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Modelling and Optimization of Photonic Crystal Fibres

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Summary

Photonic Crystal Fibres (PCFs) have attracted much attention due to their unusual optical properties, such as extra-large chromatic dispersion, small or large mode field area, wide range single mode operations, etc.

The objective of this research is to develop an accurate and efficient modelling tool for automated optimal design of PCFs. In this thesis, we have studied both electromagnetic (EM) modelling and optimization methodologies. For EM modelling of PCFs, the space filling mode (SFM) - effective index (EI) method and the finite element method (FEM) are formulated, implemented, and tested. The SFM-EI is a semi-analytical method and it is efficient, but it can only treat repeated lattice structures. The FEM is a full vectorial numerical tool which can handle complicated geometries. Generally speaking, FEM is accurate, flexible and quite efficient, however, it is still very time-consuming when used for optimal design where hundreds or thousands of individual solutions are needed. For optimization, genetic algorithms (GA) and particle swarm optimization (PSO) are studied, implemented, tested and compared. Both GA and PSO are population based stochastic optimization techniques. GA mimic the concept of biological genetics and natural evolution while PSO is inspired by social behavior and bird flocking or fish schooling.

Either the SFM-EI or the FEM can be selected as the EM solver to be combined with an optimization engine which could be either the GA or the PSO. The EM solvers and optimization engines are validated with examples. A comprehensive example of dispersion control for a PCF design presented shows the full optimal design processing.

Chapter 1

Introduction

1.1 Background and Motivation

Fibre-optic communication systems are lightwave systems that employ optical fibres for information transmission. Such systems have been deployed worldwide since 1980 and have revolutionized the technology behind telecommunications. Indeed, the lightwave technology, together with micro-electronics, is believed to be a major factor in the advent of the "information age" [1].

In recent years, Photonic Crystal Fibres (PCFs) have attracted much attention due to their tailorable geometry structures, which can therefore lead to unusual optical properties, such as extra large chromatic dispersion, small or large mode field area, a wide range single mode operation, etc [2].

PCFs, also called micro-structured fibres or holey fibres, are silica optical fibres consisting of a central defect region in a regular lattice of air holes. Together with recent spectacular technological advances in the application and fabrication of PCFs, powerful theoretical tools have been developed for modelling their electromagnetic fields. So far, there exist many approaches to solve this problem, including analytical, semi-analytical as well as numerical methods, for example, effective-index method [3], localized function method [4], plane-wave expansion method, multiple pole method

Figure 1.1: Schematic of PCFs: a silica core and the cladding with periodical lattice of air holes [2].

[5], and beam propagation method (BPM) [6], etc.

1.2 Focused Directions and Contributions

The main objective of this research is, to develop an accurate and efficient modelling tool for automated optimal design of PCFs, as shown in the Figure 1.2.

Different fibre structures would lead to different optical characteristics, in the engineering application of PCFs design, the objective is, to find the fibre geometry structure parameters which correspond to the desired optical characteristics.

Figure 1.2: Organization of the automatic design tool.

To compute the PCFs' electromagnetic fields accurately and efficiently, it is crucial to use vectorial analysis approach. Furthermore, due to the complicated geometry

structure of PCFs, by varying the size, number and position of the air holes, one can obtain PCFs with different optical properties. Therefore, because of large number of parameters and optical characteristics associated with various fibre structures, traditional optimization techniques are not allowed. We have to adopt powerful optimization tools to obtain the optimal design of PCFs.

In this thesis, the simulation method is based on vectorial analysis approach and powerful optimization tool. We adopted two analysis approaches as the simulation tool, namely vectorial space-filling mode together with effective index method (SFM-EI), which is actually a kind of effective index method, and full-vector finite element method (FEM). We also deployed two powerful optimization tools: Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO). Both GA and PSO are based on stochastic search and modelled on the mechanics of biological behavior phenomena. Then we combined the simulation method with the optimization tool to establish a platform to find the optimized geometry structure as desired. Finally a simple example is exposed to our platform to show the view of these techniques.

1.3 Organization of the Report

The outline of this report is summarized as follows. The current chapter introduces the background, motivation, focused directions, and contributions.

Chapter 2 reviews the novel photonic crystal fibres, their advantages together with their applications in optical communication system in greater details.

Chapter 3 studies two PCFs modelling methods, the so-called space-filling mode method and finite element method. The two methods are compared briefly.

Chapter 4 focuses on the two stochastic optimization tools, namely, genetic algorithms (GAs) and particle swarm optimization (PSO), their operators selection and the corresponding impact on the optimization process.

In chapter 5, numerical experiments are conducted and analyzed to solve the

problem of dispersion control in optical communication system and the results of our modelling/optimization approach.

Finally, Chapter 6 summarizes all the works in this report and identifies several future research directions. Conclusions are also drawn based on the experimental results.

Chapter 2

Photonic Crystal Fibres (PCFs)

2.1 Overview

Standard "step index" optical fibres guide light by the so-called total internal reflection, which operates only if the core has a higher refractive index than the encircling cladding, as shown in the Figure 2.1. Rays of light in the core, striking the interface with the cladding, are completely reflected. The nature of light wave dictates that guidance occurs only at certain angles, i.e., that only a small number of discrete *modes* can form. If only one mode exists, the fibre is known as "single mode".

The field of photonic crystal fibre (PCF) was founded by Russell and his colleagues in late 1990's [7],[8] by the realization of micro-structured silica fibres with a periodic array of several hundred air holes running down their length. Just like Figure 2.2, there exists two physical mechanisms to guide light in PCFs, namely Modified *total internal reflection (TIR)* and *photonic band gap (PBG)* guidance. We will inspect these two mechanisms in the next sections, respectively.

Figure 2.1: (a) Structure of step-index fibre and the profile of refractive index. (b) Light travels along the step-index fibre.

2.2 Mechanisms of Light Guidance

The large index contrast and complex structure in PCF make it difficult to treat analytically. Traditional scalar optical fibre analysis can not be used. One may have to apply Maxwell's equations to get the solutions:

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon\mathbf{E} \quad (2.1)$$

$$\nabla \times \mathbf{E} = -\mathbf{M} - j\omega\mu\mathbf{H} \quad (2.2)$$

$$\nabla \cdot \mu\mathbf{H} = 0 \quad (2.3)$$

$$\nabla \cdot \epsilon\mathbf{E} = \rho \quad (2.4)$$

Figure 2.2: Two physical mechanisms of light guiding in PCFs: (a) TIR Guidance (b) PBG Guidance[2].

where

E =electric field intensity (V/m)

H =magnetic field intensity (A/m)

J =electric current density (A/m^2)

M =magnetic current density (V/m²)

ρ =electric charge density (C/m^3)

2.2.1 Concept of Photonic Band Gap

With the periodically lattice of the air holes in the cladding region, wave propagation in PCFs will be different from the conventional optical fibres. As illustrated

in Figure 2.3, wave propagation are typically presented in the form of a propagation diagram, whose axes are the dimensionless quantities $\beta\Lambda$ and $\omega\Lambda/c$, where Λ is the inter-hole spacing and c is the speed of light in vacuum. For $\beta < kn$, light is free to

Figure 2.3: Propagation diagram of single-mode fibre and photonic band gap[9].

propagate; for $\beta > kn$, it is evanescent. For conventional fibre, guided modes appear when light is free to propagate in the doped core but is evanescent in the cladding, the narrow red strip is where the whole of optical telecommunications operates, as shown in the Figure 2.3A. The same diagram for PCF is sometimes known as a band-edge or "finger" plot. In a triangular lattice of circular air holes with an air-filling

fraction of 45%, as shown in the Figure 2.3B, in region 1, light is free to propagate in every region of the fibre (air, photonic crystal, and silica). In region 2, propagation is turned off in the air and in region 3, it is turned off in the air and photonic crystal. In region 4, light is evanescent in every region. The black fingers represent the regions where full two-dimensional photonic band gaps exist. Guided modes of a solid-core PCF form at the points such as Q, where light is free to travel in the core but unable to penetrate the photonic crystal. At point P, light is free to propagate in air but blocked from penetrating the cladding by the PBG, these are the conditions required for a hollow-core mode.

2.2.2 Modified total internal reflection

The holes in the first PCF were too small to expect a photonic band gap, so there was little point in introducing a hollow core in the center. Given that larger air-filling fractions seemed beyond reach in 1995, an obvious thing was to try a solid core. Conceptually, it was difficult to determine whether this structure would be a waveguide or not. From one perspective, it resembled a standard fibre because the average refractive index was lower outside the core. The striking property of this fibre was that the core did not ever seem to become multi-mode in the experiment, no matter how short the wavelength of the light, the guided mode always had a single strong central lobe filling the core, this behavior is called "endlessly single mode fibre" [8].

2.3 Structure Examples

Figure 2.4 shows an assortment of PCFs structures. Figure 2.4A is the scanning electron micrograph (SEM) of an endlessly single mode fibre which have been mentioned in the previous part. Figure 2.4B is the far-field optical pattern produced by Figure 2.4A when excited by red and green laser light. The fibres consists of an array

of 300 – nm air holes, spaced $2.3\text{ }\mu\text{m}$ apart, with a central solid core.

Figure 2.4: An assortment of optical (OM) and scanning electron (SEM) micrographs of PCF structures[9].

This intriguing "endlessly single-mode" behavior can be understood by viewing the array of holes as modal filter or "sieve". Because light is evanescent in the air, the holes act as strong barriers; they are the "wire mesh" of the sieve.

In Figure 2.4C, the guided modes become birefringent since the core micro-structure is deliberately made two-fold symmetric. Extremely high values of birefringence can be achieved, some 10 times larger than in conventional fibres. Unlike conventional fibres, the PCF birefringence is highly insensitive to temperature, which is very important in many applications.

The tendency for different frequencies of light to travel at different speeds is a crucial factor in the design of telecommunications systems, namely, *dispersion*. In PCF, the dispersion can be controlled with unprecedented freedom. As the holes get larger, the core becomes more and more isolated, until it resembles an isolated strand of silica glass suspended by six thin webs of glass. The "cobweb" PCF in Figure 2.4D, has an 800-nm diameter core and a dispersion zero at 560 nm.

Although the first photonic band gap fibre was reported in 1998 as the structure depicted in Figure 2.4E, hollow-core guidance had to wait until the technology had advanced to the point where larger air-filling fractions became possible. The first such fibre had a simple triangular lattice of holes and the hollow core was formed by removing seven capillaries. And the Figure 2.4F is the near-field optical micrograph (OM) of the six-leaved blue mode that appears when Figure 2.4E is excited by white light.

Optical and electron micrographs of a typical hollow-core PCF are shown in Figure 2.4G and Figure 2.4H. Launching white light into the fibre core causes them to transmit colored modes, indicating that guidance existed only in restricted bands of wavelength, coinciding with the photonic band gaps. This feature limits the range of potential applications. More recently it has been possible to greatly widen the transmission bands by fabricating a different structure, a Kagome lattice, as shown in Figure 2.4I.

2.4 Fabrication Techniques

The first challenge of PCF was to devise a fabrication method. Now let us inspect the technique in brief. After several false starts, Russell and his colleagues found silica capillaries could be stacked, fused together and drawn successfully down to PCF, as shown in the Figure 2.5. This stack-and-draw procedure proved highly versatile, allowing complex lattices to be assembled from individual stackable units

of the correct size and shape. The techniques adopted by Russell's team is said first used in the third- to first-centuries BC by the Egyptians to make mosaic glass [9]. The technique's success is largely due to the mechanical stability of the structure- the surface tension forces tend to balance out, allowing formation of highly regular lattice of holes during the drawing process.

Figure 2.5: Diagram of PCF fabrication technique[9].

As shown in the Figure 2.5, a stack of glass tubes and rods is constructed as a macroscopic "preform" with the required photonic crystal structure. It is then fused together and drawn to fibre c in two stages using a standard fibre drawing tower. To

soften the silica glass, the furnace runs at 1800 °C to 2000 °C.

