Multivariable control system design based on relative interaction energy

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Multivariable Control System Design
Based on
Relative Interaction Energy

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in fulfillment of the requirement for the degree of
Doctor of Philosophy

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Summary

For the design of *multi-input multi-output* (MIMO) system, two major tasks have to be resolved, i.e., loop pairing and controller design. For the first task, one needs to pair the input and output variables in such a way that the interactions among different loops are smallest. The other task is control system design according to paired variables. Generally, a good pairing results in relatively easy controller design. Unfortunately, both loop pairing and controller design are difficult since there are complicated interactions among different loops. Especially for controller design, adjusting controller parameters of one loop will affect the performance of others, sometimes to the extent of destabilizing the entire system. To ensure stability, many industrial controllers for multivariable processes are tuned loosely, which causes inefficient operation and higher energy costs.

In this thesis, *relative energy array* (REA) is defined by using steady-state gains and critical frequencies of the transfer function matrix elements, and used to provide a simple yet comprehensive description of loop interactions for MIMO processes. The value of REA can be conveniently calculated by control engineers since the critical frequencies can be easily obtained from given transfer function matrix. Furthermore, REA is used to derive simple *equivalent transfer functions* (ETFs) for each transfer function element under closed-loop control, which is fundamental in control system design. Consequently, the decentralized, full decoupling and block decoupling controllers can be easily designed by simply using single loop design approaches. The method is very simple, straightforward and easily understood by control engineers. The advantage of the proposed method is even more significant when applied to higher dimensional processes with complicated interaction modes. As a matter of fact, higher dimension does not increase the design difficulty. The simulation results for a variety of industrial processes show that the performance of the proposed controller design method is much better as compared with the existing approaches. The proposed method has

also been implemented in an experimental HVAC system to illustrate its effectiveness.

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Chapter 1

Introduction

1.1. Motivations

Most industrial operating units are naturally characterized by MIMO processes. However, MIMO controllers are more difficult to design even if the controllers themselves are specified with simple structures. The efficient tuning of decentralized PID controllers for multivariable processes is a very complex and time-consuming procedure. To relieve the pain of tuning MIMO systems, there has been strong interest in developing effective control system design approaches which are easy to use and effective in solving real process control problems.

Even though considerable effort has been dedicated to this problem and a great many design techniques have been proposed over recent years, there is no simple and yet systematic approach for the design of MIMO control systems. A unified and generally accepted definition is still lacking. Important issues, such as the effects of interaction on closed-loop performance, the measure of the severity of interaction, and the impact of variable pairing and controller tuning on interaction and subsequently on closed-loop performance, are far from being resolved.

Consequently, multivariable control system design and implementation continues to present a challenge to researchers as well as practical engineers alike. As a matter of fact, only very few of them have been successfully applied to solve real world problems.

1.2. Objectives

Adjusting controller parameters of one loop affects the performance of others, sometimes to the extent of destabilizing the entire system. To ensure stability, many industrial multi-loop controllers are tuned loosely, which causes inefficient operation resulting in higher energy costs and shorten the equipment life. The task of developing a satisfactory design procedure for multivariable process control is a big challenge and still remains as an open topic. This research is focused on developing a method for simple interaction measure and the decentralized/decoupling controller design for MIMO systems.

In summary, this thesis attempts to develop a practical and engineering oriented design methodology for the control of interactive MIMO systems.

1.3. Major contributions of the thesis

In this thesis, a systematic design approach for decentralized/decoupling control systems is proposed for general MIMO processes. Through intensive investigations on interaction analysis and controller tuning, several significant results are obtained. The major contributions of the thesis are summarized as follows:

- (1) A novel interaction measure and loop pairing criterion for decentralized control of multivariable processes is proposed. Both steady-state gain and critical frequency of the elements in the transfer function matrix are used to provide an effective description of loop interactions. The criterion can be conveniently applied by control engineers since the information needed can be easily obtained from the process model. It gives accurate interaction assessment even for some complex processes while the conventional *relative gain array* (RGA) based loop pairing criterion may result in a wrong conclusion.
- (2) A novel method for decentralized control system design based on the proposed interaction measure is proposed. The loop interactions are

quantified by relative gain and relative critical frequency. The interactions for a particular loop from all other closed loops are then analyzed through both steady-state gain and critical frequency variations. Consequently, the equivalent transfer function is constructed and the decentralized controllers can be independently designed by employing single loop tuning techniques. Differing from existing equivalent transfer functions, the proposed equivalent transfer function provides both gain and phase information for decentralized controller design in a simple and straightforward manner.

- (3) A novel full decoupling/block decoupling design approach based on the equivalent transfer function matrix for closed-loop system is proposed. Based on this matrix, both off-diagonal controllers and main-loop diagonal controllers can be easily designed using the existing PI/PID tuning rules. The decoupling controller obtained in this way has good set-point tracking and disturbance rejection properties. It still works with satisfactory performance even if the primary process parameters have significant variations as compared with existing decoupling designs.

1.4. Organization of the thesis

The rest of thesis is organized as follows:

Chapter 2 gives a review in the area of multivariable control system, especially in control system configuration, decentralized and decoupling control system design.

Chapter 3 presents an interaction measure and loop pairing criterion based on the concept of relative energy.

Chapter 4 presents a decentralized controller design approach based on the closed-loop equivalent transfer functions.

Chapter 5 presents a framework for full decoupling and block decoupling controller design approaches.

Chapter 6 describes the application of the proposed approach to the room

temperature control of an experimental HVAC system.

Chapter 7 concludes the thesis and suggests some possible future research topics in this field.

Chapter 2

Literature Review

2.1. Introduction

In modern processes, there are mainly two multi-level hierarchies of control functions as shown in Figure 2.1 (Qin and Badgwellb, 2003). A conventional control structure is shown in Figure 2.1(a) for n operating units, while Figure 2.1 (b) shows a MPC structure for the same n units. In both hierarchy structures, a plant-wide optimizer determines optimal steady-state settings for each unit in the plant at the top of the structure. These may be sent to local optimizers at each unit which run more frequently or consider a more detailed unit model at the plant-wide level. The unit optimizer computes an optimal economic steady-state and passes this to the dynamic constraint control system for implementation. In conventional structure (Figure 2.1(a)), it is accomplished by using a combination of PID algorithms, lead-lag (L/L) blocks and high/low select logic. While in MPC structure (Figure 2.1(b)), this combination of blocks is replaced by a single MPC controller which using dynamic constraint control to move the plant from one constrained steady-state to another while minimizing constraint violations along the way. In any hierarchy structure, however, the bottom level regulating loops are PID controller based DCS systems. As the performance of the bottom level is instrumental in plant-wide optimization, it is an important part of a multi-level hierarchy of control functions.

(a) Conventional structure

(b) MPC structure

Figure 2.1. Hierarchy of control system functions in a typical processing plant

Most industrial operating units are naturally characterized by MIMO processes. MIMO controllers are more difficult to design even if the controllers themselves (PID) are specified with simple structures. The efficient tuning of decentralized PID controllers for multivariable processes is a very complex and time-consuming procedure. Generally, a multivariable process can be controlled either by a multivariable (centralized), a decoupling or a multi-loop (decentralized) controller. Each control structure has advantages and disadvantages. For industrial multivariable processes, the choice of an appropriate control structure depends on the process characteristics, the economic value of improved control, and the available computing and process control resources. The various control structures are evaluated by taking into account the complexity of implementation, the robustness of the algorithm for parametric variations, and the performance for set-point changes and disturbance rejection.

Indeed, decentralized PI/PID controllers either in the simple multi-loop form or decoupling based multi-loop form remain the standard in process control applications (Campo and Morari, 1994), especially in large scale industrial processes, even though the performance may be inferior owing to the constraint on the controller structure. The underlying advantages responsible for the wide acceptance of decentralized control structures are:

- (1) Hardware simplicity: The cost of implementing of a decentralized control system is significantly less than that of a centralized controller. A centralized control system for an n -input, n -output plant usually consists of $n \times n$ individual *single-input single-output* (SISO) transfer functions, which significantly increases the complexity of the controller hardware. Furthermore, if the controlled and/or manipulated variables are physically far apart, a full controller could require numerous expensive communication links. Decentralized control structure, on the other hand, allows the use of already existing equipment and easy to implement and be understood.
- (2) Design and tuning simplicity: Decentralized controllers involve far fewer parameters, resulting in a significant reduction in the time and cost of tuning. For PID control of a *two-input two-output* (TITO) process, for

example, only 6 parameters need to be tuned instead of possible 12 for decoupling control and possible even more for centralized control.

- (3) Robustness: The failure tolerance is usually easier to achieve in decentralized control systems (Morari and Zafiriou, 1989). In many systems, it is important that stability and performance are preserved to some degree when some sensors or actuators fail.

Decentralized and decoupling control design consist of two stages: loop pairing and multi-loop or decoupling controller design. Considerable effort has been dedicated to this problem and a great many design techniques have been proposed in recent years. The main achievements will be briefly reviewed in the following sections after some preliminaries.

2.2. Preliminaries

A multivariable control system with n inputs and n outputs can be typically shown as in Figure 2.2, where $r_i, i=1,2,\dots,n$, are the reference inputs, $u_i, i=1,2,\dots,n$, are the manipulated variables, $d_i, i=1,2,\dots,n$, are the output disturbances, $y_i, i=1,2,\dots,n$, are the system outputs, $G(s)$ is the process transfer function matrix expressed by

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & g_{n2}(s) & \dots & g_{nn}(s) \end{bmatrix}, \quad (2.1)$$

$G_c(s)$ and $G_d(s)$ are the controller and disturbance transfer function matrix with compatible dimensions whose structures are determined by the chosen control schemes, respectively.

Figure 2.2. Closed-loop multivariable control system

In order to make use of SISO techniques, a multivariable process can be controlled by either decentralized, decoupling or block decoupling control schemes. The choice of the appropriate control scheme depends on the process characteristics, the economic value of improved control, and the available computing and process control resources. The pros and cons as well as the bases of selecting different control schemes may be stated as follows.

Decentralized control: It has a diagonal controller structure

$$G_c(s) = \begin{bmatrix} g_{c1}(s) & 0 & \dots & 0 \\ 0 & g_{c2}(s) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & g_{cn}(s) \end{bmatrix}, \quad (2.2)$$

and it is often used for processes with mild interactions. The control structure simplicity, easiness to design and implement, and loop failure tolerance (i.e., without the need for readjustment to the other parts of the control system to preserve stability in the case of a sensor failure and/or actuator failure) are the main advantages.

Full decoupling control: It has a full controller structure

$$G_c(s) = \begin{bmatrix} g_{c,11}(s) & g_{c,12}(s) & \dots & g_{c,1n}(s) \\ g_{c,21}(s) & g_{c,22}(s) & \dots & g_{c,2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{c,n1}(s) & g_{c,n2}(s) & \dots & g_{c,nm}(s) \end{bmatrix}, \quad (2.3)$$

and it is often adopted for two-dimensional closely coupled processes or systems which require higher control quality. By eliminating the undesirable cross-couplings responsible for the control loop interactions through decoupling

controller, the control system performance and robustness can be improved. The drawbacks may be the sacrifice of structure simplicity and control system integrity.

Block decoupling control: It has a block diagonal controller structure

$$G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 & \dots & 0 \\ 0 & G_{c2}(s) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & G_{cm}(s) \end{bmatrix}, \quad (2.4)$$

where $m < n$, and $G_{cl}(s)$, $l = 1 \dots m$, are controller sub-matrices. It is a compromise between the above two schemes for high dimensional processes. It only decouples those closely coupled subsystems while the whole structure is still decentralized. The integrity may be kept intact between the sub-blocks.

Define the sensitivity and complementary sensitivity functions as

$$S(s) \triangleq (I + G(s)G_c(s))^{-1}$$

and

$$T(s) \triangleq G(s)G_c(s)(I + G(s)G_c(s))^{-1}$$

respectively, where, $T(s)$ is the closed-loop transfer function from the reference inputs, r , to the system outputs, y , and $S(s)$ is the closed-loop transfer function from the output disturbances, d , to the system outputs, y .

Using the sensitivity and complementary sensitivity functions, the closed-loop transfer function of Figure 2.2 can be written as

$$Y(s) = T(s)R(s) + S(s)D(s) \quad (2.5)$$

The ultimate performance goals for the closed-loop control system to achieve are tracking set-point changes, rejecting disturbances, and robustness to parameter variations. For physical realization, $G_c(s)$ must be selected such that all poles of $T(s)$ and $S(s)$ lie in the open *left half plane* (LHP). Furthermore, for the feedback control system to have no offset for set-point or disturbance changes, we need

$$\lim_{s \rightarrow 0} T(s) = 1 \text{ and } \lim_{s \rightarrow 0} S(s) = 0.$$

2.3. Interaction analysis and loop pairing

In addition to y_j , the assigned corresponding output variable, the manipulated

variable u_i also influences other outputs in a closed-loop multivariable control system. This interaction problem is recognized as one of the main problems for the control of multivariable processes. For a system with several input and output variables, it can be configured in several different ways. Each u_j to y_i or pairing, constitutes a control configuration. For a TITO system there are two such configurations:

Configuration 1	Configuration 2
$u_1 \rightarrow y_1$	$u_1 \rightarrow y_2$
$u_2 \rightarrow y_2$	$u_2 \rightarrow y_1$

These two configurations are *mutually exclusive*, i.e., only one of them can be used at any particular time. For $n \times n$ dimensional systems, there are $n!$ possible input-output configurations.

Interaction analysis and structure selection play an important role in decentralized control systems. The major objective in selecting pairing is primarily to minimize the loop interaction so that the resulting multivariable control system mostly resembles its SISO counterparts and the subsequent controller tuning is largely facilitated by independent design (Seborg and Edger, 1989). This objective is usually accomplished by utilizing various interaction measures as tools to screen possible pairing alternatives (Bristol, 1966; Mc Avoy, 1983; Zhu and Jutan, 1993). The following aspects are important in the development of a measure for loop interactions:

- (1) Interaction should be defined on the basis of the dynamic framework, since a rigorous definition of interaction must be valid at more than just steady-state. This is clear by noticing that there is no interaction at steady-state if integral action is used and the system is stable;
- (2) Interaction should be extracted from the true dynamics of the closed-loop system without making any overly simplifying assumption about the system concerned and should have physical significance;
- (3) Interaction should be defined in terms of individual control loops in relation to corresponding interaction-free subsystems, so that direct implications for controller tuning and variable pairing can be better explored.

Some popular interaction measures and paring rules for process control will be summarized below.

2.3.1. RGA based loop paring

Consider the $n \times n$ plant $G(s)$ with input $u(s)$ and output $y(s)$ as shown in Figure 2.2, define relative gain and *relative gain array* (RGA) as (Bristol, 1966; Grosdidier and Morari, 1985; Niederlinski, 1971)

$$\lambda_{ij} = \frac{\left(\partial y_i / \partial u_j \right)_{\text{all loops open}}}{\left(\partial y_i / \partial u_j \right)_{\text{all other loops close except for loop } y_i - u_j}},$$

and

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \dots & \lambda_{nn} \end{bmatrix}.$$

respectively. Since

$$y(s) = G(s)u(s),$$

which implies

$$u(s) = G^{-1}(s)y(s).$$

Obviously, if all $y_{k, k \neq j}$ are perfectly controlled by the use of $u_{l, l \neq i}$ then the gain from u_i to y_j is given by $1/[G^{-1}]_{ji}$. The relative gain is actually the ratio of these “open-loop” and “closed-loop” gains while $s = 0$, and can be computed by (Grosdidier and Morari, 1985)

$$\Lambda = G(0) \otimes G^{-T}(0)$$

where the operator $\otimes$ is the hadamard product and G^{-T} represents the transpose of inverse of G .

The primary use of the RGA is to determine control loop pairings in multi-loop control systems, and in assessing the extent of interaction present in such systems. Further more, Niederlinski (1971) introduced *Niederlinski index* (NI). The NI is defined, also using steady-state gain matrix of a process, as below:

$$NI \equiv \frac{\det(G(0))}{\det(\bar{G}(0))} = \frac{|G(0)|}{\prod_{i=1}^n g_{ii}(0)}$$

where $\det(G(0))$ and $|G(0)|$ denote the determinant of matrix $G(0)$ and $\bar{G}(0) = \text{diag}(G(0))$. Under closed-loop conditions in all n loops, the system will be unstable for all possible values of controller parameters if the NI is negative (Mc Avoy, 1983; Seborg and Edger, 1989; Skogestad and Lundström, 1990). For NI, the following are worth noticing:

- (1) A negative NI is a necessary and sufficient condition for instability only for TITO systems; for higher dimensional systems, it is only a sufficient conditions: if it holds then the system is definitely unstable; otherwise, the system may, or may not be unstable depending on the values taken by the controller parameters;
- (2) For 2×2 systems the NI becomes:

$$NI = 1 - \xi$$

where $\xi = \frac{\lambda - 1}{\lambda}$. If $\xi > 1$, the NI is always negative; hence 2×2 systems paired with negative relative gains are always structurally unstable.

- (1) The NI is for systems with rational transfer function elements, thereby excluding time-delay systems. However, since it depends only on steady-state gains, the results of this theorem provide useful information about time-delay systems as well, although the analysis is no longer rigorous. It should be applied with caution when time delays are involved.

In summary, the rules for using the RGA and NI for loop pairing are:

- (1) Given $G(s)$, obtain steady-state gain matrix $G(0)$ and the RGA;
- (2) Tentatively choose a loop pairing based on the RGA by pairing on positive elements that are closest to 1.0;
- (3) Use NI to verify the stability status of the control configuration resulting from step 2; if the pairing is unacceptable, i.e., $NI < 0$, try another pairing;

- (4) Variables should be paired in such a way that the resulting pairing corresponds to an NI closest to 1.0. The NI interaction rule is based on empirical observations of the definition of the NI and is justified largely from the relationship between the sizes of the RGA. The NI interaction rule has been found to be capable of avoiding ambiguities in using the RGA interaction rule.

The RGA based pairing is further enhanced by taking into account integrity (Chiu and Arkun, 1990; Zhu and Jutan, 1993) as well as robustness (Zhu and Jutan, 1993) considerations, together with the use of the RGA and the NI.

The use of the RGA was popularized by Shinskey (1988), who applied it to numerous examples, including blending, energy conservation, and distillation. McAvoy (1983) published a monograph in which many of the applications of the RGA up to that date were presented. From a cost benefit point of view, the RGA method is highly effective due to the following facts:

- (1) It requires minimal process information, and it is in the form of a ratio so that an approximate process model can give useful results;
- (2) It is independent of control system tuning and process disturbances and it is simple to calculate.

Despite the availability of vast amount of literature on RGA, there are some common misconceptions. Kariwala and Hovd (2006), in their distinguished work, pointed out a number of incorrect claims and disproved some conjectures related to RGA. For example, they showed that RGA alone cannot be used as a definite measure of two-way interactions in a system, and pointed out that the RGA based results are time-tested heuristics, and that the theoretical basis for some of the results is poor. According to the authors, cross-checking of the findings using alternate methods is essential. In the absence of verification, the chosen control structure may not be optimal in the sense of missed opportunities and the closed-loop system may spring many surprises at the practicing engineer.

Indeed, the RGA based rules have been found to have deficiencies, and in some cases they lead to incorrect conclusions on how control loops should be paired and how much loop interaction exists. The possible situations are:

- (1) In MIMO systems disturbances can have a profound effect on the transient performance (Stanley *et al.*, 1985). However, the RGA does not provide any insight on this aspect;
- (2) In the case of one-way interaction, RGA does not apply even though the effect could be severe;
- (3) RGA does not consider dynamics, which may lead to incorrect loop pairings.

2.3.2. RIA based loop paring

This is a RGA-like approach. The relative interaction for loop $y_i - u_j$ is defined as the ratio of two elements: the increment of the process gain after all the other $n-1$ control loops are closed and the apparent gain in the same loop when all the other control loops are open (Bristol, 1967), i.e.,

$$\phi_{ij} = \frac{(\partial y_i / \partial u_j)_{\text{all other loops close except for loop } y_i - u_j} - (\partial y_i / \partial u_j)_{\text{all loops open}}}{(\partial y_i / \partial u_j)_{\text{all loops open}}}.$$

This concept was further developed by Zhu and Jutan (1996), where loop interaction can be exposed by investigating the response of an output to its input in the presence of interaction from other loops through the hidden loops. This situation can be best depicted by Figure 2.3. Focusing on an arbitrary control loop with u_j paired with y_i , output in response to any change in the input consists of two parallel paths: the direct forward path via the interaction free process and the parallel path via the hidden loops due to off-diagonal elements of the process.

(a) absolute interaction

(b) relative interaction

Figure 2.3. Definition of dynamic interaction in $u_j - y_i$ loop

Here, $a_{ij}(s)$ is the absolute interaction, which can be obtained by the signal flow technique (Zhu and Jutan, 1996), the relative interaction can be then represented as the ratio of the absolute interaction and the independent interaction free process as

$$\varphi_{ij}(s) = \frac{a_{ij}(s)}{g_{ij}(s)}.$$

At steady-state, assuming 1) the closed-loop system possesses integrity against any single-loop failure and 2) each controller in each control loop contains integral action, we obtain (Shinskey, 1988)

$$a_{ij}(0) = \frac{1}{G_{ji}^{-1}(0)} - g_{ij}(0).$$

Hence, at steady-state we obtain the relative interaction

$$\varphi_{ij} = \frac{1}{g_{ij}(0)G_{ji}^{-1}(0)} - 1.$$

All the individual relative interaction terms constitute a *relative interaction array* (RIA) as

$$\varphi_{ij} = [RIA]_{ij}.$$

Compared with the definition of the RGA, it is easy to give

$$\phi_{ij} = \frac{1}{\lambda_{ij}} - 1.$$

Clearly, the RIA provides an equivalent interaction measure to the RGA. Loop interaction and its implications can be briefly characterized by the RIA as follows:

- (1) $\phi_{ij} = 0 \Rightarrow$ no interaction;
- (2) $\phi_{ij} > 0 \Rightarrow$ interaction acts in the same direction as interaction-free process;
 $\phi_{ij} > 1.0 \Rightarrow$ interaction dominates over interaction-free process gain;
- (3) $\phi_{ij} < 0 \Rightarrow$ interaction acts in the reverse direction as interaction-free process gain;
- (4) $\phi_{ij} < -1.0 \Rightarrow$ reverse interaction dominates over interaction-free process gain.

