Region reaching control of robot manipulators

Wang, Dequn

2005

Wang, D. (2005). Region reaching control of robot manipulators. Master's thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/3646>

<https://doi.org/10.32657/10356/3646>

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Region Reaching Control of Robot Manipulators

Wang DeQun

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Master of Engineering

2005

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Acknowledgement

I would like to express my gratitude to my supervisor, Dr. Cheah Chien Chern, for his excellent guidance and great patience during my study. Without his invaluable suggestions, I could have never achieve all this work.

Special thanks are for my friends, whom I see as my brothers, for their encouragement, patience and understanding while I was working on this thesis.

I appreciate the School of Electrical and Electronics Engineering, Nanyang Technological University for giving me the opportunity to perform this work.

Finally, I would like to express my deepest gratitude to the intelligent robotics lab Laboratory Executives: Chia Chiu Kiat and Tee Kim Sia for their every kind of support.

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Summary

In conventional setpoint control problem of robots, the desired position is specified as a point. However, in some applications, the desired target is a region instead of a point. It is also important to note that when the desired region is very small, it reduces to a point. In this thesis, a new control concept called region reaching control for robot manipulators is proposed. In this new control concept, the desired objective can be specified as a region instead of a point. Several region reaching controllers are proposed in both joint-space and task-space. Since the desired region can be specified arbitrarily small, the region reaching control concept is also a generalization of setpoint control problem. The region reaching control concept is also useful for control of macro/mini structures. a large region (secondary region) is specified for the macro structure, and a smaller region (primary region) is specified for a mini system. It is also useful for narrowing down the many possible configurations in task-space control of redundant robots. Lyapunov-like functions are presented for the convergence analysis. Experiment results based on a SONY SCARA robot are presented to illustrate the performance of the proposed region reaching controllers.

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Chapter 1

Introduction

Robot manipulators are used in a wide spectrum of applications. Traditionally, robot manipulators are mainly used in industrial applications to increase speed of manufacturing or to reduce cost of production. The robot systems that are used in manufacturing applications perform mainly repetitive operations in a structured environment. With the rapid development in computing and sensor technologies, more sophisticated robots with high computing and sensing power can be designed.

One of the main aims of robotics control research is to regulate joint variables so that robot end effector automatically goes to a fixed point from an initial point. This is called setpoint control or regulation. In traditional set point control approaches, a desired position of the robot is specified as a point. However, in some applications, the desired target is a region rather than a point. For example, keeping an underwater robot within a minimum and maximum depth in water; parking a automatic vehicle into a specified

1.1 Literature Survey

parking region; measuring temperature from different regions of water. The objective of this research is to develop a region reaching control scheme for the robot manipulators. In this new control concept, a desired region can be specified as a region instead of a point.

1.1 Literature Survey

In 1976, Takase [1] proposed “computed torque” or “inverse dynamics” method appeared. This method calculates the left-hand side of Lagrange’s equation for a given robot arm in real time, and makes each joint actuator generates the required input torque according to computer command data. Since robot dynamics are highly nonlinear with coupling between joints, the “computed torque” or “inverse dynamics” method introduces some discrepancy between the computed torque and the actual torque. In 1981, Takegaki and Arimoto [2] developed a new approach from the viewpoint of mechanics for dynamic control of manipulator. They showed that using Lyapunov’s method that a simple proportional and differential (PD) control with compensation for only the gravity term is effective for setpoint control despite the nonlinearity and uncertainty of the dynamics. Moreover, this method is applicable even when redundancy occurs in the manipulator’s coordinates and is robust against uncertainty in physical parameters in the inertia and Coriolis and centrifugal terms except the gravity term. This work was considered as an important landmark in robot control theory and also the first paper on application of Lyapunov’s stability theory to robot control.

1.1 Literature Survey

In 1986, Arimoto and Miyazaki [4] further showed that the PD controlled robot system is exponentially stable. While the PD control law is beautifully simple, it's far from perfect. If the gravity force is not compensated properly or unknown term of disturbances exist such as friction torques and evaluation errors of physical constants, offset errors appear in the steady state. To reduce the offset errors, PID feedback scheme for position control of robot manipulators [3] was developed. In addition to the use of joint-angle position and velocity feedbacks at each joint actuator, the feedback of integration of position error is introduced to reduce the steady-state offset. The PID feedback scheme can also be modified to suit the case when the target point is described in terms of the Cartesian coordinates or any other task-oriented coordinates [2], [6].

In 1987, Slotine and Li [5] found a novel principle in adaptive control of mechanical systems which is a landmark for model-based adaptive control. Using this adaptive methodology, Tomei [7] proposed an adaptive PD controller. Different from Slotine and Li's way, Tomei used regressor only for the gravity vector instead of the whole robot dynamics. The proposed controller was designed to circumvent the problem of non zero steady-state errors in traditional PD control. The adaptive PD controller yields the global asymptotic stability of the whole system even if the inertia and gravity parameters are unknown. In common case in which only the robot payload is unknown, one integrator is sufficient to implement the adaptive PD controller, while a PID algorithm requires as many integrators as the number of the links. The

1.1 Literature Survey

convergence is ensured for any value of the proportional and derivative gain matrices. Later, Kelly [8] presented an adaptive PD controller with desired gravity compensation for set-point control of robot manipulators. By using a suitable Lyapunov function, the closed-loop global asymptotic stability can be attained.

In most applications of robots, a desired position for the end effector is usually specified in task space such as Cartesian space or visual space. However, the majority of the controllers developed in earlier years are joint-space controllers [1] – [16]. In order to move the robot end-effector to the desired position, it is required to solve the inverse kinematics problem to generate the desired position in joint space. When the control problem is formulated directly in task space [17] – [18], the need to solve the inverse kinematics problem is eliminated. Besides a very complicated process, solving the inverse kinematics leads to a problem that if the kinematics is uncertain, the desired path in joint-space cannot be generated exactly. A more practical and effective control strategy is task-space strategy where task oriented information is directly used in the feed back control law [19] – [36]. To overcome the problem of uncertain kinematics, several Approximate Jacobian setpoint controllers [37] – [40] were proposed recently. The proposed controllers do not require the exact knowledge of kinematics and Jacobian matrix that is assumed in the literature of robot control.

1.2 Motivation of Research

In the conventional setpoint control problems, the desired objective is specified as a point as illustrated in Figure 1.1. Research of robotics aims to realize some aspects of human functions into mechanical systems. It is important to observe from most human reaching movements that the desired target is a region rather than a point (see Figure 1.2). For example, placing chess pieces onto a chess board, inserting a thermometer into a patient's mouth, putting a spoon into a cup, placing an object on a tray or table. Other examples include driving a car into a parking lot, kicking or throwing a ball into a net, playing a game of darts etc. It is also important to note that if the desired region is specified very small, it reduces to a point.

Figure 1.1: Conventional Setpoint Control

1.2 Motivation of Research

Figure 1.2: Region Reaching Tasks

Besides reaching tasks, region reaching concept is also useful for control of macro/mini structures. As illustrated in figure 1.3, a large region (secondary region) is specified for an underwater vehicle (macro system) and a smaller region (primary region) is specified for a manipulator (mini system). This gives the vehicle more freedoms while the manipulator is performing various tasks.

Figure 1.3: Underwater robot system

1.2 Motivation of Research

Region reaching concept can also be used to narrow down the many possible configurations in task-space control of redundant robots or in obstacle avoidance. To solve the fixed obstacle avoidance problem, a "secondary desired region" could be defined. One of the "joints" will be moved and kept in the "secondary desired region", so that collision is avoided as shown in Fig 1.4.

Figure 1.4: Fixed obstacles avoidance by using region controller

The aim of this research is to develop a new robot control concept where the desired objective can be specified as a region as illustrated in Figure 1.5. Since the desired region could be specified arbitrarily small (see Figure 1.6), the region reaching control concept is also a generalization of setpoint control problem.

Figure 1.5: Region Reaching Control with a desired region

Figure 1.6: Region reaching control with very small desired region

1.3 Contributions of the Thesis

This thesis addresses the region reaching control problems of robot manipulators in both joint-space and task-space. In particular, robot control problem with desired target specified as a region. The main contributions of this thesis can be listed as follows:

1.4 Organization of the Thesis

- A new control concept called region reaching control for robot manipulators is proposed. In this new control concept, the desired objective can be specified as a region instead of a point. New Lyapunov-like functions are presented for the convergence analysis.
- The proposed region reaching controllers are extended to redundant robot. It can be used to narrow down the many possible configurations in task-space control.
- The proposed region reaching controllers are implemented on a SONY SCARA robot. Experiment results are presented to verify the theory.

1.4 Organization of the Thesis

The remaining of this thesis is organized as follows:

- Chapter 2 introduces the robot kinematics and dynamics.
- Chapter 3 presents the region reaching controllers and the stability analysis and extends the region reaching control concept to redundant robots.
- Chapter 4 presents the experiment setup and results.
- Chapter 5 concludes this thesis and proposes future research works.