The first convincing photonic crystal fibre structure emerged from the fibre drawing tower in November, 1995. It had a hexagonal close-packed array of small air channels and was free of any gross imperfections or defects. It was the photonic equivalent of a pure dopant- and defect-free semiconductor crystal, requiring controlled introduction of impurities to be useful. Functional defects could be precisely introduced during the stacking process, allowing fabrication of a wide range of different PCFs.

2.5 Applications

The diversity of new or improved features, beyond what conventional fibre offers, means that PCF is finding an increasing number of applications in ever-widening areas of science and technology. Let us sample a few of the more intriguing and important ones.

PCFs can have high values of nonlinearity per unit length and thus nonlinear optical devices based on PCF can be much shorter in length, and/or operate at lower power levels than devices based on conventional fibre types. PCFs thus promise practical nonlinear fibre devices for use within a variety of application areas including future high capacity optical communication systems.

The first true demonstration of the use of PCF to perform a function required within telecommunications was reported in 2001 and concerned 2R-optical regeneration and optical thresholding [10]. The PCF that was used for this demonstration is shown in Figure 2.6. A filter is set with its peak transmission wavelength offset from the central wavelength of an incoming data stream such that low power pulses propagating within the PCF generate little self-phase modulation (SPM) and are thus largely rejected by the filter. By contrast higher power pulses generate appreciable SPM (see Figure 2.6b) such that significant power is now generated at wavelengths within the filter pass-band. These spectral components are then transmitted by the

Figure 2.6: (a) SEM of small-core holey fibre used for 2R regeneration. (b) SPM broadened spectra. (c) Nonlinear transmission characteristic[11] .

filter. Such a nonlinear response can be used as a means to provide 2R regeneration since 'noise' zero bits are suppressed and noisy 'one' bits have their intensities equalized. The reshaping part of the 2R process is obtained through the narrow-band filtering process itself assuming that a filter with an appropriate phase and amplitude response matched to the incoming pulse response it used. A plot of a typical nonlinear transmission response for such a switch is shown in Figure 2.6c. This particular switch was used as an optical threshold within an Optical Division Multiple Access (OCDMA) receiver and allowed error-free performance with reduced power penalty relative to a simple linear receiver. In the earliest experiments of this approach regenerative operation of a 3.3 m switch was reported with a pulse operating power of 16 W [10].

Also using a fibre similar to the structure shown in Figure 2.6, a PCF Raman amplifier was demonstrated [11]. The 75 m long amplifier was configured for co-propagating pump and signal. The signal light was generated from a continuous wave external cavity laser tunable in the range 1600-1640 nm. The maximum peak pump power that we could launch into the holey fibre was about 6.7 W, and the maximum launched signal power was about 0 dBm. The loss of this fibre and nonlinear coefficient were measured to be 40 dB/km and $\gamma = 32 \text{ W}^{-1} \cdot \text{km}^{-1}$, respectively. The 75 m of PCF, therefore has an effective nonlinear length L_{eff} of just 54 m. Higher gains and lower noise figure are observed in this amplifier as the probe signal wavelength approaches the peak of the Raman gain curve around 1650 nm, corresponding to the peak Raman shift of 13.2 THz. The observed variation in gain with wavelength from [11] was in good agreement with the expectations based on published data on the Raman lineshape of silica.

Another dramatic example of PCF is supercontinuum generation. When ultra-short, high-energy pulses travel through a material, their frequency spectrum can experience giant broadening due to a range of interconnected nonlinear effects. Until recently this required a regeneratively amplified Ti-sapphire laser operating at 800-

nm wavelength. Pulses from the master oscillator (100-MHz repetition rate, 100 fs duration, few nJ energy) are regeneratively amplified up to about 1 mJ. Because the amplifier needs to be recharged between pulses, the repetition rate is only around 1 KHz. Thus, there was great excitement when it was discovered that highly nonlinear PCF, designed with zero chromatic dispersion close to 800 nm, displays giant spectral broadening when the 100 MHz pulse train from the master oscillator is launched into just a few cm of fibre [9], as depicted in Figure 2.7. The pulses emerge from a tiny aperture which is only about $0.5 \mu\text{m}^2$ and last only a few ps. They have the bandwidth of sunlight but are 10^4 times brighter. Not surprisingly, this source is finding many uses, e.g., in optical coherence tomography (OCT). The supercontinuum

Figure 2.7: PCFs application: supercontinuum generation[9].

turns out to consist of millions of individual frequencies, precisely separated by the repetition rate of the pump laser, as shown in Figure 2.7B. This "frequency comb" can be used to measure optical frequency to an accuracy of one part in 5.1×10^{-16} . A commercial system is already on the market, based on a compact diode-pumped fs laser source. The huge bandwidth and high spectral brightness of the supercontinuum source make it ideal for all sorts of spectroscopy. Measurements that used to take hours and involve counting individual photons can now be made in a fraction of a second. Furthermore, because the light emerges from a microscopic aperture it is uniquely easy to perform spectroscopy with very high spatial resolution.

2.6 Remarks

Photonic crystal fibres represent a next generation, radically improved version of a well-established and highly successful technology. In escaping from the confines of conventional fibre optics, PCFs have created a renaissance of new possibilities in a large number of diverse areas of research and technology, in the process irrevocably breaking many of the tenets of received wisdom in fibre optics. In the following parts, we will study the modelling and optimization of this kind of novel fibres.

Chapter 3

Electromagnetic Modelling of PCFs

3.1 Overview

As we mentioned in the previous chapter, for electromagnetic (EM) modelling of PCFs, the large index contrast between air and silica, together with complex structure of PCFs does not allow for the direct use of methods from traditional fibre theory. Especially for the novel PCF, operating by the PBG effect, the full vectorial nature of the electromagnetic waves has to be taken into account. However, for the index-guiding PCFs, a simpler scalar model, based on an effective-index of the cladding which is obtained from the so-called space filling mode in the cladding region, has proven to give a good qualitative description of the operation [3],[12]. We will be using this model to gain qualitative information about the properties of the high-index core triangular PCFs, as well as a more advanced method, based on finite element method (FEM) using hybrid edge/nodal elements with triangular shape, for accurately determining the modal dispersion properties. Roughly speaking, the simple effective-index method is a kind of analytical or semi-analytical methods while the complicated FEM is belong to numerical methods.

3.2 Space Filling Mode-Effective Index (SFM-EI) Method

3.2.1 Introduction

In order to establish a relatively simple numerical tool that could provide qualitative mode-propagation properties of high-index core triangular PCFs, Birks et al. proposed a method in which sequential use of well-established fibre tools was applied [3]. The fundamental idea behind this work is to first evaluate the periodically repeated hole-in-silica structure of the cladding and then, based on the approximate waveguide properties of this cladding structure, replace the cladding by a properly chosen effective index [2], [7]. In this model, the resulting equivalent waveguide consists of a core and a cladding region that has refractive indices n_{co} and n_{cl} , respectively. Apparently, the core region is pure silica, however, the definition of the refractive index of the micro-structured cladding region is given in terms of the propagation constant of the lowest-order mode that could propagate in the infinite cladding material. After

Figure 3.1: Basic principle of effective-index method.

that, resulting model consists of a core and cladding region with the refractive indices of n_{co} and n_{cl} . Having decided the cladding- and core- index values, we may calculate the approximate propagation properties of the PCF as for a step-index fibre [13]. The

idea is depicted as Figure 3.1.

3.2.2 Space Filling Mode in Cladding Region

The value of the cladding effective index is a very important parameter. It sets the width of the spectral range over which a holy fibre is single-mode. So far, computation of the cladding effective index has been performed by using either full-vectorial numerical techniques or approximate scalar analytical approaches. Here we show that the cladding effective index can be evaluated using a fully analytical vector approach.

The exact solutions of Maxwell's equations in a hexagonal lattice can be chased on the grounds of simple symmetry properties. And the complicated repeated lattice structured in PCFs' cladding can be simplified as shown in Figure 3.2. The circle C

Figure 3.2: Schematic diagram of the single cell in the hexagonal lattice[11].

is where the boundary conditions due to symmetry properties have to be applied. ρ and θ are transverse coordinates of a cylindrical reference frame centered on the cell axis.

In the cylindrical frame of Figure 3.2, the electromagnetic (EM) field within the

unit cell can be expressed as following:

$$\mathbf{E}_j = \vec{\varepsilon}_j(\rho, \theta) e^{-\gamma z} \quad (3.1)$$

$$\mathbf{H}_j(\rho, \theta, z) = \vec{H}_j(\rho, \theta) e^{-\gamma z} \quad (3.2)$$

where $j=1$ or 2 . $j=1$ for quantities defined in the $0 \leq \rho \leq a$ region and $j=2$ in $a \leq \rho \leq R$ [14].

The transverse fields $\{\vec{\varepsilon}_t, \vec{H}_t\}$ may be written as:

$$\vec{\varepsilon}_{tj} = -\frac{1}{\gamma^2 - \sigma_j^2} \{ \gamma \nabla_t \varepsilon_{zj} + i\omega\mu \nabla_t H_{zj} \times \hat{z} \} \quad (3.3)$$

$$\vec{H}_{tj} = -\frac{1}{\gamma^2 - \sigma_j^2} \{ \gamma \nabla_t H_{zj} + i\omega\varepsilon_j \nabla_t \varepsilon_{zj} \times \hat{z} \} \quad (3.4)$$

where $\sigma_j^2 = -\omega^2 \mu \varepsilon_j$, $\nabla_t = \hat{\rho} \partial / (\partial \rho) + \hat{\theta} / (1/\rho) \partial / \partial \theta$, and ε_{zj} , H_{zj} are two independent solutions of a 2-D Helmholtz equation (3.23) [14].

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} \quad (3.5)$$

And let us inspect the boundary condition in each cell. The boundary condition can be obtained by reflection symmetry of the hexagonal lattice. Let us focus the point P in Figure 3.3. There are two planes of symmetry, marked S_1 and S_2 , through P. Fields which exhibit either an odd or an even parity with respect to either S_1 or S_2 are unaffected if their symmetry plane is replaced either by a perfect electric or by a perfect magnetic conductor. For the longitudinal components of the electric and magnetic field this translates as follows:

Case1: $E_z(P) = 0$, if S_1 and/or S_2 are perfect electric conductors;

Case2: $H_z(P) = 0$, if S_1 and/or S_2 are perfect magnetic conductors;

Case3: $E_z(P) = 0$ and $H_z(P) = 0$ if S_1 is a perfect magnetic conductors and S_2 a perfect electric conductor, or vice-versa.

