Developing efficient processing algorithms for wideband sonar signals

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**NANYANG
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**DEVELOPING EFFICIENT PROCESSING
ALGORITHMS FOR WIDEBAND SONAR SIGNALS**

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2007

Developing Efficient Processing Algorithms for Wideband Sonar Signals

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Abstract

Due to the growing human activities in sonar environments, study of wideband signals has been attracting increasing attention to the researchers. A corresponding need of efficient processing techniques for wideband signals is also being felt. Unfortunately, a limited amount of research work on this direction has been carried out so far, partly due to the computational difficulties involved in wideband processing.

The thesis investigates the use of efficient algorithms for wideband sonar target parameter estimation. Initial study of the thesis is based on the use of Gaussian linear frequency modulated (LFM) waveforms as the transmitted signal. The Gaussian LFM signals possess very good delay-scale (or delay-Doppler) resolution properties, and are relatively easier to generate at the transmitter. Due to the computational complexity of two-dimensional (2-D) search in the ambiguity plane, an alternative one-dimensional (1-D) search technique that is based on the evaluation of the analytical expressions for the ambiguity functions is proposed for the Gaussian LFM signals.

It has been noted that the Gaussian LFM signals present linear ambiguity function characteristics in the narrowband ambiguity plane. As such, a novel transform consists on the combination of Radon transform with the narrowband ambiguity function identified as Radon-ambiguity transform (RAT) is proposed. The

possibility of estimating multiple as well as overlapping targets using RAT has been demonstrated. However, the computation of RAT is intensive, and thus, a novel, efficient technique based on a single slice RAT for delay and scale (or Doppler) estimation has been implemented. Using the dechirp technique, an efficient computation of the single slice of RAT via image rotation has also been proposed. Besides being computationally efficient, such an image rotation also aids detection in the presence of multiple-targets.

The latter part of the thesis is devoted to the extension of the work for non-Gaussian transmitted signal amplitude. This is important as the transmitted signals may suffer from amplitude distortions due to various transmitter hardware limitations. Furthermore, other system constraints may also make the sonar operator to use non-Gaussian amplitudes. Under this, use of the Hermite series expansion for efficient computation of the wideband cross-ambiguity function is proposed and the efficient 2-D local search techniques in conjunction with the wideband cross-ambiguity function have been developed.

Finally, it is demonstrated that the Hermite series decomposition can be effectively used in multiple signal transmissions to improve target detection capabilities, especially in low signal-to-noise ratio conditions. The efficient use of the Hermite decomposition to evaluate wideband spreading function by using wideband deconvolution is also presented in the process of identification of distributed targets. In all the developed algorithms, the signals are analyzed in the frequency domain; hence the algorithms are faster than time domain algorithms.

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List of Abbreviations

FT	Fourier transform
FFT	Fast Fourier transform
1-D	One-dimensional
2-D	Two-dimensional
FM	Frequency modulation
LFM	Linear frequency modulation
CAF	Cross-ambiguity function
WBCAF	Wideband cross-ambiguity function
NBCAF	Narrowband cross-ambiguity function
AWGN	Additive white Gaussian noise
RAT	Radon-ambiguity transform
SSRAT	Single slice Radon-ambiguity transform
CRLB	Cramer-Rao lower bound
SNR	Signal-to-noise ratio
dB	Decibel
STFT	Short-time Fourier transform
WVD	Wigner-Ville distribution
RWT	Radon-Wigner transform
MSE	Mean square error
LSE	Least squares estimation
AH	Associated Hermite
HR	Hermite-Rodriguez
WBSF	Wideband spreading function
NBSF	Narrowband scattering function
WSSUS	Wide sense stationary and uncorrelated scattering
AWT	Auto-wavelet transform
CWT	Cross-wavelet transform

Chapter 1

Introduction

Wideband signal analysis has attracted increasing attention due to the vast applications of sonar signals. There is a corresponding demand for efficient processing techniques for wideband signals. In this thesis, we focus on improved and efficient target parameter estimation techniques for wideband signals.

Wideband signals are currently used extensively in sonar, radar, synthetic aperture sonar and radar, ground penetrating radar (GPR), ultra wideband radar technology for through-wall motion detection, mine hunting, bottom mapping, remote sensing, medium probing and characterization, spread spectrum communication, biodynamic and multistatic signal processing, biomedical signal processing, ultrasonic imaging for medicine, and so on. GPR is one of the most promising technologies for detection and identification of buried objects. The principle of sonar was first used experimentally in World War I by the Allies (United States, Britain, and France). It has played a major role in not only war, but it has also played a vital role in civilian applications. Sonar is used on naval vessels to detect other ships like submarines, destroyers, and other anti-submarine ships. It is widely used to explore the oceans floors, and is often used to map out rock formations.

The detection and estimation [1] of signal received from target are important problems in many areas noted earlier, e.g., sonar [2–3], airborne radar [3–4], ground penetrating radar, ultra wideband radar, etc. In these applications, the signals are usually wideband, and thus the target parameters (range and velocity) are optimally estimated via wideband matched filter processing [5–6] (also known as wideband processing [6]) rather than via the simpler narrowband processing. In sonar/radar systems, the received signal usually contains information of both delay and scale (or Doppler) [10]. In order to estimate signal parameters such as range and velocity of the target, accurate estimation of delay and scale (or Doppler) from the received signal is crucial.

In active sonar system, the process of transmitting a signal into an environment and receiving the echo is generally referred to as delay-scale (or delay-Doppler) detection/estimation problem [1–4]. The received echo can be modeled in two ways, viz. narrowband and wideband models [5–6]. The selected model depends on the assumed characteristics of the received signal, and it turns out that under certain conditions, the wideband model can be approximated by the narrowband model [6]. Based on the two models, two detection methods for joint delay-scale (or delay-Doppler) estimation are considered in the literature, viz. narrowband and wideband processing [1–14]. The first is defined when the change in the received signal due to the velocity of the target is modeled as a Doppler-shift, while the latter is defined when the change in the received signal due to the velocity of the target is modeled as a time-scaling of the signal.

1.1 Motivation

Joint estimation of delay and scale (or delay and Doppler) of the signal received from target is required in many signal processing applications. The estimation of delay and Doppler for narrowband signals has received much research attention over the past several decades. When both transmitter and receiver are in motionless state and signal is reflected from a stationary target, scale factor $\alpha = 1$ and Doppler-shift $\nu = 0$ [6]. Under this condition, generalized cross-correlator is the optimum method to estimate delay [7]. In case of relative motion between target and receiver, the generalized cross-correlator is no longer a valid technique due to the mismatch of the waveforms resulting from time-scaling. Weinstein and Kletter [8] proposed incoherent processing to overcome this drawback. This technique divides the data sequence into many small segments within which the time-scalings are neglected. Although incoherent processing is computationally efficient, it only provides suboptimum estimates. The technique also has a high signal-to-noise ratio (SNR) threshold at which the delay variance deviates from the Cramer-Rao lower bound (CRLB). The standard technique for joint estimation of delay and Doppler is the cross-ambiguity function (CAF) [1], [10]. CAF jointly estimates delay and Doppler through maximization of CAF, which is also the maximum likelihood function for those two parameters when the noise is additive white Gaussian noise (AWGN). The joint estimation of delay and Doppler using the CAF is efficient in the sense that the estimation variance reaches the CRLB under high SNR [10].

Some other time-frequency representations based on narrowband signals, such as the short-time Fourier transform (STFT) and the Wigner-Ville distribution (WVD), can also be used to jointly estimate delay and Doppler. Time-frequency analysis based on spectrogram, which is the magnitude squared of the STFT, has been used in [19]. However, a crucial feature inherent in the STFT method is that the length of the window can be selected arbitrarily, but is fixed exogenously once the selection is made. To enhance time resolution, therefore, one must choose a short window; and to enhance frequency resolution, one must choose a long window, which means that the time resolution (nonstationarities) occurring within the window interval is suppressed. Hence, a longer window implies the loss of resolution along time dimension, and a shorter window implies the loss of resolution along frequency dimension. The type and the length of window are the two most important issues involved, limiting its application in practice. Studies have shown that the WVD is useful for analyzing and detecting nonstationary signals [21–24]. But for multicomponent signals, the WVD suffers from noise and interference from the cross-terms seriously, which cause difficulties in interpreting time-varying spectra. Since both cross-terms and white noise produce oscillations in the time-frequency plane, line integration through the WVD (known as WVD-based detector) may be used to emphasize slowly-varying auto-component contributions [25–26], [30]. The Radon transform of the WVD or the Radon-Wigner transform (RWT) [30–33], which is equivalent to quadrature matched filtering or the dechirp method [27–28] has been used to analyze time-varying nonstationary signals. The RWT offers a way to track straight lines in the time-frequency plane and locating maxima in a 2-D plane [32], which however is

a computationally exhaustive technique because of 2-D search. M. Wang *et al.* [36] have developed Radon-ambiguity transform (RAT)-based detector that has been shown to provide equivalent performance to the RWT-based algorithm while reducing the search space from 2-D to 1-D. However, with the use of RAT, it was assumed in [35–40] that all directions of interest pass through the origin of the ambiguity plane. Therefore, the only parameter of interest is the chirp rate (or scale) and as a result it is relatively easy to implement the estimator using a 1-D technique. Under this special assumption, delay and Doppler cannot be estimated simultaneously. However, in many practical cases, it may be necessary to estimate both delay and Doppler simultaneously. Simultaneous estimation of delay and Doppler (or scale) is one of the major objectives of this thesis. In the case of single target, estimating the range would be sufficient as range can be tracked to estimate the velocity. However, in multiple targets case, simultaneous estimation of range and velocity provides extra dimensions, which is especially useful for separating close by targets. (Targets may have same range, but different velocities, hence can be separated.) Simultaneous estimation of range and velocity also helps in classification of complex targets when they can be considered as a collection of point targets.

Although the narrowband model is used in some applications, in many other applications, however, it is not appropriate to approximate the signal by the narrowband model [10]. There are some additional assumptions involved with the narrowband model. If TB is the time-bandwidth product of the signal, c is the propagation velocity of the signal in the medium under observation, and ν is the

radial velocity of the target, then the narrowband condition can be defined as $TB \ll c/2v$ [6]. Processing under the assumption that this condition holds is termed narrowband processing [6]. When this condition is violated, wideband processing occurs. There are two basic ways that violate the narrowband condition [6]: (i) the velocity of the object, v , may be large comparable to c ; (ii) the time-bandwidth product, TB , may be very large. Although the narrowband model is generally valid for radar applications since c is quite large, it is not always valid in acoustic environment. In wideband signal processing, the bandwidth of the transmitted signal is often very large, and it is easily possible to violate the narrowband condition.

For active sonar or airborne acoustics, the speed of signal is relatively low, and the narrowband condition may not hold for rapidly moving objects [6], [10]. For example [6], in airborne acoustics, the speed of sound is $c = 340 \text{ m/s}$. If an airplane is moving with constant velocity $v = 100 \text{ m/s}$ and if $TB \approx 1$, then $c/2v \approx 1.7$ (slightly greater than 1). The narrowband condition is not valid in this case. Another example [10] is the sonar application, in which the signals usually have very large time-bandwidth product ($TB > 100$). Again, the narrowband condition is violated. Therefore, rapidly moving objects and/or high time-bandwidth product signals may violate the narrowband condition [6]. Wideband model should be introduced in applications in which the narrowband condition is violated. A good explanation of the wideband and narrowband signal models can be found in [6].

Wideband signals are usually used in active sonar systems for the detection and estimation of targets in the presence of noise. By using matched filtering techniques [6] on the received signal, these signals can provide accurate estimates of the targets' ranges and relative velocities. It is well known that the range resolution of the detection process can be improved by using short burst signals, while with continuous wave signals; range resolution can be increased only at the expense of a decrease in velocity resolution. However, using wideband signals such as linear frequency modulated (LFM) signals; by increasing the bandwidth of the transmitted signal it is possible to increase the range resolution without affecting the velocity resolution. Wideband LFM signals are used in active sonar/radar because of their excellent range and velocity resolution properties [17–18]. There are some advantages of using Gaussian windowed LFM signals. For example: (i) It is easier to generate Gaussian signal at the transmitter; (ii) The Fourier transform of Gaussian function is also Gaussian; (iii) Rectangular window, for example, has good concentration in time domain but has poor decay in frequency domain. The discontinuity of the window, in the time domain, introduces sidelobes in the frequency domain. Sidelobes may hinder to detect weak signals. Using Gaussian window function, which gradually decays toward zero, the sidelobes can be minimized. However, the mainlobe will be increased in comparison with the rectangular losing a little on the range resolution. This is the drawback of using a smooth window instead of a rectangular window.

Due to the nature of transmitted acoustic signal, traditional narrowband signal processing techniques used in radar signal processing cannot be usually used in

sonar signal processing. For example, instead of narrowband time-frequency type analysis, wideband time-scale analysis is required in wideband active processing. However, wideband signal processing is computationally intensive and limits the use in real-time applications such as in surveillance and other applications. Although wideband processing has been studied for decades, its implementation has been limited so far. Conventionally the sonar processor is designed by using the fast Fourier transform (FFT) based algorithm [15], but this algorithm is inadequate for a sonar system with large bandwidth since the FFT cannot be effectively performed for wideband processing. To overcome the disadvantage of the FFT-based algorithm, wideband processing by other alternative techniques is proposed.

Wideband cross-ambiguity function (WBCAF) evaluated via matched filter processing, provides optimal performance in the sense that it meets the CRLB. However, 2-D search makes it computationally intensive. Furthermore, the technique is not effective in separating multiple-targets especially when the targets are close to each other. Therefore, to overcome high computational cost and to deal with multiple-targets, we investigate efficient techniques for the estimation of delay and scale parameters of multiple-targets.

Signals with finite time support are frequently encountered in sonar/radar applications. It is usually necessary to have a compact description of these signals' shape and time evolution [49]. In these applications, the signals are usually broadband and thus the target parameters are optimally estimated via

WBCAF-based processing [6]. Gaussian windowed LFM signals are good candidates for the transmitted signal as they are known to possess very good resolution properties for target parameter estimation. However, this thesis also considers a more general transmitted waveform where the amplitude is not strictly a Gaussian function. This is important as even a Gaussian transmission signal may suffer from amplitude distortions due to various transmitter hardware limitations. For this purpose, the Hermite series expansions are used to approximate arbitrary shaped LFM signal. It is proposed to decompose the transmitted signal amplitude using the Hermite expansions for efficient computation and processing via the WBCAF.

Due to its Gaussian weight function, the Hermite series expansions have proven to be a useful tool in many signal processing applications. In the last few decades, a significant amount of research has been carried out on the use of Hermite functions [44–61]. However, most of the research have been on image processing [44–48], biomedical engineering [49–53], neural network [54–56], electromagnetic signals [57–58], transient signals [59], wireless communications [60], etc., and to the best of our knowledge, little work has been reported on Hermite decompositions in wideband sonar signal processing. Alsup and Whitehouse [61] have used Hermite functions to design sonar comb waveforms for delay and Doppler estimation. They have used Hermite functions as a transmitted waveform. However, our application of the Hermite series expansions is significantly different from [61] in that we have used Hermite series to

approximate an arbitrary shaped LFM signal. It is also our aim to show the use of Hermite functions as a suitable technique in wideband signal applications.

The aim is the evaluation of the WBCAF between the transmitted and received signals to estimate the sonar target parameters efficiently. As arbitrary shaped signals are used in transmission and hence in processing, an appropriate analytical model is necessary to scale a signal in time in order to compute the WBCAF. However, the major problem associated with this is that the signal is not known analytically. Since the signal is not known analytically, creating the scaled and delayed replicas (to compute WBCAF) requires sampling the signal and then applying a multirate conversion technique. As such, these techniques are not efficient because of the very high computational complexity [6]. Scaling is further complicated by the scaling factor, which is typically close to unity [6]. Therefore, a very high accuracy in scaling is needed. A wavelet transform based method has been proposed in [5] to overcome the difficulties involved with sampling and multirate conversion technique. In this approach *a priori* admissible analytic function has been chosen to create many different scaled and delayed replicas of the transmitted signal.

In this thesis, a computationally efficient technique is proposed to solve the scaling problem. The proposed technique is then applied to simultaneous delay and scale estimation for wideband signal. The Hermite decomposition is used to develop an analytical model for the arbitrary shaped LFM signals to efficiently evaluate WBCAF for parameter estimation.

A semi-active sonar/radar case will also be considered in this thesis where the transmitted signal is contaminated with AWGN and thereby making it difficult to detect the target especially when the noise level is considerably high. In semi-active case the receiver, situated far from the transmitter, receives two signals; one directly from the transmitter; other reflected from the targets. It is also our aim to consider a distributed target in the delay-scale plane. Signal processing of acoustic signals scattered from distributed targets has become an area of increasing attention to the researchers. The delay-scale domain wideband spreading function is often used to characterize the distributed target environment [67], [77–85]. This thesis investigates an efficient technique for estimating wideband spreading function using Hermite decomposition.

In many sonar/radar applications, it may be necessary to detect more than one target simultaneously. Under this scenario, delay-scale resolution is a major concern to be considered to separate multiple-targets. We also focus on the applications of Hermite decomposition to the use of multiple transmissions to improve delay-scale resolution. In a complicated environment such as the ocean, the target is composed of several objects, or a physical large object with continuum of reflectors that are often very close in delay-scale plane [68]. Therefore, one of the major problems in sonar/radar signal processing is how to deal with multiple-targets. Under these conditions, the WBCAF using single transmission often fails to separate close targets due to the noise as well as resolution effects. Multiple-targets detection and tracking have been an active research area to deal with this issue [88–91]. These methods are reported to

perform well under high SNR environment. However, the methods may exhibit poor performance when the noise level is high [88].

In this thesis, the use of multiple transmissions will be investigated to track and detect multiple-targets in noisy environment. This method could be implemented efficiently if the transmitted signal is known in analytic form. Hermite decomposition techniques will be used for this purpose.

1.2 Objectives

It is a challenging task to efficiently estimate target parameters using wideband signals. Using appropriate models, this thesis aims to investigate efficient target parameter estimation techniques. The main objectives of the thesis are as follows:

- To explore the known structures of the CAF for both wideband and narrowband processing, e.g., the characteristics of the main ridges. To study the conventional 2-D peak search methods, such as WBCAF and narrowband cross-ambiguity function (NBCAF). To investigate FFT-based narrowband processing for approximate estimation of the scale parameter.
- Investigation of an efficient technique based on RAT using dechirp technique for the estimation of delay and scale parameters of multiple-targets. Analytical verification of the effect of integration length of RAT in AWGN process is also sought.

- Demonstration of the usefulness of Hermite series expansions to decompose the arbitrary transmitted signal for efficient computation and processing via the WBCAF. The Hermite decomposition is used to develop an analytical model for the arbitrary shaped LFM signal. The estimation of spread and shift parameters associated with the Hermite expansions using different optimization techniques are investigated. Efficient 2-D peak search methods on the WBCAF are proposed to estimate the target parameters.
- Based on Hermite decomposition, a semi-active sonar/radar case is considered where the transmitted signal is also contaminated with AWGN process. An efficient technique for estimating wideband spreading function using Hermite decomposition is presented.
- This thesis also focuses on the applications of Hermite decomposition to the use of multiple transmissions. The use of multiple transmissions are investigated by transmitting arbitrary shaped LFM signals at known interval of time to track and detect multiple-targets and improve the delay-scale resolution in noisy environment.

1.3 Major Contributions of the Thesis

The following items are the original contributions stemmed from the thesis:

- An efficient algorithm based on NBCAF for estimating delay and scale parameters is proposed. An FFT-based NBCAF processing for approximate estimation of the scale parameter is presented (Chapter 2).

- A novel efficient technique based on single slice RAT using dechirp method for the estimation of delay and scale parameters of multiple-targets is proposed (Chapter 3).
- The usefulness of Hermite series expansions using arbitrary shaped LFM signal for efficient computation and processing via the WBCAF is presented. Efficient 2-D peak search methods on the WBCAF is proposed to estimate the target parameters. Newton's optimization technique and Nelder-Mead simplex algorithm are used for this purpose (Chapter 4).
- An efficient technique for estimating wideband spreading function using Hermite decomposition is proposed. The applications of Hermite decomposition to the use of multiple transmissions to track and detect multiple-targets and to improve the delay-scale resolution in noisy environment are suggested (Chapter 5).

1.4 Organization of the Thesis

The thesis is organized into six chapters as follows:

In Chapter 1, the introduction and literature survey have been reported that results in the motivation, reasoning, and the proposition of efficient techniques against the available conventional techniques in the literature.

In Chapter 2, the fundamental and basic topics related to the narrowband and wideband signal models are provided which are the preliminaries required for the research. The WBCAF and NBCAF processing are presented with simulation

results. Characteristics of the main ridge of the CAFs are also discussed. An efficient parameter estimation technique based on NBCAF is proposed. A 2-D FFT-based NBCAF algorithm is also proposed for approximate estimation of the slope of the main ridge of NBCAF in multipath environment.

In Chapter 3, the usefulness of single slice RAT for multiple-targets detection in noisy environment is demonstrated. A computationally efficient technique based on single slice RAT for simultaneous estimation of delay and scale parameters is proposed. The theoretical derivation of RAT has justified the ability of the single slice RAT-based algorithm to estimate the parameters. Simulation results are presented to show the effectiveness of the proposed technique.

In Chapter 4, Hermite series expansions, estimation of spread and shift parameters for Hermite decomposition, and efficient 2-D peak search algorithms are addressed. The analytical expressions of the Fourier transform (FT) of arbitrary shaped LFM Hermite functions have been developed. The use of the FT of arbitrary shaped LFM Hermite expansions for efficient computation of the WBCAF has been proposed. Efficient 2-D peak search methods on the WBCAF have also been proposed to estimate the target parameters. It has been shown that the proposed 2-D peak search methods are faster than the conventional methods.

In Chapter 5, an efficient technique for estimating wideband spreading function by using wideband deconvolution is presented. A technique of multiple transmissions to resolve close targets using the WBCAF is also proposed. It

appears that the multiple transmissions technique can distinctively separate multiple-targets and are also robust to noise. Simulation results have been provided to show the usefulness of the proposed method.

In Chapter 6, conclusions and some suggestions for further possible avenues of research are presented.

Chapter 2

The Use of Cross-Ambiguity Function for Wideband Signal Parameter Estimation from a Single Target

2.1 Introduction

This chapter presents issues related to wideband and narrowband signal models, wideband cross-ambiguity function (WBCAF)-, and narrowband cross-ambiguity function (NBCAF)-based delay-scale parameter estimators. In this chapter, Gaussian LFM signals are used as the transmitted burst. Gaussian LFM signals could also be used as an elementary basis function for expressing time-frequency distributions, i.e., via the Gabor-like representations [16]. Furthermore, the analysis based on Gaussian LFM signal is easier than that of the other waveforms; for instance, Rectangular LFM [36] and Hyperbolic FM waveforms [9] are not easy to analyze. As noted before, using the wideband signals such as the Gaussian LFM signals, it is also possible to increase the range resolution without sacrificing the velocity resolution.

Conventional methods based on NBCAF and WBCAF for target parameter estimation will be addressed first from Sections 2.3 to 2.5. To overcome the drawbacks associated with these methods, efficient techniques will then be proposed in Sections 2.6 and 2.7. The computational efficiency of the proposed methods will be presented in Section 2.8.

Demonstration of the main ridge characteristics of NBCAF and WBCAF, an efficient NBACF-based algorithm for estimating delay and scale, and a 2-D FFT-based algorithm to estimate the slope of the main ridge of NBCAF are the original contributions in this chapter.

2.2 Wideband Signal Model

In order to implement an estimator, a received signal model is necessary. Suppose a signal $g(t)$ transmitted from a stationary position and reflected from a stationary target. The received signal $r(t)$ will be delayed by a factor $\tau = 2R/c$, so that $r(t) = g(t - \tau)$, where R is the distance from the stationary position to the target and c is the propagation velocity of the signal in the medium under observation. For the analysis of moving targets, assume a point target at range $R(t)$ moving with a constant speed v relative to a stationary transmitter. Suppose that a transducer emits a signal $g(t)$ and that this signal is reflected off from a single object. The received signal, under ideal condition, i.e. noise free, is denoted by $r(t)$. Then the received backscatter from a single scatterer is given by

$$r(t) = g[t - \tau(t)] \quad (2.1)$$

where the amplitude of the reflection is assumed to be unity without the loss of generality and $\tau(t)$ is the round-trip travel time of the signal arriving at the receiver at time t that can be expressed in terms of the target range $R(t)$ [3] as

$$\tau(t) = \frac{2}{c} R \left[t - \frac{\tau(t)}{2} \right]. \quad (2.2)$$

In most of the applications, it can be assumed that the scatterer's radial velocity v is constant. For an assumed constant radial velocity, (2.2) can be written as [6]

$$\tau(t) = \tau + \frac{2v}{c+v} (t - \tau) \quad (2.3)$$

where τ is the time taken by the back-scattered signal to reach the sensor, which is given by

$$\tau = \frac{2R}{c-v} \quad (2.4)$$

with R being the initial range of target.