Chapter 2

Robot Dynamics and Kinematics

2.1 Introduction

In motion control design problems, a mathematical model expressing the motion of the system is necessary for the analysis. In this chapter, the properties of robot kinematics are discussed in section 2.2, the relationship between joint-space and task-space is introduced. The Jacobian matrix mapping from joint-space to task-space is also derived. Then the structure and properties of robot dynamism are reviewed in section 2.3, the model that describes the motion of robot manipulator with n degrees of freedom is presented. Kinetic and potential energies of robot are introduced. Then, by using Lagrange's equations, the dynamics of robot manipulator is derived.

2.2 Robot Kinematics

In this section, the relationship between joint space and task space is introduced. Then the Jacobian matrix mapping from joint space to task space is derived. The relation between joint space and task space can be expressed by a non-linear transformation matrix $h(q): R^n \rightarrow R^m$ [10],

$$X = h(q). \quad (2.1)$$

Then the derivative of $h(q) \in R^m$ gives the Jacobian matrix of mapping from joint-space to task-space as:

$$\dot{X} = J(q)\dot{q}, \quad (2.2)$$

where $J(q) \in R^{m \times n}$ is a manipulator Jacobian matrix of the mapping from joint space to task space.

For a 2 DOF (Degree Of Freedom) planar robot manipulator as shown in Figure 2.1, the forward kinematics from joint space to task space is given as:

$$X = \begin{cases} x = l_1 \cos(q_1) + l_2 \cos(q_1 + q_2), \\ y = l_1 \sin(q_1) + l_2 \sin(q_1 + q_2), \end{cases} \quad (2.3)$$

2.3 Robot Dynamics Equations

Figure 2.1: 2-link planar robot

where l_1, m, l_2, m are the lengths of Link 1 and Link 2 respectively, q_1, q_2 are the joint angles. The Jacobian matrix of mapping from joint-space to task-space can be derived by differentiating the kinematic equation. Hence, we have

$$\dot{X} = J(q)\dot{q}, \quad (2.4)$$

where

$$J(q) = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1) \\ l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}, \quad (2.5)$$

2.3 Robot Dynamics Equations

The equation of motion of a robot manipulator with n degree of freedom in joint space can be derived using Lagrange-Euler equation [6]. In a Lagrange-

2.3 Robot Dynamics Equations

Euler system, a generalized torque vector $\tau \in R^n$ is described as [6]:

$$\frac{d}{dt}\left(\frac{\partial L(q, \dot{q})}{\partial \dot{q}}\right) - \frac{\partial L(q, \dot{q})}{\partial q} = \tau, \quad (2.6)$$

where $q \in R^n$ is a vector of generalized joint coordinates, $L(q, \dot{q})$ is a Lagrangian of the manipulator system and can be written as:

$$L(q, \dot{q}) = K(q, \dot{q}) - P(q), \quad (2.7)$$

where $K(q, \dot{q})$ denotes kinetic energy and $P(q)$ denotes a potential energy of the manipulator. The kinetic energy of the robot manipulator with n degrees of freedom can be described by:

$$K(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q}, \quad (2.8)$$

where $M(q) \in R^{n \times n}$ denotes a positive definite inertia matrix. Substituting equations (2.7) and (2.8) into (2.6), the motion equation of robot manipulator system can be derived as [6, 10, 41]:

$$M(q)\ddot{q} + \left(\frac{1}{2}\dot{M}(q) + S(q, \dot{q})\right)\dot{q} + g(q) = \tau, \quad (2.9)$$

where,

$$S(q, \dot{q})\dot{q} = \frac{1}{2}\dot{M}(q) - \frac{1}{2}\left[\frac{\partial}{\partial q}\dot{q}^T M(q)\dot{q}\right],$$

$$g(q) = \left[\frac{\partial P(q)}{\partial q}\right]^T.$$

2.3 Robot Dynamics Equations

In the dynamic equation (2.9), $g(q) \in R^n$ is the gravitational force vector, $\tau \in R^n$ is the control inputs.

Knowledge of the properties of robot dynamics in robot control design is necessary to design simpler and more effective controllers. Two important properties for proving stability is summarized as [6]

Property 1: The inertia matrix $M(q)$ is symmetric and positive definite for all $q \in R^n$.

Property 2: $S(q, \dot{q})$ is skew-symmetric such that:

$$\dot{q}^T S(q, \dot{q}) \dot{q} = 0,$$

for all $q, \dot{q} \in R^n$.

Chapter 3

Region Reaching Controller

In this chapter, we present several region reaching control schemes for robotic manipulators. Stability analysis of the region reaching control systems are presented.

3.1 Task-Space Region Reaching Control with Single Objective Function

For illustration, we first consider a region reaching control problem where the desired region is specified by an inequality function as follows:

$$f(\delta X_0) \leq 0, \quad (3.1)$$

where $f(\delta X_0) \in R$ is a scalar function that with continuous first partial derivatives, $\delta X_0 = X - X_0$, $X_0 \in R^n$ is a reference point of the desired region. An illustration of the objective function is shown in figure 3.1.

3.1 Task-Space Region Reaching Control with Single Objective Function

Figure 3.1: Illustration of the objective function

For example, if $X = [x, y]^T \in R^2$, a desired region can be specified as a circle as:

$$f(\delta X_0) = (x - x_0)^2 + (y - y_0)^2 - r^2 \leq 0, \tag{3.2}$$

where r is the radius of the circle, (x_0, y_0) is the center of a circle as shown in Figure 3.2. If r reduces to zero, the desired region reduces to a point.

Figure 3.2: Desired region specified as a circle

3.1 Task-Space Region Reaching Control with Single Objective Function

The desired region could be also specified as a opened region as illustrated in Figure 3.3. The objective function is specified as:

$$f(\delta X_0) = x^4 + c - y \leq 0, \quad (3.3)$$

where c is a scalar. Since the workspace of the robot manipulator is limited, the opened region is actually closed by the finite work space.

Figure 3.3: Desired region specified as a opened region

The potential energy function for the region reaching controller is specified in task space as:

$$P_a(X) = \frac{k_p}{2} [\max(0, f(\delta X_0))]^2. \quad (3.4)$$

That is,

$$P_a(X) = \begin{cases} 0, & f(\delta X_0) \leq 0, \\ \frac{k_p}{2} f^2(\delta X_0), & f(\delta X_0) > 0, \end{cases} \quad (3.5)$$

3.1 Task-Space Region Reaching Control with Single Objective Function

where k_p is a positive constant. An graphical illustration of the potential energy function is shown in Figure 3.4. Partial differentiating the potential energy (3.5) with respect to X , we have,

$$\left(\frac{\partial P_a(X)}{\partial X}\right)^T = \begin{cases} 0, & f(\delta X_0) \leq 0, \\ k_p f(\delta X_0) \left(\frac{\partial f(\delta X_0)}{\partial X}\right)^T, & f(\delta X_0) > 0, \end{cases} \quad (3.6)$$

Figure 3.4: Task-space potential energy function

which can be written as,

$$\left(\frac{\partial P_a(X)}{\partial X}\right)^T = k_p \max(0, f(\delta X_0)) \left(\frac{\partial f(\delta X_0)}{\partial X}\right)^T. \quad (3.7)$$

From equation (3.7), the task-space region reaching controller is proposed as:

$$\tau = -K_v \dot{q} - J^T(q) k_p \max(0, f(\delta X_0)) \left(\frac{\partial f(\delta X_0)}{\partial X}\right)^T + g(q), \quad (3.8)$$

3.1 Task-Space Region Reaching Control with Single Objective Function

where $K_v \in R^{n \times n}$ is a positive definite velocity feedback gain matrix, $J^T(q)$ is the transpose of the Jacobian matrix, $g(q) \in R^n$ is the gravitational force vector, $\tau \in R^n$ is the control inputs. $(\frac{\partial f(\delta X_0)}{\partial X})^T$ is a gradient vector that provides direction information toward the desired region. Substituting equation (3.8) into equation (2.9), the closed-loop equation is obtained as:

$$M(q)\ddot{q} + (\frac{1}{2}\dot{M}(q) + S(q, \dot{q}))\dot{q} + J^T(q)[k_p \max(0, f(\delta X_0))(\frac{\partial f(\delta X_0)}{\partial X})^T] + K_v \dot{q} = 0. \quad (3.9)$$

A Lyapunov-like function is proposed as:

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + P_a(X), \quad (3.10)$$

where $P_a(X)$ is defined in equation (3.5). Note that V is a scalar function with continuous first partial derivatives since $f(\delta X_0)$ is continuous in X . Differentiating (3.10) with respect to time, yields:

$$\dot{V} = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{X}^T (\frac{\partial P_a(X)}{\partial X})^T. \quad (3.11)$$

Substituting equations (3.7) and (3.9) into equation (3.11), we have,

$$\begin{aligned} \dot{V} = & -k_p \max(0, f(\delta X_0))\dot{q}^T J^T(q)(\frac{\partial f(\delta X_0)}{\partial X})^T + k_p \max(0, f(\delta X_0))\dot{X}^T (\frac{\partial f(\delta X_0)}{\partial X})^T \\ & - \dot{q}^T K_v \dot{q} - \dot{q}^T S(q, \dot{q})\dot{q}. \end{aligned} \quad (3.12)$$

3.2 Task-space Region Reaching Control with Multiple Objective Functions

Since $\dot{X} = J(q)\dot{q}$, simplifying equation (3.12) and applying Property 2, $\dot{V}$ reduces to:

$$\dot{V} = -\dot{q}^T K_v \dot{q}. \quad (3.13)$$

We are now in a position to state the following Theorem:

Theorem 3.1: *The closed-loop system described by equation (3.9) gives rise to the convergence of X to the desired region $f(\delta X_0) \leq 0$ and $\dot{q}$ to 0 as $t \rightarrow \infty$.*

Proof: Since $\dot{V} = 0$ implies $\dot{q} = 0$ as $t \rightarrow \infty$, the maximum invariant set satisfy:

$J(q)^T k_p \max(0, f(\delta X_0)) \left(\frac{\partial f(\delta X_0)}{\partial X} \right)^T = 0$. Hence $\max(0, f(\delta X_0)) \left(\frac{\partial f(\delta X_0)}{\partial X} \right)^T = 0$ and $\dot{q} = 0$ if $J(q)$ is non singular. As seen from equation (3.6), this implies that $f(\delta X_0) \leq 0$ as $t \rightarrow \infty$.

