Supply contract modeling for a decentralized assembly system

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Supply Contract Modeling for a Decentralized Assembly System

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ABSTRACT

Supply chain is increasingly becoming a complex process due to global links for production and distribution. When a final product producer seeks specified quantities of parts or modules from various suppliers, delivery of the stated quantities may be constrained by suppliers' own capacity constraints and cost structures. In such cases, suppliers tend to supply the quantities that best match their optimal scenario. However, if the suppliers are given this choice without any coordination, there will be various quantities of parts received from different suppliers. This can create mismatch of components and therefore affect the potential to obtain optimum benefit from a supply chain since, in such a system, the production of final product depends on the minimum number of unique components supplied by any supplier.

On the other hand, developing long-term relationship to thwart competition is becoming a strategic mission of the companies. This will not only benefit the suppliers with assured long-term market but also the assemblers as they will be able to focus on their core products to improve their business performance. However, long-term relation advocates coordination of supply chain activities through various mechanisms so that mismatch of components is minimized and overall supply chain profit is maximized.

In this thesis, various supply contract models have been proposed as coordinating mechanisms between the suppliers and the assembler. Suppliers are considered responsible for the inventory replenishment at the assembler's premises whereas the assembler determines its purchasing quantity for each component. Channel coordination is brought about by modifying the pricing schemes between the suppliers and the assembler.

Mathematical models are formulated to determine suppliers' optimal production/delivery quantities and the assembler's optimal purchasing quantity for a single-period and a

two-period transaction with centralized or decentralized control modes, respectively. As centralized control mode refers to the situation where a single decision maker who, having access to all the information, makes decisions for the benefits of the entire system, this model can be assumed as a benchmark to test the results obtained from other models. In a decentralized control mode, all suppliers are shared with market demand information to make their production and supply decisions.

The analysis in this thesis proves that, with decentralized control mode, the assembly system cannot be coordinated with a price-only contract alone; however, a buyback mechanism can be integrated to achieve channel coordination. In particular, for the two-period model, the analysis for the centralized control mode shows that the optimal policy depends on each player's initial inventory level and the supplier with less on-hand inventory of the first period is the critical supplier of the two consecutive periods.

This research contributes in terms of modeling incentive problems for a decentralized assembly system such that the system performance is optimized and it also contributes by examining coordination mechanisms in a two-period setting. This research is expected to be useful for companies to align their complicated supply chain process for a globally optimized supply situation.

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TABLE OF CONTENTS

ABSTRACT i

ACKNOWLEDGEMENTS iii

TABLE OF CONTENTS iv

LIST OF FIGURES vii

LIST OF TABLES viii

Chapter 1 Introduction 1

 1.1 Research Background 3

 1.2 Research Objective 6

 1.3 Research Scope 8

 1.4 Research Methodology 9

 1.5 Thesis Organization 10

Chapter 2 Literature Review 12

 2.1 Multi-Echelon Inventory Models 15

 2.2 Supply Contract Models 18

 2.2.1. Contracts with Risk-Sharing Objective 19

 2.2.1.1.Quantity Flexibility Contract 20

 2.2.1.2.Commitment Contract 21

 2.2.1.3.Quantity Discount Contract 23

 2.2.2. Contracts with Channel Coordination Objective 25

 2.2.2.1.Price-Only Contract 26

 2.2.2.2.Quantity Discount Contract 27

 2.2.2.3.Buyback Contract 28

 2.2.2.4.Revenue-Sharing Contract 31

 2.2.2.5.Joint Pricing-Production Decision Making 33

 2.3 Summary 35

Chapter 3 Single-Period Supply Contract Model 40

 3.1 Assumptions for the Basic Model 40

 3.2 Model Formulation 43

3.2.1	Centralized Control Model	44
3.2.2	Price-Only Decentralized Control Model	45
3.2.3	Coordinated Decentralized Control Model.....	51
I.	Channel Coordination with Buyback Contract.....	51
II.	Channel Coordination with Revenue-Sharing Contract	55
III.	Comparison of the Buyback and Revenue-Sharing Contract.....	60
3.3	Numerical Study	60
3.4	Summary.....	66
Chapter 4	Two-Period Supply Contract Model	67
4.1	Assumptions for the model	67
4.2	The Centralized Control Model	70
4.2.1	The Last-Period Model for the Centralized Control Setting.....	70
4.2.2	Two-Period Model for the Centralized Control Setting.....	76
I.	Scenario I	77
II.	Scenario II.....	83
4.2.3	Summary to the Two-Period Centralized Model	86
4.3	The Price-Only Decentralized Contract Model	86
4.3.1	Last-Period Problem for the Price-Only Decentralized Control Setting	86
I.	The Assembler's Last-Period Problem	86
II.	The Suppliers' Last-Period Problem	88
III.	Overview of the Last-Period Problem	92
IV.	Comparison of the Last-Period Problem with Different Control Modes.....	94
4.3.2	Two-Period Model for the Price-Only Decentralized Control Setting	96
I.	Scenario I and III	97
II.	Scenario II.....	108
III.	Scenario IV	117
4.3.3	Summary to the Decentralized Price-Only Supply Contract	120
4.4	Coordinated Decentralized System.....	120
4.4.1	Last-Period Problem with Buyback Contract	121
4.4.2	Two-Period Problem with Buyback Contract	127
4.5	Discussions	134
4.6	Summary.....	137

Chapter 5	Numerical Study	144
5.1	The Last-Period Problem	146
I.	The Centralized Control Setting	146
II.	The Price-Only Decentralized Contract Setting	147
III.	The Coordinated Decentralized Control Setting.....	151
IV.	Impact of Parameters on System Performance in a Coordinated Setting	152
	a). <i>Variation in Product Selling Price</i>	153
	b). <i>Increase in the assembler's holding cost</i>	154
	c). <i>Increase in Backorder Penalty Cost</i>	155
5.2	Numerical Study for the Two-Period Problem.....	157
I.	The Centralized Control Setting	157
II.	The Price-Only Decentralized Contract Setting	158
III.	The Coordinated Decentralized Setting.....	162
	a). <i>Variation in product selling price</i>	162
	b). <i>Variation in supplier 1's wholesale price</i>	163
5.3	Summary	165
Chapter 6	Conclusions and Recommendations for Future Work	167
6.1	Research Contributions	170
6.2	Recommendation for Future Work.....	171
References		175
Appendix I	Newsvendor Model.....	183
Appendix II	Formulation Derivation	190
Appendix III	Proofs for Propositions	192
Appendix IV	Research Communication	197

LIST OF FIGURES

Figure 3-1 Comparison of quantities in the centralized and price-only decentralized setting ($p = 6$)	62
Figure 3-2 Comparison of system profits in the centralized and price-only decentralized setting ($p = 6$).....	62
Figure 3-3 Comparison of quantities in the centralized and price-only decentralized setting ($p = 7$)	63
Figure 3-4 Comparison of system profits in the centralized and price-only decentralized setting ($p = 7$).....	63
Figure 4-1 Flowchart of the analytical process for centralized control mode	139
Figure 4-2 Flowchart of the analytical process for price-only decentralized control mode ..	141
Figure 4-3 Flowchart of the analytical process for channel coordination with buyback.....	142
Figure 5-1 The impact of w_{d11} on y_{del} , y_{d01} and Q_{d01}^*	150
Figure 5-2 The impact of b_{11} on various coordinating parameters.....	151
Figure 5-3 The impact of b_{11} on expected profits.....	152
Figure 5-4 The impact of w_{d11} on optimal expected profits	161
Figure 5-5 The impact of p on optimal expected profits	163
Figure 5-6 The impact of w_{b11} on various coordinating parameters	164
Figure 5-7 The impact of w_{b11} on optimal expected profits.....	165
Figure I-1 The relationship between events and analysis sequence.....	187

LIST OF TABLES

Table 2-1 Summary of supply contracts with risk-sharing objective.....	37
Table 2-2 Summary of supply contracts with channel coordination objective	38
Table 3-1 Notations for single-period supply contract model.....	41
Table 3-2 Initial conditions for numerical study	61
Table 3-3 Centralized vs. decentralized system performance ($p = 6$)	61
Table 3-4 Channel coordination with buyback contract	64
Table 3-5 Channel coordination with revenue-sharing contract	65
Table 4-1 Notations for multi-period supply contract model.....	68
Table 4-2 Combination of suppliers' order-up-to levels after replenishment	92
Table 4-3 Scenario combinations.....	97
Table 5-1 Initial conditions for the two-period problem.....	146
Table 5-2 Results for the last-period centralized control setting	147
Table 5-3 Results for the last-period price-only decentralized control setting	148
Table 5-4 The impact of w_{d11} on system performance in a price-only decentralized setting	149
Table 5-5 The impact of p on optimal quantities and expected profits.....	153
Table 5-6 The impact of h_0 on optimal quantities and profits	154
Table 5-7 The impact of s on quantities and profits.....	156
Table 5-8 Results for the two-period centralized control setting.....	158
Table 5-9 Results for two-period price-only decentralized setting.....	159
Table 5-10 The impact of w_{d11} on optimal quantities	160
Table 5-11 The impact of p on optimal quantities (two-period coordinated setting)	163
Table I-1 Notations for simple newsvendor problem	183
Table I-2 Notations for the multi-period newsvendor model	186

Chapter 1

Introduction

Coordination in supply chain is an important factor that contributes to the supply chain efficiency and effectiveness. Recent research in supply chain management (SCM) has emphasized coordination. However, as every player in a supply chain (SC) is a decision maker for its own operation, such coordination is always hard to achieve. Hence, the achievement of a win-win situation or coordination in a SC needs the goodwill of all the participants and this can be developed by a long-term experience through an on-going relationship. Some examples of successful coordination among players along a SC include Dell Computer Corporation, Amazon.Com and Campbell Soup Company (Simchi-Levi et al, 2003).

SCM also considers topology of a system. Assembly system, which is the main process for producing final products, requires various components for the final assembly. However, these components are often supplied by the suppliers located at different locations, even beyond a country's boundary. Therefore, the final quantity to be produced from an assembly line depends on the minimum number of parts or modules received from different suppliers. The number of parts to be supplied by a supplier can depend on various factors, such as cost of production at suppliers' premises and their capacity. Therefore, if the individual interest of any supplier is not treated from a collective perspective, it is difficult to achieve optimum efficiency in a supply chain.

However, for a decentralized assembly system, which is characterized by distributed decision

making and self-interested players, suppliers might look at their individual benefits and choose to produce/deliver the number of components that could maximize their own profits. Due to the difference in their cost structures of production and the stochastic nature of market demand, the supplied numbers of parts by different suppliers do not match, which incurs mismatch of components. This kind of mismatch of components is caused by suppliers' conflicting incentives concerning component delivery, which could be improved or solved through a proper supply chain coordination mechanism (Gerchak and Wang, 2004). In practice, however, mismatch of components could also happen due to production yield loss (Gerchak et al., 1988; Gurnani et al., 1996, 2000), which could be improved or solved with the implementation of total quality management (TQM) or production process re-engineering (Gurnani et al, 2000).

Various companies attempt to solve mismatch of components incurred by incentive conflicts through long-term relationship. For example, Sony calls on global suppliers to match their specifications for a particular product with parts or modules and, if successful, be a partner with them for a minimum period of time for continuous upgrades. This requires Sony to understand the cost and capacity structure of the supplier, given the technological competence. In this case, supplier's profile needs to be taken into account by the final assembler (Sony). In the mean time, each supplier also wants to gain the maximum from the long-term relationship given their own profile. For that, the final assembler needs to share the demand and other information with their suppliers and expect that the number of parts or modules supplied by each supplier do not mismatch. The expectation for avoiding mismatch of the components is achieved by a coordination mechanism mutually agreed upon by the assembler and the suppliers. In long-term relationship (exceeding one time contract), such a coordination mechanism is reviewed continuously to optimize the supply chain efficiency.

However, it is observed that most researches in supply chain coordination are focused on developing supply contract models for single-period newsvendor settings mostly due to the fact that the variety and complexity of the contracts used in practice makes it difficult to model them for more than one period (Tsay et al., 1999). However, due to the nature of decentralized production system in practice today, multi-period supply contracts are essential.

1.1 Research Background

In recent years, the ever-shortening product life cycle, globalization and growing competition compel companies to look not only at the operations of their own but of their suppliers and distributors. Such a comprehensive understanding of the entire supply chain promotes SCM. Although there are many definitions in prevalence of SCM due to its wide coverage of multiple, fragmented disciplines (Croom et al., 2000; Stadtler, 2005), this thesis adopts the following definition of SCM: a set of approaches aiming at integrating organizational (functional) units along a SC and coordinating material, information and financial flows both in and between these different units in order to satisfy customer demand while improving the competitiveness of the entire SC (Min and Zhou, 2002; Minner, 2003; Simchi-Levi et al., 2003).

In addition to the implementation of SCM, companies use strategies such as outsourcing to expect a benefit of low(er) manufacturing or distribution costs without a substantial investment. Byrne (1996) showed that the use of outsourcing for at least some services by major corporations increased from 58% in 1992 to 86% in 1996. However, outsourcing distributes decision rights since SCs are operated with independently managed organizations or business functions always beyond an organization's boundary to purchase, produce and deliver products to satisfy customer demand (Lariviere, 1999; Zhao and Wang, 2002).

Moreover, outsourcing distributes knowledge and information of all value-added activities (Tomas and Griffin, 1996; Tsay and Lovejoy, 1999). The reality is modern SCs are operated with decentralized control mode, which assumes multiple decision makers who make local optimal decisions with local information.

However, decentralized control mode puts some limitation on SC performance: the existing organizational barrier impedes smooth and complete information sharing resulting in information asymmetry (e.g., players have private information on their cost structures) and; players always have conflicting incentives when making decisions. Without solving the problems on players' conflicting objectives and asymmetric information, it is difficult to achieve channel-wide optimality. Specifically, in the literature of operations management, the above mentioned problems associated with decentralized control mode are identified as "incentive conflicts" and "information asymmetry" (Corbett and Tang, 1999; Anupindi and Bassok, 1999).

With the associated problems, decentralization displays a negative effect on the overall performance of a SC and leads to double marginalization. Double marginalization refers to the phenomenon where a retailer and a manufacturer add successive mark-ups in setting their prices and the retailer does not account for the influence of the final price decision in the manufacturer's optimization (Spengler, 1950). Cachon (1999) states that double marginalization exists in most supply chain models because it occurs whenever the supply chain's profits are divided among two or more players and at least one of the players influences demand.

Since decentralization reduces system efficiency of a SC when benchmarked against that of

centralization (a control mode which assumes central decision making and complete information sharing), coordinating mechanisms are required for system efficiency improvement. Coordinating mechanisms are inter-replaceable with supply contract in this research, which aims at overall system profits maximization through proper contract terms or clauses.

Although supply contracts have been widely adopted to ensure the success of transactions by explicitly stipulating all the players' rights and obligations, there is not a unified definition as to what constitutes a supply contract (Tsay et al., 1999). In this research, a supply contract is referred to as a kind of coordinating mechanism with which the relationship between players could be well managed and player's individual incentive could be aligned with global optimization.

In practice, supply contract also takes the form of cooperative agreement, e.g. vendor managed inventory (VMI). In a VMI program, the manufacturer assumes control of the stock management of the retailer and determines the quantity and timing of shipment to the retailer. Examples of successful implementation of VMI initiatives can be found between P&G and Wal-Mart (Simchi-Levi et al., 2003), printer manufacturer Lexmark International (Alexander, 1998) and food manufacturer Kraft Inc. (Emigh, 1999) etc.

In an assembly system, the implementation of VMI requires information sharing between the component suppliers and the assembler; however, there is usually no information sharing among the component suppliers. Moreover, with the increasing practice of global sourcing, suppliers could spread across all the continents with different production characteristics and different cost structures. Hence, when suppliers assume responsibility of the assembler's

inventory replenishment, they would deliver, in anticipation of other suppliers' behavior, with quantities to maximize their own profits. However, these quantities could be different with each other. The difference in suppliers' delivery quantities makes the final assembly be dictated by the component with the fewest number. Such strategic behavior of the suppliers could impair system performance (measured in this research by the number of components finally purchased by the assembler and the total expected system profits). Moreover, when customer demand is stochastic, suppliers' profits would be quite uncertain because: 1) the numbers of the components delivered by other suppliers are uncertain; and 2) market demand is uncertain.

Moreover, coordination would become easier to be achieved when players tend to develop long-term relationship, represented by players' willingness and devotion to carry out multiple transactions.

1.2 Research Objective

Based on the discussions above, it is noted that:

- Most modern supply chains are operated with decentralized control mode; however, decentralized control mode distributes decision rights, knowledge, which leads to "incentive conflicts" and "information asymmetry". This finally reduces system (or supply chain) efficiency;
- Companies focusing on establishing long-term relationship need to understand the additional factors that have to be considered and the benefits of multiple transactions;
- Channel coordination is an important criterion for system performance measurement; and
- Decentralized assembly system requires coordination of various components for the final assembly.

Lariviere (1999) has grouped the literature on supply contract into two classes: first by taking a particular contract to determine the optimal actions in a contract with fixed terms; and second, by taking players' optimal policies under a contract as given and considering the adjustment of contract terms to improve or coordinate the supply chain.

As channel coordination is an important criterion for system performance measurement (Anupindi and Bassok, 1999), the second class of literature on supply contract serves the purpose of system-wide performance improvement, which is also referred to as the channel coordination purpose (Tsay et al., 1999). Channel coordination may be achieved by first identifying the intra-chain dynamics which cause the inefficiency, then modifying the structure of the relationship to more closely align individual incentives with global optimization.

Hence, the following problems are to be solved in this research:

- What is the impact of decentralization on the performance of an assembly system?
- What is the impact of lengthening the transaction period on players' behavior? What are the additional factors that need to be considered?
- Can the decentralized assembly system be coordinated through modification of the relationship between all the players?

For the first problem, the assembly system operated with a centralized control mode is analyzed. As centralized control mode targets at optimum system efficiency, the results obtained from this analysis are considered as a benchmark. Each player's best response with a price-only contract (for the decentralized control mode without coordination) is then derived. System performances, under the two control modes, are compared.

For the second problem, two-period supply contract models are formulated. Nonetheless, single-period contract models, representing one-time transactions, are examined first to act as a building block. Compared with the single-period models, factors such as non-stationary cost parameters, transfer of inventory status and expected future demand are incorporated into the two-period models. Although, only a two-period setting is formulated (mainly due to the computational complexities in the extension of time horizon), the developed models are capable of reflecting the operations of the assembly system in a generic multi-period setting by incorporating more complexities.

For the last problem, channel coordination is realized by modifying the pricing scheme between the suppliers and the assembler. Since wholesale price, in practice, represents the simplest and the most basic pricing relation between partners, a price-only contract model will be examined first to evaluate the assembly system without coordination. However, it can be demonstrated that with wholesale price alone, the decentralized assembly system cannot be coordinated. Therefore, a modified pricing scheme, which takes the form of wholesale price plus buyback price, is considered for channel coordination.

1.3 Research Scope

This research focuses on channel coordination in a two-echelon decentralized assembly system. Single-period models and two-period models are developed for analysis, respectively. Assumptions such as stochastic nature of market demand, complete sharing of demand information, stationary system parameters (except for the wholesale prices) and risk neutral and rational players are assumed for the development of the models.

For the single-period models, multiple component suppliers are considered; however, both

the suppliers and the assembler are assumed to have zero initial inventory. For the two-period models, two suppliers, each supplying a unique component for the final assembly, are considered. Further, non-zero initial inventory (leftover inventory is automatically transferred from the previous period to the next) and non-stationary wholesale prices are assumed for model development.

1.4 Research Methodology

Several methodologies have been used in this research. First of all, “agency theory” concept is used to analyze systems with decentralized control and to identify system inefficiencies as used in Porteus and Whang (1991), Whang (1995), Lee and Whang (1999), Lariviere (1999) and Tsay et al. (1999). In this theory, a supply chain is regarded as a nexus of contracts among self-interested “agents”, each maximizing its own objective. Coordinating mechanism or contract design to align the incentives of agents has been one of the prime research interests.

Analytical models are developed to evaluate system performances. Specifically, the formulated single-period models are, in essence, “newsvendor models” which describe a one-time transaction between the assembler and its suppliers. A review of the basic “newsvendor model” is given in Appendix I. For the two-period models, backward stochastic dynamic programming (DP) is applied due to its capability in making sequential decisions over time. The modeling starts from the last period when the transaction ends and rolls back to the period when the transaction starts. A review of the multi-period newsvendor model with stochastic DP in a backward manner is also provided in Appendix I.

1.5 Thesis Organization

This thesis is organized in six chapters. Outlines of each chapter are summarized as follows.

- Chapter one starts with the introduction of research motivation. Followed by that, research background is discussed. Research objectives are then proposed based on the previous analysis. Finally, research scope for the thesis is identified and research methodology applied is briefly discussed.
- Chapter two classifies relevant literature into two major categories according to different control modes of a SC. Specifically, various supply contract models are reviewed for SCs with decentralized control mode. Further, the supply contract models are classified according to their purposes served. At the end of the chapter, various contract models reviewed are summarized in a table format for easy reference.
- In chapter three, models for the assembly system operated with centralized and decentralized control in a single-period setting are formulated. Particularly, for decentralized control mode, a price-only contract model and two contract models with different coordinating mechanisms are developed. Optimal production/delivery quantities for different control modes are derived, respectively. Lastly, a numerical study is carried out to validate and to show the applicability of the proposed models.
- Chapter four extends the single-period model by considering a two-period setting. Likewise, the assembly system operated with centralized and decentralized control modes are considered. In particular, a price-only contract model and a coordinating contract model are developed for the assembly system operated with decentralized control mode. Players' optimal behavior for each control mode is determined, respectively. Moreover, a discussion on the modeling assumptions and the formulated models is presented at the end of the chapter.

- A numerical study is carried out in chapter five, which verifies and validates the models formulated in chapter four through numerical analysis. In addition, the impacts of various parameters on system performance are presented.
- Chapter six starts with introduction of the main conclusions and major contributions of this research. Then, possible future extensions are briefly discussed.

Chapter 2

Literature Review

A supply chain can be operated with two distinct control modes – the centralized and the decentralized control mode (Lee and Whang, 1999; Whang, 1995; Thomas and Griffin, 1996; Tsay et al., 1999; Giannoccaro and Pontrandolfo, 2004). With centralized control mode, it is assumed that there exists a unique central decision-maker, who has access to all the information that is relevant in its decision-making (DM) process and is granted with the absolute power to make decisions for the benefits of the entire SC. Models assuming centralized control mode generate the first-best system performance (Lee and Billington, 1993; Lariviere, 1999; Tsay et al., 1999).

However, assumption of centralized control mode of a SC is not realistic in today's globalization context (Tsay et al., 1999; Cachon, 2002; Zhao and Wang, 2002). This is due to the fact that the requirements for the implementation of centralized control mode, such as full access to information and absolute power in decision making without disobedience from other players, are often too hard to be satisfied. Hence, it is more appropriate to assume decentralized control mode. Contrary to centralized control mode, decentralized control mode considers the presence of multiple decision makers and incomplete information sharing (Anupindi and Bassok, 1999).

In practice, the incentives for having a decentralized SC are:

- Modern organizations constantly face the challenge of making timely decisions using specialized information. In this case, it is helpful to delegate decision rights to the person

on the spot who has the intimate knowledge of his or her immediate surroundings (Lee and Whang, 1999).

- Companies outsource in order to reduce costs, spread risks and to avoid substantial investment without losing their core competencies (Bowersox, 1990). However, outsourcing distributes decision rights.
- Globalization requires localized effort. Due to the differences in culture, infrastructure, politics, standards, languages and human resources, SCs extending to other countries need decentralization (Simchi-Levi, et al., 2003).

However, there are some pitfalls associated with decentralized SCs:

- Decentralization is at the expense of system efficiency, which means that the profit of a decentralized system cannot be higher than that of the corresponding centralized system (Lariviere and Porteus, 2001).
- Decentralization leads to “double marginalization” which makes it difficult to realize economics of scale (due to the reduction in the buyer’s optimal stocking level) (Simchi-Levi, et al., 2003).
- Decentralization makes it difficult to cooperate among players in a SC. While players may agree in principle to cooperate, each may face the temptation to deviate from the agreement in order to minimize (or maximize) its own costs (or profits). Hence, every player in the SC may try to game the others, unless an equilibrium can be achieved (Cachon and Zipkin, 1999).

Specifically, “incentive conflicts” and “information asymmetry” are two commonly identified problems associated with decentralized control in the literature of operations management (Anupindi and Bassok, 1999; Corbett and Tang, 1999).

Since channel coordination is an important criterion for system performance measurement (Anupindi and Bassok, 1999), coordinating mechanisms are explored to improve the performance of the investigated assembly system. In this thesis, a decentralized system will be considered as coordinated if the same level of efficiency as that of the corresponding centralized system can be achieved.

One main stream of research that assumes centralized control and is relevant to this research is multi-echelon inventory management, which will be reviewed in the following section. It is to note that the analytical models for the centralized control mode to be formulated in the next two chapters are influenced by the developments in multi-echelon inventory models.

As supply contract will be used for channel coordination, literature relates to decentralized control mode will focus on various supply contracts models. However, due to its wide coverage in topics, analytical contract models that could be applied to facilitate risk sharing or channel coordination will be reviewed.

Channel coordination is realized through negotiation of the terms governing the relationship between players in this research. This implies that the terms of the contractual relationship between the players would be treated as decision variables. However, the classical inventory models, such as the multi-echelon inventory models, treat the supply parameters (e.g. setup costs, holding and backorder penalty costs, and lead time) as exogenous environmental characteristics. Despite the differences in the multi-echelon inventory models and the supply contract models, they share something in common: both of them focus on the logistics problem – the material coordination.

2.1 Multi-Echelon Inventory Models

For the analysis of multi-echelon inventory models, two supply chain network structures are distinguished and they are treated as building blocks for the analysis of a more complicated network structure. Usually, the upstream of a chain is characterized by a convergent structure, represented by an assembly stage. After the assembly stage, the finished product is probably stored at a central depot, which supplies a number of downstream stock-points. The distribution of the finished product from the central depot to the end stock-point is characterized by a divergent structure.

Convergent and divergent systems have different research focuses. For the divergent system, special attention is given to situations where there is insufficient inventory to meet the order replenishment requirement downstream. This further leads to different assumptions of balanced or imbalanced inventory and discussions on different rationing policies (Diks et al., 1996; Diks and de Kok, 1998). Since the focus of this research is on channel coordination through supply contract, the review on multi-echelon inventory model would not be very extensive; however, classical multi-echelon inventory models, especially those considering convergent systems (e.g. the assembly system) are reviewed.

Multi-echelon inventory management is primarily about controlling the timing and quantity of material flows in a SC. A central role in the analysis of multi-echelon inventory system is the concept of echelon stock, proposed by Clark and Scarf (1960) to establish the optimality of echelon order-up-to policies in a serial system. Since echelon stock requires information concerning the inventory levels downstream (Axsäter and Juntti, 1996, 1997), models using echelon stock concept for the decomposition assume centralized control mode (Van Houtum et al., 1996; Tsay and Agrawal, 2004) and some of which are reviewed as follows.

In the seminal work of Clark and Scarf (1960) on multi-echelon inventory management in which an N -echelon serial system with stochastic customer demand, periodic replenishment, is studied (Schmidt and Nahmias, 1985; Rosling, 1989; Tsay et al., 1999). Most importantly, this research proposes the concept of “echelon-stock”. With this concept, the optimal decisions concerning the purchasing quantity at each installation of the system can be determined by decomposing the N -echelon problem into N separate single-location problems. The optimal purchasing quantity is shown to have an “order-up-to” structure. The work of Clark and Scarf (1960) has laid a foundation for the subsequent researches on multi-echelon inventory management, since most of the subsequent researches are various extensions of it.

Schwarz and Schrage (1975) extend Clark and Scarf (1960) by studying a multi-echelon assembly system under continuous review with constant demand over an infinite planning horizon. Inventory costs at each stage consist of a fixed production setup cost and a linear holding cost on “echelon inventory”. The objective is to minimize (or nearly minimize) average cost per unit time. A branch and bound algorithm is proposed for quick determination of the optimal solution.

Another extension of Clark and Scarf (1960) to an infinite-horizon case is made by Federgruen and Zipkin (1984), in which an arborescent system structure is considered. In the proposed system, inventory levels at each echelon are reviewed periodically. The analysis shows that the computations required for the infinite-horizon case are far easier than that for the finite-horizon case.

Further extension on Clark and Scarf (1960) is made by Rosling (1989) who considered a

multi-echelon assembly system with random customer demand and proportional production and stock holding costs. The author shows that an assembly system can be decomposed into a serial system under the condition of long-run balance.

Schmidt and Nahmias (1985) study a two-echelon assembly system in which an end product is assembled from two components, each of which was ordered from an external supplier. Customer demand is assumed to be stochastic. Fixed order and assembly lead times are assumed. A multi-period model is formulated with dynamic programming. The optimal assembly and order policy are determined separately through decomposition method. The authors find that due to different lead times, even with the same number of components supplied by different suppliers, the solutions would not be optimal. However, the proposed modeling concept can be used in the development of a centralized control model in a newsvendor setting.

Reviews of papers on multi-echelon inventory management can be seen in Lee and Nahmias (1993), Muckstadt and Roundy (1993), Van Houtum et al. (1996) and Diks et al. (1996). Lee and Nahmias (1993) present an overview of the major developments and results for single product inventory management. Muckstadt and Roundy (1993) review optimization based methods for a range of serial, assembly, distribution and other more general system structures with constant demand rates. Precisely, this review presents models and algorithms that can be used to determine the “reorder intervals” for each stage of the systems with different topology. Van Houtum et al. (1996) review the most important results of multi-echelon inventory management from the cost-minimization perspective. However, Diks et al. (1996) reviewed multi-echelon inventory management from the perspective of service measurement.

It is noted from the literature above that:

- Traditional multi-echelon inventory models reviewed above treat all the transactions between different players inside a system as internal activities with implicit assumptions such as complete information sharing and incentive alignment of all the players' behavior.
- Although the multi-echelon inventory models take factors such as system topology, various pricing factors, lead time into consideration, the objective function is always inventory cost minimization focused. Moreover, the derived optimal or near optimal solutions are often too complicated to be applied in practice.

The supply contract models to be reviewed next differ from the multi-echelon inventory models in that the relationship between the players, represented by the pricing scheme or by quantity commitments between players, is treated as decision variable and thus negotiable. Further, exchange of information between players is dependent on contractual terms.

2.2 Supply Contract Models

In general, the parameters over which two parties may contract upon will be context specific; however, a supply contract usually include parameters such as horizon length, pricing scheme, periodicity of ordering, quantity commitment, flexibility, delivery commitment, quality and information sharing, capturing the material, information and financial flows in a SC (Anupindi and Bassok, 1999).

In the literature of supply contract, it is noted that different supply contracts serve different purposes. Some contracts have the risk-sharing objective while some others have the channel coordination objective (Tsay et al., 1999). In the following sections, various supply contract

models will be reviewed according to their purposes. However, the same supply contract type may serve different purposes under different circumstances and, thus, will be reviewed separately.

2.2.1. Contracts with Risk-Sharing Objective

Contracts with risk-sharing objective focus on the profit allocation between the contractual players by sharing the risks arising from various sources of uncertainty. For instance, uncertainty in market demand is a commonly observed uncertainty in a SC and “phantom ordering” is the most commonly observed problem associated with it. Phantom ordering happens when a buyer deliberately inflates its demand forecasting to its supplier (to ensure supply); however, there are no restrictions imposed on the buyer’s actual purchasing quantity. In this case, the buyer transfers all the uncertainty in the demand process to the supplier.

Hence, in order to protect the supplier, buyers are always required to commit to a certain minimum purchasing quantity and are compensated with a reduced purchasing price. This practice is referred to as commitment contract (Bassok and Anupindi, 1997; Anupindi and Bassok, 1999; Urban, 2000). In addition, buyers may be required to stick to their original orders with some adjustable parameters (either upward or downward or both) indicating the possible deviations from its original order. This practice is referred to as quantity flexibility (QF) contract (Eppen and Iyer, 1997; Tsay, 1999; Tsay and Lovejoy, 1999). Both contracts focus on the determination of the best response at one end of a SC (e.g., the buyer’s optimal purchasing quantity given a contract type) rather than on both ends of a SC. Apart from these two contracts, some quantity discount models also focus on the determination of the best response at one end of a SC: from the buyer’s perspective to focus on the determination of its optimal purchasing policy given a fixed quantity discount scheme (Monahan, 1984; Lee and

Rosenblatt, 1986) or from the supplier's perspective to focus on the design of an appropriate quantity discount scheme (Corbett and de Groote, 2000).

2.2.1.1. Quantity Flexibility Contract

Quantity flexibility contracts are frequently used for components in the electronics and computer industry and have occasionally been used in the automotive industry (Lariviere, 1999).

Eppen and Iyer (1997) study the supplier-buyer relationship in a distribution channel where the buyer sells catalogue product (which is characterized by uncertain demand, long production lead time and a short selling season). A "backup agreement" is considered and the research focuses on the determination of various parameters such that the buyer's expected profit can be maximized. A two-period stochastic dynamic programming model is formulated which takes the demand update information between the periods into account. The results indicate that an increase in the backup fraction can improve the catalog company's expected profit as well as its initial commitment.

Tsay and Lovejoy (1999) study QF contract in a multi-period, rolling horizon context, which is an operational method of incorporating new demand information in the buyers' period by period replenishment schedule. Similarly, a QF arrangement, which takes the form of an upward percentage above the buyer's original orders that have to be covered by the supplier and a downward percentage that the buyer has to commit, is proposed to determine the best inventory distribution along a supply chain. In addition, the QF arrangement is able to dampen the effect of order variability throughout the entire supply chain.

Tsay (1999) considers “phantom ordering” in a two-echelon distribution channel with a supplier selling to a buyer facing newsvendor problem. The proposed contract couples the retailer’s commitment to purchase no less than a certain percentage below its forecast with the supplier’s guarantee to deliver up to a certain percentage above. Rather than assuming a passive supplier who simply accommodates the buyer’s actions, a behavior model of each player’s local incentives, with consideration of updated demand information, is formulated. The analysis shows that, with appropriate choice of the coordinating parameters (such as the wholesale price), a QF contract can achieve system-wide efficiency.

2.2.1.2. Commitment Contract

Commitment contract takes various forms such as total minimum quantity commitment contract, total minimum dollar volume commitment contract and periodic commitment with or without flexibility. Total minimum quantity commitment contract is the most flexible to the buyer, which requires that the buyer’s cumulative purchasing quantity over the entire horizon should be no less than a prescribed minimum quantity. When this kind of contract applies to a family of products, the contract takes the form of minimum dollar volume contract.

Further, a more stringent restriction could even be imposed on a buyer’s purchasing quantity in each period when a supplier is greatly affected by the uncertainty in each period. This takes the form of periodic commitment. The most stringent periodic commitment contract is periodic commitment contract without flexibility. With this contract type, all the uncertainties that lie in the demand process are transferred back to the buyer. In some cases, a buyer taking a periodic commitment contract is allowed to increase its commitment to a certain percentage above its original commitment, however, at a price premium.

Clearly, the newsvendor model (with no restrictions imposed on a buyer's purchasing quantity) and a periodic commitment contract without flexibility are two extremes. At an intermediate level, suppliers and buyers share the burden of demand uncertainty with a commitment contract having a certain level of flexibility.

Bassok and Anupindi (1997) analyze a total minimum quantity commitment contract for a distribution channel with uncertain customer demand and the buyer place orders periodically. This research derives the optimal purchasing policy for the buyer with a given total minimum quantity commitment and a discounted price. It is demonstrated that the optimal policy is characterized by the order-up-to levels of the corresponding finite horizon and a single-period standard newsvendor problem with no commitment but with discounted price.

Anupindi and Bassok (1999) review various quantity commitment models for a distribution channel with stochastic demand. It argues that quantity commitment contracts usually provide some flexibility, however at the expense of the suppliers.

Urban (2000) considers a periodic, stationary commitment contract in a distribution channel where the buyer is with limited flexibility to change its order quantity in each period but at the expense of additional penalty costs. A solution methodology is provided for certain demand distributions with stochastic and non-stationary demand rate. The deterministic scenario of the problem is investigated as well which is formulated with linear mixed-integer programming and as a network flow problem.

The literature shows that commitment contract focuses on the determination of the buyers' optimal ordering policy given a commitment contract type and the analytical models are

always formulated over a multi-period horizon representing multiple order replenishment opportunities.

2.2.1.3. Quantity Discount Contract

In practice, a supplier who offers a quantity discount to a retailer varies its wholesale price charged to the retailer according to the retailer's purchasing quantity – the larger the purchasing quantity, the lower the per unit wholesale price. Quantity discount contract acts as a mechanism to induce the retailer from placing a larger order.

Given a fixed scheme, traditional research on quantity discount contract is done from the perspective of the buyer. Hence, it is the buyer's total inventory related costs that are minimized and it is the buyer's best response to the given quantity discount terms that is determined. In addition, attention has been paid to study the quantity discount contract from the perspective of the supplier – how a supplier would determine the terms of a quantity discount scheme to maximize its own profits.

Monahan (1984) studies quantity discount contract from the perspective of the supplier. It presents an economics analysis as to how a supplier can structure the terms of an optimal quantity discount schedule to induce the buyer from placing a larger order. Customer demand, however, is assumed to be deterministic. The terms of the discount schedule are determined by a mathematical model, which anticipates the buyer's likely reaction to any of the supplier's proposals. The ultimate schedule is one which maximizes the supplier's resultant economic gain, but does so at absolutely no added cost to the buyer.

Lee and Rosenblatt (1986) extends Monahan (1984) by explicitly incorporating constraints

imposed on the amount of discount that can be offered by the supplier (that is, the amount of price discount offered to the buyer should be lower than the product's selling price); also the lot-for-lot policy (that is, whenever the buyer places orders the supplier incurs a setup cost) is relaxed. The analysis shows that the extent to which the supplier would offer a price discount to induce the buyer from placing a larger order is a function of both the setup and inventory holding costs of both the supplier and the buyer.

Both papers reviewed above focus on operating cost minimization; however, an implicit critical assumption is that the supplier has full information and, thus, is able to design the quantity discount scheme. Nonetheless, when this assumption does not hold any more, system efficiency would be affected by information asymmetry.

Corbett and de Groote (2000) study this problem by deriving the supplier's optimal quantity discount scheme with consideration of the buyer holding private information about its cost structure while the supplier having considerably higher setup costs. Two contracts are compared: the supplier's optimal contract under full information and asymmetric information. In the asymmetric information setting, revelation principle (that is, if there is an optimal contract for the supplier, there also exists an optimal contract under which the buyer will truthfully reveal its holding cost (Laffont and Tirole, 1993)) is applied to derive the solutions. The analysis shows that global efficiency can be reduced by information asymmetry; however, system performance can be improved through quantity discount. Further, the supplier's expected net costs increase under information asymmetry; whereas, the buyer's net cost decrease when it has private information.

2.2.2. Contracts with Channel Coordination Objective

With the advent of modern information technology, business communities have increased the degree of coordination among multiple plants, multiple functions and multiple organizations. Whang (1995) classifies coordination along a SC into three categories: coordination within operations, cross-functional coordination and inter-organizational coordination. Moreover, the author proposes that coordination is also dependent on the perspective from which an organization is viewed.

The inter-organizational coordination with the nexus-of-contract perspective follows the tradition of agency theory (Jensen and Meckling, 1976), in which an organization is a nexus of contracts among self-interested agents (or referred to as players in this research), each maximizing his or her personal objective. Such self serving incentive often results in poorer system performance. However, global optimal performance can be achieved if players can be coordinated through contracting on a set of transfer payments such that each player has the incentive to act in accordance with the entire supply chain's objective (Cachon, 2002).

Cachon (2002) reviews recent research (mostly analytical models) on SC coordination with contracts, in which coordination is defined as “a Nash Equilibrium in the set of SC optimal actions”, that is, no firm has a profitable unilateral deviation from the set of supply chain optimal actions. Issues, which include the coordination of a newsvendor problem with price-dependent demand, effort-dependent demand, multiple competing newsvendors, multi-stage newsvendor and information asymmetry, are reviewed. Tsay et al. (1999) review research on supply contracts according to different supply contract clauses which include: specification of decision rights, pricing, minimum purchase commitment, quantity flexibility, buyback or returns policies, allocation rules, lead time and quality.

The general goal of supply contracts with the channel coordination objective is to establish rules for material accountability or pricing structure that can guide autonomous players towards a global optimized solution (Whang, 1995; Lariviere, 1999; Tsay, 1999; Cachon and Lariviere, 2002; Wang et al., 2004a; Zou et al., 2004). However, channel coordination can also be achieved when all the players along a SC take a cooperative attitude (or referred to as the team perspective in Whang (1995)) in their decision-making process.

2.2.2.1. Price-Only Contract

The pricing scheme between two players under a price-only contract (or referred to as the wholesale price contract) is described by wholesale price only, which is probably the simplest pricing format. However, a price-only contract alone fails to coordinate a decentralized channel facing newsvendor problem due to “double marginalization” (Anupindi and Bassok, 1999).

Lariviere and Porteus (2001) consider a two-echelon distribution channel with one vendor selling to a retailer facing a newsvendor problem. The manufacturer sets the wholesale price and the retailer determines its purchasing quantity. The only contract parameter is wholesale price. The research focuses on the determinants of the wholesale price, the efficiency of the supply chain (measured by the fraction of maximum supply chain profit achieved by the decentralized system) and the division of realized supply chain profit. The analysis shows that when relative variability falls, measured by the coefficient of variation of the demand distribution, the manufacturer charges a higher wholesale price, system efficiency improves, and the manufacturer takes a larger share of chain profits. In addition, it is found out that the manufacturer could choose a lower wholesale price if a better demand forecasting could be achieved or the retailer has a higher bargain power. This research also concludes that

wholesale price alone cannot coordinate a decentralized distribution channel.

Despite the fact that a price-only contract alone cannot coordinate a decentralized channel, the contract can be used to evaluate the performance of a decentralized system (without coordination) since wholesale price represents the most basic pricing relationship between any two transactional parties.

2.2.2.2. Quantity Discount Contract

As reviewed earlier, a price-only contract leads to a reduction in the retailer's optimal stocking level due to "double marginalization"; however, the retailer might increase its purchasing volume if the supplier could share a portion of its increased profits (attained from an increased purchasing quantity of the retailer) by reducing the retailer's purchasing price. This is how a quantity discount contract can be used as a mechanism for channel coordination.

Jeuland and Shugan (1983) consider a supplier-retailer distribution channel. Customer demand is assumed to be deterministic, but retail price dependent. The authors point out that since each channel member's decision does affect other channel members' profits and actions, a lack of coordination of these decisions can lead to undesirable consequences. Several possible solutions to channel coordination, such as joint ownership, implicit understanding (that is, channel members determine their best responses based on their anticipations of others' possible behavior) and profit sharing, are discussed. However, it is demonstrated that by applying quantity discount, many weaknesses of the aforementioned strategies, such as the requirement of vertical integration for the implementation of joint ownership and the requirement for incentive design to prevent players from unilaterally deviating from global

optimal solutions, can be overcome.

Weng (1995) presents a model to integrate channel coordination and operating cost minimization in a system consisting of a supplier and a buyer. The analysis shows that quantity discount, either all-unit or incremental, does not guarantee joint profit maximization. However, by employing quantity discount and franchise fees simultaneously, channel coordination can be realized.

The literature reviewed above examines quantity discount contract in two-echelon distribution channels. However, when there is an increase in the number of echelons, players in the middle echelons can act as coordinators in offering plausible coordinating mechanisms simultaneously to their upstream suppliers and downstream buyers.