Under RIA based pairing criterion, variables should be paired in such a way so that:

- (1) All the RIA elements are closest to 0;
- (2) NI is positive;
- (3) All the RIA elements are greater than -1;
- (4) RIA elements close to -1 are avoided.

It is important to notice that both very large positive and very large negative RGA elements drive loop interaction toward the inverse direction of the original interaction-free process gain and that both very small positive and very small negative RGA elements are also, actually even more, harmful to process control. Apparently, in contrast to the RIA, the RGA exhibits discontinuous behavior when used to characterize loop interaction.

Although the above interaction rule seems to be equivalent to the RGA interaction rule, the closeness to a value (0 in RIA rule, 1.0 in RGA rule) has different implications. Specifically, the distance of the RGA elements from 1.0 may not realistically reflect the amount of interaction as noted previously, whereas the distance of the RIA does. In particular, the RGA interaction rule mainly targets at dismissing large RGA elements, which makes the robustness rule redundant, whereas the RIA interaction rule essentially aims at eliminating small RGA elements (large interaction), which complements the robustness rule.

However, the RIA does not contain any more information on the process than the RGA, although they provide valuable alternative viewpoints and smoother interpretation. The common feature of those indices is that they only use the system model at zero frequency.

2.3.3. Dynamic RGA (DRGA) based loop paring

DRGA is another approach to evaluate the interactions, which may overcome some limitations from the RGA based approaches. The first paper to define and propose an approach to solve DRGA was published by Witcher and Mc Avoy (1977). They used a transfer function model in place of the traditional steady-state model used for the RGA calculation. The denominator of the DRGA involves achieving perfect control at all frequencies, while the numerator is simply the open-loop transfer function. Other methods to calculate DRGA were also proposed (Bristol, 1979; Tung and Edgar, 1981; Gagnepain and Seborg, 1982). Since the DRGA is most valuable for screening alternate control system designs, the requirement of an extensive controller design tends to defeat the utility of these methods. A better approach to define a useful DRGA should involve relatively little user interaction in the controller design aspect of the analysis.

Recently, Mc Avoy *et al.* (2003) presented a new approach to define a DRGA. It is assumed that a dynamic model of the process is available. If the model is a transfer function model then it is converted to a state space model. The optimal proportional output feedback controller is used to define the DRGA. The scaled linear state space dynamic model is given by

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

The objective function to be minimized is defined as

$$J = \frac{1}{2} \int_0^{\infty} (y^T Q y + u^T R u) dt.$$

The optimal control problem to be solved is a linear quadratic regulator with output feedback (Levine and Athans, 1970). Here, Q and R are typically taken as

identity matrices, since u and y have already been scaled by their operating ranges. Such a choice for Q and R treats all measurements and manipulated variables equally. An output feedback matrix gives the admissible controls as

$$u = -Kx$$

where K is an optimal feedback controller.

By solving the expected value of J for the case where the initial state forcing lies on a unit sphere around the origin, the DRGA which is based on the optimal proportional controller K can be defined for the $(i, j)^{th}$ element of the DRGA as

$$\lambda_{Dij} = \frac{\partial u_i / \partial y_j \big|_{u_k \neq 0, k \neq i}}{\partial u_i / \partial y_j \big|_{u_k = 0, k \neq i}},$$

where both terms give the gain of u_i to y_j during a transient in which the process is controlled using the optimal output proportional gain matrix, K . Furthermore, from $u = -Kx$, the partial derivatives can be calculated to give the following result

$$\lambda_{Dij} = \frac{-K_{ij}}{-1/\hat{K}_{ji}} = K_{ij} \hat{K}_{ji},$$

where $\hat{K}$ is equal to K^{-1} . The following interpretation of λ_{Dij} and pairing rule is proposed. If λ_{Dij} is close to 1, then the optimal gain between u_i and y_j remains the same regardless of whether the remaining manipulated variables are changing aggressively or moving very little. If $\lambda_{Dij} \approx 1.0$, then one can interpret this result to mean that the manipulated variable under consideration does not interact with the other manipulated variables so far as y_j is concerned. Thus, pairing y_j with u_i would be one candidate for SISO control. If a manipulated variable has a negative value of λ_{Dij} , then its behavior switches sign depending on how aggressively the other manipulated variables are moving. Such a pairing should be avoided since interaction is likely to be high. Large values of λ_{Dij} also indicate large changes in the behavior of a manipulated variable, which in turn indicate that interaction is high.

Even though in all examples above, the new DRGA results in the best pairings to use and it accurately assesses the extent of interaction present, its disadvantages are also obvious:

- (1) This pairing rule is still heuristic;
- (2) Complex computing: a conversion from a transfer function model to a state space model and a linear quadratic optimal control solution are required;
- (3) The value of λ_{Dij} is sensitive to the weighting matrices, Q and R .

2.3.4. GRDG based loop paring

Another interesting dynamic interaction measure is the generalized relative dynamic gain (GRDG) (Huang *et al.*, 1994). The GRDG also takes into account the dynamics of the closed-loops. The element $\lambda_{11}(s)$ of GRDG for a TITO process is defined as:

$$\lambda_{11}(s) = \frac{g_{11}(s)g_{22}(s)\frac{r_2(s)}{y_2(s)}}{g_{11}(s)g_{22}(s)\frac{r_2(s)}{y_2(s)} - g_{12}(s)g_{21}(s)}$$

where the variables $r_2(s)$ and $y_2(s)$ are the set-point and the process output of the other loop, and $\frac{r_2(s)}{y_2(s)}$ is the desired dynamics of the second loop. The transfer functions $g_{11}(s)$, $g_{12}(s)$, $g_{21}(s)$ and $g_{22}(s)$ are the elements of the process transfer matrix $G(s)$.

The loop paring rules are as below:

- (1) Eliminate the infeasible pairings by normal RGA procedure;
- (2) Calculate interaction potentials to obtain the most feasible pairings.

Pomerleau and Pomerleau (2001) gave a practical representation of the GRDG as a function of both closed-loop bandwidths. For easier control and tuning, the specifications on the closed-loop set-point responses have to be chosen in a frequency band where interaction is reduced so the system behaves more like SISO systems. In order to do so, the closed-loop response specifications are chosen in frequency band where the GRDG is close to one since, as for RGA, it means that the interaction is low.

Because the zeros in a transfer function affect the process dynamic, the GRDG, which takes into account the dynamic part of the transfer function, is preferred to the RGA, which only considers the steady-state. However, as can be seen from $\lambda_{11}(s)$, the complexity of representation and computation of the RDGA will dramatically increase if the system dimension increases.

2.3.5. SSV based loop paring

A shortcoming of many currently available interaction measures is their limited theoretical basis. Using the notion of *structural singular value* (SSV), Grosdidier and Morari (1986) presented a dynamic interaction measure for multivariable systems under feedback with diagonal or block diagonal controllers.

Let $G(s)$ be the process, and $\tilde{G}(s)$ be a matrix consisting of the block diagonal elements of $G(s)$. A block diagonal controller

$$C(s) = \text{diag}(C_1(s), C_2(s), \dots, C_m(s))$$

is to be designed for the system

$$\tilde{G}(s) = \text{diag}(G_{11}(s), G_{22}(s), \dots, G_{mm}(s))$$

such that block diagonal closed-loop system with the transfer matrix

$$\tilde{H}(s) = \tilde{G}(s)C(s)(I + \tilde{G}(s)C(s))^{-1}$$

is stable. An interaction measure expresses the constraints imposed on the choice of the closed-loop transfer matrix $\tilde{H}(s)$ for the block diagonal system, which guarantee that the full closed-loop system

$$H(s) = G(s)C(s)(I + G(s)C(s))^{-1}$$

is stable.

Using relative error,

$$E(s) = (G(s) - \tilde{G}(s))\tilde{G}^{-1}(s),$$

which arises from the “approximation” of the full system $G(s)$ by the block diagonal system $\tilde{G}(s)$, the general interaction measure can be stated as:

Assume that $G(s)$ and $\tilde{G}(s)$ have the same RHP poles and that $\tilde{H}(s)$ is stable. Then the closed-loop system $H(s)$ is stable if

$$\sigma_{\max}(\tilde{H}(jw)) < \mu^{-1}(E(jw)) \quad \forall w, \quad (2.6)$$

where $\sigma(A)$ is the spectral radius of A , μ is the SSV suggested by Doyle (1982).

Equation (2.6) implies that for stability, the magnitude of the diagonal blocks $\tilde{H}_i(s)$ has to be constrained by the reciprocal of the SSV μ of the relative error matrix. The value of μ depends on the structure assumed for $\tilde{H}(s)$.

Although all popular interaction measures are either explicitly or implicitly based on this definition and thus it has a rigorous theoretical basis, this definition has its limitations and therefore the results should be interpreted with caution. The reason is that interaction measure is based on the block diagonal $\tilde{H}(s)$ which might not be indicative of the actual full closed-loop transfer function matrix $H(s)$. Though the definition of the interaction measure guarantees $H(s)$ to be stable it can be very badly behaved. The interaction measure might indicate “small” interactions but the performance could be arbitrarily poor. Furthermore, the following disadvantages also need to be noticed as they are obstacles for it to be understood and accepted by control engineers:

- (1) When applied to loop pairing, all pairing possibilities need to be screened and calculated to find the best one;
 - (2) The computation of SSV μ needs complicated digital optimization;
 - (3) The SSV μ gives equal preference to all the loops. In some cases this may impose an early and perhaps unnecessary roll-off for some of the $\tilde{H}_i(s)$.
- The introduction of a weighting matrix with block diagonal structure equal

to that of $\tilde{H}(s)$ can circumvent this problem. Nevertheless, this makes the application more difficult.

2.4. Multi-loop controller design

A decentralized control system consists of a set of independent SISO controllers. In process control industries, more than 95% of the control loops are of PID type and most controllers only use the proportional and integral actions (PI) (Åström and Hägglund, 1995). The popularity of these controllers comes from their simplicity of implementation, their availability in distributed control systems (DCS) and programmable logic controllers (PLC), their flexibility, and their easy acceptance by operators. According to Skogestad and Morari (1987), they have fewer tuning parameters, are easier to understand, and are more easily made fault tolerant than decoupling controllers. In practice, decentralized control structure for MIMO systems are also preferred for ease of start-up, soft automatic/manual transfer, and fault tolerance in the event of actuator or sensor failures (Lakshminarayanan *et al.*, 1997).

However, compared to SISO systems, efficient tuning of decentralized PID controllers for multivariable processes is much more complex and many controllers work without satisfactory performance. Many researchers, for example, Vinante and Luyben (1972), Wood and Berry (1973) and Ogunnaike and Ray (1979) have studied intensively on a large number of genuine decentralized control systems, which are made up of SISO controllers acting in a multi-loop fashion. Owing to inner interactions in a multivariable controlled system, a change of controller parameters of one loop will affect the performance of others. The design of SISO controllers for a MIMO process is more difficult because of these interactions and has been an open research topic for years. In particular, the design of SISO controllers for highly coupled multivariable processes often leads to poor performance because of a bad choice of manipulated variables, poor specifications and/or poor tuning of the controllers. The decentralized control scheme should not suppress those interactions, only takes them into account in controller design. Therefore, it is preferable to have a design technique which is simple, can be frequently repeated, to give good, though not optimal, performance in all the

conditions. Many design methods have been reported in the literature, which may be roughly divided into four categories:

- (1) Detuning methods;
- (2) Sequential loop closing methods;
- (3) Independent methods;
- (4) Simultaneous equation solving methods.

2.4.1. Detuning methods

In detuning methods, each controller of the single-loop control system composed of diagonal element of transfer function matrix is tuned, ignoring the process interactions from other loops. Then interactions are taken into account and each controller is detuned using some interaction measures until some stability criterion is met. If a MIMO process model is available, several detuning schemes for decentralized PI/PID controllers are available based on frequency domain analysis.

The following practical procedure is often recommended for 2×2 processes (Mc Avoy, 1983):

- (1) Use any single loop tuning rules to obtain starting values for the individual controllers based on the corresponding main process, let the controller gains be $k_{p,i}^o$,
- (2) These gains should be reduced using the following expressions that depend on the relative gain parameter λ :

$$k_{p,i} = \begin{cases} (\lambda - \sqrt{\lambda^2 - \lambda}) k_{p,i}^o & \lambda > 1.0 \\ \left| \lambda + \sqrt{\lambda^2 - \lambda} \right| k_{p,i}^o & \lambda < 1.0 \end{cases}.$$

It may still be necessary to retune these controllers after they have been put in operation, however, this will not require as much effort as if one were starting from scratch. The disadvantage of the method is that this nicely analytic formula is only suitable for 2×2 systems, no analytic solution for higher dimensional systems.

Another practical work in this field is the *biggest log modulus tuning* (BLT) method proposed by Luyben (1986). The system studied is supposed as an $n \times n$ multivariable process given in the transfer function form

$$Y = G(s)U$$

where Y and U are vectors of the controlled variables and the manipulated variables, respectively, and G is a matrix of the open-loop process transfer functions g_{ij} relating controlled variables y_i to manipulated variables u_j . These transfer functions are typically of the form

$$g_{ij}(s) = \frac{k_{ij}(\tau_{1ij}s + 1)e^{-\theta_{ij}s}}{(\tau_{2ij}s + 1)(\tau_{3ij}s + 1)(\tau_{4ij}s + 1)}.$$

The structure of the control system variable pairing has been determined such that y_i is controlled by u_i using a PI controller $g_{ci}(s)$. Thus, the feedback controller matrix $G_c(s)$ is diagonal since we are using n SISO controllers with

$$g_{ci}(s) = k_{p,i} \left(1 + \frac{1}{\tau_{i,i}s} \right).$$

The first step in the proposed procedure is to calculate the Ziegler-Nichols setting for each individual loop. The ultimate gain $k_{u,i}$, and ultimate frequency $\omega_{u,i}$ of each diagonal transfer function $g_{ii}(s)$ are calculated in the classical SISO way. Next, a factor f is assumed. Typical values vary from 2 to 5. The gains of all feedback controllers ($k_{p,i}$) are calculated by dividing the Ziegler-Nichols gain (k_{ZN}) by the factor f

$$k_{p,i} = \frac{K_{ZN,i}}{f}$$

where

$$K_{ZN,i} = \frac{k_{u,i}}{2.2}.$$

The reset times ($\tau_{i,i}$) of all controllers are calculated by multiplying the Ziegler-Nichols reset times ($\tau_{ZN,i}$) by the same factor f

$$\tau_{i,i} = f\tau_{ZN,i}$$

where

$$\tau_{ZN,j} = \frac{2\pi}{1.2\omega_{u,j}}.$$

The f factor can be considered a “detuning” factor which is applied to all loops. The larger the value is, the more stable the system will be but the more sluggish the set-point and load responses will be. Based on intuition and empirical grounds, a multivariable closed-loop log modulus is defined as

$$L_{cm} = 20 \log \left| \frac{W}{1+W} \right|$$

where

$$W(s) = -1 + \det(I + GB)(s).$$

The proposed tuning method is based on varying the factor f until the “biggest log modulus” $(L_{cm})^{\max}$, which is a measure of how far the system is from closed-loop instability, is equal to some reasonable number. Thus, this tuning method is called the biggest log modulus tuning. By a larger number of tests, the author found that the following tuning criterion appears to give good response:

$$(L_{cm})^{\max} = 2n.$$

Similar methods taking into accounts the interaction by detuning have also been addressed by Chien *et al.* (1999, 2000) and Chen and Seborg (2003). In the work of Chien *et al.*, a straightforward tuning method is derived from a controller synthesis method with a control performance specification of 5% overshoot of the servo response. Depending on the interaction natures of the multi-loop systems, controller tuning based on the diagonal elements of the model and further detuning may be necessary. The RGA is employed to detune the parameters. For systems with a relative gain array $[RGA(\lambda_{ii}) < 1]$, a detuning factor based on this information is proposed. The information needed for controller tuning purposes includes the dynamic model parameters of the diagonal elements and the process gain information of the off-diagonal elements.

The simplicity of this method is its major advantage. Nevertheless, the disadvantage lies in the fact that loop performance and stability can not be clearly defined through the detuning procedures. Generally, this class of method only provides reasonable preliminary controller settings with guaranteed closed-loop stability.

2.4.2. Sequential loop closing methods

In sequential loop closing methods, a controller is designed for a selected input-output pair and this loop is closed. Then a second controller is designed for a second pair while the first controller has been closed. The dynamic interaction of current loop is considered in the closing of next loop, and so on. In this manner, a series of controllers are designed sequentially. Usually the fastest loop is designed first. Examples of such methods are those of Mayne (1973), Chiu and Arkun (1992), and Hovd and Skogestad (1994). If the process model is not available, the relay auto-tuning approach (Åström and Hägglund, 1995; Yu, 1999) is a useful and simple method to obtain system frequency information for PI controller design. Because MIMO systems have an infinite number of ultimate points (Halevi *et al.*, 1997), the relay auto-tuning approach for SISO systems cannot be directly applied. Loh *et al.* (1993), Loh and Vasnani (1994) proposed sequential relay-feedback tests to locate the ultimate points of a MIMO system and to design PI controllers. Palmor *et al.* (1995), Halevi *et al.* (1997) used similar simultaneous relays in all of the control loops to obtain the ultimate point information. In both the sequential and simultaneous relay auto-tuning approaches, after the frequency response information has been obtained, the Z-N (Ziegler and Nichols, 1942) or modified ZN tuning rules (Shen and Yu, 1994; Tan *et al.*, 1999; Hang *et al.*, 1991) can be used to tune the PI/PID controllers. The trial-and-error methods such as iterative continuous cycling (ICC) method (Lee *et al.*, 1998) determine the PI (only) parameters sequentially by driving the system to have continuous cycling. Additional constraints are imposed to compute the controller settings so as to guarantee their nominal stability.

A typical autotuning approach proposed by Shen and Yu (1994) is summarized here. Consider an $n \times n$ system $G(s)$ with the entry $g_{ij}(s)$ under decentralized control with a known pairing as shown in Figure 2.2. Initially, a relay is placed between y_1 and u_1 while all other loops are on *manual*. The closed-loop relationship between y_1 and u_1 becomes a bit more complicated than its open-loop one (g_{11}):

$$g_{11,CL}(s) = \left(\frac{y_1}{u_1} \right)_{CL} = g_{11} \left[1 - \frac{g_{12}g_{22}^{-1}h_2g_{21}}{g_{11}} \right],$$

where the subscript CL stands for closed-loop, k_2 is the sub-matrix of the rest of the system matrix $G(s)$, and $h_2 = g_{22}k_2[I + g_{11}k_2]^{-1}$ is the complementary sensitivity function for the remaining loops. Following the relay-feedback test, a controller can be designed from the ultimate gain and ultimate frequency for loop 1. The next step is to perform relay-feedback test between y_2 and u_2 while loop 1 is on *automatic* and other loops are still on *manual*. A controller can also be designed for loop 2 following the relay-feedback test. Once the controller on the loop 2 is put on *automatic*, a relay-feedback experiment is performed between y_3 and u_3 . In this way, all loops will be tested and closed. Then, another relay-feedback experiment is performed between y_1 and u_1 again. Generally, a new set of tuning constants is found for the controller in loop 1. This procedure is repeated until the controller parameters converge. Typically, the controller parameters converge in 3 ~4 relay-feedback tests for 2×2 systems. At each stage, a modified Ziegler-Nichols formula, with a detuning factor $f = 2.5$, is employed to tune the controller parameters. It is worth noticing that the proposed MIMO autotuning concept repeats the “identification-design” procedure on equivalent SISO transfer functions.

This class of approaches has several advantages. First, it makes the problem simple by treating the MIMO system as a sequence of SISO systems which the relay-feedback system is proven useful and reliable. Second, it operates in an efficient manner. Third, it can be applied for nonlinear multivariable systems. However, the method has some fundamental disadvantages: 1) it cannot guarantee convergence even if starting in a neighborhood of the true solution; 2) loop failure tolerance is not automatically guaranteed when loops designed early fail; 3) the performance of the method depends strongly on which loop is designed first and how the first controller is designed; and 4) the iteration procedure is essential because closing the subsequent loops may alter the response of the previously designed loops. Hence, conservative design may result due to the RHP zero on the diagonal which may not be the RHP transmission zeros of the MIMO process. In some scheme like the work of Shen and Yu (1994), after all loops have been closed, the controller will be re-tuned one after another with all other loops closed with the controllers obtained in the previous step. This procedure will go on until

they converge. For higher dimensional systems, the number of relay-feedback tests will be very large for the solution to converge. Finally, the tie between the tuning procedure and the loop performance is weak.

