Blind multiuser detection techniques for DS-CDMA systems

Liu, Xin

2007

Liu, X. (2007). Blind multiuser detection techniques for DS-CDMA systems. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/39034>

<https://doi.org/10.32657/10356/39034>

Blind Multiuser Detection Techniques for DS-CDMA Systems

Liu Xin

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2007

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me and has not been submitted for a higher degree to any other University or Institute.

11/03/2007

Date

Liu Xin

LIU XIN

Acknowledgements

First of all, I would like to express my sincere thanks and great gratitude to my supervisors, Assoc/Prof. Teh Kah Chan and Assoc/Prof. Erry Gunawan. Without their intellectual guidance, constant attention and strict demands, this work would not have been completed. Their extensive knowledge, enlightened direction and continuous encouragement make my thesis work much easier. In particular, I wish to thank them for teaching me some of the philosophical aspects of research and some intangible skills, which I consider are the most invaluable knowledge I have acquired in this research program.

I would like to thank all the technicians and labmates in Communication Lab IV for their kind support and help in the past three years. I would like to express my gratitude to my family for their consistent support and encouragement during all periods of my study. Last, but not the least, I would like to thank Nanyang Technological University for all the study facilities and research scholarship it provided.

Summary

Direct-sequence code-division multiple-access (DS-CDMA) technique is a widely used digital cellular technique for personal communication, which allows multiple users to share limited bandwidth and time. In CDMA systems, the transmitted signals will experience multipath fading channel as well as additive white Gaussian noise (AWGN). Therefore, the capacity and performance of CDMA systems are affected by several types of interference, including multiple access interference (MAI), inter-symbol interference (ISI) as well as AWGN. During the past few decades, blind multiuser detection (MUD) technique has attracted more and more attention since it can suppress various types of interference and demodulate the desired signals efficiently and effectively. In this thesis, various issues regarding blind multiuser detection against MAI, ISI, and AWGN will be addressed.

Assuming the received signals are processed in a batch manner, several types of blind batch MUD algorithms will firstly be derived. The first type is a user identification and detection (UID) algorithm. It is a two-step detector, in which we first identify the active unknown interferers by comparing the estimated power levels of each admissible user. Following that, a zero-forcing (ZF) detector can be formed to suppress the MAI and ISI. The second type of batch MUD is an inter-symbol subspace-based (ISSB) minimum mean square error (MMSE) detector. This detector utilizes the inter-symbol correlation matrix formed by two successive received signal bits and does not need to estimate the background noise level. Due to the properties of this matrix, we propose a simplified

rank determination algorithm. Therefore, the proposed ISSB MMSE detector has lower complexity as compared with the existing subspace-based MMSE detector.

Following that, assuming the received signals are processed in an adaptive manner, three types of blind adaptive MUD algorithms are derived. The first two detectors are Kalman filter-based detector with different constraints, in which the first one uses the inverse filtering and the second one uses the linear prediction as the constraint, respectively. The third proposed blind adaptive MUD is a subspace-based adaptive multiuser detector utilizing the refinement-only fast subspace tracking (RO-FST) algorithm.

In addition, we also analyze the performance of the blind adaptive detector using singular value decomposition (SVD) algorithm. We derive the closed-form expressions of both analytical signal-to-noise-and-interference ratio (SINR) and bit-error-rate (BER) for differential encoding and decoding schemes. Since the exact SVD algorithm can be considered as the optimal algorithm among the SVD-based subspace tracking algorithms, these theoretical SINR and BER results can be treated as performance bounds of the blind adaptive MUD algorithms using other suboptimal SVD-based subspace tracking algorithms.

List of Abbreviations

3G	third generation
AIC	Akaike information criterion
AWGN	additive white Gaussian noise
BER	bit-error-rate
BPSK	binary phase-shift-keying
CDMA	code-division multiple-access
CF	carrier frequency
DMI	direct-matrix-inversion
DS	direct sequence
EVD	eigenvalue decomposition
FAR	false alarm rate
FDMA	frequency-division multiple-access
FH	frequency hopping
FST	fast subspace tracking
GSC	generalized sidelobe canceller
ISI	inter-symbol interference
ISSB	inter-symbol subspace-based
LMS	least mean square
LOS	line-of-sight
LP	linear prediction
MAI	multiple access interference

List of Abbreviations

MDL	minimum description length
ML	maximum-likelihood
MMSE	minimum mean square error
MOE	minimum output energy
MSE	mean square error
MUD	multiuser detection
NAHJ	noise-averaged Hermitian-Jacobi
PASTd	projection approximation subspace tracking with deflation
PDF	probability density function
PG	processing gain
PIC	parallel interference cancellation
PN	pseudo-noise
PSD	power spectral density
RLS	recursive least squares
RO	refinement-only
SB	subspace-based
SIC	successive interference cancellation
SINR	signal-to-interference-and-noise ratio
SNR	signal-to-noise ratio
SO	selective orthogonalization
SS	spread spectrum
SVD	singular value decomposition
TDMA	time-division multiple-access

List of Abbreviations

TH	time hopping
UID	user identification and detection
ZF	zero-forcing

List of Symbols

A_k	amplitude of the k th user's signal
b_k	data symbol stream of the k th user
c_k	normalized binary PN sequence of the k th user
d	dimension of signal subspace
f_c	carrier frequency
$\mathbf{g}$	channel response vector
$\mathbf{h}_k$	effective signature sequence of the k th user
$\mathbf{I}_a$	identity matrix with dimension of $a \times a$
$I_0(\cdot)$	modified Bessel function of order zero
J	estimated sample size
K	number of active users
L_k	length of the k th user's channel response
N_0	two-sided PSD of the noise
$\mathbf{n}$	AWGN vector
P	processing gain
P_e	probability of bit error
$\mathcal{Q}(\cdot, \cdot)$	generalized Marcum Q function
$\mathbf{R}$	covariance matrix of the received signals
$\mathbf{r}$	received signal vector

List of Symbols

s_k	signature waveform of the k th user
$\text{sgn}(\cdot)$	signum function
T_b	bit duration
T_c	chip duration
$\mathbf{U}^s$	signal subspace
$\mathbf{U}^n$	noise subspace
W	transmission bandwidth
σ_n^2	variance of Gaussian noise
τ_{d_k}	delay of the k th user
τ_m	maximum multipath delay spread among all the users
κ_4	fourth-order cumulant
$\Im(\cdot)$	imaginary part of a complex value
$\Re(\cdot)$	real part of a complex value
$\mathbf{\Lambda}^s$	diagonal eigenvalue matrix corresponding to signal subspace
$\mathbf{\Lambda}^n$	diagonal eigenvalue matrix corresponding to noise subspace

Table of Contents

Acknowledgements..... iii

Summary iv

List of Abbreviations vi

List of Symbols ix

Table of Contents..... xi

List of Figuresxv

List of Tables xxi

Chapter 1 Introduction1

 1.1 Motivation..... 1

 1.2 Objectives 6

 1.3 Major Contributions of the Thesis 7

 1.4 Organization of the Thesis 9

Chapter 2 DS-CDMA System Model and Literature Survey11

 2.1 DS-CDMA System Model..... 11

 2.1.1 Transmitted Baseband Signal Model 13

 2.1.2 Channel Model..... 16

 2.1.3 Received Baseband Signal Model 19

 2.2 Non-Blind Multiuser Detection Techniques..... 24

 2.2.1 Conventional Detector 25

 2.2.2 Optimal Multiuser Detector 26

 2.2.3 Suboptimal Linear Multiuser Detectors..... 27

Table of Contents

2.2.4 Subtractive Interference Cancellation Detectors	29
2.3 Blind Multiuser Detection Techniques	29
2.3.1 Blind Batch Multiuser Detectors.....	30
2.3.2 Blind Adaptive Multiuser Detectors	33
2.3.3 Fundamentals of Subspace Based MUD Techniques	36
Chapter 3 Asynchronous Blind User Identification and Detection	41
3.1 Introduction.....	41
3.2 System Model	43
3.3 Proposed LP-UID Algorithm	44
3.3.1 Identification of Unknown Interferers	44
3.3.2 Blind ZF Detector	47
3.4 Performance Analyses of the Proposed Detector.....	50
3.4.1 Perturbation Analysis of Covariance Estimation	50
3.4.2 Perturbation Analysis of Effective Signature Sequence	51
3.4.3 Covariance Matrix of the Proposed Detector.....	53
3.4.4 Performance Analyses of SINR and BER	55
3.5 Numerical Results and Discussions	58
3.6 Conclusion	71
Chapter 4 Asynchronous Blind Inter-Symbol Subspace-Based MMSE Detector.....	72
4.1 Introduction.....	72
4.2 System Model	74
4.3 Proposed ISSB-MMSE Algorithm	76
4.3.1 Inter-Symbol Correlation Matrix	76
4.3.2 Proposed Blind ISSB-MMSE Detector	77

Table of Contents

4.4 Rank Determination	79
4.5 Numerical Results and Discussions	81
4.6 Conclusion	89
Chapter 5 Asynchronous Blind Adaptive Kalman Filter-Based Multiuser Detector	90
5.1 Introduction.....	90
5.2 Kalman Filter-Based MUD Using Inverse Filtering.....	91
5.2.1 System Model	92
5.2.2 Proposed Kalman Filter-Based MUD	92
5.3 Kalman Filter-Based MUD Using Linear Prediction	97
5.3.1 System Model	97
5.3.2 New Canonical Representation.....	98
5.3.3 Proposed Kalman Filter-Based MUD	102
5.4 Numerical Results and Discussions	103
5.5 Conclusion	114
Chapter 6 Asynchronous Blind Adaptive Subspace-Based MUD and Its Perturbation Analyses	116
6.1 Introduction.....	116
6.2 Adaptive MUD Using RO-FST	119
6.2.1 System Model	119
6.2.2 Proposed Adaptive MUD Using RO-FST	120
6.3 Performance Analysis of Blind SVD-Based MUD	126
6.3.1 Adaptive MUD Using SVD.....	126
6.3.2 Perturbation Analysis of Adaptive MUD Using SVD.....	129
6.4 Numerical Results and Discussions	141

Table of Contents

6.5 Conclusion 160

Chapter 7 Conclusions and Future Works 162

7.1 Conclusions..... 162

7.2 Recommendations for Further Research..... 165

Appendix A 168

Appendix B 170

Author’s Publications..... 172

Bibliography 173

List of Figures

Fig. 2.1 DS-CDMA system model..... 12

Fig. 2.2 PSD of additive white Gaussian noise..... 16

Fig. 2.3 PDF of a Rayleigh faded random variable with different values of σ 18

Fig. 3.1 Block diagram of the proposed blind LP-UID algorithm..... 49

Fig. 3.2 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms under the equal-power situation with different SNR and $M=9$ 65

Fig. 3.3 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms as a function of M under the equal-power situation with SNR=15 dB... 65

Fig. 3.4 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms under the near-far and equal-power situations with different SNR and $M=14$ 66

Fig. 3.5 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms as a function of M under the near-far situation with SNR=5 dB. 66

Fig. 3.6 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=5$ under the equal-power situation. 67

Fig. 3.7 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=5$ under the near-far situation..... 67

Fig. 3.8 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=15$ under the equal-power situation. 68

Fig. 3.9 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=15$ under the near-far situation..... 68

Fig. 3.10 BER Comparisons between the proposed LP-UID algorithm and the existing SB-UID algorithm under different estimation sample size with $K=5$69

Fig. 3.11 BER Comparisons between the proposed LP-UID algorithm and the existing SB-UID algorithm, against different number of known interferers with $K=8$69

Fig. 3.12 Simulation and analytical SINR results of the proposed LP-UID detector with different estimation sample size.....70

Fig. 3.13 Simulation and analytical BER results of the proposed LP-UID detector with different estimation sample size.....70

Fig. 4.1 Rank estimation performance of the proposed simplified rank estimation method based on inter-symbol (IS) correlation matrix $\bar{\mathbf{R}}_r$ and covariance matrix $\mathbf{R}_r$ respectively, under the equal-power situation with $K=5$85

Fig. 4.2 Rank estimation performance of the proposed simplified rank estimation method based on inter-symbol (IS) correlation matrix $\bar{\mathbf{R}}_r$ and covariance matrix $\mathbf{R}_r$ respectively, under the near-far situation with $K=5$85

Fig. 4.3 Comparisons of SINR between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the equal-power situation with $K=5$86

Fig. 4.4 Comparisons of BER between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the equal-power situation with $K=5$86

Fig. 4.5 Comparisons of SINR between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the near-far situation with $K=5$87

Fig. 4.6 Comparisons of BER between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the near-far situation with $K=5$87

List of Figures

Fig. 4.7 Comparisons of SINR between the proposed ISSB-MMSE detector and the existing SB-MMSE detector using the proposed rank estimation algorithm under the near-far situation..... 88

Fig. 4.8 Comparisons of BER between the proposed ISSB-MMSE detector and the existing SB-MMSE detector using the proposed rank estimation algorithm under the near-far situation..... 88

Fig. 5.1 Comparisons of SINR results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the equal-power situation..... 109

Fig. 5.2 Comparisons of BER results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the equal-power situation..... 109

Fig. 5.3 Comparisons of SINR results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the near-far situation..... 110

Fig. 5.4 Comparisons of BER results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the near-far situation..... 110

Fig. 5.5 Comparisons of SINR results between the proposed Kalman filter-based MUD using inverse filtering and the LMS MUD using inverse filtering under a dynamic environment. 111

Fig. 5.6 Comparisons of SINR results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the equal-power situation..... 111

List of Figures

Fig. 5.7 Comparisons of BER results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the equal-power situation.	112
Fig. 5.8 Comparisons of SINR results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the near-far situation.	112
Fig. 5.9 Comparisons of BER results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the near-far situation.	113
Fig. 5.10 Comparisons of SINR results between the proposed Kalman filter-based MUD using linear prediction and the LMS MUD using linear prediction under a dynamic environment.	113
Fig. 5.11 Comparisons of SINR results between two proposed Kalman filter-based MUD algorithms.	114
Fig. 6.1 Block diagram of the proposed blind adaptive MMSE detector.	121
Fig. 6.2 Comparisons of SINR performance among the proposed detector using RO-FST and the existing MMSE detectors using NAHJ-FST and PASTd, respectively, with SNR=15dB.	149
Fig. 6.3 Comparisons of SINR performance among the proposed detector using RO-FST, the MMSE detectors using NAHJ-FST and PASTd, respectively, with $K=5$	149
Fig. 6.4 Comparisons of BER performance among the proposed detector using RO-FST, the MMSE detectors using NAHJ-FST and PASTd, respectively, with $K=5$	150

List of Figures

Fig. 6.5 Comparisons of SINR performance among the proposed detector using RO-FST, the MMSE detectors using NAHJ-FST and PASTd, respectively, under a dynamic environment.	150
Fig. 6.6 Comparisons of SINR performance between the proposed detector using RO-FST and the Kalman filter-based MUD using linear prediction.....	151
Fig. 6.7 Comparisons of SINR performance between the proposed detector using RO-FST and the Kalman filter-based MUD using linear prediction, under time-varying channels with $v=5\text{km/h}$	151
Fig. 6.8 Comparisons of SINR performance between the proposed detector using RO-FST and the Kalman filter-based MUD using linear prediction, under time-varying channels with $v=120\text{km/h}$	152
Fig. 6.9 Simulation and analytical MSE results of the blind DMI MMSE detector with SNR=15dB and different forgetting factors γ	152
Fig. 6.10 Simulation and analytical MSE results of the blind subspace MMSE detector with SNR=15dB and different forgetting factors γ	153
Fig. 6.11 Simulation and analytical MSE results of the blind subspace ZF detector with SNR=15dB and different forgetting factors γ	153
Fig. 6.12 Simulation and analytical MSE results of the group-blind detector with SNR=15dB and different forgetting factors γ	154
Fig. 6.13 Simulation and analytical SINR results of the blind DMI MMSE detector with SNR=15dB and different forgetting factors γ	154
Fig. 6.14 Simulation and analytical SINR results of the blind subspace MMSE detector with SNR=15dB and different forgetting factors γ	155

List of Figures

Fig. 6.15 Simulation and analytical SINR results of the blind subspace ZF detector with SNR=15dB and different forgetting factors γ .	155
Fig. 6.16 Simulation and analytical SINR results of the group-blind detector with SNR=15dB and different forgetting factors γ .	156
Fig. 6.17 Simulation and analytical SINR results of the blind subspace MMSE detector with $J=5000$ and different forgetting factors γ .	156
Fig. 6.18 Simulation and analytical BER results of the blind DMI MMSE detector with $K=5$ and different forgetting factors γ .	157
Fig. 6.19 Simulation and analytical BER results of the blind subspace MMSE detector with $K=5$ and different forgetting factors γ .	157
Fig. 6.20 Simulation and analytical BER results of the blind subspace ZF detector with $K=5$ and different forgetting factors γ .	158
Fig. 6.21 Simulation and analytical BER results of the group-blind detector with $K=5$, $\bar{K}=3$ and different forgetting factors γ .	158
Fig. 6.22 Comparisons between simulation and analytical SINR results for the blind subspace MMSE detector over a dynamic environment with SNR=15dB, $K=8$ (for $J<1000$), and $K=5$ (for $J>1000$). .	159
Fig. 6.23 Comparisons between the simulated SINR results of the proposed RO-FST based detector and analytical SINR results of the blind DMI MMSE detector using exact SVD, with $K=5$ and different forgetting factors γ .	159
Fig. 6.24 Comparisons between the simulated BER results of the proposed RO-FST based detector and analytical BER results of the blind DMI MMSE detector using exact SVD, with $K=5$ and different forgetting factors γ .	160

List of Tables

Table 3.1 Computational complexity of LP-UID, SB-UID, LP-MMSE and SB-ZF algorithms.63

Table 5.1 One-step PASTd algorithm for updating **u**..... 101

Chapter 1

Introduction

1.1 Motivation

Code-division multiple-access (CDMA) has become a promising technique as compared with other multiple-access techniques, such as frequency-division multiple-access (FDMA) and time-division multiple-access (TDMA). In FDMA and TDMA, users are multiplexed by orthogonal frequency bands and orthogonal time slots, respectively. In CDMA, however, users are multiplexed by distinct codes. All users can transmit at the same time and utilize the entire available frequency spectrum. CDMA is also known as spread-spectrum multiple-access (SSMA) or simply spread-spectrum (SS) communications because the transmission bandwidth W is much greater than the information rate R in bits/s [1].

There are three major categories of spread-spectrum techniques [2]. The first category is frequency hopping (FH) SS, in which the signal is rapidly switched pseudo-randomly between different frequencies within the hopping bandwidth, and the receiver knows beforehand where to locate the frequency at any given time. The second category is time hopping (TH) SS, in which the signal is transmitted in short bursts at different time slots pseudo-randomly, and the receiver knows beforehand when to expect the burst. The third category is direct sequence (DS) SS. In this scheme, the digital data is

directly coded at a much higher rate. The code is generated pseudo-randomly, and denoted by pseudo-noise (PN) sequence. The receiver will reproduce the same code, and correlate the received signal with that code to extract the data. DS-CDMA utilizes the DS-SS technique as its multiple-access scheme and has been commercially deployed over the past few decades. Furthermore, DS-CDMA has been adopted for the third generation (3G) mobile communication systems.

Although DS-CDMA has many attractive properties as compared with TDMA and FDMA [3]-[6], there are still some limitations for a conventional CDMA system. Multiple access interference (MAI), which refers to the interference from other users, is the main factor that limits the capacity. This interference is the result of random time offsets between signals that make it almost impossible to design the code waveforms which are completely orthogonal. Thus at the receiver, the interference will be enhanced as the number of users increases. Even if the number of interferers is not too large, some users may be received at such high signal levels that a lower-power user may be swamped out. This is called the near-far effect: users near the receiver are received at higher power levels than those far away, and those further away suffer a degradation in performance. In addition, even if users are at the same distance from the base station, there can be an effective near-far effect because some signals may be received during a deep fade. Conventional DS-CDMA systems are very sensitive to this problem and tight power control, with added complexity, is needed to combat it.

Another factor that will greatly affect the performance of DS-CDMA systems is the multipath fading. Due to multiple reflections, the received signal contains the superposition of several delayed and distorted replicas of the original transmitted signal, each with its own amplitude and phase. Due to the motion of a mobile user, the relative

phases of the received signals are continually changing. This results in very large time variations in the received signal. Such variations are referred to as Rayleigh fading (or Rician fading, if there is a direct component in addition to the reflections). If the maximum excess delay of the received replicas is smaller than one symbol duration, it will cause flat fading. On the other hand, if the maximum excess delay is larger than one symbol duration, it will cause frequency selective fading, and hence the received signal will suffer from inter-symbol interference (ISI) [7]. When designing and analyzing low-rate CDMA systems, the presence of ISI is often neglected. However for high-rate CDMA systems, the effect of ISI becomes substantial and it will greatly affect the system performance.

In a conventional CDMA system, all users interfere with each other. Potentially, significant increase in capacity and near-far resistance can theoretically be achieved if the negative effect from other users can be suppressed. Therefore, multiuser detection (MUD) techniques, in which all users are considered as signals for each other, have been proposed. In MUD, the signals from all the users are used for joint detection [8], [9]. Recently, blind MUD techniques have attracted more and more attentions [10]-[12]. The advantage of blind MUD is that it does not require knowledge of the PN sequences and propagation channels of the interferers, and it only requires a minimal amount of information about the desired signal. Thus, it is computationally efficient as compared with the non-blind MUD, and the performance of blind detectors is much better than the conventional detector [10]. Therefore, the blind multiuser detection technique is promising for DS-CDMA systems and is worthwhile to further develop it.

When the data rate of a DS-CDMA system is considerably high, the transmitted signal undergoes multipath fading over practical communication channels. The received

signal of the desired user is distorted by many types of interference, including MAI, ISI, as well as additive white Gaussian noise (AWGN). Therefore, blind MUD techniques need to be developed to suppress all the interferences. According to the manner whether the detector processes the received signals adaptively or not, blind detectors can be categorized into adaptive algorithms and batch algorithms. For the batch MUD, we will study the performance of signal-to-interference-and-noise ratio (SINR) and bit-error-rate (BER). For the adaptive MUD, in addition to SINR and BER, we will also study the performance of convergence rate and tracking capability under a dynamic environment. At the same time, computational complexity will be also taken into account because it is another important criteria for practical implementation. Therefore, how to achieve good balance between good performance and low computational complexity is the major focus of this thesis.

The blind user identification and detection (UID) algorithm enables the receiver to blindly identify the active interferers first and then demodulate the desired user's signal by taking advantage of the estimated interferers' information. The subspace-based (SB) UID algorithm was proposed in [13] over a multipath fading channel. It has been shown in [13] that for a relatively short processing interval, the UID algorithm can approach better performance than the completely blind linear detectors proposed in [14]. However, when the number of active users becomes large, the false alarm rate (FAR) of the existing subspace-based UID algorithm increases. As a consequence, its BER performance will degrade. Therefore, it is important to propose a new UID algorithm with low FAR and BER performance even when the number of active users increases.

The blind linear subspace-based detector is one of the most important blind MUD algorithms due to its robustness. However, it needs to utilize the rank determination

algorithms to estimate the rank of the signal subspace of received signals, which is computationally costly. Therefore, one of our research directions is to simplify the existing subspace-based detector without much performance degradation.

For practical DS-CDMA systems, the amplitudes and cross-correlations of the received signals may be time-variant. Therefore the batch detectors may not be able to demodulate the desired signal accurately. In this case, adopting an adaptive detection technique is necessary. The adaptive constrained minimum output energy (MOE) detectors have been proposed and shown to be computationally efficient [10], [11]. The Kalman filter-based detector presented in [15] is one of the most promising algorithms among this type of detectors due to its rapid convergence rate and good tracking capability. However, the Kalman filter-based MUD in [15] mainly focuses on an AWGN channel and its performance will degrade significantly over a multipath fading channel. Therefore, the extension of the existing Kalman filter-based MUD algorithm to a multipath fading channel with the capability of suppressing both MAI and ISI is an important and worthwhile research work.

Another type of blind adaptive MUD is based on subspace-tracking algorithms, which has better performance as compared with the constrained MOE approach. However, the optimal adaptive subspace-based MUD algorithm using the exact subspace decomposition has a high computational complexity. Therefore, it is necessary for us to reduce the complexity of the adaptive subspace-based MUD, and make it suitable to be implemented in practice.

Theoretical analyses of blind MUD can serve as a benchmark and validate the simulation results of the proposed blind MUD algorithms and provide a basis for further development of new mechanisms. For DS-CDMA systems over multipath fading

channels, performance analyses of blind batch subspace-based MUD have been studied in [16]-[18]. Therefore, extension of this analysis to adaptive subspace-based MUD is necessary for our research.

1.2 Objectives

The objectives of this thesis are to improve the performance and reduce the computational complexity of blind multiuser detection techniques for DS-CDMA systems over multipath fading channels. The objectives are listed as follows.

1). Firstly, we will design an efficient blind UID algorithm with lower FAR and BER as compared with the existing SB-UID algorithm over a multipath fading channel, even when the number of active users becomes large. In addition, we will also analyze the performance of the proposed detector in terms of both SINR and BER performance.

2). Secondly, we will propose a modified subspace-based MMSE detector over multipath fading channels. As compared with the existing subspace-based MMSE detector, the proposed algorithm is expected to have lower complexity and comparable performance.

3). Following that, we will extend the existing adaptive Kalman filter-based MUD algorithm to a multipath fading channel. The proposed algorithm is expected to have the capability of suppressing the MAI as well as ISI caused by multipath fading.

4). Lastly, we will propose a blind adaptive subspace-based MUD using the suboptimal SVD-based subspace tracking algorithm. The proposed algorithm is expected to have low complexity and high performance. In addition, we will also analyze the performance of the adaptive MUD using the singular value decomposition (SVD) based subspace tracking algorithm.

1.3 Major Contributions of the Thesis

The major contributions of this thesis can be summarized as follows.

1). A type of UID algorithm named linear prediction (LP) based UID is proposed. This scheme identifies the active interferers first and then forms a zero-forcing (ZF) detector. The existing SB-UID algorithm proposed in [13] utilizes the orthogonality between noise subspace and PN sequence of each user to identify the active users. Our proposed LP-UID scheme, however, utilizes the estimated power level of each user for identification. Compared with the existing SB-UID algorithm, the proposed LP-UID algorithm significantly improves the FAR without much increase in the computational complexity. Furthermore, numerical results show that the proposed LP-UID outperforms the SB-UID as well as the completely blind linear detectors in terms of BER performance. In addition, we also analyze the SINR and BER performance of the proposed detector.

2). A blind subspace-based minimum mean square error (MMSE) MUD called inter-symbol subspace-based (ISSB) MMSE detector is proposed. This algorithm utilizes the correlation matrix formed by the successive received symbols and does not need to estimate the background noise levels. Furthermore, we also derive a simplified rank determination algorithm to determine the dimension of signal subspace. Therefore, the proposed ISSB MMSE detector has lower complexity as compared with the existing SB MMSE detector in [18] and numerical results show that the BER performance of the proposed ISSB MMSE detector is close to that of the SB MMSE detector when we choose a comparatively large estimated sample size of received signals when the estimated sample size is large.

3). We extend the existing blind Kalman filter-based MUD to a multipath fading channel. Two different constraints are adopted for different channel models. The first constraint is obtained by the inverse filtering whereas the second constraint is obtained by the linear prediction algorithm. Numerical results show that they can achieve better performance as compared with the existing least mean square (LMS) filter-based detector.

4). A subspace-based adaptive detector is proposed. This proposed detector utilizes the refinement-only fast subspace tracking (RO-FST) algorithm [19] to adaptively track the signal and noise subspaces. To reduce the computational complexity of the exact SVD algorithm, some suboptimal subspace tracking algorithms with lower complexity have been proposed and applied to multiuser detection, such as projection approximation subspace tracking with deflation (PASTd) algorithm [20] and noise-averaged Hermitian-Jacobi fast subspace tracking (NAHJ-FST) algorithm [21]. Our proposed blind adaptive MUD utilizing the RO-FST algorithm is shown to outperform the existing blind adaptive subspace-based MUD algorithms, and approach the exact SVD-based MUD in terms of SINR and BER performance. Furthermore, the proposed RO-FST based MUD is shown to have good tracking capability under dynamic environments. At the same time, the computational complexity is at the same order of that of the PASTd and NAHJ-FST algorithms.

5). In this thesis, we also analyze the performance of blind adaptive multiuser detectors over multipath fading channels. We derive the explicit closed-form expressions of SINR and BER performance for the blind adaptive SVD-based MUD using differential encoding and decoding schemes. Since the exact SVD algorithm can be regarded as the optimal algorithm among the SVD-based subspace tracking algorithms,

the derived expressions of SINR and BER can be regarded as the upper bound and lower bound of the suboptimal blind adaptive SVD-based multiuser detectors, respectively.

1.4 Organization of the Thesis

This thesis is organized as follows. In Chapter 2, the general knowledge of DS-CDMA systems and blind multiuser detection techniques will be illustrated and the literature review will be presented. The transmitter, channel and receiver models will be introduced. Several important types of existing non-blind and blind multiuser detection algorithms are also presented.

In Chapter 3, the blind LP-UID detector over a multipath fading channel is proposed. The identification method based on comparing each user's power level is introduced. The performance comparisons between the proposed LP-UID algorithm and the existing SB-UID in terms of FAR and BER performance are presented. In addition, the BER performance of existing blind linear detectors is also presented for comparison. In this chapter, the performance analysis of the proposed detector is also derived, and simulation results are provided to verify the analytical results.

In Chapter 4, the ISSB MMSE algorithm is introduced over a multipath fading channel. A simple rank estimator for this detector is proposed. Numerical results of both ISSB MMSE and existing SB MMSE detectors are provided to show that the BER performance of the proposed ISSB MMSE detector is close to that of the SB MMSE detector when the estimated sample size is large.

In Chapter 5, we develop two types of blind adaptive Kalman filter-based multiuser detectors over multipath fading channels with different constraints. The first type uses the inverse filtering to form its constraint, whereas the second type uses linear

prediction algorithm. The performance comparisons between the proposed Kalman filter-based MUD and the existing LMS filter-based MUD in terms of BER, SINR, convergence rate, as well as tracking capability under a dynamic environment are illustrated.

In Chapter 6, the blind adaptive RO-FST based MUD algorithm is presented. Numerical results in terms of SINR and BER performance are shown and compared with that of the adaptive detector using NAHJ-FST and PASTd algorithms. The numerical result of the tracking capability for the proposed RO-FST based MUD under a dynamic environment is also presented. In this chapter, we also derive the explicit closed-form expressions of SINR and BER performance for the blind adaptive SVD-based MUD algorithm using differential encoding and decoding schemes, which can be regarded as the performance bounds of other SVD-based detectors. Simulation results are also provided to verify the analytical results.

In Chapter 7, we will give the final conclusions of this thesis and some recommendations for future work.

Chapter 2

DS-CDMA System Model and Literature Survey

2.1 DS-CDMA System Model

DS-CDMA technique is one of the most promising multiple access techniques used in wireless applications, such as the personal communications, mobile telephony, and indoor wireless networks [6], [22]. The DS-CDMA system assigns different PN sequence to different users rather than orthogonal time slots, as in TDMA, or orthogonal frequency bands, as in FDMA. All users transmit at the same time and each user is allocated the entire available frequency band. Therefore, many users can share a common bandwidth. The PN sequence of each user is approximately orthogonal to the PN sequences of all the other users. Hence, the transmitted signal to a particular user can be distinguished from one another by proper correlation. DS-CDMA systems have the potentials of being implemented with lighter protocols and higher frequency-reuse capabilities. They can support multi-rate transmissions, and can operate asynchronously over multi-path fading channels. Furthermore, DS-CDMA also has greater flexibility in channel allocation and can share bandwidth with narrow-band services. Due to the above properties, a DS-CDMA system has larger capacity than TDMA as well as FDMA systems, and has been

selected as the basic technology for the realization of the air-interface of 3G cellular communication networks [23], [24].

Fig. 2.1 DS-CDMA system model.

We assume that the data modulation is differential binary phase-shift-keying (DBPSK) throughout the thesis. The binary PN sequence has a nearly flat spectrum and information data of each user is modulated with its own PN sequence. Therefore the transmission bandwidth is much wider than the original data bandwidth. The transmitted signals propagate through communication channels and are distorted by many types of interference. At the receiver, the received signal of each user is composed of the sum of all users' signals, and is demodulated to recover the original information data. A typical DS-CDMA system model is illustrated in Fig. 2.1. The binary pulses spread by the PN sequences are defined as chips, and the number of chips per data bit is defined as the

spreading factor or processing gain (PG) of the system. In the following, we will introduce the DS-CDMA system model adopted in this thesis. We will illustrate the transmitter model, fading channel model, and receiver model sequentially. Note that since the signal carrier can be completely removed by the demodulation, we only focus on the baseband signal in the rest of this thesis.

2.1.1 Transmitted Baseband Signal Model

We consider a DS-CDMA system wherein K users simultaneously transmit DBPSK signals. Such a system applies to both the uplink (i.e., mobile to base station) and the downlink (i.e., base station to mobile) systems. For the uplink system, all users' signals are asynchronous and propagate through different dispersive channels. Therefore, the transmission delay of each user is different. As for the downlink system, all users' signals are synchronous and propagate through the same dispersive channel. Assuming that the receiver at the mobile unit has synchronized with the base station transmitter, the delay of each user is zero for the downlink system.

We first consider the downlink system. The binary information data is multiplied with the signature waveform and then up-converted to carrier frequency (CF) f_c by DBPSK modulation. Assuming there are K users in a DS-CDMA system, the k th user's baseband waveform may be expressed as

$$x_k(t) = A_k \sum_{j=-\infty}^{+\infty} b_k(j) s_k(t - jT_b) \quad (2.1)$$

where

- A_k is the amplitude of the k th user's signal, and A_k^2 is referred to as the signal power.
- T_b is the bit duration for the transmitted data.

- $s_k(t)$ is the deterministic signature waveform assigned to the k th user, and is normalized so as to have unit energy, i.e.,

$$\|s_k\|^2 = \int_0^{T_b} s_k(t) dt = 1. \quad (2.2)$$

The signature waveforms are assumed to be zero outside the interval $[0, T_b]$.

- $b_k(j)$, $-\infty < j < +\infty$ is the data symbol stream transmitted by the k th user, and may be encoded from the source data by different encoding schemes.

For the uplink system, the delay of each user is different. Therefore, an asynchronous transmitter model needs to be set up. Denoting $\tau_{d_k} \in (0, T_b)$ as the delay of the k th user, the baseband transmitted waveform is given by

$$x_k(t) = A_k \sum_{j=-\infty}^{+\infty} b_k(j) s_k(t - jT_b - \tau_{d_k}). \quad (2.3)$$

In the followings, we will introduce the characteristic of $s_k(t)$ and $b_k(j)$.

For a DS-CDMA system, the spectrum spreading is realized by multiplying the information data with the unique signature waveform of each user. Assuming the processing gain is P , the signature waveform of the k th user is given by

$$s_k(t) = \sum_{p=0}^{P-1} c_k(p) u(t - pT_c) \quad (2.4)$$

where $c_k(p)$, $T_c = T_b / P$ and $u(t)$ are the normalized binary PN sequence of the k th user, the chip duration, and the chip waveform, respectively. In this thesis, we choose the unit-amplitude rectangular waveform of duration T_c as the chip waveform $u(t)$ [25], [26], i.e.,

$$u(t) = \begin{cases} 1, & 0 \leq t < T_c, \\ 0, & \text{otherwise.} \end{cases} \quad (2.5)$$

Therefore, this chip waveform can be considered as time limited over one chip duration and will not span multiple chips.

The bandwidth of the transmitted signal is proportional to the processing gain P . Large values of P contribute to the privacy of the system because they hinder unintended receivers to unveil signature waveforms in order to eavesdrop on the transmitted information. For DS-CDMA systems, both the spreading factor P and the chip waveform are the same to all users. What distinguishes the signature waveform $s_k(t)$ from different users is the assignment of the binary PN sequence $c_k(p)$, which is expected to achieve low cross-correlations for all possible offsets with given values of P and K .

In this thesis, all the binary data streams $b_k(j)$, $-\infty < j < +\infty$, $1 \leq k \leq K$ are assumed to be binary and equiprobable. For the blind multiuser detection over fading channels, there will be phase ambiguity when determining the channel response. Therefore, differential encoding of information data, coupled with differential demodulation is strictly necessary. In differential encoding, the transmitted stream is not the data stream itself but its transition, and the rule is given by [27]

$$b_k(j) = d_k(j)b_k^*(j-1) \quad (2.6)$$

where $(\cdot)^*$ denotes the complex conjugate operation and $d_k(j)$ is the original information data stream. The differential decoder is initialized with a bit $b_k(0)$ known to the receiver, and the decision rule at the receiver becomes

$$d_k(j) = b_k(j)b_k^*(j-1). \quad (2.7)$$

The receiver uses the phase of the received signal from the preceding signaling interval as the carrier phase reference. Therefore, the differential decoder does not require the estimation of the carrier phase and can overcome the problem of phase ambiguity.

2.1.2 Channel Model

A communication channel is the physical medium that is used to send the signal from the transmitter to the receiver, and the transmitted signals may be corrupted in a random manner by a variety of possible mechanisms. In this section, we will illustrate a few channel models, including the AWGN channel and several types of fading channels.

2.1.2.1 AWGN Channel

Additive white Gaussian noise (AWGN) is commonly encountered in physical communication channels. In this channel model, the transmitted signal is corrupted by a simple addition of white Gaussian noise, which is the background noise spreading over the whole frequency spectrum of the communication channel, as shown in Fig. 2.2.

Fig. 2.2 PSD of additive white Gaussian noise.

The AWGN denoted as $n(t)$ can be expressed as a white Gaussian process with the two-sided power spectral density (PSD) level of $N_0/2$. The mean of the $n(t)$ is zero and the auto-correlation function of $n(t)$ is given by

$$E\{n(t_1)n(t_2)\} = \frac{N_0}{2} \delta(t_1 - t_2) \quad (2.8)$$

where $\delta(t)$ denotes the delta function.

2.1.2.2 Small-Scale Fading Channel Model

Even though the AWGN channel is often used as a reference channel model in digital communication systems, it is not sufficient to describe real channels considered in this thesis that suffer from small-scale fading. Small-scale fading, or simply fading is used to describe the rapid fluctuations of the amplitudes, phases, or multipath delays of a radio signal over a short period of time or travel distance. For a practical DS-CDMA system, multipath in the radio channel creates small-scale fading effects, and the received signal at any point in space may consist of a large number of waves having randomly distributed amplitudes, phases, and angles of arrival. A detailed study on the characterization of fading channels can be found in Sklar's tutorial papers [28], [29].