Munson and Rosenblatt (2001) model quantity discount in a three-echelon SC with the middle company enjoying a strong position, offering different discount schemes to its upstream supplier and downstream customer. The results show that incorporating quantity discounts by the middle echelon can significantly decrease costs compared to concentrating only on the customer side.

2.2.2.3. Buyback Contract

Buyback contract (or referred to as returns policy) has been widely used in the industries producing commodities with short shelf life (or demand life) such as newspapers, baked goods, periodicals and records (Pasternack, 1985). Buyback contracts applied in practice range from a full credit on all unsold goods to no credit for unsold items and the intermediate strategies such as full credit for partial returns or partial credit for full returns.

With a buyback contract, a supplier charges a retailer a wholesale price for each unit purchased but pays the retailer a buyback price for each unit remaining at the end of a selling season. Moreover, the buyback price is always set to be lower than the retailer's purchasing price such that the possibilities that retailers might make profits by stocking inventories can be eliminated.

Pasternack (1985) considers returns policy in a manufacturer-retailer channel facing newsvendor problem. The research shows that several current practices of returns policy are suboptimal and cannot realize channel coordination. These practices include full credit for all unsold goods offered from the manufacturer to the retailers or no returns of unsold goods. It is also demonstrated that a policy whereby a manufacturer offers a retailer full credit for a partial return of goods may achieve channel coordination, but the optimal return allowance will be a function of the retailer's demand. Therefore, such a policy cannot be optimal in a multi-retailer environment. It is proven, however, that a wholesale price plus a buyback contract can achieve channel coordination in a multi-retailer environment.

Emmons and Gilbert (1998) also consider returns policy in a manufacturer-retailer channel; however, the retailer sells "catalogue style goods". The manufacturer offers a wholesale price as well as a buyback price to the retailer prior to the selling season; the retailer determines its order quantity as well as the product selling price. Market demand is both uncertain and selling price dependent. The retailer and the manufacturer's problems are modeled separately. The results show that demand uncertainty tends to increase the retail price. In addition, for a given wholesale price, the offer to buy back excess stock tends to increase the total system profits. This research differs from Pasternack (1985) in that it treats retail price as a decision variable while the latter treats retail price as an independent variable.

Lariviere (1999) studies the impact of three contracts, namely, price-only contract, buyback contract and QF contract, on coordination of a manufacturer-retailer channel facing newsvendor problem. The analysis shows that the price-only contract fails to coordinate such a channel due to “double marginalization”. The author argues that by adjusting the price mechanism through buyback contract or adjusting a quantity scheme through quantity flexibility contract, the decentralized distribution channel can be coordinated. In addition, both schemes can result in various split of channel profit between the channel members. It is also found out that buyback contract might be superior to QF contract since it does not depend on demand distribution to realize channel coordination.

Donohue (2000) examines a manufacturer-retailer channel where the manufacturer operates two modes of production (one production mode starts long before the selling season while the other starts closer to the selling season) and the retailer has the ability to update its demand forecast as the selling season approaches. Buyback contract is considered as the coordinating mechanism. Such a problem is formulated as a two-stage newsvendor problem with consideration of demand update between periods. Coordinating conditions to align the manufacture and the retailer’s incentives with channel-wide optimization are determined, which turn out to be dependent on the degree of demand forecasting improvement between periods. Moreover, guidance in designing a wholesale price scheme that allows the channel to take full advantage of the second, faster production mode is provided.

Gerchak and Wang (2004) study the incentive coordination problem in a two-echelon decentralized assembly system with random customer demand. The system is examined with two distinct arrangements: VMI with revenue-sharing led from downstream and a wholesale-price-based arrangement led from upstream. For the first arrangement, it is found

out that as the assembler's portion of production cost increases, the total channel profit as well as the profit of the assembler's increase while the total profit of the suppliers decrease. A revenue-plus-surplus-subsidy contract is proved to be able to maximize the total system profit by paying a surplus subsidy on unsold items to each supplier. For the second arrangement, a wholesale price contract has individual suppliers choosing their wholesale prices and then the assembler determines its purchasing quantity. This scheme prevents component mismatching, but the results of this scheme are inferior to that of a revenue-sharing scheme. Further, a partially-credited buyback policy reaches the system optimum.

Gerchak and Wang (2004) are relevant to this research. A similar incentive problem concerning mismatch of components, as discussed in Gerchak and Wang (2004), is explored in this research. However, a different arrangement between the suppliers and the assembler is used. Gerchak and Wang (2004) propose revenue-plus-surplus-subsidy for channel coordination with a subsidy for suppliers' over-delivered components led from the assembler to the suppliers. However, in this research, a coordinating mechanism comprised of a wholesale price and a buyback price (led from the suppliers to the assembler) is considered for channel coordination. Further, this research extends Gerchak and Wang (2004) by considering a two-period transaction.

2.2.2.4. Revenue-Sharing Contract

Revenue-sharing contract has been widely used in the video rental industry which faces a challenging capacity problem; that is, the peak popularity of a rental title lasts only for a few weeks but the cost of a tape has traditionally been higher relative to the price of a rental. With a revenue-sharing contract, the supplier charges the retailer a per unit wholesale price; however, the retailer gives the supplier a portion of its revenue.

Dana and Spier (2001) consider revenue sharing as a coordinating mechanism in a two-echelon distribution channel with multiple retailers having perfect competition. In such a channel, the upstream supplier controls both the retailers' inventory levels and their retail prices. The analysis compares the impact of two-part tariffs (a franchise fee plus a quantity dependent cost) and revenue sharing on system performance. The comparison shows that two-part tariffs is unable to accomplish softening of downstream competition and inventory holding at the same time; however, a revenue-sharing contract, combined with a lower wholesale price charged to the retailer, aligns retailers' incentives in the vertical chain.

Cachon and Lariviere (2004) generalize Dana and Spier (2001) by studying the revenue-sharing contract in a two-echelon distribution channel with competing retailers. In a basic setting, a supplier-retailer channel facing newsvendor problem is considered. The analysis shows that revenue sharing can coordinate the decentralized channel, so do the other contracts such as buyback contract, QF contract and sales-rebate contract. In an extended setting, a price-dependent newsvendor scenario (that is, market demand is both retail price sensitive and stochastic) is considered. The analysis shows that only revenue sharing can coordinate such a setting. Further, the research extends to include multiple retailers competing for quantities (since each retailer's revenue depends on its quantity as well as the other retailers' quantities). The results indicate that a revenue-sharing contract can also coordinate such a setting while supporting arbitrary split of profit. Several limitations of revenue-sharing contract, such as the administrative costs associated with the implementation of such a contract and the coordinating issue in a competing price-dependent newsvendor setting, are briefly discussed.

Wang et al. (2004b) consider the implementation of consignment and revenue sharing in a

one-period, supplier-buyer channel. Market demand is both uncertain and retail price dependent. The buyer determines the share of the revenue offered to the supplier; the supplier decides on the production quantity as well as the selling price. The decision-making process is modeled as a Stackelberg game (the leader-follower game). The results indicate that the loss in channel profit (defined as the profit difference in percentage between a centralized channel and a decentralized channel) increases with demand price elasticity and decreases with the buyer's share of cost. In addition, the profit share extracted by the retailer decreases with demand price elasticity and increase with the buyer's share of cost.

Zou et al. (2004) consider channel coordination in a two-echelon decentralized assembly system facing uncertain market demand. The authors develop an analytical model to synchronize different processing times through strategic placement of safety stocks at each player's premises. Moreover, a revenue-sharing model is proposed for channel coordination.

2.2.2.5. Joint Pricing-Production Decision Making

Various supply contract models reviewed above take the nexus-of-contract perspective to design the coordinating mechanism. With that perspective, players along the channel are granted with some decision rights over some of the parameters such as selling price, order or production quantity and wholesale price. However, some researchers argue that, due to the growing competition and shrinking profit margins, players along a channel would consider a joint coordination of their decisions such that a better performance can be expected (Weng, 1997, 1999; Zhao and Wang, 2002). This is a cooperative coordination perspective.

Weng (1997) studies the effect of joint coordination of pricing-production/ordering decisions in a manufacturing-distribution system consisting of one manufacturer and one distributor.

The system operates to meet price-sensitive random demand with the objective of maximizing expected profits of both the manufacturer and the distributor. The coordinating model is developed in a one-period setting. The results show that a two-part tariff policy is able to achieve channel coordination; however, the manufacturer's wholesale price alone is insufficient to achieve channel coordination. Further, price sensitivity places an important role in increasing the system expected profit through coordination; namely, the more sensitive the random demand is to sales price, the more benefits that can be obtained from coordination. In addition, joint coordination leads to an increase in the distributor's order quantity.

Weng (1999) also studies joint coordination of pricing-production/ordering decisions in a two-echelon manufacturer-distributor channel; however, this research focuses on deriving the closed-form multi-attribute measures of system performance in a one-period setting. Likewise, a two-part tariff is shown to be sufficient to achieve channel coordination with a fixed payment representing different allocations of system profits. Further, the analysis shows that the more sensitive random demand is to sales price, the more benefits that can be obtained from coordination; however, the benefits from coordination depend on the attitude toward risk – the more risk-averse, the less benefit will be gained from coordination.

Zhao and Wang (2002) consider joint coordination of pricing-production/ordering decisions in a multi-period, decentralized manufacturer-distributor channel. Market demand is assumed to be deterministic and retail price sensitive. In the absence of joint coordination, both the manufacturer's and the retailer's optimal decisions are determined by developing the model as a Stackelberg game with the manufacturer acting as the leader and the distributor as the follower. The analysis shows that the Stackelberg solution itself will, in general, not lead to channel optimality. However, expanding the manufacturer's wholesale price contract with a

side-payment, the channel-optimal pricing and ordering schedule can be obtained. The different values of side-payment represent different allocation schemes of system profits.

2.3 Summary

A summary of the reviews grouped in two tables is given in Table 2-1 and Table 2-2. It is noted from the literature above that although various supply contracts, aiming at supply chain coordination, have been formulated, there are some gaps in them:

- The existing research on channel coordination through supply contracts focuses on two-echelon supplier-retailer channels (or channels representing any two consecutive nodes in a SC) or distribution channels consisting of a group of retailers. However, decentralized assembly system, which consists of multiple suppliers and requires the coordination of various components, has been studied by only a few authors. In an actual assembly system, there is more than one supplier and therefore, the model should consider the characteristics of the combination of the suppliers because the supply quantity of one supplier impacts the final production of the assembled products.
- Most of the supply contract models are formulated in a single-period newsvendor setting. Even there are some supply contract models formulated in a general multi-period setting (Bassok and Anupindi, 1997; Anupindi and Bassok, 1999; Tsay and Lovejoy, 1999; Urban, 2000), they do not focus on “channel coordination”. That is, they do not have a benchmark against which to measure if the system performance is optimized. Further, some researches focusing on channel coordination and multi-period transaction are limited by the assumption of deterministic market demand. In addition, some contract models are formulated for a two-period setting with considerations of updated demand information between periods (that is, correlated demand between periods). In other words, contract models developed for channel coordination for an assembly system by

considering multiple transactions and stochastic market demand are required.

This research, investigating coordination issues in SCM, addresses the above gaps. Specifically, two-period supply contract models are formulated to consider the incentive conflicts, the impact of changes in the cost parameters, transfer of inventory status and future demand of a decentralized two-echelon assembly system. Although, closed-form solutions are derived only for a two-period setting, the models are capable of capturing the main characteristics of a more general multi-period setting.

Table 2-1 Summary of supply contracts with risk-sharing objective

Supply Contracts with Risk-Sharing Objective:					
Contract Type	Reference	SC Topology	Modeling Horizon	Main Assumptions	Main Results
QF	Eppen and Iyer (1997)	supplier-buyer channel	two-period model	catalogue product; stochastic demand; updated information	An increase in the backup fraction can improve the catalog company's expected profit as well as its initial commitment
	Tsay (1999)	supplier-buyer channel	single-period model	phantom ordering; newsvendor setting	With appropriate choice of the wholesale price, a QF can achieve system-wide efficiency
	Tsay and Lovejoy (1999)	supplier-buyer channel	multi-period model	stochastic demand; rolling horizon setting	The QF contract, which takes the form of an upward percentage above the buyer's original orders that have to be covered by the supplier and a downward percentage that has to be committed by the buyer, is able to dampen the effect of order variability.
Commitment Contract	Bassok and Anupindi (1997)	supplier-buyer channel	multi-period model	stochastic demand; total minimum quantity commitment	The optimal policy is characterized by the order-up-to levels of the corresponding finite horizon and a single-period standard newsvendor problem with no commitment, discounted price.
	Anupindi and Bassok (1999)	supplier-buyer channel	multi-period model	stochastic demand; various commitment contracts	Quantity commitment contracts usually provide some flexibility, however at the expense of the suppliers.
	Urban (2000)	supplier-buyer channel	multi-period model	stochastic demand; periodic, stationary commitment	A solution methodology is provided for certain demand distributions with stochastic and non-stationary demand rate. The deterministic scenario of the problem is formulated with linear mixed-integer programming and as a network flow problem.
Quantity Discount Contract	Monahan (1984)	supplier-buyer channel	single-period model	deterministic demand; from the supplier's perspective	An analytical model is proposed to model the buyer's likely reaction to the supplier's proposal. The ultimate schedule is one which maximizes the supplier's resultant economic gain, but does so at absolutely no added cost to the buyer.
	Lee and Rosenblatt (1986)	supplier-buyer channel	single-period model	deterministic demand; from the supplier's perspective	It is an extension to Monahan (1984) by incorporating constraints on the amount of discount and the lot-for-lot policy. The results show that the optimal scheme is a function of both the setup and inventory holding costs of both players.
	Corbett and de Groote (2000)	supplier-buyer channel	single-period model	deterministic demand; information asymmetry	Global efficiency is reduced by information asymmetry; system performance can be improved through quantity discount. Supplier's expected net costs increase under information asymmetry; buyer's net cost decrease with private information.

Table 2-2 Summary of supply contracts with channel coordination objective

Supply Contracts with Channel Coordination Objective:					
Contract Type	Reference	SC Topology	Modeling Horizon	Main Assumptions	Main Results
Price-Only Contract	Lariviere and Porteus (2001)	supplier-buyer channel	single-period model	newsvendor setting;	Relative variability of market demand plays an important role in profit allocation and system performance improvement. Wholesale price alone cannot coordinate a decentralized channel.
	Jeuland and Shugan (1983)	supplier-buyer channel	single-period model	deterministic market demand	Quantity discount contract can be used to coordinate the proposed supplier-buyer channel. It has advantages over some coordinating mechanisms applied in the market literature.
Quantity Discount Contract	Weng (1995)	supplier-buyer channel	single-period model	newsvendor setting; integrate channel coordination and operating cost minimization	Quantity discount, either all-unit or incremental, does not guarantee joint profit maximization. Channel coordination can be realized with quantity discount plus franchise fee.
	Munson and Rosenblatt (2001)	three-echelon SC	single-period model	stochastic market demand; the middle company offers two contracts separately	Incorporating quantity discount by the middle echelon can significantly decrease costs compared to concentrating only on the customer side.
	Pasternack (1985)	manufacturer-retailer channel	single-period model	newsvendor setting	Full credit for a partial return of goods achieves coordination in a single retailer setting. A wholesale price plus buyback contract achieves coordination in a multi-retailer environment.
Buyback Contract	Emmons and Gilbert (1998)	manufacturer-retailer channel	single-period model	market demand is uncertain and price dependent	Demand uncertainty tends to increase the retail price. For a given wholesale price, the offer to buy back excess stock tends to increase the total system profits.
	Lariviere (1999)	manufacturer-retailer channel	single-period model	newsvendor setting	Wholesale price contract fails to coordinate the decentralized channel. Both buyback and QF can realize channel coordination as well as random split of channel profits. Further, QF is superior to buyback due to its independence of demand distribution.
	Donohue (2000)	manufacturer-retailer channel	two-period model	stochastic demand; two modes of production; information update	Buyback contract is able to coordinate the channel and the optimal coordinating condition is dependent on the demand forecasting improvement between periods.
	Gerchak and Wang (2004)	two-echelon assembly system	single-period model	newsvendor setting; the system operates with two arrangements	For the first arrangement, a revenue-plus-surplus-subsidy contract coordinates the channel. For the second arrangement, component mismatching is avoided; and a partially-credited buyback contract coordinates the channel.

Contract Type	Reference	SC Topology	Modeling Horizon	Main Assumptions	Main Results
Revenue-Sharing Contract	Dana and Spier (2001)	two-echelon multi-retailer channel	single-period model	stochastic demand; retailers having perfect competition	Two-part tariffs is unable to accomplish softening of downstream competition and inventory holding at the same time; however, a revenue-sharing contract, combined with a lower wholesale price charged to the retailer, aligns retailers' incentives.
	Cachon and Lariviere (2004)	two-echelon multi-retailer channel	single-period model	newsvendor problem and an extended price-dependent newsvendor setting	For the basic setting, revenue sharing and other contracts as buyback, QF and sales-rebate coordinate the channel. For the extended setting, only revenue sharing can realize coordination.
	Weng et al. (2004)	supplier-buyer channel	single-period model	uncertain and price-dependent demand; consignment arrangement	The DM process is modeled as a Stackelberg game. The results indicate that the loss in channel profit increases with demand price elasticity and decreases with the buyer's share of cost.
	Zou et al. (2004)	two-echelon assembly system	single-period model	newsvendor setting; lead time synchronization	Different processing times are synchronized through strategic placement of safety stocks and a revenue-sharing contract is able to realize channel coordination.
Joint Pricing-Production DM	Weng (1997)	manufacturer-retailer channel	single-period model	price-sensitive random demand	A two-part tariff policy is able to achieve channel coordination. The more sensitive the random demand is to sales price, the more benefits that can be obtained from coordination.
	Weng (1999)	manufacturer-retailer channel	single-period model	price-sensitive random demand; deriving closed-form multi-attribute measure of system performance	A two-part tariff policy is able to achieve channel coordination. The more sensitive the random demand is to sales price, the more benefits that can be obtained from coordination. Benefits from coordination depend on player's attitude toward risk.
	Zhao and Wang (2002)	manufacturer-retailer channel	multi-period model	deterministic, price-sensitive demand	The DM process is modeled as a Stackelberg game. In the absence of joint coordination, the Stackelberg solution does not realize channel coordination. However, a wholesale price plus side-payment is able to realize channel coordination.

Chapter 3

Single-Period Supply Contract Model

In this chapter, a model for the assembly system operated with centralized control is presented first. The results obtained from this model are used as a benchmark to assess the system performance with decentralized control. Followed by that, the assembly system operated with price-only decentralized control is considered. System inefficiencies due to conflicting incentives are analyzed by formulating individual player's best response and the results obtained from the analysis are compared with that of the centralized model. Lastly, two contract models, buyback contract and revenue sharing contract, are proposed for channel coordination. Applicability of the proposed models is highlighted by a numerical study at the end of this chapter.

3.1 Assumptions for the Basic Model

For model development, a two-echelon decentralized assembly system with one assembler and multiple component suppliers is considered. Suppliers and the assembler are engaged in a contract that lasts one period. The suppliers' optimal production/delivery quantities as well as the assembler's optimal purchasing quantity with different operational control modes are determined. Further, assumptions for the proposed assembly system are outlined below:

- Components for the final assembly are not interchangeable and each component is purchased from a different supplier.
- Each supplier has a different cost structure.
- Market demand is stochastic and it is completely shared by all the suppliers.

- Suppliers’ production/delivery decisions as well as the assembler’s purchasing decision are made before the realization of market demand.
- Any leftover inventory at the end of the selling season is disposed without incurring any additional costs (consistent with the assumption in Gerchak and Wang, 2004; Zou et al., 2004). Since the focus of this thesis is on system performance improvement through coordinating mechanism, consideration of salvage cost will only change the total expected system profit finally obtained rather than the method proposed. Hence, it is not explicitly modeled in the single-period model.
- Backordered customer demand is charged with penalty cost to represent loss of customer goodwill.
- All the costs and revenues are proportional to the number of units involved.

Cost parameters include production cost incurred by all the players, wholesale prices (in the decentralized control setting) charged by suppliers to the assembler for every unit of component purchased; shortage cost incurred by the assembler for backordered customer demand. Specifically, production cost, shortage cost and selling price are regarded as system parameters.

- All the players are risk neutral and therefore, they are profit maximizers.
- Initial inventory level at each player’s premises is assumed to be zero.
- Assembly of one unit of final product requires one unit of each component.

Notations for the single-period model are listed in Table 3-1.

Table 3-1 Notations for single-period supply contract model

N	Represent N component suppliers in the system
c	Subscript, indicates the centralized control setting
d	Subscript, indicates the decentralized control setting
b	Subscript, indicates the coordinated setting with buyback contract
r	Subscript, indicates the coordinated setting with revenue sharing
e	Subscript, represents equilibrium solution

ψ	Subscript, the assembler's order-up-to level without constraint
i	Subscript, $i = 1, 2, \dots, N$ represents the suppliers, $i = 0$ represents the assembler
ξ	Stochastic market demand (≥ 0)
λ	Mean of market demand
$f(\xi)$	The probability distribution function of market demand
$F(\xi)$	The cumulative distribution function of market demand
P	Unit selling price
w_{di}	Wholesale price in the price-only decentralized setting
w_{bi}	Wholesale price in the coordinated decentralized setting with buyback contract
w_{ri}	Wholesale price in the coordinated decentralized setting with revenue sharing contract
c_i	Unit production cost for player i , $\forall i$
s	Unit shortage cost
δ	The KKT condition multiplier
Q_c	Production/purchasing quantity in the centralized setting
Q_{di}	Production/purchasing quantity in the price-only decentralized setting for player i , $\forall i$
Q_{de}	Supplier's optimal equilibrium production/delivery quantity in the price-only decentralized setting
$Q_{d\psi}$	Assembler's optimal purchasing quantity without constraint in the price-only decentralized setting
Q_{bi}	Production/purchasing quantity in the coordinated control setting with buyback contract for player i , $\forall i$
Q_{be}	Supplier's optimal equilibrium production/delivery quantity in the coordinated control setting with buyback contract
$Q_{b\psi}$	Assembler's optimal purchasing quantity without constraint in the coordinated control setting with buyback contract
Q_{ri}	Production/purchasing quantity in the coordinated control setting with revenue-sharing contract for player i , $\forall i$
Q_{re}	Supplier's optimal equilibrium production/delivery quantity in the coordinated control setting with revenue-sharing contract
$Q_{r\psi}$	Assembler's optimal purchasing quantity without constraint in the coordinated control setting with revenue-sharing contract
π_c	Expected system profit in the centralized setting
π_{di}	Expected profit for player i in the decentralized setting
π_{bi}	Expected profit for player i in the coordinated setting with buyback contract
π_{ri}	Expected profit for player i in the coordinated setting with revenue sharing

Some shorthand expressions and some useful mathematical manipulations to simplify the deduction process are listed below,

- i.) If $a, b \in \mathbb{R}$ and $a \leq b$, then $x \perp [a, b]$ denotes the point in the interval $[a, b]$ that is close to x . More precisely,

$$x \perp [a, b] = \begin{cases} a & \text{if } x \leq a \\ x & \text{if } a < x < b \\ b & \text{if } b \leq x \end{cases}.$$

$$\text{ii.) } (a-b)^+ = \begin{cases} a-b & \text{if } a > b \\ 0 & \text{else} \end{cases}.$$

$$\text{iii.) } x \perp [a, b] = b + (a-x)^+ - (b-x)^+.$$

Proof.

$$- \text{ If } x \leq a, \text{ then } b + (a-x)^+ - (b-x)^+ = a;$$

$$- \text{ If } a < x < b, \text{ then } b + (a-x)^+ - (b-x)^+ = x;$$

$$- \text{ If } x \geq b, \text{ then } b + (a-x)^+ - (b-x)^+ = b.$$

The results that the expression of $x \perp [a, b] = b + (a-x)^+ - (b-x)^+$ satisfies the definition for $x \perp (a, b)$, which concludes the demonstration. ■

$$\text{iv.) } \frac{d \left[E_{\xi} \left\{ (x-\xi)^+ \right\} \right]}{dx} = F(x).$$

Proof.

Since $E_{\xi} \left\{ (x-\xi)^+ \right\} = x \int_0^x f(\xi) d\xi - \int_0^x \xi f(\xi) d\xi = xF(x) - \int_0^x \xi f(\xi) d\xi$, take first-order

derivative with respect to x , $E_{\xi} \left\{ (x-\xi)^+ \right\}'_x = F(x) + xf(x) - xf(x) = F(x)$. This

concludes the demonstration. ■

3.2 Model Formulation

In this section, mathematical models for the proposed assembly system operated with centralized and decentralized control modes are developed. Specifically, for the decentralized control mode, a price-only contract model and models for channel coordination are

formulated, respectively. System performance, in terms of the total expected system profits and the assembler's purchasing quantity, of the different models are measured and compared.

3.2.1 Centralized Control Model

With centralized control mode, it is assumed that there is a central decision maker (however, the central decision maker is not necessarily the assembler) who makes decisions for the benefits of the entire system. For this control mode, individual incentives are consistent with the system-wide incentive. Hence, the transactional relationship between the assembler and its component suppliers are not modeled (since they are regarded as internal activities). Moreover, with no initial on-hand inventory, all the suppliers would deliver the same number of components in the centralized setting; thus mismatch of components could be avoided. In addition, the assembler would purchase the exact number of components delivered by the component suppliers since over-delivered components are wasted. Therefore, the total expected system profits for the centralized setting can be formulated as,

$$\pi_c(Q_c) = E_\xi \left\{ p \cdot \min(\xi, Q_c) - Q_c \cdot \sum_{i=0}^N c_i - s(\xi - Q_c)^+ \right\} \quad (3-1)$$

the first term represents the expected revenue from product selling, the second term represents the total production costs and the last term represents the expected backorder penalty cost. With further mathematical manipulations, eq.(3-1) can be extended as,

$$\pi_c(Q_c) = \left(p - \sum_{i=0}^N c_i + s \right) \cdot Q_c - (p + s) \cdot E_\xi \left\{ (Q_c - \xi)^+ \right\} - s \cdot \lambda \quad (3-2)$$

It can be easily proved that $\pi_c(Q_c)$ is concave in Q_c (a negative second-order derivative); therefore, the optimal Q_c^* can be derived with the first-order condition. That is,

$$Q_c^* = F^{-1} \left(\frac{p + s - \sum_{i=0}^N c_i}{p + s} \right) \quad (3-3)$$

Replace Q_c in eq.(3-2) with Q_c^* , the optimal expected system profit is,

$$\pi_c(Q_c^*) = (p + s) \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi - s \cdot \lambda \quad (3-4)$$

The derived results in eqs.(3-3) and (3-4) will be used as a benchmark to compare with the corresponding results of the decentralized control setting.

3.2.2 Price-Only Decentralized Control Model

For this model, the pricing scheme between the component suppliers and the assembler is described by wholesale price alone to represent transfer of the ownership of components. Since the pricing scheme is characterized by wholesale price only, the developed model in this section is referred to as “price-only” decentralized control model.

The decision-making process for such a system is as follows:

1. The assembler shares market demand to all the component suppliers.
2. Suppliers determine how many components to produce based on the shared market demand as well as its anticipation of other suppliers' decisions.
3. The assembler determines its final purchasing quantities and pays each supplier for every unit of component purchased (that is, a consignment arrangement).
4. Market demand is realized and the assembler makes profit by selling the products.

In addition, assumptions on some of the cost parameters are discussed,

$$i. \quad p + s > \sum_{i=1}^N w_{di} + c_0$$

This inequality is required to make the assembler to be willing to engage in the business.

$$ii. \quad w_{di} > c_i \quad \text{for } i = 1, 2, \dots, N$$

This inequality is required to make all the component suppliers to be willing to engage in the business since it guarantees that suppliers can make profit from price markup.

Before developing the mathematical models, it is imperative to understand the suppliers' incentives and the constraints to the assembler. As mentioned, suppliers would produce to maximize their individual profits. However, due to the difference in their cost structures, the quantities delivered by each supplier could be different; therefore, the assembler can only purchase up to the quantity delivered by the "critical supplier". Moreover, suppliers are only paid for purchased components by the assembler; thus, the over-delivered components in this single-period setting are wasted. In anticipation of the possible costs from over-production, each supplier has the incentive to be the critical supplier. Such strategic behavior is expected to deteriorate system performance, resulting in reduced total expected system profits and a reduction in the assembler's purchasing quantity. In view of the constraint from the critical supplier, the assembler's purchasing quantity is expected to be,

$$Q_{d0} = \begin{cases} 0 & \text{if } \xi = 0 \\ \xi & \text{if } 0 < \xi \leq \min\{Q_{di}\} \\ \min\{Q_{di}\} & \text{if } \xi > \min\{Q_{di}\} \end{cases} \quad (3-5)$$

which can be interpreted as: when customer demand is fewer than the critical supplier's deliveries, the assembler will purchase the exact quantity to match market demand; however, when customer demand is more than the critical supplier's deliveries, the assembler can only purchase up to the critical supplier's deliveries. Specifically, eq.(3-5) can be written as,

$$Q_{d0} = \xi \perp [0, \min\{Q_{di}\}] \quad \text{for } i = 1, 2, \dots, N \quad (3-6)$$

Therefore, supplier i 's expected profits can be formulated as,

$$\begin{aligned} \pi_{di}(Q_{di}) &= w_{di} \cdot Q_{d0} - c_i \cdot Q_{di} \\ &= w_{di} \cdot [\xi \perp [0, \min\{Q_{di}\}]] - c_i \cdot Q_{di} \end{aligned} \quad (3-7)$$

Supplier i 's optimal production quantity Q_{di} can be determined as follows.

1. If $Q_{di} = \min\{Q_{di}\}$ for $i \neq 0$ (that is, if supplier i is the critical supplier), the derivatives of $\pi_{di}(Q_{di})$ with respect to Q_{di} are,

$$\begin{cases} \frac{d\pi_{di}(Q_{di})}{dQ_{di}} = (w_{di} - c_i) - w_{di} \cdot F(Q_{di}) \\ \frac{d^2\pi_{di}(Q_{di})}{dQ_{di}^2} = -w_{di} \cdot f(Q_{di}) < 0 \end{cases} \quad (3-8)$$

2. If $Q_{di} \neq \min\{Q_{di}\}$ for $i \neq 0$; that is, if supplier i is not the critical supplier; the derivatives of $\pi_{di}(Q_{di})$ with respect to Q_{di} are,

$$\frac{d\pi_{di}(Q_{di})}{dQ_{di}} = -c_i \quad \text{and} \quad \frac{d^2\pi_{di}(Q_{di})}{dQ_{di}^2} = 0$$

It is noted from the discussions in scenario 1 above that when supplier i is the critical supplier, its expected profit is concave in its production quantity (since the second-order condition is negative). Therefore, there exists a unique quantity that can maximize its expected profit. Derived from the first-order condition, the optimal quantity Q_{di}^* satisfies,

$$F(Q_{di}^*) = \frac{w_{di} - c_i}{w_{di}} \quad (3-9)$$

However, when supplier i is not the critical supplier (as shown in scenario 2), its optimal quantity cannot be derived with the traditional first-order condition (because the second-order derivative equals to zero). Whereas, the negativity of the first-order condition implies that supplier i 's expected profit decreases in its production quantity when its production quantity exceeds the critical supplier's delivery. In view of this, the supplier other than the critical supplier would have the incentive to deliberately decrease its production/delivery quantities until it equals to Q_{di}^* (that is, supplier i will deliver as if it is the critical supplier).

Proposition 3-1

All the production/delivery quantities lying within the range $[0, \min\{Q_{di}^*\}] \forall i \neq 0$ are Nash

equilibria and $Q_{di} = \min\{Q_{di}^*\}$ is the Pareto optimal solution. ■

Proofs for Proposition 3-1 are presented in Appendix III.

Constrained by $\min\{Q_{di}^*\} \forall i \neq 0$, the assembler's expected profit can be formulated as,

$$\begin{aligned} \pi_{d0}(Q_{d0}) &= E_{\xi} \left\{ p \cdot \min(\xi, Q_{d0}) - Q_{d0} \cdot \left(\sum_{i=1}^N w_{di} + c_0 \right) - s(\xi - Q_{d0})^+ \right\} \\ \text{s.t.} \quad Q_{d0} &= \xi \perp [0, \min\{Q_{di}^*\}] \end{aligned} \quad (3-10)$$

The derivatives of $\pi_{d0}(Q_{d0})$ with respect to Q_{d0} are,

$$\begin{aligned} \frac{d\pi_{d0}(Q_{d0})}{dQ_{d0}} &= \left(p - \sum_{i=1}^N w_{di} - c_0 + s \right) - (p + s)F(Q_{d0}) \\ \frac{d^2\pi_{d0}(Q_{d0})}{dQ_{d0}^2} &= -(p + s)f(Q_{d0}) < 0 \end{aligned}$$

It is noted that the assembler's expected profit is concave in its order quantity; hence, deduction of the assembler's optimal purchasing quantity can be treated as a constrained optimization problem where the Karush-Kuhn-Tucker (KKT) condition is satisfied (Hillier and Lieberman, 2002). Rewriting the constraint as $0 \leq Q_{d0}^* \leq \min\{Q_{di}^*\}$ and denote δ as the multiplier. The following equations are satisfied to derive the optimal solution,

$$\begin{cases} \left(p - \sum_{i=1}^N w_{di} - c_0 + s \right) - (p + s)F(Q_{d0}^*) - \delta \leq 0 \\ Q_{d0}^* \left\{ \left(p - \sum_{i=1}^N w_{di} - c_0 + s \right) - (p + s)F(Q_{d0}^*) - \delta \right\} = 0 \\ Q_{d0}^* - \min(Q_{di}^*) \leq 0 \\ \delta [Q_{d0}^* - \min(Q_{di}^*)] = 0 \\ Q_{d0}^* \geq 0 \\ \delta \geq 0 \end{cases} \quad (3-11)$$

Hence, the optimal solutions can be determined based on the following discussions.

1. If $Q_{d0}^* \neq 0$, $\delta \neq 0$, it can be derived that

$$\begin{cases} F(Q_{d0}^*) = F(\min\{Q_{di}^*\}) = \min\left\{\frac{w_{di} - c_i}{w_{di}}\right\} \\ \delta = \left(p - \sum_{i=1}^N w_{di} - c_0 + s\right) - (p + s) \cdot \min\left\{\frac{w_{di} - c_i}{w_{di}}\right\} \end{cases}$$

2. Otherwise,

$$\begin{cases} F(Q_{d0}^*) = \frac{p - \sum_{i=1}^N w_{di} - c_0 + s}{p + s} \\ \delta = 0 \end{cases}$$

Therefore, the assembler's optimal order quantity is,

$$Q_{d0}^* = \min\left\{F^{-1}\left(\frac{p - \sum_{i=1}^N w_{di} - c_0 + s}{p + s}\right), F^{-1}\left(\min\left\{\frac{w_{di} - c_i}{w_{di}}\right\}\right)\right\} \quad (3-12)$$

Denoting $Q_{d\psi}^* = F^{-1}\left(\frac{p - \sum_{i=1}^N w_{di} - c_0 + s}{p + s}\right)$, which represents the assembler's optimal

purchasing quantity without constraint and denoting $Q_{de}^* = F^{-1}\left(\min\left\{\frac{w_{di} - c_i}{w_{di}}\right\}\right)$, which

represents the optimal equilibrium quantity delivered by the critical supplier. Equation (3-12)

shows that if $Q_{d\psi}^* < Q_{de}^*$, the assembler will purchase $Q_{d\psi}^*$ to maximize its profits; whereas,

when $Q_{d\psi}^* > Q_{de}^*$, the assembler can only purchase up to the quantity delivered by the critical

supplier. Hence, all the players' equilibrium solutions can be summarized by Proposition 3-2.

Proposition 3-2

The equilibrium solution for the price-only decentralized control setting is,

$$\begin{aligned} & \langle Q_{d1}^*, Q_{d2}^*, \dots, Q_{dN}^*, Q_{d0}^* \rangle \\ &= \left\langle \underbrace{Q_{de}^*, \dots, Q_{de}^*}_N, \min\left\{F^{-1}\left(\frac{p - \sum_{i=1}^N w_{di} - c_0 + s}{p + s}\right), F^{-1}\left(\min\left\{\frac{w_{di} - c_i}{w_{di}}\right\}\right)\right\}\right\rangle \end{aligned} \quad (3-13)$$

Proposition 3-2 shows that, at equilibrium, it's optimal for suppliers to deliver the same number of components, which equals to Q_{de}^* ; hence, mismatch of components can be eliminated. Moreover, this could be achieved by adjusting the wholesale prices through individual negotiations between the assembler and the suppliers. In this way, the wholesale prices not only represent the pricing scheme between the suppliers and the assembler, it also acts as a coordinating mechanism (referred to as price-only contract) with which equilibrium solutions are obtained. Although, mismatch of components can be eliminated by adjusting wholesale prices, there would be leftover components, if $Q_{d\psi}^* = \min(Q_{d\psi}^*, Q_{de}^*)$, at the end of the selling season. Whereas, if $Q_{de}^* = \min(Q_{d\psi}^*, Q_{de}^*)$, the assembler would purchase all the components delivered by the suppliers to maximize its profits; hence, there would be no over-delivered components. In particular, the smaller the difference between $Q_{d\psi}^*$ and Q_{de}^* , the less inventory cost is incurred. Specifically, when $Q_{d\psi}^* = Q_{de}^*$, the total expected system profit is maximized in the price-only decentralized setting.

Compare Q_{d0}^* (in eq.(3-12)) with Q_c^* (in eq.(3-3)), it is noted that $Q_{d0}^* \neq Q_c^*$. Moreover, the total expected system profit obtained in the price-only decentralized setting is not equal to that of the centralized setting, either.

Proposition 3-3

The assembler's purchasing quantity in the price-only decentralized setting is fewer than that of the corresponding centralized setting; that is, $Q_{d0}^* < Q_c^*$. Moreover, the total expected system profit in the decentralized setting is less than that in the centralized setting. ■

Proofs for Proposition 3-3 are presented in Appendix III.

It is clearly observed from the above-mentioned analysis that the price-only contract cannot coordinate the proposed decentralized assembly system as system performance deteriorates, which is represented by a reduction in the assembler's purchasing quantity and reduced total expected system profit. Similar conclusions have been derived in the literature concerning decentralized distribution channels with one supplier and one buyer at each echelon (Lariviere, 1999; Lariviere and Porteus, 2001). Therefore, as discussed in the literature review, other mechanisms, such as buyback contract and revenue-sharing contract, are explored below for channel coordination.

3.2.3 Coordinated Decentralized Control Model

Two coordinating mechanisms, namely, buyback contract and revenue-sharing contract, are discussed for channel coordination. Optimal coordinating parameters for each contract type are determined and a comparison between the two contracts is conducted.

I. Channel Coordination with Buyback Contract

With a buyback contract, it is assumed that each supplier pays τ_i ($\forall i \neq 0$) to the assembler for every unit of final product leftover at the end of the selling season. With this contract, it does not mean that suppliers physically buy the assembler's excessive inventory back, rather suppliers partially compensate the assembler for over-produced final products as the assembler alone incurs backorder penalty cost if insufficient products are produced to satisfy market demand (Tsay, 1999).

Analogous to the analysis in the price-only decentralized model, the assembler's purchasing quantity is expected to be $Q_{b0} = \xi \perp [0, \min\{Q_{bi}\}]$ for $i \neq 0$, where $\min\{Q_{bi}\}$ represents the

critical supplier's deliveries in the coordinated decentralized setting with buyback contract.

Thus, supplier i 's expected profit can be formulated as (for $i \neq 0$),

$$\begin{aligned} \pi_{bi}(Q_{bi}) &= E_{\xi} \left\{ w_{bi} \cdot Q_{b0} - c_i \cdot Q_{bi} - \tau_i (Q_{b0} - \xi)^+ \right\} \\ \text{s.t.} \quad Q_{b0} &= \xi \perp [0, \min\{Q_{bi}\}] \end{aligned} \quad (3-14)$$

Likewise, supplier i 's optimal production quantity can be determined as follows:

1. if $Q_{b0} = Q_{bi}$; that is, if supplier i is the critical supplier; the derivatives of $\pi_{bi}(Q_{bi})$

with respect to Q_{bi} are,

$$\begin{cases} \frac{d\pi_{bi}(Q_{bi})}{dQ_{bi}} = (w_{bi} - c_i) - \tau_i \cdot F(Q_{bi}) \\ \frac{d^2\pi_{bi}(Q_{bi})}{dQ_{bi}^2} = -\tau_i \cdot f(Q_{bi}) < 0 \end{cases}$$

2. otherwise

$$\frac{d\pi_{bi}(Q_{bi})}{dQ_{bi}} = -c_i; \quad \frac{d^2\pi_{bi}(Q_{bi})}{dQ_{bi}^2} = 0$$

By analogy, it can be derived that each supplier will deliver $F^{-1}\left(\frac{w_{bi} - c_i}{\tau_i}\right)$ to maximize its

own profit. In particular, through simultaneous adjustment of w_{bi} and τ_i , all the suppliers can deliver the same number of components at equilibrium. Thus, mismatch of components is eliminated. In other words, the following expressions should be satisfied,

$$\frac{w_{b1} - c_1}{\tau_1} = \frac{w_{b2} - c_2}{\tau_2} = \dots = \frac{w_{bN} - c_N}{\tau_N} = F(Q_{be}^*) \quad (3-15)$$

where Q_{be}^* represents suppliers' optimal equilibrium delivery quantity with buyback contract.

Further, it is noted that the implementation of buyback contract requires $w_{bi} < c_i + \tau_i$.

Moreover, the assembler's expected profit can be developed as,

$$\pi_{b0}(Q_{b0}) = E_{\xi} \left\{ p \cdot \min(\xi, Q_{b0}) - Q_{b0} \cdot \left(\sum_{i=1}^N w_{bi} + c_0 \right) + (Q_{b0} - \xi)^+ \cdot \sum_{i=1}^N \tau_i - s \cdot (\xi - Q_{b0})^+ \right\} \quad (3-16)$$

$$s.t. \quad Q_{b0} = \xi \perp [0, \min\{Q_{bi}\}] \xi$$

Analogously, the assembler's optimal purchasing quantity can be derived with KKT condition. Denote δ as the multiplier, the optimal solutions are presented as follows,

1. if $Q_{b0}^* \neq 0$, $\delta \neq 0$

$$\begin{cases} Q_{b0}^* = Q_{be} \\ \delta = \left(p - \sum_{i=1}^N w_{bi} - c_0 + s \right) - \left(p - \sum_{i=1}^N \tau_i + s \right) \cdot Q_{be} \end{cases}$$

2. otherwise

$$\begin{cases} F(Q_{b0}^*) = \frac{p - \sum_{i=1}^N w_{bi} - c_0 + s}{p - \sum_{i=1}^N \tau_i + s} \\ \delta = 0 \end{cases}$$

Denote $Q_{b\psi}^* = F^{-1} \left(\frac{p - \sum_{i=1}^N w_{bi} - c_0 + s}{p - \sum_{i=1}^N \tau_i + s} \right)$, which represents the assembler's optimal

order-up-to level without constraint. Hence, the assembler's optimal purchasing quantity equals to,

$$Q_{b0}^* = \min\{Q_{b\psi}^*, Q_{be}^*\} \quad (3-17)$$

In addition, a necessary condition for channel coordination is that the assembler's purchasing quantity in the decentralized control setting with buyback contract could be the same as that in the corresponding centralized setting; that is to satisfy $Q_{b0}^* = Q_c^*$.