2.4.3. Independent design methods

In independent design methods, the design procedure is to find each controller based on the paired transfer function while satisfying some constraints due to process interactions (Grosdidier and Morari, 1986; Lee *et al.*, 2001; Skogestad and Morari, 1989; Hovd and Skogestad, 1993). In this approach the issue of interaction between loops is examined first, and sufficient conditions are derived for the individual loops that guarantee stability and robust performance. Subsequently, the SISO controllers are designed such that each loop satisfies these sufficient conditions. Most of design methods are based on pre-specifying the form of closed-loop transfer function. While this kind of approach is systematic and with stability guaranteed, a disadvantage is that this method is potentially conservative since the information on other controllers is not exploited in the design of a particular controller (Hovd and Skogestad, 1994). Some recent progress is summarized as below.

Desbiens *et al.* (1996) presented a practical frequency based tuning approach for TITO systems. For a 2×2 process $G(s)$ with a decentralized controller $G_c(s)$ as shown in Figure 2.2, first select the desired tracking closed-loop dynamics, which can be minimal-phase or nonminimal-phase, for each loop as

$$\frac{y_1(s)}{r_1(s)} = \frac{1 - \tau_{01}s}{(1 + \tau_{11}s)(1 + \tau_{21}s)}$$

and

$$\frac{y_2(s)}{r_2(s)} = \frac{1 - \tau_{02}s}{(1 + \tau_{12}s)(1 + \tau_{22}s)}.$$

Then the open-loop characteristics are deduced from tracking the closed-loop specification as

$$G_{OL1}(s) = g_{c1}(s)g_{11}(s) = \frac{1 - \tau_{01}s}{s(\tau_{11} + \tau_{21} + \tau_{01} + \tau_{11}\tau_{21}s)} \quad (2.7)$$

and

$$G_{OL2}(s) = g_{c2}(s)g_{22}(s) = \frac{1 - \tau_{02}s}{s(\tau_{12} + \tau_{22} + \tau_{02} + \tau_{12}\tau_{21}s)}. \quad (2.8)$$

By solving Equations (2.7) and (2.8), the magnitude and phase of the SISO controllers leading to the desired performances are obtained for any frequency: $g_{c1}(j\omega)$ and $g_{c2}(j\omega)$. The last step is to define the transfer functions $g_{c1}(s)$ and $g_{c2}(s)$ that best approximate the frequency responses $g_{c1}(j\omega)$ and $g_{c2}(j\omega)$. The following controller structure is used:

$$g_{ci}(s) = \frac{K_c(1 + T_is)(1 + T_{d1}s + T_{d2}s^2)}{T_is(1 + T_{f1}s + T_{f2}s^2)}.$$

An optimization method is then used to get an approximation of each corresponding transfer function of the controller while a constraint based on singular value stability robustness theorem is implemented to ensure a stable closed-loop system.

The settings obtained by this approach can lead to performances corresponding closely to the selected tracking closed-loop specification. However, the resulting controllers are not of standard PID structure and an optimization procedure is included, so it is difficult to implement and accepted by control engineers. In addition, it is not straight forward to extend the approach to 3×3 and higher dimension systems.

Lee and Edgar (2000) proposed a phase stability condition imposed on the individual loop transfer function to reduce conservatism of the independent design method with the μ -interaction measure alone. The μ -interaction measure is based on the small gain theorem. Although the μ -interaction measure is the tightest norm bound of the individual closed-loop transfer function, it is conservative because the phase information is not utilized. However, by combining various linear fractional transformations and the small gain theorem, a phase stability condition can be obtained. This condition can be used to design multi-loop control systems to reduce the conservatism of the μ -interaction measure considerably. This approach is straightforward and may result in stable multi-loop control systems that cannot be obtained by the μ -interaction measure alone. Nevertheless, some relatively complicated plots are needed during design procedure.

Chen and Soberg (2002) proposed a forward multi-loop controller design method based on Gershgorin bands. A new definition of the ultimate point is introduced for diagonal dominant MIMO systems. Analytical formulas are developed for new ultimate gains and ultimate frequencies based on the system frequency response and Gershgorin bands, a decentralized PI/PID is then designed by a modified version of the Z-N relations. This approach is systematic with classical theoretical foundation. But it cannot be directly applied to processes without diagonal dominance and the design procedure is rather complicated for practical engineers.

2.4.4. Simultaneous equation solving methods

In Simultaneous equation solving methods, the equations related to controller parameters are created according to certain performance criterion and solved by some optimization technique. Generally, design of multi-loop controller by way of simultaneous equation solving is numerically difficult. Some important works are summarized below.

Wang *et al.* (1998) presented a design method for multi-loop PI/PID controllers. Consider a stable 2×2 process as shown in Figure 2.2,

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}.$$

Assume that proper input-output pairing has been made to the process. The process is to be controlled in a negative feedback configuration by the multi-loop controller:

$$G_c(s) = \begin{bmatrix} g_{c1}(s) & 0 \\ 0 & g_{c2}(s) \end{bmatrix}.$$

The controller design objective is to find $G_c(s)$ such that both loops achieve satisfactory performance. The equivalent transfer functions $g_1(s)$ between input $u_1(s)$ and output $y_1(s)$, and $g_2(s)$ between input $u_2(s)$ and output $y_2(s)$ can be obtained as

$$g_1(s) = g_{11}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{c2}^{-1}(s) + g_{22}(s)}$$

and

$$g_2(s) = g_{22}(s) - \frac{g_{21}(s)g_{12}(s)}{g_{c1}^{-1}(s) + g_{11}(s)},$$

respectively. It is thus clear that in order to achieve a good loop performance, $g_{c1}(s)$ and $g_{c2}(s)$ should be designed for $g_1(s)$ and $g_2(s)$ instead of the plant diagonal element $g_{11}(s)$ and $g_{22}(s)$, as is done in many other methods. In this work, the modified Ziegler-Nichols is applied to the equivalent transfer function $g_1(s)$ and $g_2(s)$, i.e., the controller $g_{ci}(s), i=1,2$ is designed such that a given point on the Nyquist curve of $g_i(s), i=1,2$ is moved to a desired point. In this way, the optimum settings for the multi-loop controller can be obtained with all the benefits derived from the well-developed SISO tuning. Let

$$A_i = g_i(j\omega_i) = r_{ai} e^{j(-\pi + \varphi_{ai})}$$

be the two given points on the Nyquist curves $g_i(s), i=1,2$. They are to be moved respectively to the desired points

$$B_i = g_i(j\omega_i)g_{ci}(j\omega_i) = r_{bi} e^{j(-\pi + \varphi_{bi})}.$$

Let $g_{ci}(s), i=1,2$ be of PID type, i.e.,

$$g_{ci}(s) = k_{p,i} \left(1 + \frac{1}{\tau_{i,d}s} + \tau_{i,d}s\right), i=1,2.$$

Applying the modified Ziegler-Nichols method to each $g_{ci}(s)$ yields

$$g_{ci}(s) = k_{p,i} \left(1 + \frac{\omega_i}{\beta_i} + \alpha_i \frac{\beta_i}{\omega_i} s\right), i=1,2$$

where

$$\beta_i = \begin{cases} \frac{\tan(\varphi_{bi} - \varphi_{ai}) + \sqrt{4\alpha_i + \tan^2(\varphi_{bi} - \varphi_{ai})}}{2\alpha_i}, & \alpha_i > 0 \\ -\frac{1}{\tan(\varphi_{bi} - \varphi_{ai})}, & \alpha_i = 0 \end{cases}.$$

It follows from the phase part of the representation of B_i that

$$g_{ci}(j\omega) = k_{p,i} (1 + j \tan(\varphi_{bi} - \varphi_{ai})), i=1,2.$$

Then, the representation of B_i can be written as

$$g_i(j\omega_i)k_{p,i} = \cos(\varphi_{bi} - \varphi_{ai})r_{bi} e^{j(-\pi + \varphi_{bi})}. \quad (2.9)$$

Bringing $g_i(s), i=1,2$ and $g_{ci}(s), i=1,2$ into Equation (2.9) gives two equations which are complex and can be broken down to four real equations. Because the number of unknowns equals the number of real equations, intuitively, it is expected that the problem can be easily solved. Unfortunately this is not the case. The main difficulty lies in the nonlinearities of the equations which are further complicated by the coupling among the equations. Even though the authors presented a novel graphic iterative procedure to solve these equations with Nyquist stability constraints, there is no guarantee for the existence of solutions. In addition, the graph based iterative optimization technique may be complicated for control engineers and the extension to 3×3 or higher dimensional systems is far from straight forward if not impossible at all.

Another work of Bao *et al.* (1999) formulated the multi-loop design as a nonlinear optimization problem with matrix inequality constraints. As has been illustrated, the formulation does not include the systems that have different input delays, which unfortunately happens to be very common in MIMO process control. Simultaneous optimization for solving multi-loop controllers is also numerically difficult. The result is very much dependent on the conditions defined in the objective function. The controllers may result in instability, in case of loop failure or where loops are closed in different orders. Zhuang and Atherton (1994) investigated a PID tuning approach using an integral performance optimization procedure for a TITO system. First of all, represent the error matrix as combination of the process model, reference inputs and controller parameters. Then some integral optimum indices related to the error matrix were introduced. The optimal PID settings of decentralized controller are therefore obtained by minimizing an integral performance criterion with some constraints. Even if only for TITO systems, the calculation is relatively complicated and the existence of the optimal solution is not guaranteed. In addition, Vlachos *et al.* (1997) studied a genetic algorithm (GA) approach for optimal PI controller design. After evaluating all n objective function elements for the j^{th} set-point pattern, the results can be weighted and added together to form a single number that represents the quality of the controllers for the j^{th} set-point pattern. Finally, when all n set-point patterns have been applied to the closed-loop system, the maximum of all resulting objective function elements (indicating the worst performance) can

be selected as the final objective function value for the genetic algorithm. The crossover operator improves the accuracy of the GA according to the authors. The main advantage of the proposed method is the ability to handle arbitrary performance objectives that can be different for each of the outputs of the system. A disadvantage of the proposed method is that n simulations are required for the evaluation of the objective function for a single set of controller parameters. This significantly reduces the speed of the genetic algorithm. In addition, only a TITO simulation example is given.

2.5. Decoupling controller design

For processes with significant interactions, the performance by decentralized control may be too poor to meet specifications, or the processes may not even be stabilized due to degraded performances caused by process interactions. To improve the closed-loop performances of multi-loop control systems, decouplers are often used such that the interaction effects are eliminated, or at least significantly reduced.

Generally, decoupling is achieved by introducing an additional transfer function block or interaction compensator between the single loop controllers, and the process. The decoupler causes the control loops to act as if totally independent of each another, thus reducing the controller tuning task to that of tuning several non-interacting controllers. The design of ideal decouplers can be either simplified decoupling or generalized decoupling.

For simplified decoupling, the structure of decoupling matrix is in the form of

$$G_d(s) = \begin{bmatrix} 1 & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & 1 & \dots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1}(s) & \dots & g_{mn-1}(s) & 1 \end{bmatrix}$$

the decoupler design is to find $g_{ij}(s)$, $i \neq j$ such that the cross coupling is eliminated and $G(s)$ becomes diagonal.

For general decoupling, a diagonal matrix, $G_R(s)$, is specified, usually with the form of

$$G_R(s) = \text{Diag}[G(s)] .$$

Then the decoupler $G_I(s)$ is determined by

$$G(s)G_I(s) = G_R(s) .$$

Each method has advantages and disadvantages. For simplified decoupling, the “equivalent” open-loop decoupled system is much more complicated than the $G_R(s)$ matrix specified in the generalized decoupling design. Therefore, the tuning and closed-loop performance for the generalized decoupling is much better than for simplified decoupling. However, the price for the improved closed-loop performance in the generalized decoupling is complicated decoupler. Because of complexities in implementation or the lack of a high-quality dynamic model, it is often necessary to make use of simplifications of the ideal decoupler. In order to make the best balance between complexity and closed-loop controller performance, steady state decoupling and partial/block decoupling are often employed in practice.

Steady-state decoupling requires only the matrix of steady-state gains as a model. If each transfer function element $g_{ij}(s)$, has a steady-state gain term k_{ij} and the matrix of steady-state gains is represented by K , then the simplified steady-state decoupler for an $n \times n$ system is

$$G_I(s) = \begin{bmatrix} 1 & k_{I12} & \dots & k_{I1n} \\ k_{I21} & 1 & \dots & k_{I2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{In1} & \dots & k_{Inn-1} & 1 \end{bmatrix}$$

which will take simple, constant numerical values, and will always be realizable, as well as implementable.

In generalized steady-state decoupling, the decoupler matrix is given by

$$G_I(s) = K^{-1} K_R$$

where K_R is the steady-state version of $G_R(s)$. Since the inversion concerns only a matrix of numbers, it will always be realizable and easily implemented.

The main advantages of steady-state decoupling are that its design involves simple numerical computations, and the resulting decouplers are always realizable. As steady-state decouplers are very easy to design and implement, it is normally recommended that steady-state decoupling is the first technique to try in industry. Only if the dynamic interactions prove to be persistent will dynamic considerations be entertained. Especially, if the dynamic aspects of the transfer function elements in each row of the transfer function matrix are similar, the dynamic decoupler will be very close to its steady-state version.

Partial/block decoupling reduces the dimensionality of the problem by requiring decoupling in only a few of the control loops. When some of the loop interactions are weak or if some of the loops need not have high performance, one may consider partial decoupling. In this case, attention is focused only on a subset of the control loops where the interactions are important and high performance control is required. It is typical to consider partial decoupling for 3×3 or higher systems, as the main advantage is reduction of dimensionality.

In summary, designing a decoupler is to select elements of the compensator to satisfy one of the following objectives:

- (1) Ideal/dynamic decoupling: eliminate interactions from all loops, at every instant of time. This is an ideal decoupling objective. However, perfect decoupling is possible only if the process model is perfect which is impossible in practice. In addition, ideal dynamic decouplers may have realization problems, particularly when time delays are involved in the transfer function elements.
- (2) Steady-state decoupling: eliminate only steady-state interactions from all loops, and dynamic interactions are tolerated. The difference between dynamic decoupling and steady-state decoupling is that the former uses the complete, dynamic transfer function elements in the decoupler, and the latter uses only the steady-state gain portion of these transfer function elements. If the dynamic aspect of the transfer function elements in each row of the transfer function matrix are similar, the dynamic decoupler will be very close to its steady-state version.
- (3) Partial/block decoupling: this objective focuses attention only on the critical loops in which the interactions are strongest, leaving those with weak interactions intact. When some of the loop interactions are weak or some of the loops do not need high performance, one may consider partial decoupling. It is typical to consider partial decoupling for 3x3 or higher systems, as the main advantage is to reduce dimensions.

Some existing decoupling design approaches are summarized as below.

2.5.1. Ideal/dynamic decoupling methods

Ideal/dynamic decouplers compensate for process interactions and result in decoupled responses of controlled variables (Luyben, 1970; Waller, 1974). The solution to this problem has been studied by several researchers and various approaches, configurations and techniques have been used to achieve decoupling (see, for example, Wonham, 1979; Hautus and Heymann, 1983; Hammer and Khargonekar, 1984; Desoer and Gundes, 1986; Vardulakis, 1987; Wang, 1992). Especially, decouplers have been well established in the field of distillation control and almost all textbooks for process control deal with them (Shinskey, 1988; Seborg and Edgar, 2004).

Among all the dynamic decoupling methods, Wang *et al.* (2003) proposed a full dimensional decoupling control design for an $n \times n$ process described, as shown in Figure 2.2, via a square and non-singular transfer matrix $G(s)$ as

$$\begin{bmatrix} y_1(s) \\ \vdots \\ y_n(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & \cdots & g_{1n}(s) \\ \vdots & \ddots & \vdots \\ g_{n1}(s) & \cdots & g_{nn}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ \vdots \\ u_n(s) \end{bmatrix}$$

where $g_{ij}(s) = g_{ij0}(s)e^{-\theta_{ij}s}$, and $g_{ij0}(s)$ are strictly proper, stable scalar rational functions and θ_{ij} are non-negative constants. The process is to be controlled in a conventional negative feedback configuration by a full cross coupled multivariable controller $G_c(s) = [g_{c,ij}(s)]_{n \times n}$, where $g_{c,ij}(s) = g_{c,ij0}(s)e^{-d_{ij}s}$, d_{ij} are non-negative constants, and $g_{c,ij0}(s)$ are scalar proper rational functions. The task here is to find a $G_c(s)$ such that the closed-loop transfer matrix between the output vector y and the reference vector r :

$$H(s) = G(s)G_c(s)[I + G(s)G_c(s)]^{-1},$$

is decoupled, that is, $H(s)$ is diagonal and nonsingular. By analyzing the properties of the decoupler and the achievable performance of the decoupled system, the authors gave the following decoupling design procedure:

- (1) Based on the process characteristics, derive h_n , the objective closed-loop transfer functions from the i^{th} set-point r_i to the i^{th} output y_i for each decoupled closed-loop. The i^{th} objective open-loop transfer function q_{ri} is then given by

$$q_{ri} = \frac{h_n}{1 - h_n}$$

- (2) Match the actual loop $\tilde{g}_u k_u$ to the objective loop: $\tilde{g}_u(s)g_{c,ii}(s) = q_{ri}(s)$, to find the ideal diagonal elements of the controller as

$$g_{c,ii}^{IDEAL}(s) = \tilde{g}_u^{-1}(s)q_{ri}(s) = \frac{G^u(s)}{|G(s)|} q_{ri}(s),$$

where G^{ii} is the cofactor corresponding to g_{ij} in G , and the corresponding ideal decoupling off-diagonal elements $g_{c,ji}(s)$ of the controller as

$$g_{c,jl}^{IDEAL}(s) = \frac{G^j(s)}{G^i(s)} g_{c,ji}^{IDEAL}(s) = \frac{G^j(s)}{|G(s)|} q_{ji}(s).$$

- (3) Approximate the ideal controller $g_{c,ji}^{IDEAL}(s)$ wisely with a low-order transfer function plus possible delay so that a set of specified performance requirement is satisfied.

Even though a systematic design procedure for MIMO system design with good performance is provided, the overall design is complicated for control engineers as the model reduction technique is used in both desired performance and controller design. Furthermore, although the dynamic decoupling can work well with exact process model, it is not robust to modeling errors. This has been confirmed by Arkun *et al.* (1984) using a robustness analysis procedure based on the singular value to distillation control systems with decouplers. In addition, dynamic decouplers also require detailed process models.

2.5.2. Static decoupling methods

To avoid detailed process models, static decouplers only use process steady-state gains. Åström *et al.* (2002) showed that static decouplers are very effective for some processes when very fast controls are not required. Åström *et al.* (2002) described a simple multivariable controller consisting of a static decoupler and two single loop PI controllers. A method for finding the parameters of the controller is developed in the paper. For static decoupled transfer function matrix, the authors introduced two quantitative interaction indices k_1 and k_2 for very small s (very low frequencies). Possible detuning of the PID controllers is then quantified in terms of these indices. The interaction indices k_1 and k_2 depend on both the process and the controller and take dynamics into account. The indices introduced in the paper capture the fact that interaction can be reduced by detuning the loops. Nevertheless, the proposed interaction indices and controller design depend on the assumption that the interaction is not too severe. In addition, the approach is only developed for TITO systems.

Recently, Lee *et al.* (2005) showed that static decouplers should be applied only to the integral modes for such processes since static decouplers can cause undesirable effects on high frequency responses for some processes. Frequency

dependent relative gain arrays (Bristol, 1966) are used to determine whether static decouplers applied only on the integral modes are recommended. The main results are listed here. Consider a process of n inputs and n outputs, as shown in Figure 2.2, whose transfer function is

$$G(s) = \{g_{ij}(s), i = 1, 2, \dots, n, j = 1, 2, \dots, n\}.$$

To improve control performances of multi-loop controllers $G_c(s) = \{K_i/s + K_p\}$, where K_i and K_p are diagonal, static decouplers designed from steady-state process gains can be used as follows

$$G_c(s) = G^{-1}(0) \left(\frac{1}{s} K_i + K_p \right) = \frac{1}{s} G^{-1}(0) \begin{bmatrix} k_{i,1} & & 0 \\ & \cdots & \\ 0 & & k_{i,n} \end{bmatrix} + G^{-1}(0) \begin{bmatrix} k_{p,1} & & 0 \\ & \cdots & \\ 0 & & k_{p,n} \end{bmatrix}.$$

The static decoupler $G^{-1}(0)$ reduces process interactions at frequencies near zero. However, for some processes, it can introduce undesirable effects at high frequencies. Typically, diagonal elements of the process transfer function are faster than off-diagonal elements. As frequency increases, the magnitudes of off-diagonal elements decrease faster than those of diagonal elements and, consequently, interactions for such processes become smaller. However, when static decouplers are applied to such processes, off-diagonal elements of the decoupled process $G(s)G^{-1}(0)$ contain faster elements and their magnitudes relative to those of diagonal elements will not decrease, resulting in higher process interactions at high frequencies. There are many realistic process models, such as Wood–Berry column model, for which the static decouplers can have negative effects on process interactions at high frequencies. To avoid the disadvantage that a static decoupler causes undesirable effects on reducing process interactions at high frequencies, the authors proposed a control system where the static decoupler is applied for only the integral mode, i.e.,

$$G_c(s) = \frac{1}{s} G^{-1}(0) K_i + K_p = \frac{1}{s} G^{-1}(0) \begin{bmatrix} k_{i,1} & & 0 \\ & \cdots & \\ 0 & & k_{i,n} \end{bmatrix} + \begin{bmatrix} k_{p,1} & & 0 \\ & \cdots & \\ 0 & & k_{p,n} \end{bmatrix}.$$

Qualitatively, the integral mode $G^{-1}(0)K_i/s$ is dominant at low frequencies and its magnitude decreases as the frequency increases. Thus the static decoupler is

effective only at low frequencies and its undesirable behavior at high frequencies is avoided.

