Learning-based mathematical morphology in color image processing

Zhou, Hao

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Learning-based Mathematical Morphology in Color Image Processing

Zhou Hao

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Master of Engineering

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Summary

Mathematical Morphology is a powerful nonlinear image processing tool and is widely used in image segmentation, image filtering, shape representation, image measurement and so on. Most of the morphological operation based image processing approaches are developed for binary and grayscale images. Recently, since the color image has become the dominant image format nowadays and contains much more information than grayscale image, a problem of significant challenging in mathematical morphology is to extend basic morphological operations and their applications into color image processing area.

This thesis mainly focuses on extending the existing basic morphological operators and their applications into color image processing area, by incorporating supervised and unsupervised learning techniques. The major contributions are summarized below:

A new color pixel ordering scheme is established. Pixel ordering is the foundation of mathematical morphology. Color pixels are represented by multi-dimensional vectors, and the intensity-based pixel ordering scheme used in grayscale morphology is no longer applicable to vector ordering in color morphology. Therefore, a probabilistic framework for color pixel ordering is established, where the probability estimator is learned through supervised learning. Based on the probability-based color pixel order-

ing scheme, basic morphological operations: erosion, dilation, opening and closing are built.

Two edge detectors are developed by using of such basic color morphological operations. One simple edge detector for color images containing simple colored shapes, uses the difference between elementary dilated and eroded color images as the edge detection result, which is similar as the edge detector for binary image. Color morphological gradient vector (CMGV) is introduced for image segmentation problems in complicated color images. Supervised learning techniques are used to extract boundary information from CMGV, and it is effective when separating specific objects from the images containing other objects rather than the one of interests.

The multichannel filtering approach established on unsupervised learning-based color morphological operations for impulsive noise removal is developed in this thesis. By using color pixel ordering scheme, which is learned from the pre-estimation of impulsive noise, contaminated pixels are ordered as maximum or minimum ones within the operation window, in color erosion or dilation respectively. This character ensures that only uncontaminated color pixels are distributed as supremum or infimum in morphological operations, hence noise pixels are suppressed.

The color morphological operations and their application algorithms are tested on both synthetic and real color images. For segmentation application, color histological images are used. For image filtering application, scenery color images are used.

Abstract

Mathematical Morphology is a powerful nonlinear image processing tool and is widely used in image processing and analysis areas. Most of the morphological operations based image processing approaches are developed for binary and grayscale images. Nowadays, color image has become the dominant image format and contains more information than monochrome images. Extending mathematical morphology to color image attracts more interests recent years.

This thesis mainly focuses on extending the existing basic morphological operators and their applications into color image processing area, by incorporating supervised and unsupervised learning techniques. A probability-based color pixel ordering framework is first established, based on which fundamental color morphological operations are built. Color morphological gradient vector (CMGV) is introduced then, and new edge detectors are developed for image segmentation in complicated color images which extracts boundary information directly from CMGV. Morphological filter is also studied in this thesis, and unsupervised learning-based color morphological filter is developed for impulsive noise filtering in color image.

The effectiveness of the proposed color morphological operations and their application algorithms are tested on both synthetic and real color images.

Glossary

This list contains the important symbols and abbreviations used in the thesis.

Symbols

$\mathbf{L}$	a set
$\vee \mathbf{X}_i$	supremum in set $\mathbf{X}$
$\wedge \mathbf{X}_i$	infimum in set $\mathbf{X}$
δ	dilation
ε	erosion
$\circ$	opening
$\bullet$	closing
$\mathbf{B}$	structuring element
$f(x)$	an image
$p(x)$	probability of being an object pixel
$\mathbf{S}_n$	a set of color pixel vectors
$\nabla_{\mathbf{B}}(f)$	morphological gradient
Q	quantitative assessment for image segmentation
α	parameter of SVMOC filter
p	noise density

Abbreviations

AHMF	Adaptive Hybrid Multivariate Filter
AVDF	Adaptive Vector Directional Filter
BVDF	Basic Vector Directional Filter
CMGV	Color Morphological Gradient Vector
CR	Correspondence Ratio
DDF	Distance Directional Filter
FVD	Fuzzy Vector Directional Filter
GVDF	Generalized Vector Directional Filter
LAMO	Learning After Morphological Operation
LBMO	Learning Before Morphological Operation
LCPF	Local Information Based Filter
LDA	Linear Discriminant Analysis
MAE	Mean Absolute Error
MLE	Maximum Likelihood Estimation
MM	Mathetical Morphology
MMF	Marginal Median Filter
NCD	Normalized Color Difference
NOPNCP	Component-wise NOPNCP Filter
PCA	Principal Component Analysis
PM	Percent Match
PSNR	Peak Signal-to-Noise Ratio
SCWVDF	Switching Center Weighted Vector Directional Filter

SVM	Support Vector Machine
SVMOC	SVM Opening Closing Filter
VMF	Basic Vector Median Filter
VSF	Vector Sigma Filter

Table of Contents

Acknowledgements	i
Summary	ii
Abstract	iv
Glossary	v
Table of Contents	viii
List of Figures	x
List of Tables	xv
1 Introduction	1
1.1 General Introduction	1
1.2 Motivation and Objectives	3
1.3 Main Contribution of the Thesis	4
1.4 Organization of the Thesis	5
2 Mathematical Morphology and Extension to Color Image	7
2.1 Introduction to Lattice Theory	7
2.2 Binary and Grayscale Morphological Operations	9
2.3 Color Morphology and Applications in Color Image Processing	12
2.4 Summary of the Chapter	15

TABLE OF CONTENTS	ix
3 Probabilistic Framework for Color Pixel Ordering	16
3.1 Introduction	16
3.2 Probability Based Color Morphology	18
3.2.1 Probability Framework for Pixel Ordering	18
3.2.2 Color Vector Ordering Based on Probability Criterion	19
3.2.3 Parameter Determination for the Logistic Regression	21
3.3 Simple Edge Detector Based on Probability-based Color Morphology .	26
3.4 Experiment Results	28
3.4.1 Edge Detection	28
3.4.2 Other Applications	31
3.5 Summary of the Chapter	31
4 Color Morphological Gradient Vector	37
4.1 Morphological Gradient	38
4.1.1 Conventional Morphological Gradient	38
4.1.2 Color Morphological Gradient	39
4.2 Color Morphological Gradient Vector (CMGV)	41
4.2.1 Morphological Gradient Analysis	41
4.2.2 Color Morphological Gradient Vector	43
4.2.3 PCA-based Morphological Gradient	45
4.2.4 SVM-based Color Morphological Gradient	46
4.3 CMGV Applications in Image Segmentation	54
4.3.1 Marker-controlled Watershed for Overlapping Cell Nuclei Separation	55
4.3.2 Edge Detection	58
4.4 Summary of the Chapter	61
5 Impulsive Noise Color Image Filtering	66
5.1 Introduction	67
5.1.1 Nonlinear Color Image Filters	67

TABLE OF CONTENTS	x
5.1.2 Morphological Filters	68
5.2 Learning-based Color Morphological Filter	69
5.2.1 Difficulties in Extension of Color Morphological Filter	69
5.2.2 Learning-based Color Morphological Filter	72
5.2.3 SVM-based Color Pixel Ordering Schemes and Morphological Operations	74
5.2.4 Reconstruction	76
5.3 Simulation and Discussion	77
5.4 Summary of the Chapter	80
6 Conclusions and Recommendations	94
6.1 Conclusions	94
6.2 Recommendations for Future Research	95
Author's Publications	98
Bibliography	99

List of Figures

2.1	(a) Binary image of bladder cancer cell image. (b) Dilation of the image by an elementary structuring element. (c) Erosion of the image by an elementary structuring element.	11
2.2	(a) Grayscale image of bladder cancer cell image. (b) Dilation of the image by an elementary structuring element. (c) Erosion of the image by an elementary structuring element.	12
3.1	A typical logistic regression model curve.	19
3.2	(a) Synthetic color image. Green, blue and black samples in (b)~(e) denote color pixels of green object, dark blue object and background in (a), respectively. (b) Pixel distribution in the plane of Hue (HLS color space) and Value (HSV color space). (c) Pixel distribution in RGB color space and the projection to axis $\mathbf{a} = [0.299, 0.578, 0.11]^T$. (d) and (e) Pixel distributions in RGB color space and projections to the axis $\mathbf{a} = [-0.535, -0.428, -0.729]^T$ or $\mathbf{a} = [-0.719, -0.528, -1.000]^T$ learned by LDA and SVM techniques, respectively.	25
3.3	(a) Test color image, 5×5 square structuring element is used. (b1)~(i1) Dilated color images based on $M1 \sim M8$ respectively. (b2)~(i2) Eroded color images based on $M1 \sim M8$ respectively.	33

LIST OF FIGURES	xii
3.4 (a)~(h) Internal edges of all the objects in Fig.3.3(a) extracted based on $M1 \sim M8$, respectively.	34
3.5 (a)~(h) External edges of all the objects in Fig.3.3(a) extracted based on $M1 \sim M8$, respectively.	34
3.6 Edge detection results of specific objects of Fig.3.3(a). (a1)~(d1) Edges of green objects detected based on $M1 \sim M4$. (a2)~(d2) Edges of dark blue objects detected based on $M1 \sim M4$	35
3.7 (a)~(d) Edge detection results based on $M5 \sim M8$, respectively.	35
3.8 (a) Original color image, (b)~(i) Hole filling results by $M1$ to $M8$ respectively.	36
3.9 (a) Original noisy color image. 3×3 square structuring element is used. (b)~(i) Resultant images of opening followed by closing based on $M1$ to $M8$, respectively.	36
4.1 (a) Two overlapping cells of p53 immunohistochemistry stained bladder inverted papilloma. (b)~(c) Dilated and eroded images by 5×5 square structuring element, respectively. (d) Histogram of initial grayscale image (a). (e) Histograms of dilated image (b). (f) Histogram of eroded image (c).	43
4.2 (a) Color morphological gradient vector (CMGV). (b) Projections of CMGV on the axis with direction $\mathbf{w}_1$ and $\mathbf{w}_2$, respectively. (c) The projection of CMGV on the axis $\mathbf{w}$ learned by SVM. (d) The projection of CMGV on the axis learned by SVM, when there are other object in the color image besides the one to be segmented.	44

LIST OF FIGURES

xiii

4.3	(a) p53 immunohistochemical stained bladder tissue cell nuclei. (b) PCA-based color morphological gradient (the first principle component p_1 of CMGV), (c) The second principle component p_2 of CMGV, (d) The third principle component p_3 of CMGV.	47
4.4	(a) Two overlapping cells of p53 immunohistochemistry stained bladder inverted papilloma. (b) SVM-based (LBMO) color morphological gradient. (c) SVM-based (LAMO) color morphological gradient.	50
4.5	(a1)~(j1) Morphological gradients of G1 to G10, respectively. (a2)~(j2) Watershed results based on (a1)~(j1), respectively.	56
4.6	(a) Synthetic color image. (b) Projection of CMGV on axis $\mathbf{w}$ by LBMO SVM learning. (c) Edge detection result. (d)~(f) Color morphological gradient vectors (CMGV) of zoomed region of rectangle in (a), projected on $R-G$, $G-B$ and $B-R$ plane, respectively.	62
4.7	(a) Synthetic color image. (b) Grayscale morphological gradient. (c) SVM-based (LBMO) color morphological gradient. (d) Edge detection result based on (c).	63
4.8	(a) Histological color image of breast phyllodes tumor tissue stained by CD31. (b) Zoomed region of rectangle in (a), brown object is a microvessel to be detected. (c1) Grayscale morphological gradient. (d1) Color morphological gradient based on Euclidean distance in RGB color space. (e1) SVM-based (LBMO) color morphological gradient. (f1) SVM-based (LAMO) color morphological gradient. (c2)~(f2) Binary image obtained from (c1) to (f1), respectively. (c3)~(f3) Microvessel boundary detected based on (c2) to (f2), respectively.	64

LIST OF FIGURES	xiv
4.9 (a) Histological color image of breast phyllodes tumor tissue stained by CD31. (b) Microvessel detection result by SVM-based (LBMO) color morphological gradient. (c) Microvessel detection result by SVM-based (LAMO) color morphological gradient.	65
5.1 (a) Zoomed part of Baboon color image. (b) Pixel distribution in RGB color space of uncontaminated image. (c) Image corrupted by 30% impulsive <i>salt and pepper</i> noise. (d) Pixel distribution in RGB color space of contaminated image. Blue * samples denote impulsive noise, and red o samples denote uncontaminated pixels.	81
5.2 Initial one dimensional noisy signal and the results of each step of OC filter with flat linear structuring element of 3 neighbors.	82
5.3 (a) A 9×9 region of Baboon image corrupted by 10% impulsive <i>salt and pepper</i> noise. (b) Uncontaminated and noisy pixels distribution in RGB color space. Blue * samples denote impulsive noise, and red o samples denote uncontaminated pixels. (c) Impulsive noise locations within the black square. (d) R, G, B values of the pixels within the black square, and variance value corresponding with each pixel, $d_{dilation}$, $d_{erosion}$ values produced by the SVM classifiers for dilation and erosion, respectively. .	83
5.4 Color test images used in this paper. (a) Goldhill. (b) Lena. (c) Baboon. (d) Pepper. (e) Parrot.	83
5.5 Dependence of the quantitative measuring results MAE and NCD on the coefficient of α and percentage of the impulsive noise on Baboon image. (a) MAE, N1 (b) NCD, N1 (c) MAE, N2 (d) NCD N2.	84
5.6 (a) Zoomed part of Pepper image. (b) Image corrupted by 10% N1 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output.	84

LIST OF FIGURES

5.7 (a) Zoomed part of Baboon image. (b) Image corrupted by 10% N2 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output. 92

5.8 (a) Zoomed part of Parrots image. (b) Image corrupted by 20% N1 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output. 92

5.9 (a) Zoomed part of Lena image. (b) Image corrupted by 30% N1 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output. 93

5.10 (a) Zoomed parts of Goldhill image. (b) Image corrupted by 30% N2 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output. 93

List of Tables

4.1 Quantitative validation Q for $G1$ to $G10$ of five color images with size of 256×256 randomly selected from histological image database of bladder inverted papilloma. 57

4.2 Quantitative validation PM and CR of microvessel segmentation by using SVM-based color morphological gradients. 61

5.1 List of filters used for comparisons with SVMOC filter. 78

5.2 Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 10\%$ N1 impulsive noise. SVMOC ($\alpha = 2.0$). 85

5.3 Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 20\%$ N1 impulsive noise. SVMOC ($\alpha = 2.0$). 86

5.4 Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 30\%$ N1 impulsive noise. SVMOC ($\alpha = 2.0$). 87

5.5 Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 10\%$ N2 impulsive noise. SVMOC ($\alpha = 2.0$). 88

LIST OF TABLES

xvii

5.6	Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 20\%$ N2 impulsive noise. SVMOC ($\alpha = 2.0$).	89
5.7	Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 30\%$ N2 impulsive noise. SVMOC ($\alpha = 2.0$).	90
5.8	Quantitative validation of filters performances on noise free images. SV-MOC ($\alpha = 2.0$).	91

Chapter 1

Introduction

1.1 General Introduction

Mathematical Morphology (MM) [1], [2], [3] or simply morphology is a theory for the analysis of spatial structures. It is so called because it aims at analyzing the shape and form of objects, and the analysis is based on set theory, integral geometry, and lattice algebra. Mathematical morphology is not only a theory, but also a powerful technique for image analysis.

Mathematical morphology was developed by Matheron and Serra in the mid of 1960s, and was first applied to binary image processing and analysis. Later, it was extended to grayscale and color image processing. Nowadays, mathematical morphology has become a major approach for image processing and analysis. The uses of the morphological approach in image processing and analysis have been extended to many areas, including image segmentation, image filtering, image measurement etc.

In a morphological operation, an image is transformed by another set, known as the structuring element. The characteristics of the structuring element determine the resul-

tant image. Two fundamental morphological operators are *dilation* and *erosion*. These dual operators are the two letters of the morphological alphabet, all other operators being expressed in terms of dilations and erosions. Another two common morphological operators are *opening* and *closing*, which are defined as erosion followed by dilation and dilation followed by erosion, respectively.

The scope of the morphological approach to image analysis has been extended to a lot of problems.

Image Filtering

Image filtering can be generally considered as a neighborhood operator, which can perform a wide variety of tasks. Mathematical morphology provides us with image filters to do the tasks, for example, removal of small blobs, extraction and grouping of linear objects, correction of uneven illumination, noise filtering, and edge enhancement. The idea of these applications is based on the essence of the most morphological operators: the image structures are investigated by a small template and they are preserved or altered depending on whether the template fits or does not fit them [1].

