Behavior of corner slab-column connections in irregular flat plate floors under gravity and bidirectional lateral loading

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**NANYANG
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UNIVERSITY**

**BEHAVIOR OF CORNER SLAB-COLUMN
CONNECTIONS IN IRREGULAR FLAT PLATE
FLOORS UNDER GRAVITY AND BIDIRECTIONAL
LATERAL LOADING**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2008

**Behavior of Corner Slab-Column Connections in
Irregular Flat Plate Floors under
Gravity and Bidirectional Lateral Loading**

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School of Civil and Environmental Engineering

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SUMMARY

The structural behaviors of rectangular slab-column connections in modern flat plate structures are still unclear due to currently limited experimental data. One of the unclear aspects is the design value of transferred unbalanced moments, which are critical contributor to the connection shear stresses beside the gravity shear stresses. More accurate values of unbalanced moments can be obtained by modeling reduced slab stiffness due to slab cracking more accurately. However, currently available models such as Effective Beam Width, Equivalent Frame Model, and simplified frame analysis, which are only applicable for flat plate structures with regular column layouts, do not represent the actual behavior of the cracked slab.

The objective of this study is to improve the analysis and design of flat plate structure slab-column connections by obtaining representative values of unbalanced moments, which are important contributor to the connection's shear stresses. It is achieved by proposing reduced slab stiffness model based on the modified effective moment of inertia method. The use of the effective moment of inertia method for cracked flexural members ensures simplicity and, with appropriate use of parameters, can lead to accurate prediction of reduced stiffness of cracked flexural members. Accurate reduced slab stiffness computation is important in obtaining representative values of unbalanced moments. By using representative values of unbalanced moments, shear stresses of the connections can be predicted accurately. Unlike currently available models that are only applicable for flat plate structures with regular column layouts, the proposed model is also applicable for irregular column layouts. The proposed model has been verified using the experimental data of unbalanced moment-lateral drift relationships from both slab-column connections with square columns from past research as well as slab-column connections with rectangular columns tested in this experimental program. The accuracy of the proposed model in modeling reduced slab stiffness has been shown much better compared to applying uniform reduced slab stiffness, which is commonly used in the available models such as Effective Beam Width and Equivalent Frame Model. The proposed model of reduced slab stiffness should be useful for design engineers

dealing with flat plate slab-column connections to obtain accurate slab deflection, inelastic lateral drift, and design value of transferred unbalanced moment.

This study also investigates the behavior of flat plate structure with irregular column layouts, which is currently still unclear due to very limited experimental data. It is achieved by the experimental program involving five corner slab-column connections and four slab-column connections with 135-degree slabs with rectangular columns, which are often found in modern flat plate structures. The specimens were tested to investigate their behaviors in terms of strength, drift capacity, stiffness, ductility, and the effect of shear reinforcement. The experimental results cover the effects of bidirectional lateral load, gravity load magnitude, and the use of stud shear reinforcement (SSR). The limits of applied gravity loads for rectangular corner slab-column connections and slab-column connections with 135-degree slabs to withstand critical lateral drifts are recommended. Analysis of the experimental data using eccentric shear model by the ACI 318 gives conservative result of shear stress ratio and in general it also predicts the type of failure fairly correctly. The experimental data of unbalanced moment-lateral drift relationships were also used to verify the proposed model of reduced slab stiffness. The experimental results should be useful to design engineers and researchers dealing with modern flat plate structures, especially rectangular corner slab-column connections and slab-column connections with 135-degree slabs.

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LIST OF SYMBOLS

A_c	=	area of the critical section for punching shear
A_s	=	area of reinforcing steel in tension
A'_s	=	area of reinforcing steel in compression
A_v	=	area of shear reinforcement in each peripheral line parallel to the column face
b	=	beam or slab width
b_o	=	length of the critical perimeter at $d/2$ from the column face
b_1	=	length of the critical section perpendicular to the axis of moment
b_2	=	length of the critical section parallel to the axis of moment
c	=	column dimension parallel to the moment axis
C_c	=	concrete compression force
C_d	=	deflection amplification factor
c_t	=	column dimension perpendicular to the moment axis
C_t	=	concrete tensile force
c_u	=	dept of the neutral axis at ultimate limit state
c_x	=	column dimension parallel to x - axis
c_y	=	column dimension parallel to y - axis
c_1	=	column dimension parallel to the lateral load direction
c_2	=	column dimension perpendicular to the lateral load direction
d	=	distance between center of tension steel and the outermost concrete compression face
d'	=	distance between center of compression steel and the outermost concrete compression face
D_p	=	displacement at the maximum load
D_u	=	displacement at 80% of maximum lateral load

DR_u	=	ultimate drift ratio
D_y	=	displacement at yield of the reinforcement
E_c	=	Young's elastic modulus of concrete
E_p^*	=	slope of the plastic range of reinforcing steel embedded in concrete
E_s	=	Young's elastic modulus of steel
f_{cr}	=	tensile strength of concrete
f_c'	=	concrete compressive strength
f_n^*	=	steel stress at point of intersection between elastic and plastic lines of reinforcing steel embedded in concrete
f_r	=	modulus of rupture of concrete
f_t	=	tensile strength of concrete
f_y	=	yield strength of reinforcing steel
f_{yv}	=	specified yield strength of the shear reinforcement
f_y^*	=	reduced yield stress of reinforcing steel embedded in concrete
G_c	=	shear modulus of concrete
h	=	height of beam or slab
h_f	=	column length
h_s	=	interstory height
I	=	second moments of area of critical section
I_c	=	moment of inertia of the column
I_{cr}	=	moment of inertia of fully cracked section
I_e	=	effective moment of inertia
$I_{e_{exp}}$	=	experimental effective moment of inertia
I_{ex}	=	effective moment of inertia in the x -axis
I_{ey}	=	effective moment of inertia in the y -axis

I_g	=	gross moment of inertia
I_s	=	moment of inertia of the slab
I_t	=	transformed moment of inertia
I_u	=	moment of inertia at ultimate limit state
I_y	=	moment of inertia at yield state
K	=	stiffness matrix
kd	=	depth of the neutral axis of the section
$k_y d$	=	depth of neutral axis at yield state
LL	=	service live load
l_x	=	the projection of the critical section on the principal axis x
l_y	=	the projection of the critical section on the principal axes y
l_1	=	span length parallel to the lateral load direction
l_2	=	span length perpendicular to the lateral load direction
M_a	=	maximum applied moment
M_{cr}	=	cracking moment
M_{pr}	=	the flexural strengths of the opposite sides of the critical section
M_u	=	ultimate moment
M_{ur}	=	reduced ultimate moment
M_{ux}	=	ultimate moment about y -axis
M_{uy}	=	ultimate moment about x -axis
$M_{x\ ave}$	=	average slab moment about y -axis
M_{xy}	=	torsional moment of the slab
M_y	=	yield moment
$M_{y\ ave}$	=	average slab moment about x -axis
M_{yx}	=	yield moment about y -axis
M_{yy}	=	yield moment about x -axis

N	=	number of nodes per element
R_w	=	coefficient representing the ductility and redundancy of a building system
s	=	spacing between peripheral lines of shear reinforcements
S	=	resultant force of the reinforcing steel
t	=	slab thickness
T	=	fundamental period of the structure
v_c	=	shear stress capacity contributed by concrete
V_c	=	nominal punching shear force capacity
v_n	=	nominal shear capacity
v_s	=	shear stress capacity contributed by shear reinforcement
v_u	=	ultimate shear stress
V_u	=	ultimate shear force
α	=	portion of the slab width
α_s	=	40, 30, and 20 for interior, edge, and corner column-slab connections, respectively
α_x	=	reduced slab stiffness factor in x -axis
α_y	=	reduced slab stiffness factor in y -axis
β	=	ratio of cracked stiffness to gross-section stiffness
β_c	=	ratio of long side to short side dimensions of the column
δ	=	beam deflection
Δ	=	deflection of the flexural member
Δ_{in}	=	inelastic lateral displacement
Δ_s	=	elastic lateral displacement
ε_c	=	strain of concrete in compression
ε_{cm}	=	strain of the concrete at the outermost compression face
ε_{cp}	=	strain at peak stress

ε_{cu}	=	concrete strain at the outermost compression face at ultimate limit state
ε_s	=	strain of the reinforcing steel
ε'_s	=	strain of the compression steel
ε_{tm}	=	concrete strain at the outermost tension face
ε_{tp}	=	strain at peak tensile stress
ε_y	=	yield strain of reinforcing steel
ϕ	=	curvature of the section
ϕ_{cr}	=	cracking curvature
ϕ_u	=	ultimate curvature
ϕ_{ur}	=	reduced ultimate curvature
ϕ_{ux}	=	ultimate curvature about y -axis
ϕ_{uy}	=	ultimate curvature about x -axis
ϕ_y	=	yield curvature
ϕ_{yx}	=	yield curvature about y -axis
ϕ_{yy}	=	yield curvature about x -axis
γ_f	=	fraction of unbalanced moment transferred by flexure
γ_{vx}	=	fraction of unbalanced moment transferred by shear about x -axis
γ_{vy}	=	fraction of unbalanced moment transferred by shear about y -axis
κ	=	shear correction factor
μ_p	=	ratio between displacement at the maximum load and yield displacement
μ_u	=	ratio between displacement at 80% of the maximum load and yield displacement
ρ	=	reinforcement ratio in tension
ρ'	=	reinforcement ratio in compression
ρ_x	=	reinforcement ratio parallel to x -axis

ρ_y = reinforcement ratio parallel to y-axis

σ_c = stress of concrete in compression

ν = Poisson's ratio of concrete

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The flat plate structural system is a structural system where the slabs are directly supported on columns. Using a flat plate structure can lead to a significant reduction in structural cost because of its simple possible formwork. Other significant advantages of this type of structures are shorter construction time and story height reduction. From an architectural point of view, flat plate structures also provide flexibility in rooms and utilities arrangements. Because of these considerations, the flat plate structure is widely used for various buildings around the world.

In Singapore, the flat plate structural system is used especially for apartments, condominiums, and offices. It can be noticed that most of those buildings are flat plate structures with irregular column layouts. Elongated column sections are also commonly used in these buildings.

One of the most important issues in flat plate structures is brittle punching shear failure. This type of failure mechanism may occur due to a combination of the transfer of shear forces and unbalanced moments on the slab-column connections. Unbalanced moments which are caused by non-uniform gravity loads on the slab and lateral loads can lead to significant additional shear stresses in the slab-column connections. Thus, every slab-column connection in a flat plate structure has to have adequate strength and ductility under combination of gravity and lateral loads.

Significant research has been performed to obtain the values of punching shear capacities of the slab-column connections. ACI 318 requires that the shear stress around the critical perimeter for punching shear be calculated based on eccentric shear model due to combination of shear force and unbalanced moment. This value of shear stress must not exceed punching shear capacity of the slab.

The value of unbalanced moment as shear stress contributor especially under lateral load is another important issue in slab-column connections design. Underestimating the value of unbalanced moment may lead to punching shear

failure since the shear stress is underestimated. However, overestimating the value may lead to unnecessary use of shear reinforcement. The accuracy of the value of unbalanced moment mainly depends on the slab stiffness used in the analysis. For flat plate structures under lateral load, ACI 318 requires that the analysis must take into account the effect of slab cracking. Currently few models are available to obtain the values of unbalanced moments in the slab-column connections. Hwang and Moehle (2000) suggested uniform reduced slab stiffness in effective beam width or equivalent frame analysis to take into account the effect of slab cracking. Megally and Ghali (2000) suggested a simplified frame analysis taken from full frame analysis to obtain the design value of unbalanced moment caused by lateral load. However, those models are only applicable for flat plate structure with regular column layouts. Model that takes into account the effect of slab cracking is still not available for flat plate structure with irregular column layouts. Due to limited experimental data, structural behaviors of flat plate structure with irregular column layouts also remain unclear and require further investigations.

This study investigates the behavior of flat plate structure with irregular column layouts in term of strength, drift capacity, stiffness, ductility, and the effect of shear reinforcement. A model that represents reduced slab stiffness based on the elastic moments is also presented. The model can be used to obtain the value of unbalanced moment for punching shear design. It is verified using the values of unbalanced moments of both slab-column connection specimens with regular column layouts from past research as well as slab-column connection specimens with irregular column layouts tested in this experimental program. It can be shown that the model is applicable not only for flat plate structure with regular column layouts but also for flat plate structure with irregular column layouts.

1.2 OBJECTIVES

The objective of this study is to improve the analysis and design of flat plate structure slab-column connections by obtaining representative values of unbalanced moments, which are important contributor to the connection shear stress. It is achieved by modeling cracked slab using modified stiffness based on the elastic moments. Accurate reduced slab stiffness is important in obtaining representative

values of unbalanced moments. By using representative values of unbalanced moments, shear stress of the connections can be predicted accurately.

This study also covers the behavior of flat plate structure with irregular column layouts. To cover all aspects of irregular column layouts in one experimental program is impossible. Available experimental data for flat plate structure with irregular column layouts are also very limited. Nine single slab-column connection specimens with elongated columns were tested to investigate their behaviors. It is also expected that the data can be used for further research in this area as well as for verifying the proposed model of modified slab stiffness. Eccentric shear model which is used as a simple design guide by ACI 318 is also evaluated.

Analysis with reduced slab stiffness is verified using nine specimens tested in the experimental program and some specimens from past research. Before implementing the model in flat plate slab-column connections which are more complex, the model is verified using experimental data of deflection of under-reinforced beams and slabs from past research.

Generally the research objectives can be summarized as follows:

- to investigate the behavior and failure types of nine slab-column connection specimens tested in the experimental program
- to investigate the effect of each parameter involved in determining the strength and performance of those slab-column connection specimens
- to review the design guide to calculate the connection shear stress by ACI 318 using experimental data from tested specimens and other related data from past research
- to recommend the limit of gravity shear force to withstand specific lateral drift ratio based on experimental data
- to evaluate available models of reduced slab stiffness to obtain unbalanced moments transferred at the slab-column connections
- to provide an improved model of stiffness reduction for cracked flexural members
- to provide an improved model of reduced slab stiffness to obtain the values of unbalanced moments transferred between slabs and columns in flat plate structure with both regular and irregular column layouts.

1.3 ORGANIZATION

Literature reviews on slab-column connections and available models to obtain the unbalanced moments are presented in Chapter 2 and 3, respectively. Experimental program involving nine slab-column connection specimens, the analysis results, and the evaluation of punching shear design guide by ACI 318 are presented in Chapters 4 and 5. The proposed stiffness reduction model for cracked flexural members is presented in Chapter 6. The proposed reduced slab stiffness model to obtain the values of unbalanced moments is presented in Chapter 7. Conclusion and recommendation are presented in Chapter 8.

CHAPTER 2

SLAB-COLUMN CONNECTIONS UNDER COMBINED SHEAR FORCE AND UNBALANCED MOMENT

2.1 SLAB-COLUMN CONNECTIONS UNDER GRAVITY AND LATERAL LOADS

One major problem of a flat plate structure subjected to lateral force such as wind or earthquake is its stiffness. Practically, a lateral force-resisting system which is called the primary system is built to provide lateral stiffness of a flat plate structure. Common examples of primary systems are shear walls and core walls. A flat plate structure, including its slab-column connections, must be designed to undergo the same lateral deformation as the primary system without losing its capability to support gravity loads. Thus, the connection must be designed based on the shear forces and unbalanced moments due to gravity and lateral loads. Adequate displacement ductility has to be ensured in a flat plate structure under extreme lateral forces.

The primary system of a flat plate structure must have adequate lateral stiffness to undergo specified value of interstory drift. Interstory drift is defined as the ratio between the lateral displacement of one floor and the respective story height. Certain limit values of interstory drifts have been defined in the code to be used for designing both primary system and flat plate structure.

Generally flat plate structure is not designed to contribute lateral stiffness-resistant to the primary system. However, there are unbalanced moments that are transferred between the slabs and the columns when a flat plate structure is subjected to lateral forces. Available models to obtain design values of these unbalanced moments are presented in detail in Chapter 3. Each slab-column connection must be designed to have adequate capacity to transfer shear forces and unbalanced moments to reduce the risk of punching shear failure.

2.1.1 Lateral Displacement Ductility of Slab-Column Connection

There are no unique values of yield displacements for slab-column connections in a flat plate structure. However, yield displacement is required to

obtain the lateral displacement ductility of a flat plate structure. Pan and Moehle (1989) defined the values of yield displacements through certain procedure. Previous results from other research were analyzed to obtain displacement ductility values. The data were obtained from interior connections without shear reinforcement.

Figure 2.1 shows the procedure to obtain yield displacements defined by Pan and Moehle (1989). It begins with constructing the envelope of relationship between lateral displacement and lateral load which is idealized by an elastoplastic relation. The initial slope of the idealized relation is obtained through the point of two-thirds of maximum load. The plastic portion passes through the maximum load and the deflection at the maximum load D_p . The intersection between these two lines defines yield displacement D_y . Pan and Moehle (1989) defined displacement ductility μ_p as:

$$\mu_p = \frac{D_p}{D_y} \quad (2.1)$$

Megally and Ghali (2000a) used the displacement at 80% of maximum lateral load D_u to define displacement ductility ($\mu_u = D_u / D_y$). The author uses the ductility definition by Pan and Moehle ($\mu_p = D_p / D_y$) to present the experimental results in Chapters 4 and 5. The values of D_u for some specimens could not be obtained during testing due to limited strokes of the horizontal jacks.

2.1.2 Limit Values of Interstory Drift Ratios

Interstory drift ratio determines the design value of unbalanced moment. Models to obtain design values of unbalanced moments are presented in Chapter 3. Concrete structures subjected to lateral load such as wind or earthquake should be able to undergo an inelastic interstory drift ratio of at least 1.5% (Sozen, 1980). Pan and Moehle (1989) adopted the same value for slab-column connections and pointed out that the connections should be able to undergo drift ratio of 1.5% without punching shear failure. Interstory drift ratio of 1.5% is based on the upper limit required by the 1994 Uniform Building Code (UBC 1994). The code requires

that the limit value of elastic lateral displacement should be calculated based on the fundamental period T of a structure.

For $T < 0.7$ seconds:

$$\Delta_s \leq \frac{0.04 h_s}{R_w} \quad (2.2)$$

and
$$\Delta_s \leq 0.005 h_s \quad (2.3)$$

For $T \geq 0.7$ seconds:

$$\Delta_s \leq \frac{0.03 h_s}{R_w} \quad (2.4)$$

and
$$\Delta_s \leq 0.004 h_s \quad (2.5)$$

Inelastic lateral displacement can be calculated as:

$$\Delta_{in} = \frac{3 R_w}{8} \Delta_s \quad (2.6)$$

where:

- Δ_s = elastic lateral displacement of one floor
- h_s = interstory height
- R_w = coefficient representing the ductility and redundancy of a building system
- Δ_{in} = inelastic lateral displacement of one floor

Assuming $R_w = 8$ for building frame system with a concrete shear wall in those formulas, the upper limits for elastic and inelastic interstory drift ratios of a flat plate structure are 0.5% and 1.5%, respectively.

UBC 1997 requires that calculated inelastic interstory drift does not exceed 2.5% for structures with fundamental period less than 0.7 seconds. For structures with fundamental period of 0.7 seconds or greater, the calculated inelastic interstory drift should not exceed 2.0%. According to UBC 1997, inelastic lateral displacement can be calculated as follows:

$$\Delta_{in} = 0.7 R \Delta_s \quad (2.7)$$

where R is a dimensionless coefficient representing the deformability of the primary system. Thus, the upper limit of inelastic interstory drift of a flat plate structure

should not exceed 2.0% or 2.5%. Each slab-column connection must have adequate ductility to undergo the specified drift limit.

According to the 2003 International Building Code (IBC 2003), the upper limit of the inelastic interstory drift ratio is 2.5%. The inelastic lateral displacement Δ_{in} can be calculated from the elastic lateral displacement Δ_s as follows:

$$\Delta_{in} = \frac{C_d \Delta_s}{I_e} \quad (2.8)$$

where I_e is the occupancy importance factor of the structure and C_d is the deflection amplification factor.

In force-based design, a fully static elastic analysis is performed to obtain the elastic interstory drift of the primary system. This value is then multiplied by an amplification factor given by UBC to obtain the inelastic interstory drift that should not exceed the specified upper limit.

2.1.3 Effect of Gravity Load Magnitude on Drift Capacity

Gravity load magnitude influences ultimate interstory drift capacity of slab-column connection in a flat plate structure. This can be shown from serial tests conducted by Pan and Moehle (1989, 1992), Robertson and Durrani (1992), Wey and Durrani (1992), Islam and Park (1976), Hawkins et al. (1975), Dilger and Brown (1995), and Dilger and Cao (1991). The results of those tests are shown in Figure 2.2. In Figure 2.2, each point shows the variation of ultimate interstory drift ratio DR_u with the ratio of shear force and punching strength capacity of the connection $V_u / \phi V_c$. The best-fit curves 1, 2, and 3 represent the experimental data reported for interior specimens without shear reinforcement (curve 1), with stirrups (curve 2), and SSR (curve 3). The specimens were subjected to shear force V_u and uniaxial cyclic moment. V_u represents the maximum shear force transferred between the slab and the column at failure and V_c represents the punching shear strength of the slab-column connection. The strength reduction factor ϕ is taken equal to 1.0 because the strengths of materials are known.

Figure 2.2 also shows that the capabilities of slab-column connections to undergo specific lateral drift ratio without failure decrease with the increasing

magnitude of applied gravity loads. Lateral drift ratio of 1.5% is represented by solid horizontal line. This line intersects curve 1 (interior specimens without shear reinforcement) approximately at $V_u / \phi V_c = 0.40$. It indicates that interior slab-column connections without shear reinforcement may satisfy ultimate drift ratio $DR_u = 1.5\%$ only if V_u does not exceed $0.40 \phi V_c$. The horizontal dashed lines in Figure 2.2 represent 2.0% and 2.5% drift ratios. Those lines intersect curve 1 at $V_u / \phi V_c = 0.32$ and 0.25 , respectively. It indicates that for $DR_u = 2.5\%$ or drift limit specified by IBC 2003, the interior connections without shear reinforcement may satisfy the drift requirement only if V_u does not exceed $0.25 \phi V_c$. It also indicates that for $DR_u = 2.5\%$, the interior slab should be designed with shear reinforcement when V_u exceeds $0.25 \phi V_c$.

ACI 318 requires that punching shear capacity V_c is the smallest of:

In inch-pound units:

$$V_c = \left(2 + \frac{4}{\beta_c} \right) b_o d \sqrt{f_c'} \quad (2.9)$$

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2 \right) b_o d \sqrt{f_c'} \quad (2.10)$$

$$V_c = 4 b_o d \sqrt{f_c'} \quad (2.11)$$

In SI units:

$$V_c = \left(1 + \frac{2}{\beta_c} \right) \frac{\sqrt{f_c'} b_o d}{6} \quad (2.12)$$

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2 \right) \frac{\sqrt{f_c'} b_o d}{12} \quad (2.13)$$

$$V_c = \frac{1}{3} b_o d \sqrt{f_c'} \quad (2.14)$$

where :

- b_o = length of the critical perimeter at $d/2$ from the column face
- d = slab effective depth
- f_c' = concrete compressive strength

β_c = ratio of long side to short side dimensions of the column
 α_s = 40, 30, and 20 for interior, edge, and corner slab-column connections, respectively

Figure 2.3 shows the critical sections for punching shear at $d/2$ from the column face for interior, edge, and corner connections.

Megally and Ghali (2000) presented the relation between ultimate drift ratio and applied gravity load for edge connections as shown in Figure 2.4. Curves A and B represent the experimental data of edge slab-column connections without and with Stud Shear Reinforcement (SSR), respectively. The edge slab-column connections were subjected to shear force and cyclic moment reversals in a direction perpendicular to the slab edge.

Similar to interior slab-column connections, the capabilities of edge slab-column connections to undergo ultimate drift ratio without failure decrease with increasing gravity loads. Figure 2.4 shows that solid horizontal line ($DR_u=1.5\%$) intersects curve A (edge connections without shear reinforcement) approximately at $V_u/\phi V_c=0.50$. It indicates that for $DR_u=1.5\%$, the edge slab should be designed with shear reinforcement when $V_u > 0.50\phi V_c$. The horizontal dashed line representing $DR_u=2.5\%$ intersects curve A approximately at $V_u/\phi V_c=0.40$. It indicates that for $DR_u=2.5\%$, the edge slab should be designed with shear reinforcement when $V_u > 0.40\phi V_c$.

Curve 1 in Figure 2.2 and curve A in Figure 2.4 show that edge slab-column connections can resist relatively higher gravity shear force ratio $V_u/\phi V_c$ than interior connections under specific limits of drift ratios. Curves 2 and 3 in Figure 2.2 and curve B in Figure 2.4 representing slab-column connections with conventional stirrups or SSR are far above the horizontal dashed lines required by UBC 1997 ($DR_u=2.0\%$ and 2.5%). There is no specific limit of $V_u/\phi V_c$ for slab-column connections with shear reinforcements to satisfy the requirement of 2.0% or 2.5% drift ratio without failure. It is also shown that drift capacity of slabs with SSR is higher than slabs with conventional stirrups (see Curves 2 and 3 in Figure 2.2).

2.2 SHEAR REINFORCEMENT REQUIREMENT

Under a specific drift ratio, the slab must be designed with shear reinforcement if certain limit of $V_u / \phi V_c$ is exceeded. Megally and Ghali (2000) recommended that for interior connections, slabs should have shear reinforcements except if the value of transferred shear force V_u is less than $0.25 \phi V_c$, where ϕ is the strength reduction factor equal to 0.85 for shear based on the ACI 318-99 code. The strength reduction factor ϕ for shear and torsion was modified to 0.75 in the ACI 318-02 code, along with changes in the load factors. The amount of shear reinforcements should satisfy the minimum requirement to ensure that the slab-column connections can withstand ultimate drift ratio DR_u of 2.5%.

When DR_u does not exceed 1.5%, Megally and Ghali (2000) recommended that shear reinforcement is required when V_u exceeds $0.40 \phi V_c$ instead of $0.25 \phi V_c$. Slab-column connections can be designed without any shear reinforcements only when $V_u / \phi V_c$ satisfies the limit based on applied DR_u . The maximum shear stress v_u due to shear force V_u combined with unbalanced moment M_u should be less than ϕv_c at the critical section at $d/2$ from the column face, where v_c is the nominal stress capacity. The connections that do not satisfy these conditions should be designed with minimum shear reinforcements, such that the nominal shear strength contributed by steel v_s satisfies the following:

In inch-pound units:

$$v_s = \frac{A_v f_{yv}}{b_o s} \geq 3\sqrt{f_c'} \quad (2.15)$$

In SI units:

$$v_s = \frac{A_v f_{yv}}{b_o s} \geq \frac{1}{4} \sqrt{f_c'} \quad (2.16)$$

where :

- A_v = area of shear reinforcement in each peripheral line parallel to the column face
- s = spacing between peripheral lines of shear reinforcements ($s \leq 0.5d$ for stirrups and $s \leq 0.75d$ for SSR)

$$b_o = \text{length of critical perimeter at } d/2 \text{ from the column face}$$

$$f_{yv} = \text{specified yield strength of the shear reinforcement}$$

The distance between the column face and the outermost peripheral line of shear reinforcement should not be less than $3.5d$ (Figure 2.5). The recommended minimum value of shear reinforcement above was obtained from experiments conducted by Hawkins et al. (1975), Islam and Park (1976), Dilger and Cao (1991), Dilger and Brown (1995), Megally (1998), and Megally and Ghali (2000a). In these experiments, slab column connections with various amounts of stirrups or SSR could withstand drift ratio higher than 2.5 % without punching shear failure with relatively high $V_u / \phi V_c$. Megally and Ghali (2000) provided the design procedure for the slab-column connections based on the magnitude of drift ratio DR_u as shown in Figure 2.6.

2.3 EFFECT OF SHEAR REINFORCEMENT ON STRENGTH AND DUCTILITY OF SLAB-COLUMN CONNECTION

Increasing punching shear capability can be achieved with provisions of drop panels, shear capitals, stirrups, or SSR. Although these methods have been proven successfully to increase punching shear capability, the effect of each method on ductility of slab-column connections is substantially different.

Shear capital refers to increasing a uniform thickness of slab area on small region surrounding the column. Because the plan dimensions of shear capitals are small, usually they do not contain vertical reinforcement other than vertical reinforcement of the column. Drop panel refers to increasing the slab thickness at the area surrounding the column. The plan dimensions of drop panels are generally larger than shear capitals. Besides increasing punching shear capacity of the slab, drop panels also increase flexural stiffness of the slab. ACI 318 requires that the drop panels extend at least one-sixth of the span length in each direction and project below the slab at least one-quarter of the slab thickness in order to use the full effective depth of the drop panels for flexural strength.

Most common types of shear reinforcements in slabs are stirrups and SSR. Stirrups may not be as effective as SSR because of inefficient anchorages. Although ACI 318 specifies anchorage requirement for stirrups, it may be difficult to satisfy the requirement in slabs thinner than 250 mm. The anchorage of SSR is provided mechanically by forge heads or by a forge head at one end and welded stem to steel rail at the other end.

Megally and Ghali (2000) presented the effects of strengthening methods of punching shear capability as shown in Figure 2.7. It shows load-deflection graphs of five slab specimens with 150 mm thickness. The data were obtained from interior slab-column connections simply supported on four edges. Applied axial load on the column was transferred to the slab in the form of shear force. The slabs have the same flexural reinforcement layouts and similar material properties for concrete and reinforcements with various shear strengthening. Control slab AB-1 was not reinforced with any shear reinforcement (Mokhtar et al., 1985). The other data were obtained from specimen 1-2 with drop panel (Megally, 1998), specimen 1-1 with shear capital (Megally, 1998), specimen B with stirrups (Hammil and Ghali, 1992), and specimen AB5 with SSR (Mokhtar et al., 1985).

Within the drop panel and shear capital, the slab thickness was increased from 150 mm to 225 mm. The nominal shear strengths contributed by steel v_s were calculated using Equation (2.16). Specimens B and AB5 have v_s of 2.96 MPa and 3.13 MPa, respectively. Punching shear failure occurred at a section within the shear-reinforced zone in specimen B and at a section outside the shear-reinforced zone in specimen AB5. The initial portions of the experimental curves for specimens AB1 and AB5 are not available. However, the conclusions from these curves are not affected.

Figure 2.7 shows that the punching shear capacity of slab-column connections slightly increases with the presence of stirrups. Drop panel, shear capital, and SSR increase punching shear strength significantly. The specimens with drop panel and the shear capital experienced a brittle failure. Figure 2.7 shows that deflections at failure of slabs with drop panel and shear capital are much smaller compared to the slab with SSR which experienced ductile failure. The conventional stirrups provide insignificant increase in ductility for such a thin slab. Stirrups

might be more efficient to be used in slabs thicker than 250 mm as suggested in ACI 318. Figure 2.7 also indicates that SSR is an effective way to increase punching shear capability and ductility of the connection, even when the slab is relatively thin. Thus, SSR is effective to be used for seismic-resistant slab-column connections.

2.4 SHEAR STRESS OF SLAB-COLUMN CONNECTION

Each slab-column connection in a flat plate structure must be designed to withstand a specific drift ratio without losing its capability to support gravity loads. Thus, shear stress due to combined gravity and lateral loads on every connection must be ensured not to exceed the shear stress capacity of the connection.

ACI 318 requires that slab-column connection that transfers shear force V_u and unbalanced moment M_u should be designed based on the assumption that V_u and a fraction of unbalanced moment $\gamma_v M_u$ cause vertical shear that varies linearly along the critical section. Models to obtain the design value of unbalanced moment M_u are presented in Chapter 3. The shear stress calculation that satisfies the code requirement is given by:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux}}{I_x} y + \frac{\gamma_{vy} M_{uy}}{I_y} x \quad (2.17)$$

where:

A_c	=	area of the critical section
γ_{vx}, γ_{vy}	=	fraction of unbalanced moment transferred by shear about x and y axes, respectively
I_x, I_y	=	second moments of area of the critical section about the principal axes x and y
x, y	=	coordinates where v_u is calculated

ACI 318 defines the fraction of unbalanced moment transferred by shear as:

$$\gamma_v = 1 - \frac{1}{1 + (2/3)\sqrt{(b_1/b_2)}} \quad (2.18)$$

where:

b_1 = length of the critical section perpendicular to the axis of moment considered

b_2 = length of the critical section parallel to the axis of moment considered

Calculated shear stress v_u of the slab-column connection must not exceed the shear stress capacity v_c of the connection. Nominal shear stress capacity v_n of slab-column connection is contributed by concrete with or without shear reinforcement as follows:

For slab without shear reinforcement:

$$v_n = v_c \quad (2.19)$$

For slab with shear reinforcement:

$$v_n = v_c + v_s \quad (2.20)$$

where v_c and v_s are the shear stress capacity provided by concrete and shear reinforcement, respectively. ACI 421 limits v_n to $8\sqrt{f'_c}$ psi or $\frac{2}{3}\sqrt{f'_c}$ MPa for SSR and $6\sqrt{f'_c}$ psi or $\frac{1}{2}\sqrt{f'_c}$ MPa for stirrups. These two limits should be increased by 25% in seismic design when the shear stress due to V_u alone does not exceed $4\sqrt{f'_c}$ psi or $\frac{1}{3}\sqrt{f'_c}$ MPa.

The shear stress contributors in Equation (2.17) are the shear force V_u and the unbalanced moment transferred by shear $\gamma_v M_u$. The design value of shear force is straightforward since it can be directly obtained from the tributary area of the column. However, the design value of unbalanced moment mainly depends on the stiffness of the cracked slab. It is important to model the stiffness of the cracked slab correctly to obtain accurate value of transferred unbalanced moment. Available models to obtain the design value of unbalanced moment are presented in Chapter 3. However, these models are only applicable for flat plate structure with regular

column layouts. The proposed model that is also applicable for irregular slab-column connections is presented in Chapter 7.

ACI formulas for calculating shear stresses and punching shear capacities of the connections have been verified using a significant number of experimental data of square slab-column connections. However, the verification for rectangular slab-column connections is still questionable due to currently limited experimental data. The experimental data from the nine rectangular corner slab-column connections with column ratio of 5:1 tested in this experimental program are used to verify those formulas. The analysis results are presented in Chapters 4 and 5.

Fig. 2.1 Procedure to Obtain Displacement Ductility

Fig. 2.2 Effects of Gravity Load on Lateral Drift Capacity of Interior Slab-Column Connections (Megally and Ghali, 2000)

Fig. 2.3 Critical Sections for Punching Shear at $d/2$ from the Column Face

Fig. 2.4 Effects of Gravity Load on Drift Capacity of Edge Slab-Column Connections
(Megally and Ghali, 2000)

Fig. 2.5 Critical Sections for Punching Shear Outside the Shear-Reinforced Zone
(Megally and Ghali, 2000)

Fig. 2.6 Design Steps for Slab-Column Connections by Megally and Ghali (2000)

Fig. 2.7 Load-Deflection Curves of Slabs with Different Methods of Increasing Punching Shear Capability (Megally and Ghali, 2000)

CHAPTER 3

DESIGN VALUE OF UNBALANCED MOMENT

3.1 CONTRIBUTION OF UNBALANCED MOMENT TO THE SHEAR STRESS AT SLAB-COLUMN CONNECTION

Gravity loads mainly cause shear forces on slab column connections. Unbalanced moments, which are caused by non-uniform gravity loads on the slab and lateral load such as wind or earthquake, can lead to significant additional shear stresses on the critical shear perimeter.

The design value of unbalanced moment is an important contributor to the shear stress at slab-column connection. Shear stress calculation at the critical section due to combined shear force and unbalanced moment is presented in Chapter 2. Underestimating the magnitude of unbalanced moment in the slab-column connection design can lead to punching shear failure since the shear stress contributed by the unbalanced moment is not taken into account correctly. Overestimating the magnitude of unbalanced moment in the design can lead to uneconomical design in the form of unnecessary use of shear reinforcements or over-sized slab thickness.

There is still no consensus on obtaining the design value of unbalanced moment for punching shear design. Available methods are only applicable to model flat plate structure with regular column layouts. The effect of slab cracking is mostly included by applying uniform reduced slab stiffness which does not really represent the stiffness of the cracked slab. A method that can represent the reduced slab stiffness of the cracked slab correctly is required to obtain accurate design value of unbalanced moment. Due to popular use of flat plate structures with irregular column layouts as mention in Chapter 1, the method must also be applicable for both regular and irregular column layouts.

3.2 DESIGN REQUIREMENTS BY ACI 318

ACI 318 requires that the analysis of unbraced frames under lateral load must take into account the effects of slab cracking and reinforcement on the stiffness of the frame members. During the life of the structure, factors such as

construction loads, occupancy loads, anticipated overloads, and volume changes cause reduced slab stiffness due to cracking. When lateral loads are applied on the structure, reduced slab stiffness due to cracking increases lateral deflection. To ensure that lateral drift is not underestimated, reduced slab stiffness must be taken into account in the analysis. ACI 318 also requires that if the slab stiffness is not obtained by the analysis taking into account the effects of cracking and reinforcement, the effective moment of inertia of fully cracked section can be used for the slab members for obtaining conservative lateral drift. ACI 318 permits combining the results of the gravity load analysis with the results of the lateral load analysis.

3.3 AVAILABLE MODELS OF REDUCED SLAB STIFFNESS FOR FLAT PLATE STRUCTURE WITH REGULAR COLUMN LAYOUTS

There are few available methods to model flat plate structure with regular column layouts subjected to gravity and lateral loads. Those methods are simplified approaches for design purpose. The use of those methods is not intended to yield accurate analytical results. The effect of cracking is mostly taken into account by applying uniform reduced slab stiffness in the analysis. Specific procedure to obtain the value of unbalanced moment for connection design was suggested by Megally and Ghali (2000). In their model, a single joint is taken out from the structure to be analyzed separately under specific lateral drift. Currently there is still no specific consensus to obtain the design value of unbalanced moment.