3.2 Task-space Region Reaching Control with Multiple Objective Functions

In the previous section, the desired region is specified as a single objective function (3.1). In this section, we extend the result to the region control problem where the desired region can be specified as multiple objective functions

3.2 Task-space Region Reaching Control with Multiple Objective Functions

as follows:

$$f(\delta X_0) = \begin{bmatrix} f_1(\delta X_{01}) \\ f_2(\delta X_{02}) \\ \vdots \\ f_N(\delta X_{0N}) \end{bmatrix} \leq 0, \quad (3.14)$$

where $\delta X_{0i} = X - X_{0i}$, X_{0i} is the reference point of the i^{th} desired region, $i = 1, 2, \dots, N$, N is the total number of objective functions, $f_i(\delta X_{0i})$ are scalar functions with continuous first partial derivatives. For example, a desired region can be specified as an intersection of two circles as shown in Figure 3.5 (a). In this case, the objective functions are specified as:

$$\begin{aligned} f_1(\delta X_{01}) &= (x - x_{01})^2 + (y - y_{01})^2 - r_1^2 \leq 0, \\ f_2(\delta X_{02}) &= (x - x_{02})^2 + (y - y_{02})^2 - r_2^2 \leq 0, \end{aligned} \quad (3.15)$$

where r_1, r_2 are the radius of the two circles, (x_{01}, y_{01}) and (x_{02}, y_{02}) represent the centers of the two circles. If $f_2(\delta X_{02})$ changes to $f_2(\delta X_{02}) = r_2^2 - (x - x_{02})^2 - (y - y_{02})^2 \leq 0$, then the desired region is illustrated in Figure 3.5 (b).

Figure 3.5: Desired region as the intersection of two circles

3.2 Task-space Region Reaching Control with Multiple Objective Functions

Another example is illustrated in Figure 3.6, where the desired region is a rectangle specified by,

$$\begin{aligned} f_1(\delta X_{01}) &= (x - x_{01})^2 - a^2 \leq 0, \\ f_2(\delta X_{02}) &= (y - y_{02})^2 - b^2 \leq 0, \end{aligned} \quad (3.16)$$

such that a, b are half of the length and width of the rectangle, (x_{01}, y_{02}) represent the center of the rectangle.

Figure 3.6: Desired region as a rectangle

3.2 Task-space Region Reaching Control with Multiple Objective Functions

Figure 3.7 shows another example where the desired region is a triangle specified by,

$$\begin{aligned} f_1(\delta X_{01}) &= y - (k_3x + d_3) \leq 0, \\ f_2(\delta X_{02}) &= (k_2x + d_2) - y \leq 0, \\ f_3(\delta X_{03}) &= y - (k_1x + d_1) \leq 0, \end{aligned} \quad (3.17)$$

such that $k_1, d_1, k_2, d_2, k_3, d_3$ are constants that specifies the lines 1, 2, 3.

Figure 3.7: Desired region as a triangle

One more example of a desired region specified by multiple functions is illustrated in Figure 3.8. The multiple objective functions are specified by:

$$\begin{aligned} f_1(\delta X_0) &= (x - x_0)^2 - a^2 \leq 0, \\ f_2(\delta X_0) &= (y - y_0)^2 - a^2 \leq 0, \\ f_3(\delta X_0) &= (y - y_0)^2 - b_1^2 \leq 0, \end{aligned} \quad (3.18)$$

3.2 Task-space Region Reaching Control with Multiple Objective Functions

Figure 3.8: Region reaching within a specified boundary

In equation (3.18), the first two inequalities specified the desired target and the last inequality specified a boundary for the movement of the robot. The objective of this region reaching control problem is to regulate the robot joints, so that the end effector reaches for the desired square region while keeping the reaching movement within the specified boundary.

The potential energy function for the region reaching controller with multiple objective functions is specified as:

$$P_b(X) = \sum_{i=1}^N P_{bi}(X), \quad (3.19)$$

where,

$$P_{bi}(X) = \frac{k_{pi}}{2} [\max(0, f_i(\delta X_{0i}))]^2 \quad (3.20)$$

and k_{pi} are positive constants. Note that $P_b(X) = 0$ if and only if all the objective functions in (3.14) are satisfied. Similarly, the above equation (3.20)

3.2 Task-space Region Reaching Control with Multiple Objective Functions

can be written as:

$$P_{bi}(X) = \begin{cases} 0, & f_i(\delta X_{0i}) \leq 0, \\ \frac{k_{pi}}{2} f_i^2(\delta X_{0i}), & f_i(\delta X_{0i}) > 0, \end{cases} \quad (3.21)$$

Partial differentiating the potential energy function described by equation (3.21) with respect to X , we have,

$$\left(\frac{\partial P_{bi}(X)}{\partial X}\right)^T = \begin{cases} 0, & f_i(\delta X_{0i}) \leq 0, \\ k_{pi} f_i(\delta X_{0i}) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T, & f_i(\delta X_{0i}) > 0, \end{cases}, \quad (3.22)$$

which can be written as:

$$\left(\frac{\partial P_{bi}(X)}{\partial X}\right)^T = k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T. \quad (3.23)$$

Partial differentiating the potential energy function described by equation (3.19) with respect to X and using equation (3.23), we have,

$$\left(\frac{\partial P_b(X)}{\partial X}\right)^T = \sum_{i=1}^N \left(\frac{\partial P_{bi}(X)}{\partial X}\right)^T = \sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T. \quad (3.24)$$

From equation (3.24), the region reaching controller is proposed as:

$$\tau = -J^T(q) \sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T - K_v \dot{q} + g(q). \quad (3.25)$$

3.2 Task-space Region Reaching Control with Multiple Objective Functions

Substituting equation (3.25) into equation (2.9), the closed-loop equation is obtained as:

$$M(q)\ddot{q} + \left(\frac{1}{2}\dot{M}(q) + S(q, \dot{q})\right)\dot{q} + J^T(q)\left[\sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i}))\left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T\right] + K_v \dot{q} = 0. \quad (3.26)$$

The Lyapunov-like function is proposed as:

$$V = \frac{1}{2}\dot{q}^T M(q)\dot{q} + P_b(X), \quad (3.27)$$

where $P_b(X)$ is defined in equations (3.19) and (3.20). V is a scalar function with continuous first partial derivatives since $f_i(\delta X_{0i})$ is continuous in X .

Differentiating equation (3.27) with respect to time, yields:

$$\dot{V} = \dot{q}^T M(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{X}^T \left(\frac{\partial P_b(X)}{\partial X}\right)^T. \quad (3.28)$$

Substituting equations (3.24) and (3.26) into equation (3.28), we have,

$$\begin{aligned} \dot{V} = & -\dot{q}^T K_v \dot{q} - \dot{q}^T S(q, \dot{q})\dot{q} - \sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i}))\dot{q}^T J^T(q)\left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T \\ & + \sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i}))\dot{X}^T \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T. \end{aligned} \quad (3.29)$$

Since $\dot{X} = J(q)\dot{q}$, simplifying equation (3.29) and applying Property 2, $\dot{V}$ reduces to:

$$\dot{V} = -\dot{q}^T K_v \dot{q}. \quad (3.30)$$

3.3 Joint-Space Region Reaching Controller

The stability of the system is specified by the following Theorem:

Theorem 3.2: *The closed-loop system described by equation (3.26) gives rise to the convergence of X to the region $f(\delta X_0) = [f_1(\delta X_{01}) \dots f_N(\delta X_{0N})]^T \leq 0$ and $\dot{q}$ to 0 as $t \rightarrow \infty$.*

Proof: Since $\dot{V} = 0$ implies $\dot{q} = 0$ as $t \rightarrow \infty$, the maximum invariant set satisfy:

$J(q)^T \sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T = 0$. Hence,
 $\sum_{i=1}^N k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T = 0$ and $\dot{q} = 0$, if $J(q)$ is non singular. As seen from equation (3.22) and (3.24), this implies that $f(\delta X_0) = [f_1(\delta X_{01}) \dots f_N(\delta X_{0N})]^T \leq 0$ as $t \rightarrow \infty$.

