Nonlinear system identification method for structural health monitoring : techniques for the detection of nonlinear indicators

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Nonlinear System Identification Method for Structural Health Monitoring: Techniques for the Detection of Nonlinear Indicators

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ABSTRACT

With the progress of signal processing technologies, structural health monitoring (SHM) has received more and more attentions. The core algorithm in SHM is based on the detection of damage-sensitive indicator. In the recent decades, engineers already have the ability to deal with nonlinear problem. A literature survey of nonlinear indicators is firstly examined in the study. It is found that a successful SHM requires the monitoring technologies have their flexibility, simplicity, and, of course, accuracy. The nonparametric system identification method is a potential candidate which can meet these requirements. Therefore, several nonlinear indicators corresponding to the nonparametric system identification method are studied in this research, both from frequency and time domain analysis. In this research, the frequency-domain nonlinear indicators included: (1) Hilbert transform of frequency response function, (2) coherence function, (3) Hilbert marginal spectrum, (4) wavelet packet transform component correlation coefficient, and (5) bispectral analysis; and the time-domain nonlinear indicators included: (1) instantaneous frequency, (2) instantaneous phase difference, (3) Holder exponent, (4) discrete wavelet transform, and (5) singular spectrum analysis (SSA).

Test data from a series of shake table test to the 1-story 2-bay RC frame is generated from NCREE (National Center for Research on Earthquake Engineering), Taiwan. For these shake table tests data from two groups of specimens are analysed using the proposed nonlinear indicators. The first group of seismic response data is to consider the response from different specimen subjected to different level of

seismic excitation (TCU082). The second group of data is to examine the damage level through a series of excitation back to back on a specimen. In cooperation with the experimental data, the result shows that nonlinear indicators can provide the identification of structural nonlinearities, which include stiffness degradation and cracks. Finally, the singular spectrum analysis (SSA) technique was used to extract structural residual deformation and to eliminate the noise effect. Furthermore, the SSA method can be used to derive residual displacement using the measured acceleration signal if there is no information on displacement measurement. In this thesis, a deeper realization to nonlinear indicators can be achieved. And it is possible to execute an effective SHM and nonlinear identification of the structure directly from the measurement by using appropriate nonlinear indicators.

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CHAPTER 1 INTRODUCTION

1.1 Background

Structural system identification and damage detection have received more and more attentions in the field of civil engineering. Natural and man-made disasters and contrived destructions, e.g. the 921 Chi-Chi Earthquake in Taiwan and the 911 terrorism event in New York City, raised the awareness for the safety of structures and properties. In order to minimise the loss under such extreme events, it is urgent to develop techniques for structural health monitoring and damage assessment to give an early warning of possible structural damages. Structural system identification is one of the techniques for structural damage evaluation.

Generally, structural system identification can be classified as two categories: linear and nonlinear system identification (ID). Linear system ID is under the assumption that the system behaviour is linear before and after the presence of external excitations. In view of the vibration-based identification method, various linear damage detection schemes have been developed and are summarised in the reference (Doebeling, Farrar et al. 1998). On the other hand, structures may undergo nonlinear behaviour when they are subjected to severe external loadings. This nonlinear phenomenon may be caused by many sources, for example, geometric nonlinearity due to large displacement and the material nonlinearity when the constitutive law of material relating strain and stress is nonlinear. The nonlinearity may also arise in a damaged structure, e.g. stiffness degradation and strength deterioration of structural member, opening and closing of cracks, or loosened bolts.

Due to the complexity of nonlinear mechanics, it is difficult to have a specific model to describe all the nonlinear conditions.

Most civil engineering structures exhibit hysteresis behaviour accompanied by stiffness degradation, strength deterioration or pinching phenomenon when undergoing significant deformations induced by severe external excitations. In the past, due to limited knowledge and practicability consideration, engineers used to adopt the linear model to simplify the complicated phenomenon in the real-life. The linear model provides accurate predictions of structural behaviour only with a small amplitude response although it is not always the case (e.g. dry friction). Such assumption limited the engineering design to a very conservative regime. The fact is that any simulation using linear system assumption is doomed to fail under extreme events. Damage identification incorporated with nonlinear system identification is, therefore, demanded.

In the reference (Kerschen, Worden et al. 2006), a tutorial of discriminating linear and nonlinear and a comprehensive review of literatures related to nonlinearity identification were discussed. The main feature of nonlinear vibration is that the instantaneous frequency and damping coefficient of a system can be represented in terms of a time-varying function according to the type of nonlinearity. The time-frequency analysis provides a promising tool to solve such non-stationary signals and to estimate the model parameters of the time-varying system. The short time moving window Fourier transform (STFT) gives an inspiration to capture the temporal characteristic by utilizing the time-moving window technique. But STFT has very poor resolution on time-frequency plane and can not provide a further analysis for non-stationary and nonlinear data. Wavelet analysis is then developed

with its versatility for a better resolution. Wavelet analysis has an adjustable window technique which is capable of structural damage detection. It can also unveil discontinuities masked in response signals or to decompose the original signal into several sub-components. The versatility turns the wavelet technique into a powerful signal processing tool for structural response analysis. An overview of the applications of wavelet analysis for system identification and damage detection can be found in the reference (Peng and Chu 2004; Reda Taha, Noureldin et al. 2006).

It is believed that with the application of appropriate signal processing techniques one can identify the structural nonlinear behaviours by analysing the structural input and output signals. If the nonlinearity can be qualified as well as quantified, an early warning or an estimation of damage repair can be, therefore, achieved.

1.2 Literature Review

Two classes of system identification methods are reviewed: (1) system identification without knowing a priori model and (2) system identification with known mathematical model.

1.2.1 System Identification without Knowing a Priori Model

Signal processing techniques will be the major analytical tools for system identification without knowing a priori model. It is simple to detect the damage by implementing signal processing through the comparison between the intact data and

the damaged data. Sequentially, damage indicators are derived from the results of these signal processing techniques and can be used for structural damages assessment. The most representative damage indicator is the wavelet packet component energy which is defined as follows (Yen and Kuo 2000):

$$e_{j,n} = \sum_k c_{j,n,k}^2 \quad (1.2.1)$$

which represents the component energy at the j -th level and n -th node. Notation ' c ' represents the wavelet packet coefficient and ' k ' is the translation parameter. The damage indicators selected in the references (Sun and Chang 2002; Yam, Yan et al. 2003; Li, Sun et al. 2004; Han, Ren et al. 2005; Diao, Li et al. 2006) are all based on the component energy. The wavelet packet component energy was fed into two neural networks to determine the presence and the extent of damage respectively (Sun and Chang 2002). A damage feature called wavelet packet energy rate index (WPERI) was defined as a summation of the rate of the shift between pre- and post-damaged component energy in each node (Han, Ren et al. 2005). A combination of the wavelet technique and the neural network was also proposed (Diao, Li et al. 2006). There are other damage indicators such as the instantaneous phase (Salvino, Pines et al. 2003). The instantaneous phase is derived from a mono-component signal decomposed by Hilbert-Huang transform. The difference of the instantaneous phase between the intact system and the damaged system can be used as a damage indicator for assessing the damage severity. In summary, the limitations of this class of nonlinear indicators are: (i) intact system response is required (i.e. a baseline is needed); and (ii) the excitation in pre- and post-damaged states should be the same, e.g. a continuous constant impulse or random excitations. The advantage of this class of nonlinear indicators is that system parameters can be

ignored.

Signal processing technique may also be applied to structural response accompanied with a small discontinuity in responses. The Holder exponent was introduced to detect the singularity in structural responses (Robertson, Farrar et al. 2003). The anomalous behaviour of a bridge was observed from a long-term monitoring data by employing DWT (Discrete Wavelet Transform) and plotting cross-wavelet spectrum (Moyo and Brownjohn 2002). Several damage indicators, namely cross-wavelet power spectrum, first derivative of instantaneous frequency, and Holder exponent, were applied to a seismic response signal of an experimental reinforced concrete frame. (Ku 2005). The main assumption for this class of nonlinear indicators is that the signal must contain the occurrence of damage. The attractive advantages is that no baseline is required which make this class of method practical for the application to real data.

Another nonlinear indicators based on wavelet techniques are introduced in the reference (Staszewski 2000), which includes wavelet ridges and skeletons, damping estimation procedure and wavelet-based FRF. Basu (Basu 2005) proposed a new concept to examine the system nonlinearity by the correlation of wavelet coefficient between input data and output data. In the reference (Vanhoeacker, Schoukens et al. 2002), several simple nonlinear indicators are summarized. These indicators include the coherence function, correlation tests, and bispectral analysis, which are all potential to reveal the nonlinearity in the system.

1.2.2 System Identification with Known Mathematical Model

One of the most common ways of system identification is to give a

mathematical model to represent the structural system, either linear or nonlinear model, and based on the input and output measurements to identify the model parameters. According to the reference (Kerschen, Worden et al. 2006), the identification procedure can be roughly divided into 3 steps: (1) Detect the system nonlinearity; (2) Characterise the location, the type and the functional form of the nonlinearity; (3) Determine the model parameters of the nonlinear system. Depends on the proposed model, the nonlinear system identification methods are summarised as follows:

Equivalent linear system model

Instead of using nonlinear model to represent the nonlinear structural system, the equivalent linear system is used. Although the estimation is not exact, it provides an overview of the time-varying system parameters. Tsao (Tsao 2008) utilised the equivalent linear system to examine the nonlinearities of hysteresis systems.

Nonlinear system model

The *restoring force surface (RFS) method* is a representative of this class of system identification method. RFS method starts from Newton's second law and assume that the mass is known beforehand. The restoring force can be represented in terms of input acceleration excitation and absolute acceleration response. A restoring force surface is then constructed on a phase plane (displacement-velocity domain). Polynomial expansion like the double Chebyshev is then applied to fit the restoring surface, and the stiffness curve can be derived. RFS method is rather simple to be implemented but can not be used for hysteresis systems since the restoring force is not only in terms of displacement and velocity. Although

extension has been developed in the reference (Yar and Hammond 1987) to cover the hysteresis system such as Bouc-Wen model (Wen 1976), the procedure is complicated.

A generalisation of ARMA (Auto-Regressive Moving Average) model to nonlinear cases is called *NARMAX* (Nonlinear ARMA with eXogeneous input) *model*. NARMAX can be employed in many nonlinear models without knowing the physical meaning of the system. Another time-domain method is based on Hilbert transform which is known as *FORCEVIB* approach proposed in the reference (Feldman 1994). This approach is also feasible for a large range of system models. However, this approach is only suitable for a mono-component signal.

The Bouc-Wen model (Wen 1976) is a versatile mathematical model which can simulate most behaviours of the hysteresis system. The parameters in the Bouc-Wen model control different hysteresis behaviours such as stiffness degradation, strength deterioration and pinching effect. A wavelet-based system identification method was proposed (Chang and Shi 2008) to identify these hysteretic parameters. With absolute acceleration measurements as well as input acceleration excitation measurements, the time-varying hysteresis parameter can be tracked by the orthogonal forward regression algorithm,.

The nonlinear system identification methods mentioned above are all well-assembled schemes. However, some of them become too complicate to be implemented straightforwardly. Physical explanation is needed to be made.

1.3 Research Scope and Objectives

The nonlinearity of a reinforced concrete (RC) structure is well-known for its hysteresis behaviour when the structure is subjected to large external loading. Although the hysteresis property makes concrete a good material for energy dissipation, the mathematical model of restoring force is also very complicated. It is rather easy if the system is in its linear stage and the assumption of linearity can be made in order to identify the system parameters from the measured input and output. But it is not the case for nonlinear systems. Obstacles are the inapplicability of superposition law, the highly ‘individualistic’ nature and the unknown transfer function which connects the input and the output for most practical cases. Thus, for a RC structure, it is difficult to track the exact hysteretic behaviours by adopting an analytical model. From the system identification point of view, even if the analytical model is well-established, the required computations may be very considerable to identify the model parameters. A concept to overcome the complexity of the nonlinear system identification, with the implementation of signal processing techniques directly to the system response measurements to extract the so-called nonlinear indicators, is proposed in this study. By observing the variation of the nonlinear indicators, one can identify the nonlinear behaviours in the RC structure.

The objective of this research is to conduct a survey on several nonlinear indicators and to apply these nonlinear indicators to a series of seismic response data of RC frame to identify the degree of nonlinearity. The advantages of using nonlinear indicators have the following potentials: (1) sensitive to the class of nonlinearities in RC structures, (2) low computation effort, and (3) meaningful to physical parameters. Through this survey, it is expected that one can identify the

severity of physical structural nonlinearity and its damage severity.

The organization of this research is briefly described as follows.

Chapter1: A brief description of the research background is firstly given. Then, a literature survey on the state of the art of the existing identification techniques is presented.

Chapter2: Some basic signal processing techniques are introduced at the beginning in this chapter. Several nonlinear indicators, which have potentials for utilising in nonlinearity identification techniques, are derived theoretically. A simulation test is also employed to confirm the feasibility of these nonlinear indicators.

Chapter3: Seismic response data of a two-bay one-storey RC frame is used for identification study. The experimental design and test procedures in NCREE (The National Centre for Research on Earthquake Engineering, Taiwan) are described also. Analysis to the experimental data by the proposed indicators is demonstrated in this chapter. Following the discussion of each nonlinear indicator, a practical estimating procedure is also given.

Chapter4: Summaries and suggestions for the use of the proposed indicators will be given here. The potential research topics are indicated at the end.

CHAPTER 2 NONLINEAR INDICATORS: METHODOLOGY

2.1 General Description

From literature review, there are various nonlinear identification methods which are effective to detect nonlinear behaviours of structure. Basically, these identification methods might be classified into two categories: *parametric methods* and *non-parametric methods*. In parametric identification, mathematical model of the structure is assumed as a priori and structural model parameters are identified through optimization technique. In nonparametric identification, no (or very little) assumption is made with respect to the model structure. So the above mentioned non-parametric identification approaches do not require knowledge of structural physical model. Generally, physical model will be used for the parametric identification method.

The most commonly used parametric model of the dynamic system is to assume the structural system as the equivalent linear system with constant stiffness and damping. As for the non-parametric identification method, both time domain and frequency domain methods are used without using any physical model. From the viewpoint of non-parametric identification method, one approach is to develop the nonlinear indicators. Either time-domain or frequency-domain analysis can be used to represent the nonlinear indicators. In this study, the following indicators will be introduced:

A. Frequency Domain Non-parametric Identification Method

1. Hilbert Transform of Frequency Response Function

2. Coherence Function
3. Hilbert Amplitude Spectrum
4. WPT Component Correlation Coefficient
5. Bispectral Analysis

B. Time Domain Non-parametric Identification Method

1. Instantaneous Frequency
2. Instantaneous Phase Difference
3. Holder Exponent
4. Level-1 Detail Component of Discrete Wavelet Transform
5. Residual Component of Singular Spectrum Analysis (SSA)

To identify the degree of nonlinearity of the dynamic system from both input and output or output only signals, the indicators listed above can provide information for identifying the system nonlinearity as well as damage severity of the dynamic system. In this chapter, two basic signal processing tools will be introduced to develop the nonlinear indicator: the wavelet analysis and the Hilbert transform. Besides, in order to establish the correlation of these nonlinear indicators to the physical parameters, a numerical simulation study is conducted.