Reduced slab stiffness due to slab cracking must be represented accurately in the analysis to obtain the design value of unbalanced moment correctly. With rapid development of finite element software, it is possible to analyze a complete flat plate structure using three-dimensional nonlinear analysis to accommodate the effect of slab cracking. However, it is not practical to use nonlinear analysis of complete structure for design purpose since it involves a lot of variables and consumes a lot of time. Available models to obtain the unbalanced moments that also take into account the effect of slab cracking are described in the following

sections. These models are only applicable for flat plate structure with regular column layouts.

3.3.1 Analytical Evaluations by Hwang and Moehle

Effective Beam Width and Equivalent Frame Models are commonly used for design purpose to model the lateral load response of slab-column frames. Hwang and Moehle (2000) performed analytical evaluations of these methods. Factors such as type of connection, panel geometry, and slab cracking were included in the analytical evaluations. Suitable models for design implementation were also recommended.

Experimental data were obtained from the test of a nine-panel slab-column frame by Hwang and Moehle (2000a) as shown in Figure 3.1. A portion of the slab was designed for gravity and wind load according to ACI 318-83, whereas the remainder was designed for moment redistribution that is not permitted by the code. Gravity load tests provided the experimental data of the structural response at service load level. Bidirectional lateral load tests provided experimental data of the structural response ranging from service load level to the ultimate load level.

Figure 3.2 shows the basic concept of the Effective Beam Width Model. The slab is modeled as beam element that rotates uniformly across its transverse width. Since the actual slab does not rotate uniformly across its width, only a fraction of the transverse width of the actual slab is used as the effective beam width. The inflection lines are assumed at the slab midspan perpendicular to the lateral load direction. Slab-column connection is modeled as a rigid joint between the beam and the column.

Banchik (1987) performed finite element analysis to provide equations to obtain effective beam width b for interior, edge, and corner connections as follows: For interior connections and edge connections with bending about an axis perpendicular to the edge:

$$b = 2c_1 + \frac{l_1}{3} \quad (3.1)$$

For corner connections and edge connections with bending about an axis parallel to the edge:

$$b = c_1 + \frac{l_1}{6} \quad (3.2)$$

Hwang and Moehle (2000) recommended both equations to obtain the effective beam width for column aspect ratio c_2 / c_1 between $\frac{1}{2}$ to 2, and slab aspect ratio l_2 / l_1 greater than $\frac{2}{3}$. Variables c_1 and l_1 are the column dimension and the span length parallel to the lateral load direction. Variables c_2 and l_2 are the column dimension and the span length perpendicular to the lateral load direction.

Slab stiffness reduction is caused by restrained volume changes and applied loads. Vanderbilt and Corley (1983) recognized the need to consider stiffness reduction due to slab cracking. To take into account the effect of cracking, slab stiffness of $\frac{1}{3}$ of gross-section stiffness was recommended. For Effective Beam Width model, Hwang and Moehle found that reduced slab stiffness ratio or the ratio of cracked stiffness to gross-section stiffness β could be expressed as a function of service live load, slab geometry, and material properties as follows:

$$\beta = 5 \frac{c}{l} - 0.1 \left(\frac{LL}{40} - 1 \right) \geq \frac{1}{3} \quad (3.3)$$

where LL is the service load in lb/ft^2 .

Hwang and Moehle also provided simplified and conservative form as follows:

$$\beta = 4 \frac{c}{l} \geq \frac{1}{3} \quad (3.4)$$

The lower value of $\frac{1}{3}$ in those equations matches the recommendation by Vanderbilt and Corley (1983).

Equivalent Frame Model is illustrated in Figure 3.3. The model consists of columns, beams, and torsion beams that represent the flexural framing stiffness between the column and the slab. The full span width is used for the beam width. Torsion member is attached to accommodate different rotations along the span width. Vanderbilt and Corley (1983) recommended the length of torsion member equal to the lesser of l_2 or l_1 . Hwang and Moehle recommended modifications of the length of transverse torsion member and the effect of slab cracking in the model. Based on the finite element analysis of the specimen, they recommended the length of l_1 as the length of transverse torsion member regardless of the length of l_2 .

The effect of slab cracking can be included in the Equivalent Frame Model by reducing the torsion member stiffness, by reducing the beam stiffness, or by reducing both of them. Hwang and Moehle recommended reduction of the torsion member stiffness after observing the cracking pattern of the specimen. Under combined gravity and lateral loads, flexural cracks concentrated near the column, whereas the slab away from the column remained uncracked. The redistributed moment accumulated in the positive moment region because of the reduced stiffness of the connections. Hwang and Moehle also pointed out that this behavior could be represented only by decreasing the torsion member stiffness. They recommended reduction of the torsion member stiffness using Equation (3.3).

Figure 3.4 shows the result of the analytical evaluation by Hwang and Moehle (2000) in the form of lateral stiffness comparison between the experimental data, Effective Beam Width, and Equivalent Frame Model. Figure 3.4 shows the lateral stiffness in E-W direction of EW800, EW400, and EW200 representing lateral drifts of 0.125%, 0.25%, and 0.5%, respectively. It shows that the lateral stiffness decreases gradually with increasing lateral drift. Effective Beam Width and Equivalent Frame Model with stiffness reduction represent lateral stiffness better than elastic analysis. However, Figure 3.4 also shows that gradual reduction of lateral stiffness with increasing lateral drift cannot be represented by single value of stiffness reduction.

3.3.2 Two-Beam Analytical Model by Robertson

Robertson (1997) recognized the need to include different slab stiffness reductions in different slab locations based on the moment regions and the lateral drifts. Robertson also pointed out that Equivalent Frame Model and Effective Beam Width were mostly used in design offices with the development of computer programs. Since the equivalent column stiffness may vary from floor to floor and from one direction to the perpendicular direction, Equivalent Frame Model can only be applied to single story two-dimensional strip of slab. This model is not applicable for the analysis of a complete three-dimensional structure. The major advantage of Effective Beam Width is the possibility of modeling complete three-dimensional structure. Robertson pointed out that the analysis must accommodate

the condition where portion of the slab may be still uncracked whereas the other portion near the columns may experience substantial cracking.

Robertson (1997) analyzed the experimental data of a continuous slab-column specimen tested by Robertson and Durrani (1990) as shown in Figure 3.5. The specimen consisted of one interior and two exterior connections. It was tested under cyclic lateral displacement and 30% of the design live load. Based on the analysis of the experimental data, Robertson recommended a new model called Two-Beam Analytical Model.

With portion of the slab width $\alpha = 0.4$ and reduced slab stiffness ratio $\beta = 1/3$ in Effective Beam Width model, Robertson found that the experimental result could not be represented well. Equivalent Frame model with $\alpha = 1$ and $\beta = 1/3$ also resulted in the same result. Table 3.1 shows that the lateral drifts and the slab moments from the experimental data differ significantly from the analysis using both models. At 0.5% drift, both models do not produce the correct slab moments, whereas only the Effective Beam Width produces correct lateral drift. The Equivalent Frame Model underestimates the lateral drift by 46%. At 1.5% drift, both models do not predict the slab moments and the lateral drifts correctly.

Robertson pointed out that Equivalent Frame Model and Effective Beam Width could not represent the specimen behavior due to single coefficient of beam width and the slab stiffness reduction over the entire span. The crack pattern of the specimen showed variation of cracking in the positive and negative moment regions. Under combined gravity and lateral loads, the slab negative moment at the column face was higher than the slab positive moment at midspan. The positive moment was resisted by greater width of span compared to the negative moment that concentrated at the face of the column. Higher negative slab moment and smaller effective width resulted in lower stiffness for the slab region near the column face compared to the stiffness of the slab region at midspan. It was recommended to divide the slab element into two members with different properties joined at the inflection point. Based on the moment distribution of the specimen, the inflection point occurred at approximately one fifth of the span length from the negative moment end. To include these factors in the analysis, Robertson recommended Two-Beam Analytical model with different properties for each

moment region as shown in Figure 3.6. Table 3.2 shows the comparison between the experimental data and the analysis results using Two-Beam Analytical model. Robertson also stated that a beam consisting of two elements was adequate for unidirectional static loading. For dynamic analysis, a beam consisting of three elements with nodes at potential inflection points could be used. However, the section properties of each element and the actual location of the inflection point varied based on the lateral load directions.

The exact values of α and β used in Two-Beam Analytical model are shown in the Parts 1 and 2 of Table 3.2. The analysis results with standard values of α and β are shown in the Parts 3 and 4. Lower values of β in the negative moment region represent more substantial cracking in this region. At both 0.5% and 1.5% drifts, Two-Beam Analytical model produces good prediction of slab moments.

3.3.3 Simplified Frame Analysis by Megally and Ghali

Megally and Ghali (2000) suggested a simplified frame analysis for obtaining the design value of unbalanced moment. A single joint is taken out of the complete structure to be analyzed separately under elastic lateral displacement Δ_s . Unbalanced moments transferred between the slab and the columns are determined using simplified frame analysis as shown in Figure 3.7. The unbalanced moment produced by the analysis must not exceed specific upper limit value.

The inflection line of the slab is assumed at midspan. The inflection points of the column are assumed at the midheight of the column. The length of each span adjacent to the column is equal to l and the lengths of the column above and below the joint are equal to h_f . The slab is simply supported at the locations of inflection lines. The column ends are supported by pin support at the bottom and by roller support at the top. The moment of inertia of the column I_c must take into account the effect of torsion arm. To avoid underestimation of the transferred unbalanced moment, the moment of inertia of slab I_s is taken equal to one half of the moment of inertia of the gross section. Slab-column joint is modeled as a rigid joint. Elastic horizontal displacement Δ_s is applied at the top of the column. The unbalanced

moment due to applied horizontal displacement is equal to the sum of the two moments at the column ends at the connection. Additional unbalanced moment caused by factored vertical force during earthquake must also be considered. The value of unbalanced moment from the analysis must not exceed the specific upper limit value.

Based on finite elements analysis and experiments on slab-column connections transferring shear forces combined with moment reversals, Megally and Ghali (2000) recommended the upper limit of unbalanced moment M_u as:

$$M_u \leq \frac{M_{pr}}{\alpha_m} \quad (3.5)$$

where M_{pr} is the sum of the flexural strengths of the opposite sides of the critical section within the width of $c_x + d$ or $c_y + d$, when the transferred moment is about x or y axes, respectively. c_x and c_y are the column dimensions parallel to x and y axes, respectively. d represents the slab effective depth. The flexural strength should be based on the probable yield strength of the flexural reinforcement equal to $1.25 A_s f_y$, where f_y is the yield strength of the reinforcement and A_s represents the areas of the flexural rebars normal to the two opposite sides of the critical section. The term M_{pr}/α_m in Equation (3.5) represents the magnitude of unbalanced moment that develops the yield strength of the flexural reinforcement. The empirical coefficient α_m is expressed as:

For interior connections:

$$\alpha_m = 0.85 - \gamma_v - \left(\frac{\beta_r}{20} \right) \quad (3.6)$$

For exterior connections:

$$\alpha_m = 0.55 - \gamma_v - \left(\frac{\beta_r}{40} \right) + 10\rho \quad (3.7)$$

where γ_v is the fraction of unbalanced moment transferred by shear stress at the slab-column connection. β_r is equal to l_y/l_x or l_x/l_y when the transferred moment is about the x or y axes, respectively. l_x and l_y are the projections of the

critical section on the principal axes x and y , respectively (Figure 2.3). ρ is the ratio of tensile flexural rebars passing through the projections of the critical section in the direction of transferred moment.

3.4 REVIEW OF AVAILABLE MODELS OF REDUCED SLAB STIFFNESS FOR OBTAINING UNBALANCED MOMENTS

Currently available models for obtaining unbalanced moments are only applicable for flat plate structure with regular column layouts. Reduced slab stiffness in the analysis determines the accuracy of the design values of unbalanced moments. Slab cracking is taken into account in the available models by applying uniform value of reduced slab stiffness or by using the element with different properties on the positive and negative moment regions.

3.4.1 Effective Beam Width and Equivalent Frame Model

Effective Beam Width model is often used because of its simplicity. The formula to obtain the effective width of the slab is straightforward. Reduced slab stiffness ratio can be easily applied by reducing the effective width. It is also possible to model a complete three-dimensional structure. However, uniform value of reduced slab stiffness used in the model does not represent variation of cracking at different slab locations. Uniform reduced slab stiffness also does not represent decreasing lateral stiffness when lateral drift increases. Effective Beam Width model is not applicable for flat plate structures with irregular column layouts due to difficulty in modeling the effective width of the slab.

Equivalent Frame Model is also widely used for design purpose because of its simplicity. The formula to calculate the column stiffness taking into account the effect of the torsion arm is quite simple. Reduced slab stiffness can be easily included in the model by reducing the stiffness of the torsion arm as recommended by Hwang and Moehle (2000). However, uniform reduced slab stiffness used in the model does not represent the variation of cracking at different regions of the slab. Equivalent Frame Model is only applicable to model a single story two-dimensional strip of slab with regular column layouts.

3.4.2 Two-Beam Analytical Model

Two-Beam Analytical Model was recommended by Robertson (1997). It was verified by using experimental data from a continuous slab specimen as shown in Figure 3.5. Reduced slab stiffness is applied on the beam element with different properties for positive and negative slab moment regions separated by the inflection points.

Few factors must be considered when using Two-Beam Analytical Model. Load combinations and the directions of lateral load lead to variation of inflection line positions and reduced slab stiffness. Two-Beam Analytical Model is not applicable for a flat plate structure with irregular column layouts.

3.4.3 Simplified Frame Analysis

In the simplified frame analysis suggested by Megally and Ghali (2000), a single joint is taken out of the complete structure to be analyzed separately under elastic lateral displacement. The unbalanced moment obtained from the analysis must not exceed specific upper limit value based on the yield moment of the slab reinforcement. The moment of inertia of the cracked slab is taken equal to one half of the moment of inertia of the gross section. The uniform reduced slab stiffness ratio can be directly applied on the beam elements. The column stiffness calculation must include the stiffness of the torsion arms.

Few factors must be considered when using simplified frame analysis for obtaining the design value of unbalanced moment. Additional single joint analysis is required beside the analysis of the complete structure. Uniform reduced slab stiffness does not represent the cracking behavior of the slab. A joint may need to be modeled more than once depending on the lateral load directions. This model is not applicable for a flat plate structure with irregular column layouts.

For design purpose, it is necessary to provide a model to obtain the design value of unbalanced moment that is applicable for flat plate structures with both regular and irregular column layouts. Reduced slab stiffness that can represent the cracked slab behavior correctly must also be included in the model since it affects the accuracy of the unbalanced moment. The proposed reduced slab stiffness model for slabs is presented in Chapter 7. The model is applicable for flat plate structures

with both regular and irregular column layouts. Since available models of reduced slab stiffness are not intended to yield accurate analytical results, they cannot be used to verify the proposed model. Instead, the experimental data are used for verification. The verification of the model using the experimental data of slab deflections and the unbalanced moments of slab-column connections subjected to lateral loads is presented in Chapter 7. The basis of the model is the modified effective moment of inertia method, which is proposed in Chapter 6. Nine slab-column connections were tested in the experimental program to provide additional data to augment the currently limited experimental data of irregular slab-column connections. The experimental data are presented in Chapters 4 and 5. These experimental data, together with the experimental data from other researchers, were also used to verify the proposed model of reduced slab stiffness for slabs.

Table 3.1. Experimental and Analytical Results by Robertson (1997)

Drift (%)	Model	α	β			Slab Moment (Kip-in)				Drift (%)
			Pos	Neg. Int	Neg. Ext	West Ext.	West Int.	East Int.	East Ext.	
0.5	Experimental Results	-	-	-	-	73	-82	88	-143	0.50
	Effective Beam Width	0.4	0.33	0.33	0.33	21	-174	18	-172	0.53
	Equivalent Frame Method	1	0.33	0.33	0.33	35	-177	8	-164	0.27
1.5	Experimental Results	-	-	-	-	168	-153	214	-260	1.50
	Effective Beam Width	0.4	0.33	0.33	0.33	122	-275	118	-275	0.97
	Equivalent Frame Method	1	0.33	0.33	0.33	140	-275	106	-270	0.56

Table 3.2. Analysis Results Using Two-Beam Analytical Model by Robertson (1997)

Part	Model	α	β			Slab Moment (Kip-in)				Drift (%)
			Pos	Neg. Int	Neg. Ext	West Ext.	West Int.	East Int.	East Ext.	
1	Experimental Results	-	-	-	-	73	-82	88	-143	0.50
	Two-Beam Eff. Beam Width	0.34	1	0.09	0.24	65	-88	88	-144	0.53
	Two-Beam Equiv .Frame Method	0.45	1	0.08	0.25	71	-82	85	-147	0.45
2	Experimental Results	-	-	-	-	168	-153	214	-260	1.50
	Two-Beam Eff. Beam Width	0.4	0.5	0.08	0.2	163	-148	212	-269	1.34
	Two-Beam Equiv .Frame Method	0.4	0.5	0.08	0.18	162	-153	217	-260	1.50
3	Experimental Results	-	-	-	-	73	-82	88	-143	0.50
	Two-Beam Eff. Beam Width	0.4	1	0.1	0.25	70	-90	91	-146	0.42
	Two-Beam Equiv .Frame Method	0.4	1	0.1	0.25	83	-91	81	-143	0.48
4	Experimental Results	-	-	-	-	168	-153	214	-260	1.50
	Two-Beam Eff. Beam Width	0.4	0.5	0.1	0.2	164	-170	199	-259	1.27
	Two-Beam Equiv .Frame Method	0.4	0.5	0.1	0.2	170	-170	193	-258	1.35

Fig. 3.1 Nine-Panel Specimen by Hwang and Moehle (2000a)

Fig. 3.2 Effective Beam Width Model

Fig. 3.3 Equivalent Frame Model

Fig. 3.4 Comparison between Analytical Model and Experimental Data by Hwang and Moehle (2000)

Fig. 3.5 Continuous Slab-Column Specimen Tested by Robertson and Durrani (1990)

Fig. 3.6 Two-Beam Analytical Model by Robertson (1997)

Fig. 3.7 Simplified Frame Analysis by Megally and Ghali (2000)

CHAPTER 4:

CORNER SLAB-COLUMN CONNECTIONS SUBJECTED TO BIDIRECTIONAL LATERAL LOADING

4.1 INTRODUCTION

Experimental program involving five corner slab-column connections having column aspect ratio of 5:1 is presented in this chapter. The parameters involved were the effect of gravity load magnitude, bidirectional lateral loading, and the presence of stud shear reinforcement (SSR). One specimen was tested under gravity load. Two other specimens were tested under combined gravity and bidirectional lateral loads. Two more specimens with embedded SSR were also tested to investigate the benefit of using SSR. These two specimens were tested under combined gravity and bidirectional lateral loads. Three specimens without SSR failed under brittle punching shear while the other two specimens with SSR experienced ductile flexural failure combined with compression strut failure. The experimental results showed that the effect of bidirectional lateral load on the connection with elongated column section could still be predicted fairly accurately by the ACI eccentric shear model.

Despite a number of research that had been done in the past, research on the behavior of slab-column connections still attracts a lot of interest. The behavior of slab-column connections is fairly complex. For corner slab-column connections, the complexity increases due to the presence of moment transfer in addition to the usual gravity load. This chapter concentrates on an experimental research on corner slab-column connections with one more factor to consider, which is the connections having a column aspect ratio of 5:1. This chapter also discusses drift capacity, stiffness, and ductility in the light of the new experimental results involving five large-scale specimens.

So far, the eccentric shear model of the ACI Code is able to fulfill the need for design guide for slab-column connections. However, very little data are available for slab-column connections subjected to biaxial unbalanced moments. This chapter also verifies the suitability of the ACI method for connections with

column aspect ratio of 5:1 subjected to combined gravity and bidirectional lateral loads.

The results of an experimental program on elongated corner slab-column connections are presented. The effects of bidirectional lateral load, different magnitude of gravity load, and the use of stud shear reinforcement (SSR) are also discussed. The experimental result should be useful to design engineers and researchers dealing with modern flat plate building, especially corner slab-column connections. The experimental results are also used to verify the model to obtain the unbalanced moments presented in Chapter 7.

4.2 SPECIMEN DETAILS

All five corner slab-column connection specimens, which were modeled after a typical high rise flat plate building, have a slab thickness of 150 mm and column dimensions of 900 mm x 180 mm. The dimensions of the slab are 3.3 m by 2.34 m and the column length is 2.13 m. Figure 4.1 shows a typical specimen.

The edges of the slab are actually the midspan of slab panel and they are assumed to have zero deflection under lateral load. The edges of the slab are supported on a series of edge links along the edges as shown in Figure 4.2.

The prototype building was assumed to be subjected to dead load consisting of self-weight and 1 kPa imposed dead load and 4 kPa service live load. With design concrete strength f'_c equal to 40 MPa (actual concrete strength of each specimen is given in Table 4.1 and Appendix 2), the load combination leads to a design value of gravity shear ratio V_u/V_c equal to 0.29. This is taken as the high gravity shear ratio. If a reduced live load equal to 30% of the design live load can be assumed in a building undergoing combined gravity and lateral loads, the design value of gravity shear ratio V_u/V_c becomes equal to 0.21. This is defined as the low gravity shear ratio. Since the actual concrete strength of each specimen varies, the shear force capacity V_c of each specimen also varies. The value of V_u/V_c of each specimen depending on its actual concrete strength is shown in Table 4.1 ($f'_c=38.74$ MPa leads to $V_u/V_c = 0.21$ for SC-LD, $f'_c=47.62$ MPa leads to $V_u/V_c = 0.19$ for SC-

LDS, and $f'_c = 47.04$ MPa leads to $V_u / V_c = 0.26$ for SC-HD and SC-HDS). The shear force capacity V_c is the lowest value from the following ACI equations:

$$V_c = \left(1 + \frac{2}{\beta_c}\right) \frac{\sqrt{f'_c} b_o d}{6} \quad (4.1)$$

$$V_c = \left(\frac{\alpha_s d}{b_o} + 2\right) \frac{\sqrt{f'_c} b_o d}{12} \quad (4.2)$$

$$V_c = \frac{1}{3} b_o d \sqrt{f'_c} \quad (4.3)$$

where:

- b_o = length of critical perimeter at $d/2$ from the column face
- d = slab effective depth
- β_c = ratio of long side dimension to short side dimension of the column
- α_s = 20 for corner slab-column connections

The column strips reinforcement ratios equal to 1.07% and 1.16% were placed parallel to x and y axes, respectively. The outermost top and bottom reinforcement were placed along x -axis. At least two bottom steel bars in both directions were placed through the column to support the slab after punching shear failure. Clear concrete cover equal to 15 mm resulted in the average effective depth of the slab d_{ave} of 122mm. T13 deformed steel bars with 13mm diameter and T10 deformed steel bars with 10 mm diameter were used for top and bottom reinforcements, respectively. All top and bottom steel bars were anchored using 180-degree hooks at the edge of the slab based on the requirement by ACI 318 Clause 12.5.3. The yield stresses f_y for T13 and T10 steel bars are 525.6 MPa and 521.6 MPa, and the Young's Moduli E_s are 197.2 GPa and 192.2 GPa, respectively. The yield stresses and the Young's moduli for T13 and T10 steel bars were obtained from the laboratory test (see Appendix 2). The column was heavily reinforced with 16T20 steel bars or 3.1% of the area of column gross section since it

was designed to remain elastic. Typical slab reinforcement layout and the location of slab edge supports are shown in Figure 4.2.

Stud Shear Reinforcement (SSR) with 10mm stem and 30mm diameter head was placed in two specimens. Typical stud rail and its arrangement around the column according to the recommendation of the ACI Committee 421 are shown in Figure 4.3. One stud rail consists of five studs with the distance between column face and the first stud equal to 40 mm ($0.33 d_{ave}$) and subsequent distance between studs equal to 90 mm ($0.74 d_{ave}$).

Table 4.1 shows the properties of all specimens. Specimen SC-H was only subjected to gravity load. Specimens SC-LD and SC-HD were subjected to low and high gravity shear ratio, respectively combined with bidirectional lateral drift. Specimens with SSR, SC-LDS and SC-HDS, were subjected to low and high gravity shear ratio, respectively, combined with bidirectional lateral drift. The actual gravity shear ratios V_u/V_c for specimens SC-LD and SC-HD are 0.21 and 0.26, respectively. The actual gravity shear ratios V_u/V_c for specimens SC-LDS and SC-HDS are 0.19 and 0.26, respectively. These values were obtained from the actual concrete strength of each specimen and maintained constant during the test. The experimental results also always refer to the actual gravity shear ratios.

4.3 EXPERIMENTAL SETUP AND INSTRUMENTATION

Figure 4.4 shows typical experimental setup. This typical setup was designed for four experimental programs involving 19 rectangular slab-column connections at Nanyang Technological University (NTU), Singapore. Those experimental programs consist of five interior slab-column connections by Tan and Teng (2005), five edge slab-column connections by Anggadajaja (2006), five corner slab-column connections by the author, and four slab-column connections with 135-degree slabs by the author (see Chapter 5). The author was part of the team designing the overall experimental setup. In overall, the basic idea of the experimental setup was quite similar to that of Pan and Moehle (1992). The main components of the experimental setup are vertical hydraulic jack at the bottom of

the column, vertical edge links at the slab perimeter, horizontal hydraulic jacks at both x and y axes attached to the top of the column, and torsion restraining frame.

Slab inflection point was simulated by a series of roller supports in the form of eight vertical edge links as shown in Figure 4.5. Each vertical edge link was attached to the slab perimeter at one end and to the strong floor of the laboratory at the other end. The pin-type mechanism on each end of the edge link allowed translation and rotation about both x and y axes with negligible vertical displacement along slab inflection point.

Figure 4.6 shows the combination between steel rocker and reaction arms at the bottom of the column. Ball-bearing mechanism was provided at the bottom of the column. The bottom of the column was held in place by two horizontal reaction arms in both x and y axes. These reaction arms were used to transfer horizontal reaction forces to the reaction frames in both directions. The reaction frames were secured to the strong floor of the laboratory. The pin-type mechanism on each end of the steel reaction arm allowed rotation about both x and y axes without any horizontal translation at the column support. Vertical hydraulic jack at the bottom of the column and steel blocks placed on the slab simulated the applied gravity load. A load cell was also provided below the vertical hydraulic jack to measure applied gravity load. The vertical hydraulic jack and the load cell below it were kept in place using four vertical steel plates welded to the steel base plate which was secured to the strong floor of the laboratory.

Figure 4.7 shows two horizontal hydraulic jacks attached to the top of the column in both x and y axes. These horizontal hydraulic jacks were attached to the reaction frames and to the top of the column using pin-type mechanism. Those hydraulic jacks simulated displacement-based bidirectional lateral load. The magnitudes of the lateral load were recorded using two load cells attached to the horizontal hydraulic jacks in both directions. The reaction frames transferred horizontal reaction forces induced by the horizontal hydraulic jacks to the strong floor of the laboratory. This system was used to apply lateral displacement along one axis (the top of the column displaced along either x or y direction only, the other direction was maintained at zero) as well as lateral displacements along two axes simultaneously (lateral displacements of the same magnitude were applied to

the top of the column at both axes simultaneously). Complete loading stages will be presented in Section 4.4.

Figure 4.8 shows torsion-restraining frame attached to the slab perimeter. The idea of torsion-restraining frame followed that of Pan and Moehle (1992). The frame was attached to the edge of the slab by two torsion arms. The pin-type mechanism was used on both ends of the torsion arm. Pin-type mechanism was also used as supports of the torsion-restraining frame. This frame was secured to the strong floor of the laboratory. The torsion-restraining frame was used to prevent rigid body rotation of the slab when the displacement-based lateral loads were applied to the top of the column.

Five load cells were installed to measure applied gravity load, lateral loads, and prestressing forces. Seven LVDTs were used to measure vertical deflections of the slab. Nine wire transducers attached to the column were used to measure lateral displacements. Figure 4.9 shows a total of 61 strain gauges that were installed on the top and bottom steel bars of each specimen. Figure 4.10 shows a total of 40 additional strain gauges that were also installed on the shear studs.

4.4 SEQUENCE OF LOADING

Before applying the gravity load, the column was pre-stressed with an axial force of 205 kN using prestressing hydraulic jacks as shown in Figure 4.11. Bearing plates were provided at the top and bottom of the column for two prestressing rods. Two load cells were installed at the top of the column to measure the prestressing forces. The axial force of 205 kN represented the axial load of the column. The specimen self weight was simulated by jacking up the column until the specimen self weight was carried by the steel rocker at the bottom of the column. Gravity load equal to the specific value of low or high gravity shear force was then applied incrementally using the vertical hydraulic jack to obtain the strain profiles and the crack progress. This procedure was carried out for the specimen SC-H until failure. For the other specimens, displacement-based bidirectional lateral load was applied after reaching the specific gravity shear force ($V_u/V_c=0.26$ for specimens SC-HD and SC-HDS, $V_u/V_c=0.21$ for specimen SC-LD, and $V_u/V_c=0.19$ for specimen

SC-LDS). The gravity shear force ratios were maintained constant during lateral load test.

Figure 4.12 shows the displacement-based lateral load sequence and target drift ratio. Lateral drift ratio is defined as the ratio of the lateral displacement measured between the column ends to the height of the column. The lateral displacement sequence starts at 0% to 1% lateral drift with the incremental drift of 0.25%. For 1% to 3% lateral drifts, the incremental drift is increased to 0.5%. After reaching 3% lateral drift, the incremental drift is increased to 1%. Each lateral drift target is performed in complete two cycles. For 1.5% to 3% lateral drifts, one complete cycle of 1% lateral drift is performed before applying subsequent lateral drift target to observe the serviceability of the connection.

Figure 4.13 shows 16 loading stages and the sign convention for lateral drift, moment about x -axis M_{ux} and moment about y -axis M_{uy} . Loading stages 1, 7, 9, and 15 indicate that only the lateral drift along x -direction is applied, while the lateral drift along y -direction is set to zero. Loading stages 3, 5, 11, and 13 indicate that only the y -direction drift is applied, while the x -direction drift is set to zero. Loading stages 2, 6, 10, and 14 indicate that two lateral drifts of the same magnitude are applied in x and y directions simultaneously. The loading stages start with applying x -direction drift to the loading stage 1. While maintaining x -direction drift, y -direction drift of the same magnitude is applied to obtain loading stage 2. Loading stage 3 is obtained by reducing x -direction drift to zero while maintaining y -direction drift. Loading stage 4 is obtained by reducing y -direction drift to zero after loading stage 3. The loading loops are carried on for all 16 loading stages. Note that for corner slab-column connections, the gravity load alone automatically induces unbalanced moments.

4.5 GENERAL SPECIMEN BEHAVIOR

Specimen SC-H subjected to gravity load experienced punching shear failure with the peak gravity load equal to 115.74 kN or $V_u/V_c = 0.54$. Peak unbalanced moments were $M_{ux} = -116.4$ kN-m and $M_{uy} = -96.5$ kN-m. Flexural cracks first occurred near the column accompanied by torsion crack from the inner

column edge extending to the slab edge along the column long side. The flexural cracks continued to propagate under increasing gravity load toward the slab edge until the punching shear failure occurred. Figure 4.14 shows the punching shear failure of specimen SC-H.

Visible flexural cracks on the top surface of slab near the column occurred on specimens SC-HD (high gravity) and SC-HDS (high gravity with SSR) after applied gravity force and they continued to propagate under further loading toward the slab edge. Torsion crack occurred on the slab edge along the column long side. Punching shear failure of specimen SC-HD started to occur at loading stage 2 during the second cycle of 1.5% target drift. Sudden drop of unbalanced moment occurred at loading stages 9 and 14 during the first cycle of 2% drift when the shear perimeter became more obvious. The peak unbalanced moments were $M_{ux} = -126.4$ kN-m at -1.51% y -direction drift and $M_{uy} = -183.5$ kN-m at -1.51% x -direction drift. Figure 4.15 shows the punching shear failure of specimen SC-HD. Specimen SC-HDS experienced flexural failure combined with compression strut failure marked by severe concrete crushing on the bottom of the slab near the column as shown in Figure 4.16. The crushing of concrete started to occur at loading stage 2 during the second cycle of 2% y -direction drift. Yielding of top steel reinforcement along x -axis across the width of $c+1.5h$ occurred at loading stage 9 during the first cycle of 1.5% x -direction drift, where c is the column dimension perpendicular to the steel rebars and h is the slab thickness. The shear force induced large compression strut action focused at the bottom of the slab at the column face. The shear studs could prevent the diagonal shear cracking, but they could not prevent the compression strut failure. The combination of flexural compression and shear strut compression caused the crushing of concrete at the bottom of the slab at the face of the column. Sudden drop of unbalanced moments did not occur until loading stage 14 during second cycle of 4% x -direction drift. The test was stopped since the horizontal hydraulic jacks were out of stroke. The peak unbalanced moments were $M_{ux} = -130.4$ kN-m at -2.02% y -direction drift and $M_{uy} = -197.7$ kN-m at -3.01% x -direction drift.

Only one fine flexural crack on the top surface of the slab near the column occurred on specimens SC-LD (low gravity) and SC-LDS (low gravity with SSR) after applied gravity force. Flexural cracks continued to extend under further loading to the middle of the slab and towards the edge links. Torsion cracks also occurred on the slab edge along the column long side. Specimen SC-LD failed in punching shear that started to occur at loading stage 14 during the second cycle of 2.5% drift. Sudden drop of unbalanced moments occurred at loading stage 2 and 9 of the first cycle of 3% drift followed by complete punching shear failure at loading stage 14 during first cycle of 3% drift. The peak unbalanced moments were $M_{ux} = -112.7$ kN-m at -2.53% y -direction drift and $M_{uy} = -135.1$ kN-m at -2.48% x -direction drift. Figure 4.17 shows punching shear failure of specimen SC-LD. Specimen SC-LDS with SSR failed in flexural failure combined with compression strut failure at higher drift ratio. The failure was marked by severe concrete crushing on the bottom of the slab near the column as shown in Figure 4.18. The combination of flexural compression and shear strut compression caused the crushing of concrete at the bottom of the slab at the face of the column. The crushing of concrete started to occur at loading stage 2 during the second cycle of 3% drift. Yielding of top steel reinforcement along x -axis across the width of $c+1.5h$ occurred at loading stage 14 during the first cycle of 2.5% drift. The unbalanced moments decreased gradually until loading stage 14 during second cycle of 4% drift. The test was stopped since the hydraulic jacks were out of stroke. The peak unbalanced moments were $M_{ux} = -113.5$ kN-m at -2.50% Y -drift and $M_{uy} = -144.8$ kN-m at -3.02% X -drift.

It was observed that punching shear area of specimen SC-HD was relatively larger than punching shear area of specimen SC-LD. Two bottom steel bars along x -axis passing through the column were proven effective in supporting the slab after punching shear failure. Figures 4.19 – 4.26 show the hysteretic loops of the unbalanced moments during first and second cycles of applied lateral drifts of each specimen. Figures 4.27 – 4.30 show the unbalanced moments envelopes of each specimen where the sudden or gradual drops of unbalanced moments due to the connection failure can be observed.

4.6 EXPERIMENTAL RESULTS

Experimental results of the specimens are presented in term of drift capacity, ductility, and stiffness. The drift capacity D_p is defined as the drift ratio at the maximum unbalanced moment M_u . Figure 4.31 shows the term ductility μ_p defined by Pan and Moehle (1989). For all specimens, ductility μ_p is defined as the ratio between the drift ratio at peak unbalanced moment (not $0.8 M_u$) D_p to the drift ratio at first yield of slab reinforcement D_y or it can be expressed as $\mu_p = D_p / D_y$. Note that since the hydraulic jack stroke was limited, D_u (drift ratio at $0.8 M_u$) for specimens with SSR could not be obtained.

Stiffness is defined as the ratio between peak-to-peak values of unbalanced moments and the drift ratio obtained from the hysteretic loop as shown in Figure 4.32. Two cases of stiffness obtained from the loading stages shown in Figure 4.13 are discussed. The first case represents unidirectional lateral load where only one direction of lateral drift is applied in addition to applied gravity load. This case consists of the stiffness obtained from loading stages 1 and 9 for moment about y -axis M_{uy} and the stiffness obtained from loading stages 5 and 13 for moment about x -axis M_{ux} . The second case represents bidirectional lateral load where lateral drifts are applied in two directions in addition to applied gravity load. This case consists of the stiffness obtained from loading stages 6 and 14 for M_{uy} and the stiffness obtained from loading stages 2 and 10 for M_{ux} .

4.6.1 Effects of Bidirectional Lateral Load

Effect of bidirectional lateral load on the unbalanced moment capacity is best shown by vertical segments in the hysteretic loops as shown in Figures 4.19 – 4.26. The vertical segments show drop or increase in unbalanced moments at some loading stages when the lateral drift in the other direction is applied or released. Taking the example of specimen of SC-LDS from Figure 4.25.a, at 1.5% lateral drift M_{uy} increases from -110.6 kN-m at loading stage 9, where only x -direction drift is applied, to -117.2 kN-m at loading stage 10 where x -direction drift from

loading stage 9 is maintained and y -direction drift is applied subsequently. It shows that reduced slab stiffness is not only caused by applied lateral drift in the respective direction but also by lateral drift in the other direction.

Figures 4.33 – 4.35 show two cases of stiffness of all specimens. For x -direction drift, comparison is made between the stiffness of loading stages 1-9 shown in Figure 4.13 that represent unidirectional lateral load and loading stages 6-14 that represent bidirectional lateral load. Comparison for y -direction drift also includes the stiffness of unidirectional lateral load from loading stages 5-13 and the stiffness of bidirectional lateral load from loading stages 2-10. Taking the examples of specimens SC-LD and SC-HD, Figure 4.33 shows that the stiffness obtained from unidirectional lateral load (loading stages 1-9) is almost always higher than the stiffness obtained from bidirectional lateral load (loading stages 6-14) for x -direction drift. It shows that bidirectional lateral load reduces the stiffness on x -direction drift. However, the stiffness obtained from unidirectional lateral load (loading stages 5-13) is almost always lower than the stiffness obtained from bidirectional lateral load (loading stages 2-10) for y -direction drift. It is caused by the maximum unbalanced moment about x -axis M_{ux} that mostly occurs at loading stage 2.