A static decoupler implemented on the integral mode can improve the closed-loop performances at low frequencies with good robustness. Generally speaking, although static decouplers are simple to design and adjust in the field, they do not always provide better closed-loop performances.

2.5.3. Partial/block decoupling methods

Since diagonal decoupling generally comes at a cost, there has also been interest in related strategies which achieve some form of partial/block decoupling (Hautus and Heymann 1983, Williams and Antsaklis 1986, Linneman and Wang 1993). The other reason why block decoupling is popular is that in many applications decoupling is practiced only in the direction of the most important loop to avoid the increased sensitivity of two-way decoupling to process uncertainty (Shinsky, 1996).

Recently, Gilbert *et al.* (2003) presented a practical one-way decoupling problem using finite frequency response data. For a 2×2 system as shown in Figure 2.2, the overall controller structure in matrix notation is given by

$$D(j\omega_i)G_c(j\omega_i) = \begin{bmatrix} 1 & k_{12} \frac{\tau_{ld}j\omega_i + 1}{\tau_{lg}j\omega_i + 1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_{p,1} + \frac{k_{i,1}}{j\omega_i} & 0 \\ 0 & k_{p,2} + \frac{k_{i,2}}{j\omega_i} \end{bmatrix},$$

where $D(j\omega)$ is a decoupling matrix with a lead/lag compensator in the 1, 2 position, and $G_c(j\omega)$ is a diagonal PI controller. First, a frequency response estimate is obtained from two closed-loop tests using an identification technique based on bandpass filters. The open-loop frequency response is then calculated and used to tune PI controllers and lead-lag compensators for decoupling. For this purpose, a desired closed-loop transfer function is specified as

$$T_d(s) = \begin{bmatrix} \frac{\omega_{n1}^2 e^{-\tau_{d1}s}}{s^2 + 2\zeta_1 \omega_{n1}s + \omega_{n1}^2} & 0 \\ 0 & \frac{\omega_{n2}^2 e^{-\tau_{d2}s}}{s^2 + 2\zeta_2 \omega_{n2}s + \omega_{n2}^2} \end{bmatrix}. \quad (10)$$

The authors suggested that $\omega_{n1} = \omega_{n2}$ and $\zeta_1 = \zeta_2 = 1.0$. The optional delay

parameters τ_{d1} and τ_{d2} can be specified as zero since the advantage of working directly with frequency response data, as opposed to transfer functions, is that delay estimation is not important because its effect is included in the frequency response data. Then, perform tuning by making the real closed-loop performance T_i approach the desired frequency response T_{d_i} defined by Equation (10) at each of the harmonic frequency using the following index:

$$F_p = \frac{P}{2} \sum_l \sum_k \left[W_{k,l} \sum_{i=1}^N \left(\left| T_{d_i}^{k,l} - T_i^{k,l} \right|^2 \{A_i^l\}^2 \right) \right],$$

where the indices k and l refer to the outputs and inputs, respectively, and i refers to the i^{th} harmonic frequency of the l^{th} set-point excitation, $W_{k,l}$ are weights applied to the k^{th} output and l^{th} input, A_i^l are the weights applied to each frequency, which should be chosen to correspond to the harmonic amplitudes of a trapezoidal waveform of initial slop K and period P :

$$A_i = \frac{4}{\pi} \frac{K}{i^2} \sin\left(\frac{i}{K} \frac{2\pi}{P}\right), i = 1, 3, 5 \dots \det(I + GDG_c).$$

While the optimization is mainly carried out for user-specified performance specifications, two constraints are proposed to check. They are the stability checks for violations of the multivariable Nyquist stability margins on $\det(I + GDG_c)$, and the robustness maximization by maximizing the minimum related singular values of $\det(I + (GDG_c)^{-1})$, and $\det(I + (DG_c G)^{-1})$.

Using this approach, it is not necessary to approximate the process with a FOPDT or SOPDT model, and hence avoiding the need to treat time delays since the time delay is indirectly represented in frequency response data. However, the developed design is not so easy to apply to higher dimensional systems since the reliability of the frequency estimates may become a limitation. In addition, the existence of the acceptable solutions may not be guaranteed.

Usually, the block decoupling is easier to realize for those processes which can be grouped into some sub-systems with weak interaction among them. How to make the groups is, however, still empirical.

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Chapter 3^{*}

Interaction Analysis and Loop Pairing for Multivariable Processes

3.1. Introduction

Usually, the interactive multivariable systems can be either controlled by 1) a multivariable or centralized MIMO controller or by 2) a set of SISO decentralized controllers. Algebraic decoupling methods or optimal multivariable control theory are usually applied to obtain centralized MIMO controllers. Centralized controllers are complex and weak in control system integrity. In contrast, the decentralized control systems enjoy certain advantages: 1) the structure is easier to understand, and implement, and requires fewer parameters to tune; 2) loop failure tolerance of the resulting control system can be assured during the design phase; and 3) both hardware and software realization is simple. Therefore, they are more often used in process control applications (Grosdidier and Morari, 1987; Chiu and Arkun, 1990). However, the potential disadvantage of using the limited control structure is the deteriorated closed-loop performance caused by interactions among loops as a result of the existence of nonzero off-diagonal elements in the

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transfer function matrix (Grosdidier and Morari, 1986; He and Cai, 2004). Thus, the primary task in the design of decentralized control systems is to determine loop configuration, i.e., pair the manipulated variables and controlled variables to achieve the minimum interactions among loops so that the resulting multivariable control system mostly resembles its single-input single-output counterparts and the subsequent controller tuning is largely facilitated by SISO design techniques (Seborg and Edgar, 1989).

As mentioned in Chapter 2, the RGA based techniques for control loop configuration have been widely used in industry, including blending, energy conservation, and distillation columns, etc (Bristol, 1966; Shinskey, 1988; Wolff and Skogestad, 1995; Hansen *et al.*, 1998). The most important advantages of RGA based techniques are computational simplicity (since only process steady-state gains are involved) and scaling independence (due to its ratio nature (Grosdidier and Morari, 1985)). However, using steady-state gain alone may result in incorrect interaction measures and consequently incorrect loop pairing decisions since no dynamic information of the process is taken into consideration. Even though several pairing methods have later been proposed by using the transfer function model instead of the steady-state gain matrix to calculate RGA to consider the effects of process dynamics to overcome the limitations of RGA based loop pairing criterion, they are often controller dependent, making it more difficult to calculate and to understand by practical control engineers. (Witcher and Mc Avoy, 1977; Bristol, 1979; Tung and Edgar, 1981; Mc Avoy *et al.*, 2003).

To combine the advantages of both RGA and DRGA, the steady-state gain and bandwidth of the process transfer function elements are used in this chapter to provide a more comprehensive description for loop interactions. Then, a new loop pairing criterion based on the new interaction measure which minimizes loop interactions is proposed in terms of *relative energy array* (REA) of the process. The main advantages of the method are: 1) as compared with DRGA methods, it requires much less computation, and does not require the specification of the controller type; 2) it results in less conservative controllers when detuning factor design methods are used; and 3) it is very simple for field engineers to understand and work out pairing decision in practice. Several examples, for which the RGA

based loop pairing criterion gives an inaccurate interaction assessment, are employed to demonstrate the effectiveness of the proposed interaction measure and loop pairing criterion.

3.2. Preliminaries

Consider a multivariable system with n inputs and n outputs as shown in Figure 2.2, where $\mathbf{G}(s)$ and $\mathbf{G}_c(s)$ are given in Equation (2.1) and (2.2), respectively.

Assume that the system is open-loop stable and the process matrix $G(s)$ is nonsingular at steady-state. The loop pairing problem defines the control system structure, i.e., which of the available plant inputs is to be used to control which plant output. The most popular loop pairing method is the RGA and NI based pairing rules as below (Bristol, 1966; Grosdidier and Morari, 1985; Niederlinski, 1971).

- (1) The paired RGA elements are closest to 1.0;
- (2) All the paired RGA elements are positive;
- (3) Large RGA elements should be avoided;
- (4) Niederlinski index is positive, i.e., $NI > 0$.

One of the main advantages of these methods is that the interaction depends on only the steady-state gains. This information is easily obtained from simple identification experiments or steady-state design models. A potential weakness of these methods, however, is the same fact that they only use the steady-state gains on which the assumption of perfect loop control to determine loop pairing is based. We use the following example to illustrate this point.

Example 3.1. Consider a process given by (Mc Avoy *et al.*, 2003)

$$\mathbf{G}(s) = \begin{bmatrix} \frac{5e^{-40s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-4s}}{10s+1} & \frac{5e^{-40s}}{100s+1} \end{bmatrix}.$$

The RGA is

$$\Lambda(\mathbf{G}(0)) = \begin{pmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{pmatrix}.$$

This result implies the diagonal pairing 1-1/2-2 is a good choice. Hence, according to the RGA based loop-pairing criterion, the pairing of 1-1/2-2 should be preferred for the smaller interaction to any one loop from other closed-loop. However, Mc Avoy *et al.* used DRGA and optimal decentralized PI controllers for various configurations, and found that the diagonal pairing resulted in a poor closed-loop performance (Mc Avoy *et al.*, 2003). The off-diagonal pairing 1-2/2-1 takes advantage of the fast $g_{21}(s)$ and $g_{12}(s)$ transfer functions to achieve better response for y_2 :

- (1) there is very little interaction from y_1 to y_2 ;
- (2) the off-diagonal pairing y_2 response is significant by better than that of y_2 with diagonal pairing;
- (3) the y_1 response for the diagonal pairing is somewhat better than that of the off-diagonal pairing when y_2 is given a step set-point change.

The main reason for the poor performance of the diagonal pairing is the dynamic properties of the transfer functions. It can be easily seen that the time constants and delays of 10 and 4 of the off-diagonal elements are 10 times smaller than diagonal ones of 100 and 40. In such a case, pairing the faster loops (even with smaller steady-state gains) takes the advantage of the time scale decoupling such that seriousness of the interactions from the slower loop would be reduced.

3.3. A modified loop pairing rule

In designing decentralized controllers for multivariable processes, it is desired that the interaction measures and loop pairing will address the following issues:

- (1) The interaction measure should consider the finite bandwidth control, since the assumption of perfect control is only valid for very low frequency range.
- (2) The loop pairing decision should be controller independent such that any controller type could be designed after loop pairing.
- (3) The pairing results in minimal interaction within the interested frequency range not only statically but also dynamically.
- (4) It should be simple and easy to use for practical engineers.

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In decentralized control system design, individual loop is tuned around the critical frequency region of the transfer function which is the region around the control system bandwidth. Thus, this is the frequency region that should be focused upon when considering the effect of interactions. Therefore, two factors in the open-loop transfer functions will affect the loop pairing decision:

- (1) Steady-state gain: the steady-state gain $g_{ij}(0)$ of the transfer function reflects the effect of manipulated variable u_j to controlled variable y_i .
- (2) Response speed: response speed is accountable for the sensitivity of the controlled variable y_i to manipulated variable u_j and, consequently, the ability to reject the interactions from other loops.

In order to develop a new interaction measure which can reveal both gain and phase information, let us factorize an arbitrary element in frequency domain, $g_{ij}(j\omega)$, in $G(j\omega)$ as

$$g_{ij}(j\omega) = g_{ij}(0)g_{ij}^0(j\omega)e^{-\theta_{ij}j\omega} \quad (3.1)$$

where $g_{ij}(0)$, $g_{ij}^0(j\omega)$ and $e^{-\theta_{ij}j\omega}$ are steady-state gain, normalized transfer function ($g_{ij}^0(0)=1$) and time-delay of $g_{ij}(j\omega)$, respectively.

To use both steady-state gain and response speed information for interaction measure and loop pairing, we now define the energy of $g_{ij}(j\omega)$ as the area in the amplitude plot of $g_{ij}(j\omega)$ in the frequency domain, i.e.,

$$e_{ij} = g_{ij}(0) \int_0^{\omega_{c,ij}} |g_{ij}^0(j\omega)e^{-\theta_{ij}j\omega}| d\omega \quad (3.2)$$

where $\omega_{c,ij}$ is the critical frequency of $g_{ij}(j\omega)$.

As $|g_{ij}^0(j\omega)e^{-\theta_{ij}j\omega}|$ represents the magnitude of the transfer function at various frequencies, the physical implication of Equation (3.2) represents the effective energy transmission ratio of $g_{ij}(j\omega)$ from u_j to y_i .

In calculating e_{ij} , the critical frequency, $\omega_{c,ij}$ can be defined as either

- (1) $\omega_{c,ij} = \omega_{b,ij}$, where $\omega_{b,ij}$ for $i, j = 1, 2, \dots, n$ is the bandwidth of the transfer function $|g_{ij}^0(j\omega)e^{-\theta_{ij}j\omega}|$ and determined by the frequency where the magnitude plot of frequency response reduced to 0.707, as shown in

Figure 3.1 (a), i.e.,

$$\left| g_{ij}^0(j\omega_{c,ij})e^{-\theta_{ij}j\omega} \right| = 0.707,$$

or

- (2) $\omega_{c,ij} = \omega_{u,ij}$, where $\omega_{u,ij}$ for $i, j = 1, 2, \dots, n$ is the ultimate frequency of the transfer function $\left| g_{ij}^0(j\omega)e^{-\theta_{ij}j\omega} \right|$ and determined by the frequency where the phase plot of frequency response across $-\pi$, as shown in Figure 3.1 (b) i.e.,

$$\arg \left[g_{ij}(j\omega_{u,ij})e^{-\theta_{ij}j\omega} \right] = -\pi.$$

The value of e_{ij} is either the area covered by the curve $g_{ij}(0) - P - Q - \omega_{b,ij}$ in Figure 3.1(a) (when using $\omega_{b,ij}$ as critical frequency), or by the curve $g_{ij}(0) - Q - \omega_{u,ij}$ in Figure 3.1(b) (when using $\omega_{u,ij}$ as critical frequency).

(a) Energy based on bandwidth $\omega_{b,ij}$

(b) Energy based on ultimate frequency $\omega_{u,ij}$

Figure 3.1. Frequency response curves and the energy, e_{ij} , of $g_{ij}(j\omega)$

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Applying Equation (3.2) to all transfer function elements, the overall process energy can be written in the form of energy matrix and expressed as

$$E = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{bmatrix}.$$

To simplify the calculation, we approximate the integration of e_{ij} by a rectangular area, i.e.,

$$e_{ij} \approx g_{ij}(0)\omega_{c,ij} \quad i, j = 1, 2, \dots, n$$

which is the area of the rectangle $g_{ij}(0) - Q' - \omega_{b,ij}$ in Figure 3.1(a) (when using $\omega_{b,ij}$ as the critical frequency), or the rectangle $g_{ij}(0) - Q' - \omega_{u,ij}$ in Figure 3.1(b) (when using $\omega_{u,ij}$ as the critical frequency). Then, the energy matrix is given by

$$E = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{bmatrix} = G(0) \otimes \Omega, \quad (3.3)$$

where

$$G(0) = \begin{bmatrix} g_{11}(0) & g_{12}(0) & \cdots & g_{1n}(0) \\ g_{21}(0) & g_{22}(0) & \cdots & g_{2n}(0) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(0) & g_{n2}(0) & \cdots & g_{nn}(0) \end{bmatrix}$$

and

$$\Omega = \begin{bmatrix} \omega_{c,11} & \omega_{c,12} & \cdots & \omega_{c,1n} \\ \omega_{c,21} & \omega_{c,22} & \cdots & \omega_{c,2n} \\ \cdots & \cdots & \cdots & \cdots \\ \omega_{c,n1} & \omega_{c,n2} & \cdots & \omega_{c,nn} \end{bmatrix},$$

are the steady-state gain and the critical frequency matrices, respectively. Even though Equation (3.2) is more accurate, Equation (3.3) is easier to use. As the energy matrix is used to derive relative interaction measures, the same approximation using Equation (3.3) for both open-loop and closed-loop cases will not affect the outcomes.

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Similar to relative gain, the relative energy, ϕ_{ij} , of $g_{ij}(j\omega)$ is defined as the energy ratio of the loop $y_i - u_j$ when other loops are open or closed, i.e.,

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} = \frac{\left[\int_0^{\omega_{c,ij}} (\partial y_i / \partial u_j) d\omega \right]_{\text{all loops open}}}{\left[\int_0^{\omega_{c,ij}} (\partial y_i / \partial u_j) d\omega \right]_{\text{all other loops closed except for loop } y_i - u_j}},$$

where $\omega_{c,ij}$ and $\hat{e}_{ij}$ are the critical frequency and the energy of $\hat{g}_{ij}(s)$ when all other loops are closed, respectively.

When the relative energies are calculated for all the input/output combinations of a multivariable process, it results in *relative energy array* (REA):

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}. \quad (3.4)$$

Following the same arguments as in DRGA and RGA, if all y_k , $k \neq i$, are perfectly controlled by the use of u_l , $l \neq i$, then the energy from u_i to y_j is equal to the energy provided by $1/[E^{-1}]_{ji}$ and Equation (3.4) can be conveniently computed by

$$\Phi = E \otimes E^{-T}. \quad (3.5)$$

Since REA is also a relative interaction measure, it has similar properties as RGA and DRGA:

- (1) The value of ϕ_{ij} is a measure of the relative energy interaction expected in the i^{th} loop if its output y_i is paired with u_j .
- (2) The elements of the REA in any row or column sum up to 1, i.e.

$$\sum_{i=1}^n \phi_{ij} = \sum_{j=1}^n \phi_{ij} = 1.$$

- (3) Let $\hat{e}_{ij}$ represent the loop i area when all the other loops are closed, whereas e_{ij} represents the normal, *open-loop* area, then:

$$\hat{e}_{ij} = \frac{1}{\phi_{ij}} e_{ij}. \quad (3.6)$$

- (4) When ϕ_j is negative, it produces an energy change in y_i in response to a change in u_j when the other loops are open totally opposite in direction to that when the other loops are closed.

Furthermore, when all n loops are closed, the multi-loop system will be unstable for all possible values of controller parameters (i.e., it will be “structurally monotonically unstable”) if

$$\mathbf{NI} = \frac{|\mathbf{G}(0)|}{\prod_{i=1}^n g_{ii}(0)} < 0,$$

where $|G(0)|$ denotes the determinant of matrix $G(0)$ (Niederlinski, 1971).

Therefore, $\mathbf{NI} > 0$ is a necessary stability condition, and consequently constitutes a complementary tool to the REA in variable pairing selection.

Parallel to RGA and NI based pairing rules, the REA and NI based loop pairing rules are given below:

- (1) Corresponding REA elements are closest to 1.0.
- (2) The NI is positive.
- (3) All paired REA elements are positive.
- (4) Large REA elements should be avoided.

Remark 3.1. For control system configuration, both bandwidth and ultimate frequency can be used to find the REA. However, ultimate frequency ($\omega_{u,ij}$) is more convenient to use since it is linked to control system dynamic performance and familiar to control engineers. For the transfer function matrix with some elements without phase crossover frequencies, such as transfer functions of first order or second order without time delay, a finite time delay factor, $e^{-j\theta_c/\omega}$, can be used to multiply all elements in $G(s)$, such that ultimate frequency is available to calculate relative energy for each element. As it is a relative measure, adding a time delay to each element will not affect the relative measures. Without loss of generality, we will use ultimate frequency in our following development.

Remark 3.2. Both REA and NI play important roles for control structure selection. REA is used to measure interactions, while NI is used as a sufficient condition to rule out the closed-loop unstable pairings. Since we are using energies instead of simply steady-state gains, dynamic interactions up to the critical frequency can be

effectively reflected. Therefore, comparing REA method with RGA and DRGA methods, it has the following advantages:

- (1) In addition to steady-state gains, only critical frequency needs to be calculated in REA method, it is far easier to calculate than those DRGA methods.
- (2) REA combines both steady-state gains and critical frequencies in measuring the loop interactions, it provides better pairing results than that of RGA based pairing and comparable with DRGA ones.
- (3) REA only uses information of open-loop process transfer functions, it is controller type independent.

3.4. Case studies

In this section, we use various examples to show the effectiveness of the above REA based method in both simplicity and correctness.

Example 3.2 (Continue with Example 3.1),

$$G(s) = \begin{bmatrix} \frac{5e^{-40s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-4s}}{10s+1} & \frac{5e^{-40s}}{100s+1} \end{bmatrix}.$$

The RGA is

$$\Lambda(G(0)) = \begin{bmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{bmatrix}.$$

This result implies the diagonal pairing 1-1/2-2 is a good choice. Hence, according to the RGA based loop-pairing criterion, the pairing of 1-1/2-2 should be preferred for the smaller interaction to any one loop from other closed-loop. However, in REA approach, the energy matrix is

$$E = \begin{bmatrix} 0.0500 & 0.1000 \\ -0.5000 & 0.0500 \end{bmatrix},$$

and $\phi_{11} = 0.0476$ strongly suggests off-diagonal pairing.

Mc Avoy et al. used DRGA and optimal decentralized PI controllers for various configurations, and found that the diagonal pairing resulted in a poor closed-loop performance (Mc Avoy et al., 2003). In order to test the pairing

results for this process, two sets of optimal PI controllers were designed for diagonal and off-diagonal pairings, respectively. For each configuration, two controller gains and two integral gains are optimized for a step change in y_1 followed by a step change in y_2 using the same objective function that was used for the optimal control calculation. Equal weighting is given to the measurements and the manipulated variables. In calculating the errors for y_1 and y_2 the difference between these measurements and their set-points was used. Since the minimization involved in tuning the PI controllers is non-convex, several different starting points are used to determine the best values of the tuning parameters. After all, it is concluded that the DRGA correctly indicates that an off-diagonal pairing produces a better overall control system response than that of the diagonal pairing.

