Lightwave propagation in microstructured optical fibers

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Lightwave Propagation in Microstructured Optical Fibers

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

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Declaration

I declare that this thesis is my own work and has not been submitted in any form for another degree or diploma at any university or other institute of tertiary education. Information derived from the published and unpublished work of others has been acknowledged in the text and a list of references is given.

Yan Min

July 16, 2005

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Yan Min

Nanyang Technological University, 2005

Abstract

The first optical fiber with an array of air cylinders running along its axis was fabricated by Knight *et al.* in 1996. It has been a new era for fiber optics ever since. Such fibers can be broadly referred to as microstructured optical fibers (MOFs). Many properties of such fiber, such as dispersion, birefringence, nonlinearity and bending loss etc, can be tailored beyond what we can obtain in a conventional step-index fiber. Later experiments further achieved air-guiding with a periodic air-silica cladding. Such guidance in certain ways requires us to re-define what an optical fiber is.

Owing to the structural complexity, accurate and efficient theoretical methods are in great demand for analyzing modes propagating in such MOFs. In the first part of this thesis, several commonly used theoretical methods for studying modal properties of MOFs are presented. Though most of them are mentioned in recent publications, they have not been discussed in a consistent and rigorous manner. The merits and demerits of each method are pointed out. Possible improvements are suggested. Applications of these methods to challenging MOF problems are also addressed. In the second part, novel fiber designs are presented. The properties of the proposed fibers are theoretically predicted using mode solvers introduced in the first part. It will be shown that new MOF designs can perform significantly better than existing ones (especially those guide light with the photonic bandgap, or PBG effect). For example, a modified core will be introduced to a PBG fiber with honeycomb photonic crystal (HPC) cladding. The modified core helps to increase the PBG-guiding wavelength range (over 1000nm) and to keep the fiber single-mode. Also, air-guiding HPC fiber is for the first time demonstrated. Our modelling tells that such fiber eases the surface-mode problem. Triangular photonic crystal (TPC) used for conventional air-guiding MOF design is also generalized. And a hollow-core fiber with an improved TPC cladding is shown to be able to achieve un-disturbed PBG guidance over 350nm wavelength range. The thesis also includes some of our fabrication attempts for the air-guiding HPC fiber.

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While I was starting my project, some established researchers in the field have helped me in one way or another. Daniel Herrmann from Rsoft Design Group has given me much help as I was dealing with waveguide problems using BeamPROP and also later BandSOLVE softwares. Dr. Steven Johnson is also acknowledged for his contribution of the MIT Photonic-Bands (MPB) software package which I initially used for photonic band calculations. His answers to many questions posted on the *gmane.comp.science.photonic-bands* newsgroup are also enjoyable and beneficial to read. As I found commercial softwares are not enough for my research purposes, I started to write my own computer codes. During this period, Shanping Guo from the Old Dominion University, USA, and Henri P. Uranus from the University of Twente, the Netherlands have helped in one way or another regarding waveguide mode solvers. I should also thank my fiends in the Nanyang Technological University who were working on a similar project. They include Wang Xiaoyan, Lou Junjun, Yu Xia and Tee Chingh Wee. Guillaume Vienne from the Chinese University of Hong Kong is also acknowledged for stimulating discussions on Bragg fibers.

Certainly I should not forget the fellow researchers in NTRC. Without them (the name list is just too long to be included here), I can't imagine how boring a day would be! In addition, I need to mention people I have worked with during my four-month attachment in OFTC, University of Sydney, Australia. Especially, Dr John Canning and Dr Martijn van Eijkelenborg are acknowledged for their investments in the fabrication of the air-guiding honeycomb photonic bandgap fiber. Three experimentalists — Katja Lyytikainen, Brian Aston, and Richard Lwin — should be thanked for their involvement in the fabrications. Experiment aside, Nader Issa has introduced a lot of things to me about Australia, besides his FDM²-ABC program. It was also very enlightening to have discussions with Mr. Alexander Argyros and Dr. Ian Basset.

At last I should thank my parents for their understanding and encouragement. Support from my brother, Yan Bing, is also indispensable for the completion of this thesis.

Yan Min

Nanyang Technological University

July 2005

Preface

This thesis is a faithful record, almost in temporal order, of work I carried out during my three-year PhD study. The objective of this thesis is two fold — to fulfill requirements by the school as a PhD candidate, and more importantly, to introduce microstructured optical fibers (MOFs) as well as numerical methods for analyzing complex optical waveguides to those who are new in this field.

Part I of this thesis summarizes several state-of-the-art numerical methods for deriving modes in an optical waveguide. Though numerically solving Maxwell equations (especially the source-free wave equation) has been tackled for decades, the first appearance of MOF still fascinates and puzzles theoreticians. It was not long before classic mode solvers like plane wave method, finite difference method, finite element method, and boundary element method etc were adapted for MOFs, though some of the adaption processes are not painless. In addition, some specialized methods were also developed to particularly deal with MOFs. One of them is the multipole expansion method. All these methods, being classic or newly formulated, are able to solve MOF modes in a full-vector dependence, and their agreement appears to be very good. However, during my PhD study I have found that few of the methods are recorded in a friendly manner, and their presentations are not all flawless. Hence, I decide to include the formulations in the thesis. Detailed derivations are shown for most of the methods, from the initial wave equation to the final matrix equation. Readers should be able to write their own programs immediately after reading the thesis.

Having said that, Part I is organized as follows. In Chapter 2, the transfer matrix method (TMM) is presented. TMM is a method specially for solving modes in so-called Bragg fibers. Such a fiber's cladding is made of a stack of concentric dielectric layers. This version of TMM is similar to the method proposed by Yeh *et al.*, but yet it is improved in boundary condition as well as field representation basis in each layer. An extended TMM is also introduced which can deal with "Bragg" fibers with non-circular cladding layers. In Chapter 3, the newly proposed multipole expansion method (MEM) by White *et al.* is presented. This method treats MOFs with only circular inclusions. MEM uses Bessel and Hankel functions to represent fields. The number of basis functions involved is small, hence it can be quite efficient and accurate. In Chapter 4, the finite element method (FEM) is presented. Whereas TMM and MEM can be regarded as semi-numerical methods owing to their deployment of analytical functions, FEM is one of the most beautiful true-numerical methods. It is so general that it can almost be applied to waveguides with any transverse index profile. The advantage of FEM over some other numerical methods lies in

its ability in handling curved dielectric interfaces since it uses a triangular mesh. Radiation boundary condition is deployed to take leakage loss into consideration. In Chapter 5, the plane wave method (PWM) is presented. Different from the conventional PWM method, it calculates the propagation constant from a given wavelength. This method is very suitable for calculating modes in an infinitely extending 2D photonic crystal. The modes should have a wavevector component along the direction in which the structure is invariant. The supercell technique can be used to calculate defect modes, i.e., the core modes propagating in an MOF. In Chapter 6, the finite difference method is presented. This method solves the Maxwell equations using the well-known Yee's mesh. Perfectly matched layers are deployed to absorb all outgoing waves.

Part II mainly focuses on novel MOF designs. This part consists of four chapters. In the first chapter, Chapter 7, we give an overall introduction to MOFs. Its definition, classification, history and applications are recounted. It is hoped that this chapter gives adequate information to readers new to the topic. In Chapter 8, index-guiding MOFs are discussed. In particular, in the first section of the chapter, air holes are used in a multi-clad fiber to flatten dispersion curves; in the second section, antiguiding in a normal index-guiding MOF is theoretically demonstrated and their implications are discussed. In Chapter 9, MOFs whose guidance is achieved by the cladding photonic bandgap (PBG) are presented. Such PBG fibers are presented in two categories — those with an air core and those with a composite core. In the first category, two types of cladding photonic crystals (PCs), namely the triangular photonic crystal (TPC) and the honeycomb photonic crystal (HPC), are proposed and analyzed. Both of these PCs show their promising capability to confine light in a hollow-core. The recently fabricated air-silica Bragg fiber is also theoretically investigated. In the second category of PBG fibers, the honeycomb photonic bandgap fiber with a modified composite core design is presented. Its performance is demonstrated to be better than its traditional counterpart as proposed by Broeng *et al.* At this point, the concept of having a composite core in an MOF's cladding is generalized, and MOFs with both its core and cladding made of PCs are proposed. Such fibers are named heterostructured photonic crystal fibers (HSPCFs). HSPCFs are expected to have application in nonlinear optics, sensing etc. In Chapter 10, several fabrication attempts, using both glass and polymer, are presented.

The thesis finishes with Chapter 11, in which two parts of the thesis are concluded, followed by our proposal of future work.

As this thesis is written within a limited time period, its content as well as its organization probably have various defects. I hereby appreciate generous feedbacks. You can email your comments to myan@ieee.org.

Part I

NUMERICAL METHODS

Chapter 1

Introduction to Optical Waveguide Analysis

1.1 Introduction

Electromagnetic (EM) problems can be grouped into two categories: deterministic problems and eigenvalue problems. In a deterministic problem, one is interested in the EM field evolution with respect to time or/and space. A source (initial condition) is always present in such problems. Mathematically, such a problem results from “either an inhomogeneous differential equation or inhomogeneous boundary conditions or both” [1]. Eigenvalue problems are source-free problems. That is, there isn’t any electric or magnetic charge existing. Mathematically, such a problem results from “a homogeneous governing differential equation and homogeneous boundary conditions” [1]. Modes resonating in an optical cavity and those propagating in an optical waveguide are usually solved as eigenvalue problems. It should be noted that most eigenvalue problems can also be solved as a deterministic problem, followed by a Fourier analysis to locate resonant states (modes). But such a formulation is usually time-consuming and the result is highly dependent on how the source is positioned.

Before the emergence of microstructured optical fibers (MOFs), waveguides like conventional step-index fiber (SIF) and slab waveguide usually have simple geometries and their mode fields can be represented using known functions, *e.g.*, trigonometric and Bessel functions, etc. Due to this reason, we can derive the modes of such waveguides analytically. Furthermore, as most of these waveguides are formed by low index-contrast materials, the weakly-guiding (or scalar) approximation can be used, which further simplifies the derivation. However, modelling a MOF does not share the same advantages we had with the conventional waveguides. First, due to the fact that MOFs are composed by high index-contrast materials (normally air and silica¹), analysis of MOFs requires a full-vector formulation. Second, as the cross-section of an MOF is complex, no single analytical expression is available to represent its mode field. Instead, we have to resort

¹Some people ignore the air and hence claim the MOFs are of single-material. This is correct in the sense that the air can be vacuumed without affecting the fiber’s properties. However, we don’t encourage such definition as MOFs can be fabricated with two solid materials.

to numerical methods. Third, as all modes in a MOF are leaky, we need to take care of the radiation wave. In other words, we have to implement certain boundary condition to avoid reflection off the termination line of the problem domain.

Indeed, we did see a revolution in optical waveguide mode solvers after the emergence of MOF. Within a few years, various numerical methods, either newly formulated or adapted from some existing methods, were reported for MOF analysis. They are either specially targeted for MOF, like the *multipole expansion method* (MEM) [2, 3, 4] and *localized function method* (LFM) [5, 6], or general methods which can be tailored for MOF problems, like the *finite element method* (FEM) [7, 8, 9, 10], *beam propagation method* (BPM)[11], *boundary element method* (BEM) [12, 13, 14, 15], *finite difference method* (FDM) [16, 17, 18, 19, 20], *plane wave method* [21, 22, 23, 24, 25], and a hybrid finite difference method which discretizes radially using finite difference but angularly using Fourier decomposition (FDM²-ABC) [26, 27, 28], etc. It should be noticed that all the methods are derived from the Maxwell equations, with full considerations of the vector nature of EM wave. Therefore, for a same waveguide problem, the above-mentioned methods will converge to an identical solution when the numerical resolution (either spacial or spectral) used is high enough.

In Part I of the thesis, we would like to summarize several versatile methods for MOF analysis, namely MEM, FEM, PWM and FDM. MEM is a semi-analytical method that has very high accuracy. PWM is quite efficient for periodic structure analysis as it reduces the problem domain to a single unit cell. FEM is great to deal with complex structures with curved interface lines, and it can reduce significantly the number of unknowns with the help of modern adaptive meshing algorithm. FDM is the most straightforward method to solve the Maxwell equations, and it is simple to implement and use.

Apart from “holey” MOFs, Bragg fiber is another type of MOF which makes use of a cylindrical 1D photonic crystal to confine lightwaves. Bragg fibers usually have a very small feature size (sometimes only tens of nanometers), hence they are difficult to tackle by any numerical method. The *transfer matrix method* (TMM) was proposed by Yeh *et al.* in 1978 to analysis such types of fibers [29]. Later, this method was improved by Johnson *et al.* to allow leaky mode analysis [30]. As TMM is a near-analytical method, it is quite efficient in terms of CPU and memory usage. TMM (with a slightly different formulation) will also be introduced in Part I. Owing to its simpleness, it will be placed ahead of other methods.

Most of the methods to be presented start from vector wave equations, which are deduced from Maxwell equations. Therefore in the rest of this chapter, we will show how the wave equations in various forms can be derived, followed by a brief discussion on coordinate system selection.

1.2 Basic Equations

1.2.1 Maxwell Equations

Interaction of EM wave and matters is governed by macroscopic Maxwell equations²

$$\nabla \cdot \mathbf{B} = 0, \quad (1.1)$$

$$\nabla \cdot \mathbf{D} = \rho, \quad (1.2)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad (1.3)$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}, \quad (1.4)$$

where $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic fields, respectively; $\mathbf{D}$ and $\mathbf{B}$ are displacement and magnetic induction fields, respectively; ρ and $\mathbf{J}$ are free charge density and current density, respectively. In a linear medium $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$, where $\epsilon = \epsilon_r \epsilon_0$ and $\mu = \mu_r \mu_0$. ϵ_0 and μ_0 are the electric permittivity and magnetic permeability of vacuum, respectively. And ϵ_r and μ_r are the relative electric permittivity and relative magnetic permeability of a material, respectively. For source-free waveguide problem, $\rho = 0$ and $\mathbf{J} = 0$. Also we assume all materials are impermeable, *i.e.*, $\mu_r = 1$ is valid for all materials. Notice that the fields $\mathbf{B}$, $\mathbf{D}$, $\mathbf{E}$ and $\mathbf{H}$ are functions of space $\mathbf{r}$ and time t .

For time-harmonic problems, the electric and magnetic fields can be written as³

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{i\omega t}, \quad (1.5)$$

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r})e^{i\omega t}. \quad (1.6)$$

These expressions are also called phasor expressions of EM fields. We should bear in mind that only the real parts of the expressions are physical [32]. After substituting these two equations into Eqs. 1.1-1.4, we arrive at

$$\nabla \cdot \mathbf{H}(\mathbf{r}) = 0, \quad (1.7)$$

$$\nabla \cdot [\epsilon \mathbf{E}(\mathbf{r})] = 0, \quad (1.8)$$

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu_0 \mathbf{H}(\mathbf{r}), \quad (1.9)$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\epsilon \mathbf{E}(\mathbf{r}). \quad (1.10)$$

²In this thesis, *nabla* sign in the gradient and Laplacian operators is written in normal weight as ∇ , whereas in the curl and divergence operators it is written in blackboard bold as ∇ .

³Maxwell equations are *time reversal invariant*. That is, it makes no difference if we replace t with $-t$ in Eq. 1.5 and 1.6. However, the validity of this time reversal invariance is still in argument. Interested readers can refer to [31] and references therein for further information.

Figure 1.1: A general z -invariant waveguide.

1.2.2 Wave Equations

The four Maxwell equations are first-order differential equations with two inter-related variables $\mathbf{E}$ and $\mathbf{H}$. We can further combine them into one second-order differential equation in terms of either $\mathbf{E}$ or $\mathbf{H}$.

To do that, take curl of Eq. 1.9, and combine with Eq. 1.8, we have

$$\nabla \times \nabla \times \mathbf{E} = \epsilon_r \left(\frac{\omega}{c} \right)^2 \mathbf{E}, \quad (1.11)$$

where we have used the equality $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$. Equation 1.11 is a source-free vector wave equation in terms of the electric field. Similarly we can derive that for the magnetic field as

$$\nabla \times \nabla \times \mathbf{H} = \frac{\nabla \epsilon_r}{\epsilon_r} \times \nabla \times \mathbf{H} + \left(\frac{\omega}{c} \right)^2 \epsilon_r \mathbf{H}. \quad (1.12)$$

Note here $\mathbf{E}$, $\mathbf{H}$ and ϵ_r are simplified expressions for $\mathbf{E}(\mathbf{r})$, $\mathbf{H}(\mathbf{r})$ and $\epsilon_r(\mathbf{r})$, respectively.

Using the vector identity

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}, \quad (1.13)$$

we can further express the two wave equations in 1.11 and 1.12 as

$$\nabla^2 \mathbf{E} + \nabla \left(\frac{\nabla \epsilon_r}{\epsilon_r} \cdot \mathbf{E} \right) + k^2 \mathbf{E} = 0, \quad (1.14)$$

and

$$\nabla^2 \mathbf{H} + \frac{\nabla \epsilon_r}{\epsilon_r} \times (\nabla \times \mathbf{H}) + k^2 \mathbf{H} = 0, \quad (1.15)$$

where $k = k_0 n$. n is the refractive index of the medium and $n^2 = \epsilon_r$. k_0 is the free space wave number and $k_0 = \frac{\omega}{c}$. These two wave equations are valid for all source-free EM problems. But for structures that exhibit certain symmetry property, they can be further simplified. For example, optical fibers have infinite translational symmetry along its axial direction⁴. In such regular waveguides, a *forward-propagating* mode can be represented as

$$\mathbf{E}(x, y, z) = \mathbf{E}(x, y) \exp(j\beta z), \quad (1.16)$$

⁴Refer to Fig. 1.1, we name a z -invariant waveguide as a *regular waveguide*.

$$\mathbf{H}(x, y, z) = \mathbf{H}(x, y)\exp(j\beta z), \quad (1.17)$$

where β is the propagation constant and we define

$$n_{\text{eff}} = \frac{\beta}{k_0}. \quad (1.18)$$

n_{eff} is called the effective index of a mode. For a bounded mode, β , and in turn n_{eff} , is real. Whereas for a leaky mode they are complex, with the imaginary parts denoting the radiation loss.

Substituting Eq. 1.16 into the wave equation 1.14 will decompose the wave equation into two equations. They are

$$(\nabla_t^2 + k^2 - \beta^2)\mathbf{E}_t = -\nabla_t(\mathbf{E}_t \cdot \nabla_t \ln \epsilon_r), \quad (1.19)$$

$$(\nabla_t^2 + k^2 - \beta^2)E_z = -j\beta\mathbf{E}_t \cdot \nabla_t \ln \epsilon_r. \quad (1.20)$$

And similarly we have two wave equations for the magnetic field as

$$(\nabla_t^2 + k^2 - \beta^2)\mathbf{H}_t = (\nabla_t \times \mathbf{H}_t) \times \nabla_t \ln \epsilon_r, \quad (1.21)$$

$$(\nabla_t^2 + k^2 - \beta^2)H_z = (\nabla_t H_z - j\beta\mathbf{H}_t) \cdot \nabla_t \ln \epsilon_r. \quad (1.22)$$

The transverse fields $\mathbf{E}_t$ and $\mathbf{H}_t$ are defined as

$$\mathbf{E}_t = E_x \hat{x} + E_y \hat{y}, \quad (1.23)$$

$$\mathbf{H}_t = H_x \hat{x} + H_y \hat{y}. \quad (1.24)$$

where $\hat{x}$ and $\hat{y}$ are unit vectors along the x and y directions, respectively. E_x , E_y , E_z , H_x , H_y , H_z and ϵ_r in Eq. 1.19-1.24 are functions of x and y . That is, we have reduced a 3D problem into a 2D problem.

We notice that in wave equations 1.20 and 1.22, both the transverse and the longitudinal field components are involved, whereas in Eqs. 1.19 and 1.21 only the transverse field is involved. Thus most numerical vector mode solvers use the wave equation shown in either Eq. 1.19 or 1.21, since this would result in one third less in the number of unknowns. Equations 1.19 and 1.21 can be rearranged into a standard eigen-problem as

$$\mathcal{L}\mathbf{F} = \beta^2\mathbf{F}, \quad (1.25)$$

where the operator $\mathcal{L}$ has

$$\mathcal{L} = \nabla_t^2 + k^2 + \nabla_t(\nabla_t \ln \epsilon_r \cdot) \quad (1.26)$$

as $\mathbf{F} = \mathbf{E}_t$. And

$$\mathcal{L} = \nabla_t^2 + k^2 + \nabla_t \ln \epsilon_r \times (\nabla_t \times) \quad (1.27)$$

as $\mathbf{F} = \mathbf{H}_t$.

However, for a certain numerical method, like FEM, there is a risk in using wave equation 1.19 or 1.21. If we look carefully how Eq. 1.19 is derived, we notice that one of the Maxwell equations (Eq. 1.7) is not used at all. Due to this, there is a possibility that some artificial solutions, or so-called *spurious modes*, exist. Such modes can be easily identified by examining their field distributions.

β or n_{eff} is one of the key parameters that characterize a propagating mode. They determine the fiber's basic properties like birefringence, group velocity dispersion (GVD), and radiation loss etc. Besides β , the field distribution is another piece of information we need to know about a mode. These two are calculated as an eigenvalue and eigenvector pair in an eigen-problem formulation.

Before we move on, it should be mentioned that the magnetic vector wave equation 1.12 can be put into another form as

$$\nabla \times \left(\frac{1}{\epsilon_r} \nabla \times \mathbf{H} \right) = k_0^2 \mathbf{H}. \quad (1.28)$$

This equation is directly solved in the well-known PWM for calculating the bands of photonic crystals (PCs) [22, 33].

1.2.3 Scalar Wave Equation

For a conventional single mode fiber, the refractive index contrast between core and cladding is typically less than one percent. For such waveguides, spacial differentiation of ϵ_r function is close to zero⁵. That is, the right-hand term in the wave equations 1.19-1.22 can be dropped. This is the so-called weakly-guiding approximation, or scalar approximation. After taking this approximation, the four wave equations degenerate to a common wave equation, *i.e.*,

$$(\nabla_t^2 + k^2 - \beta^2)F = 0, \quad (1.29)$$

where F denotes either E_x , E_y , E_z , H_x , H_y or H_z .

Equation 1.29 is the so-called the Helmholtz equation, or scalar wave equation. The scalar wave equation further reduces the number of unknowns by half as compared to Eq. 1.19 and 1.21. It should be noticed that an EM wave propagating in a homogeneous material satisfies Helmholtz equation automatically.

⁵When a dielectric system is not homogeneous (piecewise for example), under the influence of an electric field, hypothetical electric charge exists at material discontinuities [34]. This can be derived from the Maxwell equation 1.8, assuming $\rho = 0$, as

$$\nabla \cdot (\epsilon \mathbf{E}) = \nabla \epsilon \cdot \mathbf{E} + \epsilon \nabla \cdot \mathbf{E} = 0.$$

Hence we have

$$\nabla \cdot \mathbf{E} = -\frac{1}{\epsilon} \nabla \epsilon \cdot \mathbf{E}.$$

The right-hand side can be treated as electric source. The value of this source is negligible when index contrast is small.

1.2.4 Interface Condition

Being aware of the field behavior across an interface between two impermeable dielectric materials helps to understand certain mode behavior in an optical waveguide. Full derivations of the interface conditions can be found in [32]. Here we summarize them as

- Magnetic field is continuous everywhere, regardless of its polarization, *i.e.*

$$H_1 = H_2. \quad (1.30)$$

- Tangential component of the electric fields is continuous, *i.e.*

$$E_t^1 = E_t^2. \quad (1.31)$$

- Normal component of the electric induction field is continuous, *i.e.*

$$D_n^1 = D_n^2, \quad (1.32)$$

or

$$\epsilon_1 E_n^1 = \epsilon_2 E_n^2. \quad (1.33)$$

1.3 Coordinate System

The vector wave equations in Subsection 1.2.2 are written in terms of the transverse and the longitudinal field components. We have deliberately expressed the transverse fields in Cartesian coordinate system in Eqs. 1.23 and 1.24. However, it should be noticed that they can be similarly expressed in cylindrical coordinate system as

$$\mathbf{E}_t = E_r \hat{r} + E_\theta \hat{\theta}, \quad (1.34)$$

$$\mathbf{H}_t = H_r \hat{r} + H_\theta \hat{\theta}, \quad (1.35)$$

where $\hat{r}$ and $\hat{\theta}$ are unit vectors along the radial and azimuthal directions, respectively.

Depending on the physical problem, a proper choice of the coordinate system can simplify the mode derivation process significantly. For fibers exhibiting infinite rotational symmetry, *e.g.* conventional SIF, a cylindrical coordinate system is definitely preferred. As in such case, the fiber appear as a 1D structure radially. The recently proposed MEM has demonstrated that cylindrical coordinate system is also quite efficient for those MOFs with only circular inclusions. This is due to the fact that modes supported in such fibers can be treated as a superposition of cylindrical waves (Bessel functions) originated from dielectric interfaces. And the number of Bessel functions is relatively small when the dielectric interfaces are all of circular shape. The cylindrical coordinate system is also employed in Nader's FDM²-ABC program [28]. A very attractive advantage for formulation in cylindrical coordinate system is that rotational symmetry can be easily

exploited, since the Bloch theorem can be naturally imposed in the azimuthal direction [35, 28]. However, in the FDM²-ABC formulation, the problem domain is not sampled uniformly. That is, the region near the origin is over-sampled, whereas the region far away from the origin is coarsely sampled. In fact, sometimes we prefer a uniform sampling, which is especially true for modelling a photonic bandgap fiber, where every cladding unit contributes equally to the forbidden photonic bandgap (PBG).

Compared to the cylindrical coordinate system, the Cartesian coordinate system is more commonly used in numerical methods. These methods include FDM, FEM, and BEM etc. However, so far only reflection symmetry is exploited in such formulations. Rotational symmetry is difficult to be incorporated. This is certainly a big waste. FDM seems to have an obvious difficulty to include rotational symmetry condition. But FEM and BEM codes should be able to incorporate the rotational symmetry, with some added complexity.

Chapter 2

Transfer Matrix Method

2.1 Introduction

The concept of Bragg fiber was first proposed by Yeh *et al.* [29] in 1978. A schematic diagram of a Bragg fiber cross-section is shown in Fig. 2.1. The cladding of a Bragg fiber is normally formed by alternating layers of two different dielectric materials. Generally speaking, lightwave confinement in a Bragg fiber is achieved by constructive reflections at concentric dielectric interfaces. Successful Bragg fiber fabrications have been reported by Fink *et al.* [36], Brechet *et al.* [37], Temelkuran *et al.* [38] and Katagiri *et al.* [39]. Depending on their composing materials and designs, such fibers can exhibit high-power handling capability [38], low bending loss [36], great dispersion tunability [40], etc. Very recently, hollow-core Bragg fibers working in the near infrared wavelength range have been fabricated by the “Photonic Bandgap Fibers & Devices Group” in MIT [41]. The air-silica “Bragg fiber” appeared in [42] has a pseudo-1D photonic crystal cladding, in which the concentric silica layers are connected by nanoscale bridges. Such imitated Bragg fiber will not be discussed in this chapter. However we will turn our attention to this special type of air-guiding fiber in Chapter 9.

Theoretical methods for analyzing modal properties of Bragg fibers can be found in [29, 43, 30, 44, 45]. In [29], Yeh *et al.* introduced a transfer matrix method (TMM). In their formulation, the mode field in each dielectric layer is expressed using Bessel functions J and Y . This method is ideal for Bragg fibers with an infinite number of cladding layers. For fibers with a finite number of cladding layers, the method treats propagating modes as quasi-modes, which are found by minimizing their radiation losses. The loss minimization process is generally complicated, especially for modes which are not purely transverse-electric (TE) or transverse-magnetic (TM). In [43], asymptotic approximations of Bessel functions are used for the field representation in outer cladding layers. This approximation actually simplifies the circular stack of alternating dielectric layers to a planar one. Hence fields in those outer layers can be separated into independent TE and TM components. The Bloch theorem can be further applied if the stack has a periodic refractive index profile. In order to give more accurate results, more and more cladding layers are required to be classified into an *effective* core region. But in doing so, the method loses its advantage in efficiency. Guo *et al.* has noticed that exact field matching

Figure 2.1: Schematic cross-section of a Bragg fiber. Core (white region) has refractive index n_0 . Light-grey layers have refractive index n_1 . Dark-grey layers have refractive index n_2 . n_1 can be equal to n_0 . Outermost layer extends infinitely far, with refractive index n_1 or n_2 .

for this method is not possible at the interface between the core region and the cladding region [45]. Also they have found a negative imaginary part for the effective modal index (n_{eff}), which might be caused by the unphysical formulation of the problem. In [30], Johnson *et al.* treat the inner layers the same way as in [29]. For the outermost cladding layer, the mode field is represented by Hankel functions $H^{(1)}$ and $H^{(2)}$, corresponding to outward-travelling and inward-travelling cylindrical waves, respectively. By enforcing a zero inward-travelling wave in the outermost layer, the leaky mode can be identified, which has a complex propagation constant β , and in turn a complex modal n_{eff} . The imaginary part of the n_{eff} value denotes the loss rate of the mode. This method makes no approximation, hence it gives the exact solution. In [44], Guo *et al.* expand the global mode field by using a series of Laguerre-Gauss functions. The expansion is not that efficient, and it needs a large number of such orthogonal functions for a waveguide of complex radial refractive index profile (e.g., Bragg fiber). Also when this method is applied to derive modes propagating in a leaky Bragg fiber, an imaginary cladding, whose index is smaller than that of the innermost core, is assumed. This assumption not only disqualifies the method from calculating the leakage loss, but also will introduce certain amount of error to the real part of the propagation constant.

Other than these analytical or semi-analytical methods, numerical methods like the beam propagation method (BPM) [37], the 2D finite-difference time-domain (FDTD) [43] and the finite element method (FEM) [10, 46] can also be used for deriving modal properties of Bragg fibers. In general, numerical methods are expensive in CPU and memory consumption, especially when the accuracy requirement is high.

In this chapter, we present a TMM method similar to that in [29] and [30] for deriving propagating modes in Bragg fibers. The mode field in each single layer is represented by

the Bessel function of the first kind J and the Hankel function of the first kind $H^{(1)}$. Such a field representation for leaky mode analysis has also appeared in [2]. The method in [30] treats the outermost layer differently from the inner layers, hence the relaying matrix for the last interface is different from those for the rest interfaces. Our TMM treats every layer indifferently, hence the transfer matrix for each interface can be computed in exactly the same manner.

In the first section, we give basic formulation for our TMM. Some simulation results are given to confirm the correctness of our formulation. In the second section, the TMM is extended for analyzing perturbed Bragg fibers whose cladding interfaces are circular but not concentric. Though actual Bragg fiber fabricated can deviate in a much more complex manner than what we present here, our result can be treated as a preliminary perturbation study of an ideal Bragg fiber. The validity of the extended TMM is assured by two full-vector numerical methods. As the cladding layers in a Bragg fiber are very thin, high accuracy can't be easily assured with a numerical method unless huge memory and a powerful processor are provided. Our semi-analytical formulation provides a neat and simpler way to tackle such problems.

2.2 TMM for Ideal Bragg Fibers

2.2.1 Theory

From Chapter 1, we know that the longitudinal electric field E_z and the longitudinal magnetic field H_z satisfy the scalar Helmholtz equation in each homogeneous material region. That is, in the i^{th} dielectric layer, the E_z field satisfies

$$[\nabla_t^2 + k_{t,i}^2]E_z = 0, \quad (2.1)$$

$k_{t,i} = \sqrt{k^2 n_i^2 - \beta^2}$ is the transverse wave number, and n_i is the refractive index, both in the i^{th} dielectric layer. $k = 2\pi/\lambda$ is the wave number in vacuum. $\beta = kn_{\text{eff}}$. H_z satisfies exactly the same equation. In the cylindrical coordinate system, Eq. 2.1 can be written as

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \theta^2} + k_{t,i}^2 E_z = 0. \quad (2.2)$$

If the cladding layers are concentric, the waveguide has continuous rotational symmetry. This suggests the E_z field can be factorized as

$$E_z = R_z(r)\Theta_z(\theta), \quad (2.3)$$

where $R_z(r)$ depends on r only and $\Theta_z(\theta)$ depends on θ only. Substitute Eq. 2.3 into Eq. 2.2, we find that the wave equation can be separated into two independent equations as [47]

$$\frac{\partial^2 \Theta_z}{\partial \theta^2} + m^2 \Theta_z = 0, \quad (2.4)$$

$$\frac{\partial^2 R_z}{\partial r^2} + \frac{1}{r} \frac{\partial R_z}{\partial r} + (k_{t,i}^2 - \frac{m^2}{r^2}) R_z = 0. \quad (2.5)$$

From Eq. 2.4, we arrive at the conclusion that the local mode field in i^{th} layer should have a $\cos(m\theta)$ or $\sin(m\theta)$ azimuthal dependence, where m is the azimuthal quantum number. Continuity of fields requires that m must be an integer. Equation 2.5 is the well-known Bessel differential equation, and its generalized solution can be expressed in terms of a set of transcendental functions (Bessel functions) [48]. Due to the variable separation, m can be fixed prior to the derivation of $R_z(r)$ from Eq. 2.5. If now we consider $(i+1)^{th}$ layer, similar equations as 2.4 and 2.5 can also be obtained. The only way to have E_z field be continuous across the interface at all azimuthal locations is to let the m value in the $(i+1)^{th}$ layer be the same as that in the i^{th} layer. The result of this analysis tells us local mode fields in all dielectric layers share the same azimuthal quantum number m .

The general solution for the E_z field in a homogeneous i^{th} layer can be expressed as

$$E_z = [A_m^{Ei} J_m(k_{t,i}r) + B_m^{Ei} H_m(k_{t,i}r)] \exp(im\theta). \quad (2.6)$$

And the longitudinal magnetic field can be similarly written as

$$K_z = [A_m^{Ki} J_m(k_{t,i}r) + B_m^{Ki} H_m(k_{t,i}r)] \exp(im\theta). \quad (2.7)$$

Notice in Eq. 2.7, we have used the normalized magnetic field $K_z = \mathcal{Z}H_z$, where $\mathcal{Z} = \sqrt{\mu_0/\epsilon_0}$ is the free space impedance. In this way, we can let the magnetic field have roughly the same amplitude as that for the electric field. In Eqs. 2.6 and 2.7, the J_m (Bessel function of the first kind of order m) term represents a *standing* wave that originates from a source (dielectric interface) outside the region of consideration; H_m (in short for $H_m^{(1)}$, or the Hankel function of the first kind of order m) term represents an *outward-travelling* cylindrical wave, which originates from a source (dielectric interface) contained within the region of consideration. Considering our Bragg fiber problem, the outermost layer extends infinitely, *i.e.*, there's no interface beyond that region. Hence the J_m term does not appear there. Also, the innermost layer does not contain any dielectric interface, hence the H_m term does not exist there. As will be shown later, these two boundary conditions will be utilized to establish a characteristic matrix equation for solving the modes. Coefficients A_m^{Ei} , B_m^{Ei} , A_m^{Ki} , B_m^{Ki} in Eq. 2.6 and 2.7, as well as β , which is contained in $k_{t,i}$ are the unknowns to be deduced, subjected to input wavelength λ , index profile and angular mode number m . Notice β here is in general a complex number.

E_θ and K_θ fields can be derived easily from E_z and K_z using

$$E_\theta = \frac{i}{k_{t,i}^2} \left(\frac{\beta}{r} \frac{\partial E_z}{\partial \theta} - k \frac{K_z}{\partial r} \right), \quad (2.8)$$

$$K_\theta = \frac{i}{k_{t,i}^2} \left(\frac{\beta}{r} \frac{\partial K_z}{\partial \theta} + k n_i^2 \frac{E_z}{\partial r} \right). \quad (2.9)$$

Figure 2.2: A general interface r_1 between layer 1 and layer 2.

Figure 2.2 shows a general interface between two dielectric layers. Across the boundary, we should have these four field continuity equations

$$K_z^1 = K_z^2, \quad (2.10)$$

$$E_\theta^1 = E_\theta^2, \quad (2.11)$$

$$E_z^1 = E_z^2, \quad (2.12)$$

$$K_\theta^1 = K_\theta^2. \quad (2.13)$$

Superscripts in Eqs. 2.10-2.13 denotes layer number.

After putting Eqs. 2.6-3.49 into Eqs. 2.10-2.13, we have the matrix equation

$$\mathbf{M}_1 \begin{bmatrix} A_m^{E1} \\ B_m^{E1} \\ A_m^{K1} \\ B_m^{K1} \end{bmatrix} = \mathbf{M}_2 \begin{bmatrix} A_m^{E2} \\ B_m^{E2} \\ A_m^{K2} \\ B_m^{K2} \end{bmatrix}, \quad (2.14)$$

where

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 0 & J_m^1 & H_m^1 \\ -\frac{m\beta}{k_{t,1}^2 r} J_m^1 & -\frac{m\beta}{k_{t,1}^2 r} H_m^1 & -\frac{ik}{k_{t,1}} J_m^1 & -\frac{ik}{k_{t,1}} H_m^1 \\ J_m^1 & H_m^1 & 0 & 0 \\ \frac{ikn_1^2}{k_{t,1}} J_m^1 & \frac{ikn_1^2}{k_{t,1}} H_m^1 & -\frac{m\beta}{k_{t,1}^2 r} J_m^1 & -\frac{m\beta}{k_{t,1}^2 r} H_m^1 \end{bmatrix}. \quad (2.15)$$

In Eq. 2.15, J_m^1 is in short for $J_m(k_{t,1}r)$. And other similar terms should be comprehended in the same way. $\mathbf{M}_2$ has the same elements as those in $\mathbf{M}_1$, except that n_1 , $k_{t,1}$ and superscript 1 should be accordingly changed to n_2 , $k_{t,2}$, and 2. Notice when $m = 0$, i.e., when the modal field has no azimuthal dependence, Eq. 2.14 can be decomposed into two matrix equations: one relates $[A_m^K, B_m^K]^T$ field components (TE fields), and the other relates $[A_m^E, B_m^E]^T$ field components (TM fields).

From Eq. 2.14, the field in layer 1 can be related to field in layer 2, across the interface r_1 , using

$$\begin{bmatrix} A_m^{E1} \\ A_m^{K1} \\ B_m^{E1} \\ B_m^{K1} \end{bmatrix} = \mathbf{M}_{r_1} \begin{bmatrix} A_m^{E2} \\ A_m^{K2} \\ B_m^{E2} \\ B_m^{K2} \end{bmatrix}, \quad (2.16)$$

where $\mathbf{M}_{r_1} = \mathbf{M}_1^{-1} \mathbf{M}_2$ is the transfer matrix.

Now we turn our attention to an N -layered Bragg fiber. Assume the fiber's layers are numbered from the innermost core layer to the outermost layer as 1, 2, ..., N , and the interfaces are referred to as $r_1, r_1, \dots, r_{(N-1)}$ outwards. After applying the relation given by Eq. 2.16 iteratively for all interfaces, we can finally arrive at

$$\begin{bmatrix} A_m^{E1} \\ B_m^{E1} \\ A_m^{K1} \\ B_m^{K1} \end{bmatrix} = \mathbf{M}_{r_1} \mathbf{M}_{r_2} \cdots \mathbf{M}_{r_{(N-1)}} \begin{bmatrix} A_m^{EN} \\ B_m^{EN} \\ A_m^{KN} \\ B_m^{KN} \end{bmatrix}. \quad (2.17)$$

We can let $\mathbf{M} = \mathbf{M}_{r_1} \mathbf{M}_{r_2} \cdots \mathbf{M}_{r_{(N-1)}}$ and assume $\mathbf{M}$ has the following elements

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}. \quad (2.18)$$

Since there is no outward-travelling wave in the innermost dielectric layer, B_m^{E1} and B_m^{K1} should be zero. Also since no standing wave exists in the outermost dielectric layer, A_m^{EN} and A_m^{KN} should be zero. With these boundary conditions in mind, Eq. 2.17 can be reduced to

$$\mathbf{M}_0 \begin{bmatrix} B_m^{EN} \\ B_m^{KN} \end{bmatrix} = 0, \quad (2.19)$$

where

$$\mathbf{M}_0 = \begin{bmatrix} m_{22} & m_{24} \\ m_{42} & m_{44} \end{bmatrix}. \quad (2.20)$$

Notice the matrix elements of $\mathbf{M}_0$ (and also $\mathbf{M}$) are functions of β . Since Eq. 2.19 has a nontrivial solution only when the determinant of $\mathbf{M}_0$ goes to zero, a β solution can be found by searching for the root of function $\mathcal{F}(\beta) = \det(\mathbf{M}_0)$. After getting the β value, we can substitute it back to Eq. 2.19 to get the field values in the outmost layer. Fields in the inner layers can be solved iteratively using Eq. 2.16.

2.2.2 Simulation Results

In this subsection, we give some simulation results for a particular Bragg fiber. The fiber is made of three materials, with (referring to Fig. 2.1 and its caption) $n_0 = 1.0$, $n_1 = 1.5$, and $n_2 = 3.0$. The optimum cladding layer thicknesses can be estimated using a simple approach as stated in [30]. The basic idea is to predict the effective transverse wavelength

Figure 2.3: Radial index profile for the Bragg fiber under consideration.

$\lambda_{t,i}$ in i^{th} layer using

$$k_{t,i}^2 = k_i^2 - k_z^2, \quad (2.21)$$

where $k_{t,i} = 2\pi/\lambda_{t,i}$, $k_i = 2\pi/\lambda \cdot n_i$, and $k_z = 2\pi/\lambda \cdot n_{\text{eff}}$. n_{eff} is the modal effective index. The targeted operation wavelength is $1.55\mu\text{m}$. Since most of the mode energy resides inside the core, n_{eff} should have a value less than but very close to n_0 . This is especially true for a Bragg fiber with a relatively large core size compared to the wavelength. In fact, we can just use $n_{\text{eff}} = n_0 = 1.0$ for a fairly accurate prediction. After getting $\lambda_{t,i}$, the optimum layer thicknesses l_i are computed as

$$l_i = \frac{\lambda_{t,i}}{4} + s \frac{\lambda_{t,i}}{2}, \quad s = 0, 1, 2, \dots \quad (2.22)$$

$s = 0$ corresponds to the fundamental Bragg condition. Other s values correspond to higher-order Bragg reflection conditions.