4.6.2 Effects of Gravity Load Magnitude

Table 4.2 shows that gravity shear force reduces the ultimate drift capacity in both axes. Specimen SC-H without any lateral load can withstand gravity shear force ratio V_u/V_c of 0.54. Specimen SC-HD (high gravity) subjected to $V_u/V_c=0.26$ has lower ultimate drift capacity compared to specimen SC-LD (low gravity) with $V_u/V_c=0.21$. Specimen SC-HD has drift capacity parallel to x -axis $D_{px}=-1.51\%$ and drift capacity parallel to y -axis $D_{py}=-1.51\%$, whereas specimen SC-LD has $D_{px}=-2.48\%$ and $D_{py}=-2.53\%$. Specimens with SSR also behave in the same manner. Specimen SC-HDS (high gravity, $V_u/V_c=0.26$ with SSR) has $D_{px}=-3.01\%$ and $D_{py}=-2.02\%$ that are lower than specimen SC-LDS (low gravity, $V_u/V_c=0.19$ with SSR) with $D_{px}=-3.02\%$ and $D_{py}=-2.50\%$.

Although specimens subjected to low gravity shear force ratio can withstand higher ultimate drift capacities, the maximum unbalanced moments that those specimens can resist are lower compared to the specimens subjected to high gravity shear force. Table 4.2 shows that specimen SC-HD has $M_{uy}=-183.5$ kN-m and $M_{ux}=-126.4$ kN-m that are higher than specimen SC-LD with $M_{uy}=-135.1$ kN-m and $M_{ux}=-112.7$ kN-m. Specimen SC-HDS has $M_{uy}=-197.7$ kN-m and $M_{ux}=-130.4$ kN-m that are also higher than specimen SC-LDS with $M_{uy}=-144.8$ kN-m and $M_{ux}=-113.5$ kN-m. Higher unbalanced moments are contributed by the axial gravity load that is applied through the center of the column that counteracts the negative unbalanced moments arising from lateral loads. This shows that taking the centroid of the critical section as the center of the shear force and unbalanced moments is correct.

Table 4.2 shows that higher gravity shear force slightly reduces ductility in both axes. It shows that the ductility of specimen SC-HD is slightly lower compared to the ductility of specimen SC-LD. Specimen SC-LD has ductility in x -axis $\mu_{px}=2.01$ and ductility in y -axis $\mu_{py}=2.06$ that are slightly higher than specimen SC-HD with $\mu_{px}=1.97$ and $\mu_{py}=1.99$. Specimen SC-HDS has $\mu_{px}=2.12$ and $\mu_{py}=2.20$ that are slightly lower than specimen SC-LDS with $\mu_{px}=2.14$ and $\mu_{py}=2.26$. Higher ductility for specimens subjected to low gravity shear force is a result of higher ultimate drift ratio of those specimens.

It is interesting to observe from Figure 4.33 that the stiffness of specimens subjected to low gravity shear force with higher ultimate drift capacity is not higher than the specimens subjected to high gravity shear force. Figure 4.33 shows that specimen SC-HD shows higher overall stiffness compared to the stiffness of specimen SC-LD for two cases of stiffness in both axes. The expected larger stiffness reduction due to higher gravity shear force is still apparent in Figure 4.33. It is shown from the steeper or larger negative tangent of stiffness curves of specimen SC-HD compared to the stiffness curves of specimen SC-LD. The larger negative tangent shows more rapid stiffness degradation due to more immense cracking.

There is insufficient data to recommend precise design limit of V_u / V_c for specimens subjected to lateral drift, since past research mostly focused on corner connections with square columns subjected to gravity load. However, there is a need for a guideline for corner connection design subjected to drift ratio of 1.5% and 2%. The data obtained from all five specimens are used to produce conservative limit of V_u / V_c . Figure 4.36 shows that to withstand 1.5% and 2% lateral drifts, V_u / V_c of corner connections with rectangular columns should be limited to 0.30 and 0.25, respectively.

4.6.3 Effects of the Use of Stud Shear Reinforcement (SSR)

Besides avoiding brittle punching shear, effective arrangement of shear studs increases unbalanced moment capacity and ultimate drift capacity as shown in Table 4.2. Maximum unbalanced moment and ultimate drift capacity are higher for specimen SC-HDS (high gravity with SSR) compared to specimen SC-HD (high gravity). For specimen SC-HDS, there are increases of 99% and 34% compared to specimen SC-HD for drift capacities parallel to x and y axes, D_{px} and D_{py} , respectively. There are also increases of 8% and 3% for M_{uy} and M_{ux} , respectively. For specimens subjected to low gravity shear force, there is an increase of 22% for D_{px} without any increase in D_{py} for specimen SC-LDS (low gravity with SSR) compared to specimen SC-LD (low gravity). There are also an increase of 7% for M_{uy} and a slight increase of 1% for M_{ux} .

Table 4.2 shows that the presence of shear studs improves ductility. Specimen SC-LDS has higher ductility compared to specimen SC-LD with increases of 6% and 10% for μ_{px} and μ_{py} in the x and y directions, respectively. The ductility of specimen SC-HDS is also higher than specimen SC-HD with increases of 8% and 11% for μ_{px} and μ_{py} , respectively.

Figures 4.34 and 4.35 show that the presence of shear studs does not significantly increase the connection stiffness. The stiffness curves of specimens SC-HDS (high gravity with SSR) and SC-HD (high gravity) almost coincide as shown in Figures 4.34 and 4.35. Specimens SC-LDS (low gravity with SSR) and

SC-LD (low gravity) also behave similarly. However, after punching shear failure at 1.5% drift, significant loss of stiffness can be seen from steeper curve of specimen SC-HD. Meanwhile, the stiffness curve of specimen SC-HDS does not show any significant loss of stiffness after 1.5% drift and only shows gradual stiffness loss due to combination of ductile flexural failure and compression strut failure until 4% drift. It shows that the presence of shear studs helps to avoid significant loss of stiffness after reaching critical target drift ratio.

4.7 ECCENTRIC SHEAR MODEL BY ACI 318

Eccentric shear model requires that the critical section is assumed at $d/2$ from the column face. Shear stress due to gravity shear force is assumed to be constant along the critical perimeter. Part of unbalanced moment M_u is transferred by shear stress $\gamma_v M_u$ and the remaining is transferred by flexure $\gamma_f M_u$. Fraction of the unbalanced moment transferred by shear and the total shear stress v_u can be calculated as follows:

$$\gamma_v = 1 - \gamma_f = 1 - \frac{1}{1 + \frac{2}{3} \sqrt{\frac{b_1}{b_2}}} \quad (4.4)$$

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux}}{I_x} y + \frac{\gamma_{vy} M_{uy}}{I_y} x \quad (4.5)$$

where:

- b_1 = length of the critical section perpendicular to the moment axis considered
- b_2 = length of the critical section parallel to the moment axis considered
- A_c = area of the critical section
- I_x, I_y = second moments of area of the critical section
- x, y = coordinates where v_u is calculated

The shear stress capacity v_n can be calculated as follows:

For slab without shear reinforcement:

$$v_n = v_c \quad (4.6)$$

where v_c is determined from $V_c / (b_o d)$. The shear force capacity V_c is obtained from Equations (4.1), (4.2), and (4.3).

For slab with shear reinforcement:

$$v_n = v_c + v_s \quad (4.7)$$

$$v_c = \frac{1}{6} \sqrt{f_c'} \quad (\text{MPa}) \quad (4.8)$$

$$v_s = \frac{f_{yv} A_v}{b_o s} \quad (\text{MPa}) \quad (4.9)$$

where:

- b_o = length of critical perimeter at $d/2$ from the column face
- d = slab effective depth
- s = spacing of shear reinforcement
- f_{yv} = yield strength of shear reinforcement
- A_v = area of one peripheral perimeter of shear reinforcement

Column 12 of Table 4.3 shows the calculated v_u / v_c of all five specimens. Note that the unbalanced moments used for the calculation are taken from combination of M_{ux} and M_{uy} at one loading stage that causes maximum shear stress v_u . Columns 13 and 14 of Table 4.3 show the ratio of unbalanced moment transferred by flexure and moment capacity $\gamma_f M_u / M_f$ in both axes. For corner connections subjected to cyclic lateral loading, ACI 318 requires that the moment capacity M_f is obtained from the smallest width between $c + 1.5h$ and $c + c_t$ for each side of the column, where c is the column dimension parallel to the moment axis, h is the slab thickness and c_t is the column dimension perpendicular to the moment axis. The maximum value of v_u / v_c , $\gamma_{fx} M_{ux} / M_{fx}$, and $\gamma_{fy} M_{uy} / M_{fy}$ controls the type of connection failure.

Table 4.3 shows that for specimens with punching shear failure, conservative values of v_u / v_c equal to 2.20, 2.02, and 2.29 are shown for specimens SC-H, SC-LD, and SC-HD, respectively. For specimens with flexural failure combined with compression strut failure, $\gamma_f M_u / M_f$ equal to 1.83 and 2.50 are obtained for specimens SC-LDS and SC-HDS, respectively. The type of failure for those specimens can be predicted accurately using the eccentric shear model. The shear stress ratios v_u / v_c at failures also show that eccentric shear model by the ACI code is quite conservative.

A total of 39 data of corner slab-column connections without shear reinforcement were collected and analyzed as shown in Table 4.4. Only corner specimens by Falamaki and Loo (1992) include rectangular columns with column ratios of 2:1 and 4:3 and corner specimens by Hammill and Ghali (1997) include a combination of gravity load and lateral drift. Column ratios of 5:1 and cyclic bidirectional lateral load are covered by specimens SC-LD and SC-HD.

Table 4.5 shows that the eccentric shear model mostly gives conservative results of shear stress ratio v_u / v_c . The mean value of v_u / v_c of those specimens is equal to 1.73. Column 9 of Table 4.5 shows observed failures of all specimens during testing. Note that flexural-punching failure often implies that flexural yielding of the slab reinforcement has occurred prior to punching shear failure. Gardner and Zhou (1996) specifically mentioned that the slab reinforcement experienced yielding due to negative moment before punching shear failure. They referred the failure as combined flexural punching shear failure. Other researchers referred the failure as ductile punching shear failure. Figure 4.37 shows $\gamma_f M_u / M_f$ that is plotted against v_u / v_c . Points that fall above the 45-degree line indicate flexural failure, whereas points that fall below the line indicate punching shear failure. Figure 4.37 shows that the eccentric shear model can predict the type of connection failure in general except for two specimens by Walker and Regan (1987) and three specimens by Falamaki and Loo (1992). Figure 4.38 shows the relation between v_u / v_c and V_u / V_c . It shows that the accuracy of the eccentric shear model is affected by the increasing gravity shear force ratio V_u / V_c . Figure 4.38 indicates that

higher gravity shear force ratio gives more conservative result of shear stress ratio v_u / v_c . It indicates the possibility of part of unbalanced moments transferred by shear stresses decreases with increasing gravity shear force ratio V_u / V_c .

4.8 CONCLUSION

From the analysis of the experimental results, it can be concluded that unidirectional and bidirectional lateral loads result in different reduced slab stiffness of the connections. Taking the examples of specimens SC-LD (low gravity) and SC-HD (high gravity), Figure 4.33 shows that the stiffness obtained from unidirectional lateral load (loading stages 1-9) is generally always higher than the connection stiffness obtained from bidirectional lateral load (loading stages 6-14) for x -direction drift. Lower connection stiffness indicates higher reduction in cracked slab stiffness.

Lower gravity load leads to higher ultimate drift capacity and ductility. However, it does not increase the unbalanced moment capacity due to counteracting moment caused by the axial load at the centroid of the critical section. Table 4.2 shows that specimen SC-HD (high gravity) has $M_{uy} = -183.5$ kN-m and $M_{ux} = -126.4$ kN-m that are higher than specimen SC-LD (low gravity) with $M_{uy} = -135.1$ kN-m and $M_{ux} = -112.7$ kN-m. The presence of stud shear reinforcement (SSR) increases the unbalanced moment and ultimate drift capacity. It also increases ductility and avoids significant loss of stiffness after reaching critical lateral drift. Specimen SC-LDS (low gravity with SSR) has higher ductility compared to specimen SC-LD (low gravity) with increases of 6% and 10% for μ_{px} and μ_{py} in the x and y directions, respectively. The ductility of specimen SC-HDS (high gravity with SSR) is also higher than specimen SC-HD (high gravity) with increases of 8% and 11% for μ_{px} and μ_{py} , respectively. Figures 4.34 and 4.35 show that the stiffness curves of specimens SC-HDS (high gravity with SSR) and SC-HD (high gravity) almost coincide. However, after punching shear failure at 1.5% drift, significant loss of stiffness can be seen from steeper curve of specimen SC-HD. Meanwhile, the stiffness curve of specimen SC-HDS does not show any significant

loss of stiffness after 1.5% drift and only shows gradual stiffness loss due to combination of ductile flexural failure and compression strut failure until 4% drift.

It is recommended that the gravity shear force ratio V_u / V_c be maintained less than 0.30 and 0.25 to achieve 1.5% and 2% target drift ratios, respectively for corner slab-column connections with rectangular columns. Analysis of the experimental data using eccentric shear model by the ACI 318 gives conservative result of shear stress ratio v_u / v_c and in general it also predicts the type of failure fairly correctly.

In addition to five corner specimens presented in this chapter, the experimental data of four slab-column connections with 135-degree slabs are presented in Chapter 5. The experimental data from all nine specimens should be useful for research area of modern flat plate structures with irregular column layouts and rectangular columns. The experimental results are also used to verify the proposed model of reduced slab stiffness presented in Chapter 7.

Table 4.1. Corner Specimens Properties

Specimen (1)	f'_c (MPa) (2)	V_u/V_c (3)	Lateral Load (4)	h (mm) (5)	d_{ave} (mm) (6)	C_1 (mm) (7)	C_2 (mm) (8)	ρ_{topx} (%) (9)	ρ_{topy} (%) (10)	ρ_{botx} (%) (11)	ρ_{boty} (%) (12)	$f_{y\ top}$ (MPa) (13)	$f_{y\ bot}$ (MPa) (14)	Shear Studs (15)
SC-H	38.74	0.54	-	150	122	900	180	1.07	1.16	0.64	0.46	525.6	521.6	-
SC-LD	38.74	0.21	Bidirectional	150	122	900	180	1.07	1.16	0.64	0.46	525.6	521.6	-
SC-HD	47.04	0.26	Bidirectional	150	122	900	180	1.07	1.16	0.64	0.46	525.6	521.6	-
SC-LDS	47.62	0.19	Bidirectional	150	122	900	180	1.07	1.16	0.64	0.46	525.6	521.6	5 studs/rail
SC-HDS	47.04	0.26	Bidirectional	150	122	900	180	1.07	1.16	0.64	0.46	525.6	521.6	5 studs/rail

Table 4.2. Experimental Results of Corner Specimens

Specimen	f'_c (MPa)	V_u/V_c	V_u (kN)	Maximum Unbalanced Moments					Ductility		Observed Failure	
				Loading Stage (5)	D_{px} (%) (6)	M_{uy} (kN-m) (7)	Loading Stage (8)	D_{py} (%) (9)	M_{ux} (kN-m) (10)			
										μ_{px} (11)		μ_{py} (12)
(1)												
SC-H	38.74	0.54	115.7	-	-	-96.5	-	-	-116.4	-	-	Punching
SC-LD	38.74	0.21	44.8	10	-2.48	-135.1	2	-2.53	-112.7	2.01	2.06	Punching
SC-HD	47.04	0.26	61.8	9	-1.51	-183.5	2	-1.51	-126.4	1.97	1.99	Punching
SC-LDS	47.62	0.19	44.8	14	-3.02	-144.8	2	-2.50	-113.5	2.14	2.26	Flexure + Compression Strut
SC-HDS	47.04	0.26	61.8	10	-3.01	-197.7	2	-2.02	-130.4	2.12	2.20	Flexure + Compression Strut

Table 4.3. Analysis of the Experimental Results of Corner Specimens

Specimen	f'_c (MPa)	V_u/V_c	V_u (kN)	Max. Unbalanced Moments				Max. Shear Stress				v_u/v_c	$\gamma_t M_{uy}/M_{fy}$	$\gamma_t M_{ux}/M_{fx}$
				Loading Stage		M_{uy} (kN-m)	Loading Stage	M_{ux} (kN-m)	Loading Stage	M_{uy} (kN-m)	M_{ux} (kN-m)			
				(5)	(6)									
(1)		(3)	(4)		(6)	(7)	(8)		(10)	(11)	(12)	(13)	(14)	
SC-H	38.74	0.54	115.7	0	-96.5	0	-116.4	0	-96.5	-116.4	2.20	1.22	0.89	
SC-LD	38.74	0.21	44.8	10	-135.1	2	-112.7	14	-116.7	-88.5	2.02	1.48	0.68	
SC-HD	47.04	0.26	61.8	9	-183.5	2	-126.4	14	-158.2	-92.7	2.29	2.00	0.71	
SC-LDS	47.62	0.19	44.8	14	-144.8	2	-113.5	10	-144.6	11.9	$v_u/v_n=0.67$	1.83	0.87	
SC-HDS	47.04	0.26	61.8	10	-197.7	2	-130.4	10	-197.7	-7.6	$V_u/V_n=0.72$	2.50	1.00	

Table 4.4. Corner Specimens Data

Researchers (1)	Specimen (2)	h (mm) (3)	d _{ave} (mm) (4)	C ₁ (mm) (5)	C ₂ (mm) (6)	ρ_{topx} (%) (7)	ρ_{topy} (%) (8)	f _{y top} (MPa) (9)	f _c [*] (MPa) (10)	V _u (kN) (11)	M _{ux} (kN-m) (12)	M _{uy} (kN-m) (13)
Zaghlool, Paiva, and Glockner (1970)	I	139.7	114.3	139.7	139.7	1.47	1.47	379.2	24.1	106.4	9.2	8.3
	III	139.7	114.3	165.1	165.1	1.47	1.47	379.2	20.7	100.6	16.1	16.0
	IV	139.7	114.3	165.1	165.1	1.47	1.47	379.2	35.8	121.5	15.8	22.0
Zaghlool and Paiva (1973)	Z-I (1)	152.4	121	178	178	1.23	1.23	379.2	32.7	74.3	19.2	19.2
	Z-II (1)	152.4	121	267	267	1.23	1.23	388.9	53.0	137.9	38.5	38.5
	Z-II (2)	152.4	121	267	267	1.65	1.65	405.4	33.4	177.2	53.4	53.4
	Z-II (3)	152.4	118	267	267	2.23	2.23	450.9	27.7	177.9	58.0	58.0
	Z-II (6)	152.4	121	267	267	1.23	1.23	381.3	33.6	82.3	38.9	38.9
	Z-III (1)	152.4	121	356	356	1.23	1.23	379.2	33.6	179.7	52.7	52.7
Ingvarsson (1974)	1	120	95.0	180	180	1.89	1.89	454.0	25.8	108.0	20.9	20.9
	2	120	95.0	180	180	0.96	0.96	454.0	33.3	76.0	16.7	16.7
	4	120	95.0	180	180	0.96	0.96	452.0	32.0	101.0	18.1	18.1
	5	120	95.0	180	180	1.45	1.45	452.0	29.2	110.0	22.1	22.1
	SC1	125	100	300	300	0.65	0.65	450.0	43.3	81.5	25.2	25.2
Walker and Regan (1987)	SC2	125	100	300	300	0.41	0.41	450.0	47.9	74.8	24.0	24.0
	SC3	125	100	300	300	0.83	0.83	450.0	37.4	74.2	31.6	31.6
	SC4	125	100	220	220	0.65	0.65	450.0	40.8	63.8	16.7	16.7
	SC5	125	100	220	220	0.93	0.93	450.0	46.5	82.2	18.8	18.8
	SC7	125	100	220	220	0.93	0.93	450.0	43.8	82.2	27.6	27.6
	SC8	85	64	160	160	0.28	0.28	595.0	37.4	33.0	4.7	4.7
	SC9	85	64	160	160	0.56	0.56	595.0	34.3	33.0	5.9	5.9
	SC11	81	60	160	160	1.85	0.70	595.0	27.2	33.0	2.2	4.6
	SC12	81	60	300	300	1.21	0.48	595.0	40.7	36.8	8.9	12.7

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Table 4.4. Cont'd

Researchers (1)	Specimen (2)	h (mm) (3)	d _{ave} (mm) (4)	c ₁ (mm) (5)	c ₂ (mm) (6)	ρ_{topx} (%) (7)	ρ_{topy} (%) (8)	f _{y top} (MPa) (9)	f _c (MPa) (10)	V _u (kN) (11)	M _{ux} (kN-m) (12)	M _{uy} (kN-m) (13)
Falamaki and Loo (1992)	W5-A	100	83.0	200	200	0.35	0.35	570.0	26.8	32.6	4.8	9.8
	M5-A	100	82.2	200	200	0.62	0.62	672.0	34.0	34.4	9.4	14.0
	W5-C	100	84.7	400	200	0.52	0.41	570.0	26.8	37.3	8.3	12.0
	M5-C	100	81.0	400	300	0.62	0.57	672.0	34.0	38.4	12.4	17.2
Hammill and Ghali (1994)	NH1	150	114	250	250	1.85	1.85	440.0	41.5	146.9	43.0	43.0
	NH2	150	114	250	250	1.85	1.85	440.0	42.2	139.1	40.2	40.2
	NH4	150	114	250	250	1.85	1.85	440.0	46.6	0.0	33.0	33.0
Desayi and Seshadri (1997)	S101	100	80.0	100	100	0.53	0.53	720.0	36.0	32.6	6.2	6.2
	S201	100	80.0	100	100	0.80	0.80	720.0	36.0	47.0	9.0	9.0
	S102	100	80.0	100	100	0.53	0.53	720.0	24.8	52.2	7.6	7.6
	S202	100	80.0	100	100	0.80	0.80	720.0	27.2	40.7	5.9	5.9
	S302	100	80.0	100	100	1.07	1.07	720.0	22.4	42.8	6.2	6.2
Gardner and Zhou (1996)	8	140	120.0	254	254	0.61	0.61	460.0	21.5	81.0	10.5	10.5
Author	SC-H	150	122.0	900	180	1.07	1.16	525.6	38.7	115.7	116.4	96.5
	SC-HD	150	122.0	900	180	1.07	1.16	525.6	47.0	61.8	92.7	158.2
	SC-LD	150	122.0	900	180	1.07	1.16	525.6	38.7	44.8	88.5	116.7

Table 4.5. Analysis of Corner Specimens Data Using Eccentric Shear Model

Researchers (1)	Specimen (2)	M_{fx} (kN-m) (3)	M_{fy} (kN-m) (4)	V_u/V_c (5)	V_u/V_c (6)	$\gamma_f M_{ux}/M_{fx}$ (7)	$\gamma_f M_{uy}/M_{fy}$ (8)	Observed Failure (9)
Zaghlool, Paiva, and Glockner (1970)	I	20.3	20.3	1.44	1.50	0.27	0.25	Punching
	III	24.0	24.0	1.31	2.08	0.40	0.40	Punching
	IV	24.0	24.0	1.20	1.87	0.39	0.55	Punching
	Z-I (1)	24.3	24.3	0.68	1.51	0.47	0.47	Flexure+Punching
Zaghlool and Paiva (1973)	Z-II (1)	34.7	34.7	0.72	1.38	0.67	0.67	Flexure+Punching
	Z-II (2)	48.5	48.5	1.16	2.38	0.66	0.66	Flexure+Punching
	Z-II (3)	69.4	69.4	1.32	2.90	0.50	0.50	Flexure+Punching
	Z-II (6)	34.0	34.0	0.54	1.62	0.69	0.69	Flexure+Punching
	Z-III (1)	39.9	39.9	0.92	1.58	0.79	0.79	Flexure+Punching
Ingvarsson (1974)	1	27.9	27.9	1.48	2.79	0.45	0.45	Punching
	2	14.2	14.2	0.91	1.92	0.71	0.71	Flexure+Punching
	4	14.1	14.1	1.24	2.21	0.77	0.77	Punching
	5	21.3	21.3	1.41	2.76	0.62	0.62	Punching
Walker and Regan (1987)	SC1	14.3	14.3	0.53	1.09	1.06	1.06	Flexure+Punching
	SC2	9.0	9.0	0.46	0.98	1.60	1.60	Punching
	SC3	18.2	18.2	0.52	1.39	1.04	1.04	Flexure+Punching
	SC4	11.9	11.9	0.55	1.19	0.84	0.84	Flexure+Punching
	SC5	17.1	17.1	0.67	1.28	0.66	0.66	Punching
	SC7	17.1	17.1	0.69	1.82	0.97	0.97	Punching
	SC8	2.0	2.0	0.66	1.15	1.44	1.44	Flexure+Punching
	SC9	3.9	3.9	0.69	1.44	0.90	0.90	Flexure+Punching
	SC11	11.1	4.2	0.83	1.16	0.31	0.25	Punching
	SC12	10.9	4.3	0.46	0.97	1.23	0.70	Punching

Table 4.5. Cont'd

Researchers (1)	Specimen (2)	M _{fx} (kN-m) (3)	M _{fy} (kN-m) (4)	V _u /V _c (5)	V _u /V _c (6)	$\gamma M_{ux}/M_{fx}$ (7)	$\gamma M_{uy}/M_{fy}$ (8)	Observed Failure (9)
Falamaki and Loo (1992)	W5-A	4.8	4.8	0.47	0.98	0.60	1.22	Torsion+Punching
	M5-A	9.9	9.9	0.45	1.33	0.57	0.85	Punching
	W5-C	5.9	11.8	0.37	0.75	0.60	0.84	Torsion+Punching
	M5-C	11.2	15.0	0.31	0.72	0.57	0.80	Torsion+Punching
Hammill and Ghali (1994)	NH1	50.1	50.1	0.98	2.06	0.51	0.51	Punching
	NH2	50.1	50.1	0.92	1.92	0.48	0.48	Punching
	NH4	50.1	50.1	0.00	1.23	0.39	0.39	Punching
Desayi and Seshadri (1997)	S101	4.9	4.9	0.73	1.93	0.77	0.77	Flexure+Punching
	S201	7.4	7.4	1.05	2.78	0.73	0.73	Flexure+Punching
	S102	4.9	4.9	1.40	2.94	0.93	0.93	Punching
	S202	7.4	7.4	1.05	2.19	0.48	0.48	Punching
	S302	9.9	9.9	1.21	2.53	0.38	0.38	Punching
Gardner and Zhou (1996)	8	18.7	18.7	0.70	0.78	0.34	0.34	Flexure+Punching
Author	SC-H	98.0	34.0	0.54	2.20	0.89	1.22	Punching
	SC-HD	98.0	34.0	0.26	2.29	0.71	2.00	Punching
	SC-LD	98.0	34.0	0.21	2.02	0.68	1.48	Punching

Fig. 4.1 Typical Corner Specimen

(a) Top Reinforcement

(b) Bottom Reinforcement

Fig. 4.2 Typical Slab Reinforcement

Fig. 4.3 Typical Stud Shear Reinforcement

(a) Column Short Side View

(b) Column Long Side View

Fig. 4.4 Elevation View of Corner Specimen Setup

Fig. 4.5 Edge Links Supports

Fig. 4.6 Bottom Column Support

Fig. 4.7 Horizontal Hydraulic Jack

Fig. 4.8 Torsion Frame

Fig. 4.9 Strain Gauges Locations

Fig. 4.10 Strain Gauges on Shear Studs

Fig. 4.11 Prestressing Hydraulic Jacks

Fig. 4.12 Lateral Drift Sequence

Fig. 4.13 Lateral Load Stage and Sign Convention

Fig. 4.14 Punching Shear Failure of SC-H

Fig. 4.15 Punching Shear Failure of SC-HD

(a) Top View

(b) Bottom View

Fig. 4.16 Combination of Flexural and Compression Strut Failures of SC-HDS

Fig. 4.17 Punching Shear Failure of SC-LD

Fig. 4.18 Combination of Flexural and Compression Strut Failures of SC-LDS

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.19 Hysteretic Loops of SC-HD during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.20 Hysteretic Loops of SC-HD during Second Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.21 Hysteretic Loops of SC-HDS during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.22 Hysteretic Loops of SC-HDS during Second Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.23 Hysteretic Loops of SC-LD during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.24 Hysteretic Loops of SC-LD during Second Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.25 Hysteretic Loops of SC-LDS during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 4.26 Hysteretic Loops of SC-LDS during Second Cycle

(a) SC-HD and SC-HDS

(b) SC-LD and SC-LDS

Fig. 4.27 Unbalanced Moment Envelope about Y-axis

(a) SC-HD and SC-HDS

(b) SC-LD and SC-LDS

Fig. 4.28 Unbalanced Moment Envelope about X-axis

Fig. 4.29 Unbalanced Moment Envelope about Y-axis for All Specimens

Fig. 4.30 Unbalanced Moment Envelope about X-axis for All Specimens

Fig. 4.31 Definition of Ductility

Fig. 4.32 Definition of Stiffness

(a) X-Direction Drift

(b) Y-Direction Drift

Fig. 4.33 Stiffness of SC-HD and SC-LD

(a) X-Direction Drift

(b) Y-Direction Drift

Fig. 4.34 Stiffness of SC-HD and SC-HDS

(a) X-Direction Drift

(b) Y-Direction Drift

Fig. 4.35 Stiffness of SC-LD and SC-LDS

Fig.4.36 Correlation between Gravity Shear Force and Drift Capacity

Fig. 4.37 Moment Ratio vs. Shear Stress Ratio

Fig. 4.38 Shear Stress Ratio vs. Gravity Shear Force

CHAPTER 5

SLAB-COLUMN CONNECTIONS WITH 135-DEGREE SLABS SUBJECTED TO BIDIRECTIONAL LATERAL LOADING

5.1 INTRODUCTION

Following the first series of experiments consisting of five corner slab-column connection specimens presented in Chapter 4, four slab-column connections with 135-degree slabs were tested in the second series of experiments. The parameters involved in the second series of experiments were the effect of gravity load magnitude, bidirectional lateral loading, and the presence of stud shear reinforcement (SSR). All four specimens were tested under combined gravity and bidirectional lateral loads resulting in punching shear failures of two specimens without SSR and ductile failures of two specimens with SSR. All four specimens were modeled after a corner slab-column connection with 135-degree slab portion that is often found in modern flat plate structures. The experimental results should provide useful data on the behavior of flat plate structures with irregular column layouts. The experimental results are also used to verify the model to obtain the unbalanced moments presented in Chapter 7. This chapter presents the experimental results of the second series of experiments in term of drift capacity, stiffness, and ductility. The verification of the suitability of eccentric shear model by the ACI code for 135-degree slab-column connections is also presented.

5.2 SPECIMENS DETAIL

All 135-degree specimens were modeled after a corner slab-column connection with 135-degree slab portion cut off from a modern flat plate structure. The boundaries of the specimen edges are shown by the shaded area in Figure 5.1. These boundaries are actually the midspan of slab panel. They are assumed to have zero deflection under lateral load. Figure 5.1 also shows the critical section for punching shear calculation at $d/2$ from the column face. Figure 5.2 shows the typical 135-degree specimen with 150 mm slab thickness and column dimensions of

900 mm x 180 mm. The column height is 2.13 m. The slab dimensions are 2.34 m by 5.1 m. A series of edge links that support the slab edges is shown in Figure 5.3. The prototype building was assumed to be subjected to self-weight, imposed dead load of 1 kPa, and service live load of 4 kPa. With the design concrete strength f'_c equal to 40 MPa, the load combination results in the design value of high gravity shear ratio with V_u/V_c equal to 0.29. The actual concrete strength of each specimen is shown in Table 5.1 and Appendix 2. Assumption of reduced live load equal to 30% of the design live load for a building under combined gravity and lateral loads results in the design value of low gravity shear ratio with V_u/V_c equal to 0.22. The actual gravity shear force ratio V_u/V_c of each specimen based on its actual concrete strength is shown in Table 5.1 ($f'_c=47.04$ MPa leads to $V_u/V_c=0.19$ for SA-LD, $f'_c=38.74$ MPa leads to $V_u/V_c=0.27$ for SA-HD, $f'_c=47.62$ MPa leads to $V_u/V_c=0.19$ for SA-LDS, and $f'_c=37.56$ MPa leads to $V_u/V_c=0.28$ for SA-HDS).

Typical slab reinforcement layout is shown in Figure 5.3. The slab was reinforced with column strips reinforcement ratios equal to 1.07% and 1.13% parallel to x and y axes, respectively. The outermost top and bottom reinforcements were placed parallel to x -axis. At least two bottom steel bars in both directions were placed through the column to support the slab after punching shear failure. Clear slab concrete cover of 15 mm resulted in the average effective depth of slab d_{ave} of 122mm. T13 deformed steel bars with 13mm diameter and T10 deformed steel bars with 10 mm diameter were used for top and bottom reinforcements, respectively. All top and bottom steel bars were anchored using 180-degree hooks at the edge of the slab. The yield stresses f_y for T13 and T10 steel bars are 525.6 MPa and 521.6 MPa, respectively. The Young's moduli E_s for T13 and T10 steel bars are 197.2 GPa and 192.2 GPa, respectively. The yield stresses and the Young's moduli for T13 and T10 steel bars were obtained from the laboratory test. The column was designed to remain elastic. It was reinforced with 16T20 longitudinal steel bars or 3.1% of the column gross section area.

Stud Shear Reinforcement (SSR) with total height of 150mm, 10mm stem and 30mm diameter head was placed in two specimens to investigate the benefit of using SSR. Typical stud rail and its arrangement around the column according to the recommendation of the ACI Committee 421 are shown in Figure 5.4. One stud rail consists of five studs with the distance between column face and the first stud equal to 40 mm ($0.33 d_{ave}$) and subsequent distance between studs equal to 90 mm ($0.74 d_{ave}$).

Table 5.1 shows the properties of all specimens. Specimen SA-LD and SA-HD were subjected to bidirectional lateral drift combined with low and high gravity shear ratios, respectively. Specimens with SSR, SA-LDS and SA-HDS, were subjected to bidirectional lateral drift combined with low and high gravity shear ratios, respectively. The actual gravity shear force ratios V_u / V_c of specimens SA-LD and SA-HD are 0.19 and 0.27, respectively. The actual gravity shear force ratios V_u / V_c of specimens SA-LDS and SA-HDS are 0.19 and 0.28, respectively. These values were obtained from the actual concrete strength of each specimen.

5.3 EXPERIMENTAL SETUP AND INSTRUMENTATION

Typical experimental setup of 135-degree specimens shown in Figure 5.5 is very similar to that of the corner specimens described in Section 4.3. Series of roller supports in the form of edge links simulated the slab inflection point. These supports allowed translation and rotation about both x and y axes with negligible vertical displacement. Combination between steel rocker and reaction arms at the bottom of the column allowed rotation about both x and y axes without any horizontal translation. Applied gravity load was simulated by the vertical hydraulic jack and steel blocks placed on the slab. Two horizontal hydraulic jacks simulated bidirectional lateral load. These horizontal hydraulic jacks were used to apply lateral displacement along one axis as well as lateral displacements of the same magnitude along two axes simultaneously. Torsion-restraining frame was used to prevent rigid body rotation of the slab under lateral load.

Applied gravity load, lateral load, and pre-stressing forces were measured by five load cells. Vertical deflections and lateral drifts were measured by eight

LVDTs and nine wire transducers, respectively. Figure 5.6 shows a total of 61 strain gauges that were installed on the top and bottom steel bars on each specimen. Figure 5.7 shows 56 additional strain gauges that were installed on the shear studs.

5.4 SEQUENCE OF LOADING

The loading sequence for corner specimens described in Chapter 4 was also used for 135-degree specimens. Before applying the gravity load, the column was pre-stressed with an axial force of 210 kN that represented axial force of the column using prestressing hydraulic jacks. Specimen self weight was simulated by jacking up the column until the specimen self weight was carried by the steel rocker. Gravity load equal to the specific value of low or high gravity shear force was then applied incrementally using the vertical jack to obtain the strain profiles and the crack progress. After reaching the specific gravity shear force ($V_u/V_c=0.19$ for specimens SA-LD and SA-LDS, $V_u/V_c=0.27$ for specimen SA-HD, and $V_u/V_c=0.28$ for specimen SA-HDS), displacement-based bidirectional lateral load was then applied. These gravity shear forces were maintained constant during lateral load test.

Figure 5.8 shows the displacement-based lateral load sequence and target drift value. The lateral displacement sequence starts at 0% to 1% lateral drift with the incremental drift of 0.25%. For 1% to 3% lateral drifts, the incremental drift is increased to 0.5%. After reaching 3% lateral drift, the incremental drift is increased to 1%. Each lateral drift target is performed in complete two cycles. For 1.5% to 3% lateral drifts, one complete cycle of 1% lateral drift is performed before applying subsequent lateral drift target to observe the serviceability of the connection. Figure 5.9 shows 16 loading stages and the sign convention for lateral drift, moment about x-axis M_{ux} and moment about y-axis M_{uy} . For all specimens, gravity load alone automatically induces unbalanced moments.

5.5 GENERAL SPECIMEN BEHAVIOR

Specimens SA-HD (high gravity) and SA-HDS (high gravity with SSR) experienced visible flexural cracks near the column after high gravity shear force was applied. The flexural cracks continued to propagate under further loading to the middle of the slab and toward the edge links. Torsion crack also occurred on the slab edge along the column long side. Punching shear crack on the top surface of the slab of specimen SA-HD started to occur at loading stage 2 during the first cycle of 2% target drift. It kept propagating with further loading and caused complete punching shear failure at loading stage 14 during the second cycle of 2% target drift. Significant drop of unbalanced moments occurred at loading stages 9 and 14 during second cycle of 2% target drift. Although punching shear crack only started to occur during first cycle of 2% target drift, the peak unbalanced moments and the corresponding drifts were $M_{ux} = -149.1$ kN-m at -1.49% y -direction drift and $M_{uy} = -153.6$ kN-m at -1.51% x -direction drift. It indicated that punching shear cracks started to develop during 1.5% target drift although significant drop of unbalanced moments only occurred during 2% target drift. Figure 5.10 shows punching shear failure of specimen SA-HD.