Example 3.3 Consider a 3×3 process (Huang *et al.*, 1994), given by

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{s+1} & \frac{e^{-s}}{s+1} \\ \frac{1.5e^{-s}}{s+1} & \frac{e^{-s}}{s+1} & \frac{-2e^{-s}}{10s+1} \\ \frac{e^{-s}}{s+1} & \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{s+1} \end{bmatrix}.$$

The RGA of the system is

$$\Lambda = \begin{bmatrix} -0.9302 & 1.1860 & 0.7442 \\ 1.1860 & 0.7442 & -0.9302 \\ 0.7442 & -0.9302 & 1.1860 \end{bmatrix}.$$

Obviously, two possible pairings 1-2/2-1/3-3 and 1-3/2-2/3-1 are comparable because all related RGA elements close to 1. Therefore, RGA pairing approach cannot determine which pairing is better.

Using REA method, the critical frequency matrix

$$\Omega = \begin{bmatrix} 0.1 & 1 & 1 \\ 1 & 1 & 0.1 \\ 1 & 0.1 & 1 \end{bmatrix},$$

and REA of the system is obtained as

$$\Phi = \begin{bmatrix} 0.0554 & 0.6977 & 0.2468 \\ 0.6977 & 0.2468 & 0.0554 \\ 0.2468 & 0.0554 & 0.6977 \end{bmatrix}.$$

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This indicates the best pairing is 1-2/2-1/3-3 because the corresponding REA elements are much closer to 1 than the other option. This was confirmed by the generalized dynamic relative gain (GDRG) approach (Huang *et al.*, 1994). However, the proposed method is much simpler.

Example 3.4 Consider a 4×4 industrial reactor/recycle system (Robinson *et al.*, 2001) given by

$$\mathbf{G}(s) = \begin{bmatrix} 0 & \frac{3.96(197s+1)}{49.4s^2+14.1s+1} & \frac{0.536(258s+1)}{83.3s^2+18.3s+1} & \frac{9.7}{24.3s+1} \\ \frac{0.00111}{s} & \frac{0.044}{46.9s^2+s} & \frac{-0.0152e^{-32s}}{s} & \frac{0.039e^{-20s}}{s} \\ 0 & \frac{-232.2}{32.2s+1} & \frac{-15.96(529s+1)}{10417s^2+204s+1} & \frac{139.2}{7.27s+1} \\ \frac{-0.582}{s} & \frac{-2.54(8.11s+1)}{6.25s^3+5s^2+s} & \frac{0.0462(45.6s+1)e^{-35s}}{306s^3+35s^2+s} & \frac{-0.0358e^{-30s}}{s} \end{bmatrix}.$$

As proposed by Arkun and Downs (1990), for processes that contain pure integrator elements (that has no steady-state) the steady-state gain and RGA of integrating processes can be calculated by letting $I = 1/s$, substituting I into the original transfer function matrix, and taking the limit $K = \lim_{s \rightarrow 0} G(s)$ in calculating the RGA

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 0.107 & 0.148 & 0.745 \\ -0.072 & 0.196 & 0.829 & 0.046 \\ 0 & 0.723 & 0.067 & 0.210 \\ 1.072 & -0.027 & -0.044 & 0 \end{bmatrix}.$$

It indicates the pairings: 1-4/2-3/3-2/4-1.

To use REA method, the bandwidth matrix of $G(s)$ and REA are obtained, respectively, as

$$\mathbf{\Omega} = \begin{bmatrix} 0 & 5.6400 & 4.3800 & 0.0410 \\ 1.4500 & 0.1750 & 1.4500 & 1.4500 \\ 0 & 0.0310 & 0.0710 & 0.1380 \\ 1.4500 & 1.2980 & 0.4550 & 1.4500 \end{bmatrix},$$

and

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 0.7856 & 0.1387 & -0.0047 \\ -0.0806 & -0.0285 & 1.2895 & -0.0244 \\ 0 & -0.0026 & -0.0007 & 0.8426 \\ 0.9438 & -0.0291 & 0.0215 & 0 \end{bmatrix}.$$

This indicates pairings: 1-2/2-3/3-4/4-1. This is the same as the pairing option given by the DRGA approaches (Mc Avoy *et al.*, 2003; Robinson *et al.*, 2001). Robinson *et al.* (2001) confirmed this conclusion and they showed that the transients produced by this pairing option are significant superior to those produced by pairing option suggested by RGA. Nevertheless, the proposed method is much more direct and the computation is much easier.

3.5. Conclusions

In this chapter, both the steady-state gain and the critical frequency of the process are used to provide a simple yet comprehensive description of loop interactions for MIMO processes. The REA can be conveniently calculated by control engineers since the bandwidth or ultimate frequency can be easily obtained from the given transfer function matrix. The effectiveness of the method is demonstrated by several examples, for which the RGA based loop pairing criterion gives an inaccurate interaction assessment, while the proposed interaction measure and loop pairing criterion provides accurate results and they are very easy to be calculated. Furthermore, the REA method is not only an effective tool for loop pairing, but also very useful in the design of the decentralized and decoupling control systems, especially, for high dimensional processes. These issues will be discussed later.

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Chapter 4^{*}

Decentralized Control System Design

4.1. Introduction

In process control industry, more than 95% of the control loops are of PI/PID type (Åström and Hägglund, 1995). This is mainly attributed to its effectiveness and relatively simple structure, which can be easily understood and implemented in practice. Consequently, the research on PID control algorithm development and their applications is still a very active area; many formulas have been derived to tune the PID controllers over the years.

Due to the high product quality and energy integration requirements, most of modern industry processes, however, are MIMO processes. For easier field implementation, it is desirable to apply well established single loop PID tuning principles to these MIMO processes (Grosdidier and Morari, 1987). However, compared with SISO counterparts, MIMO systems are more difficult to control due to the existence of interactions between different input and output variables. Adjusting controller parameters of one loop affects the performance of the others, sometimes to the extent of destabilizing the entire system. To ensure stability, many decentralized controllers in industry are tuned loosely, which causes inefficient operation and higher energy costs.

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Although considerable effort has been dedicated to this problem and many design techniques, such as sequential loop closing methods (Lee *et al.*, 1998; Loh *et al.*, 1993; Shen and Yu, 1994), detuning factor methods (Luyben, 1986; Chen and Seborg, 2003), and independent design methods (Grosdidier and Morari, 1986; Skogestad and Morari, 1989; Pomerleau and Pomerleau, 2001; Huang and Jeng, 2002; Huang *et al.*, 2003) have been proposed, decentralized control system design and implementation is still difficult to deal with for control engineers due to the lack of simple and practical approach. Since controllers interact with each other in a decentralized control system, the performance of one loop cannot be evaluated without the information of the controllers of other loops.

In this chapter, a novel equivalent transfer function method in terms of REA for multivariable processes is proposed. By considering four combination modes of gain and phase changes for a particular loop when other loops are closed, this equivalent transfer function can effectively approximate the dynamic interactions among loops. Consequently, the design of decentralized controller for MIMO processes can be converted to the design of single loop controllers. The method is simple, straightforward, easy to understand and implement. Several multivariable industrial processes with different interaction characteristics are employed to demonstrate the effectiveness and simplicity of the design method compared with the existing methods.

4.2. Equivalent transfer function

Consider an open-loop stable multivariable system as shown in Figure 2.2 with decentralized control structure, Equations (2.1) and (2.2), respectively. From Chapter 3, we have the relative energy, ϕ_{ij} , between output variable y_i and input variable u_j :

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \quad (4.1)$$

where $\hat{e}_{ij}$ is the effective energy transmission ratio between output variable y_i and input variable u_j when all other loops are closed. When the relative energy are

calculated for all the input/output combinations of a multivariable process, it results in an array of the form, i.e., REA, which can be calculated by

$$\Phi = \mathbf{E} \otimes \mathbf{E}^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix}.$$

In this chapter, we will employ this interaction measure to develop *equivalent transfer functions* (ETFs) for controller design under decentralized control structure.

Suppose that the best loop configuration has been determined and the best pair is diagonally placed in the transfer function matrix as shown in Figure 2.2. As mentioned in Chapter 3, let the energy of $\hat{g}_{ij}(s)$, $\hat{e}_{ij}$, when all other loops are closed be $\hat{e}_{ij} = \hat{g}_{ij}(0)\hat{\omega}_{c,ij}$ $i, j = 1, 2, \dots, n$, where $\hat{g}_{ij}(0)$ and $\hat{\omega}_{c,ij}$ are the steady-state gain and ultimate frequency between output variable y_i and input variable u_j when all other loops are closed, respectively. Then, from Equation (4.1)

$$\hat{g}_{ij}(0)\hat{\omega}_{c,ij} = \frac{g_{ij}(0)\omega_{c,ij}}{\phi_{ij}}. \quad (4.2)$$

By the definition of RGA, we have

$$\hat{g}_{ij}(0) = \frac{g_{ij}(0)}{\lambda_{ij}} \quad (4.3)$$

where λ_{ij} is the relative gain.

Substitute Equation (4.3) into (4.2) and rearrange to result

$$\frac{\phi_{ij}}{\lambda_{ij}} = \frac{\omega_{c,ij}}{\hat{\omega}_{c,ij}} \equiv \gamma_{ij} \quad (4.4)$$

where γ_{ij} represents the critical frequency change of loop i - j when other loops are closed, defined as *relative critical frequency*. When the relative critical frequencies are calculated for all the input/output combinations of a multivariable process, it results in an array of the form, i.e., *relative critical frequency array* (RCFA) defined as

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix}. \quad (4.5)$$

The elements of Equation (4.5) can be calculated by

$$\begin{aligned} \Gamma &\triangleq \Phi \otimes \Lambda \\ &= \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \otimes \begin{bmatrix} 1/\lambda_{11} & 1/\lambda_{12} & \cdots & 1/\lambda_{1n} \\ 1/\lambda_{21} & 1/\lambda_{22} & \cdots & 1/\lambda_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ 1/\lambda_{n1} & 1/\lambda_{n2} & \cdots & 1/\lambda_{nn} \end{bmatrix}. \end{aligned} \quad (4.6)$$

In Chapter 3, we have derived the energy for a control loop when other loops are closed. In the following, we will derive ETFs, $\hat{g}_y(s)$, using REA for loop $y_i - u_j$ when all other loops are closed.

Since control loop transfer functions when other loops closed will have similar frequency properties with when other loops open if it is well paired (Chien *et al.*, 1999), we can let the ETFs have the same structures as the corresponding open-loop transfer functions but with different parameters

$$\hat{g}_{ii}(s) = \hat{g}_{ii}(0) g_{ii}^r(s) e^{-\hat{\theta}_{ii}s} \quad (4.7)$$

where $g_{ii}^r(s)$ is defined by

$$g_{ii}^r(s) = g_{ii}^0(s) e^{-\theta_{ii}s}$$

and $\hat{\theta}_{ii}$ is the time delay of the ETF.

As the change in ultimate frequency of a control loop is generally affected by changes in both time constant and time delay when other loops are closed, and they may be reasonably represented by a linear approximation, it is reasonable to change only time delay to reflect the phase changes.

In Equation (4.7), $\hat{g}_{ii}(0)$ can be determined by using Equation (4.3), while by the definition of the ultimate frequency,

$$-\hat{\theta}_{ij} \hat{\omega}_{c,ij} + \angle g_{ij}^0(j \hat{\omega}_{c,ij}) = -\theta_{ij} \omega_{c,ij} + \angle g_{ij}^0(\omega_{c,ij}) = -\pi,$$

$\hat{\theta}_{ii}$ can be easily determined by

$$\hat{\theta}_n = \frac{\pi + \angle g_n^r(j\hat{\omega}_{c,n})}{\hat{\omega}_{c,n}} = \gamma_n \frac{\pi + \angle g_n^r(j\omega_{c,n})}{\omega_{c,n}}. \quad (4.8)$$

Notice that

- (1) $g_n^r(s)$ is usually low order transfer functions, their contribution to the phase change at low frequency range are small and can be equivalently represented by the additional time delay term.
- (2) In many decentralized control system designs, such as gain and phase margin method, individual loop is tuned around the critical frequency region of each control loop. Accurate estimation is required around critical frequency point, not who contribute to the change.

Assume the contribution of $g_n^0(s)$ in the phase change at critical frequency is small, we can let $\angle g_n^r(j\hat{\omega}_{c,n}) \approx \angle g_n^r(\omega_{c,n})$, and make further simplification to Equation (4.8) as

$$-\hat{\theta}_n \hat{\omega}_{c,n} \approx -\theta_n \omega_{c,n}$$

which results by considering Equation (4.4)

$$\hat{\theta}_n \approx \frac{\omega_{c,n}}{\hat{\omega}_{c,n}} \theta_n = \gamma_n \theta_n \quad (4.9)$$

This is the practical formula which will be used to create ETFs. Even though Equation (4.9) is less accurate than Equation (4.8), several simulation results have showed that the control system performances are comparable by the two approximations, but Equation (4.9) is much more straightforward and easier to understand than Equation (4.8).

Since it is desirable that the controlled system possesses integrity property; that is, the overall control system remained stable regardless of input and output of other control loops, $\hat{g}_n(0)$ and $\hat{\theta}_n$ in ETF must take different values for different combination of λ_n and γ_n . For the four different combinations of λ_n and γ_n , $\hat{g}_n(s)$ may take different modes shown in Figures 4.1-4.4, and are discussed below:

Case 1: $\lambda_n < 1$, $\gamma_n \leq 1$, Figure 4.1.

In this case, $\left(\frac{1}{\lambda_{ii}} - 1\right) > 0$ and $(\gamma_{ii} - 1) \leq 0$. According to Equations (4.3) and (4.9),

we have $\hat{g}_{ii}(0) > g_{ii}(0)$, and $\hat{\theta}_{ii} \leq \theta_{ii}$.

- $\hat{g}_{ii}(0) > g_{ii}(0)$. This means that the magnitude of the frequency response when the other loops closed is not less than that of when the other loops open. Since the retaliatory effect from the other loops magnifies the main effect of u_i on y_i , we need to reduce the controller gain to assure system stability. In this case, the gain is by Equation (4.3)

$$\hat{g}_{ii}(0) = g_{ii}(0) / \lambda_{ii}.$$

- $\hat{\theta}_{ii} \leq \theta_{ii}$. This means that the time delay when the other loops closed is not bigger than that of when other loops open. The reduced time delay will increase the phase margin. However, by considering the control system integrity, the time delay needs to be kept as before, i.e.,

$$\hat{\theta}_{ii} = \theta_{ii}.$$

Figure 4.1. Interaction mode with $\lambda_{ii} < 1$, $\gamma_{ii} \leq 1$

Case 2: $\lambda_{ii} < 1$, $\gamma_{ii} > 1$, Figure 4.2.

In this case, $\left(\frac{1}{\lambda_{ii}} - 1\right) > 0$ and $(\gamma_{ii} - 1) > 0$. According to Equations (4.3) and (4.9),

we have $\hat{g}_{ii}(0) > g_{ii}(0)$, and $\hat{\theta}_{ii} > \theta_{ii}$.

- $\hat{g}_u(0) > g_u(0)$. Same as in Case 1,

$$\hat{g}_u(0) = g_u(0) / \lambda_u.$$

- $\hat{\theta}_u > \theta_u$. This means that the time delay when the other loops closed is bigger than that of when the other loops open. The enlarged time delay will reduce the phase margin. In this case, the time delay is determined by Equation (4.9)

$$\hat{\theta}_u = \gamma_u \theta_u.$$

Figure 4.2. Interaction mode with $\lambda_u < 1$, $\gamma_u > 1$

Case 3: $\lambda_u \geq 1$, $\gamma_u \leq 1$, Figure 4.3.

In this case, $\left(\frac{1}{\lambda_u} - 1\right) \leq 0$ and $(\gamma_u - 1) \leq 0$. According to Equations (4.3) and (4.9),

we have $\hat{g}_u(0) \leq g_u(0)$, and $\hat{\theta}_u \leq \theta_u$.

- $\hat{g}_u(0) \leq g_u(0)$. This means that the magnitude of the frequency response when the other loops closed is smaller than that of when the other loops open. Even if the retaliatory effect from other loops acts in opposition to the main effect of u_i on y_i , we cannot enlarge the controller gain for better performance due to the system integrity consideration. Hence, the gain should be unchanged, i.e.,

$$\hat{g}_u(0) = g_u(0).$$

- $\hat{\theta}_n \leq \theta_n$. Same as in Case 1

$$\hat{\theta}_n = \theta_n.$$

Figure 4.3. Interaction mode with $\lambda_n \geq 1$, $\gamma_n \leq 1$

Case 4: $\lambda_n \geq 1$, $\gamma_n > 1$, Figure 4.4.

In this case, $\left(\frac{1}{\lambda_n} - 1\right) \leq 0$ and $(\gamma_n - 1) > 0$. According to Equations (4.3) and (4.9),

we have $\hat{g}_n(0) \leq g_n(0)$, and $\hat{\theta}_n > \theta_n$.

- $\hat{g}_n(0) \leq g_n(0)$. Same as in Case 3,

$$\hat{g}_n(0) = g_n(0).$$

- $\hat{\theta}_n > \theta_n$. Same as in Case 2,

$$\hat{\theta}_n = \gamma_n \theta_n.$$

Figure 4.4. Interaction mode with $\lambda_{ii} \geq 1$, $\gamma_{ii} > 1$

4.3. SISO controller design

To illustrate the control system design procedure based on ETFs, the gain and phase margins design approach (Wang and Cai, 2002) is presented here for the sake of completeness.

Without loss of generality, we assume that each main loop, i.e., diagonal element in the transfer function matrix is represented by a SOPDT model, which can be used to describe most of the industrial processes:

$$g_{ii}(s) = \frac{b_{0,ii}}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-\theta_{ii}s}.$$

Similarly, ETF is represented as

$$\hat{g}_{ii}(s) = \frac{\hat{g}_{ii}(0)}{a_{2,ii}s^2 + a_{1,ii}s + 1} e^{-\hat{\theta}_{ii}s},$$

The decentralized controllers can then be independently designed by single loop approaches based on corresponding ETFs. Here we employ the gain and phase margins approach. This is primarily because the frequency response method provides good performance in the face of uncertainty in both plant model and disturbances. The PID controller of each loop is supposed of the following standard form:

$$g_{c,i}(s) = k_{p,i} + \frac{k_{i,i}}{s} + k_{d,i}s = k \frac{As^2 + Bs + C}{s}$$

where $A = k_{d,i}/k$, $B = k_{p,i}/k$ and $C = k_{i,i}/k$. By letting $A = a_2$, $B = a_1$ and $C = 1$, the open-loop transfer function becomes

$$g_{c,i}(s)\hat{g}_u(s) = k \frac{\hat{g}_u(0)}{s} e^{-\hat{\theta}_u s}.$$

Denoting the gain and phase margin specifications as $A_{m,i}$ and $\Psi_{m,i}$, and their crossover frequencies as $\omega_{g,i}$ and $\omega_{p,i}$, respectively, we have

$$\arg[g_{c,i}(j\omega_{g,i})\hat{g}_u(j\omega_{g,i})] = -\pi$$

$$A_{m,i} |g_{c,i}(j\omega_{g,i})\hat{g}_u(j\omega_{g,i})| = 1$$

$$|g_{c,i}(j\omega_{p,i})\hat{g}_u(j\omega_{p,i})| = 1$$

$$\Psi_{m,i} = \pi + \arg[g_{c,i}(j\omega_{p,i})\hat{g}_u(j\omega_{p,i})]$$

By substitution and simplification to above equations, we obtain

$$\omega_{g,i}\hat{\theta}_u = \frac{\pi}{2} \quad A_{m,i} = \frac{\omega_{g,i}}{k\hat{g}_u(0)}$$

$$k\hat{g}_u(0) = \omega_{p,i} \quad \Psi_{m,i} = \frac{\pi}{2} - \omega_{p,i}\hat{\theta}_u$$

which result

$$\Psi_{m,i} = \frac{\pi}{2} \left(1 - \frac{1}{A_{m,i}}\right), \quad k = \frac{\pi}{2A_{m,i}\hat{\theta}_u\hat{g}_u(0)}$$

By this formulation, the gain and phase margin are interrelated to each other, some possible gain and phase margin selections are given in Table 4.1.

Table 4.1. Typical gain and phase margin values

$\Psi_{m,i}$	$\pi/4$	$\pi/3$	$3\pi/8$	$2\pi/5$
$A_{m,i}$	2	3	4	5

The PID parameters are given by

$$\begin{bmatrix} k_{p,i} \\ k_{i,i} \\ k_{d,i} \end{bmatrix} = \frac{\pi}{2A_{m,i}\hat{\theta}_u\hat{g}_u(0)} \begin{bmatrix} a_{1,i} \\ 1 \\ a_{2,i} \end{bmatrix}. \quad (4.10)$$

Applying Equation (4.10) for each case discussed in Section 3, we can easily obtain both ETFs and the PID parameters which are summarized in Table 4.2.