Further, if $Q_{b\psi}^* = Q_c^*$, it is derived that,

$$\frac{p - \sum_{i=1}^N w_{bi} - c_0 + s}{p - \sum_{i=1}^N \tau_i + s} = \frac{p + s - \sum_{i=0}^N c_i}{p + s} \quad (3-18)$$

With further mathematical manipulation to (3-18), it can be derived that

$$\begin{aligned}
 \frac{p - \sum_{i=1}^N w_{bi} - c_0 + s}{p - \sum_{i=1}^N \tau_i + s} &= \frac{p + s - \sum_{i=0}^N c_i}{p + s} \Rightarrow \frac{\sum_{i=1}^N (w_{bi} - \tau_i) + c_0}{p - \sum_{i=1}^N \tau_i + s} = \frac{\sum_{i=0}^N c_i}{p + s} \\
 \Rightarrow (p + s) \sum_{i=1}^N (w_{bi} - \tau_i) + c_0 \cdot (p + s) &= \left(p - \sum_{i=1}^N \tau_i + s \right) \cdot \sum_{i=0}^N c_i \\
 \Rightarrow F(Q_c^*) &= \frac{\left(p + s - \sum_{i=0}^N c_i \right)}{p + s} = \frac{\sum_{i=1}^N (w_{bi} - c_i)}{\sum_{i=1}^N \tau_i}
 \end{aligned} \tag{3-19}$$

Moreover, it can be derived from (3-15) that

$$\begin{aligned}
 w_{bi} - c_i &= F(Q_{be}) \cdot \tau_i \Rightarrow \sum_{i=1}^N (w_{bi} - c_i) = F(Q_{be}) \cdot \sum_{i=1}^N \tau_i \\
 \Rightarrow \frac{\sum_{i=1}^N (w_{bi} - c_i)}{\sum_{i=1}^N \tau_i} &= F(Q_{be}) = \frac{w_{bi} - c_i}{\tau_i} = F(Q_c^*)
 \end{aligned} \tag{3-20}$$

Hence, it can be derived from eqs.(3-18) to (3-20) that

$$Q_{b0}^* = Q_{b\psi}^* = Q_{be}^* = Q_c^* \tag{3-21}$$

The expression in eq.(3-21) leads to Proposition 3-4.

Proposition 3-4

The decentralized assembly system can be coordinated with the proposed buyback contract.

Specifically, at coordination, $Q_{b1}^* = Q_{b2}^* = \dots = Q_{bN}^* = Q_{be}^* = Q_{b\psi}^* = Q_{b0}^* = Q_c^*$. ■

Further implications from Proposition 3-4 are,

- Mismatch of components is completely eliminated since all the component suppliers deliver the same number of components.

- There will be no leftover components at the end of the selling season since the assembler purchases the exact number of components delivered by the suppliers.
- The assembler's purchasing quantity equals to that of the centralized setting.
- There exists a continuum of (w_{bi}, τ_i) , satisfying eqs.(3-15) and (3-18), that can achieve channel coordination and each pair of (w_{bi}, τ_i) represents a different allocation scheme of system profit.

Moreover, with the optimal quantities in Proposition 3-4, the assembler and each component suppliers' optimal expected profit are,

$$\begin{aligned}\pi_{b0}(Q_c^*) &= \left(p - \sum_{i=1}^N \tau_i + s \right) \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi - s \cdot \lambda \\ \pi_{bi}(Q_c^*) &= \tau_i \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi\end{aligned}\tag{3-22}$$

It can be further inferred from eq.(3-22) that,

- When system parameters are fixed, each supplier's expected profit only depends on its buyback price τ_i ; thus, each supplier can negotiate independently with the assembler concerning allocation of system profit.
- At coordination, w_{bi} and τ_i are positively related satisfying $w_{bi} = F(Q_c^*) \cdot \tau_i + c_i$. That is, the supplier who charges a higher wholesale price pays a higher buyback price. When $w_{bi} = c_i$ (for $i \neq 0$), the assembler extracts all the system profit; however, suppliers would not be willing to engage in this type of transaction.
- The supplier's expected profit is proportional to its buyback price τ_i .

II. Channel Coordination with Revenue-Sharing Contract

With a revenue-sharing contract, it is assumed that supplier i (for $i \neq 0$) charges w_{ri} to the

assembler for every unit of component purchased and the assembler shares with each supplier a fraction, ϕ_i , of its expected revenue. The assembler keeps a fraction, ϕ_0 , of its expected revenue. Specifically, it satisfies $\sum_{i=0}^N \phi_i = 1$.

Analogously, the assembler's optimal purchasing quantity in the decentralized setting with revenue-sharing contract is expected to be $Q_{r0} = \xi \perp [0, \min\{Q_{ri}\}]$ for $i \neq 0$, where $\min\{Q_{ri}\}$ is the critical supplier's optimal delivery quantity. Therefore, supplier i 's expected profit can be formulated as,

$$\pi_{ri}(Q_{ri}) = E_{\xi} \{ \phi_i p \cdot \min(\xi, Q_{r0}) + w_{ri} \cdot Q_{r0} - c_i \cdot Q_{ri} \} \quad (3-23)$$

Hence, the derivatives of $\pi_{ri}(Q_{ri})$ with respect to Q_{ri} are,

1. if $Q_{r0} = Q_{ri}$; that is, if supplier i is the critical supplier

$$\begin{cases} \frac{d\pi_{ri}(Q_{ri})}{dQ_{ri}} = (\phi_i p + w_{ri} - c_i) - \phi_i p \cdot F(Q_{ri}) \\ \frac{d^2\pi_{ri}(Q_{ri})}{dQ_{ri}^2} = -\phi_i p \cdot f(Q_{ri}) < 0 \end{cases}$$

2. otherwise

$$\frac{d\pi_{ri}(Q_{ri})}{dQ_{ri}} = -c_i < 0 \quad \text{and} \quad \frac{d^2\pi_{ri}(Q_{ri})}{dQ_{ri}^2} = 0$$

Also, by analogy, supplier i will deliver $F^{-1}\left(\frac{\phi_i p + w_{ri} - c_i}{\phi_i p}\right)$ to maximize its expected profit. In particular, mismatch of components can be eliminated by simultaneously adjusting w_{ri} and ϕ_i . In other words, the following equation should be satisfied,

$$\frac{\phi_1 p + w_{r1} - c_1}{\phi_1 p} = \frac{\phi_2 p + w_{r2} - c_2}{\phi_2 p} = \dots = \frac{\phi_N p + w_{rN} - c_N}{\phi_N p} = F(Q_{re}^*) \quad (3-24)$$

where Q_{re}^* represents suppliers' optimal delivery quantity at equilibrium.

It is noted from eq.(3-24) that channel coordination with revenue-sharing contract requires a supplier's wholesale price to be set below its production cost; that is to satisfy $w_{ri} < c_i$ (Zou et al., 2004). This is contrary to the assumption made in the price-only decentralized setting, in which suppliers make money through component selling. However, in the case of coordination with revenue-sharing contract, suppliers lose money in selling components but make money by sharing the assembler's revenue.

Further, the assembler's expected profit is,

$$\begin{aligned} \pi_{r0}(Q_{r0}) = E_{\xi} \left\{ \phi_0 p \cdot \min(\xi, Q_{r0}) - Q_{r0} \cdot \left(\sum_{i=1}^N w_{ri} + c_0 \right) - s(\xi - Q_{r0})^+ \right\} \\ \text{s.t. } Q_{r0} = \xi \perp [0, \min\{Q_{ri}\}] \end{aligned} \quad (3-25)$$

which is a constrained optimization problem and can be addressed with the KKT condition.

Denote δ as the multiplier, the optimal solutions are presented as follows,

1. if $Q_{r0} \neq 0$, $\delta \neq 0$

$$\begin{cases} Q_{r0}^* = Q_{re} \\ \delta = -\phi_0(p + s)Q_{re} + \left(\phi_0 p - \sum_{i=1}^N w_{ri} - c_0 + s \right) \end{cases}$$

2. otherwise

$$\begin{cases} F(Q_{r0}^*) = \frac{\phi_0 p - \sum_{i=1}^N w_{ri} - c_0 + s}{s + \phi_0 p} \\ \delta = 0 \end{cases}$$

Denote $Q_{r\psi}^* = F^{-1} \left(\frac{\phi_0 p - \sum_{i=1}^N w_{ri} - c_0 + s}{\phi_0 p + s} \right)$, which represents the assembler's optimal

order-up-to level without constraint. Therefore, the assembler's optimal order quantity is,

$$Q_{r0}^* = \min(Q_{r\psi}^*, Q_{re}^*) \quad (3-26)$$

Aimed at channel coordination, a necessary condition is to have $Q_{r0}^* = Q_c^*$. Moreover, with

the assumption of $\sum_{i=0}^N \phi_i = 1$, it is further derived from eq.(3-24) that,

$$\begin{aligned} F(Q_{re}^*) &= \frac{\phi_1 p + w_{r1} - c_1}{\phi_1 p} = \frac{\phi_2 p + w_{r2} - c_2}{\phi_2 p} = \dots = \frac{\phi_N p + w_{rN} - c_N}{\phi_N p} \\ &= \frac{p \sum_{i=1}^N \phi_i + \sum_{i=1}^N (w_{ri} - c_i)}{\sum_{i=1}^N \phi_i \cdot p} = \frac{(1 - \phi_0) p + \sum_{i=1}^N (w_{ri} - c_i)}{(1 - \phi_0) p} \end{aligned} \quad (3-27)$$

Further, if $Q_{re}^* = Q_c^*$, it can be further derived from eq.(3-27) that

$$\begin{aligned} F(Q_{re}^*) &= \frac{(1 - \phi_0) p + \sum_{i=1}^N (w_{ri} - c_i)}{(1 - \phi_0) p} = \frac{p + s - \sum_{i=0}^N c_i}{p + s} = F(Q_c^*) \\ &= \frac{\left[(1 - \phi_0) p + \sum_{i=1}^N (w_{ri} - c_i) \right] - \left(p + s - \sum_{i=0}^N c_i \right)}{(1 - \phi_0) p - p + s} = \frac{\phi_0 p - \sum_{i=1}^N w_{ri} - c_0 + s}{\phi_0 p + s} = F(Q_{r\psi}^*) \end{aligned} \quad (3-28)$$

Hence, it can be deduced from eqs.(3-27) and (3-28) that

$$Q_{r0}^* = Q_{re}^* = Q_{r\psi}^* = Q_c^* \quad (3-29)$$

Proposition 3-5

The decentralized assembly system can be coordinated with the proposed revenue-sharing contract. Specifically, at coordination, $Q_{r1}^* = Q_{r2}^* = \dots = Q_{rN}^* = Q_{r0}^* = Q_{re}^* = Q_{r\psi}^* = Q_c^*$. ■

The statements in Proposition 3-5 imply that,

- Mismatch of components can be completely eliminated with the proposed revenue-sharing contract as all the component suppliers deliver the same number of components.

- There will be no leftover components at the end of the selling season since the assembler purchases the exact number of components delivered by the suppliers.
- The assembler's purchasing quantity equals to that of the centralized setting.
- There exists a continuum of (w_{ri}, ϕ_i) , satisfying eqs.(3-24) and (3-28), with which channel coordination can be achieved. In addition, a different pair of (w_{ri}, ϕ_i) represents a different allocation of system profits to the supplier.

With channel coordination, the assembler and each supplier's optimal expected profit can be formulated as,

$$\begin{aligned}\pi_{r0}(Q_c^*) &= \frac{\phi_0 p - \sum_{i=1}^N w_{ri} - c_0 + s}{F(Q_c^*)} \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi - s \cdot \lambda = (\phi_0 p + s) \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi - s \cdot \lambda \\ \pi_{ri}(Q_c^*) &= \frac{\phi_i p + w_{ri} - c_i}{F(Q_c^*)} \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi = \phi_i p \cdot \int_0^{Q_c^*} \xi f(\xi) d\xi\end{aligned}\quad (3-30)$$

It can be further inferred from eq.(3-30) that,

- Either supplier's or the assembler's optimal expected profit is proportional to its share of the revenue.
- When system parameters are fixed, each supplier's expected profit is only dependent on its own share of the revenue ϕ_i ; thus, each supplier can negotiate independently with the assembler on the allocation of system profit.
- From eq.(3-24), it can be derived that, at coordination, w_{ri} and ϕ_i are negatively related satisfying $w_{ri} = c_i - [1 - F(Q_c^*)] \cdot \phi_i p$. That is, a higher revenue share results in a reduction in the corresponding wholesale price.

III. Comparison of the Buyback and Revenue-Sharing Contract

In practice, both buyback contract and revenue-sharing contract have been used to align individual incentives with system-wide optimization. In the previous two sections, both contracts are shown to be able to realize channel coordination in a decentralized assembly system through manipulations of the coordinating parameters. Moreover, some connection between the two contracts is identified when they are used as coordinating mechanisms for channel coordination.

Proposition 3-6

With a fixed selling price, for any coordinating buyback contract, characterized by $\{w_{bi}, \tau_i\}$, there exists a unique revenue-sharing contract, characterized by $\{w_{ri}, \phi_i\}$, that can generate the same performance for any realization of market demand. Specifically, the relationship between the parameters of the two contracts satisfies,

$$\begin{cases} w_{bi} = \phi_i p + w_{ri} \\ \tau_i = \phi_i p \end{cases} \quad \text{for } i = 1, 2, \dots, N \quad (3-31)$$

Proofs for Proposition 3-6 are presented in Appendix III.

3.3 Numerical Study

In this section, a numerical study is presented to highlight the results of the coordination problem in a two-echelon assembly system. The applicability of the proposed models is investigated by comparing the results in the centralized model, price-only decentralized model and the coordinated decentralized models with buyback, revenue-sharing, respectively.

Without loss of generality, it is assumed that there are two component suppliers (denoted as

$S1$, $S2$) and one assembler (denoted as A) in the system. Market demand is exponentially distributed with a mean of 1000. Initial conditions for the numerical study are listed in Table 3-2.

Table 3-2 Initial conditions for numerical study

Entity	Initial Conditions			
	c_i	p	s	λ
$S1$	0.6	---	---	1000
$S2$	0.5	---	---	
A	1.0	6	1.2	

System performances, in terms of the assembler’s purchasing quantity and the total expected system profits, of the centralized control model and price-only decentralized control model are compared based on the numerical results presented in Table 3-3.

Table 3-3 Centralized vs. decentralized system performance ($p = 6$)

w_{di}/c_i	1.5	1.55	1.6	1.65	1.7	1.75	1.8	1.85	1.9	1.95	2.0	2.05	2.1	2.15	2.2	2.25	2.3	2.35	2.4	2.45	2.5
Q_c^*	1232																				
$Q_{d\psi}^*$	1000	979	959	939	920	901	882	864	846	828	811	794	777	761	744	728	713	697	682	667	652
Q_{de}^*	405	438	470	501	531	560	588	615	642	668	693	718	742	765	788	811	833	854	875	896	916
Q_{d0}^*	405	438	470	501	531	560	588	615	642	668	693	718	742	761	744	728	713	697	682	667	652
π_c^*	1312.1																				
$\sum \pi_{di}^*$	347	434	513	585	651	711	766	816	863	906	944	981	1013	1034	968	903	841	777	715	652	589

Specifically, the derived Q_c^* and π_c^* of the centralized control model are used as a benchmark. Since focus is on the impact of decentralization on system performance in the presence of price-only contract, supplier’s delivery quantity at equilibrium Q_{de}^* is considered. Moreover, the ratio of wholesale price over corresponding production cost (>1 since $w_{di} > c_i$ in the price-only setting) is varied from 1.5 to 2.5 at an incremental step of 0.05 to evaluate the changes of Q_{de}^* , $Q_{d\psi}^*$, Q_{d0}^* and $\sum \pi_{di}^*$.

The relationship between various quantities (based on the numerical results in Table 3-3) is

presented in Figure 3-1. Moreover, a comparison between expected system profits of the two control modes is presented in Figure 3-2.

Figure 3-1 Comparison of quantities in the centralized and price-only decentralized setting ($p = 6$)

Figure 3-2 Comparison of system profits in the centralized and price-only decentralized setting ($p = 6$)

It is observed from Figure 3-1 and Figure 3-3 that $Q_{d\psi}^*$ decreases in the ratio of w_{di}/c_i ; however, Q_{de}^* increases in the ratio. Since Q_{d0}^* equals to the minimum value of $Q_{d\psi}^*$ and Q_{de}^* ; it reaches its peak value when $Q_{d\psi}^* = Q_{de}^*$. However, in the groups of data tested, the maximum Q_{d0}^* lies in the point where the gap between $Q_{d\psi}^*$ and Q_{de}^* is the smallest (in Figure 3-1, $w_{di}/c_i = 2.15$ and in Figure 3-3, $w_{di}/c_i = 2.3$). Moreover, it is shown in both

figures that the number of components purchased by the assembler in the decentralized control setting with price-only contract is fewer than that of the corresponding centralized control settings. In addition, given the same ratio, it is noted that Q_c^* and $Q_{d\psi}^*$ increase in p ; however, Q_{de}^* remains constant.

Figure 3-3 Comparison of quantities in the centralized and price-only decentralized setting ($p = 7$)

Figure 3-4 Comparison of system profits in the centralized and price-only decentralized setting ($p = 7$)

In addition, comparison between various quantities and expected system profits of the two control modes when $p = 7$ are presented in Figure 3-3 and Figure 3-4, respectively. Detailed numerical results when $p = 7$ are not presented.

It is further noted from Figure 3-2 and Figure 3-4 that $\sum \pi_{di}^*$ changes in the same way as that of Q_{d0}^* . It first increases in the ratio; then it reaches its peak value when Q_{d0}^* is maximized; after that point it starts to decrease. Similarly, an increase in the selling price leads to an increase in both π_c^* and $\sum \pi_{di}^*$ given the same ratio. However, the total expected system profits in the price-only decentralized control setting $\sum \pi_{di}^*$ is always fewer than π_c^* of the centralized setting. Hence, it can be concluded from the numerical results above that, a price-only contract alone cannot coordinate the decentralized assembly system.

Numerical study for channel coordination with buyback contract and revenue-sharing contract are presented in Table 3-4 and Table 3-5, respectively. Three different scenarios are considered for each contract type, representing changes in the coordinating parameters. It is noted from the results that both contracts can achieve the first-best system performance since both $\sum \pi_{bi}^*$ (in Table 3-4) and π_{ri}^* (in Table 3-5) equal to π_c^* of the centralized setting (in Table 3-3).

Specifically, it is noted from Table 3-4 that, with buyback contract, suppliers' expected profits increase in their buyback price τ_i , accounting for a larger portion of total expected system profit. In addition, wholesale price and the corresponding buyback price are positively related. Therefore, suppliers, with buyback contract, would negotiate with the assembler for a higher wholesale price to increase expected profits.

Table 3-4 Channel coordination with buyback contract

Entity	Scenario I				Scenario II				Scenario III			
	τ_i	w_{bi}	π_{bi}^*	$\sum \pi_{bi}^*$	τ_i	w_{bi}	π_{bi}^*	$\sum \pi_{bi}^*$	τ_i	w_{bi}	π_{bi}^*	$\sum \pi_{bi}^*$
S1	1.0	1.31	348.9	1312.1	1.5	1.66	523.3	1312.1	1.3	1.52	453.6	1312.1
S2	0.8	1.07	279.1		1.2	1.35	418.7		0.7	0.996	244.2	
A	---	---	684.1		---	---	370.1		---	---	614.3	

Table 3-5 Channel coordination with revenue-sharing contract

Entity	Scenario I				Scenario II				Scenario III			
	ϕ_i	w_{ri}	π_{ri}^*	$\sum \pi_{ri}^*$	ϕ_i	w_{ri}	π_{ri}^*	$\sum \pi_{ri}^*$	ϕ_i	w_{ri}	π_{ri}^*	$\sum \pi_{ri}^*$
S1	0.2	0.25	418.7	1312.1	0.3	0.075	628.0	1312.1	0.15	0.337	314.0	1312.1
S2	0.2	0.15	418.7		0.2	0.15	418.7		0.25	0.062	523.3	
A	0.6	---	474.7		0.5	---	265.4		0.6	---	474.8	

Moreover, it is noted from Table 3-5 that, with revenue-sharing contract, every player’s expected profit increases in its revenue share. Whereas, a supplier’s revenue share ϕ_i and its wholesale price w_{ri} are negatively related. Therefore, suppliers with revenue-sharing contract would negotiate with the assembler for a higher share in order to increase its expected profit.

It is noted from the numerical study that, in the price-only decentralized control setting, the number of components purchased by the assembler and the total expected system profits are fewer than that of the corresponding centralized setting. Moreover, the two values are maximized at the point $Q_{dy}^* = Q_{de}^*$. In addition, the total expected system profit and the number of components purchased by the assembler increase product’s selling price.

Coordinated with buyback or revenue-sharing contract, the decentralized assembly system can achieve the same total expected system profit as that in the centralized setting. In addition, it is noted that different parameter pairs represent different allocation schemes of total expected system profit. Specifically, with buyback contract, suppliers’ share of profit is proportional to their buyback price; whereas, with revenue-sharing contract, each player’s share of total expected system profit is proportional to its share of the assembler’s revenue.

3.4 Summary

Mathematical models for an assembly system operated with centralized control, price-only decentralized control in a single-period setting have been formulated. In addition, a buyback contract and a revenue-sharing contract have been developed for channel coordination of the assembly system operated with decentralized control.

The proposed models illustrate how incentive conflicts associated with decentralized control impair system performance, which is reflected by mismatch of components, a reduction in the assembler's purchasing quantity and reduced expected system profits. However, modification of the pricing schemes between the suppliers and the assembler either through buying back the assembler's leftover inventory or through participating in the revenue-sharing scheme are shown to realize channel coordination.

The single-period model can be applied to situations where only one-time transaction is desired. The proposed models presented in this chapter are used as building blocks for the multi-period contract model to be developed in the next chapter where more complexities are incorporated.

Chapter 4

Two-Period Supply Contract Model

The single-period model formulated in Chapter 3 is extended in this chapter by considering a two-period setting, for which the impact of current period's performance on the next period's performance and the expected optimal behavior of the last period are considered. The formulated two-period models can be applied to settings where there are two consecutive ordering opportunities. Moreover, the two-period analytical models form a basis for the extension to a more generic multi-period setting since they are capable of capturing most of the characteristics of the multi-period models.

Three control settings, namely the centralized control, price-only decentralized control and coordinated decentralized control, are modeled. The suppliers' optimal production quantities and the assembler's optimal purchasing quantity for each control setting are derived and compared. Likewise, the centralized control setting is studied as a benchmark and the price-only decentralized control setting is examined to explain inferior system performance brought about by incentive conflicts associated with decentralization. It is shown that channel coordination can be realized by considering a wholesale plus buyback pricing scheme. Further, non-stationary wholesale prices are considered for decentralized control.

4.1 Assumptions for the model

Assumptions for the two-period supply contract model are briefly discussed as follows.

- Market demand in each period is independent and identically distributed (IID) with a common cumulative distribution function and probability distribution function.

- All the cost parameters are stationary over the entire horizon except for the wholesale prices, which are negotiated periodically.
- All the costs (revenue) are proportional to the number of units involved.
Costs parameters include: production cost, wholesale price charged by the component suppliers to the assembler for every unit of components purchased; shortage cost incurred by the assembler, holding costs for leftover inventory at the end of each period and selling price for every unit of final product sold.
- Leftover inventory at the end of a period is automatically transferred to the next period incurring corresponding holding costs; whereas, excessive market demand is completely backlogged.
- For the sake of simplicity, it is assumed that there are only two component suppliers.
- There is only one kind of final product in the system and one unit of final product requires one unit of each component (that is, component usage rate equals to 1).
- All the players are risk neutral and are assumed to be profit maximizers.
- Suppliers' production/delivery decisions and the assembler's purchasing decision are made before the realization of market demand.

Notations for the multi-period setting are listed in Table 4-1.

Table 4-1 Notations for multi-period supply contract model

t	Subscript, indicates the period when there are t periods left
c	Subscript, indicates the centralized control setting
d	Subscript, indicates the price-only decentralized control setting
b	Subscript, indicates the coordinated decentralized setting with buyback contract
i	Subscript, $i = 0$, the assembler; $i = 1, 2$, the component suppliers
ξ_t	Market demand in period t (≥ 0)
$f(\xi_t)$	Probability distribution function of market demand in period t
$F(\xi_t)$	Cumulative distribution function of market demand in period t
λ	Mean of market demand
Q_{cit}	Player i 's production/purchasing quantity in the centralized setting in period t

Q_{dit}	Player i 's production/ purchasing quantity in the price-only decentralized setting in period t
Q_{bit}	Player i 's production/ purchasing quantity in the coordinated decentralized setting with buyback contract in period t
I_{it}	Player i 's initial inventory level at the beginning of period t , $\forall i$
P	Unit selling price
h_i	Per unit per period inventory holding cost for player i
c_i	Unit production cost for player i
s	Unit shortage cost
w_{dit}	Unit wholesale price in the price-only decentralized setting in period t for component i
w_{bit}	Unit wholesale price in the coordinated decentralized setting with buyback contract in period t for component i
S_{jt}	Order-up-to level in the centralized control setting in period t for level j , $j=1,2,3$
y_{dot}	Assembler's optimal order-up-to level without constraint in the price-only decentralized setting in period t
y_{de1}	Equilibrium order-up-to level in the price-only decentralized setting in period $t=1$
y_{di2}^I	Order-up-to level in Scenario I of the price-only decentralized setting with $w_{di2} < w_{di1}$ in period $t=2$, $i=1,2$
y_{di2}^{I2}	Order-up-to level in Scenario I of the price-only decentralized setting with $w_{di2} > w_{di1}$ in period $t=2$, $i=1,2$
y_{di2}^{IIj}	Order-up-to level in Scenario II of the price-only decentralized setting in period $t=2$, $j=1,2,3$ when supplier 1 is the critical supplier of period $t=2$
y_{d22}^{IIj}	Order-up-to level in Scenario II of the price-only decentralized setting in period $t=2$, $j=1,2$ when supplier 2 is the critical supplier of period $t=2$
Z_{jt}	Order-up-to level in the coordinated decentralized setting with buyback contract in period t for level j , $j=1,2,3$
π_{ct}	Expected system profit in the centralized setting in period t
J_{ct}	Optimal total expected profit in the centralized setting from period t till the end of the last period
M_{ct}	Total expected profit in the centralized setting from period t till the end of the last period
π_{dit}	Player i 's expected profit in the price-only decentralized setting in period t
J_{dit}	Player i 's optimal expected profit in the price-only decentralized setting from period t till the end of the last period
M_{dit}	Player i 's total expected profit in the price-only decentralized setting from period t till the end of the last period
π_{bit}	Player i 's expected profit in the coordinated decentralized setting with buyback contract in period t
J_{bit}	Player i 's optimal expected profit in the coordinated decentralized setting with buyback contract from period t till the end of the last period
M_{bit}	Player i 's total expected profit in the coordinated decentralized setting with buyback contract from period t till the end of the last period

Mathematical models for the two-echelon decentralized assembly system in a two-period setting will be formulated in the next section. In a multi-period setting, leftover inventory, if any, from the previous periods is automatically transferred to the next period and customer demand is backlogged in case of higher than expected market demand. Since each player could have some on-hand inventory at the beginning of a period, market demand might be partially or fully met from its on-hand inventory without any additional replenishment. Hence, the optimal policy could be more complex than that obtained in the single-period model due to the dependence on players' on-hand inventory.

As sequential decisions need to be made, the two-period model is formulated with probabilistic dynamic programming in a backward manner. In this thesis, the last period refers to the period when the contract ends and is denoted as $t = 1$. The analysis starts from the last period, and then moves backward until it reaches the first period.

4.2 The Centralized Control Model

With centralized control, all the decisions are made for global optimization. If each player has some initial inventory at the beginning of a period, the assembler's purchasing quantity is not necessarily to be any supplier's production quantity (which is different from the single-period model). Further, transactions between the suppliers and the assembler, with this control mode, are regarded as internal activities.

4.2.1 The Last-Period Model for the Centralized Control Setting

The expected system profit for the last period can be formulated as,

$$\pi_{cl}(Q_{cl}, I_{il}) = E_{\xi_1} \left\{ \begin{aligned} & p \cdot \min(Q_{c01} + I_{01}, \xi_1) - s(\xi_1 - Q_{c01} - I_{01})^+ - h_0(I_{01} + Q_{c01} - \xi_1)^+ \\ & - c_0 \cdot Q_{c01} - \sum_{i=1}^2 [c_i \cdot Q_{ci1} + h_i \cdot (I_{i1} + Q_{ci1} - Q_{c01})] \end{aligned} \right\} \quad (4-1)$$

$$s.t. \quad Q_{c01} \leq \min(Q_{c11} + I_{11}, Q_{c21} + I_{21})$$

terms in eq.(4-1) represent the following: the expected revenue from product selling, the assembler's expected backorder penalty cost, the assembler's holding cost, the assembler's production cost and the sum of component suppliers' production and holding costs. In addition, the constraint indicates that the assembler's purchasing quantity is subject to the number of components delivered by the critical supplier.

Further, the first-order partial derivatives of $\pi_{cl}(Q_{cl}, I_{il})$ with respect to $Q_{ci1} \forall i$ are,

$$\begin{aligned} \frac{\partial \pi_{cl}}{\partial Q_{c01}} &= (p + s - c_0 + \sum_{i=1}^2 h_i) - (p + s + h_0)F(Q_{c01} + I_{01}) \\ \frac{\partial \pi_{cl}}{\partial Q_{c11}} &= -(c_1 + h_1) < 0 \\ \frac{\partial \pi_{cl}}{\partial Q_{c21}} &= -(c_2 + h_2) < 0 \end{aligned} \quad (4-2)$$

It is noted that $\pi_{cl}(Q_{cl}, I_{il})$ is concave in Q_{c01} . Denote the zero-point to the first-order condition as S_{11} , which can be derived as,

$$S_{11} = F^{-1} \left(\frac{p + s - c_0 + \sum_{i=1}^2 h_i}{p + s + h_0} \right) \quad (4-3)$$

S_{11} represents the assembler's optimal order-up-to level without constraint. With S_{11} , the optimal policy for the last period is derived according to players' on-hand inventory. Specifically, there are two major scenarios that need to be addressed: *i*) with $I_{01} \geq S_{11}$ and *ii*) with $I_{01} < S_{11}$.

i.) If $I_{01} \geq S_{11}$

It is noted that $\left. \frac{\partial \pi_{c1}}{\partial Q_{c01}} \right|_{Q_{c01}=0} \leq 0$ when $I_{01} \geq S_{11}$; thus, it is optimal for the assembler not to

place orders. Market demand is met from the assembler's on-hand inventory. Further, the first-order partial derivatives of $\pi_{c1}(Q_{c11}, I_{11})$ with respect to Q_{c11} and Q_{c21} are negative, which represents a marginal loss of component production. This implies that it is optimal for both suppliers not to produce any additional components when the assembler does not place orders. Hence, under the given condition of $I_{01} \geq S_{11}$, it is optimal to set $Q_{c11}^* = 0 \forall i$.

ii.) If $I_{01} < S_{11}$

Since it is derived that $\left. \frac{\partial \pi_{c1}}{\partial Q_{c01}} \right|_{Q_{c01}=0} > 0$ when $I_{01} < S_{11}$, it is optimal for the assembler to ask

for replenishment. However, when suppliers' on-hand inventory is sufficient to raise the assembler's inventory level up to S_{11} , it might be optimal for suppliers not to produce additional components; otherwise, suppliers might consider extra production. In a word, suppliers' production/delivery decisions are dependent on their on-hand inventory levels.

In order to derive the optimal solutions under the assumption of $I_{01} < S_{11}$, it is assumed, without loss of generality, that $I_{11} < I_{21}$ (that is, supplier 1 is assumed to have a lower inventory level at the beginning of the last period). The following three scenarios are discussed according to various relationships between suppliers' on-hand inventory levels and the inventory needed to raise the assembler's inventory level up to S_{11} .

a.) If $S_{11} - I_{01} \leq I_{11} < I_{21}$

This represents the situation when both suppliers have sufficient on-hand inventory to raise the assembler's inventory level up to S_{11} . Therefore, it is optimal for both suppliers not to produce any additional components. That is, $Q_{c01}^* = S_{11} - I_{01}$ and $Q_{c11}^* = Q_{c21}^* = 0$.

b.) If $I_{11} < S_{11} - I_{01} \leq I_{21}$

This represents the situation when supplier 2 alone has sufficient on-hand inventory to raise the assembler's inventory level up to S_{11} . Therefore, it is optimal to set $Q_{c21}^* = 0$. Whereas, it is not direct to tell if it is optimal for supplier 1 to produce extra components. On one hand, the negativity of the first-order partial derivative indicates a marginal loss of producing component 1; on the other hand, assembly of one unit of final product might increase system revenue. Hence, supplier 1's optimal production condition depends on the trade off between system-wide inventory costs. Denote S_{21} as the order-up-to level at which the expression

$$\frac{\partial \pi_{c1}}{\partial S_{21}} = \left| \frac{\partial \pi_{c1}}{\partial Q_{c11}} \right| \quad (\text{which represents that at the inventory level of } S_{21} \text{ the marginal costs and}$$

marginal profits of producing one unit of component 1 equals) holds. Specifically, it satisfies,

$$\begin{aligned} & \left(p + s - c_0 + \sum_{i=1}^2 h_i \right) - (p + s + h_0) F(S_{21}) = c_1 + h_1 \\ \Rightarrow S_{21} &= F^{-1} \left(\frac{p + s - c_0 - c_1 + h_2}{p + s + h_0} \right) < S_{11} \end{aligned} \quad (4-4)$$

Compare S_{21} with S_{11} , it is noted that $S_{21} < S_{11}$. Moreover, the following two scenarios,

$S_{21} \leq I_{11} + I_{01}$ and $S_{21} > I_{11} + I_{01}$, are discussed.

b.1.) If $S_{21} \leq I_{11} + I_{01}$

The given condition indicates that it is optimal for supplier 1 to deliver all of its on-hand components to the assembler without additional production. Hence, it is optimal to set

$$Q_{c01}^* = I_{11} \text{ and } Q_{c11}^* = Q_{c21}^* = 0.$$

b.2.) If $S_{21} > I_{11} + I_{01}$

Given the condition, it is optimal for supplier 1 to raise its inventory level up to $S_{21} - I_{01}$ (through extra production). Therefore, it is optimal to set $Q_{c01}^* = S_{21} - I_{01}$, $Q_{c11}^* = S_{21} - I_{01} - I_{11}$, and $Q_{c21}^* = 0$.

c.) If $I_{11} < I_{21} < S_{11} - I_{01}$

This represents the situation when both suppliers have insufficient on-hand inventory to raise the assembler's inventory level up to S_{11} . Hence, it might be optimal for both suppliers to produce additional components such that the total expected system profit might be maximized.

Since component usage rate is assumed to be 1, production of one unit of additional component 2 requires the production of one unit of component 1 simultaneously (in order to avoid mismatch of components). Analogously, supplier 2's optimal production condition depends on the trade off between system-wide inventory costs. Denote S_{31} as an optimal

order-up-to level at which the expression $\frac{\partial \pi_{c1}}{\partial S_{31}} = \left| \frac{\partial \pi_{c1}}{\partial Q_{c11}} + \frac{\partial \pi_{c1}}{\partial Q_{c21}} \right|$ holds. Specifically, S_{31}

can be derived as,

$$\begin{aligned} & \left(p + s - c_0 + \sum_{i=1}^2 h_i \right) - (p + s + h_0) F(S_{31}) = c_1 + h_1 + c_2 + h_2 \\ \Rightarrow S_{31} &= F^{-1} \left(\frac{p + s - \sum_{i=0}^2 c_i}{p + s + h_0} \right) < S_{21} < S_{11} \end{aligned} \quad (4-5)$$

It is noted that S_{31} is smaller than both S_{21} and S_{11} . With S_{31} , the following four scenarios are considered for the derivation of the optimal solutions.

c.1.) If $S_{21} \leq I_{11} + I_{01}$, then it is optimal to set $Q_{c01}^* = I_{11}$, $Q_{c11}^* = Q_{c21}^* = 0$.

c.2.) If $I_{11} + I_{01} < S_{21} \leq I_{21} + I_{01}$, then it is optimal to set $Q_{c01}^* = S_{21} - I_{01}$,

$$Q_{c11}^* = S_{21} - I_{01} - I_{11} \text{ and } Q_{c21}^* = 0.$$

c.3.) If $S_{31} \leq I_{21} + I_{01} < S_{21}$

With this condition, it is optimal for supplier 2 not to produce any additional components; that is to set $Q_{c21}^* = 0$. Further, it can be derived that $I_{11} + I_{01} < I_{21} + I_{01} < S_{21}$; hence, it is optimal for supplier 1 to replenish up to supplier 2's on-hand inventory level I_{21} . Therefore, the optimal solutions are, $Q_{c01}^* = I_{21}$, $Q_{c11}^* = I_{21} - I_{11}$ and $Q_{c21}^* = 0$.

c.4.) If $S_{31} > I_{21} + I_{01}$

With this condition, it is optimal for supplier 2 to produce up to $S_{31} - I_{01}$. Therefore, the optimal solutions are $Q_{c01}^* = S_{31} - I_{01}$, $Q_{c11}^* = S_{31} - I_{01} - I_{11}$ and $Q_{c21}^* = S_{31} - I_{01} - I_{21}$.

Based on the analysis above, the optimal policy for the last period, with the assumption of $I_{11} < I_{21}$, is summarized in the following proposition.

Proposition 4-1

Given the assumption of $I_{11} < I_{21}$, the suppliers' optimal production quantities and the assembler's optimal purchasing quantity of the last period for the assembly system operated with centralized control are,

1. If $I_{01} \geq S_{11}$, then $Q_{c01}^* = Q_{c11}^* = Q_{c21}^* = 0$.
2. If $S_{11} - I_{01} \leq I_{11} < I_{21}$, then $Q_{c01}^* = S_{11} - I_{01}$, $Q_{c11}^* = Q_{c21}^* = 0$.
3. If $I_{11} < S_{11} - I_{01} \leq I_{21}$ and

$$\begin{cases} S_{21} \leq I_{11} + I_{01}, \text{ then } Q_{c01}^* = I_{11}, Q_{c11}^* = Q_{c21}^* = 0 \\ I_{11} + I_{01} < S_{21} < I_{21} + I_{01}, \text{ then } Q_{c01}^* = S_{21} - I_{01}, Q_{c11}^* = S_{21} - I_{01} - I_{11}, Q_{c21}^* = 0 \end{cases}$$

4. If $I_{11} < I_{21} < S_{11} - I_{01}$ and

$$\begin{cases} S_{21} \leq I_{11} + I_{01}, \text{ then } Q_{c01}^* = I_{11}, Q_{c11}^* = Q_{c21}^* = 0 \\ I_{11} < S_{21} - I_{01} \leq I_{21}, \text{ then } Q_{c01}^* = S_{21} - I_{01}, Q_{c11}^* = S_{21} - I_{01} - I_{11}, Q_{c21}^* = 0 \\ S_{31} \leq I_{21} + I_{01} < S_{21}, \text{ then } Q_{c01}^* = I_{21}, Q_{c11}^* = I_{21} - I_{11}, Q_{c21}^* = 0 \\ S_{31} > I_{21} + I_{01}, \text{ then } Q_{c01}^* = S_{31} - I_{01}, Q_{c11}^* = S_{31} - I_{01} - I_{11}, Q_{c21}^* = S_{31} - I_{01} - I_{21} \end{cases} \quad \blacksquare$$

The optimal solutions presented in Proposition 4-1 show that supplier 1 is the critical supplier of the last period given the assumption of $I_{11} = \min(I_{11}, I_{21})$.

4.2.2 Two-Period Model for the Centralized Control Setting

In this section, it is assumed without loss of generality that $I_{12} = \min(I_{12}, I_{22})$; that is, supplier 1 is assumed to have less initial on-hand inventory level for the first period ($t = 2$). However, given the assumption of $I_{12} = \min(I_{12}, I_{22})$, it is possible that either supplier's on-hand inventory level could be the lower at the beginning of the last period ($t = 1$) depending on their production quantities. Hence, two scenarios: 1) $I_{11} = \min(I_{11}, I_{21})$ and 2) $I_{21} = \min(I_{11}, I_{21})$ are considered.

The two scenarios mentioned above assume different relationship concerning suppliers' initial inventory level of the last period, which affect the optimal policy of the last period and consequently the optimal policy of the two-period problem. For the first scenario, the optimal solutions for the last period (presented in Proposition 4-1) can be directly applied; however, for the second scenario, the optimal solutions can be derived by analogy (based on the analysis for the first scenario). Solutions for the two-period problem will be derived for each of the scenario and will be examined for their feasibility. For scenario 1, if the solutions derived for the two-period model do not violate the assumption of $I_{11} = \min(I_{11}, I_{21})$, they

are feasible solutions and they are global optima. Similar analysis would be carried out for solutions derived for scenario 2.

I. Scenario I

For model formulation, Proposition 4-1 is simplified as follows,

Suppose $I_{11} = \min(I_{11}, I_{21})$, the optimal solutions for the last period problem with centralized control are,

1. If $I_{01} \geq S_{11}$, then $Q_{c01}^* = Q_{c11}^* = Q_{c21}^* = 0$.
2. If $S_{11} - I_{01} \leq I_{11} < I_{21}$, then $Q_{c01}^* = S_{11} - I_{01}, Q_{c11}^* = Q_{c21}^* = 0$.
3. If $\begin{cases} S_{11} > I_{11} + I_{01} \\ S_{21} \leq I_{11} + I_{01} \end{cases}$, then $Q_{c01}^* = I_{11}, Q_{c11}^* = Q_{c21}^* = 0$.
4. If $\begin{cases} S_{11} > I_{11} + I_{01} \\ I_{11} + I_{01} < S_{21} \leq I_{21} + I_{01} \end{cases}$, then $Q_{c01}^* = S_{21} - I_{01}, Q_{c11}^* = S_{21} - I_{01} - I_{11}, Q_{c21}^* = 0$.
5. If $\begin{cases} I_{11} < I_{21} < S_{11} - I_{01} \\ S_{31} \leq I_{21} + I_{01} < S_{21} \end{cases}$, then $Q_{c01}^* = I_{21}, Q_{c11}^* = I_{21} - I_{11}, Q_{c21}^* = 0$.
6. If $S_{31} > I_{21} + I_{01}$, $Q_{c01}^* = S_{31} - I_{01}, Q_{c11}^* = S_{31} - I_{01} - I_{11}, Q_{c21}^* = S_{31} - I_{01} - I_{21}$. ■

Further, the inventory transfer functions are,

$$\begin{aligned} I_{01} &= I_{02} + Q_{c02} - \xi_2 \\ I_{i1} &= I_{i2} + Q_{ci2} - Q_{c02} \quad \text{for } i = 1, 2 \end{aligned} \quad (4-6)$$

The first equation shows that the assembler's initial on-hand inventory level of the last period equals to its initial inventory level of the previous period plus its purchasing quantity minus market demand. The second expression indicates that both suppliers' initial on-hand inventory levels of the last period equal to their initial inventory levels of the last but the second period plus the quantity produced minus the quantity purchased by the assembler. In

addition, it can be further derived from the inventory transfer functions that the supplier having a lower on-hand inventory level at the beginning of period $t=1$ is the critical supplier for period $t=2$.