Specimen SA-HDS experienced failure combined with compression strut failure marked by severe concrete crushing on the bottom of the slab near the column as shown in Figure 5.11. The crushing of concrete started to occur at loading stage 2 during second cycle of 2% target drift. Yielding of top steel reinforcement along x -axis across the width of $c+1.5h$ occurred at loading stage 9 during the first cycle of 1.5% x -direction drift. The compression strut action focused at the bottom of the slab at the column face was induced by the shear force. The shear studs could prevent the diagonal shear cracking, but they could not prevent the compression strut failure. The crushing of concrete at the bottom of the slab at the face of the column was caused by the combination of flexural compression and shear strut compression. The unbalanced moments dropped gradually until immense concrete crushing occurred at loading stage 9 during second cycle of 4% target drift. The test was stopped after completing second cycle of 4% target drift since horizontal hydraulic jacks were out of stroke. The peak unbalanced moments and the

corresponding drifts were $M_{ux} = -149.5$ kN-m at -2.02% y -direction drift and $M_{uy} = -172.5$ kN-m at -2.51% x -direction drift.

Specimens SA-LD (low gravity) and SA-LDS (low gravity with SSR) only experienced one fine flexural crack on the top surface of the slab near the column after low gravity force was applied. Flexural cracks kept propagating under further loading on the slab area near the column and some of them extended to the middle of the slab and towards the edge links. Torsion cracks occurred on the slab edge along the column long side. Specimen SA-LD failed in punching shear as shown in Figure 5.12. The failure started to occur at loading stage 14 during second cycle of 2.0% target drift. At loading stage 2 during first cycle of 2.5% drift, sudden drop of unbalanced moment occurred and punching shear crack then fully developed after loading stage 14 during first cycle of 2.5% drift. The peak unbalanced moments and the corresponding drifts were $M_{ux} = -132.6$ kN-m at -2.02% y -direction drift and $M_{uy} = -135.9$ kN-m at -1.99% x -direction drift.

Specimen SA-LDS experienced flexural failure combined with compression strut failure as shown in Figure 5.13 marked by severe concrete crushing on the bottom of the slab near the column. The crushing of concrete started to occur at loading stage 2 during second cycle of 3% drift. Yielding of top steel reinforcement along x -axis across the width of $c+1.5h$ occurred at loading stage 9 during the first cycle of 1.5% x -direction drift. Unbalanced moments decreased gradually until loading stage 14 during second cycle of 4% drift followed by severe concrete crushing on the bottom slab near the column. The test was stopped since two horizontal hydraulic jacks were out of stroke. The peak unbalanced moments and the corresponding drifts were $M_{ux} = -134.6$ kN-m at -2.53% y -direction drift and $M_{uy} = -151.2$ kN-m at -2.98% x -direction drift.

Figures 5.14 – 5.21 show the hysteretic loops of the unbalanced moments during the first and second cycles of applied lateral drifts of each specimen. Figures 5.22 – 5.25 show the unbalanced moment envelopes of each specimen showing sudden or gradual drops of unbalanced moments due to connections failures.

5.6 EXPERIMENTAL RESULTS

The experimental results of 135-degree specimens are presented in term of drift capacity, ductility, and stiffness. Drift capacity D_p is defined as the drift ratio at the maximum unbalanced moment M_u . Figure 4.31 shows the definition of ductility that is adopted in Chapters 4 and 5. Ductility μ_p is defined as the ratio between the drift ratio at peak unbalanced moment D_p to the drift ratio at first yield of slab reinforcement D_y . It can be expressed as $\mu_p = D_p / D_y$. Due to limited strokes of the horizontal hydraulic jacks, drift ratio at $0.8 M_u$ or D_u of specimens with SSR could not be obtained.

Stiffness is defined as the ratio between peak-to-peak values of unbalanced moments and the drift ratio obtained from the hysteretic loop as shown in Figure 4.32. Two cases of stiffness obtained from the loading stages shown in Figure 5.9 are presented. The first case represents unidirectional lateral load where only one direction of lateral drift is applied in addition to applied gravity load. This case consists of the stiffness obtained from loading stages 1 and 9 for moment about y -axis M_{uy} and the stiffness obtained from loading stages 5 and 13 for moment about x -axis M_{ux} . The second case represents bidirectional lateral load where lateral drifts are applied in two directions in addition to applied gravity load. This case consists of the stiffness obtained from loading stages 6 and 14 for M_{uy} and the stiffness obtained from loading stages 2 and 10 for M_{ux} .

5.6.1 Effects of Bidirectional Lateral Load

The vertical segments in the hysteretic loops shown in Figures 5.14-5.21 show the effect of bidirectional lateral load on unbalanced moment capacity. They indicate drop or increase in unbalanced moments at some loading stages when the lateral drift in the other direction is applied or released. Taking the example of specimen of SA-HD from Figure 5.14, at 1.5% lateral drift M_{uy} drops from $M_{uy} = -145.7$ kN-m at loading stage 9, where only x -direction drift lateral is applied, to $M_{uy} = -135.2$ kN-m at loading stage 10, where x -direction drift from point 9 is

maintained and y -direction drift is applied subsequently. M_{ux} drops from $M_{ux} = -149.1$ kN-m at loading stage 2, where x and y directions drifts are applied, to $M_{ux} = -130.8$ kN-m at loading stage 3, where y -direction drift is maintained and x -direction drift is released. It shows that reduced slab stiffness is caused by the lateral drifts in both directions.

Both stiffness cases from unidirectional and bidirectional lateral loads are shown in Figures 5.26 – 5.28. For x -direction drift, comparison is made between the stiffness of loading stages 1-9 shown in Figure 5.9 that represent unidirectional lateral load and loading stages 6-14 that represent bidirectional lateral load. Comparison for y -direction drift includes the stiffness of unidirectional lateral load from loading stages 5-13 and the stiffness of bidirectional lateral load from loading stages 2-10. Taking the example of specimens SA-LD and SA-HD, Figure 5.26 shows that the stiffness from unidirectional lateral load (loading stages 1-9) is almost always higher than the stiffness from bidirectional lateral load (loading stages 6-14) for x -direction drift. It shows that bidirectional lateral load reduces the stiffness on x -direction drift. However, the stiffness from unidirectional lateral load (loading stages 5-13) is almost always lower than the stiffness from bidirectional lateral load (loading stages 2-10) for y -direction drift. It is caused by the maximum unbalanced moment about x -axis M_{ux} that mostly occurs at loading stage 2.

5.6.2 Effects of Gravity Load Magnitude

Table 5.2 shows that gravity shear force reduces the ultimate drift capacity in both axes. Specimen SA-HD (high gravity) with $V_u/V_c=0.27$ has lower ultimate drift capacity compared to specimen SA-LD (low gravity) with $V_u/V_c=0.19$. Specimen SA-HD has drift capacity in x -axis $D_{px}=-1.51\%$ and drift capacity in y -axis $D_{py}=-1.49\%$. Specimen SA-LD has higher drift capacities with $D_{px}=-1.99\%$ and $D_{py}=-2.02\%$. Specimen SA-LDS (low gravity, $V_u/V_c=0.19$ with SSR) has higher ultimate drift capacity than specimen SA-HDS (high gravity, $V_u/V_c=0.28$ with SSR). Specimen SA-LDS has $D_{px}=-2.98\%$ and $D_{py}=-2.53\%$ that are higher than specimen SA-HDS with $D_{px}=-2.51\%$ and $D_{py}=-2.02\%$.

Table 5.2 shows that although the specimens with lower gravity shear force have higher ultimate drift capacity, the maximum unbalanced moments that those specimens can resist are lower compared to the specimens with higher gravity shear force. Specimen SA-HD has $M_{uy}=-153.6$ kN-m and $M_{ux}=-149.1$ kN-m that are higher than specimen SA-LD with $M_{uy}=-135.9$ kN-m and $M_{ux}=-132.6$ kN-m. Specimen SA-HDS has $M_{uy}=-172.5$ kN-m and $M_{ux}=-149.5$ kN-m that are also higher than specimen SA-LDS with $M_{uy}=-151.2$ kN-m and $M_{ux}=-134.6$ kN-m. It is caused by the axial gravity load that is applied through the center of the column that counteracts the negative unbalanced moments arising from lateral loads. It also indicates that taking the centroid of critical section as the center of the shear force and unbalanced moments is correct.

Table 5.2 also shows that higher gravity shear force reduces ductility. Ductility of specimen SA-HD (high gravity) is always lower compared to that of specimen SA-LD (low gravity). Specimen SA-LD has ductility in x -axis $\mu_{px}=1.92$ and ductility in y -axis $\mu_{py}=2.26$ that are higher than specimen SA-HD with $\mu_{px}=1.56$ and $\mu_{py}=2.05$. Specimen SA-LDS (low gravity with SSR) has $\mu_{px}=1.98$ and $\mu_{py}=2.47$ that are also higher than specimen SA-HDS (high gravity with SSR) with $\mu_{px}=1.91$ and $\mu_{py}=2.35$. Higher ductility of the specimens subjected to low gravity shear force is contributed by higher drift capacity of those specimens.

Figures 5.26-5.28 show that although specimens with low gravity shear force can resist higher ultimate drift capacity, the stiffness of those specimens is not higher than the specimens with high gravity shear force. The stiffness of specimen SA-HD is higher in overall compared to the stiffness of specimen SA-LD for both cases of stiffness. Larger stiffness reduction due to higher gravity load is still apparent in Figure 5.26 shown by the steeper stiffness curve of specimen SA-HD compared to the stiffness curve of specimen SA-LD. The steeper stiffness curve shows more rapid stiffness degradation due to more immense cracking.

Table 5.2 shows that specimens SA-HD and SA-LD, which were subjected to bidirectional lateral load, can withstand target lateral drifts of 1.5% and 2%,

respectively. Based on these experimental data, the approximate maximum limit of gravity shear force ratio V_u/V_c of 135-degree slab-column connections with rectangular columns to withstand 1.5% drift ratio is around 0.25 to 0.30.

5.6.3 Effects of the Use of Stud Shear Reinforcement (SSR)

Table 5.2 shows that the presence of shear studs increases the unbalanced moment and ultimate drift capacity. Maximum unbalanced moment and drift capacity of specimen SA-HDS (high gravity with SSR) are higher than specimen SA-HD (high gravity). There are increases of 66% from -1.51% to -2.51% for drift capacity in x -axis D_{px} and 36% from -1.49% to -2.02% for drift capacity in y -axis D_{py} . There is also an increase of 12% for M_{uy} from -153.6 kN-m to -172.5 kN-m, meanwhile there is a slight increase in M_{ux} . Specimen SA-LDS (low gravity with SSR) has $D_{px}=-2.98\%$ and $D_{py}=-2.53\%$ that are higher than specimen SA-LD (low gravity) with $D_{px}=-1.99\%$ and $D_{py}=-2.02\%$. There are increases of 50% and 25% for D_{px} and D_{py} , respectively. There is an increase of 11% for M_{uy} from -135.9 kN-m to -151.2 kN-m, meanwhile there is only a slight increase in M_{ux} .

Table 5.2 also shows that the presence of shear studs improves ductility. Specimen SA-LDS (low gravity with SSR) has ductility in x -axis $\mu_{px}=1.98$ and ductility in y -axis $\mu_{py}=2.47$ that are higher than specimen SA-LD (low gravity) with $\mu_{px}=1.92$ and $\mu_{py}=2.26$. There are increases of 3% and 9% for μ_{px} and μ_{py} , respectively. Comparing specimens SA-HDS (high gravity with SSR) and SA-HD (high gravity), there are also increases of 22% and 15% for μ_{px} and μ_{py} , respectively. Specimen SA-HDS has $\mu_{px}=1.91$ and $\mu_{py}=2.35$ that are higher than specimen SA-HD with $\mu_{px}=1.56$ and $\mu_{py}=2.05$.

Figures 5.26-5.28 show that the presence of shear studs does not significantly increase the connection stiffness. It is shown that the stiffness curves of specimens SA-HDS and SA-HD almost coincide. Comparison between the

stiffness curves of specimens SA-LDS and SA-LD also indicates the same result. However, specimen SA-HD experiences significant loss of stiffness showed by the steeper stiffness curve approximately after reaching 2% lateral drift. Meanwhile, the stiffness curve of specimen SA-HDS does not show any significant loss of stiffness after reaching 2% lateral drift. Instead, it only shows gradual stiffness loss due to combination of flexural failure and compression strut failure up to 4% lateral drift. It shows that the presence of shear studs can avoid significant loss of stiffness after reaching critical lateral drift.

5.7 ECCENTRIC SHEAR MODEL BY ACI 318

Eccentric shear model was used to analyze the experimental results of 135-degree specimens. The shear stress ratios v_u / v_c of all specimens are shown in Column 12 of Table 5.3. The unbalanced moments are taken from the combination between unbalanced moment about x -axis M_{ux} and unbalanced moment about y -axis M_{uy} at one loading stage that causes maximum v_u . Columns 13 and 14 of Table 5.3 show the ratio of unbalanced moment transferred by flexure and the moment capacity $\gamma_f M_u / M_f$ about both axes. The moment capacity M_f is obtained from the smallest dimension of $c + 1.5h$ and $c + c_t$ for moment about y -axis and from the smallest dimension of $c + 3h$ and $c + 2c_t$ for moment about x -axis, where c is the column dimension parallel to the moment axis, h is the slab thickness and c_t is the column dimension perpendicular to the moment axis. The maximum value from v_u / v_c , $\gamma_{fx} M_{ux} / M_{fx}$, and $\gamma_{fy} M_{uy} / M_{fy}$ determines the type of connection failure.

Table 5.3 shows that eccentric shear model produces conservative v_u / v_c equal to 1.73 and 2.14 for specimens with punching shear failure, SA-LD and SA-HD, respectively. For specimens with combination of flexural failure and compression strut failure, eccentric shear model produces conservative $\gamma_f M_u / M_f$ equal to 1.78 and 2.04 for specimens SA-LDS and SA-HDS, respectively. The eccentric shear model can also predict the types of connection failure accurately.

5.8 CONCLUSION

It can be concluded that higher gravity load leads to lower ultimate drift capacity and ductility. However, it does not reduce the unbalanced moment capacity due to counteracting moment caused by the axial load acting at the centroid of the critical section. Specimen SA-HD (high gravity) has drift capacity in x -axis $D_{px} = -1.51\%$ with $M_{uy} = -153.6$ kN-m and drift capacity in y -axis $D_{py} = -1.49\%$ with $M_{ux} = -149.1$ kN-m. The drift capacity and maximum unbalanced moment of specimen SA-LD (low gravity) are $D_{px} = -1.99\%$ with $M_{uy} = -135.9$ kN-m and $D_{py} = -2.02\%$ with $M_{ux} = -132.6$ kN-m.

The presence of stud shear reinforcement increases the maximum unbalanced moment, drift capacity, and ductility. Maximum unbalanced moment and drift capacity of specimen SA-HDS (high gravity with SSR) are higher than those of specimen SA-HD (high gravity). There are increases of 66% for drift capacity in x -axis D_{px} and 36% for drift capacity in y -axis D_{py} . There is also an increase of 12% for M_{uy} , meanwhile there is a slight increase in M_{ux} . Maximum unbalanced moment and drift capacity of specimen SA-LDS (low gravity with SSR) are also higher than those of specimen SA-LD (low gravity). There are increases of 50% and 25% for D_{px} and D_{py} , respectively. There is an increase of 11% for M_{uy} , meanwhile there is a slight increase in M_{ux} . The ductility of specimen SA-LDS (low gravity with SSR) is higher than that of specimen SA-LD (low gravity). There are increases of 3% and 9% for ductility in x -axis μ_{px} and ductility in y -axis μ_{py} , respectively. The ductility of specimen SA-HDS (high gravity with SSR) is also higher than that of specimen SA-HD (high gravity). There are increases of 22% and 15% for μ_{px} and μ_{py} , respectively. The presence of stud shear reinforcement also avoids significant loss of stiffness after reaching critical lateral drift. Specimen SA-HD (high gravity) experiences significant loss of stiffness showed by the steeper stiffness curve approximately after reaching 2% lateral drift as shown in Figure 5.27. Meanwhile, the stiffness curve of specimen SA-HDS (high gravity with SSR) does not show any significant loss of stiffness after reaching 2% lateral

drift. Instead, it only shows gradual stiffness loss due to combination of flexural failure and compression strut failure up to 4% lateral drift. Unidirectional and bidirectional lateral loads result in different reduced slab stiffness indicated by the stiffness of the connections. Taking the example of specimens SA-LD (low gravity) and SA-HD (high gravity), Figure 5.26 shows that the stiffness from unidirectional lateral load (loading stages 1-9) is almost always higher than the stiffness from bidirectional lateral load (loading stages 6-14) for x -direction drift.

Figure 2.2 indicates that interior slab-column connections without shear reinforcement may satisfy ultimate drift ratios of 1.5% and 2% when V_u / V_c does not exceed 0.40 and 0.32, respectively. To satisfy ultimate drift ratios of 1.5% and 2%, Figure 2.4 shows that V_u / V_c of edge slab-column connections should not exceed 0.50 and 0.45, respectively. There are currently limited data of corner connections that were tested under combination of gravity and lateral loads. From a total of 39 experimental data of corner slab-column connections without shear reinforcement that were presented in Section 4.7, only corner specimens with square columns by Hammill and Ghali (1997) and corner specimens with rectangular columns by the author were tested under combination of gravity and lateral loads. Specimens NH1 and NH2 (Hammill and Ghali, 1997) did not experience failure under combination of service gravity load $V_u / V_c = 0.29$ and 3% lateral drift. The lateral drift was set to zero after reaching 3% and the gravity load was increased until the specimens experienced punching shear failures. Based on the recommendation by ACI-ASCE Committee 352 (1989), the maximum shear force acting on all types of connections should not exceed $0.40 V_c$ to ensure 1.5% drift capacity.

The experimental results of corner specimens and 135-degree specimens are presented in Section 4.6.2 and Section 5.6.2, respectively. To withstand 1.5% and 2% drift ratios, the gravity shear force ratio V_u / V_c of corner connections with rectangular columns should be limited to 0.30 and 0.25, respectively. Based on limited experimental data, the maximum limit of gravity shear force ratio V_u / V_c of 135-degree specimens is around 0.25 to 0.30 to achieve 1.5% drift ratio. Anggadajaja and Teng (2008) recommended the maximum limit of gravity shear force ratio

$V_u / V_c = 0.30$ for rectangular edge slab-column connections to withstand 1.5% drift ratio. To withstand 1.5% drift ratio, Tan and Teng (2005) recommended the maximum limit of gravity shear force ratio $V_u / V_c = 0.28$ for rectangular interior slab-column connections. These limits of gravity shear force ratio V_u / V_c for rectangular slab-column connections under combination of gravity and bidirectional lateral loads are lower than ACI-ASCE Committee 352 recommendation of $V_u / V_c = 0.40$. It can be concluded that column rectangularity causes the drift capacity of the connection to be more severely affected by the gravity shear force ratio.

The eccentric shear model by the ACI 318 gives conservative results of shear stress ratio and it can also predict the types of failure correctly. The experimental results of all nine specimens in the form of unbalanced moment-drift ratio relationships presented in Chapters 4 and 5 are used to verify the proposed model of reduced slab stiffness for slabs presented in Chapter 7.

Table 5.1. 135-Degree Specimens Properties

Specimen	f'_c (MPa)	V_u/V_c	Lateral Load	h (mm)	d _{ave} (mm)	C ₁ (mm)	C ₂ (mm)	ρ_{topx} (%)	ρ_{topy} (%)	ρ_{botx} (%)	ρ_{boty} (%)	$f_{y\ top}$ (MPa)	$f_{y\ bot}$ (MPa)	Shear Studs
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
SA-LD	47.04	0.19	Bidirectional	150	122	900	180	1.07	1.13	0.64	0.43	525.6	521.6	-
SA-HD	38.74	0.27	Bidirectional	150	122	900	180	1.07	1.13	0.64	0.43	525.6	521.6	-
SA-LDS	47.62	0.19	Bidirectional	150	122	900	180	1.07	1.13	0.64	0.43	525.6	521.6	5 studs/rail
SA-HDS	37.56	0.28	Bidirectional	150	122	900	180	1.07	1.13	0.64	0.43	525.6	521.6	5 studs/rail

Table 5.2. Experimental Results of 135-Degree Specimens

Specimen	f'_c (MPa)	V_u/V_c	V_u (kN)	Maximum Unbalanced Moments						Ductility		Observed Failure
				Loading Stage	D_{px} (%)	M_{uy} (kN-m)	Loading Stage	D_{py} (%)	M_{ux} (kN-m)	μ_{px}	μ_{py}	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
SA-LD	47.04	0.19	52.84	9	-1.99	-135.9	2	-2.02	-132.6	1.92	2.26	Punching
SA-HD	38.74	0.27	69.84	14	-1.51	-153.6	2	-1.49	-149.1	1.56	2.05	Punching
SA-LDS	47.62	0.19	52.84	14	-2.98	-151.2	2	-2.53	-134.6	1.98	2.47	Flexure + Compression Strut
SA-HDS	37.56	0.28	69.84	10	-2.51	-172.5	2	-2.02	-149.5	1.91	2.35	Flexure + Compression Strut

Table 5.3. Analysis of the Experimental Results of 135-Degree Specimens

Specimen	f'_c (MPa)	V_u/V_c	V_u (kN)	Max. Unbalanced Moments				Max. Shear Stress				V_u/V_c	$\gamma_f M_{uy}/M_{fy}$	$\gamma_f M_{ux}/M_{fx}$
				Loading Stage	M _{uy} (kN-m)	Loading Stage	M _{ux} (kN-m)	Loading Stage	M _{uy} (kN-m)	M _{ux} (kN-m)				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
SA-LD	47.04	0.19	52.84	9	-135.9	2	-132.6	14	-127.4	-126.7	1.73	1.50	0.88	
SA-HD	38.74	0.27	69.84	14	-153.6	2	-149.1	14	-153.6	-124.2	2.14	1.81	0.86	
SA-LDS	47.62	0.19	52.84	14	-151.2	2	-134.6	10	-124.4	-9.9	$V_u/V_n=0.34$		1.78	0.93
SA-HDS	37.56	0.28	69.84	10	-172.5	2	-149.5	10	-172.5	-31.1	$V_u/V_n=0.38$		2.04	1.04

Fig. 5.1 Prototype Building

Fig. 5.2 Typical 135-Degree Specimens

(a) Top Reinforcement

(b) Bottom Reinforcement

Fig. 5.3 Typical Slab Reinforcement

Fig. 5.4 Typical Stud Shear Reinforcement

(a) Column Short Side View

(b) Column Long Side View

Fig. 5.5 Elevation View of 135-Degree Specimen Setup

Fig. 5.6 Strain Gauges Locations

Fig. 5.7 Strain Gauges on Shear Studs

Fig.5.8 Lateral Drift Sequence

Fig.5.9 Lateral Load Stage and Sign Convention

Fig.5.10 Punching Shear Failure of SA-HD

Fig.5.11 Combination of Flexural and Compression Strut Failures of SA-HDS

Fig.5.12 Punching Shear Failure of SA-LD

Fig.5.13 Combination of Flexural and Compression Strut Failures of SA-LDS

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.14 Hysteretic Loops of SA-HD during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.15 Hysteretic Loops of SA-HD during Second Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.16 Hysteretic Loops of SA-HDS during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.17 Hysteretic Loops of SA-HDS during Second Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.18 Hysteretic Loops of SA-LD during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.19 Hysteretic Loops of SA-LD during Second Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.20 Hysteretic Loops of SA-LDS during First Cycle

(a) Moment about Y-axis

(b) Moment about X-axis

Fig. 5.21 Hysteretic Loops of SA-LDS during Second Cycle

(a) SA-HD and SA-HDS

(b) SA-LD and SA-LDS

Fig.5.22 Unbalanced Moment Envelope about Y-axis

(a) SA-HD and SA-HDS

(b) SA-LD and SA-LDS

Fig. 5.23 Unbalanced Moment Envelope about X-axis

Fig.5.24 Unbalanced Moment Envelope about Y-axis for All Specimens

Fig. 5.25 Unbalanced Moment Envelope about X-axis for All Specimens

(a) X-Direction Drift

(b) Y-Direction Drift

Fig. 5.26 Stiffness of SA-HD and SA-LD

(a) X-Direction Drift

(b) Y-Direction Drift

Fig. 5.27 Stiffness of SA-HD and SA-HDS

(a) X-Direction Drift

(b) Y-Direction Drift

Fig.5.28 Stiffness of SA-LD and SA-LDS

CHAPTER 6

CRACKED STIFFNESS OF FLEXURAL MEMBERS

6.1 INTRODUCTION

The accuracy of reduced slab stiffness computation is very important in order to obtain the unbalanced moments for slab-column connections correctly. The basis of the proposed model of reduced slab stiffness for slabs presented in Chapter 7 is to take into account the effect of reduced stiffness of cracked slabs through the modified effective moment of inertia method (Branson, 1963 and Teng and Branson, 1993). This chapter presents the proposed modified effective moment of inertia method for flexural members and the verification using the experimental data of beams deflections. Precise calculation of deflection can be obtained by using the actual stress-strain relationship of the materials: concrete and steel, and the interaction between the steel and concrete. These stress-strain equations must also take into account the contributions of concrete in tension and the behavior of steel embedded in concrete. The accurate procedure involves the integration of curvatures over the span of the beam which is suitable for computer program. It is also possible to use a simpler method and still accommodate some parameters representing the interactions between concrete and steel through the use of the effective moment of inertia. The use of the effective moment of inertia ensures simplicity and, with appropriate use of parameters, can lead to accurate prediction. This chapter presents two approaches: one deals with numerical integration of curvatures where the yield stress of steel embedded in concrete is differentiated from that of bare steel bars, and the second is the proposed effective moment of inertia based method where the effects of various parameters, such as; curvatures at yield and at ultimate are taken into consideration.

For deflection calculation, the stress-strain equations chosen for sectional analysis must take into account the overall behavior of the concrete in tension and compression and the behavior of reinforcing steel embedded in concrete. For concrete in compression, the available constitutive equation is generally able to well represent the behavior of concrete especially for the ascending branch. In tension,

tension stiffening effect of cracked concrete is an important factor in the deflection calculation since underestimating or overestimating it may lead to inaccurate deflection predictions. The stress-strain equation for concrete in tension must also be able to represent tension stiffening effect accurately. The behavior of reinforcing steel embedded in concrete is different from that of bare steel bars. It has been known that reinforcing steel embedded in concrete will have lower yield strength as shown by Belarbi and Hsu (1994). All those factors are described individually in this chapter.

6.2 STRESS-STRAIN EQUATION FOR CONCRETE IN COMPRESSION

The stress-strain equation adopted in this chapter for concrete in compression is based on the general equation derived by Saenz (1964). The equation is a generalized form of a simple equation proposed by Desayi and Krishnan (1964). Saenz pointed out that the generalized form provided a better prediction of the ascending branch of the stress-strain curve compared to the simple equation. The general form of the equation can be written as:

$$\sigma_c = \frac{E_c \varepsilon_c}{1 + (N - 2) \left(\frac{\varepsilon_c}{\varepsilon_{cp}} \right) + \left(\frac{\varepsilon_c}{\varepsilon_{cp}} \right)^2} \quad (6.1)$$

where

$$N = \frac{E_c \varepsilon_{cp}}{f'_c} \quad (6.2)$$

and

σ_c	=	stress of concrete in compression
ε_c	=	strain of concrete in compression
E_c	=	Young's elastic modulus of concrete
ε_{cp}	=	strain at peak stress
f'_c	=	concrete compression strength

If N is taken as constant and equal to 2, the general equation is reduced to the simple equation proposed by Desayi and Krishnan as follows:

$$\sigma_c = \frac{E_c \varepsilon_c}{1 + \left(\frac{\varepsilon_c}{\varepsilon_{cp}} \right)^2} \quad (6.3)$$

Saenz pointed out that N value varied with concrete strength. It could vary from near 4 for normal strength concrete of 1000 psi (6.9 MPa) to about 1.3 for concrete strength of 10,000 psi (69 MPa). When N was taken as constant and equal to 2, the available experimental data could not be represented well by Equation (6.1). For those data, it was shown by Saenz that the value of $N=2$ was too low for lower strength concrete and too high for higher strength concrete. It was shown by Saenz that the value of N would be correct with approximate values of ε_{cp} and E_c as follows:

$$\varepsilon_{cp} = 10^{-5} f_c'^{0.25} (31.5 - f_c'^{0.25}) \quad (6.4)$$

$$E_c = \left(\frac{10^5 f_c'^{0.5}}{1 + 0.006 f_c'^{0.5}} \right) \quad (6.5)$$

where E_c and f_c' are in psi.

6.3 STRESS-STRAIN CURVE FOR CONCRETE IN TENSION

For concrete in tension, stress-strain curve shown in Figure 6.1 introduced by Okamura et al. (1985) is adopted. This constitutive relation was later verified by Tamai et al. (1998) based on an experimental program on uniaxial tensile tests conducted at the University of Tokyo. The equations can be written as follows:

$$\text{For } \varepsilon_c \leq \varepsilon_{tp}; \sigma_c = E_c \varepsilon_c \quad (6.6)$$

$$\text{For } \varepsilon_c > \varepsilon_{tp}; \sigma_c = f_t \left(\frac{\varepsilon_{tp}}{\varepsilon_c} \right)^c \quad (6.7)$$

where:

ε_{tp} = strain at peak tensile stress

f_t = tensile strength of concrete

c = constant depending on bond characteristic, 0.4 for deformed bars

Based on 17 full-size reinforced concrete panels that they tested, Belarbi and Hsu (1994) suggested the value of f_t as follows:

$$f_t = 3.75\sqrt{f'_c} \quad (6.8)$$

where f'_c is in psi.

ACI 318 uses the same expression with the constant 4 for the shear cracking in beams. Belarbi and Hsu noted that the lower coefficient in Equation (6.8) determined from the test was probably due to relatively large dimensions of the test panels. Based on those 17 panels, the average value of ε_{tp} was found to be about 0.00008. Belarbi and Hsu also showed that the constant $c=0.4$ was the best fit for all 17 panels prior to the yielding of steel.

6.4 STRESS-STRAIN EQUATION FOR REINFORCING BAR EMBEDDED IN CONCRETE

The behavior of reinforcing bar surrounded by concrete is different from that of bare steel bar. The stress-strain curve of bare reinforcing bar is normally assumed to be elastic-perfectly plastic with a yield stress value of f_y . However, when the reinforcing bar is embedded in concrete, the stress-strain relationship becomes different due to the reduction of yield stress. Belarbi and Hsu (1994) suggested that lower value of yield stress of reinforcing bar embedded in concrete, which is called apparent yield stress f_y^* , could be expressed as:

$$\frac{f_y^*}{f_y} = 1 - 1.314 \frac{(E_s / E_c)^{0.434}}{\rho^{1.084}} \left(\frac{f_t}{f_y} \right)^{1.517} \quad (6.9)$$

where:

E_s = Young's elastic modulus of steel

ρ = reinforcement ratio

f_y = yield strength of bare steel

f_t = tensile strength of concrete

Belarbi and Hsu (1994) showed that the tensile stress-strain curve of steel embedded in concrete could be modeled by the bilinear curve shown in Figure 6.2. The first straight line represents the elastic slope E_s . The second line represents the plastic range with a slope of E_p^* . These two lines intersect at a stress level of f_n^* .

Belarbi and Hsu found that the values of E_p^* and f_n^* mainly depended on the apparent yield stress f_y^* and suggested the following relationship between the stress of the reinforcing steel f_s and the steel strain ε_s :

$$\text{For } f_s \leq f_n^*; f_s = E_s \varepsilon_s \quad (6.10)$$

$$\text{For } f_s > f_n^*; f_s = f_o^* + E_p^* \varepsilon_s \quad (6.11)$$

Belarbi and Hsu suggested that the normalized stress level of the intersection of the two lines with respect to the yield stress f_n^* / f_y and the normalized plastic modulus with respect to the plastic modulus of a bare steel bar E_p^* / E_p were approximated by linear relationships as follows:

$$\frac{f_n^*}{f_y} = 0.43 + 0.5 \frac{f_y^*}{f_y} \quad (6.12)$$

$$\frac{E_p^*}{E_p} = 3.3 - 2.5 \frac{f_y^*}{f_y} \quad (6.13)$$

The value of intercept f_o^* could be obtained as follows:

$$f_o^* = \frac{E_s - E_p^*}{E_s} f_n^* \quad (6.14)$$

According to Belarbi and Hsu, the value of plastic modulus E_p could range between 1.8% and 2.5% E_s . The value of $E_p = 2.5\% E_s$ is adopted in this chapter.

6.5 COMPLETE SECTIONAL STRESS-STRAIN RELATIONSHIP

Stress-strain analysis of a section can be performed using stress-strain equations for concrete in both compression and tension and bilinear model for reinforcing bar embedded in concrete. It is assumed that plane sections remain plane and the overall bond slip is negligible so that the average strain in steel is equal to the average strain of concrete at the same level shown in Figure 6.3.

A_{s1} and A_{s2} are the areas of steel in compression and tension, respectively. Their corresponding strains and stresses are $\varepsilon_{s1,2}$ and $\sigma_{s1,2}$. The force resultants are $S_{1,2} = \sigma_{s1,2} A_{s1,2}$. If the strain of the concrete at the outermost compression face equal to ε_{cm} , the depth of neutral axis equal to kd , and the distance between center of steels and the outermost concrete compression face equal to d_j , then from the linear strain distribution steel strains ε_{sj} at certain level can be expressed as follows:

$$\varepsilon_{sj} = \varepsilon_{cm} \frac{kd - d_j}{kd} \quad (6.15)$$

Based on the values of ε_{sj} , the steel stresses σ_{sj} are determined from Equations (6.10) and (6.11), which represent the stress-strain equation for reinforcing steel embedded in concrete.

If k_1 defines the average compressive stress and b defines the cross sectional width then the compressive stress resultant in concrete can be expressed as follows:

$$C_c = k_1 f'_c b k d \quad (6.16)$$

The concrete compressive force C_c acts at a distance $k_2 k d$ below the outermost compression face of the concrete. Similarly, if h defines the height of the cross section, the resultant of tensile stresses in concrete acts at a distance $k_4(h - kd)$ from the outermost tension face of the concrete. If k_3 represents the average tensile stress of the concrete and f_t represents the tensile strength of the concrete, the concrete tensile force C_t can be expressed as follows:

$$C_t = k_3 f_t b (h - kd) \quad (6.17)$$

The concrete strain at the outermost tension face ε_{tm} can be obtained by the strain diagram:

$$\varepsilon_{tm} = \frac{\varepsilon_{cm}(h - kd)}{kd} \quad (6.18)$$

General solutions for k_1 and k_2 are expressed as:

$$k_1 = \frac{\int_0^{\varepsilon_{cm}} \sigma_c d\varepsilon_c}{f'_c \varepsilon_{cm}} \quad (6.19)$$

$$k_2 = 1 - \frac{\int_0^{\varepsilon_{cm}} \varepsilon_c \sigma_c d\varepsilon_c}{\varepsilon_{cm} \int_0^{\varepsilon_{cm}} \sigma_c d\varepsilon_c} \quad (6.20)$$

By substituting Equation (6.1) into Equations (6.19) and (6.20), Bazant and Oh (1984) derived the solutions for k_1 and k_2 as follows:

$$k_1 = \frac{E_c}{f'_c \varepsilon_{cm}} \left[\frac{1}{2B} \ln(1 + A\varepsilon_{cm} + B\varepsilon_{cm}^2) + \frac{A}{B\sqrt{q}} \left(\tan^{-1} \frac{A}{\sqrt{q}} - \tan^{-1} \frac{A + 2B\varepsilon_{cm}}{\sqrt{q}} \right) \right] \quad (6.21)$$

$$k_2 = 1 - \frac{\left[\frac{\varepsilon_{cm}}{B} - \frac{A}{2B^2} \ln(1 + A\varepsilon_{cm} + B\varepsilon_{cm}^2) + \left(\frac{A^2 - 2B}{B^2\sqrt{q}} \right) \left(\tan^{-1} \frac{2B\varepsilon_{cm} + A}{\sqrt{q}} - \tan^{-1} \frac{A}{\sqrt{q}} \right) \right]}{\varepsilon_{cm} \left[\frac{1}{2B} \ln(1 + A\varepsilon_{cm} + B\varepsilon_{cm}^2) + \frac{A}{B\sqrt{q}} \left(\tan^{-1} \frac{A}{\sqrt{q}} - \tan^{-1} \frac{2B\varepsilon_{cm} + A}{\sqrt{q}} \right) \right]} \quad (6.22)$$

where:

$$A = \frac{1}{\varepsilon_{cp}} \left(\frac{E_c \varepsilon_{cp}}{f'_c} - 2 \right) \quad (6.23)$$

$$B = \frac{1}{\varepsilon_{cp}^2} \quad (6.24)$$

$$q = 4B - A^2 > 0 \quad (6.25)$$

Similarly, general solutions for k_3 and k_4 are expressed as:

$$k_3 = \frac{\int_0^{\varepsilon_{tm}} \sigma_c d\varepsilon_c}{f'_t \varepsilon_{tm}} \quad (6.26)$$

$$k_4 = 1 - \frac{\int_0^{\varepsilon_{tm}} \varepsilon_c \sigma_c d\varepsilon_c}{\varepsilon_{tm} \int_0^{\varepsilon_{tm}} \sigma_c d\varepsilon_c} \quad (6.27)$$

Before the concrete strain at the outermost tension face ε_{tm} reaches ε_{tp} , the relationship between f_t and ε_{tm} is linear with slope E_c (see Equation (6.6) and Figure 6.1). By substituting Equation (6.6) into Equations (6.26) and (6.27), Bazant and Oh (1984) derived the solutions for k_3 and k_4 for tensile strain of concrete $\varepsilon_{tm} \leq \varepsilon_{tp}$:

$$k_3 = \frac{E_c \varepsilon_{tm}}{2f_t} \quad (6.28)$$

$$k_4 = \frac{1}{3} \quad (6.29)$$

The descending curve in Figure 6.1 shows the relationship between f_t and ε_{tm} when ε_{tm} exceeds ε_{tp} . By substituting Equation (6.7), which expresses this relationship, into Equations (6.26) and (6.27), the author derived the solutions for k_3 and k_4 for tensile strain of concrete $\varepsilon_{tm} > \varepsilon_{tp}$:

$$k_3 = \frac{1}{f_t \varepsilon_{tm}} \left[\frac{1}{2} E_c \varepsilon_{tp}^2 + \frac{5}{3} \varepsilon_{tp}^{0.4} f_t (\varepsilon_{tm}^{0.6} - \varepsilon_{tp}^{0.6}) \right] \quad (6.30)$$

$$k_4 = 1 - \frac{\left[\frac{1}{3} E_c \varepsilon_{tp}^3 + \frac{5}{8} \varepsilon_{tp}^{0.4} f_t (\varepsilon_{tm}^{1.6} - \varepsilon_{tp}^{1.6}) \right]}{\varepsilon_{tm} \left[\frac{1}{2} E_c \varepsilon_{tp}^2 + \frac{5}{3} \varepsilon_{tp}^{0.4} f_t (\varepsilon_{tm}^{0.6} - \varepsilon_{tp}^{0.6}) \right]} \quad (6.31)$$

From equilibrium condition, normal force N and bending moment M of the cross section can be expressed as follows:

$$N = k_1 f_c' b k d + \sum_{j=1}^2 \sigma_{sj} A_{sj} - k_3 f_t b (h - k d) \quad (6.32)$$

$$M = k_1 f_c' b k d \left(\frac{h}{2} - k_2 k d \right) + \sum_{j=1}^2 \sigma_{sj} A_{sj} \left(\frac{h}{2} - d_j \right) + k_3 f_t b (h - k d) \left[\frac{h}{2} - k_4 (h - k d) \right] \quad (6.33)$$

By setting equilibrium condition for the cross section, $N = 0$, the depth of neutral axis kd for every increment of ε_{cm} can be obtained. For each kd , the curvature ϕ can then be obtained as follows:

$$\phi = \frac{\varepsilon_{cm}}{kd} \quad (6.34)$$

The beam deflection δ is calculated by integrating ϕ over the length of the beam L :

$$\delta = \int_0^L \phi m dx \quad (6.35)$$

where m represents moment distribution corresponding to a unit load at the deflection point. With the procedure described above, the moment-curvature or load-deflection relationships can be predicted even up to failure ($\varepsilon_{cm} = 0.003$). The complete procedure is given in the flowchart shown in Figure 6.4.