For the DS-CDMA system considered in this thesis, the data rate is assumed to be very high. Therefore, the chip duration is smaller than the maximum excess delay of the channel. Thus, the channel exhibits frequency-selective fading, also called multipath fading. The received signal will undergo channel-induced inter-symbol interference (ISI) distortion because many of the multipath components are resolvable by the receiver. At the same time, since the data rate is high, the channel impulse response may be assumed to change at a rate much slower than the transmitted baseband signal. Therefore, the

channel undergoes slow time-varying fading. In this case, the channel may be assumed to be static over several symbol intervals, and the propagating symbols will likely not suffer from the pulse distortion.

Fig. 2.3 PDF of a Rayleigh faded random variable with different values of σ .

In mobile radio channels, Rayleigh distribution is normally used to describe the statistical time-varying nature of the received envelope of an individual multipath component. When the received signal is made up of multiple reflective rays with no significant line-of-sight (LOS) component, the signal amplitude due to small-scale fading has a Rayleigh distribution with probability density function (PDF) given by [2]

$$P(r) = \begin{cases} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, & 0 \leq r < \infty \\ 0, & r < 0 \end{cases} \quad (2.9)$$

where σ^2 is the time-average power of the received signal, and r is the received signal envelope. Figure 2.3 shows the PDF of a Rayleigh faded random variable r with different values of σ . A detailed survey on the characterization of Rayleigh fading channels for mobile digital communication systems was given in [28] and [29]. If the channel exhibits slow time-varying fading, the Rayleigh distribution leads to a complex-valued Gaussian distribution with zero mean.

2.1.3 Received Baseband Signal Model

In this section, we will introduce the receiver model of the DS-CDMA system. The receiver models over both the AWGN and fading channels will be illustrated respectively.

2.1.3.1 Received Signal over an AWGN Channel

At the receiver, the received signal is first down-converted by a CF down-converter. The envelope of the baseband equivalent of the received signal is a superposition of K users' signals as well as AWGN, and can be expressed as

$$r(t) = \sum_{k=1}^K x_k(t) + n(t) \quad (2.10)$$

where $n(t)$ is the AWGN with variance σ_n^2 . For simplicity, we only focus on the synchronous system as shown in (2.1), and an asynchronous system with K users can then be viewed as equivalent to a synchronous system with $2K-1$ users [12], [30]. Substituting (2.1) and (2.4) into (2.10), we obtain

$$r(t) = \sum_{k=1}^K A_k \sum_{j=-\infty}^{+\infty} b_k(j) \sum_{p=0}^{P-1} c_k(p) u(t - jT_b - pT_c) + n(t). \quad (2.11)$$

We sample the received baseband signal at chip rate and the sampled output at the p th chip of the j th bit interval is given by

$$\begin{aligned} r_p(j) &= \int_{jT_b + pT_c}^{jT_b + (p+1)T_c} r(t) u(t - jT_b - pT_c) dt \\ &= \sum_{k=1}^K A_k b_k(j) c_k(p) + n_p(j) \end{aligned} \quad (2.12)$$

where $n_p(j) = \int_{jT_b + pT_c}^{jT_b + (p+1)T_c} n(t) u(t - jT_b - pT_c) dt$. Stacking P chips of $r_p(j)$ together to

form a vector and defining $\mathbf{c}_k = [c_k(0), \dots, c_k(P-1)]^T$, we obtain

$$\mathbf{r}(j) = \sum_{k=1}^K A_k b_k(j) \mathbf{c}_k + \mathbf{n}(j) \quad (2.13)$$

where we define the following P -dimensional vectors

$$\mathbf{r}(j) = \begin{pmatrix} r_0(j) \\ \vdots \\ r_{P-1}(j) \end{pmatrix}, \text{ and } \mathbf{n}(j) = \begin{pmatrix} n_0(j) \\ \vdots \\ n_{P-1}(j) \end{pmatrix}. \quad (2.14)$$

From (2.13), it can be observed that the desired user's signal is distorted by many other users' signals that are transmitted simultaneously. In practical DS-CDMA systems, the signals from different users cannot be kept orthogonal because of the random time offsets between signals. Although the signature waveforms are designed with low-correlation properties, they are not truly orthogonal. As a result, the interference between different users is inevitable and it is called multiple access interference (MAI). The existence of MAI will significantly affect the capacity and performance of DS-CDMA systems. As the number of interferers increases, the amount of MAI increases as well. Even if the number of interferers is not too large, the users' signals may have widely varying power levels. It will cause the near-far problem, i.e., the weak users may be

swamped by the strong ones, and the effects of MAI on system performance become more substantial. Such a problem may also occur when the distances between the transmitters and the receiver are different. Note that for fading channels, the different propagation effects may also cause the near-far problem even if all users are in the same distance from the receiver. For these reasons, MAI cancellation techniques are of great interest to DS-CDMA systems.

2.1.3.2 Received Signal over a Fading Channel

In this thesis, our target is to suppress MAI and ISI over fading channels. Since the data rate of CDMA systems is assumed to be very high, the channel can be modeled as a frequency-selective, slow fading channel with Rayleigh distribution. In this case, the channel statistics will change very slowly within the transmitted packet duration and the received signal at the receiver undergoes channel-induced ISI distortion, caused by multipath fading. In the followings, we will explicitly illustrate the received baseband signal over this type of fading channel.

Considering a more general case that the delay of each user is different, the output of the CF down-converter is given by

$$r(t) = \sum_{k=1}^K x_k(t) * f_k(t) + n(t) \quad (2.15)$$

where $f_k(t)$ is the unknown impulse response of the k th user's channel and $*$ denotes the convolution operation. Substituting (2.3) into (2.15), we have

$$\begin{aligned} r(t) &= \sum_{k=1}^K A_k \sum_{j=-\infty}^{+\infty} b_k(j) s_k(t - jT_b - \tau_{d_k}) * f_k(t) + n(t) \\ &= \sum_{k=1}^K A_k \sum_{j=-\infty}^{+\infty} b_k(j) \bar{h}_k(t - jT_b) + n(t) \end{aligned} \quad (2.16)$$

where

$$\begin{aligned}
 \bar{h}_k(t) &= s_k(t - \tau_{d_k}) * f_k(t) \\
 &= \sum_{p=0}^{P-1} c_k(p) u(t - pT_c - \tau_{d_k}) * f_k(t) \\
 &= \sum_{p=0}^{P-1} c_k(p) \bar{f}_k(t - pT_c). \tag{2.17}
 \end{aligned}$$

In (2.17), the function $\bar{f}_k(t)$ includes all the unknown channel characteristics and is given by

$$\bar{f}_k(t) = u(t - \tau_{d_k}) * f_k(t) \tag{2.18}$$

where τ_{d_k} is the delay of the k th user as mentioned before. Assuming that τ_m is the maximum multipath delay spread among all the users, the function $\bar{h}_k(t)$ has compact support in $[\tau_{d_k}, \tau_{d_k} + \tau_m + T_b] \subseteq [0, L_k T_b]$, where L_k is the length of the k th user's channel impulse response in terms of bit interval. The received signal is sampled at every chip interval to obtain the discrete-time signal. The sampled signal at the p th chip of the j th bit interval is then given by

$$\begin{aligned}
 r_p(j) &= \int_{jT_b + pT_c}^{jT_b + (p+1)T_c} r(t) u(t - jT_b - pT_c) dt \\
 &= \sum_{k=1}^K A_k \sum_{l=0}^{L_k-1} h_{pk}(l) b_k(j-l) + n_p(j) \tag{2.19}
 \end{aligned}$$

where $n_p(j) = \int_{jT_b + pT_c}^{jT_b + (p+1)T_c} n(t) u(t - jT_b - pT_c) dt$ and $h_{pk}(l) = h_k(lP + p)$, for $0 \leq p \leq P-1$

and $0 \leq l \leq L_k - 1$. Note that

$$h_k(m) = \begin{cases} \sum_{p=0}^{P-1} c_k(p) g_k(m-p), & 0 \leq m \leq L_k P - 2 \\ 0, & m = L_k P - 1 \end{cases} \quad (2.20)$$

and

$$g_k(m) = \int_0^{T_c} \bar{f}_k(t + mT_c) u(t) dt, \quad 0 \leq m \leq (L_k - 1)P - 1. \quad (2.21)$$

We denote $q_{g_k} = \lceil (\tau_{d_k} + \tau_m) / T_c \rceil$ as the order of channel response vector in (2.21)

in terms of chip intervals, which means that $g_k(m) = 0$ when $m > q_{g_k}$, where $\lceil \alpha \rceil$ rounds α to the nearest integer greater than or equal to α . Moreover, we also denote $q_g = \max_{k \in K} (q_{g_k})$. By stacking up P received chips over one bit duration, the received signal at the j th bit interval can be expressed as

$$\mathbf{r}(j) = \sum_{k=1}^K A_k \sum_{l=0}^{L_k-1} \mathbf{h}_k(l) b_k(j-l) + \mathbf{n}(j) \quad (2.22)$$

where $\mathbf{r}(j) = [r_0(j) \ \cdots \ r_{P-1}(j)]^T$, $\mathbf{n}(j) = [n_0(j) \ \cdots \ n_{P-1}(j)]^T$, and $\mathbf{h}_k(l)$ represents the effective signature sequence of user k , which includes the PN sequence and the channel response. Here we denote $L = \max_k L_k$, and assume a normal case that $L = 2$, i.e., $\tau_{d_k} + \tau_m < T_b$, [25], [26]. As a consequence, assuming that the first user is the desired user, the desired bit $b_1(j)$ is only included in $\mathbf{r}(j)$ and $\mathbf{r}(j+1)$, therefore only $\mathbf{h}_k(0)$ and $\mathbf{h}_k(1)$, which have the dimension of $P \times 1$, are considered and can be expressed as

$$\begin{bmatrix} \mathbf{h}_k(0) \\ \mathbf{h}_k(1) \end{bmatrix} = \mathbf{C}_k \mathbf{g}_k = \begin{bmatrix} c_k(0) & 0 & \dots & 0 \\ \vdots & c_k(0) & \ddots & \vdots \\ c_k(P-1) & \ddots & \ddots & c_k(0) \\ 0 & c_k(P-1) & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_k(P-1) \\ 0 & \dots & \dots & 0 \end{bmatrix}_{2P \times P} \begin{bmatrix} g_k(0) \\ g_k(1) \\ \vdots \\ g_k(P-1) \end{bmatrix}_{P \times 1} \quad (2.23)$$

where $\mathbf{g}_k$ is the vector of the channel response in a tapped delay line (TDL) form, and is normalized to have unit norm, according to [21], [31] and [32]. If the channel is slow time-varying fading and obeys Rayleigh distribution, $\mathbf{g}_k$ has a fixed complex standard Gaussian distribution with zero mean. Hence, (2.22) can also be expressed as

$$\mathbf{r}(j) = A_1 \mathbf{h}_1(0) b_1(j) + A_1 \mathbf{h}_1(1) b_1(j-1) + \sum_{k=2}^K A_k \sum_{l=0}^1 \mathbf{h}_k(l) b_k(j-l) + \mathbf{n}(j). \quad (2.24)$$

In (2.24), the first term corresponds to the desired user's signal, the second term corresponds to the interference caused by multipath fading, the third term corresponds to the interference caused by other active users, and the last term is the AWGN vector. Therefore, for the multipath fading channel, we need to mitigate not only MAI caused by other active users, but also the ISI caused by the desired user.

In the following sections, we will introduce some existing multiuser detection techniques that can mitigate the interference and recover the desired user's information data.

2.2 Non-Blind Multiuser Detection Techniques

Multiuser detection deals with the demodulation of the mutually interfering digital streams of information in areas such as wireless communication and high-speed data transmission. This section provides an overview of the major strategies of the MUD for

DS-CDMA systems. Both the conventional detection and several types of non-blind MUD will be introduced.

2.2.1 Conventional Detector

The conventional detector is the simplest approach to demodulate CDMA signals, and was first applied in the implementation of CDMA receivers. The conventional detector can be equivalently implemented through what is known as matched filtering [1]. Therefore, the conventional detector is often referred to as the matched filter detector. The performance of this detector depends on the properties of the correlation between the PN sequences. It requires the autocorrelation of the PN sequence to be much larger than the cross-correlation between different PN sequences. The conventional detector was originally proposed in [33] as the maximal signal-to-noise ratio solution to a radar problem. The capacity analysis of the single-user matched filter with direct-sequence signature waveforms can be found in [34].

The conventional detection is computationally simple and requires no knowledge beyond the PN sequences and timing of the desired users. However, its performance can be severely degraded in the presence of MAI, especially when the near-far problem arises. To alleviate the performance degradation, the PN sequences need to have low cross-correlation properties, and power control techniques should be utilized. Power control is to ensure that all users arrive at the receiver with the same power level. However, strict power control on mobiles is not easy to realize, and will limit the performance of the users with good channels as their transmitted powers are restricted by weak users [8].

2.2.2 Optimal Multiuser Detector

Although the conventional detector is computationally simple and requires little information, it is severely limited by the near-far problem, and even in the presence of perfect power control, its BER performance is in the orders of magnitude far from optimal. To overcome the drawback of the conventional detector, there has been great interest in improving the performance of DS-CDMA detection through the use of MUD. In MUD techniques, code and timing (and possibly amplitude and phase) information of multiple users is jointly used to better detect each individual user's signal. The important assumption is that the receiver knows the spreading codes of the multiple users *a priori*. Verdu's work published in 1986 [35] proposed and analyzed the optimal MUD, or the maximum-likelihood (ML) detection. The ML detection yields the most likely transmitted sequence, by maximizing the log-likelihood function. The derivation and analysis of the optimum MUD in the presence of frequency nonselective fading and frequency selective fading channels can be found in [36] and [37], respectively. It has been shown in [38] that under a mild condition, the near-far problem will not cause much degradation of performance if the optimum ML detection is applied.

However, the computational complexity of the optimal ML detection is exponential in the number of users, which is much higher than that of the conventional detector. Moreover, the optimal ML detector requires the knowledge of the received signal amplitudes, cross-correlations and noise level. These values, however, are not known *a priori*, and must be estimated in practice [39].

2.2.3 Suboptimal Linear Multiuser Detectors

The optimal ML detector is too complex to implement for practical DS-CDMA systems. Therefore, over the past few decades, several types of suboptimal MUD solutions have been proposed, which are more feasible to implement. Most of the proposed suboptimal multiuser detectors can be classified into two categories: linear multiuser detectors and subtractive interference cancellation detectors. For linear multiuser detectors, a linear mapping (transformation) is applied to the soft outputs of the conventional detector to produce a new set of outputs, which hopefully provide better performance. The most commonly used linear detectors are the decorrelating detector and the minimum mean square error (MMSE) detector.

1). Decorrelating Detector

The decorrelating detector (also called zero-forcing detector [10]) consists of a bank of matched filters, followed by the inverted cross-correlation matrix. After processing the output of the matched filter bank with the inversion of the cross-correlation, each output is free from interference caused by any of the other users, and only contains a noise term. Thus, the decorrelating detector can completely eliminate the MAI. The decorrelating detector was initially proposed in [40] and [41]. It was extensively analyzed by Lupas and Verdu in [38] and [42], and has been shown to have many attractive properties, such as low computational complexity and the substantial performance and capacity gains over the conventional detector.

One disadvantage of this detector is that it causes noise enhancement. The power associated with the noise term at the output of the decorrelating detector is always greater than or equal to the power associated with the noise term at the output of the conventional detector for each bit [43].

2). MMSE Detector

The MMSE detector is a linear detector that takes into account the background noise and utilizes the knowledge of the received signal power levels [44]. Based on the mean square error (MSE) criterion, the estimator is obtained by minimizing the MSE of the received signal vector. The MMSE detector strikes a balance in mitigating both AWGN and MAI, thus it generally provides better probability of error performance than the decorrelating detector. As the background noise approaches zero, the MMSE detector converges to the decorrelating detector in BER performance.

Unlike the decorrelating detector, the MMSE detector requires the estimation of the received signal amplitudes, and its performance depends on the power levels of the interfering users [44]. However, the advantage of the linear MMSE detector is that it leads itself to adaptive implementation with training sequences, as proposed by Miller [45], Madhow and Hiong [30], Rapajic and Vucetic [46], and Abdulrahman *et al.* [47]. These adaptive MMSE detectors require only a training sequence transmitted by the desired user and coarse timing information of the desired user. They can be implemented adaptively using standard algorithms such as LMS and recursive least squares (RLS). After the training phase, the detectors can continue to adapt in decision-directed mode, in which symbol decisions made by the detector are fed back for further adaptation [48]. Unlike the MMSE detector in [44], this type of detector does not require the received amplitudes. Furthermore, it does not require the PN sequences and timing of interferers. The knowledge of the PN sequence of the desired user is also not necessary, other than to initialize the algorithm [49]. Note that the adaptive MMSE detector normally has a faster convergence rate [48], as compared with the blind adaptive detector that will be introduced in the next section.

2.2.4 Subtractive Interference Cancellation Detectors

Another important group of MUD is classified as subtractive interference cancellation detectors. The basic principle underlying these detectors is the creation of separate estimates of the MAI contributed by each user in order to subtract out some or all of the MAI seen by each user. Such detectors are often implemented with multiple stages, where the expectation is that the decisions will improve at the output of the successive stages. The family of the subtractive interference cancellation detectors includes the successive interference cancellation (SIC) [50], [51] and the parallel interference cancellation (PIC) [52], [53]. Additional references can be found in three survey papers [54]-[56] that focus on these detectors.

2.3 Blind Multiuser Detection Techniques

In the previous section, we have introduced the conventional detector and some important types of non-blind MUD. The conventional detection only requires the information of the PN sequence and the timing of the desired user. On the other hand, almost all of the non-blind MUD algorithms require additional information such as the PN sequence, timing, and received signal amplitude (relative to that of the desired user) of each interfering users. Therefore, although the non-blind MUD can achieve significant performance gain, it is computational inefficient as compared with the conventional detector.

Recently, blind multiuser detection for DS-CDMA systems has attracted more and more attention. These blind techniques essentially allow one to use a linear multiuser detector for a given user with almost no knowledge beyond that required for implementation of the conventional detector for that user. As a result, these blind MUD

algorithms require almost the same information as that of the conventional detector, while the performance of the blind MUD is substantially better. According to the manner whether the detector processes the received signals adaptively or not, the existing blind MUD can be categorized into batch algorithms and adaptive algorithms. In the followings, we will introduce several types of blind batch and adaptive MUD.

2.3.1 Blind Batch Multiuser Detectors

The blind batch MUD processes the received signals packet by packet. Over one packet, it is assumed that the characteristic of the received signals remains unchanged. The subspace-based batch MMSE and zero-forcing (ZF) detectors were first proposed in [12] for an AWGN channel, and were then extended to a multipath fading channel in [14]. This type of detector performs SVD on a block of received signals or eigenvalue decomposition (EVD) on the covariance matrix by averaging the received signal samples, in order to separate the signal subspace and noise subspace. After that, both MMSE criterion and ZF criterion can be applied to form the detector. If the channel undergoes multipath fading, the channel response can be estimated by the subspace method proposed in [57]. This type of subspace-based detectors need to estimate the rank of the signal subspace, some information theoretic criteria, such as the Akaike information criterion (AIC) and minimum description length (MDL) criterion are used to determine the rank. However, these rank determination algorithms are computationally complex. Therefore, in Chapter 4 of this thesis, we will propose a modified subspace-based detector using a simplified rank determination algorithm, which has lower complexity as compared with the existing subspace-based detector. The proposed detector utilizes the inter-symbol correlation matrix, and the performance is only slightly

worse than that of the existing subspace-based detector with large estimated sample size. When the estimated sample size is comparatively small, the performance of the proposed detector will degrade due to the finite sample effect. However, since we mainly focus on the slow time-varying fading channel, it is feasible to adopt a large number of estimated sample size.

Although the performance of subspace-based blind MUD is shown to be robust, it needs to perform the subspace decomposition (SVD or EVD), which is computationally costly. Another drawback of the subspace-based approach is that it is difficult to obtain the accurate rank determination in a practical noisy environment [58]. Therefore, a blind linear prediction based MUD was proposed in [58] for multipath fading channels. The basic idea of the linear prediction (LP) approach is to directly estimate the effective signature sequence of the desired user, which includes the PN sequence and the channel response, by linear prediction algorithm [59]. At the same time, the LP algorithm can also estimate the amplitude levels of active users. After that, the MMSE and ZF detectors can be obtained by using MMSE criterion and ZF criterion, respectively. The LP algorithm avoids the need to estimate channel response, and thus does not need to perform SVD/EVD or rank estimation. Utilizing these properties, we will propose an adaptive Kalman filter-based detector, in which we adopt the LP algorithm to estimate the effective signature sequence. Furthermore, since the LP algorithm can also estimate the amplitude levels, we will utilize this property to identify the active users in Chapter 3 of this thesis.

In a CDMA uplink channel, the base station receiver has the exact knowledge of the PN sequences of all the active users within its cell, but not that of the users outside its cell. Therefore, several subspace-based algorithms for linear group-blind multiuser

detection were proposed in [31]. These detectors attempt to suppress the intracell interference by using the knowledge of the PN sequences and the estimated multipath channels of the active intracell users. On the other hand, they will suppress the intercell interference blindly. It is expected that some performance gains can be achieved over the completely blind methods that assume only the desired user's PN sequence is available.

For some cases, such as an uplink system, there are K out of P users transmitting signals. The receiver may have the information of the P admissible PN sequences, but does not know which sequences are active except the desired user's sequence. Furthermore, the channel impulse response of each active user is also unknown to the receiver. Therefore, the user identification and detection (UID) techniques for CDMA systems have been proposed, which can effectively identify the active users and demodulate the signals transmitted by them. Instead of using the completely linear blind detectors, the UID algorithm can utilize the PN sequences and the estimated channel information of all identified active interferers to form the detector. As a result, the UID algorithm is expected to outperform the completely linear blind detectors that only utilize the PN sequence of the desired user. Therefore, UID algorithm is a promising technique for such an uplink communication system. On the other hand, if the identification of active interferers is not accurate, it will significantly deteriorate the performance of the detector. Therefore, how to acquire an accurate identification of active interferers is essential for the UID algorithm. The UID algorithm was first proposed for an AWGN channel in [60], and was extended to multipath fading channels in [61] and [13]. Since the UID algorithm can utilize the information of all the active users after identification, it can outperform the completely blind linear MUD in [14] in terms of BER performance for small estimation sample size [13]. In [61], it is assumed that the receiver has the

information of the TDL coefficients of the channel so that the receiver can mask out the portion of the received signal affected by ISI. The UID algorithm proposed in [13] is a subspace-based UID, and assumes that the multipath fading channel is completely unknown and no timing information is required explicitly. It identifies the active interferers by the orthogonality property between the PN sequence and the noise subspace of the received signals. According to the property of subspace-based approach from [13] and [14], the identification performance of SB-UID algorithm in [13] degrades significantly when the number of active users increases. In order to improve the performance of the existing UID algorithm, we propose a LP based UID algorithm in Chapter 3 of this thesis. This proposed algorithm identifies the active users by comparing the estimated amplitude levels which are obtained from the LP algorithm proposed in [58]. Numerical results show that the proposed LP-UID can outperform the existing SB-UID in terms of FAR and BER. Furthermore, the proposed LP-UID can still effectively identify the active users when the number of active users is comparatively large. It should be noticed that the performance improvement for the proposed LP-UID is achieved at the expense of a slight increase in computational complexity, and details will be presented in Chapter 3.

2.3.2 Blind Adaptive Multiuser Detectors

For practical DS-CDMA systems, the correlation matrix of the received signals may be time variant, and the batch detectors may not be able to demodulate the desired signal accurately. Therefore, another important thrust for research in multiuser detection is the design of blind adaptive detectors. For the adaptive MMSE detectors proposed in [30], [45]–[47], they only require the training sequence and coarse timing information of the

desired user. When the received amplitudes and cross-correlations are slowly varying, it is possible to track those variations with the adaptive algorithm driven either by a training sequence or in decision-directed mode. However, if there is a sudden change in the CDMA channel such as some interferers are being added in or disconnected from the system, the decision-directed algorithm will use unreliable decisions in lieu of the true data and may not converge. In such a dynamic environment, these adaptive detectors need to have the capability to detect the sudden changes of channel state and send a fresh training sequence again to readjust the detectors [48], [49]. Therefore, it is important to develop a type of blind adaptive detector without training sequence, which can adaptively adjust itself and achieve satisfied performance over such a dynamic environment. Furthermore, blind reception is also of interest in broadcast or multicast settings, because it enables detectors to asynchronously tune in to a transmission of interest at any time. Achieving this using a training-based mechanism requires significant overhead for transmission of training sequences at regular intervals [48]. Therefore, the blind adaptive detector is a promising technique in the future communication systems, and has the potential to constitute the most practical receiver for future high-capacity communication systems based on DS-CDMA. An overview of blind adaptive MUD can be found in [48]. In the followings, we will introduce several types of important blind adaptive MUD algorithms.

The adaptive MMSE MUD algorithms based on a constrained minimum output energy (MOE) criterion were first proposed over an AWGN channel. Among them, the LMS detector was presented in [10] and [49], and the RLS detector was proposed in [11]. These detectors can provide satisfying performance and good tracking capability in dynamic environments. However, these methods are not targeted at detection over

multipath fading channels. To overcome this issue, some improved constrained MOE detectors have been proposed for multipath fading channels in [62]-[64]. More recently, a blind adaptive MUD based on Kalman filtering was proposed in [15]. It is shown that the Kalman filter-based MUD algorithm can outperform both LMS and RLS algorithms in convergence rate and has better tracking capability in dynamic environments. However, the Kalman filter-based MUD in [15] mainly focuses on AWGN channels and its performance will degrade over multipath fading channels. Therefore, in Chapter 5 of this thesis, we propose two types of Kalman filter based adaptive MUD over a multipath fading channel. These detectors can suppress ISI caused by multipath fading as well as MAI. They can also outperform LMS detectors in terms of SINR and BER performance. At the same time, it should be emphasized that the performance improvement of the proposed Kalman filter based detectors is achieved at the expense of higher complexity as compared with the LMS based detectors, which will be discussed in more details in Chapter 5.

Another important type of blind adaptive MUD is based on subspace tracking algorithms. As introduced before, the subspace-based MUD needs to estimate the signal and noise subspaces. Unfortunately, the exact EVD and SVD algorithms are not well suited for the adaptive application because they are not computationally efficient for updating as new data sequences arrive. Therefore, several kinds of adaptive subspace tracking algorithms have been proposed for blind adaptive MUD. These algorithms have low computational complexity and are suboptimal to the exact SVD algorithm. In [12], the blind adaptive MUD based on the projection approximation subspace tracking with deflation (PASTd) algorithm [20] was first proposed over an AWGN channel and was then extended to a multipath fading channel in [65]. However, this algorithm cannot

maintain the orthonormality of the estimated signal subspace and noise subspace, which will deteriorate its performance. In [21], a type of group-blind adaptive MUD based on the noise-averaged Hermitian-Jacobi fast subspace tracking (NAHJ-FST) algorithm was proposed for a multipath fading channel. This algorithm utilizes a selected set of Givens rotations [66] to update the signal subspace and noise subspace, and can maintain the orthonormality of signal subspace and noise subspace. However, the NAHJ-FST based detector does not utilize a complete set of Givens rotations, and thus its performance will degrade. In Chapter 6 of this thesis, we will propose a blind adaptive MUD using RO-FST algorithm. This algorithm adopts a complete set of Givens rotations to update the signal and noise subspace. Therefore, its performance is better than that of the existing NAHJ-FST based detector. Meanwhile, this proposed detector has the same level of computational complexity as the NAHJ-FST algorithm.

2.3.3 Fundamentals of Subspace Based MUD Techniques

In this thesis, the subspace based approach is frequently adopted in the proposed detectors, including batch detectors and adaptive detectors. Therefore, in this section, we will introduce some fundamentals of the subspace based MUD techniques.

2.3.3.1 Subspace Decomposition

Subspace method is an important technique that has been employed in solving various statistical problems in array signal processing, blind channel estimation and CDMA communications [67]. Typically, subspace methods are based on subspace decomposition. Assuming that $\mathbf{X}$ is an $m \times n$ matrix whose entries come from the field K ,

which is either the field of real numbers or the field of complex numbers, there exists a factorization given by [68]

$$\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H \quad (2.25)$$

where $\mathbf{U}$ is an $m \times m$ unitary matrix which contains singular vectors, $\mathbf{\Lambda}$ is an $m \times n$ matrix with nonnegative singular values on the diagonal and zeros off the diagonal, and $\mathbf{V}$ is an $n \times n$ unitary matrix which contains singular vectors. Such a factorization is called singular value decomposition (SVD). SVD is very general in the sense that it can be applied to any $m \times n$ matrix.

The eigenvalue decomposition (EVD), on the other hand, can only be applied to certain classes of square matrices. If $\mathbf{Y}$ is an $m \times m$ Hermitian matrix which is positive semi-definite, then the decomposition in (2.25) becomes EVD, which has the form

$$\mathbf{Y} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H. \quad (2.26)$$

In this case, the singular values in the $m \times m$ matrix $\mathbf{\Lambda}$ and singular vectors in the $m \times m$ unitary matrix $\mathbf{U}$ become the eigenvalues and eigenvectors, respectively.

More generally, given an SVD of matrix $\mathbf{X}$ with dimension of $m \times n$, the following two relations hold:

$$\mathbf{X}^H \mathbf{X} = \mathbf{V} \mathbf{\Lambda}^H \mathbf{U}^H \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H = \mathbf{V} (\mathbf{\Lambda}^H \mathbf{\Lambda}) \mathbf{V}^H, \quad (2.27)$$

$$\mathbf{X} \mathbf{X}^H = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H \mathbf{V} \mathbf{\Lambda}^H \mathbf{U}^H = \mathbf{U} (\mathbf{\Lambda} \mathbf{\Lambda}^H) \mathbf{U}^H. \quad (2.28)$$

The right hand sides of the above two relations describe EVD of the left hand sides. Consequently, the squares of the non-zero singular values of $\mathbf{X}$ are equal to the non-zero eigenvalues of either $\mathbf{X}^H \mathbf{X}$ or $\mathbf{X} \mathbf{X}^H$. Furthermore, the columns of $\mathbf{U}$ (left singular vectors) are eigenvectors of $\mathbf{X} \mathbf{X}^H$ and the columns of $\mathbf{V}$ (right singular vectors) are eigenvectors of $\mathbf{X}^H \mathbf{X}$ [66].

2.3.3.2 Blind Subspace-Based MUD

For the blind subspace based detector, the smoothing factor S is utilized to optimize the performance, which means that the detector processes the data by stacking up S successive $\mathbf{r}(j)$ together [14]. We define the following quantities:

$$\mathbf{r}_s(j) = \begin{bmatrix} \mathbf{r}(j) \\ \vdots \\ \mathbf{r}(j+S-1) \end{bmatrix}_{SP \times 1}, \text{ and } \boldsymbol{\theta}_{r_s} = \begin{bmatrix} \tilde{\boldsymbol{\theta}}(L-1) & \cdots & \tilde{\boldsymbol{\theta}}(0) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \tilde{\boldsymbol{\theta}}(L-1) & \cdots & \tilde{\boldsymbol{\theta}}(0) \end{bmatrix}_{SP \times K(S+L-1)}$$

where $\tilde{\boldsymbol{\theta}}(l) = [A_1 \mathbf{h}_1(l) \cdots A_K \mathbf{h}_K(l)]$, $l = 0, \dots, L-1$. At the receiver, we denote $\mathbf{R}_r$ with dimension of $SP \times SP$ as the covariance matrix of the received signal. Since the additive noise is assumed to be white and independent of the signals, from (2.22) we perform EVD on $\mathbf{R}_{r_s}$, and obtain

$$\mathbf{R}_{r_s} \triangleq E \left\{ \mathbf{r}_s(j) \mathbf{r}_s(j)^H \right\} = \boldsymbol{\theta}_{r_s} \boldsymbol{\theta}_{r_s}^H + \sigma_n^2 \mathbf{I}_{SP} = \mathbf{U}_{r_s}^s \boldsymbol{\Lambda}_{r_s}^s \mathbf{U}_{r_s}^{sH} + \sigma_n^2 \mathbf{U}_{r_s}^n \mathbf{U}_{r_s}^{nH} \quad (2.29)$$

where $\mathbf{I}_{SP}$ is the identity matrix with dimension of $SP \times SP$. $\boldsymbol{\Lambda}_{r_s}^s$ is the diagonal matrix containing the largest $K(S+L-1)$ eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{K(S+L-1)}$; whereas $\mathbf{U}_{r_s}^s$ and $\mathbf{U}_{r_s}^n$ contain the eigenvectors corresponding to signal subspace and noise subspace, respectively.

If the order of channel response vector q_g is known to us, we define

$$\bar{\mathbf{g}}_k = \begin{bmatrix} g_k(0) \\ \vdots \\ g_k(q_g) \end{bmatrix}, \text{ and } \bar{\mathbf{C}}_k = \begin{bmatrix} c_k(0) & 0 & \cdots & 0 \\ c_k(1) & c_k(0) & & \vdots \\ \vdots & c_k(1) & \ddots & 0 \\ \vdots & \vdots & \ddots & c_k(0) \\ c_k(P-1) & \vdots & & c_k(1) \\ 0 & c_k(P-1) & & \vdots \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \vdots & & c_k(P-1) \\ & & & 0 \\ 0 & \cdots & \cdots & \vdots \end{bmatrix}_{SP \times q_g} \quad (2.30)$$

According to the subspace based channel estimation approach in [57], we can utilize the orthogonality between the signal subspace and the noise subspace. Specifically, since $\mathbf{U}_{r_s}^n$ is orthogonal to the column space of $\boldsymbol{\theta}_{r_s}$, we have

$$\mathbf{U}_{r_s}^{nH} \bar{\mathbf{C}}_k \bar{\mathbf{g}}_k = \mathbf{0}. \quad (2.31)$$

Therefore, we can estimate $\bar{\mathbf{g}}_k$ by

$$\bar{\mathbf{g}}_k = \arg \min_{\|\mathbf{x}\|=1} \left\| \mathbf{U}_{r_s}^{nH} (\bar{\mathbf{C}}_k \mathbf{x}) \right\|^2 = \arg \min_{\|\mathbf{x}\|=1} \mathbf{x}^H (\bar{\mathbf{C}}_k^H \mathbf{U}_{r_s}^n \mathbf{U}_{r_s}^{nH} \bar{\mathbf{C}}_k) \mathbf{x} \quad (2.32)$$

which means that $\bar{\mathbf{g}}_k$ is determined by the eigenvector corresponding to the smallest eigenvalue of $(\bar{\mathbf{C}}_k^H \mathbf{U}_{r_s}^n \mathbf{U}_{r_s}^{nH} \bar{\mathbf{C}}_k)$. A necessary condition for such a channel estimate to be unique is that the matrix $\mathbf{U}_{r_s}^{nH} \bar{\mathbf{C}}_k$ is tall, i.e., $SP - K(S + L - 1) \geq g_q$ [31]. Since $g_q \leq (L - 1)P$, we therefore choose S to satisfy

$$SP - K(S + L - 1) \geq (L - 1)P \geq g_q. \quad (2.33)$$

That is, the smoothing factor is chosen to be $S \geq \lceil (P + K)/(P - K) \rceil (L - 1)$ [31]. Note that in (2.32), $\bar{\mathbf{g}}_k$ is determined uniquely up to a scale and phase ambiguity.

After estimating $\bar{\mathbf{g}}_k$, the corresponding subspace based MMSE and ZF detectors can be obtained by using the following forms [14]

$$\mathbf{m}_{\text{MMSE}} = \mathbf{U}_{r_s}^s \left(\Lambda_{r_s}^s \right)^{-1} \mathbf{U}_{r_s}^{sH} \bar{\mathbf{C}}_1 \bar{\mathbf{g}}_1, \quad (2.34)$$

$$\mathbf{m}_{\text{ZF}} = \mathbf{U}_{r_s}^s \left(\Lambda_{r_s}^s - \sigma_n^2 \mathbf{I}_d \right)^{-1} \mathbf{U}_{r_s}^{sH} \bar{\mathbf{C}}_1 \bar{\mathbf{g}}_1. \quad (2.35)$$

Note that for blind MUD over a multipath fading channel, differential demodulation is strictly necessary to eliminate the effect of phase ambiguity when determining the channel response [14]. In the case that q_g is unavailable to us, we can use $(L-1)P$ instead of q_g and follow the similar approach in (2.30)-(2.35) to obtain the MMSE and ZF detectors.

Chapter 3

Asynchronous Blind User

Identification and Detection

3.1 Introduction

In some applications, such as an uplink system, it may be reasonable to assume that the receiver knows the PN sequences of all the admissible users. Therefore, an important engineering task is to identify the active interferers during the observation interval from these admissible users, in order to help the system to promptly process requests and efficiently allocate channels. In such a way, system capacity can be increased. Most importantly, the information on active interferers can produce a reliable estimate of MAI, whose cancellation is the main objective of multiuser detection schemes. Several blind user identification and detection (UID) algorithms for DS-CDMA systems have been proposed. In [60], an UID algorithm over an AWGN channel was proposed. In [13] and [61], some UID algorithms were proposed over multipath fading channels where the presence of MAI and ISI may degrade the system performance. In [61], it is assumed that the receiver has the information of the TDL coefficients of the channel so that the receiver can mask out the portion of the received signal affected by ISI. The UID algorithm proposed in [13] assumes that the multipath fading channel is completely

unknown and no timing information is required explicitly. This UID algorithm can be classified as a subspace-based UID (SB-UID) algorithm as it identifies the active interferers by the orthogonality property between the PN sequence and the noise subspace of the received signals. After that, the desired user's signals can be demodulated by utilizing the information of the identified active interferers. Compared with the blind linear MUD in [14], it has been shown in [13] that this UID algorithm can provide satisfactory performance with much shorter processing intervals.

In [58], blind multiuser detectors based on the linear prediction (LP) algorithm were proposed. The LP algorithm in [59] is employed to directly estimate the effective signature sequence which consists of PN sequence and channel response. Since the LP algorithm can also be utilized to estimate the transmitted power levels of the active users, we will apply this property to develop a new type of UID named LP-UID over a multipath fading channel. The proposed detector is a two-step detector, in which we first estimate the power level of each admissible user by the LP algorithm and choose those that correspond to the M largest power levels as the M unknown active interferers, where M is the number of unknown active interferers. Following that, a ZF detector can be formed to suppress the MAI and ISI. Numerical results will show that the proposed LP-UID algorithm outperforms the existing SB-UID algorithm in terms of FAR as well as BER performance. Furthermore, the proposed LP-UID algorithm can outperform the existing blind linear detectors for the small size of the estimation samples.