The argument, $t - \tau(t)$, of the received signal, $r(t)$, is deduced from (2.3) as

$$t - \tau(t) = \frac{c-v}{c+v} (t - \tau) = \alpha(t - \tau) \quad (2.5)$$

where α is the time-scaling of the received signal relative to the transmitted signal, which is defined as

$$\alpha = \frac{c-v}{c+v}. \quad (2.6)$$

Substituting $t - \tau(t)$, (2.1) can be reduced to

$$r(t) = g[\alpha(t - \tau)]. \quad (2.7)$$

The form of the received signal in (2.7) is the wideband model. From (2.7) it can be seen that the transmitted signal is stretched or contracted by the scale factor α . Once τ and α are known, the range, R , and the velocity, v , of a target can be deduced from (2.4) and (2.6) as,

$$R = \frac{c\alpha\tau}{1+\alpha} \quad (2.8)$$

and

$$v = \frac{c(1-\alpha)}{1+\alpha}. \quad (2.9)$$

2.3 Wideband and Narrowband Processing

In order to obtain the range and velocity of a target, the received signal is correlated with hypothesized delayed and scaled (Doppler-shifted) replicas of the transmitted signal. The global maximum of the correlation output will correspond to the range and velocity of the target. Using the wideband model, the WBCAF (also known as matched-filter output or correlator output [6]) between the received signal, $x(t)$, and the transmitted signal, $g(t)$, for different values of delays (τ) and scales (α) is given by,

$$\chi_{wb}(\tau, \alpha) = \sqrt{\alpha} \int_{-\infty}^{\infty} x(t) g^*[\alpha(t - \tau)] dt \quad (2.10)$$

$$= \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} X(f) G^*(f/\alpha) e^{j2\pi f\tau} df \quad (2.11)$$

where the term $\sqrt{\alpha}$ is a normalization used in the definition such that the transmitted and received signals have the same energy [6], $X(f)$ and $G(f)$ are the FTs of $x(t)$ and $g(t)$, respectively.

The received signal $x(t)$ in (2.10) accompanied by noise is represented as,

$$x(t) = g[\alpha_0(t - \tau_0)] + n(t) \quad (2.12)$$

with $n(t)$ being the additive white Gaussian noise (AWGN) process with zero mean and standard deviation σ_n , τ_0 and α_0 represent the delay and scale parameters of the target. Note that the definitions in (2.10) and (2.11) are equivalent. Although it is possible to evaluate (2.10) in time domain, the frequency domain approach is more efficient due to the use of inverse FT as in (2.11). (In frequency domain method, the integration associated with the correlation becomes a complex conjugate multiplication, which is easier to implement.) Equation (2.11) defines the WBCAF between the transmitted and received signals. The square modulus of the WBCAF is expressed in a different form as,

$$\Psi_{wb}(\tau, \alpha) = |\chi_{wb}(\tau, \alpha)|^2 = \left| \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} X(f) G^*(f/\alpha) e^{j2\pi f\tau} df \right|^2. \quad (2.13)$$

In the delay-scale plane, $\Psi_{wb}(\tau, \alpha)$ peaks at $(\tau, \alpha) = (\tau_0, \alpha_0)$, which allows one to estimate the range and the velocity of a target. Figure 2.1 shows the block diagram of the wideband cross-correlation receiver for the estimation of delay and scale parameters.

Figure 2.1: Block diagram of the wideband cross-correlation receiver for the estimation of delay and scale parameters.

Wideband processing was considered computationally intensive for many years [6]. As such, an approximation was made so that the processing could be carried out using FT. Under the assumption $|v|/c \ll 1$, the wideband model in (2.7) can be approximated by the narrowband model [1], [6]

$$r(t) \approx g(t - \tau) e^{j2\pi v_D t}. \quad (2.14)$$

The form in (2.14) is the narrowband model for received signal. With the narrowband model, time-scaling the signal by α is approximated by Doppler-shift, v_D . For a signal with center frequency, κ , the approximation is [3], [6]

$$v_D \approx -\frac{2v}{c} \kappa \approx (\alpha - 1) \kappa. \quad (2.15)$$

Note that using (2.15), it is possible to convert the delay-scale $(\tau - \alpha)$ representation to a delay-Doppler $(\tau - v_D)$ (range-velocity) representation. Hereafter, v_D will be denoted as v .

For the narrowband model, the NBCAF between the transmitted and the received signal is defined as

$$\chi_{nb}(\tau, v) = \int_{-\infty}^{\infty} x(t) g^*(t - \tau) e^{-j2\pi v t} dt \quad (2.16)$$

$$= e^{-j2\pi v \tau} \int_{-\infty}^{\infty} X(f) G^*(f - v) e^{j2\pi f \tau} df. \quad (2.17)$$

Likewise, the square magnitude of the NBCAF is expressed as [3]

$$\Psi_{nb}(\tau, v) = |\chi_{nb}(\tau, v)|^2 = \left| \int_{-\infty}^{\infty} X(f) G^*(f - v) e^{j2\pi f \tau} df \right|^2. \quad (2.18)$$

In the NBCAF plane, $\Psi_{nb}(\tau, v)$ is centered on the target location in the delay-Doppler plane. That is, $\Psi_{nb}(\tau, v)$ peaks at $(\tau, v) = (\tau_0, v_0)$. Figure 2.2 shows the

block diagram of a narrowband cross-correlation receiver for the estimation of delay and Doppler parameters.

Figure 2.2: Block diagram of a narrowband cross-correlation receiver for the estimation of delay and Doppler parameters.

When both the transmitter and the receiver are in motionless state and the signal is reflected from a stationary target, then the scale factor $\alpha = 1$ or the Doppler-shift $v = 0$ [6]. Under this special case, both (2.11) and (2.17) reduce to the same equation, which is the standard cross-correlation used in the delay estimation theory:

$$\chi(\tau) = \int_{-\infty}^{\infty} X(f)G^*(f)e^{j2\pi f\tau} df. \quad (2.19)$$

Equations (2.1)–(2.19) are the basic formulas that will be used throughout the thesis. One could use wideband model as in (2.7) to estimate signal parameters using the WBCAF. This is called wideband processing technique and it is optimal as it meets the Cramer-Rao lower bound (CRLB) for parameter estimation. On the other hand, one could also use approximated narrowband model as in (2.14) to estimate signal parameters using the NBCAF. This technique called narrowband processing is suboptimal; however, it is more efficient than the use in the wideband processing technique.

2.4 Characteristics of WBCAF and NBCAF

In this section, the behavior of WBCAF and NBCAF of Gaussian LFM signal is discussed. For the simplicity of analysis, a point target is considered and the multipath propagation is considered to be negligible. In this chapter the analysis will be limited to only Gaussian LFM waveform as the transmitted signal. This signal provides closed form expressions for WBCAF and NBCAF, which will be shown later of this section.

The complex form of the windowed-LFM signal is given by,

$$g(t) = w(t) \exp\{j[2\pi\kappa t + \pi\beta t^2]\} \quad (2.20)$$

where $w(t)$ is an envelope, the instantaneous frequency at $t = 0$ is given by κ , and β is the frequency sweep rate. For a Gaussian envelope, (2.20) can be expressed as,

$$g(t) = \exp(-t^2 / 2\lambda^2) \exp\{j[2\pi\kappa t + \pi\beta t^2]\} \quad (2.21)$$

where λ is the amplitude shaping parameter for Gaussian window and the full width at half maximum of the Gaussian LFM signal is $T = 2.3548\lambda$. The form of the signal in (2.21) is termed as Gaussian LFM signal. Note that the Gaussian LFM signal in (2.21) is approximately time limited due to the exponential term in the amplitude. Figures 2.3 and 2.4 show a Gaussian window function and a Gaussian LFM signal, respectively.

Figure 2.3: Gaussian window function.

Figure 2.4: Gaussian LFM signal (real part).

After some algebraic manipulation, the following expressions can be obtained from (2.13) for the WBCAF, $\Psi_{wb}(\tau, \alpha)$, resulting from the Gaussian LFM signal reflected from a single target parameterized by (τ_0, α_0) ,

$$\Psi_{wb}(\tau, \alpha) = \Gamma_{wb}(\alpha) \exp \left\{ -\frac{\alpha^2 \alpha_0^2 (\alpha^2 + \alpha_0^2) (1/4\lambda^2 + \pi^2 \beta^2) / \lambda^2}{\{(\alpha^2 + \alpha_0^2)^2 / 4\lambda^4 + \pi^2 \beta^2 (\alpha^2 - \alpha_0^2)^2\}} [\tau - \mu(\alpha)]^2 \right\} \quad (2.22)$$

where

$$\mu(\alpha) = \tau_0 + \frac{2\pi^2 \beta \kappa (\alpha - \alpha_0)}{(1/4\lambda^4 + \pi^2 \beta^2)(\alpha^2 + \alpha_0^2)} \quad (2.23)$$

and

$$\Gamma_{wb}(\alpha) = \frac{\pi \alpha \exp \left\{ -\frac{\pi^2 \kappa^2 (\alpha - \alpha_0)^2 / \lambda^2}{(1/4\lambda^4 + \pi^2 \beta^2)(\alpha^2 + \alpha_0^2)} \right\}}{\left[(\alpha^2 + \alpha_0^2)^2 / 4\lambda^4 + \pi^2 \beta^2 (\alpha^2 - \alpha_0^2)^2 \right]^{1/2}}. \quad (2.24)$$

Figure 2.5 shows the contour plots of (2.13) for three point targets. The parameters of the Gaussian LFM signal used to obtain Figure 2.5 are: $\kappa = 200\text{Hz}$, $\beta = 300\text{Hz/s}$, $\lambda = 0.1262\text{s}$, and sampling frequency $F_s = 1000\text{Hz}$. It is assumed that the received signal is resulting from three equal strength point targets with parameters (τ_0, α_0) given by $(-0.1, 1.2)$, $(0, 1.0)$, and $(0.2, 0.8)$. These parameters are chosen for better visualization and demonstration. In the next chapters, more realistic target parameters will be used for simulation. (Time is measured, relative to the second target, in seconds.)

(a)

(b)

Figure 2.5: Contour plot of WBCAF in the presence of three point targets and the three main ridges of the WBCAF: (a) Without noise; (b) With 10 dB additive noise.

Similarly, using (2.18), the following expressions can be obtained for the NBCAF, $\Psi_{nb}(\tau, \nu)$, of the Gaussian LFM signal, for a single target parameterized by (τ_0, α_0) ,

$$\Psi_{nb}(\tau, \nu) = \Gamma_{nb}(\nu) \times \exp \left\{ \frac{-\alpha_0^2(1+\alpha_0^2)(1/4\lambda^4 + \pi^2\beta^2)/\lambda^2}{\{(1+\alpha_0^2)^2/4\lambda^4 + \pi^2\beta^2(1-\alpha_0^2)^2\}} [\tau - \lambda(\nu)]^2 \right\} \quad (2.25)$$

where

$$\lambda(\nu) = \tau_0 + \frac{2\pi^2\beta[\nu - \kappa(\alpha_0 - 1)]}{(1/4\lambda^4 + \pi^2\beta^2)(1+\alpha_0^2)} \quad (2.26)$$

and

$$\Gamma_{nb}(\nu) = \frac{\pi \exp \left\{ -\frac{\pi^2[\nu - \kappa(\alpha_0 - 1)]^2/\lambda^2}{(1/4\lambda^4 + \pi^2\beta^2)(1+\alpha_0^2)} \right\}}{\left[(1+\alpha_0^2)^2/4\lambda^4 + \pi^2\beta^2(1-\alpha_0^2)^2 \right]^{1/2}}. \quad (2.27)$$

Figure 2.6 shows the contour plots of $\Psi_{nb}(\tau, \nu)$, using the same received signal as in Figure 2.5. Note that both Figure 2.5 and 2.6 could be used, via a two-dimensional (2-D) peak search, to estimate the targets' range and velocity parameters. The scale parameters in Figure 2.5 can be related to the Doppler-shifts of Figure 2.6, via the relation in (2.15), i.e. $\nu = (\alpha - 1)\kappa$. Note also that obtaining Figure 2.6 is computationally less intensive than obtaining Figure 2.5, as no nonlinear stretch operations are required.

(a)

(b)

Figure 2.6: Contour plot of NBCAF of the same signal as in Figure 2.5 and the three main ridges of the NBCAF: (a) Without noise; (b) With 10 dB additive noise.

The main ridges of the contour plots are also shown in Figures 2.5 and 2.6. These main ridge characteristics of the WBCAF and NBCAF are discussed here. If the main ridge characteristics of the NBCAF are known, then efficient one-dimensional (1-D) search technique can be used to obtain the target parameters. Therefore, searching for the parameters in a 2-D plane can be simplified by searching for the parameters along the main ridge line to efficiently estimate the target parameters.

It can be seen from (2.22), that the main ridge of the WBCAF can be described by the relation $\tau = \mu(\alpha)$. That is, the peak position of $\Psi_{wb}(\tau, \alpha)$ for a given value of α is given by $\mu(\alpha)$ of (2.23). Furthermore, the amplitude variation along the main ridge is given by $\Gamma_{wb}(\alpha)$ of (2.24). Similarly, in (2.26), the function $\tau = \lambda(v)$ describes the main ridge of the NBCAF, and the amplitude variation along the main ridge is given by $\Gamma_{nb}(v)$ of (2.27). As an example, Figure 2.7 shows the behavior of (2.23) and (2.26) for the three point targets in Figure 2.5. It can be seen from Figure 2.7 that the main ridges of the plot in Figure 2.6 are straight lines whereas the main ridges of the plot in Figure 2.5 are nonlinear.

Figure 2.7: Main ridges of Figures 2.5 and 2.6.

Figures 2.8 and 2.9 show the behavior of the main ridge amplitude variation as a function of Doppler/scale and delay, respectively. In the Figures, it can be observed that the resolution of the WBCAF is better than that of the NBCAF in both Doppler/scale and delay planes. Also, around the peak, NBCAF has lesser signal strength (especially when scale is not unity). However, away from the peak position the signal strength for the NBCAF and WBCAF are of comparable magnitude. Note here that, both the ridges of NBCAF and WBCAF are shown in the same picture for better comparison. The Doppler-shift axis of Figures 2.7 and 2.8 can be converted to scale using the relation in (2.15). Figures 2.8 and 2.9 are shown only for pictorial representations of main ridges of the WBCAF and the NBCAF in the absence of noise. In the next section, however, simulation results will also be presented with additive noise to show the difference between the WBCAF and the NBCAF.

Figure 2.8: Doppler vs. Amplitude along the main ridges of Figures 2.5 and 2.6.

Figure 2.9: Delay vs. Amplitude along the main ridges of Figures 2.5 and 2.6.

2.5 Parameter Estimation based on 2-D Peak Search on WBCAF and NBCAF for Single Target

Detection and parameter estimation of a Gaussian LFM signal in a noise-free or a high signal-to-noise ratio (SNR) environment is a relatively simple task. However, the presence of noise may affect the performance of the estimator. The error performances of the 2-D peak search estimation shown here are based on WBCAF and NBCAF methods. The received signal is assumed to consist of a Gaussian LFM signal corrupted by AWGN process as given in (2.12). One desires to estimate the unknown parameters, $p = [\tau_0 \ \alpha_0]^T$. The estimate of p is given by the solution of:

$$\hat{p} = \arg \max_{\tau, \alpha} \Psi_{wb}(\tau, \alpha), \quad \text{for wideband processing} \quad (2.28)$$

$$\hat{p} = \arg \max_{\tau, \nu} \Psi_{nb}(\tau, \nu), \quad \text{for narrowband processing.} \quad (2.29)$$

2-D peak search algorithm based on NBCAF and WBCAF are used to estimate the unknown parameters. Qu Jin et al. in [10] have shown that under high SNR, the joint estimation of delay and scale using the ambiguity function is efficient in the sense that it is unbiased and the estimation variance reaches the CRLB. The mean square error (MSE) of the estimates $\hat{\tau}_0$ and $\hat{\alpha}_0$ are obtained from averaging N independent trials, defined below,

$$MSE(\hat{\tau}_0) = \frac{1}{N} \sum_{i=1}^N [\hat{\tau}_0(i) - \tau_0]^2 \quad (2.30)$$

$$MSE(\hat{\alpha}_0) = \frac{1}{N} \sum_{i=1}^N [\hat{\alpha}_0(i) - \alpha_0]^2 \quad (2.31)$$

The total SNR [9] is defined in dB as,

$$SNR = 10 \log_{10} \frac{\|x - n\|^2}{\sigma_n^2} \quad (2.32)$$

where x is the received signal, n is the AWGN process with standard deviation σ_n , $\|\cdot\|$ represents the Euclidean length of a vector.

To compare MSE with CRLB, the CRLBs for the minimum variance of delay and scale estimators are derived in Appendix A, as follows:

$$CRLB(\tau_0) = \frac{10^{-SNR/10} (\pi^2 \beta^2 + 1/4\lambda^4 + \pi^2 k^2 / \lambda^2)}{4\lambda^2 \alpha_0^2 (1/16\lambda^8 + \pi^2 \beta^2 / 2\lambda^4 + \pi^2 \kappa^2 / 4\lambda^6 + \pi^4 \beta^4)} \quad (2.33)$$

$$CRLB(\alpha_0) = \frac{\alpha_0^2 10^{-SNR/10} (1/16\lambda^8 + \pi^2 \beta^2 / 4\lambda^4)}{(1/16\lambda^8 + \pi^2 \beta^2 / 2\lambda^4 + \pi^2 \kappa^2 / 4\lambda^6 + \pi^4 \beta^4)} \quad (2.34)$$

Computer simulations are carried out for the signal model in (2.12) for single target. The parameters of the Gaussian LFM signal used are: $\kappa = 200\text{Hz}$, $\beta = 300\text{Hz/s}$ and $\lambda = 0.1262\text{s}$. Figure 2.10 shows the MSEs of the estimates of delay and scale parameters versus SNR by using WBCAF. Both the simulation results and the CRLB values are plotted in the same pictures for comparison. Under high SNR, the simulated performance agrees with CRLB. Note that, Figure 2.10 was generated for three individual targets with different parameters and all three simulation results are shown in the same plot. It is seen that the estimator shows similar performance for different values of the target parameters. It can also be observed in Figure 2.10 that the estimates are not accurate for $SNR < 10$ dB. This agrees with the observations in [9–10], [12]. (Note that the CRLB line is

shown for $\alpha = 1$ because the CRLBs are almost the same for all three point targets.)

Figure 2.11 shows the MSEs of the estimates of delay and scale parameters by using NBCAF. Figure 2.11 was generated in the same way as Figure 2.10. Scale parameter is obtained by estimating the Doppler-shift and using (2.15). In the case of NBCAF, the MSEs do not agree with CRLB for the same value of λ that has been used to generate Figure 2.10 for WBCAF. It is because the simulated results, SNR, and CRLB values vary with λ . The smaller λ gets, it satisfies the narrowband assumption $2\nu/c \ll 1/TB$, where $TB \approx \sqrt{1 + 4\pi^2 \beta^2 \lambda^4}$ is the time-bandwidth product of full width at half maximum of the Gaussian LFM signal. While for $\lambda = 0.1262s$, the MSE of estimates using the WBCAF agrees with CRLB under high SNR (Figure 2.10); the MSE of estimates using the NBCAF does not agree with CRLB (Figure 2.11). For this λ the narrowband condition is $2\nu/c \ll 0.033$. However, for $\lambda = 0.04s$, the MSE of estimates using the NBCAF agrees with CRLB that can be seen in Figure 2.12. This is because the narrowband condition, $2\nu/c \ll 0.333$, now satisfies. Therefore, for the same accuracy of the estimates, the NBCAF requires higher SNR than the WBCAF.

(a)

(b)

Figure 2.10: MSE vs. SNR using WBCAF for three individual targets. The three target parameters are shown in the legend: (a) Delay estimation; (b) Scale estimation.

(a)

(b)

Figure 2.11: MSE vs. SNR using NBCAF for three individual targets. The three target parameters are shown in the legend: (a) Delay estimation; (b) Scale estimation.

(a)

(b)

Figure 2.12: MSE vs. SNR using NBCAF for single target. The target parameters are shown in the legend: (a) Delay estimation; (b) Scale estimation.

2.6 Efficient Parameter Estimation Technique

The previous section has clearly shown the necessity of the use of WBCAF for target parameter estimation. The conventional method of determining target parameters is to compute the WBCAF, $\Psi_{wb}(\tau, \alpha)$, in the entire delay-scale $(\tau - \alpha)$ plane. The estimation procedure is accomplished by evaluating the WBCAF and searching for its peak in the whole delay-scale plane. In order to get a good estimate, the global maximum of WBCAF must be accurately located and the delay-scale plane has to be densely sampled. However, this is a computationally intensive technique as it involves a large number of computations to perform the 2-D search. Computer simulations show that the computational efficiency of 2-D search methods is very low.

For many applications encountered in sonar areas, the value of scale parameter, α , is around unity [9]. Using this fact, one can reduce the computational cost of evaluating the WBCAF by some factor. However, one still need to evaluate the function $\Psi_{wb}(\tau, \alpha)$ at $N_\tau N_\alpha$ points, where N_τ is the number of points in τ and N_α is the number of points in α [9]. The evaluation of WBCAF will further be complicated if the signal is not known analytically. In this case, creating the scaled and delayed replicas (to compute WBCAF) requires resampling the transmitted signal and which is a computationally cumbersome procedure. The resampling issue will be discussed in more detail in Chapter 4.

To avoid the exhaustive 2-D search, a computationally efficient technique for estimating the target parameters is proposed here. This is achieved by the evaluation of the NBCAF along the main ridge rather than on the entire ambiguity plane. To ease the computational load and to increase accuracy, both NBCAF and WBCAF are utilized in the procedure. The following summarizes the proposal for efficient computation of parameters for a single target. The main ridge of NBCAF, $\Psi_{nb}(\tau, \nu)$, in (2.26) lies along a straight line in $\tau - \nu$ plane given by (2.35). Thus, searching for the maximum point in a 2-D plane can be simplified by searching for the maximum point along the main ridge given by (2.35) which is as same as (2.26).

$$\tau = \tau_0 + \frac{2\pi^2 \beta [\nu - \kappa(\alpha_0 - 1)]}{(1/4\lambda^4 + \pi^2 \beta^2)(1 + \alpha_0^2)} \quad (2.35)$$

- Now, select two initial values for ν_1 and ν_2 , preferably in the vicinity of the expected scale of the target [9]. Computing the $\Psi_{nb}(\tau, \nu)$ obtains the time delays τ_1 and τ_2 which maximizes $\Psi_{nb}(\tau, \nu_1)$ and $\Psi_{nb}(\tau, \nu_2)$, respectively.
- Using ν_1 and ν_2 , and the delays τ_1 and τ_2 , it is possible to estimate the expression for the main ridge of $\Psi_{nb}(\tau, \nu)$ as given in (2.35).
- Evaluate $\Psi_{nb}(\tau, \nu)$ and perform 1-D peak search along the main ridge of the NBCAF to estimate the parameters.
- For more accurate estimation, evaluate $\Psi_{wb}(\tau, \alpha)$ and perform 1-D peak search along the main ridge of the WBCAF to estimate the parameters.

So far, it was considered that the echo reflects from a target via single path. That is, the effect of multipath reflection was considered negligible. However, in many cases the multipath reflection cannot be neglected due to the effect of channel boundary. For example, in ocean acoustics, the echo reflected from a submarine may take at least three paths to reach to the receiver, viz. via direct path, via ocean floor, and via ocean surface. The echo may be reflected from other targets as well. In this case the above procedure of determining the main ridge fails. In the next section, the use of 2-D FT of the NBCAF for parameter estimation in multipath propagation will be demonstrated which resolves this issue.

