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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

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A thesis submitted to the Nanyang Technological University
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Doctor of Philosophy

2006

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

..... 3 May 2007

Date

..... Yan Ming

Yan Ming

To my family.

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Summary

Computed Tomography (CT) is a powerful technique for producing 2D cross-sectional images of an object. Recently, multislice CT (MSCT) with a large number of detector rows is investigated. Developing reconstruction algorithms for MSCT has been one of the most important research topics in the medical imaging field.

In recent years, FDK [1] type reconstruction algorithms are actively developed and adopted in medical CT. However, there are three drawbacks of FDK type algorithms. First, FDK type algorithms suffer from reconstruction artifacts when the cone angle is large. Second, some clinical applications require high pitch value (>1.0), but the maximum achievable pitch value of the FDK algorithm is limited without extrapolation [2]. Third, conventional half scan FDK type algorithms filter the projection data once for each image reconstruction which is not computationally efficient [3].

In this thesis, based on the framework of the FDK algorithm, geometrically optimized reconstruction algorithms are proposed for image reconstruction in MSCT with a large number of detector rows. The tilted plane FDK type reconstruction algorithm (TPFR) is proposed which extends the FDK algorithm to deal with its inaccuracy and artifact problems. This is done by reconstructing images on tilted reconstruction planes which optimally fit source trajectory segments.

The TPFR is extended in cooperation with the tangential filtering method (ETPFR). The reconstruction image quality of the TPFR algorithm is improved by ETPFR algorithm. ETPFR algorithm forms a projection data set in the cone-parallel geometry using the cone-beam to cone-parallel rebinning technique. Filtering projection data along the row direction of the cone-parallel data set is equivalent to filter the data along the tangential direction. Because the Parker's weighting scheme is not needed in the reconstruction procedure with the cone-parallel projection data set,

projection data are filtered once and used to reconstruct different slices. Therefore, the ETPFR algorithm is more computationally efficient than other half scan FDK type algorithms. On the other hand, because the distance is reduced between the reconstruction plane and the x ray source trajectory segment for the optimal tilted plane, the maximum pitch value for TPFR algorithm and ETPFR algorithm is larger than that of other FDK type reconstruction algorithms based on horizontal reconstruction plane. The TPFR and ETPFR algorithm larger maximum pitch values than other FDK type algorithms and have the potential to be used in clinical applications which need high scanning speed.

The resolution, slice sensitivity profiles, noise properties and reconstruction accuracy of the TPFR algorithm and ETPFR algorithm are evaluated in the thesis. Simulation results show that the image reconstruction performance of proposed algorithms is superior to that of the conventional FDK reconstruction algorithm and tangential filtering reconstruction algorithm in slice sensitivity profiles, noise properties and reconstruction accuracy.

In some clinical applications, the gantry is tilted for scanning and reconstruction algorithms for non-gantry tilted CT are extended for the gantry tilted CT reconstruction. . These reconstruction algorithms also suffer from reconstruction artifacts with the increment of the pitch value and detector slice number.

In this thesis, we extend the TPFR algorithm to the gantry tilted geometry and formulate a new geometrically optimized reconstruction algorithm. This algorithm suppresses artifacts by reconstructing images on optimal tilted planes which effectively reduce the cone angle. It's known that filtering projection data along the helical tangential direction can reduce reconstruction artifacts associated with FDK type algorithms that filter data horizontally. We present a gantry tilted tangential filtering algorithm. The resolution, noise properties and reconstruction accuracy of these two algorithms are evaluated and compared with current gantry tilted recon-

struction algorithms [4] [5]. It is shown that the image reconstruction performance of the proposed algorithms is superior to that of the gantry tilted FDK algorithm and the gantry tilted ASSR algorithm.

Abbreviations

2D	2 dimensional
3D	3 dimensional
ASSR	Advanced single-slice rebinning algorithm
CT	Computed tomography
ETPFR	Extended tilted plane FDK type reconstruction algorithm
FDK	Feldkamp, Davis, and Kress
FWHM	Full-width half maxima
GOFTR	Geometrically optimized algorithm for gantry tilted CT reconstruction algorithm
GSR	General surface reconstruction algorithm
GTASSR	Gantry tilted ASSR algorithm
GTFDK	Gantry tilted FDK algorithm
GTTF	Gantry tilted tangential filtering reconstruction algorithm
HBP	Helical body phantom
MSCT	Multislice Computed tomography
SSP	Slice sensitive profile
SSR	Single-slice rebinning algorithm
TPFR	Tilted plane FDK type reconstruction algorithm

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Chapter 1

Introduction

1.1 Introduction of computed tomography

Computed tomography (CT), originally known as the computed axial tomography (CAT or CT scan), is a nondestructive medical imaging instrument to generate cross section images of the patient. CT images are reconstructed by the CT scanner from attenuated projection data by illuminating the patient from different directions. And the density of a point on the CT image denotes the x ray attenuation coefficient of the tissue. CT is the first imaging instrument that probes the inner depths of the body, slice by slice so to provide accurate reconstruction images of organs. The 3D display of interested organs can be rendered from reconstruction images, to examine the size, shape and nature of the organs.

The first commercial CT system was invented by Hounsfield [6] at THORN EMI Central Research Laboratory in 1971. Cormack of Tufts University independently invented a similar process. The 1979 Nobel Prize in physiology and medicine was awarded to Cormack and Hounsfield for their contributions toward the development of computer-assisted tomography, a revolutionary radiological method, particularly

for the investigation of diseases of the nervous system.

However, Hounsfield's original CT system was limited to generate transversal images of the brain. The CT system which could generate images of any parts of the body was designed by Ledley at Georgetown University. Both the above systems measured projection data with thin parallel x ray beams through 180 degrees and cost long time to reconstruct images. The fan beam CT composed of a point shape source and a curved detector was set up in 1980s. This kind of CT scanner significantly reduced the scanning time. In the late 1980s and early 1990s, the commercial helical CT was introduced [7,8]. The helical CT is a significant advancement which developed CT from a slice oriented imaging modality to an organ-oriented modality [9]. In order to further improve the volume coverage speed of CT, the multislice detector was employed in CT since the late 1990s. The scanning speed is further increased by the dual sources CT which is developed in 2005 [10,11]. Volumetric CT which aims to increase the scan width to 10-20 cm, with current prototypes using 256 detector-rows are under investigation [12]. Nowadays, CT scanners with high volume coverage abilities are widely used in challenging clinical applications such as cardiac imaging and 3D cine-angiography.

Modern CT scanner is composed of the patient table, gantry, x ray tube, high voltage generator, image reconstruction hardware/software and operation control computer. Based on the predefined position, CT scans the volume of interest under the predefined scanning protocol. During the scanning procedure, the operation control computer produces instructions to the patient table, x ray generation system and x ray detector. The x ray tube emits x ray photons and these photons pass through the body and are attenuated. These attenuated photons are detected on the detector and changed to electrical signal. The electrical signal is converted into digital signal and processed by high speed image reconstruction hardware. Finally, a sequence of images for the volume of interest is reconstructed and can be displayed

by the operation computer.

1.2 Development of CT

In early years, CT scanners reconstruct images slice by slice by employing the step and shot scanning protocol as shown in Figure 1.1. This scanning protocol is composed of data acquisition period and non-data acquisition period. In the data acquisition period, the patient table remains stationary and the x ray tube and detector rotate at a constant speed to collect projection data. In the non-data acquisition period, the x ray tube turns off, the gantry remains stationary and the table moves the patient to a new scanning position. There are two main drawbacks in the step and shot reconstruction scheme. First, in order to achieve high axial resolution, the distance between two scanning positions should be sufficiently small. During the non-data acquisition period, the patient and patient table are accelerated to a constant speed and decelerate before arriving at the new scanning position. The distance between two scanning positions is small, usually several millimeters. Therefore, the acceleration and deceleration value should be very large. Large accelerated speed will cause the patient body deformation and introduce motion artifacts, Second, because the CT scanner only scans the patient during the data acquisition period, the scanning efficiency of the step and shot CT is 50% at best. It will take a long scanning time to cover a long object. In some clinical applications, contrast agent is injected into the patient body and it may lose its effect in short time, this requires that the CT scanner could scan an organ in short time. However, the step and shot CT scanner can not meet this requirement [9].

The helical CT [13] was introduced in late 1980s and early 1990s which can overcome these difficulties. In the helical CT, x ray tube and detector rotate and collect projection data continuously while the patient is translated through the gantry at a

Figure 1.1: Illustration of Step and shot scanning protocol for single slice CT.

constant speed. The non data acquisition time is reduced in the helical CT and the scanning efficiency of the helical CT is near 100% [9]. There is no acceleration and deceleration during the scanning procedure. This simplifies the mechanical design and doesn't cause patient body deformation. Advantages of the helical CT over the conventional step and shot CT are following.

1. The helical CT increases scanning speed and eliminates the nondata acquisition period. It is possible for helical CT to scan an organ in a single breath hold time period.
2. The helical CT samples uniformly along the table feeding direction while the patient table moves at a constant speed. In contrast, conventional CT with step and shot protocol can only reconstruct images at discrete positions where slices are scanned. And it can reconstruct images at any positions in the scanned volume.

Figure 1.2: Illustration of the helical single slice CT.

The helical CT is still a single slice CT. Although it has significantly improved the volume coverage speed, it can not meet some clinical requirements. For example, in CT cardiac imaging, it requires the CT scanner to achieve motion free reconstructed images for specific cardiac phase. The CT scanner has to be accompanied with ECG (electrocardiogram) to catch up the specific cardiac phase. The data collection time is very short and it requires that the CT scanner has high spacial resolution, scanning speed and volume coverage ability. In order to achieve satisfactory resolution in the table feeding direction, single slice the helical CT has to employ a thin collimation and small pitch. Its distance per rotation is determined by the collimation thickness and pitch value. In order to increase the volume coverage speed, the single slice helical CT has to increase the gantry rotation speed. However, the gantry rotation speed has its limitation so it is difficult for the single slice helical CT to cover the interest volume in a short time. In addition, there is another example of abdomen examination in which the CT scanner scans abdomen and pelvis with scanning

Figure 1.3: Illustration of multislice CT.

length about 30-40 cm. In order to reduce motion artifacts caused by breathing, it has to finish the scanning procedure in one breath hold time. In practical, it will cost the single slice helical CT 60 to 80 seconds to finish the whole scan which has 5 mm slice thickness and rotates one cycle per second [9]. Such a long time is not feasible for the patient.

In order to achieve higher volume coverage speed, the multislice detector is employed in multislice CT (MSCT), as shown in Figure 1.3. For the example of abdomen examination which we used as an example in the former part, it has a scanning length about 30-40 cm. This examination can be finished in one breath hold time with the 16 slice CT scanner and motion artifacts can be effectively reduced. For a single slice CT, a large part of x ray photons is blocked by collimators, as shown in Figure 1.4, where the single slice detector is denoted by the shadowed zone. The MSCT uses a thicker collimation than the single slice CT does and can increase the

Figure 1.4: Comparison collimator thickness between MSCT and single slice CT.

dose usage efficiency.

Compared with the multislice helical CT, MSCT with a circular orbit [1, 14–21] remains one of the most popular configurations for cone-beam scanning. It is widely employed in clinical applications such as micro CT [22–30], dedicated breast CT [31–37] and cardiac imaging [11, 38–69] because of its minimal mechanical complexity and rotational symmetry.

1.3 Motivation and objective of the thesis

1.3.1 Review of reconstruction algorithms for MSCT

In recent years, there are many contributions to develop a new generation MSCT. Research work on MSCT are focused on several areas which include noise reduction

methods for the sinogram, raw data preprocessing technology, detector design and reconstruction algorithms.

The interaction of x ray with matter includes three mechanisms: the photoelectric effect, the Compton effect and coherent scattering. The photoelectric effect and Compton effect have more influence on the CT performance than that of the coherent scattering. In the CT scanning, the x ray is attenuated by the photoelectric absorption and is scattered by Compton effect. The photons scattered by Compton effect are considered as scatter noise in reconstruction. Such scatter noise can be reduced with post-patient collimators for fan beam CT and MSCT with equal angular surface detector. For CT with flat detector, it mainly uses various noise reduction methods to reduce scatter noise.

With the introduction of MSCT, many clinical applications need higher volume coverage ability and thinner slices. In order to meet these clinical requirements, MSCTs with more detector rows and thinner slice thickness are developed. The increase of the number of detector rows leads to the increase of the cone angle. Conventional reconstruction algorithms for single slice CT can not be used directly on MSCT and a number of algorithms have been developed for spiral MSCT image reconstruction.

CT reconstruction algorithms can be divided into two major groups which are analytical algorithms and iterative algorithms [31, 39, 70–84] / algebraic algorithms [47, 85–94]. Iterative algorithms and algebraic algorithms can be used to reduce reconstruction artifacts for the circular cone beam CT [14] and the helical CT caused by large cone angle. Iterative algorithms are used to solve reconstruction problems for other x ray medical imaging systems such as Tomosynthesis [71]. The scope of this thesis is constrained to analytical algorithms for MSCT, and further discussion about iterative and algebraic reconstruction algorithms is beyond the scope of the thesis.

Analytical reconstruction algorithms can be classified into two major categories: exactly algorithms and approximate algorithms. They are described in the following.

1.3.1.1 Exact reconstruction algorithms

Exact reconstruction algorithms reconstruct images exactly with spiral cone beam projection data. Tuy first developed an exact inverse cone beam reconstruction formula in [95] and also provided a sufficient condition for exact reconstruction condition called Tuy's condition. Tuy's condition states that an object can be reconstruct exactly when all planes intersecting the object intersect the x ray source trajectory. It was difficult to practically implement Tuy's inverse formula and more practically feasible exact reconstruction algorithms were developed by Grangeat [96] and Smith [97]. These two exact reconstruction algorithms are based on the link between the cone beam projection data and a function related to the 3D Radon transform of the object. The object is recovered from the intermediate function using the standard inverse 3D Radon transform. The difference between these two algorithms is that Smith's intermediate function is the Hilbert transform of the first derivative of the Radon transform and Grangeat's intermediate function is the first derivative of the Radon transform. In discrete implementations of these algorithms, a 3D matrix containing samples of the intermediate function requires all projection data and is filtered before further processing. It is impossible to process projection data when it is measured. To overcome this problem, Defrise and Clack [98–100] derived an exact filtered backprojection reconstruction algorithm from the Grangeat's algorithm. The Grangeat algorithm is further extended to half scan reconstruction by Lee and Wang [101–103].

All these reconstruction algorithms require that the projection data be untruncated in all directions. However, in medical CT, it is impractical to use a large detector to cover the whole patient and projection data in some directions are truncated. Tam

proposed an exact reconstruction algorithm which can use truncated projection in a special shaped window on the detector and this window is called Tam's window [104–106]. Although the Tam's algorithm can solve the truncated projection data problem, it still requires that the whole object is scanned before reconstruction. The problem of reconstructing a part of the object with projection data for the whole object is the 'long object problem' [107–111].

The long object problem is first solved by Wang *et al* [108] with the helical cone beam CT. Recently developed exact reconstruction algorithms [112–120] can reconstruct a point in the object with a 2π trajectory segment and are used to solve the long object problem. Katsevich developed a filtered backprojection exact reconstruction algorithm named as Katsevich algorithm [112]. This algorithm improved the spatial resolution by minimizing the number of interpolations along the z-direction and its filtering is shift-invariant which makes it more computationally efficient than earlier exact reconstruction algorithms. Katsevich algorithm is extended to the variable pitch value by Yu *et al.* [121]. Another popular exact reconstruction algorithm called Zou-Pan algorithm is proposed by Zou and Pan which is a backprojection filtration exact reconstruction algorithm. This algorithm requires less projection data than earlier exact reconstruction algorithms and can reconstruct images within any selected region of interest from minimum data. It allows the use of detector with a reduced size and the selection of a minimum number of rotation angles. Thus it reduces radiation dose delivered to the imaged subject [113]. It is extended to the filtered backprojection algorithm by Zou and Pan [122]. Ye *et al* [123] and Zou *et al* [124] apply the Zuo-Pan algorithm to applications with the variable pitch value. Exact reconstruction algorithms for general trajectory are first proposed by Yu *et al* [125] and the Zuo-Pan algorithm is extended to general trajectories by Xia *et al* [126] and Zuo *et al* [127]. A unified reconstruction formula based on Katsevich algorithm and Zuo-Pan algorithm is proposed by Zhao *et al.* which is applicable for general smooth scanning curves [128]. The Zou-Pan algorithm can also be applied

in PET imaging with flat detector [129].

Katsevich algorithm and Zou-Pan algorithm reconstruct a point on its corresponding π line and the filter operation of these reconstruction algorithms is applied along the π line direction. Because one point is on one and only one π line in the helical scanning geometry, filtered data for one point can not be used to reconstruct other points. So these reconstruction algorithms are computationally expensive.

1.3.1.2 Approximate reconstruction algorithms

Although computer simulation shows that exact reconstruction algorithms can provide accurate reconstruction images, these algorithms are computationally expensive and their noise characteristics and temporal resolution are still under investigation for various clinical applications [130]. In comparison with exact reconstruction algorithms, approximate CT reconstruction algorithms can provide more computationally efficiency and flexibility in dealing with non-ideal situations in clinical applications [10, 131], so they have been popularly studied and developed.

Approximate reconstruction algorithms can be divided into 2D approximate reconstruction algorithms and 3D approximate reconstruction algorithms. 2D approximate reconstruction algorithms reform 2D projection data sets from the cone beam projection data set and use 2D filtered back projection approaches to reconstruct images. A number of 2D approximate reconstruction algorithms have been formulated for low number of detector rows [132–134]. They employ various weighting and interpolation methods to reform 2D projection data sets. Noo. *et al.* [135] proposed the single slice rebinning reconstruction algorithm for MSCT. This algorithm combines the single-slice rebinning method of PET imaging with the weighting scheme of spiral CT algorithms and can obtain better reconstruction images than other weighting and interpolation reconstruction algorithms. For small number of detector rows and small cone angles, these weighting and interpolation algorithms assume

that projection rays measured on different detector rows should be parallel to each other and perpendicular to the longitudinal axis. A complete set of projection data for reconstruction can be formed by rebinning and interpolating from the cone beam projection data set. Because these algorithms do not take into account effects of cone angle, severe cone beam artifacts may occur when a considerably large number of detector rows are used.

To deal with this problem, some advanced 2D approximate reconstruction algorithms taking into account cone angle effects are developed. They are known as: NSR [136], ASSR [137, 138], AMPR [139, 140] and GSR [141]. The NSR algorithm was developed by Larson, Ruth, and Crawford [136] and a tilted plane is employed as the reconstruction plane. Projection data of the tilted plane forms a surface on the detector plane and the reconstruction plane is tilted to an optimal tilted angle which minimize the total projected area over all projection angles. Reconstruction artifacts can be reduced by reconstructing images on these tilted planes with optimal tilted angles. The ASSR was derived from the single slice rebinning reconstruction algorithm [135] and a parallel single slice projection data set for the optimal tilted plane is reformed from the cone beam projection data set. The optimal tilted plane used by ASSR can effectively reduce the projection cone angle between the x ray source trajectory and the reconstruction plane, thus it can reduce reconstruction artifacts. The idea of the ASSR algorithm is similar with the NSR algorithm and the difference between the NSR and ASSR algorithm is the minimization criterion in determining the tilt angle of the reconstruction plane. Because the x ray dose isn't used efficiently by ASSR when pitch value is small [140], AMPR is derived from ASSR algorithm to solve this problem. AMPR employs several tilted reconstruction planes and can increase x ray dose usage efficiency. GSR algorithm uses the optimal surface to minimize the cone angle to reduce artifacts. A series of intermediate images are reconstructed by optimal reconstruction of planes or surfaces. The final images are obtained by interpolation of intermediate images in the longitude

direction. However, these 2D approximate algorithms have difficulties in situations with larger scanning cone angle caused by the large slice number and high pitch value [142].

3D approximate algorithms use the cone beam projection data set directly for the reconstruction. A popular 3D approximate algorithm is known as FDK algorithm developed by Feldkamp *et al.* [1] which extends the 2D fan-beam filtered back projection algorithm to the 3D reconstruction for circular cone beam CT. The FDK algorithm is practical, feasible, computationally efficient and can provide good reconstruction images when the cone angle is small. It was extended to deal with spiral CT reconstruction by Wang *et al.* [143]. There have been methods using the 2D helical view weighting technique with the FDK algorithm to reduce reconstruction artifacts and improve dose usage. These FDK type methods are proposed to handle over scan (projection angle range larger than 360°) [144–146], full scan [142, 144–148] and short scan [149–152] reconstruction problems.

FDK type algorithms use horizontal reconstruction planes and filter projection data along the horizontal direction. With the increase of the detector slice number and pitch value, the cone angle increases which will cause serious artifacts. In order to improve the reconstruction accuracy, tangential filtering algorithms are formulated. These algorithms filter projection data along the x ray source trajectory tangential direction [142, 153–158] and can improve the reconstruction image quality significantly. The tangential filtering method was first proposed by Yan and Leahy [157] and was applied to half scan by Sourbelle and Kalender [142]. The tangential direction isn't parallel to the detector rows and the projection data set along the tangential direction is truncated at the boundary of the detector. Tang and Hsieh [159] adopted rotation filtering to avoid the projection data truncation. Because current tangential filtering methods employ horizontal reconstruction planes, the maximum pitch value for tangential filtering method is still limited. Recently,

Hu *et al.* [158] developed an FDK type reconstruction algorithm to reconstruct nutating curved surfaces which are obtained by interpolation two PI-line surfaces. The nutating curved surface is derived to satisfy Tuy's condition such that each point on the surface has potential to be exactly constructed. It is in fact an intermediate means to associate its points to the short scan helix segment and the final image of the object on cartesian grids is constructed pointwisely by using the corresponding helical segment and tangential filtering. Although simulation results of Hu's algorithm are promising, a rigorous analysis of this improvement is not derived.

There are two kinds of tangential filtering reconstruction algorithms. One method is the prototype of the tangential filtering method developed by Yan and Leahy [157] for various x ray source trajectories such as ellipse and helical trajectory, in which the projection data set is filtered along trajectory tangential direction for each projection view. Because this method weights projection data before filtering for half scan reconstruction, filtered projection for one image reconstruction can not be used by others. The other method is the cone beam to cone parallel rebinning method [142, 149, 153–156, 159, 160] for the spiral trajectory. The rebinning method reforms a cone parallel projection data set from the cone beam projection data set and because the x ray source rotates on a spiral trajectory, the row direction of the reformed data set is parallel to the trajectory tangential direction. Thus, filtering along the row direction on cone parallel data set is identical to filtering along the trajectory tangential direction [130]. Because an image can be reconstructed from the parallel projection data set in the projection range π , the projection data set isn't weighted before filtering and filtered projection data can be used for different images reconstruction, Thus, this method is more computationally efficient.

1.3.2 Motivation and objective of the thesis

The review of reconstruction algorithms for MSCT shows that the exact reconstruction algorithms can obtain more accurate reconstruction images than approximate reconstruction algorithms. However, exact reconstruction algorithms are computationally expensive and their noise characteristics and temporal resolution need further investigations [130]. Few exact reconstruction algorithms have been adopted in medical CT.

Compared with exact reconstruction algorithms, approximate reconstruction algorithms are much more computationally efficient and are more widely adopted in medical CT. With the review of approximate reconstruction algorithms in Section 1.3.1, it is known that FDK type reconstruction algorithms are computationally efficient, and can provide good reconstruction image quality when cone angle is small. Thus they are widely applied in CT scanners for these advantages. However, with the requirement of higher volume coverage speed for clinical applications, larger detector and higher scanning pitch are adopted in medical CT. With the increase of detector area and pitch value, effective cone angle for CT scanning increases and causes reconstruction artifacts in FDK type algorithms. On the other hand, in some clinical applications, in order to achieve high volume coverage, the CT scanner often employs high scanning pitch, where the maximum pitch value for FDK algorithms and their extensions is too small to meet such clinical needs. These drawbacks significantly limit the application of FDK algorithms in clinical practices. In addition, in order to reduce the scanning distance in the longitude direction and increase temporal resolution, half scan FDK type algorithms are adopted in clinical applications. Half scan FDK type algorithms multiply projection data with Parker's weights before filtering operation, in which Parker's weights depend on the location of the x ray source relative to the segment of helix used for reconstruction. Projection data are filtered once for each image reconstruction. So it is not computationally efficient [3].

Therefore, one objective of the thesis is aimed at solving these problems by formulating new approximate reconstruction algorithms. First, these algorithms could suppress reconstruction artifacts for the large cone angle case and generate accurate reconstruction images for medical CT with a large detector area and high scanning pitch. Second, these algorithms should have higher maximum pitch value than other FDK type schemes. Third, these algorithms should be computationally efficient and could be flexibly applied in various clinical applications. The resolution, noise properties and reconstruction accuracy of new algorithms should be evaluated and compared with FDK type algorithms.

Besides, in some clinical applications, gantry is tilted for scanning and reconstruction algorithms are designed for these applications. Reconstruction algorithms for gantry tilted CT are extended from algorithms for non gantry tilted CT. These reconstruction algorithms [4, 5] also suffer from reconstruction artifacts with the increase of the pitch value and detector slice number.

Consequently, another objective of the thesis is to develop new reconstruction algorithms for the gantry tilted CT. These algorithms should be computationally efficient and could provide more accurate reconstruction images than current gantry tilted reconstruction algorithms. The resolution, noise properties and reconstruction accuracy of new algorithms are evaluated and compared with current gantry tilted reconstruction algorithms.