2.1.1 Simulation Studies

Consider a single-degree-of-freedom non-stationary system which included changes in system natural frequency and the interference of impulse force (Pai and Aaron 2009). The details of the equation of motion of the system are shown as follows:

$$m\ddot{u} + c\dot{u} + rku = m(\ddot{u} + 2\zeta\omega\dot{u} + r\omega^2 u) = F$$

$$m = 3 \text{ (kg)}, \quad \zeta = 0.01, \quad \omega = 2\pi, \quad t_r = 0.05 \text{ (sec)}$$

$$r = \begin{cases} 0.98 - 0.02 \tanh[(t-10)/t_r], & \text{for } 0 < t \leq 12.5 \\ 0.94 - 0.02 \tanh[(t-15)/t_r], & \text{for } 12.5 < t \leq 17.5 \\ 0.90 - 0.02 \tanh[(t-20)/t_r], & \text{for } 17.5 < t \end{cases} \quad (2.1.1)$$

$$\text{Boundary Condition: } \begin{cases} u(0) = 5 \text{ (m)} \\ \dot{u}(0) = 0 \end{cases}$$

where $\tanh[(t-t_a)/t_r]$ represents a function changes from -1 to 1 around $t = t_a$ with a transient period defined by t_r . The external force F equals to zero except an impulse force $F = -20N$ is introduced between $25\text{sec} \leq t \leq 25.05\text{sec}$. $\Delta t = 0.05\text{sec}$ is used in this test. A normal distributed zero-mean noise with standard deviation of 0.003 is added to the response signal. The nonlinearity in this simulated system can be defined as: (1) nonlinear degradation of system frequency of 2% at 10 sec, 15 sec and 20 sec, respectively; (2) nonlinear behaviour of system response due to abrupt external loading. The displacement response and its Fourier spectrum are shown in Figure 2-1. The objective of this test is to track the variation or to detect the presence of the shifts of the system parameters by applying the nonlinear indicators presented in this chapter. Hence, one can utilise these indicators and conduct the damage assessment of the dynamic system.

2.2 Basic Signal Processing Tools

2.2.1 Fourier Transform

The Fourier transform rules over linear time-invariant signal processing since

sinusoidal waves $e^{i\omega t}$ are eigenvectors of linear time-invariant operators. A linear time-invariant operator L is entirely specified by the eigenvalues $\lambda(\omega)$:

$$\forall \omega \in \mathfrak{R}, \quad L e^{i\omega t} = \lambda(\omega) e^{i\omega t} \quad (2.2.1)$$

where $\mathfrak{R}$ refers to real numbers. To compute Lf , a signal f is decomposed as a sum of sinusoidal eigenvectors:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{i\omega t} d\omega \quad (2.2.2)$$

If f has finite energy, the theory of Fourier integrals (Mallat 1999) proves that the amplitude $\hat{f}(\omega)$ of each sinusoidal wave is the Fourier transform of f :

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (2.2.3)$$

Applying the operator L to f in Equation (2.2.2) and inserting the eigenvector expression (2.2.1):

$$Lf(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \lambda(\omega) e^{i\omega t} d\omega \quad (2.2.4)$$

The operator amplifies or attenuates each sinusoidal component $e^{i\omega t}$ of f by $\lambda(\omega)$. It is a frequency filtering of f .

As described above, the Fourier transform theory is a linear time-invariant operator, therefore, if transient phenomena are encountered, Fourier transform becomes a cumbersome tool. Only averaged result can be derived by Fourier transform whereas temporal information will loss. It is, therefore, required a time-frequency signal processing technique, namely the wavelet technique, to solve the transient problem.

2.2.2 Overview of Wavelet

Wavelet analysis is a fast-evolving signal processing tool over the past few decades. In 1984, Morlet proposed a novel concept for dealing with a resolution problem in reflection seismology. Instead of feeding high frequency pulse in equal duration, Morlet only used shorter waveform at high frequencies. This idea attracted Grossmann who devoted to the research of coherent quantum states. With the help of Grossmann, Morlet formalized the well-known continuous wavelet transform (CWT) (Grossmann and Morlet 1984). Later, Meyer and Mallat proposed the multi-resolution analysis (MRA) which is based on orthogonal wavelet basis (Mallat. 1989). Daubechies also contributed to develop wavelet theory and constructed a simple and useful orthogonal wavelet basis. Under the effort of Mallat as well as Daubechies, the theory of discrete wavelet transform (DWT) was completed (Daubechies 1988). In 1992, the wavelet packet transform (WPT), an advanced version of MRA, was proposed (Coifman and Wickerhauser 1992). The idea of wavelet theory is not totally new since similar time-frequency analysis technique, namely STFT, was already proposed by Gabor in 1946 (Gabor 1946). Nevertheless, it is a familiar subject for researchers in different areas so that wavelet theory can be completed in such a short period.

2.2.3 Continuous Wavelet Transform (CWT)

The continuous wavelet transform (CWT) is defined as follows:

$$W_{\psi} f(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \bar{\psi} \left(\frac{t-b}{a} \right) dt \quad (2.2.5)$$

In Equation (2.2.5), W_ψ is the continuous wavelet transform operator to a measurable signal $f(t)$ where the variable ' t ' usually represents time or space. $\psi(t)$ is the mother wavelet and the bar over the basis refers to the complex conjugate. The formulation can be understood that the wavelet is centred at ' b ' and spreads by dilation ' a '. The idea of CWT is fitting different scaled wavelets across the entire signal. The scaled wavelet is called the wavelet family which is derived from the mother wavelet. For example, an ' s ' scaled normalized wavelet is written as:

$$\psi_{s,u} = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right) \quad (2.2.6)$$

This wavelet member then shifts the temporal parameter ' u ' to cover the whole signal.

The result of CWT is called the wavelet coefficient. The wavelet coefficient can be viewed as the correlation of the scaled wavelet basis and the original signal in the windowed region. The higher the value of wavelet coefficient, the more similar the wavelet basis is to the original signal. One can observe the variation of wavelet coefficient from the wavelet modulus map defined as follows:

$$WM_\psi f(s,u) = |W_\psi f(s,u)| \quad (2.2.7)$$

which is, generally, a time-scale plane with different colour patterns to represent different intensities of the modulus. It should be noted that different mother wavelet function produces different CWT results. Therefore, the selection of an appropriate wavelet basis is an important issue which will be discussed in section 2.2.6.

According to Heisenberg's uncertainty principle, a higher resolution in frequency accompanies a lower time resolution and vice versa. The adaptation of wavelet analysis, a time-scale analysis, improves this problem since the scale is

inverse proportional to frequency. The switch from scale to frequency will be described in section 2.2.6.

2.2.4 Discrete Wavelet Transform (DWT)

In order to make wavelet analysis more feasible in practical application, a fast computational algorithm is required. The discrete wavelet transform (DWT) gives a promise solution. DWT is based on the orthogonal wavelet basis. In DWT, it is simply a substitution of parameters in Equation (2.2.5) by:

$$a = 2^j, b = k \cdot 2^j \quad (2.2.8)$$

The parameter 'j' refers to the decomposition level and 'k' is the shifting coefficient. DWT can be viewed as a filtering process which decomposes the signal into two components to next level. The component derived from low-pass filter is called approximation component which can describe the trend of signal:

$$c_{j,k} = 2^{-j/2} \int_R f(t) \bar{\phi}(2^{-j}t - k) dt \quad (2.2.9)$$

The scaling function $\phi(x)$ is derived from the mother wavelet $\psi(x)$ in order to reconstruct the lower-frequency part of the signal. Not every wavelet function has scaling function. Only orthogonal function has scaling function which permits the fast algorithm. On the other hand, the result of high-pass filter is the detail component:

$$d_{j,k} = 2^{-j/2} \int_R f(t) \bar{\psi}(2^{-j}t - k) dt \quad (2.2.10)$$

The multi-resolution analysis (MRA) is a technique which decomposes the approximation component to next level by DWT (Figure 2-2). Since DWT has the orthogonal property, the original signal can be reconstructed by linear combination

of all the approximation and detail components:

$$f(t) = \sum_{k=-\infty}^{\infty} c_{j,k} \phi(t) + \sum_{j=-\infty}^J \sum_{k=-\infty}^{\infty} d_{j,k} \psi(t) = A_J + \sum_{i=1}^J D_i \quad (2.2.11)$$

In Figure 2-2, ‘ A ’ indicates approximation (higher frequency components) and ‘ D ’ indicates detail (lower frequency components).

2.2.5 Wavelet Packet Transform (WPT)

There exists one problem in MRA: the resolution is low in high frequency since only approximation components are decomposed. The packet wavelet transform (WPT) is, therefore, developed to provide further decomposition at detail components (Figure 2-3). The wavelet packet is a function with three parameters:

$$\psi_{j,k}^i(t) = 2^{j/2} \psi^i(2^j t - k), \quad i = 0, 1, 2, \dots, 2^j - 1 \quad (2.2.12)$$

The parameters ‘ i ’, ‘ j ’ and ‘ k ’ refer to the modulation, scale and translation respectively. The wavelet ψ^i is obtained from the recursive relation:

$$\psi^{2i}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} h(k) \psi^i(2t - k) \quad (2.2.13)$$

$$\psi^{2i+1}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} g(k) \psi^i(2t - k) \quad (2.2.14)$$

It is noted that ψ^1 is the mother wavelet and the ψ^0 is equal to the scaling function ϕ .

Function $h(k)$ and $g(k)$ are quadrature mirror filters associated with the scaling function and wavelet basis respectively. Similarly, the reconstruction of WPT is a linear combination of decomposed components:

$$f(t) = \sum_{i=0}^{2^J-1} f_j^i(t) \quad (2.2.15)$$

The decomposed component f_j^i is the linear combination of wavelet packet function $\psi_{j,k}^i$:

$$f_j^i(t) = \sum_{k=-\infty}^{\infty} c_{j,k}^i \psi_{j,k}^i(t) \quad (2.2.16)$$

And the wavelet packet coefficient $c_{j,k}^i$ can be computed from:

$$c_{j,k}^i = \int_{-\infty}^{\infty} f(t) \psi_{j,k}^i(t) dt \quad (2.2.17)$$

providing the orthogonal condition:

$$\psi_{j,k}^m(t) \psi_{j,k}^n(t) = 0 \quad \text{if } m \neq n \quad (2.2.18),$$

It should be noted that the central frequency of wavelet component is not in ascending order resulted from the aliasing. A Paley order is, therefore, supposed to be considered if one tends to implement the WPT technique. The Paley ordering process is illustrated in Figure 2-4 so that the left node has a lower central frequency than its right node.

2.2.6 Implementation Guideline for Wavelet Analysis

In order to implement wavelet analysis correctly, some details should be reminded:

(a) Wavelet classification

Various wavelet bases have been developed in these few decades. In the commercial software MATLAB[®], it provides the most well-known wavelet basis functions. Roughly, they can be classified as followed:

1. The Morlet, Gaussian, Mexican hat, and Shannon wavelet has an infinite support. The scaling function does not exist in this class of wavelet so the DWT

and fast algorithm is not available. But the infinite regularity permits an exact reconstruction.

2. The Meyer wavelet is an orthogonal wavelet function without compact support. It implies that DWT is feasible with Meyer wavelet whereas the fast algorithm is impossible. An exact reconstruction is also possible due to the infinite regularity property.
3. The orthogonal wavelets, such as Haar, Daubechies and Coiflet wavelet, have compact supports and promise the DWT and fast algorithm. Haar wavelet is exactly the Daubechies wavelet with order 1 (denoted as DB1). With a higher order, the orthogonal wavelet has more degree of regularity. However, this class of wavelet basically has limited regularity so the exact reconstruction is not available. It is also noted that they are not symmetric except the Haar wavelet.
4. In order to improve the disadvantage of orthogonal wavelet, biorthogonal wavelet is developed. Similar to orthogonal wavelet, the DWT and fast algorithm is permitted for biorthogonal wavelet. The distinction is that biorthogonal wavelet is symmetric and it uses one scaling function and two wavelet functions. The two wavelet functions are utilised for decomposition and reconstruction respectively. Therefore, exact reconstruction is possible. The notation of a biorthogonal wavelet is typically ' $BiorN_rN_d$ ' where N_r represents the order of reconstruction wavelet and N_d is the order of decomposition wavelet. ' $Bior6.8$ ' wavelet is plotted in Figure 2-5 as an example.

(b) Selection of wavelet basis

It is emphasized that different wavelet results in different analysis result. Hence, an appropriate selection of wavelet basis is required. In practical application,

it lacks a systematic method for choosing the optimal wavelet basis. Trial and error is the most common method. Nevertheless, some guidelines can be followed while searching the optimal basis function (Torrence and Compo 1998; Ovanesova and Suárez 2004):

- (1) *Orthogonal or non-orthogonal.* If one plans to implement DWT or WPT, the selection would be limited to orthogonal wavelet basis. For engineering application, a fast algorithm is demanded. So, orthogonal wavelets, such as Daubechies and Haar, are usually adopted.
- (2) *Real or analytical.* The wavelet analysis with real wavelet function unveils discontinuities or isolated peaks in the signal. For those who want to observe the phase variation, an analytical wavelet can satisfy such requirement.
- (3) *Vanishing moment and Regularity.* The number of vanishing moment represents the largest order of polynomial expansion. Different wavelet function has different number of vanishing moment. For example, a Daubechies 4 wavelet (DB4) has 4 vanishing moments and a biorthogonal wavelet with order $N_r=6$ and $N_d=8$ (*Bior6.8*) has 6 vanishing moments. The regularity means the degree of continuous differentiability of the signal. For example, if the signal has regularity ' r ' which is an integer, the signal is r -time differentiable. The regularity ' r ' might not be an integer then the signal is n -time differentiable, where $n < r < n+1$, and its n -time derivative resembles $(x-x_0)^{r-n}$ locally around x_0 . These two properties can be extended for singularity detection such as Holder exponent (Mallat and Hwang 1992).
- (4) *Shape.* This criterion depends on the experience or the knowledge about the original signal. For example, it is suitable to fit a signal with sharp peaks by

Haar wavelet. The symmetry is another consideration when choosing the wavelet basis. A symmetric wavelet can avoid dephasing in image processing.

- (5) *Width*. After deciding a wavelet basis, there is another criterion needed to be concerned: the width of wavelet function. As mentioned in section 2.2.3, a wider time window, i.e. a wider support of wavelet function, gives higher resolution in frequency and lower resolution in time. Thus, one can increase the resolution in frequency by widening the mother wavelet. It should also be noted that the wider is the wavelet function, the more computation effort is required.

If one would like to apply the wavelet analysis for an instantaneous monitoring system, a fast algorithm is required. The orthogonal wavelet with compact support and biorthogonal wavelet are then remained. Secondly, if a good reconstruction is desired, wavelet with symmetric shapes will be selected. They are Haar and biorthogonal wavelet. The former has an inferior reconstruction due to the poor regularity. Therefore, the biorthogonal wavelet with higher order, says 'Bior6.8', is left as the optimal candidate.

(c) End-effect

If signals are finite-length sequences, the end-effect may be of concern. Since it does not assume that the signal is cyclic in wavelet analysis, error will occur at the beginning and the end point. The simplest solution is zero-padding method which creates virtual data with zero value after the end of signal in advance and removes them after the analysis.