3.3 Joint-Space Region Reaching Controller

In this section, we consider a joint-space region reaching control problem where the desired region is specified by a single inequality function as follows:

$$f(\delta q_0) \leq 0, \quad (3.31)$$

where $f(\delta q_0) \in R$ is a scalar function scalar with continuous first partial derivatives, $\delta q_0 = q - q_0$, $q_0 \in R^n$ is a reference joint angle of the desired region. For example, if $q = [q_1, q_2]^T \in R^2$, a desired joint-space region can

3.3 Joint-Space Region Reaching Controller

be specified as:

$$f(\delta q_0) = (q_1 - q_{10})^2 + (q_2 - q_{20})^2 - \mu^2 \leq 0, \quad (3.32)$$

where μ is scalar, (q_{10}, q_{20}) is the reference angle of the joint-space desired region. Note that if μ reduces to zero, the joint-space region reaching controller becomes a joint-space setpoint controller.

The potential energy function for the region reaching controller is specified in joint-space as:

$$P(q) = \frac{k_p}{2} [\max(0, f(\delta q_0))]^2. \quad (3.33)$$

That is,

$$P(q) = \begin{cases} 0, & f(\delta q_0) \leq 0, \\ \frac{k_p}{2} f^2(\delta q_0), & f(\delta q_0) > 0, \end{cases} \quad (3.34)$$

where k_p is a positive scalar. Partial differentiating the potential energy function described by equation (3.34) with respect to q , we have,

$$\left(\frac{\partial P(q)}{\partial q}\right)^T = \begin{cases} 0, & f(\delta q_0) \leq 0, \\ k_p f(\delta q_0) \left(\frac{\partial f(\delta q_0)}{\partial q}\right)^T, & f(\delta q_0) > 0, \end{cases} \quad (3.35)$$

which can be written as:

$$\left(\frac{\partial P(q)}{\partial q}\right)^T = k_p \max(0, f(\delta q_0)) \left(\frac{\partial f(\delta q_0)}{\partial q}\right)^T. \quad (3.36)$$

3.3 Joint-Space Region Reaching Controller

From equation (3.36), the region reaching controller in joint space is proposed as:

$$\tau = -K_v \dot{q} - k_p \max(0, f(\delta q_0)) \left(\frac{\partial f(\delta q_0)}{\partial q} \right)^T + g(q), \quad (3.37)$$

where $K_v \in R^{n \times n}$ is a positive definite velocity feedback gain matrix, $\tau \in R^n$ is the applied joint torque to the robot, and $g(q) \in R^n$ is the gravitational force compensation.

Substituting equation (3.37) into equation (2.9), the closed-loop equation is obtained as:

$$M(q)\ddot{q} + \left(\frac{1}{2} \dot{M}(q) + S(q, \dot{q}) \right) \dot{q} + K_v \dot{q} + k_p \max(0, f(\delta q_0)) \left(\frac{\partial f(\delta q_0)}{\partial q} \right)^T = 0. \quad (3.38)$$

The Lyapunov-like function candidate in joint space is proposed as,

$$V = \frac{1}{2} \dot{q}^T M(q) \dot{q} + P(q), \quad (3.39)$$

where $P(q)$ is defined as equation (3.33) and (3.34). V is a scalar function with continuous first partial derivatives since $f(\delta q_0)$ is continuous in q . Differentiating the Lyapunov-like function candidate with respect to time, yields:

$$\dot{V} = \dot{q}^T M(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \dot{q}^T \frac{\partial P(q)}{\partial q}. \quad (3.40)$$

Substituting equation (3.36) and (3.38) into equation (3.40), we have,

$$\begin{aligned} \dot{V} = & -\dot{q}^T K_v \dot{q} - \dot{q}^T \left[\frac{1}{2} \dot{M}(q) + S(q, \dot{q}) - \frac{1}{2} \dot{M}(q) \right] \dot{q} - k_p (\max(0, f(\delta q_0))) \dot{q}^T \left(\frac{\partial f(\delta q_0)}{\partial q} \right)^T \\ & + k_p (\max(0, f(\delta q_0))) \dot{q}^T \left(\frac{\partial f(\delta q_0)}{\partial q} \right)^T. \end{aligned} \quad (3.41)$$

3.3 Joint-Space Region Reaching Controller

Simplifying and applying **Property 2** to equation (3.41), $\dot{V}$ reduces to:

$$\dot{V} = -\dot{q}^T K_v \dot{q} \leq 0. \quad (3.42)$$

We are now in a position to state the following Theorem:

Theorem 3.3: The closed-loop system described by equation (3.38) gives rise to the convergence of q to the desired region $f(\delta q_0) \leq 0$ and $\dot{q}$ to 0 as $t \rightarrow \infty$.

Proof: Since $\dot{V} = 0$ implies $\dot{q} = 0$ as $t \rightarrow \infty$, the maximum invariant set satisfy:

$\max(0, f(\delta q_0))(\frac{\partial f(\delta q_0)}{\partial q})^T = 0$ and $\dot{q} = 0$. As seen from equation (3.35), this implies that $f(\delta q_0) \leq 0$ as $t \rightarrow \infty$.

Remark 3.1: Clearly, the joint-space region reaching controller can be similarly extended to multiple objective functions by following a similar argument as section 3.2. The joint-space objective functions can be specified as follows:

$$f(\delta q_0) = \begin{bmatrix} f_1(\delta q_{01}) \\ f_2(\delta q_{02}) \\ \vdots \\ f_N(\delta q_{0N}) \end{bmatrix} \leq 0, \quad (3.43)$$

3.4 Revisiting the Sepoint Control Problem

where $\delta q_{0i} = q - q_{0i}$, q_{0i} is the reference point of the i^{th} desired region, $i = 1, 2, \dots, N$, N is the total number of objective functions, $f_i(\delta q_{0i})$ are scalar functions with continuous first partial derivatives.

3.4 Revisiting the Sepoint Control Problem

When the desired region is specified arbitrarily small, it reduces to a point. Hence, the region control concept is a generalization of the setpoint control problem. In fact, it is interesting to note that *any closed region with arbitrary shape reduces to a point as the region reduces to zero*. In this section, we revisit the setpoint control problem from region control point of view, to gain new insights into the setpoint control problem. We show that the well known Takegaki-Arimoto regulator is a special case of the propose region reaching controller. First, consider a robot with six degree of freedoms and let $X_p = [x_1, x_2, x_3]^T$ represent the position and $X_o = [x_4, x_5, x_6]^T$ represents the orientation of the robot end effector. Let the desired regions of the position and orientation be specified as follows:

$$\begin{aligned} f_1(\delta X_0) &= \sqrt{(x_1 - x_{01})^2 + (x_2 - x_{02})^2 + (x_3 - x_{03})^2} - r_1 \leq 0, \\ f_2(\delta X_0) &= \sqrt{(x_4 - x_{04})^2 + (x_5 - x_{05})^2 + (x_6 - x_{06})^2} - r_2 \leq 0. \end{aligned} \quad (3.44)$$

3.4 Revisiting the Sepoint Control Problem

where r_1 and r_2 are nonnegative scalars. Hence

$$\begin{aligned} \left(\frac{\partial f_1(\delta X_0)}{\partial X}\right)^T &= \frac{1}{\sqrt{(x_1 - x_{01})^2 + (x_2 - x_{02})^2 + (x_3 - x_{03})^2}} \begin{bmatrix} \delta X_p \\ 0 \end{bmatrix} \\ \left(\frac{\partial f_2(\delta X_0)}{\partial X}\right)^T &= \frac{1}{\sqrt{(x_4 - x_{04})^2 + (x_5 - x_{05})^2 + (x_6 - x_{06})^2}} \begin{bmatrix} 0 \\ \delta X_o \end{bmatrix} \end{aligned} \quad (3.45)$$

where $\delta X_p = [x_1 - x_{01}, x_2 - x_{02}, x_3 - x_{03}]^T$ and $\delta X_o = [x_4 - x_{04}, x_5 - x_{05}, x_6 - x_{06}]^T$. If r_1 and r_2 reduce to zero, then $f_1(\delta X_0)$ and $f_2(\delta X_0)$ are nonnegative and hence

$$\begin{aligned} \max(0, f_1(\delta X_0)) \left(\frac{\partial f_1(\delta X_0)}{\partial X}\right)^T &= f_1(\delta X_0) \left(\frac{\partial f_1(\delta X_0)}{\partial X}\right)^T = \begin{bmatrix} \delta X_p \\ 0 \end{bmatrix} \\ \max(0, f_2(\delta X_0)) \left(\frac{\partial f_2(\delta X_0)}{\partial X}\right)^T &= f_2(\delta X_0) \left(\frac{\partial f_2(\delta X_0)}{\partial X}\right)^T = \begin{bmatrix} 0 \\ \delta X_o \end{bmatrix} \end{aligned} \quad (3.46)$$

Based on equation (3.25), the task-space region reaching controller becomes:

$$\tau = -J^T(q)K_p\delta X - K_v\dot{q} + g(q), \quad (3.47)$$

which is the Takegaki-Arimoto regulator where $\delta X = [\delta X_p^T, \delta X_o^T]^T$ and $K_p = \text{diag}(k_{p1}, k_{p1}, k_{p1}, k_{p2}, k_{p2}, k_{p2})$. Clearly, similar argument can be applied to the joint-space controller presented in **Remark 3.1**.