Figure 3.3: Schematic diagram of the periodically arrangement of hexagonal lattice[11]

Cases 1 and 2 entail a field with a quadrant-like symmetry. This is not compatible with the fundamental mode, which, as frequency $\omega \rightarrow 0$, must tend to a uniform plane wave. And analysis in [14] implies that the lattice fundamental mode must be a field of case 3. And at $\rho = a$, continuity of tangential components reads

$$\begin{aligned}
 \varepsilon_{z1}(\rho = a, \theta) &= \varepsilon_{z2}(\rho = a, \theta) \\
 H_{z1}(\rho = a, \theta) &= H_{z2}(\rho = a, \theta) \\
 \varepsilon_{\theta1}(\rho = a, \theta) &= \varepsilon_{\theta2}(\rho = a, \theta) \\
 H_{\theta1}(\rho = a, \theta) &= H_{\theta2}(\rho = a, \theta)
 \end{aligned} \tag{3.6}$$

Case 3 is the most interesting conditions because it encompasses the fundamental mode with boundary conditions to be applied at $\rho = R$, read

$$\varepsilon_{z2}(\rho = R, \theta) \equiv 0 \quad H_{z2}(\rho = R, \theta) \equiv 0 \tag{3.7}$$

After substituting (3.7) into the 2-D Helmholtz equation (3.23) together with the boundary conditions (3.6) and then a series of complicated calculation [14], we finally

obtained the two equations for different field distributions as follows [14].

$$\frac{I_{l+1}(\omega)}{I_l(\omega)} = -\frac{l}{\omega} - \frac{\omega}{2} \left(A'_l + \frac{\epsilon_{r2}}{\epsilon_{r1} B'_l} \right) - \omega \sqrt{\frac{1}{4} \left(A'_l - \frac{\epsilon_{r2}}{\epsilon_{r1} B'_l} \right)^2 + \frac{f^2(u, \omega, l)}{\epsilon_{r1}}} \quad (3.8)$$

$$\frac{I_{l-1}(\omega)}{I_l(\omega)} = \frac{l}{\omega} - \frac{\omega}{2} \left(A'_l + \frac{\epsilon_{r2}}{\epsilon_{r1} B'_l} \right) + \omega \sqrt{\frac{1}{4} \left(A'_l - \frac{\epsilon_{r2}}{\epsilon_{r1} B'_l} \right)^2 + \frac{f^2(u, \omega, l)}{\epsilon_{r1}}} \quad (3.9)$$

where the quantities called A'_l and B'_l are functions of u , and read

$$\begin{aligned} A'_l(u) &= p'_l(u), \quad B'_l(u) = q'_l(u) \quad \text{for "M" modes} \\ A'_l(u) &= q'_l(u), \quad B'_l(u) = p'_l(u) \quad \text{for "E" modes} \\ A'_l(u) &= B'_l(u) = p'_l(u) \quad \text{for "T" modes} \end{aligned} \quad (3.10)$$

with

$$p'_l(u) = \frac{P'_l(u)}{u P_l(\rho = a)} \quad (3.11)$$

$$q'_l(u) = \frac{Q'_l(u)}{u Q_l(\rho = a)} \quad (3.12)$$

$$P'_l(u) = \frac{a}{u} \frac{\partial P_l(\rho)}{\partial \rho} \Big|_{\rho = a} \quad (3.13)$$

$$Q'_l(u) = \frac{a}{u} \frac{\partial Q_l(\rho)}{\partial \rho} \Big|_{\rho = a} \quad (3.14)$$

And please note here,

$$P_l(\rho) = J_l(u\rho/a) Y_l(uR/a) - Y_l(u\rho/a) J_l(uR/a) \quad (3.15)$$

$$Q_l(\rho) = J_l(u\rho/a) Y'_l(\xi) \Big|_{\xi=uR/a} - Y_l(u\rho/a) J'_l(\xi) \Big|_{\xi=uR/a} \quad (3.16)$$

with $J_l(\cdot)$ and $Y_l(\cdot)$ the l -th order Bessel functions of the first and second kind, respectively, and $J_l(\cdot)$ the l -th order modified Bessel function of the first kind[14]. Primes denote derivatives with respect to the argument, and

$$\omega^2 = \omega^2 (n_{eff}^2 - n_1^2) a^2 / c^2 \quad (3.17)$$

$$u^2 = \omega^2(n_2^2 - n_{eff}^2)a^2/c^2 \quad (3.18)$$

finally,

$$f^2(u, \omega, l) = l^2 \left(\frac{1}{u^2} + \frac{1}{\omega^2} \right) \left(\frac{\varepsilon_{r1}}{\omega^2} + \frac{\varepsilon_{r2}}{u^2} \right) \quad (3.19)$$

and propagation constant β can be obtained by

$$\beta = (\omega/c)n_{eff} = -i\gamma \quad (3.20)$$

By analogy with step-index fibres, we will call "EH" those modes which correspond to the solutions of (3.8), "HE" those which correspond to the solutions of (3.9) [14].

To determine the space filling mode of the hexagonal lattice, by definition, among all those modes we found so far, which exhibits the highest effective index. Dispersion curves for some low-order modes are shown in the Figure 3.4. In this figure, Figure 3.4a refers to modes bounded by a perfect magnetic conductor, whereas Figure 3.4b and Figure 3.4c refers to the case of modes bounded by a perfect electric conductor, and by a surface imposing null values to the longitudinal component of both the electric and the magnetic field. In all the diagrams in Figure 3.4, solid line is for the EH_{11} mode, dashed line for the HE_{11} mode, dotted line for the TM_{01} mode and dashed-dotted line for the TE_{01} mode. We know that analogy with step-index fibres might be misleading, to be on the safe side, it should be identified by solving numerically and through the calculation in [14], the fundamental mode belongs to the "T" set. In particular, it is the $EH_{11}^{(T)}$ mode. And then, we can follow (3.8) to obtain the fundamental mode distribution in the cladding region of PCFs.

After we obtained the space filling mode in the cladding, we can therefore know the effective cladding index value. And the complicated PCFs structure can be treated simply as the conventional step-index fibre so that the optical waveguide theory in step-index fibre can be used.

Figure 3.4: Diagram of different modes propagated in the cladding cell of PCFs [14].

3.2.3 Optical Waveguide Theory

From the Maxwell's equations, the axial component of electrical field distribution E_z can be written as

$$E_z(\rho, \phi, z) = F(\rho)\Phi(\phi)Z(z) \quad (3.21)$$

And using the method of separation of variables cylindrical coordinates, finally we obtained the equations as follows:

$$d^2Z/dz^2 + \beta^2Z = 0 \quad (3.22)$$

$$d^2\Phi/d\phi^2 + m^2\Phi = 0 \quad (3.23)$$

$$\frac{d^2F}{d\rho^2} + \frac{1}{\rho} \frac{dF}{d\rho} + \left(n^2k_0^2 - \beta^2 - \frac{m^2}{\rho^2} \right) F = 0 \quad (3.24)$$

Where equation (3.22) has a solution of the form $Z = \exp(i\beta z)$, where β has the physical significance of the propagation constant. Similarly, equation (3.23) has also a solution $\Phi = \exp(im\phi)$, but the constant m is restricted to take only integer values since the field must be periodic in ϕ with a period of 2π .

Equation (3.24) is the well-known differential equation satisfied by the Bessel functions. Its general solution in the core and cladding regions can be written as follows:

$$F(\rho) = \begin{cases} AJ_m(p\rho) + A'Y_m(p\rho) & \rho \leq a, \\ CK_m(q\rho) + C'I_m(q\rho) & \rho > a, \end{cases} \quad (3.25)$$

where A, A', C and C' are constants and just like in PCF's case, J_m, Y_m, K_m and I_m are different kinds of Bessel functions. The parameters p and q are defined by

$$p^2 = n_1^2k_0^2 - \beta^2, \quad (3.26)$$

$$q^2 = \beta^2 - n_2^2k_0^2 \quad (3.27)$$

Considerable simplification occurs when we use the boundary condition that the optical field for a guided mode should be finite at $\rho = 0$ and decay to zero at $\rho = \infty$.

After considerable algebraic details, together with the boundary condition, finally leads us to the following eigenvalue equation:

$$\left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{K'_m(qa)}{qK_m(qa)} \right] \left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{n_2^2}{n_1^2} \frac{K'_m(qa)}{qK_m(qa)} \right] = \frac{m^2}{a^2} \left(\frac{1}{p^2} + \frac{1}{q^2} \right) \left(\frac{1}{p^2} + \frac{n_2^2}{n_1^2} \frac{1}{q^2} \right) \quad (3.28)$$

So far, we have already set up the entire model for EM distribution in PCF and this problem would attribute to two equations, namely, (3.8) and (3.28).

3.2.4 Simulation Results

Here we choose the following parameters which correspond to an example in [15] for verification purpose: an air-silica hexagonal lattice where the radius of the air holes is $a = 0.3 \mu\text{m}$; the distance between centers of neighboring holes (or lattice pitch) is $A = 2.3 \mu\text{m}$, that means $R = \Lambda/2 = 1.15 \mu\text{m}$. The refractive index of air and silica is taken to be $n_1 = 1.0$ and $n_2 = 1.46$, respectively.

Figure 3.5: Simulation results: diagram of effective cladding index and normalized frequency.

We listed the numerical results in Table 3.1. There are only 7 sample wavelengths, which range from 1300 nm to 1600 nm at an interval of 50 nm, been used to calculate the corresponding effective cladding index and the normalized frequency, in the term of step-index fibre. And the corresponding relationship between the normalized frequency versus the parameter obtained from pitch between air holes Λ over wavelength λ is depicted in Figure 3.6 to compare with the result from other publications and very good agreement is achieved. Please note the blue "*" in Figure 3.6 are the data extracted from [14].

Wavelength(nm)	1300	1350	1400	1450	1500	1550	1600
Effective cladding index	1.4451	1.4444	1.4438	1.4431	1.4425	1.4419	1.4413
Normalized frequency	1.4806	1.4574	1.4343	1.4115	1.389	1.3669	1.3451

quency versus the parameter obtained from pitch between air holes Λ over wavelength λ is depicted in Figure 3.6 to compare with the result from other publications and very good agreement is achieved. Please note the blue "*" in Figure 3.6 are the data extracted from [14].

Figure 3.6: Verification of our SFM-EI implementation.

Then we compare the simulation result over a wider wavelength range with the

result in publication. Result is shown in the Figure 3.7, different geometry structures would lead to different optical parameters. Also note here when the radius of air holes is relatively small, the normalized frequency could always be less than 2.405, which is the criteria for single-mode operation of step-index fibre. This is why the PCF is called "Endless single-mode operation". Comparing the result of the normalized frequency from other publication which is obtained from finite element method (FEM) by Koshiba [16], the results by these two different methods are obviously almost the same.

Figure 3.7: Simulation results: normalized frequency over a wider range of wavelength.

3.2.5 Remarks

SFM-EI method basically is an efficient approach with good accuracy, and it is also a kind of analytical method. However, we only modelled the relative simple cladding structure of PCFs, which is the repeated lattice structure, and usually this

kind of PCFs are called holey fibres, whose mechanism to guide light is modified total internal reflection (TIR). Imagining a structure with air holes of different radius, it is almost impossible for SFM-EI to solve the problem like that.

Furthermore, although we obtained the effective index by the so-called space filling mode using a full vector approach, the effective index method is a scalar and qualitative method. Information provided by this method is not good enough for various PCFs structures since they are only treated as conventional stepindex fibres.

To solve these problems, we should turn to numerical method to achieve accurate and efficient modelling results for complicated PCFs structure, and the next part we should examine one of these method — finite element method (FEM).

3.3 Finite Element Method (FEM)

3.3.1 Numerical Approaches

Scientists and engineers use several techniques when solving EM field distribution problems we described above. Loosely speaking, these techniques can be classified as experimental, analytical or numerical. Despite the expensive, time-consuming experimental methods, analytical or numerical methods are very common to use in such problems. The analytical methods is often efficient but it very suitable for problems with relatively simple geometry.

Numerical solution of EM problems started in the mid-1960s with the availability of modern high-performance computers. Since then, much effort has been expended on solving practical but complex EM-related problems for which closed form analytical solutions are either intractable or do not exist. The numerical approach has the advantage of allowing the actual work to be carried out by operators without a knowledge of higher mathematics or physics, with a resulting economy of labor on the part of the highly personnel.

Following are among numerical methods very often to use:

- 1) Finite element method (FEM)
- 2) Finite difference time-domain method (FDTD)
- 3) Method of moment (MOM)
- 4) Transmission-line modelling (TLM)

As we all know, these numerical methods give sufficient accuracy for researching or engineering purposes. FDTD, together with TLM, basically can obtain the time-domain solution. They can obtain one solution for many frequency points, in other word, they are very efficient. The beam propagation method (BPM) or full vectorial beam propagation method is actually a kind of FDTD which is widely used in PCF's modelling and can handle the 2-D and 3-D problem efficiently [6]. But the disadvantage of these methods is to produce larger meshing error compared with other approaches.

In contrast, FEM, BEM, and MOM are all solve the problems in frequency domain. Roughly speaking, BEM is actually a kind of MOM. BEM and MOM are very flexible to solve the eigenvalue problem and solve the problem based on simple mesh, but they are tried to solve the problem in terms of dense matrix instead of sparse matrix, which made the whole eigenvalue system very hard to solve. However, we can see in the later section that FEM can solve the problem in terms of sparse matrix.

Elaboration work from the literature and the research of our fellow group members [17], we believe FEM is the flexible and efficient simulation method which can be combined with optimization approaches. Therefore we choose FEM as the modelling tool of our PCF case.