1.4 Contribution of the thesis

Recently, medical CT are under rapid growth with the larger area detector. There is a critical need of reconstruction algorithms which could provide accurate reconstruction image with high temporal resolution. In this thesis, we formulate a collection of new algorithms for non-gantry tilted and gantry tilted MSCT reconstruction and

evaluate these algorithms with noise property, spatial resolution and reconstruction accuracy. These algorithms can meet requirements of some special clinical applications, such as cardiac imaging and CT angiography, in terms of computationally efficiency and image quality.

With the review of approximate reconstruction algorithms, it is known that FDK type algorithms are widely adopted in medical CT. However, FDK type algorithms suffer from reconstruction artifacts when the cone angle is large and have small maximum pitch values. This thesis makes contributions to formulate geometrically optimized algorithms to solve these problems. The reconstruction plane for TPFR algorithm is tilted to the optimal tilted angle to minimize the deviation of the construction plane from the x ray source trajectory. Reconstructing images on optimal reconstruction planes reduces reconstruction cone angle effectively, thus reduces reconstruction artifacts for the large cone angle case. Because the distance between the reconstruction plane and the x ray source trajectory for the optimal tilted plane is reduced, the maximum pitch value of TPFR algorithm is higher than that of other FDK type reconstruction algorithms with horizontal reconstruction plane. It has higher volume coverage speed than other FDK type algorithms and is suitable for clinical applications which need higher scanning speed.

TPFR algorithm forms a projection data set for each optimal tilted plane and filtered projection data for one image reconstruction can not be used for another image reconstruction. Thus it is not very computationally efficient. As we know, the reconstruction image quality can be improved by filtering the projection data along the tangential direction of x ray source trajectory for FDK type reconstruction algorithms. We can further improve TPFR algorithm by combining it with tangential filtering technique to propose the extended TPFR algorithm (ETPFR). The tangential filtering method used in ETPFR is implemented with the cone beam to cone parallel rebinning. A projection data set in cone parallel geometry is reformed and

projection data in cone parallel geometry is filtered only once before reconstruction. In addition, ETPFR algorithm which is more computationally efficient than TPFR algorithm, improves the reconstruction image quality of TPFR algorithm.

The TPFR and ETPFR algorithm reconstruct images on same reconstruction planes as ASSR [4]. However, the ASSR algorithm is a 2D approximate reconstruction algorithm and will suffer from serious reconstruction artifacts when the slice number is large. Reconstructing images with the TPFR and ETPFR on optimal tilted reconstruction planes can improve the reconstruction image quality. Similarly, the tangential filtering method can improve reconstruction quality for FDK algorithms. We combine this technique with optimal tilted technique to form the ETPFR algorithm and the ETPFR algorithm are evaluated in the thesis.

We also make contributions to solve gantry tilted reconstruction problems. Current reconstruction algorithms for gantry tilted CT suffer from reconstruction artifacts with the increase of pitch value and detector slice number. We extend TPFR algorithm to gantry tilted geometry and obtain the geometrically optimized reconstruction algorithm for gantry tilted CT (GOFTR). Moreover, we further extend the tangential filtering method to gantry tilted geometry (GTTF). These algorithms are FDK type reconstruction algorithms and provide more computationally efficiency and better reconstruction image quality than that of current gantry tilted reconstruction algorithms [4, 5].

Our research on geometrically optimized reconstruction algorithms makes contributions to solve reconstruction problems for medical CT with a large area detector. It provides an optimized scheme to improve reconstruction image quality, reconstruction efficiency and reconstruction speed of FDK type algorithms. Additionally, it provides a simple and flexible scheme which can be extended and adopted in major clinical applications for its flexibility.

Advantages of the proposed reconstruction algorithms are summarized in the fol-

lowing:

1. Better reconstruction image quality. The TPFR algorithm and ETPFR algorithm can obtain better reconstruction image quality than other FDK type reconstruction algorithms. The GOFTR algorithm and GTTF algorithm also can obtain better reconstruction image quality than current gantry tilted reconstruction algorithms
2. High computational efficiency. These reconstruction algorithms which are FDK type approximate reconstruction algorithms are more computationally efficient than exact reconstruction algorithms. The ETPFR algorithm further improve the computational efficiency of TPFR algorithm by adopting the cone parallel rebinning technique.
3. High volume coverage speed. These reconstruction algorithms reconstruct images on optimal tilted planes. Because optimal tilted planes reduce the distance between the x ray source trajectory and reconstruction plane, the maximum pitch value for these reconstruction algorithms increases. Thus, these algorithms can work at higher pitch value and can obtain higher volume coverage speed than FDK type algorithms with horizontal reconstruction planes.

In view of these advantages, our algorithms can be applied to clinical practices to provide high volume speed and accurate reconstruction image quality. The developed algorithms are listed here:

1. The TPFR algorithm in Chapter 3.
2. The ETPFR algorithm in Chapter 3.
3. The GOFTR algorithm for gantry tiled CT in Chapter 4.
4. The GTTF algorithm in Chapter 4.

1.5 Organization of the thesis

The thesis is organized as follow:

In Chapter 1, we first introduce the modern computed tomography. Current reconstruction algorithms including exact reconstruction algorithms and approximate algorithms are reviewed in this chapter. The motivation and objective of the thesis work are presented following the literature review. And contributions of the thesis work is presented.

In Chapter 2, we present the Fourier slice theory and the fan beam reconstruction algorithm are introduced following the introduction of basic concepts for CT reconstruction algorithm. Then, parallel rebinning and half scan technique which are widely used in cone beam reconstruction algorithms are introduced. Because most current approximate reconstruction algorithms are FDK type reconstruction algorithms, the conventional FDK algorithm is introduced in this chapter.

In Chapter 3, optimal plane reconstruction algorithms for spiral MSCT are presented. In this chapter, we first define the geometry of the MSCT. Then TPFR algorithm is presented based on this geometry. In order to improve the computational efficiency and reconstruction image quality, ETPFR algorithm is further presented. TPFR and ETPFR algorithms are evaluated by resolution study, noise study, SSP study and phantom reconstruction study. These algorithms are also compared with the conventional FDK algorithm and the tangential filtering reconstruction algorithm.

In Chapter 4, we present two FDK type reconstruction algorithms for gantry tilted MSCT. In this chapter, we first defined the gantry tilted geometry for gantry tilted CT. The optimal plane technique and tangential filtering technique are further extended to the gantry tilted geometry. These algorithms are evaluated by resolution study, noise study and phantom reconstruction study. We also make comparison

between proposed algorithms and current gantry tilted reconstruction algorithms.

In Chapter 5, we present the conclusion and recommendations for further research.

Chapter 2

Fundamentals of CT reconstruction

In this chapter, we introduce fundamental knowledge of reconstruction algorithms. We first introduce the nature of CT followed by the Fourier slice theorem and the parallel reconstruction algorithm for the first generation of CT. Because commercial CT scanners employ point x ray sources and fan shaped or plane detectors, fan-beam reconstruction formula will be derived from the parallel reconstruction formula. The parallel rebinning and half scan technique widely used in cone beam reconstruction algorithms are presented. Because most current approximate reconstruction algorithms are of FDK type, FDK algorithm will be introduced in this chapter. Detailed descriptions about CT reconstruction algorithms can be found in books of Herman [161], Natterer [162] and Hsieh [9].

2.1 Measurements of line integral and the Fourier slice theorem

The CT x ray source radiates x ray photons onto the detector surface through the object. The x ray beam is attenuated by the interaction with the object. We set up a cartesian coordinate system $x - y$ and define $f(x, y)$ as the object attenuation coefficient function. The incident number of x ray photons is I_0 and the detected number of x ray photons on the detector is I_d . The relationship between I_0 and I_d is [9]:

$$I_d = I_0 e^{-\int_L f(x, y) d\vec{L}}. \quad (2.1)$$

In (2.1), L is the line of the projection ray and $\int_L f(x, y) d\vec{L}$ is the linear integration of the object attenuation coefficient function along L . We can obtain the linear integral value of the object attenuation coefficient function as

$$\int_L f(x, y) d\vec{L} = \ln \frac{I_0}{I_d}. \quad (2.2)$$

The measurement of the linear integral is obtained with (2.2) and can be used for CT reconstruction. It is noted that the form of linear integral in (2.2) is an ideal condition. For practical CT, projection data collected from the detector are influenced by scatter noise, beam harden phenomena and statical noise and are subjected to complex preprocessing procedure before the logarithm operation and reconstruction.

In the parallel geometry, the object is sampled by parallel projection rays at different directions as shown Figure 2.1, where a rotation cartesian coordinate system $s - t$ for the parallel geometry is set up. The angle between the projection ray and x -axis is the projection angle β . The origin of $s - t$ has the same origin as $x - y$ coordinate system and the s -axis is parallel to the projection ray. The projection ray is defined

Figure 2.1: Illustration of the linear integration and parallel geometry for CT.

by the projection angle β and the distance to the iso centre which is denote by its t coordinate.

With the rotation coordinate system $s - t$, the line which denotes the projection ray in $s - t$ is:

$$t = y \cos \beta - x \sin \beta. \quad (2.3)$$

The linear integral of attenuation coefficients along the projection ray can be written as:

$$D(\beta, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(y \cos \beta - x \sin \beta - t) dx dy. \quad (2.4)$$

where δ is the Delta function.

The fourier transform of $D(\beta, t)$ along the t axis is:

$$P(\omega, \beta) = \int_{-\infty}^{\infty} D(\beta, t) e^{-j\omega t} dt. \quad (2.5)$$

And the two dimensional fourier transform of the object attenuation coefficient function is:

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(xu+vy)} dx dy. \quad (2.6)$$

Let $u = -\omega \sin \beta$ and $v = \omega \cos \beta$ and bring the (2.4) into the (2.5), The (2.5) and (2.6) are identical.

$$F(-\omega \sin \beta, \omega \cos \beta) = P(\omega, \beta). \quad (2.7)$$

The Fourier transform of the parallel projection data along the t axis is a slice of the two dimensional Fourier transform of the attenuation coefficient function in the Fourier space. This relationship is the Fourier slice theorem [9].

2.2 The filtered backprojection

The Fourier slice theorem provides a solution for the image reconstruction. The object attenuation coefficient function $f(x, y)$ can be obtained from $F(u, v)$ with the following two dimensional inverse Fourier transform.

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv. \quad (2.8)$$

With the Fourier slice theorem, the Fourier transform of $f(x, y)$ is obtained with the Fourier transform of projection data along the detector direction at different projection angles. Substituting the (2.5) into the inverse Fourier transform (2.8), we set up the following relationship:

$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{\infty} P(\omega, \beta) |\omega| e^{j2\pi\omega t} d\omega d\beta. \quad (2.9)$$

In order to implement (2.9), we derive its spatial domain expression as

$$f(x, y) = \int_0^\pi \int_{-t_m}^{t_m} D(t', \beta) h(t - t') dt d\beta. \quad (2.10)$$

where the projection rays are in the range $t \in [-t_m, t_m]$ and

$$h(t) = \int_{-\infty}^{\infty} |\omega| e^{j2\pi\omega t} d\omega. \quad (2.11)$$

represents the ramp filter. Because the integration in (2.11) does not exist, we deal with it by multiplying $|\omega|$ with a window function.

$$q(\omega) = \begin{cases} 1 & |\omega| < \frac{1}{2\Delta} \\ 0 & \text{else} \end{cases} \quad (2.12)$$

where Δ denotes the projection sampling interval.

$$h(t) = \int_{-\frac{1}{2\Delta}}^{\frac{1}{2\Delta}} |\omega| e^{j2\pi\omega t} d\omega. \quad (2.13)$$

The reconstruction formula for parallel geometry CT can be obtained by substituting the (2.13) into (2.10):

$$f(x, y) = \int_0^\pi \int_{-t_m}^{t_m} D(t', \beta) h(t - t') dt d\beta. \quad (2.14)$$

The discrete form of the $h(t)$ with $t = n\Delta$ is [9]:

$$h(n\Delta) = \begin{cases} \frac{1}{4\Delta^2}, & n = 0; \\ 0, & n = \text{even}; \\ \frac{-1}{(n\pi\Delta)^2}, & n = \text{odd}; \end{cases} \quad (2.15)$$

The parallel reconstruction algorithm can be implemented in discrete form by the following procedure

Figure 2.2: Illustration of equalangular fan beam geometry for CT.

1. the projection data $D(n\Delta, \beta)$ is convolved with $h(n\Delta)$ for each projection angle $\beta \in [0, \pi]$.
2. $f(x, y)$ is obtained by integrating filtered data over π .

From the implementation procedure, we find that the object attenuation coefficient function $f(x, y)$ can be reconstructed with the filtering and backprojection steps. The reconstruction algorithm with the (2.14) is called filtered backprojection algorithm.

2.3 Fan beam reconstruction algorithm

In the above section, we have introduced the parallel geometry CT and the filtered backprojection algorithm. It is a prototype geometry and is not used in commercial CT scanners. Most single slice medical CT employ fan beam geometry which is composed of a point shaped x ray source and an equalangular or equalspace detector. The fan beam geometry is shown in Figure 2.2 where β is the projection angle, θ is

the detector angle, θ_{fan} is the fan angle and R_f is the x ray source radius. The x ray source trajectory is:

$$C_s(\beta) = \begin{Bmatrix} R_f \cos \beta \\ R_f \sin \beta \\ z_s \end{Bmatrix}. \quad (2.16)$$

In this equation, z_s is the z position of the plane where the x ray source lies on.

The filtered backprojection formula for the parallel geometry can be modified into a formula for the fan beam geometry [9, 161, 163, 164]. The reconstruction formula for equalangular fan beam reconstruction is:

$$f(x, y) = \frac{1}{2} \int_0^{2\pi} \int_{-\theta_{fan}/2}^{\theta_{fan}/2} \frac{1}{L^2} D(\theta', \beta) \cos \theta' g(\theta - \theta') d\theta d\beta. \quad (2.17)$$

In this equation, L is the distance between the x ray source and $(x, y)^T$ and $g(\theta)$ is the ramp filter for equalangular fan beam geometry. The sampling interval in the projection fan is $\Delta\theta$ and the number of projection rays in the projection fan is N_f .

$$\Delta\theta = \frac{\theta_{fan}}{N_f}$$

The discrete form of $g(\theta)$ with $\theta = n\Delta\theta$:

$$g(n\Delta\theta) = \begin{cases} \frac{1}{8\Delta\theta^2}, & n = 0; \\ 0, & n = \text{even}; \\ \frac{-1}{2(\pi \sin n\Delta\theta)^2}, & n = \text{odd}; \end{cases} \quad (2.18)$$

The equal angular fan beam reconstruction algorithm can be implemented in the following discrete form:

1. The projection data $D(n\Delta\theta, \beta)$ are pre-weighted by multiplying $\cos n\Delta\theta$.
2. The weighted data are convolved with $g(n\Delta\theta)$ for each projection angle $\beta \in$

Figure 2.3: Illustration of converting parallel projections from fan beam projections.

$[0, 2\pi]$.

3. Perform weighted backprojection of each filtered data along the fan and $f(x, y)$ is obtained with (2.17).

2.4 Parallel rebinning and half-scan scanning

The reconstruction algorithm for parallel geometry is derived in Sec. 2.2, meanwhile, the fan beam reconstruction formula is derived from the parallel reconstruction formula in Sec. 2.3. In this section, the parallel rebinning method [18, 136, 137, 165–171] is introduced which reforms a parallel projection data set from the fan beam projection data set and reconstructs image with the parallel reconstruction algorithm. The process to convert from fan beam projections to parallel projections is called parallel rebinning.

The projection ray with projection angle β and detector angle θ is denoted by the dark line in Figure 2.3. This projection ray can be converted to the parallel geometry.

It is parameterized with the projection angle α and the distance from the projection ray to the rotation axis t . As shown in Figure 2.3, the in-plane correspondence between the fan beam coordinates and the parallel coordinates of the projection ray is:

$$\begin{aligned}\alpha &= \beta + \theta \\ t &= -R_f \sin \theta.\end{aligned}\tag{2.19}$$

The projection ray is parameterized by α and t and its projection data can be denoted by $D_p(\alpha, t)$. The projection data $D(\beta, \theta)$ and $D_p(\alpha, t)$ are due to the same projection ray, we have

$$D_p(\alpha, t) = D(\beta, \theta).$$

The rebinning procedure defined by (2.19) is called fan beam to parallel rebinning. This technique is also widely used in MSCT to obtain projection data set in cone parallel geometry [130, 156, 172].

As shown in (2.19), the parallel projection is composed of non-equal distant sampled projection rays. Interpolation methods are used to obtain equal distant sampled projection rays. Besson [173, 174] presented a method to deal with non-equal distant sampled projection rays without interpolation. These methods can provide sharper image than that of other interpolation methods.

With the reconstruction equation for parallel geometry (2.14), a point (x, y) can be reconstructed by projections with the projection angle range equal to π . A parallel projection data set with $\alpha \in [\alpha_i, \alpha_i + \pi]$ is reformed from the fan beam projection data set for reconstruction where α_i is the initial projection angle for the reformed projection data set. From (2.19), we can obtain $\beta = \alpha - \theta$ with the projection angle θ being in $[-\theta_{fan}/2, \theta_{fan}/2]$. In order to reform a parallel projection data set composed of projection rays with $\alpha \in [\alpha_i, \alpha_i + \pi]$ and $t \in [-R_f \sin \theta_{fan}/2, R_f \sin \theta_{fan}/2]$,

Figure 2.4: Illustration of fan beam sinogram for half scan .

the minimal projection angle range for fan beam projection data set is $\beta \in [\alpha_i - \theta_{fan}/2, \alpha_i + \pi + \theta_{fan}/2]$. Without loss of generality, we set $\alpha_i = 0$ and the length of the projection angle range for the parallel projection reforming data set is $\pi + \theta_{fan}$.

The symmetric property of the rotation sets up the following relationship:

$$D(\beta, \theta) = D(\beta + \pi - \theta, -\theta). \quad (2.20)$$

Thus, as shown in Figure 2.4, the projection data in the triangle $0 < \beta < \theta_{fan} - 2\theta$ will be redundant with projection data in the triangle $\pi - 2\theta < \beta < \pi + \theta_{fan}$. Projections in these two triangles which are marked with shadowed areas are doubly scanned.

With the derivation above, a minimal projection angle range for the fan beam reconstruction is $\pi + \theta_{fan}$. In this minimal projection data set, only a part of the projections in the shadowed areas is doubly scanned. These projections should be properly weighted before the reconstruction. Parker [175] and Crawford *et. al* [8] provided an exact formula for weighting functions and the Parker's weighting function is:

$$W(\beta, \theta) = \begin{cases} \sin^2\left(\frac{\pi}{4} \frac{\beta}{\frac{\theta_{fan}}{2} - \theta}\right), & 0 \leq \beta \leq \theta_{fan} - 2\theta \\ 1, & \theta_{fan} - 2\theta \leq \beta \leq \pi - 2\theta \\ \sin^2\left(\frac{\pi}{4} \frac{\pi + \theta_{fan} - \beta}{\theta}\right), & \pi - 2\theta \leq \beta \leq \pi + \theta_{fan} \\ 0, & \text{otherwise,} \end{cases} \quad (2.21)$$

The reconstruction formula with the minimal complete projection data set can be written as:

$$f(x, y, z) = \int_0^{\pi + \theta_{fan}} \frac{1}{L^2} \int_{-\theta_{fan}/2}^{\theta_{fan}/2} R(\beta, \theta) W(\beta, \theta) g(\bar{\theta} - \theta) d\theta d\beta \quad (2.22)$$

The reconstruction algorithm whose projection angle range length equals to $\pi + \theta_{fan}$ is called half scan or short scan algorithm. Compared with it, the reconstruction algorithm with the length of the projection angle range being 2π is called the full scan algorithm.

As we know, the table feeding distance of spiral Multislice CT per gantry rotation is determined by its slice thickness, slice number, scanning pitch and projection angle range used for the reconstruction. The slice thickness and scanning pitch are determined by clinical application requirements. The projection angle range for half scan is $\pi + \theta_{fan}$ which is smaller than that for the full scan. Thus, the x ray source moving distance in the table feeding direction for the half scan is smaller than that of the full scan that means cone angles for the half scan is smaller than the full scan. In the

scanning procedure with high scanning pitch and large detector number, half scan reconstruction algorithms will produce smaller artifacts than full scan reconstruction algorithms. Because the trajectory segment used for half scan reconstruction algorithms is shorter than that for full scan algorithms, half scan algorithms have better temporal resolution. Such merit is favorable for clinical applications which require high temporal resolution. On the other hand, full scan reconstruction algorithms employ more projection data than half scan reconstruction algorithms and have better noise performance than half scan reconstruction algorithms. Hein [176] developed a reconstruction algorithm whose projection angle range is larger than the half scan and smaller than the full scan. A modified weighting function [177] is developed to process the redundant projection data. This algorithm can increase the projection angle range to obtain better noise performance for the low scanning pitch and decrease the projection range to obtain smaller artifacts for the high scanning pitch.

Wang *et al.* [178], Silver [179] and Noo and Defrise [180] also proposed smooth sinogram windowing method to deal with the redundant projection data.

2.5 FDK reconstruction algorithm

In former sections, our discussions are limited to single slice reconstruction algorithms and the x ray source lies on the reconstruction plane. In this section, another CT configuration circular cone beam CT is introduced and one of the most popular reconstruction algorithms for this configuration is presented. The circular cone beam CT employs a multislice detector array which can be curved or flat and its x ray source rotates on a circular trajectory. With this configuration, projection dose is used more efficiently and a volume of an organ can be reconstructed in a single gantry rotation.

Figure 2.5: The geometry of the FDK algorithm .

Feldkamp, Davis, and Kress developed an approximate reconstruction algorithm for circular cone beam CT which is well known as FDK algorithm [1].

In FDK algorithm, the x ray source C_s rotates on a circular trajectory with radius R_f and the plane on which the x ray source lies is called centre plane as shown in Figure 2.5. The detector rotates around the z axis with radius R_d . A projection ray passes the point $(x, y, z)^T$ and the angle γ between the reconstruction plane and the projection ray is called cone angle. The point (x, y, z) can be reconstructed by the fan beam reconstruction algorithm with a fan beam projection data set. The FDK algorithm approximates projection data by multiplying the cone beam projection data with the cosine value of the cone angle. The projection data with projection angle β , detector angle θ and distance to the centre plane u is denoted as $D(\beta, \theta, u)$.

And the cosine value of the cone angle γ is:

$$\cos \gamma = \frac{R_f + R_d}{\sqrt{(R_f + R_d)^2 + u^2}}.$$

Thus, the approximate fan beam projection data $D_f(\beta, \theta, u)$ is obtained from $D(\beta, \theta, u)$ with:

$$D_f(\beta, \theta, u) = D(\beta, \theta, u) \cos \gamma = D(\beta, \theta, u) \frac{R_f + R_d}{\sqrt{(R_f + R_d)^2 + u^2}}. \quad (2.23)$$

With the fan beam projection data, the point $(x, y, z)^T$ can be reconstructed with (2.17) as followed.

$$f(x, y, z) = \frac{1}{2} \int_0^{2\pi} \int_{-\theta_{fan}/2}^{\theta_{fan}/2} \frac{1}{L^2} D_f(\theta', \beta, u) \cos \theta' g(\theta - \theta') d\theta d\beta. \quad (2.24)$$

Substituting (2.23) into (2.24), the reconstruction formula for FDK algorithm is obtained:

$$f(x, y, z) = \frac{1}{2} \int_0^{2\pi} \int_{-\theta_{fan}/2}^{\theta_{fan}/2} \frac{1}{L^2} \frac{R_f + R_d}{\sqrt{(R_f + R_d)^2 + u^2}} D(\theta', \beta, u) \cos \theta' g(\theta - \theta') d\theta d\beta. \quad (2.25)$$

The definition of L is identical to that of the fan beam reconstruction algorithm which is the distance between $(x, y, z)^T$ and the x ray source in centre plane.

It is noted that the FDK algorithm is an approximate cone beam reconstruction algorithm and performs well for small cone angles. Because the approximate nature of the FDK algorithm, when the cone angle is large, obvious artifacts will be produced and the shape of the reconstruction images can also be distorted.

Chapter 3

Tilted plane FDK type reconstruction algorithms for spiral cone beam CT

In the former chapter, fundamental knowledge of reconstruction problems is introduced. In this chapter, the tilted plane FDK type reconstruction algorithm for spiral cone beam CT (TPFR) is introduced. An improvement of the TPFR algorithm is further developed. These algorithms are evaluated by noise study, resolution study, SSP (Slice Sensitivity profile) study and image reconstruction accuracy study with various phantoms.

3.1 Geometry for spiral MSCT

The spiral MSCT scanning set up consists of an x ray source and a detector array forming a source-detector framework. The geometry of the source-detector framework is shown in a cartesian coordinate system $x - y - z$ as in Figure 3.1, where

Figure 3.2: In plane geometry of multislice CT scanner.

C_s denotes the x ray source and the detector array is an arched rectangular surface with the horizontal centre line l_h , vertical centre line l_v and geometric centre point C_d . It is supposed that the detector array contains M horizontally parallel detector slices and there are N_f detector cells in each detector slice. The x ray source C_s and the detector array centre point C_d move spirally around the z -axis with radius R_f and R_d , respectively.

The x ray source C_s and the horizontal centre line l_h form the centre plane of the source-detector framework which is parallel to the $x - y$ plane. On the centre plane, the projection fan angle θ_{fan} , detector angle θ and projection angle β are defined which are shown in Figure 3.2.

In the spiral scanning protocol, the x ray source in the gantry which is perpendicular

Figure 3.3: Illustration the definition of slice thickness S .

to the table feeding direction rotates at a constant speed and the patient table moves the patient across the gantry. If the patient is consider stationary, the x ray source moves on a spiral trajectory.