In this chapter, we also present a performance analysis of the proposed ZF detector, given the accurate identification of active interferers. In particular, we derive closed-form expressions for the SINR and BER performance, and numerical results will show that the analytical results match the simulation results well.

3.2 System Model

We consider an asynchronous DS-CDMA system over a multipath fading channel with K active users and P chips per bit ($K \leq P$), and the received discrete-time (sampled in chip rate) signal vector $\mathbf{r}(j)$ due to all users at the j th bit interval is given by (2.24), which is reproduced here as

$$\mathbf{r}(j) = A_1 \mathbf{h}_1(0) b_1(j) + A_1 \mathbf{h}_1(1) b_1(j-1) + \sum_{k=2}^K A_k \sum_{l=0}^1 \mathbf{h}_k(l) b_k(j-l) + \mathbf{n}(j). \quad (3.1)$$

Therefore, the desired bit $b_1(j)$ is only included in $\mathbf{r}(j)$ and $\mathbf{r}(j+1)$, and only $\mathbf{h}_k(0)$ and $\mathbf{h}_k(1)$ are considered. By stacking up $\mathbf{r}(j)$ and $\mathbf{r}(j+1)$, $\mathbf{n}(j)$ and $\mathbf{n}(j+1)$ together, we obtain

$$\mathbf{q}(j) = \begin{pmatrix} \mathbf{r}(j) \\ \mathbf{r}(j+1) \end{pmatrix}, \text{ and } \mathbf{N}(j) = \begin{pmatrix} \mathbf{n}(j) \\ \mathbf{n}(j+1) \end{pmatrix}. \quad (3.2)$$

Defining $\mathbf{H}_{k,-1} = [\mathbf{h}_k^H(1) \quad \mathbf{0}]^H$, $\mathbf{H}_{k,0} = [\mathbf{h}_k^H(0) \quad \mathbf{h}_k^H(1)]^H$, $\mathbf{H}_{k,+1} = [\mathbf{0} \quad \mathbf{h}_k^H(0)]^H$, and substituting (3.1) into (3.2), we obtain

$$\begin{aligned} \mathbf{q}(j) &= \begin{pmatrix} \mathbf{r}(j) \\ \mathbf{r}(j+1) \end{pmatrix} \\ &= A_1 \mathbf{H}_{1,0} b_1(j) + A_1 \bar{\mathbf{H}}_1 \bar{\mathbf{B}}_1(j) + \sum_{k=2}^K A_k \mathbf{H}_k \mathbf{B}_k(j) + \mathbf{N}(j) \end{aligned} \quad (3.3)$$

where

$$\mathbf{H}_k = [\mathbf{H}_{k,-1} \quad \mathbf{H}_{k,0} \quad \mathbf{H}_{k,+1}],$$

$$\bar{\mathbf{H}}_1 = [\mathbf{H}_{1,-1} \quad \mathbf{H}_{1,+1}],$$

$$\mathbf{B}_k(j) = [b_k(j-1) \quad b_k(j) \quad b_k(j+1)]^T,$$

and

$$\bar{\mathbf{B}}_1(j) = [b_1(j-1) \quad b_1(j+1)]^T.$$

In (3.3), $\mathbf{H}_{1,0}$ is the effective sequence vector corresponding to $b_1(j)$, and $\bar{\mathbf{H}}_1$ is the effective sequence matrix corresponding to the other bits of the desired user. Therefore, the first term of (3.3) represents the desired user's useful signal, and the other three terms are interference including ISI, MAI, as well as AWGN.

3.3 Proposed LP-UID Algorithm

For the proposed UID scheme, the detector has the information of P admissible PN sequences, K out of which are active in this system. Besides the desired user, we also assume that the detector knows the first $\bar{K}$ out of $K-1$ interferers' PN sequences. Note that in the case of $\bar{K} = 0$, the detector becomes a completely blind UID detector. Moreover, the effective signature sequence $\mathbf{h}_k(l)$ of each active user is not available and needs to be estimated. Therefore, the task of the proposed detector is to first identify the unknown $M = K - \bar{K} - 1$ active interferers' PN sequences among the $P - \bar{K} - 1$ admissible ones and then utilize this information to form a ZF detector.

3.3.1 Identification of Unknown Interferers

In [13], the SB-UID algorithm utilizes the orthogonality property between the admissible users' PN sequences and the noise subspace to identify the active users. In this chapter, however, we will compare the transmitted power levels of the admissible users to find out the unknown active interferers. The principle of the proposed identification algorithm is as follows. Since the inactive users do not transmit, it is reasonable to assume that the estimated power levels of these inactive users are much lower than those of the active

users. Therefore, we first assume that all the possible $P - \bar{K} - 1$ PN sequences are “active”, and then estimate their transmitted power levels respectively. The users with M largest estimated power levels will be finally identified as the unknown active interferers.

Here we resort to the LP algorithm to efficiently estimate the transmitted power levels. In the followings, we will briefly describe the LP algorithm proposed in [58] and apply it to obtain the transmitted power levels. Denoting by $\mathbf{T}_k$ with dimension of $P \times 2P$, for $\bar{K} + 2 \leq k \leq P$, as the left singular vectors of $\mathbf{C}_k$ corresponding to its zero singular values [58], we have

$$\mathbf{T}_k \mathbf{H}_{k,0} = \mathbf{T}_k \mathbf{C}_k \mathbf{g}_k = 0 \quad (3.4)$$

where $\mathbf{C}_k$ and $\mathbf{g}_k$ are defined in (2.23). From (3.3) and (3.4), it can be shown that $\mathbf{T}_k \mathbf{q}(j)$ does not contain $b_k(j)$. Stacking up $\mathbf{r}(j-1)$, $\mathbf{q}(j)$, and $\mathbf{r}(j+2)$ together, we obtain

$$\mathbf{y}(j) = \begin{bmatrix} \mathbf{r}(j-1) \\ \mathbf{q}(j) \\ \mathbf{r}(j+2) \end{bmatrix} \quad (3.5)$$

and its covariance matrix is given by

$$\mathbf{R}_y = E \{ \mathbf{y}(j) \mathbf{y}^H(j) \} = \boldsymbol{\theta}_y \boldsymbol{\theta}_y^H + \sigma_n^2 \mathbf{I}_{4P} \quad (3.6)$$

where $\mathbf{I}_{4P}$ is the identity matrix with dimension of $4P \times 4P$, $\boldsymbol{\theta}_y = [\mathbf{A}_1 \mathbf{H}_{y,1} \quad \cdots \quad \mathbf{A}_K \mathbf{H}_{y,K}]$,

and

$$\mathbf{H}_{y,k} = \begin{bmatrix} \mathbf{h}_k(1) & \mathbf{h}_k(0) & \cdots & \mathbf{0} \\ \vdots & \mathbf{h}_k(1) & \mathbf{h}_k(0) & \vdots \\ & \ddots & \mathbf{h}_k(1) & \mathbf{h}_k(0) \\ \mathbf{0} & \cdots & \mathbf{h}_k(1) & \mathbf{h}_k(0) \end{bmatrix}.$$

Note that for practical implementation, $\mathbf{R}_y$ is normally estimated by averaging over J received data given by

$$\hat{\mathbf{R}}_y = \frac{1}{J} \sum_{j=1}^J \mathbf{y}(j) \mathbf{y}^H(j). \quad (3.7)$$

Based on the LP algorithm in [58], the transmitted power level A_k^2 and the effective signature sequence $\mathbf{H}_{k,0}$ can be estimated by $\mathbf{R}_y$. We define the following four matrixes

$$\mathbf{R}_{11,k} = \mathbf{F} \mathbf{R}_y \mathbf{F}^H, \quad (3.8)$$

$$\mathbf{R}_{12,k} = \mathbf{F} \mathbf{R}_y \tilde{\mathbf{T}}_k^H, \quad (3.9)$$

$$\mathbf{R}_{22,k} = \tilde{\mathbf{T}}_k \mathbf{R}_y \tilde{\mathbf{T}}_k^H, \quad (3.10)$$

and

$$\mathbf{R}_{21,k} = \tilde{\mathbf{T}}_k \mathbf{R}_y \mathbf{F}^H, \quad (3.11)$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{2P} & \mathbf{0} \end{bmatrix}_{2P \times 4P}, \text{ and } \tilde{\mathbf{T}}_k = \begin{bmatrix} \mathbf{I}_P & \mathbf{0} \\ & \mathbf{T}_k \\ \mathbf{0} & \mathbf{I}_P \end{bmatrix}_{3P \times 4P}. \quad (3.12)$$

After some algebraic manipulations and ignoring the noise for the time being, the following matrix can be obtained [58]

$$\mathbf{\Pi}_k = A_k^2 \mathbf{H}_{k,0} \mathbf{H}_{k,0}^H = \mathbf{R}_{11,k} - \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \mathbf{R}_{21,k} \quad (3.13)$$

where $(\cdot)^+$ denotes the pseudo-inverse of a matrix. Note that in practice, $\mathbf{R}_{22,k}$ is invertible because of the presence of noise. From (3.13), it can be observed that the noise-free matrix of $\mathbf{\Pi}_k$ has a rank of 1. In practice, however, there are noise components in $\mathbf{\Pi}_k$. Therefore, the estimation of A_k^2 and $\mathbf{H}_{k,0}$ can be obtained by the

largest eigenvalue of $\mathbf{\Pi}_k$ and its eigenvector, and are denoted by $\hat{A}_k^2$ and $\hat{\mathbf{H}}_{k,0}$, respectively.

For simplicity, here we only consider the condition when L is equal to 2. In some high rate CDMA systems, the total delay may extend over several bit durations, i.e., $L > 2$. In these cases, the system model can be easily extended and the above LP algorithm is still applicable according to [58].

In our proposed UID algorithm, we scan all the $P - \bar{K} - 1$ admissible codes, and choose those that correspond to the M largest $\hat{A}_k^2$ as the unknown active users. Moreover, their corresponding $\hat{\mathbf{H}}_{k,0}$ can be utilized to form the ZF detector that will be discussed later. Note that when the interferer's power becomes stronger, the probability of identification is higher, and vice versa [13].

3.3.2 Blind ZF Detector

We denote the estimation of $\bar{\mathbf{H}}_1$ and $\mathbf{H}_k$ by $\hat{\bar{\mathbf{H}}}_1$ and $\hat{\mathbf{H}}_k$, respectively. Since all of $\mathbf{H}_{k,-1}$, $\mathbf{H}_{k,0}$, and $\mathbf{H}_{k,+1}$ are formed by $\mathbf{h}_k(0)$ and $\mathbf{h}_k(1)$ for different k , we can directly obtain $\mathbf{H}_{k,-1}$ and $\mathbf{H}_{k,+1}$ after estimating $\hat{\mathbf{H}}_{k,0}$ for all the active users, based on the LP algorithm presented in Section 3.3.1. After identifying all the unknown interferers and estimating all the active users' effective signature sequences, we may construct a ZF detector. We define

$$\bar{\mathbf{H}} = [\bar{\mathbf{H}}_1 \quad \mathbf{H}_2 \quad \cdots \quad \mathbf{H}_K]_{2P \times (3K-1)} \quad (3.14)$$

as the effective sequence matrix corresponding to all the transmitted bits excluding $b_1(j)$. In order to suppress both MAI and ISI, we need to project the received signal

$\mathbf{q}(j)$ onto the orthogonal subspace of $\bar{\mathbf{H}}$, so that the second and third terms in (3.3) can be completely eliminated and keep the first term of (3.3) to be non-zero. For this purpose, we calculate $\mathbf{U}_{\bar{\mathbf{H}}}^n$ as the left singular vectors of $\bar{\mathbf{H}}$ corresponding to its zero singular values, and the detector can be expressed by

$$\mathbf{m}_{\text{LP-UID}} = \mathbf{U}_{\bar{\mathbf{H}}}^n \mathbf{U}_{\bar{\mathbf{H}}}^{nH} \mathbf{H}_{1,0}. \quad (3.15)$$

Applying $\mathbf{m}_{\text{LP-UID}}$ on the received signal $\mathbf{q}(j)$, we can observe that the second and third terms of $\mathbf{q}(j)$ in (3.3) can be removed. By denoting $\hat{\mathbf{H}}_k$ and $\hat{\mathbf{U}}_{\bar{\mathbf{H}}}^n$ as the estimations of $\mathbf{H}_k$ and $\mathbf{U}_{\bar{\mathbf{H}}}^n$ respectively, the practical proposed ZF detector can be implemented as

$$\hat{\mathbf{m}}_{\text{LP-UID}} = \hat{\mathbf{U}}_{\bar{\mathbf{H}}}^n \hat{\mathbf{U}}_{\bar{\mathbf{H}}}^{nH} \hat{\mathbf{H}}_{1,0} \quad (3.16)$$

where $\hat{\mathbf{H}}_{1,0}$ is the estimation of $\mathbf{H}_{1,0}$. The block diagram of the proposed detector is presented in Fig. 3.1. As mentioned in Chapter 2, there is an arbitrary phase ambiguity when estimating the effective signature sequences from (3.13), and this ambiguity will greatly affect the system performance. Therefore, the differential encoding and decoding schemes are employed to solve this problem, and the decision rule becomes [69]

$$\hat{d}_1(j) = \text{sgn} \left\{ \Re \left[\left(\hat{\mathbf{m}}_{\text{LP-UID}}^H \mathbf{q}(j) \right) \left(\hat{\mathbf{m}}_{\text{LP-UID}}^H \mathbf{q}(j-1) \right)^* \right] \right\} \quad (3.17)$$

where $\Re(\cdot)$ denotes the real part of a complex value. In the above derivation, it is assumed that the total number of active users K in this system is known to us. If the number of active users K is unknown, we can resort to some existing algorithms such as the Akaike information criterion (AIC) or minimum description length (MDL) criterion to estimate it [70].

In [58], the main task of the LP algorithm is to estimate the effective signature sequence of the desired user that consists of both the PN sequence and channel response.

In our proposed algorithm, however, the LP algorithm is adopted for estimating not only the effective signature sequence, but also the transmitted power level. By comparing the transmitted power level of each admissible user, we develop a new type of UID algorithm and a ZF detector. This detector can utilize the estimated channel information of both the desired user and the active interferers. Therefore, the proposed detector is expected to outperform the existing blind linear detector in [14] and [58] in terms of BER performance for small estimation sample size, and the numerical results will verify it.

Fig. 3.1 Block diagram of the proposed blind LP-UID algorithm.

Lastly, it must be emphasized that the performance improvement of the proposed algorithm is achieved at the expense of a slight increase in computational complexity. For identifying each admissible user, if we resort to the matrix inversion lemma and recursive algorithms [66] to estimate the matrix inversion $\mathbf{R}_{22}^+(k)$ and the exact SVD to estimate A_k^2 and $\mathbf{H}_{k,0}$, the complexity of the proposed LP-UID algorithm is $O(156P^3)$. For SB-UID, it uses different smoothing factor for different number of users in order to optimize its performance [14]. For identifying each admissible user, the complexity of the SB-UID with $S=2$ and $S=4$ are $O(48P^3)$ and $O(212P^3)$ respectively, where the

complexity of SVD is obtained from [19]. Therefore, the complexity of the LP-UID algorithm is only about 3 times of that of the SB-UID with $S=2$, and is lower than the complexity of SB-UID with $S=4$.

3.4 Performance Analyses of the Proposed Detector

Due to the finite received sample size, the estimated covariance matrix $\hat{\mathbf{R}}_y$ will be perturbed, resulting in perturbation for the proposed detector in (3.15). In [71], the perturbation analysis based on the sample covariance matrix was studied, and the explicit closed-form expressions were given. Furthermore, this perturbation analysis has been applied to the subspace-based channel estimation in a CDMA system, and the perturbation analysis for the channel estimation was derived in [71]. In this section, we will utilize the perturbation analysis in [71] to develop the performance analysis of the proposed LP-UID algorithm. In particular, we will derive the explicit closed-form expressions of the SINR and BER performance, under the condition that the active interferers have been perfectly identified.

3.4.1 Perturbation Analysis of Covariance Estimation

The ideal and estimated covariance of $\mathbf{y}(j)$ are given by (3.6) and (3.7), respectively.

From these two equations, we obtain

$$E\{\hat{\mathbf{R}}_y\} = \frac{1}{J} \sum_{j=1}^J E\{\mathbf{y}(j)\mathbf{y}^H(j)\} = \mathbf{R}_y. \quad (3.18)$$

Therefore, $\hat{\mathbf{R}}_y$ is an unbiased estimate of $\mathbf{R}_y$, and we define the perturbation of $\mathbf{R}_y$ as

$$\Delta\mathbf{R}_y = \hat{\mathbf{R}}_y - \mathbf{R}_y. \quad (3.19)$$

Here we follow the perturbation analysis in [71], and focus on the covariance matrix of $\Delta \mathbf{R}_y$ in terms of a weighted matrix given by $E\{\Delta \mathbf{R}_y \mathbf{\Psi} \Delta \mathbf{R}_y\}$, where $\mathbf{\Psi}$ is a deterministic weighting matrix and can be of different forms in different applications. Note that in our system model, $\mathbf{B}(j)$ is a real-value vector and $\mathbf{N}(j)$ is a complex-value vector. After some algebraic manipulations, we can obtain from [71] that

$$E\{\Delta \mathbf{R}_y \mathbf{\Psi} \Delta \mathbf{R}_y\} = \frac{\kappa_{4b}}{J} \boldsymbol{\theta}_y \left[\mathbf{I}_{3K} \odot (\boldsymbol{\theta}_y^H \mathbf{\Psi} \boldsymbol{\theta}_y) \right] \boldsymbol{\theta}_y^H + \frac{1}{J} \text{tr}(\mathbf{R}_y \mathbf{\Psi}) \mathbf{R}_y + \frac{1}{J} \boldsymbol{\theta}_y \boldsymbol{\theta}_y^T \mathbf{\Psi}^T \boldsymbol{\theta}_y^* \boldsymbol{\theta}_y^H \quad (3.20)$$

where “ $\odot$ ” denotes the element-wise multiplication and κ_{4b} is the fourth-order cumulant κ_4 corresponding to the transmitted signal b . Note that for the real-value transmitted signal b , $\kappa_{4b} = m_{4b} - 3\sigma_b^4$ where m_{4b} and σ_b^2 are the fourth order absolute moment and variance of b , respectively. For the Gaussian noise n , its cumulant κ_{4n} is zero [71].

3.4.2 Perturbation Analysis of Effective Signature Sequence

According to (3.13), the estimation of $\hat{\boldsymbol{\Pi}}_k$ is given by

$$\hat{\boldsymbol{\Pi}}_k = \hat{\mathbf{R}}_{11,k} - \hat{\mathbf{R}}_{12,k} \hat{\mathbf{R}}_{22,k}^+ \hat{\mathbf{R}}_{21,k}. \quad (3.21)$$

Assuming $\Delta \boldsymbol{\Pi}_k$, $\Delta \mathbf{R}_{11,k}$, $\Delta \mathbf{R}_{12,k}$, $\Delta \mathbf{R}_{22,k}$, and $\Delta \mathbf{R}_{21,k}$ are the perturbations of $\boldsymbol{\Pi}_k$, $\mathbf{R}_{11,k}$, $\mathbf{R}_{12,k}$, $\mathbf{R}_{22,k}$, and $\mathbf{R}_{21,k}$ respectively, the above equation can be rewritten as

$$\begin{aligned} \hat{\boldsymbol{\Pi}}_k &= \boldsymbol{\Pi}_k + \Delta \boldsymbol{\Pi}_k \\ &= (\mathbf{R}_{11,k} + \Delta \mathbf{R}_{11,k}) - (\mathbf{R}_{12,k} + \Delta \mathbf{R}_{12,k}) (\mathbf{R}_{22,k} + \Delta \mathbf{R}_{22,k})^+ (\mathbf{R}_{21,k} + \Delta \mathbf{R}_{21,k}). \end{aligned} \quad (3.22)$$

By using the Taylor's expansion, we can obtain

$$\hat{\boldsymbol{\Pi}}_k \approx (\mathbf{R}_{11,k} + \Delta \mathbf{R}_{11,k}) - (\mathbf{R}_{12,k} + \Delta \mathbf{R}_{12,k}) (\mathbf{R}_{22,k}^+ - \mathbf{R}_{22,k}^+ \Delta \mathbf{R}_{22,k} \mathbf{R}_{22,k}^+) (\mathbf{R}_{21,k} + \Delta \mathbf{R}_{21,k}). \quad (3.23)$$

Expanding the above expression and keeping only the first-order terms, the perturbation of $\mathbf{\Pi}_k$ is given by

$$\Delta \mathbf{\Pi}_k \approx \Delta \mathbf{R}_{11,k} - \Delta \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \mathbf{R}_{21,k} + \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \Delta \mathbf{R}_{22,k} \mathbf{R}_{22,k}^+ \mathbf{R}_{21,k} - \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \Delta \mathbf{R}_{21,k}. \quad (3.24)$$

From (3.24), it can be shown that $E\{\Delta \mathbf{\Pi}_k\} = 0$. For the proposed ZF detector, the effective sequence $\mathbf{H}_{k,0}$ can be estimated from

$$\hat{\mathbf{H}}_{k,0} = \arg \max_{\|\mathbf{x}\|=1} \mathbf{x}^H \hat{\mathbf{\Pi}}_k \mathbf{x}. \quad (3.25)$$

Since the channel response of each user is normalized to have unit norm as mentioned in Chapter 2, $\hat{\mathbf{H}}_{k,0}$ can be approximated as the eigenvector corresponding to the signal subspace of $\hat{\mathbf{\Pi}}_k$ and its perturbation can be expressed as $\Delta \mathbf{H}_{k,0} \approx \hat{\mathbf{H}}_{k,0} - \mathbf{H}_{k,0}$. We denote $\mathbf{U}_{\Pi,k}^s$, $\mathbf{U}_{\Pi,k}^n$, and $\mathbf{\Omega}_{\Pi,k}^s$ as the signal subspace, the noise subspace and the diagonal eigenvalue matrix corresponding to the signal subspace of the noise-free matrix of $\mathbf{\Pi}_k$, respectively. Utilizing the perturbation analysis for subspace decomposition in [67], the perturbation of $\mathbf{H}_{k,0}$ is given by

$$\Delta \mathbf{H}_{k,0} \approx \mathbf{U}_{\Pi,k}^n \mathbf{U}_{\Pi,k}^{nH} \Delta \mathbf{\Pi}_k \mathbf{H}_{k,0} \left(\mathbf{\Omega}_{\Pi,k}^s \right)^{-1}. \quad (3.26)$$

Substituting (3.24) into (3.26), we obtain

$$\begin{aligned} \Delta \mathbf{H}_{k,0} = & \mathbf{U}_{\Pi,k}^n \left(\mathbf{U}_{\Pi,k}^n \right)^H \left(\Delta \mathbf{R}_{11,k} - \Delta \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \mathbf{R}_{21,k} + \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \Delta \mathbf{R}_{22,k} \mathbf{R}_{22,k}^+ \mathbf{R}_{21,k} \right. \\ & \left. - \mathbf{R}_{12,k} \mathbf{R}_{22,k}^+ \Delta \mathbf{R}_{21,k} \right) \mathbf{H}_{k,0} \left(\mathbf{\Omega}_{\Pi,k}^s \right)^{-1}. \end{aligned} \quad (3.27)$$

Therefore $\hat{\mathbf{H}}_{k,0}$ is an unbiased estimate of $\mathbf{H}_{k,0}$ since $E\{\Delta \mathbf{H}_{k,0}\} = 0$.

3.4.3 Covariance Matrix of the Proposed Detector

As mentioned in Chapter 2, there will be a phase ambiguity $e^{j\phi}$ for the blind channel estimation. It means that if $\hat{\mathbf{H}}_{k,0}$ is the solution to (3.25), so is $\hat{\mathbf{H}}_{k,0}e^{j\phi}$. Consequently, the ambiguity will be transferred to the estimated detectors [16]. Therefore, the perturbation of the proposed detector can be presented by

$$\begin{aligned}\hat{\mathbf{m}}_{\text{LP-UID}} &= (\mathbf{m}_{\text{LP-UID}} + \Delta\mathbf{m}_{\text{LP-UID}})e^{j\phi} \\ &= (\mathbf{U}_{\bar{\mathbf{H}}}^n + \Delta\mathbf{U}_{\bar{\mathbf{H}}}^n)(\mathbf{U}_{\bar{\mathbf{H}}}^{nH} + \Delta\mathbf{U}_{\bar{\mathbf{H}}}^{nH})(\mathbf{H}_{1,0} + \Delta\mathbf{H}_{1,0})e^{j\phi}.\end{aligned}\quad (3.28)$$

where $\Delta\mathbf{m}_{\text{LP-UID}}$ is the perturbation of the proposed detector. Expanding the above expression and keeping only the first-order terms, the first-order perturbation of $\mathbf{m}_{\text{LP-UID}}$ is given by

$$\begin{aligned}\Delta\mathbf{m}_{\text{LP-UID}} &= \hat{\mathbf{m}}_{\text{LP-UID}}e^{-j\phi} - \mathbf{m}_{\text{LP-UID}} \\ &\approx \Delta\mathbf{U}_{\bar{\mathbf{H}}}^n \mathbf{U}_{\bar{\mathbf{H}}}^{nH} \mathbf{H}_{1,0} + \mathbf{U}_{\bar{\mathbf{H}}}^n \Delta\mathbf{U}_{\bar{\mathbf{H}}}^{nH} \mathbf{H}_{1,0} + \mathbf{U}_{\bar{\mathbf{H}}}^n \mathbf{U}_{\bar{\mathbf{H}}}^{nH} \Delta\mathbf{H}_{1,0}.\end{aligned}\quad (3.29)$$

Utilizing the perturbation analysis in [67] again, $\Delta\mathbf{m}_{\text{LP-UID}}$ can be represented as

$$\Delta\mathbf{m}_{\text{LP-UID}} \approx -\bar{\mathbf{H}}^+ \Delta\bar{\mathbf{H}}^H \mathbf{U}_{\bar{\mathbf{H}}}^n \mathbf{U}_{\bar{\mathbf{H}}}^{nH} \mathbf{H}_{1,0} - \mathbf{U}_{\bar{\mathbf{H}}}^n \mathbf{U}_{\bar{\mathbf{H}}}^{nH} \Delta\bar{\mathbf{H}}(\bar{\mathbf{H}}^+)^H \mathbf{H}_{1,0} + \mathbf{U}_{\bar{\mathbf{H}}}^n \mathbf{U}_{\bar{\mathbf{H}}}^{nH} \Delta\mathbf{H}_{1,0}.\quad (3.30)$$

Defining

$$\mathbf{M}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_P \\ \mathbf{0} & \mathbf{0} \end{bmatrix}_{2P \times 2P},$$

$\mathbf{H}_k$ in (3.3) can be represented by

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{M}_1 \mathbf{H}_{k,0} & \mathbf{H}_{k,0} & \mathbf{M}_1^T \mathbf{H}_{k,0} \end{bmatrix}.\quad (3.31)$$

As a consequence, each column of $\bar{\mathbf{H}}$ and $\Delta\bar{\mathbf{H}}$ can be represented by functions of $\mathbf{H}_{k,0}$ and $\Delta\mathbf{H}_{k,0}$ for different k , respectively. Therefore, it can be found that $E\{\Delta\mathbf{m}_{\text{LP-UID}}\} = 0$

based on (3.30), which means that $\hat{\mathbf{m}}_{\text{LP-UID}}$ is an unbiased estimate of $\mathbf{m}_{\text{LP-UID}}$. From (3.30), we observe that the covariance matrix $E\{\Delta\mathbf{m}_{\text{LP-UID}}\Delta\mathbf{m}_{\text{LP-UID}}^H\}$ of the proposed detector consists of nine expectation terms. In order to derive this covariance, we need to calculate $E\{\Delta\mathbf{H}_{1,0}\Psi\Delta\mathbf{H}_{1,0}^H\}$, $E\{\Delta\bar{\mathbf{H}}\Psi\Delta\mathbf{H}_{1,0}^H\}$, $E\{\Delta\mathbf{H}_{1,0}\Psi\Delta\bar{\mathbf{H}}\}$, $E\{\Delta\bar{\mathbf{H}}^H\Psi\Delta\bar{\mathbf{H}}\}$, $E\{\Delta\bar{\mathbf{H}}\Psi\Delta\bar{\mathbf{H}}^H\}$, and $E\{\Delta\bar{\mathbf{H}}\Psi\Delta\bar{\mathbf{H}}\}$, respectively.

We first derive the expression of $E\{\Delta\mathbf{H}_{k_1,0}\Psi\Delta\mathbf{H}_{k_2,0}^H\}$. Equation (3.27) shows that there are sixteen expectation terms in it. Based on (3.8)-(3.11) and (3.19), we obtain

$$\Delta\mathbf{R}_{11,k} = \mathbf{F}\Delta\mathbf{R}_y\mathbf{F}^H, \quad (3.32)$$

$$\Delta\mathbf{R}_{12,k} = \mathbf{F}\Delta\mathbf{R}_y\tilde{\mathbf{T}}_k^H, \quad (3.33)$$

$$\Delta\mathbf{R}_{22,k} = \tilde{\mathbf{T}}_k\Delta\mathbf{R}_y\tilde{\mathbf{T}}_k^H, \quad (3.34)$$

and

$$\Delta\mathbf{R}_{21,k} = \tilde{\mathbf{T}}_k\Delta\mathbf{R}_y\mathbf{F}^H. \quad (3.35)$$

Substituting (3.32)-(3.35) into (3.27), each of the sixteen terms in $E\{\Delta\mathbf{H}_{k_1,0}\Psi\Delta\mathbf{H}_{k_2,0}^H\}$ can be represented as a function of $E\{\Delta\mathbf{R}_y\Psi\Delta\mathbf{R}_y\}$. Therefore, $E\{\Delta\mathbf{H}_{k_1,0}\Psi\Delta\mathbf{H}_{k_2,0}^H\}$ can be calculated by summing up these functions together, i.e.,

$$E\{\Delta\mathbf{H}_{k_1,0}\Psi\Delta\mathbf{H}_{k_2,0}^H\} = \sum_{i=1}^{16} f_i(E\{\Delta\mathbf{R}_y\Psi_i\Delta\mathbf{R}_y\}) \quad (3.36)$$

and each function can be calculated by (3.20) with different Ψ_i . Thus, $E\{\Delta\mathbf{H}_{1,0}\Psi\Delta\mathbf{H}_{1,0}^H\}$ can be obtained. We denote $E\{\Delta\bar{\mathbf{H}}\Psi\Delta\bar{\mathbf{H}}\} = [\mathbf{W}_1, \dots, \mathbf{W}_{3K-1}]$, where $\mathbf{W}_i$ is the i th column of $E\{\Delta\bar{\mathbf{H}}\Psi\Delta\bar{\mathbf{H}}\}$ given by

$$\mathbf{W}_i = E\{\Delta \bar{\mathbf{H}} \Psi \Delta \bar{\mathbf{H}}_i\} = \sum_{j=1}^{3K-1} E\{\Delta \bar{\mathbf{H}}_j \Psi[j, :] \Delta \bar{\mathbf{H}}_i\} \quad (3.37)$$

where $\Psi[j, :]$ denotes the j th row of Ψ . Substituting (3.27) and (3.31) into (3.37), it can be observed that $\mathbf{W}_i$ is a function of $E\{\Delta \mathbf{R}_y \Psi \Delta \mathbf{R}_y\}$ and thus can be calculated by (3.20). Furthermore, the value of $E\{\Delta \mathbf{H}_{1,0} \Psi \Delta \bar{\mathbf{H}}\}$ can be obtained in a similar manner. According to the perturbation analysis in [17], we have

$$E\{\Delta \bar{\mathbf{H}} \Psi \Delta \bar{\mathbf{H}}^H\} = \sum_{i=1}^{3K-1} \sum_{j=1}^{3K-1} \Psi[i, j] E\{\Delta \bar{\mathbf{H}}_i \Delta \bar{\mathbf{H}}_j^H\} \quad (3.38)$$

where $\Psi[i, j]$ is the (i, j) th element of Ψ . Based on (3.36) and (3.20), $E\{\Delta \bar{\mathbf{H}} \Psi \Delta \bar{\mathbf{H}}^H\}$ can be calculated. Likewise, the value of $E\{\Delta \bar{\mathbf{H}} \Psi \Delta \mathbf{H}_{1,0}^H\}$ can also be similarly obtained. Based on the derivation in [17], the (i, j) th element of $E\{\Delta \bar{\mathbf{H}}^H \Psi \Delta \bar{\mathbf{H}}\}$ is a scalar $E\{\Delta \bar{\mathbf{H}}_i^H \Psi \Delta \bar{\mathbf{H}}_j\}$, which can be written as $\text{tr}(E\{\Delta \bar{\mathbf{H}}_j \Delta \bar{\mathbf{H}}_i^H\} \Psi)$. Therefore, the value of $E\{\Delta \bar{\mathbf{H}}^H \Psi \Delta \bar{\mathbf{H}}\}$ can be obtained by (3.36) and (3.20).

Thus far, all the terms in $E\{\Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H\}$ can be represented as the function of $E\{\Delta \mathbf{R}_y \Psi \Delta \mathbf{R}_y\}$ and can be calculated by (3.20).

3.4.4 Performance Analyses of SINR and BER

In the followings, we will show that the analytical expressions of both SINR and BER performance can be expressed as a function of $E\{\Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H\}$. According to (3.3), the SINR can be defined as [17], [18]

$$\hat{\text{SINR}} = \frac{E \left\{ \left\| A_1 \hat{\mathbf{m}}_{\text{LP-UID}}^H \mathbf{H}_{1,0} \right\|^2 \right\}}{E \left\{ \left\| \hat{\mathbf{m}}_{\text{LP-UID}}^H (\mathbf{q} - A_1 \mathbf{H}_{1,0} b_1) \right\|^2 \right\}}. \quad (3.39)$$

Since $E \{ \Delta \mathbf{m}_{\text{LP-UID}} \} = 0$, substituting $\hat{\mathbf{m}}_{\text{LP-UID}} = (\mathbf{m}_{\text{LP-UID}} + \Delta \mathbf{m}_{\text{LP-UID}}) e^{j\phi}$ into (3.39) yields

$$\hat{\text{SINR}} = \frac{A_1^2 \mathbf{H}_{1,0}^H \mathbf{m}_{\text{LP-UID}} \mathbf{m}_{\text{LP-UID}}^H \mathbf{H}_{1,0} + A_1^2 \mathbf{H}_{1,0}^H E \{ \Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H \} \mathbf{H}_{1,0}}{\text{tr}(\bar{\boldsymbol{\theta}}^H \mathbf{m}_{\text{LP-UID}} \mathbf{m}_{\text{LP-UID}}^H \bar{\boldsymbol{\theta}}) + \sigma_n^2 \left\| \mathbf{m}_{\text{LP-UID}} \right\|^2 + \text{tr}(\bar{\boldsymbol{\theta}}^H E \{ \Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H \} \bar{\boldsymbol{\theta}}) + \sigma_n^2 \text{tr}(E \{ \Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H \})} \quad (3.40)$$

where $\bar{\boldsymbol{\theta}} = [A_1 \bar{\mathbf{H}}_1 \quad A_2 \mathbf{H}_2 \quad \cdots \quad A_K \mathbf{H}_K]_{2P \times (3K-1)}$. Therefore, the analytical expression of

SINR is a function of $E \{ \Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H \}$. For the analytical BER expressions, we

recall from (3.17) that the decision rule can be rewritten as [69]

$$\begin{aligned} \hat{d}_1(j) &= \text{sgn} \left\{ \Re \left[\left((\mathbf{m}_{\text{LP-UID}} + \Delta \mathbf{m}_{\text{LP-UID}})^H \mathbf{q}(j) \right) \left((\mathbf{m}_{\text{LP-UID}} + \Delta \mathbf{m}_{\text{LP-UID}})^H \mathbf{q}(j-1) \right)^* \right] \right\} \\ &= \text{sgn} \left(\frac{1}{2} x_j x_{j-1}^* + \frac{1}{2} x_j^* x_{j-1} \right) \end{aligned} \quad (3.41)$$

where $\text{sgn}(\cdot)$ is the signum function and

$$\begin{aligned} x_j &= (\mathbf{m}_{\text{LP-UID}} + \Delta \mathbf{m}_{\text{LP-UID}})^H \mathbf{q}(j) \\ &= \mathbf{m}_{\text{LP-UID}}^H A_1 \mathbf{H}_{1,0} b_1(j) + \mathbf{m}_{\text{LP-UID}}^H \left(A_1 \bar{\mathbf{H}}_1 \bar{\mathbf{B}}_1(j) + \sum_{k=2}^K \boldsymbol{\theta}_k \mathbf{B}_k(j) + \mathbf{N}(j) \right) + \Delta \mathbf{m}_{\text{LP-UID}}^H \mathbf{q}(j) \\ &= \mathbf{m}_{\text{LP-UID}}^H A_1 \mathbf{H}_{1,0} b_1(j) + \xi(j) \end{aligned} \quad (3.42)$$

where

$$\boldsymbol{\theta}_k = A_k \mathbf{H}_k. \quad (3.43)$$

Assuming that $\xi(j)$ is approximately Gaussian distributed with zero mean [16], [69], [72], the random variable x_j is itself Gaussian distributed. The expectation and variance of x_j , as well as the covariance between x_j and x_{j-1} are given by

$$\mu = E\{x_j\} = \mathbf{m}_{\text{LP-UID}}^H \mathbf{A}_1 \mathbf{H}_{1,0} b_1(j),$$

$$c(0) = \text{var}\{x_j | b_1(j)\} = \text{tr}\left[(\overline{\boldsymbol{\theta}\boldsymbol{\theta}^H} + \sigma_n^2 \mathbf{I}_{2P}) \mathbf{m}_{\text{LP-UID}} \mathbf{m}_{\text{LP-UID}}^H + \mathbf{R}_q E\{\Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H\}\right],$$

and

$$c(1) = \text{cov}\{x_j, x_{j-1} | b_1(j), b_1(j-1)\} = \text{tr}\left[\mathbf{Z}_1 E\{\Delta \mathbf{m}_{\text{LP-UID}} \Delta \mathbf{m}_{\text{LP-UID}}^H\} + \mathbf{Z}_2 \mathbf{m}_{\text{LP-UID}} \mathbf{m}_{\text{LP-UID}}^H\right],$$

where

$$\mathbf{R}_q = \boldsymbol{\theta}\boldsymbol{\theta}^H + \sigma_n^2 \mathbf{I}_{2P},$$

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_1 \quad \cdots \quad \boldsymbol{\theta}_K],$$

$$\mathbf{Z}_1 = \sum_{k=1}^K \left[A_k^2 \mathbf{H}_{k,-1} \mathbf{H}_{k,0}^H + A_k^2 \mathbf{H}_{k,0} \mathbf{H}_{k,+1}^H \right] + \sigma_n^2 \mathbf{M}_1,$$

and

$$\mathbf{Z}_2 = \sum_{k=2}^K \left[A_k^2 \mathbf{H}_{k,-1} \mathbf{H}_{k,0}^H + A_k^2 \mathbf{H}_{k,0} \mathbf{H}_{k,+1}^H \right] + \sigma_n^2 \mathbf{M}_1.$$

Following [1] and [69], the probability of bit error for differential BPSK can be obtained by

$$P_e = Q_1(a, b) - \alpha I_0(ab) e^{-(a^2 + b^2)/2} \quad (3.44)$$

where $Q_1(\cdot, \cdot)$ is the generalized Marcum Q function, $I_0(\cdot)$ is the modified Bessel function of order zero, and

$$a = |\mu| \sqrt{\frac{c(0) - \sqrt{c^2(0) - \Im^2(c(1))}}{c^2(0) - \Im^2(c(1))}}, \quad (3.45)$$

$$b = |\mu| \sqrt{\frac{c(0) + \sqrt{c^2(0) - \Im^2(c(1))}}{c^2(0) - \Im^2(c(1))}}, \quad (3.46)$$

and

$$\alpha = \frac{1}{2} \left(1 + \frac{\Re(c(1))}{\sqrt{c^2(0) - \Im^2(c(1))}} \right) \quad (3.47)$$

where $\Im(\cdot)$ denotes the imaginary part of a complex value.