(d) From scale to frequency

The parameter, scale, does not have any physical meaning. It is necessary to

convert the scale to frequency after the wavelet packet transform. For CWT, the transformation is accomplished by fitting a cosine wave with known frequency to the wavelet function at specific scale. One can find the approximated frequency f_c of specific wavelet function by adjusting the harmonic frequency to maximize the scalogram. Then, for scale a_0 with time interval dt , the transformation can be completed as followed:

$$f_0 = \frac{f_c}{(a_0 \cdot dt)} \quad (2.2.19)$$

For WPT to j -level, there are totally 2^j components with an equal bandwidth and the scale-frequency relationship is simply to divide the Nyquist frequency equally and to allot to each component. The central frequency of i th wavelet packet component is given by:

$$f_i = \frac{(2i-1)f_N}{2^{j+1}} \quad (2.2.20)$$

The central frequency f_i of i th component at level j is represented in terms of Nyquist frequency f_N , which is defined as a half of the sampling frequency.

2.2.7 Hilbert Transform (HT)

It is known that Fourier transform has a symmetric property due to the repeated information in both positive and negative frequency domain. Considering a real signal $x(t)$, to avoid the repeating calculation, the *analytical signal* $\tilde{x}(t)$ is defined with its Fourier transformation $\tilde{X}(\omega)$ equal to zero in the negative frequency region.

$$\tilde{x}(t) = x_r(t) + i x_i(t) = x_r(t) + i H\{x_r(t)\} \quad (2.2.21)$$

$$\tilde{X}(\omega) = \begin{cases} X(\omega) & , \text{for } 0 < \omega < \omega_s/2 \\ 0 & , \text{for } -\omega_s/2 < \omega < 0 \end{cases} \quad (2.2.22)$$

The operator H in Equation (2.2.21) is the Hilbert transform. The imaginary part is the Hilbert transformation of the real signal in time domain:

$$x_i(t) = H\{x_r(t)\} = \frac{1}{\pi} P \int_{-\infty}^{\infty} x_r(\tau) \frac{1}{t - \tau} d\tau \quad (2.2.23)$$

where P is the Cauchy principal value. The Hilbert transform is simply to rotate the phase angle by 90 degrees. Thus, the analytical signal has the same content as the original signal when frequency analysis is implemented. Equation (2.2.21) can also be expressed in exponential form:

$$\tilde{x}(t) = A(t) \cdot \exp(-i\theta(t)) \quad (2.2.24)$$

where A is the Hilbert amplitude and θ is the instantaneous phase:

$$A = \sqrt{x_r^2 + x_i^2} \quad (2.2.25)$$

$$\theta = \tan^{-1}\left(\frac{x_i}{x_r}\right) \quad (2.2.26)$$

The instantaneous frequency can be calculated from the first derivative of the instantaneous phase θ .

2.3 Singular Spectrum Analysis (SSA)

2.3.1 Overview of SSA

SSA is a novel non-parametric technique used in the analysis of time series and

based on multivariate statistics. SSA was firstly applied to extract tendencies and harmonic components in meteorological and geophysical time series. In the reference (Alonso, Del Castillo et al. 2005), SSA was implemented to the displacement signal to improve its derivatives, namely the velocity as well as the acceleration, by the numerical differentiation processing. It showed that SSA is a good tool to eliminate the effect of noise.

Except the extraction of tendency, SSA can be applied to smooth a noisy signal, to extract seasonality component, or to detect the singularities. Basically, SSA is capable of decomposing the original series into a sum of series, so that each component in this sum can be identified as a tendency, periodic component, or noise. Formally, SSA can be implemented to arbitrary time series. Different considerations should be concerned for different problems in hand. Detail descriptions for the application of SSA to various problems can be found in the reference (Golyandina 2001). The versatility makes SSA a potential technique for extracting damage features directly from the time series.

2.3.2 Procedures of SSA

SSA procedure consists of four steps: (1) embedding, (2) singular value decomposition (SVD), (3) grouping, and (4) reconstruction. The detail description of each step is shown in formal terms as follows:

Step1: Embedding

The method starts to produce a Hankel matrix from the time series itself by sliding a window that is shorter in length than the original series. Firstly, let $F = (f_0, f_1, \dots, f_{N-1})$ be the time series of length N . And let L be the window

length, which is an integer in $1 < L < N$. Each sliding window vector $\mathbf{X}_j$ with length of L would then be derived: $\mathbf{X}_j = (f_{j-L}, f_j, \dots, f_{j+L-2})^T, j = 1, 2, \dots, K$, where $K = N - L + 1$ is the number of columns. The matrix $\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_K]$ is a Hankel matrix:

$$\mathbf{X} = \begin{bmatrix} f_0 & f_1 & f_2 & \cdots & f_{K-1} \\ f_1 & f_2 & f_3 & \cdots & f_K \\ f_2 & f_3 & f_4 & \cdots & f_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{L-1} & f_L & f_{L+1} & \cdots & f_{N-1} \end{bmatrix} \quad (2.3.1)$$

since all elements in diagonal $i+j=\text{constant}$ are equal.

Step2: SVD of the Hankel matrix

The Hankel matrix can be represented in the form: $\mathbf{X} = \mathbf{E}_1 + \mathbf{E}_2 + \dots + \mathbf{E}_d$, where d is the number of non-zero eigenvalues of the $L \times L$ matrix $\mathbf{S} = \mathbf{X} \cdot \mathbf{X}^T$. The i -th elementary matrix, or called i -th eigentriple, are given by $\mathbf{E}_i = \sqrt{\lambda_i} \mathbf{U}_i \mathbf{V}_i^T$, where $\lambda_1, \lambda_2, \dots, \lambda_d$ are the non-zero eigenvalues of $\mathbf{S}$, in descending order, $\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_d$ are the corresponding eigenvectors, and vectors $\mathbf{V}_i$ are derived by $\mathbf{V}_i = \mathbf{X}^T \cdot \mathbf{U}_i / \sqrt{\lambda_i}$, $i = 1, 2, \dots, d$. The plot of the eigenvalues in descending order is called the singular spectrum and is essential in deciding the index from where to truncate the summation.

Step3: Grouping

This step is to decide a parameter r to reconstruct an approximate matrix of $\mathbf{X}$, i.e. $\mathbf{X} \approx \mathbf{E}_1 + \mathbf{E}_2 + \dots + \mathbf{E}_r$. The decision making procedure may varied depending on the objectives of users. For example, if one intends to derive the tendency of the structural response displacement, by which the residual displacement may be clearly indicated, r would be decided as 1, namely the first leading eigentriple.

Step4: Reconstruction

The approximate matrix is no longer a Hankel matrix, but an approximated time series may be recovered by taking the average of the diagonals. It is practical to recover the elementary time series for each elementary matrix. These elementary time series $(g_0, g_1, \dots, g_{N-1})$ is also called the principal component. If y_{ij} is used to represent the i -th row and j -th column element in any elementary matrix E , the reconstruction algorithm for each principal component can be formulated as follows:

$$g_k = \begin{cases} \frac{1}{k+1} \sum_{m=1}^{k+1} y_{m,k-m+2} & \text{for } 0 \leq k < R-1 \\ \frac{1}{R} \sum_{m=1}^R y_{m,k-m+2} & \text{for } R-1 \leq k < P \\ \frac{1}{N-k} \sum_{m=k-P+2}^{N-P+1} y_{m,k-m+2} & \text{for } P \leq k < N \end{cases} \quad (2.3.2)$$

where $R=\min(L,K)$, $P=\max(L,K)$. The smoothed time series is obtained by adding the first r principal components.

2.3.3 Selection of SSA Parameters

There are mainly two parameters needed to determine in SSA: the window length L and the number of series involved in grouping. The selection of these two parameters depends on the class of signal and the analysis required to be performed. Only few basic concepts are discussed in this section. For more detail description, one can refer to the reference (Golyandina 2001).

(a) Grouping effects

1. The Singular Spectrum can be a reference for selecting proper amounts of eigentriple.
2. For a simple series, the tendency component would have the largest value of

eigenvalue.

3. For a complex series, the Singular Spectrum may be less useful to provide any information for grouping. Some other techniques have to be introduced.

(b) Window length effects

1. The SVD results of the Hankel matrix, corresponding to window length L and $K = N-L+1$, are equivalent. Therefore, it is meaningless to do SSA with window length larger than half of the time series length.
2. It is straightforward that the larger the window length is, the more detail is the decomposition of the time series. It is actually a trade-off problem since a larger window length would require a more massive computation effort. Furthermore, over decomposed components may occur due to excessive decomposition, which may lead to the mixing amongst components interested. It is possible to improve the decomposition even with a small variation of the window length. Therefore, it is always worthwhile trying several window lengths.
3. For trend extraction problem, it requires a larger number of L since the highly strict demand of the separation from the noise and periodic components. Also, the value of L can not be too large (even for a very complex series) for the consideration of non-necessary decomposition.
4. In order to ensure the component decomposed by SSA is independent to each other, it is suggested to implement an auto-correlation test to decide a proper value of L before the SSA procedure (Tzagkarakis, Papadopouli et al. 2009). If the signal x with N data points is going to be analysed by SSA, the auto-correlation test is given by:

$$C(L) = \frac{\sum_{j=1}^N (x_{j+L} - \bar{x})(x_j - \bar{x})}{\sum_{j=1}^N (x_j - \bar{x})^2} \quad (2.3.3)$$

When the auto-correlation coefficient C crosses for the first zeros, the corresponding lag L may be the optimal candidate. However, the best L might not exactly correspond to the first zeros, but around it. It is good to try several L around the zeros.

2.4 Parametric Identification Method

2.4.1 Equivalent Linear System

Since the genuine system response may not be in linear response when subject to external loading, it is possible to use the equivalent linear system with time-varying model parameters to represent the system response by the equivalent linear system with constant model parameters within a specific time-window. In this research, the governing equation for the linear system is represented as:

$$\ddot{x}(t) + 2\omega\zeta \dot{x}(t) + \omega^2 x(t) = -\ddot{u}_g(t) \quad (2.4.1)$$

where $x(t)$ is the response displacement which is a function of time, $\ddot{u}_g(t)$ is the input acceleration, and parameters ζ and ω are modal damping and modal frequency, respectively.

The parameters of the equivalent linear system can be adjusted iteratively to represent the system response, which is the main idea of curve fitting technique. In addition to tracking the parameters along with time, a *time-varying window* with

length ‘ M ’ is employed which assumes a constant parameter in the windowed region. The error function Er is defined:

$$Er = \sum_{i=1}^M [x(t_i) - \bar{x}_i(\zeta, \omega, \dot{x}(t_i), x(t_i))]^2 \quad (2.4.2)$$

which is the difference of measured (or simulated) acceleration x and the theoretical acceleration $\bar{x}$ solved by applying *Newmark- β method* to the linear model equation. ‘ M ’ is the length of the time-window. In this case, the unknown parameters are equivalent natural frequency ω and equivalent damping ratio ζ of the equivalent linear system. The unknown parameters can be obtained by *nonlinear least-square procedure* which is based on the concept described as follows:

$$Min. Er \Rightarrow \begin{cases} \frac{\partial Er}{\partial \zeta} = 0 \\ \frac{\partial Er}{\partial \omega} = 0 \end{cases} \quad (2.4.3)$$

If the partial derivative with respect to the unknown parameter equals to zero, they correspond to the stationary point of the error function Er . In other words, the parameters ζ and ω corresponding to the minimum of the error function Er are the most optimal solutions. The optimal parameters are then served as the initial condition of the next time-window.

Based on the example shown in Equation (2.1.1), an equivalent linear system with a time-moving window with length of 1 second (40 points) is applied. The least square method is executed by the MATLAB function ‘lsqnonlin’ and the identified results are shown in Figure 2-6. The natural frequency of the equivalent linear system shows a gradual decay which does not reflect the exact variation of dominant frequency. It is important to note that the impact load at 25 second can not be revealed by using the equivalent linear system.

2.5 Frequency Domain Non-parametric Identification Method

2.5.1 Hilbert Transform of Frequency Response Function (FRF)

For a linear system, the relationship between the input and the output can be represented by the frequency response function $H(\omega)$:

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \quad (2.5.1)$$

where $Y(\omega)$ and $X(\omega)$ are the Fourier transformation of the output and the input, respectively, and $H(\omega)$ is the so-called transfer function. In the case of nonlinear system, the term ‘transfer function’ is not used; instead, ‘frequency response function’ (FRF) is used.

It is suggested to calculate the frequency response function by the power density function. The reason is that the Fourier transform of a random time series is itself random, and therefore of little use calculating transfer relationships (i.e. finding the output of a filter when the input is random). The FRF used in this research can be defined as:

$$G(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} = \text{Re}(G(\omega)) + i \text{Im}(G(\omega)) \quad (2.5.2)$$

where S_{xy} is the cross power spectral density function and S_{xx} is the auto power spectral density function of the input. The FRF $G(\omega)$ for a nonlinear system is depended on the input.

In order to detect the occurrence of nonlinearities, one can compare the FRF to

the its Hilbert transformation (Simon and Tomlinson 1984). Considering a complex FRF $G(z)$, where z is a complex variable, Cauchy's formula allows one to rewrite to:

$$G(z) = \frac{1}{2\pi i} P \oint_C G(\lambda) \frac{1}{\lambda - z} d\lambda \quad (2.5.3)$$

where C is a closed contour on the complex plane for the integration. Under the assumption of linear system, the superposition law allows one to simplify Equation (2.5.3) to:

$$G(\omega_c) = \frac{-1}{\pi i} P \int_{-\infty}^{\infty} G(\omega) \frac{1}{\omega - \omega_c} d\omega \quad (2.5.4)$$

Equation (2.5.4) is of the form of Equation (2.2.20):

$$H\{G(\omega_c)\} = HG(\omega_c) = \frac{-1}{\pi i} P \int_{-\infty}^{\infty} G(\omega) \frac{1}{\omega - \omega_c} d\omega \quad (2.5.5)$$

Equation (2.5.5) can be split into its real and imaginary parts:

$$\begin{aligned} \text{Re } HG(\omega_c) &= \frac{-1}{\pi} P \int_{-\infty}^{\infty} G(\omega) \frac{1}{\omega - \omega_c} d\omega = H\{\text{Im } G(\omega_c)\} \\ \text{Im } HG(\omega_c) &= \frac{1}{\pi} P \int_{-\infty}^{\infty} G(\omega) \frac{1}{\omega - \omega_c} d\omega = H\{\text{Re } G(\omega_c)\} \end{aligned} \quad (2.5.6)$$

When $G(\omega)$ is the Fourier transform of a causal time function, Equation (2.5.6) is tenable. In the other words, if Equation (2.5.6) is not true, the system is non-causal. It implies that nonlinearities exist in the system. In conclusion, one can determine whether the system is nonlinear by the relation described below:

$$\begin{aligned} G(\omega) &= HG(\omega) && \text{if the system is linear} \\ G(\omega) &\neq HG(\omega) && \text{if the system is nonlinear} \end{aligned} \quad (2.5.7)$$

2.5.2 Coherence Function

The coherence can be used to assess the quality of the calculation of FRF. It is also a rapid indicator to detect the presence of nonlinearities in specific frequency bands or resonant regions. The definition of coherence function is as follows:

$$\gamma_{xy}(\omega) = \frac{|S_{xy}(\omega)|^2}{S_{xx}(\omega)S_{yy}(\omega)}, \quad 0 \leq \gamma_{xy} \leq 1 \quad (2.5.8)$$

The coherence function should equal to 1 for ideal case. A low coherence can be attributed to: (1) leakage errors of discrete Fourier transform, (2) other inputs besides $X(\omega)$ contributing to the output $Y(\omega)$, or (3) a nonlinear distortion. It is noted that the coherence function should be derived from a random excitation test. It is also indicated that the coherence function is almost immune against the noise (Rauch 1992).

