Measurement of polarization effects in optical fibers and components

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Measurement of Polarization Effects in Optical Fibers and Components

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SUMMARY

With the introduction of long-distance all-optical networks, optical fiber communication systems are becoming more and more complex and incorporate a wide variety of devices and components in ever larger numbers. Thus, accurate and thorough characterization of optical devices and components is becoming increasingly important.

The optical fiber system studied here is assumed to be a linear, time-invariant and non-imaging system. There are mainly two types of properties of this kind of system: polarization related and non-polarization related. The polarization related properties include polarization dependent loss or gain (PDL/G), birefringence which is the sources of polarization mode dispersion (PMD), and depolarization. The non-polarization related properties or effects include chromatic dispersion (CD), attenuation, scattering, reflection, etc. Since, the PMD and PDL can cause pulse spreading or signal distortion, increase random fluctuations of the system signal-to-noise ratio (SNR) and lead to significant system performance degradation, the measurement of the PMD and PDL/G becomes very important. Thus, we mainly focus here on the PDL/G and PMD measurements.

This thesis covers three major aspects: basic theories for the polarization properties of an optical fiber system, PDL/G measurement, and PMD measurement.

The first major aspect focuses on the fundamental concepts. To begin with, the system classification is described. Next, a brief idea of the polarization of light and its expressions are given. Then, the description tools for optical transmission systems, Jones matrix and Mueller matrix, are introduced. For optical systems containing different effects, the transmission matrix also has different forms and properties. Based on these properties, the measurement methods for the transmission matrices can be optimized. At last, we propose a convenient method for the measurement of non-depolarization Mueller matrix and derive the criteria for optimum measurement accuracy.

Based on the knowledge introduced in the first part, the PDL/G measurement can be carried out. A review of the techniques used for PDL/G measurement is presented. We notice that all the existing measurement techniques have their advantages and shortcomings. From the point

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of view of system properties and categorization, we suggest two new measurement techniques with improved measurement efficiency and give guidelines for choosing a suitable measurement method for different conditions.

The third major aspect of this thesis relates to measurement of PMD vector. A review of PMD measurement methods is given first. According to the system polarization properties, these methods can be classified into two categories: methods for the system with and without PDL/G. In each category, the methods can further be divided into two kinds: methods based on the differential equations and methods based on the analysis of the system transmission matrices. The methods under different categories and kinds have different restrictions. Thus, we propose new methods that overcome the disadvantages of existing methods.

A computer-controlled measurement system has been developed and has been used to carry out all the above mentioned PDL/G and PMD measurement methods.

The report concludes by summarizing the major achievements and recommendations for future research work.

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ACRONYMS

CD	Chromatic Dispersion
DAQ	Data Acquisition
DAS	Differential Attenuation Slope
DGD	Differential Group Delay
DGDD	Differential Group Delay Dispersion
DFB	Distributed Feedback
DOF	Degree of Freedom
DOP	Degree of Polarization
DUT	Device Under Test
FIMMM	Flexible Input Mueller Matrix Method
FUT	Fiber under Test
GMMM	Generalized Mueller Matrix Method
JME	Jones Matrix Eigenanalysis Method
LD	Laser Diode
MMM	Mueller Matrix Method
PC	Polarization Controller
PDG	Polarization Dependent Gain
PDL	Polarization Dependent Loss
PMD	Polarization Mode Dispersion
PMF	Polarization Maintaining Fiber
PSP	Principal State of Polarization
PST	Poincaré Sphere Technique
RTM	Round Trip Method
SMF	Single-mode Fiber
SNR	Signal-to-noise Ratio

SOP	State of Polarization
TSM	Two-states Method
ULM	Unpolarized Light Method
VGMMM	Virtual Generalized Mueller Matrix Method

Chapter 1 INTRODUCTION

1.1 Motivation

With the introduction of long-distance all-optical networks, optical fiber communication systems are becoming more complex and incorporate a wide variety of devices in ever larger numbers. Thus, accurate and efficient characterization of optical devices is becoming increasingly important.

Determination of polarization dependent loss or gain (PDL/G) and polarization mode dispersion (PMD) in optical devices and optical fiber systems is very important due to their adverse effect on both analog and digital optical signals.

PDL may be caused by various inline optical components, such as isolators, couplers, filters, and switches etc., while PDG is an effect that is related to preferential gain between signal and noise in optical amplifiers. Owing to PDL and PDG, a polarized signal may be attenuated or amplified differently from unpolarized noise. This induces random fluctuations of the signal-to-noise ratio (SNR) and thus leads to a significant system performance degradation and causes performance penalty in long-distance systems [1, 2].

Polarization mode dispersion arises mainly from the birefringence of the optical fiber. Due to the residual anisotropy produced by core ellipticity or noncircularly symmetric stresses, the two orthogonally polarized HE_{11} modes propagate with different propagation constants and give rise to the input pulses being split and differentially delayed. Thus, the pulse spreading of the output data causes the transmission performance degradation through intersymbol interference for long distance high bit rate digital transmission systems [3, 4]. For an analog transmission system with single-mode fiber, the PMD causes signal distortion [4] and harmonics which may result in crosstalk between channels [5].

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For a long-distance all-optical network, the optical components may possess significant PDL/G in contrast with standard single-mode optical fiber [6]. Combined effect of PDL/G and PMD gives rise to anomalous pulse spreading and deteriorates the bit error rate through several peculiar effects, such as anomalous DGD, group velocity, and pulse spread [5, 7].

Because of the importance of the PDL/G and PMD, it is important to study the measurement methods of these polarization effects especially for systems which are complex and incorporate a wide variety of devices and components. Currently, there are several methods for measuring the PDL/G and PMD of an optical fiber system or components; however, they all have certain advantages as well as shortcomings. In this thesis, the existing methods for PDL/G and PMD measurement and their features are reviewed in some detail in chapters 3 and 4, respectively and new measurement methods are suggested that can remove some of these shortcomings.

1.2 Objective

The objective of this research work is to study the polarization related phenomena and investigate the measurement methods of PDL/G and PMD in optical fiber systems. So, there are two main objectives we aim to achieve.

The first is to investigate simple methodologies for extracting polarization and system transmission properties of the fiber systems and components. This is the basis of the PDL/G and PMD measurements.

The second objective is to examine the advantages and shortcomings of the existing methods and thus, develop fast and efficient PDL/G and PMD measurement methods based on the system properties.