2.7 The Use of 2-D FT of NBCAF to Estimate the Main Ridge Slope

Multipath reflections [14], in which one or more attenuated and delayed versions of the same transmitted signal are received, is a frequent phenomenon where a signal and its echo are present. This situation encounters in many areas like ocean acoustics, radar, geophysics, etc. [14]. This may also result from the signal reflecting off several objects to arrive at the receiver. A single point target case is considered here, where the receiver receives echoes via long range multipath. That is, the assumption that all the received echoes will have the same scale parameter with different delays. In this special case, the received signal can be modeled as,

$$x(t) = \sum_{i=0}^P a_i g[\alpha_0(t - \tau_i)] + n(t) \quad (2.36)$$

where $g(t)$ is the transmitted signal, a_i the amplitude for path i , τ_i the time delay for path i , P the number of different paths, and $n(t)$ is the AWGN process. It is assumed that $g(t)$ is known and the problem is to estimate the unknown parameter α_0 (also the slope of the main ridge of NBCAF) from the samples of the received signal.

A computationally efficient fast Fourier transform (FFT)-based NBCAF processing is proposed to estimate the slope. Applying the self transform property [4], it is noted that the 2-D FT of NBCAF passes through the origin. It can also be computed by convolving the auto-ambiguity functions of the transmitted and received signals, respectively. After some algebraic manipulation, the following analytical expression for the 2-D FT of NBCAF for single target in (2.25) can be obtained as,

$$\begin{aligned} F(u', v') &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi_{nb}(\tau, \nu) e^{-j2\pi\{\tau u' + \nu v'\}} d\tau d\nu \\ &= \frac{\pi}{\sqrt{XY}} \exp \left\{ -\frac{\pi^2 u'^2}{Y} - \frac{(\pi v' + \pi Z u')^2}{X} \right\} \end{aligned} \quad (2.37)$$

where

$$X = \frac{\pi^2 / \lambda^2}{(1/4\lambda^4 + \pi^2 \beta^2)(1 + \alpha_0^2)}, \quad Y = \frac{\alpha_0^2(1 + \alpha_0^2)(1/4\lambda^4 + \pi^2 \beta^2) / \lambda^2}{(1 + \alpha_0^2)^2 / 4\lambda^4 + \pi^2 \beta^2 (1 - \alpha_0^2)^2}, \quad Z = \frac{2\pi^2 \beta}{(1/4\lambda^4 + \pi^2 \beta^2)(1 + \alpha_0^2)}.$$

The main ridge of the function in (2.37) can be obtained by taking derivative of square modulus of $F(u', v')$ with respect to u' and set the result equal to zero to solve for v' ,

$$v' = -\frac{1 + \alpha_0^2}{2\beta\alpha_0^2} u' = -m u' \quad (2.38)$$

which means $m = \frac{1 + \alpha_0^2}{2\beta\alpha_0^2}$ is the slope of the main ridge of the function in (2.37).

Once the slope is known, the parameter α_0 can be estimated from m . The three-step procedure of estimating slope is summarized as follows:

- Compute the NBCAF.
- Perform a 2-D FT of the NBCAF.
- Estimate the slope by using least squares estimation (LSE) technique.

The square modulus of 2-D FT of NBCAF can also be computed directly from transmitted and received signals without prior computing NBCAF. The necessary steps for this are given here. The 2-D FT of NBCAF in (2.18) can be expressed as,

$$\begin{aligned} F(f, t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi_{nb}(\tau, v) e^{-j2\pi(\tau f + vt)} d\tau dv \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| \int_{-\infty}^{\infty} x(t_1) g^*(t_1 - \tau) e^{-j2\pi v t_1} dt_1 \right|^2 e^{-j2\pi(\tau f + vt)} d\tau dv \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t_1) g^*(t_1 - \tau) x^*(t_2) g(t_2 - \tau) e^{-j2\pi v(t_1 - t_2)} e^{-j2\pi(\tau f + vt)} dt_1 dt_2 d\tau dv \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(t_1) g^*(t_1 - \tau) x^*(t_1 + t) g(t_1 + t - \tau) e^{-j2\pi \tau f} dt_1 d\tau \\ &= -\left(\int_{-\infty}^{\infty} g(t_1) g^*(t_1 + t) e^{-j2\pi t_1 f} dt_1 \right)^* \times \int_{-\infty}^{\infty} x(t_2) x^*(t_2 + t) e^{-j2\pi t_2 f} dt_2 \\ &= -A_{gg}^*(f, -t) \times A_{xx}(f, -t) \end{aligned} \quad (2.39)$$

where A_{gg} and A_{xx} are the narrowband auto-ambiguity functions of transmitted signal, $g(t)$, and received signal, $x(t)$, respectively. The slope of main ridge of $|F(f, t)|^2$ is computed using LSE technique. This technique could also be applied in the case of multiple-targets which then would estimate an average of the slopes of the main ridges.

The orthogonal LSE technique is applied to estimate the gradient, which minimizes the vertical distance between the data points and a best fit straight line to the data. A mathematical procedure is given below for finding the best-fitting line to a given set of points by minimizing the sum of the squares of the offsets of the points from the line. The sum of the squares of the offsets is used instead of the offset absolute values because this allows the residuals to be treated as a continuous differentiable quantity. The residuals of the best-fit line for a set of points using unsquared perpendicular distances $d_{i,j}$ of points (x_i, y_j) are given by

$$R_{\perp} = \sum_{i=1}^m \sum_{j=1}^n d_{i,j} . \quad (2.40)$$

Figure 2.13: Perpendicular offsets.

Since the perpendicular distance from a line $y = a + bx$ to point (i, j) is given by

$$d_{i,j} = \frac{|y_j - (a + bx_i)|}{\sqrt{1 + b^2}}, \quad (2.41)$$

the function to be minimized is

$$R_{\perp} = \sum_{i=1}^m \sum_{j=1}^n W_{i,j} \frac{|y_j - (a + bx_i)|}{\sqrt{1 + b^2}} \quad (2.42)$$

where $W_{i,j}$ is a weighting function defined in (2.39). Because the absolute value function does not have continuous derivatives, minimizing $R_{\perp}$ is not amenable to analytic solution. However, if the square of the perpendicular distances

$$R_{\perp}^2 = \sum_{i=1}^m \sum_{j=1}^n W_{i,j} \frac{[y_j - (a + bx_i)]^2}{1 + b^2} \quad (2.43)$$

is minimized instead, the problem can be solved in closed form. $R_{\perp}^2$ is minimum when

$$\frac{\partial R_{\perp}^2}{\partial a} = -\frac{2}{1 + b^2} \sum_{i=1}^m \sum_{j=1}^n W_{i,j} [y_j - (a + bx_i)] = 0 \quad (2.44)$$

and

$$\frac{\partial R_{\perp}^2}{\partial b} = \frac{2}{1 + b^2} \sum_{i=1}^m \sum_{j=1}^n W_{i,j} [y_j - (a + bx_i)](-x_i) - \sum_{i=1}^m \sum_{j=1}^n W_{i,j} \frac{[y_j - (a + bx_i)]^2 (2b)}{(1 + b^2)^2} = 0 \quad (2.45)$$

After some algebraic manipulation, the solution of b can be derived as

$$b = -B \pm \sqrt{B^2 + 1} \quad (2.46)$$

where $B = \frac{\sum_{i=1}^m \sum_{j=1}^n W_{i,j} x_i^2 - \sum_{i=1}^m \sum_{j=1}^n W_{i,j} y_j^2}{\sum_{i=1}^m \sum_{j=1}^n W_{i,j} x_i y_j}$.

Figure 2.14 shows the contour plot of NBCAF of single target with one main path and four multipath. The scale and delay parameters used to generate Figure 2.14 are: $\alpha_0 = 1.0325$ and $\tau = [-0.2, 0.1, 0, 0.1, 0.2]$. In the contour plot, the echoes resulting from the same target show equal slopes. This property of NBCAF can be used for estimation of the slope (also the scale parameter, α_0). As the 2-D FT of a linear Gaussian shape results in a linear Gaussian shape at the origin, echoes from multipath reflection could be used constructively to approximately estimate the slope of the main ridge of the NBCAF. Figure 2.15 shows the 2-D FT of Figure 2.14 in the time-frequency plane. It is seen that all the paths from the same target pass through the origin. Therefore, the slope of Figure 2.15 can easily be estimated by LSE technique. The slope of a contour in the time-frequency plane is a measure of the scale parameter α_0 via (2.38). The separate peaks in Figure 2.15 are the results of interference effect due to multiple paths in Figure 2.14.

Figure 2.14: Contour plot of NBCAF of single target with four multipath.

Figure 2.15: 2-D FT of Figure 2.14.

Figure 2.16 shows the simulation result of estimating α_0 of the target in Figure 2.14. The MSE of the proposed estimator is compared with the CRLB (note that CRLB is for using single return, no multipath). Simulation result indicates that FFT-based NBCAF algorithm is effective to estimate α_0 (or slope, m) for multipath case. The same technique can also be used when there is only one return reflected from a single target, i.e., there is no multipath. Figure 2.17 shows the simulation result of estimating α_0 for this particular case. The single return parameters were selected as $(\tau_0, \alpha_0) = (0.1, 1.0325)$. Comparing Figures 2.16 with Figure 2.17, it can be seen that having multipath is beneficial for the estimation. Because the modulus of NBCAF has been used to remove the phases of the multipath signals. Furthermore, once α_0 is known by evaluating $\Psi_{wb}(\tau, \alpha)$ at $\alpha = \alpha_0$ and performing a 1-D search on τ , τ_0 can also be estimated.

Figure 2.16: MSE of scale parameter against total SNR for multipath case.

Figure 2.17: MSE of scale parameter against total SNR for single return case.

2.8 Computational Efficiency

As stated in Section 2.6, the 2-D peak search on WBCAF, $\Psi_{wb}(\tau, \alpha)$, is computationally intensive. To reduce the computational cost, an efficient technique based on the main ridge of NBCAF/WBCAF has been proposed. The proposed approach significantly speeds up the computations by reducing 2-D search problem into two 1-D searches. Therefore, instead of computing $\Psi_{wb}(\tau, \alpha)$ on $N_\tau N_\alpha$ points in a 2-D plane, one only needs to compute on $N_\tau + N_\alpha$ points.

2.9 Conclusions

This chapter addresses the issues related to the narrowband and wideband signal models, the narrowband and wideband cross-ambiguity functions, the main ridge characteristics of the ambiguity functions, and the 2-D FFT-based algorithms for wideband signal processing. Simulation results are provided throughout the chapter to illustrate the performance of the addressed methods.

It has been shown that the error performance of the WBCAF is better than that of the NBCAF in that the WBCAF is superior in estimating delay and scale parameters. It has also been shown that the 2-D peak search on the WBCAF is a computationally cumbersome procedure and limits the computational speed. To overcome the computational complexity of the WBCAF, an efficient NBCAF-based algorithm is proposed. However, NBCAF-based algorithm is suboptimal. A 2-D FFT-based NBCAF algorithm is also proposed to approximately estimate the slope of the main ridge of NBCAF in multipath environment.

The algorithm proposed in Section 2.6 is for a single target with negligible multipath effect. In Section 2.7, the multipath effect has been taken into consideration. However, both the algorithms are only suitable for single target case. In the presence of multiple-targets, there would be difficulties and limitations associated with these algorithms.

On one hand, 2-D peak search on the WBCAF is computationally intensive, and on the other hand efficient methods proposed in this chapter may not be suitable in detecting multiple-targets. To overcome the computational burden and for dealing with multiple-targets, efficient techniques based on Radon-ambiguity transform and Hermite decompositions will be explored in Chapters 3 and 4.

Chapter 3

Efficient Wideband Parameter Estimation from Multiple-Targets Using the Radon-Ambiguity Transform

3.1 Introduction

In Chapter 2, parameter estimation from a single target having single or multipath reflections has been considered. FFT-based narrowband cross-ambiguity function (NBCAF) processing algorithm described in Section 2.7 has shown an effective way of estimating scale parameter. However, real applications may encounter multiple-targets with different scale parameters (see Figure 3.1, where $\alpha = 1.0, 0.9512, 1.0788$). FFT-based NBCAF algorithm is not appropriate in this particular case as the simple LSE gives only an average estimate (as in Figure 3.2, where $\alpha_{est} = 0.9954$).

It has also been shown in Chapter 2 that with single target and additive white Gaussian noise (AWGN) the wideband cross-ambiguity function (WBCAF) of Gaussian LFM signals, evaluated via matched filter processing, provide optimal performance in the sense that it meets CRLB. As mentioned earlier, this is a computationally exhaustive technique as it involves many computations due to the

2-D search. Furthermore, the technique is not effective in separating multiple-targets especially when the targets are close to each other. To overcome the high computational cost and to deal with multiple-targets, we investigate an efficient technique based on single slice Radon-ambiguity transform (SSRAT) for the estimation of delay and scale parameters of multiple-targets. An efficient method is proposed that combines NBCAF, WBCAF, and SSRAT to estimate the delay and scale parameters of multiple LFM sonar signals, simultaneously. The computational load of direct computation of NBCAF and WBCAF algorithms with SSRAT in conjunction with 1-D searches on NBCAF and WBCAF are compared. Simulations show that our proposed algorithms are more efficient than the traditional algorithms based on direct evaluation of NBCAF or WBCAF.

As noted earlier, the square modulus of Gaussian LFM signals has high-energy centrality in the ambiguity plane and its peaks in the NBCAF fall along nearly straight lines whose slopes depend on the Doppler rates of the moving targets. These lines could be effectively detected by computing the Radon-ambiguity transform (RAT) of the NBCAF for all possible angles; however, it is also a computationally intensive procedure. It is shown in this chapter that without calculating the entire RAT, it is possible to estimate target parameters using only a single slice of the RAT, i.e. using an appropriate projection of the NBCAF. It is demonstrated that the proposed method can successfully separate overlapping targets efficiently. The efficiency is achieved due to FFT-based processing, use of SSRAT, and the use of only 1-D searches.

Figure 3.1: Contour plot of NBCAF in the presence of three point targets and the three main ridges of the NBCAF.

Figure 3.2: 2-D FT of Figure 3.1.

3.2 Motivation

The main ridge of NBCAF of Gaussian LFM signal is a straight line in the ambiguity plane, which is similar in shape to a time-frequency distribution of an LFM signal. As such, our interest here is the detection and estimation of multicomponent LFM signals; however it is relatively a difficult task in sonar areas. Time-frequency analysis based on spectrogram, which is the magnitude squared of the short-time Fourier transform (STFT), has been used in [19]. However, a crucial drawback inherent in the STFT method is that the length of the window is fixed, limiting its application in practice. Studies have shown that the Wigner-Ville distribution (WVD) is useful for analyzing and detecting nonstationary signals [21–24]. But in multicomponent signals condition, the WVD suffers from noise and interference from the cross-terms seriously, which causes difficulties in interpreting time-varying spectra. Since both cross-terms and white noise produce oscillations in the time-frequency plane, line integration through the WVD (known as WVD-based detector) may be used to emphasize the slowly-varying auto-component contributions [25–26], [30]. The Radon transform of the WVD or the Radon-Wigner transform (RWT) [30–33], which is equivalent to quadrature matched filtering or the dechirp method [27–28], has been used to analyze time-varying nonstationary signals. The RWT offers a way to track straight lines in the time-frequency plane and locating maxima in a 2-D plane [32], which however is a computationally exhaustive technique because of 2-D search. M. Wang *et al.* [36] have developed Radon-ambiguity transform (RAT)-based detector that has been shown to provide equivalent performance to the

RWT-based algorithm while reducing the search space from 2-D to 1-D only. However, with the use of RAT, it was assumed in [35–40] that all directions of interest pass through the origin of the ambiguity plane. Therefore, the only parameter of interest is the chirp rate (scale) and as a result it is relatively easy to implement the estimator using a 1-D technique. Under this special assumption, delay and scale parameters cannot be estimated simultaneously. In many practical cases, however, it may be necessary to estimate both delay and scale parameters simultaneously. Moreover, our work is significantly different from [35–40] in the sense that it deals with 2-D problems using 1-D search techniques via dechirp method to estimate both delay and scale parameters simultaneously. A sufficient amount of computer simulations are also presented.

It is noted here that the Radon transform of a time-frequency distribution produces local areas of signal concentration that facilitate interpretation of multicomponent signals [29–42]. A major advantage of Radon transform is that lines are allowed to intersect [34]. Another major advantage is that the Radon transform is robust in the presence of noise.

Two novel algorithms are proposed in this chapter. First it is shown that the SSRAT in conjunction with a 1-D search on NBCAF would result in approximate estimation in the vicinity of the true parameters, and the algorithm is efficient. Then for more accurate estimation, it is shown that the SSRAT in conjunction with a 1-D search on WBCAF closely achieves the CRLB. The resolution of RAT for bicomponent signals and the effect of the finite integration length of RAT in

AWGN have been provided. The development of SSRAT-based efficient algorithms with analytical justification is the novelty reported in this chapter.

This chapter is organized as follows. The RAT technique is presented in Section 3.3. By combining NBCAF and WBCAF, the SSRAT algorithms are proposed in Section 3.4. Section 3.5 demonstrates the radius-angle resolution and the effect of the integration length of RAT in AWGN. Computer simulations are given in Section 3.6 to show the effectiveness of the proposed techniques. Section 3.7 draws the conclusion.

3.3 Radon-Ambiguity Transform

Assume that $f(x, y)$ is a 2-D image function, then its Radon transform is defined as its line integral along a line inclined at an angle θ from the y-axis and at a distance ρ (radius) from the origin. Mathematically, it is written as [29]

$$\mathfrak{R}(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta + y \sin \theta - \rho) dx dy \quad (3.1)$$

where $-\infty < \rho < \infty$, $0 \leq \theta \leq \pi$.

The function $\mathfrak{R}(\rho, \theta)$, the Radon transform of $f(x, y)$, is the 1-D projection of $f(x, y)$ at an angle θ . In the rotated coordinate system (ρ, u) , (3.1) can be expressed as,

$$\mathfrak{R}(\rho, \theta) = \int_{-\infty}^{\infty} f(\rho \cos \theta - u \sin \theta, \rho \sin \theta + u \cos \theta) du \quad (3.2)$$

where
$$\left. \begin{aligned} \rho &= x \cos \theta + y \sin \theta \\ u &= -x \sin \theta + y \cos \theta \end{aligned} \right\} \text{ or } \left\{ \begin{aligned} x &= \rho \cos \theta - u \sin \theta \\ y &= \rho \sin \theta + u \cos \theta \end{aligned} \right.$$

The key idea shown in (3.1) and (3.2) is to integrate $f(x, y)$ along the line with parameters ρ and θ . Each point in the (ρ, θ) plane corresponds to a line in the spatial domain (x, y) . Some of the useful properties of Radon transform have been reported in [29].

Using (3.2), the Radon transform of the 2-D NBCAF, $\Psi_{nb}(\tau, v)$, is defined in a different form as,

$$\mathfrak{R}_A(\rho, \theta) = \int_{-\infty}^{\infty} \Psi_{nb}(\tau, v) du = \int_{-\infty}^{\infty} \Psi_{nb}(\rho \cos \theta - u \sin \theta, \rho \sin \theta + u \cos \theta) du \quad (3.3)$$

where $\tau = \rho \cos \theta - u \sin \theta$ and $v = \rho \sin \theta + u \cos \theta$.

Equation (3.3) defines the RAT which represents the sum of $\Psi_{nb}(\tau, v)$ along a line located at a distance $|\rho|$ from the origin and angle of inclination θ formed by the perpendicular to the line. It will be shown that the Radon transform transforms each of the lines into peaks positioned corresponding to the parameters of the lines. Figure 3.3 shows the geometrical relationship of the RAT.

Figure 3.3: Radon transform geometry for the RAT.

For Gaussian LFM signals, after some manipulation, the analytical expression of RAT in (3.3) resulting from a single target with parameters (τ_0, v_0) can be obtained as

$$\Re_A(\rho, \theta) = A \sqrt{\frac{\pi}{P}} \times e^{\left\{ -B\{\rho\beta \sin \theta - v_0\}^2 - C\{\rho \cos \theta - D\rho\beta \sin \theta + Dv_0 - \tau_0\}^2 + \frac{Q^2}{4P} \right\}} \quad (3.4)$$

with

$$A = \frac{\pi}{\left[\{1 + (v_0 / \kappa + 1)^2\}^2 / 4\lambda^4 + \pi^2 \beta^2 \{1 - (v_0 / \kappa + 1)^2\}^2 \right]^{1/2}}$$

$$B = \frac{\pi^2 / \lambda^2}{(1 / 4\lambda^4 + \pi^2 \beta^2) \{1 + (v_0 / \kappa + 1)^2\}}$$

$$C = \frac{(v_0 / \kappa + 1)^2 \{1 + (v_0 / \kappa + 1)^2\} (1 / 4\lambda^4 + \pi^2 \beta^2) / \lambda^2}{\{1 + (v_0 / \kappa + 1)^2\}^2 / 4\lambda^4 + \pi^2 \beta^2 \{1 - (v_0 / \kappa + 1)^2\}^2}$$

$$D = \frac{2\pi^2 \beta}{(1 / 4\lambda^4 + \pi^2 \beta^2) \{1 + (v_0 / \kappa + 1)^2\}}$$

$$P = \{(B + CD^2)\beta^2 \cos^2 \theta + C \sin \theta (\sin \theta + 2D\beta \cos \theta)\}$$

$$Q = (Bv_0 + CD\rho \cos \theta - CD\tau_0 + CD^2v_0)2\beta \cos \theta + (Dv_0 - D\rho\beta \sin \theta - \tau_0)2C \sin \theta + (-B\beta^2 + C - CD^2\beta)2\rho \cos \theta \sin \theta$$

For a given angle θ , the peak position of ρ of the RAT can be deduced from (3.4) as,

$$\rho_p = \tau_0 \cos \theta + v_0 \sin \theta. \quad (3.5)$$

Equation (3.5) suggests that for known (ρ_p, θ) , $\rho_p = \tau_0 \cos \theta + v_0 \sin \theta$ describes a line passing through the target parameters (τ_0, v_0) in the NBCAF plane. Using the mapping in (2.15), the corresponding line in the WBCAF plane in (3.5) can be rewritten as,

$$\rho_p = \tau_0 \cos \theta + (\alpha_0 - 1)\kappa \sin \theta. \quad (3.6)$$

It is noted here that, when more than one target exist, the targets can be effectively distinguished from each other according to the slope of each line of NBCAF respectively, as shown by Figure 3.1. The targets in Figure 3.1 can be detected by processing the entire RAT, which is shown in Figure 3.4 (left window) in the $\rho - \theta$ plane. Note that each target shows a peak in RAT plane at radius ρ_{pi} and rotation angle θ_i , where i is the number of target.

Figure 3.4: 2-D RAT of Figure 3.1 (left window) and SSRAT of Figure 3.1 at the angle $\theta_a = 135^\circ$ (right window).

Once ρ_{pi} and θ_i are determined, scale α_i and delay τ_i can be obtained from (2.26) and (3.6) as,

$$\alpha_i = \sqrt{\frac{2\pi^2\beta}{-(1/4\lambda^4 + \pi^2\beta^2)\cot\theta_i}} - 1 \quad (3.7)$$

and

$$\tau_i = \rho_{pi} \sec\theta_i - \kappa(\alpha_i - 1)\tan\theta_i. \quad (3.8)$$

The Radon transform of Figure 3.1 is calculated for all possible angles. Note that the evaluation of RAT for all possible angles is computationally intensive. In the next sections, an efficient way of computing target parameters will be demonstrated.

3.4 Efficient Parameter Estimation Techniques

In Chapter 2, it has been noted that the conventional methods are computationally cumbersome. Another possible method of determining target parameters is to compute the $\mathcal{R}_A(\rho, \theta)$ in the entire $\rho - \theta$ plane. In order to get a good estimate, the $\rho - \theta$ plane has to be densely sampled. This also involves 2-D search and it is computationally more intensive than the conventional methods. To avoid the exhaustive 2-D search on entire RAT, computationally efficient techniques for estimating the target parameters are proposed.

Based on (3.6), without calculating the entire RAT, it is possible to estimate parameters using only a SSRAT, i.e. using an appropriate projection (using approximate slope) of the NBCAF. Once the approximate slope is known, the projection angle between, θ_a , can be related to slope m via $\theta_a = -\cot^{-1} m + \pi$ (see Figure 3.3). The projection angle would approximately be equal to the

average slope of the main ridge of the NBCAF. Then a slice of the Radon transform of Figure 3.4 (left window) is calculated at the angle θ_a (right window of Figure 3.4).