3.5 Extension to Redundant Robots

In the previous sections, we have shown that the position of the robot end effector can converge to a specified region. However, the result is focusing on non-redundant robot. In this section, we extend the results to the case of redundant robot. The main idea is to specify a desired joint-space region for the redundant degree of freedom. Let us consider a redundant robot with n degree of freedom, where the end effector position vector is given as $X \in R^m (m \leq n)$. Next, we partition the joint position vector q as $[q_a, q_b]$, where $q_a \in R^{n-m}$ denotes the first $n-m$ joints and $q_b \in R^m$ denotes the last m joints. Similarly, the Jacobian matrix can be partitioned as $J(q) = [J_1(q) J_2(q)]$ such that $\dot{X} = J_1(q)\dot{q}_a + J_2(q)\dot{q}_b$. The desired region for end effector can be similarly specified as:

$$f(\delta X_0) = \begin{bmatrix} f_1(\delta X_{01}) \\ f_2(\delta X_{02}) \\ \vdots \\ f_{N_1}(\delta X_{0N_1}) \end{bmatrix} \leq 0, \quad (3.48)$$

where $\delta X_{0i} = X - X_{0i}$, X_{0i} is the reference point within the i^{th} desired region, $i = 1, 2, \dots, N_1$, N_1 is the total number of primary objective functions, $f_i(\delta X_{0i}) \in R$ are scalar functions with continuous first partial derivatives. In

3.5 Extension to Redundant Robots

addition, we specify a desired region for q_a as:

$$h(\delta q_{0j}) = \begin{bmatrix} h_1(\delta q_{01}) \\ h_2(\delta q_{02}) \\ \vdots \\ h_{N_2}(\delta q_{0N_2}) \end{bmatrix} \leq 0, \quad (3.49)$$

where $\delta q_{0j} = q_a - q_{0j}$, q_{0j} is the reference point within the j^{th} desired joint space region, $j = 1, 2, \dots, N_2$, N_2 is the total number of secondary objective functions, $h_j(\delta q_{0j}) \in R$ are scalar functions with continuous first partial derivatives. For example, consider a 3-link planner robot as shown in Figure 3.9. Suppose that the robot end-effector is required to move to a desired planner region (primary region). Then a secondary region can be defined for the first joint to confine it to a desired region as illustrated in Figure 3.9. In the presence of obstacle in the workspace, the specification of the secondary region may be useful in avoiding the obstacle. This secondary region can also be used to narrow down the many possible configurations in task-space control of redundant robot.

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Figure 3.9: Extension to Redundant Robots

The potential energy functions for the desired regions described in inequalities (3.48) and (3.49) can be specified respectively as:

$$P_p(X) = \sum_{i=1}^{N_1} \frac{k_{pi}}{2} [\max(0, f_i(\delta X_{0i}))]^2, \quad (3.50)$$

$$P_s(q_a) = \sum_{j=1}^{N_2} \frac{k_{qj}}{2} [\max(0, h_j(\delta q_{0j}))]^2, \quad (3.51)$$

where k_{pi}, k_{qj} are positive constants. Partial differentiating the potential energy function (3.50) with respect to X and using the similar arguments as in previous sections, we have,

$$\left(\frac{\partial P_p(X)}{\partial X}\right)^T = \sum_{i=1}^{N_1} k_{pi} [\max(0, f_i(\delta X_{0i}))] \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T \quad (3.52)$$

3.5 Extension to Redundant Robots

Partial differentiating the potential energy function (3.51) with respect to q_a , we have,

$$\left(\frac{\partial P_s(q_a)}{\partial q_a}\right)^T = \sum_{j=1}^{N_2} k_{qj} [\max(0, h_j(\delta q_{0j}))] \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a}\right)^T, \quad (3.53)$$

From equations (3.52) and (3.53), the task-space region reaching controller is proposed as:

$$\begin{aligned} \tau = & -K_v \dot{q} + g(q) - J^T(q) \sum_{i=1}^{N_1} k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T \\ & - \begin{bmatrix} I_{n-m} \\ 0 \end{bmatrix} \sum_{j=1}^{N_2} k_{qj} [\max(0, h_j(\delta q_{0j}))] \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a}\right)^T, \end{aligned} \quad (3.54)$$

where I_{n-m} is a $(n-m) \times (n-m)$ identity matrix, 0 is a $m \times n-m$ zero matrix. Substituting equation (3.54) into equation (2.9), the closed-loop equation is obtained as:

$$\begin{aligned} M(q)\ddot{q} + \left(\frac{1}{2}\dot{M}(q) + S(q, \dot{q})\right)\dot{q} + K_v \dot{q} + J^T(q) \left[\sum_{i=1}^{N_1} k_{pi} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X}\right)^T\right] \\ + \begin{bmatrix} I_{n-m} \\ 0 \end{bmatrix} \sum_{j=1}^{N_2} k_{qj} [\max(0, h_j(\delta q_{0j}))] \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a}\right)^T = 0. \end{aligned} \quad (3.55)$$

The Lyapunov-like function is proposed as:

$$V = \frac{1}{2} \dot{q}^T M(q) \dot{q} + P_p(X) + P_s(q_a), \quad (3.56)$$

where $P_p(X)$ and $P_s(q_a)$ are defined in equations (3.50) and (3.51). Note that V is a scalar function with continuous first partial derivatives since $f(\delta X_0)$ in

3.5 Extension to Redundant Robots

continuous in X . Differentiating equation (3.56) with respect to time, yields:

$$\dot{V} = \dot{q}^T M(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \dot{X}^T \left(\frac{\partial P_p(X)}{\partial X} \right)^T + \dot{q}_a^T \left(\frac{\partial P_s(q_a)}{\partial q_a} \right)^T. \quad (3.57)$$

Substituting equations (3.52), (3.53) and (3.55) into equation (3.57), we have,

$$\begin{aligned} \dot{V} = & - \sum_{i=1}^{N_1} k_{pi} \max(0, f_i(\delta X_{0i})) \dot{q}^T J^T(q) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T \\ & + \sum_{j=1}^{N_2} k_{qj} \max(0, h_j(\delta q_{0j})) \dot{q}_a^T \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a} \right)^T \\ & + \sum_{i=1}^{N_1} k_{pi} \max(0, f_i(\delta X_{0i}^p)) \dot{X}^T \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T \\ & - \sum_{j=1}^{N_2} k_{qj} \max(0, h_j(\delta q_{0j})) \dot{q}^T \begin{bmatrix} I_{n-m} \\ 0 \end{bmatrix} \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a} \right)^T \\ & - \dot{q}^T K_v \dot{q} - \dot{q}^T S(q, \dot{q}) \dot{q}, \end{aligned} \quad (3.58)$$

since $\dot{X} = J(q) \dot{q}$ and $\dot{q}^T \begin{bmatrix} I_{n-m} \\ 0 \end{bmatrix} = \dot{q}_a^T$, simplifying equation (3.58) and applying Property 2, $\dot{V}$ reduces to:

$$\dot{V} = -\dot{q}^T K_v \dot{q}. \quad (3.59)$$

We are now in a position to state the following Theorem:

Theorem 3.4: *The closed-loop system described by equation (3.55) gives rise to the convergence of X to the region $f(\delta X_0) = [f_1(\delta X_{01}) \dots f_{N_1}(\delta X_{0N_1})]^T \leq 0$, and q_a to the region $h(\delta q_0) = [h_1(\delta q_{01}) \dots h_{N_2}(\delta q_{0N_2})]^T \leq 0$ and $\dot{q}$ to 0 as $t \rightarrow \infty$.*

3.5 Extension to Redundant Robots

Proof: Since $\dot{V} = 0$ implies $\dot{q} = 0$ as $t \rightarrow \infty$, the maximum invariant set satisfy:

$$\sum_{i=1}^{N_1} J^T(q) \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T + \sum_{j=1}^{N_2} \begin{bmatrix} I_{n-m} \\ 0 \end{bmatrix} \max(0, h_j(\delta q_{0j})) \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a} \right)^T = 0.$$

The above equation can be written as:

$$\begin{bmatrix} I_{n-m} & J_1^T(q) \\ 0 & J_2^T(q) \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{N_2} \max(0, h_j(\delta q_{0j})) \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a} \right)^T \\ \sum_{i=1}^{N_1} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T \end{bmatrix} = 0, \quad (3.60)$$

where $J(q) = [J_1(q) \ J_2(q)]$. Hence, $\sum_{i=1}^{N_1} \max(0, f_i(\delta X_{0i})) \left(\frac{\partial f_i(\delta X_{0i})}{\partial X} \right)^T = 0$, and $\sum_{j=1}^{N_2} \max(0, h_j(\delta q_{0j})) \left(\frac{\partial h_j(\delta q_{0j})}{\partial q_a} \right)^T = 0$ and $\dot{q} = 0$ if the square matrix

$\begin{bmatrix} I_{n-m} & J_1^T(q) \\ 0 & J_2^T(q) \end{bmatrix}$ is non singular (i.e. if $J_2^T(q) \in R^{m \times n}$ is nonsingular). This finally implies that

$$f(\delta X_0) = [f_1(\delta X_{01}), \dots, f_{N_1}(\delta X_{0N_1})]^T \leq 0 \text{ and,}$$

$$h(\delta q_0) = [h_1(\delta q_{0N_2}), \dots, h_{N_2}(\delta q_{0N_2})]^T \leq 0 \text{ as } t \rightarrow \infty.$$

Remark 3.2: Clearly, region reaching control is a generalization of set-point control where the position error $\delta X = X - X_d$ converges to zero or bounded as $\|\delta X\| \leq \epsilon$ as $t \rightarrow \infty$. Using the multiple objective functions, desired region with arbitrarily shape can be specified as intersections of several regions. The proposed concept is also useful in control of macro/mini structures and redundant robot.