The lateral photonic bandgap (PBG) due to the fundamental Bragg reflection is the largest (the corresponding cladding bandgap is named primary bandgap). However, to make use of this primary bandgap to confine light at near infrared wavelength, the cladding layer widths are too small ($\sim 0.1\mu\text{m}$). The small feature sizes do not pose any problem for our theoretical calculation. But such Bragg fibers may be difficult to fabricate. Here, we use the second-order Bragg reflection condition for both layers, *i.e.*, $s = 1$, to achieve lightwave confinement. The layer thickness estimated for n_1 material is $l_1 = 1.0\mu\text{m}$, and for n_2 material $l_2 = 0.4\mu\text{m}$. Core radius is arbitrarily chosen as $7\mu\text{m}$. Ten pairs of cladding layers are used. The radial index profile of the fiber is shown in Fig. 2.3.

The concentric cladding stack can be approximated as a planar one. In Fig. 2.4 we show the lateral PBG regions possessed by the cladding, calculated using the plane wave method (PWM) [23]. Notice that the TM gap regions are “immersed” inside the TE gap regions. As predicted, the air line ($n_{\text{eff}} = 1.0$) crosses the second largest gap region around $\lambda = 1.55\mu\text{m}$.

Figure 2.4: Red regions in the background are TE PBG regions found for the Bragg fiber's periodic cladding. Green regions in the foreground are TM PBG regions. Air line is shown as the dash-dotted line.

For the selected core size, the Bragg fiber is multimode. Figure 2.5 gives the modal n_{eff} curves for TM_{01} and TE_{01} modes. The imaginary part of n_{eff} value is related to the mode's loss rate α (in dB/km) through

$$\alpha = \frac{2\pi}{\lambda} \frac{20}{\ln(10)} 10^9 \Im(n_{\text{eff}}), \quad (2.23)$$

with wavelength in micrometers. We observe from Fig. 2.5 that the loss rate for the TM_{01} and TE_{01} modes are in accord with bandgap calculation, which confirms the accuracy of the method. The TE mode has a smaller loss rate since the cladding periodic stack reflects stronger for TE light. Though not plotted in Fig. 2.5, the HE_{11} and EH_{11} modes have loss rates comparable to that of the TM_{01} mode, as they both consist of TM component. Confinement beyond the PBG regions is due to antiguiding between the innermost core and outermost cladding layer. The TE_{01} mode has its smallest leakage loss around $\lambda = 1.50\mu\text{m}$. At this wavelength, the loss difference between TM_{01} and TE_{01} is over 5 orders (64096dB/km and 0.19dB/km, respectively). Effectively speaking, the Bragg fiber can act as a single-mode fiber (SMF). Unlike a conventional index-guiding SMF, such a single-mode Bragg fiber doesn't have any polarization preference, therefore the polarization mode dispersion (PMD) can be avoided.

At $\lambda = 1.50\mu\text{m}$, the fields for TE_{01} and TM_{01} modes are shown in Fig. 2.6 and Fig. 2.7, respectively. The mode fields presented are exactly the same as the results computed using the TMM method mentioned in [30] (not shown). A larger portion of TM_{01} modal energy, as compared to TE_{01} mode, resides in the cladding composite. This gives rise to a more dispersive $\Re(n_{\text{eff}})$ curve for the TM_{01} mode [Fig. 2.5(a)].

Figure 2.5: Computed modal n_{eff} curves for TM_{01} and TE_{01} modes. The real part is shown in (a), and the imaginary part is shown in (b). TE and TM PBG regions computed in Fig. 2.4 are also shown in (a).

Figure 2.6: Computed TE_{01} mode fields at $\lambda = 1.50 \mu\text{m}$, with modal $n_{\text{eff}} = 0.99143265 + 5.12849 \times 10^{-12}i$.

Figure 2.7: Computed TM_{01} mode fields at $\lambda = 1.50\mu\text{m}$, with modal $n_{\text{eff}} = 0.99690296 + 1.76174 \times 10^{-6}i$. E_r field is not continuous since the field is not tangential to the dielectric interfaces.

Figure 2.8: A schematic Bragg-like fiber. The fiber's cladding is formed by a multilayer of dielectric materials. The dielectric interfaces are circular, but not concentric.

2.3 Extended TMM for Perturbed Bragg Fibers

2.3.1 Theory

The method mentioned in the previous section can be extended for analyzing multilayered leaky optical fibers, whose cladding interfaces are circular, but not concentric. A schematic diagram for such a fiber's cross-section is shown in Fig. 2.8.

Since the continuous rotational symmetry is lost for such a fiber, the wave equation 2.2 cannot be separated into two equations. Hence, unlike the case for ideal Bragg fibers, we cannot fix an azimuthal quantum number m prior to the radial field calculation. In other words, for a particular mode, the local mode field in each dielectric layer cannot be represented by some Bessel/Hankel function of the same order. There exists coupling between Bessel functions of different orders at each dielectric interface.

Since the dielectric interfaces now are not concentric, each dielectric layer i has its own local coordinate system whose origin coincides with the interface r_i 's center. We

Figure 2.9: Two general nonconcentric interfaces in a Bragg-like fiber.

choose the innermost layer (core)'s coordinate system as the global coordinate system. The outermost layer will share the same coordinate system with the innermost layer. A generalized solution for mode field in i^{th} layer can be expressed, in the i^{th} layer's own coordinate (r_i, θ_i) , as

$$E_z = \sum_m [A_m^{E,i} J_m(k_{t,i} r_i) + B_m^{E,i} H_m(k_{t,i} r_i)] \exp(im\theta_i), \quad (2.24)$$

$$K_z = \sum_m [A_m^{K,i} J_m(k_{t,i} r_i) + B_m^{K,i} H_m(k_{t,i} r_i)] \exp(im\theta_i), \quad (2.25)$$

where m is an integer in the range of $-\infty$ to ∞ . The field now is a Fourier-Bessel series, which has infinite number of terms. In the actual calculation, we truncate the series into a finite number of terms (from $-n$ to n).

Figure 2.9 depicts two general nonconcentric interfaces in a perturbed Bragg fiber. As the fields in two adjacent layers are expanded in two different coordinate systems, they are not term-by-term equatable at their conjunct interface. But such term-by-term equation can be performed after we transform the field in layer 2 into layer 1's coordinate system. The transformation uses the Graf's addition theorems [2].

For the E_z field in layer 2, each J (or H) term expressed in layer 1's coordinate system can be written as a series of J (or H) terms expressed in layer 2's coordinate system as

$$A_{\hat{m}}^{E2,1} J_{\hat{m}}(k_{t,2} |\mathbf{r}_1|) \exp(i\hat{m}\theta_1) = \sum_m \mathbf{J}_{12}(\hat{m}, m) A_m^{E2,2} J_m(k_{t,2} |\mathbf{r}_2|) \exp(im\theta_2), \quad (2.26)$$

and

$$B_{\hat{m}}^{E2,1} H_{\hat{m}}(k_{t,2} |\mathbf{r}_1|) \exp(i\hat{m}\theta_1) = \sum_m \mathbf{J}_{12}(\hat{m}, m) B_m^{E2,2} H_m(k_{t,2} |\mathbf{r}_2|) \exp(im\theta_2), \quad (2.27)$$

where $\hat{m}$ can be any integer from $-\infty$ to ∞ . $\mathbf{J}_{12}(\hat{m}, m)$ is the coordinate projection matrix, whose elements are calculated as [2]

$$\mathbf{J}_{12}(\hat{m}, m) = (-1)^{\hat{m}-m} J_{\hat{m}-m}(k_{t,2} |\mathbf{r}_{12}|) \exp[-i(\hat{m}-m)\arg(\mathbf{r}_{12})], \quad (2.28)$$

where $\mathbf{r}_{12}$ is a vector pointing from the center of interface r_1 to the center of interface r_2 (Fig. 2.9).

Like what we did for m , $\tilde{m}$ is given an finite integer value from $-n$ to n in order to make the problem solvable. For standing wave terms, we then have $2n+1$ equations in the form of (Eq. 2.26). Putting these $2n+1$ equations together, we have this matrix equation

$$\begin{bmatrix} A_{-n}^{E2,1} \\ \vdots \\ A_n^{E2,1} \end{bmatrix} = \mathbf{J}^{12} \begin{bmatrix} A_{-n}^{E2,2} \\ \vdots \\ A_n^{E2,2} \end{bmatrix}, \quad (2.29)$$

and a similar matrix equation can be derived for outgoing field terms as

$$\begin{bmatrix} B_{-n}^{E2,1} \\ \vdots \\ B_n^{E2,1} \end{bmatrix} = \mathbf{J}^{12} \begin{bmatrix} B_{-n}^{E2,2} \\ \vdots \\ B_n^{E2,2} \end{bmatrix}. \quad (2.30)$$

K_z field components in each layer can be transformed from one coordinate system to another in exactly the same way.

Fields in two adjacent layers across interface r_1 can then be related as

$$\begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{13} & \mathbf{M}_{14} \\ \mathbf{M}_{21} & \mathbf{M}_{22} & \mathbf{M}_{23} & \mathbf{M}_{24} \\ \mathbf{M}_{31} & \mathbf{M}_{32} & \mathbf{M}_{33} & \mathbf{M}_{34} \\ \mathbf{M}_{41} & \mathbf{M}_{42} & \mathbf{M}_{43} & \mathbf{M}_{44} \end{bmatrix} \begin{bmatrix} \mathbf{J}_{12} & 0 & 0 & 0 \\ 0 & \mathbf{J}_{12} & 0 & 0 \\ 0 & 0 & \mathbf{J}_{12} & 0 \\ 0 & 0 & 0 & \mathbf{J}_{12} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{E2,2} \\ \mathbf{B}^{E2,2} \\ \mathbf{A}^{K2,2} \\ \mathbf{B}^{K2,2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{E1,1} \\ \mathbf{B}^{E1,1} \\ \mathbf{A}^{K1,1} \\ \mathbf{B}^{K1,1} \end{bmatrix}, \quad (2.31)$$

where some representative submatrices/subvectors are defined as

$$\mathbf{M}_{11} = \begin{bmatrix} m_{11}^{-n} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & m_{11}^n \end{bmatrix}, \quad (2.32)$$

$$\mathbf{A}^{E2,2} = \begin{bmatrix} A_{-n}^{E2,2} \\ \vdots \\ A_n^{E2,2} \end{bmatrix}. \quad (2.33)$$

Derivation of matrix elements like m_{11} in Eq. (2.32) is the same as the derivation of matrix elements for $\mathbf{M}_{r-1}$ in Eq. (2.16). Equation (2.31) can be used iteratively to relate the field components in the innermost layer to the field components in the outermost layer for an N -layered Bragg fiber. Finally, if we consider the boundary conditions of $\mathbf{B}^{E1,1} = \mathbf{B}^{K1,1} = 0$ and $\mathbf{A}^{EN,N} = \mathbf{A}^{KN,N} = 0$, we can deduce a matrix equation as

$$\mathbf{M}_0 \begin{bmatrix} \mathbf{B}^{EN,N} \\ \mathbf{B}^{KN,N} \end{bmatrix} = 0, \quad (2.34)$$

where matrix $\mathbf{M}_0$ has the dimension of $2(2n+1)$. This equation can be solved in exactly the same way as what we did for the Bragg fiber to get the modal n_{eff} , and in turn, mode field components $\mathbf{A}^{Ei,i}$, $\mathbf{B}^{Ei,i}$, $\mathbf{A}^{Ki,i}$, $\mathbf{B}^{Ki,i}$.

2.3.2 Convergence and Accuracy Tests

We have carried out a convergence test for a W-fiber with a displaced core (Fig. 2.10, inset). The physical parameters of the fiber are described in the caption of Fig. 2.10. As the outer cladding has the same refractive index as the core, modes supported by the fiber are leaky due to evanescent coupling. We focus on the y -polarized HE_{11} -like fundamental mode. Figure 2.10 shows that both real and imaginary parts of the mode effective index ($n_{\text{eff}} = \beta/k_0$) converge quickly as n increases. Result calculated with $n = 6$ has a relative error of 6.6370×10^{-9} in real part and 0.0017 in imaginary part as compared to the converged value. Wavelength is at $1.55\mu\text{m}$ for the test.

Figure 2.10: Convergence of the n_{eff} value with respect to n . Inset, the fiber under test. The core has index $n_1 = 1.45$. The first cladding layer has index $n_2 = 1.42$. The outer cladding shares the same index as the core. The inner two interfaces have radii at $5\mu\text{m}$ and $10\mu\text{m}$ respectively, and their centers are displaced by $\Delta d = 3\mu\text{m}$ in y (vertical) direction.

To validate the TMM, a full-vector finite-difference method (FDM) [19] is used to derive the y -polarized HE_{11} -like mode in the same W-fiber, with the offset between two interfaces Δd varying from 0 to $3\mu\text{m}$. The results are compared with those calculated using the TMM in Fig. 2.11. The real part has a consistent relative error of $\sim 1.9 \times 10^{-5}$ between the two methods [Fig. 2.11(a)]; the imaginary part has a relative error of 0.024 to 0.095 between the two methods [Fig. 2.11(b)]. The nearly constant discrepancy in real part suggests that we might have scaled the fiber structure when the index profile is input into the FDM program as an image. $n = 11$ has been used for TMM calculations. For FDM calculations, domain is at $27 \times 27\mu\text{m}$, meshed with 300×300 grid points. Perfectly-matched layer is of a width equivalent to 12 grid points. Wavelength is at $1.55\mu\text{m}$. Results from a commercial full-vector mode solver (FemSIM v.1.0 from RSoft Design Group, Inc.) based on the finite-element method (FEM) are also presented in Fig. 2.11. The FEM results agree in general with those calculated using TMM and FDM, despite there are some convergence jerks observed in the real part.

Figure 2.11: Comparison of n_{eff} values of the HE_{11}^y -like mode computed using TMM, FDM and FEM. (a) Real part; (b) Imaginary part.

2.3.3 Simulation Results

In this subsection, we study the modal property of a perturbed Bragg fiber. The fiber is roughly the same as the Bragg fiber we looked into in the last section, except that the interfaces (except the innermost one) are displaced by a constant amount Δd . The direction of dislocations, from the second interface to the outermost interface, are along the $+x$, $+y$, $-x$, $-y$, so on and so forth.

Figure 2.12 shows variations in the n_{eff} value and the leakage loss for the TE_{01} mode¹ as Δd varies from 0 to $0.08\mu\text{m}$. Wavelength is chosen as $1.55\mu\text{m}$. We notice that, when Δd (perturbation) increases, the n_{eff} value decreases only slightly, whereas the leakage loss increases drastically. At $\Delta d = 0.08\mu\text{m}$, n_{eff} changes only by 6.86×10^{-5} (from 0.99079031 to 0.99072237); the leakage loss however becomes ~ 650000 times as large as that of the unperturbed fiber (from 0.428dB/km to 277348.8dB/km). We have plotted the TE_{01} mode at $\Delta d = 0.08\mu\text{m}$ in Fig. 2.13. It is noticed that, in Fig. 2.13(a) the transverse component E_t appears similar to that supported by an unperturbed Bragg fiber. However, a zoom-in view shows that the field close to cladding stack is not strictly parallel to the core-cladding interface [Fig. 2.13(a) inset]. Figure 2.13(b) shows that there is a sizable E_z component for the “ TE_{01} ” mode. An examination of this minor field component can reveal what causes the high leakage loss of the mode. In this particular case, the E_z field stays primarily in the cladding and it has a very large azimuthal quantum number, hence we can speculate that it is the cladding resonance which causes the high leakage loss.

We have recently presented a more thorough analysis of perturbed Bragg fibers using this extended TMM method in [49]. It is found that the core modes are likely to interact with each other at wavelength points where their dispersion curves are supposed to make

¹In fact, the mode is now a hybrid one. However, for simplicity we borrow names of the modes in an unperturbed Bragg fiber to denote those in a perturbed one owing to the proximity in modes appearances.

Figure 2.12: Variations in the effective index (a) and the radiation loss (b) of the TE_{01} mode as Δd increases. $n = 10$ is used for the calculations.

a cross. Such interaction will result in some very high radiation loss spectral regions for the TE_{01} mode.

2.4 Conclusion

The Bessel function of the first kind J and the Hankel function of the first kind $H^{(1)}$ are used to represent the mode field in each layer of a Bragg fiber. The proposed transfer matrix method treats all dielectric interfaces indifferently, which facilitates an easier implementation. As the method uses analytical functions to represent the mode field, its solution is exact.

We have also extended the method to deal with multilayered fibers whose dielectric interface are circular but not concentric. The extended transfer matrix method is much less expensive computationally compared to any numerical method. And the result can be as accurate as we wish, at a very little additional computational cost. Excellent agreement is achieved between the extended TMM and two numerical methods (FEM and FDM) for a simple test case. We believe this method can be deployed for efficient perturbation analysis of Bragg fibers.

Figure 2.13: TE_{01} mode at $1.55\mu\text{m}$ wavelength, with $\Delta d = 0.08\mu\text{m}$. (a) E_t ; (b) E_z . Axis unit: μm . The inset in (a) shows the magnified E_t field across the core-cladding boundary. In (b), red is for positive, and blue is for negative.

Chapter 3

Multipole Expansion Method

3.1 Brief History

In a paper dated back in 1892, Lord Rayleigh described a method for the solution of electrostatic problems involving lattices of spheres or arrays of cylinders. The essence of his idea is in relating the regular field in the vicinity of any scatterer to fields radiated by scatterers and external sources. In 1969, Goell published a paper on optical waveguide analysis [50], in which circular harmonics are used to represent optical fields in an arbitrary cross-sectioned waveguide. In that paper, only a single core is considered. Later, in 1973, Wijngaard published a paper on guided modes of two parallel circular dielectric rods by using the same harmonic expansion [51]. In 1994, K. M. Lo *et al.* studied waveguiding effect by multiple embedded cylinders [52]. However, in their paper the refractive indices of the cylinders are higher than that of the background material. After the invention of the microstructured optical fibers (MOFs), Lo's method was soon adapted to this class of waveguides by White *et al.* ([2] and [4]).

3.2 Formulation

Our objective is to find the propagating modes supported by a set of parallel cylinders, as shown in Fig. 3.1. From Maxwell equations, we know that for a z -invariant waveguide problem, the longitudinal electrical and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ fulfill the Helmholtz equation in a homogeneous material region. In the host material (silica in our case),

$$[\nabla^2 + (k_t^e)^2]F = 0, \quad (3.1)$$

where $F = E_z$ or K_z , and $k_t^e = \sqrt{k^2 n_e^2 - \beta^2}$. Here we use $\mathbf{K} = \mathcal{Z}\mathbf{H}$, and $\mathcal{Z} = \sqrt{\mu_0/\epsilon_0}$ is the free-space impedance. n_e is refractive index of the host material. $k = 2\pi/\lambda$ is the wave number in free space. $\beta = kn_{\text{eff}}$ is the propagation constant.

Refer to Fig. 3.1, the E_z and K_z fields near l^{th} cylinder can be expanded in Fourier-Bessel series as

$$E_z = \sum_m [A_m^{El} J_m(k_t^e r_l) + B_m^{El} H_m(k_t^e r_l)] \exp(im\theta_l), \quad (3.2)$$

Figure 3.1: Schematic diagram for a piece-wise homogeneous microstructured optical fiber. Outer circle indicates jacket boundary. Region beyond that is jacket material of index n_0 . Inner circles indicate cylindrical inclusions. Any inclusion can take a different index n_l , which can be higher or lower than their host material n_e .

$$K_z = \sum_m [A_m^{Kl} J_m(k_t^e r_l) + B_m^{Kl} H_m(k_t^e r_l)] \exp(im\theta_l). \quad (3.3)$$

In Eqs. 3.2 and 3.3, J_m is the Bessel function of order m , and H_m is the Hankel function of the first kind of order m . Each J_m term represents the incident field (or *regular field*) whose source is outside the region of consideration. In this case the sources are located on all cylinder boundaries (including jacket's) except that of the l^{th} cylinder. Each H_m represents the outgoing field whose source is on the boundary of the l^{th} cylinder. We call such field expansion as a local expansion, since it is valid just in the vicinity of the l^{th} cylinder. Notice that the expansion takes place in the l^{th} cylinder's own polar coordinate (with its origin at l^{th} cylinder's center).

Global expansion of the field in the host material can be expressed as (in the case of E_z)

$$E_z = \sum_{l=1}^{N_c} \sum_m B_m^{El} H_m(k_t^e |r_l|) \exp[im \arg(r_l)] + \sum_m A_m^{E0} J_m(k_t^e r) \exp(im\theta). \quad (3.4)$$

It says the field in the host material region is a superposition of the outgoing fields due to all the cylinders immersed in the host material, plus the regular field due to the jacket which encloses the host region. Such field expansion was originally proposed by Wijngaard, and was later proven rigorously using Green's function. Notice that the outgoing field due to each cylinder is expressed in the cylinder's local polar coordinate (with its origin at the cylinder's center).

3.2.1 Two Equalities

Equating the local expansion (Eq. 3.2) and the global expansion (Eq. 3.4) for the field near cylinder l , we arrive at

$$\sum_m A_m^{El} J_m(k_t^e r_l) \exp(im\theta_l) = \sum_{j=1, j \neq l}^{N_c} \sum_m B_m^{Ej} H_m(k_t^e r_j) \exp(im\theta_j) + \sum_m A_m^{E0} J_m(k_t^e r) \exp(im\theta). \quad (3.5)$$

Notice the H_m terms on the left hand side are cancelled. Equation 3.5 is the first equality. It tells that the regular field in the vicinity of cylinder l is due to the outgoing fields originated from the rest cylinders plus the regular field due to the jacket interface. Another equality states that the outgoing field in the vicinity of the jacket boundary (just inside the jacket material) is due to the outgoing field from all cylinders in the host material. We express it as

$$\sum_m B_m^{E0} H_m(k_t^e r) \exp(im\theta) = \sum_{j=1}^{N_c} \sum_m B_m^{Ej} H_m(k_t^e |\mathbf{r}_j|) \exp[im \arg(\mathbf{r}_j)]. \quad (3.6)$$

We now pay attention to Eq. 3.5. Notice on the right hand side of Eq. 3.5, there are $N_c + 1$ series expansions (ignore the $j \neq l$ constraint). Each expansion has $2m + 1$ terms, which are written in the cylinder j or the jacket's polar coordinate. Now we will transform these expansions into l^{th} cylinder's coordinate. Mathematically, this transformation is called a *coordinate projection*.

The coordinate projection can be derived using Graf's addition theorems. For example, one H_m expansion in Eq. 3.5 (in j^{th} cylinder's coordinate) can be transformed into cylinder l 's coordinate as

$$\sum_m B_m^{Ej} H_m(k_t^e r_j) \exp(im\theta_j) = \sum_n A_n^{Elj} J_n(k_t^e |\mathbf{r}_l|) \exp[in \arg(\mathbf{r}_l)], \quad (3.7)$$

where

$$A_n^{Elj} = \sum_m \mathcal{H}_{nm}^{lj} B_m^{Ej}, \quad (3.8)$$

$$\mathcal{H}_{nm}^{lj} = H_{n-m}(k_t^e |\mathbf{r}_{lj}|) \exp[-i(n-m)\arg(\mathbf{r}_{lj})]. \quad (3.9)$$

In Eq. 3.9, $\mathbf{r}_{lj}$ is the vector pointing from center of the l^{th} cylinder to that of the j^{th} cylinder (Fig. 3.1). Similarly, the final expansion in Eq. 3.5, which is in the jacket's coordinates (or global coordinate), can be transformed into cylinder l 's coordinate as

$$\sum_m A_m^{E0} J_m(k_t^e r) \exp(im\theta) = \sum_n A_n^{El0} J_n(k_t^e |\mathbf{r}_l|) \exp[in \arg(\mathbf{r}_l)], \quad (3.10)$$

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where

$$A_n^{El0} = \sum_m \mathcal{J}_{nm}^{l0} A_m^{E0}, \quad (3.11)$$

$$\mathcal{J}_{nm}^{l0} = (-1)^{n-m} J_{n-m}(k_t^e |\mathbf{r}_l|) \exp[-i(n-m)\arg(\mathbf{r}_l)]. \quad (3.12)$$

Now substitute Eq. 3.7 and Eq. 3.10 into Eq. 3.5, then this equality becomes

$$\sum_m A_m^{El} J_m(k_t^e |\mathbf{r}_l|) \exp[i m \arg(\mathbf{r}_l)] = \sum_{j=1, j \neq l}^{N_c} \sum_n A_n^{Elj} J_n(k_t^e |\mathbf{r}_l|) \exp[i n \arg(\mathbf{r}_l)] + \sum_n A_n^{El0} J_n(k_t^e |\mathbf{r}_l|) \exp[i n \arg(\mathbf{r}_l)]. \quad (3.13)$$

Normally we choose $m = n$ for keeping the matrices square. Since coefficients for each expansion order decouple, we can now ignore the common factor $J_n(k_t^e |\mathbf{r}_l|) \exp[i n \arg(\mathbf{r}_l)]$ in the equation. We then get a system of linear equations, which involves only expansion coefficients, as

$$\begin{aligned} A_{-m}^{El} &= A_{-n}^{El1} + \dots + A_{-n}^{ElN_c} + A_{-n}^{El0}, \\ \vdots &\quad \quad \quad \vdots \quad \quad \quad \vdots \\ A_m^{El} &= A_n^{El1} + \dots + A_n^{ElN_c} + A_n^{El0}. \end{aligned} \quad (3.14)$$

This set of equations can be equivalently written as

$$\begin{bmatrix} A_{-m}^{El} \\ \vdots \\ A_m^{El} \end{bmatrix} = \begin{bmatrix} A_{-n}^{El1} \\ \vdots \\ A_n^{El1} \end{bmatrix} + \dots + \begin{bmatrix} A_{-n}^{ElN_c} \\ \vdots \\ A_n^{ElN_c} \end{bmatrix} + \begin{bmatrix} A_{-n}^{El0} \\ \vdots \\ A_n^{El0} \end{bmatrix}, \quad (3.15)$$

Or

$$\mathbf{A}^{El} = \mathbf{A}^{El1} + \dots + \mathbf{A}^{ElN_c} + \mathbf{A}^{El0}. \quad (3.16)$$

Notice on the right hand side of Eq. 3.16, one of the column vectors has zero elements if $j = l$. Now we examine the first column vector at the right hand side of the equation. Without loss of generality, we assume $j = 1$. From Eq. 3.8, each element of this column vector can be written as

$$\begin{aligned} A_{-n}^{El1} &= \mathcal{H}_{(-n)(-m)}^{l1} B_{-m}^{E1} + \dots + \mathcal{H}_{(-n)(m)}^{l1} B_m^{E1}, \\ \vdots &\quad \quad \quad \vdots \\ A_n^{El1} &= \mathcal{H}_{(n)(-m)}^{l1} B_{-m}^{E1} + \dots + \mathcal{H}_{(n)(m)}^{l1} B_m^{E1}. \end{aligned}$$

This set of equations can be equivalently written as

$$\begin{bmatrix} A_{-n}^{El1} \\ \vdots \\ A_n^{El1} \end{bmatrix} = \begin{bmatrix} \mathcal{H}_{(-n)(-m)}^{l1} & \dots & \mathcal{H}_{(-n)(m)}^{l1} \\ \vdots & \ddots & \vdots \\ \mathcal{H}_{(n)(-m)}^{l1} & \dots & \mathcal{H}_{(n)(m)}^{l1} \end{bmatrix} \begin{bmatrix} B_{-m}^{E1} \\ \vdots \\ B_m^{E1} \end{bmatrix}. \quad (3.17)$$

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In short,

$$\mathbf{A}^{El1} = \mathbf{H}^{l1} \mathbf{B}^{E1}. \quad (3.18)$$

This equation is valid for all other j values. Notice that when $j = l$, $\mathbf{H}^{lj}$ has all zero elements.

Elements in the final single-column vector at the right hand side of Eq. 3.15 can be expressed as

$$\begin{aligned} A_{-n}^{El0} &= \mathcal{J}_{(-n)(-m)}^{l0} A_{-m}^{E0} + \cdots + \mathcal{J}_{(-n)(m)}^{l0} A_m^{E0}, \\ \vdots \quad \quad \quad \vdots \\ A_n^{El0} &= \mathcal{J}_{(n)(-m)}^{l0} A_{-m}^{E0} + \cdots + \mathcal{J}_{(n)(m)}^{l0} A_m^{E0}. \end{aligned}$$

In matrix form, the equations can be written as

$$\begin{bmatrix} A_{-n}^{El0} \\ \vdots \\ A_n^{El0} \end{bmatrix} = \begin{bmatrix} \mathcal{J}_{(-n)(-m)}^{l0} & \cdots & \mathcal{J}_{(-n)(m)}^{l0} \\ \vdots & \ddots & \vdots \\ \mathcal{J}_{(n)(-m)}^{l0} & \cdots & \mathcal{J}_{(n)(m)}^{l0} \end{bmatrix} \begin{bmatrix} A_{-m}^{E0} \\ \vdots \\ A_m^{E0} \end{bmatrix}. \quad (3.19)$$

Or we can write in a short form as

$$\mathbf{A}^{El0} = \mathbf{J}^{l0} \mathbf{A}^{E0}. \quad (3.20)$$

At this stage, we can substitute Eq. 3.18 and 3.20 into Eq. 3.16. The first equality hence becomes

$$\mathbf{A}^{El} = \sum_{j=1, j \neq l}^{N_c} \mathbf{H}^{lj} \mathbf{B}^{Ej} + \mathbf{J}^{l0} \mathbf{A}^{E0}. \quad (3.21)$$

If we apply Eq. 3.21 for all l values, we have

$$\begin{aligned} \mathbf{A}^{E1} &= \mathbf{H}^{11} \mathbf{B}^{E1} + \cdots + \mathbf{H}^{1N_c} \mathbf{B}^{E1} + \mathbf{J}^{10} \mathbf{A}^{E0}, \\ \vdots \quad \quad \quad \vdots \\ \mathbf{A}^{EN_c} &= \mathbf{H}^{N_c 1} \mathbf{B}^{E1} + \cdots + \mathbf{H}^{N_c N_c} \mathbf{B}^{E1} + \mathbf{J}^{N_c 0} \mathbf{A}^{E0}. \end{aligned}$$

This set of equations is valid also for the scaled magnetic field K_z . Putting the two sets together, we arrive at

$$\begin{aligned} \mathbf{A}^{E1} &= \mathbf{H}^{11} \mathbf{B}^{E1} + \cdots + \mathbf{H}^{1N_c} \mathbf{B}^{E1} + \mathbf{J}^{10} \mathbf{A}^{E0}, \\ \mathbf{A}^{K1} &= \mathbf{H}^{11} \mathbf{B}^{K1} + \cdots + \mathbf{H}^{1N_c} \mathbf{B}^{K1} + \mathbf{J}^{10} \mathbf{A}^{K0}, \\ \vdots \quad \quad \quad \vdots \\ \mathbf{A}^{EN_c} &= \mathbf{H}^{N_c 1} \mathbf{B}^{E1} + \cdots + \mathbf{H}^{N_c N_c} \mathbf{B}^{E1} + \mathbf{J}^{N_c 0} \mathbf{A}^{E0}, \\ \mathbf{A}^{KN_c} &= \mathbf{H}^{N_c 1} \mathbf{B}^{K1} + \cdots + \mathbf{H}^{N_c N_c} \mathbf{B}^{K1} + \mathbf{J}^{N_c 0} \mathbf{A}^{K0}. \end{aligned}$$

The first two equations ($l = 1$) can be written as

$$\begin{bmatrix} \mathbf{A}^{E1} \\ \mathbf{A}^{K1} \end{bmatrix} = \begin{bmatrix} \mathbf{H}^{11} & 0 \\ 0 & \mathbf{H}^{11} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{E1} \\ \mathbf{B}^{K1} \end{bmatrix} + \cdots + \begin{bmatrix} \mathbf{H}^{1N_c} & 0 \\ 0 & \mathbf{H}^{1N_c} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{EN_c} \\ \mathbf{B}^{KN_c} \end{bmatrix} + \begin{bmatrix} \mathbf{J}^{10} & 0 \\ 0 & \mathbf{J}^{10} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{E0} \\ \mathbf{A}^{K0} \end{bmatrix} \quad (3.22)$$

Generalizing this matrix equation for all l values, we have

$$\tilde{\mathbf{A}}^l = \tilde{\mathbf{H}}^{l1} \tilde{\mathbf{B}}^1 + \cdots + \tilde{\mathbf{H}}^{lN_c} \tilde{\mathbf{B}}^{N_c} + \tilde{\mathbf{J}}^{l0} \tilde{\mathbf{A}}^0. \quad (3.23)$$

Express this equation for $l = 1 \cdots N_c$, we get

$$\begin{aligned} \tilde{\mathbf{A}}^1 &= \tilde{\mathbf{H}}^{11} \tilde{\mathbf{B}}^1 + \cdots + \tilde{\mathbf{H}}^{1N_c} \tilde{\mathbf{B}}^{N_c} + \tilde{\mathbf{J}}^{10} \tilde{\mathbf{A}}^0, \\ &\vdots \\ \tilde{\mathbf{A}}^{N_c} &= \tilde{\mathbf{H}}^{N_c1} \tilde{\mathbf{B}}^1 + \cdots + \tilde{\mathbf{H}}^{N_cN_c} \tilde{\mathbf{B}}^{N_c} + \tilde{\mathbf{J}}^{N_c0} \tilde{\mathbf{A}}^0. \end{aligned} \quad (3.24)$$

Equivalently we can write in a higher-level matrix form

$$\begin{bmatrix} \tilde{\mathbf{A}}^1 \\ \vdots \\ \tilde{\mathbf{A}}^{N_c} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{H}}^{11} & \cdots & \tilde{\mathbf{H}}^{1N_c} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{H}}^{N_c1} & \cdots & \tilde{\mathbf{H}}^{N_cN_c} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{B}}^1 \\ \vdots \\ \tilde{\mathbf{B}}^{N_c} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{J}}^{10} \\ \vdots \\ \tilde{\mathbf{J}}^{N_c0} \end{bmatrix} \tilde{\mathbf{A}}^0. \quad (3.25)$$

In short, we have the first equality as

$$\mathbf{A} = \mathbf{H}\mathbf{B} + \mathbf{J}^{B0} \tilde{\mathbf{A}}^0. \quad (3.26)$$

In Eq. 3.26, $\tilde{\mathbf{B}}^j$ ($j = 1 \cdots N_c$, $j \neq l$) represents outgoing fields due to all other cylinders, while $\tilde{\mathbf{A}}^0$ represents regular fields contributed by the jacket.

The rest of the subsection is for the second equality, which states that the outgoing field just near the jacket boundary (inside jacket material) is contributed to by outgoing fields from all cylinders. Following the logic we had for the first equality, we should end up with a relation for coefficient $\tilde{\mathbf{B}}^0$ and $\tilde{\mathbf{B}}^j$, $j = 1 \cdots N_c$.

Again, by using a derived addition theorem after that of Graf's, we transform a Hankel function in cylinder j^{th} coordinate into a series of Hankel functions in the jacket cylinder's coordinate, which is normally the same as our global coordinate. The transformation can be expressed as

$$\sum_m B_m^{Ej} H_m(k_t^e |\mathbf{r}_j|) \exp[im \arg(\mathbf{r}_j)] = \sum_n B_n^{E0j} H_n(k_t^e |\mathbf{r}|) \exp[in \arg(\mathbf{r})], \quad (3.27)$$

where

$$B_n^{E0j} = \sum_m \mathcal{J}_{nm}^{0j} B_m^{Ej}, \quad (3.28)$$

$$\mathcal{J}_{nm}^{0j} = \mathbf{J}_{n-m}(k_t^e |\mathbf{r}_{j0}|) \exp[-i(n-m) \arg(\mathbf{r}_{j0})]. \quad (3.29)$$

Substitute Eq. 3.27 back into Eq. 3.6, we have

$$\sum_m B_m^{E0} H_m(k_t^e r) \exp(im\theta) = \sum_{j=1}^{N_c} \sum_n B_n^{E0j} H_n(k_t^e |r|) \exp[in \arg(r)]. \quad (3.30)$$

Similarly, coefficients for each order of Hankel function decouple. We get a system of linear equations as

$$\begin{aligned} B_{-m}^{E0} &= B_{-n}^{E01} + \dots + B_{-n}^{E0N_c}, \\ \vdots &\quad \quad \quad \vdots \quad \quad \quad \vdots \\ B_m^{E0} &= B_n^{E01} + \dots + B_n^{E0N_c}. \end{aligned} \quad (3.31)$$

In matrix form it is

$$\begin{bmatrix} B_{-m}^{E0} \\ \vdots \\ B_m^{E0} \end{bmatrix} = \begin{bmatrix} B_{-n}^{E01} \\ \vdots \\ B_n^{E01} \end{bmatrix} + \dots + \begin{bmatrix} B_{-n}^{E0N_c} \\ \vdots \\ B_n^{E0N_c} \end{bmatrix}, \quad (3.32)$$

Or

$$\begin{aligned} \mathbf{B}^{E0} &= \mathbf{B}^{E01} + \dots + \mathbf{B}^{E0N_c} \\ &= \sum_{j=1}^{N_c} \mathbf{B}^{E0j}. \end{aligned} \quad (3.33)$$

Now we examine the first column vector at the right hand side of the Eq. 3.32. After applying the projection derived in Eq. 3.28, we get

$$\begin{aligned} B_{-n}^{E01} &= \mathcal{J}_{(-n)(-m)}^{01} B_{-m}^{E1} + \dots + \mathcal{J}_{(-n)(m)}^{01} B_m^{E1}, \\ \vdots &\quad \quad \quad \vdots \\ B_n^{E01} &= \mathcal{J}_{(n)(-m)}^{01} B_{-m}^{E1} + \dots + \mathcal{J}_{(n)(m)}^{01} B_m^{E1}, \end{aligned} \quad (3.34)$$

or

$$\begin{bmatrix} B_{-n}^{E01} \\ \vdots \\ B_n^{E01} \end{bmatrix} = \begin{bmatrix} \mathcal{J}_{(-n)(-m)}^{01} & \dots & \mathcal{J}_{(-n)(m)}^{01} \\ \vdots & \ddots & \vdots \\ \mathcal{J}_{(n)(-m)}^{01} & \dots & \mathcal{J}_{(n)(m)}^{01} \end{bmatrix} \times \begin{bmatrix} B_{-m}^{E1} \\ \vdots \\ B_m^{E1} \end{bmatrix}. \quad (3.35)$$

In short form, it is

$$\mathbf{B}^{E01} = \mathbf{J}^{01} \mathbf{B}^{E1}. \quad (3.36)$$

This equation is valid for all other j values. Substitute Eq. 3.36 back into Eq. 3.33, we can have

$$\mathbf{B}^{E0} = \mathbf{J}^{01} \mathbf{B}^{E1} + \dots + \mathbf{J}^{0N_c} \mathbf{B}^{EN_c}. \quad (3.37)$$

For K_z field, we still have the same form of equation, which is

$$\mathbf{B}^{K0} = \mathbf{J}^{01} \mathbf{B}^{K1} + \dots + \mathbf{J}^{0N_c} \mathbf{B}^{KN_c}. \quad (3.38)$$

Put Eq. 3.37 and Eq. 3.38 together into a matrix form, we have

$$\begin{bmatrix} \mathbf{B}^{E0} \\ \mathbf{B}^{K0} \end{bmatrix} = \begin{bmatrix} \mathbf{J}^{01} & 0 \\ 0 & \mathbf{J}^{01} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{E1} \\ \mathbf{B}^{K1} \end{bmatrix} + \cdots + \begin{bmatrix} \mathbf{J}^{0N_c} & 0 \\ 0 & \mathbf{J}^{0N_c} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{EN_c} \\ \mathbf{B}^{KN_c} \end{bmatrix}. \quad (3.39)$$

If we let

$$\tilde{\mathbf{B}}^0 = \begin{bmatrix} \mathbf{B}^{E0} \\ \mathbf{B}^{K0} \end{bmatrix}, \quad \tilde{\mathbf{J}}^{0j} = \begin{bmatrix} \mathbf{J}^{01} & 0 \\ 0 & \mathbf{J}^{01} \end{bmatrix}, \quad \text{and} \quad \tilde{\mathbf{B}}^j = \begin{bmatrix} \mathbf{B}^{Ej} \\ \mathbf{B}^{Kj} \end{bmatrix}, \quad j = 1 \cdots N_c,$$

we can re-write Eq. 3.39 as

$$\begin{aligned} \tilde{\mathbf{B}}^0 &= \tilde{\mathbf{J}}^{01} \tilde{\mathbf{B}}^1 + \cdots + \tilde{\mathbf{J}}^{0N_c} \tilde{\mathbf{B}}^{N_c} \\ &= \begin{bmatrix} \tilde{\mathbf{J}}^{01} & \cdots & \tilde{\mathbf{J}}^{0N_c} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{B}}^1 \\ \vdots \\ \tilde{\mathbf{B}}^{N_c} \end{bmatrix}, \end{aligned}$$

or

$$\tilde{\mathbf{B}}^0 = \mathbf{J}^{0B} \mathbf{B}, \quad (3.40)$$

where

$$\begin{aligned} \mathbf{J}^{0B} &= \begin{bmatrix} \mathbf{J}^{01} & 0 & \cdots & \mathbf{J}^{0N_c} & 0 \\ 0 & \mathbf{J}^{01} & \cdots & 0 & \mathbf{J}^{0N_c} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \mathbf{B}^{E1} \\ \mathbf{B}^{K1} \\ \vdots \\ \mathbf{B}^{EN_c} \\ \mathbf{B}^{KN_c} \end{bmatrix}. \end{aligned} \quad (3.41)$$

3.2.2 Reflection Matrices

In the previous subsection, we were only concerned about optical fields residing in the host material. We need to consider fields inside the cylinders in order to characterize the physical problem completely. EM theory requires that across the interface of two dielectric materials, tangential fields remain continuous. As the boundary line in our problem is perfectly circular (Fig. 3.1), the E_z , E_θ and K_z , K_θ fields (in each cylinder's local coordinate) should fulfill this condition. Without loss of generality, we consider a low-index cylinder (of index n_-) immersed in a high index host material (of index n_+). See Fig. 3.2 for a schematic diagram of the structure. The cylinder has its radius at a . The origin of the coordinate system is at the center of the cylinder. The continuity equations

are

$$E_z^+ = E_z^-, \quad (3.42)$$

$$K_z^+ = K_z^-, \quad (3.43)$$

$$E_\theta^+ = E_\theta^-, \quad (3.44)$$

$$K_\theta^+ = K_\theta^-. \quad (3.45)$$

Superscript $+$ denotes the field outside the cylinder, i.e., $r > a$, and the superscript $-$ denotes the field inside the cylinder, i.e., $r < a$.