3.5 Numerical Results and Discussions

In this section, the performance of the proposed LP-UID algorithm is compared with that of the existing SB-UID algorithm in [13], the blind subspace-based ZF detector (SB-ZF) [14] and the LP-based MMSE detector (LP-MMSE) in [58]. The following numerical results are based on the differential decoding criterion. The FAR and BER at each point are obtained by averaging over 100 independent simulations, and the total number of errors accumulated is at least 100. In each simulation, the parameters of channel information for all users are randomly generated according to the multipath fading channel model presented in Section 2.1.3.2. Specifically, the delay of each path for user k is generated from a uniform distribution in $[\tau_{d_k}, \tau_{d_k} + \tau_m]$, where the delay of the k th user τ_{d_k} is generated from a uniform distribution in $[0, 10T_c]$, and the delay for each path is less than $20T_c$, i.e., $\tau_m = 20T_c$. The path gain of each path for k th user $g_k(m)$ is generated according to Rayleigh distribution. Since we assume the channel is slowly time-varying fading, $g_k(m)$ are generated from a complex Gaussian distribution with zero mean and held constant for the entire frame length. The path gains in each user's

channel are normalized according to [21], [31] and [32], so that the total path gain of each user is 1. Both the equal-power situation (i.e., all users have the same power level) and a near-far situation (i.e., the power level of users is different) are simulated. Gold sequences of length $P=31$ are adopted as the PN sequences and the sample size J is chosen to be 200 for averaging the received signals. The signal-to-noise-ratio (SNR) is defined as $\text{SNR} = A_1^2 / \sigma_n^2$ [14]. Furthermore, we choose the number of known interferers to be $\bar{K} = 0$. Therefore, the detector has no information of all the interferers ($M=K-1$). Note that for the SB-UID and SB-ZF algorithms, in order to optimize the performance, we will apply different smoothing factors S for different K based on [13] and [14].

First we compare the FAR performance of the proposed LP-UID algorithm and the existing SB-UID algorithm. Here the smoothing factor S is chosen to be 2 for the SB-UID. The FAR is defined as the ratio between the number of false identification (I_e) and the total number of unknown interferers, i.e. $\text{FAR} = E \left\{ \frac{I_e}{M} \right\}$. In comparing the FAR results, we make the same assumption as in [60] that the number of active users K is known. The equal-power situation is considered in Figs. 3.2 and 3.3, whereas the near-far situation is considered in Figs. 3.4 and 3.5, where the power level of each interferer is 10dB higher than that of the desired user. In Fig. 3.2, we consider a moderately loaded system in which the number of active interferers is $M=9$. It can be found that the FAR of the proposed LP-UID is lower than that of the SB-UID for almost all SNR values. At $\text{SNR}=15\text{dB}$, the FAR of the proposed LP-UID is lower than 10^{-3} , which is about two order of magnitude better than that of the SB-UID scheme. In Fig. 3.3, the FAR results are plotted against the number of unknown interferers M ($M=K-1$ in this case) with $\text{SNR}=15\text{dB}$. This figure shows that when M increases, the FAR of the proposed LP-UID

increases much slower than that of SB-UID, and when $M=16$, the FAR of LP-UID is still lower than 0.1, whereas the FAR of SB-UID increases to about 0.4. Similar comparison results are also presented for the near-far situation in Figs 3.4 and 3.5. In Fig. 3.4, the FAR results are plotted against different SNR with $M=14$ whereas in Fig. 3.5, the FAR results are plotted against the number of unknown interferers M with SNR=5dB. From these two figures, we observe that the FAR results of the proposed LP-UID algorithm is always lower than those of the SB-UID algorithm under the near-far situation. In Fig. 3.4, we also present the FAR results against different SNR with $M=14$ under an equal-power situation. It can be observed that the FAR results under the near-far situation are lower than that under the equal-power situation. This is because that when the interferer has stronger power, the probability of identification is also higher [13]. In Figs. 3.3 and 3.5, we can observe that the FAR of SB-UID increases much faster than that of LP-UID, when K , i.e., $(M+1)$ increases. It is because that for the subspace based algorithm, it needs to first separate the noise subspace from the received covariance matrix. According to the property of subspace-based approach as presented in Section 2.3.3.2, the dimension of the noise subspace should not be too small so that the active users can be identified successfully. Furthermore, the dimension of noise subspace is constrained by the number of active users [14]. When K becomes comparatively large, the dimension of the noise subspace will be comparatively small and thus the subspace-based approach can not work well, resulting in significant FAR performance degradation of the corresponding SB-UID algorithm. On the other hand, for the linear prediction based algorithm, unlike the subspace based algorithm, it can still effectively estimate the power level of the active users when K increases. Therefore, it can obtain more accurate identification of active users, as compared with the existing SB-UID algorithm. Based on

Figs. 3.2-3.5, it can be concluded that the proposed LP-UID is much better than the existing SB-UID in terms of FAR performance in both equal-power and near-far situations, particularly when the number of unknown users increases.

Next, we compare the BER performance of the proposed LP-UID with the existing SB-UID, SB-ZF and LP-MMSE algorithms. The AIC algorithm is adopted for estimating the number of active users K in the following simulations. The results under a lightly loaded system with $K=5$ and $S=2$ are presented first. In Fig. 3.6, the equal-power situation is considered. It can be found that the BER of the proposed LP-UID algorithm is lower than that of the SB-UID algorithm. It can also be observed that the BER performance of both the LP-UID and SB-UID algorithms is significantly lower than that of the SB-ZF and LP-MMSE algorithms when SNR is high. In fact, both the SB-ZF and LP-MMSE algorithms exhibit error floors at high SNR values. It is because the UID algorithm makes use of the channel information of all the identified active interferers whereas the other two types of blind linear detectors only know the desired user's information. Similar comparisons for the near-far situation case are shown in Fig. 3.7, where the power level of all the interferers is 10dB higher than that of the desired user. Comparing Figs. 3.6 and 3.7, we can observe that the BER under the near-far situation is lower than that under the equal-power situation. The possible reason is that when the system load is light, the near-far situation produces lower FAR as compared with the equal-power situation. Hence, the detector has more accurate channel information about the active interferers to form the ZF detector.

In Figs. 3.8 and 3.9, the BER results against SNR under a heavily loaded system with $K=15$ are presented. Here, the smoothing factor for the SB-UID and SB-ZF algorithms are chosen to be $S=4$ in order to obtain good performance according to [13]

and [14]. The equal-power situation is considered in Fig. 3.8, whereas the near-far situation is considered in Fig. 3.9. In Fig. 3.9, we assume that four interferers have the same power as the desired user, whereas the power level of the other interferers is 10dB higher. From these two figures, it can be found that even in such a heavily loaded system, our proposed LP-UID algorithm still outperforms the existing algorithms when SNR is high. In addition, we observe that in a heavily loaded system, the BER results under the near-far situation are worse than that under the equal-power situation when SNR is high. The possible reason could be that when SNR is high, the FAR results under both equal-power situation and near-far situation are comparatively low. Under such a condition, the BER performance of the detector under the near-far situation degrades due to the heavily loaded system and increased interference as compared with the equal-power situation. In Table 3.1, we present the computational complexity of the above four detectors, i.e., LP-UID, SB-UID, SB-ZF and LP-MMSE algorithms. From this table, we can observe that for identifying the active interferers, the complexity of our proposed LP-UID algorithm is about 3 times the complexity of SB-UID with $S=2$, and is lower than the complexity of SB-UID with $S=4$.

In Figs. 3.6-3.9, there are some crossovers between the curves of UID algorithms (SB-UID, LP-UID) and blind linear detectors (SB-ZF, LP-MMSE), and it is expected due to the following reasons. For the blind linear detectors, they only utilize the effective sequences of the desired user. For the UID algorithms, on the other hand, they need the effective sequences of all active users to form their detectors, and thus their BER performance is restricted by their corresponding FAR performance. When SNR is low, the FAR of UID algorithms is high, and thus the inactive users which are selected inaccurately will dramatically deteriorate the performance of detectors, and thus the BER

performance of UID algorithms may be lower than those of the blind linear detectors. However, comparing Figs. 3.6 and 3.7, we can also observe that when FAR decreases, the SNR value at the crossovers in Fig. 3.7 is smaller than those in Fig. 3.6.

	Estimating number of users	Identifying each admissible user	Forming the detector
LP-UID	$O(96P^3)$	$O(156P^3)$	$O(64P^3)$
SB-UID ($S=2$)	$O(96P^3)$	$O(48P^3)$	$O(64P^3)$
SB-UID ($S=4$)	$O(96P^3)$	$O(212P^3)$	$O(512P^3)$
LP-MMSE			$O(252P^3)$
SB-ZF ($S=2$)			$O(24P^3)$
SB-ZF ($S=4$)			$O(820P^3)$

Table 3.1 Computational complexity of LP-UID, SB-UID, LP-MMSE and SB-ZF algorithms.

In Fig. 3.10, the BER results of the proposed LP-UID algorithm and the existing SB-UID algorithm with $K=5$ and different estimated sample size J are plotted under the equal-power situation. From this figure, it can be observed that when J increases, the BER results of both algorithms decrease. We can also observe that the LP-UID algorithm always outperforms the SB-UID algorithm for the same value of J . From the above numerical results, it can be concluded that the proposed LP-UID algorithm gives better BER performance under both equal-power and near-far situations as compared with the existing SB-UID, SB-ZF and LP-MMSE algorithms, even in a heavily loaded system.

In Fig. 3.11, we provide the numerical results of BER performance against different number of known interferers $\bar{K}$. Here we assume that there are 8 equal-power users in the system. SNR and J are set to be 11dB and 200, respectively. From this figure, we can observe that the BER performance of both LP-UID and SB-UID decrease when $\bar{K}$ increases. It is because when $\bar{K}$ increases, the FAR will decrease and thus BER will also decrease. From this figure, we can also observe that the decrease in BER of the proposed LP-UID is not as significant as that of SB-UID. It is because that the FAR performance of LP-UID with $\bar{K} = 0$ is lower than that of SB-UID. Therefore when $\bar{K}$ increases, the FAR performance of LP-UID will not decrease as significantly as that of SB-UID. Consequently, the corresponding BER performance improvement of LP-UID is not as significant as that of SB-UID.

Next, we will present some numerical results to verify the analyses provided in Section 3.4. We consider the equal-power situation with $K=5$. Here the estimated sample size J is chosen to be 200 and 500 respectively. The analytical and simulated SINR and BER results are presented in Figs. 3.12 and 3.13, respectively. For comparisons, we also plot the simulated SINR and BER results for the ideal ZF detector based on (3.15), which has the knowledge of the PN sequence as well as channel response of each active user. From these two figures, we can observe that the simulation results of both SINR and BER can match well with those of the analytical results. Furthermore, it is shown that a larger value of J results in higher SINR performance and lower BER performance, which in turn can approach closer to the ideal ZF detector. From these two figures, we can conclude that the analytical expressions of SINR and BER performance are accurate for different SNR and sample size.

Fig. 3.2 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms under the equal-power situation with different SNR and $M=9$.

Fig. 3.3 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms as a function of M under the equal-power situation with SNR=15 dB.

Fig. 3.4 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms under the near-far and equal-power situations with different SNR and $M=14$.

Fig. 3.5 Comparisons of FAR between the proposed LP-UID and the existing SB-UID algorithms as a function of M under the near-far situation with SNR=5 dB.

Fig. 3.6 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=5$ under the equal-power situation.

Fig. 3.7 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=5$ under the near-far situation.

Fig. 3.8 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=15$ under the equal-power situation.

Fig. 3.9 Comparisons of BER among the proposed LP-UID, the existing SB-UID, SB-ZF, and LP-MMSE algorithms with $K=15$ under the near-far situation.

Fig. 3.10 BER Comparisons between the proposed LP-UID algorithm and the existing SB-UID algorithm under different estimation sample size with $K=5$.

Fig. 3.11 BER Comparisons between the proposed LP-UID algorithm and the existing SB-UID algorithm, against different number of known interferers with $K=8$.

Fig. 3.12 Simulation and analytical SINR results of the proposed LP-UID detector with different estimation sample size.

Fig. 3.13 Simulation and analytical BER results of the proposed LP-UID detector with different estimation sample size.

3.6 Conclusion

In this chapter, we have proposed a new type of two-step multiuser detector based on the linear prediction algorithm. The first step is to identify the unknown interferers by comparing the transmitted power levels and the second step is to construct a ZF detector to suppress both MAI and ISI. Numerical results have shown that the proposed LP-UID detector can identify the unknown active interferers more accurately than the existing SB-UID algorithm, especially when the number of unknown interferers becomes large. In addition, the proposed algorithm gives satisfactory BER performance, and outperforms the existing SB-UID algorithm significantly even in a heavily loaded system. Furthermore, numerical results have shown that the proposed LP-UID algorithm outperforms the blind linear detectors for small estimation sample size. Lastly, we have also analyzed the performance of the proposed ZF detector, and derived the closed-form expressions of the SINR and BER performance. Numerical results have shown that the analytical and simulation results can match well.

Chapter 4

Asynchronous Blind Inter-Symbol Subspace-Based MMSE Detector

4.1 Introduction

In Chapter 3, we have proposed an UID algorithm that can take advantage of the interferers' information. Under some conditions, however, the receiver may not have the information of the admissible users' PN sequences. Therefore, the receiver must process the received signal blindly. In this chapter, we will propose a blind MUD that requires only the desired user's information.

The proposed detector is expected to suppress the MAI and ISI over a multipath fading channel. Thus, we need to estimate the desired user's channel response first and then develop a MMSE detector. Recently, some approaches for joint blind multiuser detection and blind channel estimation have been presented [14], [58]. The most popular one is the subspace-based method due to its robustness. It was first proposed in [57] and was then successfully applied to CDMA systems [14], [18], [32], including MMSE detectors and ZF detectors. The subspace-based method usually performs the subspace decomposition algorithm to separate the signal subspace and noise subspace from

received signals, and needs to estimate the rank of the signal subspace. Accurate rank determination is important for blind detector and rank mismatch will affect the performance of blind detectors greatly. Specifically, rank underestimation will cause the desired signal vector to have a significant component in the estimated noise subspace, thereby leading to more errors in channel estimation. Rank underestimation will also lead to imperfect signal subspace estimation, which may reduce the interference suppression capability of the detector significantly. On the other hand, rank overestimation of the signal subspace dimension will reduce the dimension of estimated noise subspace. As a result, the accuracy of channel estimation and thus the performance of detector will be affected [73]. In [70], some likelihood-based information theoretic criteria, such as the Akaike information criterion (AIC) and minimum description length (MDL) criterion, were used to determine the rank. Recently, some works have been done to develop the techniques of rank determination [73]. However, these criteria are computationally complex.

More recently, a new kind of blind subspace-based approach, called the inter-symbol constraint-optimized detector has been proposed in [74] and [75]. The basic idea of this technique is to construct an inter-symbol correlation matrix of the adjacent received bits. Based on this inter-symbol correlation matrix, theoretically, a simplified ZF detector can be derived without using any rank estimation, and its performance is quite close to that of the subspace-based ZF detector [14]. However, the ZF detector proposed in [74] assumes that the total multipath length of the desired user is less than the delay of the interferer, which may not be true in a practical environment. Furthermore, the detector proposed in [75] utilizes the pilot-signaling channel to mask out the ISI caused by the desired user.

In this chapter, we propose a subspace-based direct-matrix-inversion (DMI) MMSE detector using the inter-symbol correlation matrix, which is capable of suppressing the interference including both MAI and ISI. In [18], the existing subspace-based DMI MMSE detector has been presented, together with the simulated and analytical SINR and BER performance. Our target is to reduce the computational complexity of the existing SB-MMSE detector. The proposed algorithm does not need to estimate the noise level to form the detector. At the same time, utilizing the property of the inter-symbol correlation matrix, we propose a new simplified rank determination algorithm. This algorithm calculates the ratios between two adjacent eigenvalues of the inter-symbol correlation matrix, and determines the rank by choosing the maximum of these ratios. As compared with the existing likelihood-based AIC and MDL algorithms that have complexities of $O(P^2)$, the proposed simplified rank determination algorithm has a lower computational complexity with $O(P)$. Therefore the proposed MMSE detector is much simpler to implement. In addition, the proposed algorithm does not require the assumption that the total multipath length of the desired user is less than the delay of the interferer in [74]. At the same time, the proposed algorithm does not require the pilot-signaling channel as in [75]. Thus the proposed algorithm turns out to be more feasible for practical implementation.

4.2 System Model

In this chapter, we consider an asynchronous DS-CDMA system over a multipath fading channel with K users and P chips per bit. Assuming that the total delay of each user is less than one bit, i.e., $\tau_{d_k} + \tau_m < T_b$. Therefore the received discrete-time (sampled in chip

rate) signal vector $\mathbf{r}(j)$ due to all users at the j th bit interval is given by (2.24), and is reproduced here as

$$\mathbf{r}(j) = A_1 \mathbf{h}_1(0) b_1(j) + A_1 \mathbf{h}_1(1) b_1(j-1) + \sum_{k=2}^K A_k \sum_{l=0}^1 \mathbf{h}_k(l) b_k(j-l) + \mathbf{n}(j). \quad (4.1)$$

We assume that the order of channel response vector q_g is known to us and $q_g \ll P$.

Therefore, we can obtain the reduced-order matrix of $\mathbf{C}_k$. Based on (2.23), $\mathbf{h}_k(0)$ and $\mathbf{h}_k(1)$ may be represented as

$$\mathbf{h}_k(0) = \bar{\mathbf{C}}_k^0 \bar{\mathbf{g}}_k \quad \text{and} \quad \mathbf{h}_k(1) = \bar{\mathbf{C}}_k^1 \bar{\mathbf{g}}_k \quad (4.2)$$

where $\bar{\mathbf{g}}_k = [g_k(0) \ \cdots \ g_k(q_g)]^T$ according to (2.30),

$$\bar{\mathbf{C}}_k^0 = \begin{bmatrix} c_k(0) & 0 & \cdots & 0 \\ c_k(1) & c_k(0) & \ddots & \vdots \\ \vdots & c_k(1) & \ddots & 0 \\ \vdots & \vdots & \ddots & c_k(0) \\ \vdots & \vdots & & \vdots \\ c_k(P-1) & c_k(P-2) & \cdots & c_k(P-q_g-1) \end{bmatrix}, \text{ and } \bar{\mathbf{C}}_k^1 = \begin{bmatrix} 0 & c_k(P-1) & \cdots & c_k(P-q_g) \\ & & \ddots & \vdots \\ \vdots & \ddots & & c_k(P-1) \\ & & \ddots & 0 \\ 0 & \cdots & & \vdots \end{bmatrix}.$$

Denoting

$$\boldsymbol{\theta}_r = [A_1 \mathbf{h}_1(0) \ \cdots \ A_K \mathbf{h}_K(0) \ A_1 \mathbf{h}_1(1) \ \cdots \ A_K \mathbf{h}_K(1)] \quad (4.3)$$

and

$$\mathbf{B}_r(j) = [b_1(j) \ \cdots \ b_K(j) \ b_1(j-1) \ \cdots \ b_K(j-1)]^T, \quad (4.4)$$

the received signal in (4.1) can be represented by

$$\mathbf{r}(j) = \boldsymbol{\theta}_r \mathbf{B}_r(j) + \mathbf{n}(j). \quad (4.5)$$

4.3 Proposed ISSB-MMSE Algorithm

In this section, we will first construct a correlation matrix based on two adjacent received bits to estimate the noise subspace, and then derive the proposed MMSE detector.

4.3.1 Inter-Symbol Correlation Matrix

Since $\mathbf{n}(j)$ is an AWGN sequence, two adjacent bits of the noise vector are uncorrelated. Therefore theoretically, we have

$$E\{\mathbf{n}(j)\mathbf{n}^H(j)\} = \sigma_n^2 \mathbf{I}_p \quad (4.6)$$

and

$$E\{\mathbf{n}(j)\mathbf{n}^H(j+1)\} = \mathbf{0}. \quad (4.7)$$

Based on (4.5)-(4.7), the covariance matrix of $\mathbf{r}(j)$ is given by

$$\mathbf{R}_r = E\{\mathbf{r}(j)\mathbf{r}^H(j)\} = \boldsymbol{\theta}_r \boldsymbol{\theta}_r^H + \sigma_n^2 \mathbf{I}_p \quad (4.8)$$

and the cross-correlation matrix between $\mathbf{r}(j)$ and $\mathbf{r}(j+1)$ is given by

$$\bar{\mathbf{R}}_r = E\{\mathbf{r}(j)\mathbf{r}(j+1)^H\} + E\{\mathbf{r}(j+1)\mathbf{r}(j)^H\} = \boldsymbol{\theta}_r \begin{bmatrix} 0 & \mathbf{I}_K \\ \mathbf{I}_K & 0 \end{bmatrix} \boldsymbol{\theta}_r^H. \quad (4.9)$$

From (4.8), it can be found that the covariance matrix $\mathbf{R}_r$ contains the signal component as well as the noise component. On the other hand, (4.9) shows that theoretically, the cross-correlation matrix $\bar{\mathbf{R}}_r$ between two adjacent symbols has no noise component, which means $\bar{\mathbf{R}}_r$ can only span the signal subspace. By applying subspace decomposition on $\bar{\mathbf{R}}_r$ and $\mathbf{R}_r$, respectively, we obtain

$$\bar{\mathbf{R}}_r = \begin{bmatrix} \mathbf{U}_{\bar{\mathbf{R}}_r}^s & \mathbf{U}_{\bar{\mathbf{R}}_r}^n \end{bmatrix} \begin{bmatrix} \Lambda_{\bar{\mathbf{R}}_r}^s & \\ & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{\bar{\mathbf{R}}_r}^s & \mathbf{U}_{\bar{\mathbf{R}}_r}^n \end{bmatrix}^H = \mathbf{U}_{\bar{\mathbf{R}}_r}^s \Lambda_{\bar{\mathbf{R}}_r}^s \mathbf{U}_{\bar{\mathbf{R}}_r}^{sH} \quad (4.10)$$

and

$$\mathbf{R}_r = \begin{bmatrix} \mathbf{U}_{R_r}^s & \mathbf{U}_{R_r}^n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_{R_r}^s & \\ & \mathbf{\Lambda}_{R_r}^n \end{bmatrix} \begin{bmatrix} \mathbf{U}_{R_r}^s & \mathbf{U}_{R_r}^n \end{bmatrix}^H = \mathbf{U}_{R_r}^s \mathbf{\Lambda}_{R_r}^s \mathbf{U}_{R_r}^{sH} + \mathbf{U}_{R_r}^n \mathbf{\Lambda}_{R_r}^n \mathbf{U}_{R_r}^{nH}. \quad (4.11)$$

In [74], it has been shown that the null space of $\mathbf{U}_{R_r}^s$ is equal to the null space of $\mathbf{U}_{R_r}^n$,

i.e.,

$$\text{null}(\mathbf{U}_{R_r}^s) = \text{null}(\mathbf{U}_{R_r}^n). \quad (4.12)$$

Therefore, the projection matrix onto the noise subspace of the received signal $\mathbf{r}(j)$ can be obtained by

$$\mathbf{S}^n = \mathbf{U}_{R_r}^n \mathbf{U}_{R_r}^{nH} = \mathbf{I}_P - \mathbf{U}_{R_r}^s \mathbf{U}_{R_r}^{sH}. \quad (4.13)$$

4.3.2 Proposed Blind ISSB-MMSE Detector

After estimating the noise subspace, the channel response $\bar{\mathbf{g}}_k$ and effective signature sequence $\mathbf{h}_k(l)$ can be obtained. Since $\mathbf{U}_{R_r}^n$ is orthogonal to $\mathbf{U}_{R_r}^s$, it is also orthogonal to $\boldsymbol{\theta}_r$. According to (4.12) and (4.13), and utilizing the subspace method [57], we obtain

$$\bar{\mathbf{g}}_k = \arg \min_{\|\mathbf{x}\|=1} \left\| \mathbf{U}_{R_r}^{nH} (\bar{\mathbf{C}}_k^0 \mathbf{x}) \right\|^2 = \arg \min_{\|\mathbf{x}\|=1} \mathbf{x}^H (\bar{\mathbf{C}}_k^{0H} \mathbf{S}^n \bar{\mathbf{C}}_k^0) \mathbf{x} \quad (4.14)$$

which means that $\bar{\mathbf{g}}_k$ can be obtained by the eigenvector corresponding to the smallest eigenvalue of $(\bar{\mathbf{C}}_k^{0H} \mathbf{S}^n \bar{\mathbf{C}}_k^0)$. Following that, $\mathbf{h}_k(0)$ and $\mathbf{h}_k(1)$ can be calculated by (4.2).

To detect the desired user's signal $b_1(j)$, we first denote

$$\bar{\mathbf{r}}(j+1) = \begin{bmatrix} r(jP) \\ \vdots \\ r(jP + q_g - 1) \end{bmatrix}. \quad (4.15)$$

Because $q_{g_k} \ll P$, we stack up the two vectors $\mathbf{r}(j)$ and $\bar{\mathbf{r}}(j+1)$ together to form $\bar{\mathbf{q}}(j)$, which is given by

$$\bar{\mathbf{q}}(j) = \begin{pmatrix} \mathbf{r}(j) \\ \bar{\mathbf{r}}(j+1) \end{pmatrix}. \quad (4.16)$$

Therefore, the covariance matrix of $\bar{\mathbf{q}}(j)$ is given by

$$\mathbf{R}_{\bar{\mathbf{q}}} = E \{ \bar{\mathbf{q}}(j) \bar{\mathbf{q}}^H(j) \} \quad (4.17)$$

and can be estimated by

$$\hat{\mathbf{R}}_{\bar{\mathbf{q}}} = \frac{1}{J} \sum_{j=1}^J \bar{\mathbf{q}}(j) \bar{\mathbf{q}}^H(j). \quad (4.18)$$

The purpose of the proposed linear inter-symbol subspace-based (ISSB) MMSE detector $\mathbf{m}_{\text{IS}}$ is to minimize the mean-square error (MSE), defined as

$$\text{MSE} = E \left\{ \left\| A_1 b_1(j) - \mathbf{m}_{\text{IS}}^H \bar{\mathbf{q}}(j) \right\|^2 \right\}. \quad (4.19)$$

After some deductions, and based on the well-known Wiener solution [66], the ideal proposed DMI MMSE detector has the form

$$\mathbf{m}_{\text{IS}} = \mathbf{R}_{\bar{\mathbf{q}}}^{-1} \bar{\mathbf{H}}_{1,0} \quad (4.20)$$

where

$$\bar{\mathbf{H}}_{1,0} = \begin{pmatrix} \mathbf{h}_1(0) \\ \bar{\mathbf{h}}_1(1) \end{pmatrix}, \text{ and } \bar{\mathbf{h}}_1(1) = \begin{bmatrix} h_1(P) \\ \vdots \\ h_1(P + q_g - 1) \end{bmatrix}. \quad (4.21)$$

Denoting $\hat{\mathbf{m}}_{\text{IS}}$ and $\hat{\bar{\mathbf{H}}}_{1,0}$ as the estimation of $\mathbf{m}_{\text{IS}}$ and $\bar{\mathbf{H}}_{1,0}$, respectively, the proposed detector can be realized by

$$\hat{\mathbf{m}}_{\text{IS}} = \hat{\mathbf{R}}_{\bar{\mathbf{q}}}^{-1} \hat{\bar{\mathbf{H}}}_{1,0}. \quad (4.22)$$

Therefore, we have proposed a subspace-based DMI MMSE detector using the inter-symbol correlation matrix, which does not need to estimate the noise level. Since there will be a phase ambiguity when performing the channel estimation based on (4.14), the differential demodulation is adopted. Theoretically, this proposed detector also does not need to estimate the rank of the signal subspace because $\bar{\mathbf{R}}_r$ in (4.10) does not contain any noise component. In practice, however, the noise component may not be strictly zero due to the finite number of received signal samples. Therefore, we still need to estimate the rank of this matrix. In the followings, we will propose a simplified rank determination algorithm by taking advantage of the property of the inter-symbol correlation matrix.

4.4 Rank Determination

In this section, we will propose a simplified empirical rank determination algorithm to separate the signal and the noise subspaces from the inter-symbol correlation matrix. We denote the estimation of $\bar{\mathbf{R}}_r$ as $\hat{\mathbf{R}}_r$, which can be expressed as

$$\hat{\mathbf{R}}_r = \frac{1}{J} \sum_{j=1}^J [\mathbf{r}(j)\mathbf{r}(j+1)^H + \mathbf{r}(j+1)\mathbf{r}(j)^H]. \quad (4.23)$$

Due to the finite received signal samples, the eigenvalue of $\hat{\mathbf{R}}_r$ corresponding to the noise subspace is not strictly zero. However, it can be found that the gap between the smallest eigenvalue of signal subspace and the largest eigenvalue of noise subspace of $\hat{\mathbf{R}}_r$ is much larger than the gaps between other adjacent eigenvalues. Denoting the ratio

$$s(i) = \frac{\lambda(i)}{\lambda(i+1)} \quad (4.24)$$

where $\lambda(i)$ is the i th eigenvalue, the dimension of signal subspace can be determined by the index that corresponds to the largest $s(i)$, i.e.,

$$\hat{d} = \arg \max_i s(i). \quad (4.25)$$

The advantage of this method is that its computational complexity is much lower than that of the traditional AIC or MDL criterion in [70]. Specifically, the AIC and MDL algorithms have computational complexities of $O(P^2)$, whereas our proposed simplified method only has a complexity of $O(P)$, according to (4.24) and (4.25).

It is worthwhile to mention that our proposed simplified rank determination algorithm is based on the fact that theoretically the eigenvalues of noise subspace of $\bar{\mathbf{R}}_r$ is zeros, resulting in the assumption that the estimated eigenvalues corresponding to the noise subspace of $\hat{\mathbf{R}}_r$ is quite small. This assumption can be satisfied by averaging the received signals using a comparatively large estimated sample size, and the numerical results in the next section will show that our proposed rank determination algorithm is accurate and reliable under this situation. On the other hand, if the number of estimated sample size is small, due to the finite sample effect, the above assumption may not be satisfied. In this case, the estimated eigenvalues of noise subspace may be much larger than zeros, and thus the difference of each gap becomes smaller, resulting in performance degradation of the proposed rank determination algorithm. In the next section, numerical results will be presented to demonstrate the performance of our proposed rank determination algorithm under different situations. Note that since we focus on the batch algorithm and the channel undergoes slow time-varying fading in this chapter, it is reasonable to require a comparatively large sample size.

4.5 Numerical Results and Discussions

In this section, simulated SINR and BER results will be presented to illustrate the performance of the proposed ISSB-MMSE detector over a multipath fading channel. In the following simulations, it is assumed that Gold sequences of length 31 are used as the PN sequences and there are five users in the system. It is assumed that the number of multipath is 5, the channel response $\bar{\mathbf{g}}_k$ of each user is generated according to the complex Gaussian distribution with zero mean, and the delay of each user is also randomly chosen to be within 0-10 chips. The SNR is defined as $\text{SNR} = A_1^2 / \sigma_n^2$. The estimated sample size J is chosen to be 500 and 5000, respectively, in order to present the performance of proposed algorithm under different situations. All of the results are obtained by averaging over 100 independent simulations. Since the number of active users is 5, the accurate rank of the signal subspace of the inter-symbol correlation matrix $\bar{\mathbf{R}}_r$ and covariance matrix $\mathbf{R}_r$ is 10.

In Figs. 4.1 and 4.2, the rank estimation performance of the proposed simplified rank determination algorithm based on the inter-symbol correlation matrix $\bar{\mathbf{R}}_r$ are presented under the equal-power situation and near-far situation, respectively. For the near-far situation, we assume that the power level of each interferer is 10dB higher than that of the desired user. For comparison purposes, we also include the performance of the proposed rank determination algorithm and the AIC algorithm [70] based on the covariance matrix $\mathbf{R}_r$. From these two figures, we observe that if the estimated sample size J is 5000, the proposed rank determination algorithm based on the matrix $\bar{\mathbf{R}}_r$ is more accurate as compared with the proposed rank determination algorithm based on the

matrix $\mathbf{R}_r$. When the SNR is larger than 10dB, the proposed rank determination algorithm on $\bar{\mathbf{R}}_r$ can accurately determine the rank of the signal subspace. From Fig. 4.2, we can observe that the AIC algorithm with $J=5000$ can estimate the rank more accurately than our proposed simplified rank determination algorithm with $J=5000$ at low SNR. It is because that even if we choose a comparatively large estimated sample size, the estimated eigenvalues of the noise subspace of $\hat{\mathbf{R}}_r$ can not be exactly zeros. Therefore, at low SNR, the gap between the smallest eigenvalues of signal subspace of $\hat{\mathbf{R}}_r$ and the largest eigenvalue of noise subspace of $\hat{\mathbf{R}}_r$ becomes smaller as compared with that at high SNR, and thus it is difficult for our proposed algorithm to accurately determine the rank. From Figs. 4.1 and 4.2, if the estimated sample sizes J is 500, we can observe that the accuracy of the proposed rank determination algorithm is reduced. The reason is that the estimated eigenvalues of noise subspace of $\hat{\mathbf{R}}_r$ may be much larger than zeros, as explained at the end of Section 4.4. In these two figures, we observe that the performance of the proposed rank determination under near-far situation is slightly better than that under equal-power situation. It is because that the eigenvalues of signal subspace of $\hat{\mathbf{R}}_r$ under near-far situation is larger than those under equal-power situation, resulting in larger gap between the smallest eigenvalue of signal subspace and the largest eigenvalue of noise subspace.

Next, we compare the SINR and BER performance between the proposed ISSB-MMSE MUD and the existing subspace-based DMI MMSE MUD in [18]. Here we apply the proposed simplified rank determination algorithm on the ISSB-MMSE detector, whereas the AIC algorithm is employed for the SB-MMSE detector. The SINR and BER results under the equal-power situation are presented in Figs. 4.3 and 4.4, respectively. In

Figs. 4.5 and 4.6, the SINR and BER results under a near-far situation are plotted, where the power level of each interferer is 10dB higher than that of the desired user. From these four figures, we observe that if J is 5000, the SINR and BER results of the proposed ISSB-MMSE detector are only slightly worse than those of the SB-MMSE detector. It is worthwhile to notice that there are small performance gaps between the proposed detector and the existing SB-MMSE detector even when SNR is comparatively large. According to Figs. 4.1 and 4.2, we have observed that our proposed rank determination algorithm on $\bar{\mathbf{R}}_r$ with $J=5000$ can accurately determine the rank when SNR increases. Therefore, the cause of this kind of performance drop is due to the inaccurate channel estimation using the null space of $\mathbf{U}_{\bar{\mathbf{R}}_r}^s$. Theoretically, as explained before, the covariance matrix of AWGN vector is an identity matrix with variance σ_n^2 , and the cross-correlation matrix of AWGN between two adjacent symbol is zero, as shown in (4.8) and (4.9), respectively. Based on this fact, it has been proven in [74] that the null space of $\mathbf{U}_{\mathbf{R}_r}^s$ is equal to the null space of $\mathbf{U}_{\bar{\mathbf{R}}_r}^s$. In practice, however, the noise components due to the finite number of received samples will affect the covariance matrix as well as cross-correlation matrix. In turn, these imperfect covariance matrix and cross-correlation matrix will lead to inaccurate null space of $\mathbf{U}_{\mathbf{R}_r}^s$ and $\mathbf{U}_{\bar{\mathbf{R}}_r}^s$, respectively. As a consequence, the subsequent channel estimation using $\mathbf{U}_{\mathbf{R}_r}^s$ and $\mathbf{U}_{\bar{\mathbf{R}}_r}^s$ will also be affected. Based on the SINR and BER performance corresponding to these two types of detector in Figs. 4.3-4.6, we observe that the robustness of channel estimation using the null space of $\mathbf{U}_{\mathbf{R}_r}^s$ is better than that using the null space of $\mathbf{U}_{\bar{\mathbf{R}}_r}^s$, when both of them suffer from the effect of finite number of noise samples. However, we can also observe

that the performance drop is acceptable when we choose a comparatively large sample size of received signals. In Figs. 4.3-4.6, if the estimated sample size J is 500, we can observe that there are obvious performance gaps between the proposed ISSB-MMSE detector and the existing SB-MMSE detector in terms of SINR and BER, due to the decrease in accuracy of rank determination and channel estimation. Therefore, we can conclude that our proposed batch ISSB-MMSE detector is suitable for situation that the estimated sample size is comparatively large, which can be satisfied in the environment that undergoes slow time-varying fading.

In Figs. 4.7 and 4.8, we compare the SINR and BER results of the proposed ISSB-MMSE detector and the SB-MMSE detector with $J=5000$, under the same near-far situation as that in Figs. 4.5 and 4.6. Both of these two detectors adopt the proposed rank determination algorithm. From these two figures, it can be observed that due to the inaccuracy of the rank estimation, the SINR and BER results of the SB-MMSE detector adopting the proposed rank determination algorithm are worse than those of the proposed ISSB-MMSE detector. Therefore, it can be concluded that it is necessary to utilize the inter-symbol correlation matrix rather than the covariance matrix, if we want to adopt the proposed simplified rank determination algorithm.

From the above numerical results, we can conclude that our proposed ISSB-MMSE detector has lower computational complexity because of the lower complexity of proposed rank determination algorithm. Since we mainly focus on the slow time-varying fading channel, we can choose a large estimated sample size, and thus the performance of the proposed ISSB-MMSE detector is only slightly degraded as compared with the existing SB-MMSE detector. It could be an interesting topic to further improve this algorithm to make it suitable for the situation with small estimated sample size.