For linear systems, the transfer function can fully represent the content of the system and the corresponding coherence function would be unity along all frequencies except the resonant frequencies. In the case of nonlinear systems, the correlation of the input and the output is weak so that the FRF derived from the measured signals can not describe the real system. As a result, it causes lower values in coherence function.

2.5.3 Hilbert Marginal Spectrum

Although the wavelet modulus map defined in Equation (2.2.3) provides a fast glance of amplitude variations on the time-frequency plane, it contains periodic fluctuations which may mislead the observation. Therefore, the Hilbert transform is

introduced to smooth the wavelet modulus map. Applying the Hilbert transform to each wavelet components (time-series) and substituting the scale by the frequency, the so-called Hilbert amplitude spectrum can be defined as:

$$HW(f, u) = |H\{WM(f, u)\}| \quad (2.5.9)$$

The reconstructed modulus map, $HW(f)$, is clearer to observe the tendency of system dominant frequencies varying with time. To examine the energy distribution in frequency domain in a specific time segment, T , one can produce the ‘Hilbert marginal spectrum’ which is defined as:

$$HM(f) = \int_T HW(f, u) du \quad (2.5.10)$$

From the example in Equation (2.1.1), the Hilbert amplitude spectrum HW of the acceleration signal is plotted in Figure 2-7, which can only show the trend of variation of the dominant frequency. Not any information about the timing of frequency decreasing and introducing of the impulse loading is available. Hilbert marginal spectrums HM for different time segments (0~10sec, 10~15sec, 15~20sec, and 20~30sec) are then plotted in Figure 2-8. The Hilbert marginal spectrum provides useful information while comparing frequency contents amongst several cases.

2.5.4 WPT Component Correlation Coefficient

As described in section 2.2.5, WPT is a level-by-level decomposition technique. If one decomposes a signal to J -level by WPT, totally 2^J components would be derived. Each component has the central frequency defined in Equation (2.2.20). The component energy can then be calculated from Equation (1.2.1).

The temporal variation of wavelet coefficient in a particular frequency band indicates shifts of energy from or to another frequency band. There are mainly two reasons for shifts of energy amongst frequency band: (1) temporal variation in frequency content of the excitation, and (2) changes of system characteristics, such as stiffness degradation or sudden occurrence of non-ductile events. The latter is the objective of nonlinearity identification. However, if one examines the output signal only, it is hard to attribute the source of the variation in the signal. It is necessary to compare the excitation and the response wavelet coefficient plot to identify the changes of system characteristics. The correlation between the excitation and the response wavelet coefficient can be observed by their plot shape. In order to quantify the correlation, a correlation coefficient of a particular wavelet packet node is defined as (Goggins, Broderick et al. 2007):

$$\rho_{xz_j} = \frac{\sum_i W_{ij}x W_{ij}z}{\sqrt{\sum_i W_{ij}x^2} \sqrt{\sum_i W_{ij}z^2}} \quad (2.5.11)$$

where $W_{ij}x$ and $W_{ij}z$ represent the wavelet coefficient at time interval b_i of band j for response and excitation, respectively. If the correlation coefficient $\rho_{xz_j} = 0$, there is no correlation. If $\rho_{xz_j} = \pm 1$, it means perfect correlation.

2.5.5 Bispectral Analysis

The Bispectral analysis techniques are used to study mode interaction behaviours and to detect the second order nonlinearity. Bispectral analysis is executed by employing the so-called bispectrum. The bispectrum is sensitive to the signal skewness and the definition is given as follows (Haroon and Adams 2007):

$$E_{YYY}(f_1, f_2) = E[Y^*(f_1)Y^*(f_2)Y(f_1 + f_2)] \quad (2.5.12)$$

where $E[\cdot]$ denotes the expectation operator and Y is the Fourier transform of the response signal. Peaks in the bispectrum measure the amount of the coupling between these three spectral components. The normalised form of Equation (2.5.12), i.e. the bicoherence, provides a more practical view for the bispectral analysis:

$$b^2(f_1, f_2) = \frac{|E[Y^*(f_1)Y^*(f_2)Y(f_1 + f_2)]|^2}{E[|Y(f_1)Y(f_2)|^2]E[|Y(f_1 + f_2)|^2]} \quad (2.5.13)$$

The main advantage is that bicoherence is restricted to vary between 0 and 1. If there is a complete statistical dependency (i.e. $Y(f_1 + f_2) = \alpha Y(f_1)Y(f_2)$, where α is a constant), the bicoherence would equal to one. If there is no statistical dependency, the bicoherence would be zero.

The bicoherence can be calculated from a signal by dividing it into K segments, applying an appropriate window to each segment to reduce leakage. It is suggested that the length of each is the square root of the length of the signal. The application of bispectral analysis for structural crack detection can be found in the reference (Rivola and White 1998). It is also important to note that the random excitation is assumed in this technique.

2.6 Time Domain Non-parametric Identification Method

2.6.1 Instantaneous Frequency

Recalling Equation (2.2.26) in section 2.2.7, for a SDOF system, the instantaneous phase can be derived from a signal by its analytical form. The first

order derivative of the phase is exactly represented by the system dominant circular frequency:

$$\omega(t) = \frac{\partial \theta(t)}{\partial t} \quad (2.6.1)$$

It is noted that the estimated frequency is exact only when the system is undamped (Ruzzene, Fasana et al. 1997). Errors will be introduced due to the effect of damping.

In the case of digital signal processing, the derivative operator in Equation is referred to a numerical differentiate technique (Boashash 1992):

$$\omega(k) = \begin{cases} \frac{1}{12}(-25\theta(k) + 48\theta(k+1) - 36\theta(k+2) + 16\theta(k+3) - 3\theta(k+4)) & \text{for } k=1,2. \\ \frac{1}{12}(\theta(k-2) - 8\theta(k-1) + 8\theta(k+1) - \theta(k+2)) & \text{for } k=3, \dots, N-2. \\ \frac{1}{12}(25\theta(k) - 48\theta(k-1) + 36\theta(k-2) - 16\theta(k-3) + 3\theta(k-4)) & \text{for } k=N-1, N. \end{cases} \quad (2.6.2)$$

where N refers to the length of the time series. In order to ignore the effect of noise, one can apply WPT at the beginning. In this study, only dominant wavelet packet components are considered to calculate the instantaneous frequency. Hence, a smoothed result could be derived. For example, if a time series is decomposed by WPT to J level, there would be 2^J nodes and the circular frequency in j -th nodes could be denoted as $\omega_{j,k}$, $j = 1, \dots, 2^J$, where k means the temporal parameter. It is also suggested to apply a time varying mean procedure to eliminate the fluctuations in identified frequency. Therefore, the instantaneous frequency is defined as (Olhede and Walden 2004) :

$$\frac{f(k)}{2\pi} = \frac{\sum_{j=1}^{J_0} A_{j,k}^2 \omega_{j,k}}{\sum_{j=1}^{J_0} A_{j,k}^2} \quad (2.6.3)$$

where J_0 is the number of nodes considered and $A_{j,k}$ is the modulus (defined in Equation (2.2.25)) of analytical signal of the j -th component.

The identification of instantaneous frequency can provide the system dominant frequency varies by only analyzing the output signal. The structural nonlinear behaviour, induced by stiffness degradation, strength deterioration and pinching effect, will cause the variation of system dominant frequency. Hence, the instantaneous frequency can provide a tendency to identify the system nonlinearity and also provide a rough assessment of damage severity.

From the example shown in Equation (2.1.1), in the beginning, the response acceleration is decomposed by DWT. The approximation component is extracted and Hilbert transform is applied. The instantaneous amplitude and frequency are plotted in Figure 2-9. A simple moving average (MA) smoothing technique is applied:

$$\hat{x}(k) = \frac{1}{m} \sum_{i=0}^{m-1} x(k+i) \quad (2.6.4)$$

where x is the original signal, $\hat{x}$ is the smoothed signal, and m is the window length which is chosen as 0.5 second in this case. It shows clearer drops of system frequency than the result derived from equivalent linear system (Figure 2-6). The identified instantaneous frequency (after smoothing) provides a better tendency than the result from equivalent linear system.

2.6.2 Instantaneous Phase Difference

The definition of instantaneous phase difference, ϕ_s , is quite simple (Pines and Salvino 2006):

$$\phi_s(t) = |\theta_I(t) - \theta_O(t)| \quad (2.6.5)$$

For a SDOF system θ_I is the unwrapped phase derived from ground motion signal and θ_O is the unwrapped phase derived from response signal.

Phase lag exists when incident waves reach different positions along the structure at different times. The main idea of damage detection by using the instantaneous phase difference is that damages in a structure will change the wave propagation pattern which may alter the energy traverse through the structure. By observing the variation of this nonlinear indicator, it is possible to identify the presence and the location of the damage.

2.6.3 Holder Exponent

Holder exponent, also known as Lipschitz exponent, is an indicator to assess the regularity of the signal at a given time point. The regularity identifies what order a function is differentiable. For example, if the function is discontinuous but bounded at $t=0$, such as a step function, the Holder exponent would be 0; if the function is discontinuous and unbounded at $t=0$, such as an impulse function, the Holder exponent would be -1. Conventionally, the Holder exponent is found from Fourier transformation of the signal which only provides global minimum regularity. With the wavelet techniques, signal regularity can be assessed at every time step.

The Holder regularity is defined as follows. Assume that a signal $f(t)$ can be

approximated locally at t_0 by a polynomial form (Robertson, Farrar et al. 2003):

$$\begin{aligned} f(t) &= c_0 + c_1(t - t_0) + \dots + c_n(t - t_0)^n + C|t - t_0|^\alpha \\ &= P_n(t - t_0) + C|t - t_0|^\alpha \end{aligned} \quad (2.6.6)$$

where P_n is a polynomial of order n and C is a coefficient. The term associated with the exponent α can be thought of as the residual that remains after fitting a polynomial of order n to the signal, or as the part of the signal that does not fit into an $n+1$ term approximation. The local regularity of a function at t_0 can then be characterised by this Holder exponent:

$$|f(t) - P_n(t - t_0)| \leq C|t - t_0|^\alpha \quad (2.6.7)$$

A higher value of α indicates a better regularity or a smoother function. In order to detect singularities, a transform is needed that ignores the polynomial part of the signal. A wavelet transform that has n vanishing moments is able to ignore polynomials up to order n :

$$\int_{-\infty}^{\infty} t^n \psi(t) dt = 0 \quad (2.6.8)$$

Transformation of Equation (2.6.8) using a wavelet with at least n vanishing moments then provides a method for extracting the values of the Holder exponent in time:

$$|Wf(u, s)| \leq Cs^\alpha \quad (2.6.9)$$

The wavelet transform of the polynomial terms is zero and so what remains is a relationship between the wavelet transform of $f(t)$ and error between the polynomial and $f(t)$. The exponent α can be calculated at a specific time point by finding the slope of the log of the wavelet modulus at that time versus the log of the scale vector s . A practical procedure of the Holder exponent is given below:

Calculation procedure of Wavelet-based Holder exponent

1. Apply CWT to the original time-series $f(t)$ to produce a wavelet modulus map, namely a CWT spectrum.
2. For every time point, draw a plot of the log of wavelet modulus versus to the log of scale.

$$\log|Wf(u, s)| = \log(C) + \alpha \log(s) \quad (2.6.10)$$

It is recommended to consider a coarser scale region (i.e. higher frequency part).

For instance, if a SDOF system with natural frequency of 5Hz, one can consider the CWT coefficients in the region from 5Hz to the Nyquist frequency.

3. Smooth the logarithmic plot by Gaussian function. (optional)
4. Determine the slope (Holder exponent) of the curve by: a) first order polynomial curve fitting, or b) the slope of the tangent line which is always above the original curve. The slope is of the analytical form:

$$m = \frac{\log|Wf(s, u)|}{\log(s)} = \alpha \quad (2.6.11)$$

where $Wf(s, u)$ refers to the wavelet coefficient, m is the slope and is equal to the holder exponent α at the temporal location u .

5. After calculating Holder exponents at each time point, a threshold can be set to extract unique singularity point. One can refer to the reference (Robertson, Farrar et al. 2003) for methods of setting up a threshold, which are statistic control process and damage detection algorithm.

From the example shown in Equation (2.1.1), the CWT is used. The wavelet function, ‘Morlet’, which has infinite vanishing moment, is selected and the scale is

set from 1 to 300 for the CWT procedure. The Nyquist frequency of the simulation test is 10Hz and the system dominant frequency is below 1 Hz. Therefore, the frequency region of 1Hz to 10Hz is concerned for the computation of Holder exponents. The curve-fitting procedure is achieved by the MATLAB function 'polyfit' with first order fitting at each time point. From Figure 2-10, it can be observed that when there is no singularity (5 sec), the wavelet coefficient curve is steeper, whereas the curve becomes plane at the time point with singularity (25 sec). Figure 2-11 shows the value of Holder exponent at all time point. Drops which refer to the singularities can be observed clearly. Both kinds of event could be detected through the Holder exponent.

2.6.4 Level-1 Detail Component of DWT

According to section 2.2.4, DWT decomposes signal into two components in each decomposition level. The level-1 detail component of DWT equals to the 'D1' in Equation (2.2.11). Due to the idea of the dyadic decomposition, the detail component contains upper half frequency band of the original signal. For example, if the sampling frequency is 200Hz, the Nyquist frequency would be the half, i.e. 100Hz. Level-1 detail component would then cover the frequency band of 50Hz to 100Hz. Generally, singularities in the signal are embedded in very high frequency bands. Level-1 detail component can reveal the presence of the singularities with a quite fast computation.

From the example shown in Equation (2.1.1), the DWT procedure adopts the mother wavelet 'bior6.8' to decompose the acceleration responses. The level-1 detail component is extracted and is plotted in Figure 2-12. It can be clearly

observed that there is a sharp peak at 25 sec when is actually the introducing of the impact force. However, the variations of system dominant frequency are too vague to detect.

2.6.5 Residual Component of SSA

The singularity of structural response signals may be induced by the crack or damage of the structure at that instant of time. As introduced in section 2.3, SSA can decompose the signal into three categories of components: tendency, oscillation, and noise component. The singularity belongs to neither the tendency nor the oscillation; therefore, the information of singularity would be embedded in the noise component. The noise component has less eigenvalue so that it always belongs to the residual series of SSA. Therefore, the application of SSA for singularity identification will aim to extract the residual component instead of the tendency component. Since the residual component contains both noise and singularity information, one main discrepancy is that the singularity shows obvious peaks in residual component whereas the noise shows minor and scattered fluctuations.

The example from Equation (2.1.1) is analyzed by SSA. The extraction criterion of SSA is the energy contribution of the residual component. The singularity signals generally have low contribution to the whole signal. After several test for different value of the extraction criterion, it is recommended that to extract the residual component with energy contribution less than 0.01%. However, if the level of noise is too high and the singularity is weak, the residual component of SSA may reveal nothing but a noisy signal. For the simulation test, one can determine the number of component extracted by observing the singular spectrum (Figure 2-13) of

the acceleration response. Since the noise level is not too high so that the result (Figure 2-14) shows a good correlation to the variation of system parameter.