1.3 Major contribution of the thesis

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1. This thesis gives detailed analysis of the properties of the system transmission Mueller matrix. For a system that has all the three polarization effects, depolarization, birefringence and PDL/G, the Mueller matrix has a size of 4×4 with 16 degrees of freedom. Four linearly independent input states of polarization (SOPs) are required for measuring the Mueller matrix. When the system has only birefringence, the degrees of freedom become 3 and two linearly independent input states of polarization are enough. However, when the system has both birefringence and PDL/G, there has been no clear understanding about the requirement of the input SOPs. Based on the Lorentz transformation property of non-depolarization Mueller matrix, we prove here that three arbitrary inputs are sufficient to measure Mueller matrix and no input parameters are redundant. A convenient way to calculate Mueller matrix with these 3 pairs of input and output SOPs is also presented.
2. Through the review of the PDL/G measurement methods, we found that it is possible to reduce the number of input SOPs to perform this measurement. Thus, two new PDL measurement methods, two-states method and unpolarized light method, are proposed, which require only two and one input SOPs respectively and improve the measurement efficiency. A comparison between different methods is also given which will help to choose a suitable measurement method under different conditions.
3. Following the same objective of improving the measurement efficiency, a new round trip method is developed for the PMD measurement when the system has no PDL/G. In the presence of PDL/G, the PMD vector becomes complex. The signal to noise ratio (SNR) for the existing differential methods is relatively low and the methods based on the system transmission matrix either cannot fully retrieve the complex PMD vector or have relatively large equipment error induced measurement uncertainty. Thus a new method, flexible input Mueller matrix method, is proposed. The proposed method can achieve better SNR and measurement efficiency.
4. All the PDL/G and PMD measurement methods need to synchronize and control tunable laser source, computer-controlled polarization controller and polarimeters. We have developed a measurement system which incorporates all the above PDL/G

and PMD measurement methods. It is a powerful and convenient tool and can be used for system measurement and also for optimizing the PDL/G and PMD value during the manufacturing of optical fiber components.

1.4 Thesis organization

This thesis is organized into five chapters. Chapter 1 presents the motivation and objectives of this work, major contributions as well as thesis organization. Chapter 2 gives the background information on the polarization of light, the representation of an optical system, the three basic polarization effects and corresponding system matrix measurements. A convenient non-depolarization Mueller matrix measurement method is presented with the requirement of input SOPs for achieving optimum accuracy. Chapter 3 discusses the PDL measurement. The definition of PDL is introduced and the existing measurement methods are reviewed with respect to their advantages and shortcomings. Based on the type of the optical system under measurement, we propose two new methods. All the methods are compared with each other and the suggestions for choosing a suitable measurement method are given. Chapter 4 explores the PMD vector measurement methods. The methods for PMD measurement are divided into two categories: methods for system without PDL and with PDL. Each part begins with a literature review. Through the analysis of the disadvantages of the existing methods, new methods are introduced. Finally, Chapter 5 gives the conclusions and future work.

Chapter 2 BASIC THEORIES ABOUT THE POLARIZATION PROPERTIES OF AN OPTICAL FIBER SYSTEM

The optical system studied here is not an optical imaging system involving lenses, prisms, etc., but an optical fiber transmission system which is linear, time-invariant and non-imaging. Here, linear means that, compared to the input, no new frequencies are generated. Therefore, the concern is the state of polarization (SOP) and the intensity of light as it passes through the optical system rather than directions of the rays in the light beam.

Before introducing the measurement methods for PDL/G and PMD in the subsequent chapters, this chapter gives the background information on the polarization of light, the representation of an optical system, the three basic polarization effects and corresponding system matrix measurements. A convenient non-depolarization Mueller matrix measurement method is also presented with the requirement of input SOPs for achieving optimum accuracy.

2.1 Representations of polarized light

Light wave is a transverse electromagnetic wave with electric and magnetic fields orthogonal to each other. The polarization of a light wave signal is defined through electric field. The electric field of a signal can be resolved into two orthogonal vector components. The state of polarization of a signal can be determined by the relative amplitude and phase of the E-field vector components. Consider a plane and monochromatic light wave traveling along the z-axis in free space so that E-field vector has only transverse components, i.e. no $\hat{z}$ component. The electric field vector is

$$\mathbf{E}(x, y, t) = \hat{x}E_x + \hat{y}E_y \quad (2.1)$$

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where $E_x = E_{0x}e^{i(\omega t - kz + \phi_x)}$ and $E_y = E_{0y}e^{i(\omega t - kz + \phi_y)}$ are the two orthogonal components of the E-field along $\hat{x}$ and $\hat{y}$ direction, ϕ_x and ϕ_y are the initial phases for these two component, ω is the angular frequency, and k is the propagation constant. For the light wave propagating in a single mode optical fiber, because of the weakly guiding condition, the z component of the electric field is very small and Eq. (2.1) is still applicable. After canceling the term $\omega t - kz$ in the E_x and E_y components, we can get the trace of the endpoint of the electric vector

$$\left(\frac{1}{E_{0x}}\right)^2 E_x^2 + \left(\frac{1}{E_{0y}}\right)^2 E_y^2 - 2\frac{E_x}{E_{0x}}\frac{E_y}{E_{0y}}\cos\delta = \sin^2\delta \quad (2.2)$$

where $\delta = \phi_x - \phi_y$ is the relative phase difference between the E_x and E_y components. The above equation represents an ellipse. When $\delta = m\pi$ ($m=0, \pm 1, \pm 2, \dots$), Eq. (2.2) becomes a linear equation, the corresponding polarization state is called linear polarization. When $E_x = E_y$ and $\delta = m\pi/2$ ($m = \pm 1, \pm 3, \pm 5, \dots$), the trace is a circle. This polarization state is called circular polarization. For other conditions, the polarization state is elliptical polarization.

2.1.1 Jones vector

In year 1941, R. Clark Jones invented a representation of polarized light [8]. The technique he developed has the advantages of being applicable to coherent beams and in terms of the electric vector itself. Writing Eq. (2.1) in column matrix form, the Jones vector is

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{bmatrix} \quad (2.3)$$

Multiplication of the Jones vector by any complex constant does not modify the state of polarization, thus it is often convenient to work with normalized Jones vectors.

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The advantage of the Jones vector is that it depicts the electric field explicitly. One can know the phase and amplitude of the electric vector directly. However, Jones vector representation is limited to the description of fully polarized light, i.e., light with a degree of polarization (defined on next page) of about 100%.