E_z and K_z fields just outside and inside cylinders respectively, can be expanded in Fourier-Bessel series as,

$$\begin{cases} E_z^+(r, \theta) = \sum_m [A_m^{E+} J_m(k_t^+ r) + B_m^{E+} H_m(k_t^+ r)] \exp(im\theta), \\ E_z^-(r, \theta) = \sum_m [A_m^{E-} J_m(k_t^- r) + B_m^{E-} H_m(k_t^- r)] \exp(im\theta). \end{cases} \quad (3.46)$$

$$\begin{cases} K_z^+(r, \theta) = \sum_m [A_m^{K+} J_m(k_t^+ r) + B_m^{K+} H_m(k_t^+ r)] \exp(im\theta), \\ K_z^-(r, \theta) = \sum_m [A_m^{K-} J_m(k_t^- r) + B_m^{K-} H_m(k_t^- r)] \exp(im\theta). \end{cases} \quad (3.47)$$

The θ components of the fields can be expressed in E_z and K_z using

$$E_\theta(r, \theta) = \frac{i}{k_t^2} \left(\frac{\beta}{r} \frac{\partial E_z}{\partial \theta} - k \frac{\partial K_z}{\partial r} \right), \quad (3.48)$$

$$K_\theta(r, \theta) = \frac{i}{k_t^2} \left(\frac{\beta}{r} \frac{\partial K_z}{\partial \theta} + kn^2 \frac{\partial E_z}{\partial r} \right), \quad (3.49)$$

From continuity (Eq. 3.42), we have

$$\sum_m [A_m^{E+} J_m(k_t^+ a) + B_m^{E+} H_m(k_t^+ a)] \exp(im\theta) = \sum_m [A_m^{E-} J_m(k_t^- a) + B_m^{E-} H_m(k_t^- a)] \exp(im\theta). \quad (3.50)$$

Hereafter, we would like to abbreviate $J_m(k_t^+ a)$ as J_m^+ . Other similar terms are treated in the same way. Since the equation is valid for all θ values, series expansion coefficients corresponding to the same order are equal on both sides. We hence have

$$A_m^{E+} J_m^+ + B_m^{E+} H_m^+ = A_m^{E-} J_m^- + B_m^{E-} H_m^-. \quad (3.51)$$

Similarly for K_z field, we have

$$A_m^{K+} J_m^+ + B_m^{K+} H_m^+ = A_m^{K-} J_m^- + B_m^{K-} H_m^-. \quad (3.52)$$

Another two equations for interior and exterior azimuthal fields are given as

$$\begin{aligned} \frac{m\beta}{k_t^{+2}a} (A_m^{E+} J_m^+ + B_m^{E+} H_m^+) + \frac{ik}{k_t^+} (A_m^{K+} J_m'^+ + B_m^{K+} H_m'^+) = \\ \frac{m\beta}{k_t^{-2}a} (A_m^{E-} J_m^- + B_m^{E-} H_m^-) + \frac{ik}{k_t^-} (A_m^{K-} J_m'^- + B_m^{K-} H_m'^-), \end{aligned} \quad (3.53)$$

Figure 3.2: External reflection matrix $\mathbf{R}^+$ relates the regular field $\mathbf{A}^+$ and outgoing field $\mathbf{B}^+$ in the host material. It is assumed that there is no outgoing field inside cylinder inclusion.

$$\begin{aligned} \frac{m\beta}{k_t^{+2}a} (A_m^{K+} J_m^+ + B_m^{K+} H_m^+) - \frac{ikn_+^2}{k_t^+} (A_m^{E+} J_m'^+ + B_m^{E+} H_m'^+) = \\ \frac{m\beta}{k_t^{-2}a} (A_m^{K-} J_m^- + B_m^{K-} H_m^-) - \frac{ikn_-^2}{k_t^-} (A_m^{E-} J_m'^- + B_m^{E-} H_m'^-). \end{aligned} \quad (3.54)$$

Refer to Fig. 3.2, we begin to derive the external reflection matrix which relates the outgoing optical fields (represented by B_m^+ coefficients) to the incoming optical fields (represented by A_m^+ coefficients). Notice within a single cylinder, there isn't any source present, so there should not be any outgoing fields existing within a single cylinder. Hence we can let all $B_m^- = 0$ during the analysis. After some mathematical manipulations, the following can be derived based on Eqs. 3.51-3.54

$$B_m^{E+} = R_m^{EE+} A_m^{E+} + R_m^{EK+} A_m^{K+}, \quad (3.55)$$

$$B_m^{K+} = R_m^{KE+} A_m^{E+} + R_m^{KK+} A_m^{K+}, \quad (3.56)$$

where

$$\begin{aligned} R_m^{EE+} &= \frac{1}{\delta_m} [(\alpha_{J-H+}^+ - \alpha_{H+J-}^-)(n_-^2 \alpha_{J-J+}^+ - n_+^2 \alpha_{J+J-}^-) - m^2 J_m^+ H_m^+ J_m^{-2} \tau^2], \\ R_m^{EK+} &= \frac{1}{\delta_m} \left[\frac{2m\tau}{\pi k a} \frac{k_t^-}{k_t^+} J_m^{-2} \right], \\ R_m^{KE+} &= -n_+^2 R_m^{EK+}, \\ R_m^{KK+} &= \frac{1}{\delta_m} [(\alpha_{J-J+}^+ - \alpha_{J+J-}^-)(n_-^2 \alpha_{J-H+}^+ - n_+^2 \alpha_{H+J-}^-) - m^2 J_m^+ H_m^+ H_m^{-2} \tau^2], \end{aligned} \quad (3.57)$$

with

$$\begin{aligned} \delta_m &= (\alpha_{H+J-}^- - \alpha_{J-H+}^+)(n_-^2 \alpha_{J-H+}^2 - n_+^2 \alpha_{H+J-}^-) + (m J_m^- H_m^+ \tau)^2, \\ \tau &= \frac{\beta}{a k_t^- k_t^+} (n_+^2 - n_-^2). \end{aligned}$$

In Eq. 3.57, term α_{J-H+}^+ is defined as

$$\alpha_{J-H+}^+ = \frac{k_t^+}{k} J_m'^- H_m^+.$$

And other terms in the same form are defined similarly.

We express Eq. 3.55 for every expansion order as

$$\begin{aligned} B_{-m}^{E+} &= R_{-m}^{EE+} A_{-m}^{E+} + R_{-m}^{EK+} A_{-m}^{K+}, \\ &\vdots \\ B_m^{E+} &= R_m^{EE+} A_m^{E+} + R_m^{EK+} A_m^{K+}. \end{aligned} \quad (3.58)$$

In a matrix form, this set of equations can be written as

$$\begin{bmatrix} B_{-m}^{E+} \\ \vdots \\ B_m^{E+} \end{bmatrix} = \begin{bmatrix} R_{-m}^{EE+} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_m^{EE+} \end{bmatrix} \begin{bmatrix} A_{-m}^{E+} \\ \vdots \\ A_m^{E+} \end{bmatrix} \quad (3.59)$$

$$+ \begin{bmatrix} R_{-m}^{EK+} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_m^{EK+} \end{bmatrix} \begin{bmatrix} A_{-m}^{K+} \\ \vdots \\ A_m^{K+} \end{bmatrix}. \quad (3.60)$$

Or in short,

$$\mathbf{B}^{E+} = \mathbf{R}^{EE+} \mathbf{A}^{E+} + \mathbf{R}^{EK+} \mathbf{A}^{K+}. \quad (3.61)$$

Similarly, we can expand equation 3.56 and write them in a matrix form as

$$\begin{bmatrix} B_{-m}^{K+} \\ \vdots \\ B_m^{K+} \end{bmatrix} = \begin{bmatrix} R_{-m}^{KE+} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_m^{KE+} \end{bmatrix} \begin{bmatrix} A_{-m}^{E+} \\ \vdots \\ A_m^{E+} \end{bmatrix} \quad (3.62)$$

$$+ \begin{bmatrix} R_{-m}^{KK+} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & R_m^{KK+} \end{bmatrix} \begin{bmatrix} A_{-m}^{K+} \\ \vdots \\ A_m^{K+} \end{bmatrix}. \quad (3.63)$$

And we can put it into matrix form as

$$\mathbf{B}^{K+} = \mathbf{R}^{KE+} \mathbf{A}^{E+} + \mathbf{R}^{KK+} \mathbf{A}^{K+}. \quad (3.64)$$

Combine 3.61 and 3.64, we have the exterior reflection matrix for the single cylinder

$$\begin{bmatrix} \mathbf{B}^{E+} \\ \mathbf{B}^{K+} \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{EE+} & \mathbf{R}^{EK+} \\ \mathbf{R}^{KE+} & \mathbf{R}^{KK+} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{E+} \\ \mathbf{A}^{K+} \end{bmatrix}, \quad (3.65)$$

or

$$\tilde{\mathbf{B}}^+ = \tilde{\mathbf{R}}^+ \tilde{\mathbf{A}}^+. \quad (3.66)$$

$\tilde{\mathbf{R}}^+$ in Eq. 3.66 is the exterior reflection matrix for a single cylinder. Notice each submatrix of the reflection matrix is diagonal, which is the direct result from the fact that the inclusion

Figure 3.3: Internal reflection matrix $\mathbf{R}^-$ relates the outgoing field $\mathbf{B}^-$ and regular field $\mathbf{A}^-$ in a cylinder inclusion. It is assumed that there is no incoming field outside the cylinder.

is perfectly circular. For other inclusion shapes, numerical treatment is need to derive this reflection matrix, and the final matrix will not be diagonal in general.

Now we turn our attention to the internal reflection matrix (see Fig. 3.3). As a microstructured fiber normally has a jacket surrounding the cylindrical inclusions, there should be reflection off of the interface between the jacket and the host material. As there is no source existing outside the jacket boundary, we can treat all $A_m^+ = 0$ in Eqs. 3.51-3.54. With this assumption, we can derive a matrix equation similar to Eq. 3.65 as

$$\begin{bmatrix} \mathbf{A}^{E-} \\ \mathbf{A}^{K-} \end{bmatrix} = \begin{bmatrix} \mathbf{R}^{EE-} & \mathbf{R}^{EK-} \\ \mathbf{R}^{KE-} & \mathbf{R}^{KK-} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{E-} \\ \mathbf{B}^{K-} \end{bmatrix}, \quad (3.67)$$

or

$$\tilde{\mathbf{A}}^- = \tilde{\mathbf{R}}^- \tilde{\mathbf{B}}^-, \quad (3.68)$$

with

$$\begin{aligned} \mathbf{R}^{EE-} &= \text{diag} (R_m^{EE-}), \\ \mathbf{R}^{EK-} &= \text{diag} (R_m^{EK-}), \\ \mathbf{R}^{KE-} &= \text{diag} (R_m^{KE-}), \\ \mathbf{R}^{KK-} &= \text{diag} (R_m^{KK-}). \end{aligned}$$

Reflection coefficients for each order of expansion are

$$\begin{aligned} R_m^{EE-} &= \frac{1}{\delta_m} [(\alpha_{J-H+}^+ - \alpha_{H+J-}^-)(n_-^2 \alpha_{H-H+}^+ - n_+^2 \alpha_{H+H-}^-) - m^2 J_m^- H_m^- H_m^{+2} \tau^2], \\ R_m^{EK-} &= \frac{1}{\delta_m} \left[\frac{2m\tau}{\pi k a} \frac{k_t^+}{k_t^-} H_m^{+2} \right], \\ R_m^{KE-} &= -n_-^2 R_m^{EK-}, \\ R_m^{KK-} &= \frac{1}{\delta_m} [(\alpha_{H-H+}^+ - \alpha_{H+H-}^-)(n_-^2 \alpha_{J-H+}^+ - n_+^2 \alpha_{H+J-}^-) - m^2 J_m^- H_m^- H_m^{+2} \tau^2]. \end{aligned} \quad (3.69)$$

3.2.3 Generalized Rayleigh Identity

In the previous two subsections, we have deduced two equalities according to Rayleigh's argument, as well as the reflection matrices which couples the E and K fields at the

material interfaces. We are ready to put these equations together and express them as a single matrix equation, i.e. the *generalized Rayleigh identity*.

We now re-state four key equations derived before. They are two equalities (Eqs. 3.26 and 3.40)

$$\mathbf{A} = \mathbf{H}\mathbf{B} + \mathbf{J}^{B0}\tilde{\mathbf{A}}^0, \quad (3.70)$$

$$\tilde{\mathbf{B}}^0 = \mathbf{J}^{0B}\mathbf{B}, \quad (3.71)$$

and two reflection matrices (Eqs. 3.66 and 3.89)

$$\tilde{\mathbf{B}}^+ = \tilde{\mathbf{R}}^+\tilde{\mathbf{A}}^+, \quad (3.72)$$

$$\tilde{\mathbf{A}}^- = \tilde{\mathbf{R}}^-\tilde{\mathbf{B}}^-. \quad (3.73)$$

In Eqs. 3.72 and 3.73, the superscripts $+$ and $-$ are indications for the exterior and interior reflections, respectively. The exterior reflection happens for all cylinder inclusions (denoted by l , $l = 1, \dots, N_c$), while the interior reflection happens only for the jacket (denoted by 0). Hence we may write 3.72 and Eqs. 3.73 using the same denotation as in Eqs. 3.70 and 3.71. They become

$$\tilde{\mathbf{B}}^l = \tilde{\mathbf{R}}^l\tilde{\mathbf{A}}^l, \quad (3.74)$$

$$\tilde{\mathbf{A}}^0 = \tilde{\mathbf{R}}^0\tilde{\mathbf{B}}^0. \quad (3.75)$$

From Eq. 3.74, we get the relationship between $\mathbf{A}$ and $\mathbf{B}$ as

$$\mathbf{B} = \mathbf{R}\mathbf{A}, \quad (3.76)$$

where

$$\mathbf{R} = \text{diag}(\tilde{\mathbf{R}}^l). \quad (3.77)$$

Pre-multiply Eq. 3.70 by $\mathbf{R}$, and with the help of Eq. 3.76, we then get

$$\mathbf{B} = \mathbf{R}\mathbf{H}\mathbf{B} + \mathbf{R}\mathbf{J}^{B0}\tilde{\mathbf{A}}^0. \quad (3.78)$$

Substituting 3.71 into 3.75, we eliminate $\tilde{\mathbf{B}}^0$. Further if we substitute Eq. 3.75 into Eq. 3.78, we can eliminate $\tilde{\mathbf{A}}^0$. Finally we arrive at

$$\mathbf{B} = \mathbf{R}\mathbf{H}\mathbf{B} + \mathbf{R}\mathbf{J}^{B0}\tilde{\mathbf{R}}^0\mathbf{J}^{0B}\mathbf{B}. \quad (3.79)$$

or

$$\mathbf{M}\mathbf{B} = 0, \quad (3.80)$$

where

$$\mathbf{M} = \mathbf{I} - \mathbf{R}(\mathbf{H} + \mathbf{J}^{B0}\tilde{\mathbf{R}}^0\mathbf{J}^{0B}), \quad (3.81)$$

and $\mathbf{I}$ is an identity matrix. Notice that if the outer jacket does not exist, the Rayleigh identity becomes

$$(\mathbf{I} - \mathbf{R}\mathbf{H})\mathbf{B} = 0. \quad (3.82)$$

Figure 3.4: Final matrix schematics for Rayleigh method formulation. Matrices are obtained for six-air-hole microstructured optical fiber. Eleven terms ($m = 5$) are used for Fourier-Bessel expansion around each cylinder. (a) $\mathbf{H}$ matrix; (b) $\mathbf{R}$ matrix; (c) $\mathbf{J}^{B0}$ matrix; (d) $\mathbf{J}^{0B}$ matrix; (e) $\hat{\mathbf{R}}^0$ matrix.

The matrix $\mathbf{M}$ has a dimension which depends on the number of cylinder inclusions in the host material, and also the number of expansion terms for field in the vicinity of each cylinder. For example, if the number of inclusions is N_c and number of expansion terms is $2m + 1$ for each cylinder, $\mathbf{M}$ is of size $N \times N$, where $N = (2m + 1) \times 2 \times N_c$. The existence of a jacket does not increase the matrix size. A non-trivial $\mathbf{B}$ solution for Eq. 3.80 requires that the coefficient matrix $\mathbf{M}$ has determinant value of zero. Notice that the matrix elements in $\mathbf{M}$ depend only on β or n_{eff} if the wavelength and dielectric function of the waveguide is given. The problem becomes a root-searching of the function $\mathcal{F}(\beta) = \det(\mathbf{M})$. Different from conventional SIF, modes in MOFs are inherently leaky. Hence we need to search for roots in a complex domain

3.3 Field Calculation

3.3.1 Field in Host Region

After we get the n_{eff} value corresponding to a particular mode in an MOF, we may substitute it back to $\mathbf{M}$ matrix, and matrix Eq. 3.80 becomes

Figure 3.5: External-to-internal transmission matrix $\mathbf{T}^{+-}$ relates the regular fields in the host material $\mathbf{A}^+$ and that in a cylinder inclusion $\mathbf{A}^-$. It is assumed that there is no outgoing field inside the cylinder inclusion.

$$\begin{bmatrix} m_{11} & \cdots & m_{1(N-1)} & m_{1N} \\ \vdots & \ddots & \vdots & \vdots \\ m_{(N-1)1} & \cdots & m_{(N-1)(N-1)} & m_{(N-1)N} \\ m_{N1} & \cdots & m_{N(N-1)} & m_{NN} \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_{(N-1)} \\ B_N \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (3.83)$$

Notice that this equation has an infinite set of $\mathbf{B}$ solutions, which are related to each other by multiplying a constant. We only need one of them. Having understand this, we set B_N to 1. Now the equation suggests we have $N - 1$ unknowns, and there are N linear equations. Any one of the equations can be treated as redundant. If we eliminate the N^{th} equation, and re-arrange the other $N_c - 1$ equations, we arrive at

$$\begin{bmatrix} m_{11} & \cdots & m_{1(N-1)} \\ \vdots & \ddots & \vdots \\ m_{(N-1)1} & \cdots & m_{(N-1)(N-1)} \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_{(N-1)} \end{bmatrix} = \begin{bmatrix} -m_{N1} \\ \vdots \\ -m_{N(N-1)} \end{bmatrix} \quad (3.84)$$

From here, unknown B s can be solved easily by matrix dividing.

Matrix $\bar{\mathbf{B}}_0$, $\bar{\mathbf{A}}_0$ and $\mathbf{A}$ can be deduced using Eq. 3.71, 3.75 and 3.70, respectively. E_z and K_z Fields in the host material can be calculated using Wijngaard's expansion (Eq. 3.4).

3.3.2 Field in Inclusions and Jacket

To calculated fields in the cylinder inclusions and also in the jacket, we have to derive two transmission matrices — the external-to-internal and the internal-to-external transmission matrices.

First of all, let us look at external-to-internal transmission (see Fig. 3.5). Like what we did for the reflection matrices, we treat a single cylinder in isolation, and assume the refractive index inside the cylinder is n_- and that outside n_+ . We still begin with the field continuity conditions across an interface, which are represented by Eqs. 3.51-3.54. Since

Figure 3.6: Internal-to-External transmission matrix $\mathbf{T}^{-+}$ associates the outgoing fields in the host material $\mathbf{B}^+$ and that in a cylinder inclusion $\mathbf{B}^-$. It is assumed that there is no regular field outside the cylinder.

there is no source existing inside a cylinder inclusion, we let B_m^{E-} and B_m^{K-} be zero. After we eliminate B_m^{E+} and B_m^{K+} , we can finally have

$$\begin{bmatrix} \mathbf{A}^{E-} \\ \mathbf{A}^{K-} \end{bmatrix} = \begin{bmatrix} \mathbf{T}^{EE+-} & \mathbf{T}^{EK+-} \\ \mathbf{T}^{KE+-} & \mathbf{T}^{KK+-} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{E+} \\ \mathbf{A}^{K+} \end{bmatrix}, \quad (3.85)$$

or

$$\tilde{\mathbf{A}}^- = \tilde{\mathbf{T}}^{-+} \tilde{\mathbf{A}}^+, \quad (3.86)$$

with

$$\begin{aligned} \mathbf{T}^{EE+-} &= \text{diag} (T_m^{EE+-}), \\ \mathbf{T}^{EK+-} &= \text{diag} (T_m^{EK+-}), \\ \mathbf{T}^{KE+-} &= \text{diag} (T_m^{KE+-}), \\ \mathbf{T}^{KK+-} &= \text{diag} (T_m^{KK+-}). \end{aligned}$$

Superscript $+-$ means the transmission happens from n_+ (outside cylinder) material to n_- material (inside cylinder). Transmission coefficients for each order of expansion are

$$\begin{aligned} T_m^{EE+-} &= \frac{1}{\delta_m} [n_+^2 (\alpha_{J+H+}^- - \alpha_{H+J+}^-) (\alpha_{H+J-}^- - \alpha_{J-H+}^+)], \\ T_m^{EK+-} &= \frac{1}{\delta_m} [im\tau J_m^- H_m^+ (\alpha_{H+J+}^- - \alpha_{J+H+}^-)], \\ T_m^{KE+-} &= n_+^2 T_m^{EK+-}, \\ T_m^{KK+-} &= -\frac{1}{\delta_m} [(\alpha_{J+H+}^- - \alpha_{H+J+}^-) (n_-^2 \alpha_{J-H+}^+ - n_+^2 \alpha_{H+J-}^-)], \end{aligned} \quad (3.87)$$

where δ_m , α_{J+H+}^- etc are defined in the same way as what we did for the reflection matrices.

For derivation of the internal-to-external transmission matrix (Fig. 3.6), we can treat A_m^{E+} and A_m^{K+} as zero. Similarly we can have

$$\begin{bmatrix} \mathbf{B}^{E+} \\ \mathbf{B}^{K+} \end{bmatrix} = \begin{bmatrix} \mathbf{T}^{EE-+} & \mathbf{T}^{EK-+} \\ \mathbf{T}^{KE-+} & \mathbf{T}^{KK-+} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{E+} \\ \mathbf{B}^{K+} \end{bmatrix}, \quad (3.88)$$

or

$$\bar{\mathbf{B}}^+ = \bar{\mathbf{T}}^{-+} \bar{\mathbf{B}}^-, \quad (3.89)$$

with

$$\begin{aligned} \mathbf{T}^{EE-+} &= \text{diag} (T_m^{EE-+}), \\ \mathbf{T}^{EK-+} &= \text{diag} (T_m^{EK-+}), \\ \mathbf{T}^{KE-+} &= \text{diag} (T_m^{KE-+}), \\ \mathbf{T}^{KK-+} &= \text{diag} (T_m^{KK-+}). \end{aligned}$$

Transmission coefficients for each order of expansion are

$$\begin{aligned} T_m^{EE-+} &= \frac{1}{\delta_m} [n_-^2 (\alpha_{J-H-}^+ - \alpha_{H-J-}^-) (\alpha_{H+J-}^- - \alpha_{J-H+}^+)], \\ T_m^{EK-+} &= \frac{1}{\delta_m} [im\tau J_m^- H_m^+ (\alpha_{H-J-}^+ - \alpha_{J-H-}^+)], \\ T_m^{KE-+} &= -n_-^2 T_m^{EK-+}, \\ T_m^{KK-+} &= \frac{1}{\delta_m} [(\alpha_{H-J-}^+ - \alpha_{J-H-}^+) (n_-^2 \alpha_{J-H+}^+ - n_+^2 \alpha_{H+J-}^-)]. \end{aligned} \quad (3.90)$$

At this stage, we can calculate regular field expansion coefficients in a cylinder inclusion (l) as

$$\bar{\mathbf{A}}^{l-} = \bar{\mathbf{T}}^{l+-} \bar{\mathbf{A}}^{l+}, \quad (3.91)$$

or globally (for all cylinder inclusions $l = 1 \cdots N_c$)

$$\mathbf{A}^- = \mathbf{T}^{+-} \mathbf{A}^+, \quad (3.92)$$

and the E_z field in the cylinder inclusion can be calculated using

$$E_z(r_l, \theta_l) = \sum_m A_m^{El-} J_m(k_t^{l-} r_l) \exp(im\theta_l). \quad (3.93)$$

Notice the field in a cylinder does not contain any outgoing field terms. The K_z field in a cylinder can be calculated in exactly the same way. In the jacket, the outgoing field expansion coefficients can be calculated as

$$\bar{\mathbf{B}}^{0+} = \bar{\mathbf{T}}^{0-+} \bar{\mathbf{B}}^{0-}, \quad (3.94)$$

hence the E_z field there can be calculated as

$$E_z^+(r_0, \theta) = \sum_m B_m^{E0+} H_m(k_t^{0+} r_0) \exp(im\theta_0), \quad (3.95)$$

K_z fields in the jacket can be calculated in the same way.

3.3.3 Other Field Components

Now we have the E_z and K_z fields in the whole transverse domain, the E_θ , K_θ , E_r , and K_r fields can be calculated using Eqs. 3.48 and 3.49, as well as

$$E_r(r, \theta) = \frac{i}{k_t^2} \left(\beta \frac{\partial E_z}{\partial r} + \frac{k}{r} \frac{\partial K_z}{\partial \theta} \right), \quad (3.96)$$

$$K_r(r, \theta) = \frac{i}{k_t^2} \left(\beta \frac{\partial K_z}{\partial r} - \frac{kn^2}{r} \frac{\partial E_z}{\partial \theta} \right). \quad (3.97)$$

Attention should be paid to different coordinate systems. We calculate the E_θ , K_θ and E_r , K_r fields for each homogeneous refractive index region, in that region's polar coordinate. To sum up, in the host material, E_θ , K_θ , E_r and K_r contributed by each cylinder is

$$E_\theta^l = -\frac{\beta}{k_t^{e2} r_l} \sum_m m B_m^{El} H_m(k_t^e r_l) \exp(im\theta_l) - \frac{ik}{k_t^e} \sum_m B_m^{Kl} H'_m(k_t^e r_l) \exp(im\theta_l), \quad (3.98)$$

$$K_\theta^l = -\frac{\beta}{k_t^{e2} r_l} \sum_m m B_m^{Kl} H_m(k_t^e r_l) \exp(im\theta_l) + \frac{ikn_e^2}{k_t^e} \sum_m B_m^{El} H'_m(k_t^e r_l) \exp(im\theta_l), \quad (3.99)$$

$$E_r^l = \frac{i\beta}{k_t} \sum_m m B_m^{El} H'_m(k_t^e r_l) \exp(im\theta_l) - \frac{k}{k_t^{e2} r_l} \sum_m B_m^{Kl} H_m(k_t^e r_l) \exp(im\theta_l), \quad (3.100)$$

$$K_r^l = \frac{i\beta}{k_t} \sum_m m B_m^{Kl} H'_m(k_t^e r_l) \exp(im\theta_l) + \frac{kn_e^2}{k_t^{e2} r_l} \sum_m B_m^{El} H_m(k_t^e r_l) \exp(im\theta_l). \quad (3.101)$$

The four fields due to the jacket reflection are

$$E_\theta^0 = -\frac{\beta}{k_t^{e2} r_0} \sum_m m A_m^{E0} J_m(k_t^e r_0) \exp(im\theta_0) - \frac{ik}{k_t^e} \sum_m A_m^{K0} J'_m(k_t^e r_0) \exp(im\theta_0), \quad (3.102)$$

$$K_\theta^0 = -\frac{\beta}{k_t^{e2} r_0} \sum_m m A_m^{K0} J_m(k_t^e r_0) \exp(im\theta_0) + \frac{ikn_e^2}{k_t^e} \sum_m A_m^{E0} J'_m(k_t^e r_0) \exp(im\theta_0), \quad (3.103)$$

$$E_r^0 = \frac{i\beta}{k_t} \sum_m m A_m^{E0} J'_m(k_t^e r_0) \exp(im\theta_0) - \frac{k}{k_t^{e2} r_0} \sum_m A_m^{K0} J_m(k_t^e r_0) \exp(im\theta_0), \quad (3.104)$$

$$K_r^0 = \frac{i\beta}{k_t} \sum_m m A_m^{K0} J'_m(k_t^e r_0) \exp(im\theta_0) + \frac{kn_e^2}{k_t^{e2} r_0} \sum_m A_m^{E0} J_m(k_t^e r_0) \exp(im\theta_0), \quad (3.105)$$

E_θ , K_θ and E_r , K_r fields in each cylinder inclusion are

$$E_\theta^l = -\frac{\beta}{k_t^{l2} r_l} \sum_m m A_m^{El} J_m(k_t^l r_l) \exp(im\theta_l) - \frac{ik}{k_t^l} \sum_m A_m^{Kl} J'_m(k_t^l r_l) \exp(im\theta_l), \quad (3.106)$$

$$K_\theta^l = -\frac{\beta}{k_t^{l2} r_l} \sum_m m A_m^{Kl} J_m(k_t^l r_l) \exp(im\theta_l) + \frac{ikn_l^2}{k_t^l} \sum_m A_m^{El} J'_m(k_t^l r_l) \exp(im\theta_l), \quad (3.107)$$

$$E_r^l = \frac{i\beta}{k_t} \sum_m A_m^{El} J'_m(k_t^l r_l) \exp(im\theta_l) - \frac{k}{k_t^2 r_l} \sum_m m A_m^{Kl} J_m(k_t^l r_l) \exp(im\theta_l), \quad (3.108)$$

$$K_r^l = \frac{i\beta}{k_t} \sum_m A_m^{Kl} J'_m(k_t^l r_l) \exp(im\theta_l) + \frac{kn_l^2}{k_t^2 r_l} \sum_m m A_m^{El} J_m(k_t^l r_l) \exp(im\theta_l), \quad (3.109)$$

As we are normally interested in field components decomposed in Cartesian coordinate system, *i.e.*, E_x , K_x and E_y , K_y , we may convert the fields using the equations

$$F_\theta^x = F_\theta \cos(\theta_l + \frac{\pi}{2}) = -F_\theta \sin(\theta_l), \quad (3.110)$$

$$F_\theta^y = F_\theta \sin(\theta_l + \frac{\pi}{2}) = F_\theta \cos(\theta_l), \quad (3.111)$$

$$F_r^x = F_r \cos(\theta_l), \quad (3.112)$$

$$F_r^y = F_r \sin(\theta_l), \quad (3.113)$$

where F denotes either the electric field E or the scaled magnetic field K . θ_l is the angle of the vector pointing from the origin to the point P in the l^{th} cylinder's polar coordinate (Fig. 3.1). F_θ^x means the x -component of the azimuthal field. We then have F_x and F_y at point P as

$$F_x = F_\theta^x + F_r^x, \quad (3.114)$$

$$F_y = F_\theta^y + F_r^y. \quad (3.115)$$

3.4 Numerical Results

The microstructured fiber we are considering in this section is borrowed from Ref. [2]. It only consists of six air holes. $\Lambda = 6.75\mu\text{m}$ and $d = 5\mu\text{m}$, as shown in Fig. 3.7. Silica and air indices are assumed to be 1.45 and 1, respectively.

Figure 3.7: A simple MOF. Hatched regions are air. Host is silica.

The determinant scan at $\lambda = 1.45\mu\text{m}$ is shown in Fig. 3.8. The scan is done by calculating the determinant of the characteristic matrix $\mathbf{M}$ at a range of input n_{eff} values. The imaginary part of this n_{eff} value is treated as zero for this scan. m is set to 5. The roots of the function $\mathcal{F}(n_{\text{eff}}) = \det(\mathbf{M})$ are denoted by the dips observed in Fig. 3.8. Hence the n_{eff} index value, and therefore the propagation constant β , of a mode can be found by searching for the exact location of a dip. Notice that the searching is not limited to the real n_{eff} values. An imaginary part of the n_{eff} , which denotes the radiation loss of the mode, should also be determined at the same time.

Figure 3.8: Determinant scan for the MOF shown in Fig. 3.7.

m	$\Re(n_{\text{eff}})$	$\Im(n_{\text{eff}})$
1	1.446106080216	5.5143585×10^{-5}
2	1.445568954412	1.6419003×10^{-6}
3	1.445421695383	2.9336203×10^{-7}
4	1.445397800570	2.7902439×10^{-8}
5	1.445395346371	3.1506891×10^{-8}
6	1.445395234644	3.1938617×10^{-8}
7	1.445395234033	3.1939333×10^{-8}
8	1.445395232474	3.1944859×10^{-8}
9	1.445395232145	3.1945548×10^{-8}
10	1.445395232132	3.1945535×10^{-8}

Table 3.1: Calculated n_{eff} value as m increases.

In Table 3.1, we have shown that the n_{eff} value quickly converges as the multipole expansion order, represented by m , increases. At $m = 5$, the real part is accurate up to seven significant digits, and the imaginary part is accurate up to two significant digits. These values are adequate enough for calculating the group velocity dispersion (GVD) and leakage loss properties of the MOF. At $m = 10$, the real part is accurate up to 11 significant digits and the imaginary is accurate up to 6 significant digits. The data presented in the convergence table is also plotted in Fig. 3.9 for a more straightforward reading.

Effective modal indices for the first seven modes are given in Table 3.2. Their corresponding McIsaac mode classes [53] as well as their degeneracy properties are also included in the table. Classification and degeneracy of modes in a complex waveguide are discussed in detail in [53]. Longitudinal and transverse field distributions of the seven modes are shown in Fig. 3.10 and 3.11, respectively. These mode patterns indicate that we can calculate modes using only a portion of the waveguide cross section with the help of appropriate boundary conditions [53].

3.5 Conclusion

MEM is a both physically and mathematically beautiful method. Its accuracy has been confirmed by a full-vector beam propagation method in [54]. As the material interfaces

Figure 3.9: Convergence test of the fundamental mode effective index. (a) Real part; (b) Imaginary part.

	$\Re(n_{\text{eff}})$	$\Im(n_{\text{eff}})$	Class	Degeneracy
1	1.445395345	3.150×10^{-8}	1, 2	2
2	1.438585801	4.986×10^{-7}	2	1
3	1.438445842	9.929×10^{-7}	5, 6	2
4	1.438366726	1.374×10^{-6}	1	1
5	1.430414400	2.220×10^{-5}	8	1
6	1.429969400	1.577×10^{-5}	3, 4	2
7	1.429255296	9.337×10^{-6}	7	1

Table 3.2: First seven modes found for the PCF shown in Fig. 3.7.

are all circular, a small number of expansion terms is required to get acceptable convergence. Significant improvement in efficiency is possible when the symmetry condition is considered, in which case only one air hole is involved in deriving the final matrix equation for an MOF exhibiting C_6 symmetry. In our derivation, we have used the same number of expansion terms for each cylinder inclusion. In fact, as pointed out by White *et al.*, we can assign priority to each air hole inclusion. Less expansion terms are used for those holes far away from the core region as they could have little interaction with the propagating modes confined in the core region. In this way, the problem size can be further reduced.

However, MEM has its disadvantages. First, it can only deal with circular air holes. This greatly limits its applicability due to the fact that only a very small fraction of MOFs fabricated have circular air holes. Second, MEM's convergence can be greatly challenged when circular inclusions are close to each other. Under such a situation, more expansion terms are necessary to get an accurate solution.

MEM has been very recently extended to deal with MOFs with non-circular air holes [55]. In the newly proposed method, the reflection matrix for each hole inclusion is calculated numerically.

Figure 3.10: E_z field plots for first seven modes found for PCF shown in Fig. 3.7. (a) HE_{11}^y mode (notice it has one degenerate HE_{11}^x mode which possesses the same modal effective index); (b) TE mode; (c) HE_{21} mode; (d) TM mode; (e) HE_{31}^1 mode; (f) EH_{11} mode; (g) HE_{31}^2 mode.

Figure 3.11: Transverse electric field distributions for first seven modes found for PCF shown in Fig. 3.7. (a) HE_{11}^y mode (notice it has one degenerate HE_{11}^x mode which possesses the same modal effective index); (b) TE mode; (c) HE_{21} mode; (d) TM mode; (e) HE_{31}^1 mode; (f) EH_{11} mode; (g) HE_{31}^2 mode.

Chapter 4

Finite Element Method

4.1 Brief Introduction

Figure 4.1 shows the cross-section of a general regular dielectric optical waveguide with a piecewise homogeneous index profile. Strictly speaking, modes supported by such an all-dielectric waveguide, or open waveguide, should be solved by considering the infinite cross-sectional domain. However, to numerically derive the modes, one has to truncate the infinite domain to a finite one, whose boundary is indicated by Γ_0 in Fig. 4.1. The finite-element method (FEM) discretizes this finite domain into a mesh formed by triangles or/and quadrangles (elements). The wave equation is then enforced separately on each element of the mesh, with discrete unknowns defined either on the element vertices or on the element edges. The final modal solution can be solved by considering the collective contributions by all elements.

Figure 4.1: Cross-section of a general all-dielectric optical waveguide.

Waveguide mode solvers that employ the FEM method can be traced back to the early 70s in the last century. However, difficulty arises when people try to apply their existing FEM codes to MOFs. The difficulty lies mainly in the boundary condition (Γ_0 in Fig. 4.1). In traditional FEM formulations, the computational domain is terminated using either *perfect electric conductor* (PEC), *perfect magnetic conductor* (PMC), or simply the zero boundary condition. These boundary conditions are adequate for deriving bound

modes, like those propagating in a conventional step-index fiber (SIF). But as almost all modes in MOFs are leaky, the above-mentioned termination methods will cause reflection off the boundary, which will give rise to errors. In fact, leakage loss (or radiation loss) is a key property of PBG-guiding MOFs. Being able to calculate radiation loss is very meaningful in designing and characterizing a PBG-guiding MOF. During recent years, a few full-vector FEM mode solvers have been proposed to explicitly deal with leakage loss of MOFs. Notably, there are three versions.

First, an edge-element-based FEM was proposed by Selleri *et al.* [9]. A *perfectly matched layer* (PML¹) is added outside the computational domain, which is further terminated by a zero boundary condition. Their formulation defines the axial field unknowns on the nodes and the transverse field unknowns on the edges. The deployment of edge-elements² helps to prevent the spurious mode problem as well as to eliminate interface integrations. The final matrix equation is derived through a variational approach, and can be expressed in terms of either the electric or the magnetic field. As both transverse and longitudinal field components are involved, the number of unknowns is $N_e + N_n$ for formulation using the first-order shape function, where N_e and N_n are the number of edges and nodes, respectively. Normally $N_e \approx 2N_n$.

Second, Saitoh *et al.* introduced a hybrid FEM in which unknowns are defined on both edge elements and node elements of the mesh (they also claims that the edge elements are curvilinear) [7]. For each triangular element, it imposes eight transverse field unknowns and six axial field unknowns. The total number of unknowns is approximately $2N_e + 3N_n$. Similar to Selleri *et al.*, they employ an anisotropic PML as an absorbing layer. The imaginary-distance beam propagation method was originally proposed to solve for modes [7]. Later they re-formulated the method for deriving modes through a generalized eigenvalue equation [60].

Third, Uranus *et al.* introduced a node-element-based FEM [61] in which unknowns are the transverse magnetic fields H_x and H_y . The number of unknowns is only $2N_n$ if a first-order shape function is used. Their method employs a transparent boundary condition where the radiation field outside the computation domain is approximated using analytical functions. We should mention that this method is almost identical to that proposed by Hernández-Figueroa *et al.* [62], except that Uranus *et al.* have used a radiation boundary condition that is one-order more accurate. The advantage of the radiation boundary condition over PML is that it can reduce the number of unknowns significantly. This is due to the fact that it does not require the boundary line to be of rectangular shape. Though such a boundary condition requires an iterative process to converge its eigen-solution, a good convergence is normally achieved in less than five iterations (a good initial guess usually reduces the number of iterations to one or two). As this method reduces the number of unknowns by at least one third as compared to the previous two

¹PML is a layer of artificial lossy material that absorbs EM waves in any frequency and impinging angle. It was proposed by Berenger [56] for truncating problem domain in *finite-difference time-domain* (FDTD) method. The originally proposed PML needs modification of Maxwell equations, which is not directly applicable to existing FEM codes. Later Pekel *et al.* [57] and Sacks *et al.* [58] re-formulated PML as diagonally anisotropic material. Their work puts PML together with FEM beautifully.

²Such edge-element formulation was first proposed by Lee *et al.* [59].

methods, hence it is very suitable for users who are constrained by their hardwares, *i.e.*, CPU and RAM. Also the method is spurious-mode-free, since the magnetic field divergence condition, $\nabla \cdot \mathbf{H} = 0$, is considered explicitly in deriving the final matrix equation.

All of the above-mentioned FEMs lead to a generalized eigenvalue equation, via either the Galerkin or the variational approach, as

$$\mathbf{A}\Phi = \beta^2 \mathbf{B}\Phi. \quad (4.1)$$

where $\mathbf{A}$ and $\mathbf{B}$ are finite element matrices, Φ (eigenvector) is a vector comprising field unknowns defined on the triangular mesh, the β^2 (eigenvalue) is the propagation constant to be solved.

It is quite obvious that the third FEM used by Uranus *et al.* is the most efficient as the number of unknowns involved is significantly less than those in the other two FEM versions. Hence in the following, we will introduce this method in full. We will also show how the reflection symmetry of a waveguide can be utilized to reduce the problem domain to one of its quadrants. An numerical example on a complex air-silica Bragg fiber will be present in the end.

4.2 Galerkin Approach

The Galerkin method is a special type of *weighted residual method*. In reference [61], Uranus *et al.* derive the final functional through the decomposed Maxwell equations in Cartesian coordinate. However, in this section, we follow a vectorial derivation procedure, which was also adopted in [63, 62]. The vector approach leads to the final functional significantly faster.