Image Segmentation

Image segmentation is the division of an image into different regions, which have their certain properties. The major tasks which can be solved using morphological transforms include separation of connected blobs, segmentation of digital elevation models, histogram based classification, feature space clustering, and extraction of grid lines.

Image Measurements

Image measurements aim at characterising the objects of an image by some numerical values. Morphology provides us with a wide variety of image measurement tools,

include pattern spectrum or granulometries, analysis of directions, texture analysis, and shape description.

Some Other Tasks

A few of other applications that morphology can be solve, for example, interpolation of contour data, minimal path detection, and unwrapping of noisy phase-maps.

1.2 Motivation and Objectives

Recently, color images become the dominant image record and are ubiquitous. When mathematical morphology operates on color image, the most used method is to transform color image into grayscale format first, and then grayscale morphological operations are applied. Such transformation loses a lot of information contained in color image, and the resultant image can not be restored to color format neither. Therefore, it is expected morphological operations can be implemented directly on color images. Extension of morphology to color images is not straightforward because color pixels are represented by vectors and the intensity-based pixel ordering scheme used in grayscale morphology is no longer applicable to vector ordering in color morphology.

Therefore, the most important issue in extension of color morphology is multi-dimensional color vector ordering. The most commonly used color vector ordering scheme is reduced-ordering or conditional-ordering based approach, which are not unified with classic binary and grayscale morphology. Hence, establishing a unified pixel ordering scheme for both monochrome and color images is the first and the most important step to develop color morphology algorithm.

Conventional morphological operations-based image processing methods are neighbor-

hood operations, and only spatial information is considered during the operations. Because no contextual information is considered, such operations may not perform well in some applications, for example segmenting specific object from the image containing other objects besides the one of interest. Learning technique is therefore expected to be incorporated into morphological operations, and helps combine contextual information into neighborhood operations.

Classical morphological image filters are effective in impulsive noise filtering, which are usually implemented using opening followed by closing or closing followed by opening. Impulsive noise pixels are usually ordered as maximum or minimum ones within the operation window, and morphological operations distribute maximum or minimum pixel within the operation window. Therefore sequentially or parallelly combination of morphological operations inevitably have the property that, while part of the noise pixels are depressed, the other part of the noise pixels are magnified meanwhile. Hence, learning technique is expected to be incorporated to distinguish noise pixels between pixel ordering and pixel distribution in morphological operations, to help alleviate such drawback and enhance the performance of morphological filters.

1.3 Main Contribution of the Thesis

Probabilistic Framework for Color Pixel Ordering

A probabilistic framework for color pixel ordering is established in the present thesis. Based on the probability-based color pixel ordering scheme, color morphological operations are built. This probabilistic framework for color pixel ordering unifies the pixel ordering both in grayscale and color images.

Color Morphological Gradient Vector

The color morphological gradient vector is introduced (CMGV). Object boundary information is extracted directly from CMGV by using supervised learning techniques. The new algorithms developed based on CMGV greatly enhance the performance of morphological operation in specific object extraction.

Impulsive Noise Color Image Filtering

An impulsive noise color image filtering approach is developed. Learning technique is contained in the filtering process, which helps avoid noise pixels are distributed as output of each morphological operation. This property enhances the morphological filter performance when filtering the images highly contaminated by impulsive noise.

1.4 Organization of the Thesis

Chapter 2 provides a review of lattice theory, which is the basis of mathematical morphology. The difficulty of extending morphology to color image is analyzed in this chapter.

Chapter 3 provides a probability framework for color pixel ordering, where the probability estimator is learned through supervised learning. Probability-based color morphological operations including dilation, erosion, opening and closing are established based on the probability-based color pixel ordering scheme. A simple edge detector based on morphological gradient and other simple applications of mathematical morphology are implemented as well to evaluate the probability-based color morphology.

Chapter 4 describes color morphological gradient vector, based on which object boundary information can be extracted directly. The newly developed edge detection algo-

rithms are tested on both synthetic and histological color images.

Chapter 5 first reviews nonlinear filter for impulsive noise filtering, and then presents a new unsupervised learning-based color morphological filter. This unsupervised learning-based image filtering approach is tested on color images and evaluated by both image result and quantitative measures.

Chapter 6 concludes the thesis with suggestions for further work.

Chapter 2

Mathematical Morphology and Extension to Color Image

In this chapter, mathematical morphology is reviewed, and the extension of mathematical morphology to color image processing and its difficulties are analyzed.

2.1 Introduction to Lattice Theory

Lattice theory is the basis of mathematical morphology. A brief introduction of some basic notions that are commonly used in lattice theory is given here [2].

A partially ordered set (poset) is a set in which a partial ordering relationship exists between its members. This partial ordering relationship is denoted by the symbol $<$ and has the following properties, where X , Y and Z denote members of the set:

$$X < Y \quad (\text{reflexivity}) \quad (2.1)$$

$$X < Y \text{ and } X > Y \text{ implies } X = Y \quad (\text{antisymmetry}) \quad (2.2)$$

$$X < Y \text{ and } Y < Z \text{ implies } X < Z \quad (\text{transitivity}) \quad (2.3)$$

A set $\mathbf{L}$ is totally ordered if the following property (which usually called pairwise ordering) also holds:

$$X < Y \text{ or } Y < X \text{ for each pair } X, Y \in \mathbf{L} \quad (2.4)$$

A is a lower bound of a poset $\mathbf{L}$ if $A < X$ for all $X \in \mathbf{L}$. Likewise, B is an upper bound of a poset $\mathbf{L}$ if $B > X$ for all $X \in \mathbf{L}$. (Note A and B are not necessarily members of $\mathbf{L}$). Let $\mathbf{A}'$ be the set of all lower bounds of $\mathbf{L}$; A is the greatest lower bound (g.l.b) of $\mathbf{L}$ if $A_0 < A$ for all $A_0 \in \mathbf{A}'$. Similarly, B is the least upper bound (l.u.b) of $\mathbf{L}$ if $B_0 < B$ for all $B_0 \in \mathbf{B}'$, where $\mathbf{B}'$ is the set of all upper bounds of $\mathbf{L}$.

A lattice is a poset in which any two elements possess a l.u.b and a g.l.b. Lattice element A is 'below' B if $A < B$. Every pair of lattice elements A and B do have a l.u.b called the supremum of A and B ; both A and B are below the supremum. Also, A and B have a g.l.b called the infimum of A and B ; the infimum is below both A and B . The supremum and infimum operators are denoted by symbols $\vee$ and $\wedge$, respectively. A lattice is complete if every subset of the lattice has a unique supremum and infimum. A complete lattice also implies that the entire lattice has a supremum and infimum, which are called the universal bounds of the lattice.

A lattice operator maps a lattice element into another lattice element. Strictly speaking, a lattice operator maps an element of one (source) lattice to an element of another (target) lattice. The target lattice may be identical to or different from the source lattice. Here, we restrict ourselves to identical source and target lattices. This means, for instance, a lattice operation on a binary image will yield a binary image.

Mathematical morphology operators are constructed based on the theory described above. Two fundamental morphological operations are dilation and erosion, they are defined below based on lattice theory:

A dilation δ is a lattice operator that distributes over supremum:

$$\delta(\vee \mathbf{X}_i) = \vee \{\delta(\mathbf{X}_i)\} \quad (2.5)$$

and an erosion ε is a lattice operator that distributes over infimum:

$$\varepsilon(\wedge \mathbf{X}_i) = \wedge \{\varepsilon(\mathbf{X}_i)\} \quad (2.6)$$

2.2 Binary and Grayscale Morphological Operations

The value of a pixel of a binary image is either 1 or 0 depending on whether the pixel belongs to an object or to its background [1]. For a binary image, the operation of supremum can be regarded as set union, the operation of infimum can be regarded as set intersection. Therefore, binary morphological operators Dilation and Erosion are defined by set union and set intersection.

$$\delta_B(\mathbf{X}) = \{x | (\hat{\mathbf{B}})_x \cap \mathbf{X} \neq \emptyset\} \quad (2.7)$$

$$\varepsilon_B(\mathbf{X}) = \{x | (\mathbf{B})_x \subseteq \mathbf{X}\} \quad (2.8)$$

where $\mathbf{X}$ is the original image, $\mathbf{B} \subseteq \mathbf{2}^{\mathbf{Z}^2}$ is the structuring element, $(\mathbf{B})_x$ and $\hat{\mathbf{B}}_x$ is the translation of $\mathbf{B}$ by x .

The dilation and erosion are both dual operations. They modify the objects shape

in a binary image via the characteristic of the structuring element (SE), for example, the size and the shape of the SE. Generally speaking, dilation enlarges the objects while erosion shrinks the objects. The Opening and Closing are two another basic morphological operations, defined as erosion followed by dilation and dilation followed by erosion respectively:

$$\mathbf{X} \circ \mathbf{B} = \delta(\varepsilon(\mathbf{X}, \mathbf{B})) \quad (2.9)$$

$$\mathbf{X} \bullet \mathbf{B} = \varepsilon(\delta(\mathbf{X}, \mathbf{B})) \quad (2.10)$$

Opening has general effects of smoothing the contour of an object, eliminating thin protrusions and breaking narrow isthmuses. Closing also tends to smooth sections of contours but it has the general effect of fusing narrow breaks and long thin gulfs, eliminating small holes, and filling gaps in the contour [4].

The range of pixel values of a grayscale image is not restricted to $\{0, 1\}$, but is extended to a larger finite set of nonnegative integers, for instance $\{0, 255\}$. For a grayscale image, the supremum and infimum operators are the pointwise maximum and minimum operators [5]:

$$f \vee g = \max\{f(x), g(x)\} \quad (2.11)$$

$$f \wedge g = \min\{f(x), g(x)\} \quad (2.12)$$

Therefore, dilation and erosion for grayscale image are defined as:

$$\delta_B(f) = \max\{f(x - i, y - j) + b(i, j) | i, j \in \mathbf{B}\} \quad (2.13)$$

$$\varepsilon_B(f) = \min\{f(x + i, y + j) - b(i, j) | i, j \in \mathbf{B}\} \quad (2.14)$$

where f denotes a grayscale image, $\mathbf{B}$ is the structuring element. Similarly, opening and closing for grayscale image are defined as:

$$f \circ \mathbf{B} = \delta(\varepsilon(f, \mathbf{B})) \quad (2.15)$$

$$f \bullet \mathbf{B} = \varepsilon(\delta(f, \mathbf{B})) \quad (2.16)$$

Generally speaking, in grayscale images, the intensity surface is altered by dilation and erosion. Dilation raises the intensity surface, while erosion drops the intensity surface. Opening removes features brighter than the background and smaller than the structuring element, while closing removes features darker than the background and smaller than the structuring element.

Erosions and dilations are the elements of mathematical morphology. These basic operations are then combined to create meaningful image processing algorithms. Some basic morphological operation based image processing applications, which are implemented in this thesis to evaluated new developed algorithms, are briefly introduced in this section.

Figure 2.1: (a) Binary image of bladder cancer cell image. (b) Dilation of the image by an elementary structuring element. (c) Erosion of the image by an elementary structuring element.

Figure 2.2: (a) Grayscale image of bladder cancer cell image. (b) Dilation of the image by an elementary structuring element. (c) Erosion of the image by an elementary structuring element.

2.3 Color Morphology and Applications in Color Image Processing

In recent years, attentions have switched to color morphology, because color image is the dominant image format nowadays and contains much more information than conventional binary or grayscale image format. However, extension of morphology to color images is not straightforward [6]. Mathematical morphology is built on dilation and erosion, and lattice theory is the foundation of the two fundamental operators, where pixel ordering is the central issue. In binary and grayscale images, pixels are represented by scalars and the order relationship of pixels can be easily established. In color images, pixels are usually represented by 3-dimensional vectors, and vector ordering is the major difficulty encountered in extending morphology to color images [7].

The generalization of morphology to its most basic elements, three key ideas are concluded [3]: an order relationship, a supremum (or infimum) based on that order, and the possibility of admitting an infinity of operands. Color images are always represented by 3D color vectors, and it is difficult to establish an unambiguous way to order two or more colors. Thus, the first two ideas are missing in color images [7], and it is

difficult to define 'color morphology'. However, it is possible that some techniques of ordering multivariate data could be used to help establish color morphology. Therefore, the extension of morphology to color image has been derived to find a proper ordering relationship for the colors in an image.

In the literature, three types of multivariate vector ordering schemes [8] have been presented for color morphology, including marginal ordering, conditional ordering and reduced ordering. In the marginal ordering scheme [7], scalar ordering is performed in the three color channels separately, and the maximum or minimum pixel is the vector comprising of the maximum or minimum values of the three color channels. That is to say, morphological operations are independently applied to each primary color channel of the image. This is also known as component-wise operation. Comer and Delp presented a component-wise ordering method, where each component of the color space is processed independently. The problem of the marginal ordering scheme is that it might create new colors in the image [9], which is unacceptable in some applications. The new color problem can be avoided if conditional ordering or reduced ordering is employed.

In the conditional ordering, images are first transformed from the commonly used RGB color space to HSV space or its relatives, and color pixel vectors are then ordered in terms of H (Hue) [10] or/and V (Value) component [11]. For vectors having the same value of the first used component, ordering is performed based on the second or/and the third component. This ordering method is also referred as Lexicographical ordering. The C-Ordering method is the most used techniques in the color morphology extension [12], [13]. The character of this ordering method is that users could manually choose the component which they regard most important as the first component. And

the components of the color space used should have concrete meanings. For example, in HSI color space, hue component relates to the dominant wavelength of the color stimulus, saturation corresponds to relative color purity, intensity present the brightness.

C-Ordering method can provide a totally ordering of color vectors, the advantage of this strategy is that users could select the component which is the most significant one for their image processing task as the first component. However, some drawbacks exist in this kind approach. The hue component in HSV color space is mostly chosen as the first component, which means a reference hue value is needed. This always leads the morphological operation to be specific color operation, for example, green dilation or red erosion. On the other hand, C-Ordering mainly depends on the first component. Although method [13] has been introduced to solved the problem, it would find that the third component still with little contribution to the ordering, which may lose information contained in the third channel.

The reduced ordering scheme maps vectors to scalars through a transform, and then orders the vectors in terms of the scalar values obtained [7], [5]. Therefore, the selection of the function used to compute the scalar value for comparison is the key issue in this ordering scheme. Mary L. Comer and Edward J. Delp [7] proposed to use a linear combination which derives luminance image from RGB color image to calculate the scalar values:

$$d(f(x, y)) = a_R f_R(x, y) + a_G f_G(x, y) + a_B f_B(x, y) \quad (2.17)$$

where $a_R = 0.299$, $a_G = 0.587$, and $a_B = 0.114$. Color vectors are then ordered according to this luminance value. They also discussed some other selection of the coefficient

of this linear combination. Different selection will enhance or suppress specific colors. For example, if the coefficients are set as $a_R = 1$, $a_G = 0$, and $a_B = 0$, the color opening would suppress objects with high red content.

Compared with the marginal and the conditional ordering, the strength of the reduced ordering scheme is that it does not create new colors to the image, and utilizes information in all color channels. However, reduced ordering is a partial ordering method, and might produce errors when two different color vectors within the window defined by structuring element are mapped to the same scalar value.

2.4 Summary of the Chapter

In this chapter, the basis of mathematical morphology: lattice theory, is firstly introduced. Then, basic binary and grayscale morphological operations are described. Detailed literature review of color pixel ordering schemes and their corresponding color morphological operations are provided as well.

Chapter 3

Probabilistic Framework for Color Pixel Ordering

3.1 Introduction

In grayscale morphology, pixel ordering is based on intensity: the higher the intensity of a pixel, the larger the pixel. An important property of the intensity-based pixel ordering scheme is that, if objects are brighter than background, dilation operation enlarges objects and erosion operation shrinks objects. This property is the foundation of many applications of mathematical morphology, such as edge detection, hole filling, noise removal etc. The root cause behind this property is the max-min filtering mechanism of dilation and erosion. The dilation operation distributes the maximum pixel intensity within the operation window, while erosion distributes the minimum pixel intensity within the operation window. Such max-min filtering property can be interpreted according to their effects as: dilation operation replaces the pixel being dilated with the one that is the most likely being an object pixel within the operation window defined by the structuring element, while erosion operation replaces the pixel being

eroded with the one that is the most unlikely being an object pixel in the window. In this sense, we may interpret pixel ordering in terms of probability criterion: the higher the probability of being an object pixel, the larger the pixel. Motivated by this interpretation, a probability framework for pixel ordering is presented.