Time Domain	FDTD	✓ One solution for many frequency points
	TLM	× Larger meshing error
Frequency Domain	MOM	✓ Flexible and simple mesh
	BEM	× Dense matrix
	FEM	✓ Sparse matrix × Spurious solutions

Table 3.2: Comparison of different numerical techniques.

3.3.2 Introduction to FEM

The finite element method (FEM) is a numerical procedure for achieving solutions to boundary-value problems. The principle of the method is to divide an entire continuous domain into a number of sub-domains in which the unknown functions are replaced by simple interpolation functions with unknown coefficients [15].

The main advantage of FEM is its capability to treat arbitrary geometry and complex material profile without altering the formulation and computer code. Moreover, the application of FEM leads to sparse matrix systems which can be solved very efficiently. FEM systems typically have $O(N)$ storage requirements, implying that the memory needed for a solution of an FEM system is proportional to the number of unknowns N . From the advantages of FEM, we can understand that FEM is very suitable for analysis of the arbitrary and complicated transverse geometry structure in PCFs. Furthermore, since one of our objects is to use together with the optimization tools, it is necessary that the modelling method should be very accurate and efficient.

The finite element analysis of any problem involves basically four steps as follows:

- discretizing the solution region into a finite number of *subregions or elements*,
- deriving governing equations for a typical element,

- assembling of all elements in the solution region and,
- solving the system of equations obtained.

We now ready to implement the finite element method in the same sequence.

3.3.3 Finite Element and Mesh Generation

There are two types of elements in finite element method, namely node-based element and edge-based element. Node-based shape functions have been extensively used in many engineering applications including scalar electromagnetic field problems. However, vector formulation often brings out numerous deficiencies in these traditional element functions. Hence, edge-based vector basis functions with unknowns associated with element edges have been derived to overcome the problems related to nodal basis [18], [19].

In node-based finite elements, the form of sought function in the element is controlled by the function values at its nodes. If the function values u_i^e at the nodes are taken as nodal variables, then the approximating function for a two-dimensional element e with p nodes has the form

$$u^e(x, y) = \sum_{i=1}^p u_i^e N_i^e(x, y) \quad (3.29)$$

Since (3.29) should be valid for any nodal variable u_i^e , the basis function $N_i^e(x, y)$ must be unity at node i and zero for all remaining nodes within the element.

However, in electromagnetic computation, we encounter serious problems when node-based elements are employed to represent vector electric or magnetic fields. First, spurious modes are observed when modelling cavity problems using node-based elements. Nodal basis functions impose continuity in all three spatial components whereas edge bases guarantee continuity only along the tangential component. This feature mimics the behavior of field components along discontinuous material boundaries. Edge-based finite elements, whose degrees of freedom are associated with the

edges and the faces of the finite element mesh, have been shown to be free of the above shortcomings. Edge basis functions were described by Whitney over 37 years ago and have been revived by Bossavit and Verite.

In PCFs modelling, we adopted the hybrid type edge elements, which is very simple but powerful. Figure 3.8 shows the lowest order mixed-type element which is

Figure 3.8: Hybrid edge-node triangular element.

composed of two types of elements: a constant edge element with three tangential unknowns, ϕ_{t1} to ϕ_{t3} , and a linear nodal (conventional Lagrange) element with three axial unknowns, ϕ_{z1} to ϕ_{z3} . Since both ϕ_t and ϕ_z are tangential to material interfaces, the tangential continuity can be straight forwardly imposed in the mixed-type element analysis. In this lowest order element the tangential component $\phi_t = \phi \cdot \mathbf{t}$ is constant along each side of triangles, it is why the edge element is called the constant edge element [20], [21].

On the other hand, the accuracy of the finite element approximation depends on the quality of approximation provided by the local functions and of the size of these elements. Naturally, a better approximation is obtained with small elements and high degree polynomials. However, the total number of the degrees of freedom (which means the order of the matrices), the computing time and, consequently, the computing cost increase with the number of elements for a given fixed polynomial

order [21].

In real problems, the required level of accuracy and the geometry or material complexity makes necessary the use of fine meshes with a large number of elements. The definition of the elements, the nodal points and their coordinates must be accomplished before any calculation. This is a very boring and tedious task even for small meshes and quite a huge one for a large, realistic mesh. Nonuniform meshes are preferred if it is known before mesh generation that the parameter of our interest varies rapidly in some parts of the solution domain. This allows a concentration of relatively small elements in the regions where the parameter changes very quickly, which is very important for the case of our PCFs with complicated geometry structure [22],[23]. Moreover, in the previous part of this report, we learn the fact the guided modes of the PCFs are localized in the close vicinity of the defect forming the core region. Hence, uniform mesh does not fit the EM field distribution of PCFs any more. In addition to the comparatively complicated geometry structure of PCFs index profile, it is appropriate to utilize the automatic mesh generation. Using the build-in functions provided by Matlab, we may solve the problem of mesh generation, as shown in Figure 3.9.

Figure 3.9 shows the mesh generation with the PCF's geometry structure parameter $d/\Lambda = 0.4$. As we can see, in the core region, the mesh is much smaller than that in the cladding, especially the region far away from the core. It is just along the trend of EM field distribution in PCFs, which changes very rapidly in the core region while keeps comparatively stable in the cladding region. Furthermore, Matlab also provides function to refine the mesh arbitrarily, which means we can refine the meshes in some specific region, for example, the air-holes composed of the first ring.

3.3.4 Basic Equations and FEM Formulation

As to the optical waveguide problems, there are mainly two analysis methods shown in Figure 3.10.

Figure 3.9: Mesh generation of PCF

Figure 3.10: Analysis methods based on the finite-element method.

Let us examine the variational method because it is a fundamental method. This method is often employed to formulate finite element solutions which has several advantages. The primary advantage is its well-established and solid foundation in physics and mathematics, which may permit a physical interpretation and provide a different portrait to a physical problem. Another advantage is that, by using the variational procedure one can clearly demonstrate the differences between the essential and natural conditions. Other advantages include convenience of description and elegance of the formulation.

In our implementation of finite element method, however, we choose another

method, namely weighted residual approximation. This method is more general and has wider application than the variational method because it is not limited to a class of variational problems.

Consider the boundary value problem

$$\mathcal{L}u = s \quad (3.30)$$

In the weighted residual method, the solution to (3.30) is approximated using the expansion functions, u_n , i.e.,

$$\tilde{u} = \sum_{n=1}^N a_n u_n \quad (3.31)$$

where a_n are the expansion coefficients. We seek to make

$$L\tilde{u} \approx s \quad (3.32)$$

Substitution of the approximate solution in the operator equation results in a residual R , which means an error in the equation, for example, like this,

$$R = L(\tilde{u} - u) = L\tilde{u} - s \quad (3.33)$$

In the weighted residual method, the weighting functions are ω_m (and please note that the weighting function here are not the same as the expansion functions u_n) so that the integral of a weighted residual of the approximation is zero, that is to say,

$$\int \omega_m R dv = 0$$

or the simple way to express this equation:

$$\langle \omega_m, R \rangle = 0 \quad (3.34)$$

If a set of so-called **weighting functions** $\{\omega_m\}$ (also known as **testing functions**) are chosen and the inner production of (3.32) can be cast into matrix form like the following forms:

$$[A][X] = [B] \quad (3.35)$$

where $A_{mn} = \langle \omega_m, Lu_n \rangle$, $B_m = \langle \omega_m, s \rangle$, $X_n = a_n$. Solving for $[X]$ in (3.35) and substituting for a_n in (3.31) gives the approximate solution to (3.30).

The Galerkin method, which is a type of weighted residual method, was developed by the Russian engineer Galerkin in 1915. In this method, basis functions are selected the same as the weighting function, i.e., $\omega_m = u_m$. When the operator is a linear differential operator of even order, the Galerkin method reduces to the Rayleigh-Ritz method. This is due to the fact that the differentiation can be transferred to the weighting functions and the resulting coefficient matrix $[A]$ will be symmetric. In order for the Galerkin method to be applicable, the operator must be of a certain type. Also, the expansion function u_n must span both the domain and the range of the operator.

Following we would utilize the Galerkin method to analyze the electromagnetic wave propagation in PCFs. For an inhomogeneous wave guiding structures, the classical wave equation which is obtained under the condition of linear, isotropic, homogeneous, source-free medium would not be appropriate, therefore we should start from Maxwell's equations (2.1) to (2.4). As the same way, taking curl of (2.1) and hence a single double-curl equation in terms of electric field equation is obtained as following.

$$\nabla \times \frac{1}{\mu_r} \nabla \times \mathbf{E} = k_0^2 \bar{\epsilon}_r \mathbf{E} \quad (3.36)$$

where $\bar{\epsilon} = \epsilon + \sigma/(j\omega)$ and ϵ as well as σ represent the permittivity and conductivity, respectively, of dielectric materials. $k_0^2 = \omega^2 \mu_0 \epsilon_0$ and $\bar{\epsilon}_r = \bar{\epsilon}/\epsilon_0$. Then we subdividing the electric field into the transverse ($\mathbf{E}_t$) and longitudinal (E_z) parts under the assumption that all the field components depend on the spatial coordinate z with the form $\exp(-\gamma z)$, with $\gamma = \alpha + j\beta$ as the complex propagation constant along the waveguide, we can achieve:

$$\mathbf{E}(x, y, z) = [\mathbf{E}_t(x, y) + \hat{z}E_z(x, y)]e^{-\gamma z} \quad (3.37)$$

In accordance with the variational principle described above, the equivalent varia-

tional problem is given by

$$\delta F(\mathbf{E}) = 0 \quad (3.38)$$

where

$$F(\mathbf{E}) = \frac{1}{2} \int_{\Omega} \left[\frac{1}{\mu_r} (\nabla \times \mathbf{E}) \cdot (\nabla \times \mathbf{E})^* - k_0^2 \varepsilon_r \mathbf{E} \cdot \mathbf{E}^* \right] d\Omega \quad (3.39)$$

And then we use the transformation of variables $\mathbf{e}_t = \gamma \mathbf{E}_t$ and $e_z = E_z$. Substituting these two formula into (3.39) and multiplying it with k_z^2 , we can achieve

$$\begin{aligned} F(\mathbf{e}) = & \frac{1}{2} \int_{\Omega} \left\{ \frac{1}{\mu_r} (\nabla_t \times \mathbf{e}_t) \cdot (\nabla_t \times \mathbf{e}_t)^* - k_0^2 \varepsilon_r \mathbf{e}_t \cdot \mathbf{e}_t^* - \right. \\ & \left. \gamma^2 \left[\frac{1}{\mu_r} (\nabla_t \mathbf{e}_z + \mathbf{e}_t) \cdot (\nabla_t \mathbf{e}_z + \mathbf{e}_t)^* - k_0^2 \varepsilon_r e_z e_z^* \right] \right\} d\Omega \end{aligned} \quad (3.40)$$

As we all know, the discretion of this for a given k_0 will result in a system with y^2 as its eigen-values, that will be the propagation constant we needed. To discrete (3.40), the area under consideration is subdivided into small elements, say N elements. The element is displayed in the previous Figure 3.8. Each element consists of six nodes, including three corner and three side points in each mesh. The three corner points, from 1 to 3 are for axial component ϕ_z (E_z or H_z) while side points for the tangential component ϕ_t (E_t or H_t) [15],[24].

The axial component ϕ_z is approximated by a complete polynomial of first order:

$$\phi_z = \sum_{i=1}^3 N_i e_{zi} = \{N(x, y)\}^T \{\phi_z\}_e = \{N\}^T \{\phi_z\}_e \quad (3.41)$$

On the other hand, the transverse components ϕ_t (including ϕ_x and ϕ_y) are approximated by :

$$\phi_t = \sum_{i=1}^3 \mathbf{N}_i e_{ti} = \{\mathbf{N}(\mathbf{x}, \mathbf{y})\}^T \{\phi_t\}_e = \{\mathbf{N}\}^T \{\phi_t\}_e \quad (3.42)$$

Here $\{\phi_z\}_e$ and $\{\phi_t\}_e$ are the nodal axial-field vector for each element and edge variables in the transversal plane for each element, respectively.