We consider waveguides that are composed by diagonally anisotropic materials. That is, the epsilon function can be written as

$$\hat{\epsilon} = \hat{\epsilon}_{tt} + \epsilon_{zz}\hat{z}\hat{z}, \quad (4.2)$$

where

$$\hat{\epsilon}_{tt} = \epsilon_{xx}\hat{x}\hat{x} + \epsilon_{xy}\hat{x}\hat{y} + \epsilon_{yx}\hat{y}\hat{x} + \epsilon_{yy}\hat{y}\hat{y}. \quad (4.3)$$

As a fiber is invariant along the z direction, a propagating mode should have a z -dependence of $\exp(j\beta z)$. By considering Eq. 4.2, the vector wave equation 1.28 can be decomposed into two equations — one for the transverse vector component and one for the longitudinal vector component. The first one is written as

$$\nabla_t \times (\epsilon_{zz}^{-1} \nabla_t \times \mathbf{H}_t) - j\beta \hat{z} \times [\hat{\epsilon}_{tt}^{-1} \cdot (\nabla_t \times \hat{z} H_z)] - \beta^2 \hat{z} \times [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{H}_t)] - k_0^2 \mathbf{H}_t = 0. \quad (4.4)$$

From the divergence theorem, *i.e.* Eq. 1.7, we have

$$H_z = -\frac{\nabla_t \cdot \mathbf{H}_t}{j\beta}. \quad (4.5)$$

If we substitute Eq. 4.5 to Eq. 4.4, we have an equation free of H_z , i.e.,

$$\nabla_t \times (\epsilon_{zz}^{-1} \nabla_t \times \mathbf{H}_t) - \hat{z} \times [\hat{\epsilon}_{tt}^{-1} \cdot \nabla_t \times (\hat{z} \nabla_t \cdot \mathbf{H}_t)] - \beta^2 \hat{z} \times [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{H}_t)] - k_0^2 \mathbf{H}_t = 0. \quad (4.6)$$

Now we apply the Galerkin procedure term-by-term to Eq. 4.6. For the first term, by multiplying (dot product for a vector) it with a weight function $\mathbf{w}_t$ and integrating across the whole cross-section, we have

$$\begin{aligned} & \int_{\Omega} \nabla_t \times (\epsilon_{zz}^{-1} \nabla_t \times \mathbf{H}_t) \cdot \mathbf{w}_t d\Omega \\ &= \int_{\Omega} \epsilon_{zz}^{-1} (\nabla_t \times \mathbf{w}_t) \cdot (\nabla_t \times \mathbf{H}_t) d\Omega - \oint_{\Gamma} \epsilon_{zz}^{-1} (\mathbf{w}_t \times \nabla_t \times \mathbf{H}_t) \cdot \hat{n} d\Gamma \\ &= \int_{\Omega} \epsilon_{zz}^{-1} (\nabla_t \times \mathbf{w}_t) \cdot (\nabla_t \times \mathbf{H}_t) d\Omega + \oint_{\Gamma} \epsilon_{zz}^{-1} (\nabla_t \times \mathbf{H}_t) \cdot (\mathbf{w}_t \times \hat{n}) d\Gamma. \end{aligned} \quad (4.7)$$

We have used during the derivation the *First Vector Green's Theorem*, i.e.,

$$\int_{\Omega} [u(\nabla \times \mathbf{a}) \cdot (\nabla \times \mathbf{b}) - \mathbf{a} \cdot (\nabla \times u \nabla \times \mathbf{b})] d\Omega = \oint_{\Gamma} u(\mathbf{a} \times \nabla \times \mathbf{b}) \cdot \hat{n} d\Gamma. \quad (4.8)$$

For the second term in Eq. 4.6, after applying weight function and performing integration, we have

$$\begin{aligned} & - \int_{\Omega} \hat{z} \times [\hat{\epsilon}_{tt}^{-1} \cdot \nabla_t \times (\hat{z} \nabla_t \cdot \mathbf{H}_t)] \cdot \mathbf{w}_t d\Omega \\ &= - \int_{\Omega} (\mathbf{w}_t \times \hat{z}) \cdot [\hat{\epsilon}_{tt}^{-1} \cdot \nabla_t \times (\hat{z} \nabla_t \cdot \mathbf{H}_t)] d\Omega \\ &= \int_{\Omega} [\nabla_t \times (\hat{z} \nabla_t \cdot \mathbf{H}_t)] \cdot [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{w}_t)] d\Omega \\ &= \int_{\Omega} \hat{z} \nabla_t \cdot \mathbf{H}_t \cdot \left\{ \nabla_t \times [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{w}_t)] \right\} d\Omega \\ & \quad + \oint_{\Gamma} \hat{z} \nabla_t \cdot \mathbf{H}_t \times [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{w}_t)] \cdot \hat{n} d\Gamma, \end{aligned} \quad (4.9)$$

where, we have used the simplified *First Vector Green's Theorem*, i.e.,

$$\int_{\Omega} \nabla \times \mathbf{a} \cdot \mathbf{b} d\Omega = \int_{\Omega} \mathbf{a} \cdot (\nabla \times \mathbf{b}) d\Omega + \oint_{\Gamma} \mathbf{a} \times \mathbf{b} \cdot \hat{n} d\Gamma. \quad (4.10)$$

For the third term in Eq. 4.6, we have

$$\begin{aligned} & - \int_{\Omega} \beta^2 \hat{z} \times [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{H}_t)] \cdot \mathbf{w}_t d\Omega \\ &= - \int_{\Omega} \beta^2 [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{H}_t)] \cdot (\mathbf{w}_t \times \hat{z}) d\Omega \\ &= \beta^2 \int_{\Omega} (\hat{z} \times \mathbf{w}_t) \cdot [\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{H}_t)] d\Omega. \end{aligned} \quad (4.11)$$

For the final term in Eq. 4.6, we have

$$- \int_{\Omega} k_0^2 \mathbf{H}_t \cdot \mathbf{w}_t d\Omega. \quad (4.12)$$

Putting all terms together, we have

$$\begin{aligned}
& \int_{\Omega} \epsilon_{zz}^{-1} (\nabla_t \times \mathbf{w}_t) \cdot (\nabla_t \times \mathbf{H}_t) d\Omega + \oint_{\Gamma} \epsilon_{zz}^{-1} (\nabla_t \times \mathbf{H}_t) \cdot (\mathbf{w}_t \times \hat{n}) d\Gamma \\
& + \int_{\Omega} \hat{z} \nabla_t \cdot \mathbf{H}_t \cdot \left\{ \nabla_t \times \left[\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{w}_t) \right] \right\} d\Omega \\
& + \oint_{\Gamma} \hat{z} \nabla_t \cdot \mathbf{H}_t \times \left[\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{w}_t) \right] \cdot \hat{n} d\Gamma \\
& + \beta^2 \int_{\Omega} (\hat{z} \times \mathbf{w}_t) \cdot \left[\hat{\epsilon}_{tt}^{-1} \cdot (\hat{z} \times \mathbf{H}_t) \right] d\Omega \\
& - \int_{\Omega} k_0^2 \mathbf{H}_t \cdot \mathbf{w}_t d\Omega = 0.
\end{aligned} \tag{4.13}$$

On each FEM mesh element, the unknown function (H_x and H_y field distributions in our case) is assumed to be well approximated by a simple function (linear, quadratic function etc). Hence integration and differentiation of the field over the whole domain, most often inhomogeneous, can be calculated as a sum of integrations and differentiations over small, homogeneous elements. The integrations and differentiations over a simple domain (e.g. a triangle) have analytical forms, which facilitates easy implementation of the algorithm on a computer.

Equation 4.13 can be discretized into

$$\begin{aligned}
& \sum_{\text{TE}} \int_{\Omega_e} \left\{ \frac{1}{\epsilon_{rzz}} \left(\frac{\partial H_y}{\partial x} \right)^2 - \frac{1}{\epsilon_{rzz}} \frac{\partial H_y}{\partial x} \frac{\partial H_x}{\partial y} - \frac{1}{\epsilon_{rzz}} \frac{\partial H_x}{\partial y} \frac{\partial H_y}{\partial x} + \frac{1}{\epsilon_{rzz}} \left(\frac{\partial H_x}{\partial y} \right)^2 \right. \\
& \quad + \frac{1}{\epsilon_{ryy}} \left(\frac{\partial H_x}{\partial x} \right)^2 + \frac{1}{\epsilon_{ryy}} \frac{\partial H_x}{\partial x} \frac{\partial H_y}{\partial y} + \frac{1}{\epsilon_{rxx}} \frac{\partial H_y}{\partial y} \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{rxx}} \left(\frac{\partial H_y}{\partial y} \right)^2 \\
& \quad \left. + n_{\text{eff}}^2 \left(\frac{k_0^2}{\epsilon_{ryy}} H_x^2 + \frac{k_0^2}{\epsilon_{rxx}} H_y^2 \right) - k_0^2 H_x^2 - k_0^2 H_y^2 \right\} d\Omega \\
& + \sum_{\text{IE}} \left\{ - \int_{\Gamma_e^1} \left(\frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_y}{\partial y} \right) dy + \int_{\Gamma_e^1} \left(\frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_y}{\partial y} \right) dx \right\} \\
& + \sum_{\text{BE}} \left\{ - \int_{\Gamma_e^2} \left(\frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_y}{\partial x} - \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_x}{\partial y} \right) dy - \int_{\Gamma_e^2} \left(\frac{1}{\epsilon_{rzz}} H_x \frac{\partial H_y}{\partial x} - \frac{1}{\epsilon_{rzz}} H_x \frac{\partial H_x}{\partial y} \right) dx \right. \\
& \quad \left. - \int_{\Gamma_e^2} \left(\frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_y}{\partial y} \right) dy + \int_{\Gamma_e^2} \left(\frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_y}{\partial y} \right) dx \right\} \\
& = 0, \tag{4.14}
\end{aligned}$$

where TE, IE, BE stand for triangular, interface, and boundary element, respectively. Notice, as implied by the Galerkin method, we have let $\mathbf{w}_t = [H_x, H_y]^T$ during the discretization. H_x and H_y are field values to be deduced. Their value over a particular triangular element can be expanded as

$$H_x = \mathbf{N}_j^T \mathbf{H}_{xj}, \tag{4.15}$$

$$H_y = \mathbf{N}_j^T \mathbf{H}_{yj}, \tag{4.16}$$

Figure 4.2: A triangle element used for first order shape function approximation.

where N_j is the so-called *basis functions*, or *shape functions*, and H_{xj} and H_{yj} are field values at certain positions on the triangular element, which are normally chosen at the vertices. For approximations using higher-order shape function, more positions need to be assigned. The final problem becomes solving field values at these special positions. Field values at other positions can be interpolated from the field values at these special points together with the shape functions. Intuitively, the finer the mesh, the more accurate the result is.

4.3 Formulation with First-order Shape Function

In the formulation with the first-order shape function, the unknown field is assumed to have a linear distribution over a domain element, *i.e.*, the field surface is flat over an element. We know that three points are adequate to define a flat surface. Hence the problem of finding field values over a continuous element domain becomes finding field values at three points.

For a general triangular element shown in Fig. 4.2, the vectors in Eqs. 4.15 and 4.16 can be written as

$$\mathbf{N}_j = \{N_1 \ N_2 \ N_3\}^T, \quad (4.17)$$

$$\mathbf{H}_{xj} = \{H_{x1} \ H_{x2} \ H_{x3}\}^T, \quad (4.18)$$

$$\mathbf{H}_{yj} = \{H_{y1} \ H_{y2} \ H_{y3}\}^T. \quad (4.19)$$

H_{x1} , H_{x2} , and H_{x3} are H_x field values at triangle's vertices 1, 2, and 3, respectively, and N_1 , N_2 , and N_3 are basis functions for the expansion, which can be expressed in terms of the triangular's area coordinate L_1 , L_2 , and L_3 (refer to [1] for definition of area coordinate).

Figure 4.3: A line element used for first order shape function approximation.

In the case of line element (Fig. 4.3), the vectors in Eqs. 4.15 and 4.16 can be written as

$$\mathbf{N}_j = \{N_1 \ N_2\}^T, \quad (4.20)$$

$$\mathbf{H}_{xj} = \{H_{x1} \ H_{x2}\}^T, \quad (4.21)$$

$$\mathbf{H}_{yj} = \{H_{y1} \ H_{y2}\}^T. \quad (4.22)$$

H_{x1} and H_{x2} are H_x field values at the line's two vertices 1 and 2, respectively. And N_1 and N_2 are basis functions for the expansion, which can be expressed in terms of the line element's length coordinate L_1 and L_2 .

For *each* area integration in equation 4.14, with substitution of Eqs. 4.15 and 4.16, we can have

$$\begin{aligned} \int_{\Omega_e} \left\{ \frac{1}{\epsilon_{rzz}} \mathbf{H}_{yj}^T \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial x} \mathbf{H}_{yj} - \frac{1}{\epsilon_{rzz}} \mathbf{H}_{yj}^T \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial y} \mathbf{H}_{xj} - \frac{1}{\epsilon_{rzz}} \mathbf{H}_{xj}^T \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial x} \mathbf{H}_{yj} \right. \\ + \frac{1}{\epsilon_{rzz}} \mathbf{H}_{xj}^T \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial y} \mathbf{H}_{xj} + \frac{1}{\epsilon_{ryy}} \mathbf{H}_{xj}^T \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial x} \mathbf{H}_{xj} + \frac{1}{\epsilon_{ryy}} \mathbf{H}_{xj}^T \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial y} \mathbf{H}_{yj} \\ + \frac{1}{\epsilon_{rxx}} \mathbf{H}_{yj}^T \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial x} \mathbf{H}_{xj} + \frac{1}{\epsilon_{rxx}} \mathbf{H}_{yj}^T \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial y} \mathbf{H}_{yj} + n_{\text{eff}}^2 \left(\frac{k_0^2}{\epsilon_{ryy}} \mathbf{H}_{xj}^T \mathbf{N}_j \mathbf{N}_j^T \mathbf{H}_{xj} \right. \\ \left. \left. + \frac{k_0^2}{\epsilon_{rxx}} \mathbf{H}_{yj}^T \mathbf{N}_j \mathbf{N}_j^T \mathbf{H}_{yj} \right) - k_0^2 \mathbf{H}_{xj} \mathbf{N}_j \mathbf{N}_j^T \mathbf{H}_{xj} - k_0^2 \mathbf{H}_{yj} \mathbf{N}_j \mathbf{N}_j^T \mathbf{H}_{yj} \right\} d\Omega = 0. \quad (4.23) \end{aligned}$$

Equation 4.23 can be decomposed into two equations.

$$\begin{aligned} \mathbf{H}_{xj}^T \int_{\Omega_e} \left\{ \frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial y} + \frac{1}{\epsilon_{ryy}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial x} - k_0^2 \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega \mathbf{H}_{xj} \\ + \mathbf{H}_{xj}^T \int_{\Omega_e} \left\{ -\frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial x} + \frac{1}{\epsilon_{ryy}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial y} \right\} d\Omega \mathbf{H}_{yj} \\ = -\mathbf{H}_{xj}^T n_{\text{eff}}^2 \int_{\Omega_e} \left\{ \frac{k_0^2}{\epsilon_{ryy}} \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega \mathbf{H}_{xj}, \quad (4.24) \end{aligned}$$

$$\begin{aligned}
& \mathbf{H}_{yj}^T \int_{\Omega_e} \left\{ -\frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial y} + \frac{1}{\epsilon_{rxx}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial x} \right\} d\Omega \mathbf{H}_{xj} \\
& + \mathbf{H}_{yj}^T \int_{\Omega_e} \left\{ \frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial x} + \frac{1}{\epsilon_{rxx}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial y} - k_0^2 \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega \mathbf{H}_{yj} \\
& = -\mathbf{H}_{yj}^T n_{\text{eff}}^2 \int_{\Omega_e} \left\{ \frac{k_0^2}{\epsilon_{rxx}} \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega \mathbf{H}_{yj}. \quad (4.25)
\end{aligned}$$

In a short form, Eqs. 4.24 and 4.25 can be written as

$$\mathbf{A}_{xx}^{\Omega_e} \mathbf{H}_{xj} + \mathbf{A}_{xy}^{\Omega_e} \mathbf{H}_{yj} = n_{\text{eff}}^2 \mathbf{B}_{xx}^{\Omega_e} \mathbf{H}_{xj}, \quad (4.26)$$

$$\mathbf{A}_{yx}^{\Omega_e} \mathbf{H}_{xj} + \mathbf{A}_{yy}^{\Omega_e} \mathbf{H}_{yj} = n_{\text{eff}}^2 \mathbf{B}_{yy}^{\Omega_e} \mathbf{H}_{yj}. \quad (4.27)$$

Or in matrix form,

$$\begin{bmatrix} \mathbf{A}_{xx}^{\Omega_e} & \mathbf{A}_{xy}^{\Omega_e} \\ \mathbf{A}_{yx}^{\Omega_e} & \mathbf{A}_{yy}^{\Omega_e} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{xj} \\ \mathbf{H}_{yj} \end{bmatrix} = n_{\text{eff}}^2 \begin{bmatrix} \mathbf{B}_{xx}^{\Omega_e} & 0 \\ 0 & \mathbf{B}_{yy}^{\Omega_e} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{xj} \\ \mathbf{H}_{yj} \end{bmatrix}. \quad (4.28)$$

The sub-matrices in Eqs. 4.26 and 4.27 are

$$\begin{aligned}
\mathbf{A}_{xx}^{\Omega_e} &= \int_{\Omega_e} \left\{ \frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial y} + \frac{1}{\epsilon_{ryy}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial x} - k_0^2 \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega, \\
\mathbf{A}_{xy}^{\Omega_e} &= \int_{\Omega_e} \left\{ -\frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial x} + \frac{1}{\epsilon_{ryy}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial y} \right\} d\Omega, \\
\mathbf{A}_{yx}^{\Omega_e} &= \int_{\Omega_e} \left\{ -\frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial y} + \frac{1}{\epsilon_{rxx}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial x} \right\} d\Omega, \\
\mathbf{A}_{yy}^{\Omega_e} &= \int_{\Omega_e} \left\{ \frac{1}{\epsilon_{rzz}} \frac{\partial \mathbf{N}_j}{\partial x} \frac{\partial \mathbf{N}_j^T}{\partial x} + \frac{1}{\epsilon_{rxx}} \frac{\partial \mathbf{N}_j}{\partial y} \frac{\partial \mathbf{N}_j^T}{\partial y} - k_0^2 \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega, \\
\mathbf{B}_{xx}^{\Omega_e} &= \int_{\Omega_e} \left\{ -\frac{k_0^2}{\epsilon_{ryy}} \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega, \\
\mathbf{B}_{yy}^{\Omega_e} &= \int_{\Omega_e} \left\{ -\frac{k_0^2}{\epsilon_{rxx}} \mathbf{N}_j \mathbf{N}_j^T \right\} d\Omega. \quad (4.29)
\end{aligned}$$

Similarly for each interface element (IE) encountered in each triangle element,

$$-\int_{\Gamma_e^1} \left\{ \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_y}{\partial y} \right\} dy + \int_{\Gamma_e^1} \left\{ \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} + \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_y}{\partial y} \right\} dx = 0. \quad (4.30)$$

Substitute Eqs. 4.15 and 4.16 into Eq. 4.30,

$$\begin{aligned}
& -\mathbf{H}_{xj}^T \int_{\Gamma_e^1} \left\{ \frac{1}{\epsilon_{ryy}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} \right\} dy \mathbf{H}_{xj} - \mathbf{H}_{xj}^T \int_{\Gamma_e^1} \left\{ \frac{1}{\epsilon_{ryy}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} \right\} dy \mathbf{H}_{yj} \\
& + \mathbf{H}_{yj}^T \int_{\Gamma_e^1} \left\{ \frac{1}{\epsilon_{rxx}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} \right\} dx \mathbf{H}_{xj} + \mathbf{H}_{yj}^T \int_{\Gamma_e^1} \left\{ \frac{1}{\epsilon_{rxx}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} \right\} dx \mathbf{H}_{yj} = 0. \quad (4.31)
\end{aligned}$$

In a matrix form, it is

$$\begin{bmatrix} \mathbf{A}_{xx}^{\Gamma_e^1} & \mathbf{A}_{xy}^{\Gamma_e^1} \\ \mathbf{A}_{yx}^{\Gamma_e^1} & \mathbf{A}_{yy}^{\Gamma_e^1} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{xj} \\ \mathbf{H}_{yj} \end{bmatrix} = 0, \quad (4.32)$$

with

$$\begin{aligned} \mathbf{A}_{xx}^{\Gamma_e^1} &= - \int_{\Gamma_e^1} \left(\frac{1}{\epsilon_{ryy}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} \right) dy, \\ \mathbf{A}_{xy}^{\Gamma_e^1} &= - \int_{\Gamma_e^1} \left(\frac{1}{\epsilon_{ryy}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} \right) dy, \\ \mathbf{A}_{yx}^{\Gamma_e^1} &= \int_{\Gamma_e^1} \left(\frac{1}{\epsilon_{rxx}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} \right) dx, \\ \mathbf{A}_{yy}^{\Gamma_e^1} &= \int_{\Gamma_e^1} \left(\frac{1}{\epsilon_{rxx}} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} \right) dx. \end{aligned} \quad (4.33)$$

For a certain triangle, with three vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) (counter-clock wise), we define

$$\begin{aligned} Q_1 &= y_2 - y_3, \\ Q_2 &= y_3 - y_1, \\ Q_3 &= y_1 - y_2, \\ R_1 &= x_3 - x_2, \\ R_2 &= x_1 - x_3, \\ R_3 &= x_2 - x_1. \end{aligned} \quad (4.34)$$

The area of the triangle is

$$S_e = \frac{1}{2} [(y_3 - y_1)(x_2 - x_1) - (x_3 - x_1)(y_2 - y_1)]. \quad (4.35)$$

The relation of the basis functions N_j and shape coordinates L_j are

$$N_1 = L_1 = \frac{Q_1(x - x_2) + R_1(y - y_2)}{2S_e}, \quad (4.36)$$

$$N_2 = L_1 = \frac{Q_2(x - x_3) + R_2(y - y_3)}{2S_e}, \quad (4.37)$$

$$N_3 = L_1 = \frac{Q_3(x - x_1) + R_3(y - y_1)}{2S_e}. \quad (4.38)$$

The differentiations of the area coordinates are

$$\frac{\partial L_1}{\partial x} = \frac{Q_1}{2S_e}, \quad (4.39)$$

$$\frac{\partial L_2}{\partial x} = \frac{Q_2}{2S_e}, \quad (4.40)$$

$$\frac{\partial L_3}{\partial x} = \frac{Q_3}{2S_e}, \quad (4.41)$$

$$\frac{\partial L_1}{\partial y} = \frac{R_1}{2S_e}, \quad (4.42)$$

$$\frac{\partial L_2}{\partial y} = \frac{R_2}{2S_e}, \quad (4.43)$$

$$\frac{\partial L_3}{\partial y} = \frac{R_3}{2S_e}. \quad (4.44)$$

And integration of the area coordinates follows this formula

$$\int_{\Omega} L_1^i L_2^j L_3^k dx dy = \frac{i!j!k!}{(i+j+k+2)!} 2S_e \quad (i, j, k = 0, 1, 2, \dots). \quad (4.45)$$

Substitute Eqs. 4.39-4.45 into sub-matrices in Eq. 4.29, we have the sub-matrices elements as

$$\begin{aligned} \mathbf{A}_{xx}^{\Omega_e}(1, 1) &= \frac{1}{\epsilon_{rzz}} \frac{R_1 R_1}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_1 Q_1}{4S_e} - k_0^2 \frac{S_e}{6}, \\ \mathbf{A}_{xx}^{\Omega_e}(1, 2) &= \frac{1}{\epsilon_{rzz}} \frac{R_1 R_2}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_1 Q_2}{4S_e} - k_0^2 \frac{S_e}{12}, \\ \mathbf{A}_{xx}^{\Omega_e}(1, 3) &= \frac{1}{\epsilon_{rzz}} \frac{R_1 R_3}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_1 Q_3}{4S_e} - k_0^2 \frac{S_e}{12}, \\ \mathbf{A}_{xx}^{\Omega_e}(2, 1) &= \frac{1}{\epsilon_{rzz}} \frac{R_2 R_1}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_2 Q_1}{4S_e} - k_0^2 \frac{S_e}{12} = \mathbf{A}_{xx}^{\Omega_e}(1, 2), \\ \mathbf{A}_{xx}^{\Omega_e}(2, 2) &= \frac{1}{\epsilon_{rzz}} \frac{R_2 R_2}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_2 Q_2}{4S_e} - k_0^2 \frac{S_e}{6}, \\ \mathbf{A}_{xx}^{\Omega_e}(2, 3) &= \frac{1}{\epsilon_{rzz}} \frac{R_2 R_3}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_2 Q_3}{4S_e} - k_0^2 \frac{S_e}{12}, \\ \mathbf{A}_{xx}^{\Omega_e}(3, 1) &= \frac{1}{\epsilon_{rzz}} \frac{R_3 R_1}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_3 Q_1}{4S_e} - k_0^2 \frac{S_e}{12} = \mathbf{A}_{xx}^{\Omega_e}(1, 3), \\ \mathbf{A}_{xx}^{\Omega_e}(3, 2) &= \frac{1}{\epsilon_{rzz}} \frac{R_3 R_2}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_3 Q_2}{4S_e} - k_0^2 \frac{S_e}{12} = \mathbf{A}_{xx}^{\Omega_e}(2, 3), \\ \mathbf{A}_{xx}^{\Omega_e}(3, 3) &= \frac{1}{\epsilon_{rzz}} \frac{R_3 R_3}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_3 Q_3}{4S_e} - k_0^2 \frac{S_e}{6}. \end{aligned} \quad (4.46)$$

$$\begin{aligned}
A_{xy}^{\Omega_e}(1, 1) &= -\frac{1}{\epsilon_{rzz}} \frac{R_1 Q_1}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_1 R_1}{4S_e}, \\
A_{xy}^{\Omega_e}(1, 2) &= -\frac{1}{\epsilon_{rzz}} \frac{R_1 Q_2}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_1 R_2}{4S_e}, \\
A_{xy}^{\Omega_e}(1, 3) &= -\frac{1}{\epsilon_{rzz}} \frac{R_1 Q_3}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_1 R_3}{4S_e}, \\
A_{xy}^{\Omega_e}(2, 1) &= -\frac{1}{\epsilon_{rzz}} \frac{R_2 Q_1}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_2 R_1}{4S_e}, \\
A_{xy}^{\Omega_e}(2, 2) &= -\frac{1}{\epsilon_{rzz}} \frac{R_2 Q_2}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_2 R_2}{4S_e}, \\
A_{xy}^{\Omega_e}(2, 3) &= -\frac{1}{\epsilon_{rzz}} \frac{R_2 Q_3}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_2 R_3}{4S_e}, \\
A_{xy}^{\Omega_e}(3, 1) &= -\frac{1}{\epsilon_{rzz}} \frac{R_3 Q_1}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_3 R_1}{4S_e}, \\
A_{xy}^{\Omega_e}(3, 2) &= -\frac{1}{\epsilon_{rzz}} \frac{R_3 Q_2}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_3 R_2}{4S_e}, \\
A_{xy}^{\Omega_e}(3, 3) &= -\frac{1}{\epsilon_{rzz}} \frac{R_3 Q_3}{4S_e} + \frac{1}{\epsilon_{ryy}} \frac{Q_3 R_3}{4S_e}.
\end{aligned} \tag{4.47}$$

$$\begin{aligned}
A_{yx}^{\Omega_e}(1, 1) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_1 R_1}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_1 Q_1}{4S_e}, \\
A_{yx}^{\Omega_e}(1, 2) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_1 R_2}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_1 Q_2}{4S_e}, \\
A_{yx}^{\Omega_e}(1, 3) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_1 R_3}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_1 Q_3}{4S_e}, \\
A_{yx}^{\Omega_e}(2, 1) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_2 R_1}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_2 Q_1}{4S_e}, \\
A_{yx}^{\Omega_e}(2, 2) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_2 R_2}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_2 Q_2}{4S_e}, \\
A_{yx}^{\Omega_e}(2, 3) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_2 R_3}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_2 Q_3}{4S_e}, \\
A_{yx}^{\Omega_e}(3, 1) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_3 R_1}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_3 Q_1}{4S_e}, \\
A_{yx}^{\Omega_e}(3, 2) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_3 R_2}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_3 Q_2}{4S_e}, \\
A_{yx}^{\Omega_e}(3, 3) &= -\frac{1}{\epsilon_{rzz}} \frac{Q_3 R_3}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_3 Q_3}{4S_e}.
\end{aligned} \tag{4.48}$$

$$\begin{aligned}
\mathbf{A}_{yy}^{\Omega_e}(1,1) &= \frac{1}{\epsilon_{rzz}} \frac{Q_1 Q_1}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_1 R_1}{4S_e} - k_0^2 \frac{S_e}{6}, \\
\mathbf{A}_{yy}^{\Omega_e}(1,2) &= \frac{1}{\epsilon_{rzz}} \frac{Q_1 Q_2}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_1 R_2}{4S_e} - k_0^2 \frac{S_e}{12}, \\
\mathbf{A}_{yy}^{\Omega_e}(1,3) &= \frac{1}{\epsilon_{rzz}} \frac{Q_1 Q_3}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_1 R_3}{4S_e} - k_0^2 \frac{S_e}{12}, \\
\mathbf{A}_{yy}^{\Omega_e}(2,1) &= \frac{1}{\epsilon_{rzz}} \frac{Q_2 Q_1}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_2 R_1}{4S_e} - k_0^2 \frac{S_e}{12} = \mathbf{A}_{yy}^{\Omega_e}(1,2), \\
\mathbf{A}_{yy}^{\Omega_e}(2,2) &= \frac{1}{\epsilon_{rzz}} \frac{Q_2 Q_2}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_2 R_2}{4S_e} - k_0^2 \frac{S_e}{6}, \\
\mathbf{A}_{yy}^{\Omega_e}(2,3) &= \frac{1}{\epsilon_{rzz}} \frac{Q_2 Q_3}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_2 R_3}{4S_e} - k_0^2 \frac{S_e}{12}, \\
\mathbf{A}_{yy}^{\Omega_e}(3,1) &= \frac{1}{\epsilon_{rzz}} \frac{Q_3 Q_1}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_3 R_1}{4S_e} - k_0^2 \frac{S_e}{12} = \mathbf{A}_{yy}^{\Omega_e}(1,3), \\
\mathbf{A}_{yy}^{\Omega_e}(3,2) &= \frac{1}{\epsilon_{rzz}} \frac{Q_3 Q_2}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_3 R_2}{4S_e} - k_0^2 \frac{S_e}{12} = \mathbf{A}_{yy}^{\Omega_e}(2,3), \\
\mathbf{A}_{yy}^{\Omega_e}(3,3) &= \frac{1}{\epsilon_{rzz}} \frac{Q_3 Q_3}{4S_e} + \frac{1}{\epsilon_{rxx}} \frac{R_3 R_3}{4S_e} - k_0^2 \frac{S_e}{6}.
\end{aligned} \tag{4.49}$$

$$\begin{aligned}
\mathbf{B}_{xx}^{\Omega_e}(1,1) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{6}, \\
\mathbf{B}_{xx}^{\Omega_e}(1,2) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{12}, \\
\mathbf{B}_{xx}^{\Omega_e}(1,3) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{12}, \\
\mathbf{B}_{xx}^{\Omega_e}(2,1) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{12}, \\
\mathbf{B}_{xx}^{\Omega_e}(2,2) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{6}, \\
\mathbf{B}_{xx}^{\Omega_e}(2,3) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{12}, \\
\mathbf{B}_{xx}^{\Omega_e}(3,1) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{12}, \\
\mathbf{B}_{xx}^{\Omega_e}(3,2) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{12}, \\
\mathbf{B}_{xx}^{\Omega_e}(3,3) &= -\frac{k_0^2}{\epsilon_{ryy}} \frac{S_e}{6}.
\end{aligned} \tag{4.50}$$

$$\begin{aligned}
\mathbf{B}_{yy}^{\Omega_e}(1,1) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{6}, \\
\mathbf{B}_{yy}^{\Omega_e}(1,2) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{12}, \\
\mathbf{B}_{yy}^{\Omega_e}(1,3) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{12}, \\
\mathbf{B}_{yy}^{\Omega_e}(2,1) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{12}, \\
\mathbf{B}_{yy}^{\Omega_e}(2,2) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{6}, \\
\mathbf{B}_{yy}^{\Omega_e}(2,3) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{12}, \\
\mathbf{B}_{yy}^{\Omega_e}(3,1) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{12}, \\
\mathbf{B}_{yy}^{\Omega_e}(3,2) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{12}, \\
\mathbf{B}_{yy}^{\Omega_e}(3,3) &= -\frac{k_0^2}{\epsilon_{rxx}} \frac{S_e}{6}.
\end{aligned} \tag{4.51}$$

For an interface element, with two vertices (x_1, y_1) and (x_2, y_2) , we define

$$\begin{aligned}
R &= x_2 - x_1, \\
Q &= y_2 - y_1.
\end{aligned} \tag{4.52}$$

And the element length is

$$L_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \tag{4.53}$$

The length coordinate is defined as

$$\begin{aligned}
L_1 &= \frac{\sqrt{(x_2 - x)^2 + (y_2 - y)^2}}{L_e}, \\
L_2 &= \frac{\sqrt{(x - x_1)^2 + (y - y_1)^2}}{L_e}.
\end{aligned} \tag{4.54}$$

The differentiation and integration of the length coordinates are computed as

$$\frac{\partial L_1}{\partial x} = -\frac{R}{L_e^2}, \tag{4.55}$$

$$\frac{\partial L_2}{\partial x} = \frac{R}{L_e^2}, \tag{4.56}$$

$$\frac{\partial L_1}{\partial y} = -\frac{Q}{L_e^2}, \tag{4.57}$$

$$\frac{\partial L_2}{\partial y} = \frac{Q}{L_e^2}, \tag{4.58}$$

and

$$\int_{\Gamma} L_1^k L_2^l dx = R \frac{k!l!}{(k+l+1)!}, \quad (4.59)$$

$$\int_{\Gamma} L_1^k L_2^l dy = Q \frac{k!l!}{(k+l+1)!}. \quad (4.60)$$

The sub-matrices elements for interface element (IE) in Eq. 4.33 are subsequently calculated as

$$\begin{aligned} \mathbf{A}_{xx}^{\Gamma_e^1}(1, 1) &= \frac{RQ}{2\epsilon_{ryy}L_e^2}, \\ \mathbf{A}_{xx}^{\Gamma_e^1}(1, 2) &= -\frac{RQ}{2\epsilon_{ryy}L_e^2} = -\mathbf{A}_{xx}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{xx}^{\Gamma_e^1}(2, 1) &= \frac{RQ}{2\epsilon_{ryy}L_e^2} = \mathbf{A}_{xx}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{xx}^{\Gamma_e^1}(2, 2) &= -\frac{RQ}{2\epsilon_{ryy}L_e^2} = -\mathbf{A}_{xx}^{\Gamma_e^1}(1, 1). \end{aligned} \quad (4.61)$$

$$\begin{aligned} \mathbf{A}_{xy}^{\Gamma_e^1}(1, 1) &= \frac{Q^2}{2\epsilon_{ryy}L_e^2}, \\ \mathbf{A}_{xy}^{\Gamma_e^1}(1, 2) &= -\frac{Q^2}{2\epsilon_{ryy}L_e^2} = -\mathbf{A}_{xy}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{xy}^{\Gamma_e^1}(2, 1) &= \frac{Q^2}{2\epsilon_{ryy}L_e^2} = \mathbf{A}_{xy}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{xy}^{\Gamma_e^1}(2, 2) &= -\frac{Q^2}{2\epsilon_{ryy}L_e^2} = -\mathbf{A}_{xy}^{\Gamma_e^1}(1, 1). \end{aligned} \quad (4.62)$$

$$\begin{aligned} \mathbf{A}_{yx}^{\Gamma_e^1}(1, 1) &= -\frac{R^2}{2\epsilon_{rxx}L_e^2}, \\ \mathbf{A}_{yx}^{\Gamma_e^1}(1, 2) &= \frac{R^2}{2\epsilon_{rxx}L_e^2} = -\mathbf{A}_{yx}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{yx}^{\Gamma_e^1}(2, 1) &= -\frac{R^2}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yx}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{yx}^{\Gamma_e^1}(2, 2) &= \frac{R^2}{2\epsilon_{rxx}L_e^2} = -\mathbf{A}_{yx}^{\Gamma_e^1}(1, 1). \end{aligned} \quad (4.63)$$

$$\begin{aligned} \mathbf{A}_{yy}^{\Gamma_e^1}(1, 1) &= -\frac{QR}{2\epsilon_{rxx}L_e^2}, \\ \mathbf{A}_{yy}^{\Gamma_e^1}(1, 2) &= \frac{QR}{2\epsilon_{rxx}L_e^2} = -\mathbf{A}_{yy}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{yy}^{\Gamma_e^1}(2, 1) &= -\frac{QR}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yy}^{\Gamma_e^1}(1, 1), \\ \mathbf{A}_{yy}^{\Gamma_e^1}(2, 2) &= \frac{QR}{2\epsilon_{rxx}L_e^2} = -\mathbf{A}_{yy}^{\Gamma_e^1}(1, 1). \end{aligned} \quad (4.64)$$

4.3.1 Boundary Condition

The computational domain is truncated using the radiating boundary condition. The differential operators used for the magnetic field H (be either x - or y -component) at the boundary Γ are, after Bayliss *et al.* [64, 1],

$$B_m(H)|_\Gamma = \mathcal{O}\left(\frac{1}{r^{2m+\frac{1}{2}}}\right), \quad (4.65)$$

where,

$$B_m = \prod_{p=1}^m \left[\frac{\partial}{\partial r} + jk_r + (2p - \frac{3}{2})/r \right], \quad (4.66)$$

and

$$\begin{aligned} k_r &= k_0 \sqrt{n^2 - n_{\text{eff}}^2} & (\text{if } n > n_{\text{eff}}), \\ k_r &= k_0 j \sqrt{n_{\text{eff}}^2 - n^2} & (\text{if } n < n_{\text{eff}}). \end{aligned} \quad (4.67)$$

The field on the boundary can then be asymptotically expressed as

$$H(r, \theta)|_\Gamma = \sum_{p=0}^{\infty} H_p(\theta) \frac{\exp(-jk_r r)}{r^{p+\frac{1}{2}}}. \quad (4.68)$$

We choose the first-order boundary operator, *i.e.*,

$$B_1 = \frac{\partial}{\partial r} + jk_r + \frac{1}{2r}. \quad (4.69)$$

And also we ignore the azimuthal variation of the field, *i.e.*, $p = 0$. We then can derive

$$\left(\frac{\partial}{\partial r} + jk_r + \frac{1}{2r} \right) H|_\Gamma = \mathcal{O}(r^{-\frac{5}{2}}), \quad (4.70)$$

or

$$\frac{\partial}{\partial r} H|_\Gamma = - \left(jk_r + \frac{1}{2r} \right) H|_\Gamma + \mathcal{O}(r^{-\frac{5}{2}}). \quad (4.71)$$

If we separate the differentiation along x - and y -directions, the following equations can be derived

$$\frac{\partial}{\partial x} H|_\Gamma = \frac{\partial}{\partial r} H|_\Gamma \frac{\partial r}{\partial x} = -\cos\theta \left(jk_r + \frac{1}{2r} \right) H|_\Gamma + \mathcal{O}(r^{-\frac{5}{2}}), \quad (4.72)$$

$$\frac{\partial}{\partial y} H|_\Gamma = \frac{\partial}{\partial r} H|_\Gamma \frac{\partial r}{\partial y} = -\sin\theta \left(jk_r + \frac{1}{2r} \right) H|_\Gamma + \mathcal{O}(r^{-\frac{5}{2}}). \quad (4.73)$$

For each interface element that is on a radiation boundary, from Eq. 4.14, we have

$$\begin{aligned}
 & - \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_y}{\partial x} dy + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_x}{\partial y} dy \\
 & - \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_x \frac{\partial H_y}{\partial x} dx + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_x \frac{\partial H_x}{\partial y} dx \\
 & - \int_{\Gamma_e^2} \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_x}{\partial x} dy - \int_{\Gamma_e^2} \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_y}{\partial y} dy \\
 & + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} dx + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_y}{\partial y} dx = 0.
 \end{aligned} \tag{4.74}$$

We already known

$$\begin{aligned}
 H_x &= \mathbf{N}_j^T \mathbf{H}_{xj}, \\
 H_y &= \mathbf{N}_j^T \mathbf{H}_{yj},
 \end{aligned} \tag{4.75}$$

and

$$\begin{aligned}
 \frac{\partial H_x|_{\Gamma}}{\partial x} &= -\hat{r} \cdot \hat{x} \left(jk_{tx} + \frac{1}{2r} \right) H_x|_{\Gamma}, \\
 \frac{\partial H_x|_{\Gamma}}{\partial y} &= -\hat{r} \cdot \hat{y} \left(jk_{tx} + \frac{1}{2r} \right) H_x|_{\Gamma}, \\
 \frac{\partial H_y|_{\Gamma}}{\partial x} &= -\hat{r} \cdot \hat{x} \left(jk_{ty} + \frac{1}{2r} \right) H_y|_{\Gamma}, \\
 \frac{\partial H_y|_{\Gamma}}{\partial y} &= -\hat{r} \cdot \hat{y} \left(jk_{ty} + \frac{1}{2r} \right) H_y|_{\Gamma}.
 \end{aligned} \tag{4.76}$$

The first integration term in Eq. 4.74 can hence be derived as

$$- \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_y}{\partial x} dy = + \frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{x} \left(jk_{ty} + \frac{1}{2r_j} \right) \mathbf{H}_{yj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy \mathbf{H}_{yj}. \tag{4.77}$$

Similarly other terms can be derived as

$$\int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_x}{\partial y} dy = - \frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{y} \left(jk_{tx} + \frac{1}{2r_j} \right) \mathbf{H}_{yj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy \mathbf{H}_{xj}, \tag{4.78}$$

$$- \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_x \frac{\partial H_y}{\partial x} dx = + \frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{x} \left(jk_{ty} + \frac{1}{2r_j} \right) \mathbf{H}_{xj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx \mathbf{H}_{yj}, \tag{4.79}$$

$$\int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_x \frac{\partial H_x}{\partial y} dx = - \frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{y} \left(jk_{tx} + \frac{1}{2r_j} \right) \mathbf{H}_{xj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx \mathbf{H}_{xj}, \tag{4.80}$$

$$- \int_{\Gamma_e^2} \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_x}{\partial x} dy = + \frac{1}{\epsilon_{ryy}} \hat{r}_j \cdot \hat{x} \left(jk_{tx} + \frac{1}{2r_j} \right) \mathbf{H}_{xj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy \mathbf{H}_{xj}, \tag{4.81}$$

$$- \int_{\Gamma_e^2} \frac{1}{\epsilon_{ryy}} H_x \frac{\partial H_y}{\partial y} dy = + \frac{1}{\epsilon_{ryy}} \hat{r}_j \cdot \hat{y} \left(jk_{ty} + \frac{1}{2r_j} \right) \mathbf{H}_{xj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy \mathbf{H}_{yj}, \tag{4.82}$$

$$\int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} dx = - \frac{1}{\epsilon_{rxx}} \hat{r}_j \cdot \hat{x} \left(jk_{tx} + \frac{1}{2r_j} \right) \mathbf{H}_{yj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx \mathbf{H}_{xj}, \tag{4.83}$$

$$\int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_y}{\partial y} dx = -\frac{1}{\epsilon_{rxx}} \hat{r}_j \cdot \hat{y} (jk_{ty} + \frac{1}{2r_j}) \mathbf{H}_{yj}^T \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx \mathbf{H}_{yj}. \quad (4.84)$$

By substituting Eqs. 4.77-4.84 into Eq. 4.74, we can get the matrix equation

$$\begin{bmatrix} \mathbf{A}_{xx}^{\Gamma_e^2} & \mathbf{A}_{xy}^{\Gamma_e^2} \\ \mathbf{A}_{yx}^{\Gamma_e^2} & \mathbf{A}_{yy}^{\Gamma_e^2} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{xj} \\ \mathbf{H}_{yj} \end{bmatrix} = 0. \quad (4.85)$$

The submatrices are computed as

$$\mathbf{A}_{xx}^{\Gamma_e^2} = -\frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{y} (jk_{tx} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx + \frac{1}{\epsilon_{ryy}} \hat{r}_j \cdot \hat{x} (jk_{tx} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy, \quad (4.86)$$

$$\mathbf{A}_{xy}^{\Gamma_e^2} = +\frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{x} (jk_{ty} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx + \frac{1}{\epsilon_{ryy}} \hat{r}_j \cdot \hat{y} (jk_{ty} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy, \quad (4.87)$$

$$\mathbf{A}_{yx}^{\Gamma_e^2} = -\frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{y} (jk_{tx} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy - \frac{1}{\epsilon_{rxx}} \hat{r}_j \cdot \hat{x} (jk_{tx} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx, \quad (4.88)$$

$$\mathbf{A}_{yy}^{\Gamma_e^2} = +\frac{1}{\epsilon_{rzz}} \hat{r}_j \cdot \hat{x} (jk_{ty} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dy - \frac{1}{\epsilon_{rxx}} \hat{r}_j \cdot \hat{y} (jk_{ty} + \frac{1}{2r_j}) \int_{\Gamma_e^2} \mathbf{N}_j \mathbf{N}_j^T dx. \quad (4.89)$$

Matrix elements for these submatrices are

$$\begin{aligned} \mathbf{A}_{xx}^{\Gamma_e^2}(1,1) &= -C_1 \frac{R}{3\epsilon_{rzz}} + C_2 \frac{Q}{3\epsilon_{ryy}}, \\ \mathbf{A}_{xx}^{\Gamma_e^2}(1,2) &= -C_1 \frac{R}{6\epsilon_{rzz}} + C_2 \frac{Q}{6\epsilon_{ryy}}, \\ \mathbf{A}_{xx}^{\Gamma_e^2}(2,1) &= -C_1 \frac{R}{6\epsilon_{rzz}} + C_2 \frac{Q}{6\epsilon_{ryy}}, \\ \mathbf{A}_{xx}^{\Gamma_e^2}(2,2) &= -C_1 \frac{R}{3\epsilon_{rzz}} + C_2 \frac{Q}{3\epsilon_{ryy}}, \end{aligned} \quad (4.90)$$

$$\begin{aligned} \mathbf{A}_{xy}^{\Gamma_e^2}(1,1) &= +C_3 \frac{R}{3\epsilon_{rzz}} + C_4 \frac{Q}{3\epsilon_{ryy}}, \\ \mathbf{A}_{xy}^{\Gamma_e^2}(1,2) &= +C_3 \frac{R}{6\epsilon_{rzz}} + C_4 \frac{Q}{6\epsilon_{ryy}}, \\ \mathbf{A}_{xy}^{\Gamma_e^2}(2,1) &= +C_3 \frac{R}{6\epsilon_{rzz}} + C_4 \frac{Q}{6\epsilon_{ryy}}, \\ \mathbf{A}_{xy}^{\Gamma_e^2}(2,2) &= +C_3 \frac{R}{3\epsilon_{rzz}} + C_4 \frac{Q}{3\epsilon_{ryy}}, \end{aligned} \quad (4.91)$$

$$\begin{aligned} \mathbf{A}_{yx}^{\Gamma_e^2}(1,1) &= -C_1 \frac{Q}{3\epsilon_{rzz}} - C_2 \frac{R}{3\epsilon_{rxx}}, \\ \mathbf{A}_{yx}^{\Gamma_e^2}(1,2) &= -C_1 \frac{Q}{6\epsilon_{rzz}} - C_2 \frac{R}{6\epsilon_{rxx}}, \\ \mathbf{A}_{yx}^{\Gamma_e^2}(2,1) &= -C_1 \frac{Q}{6\epsilon_{rzz}} - C_2 \frac{R}{6\epsilon_{rxx}}, \\ \mathbf{A}_{yx}^{\Gamma_e^2}(2,2) &= -C_1 \frac{Q}{3\epsilon_{rzz}} - C_2 \frac{R}{3\epsilon_{rxx}}, \end{aligned} \quad (4.92)$$

Figure 4.4: Perfect electric conductor condition at the left boundary.

$$\begin{aligned}
 A_{yy}^{\Gamma_s^2}(1, 1) &= +C_3 \frac{Q}{3\epsilon_{rzz}} - C_4 \frac{R}{3\epsilon_{rxx}}, \\
 A_{yy}^{\Gamma_s^2}(1, 2) &= +C_3 \frac{Q}{6\epsilon_{rzz}} - C_4 \frac{R}{6\epsilon_{rxx}}, \\
 A_{yy}^{\Gamma_s^2}(2, 1) &= +C_3 \frac{Q}{6\epsilon_{rzz}} - C_4 \frac{R}{6\epsilon_{rxx}}, \\
 A_{yy}^{\Gamma_s^2}(2, 2) &= +C_3 \frac{Q}{3\epsilon_{rzz}} - C_4 \frac{R}{3\epsilon_{rxx}},
 \end{aligned} \tag{4.93}$$

where,

$$\begin{aligned}
 C_1 &= \hat{r}_j \cdot \hat{y}(jk_{tx} + \frac{1}{2r_j}), \\
 C_2 &= \hat{r}_j \cdot \hat{x}(jk_{tx} + \frac{1}{2r_j}), \\
 C_3 &= \hat{r}_j \cdot \hat{x}(jk_{ty} + \frac{1}{2r_j}), \\
 C_4 &= \hat{r}_j \cdot \hat{y}(jk_{ty} + \frac{1}{2r_j}).
 \end{aligned} \tag{4.94}$$

In all submatrice elements, $j = 1$ or 2 , which should be the column number in the element submatrices.