Table 4.2. ETFs and PID controllers for gain and phase margin method

Mode	$\hat{g}_u(s)$	$k_{p,u}$	$k_{i,u}$	$k_{d,u}$
$\lambda_{ui} < 1,$ $\gamma_{ui} \leq 1$	$\frac{g_u(0)/\lambda_{ui}}{a_{2,u}s^2 + a_{1,u}s + 1} e^{-\theta_u s}$	$\frac{\pi \lambda_{ui} a_{1,u}}{2 A_{m,j} \theta_{ui} g_u(0)}$	$\frac{\pi \lambda_{ui}}{2 A_{m,j} \theta_{ui} g_u(0)}$	$\frac{\pi \lambda_{ui} a_{2,u}}{2 A_{m,j} \theta_{ui} g_u(0)}$
$\lambda_{ui} < 1,$ $\gamma_{ui} > 1$	$\frac{g_u(0)/\lambda_{ui}}{a_{2,u}s^2 + a_{1,u}s + 1} e^{-\gamma_{ui} \theta_u s}$	$\frac{\pi \lambda_{ui} a_{1,u}}{2 A_{m,j} \gamma_{ui} \theta_{ui} g_u(0)}$	$\frac{\pi \lambda_{ui}}{2 A_{m,j} \gamma_{ui} \theta_{ui} g_u(0)}$	$\frac{\pi \lambda_{ui} a_{2,u}}{2 A_{m,j} \gamma_{ui} \theta_{ui} g_u(0)}$
$\lambda_{ui} \geq 1,$ $\gamma_{ui} \leq 1$	$\frac{g_u(0)}{a_{2,u}s^2 + a_{1,u}s + 1} e^{-\theta_u s}$	$\frac{\pi a_{1,u}}{2 A_{m,j} \theta_{ui} g_u(0)}$	$\frac{\pi}{2 A_{m,j} \theta_{ui} g_u(0)}$	$\frac{\pi a_{2,u}}{2 A_{m,j} \theta_{ui} g_u(0)}$
$\lambda_{ui} \geq 1,$ $\gamma_{ui} > 1$	$\frac{g_u(0)}{a_{2,u}s^2 + a_{1,u}s + 1} e^{-\gamma_{ui} \theta_u s}$	$\frac{\pi a_{1,u}}{2 A_{m,j} \gamma_{ui} \theta_{ui} g_u(0)}$	$\frac{\pi}{2 A_{m,j} \gamma_{ui} \theta_{ui} g_u(0)}$	$\frac{\pi a_{2,u}}{2 A_{m,j} \gamma_{ui} \theta_{ui} g_u(0)}$

Remark 4.1. A unique problem for decentralized control of MIMO processes is the zero crossing (Cui and Jacobsen, 2002): stable or unstable zeros might be introduced into a particular control loop when other loops are closed. If an unstable zero is introduced, it will result in a phase shift to the left in the frequency domain. In order to guarantee the entire system stability, the controllers are normally conservatively designed by conventional detuning approaches (Cui and Jacobsen, 2002). By introducing the relative critical frequency, γ_{ui} , to indicate phase changes after the other loops closed, the effects of unstable zeros can be accurately estimated in each control loop. Consequently, the resultant control systems will be much less conservative.

Remark 4.2. Mathematically, the equivalent transfer function should incorporate the controllers of all other loops. To solve such a complex problem, recursive solution is required by first assigning initial controllers, then finding the equivalent loop transfer functions and designing controllers again. This process is continuous until a stable solution is obtained. To simplify the problem, both detuning and independent methods proposed so far assumes that all other closed-loops are under perfect control when design controller for a particular loop and consider only the gain change. In the proposed method, the changes are considered for both gain and frequency. As will be shown later, it is far more accurate than the current existing methods.

As can be seen, the design procedure is simple, straightforward and can be fully automated. The procedure for designing decentralized PI/PID controller for MIMO processes can be summarized as follows.

Algorithm 4.1:

- Step 1.* For a given transfer function matrix, calculate E , Φ , Λ and Γ by Equations (3.3), (3.5), (3.1) and (4.6), respectively.
- Step 2.* Select the best pairing and put them on the diagonal position according to REA, determine the equivalent transfer functions for diagonal elements by calculating the two correcting factors using Equations (4.3) and (4.9), respectively.
- Step 3.* Determine the parameters of the diagonal controllers by looking up Table 4.2.

4.4. Case studies

In this section, we apply the proposed design method to a number of industrial processes. The gain and phase margins for all examples except Example 4.2 are specified to be 3 db and $\pi/3 \text{ rad}$, respectively. Those margins are generally regarded as making the best compromise between performance and robustness in process control. The proposed design method is compared with the following popular decentralized control design approaches to show the effectiveness of the proposed design method:

- (1) The Ziegler-Nichols with detuning factor approach proposed by Mc Avoy (1983);
- (2) The biggest log modulus (BLT) tuning approach proposed by Luyben (1986);
- (3) The RGA based tuning approach proposed by Chien *et al.* (1999);
- (4) The sequential loop tuning approach proposed by Shen and Yu (1994);
- (5) The relay based auto-tuning approach proposed by Loh *et al.* (1993); and
- (6) The describing function matrix approach proposed by Loh and Vasnani (1994).

Example 4.1. Consider a process given by

$$\mathbf{G}(s) = \begin{bmatrix} \frac{5e^{-3s}}{4s+1} & \frac{2.5e^{-5s}}{15s+1} \\ \frac{-4e^{-6s}}{20s+1} & \frac{e^{-4s}}{5s+1} \end{bmatrix}.$$

This process seems easy to control. However, as can be seen below, both variable pairing and control design are difficult with conventional approaches.

The RGA, critical frequency array, REA and RFA are given respectively by

$$\Lambda = \begin{bmatrix} 0.3333 & 0.6667 \\ 0.6667 & 0.3333 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0.6308 & 0.2732 \\ 0.2856 & 0.4909 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0.6649 & 0.3351 \\ 0.3351 & 0.6649 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1.9948 & 0.5026 \\ 0.5026 & 1.9948 \end{bmatrix}.$$

It can be seen that the best pairing according to RGA (0.6667) and NI (=1.5000>0) is off-diagonal, i.e., 1-2/2-1. Using the method (1), we obtain the detuning factor $\left|0.6667 + \sqrt{0.6667^2 - 0.6667}\right| = 0.8165$, and corresponding PI controller parameters which results in an unstable system. By keeping detuning the original PI controller using the same detuning factor four times, we obtain the PI controller parameters as listed in Table 4.3. Both loops are stabilized but with very oscillatory response as shown in Figure 4.5.

Following REA (0.6649) and NI (=0.6152>0), the best pairing is diagonal, i.e., 1-1/2-2. As $\lambda_{r_i} < 1$ and $\gamma_{r_i} > 1$, the equivalent process for two loop are calculated by proposed method as

$$\frac{15.0000e^{-5.9844s}}{4s+1} \quad \text{and} \quad \frac{3.0000e^{-7.9792s}}{5s+1}$$

respectively. The resulting PI controllers and the closed-loop responses are also listed in Table 4.3 and shown in Figure 4.5, respectively, where the unit set-points change in r_1 at $t=0$ and r_2 at $t=1250$. It can be seen that the proposed method results in a better loop pairing and control system performance while the conventional detuning method (1) results in a unsatisfactory performance even detuning by four times.

Table 4.3. Controllers for Example 4.1

Controller	Proposed		Mc Avoy	
	$k_{p,i}$	$\tau_{i,i}$	$k_{p,i}$	$\tau_{i,i}$
Loop 1	0.0233	4.0000	-0.3333	18.0000
Loop 2	0.1094	5.0000	0.4800	15.0000

Figure 4.5 Closed-loop responses for Example 4.1

Example 4.2. Consider an industrial-scale polymerization reactor given by (Chien *et al.*, 1999)

$$\mathbf{G}(s) = \begin{bmatrix} \frac{22.89}{4.572s + 1} e^{-0.2s} & \frac{-11.64}{1.807s + 1} e^{-0.4s} \\ \frac{4.689}{2.174s + 1} e^{-0.2s} & \frac{5.80}{1.801s + 1} e^{-0.4s} \end{bmatrix}.$$

The time scales are in hours, so it is a quite slow process. In addition, it is easy to verify that it is not diagonally dominate. The two controlled variables are two measurements representing the reactor condition, and two manipulated variables are the set-points of two reactor feed flow loops with load disturbance as the purge flow of the reactor.

The RGA, critical frequency array, REA and RFA are given respectively by

$$\Lambda = \begin{bmatrix} 0.7087 & 0.2913 \\ 0.2913 & 0.7087 \end{bmatrix}, \Omega = \begin{bmatrix} 8.0554 & 4.1888 \\ 7.8540 & 4.3036 \end{bmatrix},$$
$$\Phi = \begin{bmatrix} 0.7193 & 0.2807 \\ 0.2807 & 0.7193 \end{bmatrix}, \Gamma = \begin{bmatrix} 1.0151 & 0.9633 \\ 0.9633 & 1.0151 \end{bmatrix}.$$

Both REA and RGA indicate diagonal pairing ($NI = 1.4111 > 0$). In this example, we have $\lambda_{ii} < 1$ and $\gamma_{ii} > 1$ for $i = 1, 2$. According to the proposed method the equivalent process for the two loop are calculated as

$$\frac{32.3003}{4.572s + 1} e^{-0.2030s} \quad \text{and} \quad \frac{8.1844}{1.801s + 1} e^{-0.4060s}$$

respectively. For the gain and phase margins of 5 and $2\pi/5$, respectively, the resulting PI controllers determined by the proposed method together with those determined by methods (2), (3) and (5) are listed in Table 4.4.

Table 4.4. Controllers for Example 4.2

Controller	Proposed		Luyben		Chien <i>et al.</i>		Loh <i>et al.</i>	
	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$
Loop 1	0.2190	4.5720	0.210	2.26	0.263	1.42	0.620	0.60
Loop 2	0.1703	1.8010	0.175	4.25	0.163	1.77	0.247	1.78

Figure 4.6 shows the closed-loop responses for different controllers, where the unit set-points change in r_1 at $t=0$ and r_2 at $t=20$. It can be seen that for this process which is not diagonally dominant, the proposed design method gives the best performance in terms of overshoot and oscillation.

Figure 4.6. Closed-loop responses for Example 4.2

To investigate the robustness of the system, we mismatch the process model by increasing all four time delays by 25%. The closed-loop responses under same controllers are shown in Figure 4.7. As can be seen, the proposed method still shows the best performance.

Figure 4.7. Closed-loop responses for Example 4.2 (model mismatched)

Example 4.3. Consider the 3x3 process given by (Loh and Vasnani, 1994)

$$G(s) = \begin{bmatrix} \frac{119e^{-5s}}{21.7s+1} & \frac{153e^{-5s}}{337s+1} & \frac{-2.1e^{-5s}}{10s+1} \\ \frac{37.0e^{-5s}}{500s+1} & \frac{76.7e^{-5s}}{28s+1} & \frac{-5.0e^{-5s}}{10s+1} \\ \frac{93.0e^{-5s}}{500s+1} & \frac{-66.7e^{-5s}}{166s+1} & \frac{-103.3e^{-5s}}{23s+1} \end{bmatrix}.$$

Loh and Vasnani used this process to verify their design for high dimensional systems. It is also not diagonally dominate process.

The RGA, critical frequency array, REA and RFA are given respectively by

$$\Lambda = \begin{bmatrix} 2.1896 & -1.1446 & -0.0449 \\ -1.3147 & 2.0677 & 0.2470 \\ 0.1252 & 0.0769 & 0.7979 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0.3290 & 0.3222 & 0.3611 \\ 0.3142 & 0.3307 & 0.3696 \\ 0.3142 & 0.3142 & 0.3491 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 2.0095 & -0.9693 & -0.0402 \\ -1.1226 & 1.8976 & 0.2250 \\ 0.1131 & 0.0717 & 0.8153 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0.9178 & 0.8468 & 0.8948 \\ 0.8539 & 0.9177 & 0.9108 \\ 0.9034 & 0.9322 & 1.0217 \end{bmatrix}.$$

The best pairing according to both RGA, REA and NI ($=0.4759 > 0$) is 1-1/2-2/3-3. For $i = 1, 2$, $\lambda_{ii} > 1$ and $\gamma_{ii} < 1$. Hence, the original transfer functions are selected as corresponding ETFs. For $i = 3$, $\lambda_{ii} < 1$ and $\gamma_{ii} > 1$. The equivalent process is calculated as

$$\frac{-129.4574e^{-5.1084s}}{23s + 1}$$

The resulting PI controllers by proposed method are listed in Table 4.5 together with those of method (6).

Table 4.5. Controllers for Example 4.3

Controller	Proposed		Loh and Vasnani	
	$k_{p,ji}$	$\tau_{i,ji}$	$k_{p,ji}$	$\tau_{i,ji}$
Loop 1	0.0191	21.7000	0.0181	14.4936
Loop 2	0.0382	28.0000	0.0335	14.4936
Loop 3	-0.0182	23.0000	-0.0260	14.4936

Figure 4.8 shows the closed-loop responses, where the unit set-points change in r_1 at $t=0$, r_2 at $t=100$ and r_3 $t=200$. As can be seen, the proposed method results in a better performance with the integral times much different from those of Loh and Vasnani.

Figure 4.8. Closed-loop responses for Example 4.3

Example 4.4. Consider the 4x4 process given by (Doukas and Luyben, 1978)

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-9.811e^{-1.59s}}{11.36s+1} & \frac{0.374e^{-7.75s}}{22.22s+1} & \frac{-2.368e^{-27.33s}}{33.3s+1} & \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} \\ \frac{5.984e^{-2.24s}}{14.29s+1} & \frac{-1.986e^{-0.71s}}{66.67s+1} & \frac{0.422e^{-8.72s}}{(250s+1)^2} & \frac{5.24e^{-60s}}{400s+1} \\ \frac{2.38e^{-0.42s}}{(1.43s+1)^2} & \frac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \frac{0.513e^{-s}}{s+1} & \frac{-0.33e^{-0.68s}}{(2.38s+1)^2} \\ \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \frac{-0.176e^{-0.48s}}{(6.9s+1)^2} & \frac{15.54e^{-s}}{s+1} & \frac{4.48e^{-0.52s}}{11.11s+1} \end{bmatrix}$$

This is a real process with complicated delay and interaction modes. The problems of some of the unrealistic pathological cases would be avoided using such kind of processes to verify design.

The RGA, critical frequency array, REA and RFA are given by

$$\Lambda = \begin{bmatrix} 0.1264 & -0.1013 & -0.0314 & 1.0063 \\ 0.0107 & 1.0935 & 0.0003 & -0.1045 \\ 0.7264 & 0.0025 & 0.1630 & 0.1081 \\ 0.1366 & 0.0054 & 0.8680 & -0.0099 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} 1.0134 & 0.2285 & 0.0757 & 0.1571 \\ 0.7306 & 2.2440 & 0.0306 & 0.0279 \\ 1.7952 & 0.6830 & 1.9635 & 1.0833 \\ 0.1551 & 0.7854 & 2.0268 & 3.1416 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 0.5237 & -0.0052 & -0.0222 & 0.5036 \\ -0.0024 & 1.0035 & 0.0000 & -0.0012 \\ 0.4719 & 0.0012 & 0.2965 & 0.2303 \\ 0.0067 & 0.0004 & 0.7256 & 0.2673 \end{bmatrix},$$

and

$$\Gamma = \begin{bmatrix} 4.1448 & 0.0511 & 0.7066 & 0.5004 \\ -0.2239 & 0.9177 & 0.0978 & 0.0113 \\ 0.6497 & 0.4888 & 1.8192 & 2.1309 \\ 0.0492 & 0.0793 & 0.8359 & -26.9057 \end{bmatrix},$$

respectively. The best pairing according to REA and NI ($=1.1814 > 0$, ideal pairing) is 1-4/2-2/3-1/4-3. This example has complicated interaction modes. For $i=1,2$, $\lambda_{ii} > 1$ and $\gamma_{ii} < 1$, the main process is selected as corresponding equivalent process. For $i=3,4$, $\lambda_{ii} < 1$ and $\gamma_{ii} < 1$, the equivalent process is calculated as

$$\frac{17.9031e^{-s}}{s+1} \text{ and } \frac{-11.2289e^{-3.79s}}{(21.74s+1)^2}$$

respectively. The resulting PI/PID controllers by the proposed method are listed in Table 4.6 together with method (2) based on the diagonal pairing. It should be noticed that different designs have much different controller parameters due to different pairing schemes.

Table 4.6. Controllers for Example 4.4

Controller	Proposed			Luyben	
	$k_{p,u}$	$\tau_{i,u}$	$\tau_{d,u}$	$k_{p,u}$	$\tau_{i,u}$
Loop 1	1.4981	2.8600	0.7150	-0.084	33
Loop 2	-24.7566	66.6700	0	-5.16	15.5
Loop 3	0.0292	1	0	0.305	17.0
Loop 4	-0.5349	43.4800	10.8700	0.529	11.2

Figure 4.9 shows the closed-loop responses, where the unit set-points change in r_1 at $t=0$, r_2 at $t=400$, r_3 $t=800$ and r_4 $t=1200$. Although there are larger overshoots in some loops, the system has better overall performance. This example illustrates that the proposed method can be easily applied to high dimensional processes with complicated interaction modes. In fact, the dimension does not increase the design difficulty.

Figure 4.9. Closed-loop responses for Example 4.4

To investigate the robustness of this higher dimensional system, we mismatch the process model by increasing all four main process steady-state gains by 70%. The closed-loop responses under same controllers are shown in Figure 4.10. It is noted that the proposed method still shows satisfactory performance (almost no change) under such a severe model variation while the other one is more oscillatory.

Figure 4.10. Closed-loop responses for Example 4.4 (model mismatched)

4.5. Conclusions

This chapter presents a novel method to construct equivalent transfer function models for decentralized control system design of multivariable interactive processes. The simplicity and effectiveness of the model is based on the energy transfer ratio of each individual transfer function which provides necessary information of gain and frequency changes when all other loops are closed. Consequently, the decentralized controllers can be obtained by simply using single loop design approaches. Simulation results for a number of industrial 2×2 , 3×3 and 4×4 processes show that the proposed method provides a better overall performance compared to other reported design approaches. The advantage of the proposed method is even more significant when applied to higher dimensional processes with complicated interaction modes. The method can be easily integrated into an auto-tuning control structure combined with some on line parameter identification module and implemented for industrial control systems.

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Chapter 5*

Decoupling Control System Design

5.1. Introduction

For MIMO processes with severe loop interactions, the decoupling control schemes are often preferred. The decoupling control usually requires two steps: 1) design of decoupler to minimize the interactions among loops; and 2) design the main loop controllers for overall system performance. The key advantages of the decoupling control approach are that: 1) it allows the use of SISO controller design methods; and 2) in case of actuator or sensor failure, it is relatively easy to stabilize the loop manually, since only one loop is directly affected by the failure. However, such a design may result in very complicated control structures, especially, when the system dimension is high. Consequently, the research activities are primarily focused on TITO processes (Shinskey, 1996; Gawthrop, 1987; Palmor *et al.*, 1995; Ho *et al.*, 2000; Wang *et al.*, 2000; Gilbert *et al.*, 2003; Chen and Seborg, 2002; Tavakoli *et al.*, 2006). For high dimensional processes, Wang *et al.* developed some methods for full dimensional PID or non-PID controllers with systematic design procedures and relatively simple system structures (Wang *et al.*, 1997; Wang and Yang, 2002; Wang *et al.*, 2003). However, due to the mathematic approximation of the decouplers for high dimensional process, the robustness of such a control system can not be

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guaranteed. Another approach is the static decoupling which can significantly improve control performances with their robust stability comparable to that of multi-loop control systems are adopted by the industry to deal with severe loop interactions for high dimensional MIMO processes (Lee *et al.*, 2005). Although static decoupling is simple to design and implement, they may not always provide satisfactory closed-loop performances if there exists large difference in dynamic characteristics among the transfer function elements.

In this chapter, a novel decoupling design method for multivariable processes is proposed. The method include three steps: 1) using the concepts of energy transmission ratio to obtain the effective relative gain, relative gain and relative frequency of a given transfer function matrix; 2) using the information obtained in the first step to obtain an equivalent transfer function matrix for closed-loop system; and 3) designing the off-diagonal controllers based on interaction analysis and the diagonal controllers for original transfer functions of main loops. Although this method is basically a full dimensional controller design, it is simple, straightforward and easy to be applied by field control engineers. The resulting controller can have satisfactory performance even when the primary process parameters change significantly. Several multivariable industrial processes with different interaction characteristics are employed to demonstrate the simplicity and robustness of the design.

5.2. General formulation of MIMO control

Consider an open-loop stable multivariable system as shown in Figure 2.2. Again, a SOPDT model is adopted to approximate each element of the process transfer function matrix, i.e.,

$$g_{ij}(s) = \frac{g_{ij}(0)}{a_{2,ij}s^2 + a_{1,ij}s + 1} e^{-\theta_{ij}s}$$

For a closed-loop controlled MIMO system, it is desirable that the forward transfer function to be of the form

$$G(s)G_c(s) \approx G(s)G^{-1}(s) = I/s \quad (5.1)$$

It is, however, not an easy task to find a suitable solution for $G^{-1}(s)$ under the fixed control structures, as it may end up with very complicated expressions,

especially, when the system dimension is high. In the following, technique for computing loop dynamic interactions are employed to find a simple description for $G^{-1}(s)$, and establish the relationship between $G^{-1}(s)$ and the closed-loop transfer function matrix.