For the case of electrical field E , substituting these into (3.40), we can obtain:

$$F = \frac{1}{2} \sum_{e=1}^N (\{e_t^e\}^T [A_{tt}^e] \{e_t^e\}^* - \gamma^2 \begin{Bmatrix} e_t^e \\ e_z^e \end{Bmatrix}^T \begin{bmatrix} B_{tt}^e & B_{tz}^e \\ B_{zt}^e & B_{zz}^e \end{bmatrix} \begin{Bmatrix} e_t^e \\ e_z^e \end{Bmatrix}^*) \quad (3.43)$$

Here we assume that the number of elements is N .

Using global notation, (3.43) can be written like this:

$$F = \frac{1}{2} \{e_t\}^T [A_{tt}] \{e_t\}^* - \frac{1}{2} \gamma^2 \begin{Bmatrix} e_t \\ e_z \end{Bmatrix}^T \begin{bmatrix} B_{tt} & B_{tz} \\ B_{zt} & B_{zz} \end{bmatrix} \begin{Bmatrix} e_t \\ e_z \end{Bmatrix}^* \quad (3.44)$$

Taking the derivative of (3.44), we can achieve the generalized eigenvalue problem as the form:

$$\begin{bmatrix} A_{tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} e_t \\ e_z \end{Bmatrix} = \gamma^2 \begin{bmatrix} B_{tt} & B_{tz} \\ B_{zt} & B_{zz} \end{bmatrix} \begin{Bmatrix} e_t \\ e_z \end{Bmatrix} \quad (3.45)$$

In (3.43), (3.45) the element matrices are given by the following formula:

$$\begin{aligned} [A_{tt}^e] &= \int_{\Omega} \left[\frac{1}{\mu_r} \{\nabla_t \times \mathbf{N}\} \cdot \{\nabla_t \times \mathbf{N}\}^T - k_0^2 \epsilon_r \{\mathbf{N}\} \cdot \{\mathbf{N}\}^T \right] d\Omega \\ [B_{tt}^e] &= \int_{\Omega} \frac{1}{\mu_r} \{\mathbf{N}\} \cdot \{\mathbf{N}\}^T d\Omega \\ [B_{tz}^e] &= [B_{zt}^e]^T = \int_{\Omega} \frac{1}{\mu_r} \{\mathbf{N}\} \cdot \{\nabla_t \mathbf{N}\}^T d\Omega \\ [B_{zz}^e] &= \int_{\Omega} \left[\frac{1}{\mu_r} \{\nabla_t \mathbf{N}\} \cdot \{\nabla_t \mathbf{N}\}^T - k_0^2 \epsilon_r \{\mathbf{N}\} \cdot \{\mathbf{N}\}^T \right] d\Omega \end{aligned} \quad (3.46)$$

By solving this eigenvalue system, we can get the eigenvalue γ^2 as well as eigenvector $\{e_t \ e_z\}^T$ which mean propagation constant and corresponding modal field distribution, respectively [25], [26].

3.3.5 Simulation Results

Dielectric-Slab-Loaded Metallic Rectangular Waveguide

To validate of the vector FEM formulation and implementation with triangular edge element presented in section 3.3, we consider the simple case of a rectangular

Figure 3.11: (a) Waveguide structure and (b) mesh structure.

waveguide with metallic walls and loaded with a dielectric slab as shown in the Figure 3.11a. Based on the mesh generation of Figure 3.11b and the FEM formulation, we can achieve the propagation constant of this structure. Compared to the known analytical solutions for this type of structure [27], [28], the excellent agreement is achieved as shown in Figure 3.12 despite relatively coarse mesh used.

Photonic Crystal Fibres

As to the case of more complicated PCF structures, we may start from a simple geometry. There are two simple geometry of PCFs under our consideration as shown in Figure 3.13, which are only one and two rings of air-holes, respectively. Where d is the hole diameter, and A represents the pitch between holes. Background material is

Figure 3.12: Simulation results in dielectric-slab-loaded metallic rectangular waveguide.

pure silica, therefore the index is $n = 1.45$. We calculated the simulation results and compared with results by other methods. Please note here that only the fundamental mode is plotted.

Figure 3.14a exhibits the real part of the complex effective index versus wavelength. From the figure, we can see clearly that whatever the number of air holes around the core region changes, the real part of complex effective index are very stable, which means under the condition of air-fraction filling is fixed ($d/\Lambda = \text{constant}$), the real part of the effective index is almost independent of the number of arrays of air holes. On the other hand, from the below sub-figure of Figure 3.14b we can learn that the propagation loss changes with wavelength significantly and rapidly. Moreover, the propagation loss of PCF with two rings of holes is drastically reduced compared with one ring.

Figure 3.14 is achieved with the same geometry parameters: $A = 2.3 \mu m$ and $d/\Lambda = 0.5$. Compared with the results in publication [15], the agreement is achieved.

Figure 3.13: Simple geometry structures of PCF with (a) one ring and (b) two rings.

Again, our FEM analysis tool is proved to be accurate and efficient.

3.3.6 Remarks

As we can see from the results, FEM is an accurate and versatile tool which can handle complicated geometries of PCFs. Even though, FEM is still much time-consuming compared with the simple effective-index method, especially when used for optimal design. In optimization, a large number of EM solutions is needed. Therefore, the choice of EM solver is case by case, determined by the request.

Figure 3.14: Wavelength dependence of (a) the real part of complex effective index and (b) the propagation loss.

Chapter 4

Two Stochastic Optimization Algorithms

4.1 Overview

For the case of optimization of the fibres with various geometry structure, generally speaking, one often starts with a given set of fibre specifications, for example, some optical property characteristics, and an initial fibre geometry design. The compute-aided analysis techniques are used to evaluate fibre performances of different structures. Fibre characteristics obtained from the analysis tools are compared with the given specifications. If the results fail to satisfy the specifications wanted, the optimization parameters of the fibre are altered in a systematic manner. The sequence of the fibre analysis, compared with the optimization performance, and modification of optimized parameters is performed iteratively until the optimum performance of the fibre is achieved. This process is known as **optimization** [29],[30]. Let us recall two main parts of the project, namely electromagnetic solver and optimization tools. Based on this study, an EM solver could be either SFM-EI approach or the FEM solver presented in Chapter 3. In this chapter, we will introduce two powerful optimization tools, Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO).

Figure 4.1: Combination of EM modelling method and optimization tool.

The problem of optimization may be formulated as minimization of a scalar objective function $U(\Phi)$, which is also known as an error function or a cost function. For the optimal design of PCFs, the formulation of $U(\Phi)$ may involve the specified and achieved values of the dispersion parameter. Optimization problems are usually formulated as minimization of $U(\Phi)$. Φ is the set of designable parameters whose values can be modified during the optimization process [31]. The elements of Φ could be the values of fibre geometry structure, say, the diameter of air holes d , the pitch between the holes Λ and even, the number of the holes. Here we study two stochastic-based search approaches, namely genetic algorithm (GA) and particle swarm optimization (PSO) as the optimization method. Now let us inspect these two approaches respectively.

4.2 Genetic Algorithms (GAS)

4.2.1 Introduction

Genetic algorithms (GAs) are stochastic-based search techniques based on concepts of biological genetics and evolution [32]. Natural evolution is a search procession for the fittest in the species space. GAs are hereby capable of arriving at an optimal solution without the explicit knowledge about the solution space. GAs are very powerful and can be used in various fields to solve problems which may be very difficult

or impossible by other methods [33],[34]. GA was introduced by Holland in 1970's but has attracted wide attention since early 1990's.

A generic algorithm starts with an initial set of random configurations and uses a process similar to biological evolution to improve upon them. The set of configurations is called the **population**. A binary bit string usually, but not necessarily, represents each parameter in the configurations. During each time of iteration procession, called a **generation**, the configurations in the current population are evaluated using some measures of fitness. The parameters of the fitter configurations have a higher probability of being selected to be the parents. A number of genetic operators such as **crossover**, **mutation** and **inversion** are then applied to the parents to generate new configurations called offspring. The offspring are next evaluated, and a new generation is formed by selecting some of the parents and offspring and rejecting others so as to keep the population size constant. As the iterative process is carried on, the average fitness of the population keeps improving. Conventionally, a genetic algorithm requires a large population size in order to explore solutions in a space as large as possible [35].

The foundation of this method is the simulated generation of a random group of individuals and track their progress as they evolve toward an optimal solution [36]. Since the algorithm has no preconceptions about the search space, it is allowed to explore without the biases that may lead to an unduly sub-optimal solution. However, one difficulty is to generate an individual that is truly representative of the population, which is known as **fitness**. That is, an instance of the problem must be properly encoded in binary form to accurately reflect all of its characteristics. The resulting binary string then becomes one of the individuals in the population. For instance, suppose that the problem being studied consisted of a formula with two parameters say, x-coordinate and y-coordinate, a possible encoding would be to convert each of the coordinates to its binary counterpart and then concatenate the two pieces. The result is a binary string suitable for later manipulation.

Following is an example of basic genetic algorithm:

- 1) Generate a random population of n chromosomes;
- 2) Assign a fitness to each individual;
- 3) Repeat the following steps until n children have been produced;
 - Choose 2 parents based on **fitness proportional selection**. This is a method of choosing parents based on their relative fitness. While every individual has the possibility of being chosen, those with higher fitness have a greater likelihood of being chosen.
 - Apply **genetic operators** to copies of the parents.
 - Produce 2 new individuals.

When using the encoding of the problem, the initial generation must be created. The size of this generation is important. If the population is too small, there will be insufficient genetic diversity. On the other hand, if it is too large, evolution will proceed slowly because of the computer resources needed and the time required to build each subsequent generation. An appropriate population size is several hundred to several thousand individuals.

In the preceding part of Genetic Algorithm we mentioned the term of fitness, the fitness of an individual is an indicator of how well suited an individual is to its current environment [37]. Fitness is established by means of a function. For every individual in the population, a level of fitness must be assigned to it. The process of evolution is predicated on the concept of "survival of the fittest" [38]. Therefore it makes sense that the more fit an individual is, the more likely it will be to be chosen as a parent to help form the next generation. This is the concept of natural selection. This is the selection scheme used in genetic algorithms. On other word, the likelihood of any individual being picked as a parent is directly proportional to its relative fitness [39].

Now let us consider the magic factor behind evolution, that is the result of genetic operators at work. The genetic operators are mainly two mechanisms: **crossover** and **mutation** as follows [32][33]. Cross-over, also known as recombination, is the direct exchange of genetic material (bits) between two parents to produce a new individual. A limited section of the chromosome (bit string) is transposed between two parents. The results in the sharing of genetic information to produce a child that has some of the characteristics of each parent. The outcome is a small change. Crossing over can occur at multiple sites along the bit string. The probability of a crossover occurring at any point is a predefined parameter of the algorithm. The second major operator is mutation. This is the flipping bits at random points along the bit string. Again, the probability of a mutation occurring at any point on the string is a function of a predefined mutation parameter. This two mechanisms are interpreted as the figure.

Figure 4.2: Interpretation of two GA operators—crossover and mutation.

4.2.2 Major GA Operators

As we mentioned previously, GA bears close resemblance to natural genetics. Its working principles follow the evolutionary process of nature. The traditional GA encodes its variables (genotype) to be operated as bit string. When combined with the time-consuming EM solver, they suffer very slow convergence. To improve the search and optimization ability of GA, many researchers have found the use of real/floating point numbers to be a better genotypic representation than binary method [36],[38]. Through conducted experiments they have found that

1. The floating point genotypic representation is faster, more consistent from run to run and provides higher precision.
2. Floating point representation is intuitively closer to the problem space.
3. For many applications in science and technology, the use of floating point representation would lead to the use of numbers with base-10 format.

Since we have viewed the above advantages indicating that using the floating point method would be more effective in searching for optimum solutions, then how about other operators, such as the crossover, mutation or mating? Here we list these three categories and several types of operators in each category as Table 4.1.

Table 4.1: Overview/summary of GA Operators.

