractive than disposal, as the inventory holding cost for the *recoverable* item (old items that are returned, and can be recovered by remanufacturing) and remanufacturing cost are lower compared to the inventory holding cost for the *serviceable* item (new items or recovered items that are as good as new) and manufacturing cost. Besides, waste reduction has received growing attention in the industrialized countries in view of depletion of landfill and incineration capacities. More and more countries have enforced environmental legislation, charging producers with responsibility for the whole product life cycle, among which, take-back obligations (to take back and recycle products) after use are typical measures taken. Moreover green image (that results from such actions) has become an important marketing element for companies (Fleischmann, Bloemhof-Ruwaard, Dekker, et al. 1997).

Hence, the proper management of reverse flows of products and materials is of considerable importance in many industries as it greatly influences economic performance and also environmental impact. New and challenging planning and control problems arise as a result, especially for product recovery management concerning remanufacturing operations where used products, after being returned to the manufacturer, are recovered (through repair, refurbishing, and/or remanufacturing) so that they are as good as new and can be re-introduced into the supply chain to fulfill the demand.

1.1 Three Key Areas in Reverse Logistics

Although the situations in which reuse occurs are diverse and can be classified according to a number of criteria, including reuse motivation, type of recovered items, form of reuse, and involved actors, there are three key areas in reverse logistics

(Fleischmann, Bloemhof-Ruwaard, et al. 1997) , namely, reverse distribution, inventory control in systems with return flows, and production planning with reuse of parts and materials.

Reverse distribution deals with the collection and transportation of used products and packages, and can take place through the original forward channel, through a separate reverse channel, or through combinations of the forward and reverse channel. A major issue in reverse distribution systems is improving channel efficiency, or in the other words, if and how forward and reverse channels should be integrated. From the distribution aspects of recycling and other reuse activities, the reverse distribution is not necessarily symmetrical to forward distribution. Hence special attention is needed for the design of the reverse distribution network, which requires modifications and extensions of the traditional network design models.

The second key area in reverse logistics is inventory management. How far traditional inventory management methods differ from inventory management within reverse logistics depends on the actors involved in the reuse activities and their respective functions. If the used products are returned to the original manufacturer and provide an alternative input resource in the fabrication of new products, the producer has two alternatives for fulfilling the demand: either ordering the required raw materials externally and manufacturing new products or remanufacturing recoverable products and bringing them back to as good as new conditions. The related cost includes the manufacturing/remanufacturing cost, and holding cost for the serviceable (i.e., new or remanufactured) items and for the recoverable (i.e., returned) items. However, the penalty cost and backordering cost may also need to be considered if the producer fails to fulfill the demand in time, as well as the disposal cost if not all of the returned items are remanufactured. The objective of inventory

management is to control external orders and the internal recovery process to guarantee a required service level and to minimize the total cost.

A third key area in reverse logistics is production planning, which deals with selection of recovery options and scheduling in a product recovery environment. Extended approaches are required for the scheduling of production activities related with production and material reuse.

The focus of our research in this thesis is on inventory management with product recovery, both for deterministic demand and stochastic demand. The impact of including the return flow into the inventory management is twofold, on the one hand, it may be cheaper to remanufacture the returned items than manufacturing new items, on the other hand, since the producer typically has little control on the returned items in terms of quantity, quality and timing, it may incur great uncertainty which may lead to higher safety stock levels and possibly increase the total cost. Compared to the traditional inventory control, the inventory management with returns differs in two aspects. First, the inventory level of the serviceable item between two manufacturing cycles is no longer necessarily decreasing, but may increase also. This is because in between the two manufacturing cycles, there may be a remanufacturing cycle which increases the inventory level of the serviceable item. Second, the two alternatives for fulfilling the demand impose an additional set of decisions to be taken. The manufacturing and remanufacturing/recovery cycles have to be coordinated. As there are now inventory of the serviceable item and recoverable item, and the transition probability of the inventory positions may depend on each other, the manufacturer has to manage a two echelon inventory system. This makes the management of inventory systems with product recovery more complex.

1.2 Relevant Processes and Products Flows

The following processes are highly related in the inventory systems with product recovery (Van der Laan 1997).

First, the manufacturing process. The activities involved in this process are procurement from outside to obtain raw materials and sub-assemblies, and production/assembly operations to create new or serviceable products.

Second, the disassembly and testing process. The returned products first enter the disassembly and testing process. During disassembly, returned products are decomposed into modules and components. While testing, it is determined whether the modules and components satisfy the quality requirements for remanufacturing. The modules and components level to which returned products are disassembled is usually predetermined by technical constraints set by product design and by production process design. Based on the outcome of the testing, recoverable item (that can be remanufactured) are separated from non-recoverables. Inputs of this process are recoverable products and outputs are two different material flows, i.e., the flow of non-recoverables and the flow of the recoverable item.

Third, the remanufacturing and testing process. This process transforms the recoverable item into serviceable products. It usually consists of repair, replacement, and cleaning operations. Furthermore, after remanufacturing, the products are tested.

Lastly, the disposal process. There are two kinds of disposal, unplanned disposal and planned disposal. Unplanned disposal refers to the disposal because the returned products do not satisfy the quality requirement. Planned disposal refers to the disposal because it's more cost effective than remanufacturing.

1.3 Research Objective and Contributions

Inventory management systems for reverse logistics involve higher complexity, compared with traditional inventory systems, as it typically consists of more stocking points to balance the inputs and outputs of the processes, and more complex cost structures such as setup costs for remanufacturing and holding cost for the recoverable item. A major characteristic of inventory systems with product recovery is that with product returns, there is growing uncertainty within the system which partly counterbalances the material savings. These aspects require more sophisticated planning and control methods than those for traditional manufacturing inventory systems.

In this thesis, we study and attempt to develop optimal policies for several inventory models for reverse logistics.

First, we consider a product recovery system with deterministic demand and return. The returned items are assumed to be as good as new and satisfy the same demands as new items after remanufactured. The relevant costs are setup costs for manufacturing new items and remanufacturing recoverable items, holding costs for recoverable (i.e. returned) items and for serviceable (manufactured new or remanufactured from recoverable) items. The objective is to minimize the long run average total costs.

The heuristics proposed by prior literature suggest integer-ratio policy, where in either one remanufacturing cycle is followed by an integer number P identical manufacturing cycles, or vice versa. The manufacturing (remanufacturing) lot sizes are identical across cycles. However these heuristics do not provide any apriori performance guarantees.

In the first essay, we analyze the apriori performance guarantee of heuristics

for the problem. First, we develop a lower bound for the problem. And evaluate the above mentioned integer ratio policy, as well as the optimal integer ratio policy. We show that the optimal integer ratio policy is at most 1.5% more than the lower bound and the heuristic policies proposed in the prior literature is at most 6% more than the lower bound.

In the second essay, we consider inventory models with product recovery with stochastic demand and return rates. All returned items are assumed to be remanufactured and recovered, and the recovered items are as good as new and satisfy the same demands as new items. Demands that are not satisfied immediately are assumed to be backlogged. In addition to setup costs for manufacturing and remanufacturing, and holding cost for the serviceable item and recoverable item, in the stochastic model there is also penalty cost for backlogging of demand.

The classes of policies suggested by past work can be broadly labeled as Push and Pull policy. Under a Push policy, as soon as there is sufficient inventory of the recoverable item, remanufacturing of a lot is commenced, irrespective of the inventory position of the serviceable item. Manufacturing of a new lot is scheduled if the inventory position of serviceable item is low and there is not enough recoverable item to remanufacture. In Pull policies, only when the inventory position of the serviceable item is sufficiently low, manufacturing or remanufacturing of a lot is commenced. Several classes of policies under this umbrella of Push and Pull policies have been suggested in the literature. In general Pull policies outperform Push policies. However all these policies suggested in the literature involve evaluating the steady state probabilities of a Markov chain with two states. Therefore even for a single set of numerical values for the policy parameters, the total cost can be evaluated only numerically. No closed form optimal solution or a systematic search

algorithm for determining the optimal solution is available. Therefore optimal policies can be evaluated only by a completely extensive numerical search.

In this thesis, two new policies are proposed, both under the class of Pull policies. Based on the recent research on cost function behavior of (r, Q) policies, optimal algorithms for these particular classes of policies are developed.

The first policy we derived is a basic (r, Q) policy, assuming the same reorder point and order quantity for both manufacturing and remanufacturing orders. The policy works as follows: as soon as the inventory position of the serviceable item drops to r , we place an order for a lot of size Q . If there is sufficient inventory of the recoverable item, then it is a remanufacturing order of size Q . Otherwise, a manufacturing order of size Q is placed to increase the inventory position of the serviceable item to $r + Q$. For this class of policies, a total cost formula is derived. An efficient optimal algorithm for this policy is developed.

The second policy we considered is an (r_p, Q_p, r_r, Q_r) policy, which has different reorder point and order quantity for manufacturing and remanufacturing orders. The policy works as follows: as soon as the inventory position of the serviceable item drops to r_r , a remanufacturing lot of size Q_r is setup if there is sufficient inventory of the recoverable item. Otherwise, we let the inventory position of the serviceable item decrease. As soon as it drops to r_p , a manufacturing order of size Q_p is placed to increase the inventory position of the serviceable item, irrespective of the inventory position of the recoverable products. An efficient optimal algorithm for this class of policies is developed.

We also compare the proposed policy with this policy. The results show that

even the (s_m, Q_m, s_r, S_r) policy determined using the optimal parameter values of our (r_p, Q_p, r_r, Q_r) policy is slightly worse than the optimal (s_m, Q_m, s_r, S_r) policy, it is much efficient.

For the deterministic model, we also propose a heuristic for a more general class of polices, where there are R remanufacturing cycles for every P manufacturing cycles. While P and R are integers, their ratio needs not be an integer value. We implement this heuristic through an innovative interleaving of the lot schedule. When the manufacturing (remanufacturing) lot sizes are kept stationary, the proposed interleaving based heuristic does not outperform the existing methods. However, when variable lot sizes are used across different cycles, the proposed heuristic generates solution with lower cost than existing methods for several problem instances.

1.4 Thesis Organization

This thesis is organized into six chapters. The outline of each chapter is summarized below.

Chapter one gives an introduction to reverse logistics and this provides the motivation for the specific models we introduce and study in this thesis.

Chapter two provides a detailed review of the literature for both the deterministic and stochastic versions of the model that we study.

Chapter three addresses the deterministic version of the problem. In this chapter, we develop the lower bound and also the analytical effectiveness of two classes of polices for the problem.

In chapter four, we consider the stochastic version of the problem and develop

optimal algorithms for two class of policies.

In chapter five, we consider the deterministic version of the problem. Two lot-sizing heuristic for manufacturing systems with product recovery are proposed. The results are numerically evaluated.

Chapter six provides concluding remarks and a summary. Possible future work and extensions are also briefly discussed.

Chapter 2. Literature Review

In this chapter, we provide a detailed review of the models for inventory control in manufacturing systems with product return and recovery. To make the essays/chapters in the thesis self-contained, literature review pertaining to the specific models that we study in this thesis will be repeated in the appropriate individual chapters.

2.1 Reverse Logistics

Reverse logistics is a process in which a manufacturer systematically accepts previously shipped (and sold) products or parts from the point for consumption for possible recycling, remanufacturing, or disposal (Shad 2000). The proposal of the European Working Group on Reverse Logistics, REVLOG, appears to be the most complete and will hence be the definition that we use. The research group defines reverse logistics as the process of planning, implementing and controlling backward flows of raw materials, in process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal. A reverse logistics system incorporates a supply chain that has been redesigned to manage the flow of products or parts destined for remanufacturing, recycling, or disposal and to use resources effectively. The objective of managing the product recovery is to recover as much of the economic and ecological value as reasonably possible, thereby reducing the ultimate quantities of waste (Thierry, Salomon, et al. 1995).

For a variety of economic, environmental, or legislative reasons, product disposal may no longer be the consumers' choice as products come to be recycled or remanufactured by the original manufacturers. Fewer products will be disposed of as

companies investigate their reusability, remanufacturability, and recyclability.

Returned products and components can be resold directly, recovered, or disposed (incinerated or landfilled). The five product recovery options are: repair, refurbishing, remanufacturing, cannibalization, and recycling (Thierry, Salomon, et al. 1995). Examples of items that may be reused directly without prior repair operations (after possibly cleaning and minor maintenance) are reusable packages such as bottles, pallets or containers. The goal of repair is to restore failed products to working order, though possibly with a loss of quality. Examples are numerous and include durable products, such as domestic appliances, industrial machines, and electronic equipment. Recycling denotes material recovery without conserving any product structures. Examples are metal recycling from scrap, glass and paper recycling, and also plastic recycling. By contrast remanufacturing conserves the product identity and seeks to bring the product back into a new condition by carrying out the necessary disassembly, overhaul, and replacement operations. Traditional examples for remanufacturing are mechanical assemblies such as aircraft engines and machine tools. The remaining sections review the literature on the issues in reverse logistics.

2.2 Reverse Distribution

Reverse distribution is the collection and transportation of used products and packages, which can take place through the original forward channel, through a separate reverse channel or through combinations of the forward and the reverse channel. The possible functions in the reverse distribution channel are: collection, testing, sorting, transportation, and processing (Pohlen and Farris 1992). Hence, the major issue is to design a distribution network which can fulfill these functions. Figure 2.1 shows a framework of reverse distribution systems combining the forward

flow from producers to customers, and the reverse flow from customers to producers. Fleischmann et. al. (1997) give an elaborate review on reverse distribution and classify the literature into two models that consider the separate reverse flow, and models that partly use the original forward network for the reverse distribution.

Figure 2.1 Framework of the Reverse Distribution System

To model separate reverse flow, several papers propose modification of traditional facility location models for the design of reverse distribution networks. Unlike the traditional location models, these models consider a convergent structure from many sources to few demand points (Ginter and Starling 1978; Batta and Chiu 1988; Mirchandani and Francis 1989).

Caruso, Colorni and Paruccini (1993) study the design of reverse flows in a solid waste management system, which includes collection, transportation, incineration, composting, recycling and disposal. A multi-objective location allocation model and some heuristics are used to plan the waste management system. The procedure results in the number and location of waste disposal plants, specification of the technology adopted, and the amount of waste processed. Other papers that have

dealt with the design of reverse distribution networks with separate reverse flows include Kroon and Vrijens (1995), Barros, Dekker and Scholten (1998), and Spengler, Puchert, Penkuhn et. al. (1997).

At present, there are only a few models in the literature that treat forward and reverse distribution simultaneously. In industrial practice, rather simple approaches are taken to integrated forward and reverse distribution. In the network design part, an additional cost component representing collection and return handling is added to the transportation costs. Transportation routes are planned completely based on the forward flow; returned products are collected along with the delivery tours. Salomon, Thierry, Hillegersberg, et. al. (1996) developed a decision support system called REVLOG for the design of distribution and collection networks. Given the facility locations, the system determines the optimal goods flows in the return network and resulting cost. By using this decision support system, different network designs can be compared. Castillo and Cochran (1996) study production and distribution planning for products delivered in reusable containers. Their model includes transportation of empty containers back to the plants, and is applied to a case study of a soft-drink company using returnable bottles.

2.3 Inventory Control in Manufacturing Systems with Return Flows

In inventory management for reverse logistics, appropriate control mechanisms are required to integrate the return flow of used products into the producers' manufacturing process. Whether the traditional inventory management methods are adequate for it depends on the actors involved in the reuse activities and their respective functions. For the case where the returned products are sent to the third party or specialized recycling companies, the recoverable products are regarded

as the only input of the inventory, which is quite similar to the traditional inventory models. However, if the used products are returned to the original producer and provide an alternative input resource in the manufacturing of new products, the situation is much different from the previous one and the inventory management problem become quite complex. A general framework for this situation is shown in Figure 2.2.

Figure 2.2 Framework of Inventory Management with Returns

In Figure 2.2, the producer meets the demand for new products and receives used products returned from the customers. There are two alternatives for fulfilling the demand. The producer either orders the required raw materials externally and fabricates new products or overhauls old products and brings them back to as good as new condition. The relevant costs include setup cost for manufacturing and remanufacturing, holding cost for the serviceable item and recoverable item, the penalty cost, and also disposal cost if not all the returned products are recovered. The objective of such inventory management systems is to control the external component orders and the internal component recovery process to guarantee a required service level and to minimize long run average cost per time unit.

Although it may be cheaper to recover an old product than to produce a new

one, the recovery policy also incurs much uncertainty, as the producer typically has little control on the returned flow in the terms of quantity, quality and timing, and may lead to higher safety stock levels or total cost to some extent. The framework above differs from traditional inventory control situations essentially in three aspects. First, as a consequence of the return flow the inventory level of serviceable products between two manufacturing cycles is no longer necessarily decreasing but may increase due to returns and remanufacturing of recoverable products. Second, the two alternatives for fulfilling the demand impose an additional set of decisions to be taken. External orders and recovery process have to be coordinated. This can be compared with a two supply mode inventory system with the special property that supply of one mode cannot fully be controlled. Third, by distinguishing the inventory of products as recoverable (returned items) and serviceable (new or remanufactured items that can meet the demand), the situation described above naturally leads to a two-echelon inventory system.

In this section, we discuss the models proposed and studied in the literature within this framework. The models mainly differ with respect to assumptions on demand and return process and on the recovery process. The literature in this area can be broadly classified into deterministic versus stochastic models.

2.3.1 Common Assumptions

Before reviewing the literature, we list the most common assumptions relating to each process within a manufacturing/inventory system with product recovery.

Demand and return process. The demand for the product and returns can occur at a constant deterministic rate or it can be stochastic. The return process can be independent of the demand or they can be correlated.

Disassembly/assembly process. Most of the models deal with single product, and each product is assumed to consist of a single component only. Therefore, no disassembly operations need to occur.

Testing process. Assumptions related are the testing capacity, the testing time, and the variable testing costs.

Remanufacturing process. Assumptions are made on the number of parallel facilities, the remanufacturing time, and the fixed and variable remanufacturing costs.

Outside procurement/manufacturing process. With respect to this process, assumptions are made on fixed and variable manufacturing costs, the leadtime, and production capacity.

Inventory process. There are two kinds of inventory in the systems, inventory of recoverable products and the inventory of new and remanufactured products. The assumptions are made on storage capacity and inventory holding costs. Besides, to fulfill the demand by the inventory of the serviceable item, customer service in terms of backorder costs per product per time unit or in terms of service level is also involved.

Disposal process. Many papers assume that there is no disposal, and that all returned products are recovered or remanufactured. When there is disposal option, there can either be a fixed cost for disposal of a batch, or a variable cost per unit or both.

In the following two sections, we separately discuss models with deterministic demand and return rates, and models with stochastic demand and return rates.

2.3.2 Deterministic Model

In the deterministic model for inventory systems with product recovery,

demand for new (or serviceable) items occurs at a deterministic and constant rate. This demand can be met with newly manufactured items or with remanufactured (or recovered) items that are considered as good as new. Items are remanufactured with recoverable products. Product return is assumed to also occur at a deterministic and constant rate. The product return rate is generally assumed to be less than (or a fraction of) the demand rate.

There are two types of inventory held in the system. First there is inventory of recoverable products that have not yet been remanufactured or recovered. The standard terminology used in the literature is to denote this as “*recoverable*” items. Then there is the inventory of finished products which are denoted as “*serviceable*” items in most of the prior literature. The serviceable item include both newly manufactured items as well as the items that are recovered or remanufactured from returned products. The remanufactured items are considered as good as new, and hence labeled together as serviceable item, that are used to meet the demand for the product.

Apart from setup costs for manufacturing and remanufacturing, the other relevant costs are holding costs for serviceable item and recoverable item (that is assumed to be incurred at a linear rate). The objective is to determine lot sizing policies for manufacturing and remanufacturing that minimize the long run average costs per unit time.

One of the first deterministic inventory models for reverse logistics was developed and studied by Schrady (1967). The production for manufacturing and remanufacturing is assumed to occur at an infinite rate (or performed by an external supplier). For this model, Schrady proposes a class of control policy with one manufacturing order followed by a fixed number of remanufacturing orders, which is

derived similar to the classical EOQ formula. In particular the optimal manufacturing (or new production) lot-size equals the EOQ formula applied to the net demand rate, i.e. demand minus return, in the case of identical holding costs for serviceable item and recoverable item.

Schrady's model was extended for finite product recovery rate by Nahmias and Rivera (1979). Nahmias and Rivera studied a deterministic model with infinite production rate similar to that of Schrady but with a finite recovery rate that is greater than the demand rate. They proposed a class of heuristic policies that is similar to Schrady's.

Koh, Hwang and Sohn (2002) extend the work of Nahmias and Rivera (1979) by allowing remanufacturing or recovery rate to be either larger or smaller than the demand rate. Again as in Nahmias and Rivera (1979) and Schrady (1967), they consider a class of policies where every manufacturing setup is followed by an integer number R of identical remanufacturing setups.

Teunter (2001) extended the work of Schrady (1967) to consider two classes of control policies: (i) where every manufacturing setup is followed by an integer number R of remanufacturing setups, and (ii) where every remanufacturing setup is followed by an integer number P of manufacturing cycles. In either class of policies, the order quantities or lot sizes used for manufacturing (and remanufacturing) are identical across different cycles. For these restricted classes of policies and with the relaxation that $R(P)$ can be non-integer, Teunter derives the optimal solution as a closed form expression. However as the number of remanufacturing (and manufacturing) setups have to be integer, a heuristic solution is derived from his closed form expression. Teunter's paper is also different from Schrady and other

earlier literature in that it assumes partial disposal, meaning not all the product returns are remanufactured. Also unlike Schrady (1967), holding cost rates for the recoverable and serviceable item are assumed to be different.