The slice thickness S is defined as the x ray beam width for one detector slice along the z axis, as shown in Fig. 3.3. The detector is composed of M detector slices, the slice thickness is S and the distance the table moving per rotation is h . Thus, the pitch p of the scanning protocol is defined:

$$p = \frac{h}{SM}. \quad (3.1)$$

The initial z position of the gantry is z_0 and the initial projection angle of the x ray

source is β_0 . Thus, the x ray source trajectory is defined:

$$C_s(\beta) = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} = \begin{pmatrix} R_f \cos \beta \\ R_f \sin \beta \\ z_0 + \frac{pSM}{2\pi}(\beta - \beta_0) \end{pmatrix}. \quad (3.2)$$

The detector array surface as a part of the source-detector framework is also rotating spirally around the z -axis and the spiral trajectory of its centre point is

$$C_d(\beta) = \begin{pmatrix} -R_d \cos \beta \\ -R_d \sin \beta \\ z_0 + \frac{pSM}{2\pi}(\beta - \beta_0) \end{pmatrix}^T. \quad (3.3)$$

With the definition of slice thickness S , let u be the height of the detector slice row at the iso-centre as shown in Figure 3.4, with $u \in [-\frac{SM}{2}, \frac{SM}{2}]$. Each x ray projection of the cone beam emitted from the x ray source can be characterized in terms of β , θ and u and its projection data is $D(\beta, \theta, u)$. The projection ray is defined by x ray source C_s and the point $(x_p, y_p, z_p)^T$ on the projection ray.

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = \begin{pmatrix} -R_f \cos(\theta + \beta) + x_s \\ -R_f \sin(\theta + \beta) + y_s \\ u + z_s \end{pmatrix}, \quad (3.4)$$

where x_p , y_p and z_p satisfy $\sqrt{(x_p - x_s)^2 + (y_p - y_s)^2} = R_f$ and $z_p = z_s + u$.

The projection ray is projected onto the detector cell of the detector array and the position of the detector cell in $x - y - z$ is:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = \begin{pmatrix} -(R_f + R_d) \cos(\theta + \beta) + x_s \\ -(R_f + R_d) \sin(\theta + \beta) + y_s \\ \frac{R_f + R_d}{R_f} u + z_s \end{pmatrix}. \quad (3.5)$$

Figure 3.4: Geometric descriptions of u , $(x_c, y_c, z_c)^T$ and $(x_p, y_p, z_p)^T$.

The geometric positions of $(x_c, y_c, z_c)^T$ and $(x_p, y_p, z_p)^T$ are shown in Figure 3.4.

3.2 Tilted plane FDK type reconstruction algorithm

3.2.1 Optimal tilted plane for reconstruction

The FDK algorithm can obtain acceptable reconstruction images when the cone angle is small. FDK reconstruction algorithms use horizontal reconstruction planes and the cone angle will increase with the detector slice number and pitch value. Clinical applications need faster volume coverage speed than before and CT scanners with a large slice number detector working at a high pitch value are used to meet clinical requirements.

Figure 3.5: Tilted construction plane.

The conventional FDK reconstruction algorithm with horizontal reconstruction plane will suffer from serious artifacts with the large cone angles. The underlying concept of image reconstruction on the tilted plane is to geometrically reduce the deviation of the scanning trajectory from the construction plane and reduce cone angles, hence, reducing the artifacts. And this concept is illustrated in Figure 3.5 where the construction plane is tilted from the horizontal plane by an angle γ to be more inline with the scanning trajectory. Thus the proposed TPFR algorithm constructs the image by first constructing images on a sequence of tilted planes and then interpolating images of the tilted planes to obtain the image on the specified plane parallel to the $x - y$ -plane. The tilted angle is optimized in order to minimize the deviation of the construction plane from the scanning trajectory to obtain best possible image reconstruction.

The construction of the geometrically optimized tilted plane is proposed as follows.

Let $\Phi = \{P_i : i = 1, 2, 3, \dots, n\}$ be the set of tilted planes with each tilted plane denoted by P_i . And let the intersection of the tilted plane P_i and the z -axis be at $z = z_i = z_0 + i\Delta z$, for $i = 1, 2, 3, \dots, n$ with $\Delta z > 0$ being a constant. Then the tilted planes in the set Φ are uniformly placed at the intersections with the z -axis. At $z = z_i$ on the z -axis, the source trajectory $C_s(\beta)$ is at the angular position $\beta = \beta_i = \beta_0 + \frac{2\pi(z_i - z_0)}{pSM}$, and the corresponding x ray source trajectory point at β_i is

$$C_s(\beta_i) = \begin{pmatrix} R_f \cos \beta_i \\ R_f \sin \beta_i \\ z_i \end{pmatrix}.$$

The two points $C_s(\beta_i)$ and $(0, 0, z_i)$ determine a straight line l_i which intersects the two points and parallel to the $x - y$ -plane. The i th tilted plane P_i , for $i = 1, 2, 3, \dots, n$, is defined such that the straight line l_i lies in the plane P_i and the plane P_i is tilted, with the straight line l_i as axis, by an angle γ_i from the $x - y$ -plane in the anticlockwise direction. Hence, each tilted plane P_i is characterized by its intersection at $z = z_i$ with the z -axis and the tilted angle γ_i from the $(x - y)$ -plane. The tilted plane P_i is:

$$z = z_i + (y \cos \beta_i - x \sin \beta_i) \tan \gamma_i. \quad (3.6)$$

The proposed TPFR algorithm is a FDK type cone beam reconstruction algorithm and it reconstructs images on a sequence of tilted planes. We now consider to improve the CT image reconstruction by optimizing tilted angles. It will turn out that different tilted planes have the same optimal tilted angle.

In the spiral scanning process as shown in Figure 3.5, the x ray source trajectory $C_s(\beta)$ determines a cylinder surface described by $x^2 + y^2 = R_f^2$ in the $x - y - z$ coordinate system such that the trajectory $C_s(\beta)$ lies on the cylinder surface. Each

tilted plane P_i defined in (3.6) intersects the cylinder surface yielding an ellipse which is expressed by

$$E_i(\beta) = \begin{pmatrix} R_f \cos \beta \\ R_f \sin \beta \\ z_i + R_f \tan \gamma_i \sin(\beta - \beta_i) \end{pmatrix}^T. \quad (3.7)$$

For each projection angle β , the distance between the x ray source and the ellipse in the z -direction is:

$$\begin{aligned} d_{se}(\beta, \gamma_i) &= \left| z_0 + \frac{pSM(\beta - \beta_0)}{2\pi} - z_i - R_f \tan \gamma_i \sin(\beta - \beta_i) \right| \\ &= \left| z_0 + \frac{pSM(\beta - \beta_0)}{2\pi} - z_0 - \frac{pSM(\beta_i - \beta_0)}{2\pi} - \right. \\ &\quad \left. R_f \tan \gamma_i \sin(\beta - \beta_i) \right| \\ &= \left| \frac{pSM(\beta - \beta_i)}{2\pi} - R_f \tan \gamma_i \sin(\beta - \beta_i) \right|, \end{aligned}$$

which is a function in the tilted angle γ_i .

The distance $d_{se}(\beta, \gamma_i)$ between that x ray source trajectory and the intersection ellipse can be used as a measure of inlineness between the x rays and the tilted plane. In Figure 3.6, the distance $d_{se}(\beta, 0^\circ)$ for the horizontal plane denoted by the solid curve and $d_{se}(\beta, 0.4^\circ)$ for a tilted plane with $\gamma_i = 0.4^\circ$ denoted by the dotted curve are illustrated, where the parameters used are $z_i = 0$, $\beta_i = 0$, $R_f = 570\text{mm}$, $\theta_{fan} = 52^\circ$, $M = 16$, $S = 1\text{mm}$ and $p = 1.0$. It is shown that the x ray source trajectory is much closer to the tilted plane with $\gamma_i = 0.4^\circ$ than to the horizontal plane with $\gamma_i = 0^\circ$. The distance $d_{se}(\beta, 0.4^\circ)$ is smaller than $d_{se}(\beta, 0^\circ)$.

Let β_m be the interval of the projection angle for each tilted plane and let

$$\bar{\beta} = \beta - \beta_i \in [-\beta_m/2, \beta_m/2]$$

Figure 3.6: Plots of $d_{se}(\beta, \gamma_i)$ with the dotted curve for $\gamma_i = 0.4^\circ$ and solid curve for $\gamma_i = 0^\circ$.

for $\beta \in [\beta_i - \beta_m/2, \beta_i + \beta_m/2]$. The projection angle range β_m is equal to $\pi + \theta_{fan}$ for the half scan reconstruction and is equal to 2π for the full scan reconstruction. The proposed TPFR algorithm geometrically minimizes the mean square error between the source trajectory $C_s(\beta)$ and the ellipse $E_i(\beta)$ in the z -direction, which is formulated by

$$J^2(\gamma_i) = \int_{-\beta_m/2}^{\beta_m/2} (R_f \sin \bar{\beta} \tan \gamma_i - \frac{pSM\bar{\beta}}{2\pi})^2 d\bar{\beta}. \quad (3.8)$$

It can be computed that

$$\begin{aligned} J^2(\gamma_i) &= R_f^2 \left(\frac{\beta_m}{2} - \sin \frac{\beta_m}{2} \cos \frac{\beta_m}{2} \right) \tan^2 \gamma_i \\ &\quad - \frac{2(\sin \frac{\beta_m}{2} - \frac{\beta_m}{2} \cos \frac{\beta_m}{2}) R_f p S M}{\pi} \tan \gamma_i \\ &\quad + \frac{p^2 S^2 \beta_m^3}{6\pi^2}. \end{aligned} \quad (3.9)$$

As a result, the optimal γ_i which minimizes the mean square error function $J^2(\gamma_i)$ in (3.8) and (3.9) is given by

$$\gamma_{iopt} = \arctan \left[\frac{(\sin \frac{\beta_m}{2} - \frac{\beta_m}{2} \cos \frac{\beta_m}{2}) p S M}{R_f \pi (\frac{\beta_m}{2} - \sin \frac{\beta_m}{2} \cos \frac{\beta_m}{2})} \right]. \quad (3.10)$$

From this equation, the optimal tilted angle γ_{ipot} for the i th tilted plane P_i is a function of β_m and p . The optimal tilted angle γ_{iopt} is independent of the position z_i of the tilted plane in the z -direction and the index i . This implies that all the tilted planes have the same optimal tilted angle. For simplicity, we will use γ_{opt} to denote γ_{iopt} . We give an example for the optimal tilted angle, the scanning parameters are $R_f = 570\text{mm}$, $\theta_{fan} = 40^\circ$, $p = 1.0$, $S = 1\text{mm}$ and $M = 64$. The optimal angle computed with (3.10) is 2.54° for half scan and 2.04° for full scan.

With the optimal tilted angle γ_{opt} and the center projection angle β_i , the optimal tilted reconstruction plane P_i can be determined by the following equation in the

$x - y - z$ coordinate system.

$$\begin{aligned} z &= z_i + (y \cos \beta_i - x \sin \beta_i) \tan \gamma_{opt} \\ &= -x \sin \beta_i \tan \gamma_{opt} + y \cos \beta_i \tan \gamma_{opt} + z_0 + \frac{pSM(\beta_i - \beta_0)}{2\pi}. \end{aligned} \quad (3.11)$$

It is noted that the above proposed tilted plane for reconstruction is based on minimization of the geometric distance between the spiral cone beam scanning trajectory and the reconstruction plane. This can result in reduced effective cone angle and, consequently, reduced reconstruction artifacts.

3.2.2 3D reconstruction data set formation

The proposed TPFR algorithm is to apply the well known FDK algorithm to each local coordinate system to reconstruct the image for the tilted plane P_i . To achieve this objective, we now reform a 3D x ray projection data set for the tilted plane P_i from the cone beam projection data set in the global coordinate system. The reformed projection data set is composed as parallel projection data rows. The reformed projection data set, in connection with the x ray source and the tilted plane, will have the same structure as original cone beam projection data set and projection data rows are parallel to the tilted plane P_i . With the reformed projection data set, the FDK reconstruction algorithm can be applied to reconstruct image of the tilted plane. Because different tilted planes have the same optimal tilted angle γ_{opt} and projections used for the reconstruction are symmetric to the tilted plane with the same range of projection angle, the developed data set reformation procedure is the same for all the tilted planes.

In the scanning process, information of the attenuation function of the projected body is projected onto the detector array by the x rays forming a set of projection data. For each x ray projected on the detector cell at x_c, y_c, z_c of the detector

array as given in (3.5) and shown in Figure 3.4, the corresponding projection datum collected from the cell is in term of β , θ and u and is denoted as $D(\beta, \theta, u)$.

For each tilted plane P_i with its centre point being denoted by o' and at z_i in the z -direction, we can set up a local coordinate system $\xi - \eta - \zeta$. With respect to the centre projection angle $\beta_i = \frac{2(z_i - z_0)\pi}{pSM} + \beta_0$, the point $C_s(\beta_i)$ on the x ray source trajectory is at z_i position in the z -direction. The straight line from o' to $C_s(\beta_i)$ is on the tilted plane P_i which determines the ξ -axis of the $\xi - \eta - \zeta$ coordinate system. The tilted plane P_i determines the $\xi - \eta$ plane of the local coordinate system where the angle between η -axis and $x - y$ plane is γ_{opt} . Further, the ζ -axis is orthogonal to the tilted plane P_i and goes through the point o' . As a result, the transformation between the local coordinate system $\xi - \eta - \zeta$ and the global coordinate system $x - y - z$ is

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = T \begin{pmatrix} x \\ y \\ z - z_i \end{pmatrix}, \quad (3.12)$$

where the transformation matrix T is

$$\begin{aligned} T &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_{opt} & \sin \gamma_{opt} \\ 0 & -\sin \gamma_{opt} & \cos \gamma_{opt} \end{pmatrix} \begin{pmatrix} \cos \beta_i & \sin \beta_i & 0 \\ -\sin \beta_i & \cos \beta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \cos \beta_i & \sin \beta_i & 0 \\ -\sin \beta_i \cos \gamma_{opt} & \cos \beta_i \cos \gamma_{opt} & \sin \gamma_{opt} \\ \sin \beta_i \sin \gamma_{opt} & -\cos \beta_i \sin \gamma_{opt} & \cos \gamma_{opt} \end{pmatrix} \end{aligned} \quad (3.13)$$

With respect to β , θ , u and $D(\beta, \theta, u)$ in the global coordinate system, we define, in the local coordinate system, β' , θ' , u' and $D'(\beta', \theta', u')$ as the projection angle, detector angle, height of the detector slice at the iso-centre and projection data set, respectively, in the local coordinate system. We aim at establishing the correspon-

dence between the x ray source and the data set $D'(\beta', \theta', u')$ in the local coordinate system such that FDK algorithm can be applied to the image reconstruction of the tilted plane.

The reformed projection data set is composed of x ray source position and reformed projection data, hence, the x ray source position in the local coordinate system is calculated for each projection angle. At each β , the x ray source position C'_s in the local coordinate system can be directly computed with the coordinate transform function

$$\begin{pmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{pmatrix} = T \begin{pmatrix} x_s \\ y_s \\ z_s - z_i \end{pmatrix}, \quad (3.14)$$

The distance $R'_f(\beta)$ from the X ray source C'_s to the ζ -axis is given by

$$R'_f(\beta) = \sqrt{\xi_s^2 + \eta_s^2}. \quad (3.15)$$

Given β and hence its corresponding C'_s , the projection angle β' is

$$\beta' = \begin{cases} 3\pi/2 & \xi_s = 0, \eta_s < 0; \\ \pi/2 & \xi_s = 0, \eta_s > 0; \\ \pi + \arctan(\eta_s/\xi_s) & \xi_s < 0; \\ \arctan(\eta_s/\xi_s) & \text{otherwise.} \end{cases} \quad (3.16)$$

For a given $D(\beta, \theta, u)$, we need to find corresponding values of β' , θ' and u' such that

$$D'(\beta', \theta', u') = D(\beta, \theta, u)$$

is reformed. For half-scan, projection data $D(\beta, \theta, u)$ is weighted with Park's weighting function and then is used to reform new projection set. The x-ray of the source

Figure 3.7: Illustration of the in plane geometry for $s - t$ coordinate system. Point F is the projection of the point (ξ_p, η_p, ζ_p) on $\xi - \eta$ plane.

which yields $D(\beta, \theta, u)$ passes a point (x_p, y_p, z_p) with the definition of u

$$\begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} = \begin{pmatrix} -R_f \cos(\theta + \beta) + x_s \\ -R_f \sin(\theta + \beta) + y_s \\ u + z_s \end{pmatrix}.$$

The local coordinate of this point is

$$\begin{pmatrix} \xi_p \\ \eta_p \\ \zeta_p \end{pmatrix} = T \begin{pmatrix} x_p \\ y_p \\ z_p - z_i \end{pmatrix}. \quad (3.17)$$

It is noted that β' has been determined by (3.16). In order to calculate θ' and u' , we introduce a 2D coordinate system $s - t$ in the $\xi - \eta$ plane, as shown in Figure

3.7, where F is the projection of the point (ξ_p, η_p, ζ_p) on the $\xi - \eta$ plane. The $s - t$ coordinate system is a rotation of the $\xi - \eta$ plane by β' around the ζ -axis such that

$$s = \xi \cos \beta' + \eta \sin \beta', \quad t = -\xi \sin \beta' + \eta \cos \beta'.$$

Given ξ_p, η_p and β' , the corresponding point (s_p, t_p) of (ξ_p, η_p) in the $s - t$ coordinate system is given by:

$$s_p = \xi_p \cos \beta' + \eta_p \sin \beta', \quad t_p = -\xi_p \sin \beta' + \eta_p \cos \beta'. \quad (3.18)$$

With respect to the $s - t$ coordinate system, the detector angle θ' can be computed from

$$\theta' = \arctan \left(\frac{-t_p}{R'_f - s_p} \right). \quad (3.19)$$

As shown in Figure 3.8, C'_s denotes the X ray source. A plane passing through C'_s and parallel to the $\xi - \eta$ plane is Ω . The point F' denotes the point (ξ_p, η_p, ζ_p) and the point F is the projection of F' on Ω plane. G is a point on the line $C'_s F$ and $|C'_s G| = R'_f$ and G' is a point on the $C'_s F'$ with $G'G$ perpendicular to Ω plane. With the definition of u , $|GG'| = |u'|$. The relationship can be setup

$$\begin{aligned} \frac{|GG'|}{|FF'|} &= \frac{|C'_s G|}{|C'_s F|} \\ \frac{u'}{\zeta_p - \zeta_s} &= \frac{R'_f}{|C'_s F|}. \end{aligned}$$

With this relationship, u' can be calculated.

$$\begin{aligned} u' &= \frac{R'_f(\zeta_p - \zeta_s)}{|C'_s F|} \\ |C'_s F| &= \sqrt{(\xi_p - \xi_s)^2 + (\eta_p - \eta_s)^2} \end{aligned} \quad (3.20)$$

where ζ_s and η_s is determined by (3.14).

Figure 3.8: Geometry of the ζ position of the ray at iso-center.

Based on the projection data set, the image of the tilted plane can be reconstructed by applying the conventional Feldkamp algorithm which is presented in the following section.

3.2.3 Tilted plane reconstruction algorithm

Let $f(x, y, z)$ be the attenuation function to be reconstructed. The conventional Equal-angular half-scan Feldkamp algorithm can be formulated as

$$f(x, y, z) = \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} \frac{1}{L^2} \int_{-\theta_{fan}/2}^{\theta_{fan}/2} R(\beta, \theta, u) W(\beta, \theta) g(\bar{\theta} - \theta) \frac{R_f}{\sqrt{R_f^2 + u^2}} d\theta d\beta, \quad (3.21)$$

where $g(\cdot)$ denotes the ramp-filter, $\bar{\theta}$ is the detector angle for projection ray passing through the point (x, y, z) ,

$$L = \sqrt{(x - x_s)^2 + (y - y_s)^2},$$

$$R(\beta, \theta, u) = D(\beta, \theta, u) R_f \cos \theta,$$

and $W(\beta, \theta)$ is the half scan weight function written as

$$W(\beta, \theta) = \begin{cases} \sin^2\left(\frac{\pi}{4} \frac{\beta - \beta_i + \frac{\beta_m}{2}}{\theta_{fan}/2 - \theta}\right), & 0 \leq \beta - \beta_i + \frac{\beta_m}{2} \leq \theta_{fan} - 2\theta, \\ 1, & \theta_{fan} \leq \beta - \beta_i + \frac{\beta_m}{2} \leq \pi - \theta_{fan}, \\ \sin^2\left(\frac{\pi}{4} \frac{\pi + \theta_{fan} - \beta + \beta_i - \frac{\beta_m}{2}}{\theta_{fan}/2 + \theta}\right), & \pi - \theta_{fan} \leq \beta - \beta_i + \frac{\beta_m}{2} \leq \pi + \theta_{fan}, \\ 0, & \text{otherwise.} \end{cases}$$

The tilted plane reconstruction algorithm is to apply the Equal-angular half-scan FDK algorithm to the reformed data set to reconstruct image of the tilted plane. For

each point (x, y, z) on the tiled plane P_i in the global coordinate system $x-y-z$, and its corresponding $(\xi, \eta, \zeta)^T = T(x, y, z)^T$ in the local coordinate system, let $f_i(x, y, z)$ be the attenuation function of the tilted plane P_i and $f'_i(\xi, \eta, \zeta)$ be the corresponding attenuation function in the local coordinate system such that $f'_i(\xi, \eta, \zeta) = f_i(x, y, z)$. Given the reformed data set $D'(\beta', \theta', u')$, the reconstruction formula for the tilted plane in the local coordinate system can be written as

$$\begin{aligned} f(x, y, z) &= f'_i(\xi, \eta, \zeta) \\ &= \int_{\beta'_l}^{\beta'_u} \frac{1}{L'^2} \int_{-\theta_{fan}/2}^{\theta_{fan}/2} R'(\beta', \theta', u') g(\bar{\theta}' - \theta') \frac{R'_f}{\sqrt{R_f'^2 + u'^2}} d\theta' d\beta', \end{aligned} \quad (3.22)$$

where

$$L' = \sqrt{(\xi - \xi_s)^2 + (\eta - \eta_s)^2}$$

$$R'(\beta', \theta', u') = D'(\beta', \theta', u') R'_f(\beta) \cos \theta',$$

and $\bar{\theta}'$ is the detector angle for projection ray passing through the point (ξ, η, ζ) in the local coordinate system $\xi - \eta - \zeta$ and it can be calculated by substituting ξ and η into (3.18) and (3.19). β'_u and β'_l is calculated by bring $\beta_i + \beta_m/2$ and $\beta_i - \beta_m/2$ into (3.16).

It follows that the tilted plane image reconstruction in the local coordinate can follow the following procedure.

1. Determine the optimal tilted angle by (3.10) with $\beta_m = \pi + \theta_{fan}$ for half-scan;
2. Forming the projection data set for the tilted plane P_i , for each projection angle $\beta \in [\beta_i - \beta_m/2, \beta_i + \beta_m/2]$ by
 - calculating the x ray source position (ξ_s, η_s, ζ_s) in local system $\xi - \eta - \zeta$ with (3.14);
 - calculating $R'_f(\beta)$ with (3.15);

- calculating β' with (3.16);
 - determining the projection data set $D'(\beta', \theta', u')$ from the corresponding projection data set $D(\beta, \theta, u)$ with (3.19) and (3.20), for half-scan $D(\beta, \theta, u)$ is weighted with Parker's weighting function [175] before reformation;
3. The optimal tilted plane P_i is determined by (3.11), and its image is constructed by using (3.22).

The reconstructed tilted plane images are further used to form images for planes perpendicular to the z -axis and this requires interpolation in the z -direction. Recall that the intersection of the tilted plane P_i and the z -axis is at $z_i = z_0 + i\Delta z$ with $\Delta z > 0$ being a constant. If the interpolation procedure does not reduce the resolution in the z -direction, the increment of the centre of tilted planes in the z -direction Δz should fulfill the following formula [137]:

$$\Delta z + 2r_{obj} \tan \gamma \sin \frac{\Delta \beta}{2} + \frac{r_{obj}}{R_f} \Delta z_{mean} \leq S, \quad (3.23)$$

where r_{obj} is the radius of the object in $x - y$ plane and

$$\Delta z_{mean} = \frac{1}{\beta_m} \int_{-\beta_m/2}^{\beta_m/2} |(R_f \tan \gamma_{opt} \bar{\beta} - \frac{pSM\bar{\beta}}{2\pi})| d\bar{\beta},$$

$$\Delta \beta = \frac{2\pi \Delta z}{pSM}.$$

Given the reconstructed image of the tilted planes P_i for $i = 1, 2, 3, \dots, n$ and a point (x, y, z) on the plane parallel to the $x - y$ plane where the image value is to be obtained by interpolation, the z -positions on the tilted planes P_i , $i = 1, 2, 3, \dots, n$, at $(x, y)^T$ and denoted by $z_{p_i}(x, y)$ can be determined by (3.11). Thus there exist two points $(x, y, z_{P_j})^T$ and $(x, y, z_{P_k})^T$ on two tilted planes P_j and P_k , respectively, at the

upper and lower sides of $(x, y, z)^T$, respectively, which are on two tilted plans and are closest to $(x, y, z)^T$. Using the obtained attenuation functions $f_j(x, y, z_{P_j})$ and $f_k(x, y, z_{P_k})$ for the tilted plans P_j and P_k , respectively, the interpolated attenuation function can be obtained using the following interpolation formula

$$f(x, y, z) = \frac{f_j(x, y, z_{P_j})(z_{P_k} - z)}{z_{P_k} - z_{P_j}} + \frac{f_k(x, y, z_{P_k})(z - z_{P_j})}{z_{P_k} - z_{P_j}}. \quad (3.24)$$

The image reconstruction of the proposed TPFR algorithm is thus complete.