One possible method to estimate an approximate angle for projection, θ_a , using 2-D FT of the NBCAF using LSE technique, which has been shown in Chapter 2. In the presence of multiple-targets this would result in the average slope. Multipath reflection, in which one or more attenuated and delayed versions of the same radiated signal are received, is a frequent occurrence in ocean acoustics. As the 2-D FT of a linear Gaussian shape results in a linear Gaussian shape at the origin, echoes from multiple-targets could be used constructively to approximately estimate the slope of the main ridge of the NBCAF. Figure 3.2 shows the 2-D FT of Figure 3.1. There it can be seen that the 2-D FT of Figure 3.1 passes through the origin. Using the approximate slope angle, θ_a , in the following section it will be demonstrated how the projection of RAT can be efficiently used to estimate multiple-targets parameters.

It is also noted that, for most of the applications encountered in sonar areas, the value of scale parameter α is around unity [9]. Therefore, an approximate estimation of slope as well as projection angle could be known *a priori*. Using this fact, the computational cost of slope estimation could be avoided. It is also possible to compute few other projections in the vicinity of the approximate angle if a single projection misses to detect all the targets. An efficient way of computing SSRAT will now be proposed.

3.4.1 Efficient Computation of SSRAT using Dechirp Technique

Computing Radon transform of NBCAF at an angle θ_a (SSRAT) is equivalent to computing the projection of rotated-NBCAF of the dechirped signal at the chirp rate. Here, rotated-NBCAF is computed by dechirping both the transmitted and received signals. The two steps procedure of computing the projection of rotated-NBCAF is as follows,

- Estimate a projection angle θ_a (using 2-D FT and the LSE technique).
- Compute the projection of rotated-NBCAF of Figure 3.1 by dechirping both the transmitted and received signals which is shown in Figure 3.5 (right window).

Figure 3.5: 2-D rotated-NBCAF of Figure 3.1 after dechirping by the slope (left window) and 1-D projection (equivalent to SSRAT) (right window).

The projection of rotated-NBCAF on Doppler axis could be computed efficiently using the dechirp technique. To do this, it is noted that the NBCAF between $y(t)$ and $z(t)$ is defined as,

$$\Psi_{nb}(\tau, \nu) = \left| \int_{-\infty}^{\infty} z(t) y^*(t - \tau) e^{-j2\pi\nu t} dt \right|^2. \quad (3.9)$$

Taking the integration on both sides of (3.9) with respect to τ (i.e. taking the projection on ν axis), we get

$$\int_{-\infty}^{\infty} \Psi_{nb}(\tau, \nu) d\tau = \int_{-\infty}^{\infty} \left| \int_{-\infty}^{\infty} z(t) y^*(t - \tau) e^{-j2\pi\nu t} dt \right|^2 d\tau$$

or

$$\begin{aligned} \Psi_{rot}(\nu) &= \int_{-\infty}^{\infty} \left| \int_{-\infty}^{\infty} z(t) y^*(t - \tau) e^{-j2\pi\nu t} dt \right|^2 d\tau \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z(t) y^*(t - \tau) z^*(t_1) y(t_1 - \tau) e^{-j2\pi\nu(t-t_1)} dt dt_1 d\tau \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} z(t) z^*(t_1) e^{-j2\pi\nu(t-t_1)} \left[\int_{-\infty}^{\infty} y(t_1 - \tau) y^*(t - \tau) d\tau \right] dt dt_1 \\ &= \int_{-\infty}^{\infty} R_y(-\Delta) e^{-j2\pi\nu\Delta} \left[\int_{-\infty}^{\infty} z(t) z^*(t - \Delta) dt \right] d\Delta \\ &= \int_{-\infty}^{\infty} R_y(-\Delta) R_z(\Delta) e^{-j2\pi\nu\Delta} d\Delta \\ &= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} |Y(\nu)|^2 e^{j2\pi\nu\Delta} d\nu \right\} \left\{ \int_{-\infty}^{\infty} |Z(\nu)|^2 e^{j2\pi\nu\Delta} d\nu \right\} e^{-j2\pi\nu\Delta} d\Delta \end{aligned} \quad (3.10)$$

where $y(t)$ and $z(t)$ are the dechirped transmitted and received signals, respectively; $Y(\nu)$ and $Z(\nu)$ are the FTs of $y(t)$ and $z(t)$, respectively. Using (3.10), the projection of rotated-NBCAF (SSRAT) could be efficiently computed using the FFT-based processing. We will now propose two efficient parameter estimation methods based on NBCAF and WBCAF.

3.4.2 Efficient Parameter Estimation using the SSRAT, NBCAF, and WBCAF

Using θ_a and the peak position v_p from (3.10), ρ_p of (3.6) can be calculated using the geometrical relation $\rho_p = v_p \sin \theta_a$ (see Figure 3.3). Figure 3.4 (right window) shows the SSRAT of Figure 3.1 by using only one slice at an angle θ_a . There, it can be clearly seen that the three peaks corresponding to three targets. Figure 3.4 (left window) is generated from the entire RAT and Figure 3.4 (right window) is generated from SSRAT but they both show the existence of three targets. From the SSRAT slice three peak positions for radius ρ_{pi} could be estimated. Then using the line in (3.6), three 1-D searches on NBCAF/WBCAF could be used to estimate three target parameters. The three search lines are shown in Figures 3.6 (a) and (b) for NBCAF and WBCAF. The necessary steps and the *type of processing* for estimating parameters from SSRAT are as follows:

- Estimate a projection angle θ_a (using 2-D FT and the LSE technique).
- Compute the SSRAT of Figure 3.1 at an angle θ_a using the traditional method (Figure 3.4, right window) or the *efficient dechirp technique* via (3.10) (Figure 3.5, right window).
- Perform a 1-D search either on Figure 3.4 (right window) or on Figure 3.5 (right window) to find the peak positions ρ_{pi} .
- Suboptimal estimation: A 1-D search on NBCAF in Figure 3.6 (a) along the line $\rho_{pi} = \tau_i \cos \theta_a + v_i \sin \theta_a$ is performed to estimate the peak positions of each target, τ_i and v_i .

- Optimal estimation: A 1-D search on WBCAF in Figure 3.6 (b) along the line $\rho_{pi} = \tau_i \cos \theta_a + (\alpha_i - 1)\kappa \sin \theta_a$ is performed to estimate the peak positions of each target, τ_i and α_i .

(a)

(b)

Figure 3.6: Contour plots of the same signal as in Figure 3.1: (a) Three search lines through the center of the NBCAF; (b) Three search lines through the center of the WBCAF.

The line search in Figure 3.6 can also be efficiently implemented without computing the whole NBCAF/WBCAF. Instead of computing the entire NBCAF/WBCAF, 1-D NBCAF/WBCAF along the straight lines indicated in Figure 3.6 could be computed. Using this, the computational cost could be reduced appreciably. However, this method is efficient only for single target case. For multiple-targets case, the targets should be close to each other and the estimation may not be very accurate. Therefore we are supposed to have some a priori information about the targets [9]. From (3.6), the equation of the search line in Figure 3.6 which passes through the peak position can be rewritten as,

$$\alpha = -\frac{\tau}{\kappa} \cot \theta + \frac{\rho_p}{k \sin \theta} + 1. \quad (3.11)$$

Using (2.10), (2.16), and (3.11), the WBCAF and NBCAF could be efficiently evaluated for a 1-D search along the main ridge in the vicinity of the approximate parameters. When (3.11) is substituted into (2.10) and (2.16), the resulting expression of 1-D WBCAF and NBCAF along the main ridge can be written as,

$$\chi_{wb}^{1-D}(\tau) = \sqrt{\alpha} \int_{-\infty}^{\infty} x(t) g^*[\alpha(t - \tau)] dt \quad (3.12)$$

$$\chi_{nb}^{1-D}(\tau) = \int_{-\infty}^{\infty} x(t) g^*(t - \tau) e^{-j2\pi vt} dt \quad (3.13)$$

where α is a function of τ defined in (3.11) and v can be obtained from the relationship in (2.15).

Figure 3.7 shows the block diagram detailing the computation sequences of the proposed algorithms.

Figure 3.7: Block diagram of the proposed algorithms.

3.5 Detection Accuracy, Resolution and the Effect of the Integration Length of RAT

3.5.1 Detection Accuracy and Resolution of RAT

As noted, the Radon-ambiguity transform is an effective technique to detect target parameters in a noisy environment. However, detection accuracy and resolution of RAT is an important factor especially for detecting multiple-targets. In this chapter RAT is used as square-law detector as it performs better than the envelope detector for stronger or equal amplitude LFM signal detection [36]. The analytical expression of RAT for a single target is given in (3.4). The 3 dB response width of ρ for a given $(\tau_0, \nu_0) = (0,0)$ can be deduced from (3.4) as,

$$\Delta\rho = \sqrt{\frac{3(1/4\lambda^4 + \pi^2\beta^2)\ln 10/2\lambda^2}{5\{2\pi^2\beta^2(1/4\lambda^4 + \pi^2\beta^2) + 1/16\lambda^8\}}}. \quad (3.14)$$

The 3 dB width can be considered as a measure of selectivity of the estimation [36]. The Thesis gives emphasis on single slice RAT algorithm and as such (3.14) is derived for a specific projection angle $\theta_a = 135^\circ$. Figure 3.8 (a) shows the

detection accuracy of single slice RAT for a single target with parameters $(\tau_0, \nu_0) = (0, 0)$, which corresponds to $(\rho, \theta_a) = (0, 135^\circ)$, where θ_a is the correct projection angle. As expected, the peak position shows the exact location of the target. Figure 3.8 (b) shows the detection accuracy of single slice RAT for the same parameters with an incorrect projection angle $\theta_a = 145^\circ$. It is seen in Figure 3.8 (b) that both the detection accuracy and the signal strength have been deteriorated. As such, an accurate estimation of the projection angle is an important factor for single slice RAT algorithm.

Now, we consider RAT for a bicomponent signal to demonstrate resolution. We define a bicomponent signal as,

$$r(t) = \sum_{i=1}^2 a_i g[\alpha_i(t - \tau_i)] \quad (3.15)$$

where $g(t)$ is the transmitted Gaussian LFM signal as defined in Chapter 2, a_i is the amplitude reflection factor for the i^{th} echo, α_i is the scale factor for the i^{th} echo, and τ_i is the delay for the i^{th} echo. The assumption $\alpha_1 = \alpha_2 = 1$ is used for single slice RAT for simplicity. After some algebraic manipulation, the analytical expression of single slice RAT for bicomponent signal in (3.15) is derived as,

$$\begin{aligned} \Re_A(\rho) = N \times e^{-2M\rho^2} \left\{ e^{-M\{\tau_1^2 + \sqrt{8}\tau_1\rho\}} + e^{-M\{\tau_2^2 + \sqrt{8}\tau_2\rho\}} \right\} + 2N \times e^{-\frac{\pi^2\lambda^2}{2}[8\beta^2\rho^2 + \tau_s^2]} \\ \times e^{-\frac{-(\tau_1^2 + \tau_2^2)(1/4\lambda^4 + \pi^2\beta^2)\lambda^2\rho^2 - \tau_a}{\sqrt{2}M\rho - \frac{1}{8\lambda^2}\tau_a}} \times \cos \left[2\pi\tau_s \left\{ \kappa + \frac{\rho}{\sqrt{2}} + \frac{1}{4}\tau_a \right\} \right] \end{aligned} \quad (3.16)$$

with $M = 2\pi^2\beta^2\lambda^2$, $N = \sqrt{4\pi^3\lambda^6}$, $\tau_a = (\tau_1 + \tau_2)$, $\tau_s = (\tau_1 - \tau_2)$.

Figure 3.9 (a) shows the simulation result of (3.16) with the correct projection angle $\theta_a = 135^\circ$, which corresponds to $\alpha_1 = \alpha_2 = 1$. The delay parameters chosen for Figure 3.9 (a) are $(\tau_1, \tau_2) = (0.01, -0.01)$. Figure 3.9 (b) shows the simulation result of (3.16) with an incorrect projection angle $\theta_a = 145^\circ$. Comparing the two figures it is seen that the two targets are accurately resolved in Figure 3.9 (a), whereas the incorrect projection in Figure 3.9 (b) not only fails to separate two targets but also reduces the signal strength.

(a)

(b)

Figure 3.8: Detection accuracy of single slice RAT for a single target: (a) Accurate projection angle; (b) Inaccurate projection angle.

(a)

Figure 3.9: Resolution of RAT for a bicomponent signal: (a) Accurate projection angle; (b) Inaccurate projection angle.

3.5.2 The Effect of the Integration Length of RAT

The effect of the integration length of RAT in AWGN is discussed here. The RAT defined in (3.3) integrates along a line of the NBCAF, $\Psi_{nb}(\tau, \nu)$, in delay-Doppler plane. To see the performance of the RAT-based detector with respect to the integration length, (3.3) can be redefined for finite integration length as,

$$\mathfrak{R}_A(\rho, \theta) = \int_{-L/2}^{L/2} \Psi_{nb}(\rho \cos \theta - u \sin \theta, \rho \sin \theta + u \cos \theta) du \quad (3.17)$$

where L is the integration length along the main ridge of NBCAF.

To derive an optimal value for L we would look into detection performances. The detail statistical derivations of RAT based detection using (3.17) are given in Appendix B. The performance of the detection can be expressed as,

$$\eta = \frac{|E(\mathfrak{R}_A | H_1) - E(\mathfrak{R}_A | H_0)|}{\left\{ \frac{1}{2} [\text{var}(\mathfrak{R}_A | H_1) + \text{var}(\mathfrak{R}_A | H_0)] \right\}^{1/2}}. \quad (3.18)$$

Thus, using the expressions (B-5)–(B-8) from Appendix B, the effect of integration length on the detection performance can be deduced as,

$$\eta = \frac{\pi^{3/4} \lambda \sqrt{1/a} \text{erf}(b/2)}{\left[\frac{N_0^2}{a} \left\{ b \text{erf}(b) + \frac{e^{-b^2}}{\sqrt{\pi}} - \frac{1}{\sqrt{\pi}} \right\} + \frac{\lambda N_0}{2e^{b^2/4}} \left\{ \frac{3L}{4} \sqrt{\frac{\pi}{a}} e^{b^2/16} \{ \text{erf}(b/4) + \text{erf}(3b/4) \} + \frac{1}{a} (1 - e^{-b^2/2}) \right\} \right]^{1/2}} \quad (3.19)$$

where $b = L\sqrt{a}$, $a = \sin^2 \theta / 2\lambda^2$, and $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$.

Figure 3.10 shows the detection performance of RAT in AWGN with the variation of integration length. The values of the parameters used in Fig. 3.10 are: $\lambda = 0.1262$ and $\theta = 135^\circ$. It can be seen that the detection performance is slowly increasing up to a certain value of L . Then the performance starts decreasing for further increasing L and it approaches to zero as L approaches to infinity. This is because the noise components spread equally in the ambiguity plane whereas the signal components concentrate around the center of the ambiguity plane. Therefore, as the integration length increases it goes away from the center of the ridge line and thereby signal terms decrease while the noise terms remain the same. To evaluate an optimum length, (3.19) is differentiated with respect to L and equate the result as zero to solve for L . An optimum value of L is obtained in an approximate form as $L_{opt} \approx 2\lambda / \sin \theta$. If T is the full width at half maximum of the Gaussian modulated signal then $T = 2.354\lambda$ (or $\lambda = 0.4248T$). Substituting the value of λ in L_{opt} we have $L_{opt} \approx 0.8496T / \sin \theta$, which is a relation between the optimum choice of L and the effective duration of the Gaussian modulation T .

Figure 3.10: Detection performance of RAT in AWGN with the variation of the integration length of RAT.

3.6 Simulation Results

The received signal $x(t)$ is modeled as a linear sum of three (can be extended to more than three) wideband Gaussian LFM signals with delay and scale parameters τ_i and α_i , respectively, and corrupted by AWGN process $n(t)$.

$$x(t) = \sum_{i=1}^3 r_i(t) + n(t) = \sum_{i=1}^3 a_i g[\alpha_i(t - \tau_i)] + n(t) \quad (3.20)$$

where $r_i(t)$ is the echo reflected by the i^{th} scatterer. The three point target parameters are $(\tau_1, \alpha_1) = (-0.06, 1.0)$, $(\tau_2, \alpha_2) = (-0.22, 0.9512)$, and $(\tau_3, \alpha_3) = (-0.08, 1.0788)$. To demonstrate the effectiveness of the algorithms three point targets have been considered where two of them are slightly overlapping to each other and the third one is also close to them (see Figure 3.1). The transmitted

signal parameters used to obtain Figure 3.1 are: $\kappa = 200\text{Hz}$, $\beta = 300\text{Hz/s}$, and $\lambda = 0.1262\text{s}$.

The computational complexity of the proposed algorithms is tabulated in Table 3.1 in comparison with that of the conventional algorithms. The same parameters and N -point FFT were used in all algorithms to make a fair comparison. N_t is the length of the signal in sampled time, N_τ is the number of points in τ direction, and N_α is the number of points in α direction. Only complex multiplications were considered for comparison. It is noted that to compute an N -point FFT, $N \log_2 N$ complex multiplications are needed. Equations (2.11) and (2.17) are used to evaluate the computational complexity for conventional WBCAF and NBCAF, respectively, in frequency domain. Equation (3.10) is used to compute 1-D projection using dechirp method. While (3.12) and (3.13) are used to evaluate three 1-D searches on WBCAF and NBCAF, respectively.

Algorithms	Computational Complexity (complex multiplications)	$N = 512$, $N_t = 1000$ $N_\tau = 500$, $N_\alpha = 500$
1. SSRAT with three 1-D searches on NBCAF (suboptimal).	$N(5 \log_2 N + 1) + N_t(3N_\tau + 2)$	1525552
2. SSRAT with three 1-D searches on WBCAF (optimal).	$N(5 \log_2 N + 1) + N_t(3N_\tau + 2) + 3N_\tau / 2$	1526302
3. Conventional 2-D NBCAF.	$N_\alpha N(3 \log_2 N + 1)$	7168000
4. Conventional 2-D WBCAF.	$N_\alpha N(3 \log_2 N + 1) + N_\alpha / 2$	7168250
5. 2-D RAT.	Computational cost of Algorithm 3 + computational cost of RAT	Much higher than the Algorithms 3 and 4

Table 3.1: Computational complexity of the proposed and conventional algorithms.

Algorithm 1 (of Table 3.1) is preferable to the applications which require less precision as it is little faster than Algorithm 2 (Table 3.1). For more accurate estimation Algorithm 2 can be used. However, both Algorithms 1 and 2 are about five times faster than the traditional methods (Algorithms 3 and 4) and much faster than the 2-D RAT technique (Algorithm 5). It is noted here that, if the results of 1-D searches (Algorithm 2) are still not satisfactory for some applications, then it is always possible to do a 2-D search in a local small area on the WBCAF to achieve the CRLB.

Figures 3.11 and 3.12 show the simulation results of the three point targets using the SSRAT in conjunction with a 1-D search on NBCAF (Algorithm 1). The MSE of the estimated values of τ and α are compared with the CRLB. As it can be seen from Figures 3.11 and 3.12, the parameters can be estimated approximately using Algorithm 1. Note that, Figure 3.11 was generated by three simulations for three individual targets and all three simulation results are shown in the same plot. That is, in each simulation a single target is present. Figure 3.12 was generated simultaneously for three combined point targets as in (3.20). (This approach was taken to investigate the resolution effect of targets by the presence of other targets.)

(a)

(b)

Figure 3.11: MSE vs. SNR of estimating parameters (individually simulated) using the SSRAT algorithm (1-D search on NBCAF): (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 3.12: MSE vs. SNR of estimating parameters (simultaneously simulated) using the SSRAT algorithm (1-D search on NBCAF): (a) Estimation of delay; (b) Estimation of scale.

The simulation results of the same signals as in Figure 3.11 using the SSRAT with a 1-D search on WBCAF (Algorithm 2) are shown in Figure 3.13. Simulations show that, for $(\tau_1, \alpha_1) = (-0.06, 1.0)$, the estimator closely approaches the CRLB in high SNR. On the other hand, for $(\tau_2, \alpha_2) = (-0.22, 0.9512)$ and $(\tau_3, \alpha_3) = (-0.08, 1.0788)$, the estimation error does not approach the CRLB. Note that, Figure 3.13 was generated for three individual targets independently and all three simulation results are shown in the same plot. Again to investigate resolution effects, Figure 3.14 shows the simulation results of three combined point targets as in (3.20). Here it can be seen that for well separated target, the estimation is about 4 dB above the CRLBs under high SNRs. However, when targets are not well separated, the estimation is about 20 dB above the CRLBs under high SNRs. This is mainly due to the bias incurred by the other targets. Note also that when $\text{SNR} < 10$ dB the target parameters can not be estimated accurately [9–10], [12]. (Note that the CRLB is shown for $\alpha = 1$ because the CRLBs are almost the same for all three point targets.)

There are two possible reasons for inaccurate estimation in Figs. 3.11 to 3.14. First reason is that the algorithms that have been used to generate Figs. 3.11 to 3.14 are the combination of narrowband and wideband processing. Narrowband processing is not as effective as wideband processing in estimating parameters for wideband signal. Second reason is that a single slice RAT algorithm has been used to generate the Figs. 3.11 to 3.14 and the estimation of projection angle may not be accurate. If one needs to achieve CRLB, then further 2-D local search on the WBCAF is required (see Figure 2.10, where CRLB has been achieved).

(a)

(b)

Figure 3.13: MSE vs. SNR of estimating parameters (individually simulated) using the SSRAT algorithm (1-D search on WBCAF): (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 3.14: MSE vs. SNR of estimating parameters (simultaneously simulated) using the SSRAT algorithm (1-D search on WBCAF): (a) Estimation of delay; (b) Estimation of scale.

3.7 Conclusions

This chapter demonstrates the usefulness of SSRAT for multiple-targets detection in noisy environment. A new and computationally efficient technique based on SSRAT for simultaneous estimation of delay and scale parameters has been presented. The proposed approach is based on the combination of NBCAF, WBCAF, and SSRAT to estimate multiple-targets parameters. The theoretical derivation of RAT has justified the ability of the SSRAT-based algorithm to estimate the parameters. Simulation results have been presented throughout the chapter to show the effectiveness of the proposed techniques. It has been demonstrated that (i) SSRAT-based algorithms are robust to noise, (ii) computationally efficient, and (iii) help to separate multiple overlapping targets.

The assumption that the projection angle to compute SSRAT was approximately known *a priori* may not be realistic in some applications. In such cases the projection angle can be estimated as noted in Chapter 2. Moreover, a Gaussian LFM signal has been used as the transmitted burst in Chapters 2 and 3. However, one may need to use non-Gaussian LFM signals for processing. It is also noted here that the WBCAF needs signal scaling if the analytic form of the transmitted signal is not available.

To address the above issues, in Chapter 4, new algorithms will be investigated for processing more realistic sonar signals with non-Gaussian and arbitrary shaped LFM signals for parameter estimation. This is important as even a Gaussian transmission signal may suffer from amplitude distortions due to various

transmitter hardware limitations. Hermite decompositions will be used in the next chapter to approximate an arbitrary shaped LFM signal. An example of a non-Gaussian envelope of an arbitrary shaped LFM signal is shown in Figure 3.15. Figures 3.16 and 3.17 have been generated the same way as the Figures 3.1 and 3.4; however, an arbitrary shaped LFM signal is used in this case. It can be seen that the proposed SSRAT algorithm can also be used for arbitrary shaped LFM signals. Next chapter attempts to elucidate the Hermite decomposition based algorithms to be used with arbitrary shaped LFM signals.

Figure 3.15: An example of a non-Gaussian type envelope of an arbitrary shaped LFM signal.

Figure 3.16: Contour plot of NBCAF of arbitrary shaped LFM signal in the presence of three point targets as in Figure 3.1.

Figure 3.17: 2-D RAT of Figure 3.16 (left window) and SSRAT of Figure 3.16 at the angle $\theta_a = 135^\circ$ (right window).

Chapter 4

Efficient Wideband Parameter Estimation Using Arbitrary Shaped LFM Signals via Hermite Decompositions

4.1 Introduction

Signals with finite time support are frequently encountered in sonar applications. It is usually necessary to have a compact description of these signals' shape and time evolution [49]. In these applications, the signals are usually broadband and thus the target parameters are optimally estimated via WBCAF-based processing [6]. As stated earlier, Gaussian LFM signals are good candidates for the transmitted signal as they are known to possess very good resolution properties for target parameter estimation. However, this chapter considers a more general transmitted waveform where the amplitude is not strictly a Gaussian function. This is important as even a Gaussian transmission signal may suffer from amplitude distortions due to various transmitter hardware limitations.