4.3.2 PEC Boundary Condition

If an MOF exhibits twofold *reflection symmetry*, its modes can be solved by considering only one quarter of the waveguide's cross-section. The symmetry lines are physically interpreted as either a *perfect electric conductor* (PEC) or a *perfect magnetic conductor* (PMC). It should be mentioned that for a conventional MOF whose air holes are placed in a triangular lattice, only $1/6$ of its cross-section is adequate for solving modes by considering the fiber's *rotational symmetry*. However, we usually ignore all rotational symmetry in a FEM formulation since such a symmetry condition is not natural in Cartesian coordinate. In the following, we will present how the PEC symmetry condition is formulated along the positive y -axis (Fig. 4.4).

The magnetic field on a PEC boundary should fulfill the conditions of

$$\begin{aligned}
 H_{\perp} &= 0, \\
 \frac{\partial H_{\parallel}}{\partial n} &= 0.
 \end{aligned} \tag{4.95}$$

In our case, they are read as

$$\begin{aligned} H_x &= 0, \\ \frac{\partial H_y}{\partial x} &= 0. \end{aligned} \quad (4.96)$$

The number of boundary integration terms in Eq. (4.74) is reduced to three

$$\begin{aligned} & \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_x}{\partial y} dx + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} dx + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_y}{\partial y} dx \\ &= \mathbf{H}_{yj}^T \frac{1}{\epsilon_{rzz}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} dy \mathbf{H}_{xj} + \mathbf{H}_{yj}^T \frac{1}{\epsilon_{rxx}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} dx \mathbf{H}_{xj} + \mathbf{H}_{yj}^T \frac{1}{\epsilon_{rxx}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} dx \mathbf{H}_{yj} \\ &= 0. \end{aligned} \quad (4.97)$$

In a matrix form, it is

$$\begin{bmatrix} \mathbf{A}_{xx}^{\Gamma_e^2} & \mathbf{A}_{xy}^{\Gamma_e^2} \\ \mathbf{A}_{yx}^{\Gamma_e^2} & \mathbf{A}_{yy}^{\Gamma_e^2} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{xj} \\ \mathbf{H}_{yj} \end{bmatrix} = 0, \quad (4.98)$$

where

$$\begin{aligned} \mathbf{A}_{xx}^{\Gamma_e^2} &= [0], \\ \mathbf{A}_{xy}^{\Gamma_e^2} &= [0], \\ \mathbf{A}_{yx}^{\Gamma_e^2} &= \frac{1}{\epsilon_{rzz}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} dy + \frac{1}{\epsilon_{rxx}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} dx, \\ \mathbf{A}_{yy}^{\Gamma_e^2} &= \frac{1}{\epsilon_{rxx}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} dx. \end{aligned} \quad (4.99)$$

The submatrix elements are calculated as

$$\begin{aligned} \mathbf{A}_{yx}^{\Gamma_e^2}(1, 1) &= -\frac{Q^2}{2\epsilon_{rzz}L_e^2} - \frac{R^2}{2\epsilon_{rxx}L_e^2}, \\ \mathbf{A}_{yx}^{\Gamma_e^2}(1, 2) &= +\frac{Q^2}{2\epsilon_{rzz}L_e^2} + \frac{R^2}{2\epsilon_{rxx}L_e^2}, \\ \mathbf{A}_{yx}^{\Gamma_e^2}(2, 1) &= -\frac{Q^2}{2\epsilon_{rzz}L_e^2} - \frac{R^2}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yx}^{\Gamma_e^2}(1, 1), \\ \mathbf{A}_{yx}^{\Gamma_e^2}(2, 2) &= +\frac{Q^2}{2\epsilon_{rzz}L_e^2} + \frac{R^2}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yx}^{\Gamma_e^2}(1, 2). \end{aligned} \quad (4.100)$$

$$\begin{aligned} \mathbf{A}_{yy}^{\Gamma_e^2}(1, 1) &= +\frac{QR}{2\epsilon_{rxx}L_e^2}, \\ \mathbf{A}_{yy}^{\Gamma_e^2}(1, 2) &= -\frac{QR}{2\epsilon_{rxx}L_e^2}, \\ \mathbf{A}_{yy}^{\Gamma_e^2}(2, 1) &= +\frac{QR}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yy}^{\Gamma_e^2}(1, 1), \\ \mathbf{A}_{yy}^{\Gamma_e^2}(2, 2) &= -\frac{QR}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yy}^{\Gamma_e^2}(1, 2). \end{aligned} \quad (4.101)$$

Figure 4.5: Perfect magnetic conductor condition at the bottom boundary.

Besides, in the final lump-sum matrices, elements in certain rows and columns, which are related to the H_x field on the PEC boundary nodes, should be manually set to zero, except the diagonal element.

4.3.3 PMC Boundary Condition

In this subsection we implement the PMC condition along the positive x -axis (Fig. 4.5). The magnetic field on a PMC boundary should fulfill the conditions of

$$\begin{aligned} H_{\parallel} &= 0, \\ \frac{\partial H_{\perp}}{\partial n} &= 0. \end{aligned} \quad (4.102)$$

In our case, they are read as

$$\begin{aligned} H_x &= 0, \\ \frac{\partial H_y}{\partial y} &= 0. \end{aligned} \quad (4.103)$$

The number of boundary integration terms in Eq. (4.74) is reduced to three

$$-\int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_y}{\partial x} dy + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rzz}} H_y \frac{\partial H_x}{\partial y} dy + \int_{\Gamma_e^2} \frac{1}{\epsilon_{rxx}} H_y \frac{\partial H_x}{\partial x} dx = 0. \quad (4.104)$$

We can write the equation into a matrix form of

$$\begin{bmatrix} \mathbf{A}_{xx}^{\Gamma_e^2} & \mathbf{A}_{xy}^{\Gamma_e^2} \\ \mathbf{A}_{yx}^{\Gamma_e^2} & \mathbf{A}_{yy}^{\Gamma_e^2} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{xj} \\ \mathbf{H}_{yj} \end{bmatrix} = 0, \quad (4.105)$$

where

$$\begin{aligned} \mathbf{A}_{xx}^{\Gamma_e^2} &= [0], \\ \mathbf{A}_{xy}^{\Gamma_e^2} &= [0], \\ \mathbf{A}_{yx}^{\Gamma_e^2} &= \frac{1}{\epsilon_{rzz}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial y} dy + \frac{1}{\epsilon_{rxx}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} dx, \\ \mathbf{A}_{yy}^{\Gamma_e^2} &= -\frac{1}{\epsilon_{rzz}} \int_{\Gamma_e^2} \mathbf{N}_j \frac{\partial \mathbf{N}_j^T}{\partial x} dy. \end{aligned} \quad (4.106)$$

Figure 4.6: (a) PMC condition at left boundary. (b) PEC at bottom boundary.

The submatrix elements are calculated as

$$\begin{aligned}
 \mathbf{A}_{yx}^{\Gamma_e^2}(1,1) &= -\frac{Q^2}{2\epsilon_{rzz}L_e^2} - \frac{R^2}{2\epsilon_{rxx}L_e^2}, \\
 \mathbf{A}_{yx}^{\Gamma_e^2}(1,2) &= +\frac{Q^2}{2\epsilon_{rzz}L_e^2} + \frac{R^2}{2\epsilon_{rxx}L_e^2}, \\
 \mathbf{A}_{yx}^{\Gamma_e^2}(2,1) &= -\frac{Q^2}{2\epsilon_{rzz}L_e^2} - \frac{R^2}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yx}^{\Gamma_e^2}(1,1), \\
 \mathbf{A}_{yx}^{\Gamma_e^2}(2,2) &= +\frac{Q^2}{2\epsilon_{rzz}L_e^2} + \frac{R^2}{2\epsilon_{rxx}L_e^2} = \mathbf{A}_{yx}^{\Gamma_e^2}(1,2).
 \end{aligned} \tag{4.107}$$

$$\begin{aligned}
 \mathbf{A}_{yy}^{\Gamma_e^2}(1,1) &= -\frac{QR}{2\epsilon_{rzz}L_e^2}, \\
 \mathbf{A}_{yy}^{\Gamma_e^2}(1,2) &= +\frac{QR}{2\epsilon_{rzz}L_e^2}, \\
 \mathbf{A}_{yy}^{\Gamma_e^2}(2,1) &= -\frac{QR}{2\epsilon_{rzz}L_e^2} = \mathbf{A}_{yy}^{\Gamma_e^2}(1,1), \\
 \mathbf{A}_{yy}^{\Gamma_e^2}(2,2) &= +\frac{QR}{2\epsilon_{rzz}L_e^2} = \mathbf{A}_{yy}^{\Gamma_e^2}(1,2).
 \end{aligned} \tag{4.108}$$

Besides, in the final lump-sum matrices, elements in certain rows and columns, which are related to the H_x field on the PMC boundary nodes, should be manually set to zero, except the diagonal element.

Other two types of symmetry conditions, *i.e.*, PMC at the left boundary and PEC at the bottom boundary (Fig. 4.6), can be derived similarly.

4.4 Formulation with Second-order Shape Function

The higher the order of the shape function, the more accurate the result will be. However, this is at the expense of solving a larger number of unknowns. It is generally considered that formulation using the second-order shape function is the most efficient in terms of accuracy and computational cost (CPU time and RAM).

For formulation using the second-order shape function, the field profile on each triangle is no longer a flat surface. Therefore we cannot represent the local field with only three

Figure 4.7: A triangular element used for second-order shape function approximation. Nodes 4, 5, 6 are the mid-points of the edges.

Figure 4.8: A line element used for second-order shape function approximation. Node 3 is chosen to be in the middle of points 1 and 2.

vertices. In fact we have to introduce another three points, which are usually chosen at the edge centers (Fig. 4.7). Notice that the node sequence affects the final global matrices.

The shape functions can be written in terms of L_1 , L_2 , L_3 as

$$\begin{aligned}
 N_1 &= L_1(2L_2 - 1), \\
 N_2 &= L_1(2L_2 - 1), \\
 N_3 &= L_1(2L_2 - 1), \\
 N_4 &= 4L_1L_2, \\
 N_5 &= 4L_2L_3, \\
 N_6 &= 4L_3L_1.
 \end{aligned} \tag{4.109}$$

Field profile on a line element is no longer straight either. Rather, it has a quadratic shape. Hence the local field on a line has to be defined by three points. The extra point is usually chosen at the center of the line element (Fig. 4.8). The corresponding shape

Figure 4.9: Sketch of an air-silica Bragg fiber. Black is for air, white is for silica. Refer to Fig. 4.11 for the fiber's dimension.

functions are

$$\begin{aligned} N_1 &= 2L_1^2 + L_1, \\ N_2 &= 2L_1^2 + 3L_1 + 1, \\ N_3 &= -4L_1^2 - 4L_1, \end{aligned} \tag{4.110}$$

where $L_1 = l_1/l_e$. l_1 is the distance from the point under study to point 1, and l_e is the length of the whole line element.

Further substitution of these shape functions as well as derivation of submatrices elements are not fully listed here, due to their huge sizes.

4.5 Numerical Example

Figure 4.9 shows an air-silica Bragg fiber structure. Such fiber represents one of the most challenging waveguide problems. The fiber is structurally very similar to the fabricated one given by Fig. 9.18 in Chapter 9. The guidance in such a fiber is achieved by the photonic bandgap effect. To be more specific, the antiresonance of the cladding silica walls (pseudo-1D photonic crystal) repel the launched light back into the hollow core. In order to model such a fiber accurately, we have to sample the cladding silica layers with a fine enough mesh. As the field variation in the core region is not that drastic (especially for low-order modes), we can use a relatively coarser mesh to sample the core region. Numerical methods like FDM use a uniform mesh, which would result in too many unknowns for an adequate sampling of the cladding region. FEM is the most suitable method for such complex problems, especially when our computing resources like processor speed and RAM are limited.

In Fig. 4.10, we discretize the problem domain (one quarter) into triangular subdomains. The number of nodes is 16,232, and the number of triangles is 32,051. Notice we have let the mesh in the cladding silica layers be denser as compared to that in the core and background cladding regions. Also the radiation boundary is of circular shape, which

Figure 4.10: Mesh of the air-silica Bragg fiber.

reduces the number of unknowns significantly as compared to using the PML boundary condition.

A rough estimation tells us the primary bandgap of the cladding should be centered around $1.5\mu\text{m}$ and the air-guided fundamental mode should have a n_{eff} value around 0.9964. However, the actual mode searching did not give us the expected result. In Fig. 4.11, we show the H_y field of a mode found at that particular wavelength. The H_x component has a very similar profile. We can treat such mode as a cladding mode though a guided core field is vaguely present. Notice that the cladding of the fiber only resembles a concentric stack of air and silica layers macroscopically. In fact each silica layer is of distinct shape. Therefore, the silica layers do not reach resonance at the same time. In this case, only the first layer (nearest to core) is in complete resonance. The calculated n_{eff} is at 0.99618. The loss is almost zero. However, it should be noticed that such mode in a fabricated fiber does not propagate long due to very rough surfaces. We will discuss such a fiber in more detail in Chapter 9.

Figure 4.11: Contour plot of the $|H_y|$ field of a mode at $\lambda = 1.5\mu\text{m}$. The contour lines are separated by 1dB, starting from 92% of the highest value (dark red). The Bragg fiber's outline is superimposed.

4.6 Conclusion and Future Work

The FEM described in this chapter has been compared to the MEM and the FDM²-ABC methods in [10]. Excellent agreement is found for the six-hole MOF which we have studied in Section 3.4 of Chapter 3.

The formulation of the above-mentioned FEM uses the Galerkin approach. Such an approach was originally considered as the only way to derive the functional for a non-self-adjoint problem. But as pointed out by Jin, such a non-self-adjoint problem can also be formulated using a modified variational approach [1].

The radiation boundary condition uses sinusoidal functions to approximate field outside the meshed domain. What's more, the angular dependance of the outside field is neglected. The error contributed by these approximations has yet to be explicitly studied, especially when the mode considered is near cutoff. It's meaningful in future to use a Fourier-Bessel series to represent outside field, as Nader *et. al* have implemented in their FDM²-ABC method. We have recently noticed that such a method was reported in 1982 by Oyamada and Okoshi [65]. However, their formulation is based on the longitudinal field components (E_z and H_z), in which spurious modes are inevitable. Its applicability to other more complex formulations is yet to be studied. In particular, the sparsity of the final eigenmatrices need to be confirmed.

Currently in all published works on the FEM, only reflection symmetry of a waveguide is used. Rotational symmetry is neglected. Therefore, the minimum waveguide portion needed to be considered cannot be less than one quarter. However, in some cases, though rare, the waveguide may exhibit only rotational symmetry. And also for some waveguides, especially common MOFs with C_{6v} group symmetry, have very high rotational symmetry. These waveguides can be more efficiently modelled by considering rotation symmetry instead of reflection symmetry. It is hoped that in the future rotational symmetry in waveguides can be exploited in FEM formulations.

Chapter 5

Plane Wave Method

5.1 Introduction

The plane wave method (PWM) was originally proposed to calculate modes supported by photonic crystals (PCs) [21, 22]. With this method, the bands as well as the bandgaps of a 2D PC can be easily computed, when the lightwave is considered to propagate in the plane-of-periodicity (or in plane). Defect modes can also be calculated with the help of the so-called supercell technique [23]. With little extra complication, PWM can also derive modes in a 2D PC with off-plane wavevectors (i.e. the wavevector is not completely in plane). In this manner, modes in PCFs can be computed (with a large enough supercell size to prevent severe coupling across the periodic boundary). There are a few drawbacks for the PWM method to be a PCF mode solver. Firstly, it employs a periodic boundary condition. This restricts the PWM from accurately calculating very leaky modes owing to the coupling between two neighboring supercells. Secondly, the supercell technique increases drastically the problem size. Besides a larger number of plane waves need to be used, the mode-folding effect increases tremendously the computation time for deriving a high-order PCF mode (especially those air-guided modes in a PBG fiber)¹. Finally, material loss cannot be included in the calculations because a complex epsilon function will make the problem non-Hermitian. However, PWM is still a powerful method to predict the performance of a PBG fiber, as it can quickly derive the cladding PC's PBG regions. These gap regions give adequate information on the fiber's PBG-guiding capability.

It is worth mentioning that the traditional PWM as depicted in [21, 22, 23] calculates the wavevector k subject to the input propagation constant β . Material dispersion cannot be easily incorporated into such a formulation². From a physical point of view, we are more interested in the β value subject to a certain input wavelength λ (or equivalently k).

¹In the PWM implemented by Johnson *et al.* [23], the lowest-energy mode is calculated first. The next high energy mode is solved based on all previously solved modes according to the orthogonality principle. However, an advanced eigensolver can be deployed to target-solve a particular high energy mode, without knowledge of the low energy modes.

²In such PWM, one inputs a β and a dielectric function $\epsilon(\mathbf{r})$ to solve for λ . It works fine for a non-dispersive medium whose dielectric function is constant regardless of the wavelength. But, when the material is dispersive, $\epsilon(\mathbf{r})$ depends on λ . Therefore we do not know the exact $\epsilon(\mathbf{r})$ value we should input. However, material dispersion can still be handled with this PWM by multiple runnings, where λ value calculated in each running determines the $\epsilon(\mathbf{r})$ value in the next running, until the calculated λ value converges.

For MOF problems in particular, there exists another version of the PWM that calculates β from λ [24, 25, 66]. Similarly, this PWM expands both mode field and dielectric function as Fourier series. A standard eigenvalue problem can finally be derived.

The second version of PWM will be presented in this chapter. Though the method has been mentioned in the literature several times, as we will show later, their formulations are not that complete.

5.2 Formulation

The PWM can be formulated using either the electric field or the magnetic field. Here only formulation using the electric field is presented. We start from the transverse vector wave equation (Eq. 1.19), *i.e.*,

$$(\nabla_t^2 + k^2 - \beta^2)\mathbf{E}_t = -\nabla_t(\mathbf{E}_t \cdot \nabla_t \ln \epsilon_r), \quad (5.1)$$

For any mode, its transverse field can be expressed as an infinite series of sinusoidal functions, *i.e.*,

$$\mathbf{E}_t = \sum_{\mathbf{G}} E_{\mathbf{G}} e^{i(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}}, \quad (5.2)$$

where, $\mathbf{G}$ is a vector in reciprocal space. For a 2D periodic dielectric structure, $\mathbf{G}$ is normally expressed in terms of primitive reciprocal lattice vectors [67]. k is the wavevector component in the plane-of-periodicity. And similarly we can expand the dielectric function as

$$\epsilon = \sum_{\mathbf{G}'} \epsilon_{\mathbf{G}'} e^{i\mathbf{G}'\cdot\mathbf{r}}, \quad (5.3)$$

and also,

$$\ln \epsilon = \sum_{\mathbf{G}'} \kappa_{\mathbf{G}'} e^{i\mathbf{G}'\cdot\mathbf{r}}, \quad (5.4)$$

If we substitute Eqs. 5.2 and 5.4 into Eq. 5.1, the terms in Eq. 5.1 become

$$\begin{aligned} \nabla_t^2 \mathbf{E}_t &= \nabla_t^2 \sum_{\mathbf{G}} E_{\mathbf{G}} e^{i(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}} \\ &= -\sum_{\mathbf{G}} |\mathbf{G} + \mathbf{k}|^2 E_{\mathbf{G}} e^{i(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}}, \end{aligned} \quad (5.5)$$

$$\begin{aligned} k^2 \mathbf{E}_t &= k_0^2 \sum_{\mathbf{G}} \epsilon_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{r}} \sum_{\mathbf{G}'} E_{\mathbf{G}'} e^{i(\mathbf{G}'+\mathbf{k})\cdot\mathbf{r}} \\ &= k_0^2 \sum_{\mathbf{G}} \sum_{\mathbf{G}'} \epsilon_{\mathbf{G}} E_{\mathbf{G}'} e^{i(\mathbf{G}+\mathbf{G}'+\mathbf{k})\cdot\mathbf{r}} \\ &= k_0^2 \sum_{\mathbf{G}} \sum_{\mathbf{G}'} \epsilon_{\mathbf{G}-\mathbf{G}'} E_{\mathbf{G}'} e^{i(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}}, \end{aligned} \quad (5.6)$$

$$\beta^2 \mathbf{E}_t = \beta^2 \sum_{\mathbf{G}} E_{\mathbf{G}} e^{(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}}, \quad (5.7)$$

$$\begin{aligned} & \nabla_t (\mathbf{E}_t \cdot \nabla_t \ln \epsilon) \\ &= \nabla_t \left[\sum_{\mathbf{G}} E_{\mathbf{G}} e^{(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}} \cdot \nabla_t \sum_{\mathbf{G}'} \kappa_{\mathbf{G}'} e^{i\mathbf{G}' \cdot \mathbf{r}} \right] \\ &= \nabla_t \left[\sum_{\mathbf{G}} \sum_{\mathbf{G}'} i\kappa_{\mathbf{G}'} (E_{G_x} G'_x + E_{G_y} G'_y) \right] e^{i(\mathbf{G}+\mathbf{G}'+\mathbf{k}) \cdot \mathbf{r}} \\ &= - \sum_{\mathbf{G}} \sum_{\mathbf{G}'} \kappa_{\mathbf{G}'} (E_{G_x} G'_x + E_{G_y} G'_y) (\mathbf{G} + \mathbf{G}' + \mathbf{k}) e^{i(\mathbf{G}+\mathbf{G}'+\mathbf{k}) \cdot \mathbf{r}} \\ &= - \sum_{\mathbf{G}} \sum_{\mathbf{G}'} \kappa_{\mathbf{G}-\mathbf{G}'} [E_{G_x} (G'_x - G_x) + E_{G_y} (G'_y - G_y)] (\mathbf{G}' + \mathbf{k}) e^{i(\mathbf{G}'+\mathbf{k}) \cdot \mathbf{r}} \\ &= - \sum_{\mathbf{G}} \sum_{\mathbf{G}'} \kappa_{\mathbf{G}-\mathbf{G}'} [E_{G'_x} (G_x - G'_x) + E_{G'_y} (G_y - G'_y)] (\mathbf{G} + \mathbf{k}) e^{i(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}}, \quad (5.8) \end{aligned}$$

where in Eq. 5.6 and 5.8, we have used $(\mathbf{G} + \mathbf{G}') \rightarrow \mathbf{G}$ substitution. For each G vector, Eq. 5.1 becomes,

$$\begin{aligned} & -|\mathbf{G} + \mathbf{k}|^2 E_{\mathbf{G}} e^{i(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}} + k_0^2 \sum_{\mathbf{G}'} \epsilon_{\mathbf{G}-\mathbf{G}'} E_{\mathbf{G}'} e^{i(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}} + \beta^2 E_{\mathbf{G}} e^{i(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}} \\ & + \sum_{\mathbf{G}'} \kappa_{\mathbf{G}-\mathbf{G}'} [E_{G'_x} (G_x - G'_x) + E_{G'_y} (G_y - G'_y)] (\mathbf{G} + \mathbf{k}) e^{i(\mathbf{G}+\mathbf{k}) \cdot \mathbf{r}} = 0. \quad (5.9) \end{aligned}$$

By decomposing $\mathbf{E}_{\mathbf{G}}$ into $E_{G_x} \hat{x} + E_{G_y} \hat{y}$, we can arrange Eq. 5.9 in a matrix form as

$$\begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_3 & \mathbf{M}_4 \end{bmatrix} \begin{bmatrix} E_{G'_x} \\ E_{G'_y} \end{bmatrix} = \beta^2 \begin{bmatrix} E_{G'_x} \\ E_{G'_y} \end{bmatrix}. \quad (5.10)$$

where,

$$\mathbf{M}_1 = -|\mathbf{G} + \mathbf{k}|^2 \mathbf{I} + k_0^2 \epsilon_{\mathbf{G}-\mathbf{G}'} - \kappa_{\mathbf{G}-\mathbf{G}'} \cdot (\mathbf{G}_x - \mathbf{G}'_x) \cdot \mathbf{G}_x, \quad (5.11)$$

$$\mathbf{M}_2 = -\kappa_{\mathbf{G}-\mathbf{G}'} \cdot (\mathbf{G}_y - \mathbf{G}'_y) \cdot \mathbf{G}_x, \quad (5.12)$$

$$\mathbf{M}_3 = -\kappa_{\mathbf{G}-\mathbf{G}'} \cdot (\mathbf{G}_x - \mathbf{G}'_x) \cdot \mathbf{G}_y, \quad (5.13)$$

$$\mathbf{M}_4 = -|\mathbf{G} + \mathbf{k}|^2 \mathbf{I} + k_0^2 \epsilon_{\mathbf{G}-\mathbf{G}'} - \kappa_{\mathbf{G}-\mathbf{G}'} \cdot (\mathbf{G}_y - \mathbf{G}'_y) \cdot \mathbf{G}_y, \quad (5.14)$$

The matrix equation 5.10 is a standard eigenvalue problem and it can be readily solved using the freely available LAPACK. Note that in previous publications [24, 25, 66], the in-plane wavevector component $\mathbf{k}$ is neglected. Hence their formulation can only compute defect modes with the aid of the supercell technique. Here we have incorporated $\mathbf{k}$ to facilitate the band calculation in a periodic dielectric system [68]. If we let $\mathbf{k} = 0$, our result is the same as those published in [24, 25, 66].

5.3 Remarks

The authors of [24, 25, 66] have used the analytical Fourier transform of a single inclusion to derive the matrix $\epsilon_{\mathbf{G}'}$ in Eq. 5.3 as well as $\kappa_{\mathbf{G}'}$ in Eq. 5.4. However the analytical Fourier transform assumes the single inclusion is immersed in an infinitely-extending background material, which ignores the existence of inclusions outside the basic cell. Hence such a representation should introduce a certain amount of discrepancy, and the discrepancy should be larger when the inclusion-to-inclusion distance becomes smaller. A further study is necessary to confirm this. On the other hand, the numerical Fourier transform (*e.g.*, FFT) treats automatically the dielectric function as periodic outside the unit cell. Hence they will give rise to reliable converged solutions.

It should be noticed that, though PWM is not able to calculate the radiation loss due to its periodic boundary condition, its agreement with other methods is acceptable when the PCF under study has a small leakage loss. This is true even for an air-guiding photonic bandgap fiber with only a few cladding layers. Modes derived using PWM and FDM for an air-core photonic bandgap fiber are compared in [69].

Chapter 6

Finite Difference Method

6.1 Introduction

Finite difference method (FDM) is the most straightforward method for solving partial differential equations since the invention of computers. The method is very general for modelling EM problems as it solves the *un-manipulated* Maxwell equations. As a mode solver, FDM is applicable to a regular waveguide with almost any index profile (see Fig. 6.1). Material anisotropy can also be easily supported since it is formulated in a full-vector basis. For the FDM to be introduced in this chapter, a perfectly-matched layer (PML) backed with a zero boundary condition is deployed to truncate the infinite cross section into a finite one (see Fig. 6.1).

Figure 6.1: Problem domain. The PML regions are shaded in grey.

FDM discretizes the problem domain, including the PML regions, into a rectangular mesh (as shown in Fig. 6.2). It then finds the field values on the mesh grids which satisfy the Maxwell equations. It's not difficult to understand that the denser the mesh, the more accurate the solution will be. The mesh employed (Fig. 6.2) is a 2D Yee's mesh [70]. Notice the grid locations for different field components are different.

Figure 6.2: Numerical grids (Yee's grid) or mesh used in the FDM.

In the next section, we will show how Maxwell equations can be discretized over the Yee's mesh. The resulting algebraic equations can then be straightforwardly put into matrix forms. It should be mentioned that the FDM to be presented was first proposed by Zhu *et al.* [18], and later extended by Guo *et al.* [19] with PML boundary condition.

6.2 Formulation

The derivation starts from the decomposed Maxwell equations instead of one vector wave equation. Assuming EM fields have $\exp(j\beta z)$ dependence, Eq. 1.9 becomes

$$\begin{aligned} jk_0\mu_{rx}H_x &= \frac{\partial E_z}{\partial y} - j\beta E_y, \\ jk_0\mu_{ry}H_y &= j\beta E_x - \frac{\partial E_z}{\partial x}, \\ jk_0\mu_{rz}H_z &= \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}. \end{aligned} \quad (6.1)$$

Notice we have included anisotropic material permeability which is necessary for the definition of PML absorbing material. Equation 1.10 becomes

$$\begin{aligned} -jk_0\epsilon_{rx}E_x &= \frac{\partial H_y}{\partial y} - j\beta H_z, \\ -jk_0\epsilon_{ry}E_y &= j\beta H_x - \frac{\partial H_z}{\partial x}, \\ -jk_0\epsilon_{rz}E_z &= \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \end{aligned} \quad (6.2)$$

Figure 6.3: (a) Unit mesh cell about a H_z node; (b) Unit mesh cell about a E_z node. Notice the dielectric constant is defined on the H_z nodes. Distance between two E_z (or H_z) nodes is Δx along x direction, and Δy along y direction.

Applying the Eq. 6.1 to a general mesh unit as shown in Fig. 6.3(a), we have

$$jk_0\mu_{rx}(i,j)H_x(i,j) = \frac{E_z(i,j+1) - E_z(i,j)}{\Delta y} - j\beta E_y(i,j), \quad (6.3)$$

$$jk_0\mu_{ry}(i,j)H_y(i,j) = j\beta E_x(i,j) - \frac{E_z(i+1,j) - E_z(i,j)}{\Delta x}, \quad (6.4)$$

$$jk_0\mu_{rz}(i,j)H_z(i,j) = \frac{E_y(i+1,j) - E_y(i,j)}{\Delta x} - \frac{E_x(i,j+1) - E_x(i,j)}{\Delta y}. \quad (6.5)$$

For each H_z node, we have three equations (Eqs. 6.3-6.5). If we number the H_z nodes in Fig. 6.2 globally from the bottom-left corner to the top-right corner, first upwards then to the left, as $1, \dots, N$, we can put three sets of equations into matrix form as

$$jk_0\mu_{rx}\mathbf{H}_x = -j\beta\mathbf{I}\mathbf{E}_y + \mathbf{U}_y\mathbf{E}_z, \quad (6.6)$$

$$jk_0\mu_{ry}\mathbf{H}_y = j\beta\mathbf{I}\mathbf{E}_x + \mathbf{U}_x\mathbf{E}_z, \quad (6.7)$$

$$jk_0\mu_{rz}\mathbf{H}_z = -\mathbf{U}_y\mathbf{E}_x + \mathbf{U}_x\mathbf{E}_y, \quad (6.8)$$

where,

$$\mu_{rx} = \text{diag}(\mu_{rx1}, \dots, \mu_{rxN}), \quad (6.9)$$

$$\mathbf{H}_x = [H_{x1}, \dots, H_{xN}]^T, \quad (6.10)$$

$$\mathbf{U}_x = \frac{1}{\Delta x} \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad (6.11)$$

$$\mathbf{U}_y = \frac{1}{\Delta y} \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \quad (6.12)$$

and other similar terms are defined in the same manner. Notice that in deriving U_x and U_y , we have assumed the number of H_z nodes is $N = 3 \times 3 = 9$ (as in Fig. 6.2), and zero boundary condition has been considered.

Similarly, applying Eq. 6.2 to a general mesh unit as shown in Fig. 6.3(b), we have

$$-jk_0\epsilon_{rx}(i,j)E_x(i,j) = \frac{H_z(i,j) - H_z(i,j-1)}{\Delta y} - j\beta H_y(i,j), \quad (6.13)$$

$$-jk_0\epsilon_{ry}(i,j)E_y(i,j) = j\beta H_x(i,j) - \frac{H_z(i,j) - H_z(i-1,j)}{\Delta x}, \quad (6.14)$$

$$-jk_0\epsilon_{rz}(i,j)E_z(i,j) = \frac{H_y(i,j) - H_y(i-1,j)}{\Delta x} - \frac{H_x(i,j) - H_x(i,j-1)}{\Delta y}. \quad (6.15)$$

We also have three matrix equations from Eq. 6.13-6.15, *i.e.*,

$$-jk_0\epsilon_{rx}\mathbf{E}_x = -j\beta\mathbf{I}\mathbf{H}_y + \mathbf{V}_y\mathbf{H}_z, \quad (6.16)$$

$$-jk_0\epsilon_{ry}\mathbf{E}_y = j\beta\mathbf{I}\mathbf{H}_x - \mathbf{V}_x\mathbf{H}_z, \quad (6.17)$$

$$-jk_0\epsilon_{rz}\mathbf{E}_z = -\mathbf{V}_y\mathbf{H}_x + \mathbf{V}_x\mathbf{H}_y, \quad (6.18)$$

in which we define

$$\mathbf{V}_x = \frac{1}{\Delta x} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}, \quad (6.19)$$

$$\mathbf{V}_y = \frac{1}{\Delta y} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \quad (6.20)$$

Notice through Eq. 6.18, we can express $\mathbf{E}_z$ in terms of $\mathbf{H}_x$ and $\mathbf{H}_y$. And if we substitute this $\mathbf{E}_z$ into Eq. 6.6 and Eq. 6.7, we have

$$k_0 \mu_{rx} \mathbf{H}_x = -\beta \mathbf{I} \mathbf{E}_y + k_0^{-1} \mathbf{U}_y \epsilon_{rz}^{-1} (-\mathbf{V}_y \mathbf{H}_x + \mathbf{V}_x \mathbf{H}_y), \quad (6.21)$$

$$k_0 \mu_{ry} \mathbf{H}_y = \beta \mathbf{I} \mathbf{E}_x - k_0^{-1} \mathbf{U}_x \epsilon_{rz}^{-1} (-\mathbf{V}_y \mathbf{H}_x + \mathbf{V}_x \mathbf{H}_y). \quad (6.22)$$

The two equations can be put into a matrix form as

$$\mathbf{M}_1 \begin{bmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{bmatrix} = \beta \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{bmatrix}, \quad (6.23)$$

where,

$$\mathbf{M}_1 = \begin{bmatrix} -k_0^{-1} \mathbf{U}_x \epsilon_{rz}' \mathbf{V}_y & k_0 \mu_{ry} + k_0^{-1} \mathbf{U}_x \epsilon_{rz}^{-1} \mathbf{V}_x \\ -(k_0 \mu_{rx} + k_0^{-1} \mathbf{U}_y \epsilon_{rz}^{-1} \mathbf{V}_y) & k_0^{-1} \mathbf{U}_y \epsilon_{rz}^{-1} \mathbf{V}_x \end{bmatrix}. \quad (6.24)$$

From Eq. 6.8, we can express $\mathbf{H}_z$ in terms of $\mathbf{E}_x$ and $\mathbf{E}_y$. If we substitute this $\mathbf{H}_z$ into Eqs. 6.16 and 6.17, we obtain

$$-k_0 \epsilon_{rx} \mathbf{E}_x = -\beta \mathbf{I} \mathbf{H}_y + k_0^{-1} \mathbf{V}_y \mu_{rz}^{-1} (-\mathbf{U}_y \mathbf{E}_x + \mathbf{U}_x \mathbf{E}_y), \quad (6.25)$$

$$-k_0 \epsilon_{ry} \mathbf{E}_y = \beta \mathbf{I} \mathbf{H}_x + k_0^{-1} \mathbf{U}_x \mu_{rz}^{-1} (-\mathbf{U}_y \mathbf{E}_x + \mathbf{U}_x \mathbf{E}_y). \quad (6.26)$$

These two equations can be similarly put as

$$\mathbf{M}_2 \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{bmatrix} = \beta \begin{bmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{bmatrix}, \quad (6.27)$$

where

$$\mathbf{M}_2 = \begin{bmatrix} k_0^{-1} \mathbf{V}_{x\|rz} \mathbf{U}_y & -(k_0 \epsilon_{ry} + k_0^{-1} \mathbf{V}_{x\|rz}^{-1} \mathbf{U}_x) \\ k_0 \epsilon_{rx} + k_0^{-1} \mathbf{V}_{x\|rz}^{-1} \mathbf{U}_y & k_0^{-1} \mathbf{V}_{y\|rz}^{-1} \mathbf{U}_x \end{bmatrix}. \quad (6.28)$$

From Eqs. 6.23 and 6.27, we can easily derive two standard eigenvalue equations as

$$\mathbf{P} \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{bmatrix} = \beta^2 \begin{bmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{bmatrix}, \quad (6.29)$$

$$\mathbf{Q} \begin{bmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{bmatrix} = \beta^2 \begin{bmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{bmatrix}, \quad (6.30)$$

where

$$\mathbf{P} = \mathbf{M}_1 \mathbf{M}_2, \quad (6.31)$$

and

$$\mathbf{Q} = \mathbf{M}_2 \mathbf{M}_1. \quad (6.32)$$

Both $\mathbf{P}$ and $\mathbf{Q}$ are sparse matrices. Hence Eqs. 6.30 and 6.29 can be readily solved using freely available ARPACK package.