In Teunter (2001), the manufacturing and remanufacturing rates were assumed to be infinite. Teunter (2004) extends the work of Teunter (2001) to consider finite production rates for the manufacturing and remanufacturing. Again two classes of policies are considered, and within these classes of policies, the optimal solution to the relaxed problem (with non-integer manufacturing or remanufacturing setups) is obtained. From this, heuristic solutions are obtained for these classes of policies.

Konstantaras and Papachristos (2006) extended the work of Teunter (2004) in two ways. First they allow for complete backordering of demand. Second they obtain exact optimal policies within the two classes of policies considered by Teunter. However they also only consider the restricted class of policies with one manufacturing (remanufacturing) setup following by an integer number of remanufacturing (manufacturing) setups. Hence the solution obtained is not guaranteed to be the global optimal.

A model equivalent to the one by Schrady but with a different control policy has been proposed by Richter (1996a, b) and Richter and Dobos (1999). Richter (1996a, b) considered a case with variable setup numbers for production and repair within some variable collection time interval. Unlike the previously mentioned papers, these set of papers allow for a disposal option. As the disposal is allowed, the returned and recoverable items can be repaired and regarded as new, or be disposed at the end of collection time interval. Therefore, the total cost involved can be analyzed as a function of disposal rate, which is shown to be convex for small and medium disposal rates and to be concave for large rates. Based on this, the existence and generation of

optimal disposal rates are discussed.

Richter and Dobos (1999) extended the work of Richter (1996a, b) evaluating the impact of a variable disposal cost on the production strategy. The disposal rate can be regarded as a function of a variable disposal cost. They showed that the production strategy and also the disposal decision are influenced by the disposal cost, which gives insight on the ecological consequences of economic pressure.

Dobos and Richter (2004) considered a similar model as Richter (1996a, b), and investigated the set of all feasible policies, namely mixed (remanufacturing with disposal alternatives) strategy, and pure (remanufacturing without disposal or disposing all the returned products) strategies. They show that under their assumptions, the mixed strategies are dominated by the pure strategies.

All these models share the drawback of optimizing the parameters of a predetermined class of control policies, without studying optimality of the particular class of policy itself.

2.3.3 Stochastic Model

In this section, we provide a review of the relevant literature that treat the demand and return as stochastic. The stochastic reverse logistics inventory models can be catalogued into two models for two kinds of systems: *repairable inventory systems*, and *inventory systems with product recovery*.

Repairable inventory systems consider the replacement of a failed item by a repaired item (or spare). Failed items are repaired and subsequently enter the spares inventory. The main characteristics of repairable inventory systems are as follows. First, a returned item is immediately replaced by issuing a repaired item (or spare). That is, every return is accompanied by a demand for the same item. Consequently,

returns do not lead to an increase of the total system inventory. The reverse also holds. That is, every demand entails a simultaneous return. In this case, demand and return flow are perfectly correlated. Second, the system is essentially closed in that the total number of items remains constant, and the repair demand arises from the items within the system. These characteristics distinguish repairable inventory systems from the more general product recovery models. The main question addressed by repairable inventory models is how many spares are needed to guarantee a certain degree of availability the system. Papers that study repairable inventory systems include Allen and D'Esopo (1968), Sherbrooke (1968), Graves (1985).

Unlike the models for repairable inventory systems, for manufacturing/inventory systems with product recovery, there are no apriori assumptions on the relation between the demand and return process. Typically they are seen as loosely coupled. This assumption is reasonable when due to time lag and a large population of customers; a returned item can rarely be assigned to a specific demand occurrence. Many models therefore assume demand and return to be independent. The goal of these models is to determine the timing/schedule and batch sizes for manufacturing and remanufacturing so that the long run average total cost is minimized.

A system where recoverable and serviceable inventory can be merged because returned products can be reused directly without any repair or remanufacturing was modeled by Cohen, Pierskalla and Nahmias (1980). They consider a finite horizon periodic review problem, and assume that a fixed share of the products issued in a given period is returned after a fixed leadtime. There are no setup costs and the objective is to optimize the tradeoff between holding costs and shortage costs. They show that under certain assumptions a one-parameter base stock policy is optimal.

However, as disposal is not considered, the optimal inventory target level will not be attainable if it is exceeded by the returns.

The work of Cohen et al. (1980) was extended by Inderfurth (1997) and Inderfurth and Van der Laan (2001) by considering the effects of non-zero leadtimes for orders and recovery. For the model considered by Inderfurth (1997), Kiesmuller and Scherer (2003) provide a method for exact computation of the optimal policy, as well as two different approximate methods. There are also a few papers dealing with multi-echelon inventory systems with returns (DeCroix, Song and Zipkin 2005; DeCroix 2006).

All the above papers address a finite horizon, periodic review problem without any fixed setup costs for manufacturing and remanufacturing. We now discuss papers that consider a continuous review system. Heyman (1977) analyzed disposal policies to optimize the trade-off between additional inventory holding costs and production cost savings. He discusses a model with independent demand and return occurrences. Remanufacturing and outside procurement/manufacturing are assumed to be instantaneous, resulting in perfect service and only one inventory to be considered. Furthermore, no setup costs for manufacturing are considered. The system is controlled by a single parameter, disposal level strategy: when the inventory level of the items exceeds a certain threshold, the returned items are disposed of. Heyman shows the equivalence of this model to a single server queueing model. For the case of Poisson distributed demands and returns he derives an explicit expression for the optimal disposal level. He also proves optimality of the one parameter policy in this case. For generally distributed demands and returns an approximation is given.

Fleischmann, Kuik and Dekker (2002) consider a similar basic model but with setup costs for manufacturing. As in Heyman (1977), returns can be reused as a new

product immediately or in other words, remanufacturing is instantaneous. Demands and returns follow a Poisson process. The return is regarded as a negative demand; hence there is no inventory of the recoverable item involved. With such assumptions, Fleischmann et al. show the optimality of a (s, Q) policy for ordering new items and point out how to determine optimal values of the control parameters.

Muckstadt and Isaac (1981) considered a similar model, but with non-zero leadtime for manufacturing. They assumed that there is no disposal and all returned items are remanufactured, and the demand and return occurrences are following Poisson distribution. The costs considered are holding costs for serviceable item, backordering costs and fixed setup costs for manufacturing. A control policy is proposed that controls manufacturing according to a traditional (s, Q) rule whereas returned products are remanufactured as soon as they are returned. An approximation procedure based on the distribution of the net inventory is presented to determine the optimal values of s and Q . In a second step their results are carried over to a two echelon warehouse-retailer model.

For the above single echelon model two alternative approximation procedures based on the distribution of the net demand during the manufacturing leadtime is proposed by Van der Laan, Dekker and Salomon (1996b). The main difference between the Muckstadt and Isaac procedure and the Van der Laan et al. procedure lies in the nature of the approximation. In the latter procedure an approximation is used on the distribution of the net demand during the manufacturing leadtime, instead of an approximation on the distribution of the net inventory. Then, they extend their models with the option to dispose the returned products and present heuristics.

Later, Van der Laan, Dekker and Salomon (1996a) present a numerical comparison of several disposal strategies. They propose a disposal strategy with four

parameters, (s_p, Q_p, s_d, N) , in which s_p represents the reorder point for manufacturing setup, and Q_p represents the manufacturing lot size. They suggested that the returned products are disposed of whenever the inventory position equals s_d or the number of products in the remanufacturing facility equals N . This strategy is numerically compared with two other strategies (s_p, Q_p, s_d) , and (s_p, Q_p, N) , with which the disposal happens only when the inventory position equals s_d or the number of products in the remanufacturing facility equals N . As we can see, the two strategies (s_p, Q_p, s_d) , and (s_p, Q_p, N) are extreme cases of strategy (s_p, Q_p, s_d, N) . They show that it is advantageous to base disposal decisions on both the inventory level of the recoverable item and an adequately defined total inventory position.

The models discussed above did not consider any setup cost for remanufacturing. Van der Laan (1997) and Van der Laan et al. (1999) were the first to consider a model with setup cost for both manufacturing and remanufacturing. The demand and return for the single item is assumed to follow a correlated Poisson process with a Coxian-2 distributed interoccurrence times. They propose what they term as “push” control strategy and “pull” control strategy. For “push” strategy, as soon as inventory of the recoverable item contains enough items for remanufacturing, they are remanufactured. For “pull” system, the present of the remanufacturing process depends on the inventory position of the serviceable item. As soon as the inventory position of the serviceable item drops to a certain level, it is continuously verified whether sufficient inventory of the recoverable item is available to increase the inventory of the serviceable item to an order upto level for remanufacturing. If

sufficient inventory of the recoverable item is available, a remanufacturing process is present. Otherwise, when the inventory position of the serviceable item drop to an order upto level for manufacturing, and still insufficient inventory of the recoverable item is available, a manufacturing process is present. These models are quite complex, as two-dimensional state space is involved to represent the inventory position of serviceable and recoverable item. And a closed form expression of limiting joint probability distribution is not available. The underlying Markov chain can be evaluated only for specific values of the policy parameters. And the optimal solutions can be obtained only by an exhaustive numerical search.

Van der Laan and Teunter (2006) present a heuristic approach for general demand and return process. They assume that manufacturing and remanufacturing have the same constant leadtime and the density function and distribution function of demand during the leadtime is known. For the policies proposed by Van der Laan (1997) and Van der Laan et al. (1999), they present simple, closed form formulae for approximately determining the best policy parameters.

Chapter 3. Heuristics with Guaranteed Performance Bounds

In this chapter, we consider a manufacturing system with product recovery. The system manufactures new products as well as remanufactures old, recoverable items. The items remanufactured with recoverable products are as good as new and satisfy the same demands as new items. The demand rate for the new items and the return rate for the old, recovered items are deterministic and constant. The relevant costs are holding costs for new (or *serviceable*) items and returned (or *recoverable*) items, and the fixed setup costs for both manufacturing and remanufacturing. No backlogging of demand is allowed. The objective is to minimize the long-run average cost per unit time. We first develop a lower bound among all classes of policies for the problem. We then show that the optimal integer ratio policy for the problem obtains a solution whose cost is at most 1.5 percent more than the lower bound. We also show that the best of integer ratio policy heuristics proposed by Teunter (2001, 2004) obtain a solution that is at most 6% more than the lower bound.

3.1 Introduction

With increasing concerns over depletion of natural resources, as well as the impact of disposal of old products on the environment, reverse logistics or closed loop supply chain has generated greater interest among both academic researchers and practitioners. One aspect of closed loop supply chain is product recovery.

Product recovery which includes repairing and refurbishing of old products as well as remanufacturing of new items with old, recoverable products is being increasingly adopted because of both environmental as well as economic considerations. Metal scrap brokers, waste paper recycling, and deposit systems for soft drink bottles are all examples (Fleischmann, Bloemhof-Ruwaard, et al. 1997).

The electronics industry is among the most important to develop closed loop supply chains for value added recovery. And large computer companies, such as IBM, have implemented process and technology change to improve the management of returns, including assisting clients with return process (Fleischmann, Van Nunen and Grave 2003).

The management and planning of manufacturing and remanufacturing for production systems with product recovery has attracted the attention of many researchers. If there are returned products that can be remanufactured into new items and the demand can be met with newly manufactured or remanufactured products, it leads to a practical lot sizing and production scheduling problem involving manufacturing and remanufacturing.

In this chapter, we consider a manufacturing system with product recovery. Demand for the new product occurs at a constant rate, and must be met without any backlogging. A portion of the old units of the products are returned by the customers. The rate of product return per unit time is assumed to be constant. All the returned units of the products are recovered and remanufactured into new items. The cost incurred includes the inventory holding costs for the new (or *serviceable*) items and the returned (or *recoverable*) items, and the fixed setup cost incurred for every batch of manufacturing and remanufacturing. The objective is to find production lot sizes for manufacturing and remanufacturing as well the production schedule for the same, so as to minimize the long-run average cost.

One of the first papers to consider the lot sizing and scheduling in manufacturing systems with product recovery is Schrady (1967). The production for manufacturing and remanufacturing is assumed to occur at an infinite rate (or performed by an external supplier). For this model, Schrady proposed a $(1, R)$ policy

where every manufacturing setup is followed by an integer number, R , of identical remanufacturing setups. The order quantity for manufacturing and remanufacturing was kept stationary.

Schrady's model was extended for finite production rate for remanufacturing by Nahmias and Rivera (1979). Nahmias and Rivera studied a deterministic model with infinite production rate for manufacturing similar to that of Schrady but with a finite rate for remanufacturing. They proposed a class of heuristic policies that is similar to Schrady's.

Koh, Hwang and Sohn (2002) extended the work of Nahmias and Rivera (1979) by allowing the remanufacturing rate to be either larger or smaller than the demand rate. Again as in Nahmias and Rivera (1979) and Schrady (1967), they consider a class of policies where every manufacturing setup is followed by an integer number R of identical remanufacturing setups.

Teunter (2001) extended the work of Schrady (1967) to consider two classes of control policies: (i) a $(1, R)$ policy where every manufacturing setup is followed by an integer number R of remanufacturing setups, and (ii) a $(P, 1)$ policy, where every remanufacturing setup is followed by an integer number P of manufacturing cycles. In either class of policies, the order quantities or lot sizes used for manufacturing (and remanufacturing) are identical across different cycles. For these restricted classes of policies and with the relaxation that R (P) can be non-integer, Teunter derived the optimal solution as a closed form expression.

In Teunter (2001), the manufacturing and remanufacturing rates were assumed to be infinite. Teunter (2004) extended the work of Teunter (2001) to consider finite production rates for the manufacturing and remanufacturing. Again two classes of policies are considered, and within these classes of policies, the optimal solution to

the relaxed problem (with non-integer manufacturing or remanufacturing cycles) is obtained. From this, heuristic solutions are obtained for these classes of policies.

All the previous papers on the problem have focused on heuristic policies or specific classes of policies without any guaranteed performance bounds. Given that the inventory level of the recoverable item depends in a complex way on the sequences of manufacturing and remanufacturing setups, optimal policies are difficult to find. Even if optimal policies can be obtained, it may be hard to implement as a repeatable cycle as each cycle might contain complex sequence of the manufacturing and remanufacturing batches. Therefore in this chapter, we attempt to find simple heuristics with apriori performance guarantees.

We first develop a lower bound among all classes of policies for the problem. Schrady (1967) and Teunter (2001, 2004) had proposed two classes of heuristic policies without any apriori performance guarantee for the problem. We develop a heuristic policy that belongs within these two classes of policies and prove that the proposed heuristic policy obtains a solution whose cost is at most 1.5% more than the lower bound among all policies for the problem.

The remainder of this chapter is organized as follows. In section 2, we develop the model, and provide the notation and initial analysis. In section 3, we develop the lower bound among all classes of policies for the problem. In section 4, we develop the optimal among the class of integer ratio policies, where the ratio of average number of manufacturing setups per time unit to average number of remanufacturing setups per time unit is either $P:1$ or $1:R$, where P, R are integers ≥ 1 . We show that this heuristic policy achieves a solution that is at most 1.015 times the lower bound. We also show that the best of the two heuristic policies developed by Teunter (2001, 2004) obtains a solution that is at most 1.06 times the lower bound. Finally we end the

paper with a few concluding remarks.

3.2 Notation and Analysis

The notation used throughout the paper is given in Table 3-1. The demand rate for the item is D . The return rate for old units of the item is fD , where $0 < f < 1$ is the return fraction. The production rate for manufacturing is p and the production rate for remanufacturing is r , with $D \leq p$, $r \leq \infty$. We further assume that all recoverable items are remanufactured, and there is no disposal of the returned items.

The relevant costs are setup cost for manufacturing, K_p (per lot), setup cost for remanufacturing, K_r (per lot), holding cost for recoverable items and for serviceable (i.e. manufactured or remanufactured) items which are incurred at rates h_r and h_s respectively. The objective is to find the lot-sizing and scheduling policy that minimizes the long-run average total cost per time unit.

Table 3-1 Notation

D	Demand rate per time unit
f	Return fraction (return rate fD per time unit)
p	Rate of production for manufacturing (of new items)
r	Rate of production for remanufacturing
K_p	Setup cost per production lot for manufacturing
K_r	Setup cost per production lot for remanufacturing
h_r	Holding cost per unit of recoverable item per time unit
h_s	Holding cost per unit of serviceable item per time unit, $h_s > h_r$
$h_s - h_r$	Echelon holding cost for the serviceable item created from recoverable
Q_p	Production lot-size for manufacturing in a stationary policy
Q_r	Production lot-size for remanufacturing in a stationary policy

$\hat{P}$	Average number of manufacturing lots setup per time unit
$\hat{R}$	Average number of remanufacturing lots setup per time unit

Since all the returned items are remanufactured, the average demand met from remanufacturing is fD and the average demand met through manufacturing new items is $(1-f)D$.

Even assuming a stationary policy, (where the lot size for every manufacturing setup is identical and equal to Q_p , and similarly the lot size for every remanufacturing setup is identical and equal to Q_r), the optimal production schedule can be difficult to obtain. For a given $\hat{P}$, $\hat{R}$, let $\frac{\hat{P}}{\hat{R}} = \frac{P}{R}$, (where P , R are integers without common factors other than 1). In this case for every P manufacturing setups there will be R remanufacturing setups. If holding cost rate, h_r , for the recoverable item were zero, then the problem of scheduling the manufacturing and remanufacturing lots become straight forward as the only relevant costs are setup cost for manufacturing/remanufacturing (which is fixed for a given $\hat{P}$, $\hat{R}$), and holding cost for the serviceable item. The inventory holding cost for the recoverable item (which is zero with $h_r = 0$) is not affected by the sequence and schedule of the manufacturing and remanufacturing setups. Therefore one could just have P manufacturing setups followed by R remanufacturing setups, with the only requirement that the total cycle time for the P manufacturing setups is $\frac{P(1-f)}{\hat{P}}$ and cycle time for R remanufacturing setups is $\frac{Rf}{\hat{R}}$. In this case for the time duration $t_p = \frac{P(1-f)}{\hat{P}}$, there are P manufacturing setups and one need to minimize the inventory holding cost for

the serviceable item for this time duration. Clearly it is optimal in this case to have identical lot size, Q_p for each manufacturing setup. Using similar arguments, it is optimal to have identical lot size Q_r for each remanufacturing setup. Also a manufacturing or remanufacturing lot would be setup only at the point when the inventory level of the serviceable item is zero.

However, if holding cost rate h_r is greater than zero, the cost of holding inventory of the recoverable item is influenced by the manufacturing and remanufacturing lot sizes as well as their production sequences. Therefore the optimal lot sizes for manufacturing and remanufacturing in general need not be stationary.

3.3 Lower Bounds

As mentioned, the relevant costs are setup costs for manufacturing and remanufacturing, and holding costs for the serviceable item and recoverable item. Assume that the average number of manufacturing and remanufacturing setups per time unit is $\hat{P}$ and $\hat{R}$ respectively. We first develop a lower bound $Z_L(\hat{P}, \hat{R})$ on the long run average cost per time unit for a given $\hat{P}, \hat{R}$.

Let

$$Z_L(\hat{P}, \hat{R}) = Z_L^S(\hat{P}, \hat{R}) + Z_L^R(\hat{P}, \hat{R}) \quad (3.1)$$

where $Z_L^S(\hat{P}, \hat{R})$ is a lower bound on the sum of the average setup cost per time unit for manufacturing and remanufacturing, and the average holding cost per time unit for the inventory of the serviceable item, and $Z_L^R(\hat{P}, \hat{R})$ is a lower bound on the average holding cost per time unit for the inventory of the recoverable item.

Ignore the cost of holding inventory of the recoverable item for the moment.

Then, as discussed earlier, for a given $(\hat{P}, \hat{R})$, the cost of holding the inventory of the serviceable item is minimized by having identical lot sizes Q_p and Q_r in each setup for manufacturing and remanufacturing. As the annual demand satisfied by manufacturing is $(1-f)D$ and by remanufacturing is fD , the corresponding lot sizes $Q_p = \frac{(1-f)D}{\hat{P}}$ and $Q_r = \frac{fD}{\hat{R}}$. Since the production rate for manufacturing is p , the average inventory of the serviceable item held during a cycle involving a single manufacturing setup is $\frac{1}{2}Q_p\left(1 - \frac{D}{p}\right)$. As the production rate for remanufacturing is r , the average inventory of the serviceable item held during a cycle involving a single remanufacturing setup is $\frac{1}{2}Q_r\left(1 - \frac{D}{r}\right)$.

Since demand satisfied by manufacturing and remanufacturing is $(1-f)D$ and fD per time unit, respectively, the total average duration of all manufacturing and remanufacturing cycles in a time unit is $(1-f)$ and f respectively. Therefore,

$$Z_L^s(\hat{P}, \hat{R}) = K_p\hat{P} + K_r\hat{R} + \frac{1}{2}h_sQ_p\left(1 - \frac{D}{p}\right)(1-f) + \frac{1}{2}h_sQ_r\left(1 - \frac{D}{r}\right)f \quad (3.2)$$

We now develop a lower bound on the average holding cost per time unit for the inventory of the recoverable item. First, consider a general sequence of several cycles of manufacturing setups followed by several cycles of remanufacturing setups. If the time duration of the manufacturing setups and the remanufacturing setups are perfectly matched, then the amount of product returns during the entire sequence of manufacturing and remanufacturing cycles should be equal to the amount required for remanufacturing during this cycle.