2.7 Chapter Summary

In this chapter, a total of 4 signal processing tools are used, and 2 kinds of nonlinear identification methods and 10 nonlinear indicators are introduced. The signal processing tools include the Fourier Transform (FT), the Wavelet techniques, the Hilbert transform (HT) and the Singular Spectrum Analysis (SSA). The leading three techniques utilise different kernel functions to transform the original function into frequency domain or time-frequency domain. The SSA technique decomposes the original time series into three kinds of component, i.e. the trend component, the harmonic component and the noise component. These four signal processing tools can are all potential to detect nonlinearities embedded in the system and can provide the feature extraction of signal induced by damage. The derivatives of them are the so-called nonlinear indicator.

In this research, the non-parametric nonlinear identification method is emphasised due to less limitations of calculation and physic-meaningful explanation. For parametric nonlinear identification method, only the equivalent linear system is introduced to give a preliminary estimation of the physical properties of the system. The nonlinear indicators used in the non-parametric nonlinear identification methods are classified into two categories: frequency-domain indicators and time-domain indicators. The frequency-domain indicators presented in this research are (1) Hilbert transform of FRF, (2) the coherence function, (3) the Hilbert

marginal spectrum, (4) the WPT component correlation coefficient, and (5) the bispectral analysis. This class of indicator is capable of revealing the presence of nonlinearities by observing the difference of the indicators from intact system and from damaged system. The time-domain indicators are (1) instantaneous frequency, (2) instantaneous phase difference, (3) the Holder exponent, (4) level-1 detail component of DWT, and (5) the residual component of SSA. Time-domain indicators give a simultaneous detection of nonlinearity.

Several nonlinear indicators are applied to the simulation data defined in Equation (2.1.1) in order to verify their feasibility for nonlinear identification. The applied indicators are the equivalent linear system, Hilbert marginal spectrum, instantaneous frequency, Holder exponent, level-1 detail component of DWT, and the residual component of SSA. A comparison of the time-domain nonlinear indicators is shown in Figure 2-15. It indicates that the instantaneous frequency is better to track the variation of dominant frequency. Holder exponent, level-1 detail component of DWT, and residual component of SSA are capable of detecting the singularities in the response signal. Both the Holder exponent and residual component of SSA can reveal the occurrence of variation of natural frequency.

A summary of the nonlinear indicators presented is given in Table 2-1. Assessments including assumption of a *priori* model, requirement of stationary random excitation, requirement of information of input signal (output only or not), and requirement of baseline information (intact system information) are concerned in the summary table. From the summary table, one can choose a proper nonlinear indicator based on the limitation and the requirement of his research.

CHAPTER 3 EXPERIMENTAL SURVEY

3.1 Design Details of the One-Story Two-Bay RC Frame

In order to study the damage severity induced by ground shaking in a real structure, a series of shake table test was taken in National Center for Research on Earthquake Engineering (NCREE) in Taiwan (Loh, Lin et al. 2010). The test specimen was a one-storey two-spanned RC frame (Figure 3-1) with an overall height of 2 m, and an approximate weight of the frame was 6454 kg. The frame consisted of a T-beam of 4.7 m $\times$ 0.7 m, as well as 3 columns with cross-section dimension of 20 cm $\times$ 20 cm. For the sketches for the beam and column cross sections, one can refer to Figure 3-2. A total of six specimens were constructed with the same design details. The specimen was subjected to a single direction excitation. For the shake table test, 12 accelerometers were used to measure the acceleration. Four lateral LVDTs (Linear Variable Differential Transformers) were instrumented to acquire displacement response and 16 small LVDTs were attached on the top of the frame as well as the beam-column connection to measure the detail deformation at the local connection. The sampling frequency for above devices is 200Hz. Twenty-four optical sensors (the sampling frequency = 100Hz) were also instrumented on the central column for the acquisition of displacement information. The configurations can be accessed from Figures 3-3 to 3-6. The input ground motion was selected from 1999 Taiwan Chi-Chi earthquake (Station: TCU082) which is shown in Figure 3-7. Six tests with different intensity level of excitation were conducted. For each shake table test, all the recording systems are triggered

with the central recording system.

Two groups of tests are going to be discussed in this chapter. For *Group1*, test results from four specimens (RCF6, RCF2, RCF4, and RCF3) with different levels of excitation are compared. The excitation level for each test was 600 gal, 840 gal, 1100 gal, and 1300 gal, respectively (Table 3-1). For *Group2* test, a series of excitation with different intensity level of excitation was applied to specimen RCF6 back to back. The test procedure was 600gal, 800gal, 1000gal, 1200gal, 1000gal, 800gal, and 600gal, back to back (Table 3-4). Related analysis of Group2 data set would be given in Section 3.5.

In the following sections, based on the experimental data, the nonlinear indicators mentioned above were applied to detect the system nonlinearity and identify the damage severity for each structure.

3.2 Preliminary Analysis of Experimental Measurements

3.2.1 Physical parameters

After the acquisition of signals from all channels, some physic-meaningful parameters could be measured directly or can be derived through some simple analysis. The absolute acceleration responses are plotted. Figure 3-8 shows the average of the measurements from accelerometers A1, A4, and A7. Figure 3-9 shows the relative displacements (which were all zero-mean processed) measured from the lateral LVDT through the calculation: $Disp. = (D1+D2)/2 - (D3+D4)/2$. And the inter-story drift ratio (Figure 3-10) could also be estimated by: $StoryDrift = |Disp. / Height| * 100\%$, where the height is 200cm for all the specimens. Figure

3-11 shows the restoring force diagram of these four specimens from four different levels of excitation. It can be observed that the hysteretic loop from RCF2, RCF4, and RCF3 show different degree of stiffness degradation.

The Fourier spectrums of the absolute acceleration are examined in Figure 3-12. It is quite evident that the dominant frequency of Fourier spectrums for the case of stronger ground motion excitation are shifted to lower frequency bands.

Figure 3-13 is the photograph of the central column of four specimens. Clear cracks can be observed by inspection. RCF3 has more cracks which are induced by severer seismic excitation. The optical sensors can record the displacement measurements. SSA technique (L=50 and extracting the first leading components with energy contribution over 99.999%) is applied to the original optical measurements. By calculating the curvature based on these de-noised signals (Figure 3-14), local damages can be revealed. The curvature is defined as (Catbas, Gul et al. 2008):

$$v''_{q,i} = \frac{v_{q_1,i} - 2v_{q_2,i} + v_{q_3,i}}{\Delta h^2} \quad (3.2.1)$$

where $v_{q_1,i}$ is the y-direction measurement from the optical sensor node q_1 , Δh^2 is the distance between two nodes (50 mm), and $v''_{q,i}$ is the curvature. Considering optical sensor node 12, node 7 and node 8, the curvature for each specimen is calculated and is compared with the inter-story drift ratio (from LVDT) in Figure 3-15 to Figure 3-18, respectively. It is observed that the inter-story drift ratio (global) is consistent to the curvature (local); therefore, the inter-story drift ratio can be used to represent the occurrence of damages in this study.

The definition of the absolute input energy is given:

$$E_I(t) = \int_0^t m \ddot{u}_t du_g \quad (3.2.2)$$

where $\ddot{u}_t$ is the absolute acceleration and u_g is the ground displacement. The plot of absolute input energy for these test cases is shown in Figure 3-19. The occurrence of potential damage may be caused by the abrupt change of absolute input energy with a short period.

The equivalent linear system method is firstly conducted to the experimental data. A Butterworth low-pass filter with cut-off frequency of 20Hz is applied to the original signal to eliminate the effect of noise. The width of the moving-time window is chosen as 1.5 seconds (300 data points). The Tukey window is used and defined as follows:

$$w(n) = \begin{cases} 1.0, & 0 \leq |n| \leq \alpha \frac{N}{2} \\ \frac{1}{2} \left(1 + \cos \left(\pi \frac{n - \alpha \frac{N}{2}}{2(1 - \alpha) \frac{N}{2}} \right) \right), & \alpha \frac{N}{2} \leq |n| \leq \frac{N}{2} \end{cases} \quad (3.2.3)$$

where the window length $L = N+1$, and the parameter α controls the shape of the window (for Hanning window while $\alpha=1$, for rectangular window $\alpha=0$). The parameter α adopted in this equivalent linear system scheme is chosen as 0.75. The initial condition for the first time window is set as $(\ddot{u}_0, \dot{u}_0, u_0) = (0, 0, 0)$. The initial conditions for each data set from a specific time-window will be equal to the identified result in the former segment of data set. The limit of identified parameters is set as $(\omega, \xi) = ([0 \ 20\pi], [0 \ 0.5])$. The identified parameters are shown in Figure 3-20. It is seen that the identified fundamental frequency of the equivalent linear system decay around 30 to 40 seconds for these four specimens. RCF3 has the most serious degradation of the fundamental natural frequency, in other words the

stiffness, which decreases from 5.8 Hz to 2.1 Hz.

Finally, the physical parameters of the four tests are summarized in Table 3-1. It can be observed that the inter-story drift ratio and the inter-story stiffness reduction ratio are proportional to the excitation level. It is confirmed that different level of damages in these four specimens has been successfully produced after the shake table test.

3.2.2 Extraction of the Residual Displacement by SSA

To extract the residual deformation of a structure from either displacement or acceleration record is an important issue since the residual deformation may occur in the structure when subjected to severe external loading. To identify this information, a trend tracking technique is required. SSA provides a good tool to extract the permanent deformation from a signal. By decomposing the relative displacement signal by SSA, the first component is extracted as the tendency of the permanent displacement. The SSA extraction scheme is listed as follows:

Tendency Extraction Scheme by SSA

1. Choose the parameter ' L '. The higher this parameter is, the smoother is the trend extracted. In this case, $L=300$ is essential.
2. Apply SSA to the measured relative displacement signal. One would derive L principal components. The first principal component can be selected from the largest eigenvalue (See Figure 3-21).
3. Plot the first principal component which shows the tendency of the permanent deformation. (Figure 3-22).

Figure 3-23 shows the identified residual displacement of these four specimens

after the shake table test. It is clear that RCF3 has the largest residual displacement whereas there is no residual displacement presence in RCF6.

3.2.3 Improvement of Numerical Differentiation Scheme by SSA

Before proceeding with any analysis of nonlinearity identification, there is still some interesting and practical application of SSA. Except the extraction of residual deformation described in the previous section, SSA can also be utilized to improve some numerical techniques, e.g. the numerical differentiation (Alonso, Del Castillo et al. 2005). A fact is that it is sometimes difficult to possess all kinds of measurements in practice. For example, only the acceleration data (or only the displacement data) is in hand at one time. Under this condition, numerical method is required if one needs the information of the other kind of measurement. To apply the SSA in the measurements, two different situations may be encountered: one is the estimation of acceleration from displacement measurement and the other one is to estimate the residual displacement from acceleration measurement. The first issue will be discussed in this section and the second issue will be discussed in the next section.

To estimate the acceleration response from displacement measurements, data collected from the optical displacement sensor (see Figure 3-6) is used. In this study, the optical sensors return their relative coordinates with the frequency of 100Hz. Hence, a series of 3D displacement measurements can be acquired. Since the specimen is constraint to 1D motion, only the displacement in y-direction is considered. In order to exhibit the application of SSA for improving the result of numerical differentiation, the optical displacement measurement at Node 32 from

RCF2 specimen is taken as an example. The analysis procedure is as follows:

Estimation of acceleration from displacement measurement supported by SSA

1. Take an auto-correlation test (based on Equation 2.3.3) and find the optimal L .
2. The grouping criterion in SSA is by choosing the leading principal components whose energy contribution is larger than 99.999%. The purpose is to discard the noise components.
3. Apply numerical differentiation method (Equation 2.6.2) twice to the SSA processed displacement signal to estimate the acceleration data.

From the auto-correlation test of displacement data (Figure 3-24), the first zeros is between $L=65$ and $L=66$. Several trials with different L are taken, and the result is shown in Table 3-2. The best candidate is $L=69$ where the corresponding RMSE is minimum (0.0313). Therefore, $L=69$ is adopted to the SSA process. The displacement signal processed by SSA and the corresponding residual series (refer to the noise component) are shown in Figure 3-25. To assess the error of the derivatives with respect to the acceleration measured from accelerometer, the root mean square error (RMSE) is used:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\frac{|\hat{x}(i) - x(i)|}{x(i)} \right)^2} \quad (3.2.4)$$

where N is the data length, x is the absolute acceleration measurement from accelerometers, and $\hat{x}$ is the derivatives from the numerical differentiation. Compared the estimated acceleration from SSA with the acceleration estimated from the original displacement data, the process of SSA plays a role of filtering process which can filter out high frequency signal from the original signal (Figure 3-26).

3.2.4 Estimation of Residual Deformation from Acceleration data by SSA

In practice, accelerations are much easier to measure than the displacements. However, the displacement data usually contains useful information to measure the structural damages. For example, the residual deformation is a straightforward physic-meaningful damage index. If it is possible to track the displacement tendency from acceleration measurement, it will be a big progress for structural health monitoring.

Estimation of displacement from acceleration data is not an easy issue. The displacement tendency is embedded in low frequency bands but may not be in a single frequency band. Therefore, few signal processing tool is capable of solving this problem. The wavelet technique is a powerful time-frequency analysis tool which decomposes the signal into several components in specific frequency bands. However, Wavelet components are actually the correlation coefficients corresponding to the pre-determined kernel, namely the mother wavelet. This property makes wavelet technique less useful in numerical differentiation for nonparametric case (i.e. without any known model, such as the Bouc-Wen model).

SSA is a potential candidate for solving this issue since it is a versatile signal processing tool and, the most important of all, it is a nonparametric technique with less parameters needed to be concerned. Figure 3-27 shows a brief flowchart of this estimation scheme. Taking the acceleration response data from specimen RCF2 as an example, the estimation scheme is described as follows:

1. In order to determine the parameter L , the auto-correlation test proposed in Equation (2.3.3) is employed. Several lag values corresponding to the zeros can

Figure 3-28). After several

trials, the L is chosen as 450.

2. Since the time-varying baseline of acceleration signal is very minor, a numerical technique is employed (Golyandina 2001): Adding a constant to the signal and then applying SSA to the modified signal; so the first principal component will be the combination of the time-varying baseline and the pre-assigned constant value. The main idea of using this technique is to amplify the eigenvalue of the baseline in the acceleration data.
3. With the preparations in Step 1 and Step 2, the first SSA processing will produce a tendency component ($A1$) and a residual component ($D1$) (Figure 3-29).
4. Employ the second SSA processing to $A1$ (with the same L , the corresponding auto-correlation test is in Figure 3-30). A residual component ($D2$) is extracted from the second SSA (Figure 3-31) since the first principal component ($A2$) has the lowest frequency which would cause error in the numerical integration.
5. The numerical integration technique based on trapezoidal rule is given:

$$\int f(t_1) dt = \frac{\Delta t}{2} (f(t_1) + f(t_2)) \quad (3.2.5)$$

By integrating $D2$ twice, the trend of displacement can be reconstructed.

Applying the above-mentioned procedures to the acceleration response data of specimen RCF2, the estimated residual deformation is shown in Figure 3-32. Data from specimen RCF3 is also analysed with parameter $L = 400$ (Figure 3-33). Although the estimated trend of displacement does not fit the exact displacement data perfectly, the estimation of the permanent deformation at the end of external excitation is good. For the development of the estimation of residual displacement

from acceleration response by SSA, more studies to the selection of parameter L and the grouping method is needed.