2.1.2 Stokes parameters

The modern representation of polarized light actually had its origins in 1852 in the work of Sir George Gabriel Stokes. It was shown that the most general beam of partially polarized light could be characterized by four quantities, the so called “Stokes parameters” [9]. The polarization state of a light wave (either natural or totally or partially polarized) can be described in terms of these quantities. A polarization state is expressed using $\mathbf{S} = (s_0 \ s_1 \ s_2 \ s_3)^T$ and the four Stokes parameters can be related to Eq. (2.1) using the following expressions [9]

$$\begin{aligned} s_0 &= \langle E_x^2 \rangle + \langle E_y^2 \rangle \\ s_1 &= \langle E_x^2 \rangle - \langle E_y^2 \rangle \\ s_2 &= \langle 2E_x E_y \cos \delta \rangle \\ s_3 &= \langle 2E_x E_y \sin \delta \rangle \end{aligned} \quad (2.4)$$

Here, superscript ‘T’ means the matrix transpose and the angular brackets ‘ $\langle \rangle$ ’ represent the time-average. s_0 represents the intensity of light beam, s_1 reflects a tendency for the linear polarization to resemble either a horizontal or a vertical state. Similarly, s_2 implies a tendency for the light to resemble a linear polarization in the direction of $+45^\circ$ or -45° . In the same way, s_3 reveals a tendency of the beam toward right- or left- circular polarization. Notice that a completely polarized light follows the relation

$$s_0^2 = s_1^2 + s_2^2 + s_3^2 \quad (2.5)$$

Different from the Jones vector, Stokes parameters can represent unpolarized light and partial polarized light. Thus, a new concept, degree of polarization (DOP), comes in.

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DOP is a scalar value between zero and one and can be expressed in terms of Stokes parameters

$$\text{DOP} = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0} \quad (2.6)$$

For fully polarized light, $\text{DOP} = 1$; for unpolarized light, $s_1^2 + s_2^2 + s_3^2 = 0$ and $\text{DOP} = 0$.

When calculating the superposition of two beams, if they are superposed incoherently, any one of the Stokes parameters of the resultant will be the sum of the corresponding parameters. For example, if a unit intensity horizontal linear polarization $(1 \ 1 \ 0 \ 0)^T$ is added to an incoherent right handed circular polarization of intensity 2, $(2 \ 0 \ 0 \ 2)^T$, the composite wave has parameters $(3 \ 1 \ 0 \ 2)^T$. It is an elliptical polarization of intensity 3. On the other hand, if the two beams are added coherently, the Stokes parameters of the combination is still calculable from the Stokes parameters of the individual beams, but is not obtained by simple addition of the parameters. Thus, the set of Stokes parameters is not a vector which is different from Jones vector. The name “Stokes vector” is just a representation which is widely used in physics.

The advantage of the Stokes parameters is that it can be measured conveniently because optical detectors can easily measure optical power but not electric field.

2.1.3 Poincaré Sphere

Every possible polarization state can be represented by a unit sphere. The unit sphere is called Poincaré Sphere which was introduced by H. Poincaré in 1892 [9]. The unit sphere is made by normalizing the three directional Stokes components $s_{1,2,3}$ by s_0 . The linear polarization states are located on the equator ($s_3 = 0$) and the right- and left- hand circular polarizations are on the north- ($s_3 = 1$) and south- ($s_3 = -1$) poles respectively as shown in Fig. 2.1. Fully polarized polarization states are located on the sphere while partial polarization states are located inside the sphere and unpolarized light is located at

the origin. The interaction of a polarized beam with an optical polarization element results in a rotation of the polarization state on the sphere.

Fig. 2.1 Poincaré sphere

2.2 Representation of an optical system

The action of an optical device (for example, a waveplate) is to transform the polarization of a beam from one state to another (with or without power change). The transformation can be expressed by a square matrix calculus. There are two kinds of calculus, namely Jones matrix and Mueller matrix.

2.2.1 Jones matrix

The Jones matrix was invented together with Jones vector by R. C. Jones [8]. Suppose that we have a polarized incident beam represented by its Jones vector $\mathbf{E}_i$, which passes

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through an optical element, emerging as a new vector $\mathbf{E}_t$. This process can be described mathematically using a 2×2 complex matrix $\mathbf{J}$ as follows

$$\mathbf{E}_t = \mathbf{J}\mathbf{E}_i \quad (2.7)$$

where

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \quad (2.8)$$

The simplest way for the determination of the Jones matrix needs three specific input states of polarization [10]: linear polarization parallel to the x-, y- axis and parallel to the bisector of the angle between the positive x- and y- axis. From the three resulting Jones vectors h , v and q , three optical-power-independent complex ratios can be formed: $k_1 = h_x / h_y$, $k_2 = v_x / v_y$, and $k_3 = q_x / q_y$. A fourth ratio $k_4 = (k_3 - k_2) / (k_1 - k_3)$ is then found. Thus the Jones matrix is given with a complex constant β [10]

$$\mathbf{J} = \beta \begin{bmatrix} k_1 k_2 & k_2 \\ k_4 & 1 \end{bmatrix} \quad (2.9)$$

If the beam passes through a series of optical elements represented by matrices $\mathbf{J}_1, \mathbf{J}_2, \dots, \mathbf{J}_n$, then

$$\mathbf{E}_t = \mathbf{J}_n \cdots \mathbf{J}_2 \mathbf{J}_1 \mathbf{E}_i \quad (2.10)$$

2.2.2 Mueller matrix

Mueller calculus is a matrix method for manipulating Stokes parameters. It was developed in 1943 by Hans Mueller [11]. The Mueller matrices $\mathbf{M}$ are applied in similar ways as the Jones matrices

$$\mathbf{S}_{Output} = \mathbf{M}\mathbf{S}_{Input}, \quad \mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \quad (2.11)$$

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where $\mathbf{S}_{Input}$ and $\mathbf{S}_{Output}$ are the input and output Stokes parameters defined in section 2.1.2. If the optical element has only birefringence, $\mathbf{M}$ can be simplified to a 3×3 matrix. Accordingly, the Stokes parameters become $\mathbf{S} = (s_1 \ s_2 \ s_3)^T$ because there is no optical intensity change after propagation of light.

There are some differences between Mueller matrix and Jones matrix [11]. Light which is unpolarized or partially polarized needs to be treated using Mueller matrix, while fully polarized light can be treated with either Mueller matrix or the Jones matrix. However, Jones matrix can give the absolute phase difference between the input and output light while Mueller matrix can not. This is because Jones matrix works with the electric-field of light and Mueller matrix works with the intensity of light. More detailed difference will be discussed in the next section.