In the long run the average quantity of products returned fD is equal to

average quantity of remanufactured products. But in a practical sequence of several cycles of manufacturing setups followed by several cycles of remanufacturing setups, this need not be so. The inventory of the recoverable item will build up at the rate fD during the sequence of cycles of manufacturing setups. And during the duration of the cycles of remanufacturing setups, the inventory of the recoverable item will deplete at rate $r - fD$ during the production and build up at rate fD when there is no production or remanufacturing. (see Figure 3.1)

Figure 3.1 Recoverable and Serviceable Inventory

As the inventory of the recoverable item is converted into inventory of the serviceable item and then sold to customers, it is good to use the echelon inventory concept to study the recoverable products inventory.

The recoverable products inventory at the inventory of the recoverable item echelon incur a holding cost of h_r . When it is converted to the inventory of the serviceable item during remanufacturing, it moves to the inventory of the serviceable item echelon and incur a additional echelon holding cost of $h_s - h_r$.

The echelon inventory graph for the recoverable products and the inventory of the serviceable item generated from recoverable products will be as follows.

Figure 3.2 Echelon Inventory Generated from Recoverable Products

Clearly, during a series of remanufacturing cycles, there has to be sufficient inventory of the recoverable item (to remanufacture the required lot size). If the inventory of the recoverable item generated during the previous sequence of manufacturing cycles plus during the current remanufacturing cycle itself is not

sufficient to make the remanufacturing lot size, then there should have been sufficient build up of inventory of the recoverable item from earlier cycles. The echelon inventory level of the recoverable item is not affected by the lot size for remanufacturing. Therefore, the lot size decisions for remanufacturing depend only on the echelon inventory of the serviceable item generated from remanufacturing.

For a fixed number of remanufacturing setups during the time duration of the sequence of remanufacturing cycles, the echelon holding cost for the inventory of the serviceable item is therefore minimized by having identical lot sizes.

Therefore, the presence of a positive holding rate h_r for recoverable item will not influence the stationarity of the lot sizes for remanufacturing (or manufacturing) in the lower bound.

Now consider the inventory graph for the echelon inventory of recoverables (which includes inventory of the recoverable item as well as inventory of the serviceable item created by remanufacturing). During the cycles of manufacturing setups, the echelon inventory level will increase at rate fD and during cycles of remanufacturing setups, the echelon inventory level will deplete at rate $D - fD$.

Figure 3.3 Echelon Inventory of the Recoverable Products

Consider a finite time horizon T (containing several sequences of remanufacturing cycles, and followed by manufacturing cycles). As we are considering a infinite horizon problem, we can assume that T is large enough for the sequences of manufacturing and remanufacturing cycles to repeat itself every T . Therefore, the average demand satisfied by remanufacturing during this period T is TfD , and correspondingly the average demand satisfied using manufactured products is $T(1-f)D$. The amount of returns generated during the period is TfD . Therefore, the beginning echelon inventory $\tilde{I}$ is equal to the ending inventory. Clearly if we move the last half triangle to the left of Figure 3.3, we have several triangles each representing a sequence of manufacturing and remanufacturing cycles.

Let $\tilde{P} = \frac{\hat{P}}{\hat{R}}$, and $\tilde{R} = \frac{\hat{R}}{\hat{P}}$. (For ease of exposition, we will assume that the

finite horizon $T = 1$, but the arguments developed below is applicable for any time horizon.) Let q_i' be the total lot size for all the remanufacturing cycles in a particular sequence i of manufacturing and remanufacturing cycles. (when $\hat{R} \leq \hat{P}$, there can be at most $\hat{R}$ such sequences with an average of $\tilde{P}$ manufacturing setups for each remanufacturing setup; and when $\hat{R} \geq \hat{P}$, there can be at most $\hat{P}$ such sequences with an average of $\tilde{R}$ remanufacturing setups for each manufacturing setup.)

Let x be the number of different sequences of manufacturing and remanufacturing setups in the time horizon T . We prove in Theorem 1 in the appendix that to minimize holding cost of the echelon inventory, all the x sequences should be identical and should have identical total lot sizes for the remanufacturing cycles and correspondingly for the manufacturing cycles in the sequence.

Case 1: $\frac{\hat{P}}{\hat{R}} = \tilde{P} \geq 1$

In this case, the average number of manufacturing setups for every remanufacturing setup is $\tilde{P}$, which need not be an integer. Hence to achieve the lowest holding cost for the echelon inventory, we might need complex sequences of manufacturing and remanufacturing setups, where the number of manufacturing cycles in a sequence need not be the same for every sequence. As proved in theorem 1, identical total lot size for the remanufacturing cycles achieve the lowest echelon inventory of the recoverable item, and therefore a lower bound on the average inventory of the recoverable item can be achieved by assuming an average cycle of $\tilde{P} = \frac{\hat{P}}{\hat{R}}$ manufacturing setups followed by one remanufacturing setup (even though this might not be a feasible schedule due to $\tilde{P}$ not being an integer). As the number of remanufacturing setups per time unit is $\hat{R}$, the remanufacturing lot size $Q_r = \frac{Df}{\hat{R}}$. In order to remanufacture this lot size Q_r , the lowest possible inventory of the recoverable item required at the beginning of a remanufacturing setup is $Q_r - \frac{Q_r}{r} Df$. This inventory goes down to zero and then increases to $Q_r - \frac{Q_r}{r} Df$ at the end of a series of manufacturing setups (or the beginning of the next remanufacturing setup). Therefore, a lower bound on the inventory holding cost for the recoverable item when $\hat{P} \geq \hat{R}$ is

$$Z_L^R(\hat{P}, \hat{R}) = \frac{1}{2} h_r Q_r \left(1 - \frac{D}{r} f \right), \quad \text{for } \hat{P} \geq \hat{R} \quad (3.3)$$

Case 2: $\frac{\hat{R}}{\hat{P}} = \tilde{R} \geq 1$

In this case, the average number of remanufacturing setups for every manufacturing setup is $\tilde{R}$. Therefore a lower bound on the average inventory of the recoverable item can be achieved by assuming an average cycle of $\tilde{R}$ remanufacturing setups followed by one manufacturing setup. Since the amount of returns generated (TDf) during the period T is equal to the quantity remanufactured, the inventory of the recoverable item would be completely used up at the end of production of the last remanufacturing cycle before a manufacturing setup. Thereafter the inventory of the recoverable item would increase.

As the remanufacturing lot size is Q_r , the time required to deplete the inventory of the remanufactured items after completion of production is $\frac{Q_r}{D} \left(1 - \frac{D}{r}\right)$, and during this time inventory of the recoverable item would build up at rate Df . Therefore, at the commencement of the manufacturing setup, the inventory level of the recoverable item is $Q_r \left(1 - \frac{D}{r}\right) f$. The manufacturing lot size is $Q_p = \frac{D(1-f)}{\hat{P}}$.

As the duration of the manufacturing cycle is $\frac{Q_p}{D}$, the total inventory of the recoverable item at the end of the manufacturing cycle is $Q_r \left(1 - \frac{D}{r}\right) f + Q_p f$. Therefore, the average inventory of the recoverable item from the end of production of the last remanufacturing cycle (before the manufacturing setup) to the beginning of the next set of $\tilde{R}$ remanufacturing cycles is $\frac{1}{2} \left(Q_r \left(1 - \frac{D}{r}\right) f + Q_p f \right)$.

To calculate the average inventory of the recoverable item from the beginning

of the first remanufacturing setup till the end of production of the $\tilde{R}th$ remanufacturing setup, we first calculate the average echelon inventory of recoverables.

At the beginning of the first remanufacturing setup, there is no inventory of the serviceable item. The echelon inventory of the recoverable item depletes at the rate $D - fD$. At the end of production of $\tilde{R}th$ remanufacturing cycle, the inventory of the recoverable item is fully exhausted, however the inventory of the serviceable item (of remanufactured items) is $Q_r \left(1 - \frac{D}{r}\right) f$. Therefore, the average echelon inventory of recoverables during the $\tilde{R}$ remanufacturing cycles is $Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} Q_p f$.

As the remanufacturing lot size is Q_r , the average inventory of the serviceable item during this period is $\frac{1}{2} Q_r \left(1 - \frac{D}{r}\right) f$.

Hence average inventory of the recoverable item during the $\tilde{R}$ remanufacturing cycles is the difference between the average echelon inventory and the average inventory of the serviceable item which is $Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} Q_p f - \frac{1}{2} Q_r \left(1 - \frac{D}{r}\right) f = \frac{1}{2} Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} Q_p f$. Therefore, average inventory of the recoverable item during both the manufacturing cycle and the $\tilde{R}$ remanufacturing cycles is the same. Therefore, the lower bound on the inventory holding cost for the recoverable item in case 2 is

$$Z_L^R(\hat{P}, \hat{R}) = \frac{1}{2} h_r \left(Q_r \left(1 - \frac{D}{r}\right) f + Q_p f \right), \quad \text{for } \hat{R} \geq \hat{P} \quad (3.4)$$

Note that since $Q_p = \frac{D(1-f)}{\hat{P}}$, and $Q_r = \frac{Df}{\hat{R}}$, in the derivation of the lower

bound, $\hat{P} \geq \hat{R}$ is equivalent to $Q_p \leq Q_r \frac{1-f}{f}$, and $\hat{R} \geq \hat{P}$ is equivalent to

$$Q_p \geq Q_r \frac{1-f}{f}.$$

Therefore, the lower bound on the total cost for a given $\hat{P}$, $\hat{R}$ (or correspondingly a given Q_p , Q_r):

$$\begin{aligned} Z_L^1(Q_p, Q_r) = & K_p \frac{D(1-f)}{Q_p} + K_r \frac{Df}{Q_r} + \frac{1}{2} h_s Q_p \left(1 - \frac{D}{p}\right) (1-f) \\ & + \frac{1}{2} h_s Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} h_r Q_r \left(1 - \frac{D}{r}\right) f, \quad \text{for } Q_p \leq Q_r \frac{1-f}{f} \end{aligned} \quad (3.5)$$

$$\begin{aligned} Z_L^2(Q_p, Q_r) = & K_p \frac{D(1-f)}{Q_p} + K_r \frac{Df}{Q_r} + \frac{1}{2} h_s Q_p \left(1 - \frac{D}{p}\right) (1-f) \\ & + \frac{1}{2} h_s Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} h_r \left(Q_r \left(1 - \frac{D}{r}\right) f + Q_p f \right), \quad \text{for } Q_p \geq Q_r \frac{1-f}{f} \end{aligned} \quad (3.6)$$

Let

$$\begin{aligned} x_p^1 &= K_p D(1-f), \quad y_p^1 = \frac{1}{2} h_s \left(1 - \frac{D}{p}\right) (1-f) \\ x_r^1 &= K_r D(1-f), \quad y_r^1 = \left(\frac{1}{2} h_s \left(1 - \frac{D}{r}\right) f + \frac{1}{2} h_r \left(1 - \frac{D}{r}\right) f \right) \frac{f}{(1-f)} \\ x_p^2 &= K_p Df, \quad y_p^2 = \frac{1}{2} h_s \left(1 - \frac{D}{p}\right) \frac{(1-f)^2}{f} + \frac{1}{2} h_r (1-f) \\ x_r^2 &= K_r Df, \quad y_r^2 = \frac{1}{2} h_s \left(1 - \frac{D}{r}\right) f + \frac{1}{2} h_r \left(1 - \frac{D}{r}\right) f \\ P &= \frac{Q_r (1-f)}{Q_p f}, \quad R = \frac{Q_p f}{Q_r (1-f)} = \frac{1}{P} \end{aligned}$$

Then, (3.5) (3.6) can be rewritten as

$$Z_L^1(P, Q_p) = \frac{x_p^1}{Q_p} + y_p^1 Q_p + \frac{x_r^1}{P Q_p} + y_r^1 P Q_p, \quad \text{for } P \geq 1 \quad (3.7)$$

$$Z_L^2(R, Q_r) = \frac{x_p^2}{R Q_r} + y_p^2 R Q_r + \frac{x_r^2}{Q_r} + y_r^2 Q_r, \quad \text{for } R \geq 1 \quad (3.8)$$

It can be seen that when $P = R = 1$, $Z_L^1 = Z_L^2$.

A lower bound on the total cost can therefore be obtained by optimizing $Z_L^1(P, Q_p)$ and $Z_L^2(R, Q_r)$ and choosing the lower of the two bounds. Note that $Z_L^1(Z_L^2)$ is convex in $Q_p(Q_r)$ for a given $P(R)$. Therefore, optimizing with respect to $Q_p(Q_r)$, we get

$$Z_L^1(P) = 2\sqrt{\left(x_p^1 + \frac{x_r^1}{P}\right)(y_p^1 + y_r^1 P)} = 2\sqrt{x_p^1 y_p^1 + x_r^1 y_r^1 + \frac{x_r^1 y_p^1}{P} + x_p^1 y_r^1 P}, \quad P \geq 1 \quad (3.9)$$

and similarly,

$$Z_L^2(R) = 2\sqrt{\left(\frac{x_p^2}{R} + x_r^2\right)(y_p^2 R + y_r^2)} = 2\sqrt{x_p^2 y_p^2 + x_r^2 y_r^2 + \frac{x_p^2 y_r^2}{R} + x_r^2 y_p^2 R}, \quad R \geq 1 \quad (3.10)$$

The terms in the square root in the above expression are convex in $P(R)$.

However, $P(R) \geq 1$. Therefore, the value of $P^*(R^*)$ that optimizes $Z_L^1(Z_L^2)$ are

$$P^* = \begin{cases} \sqrt{\frac{x_r^1 y_p^1}{x_p^1 y_r^1}}, & \text{if } \sqrt{\frac{x_r^1 y_p^1}{x_p^1 y_r^1}} > 1 \\ 1 & \text{otherwise} \end{cases} \quad (3.11)$$

$$R^* = \begin{cases} \sqrt{\frac{x_p^2 y_r^2}{x_r^2 y_p^2}}, & \text{if } \sqrt{\frac{x_p^2 y_r^2}{x_r^2 y_p^2}} > 1 \\ 1 & \text{otherwise} \end{cases} \quad (3.12)$$

The corresponding value of Q_p^* that optimizes Z_L^1 is

$$Q_p^* = \sqrt{\frac{x_p^1 + \frac{x_r^1}{P^*}}{y_p^1 + y_r^1 P^*}} \quad (3.13)$$

and the value of Q_r^* that optimizes Z_L^2 is

$$Q_r^* = \sqrt{\frac{x_r^2 + \frac{x_p^2}{R^*}}{y_r^2 + y_p^2 R^*}} \quad (3.14)$$

Correspondingly, the value of the lower bounds can be obtained by substituting (3.11) into (3.9) and (3.12) into (3.10).

The best among the two lower bounds Z_L^1 and Z_L^2 can be chosen and therefore, the lower bound on the long run average cost for the problem is

$$Z_L = \begin{cases} Z_L^1, & \text{if } Z_L^1 \leq Z_L^2 \\ Z_L^2 & \text{otherwise} \end{cases} \quad (3.15)$$

3.4 Performance bounds of Integer-Ratio policies

The prior literature (Schradly 1967; Teunter 2004) had introduced two classes of policies. In the $(P, 1)$ policy, P manufacturing setups are followed by one remanufacturing setup in repetitive cycles whereas in the $(1, R)$ policy, one manufacturing setup is followed by R remanufacturing setups.

In both cases the lot sizes for manufacturing/remanufacturing are kept stationary. As P and R are integers, the $(P, 1)$ and $(1, R)$ policies can be classified as integer ratio policies (where the ratio of the number of manufacturing setup to number of remanufacturing setup is either $\frac{P}{1}$ or $\frac{1}{R}$).

In this section, we prove that we can obtain the optimal integer ratio policy (i.e., either a $(P, 1)$ policy or a $(1, R)$ policy) whose cost is at most 1.5% more than the lower bound Z_L on the long run average cost for the problem.

As shown in Teunter (2004) for the $(P, 1)$ policy, the long run average cost function is given by the expression (3.5) with additional restriction that

$P = \frac{Q_r(1-f)}{Q_p f}$ is an integer. And for the $(1, R)$ policy, the long run average cost

function is given by expression (3.6) with the additional restriction that

$R = \frac{Q_p f}{Q_r(1-f)}$ is an integer.

As expression (3.5) ((3.6)) is equivalent to expression (3.7) ((3.8)) and therefore expression (3.9) ((3.10)), the cost for $(P, 1)$ ($(1, R)$) policy can be written as

$$TC^{(P, 1)} = 2\sqrt{x_p^1 y_p^1 + x_r^1 y_r^1 + \frac{x_r^1 y_p^1}{P} + x_p^1 y_r^1 P}, \quad P \geq 1, \text{ and integer} \quad (3.16)$$

$$TC^{(1, R)} = 2\sqrt{x_p^2 y_p^2 + x_r^2 y_r^2 + \frac{x_p^2 y_r^2}{R} + x_r^2 y_p^2 R}, \quad R \geq 1, \text{ and integer} \quad (3.17)$$

Theorem 2

(i) Let $TC^{*(P, 1)}$ be the cost correspondingly to the optimal $(P, 1)$ policy.

Then $\frac{TC^{*(P, 1)}}{Z_L^1} \leq 1.015$.

(ii) Let $TC^{*(1, R)}$ be the cost correspondingly to the optimal $(1, R)$ policy.

Then $\frac{TC^{*(1, R)}}{Z_L^2} \leq 1.015$.

Proof:

We prove the first result below. The proof for the second result is similar. As

$(TC^{*(P, 1)})^2$ is convex, and P is required to be an integer, clearly the optimal integer value of P is either $\lceil P^* \rceil$ or $\lfloor P^* \rfloor$ whichever gives the lower cost.

When the value of P^* in the lower bound Z_L^1 is binding and equal to 1 (or any

other integer value), then clearly that itself is an integer solution and therefore

$$\frac{TC^{*(P, 1)}}{Z_L^1} = 1.$$

We now prove the result for the case when P^* is non-integer. In this case, it can be shown that integer value of P that minimize $TC^{(P, 1)}$ satisfies

$$P(P-1) \leq (P^*)^2 = \frac{x_r^1 y_p^1}{x_p^1 y_r^1} \leq P(P+1) \quad (3.18)$$

From (3.18) it can be seen that

$$\frac{1}{\sqrt{2}} \leq \frac{P}{P^*} \leq \sqrt{2} \quad (3.19)$$

where the optimal P^* in the lower bound Z_L^1 is non-integer.

From (3.16) and (3.9)

$$\left(\frac{TC^{*(P, 1)}}{Z_L^1} \right)^2 = \frac{x_p^1 y_p^1 + x_r^1 y_r^1 + \frac{x_r^1 y_p^1}{P} + x_p^1 y_r^1 P}{x_p^1 y_p^1 + x_r^1 y_r^1 + \frac{x_r^1 y_p^1}{P^*} + x_p^1 y_r^1 P^*} \quad (3.20)$$

Let

$$A = x_p^1 y_p^1 + x_r^1 y_r^1 \quad (3.21)$$

$$B = \frac{x_r^1 y_p^1}{P^*} + x_p^1 y_r^1 P^* \quad (3.22)$$

$$C = \frac{x_r^1 y_p^1}{P} + x_p^1 y_r^1 P \quad (3.23)$$

Substituting $P^* = \sqrt{\frac{x_r^1 y_p^1}{x_p^1 y_r^1}}$ into expression (3.22), we get $B = 2\sqrt{x_p^1 x_r^1 y_p^1 y_r^1}$.

In Lemma 1 (appendix), we prove that $B \leq A$.

From (3.11) and the fact P^* is non-binding,

$$\frac{x_r^1 y_p^1}{P^*} = x_p^1 y_r^1 P^* \quad (3.24)$$

Therefore, from (3.24), (3.22) and (3.23),

$$\frac{C}{B} = \frac{x_r^1 y_p^1 / P}{2 x_r^1 y_p^1 / P^*} + \frac{x_p^1 y_r^1 P}{2 x_p^1 y_r^1 P^*} = \frac{1}{2} \left(\frac{P}{P^*} + \frac{P^*}{P} \right) = \frac{1}{2} \left(\varepsilon + \frac{1}{\varepsilon} \right)$$

where $\varepsilon = \frac{P}{P^*}$ and from (3.19), $\frac{1}{\sqrt{2}} \leq \varepsilon \leq \sqrt{2}$. The expression $\left(\varepsilon + \frac{1}{\varepsilon} \right)$ is convex and

therefore its maximum value in the range $\frac{1}{\sqrt{2}} \leq \varepsilon \leq \sqrt{2}$ is at $\varepsilon = \sqrt{2}$ or $\frac{1}{\sqrt{2}}$.

therefore,

$$\frac{C}{B} \leq \frac{1}{2} \left(\sqrt{2} + \frac{1}{\sqrt{2}} \right) = \frac{3}{2\sqrt{2}} = 1.06 \quad (3.25)$$

Now from (3.20), (3.21), (3.22), (3.23) and (3.25),

$$\left(\frac{TC^{*(P, 1)}}{Z_L^1} \right)^2 = \frac{A+C}{A+B} \leq \frac{A+1.06B}{A+B} \quad (3.26)$$

From Lemma 1, we know $B \leq A$. Let $B = \delta A$ where $0 < \delta \leq 1$. Then,

$$\left(\frac{TC^{*(P, 1)}}{Z_L^1} \right)^2 \leq \frac{A+1.06\delta A}{A+\delta A} = \frac{1+1.06\delta}{1+\delta} \quad (3.27)$$

It can be shown (by taking the first derivative with respect to δ), that the right hand side of (3.27) is increasing in δ . As $0 < \delta \leq 1$,

$$\left(\frac{TC^{*(P, 1)}}{Z_L^1} \right)^2 \leq \frac{1+1.06\delta}{1+\delta} \leq \frac{1+1.06}{1+1} = 1.03$$

Therefore,

$$\frac{TC^{*(P, 1)}}{Z_L^1} \leq \sqrt{1.03} = 1.015 \quad \square$$

Theorem 3

Let $TC_{IRP}^* = \min \{TC^{*(P, 1)}, TC^{*(1, R)}\}$, which is the cost of best among the

optimal $(P, 1)$ and $(1, R)$ policies. $\frac{TC_{IRP}^*}{Z_L} \leq 1.015$.