3.5 Extension to Redundant Robots

Remark 3.3: Note that a desired point cannot be specified for q_a as it may restrict the movement of the end effector or even result in conflicting motions. Using the reaching control concept and the secondary region (see equation (3.49)) allows more freedoms in q_a and hence the motion of the end effector is not constrained. In addition, invariant set theorem can now be applied directly.

Chapter 4

Experiment Results and Setup

4.1 Introduction

In this chapter, the proposed region reaching controllers in Chapter 3 are implemented on the first two joints of a SONY SCARA robot (see Figure 4.1). The details of the experiment set up are given and experiment results are presented to illustrate the performances of the region reaching controllers.

The remaining of this chapter is organized as follows: section 4.2 presents the experiment set up: section 4.3 presents the task space experiments and results: section 4.4 presents the joint space experiments and results.

4.2 Experiment Set Up

Figure 4.1: SONY SCARA robot

4.2 Experiment Set Up

4.2.1 Hardware

The block diagram of the SONY SCARA robot system are shown in Figure 4.2. The functions of various blocks are listed below:

- Elmo Digital Drivers: the Digital Drivers perform two main functions in robot systems: they are used to drive the motors on robots; and are used to build the communications between the ServoToGo I/O card and the motors.
- ServoToGo I/O Card: the ServoToGo I/O card is divided into 3 parts:

4.2 Experiment Set Up

encoder counting part, digit-to-analog converters and analog-to-digit converters. The encoder counting part is used for reading the encoders's value from the Digital Drivers. The digit-to-analog converters are used to send the control signals from the PC to Digital Drivers. The analog-to-digit converters are used to accept the safety control signals from the robot and send them to the PC.

- SONY SCARA robot: The robot consists of motors and links, encoders are attached with motors. These encoders generate a constant number of counts in every revolution. Encoders send the encoder counts value to digital drivers, then digital drivers decode the information and send to PC through encoder counting part of ServoToGo I/O Card.
- HITACHI IP 5005 image processing card: the image processing card is used to capture the image of the SONY SCARA robot end effector, so the the end effector position in visual-space is obtained.
- PII-450MHz Personal Computer (PC): the PC is the main controller in the hardware system. The PC works out the control signals based on current joint or task space position, joint angular velocities and desired target. Then the PC send the control signals to the Digital Drivers through digit-to-analog converters. Then Digital Drivers amplify the control signals and then send them to motors.

4.2 Experiment Set Up

Figure 4.2: Block diagram of robot control system

4.2.2 Software

The task-space region reaching control software can be divided into several steps as illustrated in Figure 4.3:

- Step 1: Zero or clear the encoder values of Encoders.
- Step 2: User input the desired region specifications.
- Step 3: Read encoders to current angles and current angular velocities, compute the Jacobian Matrix $J(q)$.
- Step 4: Calculate the Cartesian space position with respect to the current joint angles or read camera image from the image processing card to get current end effector task-space position values.
- Step 5: Check whether the system is within the safety zone: (a) check whether the absolute value of current joint angles are within the safety

4.2 Experiment Set Up

range; (b) check whether the absolute joint angular velocities are within the safety range. (c) check whether the current end effector position values are within the pre-defined task-space working area. If (a) or (b) or (c) is not true, then the robot will stop.

- Step 6: If and only if the system is still within the safety zone, the controller computes the applied control signals based on current task-space position values, angular velocities, $J^T(q)$ and send them to the I/O card.
- Step 7: Check whether the loop time limit has been reached. If yes, end the loop and stop the robot. If not, go to Step 3 again.

4.2 Experiment Set Up

Figure 4.3: Flow chart of the task-space region reaching control software

4.2 Experiment Set Up

The joint-space region reaching control software is very similar to task-space control software, which is illustrated in Figure 4.4:

- Step 1: Zero or clear the encoder values of Encoders.
- Step 2: User input the desired region specifications.
- Step 3: Read encoders to get current joint angles and current angular velocities.
- Step 4: Check whether the system is within the safety zone: (a) check whether the absolute value of current joint angles are within the safety range; (b) check whether the absolute joint angular velocities are within the safety range. If (a) or (b) is not true, then the robot will stop.
- Step 5: If and only if the system is still within the safety zone, the controller computes the applied control signals based on current joint angles, angular velocities and send them to the I/O card.
- Step 6: Check whether the loop time limit has been reached. If yes, end the loop and stop the robot. If not, go to Step 3 again.

4.2 Experiment Set Up

Figure 4.4: Flow chart of the joint-space region reaching control software

4.3 Task-space experiment Results

4.3.1 Cartesian-space experiment results

In this experiment, the region reaching controllers are implemented in Cartesian space. The kinematics and Jacobian matrix of the first two joints of the robot were given in equation (2.3) and (2.5). The Cartesian-space position of the robot end effector were obtained from the forward kinematics equation (2.3)). Joint angles were obtained from the incremental encoders and joint velocities were obtained from differentiation of the joint angles. The lengths of the links are $l_1 = 0.35m$, $l_2 = 0.25m$. First, a desired region is specified as a circle (see Figure 3.2) with $x_0 = 0.45\text{ m}$, $y_0 = 0.35\text{ m}$, $r = 0.05\text{ m}$ (see equation (3.2)). Figure 4.5 shows that the robot end effector converges to the desired region with the control gains set as $k_p = 150$, $K_v = \text{diag}(0.15, 0.2)$. The position of the end effector versus time is plotted in Figure 4.6. The radius of the desired region is then reduced to zero, and Figure 4.7 shows the end effector position converges to the desired point with the same gains. The position of end effector versus time is plotted in Figure 4.8. With the radius of the circle reduces to zero, the region reaching control problem reduces to a setpoint control problem. These results (Figure 4.5, Figure 4.7) illustrated that region reaching control concept is a generalization of setpoint control problem.

Next, a region reaching control experiment was done with the desired region defined by a opened region as shown in Figure 3.3 (see equation (3.3)) with $c = 0.25\text{ m}$. Figure 4.9 shows the robot end effector can converge to the

4.3 Task-space experiment Results

desired region with the control gains set as $k_p = 50$, $K_v = \text{diag}(0.15, 0.2)$. The position of end effector versus time is plotted in Figure 4.10.

Two region reaching control experiments were carried out with the desired region defined by the intersection of two circles as shown in Figure 3.5 part (a) and (b). First, the experiment was done by setting $x_{01} = 0.35m$, $y_{01} = 0.4m$, $x_{02} = 0.45m$, $y_{02} = 0.4m$, $r_1 = 0.08m$, $r_2 = 0.08m$ (see equation (3.15)). Figures 4.11 shows that the robot end effector converge to the desired region with the control gains set as $k_{p1} = 300$, $k_{p2} = 300$, $K_v = \text{diag}(0.15, 0.2)$. The position of the end effector versus time is plotted in Figure 4.12. Then the objective function $f_2(\delta X_{02})$ changes to $f_2(\delta X_{02}) = r_2^2 - (x - x_{02})^2 - (y - y_{02})^2 \leq 0$, then the desired region becomes Figure 3.5 part (b), and Figure 4.13 shows the end effector converge to the desired region with the control gains $k_{p1} = 600$, $k_{p2} = 500$, $K_v = \text{diag}(0.15, 0.2)$ and $x_{02} = 0.45m$, $y_{02} = 0.4m$, $r_1 = 0.08m$, $r_2 = 0.08m$. The position of end effector versus time is plotted in Figure 4.14.

Next, one region reaching control experiment was carried out with the desired region defined by a square as shown in Figure 3.6 in Cartesian space with $a = b = 0.08\text{ m}$, $x_{01} = 0.45\text{ m}$, $y_{02} = 0.35\text{ m}$. Figure 4.15 shows the robot end effector can converge to the desired region with the control gains set as $k_{p1} = 300$, $k_{p2} = 300$, $K_v = \text{diag}(0.15, 0.2)$. The position of end effector versus time is plotted in Figure 4.16.

Figure 4.17 shows the robot end effector can also converge to a desired region

4.3 Task-space experiment Results

specified by equation (3.17) (See Figure 3.7 also). Where $k_3 = 1, d_3 = 0.4, k_2 = 0, d_2 = 0.4, k_1 = -1, d_1 = 0.6$. The control gains were set as $k_{p1} = 30, k_{p2} = 80, k_{p3} = 70, K_v = \text{diag}(0.2, 0.5)$, The position of end effector versus time is plotted in Figure 4.18.