3.3 Frequency-Domain Identification of Shake Table Test Data

3.3.1 Hilbert Transform of Frequency Response Function

Hilbert transform is applied to the FRF to detect the presence of nonlinearity. The detailed procedure is as follows:

Calculation for HT of FRF

1. Apply a Butterworth low-pass filter with cut-off frequency of 20Hz to the original acceleration data.
2. Calculate the FRF. The ratio of the auto PSD function (S_{xx}) of input and the cross PSD function (S_{xy}) between input and output is the FRF $G(f)$. Two different approaches were applied to estimate the auto- and cross- power spectral density (PSD) function of signals: one is via correlation function and the other is the direct Fourier Transform.

(a) For the analysis of seismic response data (transient signals), the correlation approach are used. The cross PSD function is defined as:

$$S_{xy} = \int R_{xy}(\tau) e^{-i\omega\tau} d\tau \quad (3.3.1)$$

where $R_{xy}(\tau)$ is the cross-correlation function. In this study, the acceleration data from 25 second to 85 second (12000 data points) is included in the calculation, and the length of the correlation function equals to 25% of the selected signal segment ($12000 \times 0.25 = 3000$ data points). A

Tukey window (Equation (3.2.3)) of parameter $\alpha = 0.25$ is applied to the correlation function.

- (b) For the analysis of white noise response data (stationary signal), the Fourier transform method is used. The cross PSD function is defined as:

$$S_{xy}(\omega) = \frac{E[X^*(\omega)Y(\omega)]}{T} \quad (3.3.2)$$

where $X(f)$ is the Fourier transform of $x(t)$ and T is the duration. It is suggested to divide the original data into several segments. Sequentially, the PSD can be calculated based on the expectation of these segments. In this case, 8 segments with Hamming window are employed.

3. Equation (2.5.5) can be written in the form:

$$H\{G(f)\} = \text{Re}[HG(f)] + i \text{Im}[HG(f)] \quad (3.3.3)$$

where $H\{\cdot\}$ is the Hilbert transform operator. Derive the Hilbert transform of FRF, i.e. $HG(f)$, by discrete Hilbert transform. The algorithm is shown as follows (Simon and Tomlinson 1984):

$$\begin{aligned} \text{Re}[HG(f_i)] &= \frac{-2}{\pi} \sum_{k=1}^n \frac{\text{Im}[G(f_k)] f_k \Delta f}{f_k^2 - f_i^2} \\ \text{Im}[HG(f_i)] &= \frac{2f_i}{\pi} \sum_{k=1}^n \frac{\text{Re}[G(f_k)] \Delta f}{f_k^2 - f_i^2} \end{aligned} \quad (3.3.4)$$

4. An index to quantify the difference between the FRF and its Hilbert transformation is proposed:

$$DI_H = RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^J \left(\frac{|HG(f_i) - G(f_i)|}{G(f_i)} \right)^2} \quad (3.3.5)$$

where $f_l = 1\text{Hz}$ as well as $f_j = 10\text{Hz}$ defines the interested region, and n is the length of the frequency-domain data in the interested region.

Figure 3-34 shows the result of the FRF analysis from the shake table test data. Specimen RCF3 has the highest RMSE value which implies the most severe nonlinearity. Figure 3-35 and Figure 3-36 show the results of the white noise excitation data before and after the shake table test, respectively. For intact structure, the difference between the FRF and its Hilbert transformation is less and the corresponding index is also low. For the damaged system, the difference between the $G(f)$ and $HG(f)$ increases and the value of index is proportional to the level of nonlinearity.

3.3.2 Coherence Function

The FRF and the corresponding coherence functions between input and output are computed and plotted in Figure 3-37 and Figure 3-38. The computation procedure is listed as follows:

Calculation of the coherence function

1. Following the Step1 (b) of the last section, calculate the auto power spectral density function of input S_{xx} , the auto power spectral density function of output S_{yy} , and the cross power spectral density function S_{xy} .
2. Based on Equation (2.5.8) to derive the coherence function C_{xy} .
3. In order to assess the level of nonlinearity, an index is proposed:

$$DI_C = \sum_{i=I}^J (1 - C_{xy}(f_i)) \quad (3.3.6)$$

where the parameter I as well as J define the lower and upper bound of interested frequency range, in this case, $f_I = 1Hz$ and $f_J = 10Hz$.

It is clear that in Figure 3-37 the coherence functions are near unity except the

resonant frequency band. In Figure 3-38, the index shows an assemble result with the result of HT of FRF exhibits in the last section ($R_{CF3} > R_{CF6} > R_{CF2} \approx R_{CF4}$). R_{CF3} has the highest value of index which implies the highest nonlinearity level.

3.3.3 Hilbert Marginal Spectrum

The Hilbert amplitude spectrum for each test is shown from Figure 3-39 to Figure 3-42. It is difficult to observe any evident discrepancy amongst the Hilbert amplitude spectrums from the response data of these four specimens. Therefore, the Hilbert marginal spectrum is used to give a clear comparison. The detail calculations are summarized as follows:

Calculation of Hilbert marginal spectrum

1. Apply WPT (or CWT) to the absolute acceleration response data. For the WPT algorithm, the mother wavelet ‘bior6.8’ wavelet is selected and the decomposition level is 9.
2. Apply HT to every time-series components and calculate the modulus of each component.
3. In order to emphasize the main variation on the time-frequency map, a smoothing technique is used. It is suggested to introduce a simple moving average (MA) technique to each Hilbert transformed components. The window length of the MA procedure is chosen as 0.5 second (100 data points).
4. According to Equation (2.5.10), the Hilbert marginal spectrum is calculated for a specified time-window.

The Hilbert marginal spectrum of two time-windows, 28-40 second strong motion data and 40-80 second coda wave, are plotted in Figure 3-43 and Figure

3-44. The peaks in these plots will shift to lower frequency band for the case of larger excitation. Specimen RCF3 has more energy distribution in the lower frequency band and specimen RCF6 has the least. It explains that the nonlinear behaviours caused by stiffness degradation and strength deterioration would result in the shift of the wavelet packet component energy from higher frequency bands to lower frequency bands and the amount of shifting is proportional to the level of nonlinearity.

3.3.4 WPT Component Correlation

Different from the Fourier transform, wavelet technique can deal with non-stationary signal and nonlinear data. The plot of component energy of WPT (defined in Equation 1.2.1) versus to frequency can provide information of signal energy distribution in frequency domain. Figure 3-45 shows the distribution of WPT component energy of four specimens. It is noted that the unit of the vertical axis is the energy ratio (percentage) respect to the total energy. It is evident that in lower frequency band, around 1Hz to 2Hz, the energy ratio shows an ascending order amongst four specimens ($RCF6 < RCF2 < RCF4 < RCF3$). It is clear to see the phenomenon of energy transferring due to the damage level of the specimen.

A further investigation to WPT component is conducted up to level-7 decomposition to examine the correlation of the wavelet packet component of the input and of the output. The first 8 leading components (with 98% energy of the total energy) of the excitation and four responses are plotted in Figure 3-46 for comparison. The results of the correlation coefficient between input and output are listed in Table 3-3. It is observed that in some frequency bands (remarked by

boldface), the correlation coefficient for the specimen with larger excitation is closer to zero. It interprets that structural damages would reduce the correlation of excitation and response in specific frequency bands.

3.3.5 Bispectral Analysis

The application of bispectral analysis is limited to the specimens with random excitations. Therefore, the data measured from the white noise tests before and after the shake table test is used. The computation procedure is given:

Calculation of the bicoherence

1. The length of the absolute acceleration signal is 16384 with sampling rate of 200Hz. A Hamming window with length of 128 is applied to divide the original signal into 128 segments.
2. Based on Equation (2.5.13), the bicoherence is calculated by averaging the bicoherence coefficient of all the segments.
3. An index for the bicoherence is proposed:

$$DI_B = \sum E_{xxx}(f_i, f_j), \in E_{xxx}(f_i, f_j) > 0.04 \quad (3.3.7)$$

The bicoherence is plotted in Figure 3-47. It is clear that all the bicoherences for the white noise before the shake table test are very ‘clean’, i.e. near to zero, which indicates no correlations between frequency f_i and f_j . And the bicoherences for the white noise after the shake table test have some peaks which are the so-called second order nonlinear coupling. For the bicoherence for the white noise after the shake table test of RCF3, the nonlinear coupling is much more significant than others. The existing nonlinear coupling can be explained that damages occur from the data of RCF3. It is notable that clear nonlinear couplings induce at the

frequency pair (2Hz, 40Hz), (23Hz, 40Hz), and (40Hz, 40Hz).

3.4 Time-Domain Nonlinearity Identification of Shake Table Test Data

3.4.1 Instantaneous Frequency

In the time domain analysis, the instantaneous frequency can be examined directly from the measurement. The calculation procedure is as follows:

Calculation of instantaneous frequency

1. Utilise WPT to the absolute acceleration response measurement, where the decomposition level equals to 9 and the mother wavelet is 'bior6.8'.
2. Calculate the WPT component energy ratio (by MATLAB function 'wenergy'). Extract the components whose energy ratios are larger than 0.1% (Figure 3-48).
3. Apply HT to each of the extracted component. Based on Equation (2.2.25) and (2.2.26), and derive the instantaneous modulus $A_{j,t}$ and instantaneous phase (unwrapped).
4. Introduce the numerical differentiation technique (proposed in Equation (2.6.2)) to the component instantaneous phase to derive the component instantaneous frequency $\omega_{j,t}$.
5. Through Equation (2.6.3), derive the time-varying mean frequency, namely the overall instantaneous frequency $f(t)$,
6. To eliminate the fluctuations, one can apply the simple MA smoothing algorithm (Equation (2.6.4)). Here the window length of 0.5 second is used.

From the plot of instantaneous frequency, as shown in Figure 3-49, it is evident that there is a descending of frequency during 30 to 40 second for all four

specimens which indicate obvious stiffness degradation. The amount of descending of the instantaneous frequency is in the ascending order: RCF6 < RCF2 < RCF4 < RCF3. After time 40 second, the descending of frequency becomes mild. It is also noted that some drops occur at 45 second and 53 second for all specimens. These drops coincide with the peaks in inter-story drift ratio which may be related to the occurrence of strength deterioration.

3.4.2 Instantaneous Phase Difference

The procedure for the calculation of instantaneous phase difference is given:

Calculation of the instantaneous phase difference

1. Apply a Butterworth low-pass filter with cut-off frequency of 20Hz to the displacement measurements to eliminate the effect of noise.
2. Apply HT to both the relative displacement and the ground displacement to obtain their instantaneous phase (unwrapped).
3. According to Equation (2.6.5), calculate the absolute value of the instantaneous phase difference.

The unwrapped instantaneous phase of input (base displacement) and output (relative displacement) for each specimen are shown in Figure 3-50 to Figure 3-53, respectively. The unwrapped instantaneous phase difference is plotted in Figure 3-54. The figure shows that the phase difference of RCF3 is the lowest, which is the specimen subjected to severest damage. The more severe damage the more stiffness degradation and cracks may occur in the structural member, which may cause the reflection and refraction of propagation waves between the base excitations and the response.

3.4.3 Holder Exponent

Following the procedure proposed in Section 2.6.3, the Holder exponent is calculated using the CWT of the absolute acceleration response data. The frequency region from 6Hz to 50Hz is concerned to compute the Holder exponent. It is suggested that to consider the frequency bands over the natural frequency of the intact structure. Figure 3-55 compared the Holder exponent calculated by considering frequency region 6Hz to 50 Hz and 1.5 Hz to 50 Hz, respectively. It can be observed that clear singularities appear in the Holder exponent considering 6Hz to 50Hz and have a good correlation with the peaks of inter-story drift ratio.

A simple MA smoothing algorithm with window length of 0.3 sec is applied to the Holder exponent to eliminate local fluctuations. The Holder exponent for shake table test data is shown in Figure 3-56. Several extreme values or singularities in the time series of Holder exponent were observed. Some basic statistical calculations are also implemented to the Holder exponent data. In the Holder exponent data, the mean value is lower, the standard deviation is larger, and the minimum value is smaller for the specimen subjected to severe excitation. Two groups of comparison of Holder exponent is made in Figure 3-57. It is obvious that the Holder exponents of RCF6 and RCF2 are similar and their singularities would have good correlation to the inter-story drift ratio (it can be proved in section 3.4.5). The Holder exponent of RCF4 and RCF3 has more singularities which may be induced by severer damages. The Holder exponents of white noise data are plotted in Figure 3-58 and Figure 3-59. It can be observed that the Holder exponent of the white noise test before the seismic excitation has higher average and higher minimum value. Holder

exponent is an excellent indicator which can provide both the local and global nonlinearity information.

3.4.4 Level-1 Detail Component of DWT

The analysis of level-1 detail component of DWT, which is also based on 'bior6.8' wavelet, is extracted and shown in Figure 3-60. The level-1 detail component contains information of frequency band of 50 Hz to 100 Hz. There are several evident spikes in the plots which indicate the presence of singularities in the recorded absolute acceleration signal. By a simple comparison, these singularity points are coincident with those of the residual time-series extracted from SSA shown in the last section. It is noticed that for the specimen with a larger excitation, the amount of spikes increases in higher frequency band. It is believed that the singularity in the response signals is formed by the cracks induced by significant external loading.

3.4.5 Residual Component of SSA

The residual component of SSA is shown in Figure 3-61. The value of ' L ' is chosen as 300. The energy threshold for grouping in SSA is set as 99.99%, i.e. the first leading singular components with energy contribution of 99.99% are discarded and the remained component is extracted as the residual component. As mentioned in the last section, the identified results from two indicators (level-1 detail component of DWT and the residual component of SSA) are similar. It can be auxiliary works for the detection of singularities.

In order to find more detailed correlations between the nonlinear indicators and the physic-meaningful parameters, a series of comparison is made. From Figure 3-62 to Figure 3-81, several physic-meaningful parameters, including the absolute acceleration, inter-story drift ratio, absolute input energy, equivalent natural frequency and equivalent damping ratio, are compared with three nonlinear indicators, which are Holder exponent, level-1 detail component of DWT, and residual component of SSA. It is clear that the peaks in these three nonlinear indicators coincide with the maximums of inter-story drift ratio. Taking RCF6 as an example (Figure 3-63), the peaks of indicators occurs at 31sec, 35.5sec, 38 sec, 40 sec, and 51 sec when the evident local maximums are present. It is confirmed that the detail component of DWT as well as the residual component SSA can identify the occurrence of significant structural drifts which have relations with the structural stiffness degradation.

3.5 Supplementary Study on RCF6

The physical parameters of Group2 are listed in Table 3-4. It should be noted that the data acquisition of the second shake table test (800gal) failed so that all measurements of RCF6-x were missing. The measured data including absolute acceleration, inter-story drift ratio and hysteresis loops are plotted in Figure 3-82 to Figure 3-84, respectively. The peak acceleration of the damaged specimen (464 gal for RCF6-6) is much lower than the intact specimen (769 gal for RCF6-1). The inter-story drift ratio in Figure 3-83 indicates that a damaged specimen would have

a smaller drift ratio than an undamaged system under the same excitation, e.g. 1.35% for RCF6-1 and 2.01% for RCF6-6, due to the softening phenomenon. Likewise, the absolute input energy plotted in Figure 3-86 shows similar phenomenon. The absolute input energy of damaged specimen is lower than the energy of the intact specimen.