To further simplify the representation, define $L(\varphi) = s(\xi - \varphi)^+ + h_0(\varphi - \xi)^+$, which represents the expected shortage and holding costs at an inventory level of φ and ξ , market demand. Hence, the optimal expected profit for the last period centralized setting can be formulated as,

$$\begin{aligned}
 & J_{c1}(I_{01}, I_{11}, I_{21}) \\
 = & \int_0^{I_{02} + Q_{c02} - S_{11}} \left\{ p \cdot \min(I_{02} + Q_{c02} - \xi_2, \xi_1) - \sum_{i=1}^2 h_i \cdot (I_{i2} + Q_{ci2} - Q_{c02}) \right. \\
 & \quad \left. - L(I_{02} + Q_{c02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 & + \int_{I_{02} + Q_{c02} - S_{11}}^{I_{12} + Q_{c12} + I_{02} - S_{11}} \left\{ p \cdot \min(S_{11}, \xi_1) - c_0(S_{11} - I_{02} - Q_{c02} + \xi_2) - L(S_{11}) \right. \\
 & \quad \left. - \sum_{i=1}^2 h_i \cdot (I_{i2} + Q_{ci2} + I_{02} - S_{11} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 & + \int_{I_{12} + Q_{c12} + I_{02} - S_{11}}^{I_{12} + Q_{c12} + I_{02} - S_{21}} \left\{ p \cdot \min(I_{12} + Q_{c12} + I_{02} - \xi_2, \xi_1) - c_0(I_{12} + Q_{c12} - Q_{c02}) \right. \\
 & \quad \left. - h_2(I_{22} + Q_{c22} - I_{12} - Q_{c12}) - L(I_{12} + Q_{c12} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 & + \int_{I_{12} + Q_{c12} + I_{02} - S_{21}}^{I_{22} + Q_{c22} + I_{02} - S_{21}} \left\{ p \cdot \min(S_{21}, \xi_1) - c_0(S_{21} - I_{02} - Q_{c02} + \xi_2) - L(S_{21}) \right. \\
 & \quad \left. - c_1(S_{21} - I_{12} - Q_{c12} - I_{02} + \xi_2) - h_2(I_{22} + Q_{c22} + I_{02} - S_{21} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 & + \int_{I_{22} + Q_{c22} + I_{02} - S_{21}}^{I_{22} + Q_{c22} + I_{02} - S_{31}} \left\{ p \cdot \min(I_{22} + Q_{c22} + I_{02} - \xi_2, \xi_1) - c_0(I_{22} + Q_{c22} - Q_{c02}) \right. \\
 & \quad \left. - c_1(I_{22} + Q_{c22} - I_{12} - Q_{c12}) - L(I_{22} + Q_{c22} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \quad (4-7) \\
 & + \int_{I_{22} + Q_{c22} + I_{02} - S_{31}}^{\infty} \left\{ p \cdot \min(S_{31}, \xi_1) - c_0(S_{31} - I_{02} - Q_{c02} + \xi_2) - L(S_{31}) \right. \\
 & \quad \left. - \sum_{i=1}^2 c_i \cdot (S_{31} - I_{i2} - Q_{ci2} - I_{02} + \xi_2) \right\} f(\xi_2) d\xi_2
 \end{aligned}$$

Each line of the expressions in eq.(4-7) corresponds to one of the scenarios in the simplified Proposition 4-1. Further, the total expected system profit over the last two periods can be modeled as,

$$\begin{aligned}
 M_{c2}(I_{i2}, Q_{ci2}) &= E_{\xi_2} \{ \pi_{c2}(I_{i2}, Q_{ci2}) + J_{c1}(I_{01}, I_{11}, I_{21}) \} \\
 &= E_{\xi_2} \left\{ p \cdot \min(Q_{c02} + I_{02}, \xi_2) - \sum_{i=1}^2 [c_i \cdot Q_{ci2} + h_i(I_{i2} + Q_{ci2} - Q_{c02})] \right. \\
 &\quad \left. - L(Q_{c02} + I_{02}) - c_0 \cdot Q_{c02} + J_{c1}(I_{01}, I_{11}, I_{21}) \right\} \quad (4-8) \\
 \text{s.t.} \quad Q_{c02} &\leq \min(I_{12} + Q_{c12}, I_{22} + Q_{c22})
 \end{aligned}$$

Similarly, the constraint indicates that the assembler's purchasing quantity of period $t = 2$ is subject to the number of components delivered by the "critical supplier" of the period.

To derive the optimal solutions for period $t = 2$, the first-order partial derivatives of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to $Q_{ci2}, \forall i$ are examined.

1.) First-order partial derivative of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{c02} is,

$$\begin{aligned}
 \frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c02}} &= \left(p + s + \sum_{i=1}^2 h_i \right) - (p + s + h_0) F(I_{02} + Q_{c02}) \\
 &\quad + \left(p + \sum_{i=1}^2 h_i + s - c_0 \right) F(I_{02} + Q_{c02} - S_{11}) \\
 &\quad - (p + s + h_0) \int_0^{I_{02} + Q_{c02} - S_{11}} F(I_{02} + Q_{c02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-9)
 \end{aligned}$$

2.) First-order partial derivative of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{c12} is,

$$\begin{aligned}
 \frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c12}} &= -h_1 - \left(p + s + \sum_{i=1}^2 h_i - c_0 \right) \cdot F(I_{12} + Q_{c12} + I_{02} - S_{11}) \\
 &\quad + (p + s + h_2 - c_0 - c_1) F(I_{12} + Q_{c12} + I_{02} - S_{21}) \\
 &\quad - (p + s + h_0) \int_{I_{12} + Q_{c12} + I_{02} - S_{11}}^{I_{12} + Q_{c12} + I_{02} - S_{21}} F(I_{12} + Q_{c12} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-10)
 \end{aligned}$$

3.) First-order partial derivative of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{c22} is,

$$\begin{aligned}
 \frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c22}} &= -h_2 - (p + s - c_0 - c_1 + h_2) \cdot F(I_{22} + Q_{c22} + I_{02} - S_{21}) \\
 &\quad - (p + s + h_0) \int_{I_{22} + Q_{c22} + I_{02} - S_{21}}^{I_{22} + Q_{c22} + I_{02} - S_{31}} F(I_{22} + Q_{c22} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-11) \\
 &\quad + \left(p + s - \sum_{i=0}^2 c_i \right) F(I_{22} + Q_{c22} + I_{02} - S_{31})
 \end{aligned}$$

Further, second-order partial derivatives are derived for analysis. A useful expression that can

be applied to facilitate the derivation is,

$$\left[\int_0^{y_1 - y_2} F(y_1 - \xi) f(\xi) d\xi \right]_{y_1} = F(y_2) f(y_1 - y_2) + \int_0^{y_1 - y_2} f(y_1 - \xi) f(\xi) d\xi \quad (4-12)$$

A detailed demonstration to this expression is presented in Appendix II.

With eq.(4-12), the corresponding second-order partial derivatives are derived as follows.

1.) Second-order partial derivative of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{c02} is,

$$\frac{\partial^2 M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c02}^2} = -(p + s + h_0) \left[f(I_{02} + Q_{c02}) + \int_0^{I_{02} + Q_{c02} - S_{11}} f(I_{02} + Q_{c02} - \xi_2) f(\xi) d\xi \right] \quad (4-13)$$

which is negative. This implies that $M_{c2}(I_{i2}, Q_{ci2})$ is concave in Q_{c02} and its corresponding first-order partial derivative has, at most, one zero point. In addition, it is noted that

$$\frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c02}} > 0 \quad \text{when} \quad I_{02} + Q_{c02} = S_{11} \quad \text{and} \quad \frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c02}} < 0 \quad \text{when} \quad I_{02} + Q_{c02} \rightarrow \infty.$$

Thus, there is a unique zero-point to the first-order partial derivative. Denote the zero point as S_{12} , which represents the assembler's optimal order-up-to level without constraint.

2.) Second-order partial derivative of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{c12} is,

$$\frac{\partial^2 M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c12}^2} = -(p + s + h_0) \int_{I_{12} + Q_{c12} + I_{02} - S_{11}}^{I_{12} + Q_{c12} + I_{02} - S_{21}} f(I_{12} + Q_{c12} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-14)$$

which is negative. This implies that the corresponding first-order partial derivative is monotonously decreasing. Reexamine eq.(4-10), it is found out that $\frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c12}}$ is

negative over its feasible region, which represents a marginal production loss of component.

3.) Second-order partial derivative of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{c22} is,

$$\frac{\partial^2 M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c22}^2} - (p + s + h_0) \int_{I_{22} + Q_{c22} + I_{02} - S_{21}}^{I_{22} + Q_{c22} + I_{02} - S_{31}} f(I_{22} + Q_{c22} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-15)$$

which is negative. This implies that the corresponding first-order partial derivative is monotonously decreasing. Reexamine eq.(4-11), $\frac{\partial M_{c2}(I_{i2}, Q_{ci2})}{\partial Q_{c22}}$ turns out to be negative.

It is noted that the properties of $M_{c2}(I_{i2}, Q_{ci2})$ with respect to Q_{ci2} (for period $t = 2$) are quite similar to the properties of $\pi_{c1}(I_{i1}, Q_{ci1})$ with respect to Q_{ci1} ($\forall i$) for the last period, the optimal policy for period $t = 2$ can, thus, be deduced by analogy.

Proposition 4-2

Suppose S_{12}, S_{22}, S_{32} are three optimal order-up-to levels satisfying $S_{32} < S_{22} < S_{12}$. Further, the suppliers' initial inventory levels satisfy $I_{12} = \min(I_{12}, I_{22})$. Hence, the suppliers' optimal production quantities and the assembler's optimal purchasing quantity for period $t = 2$ under the assumption of $I_{11} = \min(I_{11}, I_{21})$ can be summarized as follows,

1. If $I_{02} \geq S_{12}$, then $Q_{c02}^* = Q_{c12}^* = Q_{c22}^* = 0$.
2. If $S_{12} - I_{02} < I_{12} < I_{22}$, then $Q_{c02}^* = S_{12} - I_{02}, Q_{c12}^* = Q_{c22}^* = 0$.
3. If $I_{12} < S_{12} - I_{02} < I_{22}$ and

$$\begin{cases} S_{22} \leq I_{12} + I_{02}, \text{ then } Q_{c02}^* = I_{12}, Q_{c12}^* = Q_{c22}^* = 0 \\ I_{12} + I_{02} < S_{22} < I_{22} + I_{02}, \text{ then } Q_{c02}^* = S_{22} - I_{02}, Q_{c12}^* = S_{22} - I_{02} - I_{12}, Q_{c22}^* = 0 \end{cases}$$

4. If $I_{12} < I_{22} < S_{12} - I_{02}$ and

$$\begin{cases} S_{22} \leq I_{12} + I_{02}, \text{ then } Q_{c02}^* = I_{12}, Q_{c12}^* = Q_{c22}^* = 0 \\ I_{12} < S_{22} - I_{02} \leq I_{22}, \text{ then } Q_{c02}^* = S_{22} - I_{02}, Q_{c12}^* = S_{22} - I_{02} - I_{12}, Q_{c22}^* = 0 \\ S_{32} \leq I_{22} + I_{02} < S_{22}, \text{ then } Q_{c02}^* = I_{22}, Q_{c12}^* = I_{22} - I_{12}, Q_{c22}^* = 0 \\ S_{32} > I_{22} + I_{02}, \text{ then } Q_{c02}^* = S_{32} - I_{02}, Q_{c12}^* = S_{32} - I_{02} - I_{12}, Q_{c22}^* = S_{32} - I_{02} - I_{22} \end{cases} \quad \blacksquare$$

Proof.

As similar properties are derived in the two-period setting, most of the deduction process for the optimal solutions can follow the deduction process for the last-period problem. The deduction of S_{22} and S_{32} are illustrated. S_{22} corresponds to S_{21} and it is derived by trading off system-wide inventory costs when producing component 1. Specifically, S_{22} satisfies,

$$(p+s+h_2)-(p+s+h_0)F(S_{22})+(p+s+h_2-c_0-c_1)F(S_{22}-S_{21}) \\ - (p+s+h_0) \int_0^{S_{22}-S_{21}} F(S_{22}-\xi_2)f(\xi_2)d\xi_2 = 0 \quad (4-16)$$

Further, the first-order derivative of the left-hand side of eq.(4-16) with respect to S_{22} is,

$$-(p+s+h_0) \left[f(S_{22}) + \int_0^{S_{22}-S_{21}} f(S_{22}-\xi_2)f(\xi_2)d\xi_2 \right] \quad (4-17)$$

which is negative. Hence, there is a unique S_{22} satisfying eq.(4-16).

S_{32} corresponds to S_{31} of the last period. It is also derived by trading off system-wide inventory costs when extra production of both component 1 and 2 is required. Specifically, it satisfies,

$$(p+s)-(p+s+h_0)F(S_{32})+\left(p+s-\sum_{i=0}^2 c_i\right)F(S_{32}-S_{31}) \\ - (p+s+h_0) \int_0^{S_{32}-S_{31}} F(S_{32}-\xi_2)f(\xi_2)d\xi_2 = 0 \quad (4-18)$$

Further, the first-order derivative of the left-hand side of eq.(4-18) with respect to S_{32} is,

$$-(p+s+h_0) \left[f(S_{32}) + \int_0^{S_{32}-S_{31}} f(S_{32}-\xi_2)f(\xi_2)d\xi_2 \right] \quad (4-19)$$

which is negative. Hence, there is a unique S_{32} satisfying eq.(4-18).

Since all the solutions in Proposition 4-2 are derived under the assumption of

$I_{11} = \min(I_{11}, I_{21})$ (or with the assumption of supplier 1 being the critical supplier of period $t = 2$), these solutions have to be examined to see if there is a violation of the assumption. A reexamination at all the solutions shows that with the assumption of $I_{12} = \min(I_{12}, I_{22})$, the derived solutions are consistent with the assumption of $I_{11} = \min(I_{11}, I_{21})$. Hence, all the solutions are feasible. This concludes the demonstration for Proposition 4-2. ■

II. Scenario II

With the assumption of $I_{21} = \min(I_{11}, I_{21})$, the optimal policy for the last period would be different from that proposed in Proposition 4-1; however, it can be derived by analogy. It is noted that, under the assumption of $I_{21} = \min(I_{11}, I_{21})$, the optimal policy for the last period also consists of three order-up-to levels. They are denoted as $\tilde{S}_{j1}$ ($j = 1, 2, 3$), which corresponds to S_{j1} of the scenario where $I_{11} = \min(I_{11}, I_{21})$ is assumed. Further, it is derived that $\tilde{S}_{11} = S_{11}$ and $\tilde{S}_{31} = S_{31}$. However, $\tilde{S}_{21} \neq S_{21}$ and it can be derived as,

$$\tilde{S}_{21} = F^{-1}\left(\frac{p + s - c_0 - c_2 + h_1}{p + s + h_0}\right) \quad (4-20)$$

The total expected profit over the last two periods can be modeled as,

$$\tilde{M}_{c2}(I_{i2}, \tilde{Q}_{ci2}) = E_{\xi_2} \left\{ \tilde{\pi}_{c2}(I_{i2}, \tilde{Q}_{ci2}) + \tilde{J}_{c1}(I_{01}, I_{11}, I_{21}) \right\} \quad (4-21)$$

in (4-21), $\tilde{\pi}_{c2}(I_{i2}, \tilde{Q}_{ci2})$ represents the expected profit for period $t = 2$, which has the same format as that of $\pi_{c2}(I_{i2}, Q_{ci2})$ (as formulated in eq.(4-8)). Specifically, $\tilde{J}_{c1}(I_{01}, I_{11}, I_{21})$ represents the optimal expected profit for the last period and it can be formulated as,

$$\begin{aligned}
 & \tilde{J}_{c1}(I_{01}, I_{11}, I_{21}) \\
 &= \int_0^{I_{02} + \tilde{Q}_{c02} - S_{11}} \left\{ p \cdot \min(I_{02} + \tilde{Q}_{c02} - \xi_2, \xi_1) - \sum_{i=1}^2 h_i \cdot (I_{12} + \tilde{Q}_{ci2} - \tilde{Q}_{c02}) \right. \\
 & \quad \left. - L(I_{02} + \tilde{Q}_{c02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{02} + \tilde{Q}_{c02} - S_{11}}^{I_{22} + \tilde{Q}_{c22} + I_{02} - S_{11}} \left\{ p \cdot \min(S_{11}, \xi_1) - c_0(S_{11} - I_{02} - \tilde{Q}_{c02} + \xi_2) - L(S_{11}) \right. \\
 & \quad \left. - \sum_{i=1}^2 h_i \cdot (I_{12} + \tilde{Q}_{ci2} + I_{02} - S_{11} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{22} + \tilde{Q}_{c22} + I_{02} - S_{11}}^{I_{22} + \tilde{Q}_{c22} + I_{02} - \tilde{S}_{21}} \left\{ p \cdot \min(I_{22} + \tilde{Q}_{c22} + I_{02} - \xi_2, \xi_1) - c_0(I_{22} + \tilde{Q}_{c22} - \tilde{Q}_{c02}) \right. \\
 & \quad \left. - h_1(I_{12} + \tilde{Q}_{c12} - I_{22} - \tilde{Q}_{c22}) - L(I_{22} + \tilde{Q}_{c22} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{22} + \tilde{Q}_{c22} + I_{02} - \tilde{S}_{21}}^{I_{12} + \tilde{Q}_{c12} + I_{02} - \tilde{S}_{21}} \left\{ p \cdot \min(\tilde{S}_{21}, \xi_1) - c_0(\tilde{S}_{21} - I_{02} - \tilde{Q}_{c02} + \xi_2) - L(\tilde{S}_{21}) \right. \\
 & \quad \left. - c_2(\tilde{S}_{21} - I_{22} - \tilde{Q}_{c22} - I_{02} + \xi_2) \right. \\
 & \quad \left. - h_1(I_{12} + \tilde{Q}_{c12} + I_{02} - \tilde{S}_{21} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{12} + \tilde{Q}_{c12} + I_{02} - \tilde{S}_{21}}^{I_{12} + \tilde{Q}_{c12} + I_{02} - S_{31}} \left\{ p \cdot \min(I_{12} + \tilde{Q}_{c12} + I_{02} - \xi_2, \xi_1) - c_0(I_{12} + \tilde{Q}_{c12} - \tilde{Q}_{c02}) \right. \\
 & \quad \left. - c_2(I_{12} + \tilde{Q}_{c12} - I_{22} - \tilde{Q}_{c22}) - L(I_{12} + \tilde{Q}_{c12} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \quad (4-22) \\
 &+ \int_{I_{12} + \tilde{Q}_{c12} + I_{02} - S_{31}}^{\infty} \left\{ p \cdot \min(S_{31}, \xi_1) - c_0(S_{31} - I_{02} - \tilde{Q}_{c02} + \xi_2) - L(S_{31}) \right. \\
 & \quad \left. - \sum_{i=1}^2 c_i \cdot (S_{31} - I_{12} - \tilde{Q}_{ci2} - I_{02} + \xi_2) \right\} f(\xi_2) d\xi_2
 \end{aligned}$$

With further deduction, the first-order and the second-order partial derivatives of $\tilde{M}_{c2}(I_{12}, \tilde{Q}_{c12})$ with respect to $\tilde{Q}_{c02}$ are found out to be the same as those derived in eq.(4-9) and eq.(4-13), hence are not presented.

Whereas, the first-order partial derivatives of $\tilde{M}_{c2}(I_{12}, \tilde{Q}_{c12})$ with respect to $\tilde{Q}_{c12}$ and $\tilde{Q}_{c22}$ are different and they are presented below.

$$\begin{aligned}
 \frac{\partial \tilde{M}_{c2}(I_{12}, \tilde{Q}_{c12})}{\partial \tilde{Q}_{c12}} &= -h_1 + \left(p + s - \sum_{i=0}^2 c_i \right) \cdot F(I_{12} + \tilde{Q}_{c12} + I_{02} - S_{31}) \\
 &\quad - (p + s + h_0) \int_{I_{12} + \tilde{Q}_{c12} + I_{02} - \tilde{S}_{21}}^{I_{12} + \tilde{Q}_{c12} + I_{02} - S_{31}} F(I_{12} + \tilde{Q}_{c12} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-23) \\
 &\quad - (p + s - c_0 - c_2 + h_1) \cdot F(I_{12} + \tilde{Q}_{c12} + I_{02} - \tilde{S}_{21})
 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \tilde{M}_{c2}(I_{12}, \tilde{Q}_{c12})}{\partial \tilde{Q}_{c22}} = & -h_2 - \left(p + s + \sum_{i=1}^2 h_i - c_0 \right) \cdot F(I_{22} + \tilde{Q}_{c22} + I_{02} - S_{11}) \\ & - (p + s + h_0) \int_{I_{22} + \tilde{Q}_{c22} + I_{02} - S_{11}}^{I_{22} + \tilde{Q}_{c22} + I_{02} - \tilde{S}_{21}} F(I_{22} + \tilde{Q}_{c22} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-24) \\ & + (p + s + h_1 - c_0 - c_2) \cdot F(I_{22} + \tilde{Q}_{c22} + I_{02} - \tilde{S}_{21}) \end{aligned}$$

Both first-order partial derivatives are negative and the corresponding second-order partial derivatives can be proved to be negative as well. Since similar properties are derived, the optimal policy for period $t = 2$ can be summarized by the following lemma.

Lemma 4-1

Suppose $\tilde{S}_{12}$, $\tilde{S}_{22}$ and $\tilde{S}_{32}$ are three order-up-to levels satisfying $\tilde{S}_{32} < \tilde{S}_{22} < \tilde{S}_{12}$.

Moreover, it is assumed that $I_{12} = \min(I_{12}, I_{22})$. Hence, solutions for period $t = 2$ under the assumption of $I_{21} = \min(I_{11}, I_{21})$ are,

1. If $I_{02} \geq \tilde{S}_{12}$, then $\tilde{Q}_{c02}^* = \tilde{Q}_{c12}^* = \tilde{Q}_{c22}^* = 0$.
2. If $\tilde{S}_{12} - I_{02} \leq I_{12} < I_{22}$, then $\tilde{Q}_{c02}^* = \tilde{S}_{12} - I_{02}$, $\tilde{Q}_{c12}^* = \tilde{Q}_{c22}^* = 0$.
3. If $I_{12} < \tilde{S}_{12} - I_{02} \leq I_{22}$ and

$$\begin{cases} \tilde{S}_{22} \leq I_{12} + I_{02}, \text{ then } \tilde{Q}_{c02}^* = I_{12}, \tilde{Q}_{c12}^* = \tilde{Q}_{c22}^* = 0 \\ I_{12} + I_{02} < \tilde{S}_{22} < I_{22} + I_{02}, \text{ then } \tilde{Q}_{c02}^* = \tilde{S}_{22} - I_{02}, \tilde{Q}_{c12}^* = \tilde{S}_{22} - I_{02} - I_{12}, \tilde{Q}_{c22}^* = 0 \end{cases}$$

4. If $I_{12} < I_{22} < \tilde{S}_{12} - I_{02}$ and

$$\begin{cases} \tilde{S}_{22} \leq I_{12} + I_{02}, \text{ then } \tilde{Q}_{c02}^* = I_{12}, \tilde{Q}_{c12}^* = \tilde{Q}_{c22}^* = 0 \\ I_{12} < \tilde{S}_{22} - I_{02} \leq I_{22}, \text{ then } \tilde{Q}_{c02}^* = \tilde{S}_{22} - I_{02}, \tilde{Q}_{c12}^* = \tilde{S}_{22} - I_{02} - I_{12}, \tilde{Q}_{c22}^* = 0 \\ \tilde{S}_{32} \leq I_{22} + I_{02} < \tilde{S}_{22}, \text{ then } \tilde{Q}_{c02}^* = I_{22}, \tilde{Q}_{c12}^* = I_{22} - I_{12}, \tilde{Q}_{c22}^* = 0 \\ \tilde{S}_{32} > I_{22} + I_{02}, \text{ then } \tilde{Q}_{c02}^* = \tilde{S}_{32} - I_{02}, \tilde{Q}_{c12}^* = \tilde{S}_{32} - I_{02} - I_{12}, \tilde{Q}_{c22}^* = \tilde{S}_{32} - I_{02} - I_{22} \end{cases} \quad \blacksquare$$

The derived solutions have to be examined for feasibility. Whereas, an examination at all the solutions finds out that they violate the assumption of $I_{21} = \min(I_{11}, I_{21})$. Therefore, they are infeasible solutions and have to be discarded.

4.2.3 Summary to the Two-Period Centralized Model

Based on the analysis for scenarios I and II, it can be concluded that it is global optimal for supplier 1, under the assumption of $I_{12} = \min(I_{12}, I_{22})$, to be the critical supplier of the two consecutive periods. More generally, it can be concluded that it is a global optimal for a supplier, having a lower on-hand inventory level at the beginning of period $t = 2$, to be the critical supplier of the two consecutive periods.

4.3 The Price-Only Decentralized Contract Model

In this section, the assembly system operated with decentralized control is explored. With decentralized control mode, both suppliers and the assembler are treated as individual decision-maker of their respective production/purchasing quantities for each period. Moreover, each player is assumed to be risk neutral, rational; thus, they all have the incentive to maximize their own profit. Further, the pricing scheme is represented by wholesale price alone; hence, the model to be developed is referred to as price-only decentralized contract model.

4.3.1 Last-Period Problem for the Price-Only Decentralized Control Setting

Treated as independent entities, all the players' expected profits in the last period are modeled separately. Moreover, to be consistent with the assumption for the centralized setting, the suppliers' initial inventory levels of the last period are assumed to satisfy $I_{11} = \min(I_{11}, I_{21})$.

I. The Assembler's Last-Period Problem

The assembler's expected profit for period $t = 1$ (last period) can be formulated as,

$$\pi_{d01}(I_{01}, Q_{d01}) = E_{\xi_1} \left\{ p \cdot \min(Q_{d01} + I_{01}, \xi_1) - Q_{d01} \cdot \left(\sum_{i=1}^2 w_{di1} + c_0 \right) - L(Q_{d01} + I_{01}) \right\} \quad (4-25)$$

$$s.t. \quad Q_{d01} \leq \min(Q_{d11} + I_{11}, Q_{d21} + I_{21})$$

terms of eq.(4-25) represent: the assembler's expected revenue, purchasing plus production cost and the expected holding and penalty costs. Moreover, the constraint indicates that the assembler's purchasing quantity is subject to the number of components delivered by the critical supplier.

The partial derivatives of $\pi_{d01}(I_{01}, Q_{d01})$ with respect to Q_{d01} are,

$$\frac{\partial \pi_{d01}(I_{01}, Q_{d01})}{\partial Q_{d01}} = \left(p - \sum_{i=1}^2 w_{di1} - c_0 + s \right) - (p + h_0 + s) \cdot F(Q_{d01} + I_{01}) \quad (4-26)$$

$$\frac{\partial^2 \pi_{d01}(I_{01}, Q_{d01})}{\partial Q_{d01}^2} = -(p + h_0 + s) \cdot f(Q_{d01} + I_{01})$$

The negativity of the second-order partial derivative indicates that there is, at most, one zero-point to the corresponding first-order condition. Further, it is derived that,

$$\left. \frac{\partial \pi_{d01}(I_{01}, Q_{d01})}{\partial Q_{d01}} \right|_{Q_{d01} + I_{01} = 0} = p - \sum_{i=1}^2 w_{di1} - c_0 + s > 0$$

$$\lim_{Q_{d01} + I_{01} \rightarrow \infty} \frac{\partial \pi_{d01}(I_{01}, Q_{d01})}{\partial Q_{d01}} = - \left(\sum_{i=1}^2 w_{di1} + c_0 + h_0 \right) < 0$$

Hence, it can be derived that there is one and only one zero-point to the first-order condition.

Denote the zero-point as y_{d01} , which represents the assembler's optimal order-up-to level without constraint. Specifically, y_{d01} satisfies,

$$y_{d01} = F^{-1} \left(\frac{p - \sum_{i=1}^2 w_{di1} - c_0 + s}{p + h_0 + s} \right) \quad (4-27)$$

Further, it is noted that $\left. \frac{\partial \pi_{d01}(Q_{d01}, I_{01})}{\partial Q_{d01}} \right|_{Q_{d01}=0} \leq 0$ when $I_{01} \geq y_{d01}$ and it is hence optimal for

the assembler not to purchase any additional components; however, $\left. \frac{\partial \pi_{d01}(Q_{d01}, I_{01})}{\partial Q_{d01}} \right|_{Q_{d01}=0} > 0$

when $I_{01} < y_{d01}$ and it is thus optimal for the assembler to place orders and to set $Q_{d01} > 0$.

Based on the analysis above, the assembler's optimal order quantity satisfies,

$$Q_{d01}^* = \begin{cases} 0 & \text{if } I_{01} > y_{d01} \\ \min(y_{d01} - I_{01}, \min(I_{11} + Q_{d11}, I_{21} + Q_{d21})) & \text{if } I_{01} \leq y_{d01} \end{cases} \quad (4-28)$$

which indicates that when the assembler's on-hand inventory level is higher than its optimal order-up-to level without constraint, it is optimal for the assembler not to place orders; however, when its on-hand inventory level is lower than y_{d01} , the assembler would ask for replenishment. Nonetheless, whether it is capable for the assembler to replenish up to y_{d01} depends on the relationship between $y_{d01} - I_{01}$ (the quantity required to raise the assembler's inventory level up to y_{d01}) and $\min(I_{11} + Q_{d11}, I_{21} + Q_{d21})$ (the quantity delivered by the critical supplier). Hence, it is critical to determine the suppliers' optimal production/delivery quantities in order to determine the assembler's optimal purchasing quantity.

II. The Suppliers' Last-Period Problem

For supplier i ($i = 1, 2$), its expected profit for the last period can be formulated as,

$$\pi_{di1}(Q_{di1}, I_{i1}) = w_{di1} \cdot Q_{d01} - c_i \cdot Q_{di1} - h_i(I_{i1} + Q_{di1} - Q_{d01}) \quad (4-29)$$

terms of eq.(4-29) represent: supplier's revenue from component selling, production cost and holding cost for leftover components.

It is noted from eqs.(4-25) and (4-29) that Q_{d01} and Q_{di1} are interdependent. Therefore, suppliers have to determine their optimal production quantities based on their anticipations of the assembler's optimal purchasing quantity, which has the following structure as,

$$Q_{d01} = \begin{cases} 0 & \text{if } I_{01} \geq \xi_1 \\ \xi_1 - I_{01} & \text{if } I_{01} < \xi_1 \text{ and } \min(I_{i1} + Q_{di1}) + I_{01} \geq \xi_1 \\ \min(I_{i1} + Q_{di1}) & \text{if } \min(I_{i1} + Q_{di1}) + I_{01} < \xi_1 \end{cases} \quad (4-30)$$

That is, when $I_{01} \geq \xi_1$; namely, when the assembler has sufficient on-hand inventory to meet market demand, it is optimal for the assembler not to place orders (hence, $Q_{d01} = 0$). But, when $I_{01} < \xi_1$ and $\min(I_{i1} + Q_{di1}) + I_{01} \geq \xi_1$; that is, when the assembler is able to meet customer demand after replenishment, it is optimal for the assembler just to replenish up to the customer demand level (hence, $Q_{d01} = \xi_1 - I_{01}$). Lastly, when $\min(I_{i1} + Q_{di1}) + I_{01} < \xi_1$; that is, when the assembler's inventory level after replenishment is insufficient to satisfy market demand; it is optimal for the assembler to purchase up to the critical supplier's delivery.

For the sake of modeling, the expression for Q_{d01} in eq.(4-30) can be rewritten as,

$$Q_{d01} = E_{\xi_1} \left\{ \min(I_{i1} + Q_{di1}) + (I_{01} - \xi_1)^+ - [\min(I_{i1} + Q_{di1}) + I_{01} - \xi_1]^+ \right\} \quad (4-31)$$

Replacing Q_{d01} in eq.(4-29) with the expression in eq.(4-31),

$$\begin{aligned} & \pi_{di1}(Q_{di1}, I_{i1}) \\ &= w_{di1} \cdot E_{\xi_1} \left\{ \min(I_{i1} + Q_{di1}) + (I_{01} - \xi_1)^+ - [\min(I_{i1} + Q_{di1}) + I_{01} - \xi_1]^+ \right\} - c_i \cdot Q_{di1} \\ & \quad - h_i \cdot E_{\xi_1} \left\{ I_{i1} + Q_{di1} - [\min(I_{i1} + Q_{di1}) + (I_{01} - \xi_1)^+ - [\min(I_{i1} + Q_{di1}) + I_{01} - \xi_1]^+] \right\} \end{aligned} \quad (4-32)$$

Therefore, two major scenarios, $Q_{d01} = 0$ and $Q_{d01} > 0$, are considered to determine supplier i 's optimal production quantity.

i.) If $Q_{d01} = 0$

As analyzed, this represents the scenario when market demand is directly met from the assembler's on-hand inventory. Hence, setting $Q_{d01} = 0$ in eq.(4-32), the equation becomes,

$$\pi_{di1}(Q_{di1}, I_{i1}) = -c_i \cdot Q_{di1} - h_i(I_{i1} + Q_{di1}) \quad (4-33)$$

The first-order derivative of eq.(4-33) with respect to Q_{di1} is,

$$\frac{d\pi_{di1}(Q_{di1}, I_{i1})}{dQ_{di1}} = -(c_i + h_i)$$

which is negative. The negativity of the first-order derivative implies that in the event that the assembler does not place orders, it is optimal for both suppliers not to produce any additional components. That is to set $Q_{d11} = Q_{d21} = 0$.

ii.) If $Q_{d01} > 0$

When replenishment is required by the assembler, a supplier's optimal production quantity not only depends on its on-hand inventory level but also on who is the critical supplier. Two scenarios, namely, supplier i is or is not the critical supplier, are discussed below.

a.) If $I_{i1} + Q_{di1} = \min(I_{i1} + Q_{di1})$ for $i \neq 0$

This represents the scenario when supplier i is the critical supplier. Hence, the first-order partial derivative of eq.(4-32) with respect to Q_{di1} is,

$$\frac{\partial \pi_{di1}(Q_{di1}, I_{i1})}{\partial Q_{di1}} = (w_{di1} - c_i) - (w_{di1} + h_i) \cdot F(I_{i1} + Q_{di1} + I_{01}) \quad (4-34)$$

Denote the zero-point to eq.(4-34) as y_{di1} , which satisfies,

$$y_{di1} = F^{-1}\left(\frac{w_{di1} - c_i}{w_{di1} + h_i}\right) \quad (4-35)$$

y_{di1} represents the assembler's optimal order-up-to level without constraint.

b.) If $I_{i1} + Q_{di1} \neq \min(I_{i1} + Q_{di1})$ for $i \neq 0$

This represents the scenario when supplier i is not the critical supplier. Hence, it satisfies $I_{i1} + Q_{di1} \geq \min(I_{di1} + Q_{di1})$ for $i \neq 0$. In addition, the first-order partial derivative of eq.(4-32)

with respect to Q_{di1} is,

$$\frac{\partial \pi_{di1}(Q_{di1}, I_{i1})}{\partial Q_{di1}} = -(c_i + h_i) < 0 \quad (4-36)$$

which is negative. This implies that supplier i 's expected profit decreases when it produces extra components to raise its inventory level higher than $y_{di1} - I_{01}$. Hence, it is optimal for supplier i only to raise its inventory level up to $y_{di1} - I_{01}$ despite who is the critical supplier.

Based on the discussions above, it can be derived that supplier i 's optimal production quantity depends on the relationship between I_{i1} and $y_{di1} - I_{01}$. Specifically, when $I_{i1} \geq y_{di1} - I_{01}$, it is optimal for supplier i not to produce extra components; however, when $I_{i1} < y_{di1} - I_{01}$, it is optimal for supplier i to raise its inventory level up to $y_{di1} - I_{01}$.

As each supplier's order-up-to level, y_{d11} and y_{d21} , might be different with each other, this leads to mismatch of components. Nonetheless, mismatch of components might be eliminated, in the event that both suppliers produce additional components, by adjusting the wholesale prices such that the following equation can be satisfied,

$$y_{d11} - I_{01} = y_{d21} - I_{01} \quad (4-37)$$

In other words, this is to satisfy

$$\frac{w_{d11} - c_1}{w_{d11} + h_1} = \frac{w_{d21} - c_2}{w_{d21} + h_2} = F(y_{de1}) \quad (4-38)$$

Define y_{de1} in eq.(4-38) as the equilibrium order-up-to level. Therefore, supplier i 's optimal production policy, at equilibrium, is,

$$Q_{di1}^* = \begin{cases} 0 & \text{if } I_{01} \geq y_{d01} \\ 0 & \text{if } I_{01} < y_{d01} \text{ and } I_{i1} \geq y_{del} - I_{01} \\ y_{del} - I_{01} - I_{i1} & \text{if } I_{01} < y_{d01} \text{ and } I_{i1} < y_{del} - I_{01} \end{cases} \quad (4-39)$$

that is, when $I_{01} \geq y_{d01}$, namely when the assembler does not place orders, it is optimal for supplier i not to produce any additional components, either. However, when supplier i 's on-hand inventory level is higher than $y_{del} - I_{01}$, supplier i would not produce any additional components. Lastly, when supplier i 's on-hand inventory level is lower than $y_{del} - I_{01}$, the supplier would produce $y_{del} - I_{01} - I_{i1}$ units of component.

III. Overview of the Last-Period Problem

Based on supplier i ' optimal production policy in eq.(4-39), the suppliers' order-up-to levels after replenishment have the following combinations (as shown in Table 4-2). The table shows that when suppliers have sufficient on-hand inventory to raise the assembler's inventory level up to y_{del} , their order-up-to levels (as shown in the third column of Table 4-2) after replenishment equal to their on-hand inventory levels; otherwise, their order-up-to levels equal to $y_{del} - I_{01}$. In addition, the critical supplier can be determined according to suppliers' inventory level after replenishment – the supplier having a less on-hand inventory after replenishment is the critical supplier (as shown in the fourth column of Table 4-2).

Table 4-2 Combination of suppliers' order-up-to levels after replenishment

No.	Conditions	Order-up-to Level	Critical Supplier
1	$I_{11} \geq y_{del} - I_{01}$ $I_{21} \geq y_{del} - I_{01}$	I_{11} I_{21}	Supplier 1
2	$I_{11} < y_{del} - I_{01}$ $I_{21} \geq y_{del} - I_{01}$	$y_{del} - I_{01}$ I_{21}	Supplier 1
3	$I_{11} < y_{del} - I_{01}$ $I_{21} < y_{del} - I_{01}$	$y_{del} - I_{01}$	Either

It is further noted from Table 4-2 that a supplier's order-up-to level after replenishment is not lower than $y_{del} - I_{01}$. Hence, the assembler's optimal purchasing quantity of the last period would be affected by the relationship between y_{del} and y_{d01} . That is, when $y_{d01} \leq y_{del}$, the assembler's optimization problem becomes unconstrained; hence, the assembler can replenish up to its optimal order-up-to level without constraint y_{d01} . Whereas, when $y_{d01} > y_{del}$, the assembler's optimization problem becomes constrained; hence, its optimal purchasing quantity depends on the relationship between $y_{d01} - I_{01}$ and the critical supplier's inventory level after replenishment.

Based on the analysis above, the optimal policy for the price-only decentralized setting of period $t = 1$ can be summarized as follows.

i.) When $y_{d01} \leq y_{del}$

As mentioned, the assembler's optimization problem becomes unconstrained with the given condition. Hence, it is optimal for the assembler to set its purchasing quantity as,

$$Q_{d01}^* = \begin{cases} 0 & \text{if } I_{01} \geq y_{d01} \\ y_{d01} - I_{01} & \text{if } I_{01} < y_{d01} \end{cases} \quad (4-40)$$

which has an order-up-to structure.

The assembler's optimal purchasing policy shows that suppliers are only needed to raise their inventory levels up to $y_{d01} - I_{01}$; hence, supplier i 's optimal production policy becomes,

$$Q_{di1}^* = \begin{cases} 0 & \text{if } I_{01} \geq y_{d01} \\ 0 & \text{if } I_{01} < y_{d01} \text{ and } I_{i1} \geq y_{del} - I_{01} \\ y_{d01} - I_{01} - I_{i1} & \text{if } I_{01} < y_{d01} \text{ and } I_{i1} < y_{del} - I_{01} \end{cases} \quad (4-41)$$

ii.) When $y_{d01} > y_{del}$

With the given condition, the assembler's optimization problem becomes constrained; hence,

its optimal purchasing quantity depends on the relationship between $y_{d01} - I_{01}$ and the critical supplier's inventory level after replenishment (see Table 4-2). To summarize, the assembler's optimal purchasing quantity is,

$$Q_{d01}^* = \begin{cases} 0 & \text{if } I_{01} \geq y_{d01} \\ \min(y_{d01} - I_{01}, I_{11}) & \text{if } I_{01} < y_{d01} \text{ and } I_{11} \geq y_{de1} - I_{01} \\ y_{de1} - I_{01} & \text{if } I_{01} < y_{d01} \text{ and } I_{11} < y_{de1} - I_{01} \end{cases} \quad (4-42)$$

that is, when the assembler's on-hand inventory level is higher than its optimal order-up-to level without constraint, it is optimal for the assembler not to place any orders. Whereas, when both suppliers' on-hand inventory levels are higher than $y_{de1} - I_{01}$, the assembler could purchase up to the minimum value of $y_{d01} - I_{01}$ and I_{11} (since supplier 1 is the critical supplier with the given condition). Lastly, when supplier 1's on-hand inventory level is lower than $y_{de1} - I_{01}$, it is optimal for the assembler to purchase up to $y_{de1} - I_{01}$.

With the given condition, suppliers' optimal production policy could still be described by (4-39), since the policy is derived by treating the assembler's optimization problem as a constrained one.

IV. Comparison of the Last-Period Problem with Different Control Modes

It is generally believed that the total expected system profit in the price-only decentralized system is no larger than that of the corresponding centralized control setting. A brief demonstration to the statement is to be carried out in this section based on the results obtained in the last period centralized setting (as given in Section 4.2.1) and the last period price-only decentralized setting (as given in Section 4.3.1).

Adding up all the players' expected profits, the total expected system profit for the price-only

decentralized control setting is,

$$\sum_{i=0}^2 \pi_{di1} = E_{\xi_1} \left\{ p \cdot \min(Q_{d01} + I_{01}, \xi_1) - c_0 \cdot Q_{d01} - L(Q_{d01} + I_{01}) - \sum_{i=1}^2 [c_i \cdot Q_{di1} + h_i (I_{i1} + Q_{di1} - Q_{d01})] \right\} \quad (4-43)$$

which has the same format as that for the centralized control setting (refer to eq.(4-1)). This is because the terms representing the transaction, concerning transferring ownership of the components between the assembler and the suppliers in the decentralized setting, are counteracted when adding up all the players' expected profits; whereas, such transactions between the assembler and its suppliers are not modeled in the centralized control setting. In addition, the similarity in the format of the expressions, representing total expected system profit of the two control modes, provides a method to compare the values of these two profits. Since the optimal policy for the centralized control mode is derived from the perspective of global optimization, the total expected system profit obtained in such a setting is the highest. Hence, if the optimal policy for the price-only decentralized setting is different from that of the corresponding centralized control setting, it can be concluded that the decentralized setting is inferior to that of the centralized setting. Specifically, the above-mentioned statement can be summarized in the following proposition.

Proposition 4-3

Given the same initial conditions, the total expected system profit obtained in the last period of the decentralized setting is no larger than that of the corresponding centralized setting. ■

Proofs for Proposition 4-3 are presented in Appendix III.

The conclusion in Proposition 4-3 implies that the decentralized assembly system cannot be coordinated with price-only contract alone. Similar conclusions have been derived for decentralized distribution channels (as have been reviewed in Chapter 2). In a word, system

coordination requires modification to the current pricing schemes such that individual incentives could be aligned with global optima.

In the following section, models for the assembly system operated with price-only decentralized control in a two-period setting are formulated. Wholesale prices over the two periods are non-stationary and special attention is paid to study the impact of changes in wholesale prices on system performance.