For non-depolarizing system, Jones matrix can be converted to Mueller matrix as shown in Appendix 1. The conversion from Mueller matrix to Jones matrix is not unique.

2.3 Measurement of the Stokes parameters and Mueller matrix

2.3.1 Measurement of the Stokes parameters

Since all optical detectors are square law detectors, they can only measure optical intensity but not the electrical field of the light. Thus, the Stokes parameters are immediately useful. The classical measurement of the Stokes parameters [12] is actually based on Eq. (2.4). The optical intensity $I(\theta, \phi)$ is measured after the light passing through a retarder and a polarizer. Here θ represents the angle between the polarizer transmission axis and the x-axis of the laboratory frame of reference, (x, y, z) , and ϕ is the retardance of the retarder. Then we have [12]

$$I(\theta, \phi) = \frac{1}{2} [s_0 + s_1 \cos 2\theta + s_2 \cos \phi \sin 2\theta + s_3 \sin \phi \sin 2\theta] \quad (2.12)$$

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The first three Stokes parameters are measured by removing the retarder ($\phi = 0^\circ$) and rotating the transmission axis of the polarizer to angles $\theta = 0^\circ$, $+45^\circ$, and $+90^\circ$, respectively. s_3 is measured by reinserting a quarter waveplate ($\phi = 90^\circ$) into the optical path and setting the transmission axis of the polarizer to $\theta = 45^\circ$. The respective intensities measured are then found to be

$$I(0^\circ, 0^\circ) = \frac{1}{2}[s_0 + s_1] \quad (2.13a)$$

$$I(45^\circ, 0^\circ) = \frac{1}{2}[s_0 + s_2] \quad (2.13b)$$

$$I(90^\circ, 0^\circ) = \frac{1}{2}[s_0 - s_1] \quad (2.13c)$$

$$I(45^\circ, 90^\circ) = \frac{1}{2}[s_0 + s_3] \quad (2.13d)$$

Thus, the Stokes parameters can be solved as

$$s_0 = I(0^\circ, 0^\circ) + I(90^\circ, 0^\circ) \quad (2.14a)$$

$$s_1 = I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) \quad (2.14b)$$

$$s_2 = 2I(45^\circ, 0^\circ) - I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) \quad (2.14c)$$

$$s_3 = 2I(45^\circ, 90^\circ) - I(0^\circ, 0^\circ) - I(90^\circ, 0^\circ) \quad (2.14d)$$

2.3.2 Measurement of Mueller matrix

The optical system we are studying is assumed to be a linear, non-imaging optical transmission system. There are three polarization effects for such a system: depolarization, birefringence, and polarization dependent loss or gain [13].

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A depolarizing system converts completely polarized light into partially polarized light. This kind of systems can only be described by a 4×4 Mueller matrix. A non-depolarizing system converts completely polarized light into completely polarized light. This kind of systems can be described by both Mueller matrices and Jones matrices. Most of the optical fiber systems are non-depolarizing systems.

Physically, a non-depolarizing system alters the polarization state of light by changing the amplitudes and/or the phases of the electric-field vector of the light. Thus, the non-depolarizing system can be divided into two basic types: retarders and diattenuators. Their corresponding polarization effects are birefringence and PDL. The retarder changes only the phases of the electric-field vector and can be described by a unitary Jones matrix. The diattenuator changes only the amplitudes of the electric-field vector and is described by a Hermitian Jones matrix [8, 14]. Accordingly, the retarder and diattenuator also have Mueller matrix expressions. Since retarder only changes the SOP of the light, the retarder Mueller matrix can be reduced to a size of 3×3 .

From the above description, an optical system can be expressed using a Mueller matrix. Any Mueller matrix can be decomposed into three factors using polar decomposition: a diattenuator, followed by a retarder, then followed by a depolarizer. If the optical system has all the three polarization effects, the Mueller matrix has $4 \times 4 = 16$ degrees of freedom (DOFs). For an optical system with only birefringence and PDL/G, there are 7 DOFs, including three rotations (corresponding to birefringence), three boost actions (corresponding to PDL/G) and one length compression/expansion (corresponding to attenuation/gain for unpolarized light) [13].

2.3.2.1 Measurement of Mueller matrix for depolarizing systems

The DOFs of the Mueller matrix for a system with all the 3 polarization effects are 16 and the DOFs of a set of Stokes parameters are 4 (s_0 to s_3), thus 4 input SOPs are needed to determine the matrix. Usually, $I_0(1 \ 1 \ 0 \ 0)^T$, $I_0(1 \ -1 \ 0 \ 0)^T$, $I_0(1 \ 0 \ 1 \ 0)^T$ and $I_0(1 \ 0 \ 0 \ 1)^T$ (I_0 is the input optical intensity) are used [15]. If

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the corresponding output SOPs are measured as $I_1(1 \ a_1 \ a_2 \ a_3)^T$, $I_2(1 \ b_1 \ b_2 \ b_3)^T$, $I_3(1 \ c_1 \ c_2 \ c_3)^T$ and $I_4(1 \ d_1 \ d_2 \ d_3)^T$, based on Eq. (2.11), the Mueller matrix can be calculated using the following formula