To verify the procedure, some experimental data of moment-curvature relationships from past research were analyzed. The specimens properties are shown in Table 6.1. Figure 6.5 shows the experimental versus calculated moment-curvature curves of those experimental data. The value of E_c is calculated according to ACI 318 if the test data is not available. Some beams data in the form of load-displacement relationships from past research were also analyzed. The beams properties are shown in Table 6.2. Figures 6.6 – 6.9 show the experimental and calculated load-displacement curves of those beams. Figure 6.5 – 6.9 show that the procedure involving stress-strain equations can predict the experimental results of moment-curvature and load-displacement relationships really well.

6.6 EFFECTIVE MOMENT OF INERTIA METHOD

ACI method for computing deflection is based on the effective moment of inertia as proposed by Branson (1963). Before cracking, the moment of inertia of the section I is taken equal to its gross moment of inertia I_g . After the cracking moment M_{cr} is exceeded, the effective moment of inertia I_e is used. The value of I_e must not be less than the value of the moment of inertia of the fully cracked section I_{cr} :

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^m I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^m \right] I_{cr} \leq I_g \quad (6.36)$$

with $m=3$ and M_a is the maximum moment in the member at the state deflection is computed.

The value of cracking moment M_{cr} can be calculated as follows:

$$M_{cr} = \frac{f_r I_g}{y} \quad (6.37)$$

where:

f_r = modulus of rupture of the concrete
 y = distance between extreme tension fiber and the centroid of the section

With a deflection constant for a beam K and span L , the immediate deflection can be calculated by:

$$\delta = \frac{KM_a L^2}{E_c I_e} \quad (6.38)$$

The effective moment of inertia approach has been used widely for deflection calculation due to its simplicity. However, its simplicity also means that it has limitations. Ghali (1993) noted that any member, even with a constant cross section, acted as a beam with variable rigidity because of cracking. It is impossible to find an expression for I_e that is equally accurate for all forms of variation of bending moment or reinforcement ratio over the member length. It was also shown by Ghali that the use of I_e according to ACI 318 could produce noticeable error for cases where the reinforcement ratio ρ was small and the ratio of applied moment to cracking moment M_a / M_{cr} was close to 1. It was also noted that more accuracy could be achieved by calculation of the deflection obtained from the curvatures at a number of sections.

Gilbert (1999) specifically pointed out that the accuracy of I_e for deflection prediction would be less for lightly reinforced beams and slabs with ρ less than 0.6%. The accuracy was also less when M_a was not much greater than M_{cr} since

tension stiffening might be significant. Gilbert also mentioned that the accuracy was also very sensitive to the value of f_r and that the value of f_r specified in ACI 318 often overestimated the flexural tensile strength of concrete.

Bischoff (2005) showed that the accuracy of I_e was affected by the level of service load relative to cracking moment M_a/M_{cr} and the ratio of I_g/I_{cr} . He showed that tension stiffening was overestimated as I_g/I_{cr} increased.

Al-Zaid et al (1991) pointed out that both the type and level of loading affected the estimation of the effective moment of inertia of cracked reinforced concrete beams. Al-Shaikh and Al-Said (1993) pointed out that the value of I_e was affected by the reinforcement ratio. They found that higher reinforcement ratio reduced the coefficient m in Equation (6.36). Instead of constant coefficient $m=3$ in the I_e equation, they suggested the coefficient m as a function of reinforcement ratio ρ :

$$m = 3 - 0.8\rho \quad (6.39)$$

Besides all those limitations, the I_e method is still used because it is a practical method and many of the limitations do not apply in many cases. In the next section, the author proposes some modifications to I_e formula to take into account the influences of reinforcement ratio and the level of applied load more explicitly as well as the nonlinearity near the failure load.

6.6.1 The Proposed Modified Effective Moment of Inertia Method

Figure 6.10 shows typical moment-curvature relationship of an under-reinforced beam. The two transition points in the moment-curvature curve are the cracking moment and its corresponding curvature, M_{cr} and ϕ_{cr} , and yield moment and its yield curvature, M_y and ϕ_y . Cracking moment and curvature represent the transition of the section from uncracked to cracked states before the yielding of reinforcement. Yield moment and curvature represent the transition of the cracked state before and after yielding of reinforcement. The cracking moment M_{cr} can be expressed as:

$$M_{cr} = \frac{f_r I_t}{y} \quad (6.40)$$

Cracking curvature can be expressed as:

$$\phi_{cr} = \frac{M_{cr}}{E_c I_t} \quad (6.41)$$

where:

- I_t = transformed moment of inertia taking into account the presence of reinforcing bars
- f_r = modulus of rupture of concrete
- y = distance between extreme tension fiber and the centroid of the section

The author used Figure 6.11 to derive the equations for yield moment and curvature, M_y and ϕ_y . Figure 6.11 shows typical stress-strain diagram of an under-reinforced section with compression steel A'_s and tension steel A_s . Neutral axis at yield state $k_y d$ defines the depth of neutral axis when tension steels reach yield strain ε_y . Concrete strain at the extreme compression surface ε_c and the strain of the compression steel ε'_s follow strain linearity:

$$\varepsilon_c = \varepsilon_y \frac{k_y d}{k_y d - d} \quad (6.42)$$

$$\varepsilon'_s = \varepsilon_y \frac{k_y d - d'}{k_y d - d} \quad (6.43)$$

$$\varepsilon_y = -\frac{f_y}{E_s} \quad (6.44)$$

Axial force equilibrium $\sum N = 0$ when tension steels yield is expressed as follows:

$$\frac{1}{2} b k_y d \varepsilon_c E_c + \varepsilon'_s E_s A'_s + \varepsilon_y E_s A_s = 0 \quad (6.45)$$

Substituting Equations (6.42), (6.43), and (6.44) into Equation (6.45) will result in:

$$-\frac{1}{2}b \frac{(k_y d)^2}{k_y d - d'} f_y \frac{E_c}{E_s} - f_y \frac{k_y d - d'}{k_y d - d'} A_s' - f_y A_s = 0 \quad (6.46)$$

Multiplying Equation (6.46) with $-\frac{k_y d - d'}{f_y}$ will result in quadratic equation in term of $k_y d$:

$$\frac{1}{2}b \frac{E_c}{E_s} (k_y d)^2 + (A_s + A_s') k_y d - (A_s' d' + A_s d) = 0 \quad (6.47)$$

Non-negative quadratic solution of Equation (6.47) can be written as follows:

$$k_y d = \frac{-(A_s + A_s') + \sqrt{(A_s + A_s')^2 + 2b \frac{E_c}{E_s} (A_s' d' + A_s d)}}{b \frac{E_c}{E_s}} \quad (6.48)$$

or

$$k_y = -\frac{E_s}{E_c} (\rho + \rho') + \sqrt{\left(\frac{E_s}{E_c}\right)^2 (\rho + \rho')^2 + 2 \frac{E_s}{E_c} \left(\rho' \frac{d'}{d} + \rho\right)} \quad (6.49)$$

From equilibrium condition, yield moment M_y is expressed as:

$$M_y = \frac{1}{3} b \varepsilon_c E_c (k_y d)^2 + A_s' \varepsilon_s' E_s (k_y d - d') + A_s f_y (d - k_y d) \leq M_u \quad (6.50)$$

Substituting Equations (6.42) – (6.44) into Equation (6.50) will result in:

$$M_y = \frac{1}{3} \frac{E_c}{E_s} \frac{b k_y^3 d^2}{(1 - k_y)} f_y + \frac{A_s' f_y (k_y d - d')^2}{d (1 - k_y)} + A_s f_y d (1 - k_y) \leq M_u \quad (6.51)$$

After obtaining the value of k_y from Equation (6.49), yield moment M_y can be obtained by inputting k_y into Equation (6.51). Yield curvature ϕ_y can also be obtained by inputting k_y into following equation:

$$\phi_y = \frac{f_y}{E_s (1 - k_y) d} \quad (6.52)$$

Ultimate moment and curvature, M_u and ϕ_u , are obtained using the assumptions of rectangular stress block of $0.85 f_c'$ and the concrete strain at the

outermost compression face $\varepsilon_{cu}=0.003$ as specified by the ACI code as shown in Figure 6.12. The depth of the neutral axis of a section at ultimate limit state c_u is expressed as:

$$c_u = \frac{A_s f_y - A'_s E_s \varepsilon'_s}{0.85 f'_c b \beta} \quad (6.53)$$

where β is a coefficient based on the concrete strength f'_c .

Ultimate curvature and moment, ϕ_u and M_u , are expressed as follows:

$$\phi_u = \frac{0.003}{c_u} \quad (6.54)$$

$$M_u = (A_s f_y - A'_s E_s \varepsilon'_s) \left(d - \frac{\beta c_u}{2} \right) + (A'_s E_s \varepsilon'_s) (d - d') \quad (6.55)$$

Yield curvature ϕ_y and ultimate curvature ϕ_u can be expressed in term of moment of inertia at yield and ultimate limit states I_y and I_u as follows:

$$I_y = \frac{M_y}{E_c \phi_y} \quad (6.56)$$

$$I_u = \frac{M_u}{E_c \phi_u} \quad (6.57)$$

The term I_y is identical to the term I_{cr} in Branson's equation. However, I_y is obtained from the stress-strain relationship at yield state rather than just the geometry of the section. Thus for deflection calculation using the effective moment of inertia approach, the author proposes the use of I_y instead of I_{cr} to obtain more accurate results. The value of I_t should also be used instead of I_g as far as convenient so that it reflects the transformed moment of inertia of the section better, especially for member with high reinforcement ratio. The modified equation for the effective moment of inertia proposed by the author is expressed as follows:

For $M_{cr} < M_a \leq M_y$:

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^m I_t + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^m \right] I_y \quad (6.58)$$

or

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^m I_t + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^m \right] \frac{M_y}{E_c \phi_y} \quad (6.59)$$

6.6.2 The Proposed Coefficient m

To obtain the coefficient m for the proposed modified equation for effective moment of inertia, analysis of load-deflection curves of 23 under-reinforced beams data that failed under flexure was performed. Table 6.2 shows the properties of those beams. The main parameter is the reinforcement ratio that ranges between 0.82% and 4.42%. The coefficient m can be obtained from the following procedure:

- The values of I_t , M_{cr} , ϕ_y , and M_y are obtained from the beam properties and Equations (6.40), (6.51), and (6.52).
- The load-deflection curve of one beam consists of a number of deflection data. Since the experimental deflection value Δ and the corresponding maximum applied moment M_a are available for each deflection data, the value of the experimental effective moment of inertia $I_{e_{exp}}$ of a beam with span length L can be obtained from the deflection formula below:

$$I_{e_{exp}} = \frac{kM_a L^2}{E_c \Delta} \quad (6.60)$$

where k is a constant depending on the type of loading and support conditions.

- The value of $I_{e_{exp}}$ can also be expressed in term of M_y and ϕ_y as follows:

$$I_{e_{exp}} = \frac{M_y}{E_c \phi_y} + \left(\frac{M_{cr}}{M_a} \right)^m \left(I_t - \frac{M_y}{E_c \phi_y} \right) \quad (6.61)$$

- The coefficient m for corresponding M_a can be derived from Equation (6.61) as follows:

$$m = \frac{\ln \left(\frac{I_{e_{\text{exp}}} - \frac{M_y}{E_c \phi_y}}{I_t - \frac{M_y}{E_c \phi_y}} \right)}{\ln \left(\frac{M_{cr}}{M_a} \right)} \quad (6.62)$$

- The value of m for each deflection data can be obtained by inputting the value of $I_{e_{\text{exp}}}$ from Equation (6.60) into Equation (6.62)

Table 6.3 shows the coefficient m for each beam, which is the average value of m from each deflection data. Figure 6.13 shows the relationship between the reinforcement ratios ρ (in percent) and the coefficient m for each beam data. Figure 6.13 also shows the expression for the coefficient m proposed by the author as follows:

$$m = \frac{2.5}{\rho} \leq 5 \quad (6.63)$$

The proposed equation supports the argument that higher reinforcement ratio reduces the value of m (Al-Shaikh and Al-Said, 1993). Figures 6.14 – 6.17 show the comparison between experimental deflection values of 23 beams data, deflection values obtained from Branson's ACI equation, from stress-strain equations, and from the proposed modified equations, Equations (6.59) and (6.63). Table 6.2 shows the beams properties. In general, the analysis using the stress-strain equations, which requires iteration and integration of curvature over the span length, produces the best load-displacement curves compared to the experimental data. The analysis results of the proposed modified equation, which is simpler and more straightforward, are almost as accurate as the analysis results using stress-strain equations of Section 6.5. The proposed modified equation also produces more accurate load-displacement curves compared to Branson's ACI equation.

6.6.3 The Proposed Equations for Deflection after Yielding of Reinforcement

Figure 6.10 shows that beam deflections beyond point M_y, ϕ_y are relatively difficult to predict since they increase rapidly with small increment of load. Figure 6.10 shows that the slope between point M_y, ϕ_y and point M_u, ϕ_u is very small for under-reinforced beams. In other words, ϕ_u can be several times greater than ϕ_y although M_u is not significantly greater than M_y . The author proposes that a reasonable way to calculate deflection when the applied moment M_a is larger than yield moment M_y , is by using linear interpolation between effective moment of inertia at yield state I_y and at ultimate limit state I_u as follows:

For $M_y < M_a < M_u$:

$$I_e = I_u + \frac{M_u - M_a}{M_u - M_y} (I_y - I_u) \quad (6.64)$$

For $M_a \geq M_u$:

$$I_e = I_u \quad (6.65)$$

where I_y and I_u are obtained from Equations (6.56) and (6.57), respectively.

Equation (6.64) has been verified with a limited number of beams data tested by Cox (1941) that exhibit post-yielding behavior. The properties of the beams are shown in Table 6.2. Figure 6.18 shows that Equation (6.64) produces quite reasonable predictions of post-yielding deflections. The analysis results of the proposed equation are relatively as good as the analysis results using stress-strain equations (see Section 6.5), which require iteration. Branson's ACI equation is not applicable for calculating deflections after yielding of reinforcement.

6.7 DISCUSSION OF BRANSON'S METHOD, BISCOFF'S METHOD, AND THE AUTHOR'S PROPOSED METHOD

The proposed modified effective moment of inertia method uses two transition points in the moment-curvature relationship curve: cracking moment and its corresponding curvature, M_{cr} and ϕ_{cr} , and yield moment and its yield curvature, M_y and ϕ_y . Cracking moment and curvature represent the transition of the section

from uncracked to cracked states before the yielding of reinforcement. Yield moment and curvature represent the transition of the cracked state before and after yielding of reinforcement. The yield curvature ϕ_y is then expressed in term of moment of inertia at yield I_y . The term I_y is identical to the term I_{cr} in Branson's equation. However, I_y is obtained from the stress-strain relationship at yield state rather than just the geometry of the section. Thus for deflection calculation using the effective moment of inertia approach, the author proposes the use of I_y instead of I_{cr} to obtain more accurate results. The value of I_y is generally smaller than I_{cr} with the ratio of I_y to I_{cr} increases when the reinforcement ratio ρ decreases (see Figure 6.19 in Attachment 2). Based on the best-fit of experimental data (ρ is between 0.8 and 4.42), the power $m=2.5/\rho$ for modified equation is smaller than Branson's. However, the effective moment of inertia I_e from the modified equation is not necessarily higher than that from Branson's equation since the interpolation point I_y is smaller than Branson's interpolation point I_{cr} . The proposed method has been shown to work well for beams or slabs with medium or high reinforcement ratio in Figures 6.14 - 6.18.

Branson's expression for I_e was correlated from test results of beams reinforced with steel. Most of these beams were simply supported rectangular beams having a reinforcement ratio ρ equal to 1.65% and a corresponding gross to cracked stiffness ratio I_g / I_{cr} of about 2.2. Comparison of deflections was made at a load level that was about 2.5 times the cracking value. Branson initially expected a value of 2 for the power m , but obtained a better fit with experimental data using $m=3$. Bischoff (2005) showed that the accuracy of I_e from Branson's equation was affected by the level of service load relative to cracking moment M_a / M_{cr} and the ratio of I_g / I_{cr} . He showed that tension stiffening was overestimated as I_g / I_{cr} increased. Bischoff pointed out that Branson's equation provided a reasonable estimate of deflection for beams reinforced with typical amounts of steel. Bischoff showed that the tension stiffening effect in Branson's equation was highly

dependent on both power m and the ratio of I_g / I_{cr} . The ratio of I_g / I_{cr} depended on the reinforcement ratio ρ as well as the modular ratio n . He showed that Branson's equation provided reasonable values of I_e when $\rho > 1$ with corresponding $I_g / I_{cr} < 3$. The tension stiffening effect was overestimated for values of $I_g / I_{cr} > 3$ when $m=3$.

Bischoff (2005) developed an alternative expression for I_e based on the basic concept of tension stiffening. The tensile capacity of concrete is typically neglected in strength design calculation, assuming that the steel reinforcement resists the tensile forces entirely. However, concrete continues to carry tension between the cracks through transfer of bond forces from the reinforcing steel into the concrete. The concrete contribution between cracks has an effect on the stiffness of the member, deflections, and crack widths under service load conditions. Bischoff's alternative equation for I_e , which was developed using the same approach for tensile member response, is expressed as follows (see Figure 6.20 in Attachment 2):

$$I_e = \frac{I_{cr}}{1 - \beta_c \eta (M_{cr} / M_a)} \quad (6.66)$$

where $\eta = 1 - I_{cr} / I_g$ and the tension stiffening factor $\beta_c = M_{cr} / M_a$. Figure 6.20 shows that tension stiffening is characterized by a change in deflection $\Delta\delta$ relative to the cracked member response. It cannot exceed the maximum change in deflection $\Delta\delta_{\max}$ at first cracking. The tension stiffening factor β_c represents the ratio of $\Delta\delta$ to $\Delta\delta_{\max}$. Hence, the member response due to applied moment M_a is bounded by an upper limit defined by a member response with constant tension stiffening ($\beta_c=1$) and a lower limit given by a cracked member response (I_{cr}) with no tension stiffening ($\beta_c=0$).

To obtain the contribution of tension stiffening of Branson's equation based on the reinforcement ratio, Bischoff used the parameter of tension stiffening factor derived from Branson's equation β_c as follows:

$$\beta_c = \frac{M_a / M_{cr}}{1 + \left(1 - (M_{cr} / M_a)^m\right) (I_{cr} / I_g) / (M_{cr} / M_a)^m} \quad (6.67)$$

Tension stiffening factor for the author's proposed equation can be obtained by replacing I_{cr} in Equation (6.67) with I_y .

To compare the tension stiffening factors from Branson's equation and the modified equation for low reinforcement ratios, simply supported beams with reinforcement ratios ranging from 0.3% to 1.0% were analyzed. The properties of those beams are shown in Table 6.4. Minimum reinforcement requirement by ACI 318-05 resulted in $\rho=0.33\%$ (rounded down to 0.3% for the analysis). Figure 6.21 shows that the tension stiffening is modeled reasonably well ($\beta_c \leq 1$) only when $I_g / I_{cr} \leq 3.67$ or $\rho \geq 0.7\%$ by Branson's equation since Branson calibrated his equation using beams with $I_g / I_{cr} = 2.2$. As the ratio I_g / I_{cr} is increased to 4.14 ($\rho=0.6\%$), tension stiffening starts to become excessive ($\beta_c > 1$) at lower load levels less than about twice the cracking moments ($M_a \leq 2M_{cr}$). By setting the upper limit $m=5$ ($m = \frac{2.5}{\rho} \leq 5$ in Equation (6.63)), tension stiffening is expressed reasonably well by the proposed equation, either for m governed by the upper limit of 5 or for $m < 5$ (see Figures 6.22). It indicates that the proposed equation expresses tension stiffening contribution better than Branson's equation for $\rho < 1$ and it will lead to better predictions of deflection for low reinforcement ratio.

Eight experimental data of one-way slabs reported by Gilbert and Smith (2004) were used to compare the deflection predictions of Branson's equation, Bischoff's equation, and the author's proposed equation. The reinforcement ratios of those slabs range from 0.180% to 0.841%. The properties of those slabs are shown in Table 6.5. The slabs are rectangular section and contain a single layer of longitudinal steel reinforcement. The elastic modulus of steel of 200 GPa was used in the analysis. Slabs S1 to S3 and S8 are simply supported over a span of 3.5 m. They were subjected to a single concentrated load at midspan. Slabs Z1 to Z4 are simply supported over a span of 2 m. They were subjected to two concentrated loads applied at the third span points. Figures 6.23 – 6.30 show that the proposed

method and Bischoff's equation can predict the deflections of the slabs with low reinforcement ratios better than Branson's equation. Figures 6.23 – 6.30 show that Bischoff's equation and the proposed equation by the author are an improvement of the Branson's equation in term of expressing tension stiffening contribution for beams with low reinforcement ratios.

6.8 CONCLUSION

Two methods for computing deflections were presented. The first one involved the integration of curvature and the use of actual stress-strain equations of the concrete and steel embedded in concrete. The second one was the proposed modified equation for effective moment of inertia. Both methods have been verified by experimental results and have been shown to be accurate. The proposed modified equation for effective moment of inertia has been shown to well represent the reduced stiffness of cracked beams. This method is also adopted as the basis of the proposed model of reduced slab stiffness to obtain the unbalanced moments in slab-column subassemblages presented in Chapter 7.

Table 6.1. Specimens Properties for Moment-Curvature Comparison

Test Series	f'_c (MPa)	E_c (MPa)	h (mm)	b (mm)	ρ (%)	ρ' (%)	f_y (MPa)
Bach and Graf (1917)	22.4	23700	300	1000	0.31	-	400
	22.4	23700	300	1000	0.65	-	400
	22.4	23700	300	1000	0.90	-	400
	22.4	23700	300	1000	1.32	-	400
	22.4	23700	300	1000	1.68	-	400
Sinha et al. (1964)	25.9	23899	152	71.1	1.40	-	322
Agrawal et al. (1965)	30.3	14480	152	76.2	1.31	1.31	352
Clark and Speirs (1978)	20.0	27100	204	1000	1.24	-	360
	20.0	22800	203	1000	0.79	-	360
	20.0	26300	204	1000	0.44	-	360

Table 6.2. Specimens Properties for Load-Displacement Comparison

Researcher	Beam ID.	f'_c (MPa)	E_c (MPa)	h (mm)	b (mm)	ρ (%)	f_y (MPa)	L (m)
Cox (1941)	221	21.1	21589	152	127	0.88	368	1.14
	222	21.1	21589	152	127	1.77	368	1.14
	223	21.1	21589	152	127	2.65	368	1.14
	224	21.1	21589	152	127	3.53	368	1.14
	225	21.1	21589	152	127	4.42	368	1.14
	222x5	21.1	21589	127	127	2.20	368	1.14
	222x7	21.1	21589	178	127	1.47	368	1.14
	222x8	21.1	21589	204	127	1.26	368	1.14
	222x9	21.1	21589	229	127	1.10	368	1.14
	222x10	21.1	21589	254	127	0.98	368	1.14
	222x11	21.1	21589	280	127	0.88	368	1.14
Besler and Scordelis (1963)	OA-1	22.5	22317	556	310	1.80	555	3.66
	OA-2	23.7	22890	561	305	2.26	555	4.57
	OA-3	37.6	28811	556	307	2.71	552	6.40
	A-3	35.0	27816	561	307	2.69	552	6.40
	B-3	38.7	29257	556	229	3.05	552	6.40
	C-3	35.0	27816	554	155	3.61	552	6.40
Al-Said et al. (1991)	B1-U, B2-U	38.2	29049	200	200	0.40 ρ_b	414	2.50
	B3-C, B4-C	38.2	29049	200	200	0.40 ρ_b	414	2.50
	B5-T, B6-T	38.2	29049	200	200	0.40 ρ_b	414	2.50
Al-Shaikh and Al-Said (1993)	B-L-11	33.0	26999	240	200	0.80	414	2.50
	B-N-12	33.0	26999	240	200	1.40	414	2.50
	B-H-13	33.0	26999	240	200	2.00	414	2.50

Table 6.3. Reinforcement Ratio and Coefficient m of the Beams Data

Researcher	Beam ID.	ρ (%)	m
Cox (1941)	221	0.88	2.65
	222	1.77	1.36
	223	2.65	0.95
	224	3.53	0.73
	225	4.42	0.66
	222x5	2.20	1.09
	222x7	1.47	1.39
	222x8	1.26	1.56
	222x9	1.10	2.38
	222x10	0.98	2.48
	222x11	0.88	2.55
Besler and Scordelis (1963)	OA-1	1.80	1.15
	OA-2	2.26	1.19
	OA-3	2.71	1.12
	A-3	2.69	1.00
	B-3	3.05	1.05
	C-3	3.61	0.91
Al-Said et al. (1991)	B1-U, B2-U	0.40 ρ_b	2.23
	B3-C, B4-C	0.40 ρ_b	1.67
	B5-T, B6-T	0.40 ρ_b	1.90
Al-Shaikh and Al-Said (1993)	B-L-11	0.80	2.40
	B-N-12	1.40	2.07
	B-H-13	2.00	1.41

Table 6.4. Beams Properties for Tension Stiffening Contribution Comparison

ρ (%)	f'_c (MPa)	f_y (MPa)	E_c (MPa)	E_s (MPa)	h (mm)	b (mm)	d (mm)
0.3	25	420	23500	200000	200	1000	170
0.4	25	420	23500	200000	200	1000	170
0.5	25	420	23500	200000	200	1000	170
0.6	25	420	23500	200000	200	1000	170
0.7	25	420	23500	200000	200	1000	170
0.8	25	420	23500	200000	200	1000	170
0.9	25	420	23500	200000	200	1000	170
1.0	25	420	23500	200000	200	1000	170

Table 6.5. Properties of the Slab Reported by Gilbert and Smith (2004)

Slab	f'_c (MPa)	f_y (MPa)	E_c (MPa)	E_s (MPa)	b (mm)	h (mm)	d (mm)	ρ (%)	f_t (MPa)
S1	37.3	500	26800	200000	850	110	92	0.180	3.39
S2	37.3	500	26800	200000	850	110	91	0.293	3.39
S3	37.3	500	26800	200000	850	110	90	0.463	3.39
S8	52.2	500	30700	200000	850	110	89	0.448	4.16
Z1	38.4	500	27390	200000	850	100	82	0.202	3.60
Z2	38.4	500	27390	200000	850	100	81	0.330	3.60
Z3	38.4	500	27390	200000	850	100	80	0.521	3.60
Z4	48.8	500	30500	200000	850	100	79	0.841	4.04

Fig. 6.1 Stress-Strain Curve for Concrete in Tension

Fig. 6.2 Stress-Strain Curve for Reinforcing Steel Embedded in Concrete

Fig. 6.3 Stress-Strain Analysis of a Section

Fig. 6.4 Flowchart to Obtain Moment-Curvature Relationship

(a) Experiment by Bach and Graf (1917)

(b) Experiment by Sinha et al. (1964) and Agrawal et al. (1965)

(c) Experiment by Clark and Speirs (1978)

Fig. 6.5 Moment vs. Curvature Comparison between Stress-Strain Analysis and Experiment

(a) Specimens 221, 222, 223, 224, 225, 222x5, and 222x7

(b) Specimens 222x8, 222x9, 222x10 and 222x11

Fig. 6.6 Load vs. Displacement Comparison between Stress-Strain Analysis and Experiment by Cox (1941)

(a) Specimens OA-1, OA-2

(b) Specimens OA-3, A-3

(c) Specimens B-3, C-3

Fig. 6.7 Load vs. Displacement Comparison between Stress-Strain Analysis and Experiment by Besler and Scordelis (1963)

Fig. 6.8 Load vs. Displacement Comparison between Stress-Strain Analysis and Experiment by Al-Said et al. (1991)

Fig. 6.9 Load vs. Displacement Comparison between Stress-Strain Analysis and Experiment by Al-Shaikh and Al-Said (1993)

Fig. 6.10 Typical Moment-Curvature Relationship for Under Reinforced Beams

Fig. 6.11 Stress-Strain Relationship at Yield State

Fig. 6.12 Stress-Strain Relationship at Ultimate Limit State

Fig. 6.13 Relationship between Reinforcement Ratio and the Coefficient m

(a) Specimens 221, 222, 223, 224, 225, 222x5, and 222x7

(b) Specimens 222x8, 222x9, 222x10, and 222x11

Fig. 6.14 Load-Displacement Comparison of Experimental Data by Cox (1941)

(a) Specimens OA-1 and OA-2

(b) Specimens OA-3 and A-3

(c) Specimens B-3 and C-3

Fig. 6.15 Load-Displacement Comparison of Experimental Data by Besler and Scordelis (1963)

Fig. 6.16 Load-Displacement Comparison of Experimental Data by Al-Said et al. (1991)

Fig. 6.17 Load-Displacement Comparison of Experimental Data
by Al-Shaikh and Al-Said (1993)

(a) Specimens 221 and 222

(b) Specimens 222x5 and 222x7

(c) Specimens 222x8 and 222x9

Fig. 6.18 Comparison of Load-Displacement after Yield State of Experimental Data by Cox (1941)

Fig. 6.19 Relationship between Reinforcement Ratio and I_y / I_{cr}

Fig. 6.20 Bischoff's Equation for I_e

Fig. 6.21 Tension Stiffening Factor from Branson's Equation

(a) for m not Governed by Upper Limit ($m < 5$) (b) for m Governed by Upper Limit ($m = 5$)

Fig. 6.22 Tension Stiffening Factor from the Proposed Equation

Fig. 6.23 Deflection Comparison for Slab S1

Fig. 6.24 Deflection Comparison for Slab S2

Fig. 6.25 Deflection Comparison for Slab S3

Fig. 6.26 Deflection Comparison for Slab S8

Fig. 6.27 Deflection Comparison for Slab Z1

Fig. 6.28 Deflection Comparison for Slab Z2

Fig. 6.29 Deflection Comparison for Slab Z3

Fig. 6.30 Deflection Comparison for Slab Z4

CHAPTER 7

REDUCED SLAB STIFFNESS FOR SLABS

7.1 INTRODUCTION

The magnitude of unbalanced moment transferred between the slab and the column in flat plate structures mainly depends on the cracked stiffness of the slab due to combined gravity and lateral loads. Overestimating the cracked stiffness of the slabs in the analysis may lead to underestimation of the lateral drift and overestimation of the unbalanced moment. On the other hand, underestimating the cracked slab stiffness in the analysis may lead to overestimation of the lateral drift and underestimation of the unbalanced moment. Therefore, reduced slab stiffness has to be predicted correctly to obtain accurate values of lateral drifts and unbalanced moments.

The simplest way to model the reduced slab stiffness is to apply uniform stiffness reduction over the entire slab, which is used in most available models described in Chapter 3. Although the method is simple, it does not represent the behavior of the cracked slab accurately. The stiffness of the actual cracked slab varies at different areas of the slab due to variation of slab moments, reinforcement ratios, column dimensions, and the panel shape. Therefore, a model that can take into account the reduced slab stiffness accurately is needed to obtain the correct values of unbalanced moments and drift ratios. This chapter presents the proposed model of reduced slab stiffness for slabs. Unlike currently available models such as Effective Beam Width, Equivalent Frame Model, and simplified frame analysis that are only applicable for flat plate structures with regular column layouts, the proposed model is also applicable for irregular column layouts. The proposed model can be used for obtaining unbalanced moments, drift ratios, and slab deflections under combined gravity and lateral loads. The model is verified using the experimental data of slabs deflections and the envelope curves of unbalanced moment-lateral drift relationships. The experimental data were obtained from past research as well as the experiments involving nine slab-column connections presented in Chapters 4 and 5. The proposed model for reduced slab stiffness is

based on the proposed modified effective moment of inertia method presented in Chapter 6.

7.2 THE PROPOSED MODIFIED EFFECTIVE MOMENT OF INERTIA METHOD

To obtain better prediction of deflections of flexural members, the author proposes some modifications of the Branson's ACI equation of effective moment of inertia in Chapter 6. The modified parameters include the use of moment of inertia at yield state I_y , inclusion of the state after yielding of reinforcement to the ultimate limit state, and the coefficient m based on the reinforcement ratio ρ .

Before cracking, the moment of inertia of a section is equal to its transformed moment of inertia I_t . The value of I_t should be used instead of I_g as far as convenient so that it reflects the transformed moment of inertia of the section better, especially for members with high reinforcement ratio. The author proposes the use of moment of inertia at yield state I_y instead of moment of inertia of the fully cracked section I_{cr} . The value of I_y is obtained from stress-strain relationship at yield state rather than just the geometry of the section. Essentially, the proposed modified effective moment of inertia equations after cracking are expressed as:

For $M_{cr} < M_a \leq M_y$:

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^m I_t + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^m \right] \frac{M_y}{E_c \phi_y} \quad (7.1)$$

For $M_y < M_a < M_u$:

$$I_e = I_u + \frac{M_u - M_a}{M_u - M_y} (I_y - I_u) \quad (7.2)$$

For $M_a \geq M_u$:

$$I_e = I_u \quad (7.3)$$

where :

$$m = \frac{2.5}{\rho} \leq 5 \quad (7.4)$$

$$\phi_y = \frac{f_y}{E_s(1-k_y)d} \quad (7.5)$$

$$k_y = -\frac{E_s}{E_c}(\rho + \rho') + \sqrt{\left(\frac{E_s}{E_c}\right)^2 (\rho + \rho')^2 + 2\frac{E_s}{E_c}\left(\rho'\frac{d'}{d} + \rho\right)} \quad (7.6)$$

$$M_y = \frac{1}{3} \frac{E_c}{E_s} \frac{bk_y^3 d^2}{(1-k_y)} f_y + \frac{A'_s f_y}{d} \frac{(k_y d - d')^2}{(1-k_y)} + A_s f_y d(1-k_y) \leq M_u \quad (7.7)$$

$$\phi_u = \frac{0.003}{c_u} \quad (7.8)$$

$$c_u = \frac{A_s f_y - A'_s E_s \epsilon'_s}{0.85 f'_c b \beta} \quad (7.9)$$

$$M_u = (A_s f_y - A'_s E_s \epsilon'_s) \left(d - \frac{\beta c_u}{2} \right) + (A'_s E_s \epsilon'_s) (d - d') \quad (7.10)$$

$$I_y = \frac{M_y}{E_c \phi_y} \quad (7.11)$$

$$I_u = \frac{M_u}{E_c \phi_u} \quad (7.12)$$

The proposed modified effective moment of inertia method has been shown to give good prediction of deflections of under-reinforced beams up to the ultimate limit state. Its accuracy is almost as good as the analysis using stress-strain equations which requires iteration and integration of curvature. The verification of the proposed modified equation is presented in Chapter 6.