The Fourier spectrum is shown in Figure 3-85. The spectrum keeps moving to lower frequency bands until RCF6-3 where the stiffness degradation may complete. To confirm this observation, the system frequency (Figure 3-87) estimated by equivalent linear system is examined. It is observed that the degradation of stiffness had completed at RCF6-3. After that, the equivalent system frequency turns to steady.

Applications of the coherence function and the Hilbert transform of FRF are conducted to the white noise test data of Group2. The results are plotted in Figure 3-88. Two damage indices to quantify the nonlinearity are proposed in Equation (3.3.2) and (3.3.3). Both indices for each white noise test are plotted in Figure 3-89. The trends of both damage indices are assembled. Both damage indices have the highest value at WN3, which is the white noise test after RCF6-x (800gal). After WN3, the values of damage indices decrease to a lower value and growth gradually afterwards. One reasonable explanation is that due to the feature of higher order d.o.f for the frame structure, the specimen completed some kinds of formation, such as locking of cracks, at RCF6-2. So the specimen became stable temporally at the sequential white noise test (WN4). Experiencing of the following strong excitations, the formation may be destroyed, e.g. unlocking of cracks or presence of new cracks, so that the damage indices increase afterwards.

Another frequency-domain indicator which is the WPT component correlation coefficient is applied (the component waveforms are shown in Figure 3-91). In Table 3-5, it can be observed (the boldfaces) that only slight transferring of component energy occurs for the specimens after RCF6-3. It reflected that after the RCF6-3, the structure in frequency domain becomes steady.

The results from the bispectral analysis are shown in Figure 3-92. It is clear that from WN5 to WN8 there is an evident coupling at the frequency pair (4Hz, 19Hz). It is no doubt that a clean bicoherence shown in WN1. However, WN3 and WN8 also have fewer nonlinear couplings. From Figure 3-93, the variation of damage index defined in Equation (3.3.4) gives general information of the level of second-order nonlinearity for each specimen.

The time-domain nonlinear indicators, the instantaneous phase difference and the Holder exponent, are also applied. The instantaneous phase difference is plotted in Figure 3-94. The result is basically confirmed with the stiffness degradation. The slopes of the specimens after RCF6-3 are identical (during 30~60 second). The Holder exponent for shake table test data and white noise data are plotted in Figure 3-95 and Figure 3-96, respectively. It shows a continuous changing in statistical indices. It proves that the specimen kept damaging although its natural frequency remained unchanged after RCF6-3. Another class of damage different from stiffness degradation did occur. It is observed that for RCF6-3, RCF6-4 and RCF6-5 (including WN5 to WN7), three statistical indices are identical. It is verified by a comprehensive survey with the bispectral analysis, the similarity amongst WN6 to WN8, which are the white noise tests after RCF6-4 to RCF6-6, respectively.

3.6 Chapter Summary

A series of test of two-bay one-story RC frames with different excitation levels is conducted on the shake table in NCREE. Two groups of specimen are examined respectively. The nonlinear indicators introduced in Chapter 2 are applied to the experimental data. It is proved that these nonlinear indicators are capable of reflecting variations of structural system parameters.

A brief summary of nonlinear identification in this experiment is given below:

1. Strong ground motion started at 30 second for all specimens. The excitation level is arranged as: RCF3>RCF4>RCF2>RCF6.
2. The residual displacement occurred during 38 second and 40 second. For RCF3, the second residual displacement occurred during 50 second and 53 second.
3. The HT of FRF and the coherence function can provide a fast examination for the presence of nonlinearity by comparing in the white noise test data before and after the shake table test. The analysis result shows that RCF3 has the highest level of nonlinearity.
4. For the estimation of natural frequency, the nonlinear indicators including the equivalent linear system, the Hilbert marginal spectrum, and the instantaneous frequency gives a clear insight. The natural frequency identified turned to steady after 40 second which implies the ending timing of stiffness degradation for all specimens. The stiffness degradation is inferred by instantaneous frequency and the analysis results show the degradation severity: RCF3>RCF4>RCF2>RCF6.
5. From the viewpoint of wave propagation, the WPT component correlation coefficient and the instantaneous phase difference show evident results which

imply the property changing of the structural system. The nonlinearity severity in a descending order is: $RCF3 > RCF4 > RCF2 > RCF6$.

6. For RCF3, clear nonlinear couplings induced in the bicoherence are at the frequency pair (2Hz, 40Hz), (23Hz, 40Hz), and (40Hz, 40Hz). For WN5 to WN8 in RCF6, they indicate that an obvious nonlinear coupling locates at the frequency pair (4Hz, 19Hz).
7. For singularity identification, the Holder exponent, level-1 detail component of DWT, and the residual series of SSA can give an assemble identification result. The spikes in these three indicators, especially the latter two, coincide with the local maximums of the inter-story drift ratio.
8. For level-1 detail component of DWT, and the residual series of SSA, it is observed that for the specimen with a larger excitation, the spikes of both nonlinear indicators increase. It may attribute to more cracks presence.
9. The statistical indices of Holder exponent is related to the nonlinearity level. The mean value of Holder exponent is related to the degradation of stiffness.

Summaries with quantified nonlinear indicators are given in Table 3-6 and Table 3-7 for Group1 and Group2, respectively. For white noise test data, it provides a baseline (the white noise test before shake table test) information so nonlinear indicators can indicate nonlinearity by calculating the difference between the baseline and the damaged cases. Considering the effect of noise induced in the experiment, de-noise process is needed for some nonlinear indicators. A summary of the de-noise process used to each nonlinear indicators is given in Table 3-8. In this study, low-pass filter, WPT, and SSA (for the improvement of numerical differentiation scheme) are utilised to eliminate the effect of noise.

A complementary study to this RC frame shake table test (Huang 2009) devoted to identifying the parameters in a generalised Bouc-Wen model. He indicated that stiffness degradation occurs from time 30 second to 40 second and strength deterioration occurs from time 38 to 40 second as well as time 45 second to 53 second. His result shows the correctness of the structural damage identification by the nonlinear indicators proposed in this study.

Finally, it is emphasized that the SSA technique is a potential signal processing tool for structural health monitoring. As discussed in Section 3.2, SSA is capable of extracting the residual deformation and improving the numerical differentiation. Furthermore, by using SSA, it is also possible to estimate residual deformation from acceleration response signal. In conclusion, the application of SSA is a potential topic for structural health monitoring.

CHAPTER 4 CONCLUSIONS AND RECOMMENDATIONS

4.1 Research Conclusions

The main goal of this research is to employ several nonlinear indicators for structural health monitoring and structural damage assessment. By analysing the structural response data using the proposed nonlinear indicators, one can determine the occurrence of nonlinearities and identify the structural damage severity. A series of shake table test to the 1-story 2-bay RC frame is conducted to verify the capability of using these nonlinear indicators. The analysis results can be summarized as follows:

1. Identify the system dynamic characteristics using equivalent linear system. The equivalent system natural frequency shows a trend of the variation of structural stiffness. Although it can not detect refined information of nonlinearities, it provides a simple tool to estimate the degradation ratio of stiffness. It is important to note that in conducting the identification of equivalent linear system with moving time window, the initial condition of each time window must be considered.
2. Compare the system FRF and its Hilbert transform using input and output measurements. It serves as a nonlinear indicator without using baseline to detect the system nonlinearity. Or it can be used for the case of random excitation with information from intact system as a baseline to assess the degree of nonlinearity.
3. The coherence function is to examine the correlation between input and output signals. It can resist noise corruption on the data and indicate the presence of

nonlinearity for a specific frequency band. It can also assess the damage severity by comparing the white noise test data before and after the shake table test.

4. The Hilbert marginal spectrum provides a clear view of energy distribution in the frequency domain. Peaks in Hilbert marginal spectrum move to lower frequency bands with the development of structural damage.
5. The components extracted from WPT can provide more detailed information than the original signal. The correlation between the WPT components of excitation and response can also detect the structural system nonlinearity.
6. The bicoherence shows a different view to the damage. Second-order nonlinear couplings can be revealed in the bicoherence.
7. The instantaneous frequency derived by WPT and HT techniques can also provide more refined information on system dynamic characteristics. Smoothing techniques are recommended to employ if instantaneous frequency fluctuates.
8. The instantaneous phase difference between input and output can also be used for damage detection. The damages or cracks in structure would alter the incident wave propagation paths in the structure. This implies changing in the phase difference. The higher the damage level is, the lower is the instantaneous phase difference.
9. The Holder exponent can detect the singularities of the signal and it is an advanced analysis based on the CWT technique. It is shown that these singularities have good correlation to the occurrence of peaks in inter-story drift ratio.
10. The level-1 detail component of DWT is a simple indicator to locate the

11. The other application of SSA is to extract the residual displacement. The occurrence time and the amounts of the permanent displacement can be obtained from SSA. With the help of SSA, the acceleration can be derived from the optical sensor measurements by numerical differentiation technique.
12. One more application of the SSA technique is to estimate the residual displacement from the acceleration signal. Although there is not a perfect algorithm, several trials have been made in this research. It shows that it is possible to estimate the permanent deformation from acceleration signal by using SSA.

According to the above-mentioned conclusions, a brief assessment of the feasibility for damage detection to the nonlinear indicators in this research is shown in Table 4-1. It is recommended that one can use the Hilbert transform of FRF and the coherence function to distinguish if the structure is damaged. It also suggests that if one intends to detect the singularities rapidly, the level-1 component of DWT is the best candidate. If one requires the information of both the occurrence timing and the severity of structural damage, Holder exponent is a very good nonlinear indicator. The SSA technique can be used for singularity detection or the estimation of permanent deformation.

4.2 Research Contributions and Limitations

Two sets of test data from shake table test of RC frames are examined in this research. Group1 focuses on the influence of excitation level to the structural nonlinearities. Based on the proposed nonlinear indicators, the degree of structural nonlinearity is investigated. Group2 focuses on the study of response characteristics of a structure subjected to different levels of the excitation back to back. Group2 is an emulation of most of the real-life structures which may experience different level of excitations during its life-cycle. The proposed nonlinear indicators are applied to both classes of specimens. A practical exhibition for each nonlinear indicator is demonstrated and the assessment as well as the limitation of each nonlinear indicator is discussed in this research. It is believed that one can identify the structural damage information by using the proposed nonlinear indicators from the direct measurement of structural responses. The proposed nonlinear indicators provide a tool for vibration-based damage assessment of structures.

A pioneer study in this study is the estimation of residual displacement from the acceleration signal and SSA is the main tool to achieve this objective. The proposed method to estimate permanent deformation was verified with the displacement measurement. Future study is needed to have a better estimation of deformation by using SSA method.

This research focuses on of nonlinear system identification method to a single-degree-of-freedom system. Further extensions of these nonlinear indicators to multi-degree-of-freedom system are required. The RC frame used in this study only provides one type of seismic loading. Different seismic excitation has its own feature and may cause different type of damages. This research is mainly dealing

with the nonparametric methods so less information about the amounts of variations in system parameters can be possessed.

4.3 Recommendations for Future Work

The objective of structural damage assessment is to locate the damage and to quantify the degree of damage. In this research, it shows the possibility of using nonlinear indicators to reveal structural damages. Several efforts to quantify the degree of structural damage by proposed nonlinear indicators are exhibited in this study. For example, the RMSE can be served as an index to quantify the diversity of FRF and its Hilbert transform which implies the degree of nonlinearity. It is believed that better indices can be developed to accurately quantify nonlinear indicators to represent the degree of damages.

For an accurate identification of the structural damages, it is necessary to conduct a comprehensive analysis. The combination of the proposed nonlinear indicators (nonparametric system identification methods) with parametric system identification methods or with other signal processing tools will give an accurate identification to structural damages. Another possibility is to combine the nonparametric nonlinear indicators with other parametric methods. It may provide a way to exactly quantifying of damages.

A new attempt to estimate the permanent deformation from acceleration measurement by SSA is an interesting topic. The method to extract the tendency of acceleration measurement is one important issue. In this study, the additional constant method is applied to extract the weak tendency in the acceleration data.

The value of the additional constant may influence the tendency extracted by SSA.
Another important issue is how to determine the appropriate value of parameter ' L '.
An automatic algorithm to determine ' L ' is needed. More studies are still required.

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Table 2-1 Summary of the nonlinear indicators

Nonlinear Indicators		A priori Model	Stationary Random Excitation	Require Information of Input Signal	Require Baseline Information
Equivalent Linear System		○	X	○	X
Frequency-Domain Indicator	HT of FRF	X	X	○	X
	Coherence Function	X	○	○	X
	Hilbert Marginal Spectrum	X	X	X	○
	WPT component correlation	X	X	○	○
	Bispectral Analysis	X	○	X	X
Time-Domain Indicator	Inst. Frequency	X	X	X	X
	Inst. Phase Difference	X	X	○	○
	Holder exponent	X	X	X	X
	1 st Detail of DWT	X	X	X	X
	Residual series of SSA	X	X	X	X

Table 3-1 Physical parameters measured or identified from the shake table tests (Group1)

Specimen NO.	Excitation	PGA (gal)	Max. Absolute Acceleration (gal)	Max. Inter-Story Drift Ratio (%)	Equivalent Linear System Frequency		
					Starting Freq. (Hz)	Ending Freq. (Hz)	Inter-story Stiffness Reduction ratio (%)
RCF6	White Noise	29	24	0.06	5.81	5.8	
	TCU082	622	750	1.35	5.87	2.87	69.80
	White Noise	30	15	0.04	3.24	3.14	
RCF2	White Noise	27	41	0.02	6.2	6.02	
	TCU082	815	1241	2.43	6.05	2.72	76.61
	White Noise	30	28	0.05	3.06	2.85	
RCF4	White Noise	29	46	0.03	5.68	5.68	
	TCU082	1157	1305	3.29	5.6	2.49	78.40
	White Noise	32	25	0.06	2.67	2.61	
RCF3	White Noise	24	40	0.04	5.83	5.78	
	TCU082	1287	1216	4.46	5.68	2.06	84.57
	White Noise	35	25	0.06	2.24	2.32	

Table 3-2 Analysis of the optimal L for the improvement of numerical differentiation

L	No. of components used for reconstruction	Energy ratio	RMSE
0	-	-	0.0344
25	6	0.9991	0.0327
50	9	0.9987	0.322
66	11	0.9988	0.0319
67	11	0.9987	0.0317
68	11	0.9985	0.315
69	11	0.9983	0.313
70	12	0.9993	0.325
100	16	0.9994	0.324
200	29	0.9995	0.322

Table 3-3 Wavelet packet component correlation coefficient (Group1)