Furthermore, it would be necessary for the sonar operator to use non-Gaussian amplitudes due to other system constraints. For this purpose, the Hermite series expansions are used in this chapter to approximate an arbitrary shaped LFM signal. It is proposed to decompose the transmitted signal amplitude using the Hermite expansions for efficient computation and processing via the WBCAF.

The Hermite basis functions are based on the product of Hermite polynomials and a Gaussian function. As such, the analytical expressions for the Fourier transform of LFM Hermite functions have been developed. It is proposed to use the Fourier transform of LFM Hermite functions for efficient computation of the WBCAF for target parameter estimation in active sonar processing. To demonstrate the Hermite expansion as a useful tool, we have used three test signals namely rectangular, triangular, and sinusoidal pulses. Test results show that the associated Hermite expansion performs better than the Hermite-Rodriguez expansion for the decomposition. Efficient 2-D peak search methods based on WBCAF are also proposed to estimate delay and scale parameters.

4.2 Motivation

Due to its Gaussian weight function, the Hermite series expansions have proven to be a useful tool in many signal processing applications. In the last few decades, a significant amount of research has been carried out on the use of Hermite functions [44–61]. However, most of the research have been on image processing [44–48], biomedical engineering [49–53], neural network [54–56],

electromagnetic signals [57–58], transient signals [59], wireless communications [60], etc., and to the best of our knowledge, little work have been reported on the use of Hermite decompositions in wideband sonar signal processing. Alsup and Whitehouse [61] have used Hermite functions to design sonar comb waveforms. They have used Hermite functions as a transmitted waveform. However, our application of Hermite series expansion is significantly different from [61] in that we have used Hermite series to approximate an arbitrary shaped LFM signal. It is also our aim to show the use of Hermite functions as a suitable technique in active sonar applications. As noted before, it is desirable to use LFM signals with non-Gaussian types of envelopes for transmission. The superiority of Hermite expansions over Fourier series expansion has been reported in [49]. Thus, the use of Hermite expansions to approximate arbitrary shaped LFM signals is justified.

The aim here is the evaluation of the WBCAF between the transmitted and received signals to estimate the sonar target parameters efficiently. Such processing may be required in addition to the methods noted in Chapters 2 and 3, in order to achieve CRLB. As arbitrary shaped signals are used in the transmission and hence in the processing, an appropriate analytical model is necessary to scale the signal in time to compute the WBCAF. The major problem associated with this is that the signal is not known analytically and as such creating the scaled and delayed replicas (to compute WBCAF) requires sampling and then applying a multirate conversion technique. However, these techniques are not efficient because of the very high computational complexity [6]. The multirate conversion is further complicated by the scaling factor, which is typically close to unity [6].

Therefore, a very high accuracy in scaling is needed. A wavelet transform based method has been proposed in [5] to overcome the difficulties involved with sampling and multirate conversion technique. In this approach *a priori* admissible analytic function has been chosen to create many different scaled and delayed replicas of the transmitted signal. Two cross-wavelet transforms between the hypothesized replicas of the analytic function and two received signals have been computed to obtain the output of the wideband cross-correlation receiver, which is equivalent to the WBCAF [6]. However, the method is computationally intensive as it needs to compute two cross-wavelet transforms followed by 2-D convolution and consequently a large number of scales and delays are needed for good accuracy.

In this chapter, a computationally efficient technique is proposed to solve the scaling problem. The proposed technique is then applied to simultaneous delay and scale estimation for wideband signal. The Hermite decomposition is used to develop an analytical model for the arbitrary shaped LFM signals. The zero order Hermite basis function has the form of a Gaussian distribution and all other basis functions are the derivatives of this form [52]. Therefore, closed form expressions for the transmitted signal can be easily obtained. Using these closed form expressions, algorithms are proposed to efficiently evaluate WBCAF, and subsequently use it for parameter estimation. The development of Hermite decomposition based efficient algorithms with mathematical justification is the novelty reported in this chapter. In particular, the accurate estimation of Hermite spread parameter and the derivation of closed form expressions for Fourier

transform of LFM Hermite signals for efficient parameter estimation are the main contributions.

The chapter is organized according to the following directions. Section 4.3 briefly introduces the Hermite series expansions. The estimation of spread and shift parameters associated with the expansions using different optimization techniques are described in Section 4.4. Section 4.5 presents the Hermite decomposition for simulated signals. Section 4.6 derives the analytical expressions for Hermite functions in frequency domain to efficiently evaluate the WBCAF for realistic sonar LFM signals. Efficient 2-D peak search techniques for parameter estimation using the WBCAF are explored in Section 4.7. Section 4.8 concludes the chapter.

4.3 Hermite Series Expansion

Hermite series expansion is a well known decomposition technique useful in many signal processing applications [44–61]. Usually, two approaches based on two families of orthogonal Hermite basis functions with compact time support are considered. They are namely associated Hermite and Hermite-Rodriguez functions [49]. The Hermite expansion is useful because of many desirable properties and advantages:

- The Hermite basis functions are orthogonal [49], [52].
- Hermite polynomials can easily be computed via a recurrence relation (e.g., as given in [62]).

- Any type of signal can be represented to an arbitrarily high degree of accuracy by using sufficient number of terms in the expansion [52].
- In the case of associated Hermite function, its Fourier transform is also a Hermite function except for a multiplication factor that is dependent on the order of the Hermite function.
- Hermite series is well suited for signals with finite time support [49].
- The Fourier expansion captures periodicity information and sinusoidal harmonic content of a signal, while the Hermite expansion allows signal representation in both the time and frequency domain with the same type of orthogonal basis functions and the desired resolution [Conte 1994].
- Hermite expansion helps to scale the signal easily, which can be used for efficient evaluation of the WBCAF (e.g., in the application in this chapter).

Through the Hermite expansion of an arbitrary shaped signal, 2-D peak search algorithms can be efficiently implemented for target parameter estimation in active sonar processing. A comprehensive comparison and the properties of associated Hermite and Hermite-Rodriguez functions have been reported in [49].

4.3.1 Associated Hermite Series

The associated Hermite (AH) function $z_{n,\lambda,t_c}(t)$ is defined by

$$z_{n,\lambda,t_c}(t) = \frac{1}{\sqrt{\lambda 2^n n! \sqrt{\pi}}} H_n\left(\frac{t-t_c}{\lambda}\right) \exp\left\{-\frac{(t-t_c)^2}{2\lambda^2}\right\} \quad (4.1)$$

where n is the order of the Hermite function, λ and t_c are the spread and shift parameters respectively. In (4.1), $H_n(t)$ is the n th degree Hermite polynomial generated recursively through

$$H_0(t) = 1 \quad (4.2a)$$

$$H_1(t) = 2t \quad (4.2b)$$

$$H_n(t) = 2tH_{n-1}(t) - 2(n-1)H_{n-2}(t), \quad \text{for } n \geq 2. \quad (4.2c)$$

In an explicit form the Hermite polynomial is the derivative of the Gaussian function [62] given by,

$$H_n(t) = (-1)^n e^{t^2} \frac{d^n}{dt^n} \left(e^{-t^2} \right). \quad (4.3)$$

The AH functions could also be generated recursively as,

$$z_{0,\lambda,t_c}(t) = \frac{1}{\sqrt{\lambda\sqrt{\pi}}} \exp\left\{-\frac{(t-t_c)^2}{2\lambda^2}\right\} \quad (4.4a)$$

$$z_{1,\lambda,t_c}(t) = \frac{2(t-t_c)}{\sqrt{2\lambda\sqrt{\pi}}} \exp\left\{-\frac{(t-t_c)^2}{2\lambda^2}\right\} \quad (4.4b)$$

$$z_{n,\lambda,t_c}(t) = \frac{1}{\sqrt{n}} \left[\sqrt{2}(t-t_c)z_{n-1,\lambda,t_c}(t) - \sqrt{n-1}z_{n-2,\lambda,t_c}(t) \right] \quad \text{for } n \geq 2. \quad (4.4c)$$

A set of functions $\{z_n(t)\}$ is said to be orthogonal over an interval $[-\infty, \infty]$ if

$$\int_{-\infty}^{\infty} z_j(t) z_k(t) dt = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}. \quad (4.5)$$

Any signal $w(t)$ can be represented into an AH series as,

$$\hat{w}_N(t) = \sum_{n=0}^N a_{n,\lambda,t_c} z_{n,\lambda,t_c}(t) \quad (4.6)$$

with

$$a_{n,\lambda,t_c} = \int_{-\infty}^{\infty} w(t) z_{n,\lambda,t_c}(t) dt. \quad (4.7)$$

The normalized mean square error (MSE) between the N term reconstruction $\hat{w}_N(t)$ and the original signal $w(t)$ is usually defined as,

$$MSE = \frac{\int_{-\infty}^{\infty} \{w(t) - \hat{w}_N(t)\}^2 dt}{\int_{-\infty}^{\infty} w^2(t) dt}. \quad (4.8)$$

In (4.8), the MSE decreases with the increase of N and it approaches to zero when N approaches to infinity.

4.3.2 Hermite-Rodriguez Series

The Hermite-Rodriguez (HR) function $u_{n,\lambda,t_c}(t)$ is defined by,

$$u_{n,\lambda,t_c}(t) = \frac{1}{\lambda \sqrt{2^n n! \pi}} H_n\left(\frac{t-t_c}{\lambda}\right) \exp\left(-\frac{(t-t_c)^2}{\lambda^2}\right) \quad (4.9)$$

where $H_n(t)$ are the Hermite polynomials defined in (4.3).

A set of functions $\{u_n(t)\}$ form a complete orthogonal set in the interval $[-\infty, \infty]$

if

$$\sqrt{\pi} \int_{-\infty}^{\infty} u_j(t) u_k(t) e^{t^2} dt = \begin{cases} 0, & j \neq k \\ 1, & j = k \end{cases}. \quad (4.10)$$

Any signal $w(t)$ can be expressed into an HR series as,

$$\hat{w}_N(t) = \sum_{n=0}^N b_{n,\lambda,t_c} u_{n,\lambda,t_c}(t) \quad (4.11)$$

where

$$b_{n,\lambda,t_c} = \lambda \sqrt{\pi} \int_{-\infty}^{\infty} w(t) u_{n,\lambda,t_c}(t) \exp\left(\frac{(t-t_c)^2}{\lambda^2}\right) dt. \quad (4.12)$$

The weighted normalized MSE between N term reconstruction $\hat{w}_N(t)$ and the original signal $w(t)$ is defined as,

$$MSE = \frac{\int_{-\infty}^{\infty} \{w(t) - \hat{w}_N(t)\}^2 e^{t^2/\lambda^2} dt}{\int_{-\infty}^{\infty} w^2(t) e^{t^2/\lambda^2} dt}. \quad (4.13)$$

The AH and HR functions of order zero to three for $\lambda = 1$ and $t_c = 0$ are shown in Figures 4.1 and 4.2, respectively. It can be observed from Figures 4.1 and 4.2 that the number of zero crossings is equal to the order of the waveform. Also the waveforms duration is almost the same for any order, especially for HR functions. It is important to note that the proper selection of λ and t_c are useful to obtain a good approximation of the signal with a minimum number of expansion terms.

Figure 4.1: Associated Hermite functions of order zero to three.

Figure 4.2: Hermite-Rodriguez functions of order zero to three.

4.4 Estimating the Spread and Shift Parameters for Hermite Decompositions

The procedure of expanding a function $w(t)$ through Hermite coefficients has been described in the previous section. The expansion consists of spread (λ) and shift (t_c) parameters having fixed real values. Here, we investigate the estimation of the spread and shift parameters using different optimization techniques. To expand a signal using the Hermite basis functions, it is necessary to center the expansion around a suitable $t = t_c$ [57]. This is due to the fact that the Hermite basis functions provide equal support on either side of the center of expansion [49], [57]. Therefore, centering the expansion about $t = t_c$ would require lesser terms for the Hermite expansion. The proper selection of λ is also important because it decides the amount of support in the Hermite functions. It is possible to estimate λ and t_c simultaneously using optimization algorithms. However, in this chapter we have used the centroid of square of $w(t)$ to estimate t_c . It is easy to show that such a selection of t_c minimizes the variance of $w(t)$ around t_c . An appropriate λ could be then evaluated around $t = t_c$.

Consider the minimization technique proposed in (4.14) that optimally decomposes the signal in a mean square sense. The optimization is accomplished by estimating the values of λ , t_c , C_e , C_o that minimize a cost function $\varepsilon(\varphi)$.

$$\varepsilon(\varphi) = E_e \int_{-\infty}^{\infty} \left(w_e(t) - \frac{C_e}{\sqrt{\lambda^5 \sqrt{\pi}}} e^{-(t-t_c)^2/2\lambda^2} \right)^2 dt + E_o \int_{-\infty}^{\infty} \left(w_o(t) - \frac{C_o(t-t_c)}{\sqrt{\lambda^5 \sqrt{\pi}}} e^{-(t-t_c)^2/2\lambda^2} \right)^2 dt \quad (4.14)$$

where $\varphi = [\lambda \ t_c \ C_e \ C_o]^T$ is a column vector, $w_e(t)$ and $w_o(t)$ are the even and odd parts of the signal $w(t)$ at $t = t_c$, E_e and E_o are the energy of the even and odd parts around $t = t_c$. In HR expansion case, the value of λ resulting from (4.14) has to be multiplied by $\sqrt{2}$.

Note also that if the signal $w(t)$ has either odd or even symmetry then the optimization in (4.14) becomes rather simplified. Because, $E_o = 0$ for even symmetry signals and $E_e = 0$ for odd symmetry signals.

For even symmetry signals (i.e., $E_o = 0$), the simplified form of (4.14) can be written as,

$$\varepsilon(\lambda, t_c, C) = \int_{-\infty}^{\infty} \left(w(t) - \frac{C}{\sqrt{\lambda\sqrt{\pi}}} e^{-(t-t_c)^2/2\lambda^2} \right)^2 dt \quad (4.15)$$

and for odd symmetry signals (i.e., $E_e = 0$), the simplified form of (4.14) can be written as,

$$\varepsilon(\lambda, t_c, C) = \int_{-\infty}^{\infty} \left(w(t) - \frac{C}{\sqrt{\lambda^5\sqrt{\pi}}} (t-t_c) e^{-(t-t_c)^2/2\lambda^2} \right)^2 dt. \quad (4.16)$$

The simplified approach in (4.15) or (4.16) has been used in all reported literature. However, for neither even nor odd symmetry signals (i.e., $E_e, E_o \neq 0$) it is shown later that (4.14) would be a better choice. The optimization in (4.14) can be achieved via gradient based methods and the necessary derivations are given in Appendix C.

4.4.1 Steepest-descent Method

The method of steepest-descent [52] is the simplest example of a gradient based method for minimizing an error function of several variables. Although the steepest-descent method is easy to implement, it has two main drawbacks of very slow convergence and the possibility of getting trapped in local minima. The search starts at an arbitrary point and then slides down the gradient, until the solution is close enough to the optimum point. The core of the algorithm is the following recursion formula,

$$\varphi_{k+1} = \varphi_k - \mu \nabla_{\varphi_k} \quad (4.17)$$

where ∇_{φ_k} is the vector of error gradient with respect to the unknown parameters at a given point, the subscript k refers to the iteration number, and μ is a step size which must be chosen suitably. Too large of a step would cause instability and too small of a step would result in a long convergence time. The iteration in (4.17) is repeated until it converges to the minimum of the function, $\varepsilon(\varphi)$.

4.4.2 Newton's Method

The Newton's method converges in second-order speed while the steepest-descent method converges in first-order speed. This suggests that Newton's method might converge faster; however, it is sensitive to the initial values of the parameters [52]. Newton's method requires the inversion of the Hessian matrix H , which may be computationally intensive. This method is similar to steepest-descent

method; however, there is no step-size parameter μ , instead the second order derivatives are used. Newton's method for finding an extreme point is given by,

$$\varphi_{k+1} = \varphi_k - H^{-1} \nabla_{\varphi_k} \quad (4.18)$$

where H is the Hessian matrix containing the second order derivatives of $\varepsilon(\varphi)$ with respect to the unknown parameters.

It is shown in Appendix C that the parameters C_e and C_o can be computed directly from (C-3) and (C-5), respectively, for given values of λ and t_c . Therefore, the 4-D optimization problem in (4.14) turns into a 2-D problem where λ and t_c are to be estimated. However, if the centroid of square of $w(t)$ is used as the estimation of t_c then the 2-D optimization problem in fact becomes a 1-D problem where one only needs to estimate the parameter λ . Using the optimal values of λ and t_c , the Hermite decompositions with simulated signals will be demonstrated in the next section.

4.5 Hermite Decomposition of Simulated Signals

In this section, the Hermite decomposition of simulated signals is shown using three different pulses. Figures 4.3 (a), (b), and (c) show the normalized MSE versus the number of Hermite functions for rectangular, triangular, and sinusoidal pulses, respectively when $\lambda = 25$. For all three cases it can be seen that the AH expansion exhibits better approximation than to HR expansion. In all cases, increasing the number of Hermite terms, N , the MSE decreases. The used forms

of the rectangular, triangular, and sinusoidal pulses with time duration T_s are given in (4.19), (4.20), and (4.21), respectively.

$$w(t) = \begin{cases} 1 & \text{for } |t| \leq \frac{T_s}{2} \\ 0 & \text{for } |t| > \frac{T_s}{2} \end{cases} \quad (4.19)$$

$$w(t) = \begin{cases} 1 - \frac{2}{T_s}|t| & \text{for } |t| \leq \frac{T_s}{2} \\ 0 & \text{for } |t| > \frac{T_s}{2} \end{cases} \quad (4.20)$$

$$w(t) = \begin{cases} \sin(2\pi t / T_s) & \text{for } |t| \leq \frac{T_s}{2} \\ 0 & \text{for } |t| > \frac{T_s}{2} \end{cases} \quad (4.21)$$

(a)

(b)

(c)

Figure 4.3: Normalized MSE vs. Hermite order for AH and HR expansions: (a) Rectangular pulse; (b) Triangular pulse; (c) Sinusoidal pulse.

For AH expansions, Figures 4.4 (a), (b), and (c) show the normalized MSE versus λ for different Hermite order, N . It is seen that the normalized MSE does not vary significantly with λ for $N > 8$ for all the three signals. That means, for $N > 8$ the normalized MSE lines present a wide region in which the value of λ is not critical. However, for $N < 8$ the error curves depend both on N and λ . An estimation of λ which is the best for small N may not necessarily be the best for large N [49]. However, for relatively large N the use of λ estimated from $N = 0$ or 1 shows satisfactory results.

(a)

(b)

(c)

Figure 4.4: Normalized MSE vs. lambda for different Hermite orders for AH expansion: (a) Rectangular pulse; (b) Triangular pulse; (c) Sinusoidal pulse.

For AH expansions, Figures 4.5 (a), (b), and (c) show the normalized MSE versus Hermite order for different λ . The same conclusion can be drawn from Figure 4.5 that the normalized MSE does not vary significantly with λ for $N > 8$ for all the three signals. A comparison between AH and HR functions has been presented and it has been shown that the AH function performs better than the HR function in approximating a signal. It has also been shown that for $N > 8$ the value of λ is not critical. Based on the above information we now demonstrate the AH expansion for a realistic signal.

(a)

Figure 4.5: Normalized MSE vs. Hermite order for different lambda for AH expansion: (a) Rectangular pulse; (b) Triangular pulse; (c) Sinusoidal pulse.

Figures 4.6 (a), (b), and (c) show the true and synthesized pulses for (4.19), (4.20), and (4.21), respectively. AH expansion is used in the decomposition and the truncation at $N = 10$. A threshold of MSE can be set *a priori* for the determination of the value of N , but this has not been considered in the thesis.

(a)

(b)

(c)

Figure 4.6: The original pulses and its Hermite synthesized forms: (a) Rectangular pulse; (b) Triangular pulse; (c) Sinusoidal pulse.

In the following simulations two signals containing both even and odd components and having arbitrary amplitude, expressed as in (4.22) have been used.

$$w(t) = \sum_{n=0}^3 a_n z_{n,\lambda,t_c}(t). \quad (4.22)$$

For signal set 1: $\lambda = 0.1262s$, $t_c = 0s$, and $a = [0.47 \ -0.111 \ 0.101 \ -0.274]$.

For signal set 2: $\lambda = 0.1262s$, $t_c = -0.1s$, and $a = [0.47 \ 0.411 \ 0.101 \ -0.474]$.

For the signal set 2, t_c has been estimated using the centroid of the square of the signal. λ has been estimated using even and odd separation in (4.14), using only even part in (4.15), and using only the odd part in (4.16). The estimated value of t_c using the centroid method is $t_{c_{est}} = -0.0507s$ and the three estimated values of λ are $\lambda_{est} = 0.0961s$, $0.0635s$, and $0.0773s$ using the three methods in (4.14),

(4.15), and (4.16) respectively. Figure 4.7 shows the number of iterations versus MSE for estimating λ and t_c , as C_e and C_o can be computed directly from (C-3) and (C-5), respectively (Appendix C). Equation (4.14) is used with both steepest-descent and Newton's methods for signal set 2. It is seen that the Newton's method converges faster than the steepest-descent method. Alternatively, Figure 4.8 shows the error performance contour and surface plots of the arbitrary shaped envelope for signal set 2 when the optimization is performed with respect to spread and shift parameters (2-D grid search). The estimated values of t_c and λ from the error performance plot are noted as -0.1s and 0.1s respectively. It can be seen that the estimated value of λ using the method in (4.14) closely agrees with the value using 2-D grid search. Although the estimated value of t_c using 2-D grid search is better than the value using the method in (4.14), the latter is not as computationally intensive as the 2-D grid search and hence more feasible.

Figure 4.7: MSE vs. Number of iterations for Newton's and steepest-descent methods.

Figure 4.8: Error performance contour and surface plots of the arbitrary shaped envelope for signal set 2 with respect to spread and shift parameters.

Figures 4.9 (a) and (b) show the true and synthesized signal envelopes for the three methods. Simulations show that the use of (4.14) performs better than the use of (4.15) and (4.16). The difference is most apparent in Figure 4.9 (b), where the synthesized envelope using (4.14) is closely agreed with the actual envelope. AH expansion is used in the decomposition and the truncation at $N = 10$ resulted in a residual MSE of less than 10^{-4} . Based on the result of Figure 4.9 (a), all the simulations in subsequent sections use (4.14) for the estimation of λ for Hermite decomposition.

(a)

(b)

Figure 4.9: The original envelope and its Hermite synthesized forms: (a) Signal set 1; (b) Signal set 2.

4.6 Efficient Computation of WBCAF for LFM Sonar Signals

In reference to Chapter 2, the received backscatter from the single scatterer is given by

$$r(t) = g[\alpha_0(t - \tau_0)] \quad (4.23)$$

where τ_0 and α_0 are the delay and scale parameters associated with the received signal.

As noted earlier, the transmitted signal, $g(t)$, is usually selected as an LFM signal, which is given by

$$g(t) = w(t) \exp[j(2\pi\kappa t + \pi\beta t^2)] \quad (4.24)$$

where $w(t)$ is an arbitrary shaped envelope.

The Fourier transform of $r(t)$ in (4.23) can be derived as,

$$R(f) = \frac{1}{\alpha_0} G(f / \alpha_0) e^{-j2\pi f \tau_0} \quad (4.25)$$

where $G(f)$ is the Fourier transform of $g(t)$ in (4.24). For a Gaussian envelope, which is a special case of Hermite expansion with $N = 0$, we get

$$G(f) = \sqrt{\frac{2\pi\lambda^2}{1 - j2\pi\beta\lambda^2}} \exp\left\{-\frac{2\pi^2\lambda^2(f - \kappa)^2}{1 - j2\pi\beta\lambda^2}\right\}. \quad (4.26)$$

For the general case, applying AH expansion for $g(t)$, we get

$$G_h(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} \sum_{n=0}^N G_n z_n(t) e^{j[2\pi\kappa t + \pi\beta t^2]} e^{-j2\pi f t} dt \\
 &= \sum_{n=0}^N G_n Z_n(f)
 \end{aligned} \tag{4.27}$$

where the coefficients G_n are the weights of the Hermite expansion of the transmitted signal envelope that can be computed from (4.7) as,

$$G_n = \int_{-\infty}^{\infty} w(t) z_n(t) dt \tag{4.28}$$

and $Z_n(f)$ is the Fourier transform of the LFM AH function, $z_n^{LFM}(t)$, given by

$$F\{z_n^{LFM}(t)\} = Z_n(f) = \int_{-\infty}^{\infty} z_n(t) e^{j[2\pi\kappa t + \pi\beta t^2]} e^{-j2\pi f t} dt. \tag{4.29}$$

Note that unless specifically noted, we have omitted the λ and t_c variability in (4.6) and (4.7), for simplicity.