In statistics, a variety of functions are available for probability estimation. In the present study, the logistic regression model [14], [15] is employed. For a grayscale pixel with intensity x , the probability of being an object pixel is estimated as:

$$p(x) = 1/[1 + \exp(-(ax + b))] \quad (3.1)$$

where a and b are scalar parameters. For a color pixel vector $\mathbf{x}$, the probability estimation can be obtained by simply extending the univariate logistic regression to multivariate logistic regression:

$$p(\mathbf{x}) = 1/[1 + \exp(-(\mathbf{a}^T \mathbf{x} + b))] \quad (3.2)$$

where $\mathbf{a}$ is the transforming vector with the same dimensions as $\mathbf{x}$ and b a scalar. Since the logistic regression model is a monotonic function, intensity criterion and probability criterion are equivalent in gray pixel ordering. However, introducing the probability criterion has two advantages. Firstly, the probability criterion unifies pixel ordering in grayscale and color morphology. Secondly, the supervised-learning based probability criterion ensures object pixels are larger than background pixels, regardless of the actual intensity or hue of the color pixels. This is an outstanding feature of the probability-based pixel ordering scheme because it guarantees that dilation enlarges objects and erosion shrinks objects, which makes the following processing steps be simple and unified. To determine the parameters of the logistic regression probability

estimator, we investigate three approaches in this study, including maximum likelihood estimation (MLE), Fisher's linear discriminant analysis (LDA) and support vector machines (SVMs). Based on the probability criterion, color pixel ordering scheme and color morphological operations are established in this chapter.

In order to evaluate the probability-based color morphological operations, a simple edge detector is presented based on morphological gradient. It extracts object edges simply based on the arithmetic difference between elementarily dilated and eroded color images, without any other post-processing steps, which is quite similar like morphological gradient for binary image edge detection. We show that, the probability-based color morphological operations with supervised learning techniques are effective in both internal and external edge detection, no matter how the objects are colored.

3.2 Probability Based Color Morphology

3.2.1 Probability Framework for Pixel Ordering

In grayscale morphology, transforming an image through dilation or erosion is equivalent to performing maximum or minimum filtering, where pixels are ordered based on intensity: the higher the intensity, the larger the pixel. If objects are brighter than background, morphological dilation enlarges objects while erosion shrinks objects. This means dilation converts some background pixels into object, while erosion transforms some object pixels into background. The root cause of this property is that in dilation, a pixel being dilated is replaced by the one with the maximum probability of being an object pixel within the structuring element, while in erosion, a pixel being operated is replaced by the one with the minimum probability of being an object pixel within the structuring element. In this sense, pixel ordering may be interpreted in terms of

probability criterion: the higher the probability of being an object pixel, the larger the pixel.

In the present study, logistic regression model [14], [15] is employed as probability estimator. For a gray pixel with intensity x , the probability of being an object pixel is estimated as:

$$p(x) = \frac{1}{1 + e^{-z}} \quad (3.3)$$

where $z = ax + b$. The logistic probability estimator is shown in Fig.3.1. If a pixel is mapped to a value $z > 0$, the probability is greater than 0.5, if $z < 0$, the probability is smaller than 0.5. This means the positive half of axis z corresponds to object pixels and the negative half corresponds to background. Since logistic regression model is a

Figure 3.1: A typical logistic regression model curve.

monotonic function, intensity-based pixel ordering is equivalent to probability-based ordering. However, the motivation of introducing probability criterion is to unify pixel ordering for grayscale and color images.

3.2.2 Color Vector Ordering Based on Probability Criterion

Extension of probability criterion to color vector ordering is straightforward and statistically sound. For a color pixel $\mathbf{x} = [x_1, x_2, x_3]^T$ in the chosen 3-D color space,

the probability estimation can be obtained by simply extending the univariate logistic regression in Eq.(3.3) to multivariate logistic regression given below:

$$p(\mathbf{x}) = \frac{1}{1 + e^{-(\mathbf{a}^T \mathbf{x} + b)}} \quad (3.4)$$

where $p(\mathbf{x})$ denotes the probability of color pixel $\mathbf{x}$ being an object pixel, $\mathbf{a} = [a_1, a_2, a_3]^T$ and b are parameters. For two color vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, we define:

$$\left. \begin{array}{ll} \mathbf{x}_1 > \mathbf{x}_2 & \text{if } p(\mathbf{x}_1) > p(\mathbf{x}_2) \\ \mathbf{x}_1 = \mathbf{x}_2 & \text{if } p(\mathbf{x}_1) = p(\mathbf{x}_2) \\ \mathbf{x}_1 < \mathbf{x}_2 & \text{if } p(\mathbf{x}_1) < p(\mathbf{x}_2) \end{array} \right\} \quad (3.5)$$

Based on Eq.(3.5), the supremum ($\vee$) and the infimum ($\wedge$) for a set of color pixel vectors $\mathbf{S}_n = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ are defined as:

$$\begin{aligned} \vee \mathbf{S}_n &= \vee \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \\ &= \mathbf{x}_k : \{p(\mathbf{x}_k) = \max\{p(\mathbf{x}_1), \dots, p(\mathbf{x}_n)\}\} \end{aligned} \quad (3.6)$$

$$\begin{aligned} \wedge \mathbf{S}_n &= \wedge \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \\ &= \mathbf{x}_k : \{p(\mathbf{x}_k) = \min\{p(\mathbf{x}_1), \dots, p(\mathbf{x}_n)\}\} \end{aligned} \quad (3.7)$$

Since logistic regression model is a monotonic function, pixel ordering can be performed in terms of value $z(\mathbf{x}) = \mathbf{a}^T \mathbf{x} + b$ directly in practice. Thus, we have:

$$\left. \begin{array}{ll} \mathbf{x}_1 > \mathbf{x}_2 & \text{if } z(\mathbf{x}_1) > z(\mathbf{x}_2) \\ \mathbf{x}_1 = \mathbf{x}_2 & \text{if } z(\mathbf{x}_1) = z(\mathbf{x}_2) \\ \mathbf{x}_1 < \mathbf{x}_2 & \text{if } z(\mathbf{x}_1) < z(\mathbf{x}_2) \end{array} \right\} \quad (3.8)$$

and

$$\begin{aligned} \vee \mathbf{S}_n &= \vee \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \\ &= \mathbf{x}_k : \{z(\mathbf{x}_k) = \max\{z(\mathbf{x}_1), \dots, z(\mathbf{x}_n)\}\} \end{aligned} \quad (3.9)$$

$$\begin{aligned} \wedge \mathbf{S}_n &= \wedge \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \\ &= \mathbf{x}_k : \{z(\mathbf{x}_k) = \min\{z(\mathbf{x}_1), \dots, z(\mathbf{x}_n)\}\} \end{aligned} \quad (3.10)$$

3.2.3 Parameter Determination for the Logistic Regression

Logistic regression-based probability estimation can be decomposed into two operations: the color pixel vector is first mapped to a scalar value by $z(\mathbf{x}) = \mathbf{a}^T \mathbf{x} + b$, and logistic operation is then applied to the scalar value z to produce probability estimation.

Geometrically, the mapping $z(\mathbf{x}) = \mathbf{a}^T \mathbf{x} + b$ can be interpreted as projecting point $\mathbf{x}$ in the 3-dimensional color space onto an axis whose direction is determined by $\mathbf{a}$, as shown in Fig.3.2(c)~(e). Estimating the parameters of logistic regression model is actually the search of an appropriate axis $\mathbf{a}$ and bias b to ensure the projections of object pixels are on the positive half of the axis, while background pixels the negative half

of the axis. In addition, large separation between objects and background pixels on the axis is desired. To achieve the goals, supervised learning is needed. In this study, three methods including the maximum likelihood estimation (MLE), Fisher's linear discriminant analysis (LDA) and support vector machines (SVMs) are investigated.

Maximum Likelihood Estimation (MLE)

To find $\mathbf{a}$ and b , n training pixels $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ are acquired from objects and background, and class labels $y = 1$ and $y = 0$ are assigned to object and background pixels respectively. Since the class label of a color pixel is either 1 or 0, y can be considered as a Bernoulli random variable.

The basic idea of MLE is to find such an $\mathbf{a}$ and b that maximize the likelihood function [16]:

$$L_1 = \prod_{i=1}^n \left(\frac{1}{1 + \exp(-(\mathbf{a}^T \mathbf{x}_i + b))} \right)^{y_i} \left(1 - \frac{1}{1 + \exp(-(\mathbf{a}^T \mathbf{x}_i + b))} \right)^{1-y_i} \quad (3.11)$$

or the log-likelihood

$$\log L_1 = \sum_{i=1}^n y_i \log \left(\frac{1}{1 + \exp(-(\mathbf{a}^T \mathbf{x}_i + b))} \right) + (1 - y_i) \log \left(1 - \frac{1}{1 + \exp(-(\mathbf{a}^T \mathbf{x}_i + b))} \right) \quad (3.12)$$

The likelihood function or log-likelihood function is nonlinear in $\mathbf{a}$ and b , therefore $\mathbf{a}$ and b can not be found analytically. Iterative procedure is needed to find numerical solutions.

Fisher Linear Discriminant Analysis (LDA)

Fisher's linear discriminant analysis (LDA) [16] could also be used to find suitable values for parameters of $\mathbf{a}$ and b . The basic idea of LDA is to find such an axis that

projections of object and background pixels on the axis have the largest difference, while have the smallest variance within the object and background pixels, respectively. This can be achieved by maximizing the following criterion

$$L_2 = \frac{|m_1 - m_2|^2}{\sigma_1^2 + \sigma_2^2} \quad (3.13)$$

where m_1 and m_2 are the means of object and background pixels on the axis respectively, and σ_1^2 and σ_2^2 are the variances of object and background pixels on the axis respectively.

Support Vector Machines (SVMs)

Support vector machine (SVM) is a popular pattern classification technique developed recently [16], [17]. In SVM, $\mathbf{a}$ and b are obtained by maximizing separating margin, which is inversely proportional to $\|\mathbf{a}\|$. Thus maximizing separating margin is equivalent to minimizing the following criterion:

$$L_3 = \frac{1}{2} \mathbf{a}^T \mathbf{a} \quad (3.14)$$

subject to

$$y_i(\mathbf{a}^T \mathbf{x}_i + b) \leq 1 \quad \text{for } i = 1, 2, \dots, N \quad (3.15)$$

Note the class labels for object and background pixels are set to 1 and -1 respectively in SVM.

In the above, it is assumed that object pixels could be separated from background pixels using a hyperplane $\mathbf{a}^T \mathbf{x} + b = 0$. In some applications, however, this may not be true. In such a case, we can map the color pixels from the original 3-dimensional color space into a higher-dimensional Hilbert space through a kernel function such as

Gaussian function:

$$K(\mathbf{x}, \mathbf{C}_i) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{C}_i\|^2}{2\sigma^2}\right) \quad (3.16)$$

The $\mathbf{a}$ and b are then found in the Hilbert space using SVM learning method. We name the pixel ordering based on the above four parameter estimation methods as MLE-based, LDA-based, SVM-based and GSVM-based color pixel ordering respectively.

As aforementioned, the probability-based color pixel ordering is equivalent to ordering pixels based on z , which is the projection of $\mathbf{x}$ on axis $\mathbf{a}$. In this sense, the probability-based color pixel ordering can be regarded as a reduced ordering. However, it has fundamental difference from the conventional reduced ordering. The conventional reduced ordering maps color pixel onto a fixed axis, which may not guarantee target object pixels are always larger (or smaller) than background pixels. In other words, the conventional reduced ordering-based morphological operations including dilation and erosion may not necessarily enlarge or shrink objects effectively. In our study, the axis $\mathbf{a}$ is learned from application images through supervised learning, and hence guarantees the object pixels are mapped to values larger than those of background pixels.

We next illustrate the strength of the probability criterion through an example. Fig.3.2(a) shows a synthetic image with green and dark blue objects in a light blue colored background and the distribution of objects and background pixels in the RGB color space is shown in Fig.3.2(c). In the conventional reduced-ordering scheme [7], the color vectors are projected onto a fixed axis $[0.299, 0.578, 0.114]^T$, and the projections of the green, dark blue object pixels and background pixel are 0.6029, 0.1548, 0.5218 respectively. Obviously, not all object pixels are larger than background pixels. If conditional ordering scheme is used, the distribution of pixels of Fig.3.2(a) on H-V plane

composed of Hue and Value is illustrated in Fig.3.2(b). Again, not all object pixels are larger than background pixels if Hue component or/and Value component are used in pixel ordering. As shown in Fig.3.2(d) and (e), however, object pixels are guaranteed to be larger if the color pixels are mapped to axis $\mathbf{a} = [-0.535, -0.428, -0.729]^T$ or $\mathbf{a} = [-0.719, -0.528, -1.000]^T$, which are learned using LDA and SVM respectively.

Figure 3.2: (a) Synthetic color image. Green, blue and black samples in (b)~(e) denote color pixels of green object, dark blue object and background in (a), respectively. (b) Pixel distribution in the plane of Hue (HLS color space) and Value (HSV color space). (c) Pixel distribution in RGB color space and the projection to axis $\mathbf{a} = [0.299, 0.578, 0.11]^T$. (d) and (e) Pixel distributions in RGB color space and projections to the axis $\mathbf{a} = [-0.535, -0.428, -0.729]^T$ or $\mathbf{a} = [-0.719, -0.528, -1.000]^T$ learned by LDA and SVM techniques, respectively.

3.3 Simple Edge Detector Based on Probability-based Color Morphology

Morphological gradient operator is one of the most popular and effective methods in edge detection. It is defined as the arithmetic difference between dilated and eroded images by elementary structuring element [1]:

$$\rho_B = \delta_B - \varepsilon_B \quad (3.17)$$

Eq.(3.17) was originally defined for binary and grayscale images and was later extended to color images [18], [19], [20]. The objective of morphological gradient is to enhance intensity or color difference between pixels within the structuring element to highlight object boundaries. The gradient image is usually a grayscale image, to which post-processings including noise removal, thinning, edge linking etc. are applied to produce object edges.

Basically, dilation operation converts some background pixels into object pixels while erosion converts some object pixels into background pixels. Pixels that change class labels, i.e. object or background, after erosion or dilation are boundary pixels. The internal edge pixels, which are initially on the side of object, become background pixels after elementary erosion, while the external edge pixels, which are initially on the side of background, become object pixels after elementary dilation. Based on the color morphological operations established by logistic regression model, internal object edges are the set of pixels satisfying:

$$\rho_B^-(\mathbf{x}) = \{\mathbf{x}_i \mid p(\mathbf{x}_i) > 0.5 \quad \& \quad p(\mathbf{x}_i)^\epsilon < 0.5\} \quad (3.18)$$

while external object edges are the set of pixels satisfying:

$$\rho_B^+(\mathbf{x}) = \{\mathbf{x}_i \mid p(\mathbf{x}_i) < 0.5 \quad \& \quad p(\mathbf{x}_i)^\delta > 0.5\} \quad (3.19)$$

where ϵ and δ denote the dilated and eroded color images by elementary structuring element. Object edges with two-pixel wide are defined as:

$$\rho_B(\mathbf{x}) = \{\mathbf{x}_i \mid p(\mathbf{x}_i)^\delta > 0.5 \quad \& \quad p(\mathbf{x}_i)^\epsilon < 0.5\} \quad (3.20)$$

As analyzed in Section II.B, color pixel ordering can be performed in terms of value $z(\mathbf{x})$ directly. The value of $z(\mathbf{x})$ is positive for an object pixel, while negative for a background pixel. Thus, the internal, external and two-pixel wide edges can be respectively redefined as:

$$\left. \begin{aligned} \rho_B^-(\mathbf{x}) &= \{\mathbf{x}_i \mid z(\mathbf{x}_i) > 0 \quad \& \quad z(\mathbf{x}_i)^\epsilon < 0\} \\ \rho_B^+(\mathbf{x}) &= \{\mathbf{x}_i \mid z(\mathbf{x}_i) < 0 \quad \& \quad z(\mathbf{x}_i)^\delta > 0\} \\ \rho_B(\mathbf{x}) &= \{\mathbf{x}_i \mid z(\mathbf{x}_i)^\delta > 0 \quad \& \quad z(\mathbf{x}_i)^\epsilon < 0\} \end{aligned} \right\} \quad (3.21)$$

The edge detectors presented above are basically a combination of supervised learning and morphological operations. The advantages of the proposed edge detectors are two fold. Firstly, the object class and background class can be defined according to user's interests. Thus, we could always extract objects with a specific color from the image containing multiple colored objects. However, the conventional morphological gradient-based edge detectors might produce spurious edges since the boundaries of all objects are enhanced in the gradient image. Secondly, the edge detectors produce edges by finding pixels that change class labels after dilation and erosion. This is a

simplified procedure compared with conventional morphological gradient-based edge detectors which demands post-processings such as thinning, edge linking etc.