3.3 Extended tilted plane FDK type reconstruction algorithm

The TPFR algorithm can improve reconstruction image quality by reconstructing images on optimal tilted planes. Because a projection data set is reformed for each tilted plane, filtered projection data can not be used to reconstruct other planes and it is not very computationally efficient.

It is known that the reconstruction image quality can be improved by filtering the projection data along the tangential direction of x ray source trajectory for FDK type reconstruction algorithms. In this section, we will present a reconstruction algorithm to improve the TPFR algorithm. The proposed reconstruction scheme is set up in the cone parallel geometry to reconstruct images on optimal tilted planes [2, 137]. In this scheme, cone beam projections are rearranged in the cone parallel geometry, such that filtering along the row direction in the cone parallel geometry is equivalent to filtering the cone beam projection data along the tangential direction of the x ray source trajectory [130]. With the cone parallel geometry, projection data are filtered only once and used for reconstruction of multiple image slices to provide

high computational efficiency.

As a result, the proposed scheme possesses advantages of the recent techniques of tangential filtering and optimal tilted planes. And it can provide image reconstruction with significantly improved accuracy and computational efficiency. Because the proposed scheme is based on FDK reconstruction of images on optimal tilted planes with application of the cone parallel rebinning techniques, it will be called ETPFR (extended tilted plane FDK reconstruction algorithm). The performance of ETPFR algorithm is evaluated in Sec. 3.5.

3.3.1 Projection data set rebinning

In this section, we reform a cone parallel projection data set denoted by $D_p(\alpha, l, u)$ from the cone beam projection data set denoted $D(\beta, \theta, u)$, where α is the projection angle of the projection ray, l is the distance between the projection ray and iso center and u is the height of the projection ray at iso center. As shown in Figure 3.9, the geometric relationship between the cone parallel data set and cone beam projection data set satisfies

$$\alpha = \beta + \theta \quad (3.25)$$

$$l = -R_f \sin \theta \quad (3.26)$$

For a given $D(\beta, \theta, u)$, we need to find corresponding values of α and l such that

$$D_p(\alpha, l, u) = D(\beta, \theta, u)$$

is reformed. It is noted that the 1D filtering along the row direction in the cone parallel projection becomes tangential automatically due to the row-wise fan-to-parallel rebinning. And reconstruction algorithms using the cone parallel data set

Figure 3.9: In plane geometry of the reformation of the cone parallel projection data set.

perform natural tangential filtering operations [130].

3.3.2 Reconstruction formula in cone parallel geometry

The cone parallel projection data set is formed in former section and the reconstruction formula in cone parallel geometry is expressed as

$$f(x, y, z) = \int_{\alpha_{min}}^{\alpha_{min}+\pi} \frac{R_f}{\sqrt{R_f^2 + u^2}} \tilde{D}_p(\alpha, l, u) d\alpha \quad (3.27)$$

$$\tilde{D}_p(\alpha, l, u) = D_p(\alpha, l, u) \otimes g(l), \quad (3.28)$$

where $\otimes$ denotes the convolution operation.

For an optimal tilted plane P_i in the form (3.11) with the center projection angle β_i , the initial projection angle for the reconstruction is $\alpha_{min} = \beta_i - \pi/2$ and a point

(x, y, z_{P_i}) on the P_i can be reconstructed by

$$f(x, y, z_{P_i}) = \int_{\beta_i - \pi/2}^{\beta_i + \pi/2} \frac{R_f}{\sqrt{R_f^2 + u^2}} \tilde{D}_p(\alpha, l, u) d\alpha. \quad (3.29)$$

The 1-dimensional filtering given in (3.28) indicates that the projection data are filtered without weighting. The projection data are filtered only once and used in (3.29) for different tilted planes. It is noted that half scan reconstruction algorithms with cone beam projection data set weight projection data before filtering, such as Park's weighting (Park 1982), and the weighting function depends on the helix position. Thus the filtered projection data for one slice cannot be used for reconstruction of other slices. In the above proposed processing, the filtered data using (3.29) can be used for reconstruction of different planes so to provide more efficient computing than that of half scan reconstruction algorithms with cone beam projection data set.

Given the reconstructed image of the tilted planes P_i for $i = 1, 2, 3, \dots, n$ and a point (x, y, z) on the plane parallel to the $x-y$ plane where the image value is to be obtained by interpolation, the z -positions on the tilted planes P_i , $i = 1, 2, 3, \dots, n$, at $(x, y)^T$ and denoted by $z_{P_i}(x, y)$ can be determined by (3.11). Thus there exist two points $(x, y, z_{P_j})^T$ and $(x, y, z_{P_k})^T$ on two tilted planes P_j and P_k , respectively, at the upper and lower sides of $(x, y, z)^T$, respectively, which are on two tilted plans and are closest to $(x, y, z)^T$. Using the obtained attenuation functions $f_j(x, y, z_{P_j})$ and $f_k(x, y, z_{P_k})$ for the tilted plans P_j and P_k , respectively, the interpolated attenuation function can be obtained using the following interpolation formula

$$f(x, y, z) = \frac{f_j(x, y, z_{P_j}(x, y))(z_{P_k} - z)}{z_{P_k} - z_{P_j}} + \frac{f_k(x, y, z_{P_k}(x, y))(z - z_{P_j})}{z_{P_k} - z_{P_j}}. \quad (3.30)$$

The image reconstruction of the proposed ETPFR algorithm can be completed with

the following procedure.

1. Reform cone parallel projection data set $D_p(\beta, l, u)$.
2. Determine the optimal tilted angle by (3.10) with $\beta_m = \pi + \theta_{fan}$;
3. The optimal tilted plane P_i is determined by (3.11), and its image is constructed by using (3.29).
4. After a sequence images are reconstructed on tilted planes, horizontal images are obtained by interpolation with (3.30).

3.4 Maximum pitch values of the TPFR and ETPFR algorithms

Each spiral CT reconstruction algorithm is constrained by its maximum possible pitch value. In this section we analyze the maximum possible pitch values of TPFR and ETPFR algorithms and show that they can provide greater maximum pitch value than other spiral CT algorithms.

Let r_{obj} be the radius of the object being reconstructed. The tilted plane P_i of the object intersects the cylinder with radius r_{obj} yielding an ellipse:

$$e_{obj}(\alpha) = \begin{pmatrix} r_{obj} \cos \alpha \\ r_{obj} \sin \alpha \\ z_i + r_{obj} \tan \gamma_{opt} \sin(\alpha - \beta_i) \end{pmatrix}, \quad \alpha \in [0, 2\pi]. \quad (3.31)$$

In the spiral CT scanning, each point (x, y, z) on the tilted plane P_i determines a projection x ray from the source. And its corresponding projection point on the detector array surface is characterized by β , θ and u as described in the proceeding

sections. The height of detector slice at the iso-centre u can be computed as

$$u = \frac{(z - z_s(\beta))R_f}{R}, \quad (3.32)$$

where

$$R = \sqrt{(x - R_f \cos \beta)^2 + (y - R_f \sin \beta)^2}$$

is the distance from the x ray source to the (x, y) point in the $x - y$ plane, $\beta \in [\beta_i - \beta_m/2, \beta_i + \beta_m/2]$ and $z_s(\beta)$ is the position of x ray source in the z -direction. Because the projection ray should not be projected out of the detector array surface, u must be bounded by $(M - 1)S$ and satisfy

$$|u| \leq \frac{(M - 1)S}{2}. \quad (3.33)$$

The ellipse $e_{obj}(\alpha)$ defines the boundary of P_i on the object. Thus, if each projection ray passing through a point on the ellipse is projected onto the detector array surface, then every projection ray passing through any point in the tilted plane P_i is in the range of the detector array surface. On the ellipse, we can use (3.31) and (3.32) to have

$$\begin{aligned} u &= \frac{[z_i + r_{obj} \tan \gamma_{opt} \sin(\alpha - \beta_i) - z_s(\beta)]R_f}{R}, \\ z_s(\beta) &= z_0 + \frac{pSM(\beta - \beta_0)}{2\pi}, \\ z_i &= z_0 + \frac{pSM(\beta_i - \beta_0)}{2\pi}, \\ R &= \sqrt{(r_{obj} \cos \alpha - R_f \cos \beta)^2 + (r_{obj} \sin \alpha - R_f \sin \beta)^2}. \end{aligned} \quad (3.34)$$

Substituting the optimal tilted angle γ_{opt} in (3.10) into (3.34) yields

$$u = \left[\frac{pSM(\beta_i - \beta)}{2\pi} + \frac{r_{obj}(\sin \beta_m - \beta_m \cos \beta_m)pSM \sin(\alpha - \beta_i)}{\pi R_f(\beta_m - \sin \beta_m \cos \beta_m)} \right] \frac{R_f}{R}$$

The above equation can be rewritten as

$$u = pSMV(\alpha, \beta), \quad (3.35)$$

where

$$\begin{aligned} V(\alpha, \beta) &= \frac{|\beta_i - \beta|/2 + \sin(\alpha - \beta_i)K|R_f}{\pi R(\alpha, \beta)}, \\ K &= \frac{r_{obj}(\sin \beta_m - \beta_m \cos \beta_m)}{R_f(\beta_m - \sin \beta_m \cos \beta_m)}, \\ \alpha &\in [0, 2\pi], \\ \beta &\in [\beta_i - \beta_m/2, \beta_i + \beta_m/2]. \end{aligned}$$

It follows from (3.35) that, for $|u| \leq \frac{(M-1)}{2}S$ being satisfied and $\bar{\beta} = \beta - \beta_i$, the maximum pitch value p_{max} is determined by

$$p_{max} = \frac{(M-1)}{2MV_{max}(\alpha, \bar{\beta})}, \quad (3.36)$$

where $V_{max}(\alpha, \bar{\beta})$ is the maximum value of $V(\alpha, \bar{\beta})$ for $\alpha \in [0, 2\pi]$ and $\bar{\beta} \in [-\beta_m/2, \beta_m/2]$.

For FDK type algorithms with the horizontal reconstruction plane such that $\gamma = 0$, (3.35) leads to

$$\begin{aligned} u &= \frac{pSM(\beta - \beta_i)}{2\pi R}, \\ R &= \sqrt{(r_{obj} \cos \alpha - R_f \cos \beta)^2 + (r_{obj} \sin \alpha - R_f \sin \beta)^2}, \\ \alpha &\in [0, 2\pi], \quad \beta \in [\beta_i - \beta_m, \beta_i + \beta_m]. \end{aligned} \quad (3.37)$$

Table 3.1: The maximum pitch values for TPFR and ETPFR algorithms compared with half-scan FDK algorithm and full scan FDK algorithm

M	TPFR and ETPFR	half-scan FDK	FDK
4	1.03	0.76	0.24
16	1.25	0.95	0.30
32	1.29	0.98	0.31
64	1.33	1.01	0.32

It follows that the maximum pitch value offered by Feldkamp algorithm is

$$p_{maxFDK} = \frac{2(M - 1)(R_f - r_{obj})\pi}{MR_f\beta_m}, \tag{3.38}$$

where $\beta_m = \pi + \theta_{fan}$ for half-scan Feldkamp algorithm and $\beta_m = 2\pi$ for full-scan FDK algorithm.

Table 3.1 provides a comparison of maximum pitch values between TPFR/ATFPR algorithms, half-scan FDK and full-scan FDK algorithms with $R_f = 570mm$, $R_d = 460mm$, fan angle $\theta_{fan} = 52^\circ$, $S = 1mm$, $r_{obj} = 200mm$, $\beta_m = (\pi + \theta_{fan})/2$. It is shown that TPFR and ETPFR algorithms can provide greater pitch value than half-scan FDK algorithm and full scan FDK algorithm. Hence TPFR and ETPFR algorithms have the potential to achieve higher volume coverage speed than that of other FDK type algorithms with horizontal reconstruction planes.

3.5 Performance evaluation of TPFR and ETPFR algorithms

In this section, the performance of TPFR and ETPFR algorithms is evaluated with center and off center SSP (Slice Sensitive profile), noise properties, X-Y resolution

properties and image artifacts study. The evaluation is performed using computer simulations. Parameters of the simulation are set to be: $R_f=570\text{mm}$, $\theta_{fan} = 49.8^\circ$, $S = 1\text{mm}$, $p = 1.0$. The projection number per rotation is 1024 and number of projection rays in the fan is 800.

3.5.1 Computation workload

The proposed ETPFR is an FDK type reconstruction algorithm using filtered back-projection as given in (3.28) and (3.29). The reconstruction procedure is composed of the filtering and backprojection operations, in which the projection data are filtered by (3.28) followed by the backprojection for image reconstruction. The proposed ETPFR uses the cone parallel rebinning technique to form a projection data set is reformed in cone parallel geometry. In this geometry, projection data are filtered only once and can be used for reconstruction of multiple images. In contrast, the existing TPFR algorithm reforms a projection data and performs the filtering operation for each optimal tilted plane, in which the filtered projection data set for reconstruction of one image is not applicable to another.

Let N_f be the number of projections per gantry rotation and N be the number of images to be reconstructed. Each filtering at one projection angle position is considered to be one filtering operation. The amount of filtering operations for construction of one tilted plane image of the proposed ETPFR can be computed by

$$Num_{ETPFR} = N_f \left(\frac{N \Delta z}{pSM} + \frac{\pi + \theta_{fan}}{2\pi} \right). \quad (3.39)$$

That of the existing TPFR algorithm is:

$$Num_{TPFR} = N_f N \frac{\pi + \theta_{fan}}{2\pi}. \quad (3.40)$$

Table 3.2: Comparison of amount of filtering operations for 10 images between FDK, TPFR and ETPFR

	number of projections	filtering operations
FDK	704	6534
TPFR	704	6534
ETPFR	704	704

Table 3.2 provides a comparison of computation load in terms of the amount of filtering operations between half scan FDK, TPFR and ETPFR algorithms, where parameters of the scanning protocol are set to be: $M = 96$, $R_f=570\text{mm}$, $\theta_{fan} = 49.8^\circ$, $S = 1\text{mm}$, $p = 1.0$ and $N_f = 1024$. The amount of filtering operations are for 10 reconstructed images with $\Delta z = 0.5\text{mm}$. It is clearly shown that ETPFR is significantly efficient in computation in terms of filtering operations than TPFR.

3.5.2 The Slice sensitive profile (SSP) study

The Slice Sensitive Profile (SSP) is measured as an indication of the spatial resolution in longitudinal direction (z resolution) for medical imaging systems. Similar to the in plane resolution (x-y resolution), the SSP describes the system response to the Dirac delta function [9]. The Dirac delta function is approximated by thin disc whose thickness is much smaller than the slice thickness. The z resolution also varies for different scan fields. In order to measure the SSPs at different scan fields, a computer simulated five disc phantom is employed in this study.

The disk is a cylinder with its longitude axis parallel to z-axis. The radius of the cylinder is 10mm and the length is 0.1mm. The centers of the five disks are (0,0,0), (100,0,0), (-100,0,0), (0,100,0) and (0,-100,0) (mm). With this five-disk model, both center SSPs and off-center SSPs are evaluated. SSPs of FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm are shown in Figure 3.10.1- Figure 3.10.4 respectively. The mean value and deviation of full-width half maxima (FWHM) of different algorithms with $p=1.0$, $M=64$ and $S=0.625\text{mm}$ are

Table 3.3: The mean value and deviation of full-width half maxima (FWHM) for the TPFR and ETFR algorithm compared with FDK algorithm and tangential filtering algorithm

FWHM	FDK	Tangential filtering	TPFR	ETPFR
Mean	1.08mm	0.832mm	0.821mm	0.816
Deviation	0.18mm	0.037mm	0.027mm	0.007

calculated in Table 3.3.

As shown in Figure 3.10.1, SSPs for FDK algorithm are displaced and broadened and the effect is reduced by the TPFR algorithm and Tangential filtering algorithm as shown in Figure 3.10.3 and 3.10.2. The TPFR algorithm and Tangential filtering algorithm can improve spatial resolution of FDK algorithm in z direction. SSPs of the ETPFR algorithm have less displacement and broaden effect than those of the TPFR algorithm and Tangential filtering algorithm as shown in Figure 3.10.4. ETPFR has better spatial resolution in the z direction than that of FDK algorithm, Tangential filtering algorithm and TPFR algorithm. As shown in Table 3.3, the FWHM deviation of ETPFR algorithm is small, so it has more stable spatial resolution in the z direction than that of the other algorithms.

3.5.3 X-Y resolution study

We evaluate the x-y resolution with the resolution phantom and small resolution phantom. The resolution phantom is composed of 36 objects and every 3 objects are considered as a group and objects in a group which have the same size are placed parallel to each other. In Table 3.4, the size of objects, the centre of each group and the distance between each object d in the group are listed. Parameters of the small resolution phantom are listed in Table 3.5.

The resolution phantom is used to evaluated the resolution performance of reconstruction algorithms with large field of interest (FOI). The radius of the resolution

3.10.1

3.10.2

3.10.3

3.10.4

Figure 3.10: Center and off-center SSPs for FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with $p = 1.0$, $M=64$ and $S=0.625\text{mm}$.

3.11.1

3.11.2

3.11.3

3.11.4

Figure 3.11: Images for resolution phantom with $z = 0$, $M = 64$ and $p = 1.0$. (300-900HU), reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively.

3.12.1

3.12.2

3.12.3

3.12.4

Figure 3.12: Images for resolution phantom with $z = 0$, $M = 64$ and $p = 1.2$. (300-900HU), reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively.

3.13.1

3.13.2

3.13.3

3.13.4

Figure 3.13: Images for small resolution phantom with $z = 0$, $M = 64$ and $p = 1.0$. (300-900HU), reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively.

3.14.1

3.14.2

3.14.3

3.14.4

Figure 3.14: Images for small resolution phantom with $z = 0$, $M = 64$ and $p = 1.2$. (300-900HU), reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively.

phantom is 200mm and the reconstruction image size is 600×600 . The size of each of each pixel in the reconstruction image is $0.66\text{mm} \times 0.66\text{mm}$. Images at $z = 0\text{mm}$ are reconstructed at $p = 1.0$ and $p = 1.2$ to evaluate resolution of reconstruction algorithms. As shown in Figure 3.11, Figure 3.11.1- Figure 3.11.4 are images for resolution phantom with $p = 1.2$ reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively. As shown in Figure 3.12, Figure 3.12.1- Figure 3.12.4 are images for resolution phantom with $p = 1.2$ reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively.

The small resolution phantom is used to evaluated the resolution performance of reconstruction algorithms with small field of interest (FOI). The radius of the resolution phantom is 100mm and the reconstruction image size is 600×600 . The size of each of each pixel in the reconstruction image is $0.33\text{mm} \times 0.33\text{mm}$. Images at $z = 0\text{mm}$ are reconstructed at $p = 1.0$ and $p = 1.2$ to evaluate resolution of reconstruction algorithms. As shown in Figure 3.13, Figure 3.13.1- Figure 3.13.4 are images for resolution phantom with $p = 1.0$ reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively. As shown in Figure 3.14, Figure 3.14.1- Figure 3.14.4 are images for resolution phantom with $p = 1.2$ reconstructed by FDK algorithm, Tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively.

TPFR algorithm and ETPFR algorithm employ more interpolation operations than FDK algorithm and tangential filtering algorithm. The interpolation operation degrades the spatial resolution of reconstruction algorithms. However, TPFR and ETPFR algorithm provide more accurate reconstruction image quality than that of FDK algorithm and tangential filtering algorithm. No obvious degrade in spatial resolution is shown in the X-Y resolution study.

Table 3.4: Parameters for resolution phantom (mm)

No.	$l \times w \times h$	centre	d	No.	$l \times w \times h$	centre	d
1	$48 \times 24 \times 100$	(0,164,0)	36	7	$24 \times 48 \times 100$	(-164,0,0)	36
2	$48 \times 16 \times 100$	(0,104,0)	24	8	$16 \times 48 \times 100$	(-104,0,0)	24
3	$48 \times 8 \times 100$	(0,44,0)	12	9	$8 \times 48 \times 100$	(-44,0,0)	12
4	$48 \times 4 \times 100$	(0,-44,0)	6	10	$4 \times 48 \times 100$	(44,0,0)	6
5	$48 \times 2 \times 100$	(0,-104,0)	3	11	$2 \times 48 \times 100$	(104,0,0)	3
6	$48 \times 1.2 \times 100$	(0,-164,0)	1.8	12	$1.2 \times 48 \times 100$	(164,0,0)	1.8

Table 3.5: Parameters for small resolution phantom

No.	$l \times w \times h$	centre	d	No.	$l \times w \times h$	centre	d
1	$24 \times 12 \times 100$	(0,82,0)	18	7	$12 \times 24 \times 100$	(-82,0,0)	18
2	$24 \times 8 \times 100$	(0,52,0)	12	8	$8 \times 24 \times 100$	(-52,0,0)	12
3	$24 \times 4 \times 100$	(0,22,0)	6	9	$4 \times 24 \times 100$	(-22,0,0)	6
4	$24 \times 2 \times 100$	(0,-22,0)	3	10	$2 \times 24 \times 100$	(22,0,0)	3
5	$24 \times 1 \times 100$	(0,-52,0)	2	11	$1 \times 24 \times 100$	(52,0,0)	2
6	$24 \times 0.6 \times 100$	(0,-82,0)	0.9	12	$0.6 \times 24 \times 100$	(82,0,0)	0.9

3.5.4 Noise study

Poisson noise is added to the projection data of a water cylinder phantom with a radius of 200mm. The noise levels of the reconstructed images at $z = 0mm$ by FDK, Tangential filtering, TPFR, ETPFR and Zou-Pan algorithms are evaluated and the reconstruction parameters are $M = 64$, $S = 1mm$, and $p = 1.0$. Five 10×10 pixels regions are used to evaluate noise and the centres of the five regions are (0,0,0), (150,150,0), (-150,150,0), (150,-150,0) and (-150,150,0). It is known that for I_0 being the incident x ray photon number, the received photon number I in terms of the projection data $D(\beta, \theta, u)$ is:

$$I = I_0 e^{-D(\beta, \theta, u)}.$$

We add poisson noise to the received photon number such that

$$I_n = \text{Poisson}(I) = \text{Poisson}(I_0 e^{-D(\beta, \theta, u)}) \quad (3.41)$$

Table 3.6: Mean values of the standard derivations of regions reconstructed by FDK, Tangential filtering, TPFR and ETPFR, Zou-Pan (HU)

Methods	ISO	Left Up	Right up	Left down	Right down
FDK	11.67	11.61	14.22	11.06	10.98
Tangential filtering	10.54	10.55	11.26	10.55	10.55
TPFR	9.51	10.23	11.34	9.57	9.61
ETPFR	9.78	9.98	9.72	10.03	9.88
Zuo-Pan	9.54	9.67	9.37	9.58	9.74

where $Poisson(.)$ is the function adding poisson noise to I and I is the mean value of the Poisson distribution. We can derive the projection data with noise with:

$$D_n(\beta, \theta, u) = \ln \frac{I_0}{I_n}. \tag{3.42}$$

The water phantom are reconstructed with noised projection data for 30 times. The mean values of standard derivations of in five regions are calculated and listed in the Table 3.6. As shown in this table, the TPFR and ETPFR algorithms have smaller standard derivations than FDK and Tangential filtering algorithms. Compared with the exact algorithm- Zou-Pan algorithm, the TPFR and ETPFR algorithms have larger standard derivations. However, the standard derivation of the ETPFR algorithm is close to the Zou-Pan algorithm.

3.5.5 Reconstruction accuracy

The reconstruction accuracy of the TPFR and ETPFR algorithms is evaluated with digital phantoms. And reconstruction results are compared with exact and approximate algorithms.

Turbell clock phantom is designed by Turbell and used for comparison of image artifacts of different reconstruction algorithms [165]. Simulation results for Turbell clock phantom at different pitch value are shown in Figure 3.15 and Figure 3.16. In Figure 3.15, Figure 3.15.2- Figure 3.15.6 are reconstructed by Zou-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively, with $M = 96$ and $p = 1.0$. In Figure 3.16, Figure 3.16.2- Figure 3.16.6 are reconstructed by Zou-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively, with $M = 96$ and $p = 1.2$. It is clearly demonstrated that the proposed ETPFR has much less artifacts in the images than that of the other algorithms at different pitch values.

The helical body phantom (HBP) [130] is employed to evaluate the performance of the ETPFR algorithm. HBP phantom is configured to simulate the human thorax and it can highlight artifacts caused by cone angle and projection data inconsistent in spiral scanning [181]. As shown in Figure 3.17, images Figure 3.17.2- Figure 3.17.6 of HBP Phantom are reconstructed by Zou-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively, with $p = 1.0$, $M=96$ and $S=1\text{mm}$. As shown in Figure 3.18, images Figure 3.18.2- Figure 3.18.6 of HBP Phantom are reconstructed by Zou-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm, respectively, with $p = 1.2$, $M=96$ and $S=1\text{mm}$. The image reconstructed by ETPFR algorithm has less streak and shadow artifacts than images reconstructed by other approximate reconstruction algorithms. Reconstruction images show that the ETPFR algorithm performs comparably with the Zou Pan algorithm, one of latest exact reconstruction

Figure 3.15: Images of the Clock phantom. 3.15.1 is original image and 3.15.2-3.15.6 are reconstructed by Zuo-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with $p = 1.0$, $M=96$ and $S=1\text{mm}$ (350-450HU).

3.16.1

3.16.2

3.16.3

3.16.4

3.16.5

3.16.6

Figure 3.16: Images of the Clock phantom. 3.16.1 is original image and 3.16.2-3.16.6 are reconstructed by Zuo-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with $p = 1.0$, $M=96$ and $S=1\text{mm}(350\text{-}450\text{HU})$.