Substituting for $z_n(t)$ from (4.1), (4.29) yields

$$\begin{aligned}
 Z_n(f) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{\lambda 2^n n! \sqrt{\pi}}} H_n\left(\frac{t-t_c}{\lambda}\right) \exp\left\{-\frac{(t-t_c)^2}{2\lambda^2}\right\} e^{j[2\pi\kappa t + \pi\beta t^2]} e^{-j2\pi f t} dt \\
 &= \frac{e^{j\pi t_c(2\kappa-2f+\beta t_c)}}{\sqrt{\lambda 2^n n! \sqrt{\pi}}} e^{q^2/4p} \int_{-\infty}^{\infty} H_n(t/\lambda) e^{-p(t-q/2p)^2} dt
 \end{aligned} \tag{4.30}$$

where $p = \frac{1}{2\lambda^2} - j\pi\beta$ and $q = j2\pi(\kappa - f + \beta t_c)$.

Using the integral formula in [63] (p. 488), (4.30) can be written as,

$$Z_n(f) = \frac{e^{j\pi t_c(2\kappa-2f+\beta t_c)}}{\sqrt{\lambda 2^n n! \sqrt{\pi}}} \frac{\sqrt{\pi}(p-1/\lambda^2)^{n/2}}{p^{(n+1)/2}} e^{q^2/4p} H_n\left\{\frac{q}{2\lambda p} \sqrt{\frac{p}{(p-1/\lambda^2)}}\right\}. \tag{4.31}$$

Substituting for p and q in (4.31) results

$$\begin{aligned}
 Z_n(f) &= (-j)^n \sqrt{\frac{2\lambda\sqrt{\pi}}{2^n n!}} \frac{(1 + j2\pi\beta\lambda^2)^{n/2}}{\{1 - j2\pi\beta\lambda^2\}^{(n+1)/2}} e^{\frac{j\pi t_c(2\kappa - 2f + \beta t_c) - 2\pi^2\lambda^2(f - \kappa)^2}{1 - j2\pi\beta\lambda^2}} H_n \left\{ \frac{2\pi\lambda(f - \kappa - \beta t_c)}{(1 + 4\pi^2\beta^2\lambda^4)^{1/2}} \right\} \\
 &= (-j)^n \sqrt{\frac{2\lambda\sqrt{\pi}}{2^n n!}} \frac{e^{\{j\phi_1 + A(\cos\phi_2 + j\sin\phi_2)\}}}{(1 + 4\pi^2\beta^2\lambda^4)^{1/4}} e^{\left\{ \frac{2\pi^2\lambda^2(f - \kappa)^2(\cos\phi_3 + j\sin\phi_3)}{(1 + 4\pi^2\beta^2\lambda^4)^{1/2}} \right\}} H_n \left\{ \frac{2\pi\lambda(f - \kappa - \beta t_c)}{(1 + 4\pi^2\beta^2\lambda^4)^{1/2}} \right\}
 \end{aligned} \tag{4.32}$$

where $A = \frac{\pi t_c(2\kappa - 2f + \beta t_c)}{\sqrt{1 + 4\pi^2\beta^2\lambda^4}}$, $\phi_1 = \left(n + \frac{1}{2}\right)\phi_3$, $\phi_2 = \frac{\pi}{2} + \phi_3$, $\phi_3 = \tan^{-1}(2\pi\beta\lambda^2)$.

Note that when $\beta = 0$ and $t_c = 0$, (4.32) can be rewritten as,

$$Z_n(f) = (-j)^n \sqrt{\frac{2\lambda\sqrt{\pi}}{2^n n!}} H_n[2\pi\lambda(f - \kappa)] e^{-2\pi^2\lambda^2(f - \kappa)^2}. \tag{4.33}$$

Furthermore when $\beta = 0$, $t_c = 0$, and $\kappa = 0$, i.e. the AH function, (4.32) can be rewritten as,

$$\begin{aligned}
 Z_n(f) &= (-j)^n \sqrt{\frac{2\lambda\sqrt{\pi}}{2^n n!}} H_n(2\pi\lambda f) e^{-(2\pi\lambda f)^2/2} \\
 &= (-j)^n z_{n,\eta}(f)
 \end{aligned} \tag{4.34}$$

where $\eta = 1/2\pi\lambda$. That is, except for factor $(-j)^n$, the Fourier transform of an AH function of spread λ is also an AH function of spread $1/2\pi\lambda$, which is the property of isomorphism between a function and its transform [44], [49].

In a similar manner, the Fourier transform of the LFM HR function, $u_n^{LFM}(t)$, can be expressed as,

$$\begin{aligned}
 F\{u_n^{LFM}(t)\} &= U_n(f) = \int_{-\infty}^{\infty} u_n(t) e^{j[2\pi\kappa t + \pi\beta t^2]} e^{-j2\pi f t} dt \\
 &= \int_{-\infty}^{\infty} \frac{1}{\lambda\sqrt{2^n n!}\pi} H_n\left(\frac{t-t_c}{\lambda}\right) \exp\left\{-\frac{(t-t_c)^2}{\lambda^2}\right\} e^{j[2\pi\kappa t + \pi\beta t^2]} e^{-j2\pi f t} dt \\
 &= \frac{(-j)^n (j\pi\beta\lambda^2)^{n/2} \exp(j\phi_1 + Ae^{j\phi_4})}{\sqrt{2^n n!} \{1 + \pi^2\beta^2\lambda^4\}^{(n+1)/4}} H_n\left\{\frac{\pi(f-\kappa-\beta t_c)e^{-j\phi_2}}{(\pi^2\beta^2 + \pi^4\beta^4\lambda^4)^{1/4}}\right\} e^{\frac{\pi^2\lambda^2(f-\kappa)^2 e^{j\phi_3}}{(1+\pi^2\beta^2\lambda^4)^{1/2}}}
 \end{aligned} \tag{4.35}$$

where $A = \frac{\pi t_c(2\kappa - 2f + \beta t_c)}{\sqrt{1 + \pi^2\beta^2\lambda^4}}$, $\phi_1 = \left(\frac{n+1}{2}\right)\phi_3$, $\phi_2 = \frac{1}{2}\tan^{-1}\left(\frac{1}{\pi\beta\lambda^2}\right)$,

$$\phi_3 = \tan^{-1}(\pi\beta\lambda^2), \quad \phi_4 = \frac{\pi}{2} + \phi_3.$$

Using (4.32), the WBCAF defined in (2.13) can be re-expressed using the Hermite expansion as,

$$\Psi_{wb}(\tau, \alpha) = \left| \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} \sum_{n=0}^N R(f) G_n Z_n^*(f/\alpha) e^{j2\pi f \tau} df \right|^2 \tag{4.36}$$

where $R(f)$ is the Fourier transform of the received signal, $r(t)$, without noise.

The procedure of computing the WBCAF in (4.36) efficiently using the Hermite expansion of the signal can be then outlined as follows:

- Obtain the amplitude envelope, $w(t)$, and estimate the optimal values of spread parameter, λ_{opt} , and the shift parameter, $t_{c_{opt}}$, using (4.14).

- Compute the Hermite decomposition coefficients, G_n , of the envelope, $w(t)$, using (4.28).
- Evaluate the Fourier transform of the LFM AH function, $Z_n(f)$, in (4.32).
- Using the Fourier transform of received signal, $R(f)$, evaluate the WBCAF via (4.36).

Using the efficiently computed WBCAF, in the following section, we will further propose two efficient 2-D search techniques to estimate the target parameters.

4.7 Efficient 2-D Search for Parameter Estimation

In the following simulations, the signal parameters used to generate $g(t)$ are: the carrier frequency $\kappa = 200\text{Hz}$, the sampling frequency $F_s = 1000\text{Hz}$, and bandwidth $B = 300\text{Hz}$. For realistic sonar signals, the carrier frequency is around 10kHz [64]. This needs a sampling frequency of more than 20kHz . Sampling frequency can also be reduced using the baseband processing. The sampling frequency in this case is related to B , the swept bandwidth of the chirp, because in many cases the chirp is more or less bandlimited to a frequency extent of B . Therefore, it is convenient to let $F_s = pB$, where $p \geq 1$ represents the sampling factor. In this simulation $p = 3.33$.

4.7.1 Efficient 2-D Search on WBCAF for Gaussian LFM Signal

In Chapter 2, it has been mentioned that the conventional method of determining target parameters is to compute the WBCAF, $\Psi_{wb}(\tau, \alpha)$, and searching for its peak in the entire delay-scale plane. However, this is a computationally intensive procedure.

A faster alternative is by iterative searches. Based on (4.36), in the following an efficient iterative search (2-D peak search) technique will be investigated for parameters estimation. That is, the following optimization is considered which also allows differentiation with respect to τ and α .

$$\max_{(\tau, \alpha)} \left\{ \Psi_{wb}(\tau, \alpha) = \left| \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} \sum_{n=0}^N R(f) G_n Z_n^*(f / \alpha) e^{j2\pi f \tau} df \right|^2 \right\}. \quad (4.37)$$

At first a Gaussian LFM signal will be used where the envelope $w(t)$ in (4.24) is a Gaussian function. In this case $G_h(f)$ in (4.27) can be expressed exactly as in (4.26) with zero MSE. The Newton's optimization technique will be used to maximize (4.37) as it converges faster than the steepest-descent method. As can be seen from (4.18) the Newton's method requires to evaluate the gradients and Hessian of the objective function. As shown in Appendix D, due to the Hermite decomposition, the Hessian and gradients can be easily computed. Note however, that these are susceptible to the presence of noise. As an alternative, the Nelder-Mead simplex algorithm can also be used as a direct search method [65]. The method requires only function evaluations and does not rely on numerical or

analytic gradients or even on continuity of the objective function. However, the convergence rate of Nelder-Mead simplex method may be slower than the Newton's method.

Nelder-Mead method is also computationally simple. If m is the number of parameters, a simplex in m -dimensional space is characterized by the $m+1$ distinct vectors that are its vertices. For two variables, a simplex is a triangle, and the method is a pattern search that compares function values at the three vertices of a triangle. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the convergence criteria are met. A single generic iteration of the Nelder-Mead algorithm consists of the following steps [65]:

1. Order: Order the $m+1$ vertices as $x_1, x_2, \dots, x_{m+1}$ to satisfy $f(x_1) \leq f(x_2) \leq \dots \leq f(x_{m+1})$, where $f(\bar{x})$ is the objective function parameterized by the vector $\bar{x}$.

2. Reflect: Compute the *reflection* point x_r from

$$x_r = \bar{x} + \rho(\bar{x} - x_{m+1}) \quad (4.38)$$

where $\bar{x} = \sum_{i=1}^m x_i / m$ is the centroid of the m best points. Evaluate $f(x_r)$. If

$f(x_1) \leq f(x_r) < f(x_m)$, accept the reflected point x_r and terminate the iteration.

3. Expand: If $f(x_r) < f(x_1)$, compute the *expansion* point x_e

$$x_e = \bar{x} + \chi(x_r - \bar{x}) \quad (4.39)$$

where χ is an expansion coefficient and it is set to $\chi > 1$. Evaluate $f(x_e)$. If $f(x_e) < f(x_r)$, accept x_e and terminate the iteration; otherwise {if $f(x_e) \geq f(x_r)$ }, accept x_r and terminate the iteration.

4. Contract: If $f(x_r) \geq f(x_m)$, perform a *contraction* between $\bar{x}$ and x_{m+1}

$$x_c = \bar{x} + \gamma(x_r - \bar{x}) \quad (4.40)$$

where γ is a contraction coefficient and it is set to $\gamma < 1$. Evaluate $f(x_c)$. If $f(x_c) < f(x_{m+1})$, accept x_c and terminate the iteration; otherwise, go to step 5 .

5. Shrink Simplex: Evaluate f at the m new vertices for $i = 2, \dots, m+1$.

$$v_i = x_1 + \sigma(x_i - x_1). \quad (4.41)$$

For the four coefficients, the standard values reported in the literature are: $\rho = 1$, $\chi = 2$, $\gamma = 0.5$, and $\sigma = 0.5$. Further details of the Nelder-Mead method can be found in [65–66].

Iterative methods need to provide initial values of the parameters to be estimated. A very fast method has been proposed in Section 2.6 for approximately estimating initial values. If the cost function in (4.37) is not a linearly increasing function, the search for a maximum could yield a local maximum, depending on the initial values. To increase the chance of finding the global maximum, it is also possible to obtain several estimations for different initial values and select the one that gives smallest estimation error.

In the following, simulation results are shown as a means of comparison between the Newton's method and the Nelder-Mead method. The initial values can be obtained either from *a priori* knowledge or from the techniques reported in Chapters 2 and 3. Note however that it is not our aim to comprehensively compare Newton's and Nelder-Mead methods; instead it will be shown that these methods may be suitable for our application. Figure 4.10 shows the number of iterations versus the estimation of parameters based on (4.37) for noise-free case. The values of a point target parameters chosen for the simulation are $(\tau_0, \alpha_0) = (0.1, 1.05)$. It is seen that after 12 iterations the Newton's method provides accurate estimation while Nelder-Mead method takes 25 iterations. Figures 4.11 and 4.12 show the estimation of the same target parameters when $\text{SNR} = 30$ dB and 60 dB, respectively. With $\text{SNR} = 30$ dB, it can be seen that Newton's method can estimate parameters within 12 iterations, whereas the Nelder-Mead method fails in this case. With $\text{SNR} = 60$ dB, both the methods show the same performance as in the noise-free case.

(a)

(b)

Figure 4.10: Number of iterations vs. estimated values without noise for a Gaussian LFM signal: (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 4.11: Number of iterations vs. estimated values with SNR = 30 dB for a Gaussian LFM signal: (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 4.12: Number of iterations vs. estimated values with SNR = 60 dB for a Gaussian LFM signal: (a) Estimation of delay; (b) Estimation of scale.

4.7.2 Efficient 2-D Search on WBCAF for Arbitrary Shaped LFM Signal

In this section the same technique described in the preceding section will be applied for an arbitrary shaped LFM signal. Unlike previously, the Hermite decomposition is now approximated. However, the Hermite decomposition helps to evaluate the WBCAF efficiently via the use of (4.32). Figure 4.13 shows the number of iterations versus the estimation of parameters for the arbitrary shaped LFM signal set 2 in (4.22) for noise-free case. The same target parameters that were used in Figure 4.10 have been used in the simulation to obtain Figure 4.13. Simulations show that after 10 iterations the Newton’s method gives accurate estimation while Nelder-Mead method takes around 20 iterations to give the same result. Figures 4.14 and 4.15 show the estimation of the parameters when SNR = 30 dB and 60 dB, respectively. As before, the same conclusion can be drawn between the Newton’s and Nelder-Mead methods in the presence of noise.

(a)

(b)

Figure 4.13: Number of iterations vs. estimated values without noise for an arbitrary shaped LFM signal: (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 4.14: Number of iterations vs. estimated values with SNR = 30 dB for an arbitrary shaped LFM signal: (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 4.15: Number of iterations vs. estimated values with SNR = 60 dB for an arbitrary shaped LFM signal: (a) Estimation of delay; (b) Estimation of scale.

Figures 4.10–4.15 demonstrate that the Newton’s method is superior in convergence and also robust to low SNR in comparison with Nelder-Mead method. However, Newton’s method is more complicated than Nelder-Mead method as the gradients and Hessian are required for the algorithm. Therefore, for further comparison we have evaluated the algorithm runtimes. The computational efficiency of the proposed algorithms is tabulated in Table 4.1 in comparison with that of the conventional 2-D search algorithm. Simulations were carried out on a Pentium 4 machine with 1.9 GHz speed and 256 MB of RAM. Same target parameters and 1024-point FFTs were used to make a fair comparison between methods.

Algorithms	Average runtime (sec)
1. Proposed 2-D search using Newton’s method.	0.3018
2. Proposed 2-D search using Nelder-Mead method.	0.5155
3. Conventional 2-D search using WBCAF.	2.8600

Table 4.1: Computational speed of the proposed and conventional algorithms.

As expected, it is seen in Table 4.1 that the Newton’s method is 1.7 times faster than the Nelder-Mead method as the former method requires less iteration than the latter one. Although the Nelder-Mead and the Newton’s method appears to have similar runtimes the Nelder-Mead method shows poor performance when $\text{SNR} \leq 30$ dB. Newton’s method, on the other hand, exhibits much better performance than Nelder-Mead method in the presence of noise. When the proposed 2-D search methods (Algorithms 1 and 2 of Table 4.1) are compared with the conventional 2-D method (Algorithm 3 of Table 4.1) it can be seen that the proposed methods are about ten times faster than the conventional method. By proper selection of the initial values of the parameters close to the true values using a priori knowledge the run-times of Algorithms 1 and 2 may be reduced and the divergence problems could also be avoided. However, it may not be true always.

4.8 Conclusions

This chapter addresses the Hermite series expansion, estimation of spread and shift parameters for Hermite decompositions, and efficient 2-D peak search algorithms. The Hermite series decomposition has been applied to approximate an arbitrary shaped LFM signal. The analytical expressions of the FT of LFM Hermite functions have been developed. The use of the FT of LFM Hermite expansions for efficient computation of the WBCAF has been proposed. Efficient 2-D peak search methods on the WBCAF have also been proposed to estimate the point target parameters. Examples and simulation results are presented to illustrate the effectiveness of the proposed methods. It has been shown that our proposed 2-D peak search methods are faster than the conventional methods. However, the disadvantage of the proposed 2-D peak search methods based on Newton's and Nelder-Mead simplex algorithms is that it is only suitable for a point scatterer. In the next chapter, it is intended to address extended target or multiple point scatterers.

Chapter 5

Efficient Wideband Spreading

Function Estimation Using Arbitrary

Shaped LFM Signals

5.1 Introduction

So far, active sonar applications have been considered where the transmitted signal is known at the receiver and is not corrupted with AWGN. In this chapter, a semi-active case is considered where the transmitted signal may be approximately known and is also contaminated with AWGN and thereby making it difficult to detect target especially when the noise level is considerably high. It is also our aim to consider a more realistic case where a target is distributed in delay-scale plane. In loose sense, for simplicity, we will use the notion *distributed target* as a collection of point targets [87].

In active signal processing areas such as sonar/radar, information such as range and velocity of a target can be obtained by transmitting a signal and processing the received echo. Signal processing of acoustic signals scattered from distributed

target has become a growing area of interest to the researchers. The delay-scale domain wideband spreading function (WBSF) is often used for characterizing the distributed target environments [67], [77–87]. This chapter investigates an efficient technique for estimating WBSF using Hermite decomposition. In realistic sonar/radar, it may be necessary to detect more than one target simultaneously. Under this scenario, delay-scale resolution is a major concern to be considered to separate multiple-targets. As such, this chapter also focuses on the applications of Hermite decomposition to the use of multiple transmissions to improve delay-scale resolution. Simulation results are presented to justify the usefulness of the proposed methods.

For the active sensing case, a signal is transmitted to the medium, reflects from the object and is received at the receiver. The signal also gets scattered from other objects in the medium, causing multiple-replicas of the transmitted signal to arrive at the receiver at different times. The received signal is then processed by correlating it with the hypothesized versions of the transmitted signal. In the previous chapters the NBCAF and the WBCAF were used as the means of correlator for estimating point target parameters. However, the WBSF is evaluated in this Chapter for the identification of distributed target.

This chapter aims to characterize a distributed target rather than a point target with varying ranges and velocities. Some applications of distributed target can be pointed out to sonar, radar, medical imaging, oceanography, tomography, remote sensing, etc. [67–87]. A distributed target spreads in delay-scale plane as

compared to a point target, which is associated with only particular delay-scale. In a complicated environment such as the ocean, the target is composed of several objects, or a physical large object with continuum of reflectors that are often very close in delay-scale plane. The medium itself can also cause spread in delay and scale. This phenomenon is referred to as multipath effect. In most cases the propagation and scattering is not ideal and the signal becomes spread in delay and scale so that it no longer resembles a replica of the transmitted waveform. This phenomenon usually occurs in two ways. The signal can be reflected from an elongated scatterer that may have moving components, or propagation involves multipath and intermediate reflections from channel boundaries [68].

Like Chapter 4, this Chapter also considers a general transmitted waveform where the amplitude is a non-Gaussian function and uses an arbitrary shaped LFM signal. Hermite series expansion technique has been proposed in Chapter 4 to develop an analytical model for the arbitrary shaped LFM signals. Using the analytical model, an algorithm is proposed to efficiently estimate WBSF. The closed form expressions are also used for multiple transmissions to improve delay-scale resolution of WBCAF for multiple-targets environments.

5.2 Narrowband Scattering Function

Consider narrowband scattering in the active imaging case. The narrowband model of the received signal, $r(t)$, is expressed as an integral over all possible delays and Doppler-shifts [69–72].

$$r(t) = \iint D_{nb}(\tau, \nu) h(t - \tau) e^{j2\pi\nu t} d\nu d\tau \quad (5.1)$$

where $h(t)$ is the transmitted signal, $D_{nb}(\tau, \nu)$ is the narrowband weighting function [6], and τ and ν are the propagation delay and Doppler-shift, respectively. Equation (5.1) represents the sum of delayed and Doppler-shifted replicas of the transmitted signal.

Using the narrowband model, the NBCAF is defined as the correlation between the received signal, $r(t)$, and the distorted version of the transmitted signal, $h(t)$, for different values of delays and Doppler-shifts,

$$\chi_{nb}(\tau', \nu') = \int_{-\infty}^{\infty} r(t) h^*(t - \tau') e^{-j2\pi\nu' t} dt \quad (5.2)$$

where $h(t)$ is the transmitted signal for semi-active case in the presence of noise.

Equation (5.2) peaks at values of (τ'_0, ν'_0) associated with sources in the environment. The direct estimate [78] of the narrowband scattering function (NBSF), $S_{nb}(\tau', \nu')$, can be obtained from (5.2) as,

$$S_{nb}(\tau', \nu') = |\chi_{nb}(\tau', \nu')|^2 = \left| \int_{-\infty}^{\infty} r(t) h^*(t - \tau') e^{-j2\pi\nu' t} dt \right|^2. \quad (5.3)$$

However, the resolution of direct estimation is low [78]. To obtain a high resolution estimate of NBSF, a deconvolution technique could be used. Substituting (5.1) into (5.2) yields

$$\begin{aligned} \chi_{nb}(\tau', \nu') &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{nb}(\tau, \nu) h(t - \tau) e^{j2\pi\nu t} h^*(t - \tau') e^{-j2\pi\nu' t} d\nu d\tau dt \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{nb}(\tau, \nu) \chi_{hh}(\tau' - \tau, \nu' - \nu) e^{-j2\pi\tau(\nu' - \nu)} d\nu d\tau \end{aligned} \quad (5.4)$$

where χ_{hh} is the narrowband auto-ambiguity function (NBAAF) of $h(t)$.

To determine the intensity of the reflection, the second order statistics, $E[|\chi_{nb}(\tau', \nu')|^2]$, is computed, where $E[\cdot]$ denotes expected value. This yields [6],

$$\begin{aligned} p(\tau', \nu') &= E[|\chi_{nb}(\tau', \nu')|^2] = E[\chi_{nb}(\tau', \nu')\chi_{nb}^*(\tau', \nu')] \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[D_{nb}(\tau, \nu)D_{nb}^*(\tau', \nu')] \chi_{hh}(\tau' - \tau, \nu' - \nu) \\ &\quad \times \chi_{hh}(\tau' - \tau_1, \nu' - \nu_1) e^{-j2\pi\tau(\nu' - \nu)} e^{-j2\pi\tau_1(\nu' - \nu_1)} d\nu d\nu_1 d\tau d\tau_1 \end{aligned} \quad (5.5)$$

This quadruple integral may be very complicated to evaluate. Therefore, one of the two approaches may be taken. The first approach is to return to (5.4) and perform a deconvolution. The second approach is to invoke assumption on the objects in the environment that allow (5.5) to be simplified. The assumptions are the wide sense stationary and uncorrelated scattering (WSSUS) assumptions, and they imply that the signals being processed are statistically stationary and that the scatterers are uncorrelated [5–6], [67]; that is,

$$E[D_{nb}(\tau, \nu)D_{nb}^*(\tau', \nu')] = E[|D_{nb}(\tau, \nu)|^2] \delta(\tau - \tau') \delta(\nu - \nu') \quad (5.6)$$

Inserting (5.6) into (5.5) yields

$$\begin{aligned} p(\tau', \nu') &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{nb}(\tau, \nu) |\chi_{hh}(\tau' - \tau, \nu' - \nu)|^2 d\nu d\tau \\ &= S_{nb}(\tau', \nu') ** \Psi_{hh}(\tau', \nu') \end{aligned} \quad (5.7)$$

where $S_{nb}(\tau', \nu') = E[|D_{nb}(\tau', \nu')|^2]$ is the NBSF, $\Psi_{hh}(\tau', \nu')$ is the square modulus of NBAAF, and $**$ denotes 2-D convolution. Deconvolving (5.7) for $S_{nb}(\tau', \nu')$ gives high resolution estimate [78]. Equation (5.7) can also be expressed in FT domain as,

$$P(f, t) = S_{nb}(f, t) \Psi_{hh}(f, t) \quad (5.8)$$

where $P(f, t)$, $S_{nb}(f, t)$, and $\Psi_{hh}(f, t)$ are the 2-D FTs of $p(\tau', \nu')$, $S_{nb}(\tau', \nu')$, and $\Psi_{hh}(\tau', \nu')$ respectively.