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2}\left(\frac{I_1}{I_0} + \frac{I_2}{I_0}\right) & \frac{1}{2}\left(\frac{I_1}{I_0} - \frac{I_2}{I_0}\right) & \frac{I_3}{I_0} - \frac{1}{2}\left(\frac{I_1}{I_0} + \frac{I_2}{I_0}\right) & \frac{I_4}{I_0} - \frac{1}{2}\left(\frac{I_1}{I_0} + \frac{I_2}{I_0}\right) \\ \frac{1}{2}\left(\frac{a_1 I_1}{I_0} + \frac{b_1 I_2}{I_0}\right) & \frac{1}{2}\left(\frac{a_1 I_1}{I_0} - \frac{b_1 I_2}{I_0}\right) & \frac{c_1 I_3}{I_0} - \frac{1}{2}\left(\frac{a_1 I_1}{I_0} + \frac{b_1 I_2}{I_0}\right) & \frac{d_1 I_4}{I_0} - \frac{1}{2}\left(\frac{a_1 I_1}{I_0} + \frac{b_1 I_2}{I_0}\right) \\ \frac{1}{2}\left(\frac{a_2 I_1}{I_0} + \frac{b_2 I_2}{I_0}\right) & \frac{1}{2}\left(\frac{a_2 I_1}{I_0} - \frac{b_2 I_2}{I_0}\right) & \frac{c_2 I_3}{I_0} - \frac{1}{2}\left(\frac{a_2 I_1}{I_0} + \frac{b_2 I_2}{I_0}\right) & \frac{d_2 I_4}{I_0} - \frac{1}{2}\left(\frac{a_2 I_1}{I_0} + \frac{b_2 I_2}{I_0}\right) \\ \frac{1}{2}\left(\frac{a_3 I_1}{I_0} + \frac{b_3 I_2}{I_0}\right) & \frac{1}{2}\left(\frac{a_3 I_1}{I_0} - \frac{b_3 I_2}{I_0}\right) & \frac{c_3 I_3}{I_0} - \frac{1}{2}\left(\frac{a_3 I_1}{I_0} + \frac{b_3 I_2}{I_0}\right) & \frac{d_3 I_4}{I_0} - \frac{1}{2}\left(\frac{a_3 I_1}{I_0} + \frac{b_3 I_2}{I_0}\right) \end{pmatrix} \quad (2.15)$$

2.3.2.2 Measurement of Mueller matrix for non-depolarizing systems

i. For an optical system with only birefringence and PDL/G

It has been known that three well-defined input polarization states are enough to measure Mueller matrix in this case. But it is still questionable if three arbitrary input polarization states are sufficient. The following part will strictly prove that three arbitrary inputs are sufficient to measure Mueller matrix and no input parameters are redundant. A convenient way to calculate Mueller matrix with these 3 pairs of input and output SOPs is also presented [16].

Before discussing the measurement of the Mueller matrix, we need to derive some relations between the input and output SOPs. It has been pointed out theoretically and demonstrated experimentally that Mueller matrix of such a system satisfies the Lorentz transformation, i.e., Lorentz Group $O(3,1)$ [17, 18]. It is therefore convenient to discuss the polarization issues in the 4-dimensional (4D) Minkowski space. Much like those in the Special Relativity [19], the 4 Stokes parameters are rewritten as a 4D complex

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vector $\vec{S} = (is_0 \ s_1 \ s_2 \ s_3)^T$, where $i = \sqrt{-1}$ and superscript T denotes matrix transposition. Complex number representation is used as a complex matrix is more appropriate to describe a Lorentz transformation. Then the traditional 4×4 Mueller matrix $\mathbf{M}$ is rewritten as

$$\tilde{\mathbf{M}} = \begin{pmatrix} m_{11} & im_{12} & im_{13} & im_{14} \\ -im_{21} & m_{22} & m_{23} & m_{24} \\ -im_{31} & m_{32} & m_{33} & m_{34} \\ -im_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \quad (2.16)$$

It can be noted that $|\tilde{\mathbf{M}}| = |\mathbf{M}|$, where $|\cdot|$ denotes the determinant of a square matrix or the magnitude of a 3D vector. Based on this expression, the new Mueller matrix $\tilde{\mathbf{M}}$, as a direct property of Lorentz Group, should meet [18, 20]

$$\tilde{\mathbf{M}}^T \tilde{\mathbf{M}} = \sqrt{|\tilde{\mathbf{M}}|} \mathbf{I} \quad (2.17)$$

Here $\mathbf{I}$ is an 4×4 identity matrix.

From Eq. (2.17), two pairs of inputs and outputs $\vec{S}_{\text{out}} = \tilde{\mathbf{M}} \vec{S}_{\text{in}}$ and $\vec{T}_{\text{out}} = \tilde{\mathbf{M}} \vec{T}_{\text{in}}$ should fulfill

$$\vec{S}_{\text{out}} \cdot \vec{T}_{\text{out}} = \sqrt{|\tilde{\mathbf{M}}|} \vec{S}_{\text{in}} \cdot \vec{T}_{\text{in}} \quad (2.18)$$

A simple case of Eq.(2.18) is [20]

$$\vec{S}_{\text{out}} \cdot \vec{S}_{\text{out}} = \sqrt{|\tilde{\mathbf{M}}|} \vec{S}_{\text{in}} \cdot \vec{S}_{\text{in}} \quad (2.19)$$

In an optical system where only birefringence exists, the scalar triple product of three 3D vectors (3D volume) is conservative, which describes the handedness (chirality) of the system. In analogy, a factor that denotes the handedness of an optical system with both birefringence and PDL/G should be found out. Similar to the definition of scalar triple

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product, the scalar quadruple product (4D volume) of four 4D Stokes vectors $\vec{S}, \vec{T}, \vec{U}, \vec{V}$ is defined as

$$Vol_{4D} = \begin{vmatrix} i s_0 & s_1 & s_2 & s_3 \\ i t_0 & t_1 & t_2 & t_3 \\ i u_0 & u_1 & u_2 & u_3 \\ i v_0 & v_1 & v_2 & v_3 \end{vmatrix} \quad (2.20)$$

With Eq. (2.17), it can be demonstrated that

$$Vol_{4D}^{out} = |\tilde{\mathbf{M}}| Vol_{4D}^{in} \quad (2.21)$$

Based on the above relations, we can strictly prove that three arbitrary inputs are sufficient to measure non-depolarizing Mueller matrix and no input parameters are redundant.

A 4D Stokes vector should have 4 independent parameters, thus 3 inputs generate 12 DOFs in general. But some DOFs will be cancelled out due to symmetric property of the optical system. Assume that there are three inputs $\vec{S}_{in}, \vec{T}_{in}, \vec{U}_{in}$ and their corresponding outputs $\vec{S}_{out}, \vec{T}_{out}, \vec{U}_{out}$. Based on Eq. (2.18), three equations are obtained as

$$\begin{cases} \vec{S}_{out} \cdot \vec{T}_{out} = \sqrt{|\tilde{\mathbf{M}}|} \vec{S}_{in} \cdot \vec{T}_{in} \\ \vec{S}_{out} \cdot \vec{U}_{out} = \sqrt{|\tilde{\mathbf{M}}|} \vec{S}_{in} \cdot \vec{U}_{in} \\ \vec{U}_{out} \cdot \vec{T}_{out} = \sqrt{|\tilde{\mathbf{M}}|} \vec{U}_{in} \cdot \vec{T}_{in} \end{cases} \quad (2.22)$$

Then three Mueller-matrix-independent equations in turn are deduced from Eq. (2.22)

$$\begin{cases} (\vec{S}_{in} \cdot \vec{U}_{in}) (\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in}) (\vec{S}_{out} \cdot \vec{U}_{out}) \\ (\vec{U}_{in} \cdot \vec{T}_{in}) (\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in}) (\vec{U}_{out} \cdot \vec{T}_{out}) \\ (\vec{U}_{in} \cdot \vec{T}_{in}) (\vec{S}_{out} \cdot \vec{U}_{out}) = (\vec{S}_{in} \cdot \vec{U}_{in}) (\vec{U}_{out} \cdot \vec{T}_{out}) \end{cases} \quad (2.23)$$

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In Eq. (2.23), the third equation can be derived by using the other two equations; hence only two equations are independent. This implies that there are only 10 DOFs left now. In the remaining part of this section, we will consider four combinations of three input polarization states.