Proof:

Case 1: $Z_L = Z_L^1$

In this case, we pick the best $(P, 1)$ policy. Clearly from theorem 2,

$$\frac{TC_{IRP}^*}{Z_L} = \frac{TC_{IRP}^*}{Z_L^1} \leq 1.015. \text{ As } TC_{IRP}^* \leq TC^{*(P, 1)}, \text{ the result follows.}$$

Case 2: $Z_L = Z_L^2$

In this case, we would pick the best $(1, R)$ policy. The proof is then similar to case 1. □

Teunter (2004) had developed heuristic algorithms for finding $(P, 1)$ and $(1, R)$ policies for the problem. However no apriori performance bounds were provided.

3.5 Other Bounds

Instead of writing Z_L^1 correspondingly $TC^{(P, 1)}$ as in (3.9), we could write

$$Z_L^1(Q_p, Q_r) = \frac{x_p^1}{Q_p} + y_p^1 Q_p + \frac{x_r^1 f}{Q_r(1-f)} + y_r^1 Q_r \frac{1-f}{f} \quad (3.28)$$

Then Q_p and Q_r are separable with the constraint that $Q_r \geq \frac{Q_p f}{1-f}$ for the

lower bound and $\frac{Q_r(1-f)}{Q_p f}$ is an integer for the $(P, 1)$ policy.

Then the optimal Q_p^* and Q_r^* to minimize the lower bound Z_L^1 is

$$Q_p^* = \sqrt{\frac{x_p^1}{y_p^1}} \quad Q_r^* = \begin{cases} \sqrt{\frac{x_r^1}{y_r^1}} \frac{f}{1-f} & \text{if } \sqrt{\frac{x_r^1}{y_r^1}} \geq Q_p^* \\ \sqrt{\frac{x_p^1}{y_p^1}} & \text{otherwise} \end{cases} \quad (3.29)$$

Let $P^* = \frac{Q_r^*(1-f)}{Q_p^*f}$. A heuristic $(P, 1)$ policy can be obtained by choosing

Q_p^* as in the (3.29) above and choose $P = \lfloor P^* \rfloor$ or $\lceil P^* \rceil$ whichever gives the lowest cost. Similarly a heuristic $(1, R)$ policy can also be obtained. Teunter(2004) suggests using the above heuristic. While Teunter does not provide any apriori bounds for his heuristics, using proofs that are somewhat similar to that of Theorems 2 and 3, one can show that best of his $(P, 1)$ or $(1, R)$ heuristic policy obtains a solution that is not more than 1.06 times the lower bound.

3.6 Numerical Examples

As we compare our solution with those proposed by previous literature, the parameters are kept same as in Teunter (2004) without losing generality. As we only consider the problem with demand rate less than production rate of manufacturing and remanufacturing, the demand rate d is set to be 1000, production rate of manufacturing new products p is set to be 5000, and the remanufacturing rate r is set to be 3000. And in most cases, the fixed setup cot for remanufacturing and holding cost rate for the recoverable item are less compared to the fixed setup for manufacturing and holding cost rate for the serviceable item. Therefore, the fixed cost for the manufacturing setup K_p is 20, and the remanufacturing setup K_r is 5. Holding cost for the inventory of the serviceable item h_s is 10, and the inventory

holding cost for the recoverable item h_r is 2.

The parameters used in the numerical examples are as follows.

Table 3-2 Parameter Settings for the Example

d	p	r	K_p	K_r	h_s	h_r
1000	5000	3000	20	5	10	2

The return rate f used in the examples are (0.15, 0.25, 0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95).

The cost of the lower bound, the cost of the solution obtained by our proposed optimal integer policy and that of Teunter's heuristic policy are provided in Table 3-3.

Table 3-3 Cost of Lower Bound and Integer Policy

f	Lower bound (A)	Cost of the optimal integer policy (B)	Percent difference ((B-A)/B)	Cost of Teunter's heuristic policy (C)	Percent difference ((C-A)/A)
0.15	546.7871	547.0375	0.0458%	547.0719	0.0521%
0.25	517.8055	518.411	0.1169%	518.545	0.1428%
0.35	490.6581	490.6628	0.0010%	490.6816	0.0048%
0.45	468.7466	472.7579	0.8557%	483.5825	3.1650%
0.55	446.4243	446.6542	0.0515%	447.0843	0.1479%
0.65	423.4307	423.458	0.0064%	423.4935	0.0148%
0.75	399.2149	399.6874	0.1184%	400.1046	0.2229%
0.85	372.3254	372.3957	0.0189%	372.4343	0.0293%
0.95	336.5239	336.5264	0.0007%	336.527	0.0009%

From the results shown in Table 3-3, we can see that the performance of the optimal ratio policy is within 1.5% of the lower bound (worst case 0.8557%), and the

performance of Teunter's heuristic policy within 6% of the lower bound (worst case 3.165%).

3.7 Conclusion

In this chapter, we consider manufacturing systems with product recovery. All the previous papers on the problem have focused on heuristic policies or specific classes of policies without any guaranteed performance bounds. In this chapter, we develop simple heuristics with apriori performance guarantees.

We first develop a lower bound among all classes of policies for the problem, and then prove that the cost of optimal integer ratio policy (best of $(P, 1)$ or $(1, R)$ policy) obtains a solution whose cost is at most 1.5 percent more than the lower bound.

We also show that the cost of the best of heuristics policy developed by Teunter (2001, 2004) is at most six percent above the lower bound.

Chapter 4. Stochastic Inventory Model

In this chapter, we consider a continuous review inventory model for a manufacturing system with product recovery. Prior literature has proposed several control policies. However, evaluating the underlying steady state probabilities of the inventory positions for even a particular value of the policy parameter is complex. Therefore, the optimal policy can be determined only by exhaustive enumeration. In this chapter, we propose two new classes of policies. Using the properties of the cost function, we are able to develop optimal or near optimal algorithms for the suggested policies. We also provide a comparison of these two policies.

4.1 Introduction

For a variety of economic, environmental, or legislative reasons, reverse logistics has been receiving increasing attention both from the industrial practitioners as well as from scientific researchers. Proper management of reverse flow of products and materials is of considerable importance in many industries. New and challenging planning and control problems arise as a result, especially for product recovery management and remanufacturing operations where used products that are returned are recovered and remanufactured so that they are as good as new and can be re-introduced into the supply chain to fulfill the demand.

If there are returned products available that can be remanufactured into new items and the demand for new items can be met with newly manufactured products or remanufactured products, it leads to a practical lot sizing and production scheduling problem involving manufacturing and remanufacturing.

In this chapter, we consider a continuous review model for a manufacturing system with product recovery. The system manufactures new items as well as

remanufactures the recoverable items, and the manufacturing rate and remanufacturing rate are infinite. The remanufactured items are as good as new and satisfy the same demand as the new item. Any unfulfilled demand is backordered. All the returned products are remanufactured into new items. The demand rate for the new item and the return rate for the recoverable item are stochastic. The relevant costs are holding cost for the new (or *serviceable*) item and returned (or *recoverable*) item, setup costs for both manufacturing and remanufacturing, and also penalty cost for backordered demand. The problem is to determine the timing and the lot sizes for the manufacturing and remanufacturing setups so as to minimize the long-run average cost per time unit.

Van der Laan(1997) and Van der Laan, Salomon and Dekker (1999) have proposed two classes of control policies, which they call “push” and “pull” strategy for the problem.

The so called “push” policy involves three parameters (s_m, Q_m, Q_r) and works as follows: as soon as the inventory level of the recoverable item reaches Q_r , a remanufacturing lot is setup, reducing the inventory of the recoverable item to zero, and increasing the inventory position of the serviceable item by Q_r . A manufacturing lot of size Q_m is setup whenever the inventory position of the serviceable item drops to the level s_m .

The so called “pull” policy involves four parameters (s_m, Q_m, s_r, S_r) and works as follows: as soon as the inventory position of the serviceable item drops to the level s_r , it is continuously verified whether sufficient on-hand inventory of the recoverable item is available to increase the inventory position of the serviceable item from the current level I_s to the level S_r . If sufficient inventory of the recoverable

item is present, then a remanufacturing lot of size $(S_r - I_s)$ is setup. However, when the inventory position of the serviceable item drops to s_m and the inventory of the recoverable item is not sufficient to increase the inventory position of the serviceable item to S_r , a manufacturing order of size Q_m is placed.

The models for these policies involve a two-dimensional state space and a closed form expression of the steady state joint probability distribution is not available. Therefore, it is difficult to analyze the system behavior and find the particular set of policy parameters under which the expected cost is minimized.

Hence, the optimal policy can be determined only by exhaustive enumeration of the expected cost function for each possible set of values of the policy variables.

In this chapter, we attempt to develop policies where the structure of the expected cost function can be analyzed and therefore efficient optimal algorithms for the policy can be developed.

Specifically, we first derive an (r, Q) policy, where the same reorder point r and order quantity Q are used for both manufacturing and remanufacturing lots. The policy works as follows: as soon as the inventory position of the serviceable item drops to the level r , a remanufacturing lot of size Q is setup if there is sufficient inventory of the recoverable item; Otherwise, a manufacturing order of size Q is placed.

Based on this policy, we further consider an (r_p, Q_p, r_r, Q_r) policy, which has different reorder points and order lot sizes for manufacturing and remanufacturing. The policy works as follows: as soon as the inventory position of the serviceable item reaches the level r_r , a remanufacturing lot of size Q_r is setup if there is sufficient inventory of the recoverable item. Otherwise, we let the inventory position of the

serviceable item decrease further. As it drops to the level r_p , a manufacturing order of size Q_p is placed. Note that, in this policy, when the inventory position of the serviceable item drops to the level r_p , a remanufacturing option is not considered at all irrespective of the inventory level of the recoverable item.

For both these classes of policies, we develop optimal algorithms. Then a numerical study is carried out, to evaluate the effectiveness of these policies.

In the remainder of this section, we provide a review of the relevant literature that treat the demand and return as stochastic. The stochastic reverse logistics inventory models can be catalogued into two models for two kinds of systems: *repairable inventory systems*, and *inventory systems with product recovery*.

Repairable inventory systems consider the replacement of a failed item by a repaired item (or spare). Failed items are repaired and subsequently enter the spares inventory. The main characteristics of repairable inventory systems are as follows. First, a returned item is immediately replaced by issuing a repaired item (or spare). That is, every return is accompanied by a demand for the same item. Consequently, returns do not lead to an increase of the total system inventory. The reverse also holds. That is, every demand entails a simultaneous return. In this case, demand and return flow are perfectly correlated. Second, the system is essentially closed in that the total number of items remains constant, and the repair demand arises from the items within the system. These characteristics distinguish repairable inventory systems from the more general product recovery models. The main question addressed by repairable inventory models is how many spares are needed to guarantee a certain degree of availability the system. Papers that study repairable inventory systems include Allen and D'Esopo (1968), Sherbrooke (1968), and Graves (1985).

Unlike the models for repairable inventory systems, for manufacturing/inventory systems with product recovery, there are no apriori assumptions on the relation between the demand and return process. Typically they are seen as loosely coupled. This assumption is reasonable when due to time lag and a large population of customers; a returned item can rarely be assigned to a specific demand occurrence. Many models therefore assume demand and return to be independent. The goal of these models is to determine the timing/schedule and batch sizes for manufacturing and remanufacturing so that the long run average total cost is minimized.

A system where recoverable and serviceable inventory can be merged because returned products can be reused directly without any repair or remanufacturing was modeled by Cohen, Pierskalla and Nahmias (1980). They consider a finite horizon periodic review problem, and assume that a fixed share of the products issued in a given period is returned after a fixed leadtime. There are no setup costs and the objective is to optimize the tradeoff between holding costs and shortage costs. They show that under certain assumptions a one-parameter base stock policy is optimal. However, as disposal is not considered, the optimal inventory target level will not be attainable if it is exceeded by the returns.

The work of Cohen et al. (1980) was extended by Inderfurth (1997) and Inderfurth and Van der Laan (2001) by considering the effects of non-zero leadtimes for orders and recovery. For the model considered by Inderfurth (1997), Kiesmuller and Scherer (2003) provide a method for exact computation of the optimal policy, as well as two different approximate methods. There are also a few papers dealing with multi-echelon inventory systems with returns (DeCroix, Song, et al. 2005; DeCroix 2006).

All the above papers address a finite horizon, periodic review problem without any fixed setup costs for manufacturing and remanufacturing. We now discuss papers that consider a continuous review system. Heyman (1977) analyzed disposal policies to optimize the trade-off between additional inventory holding costs and production cost savings. He discusses a model with independent demand and return occurrences. Remanufacturing and outside procurement/manufacturing are assumed to be instantaneous, resulting in perfect service and only one inventory to be considered. Furthermore, no setup costs for manufacturing are considered. The system is controlled by a single parameter, disposal level strategy: when the inventory level of the items exceeds a certain threshold, the returned items are disposed of. Heyman shows the equivalence of this model to a single server queueing model. For the case of Poisson distributed demands and returns he derives an explicit expression for the optimal disposal level. He also proves optimality of the one parameter policy in this case. For generally distributed demands and returns an approximation is given.

Fleischmann, Kuik and Dekker (2002) consider a similar basic model but with setup costs for manufacturing. As in Heyman (1977), returns can be reused as a new product immediately or in other words, remanufacturing is instantaneous. Demands and returns follow a Poisson process. The return is regarded as a negative demand; hence there is no inventory of the recoverable item involved. With such assumptions, Fleischmann et al. show the optimality of a (s, Q) policy for ordering new items and point out how to determine optimal values of the control parameters.

Muckstadt and Isaac (1981) considered a similar model, but with non-zero leadtime for manufacturing. They assumed that there is no disposal and all return items are remanufactured, and the demand and return occurrences are following Poisson distribution. The costs considered are holding costs for serviceable item,

backordering costs and fixed setup costs for manufacturing. A control policy is proposed that controls manufacturing according to a traditional (s, Q) rule whereas recoverable products are remanufactured as soon as they are returned. An approximation procedure based on the distribution of the net inventory is presented to determine the optimal values of s and Q . In a second step their results are carried over to a two echelon warehouse-retailer model.

For the above single echelon model two alternative approximation procedures based on the distribution of the net demand during the manufacturing leadtime is proposed by Van der Laan, Dekker and Salomon (1996b). The main difference between the Muckstadt and Isaac procedure and the Van der Laan et al. procedure lies in the nature of the approximation. In the latter procedure an approximation is used on the distribution of the net demand during the manufacturing leadtime, instead of an approximation on the distribution of the net inventory. Then, they extend their models with the option to dispose the returned products and present heuristics.

Later, Van der Laan, Dekker and Salomon (1996a) present a numerical comparison of several disposal strategies. They propose a disposal strategy with four parameters, (s_p, Q_p, s_d, N) , in which s_p represents the reorder point for manufacturing setup, and Q_p represents the manufacturing lot size. They suggested that the returned products are disposed of whenever the inventory position equals s_d or the number of products in the remanufacturing facility equals N . This strategy is numerically compared with two other strategies (s_p, Q_p, s_d) , and (s_p, Q_p, N) , with which the disposal happens only when the inventory position equals s_d or the number of products in the remanufacturing facility equals N . As we

can see, the two strategies (s_p, Q_p, s_d) , and (s_p, Q_p, N) are extreme cases of strategy (s_p, Q_p, s_d, N) . They show that it is advantageous to base disposal decisions on both the inventory level of the recoverable item and an adequately defined total inventory position.

The models discussed above did not consider any setup cost for remanufacturing. Van der Laan (1997) and Van der Laan et al. (1999) were the first to consider a model with setup cost for both manufacturing and remanufacturing. The demand and return for the single item is assumed to follow a correlated Poisson process with a Coxian-2 distributed interoccurrence times. They propose a so called push policy (s_m, Q_m, Q_r) and a so called pull policy (s_m, Q_m, s_r, S_r) , which have been discussed at the beginning of this chapter. As discussed, because of the complexity two-dimensional state space, the optimal for both the suggested policies can be determined only by exhaustive enumeration.

Van der Laan and Teunter (2006) present a heuristic approach for general demand and return process. They assume that manufacturing and remanufacturing have the same constant leadtime and the density function and distribution function of demand during the leadtime is known. For the policies proposed by Van der Laan (1997) and Van der Laan et al. (1999), they present simple, closed form formulae for approximately determining the best policy parameters.

4.2 Model Analysis and Notation

We consider a continuous review inventory model with product recovery and stochastic demand and return. The demand and return are driven by independent Poisson processes with rates λ_d and λ_r ($\lambda_r < \lambda_d$), respectively. All the returned

products are remanufactured. The manufacturing rate and remanufacturing rate are infinite. Setup cost for each manufacturing order is K_p , and setup cost for each remanufacturing order is K_r . Holding cost for the new or *serviceable* item is incurred at rate h_s and holding cost for the returned or *recoverable* item is incurred at rate h_r . Demands not met immediately from stock are assumed to be backordered, and a penalty cost at rate p per unit, per unit time is incurred. The leadtime for manufacturing and remanufacturing is L . As in most stochastic inventory models, it is assumed that the entire lot of finished product enter the serviceable item inventory together after the lead time.

Some of the papers in the literature have assumed non-identical leadtimes for manufacturing and remanufacturing. However, as pointed out by Inderfurth and Van der Laan (2001) and Van der Laan and Teunter (2006), it is difficult to write down the expression for steady state probabilities of the inventory level when the leadtimes are non-identical, even when the probabilities for inventory position can be determined. Moreover, with non-identical leadtimes, there is also a possibility that they can cross. Therefore, in this chapter, we assume that leadtimes for manufacturing and remanufacturing are identical. When the leadtimes are not identical, the algorithms developed in this chapter can possibly be used as heuristics.

The objective is to minimize the long run average total cost over an infinite planning horizon. The costs involved are setup cost for manufacturing, setup cost for remanufacturing, penalty cost for backordered demand, and holding costs for the inventory of the serviceable item and the recoverable item.

The notation used is listed below.

Table 4-1 Notation

λ_d	Demand rate
λ_r	Return rate, $\lambda_r < \lambda_d$
L	Leadtime for manufacturing/remanufacturing
K_p	Setup cost per production lot for manufacturing
K_r	Setup cost per production lot for remanufacturing
h_r	Holding cost per unit of recoverable (i.e., returned) item per unit time
h_s	Holding cost per unit of serviceable (i.e., new) item per unit time
p	Backordering cost (per unit, per unit time) for each unit of demand not met immediately
IP_s	Inventory position of the serviceable item. The inventory position of the serviceable item is defined as the on hand inventory of the serviceable item, plus the number under production (WIP) through manufacturing or remanufacturing, minus the backordered demand.
IL_s	Inventory level of the serviceable item. The inventory level of the serviceable item is defined as the on hand inventory of the serviceable item minus the backordered demand.
IL_r	On hand inventory level of the recoverable item.
B	Number of units of backordered demand
$(x)^+$	$= \max(x, 0)$
$(x)^-$	$= \max(-x, 0)$

In the long run, the average demand per time unit met from remanufacturing is λ_d and that from manufacturing is $\lambda_d - \lambda_r$. Therefore, the long run average system cost under a particular control policy (ϕ) is,

$$TC(\phi) = \frac{K_p(\lambda_d - \lambda_r)}{Q_p} + \frac{K_r\lambda_r}{Q_r} + h_s E(IL_s)^+ + h_r E(IL_r) + p E(IL_s)^- \quad (4.1)$$

Given the inventory level of the serviceable item IL_s , holding cost is $h_s (IL_s)^+$

and the backorder cost is $p(IL_s)^-$. Using $x^+ - x^- = x$, we can express the total cost in different ways

$$TC(\phi) = \frac{K_p(\lambda_d - \lambda_r)}{Q_p} + \frac{K_r\lambda_r}{Q_r} + (h_s + p)E(IL_s)^+ + h_rE(IL_r) - pE(IL_s) \quad (4.2)$$

In the following two sections, we propose and develop optimal algorithm for two policies: the (r, Q) policy and (r_p, Q_p, r_r, Q_r) policy.

4.3 (r, Q) Policy

For the (r, Q) policy, we assume the same reorder point (r) and order quantity (Q) for both manufacturing and remanufacturing lots. As soon as the inventory position of the serviceable item drops to the level r , we place an order for a lot of size Q . If there is sufficient inventory of the recoverable item, we place a remanufacturing order; if not, we place an order for manufacturing new products. We analyze the cost function and develop an algorithm to determine the optimal among this class of policy.