Fig. 4.1 Rank estimation performance of the proposed simplified rank estimation method based on inter-symbol (IS) correlation matrix $\bar{\mathbf{R}}_r$ and covariance matrix $\mathbf{R}_r$ respectively, under the equal-power situation with $K=5$.

Fig. 4.2 Rank estimation performance of the proposed simplified rank estimation method based on inter-symbol (IS) correlation matrix $\bar{\mathbf{R}}_r$ and covariance matrix $\mathbf{R}_r$ respectively, under the near-far situation with $K=5$.

Fig. 4.3 Comparisons of SINR between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the equal-power situation with $K=5$.

Fig. 4.4 Comparisons of BER between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the equal-power situation with $K=5$.

Fig. 4.5 Comparisons of SINR between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the near-far situation with $K=5$.

Fig. 4.6 Comparisons of BER between the proposed ISSB-MMSE detector and the existing SB-MMSE detector under the near-far situation with $K=5$.

Fig. 4.7 Comparisons of SINR between the proposed ISSB-MMSE detector and the existing SB-MMSE detector using the proposed rank estimation algorithm under the near-far situation.

Fig. 4.8 Comparisons of BER between the proposed ISSB-MMSE detector and the existing SB-MMSE detector using the proposed rank estimation algorithm under the near-far situation.

4.6 Conclusion

In this chapter, a blind batch DMI MMSE multiuser detector utilizing the inter-symbol correlation matrix has been proposed over a multipath fading channel. This detector is based on the fact that two different AWGN vectors are theoretically uncorrelated. In addition, we have also proposed a simplified rank determination algorithm that has lower computational complexity as compared with the existing AIC and MDL algorithms. Simulation results have shown that the performance of the proposed ISSB-MMSE detector is quite close to that of the existing SB-MMSE detector, when the estimated sample size is large. Hence this proposed batch detector has low computational complexity, and is feasible over a slow time-varying fading channel.

Chapter 5

Asynchronous Blind Adaptive Kalman Filter-Based Multiuser Detector

5.1 Introduction

In the previous two chapters, we have proposed two blind batch MUD algorithms for DS-CDMA systems over multipath fading channels. Simulation results have shown that they can effectively mitigate the interference including MAI and ISI. For some practical CDMA systems, however, the parameters of the received signals may vary with time. Under this condition, the batch algorithm may not be able to effectively detect the desired user's information. Therefore, the adaptive MUD algorithm is required since it can update its parameters adaptively based on the received signals.

The adaptive MOE based MUD algorithm was first proposed in [10] and [11] over an AWGN channel and was extended to a multipath fading channel in [58], [62]-[64], including the LMS and RLS detectors. This kind of adaptive MUD is computationally efficient. However, the main drawback is that the rate of convergence is

relatively slow. Recently, the Kalman filter-based method has been paid more and more attention. The Kalman filter is known to be a linear minimum variance state estimator, which also satisfies the MMSE criterion. A distinctive feature of the Kalman filter is that its mathematical formulation is described in terms of state-space concepts. Another novel feature of the Kalman filter is that it is computed recursively and can be applied without modification to both stationary and dynamic environments [66]. The blind adaptive Kalman filter-based MUD was first proposed in [15], and has been shown to have rapid convergence rate and good tracking capability as compared with the LMS and RLS algorithms.

However, the Kalman filter-based MUD in [15] mainly focuses on mitigating the MAI over an AWGN channel. For a practical multipath fading channel, this detector may not have the capability to eliminate the ISI. Therefore, it is necessary and worthwhile to extend the Kalman filter-based MUD to a multipath fading channel. In this chapter, we will propose two adaptive MUD algorithms based on the Kalman filtering that can suppress MAI and ISI over a multipath fading channel, with different constraints. In Section 5.2, we will utilize the inverse filtering technique in [62] to form the constraint whereas in Section 5.3, we will apply the linear prediction algorithm in [58] to form the constraint.

5.2 Kalman Filter-Based MUD Using Inverse Filtering

In this section, we will propose an adaptive Kalman filter over a multipath fading channel. The inverse filtering criteria is adopted to form the constraint, and we need to create a canonical representation to satisfy this constraint. In the followings, we will introduce the system model first.

5.2.1 System Model

Here we consider an asynchronous DS-CDMA system over a multipath fading channel with K users and P chips per bit ($K < P$). The multipath fading channel is modeled by a TDL form as described in Chapter 2. For the proposed Kalman filter-based MUD using inverse filtering, we assume that the first path is the main path, which means that the amplitude of the first path is much higher than that of the other paths. Also, the timing information of this path is assumed to be available [62], which means that the delay of the desired user's first path is zero. Hence, the channel coefficient $g_1(0)$ is not zero and is much higher than $g_1(i)$, for $i = 1, \dots, q_g$. The sampled signal at the p th chip of the j th bit interval is given by (2.19) and is reproduced here as

$$r_p(j) = \sum_{k=1}^K A_k \sum_{l=0}^{L_k-1} h_{pk}(l) b_k(j-l) + n_p(j). \quad (5.1)$$

5.2.2 Proposed Kalman Filter-Based MUD

In order to recover $b_1(j)$ from the received signal $r_p(j)$, the interference from the other users needs to be suppressed. If we restrict ourselves to linear solutions, then the receiver's design problem is equivalent to determine a filter given by

$$\mathbf{m}_{opt} = [m(0) \quad m(1) \quad \dots \quad m(q_f)]^T \quad (5.2)$$

where q_f is the filter order. We define the vector

$$\tilde{\mathbf{r}}(j) = [r(jP + P - 1) \quad r(jP + P - 2) \quad \dots \quad r(jP + P - 1 - q_f)]^T \quad (5.3)$$

where $r(jP + p) = r_p(j)$, then the output of the detector is

$$\hat{d}_1(j) = \text{sgn} \left\{ \Re \left[\left(\mathbf{m}_{opt}^H \tilde{\mathbf{r}}(j) \right) \left(\mathbf{m}_{opt}^H \tilde{\mathbf{r}}(j-1) \right)^* \right] \right\}. \quad (5.4)$$

where the differential demodulation is adopted. The parameter $\mathbf{m}_{opt}$ of the receiver filter is typically designed by minimizing certain cost functions. Thus if we choose the MMSE criterion here, we have [62]

$$J_{\text{MSE}} = E \left\{ \left\| A_1 b_1(j) - \mathbf{m}_{opt}^H \tilde{\mathbf{r}}(j) \right\|^2 \right\}. \quad (5.5)$$

In practice, the estimated transmit signal $\hat{b}_1(j)$ is not available, and the only information available for the receiver is the desired user's PN sequence $c_1(p)$. Therefore, we use the MOE criterion instead of the MSE criterion to set up a cost function given by [62]

$$J_{\text{MOE}} = E \left\{ \left\| \mathbf{m}_{opt}^H \tilde{\mathbf{r}}(j) \right\|^2 \right\}. \quad (5.6)$$

For an adaptive algorithm, $\mathbf{m}(j)$ can be used to represent an adaptive process to achieve its optimal vector $\mathbf{m}_{opt}$ at the j th bit interval. It is clear from (5.6) that this cost function is minimized when $\mathbf{m}(j) = 0$, which is not the desired solution. In order to avoid this trivial solution, we need to set up a constraint for $\mathbf{m}(j)$.

In [15], a canonical representation was proposed based on the framework of the well-known generalized sidelobe canceller (GSC). By constraining the linear detector $\mathbf{m}(j)$ to present a unit response to the PN sequence of the first user, we have

$$\mathbf{m}(j) = \mathbf{c}_1 - \mathbf{S}_{\mathbf{c}_1}^n \mathbf{w}(j) \quad (5.7)$$

where $\mathbf{w}(j)$ is the adaptive part of $\mathbf{m}(j)$, and the columns of the matrix $\mathbf{S}_{\mathbf{c}_1}^n$ span the null space of $\mathbf{c}_1$. Hence it can be shown that this representation satisfies the following constraint

$$\mathbf{m}^T(j)\mathbf{c}_1 = 1. \quad (5.8)$$

This canonical representation is based on an AWGN channel. In a multipath fading channel, however, the above constraint is not meaningful and will not provide the desired results because of the presence of ISI. In order to derive an appropriate constraint for the multipath fading channel, we need to modify the existing constraint proposed in [15]. Here we adopt the inverse filtering criteria proposed in [62] to set up a new canonical representation of $\mathbf{m}(j)$ to satisfy the new constraint.

We denote by $v_k(i)$ as the convolution between the PN sequence $\mathbf{c}_k$ and the receiver filter $\mathbf{m}$, we have

$$v_k(i) = \sum_{p=0}^{q_f} c_k(i-p)m(p). \quad (5.9)$$

We may set up the following constraint [62]

$$v_1(jP + P - 1 - n) = \beta \delta(j - j_0) \delta(n) \quad j = 0, \dots, \tilde{L} \quad n = 0, \dots, q_g \quad (5.10)$$

where $\tilde{L} = \lfloor (q_f + q_g) / P \rfloor$, β is a nonzero scalar, $\delta(n)$ is the delta function, and j_0 is the bit delay for detection [62]. Defining

$$\mathbf{v}_1 = \left\{ v_1(P-1) \quad \dots \quad v_1(P-1-q_g) \quad v_1(2P-1) \quad \dots \quad v_1(2P-1-q_g) \quad \dots \quad v_1((\tilde{L}+1)P-1) \quad \dots \quad v_1((\tilde{L}+1)P-1-q_g) \right\}^T, \quad (5.11)$$

the constraint in (5.10) can be written in a matrix form as

$$\mathbf{v}_1 = \tilde{\mathbf{C}}_1 \mathbf{m} = \beta \mathbf{e}_{j_0} \quad (5.12)$$

where $\mathbf{e}_{j_0} = [0 \ \cdots \ 0 \ 1 \ 0 \ \cdots]^T$ has an element one at the $j_0 + 1$ position and zero elsewhere. In addition, the matrix $\tilde{\mathbf{C}}_1$ with dimension of $(\tilde{L}+1)(q_g+1) \times (q_f+1)$ can be represented as

$$\tilde{\mathbf{C}}_1 = \begin{bmatrix} c_1(P-1) & \cdots & c_1(P-1-q_f) \\ \vdots & & \vdots \\ c_1(P-1-q_g) & \cdots & c_1(P-1-q_g-q_f) \\ \vdots & & \vdots \\ c_1[(\tilde{L}+1)P-1] & \cdots & c_1[(\tilde{L}+1)P-1-q_f] \\ \vdots & & \vdots \\ c_1[(\tilde{L}+1)P-1-q_g] & \cdots & c_1[(\tilde{L}+1)P-1-q_g-q_f] \end{bmatrix}. \quad (5.13)$$

Note that the matrix $\tilde{\mathbf{C}}_1$ is of full rank if $q_g < P$ [62]. Thus, from the above analysis, we can set up a new canonical representation over the multipath channel given by

$$\mathbf{m}(j) = \mathbf{m}(0) - \mathbf{S}_{\tilde{\mathbf{C}}_1}^n \mathbf{w}(j) \quad (5.14)$$

where $\mathbf{w}(j)$ is the adaptive part of $\mathbf{m}(j)$, and $\mathbf{m}(0) = \tilde{\mathbf{C}}_1^+ \mathbf{v}_1$ is the starting vector of $\mathbf{m}(j)$. Note that $\mathbf{S}_{\tilde{\mathbf{C}}_1}^n$ represents the projection matrix onto the null space of the matrix $\tilde{\mathbf{C}}_1$, which means that $\mathbf{S}_{\tilde{\mathbf{C}}_1}^n \tilde{\mathbf{C}}_1^T = \mathbf{0}$, and $\mathbf{S}_{\tilde{\mathbf{C}}_1}^n$ can be obtained by

$$\mathbf{S}_{\tilde{\mathbf{C}}_1}^n = \mathbf{I} - \mathbf{S}_{\tilde{\mathbf{C}}_1}^s \quad (5.15)$$

where $\mathbf{S}_{\tilde{\mathbf{C}}_1}^s = \tilde{\mathbf{C}}_1^T (\tilde{\mathbf{C}}_1 \tilde{\mathbf{C}}_1^T)^{-1} \tilde{\mathbf{C}}_1$. It can be shown that this new canonical representation in (5.14) is guaranteed to satisfy the constraint in (5.12). Indeed, $\mathbf{m}(j)$ always satisfies (5.12) since the second term in the right-hand side lies in the null space of $\tilde{\mathbf{C}}_1$, i.e.,

$$\tilde{\mathbf{C}}_1 \mathbf{m}(j) = \tilde{\mathbf{C}}_1 [\mathbf{m}(0) - \mathbf{S}_{\tilde{\mathbf{C}}_1}^n \mathbf{w}(j)] = \tilde{\mathbf{C}}_1 \mathbf{m}(0) - \mathbf{0} = \mathbf{v}_1. \quad (5.16)$$

After establishing the new canonical representation, the processing equation and the measurement equation of the Kalman filter algorithm can be obtained. For a slow time-

varying DS-CDMA system considered here, an important fact is that its optimum linear detector or tap-weight vector $\mathbf{m}_{opt}(j)$ is approximately time-invariant [15], i.e.,

$$\mathbf{m}_{opt}(j+1) \approx \mathbf{m}_{opt}(j). \quad (5.17)$$

Letting $\mathbf{w}_{opt}(j)$ be the adaptive part corresponding to $\mathbf{m}_{opt}(j)$, we can set up the processing equation for Kalman filtering given by

$$\mathbf{w}_{opt}(j+1) = \mathbf{w}_{opt}(j). \quad (5.18)$$

According to (5.6), defining $e(j) = \tilde{\mathbf{r}}^H(j)\mathbf{m}(j)$ and using (5.14), we obtain

$$e(j) = \tilde{\mathbf{r}}^H(j)\mathbf{m}(0) - \tilde{\mathbf{r}}^H(j)\mathbf{S}_{\tilde{\mathbf{C}}_1}^n \mathbf{w}(j). \quad (5.19)$$

Assuming that $\mathbf{w}$ approaches $\mathbf{w}_{opt}$, and defining

$$\tilde{y}(j) = \tilde{\mathbf{r}}^H(j)\mathbf{m}(0), \text{ and } \mathbf{d}^H(j) = \tilde{\mathbf{r}}^H(j)\mathbf{S}_{\tilde{\mathbf{C}}_1}^n, \quad (5.20)$$

equation (5.19) can be rewritten as the following measurement equation

$$\tilde{y}(j) = \mathbf{d}^H(j)\mathbf{w}_{opt}(j) + e_{opt}(j). \quad (5.21)$$

Thus, both the processing equation (5.18) and the measurement equation (5.21) construct the dynamical system model of the desired user, and they are the basis for the Kalman filtering algorithm. Following that, the standard Kalman filtering algorithm can be used to obtain the adaptive part of the receiver filter [15], [66]:

$$\tilde{\mathbf{g}}(j) = \mathbf{K}(j, j-1)\mathbf{d}(j) \left[\mathbf{d}^H(j)\mathbf{K}(j, j-1)\mathbf{d}(j) + \xi_{\min} \right]^{-1}, \quad (5.22)$$

$$\mathbf{K}(j+1, j) = \mathbf{K}(j, j-1) - \tilde{\mathbf{g}}(j)\mathbf{d}^H(j)\mathbf{K}(j, j-1), \quad (5.23)$$

and

$$\hat{\mathbf{w}}_{opt}(j) = \hat{\mathbf{w}}_{opt}(j-1) + \tilde{\mathbf{g}}(j) \left[\tilde{y}(j) - \mathbf{d}^H(j)\hat{\mathbf{w}}_{opt}(j-1) \right]. \quad (5.24)$$

Note that $\hat{\mathbf{w}}_{opt}(j)$ is the estimation of $\mathbf{w}_{opt}(j)$, $\tilde{\mathbf{g}}(j)$ and $\mathbf{K}(j+1, j)$ are the Kalman gain and the correlation matrix of the error, respectively. For initialization, we set $\mathbf{K}(1, 0) = \mathbf{I}$. As shown in [15], $\xi_{\min} = \text{var}\{e_{opt}(j)\} \approx A_1^2 + \text{MSE}\{\mathbf{m}(j)\}$, and $\text{MSE}\{\mathbf{m}(j)\}$ is much smaller than A_1^2 , thus $\xi_{\min} \approx A_1^2$. Finally, the receiver $\mathbf{m}(j)$ is updated according to (5.14).

5.3 Kalman Filter-Based MUD Using Linear Prediction

In Section 5.2, we have proposed a blind MUD algorithm based on the Kalman filter, which can eliminate the MAI and ISI over a multipath fading channel. However it assumes that the first path of the desired user is the main path, and requires the timing information of this path. Therefore, this algorithm cannot work well if these assumptions are not satisfied. In this section, we will set up a new canonical representation based on the linear prediction algorithm proposed in [58] and develop a blind Kalman filtering algorithm that does not need these assumptions. In this algorithm the channel estimation and the detector vector can be updated at the same time in every iteration, thus it can approach its convergence very fast and it is not necessary to use a large number of symbols to obtain an accurate estimate of the channel impulse response offline.

5.3.1 System Model

We consider an asynchronous DS-CDMA system over a multipath fading channel with K active users and P chips per bit. It is assumed that the total delay is less than one bit.

Therefore the received discrete-time (sampled in chip rate) signal vector $\mathbf{q}(j)$ due to all users at the j th and $(j+1)$ th bit intervals is given by (3.3), and is reproduced here as

$$\begin{aligned} \mathbf{q}(j) &= \begin{pmatrix} \mathbf{r}(j) \\ \mathbf{r}(j+1) \end{pmatrix} \\ &= A_1 \mathbf{H}_{1,0} b_1(j) + A_1 \bar{\mathbf{H}}_1 \bar{\mathbf{B}}_1(j) + \sum_{k=2}^K A_k \mathbf{H}_k \mathbf{B}_k(j) + \mathbf{N}(j). \end{aligned} \quad (5.25)$$

5.3.2 New Canonical Representation

As mentioned before, the canonical representation in (5.7) does not satisfy the requirement of MUD over a multipath fading channel. Comparing the system model given by (2.13) and (5.25), which correspond to the AWGN channel and multipath fading channel, respectively, we can conclude that the new canonical representation has the form of

$$\mathbf{m}(j) = \mathbf{H}_{1,0} - \mathbf{S}_{\mathbf{H}_{1,0}}^n \mathbf{w}(j) \quad (5.26)$$

where the columns of $\mathbf{S}_{\mathbf{H}_{1,0}}^n$ represent the projection matrix onto the null space of $\mathbf{H}_{1,0}$.

This new canonical representation satisfies the following constraint,

$$\mathbf{m}^H(j) \mathbf{H}_{1,0} = \mathbf{H}_{1,0}^H \mathbf{H}_{1,0}. \quad (5.27)$$

Since the channel response of each user is normalized to have unit norm as mentioned in Chapter 2, (5.27) becomes

$$\mathbf{m}^H(j) \mathbf{H}_{1,0} \approx 1. \quad (5.28)$$

From (5.26), we observe that it is necessary to estimate $\mathbf{H}_{1,0}$ and $\mathbf{S}_{\mathbf{H}_{1,0}}^n$ for the new canonical representation. Here, we resort to the linear prediction algorithm in [58] to

efficiently estimate $\mathbf{H}_{1,0}$. Denoting by $\mathbf{T}_1$ with dimension of $P \times 2P$, as the left singular vectors of $\mathbf{C}_1$ corresponding to its zero singular values [58], we have

$$\mathbf{T}_1 \mathbf{H}_{1,0} = \mathbf{T}_1 \mathbf{C}_1 \mathbf{g}_1 = 0 \quad (5.29)$$

where $\mathbf{C}_1$ and $\mathbf{g}_1$ have been defined in (2.23). Therefore it can be shown that $\mathbf{T}_1 \mathbf{q}(j)$ does not contain $b_1(j)$. Stacking up $\mathbf{r}(j-1)$, $\mathbf{q}(j)$, and $\mathbf{r}(j+2)$ together, we obtain a vector

$$\tilde{\mathbf{y}}(j) = \tilde{\mathbf{T}}_1 \begin{bmatrix} \mathbf{r}(j-1) \\ \mathbf{q}(j) \\ \mathbf{r}(j+2) \end{bmatrix} \quad (5.30)$$

where $\tilde{\mathbf{T}}_1$ is given by (3.12). It is possible to extract $b_1(j)$ from $\tilde{\mathbf{y}}(j)$ using linear prediction algorithm. Considering that $\mathbf{q}(j)$ contains $b_1(j)$, whereas $\tilde{\mathbf{y}}(j)$ does not, we define the following linear prediction problem

$$\boldsymbol{\varepsilon}(j) = \mathbf{q}(j) - \mathbf{M} \tilde{\mathbf{y}}(j) \quad (5.31)$$

where $\mathbf{M}$ is an unknown linear transformation matrix to be resolved. The above equation can also be written as

$$\boldsymbol{\varepsilon}(j) = \begin{bmatrix} \mathbf{I} & -\mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{q}(j) \\ \tilde{\mathbf{y}}(j) \end{bmatrix}. \quad (5.32)$$

We define the following covariance matrixes:

$$\mathbf{R}_{11} = E \{ \mathbf{q}(j) \mathbf{q}^H(j) \}, \quad (5.33)$$

$$\mathbf{R}_{12} = E \{ \mathbf{q}(j) \tilde{\mathbf{y}}^H(j) \}, \quad (5.34)$$

$$\mathbf{R}_{21} = E \{ \tilde{\mathbf{y}}(j) \mathbf{q}^H(j) \}, \quad (5.35)$$

and

$$\mathbf{R}_{22} = E\{\tilde{\mathbf{y}}(j)\tilde{\mathbf{y}}^H(j)\}. \quad (5.36)$$

It can be verified that (5.33)-(5.36) are equivalent to (3.8)-(3.11) for $k=1$. Ignoring the noise for the time being, the optimal linear prediction matrix $\mathbf{M}$ that minimizes $E\{\boldsymbol{\varepsilon}(j)\boldsymbol{\varepsilon}^H(j)\}$ is given by [58]

$$\mathbf{M} = \mathbf{R}_{12}\mathbf{R}_{22}^+. \quad (5.37)$$

The minimum $E\{\boldsymbol{\varepsilon}(j)\boldsymbol{\varepsilon}^H(j)\}$ becomes

$$E\{\boldsymbol{\varepsilon}(j)\boldsymbol{\varepsilon}^H(j)\} = A_1^2\mathbf{H}_{1,0}\mathbf{H}_{1,0}^H = \mathbf{R}_{11} - \mathbf{R}_{12}\mathbf{R}_{22}^+\mathbf{R}_{21} \quad (5.38)$$

and

$$\boldsymbol{\varepsilon}(j) = \mathbf{q}(j) - \mathbf{R}_{12}\mathbf{R}_{22}^+\tilde{\mathbf{y}}(j) = A_1\mathbf{H}_{1,0}b_1(j). \quad (5.39)$$

Note that in practice, $\mathbf{R}_{22}$ is invertible because of the presence of noise. From (5.38), it is clear that $A_1^2\mathbf{H}_{1,0}\mathbf{H}_{1,0}^H$ has a rank of 1. Let $\mathbf{u}$ be its normalized left singular vector corresponding to its non-zero singular value and $\|\mathbf{u}\|=1$, then $\mathbf{u}$ is an estimation of $\mathbf{H}_{1,0}$. Subsequently, a new canonical representation for the Kalman filter can be obtained according to (5.26) and is given by

$$\mathbf{m}(j) = \mathbf{u} - \mathbf{S}_u^n \mathbf{w}(j) \quad (5.40)$$

where $\mathbf{S}_u^n$ is the projection matrix onto the null space of $\mathbf{u}$, and $\mathbf{u}$ satisfies

$$\mathbf{m}^H(j)\mathbf{u} = \mathbf{u}^H\mathbf{u} = 1. \quad (5.41)$$

In this chapter, since we focus on the adaptive detector, it is necessary to estimate the effective sequence adaptively. In the followings, we will propose an adaptive algorithm to update $\mathbf{H}_{1,0}$. Firstly, $\mathbf{R}_{12}$ can be updated by using the following equation,

$$\mathbf{R}_{12}(j) = \gamma\mathbf{R}_{12}(j-1) + \mathbf{q}(j)\tilde{\mathbf{y}}^H(j). \quad (5.42)$$

Next, $\mathbf{R}_{22}^+$ can also be updated by using the matrix inversion lemma and recursive algorithms in [66]

$$\mathbf{R}_{22}^+(j) = \gamma^{-1} \mathbf{R}_{22}^+(j-1) - \gamma^{-1} t(j) \tilde{\mathbf{y}}^H(j) \mathbf{R}_{22}^+(j-1) \quad (5.43)$$

where

$$t(j) = \frac{\gamma^{-1} \mathbf{R}_{22}^+(j-1) \tilde{\mathbf{y}}(j)}{1 + \gamma^{-1} \tilde{\mathbf{y}}^H(j) \mathbf{R}_{22}^+(j-1) \tilde{\mathbf{y}}(j)}$$

where $0 < \gamma < 1$ is the forgetting factor. Subsequently, $\mathbf{M}$ can be calculated by (5.37). In order to reduce complexity, we resort to some subspace tracking algorithms to update the estimated effective sequence $\mathbf{u}$. Here we choose the PASTd algorithm proposed in [20]. Since $\mathbf{u}$ is the eigenvector corresponding to the largest eigenvalue of $E\{\boldsymbol{\varepsilon}(j)\boldsymbol{\varepsilon}^H(j)\}$, we can utilize the one-step PASTd algorithm for updating $\mathbf{u}$. Table 5.1 summarizes the one-step PASTd algorithm.

Updating the largest eigenvalue and the corresponding eigenvectors of signal subspace $\{\lambda, \mathbf{u}\}$	
FOR	$j=1:J$
	$\mathbf{x}(j) = \boldsymbol{\varepsilon}(j)$
	$z(j) = \mathbf{u}^H(j-1)\mathbf{x}(j)$
	$\lambda(j) = \gamma\lambda(j-1) + \ z(j)\ ^2$
	$\mathbf{u}(j) = \mathbf{u}(j-1) + [\mathbf{x}(j) - \mathbf{u}(j-1)z(j)]z^H(j)/\lambda(j)$
END	

Table 5.1 One-step PASTd algorithm for updating $\mathbf{u}$.

5.3.3 Proposed Kalman Filter-Based MUD

As mentioned in Section 5.2, for a slow time-varying CDMA system, an important fact is that its optimum linear detector vector $\mathbf{m}(j)$ is approximately time-invariant, namely $\mathbf{m}_{opt}(j+1) \approx \mathbf{m}_{opt}(j)$. Let $\mathbf{w}_{opt}(j)$ be the adaptive part corresponding to $\mathbf{m}_{opt}(j)$ in (5.40), and we can set up the processing equation for Kalman filtering given by

$$\mathbf{w}_{opt}(j+1) = \mathbf{w}_{opt}(j). \quad (5.44)$$

Defining $e(j) = \mathbf{q}^H(j)\mathbf{m}(j)$ and substituting (5.40) into $e(j)$, we can obtain

$$e(j) = \mathbf{q}^H(j)\mathbf{u} - \mathbf{q}^H(j)\mathbf{S}_u^n \mathbf{w}(j). \quad (5.45)$$

Assuming that $\mathbf{w}$ approaches $\mathbf{w}_{opt}$, and defining

$$\tilde{y}(j) = \mathbf{q}^H(j)\mathbf{u}, \text{ and } \mathbf{d}^H(j) = \mathbf{q}^H(j)\mathbf{S}_u^n, \quad (5.46)$$

equation (5.45) can be rewritten as the following measurement equation

$$\tilde{y}(j) = \mathbf{d}^H(j)\mathbf{w}_{opt}(j) + e_{opt}(j). \quad (5.47)$$

After obtaining both the processing equation (5.44) and the measurement equation (5.47), the standard Kalman filtering algorithm can be adopted to update the adaptive part $\mathbf{w}(j)$ of the receiver filter following (5.22)-(5.24), and $\mathbf{m}(j)$ can be updated according to (5.40). The complete algorithm is summarized as follows:

- 1). Update $\mathbf{w}(j)$ and $\mathbf{m}(j)$ by using the standard Kalman filtering algorithm following (5.22)-(5.24) and (5.40).
- 2). At the same time, the estimated effective sequence $\mathbf{u}$ is updated recursively based on (5.39) and Table 5.1. This process can last for about 500 symbols, after that, we can

obtain a nearly perfect estimation of $\mathbf{u}$, and there is no need to repeat this process any more.

The choice of using 500 symbols as the estimated sample size in estimating $\mathbf{u}$ is for reducing the computational complexity, and obtaining a fairly accurate $\mathbf{u}$. If we increase this estimated sample size, the estimation of $\mathbf{u}$ will be more accurate since the effect of finite sample size is reduced. However, the complexity will increase.

Note that the performance improvement of the proposed Kalman filter based detector is achieved at the expense of higher complexity. In [15], it has been shown that the Kalman filter algorithm has a computational complexity of $O(P^2)$ per iteration, and that of the adaptive LMS algorithm is $O(P)$. In addition, although there is an inverse matrix operation in (5.22), the term in the square bracket is a scalar, so there is no need to perform any matrix inversion.

5.4 Numerical Results and Discussions

In this section, several simulation results will be provided to illustrate the performance of these two proposed Kalman filter-based MUD algorithms. Gold sequences of length 31 are used as the PN sequences and the differential demodulation is adopted. Firstly, we will present the Kalman filter-based MUD using inverse filtering. The performance of the corresponding existing LMS MUD using inverse filtering proposed in [62] is also presented for comparison. It is assumed that the bit delay for detection is 0, and the channel gain of each user is generated according to a fixed complex Gaussian distribution with zero mean. It is assumed that the number of multipath is 5 and the first path is the dominant path, and its fading coefficient is 10 times of that of the other paths.

The delay of each user is randomly chosen to be within 0-10 chips. We assume that the timing information of the first path of the desired user is available, and thus the delay of the desired user's first path is zero. The SNR is defined according to [14], i.e., $\text{SNR} = A_1^2 / \sigma_n^2$, and the step-size of the LMS algorithm is set to be 0.0003 [15].

Firstly, we compare the SINR and BER results of the proposed Kalman filter-based MUD with that of the LMS MUD under an equal-power situation with $K=5$. The simulation results are obtained after 5000 iterations when both algorithms have approached their convergence respectively. The SINR and BER results are presented in Figs. 5.1 and 5.2, respectively. From Fig. 5.1, we can observe that the SINR of the proposed Kalman filter-based MUD is higher than that of the LMS MUD, especially when SNR is high. From Fig. 5.2, it can be found that the BER of the proposed Kalman filter-based MUD is lower than that of the LMS MUD, especially when SNR is high. At SNR=13dB, the BER of the LMS MUD is about 10^{-3} , whereas the BER of the proposed Kalman filter-based MUD is only slightly above 10^{-4} . Therefore it can be concluded that the SINR and BER results of the proposed Kalman filter-based MUD using inverse filtering are better than those of the existing LMS MUD under the equal-power situation.

Next, we compare the SINR performance of the proposed Kalman filter-based MUD with that of the existing LMS MUD under a near-far situation. It is assumed that there are five active users and the power level of each interferer is 10dB higher than that of the desired user. In Figs. 5.3 and 5.4, the SINR and BER results are presented, respectively. Again, we can observe that both of the SINR and BER performance of the proposed Kalman filter-based MUD is better than that of the existing LMS MUD. Therefore, we can conclude that the proposed Kalman filter-based MUD outperforms the existing LMS MUD under a near-far situation. In Figs. 5.1 and 5.2, we can observe that

the performance gaps between our proposed Kalman filter based detector and LMS detector are small, when SNR is low. This phenomenon can be explained in two aspects. The first one is when SNR is low, AWGN will deteriorate the performance of detectors. As a result, the performance of both Kalman filter-based detector and LMS detector are not satisfied. The second aspect is that the environment in Figs. 5.1 and 5.2 is equal-power situation, thus MAI under this situation is smaller than that of the near-far situation considered in Figs. 5.3 and 5.4. In this case, the advantage of Kalman filter-based detector in Figs. 5.1 and 5.2 is not as obvious as that in Figs. 5.3 and 5.4. Because of the above two aspects, the performance gaps between two types of detectors in Figs. 5.1 and 5.2 is not so obvious, and is closer than those in Figs. 5.3 and 5.4, when SNR is low. According to Figs. 5.1-5.4, if there is a dominant path and the delay of the desired user's first path is available, the LMS detector is recommended for its lower computational complexity under low SNR condition. However, when SNR increases, the proposed Kalman filter based detector will outperform the LMS detector. In this case, the Kalman filter based detector is recommended for its better performance.

In Fig. 5.5, the SINR performance of the proposed Kalman filter-based MUD using inverse filtering is compared with that of the existing LMS algorithm under a dynamic environment with SNR=15dB. When the number of iterations J is less than 2000, there are five equal-power users in the system. At $J=2000$, five new users are added into this system, and each of them has a power level of 10dB higher than that of the desired user, whereas at $J=3500$, three out of these new users are disconnected from this system. Fig. 5.5 shows that the proposed Kalman filter-based detector can rapidly approach its convergence. When the number of users is changed, it can rapidly re-converge again, and the final SINR is almost the same as before. On the other hand, the

existing LMS algorithm shows a much poorer SINR performance under this dynamic environment. The major disadvantage of the LMS algorithm is that since the convergence rate depends on the spread of eigenvalues [15], [66]. If the environment changes, such as the number of users changes, the LMS algorithm may not be able to approach its convergence again. Therefore, we can conclude that the proposed Kalman filter-based MUD has good tracking capability under a dynamic environment.

Next, we will present the performance of the Kalman filter-based MUD using linear prediction. The channel gain of each user is generated according to a fixed complex Gaussian distribution with zero mean, and the delay of each user is randomly chosen to be within 0-10 chips. Note that for this type of Kalman filter-based MUD, we assume that the delay of the desired user's first path is not available to us and there is no dominant path, which is different from the assumption for the Kalman filter-based MUD using inverse filtering. The results are compared with those of the corresponding existing LMS MUD using linear prediction and the step size is chosen to be 0.0001 [58]. The forgetting factor γ for estimating $\mathbf{u}$ is chosen to be 0.998. Note that a larger γ will make the estimation of $\mathbf{u}$ approaching its exact value faster. If the system is time variant, however, a larger γ will cause the estimation of $\mathbf{u}$ to re-converge slower, because the system is difficult to "forget" the previous behavior [66].

In Figs. 5.6 and 5.7, the SINR and BER results of the Kalman filter-based MUD and the LMS MUD are presented respectively, for the equal-power situation with $K=5$. These results are obtained after 5000 iterations when each of the algorithms approaches its convergence. The results of the batch MMSE MUD based on linear prediction proposed in [58] are also provided for comparisons. From these two figures, we observe that both of the SINR and BER results of the proposed Kalman filter-based MUD are

worse than those of the batch MMSE MUD but are much better than those of the LMS MUD. At SNR= 15dB, the BER of the proposed Kalman filter-based MUD is below 10^{-3} , whereas the BER of the LMS MUD is above 10^{-2} . Therefore, we can conclude that the proposed Kalman filter-based MUD outperforms the existing LMS MUD under the equal-power situation. Note that although these two figures show that the performance of the Kalman filter-based MUD is worse than the batch MMSE detector, the batch algorithm cannot update its parameters adaptively. Therefore the proposed adaptive MUD is more suitable for a dynamic environment.

Next, the near-far situation is considered. It is assumed that there are 5 active users and the power level of each interferer is 10 dB higher than that of the desired user. The SINR and BER results are presented in Figs. 5.8 and 5.9, respectively. From these two figures, we observe that the proposed Kalman filter-based MUD outperforms the existing LMS MUD under the near-far situation in both SINR and BER performance. Furthermore, according to Figs. 5.6-5.9, we can observe that there are obvious performance gaps between these two types of detectors, when SNR is low. Therefore, the Kalman filter-based detector is preferred for its better performance, when the delay of the desired user is not available and there is no dominant path.

In Fig. 5.10, we study the tracking capability of the proposed Kalman filter-based MUD in a dynamic environment with SNR=15dB. At $J < 2000$, there are five equal-power users in the system. At $J = 2000$, five users with a power level of 10 dB higher than that of the desired user are added into the system. At $J = 3500$, three out of these new users are disconnected. From this figure, we observe that the proposed Kalman filter-based MUD can approach its convergence rapidly. When the system load is changed, it can re-converge rapidly and achieve almost the same SINR as before. On the other hand, the

LMS MUD shows a much poorer performance under this dynamic environment. Therefore, we can conclude that the proposed Kalman filter-based MUD using linear prediction has a good tracking capability under a dynamic environment.

Lastly, we compare the performance of the two types of proposed Kalman filter-based MUD algorithms. It is assumed that there is no dominant path in the multipath fading channel. In order to ensure that the Kalman filter-based MUD using inverse filtering can work properly, we assume that the timing information of the first path is available. Fig. 5.11 provides the SINR performance of these two types of Kalman filter-based MUD with SNR=15dB. It can be found that under such a channel model, the Kalman filter-based MUD using inverse filtering presents a worse performance as compared with the Kalman filter-based MUD using linear prediction. The reason is that the Kalman filter-based MUD using linear prediction can utilize the channel response of all paths, whereas the Kalman filter-based MUD using inverse filtering can only utilize the channel response of the first path.

Fig. 5.1 Comparisons of SINR results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the equal-power situation.

Fig. 5.2 Comparisons of BER results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the equal-power situation.

Fig. 5.3 Comparisons of SINR results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the near-far situation.

Fig. 5.4 Comparisons of BER results between the proposed Kalman filter-based MUD using inverse filtering and LMS MUD using inverse filtering under the near-far situation.

Fig. 5.5 Comparisons of SINR results between the proposed Kalman filter-based MUD using inverse filtering and the LMS MUD using inverse filtering under a dynamic environment.

Fig. 5.6 Comparisons of SINR results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the equal-power situation.

Fig. 5.7 Comparisons of BER results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the equal-power situation.

Fig. 5.8 Comparisons of SINR results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the near-far situation.

Fig. 5.9 Comparisons of BER results among the proposed Kalman filter-based MUD using linear prediction, LMS MUD using linear prediction and batch MMSE MUD under the near-far situation.

Fig. 5.10 Comparisons of SINR results between the proposed Kalman filter-based MUD using linear prediction and the LMS MUD using linear prediction under a dynamic environment.

Fig. 5.11 Comparisons of SINR results between two proposed Kalman filter-based MUD algorithms.