The NBSF can be estimated by calculating the inverse FT of (5.8) as,

$$S_{nb}(\tau, \nu) = F^{-1} \left\{ \frac{P(f, t)}{\Psi_{hh}(f, t)} \right\}. \quad (5.9)$$

However, the estimation becomes infinite if the denominator, $\Psi_{hh}(f, t) = 0$. This imposes severe restrictions on signal design. In order to solve this problem, thresholding method and its derivatives have been approached in [69]. Kay and Doyle [70] have also proposed a technique to alleviate zero division problems. The direct method using high resolution waveforms is an efficient approach to estimate NBSF. But the problem with direct approach is that it is plagued by the self-clutter problem caused by the volume conservation property of the ambiguity functions [71]. In [68], [71], they have proposed a new technique applicable to the estimation of NBSF that is effective in reducing self-clutter as well as volume conservation effects. A number of methods exist to perform the deconvolution, direct and iterative. A direct measurement approach using specially designed

waveform pairs and twin receivers is given in [68], [71]. Iterative methods are computationally intensive. However, the advantage of iterative method is that the iteration can be stopped before an excessive amount of noise amplification has occurred [69], [73].

For narrowband transmitted signals, NBSF can be estimated by computing the outputs of narrowband cross-correlation receivers. However, in many cases like ocean acoustics the narrowband model is not appropriate. Moreover, the WSSUS conditions assume wide-sense stationarity in both time and frequency, which are not strictly true for the ocean medium. In such scenario this chapter concentrates on a more general representation of the received signal which is given by the wideband model.

5.3 Wideband Spreading Function

In environment with multiple scatterers, the delay, τ , and scale, α , are assumed to be distributed according to the WBSF so that the received signal is written as [5], [77], [83], [85]:

$$r(t) = \iint S_{wb}(\tau, \alpha) \frac{1}{\sqrt{|\alpha|}} h\left(\frac{t - \tau}{\alpha}\right) \frac{d\alpha d\tau}{\alpha^2} \quad (5.10)$$

where $S_{wb}(\tau, \alpha)$ is the WBSF associated with each delayed and scaled version of the transmitted signal. Equation (5.10) is the wideband model of the received signal for distributed target. The signal model in (5.10) assumes that the narrowband condition is violated.

By correlating the received signal, $r(t)$, with the hypothesized replicas of the transmitted signal, $h(t)$, the WBCAF is defined in a different form as,

$$WC(\tau', \alpha') = \frac{1}{\sqrt{|\alpha'|}} \int_{-\infty}^{\infty} r(t) h^* \left(\frac{t - \tau'}{\alpha'} \right) dt. \quad (5.11)$$

Equation (5.11) is identical to the cross-wavelet transform (CWT), $CWT_h r(\tau', \alpha')$, of the signal $r(t)$ with respect to the signal $h(t)$, which is assumed to be a mother wavelet. For this interpretation, the transmitted signal $h(t)$ should satisfy the admissibility condition of being a mother wavelet, i.e.,

$$c_h = \int_{-\infty}^{\infty} \frac{|H(\omega)|^2}{|\omega|} d\omega < \infty \quad (5.12)$$

where c_h is the admissibility constant and $H(\omega)$ is the FT of $h(t)$.

The direct estimate of the WBSF can be obtained from (5.11) in FT domain as,

$$S_{wb}(\tau', \alpha') = CWT_h r(\tau', \alpha') = \sqrt{\alpha'} \int_{-\infty}^{\infty} R(f) H^*(\alpha' f) e^{j2\pi f \tau'} df. \quad (5.13)$$

Although the direct method is computationally efficient, it however exhibits low resolution [78]. Because of the low resolution of direct method, a wideband deconvolution technique will be used to obtain a high resolution estimate of WBSF. When the wideband signal model in (5.10) is inserted into (5.11), the resulting output is expressed as,

$$CWT_h r(\tau', \alpha') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{wb}(\tau, \alpha) h \left(\frac{t - \tau}{\alpha} \right) h^* \left(\frac{t - \tau'}{\alpha'} \right) \frac{dtd\alpha d\tau}{\alpha^2 \sqrt{|\alpha \alpha'|}}$$

$$\begin{aligned}
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{wb}(\tau, \alpha) AWT_h h\left(\frac{\tau' - \tau}{\alpha}, \frac{\alpha'}{\alpha}\right) \frac{d\alpha d\tau}{\alpha^2} \\
 &= S_{wb}(\tau', \alpha') *_{wb} AWT_h h(\tau', \alpha')
 \end{aligned} \tag{5.14}$$

where $*_{wb}$ denotes wideband convolution and $AWT_h h(\tau', \alpha')$ is the auto-wavelet transform (AWT) of the transmitted signal $h(t)$. It can be seen in (5.14) that the WBSF is blurred by the AWT of the transmitted signal. Therefore, the resolution limitation is determined by the AWT of the transmitted signal. Since finite energy signals cannot have delta functions as AWT, these correlator outputs are not exact replicas as they are blurred by the AWT of the transmitted signal [6].

In [77], Naparst has proposed a method to reconstruct $S_{wb}(\tau', \alpha')$ for the affine group. However, Naparst's method failed to work in some cases. Naparst did not consider noise effects and the sensitivity to changing environment. The method is also dependent upon the choice of the signal. There are various other iterative techniques such as gradient search algorithms [67], least-square methods [79], and regularization techniques such as the method of Tikhonov [79] are also available. However, the Tikhonov method has some drawbacks of being computationally intensive with no guaranty of convergence [79]. To overcome the above drawbacks, wideband deconvolution [5], [83] of (5.14) has been used to estimate $S_{wb}(\tau', \alpha')$.

$$\begin{aligned}
 S_{wb}(\tau, \alpha) &= \frac{1}{c_g} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} CWT_h r(\tau', \alpha') AWT_h^* h\left(\frac{\tau' - \tau}{\alpha}, \frac{\alpha'}{\alpha}\right) \frac{d\tau' d\alpha'}{\alpha'^2} \\
 &= CWT_h r(\tau, \alpha) *_{wb} AWT_h^* h(\tau, \alpha).
 \end{aligned} \tag{5.15}$$

Note that if $h(t)$ satisfies (5.12), then the right hand side of (5.15) is equal to $CWT_h r(\tau, \alpha)$ (a property of CWT [81]). Therefore, if $h(t)$ is available in analytic form, $S_{wb}(\tau, \alpha)$ can be easily computed using (5.13) rather than (5.15). Once an estimate of $S_{wb}(\tau, \alpha)$ is obtained, then estimates of (τ, α) that cause $|S_{wb}(\tau, \alpha)|^2$ to peak may be determined, thereby characterizing the objects in the environment.

5.4 A Collection of Point Targets

As stated earlier, a semi-active case will be considered in this Chapter where the transmitted signal is available only as a noise corrupted version [86]. Under this scenario, the transmitted signal is not known analytically. In semi-active case the receiver, situated far from the transmitter, receives two signals; one directly from the transmitter; other reflected from the targets. For single target, the two received signal models can be written as,

$$\begin{cases} r_1(t) = g(t) + n_1(t) \\ r_2(t) = a g\left(\frac{t - \tau_0}{\alpha_0}\right) + n_2(t) \end{cases} \quad (5.16)$$

where $g(t)$ is the original source signal, $n_1(t)$ and $n_2(t)$ are assumed to be zero-mean AWGN processes.

In semi-active case, $g(t)$ can be known only in an approximate form. Even in active case, the Hermite expansion may not be exact if enough Hermite terms are not used. Therefore, $h(t)$, an approximation to $g(t)$ obtained from $r_1(t)$, will be used as the mother wavelet to compute (5.15). Using analytical expression for

$h(t)$ and using (5.15), an algorithm will be proposed to efficiently compute WBSF. This is achieved via Hermite decomposition of $r_1(t)$ that has been demonstrated in Chapter 4 (see (4.23) to (4.32)). Using the two received signals expressed in (5.16), (5.15) can be rewritten as,

$$S_{wb}(\tau, \alpha) = CWT_h r_2(\tau, \alpha) *_{wb} CWT_h^* r_1(\tau, \alpha) \quad (5.17)$$

where $CWT_h r_2(\tau, \alpha)$ and $CWT_h r_1(\tau, \alpha)$ are the two CWTs of the two received signals with respect to $h(t)$ (distorted version of $g(t)$).

Although the wideband model in (5.10) is selected in integration form, we will however use discrete form for simplicity. In most of the cases the number of real scattering units is usually finite and the distribution of these units is not continuous [87]. Therefore, we will limit our application only for distributed targets and thereby (5.10) can be written as the sum of the reflected signals [87],

$$r_2(t) = \sum_{i=1}^M a_i g\left(\frac{(t - \tau_i)}{\alpha_i}\right) + n_2(t) \quad (5.18)$$

where a_i is the amplitude reflection factor for the i^{th} echo, α_i is the scale factor for the i^{th} echo, and τ_i is the delay for the i^{th} echo.

5.5 The Use of Hermite Expansion for WBSF Estimation and Results

Now, using the Hermite expansions as in Chapter 4, (5.17) can be expressed as,

$$S_{wb}(\alpha, \tau) = \left\{ \sqrt{\alpha} \int_{-\infty}^{\infty} \sum_{n=0}^N R_2(f) H_n Z_n^*(\alpha f) e^{j2\pi f \tau} df \right\}^*_{wb} \left\{ \sqrt{\alpha} \int_{-\infty}^{\infty} \sum_{m=0}^N R_1(f) H_m Z_m^*(\alpha f) e^{j2\pi f \tau} df \right\}^* \quad (5.19)$$

where $R_1(f)$ and $R_2(f)$ are the FTs of the two received signals $r_1(t)$ and $r_2(t)$, respectively, and H_n are the Hermite coefficients of the signal $h(t)$, an approximation to transmitted signal envelope.

As $Z_n(f)$ can be obtained in closed form, it is easier to evaluate (5.19). The procedure of estimating WBSF in (5.19) efficiently using the Hermite expansion of the signal can be outlined as follows:

- Obtain the amplitude envelope, $h(t)$, and estimate the optimal values of spread and the shift parameters as described in Chapter 4.
- Compute the Hermite decomposition coefficients, H_n , of the envelope $h(t)$ as stated in Chapter 4.
- Evaluate the FT of the arbitrary shaped LFM Hermite function, $Z_n(f)$.
- Using the FTs of the two received signals $R_1(f)$ and $R_2(f)$, evaluate the WBSF in (5.19).

Figures 5.1 (a) and (b) show the estimation of WBSF obtained using direct method and deconvolution method, respectively, for the same arbitrary shaped LFM signal which has been used in Chapter 4 (signal set 2) for noise-free case in the presence of five targets. Figures 5.2 (a) and (b) show the estimation of WBSF generated the same way as Figure 5.1 but for semi-active case with AWGN process. The SNR of the received signals $r_1(t)$ and $r_2(t)$ were considered 30 dB and 15 dB respectively. While the five targets can be clearly seen for both the methods, the deconvolution method appears to be slightly better than the direct method in resolving targets [78]. Note that the aim here is to estimate the spreading function using Hermite decomposition, not to compare between direct and deconvolution methods.

Although the WBSF can distinguish multiple-targets, the delay-scale resolution issue could be a serious concern when the targets are very close to each other. To improve the delay-scale resolution, the multiple transmissions technique is presented in the next section.

(a)

(b)

Figure 5.1: 3-D plots of WBSF: (a) Direct method; (b) Deconvolution method.

(a)

(b)

Figure 5.2: 3-D plots of WBSF for semi-active case: (a) Direct method; (b) Deconvolution method.

5.6 Multiple Transmissions

One of the major problems in sonar/radar signal processing is how to deal with multiple-targets. In realistic sonar/radar, it may be necessary to detect more than one targets simultaneously. The targets may also be close to each other. Under these conditions, the WBCAF using single transmission often fails to separate close targets due to noise as well as resolution effects. Multiple-targets detection and tracking has been an active research area to deal with this issue [88–91]. Although these methods perform well under high SNR environment; however, the methods may exhibit poor performance when the noise level is high.

Here, the use of multiple transmissions will be investigated by transmitting arbitrary shaped LFM signal (the same pulse shape has been used for every pulse) at known intervals of time to track and detect multiple-targets and to improve delay-scale resolution in a noisy environment. The target velocity can be computed for known values of scale. Then the target velocity can be used to correct for the delay for every scale and subsequently align all the WBCAFs. The assumption is that the radial velocity remains constant from ping to ping. Therefore, the method would also work with multiple targets. If the initial velocity remains constant then each constant velocity line of the WBCAF can be delayed by an appropriate amount in time so that the peaks add constructively. This method could be implemented efficiently if the transmitted signal is known in analytical form. Therefore, Hermite decomposition technique will be used to have an analytical model of the transmitted signal. The purpose is to improve the effective targets energy.

For known value of scale parameter, α , the velocity of the target, v , can be computed as $v = c(1 - \alpha)/(1 + \alpha)$. Then for every α , a delay correction term, $e^{-j2\pi f v p T_{PRI}^2/c}$, is introduced, where $p = 0, 1, 2, \dots$ p is the ping number, T_{PRI} is the pulse repetition interval, and the transmitter is transmitting multiple pings at regular intervals. Then the WBCAF is computed for all the received signals and added together to get the final WBCAF. The frequency domain output of the WBCAF for multiple transmissions using straight incoherent averaging and Hermite expansion is given by

$$\Psi_{wb}^{MT}(\tau, \alpha) = \sum_{p=0}^P \left| \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} \sum_{n=0}^N X(f) H_n Z_n^*(f/\alpha) e^{j2\pi f(\tau + v p T_{PRI}^2/c)} df \right|^2 \quad (5.20)$$

where $X(f)$ is the FT of the received signal and $Z_n(f)$ is the FT of the associated Hermite function.

In order to demonstrate the multiple transmissions processing of the moving target, the following parameters for the transmitted signal are used: $c = 1500$ m/s, $T_{PRI} = 1.0$ s. The target parameters of the five received signals are: $(\tau_1, v_1) = (-0.1, 36.57)$, $(\tau_2, v_2) = (-0.05, -36.51)$, $(\tau_3, v_3) = (0, 0)$, $(\tau_4, v_4) = (0.05, 16.22)$, $(\tau_5, v_5) = (0.1, -16.24)$.

For demonstration we have generated two pictures for arbitrary shaped LFM signals. The center windows of Figures 5.3 (a) and (b) show the contour plots of (5.20) for single transmission and multiple transmissions cases, respectively, in an ideal case (i.e. both the transmitted and the received signals are noise-free). There

are ten pings ($P = 9$) used to generate Figure 5.3 (b). The top and right windows of each plot show the projection on delay and scale axes respectively. For single transmission case, it can be seen that the targets are not well separated in delay-scale plane; while the five targets are distinctively separated in delay-scale plane for multiple transmissions case. The results will be even worse for single transmission case if the targets are more close to each other and contaminated with noise.

Figures 5.4 (a) to (c) show the simulation results for active case (i.e., $h(t) = g(t)$) with different pings for arbitrary shaped LFM signals (the SNR used for the received signal, $r_2(t)$, is 8 dB). While Figures 5.5 (a) to (c) show the simulation results for semi-active case (i.e., $h(t) \approx g(t)$) with different pings for the same arbitrary shaped LFM signals as in Figures 5.4 (the SNR used for the two received signals $r_1(t)$ and $r_2(t)$ are 40 dB and 8 dB respectively). It is seen that the multiple transmissions provides much better delay-scale resolution and possess higher noise immunity than the use of a single transmission for both active and semi-active cases. This is because of the fact that increasing the pings adds up the signal terms because of the rotation of the WBCAF and as such the detection capability is also increased.

Figure 5.3: Contour plots of WBCAF: (a) Single transmission; (b) Multiple transmissions.

(a)

(b)

(c)

Figure 5.4: 3-D plots of WBCAF for arbitrary shaped LFM signals (active case): (a) Single transmission; (b) Multiple transmissions ($P=7$); (c) Multiple transmissions ($P=14$).

(a)

(b)

(c)

Figure 5.5: 3-D plots of WBCAF for arbitrary shaped LFM signals (semi-active case): (a) Single transmission; (b) Multiple transmissions ($P=7$); (c) Multiple transmissions ($P=14$).

Although the benefit of using multiple transmissions is obvious from Figures 5.3 to 5.5, the error performance the MSE of estimation is also compared with the CRLB for better judgment. Figures 5.6, 5.7, and 5.8 show the MSE of estimating delay and scale for $P = 0$, $P = 4$, and $P = 9$, respectively. It can be seen that the error performance increases with the increase in the number of transmitted signals. When Figure 3.14 is compared with the Figures 5.6, 5.7, and 5.8 it can be seen that the error performance of Figure 3.14 is better than that of Figure 5.6 (for $P = 0$). However, for higher value of P the error performance of Figure 5.8 (for $P = 9$) is better than that of Figure 3.14. This justifies the effectiveness of multiple transmissions.

(a)

(b)

Figure 5.6: MSE in the estimation of parameters (simultaneously simulated) using single transmission: (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 5.7: MSE in the estimation of parameters (simultaneously simulated) using multiple transmissions ($P=4$): (a) Estimation of delay; (b) Estimation of scale.

(a)

(b)

Figure 5.8: MSE in the estimation of parameters (simultaneously simulated) using multiple transmissions algorithm ($P=9$): (a) Estimation of delay; (b) Estimation of scale.

5.7 Conclusions

An efficient technique for wideband deconvolution to estimate WBSF using Hermite decomposition has been presented in this chapter. A technique of multiple transmissions to resolve close targets in delay-scale plane using the WBCAF has also been proposed. It appears that the multiple transmissions technique can distinctively separate multiple-targets. Simulation results have been provided to show the usefulness of the proposed methods.

Chapter 6

Conclusions and Recommendations for Further Research

6.1 Conclusions

The work investigated in this thesis has been focused to develop efficient processing algorithms for wideband signals. A substantial amount of literature survey on the related topics has been performed. The literature survey has motivated us to seek for efficient techniques against the available conventional techniques for the processing of wideband signals. To summarize the main contributions of this thesis, we point out the following aspects.

The fundamental and basic topics related to the narrowband and wideband signal models are studied. The wideband and narrowband processing have been presented with simulation results. An efficient algorithm based on narrowband ambiguity functions for estimating delay and scale parameters has been proposed. An FFT-based algorithm has also been proposed to approximately estimate the slope of the main ridge of NBCAF in multipath environment.

The usefulness of Radon-ambiguity transform for multiple-targets detection in noisy environment has been demonstrated. A computationally efficient technique

based on single slice Radon-ambiguity transform using dechirp method for simultaneous estimation of delay and scale parameters has been proposed. The proposed algorithms are about five times faster than the traditional methods and much faster than the 2-D Radon-ambiguity transform technique.

The Hermite series expansions, estimation of spread and shift parameters for Hermite decomposition, and efficient 2-D peak search algorithms have been presented. An algorithm has been proposed by using the Hermite expansions for arbitrary shaped LFM signals for efficient computation of the wideband cross-ambiguity function. Efficient 2-D peak search methods have also been proposed to estimate the target parameters. Newton's optimization technique and Nelder-Mead simplex algorithm are used for this purpose. It has been shown that our proposed 2-D peak search methods are about ten times faster than the conventional methods.

An efficient technique for estimating wideband spreading function for distributed targets using Hermite decomposition is presented. Finally, a technique using multiple transmissions has been used to improve delay-scale resolution of close targets using the wideband cross-ambiguity function. Simulation results show that the multiple transmissions technique can distinctively separate multiple-targets with much better resolution and higher noise immunity as compared to the use of a single transmission. The main advantage of all the proposed algorithms is that the signals are analyzed in the frequency domain; hence the algorithms are faster than the time domain algorithms.

6.2 Recommendations for Further Research

The possible future works are mentioned in the following manner.

The proposed algorithm using Radon-ambiguity transform has been implemented for a single slice RAT. The performance of estimation could be improved by further performing a 2-D search around a small area of the estimated parameters.

One drawback of the proposed 2-D peak search methods based on Newton's and Nelder-Mead simplex algorithms is that the algorithms are suitable for a single target. The algorithms could be extended for multiple targets.

Although the multiple transmissions technique can distinctively separate multiple-targets in delay-scale plane, the estimation performance for the delay parameter does not change significantly with the number of multiple transmissions. The reason behind this drawback could be sorted out for further improvement.

In this thesis, Gaussian windowed LFM signals are mainly considered because of their excellent range and velocity resolution properties. Moreover, Hermite decomposition techniques have been used to decompose arbitrary shaped LFM signals. The possible extension of the work can be carried out for some specific signals such as non-linear hyperbolic FM signals and rectangular LFM signals.

Although additive white Gaussian noise has been used throughout the thesis, it would be an interesting task to test the algorithms in the presence of colored

noise, and the performance of the proposed methods could also be evaluated in this case.

From the point of view of active sonar, the scattering of acoustic energy from a target not only reveals the information about its range and velocity but can also be used to identify the target [79–80], [93–94]. The identification of more realistic targets in some other applications (e.g., spherical/cylindrical/cone shaped shell in sonar application, ground penetrating radar in land mine detection, ultra wideband radar in motion detection) could be an attractive area for further research.

Investigating the factors that shaped the evolution of the variety of signals produced by a bat is a broad field of study, involving acoustics, engineering, mathematics, physiology and ecology. Echolocation is a method of sensory perception by which certain animals orient themselves to their surroundings, detect obstacles, communicate with others, and find food. The bat can determine an object's size, shape, direction, distance, and motion by transmitting a hyperbolic FM signal [92]. They can even tell the size, shape, texture, and wing vibrations of a tiny insect from its echo. There are still many unanswered questions about the acoustic abilities of bats. However, research into echolocation is already yielding practical benefits for humans. For example, sonar is a form of echolocation (from a bat's perspective) that is widely used for navigation, tracking aircraft, ships, submarines and missiles, and for forecasting weather. The analysis of bat echolocation technique could be another interesting future work.

Author's Publications

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Appendix A

DERIVATION OF CRAMER-RAO LOWER BOUNDS OF THE PARAMETERS FOR GAUSSIAN LFM SIGNAL

Let us suppose the data model as,

$$x(t) = g(t; \varepsilon) + n(t), \quad -\infty \leq t \leq \infty \quad (\text{A-1})$$

with

$$g(t; \varepsilon) = ae^{j\phi} e^{-\alpha^2(t-\tau)^2/2\lambda^2} e^{j[2\pi\kappa\alpha(t-\tau) + \pi\beta\alpha^2(t-\tau)^2]} \quad (\text{A-2})$$

where $n(t)$ is the AWGN process with standard deviation σ_n and $\varepsilon = [a, \phi, \alpha, \tau]^T$ are the unknown but deterministic parameters. Now, the two parameters to be estimated are the delay τ and the scale α , which form the parameter vector ε_1 such that $\varepsilon_1 = [\alpha, \tau]^T$.

The Fisher information matrix is given by

$$F(\varepsilon) = \frac{2}{\sigma_n^2} \text{Re} \left\{ \int_{-\infty}^{\infty} \frac{\partial g^H(t; \varepsilon)}{\partial \varepsilon} \frac{\partial g(t; \varepsilon)}{\partial \varepsilon} dt \right\} \quad (\text{A-3})$$

where $\text{Re}(u)$ denotes the real part of u and H denotes the complex conjugate transpose. Note that instead of using discrete form in [9], [95], the continuous form has been used to derive the closed form expressions of the elements of the Fisher information matrix.