6.3 PML Absorbing Boundary Condition

Referring to Fig. 6.1, the PML regions are considered to be one part of the computational domain, except that their material properties (permittivity and permeability) are computed *in situ* to avoid reflection as well as to attenuate lightwave [19]. The permittivity and permeability are represented as two complex tensors as [71]

$$\bar{\bar{\epsilon}}_r = \epsilon_r \mathbf{\Lambda}, \quad (6.33)$$

$$\bar{\bar{\mu}}_r = \mu_r \mathbf{\Lambda}, \quad (6.34)$$

where for vertical PML regions (regions II and IV in Fig 6.1),

$$\mathbf{\Lambda} = \begin{bmatrix} \frac{1}{s} & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix}. \quad (6.35)$$

For horizontal PML regions (regions I and III in Fig 6.1),

$$\mathbf{\Lambda} = \begin{bmatrix} s & 0 & 0 \\ 0 & \frac{1}{s} & 0 \\ 0 & 0 & s \end{bmatrix}. \quad (6.36)$$

For PML regions at the corners (regions V, VI, VII and VIII in Fig 6.1), the Λ value is the product of Λ matrices belonging to two neighboring PML regions. Hence they have

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s^2 \end{bmatrix}. \quad (6.37)$$

s is defined as

$$s = 1 - j\alpha, \quad (6.38)$$

where α attributes to absorbtion loss, and its profile can be computed as

$$\alpha = \alpha_{\max} \left(\frac{\delta l}{l} \right)^m. \quad (6.39)$$

In Eq. 6.39, $\alpha_{\max}$ is the maximum α value desired, δl is the normal distance from the interested point to the PML boundary, and l is the PML layer width. m determines the absorption profile. Normally a parabolic profile ($m = 2$) is sufficient in most cases.

6.4 Remarks

As FDM uses a rectangular mesh, it is not as straightforward as in FEM to represent curved material interfaces. It has been found that a binary staircase approximation of the curved interface will result in a very slow convergence speed. However, FDM still can successfully deal with curved interfaces using the index averaging technique. A simple index averaging technique computes the refractive index of a mesh cell crossing a material interface as (see for example [18])

$$\epsilon(i, j) = f_a \epsilon_a + f_b \epsilon_b, \quad (6.40)$$

where f_a and f_b are area fractions of material a and b in the mesh cell, respectively, and $f_a + f_b = 1$. A more advanced index averaging technique using the Ampere's Law has also been described by Guo *et al.* in [19].

In [18] and [19], the result computed using this version of FDM is found to be in excellent agreement with the semi-analytical MEM, both for a leaky MOF ([19]) and for a not-so-leaky MOF ([18]). Convergence test of this FDM is also presented in Refs [18] and [19]. Generally speaking, the FDM converges quickly with increasing mesh resolution especially when an index-averaging is used.

In FEM, one needs to calculate the submatrices on each mesh element before forming the global matrices. The calculation of the submatrices can sometimes be very time-consuming, especially when a high-order shape function is adopted. FDM has the advantage of forming the global matrix almost directly. Its computation speed is, in general, found to be faster, though the problem size (the number of unknowns) tends to be larger as compared to using FEM. In addition, FDM allows the index profile to be input as a

bitmap image at any color depth. This facilitates easy analysis of waveguides with complex index profiles. In fact, scanning electron micrograph (SEM) of the cross-section of an actual fabricated fiber is directly recognizable by our FDM program. With FEM, for a complex waveguide, one could easily spend (to the author's knowledge) tens of minutes in the meshing step alone. Due to these factors, this FDM is widely used in the second part of this thesis, especially for deriving defect modes propagating in an air-core photonic bandgap fiber with a complex index profile.

FDM, however, has an obvious drawback. As it uses a uniform mesh, it tends to result in a very large number of unknowns. But we have noticed that a 300×300 numerical resolution is adequate enough for most problems, and such a problem size is manageable on a personal computer. Also, during our numerical tests, reasonably good accuracy is achieved for the hollow-core air-silica photonic bandgap fiber [72], which suggests the index-averaging can well take those fine features into account.

Part II

**MICROSTRUCTURED
OPTICAL FIBERS**

Chapter 7

MOF: An Overall Introduction

7.1 Definition and Varieties

Optical fibers with air holes running along their axes were reported by Kaiser *et al.* in 1974 [73] and Okoshi *et al.* in 1982 [74]. However, it was not until the so-called photonic crystal fiber demonstrated by Knight *et al.* [75] in 1996, such fibers with an air-silica composite cladding are seriously considered by researchers around the world. To date, lots of fibers with air-hole inclusions have been reported. In fact, the “holey” fibers have been experiencing constant alterations in structure and even in material for different application purposes. Some of the fibers deviate much in structure from that in the very first publication. To address these fibers, several names have been coined — they include *photonic crystal fiber* (PCF), *holey fiber* (HF), *microstructured optical fiber* (MOF), and *photonic bandgap fiber* (PBGF). “PCF”, “HF”, and “MOF” are defined from a structure point of view, whereas “PBGF” are defined from an optical-property point of view. Among the names, MOF is the most general one. In fact, MOF can be used to address all fibers which have their feature size at micrometer (or submicrometer) scale. The remaining names can be considered as subsets of MOF, and they can’t cover rigorously all MOFs reported in general. For example, “PCF” is not appropriate for addressing fibers which lack of a periodic cladding; “HF” is not appropriate for fibers made of all solid materials; “PBGF” is certainly only applicable for those guiding light using a cladding photonic bandgap (PBG).

Figure 7.1 and 7.2 show novel MOFs made or proposed during recent years. While the claddings of the PBG fibers (except the air-silica Bragg fiber) shown in Fig. 7.2 are observed to adhere to the term “photonic crystal (PC)” faithfully, index-guiding fibers sometimes can have nothing to do with photonic crystals (for example, the air-clad fiber). Index-guiding MOFs *not* presented in Fig. 7.1 include the hole-assisted MOFs [76, 77], the double-clad index-guiding MOF [78], and the “graded-index” MOF [79]. PBG-guiding MOFs *not* presented in Fig. 7.2 include the hollow-core Bragg fiber [38] and the solid-core Bragg fiber [39]. Heterostructured PCFs [80] are not shown in either figures.

Having mentioned all these MOF varieties, we will, however, primarily limit our discussion to MOFs with a periodic cladding in this thesis. Hence in certain places an MOF can be also referred to as a PCF without ambiguity.

Figure 7.1: (a) An index-guiding PCF [75]. (b) An MOF with six holes [81]. (c) The so-called air-clad fiber, which has very large numerical aperture [78]. (d) A high-birefringent MOF [82]. (e) An MOF with elliptic cladding air holes [83]. (f) A index-guiding PCF with low-index rods in cladding [84]. Black is for air, white is for silica. In panel (f), grey is for a solid material whose index is smaller than that of the background.

Figure 7.2: (a) A PBGF whose cladding is of triangular lattice [72]. (b) A PBGF whose cladding is of honeycomb lattice [85]. (c) A traditional honeycomb PBGF [86]. (d) An improved honeycomb PBGF [87]. (e) An air-silica Bragg fiber [88]. (f) An all-solid PBGF [89]. Black is for air, white is for silica. In panel (f), white is for a solid material whose index is larger than that of the background.

Figure 7.3: Schematic dispersion diagrams for: (a) A conventional SIF with a low-index homogeneous cladding and a high-index homogeneous core; (b) A fiber with photonic crystal cladding and a homogeneous core. Regions filled with horizontal lines are lightwave states forbidden in the cladding material. Region filled with vertical lines are lightwave states allowed in the core material. Their common regions (grid-shaded regions) are where confined states (allowed in the core but not allowed in the cladding) are possible. n_{core} , n_{clad} , n_{fsm} are assumed to be constant with respect to wavelength.

7.1.1 Index-guiding *v.s.* PBG-guiding

In fact, the conventional step-index fiber (SIF) can be considered as a PBG-guiding fiber in the sense that the propagation constant of a core mode in an SIF cannot be supported by the its cladding. The bandgap region used for index-guided modes is the spectral region below the cladding's radiation line $\beta = kn_{clad}$ in β - k plot [90]. In this region, the cladding can't support any propagating mode, as the lightwave (wavelength λ) in a homogeneous material (refractive index n) should have its largest propagation constant at $\beta = \frac{2\pi}{\lambda}n$. And this largest propagation constant happens when the light propagates as a plane wave along the z direction. Any other mode of propagation will decrease its propagation constant.

A high-index material has its radiation line $\beta = kn_{high}$ positioned lower than that for a low-index material $\beta = kn_{low}$ in a β - k plot. Referring to Fig. 7.3, when a high-index rod (core) is introduced into an infinitely-extending low-index material (cladding), there exists a spectral region in the β - k plot (enclosed by $\beta = kn_{core}$ and $\beta = kn_{clad}$) in which

light is only allowed in the core but not in the background cladding material. This is how an SIF works.

Now as the homogeneous background material is replaced by a 2D photonic crystal, the cladding's radiation line becomes $\beta = kn_{\text{fsm}}$, where n_{fsm} is the effective index of the fundamental space-filling mode (FSM) of the crystal structure. The main difference between a composite and a homogeneous material is that, other than the region below its radiation line, there are possibly some small regions above the radiation line, in which light can't propagate in the crystal [Fig. 7.3(b)]. These discrete regions are the photonic bandgaps of the crystal. Indeed, we can make use of these small regions to confine light in a core material whose index is smaller than n_{fsm} . This is how a photonic bandgap fiber works. If the core index is larger than n_{fsm} , it is not difficult to imagine that both index- and bandgap-guided modes can co-propagate in the fiber [90, 91].

7.1.2 PBG-guiding *v.s.* Guidance by Antiresonant-reflection

To differentiate between these two terms, we have to recall the history of the *antiresonant-reflecting optical waveguide* (ARROW).

In 1970 Ash proposed to replace the homogeneous substrate with a multilayered substrate in a slab waveguide to guide surface waves [92]¹. This structure was then theoretically treated by A. J. Fox in 1974 [93]. Both papers call such a waveguide grating waveguide. However, Fox did not show wave guidance in a low-index material. Two years later, Yeh *et al.* theoretically demonstrated, by use of the Floquet-Bloch theorem, guidance in a slab waveguide with a layered cladding and a low-index (air) core [94]. Such a waveguide is named by Yeh *et al.* as Bragg waveguide. In this paper, very importantly, they used a third material in the core region (air), and the cladding is a periodic structure made of two solid materials. Half a year later, Cho from Bell Laboratories, in collaboration with Yariv and Yeh, experimentally confirmed confined propagation in such waveguides [95]. In 1978, Yeh *et al.* went one step further from the slab-type waveguides, and proposed to use multi-layered cylinders to propagate light in an air column [29]. Such a fiber is named as a Bragg fiber. Both the slab-type Bragg waveguide and the Bragg fiber were not investigated further until eight years later in 1986 when Duguay *et al.* fabricated a Bragg waveguide [96] on a silicon wafer. Their waveguide is however formed by two material, *i.e.*, the core material is also one of the cladding materials. But Duguay *et al.* renamed the waveguide as “antiresonant-reflecting optical waveguide” (ARROW) and attributed the guidance to the antiresonance of the high-index layer (analogous to a Fabry-Perot resonator) placed adjacent to the low-index core. To some extent, this paper revives the research on such waveguides. A 2D ridge-type ARROW was also fabricated by Freye *et al.* in 1994 [97].

Research on waveguides with a layered cladding, or ARROW waveguides, should really be considered as pioneer work on photonic crystal waveguides (bandgap guidance). In fact, the adoption of periodic claddings in ARROW waveguides can easily lead to the concept of

¹This paper is cited in [93], but is not traceable.

Figure 7.4: A traditional ARROW waveguide. Refractive index $n_2 > n_1$. The big arrow denotes light launched into the core region. Notice that the core can be made of a third material of different index (normally lower than both n_1 and n_2) [94]. k_z is the wavevector component in the longitudinal direction, which can be more commonly written as β . k_x is the wavevector component in the lateral direction. Notice that for a particular propagating mode, k_x has different values in layers with different refractive indices, whereas the same k_z value is shared in all layers.

photonic crystals (which were only proposed formally in 1987). However, as the authors did not generalize the *layered dielectric media* to other dimensions, their impact was limited.

Let's review the traditional ARROW model made popular by Duguay *et al.*. Referring to the simplest slab-type ARROW shown in Fig. 7.4, the model states that properties of the high-index layers around the core determines the guidance in the low-index core region. When the high-index cladding layers are in resonance with the core mode, light is relayed outwards laterally by the high-index layers. When the layers are not in resonance (in antiresonance) with the core mode, light is rejected and confinement in low-index core region is achieved. The resonance and antiresonance conditions can be quantitatively determined by calculating the lateral phase variation in a high-index layer (Fig. 7.4), as

$$\phi_x = k_0 \sqrt{n_2^2 - n_1^2} \cdot d_2, \quad (7.1)$$

where d_2 is the width of the high-index layers. If ϕ_x is equal to an odd multiple of $\pi/2$, the high-index layers are considered to be in an antiresonant state with the core mode. Or we can say that Bragg reflection condition is met along the x -direction. The core mode hence has the least radiation loss under this condition. If ϕ_x is equal to an even multiple of $\pi/2$, the high-index layers are considered to be in a resonant state with the core mode. The core mode experiences rapid dissipation after being launched.

This traditional ARROW model does explain successfully the light confinement in a lower-index core material. It even does not acknowledge the cladding to be periodic (which implies that we might have overlooked certain things as nowadays we explicitly use a periodic PC cladding to confine light in a lower index material). Also the model can accurately predict the exact highest-loss and lowest-loss wavelength points by considering the high-index layers in cladding to be in complete resonance and antiresonance, respectively. However, in the ARROW model proposed by Duguay *et al.*, the role of the

low-index layers in cladding region has been ignored. Antiresonance of the low-index layers also contributes to the confinement of lightwave in the core region². In other words, to better confine light in the core region, we need to tune the widths of both the high- and the low-index layers in cladding so that they meet their anti-resonance conditions together. In fact, ARROW is indeed just another name for regular waveguides whose guidance is achieved by the photonic bandgap effect. Antiresonance of the cladding composite gives rise to a forbidden bandgap. We can establish an equation

$$\begin{aligned} \text{cladding bandgap} &= \text{completely antiresonant state of cladding} \\ &+ \text{partially antiresonant states of cladding.} \end{aligned} \quad (7.2)$$

The ARROW model is straightforward for slab-type waveguides shown in Fig. 7.4. Simple equations can be written down to determine wavelength points where the lowest and the highest leakage loss happen. However, it should be mentioned that such equations are difficult, if not impossible, to be written down analytically for a 2D photonic crystal waveguide. For example, for the PBG fiber shown in Fig. 7.2(a), it is not easy to determine the least-leaky wavelengths with some analytical expression. For the all-solid PBG fiber shown in Fig. 7.2(f), though the resonance conditions for the cladding high-index rods can be written down approximately using certain expression [98, 99], the resonance condition for the low-index cladding region cannot. Especially, in such a 2D waveguide, a full transverse bandgap of the cladding is necessary for confining light. Hence we have to consider all possible wavevector orientations in the transverse plane for analyzing resonance and antiresonance conditions.

The term “photonic crystal” gives us the impression that photonic bandgaps can only exist in periodic cladding structures. However, a consequence of the ARROW model is that the cladding of an PBG waveguide does not need to be periodic. Referring to Fig. 7.4, the high-index cladding layers can have a certain width to cause a $\pi/2$ lateral phase variation, and they can also be thicker to introduce a $3\pi/2$ lateral phase variation. Besides, as far as the optimum confinement at a particular wavelength is concerned, the periodicity requirement of the cladding composite is automatically lifted in the case of Bragg fiber (and any other 2D PBG waveguide). This is due to the fact that the mode field in a Bragg fiber is represented by aperiodic Bessel functions³ [29].

²The antiresonance of the low-index layers has a relatively small contribution as compared to the high-index layers to the confinement of lightwave. This is due to the fact that only a small fraction of the cladding field resides in the low-index cladding layers.

³For a Bragg fiber or a general 2D PBG fiber, we would however be better off by using a periodic cladding in practice. There are several reasons. First, aperiodic cladding can only enhance guidance at a single wavelength point, not the whole cladding PBG window. And improvement in PBG guidance can always be achieved by including more cladding layers. Second, periodic structures is easier to fabricate as compared to aperiodic structures. And theoretically, a periodic structure facilitates the deployment of the Bloch theorem in the bandgap calculation.

Figure 7.5: Composition of a PCF. Black is for air, white is for silica. TPC stands for “triangular photonic crystal” (air holes are arranged in a triangular lattice). HPC stands for “honeycomb photonic crystal” (air holes are arranged in a honeycomb lattice).

7.2 PCF Classification

Figure 7.5 shows the possible configurations of a PCF. This figure has generalized PCF to those made of a PC cladding and a PC core, or the heterostructured PCF [80]. Though we only consider PCs made of two materials, it should be kept in mind that more materials can be involved. Structurally, only PCs exhibiting C_{6v} symmetry are presented owing to their ease of fabrication. Depending on the fabrication method, other-latticed PCs should be able to exist.

Naming the microstructured optical fibers can be annoying, especially if there are uncertainties in their component materials. However, if we assume the materials are by default air and silica, a particular two-material fiber in Fig. 7.5 can be referred to by mentioning the cladding and core structures. For example, if the cladding is a TPC and core is an HPC, then the fiber can be referred to as a TPC-HPC fiber; for a fiber with a TPC cladding and a silica core, we can name it as a solid-core TPC fiber; for a fiber with a TPC cladding and an air core, we can name it as an air-core TPC fiber, etc.

Besides those shown in Fig. 7.5, we should mention that an MOF can have a homogeneous cladding and a PC core [100].

Triangular *v.s.* Honeycomb

These two terms are sometimes confusing. Early PCFs are made of circular air holes in a silica background. Hence, we normally tell a PCF’s cladding type according to the placement of air holes. However, as PCF design becomes more complicated, such as the air-guiding TPC fiber and the HPC fiber shown in Fig. 7.2(a) and (b), the fiber cladding looks more like the silica rods immersed in the air background [85, 101]. If we define the lattice according to position of the silica pillars, traditional TPC (HPC) becomes HPC (TPC).

Addressing the lattice pattern of a PC can also be ambiguous without specifying corresponding lattice vectors. For a traditional HPC, if we use a second-level unit-cell, the crystal becomes a TPC [80].

In this thesis, we stick to the traditional naming convention, *i.e.*, a crystal structure is defined by the air hole positions, though the air holes may not be circular. Also, the primitive basis vectors are normally used for defining the crystal's lattice pattern, unless otherwise stated.

7.3 Technology Review and Current Trends

7.3.1 Index-guiding MOF

Technology Review

The possibility of fabricating a fiber whose cladding is made of photonic crystal was first proposed in [102]⁴. In this paper, Birks *et al.* theoretically showed that full 2D photonic bandgaps exist in an air-silica (holes-in-silica) photonic crystal when the off-plane wavevector component is large enough. And they suggest “a new type of optical fibre, the PBG fibre, is possible” by using “a periodic air/silica cladding surrounding a uniform core”. “The core can be solid silica or a hollow air region”. One year later, the same group published a paper on the first fabricated photonic crystal fiber [75]. As the central defect is silica, the fiber does not guide light using the PBG effect. Though Knight *et al.* have included the term “photonic crystal” in naming the waveguide, the fiber physically is equivalent to a conventional step-index fiber. Lack of PBG guidance, however, does not prevent such a fiber from being exceptional. Knight *et al.* identifies an interesting property, *i.e.*, it has a very broad single-mode operation wavelength range (458nm~1550nm for the fiber reported, with $\Lambda = 2.3\mu\text{m}$ and $d/\Lambda = 0.15$). Their work attracted immediately lots of attention and related studies were carried out by different research groups around the world. More special waveguiding properties in such fibers were reported. In year 1998, Mogilevtsev *et al.* theoretically studied the group velocity dispersion (GVD) of PCFs [104], and found some novel dispersion properties, such as the zero-dispersion point at short wavelength and the large normal dispersion value at 1550nm. These novel dispersion properties can be easily achieved by adjusting the fiber parameters (Λ and d). The dispersion tuning in such fibers was later studied by other researchers. To name a few, dispersion-flatten PCFs were studied in [105, 106, 107, 108, 109], dispersion-compensating PCFs were studied in [110, 111, 112, 113]. Remarkable birefringence property in PCFs was also reported [114, 115, 116, 82]. Nonlinear optics in PCFs is also demonstrated to be promising as the fibers can be fabricated with an

⁴Here we would like to draw attention to the paper published in 1978 by Yeh *et al.* on the Bragg fiber [29]. Strictly speaking, their work should be regarded as the first paper on *fiber* design employing a photonic crystal (1D) cladding. However, they didn't generalize the idea to 2D photonic crystal waveguides. Their work was put into practice by a group of people in MIT about 20 years later (for example, see [36] and [38]). This is quite similar to the case that 1D photonic crystal was first explained by Lord Rayleigh in 1887, but revived by E. Yablonovitch [103] and S. John [33] 100 years later. There are always some people ahead of time.

ultrahigh nonlinearity using large-sized cladding air holes and a small silica core. Among all nonlinear effects, third-harmonic generation in PCF can be found in [117], four-wave mixing can be found in, for example, [118, 119], the Raman effect can be found in [120, 121, 122], and the supercontinuum phenomenon can be found in, for example, [123, 124, 125], etc.

It should be mentioned that besides using silica, other materials are also investigated for index-guiding PCF designs. The materials include high-lead silicate glass, tellurite glass, gallium lanthanum chalcosulfide glass, and also polymethyl-methacrylate (PMMA). The first three types of glasses have quite low softening temperature, hence they are also called soft glasses. Using these soft glasses to fabricate PCFs has been summarized in [126]. Two thermally-matched soft glasses have also been used to form an index-guiding PCF [84]. The first polymer PCF was reported in [127].

Trends

Though propagation loss of an index-guiding MOF has been reduced to 0.48 dB/km [128], we don't see immediate necessity to deploy such fiber into a transmission system. There are several reasons for this. First of all, the loss is still higher than that in conventional step-index fibers; second, MOFs are certainly more expensive as compared to the commercial step-index fibers⁵; and third, compatibility (such as splicing) with existing system is still a problem. Index-guiding MOF can indeed be tailored to have some excellent properties in certain aspects, but not all aspects. In the next few years, application of such fibers would, most probably, still be limited to subsystems or optical devices. In particular, exploration of the high nonlinearity and high birefringence properties in MOFs would be likely to attract fair amount of attention. Its deployment in fiber lasers will also be a very meaningful research topic.

7.3.2 PBG-guiding MOF

Technology Review

The first PCF whose guidance is due to the existence of its cladding PC's photonic bandgap (PBG) was reported in 1998 by Knight *et al.* [129]. However, the propagating light concentrates in the silica portion of the core region, which is inevitable as the core is made of an air-silica composite [80]. The first *air-guiding* PBG fiber was reported in 1999 by Cregan *et al.* [130]. The transmission spectrum is measured using a short fiber of only a few centimeters. It is claimed to be single-mode, but the authors didn't convincingly prove their point. The fabrication technique greatly improved in the following several years. By 2003, loss for air-guiding PCF was greatly reduced to 13dB/km at 1500nm by Corning [72]. And in 2004, air-guiding fiber loss as small as 1.2 dB/km was reported [131, 132].

⁵MOF uses only pure silica, therefore its preform is cheaper in terms of material as compared to conventional SIFs where doping is involved. However, stacking of preform bundles is labor-intensive (there is currently no better way to prepare the preform) and the uncertainties involved during fabrication (dust contamination, fluctuation in pressure controlling etc) is higher than that for conventional fiber. Unless high-level automation is realized, it is reasonable to conclude that MOFs are more expensive.

Air-guiding PBG fiber was originally proposed for ultra-low loss optical communication. In addition, it was said that signal propagating in such a fiber is “free of group velocity dispersion”. However, to date, we have achieved neither of these two goals. The loss is still high compared to conventional step-index fibers. The group velocity dispersion is quite large, especially near bandgap edges [60]. Recently it is found that there are uncertainties in such a fiber’s polarization mode dispersion (PMD) [133]. What’s more, a true low-loss, single-mode, air-guiding PBG fiber has never been fabricated. All these factors would prevent such fiber from being deployed in along-haul transmission link at this moment. However, due to the unique hollow-core feature, it has thrived in areas like high-power pulse delivery [134, 135], nonlinear optics in gases [136, 137], atom guiding [138], and sensing applications [139], etc.

Besides hollow-core PBG fibers, there exist several types of solid-core PBG fibers. Theoretical work on these can be traced to [140] and [99]. Experimentally, two soft glasses have been used to fabricate a PBG fiber by Luan *et al.* [89]. Doped silica has also been used to form scatterers in the cladding region [141]. It was observed that such low-index contrast PBG fiber has a wider transmission window [141]. However, no particular application has so far been identified using these types of fibers, largely due to the fact that light is confined in a solid core, which is quite analogous to conventional a step-index fiber.

Besides using 2D PC to form a PBG fiber’s cladding, cylindrical-1D PC was also exploited for confining light in a hollow core. The hollow-core Bragg fiber, or Omniguide, was first fabricated by Fink *et al* [36]. The same group has demonstrated using Bragg fiber to deliver high-power CO₂ laser beam [38]. Recently, they have produced such a fiber with a transmission window in the near infrared wavelength range (0.85 to 2.28 μ m) [41]. Vienne *et al.* fabricated the so-called air-silica Bragg fiber whose cladding resembles a cylindrical-1D PC [88]. Very similar Bragg fiber has also been fabricated very recently by Agyros *et al.* using air and polymer [142]. However, the air-silica Bragg fiber suffers from a very fragmented transmission window [143]. Whereas Bragg fibers made by Fink’s group have three materials (core is made of air, cladding is made of soft glass and polymer bi-layers), people have also tried to fabricate Bragg fibers using two solid materials (core material is also one of the cladding materials with lower index), as in [37, 39].

Trends

The traditional air-guiding PBG fiber uses a TPC cladding. Such 2D PC was recently generalized by Yan *et al* [101]. It is found that by slightly modifying the PC structure, it is possible to tune the bandgap favorably for different purposes, *e.g.* smaller leakage or even true single-mode air-guiding etc. A hollow-core fiber with such a PC cladding was studied theoretically in [144]. The fiber has very a wide surface-mode-free PBG guiding wavelength range and it promotes single-mode operation. Technologically speaking, fabrication of such a fiber does not impose any additional difficulty as compared to a conventional air-guiding PBG fiber. It will be very interesting to see such a fiber realized in the near future.

An HPC cladding was not considered to achieve air guiding until its capability was theoretically validated by Yan *et al.* [145]. Such a fiber's leakage loss was later studied in [85]. The hollow-core of such a fiber can be less than $6\mu\text{m}$ in diameter, which suggests single-mode operation is possible. Also the surface-mode problem has been completely mitigated in such fibers. Therefore, HPCF can be a competitive alternative to existing air-guiding PBG fibers if they can be fabricated successfully.

As we have mentioned, the first fabricated honeycomb PBG fiber has an air-silica composite core [129]. The idea of using two different pieces of PCs to construct a fiber (core and cladding are both composite) was put forward by Yan *et al.* [80]. Depending on the design, the heterostructured PCFs can achieve either PBG-guiding or index-guiding. The fibers are expected to perform better in applications like gas or liquid sensing. They can also be used to exploit the phenomenon of discrete soliton propagation [146]. Such fibers deserve more theoretical and experimental studies.

Compared to air-silica PBGFs, Bragg fiber suffers significantly higher radiation loss. New types of materials with high index-contrast, low absorption, and matched thermal properties (softening temperatures, expansion coefficients, viscosity) as well as chemical properties (non-diffusive into each other) need to be identified to fabricate more robust hollow-core Bragg fibers. Preferably, the guidance wavelength should be in the telecommunication window, i.e. 1550nm wavelength. For air-silica Bragg fiber fabricated by Vienne *et al.*, more theoretical work is necessary to understand its leakage loss. It might be possible to prevent the fragmentation of the transmission window by an improved design. If that is true, we can reduce the overall leakage loss by increasing the number of air-hole rings in the cladding.

Application-wise, high-power pulse or beam delivery will still be an important direction to pursue [147]. Nonlinear optics in such fibers with gas- or liquid-filled core will also be very interesting. High dispersion in air-core PBG fibers, especially near two bandgap edges, have been utilized to compress optical pulses so that the output pulses can have a very-high peak power [148]. We believe further exploration of the dispersion property, together with the fiber's high-power handling capability, can result in more novel optical devices and sub-systems.

Chapter 8

Index-guiding MOFs

The index-guiding MOF, since its introduction in 1996 [75], has evolved into many forms, as illustrated by Fig. 7.1 in the previous chapter. Most of them have been commercialized and have found specific applications. Conversely, the PBG fibers shown in Fig. 7.2 face many uncertainties in terms of application at this moment. Indeed, one may conclude that introducing air holes as a component material into optical fiber design is almost as significant as introducing the photonic bandgap (PBG) effect. Due to the large index contrast between air and silica, many properties unachievable through doping can be realized easily by utilizing air holes.

In this chapter, we would like to present results from our studies on this index-guiding MOF category. The chapter is divided into two sections. In the first section, dispersion management by using both doping and air holes will be discussed. It is found that very small dispersion values (less than 5ps/km/nm) and near-zero dispersion slope over both C-band and L-band can be achieved. In the second section, we will theoretically demonstrate that antiguided modes are supported by the overall microstructure region in an index-guiding MOF. Such modes can affect performance of optical devices made from such fibers.

8.1 Dispersion Management with Hole-assisted Multiring MOF

8.1.1 Introduction

Hasegawa *et al.* have recently fabricated a class of hole-assisted lightguide fibers (HALFs) and some superior properties such as larger anomalous dispersion at 1550nm (as compared to a conventional fiber) and smaller loss (as compared to conventional index-guiding PCF) are observed experimentally [76]. But further clarification on such hybrid applications of both air holes and doping in fiber design is not stated in that paper. Here we extend the idea and apply air holes in a multi-ring fiber ([149], [150] and [151]) to study their effect on the fiber's dispersion property in general. We propose a novel holey fiber design where only a small number of air holes are employed. The holes are not to help confine lightwave, but to modify the fiber's dispersion characteristics. Like conventional fiber, doping is

Figure 8.1: (a) Schematic index profile for the hole-assisted fiber. Gray level is in accord with refractive index level. (b) Effects of holes on dispersion property of hole-assisted fiber ($D_{\text{hole}} = 8\mu\text{m}$).

responsible for lightwave guidance. Flat dispersion values no larger than 5ps/km/nm around 1550nm are realized over a large wavelength span. We refer to our design as *hole-assisted multi-ring fiber (HAMRF)*.

8.1.2 Analysis

When air holes are introduced into a conventional fiber near the edge of its fundamental mode field, their effect on the fiber's dispersion property is shown in Fig. 8.1(b). Referring to Fig. 8.1(a), the fiber involved in calculations has following parameters: doped core diameter $\phi_{\text{core}} = 9.0\mu\text{m}$, background index $n_b = 1.45$, index difference $\Delta = 0.4$ (in percentage). Fig. 8.1(a) shows the hole distribution. These holes can be characterized by: hole size ϕ_{hole} , distance from hole center to core center D_{hole} , and number of holes N . Material dispersion is included in all dispersion curves presented in this section. Fig. 8.1(b) shows that, for a certain range of wavelength ($1.2 - 2.1\mu\text{m}$), the dispersion curve moves upwards when holes are present and moves further up when hole size gets bigger. Dispersion values at wavelengths shorter than $1.2\mu\text{m}$ are not affected much since the holes sample very little optical field at short wavelengths. At wavelengths longer than $2.1\mu\text{m}$, the dispersion curves for holey structures become suspectable since the fundamental mode effective index (n_{eff}) is nearly equal to the background index 1.45, which indicates difficult

mode confinement. From Fig. 8.1(b) it should not be difficult for us to speculate that the effect of moving up on dispersion will diminish when the holes are getting further away from the mode field. Hence, by tuning ϕ_{hole} and D_{hole} , we can modify dispersion values around our interested wavelength favorably. In Fig. 8.1(b), it is also noticed that the dispersion slope does not change much due to the surrounding air holes. Hence simple designs like the one shown in Fig. 8.1(a) will not give us a flat dispersion curve.

Multi-clad fibers or multi-ring fibers have long before been investigated for dispersion flattening and zero-dispersion-wavelength shifting ([149], [150] and [151]). These fibers, depending on their respective designs, have either been deployed as new generation transmission fibers or as dispersion compensating fibers. Referring to Fig. 8.2(a), a tri-clad (TC) fiber has the following parameters: core radius R_1 , first two cladding radii R_2 , and R_3 , and refractive index differences Δ_1 , Δ_2 , Δ_3 (in percentage, with respect to n_b). Basically the presence of the second clad ring, which is of slightly higher index, will drag the dispersion curve down at a certain wavelength range due to field coupling between this ring and the central core [152]. This philosophy was recently deployed in the dispersion-compensating PBG fiber design [40].

Figure 8.2: (a) Schematic index profile of a tri-clad fiber; (b) Schematic index profile of a TC fiber with its central core region depressed.

The schematic index profile of the multi-ring fiber, on which our HAMRF will be based, is shown in Fig. 8.2(b). The central index depression helps to increase effective core area as well as to reduce the cut-off wavelength of the second-order mode. Our proposed holey structure includes air holes into the second clad ring of the fiber [the complete index profile is schematically shown in Fig. 8.3(a)]. The second clad ring is selected so that holes are positioned at the field edge of the fundamental mode. Dispersion values at certain wavelength range will thus move up as we showed above. In addition, the optical field at the second cladding is far less strong than in the core region, therefore both imperfections at the air-silica interface and water contamination in the air holes will have weaker perturbations on the fundamental mode field. Dispersion flattening can be achieved in two steps: 1) to find a certain multi-ring fiber index profile with its dispersion

property likely to be modified favorably by adding holes; 2) to introduce holes into the profile and adjust the dispersion curve by varying ϕ_{hole} , D_{hole} , and N .

Figure 8.3: (a) Our proposed HAMRF, whose parameters are: $R_1 = 3.90 \mu\text{m}$, $R_0/R_1 = 0.41$, $R_2/R_1 = 1.90$, $R_3/R_1 = 3.80$, $\Delta_0 = -0.10$, $\Delta_1 = 0.48$, $\Delta_2 = -0.33$, $\Delta_3 = 0.1$, $D_{\text{hole}} = (R_2 + R_3)/2 \times 1.2$, $\phi_{\text{hole}} = 1.8 \mu\text{m}$, $n_b = 1.45$. (b) Effective index curves for a multi-ring fiber with and without air holes present. Full-vector BPM results, which agree well with scalar ones, are also shown.

8.1.3 Design and Discussion

The scalar beam propagation method (BPM) is used for most of our simulations due to its efficiency and accuracy. The scalar approximation is valid since our design is essentially a weakly-guiding optical waveguide [this argument will be confirmed later in Fig. 8.3(b)].

The proposed HAMRF is shown in Fig. 8.3(a). Figure 8.3(b) gives our fiber's modal n_{eff} curve as a function of wavelength (curve with open circles). Also in the figure is the modal n_{eff} curve for the same fiber but without air holes (solid curve). Comparing the two curves, we see that, for the solid curve, the modal n_{eff} approaches n_b (1.45) at a slower rate across a wider wavelength region. This "slow-down" in the change in n_{eff} can be qualitatively explained this way: as wavelength increases, coupling between the second clad ring and core becomes more significant; the optical field in the second cladding ring

Figure 8.4: Dispersion curves. Solid curve: our proposed HAMRF but without air holes. Circled curves: our HAMRF. Dashed curves: similar HAMRFs with ϕ_{hole} value chosen at $1.4\mu\text{m}$ (bottom) and $2.2\mu\text{m}$ (top). Dotted curves: similar HAMRFs with D_{hole} increased by $1\mu\text{m}$ (bottom) and decreased by $1\mu\text{m}$ (top). Inset shows the fundamental mode field distribution at $\lambda = 1.55\mu\text{m}$.

soon overweighs that in the first clad ring; since the second clad ring has a slightly higher index than the first clad ring, this will slow down the decrease of n_{eff} . If air holes are present, this “slow-down” effect is alleviated due to a portion of the optical field residing in air holes at long wavelength region. Calculation of dispersion value involves second derivative of the modal n_{eff} curve, hence a large slope change observed for the solid n_{eff} curve will give rise to a large dispersion value. This point is confirmed in Fig. 8.4 (in the solid curve). On the other hand, the steady and small slope change observed for the circled curve in Fig. 8.3(b) suggests a flat near-zero dispersion curve. The circled curve in Fig. 8.4 confirms this point. A close study of our HAMRF dispersion curve tells us that a very flat dispersion value around 3.0ps/km/nm is achieved for the wavelength range $1.53 - 1.65\mu\text{m}$ (including both C-band and L-band). This is almost ideal for wavelength-division multiplexing (WDM) optical communication systems. In particular, it eliminates the need for second-order dispersion (dispersion slope) compensation for ultra-high speed data transmission. At $\lambda = 1.55\mu\text{m}$, we get an effective core area of $80.3\mu\text{m}^2$. This is greater than that of Corning’s commercial LEAF fiber ($72\mu\text{m}^2$). In Fig. 8.4, we also show the effects of ϕ_{hole} (dashed curve) and D_{hole} (dotted curve) on dispersion curve. The results are very similar to what we have discussed in Fig. 8.1(b).

When the wavelength is small, the LP_{11} -like second-order mode [Fig. 8.5(a)] can propagate in the waveguide. This second-order mode disappears around $\lambda = 1.1\mu\text{m}$. As wavelength increases further, the second-order mode evolves into the LP_{02} -like [Fig. 8.5(b)] mode. This LP_{02} -like mode has a very large confinement loss (Table 8.1), which can be quantified using

$$\alpha = \frac{2\pi}{\lambda} \frac{20}{\ln(10)} 10^9 \Im(n_{eff}), \quad (8.1)$$

Figure 8.5: Second-order modes: (a) LP_{11} -like at $\lambda = 1.0\mu\text{m}$; (b) LP_{02} at $\lambda = 1.45\mu\text{m}$, which is very leaky. Notice (a) is a zoom-in plot.

where λ denotes the operating wavelength in micrometers, and $\Im(o)$ means imaginary part. The computed result is in dB per kilometer. Thus, effectively, our design has a single-mode operation over both the C-band and L-band. We have also looked into the possible birefringence caused by imperfections in the fabrication technology. Fig. 8.6 gives a particular case study where two opposing air-holes are positioned by an angular offset (θ) to their original places. The birefringence caused at $\theta = 5^\circ$ is about 1.1×10^{-8} (at $\lambda = 1.55\mu\text{m}$). Due to this birefringence, a pulse will be broadened by 0.32ps for every kilometer length of propagation, which is not very significant comparing to 0.1 – 1ps/km^{1/2} polarization mode dispersion (PMD) for a normal long-haul transmission fiber. Simulations show that at $\theta = 5^\circ$, dispersion does not deviate more than 0.1ps/km/nm from that at $\theta = 0^\circ$ (for the flat region).

Wavelength (μm)	1.25	1.35	1.45	1.55	1.65
Loss (dB/km)	468	1745	7617	18916	44994

Table 8.1: Loss for the second-order mode.

A similar dispersion property is shown in Fig. 8.4 was found to be achievable if number of air holes N is chosen as 6. The only drawback of such design is the air holes are of smaller dimension, which makes our fiber more demanding for precision technology during the fabrication process.

8.1.4 Conclusion

We have proposed a hole-assisted multi-ring fiber which exhibits very small dispersion values and near-zero dispersion slope over both C-band and L-band. This type of fiber can find applications in WDM optical communication systems. Further studies such as its bending loss, mechanical stability, and feasibility in fabrication still need to be carried out in order to get a complete characterization of such type of fiber.

It is noticed that the proposed HAMRF was investigated further by Uranus in his thesis [153] by using a vector finite element method. In his simulation, there are some slight deviations in Δ_1 , D_{hole} and ϕ_{hole} parameter values. Therefore a quantitative comparison is not readily available. However, with a very close parameter setting (Uranus has used $\Delta_1 = 0.45\%$, $D_{\text{hole}} = (R_2 + R_3)/2 \times 1$ and $\phi_{\text{hole}} = 2.2\mu\text{m}$, whereas our parameter

Figure 8.6: (a) Schematic diagram for holes' angular offset placement. (b) Resulted birefringence values when a pair of air holes is positioned by an angular offset θ to their original places.

values are $\Delta_1 = 0.48\%$, $D_{\text{hole}} = (R_2 + R_3)/2 \times 1.2$ and $\phi_{\text{hole}} = 2\mu\text{m}$, he has observed less than 4ps/km/nm group velocity dispersion (GVD) over L-band telecommunication window, which is in agreement with our conclusion that around 3.0ps/km/nm GVD is achieved for both C- and L-bands.

8.2 Antiguiding in MOFs

8.2.1 Antiguiding Nature of MOF

Figure 8.7(a) shows the cross-section of a commercial index-guiding MOF. The parameters are measured as: hole diameter $d = 2.3\mu\text{m}$, and hole-to-hole distance $\Lambda = 6.3\mu\text{m}$. Refractive indices for air and pure silica are assumed to be 1 and 1.45, respectively. Modal properties of such a fiber have been numerically analyzed. A very interesting result is that an index-guiding MOF can be single-mode over an extended wavelength range, provided its d is relatively small with respect to Λ . For the fiber shown in Fig. 8.7(a), its effective V number (V_{eff}) [154] as a function of normalized frequency Λ/λ is plotted in Fig. 8.7(b) (red curve), assuming the air-silica composite cladding extends infinitely far. V_{eff} here is calculated using

Figure 8.7: (a) An endlessly single-mode MOF fabricated by Crystal Fiber A/S. (b) The V_{eff} variation with respect to normalized wavelength λ/Λ . V_{eff} for fibers with different d/Λ values are also shown in (b).

$$V_{\text{eff}} = \frac{2\pi a}{\lambda} \sqrt{n_{\text{silica}}^2 - n_{\text{fsm}}^2}. \quad (8.2)$$

a is the effective core radius. We used $a = 0.525\Lambda$. n_{fsm} is the effective index of the fundamental space-filling mode (FSM) in the cladding photonic crystal when the wave is propagating along the air hole axis. This chosen a value will give rise to a single-mode cut-off V_{eff} valued at 2.18. The same figure also shows V_{eff} curves for other fibers characterized by different d/Λ values. We may conclude from the plot that the fiber in Fig. 8.7(a) is endlessly single-mode.

However, recent experiments done by Eggleton *et al.* [155, 81] have shown that when a fiber Bragg grating (FBG) is written in the core region of such a “single-mode” MOF, the transmission spectrum shows several dips other than only one dip. Each of these dips suggests a mode that is supported by the fiber. By comparing the spectrum with that obtained for a FBG written in a conventional single-mode fiber, they conclude that the dips at short wavelengths are due to coupling from core mode to backward-propagating “cladding modes”. Field patterns of these modes are experimentally recorded by tuning the input wavelength at those dip positions. In this section, we will show that these

“cladding modes” are not actually comparable to those in conventional single-mode fibers for the MOF’s peculiar index profile. Conventional step-index fiber has only one cladding. However, an index-guiding MOF with a finite photonic crystal cladding, as shown in Fig. 8.7(a), has two layers of claddings. The first cladding is the region consisting of the air-silica photonic crystal, and the second cladding is the outer homogeneous silica region. Therefore, besides the traditional cladding modes that are confined by the interface between the outer silica cladding and the surrounding air, there should exist another type of “cladding” modes due to reflection off the interface between the two layers of claddings. As the PC cladding has an effective index lower than the silica index, these overlooked “cladding” modes are of antiguiding nature, and they are therefore inherently very leaky. Besides the antiguided mode, we will show that the composite PC cladding is likely to confine modes by the PC’s bandgap, though such fibers have been commonly known as index-guiding fibers. An approximation of the microstructured fiber into a double-clad fiber will be made to facilitate easy calculation and understanding of the antiguiding modes.