Figure 3.17: Images of the HBP phantom. 3.17.1 is original image and 3.17.2-3.17.6 are reconstructed by Zuo-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with $p = 1.0$, $M=96$ and $S=1\text{mm}(350\text{-}450\text{HU})$.

Figure 3.18: Images of the HBP phantom. 3.18.1 is original image and 3.18.2-3.18.6 are reconstructed by Zuo-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with $p = 1.0$, $M=96$ and $S=1\text{mm}(350\text{-}450\text{HU})$.

Figure 3.19: Images of the HBP phantom with noise. 3.19.1 is original image and 3.19.2 -3.19.6 are reconstructed by Zuo-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with $p = 1.2$, $M=96$ and $S=1\text{mm}(350\text{-}450\text{HU})$.

algorithms, from the perspective of reconstruction accuracy.

The reconstruction accuracy of the TPFR and ETPFR algorithms is evaluated with projection data with noise. We add poisson noise to the received photon number such that

$$I_n = \text{Poisson}(I) = \text{Poisson}(I_0 e^{-D(\beta, \theta, u)})$$

where $\text{Poisson}(\cdot)$ is the function adding poisson noise to I and I is the mean value of the Poisson distribution. We can derive the projection data with noise with:

$$D_n(\beta, \theta, u) = \ln \frac{I_0}{I_n}.$$

As shown in Figure 3.19, images Figure 3.19.2- Figure 3.19.6 of HBP Phantom are reconstructed by Zou-Pan algorithm, FDK algorithm, tangential filtering algorithm, TPFR algorithm and ETPFR algorithm with projection data with noise. Simulation results show that TPFR and ETPFR algorithms have smaller noise level and artifacts than other approximate reconstruction algorithms. The ETPFR algorithm performs comparably with the Zou Pan algorithm from the perspective of noise level and reconstruction artifacts.

3.6 Summary

The reconstruction plane for TPFR algorithm is tilted to the optimal tilted angle to minimize the deviation of the construction plane from the x ray source trajectory. Reconstructing images on optimal reconstruction planes reduce reconstruction cone angle effectively, thus reduce reconstruction artifacts for the large cone angle case. Because the distance between the reconstruction plane and x ray source trajectory for the optimal tilted plane is reduced, the maximum pitch value for the TPFR algorithm is higher than that of other FDK type reconstruction algorithms with

horizontal reconstruction plane. The TPFR algorithm has the potential to work at higher volume coverage speed than other FDK type algorithms and may be suitable for clinical applications which more concern about scanning speed.

We further improve the TPFR algorithm to the ETPFR algorithm by combining the TPFR algorithm with tangential filtering technique. The ETPFR algorithm reconstructs images on optimal tilted planes identical to the TPFR algorithm with the tangential filtering method. The tangential filtering method used in the ETPFR is implemented with the cone beam to cone parallel rebinning. A projection data set in the cone parallel geometry is reformed and projection data in cone parallel geometry are filtered only once before reconstruction. The ETPFR algorithm which is more computationally efficient than the TPFR algorithm improves the reconstruction image quality of the TPFR algorithm.

The SSPs study shows that the ETPFR has better spatial resolution in the z direction than that of FDK algorithm, Tangential filtering algorithm and TPFR algorithm. Although the TPFR and ETPFR algorithm employ more interpolation operations than FDK algorithm and tangential filtering algorithm, these algorithms doesn't show obvious degrade in X-Y resolution. The TPFR and ETPFR algorithm have better noise property than FDK algorithm and tangential filtering algorithm. The ETPFR algorithm provides more accurate reconstruction image quality than other reconstruction algorithms in the reconstruction accuracy study. Simulation results also show that the ETPFR algorithm performs comparably with the Zou Pan algorithm, one of latest exact reconstruction algorithms, from the perspective of noise property and reconstruction accuracy.

Chapter 4

Approximate reconstruction algorithms for gantry tilted MSCT

We have introduced two approximate reconstruction algorithms in former chapters. These algorithms are developed for clinical applications with the normal gantry position. In some clinical applications, it is required that the gantry be tilted. For example, in order to avoid exposure of eyes to x rays, the gantry is tilted during the head scanning procedure. Another example is that in some head scanning clinical applications, teeth are not scanned to reduce artifacts and increase contrast. This is because that the x ray attenuation coefficient of teeth is high and it will result beam harden artifacts in reconstruction images. To meet such special requirements, a number of algorithms have been developed for the gantry tilted helical MSCT image reconstruction [4, 5, 182, 183].

Among these algorithms, Kachelriess and Fuchs [4] developed a gantry tilted reconstruction algorithm based on the earlier developed 2D algorithm ASSR [137]. Hein *et al.* [5] developed a gantry tilted reconstruction algorithm based on the 3D FDK algorithm [1, 143]. In the gantry tilted FDK algorithm in Hein *et al.* [5], the reconstruction plane is perpendicular to the rotating axis of the CT scanning. Thus the

scanning cone angle increases with the pitch value and slice number which can lead to unavoidable artifacts. Recently, Noo *et al.* [183] developed a general framework which can be applied to the gantry tilted CT for exact [96, 112] and approximate image reconstruction.

FDK type algorithms for approximate reconstruction using projection data filtered in the horizontal direction suffer from considerable artifacts due to large scanning cone angle in both normal and gantry tilted MSCT imaging. Recently, some improvements have been made for FDK type algorithms. The tilted plane technique is combined with FDK algorithm to reduce cone angle and further reduce artifacts for the large cone angle case [2]. Several methods reduce artifacts by filtering projection data along the tangential direction of the helix [142, 155, 157]. Such a technique was earlier proposed by Yan and Leahy [157] and further improved by Sourbelle and Kalender [142] for short scan FDK type algorithms. The improved FDK type algorithm can achieve better image quality than that of the conventional FDK algorithm.

In this chapter, we will present two approximate reconstruction algorithms for the gantry tilted CT. One is the geometrically optimized reconstruction algorithm for gantry tilted CT which is an extension of TPFR algorithm [2] and reconstructs images on geometrically optimized planes to reduce artifacts. The other is the tangential filtering algorithm for gantry tilted CT which reduces reconstruction artifacts by filtering projection data along the x ray source trajectory tangential direction.

In this chapter, the geometry of the gantry tilted CT is introduced first. The proposed reconstruction algorithms are introduced based on this geometry and they are evaluated by SSP study, X-Y resolution study, noise study and image reconstruction accuracy study with various phantoms.

Figure 4.1: Geometry for gantry tilted multislice CT scanner.

4.1 Geometry for gantry tilted helical MSCT

The helical MSCT scanning set up consists of an x ray source and a detector array forming a source-detector framework. The geometry of the source-detector framework in a global cartesian coordinate system $x - y - z$ is shown in Figure 4.1, where C_s denotes the x ray source, the detector array is a rectangular surface with a geometric centre C_d . The source-detector framework defines a rotating cartesian coordinate system $s - l - u$ with the x ray source C_s as the origin, the straight line $C_s - C_d$ being the s axis, the l axis being parallel to the horizontal lines and the u axis being parallel to the vertical lines of the rectangular detector surface.

The $s - l$ plane containing C_s and C_d is called the centre plane and its geometry is

Figure 4.2: In plane geometry for gantry tilted multislice CT scanner.

shown in Figure 4.2, where β is the projection angle. The distances from the rotating centre point o' to the detector and the x ray source are R_d and R_f , respectively. In the gantry tilted scanning process, the source-detector framework rotates around the axis u' which is parallel to the u axis and intersects the z axis at o' as shown in Figure 4.1. The gantry tilted angle is represented by μ between the z and u' axes. The projection angle β is defined in the $s-l$ plane as shown in Figure 4.2.

For gantry tilted CT, the x ray source trajectory can be considered as a combination of two motions: the x ray source rotates on the tilted circular trajectory with a radius R_f and the center of the circle moves straightforward along the table feeding direction. As a result, the x ray source trajectory is the sum of the circular rotation trajectory on the centre plane and the linear translation in the z direction in the

following.

$$\begin{aligned}
 C_s(\beta) &= \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \mu & -\sin \mu \\ 0 & \sin \mu & \cos \mu \end{pmatrix} \begin{pmatrix} R_f \cos \beta \\ R_f \sin \beta \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_0 + \frac{pSM}{2\pi}(\beta - \beta_0) \end{pmatrix} \\
 &= \begin{pmatrix} R_f \cos \beta \\ R_f \cos \mu \sin \beta \\ z_0 + \frac{pSM}{2\pi}(\beta - \beta_0) + R_f \sin \mu \sin \beta \end{pmatrix}, \tag{4.1}
 \end{aligned}$$

where β_0 is the initial projection angle, z_0 is the initial z position of the centre of the gantry, p is the pitch value of the helical cone beam scanning, S is the slice thickness and M is the number of detector slice.

Because the projection ray from the x ray source hits on the end point on the detector, it can be denoted by the x ray source position and a point on the projection ray. The x ray source position is determined by the projection angle and the point on the projection ray is determined by its coordinates in the rotating coordinate system $s-l-u$. In Figure 4.3, a projection ray passes a point $Q = (R_f, l, u)^T$ and its projection onto the centre plane is Q' . The distance between Q and Q' is the height of the projection ray at iso-centre. For each projection angle β , the projection ray can be denoted by coordinates of Q in $l-u$. As a result, the projection data set collected from the detector array is dependent on β , l and u , and is denoted by $D(\beta, l, u)$. The projection ray can also be denoted by coordinates of the end point of the projection ray as shown in Section 4.3.2.

The projection data set $D(\beta, l, u)$ is defined in the local coordinate system $s-l-u$. In order to obtain the transformation function between $s-l-u$ and $x-y-z$,

we introduce a temporary cartesian coordinate system $x' - y' - z'$ with the origin denoted by o' . The axis x' is parallel to x axis and the angle between z' axis and z axis is μ . Its relation to the global coordinate system $x - y - z$ satisfies

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \mu & -\sin \mu \\ 0 & \sin \mu & \cos \mu \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_{o'} \end{pmatrix} = T_\mu \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_{o'} \end{pmatrix} \quad (4.2)$$

The $x' - y' - z'$ can be obtained from the local coordinate system $s - l - u$ by coordinate system transformations. Its relation to $s - l - u$ is:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -\cos \beta & -\sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s - R_f \\ l \\ u \end{pmatrix} = T_\beta \begin{pmatrix} s - R_f \\ l \\ u \end{pmatrix} \quad (4.3)$$

With coordinate system transformations defined by (4.2) and (4.3), we can obtain the relation between $x - y - z$ and $s - l - u$:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = T_\mu T_\beta \begin{pmatrix} s - R_f \\ l \\ u \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_{o'} \end{pmatrix} \quad (4.4)$$

In this section, we have introduced the geometry of the gantry tilted CT. The x ray source trajectory in global coordinate system $x - y - z$ is derived and the projection data set is defined in the rotating coordinate system $s - l - u$. And the transformation between $s - l - u$ and $x - y - z$ is also derived. These definitions and derivations will be used in following reconstruction algorithms.

Figure 4.3: Definition of the height of the projection ray at iso-centre.

4.2 Geometrically optimized reconstruction algorithm for gantry tilted CT

In this section, we present a 3D approximation algorithm for the gantry tilted CT image reconstruction based on the 3D FDK algorithm which is an extension of the tilted plane FDK reconstruction algorithm (TPFR) [2]. Observing that artifacts in FDK type algorithms for both gantry tilted and non-gantry-tilted MSCT are caused by the approximate nature of FDK algorithms when the cone angle is large which represents deviation of the scanning trajectory from the construction plane, the underlying concept of the proposed algorithm is to reconstruct the CT image on planes which are adjusted from the conventional planes to be more inline with the scanning trajectory. These reconstruction planes are obtained by geometrical optimization. The application of FDK algorithm in the proposed algorithm to the

obtained optimal reconstruction planes can effectively reduce the scanning cone angle leading to significant reduction of artifacts.

The geometrical optimization in the proposed algorithm adaptively minimizes the effective projection cone angle with respect to different pitches, number of detector rows and slice thickness. To obtain image on the optimal reconstruction plane, the projection data collected from the detector array is reformed to form a new 3D data set such that standard FDK reconstruction procedure can be applied to the optimal plane. Each optimal plane has a corresponding helix segment and is reconstructed after one filtering process. Reconstructed images on the optimal planes are further interpolated to obtain images on the rectangular grid.

4.2.1 Geometrical optimization of reconstruction planes

The proposed gantry tilted MSCT algorithm constructs the CT image by first constructing images on a sequence of geometrically optimized planes and then interpolating images of these planes to obtain the image on the specified horizontal plane in the $x - y - z$ coordinate system. Let $\psi = \{P_i : i = 1, 2, 3, \dots, n\}$ be a set of image planes whose geometrical position and orientation are to be optimized. The optimization of each P_i is proposed as follows. Let the intersection of the plane P_i and the z -axis be at point $o_i = (0, 0, z_i)^T$. $z_i = z_0 + i\Delta z$, for $i = 1, 2, 3, \dots, n$ with $\Delta z > 0$ being a constant. Then the planes in the set ψ are uniformly placed at the intersections with the z -axis. With respect to the plane P_i , a temporary coordinate system $\xi - \eta - \zeta$ can be defined such that o_i is the origin, the ξ axis is parallel to the $x - y$ plane and P_i is on the $\xi - \eta$ plane of the temporary coordinate system. The angle between the ξ -axis and the x -axis is denoted by ϕ_i and the angle between the η -axis and the $x - y$ plane is denoted by γ_i . The angles ϕ_i and γ_i are to be determined by optimization. And the transformation between the temporary

coordinate system $\xi - \eta - \zeta$ and the global coordinate system $x - y - z$ is

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = T \begin{pmatrix} x \\ y \\ z - z_i \end{pmatrix}$$

$$T = \begin{pmatrix} \cos \phi_i & \sin \phi_i & 0 \\ -\sin \phi_i \cos \gamma_i & \cos \phi_i \cos \gamma_i & \sin \gamma_i \\ \sin \phi_i \sin \gamma_i & -\cos \phi_i \sin \gamma_i & \cos \gamma_i \end{pmatrix}. \quad (4.5)$$

For the reconstruction plane P_i centered at o_i , there exists a projection angle $\beta_i = \beta_0 + \frac{2\pi i \Delta z}{pSM}$ of the x ray source trajectory such that the point $C_s(\beta_i)$ on the x ray source trajectory is at z_i , the same level as o_i , in the z -direction. We suppose that the interval of the helix segment used for reconstructing each tilted plane P_i is of length β_m and is centered at β_i for $\beta \in [\beta_i - \beta_m/2, \beta_i + \beta_m/2]$.

The x ray source trajectory (4.1) forms an ellipse cylinder and the tilted plane P_i intersects the ellipse cylinder yielding an intersection curve as shown in Figure 4.4 which is expressed by

$$E_s(\beta) = \begin{pmatrix} R_f \cos \beta \\ R_f \sin \beta \cos \mu \\ z_i - R_f \sin \phi_i \tan \gamma_i \cos \beta + R_f \cos \mu \tan \gamma_i \cos \phi_i \sin \beta \end{pmatrix}. \quad (4.6)$$

At each projection angle $\beta \in [\beta_i - \beta_m/2, \beta_i + \beta_m/2]$, the distance between the x ray source and the intersection curve in the z -direction is:

$$\begin{aligned} d_{se}(\beta, \gamma_i, \phi_i) &= \left| z_0 + \frac{pSM}{2\pi}(\beta - \beta_0) + R_f \sin \mu \sin \beta - z_i \right. \\ &\quad \left. + R_f \sin \phi_i \tan \gamma_i \cos \beta - R_f \cos \mu \tan \gamma_i \cos \phi_i \sin \beta \right| \\ &= \left| \frac{pSM(\beta - \beta_i)}{2\pi} - R_f \cos \mu \tan \gamma_i \cos \phi_i \sin \beta \right. \\ &\quad \left. + R_f \sin \mu \sin \beta + R_f \sin \phi_i \tan \gamma_i \cos \beta \right| \end{aligned} \quad (4.7)$$

The geometric relationship between the x ray source trajectory and the intersection curve on the cylinder is shown in Figure 4.4.

The concept of image reconstruction on the geometrically optimized planes is motivated by the fact that image artifacts in the conventional FDK algorithm are caused by the large intersection angle between the x rays and the image reconstruction plane due to the spiral cone beam x ray projection. When the image reconstruction plane is adjusted to be more lined up with the emitted x rays from the source, improved image can be expectably obtained [2]. In view of Figure 4.4, it is geometrically clear that the effective cone angle of the gantry tilted MSCT scanning with respect to the reconstruction plane P_i can be reduced by reducing the distance $d_{se}(\beta, \gamma_i, \phi_i)$ between the x ray source trajectory and the intersection curve, so that $d_{se}(\beta, \gamma_i, \phi_i)$ can be used as a measure of inlineness between the x rays and the image reconstruction plane. Based on this observation, the proposed gantry tilted MSCT reconstruction algorithm minimizes the mean square error between the source trajectory $C_s(\beta)$ and the intersection curve $E_s(\beta)$ in the z -direction, which is formulated by

$$\begin{aligned}
 J^2 &= \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} d_{se}(\beta, \gamma_i, \phi_i)^2 d\beta \\
 &= A \sin^2 \phi_i \tan^2 \gamma_i + B \cos^2 \phi_i \tan^2 \gamma_i - C \sin \phi_i \tan \gamma_i + D \cos \phi_i \tan \gamma_i \\
 &\quad - E \sin 2\phi_i \tan \gamma_i + \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} \left(\frac{pSM(\beta - \beta_i)}{2\pi} + R_f \sin \mu \sin \beta \right)^2 d\beta \quad (4.8)
 \end{aligned}$$

Figure 4.4: Geometry of the optimal plane

where

$$\begin{aligned}
 A &= \frac{R_f^2(\beta_m + \cos 2\beta_i \sin \beta_m)}{2} \\
 B &= \frac{R_f^2 \cos^2 \mu (\beta_m - \cos 2\beta_i \sin \beta_m)}{2} \\
 C &= \frac{pSMR_f}{\pi} \left[2 \sin \beta_i \sin \frac{\beta_m}{2} - \beta_m \sin \beta_i \cos \frac{\beta_m}{2} \right] - R_f^2 \sin \mu \sin 2\beta_i \sin \beta_m \\
 D &= \frac{pSMR_f \cos \mu}{\pi} \left[-2 \cos \beta_i \sin \frac{\beta_m}{2} + \beta_m \cos \beta_i \cos \frac{\beta_m}{2} \right] \\
 &\quad - R_f^2 \sin \mu \cos \mu (\beta_m - \cos 2\beta_i \sin \beta_m) \\
 E &= \frac{R_f^2 \cos \mu \sin 2\beta_i \sin \beta_m}{2}
 \end{aligned}$$

The mean square error J^2 is a function in ϕ_i and γ_i . For minimizing J^2 , we consider

$$\frac{\partial J^2}{\partial \gamma_i} = 0 \quad (4.9)$$

$$\frac{\partial J^2}{\partial \phi_i} = 0. \quad (4.10)$$

The solutions for (4.9) and (4.10) yield $\tan \phi_i$ and $\tan \gamma_i$ in the following.

$$\tan \phi_i = \frac{BC - ED}{EC - AD} \quad (4.11)$$

$$\tan \gamma_i = \frac{C \sin \phi - D \cos \phi}{2(A \sin^2 \phi + B \cos^2 \phi - E \sin 2\phi)}. \quad (4.12)$$

The angles ϕ_i and γ_i which geometrically optimize the reconstruction plane P_i can be computed from (4.11) and (4.12). As a result, the geometrically optimized reconstruction plane P_i can be expressed by:

$$z = z_i - x \tan \gamma_i \sin \phi_i + y \tan \gamma_i \cos \phi_i. \quad (4.13)$$

It turns out that the above obtained expression for the geometrically optimized re-

construction plane is applicable to all planes in the set $\psi = \{P_i : i = 1, 2, 3, \dots, n\}$.

4.2.2 Reconstruction using geometrically optimized planes

4.2.2.1 3D Reconstruction data set formation

For reconstruction of the image on the optimal plane P_i , we now reorganize the obtained $D(\beta, l, u)$ into a new data set in the temporary coordinate system $\xi - \eta - \zeta$ as defined in (4.5). The newly organized data set will enable the conventional FDK algorithm to be applied to reconstruct the optimal plane P_i with effectively reduced cone angle.

Under the coordinate system transformation (4.5), the x ray source position for each projection angle β in the $\xi - \eta - \zeta$ coordinate system is determined by

$$\bar{C}_s(\beta) = \begin{pmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{pmatrix} = T \begin{pmatrix} x_s \\ y_s \\ z_s - z_i \end{pmatrix} = T \begin{pmatrix} R_f \cos \beta \\ R_f \sin \beta \cos \mu \\ \frac{pSM(\beta-\beta_i)}{2\pi} + R_f \sin \mu \sin \beta \end{pmatrix}. \quad (4.14)$$

The distance $\bar{R}_f(\beta)$ from the x ray source $\bar{C}_s(\beta)$ to the ζ -axis is

$$\bar{R}_f(\beta) = \sqrt{\xi_s^2 + \eta_s^2}. \quad (4.15)$$

In the temporary coordinate system $\xi - \eta - \zeta$, we further introduce a temporary center plane which is parallel to the $\xi - \eta$ plane and passes through the x ray source. Following from the definition of the height of the projection ray u with respect to the centre plane, the projection intersects with $\eta - \zeta$ on a point and the $\eta - \zeta$ coordinates of the point are $\bar{l}$ and $\bar{u}$. As a result, the temporary $\bar{\beta}$, $\bar{l}$ and $\bar{u}$ can be used to represent a new data set $\bar{D}(\bar{\beta}, \bar{l}, \bar{u})$ in $\xi - \eta - \zeta$, to which the conventional CT algorithms can be applied to reconstruct the optimal plane P_i .

Figure 4.5: Illustration of $\bar{l}$ and $\bar{u}$

The temporary projection angle $\bar{\beta}$ is given by

$$\bar{\beta} = \begin{cases} 3\pi/2 & \xi_s = 0, \eta_s < 0; \\ \pi/2 & \xi_s = 0, \eta_s > 0; \\ \pi + \arctan(\eta_s/\xi_s) & \xi_s < 0 \\ \arctan(\eta_s/\xi_s) & \text{otherwise} \end{cases} \quad (4.16)$$

Referring to Figure 4.3, the projection ray passing through the point $Q = (R_f, l, u)^T$ yields the projection data $D(\beta, l, u)$. The corresponding expression in the temporary coordinate system $\xi - \eta - \zeta$ is determined by (4.4) and (4.5) in the following

$$\begin{pmatrix} \xi_p \\ \eta_p \\ \zeta_p \end{pmatrix} = T \left\{ T_\mu T_\beta \begin{pmatrix} 0 \\ l \\ u \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ z_0 - z_i + \frac{pSM(\beta - \beta_0)}{2\pi} \end{pmatrix} \right\}. \quad (4.17)$$

As shown in Figure 4.5, the point $\bar{Q} = (\xi_p, \eta_p, \zeta_p)^T$ is the corresponding point of Q in $\xi - \eta - \zeta$. The point $\bar{Q}'$ is its projection on the center plane. The center plane insets the ζ axis at point $\bar{o}$ and $|\bar{o}C_s|$ is equal to $\bar{R}_f$. The straight line $\bar{Q}'\bar{F}$ is perpendicular to the line $C_s\bar{o}$. With coordinate system transformation, we can obtain

$$|\bar{o}\bar{F}| = \bar{R}_f(\beta) - \xi_p \cos \bar{\beta} - \eta_p \sin \bar{\beta},$$

$$|\bar{Q}'\bar{F}| = \eta_p \cos \bar{\beta} - \xi_p \sin \bar{\beta}.$$

The $\bar{l}$ and the temporary height of projection ray at iso-center $\bar{u}$ of the projection ray passing through the point $\bar{Q}$ can be determined by

$$\bar{l} = \frac{|\bar{Q}'\bar{F}|\bar{R}_f}{|\bar{o}\bar{F}|} = \frac{\bar{R}_f(\beta)(\eta_p \cos \bar{\beta} - \xi_p \sin \bar{\beta})}{\bar{R}_f(\beta) - \xi_p \cos \bar{\beta} - \eta_p \sin \bar{\beta}} \quad (4.18)$$

$$\bar{u} = \frac{(\zeta_p - \zeta_s)\bar{R}_f}{|\bar{o}\bar{F}|} = \frac{(\zeta_p - \zeta_s)\bar{R}_f(\beta)}{\bar{R}_f(\beta) - \xi_p \cos \bar{\beta} - \eta_p \sin \bar{\beta}}. \quad (4.19)$$

The above obtained $\bar{\beta}$, $\bar{l}$ and $\bar{u}$ characterizes the projection ray passing through the point Q with respect to the temporary centre plane in the temporary coordinate system $\xi - \eta - \zeta$. Since the projection ray passing through the point Q results in the projection data $D(\beta, l, u)$ measured from the detector array, the data $\bar{D}(\bar{\beta}, \bar{l}, \bar{u})$ determined by $\bar{\beta}$, $\bar{l}$ and $\bar{u}$ must satisfy $\bar{D}(\bar{\beta}, \bar{l}, \bar{u}) = D(\beta, l, u)$. This establishes a transformation of the data set $D(\beta, l, u)$ to the new data set $\bar{D}(\bar{\beta}, \bar{l}, \bar{u})$ for reconstruction of the optimal plane P_i in the temporary coordinate system $\xi - \eta - \zeta$.