7.3 THE PROPOSED REDUCED SLAB STIFFNESS FOR SLABS

A plate-bending element includes three components consisting of two-way out of plane plate rotational stiffness and a translational stiffness in the direction normal to the plane of the element. A displacement-based plate-bending element may have either quadrilateral or triangular shape as shown in Figure 7.1. At each joint, a plate-bending element activates three degrees of freedom consisting of two rotations and one transverse translation. The stresses and internal forces of the element are extrapolated to its nodes using Gaussian numerical integration.

7.3.1 Material Model before Slab Cracking

Before cracking, the uncracked concrete is represented by an isotropic elastic material. Stiffness matrix of an isotropic elastic material based on Mindlin formulation for plate bending relates deformations in x , y , and z axes and their respective internal forces. The stiffness matrix can be divided into two parts. The first part K_b relates bending moments and curvatures. The second part K_s corresponds to the transverse shear. The stiffness matrix of an isotropic material can be expressed as:

$$K = \left[\begin{array}{c|c} K_b & 0 \\ \hline 0 & K_s \end{array} \right] \quad (7.13)$$

$$K = \left[\begin{array}{ccc|cc} \frac{E_c I_t}{1-\nu^2} & \frac{E_c I_t \nu}{1-\nu^2} & 0 & 0 & 0 \\ \frac{E_c I_t \nu}{1-\nu^2} & \frac{E_c I_t}{1-\nu^2} & 0 & 0 & 0 \\ 0 & 0 & G_c I_t & 0 & 0 \\ \hline 0 & 0 & 0 & G_c t \kappa & 0 \\ 0 & 0 & 0 & 0 & G_c t \kappa \end{array} \right] \quad (7.14)$$

where I_t is the transformed moment of inertia per unit width, t is the slab thickness, ν is the Poisson's ratio of concrete, G_c is shear modulus of concrete, and $\kappa=5/6$ is a shear correction factor for homogeneous isotropic material taking into account the thickness direction variation of transverse shear strain.

The relationship between forces and deformations can be expressed as:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{bmatrix} = K \begin{bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \\ \phi_x \\ \phi_y \end{bmatrix} \quad (7.15)$$

Linear elastic analysis is used to calculate slab moments M_x , M_y , and M_{xy} at the element nodes. To include the torsional effects in the formulation, the torsional moments M_{xy} are taken into account by combining them with M_x and M_y to

produce total flexural moments. Average moments $M_{x\text{ave}}$ and $M_{y\text{ave}}$ are then calculated for each element as follows:

$$M_{x\text{ave}} = \frac{1}{N} \times \sum_N |M_x + M_{xy}| \quad (7.16)$$

$$M_{y\text{ave}} = \frac{1}{N} \times \sum_N |M_y + M_{xy}| \quad (7.17)$$

where N is the number of nodes per element.

7.3.2 The Proposed Material Model after Slab Cracking

When average moment $M_{x\text{ave}}$ or $M_{y\text{ave}}$ exceeds M_{cr} , the element cracks in the respective direction. It results in the assumption that the concrete now behaves as orthotropic material in the x , y , and z axes. After cracking, the orthotropic stiffness matrix of the concrete must take into account the effect of stiffness reductions in the x and y axes. These stiffness reductions are incorporated into the orthotropic stiffness matrix by using reduced slab stiffness factors in the x and y axes, α_x and α_y .

Scanlon and Murray (1982) used the effective stiffness formulation for the reductions of the stiffness matrix components in the x and y directions without taking into account the torsional effects. Polak (1996) also used the effective stiffness formulation for obtaining stiffness reduction factors in the x and y directions. The use of the effective torsional rigidity for the torsional stiffness reduction in cracked section analysis was introduced by Tavio and Teng (2004). The author uses his proposed modified equations for effective moment of inertia to calculate reduced slab stiffness factors in the x and y axes, α_x and α_y . After cracking, the average moments $M_{x\text{ave}}$ and $M_{y\text{ave}}$ are used to calculate the effective moment of inertia I_{ex} and I_{ey} in the x and y axes. The proposed modified equations for effective moment of inertia in the x and y axes based on Equations (7.1) – (7.3) can be expressed as:

For $M_{cr} < M_{x\text{ave}} \leq M_{yx}$:

$$I_{ex} = \left(\frac{M_{cr}}{M_{x\text{ave}}} \right)^{m_x} I_t + \left[1 - \left(\frac{M_{cr}}{M_{x\text{ave}}} \right)^{m_x} \right] \frac{M_{yx}}{E_c \phi_{yx}} \quad (7.18)$$

$$m_x = \frac{2.5}{\rho_x} \leq 5 \quad (7.19)$$

$$\phi_{yx} = \frac{f_y}{E_s (1 - k_{yx}) d_x} \quad (7.20)$$

$$k_{yx} = -\frac{E_s}{E_c} (\rho_x + \rho'_x) + \sqrt{\left(\frac{E_s}{E_c} \right)^2 (\rho_x + \rho'_x)^2 + 2 \frac{E_s}{E_c} \left(\rho'_x \frac{d'_x}{d_x} + \rho_x \right)} \quad (7.21)$$

$$M_{yx} = \frac{1}{3} \frac{E_c}{E_s} \frac{b_x k_{yx}^3 d_x^2}{(1 - k_{yx})} f_y + \rho'_x b_x f_y \frac{(k_{yx} d_x - d'_x)^2}{(1 - k_{yx})} + \rho_x b_x f_y d_x^2 (1 - k_{yx}) \leq M_{ux} \quad (7.22)$$

For $M_{cr} < M_{y\text{ave}} \leq M_{yy}$:

$$I_{ey} = \left(\frac{M_{cr}}{M_{y\text{ave}}} \right)^{m_y} I_t + \left[1 - \left(\frac{M_{cr}}{M_{y\text{ave}}} \right)^{m_y} \right] \frac{M_{yy}}{E_c \phi_{yy}} \quad (7.23)$$

$$m_y = \frac{2.5}{\rho_y} \leq 5 \quad (7.24)$$

$$\phi_{yy} = \frac{f_y}{E_s (1 - k_{yy}) d_y} \quad (7.25)$$

$$k_{yy} = -\frac{E_s}{E_c} (\rho_y + \rho'_y) + \sqrt{\left(\frac{E_s}{E_c} \right)^2 (\rho_y + \rho'_y)^2 + 2 \frac{E_s}{E_c} \left(\rho'_y \frac{d'_y}{d_y} + \rho_y \right)} \quad (7.26)$$

$$M_{yy} = \frac{1}{3} \frac{E_c}{E_s} \frac{b_y k_{yy}^3 d_y^2}{(1 - k_{yy})} f_y + \rho'_y b_y f_y \frac{(k_{yy} d_y - d'_y)^2}{(1 - k_{yy})} + \rho_y b_y f_y d_y^2 (1 - k_{yy}) \leq M_{uy} \quad (7.27)$$

For $M_{yx} < M_{x\text{ave}} < M_{ux}$:

$$I_{ex} = I_{ux} + \frac{M_{ux} - M_{x\text{ave}}}{M_{ux} - M_{yx}} (I_{yx} - I_{ux}) \quad (7.28)$$

$$I_{ux} = \frac{M_{ux}}{E_c \phi_{ux}} \quad (7.29)$$

$$I_{yx} = \frac{M_{yx}}{E_c \phi_{yx}} \quad (7.30)$$

$$\phi_{ux} = \frac{0.003}{c_{ux}} \quad (7.31)$$

$$c_{ux} = \frac{\rho_x d_x f_y - \rho'_x d'_x E_s \epsilon'_{sx}}{0.85 f'_c \beta} \quad (7.32)$$

$$M_{ux} = \left(\rho_x b_x d_x f_y - \rho'_x b_x d'_x E_s \epsilon'_{sx} \right) \left(d_x - \frac{\beta c_{ux}}{2} \right) + \left(\rho'_x b_x d'_x E_s \epsilon'_{sx} \right) (d_x - d'_x) \quad (7.33)$$

For $M_{yy} < M_{y\text{ave}} < M_{uy}$:

$$I_{ey} = I_{uy} + \frac{M_{uy} - M_{y\text{ave}}}{M_{uy} - M_{yy}} (I_{yy} - I_{uy}) \quad (7.34)$$

$$I_{uy} = \frac{M_{uy}}{E_c \phi_{uy}} \quad (7.35)$$

$$I_{yy} = \frac{M_{yy}}{E_c \phi_{yy}} \quad (7.36)$$

$$\phi_{uy} = \frac{0.003}{c_{uy}} \quad (7.37)$$

$$c_{uy} = \frac{\rho_y d_y f_y - \rho'_y d'_y E_s \epsilon'_{sy}}{0.85 f'_c \beta} \quad (7.38)$$

$$M_{uy} = \left(\rho_y b_y d_y f_y - \rho'_y b_y d'_y E_s \epsilon'_{sy} \right) \left(d_y - \frac{\beta c_{uy}}{2} \right) + \left(\rho'_y b_y d'_y E_s \epsilon'_{sy} \right) (d_y - d'_y) \quad (7.39)$$

For $M_{x\text{ ave}} \geq M_{ux}$:

$$I_{ex} = I_{ux} \quad (7.40)$$

$$I_{ux} = \frac{M_{ux}}{E_c \phi_{ux}} \quad (7.41)$$

For $M_{y\text{ ave}} \geq M_{uy}$

$$I_{ey} = I_{uy} \quad (7.42)$$

$$I_{uy} = \frac{M_{uy}}{E_c \phi_{uy}} \quad (7.43)$$

where:

$M_{x\text{ ave}}$	=	slab average moment about y-axis
$M_{y\text{ ave}}$	=	slab average moment about x-axis
M_{yx}, ϕ_{yx}	=	slab yield moment and curvature about y-axis
M_{yy}, ϕ_{yy}	=	slab yield moment and curvature about x-axis
M_{ux}, ϕ_{ux}	=	slab ultimate moment and curvature about y-axis
M_{uy}, ϕ_{uy}	=	slab ultimate moment and curvature about x-axis
ρ_x	=	slab reinforcement ratio parallel to x-axis
ρ_y	=	slab reinforcement ratio parallel to y-axis
d_x, d'_x	=	distance from the outermost compression surface of the concrete to the rebars in tension and compression, respectively, parallel to x-axis
d_y, d'_y	=	distance from the outermost compression surface of the concrete to the rebars in tension and compression, respectively, parallel to y-axis
b_x, b_y	=	slab unit width perpendicular to the x and y axes, respectively
$k_{yx}d_x, k_{yy}d_y$	=	depth of the neutral axis at yield state for M_{yx} and M_{yy} , respectively

c_{ux}, c_{uy} = depth of the neutral axis at ultimate limit state for M_{ux} and M_{uy} , respectively

Reduced slab stiffness factors in the x and y axes, α_x and α_y , are then calculated based on the effective moment of inertia as follows:

$$\alpha_x = \frac{I_{ex}}{I_t} \quad (7.44)$$

$$\alpha_y = \frac{I_{ey}}{I_t} \quad (7.45)$$

The stiffness matrix components in Equation (7.14) are modified using reduced slab stiffness factors α_x and α_y . Since the concrete now behaves as an orthotropic material, its flexural stiffness $E_c I_t$ needs to be updated in the x and y axes. Elastic modulus of concrete E_c in the flexural stiffness $E_c I_t$ is updated using reduced slab stiffness factors, α_x and α_y , in both axes as follows:

$$E_x = \alpha_x E_c \quad (7.46)$$

$$E_y = \alpha_y E_c \quad (7.47)$$

The Poisson's ratio of concrete ν is also updated in both axes using reduced slab stiffness factors as follows:

$$\nu_x = \alpha_x \nu \quad (7.48)$$

$$\nu_y = \alpha_y \nu \quad (7.49)$$

Polak (1996) pointed out that the shear modulus of concrete should be retained at a value greater than zero to realistically model the behavior of the concrete after cracking. Decreasing magnitude of shear stiffness $G_c t \kappa$ after cracking is represented by the modified shear stiffness which is updated in the orthotropy directions, G_{xz} and G_{yz} , using reduced slab stiffness factors as follows:

$$G_{xz} = \alpha_x G_c \quad (7.50)$$

$$G_{yz} = \alpha_y G_c \quad (7.51)$$

The torsional stiffness $G_c I_t$ after cracking is represented by G_{xy} , which is updated using reduced slab stiffness factors in the x and y axes, as follows:

$$G_{xy} = \alpha_x \alpha_y G_c \quad (7.52)$$

The analysis of the cracked elements is then repeated using the updated stiffness matrix representing the reduced slab stiffness due to slab cracking. The modified stiffness matrix can be expressed as:

$$K = \begin{bmatrix} \frac{\alpha_x E_c I_t}{1 - \alpha_x \alpha_y \nu^2} & \frac{\alpha_x \alpha_y E_c I_t \nu}{1 - \alpha_x \alpha_y \nu^2} & 0 & 0 & 0 \\ \frac{\alpha_x \alpha_y E_c I_t \nu}{1 - \alpha_x \alpha_y \nu^2} & \frac{\alpha_y E_c I_t}{1 - \alpha_x \alpha_y \nu^2} & 0 & 0 & 0 \\ 0 & 0 & \alpha_x \alpha_y G_c I_t & 0 & 0 \\ 0 & 0 & 0 & \alpha_x G_c t \kappa & 0 \\ 0 & 0 & 0 & 0 & \alpha_y G_c t \kappa \end{bmatrix} \quad (7.53)$$

or

$$K = \begin{bmatrix} \frac{E_x I_t}{1 - \nu_x \nu_y} & \frac{E_y I_t \nu_x}{1 - \nu_x \nu_y} & 0 & 0 & 0 \\ \frac{E_x I_t \nu_y}{1 - \nu_x \nu_y} & \frac{E_y I_t}{1 - \nu_x \nu_y} & 0 & 0 & 0 \\ 0 & 0 & G_{xy} I_t & 0 & 0 \\ 0 & 0 & 0 & G_{xz} t \kappa & 0 \\ 0 & 0 & 0 & 0 & G_{yz} t \kappa \end{bmatrix} \quad (7.53a)$$

The components of the updated stiffness matrix in Equation (7.53a) consist of modified moduli of elasticity E_x and E_y , modified shear moduli G_{xz} , G_{yz} , and G_{xy} , and modified Poisson's ratios ν_x and ν_y . Transformed moment of inertia I_t , slab thickness t , and shear correction factor κ are unchanged. The updated stiffness matrix in Equations (7.53) and (7.53a), which is based on the proposed modified equations for effective moment of inertia, does not include any failure criteria.

The plate-bending elements described in this section are based on the Mindlin formulation for plate bending, which includes the effects of transverse shear deformation. Generally, shear deformations tend to be important when the thickness is greater than about one-tenth to one-fifth of the span. They can also be quite significant in the vicinity of the bending stress concentrations, such as near

sudden changes in thickness or support conditions, and near holes or re-entrant corners. The finite element software SAP2000 was used for obtaining the analytical results of the proposed method. The program enables the use of Mindlin-type elements without compromising accuracy for both thin and thick plates since it performs reduced integration for the stiffness matrix formulation of these elements. The finite element model was modeled and analyzed using elastic analysis in SAP2000. The elastic moment output and the components of the isotropic stiffness matrix of the slab were exported into Microsoft Excel spreadsheets. The author used the user-defined subroutines in the form of Microsoft Excel Visual Basic for Applications (VBA) to obtain the average elastic moment of each slab element. Based on the average elastic moment, the reduced slab stiffness factors of each slab element were then calculated using the author's proposed method of effective moment of inertia. The author used the user-defined subroutines to modify the stiffness matrix components based on the reduced slab stiffness factors and to create a new Microsoft Excel file consisting of the modified stiffness matrix components of each slab element, which became orthotropic material after cracking. This new file was imported to the existing SAP2000 finite element model to obtain the analytical results of the cracked slabs.

Convergence criteria can take the forms of loads, displacements, or energies (Bathe, 1996). In this chapter, the convergence was checked using *h*-refinement method (mesh refinement) with a displacement convergence tolerance of 5%. The *h*-refinement method (mesh refinement) was used to obtain the element sizes for finite element models. From 38 data of single-panel slab deflection and isolated slab-column connections in Chapter 7, *h*-refinement method was performed on 13 specimens (see Appendix 5). Those specimens were selected so that at least one specimen represents the geometries and properties of one set of specimens conducted in one experimental program either by the author or by the other researchers in the literature review. For slab deflection data, the deflection value due to midspan moment of $2M_{cr}$ was used as the reference of convergence. For slab-column connections subjected to lateral loads, the lateral displacement value due to lateral load at 1.5% lateral drift was used as the reference of convergence. The single-panel slabs in the *h*-refinement study are one-way and two-way slabs

tested by Jofriet and McNeice (1971) and the slab A3 tested by Aghayere and MacGregor (1990). The slab C1 tested by Ghoneim and MacGregor (1994) has similar geometries and properties as the slab A3. Ten slab-column connections were included in the h -refinement study. From six specimens tested by Robertson (1990), specimens 2C (specimens 6LL, 7L, and 5SO are similar in geometries and properties), 8I, and 9E were selected. Specimen MG-2A (specimens MG-3 and MG-6 are similar in geometries and properties) tested by Megally and Ghali (2000a) and specimen 2C (specimens 2CS, 3SL, and 4HS are similar in geometries and properties) tested by Robertson et al. (2002) were also selected for the h -refinement study. Specimen 1 (specimens 2, 3, and 4 are similar in geometries and properties) tested by Pan and Moehle (1992) was included as well. For slab-column connections tested at NTU-Singapore, specimen E1H (specimens E2H, E12H, and E12L are similar in geometries and properties) tested by Anggadajaja (2006) and specimen YL-L1 (specimens YL-H1V, YL-H2V, YL-H2, and YL-L2 are similar in geometries and properties) tested by Tan and Teng (2005) were included in the h -refinement study. Specimen SC-HD (specimens SC-LD, SC-HDS, and SC-LDS are similar in geometries and properties) and specimen SA-LD (specimens SA-HD, SA-HDS, and SA-LDS are similar in geometries and properties) tested by the author were included as well.

The h -refinement study of those 13 specimens is presented in Appendix 5. For all finite element models, the author limited the maximum element size of $2.5h$ between support to support in both directions, where h is the slab thickness. Mesh refinement was performed until reasonably fine element size of $1h$. The h -refinement method in Appendix 5 shows that the displacement results of finite element models with the maximum element size of $2.5h$ only deviate less than 5% of those obtained from maximum element size of $1h$. Thus, maximum element size of $2.5h$ is recommended for obtaining credible results from finite element models with the proposed reduced slab stiffness. Based on the results of h -refinement study, the author also used maximum element size of $2.5h$ for multi-panel slab specimens. The maximum element size of $2.5h$ is recommended for finite element analysis of flat plate structures under normal loading conditions such as office and residential

buildings. It may not be suitable for loading conditions that require very thick slabs such as transfer floors.

7.3.3 Verification Using the Experimental Data of Single-Panel Slab Deflections

The analysis using the proposed reduced slab stiffness was verified using the experimental data of slabs deflections. Two slabs tested by Jofriet and McNeice (1971) were used to verify the analysis. The first one is one-way slab with 0.8% reinforcing steel subjected to two point loads. The second one is a two-way slab supported at its corners with 0.85% reinforcing steel in the form of isotropic mesh. It was subjected to a point load at the center of the slab. The slabs properties are shown in Table 7.1. Modulus of rupture $f_r = 0.7\sqrt{f'_c}$ (MPa) based on ACI 318 was used in the analysis. Figure 7.2 shows finite element meshes of both slabs. Figure 7.3 shows the center deflection comparison between the experimental data of the one-way slab and the results of the analysis using the proposed reduced slab stiffness. Figure 7.3 also shows the deflections comparisons for the two-way slab at four locations.

Two slabs subjected to uniform transverse loading tested at University of Alberta were also used to verify the analysis using the proposed reduced slab stiffness. The first one is the slab A3 tested by Aghayere and MacGregor (1990) and the second one is the slab C1 tested by Ghoneim and MacGregor (1994). Both slabs were simply supported along the edges. They were subjected to nine point loads simulating uniform transverse load. Based on the test results, the tensile strength of concrete of slab A3 is 2.80 MPa and the modulus of rupture of slab C1 is 3.46 MPa. The holding down of the slab corners of slabs A3 and C1 during testing permitted the development of anticlastic corner surfaces. These support conditions caused cracks perpendicular to the diagonal on the top surfaces of the slab corners in addition to flexural cracks along the diagonal on the bottom surfaces near the corners. The twisting moments near the corners also caused torsional cracks on the edges of the slab near the corners. At failure, crushing of concrete occurred on the top surfaces near the corners. The tests were terminated because of excessive deflection at the center and rotation at the supports.

Typical finite element mesh of both slabs is shown in Figure 7.4. The slabs properties are shown in Table 7.1. The comparisons of the center deflections obtained from the analysis using the proposed reduced slab stiffness and from the experimental data are shown in Figure 7.5. Figures 7.3 and 7.5 show that the analysis using the proposed reduced slab stiffness can predict the slabs deflections accurately.

7.3.4 Verification Using the Experimental Data of Multi-Panel Slab Deflections

The analysis using the proposed reduced slab stiffness was also verified using the experimental data of 7-column irregular flat plate floor tested at Nanyang Technological University, Singapore (Tan and Teng, 2001). The slab was subjected to uniform transverse load that was represented by point loads throughout the slab. The loads were applied through spreader beams transferring vertical loads from the hydraulic jacks. Figure 7.6 shows the geometry of the 7-column specimen. The slab dimensions are 6.3 m by 5.1 m. The slab thickness is 100 mm. Three types of steel bar were used as reinforcing steels. Table 7.2 shows the 7-column specimen properties. In Table 7.2, the elastic moduli of T10 and R10 steel rebars were determined from the tensile load-strain relationship from the laboratory test (four samples for T10 and three samples for R10). There was no laboratory test conducted for obtaining the properties of the wire mesh diameter 7mm (A193). The properties of wire mesh A193 were obtained from the mill certificate. Since the elastic modulus of T10 rebars obtained from the laboratory test was substantially lower than elastic modulus of reinforcing steel, which approximately ranges from 190 GPa to 200 GPa, the author used the value of 190 GPa in the analysis. Figure 7.7 shows the arrangement of the top and bottom rebars. Modulus of rupture $f_r = 0.33\sqrt{f'_c}$ (MPa) was assumed in the analysis as suggested by Scanlon and Murray (1982) to take into account the effect of restrained shrinkage in continuous or multi-panel slabs.

Finite element mesh of the 7-column specimen is shown in Figure 7.8. The deflection comparisons between the analysis using the proposed reduced slab stiffness and the experimental data at 16 locations on the slab are shown in Figure

7.9. It shows that the deflections at those locations can be predicted well by the results of the analysis using the proposed reduced slab stiffness up to the ultimate limit state.

7.4 THE PROPOSED REDUCED SLAB STIFFNESS BEYOND ULTIMATE LIMIT STATE FOR SLAB-COLUMN CONNECTIONS

The slab-column connections of a flat plate structure must be designed to undergo the same lateral drift as the primary system such as shear wall or core wall without losing their capability to support the gravity load. The combination of gravity load and lateral drift may lead to high negative moments and severe cracks at the slab areas near the columns. Although the outermost compression surface of the slabs near the columns may experience crushing of concrete, the structure is still able to withstand the gravity load until it experiences failure as a system at specified design lateral drift.

Based on the ACI code, crushing of concrete representing ultimate limit state occurs when the outermost compression strain exceeds 0.003. The strain limit of 0.003 is used to calculate ultimate moment M_u , ultimate curvature ϕ_u , and the effective moment of inertia at ultimate limit state I_u . Equations (7.40) - (7.43) show that if the average slab moments obtained from elastic analysis exceed M_u , the effective moment of inertia is assumed as constant value I_u . Thus, after crushing of concrete the cracked stiffness of the slab is assumed to be constant. However, the actual cracked slabs especially near the columns experience drop of stiffness once crushing of concrete occurs. The cracked stiffness keeps decreasing to a certain limit until flat plate structure experiences failure at the specified lateral drift. Due to the drop of stiffness after crushing of concrete, the actual effective moment of inertia of the slab beyond ultimate limit state might be much smaller than I_u .

According to the IBC 2003, the upper limit of the inelastic drift ratio is 2.5%. Combined with the slab moments due to gravity load, the lateral drift of 2.5% may lead to high negative moments that cause crushing of concrete on the slab areas near the columns. To obtain accurate design values of unbalanced moments, the structural analysis must be carried out using the appropriate reduced slab stiffness.

Based on typical lateral load-displacement hysteretic response of the slab-column connections experiencing punching shear failure, Hueste and Wight (1997) pointed out that it was necessary to modify the rotational stiffness to take into account the softening of the connection. Figure 7.10a shows the stiffness reduction model for displacement-based analysis by Hueste and Wight (1997). It shows that the rotation is held constant as the moment capacity $M_{allowable}$ is reduced to 10% once the allowable rotation $\phi_{allowable}$ is exceeded. The reduced stiffness is represented by the slope from point of origin to the point $(\phi_{allowable}, 0.1M_{allowable})$.

To take into account the effect of drop of stiffness beyond ultimate limit state, the moment-curvature relationship as shown in Figure 7.10b is proposed for calculating the reduced slab stiffness after the outermost compression strain ε_{cu} exceeds 0.003. The proposed model is intended to be used for force-based analysis to avoid overestimation of displacement prediction due to slight increment of moment beyond ultimate moment M_u . The drop of stiffness beyond ultimate limit state is obtained by setting reduced ultimate curvature $\phi_{ur} = 1.1\phi_u$ and reduced ultimate moment $M_{ur} = 0.1M_u$. The proposed equations for effective moment of inertia based on moment-curvature relationship that takes into account the effect of reduced slab stiffness beyond ultimate limit state ($\varepsilon_{cu} \geq 0.003$) are expressed as follows:

For $M_a \geq M_u$ and $\phi_u \leq \frac{M_a}{M_u} \times \phi_u < \phi_{ur}$:

$$I_e = \frac{\left(\frac{\phi_{ur} - \frac{M_a}{M_u} \times \phi_u}{\phi_{ur} - \phi_u} \right) \times 0.9M_u + 0.1M_u}{E_c \frac{M_a}{M_u} \phi_u} \quad (7.54)$$

For $M_a \geq M_u$ and $\frac{M_a}{M_u} \times \phi_u \geq \phi_{ur}$:

$$I_e = \frac{M_{ur}}{E_c \phi_{ur}} \quad (7.55)$$

where $\phi_{ur} = 1.1\phi_u$ and $M_{ur} = 0.1M_u$.

The proposed equations consider the reduced stiffness of the slab element due to flexural cracks. They do not take into account the effect of stiffness degradation due to shear cracks and cyclic lateral loads. In the slab finite element formulation, the proposed moment-curvature relationship beyond ultimate limit state is used to calculate the effective moment of inertias, I_{ex} and I_{ey} , in x and y axes based on the average moments of the slab element, $M_{x\text{ave}}$ and $M_{y\text{ave}}$. Instead of using Equations (7.40) - (7.43), the proposed equations for reduced slab stiffness for slab-column connections beyond ultimate limit state are expressed as follows:

For $M_{x\text{ave}} \geq M_{ux}$ and $\phi_{ux} \leq \frac{M_{x\text{ave}}}{M_{ux}} \times \phi_{ux} < \phi_{urx}$:

$$I_{ex} = \frac{\left(\frac{\phi_{urx} - \frac{M_{x\text{ave}}}{M_{ux}} \times \phi_{ux}}{\phi_{urx} - \phi_{ux}} \right) \times 0.9M_{ux} + 0.1M_{ux}}{E_c \frac{M_{x\text{ave}}}{M_{ux}} \phi_{ux}} \quad (7.56)$$

For $M_{x\text{ave}} \geq M_{ux}$ and $\frac{M_{x\text{ave}}}{M_{ux}} \times \phi_{ux} \geq \phi_{urx}$:

$$I_{ex} = \frac{M_{urx}}{E_c \phi_{urx}} \quad (7.57)$$

where $\phi_{urx} = 1.1\phi_{ux}$ and $M_{urx} = 0.1M_{ux}$

For $M_{y\text{ ave}} \geq M_{uy}$ and $\phi_{uy} \leq \frac{M_{y\text{ ave}}}{M_{uy}} \times \phi_{uy} < \phi_{ury}$:

$$I_{ey} = \frac{\left(\frac{\phi_{ury} - \frac{M_{y\text{ ave}}}{M_{uy}} \times \phi_{uy}}{\phi_{ury} - \phi_{uy}} \right) \times 0.9M_{uy} + 0.1M_{uy}}{E_c \frac{M_{y\text{ ave}}}{M_{uy}} \phi_{uy}} \quad (7.58)$$

For $M_{y\text{ ave}} \geq M_{uy}$ and $\frac{M_{y\text{ ave}}}{M_{uy}} \times \phi_{uy} \geq \phi_{ury}$:

$$I_{ey} = \frac{M_{ury}}{E_c \phi_{ury}} \quad (7.59)$$

where $\phi_{ury} = 1.1\phi_{uy}$ and $M_{ury} = 0.1M_{uy}$

The effective moment of inertias of the slab element in both axes, I_{ex} and I_{ey} , are then used to calculate reduced slab stiffness factors, α_x and α_y . These factors are used to update the stiffness matrix of the slab element. The proposed equations for reduced slab stiffness beyond ultimate limit state are not intended for detecting failure. The proposed equations are used for reducing the stiffness of the slab element which is subjected to elastic moments that are larger than ultimate moments. Essentially the proposed model of reduced slab stiffness based on the proposed modified equations for effective moment of inertia does not include any failure criteria. The proposed model is intended for obtaining complete load-deformation relationships or deformation at certain magnitude of load of slab-column connections before failures. The failure criterion specified by codes such as punching shear capacity can be used for detecting failures of slab-column connections. This will be shown in the next section, which also presents the use of the proposed model to obtain complete unbalanced moment-drift ratio relationships of slab-column connections.

7.5 VERIFICATION OF THE PROPOSED MODEL OF REDUCED SLAB STIFFNESS USING THE EXPERIMENTAL DATA OF SLAB-COLUMN CONNECTIONS

The experimental data of flat plate slab-column connections specimens in the form of unbalanced moment-lateral drift relationships were used to verify the proposed model of reduced slab stiffness. Most specimens are isolated single slab-column connections subjected to combined gravity and lateral loads. The analysis of each specimen was carried out using updated stiffness matrix that was modified using the proposed reduced slab stiffness. The comparisons of unbalanced moment-lateral drift relationships between the analysis results and the experimental data can be divided into two groups. The first group consists of the comparisons for specimens subjected to combination of gravity and unidirectional cyclic lateral loads. The second group consists of the comparisons for specimens subjected to combination of gravity and bidirectional cyclic lateral loads.

Firstly, force-based elastic finite element analysis under combined gravity and lateral loads is carried out to obtain the elastic average moments of slab elements, $M_{x\text{ave}}$ and $M_{y\text{ave}}$. The effective moment of inertias of each element, I_{ex} and I_{ey} , are calculated based on the elastic average moments. Reduced slab stiffness factors, α_x and α_y , are obtained based on the proposed modified effective moment of inertia equations. These factors are used to update the components of the stiffness matrix. Slab elements within the columns regions are kept elastic. The analysis is then repeated using the updated stiffness matrix representing the reduced slab stiffness under the same magnitude of gravity and lateral loads. The unbalanced moment-lateral drift relationships from the analysis using the proposed reduced slab stiffness are compared to the experimental data in the form of envelope curves. Note that the predictions obtained from the analysis are for monotonic lateral loading, while the experimental results are from cyclic lateral loading. Hence, it would be expected that the predictions would result in higher slab stiffness compared to the experimental results. The results of the analysis using the proposed reduced slab stiffness are also compared to the analysis results using uniform reduced slab stiffness $\beta = 1/3$ of gross section stiffness recommended by

Vanderbilt and Corley (1983), which is mostly used in currently available models such as Effective Beam Width and Equivalent Frame Model. The punching shear failure criterion by the ACI code is indicated in the unbalanced moment-lateral drift curves. It shows the magnitude of transferred unbalanced moment that causes the punching shear failure of the slab-column connection.

7.5.1 Specimens under Unidirectional Lateral Load by Robertson (1990)

A total of six slab-column connections specimens were tested by Robertson (1990). Four specimens, 2C, 5SO, 6LL, and 7L, are two-span flat plate subassemblies subjected to unidirectional lateral load in E-W direction combined with shear force V_g . Specimens 8I and 9E are isolated interior and edge slab-column connections, respectively. They were subjected to combination of shear force and unidirectional lateral load in E-W direction. The slab thickness of all specimens is 114 mm. The typical column dimensions are 254 mm x 254 mm. The typical column height is 1.54 m. The slab dimensions of specimens 2C, 5SO, 6LL, and 7L are 6.05 m by 1.98 m. The span length between columns centerlines is 2.90 m. Specimen 5SO has 254mm additional slab overhangs on both edges of the exterior spans. The slab dimensions of specimen 8I are 2.90 m by 1.98 m representing isolated interior connection from the typical two-span specimen. The slab dimensions of specimen 9E are 1.58 m by 1.98 m representing isolated corner connection from the typical two-span specimen. For all specimens, the average effective depth of the slab d_{ave} is 91.4 mm. Figure 7.11 shows typical reinforcement layouts of the two-span specimens. Within the width of $c + 3h$, the top reinforcement ratio ρ_c is 0.83%. Outside this width the top and bottom reinforcement ratios ρ_m are 0.31%. Table 7.3 shows the specimens properties. In Table 7.3, the steel bars used as the main reinforcement was Grade 60 Type 2 deformed steel bars with minimum yield strength of 60 ksi (414 MPa). The steel bars were obtained from the same supplier but delivered in two different loads. Coupons of each bar size were taken for each bar size from each delivery and tested for tensile strength. Five samples were taken for each batch for the tensile test. The elastic moduli obtained from the laboratory test for first and second batches are

27,400 ksi (188,923 MPa) and 29,460 ksi (203,127 MPa), respectively. The author still used the actual value of elastic modulus of the first batch in the analysis since it was only slightly lower than the elastic modulus of reinforcing steel, which approximately ranges from 190 GPa to 200 GPa. The modulus of rupture $f_r = 0.7\sqrt{f'_c}$ (MPa) based on ACI 318 was used in the analysis.

Figures 7.12 – 7.15 show the finite element mesh of each specimen. Figures 7.16 – 7.21 show the comparisons of lateral force-drift ratio relationships between the results of the analysis using the proposed reduced slab stiffness, the analysis using $\beta = 1/3$ of gross section stiffness, and the experimental data. The unbalanced moment is represented by the lateral force. Figures 7.16 – 7.21 show that the analysis using the proposed reduced slab stiffness can predict the lateral force-drift ratio relationships fairly accurate up to the failure state. They also show that the linear line obtained from the analysis using uniform reduced slab stiffness $\beta = 1/3$ does not represent the lateral force-drift ratio relationships correctly. Taking example of specimen 2C in Figure 7.16, the linear line intersects the experimental curve at 2.3% lateral drift. It underestimates the lateral force for the lateral drifts below 2.3% and overestimates the lateral force for the lateral drifts beyond 2.3%. It supports the argument by Robertson (1997) that uniform reduced slab stiffness does not represent the actual behavior of the cracked slab. Figures 7.16 – 7.21 also show that the punching shear failure criterion by the ACI code is conservative. It indicates that if the analysis uses punching shear failure criterion by the ACI code, it will produce lateral force-drift ratio curve that terminates at lower lateral force than actual failure of lateral force-drift ratio curve from the experimental data.

7.5.2 Specimens under Unidirectional Lateral Load by Megally and Ghali (2000a)

Five edge slab-column connections specimens were tested under combination of gravity load V_g and unidirectional lateral load in N-S direction by Megally and Ghali (2000a). Specimen MG-2A without any shear reinforcement was used as the control specimen. Four specimens with studs shear reinforcement, MG-3, MG-4, MG-5, and MG-6, were tested with spacing between shear studs and

transferred gravity shear force as variables. The slab dimensions of all specimens are 1.90 m by 1.35 m. The slab thickness is 152 mm. The column dimensions are 254 mm x 254 mm. The column height is 1.55 m. The unbalanced moment-lateral drift relationship of specimen MG-2A was used as comparison with the results of the analysis using the proposed reduced slab stiffness and uniform reduced slab stiffness $\beta = 1/3$. The unbalanced moment-lateral drift relationships of specimens MG-3 and MG-6, which were subjected to the same value of gravity shear force as specimen MG-2A, were also plotted to show the increase of lateral drift capacity due to the presence of shear studs. Specimen MG-3 has larger spacing between shear studs of $0.75d$ compared to specimen MG-6 with $0.44d$. The properties of specimens MG-2A, MG-3, and MG6 are shown in Table 7.4. Modulus of rupture $f_r = 0.7\sqrt{f'_c}$ (MPa) and elastic modulus of concrete $E_c = 4700\sqrt{f'_c}$ (MPa) based on ACI 318 were used in the analysis. Figure 7.22 shows the typical rebars layouts of the specimens. Figure 7.23 shows finite element mesh of specimen MG-2A. Comparison between unbalanced moment-lateral drift relationship between the experimental data and the analysis results is shown in Figure 7.24. It shows that the analysis using the proposed reduced slab stiffness can predict unbalanced moment-lateral drift relationship fairly accurate up to the failure state. The analysis using uniform reduced slab stiffness does not represent the relationship correctly. It is also shown that the punching shear criterion for specimen MG-2A (without shear reinforcement) specified by the ACI code is conservative.