Figure 8.8: (a) cross-section of the MOF under study. Black-shaded circles represent air-holes. Inset, zoom-in view of the PCF core region. Hexagonal region enclosed by the dash-dotted line is treated as the core region. (b) A simple antiguide approximation of the MOF shown in (a). The refractive index of the gray region is equal to the effective index of fundamental space-filling mode of the MOF’s cladding PC in (a). The defect (central white circle) has area equal to that of the region bounded by the dot-dashed line in (a) inset.

Our theoretical simulation treats a fiber as depicted by Fig. 8.8(a). We have added 18 more air holes in the sixth ring as compared to the fiber shown in Fig. 8.7(a). This is to make the overall holey region have a near-circular shape. Such structure can be more accurately approximated by a simple antiguide model. Although modal studies for such a fiber have appeared in many publications, none of them has noticed its antiguiding nature. In our analysis, the periodic air-silica composite is treated as the fiber’s core, with a defect in the center, and the surrounding bulk-silica is the cladding. Its equivalent simple antiguide model is shown in Fig. 8.8(b). The simple antiguide has core (grey region) index equal to the effective index of the longitudinally-propagating FSM in the PC. As we only

focus on $\lambda = 1.55\mu\text{m}$, the core index is 1.4443. The defect (central white circle) has area equal to that of the region bounded by the dot-dashed line in Fig. 8.8(a) inset. Note that such approximation is valid only if we are interested in a few low-order modes, and polarization information is not of concern. Also, this approximation disregards the fine structure and symmetry of the composite core. Fig. 8.9 shows the four least leaky modes for this simplified antiguide. We have used a scalar transfer-matrix method to derive these modes [156]. Their modal n_{eff} values are given in Table 8.2. It is noticed that except for mode-*a*, other modes are highly leaky.

Figure 8.9: Mode field distribution of: (a) LP_{01} -like (b) LP_{11} -like (c) LP_{02} -like (d) LP_{21} -like modes.

Mode	Effective Index	Loss (dB/m)
a	$1.446121 + 4.277 \times 10^{-11}i$	0.0015
b	$1.444054 + 2.711 \times 10^{-5}i$	954.5
c	$1.444087 + 2.928 \times 10^{-5}i$	1030.9
d	$1.443831 + 5.329 \times 10^{-5}i$	1876.3

Table 8.2: Effective indices and confinement losses for four modes in Fig. 8.9.

Now we turn our attention to the MOF shown in Fig. 8.8(a). Scalar beam-propagation method (BPM), with a correlation mode solver [11], will be used to derive modes supported by the MOF. The qualitative agreement between the scalar BPM results and the experiment results for air-silica MOFs can be found in [155, 157, 81]. Fig. 8.10 gives the mode spectrum and two dominant modes (mode-*a* and *e*) when the MOF is excited with a symmetrically launched source. Whereas mode-*a* can find its analogue in a simple antiguide model [Fig. 8.9(a)], mode-*e* hasn't an equivalent mode in the simple antiguide. In fact, as we have revealed in a very recent study, mode-*e* is owing to the bandgap possessed by the PC cladding layer [90]. Eggleton et al. have misidentified these bandgap-guided modes as cladding modes [155]. Comparing mode-*a* with the mode shown in Fig. 8.9(a), the difference in their real n_{eff} values is 0.000768. Mode-*b*, *c* and *d* shown in the spectrum are too lossy to be captured by the correlation mode solver. But as we will show later, they are dominating modes when the excitation is launched asymmetrically. n_{eff} values for mode-*a* and *e* are $1.446889 + 1.633 \times 10^{-9}i$ and $1.437310 + 7.426 \times 10^{-5}i$, corresponding to a loss of 0.0575dB/m and 2614.7dB/m, respectively.

Fig. 8.11 shows the mode spectrum when the MOF is excited with an offset source. Mode-*a*, *b*, *c*, *d* dominate the spectrum. From their mode field plots, we observe that mode-*a*, *b*, *c*, *d* are antiguided modes, whose fields are based on the fundamental SFM of the cladding PC. They are called LP_{11} -, LP_{21} -, LP_{31} -, LP_{22} -like modes, respectively.

Figure 8.10: Upper panel shows the mode spectrum of the MOF excited by a rectangular-shaped launching field of width Λ . Launching position is at $x = 0\mu\text{m}$, $y = 0\mu\text{m}$. Only mode- a and e are shown. Mode- b , c , d corresponding to mode- a , b , c in the next figure, where the excitation is asymmetric. Fiber length here is $2^{14}\mu\text{m}$.

Mode- e , which is the same as that in Fig. 8.10, is still excited, but with a very small power fraction. Mode- f and g are LP_{01} - and LP_{11} -like modes that are based on a high-order SFM of the cladding PC.

Modes excited in Fig. 8.11 have n_{eff} values and corresponding losses given in Table 8.3. Mode- a is comparable to the mode- b in Fig. 8.9. They have a n_{eff} difference of $0.00035 - 0.00000693i$. Mode- b is comparable to the mode- d in Fig. 8.9. They have a n_{eff} difference of $0.000394 + 0.00002711i$. Except the mode- a in Fig. 8.10, all other modes supported by the MOF are extremely lossy. They can only propagate for about $1 \sim 4\text{cm}$ in this fiber before their power drops by 2 orders. So effectively, as far as an optical communication system is concerned, the MOF is a single-mode fiber, and it remains single-mode for an extended wavelength range for reasons explained in [154].

Mode	Effective Index	Loss (dB/m)
a	$1.444404 + 2.018 \times 10^{-5}i$	710.5
b	$1.444225 + 2.618 \times 10^{-5}i$	921.8
c	$1.443962 + 2.752 \times 10^{-5}i$	969.0
d	$1.443560 + 2.115 \times 10^{-5}i$	744.7
f	$1.425408 + 1.405 \times 10^{-5}i$	494.7
g	$1.425184 + 3.734 \times 10^{-5}i$	1314.7

Table 8.3: Effective indices and confinement losses for six modes in Fig. 8.11.

Figure 8.11: Upper panel shows the mode spectrum of the MOF excited by a rectangular-shaped launching field of width Λ . Launching position is at $x = 2.5\Lambda$, $y = 0\mu\text{m}$. Six mode profiles show mode-*a*, *b*, *c*, *d*, *f*, *g* denoted in the spectrum. Mode-*e* is the same as that in Fig. 8.10.

8.2.2 Discussion

We have shown in the previous subsection that antiguided modes exist in an index-guiding MOF with a finite photonic crystal cladding. Some modes in such an MOF can be easily understood by approximating the MOF into a simple double-clad fiber. However, such a simplified model has neglected certain modes supported by the MOF, especially the bandgap-guided mode (mode-*e* in Fig. 8.10). Having known that the modes varieties in an MOF are more complex than in a step-index fiber (SIF), in the following we will discuss the possible implications by the abnormal modes propagation in such fibers. Special attention will be paid to the antiguided modes. Interested readers can refer to Ref. [90] for a more complete calculation as well as discussion on the bandgap-guided modes in such MOFs.

Antiguided modes in an MOF are not equivalent to cladding modes in a conventional single-mode SIF. In SIF, cladding modes cannot be excited easily if the light source is intentionally coupled into the core region within an acceptance angle. Possible irregularities in the fiber cable and interconnects, which act as mode scramblers, can redistribute energy from the propagating core mode into cladding modes. However, current SIF fabrication technology allows little variation on core shape along the axial direction, and state-of-the-art fiber splicers can splice fibers with negligible loss. So cladding modes in SIF do not pose a big problem in current optical communication system. Compared to SIFs, MOFs are much more sensitive to structural perturbations due to the high-index contrast materials used. Even if we assume the MOFs can be fabricated with no longitudinal variations, antiguided modes can still be easily generated at link interconnects (splicing points etc) due to modal mismatch. Antiguided modes are inherently leaky and they usually have a very small overlapping ratio with the fundamental mode, hence the energy redistribution is irreversible. More considerations, especially on solving the MOF compatibility issues with existing optical devices and sub-systems, need to be carried out before we can employ the MOF as shown in Fig. 8.8(a) into long-distance optical communication systems.

Cladding modes in SIFs become important when we are analyzing fiber Bragg gratings (FBGs) and long-period gratings (LPGs). An index change along a fiber's core will perturb and thus redistribute energy among all possible modes, including cladding modes. If any mode has a propagation constant that fulfills the phase matching condition of the grating [158], it will be constructively enhanced for each scattering off an index irregularity as the light propagates through the grating, until the light at this wavelength is completely reflected (in the case of an FBG) or deflected into the cladding region (in the case of an LPG). Energy loss caused by a grating can be observed clearly as abrupt dips in the grating's transmission spectrum. A normal FBG is usually designed to reflect the forward-propagating core mode into the backward-propagating core mode, thus only one channel of information can be filtered or dropped. When the index modulation in the FBG is big or the grating is tilted, cladding modes also come into play, which in turn will cause reflection in channels at slightly shorter wavelengths. However, it is reported that such losses can be eliminated by employing special-designed cladding-mode suppression fibers [159] for FBG fabrication. Similarly when an FBG is written in an MOF similar to that shown in Fig. 8.8(a), its transmission spectrum has several dips [81]. Those dips at short wavelengths correspond to high-order resonances where the forward-propagating core mode is coupled to the backward-propagating high-order modes. The high-order modes include the bandgap-guiding modes, antiguided modes, as well as cladding modes supported by the outer silica layer. The reflected bandgap-guided modes are recorded experimentally by Eggleton *et al.* in [155, 81]. However Eggleton *et al.* only showed those bandgap-guided leaky modes, and antiguided modes are not shown¹. It is possible that the antiguided modes are too leaky to be captured. Also, most antiguided modes have

¹We suspect that the resonance *B* in Fig. 2(a) of the paper [155] is due to an antiguided mode. Its position is in the middle of the fundamental Bragg wavelength and the high-order Bragg wavelengths where the core mode is reflected as bandgap-guided modes [resonances *C* and *D* in Fig. 2(a) of the same reference].

very small field overlapping with the index-guided core mode, therefore their resonances are not that strong. It has been found that the separation in wavelength between the fundamental Bragg resonance and higher-order Bragg resonances is shorter in an MOF FBG than in an SIF FBG [81]. Hence an FBG in MOF tends to reflect more than one channel in WDM systems.

Chapter 9

PBG-guiding MOFs

In this chapter, we will discuss PCFs whose guidance is achieved by the photonic bandgap (PBG) possessed by the cladding photonic crystal. The PCFs are grouped into two categories according to their core compositions. The first category has an air-core; and the second has an air-silica composite core. Though we consider commonly-used air and silica as two component materials, our findings are in general applicable to any other two types of materials.

Hollow-core PCFs have attracted a considerable amount of research interest during last few years. Lightwave delivery in a hollow-core PCF fiber is of significant importance for applications like laser-beam handling [38], nonlinear optics in gases [137], sensing [139], atom/particle guiding [138], and it even holds promise for low-loss optical communications. Current air-guiding PCFs have their cladding made from a stable bundle of thin silica tubes [72]. After the interstitial gaps between tubes are collapsed during preform fusing, the air holes in the cladding are distributed according to a triangular lattice. We name such cladding PC as TPC, and the resulting fiber, as shown in Fig. 7.2(a), as TPCF. This conventional TPCF has two disadvantages: surface-mode interference and multimode operation. Though theoretically, there does exist a design which eliminates the surface-mode problem [160], such a fiber hasn't been fabricated, largely due to difficulties in preparing the preform and maintaining the core shape during drawing. In the first section of this chapter, we will present an improved TPC which can be deployed in the cladding to make a potentially better air-guiding fiber. Also, we propose for the first time to use the honeycomb photonic crystal (HPC) (air holes distributed in a honeycomb lattice) to achieve air guiding. We demonstrate theoretically that such an HPC fiber (HPCF) has promising waveguiding ability. In addition, we will also study theoretically a new photonic bandgap fiber fabricated recently by Vienne *et al.* [88] — the so-called air-silica Bragg fiber.

In the second section of this chapter, we consider PBG fibers with a composite core. The conventional honeycomb PBG fiber was proposed by Broeng *et al.* [86] and experimentally demonstrated by Knight *et al.* [129]. We will demonstrate in this section that the performance of such a honeycomb PBG fiber can be improved with a modified composite core design. Finally, we will generalize the composite-core PCF to those made of

Figure 9.1: The shaded area is the photonic bandgap region of the PC cladding of the hollow-core PCF described in Ref. [72]. Inset, the PC; Black regions represent air.

two pieces of PCs. Such fibers can be broadly referred to as *heterostructured photonic crystal fiber* (HSPCF).

9.1 Air-guiding Photonic Bandgap Fiber

9.1.1 Air-guiding with Triangular Air-Silica PC

Improved TPC

Current air-guiding photonic bandgap (PBG) fiber [72] has its photonic crystal (PC) cladding made from a stack of thin silica tubes. The tubes are arranged in a triangular lattice pattern, which is the easiest way to get a stable preform bundle. During fiber drawing, positive air pressure (with respect to room pressure) is applied inside the tubes, and negative air pressure is applied outside tubes (but within jacket tube) [161]. Such pressure control helps to close interstitial air gaps between tubes. The final air-silica PC will have its air-holes in a hexagonal shape [72, 162] (Fig. 9.1, inset). The cladding's primary PBG region for the fiber described in Ref. [72] is shown in Fig. 9.1.

Three conclusions can be drawn from Fig. 9.1. First, if we are to make a hollow-core photonic crystal fiber (PCF) with such a PC, the PBG-guiding wavelength range will be roughly 0.272 to 0.365 (normalized to the cladding hole-to-hole distance, or pitch Λ), which is the span where the air line crosses the gap region [145] (denoted g_1 in Fig. 9.1). In other words, the pitch of the cladding would be ~ 3 times as large as the wavelength. During normal PCF fabrication, usually at least seven cladding basic units are removed to form a hollow core [72, 161]. This gives rise to an air-core whose radius is ~ 4.5 times wavelength. Such a core size would be likely to result in multi-mode waveguiding [72]. Second, the gap region in Fig. 9.1 penetrates barely beyond the air line, to just $n_{\text{eff}} = 0.968$. This suggests that the air-guiding ability of such a PBG fiber is relatively vulnerable to fabrication uncertainties. Also, as explained in Ref. [145], the weak penetration would cause air guidance in a small-core PCF to suffer relatively high radiation loss. This adds a further barrier to our achieving single-mode operation with such cladding. Finally, surface modes

Figure 9.2: (a) Sample cladding PC with $s = 0.02\Lambda$, $r = 0.12\Lambda$ and $\theta = 40^\circ$, whose designations can be found in Fig. 9.3. (b) Zoom-in view of a concentrated silica block.

Figure 9.3: One portion ($\frac{1}{6}$) of a cladding PC unit. The region to the left of the thick line is air.

tend to coexist with such cladding PC [72, 162, 131]. The origin of the surface modes lies in the silica bridge that divides air core and cladding PC. The bridge guides the optical field on its own. The guided modes can stay either above the air line with an *evanescent* field outside the bridge body, or below the air line with a *radiating* field outside the bridge body. Such modes will interfere with the desired air-guided modes if they are located below the air line and inside cladding's gap region. Although in Ref. [160] West *et al.* have proposed some alternative core design that makes the PCF surface-mode-free, however, as we pointed out before, the actual preform preparation would be too difficult.

Here we will show that the PC presented in Fig. 9.1 is really a special case of one class of PC. The various structural parameters can be modified such that the PC and in turn the air-core PCF will perform better in one or even more aspects. For example, we can make the bandgap region penetrate farther beyond the air line so that the PCF will suffer less radiation loss, and we can also shift the span g_1 to a longer-wavelength position so that the fiber can be fabricated with a smaller core size to facilitate single-mode air-guiding, etc. We also briefly comment on the impact of our findings on surface mode mitigation. Finally, preform stacking for making such a PC is presented.

The proposed two-dimensional cladding PC is shown schematically in Fig. 9.2(a). The concentrated silica blocks are exaggerated compared to those in a conventional PC. It is not difficult to see that the PC resembles silica rods distributed in an air background

Figure 9.4: Variation of spans as the PC parameter θ is varied. Refer to Fig. A.1-A.3 in Appendix A for the exact TPCs used for calculation.

Figure 9.5: Span-width-to-mid-span ratio as θ is varied.

according to a honeycomb lattice pattern. We refer to those concentrated silica blocks as pillars, whose size can be denoted r [Fig. 9.2(b)]. Generally speaking, these pillars are responsible for the lateral PBG possessed by the PC. A portion of the PC's basic unit is illustrated in Fig. 9.3. Following the designations, the PC can be characterized by four parameters: hole-to-hole distance Λ ; hole size d ; concave (with respect to air hole) arc radius r ; and concave arc angle θ . We have assumed that the center of the concave arc O_2 coincides with the corner point of the basic unit. The veins that connect silica pillars have thickness $s = \Lambda - d$. The tapering from silica pillars to narrow veins is denoted by the two convex arcs, whose lengths are actually determined by θ for a fixed r . In what follows, the variation of lateral PBG regions with respect to d (or s), r , and θ will be presented. PBG calculation is done by a plane wave method [23]. Primitive basis vectors are used, and the number of plane waves is always 256×256 .

We are interested in two things when we examine a PBG region of a PC cladding, *i.e.*, the wavelength span over which it overlaps the air line and to what extent it penetrates

beyond the air line. Similarly to Ref. [85], we present only the first point. Generally speaking, penetration of the PBG region beyond the air line depends on the value of r/Λ . At two extreme r/Λ values (0 and 0.29) the PC appears as a homogeneous bulk material (air and silica, respectively). Hence we expect the deepest penetration to occur when r/Λ lies between these two extremes.

In Fig. 9.4, we record the variation of span g_1 as θ is varied. Each shaded patch in Fig. 9.4 is referred to as a span map. Notice that silica pillars look more like circular rods when θ increases. The dark-gray region is for a PC that has $s = 0.02\Lambda$ and $r = 0.0456\Lambda$. At $\theta = 0^\circ$, this PC corresponds to that shown in the inset of Fig. 9.1. As θ increases, the wavelength span shifts to a slightly lower position and the span length becomes smaller. Because the wavelength here is normalized to Λ , a smaller span length does not necessarily indicate a smaller wavelength range of PBG guidance. The mid-gray region is the span map for a PC that has $s = 0.02\Lambda$ and $r = 0.085\Lambda$. At $\theta = 0^\circ$, this PC has circular air holes. Compared with that in the previous case, the span shifts up to a higher position. This tells us that a bigger pillar size allows us to design a cladding PC with a smaller Λ , which is advantageous for achieving single-mode air guiding. The span width is expanded at the same time, which suggests that an increase in pillar size may increase the PBG-guidance wavelength range. The light-gray span map is for a PC that has $s = 0.05\Lambda$ and $r = 0.085\Lambda$. The span further shifts itself to a higher position, but this time it appears narrower in width. We would say that veins should always be kept as thin as possible in order to get a wide PBG-guiding wavelength range. A smaller PC pitch can be made possible through increasing the pillar size instead of deploying thicker veins. In Fig. 9.5, we derive from Fig. 9.4 the span-width-to-mid-span ratio as a variation of θ value. This ratio is a direct indication of the PBG-guiding wavelength range. We see that with $s = 0.02\Lambda$, $r = 0.085\Lambda$, and $\theta = 80^\circ$ the air-guiding wavelength range would be $\sim 0.39\lambda$, where λ is the desired mid-gap wavelength. This range is 1.34 times as broad as what can be achieved by the PC shown in Fig. 9.1, which has $s = 0.02\Lambda$, $r = 0.0456\Lambda$, and $\theta = 0^\circ$.

In Fig. 9.6 we show the variation of the span as r is changed. Span regions that are due to secondary PBG [85] (the lowest four patches in Fig. 9.6) are also recorded as they appear promising for air guiding. The near-black span regions are for the ideal rods-in-air PC, which has the best performance compared to the rest since the spans are low in wavelength position and wide in width. Dark-gray span regions are for a PC that has $s = 0.02\Lambda$ and $\theta = 40^\circ$. The span regions shift upwards compared with those of an ideal PC. Two factors contribute to this shift. One is the presence of a vein. The other is the fact that the size of a pillar is larger than that of a rod, though they are denoted by the same r value. Mid-gray span regions are for PCs with $s = 0.05\Lambda$ and $\theta = 40^\circ$. As the vein becomes thicker, the span regions shift themselves to higher wavelength positions. At the same time, they become narrower in width, in agreement with the conclusion we drew from Fig. 9.4. The light-gray span regions are for PCs with $s = 0.05\Lambda$ and $\theta = 30^\circ$. The span regions further shift upwards and shrink in size.

From Fig. 9.6 and Fig. 9.4, we would see that a variation in r value will affect the span more significantly than a variation in θ value. In fact, the penetration of the gap

Figure 9.6: Variation of spans as the PC parameter r is varied. Refer to Fig. A.4-A.6 in Appendix A for the exact TPCs used for calculation.

Figure 9.7: Schematic diagram of preform formation for the proposed cladding PC. Hatched regions are silica.

region beyond the air line also depends mainly on r . In Fig. 9.4, the deepest penetration of the PBG region happens when $s = 0.02\Lambda$, $r = 0.085\Lambda$, and $\theta = 0^\circ$, with $n_{\text{eff}} = 0.937$. In Fig. 9.6, the deepest penetration happens when $s = 0.02\Lambda$, $\theta = 40^\circ$, and $r = 0.144\Lambda$, with $n_{\text{eff}} = 0.922$ (considering only the primary gap region, excluding the rods-in-air PC).

We would like to point out that the proposed PC design provides more flexibility in suppressing the surface mode problem that is present in current commercial air-guiding PBG fibers. The silica ring introduced between PC cladding and the air core guides many modes, whose dispersion curves can be considered irrelevant to the cladding PC. By choosing s , r , and θ properly we can shift the cladding PBG region so that dispersion curves of the modes supported by the silica ring do not reside in the portion of the gap region below the air line. In this way, the air-guided core modes would be free from surface mode interference.

Figure 9.8: (a) Air-guiding PCF reported in [72]. (b) Proposed air-guiding PCF. (c) Ideal rods-in-air PCF. Black is for air.

The PC proposed can be drawn from a preform that is stacked as shown in Fig. 9.7. A solid silica rod is inserted at the confluence of three silica tubes. The air-pressure control should be the same as that in conventional PCF fabrication; *i.e.*, positive air pressure is applied inside the tubes and negative pressure applied outside [161]. We notice that in [163], Broeng *et al.* mentioned such preform stacking. But, as they did not mention any pressure control, the final product is quite different from ours.

Improved Air-guiding TPCF

Having presented the improved (or generalized), here we would like to study specifically a hollow-core PCF [Fig. 9.8(b)] whose cladding is made of such a PC cladding. We will show that the air-guided mode in our proposed fiber has an unperturbed transmission over a 350nm wavelength range, owing to the fact that the surface modes stay very close to the bandgap edge. Potential single-mode operation with this type of hollow-core PCF is also suggested.

In Fig. 9.8 we show three air-guiding PBG fibers. In fact, both cladding PCs in Fig. 9.8(a) and (b) are derived from the rods-in-air PC shown in Fig. 9.8(c). The guiding mechanisms of three fibers are of no difference — all of them can guide light with a cladding bandgap that exists between the PCs 4th and 5th bands (computed using the plane-wave method with primitive basis vectors), and we find the mode profiles in low-order bands are equivalent for the three PCs. For the particular fiber shown in Fig. 9.8(b), its cladding has $\Lambda = 2.6\mu\text{m}$, $d = 0.98\Lambda$ ($s = 0.02\Lambda$), $r = 0.14\Lambda$, and $\theta = 40^\circ$ (refer to Fig. 9.3). The core is formed by removing 12 silica pillars. Though we can easily get rid of the surface-mode problem theoretically by using a design rule suggested in [164], we stick to a practical core shape [Fig. 9.8(b)] to facilitate easy stacking and pressurization. Extra silica veins surrounding the air core are of thickness s .

The photonic bandgap (PBG) region possessed by the cladding PC is shown in Fig. 9.9(a) by the white patch. It is noticed that the region is extending beyond the air line to $n_{\text{eff}} = 0.922$ (not shown), which is significantly smaller than the value achievable with the PC in Fig. 9.8(a) (0.968). This feature allows us to design air-guiding fibers with smaller core and/or lower-loss air-guiding PCFs. We then use the FDM mode solver as presented in Chapter 6 to compute the guided defect modes in a full fiber with four air-hole rings in the cladding. Numerical resolution is $dx = dy = 0.12\mu\text{m}$ with 11×11

Figure 9.9: Dispersion (a) and loss (b) curves of the defect modes. Photonic bandgap region of the cladding PC is represented by the white patch in (a).

sub-grid index averaging. Perfectly matched layers have a 12-grid thickness. The two degenerate fundamental air-guided modes (HE_{11} -like) are shown by the thick solid curve in Fig. 9.9(a). It is found that the modes are un-disturbed in $1.35 \sim 1.70 \mu m$ wavelength range. Their loss spectrum is shown by the thick curve in Fig. 9.9(b). Minimum loss is about 1 dB/m.

The $|E_x|$ field distributions of modes at point A ($\lambda = 1.55 \mu m$, air-guided mode) and B ($\lambda = 1.3 \mu m$, surface mode) are shown in Fig. 9.10(a) and (b), respectively. The core mode at $1.55 \mu m$ is well confined, and has leakage loss of 1.2 dB/m, which will decrease to 0.053 dB/m and 0.003 dB/m when the number of rings in cladding is five and six, respectively. Though the six-ring loss value is higher than that for the fiber given in Fig. 9.8(a) [60], it should be reminded that our core size is significantly smaller (diameter $\sim 7.4 \mu m$ v.s. $\sim 13.6 \mu m$). Loss should decrease if additional pillars are removed in core region.

The proposed fiber is still multimode, largely because the gap region extends quite far beyond the air line. The dispersion curves of the second-order modes (TE_{01} -, two HE_{21} - and TM_{01} -like modes) are shown in Fig. 9.9(a) by four thin solid lines. Their loss values (in dB/m), represented by the thin lines in Fig. 9.9(b), are about 30 times higher than that of the fundamental modes. It should be noticed that, due to the small core size, the dispersion curves of the fundamental and second-order mode groups in Fig. 9.9(a) stay

Figure 9.10: Air-guided mode at $\lambda = 1.55\mu\text{m}$ (a) and surface mode at $\lambda = 1.30\mu\text{m}$ (b). Contour lines are in 1-dB separation.

further apart as compared to the fiber reported in [72]. This means the coupling between the two mode groups is smaller. With careful excitation, we can achieve single-mode operation for applications like laser beam delivery, in which severe fiber bending can be readily avoided.

Surface modes are denoted by dotted lines in Fig. 9.9. An obvious advantage of our fiber is that the surface modes stay very close to the bandgap edge. We attribute this to the similarity between silica pillars nearby the core and those in the cladding. It is observed from Fig. 9.10(b) that a nodal line appears in each silica pillar adjacent to the core. Such surface modes are similar to the 5th-band bulk cladding mode. They are pulled into gap region because the pillars near the core have slightly more silica than those in the cladding PC. Hence their modal energy is lower (higher in n_{eff}). By varying the cladding PC parameters, we should be able to further reduce the impact of these modes, *i.e.*, to push them closer to the gap region boundary.

9.1.2 Air-guiding with Honeycomb Air-silica PC

In the literature, a honeycomb PC (HPC), which has air-holes arranged in a honeycomb lattice, was not considered in designing air-guiding PBG fibers. One reason is that one can easily utilize HPC's vast PBG regions found below the air line in β - k space by introducing an air-silica composite core [87]. In [145], Yan *et al.* brought forward the possibility to guide light in a hollow core with a relatively large air-fraction (f_{air}) HPC cladding. Such a fiber is referred to as a HPCF. In this subsection, we give the design guideline for such air-guiding fibers. In particular, modal properties of two typical fiber structures will be presented.

HPC

Figure 9.11(a) shows the cross-section of a fiber whose cladding is made of triangularly-latticed silica rods distributed in an air background. The core is formed by omitting a silica rod. Though the structure with unsupported rods in air is too idealistic in practice, it nevertheless has theoretical interest. Waveguiding in such a microstructured fiber has

Figure 9.11: (a) Air-guiding PCF with rods-in-air cladding. Inset shows a cladding unit. (b) Air-guiding HPCF with honeycomb cladding. Inset shows a cladding unit.

Figure 9.12: Gap map calculated from a honeycomb cladding with $d = 0.5\Lambda$ and $s = 0.02\Lambda$.

been explained by considering scattering incurred by individual cladding cylinders [140]. In fact, we can well understand the mechanism by calculating the lateral PBG map of the cladding structure [145]. Figure 9.11(b) shows a practical PCF design whose cladding PC is analogous to that given in Fig. 9.11(a). The “rods” (referred to as pillars hereafter) are now connected by narrow silica veins. At another glance, we know the cladding is really nothing but traditionally so-called honeycomb PC. The core is formed by omitting seven cladding units. The cladding units adjacent to the core have their shapes slightly altered. The parameters that characterize the cladding are Λ , pillar “diameter” d , and vein thickness s . The cladding in Fig. 9.11(b) is expected to have similar photonic bandgap regions as that in Fig. 9.11(a), due to their structural proximity.

In Fig. 9.12, we show the gap map found for a honeycomb cladding with $d = 0.5\Lambda$ and $s = 0.02\Lambda$. The map is in $n_{\text{eff}}-\lambda$ plot (λ is normalized) for a more straightforward reading. Only the first four bandgaps are recorded as the rest are too narrow. We should mention that a slightly random placement of pillars would not affect the bandgap regions much.

Figure 9.13: The variation of the spans with respect to pillar diameter d . Refer to Fig. B.1 and B.2 in Appendix B for exact TPCs used for calculation.

This is due to the fact that the lower bands have their mode energy mostly localized inside pillars, and in turn their frequencies shift little from the periodic case. Ignoring the two central narrow gap regions, we notice that the air line ($n_{\text{eff}} = 1$) crosses both the fundamental (upper) and secondary (lower) gap regions. The spans of air line over fundamental and secondary gap regions are denoted as g_1 and g_2 , respectively. These two spans act as an indication of the spectral ranges within which air-guiding is possible when such an HPC is adopted as a hollow-core fiber's cladding [145]. If we record g_1 and g_2 as we vary pillar diameter d , we can get a map of the span as shown in Fig. 9.13 by the dark gray regions. Similar span maps are derived for an HPC with $s = 0.05\Lambda$ (light gray regions), and also a rods-in-air PC (black regions). As we assume all air-holes for the honeycomb cladding have convex boundaries, the span calculation stops at a certain d value, at which the corresponding cladding air holes are circular. In all calculations, $n_{\text{silica}} = 1.45$ and $n_{\text{air}} = 1$.

Generally speaking, it is advantageous to have a lower span position in Fig. 9.13 since that would give us a relatively large Λ value for the fiber to operate around a fixed wavelength. Span regions beyond $\lambda/\Lambda = 1$ may not be practical, for Λ would be shorter than the guiding wavelength. As d increases, the span width (in the unit of λ/Λ) increases first and then decreases, while the span position always increases. As the vein gets thicker, the spans get narrower and their positions in Fig. 9.13 become higher. The conclusion can be drawn that a narrower silica vein is always advantageous for making a better air-guiding PBG waveguide, provided that we don't take mechanical properties into consideration. Nevertheless, we still can get air-guiding with relatively thick silica vein. The choice of d depends on several factors, *e.g.*, the gap region we would like to operate in, the vein width we decided on, and also the Λ we are happy with in fabrication.

Figure 9.14: (a) Dispersion curve for a HPCF with $s = 0.05\Lambda$, $d = 0.4\Lambda$, and six cladding rings. The cladding's fundamental PBG region is shaded in the background. (b) Radiation loss for the HPCF with six and eight cladding rings. Inset gives the HPCF with two cladding rings.

Air-guiding HPCF

In the following we will present two fiber designs which utilize either fundamental or secondary bandgap region for air-guiding. The FDM presented in Chapter 6 is used for calculating defect propagating modes. A perfectly-matched layer (PML) backed with Dirichlet boundary condition is employed to truncate an infinite domain to a finite one. Though the fibers presented may have multiple defect modes, only the property of the fundamental mode will be given.

The first honeycomb fiber [inset of Fig. 9.14(b)] has cladding parameter values of $s = 0.05\Lambda$ and $d = 0.4\Lambda$. According to Fig. 9.13, the fundamental bandgap region of the fiber's cladding crosses the air line from 0.8 to 1.02 wavelength range (normalized to Λ). Hence Λ is chosen at $1.7\mu\text{m}$ in order for the bandgap to include $1.55\mu\text{m}$. In Fig. 9.14(a), we give the dispersion curve [$\Re(n_{\text{eff}})$ - λ relation] for the fundamental mode. Six cladding rings are used. In Fig. 9.14(b), we give the radiation loss of the fundamental mode with six, and eight cladding rings. It is observed that the loss is quite high, but as suggested by the two curves, additional rings would reduce the loss drastically. Since the loss decreases almost linearly in logarithmic scale with respect to the cladding ring number [60], an extrapolation suggests that sixteen rings are necessary to reduce the loss below 1dB/km.

Figure 9.15: (a) Dispersion curve for a HPCF with $s = 0.02\Lambda$, $d = 0.5\Lambda$, and six cladding rings. The cladding's secondary PBG region is shaded in the background. (b) Radiation loss for the HPCF with four and six cladding rings. Inset gives the HPCF with two cladding rings.

An advantage of this design is its wide bandgap guiding wavelength range, which is roughly 400nm (from $1.4\mu\text{m}$ to $1.8\mu\text{m}$). Such a wide operating wavelength range is comparable to the air-guiding TPCF with $f_{\text{air}} = 0.94$ [72]. Here in the six-ring HPCF calculation, FDM uses $\Delta x = \Delta y = 0.068\mu\text{m}$ grid resolution with index averaging. The PML has a width of 12 grid points with a quadratic α -profile and $\alpha_{\text{max}} = 28$. For eight-ring HPCF, $\Delta x = \Delta y = 0.082\mu\text{m}$ with the PML conditions unchanged.

The second HPCF has a cladding with $s = 0.02\Lambda$ and $d = 0.5\Lambda$. In Fig. 9.13, the cladding's secondary bandgap region crosses the air line from 0.54 to 0.62 wavelength range. Λ is now chosen at a larger value of $2.7\mu\text{m}$. Figure 9.15 shows that the PBG-guiding wavelength range is approximately 200nm (from $1.45\mu\text{m}$ to $1.65\mu\text{m}$). The real part of n_{eff} given in Fig. 9.15(a) has larger values as compared to that in Fig. 9.14(a), due to the fact that the fundamental mode has a larger field diameter in this second HPCF [145]. The two loss curves in Fig. 9.15(b) are calculated with four and six cladding rings. An estimation suggests that twelve cladding rings are required for a 1dB/km lowest loss.

The fundamental modes in both proposed HPCFs are similar to the HE_{11} mode in a conventional step-index fiber. In Fig. 9.16 we give the $|E_x|$ field distribution for the HE_{11}^x -like modes supported by two fibers. In both cases, the wavelength is chosen as $1.55\mu\text{m}$.

An obvious difference between the two fields is that a nodal line appear in the pillars of the second HPCF. Further knowledge of the field distributions can be obtained by examining the field cuts along $x = 0$ line, as shown here in Fig. 9.17. It is noticed that for the first HPCF, the field in a single cladding pillar has roughly a $\frac{\pi}{2}$ phase variation. This is analogous to the fundamental Bragg reflection condition in 1D photonic crystals. For the second HPCF, the field in a single cladding pillar has roughly a $\frac{3\pi}{2}$ phase variation, which corresponds to the second-order Bragg reflection condition.

Figure 9.16: (a) $|E_x|$ field for the HE_{11}^x -like fundamental mode supported by fiber given in Fig. 9.14. (b) Same field for fiber given in Fig. 9.15. Contour lines have 1dB separation.

Figure 9.17: (a) and (b) are field cuts along $x = 0$ line in Fig. 9.16(a) and (b), correspondingly. The solid vertical line pairs indicate silica pillars. Areas to the right of the dotted lines are background silica regions.

Figure 9.18: Sample air-silica Bragg fiber fabricated in Crystal-fibre A/S, Denmark (Vienne *et al.* [143]).

9.1.3 Air-silica Bragg Fiber

Recently, Vienne *et al.* have fabricated some interesting hollow-core fibers [88, 143] which structurally resemble the Bragg fiber proposed by Yeh *et al.* [29] (Fig. 9.18). Such a fiber is given the name “air-silica Bragg fiber”. The pseudo-1D PC in the cladding is made of concentric silica walls, which are connected by nanoscale silica bridges. Despite the fact that an ideal Bragg fiber with a similar index profile (without nano-bridges and with smooth silica ring surfaces) can achieve unperturbed PBG guidance over 1000nm wavelength range (minimum radiation loss as small as 0.01dB/m) with only three cladding periods [88], the actual fibers fabricated by Vienne *et al.* suffer from high leakage loss and a fragmented transmission spectrum. In this subsection, we theoretically study the waveguiding properties of such Bragg fibers.

Lateral Bandgap of a Pseudo-1D PC

It is always useful to derive the photonic bandgaps (PBGs) of the cladding structure. The PBGs experienced by a radially propagating wave in a concentric stack of dielectric layers can be approximated as those experienced by a lateral propagating wave in a 1D photonic crystal (PC) [38]. In Fig. 9.19 we show a PC block that is very close to the cladding of the fiber given in Fig. 9.18, with some alterations added to make the composite periodic. The alterations should not affect the significance of the result, since our major concern is in fact whether the non-flat silica walls are able to create a neat lateral PBG. The PC must be studied in 2D due to its finite translational symmetry along the vertical direction. The basic unit is depicted in Fig. 9.20. The PC is formed by placing two types of air holes in a silica background. Referring to Fig. 9.20(b), the bigger air holes have parameters as $d_x = 3.3\mu\text{m}$, $d_y = 2.61\mu\text{m}$, $r_s = 3.96\mu\text{m}$, $\theta = 32^\circ$, $\Lambda_x = 3.5\mu\text{m}$, and $\Lambda_y = 2.64\mu\text{m}$. The smaller air holes have parameters as $d_x = 3.28\mu\text{m}$, $d_y = 1.95\mu\text{m}$, $r_s = 2.62\mu\text{m}$, $\theta = 38^\circ$, $\Lambda_x = 3.5\mu\text{m}$ and $\Lambda_y = 1.98\mu\text{m}$. Λ_x is the air hole center-to-center distance along x direction, and Λ_y is the air hole center-to-center distance along y direction. The plane

Figure 9.19: (a) Approximated pseudo-1D PC. (b) Equivalent 1D PC which has flat wall surfaces. The 1D PC has silica wall thickness as $0.404\mu\text{m}$ and period $\Lambda_x = 3.5\mu\text{m}$. Black regions represent air, and white regions are silica. k_z is the wavevector component parallel to walls. k_t is the wavevector component perpendicular to walls.

Figure 9.20: (a) Basic unit used for simulation. (b) Schematic diagram for an air hole in the 2D PC shown in (a).

wave method (PWM) [23] is used for our calculation. Refractive index for silica is chosen as 1.45, and that for air is 1.

Referring to Fig. 9.19, when calculating the bandgap, an offset wavevector k_z is assumed parallel to the silica walls, which is equivalent to the modal propagation constant β in waveguide theory. Subject to this k_z , lateral bandgap are found for transverse wavevectors k_t that are perpendicular to the periodic silica walls. The calculated lateral gap map for the 2D PC is shown in Fig. 9.21 as grey-shaded regions. There are numerous higher-order gap regions found to the left of the plotted ones, but we don't show them as they stay above the air line. An obvious feature of the gap map is that the gap regions are in closely-packed small pieces though there are plenty of them. On the other hand, an ideal PC shown in Fig. 9.19(b) has a large primary gap region, as indicated by the thick dashed line in Fig. 9.21. The overall sum of the grey gap regions approximates very well to the fundamental gap region of the ideal 1D PC. It is not difficult to conclude that the existence of some extra cladding modes is responsible for breaking the fundamental gap region of an ideal 1D PC into many small pieces as the silica walls become non-flat. The small bandgap regions are less effective for guiding light, which is especially true when

Figure 9.21: Grey-shaded close regions are gap regions found for the 2D PC shown in Fig. 9.19(a). The region bounded by the dashed line is the fundamental gap region found for the PC shown in Fig. 9.19(b).

Figure 9.22: (a) The air-silica Bragg fiber under study. White region is silica, and black regions are air. (b) The boundary lines of the microstructured and a similar ideal Bragg fiber.

fabrication uncertainties come into play. The presence of nano-bridges that connect neighboring two silica walls does not affect the bandgap sizes and positions significantly, as the field energy supported by these bridges are trivial for modes in the first 80 or so bands of the PC.

Study of a Practical Air-silica Bragg Fiber

Theoretically, we certainly desire more flat walls as it will make the cladding look closer to that in an ideal Bragg fiber. However, as the nano-bridges can't be removed, the non-flatness of the cladding silica walls is inevitable. Considering the current stack-and-draw method [143], we can easily achieve more flat walls by using more nano-veins to connect the silica layers. In this part, we will theoretical study an improved air-silica Bragg fiber which can be very likely fabricated.

Figure 9.22(a) shows the cross-section of the Bragg fiber under study. The number of air holes in inner, middle and outer air-ring is 50, 60 and 70, respectively. The silica rings are roughly $0.37\mu\text{m}$ thick, and the bridges that connect the rings are about 40nm

Figure 9.23: (a) Loss spectra for four lowest-order modes supported by the ideal Bragg fiber. Four solid lines are for, from bottom to top, TE_{01} , HE_{11} , HE_{21} and TM_{01} modes, respectively. The curve with open circles is for the HE_{11} mode in the microstructured Bragg fiber shown in Fig. 9.22(a). (b) Loss spectra and (c) effective mode indices for three lowest-order modes supported by the Microstructured Bragg fiber shown in Fig. 9.22(a).

thick. The first, second and the third rings start at 10, 12 and $14\mu\text{m}$ from the center, respectively. In Fig. 9.22(b), the air-silica interface lines are drawn. We have also included the interface lines of an ideal Bragg fiber whose silica rings are of $0.37\mu\text{m}$ thickness. It is noted that all air holes have concave boundary lines with respect to themselves, as a result of pressurization during fiber drawing. Also, the air holes are approximately of the same size, which is advantageous for maintaining the cladding microstructure during pressurization.