Mating Schemes	Crossover Operators	Mutation Operators
Best-Mate-Worst	Intrapolation	Boundary
Adjacent-Fitness-Pair	Extrapolation	Uniform
Emperor-Selective	Intra-Extrapolation	Non-uniform

4.2.2.1 Mating Operators

Let us examine the mating schemes first. The purpose of the mating operators is to give more chances for reproduction of those individuals which are the fittest. This is because fitter individuals have higher chances of producing better offspring, thereby increasing the overall fitness so that the probability of convergence to reach an optimal solution would be higher. And there are mainly three different mating schemes interpreted as Figure 4.3.

Figure 4.3: Interpretation of different mating schemes[38]

Best-Mate-Worst (BMW) BMW effectively spreads the superior genetic material in the parent population. It is maximally disruptive, but weaker parents with desirable traits do get the chance to produce offspring with stronger parents. In BMW, the parents in the population are first sorted out and listed in descending order according to their fitness value. The best individual gets to mate with the worst individual and the second best with the second worst and so forth. This process reduces the differences in fitness between the best and worst individuals.

Adjacent-Fitness-Pairing (AFP) AFP mates two parents with adjacent fitness values. In AFP, the parents in the population are first sorted out in descending order according to their fitness values. Then the best individual gets to mate with the

adjacent second best individual and the third best mates with the adjacent fourth best and so forth. It is highly conservative of genetic formation but may result in premature convergence. However, AFP ensures the union of strong individuals whose offspring may prove to be fitter than their parents. This operator does not allow any individual to breed twice.

Emperor Selective (EMS) The best individual in the parent population gets to mate with every other even-numbered individual in the population. If one or more near-solutions are added to an initial population of random individuals, EMS usually yields the best chromosome as compared to BMW and AFP. It is the only method that allows the fittest individual to procreate freely with other numbers in the rest of the population. In EMS, the best fitness item is retained, every other even-numbered items are linked with the best fitness individual. As illustrated in this figure, the parent with the best fitness is paired with next best even-numbered individuals namely the second, fourth and sixth individuals.

Roulette Wheel (ROU) This operator, based on proportion of relative fitness, selects parents based on the ratio of the fitness value of a parent to the total fitness value of the parent population. This mating operator can be viewed as allocating pie-shaped slices on a roulette wheel to individuals, with each slice proportional to the individual's fitness. Selection of an individual to be the parent can be viewed as a spin of the wheel, with the winning individual being the one in whose slice the roulette spinner finally rests [4]. Hence, the population with the highest fitness will take up the biggest portion of the wheel and there is a higher probability for them to be selected. The least fit individuals have very little chance of being selected.

Figure 4.4 is an illustration of such mating process. Each individual is allocated a slice with sector angle $= \frac{fitness}{TotalFitness} \times 2\pi$ of a roulette wheel. The roulette wheel is spun and an individual is selected if a random number between 0 and 2π falls in the sector corresponding to the individual. Thus, every two parents thus selected mate together.

Figure 4.4: Illustration of ROU mating scheme.

4.2.2.2 Crossover Operators

Now let us take a look at the crossover operators. As we mentioned previously, crossover occurs when two parents exchange parts of their corresponding chromosomes. It recombines the genetic material in two parent chromosomes to produce two offspring and this would help the genetic algorithm to converge. There are mainly three different types of crossover operators.

Interpolation Crossover (IPX) The Interpolation Crossover operator takes two parents, P1 and P2, to produce two offspring, C1 and C2, that lie between the two parents. The offspring have equal probability to lie on anywhere within the range, α , of the parents, as depicted in Figure 4.5.

Here the range of the parents, Δ , is defined by

$$\Delta = (P2 - P1), \text{ where } P2 > P1.$$

Figure 4.5: IPX operation, showing region where offspring can lie.

And IPX is implemented using the following two equations:

$$\begin{aligned} C1 &= P1 + \alpha * \Delta \\ C2 &= P2 - \alpha * \Delta \end{aligned} \quad (4.1)$$

where α is a randomly chosen number between 0 and 1.

Extrapolation Crossover (EPX) The Extrapolation Crossover takes two parents, P1 and P2, to produce two offspring, C1 and C2, that lie outside the range, α , of the two parents. The offspring have equal probability to lie within the range α , extended in both directions from P1 and P2. C1 will then lie on the same side as P1 and C2 on the same side as P2, as interpreted in Figure 4.6

Here the range, Δ , of the parents is defined by

Figure 4.6: EPX operation showing region where C1,C2 can lie .

$\Delta = (P2 - P1)$, where $P2 > P1$.

EPX is implemented using the following two equations:

$$\begin{aligned} C1 &= P1 + \alpha * \Delta \\ C2 &= P2 - \alpha * \Delta \end{aligned} \quad (4.2)$$

where α is a randomly chosen number between 1 and 2.

Interpolation and Extrapolation Crossover (IEX) The Interpolation and Extrapolation Crossover is a hybrid operator which combines the effects of IPX and EPX. It takes two parents, P1 and P2, to produce two offspring, C1 and C2, which can lie both within and outside the range, A , of the two parents. The offspring have equal probability to lie within the range A , extended in both directions from the mid-point of P1 and P2. C1 will lie on the same side as P1 and C2 on the same side as P2, as shown in the Figure 4.7.

Compared with the crossover operators, mutation normally plays a secondary role in evolution , therefore here mutation operator may be omitted.

4.2.3 Test Functions

In order to determine the best GA operators for our application, it is of course to use our fitness evaluation function (which will be introduced in the next chapter) as

Figure 4.7: IEX operation, showing region where offspring can lie.

the test function. However, the fitness evaluation function is deduced from the EM simulation methods we established and , although are very efficient, still suffer from time-consuming. Therefore we choose a range of standard test functions (De Jong's Functions) to determine the performance of the GA operators. These functions have the following features:

1. They consist exclusively of functions that are scalable with respect to their dimension M , namely the number of unknowns in the problem can be changed when desired;
2. include a unimodal (single peaked), continuous function for comparison of convergence velocity;
3. include a step function with several flat plateau of different heights in order to test the behavior of the algorithm in case of the absence of any local gradient information;
4. cover multimodal (multi-peaked) functions of differing complexity.

In the following section, we list 4 forms of De Jong's functions, which are all used in our test.

4.2.3.1 DeJong's Function 1/Sphere Model

Figure 4.8: DeJong's Function 1 (Sphere Model) with indication of global minima.

$$f_1(x) = \sum_{i=1}^n x_i^2, \quad -5.12 \leq x_i \leq 5.12 \quad (4.3)$$

$$\min f_1 = f_1(0, \dots, 0) = 0$$

Here f_1 is a continuous, smooth and unimodal function. It is symmetrical and GAs should not have any difficulty locating its global minima. The performance of f_1 is a measure of the general efficiency of the genetic algorithm. And the diagram of DeJong 1 Function is as shown in Figure 4.8.

Figure 4.9: DeJong's Function 2 (Rosenbrock's Function) with indication of global minima.

4.2.3.2 DeJong's Function 2/Generalised Rosenbrock's Function

$$f_2(x) = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2), \quad -5.12 \leq x_i \leq 5.12 \quad (4.4)$$

$$\min f_2 = f_2(1, \dots, 1) = 0$$

f_2 is a continuous, unimodal, bi-quadratic function of two variables. It has a very narrow ridge which runs around a parabola. Algorithms which are not able to discover good directions underperform in this problem. Its diagram is shown in Figure 4.9.

4.2.3.3 DeJong's Function 3/Step Function

$$f_3(x) = 6n + \sum_{i=1}^n \text{floor}(x_i), \quad -5.12 \leq x_i \leq 5.12 \quad (4.5)$$

$$\min f_3 = f_3([-5.12, -5), \dots, [-5.12, -5)) = 0$$

Figure 4.10: DeJong's Function 3 (Step Function) with indication of global minima.

f_3 is a simple linear but discontinuous function, which consists of many small plateaus. Due to this characteristic, f_3 has many local optima and is representative of the problem of flat surfaces. Flat surfaces are obstacles for optimization algorithms, because they do not give any information as to which direction is favorable. Unless an algorithm has variable step sizes, it can become trapped in one of the flat plateau surfaces and its diagram is Figure 4.10.

4.2.3.4 DeJong's Function 5/Shekel's Foxholes

The DeJong's function 4 is not able to provide the needed global minimum and would not be of value in further testing work, therefore we skipped function 4 and directly move to function 5.

$$f_5(x) = \frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \frac{1}{i=12(x_i - a_{ij})^6}}, -65.536 \leq x_i \leq 65.536 \quad (4.6)$$

$$\mathbf{min} f_5 = f_5(-32, \dots, -32) = 0$$

Figure 4.11: DeJong's Function 5 (Shekel's Foxholes) with indication of global minima.

f_5 is a continuous, non-linear, multimodal function. It comprises a large plateau with 25 holes, and hence has many (24) local minima, but one global minima. Many standard algorithms become trapped stuck in the first peak they find. Fig 4.11 shows the diagram of Function 5.

Of the range of five DeJong's Functions, four of these Functions (Nos. 1, 2, 3 and 5) can provide the required global minima and would be sufficient and able to provide a definitive guide in further testing regarding the performance of the Toolbox.

4.2.4 Comparison of Major GA Operators

The test runs carried out cover two areas:

- Using mating operators with crossover only.
- Using mating operators with crossover.

For each set of test runs, DeJong's Functions 1, 2, 3 and 5 are used to ascertain the robustness of the various operators in the Toolbox. The different operators will be tested, varying the population size and the number of generations. For each GA operator combination, a number of independent runs (100 Nos.) would be effected, with measurements of the mean and best fitness values.

The varying of population sizes and number of generations are the variables used for testing at moderate, small and large values which are

- a At 100, 10 and 1000 respectively for population size and;
- a at 100, 50 and 500 respectively for number of generations.

Five different settings of population size and number of generations have been tested and these are shown in Table 4.2.

Table 4.2: Different settings of population size and generation number.

Setting	Population(Pop)	Generation Number(Gen)
1	Moderate(100)	Moderate(100)
2	Small(10)	Small(50)
3	Small(10)	Large(500)
4	Large(1000)	Small(50)
5	Large(1000)	Large(500)

By applying the test runs to the range of operators, for each of the three types of GAs, it would be possible to assess the robustness of the operators and determine the GA most suited to handle application problems.

4.2.4.1 Selection of mating operators

The test runs made were for Settings 1, 3 and 4. The mating operators were applied for each crossover operator and the mutation operator was not employed to eliminate the mutation effect and the result is shown in Figure 4.12.

From the Figure 4.12, a brief discussion on these mating operators is presented as following:

1. For moderate to large populations, EMS consistently shows very good performance for all test functions and this is achieved by allowing the fittest individual developed up to that point to procreate freely with other individuals in the population, resulting in faster convergence.
2. For these populations, TOU gives the second best performance also with all test functions. It can give this performance as the mating pool would contain the fitter individuals which survive the tournament process and this would result in faster convergence.
3. For small populations (Setting 3), BMW gives the best performance for all test functions. This is followed by EMS, when generation number is below 200, EMS converges at a faster rate. When generation number exceeds 200, the rate of convergence for EMS slows down but that for BMW remains at a constant rate. At higher generation numbers, the global minimum would be from BMW. By comparison, AFP and ROU generally give poorer performance than the other mating operators, for the various test functions and settings involved.

4.2.4.2 Selection of crossover operators

Test runs on the DeJong's functions with crossover were carried out using each mating operator and also in the absence of mutation so as to eliminate its effect. Different values for population sizes and number of generations were used, as listed in Table 4.2.

The off-line performance measurements of test functions - together with different crossover operators at Settings 1, 3 and 4 - are presented respectively. The overall performance of all crossover operators (on the four test functions) at five different settings, is presented in Figure 4.13.

Figure 4.12: Overall performance of mating operators for DeJong's Functions.

Figure 4.13: Overall performance of crossover operators for DeJong's Functions.

From the results, we can clearly understand that the crossover operators (IEX, EPX, EIR, DER and SBX) demonstrate good performance. They produce offspring which can take a broad range of values. This is achieved by using different forms of recombination of parent values. IEX, which is also based on this concept too, produces offspring that fall within a restricted range. Hence, the performance of the crossover operators depends on the range within which the offspring can lie and the type of operations being performed.