7.5.3 Specimens under Unidirectional Lateral Load by Robertson et al. (2002)

Four isolated interior slab-column connections were tested by Robertson et al. (2002). The typical slab dimensions are 3 m by 2.75 m. The slab thickness is 115 mm. The column dimensions are 250 mm x 250 mm. The column height is 1.525 m. All specimens were subjected to combination of gravity shear force V_u and unidirectional lateral load in N-S direction. The test parameter was the types of shear reinforcement. Specimen 1C without any shear reinforcement was selected as control specimen. Closed hoop stirrups were used for specimen 2CS, single leg

stirrups were used for specimen 3SL, and shear studs were used for specimen 4HS. Typical rebars layouts are shown in Figure 7.25. Top reinforcement ratio of 0.70% and bottom reinforcement ratio of 0.42% were placed within the width of $c + 3h$. Within the column strips, the reinforcement ratios of 0.49% and 0.38% were placed at the top and bottom layers, respectively. Top and bottom reinforcement ratios of 0.33% were placed outside the column strips. Table 7.5 shows the specimens properties. Elastic modulus of concrete $E_c = 4700\sqrt{f'_c}$ (MPa) based on ACI 318 was used in the analysis. Figure 7.26 shows finite element mesh of specimen 1C. The comparison of the lateral force-drift ratio relationship between the results of the analysis using the proposed reduced slab stiffness, the analysis using uniform reduced slab stiffness $\beta=1/3$, and the experimental data of specimen 1C is shown in Figure 7.27. The unbalanced moment is represented by the lateral force. The lateral force-drift ratio curves from specimens 2CS, 3SL, and 4HS were also plotted to show the increase of lateral drift capacity due to the presence of shear reinforcements. Figure 7.27 shows that the analysis using the proposed reduced slab stiffness produces a good prediction of lateral force-drift ratio relationship. The result of the analysis using uniform reduced slab stiffness does not represent the lateral force-drift ratio relationship correctly. It also shows that the punching shear criterion of the ACI code for specimen without shear reinforcement is fairly accurate.

7.5.4 Specimens under Bidirectional Lateral Load by Pan and Moehle (1992)

Four interior slab-column connections specimens were tested by Pan and Moehle (1992). Specimens 1 and 2 were subjected to high gravity load with gravity shear stress $v_u = 0.12\sqrt{f'_c}$ (MPa). Specimens 3 and 4 were subjected to low gravity load with gravity shear stress $v_u = 0.07\sqrt{f'_c}$ (MPa). Specimens 1 and 3 were tested under unidirectional lateral load in E-W direction. Specimen 2 and 4 were tested under bidirectional lateral load. All specimens were constructed identical. The slab dimensions are 3.97 m by 3.97 m. The slab thickness is 122 mm. The column dimensions are 274 mm x 274 mm. The column height is 1.83 m. Table 7.6 shows the specimens properties. Typical rebars layouts are shown in Figure 7.28.

The lateral force-drift ratio relationships from both unidirectional and bidirectional lateral loads were used as comparison between the analysis using the proposed reduced slab stiffness and the experimental data. The unbalanced moment is represented by the lateral force. Figure 7.29 shows typical finite element mesh of all specimens. Figures 7.30 – 7.33 show the comparisons of lateral force-drift ratio relationships obtained from the experimental data and the analysis using the proposed reduced slab stiffness. The results of the analysis using uniform reduced slab stiffness $\beta=1/3$ are also shown. Figures 7.30 – 7.33 show that the analysis using the proposed reduced slab stiffness can predict lateral force-drift ratio relationships from both unidirectional and bidirectional lateral loads fairly accurate. The linear relationship obtained from the analysis using uniform reduced slab stiffness does not represent the actual lateral force-drift ratio relationship correctly. The linear line underestimates the lateral forces below point of intersect with the experimental curve and overestimates the lateral forces beyond it. The effect of bidirectional lateral load is indicated by lower envelope curves for bidirectional lateral load compared to envelope curves for unidirectional lateral load. The stiffness reduction of specimens subjected to bidirectional lateral load is also more prominent compared to that of specimens subjected to unidirectional lateral load. Taking example of specimen 1 (high gravity, unidirectional lateral load) and specimen 2 (high gravity, bidirectional lateral load) in Figure 7.30, the envelope curve of specimen 1 is higher than the envelope curve of specimen 2 under the same lateral load path (1, 2, 13, 14 or E-W). This effect is not taken into account in the analysis using the proposed reduced slab stiffness. Figures 7.30 – 7.33 also show that the punching shear failure criterion of the ACI code is fairly accurate.

7.5.5 Specimens under Bidirectional Lateral Load Tested at NTU – Singapore

A total of 19 rectangular slab-column connections were tested under three experimental programs at Nanyang Technological University (NTU), Singapore. The first experimental program consisted of five rectangular edge slab-column connections tested by Anggadaja (2006). The second experimental program consisted of five interior slab-column connections tested by Tan and Teng (2005).

The third experimental program, which is described in Chapters 4 and 5, consisted of five rectangular corner slab-column connections and four slab-column connections with 135-degree slabs tested by the author.

Five edge slab-column connections were tested by Anggadajaja (2006). All specimens were constructed identical. The slab dimensions are 4 m by 2.9 m. The slab thickness is 135 mm. The rectangular column dimensions are 900 mm x 180 mm. The typical column height is 2.7 m. Table 7.7 shows all specimens properties. The average concrete strength f'_c is 34 MPa. Modulus of rupture $f_r = 0.7\sqrt{f'_c}$ (MPa) based on ACI 318 was used in the analysis. T13 or 13 mm diameter deformed rebars and T10 or 10 mm diameter deformed rebars were used as the top and bottom slab reinforcements, respectively. The tension test of the steel bars resulted in the yield stresses f_y of 520 MPa and 530 MPa and Young's moduli of 177 GPa and 185 GPa for T10 and T13 rebars, respectively. The top reinforcement ratio of 1.1% was placed within the width of $c + 3h$ in both x and y axes. Top rebars in N-S direction or stronger column direction were placed at the outermost layer. The uniform reinforcement ratio of 0.33% was placed at the bottom layers in both x and y axes. Figure 7.34 shows the typical reinforcement layouts.

Specimen E0U was subjected to gravity load only. Specimens E1H and E2H were subjected to high gravity shear force $V_u/V_c = 0.31$ and unidirectional lateral load in N-S and E-W directions, respectively. Specimen E12L was subjected to low gravity shear force $V_u/V_c = 0.16$ and bidirectional lateral load. Specimen E12H was subjected to bidirectional lateral load combined with high gravity shear force $V_u/V_c = 0.31$. The unbalanced moment-lateral drift relationships of specimens E1H, E2H, E12H, and E12L were used as comparisons with the results of the analysis using the proposed reduced slab stiffness. Figure 7.35 shows the typical finite element mesh of the specimens. Figures 7.36 – 7.43 show that the analysis using the proposed reduced slab stiffness can predict the unbalanced moment-lateral drift relationships of both unidirectional and bidirectional lateral loads fairly accurate. The linear line obtained from the analysis using uniform reduced slab stiffness does not represent the actual unbalanced moment-lateral drift relationship.

The punching shear failure criterion of the ACI code is found to be conservative except for specimen E12L under unidirectional lateral load in E-W direction.

A total of five rectangular interior slab-column connections were tested by Tan and Teng (2005). The typical slab dimensions are 4.5 m by 3.5 m. The slab thickness is 150 mm. The column dimensions are 900 mm x 180 mm. The typical column height is 2.25 m. The concrete strength f'_c is 40 MPa. The yield stresses f_y of the steel bars are 520 MPa for T10 (10 mm diameter) and 530 MPa for T13 (13 mm diameter). The slab was reinforced with top reinforcement ratio of 1.2% within the width of $c + 3h$ and bottom reinforcement ratio of 0.4%. The rebars parallel to y -axis were placed at the outermost layer. Shear studs with overall height of 150 mm and 10 mm stem diameter were used as shear reinforcement in two specimens. The typical rebars layouts are shown in Figure 7.44.

Specimen YL-L1 was tested under low gravity shear force $V_u/V_c = 0.17$ and unidirectional lateral load in weaker column direction parallel to the x -axis. Specimen YL-H1V with stud shear reinforcement (SSR) was tested under high gravity shear force $V_u/V_c = 0.28$ and unidirectional lateral load parallel to the x -axis. Specimens YL-H2 and YL-H2V (with SSR) were tested under high gravity shear force $V_u/V_c = 0.28$ and bidirectional lateral load. Specimen YL-L2 was tested under low gravity shear force $V_u/V_c = 0.17$ and bidirectional lateral load. Table 7.8 shows the specimens properties. Modulus of rupture of concrete $f_r = 0.7\sqrt{f'_c}$ (MPa) and elastic modulus of concrete $E_c = 4700\sqrt{f'_c}$ (MPa) based on ACI 318 were used in the analysis. Figure 7.45 shows typical finite element mesh of the specimens. Figures 7.46 - 7.49 show the comparisons of the unbalanced moment-lateral drift relationships between the experimental data and the analysis results using the proposed reduced slab stiffness. The unbalanced moment-lateral drift curves of the specimens with SSR were also plotted to show the increase in the lateral drift capacity due to the presence of shear studs. Figures 7.46 - 7.49 show that the analysis using the proposed reduced slab stiffness predicts the unbalanced moment-lateral drift relationships conservatively. The analysis using uniform reduced slab stiffness $\beta=1/3$ only produces one correct value of unbalanced

moment at the intersect point with the experimental curve. They also show that the maximum unbalanced moments of the specimens without shear studs subjected to unidirectional lateral load are mostly overestimated by the punching shear failure criterion of the ACI code.

Five corner slab-column connections were tested by the author in the experimental program presented in Chapter 4. Figures 4.2 shows the typical rebars layouts of the corner specimens. The properties of the corner specimens are shown in Tables 4.1. Corner specimens consisted of three specimens without shear reinforcement, specimens SC-H, SC-LD, and SC-HD, and two specimens with studs shear reinforcement (SSR), specimens SC-LDS and SC-HDS.

Specimen SC-H was tested under gravity load only. Specimen SC-LD was tested under combined bidirectional lateral load and low gravity shear force $V_u / V_c = 0.21$. Specimen SC-LDS (with SSR) was tested under combined bidirectional lateral load and low gravity shear force $V_u / V_c = 0.19$. Specimens SC-HD and SC-HDS (with SSR) were tested under bidirectional lateral load and high gravity shear force $V_u / V_c = 0.26$. Young's Moduli of concrete are 26.6 GPa for specimens SC-H and SC-LD; 28.1 GPa for specimens SC-HD and SC-HDS; and 28.3 GPa for specimen SC-LDS. Modulus of rupture of concrete $f_r = 0.7\sqrt{f'_c}$ (MPa) based on ACI 318 was used in the analysis. Figure 7.50 shows typical finite element mesh of corner specimens. Figures 7.51 - 7.58 show the comparisons of the unbalanced moment-lateral drift relationships between the experimental data of specimens SC-LD, SC-LDS, SC-HD, and SC-HDS and the results of the analysis using the proposed reduced slab stiffness. They also show the increase in the lateral drift capacity due to the presence of shear studs. Figures 7.51 - 7.58 show that the analysis using the proposed reduced slab stiffness can predict the unbalanced moment-lateral drift relationships of those specimens fairly accurate. The analysis using uniform reduced slab stiffness $\beta = 1/3$ does not represent the actual envelope curves. They also show that the punching shear failure criterion of the ACI code is conservative.

A total of four slab-column connections with 135-degree slabs were tested by the author. The experimental data are presented in Chapter 5. Figures 5.3 shows

the typical rebars layouts of 135-degree specimens. The properties of 135-degree specimens are shown in Tables 5.1. The specimens consisted of two specimens without shear reinforcement, specimens SA-LD and SA-HD, and two specimens with studs shear reinforcement (SSR), specimens SA-LDS and SA-HDS.

Specimens SA-LD and SA-LDS (with SSR) were subjected to combination of bidirectional lateral load and low gravity shear force $V_u / V_c = 0.19$. Specimens SA-HD and SA-HDS (with SSR) were subjected to combination of bidirectional lateral load and high gravity shear forces $V_u / V_c = 0.27$ and 0.28 , respectively. Young's Moduli of concrete are 26.6 GPa for specimen SA-HD; 28.1 GPa for specimen SA-LD; 26.1 GPa for specimen SA-HDS; and 28.3 GPa for specimen SC-LDS. Modulus of rupture of concrete $f_r = 0.7\sqrt{f'_c}$ (MPa) based on ACI 318 was used in the analysis.

Figure 7.59 shows typical finite element mesh of 135-degree specimens. Figures 7.60 – 7.67 show the comparisons of the unbalanced moment-lateral drift relationships between the experimental data and the results of the analysis using the proposed reduced slab stiffness. The unbalanced moment-lateral drift curves of the specimens with SSR were also plotted to show the increase in the lateral drift capacity due to the presence of shear studs. Figures 7.60 – 7.67 show that the unbalanced moment-lateral drift relationships of those specimens can be predicted fairly accurate by the analysis using the proposed reduced slab stiffness. The analysis using uniform reduced slab stiffness $\beta = 1/3$ does not represent the actual relationships correctly. They also show that the punching shear failure criterion of the ACI code is conservative.

7.5.6 Multi-Panel Specimen under Bidirectional Lateral Load by Hwang and Moehle (2000a)

A nine-panel flat plate frame was tested by Hwang and Moehle (2000a). The plan view of the specimen and each column dimension are shown in Figure 3.1. The slab dimensions are 324 in. (8.23 m) by 216 in. (5.49 m). The dimensions of the columns are 6.4 in. x 6.4 in. (162.6 mm x 162.6 mm), 9.6 in. x 9.6 in. (243.8 mm x 243.8 mm), 9.6 in. x 4.8 in. (243.8 mm x 121.9 mm), and 12.8 in. x 6.4 in. (325.1

mm x 162.6 mm). The slab thickness is 3.2 in. (81.3 mm). The columns extended 12 in. (304.8 mm) above the slab and 48 in. (1219.2 mm) below the slab. Table 7.9 shows the properties of the nine-panel specimen. The slab reinforcement was from three different heats. Slab rebars with yield strength f_y of 444 MPa were placed in the column strip in East-West (E-W) direction. Slab rebars with yield strength f_y of 456 MPa were placed in the column strip in North-South (N-S) direction. Slab rebars with yield strength f_y of 487 MPa were placed in the remaining locations.

The slab is symmetric about a centreline in E-W direction as shown in Figure 3.1. The slab portion below the centreline was designed based on the shear and moment design requirements of ACI 318-83, whereas the remainder was designed for moment redistribution from the more heavily stressed negative moment regions around the columns to the positive moment regions of the column strips. The slab reinforcement layouts are shown in Figure 7.68. Figure 7.69 shows the finite element mesh of the specimen.

The specimen was subjected to combination of imposed gravity load of 78 psf (3.73 kPa) and bidirectional lateral load in E-W and N-S directions. Lateral drift was first imposed in N-S direction for two cycles and then it returned to zero drift. The same sequence was immediately applied in E-W direction. This procedure was repeated with increasing lateral drift. Figures 7.70 and 7.71 show that the analysis using the proposed reduced slab stiffness can predict the lateral force-drift ratio relationships fairly accurate up to the failure state. They also show that the linear line obtained from the analysis using uniform reduced slab stiffness $\beta = 1/3$ does not represent the lateral force-drift ratio relationships correctly.

7.6 CONCLUSION

The accuracy of reduced slab stiffness computation due to slab cracking in the analysis is very important for obtaining representative values of transferred unbalanced moments at the slab-column connections. The proposed modified equations for effective moment of inertia for slabs were presented in this chapter. These equations are used for obtaining reduced slab stiffness factors of cracked slab. These factors are used to update the stiffness matrix components of the slab

element after cracking. The model of drop of stiffness beyond ultimate limit state for slab element is also proposed in the calculation of slab effective moment of inertia for slab-column connections. Generally the proposed model of reduced slab stiffness does not include any failure criteria. It is intended for obtaining accurate reduced slab stiffness for complete load-deformation relationships of slab-column connections before failures. Unlike currently available models such as Effective Beam Width, Equivalent Frame Model, and simplified frame analysis which are only applicable for flat plate structures with regular column layouts, the proposed model of reduced slab stiffness is also applicable for irregular column layouts. The accuracy of the proposed model in modeling reduced slab stiffness has been shown much better compared to applying uniform reduced slab stiffness, which is commonly used in the available models. It should also be noted that the analytical results of the finite element nonlinear analysis of layered plates with proper material model would be better than the proposed model, although currently its computational efficiency is not suitable for design purpose. The proposed model of reduced slab stiffness, which has relatively high computational efficiency, should be useful for design engineers dealing with flat plate slab-column connections in term of obtaining accurate slab deflection, inelastic lateral drift, and design value of transferred unbalanced moment.

Table 7.1. Properties of One-Way Slab, Two-Way Slab, Slab A3, and Slab C1

Specimen	Dimension w x l x t (mm)	f'_c (MPa)	E_c (MPa)	ρ_x (%)	ρ_y (%)
One-Way Slab (Jofriet and McNeice, 1971)	305 x 864 x 44	39.3	28270	-	0.80
Two-Way Slab (Jofriet and McNeice, 1971)	914 x 914 x 44	38.0	28623	0.85	0.85
Slab A3 (Aghayere and MacGregor, 1990)	1830 x 1830 x 65	32.2	23150	0.34	0.40
Slab C1 (Ghoneim and MacGregor, 1994)	1830 x 1830 x 68	25.2	21300	0.38	0.38

Table 7.2. Properties of 7-Column Specimen

Specimen	Dimension w x l x t (mm)	f'_c (MPa)	E_c (MPa)	Rebars Type (dia.)	f_y (MPa)	E_s (MPa)
7-Column Specimen (Tan and Teng, 2001)	6300 x 5100 x 100	40.7	27100	T10 (10 mm.)	518	177000
				R10 (10 mm.)	372	196000
				A193 (7 mm)	583	198000

Table 7.3. Properties of the Specimens Tested by Robertson (1990)

Specimen	Type	f'_c (MPa)	E_c (MPa)	f_y (MPa)	E_s (MPa)	ρ_c (%)	ρ_m (%)	d_{ave} (mm)	V_g (kPa)
2C	Two-Span	33.0	27011	500.2	188923	0.83	0.31	91.4	6.7
5SO	Two-Span	38.0	28959	500.2	188923	0.83	0.31	91.4	6.7
6LL	Two-Span	32.2	26670	524.9	203127	0.83	0.31	91.4	20.1
7L	Two-Span	30.8	26063	524.9	203127	0.83	0.31	91.4	13.7
8I	Single Col	39.3	29465	524.9	203127	0.83	0.31	91.4	6.7
9E	Single Col	39.3	29465	524.9	203127	0.83	0.31	91.4	6.7

Table 7.4. Properties of Specimens Tested by Megally and Ghali (2000a)

Slab	f'_c (MPa)	E_c (MPa)	f_y (MPa)	E_s (MPa)	ρ_{topx} CS (%)	ρ_{topy} CS (%)	ρ_{botx} (%)	ρ_{boty} (%)	h (mm)	V_g (kN)
MG-2A	31.6	26418	377.2	200000	1.29	1.58	0.64	0.95	152	120.1
MG-3	33.6	27252	377.2	200000	1.29	1.58	0.64	0.95	152	120.1
MG-6	30.1	25787	377.2	200000	1.29	1.58	0.64	0.95	152	120.1

Table 7.5. Properties of Specimens Tested by Robertson et al. (2002)

Slab	f'_c (MPa)	f_r (MPa)	E_c (MPa)	f_y (MPa)	f_{yv} (MPa)	$\rho_{top\ c+3h}$ (%)	$\rho_{top\ CS}$ (%)	ρ_{bot} (%)	h (mm)	V_u/V_c
1C	35.4	4.49	27964	420	465	0.70	0.49	0.33	115	0.25
2CS	31.4	3.54	26337	420	465	0.70	0.49	0.33	115	0.27
3SL	43.4	4.33	30963	420	465	0.70	0.49	0.33	115	0.23
4HS	38.2	3.49	29049	420	465	0.70	0.49	0.33	115	0.24

Table 7.6. Properties of Specimens Tested by Pan and Moehle (1992)

Slab	f'_c (MPa)	E_c (MPa)	f_t (MPa)	f_y (MPa)	$\rho_{top\ CS}$ (%)	ρ_{bot} (%)	h (mm)	v_u (MPa)	Lateral Load
1	33.3	25939	3.93	472	0.76	0.25	122	$0.12\sqrt{f'_c}$	Unidirectional E-W
2	33.3	25939	3.93	472	0.76	0.25	122	$0.12\sqrt{f'_c}$	Bidirectional
3	31.4	24167	3.72	472	0.76	0.25	122	$0.07\sqrt{f'_c}$	Unidirectional E-W
4	31.4	24167	3.72	472	0.76	0.25	122	$0.07\sqrt{f'_c}$	Bidirectional

Table 7.7. Properties of Specimens Tested by Anggadajaja (2006)

Slab	f'_c (MPa)	E_c (MPa)	f_y (MPa)	$\rho_{top\ c+3h}$ (%)	ρ_{bot} (%)	h (mm)	V_u/V_c	Lateral Load
E1H	34	26000	520(T10), 530(T13)	1.10	0.33	135	0.31	Unidirectional N-S
E2H	34	26000	520(T10), 530(T13)	1.10	0.33	135	0.31	Unidirectional E-W
E12L	34	26000	520(T10), 530(T13)	1.10	0.33	135	0.16	Bidirectional
E12H	34	26000	520(T10), 530(T13)	1.10	0.33	135	0.31	Bidirectional
E0U	34	26000	520(T10), 530(T13)	1.10	0.33	135	0.77	-

Table 7.8. Properties of Specimens Tested by Tan and Teng (2005)

Slab	f'_c (MPa)	f_y (MPa)	$\rho_{top\ c+3h}$ (%)	ρ_{bot} (%)	h (mm)	V_u/V_c	Lateral Load	Shear Studs
YL-L1	40	520(T10), 530(T13)	1.20	0.40	150	0.17	Unidirectional (x-dir)	No
YL-H2	40	520(T10), 530(T13)	1.20	0.40	150	0.28	Bidirectional	No
YL-L2	40	520(T10), 530(T13)	1.20	0.40	150	0.17	Bidirectional	No
YL-H2V	40	520(T10), 530(T13)	1.20	0.40	150	0.28	Bidirectional	Yes
YL-H1V	40	520(T10), 530(T13)	1.20	0.40	150	0.28	Unidirectional (x-dir)	Yes

Table 7.9. Properties of Nine-Panel Specimen Tested by Hwang and Moehle (2000a)

Specimen	Dimension w x l x t	f'_c (MPa)	E_c (MPa)	f_t (MPa)	Rebars Type (dia.)	f_y (MPa)
Nine-Panel Specimen (Hwang and Moehle, 2000a)	8.23mx5.49mx81.3mm	21.8	17860	2.6	No.2 (6.35 mm.) for Col Strip E-W	444
					No.2 (6.35 mm.) for Col Strip N-S	456
					No.2 (6.35 mm) for Remaining Locations	487

Fig. 7.1 Plate-Bending Elements

Fig. 7.2 Finite Element Meshes of the Slabs Tested by Jofriet and McNeice (1971)

Fig. 7.3 Comparisons between the Analysis Using the Proposed Reduced Slab Stiffness and the Experimental Data of Slabs Tested by Jofriet and McNeice (1971)

Fig. 7.4 Typical Finite Element Mesh of Slabs A3 and C1

Fig. 7.5 Comparisons between the Analysis Using the Proposed Reduced Slab Stiffness and the Experimental Data of Slabs A3 and C1

Fig. 7.6 Geometry of 7-Column Specimen

Fig. 7.7 Rebars Layouts of 7-Column Specimen

Fig. 7.8 Finite Element Mesh of 7-Column Specimen

Fig. 7.9a Displacement Comparisons at Points D1, D2, D3, D4, D5, and D6

Fig. 7.9b Displacement Comparisons at Points D7, D8, D9, D10, and D11

Fig. 7.9c Displacement Comparisons at Points D12, D13, D14, D15, and D16

(a) Model of Drop of Stiffness for Displacement-Based Analysis
by Hueste and Wight (1997)

(b) The Proposed Model of Drop of Stiffness for Force-Based Analysis

Fig. 7.10 Moment-Curvature Relationship beyond Ultimate Limit State

(a) Top Reinforcement

(b) Bottom Reinforcement

Fig. 7.11 Typical Rebars Layouts of Two-Span Specimens Tested by Robertson (1990)

Fig. 7.12 Finite Element Mesh of Specimens 2C, 6LL, and 7L

Fig. 7.13 Finite Element Mesh of Specimen 5SO

Fig. 7.14 Finite Element Mesh of Specimen 8I

Fig. 7.15 Finite Element Mesh of Specimens 9E

Fig. 7.16 Lateral Force vs. Drift Ratio Comparison of Specimen 2C

Fig. 7.17 Lateral Force vs. Drift Ratio Comparison of Specimen 5SO

Fig. 7.18 Lateral Force vs. Drift Ratio Comparison of Specimen 6LL

Fig. 7.19 Lateral Force vs. Drift Ratio Comparison of Specimen 7L

Fig. 7.20 Lateral Force vs. Drift Ratio Comparison of Specimen 8I

Fig. 7.21 Lateral Force vs. Drift Ratio Comparison of Specimen 9E

(a) Top Rebars

(b) Bottom Rebars

Fig. 7.22 Typical Rebars Layouts of Specimens Tested by Megally and Ghali (2000a)

Fig. 7.23 Finite Element Mesh of Specimen MG-2A

Fig. 7.24 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens
Tested by Megally and Ghali (2000a)

(a) Top Rebars

(b) Bottom Rebars

Fig. 7.25 Typical Rebars Layouts of Specimens Tested by Robertson et al. (2002)

Fig. 7.26 Finite Element Mesh of Specimen 1C

Fig. 7.27 Lateral Force vs. Drift Ratio Comparisons of Specimens Tested by Robertson et al. (2002)

(a) Top Rebars

(b) Bottom Rebars

Fig. 7.28 Typical Rebars Layouts of Specimens Tested by Pan and Moehle (1992)

Fig. 7.29 Typical Finite Element Mesh of Specimens Tested by Pan and Moehle (1992)

Fig. 7.30 Lateral Force vs. Drift Ratio Comparisons of Specimens 1 and 2 in E-W Direction

Fig. 7.33 Lateral Force vs. Drift Ratio Comparisons of Specimen 4 in N-S direction

Fig. 7.34 Typical Rebars Layouts of Specimens Tested by Anggadajaja (2006)

Fig. 7.35 Typical Finite Element Mesh of Specimens Tested by Anggadajaja (2006)

Fig. 7.36 Unbalanced Moment vs. Drift Ratio Comparison of Specimen E12L under Unidirectional Lateral Load in E-W Direction

Fig. 7.37 Unbalanced Moment vs. Drift Ratio Comparison of Specimen E12L under Unidirectional Lateral Load in N-S Direction

Fig. 7.38 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens E12H and E2H under Unidirectional Lateral Load in E-W Direction

Fig. 7.39 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens E12H and E1H under Unidirectional Lateral Load in N-S Direction

Fig. 7.40 Unbalanced Moment vs. Drift Ratio Comparison of Specimen E12L under Bidirectional Lateral Load in E-W Direction

Fig. 7.41 Unbalanced Moment vs. Drift Ratio Comparison of Specimen E12L under Bidirectional Lateral Load in N-S Direction

Fig. 7.42 Unbalanced Moment vs. Drift Ratio Comparison of Specimen E12H under Bidirectional Lateral Load in E-W Direction

Fig. 7.43 Unbalanced Moment vs. Drift Ratio Comparison of Specimen E12H under Bidirectional Lateral Load in N-S Direction

(a) Top Rebars

(b) Bottom Rebars

Fig. 7.44 Typical Rebars Layouts of Specimens Tested by Tan and Teng (2005)

Fig. 7.45 Typical Finite Element Mesh of Specimens Tested by Tan and Teng (2005)

Fig. 7.46 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens YL-L series in X- Direction

Fig. 7.47 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens YL-L series in Y-Direction

Fig. 7.48 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens YL-H series in X-Direction

Fig. 7.49 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens YL-H series in Y-Direction

Fig. 7.50 Typical Finite Element Mesh of Corner Specimens

Fig. 7.51 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens SC-H series under Unidirectional Lateral Load in X-Direction

Fig. 7.52 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SC-H series under Unidirectional Lateral Load in Y-Direction

Fig. 7.53 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SC-L series under Unidirectional Lateral Load in X-Direction

Fig. 7.54 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SC-L series under Unidirectional Lateral Load in Y-Direction

Fig. 7.55 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SC-H series under Bidirectional Lateral Load in X-Direction

Fig. 7.56 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SC-H series under Bidirectional Lateral Load in Y-Direction

Fig. 7.57 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SC-L series under Bidirectional Lateral Load in X-Direction

Fig. 7.58 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens SC-L series under Bidirectional Lateral Load in Y-Direction

Fig. 7.59 Typical Finite Element Mesh of 135-Degree Specimens

Fig. 7.60 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SA-H series under Unidirectional Lateral Load in X-Direction

Fig. 7.61 Unbalanced Moment vs. Drift Ratio Comparisons

of Specimens SA-H series under Unidirectional Lateral Load in Y-Direction

Fig. 7.62 Unbalanced Moment vs. Drift Ratio Comparisons

of Specimens SA-L series under Unidirectional Lateral Load in X-Direction

Fig. 7.63 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens SA-L series under Unidirectional Lateral Load in Y-Direction

Fig. 7.64 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens SA-H series under Bidirectional Lateral Load in X-Direction

Fig. 7.65 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SA-H series under Bidirectional Lateral Load in Y-Direction

Fig. 7.66 Unbalanced Moment vs. Drift Ratio Comparisons
of Specimens SA-L series under Bidirectional Lateral Load in X-Direction

Fig. 7.67 Unbalanced Moment vs. Drift Ratio Comparisons of Specimens SA-L series under Bidirectional Lateral Load in Y-Direction

(a) Top Rebars

(b) Bottom Rebars

Fig. 7.68 Rebars Layouts of Nine-Panel Specimen Tested by Hwang and Moehle (2000a)

Fig. 7.69 Finite Element Mesh of Nine-Panel Specimen

Tested by Hwang and Moehle (2000a)

Fig. 7.70 Lateral Force vs. Drift Ratio Comparisons of

Nine-Panel Specimen in N-S Direction

Fig. 7.71 Lateral Force vs. Drift Ratio Comparisons of
Nine-Panel Specimen in E-W Direction

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

Experimental program involving five rectangular corner slab-column connections and four slab-column connections with 135-degree slabs, which are often found in modern flat plate structures, were presented. The experimental results cover the effects of bidirectional lateral load, gravity load magnitude, and the use of stud shear reinforcement (SSR). Five specimens without SSR failed under brittle punching shear while the other four specimens with SSR experienced ductile flexural failure. From the analysis of the experimental results, it can be concluded that higher gravity load leads to lower ultimate drift capacity and ductility. Unidirectional and bidirectional lateral loads result in different reduced slab stiffness of the connections. The experimental results show that the presence of stud shear reinforcement (SSR) increases the maximum unbalanced moment, drift capacity, and ductility. It also avoids significant loss of stiffness after reaching critical lateral drift. Analysis of the experimental data using eccentric shear model by the ACI 318 gives conservative result of shear stress ratio and in general it also predicts the type of failure fairly correctly.

The limit of gravity shear force ratio of rectangular corner slab-column connections and rectangular slab-column connections with 135-degree slabs to withstand critical lateral drift is lower than that of ACI-ASCE Committee 352 recommendation. It indicates that column rectangularity causes the drift capacity of the connection to be more severely affected by the gravity shear force ratio.

The accuracy of reduced slab stiffness due to slab cracking in the analysis is very important for obtaining the values of transferred unbalanced moments, which are important contributor to shear stresses of slab-column connections. The model of reduced slab stiffness, which is based on the modified equations for effective moment of inertia, was proposed. The proposed modified effective moment of inertia method itself has been shown to well represent the reduced stiffness of flexural members almost as accurately as the stress-strain equations involving the

integration of curvature and the use of actual stress-strain equations of the concrete and steel embedded in concrete. The accuracy of the proposed model of reduced slab stiffness has been shown much better compared to applying uniform reduced slab stiffness, which is commonly used in the available models such as Effective Beam Width, Equivalent Frame Model, and simplified frame analysis. The proposed model of reduced slab stiffness does not include any failure criteria. It is intended for obtaining accurate reduced slab stiffness for complete load-deformation relationships of slab-column connections before failures. Unlike currently available models which are only applicable for flat plate structures with regular column layouts, the proposed model of reduced slab stiffness is also applicable for irregular column layouts. The proposed model should be useful to design engineers for obtaining accurate slab deflection, inelastic lateral drift, and design value of transferred unbalanced moment of flat plate slab-column connections.

8.2 RECOMMENDATIONS

Flat plate structures with irregular column layouts and elongated columns sections are commonly used in the modern buildings due to architectural requirements. Unfortunately, the experimental data of their slab-column connections are still very limited. It leads to unclear aspects of their behavior under the combination of gravity and lateral loads.

The experimental data obtained from this experimental program and the proposed model of reduced slab stiffness should be useful for design engineers or researchers dealing with modern flat plate structures with irregular column layouts. However, there are still no available experimental data that cover various types of slab-column connections in modern flat plate structures, in term of slab and column section geometries. Thus, extensive research of irregular slab-column connections, which is still a very broad and challenging area, is recommended.

REFERENCES

ACI-ASCE Committee 352 (1989), Recommendations for Design of Slab-Column Connections in Monolithic Reinforced Concrete Structures (ACI 352.1R-89), American Concrete Institute, Michigan, 22 pp.

ACI Committee 318 (2005), Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (318R-05), American Concrete Institute, Farmington Hills, Michigan, 430 pp.

ACI Committee 421 (1999), Shear Reinforcement for Slabs (ACI 421.1R-99), American Concrete Institute, Farmington Hills, Michigan, 15 pp.

Aghayere, A. O., and MacGregor, J. G. (1990), "Tests of Reinforced Concrete Plates Under Combined In-Plane and Transverse Load" ACI Structural Journal, V. 87, No. 6, pp. 615-622.

Agrawal, G. L., Tulin, L. G., and Gerstle, K. H. (1965), "Response of Doubly Reinforced Concrete Beams to Cyclic Loading," ACI Journal, Proceedings V. 62, No. 7, pp. 823-835.

Al-Shaikh, A. H. and Al-Zaid, R. Z. (1993), "Effect of Reinforcement Ratio on the Effective Moment of Inertia of Reinforced Concrete Beams," ACI Structural Journal, V. 90, No. 2, pp. 144-149.

Al-Zaid, R. Z., Al-Shaikh, A. H., and Abu-Hussein, M. M. (1991), "Effect of Loading Type on the Effective Moment of Inertia of Reinforced Concrete Beams," ACI Structural Journal, V. 88, No. 2, pp. 184-190.

Anggadajaja, E. (2006), "Punching Shear Capacity of Slab-Column Connections Subjected to Gravity and Biaxial Cyclic Lateral Loading," Ph.D. Thesis, School of Civil and Environmental Engineering, Nanyang Technological University, Singapore.

Anggadajaja, E. and Teng, S. (2008), "Edge-Column Slab Connections Under Gravity and Lateral Loading," ACI Structural Journal, V. 105, No. 5, pp. 541-551.

Bach, C. and Graf, O. (1917), "Tests on Reinforced Concrete Beams for Obtaining Moment-Curvature Relationships," Bulletin No. 38, Deutscher Ausschuss für Eisenbeton, Berlin, pp. 38-41.

Banchik, C. A. (1987), "Effective Beam Width Coefficients for Equivalent Frame Analysis of Flat-Plate Structures," M.E. Thesis, University of California at Berkeley, California, 56 pp.

Bathe, K. J. (1996), "Finite Element Procedures," Prentice-Hall Inc., New Jersey.

Bazant, Z. P. and Oh, B. H. (1984), "Deformation of Progressively Cracking Reinforced Concrete Beams," ACI Journal, Proceedings V. 81, No. 3, pp. 268-278.

Belarbi, A. and Hsu, T. T. C. (1994), "Constitutive Laws of Concrete in Tension and Reinforcing Bars Stiffened by Concrete," ACI Structural Journal, V. 91, No. 4, pp. 465-474.

Bischoff, P. H. (2005), "Reevaluation of Deflection Prediction for Concrete Beams Reinforced with Steel and Fiber Reinforced Polymer Bars," Journal of Structural Engineering, Vol. 131, No. 5, pp. 752-767.

Branson, D. E. (1963), "Instantaneous and Time-Dependent Deflection of Simple and Continuous Reinforced Concrete Beams," HPR Publication No.7, Part 1, Alabama Highway Department, Bureau of Public Road, pp. 1-78.

Bresler, B. and Scordelis, A. C. (1963), "Shear Strength of Reinforced Concrete Beams," ACI Journal, Proceedings, V. 60, No. 1, pp. 51-72.

Clark, L. A., and Speirs, D. M. (1978), "Tension Stiffening in Reinforced Concrete Beams and Slabs under Short-Term Load," Technical Report No. 42.521, Cement and Concrete Association, London, 20 pp.

Computers and Structures, Inc. (1998), SAP2000: Integrated Finite Element Analysis and Design of Structures, Computer and Structures, Inc., California.

Cook, R. D., Malkus, D. S., and Plesha, M. E. (2002), Concepts and Applications of Finite Element Analysis, Wiley, New York.

Cox, K. C. (1941), "Tests of Reinforced Concrete Beams with Recommendations for Attaining Balanced Design," ACI Journal, Proceedings V. 38, No. 1, pp. 65-80.

Desayi P., and Krishnan, S. (1964), "Equation for Stress–Strain Curve of Concrete," ACI Journal, Proceedings V. 61, No3, pp. 345–350.

Desayi, P., and Seshadri, K. H. (1997), "Punching Shear Strength of Flat Slab Corner Column Connections. Part 1. Reinforced Concrete Connections," Proceedings of the Institution of Civil Engineers, Structures and Buildings, 122, pp. 10-20.

Dilger, W. H., and Brown, S. J. (1995), "Earthquake Resistance of Slab-Column Connections", Festschrift Prof. Dr. Hugo Bachmann Zum 60. Geburtstag, Institut für Baustatik und Konstruktion, ETH Zürich, pp. 22-27.

Dilger, W. H., and Cao, H. (1991), "Behavior of Slab-Column Connections Under Reversed Cyclic Loading," Proceedings of the 2nd International Conference of High-Rise Buildings, China.