3.4 Experiment Results

To validate the effectiveness of the proposed pixel ordering schemes and edge detectors, both synthetic and real color images are used. Comparative study of the following color pixel ordering schemes on synthetic color images is first performed.

M 1 MLE-based color pixel ordering.

M 2 LDA-based color pixel ordering.

M 3 SVM-based color pixel ordering.

M 4 GSVM-based color pixel ordering.

M 5 Luminance-based color pixel ordering [7].

M 6 Conditional ordering in HLS color space [10]. $Hue = 0$ is used as origin hue value.

M 7 Conditional ordering in HSV color space [11].

M 8 Combination of reduced ordering and conditional ordering [5]. $\alpha = [0 \ 0 \ 0]^T$ is used as reference point.

Then, applications in hole filling, noise removal were also implemented.

3.4.1 Edge Detection

Edge detection is one of the most important applications of mathematical morphology. In the present study, two scenarios were investigated. One scenario is to detect internal

or external edges of multiple objects in a color image, and another is to detect the edge of a specific colored object from the color image containing multiple objects with various colors.

Internal/External Object Edge Detection

The key issue of internal or external object edge detection using morphological operations is to erode or dilate the target objects synchronously, which requires all target object pixels are larger (or smaller) than background pixels. Fig.3.3(a) is an exemplar image containing objects of two different colors. Assume our objective is to detect internal edges of all the objects in the image. To estimate the parameters in the logistic regression model, we collected pixels from objects of both colors and assigned class label 1 to these pixels. Background pixels were also collected and class label 0 or -1 was assigned. Based on pixel ordering schemes $M1 \sim M8$, the images after dilation and erosion with a 5×5 square structuring element are shown in Fig.3.3 respectively. Fig.3.3(b1)~(e1) and Fig.3.3(b2)~(e2) show that morphological operations based on $M1 \sim M4$ synchronously dilate and erode the two different colored objects, while Fig.3.3(f1)~(i1) and Fig.3.3(f2)~(i2) show that morphological operations based on $M5 \sim M8$ fail to dilate or erode some objects in the image. The cause of unsatisfactory results of $M5 \sim M8$ is that not all target object pixels are larger (or smaller) than background pixels in these pixel ordering schemes. However, our probability-based color pixel ordering ensures the object pixels are larger than background pixels and hence enlarges and shrinks objects correctly.

The internal edges of the green and dark blue objects detected using Eq.(3.18) based on $M1 \sim M4$ are shown in Fig.3.4(a)~(d), respectively. Internal edge detection results of $M5 \sim M8$ using internal morphological gradient [1] are illustrated in Fig.3.4(e)~(h),

respectively. Fig.3.4(a)~(d) show that the edge detectors based on color pixel ordering scheme $M1 \sim M4$ extract the internal edges of all objects correctly, while Fig.3.4(e) and (g) show that $M5$ and $M7$ -based detectors extracted the external edges of dark blue objects, and Fig.3.4(f) and (h) show that $M6$ and $M8$ -based detectors extract external edges of green objects. External edges detected based on $M1 \sim M8$ are shown in Fig.3.5(a)~(h), respectively. Again, $M1 \sim M4$ -based edge detectors produced superior results to $M5 \sim M8$ -based edge detectors.

Specific Object Extraction

In this experiment, we investigated the ability of the probability-based mathematical morphology to detect the edge of objects with a specific color from the image containing multiple colored objects.

We first collected learning samples. If the goal is to detect the edges of green objects, then pixels from green objects are defined as object pixels and are assigned class label 1, pixels from all other objects as well as background are defined as background pixels and are assigned class label 0 or -1 . Similarly, we can define object and background training samples if the edges of dark blue objects are to be detected. Learning the parameters of logistic regression model based on MLE, LDA and SVMs, and the final edge detection results are shown in Fig.3.6, where (a1)~(d1) show the edges of green objects, (a2)~(d2) show the edges of dark blue objects. The edge detection results based on $M5 \sim M8$ are shown in Fig.3.7(a)~(d). Obviously, edges of unwanted object are also extracted. This is because the conventional morphological gradient-based edge detection algorithm can not effectively distinguish different colored objects,

3.4.2 Other Applications

Hole Filling

Hole filling is another important application of mathematical morphology [1]. Fig.3.8(a) shows a synthetic color image containing objects with holes. The objective here is to fill the holes that are not connected to the image border. The hole filling results using pixel ordering schemes $M1 \sim M4$ -based morphological operations are shown in Fig.3.8(b)~(e). However, as shown in Fig.3.8(f)~(i), some objects were wrongly regarded as holes and removed by $M5 \sim M8$ -based color morphological operations. This is because the conventional reduced ordering and conditional ordering schemes can not guarantee the background pixels are always the smallest in the image. However, the probability-based methods learn the parameters from the given color image and therefore guarantee the background pixels are smaller than object pixels.

Noisy Particulate Removal

Morphological opening followed by closing is often used to remove noise, particularly when noisy particulates are contained in images. Particulates with various colors are randomly added to the synthetic image of Fig.3.3(a). Resultant images based on $M1 \sim M4$ demonstrated in Fig.3.9(b)~(e) show that the noisy particulates are completely eliminated.

3.5 Summary of the Chapter

In this chapter, we have proposed supervised learning and probability criterion for pixel ordering. Logistic regression model is selected as probability estimator. It is because the typical logistic regression model curve could effectively simulate the pixel ordering in mathematical morphology. And it is assumed that objects and background pixels in

the given image can be linearly segmented. For more complicated cases, for example, objects and background pixels are nonlinear separable in an image, or images contain textured scenes, slowly changing intensities, more sophisticated probability estimation models or algorithms must be used.

In the framework of probability, ordering of gray pixels and color pixels is unified. The supervised learning-based parameter estimation for probability estimator guarantees the object pixels are always larger than background pixels, and this ensures synchronous enlargement and shrinking of objects by dilation and erosion. In addition, the proposed edge detector greatly simplifies the morphological gradient-based edge detection algorithms. Experiment studies on synthetic color images have demonstrated the good performance of our methods.

Figure 3.3: (a) Test color image, 5×5 square structuring element is used. (b1)~(i1) Dilated color images based on $M1 \sim M8$ respectively. (b2)~(i2) Eroded color images based on $M1 \sim M8$ respectively.

Figure 3.4: (a)~(h) Internal edges of all the objects in Fig.3.3(a) extracted based on $M1 \sim M8$, respectively.

Figure 3.5: (a)~(h) External edges of all the objects in Fig.3.3(a) extracted based on $M1 \sim M8$, respectively.

Figure 3.6: Edge detection results of specific objects of Fig.3.3(a). (a1)~(d1) Edges of green objects detected based on $M1 \sim M4$. (a2)~(d2) Edges of dark blue objects detected based on $M1 \sim M4$.

Figure 3.7: (a)~(d) Edge detection results based on $M5 \sim M8$, respectively.

Figure 3.8: (a) Original color image, (b)~(i) Hole filling results by $M1$ to $M8$ respectively.

Figure 3.9: (a) Original noisy color image. 3×3 square structuring element is used. (b)~(i) Resultant images of opening followed by closing based on $M1$ to $M8$, respectively.

Chapter 4

Color Morphological Gradient Vector

Probability framework for color pixel ordering is presented in the previous chapter and fundamental color morphological operations based on the ordering scheme are also given. Solving an image analysis problem with morphological operators needs to combine and integrate these fundamental operations reasonably and artfully. Morphological gradient is one of such subtly established useful morphological operators, based on which edge detection, image segmentation and watershed algorithms are implemented. Direct extension of morphological gradient definition to color images, for example the simple edge detector presented in the previous chapter, may be not enough. Because more information is contained in color image than monochrome image, when analyzing the color image containing complex content, it may not produce expected results. In this chapter, two color morphological gradient operators are presented. They respectively incorporate feature selection and supervised learning techniques, to extract useful boundary information directly from the vectors, which are the arithmetic difference between elementary dilated and eroded color images. The effectiveness of the

proposed color morphological gradients are tested using both synthetic and real color images.

4.1 Morphological Gradient

4.1.1 Conventional Morphological Gradient

Erosions and dilations are the letters of the morphological alphabet. These letters are combined to create the words of the morphological language. Solving an image analysis problem with morphological operators consists of building meaningful sentences using the morphological vocabulary [1]. Morphological gradient, one of the most significant words of morphological language [1], is the foundation to establish meaningful approaches to image analysis problems, such as edge detection and marker-controlled watersheds segmentation algorithms. The standard morphological gradient $\nabla_{\mathbf{B}}(f)$ [21] for binary or grayscale image is defined as the arithmetic difference between dilated $\delta_{\mathbf{B}}(f)$ and eroded $\varepsilon_{\mathbf{B}}(f)$ images by elementary structuring element $\mathbf{B}$:

$$\nabla_{\mathbf{B}}(f) = \delta_{\mathbf{B}}(f) - \varepsilon_{\mathbf{B}}(f) \quad (4.1)$$

Half gradients, which are used to detect either the internal or the external boundary of an edge are also commonly used. These gradients are one pixel wide of a step edge [1].

$$\nabla_{\mathbf{B}}^{-}(f) = f - \varepsilon_{\mathbf{B}}(f) \quad (4.2)$$

$$\nabla_{\mathbf{B}}^{+}(f) = \delta_{\mathbf{B}}(f) - f \quad (4.3)$$

The operators highlight the boundaries of the objects in the image to be segmented by enhancing the differences between pixels inside the structuring element [18].

4.1.2 Color Morphological Gradient

For color images, the morphological gradients are usually associated with some color difference defined in the chosen color space using a certain measurement [22], [19], [20], [23], [24]. In literatures, several color morphological gradients were proposed:

1 *Component-wise gradient*:

$$\nabla_{\mathbf{B}}^C(f) = \vee\{\nabla_{\mathbf{B}}(f_1), \nabla_{\mathbf{B}}(f_2), \nabla_{\mathbf{B}}(f_3)\} \quad (4.4)$$

where $\nabla_{\mathbf{B}}(f_1), \nabla_{\mathbf{B}}(f_2), \nabla_{\mathbf{B}}(f_3)$ is the monochrome morphological gradient in each color image component.

2 *Hue circular gradient*: Given color image h defined in the HSI family of color models, hue circular gradient is defined as

$$\nabla_{\mathbf{B}}^{\theta}(h)(\mathbf{x}) = \vee d_h(h(\mathbf{x}), h(\mathbf{y})) - \wedge d_h(h(\mathbf{x}), h(\mathbf{y})) \quad (4.5)$$

where $y \in \{B_x - \{x\}\}$, d_h denotes the hue distance between θ_1 and θ_2 defined as:

$$d_h(\theta_1, \theta_2) = \wedge\{|\theta_1 - \theta_2|, 360 - |\theta_1 - \theta_2|\}.$$

3 *Saturation weighting-based gradient*:

$$\nabla_{\mathbf{B}}^S(f) = f_S \nabla_{\mathbf{B}}^{\theta} f_h + f_S^c \nabla_{\mathbf{B}} f_L \quad (4.6)$$

where L denotes the component representing color lightness in the HSI family or perceptually uniform color spaces.

4 *Sum of weighted gradient*:

$$\nabla_{\mathbf{B}}^S(f) = \mu_a \nabla_{\mathbf{B}}^{\theta}(f_1) + \mu_b \nabla_{\mathbf{B}}(f_2) + \mu_c \nabla_{\mathbf{B}}(f_3) \quad (4.7)$$

where f_1 represents H component and f_2, f_3 represent the other two component in HSI family color models, μ_a, μ_b and μ_c are the weights.

5 Euclidean distance gradient:

$$\nabla_{\mathbf{B}}^E(f)(\mathbf{x}) = d_E(\delta_{\mathbf{B}}(f)(\mathbf{x}) - \varepsilon_{\mathbf{B}}(f)(\mathbf{x})) \quad (4.8)$$

where $d_E(\delta_{\mathbf{B}}(f)(\mathbf{x}) - \varepsilon_{\mathbf{B}}(f)(\mathbf{x}))$ are the Euclidean distances between color pixels $\mathbf{x}$ in dilated and eroded color images in the chosen color space. Chromatic gradient and Perceptual gradient in perceptually uniform color spaces are defined based on $\nabla_B^E(f)$.

More detailed definitions and descriptions could be found in [18], [19] and [25].

These color morphological gradients aim at enhancing the variations of colors inside a structuring element to highlight the object boundary. Results of these color morphological gradient operators are grayscale images, where each pixel is the color difference between corresponding pixels with the same coordinates in dilated and eroded color images. Using these color morphological gradients might not always obtain expected results. For example, in some object detection applications, it is often expected to separate a specific object from the image that contains multiple objects besides the one of interest. Since the conventional color morphological gradients enhance the boundaries of all the objects synchronously, they may not effectively distinguish the edges of different objects, hence produce unexpected results.

In this chapter, firstly, conventional grayscale morphological gradient is analyzed, and the concept of color morphological gradient vector (CMGV) is introduced. We show that, color morphological gradients are actually the modulus or the combinations of

some components of CMGV, the computation of color morphological gradient is actually the process of extracting information from CMGV, and a better way to calculate color morphological gradient is to incorporate more sophisticated information extraction techniques or compute according to the given image segmentation targets. Then, two new color morphological gradient operators are developed: PCA-based color morphological gradient and SVM-based color morphological gradient. The essence of the two methods is to incorporate feature selection techniques (principle component analysis, PCA) and supervised learning techniques (support vector machine, SVM) into the calculation of color morphological gradient respectively, aiming at extracting useful boundary information directly from color morphological gradient vector in accordance with image segmentation targets. The advantage of PCA-based color morphological gradient is that it contains the most information among the linear transformations of CMGV, which helps enhance the contrast in final gradient image. The advantage of SVM-based color morphological gradient is that it is computed in accordance with the given image segmentation target, which helps only enhance the expected object boundary while suppress others. In the study of SVM-based color morphological gradient, two ways for supervised learning are presented and analyzed: learning before morphological operation (LBMO) and learning after morphological operation (LAMO). The experiment studies on both synthetic and histological color images show the good performance of the two new color morphological gradient operators.

4.2 Color Morphological Gradient Vector (CMGV)

4.2.1 Morphological Gradient Analysis

Transforming an image through dilation and erosion is actually a pixel moving process. If all pixels belonging to objects are considered as one class, and all background

pixels are considered as another class, dilation generally makes pixels move toward the object class, while erosion makes pixels move toward the background class. In other words, dilation makes more pixels become object pixels, while erosion makes more pixels become background pixels. As demonstrated in Fig.4.1, grayscale image of immunohistochemically stained bladder cells are dilated and eroded, respectively, by a 5×5 square structuring element. From the corresponding histograms, compared with untransformed image, we can see that more pixels assemble in the histogram bins of object pixels in dilated image, while more pixels assemble in the histogram bins of background pixels in eroded image. This is because grayscale morphological dilation moves pixels along the gray value axis to brighter side, and erosion moves pixels along the gray value axis by the contrary direction.

If we define object edge is of two pixel wide, and elementary structuring element is used, dilation makes edge pixels which are initially on the background side become object pixels, while erosion makes edge pixels which are initially on the object side become background pixels. That is to say, during the elementary dilation and erosion, edge pixels move across different classes while regional pixels move within the same class. Assume gray pixel J has gray value J_D in elementary dilated image and gray value J_E in elementary eroded image, we define the distance between J_D and J_E on gray value axis is the moving distance of pixel J during elementary dilation and erosion, and the moving direction is pointing from J_E to J_D . This moving distance is actually the arithmetic difference between elementary dilated and eroded gray pixel J , which is the morphological gradient of gray pixel J . Without loss of generality, it is easily to find that boundary pixels usually have longer moving distance than regional pixels on the gray value axis, therefore, boundary pixels have bigger gradient values than regional pixels. This is the basic reason that grayscale morphological gradient

Figure 4.1: (a) Two overlapping cells of p53 immunohistochemistry stained bladder inverted papilloma. (b)~(c) Dilated and eroded images by 5×5 square structuring element, respectively. (d) Histogram of initial grayscale image (a). (e) Histograms of dilated image (b). (f) Histogram of eroded image (c).

defined in Eq.(4.3) could highlight the object boundary in grayscale images.