Figure 4.19 shows another region reaching experimental result with control within specified boundary. The desired region was specified by equation (3.18) (see Figure 3.8 also) with one more objective function $f_4(\delta X_0) = (y - y_0)^2 - b_2^2 \leq 0$ added. In the experiment, $a = 0.03 \text{ m}, x_0 = -0.27 \text{ m}, y_0 = 0.27 \text{ m}, b_2 = 0.03 \text{ m}, b_1 = 0.07 \text{ m}$. The control gains are set as $k_{p1} = 400, k_{p2} = 400, k_{p3} = 50000, k_{p4} = 50000, K_v = \text{diag}(0.15, 0.2)$. Note that high gains were set for the boundary to bound the end effector in the specified boundary. Since the end effector started in the specified boundary, the high gains had no effect on the end effector while it is inside the boundary. The position of end effector versus time is plotted in Figure 4.20. Figure 4.21 and Figure 4.22 show the experiment results without specifying a desired boundary for the end effector reaching movement.

4.3 Task-space experiment Results

Figure 4.5: Region reaching control with specified region specified as a circle in Cartesian space

Figure 4.6: Position of the end effector

4.3 Task-space experiment Results

Figure 4.7: Region reaching control with specified region specified as a point in Cartesian space

Figure 4.8: Position of the end effector

4.3 Task-space experiment Results

Figure 4.9: Region reaching control with specified region specified as a opened region in Cartesian space

Figure 4.10: Position of the end effector

4.3 Task-space experiment Results

Figure 4.11: Region reaching control with specified region specified as Figure 3.5 part(a) in Cartesian space

Figure 4.12: Position of the end effector

4.3 Task-space experiment Results

Figure 4.13: Region reaching control with specified region specified as Figure 3.5 part(b) in Cartesian space

Figure 4.14: Position of the end effector

4.3 Task-space experiment Results

Figure 4.15: Region reaching control with specified region as a square in Cartesian space

Figure 4.16: Position of the end effector

4.3 Task-space experiment Results

Figure 4.17: Region reaching control with desired region specified as a triangle in Cartesian space

Figure 4.18: Position of the end effector

4.3 Task-space experiment Results

Figure 4.19: Region reaching control with desired region specified in figure 3.8 in Cartesian space

Figure 4.20: Position of the end effector

4.3 Task-space experiment Results

Figure 4.21: Region reaching control with desired region specified as a small square in Cartesian space

Figure 4.22: Position of the end effector

Another examples is the desired region specified as an intersection of an opened region and a circle as illustrated in Figure 4.23. The objective func-

4.3 Task-space experiment Results

tions are specified as:

$$\begin{aligned} f_1(\delta X_0) &= x^4 + c - y \leq 0, \\ f_2(\delta X_{01}) &= (x - x_0)^2 + (y - y_0)^2 - r^2 \leq 0, \end{aligned} \quad (4.1)$$

where c is a scalar, (x_0, y_0) is the center of the circle, r is the radius of the circle. Note that the desired region is closed in this case.

Figure 4.23: Desired region specified as a opened region and a circle

Figure 4.24 shows the robot end effector can also converge to a desired region specified by equation (4.1) (See Figure 4.23 also) with $c = 0.25$, $x_0 = 0.35$, $y_0 = 0.4$, $r = 0.08$ m. The control gains were set as $k_{p1} = 50$, $k_{p2} = 150$, $K_v = \text{diag}(0.15, 0.2)$, The position of end effector versus time is plotted in Figure 4.25.

4.3 Task-space experiment Results

Figure 4.24: Region reaching control with desired region specified as the intersection of a opened region and a circle in Cartesian space

Figure 4.25: Position of the end effector

4.3 Task-space experiment Results

The desired region is next specified as an intersection of an opened region and a square as illustrated in Figure 4.26. The objective functions are specified as:

$$\begin{aligned} f_1(\delta X_0) &= y - (\cos(x) - h) \leq 0, \\ f_2(\delta X_{01}) &= (x - x_{01})^2 - 0.25sl^2 \leq 0, \\ f_3(\delta X_{02}) &= (y - y_{02})^2 - 0.25sl^2 \leq 0, \end{aligned} \quad (4.2)$$

where h is scalar constant, (x_{01}, y_{02}) is the center of the square, sl is the side length of the square.

Figure 4.26: Desired region specified as a opened region and a square

Figure 4.27 shows the robot end effector can also converge to a desired region specified by equation (4.2) (See Figure 4.26 also) with $h = 0.7$, $x_0 = 0.25$, $y_0 = 0.25$, $sl = 0.16$ m. The control gains were set as $k_{p1} = 200$, $k_{p2} = 300$, $k_{p3} = 300$, $K_v = \text{diag}(0.15, 0.2)$, The position of end effector versus time is plotted in Figure 4.28.

4.3 Task-space experiment Results

Figure 4.27: Region reaching control with desired region specified as the intersection of a opened region and a square in Cartesian space

Figure 4.28: Position of the end effector

4.3 Task-space experiment Results

The desired region could be also be specified as the intersection of two opened regions to form a closed region as shown in Figure 4.29. The objective functions are specified as:

$$\begin{aligned} f_1(\delta X_0) &= y - (\cos(x) - h) \leq 0, \\ f_2(\delta X_{01}) &= x^4 + c - y \leq 0, \end{aligned} \tag{4.3}$$

where h, c are scalar constants.

Figure 4.29: Desired region specified as the intersection of two opened regions

Figure 4.30 show the robot end effector can also converge to the desired region specified by equation (4.3) (See Figure 4.29 also) with $h = 0.7, c = 0.25$. The control gains were set as $k_{p1} = 200, k_{p2} = 50, K_v = \text{diag}(0.15, 0.2)$. The position of end effector versus time is plotted in Figure 4.31.

4.3 Task-space experiment Results

Figure 4.30: Region reaching control with desired region specified as the intersection of two opened regions in Cartesian space

Figure 4.31: Position of the end effector

4.3 Task-space experiment Results

Figure 4.32 shows an desired region specified as a ring where the objective functions are specified as:

$$\begin{aligned} f_1(\delta X_{01}) &= r_1^2 - (x - x_{01})^2 - (y - y_{01})^2 \leq 0, \\ f_2(\delta X_{02}) &= (x - x_{01})^2 + (y - y_{01})^2 - r_2^2 \leq 0, \end{aligned} \quad (4.4)$$

where x_{01}, y_{01} is the center of the two circles, $r_2 \geq r_1$ are the radius of these two circles.

Figure 4.32: Desired region specified as the region between two circles with same center

Figure 4.33 and 4.34 show the robot end effector can also converge to a desired region specified by equation (4.4) (See Figure 4.32 also) with $x_{01} =$

4.3 Task-space experiment Results

0.45, $y_{01} = 0.35$, $r_1 = 0.08m$, $r_2 = 0.24m$. Figure 4.33 is a magnified version of Figure 4.33. The control gains were set as $k_{p1} = 2000$, $k_{p2} = 200$, $K_v = diag(0.15, 0.2)$. Since the end effector starting point is out of the small circle, high gain can be set for objective function $f_1(\delta X_{01})$ (move away from small circle). The position of end effector versus time is plotted in Figure 4.35. Then the starting point of the end effector was set in the small circle. Figure 4.36 shows the the robot end effector can also converge to the ring region with the same control gains. Figure 4.37 is a magnified version of Figure 4.36. The position of end effector versus time is plotted in Figure 4.38. Note that the robot end effector could not start from the center of the ring.

Figure 4.33: Region reaching control with desired region specified as a ring region in Cartesian space

4.3 Task-space experiment Results

Figure 4.34: Region reaching control with desired region specified as a ring region in Cartesian space

Figure 4.35: Position of the end effector

4.3 Task-space experiment Results

Figure 4.36: Region reaching control with desired region specified as a ring region in Cartesian space

Figure 4.37: Region reaching control with desired region specified as a ring region in Cartesian space

4.3 Task-space experiment Results

Figure 4.38: Position of the end effector

4.3.2 Visual space experiment results

In this experiment, a CCD camera is used to measure the position of the robot end effector in visual space. The image Jacobian matrix from Cartesian space to visual space is given by

$$J_I = \frac{f}{z-f} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}, \quad (4.5)$$

where β_1, β_2 denote the scaling factors in pixels/m, z is the perpendicular distance between the robot and the camera, f is the focal length of the camera. The Jacobian matrix from joint space to visual space is hence given

4.3 Task-space experiment Results

as [37]- [39]

$$J(q) = J_I(q)J_m(q), \quad (4.6)$$

where $J_m(q)$ is the Jacobian matrix from joint space to Cartesian Space (see equation (2.5)). As seen from equation (4.5), J_I is just a constant matrix in this experiment. In the visual space experiment, we first designed two region reaching controllers with the desired region specified as a single circle as shown in Figure 3.2. The experiment was done for a circle with $x_0 = 75 \text{ pixels}$, $y_0 = 210 \text{ pixels}$, $r = 15 \text{ pixels}$ (see equation (3.2)). The constant image Jacobian matrix J_I is set as an identity matrix since it is only a scaling matrix on the feedback gains. Figure 4.39 shows that the robot end effector converges to the desired region with the control gains set as $k_p = 0.35$, $K_v = \text{diag}(0.5, 0.5)$. The position of the end effector versus time is plotted in Figure 4.40.