4.3.2 Two-Period Model for the Price-Only Decentralized Control Setting

Inventory transfer functions for the price-only decentralized control setting are,

$$\begin{aligned} I_{01} &= I_{02} + Q_{d02} - \xi_2 \\ I_{11} &= I_{12} + Q_{d12} - Q_{d01} \\ I_{21} &= I_{22} + Q_{d22} - Q_{d01} \end{aligned} \quad (4-44)$$

Without loss of consistency, supplier 1 is assumed to have less on-hand inventory at the beginning of period $t = 2$; that is, $I_{12} = \min(I_{12}, I_{22})$. Given this assumption, it has been proved in the centralized setting that it is global optimal for suppliers to produce and deliver in such a way that supplier 1 is the critical supplier of the two consecutive periods.

Contrary to the centralized control mode, decisions under decentralized control mode are made for local optimization. Hence, suppliers could act so strategic that either supplier might be the critical supplier of period $t = 2$. Further, it can be derived from the inventory transfer functions in eq.(4-44) that the supplier having a lower on-hand inventory level at the beginning of period $t = 1$ is the critical supplier for period $t = 2$.

Moreover, the optimal policy for the last period takes on different formats depending on the properties of the assembler's last-period optimization problem; whereby, the optimal policy for the two-period problem is affected.

Based on the analysis above, it is noted that determination of the optimal policy for period $t = 2$ requires considering the relationship between y_{d01} and y_{del} as well as the relationship between suppliers' on-hand inventory level at the beginning of period $t = 1$. Hence, the following four scenarios (as given in Table 4-3) are considered for the two-period price-only decentralized control setting.

Table 4-3 Scenario combinations

	$I_{11} \leq I_{21}$	$I_{11} > I_{21}$
$y_{d01} \leq y_{del}$	Scenario I	Scenario III
$y_{d01} > y_{del}$	Scenario II	Scenario IV

I. Scenario I and III

For both scenarios, the assembler's last-period problem is treated as a non-constrained optimization problem (with the assumption of $y_{d01} \leq y_{del}$). In addition, it is noted that the deduction of the optimal policy for period $t = 1$ does not require any specific assumption on the suppliers' initial inventory levels of the period. In other words, the assembler's optimal purchasing quantity described by eq.(4-40) and the supplier's optimal production/delivery policy described by eq.(4-41) (derived with the assumption of $I_{11} \leq I_{21}$) are still the optimal solutions under the assumption of $I_{11} > I_{21}$.

Since the optimal policy for the last period ($t = 1$) is the same for both scenarios, all the players' two-period objective functions for the two scenarios are the same. Derivation of the

optimal solutions for the two scenarios is done together.

However, each supplier's on-hand inventory level at the beginning of period $t = 1$ could be the lower; deduction of the optimal solutions would first treat both suppliers as if they are the critical supplier. The derived solutions are then examined for their feasibility: for solutions satisfying $I_{12} + Q_{d12} < I_{22} + Q_{d22}$, they are the optimal solutions for scenario I; whereas, solutions satisfying $I_{12} + Q_{d12} = I_{22} + Q_{d22}$ (the same order-up-to level) are optimal solutions for either scenario; lastly, solutions satisfying $I_{12} + Q_{d12} > I_{22} + Q_{d22}$ are the optimal solutions for scenario III.

a.) The Assembler's Two-Period Problem

With its optimal purchasing policy in eq.(4-40), the assembler's optimal expected profits for the last period is,

$$\begin{aligned}
 & J_{d01}(I_{01}) \\
 &= \int_0^{I_{02} + Q_{d02} - y_{d01}} \left\{ p \cdot \min(I_{02} + Q_{d02} - \xi_2, \xi_1) - L(I_{02} + Q_{d02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{02} + Q_{d02} - y_{d01}}^{\infty} \left\{ p \cdot \min(y_{d01}, \xi_1) - L(y_{d01}) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (y_{d01} - I_{02} - Q_{d02} + \xi_2) \right\} f(\xi_2) d\xi_2
 \end{aligned} \quad (4-45)$$

In eq.(4-45), the first line represents the assembler's expected profit when it does not place orders, the second line represents its expected profit when it replenishes up to y_{d01} .

Hence, the assembler's expected profit for the last two periods is,

$$\begin{aligned}
 M_{d02}(I_{02}, Q_{d02}) &= E_{\xi_2} \left\{ \pi_{d02}(I_{02}, Q_{d02}) + J_{d01}(I_{01}) \right\} \\
 &= E_{\xi_2} \left\{ p \cdot \min(Q_{d02} + I_{02}, \xi_2) - \left(\sum_{i=1}^2 w_{di2} + c_0 \right) Q_{d02} - L(Q_{d02} + I_{02}) + J_{d01}(I_{01}) \right\} \\
 &\quad s.t. \quad Q_{d02} \leq \min(I_{12} + Q_{d12}, I_{22} + Q_{d22})
 \end{aligned} \quad (4-46)$$

As before, the constraint indicates that the assembler's purchasing quantity for period $t = 2$ is subject to the number of components delivered by the critical supplier of that period.

The first and the second order partial derivatives of $M_{d02}(I_{02}, Q_{d02})$ with respect to Q_{d02} are,

$$\begin{aligned} \frac{\partial M_{d02}(I_{02}, Q_{d02})}{\partial Q_{d02}} = & -(p + s + h_0) \int_0^{I_{02} + Q_{d02} - y_{d01}} F(I_{02} + Q_{d02} - \xi_2) f(\xi_2) d\xi_2 \\ & + \left(p - \sum_{i=1}^2 w_{di2} + \sum_{i=1}^2 w_{di1} + s \right) - (p + h_0 + s) F(Q_{d02} + I_{02}) \quad (4-47) \\ & + \left(p + s - \sum_{i=1}^2 w_{di1} - c_0 \right) \cdot F(I_{02} + Q_{d02} - y_{d01}) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 M_{d02}(I_{02}, Q_{d02})}{\partial Q_{d02}^2} = & -(p + s + h_0) \left[f(I_{02} + Q_{d02}) + \int_0^{I_{02} + Q_{d02} - y_{d01}} f(I_{02} + Q_{d02} - \xi_2) f(\xi_2) d\xi_2 \right] \quad (4-48) \end{aligned}$$

It is noted that the second-order partial derivative is negative, which implies that the corresponding first-order partial derivative is monotonously decreasing and it has, at most, one zero-point. Furthermore, it is derived that

$$\left. \frac{\partial M_{d02}(I_{02}, Q_{d02})}{\partial Q_{d02}} \right|_{Q_{d02} + I_{02} = 0} = p + s - \sum_{i=1}^2 w_{di2} + \sum_{i=1}^2 w_{di1} > 0$$

and

$$\lim_{Q_{d02} + I_{02} \rightarrow \infty} \frac{\partial M_{d02}(I_{02}, Q_{d02})}{\partial Q_{d02}} = -2h_0 - \sum_{i=1}^2 w_{di2} - c_0 < 0$$

Therefore, there is one and only one zero-point to eq.(4-47), denoted as y_{d02} , which represents the assembler's optimal order-up-to level without constraint. By analogy, it can be derived that it is optimal for the assembler not to place orders when $I_{02} \geq y_{d02}$; whereas, it is optimal for the assembler to set $Q_{d02} > 0$ when $I_{02} < y_{d02}$.

b.) Suppliers' Two-Period Problem

With the optimal production policy in (4-41), supplier i 's optimal expected profit for the last period can be formulated as follows ($\forall i \neq 0$),

$$\begin{aligned}
 & J_{di1}(I_{i1}) \\
 &= \int_0^{I_{02}+Q_{d02}-y_{d01}} -h_i(I_{i2}+Q_{di2}-Q_{d02})f(\xi_2)d\xi_2 \\
 &+ \int_{I_{02}+Q_{d02}-y_{d01}}^{I_{i2}+Q_{di2}+I_{02}-y_{d01}} \{w_{di1}(y_{d01}-I_{02}-Q_{d02}+\xi_2)-h_i(I_{i2}+Q_{di2}+I_{02}-y_{d01}-\xi_2)\}f(\xi_2)d\xi_2 \\
 &+ \int_{I_{i2}+Q_{di2}+I_{02}-y_{d01}}^{\infty} \{w_{di1}(y_{d01}-I_{02}-Q_{d02}+\xi_2)-c_i(y_{d01}-I_{i2}-Q_{di2}-I_{02}+\xi_2)\}f(\xi_2)d\xi_2
 \end{aligned} \quad (4-49)$$

In eq.(4-49), the first line represents supplier i 's expected profit when the assembler does not place orders (that is, $Q_{d01}=Q_{di1}=0$); the second line represents supplier i 's expected profit when both suppliers have sufficient on-hand inventory to raise the assembler's inventory level up to y_{d01} (that is, $Q_{d01}=y_{d01}-I_{01}$, $Q_{di1}=0$); the last line represents supplier i 's expected profit when it produces extra components to raise its inventory level up to $y_{d01}-I_{01}$ (that is, $Q_{d01}=y_{d01}-I_{01}$, $Q_{di1}=y_{d01}-I_{01}-I_{i1}$).

Further, supplier i 's total expected profits for the last two periods can be formulated as,

$$\begin{aligned}
 M_{di2}(I_{i2}, Q_{di2}) &= E_{\xi_2} \{ \pi_{di2}(I_{i2}, Q_{di2}) + J_{di1}(I_{i1}) \} \\
 &= E_{\xi_2} \{ w_{di2} \cdot Q_{d02} - c_i \cdot Q_{di2} - h_i(I_{i2} + Q_{di2} - Q_{d02}) + J_{di1}(I_{01}, I_{i1}) \}
 \end{aligned} \quad (4-50)$$

By analogy, the assembler's purchasing quantity for period $t=2$ is expected to be,

$$Q_{d02} = E_{\xi_2} \left\{ \min(I_{i2} + Q_{di2}) + (I_{02} - \xi_2)^+ - \left[\min(I_{i2} + Q_{di2}) + I_{02} - \xi_2 \right]^+ \right\} \quad (4-51)$$

in which $\min(I_{i2} + Q_{di2})$ for $i \neq 0$ represents the critical supplier's delivery for period $t=2$.

Moreover, when supplier i is not the critical supplier of the period, it is derived that,

$$\frac{\partial M_{di2}(I_{i2}, Q_{di2})}{\partial Q_{di2}} = -(c_i + h_i)F(I_{i2} + Q_{di2} + I_{02} - y_{d01}) - h_i$$

which is negative. This indicates that supplier i 's expected profit decreases when it produces

extra components to raise its inventory level higher than the critical supplier's delivery. In anticipation of this, a supplier would determine its optimal production quantity by treating itself as the critical supplier. Detailed analysis to each supplier's optimal production/delivery policy is presented below.

b.1.) Supplier 1's Two-Period Problem

Suppose supplier 1 is the critical supplier of period $t = 2$; eq.(4-51) can be rewritten as,

$$Q_{d02} = E_{\xi_2} \left\{ (I_{12} + Q_{d12}) + (I_{02} - \xi_2)^+ - [(I_{12} + Q_{d12}) + I_{02} - \xi_2]^+ \right\} \quad (4-52)$$

in which Q_{d02} is a function of Q_{d12} satisfying $\frac{\partial Q_{d02}}{\partial Q_{d12}} = 1 - F(I_{12} + Q_{d12} + I_{02})$. Hence, the

first-order partial derivative of $M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})$ with respect to Q_{d12} is,

$$\frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = \frac{\partial M_{d12}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} + \frac{\partial M_{d12}}{\partial Q_{d12}} \quad (4-53)$$

For the sake of exposition, terms of (4-53) are presented as follows,

$$\frac{\partial M_{d12}}{\partial Q_{d12}} = -(c_1 + h_1) \cdot F(I_{12} + Q_{d12} + I_{02} - y_{d01}) - h_1 \quad (4-54)$$

$$\begin{aligned} \frac{\partial M_{d12}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} &= [(w_{d12} - w_{d11} + h_1) + (w_{d11} + h_1) F(I_{02} + Q_{d02} - y_{d01})] \\ &\quad \times [1 - F(I_{12} + Q_{d12} + I_{02})] \end{aligned} \quad (4-55)$$

$$\begin{aligned} \frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} &= [(w_{d12} + h_1) - w_{d11} + (w_{d11} + h_1) \cdot F(I_{02} + Q_{d02} - y_{d01})] \\ &\quad \times [1 - F(I_{12} + Q_{d12} + I_{02})] - h_1 \\ &\quad - (c_1 + h_1) \cdot F(I_{12} + Q_{d12} + I_{02} - y_{d01}) \end{aligned} \quad (4-56)$$

As analyzed, the assembler will not ask for replenishment with its on-hand inventory level I_{02} being higher than y_{d02} ; under such a circumstance, supplier 1's optimal production quantity can be determined by setting $Q_{d02} = 0$ in eq.(4-56). Hence, eq.(4-56) becomes,

$$\left. \frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} \right|_{Q_{d02}=0} = -h_1 - (c_1 + h_1) \cdot F(I_{12} + Q_{d12} + I_{02} - y_{d01}) < 0$$

which is negative. This implies that it is optimal for supplier 1 not to produce any additional components when the assembler does not place orders; whereas, when the assembler asks for replenishment with its on-hand inventory level satisfying $I_{02} < y_{d02}$, supplier 1's optimal production policy would depend on its wholesale prices over the two periods. A comparison between supplier 1's wholesale prices over the last two periods is made as follows,

i.) If $w_{d11} = w_{d12}$

This represents the scenario when wholesale prices over the two periods are stationary. With this condition, the sign of the first-order derivative, as formulated in eq.(4-56), can be determined based on the following discussions,

i.a.) If $I_{12} + Q_{d12} + I_{02} \leq y_{d01}$, then

$$\frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = -h_1 \cdot F(I_{12} + Q_{d12} + I_{02}) < 0$$

i.b.) If $I_{02} + Q_{d02} < y_{d01} < I_{12} + Q_{d12} + I_{02}$, then

$$\frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = -h_1 \cdot F(I_{12} + Q_{d12} + I_{02}) - (c_1 + h_1) \cdot F(I_{12} + Q_{d12} + I_{02} - y_{d01}) < 0$$

i.c.) If $I_{02} + Q_{d02} \geq y_{d01}$, then

$$\begin{aligned} & \frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} \\ &= -(c_1 + h_1) \cdot F(I_{12} + Q_{d12} + I_{02} - y_{d01}) + (w_{d11} + h_1) F(I_{02} + Q_{d02} - y_{d01}) \\ & \quad - (w_{d11} + h_1) F(I_{02} + Q_{d02} - y_{d01}) F(I_{12} + Q_{d12} + I_{02}) - h_1 F(I_{12} + Q_{d12} + I_{02}) \\ & \quad \because F(y_{d01}) = \frac{w_{d11} - c_1}{w_{d11} + h_1} \\ & \Rightarrow \frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} < (c_1 + h_1) [F(I_{02} + Q_{d02} - y_{d01}) - F(I_{12} + Q_{d12} + I_{02} - y_{d01})] \\ & \quad - h_1 F(I_{12} + Q_{d12} + I_{02}) < 0 \end{aligned}$$

The above analysis shows that the first-order partial derivative is negative over its entire feasible domain. Hence, it is optimal for supplier 1 not to produce any additional components 1 and to set $Q_{d12} = 0$. Moreover, it is derived from eq.(4-55) that

$$\left. \frac{\partial M_{d12}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \right|_{w_{d11}=w_{d12}} = [h_1 + (w_{d11} + h_1)F(I_{02} + Q_{d02} - y_{d01})][1 - F(I_{12} + Q_{d12} + I_{02})] > 0$$

which is positive. Hence, it is optimal for supplier 1 to deliver all of its on-hand components to the assembler without additional production.

ii.) If $w_{d12} < w_{d11}$

This represents the scenario when there is a markup of supplier 1's wholesale price in the last period. If the wholesale price can be expected to rise, supplier 1 would try to deliver/sell as many components as possible in the last period. To derive supplier 1's optimal production/deliver policy under such a condition, the following scenarios are considered,

ii.a.) If $I_{12} + Q_{d12} + I_{02} \leq y_{d01}$, then

$$\frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = (w_{d12} - w_{d11})[1 - F(I_{12} + Q_{d12} + I_{02})] - h_1 \cdot F(I_{12} + Q_{d12} + I_{02}) < 0$$

ii.b.) If $I_{02} + Q_{d02} < y_{d01} < I_{12} + Q_{d12} + I_{02}$, then

$$\begin{aligned} \frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} &= (w_{d12} - w_{d11})[1 - F(I_{12} + Q_{d12} + I_{02})] - h_1 \cdot F(I_{12} + Q_{d12} + I_{02}) \\ &\quad - (c_1 + h_1)F(I_{12} + Q_{d12} + I_{02} - y_{d01}) < 0 \end{aligned}$$

ii.c.) If $I_{02} + Q_{d02} \geq y_{d01}$

$$\begin{aligned} \frac{\partial M_{d12}(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} &= [(w_{d12} + h_1) - w_{d11} + (w_{d11} + h_1)F(I_{02} + Q_{d02} - y_{d01})] \\ &\quad \times [1 - F(I_{12} + Q_{d12} + I_{02})] - h_1 - (c_1 + h_1)F(I_{12} + Q_{d12} + I_{02} - y_{d01}) \\ &< (h_1 + c_1)[F(I_{02} + Q_{d02} - y_{d01}) - F(I_{12} + Q_{d12} + I_{02} - y_{d01})] \\ &\quad + (w_{d12} - w_{d11})[1 - F(I_{12} + Q_{d12} + I_{02})] - h_1 \cdot F(I_{12} + Q_{d12} + I_{02}) < 0 \end{aligned}$$

The above analysis shows that the first-order partial derivative is negative over its entire feasible domain. Hence, it is optimal for supplier 1 not to produce any additional components 1 and to set $Q_{d12} = 0$.

It is not quite straightforward to decide whether it is optimal for supplier 1 to deliver its on-hand inventory to the assembler; hence, the following scenarios are considered.

ii.1.) If $w_{d12} + h_1 - w_{d11} \geq 0$

That is, if supplier 1's holding cost is higher than the markup value of the wholesale prices in the last two periods, it is optimal for supplier 1 to deliver all of its on-hand components to the assembler (since it can be derived that $\frac{\partial M_{d12}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} > 0$).

ii.2.) If $w_{d12} + h_1 - w_{d11} < 0$

When supplier 1's holding cost is smaller than the markup value of the wholesale prices, its delivery decision cannot be determined directly. Denote y_{d12}^I as the order-up-to level satisfying eq.(4-55),

$$\left[w_{d12} + h_1 - w_{d11} + (w_{d11} + h_1) F(y_{d12}^I - y_{d01}) \right] [1 - F(y_{d12}^I)] = 0$$

If $y_{d12}^I \geq I_{12} + I_{02}$, it is optimal for supplier 1 to keep all of its on-hand components to the next period; whereas, if $y_{d12}^I < I_{12} + I_{02}$, it is optimal for supplier 1 to deliver all of its on-hand components to the assembler.

iii.) If $w_{d12} > w_{d11}$

This represents the scenario when there is a markdown of supplier 1's wholesale price in the last period. If the wholesale price can be expected to decrease, supplier 1 has the motivation to deliver/sell as many components as possible in period $t = 2$. Further, supplier 2's optimal

production condition can be determined by denoting $y_{d12}^{I_2}$ as the order-up-to level satisfying eq.(4-56),

$$(w_{d12} - w_{d11}) + (w_{d11} - c_1)F(y_{d12}^{I_2} - y_{d01}) - \left[(w_{d12} + h_1) - w_{d11} + (w_{d11} + h_1)F(y_{d12}^{I_2} - y_{d01}) \right] \cdot F(y_{d12}^{I_2}) = 0$$

In addition, the sign of $\frac{\partial M_{d12}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}}$ is always positive with $w_{d12} > w_{d11}$. Therefore, if

$y_{d12}^{I_2} \leq I_{12} + I_{02}$, it's optimal for supplier 1 to deliver all of its on-hand inventory without producing any additional components (that is, $Q_{d02} = 0$); whereas, if $y_{d12}^{I_2} > I_{12} + I_{02}$, it's optimal for supplier 1 to produce up to $y_{d12}^{I_2} - I_{02}$ and to deliver all of its on-hand components after replenishment. Hence, given the condition of $w_{d12} > w_{d11}$, supplier 1's optimal production policy can be described as,

$$Q_{d12}^* = \begin{cases} 0 & \text{if } I_{02} \geq y_{d02} \\ 0 & \text{if } I_{02} < y_{d02} \text{ and } y_{d12}^{I_2} \leq I_{02} + I_{12} \\ y_{d12}^{I_2} - I_{02} - I_{12} & \text{if } I_{02} < y_{d02} \text{ and } y_{d12}^{I_2} > I_{02} + I_{12} \end{cases} \quad (4-57)$$

b.2.) Supplier 2's Two-Period Problem

Suppose supplier 2 is the critical supplier of period $t = 2$. With its total expected profit for the last two periods formulated in eq.(4-50) (that is, $M_{d22}(Q_{d02}(Q_{d22}), Q_{d22})$), supplier 2's optimal production/delivery quantity can be deduced by analogy. Further, the following partial derivatives are derived for analysis.

$$\begin{aligned} \frac{\partial M_{d22}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} &= \left[(w_{d22} - w_{d21} + h_2) + (w_{d21} + h_2)F(I_{02} + Q_{d02} - y_{d01}) \right] \\ &\quad \times \left[1 - F(I_{22} + Q_{d22} + I_{02}) \right] \end{aligned} \quad (4-58)$$

and

$$\begin{aligned} \frac{\partial M_{d22}(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d22}} &= [w_{d22} + h_2 - w_{d21} + (w_{d21} + h_2) \cdot F(I_{02} + Q_{d02} - y_{d01})] \\ &\quad \times [1 - F(I_{22} + Q_{d22} + I_{02})] - h_2 \\ &\quad - (c_2 + h_2) \cdot F(I_{22} + Q_{d22} + I_{02} - y_{d01}) \end{aligned} \quad (4-59)$$

As before, the relationship between supplier 2's wholesale prices over the last two periods is discussed.

i.) If $w_{d22} = w_{d21}$

When supplier 2's wholesale prices are stationary over the two periods, it can be derived by

analogy that $\frac{\partial M_{d22}(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d22}} < 0$. The negativity of the first-order partial derivative

indicates that it is optimal for supplier 2 not to place any orders. Further, it is derived that

$\left. \frac{\partial M_{d22}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} \right|_{w_{d11}=w_{d12}} > 0$. Hence, it is optimal for supplier 2 to deliver all of its on-hand

components to the assembler (without any additional production).

ii.) If $w_{d22} < w_{d21}$

When there is a markup of supplier 2's wholesale price in the last period, it can be deduced by analogy that it is optimal for supplier 2 not to produce any additional components (since the first-order partial derivative of its two-period expected profits with respect to its production quantity is negative). Whereas, supplier 2's optimal delivery conditions can be determined as follows.

ii.a.) If $w_{d22} + h_2 - w_{d21} \geq 0$

In the event that supplier 2's holding cost is not lower than the markup value of the wholesale prices, it is optimal for supplier 2 to deliver all of its on-hand components to the assembler

(since it can be derived that $\frac{\partial M_{d22}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} > 0$).

ii.b.) If $w_{d22} + h_2 - w_{d21} < 0$

In the event that supplier 2's holding cost is lower than the markup of the wholesale prices, supplier 2's optimal delivery condition can be determined by denoting $y_{d22}^{I_1}$ as the order-up-to level satisfying eq.(4-58),

$$\left[w_{d22} + h_2 - w_{d21} + (w_{d21} + h_2) F(y_{d22}^{I_1} - y_{d01}) \right] \left[1 - F(y_{d22}^{I_1}) \right] = 0$$

If $y_{d22}^{I_1} \geq I_{22} + I_{02}$, it is optimal for supplier 2 to keep all of its on-hand components until the next period; whereas, if $y_{d22}^{I_1} < I_{22} + I_{02}$, it is optimal for supplier 2 to deliver all of its on-hand components.

iii.) If $w_{d22} > w_{d21}$

This represents a decrease in supplier 2's wholesale price in the last period. Denote $y_{d22}^{I_2}$ as an order-up-to level satisfying eq.(4-59),

$$\begin{aligned} & (w_{d22} - w_{d21}) + (w_{d21} - c_2) F(y_{d22}^{I_2} - y_{d01}) \\ & - \left[(w_{d22} + h_2) - w_{d21} + (w_{d21} + h_2) F(y_{d22}^{I_2} - y_{d01}) \right] \cdot F(y_{d22}^{I_2}) = 0 \end{aligned}$$

In addition, it can be derived that the sign of $\frac{\partial M_{d22}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}}$ is always positive. Hence, if

$y_{d22}^{I_2} \leq I_{22} + I_{02}$, it's optimal for supplier 2 to deliver all of its on-hand components without any additional production; however, if $y_{d22}^{I_2} > I_{22} + I_{02}$, it is optimal for supplier 2 to produce up to $y_{d22}^{I_2} - I_{02}$ and to deliver all of its on-hand components after production.

c.) Overview of the Optimal policy for Scenario I and III

For Scenario I and III, the assembler's optimization problem for the last period is non-constrained; hence, analysis for the two scenarios is done together. Moreover, given assumption on suppliers' initial on-hand inventory level for period $t = 2$, the suppliers' optimal production quantities are determined by treating them as the critical supplier of the

period. The analysis above shows that suppliers' optimal production/delivery policies depend on their wholesale prices over the last periods when they are non-stationary. With optimal solutions derived for each supplier, if the derived solutions can lead to $I_{11} = \min(I_{11}, I_{21})$, they are the optimal solutions for Scenario I; otherwise, they are optimal solutions for Scenario III. Since the assembler's optimal purchasing quantity for period $t = 2$ is constrained by the critical supplier's delivery, it is described as follows,

$$Q_{d02}^* = \begin{cases} 0 & \text{if } I_{02} \geq y_{d02} \\ \min(y_{d02} - I_{02}, \min(I_{12} + Q_{d12}^*, I_{22} + Q_{d22}^*)) & \text{else} \end{cases} \quad (4-60)$$

II. Scenario II

For Scenario II, the assembler's last-period problem is a constrained optimization problem. However, the assumption of $I_{12} = \min(I_{12}, I_{22})$ does not determine the critical supplier for period $t = 2$; hence, each supplier's best response would first be determined by treating them as the critical supplier of the period. Finally, the derived solutions are examined for their feasibility. Only the solutions that satisfy $I_{11} = \min(I_{11}, I_{21})$ are feasible solutions; otherwise, they are infeasible and have to be discarded. As before, non-stationary wholesale prices are considered.

a). The Assembler's Two-Period Problem

With its optimal purchasing policy in eq.(4-42), the assembler's optimal expected profit for the last period can be formulated as,

$$\begin{aligned}
 & J_{d01}''(I_{01}) \\
 &= \int_0^{I_{02}+Q_{d02}-y_{d01}} \left\{ p \cdot \min(I_{02}+Q_{d02}-\xi_2, \xi_1) - L(I_{02}+Q_{d02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{02}+Q_{d02}-y_{d01}}^{I_{12}+Q_{d12}+I_{02}-y_{d01}} \left\{ p \cdot \min(y_{d01}, \xi_1) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (y_{d01} - I_{02} - Q_{d02} + \xi_2) - L(y_{d01}) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{12}+Q_{d12}+I_{02}-y_{d01}}^{I_{12}+Q_{d12}+I_{02}-y_{de1}} \left\{ p \cdot \min(I_{12}+Q_{d12}+I_{02}-\xi_2, \xi_1) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (I_{12}+Q_{d12}-Q_{d02}) \right. \\
 &\quad \left. - L(I_{12}+Q_{d12}+I_{02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{12}+Q_{d12}+I_{02}-y_{de1}}^{\infty} \left\{ p \cdot \min(y_{de1}, \xi_1) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (y_{de1} - I_{02} - Q_{d02} + \xi_2) - L(y_{de1}) \right\} f(\xi_2) d\xi_2
 \end{aligned} \tag{4-61}$$

In eq.(4-61), the first line represents the assembler's expected profit when its on-hand inventory level I_{01} is higher than y_{d01} ; hence, it does not place orders (that is to set $Q_{d01}^* = 0$); the second line and the third line represent the assembler's expected profit when both suppliers do not produce extra components and it is optimal to set $Q_{d01}^* = y_{d01} - I_{01}$ and $Q_{d01}^* = I_{11}$, respectively; the last line represents the assembler's expected profit when the assembler's inventory level is raised up to $y_{de1} - I_{01}$.

Therefore, the assembler's total expected profits for the last two periods can be formulated as,

$$M_{d02}''(I_{02}, Q_{d02}) = \pi_{d02}''(I_{02}, Q_{d02}) + J_{d01}''(I_{01}) \tag{4-62}$$

where $\pi_{d02}''(I_{02}, Q_{d02})$ has the same expression as $\pi_{d02}(I_{02}, Q_{d02})$ (as formulated in (4-46)).

Further, the first and the second-order partial derivatives of $M_{d02}''(I_{02}, Q_{d02})$ with respect to

Q_{d02} are

$$\begin{aligned}
 \frac{\partial M_{d02}''(I_{02}, Q_{d02})}{\partial Q_{d02}} &= -(p+s+h_0) \int_0^{I_{02}+Q_{d02}-y_{d01}} F(I_{02}+Q_{d02}-\xi_2) f(\xi_2) d\xi_2 \\
 &+ \left(p+s - \sum_{i=1}^2 w_{di1} - c_0 \right) F(I_{02}+Q_{d02}-y_{d01}) + (p+s) \\
 &\quad - (p+h_0+s) F(Q_{d02}+I_{02}) + \sum_{i=1}^2 (w_{di1} - w_{di2})
 \end{aligned} \tag{4-63}$$

and

$$\frac{\partial^2 M_{d02}''(I_{02}, Q_{d02})}{\partial Q_{d02}^2} = - \left[f(Q_{d01} + I_{01}) + \int_0^{I_{02} + Q_{d02} - y_{d01}} f(I_{02} + Q_{d02} - \xi_2) f(\xi_2) d\xi_2 \right] \times (p + h_0 + s) \quad (4-64)$$

Compare eq.(4-63) with eq.(4-47), eq.(4-64) with eq.(4-48) respectively, it is noted that they are identical. Hence, the assembler's optimal order-up-to level without constraint of this scenario is the same as that for Scenario I and III and it is still denoted as y_{d02} .

b). Supplier 1's Two-Period Problem

With its optimal purchasing policy in eq.(4-39), supplier 1's optimal expected profits for the last period can be formulated as follows,

$$\begin{aligned} & J_{d11}''(I_{01}, I_{11}) \\ &= \int_0^{I_{02} + Q_{d02} - y_{d01}} -h_1(I_{12} + Q_{d12} - Q_{d02}) f(\xi_2) d\xi_2 \\ &+ \int_{I_{02} + Q_{d02} - y_{d01}}^{I_{12} + Q_{d12} + I_{02} - y_{d01}} \{w_{d11}(y_{d01} - I_{02} - Q_{d02} + \xi_2) - h_1(I_{12} + Q_{d12} + I_{02} - y_{d01} - \xi_2)\} f(\xi_2) d\xi_2 \quad (4-65) \\ &+ \int_{I_{12} + Q_{d12} + I_{02} - y_{d01}}^{I_{12} + Q_{d12} + I_{02} - y_{de1}} w_{d11}(I_{12} + Q_{d12} - Q_{d02}) f(\xi_2) d\xi_2 \\ &+ \int_{I_{12} + Q_{d12} + I_{02} - y_{de1}}^{\infty} \{w_{d11}(y_{de1} - I_{02} - Q_{d02} + \xi_2) - c_1(y_{de1} - I_{12} - Q_{d12} - I_{02} + \xi_2)\} f(\xi_2) d\xi_2 \end{aligned}$$

In eq.(4-65), the first line represents supplier 1's expected profit when the assembler does not place orders, hence, $Q_{d01}^* = Q_{d11}^* = 0$; the second line and the third line represent supplier 1's expected profit when neither of the suppliers produces extra components and the assembler's purchasing quantity Q_{d01}^* equals to $y_{d01} - I_{01}$ and I_{11} , respectively; the last line represents supplier 1's expected profit when its inventory level is raised up to y_{de1} (that is to set $Q_{d01}^* = y_{de1} - I_{01}$ and $Q_{d11}^* = y_{de1} - I_{01} - I_{11}$).

Therefore, supplier 1's total expected profits over the last two periods can be formulated as,

$$M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12}) = \pi_{d12}''(I_{12}, Q_{d12}) + J_{d11}''(I_{11}) \quad (4-66)$$

where $\pi_{d12}''(I_{12}, Q_{d12})$ is the same as $\pi_{d12}(I_{12}, Q_{d12})$ (as formulated in eq.(4-50)).

Suppose supplier 1 is the critical supplier of period $t = 2$, the assembler's purchasing quantity Q_{d02} is, thus, modeled as a function of supplier 1's production quantity Q_{d12} . Specifically, the first-order partial derivative of $M_{d12}''(I_{12}, Q_{d12})$ with respect to Q_{d12} satisfies,

$$\frac{\partial M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = \frac{\partial M_{d12}''}{\partial Q_{d12}} + \frac{\partial M_{d12}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \quad (4-67)$$

For the sake of exposition, terms of eq.(4-67) are presented as follows,

$$\frac{\partial M_{d12}''}{\partial Q_{d12}} = -(w_{d11} + h_1)F(I_{12} + Q_{d12} + I_{02} - y_{d01}) - h_1 + (w_{d11} - c_1)F(I_{12} + Q_{d12} + I_{02} - y_{d01}) \quad (4-68)$$

$$\begin{aligned} \frac{\partial M_{d12}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} = & \left[(w_{d11} + h_1)F(I_{02} + Q_{d02} - y_{d01}) - w_{d11} + w_{d12} + h_1 \right] \\ & \times [1 - F(I_{12} + Q_{d12} + I_{02})] \end{aligned} \quad (4-69)$$

$$\begin{aligned} \frac{\partial M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = & \left[(w_{d11} + h_1)F(I_{02} + Q_{d02} - y_{d01}) - w_{d11} + w_{d12} + h_1 \right] \\ & \times [1 - F(I_{12} + Q_{d12} + I_{02})] \\ & - (w_{d11} + h_1)F(I_{12} + Q_{d12} + I_{02} - y_{d01}) \\ & + (w_{d11} - c_1)F(I_{12} + Q_{d12} + I_{02} - y_{d01}) - h_1 \end{aligned} \quad (4-70)$$

Particularly, it is noted that the first-order partial derivative in eq.(4-69) is the same as that in eq.(4-55). Hence, the previous analysis can be directly applied. Likewise, the relationship between supplier 1's wholesale prices over the last two periods is discussed in order to determine its optimal production/delivery policy.

i.) If $w_{d11} = w_{d12}$

With constant wholesale prices over the two periods, eq.(4-70) can be rewritten as,

$$\begin{aligned} \frac{\partial M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}} = & \left[(w_{d11} + h_1) F(I_{02} + Q_{d02} - y_{d01}) + h_1 \right] \left[1 - F(I_{12} + Q_{d12} + I_{02}) \right] \\ & - (w_{d11} + h_1) F(I_{12} + Q_{d12} + I_{02} - y_{d01}) \\ & + (w_{d11} - c_1) F(I_{12} + Q_{d12} + I_{02} - y_{d01}) - h_1 \end{aligned} \quad (4-71)$$

It is noted from eq.(4-71) that $\frac{\partial M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}}$ cannot be proved to be monotonous

rigorously; however, it is possible that there exists a zero-point to eq.(4-71), denoted as y_{d12}'' .

Moreover, it can be deduced that $\left. \frac{\partial M_{d12}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \right|_{w_{d11}=w_{d12}} > 0$. Hence, it is optimal for supplier

1 to replenish up to $y_{d12}'' - I_{02}$ and deliver all of its on-hand inventory after replenishment when $y_{d12}'' > I_{02} + I_{12}$; whereas, it is optimal for supplier 1 not to produce any additional components and only deliver its on-hand components to the assembler when $y_{d12}'' \leq I_{02} + I_{12}$.

ii.) If $w_{d11} < w_{d12}$

With a markdown in supplier 1's wholesale price in the last period, it is noted that

$\left. \frac{\partial M_{d12}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \right|_{w_{d11} < w_{d12}} > 0$. Similarly, $\frac{\partial M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}}$ cannot be proved to be

monotonous rigorously. Whereas, it is possible that there exists a zero-point to the first-order condition, denoted as y_{d12}'' . Hence, it is optimal for supplier 1 to replenish up to $y_{d12}'' - I_{02}$ and deliver all of its on-hand inventories after replenishment when $y_{d12}'' > I_{12} + I_{02}$; however, it is optimal for supplier 1 not to produce any additional components and only deliver its on-hand components when $y_{d12}'' \leq I_{02} + I_{12}$.

iii.) If $w_{d11} > w_{d12}$

Like in the previous two scenarios, $\frac{\partial M_{d12}''(Q_{d02}(Q_{d12}), Q_{d12})}{\partial Q_{d12}}$ cannot be proved to be

monotonous rigorously with the given condition. However, there might exist a zero-point to the first-order condition, which is denoted as y_{d12}^{II} . Moreover, $\left. \frac{\partial M_{d12}^{II}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \right|_{Q_{d02}=I_{12}}$ is not monotonous as well. Hence, supplier 1's optimal production/delivery condition can be determined as follows.

$$\text{iii.a). If } \left. \frac{\partial M_{d12}^{II}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \right|_{\begin{cases} Q_{d12}=0 \\ Q_{d02}=I_{12} \end{cases}} > 0$$

This represents a positive profit margin when supplier 1 delivers all of its on-hand components without extra production. Hence, If $y_{d12}^{II} > I_{12} + I_{02}$, it is optimal for supplier 1 to produce up to $y_{d12}^{II} - I_{02}$ and deliver all of its components after replenishment; otherwise, it is optimal for supplier 1 to deliver all of its on-hand components without extra production.

$$\text{iii.b). If } \left. \frac{\partial M_{d12}^{II}}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d12}} \right|_{\begin{cases} Q_{d12}=0 \\ Q_{d02}=I_{12} \end{cases}} \leq 0$$

This represents a negative profit margin when supplier 1 delivers all of its on-hand components without production. Hence, If $y_{d12}^{II} > I_{12} + I_{02}$, it is optimal for supplier 1 to produce up to $y_{d12}^{II} - I_{02}$ and deliver all of its components after replenishment; otherwise, it is optimal for supplier 1 to keep all of on-hand inventories until the next period.

c). Supplier 2's Two-Period Problem

With its optimal production policy in eq.(4-39), supplier 2's optimal expected profit for the last period can be formulated as,

$$\begin{aligned}
 & J_{d21}''(I_{21}) \\
 &= \int_0^{I_{02}+Q_{d02}-y_{d01}} -h_2(I_{22}+Q_{d22}-Q_{d02})f(\xi_2)d\xi_2 \\
 &+ \int_{I_{02}+Q_{d02}-y_{d01}}^{I_{12}+Q_{d12}+I_{02}-y_{d01}} \{w_{d21}(y_{d01}-I_{02}-Q_{d02}+\xi_2)-h_2(I_{22}+Q_{d22}+I_{02}-y_{d01}-\xi_2)\}f(\xi_2)d\xi_2 \\
 &+ \int_{I_{12}+Q_{d12}+I_{02}-y_{d01}}^{I_{12}+Q_{d12}+I_{02}-y_{de1}} \{w_{d21}(I_{12}+Q_{d12}-Q_{d02})-h_2(I_{22}+Q_{d22}-I_{12}-Q_{d12})\}f(\xi_2)d\xi_2 \\
 &+ \int_{I_{12}+Q_{d12}+I_{02}-y_{de1}}^{I_{22}+Q_{d22}+I_{02}-y_{de1}} \{w_{d21}(y_{de1}-I_{02}-Q_{d02}+\xi_2)-h_2(I_{22}+Q_{d22}+I_{02}-y_{de1}-\xi_2)\}f(\xi_2)d\xi_2 \\
 &+ \int_{I_{22}+Q_{d22}+I_{02}-y_{de1}}^{\infty} \{w_{d21}(y_{de1}-I_{02}-Q_{d02}+\xi_2)-c_2(y_{de1}-I_{22}-Q_{d22}-I_{02}+\xi_2)\}f(\xi_2)d\xi_2
 \end{aligned} \tag{4-72}$$

In eq.(4-72), the first line represents supplier 2's expected profit when the assembler does not place orders, hence, $Q_{d01}^* = Q_{d21}^* = 0$; the second and the third line represent supplier 2's expected profit when supplier 2 does not produce additional components while the assembler's purchasing quantity equals to $y_{d01} - I_{01}$ and I_{11} respectively; the last two lines represent supplier 2's expected profit when the assembler replenishes up to y_{de1} and supplier 2's production quantity equals to zero and $y_{de1} - I_{01} - I_{21}$ respectively.

Therefore, supplier 2's total expected profits over the last two periods can be formulated as,

$$M_{d22}''(I_{22}, Q_{d22}) = \pi_{d22}''(I_{22}, Q_{d22}) + J_{d21}''(I_{21}) \tag{4-73}$$

where $\pi_{d22}''(I_{22}, Q_{d22})$ is the same as $\pi_{d22}(I_{22}, Q_{d22})$ (as formulated in eq.(4-50)).

Suppose supplier 2 is the critical supplier of period $t = 2$, Q_{d02} is thus modeled as a function of supplier 2's production quantity Q_{d22} . Moreover, the first-order partial derivative satisfies,

$$\frac{\partial M_{d22}''(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d22}} = \frac{\partial M_{d22}''}{\partial Q_{d22}} + \frac{\partial M_{d22}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} \tag{4-74}$$

For the sake of explanation, terms in eq.(4-74) are presented as follows,

$$\frac{\partial M_{d22}''}{\partial Q_{d22}} = -(h_2 + c_2)F(I_{22} + Q_{d22} + I_{02} - y_{de1}) - h_2 \quad (4-75)$$

$$\begin{aligned} \frac{\partial M_{d22}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} &= \left[(h_2 + w_{d21})F(I_{02} + Q_{d02} - y_{d01}) - w_{d21} + w_{d22} + h_2 \right] \\ &\times \left[1 - F(I_{22} + Q_{d22} + I_{02}) \right] \end{aligned} \quad (4-76)$$

and

$$\begin{aligned} \frac{\partial M_{d22}''(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d12}} &= \left[(w_{d22} + h_2 - w_{d21}) + (w_{d21} + h_2)F(I_{02} + Q_{d02} - y_{d01}) \right] \\ &\times \left[1 - F(I_{22} + Q_{d22} + I_{02}) \right] - h_2 - (c_2 + h_2)F(I_{22} + Q_{d22} + I_{02} - y_{de1}) \end{aligned} \quad (4-77)$$

Compare eq.(4-77) with eq.(4-59), it is noted that the two expressions have almost the same format except for the last term (with y_{d01} in eq.(4-59), y_{de1} in eq.(4-77)). Therefore, supplier 2's optimal production/delivery policy can be determined by analogy. That is, it's optimal for supplier 2 not to produce any extra components when the assembler does not place orders (namely, $I_{02} \geq y_{d02}$). However, when the assembler asks for replenishment, supplier 2's optimal policy depends on its wholesale prices of the last two periods, as discussed below.

i.) If $w_{d22} = w_{d21}$

With stationary wholesale prices, it can be derived that $\left. \frac{\partial M_{d22}''(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d12}} \right|_{w_{d11}=w_{d12}} < 0$.

This indicates that it is optimal for supplier 2 not to produce any additional components (that

is, $Q_{d22}^* = 0$). Moreover, it is noted that $\left. \frac{\partial M_{d22}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} \right|_{w_{d11}=w_{d12}} > 0$; hence, it is optimal for

supplier 2 to deliver all of its on-hand components to the assembler.

ii.) If $w_{d22} < w_{d21}$

With a markup in wholesale prices in period $t=1$, $\left. \frac{\partial M_{d22}''(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d22}} \right|_{w_{d22} < w_{d21}} < 0$;

thus, it's optimal for supplier 2 not to produce any additional components (that is, $Q_{d22}^* = 0$).