4.2.2.2 Optimal plane reconstruction

With the projection data set formed in the preceding section, we present an algorithm in this section for reconstruction of the optimal plane P_i . Recall that the conventional FDK algorithm for reconstruction of the object $f(x, y, z)$ can be written as

$$f(x, y, z) = \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} \frac{R_f^2}{(x - x_s(\beta))^2 + (y - y_s(\beta))^2} \int \frac{l R_f D(\beta, l, u(\beta)) g(l'(\beta) - l)}{\sqrt{(R_f^2 + l^2)(R_f^2 + u(\beta)^2)}} d\theta d\beta \quad (4.20)$$

Following from this algorithm and applying it to the optimal plane $P_i = P_i(\xi, \eta, \zeta)$, in the temporary coordinate system $\xi - \eta - \zeta$, we propose the following reconstruction formula

$$P_i(\xi, \eta, \zeta) = \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} \frac{\bar{R}_f(\beta)^2}{(\xi - \xi_s(\beta))^2 + (\eta - \eta_s(\beta))^2} \int \frac{\bar{l} \bar{D}(\bar{\beta}(\beta), \bar{l}, \bar{u}(\beta)) g(\bar{l}'(\beta) - \bar{l})}{\sqrt{(\bar{R}_f^2 + \bar{l}^2)(\bar{R}_f(\beta)^2 + \bar{u}(\beta)^2)}} d\bar{l} d\beta \quad (4.21)$$

where the height $\bar{u}(\beta)$ can be calculated by using (4.19), $\bar{l}'(\beta)$ represents the detector angle in the temporary coordinate system $\xi - \eta - \zeta$ in the convolutional integration

and can be calculated by using (4.18).

The optimal plane reconstruction procedure is to reconstruct the image of the optimal plane P_i by applying the conventional Equal-angular FDK type back projection algorithm to the reorganized data set $\bar{D}(\bar{\beta}, \bar{\theta}, \bar{u})$ in the temporary $\xi - \eta - \zeta$ coordinate systems. Based on the reconstruction formula (4.21), we can summarize the construction procedure in the following.

1. Determine the z position z_i , centre of the optimal plane P_i , its centre projection angle β_i , and calculate the optimal tilted angle using (4.11) and (4.12). For half-scan reconstruction, $\beta_m = \pi + \theta_{fan}$.
2. For each projection data denoted by $D(\beta, l, u)$, find its corresponding projection $\bar{D}(\bar{\beta}, \bar{l}, \bar{u})$ to set up the reformed projection data set for the optimal plane P_i . For half-scan, $D(\beta, \theta, u)$ is weighted with Parker's weighting before reformation as follows
 - calculate $\bar{\beta}$ and $\bar{R}_f(\beta)$, by using (4.16) and (4.15), respectively.
 - calculate $\bar{l}$ and $\bar{u}$, by using (4.18) and (4.19), respectively.
3. The optimal reconstruction plane is determined by (4.13). For each point $(x, y, z(x, y))$ on the optimal reconstruction plane, its corresponding point (ξ, η, ζ) in the temporary coordinate system $\xi - \eta - \zeta$ is determined by (4.5).
4. The optimal plane P_i is constructed by (4.21).

The reconstructed optimal planes are then used to form images for the horizontal planes parallel to the $x - y$ plane. This is carried out by interpolating the planes in $\psi = \{P_i : i = 1, 2, 3, \dots, n\}$. Consider that a horizontal plane $f(x, y, z_h)$ is to be interpolated at $z = z_h$. For each point (x, y) on this plane, there exist two most adjacent points (x, y, z_{hi}) and (x, y, z_{hj}) on two optimal planes P_i and P_j ,

respectively, such that $z_{hi} \leq z_h \leq z_{hj}$ is satisfied. Given (x, y) , the values $z_{hi}(x, y)$ and $z_{hj}(x, y)$ can be computed by (4.32). Then the image value for $f(x, y, z)$, $z = z_h$, can be interpolated using the following formula.

$$f(x, y, z) = \frac{P_i(x, y, z_{hi}(x, y))I(z - z_{hj})}{I(z - z_{hj}) + I(z - z_{hi})} + \frac{P_j(x, y, z_{hj}(x, y))I(z - z_{hi})}{I(z - z_{hj}) + I(z - z_{hi})} \quad (4.22)$$

where $I(\cdot)$ is an interpolation weights window which can be specified according to different clinical requirements.

It is noted that, for different points at (x, y) of the interpolated plane, the corresponding optimal planes used for interpolation can be different. And the increment Δz of the optimal planes P_i and P_{i+1} can affect the resolution of the interpolated image in the z -direction.

The proposed algorithm also can be implemented with an equal angular curved surface detector.

4.3 The cone beam reconstruction algorithm for gantry-tilted CT using tangential filtering

In this section another reconstruction algorithm with the tangential filtering technique for gantry tilted CT is introduced. It is known that, images reconstructed by FDK algorithms suffer from serious artifacts when cone angle is large. Algorithms have been developed to reduce artifacts by filtering projection data along the tangential direction of the helix [157] [142] [155]. Such a technique was earlier proposed by Yan and Leahy [157] and further improved by Sourbelle and Kalender [142] for short scan FDK type algorithms. The improved FDK type algorithm can achieve better image quality than that of the conventional FDK algorithm.

Motivated by the observation that artifacts for FDK type algorithms can be effectively reduced by filtering projection data along the helix tangential direction, the proposed algorithm extends the existing tangential filtering technique to present a 3D FDK type approximation algorithm for gantry tilted helical MSCT.

To deal with the complicated gantry tilted geometry and large scanning cone angle, our proposed algorithm first applies the ramp filtering along the trajectory tangential direction to reconstruct a sequence of image planes which are perpendicular to the rotating axis. The final horizontal image planes are then obtained by interpolating the tilted image planes.

4.3.1 Coordinate system transformations for tangential filtering

The geometry of the gantry tilted CT has been defined in Section 4.1. In this part, the trajectory tangential direction is derived and a local coordinate system is setup with respect to the trajectory tangential direction. Transformations between coordinate systems are derived.

With the x ray source trajectory given in (4.1), we take the derivative with respect to β to obtain its tangential direction at C_s in the global coordinate system represented in the following form

$$\begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} = \frac{dC_s(\beta)}{d\beta} = \begin{pmatrix} -R_f \sin \beta \\ R_f \cos \mu \cos \beta \\ h + R_f \sin \mu \cos \beta \end{pmatrix}, \quad (4.23)$$

where $h = \frac{pSM}{2\pi}$. The x ray source trajectory and its tangential direction at C_s are shown in Figure 4.6.

Figure 4.6: Illustration of conventional filter direction and tangential filter direction.

It follows from the geometric relation between the global $x - y - z$ and the rotating $s - l - u$ coordinate system, as shown in Figure 4.1 and Figure 4.2, that the transformation of the tangential direction $(x_t, y_t, z_t)^T$ as given in (4.23) to the rotating $s - l - u$ coordinate system can be obtained by first rotating this vector by an angle μ around the x axis followed by a rotation of angle $\beta - \pi$ around the u axis. This results in the tangential direction of the trajectory in the rotating $s - l - u$ coordinate system, denoted by $(s_t, l_t, u_t)^T$, in the following.

$$\begin{aligned} \begin{pmatrix} s_t \\ l_t \\ u_t \end{pmatrix} &= \begin{pmatrix} \cos(\beta - \pi) & \sin(\beta - \pi) & 0 \\ -\sin(\beta - \pi) & \cos(\beta - \pi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \mu & \sin \mu \\ 0 & -\sin \mu & \cos \mu \end{pmatrix} \begin{pmatrix} -R_f \sin \beta \\ R_f \cos \mu \cos \beta \\ h + R_f \sin \mu \cos \beta \end{pmatrix} \\ &= \begin{pmatrix} -h \sin \beta \sin \mu \\ R_f + h \sin \mu \cos \beta \\ h \cos \mu \end{pmatrix}. \end{aligned} \quad (4.24)$$

To implement the tangential filtering for the gantry tilted scanning projection data, we further use the tangential direction $(s_t, l_t, u_t)^T$ to set up a tangential cartesian coordinate system $\xi - \eta - \zeta$. The origin of this coordinate system is the x ray source C_s , the η axis is defined by the tangential direction of the x ray source trajectory at C_s , as given in (4.24), the ζ axis is parallel to the detector plane and the $\xi - \zeta$ plane is orthogonal to the η axis.

Using the tangential direction $(s_t, l_t, u_t)^T$ in (4.24), we introduce two angular quantities σ and γ as follows

$$\begin{aligned} \sigma &= \arcsin \left(\frac{s_t}{\sqrt{s_t^2 + l_t^2}} \right) \\ &= \arcsin \left(\frac{-h \sin \mu \sin \beta}{\sqrt{R_f^2 + 2hR_f \sin \mu \cos \beta + h^2 \sin^2 \mu}} \right), \end{aligned} \quad (4.25)$$

$$\begin{aligned}
\gamma &= \arctan \left(\frac{u_t}{\sqrt{s_t^2 + l_t^2}} \right) \\
&= \arctan \left(\frac{h \cos \mu}{\sqrt{R_f^2 + 2hR_f \sin \mu \cos \beta + h^2 \sin^2 \mu}} \right). \quad (4.26)
\end{aligned}$$

Following from its definition, the $\xi - \eta - \zeta$ coordinate system can be obtained by first rotating the $s - l - u$ coordinate system around the u axis by an angle σ to obtain an intermediate coordinate system $\xi - \tilde{\eta} - u$ followed by rotating the $\xi - \tilde{\eta} - u$ coordinate system around the ξ axis by an angle γ . Such coordinate rotations and transformation are illustrated in Figure 4.7. It follows that the transformation between the $s - l - u$ and $\xi - \eta - \zeta$ coordinate systems is given by

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = T \begin{pmatrix} s \\ t \\ u \end{pmatrix}, \quad (4.27)$$

where the transformation matrix T is

$$\begin{aligned}
T &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} \cos \sigma & \sin \sigma & 0 \\ -\sin \sigma & \cos \sigma & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos \sigma & \sin \sigma & 0 \\ -\sin \sigma \cos \gamma & \cos \sigma \cos \gamma & \sin \gamma \\ \sin \sigma \sin \gamma & -\cos \sigma \sin \gamma & \cos \gamma \end{pmatrix}. \quad (4.28)
\end{aligned}$$

4.3.2 Data reformation for tangential filtering

In conventional FDK type algorithms, the ramp filtering is performed on detector rows and the filtering direction is parallel to the l axis on the detector surface. Moti-

Figure 4.7: Illustration of rotation angles between $s-l-u$ and $\xi-\eta-\zeta$.

vated by the tangential filtering technique for CT without gantry tilting, we propose in here that the ramp filtering is performed in the tangential direction of the x ray source trajectory of the gantry tilted CT as shown in Figure 4.6. For this purpose, the projection data are reformed such that rows of the reformed projection data set are parallel to the tangential direction of the x ray source trajectory. The reformed projection data set enables the tangential filtering technique for the conventional CT reconstruction being applied.

The reformed data set is built up by introducing a virtual detector array surface in the tangential coordinate system $\xi-\eta-\zeta$ and transforming the projection data from the real detector array surface in the rotating $s-l-u$ coordinate system to the virtual detector array surface. The virtual detector array surface is placed on the plane $\xi = (R_f + R_d)$, as shown in Figure 4.8, with the rows being parallel to the η axis.

Figure 4.8: Illustration of the relationship between $(\bar{\xi}, \bar{\eta}, \bar{\zeta})^T$ and $(R_f + R_d, \eta, \zeta)^T$ in coordinate system $\xi - \eta - \zeta$.

As shown in Figure 4.8, each projection ray radiates on the real detector array at the point $((R_f + R_d), l_p, u_p)^T$ in the $s-l-u$ coordinate also radiates on a point $((R_f + R_d), \eta_p, \zeta_p)^T$ on the virtual detector array. Let the $\xi-\eta-\zeta$ coordinate of the point $((R_f + R_d), l_p, u_p)^T$ on the real detector array surface be $(\bar{\xi}_p, \bar{\eta}_p, \bar{\zeta}_p)^T$. The transformation from $((R_f + R_d), l_p, u_p)^T$ to $(\bar{\xi}_p, \bar{\eta}_p, \bar{\zeta}_p)^T$ is determined by the coordinate transformation matrix T in (4.28), i.e.

$$\begin{pmatrix} \bar{\xi}_p \\ \bar{\eta}_p \\ \bar{\zeta}_p \end{pmatrix} = T \begin{pmatrix} (R_f + R_d) \\ l_p \\ u_p \end{pmatrix}. \quad (4.29)$$

In view of Figure 4.8, the coordinate $(\eta_p, \zeta_p)^T$ of the virtual detector cell can be expressed, in terms of that of the real detector cell in the $\xi-\eta-\zeta$ coordinate system, as

$$\eta_p = \frac{R_f + R_d}{\bar{\xi}_p} \bar{\eta}_p, \quad \zeta_p = \frac{R_f + R_d}{\bar{\xi}_p} \bar{\zeta}_p. \quad (4.30)$$

This, together with (4.29), can determine the coordinate of the virtual detector cell $(\eta_p, \zeta_p)^T$ in the tangential coordinate system $\xi-\eta-\zeta$ from a given real detector cell at $((R_f + R_d), l_p, u_p)^T$ in the rotating coordinate system.

Let the projection datum collected from the real detector cell at $((R_f + R_d), l_p, u_p)^T$ in the $s-l-u$ coordinate at the projection angle β be denoted by $D(\beta, l_p, u_p)$ and the corresponding projection datum on the virtual detector cell at $((R_f + R_d), \eta_p, \zeta_p)^T$ in the $\xi-\eta-\zeta$ coordinate by the same projection ray be denoted by $D_v(\beta, \eta_p, \zeta_p)$. Since $D(\beta, l_p, u_p)$ and $D_v(\beta, \eta_p, \zeta_p)$ are due to the same projection ray, we have

$$D_v(\beta, \eta_p, \zeta_p) = D(\beta, l_p, u_p). \quad (4.31)$$

This can be used to obtain the reformed projection data set following from the coordinate transform from each real detector cell at $((R_f + R_d), l_p, u_p)^T$ in the rotating

coordinate system to the corresponding virtual detector cell at $((R_f + R_d), \eta_p, \zeta_p)^T$ in the tangential coordinate system.

4.3.3 Reconstruction and interpolation

Because the gantry is tilted and final images should be on horizontal planes in the global $x - y - z$ coordinate system, we first reconstruct images on a sequences of intermediate planes parallel to the centre plane and then interpolate them to obtain the horizontal images. The introduction of the intermediate planes effectively reduces the projection cone angle in comparison with constructing directly the horizontal planes. Let $\psi = \{P_i : i = 1, 2, 3, \dots, n\}$ be the set of the intermediate planes and the intersection of each tilted plane P_i and the z -axis be at point $o_i = (0, 0, z_i)^T$, with $z_i = z_0 + i\Delta z$, $i = 1, 2, 3, \dots, n$, and $\Delta z > 0$. The projection angle for the x ray source being at z_i in the z direction is

$$\beta_i = \beta_0 + \frac{2\pi i \Delta z}{pSM}.$$

And the equation for the intermediate plane P_i is

$$z = z_i + y \tan \mu. \quad (4.32)$$

The ramp filtering is along the η direction which is defined as the tangential direction of the source trajectory. Let $g(\cdot)$ denote the ramp filter. Applying it to the reformed data set on the virtual detector array in the tangential direction of the source trajectory and performing standard cone beam weighting yields

$$\tilde{D}(\beta, \eta', \zeta) = \int \frac{R_f D_v(\beta, \eta, \zeta)}{\sqrt{\eta'^2 + R_f^2 + \zeta^2}} g(\eta' - \eta) d\eta. \quad (4.33)$$

Then the reconstruction formula is derived from Yan's [157] paper for tangential filtering reconstruction.

$$\begin{aligned}
 f_i(x, y, z) &= \frac{1}{2} \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} \frac{R_f \left\| \frac{dC_s(\beta)}{d\beta} \right\| \tilde{D}(\beta, \eta'(\beta), \zeta(\beta))}{(x \cos \beta + y \sin \beta - R_f)^2} d\beta \\
 &= \frac{1}{2} \int_{\beta_i - \beta_m/2}^{\beta_i + \beta_m/2} \frac{R_f \sqrt{R_f^2 + 2hR_f \sin \mu \cos \beta + h^2} \tilde{D}(\beta, \eta'(\beta), \zeta(\beta))}{(x \cos \beta + y \sin \beta - R_f)^2} d\beta \quad (4.34)
 \end{aligned}$$

where $\|\cdot\|$ denotes Euclidean norm and β_m is the length of the projection angle interval.

Given the reconstructed images of the tilted planes P_i , for $i = 1, 2, 3, \dots, n$, and a point $(x, y, z)^T$ on a plane parallel to the x - y plane where the image is to be obtained by interpolation, the z -positions of the intermediate planes P_i , $i = 1, 2, 3, \dots, n$, at $(x, y)^T$ and denoted by $z_{P_i}(x, y)$ can be determined by (4.32). Thus there exist two points $(x, y, z_{P_j})^T$ and $(x, y, z_{P_k})^T$ on two tilted planes P_j and P_k , respectively, at the upper and lower sides of $(x, y, z)^T$, respectively, which are closest to $(x, y, z)^T$. Using the obtained attenuation functions $f_j(x, y, z_{P_j})$ and $f_k(x, y, z_{P_k})$ for planes P_j and P_k , respectively, the interpolated attenuation function can be obtained using the following interpolation formula

$$f(x, y, z) = \frac{f_j(x, y, z_{P_j}(x, y))I(z_{P_k} - z) + f_k(x, y, z_{P_k}(x, y))I(z - z_{P_j})}{I(z_{P_k} - z) + I(z - z_{P_j})} \quad (4.35)$$

where $I(\cdot)$ is an interpolation weights window which can be specified according to different clinical requirements.

We can now summarize the proposed reconstruction procedure in the following.

1. Determine the z position z_i , centre of the optimal plane P_i , its centre projection angle β_i , and obtain the reconstruction plane using (4.32). For half-scan, $\beta_m = \pi + \theta_{fan}$.

2. Obtain the reformed projection data set on the virtual detector array using (4.30).
3. Reconstruct the image on the intermediate planes P_i using (4.34).
4. Obtain the horizontal images by interpolating the intermediate images.

4.4 Performance evaluation of GOFTR and GTTF algorithms

In this section, the performance of the GPFTR and GTTF algorithms is evaluated with center and off center SSP (Slice Sensitive profile), noise properties, X-Y resolution properties and image artifacts study. The evaluation is performed using computer simulations. Parameters of the simulation are set to be: $R_f=570\text{mm}$, $S = 1\text{mm}$, $\mu = 30^\circ$ and detector cell width 1.5mm . The projection number per rotation is 1024 and number of projection rays in the fan is 800.

4.4.1 The slice sensitive profile (SSP) study

The Slice Sensitive Profile (SSP) is measured as an indication of the spatial resolution in longitudinal direction (z resolution) for medical imaging systems. Similar to the in plane resolution (x-y resolution), the SSP describes the system response to the Dirac delta function [9]. The Dirac delta function is approximated by thin discs whose thickness is much smaller than the slice thickness. The z resolution also varies for different scan fields. In order to measure the SSPs at different scan fields, a five disc phantom is employed in this study.

The disk is a cylinder with its longitude axis parallel to z-axis. The radius of the cylinder is 10mm and the length is 0.1mm. The centers of the five disks are

Table 4.1: The mean value and deviation of full-width half maxima (FWHM) GTASSR, GTFDK, GTTF and GOFTR algorithms

FWHM	GTFDK	GTASSR	GTTF	GOFTR
Mean	0.853mm	1.14mm	0.832mm	0.829
Deviation	0.0518mm	0.0506mm	0.0365mm	0.00401

(0,0,0), (100,0,0), (-100,0,0), (0,100,0) and (0,-100,0) (mm). With this five-disk model, both center SSPs and off-center SSPs are evaluated. SSPs of the GTASSR, GTFDK, GTTF and GOFTR algorithms are shown in Figure 4.9.2- Figure 4.9.4 respectively. The mean value and deviation of full-width half maxima (FWHM) of different algorithms with $p=1.0$, $M=64$ and $S=0.625\text{mm}$ are calculated in Table 4.1.

As shown in Figure 4.9.1 and 4.9.2, SSPs for the GTASSR and GTFDK algorithm are displaced and broadened and these effects are reduced by the GTTF and GOFTR algorithms as shown in Figure 4.9.3 and 4.9.4. The GTTF and GOFTR algorithms can improve spatial resolution of the GTFDK algorithm in z direction. As shown in Table 4.1, the FWHM mean and deviation of the GTTF and GOFTR algorithms are smaller than the GTFDK and GTASSR algorithms. The GTTF and GOFTR algorithms have more stable spatial resolution in z direction than other algorithms.

4.4.2 Reconstruction under noise

Poisson noise is added to the projection data of a water cylinder phantom with a radius of 200mm . The noise levels of the reconstructed images at $z = 0\text{mm}$ by GTFDK, GTASSR, GTTF and GOFTR algorithms are evaluated and the reconstruction parameters are $M = 64$, $S = 1\text{mm}$, $\mu = 30^\circ$, and $p = 1.0$. Five 10×10 pixels regions are used to evaluate noise and the centres of the five regions are (0,0,0), (150,150,0), (-150,150,0), (150,-150,0) and (-150,-150,0). It is known that for I_0 being the incident x ray photon number, the received photon number I in terms

4.9.1

4.9.2

4.9.3

4.9.4

Figure 4.9: Center and off-center SSPs for the GTASSR, GTFDK, GTTF and GOFTR with $p = 1.0$, $M=64$ and $S=0.625\text{mm}$.

Table 4.2: Mean values of the standard derivations of regions reconstructed by GTFDK, GTASSR, GTTF and GOFTR(HU)

Methods	ISO	Left Up	Right up	Left down	Right down
GTFDK	11.67	11.61	14.22	11.06	10.98
GTASSR	11.84	11.76	12.34	11.36	11.48
GTTF	10.54	10.55	11.26	10.55	10.55
GOFTR	9.51	10.23	11.34	9.57	9.61

of the projection data $D(\beta, \theta, u)$ is:

$$I = I_0 e^{-D(\beta, \theta, u)}.$$

We add poisson noise to the received photon number such that

$$I_n = Poisson(I) = Poisson(I_0 e^{-D(\beta, \theta, u)}). \tag{4.36}$$

where $Poisson(.)$ is the function adding poisson noise to I and I is the mean value of the Poisson distribution. We can derive the projection data with noise as follows

$$D_n(\beta, \theta, u) = \ln \frac{I_0}{I_n}. \tag{4.37}$$

The water phantom are reconstructed with noised projection data for 30 times. The mean values of standard derivations of in five regions are calculated and listed in the Table 4.2. As shown in this table, the GTTF and GOFTR algorithms have smaller standard derivations than the GTFDK and GTASSR algorithms.

4.4.3 X-Y resolution

We evaluate the x-y resolution with the resolution phantom and small resolution phantom defined in Section 3.5.3.

The resolution phantom is used to evaluated the resolution performance of recon-

struction algorithms with large field of interest (FOI). The radius of the resolution phantom is $200mm$ and the reconstruction image size is 600×600 . The size of each of each pixel in the reconstruction image is $0.66mm \times 0.66mm$. Images at $z = 0mm$ are reconstructed at $p = 1.2$ to evaluate resolution of reconstruction algorithms. As shown in Figure 4.10, Figure 4.10.1- Figure 4.10.4 are images for resolution phantom with $p = 1.2$ reconstructed by the GTASSR, GTFDK, GTTF and GOFTR algorithms, respectively.

The small resolution phantom is used to evaluated the resolution performance of reconstruction algorithms with small field of interest (FOI). The radius of the resolution phantom is $100mm$ and the reconstruction image size 600×600 . The size of each of each pixel in the reconstruction image is $0.33mm \times 0.33mm$. Images at $z = 0mm$ are reconstructed at $p = 1.2$ to evaluate resolution of reconstruction algorithms. As shown in Figure 4.11, Figure 4.11.1- Figure 4.11.4 are images for resolution phantom with $p = 1.2$ reconstructed by the GTASSR, GTFDK, GTTF and GOFTR algorithms, respectively.

The GOFTR and GTTF algorithm employ more interpolation operations than the GTFDK algorithm. The interpolation operation degrades the spatial resolution of reconstruction algorithms. However, the GOFTR and GTTF algorithm provide more accurate reconstruction image quality than that of the GTFDK algorithm. No obvious degrade in spatial resolution is shown in the x-y resolution study.

4.4.4 Reconstruction image accuracy study

The Turbell clock phantom is designed by Turbell and used for comparison of image artifacts of different reconstruction algorithms. The simulation results for Turbell clock phantom are shown in Figure 4.12, Figure 4.12.1- 4.12.4 are reconstructed by the GTTF, GOFTR, GTFDK and GTASSR, respectively, with $M = 64$. It is clearly

4.10.1

4.10.2

4.10.3

4.10.4

Figure 4.10: Images for resolution phantom with $z = 0$, $M = 64$ and $p = 1.2$. (300-900HU)

4.11.1

4.11.2

4.11.3

4.11.4

Figure 4.11: Images for small resolution phantom with $z = 0$, $M = 64$ and $p = 1.2$. (300-900HU)

4.12.1

4.12.2

4.12.3

4.12.4

Figure 4.12: Images for the plane $z=0\text{mm}$ of Clock phantom generated with gantry tilted tangential filtering, GOFTR, gantry tilted Feldkamp and gantry tilted ASSR with $\mu = 30^\circ$, $S=1\text{mm}$, $p=1.0$ $M=64$ and $M=96$ (350-450HU).

demonstrated that the proposed GOFTR has much less artifacts in the images than that of the other algorithms.