The inventory graphs for the (r, Q) policy is shown in Figure 4.1. The long run average cost under this policy is:

$$TC(r, Q) = \frac{K_p(\lambda_d - \lambda_r) + K_r\lambda_r}{Q} + h_sE(IL_s)^+ + h_rE(IL_r) + pE(IL_s)^- \quad (4.3)$$

It is well known that under an (r, Q) policy for Poisson demands, the inventory position of the serviceable item is uniformly distributed between $r+1, \dots, r+Q$. The steady state probabilities of the inventory level (back orders) can be evaluated after the leadtime. Therefore the expected inventory level and back orders is given by

Figure 4.1 Inventory Graph corresponding to the (r, Q) Policy

$$E(IL_s)^+ = \sum_{i>0} i \Pr\{IL_s = i\} = \frac{1}{Q} \sum_{y=r+1}^{r+Q} \sum_{d=0}^y P_{LTd}(d)(y-d) \quad (4.4)$$

$$E(IL_s)^- = \sum_{i<0} i \Pr\{IL_s = i\} = \frac{1}{Q} \sum_{y=r+1}^{r+Q} \sum_{d=y+1}^{+\infty} P_{LTd}(d)(d-y) \quad (4.5)$$

in which, $P_{LTd}(d)$ represents the probability that the demand during the manufacturing/remanufacturing leadtime is d .

Therefore,

$$\begin{aligned}
 TC(r, Q) &= \frac{K_p(\lambda_d - \lambda_r) + K_r\lambda_r}{Q} + h_s E(IL_s)^+ + h_r E(IL_r) + p E(IL_s)^- \\
 &= \frac{K_p(\lambda_d - \lambda_r) + K_r\lambda_r}{Q} + h_r E(IL_r) \\
 &\quad + \frac{1}{Q} \sum_{y=r+1}^{r+Q} \left(h_s \sum_{d=0}^y P_{LTd}(d)(y-d) + p \sum_{d=y+1}^{+\infty} P_{LTd}(d)(d-y) \right) \\
 &= \frac{K_p(\lambda_d - \lambda_r) + K_r\lambda_r}{Q} + h_r E(IL_r) \\
 &\quad + \frac{1}{Q} \sum_{y=r+1}^{r+Q} \left((h_s + p) \sum_{d=0}^y P_{LTd}(d)(y-d) + p \sum_{d=0}^{+\infty} P_{LTd}(d)(d-y) \right) \\
 &= \frac{K_p(\lambda_d - \lambda_r) + K_r\lambda_r}{Q} + h_r E(IL_r) \\
 &\quad + \frac{1}{Q} \sum_{y=r+1}^{r+Q} \left((h_s + p) \sum_{d=0}^y P_{LTd}(d)(y-d) + p(\lambda_d L - y) \right)
 \end{aligned} \tag{4.6}$$

Observing the equation (4.6), let

$$\begin{aligned}
 G(y) &= (h_s + p) \sum_{d=0}^y P_{LTd}(d)(y-d) + p(\lambda_d L - y) \\
 &= (h_s + p) \sum_{d=0}^{y-1} P_{LTd}(d)(y-d) + p(\lambda_d L - y) \\
 &= (h_s + p) \sum_{d=0}^{y-1} P_d + p(\lambda_d L - y)
 \end{aligned} \tag{4.7}$$

where P_d represents the probability that during the leadtime the demand is less than or equal to d . Then,

$$TC(r, Q) = \frac{K_p(\lambda_d - \lambda_r) + K_r\lambda_r}{Q} + h_r E(IL_r) + \frac{1}{Q} \sum_{y=r+1}^{r+Q} G(y) \tag{4.8}$$

If we were to ignore the second term representing the inventory holding cost for the recoverable item in (4.8) above, the total cost replaces to that of a standard single item continuous inventory model with (r, Q) policy. Using the fact that $-G(y)$ is unimodal, the optimal (r, Q) values for this modified problem can be determined using the efficient algorithm of Federgruen and Zheng (1992). The

(r, Q) values obtained by using the Federgruen and Zheng algorithm can be used as the initial solution values (r', Q') for our problem.

Notice that, the value of the reorder point r for our problem only depends on the lot size Q and not on the inventory of the recoverable item. With the initial policy values r' and Q' , if holding cost of the inventory of the recoverable item is included, the total cost will increase. As the lot size Q is the same for manufacturing and remanufacturing, on average, a sufficient inventory of the recoverable item will be buildup for a remanufacturing lot only after $\left(\frac{\lambda_d - \lambda_r}{\lambda_r}\right)$ manufacturing cycles plus one remanufacturing cycle. Increasing the lot size Q will not affect the ratio, and will only increase the average inventory of the recoverable item. Therefore the buildup of inventory of the recoverable item is reduced by having more frequent or shorter cycles, which is facilitated by having a lower order quantity Q . Hence, the average inventory of the recoverable item $E(IL_r)$ will decrease as the ordering quantity Q decreases. Therefore, the total cost function including inventory holding cost for the recoverable item may decrease with an order quantity that is lower than the best ordering quantity Q' determined without considering the inventory holding cost for the recoverable item. The reorder point will also change accordingly. Therefore, after including the inventory holding cost of the recoverable item, the optimal (r^*, Q^*) can be determined from the initial (r', Q') by searching for the lower values of Q .

We first focus on finding the average inventory of the recoverable item. Let $g(j) = \Pr\{I_r = j, j = 0, 1, 2, \dots, +\infty\}$, represents the probability that the steady state inventory level of the recoverable item equal to j .

$$g(j) = \Pr\{I_r = j\} = \sum_{i=r+1}^{r+Q} \Pr\{I_r = j, I_s = i\} \quad (4.9)$$

Note that, $\{I_r = j, I_s = i\}$ is a two dimensional continuous time Markov process on $[0, +\infty) \times [r+1, r+Q]$ in the steady state. Transition rate $\pi_{j,i}(m, n)$ represents the rate of transition from state $\{I_r = j, I_s = i\}$ to $\{I_r = m, I_s = n\}$.

$$\pi_{j,i}(m, n) = \begin{cases} \lambda_r & m = j+1, n = i \\ \lambda_d & m = j, n = i-1, \text{ for } i > r+1 \\ \lambda_d & m = j-Q, n = r+Q, \text{ for } i = r+1, j \geq Q \\ \lambda_d & m = j, n = r+Q, \text{ for } i = r+1, j < Q \end{cases} \quad (4.10)$$

The first case in (4.10) occurs when there is arrival of a product return. The second case occurs when there is a demand occurrence, but the inventory position of the serviceable is greater than $r+1$. The third case occurs when the inventory position of the serviceable item at the point just before demand occurrence is $r+1$, and the recoverable item inventory is greater than or equal to Q . The fourth case occurs when the inventory position of the serviceable item just before demand occurrence is $r+1$ and the recoverable item inventory is less than Q . The bubble diagram for the transition probability of the two-state Markov Chain is given below.

Figure 4.2 Inventory Positions of the Serviceable and Recoverable item

The average inventory of the recoverable item is $E(IL_r) = \sum_{j=0}^{+\infty} jg(j)$. From the bubble diagram, we know that the probability $g(j) = \Pr\{I_r = j\}$, depends on manufacturing and remanufacturing lot size Q . As $\lambda_r < \lambda_d$, the inventory level of the recoverable item is unlikely to be times higher than the inventory position of the serviceable item in the steady state (unless there was a huge initial inventory of the recoverable item). Therefore, in the steady state, the probability $g(j)$ is decreasing as j increases. Although j can be infinite, for j greater than a sufficiently large j , the value of $g(j)$ is bound to be extremely small and can be assumed to zero without losing generality.

Unfortunately, a closed form expression $g(j)$ for any given j is not available.

To get the value of $g(j)$, following algorithm is developed:

- For a given Q , we first assume $j = I_r$ can only be in state $0, \dots, Q$. Based on this assumption and the fact that the total probability of all states should add to one, we find the probability for each state $\{I_r = j, I_s = i\}$, $j = 0, \dots, Q$, $i = r+1, \dots, r+Q$.
- We then relax the assumption, and assume that I_r can be states $0, \dots, 2Q$. Using the initial values of the steady state probabilities obtained from the previous iteration, the steady state probabilities can now be calculated for the expanded state space.
- The assumption is further relaxed so that I_r can be in states $0, \dots, 3Q, 0, \dots, 4Q, \dots$ etc, and for each iteration, the steady state probability is calculated for the expanded state space. We stop the iterations when change in the

value of the steady state probabilities is negligible.

Using the above iterative procedure, the average inventory level of the recoverable item $E(IL_r) = \sum_{j=0}^{+\infty} jg(j)$ can be calculated for any given Q .

Note that the value of the probability $g(j)$ of the inventory level of the recoverable item depends only on Q and not on the reorder level r . The initial value (r', Q') was optimal if holding cost of the recoverable inventory is ignored. As argued earlier, $E(IL_r)$ is increasing in Q . Therefore, the optimal $Q \leq Q'$, the optimal (r^*, Q^*) can be found by searching for values of $Q \leq Q'$. The algorithm to find the optimal (r, Q) policy is as follows,

Algorithm for determining optimal (r, Q) policy for manufacturing system with product recovery:

1. Let $h_r = 0$. Find the optimal (r, Q) policy using the algorithm of Federgruen and Zheng (1992) for minimizing the remaining cost elements. Let the policy values be (r', Q') .
2. Set holding cost rate for the recoverable item to the original value, and calculate the total cost including inventory holding cost for the recoverable item given $Q = Q'$.
3. Search for the minimum total cost by decreasing the order quantity Q from Q' in steps on one until no further reduction in the total cost is possible [Note that when Q is decreased, the reorder point r might also change (increase).]

4.4 (r_p, Q_p, r_r, Q_r) Policy

With the insights on the cost function for the (r, Q) policy, we now consider a policy with different reorder points and order quantities for manufacturing and remanufacturing setup, i.e., (r_p, Q_p, r_r, Q_r) policy.

The policy works as follows: as soon as the inventory position of the serviceable item reaches the level r_r , a remanufacturing lot of size Q_r is setup if there is sufficient inventory of the recoverable item (i.e. if $IL_r \geq Q_r$). If a remanufacturing lot is ordered, this raises the inventory position of the serviceable item to $r_r + Q_r$. Otherwise, we let the inventory position of the serviceable item decrease further. As soon as it drops to the level r_p ($r_p \leq r_r$), a manufacturing order of size Q_p is placed. Note that in this policy a remanufacturing option is considered only at the reorder point r_r . Below this inventory position, only the manufacturing option is considered at the reorder point r_p . This makes the analysis of the cost function tractable.

Note that if we were to allow remanufacturing for inventory position I_s of the serviceable item in the range $r_p \leq I_s < r_r$, with remanufacturing lot size $Q_r + (r_r - I_s)$, then this policy is the same as the (s_m, Q_m, s_r, S_r) of Van der Laan et al. (1999) with $s_m = r_p$, $Q_m = Q_p$, $s_r = r_r$, and $S_r = r_r + Q_r$. While it is difficult to derive the optimal parameters of the (s_m, Q_m, s_r, S_r) policy in an efficient way, we can use optimal parameters of (r_p, Q_p, r_r, Q_r) policy as the parameters of the (s_m, Q_m, s_r, S_r) .

Similar to Van der Laan (1997) and Van der Laan et al. (1999), an implicit

assumption needed here is that the remanufacturing setup cost K_r is lower than the manufacturing setup cost K_p . Otherwise, it would makes sense to have $Q_p < Q_r$ and therefore $r_r < r_p$. If K_r is much higher than K_p , then the policy would have to be modified accordingly.

The graph of the inventory position of the serviceable item and inventory level of the recoverable item under this policy is given in Figure 4.3.

Figure 4.3 Inventory Graph corresponding to the (r_p, Q_p, r_r, Q_r) Policy

The long run average cost under this policy is:

$$TC(r_p, Q_p, r_r, Q_r) = \frac{K_p(\lambda_d - \lambda_r)}{Q_p} + \frac{K_r\lambda_r}{Q_r} + h_s E(IL_s)^+ + h_r E(IL_r) + p E(IL_s)^- \quad (4.11)$$

As the expected demand per time unit satisfied from remanufacturing and manufacturing are respectively λ_r and $\lambda_d - \lambda_r$, the total setup cost is given by the first

two terms of (4.11). The inventory holding cost for the serviceable item and the penalty cost for backordering depend on both the reorder points r_p and r_r , and the lot sizes Q_p and Q_r . The reorder point r_p (or r_r) also depends on the lot size Q_p (or Q_r), which makes analysis of the cost function complicated. Intuitively, the inventory holding cost for the recoverable item depends only on manufacturing and remanufacturing lot sizes Q_p and Q_r , but it is difficult to find the probability distribution for each state of the inventory level of the recoverable item.

To calculate the total cost $TC(r_p, Q_p, r_r, Q_r)$, we first need to determine the steady state distribution of the inventory level of the serviceable item and from this the terms $h_s E(IL_s)^+ + pE(IL_s)^-$. Let $f(y)$, (where $r_p + 1 \leq y \leq r_p + Q_p$) be the steady state probability that the inventory position of the serviceable item is y .

Then $P\{IL_s = i\} = \sum_{y=r_p+1}^{r_p+Q_p} f(y) g_{LT}(y-i)$, where $g_{LT}(y-i)$ is the probability that the

lead time demand is $y-i$. As the demand rate follows a Poisson process,

$$g_{LT}(j) = e^{-\lambda_d L} \frac{(\lambda_d L)^j}{j!}, \quad j = 0, 1, \dots$$

Using standard methods in inventory theory and similar to the derivation of (4.8), the expected penalty and holding cost for the serviceable item can be written as

$$\begin{aligned} h_s E(IL_s)^+ + pE(IL_s)^- &= (h_s + p) E(IL_s)^+ - pE(IL_s) \\ &= \sum_{y=r_p+1}^{r_p+Q_p} f(y) G(y) \end{aligned} \quad (4.12)$$

where $G(y)$ is as defined in (4.7). Under the (r, Q) policy the inventory position of

the serviceable item followed a uniform distribution with probability $\frac{1}{Q}$ for states

$r+1$ to $r+Q$. For the (r_p, Q_p, r_r, Q_r) policy, this does not hold. However we can derive the steady state probability $f(y)$ using the bubble diagram for the two-state Markov Chain show below in Figure 4.4.

Figure 4.4 Inventory Position of the Service Item

At every state other than $r_p + 1$ and $r_r + 1$, the transition to the left state occurs with rate λ_d (when demand occurs). At $r_p + 1$, when demand occurs, it transits to state $r_p + Q_p$. At $r_r + 1$, when the demand occurs (at rate λ_d), the transition occurs to either state $r_r + Q_r$ (if it is a remanufacturing order) or to state r_r (if the next order is going to be manufacturing order).

As the total number of manufacturing and remanufacturing orders in a time unit are $\frac{\lambda_d - \lambda_r}{Q_p}$ and $\frac{\lambda_r}{Q_r}$, this leads the transition rates shown in the bubble diagram.

Solving the equations corresponding to the bubble diagram, we get

$$f(y) = \begin{cases} \frac{1}{Q_p} + \frac{\lambda_r}{\lambda_d} \left(\frac{1}{Q_r} - \frac{1}{Q_p} \right) & y = r_r + 1, \dots, r_r + Q_r \\ \frac{\lambda_d - \lambda_r}{\lambda_d} \frac{1}{Q_p} & \text{others} \end{cases} \quad (4.13)$$

Using (4.12) and (4.13), we can write the total cost for the (r_p, Q_p, r_r, Q_r) policy as

$$\begin{aligned} TC(r_p, Q_p, r_r, Q_r) &= \frac{K_p(\lambda_d - \lambda_r)}{Q_p} + \frac{K_r \lambda_r}{Q_r} + h_s E(IL_s)^+ + h_r E(IL_r) + p E(IL_s)^- \\ &= \frac{K_p(\lambda_d - \lambda_r)}{Q_p} + \frac{K_r \lambda_r}{Q_r} + h_r E(IL_r) + \sum_{y=r_p+1}^{r_p+Q_p} G(y) f(y) \\ &= \frac{K_p(\lambda_d - \lambda_r)}{Q_p} + \frac{K_r \lambda_r}{Q_r} + h_r E(IL_r) + \frac{1}{Q_p} \frac{\lambda_d - \lambda_r}{\lambda_d} \sum_{y=r_p+1}^{r_p+Q_p} G(y) + \frac{1}{Q_r} \frac{\lambda_r}{\lambda_d} \sum_{y=r_r+1}^{r_r+Q_r} G(y) \end{aligned} \quad (4.14)$$

If holding cost for the recoverable item is ignored (i.e., letting $h_r = 0$), expression (4.14) reduces to the sum two separate continuous review inventory cost functions involving two different (r, Q) policies, i.e.,

$$\begin{aligned} TC(r_p, Q_p, r_r, Q_r) &= \frac{\lambda_d - \lambda_r}{\lambda_d} C_{(r, Q)}(K_p) + \frac{\lambda_r}{\lambda_d} C_{(r, Q)}(K_r) + h_r E(IL_r) \end{aligned} \quad (4.15)$$

where $C_{(r, Q)}(K)$ is the cost of the (r, Q) policy for a single item inventory problem with demand rate λ_d , setup cost K , holding cost h_s and penalty cost p .

Using Federgruen and Zheng (1992) algorithm, the optimal $C_{(r, Q)}^*(K_p)$ and $C_{(r, Q)}^*(K_r)$ can be determined to find an initial value (r_p', Q_p', r_r', Q_r') for the policy.

When the inventory holding cost for the recoverable item is considered, the

total cost will of course increase. Unlike in the (r, Q) policy, the number of manufacturing cycles needed to buildup sufficient inventory of the recoverable item for remanufacturing is $M = \frac{Q_r(\lambda_d - \lambda_r)}{Q_p \lambda_r}$. While reducing Q_r can reduce the average inventory of the recoverable item, having M to be an integer possibly also facilitate this decrease. Therefore the total cost can possibly decrease by increasing or decreasing Q_r and similarly by increasing or decreasing Q_p . However there is only a need to searching for values of Q_p and Q_r such that the ratio $\frac{Q_r(\lambda_d - \lambda_r)}{Q_p \lambda_r}$ reaches the next highest and the next lowest integer. Beyond this range one need to only search for decreasing values of Q_r until no further decrease in total cost is possible. Of course, when $Q_r(Q_p)$ is changed, the corresponding reorder point $r_r(r_p)$ may also change.

The algorithm to find the optimal solution for the (r_p, Q_p, r_r, Q_r) policy is as follows,

Statement of Algorithm for the (r_p, Q_p, r_r, Q_r) Policy:

1. Without considering inventory holding cost for the recoverable item, (i.e., let $h_r = 0$), determine initial (r'_p, Q'_p, r'_r, Q'_r) by optimizing two corresponding single item inventory problem, using the Federgruen and Zheng (1992) algorithm.
2. Let $M = \frac{Q_r(\lambda_d - \lambda_r)}{Q_p \lambda_r}$, $\alpha = \lceil M \rceil / M$, AND $\beta = \lfloor M \rfloor / M$. If $\lfloor M \rfloor = 0$, let $\beta = 1$. Set h_r back to its original value. Search for the total cost exhaustively for

all values of Q_p and Q_r in the range $\left\lfloor \frac{Q_p'}{\alpha} \right\rfloor$ to $\left\lceil \frac{Q_p'}{\beta} \right\rceil$ and $\left\lfloor \beta Q_r' \right\rfloor$ to $\left\lceil \alpha Q_r' \right\rceil$.

Find the value of Q_p and Q_r that gives the lowest cost among these values.

3. Search for further decrease in total costs by decreasing Q_r by one each time, until no further improvement is possible.

We now discuss how holding cost of the recoverable item can be calculated for the above algorithm for a given (r_p, Q_p, r_r, Q_r) . As in the (r, Q) policy, the state of the inventory position of the serviceable item and inventory level of the recoverable item are related and can be represented by a two-state Markov Chain. The corresponding bubble diagram is given in Figure 4.5.

The transition rate from each state to another in the Markov Chain is given by the following expression

$$\pi_{j,i}(m, n) = \begin{cases} \lambda_r & m = j+1, \quad n = i \\ \lambda_d & m = j, \quad n = i-1, \\ \left(\text{for } i_r > r+1, \text{ or } r_p+1 < i \leq r_r, \text{ or } (i = r_r+1 \text{ and } j < Q_r) \right) \\ \lambda_d & m = j - Q_r, \quad n = r_r + Q_r, \text{ for } (i = r_r+1 \text{ and } j \geq Q_r) \\ \lambda_d & m = j, \quad n = r_p + Q_p, \text{ for } i = r_p+1 \end{cases}$$

Similar to iterative procedure discussed earlier for the (r, Q) policy case, the steady state probabilities for each state can be calculated and from this $h_r E(IL_r)$ can be evaluated.

Figure 4.5 Inventory Positions of the Serviceable and Recoverable item

4.5 Numerical Study

In this section, we carry out a small numerical study to evaluate the performance of the (r, Q) and (r_p, Q_p, r_r, Q_r) policies that we have developed.

The parameter settings for the numerical example are summarized in Table 4-2.

Table 4-2 Parameter Settings

λ_d	λ_r	p	K_p	K_r	h_s	h_r	L
0.8	0.2	3000	100	20	10	5	5

Table 4-3 gives the optimal parameter values as well as total cost of the two policies for both the case where $h_r = 0$ and $h_r = 5$.