4.2.2 Color Morphological Gradient Vector

It is similar in color morphology. Color pixels correspond with points in 3D color space, different colors correspond with different locations in the space. Color pixels move toward the area where background pixels assemble during erosion operation, while color pixels move toward the area where object pixels assemble during dilation operation. Assume color pixel $\mathbf{P}$ has color values $\mathbf{P}_D$ and $\mathbf{P}_E$ in color dilated and eroded images

respectively, in 3D color space, the arithmetic difference between dilated color pixel P_D and eroded color pixel P_E therefore becomes a 3D vector, pointing from P_E to P_D , as shown in Fig.4.2(a). We define such vector as *color morphological gradient vector*

Figure 4.2: (a) Color morphological gradient vector (CMGV). (b) Projections of CMGV on the axis with direction w_1 and w_2 , respectively. (c) The projection of CMGV on the axis w learned by SVM. (d) The projection of CMGV on the axis learned by SVM, when there are other object in the color image besides the one to be segmented.

(CMGV). The modulus of CMGV, which is the moving distance of color pixel during elementary dilation and erosion operation, is the most used color morphological gradient defined in Eq.(4.8). Similarly as on grayscale axis, in 3D color space, boundary pixels moving across different classes usually have longer moving distance than regional pixels

moving within the same classes, hence have bigger gradient values than regional pixels. The modulus computation is a kind of transform of CMGV, other previously developed color morphological gradients are also defined and computed based on some transforms of CMGV. Such transforms contain useful information extracted from CMGV, which highlights the boundaries in morphological gradient images. Therefore, color morphological gradient computation is actually a process of extracting information from CMGV by applying some transformation.

CMGV contains both strength and direction information. The previously proposed color morphological gradients use only the strength information of CMGV by applying some fixed transform. Since fixed transforms of CMGV do not consider the characteristics of the given color images and the given image segmentation targets, they are not adaptive and might not produce optimal color morphological gradient for the following steps of image segmentation and edge detection. On the other hand, the previously proposed color morphological gradients lose the direction information of CMGV. One of the disadvantages is that only using the strength information may not easily distinguish the boundaries belonging to different objects in one color image. If only some specific object is to be detected in the color image containing various objects, unexpected result might be produced. In order to overcome the drawbacks, two new color morphological gradients are presented.

4.2.3 PCA-based Morphological Gradient

The previously proposed color morphological gradients are actually the modulus or the combination of some components of CMGV in the chosen color spaces. The modulus of CMGV, or the combination of some components of CMGV, can be considered as a feature of CMGV. The calculation of color morphological gradient thus can be

considered as a feature selection process of CMGV. Therefore, in the present study, feature selection technique, principle component analysis (PCA) is incorporated, and the PCA-based color morphological gradient is developed.

Principle component analysis (PCA) is an unsupervised feature selection technique for forming new variables that are uncorrelated with each other by maximizing the rate of decrease of their variances [16], [17], [14]. The new variables are linear composites of the original variables, and called principle components. The first principle component accounts for the maximum variance in the data [14], which can also be viewed as taking the most information of the data. For a color image with size of $m \times n$ in RGB color space, CMGV produced by reduced ordering-based color morphology [7], are also $m \times n$ 3D vectors. If the three principle components of PCA transform of CMGV are denoted as $p1$, $p2$ and $p3$ respectively, we define the first principle component $p1$ of CMGV as *PCA-based color morphological gradient*. Fig.4.3(b)~(d) show the results of PCA-based color morphological gradient, as well as $p2$ and $p3$ obtained from CMGV of a color image containing two overlapping cell nuclei. They are all grayscale images, we can see PCA-based color morphological gradient displays clear boundary of the cell nuclei.

4.2.4 SVM-based Color Morphological Gradient

When color image contains multiple objects, or various background, the object pixels and background pixels have various locations in the chosen color space. During color erosion, boundary pixels move toward the locations where their neighboring background pixels assemble, while during color dilation, boundary pixels move by the the contrary directions. So, movements of boundary pixels belonging to different objects during morphological dilation and erosion must have various moving directions. In other

Figure 4.3: (a) p53 immunohistochemical stained bladder tissue cell nuclei. (b) PCA-based color morphological gradient (the first principle component p_1 of CMGV), (c) The second principle component p_2 of CMGV, (d) The third principle component p_3 of CMGV.

words, CMGV of different object boundary pixels have their own unique directions. Therefore, the direction information of CMGV is an important clue to distinguish different boundary pixels and different objects. As an example shown in Fig.4.6(d)~(f), in the boundary regions of the two colored objects, the projections of CMGV on R - G , G - B and B - R planes display various directions. Hence, such direction difference among CMGV belonging to different objects can be used to distinguish these various objects. In the present study, we analyzed two ways to highlight the expected objects boundary while suppress the boundary of other objects by applying supervised learning techniques in calculating color morphological gradient from CMGV.

Learning Before Morphological Operations

For the most used color morphology based on reduced ordering scheme [7], [8], color pixels ordering and supremum/infimum distribution during the morphological operations are all based on the predefined scalar measurement, which is calculated by some transform. If the transform is linear and denoted as

$$\mathbf{w}^T \mathbf{x} + b \quad (4.9)$$

where $\mathbf{x}$ are 3D color vectors, $\mathbf{w}$ and b are the parameters of the transform, the main directions of color pixel movement during color morphological dilation and erosion are along $\mathbf{w}$. If the moving direction of the object pixels to be segmented is similar with $\mathbf{w}$, and the moving directions of other objects pixels are quite different, or even contrary to $\mathbf{w}$, the projection of CMGV on the axis with direction $\mathbf{w}$ can easily distinguish the boundary of expected object from the boundaries of other objects. In the present study, we use the projection of CMGV on an axis, whose direction $\mathbf{w}$ is used as scalar measurement in reduced ordering-based color morphology, and learned according to the image segmentation targets, as another color morphological gradient. Since the projection contains both strength and direction information of CMGV, and the way of projection is learned according to image segmentation targets, such color morphological gradient contains more useful information, and leads better results than the previous ones.

In order to use the projection of CMGV on $\mathbf{w}$ as color morphological gradient, we must ensure that in the chosen color space, the projection of object pixels and background pixels on the axis $\mathbf{w}$ must be well separated. For the scenario that only some specific object is expected to be separated, we can define object pixels include only the pixels

to be segmented, and background pixels include both image background pixels and the object pixels to be neglected. As shown in Fig.4.2(b), assume the pink region and blue region denote object and background pixels, respectively, in RGB color space. $Pixel_D$ and $Pixel_E$ denote the boundary pixel P in elementary dilated and eroded color images, respectively. If the transform in reduced ordering is $\mathbf{w}_1$, the projection of CMGV of P on the axis $\mathbf{w}_1$ is so small, that may not be bigger enough than that of regional pixels, and fail to enhance the object boundary. However, if the transform is $\mathbf{w}_2$, the projection ED is big enough in the final gradient image to show boundary information, and also contains the direction information of the pixel movement. Meanwhile, from the figure, we find that the projections of object pixels and background pixels on axis $\mathbf{w}_2$ can be well separated, but the projections on $\mathbf{w}_1$ can not. That is to say, if the transform $\mathbf{w}$ can be learned to ensure that the object and background are well separated in the chosen color space, the projection of CMGV of boundary pixels on the axis $\mathbf{w}$ should be much bigger than that of regional pixels, and can be used as color morphological gradient.

In the present study, support vector machine (SVM) [16], [17] is chosen to learn such a transform. In SVM, $\mathbf{w}$ and b are obtained by maximizing separating margin between the two classes, which is equivalent to minimizing the following criterion:

$$L = \frac{1}{2} \mathbf{w}^T \mathbf{w} \quad (4.10)$$

subject to

$$y_i(\mathbf{w}^T \mathbf{x}_i + b) \leq 1 \quad \text{for } i = 1, 2, \dots, N \quad (4.11)$$

Note the class labels for object and background pixels are set to 1 and -1 respectively in SVM. Because the transform $\mathbf{w}$ is learned before morphological operations and used

for color pixel ordering in reduced ordering-based color morphology, we define this projection of CMGV on the axis as *SVM-based (LBMO) color morphological gradient*. Fig.4.2(c) graphs the projection of CMGV on the axis $\mathbf{w}$, which is perpendicular to the SVM separating hyperplane. Since SVM has the ability to adjust the direction of the axis according to the object and background pixels distribution in color space, using the projection of CMGV on such axis $\mathbf{w}$ could ensure pixels on the expected boundary have bigger morphological gradient values than other pixels in various situations. Fig.4.4(b) shows the result of SVM-based (LBMO) color morphological gradient of the two overlapping cell nuclei.

Figure 4.4: (a) Two overlapping cells of p53 immunohistochemistry stained bladder inverted papilloma. (b) SVM-based (LBMO) color morphological gradient. (c) SVM-based (LAMO) color morphological gradient.

Using the projection of CMGV on axis $\mathbf{w}$ learned by SVM as color morphological

gradient has great advantages when separating specific object from the color images containing multiple objects besides the one of interest. Take the scenario in which there are two types objects in a color image but only one of them is to be segmented for example. As shown in Fig.4.2(d), in RGB color space, assume *Object 1* denote the pixels belonging to the object to be separated, and *Object 2* denote the pixels belonging to the object to be neglected. $Pixel_1$, $Pixel_2$ are two boundary pixels belonging to *Object 1* and *Object 2* respectively. The moving distance of the two pixels during dilation and erosion operations are nearly the same as shown in the figure, thus, the modulus of CMGV can not distinguish the two boundary pixels belonging to different objects. If consider both *Object 2* and *Background* as background class, and *Object 1* as object class, the axis, which is perpendicular to the SVM separating hyperplane that separates the two classes, can be learned as demonstrated in Fig.4.2(d). And using SVM-based (LBMO) color morphological gradient, which is the projection of CMGV on the axis $\mathbf{w}$, makes $Pixel_1$ have much bigger gradient value than $Pixel_2$. Therefore, separating the boundary of *Object 1* from SVM-based (LBMO) color morphological gradient image becomes much easier than only using the modulus of CMGV. Fig.4.8 shows several morphological gradients of a color histological image, in which the brown microvessel is to be segmented. Compared with other morphological gradient, from Fig.4.8(e) we can see that the boundary of microvessel are much brighter than the boundary of cells tissue in SVM-based (LBMO) color morphological gradient.

The SVM-based (LBMO) color morphological gradient presented above mainly contains three steps in the computation.

Step 1 Transforming color pixels into scalar values for color pixel ordering in the morphological operations by the linear transform $\mathbf{w}$. The transform $\mathbf{w}$ is learned according to image segmentation target by SVM.

Step 2 Calculating arithmetic difference between elementary dilated and eroded color images as CMGV.

Step 3 Linearly projecting CMGV obtained in Step 2 on previously learned axis $\mathbf{w}$.

Since all the calculations are linear, the computation process can be reorganized as learning the linear transform $\mathbf{w}$ first according to image segmentation target, and then transform color image into grayscale image by such transform $\mathbf{w}$, finally grayscale morphological gradient is calculated. Therefore, the result of SVM-based (LBMO) color morphological gradient is equal to part of K. Z. Mao and Peng Zhao's work presented in [26].

In the above, it is assumed that pixels of the object to be segmented could be separated from other pixels, which include both background and other objects to be neglected, by using a linear SVM hyperplane. In some applications, however, this situation may not be true. In such a case, we can map the color pixels from the original 3D color space into a higher-dimensional Hilbert space through a kernel function such as Gaussian function:

$$K(\mathbf{x}, \mathbf{C}_i) = \exp \left(- \frac{\|\mathbf{x} - \mathbf{C}_i\|^2}{2\sigma^2} \right) \quad (4.12)$$

The linear separating hyperplane and axis direction $\mathbf{w}$ are then found in the Hilbert space using SVM learning method.

Learning After Morphological Operations

The previously described SVM-based (LBMO) color morphological gradient calculates the gradient by projecting CMGV on the axis $\mathbf{w}$, which is learned for reduced ordering morphological operations. A similar color morphological gradient can be calculated by projecting CMGV on another axis, which is learned after morphological operations. It

is named as *SVM-based (LAMO) color morphological gradient* here to distinguish from the previous one.

CMGV is vector in the chosen color space, therefore, CMGV itself can be considered as data in the 3D color space. If CMGV of boundary pixels belonging to the objects to be segmented are considered as one class, CMGV of other pixels, including both boundary pixels belonging to other objects to be neglected and all the regional pixels, are considered as another class, a separating hyperplane can be learned by SVM to separate the two classes. The distance from CMGV to the separating hyperplane is defined as SVM-based (LAMO) color morphological gradient. Fig.4.4(c) shows the result of SVM-based (LAMO) color morphological gradient of two overlapping cell nuclei. Fig.4.8(f) shows the result of SVM-based (LAMO) color morphological gradient of microvessel image. For the scenario that the two classes can not be linearly separated, a similar way to map pixels into higher-dimensional Hilbert space through a kernel function can be used as in LBMO.

The essence of the two SVM-based color morphological gradients is trying to combine direction information of CMGV into color morphological gradient computation to increase the amount of boundary information in color morphological gradient. And using supervised learning technique as a method to extract direction information from CMGV makes the calculation of color morphological gradient be pertaining to the image segmentation target, which enhances the expected boundary only and effectively suppress other boundaries to be neglected. This character enables SVM-based color morphological gradient is useful and effective in a lot of situations.

4.3 CMGV Applications in Image Segmentation

To validate the effectiveness of the proposed PCA-based color morphological gradient and SVM-based color morphological gradient, both synthetic and real color images are used. Comparative study of the following color morphological gradients, as well as the conventional grayscale morphological gradient is performed.

G 1 PCA-based color morphological gradient.

G 2 SVM-based (LBMO) color morphological gradient

G 3 SVM-based (LAMO) color morphological gradient

G 4 Classic grayscale morphological gradient.

G 5 Chromatic gradient in $L^*a^*b^*$ color space.

G 6 Component-wise gradient in RGB color space.

G 7 Hue gradient in HSI color space.

G 8 Saturation weighting-based gradient in HLS color space.

G 9 Sum of weighted gradients in HSI color space.

G 10 Euclidean distance gradient in RGB color space.

Two applications: marker-controlled watershed and edge detection are studied.

Because G 2 and G 3 are supervised learning-based color morphological gradient operators, learning samples should be collected first. For LBMO, pixels of the objects to be segmented are defined as object pixels and assigned class label 1, pixels of other objects as well as background are defined as background pixels and assigned class label -1 . And then SVM-based (LBMO) color morphological gradient operator is established.

Similarly, for LAMO, boundary pixels of the objects to be segmented are defined as one class and assigned class label 1, all the other pixels are defined as another class and assigned class label -1 , and then SVM-based (LAMO) color morphological gradient operator is built.

4.3.1 Marker-controlled Watershed for Overlapping Cell Nuclei Separation

The watershed transform is a well known morphological operation-based image segmentation algorithm, especially for separating the overlapping objects. Usually, the watershed implements on the morphological gradient of the image to be segmented. Consequently, the segmentation result is significantly influenced by the morphological gradient used for watershed computation [27]. Overlapping cell nuclei separation is an important problem in histological image analysis. Effectively and accurately separating and grading cell nuclei in the color histological image is the key issue to develop automatic cell analysis system [26]. Among the developed cell nuclei segmentation methods, marker-controlled watershed-based segmentation algorithm is the most effective and popular one. It is usually implemented on the image that is created by imposing markers on the morphological gradient image. The accuracy of watershed separation depends on the quality of both markers and gradients. In the present study, we impose the same and manually detected markers onto different color morphological gradient images to compare the effectiveness of the gradients listed above in overlapping cell nuclei separation.

Fig.4.5(a1)~(j1) show the color morphological gradient results of $G1 \sim G10$ respectively. We can see that (a1) and (b1) demonstrate more clear gradient image than others. And Fig.4.5(a2)~(j2) show the watershed results by using the correspond-

ing gradient image. PCA-based and SVM-based color morphological gradients produce accurate watershed lines describing the cell nuclei boundary and separating the two overlapping nuclei. However, because the boundary region is difficult to define and the training data for LAMO learning is therefore hard to collect, the LAMO of SVM-based color morphological gradient does not perform as well as LBMO. Classic grayscale morphological gradient (G4) and Euclidean distance gradient (G10) also produce good nuclei boundaries, but the watershed lines which separate the two nuclei has small branches (as shown in Fig.4.5(d2)) and lies in inaccurate position (as shown in Fig.4.5(j2)), respectively. Besides subjective evaluation based on human visual inspec-

Figure 4.5: (a1)~(j1) Morphological gradients of G1 to G10, respectively. (a2)~(j2) Watershed results based on (a1)~(j1), respectively.

tion, quantitative assessment is also performed based on the following criterion [28]:

$$Q = \frac{1}{1000(N \times M)} \sqrt{R} \times \sum_{i=1}^R [\frac{e_i^2}{1 + \log A_i} + (\frac{R(A_i)}{A_i})^2] \tag{4.13}$$

where the segmented image is of size $N \times M$, R is number of regions in the segmented image, e_i and A_i are the average color error and area of the i th region, $R(A_i)$ is number of regions with area A . The smaller Q value means better segmentation result. For the cell segmentation problem, we revise Eq.(4.13) as:

$$Q = \frac{1}{10^6 \sum_{i=1}^R A_i} \sqrt{R} \times \sum_{i=1}^R [\frac{e_i^2}{1 + \log A_i}] \tag{4.14}$$

where 1000 and 10^6 in Eq.(4.13) and Eq.(4.14) are the coefficients to scale the Q value for convenient comparison. Table 4.1 also shows PCA-based and SVM-based color morphology gradients leading to better separation results than other color morphological gradients.