1) Three completely polarized inputs

It has been known that the output is also completely polarized for an optical system with both birefringence and PDL/G, subjected to a completely polarized input [21]. This means that 3 DOFs will not take effect among three inputs ($s_0^2 = s_1^2 + s_2^2 + s_3^2$ for each input); therefore only 7 DOFs exist for three completely polarized inputs. As has been mentioned previously, the Mueller matrix also has 7 DOFs. Hence the Mueller matrix can be determined explicitly with three completely polarized inputs and there are no redundant independent input parameters.

2) Two completely polarized and one partially polarized (including unpolarized) inputs

For two completely polarized inputs, 2 DOFs are cancelled out and hence only 8 DOFs are left for this combination of inputs. Assuming $\vec{S}_{in}$ is the partially polarized input, from Eq. (2.19) and Eq. (2.22), two Mueller-matrix-independent equations are formed

$$\begin{cases} (\vec{S}_{in} \cdot \vec{S}_{in}) (\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in}) (\vec{S}_{out} \cdot \vec{S}_{out}) \\ (\vec{S}_{in} \cdot \vec{S}_{in}) (\vec{S}_{out} \cdot \vec{U}_{out}) = (\vec{S}_{in} \cdot \vec{U}_{in}) (\vec{S}_{out} \cdot \vec{S}_{out}) \end{cases} \quad (2.24)$$

Only one equation is independent in Eq. (2.24), which may be confirmed by comparing Eq. (2.24) and the first equation of Eq. (2.23). Effectively this reduces the number of DOF to 7.

3) One completely polarized and two partially polarized (including unpolarized) inputs

For one completely polarized input, 1 DOF is cancelled out and then 9 DOFs are left for three inputs. Assuming $\vec{S}_{in}$ and $\vec{T}_{in}$ are the partially polarized inputs, also from Eq. (2.19) and Eq. (2.22), there are four Mueller-matrix-independent equations

$$\begin{cases}
(\vec{S}_{in} \cdot \vec{S}_{in})(\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in})(\vec{S}_{out} \cdot \vec{S}_{out}) \\
(\vec{S}_{in} \cdot \vec{S}_{in})(\vec{S}_{out} \cdot \vec{U}_{out}) = (\vec{S}_{in} \cdot \vec{U}_{in})(\vec{S}_{out} \cdot \vec{S}_{out}) \\
(\vec{T}_{in} \cdot \vec{T}_{in})(\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in})(\vec{T}_{out} \cdot \vec{T}_{out}) \\
(\vec{T}_{in} \cdot \vec{T}_{in})(\vec{T}_{out} \cdot \vec{U}_{out}) = (\vec{T}_{in} \cdot \vec{U}_{in})(\vec{T}_{out} \cdot \vec{T}_{out})
\end{cases} \quad (2.25)$$

Also by comparing with Eq. (2.23), only two equations in Eq. (2.25) are independent. That is, Eq. (2.25) cancels out two DOFs. Again, 7 DOFs are left for this combination of inputs.

4) Three partially polarized (including unpolarized) inputs

Six Mueller-matrix-independent equations can be obtained from Eq. (2.19) and Eq. (2.22), in this case

$$\begin{cases}
(\vec{S}_{in} \cdot \vec{S}_{in})(\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in})(\vec{S}_{out} \cdot \vec{S}_{out}) \\
(\vec{S}_{in} \cdot \vec{S}_{in})(\vec{S}_{out} \cdot \vec{U}_{out}) = (\vec{S}_{in} \cdot \vec{U}_{in})(\vec{S}_{out} \cdot \vec{S}_{out}) \\
(\vec{T}_{in} \cdot \vec{T}_{in})(\vec{S}_{out} \cdot \vec{T}_{out}) = (\vec{S}_{in} \cdot \vec{T}_{in})(\vec{T}_{out} \cdot \vec{T}_{out}) \\
(\vec{T}_{in} \cdot \vec{T}_{in})(\vec{T}_{out} \cdot \vec{U}_{out}) = (\vec{T}_{in} \cdot \vec{U}_{in})(\vec{T}_{out} \cdot \vec{T}_{out}) \\
(\vec{U}_{in} \cdot \vec{U}_{in})(\vec{U}_{out} \cdot \vec{S}_{out}) = (\vec{U}_{in} \cdot \vec{S}_{in})(\vec{U}_{out} \cdot \vec{U}_{out}) \\
(\vec{U}_{in} \cdot \vec{U}_{in})(\vec{U}_{out} \cdot \vec{T}_{out}) = (\vec{U}_{in} \cdot \vec{T}_{in})(\vec{U}_{out} \cdot \vec{U}_{out})
\end{cases} \quad (2.26)$$

By comparing with Eq. (2.23), three equations in Eq. (2.26) are found to be independent. So Eq. (2.26) cancels out three DOFs, which also results in 7 DOFs.

From the above analysis, one conclusion can be drawn that three arbitrary input polarization states are sufficient but not redundant for the determination of Mueller matrix for a system with only birefringence and PDL/G. In fact, it can also be demonstrated that two input polarization states only generate 6 independent input parameters in such an optical system by following a similar analysis. Hence we can not use only two input polarization states to determine the Mueller matrix in an optical fiber link with PDL/G.