In Fig 9.23(a), we show the loss spectra of four lowest-order modes supported by the ideal Bragg fiber [Fig. 9.22(b)]. We have deployed the transfer matrix method (TMM) presented in Chapter 2 for calculating the modes. Our emphasis is on the HE_{11} mode and TE_{01} mode. The HE_{11} mode is the most useful mode for either delivering high-power laser beam or carrying optical signal, since it is well matched to laser beams and therefore can be readily excited. The TE_{01} mode has the minimum theoretical loss and has been proposed as the basis for transmission of optical signals with only a single polarization eigenstate [165]. It is noticed from Fig. 9.23(a) that the TE_{01} mode experiences the smallest leakage and has the widest transmission window, due to the fact that TE modes see the largest

Figure 9.24: (a) TE_{01} mode at $\lambda = 1.3\mu\text{m}$; (b) Quasi- HE_{21} mode at $\lambda = 1.62\mu\text{m}$; (c) HE_{21} mode at $\lambda = 2.0\mu\text{m}$; (d) $|E_x|$ field of the TE_{01} mode at $\lambda = 2.12\mu\text{m}$.

cladding PBG region. The remaining three modes consist of TM field components (E_z , E_r and H_θ), hence their transmission windows are determined by the cladding's TM bandgap, which is narrower than that of TE. For reference, we also show the loss curve for the HE_{11} mode supported by the practical Bragg fiber given in Fig. 9.22(a). The loss curves of the three lowest-order modes (HE_{11} , TE_{01} and HE_{21}) supported by the microstructured Bragg fiber are shown in Fig. 9.23(b). The finite difference method presented in Chapter 6 is used for deriving the curves. It is noticed that the cladding forbidden gap has been shifted to longer wavelength position, since there is more silica in the microstructured fiber [Fig. 9.22(b)] and hence an overall rise in effective index. The loss value (in dB/m) is almost increased by 100 times compared to the ideal fiber and the transmission window becomes narrower. In addition, we did not find a well-behaved TE_{01} mode loss spectrum. This is surprisingly different from the result calculated from the ideal Bragg fiber using TMM. Referring to Fig. 9.23(b) and (c), the TE_{01} mode at short wavelength transforms into the HE_{21} mode as wavelength increases. Another transition from HE_{21} mode to TE_{01} mode happens in the same wavelength region. In other words, there is an anticrossing between HE_{21} and TE_{01} modes. And since the two modes are nearly degenerate, the anticrossing behaviour happens over a relatively long wavelength span. As a result, TE_{01} mode suffers from relatively high loss in the middle of the bandgap. We show in Fig. 9.24(a)-(c) the field transition of TE_{01} mode at $1.3\mu\text{m}$ wavelength to quasi- HE_{21} mode at $1.62\mu\text{m}$ wavelength and HE_{21} mode at $2.0\mu\text{m}$ wavelength.

Another point to which we would like to pay special attention is that the loss spectra in Fig. 9.23(b) exhibit many sharp rejection notches. These narrow high-loss regions can account for the fragmentation of the transmission window observed by Vienne *et al.* A zoom-in modal search reveals that these points correspond to resonances of the cladding structure (or anticrossing points between cladding modes and the core mode). Figure 9.24(d) shows $|E_x|$ field of the TE_{01} mode at $2.12\mu\text{m}$ wavelength. The cladding is nearly in resonance with the guided core mode. A close look shows that the field in the cladding shares roughly the same periodicity with the silica rings. This suggests that the extra azimuthal periodicity (or roughness) introduced due to the presence of nano-bridges increases sharply the number of cladding modes. These modes are likely to be located inside the bandgap of perfect 1D PC. Since these cladding modes are highly leaky, they would relay field energy out of core at their resonance points. This leakage mechanism is especially severe for Bragg fibers made from stack-and-draw method introduced by Vienne *et al.* for two reasons. First, unlike a periodic 2D PC, the cladding silica rings in the fiber fabricated by Vienne *et al.* do not have a common azimuthal period, and even not a common thickness. Hence each silica ring would have different resonance frequencies. This will further increase the fragmentation of the bandgap region. Second, fabrication uncertainty will broaden the spikes found in Fig. 9.23(b).

In conclusion, we have theoretically studied the property of a hollow-core air-silica Bragg fiber, which has a pseudo 1D photonic crystal in its cladding. Two main factors contribute to its poorer performance compared to an ideal Bragg fiber. One is the anticrossing of the HE_{21} mode and the TE_{01} mode, and the other is the cladding mode resonance. It is the second factor that deteriorates the cladding bandgap considerably. Consequently, such fibers need to be carefully engineered in order for it to be a competitive alternative to those conventional air-guiding PCFs [72, 85]. New preform preparation and pressurization methods should be developed to fabricate such fibers with few bridges (large azimuthal period) whilst maintaining the silica rings in concentric circular shapes.

9.2 Composite-core Photonic Bandgap Fiber

9.2.1 Honeycomb PBG fiber

Past research efforts basically focused on two types of PBG fibers. The first type has its cladding made of large air holes arranged in a triangular lattice, and its core made of air [166]. The second type has its cladding made of relatively smaller air holes placed in a honeycomb lattice, with its core a composite of air and silica (an extra hole is placed in the center of core unit) [86]. PBG fibers of the first type have already been commercialized. The honeycomb PBG fiber, as far as we know, has not been commercialized yet, largely due to its non-air-guiding feature and its modal mismatch with a conventional optical fiber. In this subsection, however, we would like to draw attention to this honeycomb PBG fiber, but with a modified core design. PBG guidance over 1000nm wavelength range can be easily achieved with our proposed core design. Furthermore, the fiber tends to be single-mode as compared to a conventional core design.

Figure 9.25: (a) Cladding unit for honeycomb PBG fiber. (b) Conventional core unit for honeycomb PBG fiber. (c) Our proposed core design. Grey regions represent air. White region is silica.

Figure 9.26: (a) Schematic diagram for a conventional honeycomb PBG fiber. (b) Our proposed honeycomb fiber design.

The honeycomb PBG fiber has a photonic crystal (PC) cladding unit shown in Fig. 9.25(a). The full cladding structure is schematically shown in Fig. 9.26. PBG regions possessed by the cladding structure are given in Fig. 9.27. We assume silica and air refractive indices are 1.45 and 1, respectively. r_0/Λ is chosen at 0.204, corresponding to an air-fraction (f_{air}) of 30%. The gap map is derived by recording the lateral (in the plane-of-periodicity) PBGs found for the structure as the longitudinal (out of plane-of-periodicity) propagation constant β , which is treated as an offset of wave vector $\mathbf{k}$, is varied for each calculation. A plane wave expansion of both the dielectric function and the mode field, together with periodic boundary condition are employed in the formulation [23]. From the gap map, we observe two dominant gap regions. We name the upper one as the primary gap region, and the lower one as the secondary gap region. These gap regions can be used for confining modes with a proper core design. PBG regions appearing below the secondary gap region can be neglected since they are too narrow.

Considering two dielectric materials A and B, a waveguide is formed by placing a cylinder made of B into an infinitely extending A material. When a mode is confined in the waveguide's core, its n_{eff} curve (defect mode dispersion curve, or simply defect line), should stay below material B's radiation line (in a $n_{eff}-\lambda$ plot). For the fundamental defect mode, the corresponding radiation line is the dispersion curve for the fundamental space-filling mode (SFM) of the bulk core material, and we name this radiation line as the primary radiation line. Similarly we have a secondary radiation line, which is obtained

by computing the second-order SFM [100]. The statement above is true for A and B to be either a homogeneous or a composite material. Notice that the radiation line for a homogeneous material is just the material refractive index curve.

Referring to Fig. 9.25(b), with a conventional core design and $r_{\text{core}}/r_0 = 0.4$, we get primary and secondary radiation lines for the core composite as shown by the upper and lower solid curves, respectively, in Fig. 9.27(a). The primary radiation line crosses the primary gap region from 0.20 to 0.67 (normalized wavelength with respect to Λ). Following our discussion in the previous paragraph, the fundamental defect mode, if there is any, should be found inside the gap region just below that solid line. Indeed, using the supercell technique, two degenerate modes are found as shown by the two overlapping lines with open circles (in the primary gap region). These two modes are similar to the HE_{11}^x and HE_{11}^y modes in conventional fibers. They reflect the fiber's C_{6v} symmetry in combination [53]. Similar honeycomb fiber designs have appeared in literature like that in [86]. However, with the core design mentioned above, we notice there are four other defect lines in the secondary gap region, as shown by the lower lines with open circles (not very well resolved). They correspond to the TE_{01} , two HE_{21} , and TM_{01} modes in circular-core fibers. The presence of these four second-order modes can be predicted since the secondary radiation line for the composite core stays just above the secondary gap region of the PC cladding. Thus second-order defect modes should be able to propagate in that gap region.

Still with a conventional core unit, we further increase the r_{core} value to $r_{\text{core}}/r_0 = 1.0$. This core design pushes the secondary radiation line [lower dashed line in Fig. 9.27(a)] down below the secondary gap region of the PC cladding. But now the primary radiation line (upper dashed line) crosses the gap regions in a less economical way: it stays in different gap regions for two short wavelength ranges, instead of staying inside a single gap region for a long wavelength range. In fact, numerical simulation tells us that no defect mode is supported by the primary gap. Within the secondary gap region, second-order defect modes appear (not shown), partly because coupling between the core and cladding is very strong. Generally speaking, the secondary gap region with the current f_{air} value is relatively narrow in width, so losses of defect modes supported by a finite cladding should be high.

We now introduce our modified core design which outperforms the conventional designs discussed above. Referring to Fig. 9.25(c), our core unit involves a highly concentrated silica region. Six surrounding air regions are slightly enlarged to make the unit's average index lower than that of the cladding unit. Here we employ $d_{\text{core}}/\Lambda = 0.404$ and r_{core} is calculated to ensure the same thickness for silica bridges that connect to cladding. Such a core unit is analogous to the cladding unit in shape. This suggests the core composite has roughly the same dispersion property as that of the cladding composite. We have calculated the first two radiation lines for the bulk core composite [Fig. 9.27(b)]. Notice the separation between these two lines is bigger than that in Fig. 9.27(a). This is due to the fact that, to create a mode nodal line in a concentrated high-index core requires a higher energy than that in a non-concentrated one. Conventional honeycomb fiber uses a

ring-like silica core, and the nodal line in this case can be easily created with little extra energy. We found the primary radiation line stays in the primary gap region for a very long wavelength range (0.2 to 0.8). In fact, the fiber now has a well-confined fundamental defect mode across the whole wavelength range as shown in Fig. 9.27(b). No high-order defect core mode is found in the secondary gap region. The H_x field distributions for the HE_{11}^y -like mode at $\beta\Lambda = 14.286$ and 7.692 are shown in Fig. 9.28(a) and (b), respectively. The mode is better confined at shorter wavelength, and extends more energy into the cladding region at longer wavelengths.

Figure 9.27: (a) Gap map found in transverse direction for the PC cladding of the fibers shown in Fig. 9.26. 128×128 plane waves are used for all gap map calculations. No propagating modes are supported by the cladding in the shaded spectral regions. Upper and lower solid lines are respectively the primary and secondary radiation lines for a PC made of conventional the core unit [Fig. 9.25(b)], with an extra hole sized at $r_{\text{core}}/r_0 = 0.4$. Two dashed lines are the similar curves, but for $r_{\text{core}}/r_0 = 1.0$. Lines with circles are defect core modes supported by the fiber [Fig. 9.26(a)] as $r_{\text{core}}/r_0 = 0.4$. All defect modes are calculated with 5×6 supercell size and 256×256 plane waves, unless otherwise mentioned. (b) Gap map for the same honeycomb cladding. Solid lines are radiation lines for bulk core material whose unit is shown in Fig. 9.25(c), with $d_{\text{core}}/\Lambda = 0.404$. Lines with circles are defect core modes supported by the fiber [Fig. 9.26(b)].

The gap map for a honeycomb cladding with $f_{\text{air}} = 53.4\%$ ($r_0/\Lambda = 0.271$) is shown in Fig. 9.29. The honeycomb cladding now resembles silica rods in a triangular lattice immersed in air [167]. With conventional core design and $r_{\text{core}}/r_0 = 0.3$, we get the defect lines shown in Fig. 9.29(a). Second-order modes appear for $\lambda/\Lambda < 0.62$. With a modified core design and $d_{\text{core}}/\Lambda = 0.433$, we have defect modes shown in Fig. 9.29(b). It should be pointed that the second-order modes in Fig. 9.29(b) can easily be suppressed by pushing down the secondary radiation line a little bit. But this will result in a smaller effective defect mode area. Noticeably, the fundamental defect mode stays almost exactly in the middle of the primary gap. Together with the wide gap possessed by the cladding, our fiber design should give us very low confinement loss and bending loss with moderate number of rings of cladding units.

The drawback of small effective core area for the fiber shown in the inset of Fig. 9.29(b) can be eased by replacing six cladding units in the vicinity of central core unit with six core

Figure 9.28: H_x field distributions for the HE_{11}^y mode (absolute value) supported by the fiber at at (a) $\beta\Lambda = 14.286$ (with $\lambda/\Lambda = 0.590$) and (b) $\beta\Lambda = 7.692$ (with $\lambda/\Lambda = 0.997$). $\Lambda = 3\mu m$ is used for the plot.

Figure 9.29: (a) Gap map derived for a honeycomb cladding with $f_{\text{air}} = 53.4\%$. Lines with circles are for defect core modes supported by a conventional core design (inset) with $r_{\text{core}}/r_0 = 0.3$. Two solid lines are for primary and secondary radiation lines. (b) Defect core modes supported by our modified core design (inset) with $d_{\text{core}}/\Lambda = 0.433$.

Figure 9.30: H_y field of the lowest defect mode (absolute value) for a fiber with its central seven units be core units. Fiber parameters are the same as that for fiber shown in the inset of Fig. 9.29(b). The mode is computed at $\beta\Lambda = 15.385$ (with $\lambda/\Lambda = 0.698$). 7×8 supercell and 416×416 plane waves are used. $\Lambda = 3\mu m$ is used for the plot.

units. But, unlike air-guiding PBG fibers, the fundamental mode group now consists of fourteen nearly degenerate modes instead of two. These modes are conventionally called supermodes. For each of these modes, the field tends to be HE_{11} -like within a single core unit. This will cause mode energy be transferred from one core unit to another in a complicated manner due to inter-modal interference [51]. The H_y field of the lowest defect mode at $\beta\Lambda = 15.385$ is shown in Fig. 9.30.

In conclusion to this subsection, we have theoretically analyzed honeycomb air-silica optical fibers with a modified core design. The improved core has, unlike the conventional core design, a concentrated silica region. Such a core ensures that the fiber's defect core mode has similar dispersion behavior as that of the cladding material, which helps to achieve PBG guidance for a longer wavelength range. A Gaussian-like fundamental mode also facilitates coupling between this class of fibers to a conventional fiber. We also have suggested to design a PBG fiber with large effective core area by using multiple core units. Compared to air-guiding PBG fiber, the honeycomb fiber we propose should exhibit the same properties, like small bending loss, etc. Due to the fact that the honeycomb fiber operates above the air line ($n_{\text{eff}} = 1$), the defect mode field is mostly confined in silica region. This makes the honeycomb fiber with our modified core design have a similar dispersion characteristics as an index-guiding microstructured air-silica fiber [75], especially at small wavelength.

9.2.2 Heterostructured PCFs

Conventional photonic crystal fiber designs employ a microstructured cladding consisting of two materials (*e.g.*, air and silica), and a *homogeneous* (silica or air) core [75, 166]. Index-guiding PCF is realized if the core is made of the higher-index material. The guidance mechanism for such a fiber is not new compared to a conventional step-index fiber (SIF). That is, the guided modes have their dispersion curves below the radiation line of the cladding material in β - k plot (β is x -axis value). And such fiber does not require a strict periodic photonic crystal (PC) cladding. With a PC cladding, we could possibly guide light in a core made of lower-index material, if the cladding PC possesses some photonic bandgap (PBG) regions above the radiation line of the core material in the β - k plot. The PBG regions of cladding PC would forbid certain modes supported by the core from leaking out. During the last few years, both index- and PBG-guiding PCFs have been studied extensively. It is appropriate to conclude that the introduction of a composite (especially those made of two materials with contrasty refractive indices) into the cladding design has opened a new chapter in the field of optical fiber development. In this subsection, we would like to introduce microstructured optical fibers (MOFs) whose core and cladding are both composites (Fig. 9.31). For simplicity, we let the index variation in both the core and the cladding be periodic, *i.e.*, both composites are PCs. Furthermore, the core and cladding PCs share the same basis vectors. The matched lattices help to reduce the number of surface modes localized on the core-cladding interface. We refer to such fibers as heterostructured PCFs (HSPCFs). Structurally, the evolution from conventional SIF to PCF to HSPCF follows a rational logic. Fabrication of HSPCF does not pose any additional difficulty than fabrication of normal PCFs. Optically, as the light would be confined in a composite core of multiple basic units, the waveguide will certainly no exhibit single-mode operation due to the modal folding effect [87]. The mode field distribution would also deviate much from homogeneous-core optical fibers. For example, the fundamental mode may have distributed peaks instead of a single Gaussian profile (Fig. 9.30). It should be noticed that a honeycomb PBG fiber proposed several years back has a composite core [86], but only with one unit. In this subsection, the definition and variations of HSPCF will be discussed in detail. Preliminary study on modal property of such MOFs will also be presented. Some of their potential applications are suggested. Without loss of generality, we choose air (refractive index 1) and pure silica (refractive index 1.45) as the two composing materials, unless otherwise stated.

We are interested in HSPCFs whose cross-sections exhibit C_{6v} group symmetry, as such a structure is relatively easy to be fabricated. Based on this requirement, we know that both triangular-latticed PC (TPC) and honeycomb-latticed PC (HPC) can be used for composing our microstructured fibers. Fig. 9.31 shows four possible heterostructured PCF (HSPCF) designs. According to their cladding and core structures, HSPCFs given in subfigures (a) to (d) are named as TPC-TPC, TPC-HPC, HPC-TPC and HPC-HPC fibers, respectively. It should be noticed that not all four fibers have their cladding and core lattice-matched at the *primitive*-basis-vector level. For example, in the case of TPC-HPC fiber, the primitive basis vectors of its core are matched to cladding TPC's next-level basis

Figure 9.31: Four possible lattice-matched heterostructured PCF designs. (a) TPC-TPC fiber; (b) TPC-HPC fiber; (c) HPC-TPC fiber; (d) HPC-HPC fiber;

Figure 9.32: (a) A *primitive* basic unit of HPC. (b) Region bounded by dashed line is the *primitive* basic unit of TPC. The whole plot is a basic unit at next level. (c) TPC *primitive* basic unit which has hexagonal air hole shape. The white region bounded by dot-dashed line, or unit boundary, has refractive index either at 1.79 or 1.54, depending on the type of silicate glass used (see text).

vectors [refer to Fig. 9.32(a) and (b)]. We will concentrate in this work on TPC-TPC fiber with the PBG-guiding mechanism. Their modal properties will be analyzed rigorously, firstly approximately by examining optical properties of two bulk PCs, and then using an explicit mode solver. Plane wave method (PWM) [23] and the FDM presented in Chapter 6 are two numerical tools that we rely on. Modal properties of the other three fibers can be derived following the same strategy.

The first TPC-TPC fiber under study has a cladding air-hole diameter $d_1 = 0.8\Lambda$, and core air-hole diameter $d_2 = 0.9\Lambda$, where Λ is hole-to-hole distance or pitch. With the PWM, the lateral PBG map of the cladding TPC can be derived as shown in Fig. 9.33 (grey-shaded regions). Only gaps found among the first eight bands are shown. Three solid lines in Fig. 9.33 are the first three space-filling modes (SFMs) supported by the core TPC. These modes are computed by calculating mode frequencies at Γ reciprocal lattice point. It is seen that the fundamental SFM (upper solid line) crosses the fundamental gap region (upper shaded region) for a sizable wavelength range, roughly from 0.38 to 0.56 (normalized respect to Λ). Such crossing suggests possible PBG guidance [87] within that

wavelength span. Other two SFMs do not cross any gap regions, hence only fundamental-SFM-like core modes will be supported by the proposed TPC-TPC fiber.

Figure 9.33: Grey regions are PBG regions found for cladding PC of the first TPC-TPC fiber. Solid lines are three lowest-order SFMs of core PC. Nearly degenerate modes are omitted. Line with open circles is dispersion curve for one of the defect core modes supported by a six-ring fiber.

Figure 9.34: E_x (major) field of the defect mode found for the first TPC-TPC fiber at $\lambda = 1.55\mu\text{m}$. Contour lines have 1dB separation.

The actual fiber we used for simulation has six rings of air holes. The inner three rings are for the core, the rest three are for the cladding. Λ is chosen as $3.0\mu\text{m}$. As the core TPC has nineteen primitive basic units, modal folding will happen, which will increase the number of defect modes [87]. The dispersion curve for the defect mode with the largest n_{eff} value is shown by the curve with open circles in Fig. 9.33. The FDM method is employed for deriving the curve. Fig. 9.34 shows the mode's major E field component at $1.55\mu\text{m}$ wavelength. Similar to the fundamental SFM of core TPC, the field is mostly

concentrated in the silica region. It should have globally a Gaussian-like profile, which would be more apparent if we enlarge the core size (*i.e.*, include more core units). Apart from this mode, we notice there are also some other core modes. They are of the same akin, *i.e.* they are all derived from the fundamental SFM of the core PC, but with nodal lines introduced differently. These guided modes, although whose localizations are due to the existence of cladding PBG, are *locally* index-guided within the regime of core.

It is known that, at certain high-frequency bands, an infinitely extended air-holes-in-silica PC is able to guide modes whose energy is mostly supported by the low index material (*i.e.* air). The lowest-order antiguided mode is associated with a n_{eff} value that can roughly be estimated from the wavelength and individual air-hole size using

$$\frac{2\pi}{\lambda} r \sqrt{n_{\text{air}}^2 - n_{\text{eff}}^2} \approx 2.405, \quad (9.1)$$

where r is hole radius. The equation is derived by assuming the PC as an array of antiguides. The fundamental antiguided mode supported by the bulk PC would have distributed peaks located in air holes. The field profile within a single air hole could be approximated by the Bessel J function of zeroth order. In addition, the first root of the Bessel function is near the air-silica interface. Hence equation (9.1) is deduced.

In the rest of this subsection, we will show that a properly designed TPC-TPC fiber is able to support modes that are locally antiguided within the regime of core. Before doing so, we should be aware that such design is even more expensive than designing an air-guiding PBG fiber. If air holes in core region have $d = 3\mu\text{m}$, according to Eq. (9.1), the guided core mode will have $n_{\text{eff}} \approx 0.92$ at $\lambda = 1.55\mu\text{m}$, while fundamental mode of a commonly known air-guiding PBG fiber (with core size equivalent to seven basic units) has $n_{\text{eff}} \approx 0.99$. PBG region at such a small n_{eff} value is difficult to be achieved by a normal air-silica cladding PC. In order to illustrate our idea, here we choose a silicate glass (refractive index 1.79) for cladding TPC. Core TPC is made of another silicate glass (index 1.54) that thermally matches with cladding glass [89]. Both PCs share the same geometrical parameters. Their air holes are of hexagonal shape with, referring to Fig. 9.32(c), $d = 0.98\Lambda$ and $r_c = 0.3\Lambda$. Fig. 9.35 gives lateral PBG map of the cladding TPC. Gap region can be observed around $n_{\text{eff}} = 0.85$. Dispersion curves for the first five SFMs of core TPC are shown by solid lines in Fig. 9.35. It is noticed that the fourth one, which corresponds to an antiguided mode of the core PC, crosses a cladding gap region. Hence we predict that it is possible to get a well-confined mode whose energy mostly stays in core air holes.

Similar to previous design, the actual fiber we choose is of six rings of air holes. The central three rings are for the core. Λ is $3.5\mu\text{m}$. Using the FDM method, we found there are indeed localized modes whose energy mostly resides in air holes of the core TPC (shown by open circles in Fig. 9.35). As the gap region used for confinement is relatively narrow, the mode tends to penetrate more into cladding region as compared to the previous case. This would, in turn, cause the guided mode to have a slightly higher n_{eff} than that of the corresponding core SFM. Major E field of the defect mode at $\lambda = 1.55\mu\text{m}$ is shown in Fig. 9.36. It is noticed that field maximum is located in central air hole, and the modal field

Figure 9.35: Grey regions are PBG regions found for cladding of the second TPC-TPC fiber (parameters given in text). Solid lines are first five SFMs of core TPC. Line with open circles is dispersion curve for one of the defect core modes found for a six-ring heterostructured PCF.

hardly penetrates beyond fourth ring (the first cladding ring), especially along the vertical direction.

We have only shown dispersion curve for one core mode in Fig. 9.33 and 9.35. This is due to the fact that the core modes stay very close to each other and sometimes there are modal anti-crossings, which make mode searching and sorting very hard for our current solver. Also, although we have chosen two lattice-matched PCs in designing such fibers, surface modes still appear during calculation (the problem is even more severe when two unmatched lattices are used). The existence of surface modes makes mode searching even harder.

During our study of HPC-HPC PBG fiber in the last subsection, we did not notice any surface modes. This might suggest that, in addition to lattice-matching, core and the cladding PCs should be terminated in such a way that the termination line should encounter minimum high-index material in order to prevent serious surface mode problem. We expect that a rigorous study of HSPCF would help us understand surface mode problem in PBG fibers better.

In conclusion, we have studied lightwave guidance in heterostructured PCFs. It is shown that the core mode, which is due to PBG region possessed by cladding PC, can be either index-guided or antiguided locally by the core PC. In [146], Neshev et al. observed that a 2D waveguide array is able to support discrete vortex solitons. We believe some HSPCFs, like those shown in Fig. 9.31(b) and (d), are valuable in nonlinear optics applications. Also, HSPCFs can be used to make better gas or liquid sensors because mode field has larger overlapping coefficient with the air-hole regions as compared to conventional PCFs.

Figure 9.36: E_x (major) field of the defect mode found for the second TPC-TPC fiber at $\lambda = 1.55\mu\text{m}$. Contour lines have 1dB separation.

9.3 Discussion

In the first section of this Chapter, we have mainly introduced two types of air-guiding PCFs, i.e. TPCF and HPCF. Their cladding (TPC and HPC) photonic bandgaps are summarized in Fig. 9.6 and 9.13, respectively. We now convert the patches (for the first bandgap only) given in Fig. 9.6 and 9.13 into a more straightforward form as shown in Fig. 9.37. The y -axis value is obtained by taking the ratio of the span width to the mid-span wavelength. The value is interpreted as the normalized PBG-guiding wavelength range, and the normalization is with respect to the mid-gap wavelength.

In fact, as both TPC and HPC asymptotically approach a rods-in-air photonic crystal (in different lattices however) when the silica veins are very thin, therefore their ultimate abilities in propagating light in an air core can be known by examining the performances of their corresponding rods-in-air photonic crystals. If we look at the black lines in Fig. 9.37(a) and (b). They have very similar maximum and minimum values. And their slopes are quite similar too (both slopes are almost linear). Therefore, in theory, the air-guiding wavelength ranges achievable by a TPCF and an HPCF should be comparable. The radiation loss can always be reduced by adding more cladding layers, so it is not a significant issue.

However, at this moment we believe the HPCF is more feasible in realizing surface-mode free, single-mode air-guiding PBG fibers. The pillar-to-pillar distance in an HPCF is smaller than that in a TPCF for their operation around a common wavelength point. Therefore the core can have a smaller size in an HPCF than in a TPCF. Remember that a smaller core promotes single-mode operation. In addition, unlike in a TPCF [Fig. 9.8(b)], the formation of the core in an HPCF does not require any additional silica bridge [and Fig. 9.11(b)]. The extra silica bridge that divides the air core and the photonic crystal cladding has been confirmed as one key factor that causes surface-mode problem

Figure 9.37: (a) and (b) are the normalized air-guiding wavelength range for the TPC shown in Fig 9.6 and the HPC shown in Fig. 9.13, respectively. The normalization is with respect to the mid-gap wavelength. The gray shading of the markers in (a) and (b) are corresponding to those in Fig. 9.6 and 9.13, respectively. Notice the two pitches (Λ) are different in (a) and (b).

[72, 162, 168]. Therefore, an HPCF should perform better in suppressing the surface-mode problem. However the “surface-mode free” property has to be confirmed experimentally.

Chapter 10

Fabrication of Microstructured Optical Fibers

This chapter describes our attempts in fabricating the proposed air-guiding honeycomb photonic crystal fiber (HPCF) [85] (or Section 9.1.2 of this thesis). The fiber to be fabricated is based on the second fiber mentioned in reference [85]. The number of rings is three to four. The fiber is supposed to have a final look as given in Fig. 10.1 with outer diameter at $125\mu\text{m}$. Pillar-to-pillar distance, or pitch is $2.7\mu\text{m}$.

10.1 Fabrication with Silica

There are two immediate stacking methods which can lead to triangular air hole shapes according to current fabrication technology [161, 163]. The first preform stacking method is given in Fig. 10.2. Pitch (Λ) of such structure is equal to the outer diameter of the tubes, *i.e.*, $\Lambda = d_{\text{out}}$ ($d_{\text{out}} = 2r_{\text{out}}$). The inner diameter should have $d_{\text{in}} = 0.89\Lambda$ ($d_{\text{in}} = 2r_{\text{in}}$). The rod should have diameter $d_{\text{rod}} = 0.84\Lambda$. All three parameters can be scaled proportionally to larger values. The air-filling fraction for the preform is $f_{\text{air}} = 66.1\%$. For a three-ring fiber, the number of tubes needed is 126 and that for rods is 54. We also need a larger tube that creates the hollow core. The tube has a outer diameter of 5.16Λ and the thickness should be kept as thin as possible to avoid field localization at the core-cladding interface. It should be noticed that those small rods (whose diameter is 0.205Λ) do not run through the whole preform length. They act as the support between hollow tubes and solid rods only at both ends of the preform.

The second proposed preform stacking is given in Fig. 10.3. We define pitch similarly as $\Lambda = d_{\text{out}}$. The inner diameter of the thin tubes should have $d_{\text{in}} = 0.89\Lambda$. The central *thick* tube has outer diameter $d'_{\text{out}} = 1\Lambda$, and its inner diameter is $d'_{\text{in}} = 0.54\Lambda$. All four parameters can be scaled proportionally to larger values. The air-filling fraction for the preform is also $f_{\text{air}} = 66.1\%$. For a three-ring fiber, the number of thin tubes needed is 126 and that for thick tubes is 54. We need a larger tube that creates the hollow core. The tube has a outer diameter of $5 \times \Lambda$ and the thickness should be kept as thin as possible. The capillaries with thicker walls are to be collapsed using some particular pressurization method.

Figure 10.1: Final look of the fiber to be fabricated (3 cladding rings). Outer diameter is at $125\mu\text{m}$. Black regions are air holes, and white area is silica.

Figure 10.2: First preform stacking method. Shaded regions are silica, white regions are air. The dimensions are drawn strictly according to the numbers given in the text.

Figure 10.3: Second preform stacking method. Shaded regions are silica, white regions are air. The dimensions are drawn strictly according to the numbers given in the text.

In later experimental trials, we adopt the second stacking method since it facilitates easier stacking.

Our preform stacking is sketched in Fig. 10.4. The number of rings of pillars in the cladding is about 3.5. The dimensions are calculated carefully, as denoted in the sketch. It should give rise to $\sim 99\%$ space packing if the capillaries are given their sizes accordingly. However, there always exist discrepancies for the actual tubes produced. In actual stacking, the main capillaries ($\varnothing 1110\mu\text{m}$ thin) have an outer diameter $\varnothing 1100\mu\text{m}$ instead of $\varnothing 1110\mu\text{m}$. And the core tube is smaller by 100 to $350\mu\text{m}$ in outer diameter for different stackings.

The stacked preform has its side view given in Fig. 10.5. The paste is applied as follows: the capillaries with thin wall are to be plugged on one side, while the capillaries with thick wall are to be plugged on another. The core tube is plugged on the same side as those with thin walls.

Figure 10.6(a) shows one end view of the preform where thin-wall capillaries are plugged. Figure 10.6(b) shows the other end view of the preform where thick-wall capillaries are plugged. It is observed that both ends exhibit regular triangular pattern. So we are quite certain that there is no twist of capillaries etcetera happening during stacking.

The preform is fused and drawn into a cane of diameter about 3 to 4mm, as shown under an optical microscope in Fig. 10.7. For this particular attempt, the central core tube of its preform is broken into two halves and they are to be pulled out during fusing and pressurization. However such a procedure is problematic for maintaining a hollow core. Indeed we see from the microscope image that nearby silica tubes collapse into core region. Also we notice silica veins in the cladding region are too thick, which will lead to the disappearance of the photonic bandgap at the air line level.

The cane shown in Fig. 10.7 was not further drawn into fiber. At the time of writing this thesis, another similar preform as shown in Fig. 10.5 is undergoing fabrication.

Figure 10.4: Sketch of the preform stacking.

Figure 10.5: Side view of the preform stacked.

Figure 10.6: (a) End view of the preform stacked. The capillaries with blue ends have a thin wall, and positive pressure is applied within them during fusing and drawing. (b) The other end view of the preform. Capillaries with blue ends have a thick wall, and they are collapsed during fusing.

Figure 10.7: Cane drawn from the preform. Notice the left portion isn't as bad as it looks. The image there is not that clear as the cane is not cleaved properly, *i.e.* the surface there is not as flat.

Figure 10.8: (a) Cross-section of a HPCF. $d/\Lambda = 0.97$. Air core is formed by removing 7 cladding units. Black is for air, white is for polymer. (b) Cross-section of a HPCF. $d/\Lambda = 0.97$. Air core is formed by removing 19 cladding units. Black is for air, white is for polymer.

10.2 Fabrication with Polymer

The polymer we deployed is polymethyl-methacrylate (PMMA). Fabrication of microstructured air-polymer fibers employs a very different preform preparation procedure as compared to that for the air-silica fibers. Basically, drilling holes in a bulk polymer cylinder is used for preparation of the preform. Though air holes drilled are always of circular shape, their appearance can be changed by pressurization during drawing.

Possible preforms for the hollow-core air-polymer photonic bandgap fiber is shown in Fig. 10.8. For Fig. 10.8(a), hole diameter (d) to hole-to-hole (Λ) ratio is 0.97. The central core is a big circle with its diameter 5 times as long as the pitch. This fiber has 4.5 cladding rings (by counting the silica pillars). It has 247 air holes in total. For Fig. 10.8(b), d to (Λ) ratio is also at 0.97. The central core has a bigger diameter which is 8.3 times as long as the pitch. This fiber has 3.5 cladding rings (by counting the silica pillars). It has 223 air holes in total. The bandgap fiber with a big core should suffer smaller loss subject to the same number of cladding rings.

For using air-silica material, the final hole-to-hole distance is about $1.3\mu\text{m}$ in order for the fiber to function at $1.55\mu\text{m}$ wavelength region. The dimension for the air-polymer fiber should not vary too much. However, smaller dimensions are preferred due to the fact that PMMA is the most transparent at visible wavelength.

The preform shown in Fig. 10.8(b) was chosen for our first trial. The drilled preform is shown in Fig. 10.9. However, at the time of writing this thesis, this preform has not been drawn into an actual fiber.

Figure 10.9: (a) Side view of the PMMA preform. (b) End view of the PMMA preform.

Chapter 11

Conclusions and Future Work

11.1 Conclusions

The knowledge of modes propagating in a fiber is very important for understanding the waveguide's many properties, such as singlemodeness, birefringence (and in turn beat-length, polarization-dependent loss), group velocity dispersion, radiation loss, etc. In Part I of this thesis, theoretical tools used to quantitatively derive modes in MOFs are presented. The methods introduced, namely TMM, MEM, FEM, PWM and FDM, are all of full-vector nature, therefore they will give rise to identical converged solutions when the numerical resolutions used are high enough. Some of the methods, like TMM and MEM, are closely linked to the physical processes occurring in an actual MOF, *i.e.*, reflections and transmissions of light upon encountering dielectric interfaces, hence they also help us to understand such waveguides qualitatively. Other methods like FEM, PWM and FDM are full-numerical, hence they can be applied to a regular waveguide with almost any index profile.

In Part II of the thesis, we mainly presented novel MOF designs. The proposed fibers are of either index- or PBG-guiding type. In the index-guiding category, it is demonstrated that air holes can be used in a multi-clad fiber to achieve a flat dispersion curve over both C and L WDM transmission windows. In the PBG-guiding category, two types of air-silica photonic crystals (PCs) are proposed or generalized to achieve superior air guiding. They are respectively the triangular PC (TPC) and the honeycomb PC (HPC). It is believed that the two proposed PCs will help to produce significantly better air-guiding PBG fibers. Besides air-guiding PBG fibers, the honeycomb PCF with an improved composite core and the heterostructured PCFs are also proposed. The heterostructured PCFs should have applications in sensing and nonlinear optics.

Part II also includes our studies on commercialized or newly fabricated MOFs. We noticed that antiguided modes are supported by the overall holey structure in an index-guiding MOF. Also, modal properties of an air-silica Bragg fiber recently fabricated are likely to have a fragmented transmission window. It is believed that our findings are useful for designing better MOFs.

11.2 Future Work

There are several research recommendations to be carried out. One of them is to design an *air-guiding, true-single-mode, surface-mode-free, low-loss, low-dispersion, easily realizable* PBG fiber. Achieving such goal needs coordination of several key parameters, including the cladding PC type, core size, core-cladding interface definition, etc. Besides a theoretical study, iterative experimental trials are also necessary for accomplishing the task. The ultimate application of air-core PBG fibers in the telecommunication area depends on the outcome of the study.

The second task is to study in depth the various waveguiding properties of the so-called air-silica Bragg fiber. From our preliminary study, such a fiber has an unavoidable fragmented transmission window. How to mitigate or reduce the effect of fragmentation remains unknown at this time. Some guidelines on designing better air-silica Bragg fiber are of immediate interest.

The third task is to investigate the performance of a PBG fiber with aperiodic cladding structure. It is believed that light confinement in a defective site of a regular waveguide does not require a periodic cladding. A periodic cladding certainly cannot give us PBG guidance with smallest radiation loss in a 2D waveguide. Also, other waveguiding properties like dispersion in an air-guiding PBG fiber can probably be improved by using an aperiodic cladding.

On the fabrication side, currently MOFs are mainly produced using a stack-and-draw method. Preform preparation by stacking silica tubes makes the fabrication process less likely to be deterministic, especially if we want the air holes to be non-circular. That is, from the same bundle of tubes and the same set of drawing parameters, we can't always get two identical fibers (and the deviation should be larger than what we observed in the conventional SIF fabrication). This is due to the fact that two bundles of silica tubes may be mechanically very different, although they appear analogous macroscopically. Also, the stack-and-draw method makes the production labor-intensive, which indirectly makes such fibers more expensive. New preform preparation methods should be exploited to minimize uncertainties during fiber fabrication. Preform preparation using machine drilling was reported in [169]. However, drilling holes in pure silica is very time-consuming due to silica's rigidity. In the future, it might be meaningful to resort to laser ablation technique in drilling such air holes. Extrusion technique has been reported to prepare MOF preforms using soft glass [126] as well as polymer [170]. Its applicability to fused silica MOF preforms are yet to be studied. Recently Falkenstein et al. reported an interesting MOF preform preparation technique, in which they first fuse two different types of glasses, and then chemically etch one of them away [171]. This method forms good-shaped preforms. However the purity of the final preform need to be rigorously characterized.

APPENDICES

Appendix A

Triangular Photonic Crystals

Figure A.1: TPCs with $s = 0.02\Lambda$ and $r = 0.0456\Lambda$.

Figure A.2: TPCs with $s = 0.02\Lambda$ and $r = 0.085\Lambda$.

Figure A.3: TPCs with $s = 0.05\Lambda$ and $r = 0.085\Lambda$.

Figure A.4: TPCs with $s = 0.02\Lambda$ and $\theta = 40^\circ$.

Figure A.5: TPCs with $s = 0.05\Lambda$ and $\theta = 40^\circ$.

Figure A.6: TPCs with $s = 0.05\Lambda$ and $\theta = 30^\circ$.

Appendix B

Honeycomb Photonic Crystals

Figure B.1: HPCs with $s = 0.02\Lambda$.

Figure B.2: HPCs with $s = 0.05\Lambda$.

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Journal publications

1. M. Yan and P. Shum, "Guidance varieties in photonic crystal fibers," *J. Opt. Soc. Am. B*, accepted, 2006.
2. M. Yan and P. Shum, "Analysis of perturbed Bragg fibers with an extended transfer matrix method," *Opt. Express*, accepted, 2006.
3. X. Yu, M. Yan, L. W. Luo and P. Shum, "Highly birefringent all-solid photonic bandgap fiber with elliptical cladding rods," *IEEE. Photon. Technol. Lett.*, accepted, 2006.
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15. M. Yan and P. Shum, "Antiguiding in microstructured optical fibers," *Opt. Express*, vol. 12, pp. 104–116, January 2004.
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1. L. W. Luo, M. Yan, X. Yu, and P. Shum, "Birefringent all-solid photonic bandgap fiber with elliptical cladding rods," *4th International Conference on Optical Communications and Networks (ICOON)*, Bangkok, Thailand, 2005 (best student paper award).
2. M. Yan and P. Shum, "Air-guiding photonic bandgap fiber with improved triangular air-silica photonic crystal cladding," *European Conference on Optical Communication (ECOC)*, Glasgow, Scotland, 2005.
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Presentations

1. M. Yan, "Photonic crystal fiber," postgraduate seminar, School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, February 2006.

2. M. Yan, "*From optical fiber to photonic crystal fiber*," the First Colloquium on Optical Communication Technology and Market, Infostone Communication Consultant (Shenzhen) Co. Ltd., Shenzhen, China, December 2005.
3. M. Yan, "*Air-guiding photonic crystal fiber*," Optical Fiber Technology Centre (OFTC) departmental seminar, University of Sydney, Australia, May 2005.
4. M. Yan, "*Theoretical analysis of microstructured air-silica fibers*," College of Material Science and Engineering, South China University of Science and Technology, Guangzhou, China, December 2004.
5. M. Yan, "*Theoretical analysis of microstructured air-silica fibers*," Center for Advanced Research in Photonics, The Chinese University of Hong Kong, Hong Kong, November 2004.
6. M. Yan, "*Photonic crystal and photonic crystal fiber*," Institute of Optical Communication and Engineering Research, Nanjing University, China, May 2004.