When a MIMO control system is closed, there exist interactions among loops as a result of the existence of non-zero off-diagonal elements in the transfer function matrix. The interactions can be dynamically measured by the dynamic relative gain defined by (Bristol, 1966):

$$\lambda_{ij}(s) = g_{ij}(s) / \hat{g}_{ij}(s) \quad (5.2)$$

where $\hat{g}_{ij}(s)$ is the equivalent transfer function of $g_{ij}(s)$ when all other loops are closed. For overall system, Equation (5.2) can be written in a matrix form which results in the dynamic relative gain array (DRGA),

$$\Lambda(s) = \begin{bmatrix} \lambda_{11}(s) & \lambda_{12}(s) & \dots & \lambda_{1n}(s) \\ \lambda_{21}(s) & \lambda_{22}(s) & \dots & \lambda_{2n}(s) \\ \dots & \dots & \dots & \dots \\ \lambda_{n1}(s) & \lambda_{n2}(s) & \dots & \lambda_{nn}(s) \end{bmatrix} \quad (5.3)$$

Substituting Equation (5.2) into (5.3) results in

$$\begin{aligned} \Lambda(s) &= \begin{bmatrix} g_{11}(s)/\hat{g}_{11}(s) & g_{12}(s)/\hat{g}_{12}(s) & \dots & g_{1n}(s)/\hat{g}_{1n}(s) \\ g_{21}(s)/\hat{g}_{21}(s) & g_{22}(s)/\hat{g}_{22}(s) & \dots & g_{2n}(s)/\hat{g}_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s)/\hat{g}_{n1}(s) & g_{n2}(s)/\hat{g}_{n2}(s) & \dots & g_{nn}(s)/\hat{g}_{nn}(s) \end{bmatrix} \\ &= \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & g_{n2}(s) & \dots & g_{nn}(s) \end{bmatrix} \otimes \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{12}(s) & \dots & 1/\hat{g}_{1n}(s) \\ 1/\hat{g}_{21}(s) & 1/\hat{g}_{22}(s) & \dots & 1/\hat{g}_{2n}(s) \\ \dots & \dots & \dots & \dots \\ 1/\hat{g}_{n1}(s) & 1/\hat{g}_{n2}(s) & \dots & 1/\hat{g}_{nn}(s) \end{bmatrix} \end{aligned} \quad (5.4)$$

where the operator $\otimes$ is the hadamard product. Since $\hat{g}_{ij}(s)$ is controller dependant, it is impossible to compute $\Lambda(s)$ without first knowing the controller parameters. By assuming the process is under perfect control, however, a simple computational algorithm for $\Lambda(s)$ can be obtained to calculate the relative gains at each frequency point (Grosdidier and Morari, 1985):

$$\begin{aligned}\Lambda(s) &= \mathbf{G}(s) \otimes \mathbf{G}^{-T}(s) \\ &= \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \otimes \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}^{-T} \quad (5.5)\end{aligned}$$

Comparing Equations (5.4) and (5.5), we have

$$\mathbf{G}^{-T}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix}^{-T} = \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{12}(s) & \cdots & 1/\hat{g}_{1n}(s) \\ 1/\hat{g}_{21}(s) & 1/\hat{g}_{22}(s) & \cdots & 1/\hat{g}_{2n}(s) \\ \cdots & \cdots & \cdots & \cdots \\ 1/\hat{g}_{n1}(s) & 1/\hat{g}_{n2}(s) & \cdots & 1/\hat{g}_{nn}(s) \end{bmatrix},$$

which implies

$$\mathbf{G}^{-1}(s) = \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{21}(s) & \cdots & 1/\hat{g}_{n1}(s) \\ 1/\hat{g}_{12}(s) & 1/\hat{g}_{22}(s) & \cdots & 1/\hat{g}_{n2}(s) \\ \cdots & \cdots & \cdots & \cdots \\ 1/\hat{g}_{1n}(s) & 1/\hat{g}_{2n}(s) & \cdots & 1/\hat{g}_{nn}(s) \end{bmatrix}. \quad (5.6)$$

As a stable and well designed decoupling control system asymptotically approaches the perfect control, Equation (5.6) provides the fundamental relationship between open and closed-loop transfer functions. Consequently, the problem of designing PI/PID controller for MIMO processes is converted to:

- (1) Find the best loop pairings, place them on the diagonal position, and obtain the equivalent transfer function $\hat{g}_{ij}(s)$ by taking loop interactions into account. This topic has been discussed in Chapter 3 and Chapter 4.
- (2) Determine the structure and parameters of $g_{c,ij}(s)$ such that the interactions among loops are minimized and the desired control system performance is obtained. This will be discussed in the following sections.

5.3. Full decoupling control

By full dimensional controller structure, Equations (2.1), (2.3), (5.1) and (5.6), we can easily establish the relation between the controller elements and the ETFs as

$$G_c(s) = \begin{bmatrix} g_{c,11}(s) & g_{c,12}(s) & \dots & g_{c,1n}(s) \\ g_{c,21}(s) & g_{c,22}(s) & \dots & g_{c,2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{c,n1}(s) & g_{c,n2}(s) & \dots & g_{c,nm}(s) \end{bmatrix} = \frac{1}{s} \begin{bmatrix} 1/\hat{g}_{11}(s) & 1/\hat{g}_{21}(s) & \dots & 1/\hat{g}_{n1}(s) \\ 1/\hat{g}_{12}(s) & 1/\hat{g}_{22}(s) & \dots & 1/\hat{g}_{n2}(s) \\ \dots & \dots & \dots & \dots \\ 1/\hat{g}_{1n}(s) & 1/\hat{g}_{2n}(s) & \dots & 1/\hat{g}_{nn}(s) \end{bmatrix} \quad (5.7)$$

Consequently, the problem of design the PI/PID controller for MIMO processes is converted to find $G_c(s)$ such that Equation (5.7) hold. This can be divided into two sub-tasks:

- (1) **Off-diagonal controllers:** The main task of the off-diagonal controllers is to minimize the interactions among loops. According to Equation (5.7), we have

$$s g_{c,ji}(s) = \frac{1}{\hat{g}_{ij}(s)} \Rightarrow g_{c,ji}(s) \hat{g}_{ij}(s) = 1/s. \quad (5.8)$$

Image that $g_{c,ji}(s) \hat{g}_{ij}(s)$ is the forward transfer function of an *fictitious closed-loop control system*, the control objective, $g_{c,ji}(s) \hat{g}_{ij}(s) = 1/s$, is to obtain an ideal control for this loop. As the ideal control cannot be realized in practice, the PI/PID controller structure, such as, the gain and phase margin approach described in Chapter 4 can be used to design the controllers.

- (2) **Diagonal controllers:** The diagonal controllers are to provide the desired performance of the closed-loop control system. As the interactions among the loops have already been dealt with by the off-diagonal controllers, the original transfer functions of diagonal elements after loop pairing are used for the controller design. Again, the gain and phase margin approach in Chapter 4 is one of the options to be used for designing the controllers.

Remark 5.1. Theoretically, any SISO controller design approach can be employed. This is because the interaction is already approximately considered into the equivalent transfer functions. The gain and phase margins approach is selected because it provides good performance in terms of robustness with respect to the uncertainties in both process model and disturbance, and might be more acceptable by process control engineers.

It is worth noting that the only difference between off-diagonal and diagonal controllers is the proportional gain. For off-diagonal proportional gains, they are corrected by both relative gain and relative frequency factors, while the original

parameters are used for diagonal ones. In addition, it should be noticed that the control performance may not be met well for some high dimensional systems since a fixed PI/PID structure is adopted.

In summary, the whole design procedure is very simple, straightforward and can be fully automated. The general procedure for designing full dimensional PI/PID controller for MIMO processes can be summarized as follows.

Algorithm 5.1:

- Step 1.* For a given transfer function matrix, calculate E , Φ , Λ and Γ by Equations (3.3), (3.5), (3.1) and (4.6), respectively.
- Step 2.* Select the best pairing and put them on the diagonal position according to REA, determine the equivalent transfer functions for off-diagonal elements by calculating the two correcting factors using Equations (4.3) and (4.9), respectively.
- Step 3.* Determine the parameters of the off-diagonal decoupling controllers and main loop diagonal controllers by Equation (4.10) with modified and original process parameters, respectively.

5.4. Block decoupling control

The block decoupling controllers balance the high performance given by fully multivariable controllers and the easy implementation and maintenance associated with fully decentralized controllers. The use of block decoupling control is also because of the fact that in most industrial processes, the interactions are limited in scope and do not include the full scope of the plant.

For block decoupling controller structure, Equations (2.1) and (2.4), to identify the structure of $G_c(s)$, a threshold, ρ , is to be specified by the designer. If

$$|\phi_{kl}| > \rho, \quad k, l = 1, 2, \dots, n, \quad k \neq l,$$

the k^{th} and l^{th} loops are considered to be closely coupled under the ρ criterion, even if $|\phi_{lk}| < \rho$. Once the structure of $G_c(s)$ is determined, rearrange the columns and rows of the transfer function matrix such that $\hat{G}(s)$ can be partitioned as

$$\hat{G}(s) = \begin{bmatrix} \hat{G}_1(s) & * & \dots & * \\ * & \hat{G}_2(s) & \dots & \vdots \\ \vdots & \dots & \dots & * \\ * & \dots & * & \hat{G}_m(s) \end{bmatrix},$$

where $\hat{G}_l(s)$, $l=1\dots m$ are sub-matrices of closed-loop ETFs with the same dimensions as $G_{cl}(s)$. The block decoupling control starts from the 2×2 controller sub-blocks, let

$$G_{ck}(s) = \begin{bmatrix} g_{c,pp}(s) & g_{c,pp+1}(s) \\ g_{c,p+1p}(s) & g_{c,p+1p+1}(s) \end{bmatrix},$$

where $p+1 \leq n$. Correspondingly

$$\hat{G}_k(s) = \begin{bmatrix} \hat{g}_{pp}(s) & \hat{g}_{pp+1}(s) \\ \hat{g}_{p+1p}(s) & \hat{g}_{p+1p+1}(s) \end{bmatrix},$$

and

$$\Phi_k = \begin{bmatrix} \phi_{pp} & \phi_{pp+1} \\ \phi_{p+1p} & \phi_{p+1p+1} \end{bmatrix}.$$

Then, for perfect control of the sub-block $\hat{G}_k(s)$, it requires

$$G_{ck}(s) \approx \hat{G}_k(s).$$

The overall design of block decoupling control is summarized as follows.

Algorithm 5.2:

Step 1. Design decouplers for those 2×2 sub-blocks according to design procedures for off-diagonal decoupling controllers using Equation (5.8).

Step 2. Recalculate ϕ_{pp} and ϕ_{p+1p+1} . As the off-diagonal elements have been dealt with by decoupling controllers, ϕ_{pp} and ϕ_{p+1p+1} are to be updated by

$$\begin{aligned} \phi_{pp} &:= \phi_{pp} + \phi_{pp+1} \\ \phi_{p+1p+1} &:= \phi_{p+1p+1} + \phi_{p+1p} \end{aligned} \tag{5.9}$$

to result in a less conservative decentralized control design.

Step 3. Approximate the ETFs of the diagonal elements using Equation (4.7) in the sub-block $\hat{G}_k(s)$ by the new ϕ_{pp} and ϕ_{p+1p+1} obtained in Equation (5.9).

Step 4. Design the decentralized controller for the diagonal elements following the principles given in Chapter 4.

Remark 5.2. In decoupling control system design, Equation (4.7) is directly used to find the ETFs for off-diagonal elements, $\hat{g}_{ij}(s)$, $i \neq j$, using λ_{ij} and γ_{ij} , since the purpose of the off-diagonal ETFs is to design decoupling controllers to improve the system performance while control system integrity is not our main concern here. The no-causal elements, $e^{\hat{\theta}_{ij}s}$, which may appear in ETFs for some i, j , can be approximated by transfer function zeros using first order Taylor series in the decoupling controller design. Furthermore, since the off-diagonal decoupling controllers are PID type instead of simply the ratio of transfer function elements as in conventional decoupling controller design, the robustness improvement of the control system can be expected.

5.5. Case studies

In this section, we apply the proposed decoupling design methodology to a number of industrial processes. All controllers are designed by gain and phase margin method given in Chapter 4 and gain and phase margins are assigned to be $5dB$ and $2\pi/5rad$, respectively. They are compared with the following decoupling control approaches to show the effectiveness of the proposed designs:

- (1) The Gershgorin bands based decoupling approach proposed by Chen and Soberg (2002);
- (2) The integral decoupling approach proposed by Lee *et al.* (2005).

The effectiveness of the designed full decoupling and block decoupling control schemes are measured by:

- (1) Column dominance index (Chen and Seborg, 2003):

$$d_{c,j}(\omega) = \frac{|\tilde{G}_j(j\omega)| - \sum_{i=1, i \neq j}^n |\tilde{G}_i(j\omega)|}{|\tilde{G}_j(j\omega)|}, j = 1, 2, \dots, n, \quad (5.10)$$

where $\tilde{\mathbf{G}}(j\omega)$ is the forward transfer function matrix of the MIMO process (original or controlled), $d_{c,j}(\omega) = 1$ implies ideal decoupling (diagonal).

- (2) Robustness index: let the process input uncertainty be Δ , which is used to describe modeling errors, then, the closed-loop system is stable if and only if (Morari and Zafiriou, 1989)

$$\bar{\sigma}[H(j\omega)] < \frac{1}{\bar{\sigma}[\Delta(j\omega)]}, \forall \omega,$$

where $\bar{\sigma}[\bullet]$ is the maximum singular value of $\bullet$ and H is the complementary sensitivity function of the closed-loop system. A design method is more robust if the robustness index is smaller within interested frequency range.

Example 5.1. Consider an industrial-scale polymerization reactor (Chien *et al.*, 1999)

$$\mathbf{G}(s) = \begin{bmatrix} \frac{22.89}{4.572s+1}e^{-0.2s} & \frac{-11.64}{1.807s+1}e^{-0.4s} \\ \frac{4.689}{2.174s+1}e^{-0.2s} & \frac{5.80}{1.801s+1}e^{-0.4s} \end{bmatrix}.$$

For this process, the calculation of open-loop transfer function matrix gives

$$E = \begin{bmatrix} 184.3873 & -48.7575 \\ 36.8273 & 24.9606 \end{bmatrix}, \Lambda = \begin{bmatrix} 0.7087 & 0.2913 \\ 0.2913 & 0.7087 \end{bmatrix}, \Omega = \begin{bmatrix} 8.0554 & 4.1888 \\ 7.8540 & 4.3036 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0.7193 & 0.2807 \\ 0.2807 & 0.7193 \end{bmatrix} \text{ and } \Gamma = \begin{bmatrix} 1.0151 & 0.9633 \\ 0.9633 & 1.0151 \end{bmatrix}.$$

Both REA and RGA pairing rules indicate diagonal pairing (NI = 1.4111 > 0).

Full Decoupling Control: By the proposed method, the full dimensional decoupling controller is

$$G_c(s) = \begin{bmatrix} 0.3137(1 + \frac{1}{4.5720s}) & 0.1336(1 + \frac{1}{2.1740s}) \\ -0.0224(1 + \frac{1}{1.8070s}) & 0.2439(1 + \frac{1}{1.8010s}) \end{bmatrix}.$$

The designed decoupling controllers are evaluated by

- (1) The column dominance index of the decoupled and the original processes are calculated from Equation (5.10) and shown in Figure 5.1, which indicates that the original process is indeed not diagonally dominant; it is

actually off-diagonally dominant since $d_{c2}(\omega)$ equates to -1 for entire frequency range, while the degree of diagonal dominance has been substantially increased by the proposed method.

Figure 5.1. Column dominance for Example 5.1

- (2) The closed-loop system responses for the proposed *REA based full decoupling controller* (REA-FDC) are shown in Figure 5.2, where the unit set-points change in r_1 at $t=0$ and r_2 at $t=40$, load disturbance injected at $t=80$, respectively. For comparison, the results of the decoupling control given by Chen and Soberg (2002) are also shown in the same figure. The proposed design method gives better overall performance.

Figure 5.2. Closed-loop responses for Example 5.1 (nominal process)

(3) The value $\bar{\sigma}[H(j\omega)]$ of different design approaches are shown in Figure 5.3, the proposed method gives better robustness, and the maximum singular value increases slowly with the frequency.

Figure 5.3. Singular value based robustness index for Example 5.1

(4) The robustness of the different methods is further investigated by increasing all four time delays by 70%. The closed-loop responses for the two control schemes are shown in Figure 5.4. It is obvious that the performance of the proposed control method is still satisfactory while the

other one becomes oscillatory.

Figure 5.4. Closed-loop responses for Example 5.1 (model mismatched)

Example 5.2. Consider a 3x3 process given by (Ogunnaike and Ray, 1979)

$$G(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}.$$

The calculation of open-loop transfer function matrix results

$$E = \begin{bmatrix} 0.0985 & -0.0706 & -0.0005 \\ 0.3415 & -0.4720 & -0.0014 \\ -4.2552 & 4.2385 & 0.0701 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 2.0084 & -0.7220 & -0.2864 \\ -0.6460 & 1.8246 & -0.1786 \\ -0.3624 & -0.1026 & 1.4650 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} 0.1493 & 0.1157 & 0.1104 \\ 0.3077 & 0.2000 & 0.1410 \\ 0.1227 & 0.0917 & 0.0805 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 2.4267 & -1.1510 & -0.2758 \\ -0.8244 & 1.9746 & -0.1502 \\ -0.6023 & 0.1764 & 1.4259 \end{bmatrix}$$

$$\text{and } \Gamma = \begin{bmatrix} 1.2083 & 1.5942 & 0.9628 \\ 1.2762 & 1.0822 & 0.8409 \\ 1.6621 & -1.7197 & 0.9733 \end{bmatrix}.$$

The best pairing according to REA and NI ($=0.3859>0$) is 1-1/2-2/3-3.

Block Decoupling Control: Since $\phi_{12} = -1.1510$ and $\phi_{21} = -0.8244$ are large in amplitudes, block decoupling control scheme is preferred. The proposed *REA based block decoupling controller* (REA-BDC) can be easily obtained as

$$G_c(s) = \begin{bmatrix} 1.5911(1 + \frac{1}{6.7000s}) & -0.0716(1 + \frac{1}{3.2500s}) & 0 \\ 0.5758(1 + \frac{1}{8.6400s}) & -0.2679(1 + \frac{1}{5.0000s}) & 0 \\ 0 & 0 & 4.4831(1 + \frac{1}{12.4150s}) \end{bmatrix}.$$

The closed-loop responses are shown in Figure 5.5, where the unit set-points change in r_1 at $t=0$, r_2 at $t=500$ and r_3 at $t=1000$. The responses of full decoupling control given by Lee *et al.* (2005) are also shown in the same figure. It is clear that the proposed method has a better performance.

Figure 5.5. Closed-loop responses for Example 5.2 (nominal process)

To investigate the robustness of the system, we mismatch the process model by increasing steady-state gain for loop 1, time delay for loop 2, and time constants for loop 3 all by 40%. The closed-loop responses under same controllers are shown in Figure 5.6. In this case, the proposed method still gives satisfactory performance while the other method is oscillatory even it has satisfactory nominal performance under exact model. In fact, further investigation shows that the proposed controller can even work for 100% model parameter mismatches while the other one is unstable.

Figure 5.6. Closed-loop responses for Example 5.2 (model mismatched)

Example 5.3. Consider a 4x4 process given by (Doukas and Luyben, 1978)

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-9.811e^{-1.59s}}{11.36s+1} & \frac{0.374e^{-7.75s}}{22.22s+1} & \frac{-2.368e^{-27.33s}}{33.3s+1} & \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} \\ \frac{5.984e^{-2.24s}}{14.29s+1} & \frac{-1.986e^{-0.71s}}{66.67s+1} & \frac{0.422e^{-8.72s}}{(250s+1)^2} & \frac{5.24e^{-60s}}{400s+1} \\ \frac{2.38e^{-0.42s}}{(1.43s+1)^2} & \frac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \frac{0.513e^{-s}}{s+1} & \frac{-0.33e^{-0.68s}}{(2.38s+1)^2} \\ \frac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \frac{-0.176e^{-0.48s}}{(6.9s+1)^2} & \frac{15.54e^{-s}}{s+1} & \frac{4.48e^{-0.52s}}{11.11s+1} \end{bmatrix}.$$

The calculation of open-loop transfer function matrix results

$$E = \begin{bmatrix} -9.9426 & 0.0855 & -0.1793 & -1.7750 \\ 4.3719 & -4.4566 & 0.0129 & 0.1463 \\ 4.2726 & 0.0139 & 1.0073 & -0.3575 \\ -1.7531 & -0.1382 & 31.4970 & 14.0743 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 0.1264 & -0.1013 & -0.0314 & 1.0063 \\ 0.0107 & 1.0935 & 0.0003 & -0.1045 \\ 0.7264 & 0.0025 & 0.1630 & 0.1081 \\ 0.1366 & 0.0054 & 0.8680 & -0.0099 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} 1.0134 & 0.2285 & 0.0757 & 0.1571 \\ 0.7306 & 2.2440 & 0.0306 & 0.0279 \\ 1.7952 & 0.6830 & 1.9635 & 1.0833 \\ 0.1551 & 0.7854 & 2.0268 & 3.1416 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0.5237 & -0.0052 & -0.0222 & 0.5036 \\ -0.0024 & 1.0035 & 0.0000 & -0.0012 \\ 0.4719 & 0.0012 & 0.2965 & 0.2303 \\ 0.0067 & 0.0004 & 0.7256 & 0.2673 \end{bmatrix}$$

$$\text{and } \Gamma = \begin{bmatrix} 4.1448 & 0.0511 & 0.7066 & 0.5004 \\ -0.2239 & 0.9177 & 0.0978 & 0.0113 \\ 0.6497 & 0.4888 & 1.8192 & 2.1309 \\ 0.0492 & 0.0793 & 0.8359 & -26.9057 \end{bmatrix},$$

The best pairing according to REA and NI ($=1.1814>0$, ideal pairing) is 1-4/2-2/3-1/4-3.