5.5 Conclusion

In this chapter, we have extended the existing blind Kalman filtering algorithm to a multipath environment and proposed two kinds of Kalman filter-based MUD algorithms using inverse filtering and linear prediction, respectively. For the proposed Kalman filter-based MUD using inverse filtering, we assume that the first path of the desired user is the dominant path, whose amplitude is much higher than that of the other paths. Also, it is assumed that the delay of this path is available. For the proposed Kalman filter-based MUD using linear prediction, we assume that there is no dominant path and the delay of each path is not available to us. Simulation results have shown that both of the proposed Kalman filter-based MUD algorithms can outperform the existing LMS MUD algorithms in terms of SINR and BER performance. Furthermore, simulation results have also

shown that these two Kalman filter-based MUD algorithms have good tracking capability under a dynamic environment.

Chapter 6

Asynchronous Blind Adaptive Subspace-Based MUD and Its Perturbation Analyses

6.1 Introduction

In Chapter 5, we have proposed two adaptive Kalman filter-based MUD algorithms over multipath fading channels. These two detectors can be classified as the constrained MOE criterion because they minimize the output energy of a linear filter subject to a constraint. This type of detector is computationally efficient to implement. However, there is a significant gap between the performances of the blind MOE detector and that of the true MMSE detector [14], [58]. Another type of promising adaptive MUD approach is the subspace-based method. This type of detector performs the subspace decomposition, i.e., SVD or EVD, to separate the signal subspace and noise subspace of the received signals, and then forms the detector. Unfortunately, the exact SVD or EVD algorithm is not well suited for the adaptive application because it is computationally complex to update as new data arrives. Therefore, many low-complexity adaptive subspace tracking algorithms have been proposed, and applied to blind MUD algorithms [12], [21], [65], [76]-[78].

Assuming no multipath fading, the projection approximation subspace tracking with deflation (PASTd) algorithm [20], fast approximate subspace tracking (FAST) algorithm [79], and selective orthogonalization (SO) Lanczos algorithm [80], [68] have been applied for adaptive multiuser detection in [12], [76], and [77] respectively. These subspace tracking algorithms have low complexity and are suboptimal to the exact SVD algorithm. However, most of these algorithms do not take the ISI into account. In [65], a type of multiuser detector using the PASTd algorithm over multipath fading channels was proposed. In [21], the noise-averaged Hermitian-Jacobi fast subspace tracking (NAHJ-FST) based MUD was proposed to jointly reduce the effects of MAI and ISI together. The subspace tracking algorithm in [21] is a member of the SVD-based algorithm and utilizes a set of selective Givens rotations [66] instead of a complete set of Givens rotations when updating the signal subspace. Therefore, it has the advantage of maintaining the orthonormality of subspace and low computational complexity. However, since the NAHJ-FST algorithm does not adopt a complete set of Givens rotations, it cannot annihilate all of the off-diagonal elements in the diagonal matrix when performing the subspace decomposition, and thus the performance of the NAHJ-FST based detector will be degraded.

In [19], another SVD-based adaptive subspace tracking algorithm named refinement-only fast subspace tracking (RO-FST) algorithm was proposed. This algorithm averages the noise singular values and effectively sphericalizes the noise subspace. After that, it computes an orthonormal basis by using a complete set of Givens rotations for the signal subspace of the data matrix adaptively. This algorithm has the same level of computational complexity as the NAHJ-FST algorithm. Meanwhile, this algorithm only attempt to transform the received data matrix to an upper triangular

matrix instead of a diagonal matrix, by using a complete set of Givens rotations. Therefore, this algorithm does not need to explicitly compute the full set of signal eigenvalues, and is particularly efficient for adaptive subspace tracking [19], [21]. In this chapter, we will utilize the RO-FST algorithm to develop a low-complexity, high-performance blind adaptive MMSE detector. The proposed detector focuses on suppressing MAI and ISI over a multipath fading channel. Simulation results will show that the proposed detector has good performance and tracking capability even in a dynamic environment. Furthermore, since the RO-FST algorithm utilizes a complete set of Givens rotations, numerical results will show that the proposed detector outperforms the existing NAHJ-FST based detector.

Another contribution of this chapter is that we also analyze the performance of the proposed adaptive detector from a perturbation perspective when the covariance matrix of the received signal is not perfectly estimated due to the finite number received data samples. The theoretical analyses of the subspace-based MUD algorithms have been presented in some recent works [16]-[18], [72]. In [71], perturbation analysis of the covariance of the weighted sample covariance matrix was studied and the explicit closed-form expressions were provided. Following that, [17] and [18] utilize these expressions to analyze the SINR and BER performance of several blind multiuser detectors. However, these analytical results are mainly based on the batch algorithms.

In this chapter, we will focus on the perturbation analysis of SVD-based blind adaptive MUD algorithms over a multipath fading channel. For adaptive algorithms, we utilize a forgetting factor γ to minimize the effect of the previous received signals. It is important to study the effects of γ as it will affect the performance of the proposed detector greatly. We will first derive closed-form expressions for the covariance of the

weighted sample covariance matrix under the adaptive version and then analyze the performance of several blind adaptive linear detectors including DMI MMSE, subspace MMSE, subspace ZF, and group-blind detectors. The SINR and BER performance of each detector based on the differential detection will be studied as the performance measures. Since these analyses are based on the SVD algorithm, the analytical results can also be considered as the performance bounds for other SVD-based algorithms including the proposed adaptive RO-FST based MUD. Numerical results will show that the analytical results match the simulation results of the proposed adaptive RO-FST based MUD well.

6.2 Adaptive MUD Using RO-FST

In this section, we will propose a blind adaptive SVD-based MUD algorithm. It adopts the RO-FST algorithm to track the signal subspace of the received signal and then forms a DMI MMSE detector. In the followings, we will introduce the system model first.

6.2.1 System Model

We consider an asynchronous DS-CDMA system with K active users and P chips per bit. Assuming that the channel is distorted by multipath fading, the received discrete-time signal vector $\mathbf{r}(j)$ due to all users at the j th bit interval is given by (2.24). Assuming that the total delay of each user is less than one bit, i.e., $L=2$, and by stacking up $\mathbf{r}(j)$ and $\mathbf{r}(j+1)$ together, we obtain equation (3.3), which is reproduced here as

$$\mathbf{q}(j) = \begin{pmatrix} \mathbf{r}(j) \\ \mathbf{r}(j+1) \end{pmatrix}$$

$$= A_1 \mathbf{H}_{1,0} b_1(j) + A_1 \bar{\mathbf{H}}_1 \bar{\mathbf{B}}_1(j) + \sum_{k=2}^K A_k \mathbf{H}_k \mathbf{B}_k(j) + \mathbf{N}(j). \quad (6.1)$$

6.2.2 Proposed Adaptive MUD Using RO-FST

There are two commonly used linear detectors for blind multiuser detection, namely, the ZF detector and the MMSE detector. For the ZF detector, although it can suppress MAI and ISI, it will also enhance noise level [49]. In a practical environment, where noise is significant, the MMSE detector will be more desirable. Therefore, in this chapter, we aim to develop a blind MMSE multiuser detector.

Denoting $\mathbf{B}(j) = [\mathbf{B}_1^T(j), \dots, \mathbf{B}_K^T(j)]^T$ and $\boldsymbol{\theta} = [\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_K]$, where $\boldsymbol{\theta}_k$ is defined in (3.43), (6.1) can be rewritten as

$$\mathbf{q}(j) = \boldsymbol{\theta} \mathbf{B}(j) + \mathbf{N}(j) \quad (6.2)$$

and the covariance matrix of $\mathbf{q}(j)$ can be obtained by

$$\mathbf{R}_q = E\{\mathbf{q}(j)\mathbf{q}^H(j)\} = \boldsymbol{\theta} \boldsymbol{\theta}^H + \sigma_n^2 \mathbf{I}_{2P}. \quad (6.3)$$

Based on the well-known Wiener solution [66] to minimize the MSE, the detector $\mathbf{m}$ has a form of

$$\mathbf{m} = \mathbf{R}_q^{-1} \mathbf{H}_{1,0}. \quad (6.4)$$

From (6.4), we observe that the proposed detector can be classified as the DMI MMSE detector [18]. Note that the expression in (6.4) is the ideal MMSE detector. For blind multiuser detection, both $\mathbf{R}_q$ and $\mathbf{H}_{1,0}$ are not available and need to be estimated.

Therefore, the blind MMSE multiuser detector is implemented as

$$\hat{\mathbf{m}} = \hat{\mathbf{R}}_q^{-1} \hat{\mathbf{H}}_{1,0} \quad (6.5)$$

where $\hat{\mathbf{R}}_q$ and $\hat{\mathbf{H}}_{1,0}$ are the estimation of $\mathbf{R}_q$ and $\mathbf{H}_{1,0}$, respectively.

Fig. 6.1 Block diagram of the proposed blind adaptive MMSE detector.

The structure of the proposed blind adaptive MMSE detector is shown in Fig. 6.1. The received signal $\mathbf{q}(j)$ is fed into a subspace tracker to adaptively estimate the signal subspace and noise subspace, and the output is used to estimate the effective sequence $\mathbf{H}_{1,0}$. On the other hand, the estimation of $\mathbf{R}_q^{-1}$ is updated by $\mathbf{q}(j)$ in each iteration. Following that, the blind MMSE detector is constructed based on (6.5) to obtain the information data of the desired user. For an adaptive algorithm, at the J th symbol interval, the covariance matrix $\hat{\mathbf{R}}_q$ can be estimated by

$$\hat{\mathbf{R}}_q = \frac{1-\gamma}{1-\gamma^J} \sum_{j=1}^J \gamma^{J-j} \mathbf{q}(j) \mathbf{q}(j)^H \quad (6.6)$$

where $0 < \gamma < 1$ is a forgetting factor which is chosen to be close to but less than 1. This factor is intended here to ensure that in a dynamic environment, data in the distant past are “forgotten”. Denoting

$$\mathbf{Q}(j) = [\mathbf{q}(1) \ \cdots \ \mathbf{q}(j)] \text{ and } \mathbf{\Gamma}(j) = \text{diag}(\gamma^{(j-1)/2}, \dots, \gamma^{1/2}, 1), \quad (6.7)$$

equation (6.6) can be expressed as

$$\hat{\mathbf{R}}_q = \frac{1-\gamma}{1-\gamma^j} [\mathbf{Q}(j)\mathbf{\Gamma}(j)][\mathbf{Q}(j)\mathbf{\Gamma}(j)]^H. \quad (6.8)$$

For the proposed blind adaptive detector, we need to first estimate the signal subspace and noise subspace based on the received signal. Here, we employ the RO-FST algorithm in [19] because of its low computational complexity and high efficiency. Based on (6.7), at the $(j-1)$ th bit interval, we have

$$\mathbf{\Gamma}(j-1)\mathbf{Q}(j-1)^H = \mathbf{V}_Q(j-1)\mathbf{\Lambda}_Q(j-1)[\mathbf{U}_Q^s(j-1), \mathbf{U}_Q^n(j-1)]^H \quad (6.9)$$

where $\mathbf{V}_Q(j-1)$ is a $(j-1) \times 2P$ orthonormal matrix, $\mathbf{\Lambda}_Q(j-1)$ is a $2P \times 2P$ upper triangular matrix, $\mathbf{U}_Q^s(j-1)$ is a $2P \times d$ orthonormal matrix corresponding to the signal subspace, and $\mathbf{U}_Q^n(j-1)$ is a $2P \times (2P-d)$ orthonormal matrix corresponding to the noise subspace. Note that d is the rank of the signal subspace, and is assumed to be $3K$ for this asynchronous case [14]. At the j th iteration, the RO-FST algorithm is performed as follows:

- 1) Compute a vector of the j th received signal $\mathbf{q}(j)$ under the basis of $\mathbf{U}_Q^s(j-1)$:

$$\mathbf{q}_s(j) = \mathbf{U}_Q^{sH}(j-1)\mathbf{q}(j). \quad (6.10)$$

- 2) Project $\mathbf{q}(j)$ onto the noise subspace: $\mathbf{u}_n = [\mathbf{q}(j) - \mathbf{U}_Q^s(j-1)\mathbf{q}_s(j)] / \mu(j)$, where $\mu(j)$ is the norm of $[\mathbf{q}(j) - \mathbf{U}_Q^s(j-1)\mathbf{q}_s(j)]$.

- 3) Let $\mathbf{\Lambda}_Q^s(j-1)$ be the d principal sub-matrix of $\mathbf{\Lambda}_Q(j-1)$, and construct a modified matrix $\mathbf{\Lambda}_m(j)$ given by

$$\Lambda_m(j) = \left[\begin{array}{ccc|c} \gamma^{1/2} \Lambda_Q^s(j-1) & & & 0 \\ & & & \vdots \\ & & & 0 \\ - & - & - & - \\ 0 & \dots & 0 & \gamma^{1/2} \bar{\sigma}_n(j-1) \\ - & - & - & - \\ & \mathbf{q}_s^H(j) & & \mu(j) \end{array} \right]. \quad (6.11)$$

4) Apply a sequence of $d+1$ Givens rotations Θ_1 on $\Lambda_m(j)$ to make it upper triangular:

$$\tilde{\Lambda}_m(j) = \Theta_1^{d+1} \dots \Theta_1^1 \Lambda_m(j) \quad (6.12)$$

and let $\Lambda_u(j)$ be the first $d+1$ rows of $\tilde{\Lambda}_m(j)$, i.e., drop the $(d+2)$ th row of $\tilde{\Lambda}_m(j)$ after it is zeroed.

5) Post-multiply a sequence of d Givens rotations Θ_2 on $\Lambda_u(j)$ to drive the off-diagonal elements on the $(d+1)$ th column closer to zero, and then pre-multiply a sequence of d Givens rotations Θ_3 on $\Lambda_u(j)$ to make it upper triangular again:

$$\tilde{\Lambda}_u(j) = \Theta_3^d \dots \Theta_3^1 \Lambda_u(j) \Theta_2^1 \dots \Theta_2^d. \quad (6.13)$$

6) Let $\mathbf{U}_Q^s(j)$ be the first d columns of $[\mathbf{U}_Q^s(j-1); \mathbf{u}_n] \Theta_2^1 \dots \Theta_2^d$.

7) Re-average the noise power as

$$\bar{\sigma}_n^2(j) = \frac{(2P-d-1)\gamma\bar{\sigma}_n^2(j-1) + \left| (\tilde{\Lambda}_u)_{d+1,d+1}^2 \right|}{2P-d} \quad (6.14)$$

and let $\Lambda_Q^s(j)$ be the d principal sub-matrix of $\tilde{\Lambda}_u(j)$, then update $\Lambda_Q(j)$ to be

$$\left[\begin{array}{cc} \Lambda_Q^s(j) & 0 \\ 0 & \bar{\sigma}_n(j) \mathbf{I}_{(2P-d)} \end{array} \right].$$

By using the above subspace tracking algorithm, the signal subspace $\mathbf{U}_Q^s(j)$ can be updated in each iteration, and its orthonormality can be maintained. In a dynamic environment, where the rank of the signal subspace may change, the blind detector is required to dynamically track the rank. In [19], an efficient rank tracking algorithm was proposed for the RO-FST algorithm. In each iteration, this rank tracking algorithm determines whether the rank of the signal subspace is changed. If the rank is changed, this algorithm automatically alters the size of the signal subspace, as addressed explicitly in [19].

Here we utilize the subspace method in [57] to estimate $\mathbf{H}_{1,0}$. Since the matrix $\mathbf{S}_Q^n = \mathbf{I}_{2P} - \mathbf{U}_Q^s \mathbf{U}_Q^{sH}$ is orthogonal to the signal subspace $\mathbf{U}_Q^s$, $\mathbf{S}_Q^n$ is orthogonal to any column vector in $\boldsymbol{\theta}$, thus we have

$$\mathbf{g}_1^H \mathbf{C}_1^H \mathbf{S}_Q^n \mathbf{C}_1 \mathbf{g}_1 = 0. \quad (6.15)$$

Therefore, the channel response $\mathbf{g}_1$ is the right null vector of the matrix $\mathbf{C}_1^H \mathbf{S}_Q^n \mathbf{C}_1$, and can be estimated by computing the eigenvector corresponding to the smallest eigenvalue of $\mathbf{C}_1^H \mathbf{S}_Q^n \mathbf{C}_1$. To reduce the computational complexity, we can utilize the Kalman filter to sequentially estimate $\mathbf{g}_1$ and $\mathbf{H}_{1,0}$ [21], [65].

As shown in (6.5), we need to compute the inverse matrix of $\hat{\mathbf{R}}_q$. In order to keep the computational complexity in a comparatively low level, we adopt the matrix inversion lemma and recursive algorithms in [66] to update $\hat{\mathbf{R}}_q^{-1}$ adaptively as given by

$$\hat{\mathbf{R}}_q^{-1}(j) = \gamma^{-1} \hat{\mathbf{R}}_q^{-1}(j-1) - \gamma^{-1} t(j) \mathbf{q}^H(j) \hat{\mathbf{R}}_q^{-1}(j-1) \quad (6.16)$$

where

$$t(j) = \frac{\gamma^{-1} \hat{\mathbf{R}}_q^{-1}(j-1) \mathbf{q}(j)}{1 + \gamma^{-1} \mathbf{q}^H(j) \hat{\mathbf{R}}_q^{-1}(j-1) \mathbf{q}(j)}. \quad (6.17)$$

Thus, the complete proposed adaptive MMSE detector can be summarized as follows. For every snapshot $\mathbf{q}(j)$, we first use the RO-FST algorithm to estimate the signal subspace, and then update the estimation of the channel response $\hat{\mathbf{g}}_1$ and effective signature sequence $\hat{\mathbf{H}}_{1,0}$. Next we adopt (6.16) to update $\hat{\mathbf{R}}_q^{-1}$ and finally use (6.5) to update the coefficient of the detector $\hat{\mathbf{m}}$.

Next, we will compare the computational complexity of the proposed adaptive subspace tracking algorithm and other existing algorithms. From [19], the computational complexity of the RO-FST algorithm is $O(LPd)$. The computational complexity of the existing subspace tracking algorithms varies from $O(LPd)$ to $O(L^3P^3)$ per iteration [21]. For example, the NAHJ-FST algorithm in [21] and the PASTd algorithm in [20] have a complexity of $O(LPd)$, the FAST algorithm in [76] has a complexity of $O(d^3)$ and the SO Lanczos algorithm in [77] has a complexity of $O(LPd^2)$, whereas the complexity of the exact SVD algorithm is $O(L^3P^3)$ [21]. Therefore the RO-FST algorithm has a relatively low complexity among all the subspace tracking algorithms.

The RO-FST algorithm belongs to the family of SVD-based subspace tracking algorithms, and its performance is suboptimal to the exact SVD algorithm. One of the important features of the RO-FST algorithm is that it keeps $\mathbf{\Lambda}$ in an upper triangular form and does not attempt to diagonalize it, which is particularly efficient since it does not need to compute a full set of eigenvalues [21]. Note that for the proposed blind MMSE detector, we do not utilize the information of eigenvalues, and hence we do not

need to diagonalize Λ . Another feature of this algorithm is that it does not require an initial decomposition. When the iteration process commences, we only need to set $[\mathbf{U}_Q^s(0), \mathbf{U}_Q^n(0)] = \mathbf{I}_{2P}$ and $\Lambda_Q(0) = 0$. Then as $\mathbf{q}(1)$ comes in, the updating procedure can be applied in the usual way. This feature ensures that no special initialization mechanism needs to be constructed.

6.3 Performance Analysis of Blind SVD-Based MUD

In this section, we will derive the explicit expressions of various performance measures for the blind adaptive MUD using SVD, including the DMI MMSE detector, subspace MMSE detector, subspace ZF detector and group-blind detector. We will derive the closed-form expressions for the SINR and BER performance. Since these analyses are based on the exact SVD algorithm, the analytical results can also be considered as the performance bounds for other SVD-based algorithms including the proposed adaptive RO-FST based detector. Numerical results will show that the analytical results match the simulation results of the proposed adaptive MUD well.

6.3.1 Adaptive MUD Using SVD

As mentioned in Section 6.2, the received signal and its covariance matrix are represented by (6.2) and (6.3), respectively. By performing the subspace decomposition on $\mathbf{R}_q$, we have

$$\mathbf{R}_q = \mathbf{U}_q^s \Lambda_q^s \mathbf{U}_q^{sH} + \sigma_n^2 \mathbf{U}_q^n \mathbf{U}_q^{nH} \quad (6.18)$$

where Λ_q^s is the diagonal matrix containing the largest $3K$ eigenvalues of $\mathbf{R}_q$; whereas

$\mathbf{U}_q^s$ and $\mathbf{U}_q^n$ contain the eigenvectors corresponding to signal subspace and noise

subspace, respectively. Since $\mathbf{U}_q^n$ is orthogonal to $\mathbf{U}_q^s$, it is also orthogonal to $\boldsymbol{\theta}_k$. Similar as (6.15), $\mathbf{g}_k$ can be obtained by the following equation

$$\mathbf{g}_k = \arg \min_{\|\mathbf{x}\|=1} \|\mathbf{U}_q^n (\mathbf{C}_k \mathbf{x})\|^2 = \arg \min_{\|\mathbf{x}\|=1} \mathbf{x}^H \underbrace{(\mathbf{C}_k^H \mathbf{U}_q^n \mathbf{U}_q^{nH} \mathbf{C}_k)}_{\mathbf{G}_k} \mathbf{x}. \quad (6.19)$$

For an adaptive algorithm, the covariance matrix $\mathbf{R}_q$ at the J th bit interval is estimated by (6.6) and it can be shown that

$$E\{\hat{\mathbf{R}}_q\} = \frac{1-\gamma}{1-\gamma^J} \sum_{j=1}^J \gamma^{J-j} E\{\mathbf{q}(j)\mathbf{q}(j)^H\} = \mathbf{R}_q. \quad (6.20)$$

Therefore, $\hat{\mathbf{R}}_q$ is an unbiased estimate of $\mathbf{R}_q$. Correspondingly, the subspace decomposition of $\hat{\mathbf{R}}_q$ is given by

$$\hat{\mathbf{R}}_q = \hat{\mathbf{U}}_q^s \hat{\mathbf{\Lambda}}_q^s \hat{\mathbf{U}}_q^{sH} + \hat{\mathbf{U}}_q^n \hat{\mathbf{\Lambda}}_q^n \hat{\mathbf{U}}_q^{nH} \quad (6.21)$$

where $\hat{\mathbf{\Lambda}}_q^n$ is diagonal eigenvalue matrix corresponding to noise subspace of $\hat{\mathbf{R}}_q$. Note that for blind channel estimation, (6.19) only specifies $\mathbf{g}_k$ up to a phase ambiguity $e^{j\phi}$. It means that if $\mathbf{g}_k$ is the solution to (6.19), so is $\mathbf{g}_k e^{j\phi}$. Consequently, this ambiguity will be transferred to the estimated detectors [16]. Denoting $\mathbf{m}$ and $\Delta\mathbf{m}$ as the ideal blind adaptive multiuser detector and its perturbation, respectively, the corresponding estimated detector is given by

$$\hat{\mathbf{m}} = (\mathbf{m} + \Delta\mathbf{m}) e^{j\phi}. \quad (6.22)$$

For a linear detector, the phase ambiguity $e^{j\phi}$ will greatly degrade its performance. Therefore, the differential detection schemes are implemented at the receiver, and the decision rule is given by (3.17) and reproduced here as

$$\hat{d}(j) = \text{sgn} \left\{ \Re \left[\left(\hat{\mathbf{m}}^H \mathbf{q}(j) \right) \left(\hat{\mathbf{m}}^H \mathbf{q}(j-1) \right)^* \right] \right\}. \quad (6.23)$$

Next, we will introduce four types of blind subspace-based multiuser detectors.

1). DMI MMSE Detector

An ideal DMI MMSE detector is given by [18]

$$\mathbf{m}_{\text{DMI}} = \mathbf{R}_q^{-1} \mathbf{C}_1 \mathbf{g}_1. \quad (6.24)$$

Due to the finite number of received data samples, the practical DMI MMSE detector can be implemented as

$$\hat{\mathbf{m}}_{\text{DMI}} = \left(\hat{\mathbf{R}}_q^{-1} \mathbf{C}_1 \hat{\mathbf{g}}_1 \right) e^{j\phi}. \quad (6.25)$$

2). Subspace MMSE Detector

An ideal subspace MMSE detector is given by [14]

$$\mathbf{m}_{\text{MMSE}} = \mathbf{U}_q^s \left(\mathbf{\Lambda}_q^s \right)^{-1} \mathbf{U}_q^{sH} \mathbf{C}_1 \mathbf{g}_1. \quad (6.26)$$

Since there is only a finite number of received data samples available, the practical subspace MMSE detector can be implemented as

$$\hat{\mathbf{m}}_{\text{MMSE}} = \left(\hat{\mathbf{U}}_q^s \left(\hat{\mathbf{\Lambda}}_q^s \right)^{-1} \hat{\mathbf{U}}_q^{sH} \mathbf{C}_1 \hat{\mathbf{g}}_1 \right) e^{j\phi}. \quad (6.27)$$

3). Subspace ZF Detector

Based on [14], an ideal subspace ZF detector is defined as

$$\mathbf{m}_{\text{ZF}} = \mathbf{U}_q^s \left(\mathbf{\Lambda}_q^s - \sigma_n^2 \mathbf{I}_d \right)^{-1} \mathbf{U}_q^{sH} \mathbf{C}_1 \mathbf{g}_1 \quad (6.28)$$

where d is the rank of the signal subspace $\mathbf{U}_q^s$. Based on estimated parameters, the practical subspace ZF detector is given by

$$\hat{\mathbf{m}}_{\text{ZF}} = \left(\hat{\mathbf{U}}_q^s \left(\hat{\mathbf{\Lambda}}_q^s - \hat{\sigma}_n^2 \mathbf{I}_d \right)^{-1} \hat{\mathbf{U}}_q^{sH} \mathbf{C}_1 \hat{\mathbf{g}}_1 \right) e^{j\phi}. \quad (6.29)$$

4). Group-Blind Detector

Assuming $\bar{K}$ users' PN sequences are known to the detector and denoting $\boldsymbol{\theta}_{\bar{K}} = [\mathbf{H}_1, \dots, \mathbf{H}_{\bar{K}}]$, the ideal group-blind detector is given by [31]

$$\mathbf{m}_{\text{GB}} = \mathbf{U}_{\mathbf{q}}^s (\boldsymbol{\Lambda}_{\mathbf{q}}^s)^{-1} \mathbf{U}_{\mathbf{q}}^{sH} \boldsymbol{\theta}_{\bar{K}} \left[\boldsymbol{\theta}_{\bar{K}}^H \mathbf{U}_{\mathbf{q}}^s (\boldsymbol{\Lambda}_{\mathbf{q}}^s)^{-1} \mathbf{U}_{\mathbf{q}}^{sH} \boldsymbol{\theta}_{\bar{K}} \right]^{-1} \mathbf{e}_1 \quad (6.30)$$

where $\mathbf{e}_1$ is a vector of length $\bar{K}$ whose elements are zeros, except the first element which is 1. The group-blind detector can be realized by

$$\hat{\mathbf{m}}_{\text{GB}} = \left(\hat{\mathbf{U}}_{\mathbf{q}}^s (\hat{\boldsymbol{\Lambda}}_{\mathbf{q}}^s)^{-1} \hat{\mathbf{U}}_{\mathbf{q}}^{sH} \hat{\boldsymbol{\theta}}_{\bar{K}} \left[\hat{\boldsymbol{\theta}}_{\bar{K}}^H \hat{\mathbf{U}}_{\mathbf{q}}^s (\hat{\boldsymbol{\Lambda}}_{\mathbf{q}}^s)^{-1} \hat{\mathbf{U}}_{\mathbf{q}}^{sH} \hat{\boldsymbol{\theta}}_{\bar{K}} \right]^{-1} \mathbf{e}_1 \right) e^{j\phi}. \quad (6.31)$$

6.3.2 Perturbation Analysis of Adaptive MUD Using SVD

In this section, we will provide the closed-form expressions of both SINR and BER performance for the four types of blind adaptive MUD using SVD. It will be shown that these two performance indicators can be expressed by a function of the covariance matrix $E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$. This matrix, in turn, can be represented by a function of the covariance matrix $E\{\Delta \mathbf{R}_{\mathbf{q}} \boldsymbol{\Psi} \Delta \mathbf{R}_{\mathbf{q}}\}$, where $\Delta \mathbf{R}_{\mathbf{q}}$ is the perturbation of $\mathbf{R}_{\mathbf{q}}$ at the J th bit interval and $\boldsymbol{\Psi}$ is a deterministic weighting matrix, which can be of different forms in different applications. According to (6.6), the adaptive algorithm introduces a forgetting factor γ to minimize the effect caused by the data in the distant past. Therefore, this factor will affect the estimated covariance matrix $\hat{\mathbf{R}}_{\mathbf{q}}$. On the other hand, the factor of finite sample size J is also treated as the perturbation of the estimated $\hat{\mathbf{R}}_{\mathbf{q}}$ [17], [71]. Therefore, our task is to first study the effect of γ and J on $E\{\Delta \mathbf{R}_{\mathbf{q}} \boldsymbol{\Psi} \Delta \mathbf{R}_{\mathbf{q}}\}$. Following that, we will derive the covariance matrix of the four types of adaptive subspace-based detectors. Finally, we will derive the analytical SINR and BER expressions as a function of

$E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$. Moreover, these analytical results can be treated as performance bounds of other SVD-based algorithms [21].

6.3.2.1 Perturbation Analysis Based on Sample Covariance Matrix

The ideal and the estimated values of covariance of $\mathbf{q}(j)$ have been given in (6.3) and (6.6), respectively. Thus at the J th bit interval, the perturbation of covariance matrix $\mathbf{R}_q$ becomes

$$\Delta \mathbf{R}_q = \hat{\mathbf{R}}_q - \mathbf{R}_q. \quad (6.32)$$

Here we focus on the covariance of $\Delta \mathbf{R}_q$ in terms of a weighted matrix given by

$$\boldsymbol{\eta}(J, \boldsymbol{\Psi}) = E\{\Delta \mathbf{R}_q \boldsymbol{\Psi} \Delta \mathbf{R}_q\}. \quad (6.33)$$

For analyzing the covariance of $\Delta \mathbf{R}_q$ over an adaptive version, we first consider the following general expression

$$\boldsymbol{\lambda}(l, u, J, \boldsymbol{\Psi}) = E\left\{\left(\sum_{j=l}^u \gamma^{J-j} \mathbf{q}(j) \mathbf{q}(j)^H\right) \boldsymbol{\Psi} \left(\sum_{j=l}^u \gamma^{J-j} \mathbf{q}(j) \mathbf{q}(j)^H\right)\right\}. \quad (6.34)$$

From (6.1), it can be observed that $b_k(j)$ is included in $\mathbf{q}(j-1)$, $\mathbf{q}(j)$, and $\mathbf{q}(j+1)$, therefore, $\mathbf{q}(j) \mathbf{q}(j)^H$ is correlated with $\mathbf{q}(j+2) \mathbf{q}(j+2)^H$, $\mathbf{q}(j+1) \mathbf{q}(j+1)^H$, $\mathbf{q}(j-1) \mathbf{q}(j-1)^H$, and $\mathbf{q}(j-2) \mathbf{q}(j-2)^H$. Based on this fact, the following proposition can be obtained.

Proposition 1: Assuming that the CDMA system is modeled based on (6.1), $\boldsymbol{\lambda}(l, u, J, \boldsymbol{\Psi})$ has the following closed-form expression

$$\boldsymbol{\lambda}(l, u, J, \boldsymbol{\Psi}) = \frac{\gamma^{2(J-u)} (1 - \gamma^{2u-2l+2})}{1 - \gamma^2} E_1(\boldsymbol{\Psi}) + \frac{\gamma^{2J-2u+1} (1 - \gamma^{2u-2l})}{1 - \gamma^2} (E_2(\boldsymbol{\Psi}) + E_2^H(\boldsymbol{\Psi}^H))$$

$$\begin{aligned}
 & + \frac{\gamma^{2J-2u+2} (1 - \gamma^{2u-2l-2})}{1 - \gamma^2} (E_3(\Psi) + E_3^H(\Psi^H)) \\
 & + \frac{2(\gamma^{2J-2u+3} - \gamma^{2J-u-l+1} - \gamma^{2J-u-l+2} + \gamma^{2J-2l})}{(1 - \gamma)^2 (1 + \gamma)} \mathbf{R}_q \Psi \mathbf{R}_q
 \end{aligned} \quad (6.35)$$

where

$$E_1(\Psi) = E\{\mathbf{q}(j)\mathbf{q}(j)^H \Psi \mathbf{q}(j)\mathbf{q}(j)^H\}, \quad (6.36)$$

$$E_2(\Psi) = E\{\mathbf{q}(j)\mathbf{q}(j)^H \Psi \mathbf{q}(j-1)\mathbf{q}(j-1)^H\}, \quad (6.37)$$

and

$$E_3(\Psi) = E\{\mathbf{q}(j)\mathbf{q}(j)^H \Psi \mathbf{q}(j-2)\mathbf{q}(j-2)^H\}. \quad (6.38)$$

Proof: Please refer to Appendix A.

In the followings, we will derive the expressions for $E_1(\Psi)$, $E_2(\Psi)$, and $E_3(\Psi)$. $E_1(\Psi)$ is the auto-correlation matrix of $\mathbf{q}(j)\mathbf{q}(j)^H$ in terms of Ψ . Note that in our system model, $\mathbf{B}(j)$ is a real-value vector and $\mathbf{N}(j)$ is a complex-value vector.

After some algebraic manipulations, it can be verified that [71]

$$E_1(\Psi) = \kappa_{4b} \boldsymbol{\theta} [\mathbf{I}_{3K} \odot (\boldsymbol{\theta}^H \Psi \boldsymbol{\theta})] \boldsymbol{\theta}^H + \mathbf{R}_q \Psi \mathbf{R}_q + \text{tr}(\mathbf{R}_q \Psi) \mathbf{R}_q + \boldsymbol{\theta} \boldsymbol{\theta}^T \Psi^T \boldsymbol{\theta}^* \boldsymbol{\theta}^H \quad (6.39)$$

where κ_{4b} is the fourth-order cumulant κ_4 as defined in Chapter 3.

Proposition 2: If the CDMA system is modeled based on (6.1), then the expressions of

$E_2(\Psi)$ and $E_3(\Psi)$ are given by

$$E_2(\Psi) = \kappa_{4b} \boldsymbol{\theta} [\mathbf{D}_2 \odot (\boldsymbol{\theta}^H \Psi \boldsymbol{\theta})] \boldsymbol{\theta}^H + \mathbf{R}_q \Psi \mathbf{R}_q + \text{tr}(\mathbf{Z}_1^H \Psi) \mathbf{Z}_1 + \boldsymbol{\theta} \mathbf{D}_2 \boldsymbol{\theta}^T \Psi^T \boldsymbol{\theta}^* \mathbf{D}_2 \boldsymbol{\theta}^H \quad (6.40)$$

and

$$E_3(\Psi) = \kappa_{4b} \theta \left[\mathbf{D}_3 \odot (\theta^H \Psi \theta) \right] \theta^H + \mathbf{R}_q \Psi \mathbf{R}_q + \text{tr}(\mathbf{D}_3^T \theta^H \Psi \theta) \theta \mathbf{D}_3 \theta^H + \theta \mathbf{D}_3 \theta^T \Psi^T \theta^* \mathbf{D}_3 \theta^H, \quad (6.41)$$

respectively, where

$$\mathbf{Z}_1 = \theta \mathbf{D}_2 \theta^H + \sigma_n^2 \mathbf{D}_1,$$

$$\mathbf{D}_1 = \begin{pmatrix} \mathbf{0} & \mathbf{I}_P \\ \mathbf{0} & \mathbf{0} \end{pmatrix}_{2P \times 2P},$$

$$\mathbf{D}_2 = [\mathbf{a}_1, \dots, \mathbf{a}_{3K}]_{3K \times 3K},$$

and

$$\mathbf{D}_3 = [\boldsymbol{\beta}_1, \dots, \boldsymbol{\beta}_{3K}]_{3K \times 3K}.$$

Note that

$$\boldsymbol{\alpha}_l^T = \begin{cases} \left[0, \dots, 0, \underset{(l-1)\text{th}}{1}, 0, \dots, 0 \right]_{1 \times 3K} & l \neq 3k-2, \quad k=1, \dots, K \\ \left[0, \dots, 0, \dots, 0 \right]_{1 \times 3K} & l = 3k-2, \quad k=1, \dots, K \end{cases} \quad (6.42)$$

and

$$\boldsymbol{\beta}_l^T = \begin{cases} \left[0, \dots, 0, \underset{(l-2)\text{th}}{1}, 0, \dots, 0 \right]_{1 \times 3K} & l = 3k, \quad k=1, \dots, K \\ \left[0, \dots, 0, \dots, 0 \right]_{1 \times 3K} & l \neq 3k, \quad k=1, \dots, K \end{cases}. \quad (6.43)$$

Proof: Please refer to Appendix B.

Substituting (6.39)-(6.41) into (6.35), the explicit closed-form expression of $\lambda(l, u, J, \Psi)$ can be obtained. In a static environment, the number of users is fixed, thus the covariance $\mathbf{R}_q$ of the received signal is fixed. In this case, by utilizing (6.6), $\boldsymbol{\eta}(J, \Psi)$ can be obtained as

$$\boldsymbol{\eta}(J, \Psi) = E \left\{ \left(\hat{\mathbf{R}}_q - \mathbf{R}_q \right) \Psi \left(\hat{\mathbf{R}}_q - \mathbf{R}_q \right) \right\}$$

$$= \left(\frac{1-\gamma}{1-\gamma^J} \right)^2 E \left\{ \left(\sum_{j=1}^J \gamma^{J-j} \mathbf{q}(j) \mathbf{q}(j)^H \right) \Psi \left(\sum_{j=1}^J \gamma^{J-j} \mathbf{q}(j) \mathbf{q}(j)^H \right) \right\} - \mathbf{R}_q \Psi \mathbf{R}_q. \quad (6.44)$$

Based on (6.34) and let $l = 1$ and $u = J$, (6.44) can be rewritten as

$$\boldsymbol{\eta}(J, \Psi) = \left(\frac{1-\gamma}{1-\gamma^J} \right)^2 \lambda(1, J, J, \Psi) - \mathbf{R}_q \Psi \mathbf{R}_q. \quad (6.45)$$

From (6.45), it can be observed that for a fixed J , different forgetting factor γ will result in different $\boldsymbol{\eta}(J, \Psi)$. Furthermore, it can be verified that when the number of iterations J approaches infinity, the limit of $\boldsymbol{\eta}(J, \Psi)$ will have the following form

$$\begin{aligned} \lim_{J \rightarrow +\infty} \boldsymbol{\eta}(J, \Psi) &= \frac{(1-\gamma)}{(1+\gamma)} \left[E_1(\Psi) + \gamma (E_2(\Psi) + E_2^H(\Psi^H)) \right. \\ &\quad \left. + \gamma^2 (E_3(\Psi) + E_3^H(\Psi^H)) - (1 + 2\gamma + 2\gamma^2) \mathbf{R}_q \Psi \mathbf{R}_q \right]. \end{aligned} \quad (6.46)$$

This property shows that for adaptive algorithms, $\boldsymbol{\eta}(J, \Psi)$ will not converge to zero even when J approaches infinity. Note that for a batch algorithm, conversely, the covariance of $\Delta \mathbf{R}_q$ in terms of a weighted matrix will converge to zero when J is sufficiently large [71].