The elements of the Fisher information matrix $F(\varepsilon)$ were derived as follows:

$$\begin{aligned}
 [F(\varepsilon)]_{11} &= \frac{2}{\sigma_n^2} \frac{E_0}{\alpha}; & [F(\varepsilon)]_{12} &= 0; & [F(\varepsilon)]_{13} &= -\frac{2}{\sigma_n^2} \frac{aE_0}{2\alpha^2}; \\
 [F(\varepsilon)]_{14} &= 0; & [F(\varepsilon)]_{22} &= \frac{2}{\sigma_n^2} \frac{a^2 E_0}{\alpha}; & [F(\varepsilon)]_{23} &= \frac{2}{\sigma_n^2} \frac{\pi a^2 \beta \lambda^2 E_0}{\alpha^2}; \\
 [F(\varepsilon)]_{24} &= -\frac{2}{\sigma_n^2} 2\pi a^2 \kappa E_0; & [F(\varepsilon)]_{33} &= \frac{2}{\sigma_n^2} \frac{a^2 E_0}{\alpha^3} \left\{ \frac{3}{4} + 2\pi^2 \lambda^2 \kappa^2 + 3\pi^2 \beta^2 \lambda^4 \right\}; \\
 [F(\varepsilon)]_{34} &= -\frac{2}{\sigma_n^2} \frac{4\pi^2 a^2 \kappa \beta \lambda^2 E_0}{\alpha}; & [F(\varepsilon)]_{44} &= \frac{2}{\sigma_n^2} a^2 \alpha E_0 \left\{ 1/2\lambda^2 + 4\pi^2 \kappa^2 + 2\pi^2 \beta^2 \lambda^2 \right\}.
 \end{aligned}$$

with $E_0 = \sqrt{\pi \lambda^2}$.

The CRLBs of scale parameter α and time-delay τ have been obtained from the inverse of Fisher information matrix as,

$$CRLB(\alpha) = [F^{-1}(\varepsilon)]_{33} = \frac{\alpha^2 10^{-SNR/10} (1/16\lambda^8 + \pi^2 \beta^2 / 4\lambda^4)}{(1/16\lambda^8 + \pi^2 \beta^2 / 2\lambda^4 + \pi^2 \kappa^2 / 4\lambda^6 + \pi^4 \beta^4)} \quad (A-4)$$

$$CRLB(\tau) = [F^{-1}(\varepsilon)]_{44} = \frac{10^{-SNR/10} (\pi^2 \beta^2 + 1/4\lambda^4 + \pi^2 \kappa^2 / \lambda^2)}{4\lambda^2 \alpha^2 (1/16\lambda^8 + \pi^2 \beta^2 / 2\lambda^4 + \pi^2 \kappa^2 / 4\lambda^6 + \pi^4 \beta^4)} \quad (A-5)$$

where SNR is defined in dB as,

$$SNR = 10 \log_{10} \left(\frac{a^2 E_s}{\sigma_n^2} \right) \quad (A-6)$$

with E_s is the energy of the signal.

Appendix B

THE EFFECT OF THE INTEGRATION LENGTH OF RADON- AMBIGUITY TRANSFORM

The following assumptions are made to derive the means and variances of the statistic in (3.3), $\mathfrak{R}_A(\rho, \theta)$. Assume that the complex random variables n_i are jointly Gaussian with zero mean and $E\{n_i n_j^*\} = \Lambda_{ij}$. Then the following properties for complex random variables can be noted [43]:

$$E\{n_1 n_2^* n_3 n_4^*\} = \Lambda_{12} \Lambda_{34} + \Lambda_{14} \Lambda_{23} \quad (\text{B-1})$$

$$E\{n(t_1) n^*(t_2)\} = N_0 \delta(t_1 - t_2) \quad (\text{B-2})$$

$$E\{n(t_1) n(t_2)\} = E\{n^*(t_1) n^*(t_2)\} = 0 \quad (\text{B-3})$$

$$E\{n(t_1) n(t_2) n(t_3)\} = 0. \quad (\text{B-4})$$

Using the above properties, the means and variances of the statistic, $\mathfrak{R}_A(\rho, \theta)$, for the finite integration length L with complex AWGN process can be derived. The hypotheses are:

$$\begin{cases} H_0 : & x(t) = n(t) \\ H_1 : & x(t) = g(t) + n(t) \end{cases}$$

where $g(t)$ is defined in (2.21).

$$E(\mathfrak{R}_A | H_0) = E\left\{ \frac{1}{L} \int_{L/2}^{L/2} \left| \int_{-\infty}^{\infty} n(t) g^*(t - \tau) e^{-j2\pi \nu t} dt \right|^2 du \right\}$$

$$\begin{aligned}
 &= \frac{1}{L} \int_{-L/2}^{L/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[n(t)n^*(t_1)]g^*(t-\tau)g(t_1-\tau)e^{-j2\pi v(t-t_1)} dt dt_1 du \\
 &= \frac{1}{L} \int_{-L/2}^{L/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} N_0 \delta(t-t_1)g^*(t-\tau)g(t_1-\tau)e^{-j2\pi v(t-t_1)} dt dt_1 du \\
 &= \frac{1}{L} \int_{-L/2}^{L/2} \int_{-\infty}^{\infty} N_0 g^*(t-\tau)g(t-\tau) dt du \\
 &= N_0 \lambda \sqrt{\pi} .
 \end{aligned} \tag{B-5}$$

$$\begin{aligned}
 E(\mathfrak{R}_A | H_1) &= E \left\{ \frac{1}{L} \int_{-L/2}^{L/2} \left| \int_{-\infty}^{\infty} [g(t) + n(t)]g^*(t-\tau)e^{-j2\pi v t} dt \right|^2 du \right\} \\
 &= \frac{1}{L} \int_{-L/2}^{L/2} \left| \int_{-\infty}^{\infty} g(t)g^*(t-\tau)e^{-j2\pi v t} dt \right|^2 du \\
 &\quad + \frac{1}{L} \int_{-L/2}^{L/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[n(t)n^*(t_1)]g^*(t-\tau)g(t_1-\tau)e^{-j2\pi v(t-t_1)} dt dt_1 du \\
 &= \frac{\pi \lambda^2}{L} \int_{-L/2}^{L/2} e^{-a u^2} du + N_0 \lambda \sqrt{\pi} \\
 &= \frac{\pi \lambda^2}{L} \sqrt{\frac{\pi}{a}} \operatorname{erf}(L\sqrt{a}/2) + N_0 \lambda \sqrt{\pi}
 \end{aligned} \tag{B-6}$$

where $a = \sin^2 \theta / 2 \lambda^2$.

$$\begin{aligned}
 \operatorname{var}(\mathfrak{R}_A | H_0) &= E \left\{ \frac{1}{L} \int_{-L/2}^{L/2} \left| \int_{-\infty}^{\infty} n(t)g^*(t-\tau)e^{-j2\pi v t} dt \right|^2 du - E(\mathfrak{R}_A | H_0) \right\}^2 \\
 &= \frac{1}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[n(t)n^*(t_1)n(t_2)n^*(t_3)]g^*(t-\tau)g(t_1-\tau) \\
 &\quad \times g^*(t_2-\tau_1)g(t_3-\tau_1)e^{-j2\pi v(t-t_1)}e^{-j2\pi v_1(t_2-t_3)} dt dt_1 dt_2 dt_3 du du_1 - \pi N_0^2 \lambda^2 \\
 &= \frac{N_0^2}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t-\tau_1)g^*(t-\tau)g(t_1-\tau)g^*(t_1-\tau_1)e^{-j2\pi(v-v_1)t}e^{j2\pi(v-v_1)t_1} dt dt_1 du du_1 \\
 &= \frac{\pi \lambda^2 N_0^2}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} e^{-a(u-u_1)^2} du du_1
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\pi \lambda^2 N_0^2}{2L^2} \sqrt{\frac{\pi}{a}} \int_{-L/2}^{L/2} [\text{erf}\{\sqrt{a}(L/2+u)\} + \text{erf}\{\sqrt{a}(L/2-u)\}] du \\
 &= \frac{\pi^{3/2} \lambda^2 N_0^2}{aL^2} \left\{ L\sqrt{a} \text{erf}(L\sqrt{a}) + \frac{e^{-aL^2}}{\sqrt{\pi}} - \frac{1}{\sqrt{\pi}} \right\} \quad (\text{B-7})
 \end{aligned}$$

$$\begin{aligned}
 \text{var}(\mathfrak{R}_A | H_1) &= E \left\{ \frac{1}{L} \int_{-L/2}^{L/2} \left| \int_{-\infty}^{\infty} [g(t) + n(t)] g^*(t-\tau) e^{-j2\pi v\tau} dt \right|^2 du - E(\mathfrak{R}_A | H_1) \right\}^2 \\
 &= \text{var}(\mathfrak{R}_A | H_0) + \frac{2\pi^{3/2} \lambda^3 N_0}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} e^{-a(u^2 - uu_1 + u_1^2)} du du_1 \\
 &= \text{var}(\mathfrak{R}_A | H_0) + \frac{\pi^2 \lambda^3 N_0}{\sqrt{a} L^2} \int_{-L/2}^{L/2} e^{-3au^2/4} \{ \text{erf}\{(L+u)\sqrt{a}/2\} + \text{erf}\{(L-u)\sqrt{a}/2\} \} du \\
 &= \text{var}(\mathfrak{R}_A | H_0) + \frac{\pi^{3/2} \lambda^3 N_0}{L^2 e^{aL^2/4}} \left\{ \frac{3L}{4} \sqrt{\frac{\pi}{a}} e^{aL^2/16} \{ \text{erf}(L\sqrt{a}/4) + \text{erf}(3L\sqrt{a}/4) \} + \frac{1}{a} (1 - e^{-aL^2/2}) \right\} \quad (\text{B-8})
 \end{aligned}$$

From (B-5)–(B-8), for simplicity it was assumed that $\rho = 0$ while $\tau = -u \sin \theta$ and $v = u \cos \theta$.

Appendix C

Any function $w(t)$ can be written as a linear combination of an even part and an odd part around any $t = t_c$, as:

$$w(t) = w_e(t) + w_o(t) \quad (\text{C-1})$$

$$\text{where } w_e(t) = \frac{1}{2} \{w(t) + w(2t_c - t)\} \text{ and } w_o(t) = \frac{1}{2} \{w(t) - w(2t_c - t)\}$$

Making the derivative of (4.14) with respect to C_e for fixed λ , t_c , and C_o , equal to zero, we get

$$\frac{\partial \varepsilon(\varphi)}{\partial C_e} = -\frac{2E_e}{\sqrt{\lambda}\sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ w_e(t) - \frac{C_e}{\sqrt{\lambda}\sqrt{\pi}} e^{-(t-t_c)^2/2\lambda^2} \right\} e^{-(t-t_c)^2/2\lambda^2} dt = 0 \quad (\text{C-2})$$

$$\therefore C_e = \frac{1}{\sqrt{\lambda}\sqrt{\pi}} \int_{-\infty}^{\infty} w_e(t) e^{-(t-t_c)^2/2\lambda^2} dt \quad (\text{C-3})$$

Making the derivative of (4.14) with respect to C_o for fixed λ , t_c , and C_e , equal to zero, we get

$$\frac{\partial \varepsilon(\varphi)}{\partial C_o} = -\frac{2E_o}{\sqrt{\lambda^5}\sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ w_o(t) - \frac{C_o}{\sqrt{\lambda^5}\sqrt{\pi}} (t-t_c) e^{-(t-t_c)^2/2\lambda^2} \right\} (t-t_c) e^{-(t-t_c)^2/2\lambda^2} dt = 0 \quad (\text{C-4})$$

$$\therefore C_o = \frac{2}{\sqrt{\lambda}\sqrt{\pi}} \int_{-\infty}^{\infty} (t-t_c) w_o(t) e^{-(t-t_c)^2/2\lambda^2} dt \quad (\text{C-5})$$

C_e and C_o can be computed directly from (C-3) and (C-5), respectively, for any value of λ and t_c .

First and second derivatives of (4.14) with respect to λ are now given by

$$\begin{aligned} \frac{\partial \varepsilon(\varphi)}{\partial \lambda} = & -\frac{2E_e C_e}{\sqrt{\lambda^3} \sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ \frac{1}{\lambda^2} (t-t_c)^2 - \frac{1}{2} \right\} w_e(t) e^{-(t-t_c)^2/2\lambda^2} dt \\ & - \frac{2E_o C_o}{\sqrt{\lambda^7} \sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ \frac{1}{\lambda^2} (t-t_c)^2 - \frac{5}{2} \right\} (t-t_c) w_o(t) e^{-(t-t_c)^2/2\lambda^2} dt - \frac{E_o C_o^2}{\lambda^3} \end{aligned} \quad (C-6)$$

$$\begin{aligned} \frac{\partial^2 \varepsilon(\varphi)}{\partial \lambda^2} = & -\frac{2E_e C_e}{\sqrt{\lambda^5} \sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ \frac{1}{\lambda^4} (t-t_c)^4 - \frac{4}{\lambda^2} (t-t_c)^2 + \frac{3}{4} \right\} w_e(t) e^{-(t-t_c)^2/2\lambda^2} dt + \frac{3E_o C_o^2}{\lambda^4} \\ & - \frac{2E_o C_o}{\sqrt{\lambda^9} \sqrt{\pi}} \int_{-\infty}^{\infty} \left\{ \frac{1}{\lambda^4} (t-t_c)^4 - \frac{8}{\lambda^2} (t-t_c)^2 + \frac{35}{4} \right\} (t-t_c) w_o(t) e^{-(t-t_c)^2/2\lambda^2} dt \end{aligned} \quad (C-7)$$

First and second derivatives of (4.14) with respect to t_c are now given by

$$\frac{\partial \varepsilon(\varphi)}{\partial t_c} = \frac{-2}{\sqrt{\lambda^5} \sqrt{\pi}} \int_{-\infty}^{\infty} \left[E_e C_e (t-t_c) w_e(t) + E_o C_o \left\{ \frac{1}{\lambda^2} (t-t_c)^2 - 1 \right\} w_o(t) \right] e^{-(t-t_c)^2/2\lambda^2} dt \quad (C-8)$$

$$\begin{aligned} \frac{\partial^2 \varepsilon(\varphi)}{\partial t_c^2} = & -\frac{2}{\sqrt{\lambda^5} \sqrt{\pi}} \int_{-\infty}^{\infty} \left[E_e C_e \left\{ \frac{1}{\lambda^2} (t-t_c)^2 - 1 \right\} w_e(t) \right. \\ & \left. + \frac{E_o C_o}{\lambda^2} \left\{ \frac{1}{\lambda^2} (t-t_c)^2 - 3 \right\} (t-t_c) w_o(t) (t-t_c) w_o(t) \right] e^{-(t-t_c)^2/2\lambda^2} dt \end{aligned} \quad (C-9)$$

Partial derivative of (4.14) with respect to λ and t_c is given by

$$\begin{aligned} \frac{\partial^2 \varepsilon(\varphi)}{\partial t_c \partial \lambda} = & -\frac{2}{\sqrt{\lambda^7} \sqrt{\pi}} \int_{-\infty}^{\infty} \left[E_e C_e (t-t_c) \left\{ \frac{1}{\lambda^2} (t-t_c)^2 - \frac{5}{2} \right\} w_e(t) \right. \\ & \left. + E_o C_o \left\{ \frac{1}{\lambda^4} (t-t_c)^4 - \frac{11}{2\lambda^2} (t-t_c)^2 + \frac{5}{2} \right\} w_o(t) \right] e^{-(t-t_c)^2/2\lambda^2} dt \end{aligned} \quad (C-10)$$

Suppose $\varphi = [\lambda, t_c]^T$ are the parameters needed to be estimated. Then the gradient

$$\nabla_{\varphi} = \begin{bmatrix} \frac{\partial \varepsilon(\varphi)}{\partial \lambda} & \frac{\partial \varepsilon(\varphi)}{\partial t_c} \end{bmatrix}^T \quad (C-11)$$

and the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 \varepsilon(\varphi)}{\partial \lambda^2} & \frac{\partial^2 \varepsilon(\varphi)}{\partial t_c \partial \lambda} \\ \frac{\partial^2 \varepsilon(\varphi)}{\partial \lambda \partial t_c} & \frac{\partial^2 \varepsilon(\varphi)}{\partial t_c^2} \end{bmatrix}. \quad (C-12)$$

Appendix D

Equation (4.37) can be rewritten as,

$$\Psi_{wb}(\tau, \alpha) = \frac{1}{\alpha} \chi(\tau, \alpha) \chi^*(\tau, \alpha) \quad (\text{D-1})$$

where the sign $*$ is the complex conjugate, $\chi(\tau, \alpha) = \int_{-\infty}^{\infty} R(f) G_h^*(f/\alpha) e^{j2\pi f\tau} df$,

$R(f)$ and $G_h(f)$ are defined in (4.25) and (4.27), respectively.

The partial derivatives of (D-1) with respect to τ and α are given as follows:

$$\frac{\partial \Psi_{wb}(\tau, \alpha)}{\partial \tau} = \frac{1}{\alpha} \left\{ \chi \frac{\partial \chi^*}{\partial \tau} + \chi^* \frac{\partial \chi}{\partial \tau} \right\} \quad (\text{D-2})$$

$$\frac{\partial^2 \Psi_{wb}(\tau, \alpha)}{\partial \tau^2} = \frac{1}{\alpha} \left\{ \chi \frac{\partial^2 \chi^*}{\partial \tau^2} + 2 \frac{\partial \chi}{\partial \tau} \frac{\partial \chi^*}{\partial \tau} + \chi^* \frac{\partial^2 \chi}{\partial \tau^2} \right\} \quad (\text{D-3})$$

$$\frac{\partial \Psi_{wb}(\tau, \alpha)}{\partial \alpha} = \frac{1}{\alpha} \left\{ \chi \frac{\partial \chi^*}{\partial \alpha} + \chi^* \frac{\partial \chi}{\partial \alpha} \right\} - \frac{\chi \chi^*}{\alpha^2} \quad (\text{D-4})$$

$$\frac{\partial^2 \Psi_{wb}(\tau, \alpha)}{\partial \alpha^2} = \frac{1}{\alpha} \left\{ \chi \frac{\partial^2 \chi^*}{\partial \alpha^2} + 2 \frac{\partial \chi}{\partial \alpha} \frac{\partial \chi^*}{\partial \alpha} + \chi^* \frac{\partial^2 \chi}{\partial \alpha^2} \right\} - \frac{2}{\alpha^2} \left\{ \chi \frac{\partial \chi^*}{\partial \alpha} + \chi^* \frac{\partial \chi}{\partial \alpha} \right\} + \frac{2\chi \chi^*}{\alpha^3} \quad (\text{D-5})$$

$$\frac{\partial^2 \Psi_{wb}(\tau, \alpha)}{\partial \alpha \partial \tau} = \frac{1}{\alpha} \left\{ \chi \frac{\partial^2 \chi^*}{\partial \alpha \partial \tau} + \frac{\partial \chi}{\partial \tau} \frac{\partial \chi^*}{\partial \alpha} + \frac{\partial \chi^*}{\partial \tau} \frac{\partial \chi}{\partial \alpha} + \chi^* \frac{\partial^2 \chi}{\partial \alpha \partial \tau} \right\} - \frac{1}{\alpha^2} \left\{ \chi \frac{\partial \chi^*}{\partial \tau} + \chi^* \frac{\partial \chi}{\partial \tau} \right\} \quad (\text{D-6})$$

For a Gaussian LFM signal (i.e., $G_h(f) = G(f)$) the following results.

$$\frac{\partial \chi}{\partial \tau} = \int_{-\infty}^{\infty} R(f) G^*(f/\alpha) e^{j2\pi f\tau} (j2\pi f) df \quad (D-7)$$

$$\frac{\partial^2 \chi}{\partial \tau^2} = \int_{-\infty}^{\infty} R(f) G^*(f/\alpha) e^{j2\pi f\tau} (j2\pi f)^2 df \quad (D-8)$$

$$\frac{\partial \chi}{\partial \alpha} = \int_{-\infty}^{\infty} R(f) G^*(f/\alpha) e^{j2\pi f\tau} \left\{ \frac{4\pi^2 \lambda^2 f(f/\alpha - \kappa)}{\alpha^2 (1 + j2\pi\beta\lambda^2)} \right\} df \quad (D-9)$$

$$\frac{\partial^2 \chi}{\partial \alpha^2} = \int_{-\infty}^{\infty} R(f) G^*(f/\alpha) e^{j2\pi f\tau} \left\{ \frac{16\pi^4 \lambda^4 f^2 (f/\alpha - \kappa)^2}{\alpha^4 (1 + j2\pi\beta\lambda^2)^2} - \frac{4\pi^2 \lambda^2 f(3f/\alpha - 2\kappa)}{\alpha^3 (1 + j2\pi\beta\lambda^2)} \right\} df \quad (D-10)$$

$$\frac{\partial^2 \chi}{\partial \alpha \partial \tau} = \int_{-\infty}^{\infty} R(f) G^*(f/\alpha) e^{j2\pi f\tau} \left\{ \frac{j8\pi^3 \lambda^2 f^2 (f/\alpha - \kappa)}{\alpha^2 (1 + j2\pi\beta\lambda^2)} \right\} df \quad (D-11)$$

For an arbitrary shaped LFM signal, via the Hermite decomposition, the following results.

$$\frac{\partial \chi}{\partial \tau} = \int_{-\infty}^{\infty} (j2\pi f) R(f) G_h^*(f/\alpha) e^{j2\pi f\tau} df \quad (D-12)$$

$$\frac{\partial^2 \chi}{\partial \tau^2} = \int_{-\infty}^{\infty} (j2\pi f)^2 R(f) G_h^*(f/\alpha) e^{j2\pi f\tau} df \quad (D-13)$$

$$\frac{\partial \chi}{\partial \alpha} = \int_{-\infty}^{\infty} \frac{2f}{\alpha^2} R(f) \sum_n G_n A^* D e^{-jE(2\kappa - 2f/\alpha + \beta t_c) - B(f/\alpha - \kappa)^2 + j2\pi f\tau} df \quad (D-14)$$

$$\begin{aligned} \frac{\partial^2 \chi}{\partial \alpha^2} = \int_{-\infty}^{\infty} \frac{2f}{\alpha^4} R(f) \sum_n G_n A^* & \left[\{-j2Ef + 2Bf(f/\alpha - \kappa)\} D - 2\alpha D + \alpha^2 D' \right] \\ & \times \exp \left\{ -jE(2\kappa - 2f/\alpha + \beta t_c) - B(f/\alpha - \kappa)^2 + j2\pi f\tau \right\} df \end{aligned} \quad (D-15)$$

$$\frac{\partial^2 \chi}{\partial \tau \partial \alpha} = \int_{-\infty}^{\infty} \frac{j4\pi f^2}{\alpha^2} R(f) \sum_n G_n A^* D e^{-jE(2\kappa - 2f/\alpha + \beta t_c) - B(f/\alpha - \kappa)^2 + j2\pi f\tau} df \quad (D-16)$$

where

$$A = (-j)^n \sqrt{\frac{2\lambda\sqrt{\pi}}{2^n n!}} \frac{(1 + j2\pi\beta\lambda^2)^{n/2}}{\{1 - j2\pi\beta\lambda^2\}^{(n+1)/2}}, \quad B = \frac{2\pi^2\lambda^2}{1 + j2\pi\beta\lambda^2}, \quad C = \frac{2\pi\lambda}{\sqrt{1 + 4\pi^2\beta^2\lambda^4}},$$

$$E = \frac{\pi t_c}{1 + j2\pi\beta\lambda^2},$$

$$D = \{-jE + B(f/\alpha - \kappa)\}H_n[C(f/\alpha - \kappa - \beta t_c)] - nCH_{n-1}[C(f/\alpha - \kappa - \beta t_c)],$$

and

$$D' = -\frac{f}{\alpha^2} \left\{ B H_n[C(f/\alpha - \kappa)] + 2nBC(f/\alpha - \kappa)H_{n-1}[C(f/\alpha - \kappa - \beta t_c)] \right. \\ \left. - 2n(n-1)C^2 H_{n-2}[C(f/\alpha - \kappa - \beta t_c)] \right\}$$

The following formula has been used in the derivations (D-14)–(D-16).

$$\frac{\partial^m H_n[a(x/\lambda - \tau)]}{\partial \lambda^m} = -\frac{ax2^m n!}{\lambda^2 (n-m)!} H_{n-m}[a(x/\lambda - \tau)] \quad (D-17)$$