Block Decoupling Control: Since the closely coupled loops are 1 and 3, $\phi_{13} = 0.5237$ and $\phi_{31} = 0.2673$, block decoupling control scheme is selected. By proposed method, the REA-BDC is designed as

$$G_c(s) = \begin{bmatrix} -0.3983(1 + \frac{1}{43.4800s} + 10.8700s) & 0 & -0.3380(1 + \frac{1}{47600s} + 1.1900s) & 0 \\ 0 & -148540(1 + \frac{1}{66.6700s}) & 0 & 0 \\ -0.0070(1 + \frac{1}{11.3600s}) & 0 & 0.8913(1 + \frac{1}{28600s} + 0.7150s) & 0 \\ 0 & 0 & 0 & 0.0322(1 + \frac{1}{s}) \end{bmatrix}$$

Figure 5.7 shows the closed-loop responses, where the unit set-points change in r_1 at $t=0$, r_2 at $t=200$, r_3 at $t=400$ and r_4 at $t=600$. Since we cannot find any existing decoupling control scheme in literatures for this process, we compare the responses with the *REA based decentralized controller* (REA-DCC) proposed in Chapter 4, which are shown in the same figure. As can be seen, the REA-BDC method has better overall performances.

Figure 5.7. Closed-loop responses for Example 5.3 (nominal process)

To investigate the robustness of the system, we mismatch the process model by increasing all steady-state gains by 50% and all time delays by 20%. The closed-loop responses under same controllers are shown in Figure 5.8. In this case, the REA-BDC still gives satisfactory performance while the REA-DCC is more oscillatory.

Figure 5.8. Closed-loop responses for Example 5.3 (model mismatched)

5.6. Conclusions

In this chapter, a novel full dimensional PI/PID controller for MIMO processes is proposed which consists of diagonal main controllers and off-diagonal decoupling controllers. Based on the equivalent transfer function matrix, both off-diagonal decoupling controllers and main loop diagonal controllers can be easily designed using the existing PI/PID tuning rules. The design procedure is very simple and straightforward, which can be easily understood and implemented by field control engineers. The simulation results for a number of industrial processes show that the proposed method gives better or comparable nominal system performance to other reported techniques. Furthermore, due to the implementation of PI/PID controllers for decoupling loops, the proposed method has good robustness and still works with satisfactory performance even if the primary process parameters have significant variations while other designs become oscillatory or unstable.

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Chapter 6^{*}

Application to an Experimental HVAC System

6.1. Introduction

The HVAC has now become an indispensable part of our life. Different kinds of HVAC systems have been installed in housing estates, commercial complexes, office buildings, and industrial estates, etc. These systems are playing an important role in providing thermal conditions for people and appliances inside the air-conditioned space. With the wide usage of air-conditioning system, the electricity usage for space air-conditioning amounts up to 30% of the total electrical energy consumption for countries with mild climate (Yaqub and Zubair, 2001; Shoureshi, 1993). As Singapore is a tropical city country with an annual average ambient temperature of 29.4°C and an annual average ambient relative humidity of 85%, the indoor thermal condition of buildings is very important since people live and work in buildings with air-conditioning systems running almost all the time. Therefore, it is very important to increase the system efficiency and improve interior environmental control in a more cost-effective, high quality manner. The research in the area of control of air-conditioning system has, therefore, a great impact to the economy and environment.

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As shown in Figure 6.1, an HVAC system consists of indoor air loop, chilled water loop, refrigerant loop, condenser water loop, and outdoor air loop. Each component involves several variables, the working principle and system analysis can be very complicated. To minimize the energy consumption while meeting the human comfort requirement, an accurate mathematical model for each component is needed to describe its working conditions. Based on the models, optimization problem for buildings can be formulated as minimization of the energy cost with the constraints that all the requirements on air quality in the building such as room temperature, humidity, air movement and oxygen concentration be met. While for every set-point change determined by the optimization algorithm, the control system is to make sure that each device can reach its new set-point smoothly and promptly.

Figure 6.1. Scheme of a typical building HVAC system

HVAC systems require control of environmental variables such as pressure, temperature, humidity, etc. Over the years, different control strategies have been developed to control fully or partially the HVAC process operation (Teeter and Chow, 1998; Liu and Dexter, 2001; Sousa *et al.*, 1997; Ghiaus, 2001). However, as in other process control applications, classical control techniques, such as on-off switching controller (thermostats) and PID controllers are still widely used in practice due to easier implementation, low cost and reliable performance etc (Seem, 1998). However, the PID tuning procedure can be a time-consuming, expensive and difficult task (Pinnella *et al.*, 1986; Krakow and Lin, 1995). Some control engineers may take 1 to 3 days to search for a workable PID controller setting for an air pressure loop in a building. For a temperature loop, the time spent on PID parameter tuning may be even longer. For many processes, their

inputs and outputs are cross-coupled, i.e., one process input can influence other outputs of the process. For example, if the chilled water supply is used to regulate the air humidity and the room damper controls the room temperature, then changing the valve position of the chilled water supply will affect both humidity and room temperature. Also, changing one room damper to regulate the room temperature will also affect the temperature of another room. For these kinds of multivariable processes, if independent loop control is adopted without taking into account the coupling, the resulting performance is usually poor. To enhance performance, the control design should address the coupling issue, which makes loop tuning more difficult. As a result, many HVAC loops are poorly tuned (Pinnella *et al.*, 1986). A poorly tuned control system may waste energy, reduce occupant comfort, and cause excessive and unnecessary wear of actuators.

The Ziegler and Nichols methods (Ziegler and Nichols, 1942) are classical PID tuning approaches which are widely accepted in industries. But those methods have shortcomings such as long testing time and limited control performance. It is more suitable for the initial setting of a PID controller. Recently, some complex control strategies based on the classical control concepts have been proposed in attempting to improve the system performance. Among those works, Pinnella *et al.* (1986) proposed a self-tuning method of an integral only controller. Brandt (1986) applied the self-tuning control in an HVAC system. Nesler (1986) developed a computer-based auto- and self-tuning controller, where the open-loop step test of the Ziegler and Nichols method was used to derive a process model and a PID controller. Krakow and Lin (1995) designed their PI controller based on the analytical models of HVAC components. Salsbury (1998) described a feedforward controller as a supplement of the conventional PI feedback controller based on simplified physical models. Kasahara *et al.* (1999) proposed a robust PID control scheme to deal with the model uncertainty caused by changing characteristics of the plant. Bi *et al.* (2000) developed an advanced auto-tuning PID controller for both temperature and pressure control loops. However, the above literatures are mainly focused on the single loop PID design, due to the interactions among individual loops many controllers are tuned conservatively in order to make the overall system stable. This may result in a unsatisfactory performance and waste of energy. In this chapter, the REA based MIMO techniques are applied to analysis and control design of an experimental HVAC

system, which attempts to replace independent PID controller tuning/re-tuning with systematic MIMO system design.

6.2. Four-room air temperature control system

The four-room air temperature control application for HVAC system is tested on an experimental centralized HVAC system. The system has three chillers, three sections with three *air handling units* (AHUs), three cooling towers and flexible partitions up to twelve rooms. All motors (fans, pumps and compressors) are equipped with *variable speed drives* (VSD). The system is made very flexible to configure these three units to form different schemes. The portrait of the experimental system is shown in Figure 6.2, where number 1, 2, 3 and 4 indicate the components of computer controller, VAV air-conditioning system, signal process board (AI/AO) and signal transmission cables, respectively. The software environment is Matlab/Simulink V6.5. In the system, the outside air is mixed with the building return air. The mixed air, i.e., supply air is sucked through the cooling coil by a supply fan. The cooled air is then supplied to different zones. The temperature of the off-coil air can be maintained at a pre-specified value by the regulating valve of chilled water supply or return. To ensure each control loop to work properly, the supply air pressure needs to be kept at a constant value. This is realized by regulating the supply air fan speed. The temperature of each room can be fine tuned by the corresponding damper of VAV box. All motor-driven fans, pumps and compressors in the system are controlled by VSDs.

Figure 6.2. Experimental centralized HVAC system

Here we consider the temperature control problem of four neighboring rooms of one zone. The schematic diagram of the four-room multivariable system is shown as in Figure 6.3.

In this system, four temperatures are controlled by regulating the position of four *variable air volume* (VAV) dampers. When the VAV damper opens wider, more cooling air enters the room and the room temperature will drop, and vice versa.

Figure 6.3. Four-room HVAC system (Top View)

In the system, there are one air pressure sensor, P_A , in the duct, and four temperature sensors, T_{R1} , T_{R2} , T_{R3} and T_{R4} , located in Rooms 1, 2, 3 and 4, respectively. In addition, there are four supply air control dampers, DM_1 , DM_2 , DM_3 and DM_4 , and four exhausted air vents, VT_1 , VT_2 , VT_3 and VT_4 on the room ceilings of Rooms 1, 2, 3 and 4, respectively. Due to the cross coupling, change of any damper position will cause temperature offset for all four rooms. Therefore, it forms an interactive 4×4 process. The main control objective is to maintain the conditioned space temperature of each room at the set-point value. The manipulating inputs for the system are the positions of the four dampers, DM_1 , DM_2 , DM_3 and DM_4 .

6.3. Modeling and simulations

The dynamics of the temperature control of the four-room process is modeled by using closed-loop identification method (Mei *et al.*, 2005), the parameters of the transfer function matrix are obtained as:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} \frac{-0.098}{122s+1}e^{-17s} & \frac{-0.036}{149s+1}e^{-27s} & \frac{-0.014}{158s+1}e^{-32s} & \frac{-0.017}{155s+1}e^{-30s} \\ \frac{-0.043}{147s+1}e^{-25s} & \frac{-0.092}{130s+1}e^{-16s} & \frac{-0.011}{156s+1}e^{-33s} & \frac{-0.012}{157s+1}e^{-34s} \\ \frac{-0.012}{153s+1}e^{-31s} & \frac{-0.016}{151s+1}e^{-34s} & \frac{-0.102}{118s+1}e^{-16s} & \frac{-0.033}{146s+1}e^{-26s} \\ \frac{-0.013}{156s+1}e^{-32s} & \frac{-0.015}{159s+1}e^{-31s} & \frac{-0.029}{144s+1}e^{-25s} & \frac{-0.108}{128s+1}e^{-18s} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix},$$

where u_1 , u_2 , u_3 and u_4 , the positions of four dampers, are manipulating variables, and T_1 , T_2 , T_3 and T_4 , temperatures of four rooms, are controlled variables, respectively.

The calculation of the above open-loop transfer function matrix results in

$$E = \begin{bmatrix} -0.0092 & -0.0023 & -0.0007 & -0.0009 \\ -0.0108 & -0.0095 & -0.0006 & -0.0006 \\ -0.0007 & -0.0008 & -0.0103 & -0.0021 \\ -0.0007 & -0.0008 & -0.0091 & -0.0100 \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} 1.2207 & -0.2051 & -0.0053 & -0.0103 \\ -0.1947 & 1.2198 & -0.0136 & -0.0116 \\ -0.0106 & -0.0085 & 1.1095 & -0.0904 \\ -0.0154 & -0.0062 & -0.0907 & 1.1124 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 1.3874 & -0.3899 & 0.0028 & -0.0003 \\ -0.3804 & 1.3859 & -0.0049 & -0.0006 \\ 0.0018 & -0.0026 & 1.2251 & -0.2242 \\ -0.0088 & 0.0066 & -0.2229 & 1.2251 \end{bmatrix},$$

$$\text{and } \Gamma = \begin{bmatrix} 1.1366 & 1.9008 & -0.5321 & 0.0329 \\ 1.9541 & 1.1362 & 0.3641 & 0.0508 \\ -0.1673 & 0.3111 & 1.1041 & 2.4787 \\ 0.5732 & -1.0673 & 2.4577 & 1.1014 \end{bmatrix}.$$

The best pairing according to both RGA and REA and NI ($=0.7267>0$) are diagonal. In order to find the proper control strategy, several options are considered below, where the gain and phase margins are specified as $5db$ and $2\pi/5rad$, respectively.

Decentralized Control: For $i=1,2,3,4$, $\lambda_{ii}>1$ and $\gamma_{ii}>1$, by the method proposed in Chapter 4, the equivalent diagonal transfer functions are calculated as

$$\frac{-0.098}{122s+1}e^{-19.3223s}, \frac{-0.092}{130s+1}e^{-18.1786s}, \frac{-0.102}{118s+1}e^{-17.6659s} \text{ and } \frac{-0.108}{128s+1}e^{-19.8248s},$$

respectively. The resulting REA-DCC is listed in Table 6.1 together with the controllers designed by RGA base approach (Chien *et al.*, 1999) for comparison.

Table 6.1. Decentralized controllers for HVAC system

Controller	REA-DCC		RGA based	
	$k_{p,ii}$	$\tau_{i,ii}$	$k_{p,ii}$	$\tau_{i,ii}$
Loop 1	-20.2407	122.0000	-32.2705	113.1520
Loop 2	-24.4200	130.0000	-38.9186	106.4960
Loop 3	-20.5729	118.0000	-31.8627	106.4960
Loop 4	-18.7813	128.0000	-29.0158	119.8080

Decoupling Control: Since the closely coupled loops are loop 1 and loop 2 with $\phi_{12} = -0.3899$ and $\phi_{21} = -0.3804$, and loop 3 and loop 4 with $\phi_{34} = -0.2242$ and $\phi_{43} = -0.2229$, respectively, the block decoupling control scheme is adopted. By the method proposed in Chapter 5, the REA-BDC is designed as

$$G_i(s)=\begin{bmatrix} -23.7863(1+\frac{1}{122.0000s}) & 4.0161(1+\frac{1}{149.0000s}) & 0 & 0 \\ 5.5372(1+\frac{1}{147.0000s}) & -28.9986(1+\frac{1}{130.0000s}) & 0 & 0 \\ 0 & 0 & -23.5699(1+\frac{1}{118.0000s}) & 2.2449(1+\frac{1}{146.0000s}) \\ 0 & 0 & 2.0010(1+\frac{1}{144.0000s}) & -21.5432(1+\frac{1}{128.0000s}) \end{bmatrix}$$

Figure 6.4 shows the closed-loop responses, where the unit set-points change in r_1 at $t=0$, r_2 at $t=1000$, r_3 at $t=2000$ and r_4 at $t=3000$. As can be seen, the REA-DCC is better than the RGA based method which has some overshoot, the REA-BDC method has the best overall performances.

Figure 6.4. Closed-loop responses of four-room system

In order to compare robustness of different control schemes, we mismatch the process model by increasing all 16 steady-state gains, 16 time constants and 16 time delays by 50%. With the controller parameters of both REA-DCC and REA-BDC kept the same as before, the closed-loop responses are shown in Figure 6.5, where the unit set-points change in r_1 at $t=0$, r_2 at $t=1000$, r_3 at $t=2000$ and r_4 at $t=3000$. It shows that even under such a severe model mismatch, both the REA-DCC and the REA-BDC still give satisfactory dynamic performance.

Figure 6.5. Closed-loop responses of four-room system (model mismatched)

The above simulations confirm that the block decoupling control scheme is superior in both nominal performance and robustness, but it is not very significant compared with the decentralized control scheme. In addition, the uncertainty of this pilot plant may not be so large. Therefore, considering the simplicity of implementation and loop failure tolerance issues, it is decided that the REA-DCC be implemented in the 4-room HVAC system.

6.4. Experiment results

The experiment is conducted under the following conditions: The chilled water supply temperature is fixed and the cooling load variation is achieved through the air and water flow rates. In the VAV system, the supply air temperature is controlled at fixed set-point value and the speed of supply fan varies according to the time varying cooling load. All those are realized by separate controllers and a hierarchical two-level multi-model control strategy is applied to the supply air temperature control (He *et al.*, 2005).

Figures 6.6 and 6.7 show the performance of the multi-loop control system, in which the set-points values change from 22 °C to 25 °C at $t=0$, $t=1000$, $t=2000$ and $t=3000$ second, respectively. At time $t=4000$ second, the disturbance is introduced by opening the door of ROOM 2 with some extent. As can be seen, both the set-point tracking performance and the disturbance rejection performance are satisfactory.

Figure 6.6. Real-time closed-loop responses of four-room system

Figure 6.7. Real-time manipulated variables of four-room system

6.5. Conclusions

In this chapter, the proposed decentralized and decoupling design methods are used to analyze and design an experimental HVAC system. From simulations and real-time experiments, it is shown that the proposed control design works well in the experimental HVAC system even if with significant model uncertainty. This makes it possible at some extent to replace the tedious and poor manual tuning/re-tuning of independent loop PID controllers in HVAC systems. Since many processes in other industries have similar characteristics as those in HVAC systems, therefore, it is conjectured that the proposed design approach can be applied to the analysis and design of various industrial multivariable control systems.

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Chapter 7

Conclusion and Recommendations

7.1. Conclusion

In this thesis, in terms of relative interaction energy, a comprehensive engineering oriented methodology for MIMO control system design is presented which includes: interaction analysis, loop pairing, design of decentralized, full decoupling and block decoupling controllers. The design procedure is very simple and effective, which can be easily understood, and can guide control engineers in applications. The main works in this thesis include:

- (1) A novel interaction measure and loop pairing criterion is proposed which can accurately reflect dynamic loop interactions under finite bandwidth control. In this criterion, both steady-state gain and critical frequency of the process are used to provide a simple yet comprehensive description of loop interactions for MIMO processes.
- (2) A novel design method for decentralized control system based on equivalent transfer functions is proposed. The loop interactions are quantified by two elements, i.e., relative gain and relative critical frequency. The interaction effects for a particular loop from all other closed-loops are then analyzed through both steady-state gain and critical frequency variations, and directly incorporated into the equivalent transfer functions in four ways in a simple and straightforward manner. Hence,

different equivalent transfer functions for different interaction modes are derived such that the control system integrity can be preserved, and the existing single loop tuning techniques can be employed.

- (3) A novel engineering oriented decoupling control design method for multivariable systems is proposed. An equivalent relationship between open-loop and closed-loop transfer functions is established. For control system design, individual loop controllers can then be directly designed based on only one element in the closed-loop equivalent transfer function matrix. Under this framework, the full decoupling or block decoupling controller can be designed under the same formula as required.

The numerical simulations on several multivariable industrial processes with different interaction characteristics and the experiments on a temperature control system of an experimental HVAC system demonstrate the simplicity and effectiveness of the design method.

The significances of the work of the thesis are: 1) the proposed loop pairing and control system design technique results in a better control system performance compared with the existing methods; 2) all control designs are based on single loop control concept and all the existing single loop PI/PID tuning rules can be directly used; 3) the control system is robust, it still works with satisfactory performance even under severe model mismatches; and 4) the method is even more significant when applied to higher dimensional processes with complicated interaction modes.

7.2. Recommendations

In this thesis, an engineering oriented methodology for MIMO control system design is proposed. To further develop the theoretic foundation of the method and make it fully accepted, some possible issues for future research in this area are briefly described below.

- (1) As demonstrated, REA is an effective interaction measure. However, the further investigation about REA is still attractive. The graphical interpretation of REA may be one of the most significant aspects, which can

provide a visual means of summarizing interaction information. Such a graphical approach can be more convenient and straightforward in control system analysis and design.

- (2) Another interesting issue is the influence of process model uncertainty on REA analysis. Analytical expressions for REA uncertainty bounds like worst-case bounds, which concern model accuracy requirements and the robustness of decentralized control systems, will be useful for control system design.
- (3) Closing the loop around one subsystem may introduce RHP transmission zeros in other subsystems. If such zeros exist, and they are more severe than the RHP zeros of the overall system, they will represent performance limitations specific to the use of decentralized control. Hence, the investigation on the performance limitation in terms of REA is an active topic.
- (4) Generally the performance for set-point change and load disturbance are conflict in a simple PID loop. This can be improved by two stage or two degree of freedom PID control. The inner P controller is designed first. Then the outer controller is designed. However, the similar two stage approach for multi-loop system is still challenging. Hence, the REA based two stage decentralized controller design methodology would be attractive.
- (5) Extensive comparison with other MIMO design approaches in terms of time domain or frequency domain specification indices, and implementation of decentralized/decoupling control scheme for the overall HVAC pilot plant (3 sections and 12 rooms) are significant to further verifying the proposed methods.

Author's Publications

- (1) Qiang Xiong, Wen-Jian Cai and Mao-Jun He, Equivalent transfer function method for PI/PID controller design of MIMO processes, *Journal of Process Control* 17 (2007) 665-673.
- (2) Qiang Xiong and Wen-Jian Cai, Effective transfer function method for decentralized control system design of multi-input multi-output processes, *Journal of Process Control* 16 (2006) 773-784.
- (3) Qiang Xiong, Wen-Jian Cai, Mao-Jun He and Ming He, Decentralized control system design for multivariable processes--A novel method based on effective relative gain array, *Industrial and Engineering Chemistry Research* 45 (2006) 2769-2776.
- (4) Qiang Xiong, Wen-Jian Cai and Mao-Jun He, A practical loop pairing criterion for multivariable processes, *Journal of Process Control* 15 (2005) 741-747.
- (5) Qiang Xiong, Wen-Jian Cai and Ming He, A practical decentralized PID auto-tuning method for TITO systems under closed-loop control, *International Journal of Innovative Computing, Information and Control* 2 (2006) 1-18.
- (6) Qiang Xiong and Guang-Yu Jin, Iterative decentralized PID tuning based on gain and phase margins for TITO systems, *International Journal of Innovative Computing, Information and Control*, Accepted in press.
- (7) Hua Mei, Shao-Yuan Li, Wen-Jian Cai and Qiang Xiong, Decentralized closed-loop parameter identification for multivariable processes from step responses, *Mathematics and Computers in Simulation* 68 (2005) 171-192.
- (8) Shao-Yuan Li, Wen-Jian Cai, Hua Mei and Qiang Xiong, Robust decentralized parameter identification for two-input two-output process

from closed-loop step responses, *Control Engineering Practice* 13 (2005) 519–53.

- (9) Qiang Xiong and Wen-Jian Cai, An engineering interaction measure with application in vapor compression cycle control, *Proceedings of the 9th International Conference on Control, Automation, Robotics and Vision*, December 5 – 8, 2006, Singapore.