Image Index	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10
01	5.97	5.95	6.03	6.65	19.43	6.04	8.47	15.34	7.09	7.35
02	5.54	5.53	5.57	6.34	96.02	5.57	12.46	89.34	8.52	8.20
03	3.36	3.29	3.38	3.55	30.51	3.46	7.23	24.41	4.55	4.18
04	2.83	2.78	2.83	3.05	81.87	2.96	6.42	73.92	3.04	4.90
05	0.62	0.60	0.63	0.70	13.69	0.64	1.08	7.86	0.68	0.43
average	3.66	3.63	3.69	4.06	48.30	3.73	7.13	42.17	4.78	5.012

Table 4.1: Quantitative validation Q for $G1$ to $G10$ of five color images with size of 256×256 randomly selected from histological image database of bladder inverted papilloma.

4.3.2 Edge Detection

Edge detection is another the most important application of morphological gradient. Since the conventional morphological gradients just enhance the differences between pixels inside the structuring element, they highlight the boundaries of all the objects in the image synchronously, thus, may not perform well when only some type of the objects is to be segmented. In this experiment, we investigate the ability of the SVM-based color morphological gradient to detect the edge of some specific objects from the image containing multiple colored objects. Both synthetic and real color images are studied. In synthetic color image test, since there is no obvious and enough boundary regions for training sample collection, only SVM-based (LBMO) color morphological gradient is tested.

Synthetic Color Image Test

Fig.4.6(a) shows a synthetic color image containing two colored objects. Assume the purple colored objects is to be segmented and be considered as object class, both blue colored objects and background be considered as background class. Then, the SVM hyperplane separating the two classes and the axis $\mathbf{w}$ are learned. Fig.4.6(d)~(f) demonstrate the projections of CMGV on $R-G$, $G-B$ and $B-R$ plane respectively. We can see that pixels belonging to different object boundary have different moving directions. Fig.4.6(b) shows the SVM-based (LBMO) color morphological gradient, in which only the boundary of purple colored object is enhanced while the boundary of blue colored object is effectively suppressed, and Fig.4.6(c) shows the final edge detection result based on Fig.4.6(b).

Fig.4.7(a) shows another synthetic color image, in which the green object pixels are non-linear separable with both the yellow object pixels and pink background pixels in RGB

color space. It is created based on the well know banana set [29]. Color pixels are first mapped into higher Hilbert space by Gaussian function given in Eq.(4.12), and then similarly with linear case described above, axis $\mathbf{w}$ is learned. Fig.4.7(b) and (c) demonstrate the conventional grayscale morphological gradient and SVM-based (LBMO) color morphological gradient respectively. We can see that SVM-based (LBMO) color morphological gradient produces clear boundary of the green object and leads accurate edge detection result as shown in Fig.4.7(d).

Microvessel Detection

Microvessel detection is another important issue in automatic histological image analysis. Fig.4.8(a) and (b) show a histological color image of breast phyllodes tumor tissue stained by CD31 and a zoomed region containing microvessel. Microvessel detection here is a typical problem of separating specific object from the color image containing multiple objects, in which conventional morphological gradient can not produce expected results. SVM-based color morphological gradients are shown in Fig.4.8(e1) and (f1), we can see that microvessel boundary is much brighter than both background and cell tissues. Binary images shown in Fig.4.8(c2)~(f2), which are obtained directly from Fig.4.8(c1), (d1), (e1) and (f1) by Otsu's binarization method [30], also show that better contrast is presented in SVM-based color morphological gradients than that in both grayscale and Euclidean distance-based morphological gradients. Fig.4.8(c3)~(f3) demonstrate the microvessel detection results from the corresponding binary images using the same post processing and edge detection methods. Compared with the correct microvessel detection results shown in Fig.4.8(e3) and (f3), which are obtained from SVM-based color morphological gradients, several cell nuclei is mistakenly segmented as microvessel in Fig.4.8(c3) and (d3). Fig.4.9 shows the microvessel detection results of another histological color image, good results are still presented.

However, because of the same reason as described above, that the training set for LAMO learning is difficult to collect, we can see that in Fig.4.8(f1), boundary of cell tissue is not suppressed as well as Fig.4.8(e1).

Quantitative validations are also performed in microvessel detection problem. Since the main objective in the microvessel segmentation is to calculate the microvessel density, where the area of the segmented region is a significant factor in the process. We define ground truth (GT, real object pixels), true positives (TPs, object pixels found algorithmically), false positives (FPs, pixels isolated as object but not within in GT) and false negatives (FNs, GT pixels not found algorithmically). GT equals the summation of TPs and FNs. Thus, two measures: percent match (PM) and correspondence ratio (CR) [31], [32] are used in the study:

$$PM = \frac{TPs}{GT} \times 100\% \quad (4.15)$$

$$CR = \frac{TPs - 0.5FPs}{GT} \times 100\% \quad (4.16)$$

The higher the values of PM and CR, the better the algorithm performs. Table 4.2 demonstrate the PM and CR evaluation results of five histological color images of size 1392×1040 randomly selected from angiogenesis analysis database of breast phyllodes tumor. From both image and statistic results, we can see that microvessel detection based on SVM-based color morphological gradient produce satisfied and acceptable segmentations.

Image index	PM (LBMO)	CR (LBMO)	PM (LAMO)	CR (LAMO)
1	99.65%	99.65%	98.63%	97.71%
2	99.79%	99.78%	98.75%	97.70%
3	99.68%	99.53%	98.49%	97.65%
4	95.43%	95.43%	98.45%	97.50%
5	99.89%	99.85%	98.81%	97.83%
average	98.88%	98.84%	98.63%	97.68%

Table 4.2: Quantitative validation *PM* and *CR* of microvessel segmentation by using SVM-based color morphological gradients.

4.4 Summary of the Chapter

In this chapter, we first analyzed convention morphological gradient and introduced the concept of color morphological gradient vector (CMGV). Then we analyzed the color morphological gradient computation, which is actually a process of extracting useful information from CMGV. Based on these analysis, we proposed two new methods to calculate color morphological gradient from CMGV. In PCA-based color morphological gradient, feature selection technique principle component analysis (PCA) is used to extract the most information from CMGV. And in SVM-based color morphological gradient, supervised learning method support vector machine (SVM) is incorporated so that the color morphological gradient is calculated according to the given image segmentation target. In SVM-based color morphological gradient, both learning before morphological operation (LBMO) and learning after morphological operation (LAMO) are studied and analyzed. Compared with conventional morphological gradients, the two new color morphological gradient operators can effectively produce more accurate image segmentation and edge detection results. Experiment studies have demonstrated the good performance of our methods.

Figure 4.6: (a) Synthetic color image. (b) Projection of CMGV on axis $\mathbf{w}$ by LBMO SVM learning. (c) Edge detection result. (d)~(f) Color morphological gradient vectors (CMGV) of zoomed region of rectangle in (a), projected on $R-G$, $G-B$ and $B-R$ plane, respectively.

Figure 4.7: (a) Synthetic color image. (b) Grayscale morphological gradient. (c) SVM-based (LBMO) color morphological gradient. (d) Edge detection result based on (c).

Figure 4.8: (a) Histological color image of breast phyllodes tumor tissue stained by CD31. (b) Zoomed region of rectangle in (a), brown object is a microvessel to be detected. (c1) Grayscale morphological gradient. (d1) Color morphological gradient based on Euclidean distance in RGB color space. (e1) SVM-based (LBMO) color morphological gradient. (f1) SVM-based (LAMO) color morphological gradient. (c2)~(f2) Binary image obtained from (c1) to (f1), respectively. (c3)~(f3) Microvessel boundary detected based on (c2) to (f2), respectively.

Figure 4.9: (a) Histological color image of breast phyllodes tumor tissue stained by CD31. (b) Microvessel detection result by SVM-based (LBMO) color morphological gradient. (c) Microvessel detection result by SVM-based (LAMO) color morphological gradient.

Chapter 5

Impulsive Noise Color Image Filtering

In the previous chapters, fundamental color morphological operations and morphological color image segmentation are discussed. Actually, noise filtering is another important application of mathematical morphology. Morphological filter is a kind of morphological operation-based nonlinear filter. It is effective in impulsive noise filtering and has been extensively studied in the past two decades. In this chapter, a new multichannel filtering approach established on learning-based color morphological operations for impulsive noise removal in color image is presented. By using the color pixel ordering scheme learned from the pre-estimation of impulsive noise, contaminated pixels are ordered as maximum ones in erosion operation or minimum ones in dilation operation, respectively. This characteristic ensures that only uncontaminated color pixels are distributed in morphological operations, hence noisy pixels are suppressed. Reconstruction is followed to alleviate the blurring and bias effects of morphological operations and to preserve image features. The presented filtering approach greatly enhances the performance of morphological operation-based filters, especially

in color image highly corrupted by impulsive noise. Experiments and comparisons of the new filter with classical filters, such as basic vector median filter (VMF), basic vector directional filter (BVDF), NOPNCP filter etc, are performed to demonstrate the effectiveness of the proposed color image filtering algorithm.

5.1 Introduction

5.1.1 Nonlinear Color Image Filters

Impulsive noise is often encountered during image transmission process [33]. The presence of impulsive noise in an image will introduce errors in the information acquisitions and produce undesirable effects in the subsequent image processing steps [34]. In general, optimum reception schemes designed for Gaussian noise environments perform very poorly when impulsive noise is present [35]. As a result, filtering the noisy image to suppress the noise and meanwhile preserve the image features, such as edges, textures and other details, is a necessary and significant pre-processing step before any image processing applications.

Nowadays, color images are ubiquitous, which provide more information than monochrome image in pattern classification, computer vision and image understanding applications etc, and color image denoising has attracted increasing attentions [34]. The most convenient way to filtering color image is to directly extend conventional grayscale image filtering approaches [36], [37], [38], [39], [40] to each dimension of the color image. However, performing noise filtering in each channel separately might introduce new color artifacts in resultant images. Therefore, the major efforts in color image filtering are focused on vector-based methods.

The main class of vector-based nonlinear filters is rank-order filters. The classical vector median filter (VMF) [41] outputs the color vector contained within the operation window with a sum of distance to all observations which is minimum. Vector directional filter, in which the sum of angles to each of the other vectors is used to order color vectors in stead of distance measurement, is another kind of rank-order filters, including Basic vector directional filters (BVDF) [42], general directional filters (GVDF) [43] and directional-distance filters (DDF) [44]. Filters that based on order statistics theory were also extensively studied [45]. To achieve ideal noise suppression and image detail preservation, weighted color vectors in stead of original ones are usually used in pixel ordering [46], [47] [48]. Adaptive vector filters, including adaptive nearest neighbor multichannel filters [49], fuzzy member functions based filters [50], multichannel distance filters [51], statistically computed neighborhood based filters [52], etc., in which the output is a weighted combination of all color vectors within the operation window, were also formulated. The key issue in this kind of nonlinear filters is optimal weight determination. Recently, it was found that carrying out noise detection first in the filtering process could improve the performance of vector-based filters [53], and a number of filters based on switching filtering schemes [53], [54], [55], [56] were proposed.

5.1.2 Morphological Filters

Morphological filter is another kind of effective nonlinear filters [33], [34] established on the fundamental morphological operations of dilation, erosion, opening and closing [1], which is also a subclass of ranked-order filters. By parallel or sequential combining the operations, for example OC (opening followed by closing) and CO (closing followed by opening) filters, morphological filters are effective in removing impulsive noise [57] [57], [58], [59], [60]. Dilation and erosion operations are actually a max/min filtering process: dilation distributes the maximum pixel within the operation window

defined by structuring element, while erosion distributes the minimum pixel within the window. Since impulsive noise is usually quite larger (positive noise) or smaller (negative noise) than the uncontaminated pixels, during the max/min filtering, negative/positive impulsive noise can be removed. This essence determines that ordering noise pixels to be the maximum or minimum ones is the key issue in morphological filters design.

In this chapter, the extension of morphological filters to color images and the drawbacks are first analyzed. Then a new multichannel filtering approach for impulsive noise removal established on learning-based color morphological operations is presented. The color pixel ordering scheme used in the color morphological operations is learned from the pre-estimation of impulsive noise of the given noisy image. The noisy pixels are ordered as maximum ones in erosion and minimum in dilation. After each dilation or erosion operation, an image reconstruction step is followed, with the goal of preserving image features. In reconstruction, color pixels in the transformed image, which are estimated as uncontaminated ones, are replaced by their original values in the original image. By combining learning-based color morphological operations with reconstruction, the proposed morphological operation-based filter produces satisfied results.

5.2 Learning-based Color Morphological Filter

5.2.1 Difficulties in Extension of Color Morphological Filter

Because color pixels are represented by vectors, the scalar-based pixel ordering scheme used in grayscale morphology is no longer applicable to order noisy color pixels as the maximum or minimum ones within the operation window. Take zoomed part of color Baboon image corrupted by 30% impulsive *salt and pepper* noise model as an

example. As shown in Fig.5.1, noisy pixels distribute around uncontaminated ones in RGB color space. The most commonly used reduced ordering-based [7] and conditional ordering-based [10], [11] color morphology can not ensure that noisy pixels are always the largest or smallest ones within the operation window. Moreover, morphological filters are biased filters. For example, OC filter is strongly biased toward small output values, while CO filter is biased toward large output values [33]. Hence, morphological reconstruction [61], [58] is usually used to preserve image details and refine filtering results. However, it is difficult to extend the concepts of geodesic transform [1] and morphological reconstruction to color images.

On the other hand, the essence of the max/min filtering that makes morphological filters be effective in the impulsive noise removal is also the fatal drawback. That is positive/negative impulsive noise will be enhanced while negative/positive impulsive noise being suppressed [33]. This is the reason why morphological filter does not perform well in filtering highly corrupted images. Take one dimensional noisy signal shown in Fig.5.2 as an example, where red $\triangle$ and blue ∇ denote positive and negative impulsive noise respectively. If OC filter and flat linear structuring element with 3 neighbors are used to do filtering, conventional morphological OC filter can not remove the noise as shown in Fig.5.2. This is because, while one kind of impulsive noise is suppressed, the other kind is enhanced in each step. This problem can be alleviated if larger structuring elements are used. However, using large and fixed structuring element will cause distorting or removing significant image details. Hence, recent researches on morphological filters mainly focus on selecting structuring elements that are large enough to include noise features to be removed and adaptive enough to the image features to be preserved [59]. NOP and NCP filters are developed by searching optimal output within an adaptive neighborhood of the operated pixel [62], [60], [63].

GA techniques [64] and fuzzy methods [59] have also been introduced into the optimal structuring element selection.

Basically, the key issue in designing morphological filters for color image filtering is to order noisy pixels as the maximum/minimum ones within the operation window, so as to remove them through erosion/dilation operations respectively. Moreover, in order to produce good filtering results and enhance the performance of morphological filters in highly corrupted images, it is expected that few of noisy pixels are distributed during the operations. In other words, it is expected that the maximum or minimum pixel distributed respectively in dilation or erosion is the uncontaminated one. Therefore, the more noisy pixels are ordered as maximum in erosion operation or minimum in dilation operation, the better filtering results can be produced. On the other hand, in order to overcome the blurring and bias effects during the filtering process, reconstruction is needed after every morphological operation.