Further, supplier 2's optimal delivery condition, under the condition of $w_{d22} < w_{d21}$, is discussed below.

ii.a). If $w_{d22} + h_2 - w_{d21} \geq 0$

It can be easily derived that $\left. \frac{\partial M_{d22}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} \right|_{w_{d22} < w_{d21}} > 0$. Hence, it is optimal for supplier 2 to

deliver all of its on-hand components to the assembler.

ii.b). If $w_{d22} + h_2 - w_{d21} < 0$

Supplier 2's optimal delivery policy can be determined by denoting $y_{d22}^{H_1}$ as an order-up-to level satisfying eq.(4-76), which is presented below

$$\left[(w_{d22} + h_2 - w_{d21}) + (w_{d21} + h_2) F(y_{d22}^{H_1} - y_{d01}) \right] [1 - F(y_{d22}^{H_1})] = 0$$

If $y_{d22}^{H_1} \geq I_{22} + I_{02}$, it is optimal for supplier 2 to keep all of its on-hand components until the next period; whereas, if $y_{d22}^{H_1} < I_{22} + I_{02}$, it is optimal for supplier 2 to deliver all of its on-hand components.

iii.) If $w_{d22} > w_{d21}$

With a markdown in wholesale price in period $t=1$, $\left. \frac{\partial M_{d22}''(Q_{d02}(Q_{d22}), Q_{d22})}{\partial Q_{d22}} \right|_{w_{d22} > w_{d21}}$

cannot be proved to be not monotonous rigorously. Denote $y_{d22}^{H_2}$ as an order-up-to level satisfying eq.(4-77) (as presented below),

$$\left[(w_{d22} + h_2 - w_{d21}) + (w_{d21} + h_2) F(y_{d22}^{H_2} - y_{d01}) \right] [1 - F(y_{d22}^{H_2})] - h_2 - (c_2 + h_2) F(y_{d22}^{H_2} - y_{del}) = 0$$

Moreover, it can be derived that $\left. \frac{\partial M_{d22}''}{\partial Q_{d02}} \cdot \frac{\partial Q_{d02}}{\partial Q_{d22}} \right|_{w_{d22} > w_{d21}} > 0$. Hence, it's optimal for supplier 2

to produce up to $y_{d22}'' - I_{02}$ and deliver all of its on-hand inventory (after replenishment) if $y_{d22}'' > I_{22} + I_{02}$; otherwise, it's optimal for supplier 2 to deliver all of its on-hand components without extra production.

d). Overview of the Optimal Policy for Scenario II

For this scenario, each supplier's best response has been derived by treating them as the critical supplier of period $t = 2$. However, not all the solutions are feasible. Only the solutions that do not violate the assumption of $I_{11} = \min(I_{11}, I_{21})$ are feasible; otherwise, they are infeasible solutions and have to be discarded. In the event that both suppliers deliver the same number of components, they can be regarded as feasible solutions either for Scenario II or Scenario IV. As before, the assembler's optimal purchasing quantity is constrained by the critical supplier's delivery and can still be depicted by eq.(4-60).

III. Scenario IV

As for Scenario II, the assembler's last-period problem is a constrained optimization problem for Scenario IV. However, the derived optimal policy for the last period has to be modified accordingly to adapt to the assumption of supplier 2 having less initial on-hand inventory. Analogously, each supplier's best response will be determined by treating them as critical supplier respectively. Also, the derived solutions need to be examined for their feasibility.

a). The Assembler's Two-Period Problem

When supplier 2 is assumed to have less on-hand inventory at the beginning of period $t = 1$,

the assembler's optimal purchasing quantity for period $t = 1$ can be described as,

$$Q_{d01}^* = \begin{cases} 0 & \text{if } I_{01} \geq y_{d01} \\ \min(y_{d01} - I_{01}, I_{21}) & \text{if } I_{01} < y_{d01} \text{ and } I_{21} \geq y_{de1} - I_{01} \\ y_{de1} - I_{01} & \text{if } I_{01} < y_{d01} \text{ and } I_{21} < y_{de1} - I_{01} \end{cases} \quad (4-78)$$

That is, when $I_{01} \geq y_{d01}$, namely when the assembler's on-hand inventory level is higher than its optimal order-up-to level without constraint, it's optimal for the assembler not to place orders (that is, $Q_{d01} = 0$). However, when $I_{01} < y_{d01}$ and $I_{21} \geq y_{de1} - I_{01}$, namely when neither supplier produces additional components, it's optimal for the assembler to purchase up to the minimum value of $y_{d01} - I_{01}$ and I_{21} . Lastly, when $I_{01} < y_{d01}$ and $I_{21} < y_{de1} - I_{01}$, namely when supplier 2 replenishes up to $y_{de1} - I_{01}$, it's optimal for the assembler to purchase up to supplier 2's on-hand inventory level after replenishment.

Therefore, the assembler's expected profit over the two periods is,

$$M_{d02}^{IV}(I_{02}, Q_{d02}) = \pi_{d02}^{IV}(I_{02}, Q_{d02}) + J_{d01}^{IV}(I_{01}) \quad (4-79)$$

where $\pi_{d02}^{IV}(I_{02}, Q_{d02})$ is the same as $\pi_{d02}(I_{02}, Q_{d02})$ (eq.(4-50)) and $J_{d01}^{IV}(I_{01})$ equals to

$$\begin{aligned} & J_{d01}^{IV}(I_{01}) \\ &= \int_0^{I_{02} + Q_{d02} - y_{d01}} \left\{ p \cdot \min(I_{02} + Q_{d02} - \xi_2, \xi_1) - L(I_{02} + Q_{d02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\ &+ \int_{I_{02} + Q_{d02} - y_{d01}}^{I_{22} + Q_{d22} + I_{02} - y_{d01}} \left\{ p \cdot \min(y_{d01}, \xi_1) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (y_{d01} - I_{02} - Q_{d02} + \xi_2) - L(y_{d01}) \right\} f(\xi_2) d\xi_2 \\ &+ \int_{I_{22} + Q_{d22} + I_{02} - y_{d01}}^{I_{22} + Q_{d22} + I_{02} - y_{de1}} \left\{ p \cdot \min(I_{22} + Q_{d22} + I_{02} - \xi_2, \xi_1) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (I_{22} + Q_{d22} - Q_{d02}) \right. \\ &\quad \left. - L(I_{22} + Q_{d22} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\ &+ \int_{I_{22} + Q_{d22} + I_{02} - y_{de1}}^{\infty} \left\{ p \cdot \min(y_{de1}, \xi_1) - \left(\sum_{i=1}^2 w_{di1} + c_0 \right) (y_{de1} - I_{02} - Q_{d02} + \xi_2) - L(y_{de1}) \right\} f(\xi_2) d\xi_2 \end{aligned} \quad (4-80)$$

which represents the assembler's optimal expected profit for the last period, In eq.(4-80), the first line represents the assembler's expected profit when it does not place orders; the second

and the third line represent the assembler's expected profit when the assembler purchases up to $y_{d01} - I_{01}$ and I_{21} respectively; the last line represents the assembler's expected profit when it purchases up to the critical supplier's inventory level after replenishment.

Further, the first-order partial derivative of $M_{d02}^{IV}(I_{02}, Q_{d02})$ with respect to Q_{d02} is,

$$\begin{aligned} \frac{\partial M_{d02}^{IV}(I_{02}, Q_{d02})}{\partial Q_{d02}} = & -(p + s + h_0) \int_0^{I_{02} + Q_{d02} - y_{d01}} F(I_{02} + Q_{d02} - \xi_2) f(\xi_2) d\xi_2 \\ & + \left(p + s - \sum_{i=1}^2 w_{di1} - c_0 \right) F(I_{02} + Q_{d02} - y_{d01}) + (p + s) \\ & - (p + h_0 + s) F(Q_{d02} + I_{02}) + \sum_{i=1}^2 (w_{di1} - w_{di2}) \end{aligned} \quad (4-81)$$

Compare eq.(4-81) with eq.(4-63) (the corresponding first-order partial derivative for Scenario II), it is noted that the two expressions are identical. Further, it is noted that the assembler has a unique optimal order-up-to level without constraint for all the four scenarios, which is denoted as y_{d02} .

b). Suppliers' Two-Period Problem

Although both suppliers' optimal production/delivery policies for the last period are different from that for Scenario II due to different assumptions on the suppliers' initial inventory levels for period $t = 1$. However, their optimal policies for the last period can be derived by simply replacing all the parameters (of the optimal policies for Scenario II) for supplier 1 with the corresponding parameters for supplier 2 and replace all the parameters for supplier 2 with the corresponding parameters for supplier 1. The resultant expressions turn out to be so similar that the analysis for supplier 1 in Scenario II can be directly applied to analyze supplier 2 of the current scenario. So does the analysis for supplier 2 in Scenario II. Hence, detailed analysis for this scenario is not presented to avoid redundancy.

c). Overview of the Optimal Production/Ordering Policies

Contrary to the statements made in Scenario II, only the solutions that do not conflict with the assumption of $I_{21} = \min(I_{11}, I_{21})$ are feasible; otherwise, they are infeasible solutions and have to be discarded. In addition, the assembler's optimal purchasing quantity is still constrained by the critical supplier's delivery; hence, its solution structure can still be described by eq.(4-60).

4.3.3 Summary to the Decentralized Price-Only Supply Contract

In Section 4.3, models for the assembly system operated with price-only decentralized control in a two-period setting have been developed. Specifically, it has been proved in the last-period model that the decentralized assembly system cannot be coordinated by price-only contract alone. For the two-period model, the optimal solutions are discussed for scenarios where the assembler's optimization problem for the last period is treated as constrained or non-constrained and either supplier 1 or supplier 2 is assumed to be the critical supplier for period $t = 2$. Applicability of the two-period model will be further highlighted by a numerical study in the next chapter.

4.4 Coordinated Decentralized System

In this section, coordinating mechanisms are explored to improve the performance of the assembly system operated with decentralized control. The proposed coordinating mechanism should be able to align the individual incentive with system-wide optimization. This could be achieved by revising the pricing schemes between the assembler and its component suppliers (characterized by wholesale price in the price-only decentralized setting). In this section, a wholesale plus buyback price contract is studied (for short, is referred to as buyback contract

in the following analysis), which can be proved to realize channel coordination.

With this contract, it is assumed that supplier i pays b_{it} (> 0) to the assembler for every unit of leftover final products at the end of a period. Instead, the assembler pays a wholesale price of w_{bit} for every unit of components purchased. Therefore, the coordinating contract is characterized by two parameters, (w_{bit}, b_{it}) , namely the wholesale price and the buyback price.

4.4.1 Last-Period Problem with Buyback Contract

With the proposed buyback contract, the assembler's expected profit for period $t = 1$ is,

$$\pi_{b01}(I_{01}, Q_{b01}) = E_{\xi_1} \left\{ p \cdot \min(Q_{b01} + I_{01}, \xi_1) - Q_{b01} \cdot \left(\sum_{i=1}^2 w_{bi1} + c_0 \right) - L(Q_{b01} + I_{01}) + (b_{11} + b_{21})(Q_{b01} + I_{01} - \xi_1)^+ \right\} \quad (4-82)$$

$s.t. \quad Q_{b01} \leq \min(Q_{b11} + I_{11}, Q_{b21} + I_{21})$

terms in eq.(4-82) represent the assembler's expected revenue from product selling, purchasing and production costs of components, expected holding and shortage costs and buyback compensation for leftover final products at the end of the last period. Further, the assembler's purchasing quantity is subject to the number of components delivered by the critical supplier.

Hence, the first-order partial derivative of $\pi_{b01}(I_{01}, Q_{b01})$ with respect to Q_{b01} is,

$$\frac{\partial \pi_{b01}(I_{01}, Q_{b01})}{\partial Q_{b01}} = \left(p - \sum_{i=1}^2 w_{bi1} - c_0 + s \right) - (p + s + h_0 - b_{11} - b_{21}) F(Q_{b01} + I_{01}) \quad (4-83)$$

Denote $Z_{11} = F^{-1} \left(\frac{p - \sum_{i=1}^2 w_{bi1} - c_0 + s}{p + s + h_0 - b_{11} - b_{21}} \right)$, which is the zero-point of eq.(4-83). Z_{11}

represents the assembler's optimal order-up-to level without constraint. Further, it is noted

that when $I_{01} \geq Z_{11}$, $\left. \frac{\partial \pi_{b01}(I_{01}, Q_{b01})}{\partial Q_{b01}} \right|_{Q_{b01}=0} \leq 0$; hence, it is optimal for the assembler not to

place orders. Whereas, when $I_{01} < Z_{11}$, $\left. \frac{\partial \pi_{b01}(I_{01}, Q_{b01})}{\partial Q_{b01}} \right|_{Q_{b01}=0} > 0$; hence, it is optimal for the

assembler to ask for replenishment.

Further, supplier i 's ($i = 1, 2$) expected profit for the last period can be modeled as,

$$\pi_{bi1}(w_{bi1}, I_{i1}) = E_{\xi_i} \left\{ w_{bi1} \cdot Q_{b01} - c_i \cdot Q_{bi1} - h_i (I_{i1} + Q_{bi1} - Q_{b01}) - b_{i1} (Q_{b01} + I_{01} - \xi_i)^+ \right\} \quad (4-84)$$

terms in eq.(4-84) represent supplier i 's revenue from selling component to the assembler, production costs, expected holding costs, buyback price paid to the assembler for leftover final products.

Without loss of consistency, it is assumed that $I_{11} = \min(I_{11}, I_{21})$. Further, it is noted that

$\pi_{b11}(w_{b11}, I_{11})$ is a function of both Q_{b01} and Q_{b11} ; hence, the first-order partial derivatives of $\pi_{b11}(w_{b11}, I_{11})$ with respect to the two quantities are,

$$\frac{\partial \pi_{b11}(w_{b11}, I_{11})}{\partial Q_{b01}} = (w_{b11} + h_1) - b_{11} \cdot F(Q_{b01} + I_{01}) \quad (4-85)$$

and

$$\frac{\partial \pi_{b11}(w_{b11}, I_{11})}{\partial Q_{b11}} = -(c_1 + h_1) \quad (4-86)$$

It is noted from eq.(4-85) that $\pi_{b11}(w_{b11}, I_{11})$ is concave in Q_{b01} ; however, it is noted from eq.(4-86) that there is a marginal loss of producing component 1. Hence, it can be derived that when I_{11} is large enough, it is optimal for supplier 1 not to produce any extra

components; however, when I_{11} is relatively small, supplier 1 would determine its optimal policy by trading off its various inventory costs. Denote Z_{21} as the order-up-to level at which supplier 1's expected profit is maximized. Specifically, it can be derived as

$$\begin{aligned} (w_{b11} + h_1) - b_{11} \cdot F(Z_{21}) &= c_1 + h_1 \\ \Rightarrow Z_{21} &= F^{-1}\left(\frac{w_{b11} - c_1}{b_{11}}\right) \end{aligned} \quad (4-87)$$

Further, a necessary condition can be derived from eq.(4-87), that is, $w_{b11} < c_1 + b_{11}$. This condition indicates that supplier 1's wholesale price should be smaller than the sum of its production cost and the buyback price it pays to the assembler.

By analogy, supplier 2 would determine its optimal policy by trading off its various inventory costs as well. Denote Z_{31} as the optimal order-up-to level at which supplier 2's expected profit is maximized. Specifically, it can be derived from,

$$\begin{aligned} (w_{b21} + h_2) - b_{21} \cdot F(Z_{31}) &= c_2 + h_2 \\ \Rightarrow Z_{31} &= F^{-1}\left(\frac{w_{b21} - c_2}{b_{21}}\right) \end{aligned} \quad (4-88)$$

Similarly, a necessary condition that can be drawn from eq.(4-88) is to have $w_{b21} - c_2 < b_{21}$.

Since the purpose of this research is to show the possibility that channel coordination can be attained with the proposed buyback contract, it is necessary to have $Z_{11} > Z_{21} > Z_{31}$. As (w_{b11}, b_{11}) are negotiable coordinating parameters, the necessary condition can be satisfied through proper adjustment of these parameters. As a result, supplier 1's optimal production policy can be described as,

$$Q_{b11}^* = \begin{cases} 0 & \text{if } I_{01} \geq Z_{11} \\ 0 & \text{if } I_{01} < Z_{11} \text{ and } I_{11} \geq Z_{21} - I_{01} \\ \min(Z_{21} - I_{01}, I_{21}) - I_{11} & \text{if } Z_{31} < I_{11} + I_{01} < Z_{21} \\ Z_{31} - I_{01} - I_{11} & \text{if } I_{21} + I_{01} < Z_{31} \end{cases} \quad (4-89)$$

That is, when $I_{01} \geq Z_{11}$, namely when the assembler does not place orders, it is optimal for supplier 1 not to produce any extra components. However, when $I_{01} < Z_{11}$ and $I_{11} > Z_{21} - I_{01}$, namely when the assembler asks for replenishment while supplier 1's on-hand inventory level is higher than its optimal order-up-to level $Z_{21} - I_{01}$, supplier 1 will not produce any extra components. Further, when $Z_{31} < I_{11} + I_{01} < Z_{21}$, namely when supplier 1 produces and supplier 2 does not, supplier 1 would produce up to $\min(Z_{21} - I_{01}, I_{21})$. In particular, when $Q_{b11}^* = I_{21}$, mismatch of components is eliminated. Lastly, when $Z_{31} < I_{21} + I_{01}$, namely when supplier 2 produces to raise its inventory level up to $Z_{31} - I_{01}$, it is optimal for supplier 1 to replenish up to supplier 2's inventory level after replenishment.

Further, supplier 2's optimal production policy can be described as,

$$Q_{b21}^* = \begin{cases} 0 & \text{if } I_{01} \geq Z_{11} \\ 0 & \text{if } I_{01} < Z_{11} \text{ and } I_{21} \geq Z_{31} - I_{01} \\ Z_{31} - I_{01} - I_{21} & \text{if } I_{01} < Z_{11} \text{ and } I_{21} < Z_{31} - I_{01} \end{cases} \quad (4-90)$$

That is, when $I_{01} \geq Z_{11}$, namely when the assembler does not place orders, it is optimal for supplier 2 not to produce any extra components. However, when $I_{01} < Z_{11}$ and $I_{21} > Z_{31} - I_{01}$; that is, when the assembler asks for replenishment and supplier 2's on-hand inventory level is higher than its optimal order-up-to level $Z_{31} - I_{01}$, supplier 2 will not produce any extra component. Lastly, when $I_{21} < Z_{31} - I_{01}$, namely when supplier 2's on-hand inventory level is lower than its optimal order-up-to level, supplier 2 will replenish up to $Z_{31} - I_{01}$.

Since the assembler's optimal order quantity is subject to the critical supplier's deliveries, its optimal order quantity is,

$$Q_{b01}^* = \begin{cases} 0 & \text{if } I_{01} \geq Z_{11} \\ \min(Z_{11} - I_{01}, I_{11}) & \text{if } I_{01} < Z_{11} \text{ and } I_{11} \geq Z_{21} - I_{01} \\ \min(Q_{b11}^* + I_{11}, Q_{b21}^* + I_{21}) & \text{if } I_{01} < Z_{11} \text{ and } I_{11} < Z_{21} - I_{01} \end{cases} \quad (4-91)$$

Based on the optimal policy derived for each player above, the optimal policy for the entire system is summarized in Lemma 4-2.

Lemma 4-2

Suppose $I_{11} = \min(I_{11}, I_{21})$, the optimal policy for the last period of the decentralized assembly system with buyback contract is,

1. If $I_{01} \geq Z_{11}$, then $Q_{b01}^* = Q_{b11}^* = Q_{b21}^* = 0$.
2. If $Z_{11} - I_{01} \leq I_{11} < I_{21}$, then $Q_{b01}^* = Z_{11} - I_{01}, Q_{b11}^* = Q_{b21}^* = 0$.
3. If $\begin{cases} Z_{11} > I_{11} + I_{01} \\ Z_{21} \leq I_{11} + I_{01} \end{cases}$, then $Q_{b01}^* = I_{11}, Q_{b11}^* = Q_{b21}^* = 0$.
4. If $\begin{cases} Z_{11} > I_{11} + I_{01} \\ I_{11} + I_{01} < Z_{21} \leq I_{21} + I_{01} \end{cases}$, then $Q_{b01}^* = Z_{21} - I_{01}, Q_{b11}^* = Z_{21} - I_{01} - I_{11}, Q_{b21}^* = 0$.
5. If $\begin{cases} I_{11} < I_{21} < Z_{11} - I_{01} \\ Z_{31} \leq I_{21} + I_{01} < Z_{21} \end{cases}$, then $Q_{b01}^* = I_{21}, Q_{b11}^* = I_{21} - I_{11}, Q_{b21}^* = 0$.
6. If $Z_{31} > I_{21} + I_{01}$, $Q_{b01}^* = Z_{31} - I_{01}, Q_{b11}^* = Z_{31} - I_{01} - I_{11}, Q_{b21}^* = Z_{31} - I_{01} - I_{21}$. ■

Compare Lemma 4-2 with the simplified Proposition 4-1, it is noted that the two policies have the same solution structure. Specifically, if $Z_{11} = S_{11}$, $Z_{21} = S_{21}$ and $Z_{31} = S_{31}$, the proposed policy in Lemma 4-2 is the same as that in the simplified Proposition 4-1. In this case, the decentralized assembly system achieves the first-best performance. In other words,

the following equations should be satisfied for channel coordination,

$$\begin{cases} \frac{p - \sum_{i=1}^2 w_{bi1} - c_0 + s}{p + s + h_0 - b_{11} - b_{21}} = \frac{p + s - c_0 + \sum_{i=1}^2 h_i}{p + s + h_0} \\ \frac{w_{b11} - c_1}{b_{11}} = \frac{p + s - c_0 - c_1 + h_2}{p + s + h_0} \\ \frac{w_{b21} - c_2}{b_{21}} = \frac{p + s - \sum_{i=0}^2 c_i}{p + s + h_0} \end{cases} \quad (4-92)$$

With further mathematical manipulations, coordinating parameters of the proposed buyback contract, (w_{bi1}, b_{i1}) for $i = 1, 2$ satisfy the following equations,

$$\begin{cases} w_{b11} = F(S_{21})b_{11} + c_1 \\ w_{b21} = F(S_{31})b_{21} + c_2 \\ (h_1 + c_1)b_{11} + b_{21} \cdot \sum_{i=1}^2 (h_i + c_i) = (p + s + h_0) \sum_{i=1}^2 (h_i + c_i) \end{cases} \quad (4-93)$$

It can be further inferred from eq.(4-93) that, at coordination, it is optimal for suppliers to set their wholesale prices to be higher than their production costs. In addition, the last equation in eq.(4-93) can be rewritten as,

$$b_{21} = (p + s + h_0) - b_{11} \cdot \frac{(h_1 + c_1)}{\sum_{i=1}^2 (h_i + c_i)}$$

which shows that, at coordination, b_{21} and b_{11} are reversely related. Moreover, the slope of the equation depends on supplier 1's share of the total holding costs and the production costs of both suppliers, which consequently affects the extent of the relationship.

Proposition 4-4

The decentralized assembly system can be coordinated with the proposed two-parameter buyback contract. Specifically, the coordinating parameters satisfy eq.(4-93). ■

Proofs for Proposition 4-4 are presented in Appendix III.

The analysis shows that there exists a continuum of (w_{bi1}, b_{i1}) pairs that can lead to channel coordination and different pairs represent different allocation schemes of the system profit. Further, the implementation of the proposed contract requires that it is a common knowledge about each player's on-hand inventory levels. Although, each supplier negotiates independently with the assembler concerning profit allocation, the contract is implemented in a joint-decision-making context (since the condition of $Z_{11} > Z_{21} > Z_{31}$ is required for channel coordination).

4.4.2 Two-Period Problem with Buyback Contract

For the two-period setting, it is assumed, without loss of consistency, that $I_{12} = \min(I_{12}, I_{22})$. Moreover, it has been proved in the centralized two-period setting that, given the same assumption, it is global optimal to have supplier 1 being the critical supplier of period $t = 2$. Consequently, supplier 1's initial inventory level for period $t = 1$ satisfies $I_{11} = \min(I_{11}, I_{21})$. Set the results obtained in the centralized control setting as a benchmark, only supplier 1 is assumed to be the critical supplier of period $t = 2$ in the following analysis. Likewise, the inventory transfer functions for the two-period problem are,

$$\begin{aligned} I_{01} &= I_{02} + Q_{b02} - \xi_2 \\ I_{i1} &= I_{i2} + Q_{bi2} - Q_{b01} \quad (\text{for } i \neq 0) \end{aligned} \quad (4-94)$$

I. The Assembler's Two-Period Problem

Define $\tilde{L}(\varphi) = s(\xi - \varphi)^+ + (h_0 - b_{11} - b_{21})(\varphi - \xi)^+$, which represents the assembler's expected shortage costs and costs associated with excessive inventories in period $t = 1$. Further, the assembler's expected profit over the last two periods can be formulated as,

$$\begin{aligned}
 M_{b02}(I_{02}, Q_{b02}) &= E_{\xi_2} \{ \pi_{b02}(I_{02}, Q_{b02}) + J_{b01}(I_{01}) \} \\
 &= E_{\xi_2} \left\{ p \cdot \min(Q_{b02} + I_{02}, \xi_2) - Q_{b02} \cdot \left(\sum_{i=1}^2 w_{bi2} + c_0 \right) - s(\xi - Q_{b02} - I_{02})^+ \right. \\
 &\quad \left. - (h_0 - b_{12} - b_{22})(Q_{b02} + I_{02} - \xi)^+ + J_{b01}(I_{01}) \right\} \quad (4-95) \\
 \text{s.t.} \quad Q_{b02} &\leq \min(Q_{b12} + I_{12}, Q_{b22} + I_{22})
 \end{aligned}$$

Specifically, $J_{b01}(I_{01})$ represents the assembler's optimal expected profit in the last period,

$$\begin{aligned}
 &J_{b01}(I_{01}) \\
 &= \int_0^{I_{02} + Q_{b02} - S_{11}} \left\{ p \cdot \min(I_{02} + Q_{b02} - \xi_2, \xi_1) - \tilde{L}(I_{02} + Q_{b02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &\quad + \int_{I_{02} + Q_{b02} - S_{11}}^{I_{12} + Q_{b12} + I_{02} - S_{11}} \left\{ p \cdot \min(S_{11}, \xi_1) - \left(\sum_{i=1}^2 w_{bi1} + c_0 \right) (S_{11} - I_{02} - Q_{b02} + \xi_2) - \tilde{L}(S_{11}) \right\} f(\xi_2) d\xi_2 \\
 &\quad + \int_{I_{12} + Q_{b12} + I_{02} - S_{11}}^{I_{12} + Q_{b12} + I_{02} - S_{21}} \left\{ p \cdot \min(I_{12} + Q_{b12} + I_{02} - \xi_2, \xi_1) - \left(\sum_{i=1}^2 w_{bi1} + c_0 \right) (I_{12} + Q_{b12} - Q_{b02}) \right. \\
 &\quad \left. - \tilde{L}(I_{12} + Q_{b12} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &\quad + \int_{I_{12} + Q_{b12} + I_{02} - S_{21}}^{I_{22} + Q_{b22} + I_{02} - S_{21}} \left\{ p \cdot \min(S_{21}, \xi_1) - \left(\sum_{i=1}^2 w_{bi1} + c_0 \right) (S_{21} - I_{02} - Q_{b02} + \xi_2) - \tilde{L}(S_{21}) \right\} f(\xi_2) d\xi_2 \\
 &\quad + \int_{I_{22} + Q_{b22} + I_{02} - S_{21}}^{I_{22} + Q_{b22} + I_{02} - S_{31}} \left\{ p \cdot \min(I_{22} + Q_{b22} + I_{02} - \xi_2, \xi_1) - \left(\sum_{i=1}^2 w_{bi1} + c_0 \right) (I_{22} + Q_{b22} - Q_{b02}) \right. \\
 &\quad \left. - \tilde{L}(I_{22} + Q_{b22} + I_{02} - \xi_2) \right\} f(\xi_2) d\xi_2 \\
 &\quad + \int_{I_{22} + Q_{b22} + I_{02} - S_{31}}^{\infty} \left\{ p \cdot \min(S_{31}, \xi_1) - \left(\sum_{i=1}^2 w_{bi1} + c_0 \right) (S_{31} - I_{02} - Q_{b02} + \xi_2) - \tilde{L}(S_{31}) \right\} f(\xi_2) d\xi_2 \quad (4-96)
 \end{aligned}$$

each line of the expressions in eq.(4-96) corresponds to one of the scenarios in Lemma 4-2.

Moreover, the first-order and second-order partial derivatives with respect to Q_{b02} are,

$$\begin{aligned}
 \frac{\partial M_{b02}(I_{02}, Q_{b02})}{\partial Q_{b02}} &= \left[p + s + \sum_{i=1}^2 (w_{bi1} - w_{bi2}) \right] + \left(p + s - \sum_{i=1}^2 w_{bi1} - c_0 \right) \cdot F(I_{02} + Q_{b02} - S_{11}) \\
 &\quad - (p + s + h_0 - b_{11} - b_{21}) \int_0^{I_{02} + Q_{b02} - S_{11}} F(I_{02} + Q_{b02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-97) \\
 &\quad - (p + s + h_0 - b_{12} - b_{22}) \cdot F(Q_{b02} + I_{02})
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial^2 M_{b02}(I_{02}, Q_{b02})}{\partial Q_{b02}^2} &= -(p + s + h_0 - b_{11} - b_{21}) \int_0^{I_{02} + Q_{b02} - S_{11}} f(I_{02} + Q_{b02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-98) \\
 &\quad - (p + s + h_0 - b_{12} - b_{22}) f(Q_{b02} + I_{02})
 \end{aligned}$$

which is uncertain. However, in case of a markdown in wholesale prices in the last period, there exists a zero-point, denoted as Z_{12} , to the first-order condition. Moreover, it is optimal

for the assembler to place order when its on-hand inventory level is lower than Z_{12} .

II. Supplier 1's Two-Period Problem

For the sake of representational simplicity, denote $V(\varphi) = (\xi - \varphi)^+ = \int_0^\varphi F(\xi) d\xi$. Supplier

1's optimal expected profit for the last period, $J_{b11}(I_{01}, I_{11})$, can be modeled as,

$$\begin{aligned}
 & J_{b11}(I_{01}, I_{11}) \\
 &= \int_0^{I_{02}+Q_{b02}-S_{11}} \left\{ -h_1(I_{12}+Q_{b12}-Q_{b02}) - b_{11} \cdot V(I_{02}+Q_{b02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{02}+Q_{b02}-S_{11}}^{I_{12}+Q_{b12}+I_{02}-S_{11}} \left\{ w_{b11}(S_{11}-I_{02}-Q_{b02}+\xi_2) \right. \\
 &\quad \left. - h_1(I_{12}+Q_{b12}+I_{02}-\xi_2-S_{11}) - b_{11} \cdot V(S_{11}) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{12}+Q_{b12}+I_{02}-S_{11}}^{I_{12}+Q_{b12}+I_{02}-S_{21}} \left\{ w_{b11}(I_{12}+Q_{b12}-Q_{b02}) - b_{11} \cdot V(I_{12}+Q_{b12}+I_{02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{12}+Q_{b12}+I_{02}-S_{21}}^{I_{22}+Q_{b22}+I_{02}-S_{21}} \left\{ w_{b11}(S_{21}-I_{02}-Q_{b02}+\xi_2) \right. \\
 &\quad \left. - c_1(S_{21}-I_{12}-Q_{b12}-I_{02}+\xi_2) - b_{11} \cdot V(S_{21}) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{12}+Q_{b12}+I_{02}-S_{21}}^{I_{22}+Q_{b22}+I_{02}-S_{31}} \left\{ w_{b11}(I_{22}+Q_{b22}-Q_{b02}) - c_1(I_{22}+Q_{b22}-I_{12}-Q_{b12}) \right. \\
 &\quad \left. - b_{11} \cdot V(I_{22}+Q_{b22}+I_{02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\
 &+ \int_{I_{22}+Q_{b22}+I_{02}-S_{31}}^{\infty} \left\{ w_{b11}(S_{31}-I_{02}-Q_{b02}+\xi_2) \right. \\
 &\quad \left. - c_1(S_{31}-I_{12}-Q_{b12}-I_{02}+\xi_2) - b_{11} \cdot V(S_{31}) \right\} f(\xi_2) d\xi_2 \quad (4-99)
 \end{aligned}$$

Further, its total expected profit over the last two periods is,

$$\begin{aligned}
 & M_{b12}(w_{b12}, I_{12}) = E_{\xi_2} \left\{ \pi_{b12}(w_{b12}, I_{12}) + J_{b11}(I_{01}, I_{11}) \right\} \\
 &= E_{\xi_2} \left\{ w_{b12} \cdot Q_{b02} - c_1 \cdot Q_{b12} - h_1(I_{12}+Q_{b12}-Q_{b02}) - b_{12}(Q_{b02}+I_{02}-\xi_2)^+ + J_{b11}(I_{01}, I_{11}) \right\} \quad (4-100)
 \end{aligned}$$

Hence, the first order partial derivatives of $M_{b12}(w_{b12}, I_{12})$ with respect to Q_{b02} are,

$$\begin{aligned}
 \frac{\partial M_{b12}(w_{b12}, I_{12})}{\partial Q_{b02}} &= (w_{b12} + h_1 - w_{b11}) - b_{12}F(Q_{b02} + I_{02}) \\
 &\quad + (h_1 + w_{b11})F(I_{02} + Q_{b02} - S_{11}) \\
 &\quad - b_{11} \int_0^{I_{02}+Q_{b02}-S_{11}} F(I_{02} + Q_{b02} - \xi_2) f(\xi_2) d\xi_2 \quad (4-101)
 \end{aligned}$$

Further, the first-order partial derivatives of $M_{b12}(w_{b12}, I_{12})$ with respect to Q_{b12} are,

$$\begin{aligned} \frac{\partial M_{b12}(w_{b12}, I_{12})}{\partial Q_{b12}} = & -b_{11} \int_{I_{12}+Q_{b12}+I_{02}-S_{11}}^{I_{12}+Q_{b12}+I_{02}-S_{21}} F(I_{12}+Q_{b12}+I_{02}-\xi_2) f(\xi_2) d\xi_2 \\ & - (w_{b11} + h_1) F(I_{12}+Q_{b12}+I_{02}-S_{11}) - h_1 \\ & + (w_{b11} - c_1) F(I_{12}+Q_{b12}+I_{02}-S_{21}) \end{aligned} \quad (4-102)$$

By analogy, denote Z_{22} (corresponding to Z_{21} in the last-period problem) as the optimal order-up-to level at which supplier 1's expected profit is maximized and Z_{22} satisfies,

$$\begin{aligned} -b_{12} \cdot F(Z_{22}) + (w_{b11} - c_1) F(Z_{22} - S_{21}) + (w_{b12} - w_{b11}) \\ - b_{11} \int_0^{Z_{22}-S_{21}} F(Z_{22}-\xi_2) f(\xi_2) d\xi_2 = 0 \end{aligned} \quad (4-103)$$

Take the first-order derivative to the left-hand side of eq.(4-103) with respect to Z_{22} ,

$$-b_{12} \cdot f(Z_{22}) - b_{11} \cdot \int_0^{Z_{22}-S_{21}} f(Z_{22}-\xi_2) f(\xi_2) d\xi_2 \quad (4-104)$$

which is negative. Hence, it implies that the left-hand side of eq.(4-103) is monotonous and there is at most one zero-point satisfying eq.(4-103).

III. Supplier 2's Two-Period Problem

Supplier 2's optimal expected profit for the last period can be formulated as follows,

$$\begin{aligned} & J_{b21}(I_{21}) \\ = & \int_0^{I_{02}+Q_{b02}-S_{11}} \left\{ -h_2(I_{22}+Q_{b22}-Q_{b02}) - b_{21} \cdot V(I_{02}+Q_{b02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\ & + \int_{I_{02}+Q_{b02}-S_{11}}^{I_{12}+Q_{b12}+I_{02}-S_{11}} \left\{ w_{b21}(S_{11}-I_{02}-Q_{b02}+\xi_2) \right. \\ & \quad \left. - h_2(I_{22}+Q_{b22}+I_{02}-\xi_2-S_{11}) - b_{21} \cdot V(S_{11}) \right\} f(\xi_2) d\xi_2 \\ & + \int_{I_{12}+Q_{b12}+I_{02}-S_{11}}^{I_{12}+Q_{b12}+I_{02}-S_{21}} \left\{ w_{b21}(I_{12}+Q_{b12}-Q_{b02}) - h_2(I_{22}+Q_{b22}-I_{12}-Q_{b12}) \right. \\ & \quad \left. - b_{21} \cdot V(I_{12}+Q_{b12}+I_{02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\ & + \int_{I_{12}+Q_{b12}+I_{02}-S_{21}}^{I_{22}+Q_{b22}+I_{02}-S_{21}} \left\{ w_{b21}(S_{21}-I_{02}-Q_{b02}+\xi_2) \right. \\ & \quad \left. - h_2(I_{22}+Q_{b22}+I_{02}-\xi_2-S_{21}) - b_{21} \cdot V(S_{21}) \right\} f(\xi_2) d\xi_2 \\ & + \int_{I_{22}+Q_{b22}+I_{02}-S_{21}}^{I_{22}+Q_{b22}+I_{02}-S_{31}} \left\{ w_{b21}(I_{22}+Q_{b22}-Q_{b02}) - b_{21} \cdot V(I_{22}+Q_{b22}+I_{02}-\xi_2) \right\} f(\xi_2) d\xi_2 \\ & + \int_{I_{22}+Q_{b22}+I_{02}-S_{31}}^{\infty} \left\{ w_{b21}(S_{31}-I_{02}-Q_{b02}+\xi_2) \right. \\ & \quad \left. - c_2(S_{31}-I_{22}-Q_{b22}-I_{02}+\xi_2) - b_{21} \cdot V(S_{31}) \right\} f(\xi_2) d\xi_2 \end{aligned} \quad (4-105)$$

Further, supplier 2's expected profits over the last two periods is modeled as,

$$\begin{aligned} M_{b22}(w_{b22}, I_{22}) &= \pi_{b22}(w_{b22}, I_{22}) + J_{b21}(I_{21}) \\ &= w_{b22} \cdot Q_{b02} - c_2 \cdot Q_{b22} - h_2(I_{22} + Q_{b22} - Q_{b02}) - b_{22}(Q_{b02} + I_{02} - \xi_2)^+ + J_{b21}(I_{11}) \end{aligned} \quad (4-106)$$

The first-order partial derivatives of $M_{b22}(w_{b22}, I_{22})$ with respect to Q_{b02} and Q_{b22} are,

$$\begin{aligned} \frac{\partial M_{b22}(w_{b22}, I_{22})}{\partial Q_{b02}} &= (w_{b22} + h_2 - w_{b21}) - b_{22}F(Q_{b02} + I_{02}) \\ &\quad + (w_{b21} + h_2)F(I_{02} + Q_{b02} - S_{11}) \\ &\quad - b_{21} \int_0^{I_{02} + Q_{b02} - S_{11}} F(I_{02} + Q_{b02} - \xi_2) f(\xi_2) d\xi_2 \end{aligned} \quad (4-107)$$

and

$$\begin{aligned} \frac{\partial M_{b22}(w_{b22}, I_{22})}{\partial Q_{b22}} &= -(h_2 + w_{b21})F(I_{22} + Q_{b22} + I_{02} - S_{21}) - h_2 \\ &\quad + (w_{b21} - c_2)F(I_{22} + Q_{b22} + I_{02} - S_{31}) \\ &\quad - b_{21} \int_{I_{22} + Q_{b22} + I_{02} - S_{21}}^{I_{22} + Q_{b22} + I_{02} - S_{31}} F(I_{22} + Q_{b22} + I_{02} - \xi_2) f(\xi_2) d\xi_2 \end{aligned} \quad (4-108)$$

It is deduced by analogy that there exists an optimal order-up-to level at which supplier 2's expected profit over the last two periods could be maximized. Denote the order up to level as Z_{32} which corresponds to Z_{31} in the last period. Specifically, Z_{32} satisfies,

$$\begin{aligned} &(w_{b22} - w_{b21}) - b_{22}F(Z_{32}) + (w_{b21} + h_2)F(Z_{32} - S_{11}) \\ &\quad + (w_{b21} - c_2)F(Z_{32} - S_{31}) - (h_2 + w_{b21})F(Z_{32} - S_{21}) \\ &\quad - b_{21} \int_0^{Z_{32} - S_{11}} F(Z_{32} - \xi_2) f(\xi_2) d\xi_2 - b_{21} \int_{Z_{32} - S_{21}}^{Z_{32} - S_{31}} F(Z_{32} - \xi_2) f(\xi_2) d\xi_2 = 0 \end{aligned} \quad (4-109)$$

Further, take first-order derivative to the left-hand side of eq.(4-109) with respect to Z_{32} ,

$$\begin{aligned} &-b_{22} \cdot f(Z_{32}) + [(w_{b21} + h_2) - b_{21} \cdot F(S_{11})] f(Z_{32} - S_{11}) \\ &\quad + [b_{21}F(S_{21}) - (h_2 + w_{b21})] f(Z_{32} - S_{21}) - b_{21} \int_{Z_{32} - S_{21}}^{Z_{32} - S_{31}} f(Z_{32} - \xi_2) f(\xi_2) d\xi_2 \\ &\quad - b_{21} \cdot \int_0^{Z_{32} - S_{11}} f(Z_{32} - \xi_2) f(\xi_2) d\xi_2 \end{aligned} \quad (4-110)$$

It is further derived that if parameters could satisfy $F(S_{21}) < \frac{w_{b21} + h_2}{b_{21}} < F(S_{11})$, eq.(4-110)

is negative; hence, there is, at most, one zero-point satisfying eq.(4-109).

IV. Channel Coordination for the Two-Period Problem

To coordinate the decentralized system, it is necessary to have,

$$\begin{cases} Z_{12} = S_{12} \\ Z_{22} = S_{22} \\ Z_{32} = S_{32} \end{cases} \quad (4-111)$$

Therefore, supplier 1's optimal production policy for period $t = 2$, under the assumptions of $I_{12} \leq I_{22}$ and $I_{11} \leq I_{21}$, can be described as,

$$Q_{b12}^* = \begin{cases} 0 & \text{if } I_{02} \geq Z_{12} \\ 0 & \text{if } I_{02} < Z_{12} \text{ and } I_{12} \geq Z_{22} - I_{02} \\ \min(Z_{22} - I_{02}, I_{22}) - I_{12} & \text{if } Z_{32} < I_{12} + I_{02} < Z_{22} \\ Z_{32} - I_{02} - I_{12} & \text{if } I_{22} + I_{02} < Z_{32} \end{cases} \quad (4-112)$$

Moreover, supplier 2's optimal production policy can be expressed as,

$$Q_{b22}^* = \begin{cases} 0 & \text{if } I_{02} \geq Z_{12} \\ 0 & \text{if } I_{02} < Z_{12} \text{ and } I_{22} \geq Z_{32} - I_{02} \\ Z_{32} - I_{02} - I_{22} & \text{if } I_{02} < Z_{12} \text{ and } I_{22} < Z_{32} - I_{02} \end{cases} \quad (4-113)$$

Since the assembler's optimal purchasing quantity is subject to the quantity delivered by the critical supplier, its optimal policy can be described as,

$$Q_{b02}^* = \begin{cases} 0 & \text{if } I_{02} \geq Z_{12} \\ \min(Z_{12} - I_{02}, I_{12}) & \text{if } I_{02} < Z_{12} \text{ and } I_{12} \geq Z_{22} - I_{02} \\ \min(Z_{22} - I_{02}, I_{22}) & \text{if } Z_{32} \leq I_{12} + I_{02} < Z_{22} \\ Z_{32} - I_{02} - I_{12} & \text{if } I_{22} + I_{02} \geq Z_{32} \end{cases} \quad (4-114)$$

Based on the optimal policies for each player, the optimal production/purchasing policy for the entire system can be summarized by the following lemma.