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For the determination of Mueller matrix, at least three inputs have to be used, and seven independent equations can be obtained from three input-output pairs. Apart from these, nine independent bilinear equations have been proposed to express relations of Mueller matrix elements based on Mueller matrix properties [20]. Then an equation group consisting of 16 independent equations can be solved to determine all elements of the Mueller matrix in principle. However, such a method will lead to complicated calculations since some equations are nonlinear [20]. To better tackle this problem, three arbitrary inputs $\vec{S}_{in}, \vec{T}_{in}, \vec{U}_{in}$ are launched and their corresponding outputs $\vec{S}_{out}, \vec{T}_{out}, \vec{U}_{out}$ are measured in the first step. Then we can calculate (not measure) any other output $\vec{V}_{out}$ corresponding to an arbitrary assumed input $\vec{V}_{in}$ according to the following steps.

By using Eq. (2.18) and two measured input-output pairs, $\sqrt{|\tilde{\mathbf{M}}|}$ is obtained. From Eq. (2.18) and (2.21), four linear independent equations are obtained as

$$\begin{cases} \vec{S}_{out} \cdot \vec{V}_{out} = \sqrt{|\tilde{\mathbf{M}}|} \vec{S}_{in} \cdot \vec{V}_{in} \\ \vec{T}_{out} \cdot \vec{V}_{out} = \sqrt{|\tilde{\mathbf{M}}|} \vec{T}_{in} \cdot \vec{V}_{in} \\ \vec{U}_{out} \cdot \vec{V}_{out} = \sqrt{|\tilde{\mathbf{M}}|} \vec{U}_{in} \cdot \vec{V}_{in} \\ Vol_{4D}^{out} = |\tilde{\mathbf{M}}| Vol_{4D}^{in} \end{cases} \quad (2.27)$$

By solving Eq. (2.27), $\vec{V}_{out}$ can be calculated. The condition that Eq. (2.27) can be solvable is

$$\begin{vmatrix} i s_{out 0} & s_{out 1} & s_{out 2} & s_{out 3} \\ i t_{out 0} & t_{out 1} & t_{out 2} & t_{out 3} \\ i u_{out 0} & u_{out 1} & u_{out 2} & u_{out 3} \\ A_{out 0} & A_{out 1} & A_{out 2} & A_{out 3} \end{vmatrix} = A_{out 0}^2 + A_{out 1}^2 + A_{out 2}^2 + A_{out 3}^2 \neq 0 \quad (2.28)$$

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Here $A_{\text{out}0} = -\begin{vmatrix} S_{\text{out}1} & S_{\text{out}2} & S_{\text{out}3} \\ t_{\text{out}1} & t_{\text{out}2} & t_{\text{out}3} \\ u_{\text{out}1} & u_{\text{out}2} & u_{\text{out}3} \end{vmatrix}$, $A_{\text{out}1} = \begin{vmatrix} iS_{\text{out}0} & S_{\text{out}2} & S_{\text{out}3} \\ it_{\text{out}0} & t_{\text{out}2} & t_{\text{out}3} \\ iu_{\text{out}0} & u_{\text{out}2} & u_{\text{out}3} \end{vmatrix}$, $A_{\text{out}2} = -\begin{vmatrix} iS_{\text{out}0} & S_{\text{out}1} & S_{\text{out}3} \\ it_{\text{out}0} & t_{\text{out}1} & t_{\text{out}3} \\ iu_{\text{out}0} & u_{\text{out}1} & u_{\text{out}3} \end{vmatrix}$

and $A_{\text{out}3} = \begin{vmatrix} iS_{\text{out}0} & S_{\text{out}1} & S_{\text{out}2} \\ it_{\text{out}0} & t_{\text{out}1} & t_{\text{out}2} \\ iu_{\text{out}0} & u_{\text{out}1} & u_{\text{out}2} \end{vmatrix}$. This is the direct requirement regarding the output

polarization states. As for the input polarization states, we prove in Appendix 2 that: 1) Eq. (2.27) can be solved when the three 3-D input Stokes vectors are not linearly superposed in Stokes space if all inputs are completely polarized; 2) The optimum accuracy of Eq. (2.27) will be achieved when the three 3-D input Stokes vectors are coplanar, the angles between these vectors are 120 degrees and all inputs are completely polarized.

The above discussion means that if three input-output pairs are known by measurement, any other input-output pair can be calculated. Because all equations in Eq. (2.27) are linear, the calculation will be simple. Previously, people have used four pre-determined inputs to measure Mueller matrix, which are $I_0(1 \ 1 \ 0 \ 0)^T$, $I_0(1 \ -1 \ 0 \ 0)^T$, $I_0(1 \ 0 \ 1 \ 0)^T$ and $I_0(1 \ 0 \ 0 \ 1)^T$ (I_0 is the input optical power) [15]. In our method, firstly, three outputs $\vec{S}_{\text{out}}$, $\vec{T}_{\text{out}}$, $\vec{U}_{\text{out}}$ corresponding to three inputs $\vec{S}_{\text{in}}$, $\vec{T}_{\text{in}}$, $\vec{U}_{\text{in}}$ are measured. Then four outputs $I_1(1 \ a_1 \ a_2 \ a_3)^T$, $I_2(1 \ b_1 \ b_2 \ b_3)^T$, $I_3(1 \ c_1 \ c_2 \ c_3)^T$ and $I_4(1 \ d_1 \ d_2 \ d_3)^T$ corresponding to the above-mentioned four pre-determined inputs are calculated using Eq. (2.27). Finally the Mueller matrix under investigation can be achieved using Eq. (2.15).

This is a general approach, which can employ three arbitrary inputs in principle. But its accuracy is affected by various errors induced by equipment and fiber system under test. The optimum accuracy depends on the choice of the three inputs in real measurements. The accuracy of Eq. (2.15) completely relies on the accuracy of four calculated outputs corresponding to four pre-determined inputs; and finally in turn totally depends on the choice of $\vec{S}_{\text{in}}$, $\vec{T}_{\text{in}}$, $\vec{U}_{\text{in}}$ in Eq. (2.27). We have derived in Appendix 2 the conditions for

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solving Eq. (2.27) and also for achieving optimum accuracy. It should be noticed that the conditions are also the requirements for the overall Mueller matrix method to be valid.