As we can see from Table 4-3, for each policy, when the inventory holding cost for the recoverable item increases, the optimal ordering quantity decreases, which is exactly the same as the insights shown previously.

Table 4-3 Results for the Numerical Example

	(r, Q) policy		(r_p, Q_p, r_r, Q_r) policy	
	Without recoverable inventory cost (i.e. $h_r = 0$)	With recoverable inventory cost (i.e. $h_r = 5$)	Without recoverable inventory cost (i.e. $h_r = 0$)	With recoverable inventory cost (i.e. $h_r = 5$)
$r^*(r_p^*)$	8	9	8	8
$Q^*(Q_p^*)$	5	3	5	7
r_r^*	-	-	9	9
Q_r^*	-	-	3	2
Total cost	93.74	103.58	92.99	98.96

Comparing the results for the two policies, (r_p, Q_p, r_r, Q_r) policy

outperforms the (r, Q) policy by more than 3% in this example.

The outperformance of (r_p, Q_p, r_r, Q_r) policy is intuitive, as (r, Q) policy represents an extreme case of (r_p, Q_p, r_r, Q_r) policy when the reorder points for manufacturing and remanufacturing processes are the same $r_p = r_r$, and the order lot sizes for the two processes are equal $Q_p = Q_r$.

For the (r_p, Q_p, r_r, Q_r) policy, if remanufacturing is allowed in the (r_p, Q_p, r_r, Q_r) policy for inventory positions I_s in the range $r_p < I_s \leq r_r$, then the policy is effectively the same as the so called pull policy (s_m, Q_m, s_r, S_r) proposed by Van der Laan et al. (1999). We also compare the proposed policy with this policy.

Table 4-4 Results for Comparison

	(s_m, Q_m, s_r, S_r) policy proposed by Van Der Lann et al. (1999)	(s_m, Q_m, s_r, S_r) policy adopting the parameters from the (r_p, Q_p, r_r, Q_r) policy
s_m	8	8
Q_m	6	7
s_r	9	9
S_r	11	11
Total cost	78.61	81.93

It takes quite a long time to determine the optimal parameters for the (s_m, Q_m, s_r, S_r) policy proposed by Van der Laan et al. (1999), as it is based on an exhaustive search. However, adopting the optimal parameters based on our

(r_p, Q_p, r_r, Q_r) policy provides a much easier way to calculate the total cost for the (s_m, Q_m, s_r, S_r) policy.

Our simulation results show that the optimal (s_m, Q_m, s_r, S_r) policy works only slightly better than the (s_m, Q_m, s_r, S_r) policy determined using the optimal parameter values of our (r_p, Q_p, r_r, Q_r) policy.

4.6 Summary

In this chapter, we consider a continuous review inventory model for manufacturing systems with product recovery. The demand and return are independent stochastic processes. Due to the absence of a nice structure for the cost function, previous works rely on exhaustive enumeration or finding near optimal solutions.

In our study, two policies are proposed for the problem, i.e., (r, Q) policy and (r_p, Q_p, r_r, Q_r) policy.

For the (r, Q) policy, we assume the same reorder point (r) and order quantity (Q) for both manufacturing and remanufacturing lots. As soon as the inventory position of the serviceable item drops to the level r , we place an order for lot of size Q . If there is sufficient inventory of the recoverable item, we place a remanufacturing order; if not, we place an order for manufacturing new products.

For the (r_p, Q_p, r_r, Q_r) policy, we assume different reorder points and batch sizes for manufacturing and remanufacturing processes. It works as follows: as soon as the inventory position of the serviceable item reaches the level r_r , a remanufacturing lot of size Q_r is setup if there is sufficient inventory of the

recoverable item (i.e. if $IL_r \geq Q_r$). If a remanufacturing lot is ordered, this raises the inventory position of the serviceable item to $r_r + Q_r$. Otherwise, we let the inventory position of the serviceable item decrease further. As soon as it drops to the level r_p ($r_p \leq r_r$), a manufacturing order of size Q_p is placed.

We develop efficient algorithm for the two policies. If remanufacturing is allowed in the (r_p, Q_p, r_r, Q_r) policy for inventory positions I_s in the range $r_p < I_s \leq r_r$, then the policy is effectively the same as the so called pull policy (s_m, Q_m, s_r, S_r) proposed by Van der Laan et al. (1999). We also compare the proposed policy with this policy. The results show that even the (s_m, Q_m, s_r, S_r) policy determined using the optimal parameter values of our (r_p, Q_p, r_r, Q_r) policy is slightly worse than the optimal (s_m, Q_m, s_r, S_r) policy, it is much efficient.

Future research can possibly consider developing efficient optimal algorithms for the problem when the demand and return are correlated.

Chapter 5. Heuristics for Manufacturing Systems with Product Recovery

In this chapter, we consider the deterministic model of the manufacturing system with product recovery. Two new lot-sizing heuristics are developed in this chapter with the objective of improving upon the existing heuristics for the problem. We then compare and numerically evaluate the proposed heuristics with the existing ones proposed in literature.

5.1 Introduction

We consider a manufacturing system with product recovery. The system manufactures new items as well as remanufactures the recoverable items. The remanufactured/recovered items are as good as new and satisfy the same demands as new items. The demand rate for new items and return rate for the recoverable item are deterministic and constant. The relevant costs are linear holding costs for the new/serviceable item and the recoverable item, and fixed setup costs for both manufacturing and remanufacturing processes. No backlogging of demand is allowed. The problem is to determine the timing and the lot sizes for the manufacturing and remanufacturing setups so as to minimize the long-run average cost per time unit.

Teunter (2004) proposes two types of policies for the problem. In the $(1, R)$ policy, in each cycle, there is one manufacturing setup followed by R remanufacturing setups and in the $(P, 1)$ policy, each cycle has P manufacturing setups and one remanufacturing setup. Teunter develops heuristics to evaluate the cost of the $(1, R)$ and $(P, 1)$ policy and recommends choosing the better among the two policies. In the above policies, P and R are restricted to be integers to facilitate

implementation and this may lead to adjustment in the lot sizes and an increase in cost.

In this chapter, we consider a class of policies (P, R) where on average, the ratio of the number of manufacturing setups to the number of remanufacturing setups per unit time is P/R . In other words, for every P manufacturing setups over time, there will be R remanufacturing setups. Rather than have P manufacturing setups followed by R remanufacturing setups, we interleave (or intersperse) the setups of the manufacturing lots and the remanufacturing lots in such a way that the buildup of the inventory of the recoverable item is minimized. We develop two interleaving based (P, R) policy heuristics for the problem. In the first heuristic, the manufacturing lot size, Q_p , as well as the remanufacturing lot size, Q_r , are always kept fixed over time. In the second heuristic, we allow the lot size for remanufacturing, Q_r to be different in different setups in a cycle. The reason that we do not consider variable lot size Q_p for different manufacturing setups is to minimizing holding cost for the serviceable item. We then compare and numerically evaluate the proposed heuristics with the best of the $(1, R)$ and $(P, 1)$ heuristic.

One of the first deterministic inventory models for reverse logistics was developed and studied by Schrady (1967). The production for manufacturing and remanufacturing is assumed to occur at an infinite rate (or performed by an external supplier). For this model, Schrady proposes a class of control policy with one manufacturing order followed by a fixed number of remanufacturing orders, which is derived similar to the classical EOQ formula. In particular the optimal manufacturing (or new production) lot-size equals the EOQ formula applied to the net demand rate, i.e. demand minus return, in the case of identical holding costs for the serviceable item and recoverable item.

Schrady's model was extended for finite product recovery rate by Nahmias and Rivera (1979). Nahmias and Rivera studied a deterministic model with infinite production rate similar to that of Schrady but with a finite recovery rate that is greater than the demand rate. They proposed a class of heuristic policies that is similar to Schrady's.

Koh, Hwang and Sohn (2002) extend the work of Nahmias and Rivera (1979) by allowing remanufacturing or recovery rate to either larger or smaller than the demand rate. Again as in Nahmias and Rivera (1979) and Schrady (1967), they consider a class of policies where every manufacturing setup is followed by an integer number R of identical remanufacturing setups.

Teunter (2001) extended the work of Schrady (1967) to consider two classes of control policies: (i) where every manufacturing setup is followed by an integer number R of remanufacturing setups, and (ii) where every remanufacturing setup is followed by an integer number P of manufacturing cycles. In either class of policies, the order quantities or lot sizes used for manufacturing (and remanufacturing) are identical across different cycles. For these restricted classes of policies and with the relaxation that $R(P)$ can be non-integer, Teunter derives the optimal solution as a closed form expression. However as the number of remanufacturing (and manufacturing) setups have to be integer, a heuristic solution is derived from his closed form expression. Teunter's paper is also different from Schrady and other earlier literature in that it assumes partial disposal, meaning not all the product returns are remanufactured. Also unlike Schrady (1967), holding cost rates for the recoverable and serviceable item are assumed to be different.

In Teunter (2001), the manufacturing and remanufacturing rates were assumed

to be infinite. Teunter (2004) extends the work of Teunter (2001) to consider finite production rates for the manufacturing and remanufacturing. Again two classes of policies are considered, and within these classes of policies, the optimal solution to the relaxed problem (with non-integer manufacturing or remanufacturing setups) is obtained. From this, heuristic solutions are obtained for these classes of policies.

Konstantaras and Papachristos (2006) extended the work of Teunter (2004) in two ways. First they allow for complete backordering of demand. Second they obtain exact optimal policies within the two classes of policies considered by Teunter. However they also only consider the restricted class of policies with one manufacturing (remanufacturing) setup following by an integer number of remanufacturing (manufacturing) setups. Hence the solution obtained is not guaranteed to be the global optimal.

A model equivalent to the one by Schrady but with a different control policy has been proposed by Richter (1996a, b) and Richter and Dobos (1999). Richter (1996a, b) considered a case with variable setup numbers for production and repair within some variable collection time interval. Unlike the previously mentioned papers, these set of papers allow for a disposal option. As the disposal is allowed, the returned items can be repaired and regarded as new, or be disposed at the end of collection time interval. Therefore, the total cost involved can be analyzed as a function of disposal rate, which is shown to be convex for small and medium disposal rates and to be concave for large rates. Based on this, the existence and generation of optimal disposal rates are discussed.

Richter and Dobos (1999) extended the work of Richter (1996a, b) evaluating the impact of a variable disposal cost on the production strategy. The disposal rate can be regarded as a function of a variable disposal cost. They showed that the production

strategy and also the disposal decision are influenced by the disposal cost, which gives insight on the ecological consequences of economic pressure.

Dobos and Richter (2004) considered a similar model as Richter (1996a, b), and investigated the set of all feasible policies, namely mixed (remanufacturing with disposal alternatives) strategy, and pure (remanufacturing without disposal or disposing all the returned products) strategies. They show that under their assumptions, the mixed strategies are dominated by the pure strategies.

All these models share the drawback of optimizing the parameters of a predetermined class of control policies, without studying optimality of the particular class of policy itself.

5.2 Model Description and Notation

The notation used throughout the paper is given in Table 5-1.

Table 5-1 Notation

d	Demand rate
f	Return fraction (return rate fd)
p	Rate of production for manufacturing (of new items)
r	Rate of production for remanufacturing or recovery
K_p	Setup cost per production lot for manufacturing
K_r	Setup cost per lot for remanufacturing
h_r	Holding cost per recoverable item per time unit
h_s	Holding cost per serviceable item per time unit
Q_p	Production lot-size (or lot size for manufacturing)
Q_r	Recovery lot-size (or lot size for remanufacturing)
$\hat{P}$	Average number of manufacturing setups per time unit. Note that $\hat{P}$ need

	not be an integer.
$\hat{R}$	Average number of remanufacturing setups per time unit. Note that $\hat{R}$ need not be an integer.
P	Number of manufacturing setups for every R remanufacturing setups. Note that P and R are integers
$TC(Q_p, Q_r)$	Total cost with lot sizes Q_p and Q_r

We consider a deterministic lot sizing and scheduling model for a manufacturing system with product recovery. The system faces external demand for a product at a constant rate d . The demand for the product can be satisfied with newly manufactured products or with products recovered or remanufactured from old, recoverable products. The remanufactured products are considered as good as new. The product return takes place at a constant rate fd , where $f < 1$ is the return fraction. It is assumed that all the returned products can be recovered and remanufactured and that there is no disposal of product returns without recovery. The rate of production for manufacturing new products is p , and the rate of production for remanufacturing the recoverable item is r . To simplify the problem without losing generality, we assume that both p and r are greater than the demand rate d . That is $d \leq p$, $r \leq \infty$. The inventory of manufactured or remanufactured products or *serviceable* items incur a holding cost at rate h_s . The inventory of *recoverable items* (returned products that have not yet been remanufactured) incur a holding cost at rate h_r . Setup cost per lot size for manufacturing the items is K_p and setup cost per lot size for remanufacturing the items is K_r . The objective is to find lot sizes and production schedule for manufacturing and remanufacturing that minimize the long

run average cost (comprised of production setup costs and inventory holding costs).

5.3 Model Analysis

As there is no disposal of returns, the average quantity remanufactured per time unit is Df . Therefore, the average quantity manufactured per time unit = $D(1-f)$. If the production lot size per manufacturing setups is Q_p , the average number of manufacturing setups per time unit is

$$\hat{P} = \frac{D(1-f)}{Q_p} \quad (5.1)$$

Similarly, if the lot size per setup for remanufacturing is Q_r , the average of remanufacturing setups per time unit is

$$\hat{R} = \frac{Df}{Q_r} \quad (5.2)$$

As the ratio of number of manufacturing setups to remanufacturing setups per unit time is $\frac{P}{R}$, from (5.1) and (5.2),

$$\frac{P}{R} = \frac{\hat{P}}{\hat{R}} = \frac{D(1-f)Q_r}{DfQ_p} = \frac{(1-f)Q_r}{fQ_p} \quad (5.3)$$

If the choice of the value of Q_p and Q_r leads to the ratio of manufacturing setups to remanufacturing setups ($\frac{P}{R}$) to be an integer > 1 , then the optimal solution will be to schedule one remanufacturing setup with lot size Q_r followed by $\frac{P}{R}$ manufacturing setups each with lot size Q_p .

On the other hand, if the choice of the value of Q_p and Q_r leads to the ratio $\frac{R}{P}$ to be an integer > 1 , the optimal solution will be to schedule one

manufacturing setups with lot size Q_p followed by R/P remanufacturing setups, each with lot size Q_r .

Whether the ratio P/R (or R/P) is integer depends on the parameters of the problem, e.g., setup cost K_p and K_r , and holding cost rate, h_r and h_s .

When either P or $R=1$, we have the $(P, 1)$ or $(1, R)$ policy, where the scheduling of the manufacturing and remanufacturing setups is straight forward. Specifically in the $(P, 1)$ policy we have 1 remanufacturing setup (of a lot size Q_r) followed by P manufacturing setups (each of lot size Q_p) in a cycle. The inventory graph (for the serviceable item and recoverable item) and therefore the total cost can be derived without any difficulty as shown in Teunter (2004).

$$\begin{aligned}
 & TC^{(P, 1)}(Q_p, Q_r) \\
 &= \frac{K_p d(1-f)}{Q_p} + \frac{K_r df}{Q_r} + \frac{1}{2} h_s ((1-f)(1-d/p)Q_p + f(1-d/r)Q_r) + \frac{1}{2} h_r (1-fd/r)Q_r
 \end{aligned}
 \tag{5.4}$$

Similarly, in the $(1, R)$ policy, one manufacturing setup (of lot size Q_p) can be scheduled followed by R remanufacturing setups (each of lot size Q_r) in a cycle. Again the total cost can be derived as shown in Teunter (2004).

$$\begin{aligned}
 & TC^{(1, R)}(Q_p, Q_r) \\
 &= \frac{K_p d(1-f)}{Q_p} + \frac{K_r df}{Q_r} + \frac{1}{2} h_s ((1-f)(1-d/p)Q_p + f(1-d/r)Q_r) + \frac{1}{2} h_r (fQ_p + f(1-d/r)Q_r)
 \end{aligned}
 \tag{5.5}$$

Note that the optimal Q_p and Q_r that minimize the total cost as (5.4) or (5.5), still need to satisfy (5.3), i.e. the resulting value of P and R obtained by (5.3) need to be integers.

This gives the optimal solution to $(P, 1)$ policy as

$$\begin{aligned} Q_p^{(P, 1)} &= \sqrt{\frac{2K_p d}{h_s(1-d/p)}} \\ Q_r^{(P, 1)} &= \sqrt{\frac{2K_r df}{h_s f(1-d/r) + h_r(1-f d/r)}} \end{aligned} \quad (5.6)$$

And the optimal solution for the $(1, R)$ policy is

$$\begin{aligned} Q_p^{(1, R)} &= \sqrt{\frac{2K_p d(1-f)}{h_s(1-f)(1-d/p) + h_r f}} \\ Q_r^{(1, R)} &= \sqrt{\frac{2K_r d}{(h_s + h_r)(1-d/r)}} \end{aligned} \quad (5.7)$$

The resulting P (for the $(P, 1)$ policy) and R (for the $(1, R)$ policy) is then

$$\begin{aligned} P^{(P, 1)} &:= \frac{Q_r^{(P, 1)}(1-f)}{Q_p^{(P, 1)} f} \\ R^{(1, R)} &:= \frac{Q_p^{(1, R)} f}{Q_r^{(1, R)}(1-f)} \end{aligned} \quad (5.8)$$

The solution for P (R) obtained above need not be an integer. Teunter (2004) therefore suggests to modify optimal remanufacturing lot size Q_r for $(P, 1)$ policy or optimal manufacturing lot size Q_p for $(1, R)$ policy to ensure P (or R) is integer.

The modified P and Q_r for $(P, 1)$ policy suggested by Teunter (2004) is

$$\begin{aligned} \tilde{P}^{(P, 1)} &:= \max\{1, \lceil P^{(P, 1)} \rceil\} \\ \tilde{Q}_r^{(P, 1)} &:= \frac{\tilde{P}^{(P, 1)} Q_p^{(P, 1)} f}{1-f} \end{aligned} \quad (5.9)$$

The modified R and Q_p for the for $(1, R)$ policy suggested by Teunter (2004) is

$$\begin{aligned}\tilde{R}^{(1, R)} &:= \max\{1, \lceil R^{(1, R)} \rceil\} \\ \tilde{Q}_p^{(1, R)} &:= \frac{\tilde{R}^{(1, R)} Q_r^{(1, R)} (1-f)}{f}\end{aligned}\quad (5.10)$$

In the policy that we propose in this chapter, both P and R could be greater than 1. That is for every P manufacturing setups, there are R remanufacturing setups. Unlike the $(P, 1)$ policy or $(1, R)$ policy, the scheduling of the lot sizes is not straightforward here. One could of course have P manufacturing setups back to back followed by R remanufacturing setups. But this would result in too much buildup of the returns or the inventory of the recoverable item. Therefore we suggest interleaving (or interspersing) the setups of the manufacturing lots and the remanufacturing lots in such a way that the buildup of the inventory of the recoverable item is minimized.

Assume that we start a new manufacturing or remanufacturing setup only when the inventory of the serviceable item is zero*. Setup costs and holding cost for the inventory of the serviceable item should be the same as in (5.4) and (5.5).

Observing the total cost formula (5.4) and (5.5), the cost function excluding holding cost for the recoverable item are the same. Therefore, let INV_r represent the average inventory level of the recoverable item. Then, clearly the total cost function of the (P, R) policy is

$$\begin{aligned}TC^{(P, R)}(Q_p, Q_r) \\ = \frac{K_p d(1-f)}{Q_p} + \frac{K_r df}{Q_r} + \frac{1}{2} h_s ((1-f)(1-d/p)Q_p + f(1-d/r)Q_r) + h_r INV_r\end{aligned}\quad (5.11)$$

* Note that such a zero (serviceable) inventory policy need not necessarily be optimal, especially if holding cost rate of the inventory of the recoverable item, h_r , is very high.

To solve the problem, we first use the models suggested by Teunter, i.e., we calculate the Q_p and Q_r , assuming the policy followed is either $(P, 1)$ or $(1, R)$. The value of P and R (as well as P/R) obtained by expression (5.4) and (5.5) need not be integer. Let the ratio of P/R be calculated to 2 decimal places. Then a feasible value (that satisfied (5.3)) of P and R to use with the obtained value of Q_p and Q_r , could be $R=100$, and $P=P/R*100$. For example, if $P/R=2.53$, $R=100$, and $P=253$. On the other hand if $P/R=0.87$, $R=100$, and $P=87$. Of course if the value of P/R has lesser decimal places, R needs to be only 10.

We discuss a little later, how for a given P and R , the manufacturing and remanufacturing setups can be interleaved such that there is the least buildup of the inventory of the recoverable item. (note that for inventory of the serviceable item, we are following a zero inventory policy.) Once the exact schedule of the interleaving of the P manufacturing and R remanufacturing setups in a cycle is worked out, the exact value of INV_r can be calculated as function of Q_p and Q_r . Note that Q_p and Q_r still obey constraint (5.3) and therefore the entire total cost function $TC^{(P, R)}(Q_p, Q_r)$ can be expressed as a function of Q_p and Q_r . While it is not possible to give a general closed form expression for $TC^{(P, R)}(Q_p, Q_r)$, for specific values of P and R , and with the interleaving schedule worked out, the total cost can be derived in a closed form and optimized numerically, i.e., the total cost $TC^{(P, R)}(Q_p, Q_r)$ can be expressed as

$$TC^{(P, R)}(Q_p, Q_r) = \frac{A}{Q_r} + (B + C)Q_r \quad (5.12)$$

Where $A = fd\left(K_p \frac{P}{R} + K_r\right)$, $B = \frac{1}{2}h_s\left(\frac{(1-f)^2}{f}\left(1-\frac{d}{p}\right)\frac{R}{P} + f\left(1-\frac{d}{r}\right)\right)$, C is

numerically calculated for the specific interleaving schedule.