Lemma 4-3

Suppose Z_{12}, Z_{22}, Z_{32} are three optimal order-up-to levels satisfying $Z_{32} < Z_{22} < Z_{12}$. Further, it is assumed that $I_{12} \leq I_{22}$. Hence, the optimal production/purchasing policy for the entire

system with the proposed buyback contract can be described as,

1. If $I_{02} \geq Z_{12}$, then $Q_{b02}^* = Q_{b12}^* = Q_{b22}^* = 0$.
2. If $Z_{12} - I_{02} < I_{12} < I_{22}$, then $Q_{b02}^* = Z_{12} - I_{02}, Q_{b12}^* = Q_{b22}^* = 0$.
3. If $\begin{cases} Z_{12} > I_{12} + I_{02} \\ Z_{22} \leq I_{12} + I_{02} \end{cases}$, then $Q_{b02}^* = I_{12}, Q_{b12}^* = Q_{b22}^* = 0$.
4. If $\begin{cases} Z_{12} > I_{12} + I_{02} \\ I_{12} + I_{02} < Z_{22} < I_{22} + I_{02} \end{cases}$, then $Q_{b02}^* = Z_{22} - I_{02}, Q_{b12}^* = Z_{22} - I_{02} - I_{12}, Q_{b22}^* = 0$.
5. If $\begin{cases} I_{12} < I_{22} < Z_{12} - I_{02} \\ Z_{32} < I_{22} + I_{02} < Z_{22} \end{cases}$, then $Q_{b02}^* = I_{22}, Q_{b12}^* = I_{22} - I_{12}, Q_{b22}^* = 0$.
6. If $Z_{32} > I_{22} + I_{02}$, $Q_{b02}^* = Z_{32} - I_{02}, Q_{b12}^* = Z_{32} - I_{02} - I_{12}, Q_{b22}^* = Z_{32} - I_{02} - I_{22}$. ■

Hence, if eq.(4-111) is satisfied, it can be concluded that channel coordination could be attained with the proposed buyback contract.

Proposition 4-5

The proposed buyback contract can be applied to coordinate the decentralized assembly system in a two-period setting with the assumption of $I_{12} \leq I_{22}$. ■

Proofs for Proposition 4-5 are presented in Appendix III.

With channel coordination, the coordinating parameters can be determined by replacing Z_{i2} with S_{i2} (for $i = 1, 2, 3$) in eqs.(4-97), (4-103) and (4-109) respectively. Further, it is noted that there is a continuum of parameter pairs that satisfy the above-mentioned three equations and each pair of which represent a different system profit allocation schemes.

It is noted from the analysis above that, expressions for the order-up-to levels in a two-period

setting are generally complex (unlike those for the last-period model) due to the increase in the time periods considered. Values of these order-up-to levels for each control mode can be derived through the numerical methods applied in the next chapter, which will help to further highlight the applicability of the proposed two-period models as well.

4.5 Discussions

A few of the assumptions made in developing the model are discussed here as it is easier to refer to the results of the model in the discussion being presented below.

1. Only two component suppliers are considered for the modeling

As discussed in the literature review, most of the supply contracts on channel coordination are formulated in a newsvendor setting due to the increased complexities when considering a longer time horizon. It can be seen that the development of a two-supplier, two-period contract model is quite complex since derivation of the analytical solutions requires comparing suppliers' initial inventory levels of both periods in order to determine the critical supplier of each period.

When two suppliers are assumed, their initial inventory levels for the last period only have two possible combinations, that is $I_{11} \leq I_{21}$ and $I_{11} > I_{21}$; however, when three suppliers are considered, their initial inventory levels of the last period have six possible combinations. Rolling back from the last period to the first period, even when supplier 1 is assumed to have less initial on-hand inventory, assumptions concerning supplier 2 and 3's initial on-hand inventory of the period have to be made. Moreover, derivation of the optimal solution for the first period requires consideration of all the possible scenarios of the last period. Therefore, as the number of suppliers increases, the complexity of the problem would increase in terms of analytical derivation. However, the problem nature and the concepts such as critical supplier

and on-hand inventory would remain the same.

2. System costs are assumed to be stationary and quantity-dependent

As have been reviewed in Chapter 2 that most of the research on multi-echelon inventory management treats environmental factors, such as production cost, shortage cost and holding cost, as external variables. However, supply contract models treat factors relating to relationship management as decision variables, which cannot be easily modeled due to the difficulty in analyzing players' conflicting incentives. Hence, multi-echelon inventory models are usually more complex than supply contract models by considering factors such as lead time, non-stationary costs parameters, and finite/infinite horizon.

In this thesis, assumptions of stationary and quantity dependent item costs are made. However, wholesale price and buyback price are assumed to be non-stationary since they are treated as decision variables, thus are negotiable. Similar assumptions are also made in researches such as Bassok and Anupindi (1997), Tsay and Lovejoy (1999), Anupindi and Bassok (1999), Urban (2000) and Zhao and Wang (2002). Moreover, much of the attention of this research is paid to contract design and the determination of coordinating parameters; hence, when non-stationary (system) cost parameters are assumed, the problem nature is not changed except that more intensive mathematical manipulations are required. Therefore, determined by research focus, only stationary and proportional cost parameters are assumed.

3. Players in the decentralized assembly system are assumed to be risk neutral

It is assumed in the thesis that all the players in the decentralized assembly system are rational as they generally take risk-neutral positions and this assumption is also consistent with the existing literature on supply contract modeling. With this assumption, the objective

of the entire SC would be to maximize system-wide expected profits; hence, it is natural to define a SC to be coordinated if the system-wide expected profits is maximized and each rational player's reservation profit is met.

However, when players' reaction toward risk is taken into account, many difficulties are brought about and are briefly discussed below:

- a). When one or more players in the supply chain are risk averse, the objective function of the entire supply chain would be difficult to formulate (Tsay et al., 1999; Gan et al., 2004).
- b). Different players might have different reactions towards risk in a supply chain. Moreover, even when players have the same reaction towards risk, their preference levels could be different (Agrawal and Seshadri, 2000).

Despite the difficulties, supply contracts may be a tool to reconcile players' different preferences towards risk. Agrawal and Seshadri (2000) design a contract that increases a two-echelon distribution channel's order quantity to the optimal level in the risk-neutral case by having the risk-neutral agent assume all the risk (i.e., risk sharing). Gan et al. (2004) consider risk averse by defining coordination as "Pareto-optimal" joint action according to the concepts used in the group decision theory. Since the problem nature would be changed when incorporating players' reaction toward risks, only rational (namely, risk-neutral) players are assumed for modeling.

4. Each supplier supplies only one type of component for the final assembly

It is assumed that there are only two suppliers with each supplying a unique component for the final assembly. This corresponds to the real cases where suppliers, despite their

capabilities in producing various products, only supply one particular product for the final assembly of a system. However, this assumption could also be relaxed and the following two scenarios have to be considered: 1) components supplied by each supplier are unique and; 2) components supplied by each supplier are not unique.

For the first scenario, the problem nature studied in this thesis is not changed and can be easily extended. For the second scenario, the problem nature is changed and becomes what is referred to as “dual sourcing”. This relates to the sourcing strategy that the assembler takes. Dual sourcing has been used in practice to guard against uncertainties, such as supplier’s product yield loss (Elmaghraby, 2000; Gurnani et al., 2000), lead time uncertainty (Ramasesh et al., 1991), in the supply process. As the focus of this research is on supply contracts for incentive alignment, this scenario is not incorporated in this thesis.

4.6 Summary

Two-period models representing two consecutive ordering opportunities are proposed for an assembly system operated with centralized control, price-only decentralized control and coordinated decentralized control with buyback contract, respectively. For each control mode, optimal solutions are developed. The system performance, measured by the assembler’s purchasing quantity and the total expected system profits over the periods, obtained for the centralized control setting is used as a benchmark and the performances of the other two models are examined vis-à-vis the benchmark.

Although, only two-period models are formulated, the developed models are capable of capturing most of the characteristics of a general multi-period setting as the impacts of leftover inventory from previous periods, expected future demand as well as the changes in

players' cost parameters (e.g., the wholesale prices) on present decisions are studied. As a summary, analytical processes of the formulated models with different control modes are presented by the following three flowcharts.

As shown by Figure 4-1, analysis to the centralized control model starts from the last period ($t = 1$). Either of the suppliers can have less on-hand inventory; however, for analysis purpose, supplier 1 is assumed to have less on-hand inventory at the beginning of the last period. The optimal policy is derived from the perspective of global optimization and the analytical results are presented in Proposition 4-1. To develop optimal solutions for the two-period setting, it is necessary to examine both conditions of $I_{11} \leq I_{21}$ and $I_{11} > I_{21}$ and to obtain the optimal solutions for each of the conditions. However, the analytical deduction for $I_{11} > I_{21}$ is not presented as they can be derived by analogy as shown in Scenario II of Section 4.2.2.

Separated by a dotted line, analysis to the two-period model starts with an assumption of $I_{12} \leq I_{22}$. Since it is still hard to tell who is the critical supplier of period at $t = 2$, two scenarios are considered. In Scenario I, supplier 1 is assumed to be the critical supplier and in Scenario II, supplier 2 is assumed to be the critical supplier. Modeled with dynamic programming, the solutions derived in each scenario are then tested for their feasibility to see if they conflict with the assumption on the critical supplier for period $t = 2$. The results show that since supplier 1 is assumed to have less on-hand inventory at the beginning of period $t = 2$, only the solutions derived in Scenario I are feasible (the solutions do not violate the assumption that supplier 1 has less on-hand inventory at the beginning of period $t = 1$); hence are the global optimal solutions for the two-period centralized control model.

Figure 4-1 Flowchart of the analytical process for centralized control mode

Flowchart for the analytical process of the price-only decentralized setting is presented in Figure 4-2. As before, the analysis starts from the last period and uses the same assumption on suppliers' initial inventory levels of period $t = 1$. Unlike the centralized control setting, each player is an individual decision maker in a decentralized control setting. Hence, every player's optimal policy in the last period is derived accordingly. However, the assembler's

optimization problem for the last period could be a constrained (when $y_{del} < y_{d01}$) or a non-constrained one (when $y_{del} \geq y_{d01}$) depending on the relationship between y_{d01} and y_{del} , which finally affects both the assembler's and the suppliers' optimal policies of the last period. Analogously, the derived optimal policies for the last period are used in the two-period model to calculate the optimal expected profit of the last period.

For the two-period problem, the assumption in the centralized setting on suppliers' initial inventory level at period $t = 2$ is considered for consistency. Since suppliers' behavior could be so strategic in the price-only decentralized setting that either supplier might be the critical supplier of period $t = 2$. In addition, analyses should also consider and differentiate the assembler's last-period problem by treating it as either a constrained or an unconstrained optimization problem. Four scenarios are considered and every player's best response is derived for each scenario. Non-stationary wholesale prices are considered to represent price markdown or markup.

The analysis shows that changes in the wholesale prices over the periods have great impact on the suppliers' optimal production/delivery decisions. The derived solutions are further examined for their feasibility. Scenario I and III are considered together since they share the same optimal policy for the last period. For solutions derived in these two scenarios, if they do not conflict with the assumption of $I_{11} = \min(I_{11}, I_{21})$, they are the optimal solutions for Scenario I; otherwise, they are feasible solutions for Scenario III. Moreover, Scenario II and IV are analyzed separately since they have different optimal policy for period $t = 1$. For Scenario II, if the derived solutions do not violate the assumption of supplier 1 being the critical supplier of period $t = 2$, they are the optimal solutions; otherwise, they are infeasible solutions. Similarly, for Scenario IV, only the solutions that are compatible with the

assumption of supplier 2 being the critical supplier of period $t = 2$ are feasible.

Figure 4-2 Flowchart of the analytical process for price-only decentralized control mode

Figure 4-3 Flowchart of the analytical process for channel coordination with buyback

Analytical process for channel coordination with buyback contract is shown by Figure 4-3. For consistency, the same assumptions on suppliers' initial inventory levels as those in the two-period centralized setting are used as well. Starting from the last period, each player's

optimal policy with buyback contract is discussed. The analysis shows that if the three critical order-up-to levels derived in this setting equal to their corresponding order-up-to levels in the centralized setting, that is if $Z_{j1} = S_{j1}$ for $j = 1, 2, 3$, the decentralized channel can be coordinated with the proposed buyback contract for the last period. Aimed at channel coordination, only the scenario of supplier 1 being the critical supplier of period $t = 2$ is considered (since with the assumption of $I_{12} = \min(I_{12}, I_{22})$, it has been proved in the centralized setting that it is global optimal for supplier 1 to be the critical supplier of period $t = 2$). Each player's best response in period $t = 2$ is analyzed. The results indicate that, if $Z_{j2} = S_{j2}$ for $j = 1, 2, 3$, the decentralized system in a two-period setting can be coordinated with buyback contract as well. In addition, there exists a continuum of coordinating parameter pairs that can realize channel coordination and each pair of which represents a different allocation scheme of system profit.

From the above, it can be seen that various scenarios can occur in real life contract situations and they can be feasible or infeasible based on the assumption on suppliers' initial inventory levels at period $t = 2$ as well as on the way how wholesale prices change over the periods. If the scenario is feasible, it can be further analyzed for its optimality. An optimal model can then be compared with the centralized setting to validate if the solutions are correct. Two performance measurement criteria are applied: the assembler's purchasing quantity and the total expected system profit. If the proposed contract can generate the same purchasing quantity by the assembler as well as the same total expected system profit over the two periods, it can be concluded that the proposed contract model is capable of realizing channel coordination. In addition, the numerical study presented in Chapter 5 highlights the applicability of the contract models developed in this chapter.

Chapter 5

Numerical Study

A numerical study is carried out in this chapter to highlight the applicability and to validate the proposed two-period contract models. Specifically, the numerical study will be done to the last-period and the two-period problem, respectively.

Contract models for the last-period are examined first by considering different control settings. For the centralized control setting, a simulation model is developed to derive the optimal numerical solutions (through numerical methods such as enumeration), which maximize the objective function (as mentioned earlier, the objective function is, in essence, a newsvendor model; hence, there always exists an optimal solution). The simulation model is verified by first running the program for a small scaled problem (which is not presented in this thesis). With the verified simulation model, the derived numerical results are then compared with the analytical solutions, which are derived according to Proposition 4-1. If the two solutions are identical, the centralized control model is considered as validated. In addition, eight scenarios of initial inventory levels are assumed and each of which corresponds to one of the scenarios of Proposition 4-1.

For the decentralized control setting, the price-only contract model is examined by assuming wholesale prices between the suppliers and the assembler and by taking the same initial inventory levels as assumed for the centralized control setting. Since, in this case, distributed decision making is considered and each player has its own incentive to maximize its own profits, the decentralized assembly system with price-only contract alone is expected to

produce inferior performance. Moreover, for decentralized control (with price-only contract), the system performance is determined by deriving each player's best response, which includes the suppliers' production quantities and the assembler's purchasing quantity. Benchmarked against the system performance of the corresponding centralized control setting, a reduction in the assembler's purchasing quantity and a decrease in the total expected system profits can be observed. This would validate the conclusion that the decentralized assembly system cannot be coordinated with price-only contract alone. In addition, the impacts of wholesale prices are examined by varying their values.

For channel coordination with buyback contract, the same level of efficiency as that of the centralized control setting is achieved, which validates the proposed contract models. Specifically, attention is paid to the determination of the coordinating parameters. The numerical study shows that various coordinating parameter pairs can realize channel coordination, which represents different profit allocation schemes. In addition, the impacts of other cost parameters on system performance in a coordinated setting are examined.

Numerical study for the two-period models is done in a similar way as that for the single-period models. To validate the analytical solution structures in Proposition 4-2, eight different scenarios of initial inventory levels are assumed. Wholesale prices over the two periods are assumed for the decentralized control models. With the given assumptions, the optimal order-up-to level (without constraint) and the equilibrium order-up-to level of the last period, y_{d01} and y_{de1} , are derived. The relationship between the two values determines the characteristics of the last-period problem, thereby players' optimal behavior of the first period. Similarly, for channel coordination with buyback contract, determination of the coordinating parameters is done by taking the centralized control setting as a reference.

The proposed models can be used in assembly systems operated with VMI, which is a commonly observed practice in real life. With VMI, players take cooperative attitude and are willing to develop a long-term relationship. In this case, suppliers are shared with market demand information and are responsible for the delivery of different components (in an assembly system).

To adapt to the formulated models, two component suppliers ($S1$ and $S2$) and an assembler (A) are considered for the numerical study. Market demand is assumed to be exponentially distributed with a mean (per period) of 1000. Initial conditions for the numerical study are listed in Table 5-1.

Table 5-1 Initial conditions for the two-period problem

Entity	Initial Conditions				
	c_i	p	s	h_i	λ
$S1$	0.6	---	---	0.1	1000
$S2$	0.5	---	---	0.08	
A	1.0	8	1.2	0.12	

5.1 The Last-Period Problem

It is assumed, without loss of generality, that supplier 1 has less on-hand inventory at the beginning of the last period.

I. The Centralized Control Setting

For the given initial conditions in Table 5-1, the optimal order-up-to levels, $S_{it} \forall i$, are developed according to eqs.(4-3) to (4-5), respectively. Based on the developed S_{it} values, eight groups of initial inventory levels are then assumed and each corresponds to one of the scenarios in Proposition 4-1. For each scenario, the optimal numerical results (as shown in Table 5-2) for Q_{c11}^* , Q_{c21}^* , Q_{c01}^* and π_{c1}^* are calculated through enumeration. For validation,

the optimal numerical solutions are compared with the corresponding optimal analytical solutions, which are derived, with the same assumption on initial inventory level, according to Proposition 4-1. The comparison shows that the two solutions are identical. This validates the proposed analytical solution structures in Proposition 4-1.

Table 5-2 Results for the last-period centralized control setting

Scenario	Initial Inventory Levels			Optimal Quantities			π_{cl}^*
	I_{1l}	I_{2l}	I_{0l}	Q_{c1l}^*	Q_{c2l}^*	Q_{c0l}^*	
1	200	250	2350	0	0	0	6909.2
2	2050	2120	300	0	0	1994	4895.0
3	1500	2050	275	0	0	1500	4783.4
4	800	2050	275	662	0	1462	4364.6
5	800	1300	500	437	0	1237	4766.6
6	1300	1350	500	0	0	1300	5059.4
7	600	1100	500	500	0	1100	4646.3
8	600	800	500	335	135	935	4525.1
S values	S_{1l}	2294	S_{2l}	1737	S_{3l}	1435	

The numerical results for the centralized control setting (presented in Table 5-2) will be used as a benchmark to evaluate the subsequent results in the price-only decentralized setting as well as the coordinating effect of the buyback contract model.

II. The Price-Only Decentralized Contract Setting

For comparison, the same initial on-hand inventory levels as that in the corresponding centralized control setting are assumed. Detailed numerical results are presented in Table 5-3. It has been analyzed in Section 4.3.1 that mismatch of components can be eliminated, when both suppliers choose to produce extra components, by adjusting the wholesale prices simultaneously such that both suppliers would produce up to $y_{del} - I_{0l}$. With this contract, wholesale price not only represents the basic relationship between the suppliers and the assembler, but acts as a coordinating mechanism for elimination of mismatch of components.

Let w_{d11} (the wholesale price charged by supplier 1 to the assembler) be 1.4. Then, from eq.(4-38), $w_{d21}=1.16$ and $y_{de1}=762$. Similarly, from eq.(4-27), the assembler's optimal order-up-to level without constraint $y_{d01}=928$.

Table 5-3 Results for the last-period price-only decentralized control setting

Scenario	Initial Inventory Levels			Optimal Quantities			Expected Profits			
	I_{11}	I_{21}	I_{01}	Q_{d11}^*	Q_{d21}^*	Q_{d01}^*	π_{d11}	π_{d21}	π_{d01}	$\sum \pi_{di1}$
1	200	250	2350	0	0	0	-20	-20	6949.2	6909.2
2	2050	2120	300	0	0	628	737	610.9	2086.6	3434.5
3	1500	2050	275	0	0	653	829.5	647.6	1997.5	3474.6
4	800	2050	275	0	0	653	899.5	647.6	1997.5	3544.6
5	800	1300	500	0	0	428	562	427.9	2799.1	3789.1
6	1300	1350	500	0	0	428	512	423.9	2799.1	3735.1
7	600	1100	500	0	0	428	582	443.9	2799.1	3825.1
8	600	800	500	0	0	428	582	467.9	2799.1	3849.1

As $y_{d01} > y_{de1}$ with the given initial condition; the assembler's last-period optimization problem is a constrained one. Therefore, the suppliers' optimal production/delivery policies and the assembler's optimal purchasing policy can be obtained according to the optimal policy summarized in Section 4.3.1. In addition, for each group, the assembler's expected profit is derived according to eq.(4-25) and the suppliers' expected profits are derived according to eq.(4-29). In Table 5-3, $\sum \pi_{di1}$ is the total expected profit for the entire system, which is derived by summing up all the players' expected profits.

Comparing Table 5-3 with Table 5-2, it is noted that $\sum \pi_{di1} < \pi_{c1}^*$ holds for each scenario except for the first scenario. For the first scenario, the assembler, with the given initial inventory level, does not need to place any orders in both control settings. The assembler makes profit by selling its on-hand inventory. As to other scenarios, it is noted that the number of components purchased by the assembler for the price-only decentralized setting is fewer than that of the corresponding centralized setting, so is the total expected system profit.

Therefore, the numerical results comply with the conclusion that the system performance in the last-period price-only decentralized setting is no better than that of the centralized setting. Based on this observation, it can be concluded that the decentralized assembly system cannot be coordinated with a price-only contract alone.

Since the values of the wholesale prices determine y_{d01} and y_{de1} , thereby the characteristics of the last-period problem, an examination of the impacts of various wholesale prices on the order-up-to levels, the optimal production/purchasing quantities and each player's expected profits is conducted. Initial inventory level corresponding to the eighth scenario of Table 5-3 is used. The values of w_{d11} are changed from 1.0 to 2.0 at an incremental step of 0.1. Detailed numerical results are presented in Table 5-4.

Table 5-4 The impact of w_{d11} on system performance in a price-only decentralized setting

Initial inventory		$I_{11} = 600, I_{21} = 800, I_{01} = 500$									
No.	w_{d11}	w_{d21}	y_{de1}	y_{d01}	Optimal Quantities			Expected Profits			
					Q_{d11}^*	Q_{d21}^*	Q_{d01}^*	π_{d11}	π_{d21}	π_{d01}	$\sum \pi_{di1}$
1	1.0	0.83	452	1150	0	0	600	600.0	482.9	3186.8	4269.6
2	1.1	0.91	539	1090	0	0	590	648.0	522.6	3077.2	4247.9
3	1.2	1.00	619	1033	0	0	533	632.9	510.1	2974.6	4117.6
4	1.3	1.08	693	979	0	0	479	610.6	491.6	2882.1	3984.3
5	1.4	1.16	762	928	0	0	428	582.0	467.9	2799.1	3849.1
6	1.5	1.25	827	880	0	0	380	548.0	439.8	2725.3	3713.0
7	1.6	1.33	887	834	0	0	334	507.8	406.5	2660.0	3574.3
8	1.7	1.41	944	790	0	0	290	462.0	368.5	2603.1	3433.6
9	1.8	1.49	999	747	0	0	247	409.3	324.8	2554.0	3288.1
10	1.9	1.58	1050	707	0	0	207	354.0	279.0	2512.5	3145.5
11	2.0	1.66	1099	668	0	0	168	292.8	228.3	2478.3	2999.4

It is observed from Table 5-4 that w_{d21} and y_{de1} increase in w_{d11} ; whereas, y_{d01} decreases in w_{d11} . In addition, it is noted from Figure 5-1 that, when w_{d11} is relatively low (before the intersection of the two lines for y_{de1} and y_{d01}), $y_{d01} > y_{de1}$; the assembler's last-period problem is a constrained optimization problem. Since both suppliers' on-hand inventory levels are already high enough, with the given assumption, to raise the assembler's

inventory level up to y_{d01} (without any extra production), the assembler's optimal purchasing quantity equals to $y_{d01} - I_{01}$ (except for the first group where, constrained by supplier 1's deliveries, the assembler can only purchase up to 600). However, when w_{d11} is relatively higher (after the intersection), $y_{d01} < y_{del}$; the assembler's last-period problem becomes a non-constrained optimization problem. Also, its inventory level can be raised up to y_{d01} and the assembler's optimal purchasing quantity equals to $y_{d01} - I_{01}$ as well. Since the assembler's purchasing quantity equals to $y_{d01} - I_{01}$ when w_{d11} varies from 1.0 to 2.0, Q_{d01}^* decreases in w_{d11} . That is, an increase in suppliers' wholesale price leads to a reduction in the assembler's purchasing quantity.

Figure 5-1 The impact of w_{d11} on y_{del} , y_{d01} and Q_{d01}^*

Further, it is noted that both suppliers' expected profits do not increase in w_{d11} constantly. This is mainly due to the decrease in the number of component purchased by the assembler; whereby, the assembler's expected profit as well as the total expected system profit also decreases. The analysis implies that an increase in supplier's wholesale price discourages the assembler from purchasing a larger volume of components which consequently deteriorates system performance in terms of total expected profits. Hence, in practice, the assembler would turn to independent negotiation with the suppliers to seek a proper wholesale price

adjustment such that not only its own profit but also the entire system profit can be improved.

III. The Coordinated Decentralized Control Setting

The optimal production/purchasing policy for the coordinated setting with buyback contract consists of several scenarios and the numerical analysis for every scenario would be similar. Therefore, the determination of coordinating parameters and the examination of their impacts on system performance are based on the assumption of initial on-hand inventory level as given in scenario 8 of Table 5-3.

Figure 5-2 The impact of b_{11} on various coordinating parameters

Supplier 1’s buyback price b_{11} is assigned for the numerical study conducted here and its value is varied from 1.0 to 5.0 at an incremental step of 0.5. The relationship between b_{11} and the other coordinating parameters are presented in Figure 5-2 and they are calculated according to eq.(4-93). It is noted from Figure 5-2 that b_{11} and b_{21} are reversely related; however, w_{b11} and b_{11} are positively related. This implies that the supplier asking for a higher wholesale price needs to pay for a higher buyback price as well.

In addition, suppliers’ expected profits π_{bi1} (for $i = 1, 2$) are determined by eq.(4-84) and the

assembler's expected profit π_{b01} is determined by eq.(4-82). The total expected system profit $\sum \pi_{bi1}$ (as shown in Figure 5-3) is the sum of all the players' expected profits.

Figure 5-3 The impact of b_{11} on expected profits

Moreover, it is noted from Figure 5-3 that both suppliers' expected profits are proportional to their buyback price. Specifically, supplier 1's expected profits increases while supplier 2's expected profit decreases when b_{11} increases. Therefore, it can be inferred from Figure 5-2 and Figure 5-3 that a larger buyback price represents a larger portion of total expected system profits. In view of this, suppliers would negotiate with the assembler for a higher wholesale price since its wholesale price and buyback price are positively related at channel coordination. Contrary to the supplier, the assembler would ask for a reduced wholesale price to increase its share of the total expected system profits.

IV. Impact of Parameters on System Performance in a Coordinated Setting

In this section, the impacts of various system parameters, such as selling price, holding cost, penalty cost, on the assembler's optimal purchasing quantity as well as the total expected system profits are examined. Without loss of generality, initial inventory level is the same as that given in scenario 8 of Table 5-3.

It is noted from the numerical results presented below that, given the same cost parameters (as that in Table 5-1), the assembler’s purchasing quantity Q_{b01} and the total expected system profits $\sum \pi_{bi1}$ are the same as those in the corresponding centralized control setting. This implies that the decentralized assembly system can optimize its performance with a coordinating mechanism such as the buyback contract.

a). Variation in Product Selling Price

A sensitivity analysis with respect to selling price p on system performance in the coordinated decentralized setting is conducted. The selling price p is increased from 7.0 to 11.0 and b_{11} is assigned to be 4. The detailed numerical results are shown in Table 5-5.

Table 5-5 The impact of p on optimal quantities and expected profits

No.	p	b_{21}	w_{b11}	w_{b21}	S Values			Optimal Quantities			Expected Profits			
					S_{11}	S_{21}	S_{31}	Q_{b11}	Q_{b21}	Q_{b01}	π_{b11}	π_{b21}	π_{b01}	$\sum \pi_{bi1}$
1	7.0	6.13	3.81	5.00	2181	1624	1321	221	21	821	645.2	486.1	2645.8	3777.0
2	7.5	6.63	3.86	5.46	2239	1682	1380	280	80	880	699.0	579.3	2869.2	4147.5
3	8.0	7.13	3.90	5.93	2294	1737	1435	335	135	935	749.1	679.9	3096.0	4525.1
4	8.5	7.63	3.93	6.41	2346	1790	1487	387	187	987	796.6	788.0	3324.4	4909.1
5	9.0	8.13	3.96	6.88	2396	1839	1537	437	237	1037	840.5	904.2	3554.2	5298.8
6	9.5	8.63	3.99	7.36	2443	1887	1584	484	284	1084	882.4	1025.5	3785.9	5693.8
7	10.0	9.13	4.02	7.84	2488	1932	1629	529	329	1129	921.3	1153.0	4019.2	6093.5
8	10.5	9.63	4.05	8.32	2532	1975	1672	572	372	1172	958.0	1285.7	4253.9	6497.5
9	11.0	10.13	4.07	8.81	2573	2017	1714	614	414	1241	993.3	1425.0	4487.2	6905.5

It is noted from Table 5-5 that all the S values (at coordination, the Z values equal to the S values) increase with the increase in product selling price; hence, this leads to an increase in the inventory level of the entire system (The optimal policy summarized in Proposition 4-1 indicates that the suppliers’ optimal production quantities and the assembler’s optimal purchasing quantity greatly depend on the three S values. Hence, the increase in all the S values lead to the increase in all the corresponding optimal quantities, thereby, the inventory level of the entire system). Moreover, b_{21} , w_{b11} and w_{b21} also increase in p (refer to

eq.(4-93)). This implies that, in the event of a markup in product selling price, suppliers have the incentive to ask for a higher wholesale price in order to increase their own profits. Furthermore, all the players' expected profits increase. Suppliers benefit from a larger volume of components purchased by the assembler as well as an increase in its wholesale price; however, the assembler benefits from increased selling price and an increased number of product finally assembled.

b). Increase in the assembler's holding cost

In this section, a sensitivity analysis with respect to the assembler's holding cost h_0 on system performance is conducted. The holding cost is increased from 0.1 to 0.5 at an incremental step of 0.05. In addition, supplier 1's buyback price b_{11} is assigned to be 4. Detailed numerical results are shown in Table 5-6.

Table 5-6 The impact of h_0 on optimal quantities and profits

No.	h_0	b_{21}	w_{b11}	w_{b21}	S Values			Optimal Quantities			Expected Profits			
					S_{11}	S_{21}	S_{31}	Q_{b11}	Q_{b21}	Q_{b01}	π_{b11}	π_{b21}	π_{b01}	$\sum \pi_{b11}$
1	0.10	7.11	3.90	5.931	2313	1748	1442	342	142	942	758.1	690.2	3090.3	4538.6
2	0.15	7.16	3.89	5.938	2266	1723	1424	324	124	924	737.2	663.6	3104.3	4505.5
3	0.20	7.21	3.87	5.948	2221	1698	1408	308	108	908	716.6	639.8	3115.7	4472.1
4	0.25	7.26	3.85	5.957	2178	1675	1392	292	92	892	697.4	615.6	3126.7	4439.7
5	0.30	7.31	3.83	5.966	2138	1652	1376	276	76	876	678.1	591.2	3138.8	4408.0
6	0.35	7.36	3.82	5.973	2100	1631	1360	260	60	860	660.0	566.5	3150.4	4376.9
7	0.40	7.41	3.80	5.981	2063	1609	1345	245	45	845	641.5	543.1	3161.7	4346.3
8	0.45	7.46	3.78	5.991	2028	1589	1331	231	31	831	624.6	521.1	3170.6	4316.3
9	0.50	7.51	3.77	6.000	1994	1569	1317	217	17	817	607.7	498.8	3180.3	4286.8

It is noted from Table 5-6 that all the S values decrease with the increase in h_0 , thereby, resulting in a reduction in the inventory level of the entire system. Therefore, the suppliers' optimal production quantities and the assembler's optimal purchasing quantity all decrease. This implies that an increase in the holding cost downstream might motivate the assembler to purchase a smaller volume of components in order to reduce its on-hand inventory level. In addition, b_{21} increases with the same step as that of h_0 (refer to eq.(4-93)) when b_{11} is

constant; whereas, w_{b11} decreases while w_{b21} increases (refer to eq.(4-93)).

Moreover, the numerical results show that supplier 1's expected profit decreases with h_0 ; this is because the decrease in the assembler's holding cost reduces supplier 1's wholesale price and the assembler purchases a smaller number of components at the same time. Supplier 2's expected profit also decreases; this is because a slight increase in its wholesale price cannot offset the decrease in its expected revenue from component selling. However, the decreasing rate of supplier 1's expected profit is higher than that of supplier 2's because supplier 2 has to pay for an increasing buyback price. However, the assembler's expected profit increases slightly since it benefits from a decrease in its wholesale price and a higher buyback price for leftover final products. Viewed from the system perspective, the entire system performance deteriorates with an increase in the assembler's holding cost. This is because the decrease in the assembler's purchasing quantity reduces the expected number of product finally assembled and, thereby, reduces the total expected system profit.

c). Increase in Backorder Penalty Cost

The sensitivity analysis is with respect to the impact of the assembler's penalty cost s on system performance. As shown in Table 5-7, the s value is varied from 1.0 to 2.0. In addition, supplier 1's buyback price b_{11} is assigned to be 4.

It is noted from Table 5-7 that an increase in the assembler's penalty cost leads to an increase in all the S values, thereby an increase in suppliers' optimal production quantities and the assembler's optimal purchasing quantity. Specifically, when there is a 10% increase in the penalty cost, there is an averagely increase of 1.2% in Q_{b01} . This is because an increase in the assembler's backorder penalty cost might motivate the assembler from purchasing a

larger volume of components; thus, more final products would be produced and the probability of having higher than expected demand is reduced.

Table 5-7 The impact of s on quantities and profits

No.	s	b_{21}	w_{b11}	w_{b21}	S Values			Optimal Quantities			Expected Profits			
					S_{11}	S_{21}	S_{31}	Q_{b11}	Q_{b21}	Q_{b01}	π_{b11}	π_{b21}	π_{b01}	$\sum \pi_{bi1}$
1	1.0	6.93	3.881	5.745	2272	1716	1413	313	113	913	729.8	638.2	3205.3	4573.2
2	1.1	7.03	3.889	5.839	2283	1727	1424	324	124	924	739.8	658.8	3150.4	4549.0
3	1.2	7.13	3.896	5.934	2294	1737	1435	335	135	935	749.1	679.9	3096.0	4525.1
4	1.3	7.23	3.904	6.028	2305	1748	1445	345	145	945	758.8	700.0	3042.6	4501.4
5	1.4	7.33	3.911	6.123	2315	1759	1456	356	156	956	768.8	722.1	2987.1	4477.9
6	1.5	7.43	3.918	6.217	2326	1769	1466	366	166	966	777.8	743.1	2933.9	4454.7
7	1.6	7.53	3.925	6.313	2336	1779	1477	377	177	977	787.0	766.1	2878.6	4431.8
8	1.7	7.63	3.932	6.407	2346	1790	1487	387	187	987	796.6	788.0	2824.4	4409.1
9	1.8	7.73	3.939	6.502	2356	1800	1497	397	197	997	805.6	810.4	2770.6	4386.6
10	1.9	7.83	3.945	6.597	2366	1810	1507	407	207	1007	814.5	833.2	2716.6	4364.3
11	2.0	7.93	3.952	6.692	2376	1820	1517	417	217	1017	823.4	856.4	2662.5	4342.2

In addition, from (4-93), when b_{11} is constant, b_{21} increases at the same step as that of s . Further, there is an average increase of 0.2% in w_{b11} when there is a 10% increase in s ; however, there is an average increase of 1.65% in w_{b21} given the same percentage increase in s . This implies that when the assembler increases its inventory level, the assembler might ask for a higher buyback price for leftover inventory due to over-production (as a kind of compensation).

Further, the numerical results show that both suppliers' expected profits increase, which is the result of the tradeoff between their increased revenue from component selling and their increased buyback price paid for leftover final products. However, the assembler's expected profit decreases because a higher on-hand inventory level is maintained to reduce the probability of stock out. The total expected system profit decreases as well due to the increase in the backorder penalty cost.

5.2 Numerical Study for the Two-Period Problem

As mentioned earlier, expressions for the variables of the two-period model, such as the S values of the centralized control setting, the assembler's optimal order-up-to level without constraint in the price-only decentralized setting and the coordinating parameters in the coordinated decentralized setting with buyback contract, are generally complex due to the increase in the number of periods considered. These variables are computed here to show the applicability of the proposed two-period models.

In addition, numerical study for the assembly system operated with centralized control, price-only decentralized control and coordinated decentralized control with buyback contract will be conducted. Consistent with the assumption in the model formulation part, supplier 1 is assumed to have less on-hand inventory at the beginning of period $t = 2$. Initial conditions, concerning cost parameters, for the two-period problem is assumed to be the same as that for the last-period problem (as given in Table 5-1).

I. The Centralized Control Setting

With the given initial conditions in Table 5-1, the S values for the two-period setting are attained based on eqs.(4-9), (4-16) and (4-18), respectively. Analogously, for validation of the analytical solution structures in Proposition 4-2, assumptions on players' initial on-hand inventory are made to include the eight different scenarios in Proposition 4-2 (as given in Table 5-8).

The optimal quantities presented in Table 5-8 are derived through enumeration and they are compared with the corresponding analytical solutions derived according to Proposition 4-2.

Table 5-8 Results for the two-period centralized control setting

Scenario	Initial Inventory Levels			Optimal Quantities			M_{c2}^*
	I_{12}	I_{22}	I_{02}	Q_{c12}^*	Q_{c22}^*	Q_{c02}^*	
1	600	700	3750	0	0	0	14151
2	2800	2850	800	0	0	2736	11378
3	2200	2900	700	0	0	2200	11140
4	2100	2200	800	0	0	2100	11273
5	1800	2900	700	348	0	2148	10902
6	1500	2100	800	548	0	2048	10916
7	800	1780	800	980	0	1780	10445
8	600	1100	800	1103	603	1703	9989.9
S values	S_{12}	3536	S_{22}	2848	S_{32}	2503	

The comparison shows that the optimal numerical solutions and the derived analytical solutions are identical. This validates the optimal solution structures in Proposition 4-2. With the derived optimal quantities, the total expected system profit for the two-period centralized system can be calculated accordingly (refer to eq.(4-8)). Similarly, system performance of the centralized control setting is used as a reference for evaluation of the decentralized control setting with/without coordination.

II. The Price-Only Decentralized Contract Setting

The same initial inventory levels as those for the centralized control setting are assumed for comparison (as shown in Table 5-9). Supplier 1’s wholesale price for the last period w_{d11} is assigned to be 1.5; supplier 2’s wholesale price for the last period $w_{d21}=1.25$ (refer to eq.(4-38) when wholesale prices are adjusted simultaneously to eliminate mismatch of components). Further, it is derived that $y_{d01}=880$ (refer to eq.(4-27)) and $y_{de1}=827$ (refer to eq.(4-35)).

Table 5-9 Results for two-period price-only decentralized setting

No.	Initial Inventory levels			Optimal Quantities			Expected Profits			
	I_{12}	I_{22}	I_{02}	Q_{d12}	Q_{d22}	Q_{d02}	M_{d12}	M_{d22}	M_{d02}	$\sum M_{di2}$
1	600	700	3750	0	0	0	-31.5	-36.5	13938	13870
2	2800	2850	800	0	0	673	1658.3	1893.3	6038.8	9590.4
3	2200	2900	700	0	0	773	1982.8	2124.9	5508.4	9616.2
4	2100	2200	800	0	0	673	1791.2	1961.2	6038.4	9790.8
5	1800	2900	700	0	0	773	2038.1	2120.5	5508.1	9666.7
6	1500	2100	800	0	0	673	1867.2	1962.7	6037.8	9863.3
7	800	1780	800	0	0	673	1841.5	1969.7	6036.5	9847.7
8	600	1100	800	0	0	600	1707.4	1849.1	5953.4	9509.9

It is noted from the derived results that $y_{d01} > y_{del}$; hence, the assembler's optimization problem for the last period is a constrained optimization problem. In addition, suppliers' wholesale prices for period $t = 2$ are assumed to be 2 and 2.3, respectively. Hence, the assembler's optimal order-up-to level for period $t = 2$ $y_{d02} = 1473$ can be derived according to eq.(4-47).

Since the assembler's last-period optimization problem is a constrained one, derivation of the optimal policy can follow the analysis in Scenario II and IV of Section 4.3.2. It turns out that the optimal solution does not conflict with the assumption of supplier 1 being the critical supplier of period $t = 2$ (the assumption made in Scenario II). Since both suppliers' deliveries are sufficient to raise the assembler's inventory level up to y_{d02} (except for the last scenario), the assembler's purchasing quantity equals to $y_{d02} - I_{02}$. However, for the last scenario, supplier 1's delivery quantity equals to 600, it is, thus, optimal for the assembler to purchase 600 units of components from the suppliers. With optimal quantities derived for each player, their expected profit and the total expected system profit can be derived accordingly.

A comparison of the optimal quantities in the centralized and price-only decentralized setting

shows that both the assembler's purchasing quantity and the total expected system profit in the price-only decentralized setting are fewer than the corresponding values of the centralized setting. That is to say, the price-only contract alone cannot coordinate the two-period decentralized assembly system.

Similarly, wholesale price is varied to examine its impact on the order-up-to levels, the optimal quantities and the expected profit of each player. Supplier 1's wholesale price w_{d11} is changed from 1 to 3 at an incremental step of 0.2. Suppliers' wholesale prices of period $t = 2$ are still assumed to be 2 and 2.3, respectively. Player's initial inventory level corresponding to the last scenario of Table 5-9 is applied.

Table 5-10 The impact of w_{d11} on optimal quantities

No.	w_{d11}	w_{d21}	y_{d01}	y_{del}	y_{d02}	Optimal Quantities		
						Q_{d12}	Q_{d22}	Q_{b02}
1	1.0	0.83	1150	452	1273	308	0	473
2	1.2	1.00	1033	619	1368	212	0	568
3	1.4	1.16	928	762	1442	42	0	642
4	1.6	1.33	834	887	1501	0	0	600
5	1.8	1.49	747	999	1551	0	0	600
6	2.0	1.66	668	1099	1594	0	0	600
7	2.2	1.83	594	1190	1631	0	0	600
8	2.4	1.99	525	1273	1664	0	0	600
9	2.6	2.16	461	1350	1693	0	0	600
10	2.8	2.32	401	1421	1720	0	0	600
11	3.0	2.49	344	1488	1744	0	0	600

With the given conditions, supplier 2's wholesale price w_{d21} and the equilibrium order-up-to level y_{del} is calculated according to eq.(4-38). The assembler's optimal order-up-to level without constraint of period $t = 1$ and $t = 2$ are calculated according to eqs.(4-27) and (4-47), respectively. As analyzed in the last-period price-only decentralized setting, an increase in w_{d11} leads to an increase in w_{d21} and y_{del} ; whereas, a decrease in y_{d01} . Within the groups of data examined, it is noted that $y_{del} < y_{d01}$ when $w_{d11} \leq 1.4$; hence the

assembler's last-period optimization problem is a constrained optimization problem. However, $y_{del} > y_{d01}$ when $w_{d11} > 1.4$ and the assembler's last-period optimization problem becomes an unconstrained one. The numerical results indicate that the optimal quantities derived for the first three groups are optimal solutions for Scenario II while the remaining optimal quantities are optimal solutions for Scenario I. Moreover, in both cases, supplier 1 is the critical supplier for period $t = 2$. Consequently, the assembler's optimal purchasing quantity can be derived.