The thorax phantom is further used for simulation which is designed by referring to human thorax consisting of many important organs and often scanned in CT examination. These organs include lungs, heart, aorta, ribs, spine, sternum, and shoulders. The phantom definitions are from a world phantom database FORBILD. This kind of body phantoms are more realistic in modeling the real patient CT scan, thus are more feasible in examining the reconstructed image quality. As shown in Figure 4.13 and Figure 4.14, images of Thorax phantom at $z = 0mm$ are reconstructed by the GTTF, GOFTR, GTFDK and GTASSR with $M = 96$, $S = 1mm$, $p = 1.0$ and $\mu = 30^\circ$. Images reconstructed by the GTFDK and GTASSR show obvious artifacts. In the zoomed reconstructed images in the right column of Figure 4.13 and Figure 4.14, clear artifacts are shown around ribs in the images reconstructed by the GTFDK and GTASSR algorithms. In contrast, the GOFTR algorithm provides more satisfactory image quality and more accurate reconstruction.

4.5 Summary

In this chapter, we presented two FDK type reconstruction algorithms for gantry tilted CT. The objective of proposed algorithms is to suppress artifacts for the large cone angle case in the gantry tilted CT scanning. To this objective, we developed the GOFTR algorithm and GTTF algorithm. The geometrically optimized scheme (GOFTR) is developed which can significantly reduce the effective projection cone angle by minimizing the distance between the tilted plane and the x ray source trajectory. For reconstruction of the optimal planes, a 3D projection data set is reformed such that the conventional FDK algorithm is applicable for the image reconstruction. We also present another reconstruction algorithm for gantry tilted

4.13.1

4.13.2

4.13.3

4.13.4

Figure 4.13: Images for the plane $z=0\text{mm}$ of Thorax phantom generated with the GTTF and GOFTR algorithm and scanning parameters are $p = 1.0$, $\mu = 30^\circ$, $S = 1\text{mm}$ and $M=96$ (520-1212HU). The right column shows zoomed reconstruction images.

4.14.1

4.14.2

4.14.3

4.14.4

Figure 4.14: Images for the plane $z=0\text{mm}$ of Thorax phantom generated with the GTFDK and GTASSR and with scanning parameters are $p = 1.0$, $\mu = 30^\circ$, $S = 1\text{mm}$ and $M=96$ (520-1212HU). The right column shows zoomed reconstruction images.

CT. This algorithm is based on the idea of filtering the 3D projection data and the filtering direction is varying and dependent on the CT parameters and the gantry tilted angle. As a result, our proposed gantry tilted reconstruction algorithms can provide more scanning flexibility in clinical CT scanning and is efficient in computation.

In comparison with the existing 2D and 3D algorithms for gantry tilted CT image reconstruction. Our algorithms can provide better resolution in the z direction than that of the GTFDK and GTASSR algorithms. The noise property and reconstruction image quality of proposed algorithms are evaluated using computer simulations in comparison with the GTFDK algorithm and GTASSR algorithm. The simulation results have shown advantages of our proposed algorithm.

Chapter 5

Conclusions and recommendations

5.1 Conclusions

In this thesis, new approximate reconstruction algorithms for MSCT with a large number of detector rows are proposed. These algorithms combine the optimal tilted plane technique [2, 137] with the FDK algorithm and tangential filtering technique to improve reconstruction image quality. Advantages of the proposed algorithms are summarized in the following

1. Accurate reconstruction image quality. The TPFR algorithm and its extended algorithm have better image quality compared with the FDK algorithm and tangential filtering algorithm. These algorithms can obtain accurate image quality with a large number of detector rows e.g. 96 and high pitch value.
2. High computational efficiency. These reconstruction algorithms, FDK type approximate reconstruction algorithms, are more computationally efficient than exact reconstruction algorithms. Because the ETPFR algorithm reforms a cone parallel projection data set, projection data are filtered once and the filtered data can be used for reconstruction of different images. The ETPFR

is more computationally efficient than other FDK algorithms with the cone beam projection data set.

3. Large maximum pitch value. These reconstruction algorithms reconstruct images on optimal tilted planes. Because optimal tilted planes can reduce the distance between the x ray source trajectory and the reconstruction plane, the maximum pitch value of these reconstruction algorithms increases. These algorithms work at higher pitch value than FDK type algorithms with horizontal reconstruction planes and can potentially increase volume coverage speed.

In this thesis, we also developed two reconstruction algorithms for gantry tilted CT. The GOFTR is the extension of the TPFR algorithm to gantry tilted geometry. The GTTF is the application of the tangential filtering method to gantry tilted geometry. It is shown that the image reconstruction performance of these proposed algorithms is superior to that of current gantry tilted reconstruction algorithms such as GTFDK and GTASSR.

These new algorithms proposed in the thesis are FDK type algorithms which are widely adopted by the medical CT and can be implemented following implementation procedures introduced in former chapters. However, for a real CT reconstruction algorithms, in order to reconstruct an image from real projection data, noise reduction and projection data preprocessing techniques are also important. These algorithms also need to be extended for different clinical imaging applications. These research topics are beyond the scope of the thesis and recommended for further research work.

5.2 Recommendations for further research

Some future research work following from the work of this thesis are suggested in the following.

1. More comparisons of proposed algorithms with other reconstruction algorithms. In the thesis, we have compared the ETPFR algorithm with the FDK algorithm and tangential filtering algorithm. In the future, we will further evaluate and compare the ETPFR algorithm with latest exact and approximate algorithms.

In the thesis, reconstruction accuracy of the ETPFR is evaluated with noiseless projection data. In the future, we will evaluate the ETPFR algorithm with projection data containing noise and compare it with exact and approximate reconstruction algorithms. We will also evaluate its computation load, noise property and resolution together. This research can provide important evaluation for its feasibility in clinical applications. It is also important for development of reconstruction algorithms.

2. Application issues of the proposed algorithms. MSCT is one of the most important imaging devices for cardiac imaging applications because of its high volume coverage speed and spatial resolution.

The TPFR and ETPFR algorithm can provide good image quality with large area detector and high volume coverage speed. These algorithms can be adopted in cardiac imaging for medical CT with a large area detector. ECG signal is used by helical CT to guide the selection of projection data for the image reconstruction at the desired cardiac phase. However, the ECG-based gating procedure delivers an interrupt helical projection data set. The ETPFR algorithm should be extended to reconstruct images with the interrupt data set. In addition, the helical CT with a larger area detector provides more re-

dundant projection data. The ETPFR algorithm should be extended to deal with the redundant data. Thus, the imaging performance of the cardiac CT system can be improved.

3. Noise reduction in CT with large area detector. It is known that reconstruction image quality for MSCT is influenced by many factors such as reconstruction algorithms, raw data preprocessing technique and signal detection hardware. Recently, exact reconstruction algorithms have been developed for MSCT. They provide accurate reconstruction image with noiseless projection data. In the practical medical CT, projection data are corrupted by statistic noise and scatter noise. Especially, in recent years, large area detectors are widely adopted in the medical CT and scatter noise for the large area detector is much larger than that of the small area detector. Thus there is a critical need of noise reduction methods for medical CT with a large area detector. On the other hand, image quality for low dose CT is degraded due to noise and noise reduction technique is also an important aspect to improve reconstruction image quality for low dose CT [184]. The CT noise model is setup by Whiting [185] *et al* and Elbakri [81] *et al*. The scatter noise correction method with blocker array is developed by Zhu *et al* [186] to estimate scatter noise properties and reduce scatter noise. Penalized weighted least-squares methods are proposed to reduce noise [187–190].

In the future, precise noise model for medical CT with a large area detector should be formulated. Based on the model, more efficient noise reduction methods should be developed. Developing noise reduction methods for medical CT with large area detector is an important research topic for medical CT.

4. Reconstruction algorithms for circular dual sources CT. Circular cone beam CT has been popularly employed in clinical applications such as dedicated breast CT and cardiac imaging [9] because of its mechanical simplicity. But

it cannot reconstruct exact images since the circular orbit does not satisfy the sufficient condition for exact reconstruction (Tuy's condition) [95].

In recent years, dual source cone beam CT scanners are developed and applied to clinical applications [10, 11]. One of advantages of the dual source CT is in the fast scanning by simultaneous operation of the dual sources, so clear temporal resolution of objects in motion, such as cardiac imaging, can be captured. Current dual source circular cone beam CT scanners place two x ray source on the same transversal plane, which still suffer from problems due to the large cone angle.

In order to solve this problem, we propose to place two x ray sources on different positions along the rotating axis. This configuration reduces the cone angle effectively for the dual sources CT and acquires more reductant projection data. Reconstruction algorithms should be developed for this configuration. They can use the information to provide more accurate reconstruction images and deal with reductant projection data to reduce noise. In addition, current advantages of circular reconstruction algorithms [14, 17, 152, 191, 192] can also be extended in the dual sources geometry to improve reconstruction image quality.

Author's publications

Journal Paper

1. M. Yan, C. Zhang, H. Liang, "An approximate cone beam reconstruction algorithm for gantry-tilted CT using tangential filtering," The International Journal of Biomedical Imaging, accepted, 2006.
2. M. Yan, C. Zhang, "Tilted Plane Feldkams Type Reconstruction Algorithm for Spiral Cone Beam CT," Medical Physics, Vol. 32, 3455-3467, 2005.

Conference Paper

1. M. Yan, C. Zhang, H. Liang, "A new scheme and reconstruction algorithm for dual source circular CT," The 28th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, 2006.
2. M. Yan, C. Zhang, H. Liang, "An approximate cone beam reconstruction algorithm for gantry-tilted CT," The SPIE International Symposium Medical Imaging, 2006.
3. M. Yan, C. Zhang, H. Liang, "Gantry tilted Tilted Plane Feldkamp Type Reconstruction Algorithm," The 27th Annual International Conference of the IEEE Engineering in Medicine and Biology Society, 2005.
4. M. Yan, C. Zhang, "Evaluation of Slice Sensitivity Profiles for TPFR Algorithm," International Bioengineering Conference, Singapore, 432-435, 2004.
5. M. Yan, C. Zhang, "Tilted plane Feldkamp type reconstruction algorithm for spiral cone-beam CT," The 8th International Conference on Control, Automation, Robotics and Vision, 419-424, 2004.

Bibliography

- [1] L. A. Feldkamp, L. C. Davis, and J. W. Kress, "Practical cone-beam algorithm," *J. Opt. Soc. Am.*, vol. 1, no. 6, pp. 612–619, 1982.
- [2] M. Yan and C. Zhang, "Tilted plane feldkamp type reconstruction algorithm for spiral cone beam ct," *Med. Phys.*, vol. 32, no. 11, pp. 3455–3467, 2005.
- [3] H. Kudo, T. Rodet, F. Noo, and M. Defrise, "Exact and approximate algorithms for helical cone-beam ct," *Phys. Med. Biol.*, vol. 49, pp. 2913–2931, 2004.
- [4] M. Kachelreiss and T. Fuchs, "Advanced single-slice rebinning for tilted spiral cone-beam ct," *Med. Phys.*, vol. 28, no. 6, pp. 1033–1042, 2001.
- [5] I. Hein, K. Taguchi, M. D. Silver, M. Kazama, and I. Mori, "Feldkamp-based cone-beam reconstruction for gantry-tilted helical multislice ct," *Med. Phys.*, vol. 30, no. 12, pp. 3233–3242, 2003.
- [6] N. Hounsfield, "Historical notes on computerized axial tomography," *J. Can. Asso. Rad.*, vol. 27, pp. 135–142, 1976.
- [7] P. Vok, H. Jung, and W. Kallender, "Single-breath-hold spiral volumetric ct by continuous patient translation and scanner rotation," *Radiology*, vol. 173P, p. 377, 1989.
- [8] C. R. Crawford and K. F. King, "Computed tomography scanning with simultaneous patient translation," *Med. Phys.*, vol. 17, no. 6, pp. 967–982, 1990.
- [9] J. Hsieh, *Computed tomography- principles, design, artifacts and recent advances*. SPIE PRESS, 2003.
- [10] S. Achenbach, D. Ropers, A. Kuettner, T. Flohr, B. Ohnesorge, H. Bruder, H. Theessen, M. Karakaya, W. G. Daniel, W. Bautz, W. A. Kalender,

- and K. Anders, "Contrast-enhanced coronary artery visualization by dual-source computed tomography: initial experience," *European Journal of Radiology*, vol. 57, no. 2, pp. 331–335, 2006.
- [11] H. Bruder, K. Stierstorfer, C. McCollough, R. Raupach, M. Petersilka, C. S. M. Grasruck, B. Ohnesorge, and T. Flohr, "Design considerations in cardiac ct," *Proceedings of SPIE*, vol. 6142, pp. 61420H–1–61420H–13, 2006.
- [12] T. Pana, T. Lee, E. Rietzel, and G. T. Y. Chen, "4d-ct imaging of a volume influenced by respiratory motion on multi-slice ct," *Med. Phys.*, vol. 31, pp. 333–340, 2003.
- [13] I. Mori, "Computerized tomographic apparatus utilizing a radiation source," *US Patent 4630202*, 1986.
- [14] K. Zeng, Z. Chen, L. Zhang, and G. Wang, "An error-reduction-based algorithm for cone-beam computed tomography," *Med. Phys.*, vol. 31, no. 12, pp. 3206–3212, 2004.
- [15] X. Pan, D. Xia, Y. Zou, and L. Yu, "An unified analysis of fbp-based algorithms in helical cone-beam and circular cone and fan beam scans," *Phys. Med. Biol.*, vol. 49, no. 12, pp. 2717–2731, 2004.
- [16] H. Hu, "An improved cine-beam reconstruction algorithm for the circular orbit," *Scanning*, vol. 18, pp. 572–581, 2004.
- [17] H. Yang, M. Li, K. Koizumi, and H. Kudo, "A fbp-type cone-beam reconstruction algorithm with radon space interpolation ability for axially truncated data from a circular orbit," pp. 401–404, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [18] M. Grass, T. Kohler, and R. Proksa, "3d cone-beam ct reconstruction for circular trajectories," *Phys. Med. Biol.*, vol. 45, no. 2, pp. 329–347, 2000.
- [19] L. Li, Z. Chen, L. Zhang, Y. Xing, and K. Kang, "A backprojection filtered image reconstruction algorithm for circular cone-beam ct," vol. 6142, p. 61424N, SPIE, 2006.
- [20] U. van Stevendaal, P. Koken, P. G. C. Begemann, R. Koester, G. Adam, and M. Grass, "Ecg gated continuous circular cone-beam multi-cycle reconstruction for in-stent coronary artery imaging: a phantom study," vol. 6142, p. 61420L, SPIE, 2006.

- [21] A. Ziegler and T. Kohler, "Maximum likelihood reconstruction of circular cone-beam ct data," vol. 6142, p. 614246, SPIE, 2006.
- [22] N. Black and J. Gregor, "Monte carlo simulation of scatter in a cone-beam micro-ct scanner," pp. 207–210, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [23] J. M. Boone, "Dedicated breast ct for breast cancer screening," vol. 682, pp. 3–8, AIP, 2003.
- [24] H. Fu, H. Chueh, W. Tasi, and J. Chen, "Development of 3d cone-beam micro-ct reconstruction," pp. 211–214, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [25] G. Baldazzi, D. Bollini, M. Gambaccini, R. Golfieri, P. Lollini, A. Margotti, S. Masetti, G. Nicoletti, G. Pancaldi, L. Roma, P. Rossi, and M. Zuffa, "Development of a k-edge micro ct for the study of tumor angiogenesis in small animals," vol. 6142, p. 614241, SPIE, 2006.
- [26] X. Liu and A. Sasov, "Cluster reconstruction strategies for microct/nanoct scanners," pp. 215–218, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [27] B. Bodmann, M. Papadakis, D. J. Kouri, S. Gertz, P. Cherukuri, D. Vela, G. Gladish, D. Cody, I. Abodashy, J. Conyers, J. Willerson, and S. Casscells, "Frame isotropic multiresolution analysis for micro ct scans of coronary arteries," vol. 5914, p. 59141O, SPIE, 2005.
- [28] T. Yamamoto, M. Kubo, Y. Kawata, N. Niki, E. Matsui, H. Ohamatsu, and N. Moriyama, "Visualization and quantitative analysis of lung microstructure using micro ct images," vol. 5369, pp. 431–438, SPIE, 2004.
- [29] N. Niki, Y. Kawata, M. Fujii, R. Kakinuma, N. Moriyama, Y. Tateno, and E. Matsui, "Image analysis of pulmonary nodules using micro ct," vol. 4322, pp. 718–725, SPIE, 2001.
- [30] T. Benson and J. Gregor, "Three-dimensional focus of attention for iterative cone-beam micro-ct reconstruction," pp. 219–222, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.

- [31] L. Chen, C. Shaw, . Lai, M. Altunbas, T. Wang, S. Tu, and X. Liu, "Comparison of full-scan and half-scan for cone beam breast ct imaging," vol. 6142, p. 61424M, SPIE, 2006.
- [32] D. Yang, R. Ning, D. Conover, and Y. Yu, "Reconstruction implementation based on a flat panel detector cone beam breast ct," vol. 6142, p. 61424H, SPIE, 2006.
- [33] R. Ning, D. Conover, Y. Yu, Y. Zhang, W. Cai, X. Lu, and R. Betancourt-Benitez, "A novel cone beam breast ct scanner: preliminary system evaluation," vol. 6142, p. 614211, SPIE, 2006.
- [34] X. Liu, C. Shaw, T. Wang, L. Chen, M. Altunbas, and S. Kappadath, "An accurate scatter measurement and correction technique for cone beam breast ct imaging using scanning sampled measurement (ssm) technique," vol. 6142, p. 614234, SPIE, 2006.
- [35] J. Boone, A. Kwan, J. A. Seibert, N. Shah, K. Lindfors, and T. Nelson, "Technique factors and their relationship to radiation dose in pendant geometry breast ct," *Med. Phys.*, vol. 32, no. 12, pp. 3767–3776, 2005.
- [36] A. Kwan, J. Boone, and N. Shah, "Evaluation of x-ray scatter properties in a dedicated cone-beam breast ct scanner," *Medical Physics*, vol. 32, no. 9, pp. 2967–2975, 2005.
- [37] A. Kwan, N. Shah, G. Burkett, J. Seibert, K. Lindfors, T. Nelson, and J. Boone, "Progress in the development of a dedicated breast ct scanner," vol. 5368, pp. 304–310, SPIE, 2004.
- [38] M. Kachelriess, M. Knaup, and W. A. Kalender, "Multithreaded cardiac ct," *Med. Phys.*, vol. 33, no. 7, pp. 2435–2447, 2006.
- [39] M. Jiang and G. Wang, "Convergence studies on iterative algorithms for image reconstruction," *IEEE Med. Imag.*, vol. 22, pp. 569–579, 2003.
- [40] R. Guerra, A. Codreanu, Y. Ponvianne, R. Grosjean, and J. Felblinger, "Optimized time window for cardiac ct reconstruction based on doppler tissue imaging (dti) data," vol. 6142, p. 61420K, SPIE, 2006.
- [41] K. Taguchi, W. P. Segars, G. S. K. Fung, and B. M. W. Tsui, "Toward time resolved 4d cardiac ct imaging with patient dose reduction: estimating the global heart motion," vol. 6142, p. 61420J, SPIE, 2006.

- [42] W. P. Segars, K. Taguchi, G. S. K. Fung, E. K. Fishman, and B. M. W. Tsui, "Effect of heart rate on ct angiography using the enhanced cardiac model of the 4d ncat," vol. 6142, p. 61420I, SPIE, 2006.
- [43] R. Manzke, "Cardiac cone-beam ct," *Med. Phys.*, vol. 32, no. 10, pp. 3227–3227, 2005.
- [44] J. Li, J. Hsieh, K. Mohr, and D. Okerlund, "Evaluation of an adaptive filtering algorithm for ct cardiac imaging with ekg modulated tube current," vol. 5745, pp. 979–982, SPIE, 2005.
- [45] D. Matthews and D. Heuscher, "Measuring temporal resolution of cardiac ct reconstructions," vol. 5745, pp. 618–625, SPIE, 2005.
- [46] J. Sablayrolles, E. Cesmeli, L. Mintandjian, O. Adda, and D. Dessalles-Martin, "Wide coverage by volume ct: benefits for cardiac imaging," vol. 5745, pp. 324–331, SPIE, 2005.
- [47] T. Nielsen, R. Manzke, R. Proksa, and M. Grass, "Cardiac cone-beam ct volume reconstruction using art," *Med. Phys.*, vol. 32, no. 4, pp. 851–860, 2005.
- [48] R. Manzke, T. Kohler, T. Nielsen, D. Hawkes, and M. Grass, "Automatic phase determination for retrospectively gated cardiac ct," *Med. Phys.*, vol. 31, no. 12, pp. 3345–3362, 2004.
- [49] A. Zapasnik and J. Ruminski, "Multislice ct in functional cardiac exploration," vol. 5505, pp. 164–173, SPIE, 2004.
- [50] R. Manzke, M. Grass, T. Nielsen, T. Koehler, and D. Hawkes, "Automatic phase point determination for cardiac ct imaging," vol. 5370, pp. 690–700, SPIE, 2004.
- [51] U. Stevendaal, P. Begemann, J. Borgert, R. Grewer, S. Krueger, H. Timinger, and M. Grass, "Determination of the temporal and spatial resolution in retrospectively gated cardiac cone-beam ct," vol. 5370, pp. 2015–2025, SPIE, 2004.
- [52] X. Tang, T. Pan, and K. Sasaki, "Ct cardiac imaging: evolution from 2d to 3d backprojection," vol. 5369, pp. 44–50, SPIE, 2004.

- [53] R. Manzke, M. Grass, T. Nielsen, G. Shechter, and D. Hawkes, "Adaptive temporal resolution optimization in helical cardiac cone beam ct reconstruction," *Med. Phys.*, vol. 30, no. 12, pp. 3072–3080, 2003.
- [54] M. Vembar, M. J. Garcia, D. J. Heuscher, R. Haberl, D. Matthews, G. E. Bohme, and N. L. Greenberg, "A dynamic approach to identifying desired physiological phases for cardiac imaging using multislice spiral ct," *Med. Phys.*, vol. 30, no. 7, pp. 1683–1693, 2003.
- [55] H. Assen, R. Geest, M. Danilouchkine, H. Lamb, J. Reiber, and B. Lelieveldt, "Three-dimensional active shape model matching for left ventricle segmentation in cardiac ct," vol. 5032, pp. 384–393, SPIE, 2003.
- [56] G. Shechter, G. Naveh, A. Altman, R. Proksa, and M. Grass, "Cardiac image reconstruction on a 16-slice ct scanner using a retrospectively ecg-gated multicycle 3d back-projection algorithm," vol. 5032, pp. 1820–1828, SPIE, 2003.
- [57] E. Cesmeli, P. Edic, M. Iatrou, J. Hsieh, R. Gupta, and A. Pfoh, "Cardiac-state-driven ct image reconstruction algorithm for cardiac imaging," vol. 4684, pp. 779–789, SPIE, 2002.
- [58] M. Iatrou, P. Edic, H. Cline, A. Ishaque, E. Cesmeli, and A. Pfoh, "Four-dimensional x-ray attenuation model of the human heart and the coronary vasculature and its use for the assessment of cardiac ct imaging technology," vol. 4682, pp. 153–162, SPIE, 2002.
- [59] C. Yu, Y. Ching, and S. Chen, "Reconstruction of 3d geometric models of pulmonary arteries from cardiac ct images," vol. 4683, pp. 492–499, SPIE, 2002.
- [60] E. Cesmeli, P. Edic, M. Iatrou, and A. Pfoh, "Novel reconstruction algorithm for multiphasic cardiac imaging using multislice helical ct," vol. 4320, pp. 645–654, SPIE, 2001.
- [61] J. Hsieh, J. Mayo, K. Acharya, and T. Pan, "Adaptive phase-coded reconstruction for cardiac ct," vol. 3978, pp. 501–508, SPIE, 2000.
- [62] J. Tamez-Pena and C. Chen, "Local force model for cine ct cardiac dynamics analysis," vol. 3033, pp. 334–345, SPIE, 1997.
- [63] J. Tamez-Pena, C. Chen, and K. Parker, "Local force model for cardiac dynamics analysis based on ct volumetric image sequences," vol. 2962, pp. 2–13, SPIE, 1997.

- [64] M. Kachelriess, S. Ulzheimer, and W. A. Kalender, "Ecg-correlated image reconstruction from subsecond multi-slice spiral ct scans of the heart," *Medical Physics*, vol. 27, no. 8, pp. 1881–1902, 2000.
- [65] J. D. Pearlman and Z. Yaseen, "Multiplexed space-time maps for time series data visualization: application to 4d cardiac imaging," vol. 3298, pp. 153–159, SPIE, 1998.
- [66] J. G. Tamez-Pena, C. W. Chen, and K. J. Parker, "Cardiac dynamic analysis using hierarchical shape models and gaussian curvature recovery: an integrated approach," vol. 2727, pp. 904–915, SPIE, 1996.
- [67] C. W. H. III and E. A. Hoffman, "Cardiac gated ventilation," vol. 2433, pp. 303–308, SPIE, 1995.
- [68] W. E. Higgins and E. L. Ritman, "3-d image-enhancement technique for volumetric cardiac images," vol. 1245, pp. 159–170, SPIE, 1990.
- [69] S. C. Moore and P. F. Judy, "Cardiac computed tomography using redundant-ray prospective gating," *Med. Phys.*, vol. 14, no. 2, pp. 193–196, 1987.
- [70] X. L. Xu, J.-S. Liow, and S. C. Strother, "Iterative algebraic reconstruction algorithms for emission computed tomography: A unified framework and its application to positron emission tomography," *Med. Phys.*, vol. 20, pp. 1675–1684, 1993.
- [71] Y. Zhang, H. Chan, B. Sahiner, J. Wei, M. M. Goodsitt, L. M. Hadjiiski, J. Ge, and C. Zhou, "Tomosynthesis reconstruction using the simultaneous algebraic reconstruction technique (sart) on breast phantom data," vol. 6142, *Medical Imaging 2006: Physics of Medical Imaging*, SPIE, 2006.
- [72] S. Matej, J. A. Fessler, and I. G. Kazantsev, "Iterative tomographic image reconstruction using fourier-based forward and back-projectors," *IEEE Med. Imag.*, vol. 23, no. 4, pp. 401–412, 2005.
- [73] A. Sitek, R. Huesman, and G. Gullberg, "Tomographic iterative reconstruction using unconstrained grids," pp. 275–278, *International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine*, 2005.
- [74] D. Langan, B. Claus, P. Edic, R. Vaillant, B. D. Man, S. Basu, and M. Iatrou, "An iterative algorithm for soft tissue reconstruction from truncated flat panel projections," vol. 6142, p. 614224, SPIE, 2006.