4.2.5 Flowchart of EI-GA Approach

Based on the introduction of GAs in the previous chapter together with the discussion on the selection of GAs' operators, the flow chart of the EI-GA approach is showed as Figure 4.14. Of course, the EM modelling method here we applied is Effective Index method and can be replaced by other methods such as FEM we presented also. From the flowchart, we can know that a generic algorithm starts with an initial

Figure 4.14: Flow chart of the EI/GA optimization algorithm.

set of random configurations and uses a process similar to biological evolution to improve upon them. During each time of iteration process, which is called a generation, the configurations in the current population are evaluated using some measures of

fitness. The offspring are next evaluated, and a new generation is formed by selecting some of parents and offspring and rejecting others so as to keep the population size constant. As the iterative process is carried on, the average fitness of the population keeps improving till termination conditions are satisfied.

4.3 Particle Swarm Optimization (PSO)

4.3.1 Introduction

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling, depicted as Figure 4.15.

Figure 4.15: Swarm intelligence in nature[40].

PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA) [40]. The system is initialized with a population of random

solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. The detailed information will be given in following sections.

Compared to GA, the advantages of PSO are that PSO is easy to implement and there are few parameters to adjust. PSO has been successfully applied in many areas: function optimization, artificial neural network training, fuzzy system control, and other areas where GA can be applied.

4.3.2 The Algorithms

As stated before, PSO simulates the behaviors of bird flocking. Suppose the following scenario: a group of birds are randomly searching food in an area. There is only one piece of food in the area being searched. All the birds do not know where the food is. But they know how far the food is in each iteration. So what's the best strategy to find the food? The effective one is to follow the bird which is nearest to the food.

PSO learned from the scenario and used it to solve the optimization problems. In PSO, each single solution is a "bird" in the search space. We call it "particle". All of particles have fitness values which are evaluated by the fitness function to be optimized, and have velocities which direct the flying of the particles. The particles fly through the problem space by following the current optimum particles [40],[41].

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. In every iteration, each particle is updated by following two "best" values. The first one is the best solution (fitness) it has achieved so far. The fitness value is also stored. This value is called *pbest*. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called *gbest*. When a particle takes part of the population as its topological neighbors, the best

value is a local best and is called lbest.

After finding the two best values, the particle updates its velocity and positions with following equations.

$$\begin{aligned}
 v[i] &= v[i] + c1 * rand() * (pbest[i] - present[i]) \\
 &\quad + c2 * rand() * (gbest[i] - present[i]) \quad (a) \\
 present[i] &= present[i] + v[i] \quad (b)
 \end{aligned}$$

$v[i]$ is the particle velocity, $present[i]$ is the current particle (solution). $pbest[i]$ and $gbest[i]$ are defined as stated before. $rand()$ is a random number between (0,1). $c1, c2$ are learning factors, usually $c1 = c2 = 2$.

Particles' velocities on each dimension are clamped to a maximum velocity V_{max} . If the sum of accelerations would cause the velocity on that dimension to exceed V_{max} , which is a parameter specified by the user. Then the velocity on that dimension is limited to V_{max} .

The pseudo code of the procedure is as follows [40]:

For each particle

 Initialize particle

END

Do

 For each particle

 Calculate fitness value

 If fitness value is better than the best fitness value (pBest) in history

 set current value as the new pBest

 End

 Choose the particle with the best fitness value as the gBest

For each particle

 Calculate particle velocity according equation (a)

Update particle position according equation (b)

End

While maximum iterations or minimum error criteria is not attained

4.3.3 PSO Parameter Control

From the above case, we can learn that there are two key steps when applying PSO to optimization problems: the representation of the solution and the fitness function. One of the advantages of PSO is that PSO take real numbers as particles. It is not like GA, which needs to change to binary encoding, or special genetic operators have to be used. For example, we try to find the solution for $f(x) = x_1^2 + x_2^2 + x_3^2$, the particle can be set as (x_1, x_2, x_3) , and fitness function is $f(x)$. Then we can use the standard procedure to find the optimum. The searching is a repeat process, and the stop criteria are that the maximum iteration number is reached or the minimum error condition is satisfied.

There are not many parameter need to be tuned in PSO. Here is a list of the parameters and their typical values.

- The number of particles: the typical range is 20 - 40. Actually for most of the problems 10 particles is large enough to get good results. For some difficult or special problems, one can try 100 or 200 particles as well.
- Dimension of particles: It is determined by the problem to be optimized.
- Range of particles: It is also determined by the problem to be optimized, you can specify different ranges for different dimension of particles.
- V_{max} : it determines the maximum change one particle can take during one iteration. Usually we set the range of the particle as the V_{max} for example, the particle (x_1, x_2, x_3) X_1 belongs $[-10, 10]$, then $V_{max} = 20$

- Learning factors: c_1 and c_2 usually equal to 2. However, other settings were also used in different papers. But usually c_1 equals to c_2 and ranges from $[0, 4]$
- The stop condition: the maximum number of iterations the PSO execute and the minimum error requirement. This stop condition depends on the problem to be optimized.

4.3.4 Comparisons Between GA and PSO

In this part we do a brief comparison between this two stochastic based searching techniques. Let us first take a look at the common points of these two techniques, which are also shared by most of evolutionary techniques:

1. Random generation of an initial population.
2. Reckoning of a fitness value for each subject. It will directly depend on the distance to the optimum.
3. Reproduction of the population based on fitness values.
4. If requirements are met, then stop. Otherwise go back to 2.

From the procedure, we can learn that PSO shares many common points with GA. Both algorithms start with a group of a randomly generated population, both have fitness values to evaluate the population. Both update the population and search for the optimum with random techniques. Both systems do not guarantee success.

However, PSO does not have genetic operators like crossover and mutation. Particles update themselves with the internal velocity. They also have memory, which is important to the algorithm [41].

Compared with genetic algorithms (GAs), the information sharing mechanism in PSO is significantly different. In GAs, chromosomes share information with each other. So the whole population moves like a one group towards an optimal area.

In PSO, only *gBest* (or *lBest*) gives out the information to others. It is a one-way information sharing mechanism. The evolution only looks for the best solution. Compared with GA, all the particles tend to converge to the best solution quickly even in the local version in most cases.

Chapter 5

Dispersion Control for Photonic Crystal Fibres Design

In the previous chapters, we have already presented the two modelling methods (SFM-EI and FEM), as well as two optimization algorithms (GA and PSO), respectively. In this chapter, we will combine these techniques for automated optimal design of PCFs.

5.1 Dispersion in Optical Fibres

We first study the optical parameter, dispersion, in PCF design. An optical signal becomes increasingly distorted as it travels along a fibre. This distortion is a consequence of many factors, one of the main factors is **intramodal dispersion**. These distortion effects can be explained by examining the behavior of the group velocities of the guided modes, where the group velocity is the speed at which energy in particular mode travels along the fibre. There are two main causes of intramodal dispersion: one is *chromatic dispersion*, it mainly arises from material (for example, silica in PCF) and the other is *waveguide dispersion*, which associated with the optical power propagating in the cladding region. Compared with the material dispersion

arising from the material composed of the fibre, the waveguide dispersion would be effected significantly by the geometry structure. Therefore we will focus on the waveguide dispersion parameters and assume the refractive index of the material is independent of wavelength, in other words, the refractive index n is not a function of wavelength λ .

5.1.1 Step Index Fibres.

For waveguide dispersion in single mode fibres, normalized propagation constant b may be introduced to describe the group delay in the fibre and defined as [42]:

$$b = 1 - \left(\frac{ua}{V}\right)^2 = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2} \quad (5.1)$$

where normalized frequency V can be expressed as [42]:

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi a}{\lambda}NA \quad (5.2)$$

(5.1) may be approximated as [42]:

$$b \approx \frac{\beta/k - n_2}{n_1 - n_2} \quad (5.3)$$

Therefore β may be expressed as:

$$\beta \approx n_2 k(b\Delta + 1) \quad (5.4)$$

where $A = (n_1 - n_2)/n_1$. From (5.4), we can obtain the group delay arising from waveguide dispersion τ_{wg} as following [42]:

$$\tau_{wg} = \frac{L}{c} \frac{d\beta}{dk} = \frac{L}{c} \left[n_2 + n_2 \Delta \frac{d(kb)}{dk} \right] \quad (5.5)$$

After a series of calculation in [42], we finally yields the parameters to express the waveguide dispersion $D_{wg}(\lambda)$ [42]:

$$D_{wg}(\lambda) = -\frac{n_2 \Delta}{c} \frac{1}{\lambda} \left[V \frac{d^2(Vb)}{dV^2} \right] \quad (5.6)$$

Figure 5.1: Relationship between effective mode index and normalized frequency in step index fibre.

Figure 5.1 shows an example of the relationship between the Effective mode index (β/k) versus normalized frequency V . Figure 5.2 describes the relationship between the group delay of waveguide dispersion in step-index fibre, including normalized propagation constant b , $d(Vb)/dV$, $Vd^2(Vb)/dV^2$ versus the normalized frequency V . Please note here the parameters are all obtained from the SFM-EI method in this study. Both of the results are obtained under the following parameters in SI fibres: The refractive index of the core $n_1 = 1.452$, the refractive index of the cladding $n_2 = 1.447$, while the radius of the core region is $a = 5 \text{ pm}$, and wavelength range from $1.3 \text{ }\mu\text{m}$ to $5.0 \text{ }\mu\text{m}$.

Compared with the results in optical fibre theory textbooks [1],[42], they agree very well. This, once again, proved the validity and accuracy of the method and computing codes.

Figure 5.2: Mode propagation constant and dispersion parameters in single mode fibres.

5.1.2 Photonic Crystal Fibres

As to the definition of waveguide dispersion in PCF, there is somehow different from the single mode fibre. According to the definition in [27], the group velocity dispersion or, simply the dispersion parameter $D(\lambda)$ of PCF can be directly calculated from the modal effective index $n_{eff}(\lambda)$ of the fundamental mode over a range of wavelength

$$D(\lambda) = -\frac{\lambda d^2 n_{eff}(\lambda)}{cd\lambda^2} \quad (5.7)$$

where the effective refractive index of the mode is $n_{eff} = \beta[\lambda, n_m(\lambda)]/k_0$, β is the propagation constant, $k_0 = 2\pi/\lambda$ is the free-space wave number and $n_m(\lambda)$ is the chromatic dispersion of the material (silica in this PCF case). In first order approxi-

mation the real dispersion $D(\lambda)$ can be written as :

$$D(\lambda) \approx D_g(\lambda) + D_m(\lambda) \quad (5.8)$$

where D_g , the dispersion due to the fibre geometry, is given by:

$$D_g(\lambda) = -\frac{\lambda}{c} \frac{d^2 n_g}{d\lambda^2} \quad (5.9)$$

n_g is the modal refractive index evaluated by consideration of the fact that the material refractive index is wavelength independent, $n = \beta(\lambda)/k_0$, and

$$D_m(\lambda) = -\frac{\lambda}{c} \frac{d^2 n_m}{d\lambda^2} \quad (5.10)$$

is the material dispersion.

Since we consider air-silica PCFs, the chromatic dispersion of silica is a constant of the problem, and consequently this assumption fixes the value of the material dispersion. All our design procedure is based on the efficient control of the geometrical dispersion [27].

Figure 5.3: Diagram of PCF's geometry structure to be optimized.

5.1.3 Designing Objective with Its Applications

Basically, our objective is to develop an accurate and efficient modelling tool for automated optimal design of PCFs. In the engineer application, one can find the fibre geometry structure parameters correspond to the desired optical properties, which could be found among the various applications of photonic crystal fibres described in chapter 2. As we know, controllability of dispersion in PCFs is a very important problem for practical applications to optical communication systems, dispersion compensation and nonlinear optics. PCF can be used to combine with conventional fibres or other optical telecommunication devices to realize effective dispersion management [9]. In order to show the validity and power of automated tool we developed, an example is presented. The designing aim is to obtain the PCF geometry structure so as to have a as flat as possible dispersion curve in optical telecommunication wavelength window, which is often desirable in WDM applications [4]. Furthermore, if obtaining a window of flattened positive dispersion centered around 800 nm and high nonlinear property, the stabilization of ultrashort soliton pulses could be achieved, which is widely used in the supercontinuum generation, or supercontinuum light source [9].