In the color image corrupted by impulsive noise, if uncontaminated pixels are considered as one class and noisy pixels as another class, distinguishing uncontaminated pixels and noisy pixels can be regarded as a two-class discrimination problem. As shown in the examples in Fig.5.1(d) and Fig.5.3(b), the two classes are usually nonlinear separable. A discrimination function, whose decision values are positive for uncontaminated pixels and negative for noisy pixels, can be learned to distinguish the two classes. And using such decision values as color pixel ordering measure in dilation operation will make sure that maximum pixels within the operation window are uncontaminated ones and hence noisy pixels will not be distributed. Similarly, a discrimination function, whose decision values are negative for uncontaminated and positive for noisy pixels, can be learned to distinguish the two classes. And using such decision values as color pixel

ordering measure in erosion operation will ensure the minimum pixels within the operation window are uncontaminated ones and therefore noisy pixels will not be distributed during erosion operation.

To implement the ordering scheme described above, we need to train a discriminative function first. In this study, nonlinear support vector machine (SVM) is selected. The decision values produced by the learned SVM classifiers, which are the distances between color pixels and the separating hyperplane, are a good choice as color pixel ordering measure. Note here that the learned SVM classifier has to produce positive decision values for uncontaminated pixels and negative decision values for noisy pixels in dilation operation, while produce negative decision values for uncontaminated pixels and positive decision values for noisy pixels in erosion operation. After each dilation or erosion transform, reconstruction is followed to replace the pixels that are estimated as uncontaminated ones with their pixel values in the original image.

5.2.2 Learning-based Color Morphological Filter

Pre-Estimation of Impulsive Noise

SVM is a popular pattern classification technique developed recently [16]. Given a training set of instance-label pairs (x_i, y_i) , $i = 1, \dots, l$, where $x_i \in R^n$ is data instance and $y \in \{1, -1\}^l$ is target value, the support vector machines [65] require the solution of the following optimization:

$$\begin{aligned} \min \quad & \frac{1}{2}\omega^T\omega + C\sum_{i=1}^l\xi_i \\ \text{subject to} \quad & y_i(\omega^T\phi(x_i) + b) > 1 - \xi_i \\ & \xi_i > 0 \end{aligned} \tag{5.1}$$

where C is a given smoothness constraint and ϕ is the function that maps x_i into higher dimensional space. And the decision value which predicts target value of data instance x in testing set is given as:

$$d = \omega^T \phi(x) + b \quad (5.2)$$

where d is actually the distance between separating hyperplane and the testing data instance. The use of SVM here is to provide a nonlinear transformation that maps noisy pixels to be extreme ones in the operation window and ensure the pixels distributed in operations are uncontaminated ones. Since SVM is a supervised learning-based technique, pre-estimation of impulsive noise is first performed to provide training set for SVM learning.

Let $\mathbf{S}_i$ denote pixels within a window W of size m (m is an odd number and bigger than structuring element) centered at the pixel $\mathbf{x}_i$ being operated, the mean vector and variance of $\mathbf{S}_i$ are calculated as:

$$\boldsymbol{\mu} = \frac{1}{m \times m} \sum_{k=1}^{m \times m} \mathbf{x}_k \quad \text{and} \quad \sigma^2 = \frac{1}{m \times m} \sum_{k=1}^{m \times m} \|\mathbf{x}_k - \boldsymbol{\mu}\|^2 \quad (5.3)$$

where $\mathbf{x}_k$ denotes pixel within W . Assume $\forall \mathbf{x}_j \in \mathbf{S}_i$ is the mean vector and its corresponding variance is:

$$\sigma_j^2 = \frac{1}{m \times m} \sum_{k=1}^{m \times m} \|\mathbf{x}_k - \mathbf{x}_j\|^2 \quad (5.4)$$

If a pixel is a noisy pixel, it must be far away from the majority of other pixels in $\mathbf{S}_i$. In other words, if a noisy pixel is used as mean vector of $\mathbf{S}_i$, the corresponding variance must be bigger than σ^2 . Fig.5.3 shows a 9×9 region of Baboon image corrupted by 10% impulsive *salt and pepper* noise. From Fig.5.3(d), we can see that the variance value

calculated by using impulsive noise as mean vector is quite larger than the variance calculated by using the mean vector of $\mathbf{S}_i$ ($\sigma = 0.15$), and also quite larger than the variance values calculated by using uncontaminated pixel as mean vector. Therefore, we have:

$$\mathbf{x}_j \text{ is a noisy pixel} \iff \sigma_j^2 > \alpha \sigma^2 \quad (5.5)$$

where α is the parameter determined by users. By using Eq.(5.3)~Eq.(5.5), pixels in a color image are classified into uncontaminated or noisy pixels. These pixels will be used as training set for the SVM-based color pixel ordering and image reconstruction in following steps.

5.2.3 SVM-based Color Pixel Ordering Schemes and Morphological Operations

If pixel $\mathbf{x}_i$ is to be dilated, class labels +1 and -1 are first assigned to uncontaminated and noisy pixels within window W centered at $\mathbf{x}_i$. A SVM classifier is then trained based on the pixels and their labels. The obtained SVM classifier is finally used to produce decision values $d_{dilation}$, which are the distances from each pixel to the separating hyperplane between uncontaminated and noisy pixels, for the pixels within the structuring element B . For two color vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ within B , they are ordered as:

$$\left. \begin{array}{ll} \mathbf{x}_1 > \mathbf{x}_2 & \text{if } d_{dilation}(\mathbf{x}_1) > d_{dilation}(\mathbf{x}_2) \\ \mathbf{x}_1 = \mathbf{x}_2 & \text{if } d_{dilation}(\mathbf{x}_1) = d_{dilation}(\mathbf{x}_2) \\ \mathbf{x}_1 < \mathbf{x}_2 & \text{if } d_{dilation}(\mathbf{x}_1) < d_{dilation}(\mathbf{x}_2) \end{array} \right\} \quad (5.6)$$

And the supremum ($\vee$) for the set of color vectors $\mathbf{S}_n = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ within B is the color vector whose $d_{dilation}$ is the maximum of all:

$$\begin{aligned}\vee \mathbf{S}_n &= \vee \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \\ &= \mathbf{x}_k : \{d_{dilation}(\mathbf{x}_k) = \max\{d_{dilation}(\mathbf{x}_1), d_{dilation}(\mathbf{x}_2), \dots, d_{dilation}(\mathbf{x}_n)\}\} \quad (5.7)\end{aligned}$$

Similarly, for erosion operation, within window W , label -1 and $+1$ are assigned to uncontaminated and noisy pixels, respectively. Then a SVM classifier is trained. The obtained SVM classifier is used on the pixels within B to produce decision values $d_{erosion}$, which are the distances from each pixel to the separating hyperplane between uncontaminated and noisy pixels. Finally two color pixels $\mathbf{x}_1$ and $\mathbf{x}_2$ within B are ordered as:

$$\left. \begin{aligned} \mathbf{x}_1 > \mathbf{x}_2 & \text{ if } d_{erosion}(\mathbf{x}_1) > d_{erosion}(\mathbf{x}_2) \\ \mathbf{x}_1 = \mathbf{x}_2 & \text{ if } d_{erosion}(\mathbf{x}_1) = d_{erosion}(\mathbf{x}_2) \\ \mathbf{x}_1 < \mathbf{x}_2 & \text{ if } d_{erosion}(\mathbf{x}_1) < d_{erosion}(\mathbf{x}_2) \end{aligned} \right\} \quad (5.8)$$

And the infimum ($\wedge$) for the set of color vectors $\mathbf{S}_n = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$ within B is the color pixel whose $d_{erosion}$ is the minimum of all:

$$\begin{aligned}\wedge \mathbf{S}_n &= \wedge \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\} \\ &= \mathbf{x}_k : \{d_{erosion}(\mathbf{x}_k) = \min\{d_{erosion}(\mathbf{x}_1), d_{erosion}(\mathbf{x}_2), \dots, d_{erosion}(\mathbf{x}_n)\}\} \quad (5.9)\end{aligned}$$

The values of $d_{dilation}$ and $d_{erosion}$ of pixels within the black square in Fig.5.3(a) produced by SVM classifiers, which are learned in the way as described above, are shown in Fig.5.3(d). We can see that both supremum and infimum are uncontaminated color pixels. This means noisy pixels within B will be suppressed, and will not be distributed neither in dilation nor erosion operation.

5.2.4 Reconstruction

Based on the impulsive noise pre-estimation of the whole image in the first step, if label $+1$ is assigned to all the uncontaminated pixels while -1 is assigned to all the noisy pixels, a SVM classifier that gives $d_i = +1$ to uncontaminated pixel and $d_i = -1$ to noisy pixel can be learned. Then, after each morphological operation the pixel is reconstructed by:

$$\mathbf{x}_i^e = \begin{cases} \mathbf{x}_i & \text{if } d_i = 1 \\ \mathbf{x}_i' & \text{if } d_i = -1 \end{cases} \quad (5.10)$$

where $\mathbf{x}_i^e$, $\mathbf{x}_i$, $\mathbf{x}_i'$ are the pixel in reconstructed, untransformed and transformed color images, respectively. d_i denotes class label of pixel $\mathbf{x}_i$ produced by the learned SVM classifier.

The reconstruction method defined in Eq.(5.10) is totally different from the conventional geodesic reconstruction of mathematical morphology. Eq.(5.10) means that in the morphological transformed image, the pixel which is estimated as an uncontaminated pixel by the SVM classifier is replaced by its pixel value in the untransformed image, while the pixel which is estimated as a noisy pixel by the SVM classifier is kept unchanged. The essence of this reconstruction method is to preserve image features by keeping uncontaminated pixels unchanged in the transformed image.

In every use of the two fundamental morphological operations, noise pre-estimation, learning-based pixel ordering, and image reconstruction are performed to produce the final transformed image. By parallel or sequential combining the operations, filters can be established to restore the color image corrupted by impulsive noise. In this paper, the classical morphological filter structure opening followed by closing is used to establish color image filter and named as SVMOC filter in the following sections.

5.3 Simulation and Discussion

In order to evaluate the performance of the proposed SVMOC filter, two types of impulsive noise models [34] are examined in the presented study.

N1 Impulsive *salt and pepper* noise: Let $\mathbf{x} = \{x_R, x_G, x_B\}$ denote the original pixel and let $\mathbf{x}^n$ denote the corrupted pixel, o_1, o_2, o_3 are independent values and equal to 0 or 255. Then the image pixels are distorted according to:

$$\mathbf{x}^n = \begin{cases} \{o_1, x_G, x_B\} & \text{with probability } p_1 p \\ \{x_R, o_2, x_B\} & \text{with probability } p_2 p \\ \{x_R, x_G, o_3\} & \text{with probability } p_3 p \\ \{o_1, o_2, o_3\} & \text{with probability } p_\Sigma p \end{cases} \quad (5.11)$$

where p is the impulsive noise degree of contamination, and $p_\Sigma = 1 - p_1 - p_2 - p_3$.

N2 Impulsive *uniform* noise: where the contamination components o_1, o_2, o_3 in Eq.(5.11) are random variables in the interval $[0, 255]$.

The experiments and comparisons in this paper are performed in Matlab programs. And both the two noise models are simulated based on the Matlab function:

$$\text{imnoise}(\text{Image}, \text{'salt \& pepper'}, D) \quad (5.12)$$

where $D = p$ is the noise density. Color images Lena, Baboon, Parrots, Pepper and Goldhill are examined in the present study. Besides SVMOC filter, some multichannel filters listed below are also simulated and compared.

Three quantitative measures: *peak signal-to-noise ratio* (PSNR), *mean absolute error* (MAE), and *normalized color difference* (NCD) are used to assess the performance of

AHMF	Adaptive hybrid multivariate filter [66]
AVDF	Adaptive vector directional filter [50]
BVDF	Basic vector directional filter [42]
DDF	Distance directional filter [44]
FVD	Adaptive fuzzy vector directional magnitude rational hybrid filter [67]
GVDF	Generalized vector directional filter_DW_αTM filter [43]
LCPF	Local information based filter [39]
MMF	Marginal median filter [68]
NOPNCP	Component-wise NOPNCP filter [60]
SCWVDF	Switching center weighted vector directional filter [54]
VMF	Basic vector median filter [41]
VSF	Vector sigma filter [47]

Table 5.1: List of filters used for comparisons with SVMOC filter.

SVMOC filter and the comparisons. They are defined as follows:

$$PSNR = 20 \log \left(\frac{255}{\sqrt{\frac{1}{NQ} \sum_{i=1}^N \sum_{q=1}^Q \left(x_{iq}^e - x_{iq}^o \right)^2}} \right) \quad (5.13)$$

$$MAE = \frac{\sum_{i=1}^N \sum_{q=1}^Q |x_{iq}^e - x_{iq}^o|}{NQ} \quad (5.14)$$

where N is the total number of pixels in a color image, x_{iq}^e , x_{iq}^o are the q th component of pixel $\mathbf{x}_i$ in filtered and uncontaminated color images, respectively. Here $Q = 3$ for

RGB color space.

$$NCD = \frac{\sum_{i=1}^N \Delta E_{Luv}}{\sum_{i=1}^N E_{Luv}} \quad (5.15)$$

where $\Delta E_{Luv} = [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{\frac{1}{2}}$ denotes the perceptual color error and $E_{Luv} = [(L^*)^2 + (u^*)^2 + (v^*)^2]^{\frac{1}{2}}$ is the norm or magnitude of the uncorrupted image pixel in the $L^*u^*v^*$ color space.

In the first step of pre-estimation of impulsive noise, the parameter α defined in Eq.(5.5) controls how many pixels are estimated as noisy pixel. Generally, small α makes more pixels be estimated as noisy pixel, while big α causes fewer pixels are estimated as noisy pixels. Fig.5.5 shows the quantitative evaluation MAE and NCD results of performing SVMOC filter on Baboon image by selecting different values of α ranging from 0.5 to 5. We can see that sub-optimal results can be achieved when α is selected between 1.5 to 2.5. We have also investigated SVMOC filter performances by selecting different α values on other images, including Lena, Parrots, Pepper etc. Similarly, sub-optimal results are obtained when choosing α between 1.5 to 2.5. In the present experiments and comparisons studies, $\alpha = 2.0$ is used in SVMOC filter.

Table 5.2 to Table 5.7 show the quantitative results of the filters listed above of filtering three color images corrupted by N1 and N2 impulsive noise of various densities. It is obviously that the performance of SVMOC filter are superior to that of most of other filters. In some situations, the component-wise filters produced the best PSNR or MAE results, however, the NCD value clearly shows that the component-wise filters do not preserve colors well. In order to evaluate the capability of image detail preservation of the filters, we also performed filtering on the noise free images, and the results are shown in Table 5.8. The good capability of image feature preservation of SVMOC filters is obvious.

Fig.5.6~Fig.5.10 allow visual inspection of the results of SVMOC filter and some classical color image filters, including MMF, VMF and BVDF. The capability of noise suppression and feature preservation is the key factor to evaluate the performance of color image filters. From the image results, we can see that SVMOC filter produced comparative results with the classical filters in the noise attenuation. SVMOC filter performs best in preserving the edge and structural features. For example, the edges in Fig.5.6(c), the textures around the baboon and the parrot eyes in Fig.5.7(c) and Fig.5.8(c) are well preserved. When the density of impulsive noise is high, such as 30%, from Fig.5.9 and Fig.5.10, it is obviously to see that SVMOC performs best both in noise attenuation and image feature preservation.

5.4 Summary of the Chapter

In this chapter, a new morphological operation-based filtering approach for impulsive noise suppression in color image is developed. The morphological operations used in the proposed color image filter are learning-based operations, in which color pixel ordering scheme is learned according to the pre-estimation of uncontaminated and noisy pixels. By employing image reconstruction following each morphological operation, features of the original image are effectively preserved. The proposed color morphological operation-based filter performs comparatively with other filters when the density of the impulsive noise of the images is low, and outperforms other filters when the images are highly corrupted.

Figure 5.1: (a) Zoomed part of Baboon color image. (b) Pixel distribution in RGB color space of uncontaminated image. (c) Image corrupted by 30% impulsive *salt and pepper* noise. (d) Pixel distribution in RGB color space of contaminated image. Blue * samples denote impulsive noise, and red o samples denote uncontaminated pixels.

Figure 5.2: Initial one dimensional noisy signal and the results of each step of OC filter with flat linear structuring element of 3 neighbors.

Figure 5.3: (a) A 9×9 region of Baboon image corrupted by 10% impulsive *salt and pepper* noise. (b) Uncontaminated and noisy pixels distribution in RGB color space. Blue * samples denote impulsive noise, and red o samples denote uncontaminated pixels. (c) Impulsive noise locations within the black square. (d) R, G, B values of the pixels within the black square, and variance value corresponding with each pixel, $d_{dilation}$, $d_{erosion}$ values produced by the SVM classifiers for dilation and erosion, respectively.

Figure 5.4: Color test images used in this paper. (a) Goldhill. (b) Lena. (c) Baboon. (d) Pepper. (e) Parrot.