- [75] J. Thibault, C. Bouman, K. Sauer, and J. Hsieh, "A recursive filter for noise reduction in statistical iterative tomographic imaging," vol. 6065, p. 60650X, SPIE, 2006.
- [76] X. Wan, A. Yin, Y. Gao, X. He, X. Chen, X. Cheng, W. Zou, and S. Yu, "Discrete iterative optical computed tomography algorithm for reconstructions comprising opaque objects," *Optical Engineering*, vol. 44, no. 11, pp. 118001–118006, 2005.
- [77] J. Zhang, J. Sillanpaa, J. Chang, H. Amols, G. Mageras, and M. Anastasio, "Comparison of analytic and iterative algorithms for image reconstruction in gated megavoltage ct," *Med. Phys.*, vol. 32, no. 6, pp. 1941–1941, 2005.
- [78] J. Thibault, C. B. K. Sauer, and J. Hsieh, "Three-dimensional statistical modeling for image quality improvements in multi-slice helical ct," pp. 271–274, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [79] J. Hsieh, "Iterative approach to partial-volume artifact reduction in ct," vol. 3338, pp. 338–346, SPIE, 1998.
- [80] T. R. McNutt, T. R. Mackie, and B. R. Paliwal, "Analysis and convergence of the iterative convolution/superposition dose reconstruction technique for multiple treatment beams and tomotherapy," *Medical Physics*, vol. 24, no. 9, pp. 1465–1476, 1997.
- [81] I. A. Elbakri and J. A. Fessler, "Efficient and accurate likelihood for iterative image reconstruction in x-ray computed tomography," vol. 5032, pp. 1839–1850, SPIE, 2003.
- [82] J. A. Fessler, I. A. Elbakri, P. Sukovic, and N. H. Clinthorne, "Maximum-likelihood dual-energy tomographic image reconstruction," vol. 4684, pp. 38–49, SPIE, 2002.
- [83] I. A. Elbakri and J. A. Fessler, "Statistical x-ray-computed tomography image reconstruction with beam-hardening correction," vol. 4322, pp. 1–12, SPIE, 2001.
- [84] J. Sunnegadh, P. Danielsson, and S. Magnusson, "Iterative improvement of non-exact reconstruction in cone-beam ct," pp. 279–283, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.

- [85] R. Gordon, R. Bender, and G. T. Herman, "Algebraic reconstruction techniques (art) for three-dimensional electron microscopy and x-ray photography," *J. Theor. Biol.*, vol. 29, pp. 471–481, 1970.
- [86] A. H. Andersen and A. C. Kak, "Simultaneous algebraic reconstruction technique (sart). a superior implementation of the art algorithm," *Ultrason. Imaging*, vol. 6, pp. 81–94, 1984.
- [87] G. L. Zeng, G. T. Gullberg, P. E. Christian, and D. Gagnon, "Cone-beam iterative reconstruction of a segment of a long object," *IEEE Nuclear Science*, vol. 49, no. 1, p. 37C41, 2002.
- [88] G. Wang, D. L. Snyder, J. A. OSullivan, and M. W. Vannier, "Iterative deblurring for ct metal artifact reduction," *IEEE Med. Imag.*, vol. 15, p. 657C 663, 1996.
- [89] J. Wang and Y. Zheng, "Frequency domain simultaneous algebraic reconstruction techniques: algorithm and convergence," vol. 5674, pp. 344–353, SPIE, 2005.
- [90] Y. Zheng and H. Li, "Generalized algebraic reconstruction techniques," vol. 4792, pp. 19–25, SPIE, 2002.
- [91] F. Sun, J. Liu, and B. Zhu, "Algebraic reconstruction techniques in ct and their implementation," vol. 4549, pp. 155–160, SPIE, 2001.
- [92] K. Mueller and R. Yagel, "Use of graphics hardware to accelerate algebraic reconstruction methods," vol. 3659, pp. 615–625, SPIE, 1999.
- [93] T. Yuasa, Y. Watanabe, B. Devaraj, T. Akatsuka, and H. Inaba, "Algebraic reconstruction for transmission optical ct based on coherent detection imaging method," vol. 3598, pp. 242–252, SPIE, 1999.
- [94] H. Guan, "Multilevel algebraic reconstruction technique for x-ray computed tomography," *Medical Physics*, vol. 24, no. 3, pp. 473–473, 1997.
- [95] H. K. Tuy, "An inversion formula for cone-beam reconstruction," *SIAM J, Appl. Math.*, vol. 43, no. 3, pp. 546–562, 1983.
- [96] P. Grangeat, "Mathematical framework of cone beam 3d reconstruction via the first derivative of the radon transform," *Lecture Notes in Mathematics* 1497, pp. 66–97, 1992.

- [97] B. D. Smith, "Image reconstruction from cone-beam projections: Necessary and sufficient conditions and reconstruction methods," *IEEE Trans. Med. Imag.*, vol. MI-4, no. 3, pp. 14–25, 1985.
- [98] M. Defrise and R. Clack, "A cone-beam reconstruction algorithm using shift-variant filtering and cone-beam backprojection," *IEEE Trans. Med. Imag.*, vol. 13, no. 1, pp. 186–195, 1994.
- [99] M. Defrise and F. Noo, "Reconstruction of truncated cone-beam projections using the frequency-distance relation," pp. 32–35, International Meeting on Fully Bibliography Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 1997.
- [100] M. Defrise, F. Noo, and H. Kudo, "Quasi-exact region-of-interest reconstruction from helical cone-beam data," vol. 2, pp. 1101–1103, IEEE Medical Imaging, 1999.
- [101] S. Lee and G. Wang, "A helical half-scan cone-beam ct algorithm for short object geometry in the grangeat framework," *Med. Phys.*, vol. 35, no. 1, pp. 4–16, 2004.
- [102] S. Lee and G. Wang, "Grangeat-type half-scan algorithm for cone beam ct," vol. 5030, pp. 980–1000, SPIE, 2003.
- [103] S. Lee and G. Wang, "A grangeat-type half-scan algorithm for cone-beam ct," *Med. Phys.*, vol. 30, no. 4, pp. 689–700, 2003.
- [104] K. Tam, "Three-dimensional computerized tomography scanning methods and system for large objects with smaller area detectors," *US Patent 5,390,112*, 1995.
- [105] K. C. Tam, S. Samarasekera, and F. Sauer, "Exact cone-beam ct with a spiral scan," International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 1997.
- [106] K. C. Tam, G. Lauritsch, K. Sourbelle, F. Sauer, and B. Ladendorf, "Exact (spiral + circles) scan region-of-interest cone beam reconstruction via back-projection," *IEEE Trans. Med. Imag.*, vol. 19, no. 5, pp. 376–383, 2000.
- [107] M. Defrise, F. Noo, and H. Kudo, "A solution to the long object problem in helical cb tomography," *Phys. Med. Biol.*, vol. 45, pp. 623–643, 2000.

- [108] S. Lee and G. Wang, "Grangeat-type helical half-scan reconstruction of a long object," vol. 5535, pp. 522–530, SPIE, 2004.
- [109] T. Gomi, K. Koshida, and T. Miyati, "Cone-beam ct reconstruction using a nonlinear weighted filtered backprojection from half-scan data," vol. 6142, p. 61424Y, SPIE, 2006.
- [110] G. Lauritsch, K. C. Tam, K. Sourbelle, and S. Schaller, "Exact local regions-of-interest reconstruction in spiral cone-beam filtered-backprojection ct: numerical implementation and first image results," vol. 3979, pp. 520–532, SPIE, 2000.
- [111] K. C. Tam, "Exact local regions-of-interest reconstruction in spiral cone-beam filtered-backprojection ct: theory," vol. 3979, pp. 506–519, SPIE, 2000.
- [112] A. I. Katsevich, "Theoretically exact filtered backprojection-type inversion algorithm for spiral ct," *J. Appl. Math.*, vol. 62, no. 6, pp. 2012–2026, 2002.
- [113] Y. Zhou and X. Pan, "Exact image reconstruction on pi-lines from minimum data in helical cone-beam ct," *Phys. Med. Biol.*, vol. 49, no. 6, pp. 941–959, 2004.
- [114] A. K. K. Taguchi and A. A. Zamyatin, "Performance evaluation of exact and approximate cone-beam helical reconstruction algorithms," pp. 23–27, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [115] K. T. A. Katsevich and A. A. Zamyatin, "Formulation of four katsevich algorithms in native geometry," pp. 323–327, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [116] A. Katsevich, S. Basu, and J. Hsieh, "Further generalization of the exact fbp 1pi algorithm for helical ct," pp. 328–332, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [117] E. Sidky, Y. Zou, and X. Pan, "A minimum data fbp-type algorithm for image reconstruction in cone-beam ct," pp. 291–295, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.

- [118] Y. Wei, J. Hsieh, and G. Wang, "General formula for fan-beam computed tomography," *Physical Review Letters*, vol. 95, no. 25, p. 258102, 2005.
- [119] Y. Ye and G. Wang, "Filtered backprojection formula for exact image reconstruction from cone-beam data along a general scanning curve," *Med. Phys.*, vol. 32, no. 1, pp. 42–48, 2005.
- [120] Y. Ye, S. Zhao, H. Yu, and G. Wang, "Exact reconstruction for cone-beam scanning along nonstandard spirals and other curves," vol. 5535, pp. 293–300, SPIE, 2004.
- [121] H. Yu, Y. Ye, and G. Wang, "Katsevich-type algorithms for variable radius spiral cone-beam ct," vol. 5535, pp. 550–557, SPIE, 2004.
- [122] Y. Zhou and X. Pan, "Image reconstruction on pi-lines by use of filtered backprojection in helical cone-beam ct," *Phys. Med. Biol.*, vol. 49, no. 12, pp. 2717–2731, 2004.
- [123] Y. Ye, J. Zhu, and G. Wang, "Minimum detection windows, pi-line existence and uniqueness for helical cone-beam scanning of variable pitch," *Med. Phys.*, vol. 31, no. 3, pp. 566–572, 2004.
- [124] Y. Zou, X. Pan, D. Xia, and G. Wang, "Pi-line-based image reconstruction in helical cone-beam computed tomography with a variable pitch," *Med. Phys.*, vol. 32, pp. 2639–2648, 2005.
- [125] H. Yu, S. Zhao, Y. Ye, and G. Wang, "Exact bpf and fbp algorithms for nonstandard saddle curves," *Med. Phys.*, vol. 32, pp. 3305–3312, 2005.
- [126] D. Xia, Y. Zou, L. Yu, E. Sidky, and X. Pan, "Image reconstruction for scanning configurations with tilted helical and saddle trajectories," pp. 347–351, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [127] E. S. Y. Zou, X. Pan, "Image reconstruction for source trajectories with kinks," pp. 203–207, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [128] S. Zhao, H. Yu, and G. Wang, "A unified framework for exact cone-beam image reconstruction formulas," *Med. Phys.*, vol. 32, no. 6, pp. 1712–1721, 2005.

- [129] C. Kao, Y. Zou, and X. Pan, "Exact reconstruction in positron emission tomography with panel-based detectors," pp. 264–268, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [130] X. Tang, J. Hsieh, R. A. Nilsen, S. Dutta, D. Samsonov, and A. Hagiwara, "A three-dimensional-weighted cone beam filtered backprojection (cb-fbp) algorithm for image reconstruction in volumetric helical scanning," *Phys. Med. Biol.*, vol. 51, pp. 855–874, 2006.
- [131] R. Manzke, M. Grass, and D. Hawkes, "Artifact analysis and reconstruction improvement in helical cardiac cone beam ct," *IEEE Trans. Med. Imag.*, vol. 23, no. 9, pp. 1150–1164, 2004.
- [132] K. Taguchi and H. Aradate, "Algorithm for image reconstruction in multi-slice helical ct," *Med. Phys.*, vol. 25, no. 4, pp. 550–561, 1998.
- [133] H. Hu, "Multi-slice helical ct: Scan and reconstruction," *Med. Phys.*, vol. 25, no. 1, pp. 5–18, 1999.
- [134] S. Schaller, T. Flohr, K. Klingensbeck, J. Krause, and W. A. K. T. Fuchs, "Spiral interpolation algorithm for multislice spiral ct-part i: Theory," *IEEE Trans. Med. Imag.*, vol. 19, no. 9, pp. 822–834, 2000.
- [135] F. Noo, M. Defrise, and R. Clackdoyle, "Single-slice rebinning method for helical cone-beam ct," *Phys. Med. Biol.*, vol. 44, no. 2, pp. 561–570, 1999.
- [136] G. Larson, C. Ruth, and C. Crawford, "Nutating slice ct image reconstruction," *Patent Application WO 98/44847*, pp. 66–97, 1998.
- [137] M. Kachelreiss, S. Schaller, and W. Kalender, "Advanced single slice rebinning in cone-beam spiral ct," *Med. Phys.*, vol. 27, no. 4, pp. 754–772, 2000.
- [138] H. Bruder, M. Kachelreiss, S. Schaller, and T. Mertelmeier, "Performance of approximate cone-beam reconstruction in multi-slice computed tomography," pp. 541–554, SPIE Medical Imaging, 2000.
- [139] S. Schaller, K. Stierstorfer, and T. F. H. Bruder, M. Kachelreiss, "Novel approximate approach for high-quality image reconstruction in helical cone beam ct at arbitrary pitch," *Proc. SPIE*, vol. 4432, pp. 113–127, 2001.

- [140] T. Flohr, K. Stierstorfer, H. Bruder, J. Simon, and S. S. A. Polacin, "Image reconstruction and image quality evaluation for a 16-slice ct scanner," *Med. Phys.*, vol. 30, no. 5, pp. 832–844, 2003.
- [141] L. Chen, Y. Liang, and D. J. Heuscher, "General surface reconstruction for cone-beam multislice spiral computed tomography," *Med. Phys.*, vol. 30, no. 12, pp. 2804–2821, 2003.
- [142] K. Sourbelle and W. A. Kalender, "Generalisation of feldkamp reconstruction for clinical spiral cone beam ct," *Proceedings of the 2003 Meeting on Fully 3D Image Reconstruction in Radiology and Nuclear Medicine, France*, 2003.
- [143] G. Wang, T. Lin, P. Cheng, and D. M. Schiozaki, "A general cone-beam reconstruction algorithm," *IEEE Trans. Med. Imag.*, vol. 12, no. 3, pp. 486–496, 1993.
- [144] M. D. Silver, K. Taguchi, and K. S. Han, "Field-of-view dependent helical pitch in multi-slice ct," *Proc. SPIE*, vol. 4320, p. 839C850, 2001.
- [145] K. Taguchi, B. Chiang, and M. D. Silver, "New weighting scheme for cone-beam helical ct to reduce the image noise," *7th Int. Conf. on Fully 3D Reconstruction in Radiology and Nuclear Medicine, France*, 2003.
- [146] K. Taguchi, B. S. Chiang, and M. D. Silver, "A new weighting scheme for cone-beam helical ct to reduce the image noise," *Phys. Med. Biol.*, vol. 49, no. 5, pp. 2351–2364, 2004.
- [147] X. Tang, J. Hsieh, R. Nilsen, A. Hagiwara, J. Thibault, and E. Drapkin, "A three-dimensional weighted cone beam filtered backprojection (cb-fbp) algorithm for image reconstruction in volumetric ct under a circular source trajectory," *Phys. Med. Biol.*, vol. 50, pp. 3889–3905, 2005.
- [148] X. Tang, J. Hsieh, R. A. Nilsen, and S. M. McOlash, "Helical and axial cone beam reconstruction algorithms using a general window function," *IEEE Med. Imag. Conf. Rec.*, Puerto Rico, 2005.
- [149] P. E. Danielsson, P. P. Edholm, J. Eriksson, and M. Magnusson-Seger, "Towards exact 3d-reconstruction for helical cone-beam scanning of long objects: a new arrangement and a new completeness condition," pp. 25–28, *Meeting on Fully Three-dimensional Image Reconstruction in Radiology and Nuclear Medicine*, Pittsburgh, 1997.

- [150] H. Turbell and P. E. Danielsson, "An improved pi-method for reconstruction from helical cone-beam projections," Meeting on Fully Three-dimensional Image Reconstruction in Radiology and Nuclear Medicine, Seattle WA, USA, 1999.
- [151] Y. Liu, H. Liu, Y. Wang, and G. Wang, "Half-scan cone-beam ct fluoroscopy with multiple x-ray sources," *Med. Phys.*, vol. 28, no. 7, pp. 1466–1471, 2001.
- [152] L. Yu, D. Xia, Y. Zou, X. Pan, C. A. Pelizzari, and P. Munro, "Image reconstruction from truncated data in circular cone-beam ct and its applications to radiotherapy," pp. 391–395, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [153] H. Tuy, "3d image reconstruction for helical partial cone beam scanners using wedge beam transform," *US Patent 6104775*, 2000.
- [154] H. Tuy, "Methods and apparatus for cone-tilted parallel sampling and reconstruction," *US Patent 6263040 B1*, 2001.
- [155] M. Kachelriess, M. Knaup, and W. A. Kalender, "Extended parallel backprojection for standard three-dimensional and phase-correlated four-dimensional axial and spiral cone-beam ct with arbitrary pitch, arbitrary cone-angle, and 100no. 6, pp. 1623–1641, 2004.
- [156] K. Stierstorfer, A. Rauscher, J. Boese, H. Bruder, S. Schaller, and T. Flohr, "Weighted fbpa simple approximate 3d flbp algorithm for multislice spiral ct with good dose usage for arbitrary pitch," *Phys. Med. Biol.*, vol. 49, no. 11, pp. 2209–2218, 2004.
- [157] X. Yan and R. M. Leahy, "Cone-beam tomography with circular, elliptical and spiral orbits," *Phys. Med. Biol.*, vol. 37, pp. 493–506, 1992.
- [158] J. Hu, K. Tam, and J. Qi, "An approximate short scan helical fdk cone beam algorithm based on nutating curved surfaces satisfying the tuys condition," *Med. Phys.*, vol. 32, no. 6, pp. 1529–1536, 2005.
- [159] X. Tang and J. Hsieh, "A filtered backprojection algorithm for cone beam reconstruction using rotational filtering under helical source trajectory," *Med. Phys.*, vol. 31, no. 11, p. 2949C2960, 2004.
- [160] S. Schaller, T. Flohr, and P. Steffen, "New efficient fourier-reconstruction method for approximate image reconstruction in spiral cone-beam ct small

- cone angles," vol. 3032, p. 213C224, Proc. SPIE, Sandiego CA, USA, IEEE, 1997.
- [161] T. Herman, *Image reconstruction from projections: the fundamentals of Computerized Tomography*. Academic Press, 1980.
- [162] F. Natterer, *The Mathematics of Computerized Tomography*. John Wiley and Sons, 1986.
- [163] A. V. Lakshminarayanan, *Reconstruction from divergent ray data. Technical Report 92*. Department of Computer Science, State University of New York at Buffalo, 1975.
- [164] G. T. Herman and A. Naparstek, "Fast image reconstruction based on a radon inversion formula appropriate for rapidly collected data," *J. Appl. Math.*, vol. 33, pp. 511–533, 1977.
- [165] H. Turbell and P. E. Danielsson, "Fast feldkamp reconstruction," pp. 311–314, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 1999.
- [166] S. Schaller, "Practical image reconstruction for cone beam computed tomography," Ph. D. thesis, 1998.
- [167] D. Heuscher, "Helical cone beam scans using oblique 2d surface reconstruction," pp. 204–207, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 1999.
- [168] M. D. Silver, "Practical limits to high helical pitch, cone-beam computed tomography," pp. 44–47, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 1997.
- [169] P. E. Danielsson, P. Edholm, J. Eriksson, M. Magnusson-Seger, and H. Turbell, "Helical cone-beam scanning and reconstruction of long objects using 180 degrees exposure," *Patent Application PCT/SE 98/00029*, 1998.
- [170] G. L. Zeng and G. T. Gullberg, "Short-scan cone beam algorithm for circular and non-circular detector orbits," vol. 1233, pp. 453–463, SPIE Medical Imaging IV: Image Processing, 1990.
- [171] G. L. Zeng and G. T. Gullberg, "Short-scan fan beam algorithm for non-circular detector orbits," vol. 1445, pp. 332–340, SPIE Medical Imaging V: Image Processing, Technical Conference, 1991.

- [172] T. Kohler, R. Proksa, C. Bontus, M. Grass, and J. Timmer, "Artifact analysis of approximate helical cone-beam ct reconstruction algorithms," *Med. Phys.*, vol. 29, no. 1, pp. 51–64, 2002.
- [173] G. Besson, "Ct fan-beam parameterizations leading to shift-invariant filtering," *Inverse Problems*, vol. 12, no. 6, pp. 818–833, 1996.
- [174] H. Bruder, K. Stierstorfer, and T. Flohr, "Ct reconstruction from fan-parallel data," pp. 8–14, *IEEE Medical Imaging*, 1998.
- [175] D. L. Parker, "Optimal short scan convolution reconstruction for fan beam ct," *Med. Phys.*, vol. 9, no. 2, pp. 254–257, 1982.
- [176] I. Mori and T. Suzuki, "Development of advanced multislice ct scanner acquisition," *Med. Rev.*, vol. 78, pp. 1–7, 2001.
- [177] M. D. Silver, "A method for including redundant data in computed tomography," *Med. Phys.*, vol. 27, no. 4, pp. 773–774, 2000.
- [178] G. Wang, Y. Liu, T. Lin, and P. Cheng, "Half-scan cone-beam x-ray microtomography formula," *Journal of Scanning Microscopies*, vol. 16, pp. 216–220, 1994.
- [179] M. D. Silver, "3high-helical-pitch, cone-beam computed tomography," *Phys. Med. Biol.*, vol. 43, no. 4, pp. 847–855, 1998.
- [180] F. Noo, H. Kudo, and M. Defrise, "Approximate short-scan filtered-back-projection for helical cb reconstruction," vol. 1445, pp. 8–14, *IEEE Medical Imaging*, 1998.
- [181] X. Tang, "Matched view weighting in tilted-plane-based reconstruction algorithms to suppress helical artifacts and optimize noise characteristics," *Med. Phys.*, vol. 30, no. 11, pp. 2912–2918, 2003.
- [182] J. Hsieh, "Tomographic reconstruction for tilted helical multislice ct," *IEEE Trans. Med. Imag.*, vol. 19, no. 9, pp. 864–872, 2000.
- [183] F. Noo and H. K. M. Defrise, "General reconstruction theory for multislice x-ray computed tomography with a gantry tilt," *IEEE Trans. Med. Imag.*, vol. 23, no. 9, pp. 1109–1116, 2004.

- [184] J. Wang, T. Li, H. Lu, and Z. Liang, "Noise reduction for low-dose helical ct by 3d penalized weighted least-squares sinogram smoothing," vol. 6142, SPIE Medical Imaging 2005, 2006.
- [185] B. R. Whiting, P. Massoumzadeh, O. A. Earl, J. A. O'Sullivan, D. L. Snyder, and J. F. Williamson, "Properties of preprocessed sinogram data in x-ray computed tomography," *Medical Physics*, vol. 33, no. 9, pp. 3290–3303, 2006.
- [186] L. Zhu, N. Strobil, and R. Fahrig, "X-ray scatter correction for cone-beam ct using moving blocker array," vol. 5745, SPIE Medical Imaging 2005, 2005.
- [187] T. Li, J. Wang, J. Wen, X. Li, H. Lu, J. Hsieh, and Z. Liang, "Snr-weighted sinogram smoothing with improved noise-resolution properties for low-dose x-ray computed tomography," vol. 5370, SPIE Medical Imaging 2004, 2004.
- [188] T. Li, X. Li, J. Wang, J. Wen, H. Lu, J. Hsieh, and Z. Liang, "Nonlinear sinogram smoothing for low-dose x-ray ct," *IEEE Trans. Nuclear Science*, vol. 51, pp. 2505–2513, 2004.
- [189] H. Lu, I. Hsiao, X. Li, and Z. Liang, "Noise properties of low-dose ct projections and noise treatment by scale transformations," IEEE NSS-MIC, 2001.
- [190] H. Lu, X. Li, and Z. Liang, "Analytical noise treatment for low-dose ct projection data by penalized weighted least-square smoothing in the k-l domain," vol. 4682, SPIE Medical Imaging 2002, 2002.
- [191] G. C. E. Nett, T. Zhuang, "A cone-beam fbp reconstruction algorithm for short-scan and super-short-scan source trajectories," pp. 396–400, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.
- [192] H. Bruder, K. Stierstorfer, and T. Flohr, "Dynamic cardiac ct imaging using detectors with large cone angle," pp. 405–408, International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, 2005.