Thus the optimal Q_p and Q_r for the general (P, R) policy can be determined.

We next discuss how the manufacturing and remanufacturing setups can be interleaved for the case when $P \geq R$. (The scheduling of setups for $P < R$ can be implemented in a similar way). Given that $P \geq R$, let

$$P = yR + x \quad (5.13)$$

y and x are both integers, with $y \geq 1$, $0 \leq x < R$. (for example, if $P = 253$, and $R = 100$, then $y = 2$, $x = 53$.)

If $x = 0$, then it means that the ratio of manufacturing setups to remanufacturing setups is an integer, therefore one can use the $(P, 1)$ policy with $P = y$. Therefore, Teunter's (2004) model can be regarded as special case of our model, as our model considers a more general lot sizing and scheduling policy.

If $x = 0$, then clearly every sequence or cycle will have y manufacturing followed by one remanufacturing, or a $(y, 1)$ policy.

If $0 < x < R$, clearly in some sub sequences it is possible to have more than y manufacturing setups followed by one remanufacturing setup. To avoid buildup of the inventory of the recoverable item, we want the manufacturing setups spread out as evenly as possible, so that the remanufacturing setups are sequenced often enough. It would make sense to have x sub cycles with $(y+1)$ manufacturing setups followed by 1 remanufacturing setup and $(R-x)$ sub cycles with y manufacturing setups followed by 1 remanufacturing setup. The cycle is as follows,

$$\left\{ \underbrace{\overbrace{P \dots PR}^y, \dots, \overbrace{P \dots PR}^y}_{R-x}, \underbrace{\overbrace{P \dots PR}^{y+1}, \dots, \overbrace{P \dots PR}^{y+1}}_x \right\}$$

$$\left\{ \underbrace{PPR, \dots, PPR}_{47}, \underbrace{PPPR, \dots, PPPR}_{53} \right\}$$

Figure 5.1

For example, when $P = 253$, $R = 100$, $y = 2$ and $x = 53$, we will have a total cycle comprised of 53 sub cycles with $3P+1R$ and $100-53=47$ sub cycles with $2P+1R$ as shown in Figure 5.1 below. (Note that the total manufacturing setups and remanufacturing setups in the 100 sub cycles add up to 253 and 100.)

To work out the optimal interleaving we need to know the exact value of x and $R-x$. For example, if $x = R - x = \frac{R}{2}$, then every $(y+1)PR$, interleaving yPR , if $x > R-x$, clearly in some cases we will have multiple $(y+1)PR$ s followed by 1 yPR . The exact way in which it has been interleaved can be worked out for a specific numerical example. But a simpler logic or algorithmic step to do this is as follows. Assume that there is zero initial inventory of the recoverable item. The first sub cycle should be $(y+1)PR$ sub cycle. Thereafter, based on the available inventory of the recoverable item, we can decide whether to schedule a $(y+1)PR$ sub cycle or yPR sub cycle. If the inventory of the recoverable item at the beginning of the sub cycle is sufficient enough for a yPR sub cycle, we will schedule that, if not we will schedule $(y+1)PR$ sub cycle. Therefore, the whole schedule for the interleaving of the manufacturing and remanufacturing setups can dynamically be determined through simulation of an entire cycle consisting of P manufacturing setups and R remanufacturing setups. For each sub cycle (of $(y+1)PR$ or yPR) given an initial

inventory, we can analytically calculate the average inventory of the recoverable item for that sub cycle. Therefore, for an entire cycle, we can simulate and numerically calculate the average inventory for a given Q_r as CQ_r . (In other words the multiple C in (5.12) can be calculated numerically). Once this is done, the optimal Q_r is available.

We now discuss how to calculate the average inventory of the recoverable item for a particular sub cycle.

Figure 5.2 shows the serviceable and inventory of the recoverable item during

the sub cycle $\overbrace{P \cdots P}^{y+1}R$. As the sub cycle consist of $y+1$ manufacturing setups followed by one remanufacturing setup, the time duration of the sub cycle is

$\frac{Q_r}{d} + (y+1)\frac{Q_p}{d}$. The time duration until the commencement of the remanufacturing

setup is $(1 - d/r)\frac{Q_r}{d} + (y+1)\frac{Q_p}{d}$. As the product return rate is fd , the increase in the

inventory of the recoverable item during the $(y+1)$ manufacturing setups is

$$fd * \left((1 - d/r)\frac{Q_r}{d} + (y+1)\frac{Q_p}{d} \right).$$

Figure 5.2 Serviceable and Recoverable Inventory During Sub Sequence $\overbrace{P \cdots PR}^{y+1}$

The inventory of the recoverable item decreases during the remanufacturing time $\frac{Q_r}{r}$ at rate $r - fd$. Therefore, the total increase in the inventory of the recoverable item during this sub cycle is,

$$fd * \left((1 - d/r) \frac{Q_r}{d} + (y+1) \frac{Q_p}{d} \right) - (r - fd) \frac{Q_r}{r} = \frac{R-x}{P} (1-f) Q_r \quad (5.14)$$

Similarly, we look at the sub cycle of $\overbrace{P \cdots PR}^y$ with y number of manufacturing setups followed by one remanufacturing setup.

Figure 5.3 Serviceable and Recoverable Inventory During Sub Sequence $\overbrace{P \cdots PR}^y$

Figure 5.3 shows the serviceable and recoverable inventory during the sub cycle $\overbrace{P \cdots PR}^y$. As the sub cycle consist of y manufacturing setups followed by one

remanufacturing setup, the time duration of the sub cycle is $\frac{Q_r}{d} + y \frac{Q_p}{d}$. The time

duration until the commencement of the remanufacturing setup is $(1-d/r) \frac{Q_r}{d} + y \frac{Q_p}{d}$.

As the product return rate is fd , the increase in the inventory of the recoverable item

during the y manufacturing setups is $fd * \left((1-d/r) \frac{Q_r}{d} + y \frac{Q_p}{d} \right)$.

The inventory of the recoverable item decreases during the remanufacturing

time $\frac{Q_r}{r}$ at rate $r - fd$. Therefore, the total decrease in the inventory of the recoverable item during this sub cycle is,

$$(r - fd)\frac{Q_r}{r} - fd * \left((1 - d/r)\frac{Q_r}{d} + y\frac{Q_p}{d} \right) = \frac{x}{P}(1 - f)Q_r \quad (5.15)$$

From the above we see that the inventory of the recoverable item increase by

$$\frac{R - x}{P}(1 - f)Q_r \text{ during the sub cycle } \overbrace{P \cdots PR}^{y+1} \text{ and decrease by } \frac{x}{P}(1 - f)Q_r \text{ during the}$$

sub cycle $\overbrace{P \cdots PR}^y$, and the incremental and decreasing amount are linear function of Q_r with a positive coefficient.

As discussed above, since the inventory level of the recoverable item should be non-negative during the whole cycle including the beginning and the end, the cycle

should begin with a sub cycle $\overbrace{P \cdots PR}^{y+1}$, during which the inventory of the recoverable

item will increase. Whether it should be followed by a sub cycle $\overbrace{P \cdots PR}^{y+1}$ or

$\overbrace{P \cdots PR}^y$ depends on the buildup of the inventory. If it's greater than the decrease

caused by the sub cycle $\overbrace{P \cdots PR}^y$, the following sub cycle should be $\overbrace{P \cdots PR}^y$,

otherwise a sub cycle $\overbrace{P \cdots PR}^{y+1}$ will be scheduled. At each end of sub cycle, the

buildup of the inventory of the recoverable item is calculated and this provides the

information to decide what the next sub cycle should be. The order of each sequence

is also recorded and updated. And the total duration of the entire cycle with P

manufacturing setups and R remanufacturing setups is,

$$\text{time duration} = P \frac{Q_p}{d} + R \frac{Q_r}{d} = \frac{(1-f)}{f} R \frac{Q_r}{d} + R \frac{Q_r}{d} = \frac{RQ_r}{df} \quad (5.16)$$

After scheduling all the sub cycles, we can calculate the average inventory of the recoverable item INV_r according to the sequence of the sub cycles.

We always have a $(y+1)PR$ sub cycle first*. Therefore given the discussion above we can calculate the buildup and drawdown in inventory of the recoverable item during the entire cycle as well the average inventory for the entire cycle as a linear function CQ_r . (In other words the multiple C in (5.12) can be calculated numerically). Once C is numerically evaluated, the optimal Q_r can be determined using expression (5.12).

Note that, we did not consider the situation where the increase in the inventory of the recoverable item during the sub cycle $\overbrace{P \cdots P}^{y+1}R$ and decrease in the inventory of the recoverable item during the sub cycle $\overbrace{P \cdots P}^yR$ are the same. Observing the equations (5.14) and (5.15), if the increase and decrease are the same, we have,

$$R - x = x \quad (5.17)$$

* However, the buildup of the inventory of the recoverable item during the time duration of $(y+1)$ manufacturing setups (and corresponding consumption of the inventory of the serviceable item), might not be enough to schedule the remanufacturing lot of Q_r , (if the production rate of remanufacturing is very high.) therefore, based on the time duration we can calculate the initial inventory of recoverable required at the beginning of the entire cycle as a multiple of Q_r .

Substituting the relation suggested by (5.18) into (5.13),

$$P = yR + x = (2y + 1)x \quad (5.18)$$

Therefore, $P(P, R)$ policy is indeed a $P(2y+1, 2)$ policy, with one sub

cycle $\overbrace{P \cdots PR}^{y+1}$ and one sub cycle $\overbrace{P \cdots PR}^y$.

Summarizing in the table below, the interleaving algorithm to minimize the inventory of the recoverable item is,

Table 5-2 Interleaving Algorithm

Step 1:	Use the models proposed by Teunter to initial value of ratio $\frac{P}{R}$. Using this ratio up to two decimals, determining an integer value for P and R .
Step 2:	Calculate the number of sub cycle $\overbrace{P \cdots PR}^{y+1}$ and number of sub cycle $\overbrace{P \cdots PR}^y$, and also the increase or decrease in the inventory of the recoverable item during each sub cycle.
Step 3:	Let i and j represent the number of sub cycles $\overbrace{P \cdots PR}^{y+1}$ and $\overbrace{P \cdots PR}^y$ waiting for scheduling. Initially, $i = x$, and $j = R - x$. If $x = 0$, it is $(P, 1)$ policy with $P = y$, go to the final step. Otherwise, assign the first sub cycle as $\overbrace{P \cdots PR}^{y+1}$, set $i = i - 1$, calculate the inventory level of the recoverable item (as a multiple of Q_r) and update the information of the order of the sequence.
Step 4:	If i and j are not all equal to zero, compare the previous inventory level of the recoverable item (as a multiple of Q_r) with the decrease in inventory of the recoverable item during sub cycle $\overbrace{P \cdots PR}^y$. If inventory is greater than or equal to the decrease during a $\overbrace{P \cdots PR}^y$ sub

<p>cycle, then set $j = j - 1$, and schedule a sub cycle $\overbrace{P \cdots PR}^y$. Otherwise, set $i = i - 1$, and schedule a sub cycle $\overbrace{P \cdots PR}^{y+1}$. Calculate the inventory level of the recoverable item (as a multiple of Q_r) and update the information of the order of the sequence.</p>
<p>Step 5: With the information of the order of sub cycle, calculate the average inventory of the recoverable item INV_r, as a linear function of Q_r. (i.e., determine the value for C)</p>

5.4 Numerical Study

We now compare our proposed (P, R) policy with $(P, 1)$ and $(1, R)$ policies through a numerical study.

As we compare our solution with those proposed by previous literature, the parameters are kept same as in Teunter (2004). As we only consider the problem with demand rate less than production rate of manufacturing and remanufacturing, the demand rate d is set to be 1000, production rate of manufacturing new products p is set to be 5000, and the remanufacturing rate r is set to be 3000. And in most cases, the fixed setup cost for remanufacturing and holding cost rate for the recoverable item are less compared to the fixed setup for manufacturing and holding cost rate for the serviceable item. Therefore, the fixed cost for the manufacturing setup K_p is 20, and the remanufacturing setup K_r is 5. Holding cost for the inventory of the serviceable item h_s is 10, and the inventory holding cost for the recoverable item h_r is 2. The return rate f used in the study is $(0.05, 0.1, 0.15, 0.2)$.

The parameters mentioned above are summarized and shown in Table 5-3.

Table 5-3 Parameter Settings for the Example

d	p	r	K_p	K_r	h_s	h_r
1000	5000	3000	20	5	10	2

The cost of the solution obtained by our proposed (P, R) policy and that of Teunter's policy (best of $(P, 1)$ and $(1, R)$ policies) is provided in Table 5-4.

Table 5-4 Cost of (P, R) Policy

f	Original P/R	Best of $(P, 1)$ & $(1, R)$ policy		Proposed (P, R) policy	
		Policy value (P, R)	Total cost	Policy value (P, R)	total cost
0.05	3.96	(4,1)	571.31	(99,25)	587.53
0.1	2.5	(3,1)	560.97	(5,2)	563.63
0.15	1.82	(2,1)	547.07	(91,50)	565.4
0.2	1.41	(1,1)	537.4	(141,100)	562.72

From the results shown in Table 5-4, the proposed (P, R) policy does not outperform Teunter's best of $(P, 1)$ and $(1, R)$ policies. For $f = 0.1, 0.2$, the total cost for our model is less than one percent higher, compared with Teunter's policy, while for $f = 0.05, 0.15$, the total cost of our model is about 2 or 3 percent more than those of Teunter's. When holding cost rate for the recoverable item is zero $h_r = 0$, our proposed (P, R) policy always outperforms $(P, 1)$ and $(1, R)$ policies. The results with holding cost rate for the recoverable item $h_r = 0$ are shown in Table 5-5.

To understand the numerical results further, we compare each individual cost

elements with the cost of Teunter's policy*. For the cases $f = 0.15, 0.2$ in the above examples in Table 5-4, the result is shown in Table 5-6. From the table we can see that, while the total of setup cost and the inventory holding cost for the serviceable item may decrease with the proposed (P, R) policy, with interleaving algorithm, holding cost for the inventory of the recoverable item are not guaranteed to decrease, compared Teunter's policy.

Table 5-5 Cost of Alternative Policies with Zero Holding Cost for Recoverables

f	Original P/R	Best of $(P, 1)$ & $(1, R)$ policy		Proposed (P, R) policy	
		Policy value (P, R)	Total cost	Policy value (P, R)	Total cost
0.05	10.41	(10,1)	550.32	(1041,100)	550.31
0.1	4.93	(5,1)	534.94	(493,100)	534.94
0.15	3.10	(3,1)	519.58	(31,10)	519.56
0.2	2.19	(2,1)	504.40	(219,100)	504.19

One possible reason is that our model suggests longer cycle time, and although the interleaving algorithm helps in decreasing the inventory of the recoverable item (compared with no interleaving), the total cost is unlikely to be less than that of Teunter's policy.

Table 5-6 Individual Cost Comparison

f=0.15				
	Setup cost for manufacturing	Setup cost for remanufacturing	Holding cost for serviceable inventory	Holding cost for recoverable inventory

* For the rest of discussion, when we say Teunter's policy, we mean the lowest cost policy among the $(P, 1)$ and $(1, R)$ policies calculated according to Teunter's algorithm.

Teunter's model P(2,1)	240.42	30.05	252.89	23.71
our model P(91, 50)	248.18	34.09	243.89	39.23
f=0.2				
	Setup cost for manufacturing	Setup cost for remanufacturing	Holding cost for serviceable inventory	Holding cost for recoverable inventory
Teunter's model P(1,1)	226.27	56.57	238.06	16.5
our model P(141, 100)	238.98	42.37	229.97	51.39

To improve our results, instead of considering a general (P, R) policy, we consider a more restrictive policy where R is allowed to be only 1 or 2 when $P \geq R$ or P is allowed to be only 1 or 2 when $P < R$.

Let $\alpha = P/R$ represents the initial ratio obtained using expression (5.8). For ease of presentation, we assume that $\alpha > 1$. We then compare two policies, namely a) $P = 2\lfloor \alpha + 0.5 \rfloor + 1, R = 2$, b) $P = 2\lfloor \alpha + 0.5 \rfloor - 1, R = 2$. For example, if $\alpha = 2.2$, we compare two policies $(P = 5, R = 2)$ and $(P = 3, R = 2)$, and if $\alpha = 3.6$, we compare two policies $(P = 9, R = 2)$ and $(P = 7, R = 2)$.

With this assumption, the cycle consists two sub cycles only, specifically, one $\overbrace{P \cdots P}^{y+1} R$ and one $\overbrace{P \cdots P}^y R$. This assumption simplifies the algorithm we proposed and decreases the time duration of the entire cycle in most situations.

Table 5-7 Cost of Compare $(P, 2)$ Policy

		Best of $(P, 1)$ & $(1, R)$ policy		Proposed (P, R) policy		$(P, 2)$ policy	
f	Original P/R	Policy value (P, R)	Total cost	Policy value (P, R)	Total cost	Policy value (P, R)	Total cost
0.05	3.96	(4,1)	571.31	(99, 25)	587.53	(9, 2)	573.32

0.1	2.5	(3,1)	560.97	(5,2)	563.63	(5,2)	563.63
0.15	1.82	(2,1)	547.07	(91, 50)	565.4	(3,2)	553.29
0.2	1.41	(1,1)	537.4	(141,100)	562.72	(3,2)	539.63

From the results shown in Table 5-7, we can see that the modified $(P, 2)$ or $(2, R)$ policy with interleaving is better than the original (P, R) policy. However, it does not outperform the best of $(P, 1)$ and $(1, R)$ policies, although the excess cost is quite negligible.

5.5 Model with Variable Remanufacturing Lot Sizing

We therefore consider improving the suggested $(P, 2)$ or $(2, R)$ policy with variable lot sizes. In the case of $(P, 2)$, we fix Q_p but use different Q_r for the two different sub cycles. In the case of $(2, R)$, we also fix Q_p but use different Q_r for the two different sub cycles. We fix Q_p instead of Q_r to maintain lower inventory level of the serviceable item.

For ease of presentation, we only discuss the $(P, 2)$ policy below. (Note that when $R > P$, we should use the $(2, R)$ policy.) Under the modified $(P, 2)$ policy with variable remanufacturing lot size, we will have two remanufacturing lot sizing, Q_{r1} and Q_{r2} . Specifically, for the modified $(P, 2)$ policy, the cycle is $\left\{ \overbrace{P \cdots P}^{y+1} R_1, \overbrace{P \cdots P}^y R_2 \right\}$. Intuitively, Q_{r1} for the sub cycle $\overbrace{P \cdots P}^{y+1} R_1$ is greater than

Q_{r2} for the sub cycle $\overbrace{P \cdots P}^y R$.

First, we analyze $(P, 2)$ policy.

Figure 5.4 shows the serviceable and recoverable inventory for $(P, 2)$ policy with $P = 5$. With constant remanufacturing lot sizing, at the end of sub cycle $\overbrace{P \cdots P}^{y+1}R$, the inventory of the recoverable item is greater than zero. The inventory reaches zero only at the end of the second remanufacturing process.

For $(P, 2)$ policy, a closed form of average inventory of the recoverable item INV_r , which is equal to the area divided by the time duration of the entire cycle, is available.

The total time duration is $\frac{RQ_r}{d} + \frac{PQ_p}{d} = \frac{2Q_r}{df}$, and the area is,

$$\begin{aligned} & \frac{f}{2d} Q_r^2 \left[1 - \frac{d}{r} + \frac{2(1-f)(y+1)}{fP} \right]^2 + \frac{Q_r^2}{r^2} (r - fd) \\ & + \frac{f}{2d} Q_r^2 \left[1 - \frac{d}{r} + \frac{2y(1-f)}{fP} \right]^2 + \frac{Q_r^2 (1-f)(f+2y)}{dfP^2} \end{aligned}$$

Therefore,

Figure 5.4 Serviceable and Recoverable Inventory for $(P, 2)$ Policy with $P = 5$

$$\begin{aligned}
 INV_r = & \frac{f^2}{4d} Q_r \left[1 - \frac{d}{r} + \frac{2(1-f)(y+1)}{fP} \right]^2 + \frac{Q_r}{2r^2} df(r-fd) \\
 & + \frac{f^2}{4d} Q_r \left[1 - \frac{d}{r} + \frac{2y(1-f)}{fP} \right]^2 + \frac{Q_r(1-f)(f+2y)}{2P^2}
 \end{aligned} \tag{5.19}$$

Substituting (5.19) into (5.11), as the value for P is already known, the optimal lot sizing for remanufacturing is available.

Similarly, for modified $(P, 2)$ policy with variable lot sizing for remanufacturing, the serviceable and inventory of the recoverable item with $P = 5$ are shown in Figure 5.5. Compared with $(P, 2)$ Policy, the inventory of the recoverable

item reaches zero at the end of sub cycle $\overbrace{P \cdots P}^{y+1} R$, as well as sub cycle $\overbrace{P \cdots P}^{y+1} R$.