Figure 5.5: Dependence of the quantitative measuring results MAE and NCD on the coefficient of α and percentage of the impulsive noise on Baboon image. (a) MAE, N1 (b) NCD, N1 (c) MAE, N2 (d) NCD, N2.

Figure 5.6: (a) Zoomed part of Pepper image. (b) Image corrupted by 10% N1 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output.

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy	19.34	4.93	0.119	19.16	4.98	0.093	18.87	5.32	0.094
AHMF	29.79	5.20	0.048	32.34	3.29	0.037	30.71	4.06	0.045
AVDF	28.62	6.96	0.059	31.59	4.44	0.048	29.71	5.56	0.057
BVDF	26.04	8.52	0.061	30.68	4.26	0.043	26.41	6.29	0.056
DDF	28.29	6.59	0.055	31.73	3.78	0.041	29.83	4.91	0.051
FVD	27.40	7.25	0.062	29.06	5.04	0.053	28.24	5.96	0.062
GVDF	27.56	7.52	0.062	31.56	4.08	0.043	29.18	5.48	0.054
LCPF	29.13	3.63	0.055	30.08	2.77	0.039	31.62	2.32	0.037
MMF	28.81	6.18	0.061	32.75	3.32	0.042	30.94	4.36	0.052
NOPNCP	30.04	2.31	0.038	28.65	1.39	0.025	29.02	1.65	0.028
SCWVDF	27.93	3.56	0.037	28.83	2.84	0.026	27.65	2.54	0.029
VMF	28.55	6.40	0.056	32.00	3.64	0.041	30.12	4.77	0.051
VSF	28.98	1.57	0.023	29.36	1.47	0.020	27.93	1.47	0.021
SVMOC	32.74	1.31	0.015	32.62	1.11	0.014	30.33	1.47	0.018

Table 5.2: Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 10\%$ N1 impulsive noise. SVMOC ($\alpha = 2.0$).

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy	16.61	9.20	0.218	16.51	8.67	0.161	16.78	8.71	0.151
AHMF	28.50	6.04	0.055	31.23	3.90	0.043	29.75	4.65	0.050
AVDF	28.14	7.26	0.067	30.74	4.89	0.054	29.08	5.94	0.062
BVDF	25.81	8.63	0.063	29.57	4.67	0.046	25.88	6.52	0.059
DDF	27.89	6.84	0.058	31.23	4.05	0.044	29.52	5.13	0.053
FVD	26.79	7.62	0.066	28.49	5.38	0.056	27.70	6.29	0.065
GVDF	26.08	8.41	0.082	30.9	4.39	0.046	28.77	5.71	0.057
LCPF	29.19	3.32	0.055	30.08	2.54	0.038	31.39	2.12	0.036
MMF	28.55	6.32	0.065	32.38	3.49	0.045	30.61	4.49	0.053
NOPNCP	26.63	3.12	0.062	25.06	2.33	0.045	26.25	2.40	0.043
SCWVDF	27.15	4.28	0.043	28.00	3.47	0.032	25.66	3.43	0.039
VMF	28.05	6.71	0.060	31.50	3.93	0.044	29.75	4.98	0.053
VSF	23.89	3.05	0.055	24.70	2.81	0.045	23.63	2.95	0.046
SVMOC	30.59	2.26	0.023	31.66	1.77	0.020	29.53	2.14	0.025

Table 5.3: Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 20\%$ N1 impulsive noise. SVMOC ($\alpha = 2.0$).

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy	15.23	12.70	0.298	15.19	12.53	0.230	15.24	12.33	0.210
AHMF	27.76	6.51	0.060	29.76	4.50	0.049	28.42	5.33	0.058
AVDF	27.42	7.69	0.077	29.38	5.5	0.063	27.96	6.58	0.072
BVDF	25.23	8.90	0.066	28.03	5.14	0.050	24.89	7.06	0.065
DDF	27.48	7.03	0.061	30.30	4.38	0.047	28.494	5.50	0.057
FVD	25.93	8.20	0.073	27.61	5.72	0.060	26.79	6.75	0.070
GVDF	27.19	7.54	0.064	29.75	4.71	0.050	27.60	6.12	0.063
LCPF	29.18	3.13	0.056	29.80	2.42	0.039	30.90	2.06	0.036
MMF	28.41	6.42	0.068	31.87	3.67	0.049	30.13	4.68	0.057
NOPNCP	24.70	4.00	0.091	22.61	3.49	0.070	23.28	3.59	0.068
SCWVDF	26.32	5.03	0.050	26.87	4.15	0.038	24.74	4.17	0.047
VMF	27.69	6.91	0.062	30.69	4.27	0.046	28.99	5.34	0.057
VSF	21.02	4.97	0.099	21.51	4.75	0.0816	21.09	4.70	0.075
SVMOC	30.15	2.93	0.025	30.05	2.52	0.028	28.20	2.92	0.034

Table 5.4: Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 30\%$ N1 impulsive noise. SVMOC ($\alpha = 2.0$).

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy	19.18	5.06	0.121	19.66	5.15	0.097	19.19	4.97	0.087
AHMF	29.72	5.25	0.049	32.25	3.34	0.038	30.92	3.98	0.043
AVDF	28.64	6.94	0.059	31.65	4.46	0.049	29.87	5.52	0.057
BVDF	26.09	8.50	0.061	30.57	4.37	0.043	27.26	6.13	0.055
DDF	28.26	6.61	0.056	31.70	3.82	0.041	30.05	4.86	0.051
FVD	27.47	7.24	0.062	29.04	5.04	0.053	28.47	5.86	0.061
GVDF	28.11	7.14	0.056	31.64	4.12	0.043	29.44	5.40	0.054
LCPF	29.07	3.67	0.055	30.06	2.78	0.039	31.68	2.32	0.037
MMF	28.81	6.17	0.061	32.72	3.35	0.043	31.08	4.33	0.051
NOPNCP	29.91	2.35	0.039	29.03	1.41	0.026	28.79	1.65	0.028
SCWVDF	27.99	3.55	0.037	28.66	2.90	0.026	27.94	2.51	0.029
VMF	28.47	6.43	0.057	31.91	3.70	0.041	30.33	4.73	0.051
VSF	29.04	1.56	0.023	28.93	1.54	0.021	28.04	1.48	0.021
SVMOC	33.49	1.27	0.012	33.63	1.10	0.012	30.61	1.55	0.017

Table 5.5: Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 10\%$ N2 impulsive noise. SVMOC ($\alpha = 2.0$).

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy	16.77	8.87	0.210	16.76	8.85	0.165	16.67	8.90	0.153
AHMF	28.78	5.87	0.053	31.01	3.97	0.044	29.64	4.68	0.051
AVDF	28.27	7.20	0.065	30.66	4.89	0.054	28.97	5.99	0.063
BVDF	25.86	8.67	0.063	29.98	4.56	0.045	26.14	6.56	0.059
DDF	27.94	6.82	0.057	31.35	4.00	0.043	29.37	5.19	0.053
FVD	26.95	7.57	0.065	28.58	5.32	0.055	27.67	6.29	0.065
GVDF	27.97	7.25	0.058	30.93	4.33	0.045	28.75	5.70	0.057
LCPF	29.23	3.32	0.055	30.11	2.53	0.038	31.36	2.11	0.036
MMF	28.65	6.29	0.064	32.32	3.49	0.045	30.57	4.52	0.054
NOPNCP	26.63	3.07	0.060	25.14	2.36	0.046	25.87	2.47	0.044
SCWVDF	27.16	4.27	0.043	27.86	3.48	0.032	25.99	3.36	0.038
VMF	28.19	6.62	0.058	31.47	3.91	0.043	29.58	5.07	0.054
VSF	24.27	2.94	0.053	24.56	2.86	0.046	24.01	2.85	0.044
SVMOC	31.25	2.19	0.019	31.00	1.88	0.023	30.30	2.04	0.023

Table 5.6: Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 20\%$ N2 impulsive noise. SVMOC ($\alpha = 2.0$).

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
Noisy	15.44	12.16	0.285	15.14	12.76	0.233	15.17	12.46	0.214
AHMF	28.13	6.32	0.058	29.69	4.49	0.049	28.64	5.241	0.05742
AVDF	27.67	7.57	0.074	29.41	5.53	0.064	27.98	6.549	0.071
BVDF	25.54	8.77	0.065	28.59	5.07	0.049	25.32	6.87	0.063
DDF	27.7	6.92	0.059	30.27	4.38	0.049	29.07	5.34	0.055
FVD	26.40	7.90	0.070	27.64	5.76	0.061	26.98	6.60	0.069
GVDF	27.48	7.43	0.062	29.96	4.71	0.049	27.85	6.06	0.062
LCPF	29.15	3.15	0.057	29.88	2.45	0.039	30.94	2.05	0.036
MMF	28.45	6.40	0.067	31.83	3.65	0.048	30.17	4.64	0.056
NOPNCP	24.41	3.97	0.088	22.13	3.76	0.074	22.92	3.71	0.070
SCWVDF	26.24	5.07	0.050	26.90	4.12	0.038	24.32	4.29	0.048
VMF	27.99	6.76	0.060	30.63	4.26	0.046	29.27	5.25	0.056
VSF	20.99	5.01	0.100	21.65	4.68	0.080	20.86	4.88	0.077
SVMOC	30.09	2.91	0.025	29.93	2.58	0.028	28.23	2.83	0.035

Table 5.7: Quantitative validation of filters performances on Goldhill, Lena and Pepper images corrupted by $p = 30\%$ N2 impulsive noise. SVMOC ($\alpha = 2.0$).

Filters	Goldhill			Lena			Pepper		
	PSNR	MAE	NCD	PSNR	MAE	NCD	PSNR	MAE	NCD
AHMF	30.43	4.21	0.046	31.53	3.28	0.035	32.94	2.78	0.035
AVDF	27.79	7.00	0.065	28.84	5.59	0.055	30.11	4.90	0.056
BVDF	25.33	8.15	0.069	28.31	5.30	0.050	28.66	5.12	0.055
DDF	27.61	6.23	0.062	28.87	4.85	0.049	30.33	4.20	0.049
FVD	26.94	6.82	0.071	27.51	5.79	0.058	28.74	5.19	0.061
GVDF	27.28	7.02	0.065	28.84	5.21	0.051	29.96	4.72	0.054
LCPF	28.70	4.41	0.054	29.81	3.42	0.042	31.48	2.91	0.042
MMF	28.16	5.83	0.064	29.25	4.53	0.050	30.87	3.94	0.051
NOPNCP	34.81	1.47	0.019	36.87	0.98	0.013	38.07	0.84	0.013
SCWVDF	29.24	2.52	0.028	29.85	2.069	0.019	31.53	1.43	0.018
VMF	27.98	5.99	0.063	29.04	4.68	0.049	30.57	4.08	0.050
VSF	34.84	0.85	0.009	35.12	0.84	0.008	37.04	0.56	0.007
SVMOC	313.07	0	0	313.07	0	0	313.07	0	0

Table 5.8: Quantitative validation of filters performances on noise free images. SVMOC ($\alpha = 2.0$).

Figure 5.7: (a) Zoomed part of Baboon image. (b) Image corrupted by 10% N2 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output.

Figure 5.8: (a) Zoomed part of Parrots image. (b) Image corrupted by 20% N1 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output.

Figure 5.9: (a) Zoomed part of Lena image. (b) Image corrupted by 30% N1 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output.

Figure 5.10: (a) Zoomed parts of Goldhill image. (b) Image corrupted by 30% N2 noise. (c) SVMOC filter output. $\alpha = 2.0$. (d) MMF output. (e) VMF output. (f) BVDF output.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

The key issue to extend color morphological operations and applications is to find a proper color vector ordering scheme, in which various colors could be ordered based on a unified ordering framework. With suitable and effective color pixel ordering scheme, color morphological operations can be used in various applications similar as classic grayscale and binary morphology.

Probability frame work for color pixel ordering is first established in this thesis. It defines the ranking scheme according to the given image processing and analysis targets. Based on the probability ordering scheme, all of the objects of interest are warranted to be enlarged during dilation and shrunk during erosion. This critical property ensures some famous morphological operators are still effective in color image processing area, such as morphological gradient. Based on probability based color pixel ordering scheme, several classic morphology applications, including internal/external edge detectors, hole filling and etc. are implemented in this thesis.

The incorporation of probability frame work into color pixel ordering makes color morphological operations become learning-based approaches, which means morphological image processing algorithms may become effective in some area where only supervised learning-based image processing methods can be used. Conventionally, morphological gradient is effective in object detection and boundary extraction. However, it is neighborhood based operation, which does not consider contextual information and extracts all the objects in image synchronously. By incorporating supervised learning techniques, gradient information is extracted directly from color morphological gradient vectors (CMGV), according to object detection targets. The improvement makes morphological gradient can extract specific object from color image containing multiple objects.

Impulsive noise filtering is another most important application of morphological operations. The essence of morphological filters makes it can not perform well when dealing with highly corrupted image. By using learning-based color morphological operations, and incorporating other learning techniques to pre-estimate noise and reconstruct filtered image, the new developed color morphological filters perform superior than conventional ones.

6.2 Recommendations for Future Research

Color Watershed

Watershed is one of the most important mathematical morphology algorithms, which is powerful in image segmentation, especially in overlapping objects separation. It is initially introduced and defined based on geometrical concept by water raising and merging: starting from the markers, the water will progressively flood the catchment

basins of the image, boundaries are raised at the places where the waters coming from two different markers would merge [1].

However, color images can not be considered as landscape as grayscale images, which makes water raising and merging can not be directly extended to color images. The most of color watershed algorithms [69], [70], [71] developed recently actually first detect markers marks every object to be segmented, and then region growing is applied by considering all the components in color images. Or transform color image to grayscale one, and then grayscale watershed is applied.

When extending grayscale watershed to color image segmentation, a similar water raising approach can be used. For grayscale images, water raises linearly by a single direction on the gray value axis. Since color pixels are usually presented by 3D color vectors, the process of water raising must be performed in 3D color space, and raises nonlinearly as spherical wave. Therefore, a measurement to control the way by which the water raises in 3D color space must be defined first, the starting point **S** and ending point **E** for water raising are marked in 3D color space based on the measurement. Then, water can be raised from **S** to **E** like spherical wave in the 3D color space.

Such water raising in 3D color space also incorporates supervised learning techniques, in which raising starting and ending points, and water raising direction are all learned according to image segmentation target. Since color raising in 3D color space has unlimited directions to choose, combining learning process into watershed algorithm makes color water raising be adaptive to various situations and be effective when dealing with irregular colored objects.

Other Applications

A well developed set of color morphological operations needs to implement its operations into all of mathematical morphology applications. Besides those presented in this thesis, the following applications have to be developed.

- **Geodesic Transformations.** All morphological transformations discussed in this thesis operate on one input image with structuring elements. Geodesic transformations is another set of morphological operations, which is the basement of morphological reconstruction operations. It is to consider two input images [1]: morphological operation is first applied to the first image and it is then forced to remain either greater or lower than the second image. To extend and develop color geodesic transformations are important and may help extend color watershed algorithms.
- **Hit-or-miss operations and Skeletons.** Hit-or-miss transformations involve structuring elements composed of two sets: the first has to fit the object under study and the second has to miss it. The most famous applications of hit-or-miss transformations is thickenings and thinnings. Skeleton is derived from thinnings and mostly used in shape representation and object analysis. To extend and define color hit-or-miss transformations may help color image analysis and measurement.
- **Top-hats and Granulometries.** Top-hats and granulometries are two useful image content analysis tools. To extend them to color image area, the color of various objects become one of the content of the image. Therefore, adding color information into top-hats and granulometries is the key issue in color image extension and may help perfect existing color morphological operations as well.

Author's Publications

Journals

1. Hao Zhou, and K. Z. Mao , “Color Morphological Gradient Vector and Applications in Color Image Segmentation,” submitted to *Pattern Recognition*, 2006.
2. Hao Zhou, and K. Z. Mao, “An Impulsive Noise Color Image Filter Using Learning-based Color Morphological Operations,” accepted by *Digital Signal Processing*, 2007.
3. K. Z. Mao, Hao Zhou, and Puay-Hoon Tan, “Supervised Learning and Probability Criterion for Pixel Ordering in Color Morphology,” submitted to *Image and Vision Computing*, 2005.

Conferences

1. Hao Zhou, and K. Z. Mao, “Adaptive Successive Erosion-based Cell Image segmentation for p53 Immunohistochemistry in Bladder Inverted Papilloma,” *The 27th Annual International Conference of the IEEE EMBS*, Shanghai, China, September 2005.

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107

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