In the case of a dynamic environment, users may be randomly added into or removed from the system. As a result, the covariance of the received signal $\mathbf{R}_q$ is not fixed. Consider a dynamic system where initially the received signal is $\mathbf{q}_1(j)$ and the covariance of $\mathbf{q}_1(j)$ is $\mathbf{R}_{q_1}$, at the J_c th interval, some users are added into or removed from this system such that the received signal becomes $\mathbf{q}_2(j)$ and the covariance of $\mathbf{q}_2(j)$ is $\mathbf{R}_{q_2}$. Therefore after the J_c th interval, $\Delta \mathbf{R}_q = \hat{\mathbf{R}}_q - \mathbf{R}_{q_2}$ and $\boldsymbol{\eta}(J, \Psi)$ can be approximated by

$$\begin{aligned} \eta(J, \Psi) = E \left\{ \left[\frac{1-\gamma}{1-\gamma^J} \left(\sum_{j=1}^{J_c} \gamma^{J-j} \mathbf{q}_1(j) \mathbf{q}_1^H(j) + \sum_{j=J_c+1}^J \gamma^{J-j} \mathbf{q}_2(j) \mathbf{q}_2^H(j) \right) - \mathbf{R}_{\mathbf{q}_2} \right] \right. \\ \left. \cdot \Psi \left[\frac{1-\gamma}{1-\gamma^J} \left(\sum_{j=1}^{J_c} \gamma^{J-j} \mathbf{q}_1(j) \mathbf{q}_1^H(j) + \sum_{j=J_c+1}^J \gamma^{J-j} \mathbf{q}_2(j) \mathbf{q}_2^H(j) \right) - \mathbf{R}_{\mathbf{q}_2} \right] \right\}. \end{aligned} \quad (6.47)$$

Substituting (6.34) into (6.47), we have

$$\begin{aligned} \eta(J, \Psi) = \left(\frac{1-\gamma}{1-\gamma^J} \right)^2 \left[\lambda_1(1, J_c, J, \Psi) + \lambda_2(J_c+1, J, J, \Psi) \right] \\ + \frac{\gamma^{J-J_c} - \gamma^{2J-2J_c} - \gamma^J + \gamma^{2J-J_c}}{(1-\gamma^J)^2} (\mathbf{R}_{\mathbf{q}_1} \Psi \mathbf{R}_{\mathbf{q}_2} + \mathbf{R}_{\mathbf{q}_2} \Psi \mathbf{R}_{\mathbf{q}_1}) + \frac{2\gamma^{J-J_c} - \gamma^J - 1}{1-\gamma^J} \mathbf{R}_{\mathbf{q}_2} \Psi \mathbf{R}_{\mathbf{q}_2} \end{aligned} \quad (6.48)$$

where $\lambda_1(1, J_c, J, \Psi)$ and $\lambda_2(J_c+1, J, J, \Psi)$ correspond to the systems of $\mathbf{q}_1(j)$ and $\mathbf{q}_2(j)$, respectively. After some algebraic manipulations, it can be verified that when the number of iterations J approaches infinity, the limit of (6.48) has a similar form as (6.46) by replacing $\mathbf{R}$ with $\mathbf{R}_{\mathbf{q}_2}$, i.e.,

$$\begin{aligned} \lim_{J \rightarrow +\infty} \eta(J, \Psi) = \frac{(1-\gamma)}{(1+\gamma)} [E_1(\Psi) + \gamma(E_2(\Psi) + E_2^H(\Psi^H))] \\ + \gamma^2(E_3(\Psi) + E_3^H(\Psi^H)) - (1+2\gamma+2\gamma^2) \mathbf{R}_{\mathbf{q}_2} \Psi \mathbf{R}_{\mathbf{q}_2} \end{aligned} \quad (6.49)$$

where $E_1(\Psi)$, $E_2(\Psi)$, and $E_3(\Psi)$ are all corresponding to the system of $\mathbf{q}_2(j)$.

Equation (6.49) shows that the effect of the previous received signal will vanish as J approaches infinity. In the followings, we will make use of the above results to derive the covariance matrix of different detectors.

6.3.2.2 Perturbation Analysis Based on Blind Adaptive MUD

Let $\mathbf{S}_\theta$ be the noise-free data covariance matrix given by

$$\mathbf{S}_\theta = \boldsymbol{\theta}\boldsymbol{\theta}^H = \mathbf{U}_q^s \boldsymbol{\Omega}_\theta \mathbf{U}_q^{sH} \quad (6.50)$$

where $\boldsymbol{\Omega}_\theta$ is the diagonal matrix of the eigenvalue corresponding to the signal subspace of $\mathbf{S}_\theta$. According to (6.19), the first-order perturbation of channel estimation $\hat{\mathbf{g}}_k$ is given by [71]

$$\Delta \mathbf{g}_k \approx \mathbf{G}_k^+ \mathbf{C}_k^H \mathbf{U}_q^n \mathbf{U}_q^{nH} \Delta \mathbf{R}_q \mathbf{S}_\theta^+ \mathbf{C}_k \mathbf{g}_k. \quad (6.51)$$

Next, we will derive $E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$ for different kinds of detectors.

1). DMI MMSE Detector

According to (6.24) and (6.25), the first-order perturbation of the DMI MMSE detector can be obtained as

$$\Delta \mathbf{m}_{\text{DMI}} = \hat{\mathbf{m}}_{\text{DMI}} e^{-j\phi} - \mathbf{m}_{\text{DMI}} \approx \mathbf{R}_q^{-1} \mathbf{C}_1 \Delta \mathbf{g}_1 - \mathbf{R}_q^{-1} \Delta \mathbf{R}_q \mathbf{R}_q^{-1} \mathbf{C}_1 \mathbf{g}_1 \quad (6.52)$$

where we used the Taylor's expansion $(\mathbf{R}_q + \Delta \mathbf{R}_q)^{-1} \approx \mathbf{R}_q^{-1} - \mathbf{R}_q^{-1} \Delta \mathbf{R}_q \mathbf{R}_q^{-1}$. According to (6.20) and (6.51), $\Delta \mathbf{R}_q$ and $\Delta \mathbf{g}_1$ have zero mean. Therefore, $\Delta \mathbf{m}_{\text{DMI}}$ also has zero mean, which means that $\hat{\mathbf{m}}_{\text{DMI}} e^{-j\phi}$ is an unbiased estimate of $\mathbf{m}_{\text{DMI}}$. The covariance matrix of the estimated detector can be represented by

$$\begin{aligned} E\{\Delta \mathbf{m}_{\text{DMI}} \Delta \mathbf{m}_{\text{DMI}}^H\} &= E\left\{\left(\mathbf{R}_q^{-1} \mathbf{C}_1 \Delta \mathbf{g}_1 - \mathbf{R}_q^{-1} \Delta \mathbf{R}_q \mathbf{R}_q^{-1} \mathbf{H}_{1,0}\right)\left(\mathbf{R}_q^{-1} \mathbf{C}_1 \Delta \mathbf{g}_1 - \mathbf{R}_q^{-1} \Delta \mathbf{R}_q \mathbf{R}_q^{-1} \mathbf{H}_{1,0}\right)^H\right\} \\ &= \mathbf{R}_q^{-1} \left\{E\left\{\mathbf{C}_1 \Delta \mathbf{g}_1 \Delta \mathbf{g}_1^H \mathbf{C}_1^H\right\} - E\left\{\mathbf{C}_1 \Delta \mathbf{g}_1 \mathbf{H}_{1,0}^H \mathbf{R}_q^{-1} \Delta \mathbf{R}_q\right\} - E\left\{\Delta \mathbf{R}_q \mathbf{R}_q^{-1} \mathbf{H}_{1,0} \Delta \mathbf{g}_1^H \mathbf{C}_1^H\right\}\right. \\ &\quad \left.+ E\left\{\Delta \mathbf{R}_q \mathbf{R}_q^{-1} \mathbf{H}_{1,0} \mathbf{H}_{1,0}^H \mathbf{R}_q^{-1} \Delta \mathbf{R}_q\right\}\right\} \mathbf{R}_q^{-1}. \end{aligned} \quad (6.53)$$

The above equation illustrates that $E\{\Delta \mathbf{m}_{\text{DMI}} \Delta \mathbf{m}_{\text{DMI}}^H\}$ consists of four expectation terms. By substituting (6.51) into (6.53), it can be observed that each of these expectation terms can be represented by $E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}$. Therefore, $E\{\Delta \mathbf{m}_{\text{DMI}} \Delta \mathbf{m}_{\text{DMI}}^H\}$ can be obtained by utilizing (6.45) or (6.48).

2). Subspace MMSE Detector

By taking the difference between (6.26) and (6.27), the first-order perturbation of the subspace MMSE detector is given by

$$\begin{aligned} \Delta \mathbf{m}_{\text{MMSE}} &= \hat{\mathbf{m}}_{\text{MMSE}} e^{-j\phi} - \mathbf{m}_{\text{MMSE}} \\ &= \Delta \mathbf{U}_q^s (\Lambda_q^s)^{-1} \mathbf{U}_q^{sH} \mathbf{C}_1 \mathbf{g}_1 - \mathbf{U}_q^s (\Lambda_q^s)^{-1} \Delta \Lambda_q^s (\Lambda_q^s)^{-1} \mathbf{U}_q^{sH} \mathbf{C}_1 \mathbf{g}_1 \\ &\quad + \mathbf{U}_q^s (\Lambda_q^s)^{-1} \Delta \mathbf{U}_q^{sH} \mathbf{C}_1 \mathbf{g}_1 + \mathbf{U}_q^s (\Lambda_q^s)^{-1} \mathbf{U}_q^{sH} \mathbf{C}_1 \Delta \mathbf{g}_1 \end{aligned} \quad (6.54)$$

where we use the Taylor's expansion $(\Lambda_q^s + \Delta \Lambda_q^s)^{-1} \approx (\Lambda_q^s)^{-1} - (\Lambda_q^s)^{-1} \Delta \Lambda_q^s (\Lambda_q^s)^{-1}$.

Utilizing the perturbation analysis for subspace decomposition [67], we obtain

$$\Delta \mathbf{U}_q^s = \mathbf{U}_q^n \mathbf{U}_q^{nH} \Delta \mathbf{R}_q \mathbf{U}_q^s \mathbf{\Omega}_0^{-1}, \text{ and } \Delta \Lambda_q^s = \mathbf{U}_q^{sH} \Delta \mathbf{R}_q \mathbf{U}_q^s. \quad (6.55)$$

By denoting $\omega_1 = \mathbf{U}_q^n \mathbf{U}_q^{nH}$, $\omega_2 = \mathbf{U}_q^s \mathbf{\Omega}_0^{-1} \mathbf{U}_q^{sH}$, $\omega_3 = \mathbf{U}_q^s (\Lambda_q^s)^{-1} \mathbf{U}_q^{sH}$, $\omega_4 = \mathbf{C}_1 \mathbf{G}_1^+ \mathbf{C}_1^H$, and $\omega_5 = \mathbf{S}_0^+ \mathbf{C}_1 \mathbf{g}_1$, (6.54) becomes

$$\Delta \mathbf{m}_{\text{MMSE}} = \omega_1 \Delta \mathbf{R}_q \omega_2 \omega_3 \mathbf{H}_{1,0} - \omega_3 \Delta \mathbf{R}_q \omega_3 \mathbf{H}_{1,0} + \omega_3 \omega_4 \omega_1 \Delta \mathbf{R}_q \omega_5 \quad (6.56)$$

where we use the fact that $\mathbf{U}_q^{nH} \mathbf{C}_1 \mathbf{g}_1 = 0$. Since $\Delta \mathbf{R}_q$ has zero mean, $\Delta \mathbf{m}_{\text{MMSE}}$ also has zero mean, which means that $\hat{\mathbf{m}}_{\text{MMSE}} e^{-j\phi}$ is an unbiased estimate of $\mathbf{m}_{\text{MMSE}}$. From (6.56), we can deduce that the covariance $E\{\Delta \mathbf{m}_{\text{MMSE}} \Delta \mathbf{m}_{\text{MMSE}}^H\}$ consists of nine expectation

terms. Each of these expectation terms is a function of $E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}$ and can be calculated by (6.45) or (6.48).

3). Subspace ZF Detector

For the subspace ZF detector, σ_n^2 is estimated by

$$\hat{\sigma}_n^2 = \frac{1}{2P-d} \text{tr}(\hat{\Lambda}_q^n). \quad (6.57)$$

Utilizing the perturbation analytical results for subspace decomposition [67], the perturbation of $\mathbf{m}_{ZF}$ becomes

$$\begin{aligned} \Delta \mathbf{m}_{ZF} &= \hat{\mathbf{m}}_{ZF} e^{-j\phi} - \mathbf{m}_{ZF} \\ &= \omega_1 \Delta \mathbf{R}_q \omega_2 \omega_2^H \mathbf{H}_{1,0} - \omega_2 \Delta \mathbf{R}_q \omega_2^H \mathbf{H}_{1,0} + \left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i] \right) \mathbf{I}_{2P} \omega_2 \omega_2^H \mathbf{H}_{1,0} \\ &\quad + \omega_2 \omega_4 \omega_1 \Delta \mathbf{R}_q \omega_5 \end{aligned} \quad (6.58)$$

where $\mathbf{U}_q^n[:,i]$ denotes the i th column of $\mathbf{U}_q^n$. Equation (6.58) shows that $\hat{\mathbf{m}}_{ZF} e^{-j\phi}$ is an unbiased estimate of $\mathbf{m}_{ZF}$. It can be observed that the covariance $E\{\Delta \mathbf{m}_{ZF} \Delta \mathbf{m}_{ZF}^H\}$ includes the expectation terms of

$$E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}, \quad (6.59)$$

$$E\left\{\left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i]\right) \mathbf{I}_{2P} \Psi \mathbf{I}_{2P} \left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i]\right)\right\}, \quad (6.60)$$

and

$$E\left\{\left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i]\right) \mathbf{I}_{2P} \Psi \mathbf{I}_{2P} \left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i]\right)\right\}. \quad (6.61)$$

We denote $\mathbf{W}_p$ as the p th row of (6.60), thus $\mathbf{W}_p$ can be obtained by

$$\mathbf{W}_p = \frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] E\{\Delta \mathbf{R}_q \mathbf{U}_q^n[:,i] \Psi[p,:]\Delta \mathbf{R}_q\} \quad (6.62)$$

where $\Psi[p,:]$ is the p th row of Ψ . It can be observed that (6.62) is a function of $E\{\Delta \mathbf{R}_q \bar{\Psi} \Delta \mathbf{R}_q\}$, where $\bar{\Psi} = \mathbf{U}_q^n[:,i] \Psi[p,:]$. Furthermore, we have

$$\begin{aligned} & E\left\{\left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i]\right) \mathbf{I}_{2P} \Psi \mathbf{I}_{2P} \left(\frac{1}{2P-d} \sum_{i=1}^{2P-d} \mathbf{U}_q^{nH}[:,i] \Delta \mathbf{R}_q \mathbf{U}_q^n[:,i]\right)\right\} \\ &= \left[\left(\frac{1}{2P-d}\right)^2 \sum_{i_1=1}^{2P-d} \sum_{i_2=1}^{2P-d} \mathbf{U}_q^{nH}[:,i_1] E\{\Delta \mathbf{R}_q \mathbf{U}_q^n[:,i_1] \mathbf{U}_q^{nH}[:,i_2] \Delta \mathbf{R}_q\} \mathbf{U}_q^n[:,i_2]\right] \mathbf{I}_{2P} \Psi \mathbf{I}_{2P} \end{aligned} \quad (6.63)$$

which is also a function of $E\{\Delta \mathbf{R}_q \bar{\Psi} \Delta \mathbf{R}_q\}$, where $\bar{\Psi} = \mathbf{U}_q^n[:,i_1] \mathbf{U}_q^{nH}[:,i_2]$. Therefore, all the terms of $E\{\Delta \mathbf{m}_{ZF} \Delta \mathbf{m}_{ZF}^H\}$ can be expressed as a function of $E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}$ and can be calculated by (6.45) or (6.48).

4). Group-Blind Multiuser Detector

Denoting $\mathbf{X}_{GB} = \boldsymbol{\theta}_{\bar{K}}^H \mathbf{U}_q^s (\boldsymbol{\Lambda}_q^s)^{-1} \mathbf{U}_q^{sH} \boldsymbol{\theta}_{\bar{K}}$ and based on (6.30) and (6.31), the first-order perturbation of $\mathbf{m}_{GB}$ becomes

$$\begin{aligned} \Delta \mathbf{m}_{GB} &= \hat{\mathbf{m}}_{GB} e^{-j\phi} - \mathbf{m}_{GB} \\ &= \omega_1 \Delta \mathbf{R}_q \omega_2 \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \mathbf{e}_1 - \omega_3 \Delta \mathbf{R}_q \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \mathbf{e}_1 + \omega_3 \Delta \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \mathbf{e}_1 \\ &\quad - \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \Delta \boldsymbol{\theta}_{\bar{K}}^H \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \mathbf{e}_1 + \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \boldsymbol{\theta}_{\bar{K}}^H \omega_3 \Delta \mathbf{R}_q \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \mathbf{e}_1 \\ &\quad - \omega_3 \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \boldsymbol{\theta}_{\bar{K}}^H \omega_3 \Delta \boldsymbol{\theta}_{\bar{K}} \mathbf{X}_{GB}^{-1} \mathbf{e}_1 \end{aligned} \quad (6.64)$$

where we use the Taylor's expansion $(\mathbf{X}_{GB} + \Delta \mathbf{X}_{GB})^{-1} \approx \mathbf{X}_{GB}^{-1} - \mathbf{X}_{GB}^{-1} \Delta \mathbf{X}_{GB} \mathbf{X}_{GB}^{-1}$ and the fact that $\mathbf{U}_q^{nH} \mathbf{C}_k \mathbf{g}_k = 0$. The perturbation of $\boldsymbol{\theta}_{\bar{K}}$ can be represented as

$$\Delta \boldsymbol{\theta}_{\bar{K}} = [\Delta \mathbf{H}_1, \dots, \Delta \mathbf{H}_{\bar{K}}] = [\mathbf{C}_1 \Delta \mathbf{g}_1, \dots, \mathbf{C}_{\bar{K}} \Delta \mathbf{g}_{\bar{K}}]. \quad (6.65)$$

From (6.51), $\Delta \mathbf{g}_k$ has zero mean and it can be shown that $\hat{\mathbf{m}}_{\text{GB}} e^{-j\phi}$ is an unbiased estimate of $\mathbf{m}_{\text{GB}}$. According to (6.64), in order to calculate the covariance $E\{\Delta \mathbf{m}_{\text{GB}} \Delta \mathbf{m}_{\text{GB}}^H\}$, we need to calculate the terms $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \mathbf{R}_q\}$, $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\}$, $E\{\Delta \mathbf{R}_q \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\}$, $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}^H\}$, and $E\{\Delta \boldsymbol{\theta}_{\bar{K}}^H \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\}$. Based on (6.65) we have

$$E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \mathbf{R}_q\} = \sum_{k=1}^{\bar{K}} E\{\mathbf{C}_k \Delta \mathbf{g}_k \boldsymbol{\Psi}[k, :] \Delta \mathbf{R}_q\}. \quad (6.66)$$

Substituting (6.51) into (6.66), we obtain

$$E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \mathbf{R}_q\} = \sum_{k=1}^{\bar{K}} \mathbf{C}_k \mathbf{G}_k^+ \mathbf{C}_k^H \boldsymbol{\omega}_1 E\{\Delta \mathbf{R}_q \mathbf{S}_0^+ \mathbf{C}_k \mathbf{g}_k \boldsymbol{\Psi}[k, :] \Delta \mathbf{R}_q\} \quad (6.67)$$

which is a function of $E\{\Delta \mathbf{R}_q \bar{\boldsymbol{\Psi}} \Delta \mathbf{R}_q\}$ where $\bar{\boldsymbol{\Psi}} = \mathbf{S}_0^+ \mathbf{C}_k \mathbf{g}_k \boldsymbol{\Psi}[k, :]$. We denote $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\} = [\bar{\mathbf{W}}_1, \dots, \bar{\mathbf{W}}_{\bar{K}}]$, where $\bar{\mathbf{W}}_k$ is the k th column of $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\}$, thus $\bar{\mathbf{W}}_k$ can be represented by $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \mathbf{C}_k \mathbf{G}_k^+ \mathbf{C}_k^H \boldsymbol{\omega}_1 \Delta \mathbf{R}_q\} \mathbf{S}_0^+ \mathbf{C}_k \mathbf{g}_k$ which has a similar form as (6.67).

For $E\{\Delta \mathbf{R}_q \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\}$, we denote $E\{\Delta \mathbf{R}_q \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\} = [\boldsymbol{\Xi}_1, \dots, \boldsymbol{\Xi}_{\bar{K}}]$ where $\boldsymbol{\Xi}_k$ is the k th column of $E\{\Delta \mathbf{R}_q \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}\}$, therefore $\boldsymbol{\Xi}_k$ can be obtained by

$$\boldsymbol{\Xi}_k = E\{\Delta \mathbf{R}_q \boldsymbol{\Psi} \mathbf{C}_k \mathbf{G}_k^+ \mathbf{C}_k^H \boldsymbol{\omega}_1 \Delta \mathbf{R}_q\} \mathbf{S}_0^+ \mathbf{C}_k \mathbf{g}_k \quad (6.68)$$

which is a function of $E\{\Delta \mathbf{R}_q \bar{\boldsymbol{\Psi}} \Delta \mathbf{R}_q\}$, where $\bar{\boldsymbol{\Psi}} = \boldsymbol{\Psi} \mathbf{C}_k \mathbf{G}_k^+ \mathbf{C}_k^H \boldsymbol{\omega}_1$. In addition, $E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}^H\}$ can be represented as

$$E\{\Delta \boldsymbol{\theta}_{\bar{K}} \boldsymbol{\Psi} \Delta \boldsymbol{\theta}_{\bar{K}}^H\} = \sum_{k_2=1}^{\bar{K}} \sum_{k_1=1}^{\bar{K}} \mathbf{C}_{k_1} E\{\Delta \mathbf{g}_{k_1} \boldsymbol{\Psi}[k_1, k_2] \Delta \mathbf{g}_{k_2}^H\} \mathbf{C}_{k_2}^H \quad (6.69)$$

where $\Psi[k_1, k_2]$ is the element at the k_1 th row and k_2 th column of Ψ . By substituting (6.51) into (6.69), it can be shown that (6.69) is also a function of $E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}$. Finally the (i, j) th element of $E\{\Delta \boldsymbol{\theta}_{\bar{k}}^H \Psi \Delta \boldsymbol{\theta}_{\bar{k}}\}$ is a scalar $E\{\Delta \boldsymbol{\theta}_{\bar{k}}[:, i]^H \Psi \Delta \boldsymbol{\theta}_{\bar{k}}[:, j]\}$, which can be written as $\text{tr}\left(E\{\Delta \boldsymbol{\theta}_{\bar{k}}[:, j] \Delta \boldsymbol{\theta}_{\bar{k}}[:, i]^H\} \Psi\right)$. Therefore, it is also a function of $E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}$.

From the above analyses, it has been shown that all the terms of $E\{\Delta \mathbf{m}_{\text{GB}} \Delta \mathbf{m}_{\text{GB}}^H\}$ can be expressed as a function of $E\{\Delta \mathbf{R}_q \Psi \Delta \mathbf{R}_q\}$ and can be calculated by (6.45) or (6.48). Thus far, we have derived the closed-form expressions of $E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$ for all four types of detectors. Consequently, the MSE of each detector can be obtained by $E\{\|\Delta \mathbf{m}\|^2\} = \text{tr}\left(E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}\right)$. In the followings, we will show that the analytical perturbation of both SINR and BER can be represented by a function of $E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$.

6.3.2.3 Performance Analysis of SINR and BER

According to (6.1), the SINR is defined as [17], [18]

$$\hat{\text{SINR}} = \frac{E\left\{\|A_1 \hat{\mathbf{m}}^H \mathbf{H}_{1,0}\|^2\right\}}{E\left\{\|\hat{\mathbf{m}}^H (\mathbf{q} - A_1 \mathbf{H}_{1,0} b_1)\|^2\right\}}. \quad (6.70)$$

Since $E\{\Delta \mathbf{m}\} = 0$, the estimated SINR is given by (3.40) and reproduced here as

$$\hat{\text{SINR}} = \frac{A_1^2 \mathbf{H}_{1,0}^H \mathbf{m} \mathbf{m}^H \mathbf{H}_{1,0} + A_1^2 \mathbf{H}_{1,0}^H E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\} \mathbf{H}_{1,0}}{\text{tr}(\bar{\boldsymbol{\theta}}^H \mathbf{m} \mathbf{m}^H \bar{\boldsymbol{\theta}}) + \sigma_n^2 \|\mathbf{m}\|^2 + \text{tr}(\bar{\boldsymbol{\theta}}^H E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\} \bar{\boldsymbol{\theta}}) + \sigma_n^2 \text{tr}(E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\})} \quad (6.71)$$

where $\bar{\mathbf{0}} = \begin{bmatrix} A_1 \bar{\mathbf{H}}_1 & A_2 \mathbf{H}_2 & \cdots & A_K \mathbf{H}_K \end{bmatrix}_{2P \times (3K-1)}$. For the BER performance, the closed-form expression corresponding to the differential BPSK is given by (3.44) and reproduced here as

$$P_e = Q_1(a, b) - \alpha I_0(ab) e^{-(a^2+b^2)/2} \quad (6.72)$$

where a , b and α have been defined in (3.45)-(3.47).

6.4 Numerical Results and Discussions

In this section, several simulations will be conducted to study the performance of the proposed RO-FST based MMSE detector and verify the analyses provided in Section 6.3. The numerical results are obtained by averaging over 100 independent simulations. Gold sequences of length 31 are used as the PN sequences. In each simulation, the parameters of channel information for all users are randomly generated according to the multipath fading channel model presented in Section 2.1.3.2. Specifically, the delay of each path for user k is generated from a uniform distribution in $[\tau_{d_k}, \tau_{d_k} + \tau_m]$, where the delay of the k th user τ_{d_k} is generated from a uniform distribution in $[0, 10T_c]$, and the maximum multipath delay spread for all user $\tau_m = 20T_c$. The path gain of each path for k th user $g_k(m)$ is generated according to a complex Gaussian distribution with zero mean and is assumed to be the same for the entire frame length. The path gains in each user's channel are normalized so that the total path gain of each user is 1 [21], [31], [32]. It is assumed that the delay of each user is not available to us. For a comparatively rapid convergence, we choose the forgetting factor γ to be 0.998 [66].

In Fig. 6.2, we compare the SINR performance of the proposed RO-FST based DMI MMSE detector with the subspace MMSE detectors using two existing subspace tracking algorithms, i.e., the NAHJ-FST and PASTd algorithms with SNR=15dB. It is assumed that there are five equal-power active users in the system and the first user is the desired user. The NAHJ-FST algorithm in [21] is a partial implementation of the well-known symmetric Jacobi SVD algorithm [66] and has the same order of computational complexity as the RO-FST algorithm, i.e., $O(LPd)$. However the NAHJ-FST algorithm cannot ensure to annihilate all of the off-diagonal elements since it does not utilize a complete set of Givens rotations. Therefore implementing the NAHJ-FST algorithm will give rise to worse performance. The PASTd algorithm was applied to CDMA systems over a multipath fading channel in [65], in which it is assumed that the delay of the desired user is available to us. The advantage of the PASTd algorithm is its low computational complexity. However, this algorithm cannot maintain the orthonormality of the estimated signal subspace and noise subspace. Therefore the performance of this detector will be deteriorated. From Fig. 6.2, we observe that the proposed algorithm can approach its convergence rapidly. After about 500 iterations, the value of SINR of the proposed detector is above 10dB. Note that although all of these three detectors are based on the MMSE criteria, the performance of the DMI MMSE detector is theoretically worse than that of the subspace MMSE detector. This is because for the DMI MMSE detector, it has an additional term $\hat{\mathbf{U}}_q^n (\hat{\mathbf{\Lambda}}_q^n)^{-1} \hat{\mathbf{U}}_q^{nH} \mathbf{C}_1 \hat{\mathbf{g}}_1$ after $\hat{\mathbf{R}}_q$ is decomposed as compared with the subspace MMSE detector [16], [17]. This noise component will cause a large error at high SNR because of $(\hat{\mathbf{\Lambda}}_q^n)^{-1}$ and nonorthogonality of $\mathbf{C}_1 \hat{\mathbf{g}}_1$ to the estimated noise subspace $\hat{\mathbf{U}}_q^n$. From Fig. 6.2, however, it can be observed that because of

the robustness of the RO-FST algorithm, the SINR performance of the proposed DMI MMSE detector is better than that of the subspace MMSE detectors using NAHJ-FST and PASTd algorithms.

In Fig. 6.3, we present the SINR performance under different SNR levels, which is obtained after 5000 iterations when each of the adaptive algorithms has approached their convergence respectively. This figure shows that the proposed DMI MMSE detector can outperform the other two existing subspace MMSE detector even for a low SNR level. From these two figures, we can conclude that the proposed RO-FST based DMI MMSE detector has better SINR performance as compared with the MMSE detectors using NAHJ-FST and PASTd algorithms.

In Fig. 6.4, the BER results of these three detectors are provided. We calculate the BER after 5000 iterations. This figure shows that the proposed RO-FST based detector can provide better BER performance as compared with the detectors using the NAHJ-FST and PASTd algorithms. At SNR=14dB, the BER of the proposed algorithm is about 10^{-5} , whereas the BER of the NAHJ-FST algorithm is about 10^{-4} . Therefore, it can be concluded that the BER performance of the proposed algorithm is better than that of the NAHJ-FST and PASTd algorithms.

In Fig. 6.5, we illustrate the tracking capability of the proposed adaptive detector under a dynamic environment with a time-varying number of active users and SNR=15dB. When the number of iterations is less than 2000, there are five equal-power users in the system. At $J=2000$, five new users are added into this system, and each of them has the power level of 10dB higher than that of the desired user. At $J=3500$, three out of these new users are disconnected from this system. Fig. 6.5 shows that the proposed adaptive detector can rapidly approach its convergence again, and convergence

rate is faster than other two detectors. The final SINR of the proposed adaptive detector is almost the same as before. Therefore, the proposed RO-FST detector has good tracking capability under a dynamic environment. In addition, this simulation also shows that the proposed RO-FST detector can work well under a near-far situation. For comparison, we also provide the SINR performance of subspace MMSE detectors using NAHJ-FST and PASTd algorithms in Fig. 6.5. It can be observed that the SINR performance and tracking capabilities of these two detectors are poorer than those of the proposed RO-FST based detector.

In Fig. 6.6, we compare the SINR performance of the proposed RO-FST MUD with that of the proposed Kalman filter-based MUD using linear prediction proposed in Chapter 5. It is assumed that there are five equal-power users in the system and SNR is set to be 15 dB. We observe that the proposed RO-FST MUD can achieve higher SINR than the Kalman filter-based MUD. However, it should be noted that the advantage of the Kalman filter-based MUD is that it does not need to estimate the rank of the signal subspace of received signals. When the number of users is changed, the Kalman filter-based MUD also does not need to monitor the change of the rank. Note that the channel considered in Fig. 6.6 undergoes very slow time-varying multipath fading with Rayleigh distribution, which means that it is assumed that the parameter of channel response remains constant during the whole transmission packet duration. If we consider the time-varying channel with fast fading rate, however, the performance of the blind adaptive MUD will be deteriorated. Since the RO-FST MUD and the Kalman filter-based MUD only have the information of the desired user's PN sequence and do not rely on any training sequence, the multipath fading may be a fundamental limit on our proposed blind adaptive detectors and there will be obvious performance degradation [48]. In Figs.

6.7 and 6.8, we present the SINR performance of these two adaptive detectors under time-varying multipath fading channels, in which the fading channel coefficients are generated by the modified Jakes' simulator [81]. In Fig. 6.7, it is assumed that signals are transmitted at rate 10 Ksymbols/s using a 900 MHz carrier from a vehicle moving at 5km/h. That is, the Doppler frequency times the symbol interval is about 4.17×10^{-4} . From this figure, we can observe that both the RO-FST MUD and the Kalman filter-based MUD appear performance drop as compared with Fig. 6.6. In Fig. 6.8, we consider a more severe time-varying environment by increasing the velocity of vehicle to 120 km/h, which means that the Doppler frequency times the symbol interval 0.01. From this figure, we can observe that both of these two blind adaptive detectors can no longer work effectively and SINR is below 0 dB. Therefore, we can conclude that fast time-varying fading may significantly deteriorate the performance of the proposed blind adaptive MUD. Due to the lack of useful information, it seems difficult for the blind MUD to combat the fast time-varying fading. Therefore, designing robust blind adaptive detector which is validated in severely time-varying environments remains an important topic for future research. One possible solution is to develop faster tracking algorithms which can track the desired user's channel rapidly.

Next, we will provide some numerical results to verify the analytical SINR and BER expressions derived in Section 6.3. Here we assume that there are five equal-power active users in the system. For the group-blind detector, the number of known active user is set to be $\bar{K} = 3$. The forgetting factor γ is chosen to be 0.995, 0.998, and 0.9998, respectively.

Firstly, we will verify the MSE of different detectors by utilizing different forgetting factor γ . As mentioned in Section 6.3, the SINR and BER expressions can be

represented as a function of $E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$, and the MSE is obtained by calculating the trace of $E\{\Delta \mathbf{m} \Delta \mathbf{m}^H\}$. Although the MSE cannot directly reveal the SINR and BER performance, it is meaningful to present the MSE performance of the detectors to verify our analysis and simulation on MSE. We adopt the normalized norm of $E\{\|\Delta \mathbf{m}\|^2\}$ as the performance measure, which is given by

$$\varepsilon = \frac{E\{\|\Delta \mathbf{m}\|^2\}}{\|\mathbf{m}\|^2} = \frac{E\{\|\hat{\mathbf{m}}\|^2\} - \|\mathbf{m}\|^2}{\|\mathbf{m}\|^2} \quad (6.73)$$

where we use the fact that $E\{\hat{\mathbf{m}}\} = \mathbf{m}e^{j\phi}$. The MSE results of the DMI MMSE, subspace MMSE, subspace ZF, and group-blind detectors are shown in Figs. 6.9-6.12, respectively, with SNR=15dB. From these four figures, we observe that the simulation results match well with the corresponding analytical results, and the MSE decreases as the number of samples J increases. Furthermore, we observe that with a fixed J and as γ approaches 1, the MSE will decrease.

In Figs. 6.13- 6.16, we present the analytical and simulated SINR results of the four kinds of detectors, respectively. The SINR results are plotted against the number of iterations J with SNR=15dB and $\gamma = 0.995$, 0.998, and 0.9998, respectively. From these four figures, we observe that the simulated SINR results for each detector can match well with that of the analytical results, and a larger γ results in higher SINR. In Fig. 6.17, the analytical and simulated SINR results of the subspace MMSE detector are plotted against different SNR levels. Here we choose J to be 5000. It is shown that the analytical SINR results match with the simulated results well even under low SNR conditions. We also observe that a larger γ results in better SINR performance. Similar conclusions can be

obtained for the other three detectors though the corresponding numerical results are not presented here. Thus, we have validated the theoretical analyses of SINR performance of the blind adaptive detector as presented in Section 6.3.

In Figs. 6.18-6.21, we present the analytical and simulated BER results versus SNR for the four detectors, respectively. Here we choose $J=5000$, $K=5$, and $\gamma=0.995$, 0.998 , and 0.9998 . We observe from these four figures that the simulation results and the analytical results agree with each other for all the four detectors and the BER performance improves as the forgetting factor γ increases.

In Fig. 6.22, we present the theoretical and simulated SINR results of the blind adaptive subspace MMSE detector under a dynamic environment with $\gamma=0.998$ and $\text{SNR}=15\text{dB}$. It is assumed that there are 8 equal-power users when $J<1000$ and at $J=1000$, three of them are dropped from this system. We observe from Fig. 6.22 that the adaptive subspace MMSE detector can rapidly approach its convergence again and both the simulation results and the analytical results match well with each other. The above observation is also valid for the other three types of detectors though the corresponding results are not shown in this chapter. The analytical result for $\gamma=0.9998$ is also presented in Fig. 6.22 to illustrate the effects of different γ on the SINR performance. By comparing the analytical results with γ equals to 0.998 and 0.9998 , respectively, it can be observed that although higher γ will result in better SINR in a static environment, it will cost a relatively longer time for the system to re-converge when the number of users in the system changes. This is because a larger γ will make it more difficult to “forget” the previous system behavior for the detector, which results in slow system

convergence due to system alteration. Therefore, in a dynamic environment, γ should be set to a moderate value for good tracking capability [66].

The analyses in Section 6.3 are based on the exact SVD subspace tracking algorithm, which has the optimal performance among all the SVD-based subspace tracking algorithms. Therefore, the proposed analytical results can be served as performance bounds of other SVD-based blind adaptive subspace MUD algorithms. The simulated SINR and BER results of the proposed RO-FST based MMSE detector and the corresponding analytical results of the DMI MMSE detector using exact SVD algorithm are presented in Figs. 6.23 and 6.24, respectively, with $K=5$ and different forgetting factors γ . From these two figures, we observe that both of the simulated SINR and BER results of the proposed detector are close to the corresponding analytical results of the DMI MMSE detector using exact SVD algorithm. Thus we conclude that the proposed analytical SINR and BER expressions can be served as performance bounds of other blind adaptive SVD-based MUD algorithms.

Fig. 6.2 Comparisons of SINR performance among the proposed detector using RO-FST and the existing MMSE detectors using NAHJ-FST and PASTd, respectively, with SNR=15dB.

Fig. 6.3 Comparisons of SINR performance among the proposed detector using RO-FST, the MMSE detectors using NAHJ-FST and PASTd, respectively, with $K=5$.

Fig. 6.4 Comparisons of BER performance among the proposed detector using RO-FST, the MMSE detectors using NAHJ-FST and PASTd, respectively, with $K=5$.

Fig. 6.5 Comparisons of SINR performance among the proposed detector using RO-FST, the MMSE detectors using NAHJ-FST and PASTd, respectively, under a dynamic environment.