Next, two region reaching control experiments were carried out with the desired region defined by the intersection of two circles as shown in Figure 3.5 part (a). First, the experiment was done by setting $x_{01} = 75$, $y_{01} = 210$, $x_{02} = 70$, $y_{02} = 200$, $r_1 = 20$, $r_2 = 15 \text{ pixels}$ (see equation (3.15)). Figures 4.41 shows that the robot end effector converge to the desired region with the control gains set as $k_{p1} = 0.35$, $k_{p2} = 0.25$, $K_v = \text{diag}(0.5, 0.5)$. The position of the end effector versus time is plotted in Figure 4.42. The radius of the two circles were then changed to $r_1 = 10 \text{ pixels}$, $r_2 = 15 \text{ pixels}$, and Figure 4.43 shows the end effector converge to the desired region with different radius and using the same control gains. The position of end effector versus time is plotted in Figure 4.44.

4.3 Task-space experiment Results

Next, Figure 4.45 shows the robot end effector can also converge to a desired region specified as a square as in equation (3.16) (see Figure 3.6 also). With the control gains set as $k_{p1} = 1.6, k_{p2} = 1.2, K_v = \text{diag}(4, 0.3)$. In the experiment, $a = b = 15, x_{01} = 75, y_{02} = 210$. The position of end effector versus time is plotted in Figure 4.46.

Last, figure 4.47 shows the robot end effector can also converge to a desired *opened region* specified as:

$$\begin{aligned} f_1(\delta X_{01}) &= d_1 - y \leq 0, \\ f_2(\delta X_{02}) &= x - d_2 \leq 0, \end{aligned} \tag{4.7}$$

where $d_1 = 210$ pixels, $d_2 = 75$ pixels. The control gains were set as $k_{p1} = 10, k_{p2} = 10, K_v = \text{diag}(1, 0.3)$. The position of end effector versus time is plotted in Figure 4.48. Note this desired region is also bounded by the finite robot work space.

4.3 Task-space experiment Results

Figure 4.39: Region reaching control with desired region specified as a circle in visual space

Figure 4.40: Position of the end effector

4.3 Task-space experiment Results

Figure 4.41: Region reaching control with desired region specified as an intersection of two circles in visual space

Figure 4.42: Position of the end effector

4.3 Task-space experiment Results

Figure 4.43: Region reaching control with desired region specified as an intersection of two circles in visual space

Figure 4.44: Position of the end effector

4.3 Task-space experiment Results

Figure 4.45: Region reaching control with desired region specified as a square in visual space

Figure 4.46: Position of the end effector

4.3 Task-space experiment Results

Figure 4.47: Region reaching control-open region

Figure 4.48: Position of the end effector

4.4 Joint-space experiment Results

For the joint-space region reaching control experiment, the desired region is specified by equation (3.32). This joint-space region reaching control objective can be rewritten as $\sqrt{(q_1 - q_{10})^2 + (q_2 - q_{20})^2} \leq \mu$. So the root mean square error $RMSE_q$ is defined as:

$$RMSE_q = \sqrt{(q_1 - q_{10})^2 + (q_2 - q_{20})^2} \quad (4.8)$$

the parameters are set as $q_{10} = 0.4$ radian, $q_{20} = 0.6$ radian, $\mu = 0.08$ radian. Figure 4.49 shows that the robot current joint angles versus time with the control gains set as $k_p = 40$, $K_v = \text{diag}(0.15, 0.2)$. The root mean square error $RMSE_q$ is shown in Figure 4.50. Then the tolerance μ is increased to 0.15 radian, and Figure 4.51 shows that the robot current joint angles versus time with the same gains. The root mean square error $RMSE_q$ is shown in Figure 4.52.

Last, the desired region is specified by multiple objective functions:

$$\begin{aligned} f_1(\delta q_{10}) &= (q_1 - q_{10})^2 - \mu_1^2 \leq 0, \\ f_2(\delta q_{20}) &= (q_2 - q_{20})^2 - \mu_2^2 \leq 0, \end{aligned} \quad (4.9)$$

where, $q_{10} = 0.4$ radian, $q_{20} = 0.6$ radian, $\mu_1 = \mu_2 = 0.05$ radian, and Figure 4.53 shows that the robot current joint angles versus time with the control gains set as $k_{p1} = 100, k_{p2} = 75$, $K_v = \text{diag}(0.15, 0.2)$. The values of $\max(0, f_1(\delta q_{10}))$ and $\max(0, f_2(\delta q_{20}))$ are shown in Figure 4.54. Then the tolerance μ_1, μ_2 are increase to 0.1 radian, and Figure 4.55 shows that the

4.4 Joint-space experiment Results

robot current joint angles versus time with the same gains. The values of $\max(0, f_1(\delta q_{10}))$ and $\max(0, f_2(\delta q_{20}))$ are shown in Figure 4.56.

Figure 4.49: Joint-space region reaching control with specified region specified by equation (3.32)

Figure 4.50: Joint-space root mean square errors

4.4 Joint-space experiment Results

Figure 4.51: Joint-space region reaching control with specified region specified by equation (3.32)

Figure 4.52: Joint-space root mean square errors

4.4 Joint-space experiment Results

Figure 4.53: Joint-space region reaching control with specified region specified by equation (4.9)

4.4 Joint-space experiment Results

Figure 4.55: Joint-space region reaching control with specified region specified by equation (4.9)

Figure 4.56: value of $\max(0, f_1(\delta q_{10}))$ or $\max(f_2(\delta q_{20}))$

Chapter 5

Conclusion and future works

5.1 Conclusions

In this thesis, a new class of controllers called region reaching controllers are developed. Both joint and task space region reaching control problem are considered. Lyapunov-like functions have been presented for the stability analysis of the region reaching controllers. Experiment results are presented to show the effectiveness of the proposed controllers. Some conclusions and contributions of this report are summarized as follows:

- A new control concept called region reaching control is proposed. In this new control concept, the control objective can be specified as a region instead of a point. Since the desired region can be specified arbitrarily small, the region control concept is also a generalization of setpoint control problem.
- A new sets of potential energy functions was presented for region reach-

5.2 Future Research

ing control. It was shown that there exist a region where the potential energy is equal to zero.

- Region reaching controllers are developed in both task and joint space. A new set of Lyapunov-like functions were presented for the stability analysis of the region reaching controllers. It was shown that the asymptotic convergence can be guaranteed.
- The proposed controllers were implemented on the first two joints of a SONY SCARA robot. Experiment results illustrated the effectiveness of the proposed controllers.
- In this thesis, it is assumed that the kinematic and Jacobian matrix of the robot is known exactly. Since the Cartesian-space position of the robot is obtained from the forward kinematics equation, the proposed Cartesian-space controllers are sensitive to the kinematics uncertainty from joint space to Cartesian space. However, the proposed visual-space controllers can be extended to approximate Jacobian controllers as in [37]- [39] to overcome the kinematic and Jacobian uncertainties. The stability conditions of the approximate Jacobian controllers are given in [37]- [39].

5.2 Future Research

This research has develop a new region reaching control concept where the desired objective is specified as a region. The experiment has shown the effectiveness of the proposed controllers. Future research works are suggested as follows:

5.2 Future Research

- The proposed region reaching controllers assumed that the gravitational force and kinematics of the robot is known exactly, while in some applications, it is difficult to get exact robot dynamics and kinematics. Future work will be devoted to the development of an adaptive region reaching control law to overcome the uncertainties in the gravity force and kinematics.
- In actual implementation of the setpoint controllers, it is very difficult to control the end effector to go to the desired point with zero steady state error even if a PID controller is used. That is, there exist a small error in the final stage. If the PID controller is not turned off after finishing the operation, then the the integration part will become very large after sometime. If a region reaching PID controllers are used instead of a PID controller, then a desired region could be specified arbitrarily small. So that it becomes a desired point with specified tolerance. Hence, integration control signal will reach a constant value when position error is smaller than the specified tolerance. Future research works will be devoted to the development of region reaching PID controllers.
- Region reaching controllers could also be used to solve some obstacle avoidance problem of redundant robots. For redundant robots, a secondary joint space desired region could be specified for the redundant degree of freedom. The idea is to specify a boundary for the redundant degree of freedom, so that we could control the robot to avoid some obstacles. Future works will be devoted to implementing the proposed controllers on a redundant robot for obstacle avoidance.

5.2 Future Research

- Since the function $\max(0, f(\delta X_0))$ is used in the proposed controllers, the control action may stop at the boundary of the desired region. Instead of specifying a tighter boundary, it is also interesting to investigate the possibility of using optimal control to find an ideal area in the desired region. A non zero value or equation can be used in the max function to explore this possibility.

Author's Publications

C.C. Cheah and D.Q. Wang, Region Reaching Control of Robot Manipulators, IEEE Int. Conference on Robotics and Automation, (Barcelona, Spain) 2005.

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