In addition, it is noted from Figure 5-4 that the assembler's expected profit reaches its peak value at $w_{d11} = 1.4$ where the number of components purchased by the assembler is the most among the groups of data tested. Both suppliers' expected profits increase in w_{d11} . Specifically, the increasing magnitudes of suppliers' expected profits are larger before $w_{d11} = 1.4$, which is the result of an increase in the assembler's purchasing quantity as well as an increase in both suppliers' wholesale prices.

Figure 5-4 The impact of w_{d11} on optimal expected profits

III. The Coordinated Decentralized Setting

The numerical study carried out in this section focuses on the determination of the coordinating parameters of the proposed buyback contract. For the sake of comparison, players' initial inventory level corresponding to the last scenario of Table 5-8 is used. Specifically, the impacts of some of the modeling parameters, such as product selling price and supplier's wholesale price, are examined.

As discussed, system performance, in this thesis, is measured in terms of the assembler's purchasing quantity and the total expected system profits. It is noted from the numerical results presented below that when the same cost parameters are assumed, the number of components purchased by the assembler Q_{b02} and the total expected system profits $\sum M_{bi2}$ are the same as those of the two-period centralized setting. The equality validates the proposed buyback contract model.

a). Variation in product selling price

Selling price p is varied from 5 to 10 at an incremental step of 0.5; supplier 1's wholesale prices over the two periods are assigned as $w_{b11} = 3$ and $w_{b12} = 3.5$, respectively. Similar to the sensitivity analysis for the last period problem (as conducted in Section 5.1), an increase in the selling price leads to an increase in the values of all the coordinating parameters.

The numerical results (presented in Table 5-11) show that all the S values (at coordination, the Z values and the S values are inter-replaceable) increase in p , thereby increases the inventory level of the entire system – the suppliers' optimal production quantities as well as the assembler's optimal purchasing quantity increase. This indicates that an increase in product selling price motivates the assembler to purchase a larger volume of components,

whereby, the suppliers' production quantities are increased.

Table 5-11 The impact of p on optimal quantities (two-period coordinated setting)

No.	p	S Values			Optimal Quantities		
		S_{12}	S_{22}	S_{32}	Q_{b12}	Q_{b22}	Q_{b32}
1	5.0	3148	2460	2115	715	215	1315
2	5.5	3224	2536	2191	791	291	1391
3	6.0	3295	2607	2262	862	362	1462
4	6.5	3361	2673	2328	928	428	1528
5	7.0	3423	2735	2390	990	490	1590
6	7.5	3481	2793	2448	1048	548	1648
7	8.0	3536	2848	2503	1103	603	1703
8	8.5	3589	2900	2555	1155	655	1755
9	9.0	3638	2950	2605	1205	705	1805
10	9.5	3685	2997	2652	1252	752	1852
11	10.0	3731	3043	2698	1298	798	1898

Moreover, it is noted from Figure 5-5 that all the players' expected profits increase in p . Specifically, there is a 2.4% increase in supplier 1's expected profit when there is a 10% increase in the p values; whereas, there is a 3.1% increase in supplier 2's expected profit with the same increasing magnitude in p .

Figure 5-5 The impact of p on optimal expected profits

b). Variation in supplier 1's wholesale price

Supplier 1's wholesale price for the last period is varied from 2 to 3.6 at an increasing step of

0.2. In addition, supplier 1’s wholesale price w_{b12} is assumed to be 4. This represents the scenario when there is a markdown in wholesale price in the last period. It is noted from Figure 5-6 that b_{12} decreases with the increase in w_{b11} while w_{b22} and b_{22} increase.

Moreover, it is noted from Figure 5-7 that supplier1’s expected profit M_{b12} increases in w_{b11} since an increase in w_{b11} leads to a reduction in its buyback price b_{12} (as shown in Figure 5-6). However, supplier 2’s expected profit decreases in w_{b11} as a result of the tradeoff between its inventory costs – increased revenue from an increase in its wholesale price w_{b22} (as shown in Figure 5-6) and a higher buyback price b_{22} for the assembler’s leftover products (as shown in Figure 5-6). The assembler’s expected profit decreases as well due to an increase in the wholesale price charged by supplier 2. Since wholesale price is regarded as an internal coordinating parameter, variation of its value, at channel coordination, will not change the total expected system profit obtained and the assembler’s optimal purchasing quantity. Moreover, different coordinating parameter pairs represent different profit allocation schemes among the players in the system.

Figure 5-6 The impact of w_{b11} on various coordinating parameters

Figure 5-7 The impact of w_{b11} on optimal expected profits

Further, it is noted that suppliers' expected profits are not proportional to their buyback prices, which is different from the conclusion of the single-period model. This is due to player's on-hand initial inventory level at each period.

5.3 Summary

A numerical study has been done to the single-period and the two-period models, respectively. For both models, analytical models for the centralized control setting are validated by comparing the derived optimal numerical solutions with the optimal analytical solutions. The validated centralized models are used as references for performance evaluation of the decentralized control setting. The price-only contract models are examined by assuming the same initial conditions and wholesale prices over the periods. However, the decentralized assembly system cannot be coordinated with a wholesale price contract alone due to double marginalization, which results in an inferior system performance. This also validates the conclusion proposed in the preceding chapter. For the coordinating contract models, focus is laid on the determination of various coordinating parameters. Since the same level of efficiency as that of the corresponding centralized control setting can be achieved, this validates the proposed buyback contract models.

However, for the implementation of the buyback contract, the up-to-date information concerning the assembler's leftover inventory at the end of each period is required as it will have the bearing on the allocation of system profits to each player. Normally, it could be biased toward the assembler, which is not preferred by the suppliers.

Chapter 6

Conclusions and Recommendations for Future Work

Various researches have stressed the importance of supply chain efficiency improvement in a decentralized control setting with various practices such as supply contract and establishment of long-term relationship. The focus of this thesis is on the determination of suppliers' optimal component delivery quantities and the assembler's optimal purchasing quantity in a decentralized assembly system facing stochastic market demand.

Some of the previous researchers have conducted research on decentralized assembly system by considering a single transaction type of business and newsvendor models have proved their applicability in such settings. However, due to the increasing practice of outsourcing, business transactions take place between the players usually for more than one period. In such cases, determination of the optimal quantities in a decentralized assembly system requires incorporation of the changes in cost parameters, transfer of inventory status.

In addition, with a decentralized control mode, the optimal quantities should be determined by considering suppliers' incentives concerning production/delivery and the assembler's incentive concerning procurement. Since suppliers are different in their cost structures and market demand is uncertain, the quantities that maximize their own expected profits would be different with each other; hence, resulting in a mismatch of components, thereby an inferior system performance. However, the decentralized assembly system can be coordinated to gain its maximum efficiency through coordinating mechanism with consideration of more than one transaction between the players.

The thesis considers only two suppliers and a two-period supply contract for channel coordination to demonstrate the complications and implications of the model in a more-than-one-period transaction setting. It is seen that even with such assumptions, the models are already quite complex. However, the model essentially captures the characteristics of supplies from different suppliers in a decentralized assembly setting. Therefore, the extension of the model may not provide new insights but will consolidate the findings made through a two-period model.

The model also shows that transaction time period plays an important role in the determination of players' behavior. In a one-period transaction setting, suppliers attempt to avoid over delivery to the assembler. However, if two consecutive transactions are considered, players' behavior of the first period can be seen to affect their decisions in the following period.

For the centralized control setting, it is global optimal for the supplier with less on-hand inventory of the first period to be the critical supplier of the two consecutive periods. This is an important conclusion obtained from the work done in this thesis. The model shows that each player's decision on quantities for a period of transaction is dependent on their on-hand inventory level at the beginning of the period. As most suppliers do carry inventory to cushion order quantities, this finding consolidates the market behavior.

The study also consolidates the earlier findings that a decentralized control setting produces an inferior system performance compared with that of a centralized setting. Further, each player acts to maximize its own expected profits based on its on-hand inventory as well as its wholesale price for a period; hence, the supplier, who is the critical supplier of the first period,

may not necessarily be the critical supplier in the following period. This is another important conclusion obtained from the study. The analysis also shows that, similar to the single-period transactions, the two-period transactions cannot be coordinated by wholesale price alone. Therefore, a coordinating mechanism is required to coordinate a multi-period transaction setting to achieve the same level of efficiency as that of the centralized control mode. This can be done by sharing the costs of the assembler's leftover inventory at the end of each period (i.e., a buyback contract).

Various simulation studies done in Chapter 5 further highlight the findings obtained through the analytical solutions. Numerical study for the price-only decentralized control setting shows that, given the same initial conditions, there is a reduction in the assembler's purchasing quantity (thereby a reduced number of products finally assembled) and less expected system profits if compared with that of the centralized control setting. Moreover, an increase in the suppliers' wholesale price would discourage the assembler from placing a larger order. Hence, in practice, the assembler would turn to independent negotiations with the suppliers to seek a proper wholesale price. Specifically, the values of wholesale price would affect the characteristics of the last-period problem, making the assembler's purchasing quantity be constrained or not constrained by the suppliers' deliveries. For the coordinated contract models with buyback price, the numerical study shows that various coordinating parameter pairs can realize channel coordination with each pair of which represents a different profit allocation scheme. In addition, for each supplier, its wholesale price, expected profits and buyback price are positively related; however, the buyback prices of each supplier are reversely related. This implies that the supplier who charges for a higher wholesale price to the assembler has to pay for a higher buyback price to the assembler as a kind of compensation. Hence, given the fixed total expected system profits, each supplier would ask

for a higher wholesale price to the assembler in order to account for a larger portion of the total expected profits. The model is representative of the decentralized assembly system with suppliers located at different places and operated with VMI. The proposed model is easy for implementation in the situation governed by the model.

6.1 Research Contributions

The research contribution in this thesis can be grouped into the following points:

1. This thesis complements existing literature by studying the incentive problem and the associated coordination problem in a decentralized assembly system with random customer demand. When components supplied by different suppliers for the final assembly are unique and components usage rate is fixed, the incentive conflicts is reflected by “mismatch of components” as the suppliers tend to maximize their own profits in deciding on the production or delivery quantities. Mismatch of components leads to increased inventory costs, a smaller purchasing order from the assembler and reduced system efficiency. The research proposes a coordinating method to solve such an incentive conflict problem by considering buyback contract and revenue-sharing contract models. It also proves that, similar to the research on decentralized distribution system, the conventional pricing scheme, namely wholesale price, cannot coordinate the decentralized assembly system.
2. The study of the impacts of inventory and non-stationary costs in a two-period supply contract model is another contribution made in this research. This model helps to understand that the information on inventory levels at a player’s premises can swing the coordinating mechanism results one way or the other. In addition, this thesis provides a method to determine each player’s best response when suppliers’ wholesale

prices are non-stationary and the assembler's purchasing quantity is constrained or not constrained by the critical supplier's delivery.

6.2 Recommendation for Future Work

The proposed model can have a few extensions as mentioned below.

- I. One recommendation could be to extend the proposed two-period model to a N -period setting so that it becomes more generic for multi-period contracts. However, it is noted that the complexities of the problem are greatly increased when the number of periods is increased from 1 to 2. This increased complexity is partly due to the extended solution space when comparison of suppliers' on-hand initial inventory levels of each period is required to determine the critical supplier and each player's optimal behavior and also due to the assumption of non-stationary wholesale price. In addition, with backward DP, it is already difficult to determine the optimal expected profit of the last period due to the complex solution structure derived. Hence, more intensive mathematical manipulations are required to determine the properties of the two-period objective function (e.g. concavity, monotony) due to the increased complexities.

Therefore, it is easy to predict that, with an increase in the number of periods, the problem would become more complex with requirement of more intensive mathematical manipulations. For example, the mathematical model for the centralized control mode covering a general number of periods can be formulated as follows:

$$\begin{aligned}
 M_{cN}(I_{iN}, Q_{ciN}) &= E_{\xi_N} \{ \pi_{cN}(I_{iN}, Q_{ciN}) + J_{cN}(I_{0N}, I_{1N}, I_{2N}) \} \\
 &= E_{\xi_N} \left\{ p \cdot \min(Q_{c0N} + I_{0N}, \xi_N) - \sum_{i=1}^2 [c_i \cdot Q_{ciN} + h_i(I_{iN} + Q_{ciN} - Q_{c0N})] \right\} \\
 &\quad - L(Q_{c0N} + I_{0N}) - c_0 \cdot Q_{c0N} + J_{cN}(I_{0N}, I_{1N}, I_{2N}) \\
 \text{s.t.} \quad &Q_{c0N} \leq \min(I_{1N} + Q_{c1N}, I_{2N} + Q_{c2N})
 \end{aligned}$$

For this expression, as mentioned, the major difficulty lies in the determination of $J_{cN}(I_{0N}, I_{1N}, I_{2N})$ due to an extended solution space incurred by an increase in the length of the horizon.

- II. In this thesis, a two-echelon assembly system has been considered; however, the structure of the assembly system, in practice, could be much more complex with tiered suppliers supplying various components, sub-assemblies for the final assembly. However, when these tiered suppliers are combined in one model, the complexity of the model would increase further.

Munson and Rosenblatt (2001) consider a three-level supply chain with the middle supplier enjoying a strong position to dictate the production quantities of its upstream supplier and the purchasing quantity of its downstream retailer by providing two different quantity discounts. The results show that incorporating quantity discounts by the middle echelon can significantly decrease costs compared to concentrating only at the retailer side. However, in this research, customer demand is assumed to be deterministic and the retailer determines its purchasing quantity according to its EOQ value. Moreover, the examined SC has a serial structure; hence, unlike the assembly system, it does not require considering coordination of various components for the final assembly. In addition, the objective of the research is supplier's profit maximization focused, rather than channel coordination focused.

However, for the assembly system considered in this thesis, it can be divided into two sub-chains with each having a similar structure as that considered in Munson and Rosenblatt (2001). Compared with the proposed models, when tiered suppliers are

combined into one model, the modeling would become more complex since the most upstream suppliers' production/delivery quantities are uncertain. The supplier in the middle echelon has to determine its purchasing quantity and delivery quantity in anticipation of other component suppliers' delivery quantity to the assembler as well as its own supplier's delivery quantity. When the middle echelon supplier enjoys a strong position, the modeling might be easier since the supplier can dictate its upstream supplier's delivery quantity. However, incorporating tiered supplier needs further exploration and is an interesting direction for future research.

III. In order to derive analytical solutions for the proposed two-period models, customer demand between the two periods are assumed to be independent and identically distributed. However, due to the use of the information technology, assemblers may have the capability to forecast demand as continuously as possible. In such a case, a Bayesian update might be applied to represent the demand forecast. However, this makes the demand distribution different in different periods. This adds complexities in deriving the optimal solutions since convolution is required and therefore needs further investigation.

IV. It is perceived that the model can also be extended to include mass customization. The model proposed here implicitly considers only one type of product or minimal changes in the product design since a change in the product design would mean a change in the cost structure and the available capacity with the suppliers. It considers made to stock (MTS) type of production system rather than made to order (MTO) type of production system in mass customization (Eastwood, 1996; Kotha, 1996). The change in product portfolio in mass customization as shown in Jiao et al. (2003) and Drejer and Gudmundsson (2002) provides greater challenge for modeling. Nevertheless, it can be assumed that such

changes are limited to one order and therefore, variation in orders in different periods can be tackled while considering the demand in different periods. In this case buyback can also become an important aspect because the components supplied in one period may not render useful in the next period. Therefore, model needs to be extended by understanding the impact of obsolescence in inventory, wholesale price to be paid to the supplier and the buyback clauses.

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Appendix I Newsvendor Model

The Basic Newsvendor Model

As the name implies, the classical newsvendor problem (or referred to as single-period problem) is to determine the proper newspaper quantity that maximizes the newsvendor's expected profits under probabilistic demand. The newsvendor model is reflective of many real life situations and is often used to aid decision making in the fashion industry or products with short life cycle (shelf life) such as records, dairy products.

Two approaches have been applied to address the newsvendor problem. In the first approach, the expected costs of overestimating and underestimating demand are minimized. In the second approach, the expected profit is formulated and maximized. Both approaches yield the same results (Geunes et al., 2001). However, the second approach is widely applied in the literature on supply contract, probably due to the assumption that all the players in the decentralized channel are risk neutral and are profit maximizers.

The second approach is briefly reviewed as follows. Notations for the modeling are presented in Table I-1 and are consistent with those in the previous chapters.

Table I-1 Notations for simple newsvendor problem

ξ	Stochastic market demand
$f(\xi)$	The probability distribution function of demand
$F(\xi)$	The cumulative distribution function of demand
λ	Mean of demand
p	Unit retail price
c	Unit production cost
τ	Unit salvage cost
s	Unit penalty cost
Q	Optimal order quantity
π	Expected profit

Before formulating the problem, a shorthand expression is introduced,

- For $x \in \mathfrak{R}$, $(x)^+ = \max(x, 0)$, $\mathfrak{R}$ represents the real set.

Therefore, the expected profit can be formulated as,

$$\pi(Q) = E_{\xi} \left\{ p \cdot \min(\xi, Q) - c \cdot Q - s \cdot (\xi - Q)^+ + \tau \cdot (Q - \xi)^+ \right\} \quad (1-1)$$

the first term represents the expected revenue, the second term is the total production costs, the third term is the expected penalty cost and the last term is the profit from salvaging the leftover products at the end of the selling season. The value of $\pi(Q)$ can be derived with integration by parts and the equation can be extended as,

$$\begin{aligned} \pi(Q) &= p \left[\int_0^Q \xi f(\xi) d\xi + Q \int_Q^{\infty} f(\xi) d\xi \right] - c \cdot Q \\ &\quad - s \int_Q^{\infty} (\xi - Q) f(\xi) d\xi + \tau \int_0^Q (Q - \xi) f(\xi) d\xi \\ &= p \left(Q - \int_0^Q F(\xi) d\xi \right) - c \cdot Q - s \left[E(\xi) + \int_0^Q F(\xi) d\xi - Q \right] + \tau \cdot \int_0^Q F(\xi) d\xi \\ &= (p - c + s)Q - (p + s - \tau) \int_0^Q F(\xi) d\xi \end{aligned} \quad (1-2)$$

Hence, the first-order and the second-order derivatives of $\pi(Q)$ with respect to Q are,

$$\frac{d\pi(Q)}{dQ} = (p - c + s) - (p + s - \tau)F(Q) \quad (1-3)$$

$$\frac{d^2\pi(Q)}{dQ^2} = -(p + s - \tau)f(Q) < 0 \quad (1-4)$$

Since eq.(1-4) is negative, $\pi(Q)$ is concave in Q , which implies that there exists a unique Q that maximizes $\pi(Q)$. Moreover, the optimal Q satisfies the first-order condition and can be derived as,

$$Q^* = F^{-1} \left(\frac{p - c + s}{p - \tau + s} \right) \quad (1-5)$$

The optimal Q^* , in some literature, is referred to as the “critical value” of the newsvendor problem. Moreover, $F(Q^*)$ represents the probability that customer demand does not exceed the order quantity level (which is also referred to as the α -type service level).

In particular, denote $c_u = p - c + s$, which represents the underage cost; denote $c_o = c - \tau$, which represents the overage cost. Hence, eq.(1-5) can be rewritten as,

$$Q^* = F^{-1}\left(\frac{c_u}{c_u + c_o}\right) \quad (1-6)$$

The result can be derived from minimizing the “underage” and “overage” costs as well.

The mathematical model formulated above is from a single-player’s perspective. Cachon (2002) gives a review of the newsvendor model in a two-echelon distribution channel with one manufacturer selling to a retailer facing stochastic market demand. The pricing scheme between the manufacturer and the retailer is modeled by wholesale price to represent transfer the ownership of a good. The mathematical models representing the manufacturer and the retailer’s expected profit can be developed in a similar way as that for eq.(1-1). Further, the manufacture incurs production cost and makes profit from product selling to the retailer; the retailer incurs purchasing cost, penalty costs for unsatisfied customer demand and makes profits by selling the product to the market and salvaging the leftover products.

Although, in most of the cases, newsvendor models are applied to represent one-time transactions, they have been extended to various settings (Khouja, 1999; Petruzzi and Dada 1999; Geunes et al., 2001; Weng, 2004).

Multi-Period Newsvendor Model with DP

In this section, the basic newsvendor model (i.e., a single-period model) extended to a multi-period setting with stochastic dynamic programming (DP) in a reverse manner is explored. DP has been widely applied to make sequential decisions in practice.

With backward DP, the analysis will start from the last period when the transaction ends. The objective function for the last period is formulated and the optimal solution for the last period is derived. Then the analysis rolls back to the last but the second period, which is linked with the last period through inventory transfer functions. Similarly, the objective function representing the total expected profit over the two periods is formulated and the optimal solutions are derived accordingly. Analogously, the analysis keeps rolling back to the preceding periods until it reaches the first period when the transaction starts.

To illustrate, a multi-period newsvendor model formulated with DP is presented and the notations for the modeling are shown in Table I-2.

Table I-2 Notations for the multi-period newsvendor model

t	Subscript, indicates the period where there are t periods left
ξ_t	Stochastic market demand of period t
$f(\xi_t)$	The probability distribution function of demand in period t
$F(\xi_t)$	The cumulative distribution function of demand in period t
λ	Mean of demand
p	Unit retail price
c	Unit production cost
h	Unit holding cost
s	Unit penalty cost
Q_t	Optimal order quantity of period t
I_t	Initial on-hand inventory level of period t
π_t	Expected profit in period t
J_t	Optimal expected profit from period t till the end
M_t	Total expected profit from period t till the end

1. $t = 1$

As explained earlier, the analysis starts from the last period where $t = 1$. For illustration, the

relationship between events sequence and the analysis sequence is described by Figure I-1.

Figure I-1 The relationship between events and analysis sequence

Similar assumptions as those made in Appendix I are used. Hence, the expected profit for period $t = 1$ can be formulated as,

$$\begin{aligned}\pi_1(Q_1) &= E_{\xi_1} \left\{ p \cdot \min(\xi_1, I_1 + Q_1) - c \cdot Q_1 - s [\xi_1 - (I_1 + Q_1)]^+ - h [(I_1 + Q_1) - \xi_1]^+ \right\} \\ &= (p - c + s)(I_1 + Q_1) - (p + s + h) \int_0^{Q_1 + I_1} F(\xi_1) d\xi_1 + c \cdot I_1\end{aligned}\quad (1-7)$$

Hence, the first-order derivative of $\pi_1(Q_1)$ with respect to Q_1 is,

$$\frac{d\pi_1(Q_1)}{dQ_1} = (p - c + s) - (p + s + h)F(I_1 + Q_1)\quad (1-8)$$

From the first-order derivative, denote

$$y_1^* = F^{-1}\left(\frac{p + s - c}{p + s + h}\right)\quad (1-9)$$

which represents the optimal order-up-to level of the last period. Hence, the optimal order quantity Q_1^* can be derived as,

$$Q_1^* = \begin{cases} 0 & \text{if } I_1 > y_1^* \\ y_1^* - I_1 & \text{if } I_1 \leq y_1^* \end{cases}\quad (1-10)$$

2. $t = 2$

The last two periods are linked by the inventory transfer function, which is modeled as follows,

$$I_1 = I_2 + Q_2 - \xi_2\quad (1-11)$$

With eq.(1-11), eq.(1-10) can be rewritten as,

$$Q_1^* = \begin{cases} 0 & \text{if } \xi_2 < I_2 + Q_2 - y_1^* \\ y_1^* - (I_2 + Q_2 - \xi_2) & \text{if } \xi_2 \geq I_2 + Q_2 - y_1^* \end{cases} \quad (1-12)$$

In addition, define $L(\varphi) = E_{\xi} \{s(\xi - \varphi)^+ + h(\varphi - \varphi)^+\}$, which represents the expected shortage and holding cost at an inventory level of φ . Therefore, the optimal expected profit of the last period can be formulated as,

$$J_1(I_2, Q_2) = \int_0^{I_2 + Q_2 - y_1^*} \{p \cdot \min(\xi_1, I_1) - L(I_2 + Q_2 - \xi_2)\} f(\xi_2) d\xi_2 \\ + \int_{I_2 + Q_2 - y_1^*}^{\infty} \{p \cdot \min(\xi_1, y_1^*) + c \cdot (I_2 + Q_2 - y_1^*) - c \cdot \xi_2 - L(y_1^*)\} f(\xi_2) d\xi_2 \quad (1-13)$$

Therefore, the total expected profit of the last two periods is,

$$M_2(I_2, Q_2) = E_{\xi_2} \{p \cdot \min(\xi_2, I_2 + Q_2) - c \cdot Q_2 - L(I_2 + Q_2)\} + J_1(I_2, Q_2) \quad (1-14)$$

The first-order derivative of $M_2(I_2, Q_2)$ with respect to Q_2 is,

$$\frac{dM_2(I_2, Q_2)}{dQ_2} = (p + s) - (p + s + h)F(I_2 + Q_2) + (p + s - c)F(I_2 + Q_2 - y_1^*) \\ - (p + s + h) \int_0^{I_2 + Q_2 - y_1^*} F(I_2 + Q_2 - \xi_2) f(\xi_2) d\xi_2 \quad (1-15)$$

Therefore, the second-order derivative of $M_2(I_2, Q_2)$ with respect to Q_2 is,

$$\frac{d^2 M_2(I_2, Q_2)}{dQ_2^2} = -(p + s + h) \left[f(I_2 + Q_2) + \int_0^{I_2 + Q_2 - y_1^*} f(I_2 + Q_2 - \xi_2) f(\xi_2) d\xi_2 \right] \quad (1-16)$$

It shows from eq.(1-16) that the second-order derivative is negative. Moreover, it can be

derived from eq.(1-15) that $\left. \frac{dM_2(I_2, Q_2)}{dQ_2} \right|_{I_2 + Q_2 = y_1^*} = c > 0$ and $\lim_{I_2 + Q_2 \rightarrow \infty} \left. \frac{dM_2(I_2, Q_2)}{dQ_2} \right| < 0$.

Hence, there exists an optimal order-up-to level, denoted as y_2^* , that maximizes $M_2(I_2, Q_2)$, similar to the conclusion derived for the last-period problem.

3. $t = n$

Analogously, a complete dynamic programming that covers a general number of periods can be formulated as follows,

$$M_n(I_n, Q_n) = E_{\xi_n} \{ p \cdot \min(\xi_n, I_n + Q_n) - c \cdot Q_n - L(I_n + Q_n) \} + J_{n-1}(I_n, Q_n) \quad (1-17)$$

Appendix II Formulation Derivation

Compute the first-order derivative of $\int_0^{y_1 - y_2^*} F(y_1 - \xi) f(\xi) d\xi$ with respect to y_1 . ξ , is the independent variable.

Proof.

Suppose $F(y_1 - \xi)$ is continuous and is n th-order differentiable. Hence, it can be expanded with Taylor Series as,

$$F(y_1 - \xi) = \sum_i c_i (y_1 - \xi)^i \quad (2-1)$$

c_i , represent the constant term. Thus,

$$\begin{aligned} \int_0^{y_1 - y_2^*} F(y_1 - \xi) f(\xi) d\xi &= \int_0^{y_1 - y_2^*} \sum_i c_i (y_1 - \xi)^i f(\xi) d\xi \\ &= \int_0^{y_1 - y_2^*} \left[\sum_i F_{1i}(y_1) F_{2i}(\xi) + F_3(\xi) + F_4(y_1) \right] f(\xi) d\xi \end{aligned} \quad (2-2)$$

The second line in eq.(2-2) represent the polynomial expansions. Further,

$$\begin{aligned} &\left[\int_0^{y_1 - y_2^*} F(y_1 - \xi) f(\xi) d\xi \right]_{y_1} \\ &= \left[\sum_i F_{1i}(y_1) \right] \cdot \int_0^{y_1 - y_2^*} F_{2i}(\xi) f(\xi) d\xi + \sum_i F_{1i}(y_1) \cdot F_{2i}(y_1 - y_2^*) f(y_1 - y_2^*) \\ &\quad + F_3(y_1 - y_2^*) f(y_1 - y_2^*) + [F_4(y_1)]' \cdot \int_0^{y_1 - y_2^*} f(\xi) d\xi + F_4(y_1) \cdot f(y_1 - y_2^*) \end{aligned} \quad (2-3)$$

According to eq.(2-2), part of the terms in eq.(2-3) can be rewritten as,

$$\begin{aligned} &\left[\sum_i F_{1i}(y_1) \cdot F_{2i}(y_1 - y_2^*) + F_3(y_1 - y_2^*) + F_4(y_1) \right] \cdot f(y_1 - y_2^*) \\ &= F(y_2^*) \cdot f(y_1 - y_2^*) \quad (\text{set } \xi = y_1 - y_2^*) \end{aligned} \quad (2-4)$$

Moreover, the remaining two terms in eq.(2-3) can be rewritten as,

$$\begin{aligned}
 & \left[\sum_i F_{li}(y_1) \right]' \cdot \int_0^{y_1-y_2^*} F_{2i}(\xi) f(\xi) d\xi + [F_4(y_1)]' \cdot \int_0^{y_1-y_2^*} f(\xi) d\xi \\
 &= \int_0^{y_1-y_2^*} \left\{ \left[\sum_i F_{li}(y_1) \right]' \cdot F_{2i}(\xi) + [F_4(y_1)]' \right\} f(\xi) d\xi
 \end{aligned} \tag{2-5}$$

According to eqs.(2-1) and (2-2), it can be further derived that

$$\begin{aligned}
 F(y_1 - \xi) &= \sum_i c_i (y_1 - \xi)^i = \sum_i F_{li}(y_1) F_{2i}(\xi) + F_3(\xi) + F_4(y_1) \\
 \Rightarrow [F(y_1 - \xi)]'_{y_1} &= \left[\sum_i F_{li}(y_1) \right]' \cdot F_{2i}(\xi) + [F_4(y_1)]'
 \end{aligned} \tag{2-6}$$

With eq.(2-6), eq.(2-5) becomes,

$$\begin{aligned}
 & \left[\sum_i F_{li}(y_1) \right]' \cdot \int_0^{y_1-y_2^*} F_{2i}(\xi) f(\xi) d\xi + [F_4(y_1)]' \cdot \int_0^{y_1-y_2^*} f(\xi) d\xi \\
 &= \int_0^{y_1-y_2^*} [F(y_1 - \xi)]'_{y_1} \cdot f(\xi) d\xi = \int_0^{y_1-y_2^*} f(y_1 - \xi) \cdot f(\xi) d\xi
 \end{aligned} \tag{2-7}$$

Therefore, based on the expressions in eqs.(2-4) and (2-7), it can be derived that,

$$\begin{aligned}
 & \left[\int_0^{y_1-y_2^*} F(y_1 - \xi) f(\xi) d\xi \right]'_{y_1} \\
 &= F(y_2^*) \cdot f(y_1 - y_2^*) + \int_0^{y_1-y_2^*} f(y_1 - \xi) f(\xi) d\xi
 \end{aligned} \tag{2-8}$$

This concludes the derivation for this section. ■

Appendix III Proofs for Propositions

1. Proofs for Proposition 3-1

A Nash equilibrium is a profile of strategies with which each player's strategy is an optimal response to the other player's strategies (Fudenberg and Tirole, 1993). In other words, when solutions form Nash equilibrium, no player in the system could increase its profits by unilaterally deviating from the equilibrium.

Since Q_{di}^* is supplier i 's optimal delivery quantity, $\min\{Q_{di}^*\} \forall i \neq 0$ represents the critical supplier's delivery. Therefore, all the suppliers can only be paid up to the quantity of $\min\{Q_{di}^*\}$ even if Q_{di}^* is the quantity that could maximize their expected profits. Hence, suppliers do not benefit from delivering more than $\min\{Q_{di}^*\}$.

In case a supplier deliberately delivers a quantity lower than $\min\{Q_{di}^*\}$, the supplier becomes the new "critical supplier" and the assembler's purchasing quantity is limited by the supply of the new critical supplier. Therefore, it is concluded that all the quantities placed within $[0, \min\{Q_{di}^*\}]$ for $i \neq 0$ are Nash equilibria.

However, $\min\{Q_{di}^*\} \forall i \neq 0$ is the Pareto optimal solution. It is noted from eq.(3-8) that supplier i 's first-order derivative is positive when $Q_{di} < Q_{di}^*$; hence, its expected profit increases in its production quantity when $Q_{di} \leq \min(Q_{di}^*)$. Since suppliers can only sell up to $\min(Q_{di}^*)$, this concludes the demonstration for Proposition 3-1 ■

2. Proofs for Proposition 3-3

From eqs.(3-12) and (3-3), it is noted that $\frac{p - \sum_{i=1}^N w_{di} - c_0 + s}{p + s} < \frac{p - \sum_{i=1}^N c_i + s}{p + s}$ with the assumption of $w_{di} > c_i$. Hence, it is derived that $Q_{d\psi}^* < Q_c^*$. The following two scenarios are considered,

- i.) if $Q_{d\psi}^* \leq Q_{de}^*$, then $Q_{d0}^* = Q_{d\psi}^* < Q_c^*$;
- ii.) if $Q_{d\psi}^* > Q_{de}^*$, then $Q_{d0}^* = Q_{de}^* \leq Q_{d\psi}^* < Q_c^*$.

Therefore, it can be concluded that $Q_{d0}^* < Q_c^*$, which represents a reduction in the assembler's purchasing quantity in the price-only decentralized control setting.

The expected system profit in the price-only decentralized setting can be formulated as,

$$\sum_{i=0}^N \pi_{di} = E_{\xi} \left\{ p \cdot \min(\xi, Q_{d0}^*) - \sum_{i=0}^N c_i \cdot Q_{d0}^* - s(\xi - Q_{d0}^*)^+ - \sum_{i=1}^N c_i \cdot (Q_{di}^* - Q_{d0}^*) \right\}$$

the first three terms have the same format as that in the total expected profit for the

centralized setting (eq.(3-1)). Denote $\pi(Q) = E_{\xi} \left\{ p \cdot \min(\xi, Q) - \sum_{i=0}^N c_i \cdot Q - s(\xi - Q)^+ \right\}$. It is

noted that $\pi(Q)$ increases in Q when $Q < Q_c^*$ (because the first-order derivative of

$\pi(Q)$ with respect to Q is positive in the range of $(0, Q_c^*)$). As it has been proved that

$Q_{d0}^* < Q_c^*$ and the value of $\sum_{i=1}^N c_i \cdot (Q_{di}^* - Q_{d0}^*)$ is positive, it can be inferred that the total

expected profit in the decentralized setting is less than that of the centralized setting. This

concludes the demonstration for Proposition 3-3. ■

3. Proofs for Proposition 3-6

Replacing $\{w_{bi}, \tau_i\}$ in eqs.(3-14) and (3-16) with the expressions in eq.(3-31); then, comparing the consequent results with the expressions in eqs.(3-23) and (3-25). It is found out that the same expressions can be attained. This concludes the demonstration for Proposition 3-6. ■

4. Proofs for Proposition 4-3

It can be easily derived that $y_{d01} < S_{31} < S_{21} < S_{11}$ under the assumption of $w_{d1} > c_i$ ($i=1,2$). However, there is not a fixed relationship between y_{de1} and S_{i1} ($\forall i$) because y_{de1} depends on the negotiable wholesale prices between the assembler and its suppliers.

Set the optimal policy derived for the centralized setting as a benchmark, the corresponding optimal policy for the price-only decentralized setting can be derived accordingly given the same initial inventory level. Since it is always optimal to set $Q_{d01} = Q_{d11} = Q_{d21} = 0$ when $I_{01} \geq y_{d01}$, in order to avoid repetition, only the optimal policy under the condition of $I_{01} < y_{d01}$ is presented in all the following scenarios except for the first scenario.

i.) If $I_{01} \geq S_{11}$

Since $I_{01} \geq S_{11} > y_{d01}$, then it is derived that $Q_{d11} = Q_{c11} = 0$ (for $i=0,1,2$).

ii.) If $S_{11} - I_{01} \leq I_{11} < I_{21}$

$$\text{when } I_{01} < y_{d01} \Rightarrow \begin{cases} \text{if } y_{d01} \leq y_{de1}, Q_{d01} = y_{d01} - I_{01}, Q_{d11} = Q_{d21} = 0 \\ \text{if } y_{d01} > y_{de1}, Q_{d01} = y_{d01} - I_{01}, Q_{d11} = Q_{d21} = 0 \end{cases}$$

iii.) If $S_{21} \leq I_{11} + I_{01} < S_{11}$

$$\text{when } I_{01} < y_{d01} \Rightarrow \begin{cases} \text{if } y_{d01} \leq y_{de1}, Q_{d01} = y_{d01} - I_{01}, Q_{d11} = Q_{d21} = 0 \\ \text{if } y_{d01} > y_{de1}, Q_{d01} = y_{d01} - I_{01}, Q_{d11} = Q_{d21} = 0 \end{cases}$$

iv.) If $S_{11} > I_{11} + I_{01}$ and $I_{11} + I_{01} < S_{21} < I_{21} + I_{01}$

when $I_{01} < y_{d01} \Rightarrow$

$$\left\{ \begin{array}{l} \text{if } y_{d01} \leq y_{de1} \Rightarrow Q_{d01} = y_{d01} - I_{01}, Q_{d21} = 0, \text{ and } \begin{cases} \text{if } I_{11} \geq y_{d01} - I_{01}, Q_{d11} = 0 \\ \text{if } I_{11} < y_{d01} - I_{01}, Q_{d11} = y_{d01} - I_{01} - I_{11} \end{cases} \\ \text{if } y_{d01} > y_{de1} \Rightarrow Q_{d21} = 0, \begin{cases} \text{if } I_{11} \geq y_{d01} - I_{01}, Q_{d01} = y_{d01} - I_{01}, Q_{d11} = 0 \\ \text{if } \begin{cases} y_{de1} - I_{01} < I_{11} < y_{d01} - I_{01}, Q_{d01} = I_{11}, Q_{d11} = 0 \\ I_{11} < y_{de1} - I_{01} < y_{d01} - I_{01}, Q_{d01} = y_{de1} - I_{01}, Q_{d11} = y_{de1} - I_{01} - I_{11} \end{cases} \end{cases} \end{cases} \right.$$

v.) If $I_{11} < I_{21} < S_{11} - I_{01}$ and $S_{31} < I_{21} + I_{01} < S_{21}$

when $I_{01} < y_{d01} \Rightarrow$

$$\left\{ \begin{array}{l} \text{if } y_{d01} \leq y_{de1} \Rightarrow Q_{d01} = y_{d01} - I_{01}, Q_{d21} = 0, \text{ and } \begin{cases} \text{if } I_{11} \geq y_{d01} - I_{01}, Q_{d11} = 0 \\ \text{if } I_{11} < y_{d01} - I_{01}, Q_{d11} = y_{d01} - I_{01} - I_{11} \end{cases} \\ \text{if } y_{d01} > y_{de1} \Rightarrow Q_{d21} = 0, \begin{cases} \text{if } I_{11} \geq y_{d01} - I_{01}, Q_{d01} = y_{d01} - I_{01}, Q_{d11} = 0 \\ \text{if } \begin{cases} y_{de1} - I_{01} < I_{11} < y_{d01} - I_{01}, Q_{d01} = I_{11}, Q_{d11} = 0 \\ I_{11} < y_{de1} - I_{01} < y_{d01} - I_{01}, Q_{d01} = y_{de1} - I_{01}, Q_{d11} = y_{de1} - I_{01} - I_{11} \end{cases} \end{cases} \end{cases} \right.$$

vi.) If $I_{11} + I_{01} < I_{21} + I_{01} < S_{31}$

when $I_{01} < y_{d01} \Rightarrow$

$$\left\{ \begin{array}{l} \text{if } y_{d01} \leq y_{de1} \Rightarrow Q_{d01} = y_{d01} - I_{01}, \begin{cases} \text{if } I_{11} \geq y_{d01} - I_{01} \Rightarrow Q_{d11} = Q_{d21} = 0 \\ \text{if } I_{11} < y_{d01} - I_{01} < I_{21} \Rightarrow Q_{d11} = y_{d01} - I_{01} - I_{11}, Q_{d21} = 0 \\ \text{if } I_{11} < I_{21} < y_{d01} - I_{01} \Rightarrow \begin{cases} Q_{d11} = y_{d01} - I_{01} - I_{11} \\ Q_{d21} = y_{d01} - I_{01} - I_{21} \end{cases} \end{cases} \\ \text{if } y_{d01} > y_{de1} \Rightarrow \begin{cases} \text{if } I_{11} \geq y_{d01} - I_{01} \Rightarrow Q_{d01} = y_{d01} - I_{01}, Q_{d11} = Q_{d21} = 0 \\ \text{if } I_{21} < y_{de1} - I_{01} < y_{d01} - I_{01} \Rightarrow \begin{cases} Q_{d01} = y_{de1} - I_{01} \\ Q_{d11} = y_{de1} - I_{01} - I_{11} \\ Q_{d21} = y_{de1} - I_{01} - I_{21} \end{cases} \\ \text{if } I_{11} < y_{de1} - I_{01} < y_{d01} - I_{01} < I_{21} \Rightarrow \begin{cases} Q_{d01} = y_{de1} - I_{01} \\ Q_{d11} = y_{de1} - I_{01} - I_{11} \\ Q_{d21} = 0 \end{cases} \\ \text{if } y_{de1} - I_{01} < I_{11} < y_{d01} - I_{01} < I_{21} \Rightarrow Q_{d01} = I_{11}, Q_{d11} = Q_{d21} = 0 \end{cases} \end{array} \right.$$

It is noted from the above that, for each scenario, the derived optimal policy for the price-only decentralized control setting is different from that of the corresponding centralized control setting (refer to the simplified Proposition 4-1) except for the first scenario when $I_{01} \geq S_{11}$. Under the condition of $I_{01} \geq S_{11}$, the same optimal policy can be derived for both control settings; hence, the same total expected system profit can be generated. Therefore, it

can be deducted that the total expected system profit for the price-only decentralized setting is no larger than that of the centralized setting, which concludes the demonstration for Proposition 4-3. ■

5. Proofs for Proposition 4-5

With eq.(4-111) and Lemma 4-3, the assembly system with decentralized control mode has the same optimal policy as that for the centralized setting (as given in Proposition 4-2). This concludes the demonstration for Proposition 4-5. ■

Appendix IV Research Communication

- Zou, X., Pokharel, S, Piplani, R. (2004). Channel coordination in an assembly system facing uncertain demand: synchronizing process time and delivery quantity. International Journal of Production Research, Vol. 42, No. 22, pp. 4673-4689.
- Zou, X., Pokharel, S, Piplani, R. (2005). A contract model for a decentralized assembly system. Sixteenth Annual Conference of POMS, April 29 - May 2, 2005, Chicago, IL.
- Zou, X., Pokharel, S, Piplani, R. (2005), Coordination in a two-echelon decentralized assembly system: impact of wholesale price and buyback contract, Sixth asia-pacific industrial engineering and management systems conference, Philippines.