25 - 30 sec (Bandwidth(Hz))	Band1 (0 - 0.8)	Band2 (0.8 - 1.6)	Band3 (1.6-2.3)	Band4 (2.3 - 3.1)	Band5 (3.1 - 3.9)	Band6 (3.9 - 4.7)	Band7 (4.7 - 5.5)	Band8 (5.5 - 6.2)
RCF6	0.90	1.00	0.98	0.78	0.97	0.27	0.95	-0.69
RCF2	1.00	1.00	0.95	0.68	0.98	0.60	0.98	-0.39
RCF4	1.00	1.00	0.82	0.59	0.93	0.63	0.58	-0.71
RCF3	1.00	0.99	0.82	0.65	0.83	0.75	0.37	-0.65
30 - 40 sec (Bandwidth(Hz))	Band1 (0 - 0.8)	Band2 (0.8 - 1.6)	Band3 (1.6-2.3)	Band4 (2.3 - 3.1)	Band5 (3.1 - 3.9)	Band6 (3.9 - 4.7)	Band7 (4.7 - 5.5)	Band8 (5.5 - 6.2)
RCF6	0.97	0.99	0.94	0.81	0.33	-0.63	-0.82	-0.90
RCF2	1.00	0.97	0.94	0.37	0.44	-0.63	-0.83	-0.91
RCF4	0.99	0.96	0.88	-0.09	0.18	-0.38	-0.91	-0.94
RCF3	0.99	0.95	0.79	-0.04	0.09	-0.57	-0.92	-0.87
40 - 75 sec (Bandwidth(Hz))	Band1 (0 - 0.8)	Band2 (0.8 - 1.6)	Band3 (1.6-2.3)	Band4 (2.3 - 3.1)	Band5 (3.1 - 3.9)	Band6 (3.9 - 4.7)	Band7 (4.7 - 5.5)	Band8 (5.5 - 6.2)
RCF6	0.94	0.99	0.90	0.49	-0.47	-0.68	-0.82	-0.85
RCF2	1.00	0.98	0.81	-0.02	-0.65	-0.50	-0.86	-0.92
RCF4	0.99	0.96	0.69	-0.21	-0.78	-0.38	-0.88	-0.82
RCF3	0.99	0.88	0.52	-0.59	-0.86	-0.44	-0.89	-0.83

Table 3-4 Physical parameters measured or identified from the shake table tests (Group2)

Specimen NO.	Excitation	PGA (gal)	Max. Absolute Acceleration (gal)	Max. Inter-Story Drift Ratio	Equivalent Linear System Frequency		
					StartFreq. (Hz)	EndFreq. (Hz)	Inter-story Stiffness Reduction ratio (%)
WN1	White Noise	29	24	0.06	5.82		0.00
RCF6-1	TCU082	622	769	1.35	5.87	2.87	71.26
WN2	White Noise	30	18	0.04	3.12		
RCF6-x	TCU082	800	-	-	-	-	81.99
WN3	White Noise	32	13	0.05	2.47		
RCF6-2	TCU082	1041	745	2.69	2.6	2.45	78.95
WN4	White Noise	30	15	0.06	2.67		
RCF6-3	TCU082	1103	766	2.93	2.35	2.35	80.50
WN5	White Noise	32	14	0.05	2.57		
RCF6-4	TCU082	1004	711	2.88	2.35	2.36	80.50
WN6	White Noise	27	15	0.07	2.57		
RCF6-5	TCU082	803	609	2.45	2.36	2.39	81.25
WN7	White Noise	30	13	0.06	2.52		
RCF6-6	TCU082	582	464	2.01	2.42	2.43	81.70
WN8	White Noise	32	13	0.06	2.49		

Table 3-5 Wavelet packet component correlation coefficient (Group2)

25 - 30 sec	Band1	Band2	Band3	Band4	Band5	Band6	Band7	Band8
(Bandwidth(Hz))	(0 - 0.8)	(0.8 - 1.6)	(1.6-2.3)	(2.3 - 3.1)	(3.1 - 3.9)	(3.9 - 4.7)	(4.7 - 5.5)	(5.5 - 6.2)
RCF6-1	0.90	1.00	0.98	0.78	0.97	0.27	0.95	-0.69
RCF6-2	0.92	0.98	0.86	0.46	-0.74	-0.91	-0.69	-0.93
RCF6-3	0.48	0.96	0.81	0.42	-0.90	-0.91	-0.46	-0.86
RCF6-4	1.00	0.93	0.77	0.35	-0.94	-0.93	-0.32	-0.78
RCF6-5	1.00	0.92	0.74	0.32	-0.93	-0.88	-0.03	-0.76
RCF6-6	1.00	0.91	0.73	0.23	-0.87	-0.83	0.00	-0.77
30 - 40 sec	Band1	Band2	Band3	Band4	Band5	Band6	Band7	Band8
(Bandwidth(Hz))	(0 - 0.8)	(0.8 - 1.6)	(1.6-2.3)	(2.3 - 3.1)	(3.1 - 3.9)	(3.9 - 4.7)	(4.7 - 5.5)	(5.5 - 6.2)
RCF6-1	0.97	0.99	0.94	0.81	0.33	-0.63	-0.82	-0.90
RCF6-2	0.97	0.95	0.84	-0.41	0.22	-0.45	-0.97	-0.85
RCF6-3	0.84	0.88	0.65	-0.64	-0.18	-0.56	-0.95	-0.86
RCF6-4	0.99	0.86	0.57	-0.55	-0.47	-0.71	-0.93	-0.84
RCF6-5	0.99	0.84	0.51	-0.51	-0.59	-0.78	-0.90	-0.86
RCF6-6	0.99	0.82	0.42	-0.51	-0.67	-0.85	-0.88	-0.90
40 - 75 sec	Band1	Band2	Band3	Band4	Band5	Band6	Band7	Band8
(Bandwidth(Hz))	(0 - 0.8)	(0.8 - 1.6)	(1.6-2.3)	(2.3 - 3.1)	(3.1 - 3.9)	(3.9 - 4.7)	(4.7 - 5.5)	(5.5 - 6.2)
RCF6-1	0.94	0.99	0.90	0.49	-0.47	-0.68	-0.82	-0.85
RCF6-2	0.99	0.97	0.63	-0.26	-0.74	-0.30	-0.80	-0.81
RCF6-3	0.80	0.92	0.52	-0.67	-0.74	-0.22	-0.82	-0.83
RCF6-4	0.99	0.90	0.47	-0.79	-0.77	-0.21	-0.86	-0.85
RCF6-5	0.99	0.89	0.44	-0.83	-0.80	-0.17	-0.87	-0.86
RCF6-6	0.99	0.86	0.39	-0.86	-0.81	-0.14	-0.90	-0.90

Table 3-6 Summary of nonlinearity identification (Group 1)

Nonlinear indicators		RCF6	RCF2	RCF4	RCF3
<i>Shake table test (without baseline)</i>					
Peak Absolute Acceleration	(gal)	769	1241	1305	1216
Peak Drift Ratio	(%)	1.35	2.43	3.29	4.46
HT of FRF	(DI _H)	3.34	2.2	1.68	3.54
Holder exponent	Mean	2.2	2.25	1.92	1.03
Holder exponent	Std.	0.48	0.48	0.48	0.55
Holder exponent	Min. value	0.7	0.63	0.39	-0.76
<i>White noise test (with baseline): ($DI_{before} - DI_{after} / DI_{before}$)</i>					
Stiffness degradation ratio	(%)	69.8	76.61	78.4	84.57
HT of FRF	(DI _H)	1.33	1.87	1.20	3.03
Coherence function	(DI _C)	4.20	5.86	2.60	13.51
Bispectral analysis	(DI _B)	1.50	2.34	4.40	23.75
Holder exponent	Mean	0.51	0.58	0.65	0.82

Table 3-7 Summary of nonlinearity identification (Group 2)

Shake table test							
Nonlinear indicators		RCF6-1	RCF6-2	RCF6-3	RCF6-4	RCF6-5	RCF6-6
Peak Absolute Acceleration	(gal)	769	745	766	711	609	464
Peak Drift Ratio	(%)	1.35	2.69	2.93	2.88	2.45	2.01
Holder exponent	Mean	2.2	1.83	1.65	1.62	1.55	1.48
Holder exponent	Std.	0.48	0.48	0.49	0.52	0.52	0.54
Holder exponent	Min. value	0.7	0.09	-0.08	-0.12	-0.09	-0.57

White noise test ($(WN_i - WN_I) / WN_I$)									
Nonlinear indicators		WN1	WN2	WN3	WN4	WN5	WN6	WN7	WN8
Stiffness degradation ratio	(%)	0.00	71.26	81.99	78.95	80.5	80.5	81.25	81.7
HT of FRF	(DI _H)	0.00	0.27	1.18	0.82	0.45	0.55	0.36	1.55
Coherence function	(DI _C)	0.00	4.95	11.04	4.50	6.78	7.50	7.62	15.43
Bispectral analysis	(DI _B)	0.00	1.54	0.40	1.64	0.59	1.36	0.87	0.18
Holder exponent	Mean	0.00	0.51	0.72	0.61	0.68	0.58	0.67	0.72

Table 3-8 De-noise process for each nonlinear indicator

Nonlinear Indicators		De-noise Technique	Description
Equivalent Linear System		Low-pass filter	Butterworth filter (cut-off frequency = 20 Hz.)
Frequency-Domain Indicator	HT of FRF	Low-pass filter	Butterworth filter (cut-off frequency = 20 Hz.)
	Coherence Function	Low-pass filter	Butterworth filter (cut-off frequency = 20 Hz.)
	Hilbert Marginal Spectrum	None	Higher frequency band components do not influence the analysis result.
	WPT component correlation	None	Only examine the first few leading WPT component which do not include high frequency bands.
	Bispectral Analysis	None	Since nonlinear coupling usually appears at higher frequency pairs, the de-noise process is not applied.
Time-Domain Indicator	Inst. Frequency	WPT	Utilising WPT, extract the components with energy ratio over 0.1%.
	Inst. Phase Difference	Low-pass filter	Butterworth filter (cut-off frequency = 20 Hz.)
	Holder exponent	None	Singularities are embedded in noise components so that the de-noise process is not applied.
	1 st Detail of DWT	None	Singularities are embedded in noise components so that the de-noise process is not applied.
	Residual series of SSA	None	Singularities are embedded in noise components so that the de-noise process is not applied.

Table 4-1 Assessment of damage detection capability for the nonlinear indicators

Nonlinear Indicators		Computational Effort	Nonlinearity Presence	Nonlinearity Severity	Damage Class
Equivalent Linear System		Medium	○	○	△
Frequency-Domain Indicator	HT of FRF	Low	○	○	X
	Coherence Function	Low	○	○	X
	Hilbert Marginal Spectrum	Low	○	△	△
	WPT component correlation	Low	○	△	△
	Bispectral Analysis	High	○	○	○
Time-Domain Indicator	Inst. Frequency	Low	○	○	△
	Inst. Phase Difference	Low	○	△	△
	Holder exponent	Medium	○	○	△
	1 st Detail of DWT	Low	○	△	△
	Residual series of SSA	Low	○	△	△

‘○’: Good, ‘△’: Possible, ‘X’: Impossible

Figure 2-1 Displacement response and the corresponding Fourier spectrum of the simulation test

Figure 2-2 MRA tree

Figure 2-3 WPT tree

Figure 2-4 Paley-ordered WPT tree

Figure 2-5 'Biorthogonal 6.8' wavelet

Figure 2-6 Equivalent linear system frequency and damping ratio

Figure 2-7 Hilbert amplitude spectrum of the simulation test

Figure 2-8 Hilbert Marginal Spectrum of the simulation test

Figure 2-9 Amplitude and instantaneous frequency of the simulation test

Figure 2-10 Wavelet coefficient curve in the scale domain at 5 sec and 25 sec.

Figure 2-11 The response acceleration and the Holder exponent of the simulation test

Figure 2-12 Level-1 detail components for the simulation test

Figure 2-13 The singular spectrum with L=300 for the simulation test

Figure 2-14 The reconstructed component and the residual component of SSA for the simulation test

Figure 2-15 Comparison of time-domain nonlinear indicators

Figure 3-1 The two-bay one story RC frame

Figure 3-2 The cross section of elements: (a) Beam: top reinforce 3 #3, bottom reinforce 2 #3, stirrup #3@10cm (2 ends) and #3@15cm (mid-span); (b) side columns: longitude bar 4 #4 and stirrup #3@10cm; (c) central column: longitude bar 8 #4 and stirrup #3@10cm.

Figure 3-3 Configuration of the accelerometer channels

Figure 3-4 Configuration of the LVDT

Figure 3-5 Configuration of small LVDT

Figure 3-6 Configuration of optical sensor

Figure 3-7 Excitation: the seismic record of the ChiChi earthquake

Figure 3-8 Absolute acceleration measured from each specimen

Figure 3-9 The relative displacement

Figure 3-10 Inter-story drift ratio

Figure 3-11 The hysteresis loop

Figure 3-12 The Fourier spectrum

Figure 3-13 Photographs of the specimens after the seismic excitation

Figure 3-14 Measurements from optical sensors

Figure 3-15 RCF6: Comparison of inter-story drift ratio (LVDT) and the curvature (Optic)

Figure 3-16 RCF2: Comparison of inter-story drift ratio (LVDT) and the curvature (Optic)

Figure 3-17 RCF4: Comparison of inter-story drift ratio (LVDT) and the curvature (Optic)

Figure 3-18 RCF3: Comparison of inter-story drift ratio (LVDT) and the curvature (Optic)

Figure 3-19 Absolute input energy

Figure 3-20 Equivalent linear system

Figure 3-21 SSA (L=300) components for each specimen

Figure 3-22 Residual deformation estimated by SSA

Figure 3-23 Comparison of residual deformation

Figure 3-24 The auto-correlation test for optical displacement at Node 32y

Figure 3-25 The Singular spectrum and the reconstructed displacement at Node 32y and the residual series

Figure 3-26 A comparison between the estimated acceleration without SSA and with SSA processed

Figure 3-27 Scheme of the estimation of residual displacement from acceleration by SSA

Figure 3-28 Auto-correlation test of Acceleration data (RCF2)

Figure 3-29 The result of first SSA to the original acceleration signal (RCF2)

Figure 3-30 Auto-correlation test of A1 (RCF2)

Figure 3-31 The result of second SSA to A1 (RCF2)

Figure 3-32 Estimation of residual deformation from acceleration signal by SSA (RCF2)

Figure 3-33 Estimation of residual deformation from acceleration signal by SSA (RCF3)

Figure 3-34 Comparison of FRF (blue solid line) and its Hilbert transform (black dotted line) for the shake table test data

Figure 3-35 Comparison of FRF (blue solid line) and its Hilbert transform (black dotted line) for the white noise test data before the shake table test

Figure 3-36 Comparison of FRF (blue solid line) and its Hilbert transform (black dotted line) for the white noise test data after the shake table test

Figure 3-37 FRF and coherence function of the white noise data before the shake table test

Figure 3-38 FRF and coherence function of the white noise data after the shake table test

Figure 3-39 Hilbert amplitude spectrum for RCF6

Figure 3-40 Hilbert amplitude spectrum for RCF2

Figure 3-41 Hilbert amplitude spectrum for RCF4

Figure 3-42 Hilbert amplitude spectrum for RCF3

Figure 3-43 Hilbert marginal spectrum at time period 28-40sec

Figure 3-44 Hilbert marginal spectrum at time period 40-80sec

Figure 3-45 WPT component energy distribution

Figure 3-46 WPT waveforms in the first 8 leading bands

Figure 3-47 Bicoherence

Figure 3-48 WPT component energy distribution for estimation of instantaneous frequency

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Figure 3-54 Instantaneous Phase Difference

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Figure 3-57 Comparison of Holder exponent

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Figure 3-80 Singularity detection: compared with equivalent frequency (RCF3)

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Figure 3-82 Absolute Acceleration (Group 2)

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Figure 3-90 Wavelet packet component energy (Group2)

Figure 3-91 WPT waveforms in the first 8 leading bands (group2)

Conti. Figure 3-91 WPT waveforms in the first 8 leading bands (group2)

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Figure 3-93 Damage index for bispectral index (Group2)

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Figure 3-95 Holder exponent (Group 2)

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