Fig. 6.6 Comparisons of SINR performance between the proposed detector using RO-FST and the Kalman filter-based MUD using linear prediction.

Fig. 6.7 Comparisons of SINR performance between the proposed detector using RO-FST and the Kalman filter-based MUD using linear prediction, under time-varying channels with $v=5\text{km/h}$.

Fig. 6.8 Comparisons of SINR performance between the proposed detector using RO-FST and the Kalman filter-based MUD using linear prediction, under time-varying channels with $v=120\text{km/h}$.

Fig. 6.9 Simulation and analytical MSE results of the blind DMI MMSE detector with $\text{SNR}=15\text{dB}$ and different forgetting factors γ .

Fig. 6.10 Simulation and analytical MSE results of the blind subspace MMSE detector with SNR=15dB and different forgetting factors γ .

Fig. 6.11 Simulation and analytical MSE results of the blind subspace ZF detector with SNR=15dB and different forgetting factors γ .

Fig. 6.12 Simulation and analytical MSE results of the group-blind detector with SNR=15dB and different forgetting factors γ .

Fig. 6.13 Simulation and analytical SINR results of the blind DMI MMSE detector with SNR=15dB and different forgetting factors γ .

Fig. 6.14 Simulation and analytical SINR results of the blind subspace MMSE detector with SNR=15dB and different forgetting factors γ .

Fig. 6.15 Simulation and analytical SINR results of the blind subspace ZF detector with SNR=15dB and different forgetting factors γ .

Fig. 6.16 Simulation and analytical SINR results of the group-blind detector with SNR=15dB and different forgetting factors γ .

Fig. 6.17 Simulation and analytical SINR results of the blind subspace MMSE detector with $J=5000$ and different forgetting factors γ .

Fig. 6.18 Simulation and analytical BER results of the blind DMI MMSE detector with $K=5$ and different forgetting factors γ .

Fig. 6.19 Simulation and analytical BER results of the blind subspace MMSE detector with $K=5$ and different forgetting factors γ .

Fig. 6.20 Simulation and analytical BER results of the blind subspace ZF detector with $K=5$ and different forgetting factors γ .

Fig. 6.21 Simulation and analytical BER results of the group-blind detector with $K=5$, $\bar{K}=3$ and different forgetting factors γ .

Fig. 6.22 Comparisons between simulation and analytical SINR results for the blind subspace MMSE detector over a dynamic environment with SNR=15dB, $K=8$ (for $J < 1000$), and $K=5$ (for $J > 1000$).

Fig. 6.23 Comparisons between the simulated SINR results of the proposed RO-FST based detector and analytical SINR results of the blind DMI MMSE detector using exact SVD, with $K=5$ and different forgetting factors γ .

Fig. 6.24 Comparisons between the simulated BER results of the proposed RO-FST based detector and analytical BER results of the blind DMI MMSE detector using exact SVD, with $K=5$ and different forgetting factors γ .

6.5 Conclusion

In this chapter, a blind adaptive DMI MMSE multiuser detector with fast convergence has been proposed over a multipath CDMA channel. The proposed detector can be classified as a SVD-based algorithm because it utilizes the RO-FST algorithm to adaptively track the signal subspace of received signals. Simulation results have shown that the proposed detector works well in a dynamic environment and its performance is better than that of the MMSE detector using the existing subspace tracking algorithm. At the same time, the proposed detector has a relatively low computational complexity. In addition, we have analyzed the performance of several types of blind SVD-based multiuser detectors, including DMI MMSE, subspace MMSE, subspace ZF and group-blind detectors. These analyses are applicable for adaptive algorithms and we take the ISI

due to multipath fading into consideration. We have also considered the effect of the forgetting factor and finite sample size on the performance of different types of detectors. The corresponding closed-form expressions of SINR and BER performance have been provided. The corresponding numerical results have also been presented and it has been shown that the analytical results and simulation results match quite well for both static and dynamic environments. Since these analytical results are optimal among the detectors using SVD-based subspace tracking algorithms, they can be applied to analyze the performance of other blind adaptive SVD-based MUD algorithms as well.

Chapter 7

Conclusions and Future Works

7.1 Conclusions

This thesis deals with the problem of the blind multiuser detection for DS-CDMA systems over multipath fading channels. The communication channel is assumed to undergo frequency-selective, slow time-varying fading with Rayleigh distribution. Several batch and adaptive MUD algorithms have been proposed in order to suppress the interference including MAI and ISI due to multipath fading. These proposed algorithms have been shown to improve the performance and/or reduce the computational complexity of the multiuser detection.

A user identification and detection scheme named LP-UID has been proposed for the DS-CDMA system over multipath fading channels. The proposed scheme has two steps, in which the first step is to efficiently identify the unknown active interferers and the second step is to form a ZF detector to obtain the desired user's signals. Unlike the existing SB-UID algorithm, we resort to the linear prediction algorithm to estimate each admissible user's power level and the effective sequence that consists of the information of the PN sequence and the channel response. Following that, we select those with large estimated power levels as the active users. Numerical results have shown that the proposed UID algorithm can identify the unknown active users more accurately than the

existing SB-UID algorithm, resulting in better FAR and BER performance. It has also been shown that the proposed LP-UID algorithm outperforms the existing blind linear detectors for a small estimation sample size. In addition, we have provided the corresponding performance analyses of the proposed ZF detector, and the closed-form expressions for the SINR and BER performance have been derived. Numerical results have shown that the analytical results can match the simulation results well.

To reduce the computational complexity of the existing blind subspace-based MUD algorithm, we have proposed a modified inter-symbol subspace-based (ISSB) DMI MMSE detector for the DS-CDMA system over a multipath fading channel. Since two adjacent bits of an AWGN vector are uncorrelated, we form an inter-symbol correlation matrix of the adjacent received symbols. Theoretically, this correlation matrix does not contain any noise components and can only span the signal subspace. Based on this property, we propose a simplified rank determination algorithm that has lower computational complexity as compared with the existing rank determination algorithms, such as the AIC and MDL algorithms. Simulation results have shown that the proposed ISSB MMSE algorithm is only slightly worse than that of the existing subspace-based DMI MMSE algorithm that adopts the existing rank determination algorithm, if the estimated sample size is comparatively large. Therefore, we concluded that our proposed ISSB-MMSE detector has lower computational complexity and does not significantly degrade the performance as compared with the existing SB-MMSE detector, over a slow time-varying fading channel.

In a practical environment, some parameters of the received signals, such as the cross-correlation, may be time-variant. In this thesis, we have proposed two blind Kalman filter-based MUD algorithms over multipath fading channels in order to suppress

the MAI as well as ISI. The first proposed Kalman filter-based MUD algorithm utilizes the inverse filtering criterion to form the constraint, which only focuses on the dominant path of the desired user, and forces the receiver response of other paths to zero. From the numerical results, it can be clearly found that the performance of this algorithm is better than that of the corresponding LMS-based algorithm using inverse filtering in BER, SINR and the tracking capability in a dynamic system.

The above adaptive Kalman filter-based MUD algorithm only utilizes one selected tap of the channel response, and will cause performance degradation when the magnitude of this tap is not high enough. Thus we have proposed a second type of Kalman filter-based MUD algorithm, where the estimated effective sequence which consists of the PN sequence and all channel response taps is utilized to form the constraint. In this algorithm, the channel estimation and the detector can be updated in every iteration, which results in very fast convergence. Numerical results have shown that the second proposed blind Kalman filter-based MUD algorithm outperforms the existing LMS detector using linear prediction in terms of the SINR, BER as well as the tracking capability under a dynamic environment. Furthermore, numerical results have shown that the second proposed Kalman filter-based MUD algorithm outperforms the first one due to the utilization of the all the channel response taps. Therefore, we concluded that the proposed adaptive Kalman filter-based MUD algorithms have good performance under a dynamic environment over multipath fading channels.

In this thesis, we have also proposed a subspace-based adaptive DMI MMSE detector. This algorithm utilizes the RO-FST algorithm to adaptively update the signal subspace and noise subspace. Numerical results have shown that the performance of the proposed RO-FST based detector can approach that of the adaptive DMI MMSE detector

using the exact SVD algorithm, and is better than that of the existing adaptive subspace MMSE detector using NAHJ-FST and PASTd algorithms. This proposed adaptive MUD is also shown to have good tracking capability in a dynamic environment. Therefore, we concluded that the proposed adaptive RO-FST based MUD is a promising technique that can suppress both the MAI and ISI under a dynamic environment over multipath fading channels.

Lastly, we have also analyzed the performance of adaptive MUD using the exact SVD algorithm, including the DMI MMSE detector, the subspace MMSE detector, the subspace ZF detector as well as the group-blind detector. Assuming that the differential BPSK modulation is adopted, we have provided the explicit closed-form expressions of the analytical SINR and BER performance over a multipath fading channel. Numerical results have shown that the analytical results and the simulation results match well under both static and dynamic environments. Furthermore, since the exact SVD algorithm can be considered as the optimal algorithm among the existing SVD-based subspace-tracking algorithms, these analytical results can be regarded as the performance bounds of other adaptive SVD-based MUD algorithms. Numerical results have also shown that the analytical SINR and BER results of the DMI MMSE detector can match well with the simulation results of the proposed adaptive RO-FST based MUD. Therefore we concluded that the proposed theoretical analyses of blind MUD can serve as a benchmark and provide a basis for further developing of new mechanisms.

7.2 Recommendations for Further Research

The communication channel considered in this thesis is assumed to be slow time-varying, which means that the parameter of channel response remains constant during the whole

transmission packet duration. Hence, the proposed algorithms are mainly applicable to the DS-CDMA systems over slow fading channels. Actually, almost all of the existing blind multiuser detection techniques are mainly based on the slow fading channel. Although they can handle dynamic environments with varying number of active users, the possible fundamental limit of this type of detectors is the capability of combating the time variations over multipath fading channel [48], [82]. If the multipath channel undergoes fast time-varying fading, the performance of blind adaptive detectors may be significantly degraded, as presented in Chapter 6 of this thesis. Therefore, it is a major challenge and important further research direction to develop robust blind adaptive MUD techniques to combat severe channel time variations.

All the proposed algorithms presented in this thesis assume that the channel background noise is AWGN. However, in many physical channels where multiuser detection may be applied, the background noise may be correlated due to the interference from some narrowband sources. Some multiuser detection techniques have been proposed to address this problem [31], [73]. In [31], a subspace-based group-blind MUD algorithm was proposed over a multipath fading channel with correlated noise. In [73], an efficient rank estimation method suitable for both white noise and correlated noise was proposed. Therefore, the extension of these proposed MUD algorithms to a correlated noise environment is an important future research direction.

The space-time processing technique can substantially enhance the capacity of DS-CDMA systems [83], [84]. This technique utilizes the exploitation of spatial diversity using multiple transmit and receive antennas, and with some form of space-time coding. Recently, some works have been done to apply space-time processing to blind multiuser detection [32]. In this thesis, we mainly focus on the single transmit and receive antenna

condition. Therefore, it is also an interesting and important research direction to combine the proposed MUD algorithms with the space-time processing technique.

Appendix A

Based on (6.36)-(6.38) and using the symmetrical property, we have

$$E\left\{\mathbf{q}(j)\mathbf{q}(j)^H \Psi \mathbf{q}(j+1)\mathbf{q}(j+1)^H\right\} = E_2^H(\Psi^H) \quad (\text{A.1})$$

and

$$E\left\{\mathbf{q}(j)\mathbf{q}(j)^H \Psi \mathbf{q}(j+2)\mathbf{q}(j+2)^H\right\} = E_3^H(\Psi^H). \quad (\text{A.2})$$

Therefore, $\lambda(l, u, J, \Psi)$ can be represented by

$$\begin{aligned} \lambda(l, u, J, \Psi) &= E\left\{\left(\sum_{j=l}^u \gamma^{J-j} \mathbf{q}(j)\mathbf{q}(j)^H\right) \Psi \left(\sum_{j=l}^u \gamma^{J-j} \mathbf{q}(j)\mathbf{q}(j)^H\right)\right\} \\ &= \left(\gamma^{2(J-l)} + \gamma^{2(J-l-1)} + \dots + \gamma^{2(J-u)}\right) E_1(\Psi) \\ &\quad + \left(\gamma^{J-l} \gamma^{J-l-1} + \gamma^{J-l-1} \gamma^{J-l-2} + \dots + \gamma^{J-u+1} \gamma^{J-u}\right) \left(E_2(\Psi) + E_2^H(\Psi^H)\right) \\ &\quad + \left(\gamma^{J-l} \gamma^{J-l-2} + \gamma^{J-l-1} \gamma^{J-l-3} + \dots + \gamma^{J-u+2} \gamma^{J-u}\right) \left(E_3(\Psi) + E_3^H(\Psi^H)\right) \\ &\quad + \left[\left(\gamma^{J-l} \gamma^{J-l-3} + \gamma^{J-l} \gamma^{J-l-4} + \dots + \gamma^{J-l} \gamma^{J-u}\right) \right. \\ &\quad \left. + \left(\gamma^{J-l-1} \gamma^{J-l-4} + \gamma^{J-l-1} \gamma^{J-l-5} + \dots + \gamma^{J-l-1} \gamma^{J-u}\right) \right. \\ &\quad \left. + \dots + \left(\gamma^{J-u} \gamma^{J-l} + \gamma^{J-u} \gamma^{J-l-1} + \dots + \gamma^{J-u} \gamma^{J-u+3}\right)\right] \mathbf{R}_q \Psi \mathbf{R}_q. \end{aligned} \quad (\text{A.3})$$

After some algebraic manipulations, it can be shown that

$$\gamma^{2(J-l)} + \gamma^{2(J-l-1)} + \dots + \gamma^{2(J-u)} = \frac{\gamma^{2(J-u)} (1 - \gamma^{2u-2l+2})}{1 - \gamma^2}, \quad (\text{A.4})$$

$$\gamma^{J-l} \gamma^{J-l-1} + \gamma^{J-l-1} \gamma^{J-l-2} + \dots + \gamma^{J-u+1} \gamma^{J-u} = \frac{\gamma^{2J-2u+1} (1 - \gamma^{2u-2l})}{1 - \gamma^2}, \quad (\text{A.5})$$

Appendix A

$$\gamma^{J-l}\gamma^{J-l-2} + \gamma^{J-l-1}\gamma^{J-l-3} + \dots + \gamma^{J-u+2}\gamma^{J-u} = \frac{\gamma^{2J-2u+2}(1-\gamma^{2u-2l-2})}{1-\gamma^2}, \quad (\text{A.6})$$

and

$$\begin{aligned} & \left(\gamma^{J-l}\gamma^{J-l-3} + \gamma^{J-l}\gamma^{J-l-4} + \dots + \gamma^{J-l}\gamma^{J-u} \right) \\ & + \left(\gamma^{J-l-1}\gamma^{J-l-4} + \gamma^{J-l-1}\gamma^{J-l-5} + \dots + \gamma^{J-l-1}\gamma^{J-u} \right) \\ & + \dots + \left(\gamma^{J-u}\gamma^{J-l} + \gamma^{J-u}\gamma^{J-l-1} + \dots + \gamma^{J-u}\gamma^{J-u+3} \right) \\ & = \frac{2(\gamma^{2J-2u+3} - \gamma^{2J-u-l+1} - \gamma^{2J-u-l+2} + \gamma^{2J-2l})}{(1-\gamma)^2(1+\gamma)}. \end{aligned} \quad (\text{A.7})$$

Therefore, $\lambda(l, u, J, \Psi)$ can be simplified to (6.35). □

Appendix B

Substituting (6.2) into (6.37), we can obtain

$$\begin{aligned} E_2(\Psi) = & \sigma_n^2 \mathbf{\theta} \mathbf{\theta}^H \Psi + \sigma_n^2 \text{tr}(\mathbf{D}_1^T \Psi) \mathbf{\theta} \mathbf{D}_2 \mathbf{\theta}^H + \sigma_n^2 \text{tr}(\mathbf{D}_2^T \mathbf{\theta}^H \Psi \mathbf{\theta}) \mathbf{D}_1 + \sigma_n^2 \Psi \mathbf{\theta} \mathbf{\theta}^H \\ & + \mathbf{\theta} E\{\mathbf{B}(j) \mathbf{B}^T(j) \mathbf{\theta}^H \Psi \mathbf{\theta} \mathbf{B}(j-1) \mathbf{B}^T(j-1)\} \mathbf{\theta}^H \\ & + E\{\mathbf{N}(j) \mathbf{N}^H(j) \Psi \mathbf{N}(j-1) \mathbf{N}^H(j-1)\}. \end{aligned} \quad (\text{B.1})$$

To evaluate the second last term in (B.1), we use a similar method as in [71] where

$$\begin{aligned} & E\{\mathbf{B}(j) \mathbf{B}^T(j) \mathbf{\theta}^H \Psi \mathbf{\theta} \mathbf{B}(j-1) \mathbf{B}^T(j-1)\} \\ & = \text{unvec}\left[E\left\{\left(\mathbf{B}(j-1) \otimes \mathbf{B}(j)\right)\left(\mathbf{B}(j-1) \otimes \mathbf{B}(j)\right)^T\right\} \text{vec}\left(\mathbf{\theta}^H \Psi \mathbf{\theta}\right)\right] \end{aligned} \quad (\text{B.2})$$

where “ $\otimes$ ” denotes Kronecker product, and the operation “vec” is to successively stack columns of a matrix into a big column vector. After some algebraic manipulations, it can be shown that

$$E\left\{\left(\mathbf{B}(j-1) \otimes \mathbf{B}(j)\right)\left(\mathbf{B}(j-1) \otimes \mathbf{B}(j)\right)^T\right\} = \kappa_{4b} \mathbf{X}_1 + \mathbf{X}_2 + \text{vec}(\mathbf{D}_2) \text{vec}^T(\mathbf{D}_2) + \mathbf{I} \quad (\text{B.3})$$

where $\mathbf{X}_1 = \text{diag}\{\mathbf{\alpha}_1 \mathbf{\alpha}_1^T, \dots, \mathbf{\alpha}_{3K} \mathbf{\alpha}_{3K}^T\}$ and $\mathbf{X}_2 = [\mathbf{\alpha}_j \mathbf{\alpha}_i^T]_{3K \times 3K}$ has been partitioned into

$3K \times 3K$ sub-matrices with the (i, j) th sub-matrix being $\mathbf{\alpha}_j \mathbf{\alpha}_i^T$. Substituting (B.3) into (B.2), we obtain

$$E\{\mathbf{B}(j) \mathbf{B}^T(j) \mathbf{\theta}^H \Psi \mathbf{\theta} \mathbf{B}(j-1) \mathbf{B}^T(j-1)\} = \mathbf{\rho} + \text{tr}(\mathbf{D}_2^T \mathbf{\theta}^H \Psi \mathbf{\theta}) \mathbf{D}_2 + \mathbf{\theta}^H \Psi \mathbf{\theta}. \quad (\text{B.4})$$

Note that

$$\mathbf{\rho} = \kappa_{4b} \text{unvec}\left[\mathbf{X}_1 \text{vec}(\mathbf{W})\right] + \text{unvec}\left[\mathbf{X}_2 \text{vec}(\mathbf{W})\right] \text{ and } \mathbf{W} = \mathbf{\theta}^H \Psi \mathbf{\theta}$$

where “unvec” is the reverse operation of “vec”. By expressing the matrix $\mathbf{W}$ explicitly as $[\mathbf{w}_1, \dots, \mathbf{w}_{3K}]$ and then applying the properties of “vec” and “unvec”, we have [85]

$$\text{unvec}[\mathbf{X}_1 \text{vec}(\mathbf{W})] = \mathbf{D}_2 \odot \mathbf{W}. \quad (\text{B.5})$$

Based on the definition of $\mathbf{x}_2$, the i th subvector of vector $\mathbf{X}_2 \text{vec}(\mathbf{W})$ is $\alpha_1 \alpha_i^T \mathbf{w}_1 + \dots + \alpha_{3K} \alpha_i^T \mathbf{w}_{3K}$, which can also be represented as $\alpha_1 \mathbf{w}_1^T \alpha_i + \dots + \alpha_{3K} \mathbf{w}_{3K}^T \alpha_i$. Since $[\alpha_1, \dots, \alpha_{3K}] = \mathbf{D}_2$, the i th subvector of $\mathbf{X}_2 \text{vec}(\mathbf{W})$ becomes $\mathbf{D}_2 \mathbf{W}^T \alpha_i$. Following that, we can obtain

$$\text{unvec}[\mathbf{X}_2 \text{vec}(\mathbf{W})] = \mathbf{D}_2 \mathbf{W}^T \mathbf{D}_2. \quad (\text{B.6})$$

Substituting (B.5) and (B.6) into (B.4) yields

$$\begin{aligned} & E\{\mathbf{B}(j) \mathbf{B}^T(j) \boldsymbol{\theta}^H \boldsymbol{\Psi} \boldsymbol{\theta} \mathbf{B}(j-1) \mathbf{B}^T(j-1)\} \\ &= \kappa_{4b} \mathbf{D}_2 \odot (\boldsymbol{\theta}^H \boldsymbol{\Psi} \boldsymbol{\theta}) + \mathbf{D}_2 \boldsymbol{\theta}^T \boldsymbol{\Psi}^T \boldsymbol{\theta}^* \mathbf{D}_2 + \text{tr}(\mathbf{D}_2^T \boldsymbol{\theta}^H \boldsymbol{\Psi} \boldsymbol{\theta}) \mathbf{D}_2 + \boldsymbol{\theta}^H \boldsymbol{\Psi} \boldsymbol{\theta} \end{aligned} \quad (\text{B.7})$$

By using a similar method and applying $\kappa_{4n} = 0$ [71], the last term in (B.1) can be simplified to

$$E\{\mathbf{N}(j) \mathbf{N}^H(j) \boldsymbol{\Psi} \mathbf{N}(j-1) \mathbf{N}^H(j-1)\} = \sigma_n^4 \text{tr}(\mathbf{D}_1^T \boldsymbol{\Psi}) \mathbf{D}_1 + \sigma_n^4 \boldsymbol{\Psi}. \quad (\text{B.8})$$

Substituting (B.7) and (B.8) into (B.1), and applying $\mathbf{R}_q = \boldsymbol{\theta} \boldsymbol{\theta}^H + \sigma_n^2 \mathbf{I}_{2p}$, $\mathbf{Z}_1 = \boldsymbol{\theta} \mathbf{D}_2 \boldsymbol{\theta}^H + \sigma_n^2 \mathbf{D}_1$, the closed-form expression of $E_2(\boldsymbol{\Psi})$ in (6.40) can be obtained. For $E_3(\boldsymbol{\Psi})$, by using a similar deduction and utilizing $E\{\mathbf{N}(j) \mathbf{N}^H(j-2)\} = \mathbf{0}$, we can obtain (6.41). $\square$

Author's Publications

1. X. Liu, K. C. Teh, and E. Gunawan, "Blind adaptive Kalman filter-based multiuser detector over a multipath fading channel", *IEEE Commun. Lett.*, vol. 8, pp. 342-344, June 2004.
2. X. Liu, K. C. Teh, and E. Gunawan, "Performance analysis of blind adaptive subspace multiuser detectors over multipath fading channels", accepted for publication in *IEEE Trans. Veh. Technol.*.
3. X. Liu, K. C. Teh, and E. Gunawan, "Interference identification and blind multiuser detection for asynchronous CDMA systems with multipath fading", accepted for publication in *IEEE Trans. Commun.*.
4. X. Liu, K. C. Teh, and E. Gunawan, "A blind adaptive MMSE multiuser detector over multipath CDMA channels and its perturbation analysis," submitted to *IEEE Trans. Wireless Commun.* under second review.

Bibliography

- [1] J. G. Proakis, *Digital Communications*, 4th ed. Boston, MA: McGraw Hill, 2001.
- [2] T. S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall, 1996.
- [3] A. J. Viterbi, "The orthogonal-random waveform dichotomy for digital mobile personal communications," *IEEE Pers. Commun*, pp. 18-24, 1st qtr., 1994.
- [4] W. C. Y. Lee, "Overview of cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 291-302, May 1991.
- [5] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Vitervi, L. A. Weaver, and C. E. Wheatley III, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 303-312, May 1991.
- [6] R. L. Pickholtz, L. B. Milstein, and D. L. Schilling, "Spread spectrum for mobile communications," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 313-322, May 1991.
- [7] G. Turin, "The effects of multipath and fading on the performance of direct sequence CDMA systems," *IEEE J. Select. Areas Commun.*, vol. 2, pp. 597-603, Jul. 1984.
- [8] A. Duel-Hallen, J. Holtzman, and Z. Zvonar, "Multiuser detection for CDMA systems," *IEEE Pers. Commun*, vol. 2, pp. 46-58, Apr. 1995.
- [9] S. Moshavi, "Multi-user detection for DS-CDMA communications," *IEEE Commun. Mag.*, vol. 34, pp. 124-136, Oct. 1996.
- [10] M. Honig, U. Madhow, and S. Verdú, "Blind multiuser detection," *IEEE Trans. Inform. Theory*, vol. 41, pp. 944-960, Jul. 1995.
- [11] H. V. Poor and X. Wang, "Code-aided interference suppression for DS/CDMA communications-Part II: parallel Blind adaptive implementations," *IEEE Trans. Commun.*, vol. 45, pp. 1112-1122, Sep. 1997.

- [12] X. Wang and H. V. Poor, "Blind multiuser detection: a subspace approach," *IEEE Trans. Inform. Theory*, vol. 44, pp. 677-690, Mar. 1998.
- [13] F. Bandiera, G. Ricci, and M. K. Varanasi, "Blind multiuser detection over highly dispersive CDMA channels," *IEEE Trans. Commun.*, vol. 52, pp. 1377-1387, Aug. 2004.
- [14] X. Wang and H. V. Poor, "Blind equalization and multiuser detection in dispersive CDMA channels," *IEEE Trans. Commun.*, vol. 46, pp. 91-103, Jan. 1998.
- [15] X. D. Zhang and W. Wei, "Blind adaptive multiuser detection based on Kalman filtering," *IEEE Trans. Signal Processing*, vol. 50, pp. 87-95, Jan. 2002.
- [16] A. Høst-Madsen, X. Wang, and S. Bahng, "Asymptotic analysis of blind multiuser detection with blind channel estimation," *IEEE Trans. Signal Processing*, vol. 52, pp. 1722-1738, Jun. 2004.
- [17] Z. Xu, "Effects of imperfect blind channel estimation on performance of linear CDMA receivers," *IEEE Trans. Signal Processing*, vol. 52, pp. 2873-2884, Oct. 2004.
- [18] Z. Xu and X. Wang, "Large-sample performance of blind and group-blind multiuser detectors: a perturbation perspective," *IEEE Trans. Inform. Theory*, vol. 50, pp. 2389-2401, Oct. 2004.
- [19] D. J. Rabideau, "Fast, rank-adaptive subspace tracking," *IEEE Trans. Signal Processing*, vol. 44, pp. 2229-2244, Sep. 1996.
- [20] B. Yang, "Projection approximation subspace tracking," *IEEE Trans. Signal Processing*, vol. 44, pp. 95-107, Jan. 1995.
- [21] D. Reynolds and X. Wang, "Adaptive group-blind multiuser detection based on a new subspace tracking algorithm," *IEEE Trans. Commun.*, vol. 49, pp. 1135-1141, Jul. 2001.
- [22] R. Kohno, R. Meidan, and L. Milstein, "Spread spectrum access methods in wireless communications," *IEEE Commun. Mag.*, vol. 33, pp. 58-67, Jan. 1995.
- [23] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for next generation mobile communication systems," *IEEE Commun. Mag.*, vol. 36, pp. 56-69, Sep. 1998.

- [24] E. Dahlman, P. Beming, J. Knutsson, F. Ovesjo, M. Persson, and C. Roobol, "WCDMA the radio interface for future mobile multimedia communications," *IEEE Trans. Veh. Technol.*, vol. 47, pp. 1105-1118, Nov. 1998.
- [25] S. Buzzi, M. Lops, and H. V. Poor, "Blind adaptive joint multiuser detection and equalization in dispersive differentially encoded CDMA channels," *IEEE Trans. Signal Processing*, vol. 51, pp. 1880-1893, Jul. 2003.
- [26] S. Buzzi, A. De Maio, and M. Lops, "Code-aided blind adaptive new user detection in DS/CDMA systems with fading time-dispersive channels," *IEEE Trans. Signal Processing*, vol. 51, pp. 2637-2649, Oct. 2003.
- [27] F. Xiong, *Digital Modulation Techniques*. Artech House, 2000.
- [28] B. Sklar, "Rayleigh fading channels in mobile digital communication systems part I: Characterization," *IEEE Commun. Mag.*, pp. 136-146, Sep. 1997.
- [29] B. Sklar, "Rayleigh fading channels in mobile digital communication systems part II: Mitigation," *IEEE Commun. Mag.*, pp. 148-155, Sep. 1997.
- [30] U. Madhow and M. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Commun.*, vol. 42, pp. 3178-3188, Dec. 1994.
- [31] X. Wang and A. Høst-Madsen, "Group-blind multiuser detection for uplink CDMA," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1971-1984, Nov. 1999.
- [32] D. Reynolds, X. Wang, and H. V. Poor, "Blind adaptive space-time multiuser detection with multiple transmitter and receiver antennas," *IEEE Trans. Signal Processing*, vol. 50, pp. 1261-1276, Jun. 2002.
- [33] D. North, "Analysis of the factors which determine signal/noise discrimination in radar," in *Report PPR-6C, RCA*, (Princeton, NJ), Jun. 1943.
- [34] J. Savage, "Signal detection in the presence of multiple-access noise," *IEEE Trans. Inform. Theory*, vol. 20, pp. 42-49, Jan. 1974.

- [35] S. Verdú, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 32, pp. 85-96, Jan. 1986.
- [36] Z. Zvonar and D. Brady, "Multiuser detection in single-path fading channels," *IEEE Trans. Commun.*, vol. 42, pp. 1729-1739, Feb. 1994.
- [37] S. Vasudevan and M. Varanasi, "Optimum diversity combiner based multiuser detection for time-dispersive Rician fading CDMA channels," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 580-592, May 1994.
- [38] R. Lupas and S. Verdú, "Linear multiuser detectors for synchronous code-division multiple access channels," *IEEE Trans. Inform. Theory*, vol. 35, pp. 123-136, Jan. 1989.
- [39] Z. Xie, "Joint signal detection and parameter estimation in multiuser communications," *IEEE Trans. Commun.*, vol. 40, pp. 1208-1216, Aug. 1993.
- [40] K. S. Schneider, "Optimum detection of code division multiplexed signals," *IEEE Trans. Aerospace Elect. Sys.*, vol. 15, pp. 181-185, Jan. 1979.
- [41] R. Kohno, M. Hatori, and H. Imai, "Cancellation techniques of co-channel interference in asynchronous spread spectrum multiple access systems," *Elect. and Commun. In Japan*, vol. 66-A, pp.416-423, May 1983.
- [42] R. Lupas and S. Verdú, "Near-far resistance of multi-user detectors in asynchronous channels," *IEEE Trans. Commun.*, vol. 38, pp. 496-508, Apr. 1990.
- [43] R. Lupas and S. Verdú, "Asymptotic efficiency of linear multiuser detectors," in *Proc. 25th Conf. On Decision and Control*, Athens, Greece, Dec. 1986, pp. 116-128.
- [44] Z. Xie, R. T. Short, and C. K. Rushforth, "A family of suboptimum detectors for coherent multiuser communications," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 683-690, May 1990.
- [45] S. L. Miller, "An adaptive direct-sequence code-division-multiple-access receiver for multiuser interference rejection," *IEEE Trans. Commun.*, vol. 43, pp.1746-1755, Feb.-Apr. 1995.

- [46] P. B. Rapajic and B. S. Vucetic, "Adaptive receiver structures for asynchronous CDMA systems," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 685-697, May 1994.
- [47] M. Abdulrahman, A. U. H. Sheikh, and D. D. Falconer, "Decision feedback equalization for CDMA in indoor wireless communications," *IEEE J. Select. Areas Commun.*, vol. 12, pp. 698-706, May 1994.
- [48] U. Madhow, "Blind adaptive interference suppression for direct-sequence CDMA," *Proc. IEEE*, vol. 86, pp. 2049-2069, Oct. 1998.
- [49] S. Verdú, *Multiuser Detection*, Cambridge University Press, 1998.
- [50] A. J. Viterbi, "Very low rate convolution codes for maximum theoretical performance of spread-spectrum multiple-access channels," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 641-649, May 1990.
- [51] R. Kohno, H. Imai, M. Hatori, and S. Pasupathy, "Combination of an adaptive array antenna and a canceller of interference for direct-sequence spread-spectrum multiple-access system," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 675-682, May 1990.
- [52] M. K. Varanasi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access communications," *IEEE Trans. Commun.*, vol. 38, pp. 509-519, Apr. 1990.
- [53] R. Kohno, H. Imai, M. Hatori, and S. Pasupathy, "An adaptive canceller of cochannel interference for spread-spectrum multiple-access communication networks in a power line," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 691-699, May 1990.
- [54] R. Kohno, "Spatial and temporal filtering for co-channel interference in CDMA," *Code Division Multiple Access Communications*, S. G. Glisic and P. A. Leppanen, eds., The Netherlands: Kluwer, 1995, pp. 117-146.
- [55] J. M. Holtzman, "DS/CDMA successive interference cancellation," in *Proc. IEEE Int'l. Symp. On Spread Spectrum Techniques and Appl. 1994*, Oulu, Finland, Jul. 1994, pp. 69-78.

- [56] S. Moshavi, "Survey of Multi-user detection for DS-CDMA systems," Bellcore pub., IM-555, Aug. 1996.
- [57] S. E. Bensley and B. Aazhang, "Subspace-based channel estimation for code division multiple access communication systems," *IEEE Trans. Commun.*, vol. 44, pp. 1009-1020, Aug. 1996.
- [58] X. Li and H. H. Fan, "Direct blind multiuser detection for CDMA in multipath without channel estimation," *IEEE Trans. Signal Processing*, vol. 49, pp. 63-73, Jan. 2001.
- [59] X. Li and H. H. Fan, "direct estimation of blind zero-forcing equalizers based on second order statistics," *IEEE Trans. Signal Processing*, vol. 48, pp. 2211-2218, Aug. 2000.
- [60] W. C. Wu and K. C. Chen, "Identification of active users in synchronous CDMA multiuser detection," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1723-1735, Dec. 1998.
- [61] G. Ricci, M. K. Varanasi, and A. De Maio, "Blind multiuser detection via interference identification," *IEEE Trans. Commun.*, vol. 50, pp. 1172-1181, Jul. 2002.
- [62] M. K. Tsatsanis, "Inverse filtering criteria for CDMA systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 102-112, Jan. 1997.
- [63] M. K. Tsatsanis and Z. Xu, "Performance analysis of minimum variance CDMA receivers," *IEEE Trans. Signal Processing*, vol. 46, pp. 3014-3022, Nov. 1998.
- [64] Z. Xu and M. K. Tsatsanis, "Blind adaptive algorithms for minimum variance CDMA receivers," *IEEE Trans. Commun.*, vol. 49, pp. 180-194, Jan. 2001.
- [65] X. Wang and H. V. Poor, "Blind adaptive joint suppression of MAI and ISI in dispersive CDMA channels," in *Proc. Asilomar Conf. Signals, Systems, and Computers*, vol. 2, Nov. 1997, pp. 1013-1017.
- [66] S. Haykin, *Adaptive Filter Theory*, 4th Ed. Prentice Hall, 2002.
- [67] Z. Xu, "Perturbation analysis for subspace decomposition with applications in subspace-based algorithms," *IEEE Trans. Signal Processing*, vol. 50, pp. 2820-2830, Nov. 2002.

- [68] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd Ed., The John Hopkins University Press, 1996.
- [69] S. Buzzi, E. Grossi, and M. Lops, "Timing-free blind multiuser detection for multicarrier DS/CDMA systems with multiple antennae," *EURASIP Journal on Applied Signal Processing*, pp. 613-628, May 2004.
- [70] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, pp. 387-392, Apr. 1985.
- [71] Z. Xu, "On the second order statistics of the weighted sample covariance matrix," *IEEE Trans. Signal Processing*, vol. 51, pp. 527-534, Feb. 2003.
- [72] A. Host-Madsen and X. Wang, "Performance of blind and group-blind multiuser detection," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1849-1872, Jul. 2002.
- [73] Z. Xu and R. S. Cheng, "A robust rank estimation algorithm of group-blind MMSE multiuser detectors for CDMA systems," *IEEE Trans. Commun.*, vol. 51, pp. 547-552, Apr. 2003.
- [74] G. Zhang and G. Bi, "New constraint-optimized multiuser detector for asynchronous CDMA channels with multipath," in *Proc. IEEE 8th International Conf. On ICCS*, Nov. 2002, vol. 2, pp. 1116-1119.
- [75] G. Zhang, G. Bi, and L. Zhang, "Blind multiuser detection for asynchronous MC-CDMA systems without channel estimation," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 1001-1013, Jul. 2004.
- [76] H. Yang and X. D. Zhang, "A singular values and singular vectors based blind multiuser detector," in *Proc. International Conference on Communication Technology*, vol. 2, Aug. 2000, pp. 1336-1340.
- [77] A. J. Caamano, D. Segovia-Vargas, and J. Ramos, "Blind adaptive Krylov subspace multiuser detection," in *Proc. IEEE Vehicular Technology Conference*, vol. 4, Oct. 2001, pp. 2338-2341.

- [78] X. Wang and H. V. Poor, "Blind adaptive multiuser detection in multipath CDMA channels based on subspace tracking," *IEEE Trans. Signal Processing*, vol. 46, pp. 3030-3044, Nov. 1998.
- [79] E. C. Real, D. W. Tufts, and J. W. Cooley, "Two algorithms for fast approximate subspace tracking," *IEEE Trans. Signal Processing*, vol. 47, pp. 1936-1945, Jul. 1999.
- [80] G. Xu and T. Kailath, "Fast subspace decomposition," *IEEE Trans. Signal Processing*, vol. 42, pp. 539-551, Mar. 1994.
- [81] P. Dent, G. E. Bottomley, and T. Croft, "Jakes fading model revisited," *Electron. Lett.*, vol. 29, pp. 1162-1163, Jun. 1993.
- [82] L. J. Zhu, U. Madhow, "Adaptive interference suppression for direct sequence CDMA over severely time-varying channels," in *Proc. IEEE Globecom'97*, pp. 917-922.
- [83] G. J. Forschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol.6, pp. 311-335, Mar. 1998.
- [84] I. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, pp. 585-595, Nov.-Dec. 1999.
- [85] P. Lancaster and M. Tismenetsky, *The Theory of Matrices*, 2nd ed. San Diego, CA: Academic, 1985.