Elgabry, A. A., and Ghali, A. (1987), "Test on Concrete Slab-Column Connections with Stud Shear Reinforcement Subjected to Shear-Moment Transfer," ACI Structural Journal, V. 84, No. 5, pp. 433-442.

Elgabry, A. A., and Ghali, A. (1990), "Design of Stud-Shear Reinforcement for Slabs," ACI Structural Journal, V. 87, No. 3, pp. 350-361.

Elgabry, A. A., and Ghali, A. (1996), "Moment Transfer by Shear in Slab-Column Connections," ACI Structural Journal, V. 93, No. 2, pp. 187-196.

Elgabry, A. A., and Ghali, A. (1996a), "Transfer of Moments between Columns and Slabs: Proposed Code Revisions," ACI Structural Journal, V. 93, No. 1, pp. 56-61.

Falamaki, M., and Loo, C. Y. (1992), "Punching Shear Tests of Half-Scale Reinforced Concrete Flat-Plate Models with Spandrel Beams," ACI Structural Journal, V. 89, No. 3, pp. 263-271.

Gardner, N. J., and Shao, X. Y. (1996), "Punching Shear of Continuous Flat Reinforced Concrete Slabs," ACI Structural Journal, V. 93, No. 2, pp. 218-228.

Ghali, A. (1993), "Deflection of Reinforced Concrete Member: A Critical Review," ACI Structural Journal, V. 90, No. 4, pp. 364-373.

Ghoneim, M. G., and MacGregor, J. G. (1994), "Tests of Reinforced Concrete Plates Under Combined Inplane and Lateral Loads," ACI Structural Journal, V. 91, No. 1, pp. 19-30.

Gilbert, I. R. (1999), "Deflection Calculation for Reinforced Concrete Structures - Why We Sometimes Get It Wrong," ACI Structural Journal, V. 96, No. 6, pp. 1027-1033.

Gilbert, I. R., and Smith, S. T. (2004), "Strain Localization and its Impact on the Ductility of RC Slabs Containing 500 MPa Reinforcement." *Proc., 18th Australasian Conf. on the Mechanic of Structures and Materials (ASMSM18)*, Australia, pp. 811-817.

Hammill, N., and Ghali, A. (1994), "Punching Shear Resistance of Corner Slab-Column Connections," ACI Structural Journal, V. 91, No. 6, pp. 697-707.

Hawkins, N. M., Mitchell, D., and Hanna, S. N. (1975), "Effects of Shear Reinforcement on the Reversed Cyclic Loading Behavior of Flat Plate Structures," Canadian Journal of Civil Engineering, V. 2, pp. 572-582.

Hueste, M.D. and Wight, J. K. (1999), "A Nonlinear Punching Shear Failure Model for Interior Slab-Column Connections," ASCE Journal of Structural Engineering, V. 125, No. 9, pp. 997-1008.

Hwang, S. J., and Moehle, J. P. (2000), "Models for laterally Loaded Slab-Column Frame," ACI Structural Journal, V. 97, No. 1, pp. 345-353.

Hwang, S. J., and Moehle, J. P. (2000a), "Vertical and Lateral Load Tests of Nine-Panel Flat-Plate Frame," ACI Structural Journal, V. 97, No. 1, pp. 193-204.

Ingvarsson, H. (1974), "Experimentellt Studium Av Betongplattor Understodda Av Hornpelare (Experimental Study of Concrete Slabs Supported by Corner Columns)," Meddelande Nr. 111, Institutionen For Byggnadsstatik, Kungliga Tekniska Hogskolon, Stockholm, 26 pp.

International Code Council (2003), International Building Code, IBC-03, Ill., 655 pp.

International Conference of Building Officials (1994), Uniform Building Code, 1994 Edition, Whittier, California.

International Conference of Building Officials (1997), Uniform Building Code, 1997 Edition, Whittier, California.

Islam, S., and Park, R. (1976), "Tests on Slab-Column Connections with Shear and Unbalanced Flexure," Journal of Structural Division, ASCE, V. 102, ST3, pp. 549-568.

Jofriet, J. C., and McNeice, G. M. (1971), "Finite Element Analysis of Reinforced Concrete Slabs," Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, V. 97, No. ST3, pp. 785-806.

Kinnunen, S., and Nylander, H. (1960), "Punching of Concrete Slabs without Shear Reinforcement," Transaction, No. 158, Royal Institute of Technology, Stockholm, 112 pp.

Lim, F. K., and Rangan, B. V. (1995), "Studies of Concrete Slabs with Stud Shear Reinforcement in Vicinity of Edge and Corner Columns," ACI Structural Journal, V. 92, No. 5, pp. 515-525.

MacGregor, J. G., and Bartlett, F. M. (2000), Reinforced Concrete: Mechanic and Design, Prentice-Hall Canada Inc., Scarborough, Ontario.

Megally, S. (1998), Punching Shear Resistance of Concrete Slabs to Gravity and Earthquake Forces, Ph.D. Dissertation, Department of Civil Engineering, University of Calgary, Calgary, Alberta, 468 pp.

Megally, S., and Ghali, A. (1994), "Design Considerations for Slab-Column Connections in Seismic Zones," ACI Structural Journal, V. 91, No. 3, pp. 303-314.

Megally, S., and Ghali, A. (2000), "Punching Shear Design of Earthquake-Resistant Slab-Column Connections," ACI Structural Journal, V. 97, No. 5, pp. 720-730.

Megally, S., and Ghali, A. (2000a), "Seismic Behavior of Edge Column-Slab Connections with Stud Shear Reinforcement," ACI Structural Journal, V. 97, No. 1, pp. 53-60.

Megally, S., and Ghali, A. (2000b), "Punching of Concrete Slabs Due to Column's Moment Transfer," ASCE Journal of Structural Division, V. 126, No. 2, pp. 180-189.

Moehle, J. P. (1988), "Strength of Slab-Column Edge Connections," ACI Structural Journal, V. 85, No. 11, pp. 89-98.

Mokhtar, A.S., Ghali, A., and Dilger, W.H. (1985), "Stud Shear Reinforcement for Flat Concrete Plates," ACI Journal, V. 82, No. 5, pp. 676-683.

Nawy, E.G. (2003), Reinforced Concrete: A Fundamental Approach, Prentice-Hall Inc., New York.

Okamura, H., Maekawa, K., and Sivasubramaniyam, S. (1985), "Verification of Modeling for Reinforced Concrete Finite Element," Proc. of Japan-US Seminar on Finite Element Analysis of Reinforced Concrete Structures, ASCE, pp.528-543.

Pan, A., and Moehle, J. P. (1989), "Lateral Displacement Ductility of reinforced Concrete Flat Plates," ACI Structural Journal, V. 86, No. 3, pp. 250-258.

Pan, A., and Moehle, J. P. (1992), "Experimental Study of Slab-Column Connections," ACI Structural Journal, V. 89, No. 6, pp. 250-258.

Park, R., and Gamble, W. L. (2000), Reinforced Concrete Slabs, Wiley, New York.

Paulay, T., and Priestley, M. J. N. (1992), Seismic Design of Reinforced Concrete and Masonry Buildings, Wiley, Canada.

Polak, M. A. (1996), "Effective Stiffness Model for Reinforced Concrete Slabs," Journal of Structural Engineering, V. 122, No. 9, pp. 1025-1030.

Robertson, I. N. (1990), "Seismic Response of Connections in Indeterminate Flat-Slab Subassemblies," Ph.D. Dissertation, Department of Civil Engineering, Rice University, Houston.

Robertson, I. N. (1997), "Analysis of Flat Slab Structures Subjected to Combined Lateral and Gravity Loads," ACI Structural Journal, V. 94, No. 6, pp. 723-729.

Robertson, I. N., and Durrani, A. J. (1990), "Seismic Response of Connections in Indeterminate Flat-Slab Subassemblies," Structural Report No. 41, Rice University, Houston.

Robertson, I. N., and Durrani, A. J. (1991), "Gravity Load effect on Seismic Behavior of Exterior Slab-Column Connections," ACI Structural Journal, V. 88, No. 3, pp. 255-267.

Robertson, I. N., and Durrani, A. J. (1992), "Gravity Load effect on Seismic Behavior of Interior Slab-Column Connections," ACI Structural Journal, V. 89, No. 1, pp. 37-45.

Robertson, I. N., Kawai, T., Lee, J., and Enomoto, B. (2002), "Cyclic Testing of Slab-Column Connections with Shear Reinforcement," ACI Structural Journal, V. 99, No. 5, pp. 605-613.

Saenz, L. P. (1964), "Discussion of Equation for the Stress-Strain Curve of Concrete by Prakash Desayi and S. Krishnan", ACI Journal, Proceedings V. 61, No. 9, pp. 1229-1235.

Scanlon, A. and Murray, D. W. (1982), "Practical calculation of Two-Way Slab Deflections," Concrete International: Design & Construction, V. 4, No. 11, pp. 43-50.

Sinha, B. P., Gerstle, K. H., and Tulin, L. G. (1964), "Response of Singly Reinforced Concrete Beams of Cyclic Loading," ACI Journal, Proceedings V. 61, No. 9, pp. 1021-1038.

Sozen, M. A. (1980), "Review of Earthquake Response of Reinforced Concrete Buildings with View to Drift Control," State-of-the-Art in Earthquake Engineering, Seventh World Conference on Earthquake Engineering, Istanbul, pp. 119-174.

Tamai, S., Shima, H., Izumo, J., and Okamura, H. (1988), "Average Stress-Strain Relationship in Post Yield Range of Steel Bar in Concrete," Concrete Library, JSCE, No. 11, pp. 117-129.

Tan, H. N., and Teng, S. (2001), "Concrete Flat Plate Floors with Irregular Column Layout: Flexural Design and Test of a 7-Column Floor Specimen," Final Report: BCA-NTU Joint Research on Flat-Plate Structures – Phase 2A, Nanyang Technological University, Singapore.

Tan, Y., and Teng, S. (2005), "Interior Slab-Rectangular Column Connections under Biaxial Lateral Loadings," ACI SP-232-9, American Concrete Institute, Farmington Hills, Michigan, pp. 147-174.

Taranath, S. B. (1988), Structural Analysis and Design of Tall Buildings, McGraw-Hill, New York.

Taranath, S. B. (2005), Wind and Earthquake Resistant Buildings: Structural Analysis and Design, Marcel Dekker, New York.

Tavio, and Teng, S. (2004), "Effective Torsional Rigidity of Reinforced Concrete Members," ACI Structural Journal, V. 101, No. 2, pp. 252-260.

Teng, S. and Branson, D. E. (1993), "Initial and Time-Dependent Deformation of Progressively Cracking Nonprestressed and Partially Prestressed Concrete Beams," ACI Structural Journal, V. 90, No. 5, pp. 480-488.

Teng, S., Cheong, H. K., Kuang, K. L., Geng, J. Z. (2004), "Punching Shear Strength of Slabs With Openings and Supported on Rectangular Columns," ACI Structural Journal, V. 101, No. 67, pp. 678-687.

Vanderbilt, M. D., and Corley, W. G. (1983), "Frame Analysis of Concrete Building," Concrete International, V. 5, No. 12, pp. 33-43.

Walker, P. R., and Regan, P. E. (1987), "Corner Column-Slab Connections in Concrete Flat Plates," Journal of Structural Engineering, ASCE, V. 113, No. 4, pp. 704-720.

Wey, E. H., and Durrani, A. J. (1992), "Seismic Response of Interior Slab-Column Connections with Shear Capitals," ACI Structural Journal, V. 89, No. 6, pp. 682-691.

Wood, R. H. (1968), "The Reinforcement of Slabs in Accordance with a Predetermined Field of Moments," Concrete, Vol. 2, No. 2, pp. 69-79.

Wood, R. H., and Armer, S. T. (1968), "The Theory of the Strip Method for Design of Slabs," Proceedings of Inst. Civil Eng., Vol. 41, No. 10, pp. 285-311.

Zaghlool, E. R. F, and de Paiva, H. A. R. (1973), "Test of Flat Plate Corner Column-Slab Connections," Journal of Structural Division, Proceedings of ASCE, V. 99, No. ST3, pp. 551-572.

Zaghlool, E. R. F, de Paiva, H. A. R., and Glockner, P. G. (1970), "Test of Reinforced Concrete Flat Plate Floors," Journal of the Structural Division, Proceedings of ASCE, V. 96, No. ST3, pp. 487-507.

APPENDIX I

- *EXPERIMENTAL SETUP*

Corner Specimen Setup – Column Short Side View

Corner Specimen Setup – Column Long Side View

135-Degree Specimen Setup – Column Short Side View

135-Degree Specimen Setup – Column Long Side View

Edge Link Supports

Connection between Edge Link and Strong Floor

Connection between Edge Link and Bottom Slab Surface

Bottom Column Support and Vertical Hydraulic Jack

Reaction Arm between Reaction Frame and Column Base

Connection between Reaction Arm and Column Base

Connection between Reaction Arm and Reaction Frame

Horizontal Hydraulic Jack between Reaction Frame and the Top of the Column

Connection between Horizontal Hydraulic Jack and the Top of the Column

Connection between Horizontal Hydraulic Jack and Reaction Frame

Prestressing Rods and Hydraulic Jacks on Top of the Column

Torsion Frame and Attached Torsion Arms

Connection between Torsion Arm and Slab Edge

Connection between Torsion Frame and Torsion Arm

Connection between Torsion Frame and Strong Floor

Steel Blocks Simulating Gravity Loads

Complete Setup of Specimen SC-H

Complete Setup of Specimen SC-HD

Complete Setup of Specimen SC-LD

Complete Setup of Specimen SC-HDS

Complete Setup of Specimen SC-LDS

Complete Setup of Specimen SA-HD

Complete Setup of Specimen SA-LD

Complete Setup of Specimen SA-HDS

Complete Setup of Specimen SA-LDS

APPENDIX II

- *TEST RESULTS OF CONCRETE AND
STEEL REBARS SAMPLES*

Concrete Test Result of Batch I

Sample Type	Sample ID.	Age (days)	Weight (gr)	Density (kN/m ³)	Force (kN)	f _{cu} (MPa)	f' _c (MPa)
Cube 15x15x15 cm ³	I-01 CUBE	8	8101	23.55	952	42.31	33.85
	I-02 CUBE	8	8049	23.40	978	43.47	34.77
	I-03 CUBE	14	8002	23.26	1085	48.22	38.58
	I-04 CUBE	14	8035	23.36	1142	50.76	40.60
	I-05 CUBE	21	7897	22.95	1198	53.24	42.60
	I-06 CUBE	21	7865	22.86	1078	47.91	38.33
	I-07 CUBE	28	7889	22.93	1039	46.18	36.94
	I-08 CUBE	28	7948	23.10	1194	53.07	42.45
	I-09 CUBE	28	7940	23.08	1177	52.31	41.85
	I-10 CUBE	42	7911	22.99	1093	48.58	38.86
	I-11 CUBE	42	7945	23.09	1190	52.89	42.31
	I-12 CUBE	50	7983	23.20	1229	54.62	43.70
	I-13 CUBE	50	7909	22.99	1235	54.89	43.91
	I-14 CUBE	57	7945	23.09	1170	52.00	41.60
	I-15 CUBE	57	7945	23.09	1238	55.02	44.02
Cylinder 15 cm dia.x 30cm	I-01 CYL	8	12269	22.70	577	40.83	32.66
	I-02 CYL	8	12184	22.55	536	37.91	30.33
	I-03 CYL	14	12224	22.62	673	47.62	38.10
	I-04 CYL	14	12196	22.57	675	47.77	38.22
	I-05 CYL	21	12163	22.51	711	50.26	40.21
	I-06 CYL	21	12138	22.46	698	49.35	39.48
	I-07 CYL	28	11917	22.05	637	45.07	36.05
	I-08 CYL	28	12024	22.25	629	44.51	35.61
	I-09 CYL	28	12130	22.45	552	39.04	31.23
	I-10 CYL	37	12269	22.70	685	48.45	38.76
	I-11 CYL	37	12184	22.55	650	45.98	36.78
	I-12 CYL	37	12224	22.62	640	45.27	36.22
	I-13 CYL	37	12196	22.57	647	45.77	36.61
	I-14 CYL	53	12000	22.21	642	45.39	36.31

Concrete Test Result of Batch II

Sample Type	Sample ID.	Age (days)	Weight (gr)	Density (kN/m ³)	Force (kN)	f _{cu} (MPa)	f' _c (MPa)
Cube 15x15x15 cm ³	II-01 CUBE	3	7993	23.23	925	41.11	32.89
	II-02 CUBE	3	7984	23.21	995	44.22	35.38
	II-03 CUBE	7	7949	23.11	1193	53.02	42.42
	II-04 CUBE	7	7981	23.20	1102	48.98	39.18
	II-05 CUBE	14	7969	23.16	1267	56.31	45.05
	II-06 CUBE	14	7989	23.22	1285	57.11	45.69
	II-07 CUBE	22	8085	23.50	1347	59.87	47.89
	II-08 CUBE	22	7880	22.90	1334	59.29	47.43
	II-09 CUBE	28	7955	23.12	1346	59.82	47.86
	II-10 CUBE	28	7880	22.90	1309	58.18	46.54
	II-11 CUBE	28	7956	23.13	1356	60.27	48.21
	II-12 CUBE	35	7941	23.08	1409	62.62	50.10
	II-13 CUBE	35	7951	23.11	1416	62.93	50.35
	II-14 CUBE	43	7885	22.92	1430	63.56	50.84
	II-15 CUBE	43	7914	23.00	1468	65.24	52.20
Cylinder 15 cm dia.x 30cm	II-01 CYL	3	12152	22.49	549	38.81	31.05
	II-02 CYL	3	12298	22.76	463	32.77	26.22
	II-03 CYL	28	12173	22.53	923	65.29	52.23
	II-04 CYL	28	12015	22.23	873	61.75	49.40
	II-05 CYL	35	12090	22.37	861	60.90	48.72
	II-06 CYL	35	12129	22.44	745	52.70	42.16
	II-07 CYL	43	12152	22.49	863	61.04	48.84
	II-08 CYL	43	12298	22.76	822	58.14	46.52
	II-09 CYL	43	12173	22.53	832	58.85	47.08
	II-10 CYL	43	12015	22.23	698	49.37	39.50

Concrete Test Result of Batch III

Sample Type	Sample ID.	Age (days)	Weight (gr)	Density (kN/m ³)	Force (kN)	f _{cu} (MPa)	f' _c (MPa)
Cube 15x15x15 cm ³	III-01 CUBE	28	8085	23.50	1137	50.53	40.42
	III-02 CUBE	28	7880	22.90	1254	55.72	44.58
	III-03 CUBE	28	7955	23.12	1226	54.49	43.60
	III-04 CUBE	28	7880	22.90	1134	50.39	40.31
Cylinder 15 cm dia.x 30cm	III-01 CYL	28	12015	22.23	634	44.82	35.85
	III-02 CYL	28	12090	22.37	655	46.30	37.04
	III-03 CYL	28	12129	22.44	679	48.04	38.43
	III-04 CYL	28	12152	22.49	580	41.04	32.83
	III-05 CYL	35	12298	22.76	575	40.67	32.53
	III-06 CYL	35	12173	22.53	606	42.89	34.31
	III-07 CYL	35	12015	22.23	588	41.62	33.30
Average =						46.95	37.56

Concrete Test Result of Batch IV

Sample Type	Sample ID.	Age (days)	Weight (gr)	Density (kN/m ³)	Force (kN)	f _{cu} (MPa)	f' _c (MPa)
Cube 15x15x15 cm ³	IV-01 CUBE	3	8101	23.55	818	36.36	29.08
	IV-02 CUBE	3	8049	23.40	787	34.98	27.98
	IV-03 CUBE	3	8002	23.26	685	30.44	24.36
	IV-04 CUBE	14	8035	23.36	1269	56.40	45.12
	IV-05 CUBE	14	7897	22.95	1278	56.80	45.44
	IV-06 CUBE	14	7865	22.86	1186	52.71	42.17
	IV-07 CUBE	20	7889	22.93	1371	60.93	48.75
	IV-08 CUBE	20	7948	23.10	1352	60.09	48.07
	IV-09 CUBE	20	7940	23.08	1390	61.78	49.42
	IV-10 CUBE	27	7911	22.99	1121	49.82	39.86
	IV-11 CUBE	27	7945	23.09	1138	50.58	40.46
	IV-12 CUBE	27	7983	23.20	1195	53.11	42.49
	IV-13 CUBE	37	7909	22.99	1352	60.09	48.07
	IV-14 CUBE	37	7945	23.09	1331	59.16	47.32
	IV-15 CUBE	37	7945	23.09	1385	61.56	49.24
Cylinder 15 cm dia.x 30cm	IV-01 CYL	16	12269	22.70	871	61.61	49.29
	IV-02 CYL	16	12184	22.55	565	39.97	31.97
	IV-03 CYL	16	12224	22.62	816	57.72	46.18
	IV-04 CYL	22	12196	22.57	858	60.69	48.55
	IV-05 CYL	22	12163	22.51	812	57.44	45.95
	IV-06 CYL	22	12138	22.46	672	47.53	38.03
	IV-07 CYL	28	11917	22.05	700	49.51	39.61
	IV-08 CYL	28	12024	22.25	913	64.58	51.67
	IV-09 CYL	28	12130	22.45	966	68.33	54.66
	IV-10 CYL	33	12269	22.70	960	67.91	54.32
	IV-11 CYL	33	12184	22.55	741	52.42	41.93
	IV-12 CYL	33	12224	22.62	868	61.40	49.12
	IV-13 CYL	36	12196	22.57	932	65.93	52.74
	IV-14 CYL	36	12000	22.21	940	66.49	53.19
	IV-15 CYL	36	12000	22.21	962	68.05	54.44

Correlation between Concrete Cylinder Strength f'_c and Age of Testing from Batch 1

Correlation between Concrete Cylinder Strength f'_c and Age of Testing from Batch 2

Correlation between Concrete Cylinder Strength f'_c and Age of Testing from Batch 4

Test Results for Young's Modulus of Concrete

Concrete Batch	Sample	Young's Modulus E_c (MPa)	Average Young's Modulus (MPa)
Batch 1	Sample 1-1	21424	26555
	Sample 1-2	25076	
	Sample 1-3	33135	
	Sample 1-4	26584	
Batch 2	Sample 2-1	29286	28081
	Sample 2-2	27625	
	Sample 2-3	28832	
	Sample 2-4	26581	
Batch 3	Sample 3-1	27383	26149
	Sample 3-2	24915	
Batch 4	Sample 4-1	29053	28255
	Sample 4-2	27457	

Concrete Properties of the Specimens

Specimen	Concrete Batch	f'_c (MPa)	E_c (MPa)	Note
SC-H	I	38.74	26555	$f'_c = 3.2874 \ln(28) + 27.783$ (MPa)
SC-LD	I	38.74	26555	$f'_c = 3.2874 \ln(28) + 27.783$ (MPa)
SA-HD	I	38.74	26555	$f'_c = 3.2874 \ln(28) + 27.783$ (MPa)
SC-HD	II	47.04	28081	$f'_c = 6.1515 \ln(28) + 26.539$ (MPa)
SC-HDS	II	47.04	28081	$f'_c = 6.1515 \ln(28) + 26.539$ (MPa)
SA-LD	II	47.04	28081	$f'_c = 6.1515 \ln(28) + 26.539$ (MPa)
SA-HDS	III	37.56	26149	f'_c = Average of Conc. Strength of Batch 3
SC-LDS	IV	47.62	28255	$f'_c = 8.7262 \ln(28) + 18.544$ (MPa)
SA-LDS	IV	47.62	28255	$f'_c = 8.7262 \ln(28) + 18.544$ (MPa)

Test Results of T10 Rebar

T10 Rebar (10mm dia.)	Area (mm ²)	Yield Force (kN)	f_y (MPa)	Yield Strain ($\times 10^{-3}$)	Young's Modulus E_s (MPa)
Sample T10-1	78.5	40.7	518.2	2.790	185738
Sample T10-2	78.5	41.0	522.0	2.620	199247
Sample T10-3	78.5	41.2	524.6	2.740	191472
		Average =	521.6	2.717	192152

Test Results of T13 Rebar

T13 Rebar (13mm dia.)	Area (mm ²)	Yield Force (kN)	f_y (MPa)	Yield Strain ($\times 10^{-3}$)	Young's Modulus E_s (MPa)
Sample T13-1	132.7	70.0	527.4	2.687	196270
Sample T13-2	132.7	69.5	523.6	2.592	202010
Sample T13-3	132.7	69.8	525.9	2.720	193335
		Average =	525.6	2.666	197205

APPENDIX III

- *CORNER SPECIMENS TESTING*
- *135-DEGREE SPECIMENS TESTING*

SPECIMEN SC-HD

Top Surface Cracks after Applied Gravity Load

Top Surface Cracks after Completion of 0.25% Drift

Top Surface Cracks after Completion of 0.5% Drift

Torsion Cracks at Slab Edge along Column Long Side after Completion of 0.5% Drift

Top Surface Cracks after Completion of 0.75% Drift

Top Surface Cracks after Completion of 1.0% Drift

Top Surface Cracks after Completion of 1.5% Drift

Torsion and Punching Shear Cracks at Slab Edge along Column Long Side after Completion of 1.5% Drift

Cracks at Slab Edge along Column Short Side after Completion of 1.5% Drift

Punching Shear Crack after Completion of 2.0% Drift

Punching Shear Crack after Completion of 2.0% Drift

*Punching Shear Crack at Slab Edge along Column Long Side
after Completion of 2.0% Drift*

Top View of Punching Shear Crack after Completion of 2.0% Drift

SPECIMEN SC-HDS

Top Surface Cracks after Applied Gravity Load

Top Surface Cracks after Completion of 0.25% Drift

Top Surface Cracks after Completion of 0.75% Drift

Top Surface Cracks after Completion of 1.0 % Drift

Top Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Top Surface Cracks after Completion of 2.5 % Drift

Top Surface Cracks after Completion of 3.0 % Drift

Cracks at Slab Edge along Column Short Side after Completion of 3.0% Drift

Cracks at Slab Edge along Column Long Side after Completion of 3.0% Drift

Crushing of Concrete at Bottom Slab Surface after Completion of 3.0% Drift

SPECIMEN SC-LD

Top Surface Cracks after Completion of 0.5 % Drift

Top Surface Cracks after Completion of 0.75 % Drift

Top Surface Cracks after Completion of 1.0 % Drift

Bottom Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Bottom Surface Cracks after Completion of 2.0 % Drift

Top Surface Cracks after Completion of 2.5 % Drift

Punching Shear Crack at Slab Edge along Column Long Side after Completion of 2.5 % Drift

Bottom Surface Cracks after Completion of 2.5 % Drift

Top Surface Cracks after Completion of 3.0 % Drift

Punching Shear Crack at Slab Edge along Column Short Side after Completion of 3.0 % Drift

Punching Shear Crack at Top Slab Surface after Completion of 4.0 % Drift

SPECIMEN SC-LDS

Top Surface Cracks after Applied Gravity Load

Top Surface Cracks after Completion of 0.25 % Drift

Top Surface Cracks after Completion of 0.5 % Drift

Top Surface Cracks after Completion of 0.75 % Drift

Top Surface Cracks after Completion of 1.0 % Drift

Top Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Top Surface Cracks after Completion of 2.5 % Drift

Top Surface Cracks after Completion of 3.0 % Drift

Punching Shear Crack at Slab Edge along Column Long Side after Completion of 3.0 % Drift

Bottom Surface Cracks after Completion of 3.0 % Drift

Crushing of Concrete at Bottom Slab Surface after Completion of 4.0% Drift

SPECIMEN SC-H

Top Surface Cracks at Shear Force of 40 kN

Top Surface Cracks at Shear Force of 55 kN

Top Surface Cracks at Shear Force of 90 kN

Punching Shear Crack at Top Slab Surface at Peak Shear Force of 115.74 kN

Cracks at Slab Edge along Column Long Side at Peak Shear Force of 115.74 kN

Cracks at Slab Edge along Column Short Side at Peak Shear Force of 115.74 kN

Top Surface Cracks at Peak Shear Force of 115.74 kN

SPECIMEN SA-HD

Top Surface Cracks after Applied Gravity Load

Top Surface Cracks after Completion of 0.5 % Drift

Top Surface Cracks after Completion of 0.75 % Drift

Top Surface Cracks after Completion of 1.0 % Drift

Top Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Top Surface Cracks after Completion of 2.5 % Drift

Punching Shear Crack at Top Slab Surface after Completion of 2.5 % Drift

Punching Shear Crack at Slab Edge along Column Long Side after Completion of 2.5 % Drift

SPECIMEN SA-HDS

Top Surface Cracks after Applied Gravity Load

Top Surface Cracks after Completion of 0.25 % Drift

Top Surface Cracks after Completion of 0.5 % Drift

Top Surface Cracks after Completion of 0.75 % Drift

Top Surface Cracks after Completion of 1.0 % Drift

Top Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Top Surface Cracks after Completion of 2.5 % Drift

Cracks at Slab Edge along Column Long Side after Completion of 2.5 % Drift

Top Surface Cracks after Completion of 3.0 % Drift

SPECIMEN SA-LD

Top Surface Cracks after Completion of 0.25 % Drift

Top Surface Cracks after Completion of 0.5 % Drift

Top Surface Cracks after Completion of 0.75 % Drift

Top Surface Cracks after Completion of 1.0 % Drift

Top Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Punching Shear Crack at Top Slab Surface after Completion of 2.5 % Drift

Punching Shear Crack at Slab Edge along Column Long Side after Completion of 2.5 % Drift

Punching Shear Crack at Top Slab Surface after Completion of 2.5 % Drift

Punching Shear Crack at Top Slab Surface after Completion of 3.0 % Drift

Top Surface Cracks after Completion of 3.0 % Drift

SPECIMEN SA-LDS

Top Surface Cracks after Completion of 0.25 % Drift

Top Surface Cracks after Completion of 0.5 % Drift

Top Surface Cracks after Completion of 0.75 % Drift

Top Surface Cracks after Completion of 1.0 % Drift

Top Surface Cracks after Completion of 1.5 % Drift

Top Surface Cracks after Completion of 2.0 % Drift

Top Surface Cracks after Completion of 2.5% Drift

Cracks at Slab Edge along Column Long Side after Completion of 2.5 % Drift

Top Surface Cracks after Completion of 3.0% Drift

Bottom Surface Cracks after Completion of 3.0% Drift

Cracks at Slab Edge along Column Long Side after Completion of 3.0 % Drift

Top Surface Cracks after Completion of 4.0% Drift

Top Surface Cracks after Completion of 4.0% Drift

Crushing of Concrete at Bottom Slab Surface after Completion of 4.0% Drift

Crushing of Concrete at Bottom Slab Surface after Completion of 4.0% Drift

APPENDIX IV

- *STRAIN PROFILES OF SLAB REBARS*
- *STRAIN PROFILES OF STUD SHEAR REINFORCEMENT (SSR)*

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SC-LD

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 9 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 9 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 14 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 14 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 13 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LD at Loading Stage 13 (Drift $> 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SC-LDS

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 9 ($\text{Drift} \leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 9 ($\text{Drift} > 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 14 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 14 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 13 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-LDS at Loading Stage 13 (Drift $> 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SC-HD

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 9 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 9 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 14 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 14 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 13 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HD at Loading Stage 13 (Drift $> 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SC-HDS

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 9 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 9 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 14 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 14 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 13 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SC-HDS at Loading Stage 13 (Drift $> 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SC-H

Rebars Strain Profiles of Specimen SC-H due to Gravity Load

Rebars Strain Profiles of Specimen SC-H due to Gravity Load

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SA-LD

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 9 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 9 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 14 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 14 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 13 ($\text{Drift} \leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LD at Loading Stage 13 ($\text{Drift} > 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SA-LDS

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 9 ($\text{Drift} \leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 9 ($\text{Drift} > 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 14 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 14 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 13 ($\text{Drift} \leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-LDS at Loading Stage 13 ($\text{Drift} > 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SA-HD

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 9 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 9 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 14 ($\text{Drift} \leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 14 ($\text{Drift} > 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 13 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HD at Loading Stage 13 (Drift $> 1.5\%$)

STRAIN PROFILES OF SLAB REBARS
SPECIMEN SA-HDS

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 9 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 9 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 14 ($\text{Drift} \leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 14 ($\text{Drift} > 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 2 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 2 (Drift $> 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 13 (Drift $\leq 1.5\%$)

Rebars Strain Profiles of Specimen SA-HDS at Loading Stage 13 (Drift $> 1.5\%$)

STRAIN PROFILES OF SSR
SPECIMEN SA-HDS

Studs Rail SB1 at Loading Stage 9

Studs Rail SB2 at Loading Stage 9

Studs Rail SB3 at Loading Stage 9

Studs Rail SB4 at Loading Stage 9

Studs Rail SB5 at Loading Stage 9

Studs Rail SB6 at Loading Stage 9

Studs Rail SB7 at Loading Stage 9

Studs Rail SB1 at Loading Stage 14

Studs Rail SB2 at Loading Stage 14

Studs Rail SB3 at Loading Stage 14

Studs Rail SB4 at Loading Stage 14

Studs Rail SB5 at Loading Stage 14

Studs Rail SB6 at Loading Stage 14

Studs Rail SB7 at Loading Stage 14

Studs Rail SB1 at Loading Stage 2

Studs Rail SB2 at Loading Stage 2

Studs Rail SB3 at Loading Stage 2

Studs Rail SB4 at Loading Stage 2

Studs Rail SB5 at Loading Stage 2

Studs Rail SB6 at Loading Stage 2

Studs Rail SB7 at Loading Stage 2

Studs Rail SB1 at Loading Stage 13

Studs Rail SB2 at Loading Stage 13

Studs Rail SB3 at Loading Stage 13

Studs Rail SB4 at Loading Stage 13

Studs Rail SB5 at Loading Stage 13

Studs Rail SB6 at Loading Stage 13

Studs Rail SB7 at Loading Stage 13

STRAIN PROFILES OF SSR
SPECIMEN SA-LDS

Studs Rail SB1 at Loading Stage 9

Studs Rail SB2 at Loading Stage 9

Studs Rail SB3 at Loading Stage 9

Studs Rail SB4 at Loading Stage 9

Studs Rail SB5 at Loading Stage 9

Studs Rail SB6 at Loading Stage 9

Studs Rail SB7 at Loading Stage 9

Studs Rail SB1 at Loading Stage 14

Studs Rail SB2 at Loading Stage 14

Studs Rail SB3 at Loading Stage 14

Studs Rail SB4 at Loading Stage 14

Studs Rail SB5 at Loading Stage 14

Studs Rail SB6 at Loading Stage 14

Studs Rail SB7 at Loading Stage 14

Studs Rail SB1 at Loading Stage 2

Studs Rail SB2 at Loading Stage 2

Studs Rail SB3 at Loading Stage 2

Studs Rail SB4 at Loading Stage 2

Studs Rail SB5 at Loading Stage 2

Studs Rail SB6 at Loading Stage 2

Studs Rail SB7 at Loading Stage 2

Studs Rail SB1 at Loading Stage 13

Studs Rail SB2 at Loading Stage 13

Studs Rail SB3 at Loading Stage 13

Studs Rail SB4 at Loading Stage 13

Studs Rail SB5 at Loading Stage 13

Studs Rail SB6 at Loading Stage 13

Studs Rail SB7 at Loading Stage 13

STRAIN PROFILES OF SSR
SPECIMEN SC-HDS

Studs Rail SA1 at Loading Stage 9

Studs Rail SA2 at Loading Stage 9

Studs Rail SA3 at Loading Stage 9

Studs Rail SA4 at Loading Stage 9

Studs Rail SA5 at Loading Stage 9

Studs Rail SA1 at Loading Stage 14

Studs Rail SA2 at Loading Stage 14

Studs Rail SA3 at Loading Stage 14

Studs Rail SA4 at Loading Stage 14

Studs Rail SA5 at Loading Stage 14

Studs Rail SA1 at Loading Stage 2

Studs Rail SA2 at Loading Stage 2

Studs Rail SA5 at Loading Stage 2

Studs Rail SA1 at Loading Stage 13

Studs Rail SA2 at Loading Stage 13

Studs Rail SA3 at Loading Stage 13

Studs Rail SA4 at Loading Stage 13

Studs Rail SA5 at Loading Stage 13

STRAIN PROFILES OF SSR
SPECIMEN SC-LDS

Studs Rail SA3 at Loading Stage 9

Studs Rail SA4 at Loading Stage 9

Studs Rail SA5 at Loading Stage 9

Studs Rail SA1 at Loading Stage 14

Studs Rail SA2 at Loading Stage 14

Studs Rail SA5 at Loading Stage 14

Studs Rail SA1 at Loading Stage 2

Studs Rail SA2 at Loading Stage 2

Studs Rail SA3 at Loading Stage 2

Studs Rail SA4 at Loading Stage 2

Studs Rail SA5 at Loading Stage 2

Studs Rail SA1 at Loading Stage 13

Studs Rail SA2 at Loading Stage 13

Studs Rail SA5 at Loading Stage 13

APPENDIX V

- *H-REFINEMENT METHOD*

h-Refinement of One-Way Slab Tested by Jofriet and McNeice (1971)

h-Refinement of Two-Way Slab Tested by Jofriet and McNeice (1971)

h-Refinement of Slab A3 Tested by Aghayere and MacGregor (1990)

h-Refinement of Specimen 2C Tested by Robertson (1990)

h-Refinement of Specimen 8I Tested by Robertson (1990)

h-Refinement of Specimen 9E Tested by Robertson (1990)

h-Refinement of Specimen MG-2A Tested by Megally and Ghali (2000a)

h-Refinement of Specimen 1C Tested by Robertson et al. (2002)

h-Refinement of Specimen 1 Tested by Pan and Moehle (1992)

h-Refinement of Specimen E1H Tested by Anggadajaja (2006)

h-Refinement of Specimen YL-L1 Tested by Tan and Teng (2005)

h-Refinement of Specimen SC-HD Tested by the Author

h-Refinement of Specimen SA-LD Tested by the Author