5.2 Fitness Evaluation

As we mentioned previously, our design aim is to obtain the PCF geometry structure so as to have a as flat as possible dispersion curve, and in this example we focus on the telecommunication wavelength window of optical fibre communication system: $1.3\ \mu m - 1.6\ \mu m$.

According to the conventional optical waveguide theory studied previously, we may adopt the second order of the mode effective index n_{eff} to denote the waveguide dispersion introduced above ranging from $1.3\ \mu m - 1.6\ \mu m$ with 150 sample wavelengths and then calculate the standard deviation across these 150 wavelengths, as

depicted in (5.11):

$$F = \left(\frac{1}{150-1} \sum_{i=1}^{150} (D_i - \bar{D})^2 \right)^{1/2} \quad (5.11)$$

where F represents the fitness value, D_i the dispersion value at the i_{th} wavelength and $\bar{D}$ the average dispersion value over the entire range,

$$\bar{D} = \frac{1}{150} \sum_{i=1}^{150} D_i \quad (5.12)$$

A PCF geometry structure to be optimized is the triangular lattice which has been presented in the Chapter 2 and is depicted as Figure 5.3, here the radius of each air hole is the same.

Figure 5.4: Dispersion curve of different PCFs' structures[4].

Here we also calculated the fitness values at different radius and then compared with the results in publications [4]. We adopted the SFM-EI method to analyze the dispersion curve of this triangular PCF with relatively simpler structure. Please note

Figure 5.5: Fitness level with one PCFs' geometry parameter.

that there is only one parameters to be optimized, which is the radius of the air holes while the distance between the air holes, the parameter Λ , is fixed to $2.3 \mu m$.

We investigated 50 different d/Λ values ranging from 0.28 to 0.35, corresponding to the value range in [4]. In other words, we calculated 50 fitness values with the radius of air-holes ranging from $0.322 \mu m$ to $0.4025 \mu m$.

Figure 5.4 depicts the result of Group Velocity Dispersion (GVD) curve of PCFs in [4], which is obtained from localized function method instead of SFM-EI method we applied here.

The fitness values against d/Λ are shown in Figure 5.5, where the lowest fitness level at $\Lambda = 0.32$ corresponds to the flattest dispersion curve in Figure 5.4, showing the agreement between these two methods.

Please note that results in Figure 5.5 is obtained by fixing the Λ value and varies other parameters. Here rises the problem, what may happen if we change the Λ value and the radius of the air holes simultaneously? To examine this, we also calculate the fitness profile with both radius a and the pitch value Λ , depicted in Figure 5.6

and Figure 5.7. In 2-D Figure 5.6 the optimal solution, which has the smallest fitness

Figure 5.6: 2-D display of fitness profile of two PCPs' geometry parameters.

level, corresponds to the dark color, while in 3-D Figure 5.7, the optimal solution corresponds to the peak point. The fitness profile is obtained by computing 100 x 100 points by the iteration method with A from $2.25 \mu m$ to $2.35 \mu m$ and the radius value from $0.322 \mu m$ to $0.4025 \mu m$ with equal steps, respectively. This method although is quite straight-forward, but it is very time-consuming (the computation time is about 50000 sec.) and the complexity of the fitness profile shows that the effect will not be good if we use (refined) grid searching. Here the best fitness value (the smallest one) we found is $F \approx -24.28$ dB. To achieve better results, we should turn to an optimization tool. In the next section, we combine SFM-EI and GA for optimal design experiment.

Figure 5.7: 3-D display of fitness profile of two PCFs' geometry parameters.

5.3 Chromosome Construction

Before applying GA to PCF dispersion control, it is necessary to understand the construction of the chromosomes. Unlike most GA applications using binary coding and binary genetic operations, the proposed implementation adopts direct expression of original decimal parameters, which is, a chromosome is just a real number or complex number vector associated with the model. For our PCF optimal design, the structure of a chromosome for the fibre design optimization is just the number, location, the size of the air holes and the distance between the holes. Therefore, for one parameter optimization (for example, radius of the air holes), the chromosome structure should be this:

$$c = (a_1 \ a_2 \ \cdots \ a_i \ \cdots \ a_N) \quad (5.13)$$

while for two parameters (for example, radius and pitch),

$$c = \begin{pmatrix} a_1 & \Lambda_1 \\ a_2 & \Lambda_2 \\ \vdots & \vdots \\ a_i & \Lambda_i \\ \vdots & \vdots \\ a_N & \Lambda_N \end{pmatrix} \quad (5.14)$$

where N is the number of the air holes. In this way, no binary coding and decoding are needed so that the computation complexity and time can be reduced. Based on our study, direct real decimal number coding is more effective than binary coding.

5.4 Optimization Results

5.4.1 Genetic Algorithms

For selection of GAs operators, we take the reference of comparison studies in Chapter 4. As to the mating operators, without doubt, for moderate to large populations, EMS consistently shows very good performance for all test functions. It is quite strait-forward because EMS is the only method that allows the fittest chromosome to procreate freely with the rest of the population. In other words, EMS mating scheme allows the fittest individual to procreate freely with individuals of characteristics, resulting in a greater diversity and faster convergence. Therefore in our application we choose EMS as the mating operator. As to the crossover operator, from the results we know that almost all the crossover operators except IPX can demonstrate good performance. And also, IEX can produce offspring that fall within a restricted range. As a result, the performance of the crossover operators depends on the range within which the offspring can lie and the type of operations being performed. Since in our application, it is necessary to produce the offspring falling within a restricted range,

IEX is a suitable crossover operator.

With EMS mating operator, IEX crossover operator and population size of 60, we applied the established automatic design tool and obtained the results shown in Figure 5.8. From the figure we can see that the fitness level reduced significantly and efficiently.

Figure 5.8: Optimization process by selected GA operators.

And we also listed the optimal results by GA as follows:

$$RES_{GA} = \begin{cases} a = 3.871879297012695e - 001 & (\mu m.) \\ \Lambda = 2.319393916368060e + 000 & (\mu m.) \\ F \approx -26.45 \text{ dB} \end{cases} \quad (5.15)$$

where RES_{GA} represents that this result is obtained by GA, a is the radius of the air holes, Λ stands for the distance between the air holes and F means fitness level.

5.4.2 Particle Swarm Optimization

As mentioned in Chapter 4, PSO shares some similarities with GAs, such as the definition of the fitness evaluation and the construction of chromosome. However, unlike using such biological based operators as mating and crossover to update chromosomes, PSO updates particles (chromosome) using scaled group movement based on gBest (or lBest). Therefore the evolution in PSO only looks for the best solution. Compared with GA, all the particles tend to converge to the better solution faster.

As to the parameters setup of PSO, we listed in the following Table 5.1

Table 5.1: PSO parameter

Swarmsize	20
Maximum iteration number	500
V_{max} (maximum change one particle can take during one iteration)	100
Learning factor	$c1 = c2 = 2$

And the PSO optimization result is:

$$RES_{PSO} = \begin{cases} a = 3.874714319273340e - 001(\mu m) \\ \Lambda = 2.320855320111557e + 000(\mu m) \\ F \approx -27.95 \text{ dB} \end{cases} \quad (5.16)$$

where RES_{PSO} represents this optimal results is obtained by PSO, a is the radius of the air holes, Λ stands for the distance between the air holes and F means fitness level.

In this example, comparing (5.15) and (5.16), PSO is slightly better than GA. More importantly, the computation time by PSO is less than GA. According to our records, the computation time by GA is about 24018.360 sec while the computation time by PSO is about 17172.430 sec, almost 70 percent of GA's time. (Hardware setup: Intel Pentium IV 2.8 GHz (HT) CPU, 512MB RAM)

Chapter 6

Conclusions and Future Recommendations

6.1 Conclusions

Photonic Crystal Fibres (PCFs), also called micro-structured fibres, was founded by Russell and his colleagues in late 1990's. PCFs have attracted much attention due to their unusual optical properties, such as extra-large dispersion, small or large mode area, wide range single mode operations.

The objective of this research is to develop an accurate and efficient modelling tool for automated optimal design of PCFs. Different PCF structures would lead to different optical characteristics, in the engineering application, the design aim of PCFs is to find the fibre geometry structure parameters correspond to the desired optical characteristics.

For EM modelling of PCFs, the large index contrast between air and silica, together with the complex structure of PCFs does not allow for the direct use of methods from traditional optical waveguide theory. Especially for the novel PCFs operated by the PBG effect, the full vectorial nature of the EM waves has to be taken into account. In this thesis, we have studied the space filling mode (SFM)- Effective Index

(EI) method and the finite element method (FEM).

The principle idea of SFM-EI is, first to evaluate the effective cladding index of the cladding region based on the space filling mode in the cladding, then to treat PCF as conventional step index fibre. The SFM-EI is a semi-analytical method and it is efficient with good accuracy, however, it can only treat repeated lattice structures which guide light by the so-called modified total internal reflection (TIR).

The principle of FEM is to divide an entire continuous domain into a number of sub-domains in which the unknown functions are replaced by simple interpolation functions with unknown coefficients. In the FEM formulation, the hybrid edge/nodal elements have been adopted, where the edge and nodal elements are for the transverse and longitudinal parts of the EM field, respectively. The FEM is a full vectorial numerical tool which can handle complicated geometries. Generally speaking, FEM is accurate, flexible and quite efficient, however, it is still very time-consuming when used for optimal design where hundreds or thousands of individual solutions are needed.

For optimization, to achieve better results for PCFs optimal design, we have studied two stochastic-based optimization algorithms, namely genetic algorithms (GAs) and particle swarm optimization (PSO).

GAs mimic the concept of biological genetics and natural evolution. Natural evolution is a search procession for the fittest in the species space, hereby GAs are capable of arriving at an optimal solution. The foundation of GAs is to simulate a generation of a random group of individuals and to track their progress as they evolve toward an optimal solution. To determine the performance of major GA operators, different combinations of GA operators are tested and results are compared. EMS and IEX are selected as the mating scheme and crossover operator for the GA application, respectively.

PSO is inspired by social behavior and bird flocking or fish schooling. Both GAs and PSO are population based stochastic optimization techniques. They share some

similarities such as random generation of an initial population and reproduction of the population based on fitness level, etc. However, in PSO the particles fly through the problem space by following the current optimum particle.

Either the SFM-EI or the FEM can be selected as the EM solver to be combined with an optimization engine which could be either the GA or the PSO to form an automated tool for optimal design of PCFs. A comprehensive example of dispersion control for a PCF design is conducted and analyzed. In this example, SFM-EI is combined with GA and PSO respectively for optimal design experiment to obtain PCF structure so as to have a dispersion curve as flat as possible and the results by GA and PSO are compared briefly.

6.2 Future Recommendations

Despite the results we have achieved, there still have some issues to be completed. First of all, as mentioned previously, SFM-EI method can only handle PCFs with relatively simple cladding structures, for example the repeated lattice structure. It is very difficult to use SFM-EI method to analyze such optical properties as bending loss and attenuation caused by complicated geometry imperfection. We may have to use more flexible 2D and 3D FEM to solve some of the above mentioned problems. And also, nonlinearity could be analyzed by trivial iteration of changing linear models.

Secondly, our research just focuses on the establishment of algorithms which will ultimately developed a fully automated optimization technique for the design of photonic crystal fibres. From literature and our experience, also in the results given in chapter 5, PSO seems more efficient than GA for some applications. However, this does not mean PSO is more efficient than GA for all problems. There would not be a clear general answer and it depends on problems and choice of parameters of PSO and GA. Anyway, some future researches are needed. The more flexible 2D and 3D FEM which can handle complicated geometry structures are necessary for conducting

such extensive and complete researches.

Thirdly, after an accurate and efficient modelling tool for automated optimal design of PCFs is established, other PCF structure parameters, such as size and position of air holes, or even the shape and number of air holes can be optimized in PCFs optimal design. It provides directions for designing and manufacturing novel PCFs.

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