Figure 5.5 Serviceable and Recoverable Inventory for Modified $(P, 2)$ Policy with $P = 5$

With different remanufacturing lot sizing, $P = 2y + 1$, and the time duration of

the cycle is $\frac{Q_{r1} + Q_{r2}}{d} + \frac{PQ_p}{d} = \frac{Q_{r1} + Q_{r2}}{fd}$. The area is,

$$\begin{aligned} & \frac{r - fd}{2dr} Q_{r1} \left[\left(1 - \frac{d}{r} \right) Q_{r2} + Q_{r1} + (y + 1) \frac{1 - f}{Pf} (Q_{r1} + Q_{r2}) \right] \\ & + \frac{r - fd}{2dr} Q_{r2} \left[\left(1 - \frac{d}{r} \right) Q_{r1} + Q_{r2} + y \frac{1 - f}{Pf} (Q_{r1} + Q_{r2}) \right] \end{aligned}$$

Therefore,

$$\begin{aligned} INV_r = & \frac{f(r - fd)}{2r} \frac{Q_{r1}}{Q_{r1} + Q_{r2}} \left[\left(1 - \frac{d}{r} \right) Q_{r2} + Q_{r1} + (y + 1) \frac{1 - f}{Pf} (Q_{r1} + Q_{r2}) \right] \\ & + \frac{f(r - fd)}{2r} \frac{Q_{r2}}{Q_{r1} + Q_{r2}} \left[\left(1 - \frac{d}{r} \right) Q_{r1} + Q_{r2} + y \frac{1 - f}{Pf} (Q_{r1} + Q_{r2}) \right] \end{aligned} \quad (5.20)$$

Observing Figure 5.5, the initial inventory of the recoverable item should be equal to the inventory of the recoverable item at the end of the cycle. Hence, the relation between Q_{r1} and Q_{r2} is,

$$\begin{aligned}\frac{Q_{r1}}{r}(r - fd) &= fd * \left((y+1) \frac{Q_p}{d} + \left(1 - \frac{d}{r} \right) \frac{Q_{r2}}{d} \right) \\ &= fd * \left(\frac{(y+1)(1-f)}{(2y+1)fd} (Q_{r1} + Q_{r2}) + \left(1 - \frac{d}{r} \right) \frac{Q_{r2}}{d} \right)\end{aligned}$$

And therefore,

$$Q_{r2} = \frac{1 - \frac{fd}{r} - \frac{(y+1)(1-f)}{(2y+1)}}{f \left(1 - \frac{d}{r} \right) + \frac{(y+1)(1-f)}{(2y+1)}} Q_{r1} \quad (5.21)$$

Note that, as variable lot size for remanufacturing is considered, the total cost function is no longer the same as (5.11). The new total cost function is shown below,

$$TC^{(P, R)}(Q_p, Q_r) = \frac{K_p d(1-f)}{Q_p} + \frac{2K_r df}{Q_{r1} + Q_{r2}} + h_s INV_s + h_r INV_r \quad (5.22)$$

where INV_s represents the average inventory for the serviceable item, and

$$INV_s = \frac{(1-f)^2}{2(2y+1)f} \left(1 - \frac{d}{p} \right) (Q_{r1} + Q_{r2}) + \frac{f \left(1 - \frac{d}{r} \right)}{2} \frac{Q_{r1}^2 + Q_{r2}^2}{Q_{r1} + Q_{r2}} \quad (5.23)$$

Substituting (5.20), (5.21) and (5.23) into (5.22), the total cost can be expressed as a convex function of just Q_{r1} , which can then be determined by optimization. To summarize, the algorithm for modified $(P, 2)$ policy with variable remanufacturing lot size is shown in Table 5-8.

Table 5-8 Interleaving Algorithm Modified $(P, 2)$ Policy

Step 1:	Use the models proposed by Teunter to determine the initial value of ratio P/R . Set $R = 2$, and determine the possible integer values for P .
Step 2:	Calculate the optimal lot sizes for remanufacturing for each policy modified $(P, 2)$ policy.
Step 3:	Compare the total cost for the modified $(P, 2)$ policies, and choose the best value of P , as well as the optimal lot sizes for remanufacturing.

Using the same parameters in the above examples in Table 5-4, we compare the total costs for modified $(P, 2)$ or $(2, R)$ policy with the best of $(P, 1)$ and $(1, R)$ policy proposed by Teunter.

Table 5-9 Cost of Modified $(P, 2)$ or $(2, R)$ Policy

f	Original P/R or R/P	Best of $(P, 1)$ & $(1, R)$ policy		Best of $(P, 2)$ & $(2, R)$ policy		Best of Modified $(P, 2)$ & $(2, R)$ policy	
		Policy value (P, R)	Total cost	Policy value (P, R)	Total cost	Policy value (P, R)	Total cost
0.05	3.96	P(4,1)	571.31	P(9, 2)	573.32	P(9, 2)	571.78
0.1	2.5	P(3,1)	560.97	P(5,2)	563.63	P(5,2)	560.88
0.15	1.82	P(2,1)	547.07	P(3,2)	553.29	P(3,2)	549.91
0.2	1.41	P(1,1)	537.4	P(3,2)	539.63	P(3,2)	535.29
0.45	1.49	P(1,2)	483.58	P(2,3)	478.33	P(2,3)	471.79
0.5	1.79	P(1,2)	459.62	P(2,3)	469.04	P(2,3)	462.48
0.6	2.56	P(1,3)	438.41	P(2,5)	442.18	P(2,5)	436.42
0.7	3.71	P(1,4)	412.14	P(2,7)	417.13	P(2,7)	412.46
0.8	5.66	P(1,6)	386.55	P(2,11)	389.87	P(2,11)	386.63
0.9	9.98	P(1,10)	356.54	P(2,21)	358.37	P(2,21)	356.72

From the table, we can see that, the modified $(P, 2)$ or $(2, R)$ policy are better than $(P, 2)$ or $(2, R)$ policy, and in some cases, e.g.,

$f = 0.1, 0.2, 0.45, 0.6$, the modified $(P, 2)$ or $(2, R)$ policy outperform the best of $(P, 1)$ and $(1, R)$ policy of Teunter (2004). Especially for the case $f = 0.45$, the cost for the modified $(P, 2)$ or $(2, R)$ policy is 471.79, and the cost for the best of $(P, 1)$ and $(1, R)$ policy is 483.58. When the original (unconstrained) P/R (or R/P) ratio is close to $x.5$, where x is an integer greater than 1, then the proposed $(P, 2)$ or $(2, R)$ policy seems to dominate the policies of Teunter (2004).

5.6 Summary

In this chapter, we proposed two new lot-sizing and scheduling policies, with an innovative interleaving scheme. When holding cost for the recoverable item is not considered, i.e., let holding cost rate for the recoverable item to be zero, our policy clearly outperform the policies proposed by previous literature. While holding cost for the recoverable item is considered, the general (P, R) policy does not seem to outperform the earlier policy proposed by Teunter. We then develop a modified $(P, 2)$ (or $(2, R)$) policy (where variable lot sizes are used in sub cycle of the production cycle), which in most cases, performs almost as well as existing methods for the problem, and in some cases, outperforms them.

Chapter 6. Summary and Conclusions

In the past decade, reverse logistics has been receiving increasing attention both from the industrial practitioners as well as from scientific researchers. For a variety of economic, environmental, or legislative reasons, product disposal may no longer be the consumers' choice as products come to be recycled or remanufactured by the original manufacturers.

In this thesis, we develop improved solutions and algorithms for several inventory models for reverse logistics.

We consider a manufacturing system with product recovery. It is assumed that the remanufactured/recovered item is as good as new and satisfies the same demand as a new item. The relevant costs are linear holding costs for the new/serviceable item and the recoverable item, and fixed setup costs for both manufacturing and remanufacturing processes. We first consider the deterministic model where demand rate for the new item and return rate for the recoverable item are constant.

All the previous work on the problem has focused on heuristics without any guaranteed performance bound. We first develop a lower bound for the problem. We then show that an existing integer ratio policy is guaranteed to obtain a solution within 6% of the lower bound. We also show that the optimal integer ratio policy that we propose is guaranteed to obtain a solution within 1.5% of the lower bound.

We then consider the stochastic version of the model with continuous review and Poisson demand and return rate. In this case, the relevant costs also include penalty costs for back ordering.

Previous literature suggests several continuous review inventory policies. However for all these policies, the steady state probabilities and expected cost cannot be determined as a closed form expression in term of the policy variables. Therefore,

the optimal among these policies can only be determined by exhaustive enumeration. Our objective is to consider simpler classes of policies where we are able to develop efficient algorithms for determining the optimal or near optimal among the particular class of policy.

The first policy we considered is the (r, Q) policy, where the reorder point and batch size is identical for manufacturing and remanufacturing. As a two-dimensional state space is involved to represent the inventory position of serviceable item and the inventory level of the recoverable item, and a closed form expression of limiting joint probability distribution is not available. We develop an efficient algorithm for determining the steady state probability of the inventory position of the recoverable item. We are then able to develop an efficient algorithm is developed for determining the optimal (r, Q) policy.

Based on this policy, we further consider an (r_p, Q_p, r_r, Q_r) policy, which has different reorder points and batch sizes for manufacturing and remanufacturing. Here again we exploit the structure of the cost function to develop an efficient algorithm to determine the optimal among these classes of policies. We also compare the proposed policy with this policy. The results show that even the (s_m, Q_m, s_r, S_r) policy determined using the optimal parameter values of our (r_p, Q_p, r_r, Q_r) policy is slightly worse than the optimal (s_m, Q_m, s_r, S_r) policy proposed by previous literature, it is much efficient.

For the deterministic model, we also propose a new (more general) class of policy, which consist an integer number, P of manufacturing setups and an integer number R of remanufacturing setups in a repeatable production cycle. To control/minimize holding cost for the recoverable item, an innovative interleaving

algorithm is developed. We also consider variable lot sizes. In case holding cost rate for the recoverable item is zero, our policy outperforms the existing policies proposed in the literature. When holding cost for the recoverable item is non-zero, our policy does better than existing policies for problems, when the P/R value is right in between two integer values (i.e. close to $x.5$, where x is an integer).

The research in this thesis contributes to the literature as follows,

- For the deterministic model, we develop new lower bound for the problem. Based on the lower bound, we propose an optimal ratio policy, which is within 1.5% more than the lower bound. And we also evaluate the existing policies proposed by previous literature, and prove that, they are within 6% more than the lower bound.
- For the stochastic model, as a two-dimensional state space and a closed form expression of steady state joint probability distribution is not available, the existing policies tend to determine the optimal by exhaustive enumeration. However, we propose two new policies. Using the recent research on cost function behavior of (r, Q) policies, we are able to develop efficient optimal algorithms for these particular classes of policies.
- For the deterministic model, we also propose a new (more general) class of policy, which does slightly better than existing policies for certain problem instances.

Possible extensions of our work in the future are as follows. In our models we assume that all returns can be recovered without any additional work and go into the inventory of the recoverable item. Future work could consider addition of a recovery stage where returned items have to be recovered in lots before it can be

remanufactured.

In practice, the returns are accumulated at different collection centers. This makes the inventory of the recoverable item available at multiple locations, with correspondingly setup cost for transfer to the upper echelon for recovery/remanufacturing.

Both the above extensions could be considered for both deterministic and stochastic demand rate. Finally future work could also study problems involving non-stationary demand and return rates.

Appendix

Theorem 1

In a finite horizon T , let there be x sequences of manufacturing cycles (followed by remanufacturing cycles). The holding cost of the echelon inventory (of recoverable items and serviceable items created from recoverable items) for the period T is minimized by having all the x sequences identical with total remanufacturing quantity (sum of lot sizes in all the remanufacturing setups) in each sequence $= \bar{Q}f/(1-f)$, and correspondingly the total manufacturing quantity in each sequence $= \bar{Q}$.

Proof:

Figure 1: Echelon Inventory of the Recoverable Products

As already shown, the production lot sizes do not impact the echelon inventory of the recoverable items (recoverable items and serviceable items created by remanufacturing). The other inventory costs in the lower bound are minimized by having stationary lot sizes Q_p and Q_r for manufacturing and remanufacturing. Given that lot size for remanufacturing is Q_r , at the end of the previous remanufacturing

sequence $i-1$, the echelon inventory of recoverable should at least be $Q_r \left(1 - \frac{D}{r}\right) f$.

Let the total lot size for manufacturing in the i th sequence be q_i^p . Then the echelon inventory of recoverable a_i , at the end of the manufacturing cycles in the i th sequence is $a_i = q_i^p(1-f) + Q_r \left(1 - \frac{D}{r}\right) f$. Therefore, the average echelon inventory of

recoverables in the i th sequence is $Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} q_i^p(1-f)$.

As the length of the i th sequence of remanufacturing cycles should be large enough to use up all the recoverable products generated, the duration of the i th sequence for manufacturing and remanufacturing is $t_i = \frac{q_i^p}{D} + \frac{q_i^p f}{D} = \frac{q_i^p(1+f)}{D}$.

The average echelon inventory for the horizon T over x sequences is

$$\begin{aligned} & \sum_{i=1}^x t_i \left(Q_r \left(1 - \frac{D}{r}\right) f + \frac{1}{2} q_i^p(1-f) \right) \\ &= T Q_r \left(1 - \frac{D}{r}\right) f + \sum_{i=1}^x \frac{1}{2D} q_i^{p^2} (1-f^2) \\ & s.t. \quad \sum_{i=1}^x \frac{q_i^p(1+f)}{D} = T, \\ & \quad q_i^p \geq 0 \end{aligned}$$

Using the Kuhn-Tucker conditions, it is easy to show that $q_i^p = \bar{Q} = \frac{TD}{x}$, for each of the x sequences and correspondingly, the total remanufacturing quantity in each sequence is $\frac{\bar{Q}f}{1-f}$, and this proves the result. $\square$

Lemma 1: Let A and B be defined according (3.21) and (3.22) respectively, then $A \geq B$.

Proof:

Let $\alpha = \sqrt{x_p^1 y_p^1}$ and $\beta = \sqrt{x_r^1 y_r^1}$,

Then $A = \alpha^2 + \beta^2$, and $B = 2\alpha\beta$.

Therefore, $A - B = \alpha^2 + \beta^2 - 2\sqrt{\alpha\beta} = (\alpha - \beta)^2 \geq 0$,

and $A \geq B$.

□

REFERENCES

- Allen, S. G. and D. A. D'Esopo (1968). "An ordering policy for repairable stock items" Operations Research **16**(3): 669-674.
- Barros, A. I., R. Dekker and V. Scholten (1998). "A two-level network for recycling sand: A case study" European Journal of Operational Research **110**(2): 199-214.
- Batta, R. and S. Chiu (1988). "Optimal obnoxious paths on a network: transportation of hazardous materials." Operations Research **36**: 84-92.
- Caruso, C., A. Colorni and M. Paruccini (1993). "The regional urban solid waste management system: a modelling approach." European Journal of Operational Research **70**: 16-30.
- Castillo, E. D. and J. K. Cochran (1996). "Optimal short horizon distribution operations in reusable container systems." The Journal of the Operational Research Society **47**(1): 48-60.
- Cohen, M. A., W. P. Pierskalla and S. Nahmias (1980). "A dynamic inventory system with recycling." Naval Research Logistics Quarterly **27**(2): 289-296.
- DeCroix, G., J.-S. Song and P. Zipkin (2005). "A series system with returns: stationary analysis." Operations Research **53**(2): 350-362.
- DeCroix, G. A. (2006). "Optimal policy for a multiechelon inventory system with remanufacturing." Operations Research **54**(3): 532-543.
- Dobos, I. and K. Richter (2004). "An extended production/recycling model with stationary demand and return rates." International Journal of Production Economics **90**(3): 311-323.
- Dowlatsahi, S. (2005). "A strategic framework for the design and implementation of remanufacturing operations in reverse logistics." International Journal of Production Research **43**(16): 3455 - 3480.
- EUROSTAT (1997). "Panorama of EU industry."
- Federgruen, A. and Y. Zheng (1992). "An efficient algorithm for computing an optimal (r,Q) policy." Operations Research **40**(4): 808.
- Fleischmann, M., J. M. Bloemhof-Ruwaard, R. Dekker, et al. (1997). "Quantitative models for reverse logistics: a review" European Journal of Operational Research **103**(1): 1-17.
- Fleischmann, M., R. Kuik and R. Dekker (2002). "Controlling inventories with stochastic item returns: a basic model." European Journal of Operational Research **138**(1): 63-75.

Fleischmann, M., J. A. E. Van Nunen and B. Grave (2003). "Integrating closed-loop supply chains and spare-parts management at IBM." Interfaces **33**(6): 44-56.

Ginter, P. and J. Starling (1978). "Reverse distribution channels for recycling." California Management Review **20**(3): 73-82.

Graves, S. C. (1985). "A multi-echelon inventory model for a repairable item with one-for-one replenishment." Management Science **31**(10): 1247-1256.

Heyman, D. P. (1977). "Optimal disposal policies for a single-item inventory system with returns." Naval Research Logistics Quarterly **24**: 385-405.

Inderfurth, K. (1997). "Simple optimal replenishment and disposal policies for a product recovery system with leadtimes " OR Spectrum **19**(2): 111-122.

Inderfurth, K. and E. A. Van der Laan (2001). "Leadtime effects and policy improvement for stochastic inventory control with remanufacturing." International Journal of Production Economics **71**(1-3): 381-390.

Kiesmüller, G. P. and C. W. Scherer (2003). "Computational issues in a stochastic finite horizon one product recovery inventory model." European Journal of Operational Research **146**(3): 553-579.

Koh, S.-G., H. Hwang, K.-I. Sohn, et al. (2002). "An optimal ordering and recovery policy for reusable items." Computers & Industrial Engineering **43**(1-2): 59-73.

Konstantaras, I. and S. Papachristos (2006). "Lot-sizing for a single-product recovery system with backordering." International Journal of Production Research **44**(10): 2031 - 2045.

Kroon, L. and G. Vrijens (1995). "Returnable containers: an example of reverse logistics." International Journal of Physical Distribution & Logistics Management **25**(2): 56-68.

Mirchandani, P. and R. Francis (1989). Discrete location theory. New York, Wiley.

Muckstadt, J. A. and M. H. Isaac (1981). "An analysis of single item inventory systems with returns." Naval research logistics quarterly **28**: 237-254.

Nahmias, S. and H. Rivera (1979). "A deterministic model for a repairable item inventory system with a finite repair rate." International Journal of Production Research **17**(3): 215-221.

Pohlen, T. L. and M. T. I. Farris (1992). "Reverse logistics in plastics recycling." International Journal of Physical Distribution & Logistics Management **22**(7): 35-47.

Richter, K. (1996a). "The EOQ repair and waste disposal model with variable setup numbers." European Journal of Operational Research **95**(2): 313-324.

Richter, K. (1996b). "The extended EOQ repair and waste disposal model." International Journal of Production Economics **45**(1-3): 443-447.

Richter, K. and I. Dobos (1999). "Analysis of the EOQ repair and waste disposal problem with integer setup numbers." International Journal of Production Economics **59**(1-3): 463-467.

Rogers, D. S. and R. S. Tibben-Lembke (1999). Going backwards: reverse logistics trends and practices. Pittsburgh.

Salomon, M., M. Thierry, J. V. Hillegersberg, et al. (1996). Distribution logistics and return flow optimization. Management Report Series, Erasmus University Rotterdam, Netherlands.

Schrady, D. A. (1967). "A deterministic inventory model for repairable items." Naval Research Logistics Quarterly **14**: 391-398.

Shad, D. (2000). "Developing a theory of reverse logistics." Interfaces **30**(3): 143.

Sherbrooke, C. C. (1968). "Metric: a multi-echelon technique for recoverable item control." Operations Research **16**(1): 122-141.

Spengler, T., H. Püchert, T. Penkuhn, et al. (1997). "Environmental integrated production and recycling management." European Journal of Operational Research **97**: 308-326.

Teunter, R. H. (2001). "Economic ordering quantities for recoverable item inventory systems." Naval Research Logistics Quarterly **48**(6): 484 - 495.

Teunter, R. H. (2004). "Lot-sizing for inventory systems with product recovery." Computers & Industrial Engineering **46**(3): 431-441.

Thierry, M., M. Salomon, J. Van Nunen, et al. (1995). "Strategic issues in product recovery management." California Management Review **37**(2): 114.

Van der Laan, E. A. (1997). The effects of remanufacturing on inventory control. the Netherlands, Erasmus University Rotterdam. **Ph. D.**

Van der Laan, E. A., R. Dekker and M. Salomon (1996a). "Product remanufacturing and disposal: A numerical comparison of alternative control strategies." International Journal of Production Economics **45**(1-3): 489-498.

Van der Laan, E. A., R. Dekker, M. Salomon, et al. (1996b). "An (s, Q) inventory model with remanufacturing and disposal." International Journal of Production Economics **46-47**: 339-350.

Van der Laan, E. A., M. Salomon, R. Dekker, et al. (1999). "Inventory control in hybrid systems with remanufacturing." Management Science **45**(5): 733-747.

Van der Laan, E. A. and R. H. Teunter (2006). "Simple heuristics for push and pull remanufacturing policies." European Journal of Operational Research **175**(2): 1084-1102.