ii. In an optical system with pure birefringence

Under this condition, Mueller matrix has only 3 DOFs, which represent three rotations in Stokes space. Optical power and degree of polarization (DOP) are not changed for a light passing through such a system. Further, it has been noticed that the dot product of two 3-dimensional (3D) Stokes vectors and the scalar triple product (representing 3D volume) of three 3D Stokes vectors are conservative in a pure birefringent optical system. The conservation of the dot product as well as 3D volume has been used to measure Mueller matrices of single-mode optical fibers employing two input polarization states as given by R. M. Jopson [22]. Mueller matrix for a birefringent system reduces to 3×3 . If the three input SOPs are $\vec{S}_1 = [1 \ 0 \ 0]^T$, $\vec{S}_2 = [0 \ 1 \ 0]^T$, $\vec{S}_3 = [0 \ 0 \ 1]^T$ and the corresponding outputs are $\vec{T}_1 = [t_{11} \ t_{12} \ t_{13}]^T$, $\vec{T}_2 = [t_{21} \ t_{22} \ t_{23}]^T$, $\vec{T}_3 = [t_{31} \ t_{32} \ t_{33}]^T$, we have the relation

$$[\vec{T}_1 \ \vec{T}_2 \ \vec{T}_3] = \mathbf{M} [\vec{S}_1 \ \vec{S}_2 \ \vec{S}_3] \quad (2.29)$$

Thus we have

$$\mathbf{M} = [\vec{T}_1 \ \vec{T}_2 \ \vec{T}_3] = \begin{bmatrix} t_{11} & t_{21} & t_{31} \\ t_{12} & t_{22} & t_{32} \\ t_{13} & t_{23} & t_{33} \end{bmatrix} \quad (2.30)$$

In the measurement, two linear SOPs $\vec{S}_1$ and $\vec{S}_a$ are launched and the corresponding output are $\vec{T}_1$ and $\vec{T}_a$, respectively. $\vec{S}_a$ may not be $\vec{S}_2$, but optimum accuracy would be achieved for $\vec{S}_a = \vec{S}_2$. Using $\vec{S}_1$ and $\vec{S}_a$, we can get $\vec{S}_3$ and $\vec{S}_2$ using

$$\begin{aligned} \vec{S}_3 &= k\vec{S}_1 \times \vec{S}_a \\ \vec{S}_2 &= \vec{S}_3 \times \vec{S}_1 \end{aligned} \quad (2.31)$$

where the constant k is chosen to assure $|\bar{S}_3| = 1$.

Since the dot product of two 3D Stokes vectors is conservative, $\bar{T}_3$ and $\bar{T}_2$ can also be calculated using the same procedure as shown in Eq. (2.31). Finally, $\mathbf{M}$ is calculated using Eq. (2.30).

2.4 Conclusion

In this chapter, the system under study is introduced, which is linear, time-invariant and non-imaging optical fiber system. There are three polarization effects in this kind of systems, depolarization, birefringence, and polarization dependent loss or gain. For a system with different polarization effects, its transmission matrix presents different symmetry with different degree of freedoms. Thus, the requirement on the number of input SOPs for measuring the transmission matrix is also different. Based on the fact that non-depolarizing system satisfies the Lorentz transformation, we present a convenient system transmission Mueller matrix measurement method with the requirement of input SOPs for achieving optimum accuracy. With this background knowledge, we can proceed to PDL/G and PMD measurements which are described in the following chapters.

Chapter 3 POLARIZATION DEPENDENT TRANSMISSION MEASUREMENT

3.1 Literature review

Polarization dependent loss or gain (PDL/G) in optical devices is very important in an optical fiber communication system due to its adverse effect on both analogue and digital optical signals [23, 24]. The combined effects of PDL/PDG and polarization mode dispersion (PMD) give rise to anomalous pulse spreading and deteriorate the bit error rate [25].

Polarization dependent transmission (gain or loss) is a measure of the peak-to-peak difference in transmission of an optical component or system with respect to all possible states of polarization. It is the ratio of the maximum and the minimum transmission

$$\text{PDL(or PDG)} \equiv 10 \log_{10} \left(\frac{T_{\max}}{T_{\min}} \right) \quad (3.1)$$

where $T_{\max}$ and $T_{\min}$ are the transmission extrema.

In any optical fiber system, there exist two orthogonal input polarization states which correspond to maximum and minimum transmission [13]. Here, the orthogonality is in Cartesian coordinates, while in Stokes space, the orthogonality is defined as their inner product being -1, e.g., $(s_1 \ s_2 \ s_3)^T$ and $(-s_1 \ -s_2 \ -s_3)^T$. The angle between these two polarization states is 180° on the Poincaré sphere. To calculate the system PDL/G, there are two cases to be considered. Firstly, if the two input SOPs are aligned with the two transmission extrema states, the output SOPs will still be orthogonal. The length of output Stokes vectors (optical intensity) will be shorter than that of the input SOPs. In this case, PDL/G is simply the intensity difference (in dB) between the two output SOPs. This situation is shown in Figs. 3.1 (a) and (b). Secondly, if the two input SOPs are

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orthogonal but not aligned with the transmission extrema states, the output SOPs will not be orthogonal. This situation is shown in Figs. 3.2 (a) and (b). As will be shown later, one can calculate the system PDL/G using the Stokes parameters of the output light.

Fig. 3.1 Poincaré sphere plot when input SOPs are aligned with the transmission extrema states; (a) Two orthogonal input SOPs and (b) Two corresponding orthogonal output SOPs.

Fig. 3.2 Poincaré sphere plot when input SOPs are not aligned with the transmission extrema states; (a) Two orthogonal input SOPs; (b) Two corresponding output SOPs.

Chapter 3: Polarization Dependent Transmission Measurement

There exist a few techniques to measure PDL/G. These include deterministic all-states, pseudorandom all-states, and deterministic fixed-states techniques. The first two techniques sample a large number of SOPs over the entire polarization-state space in a repeatable or pseudorandom way, respectively [26]. The deterministic fixed-states techniques employ two [27, 28], three [29] or four [26, 30] well-defined input SOPs to derive PDL/G. For convenience, we call these fixed-states methods as two-states method, Jones matrix method and Mueller matrix method, respectively. Hereafter, only PDL will be mentioned because all the measurement methods are applicable for both PDL and PDG.

These measurement methods are not independent. They are all based on the system transmission property which can be described by Mueller matrix. Thus, for different transmission systems, different types of PDL measurement methods have been proposed. The following passages will introduce these PDL measurement methods, bring out the relationship between different PDL measurement methods and provide general guidelines for choosing a suitable method for different situations. From the point of view of system properties and categorization, we also suggest two new measurement techniques with improved measurement efficiency: two-states method and unpolarized light method.

3.1.1 PDL measurement for a system with all the three polarization effects

There are three methods which can be used under this condition: Mueller matrix method, deterministic and pseudorandom all-states method.

For a system with all the three polarization effects, its transmission property can be described by a 4×4 Mueller matrix M with 16 degrees of freedom. In order to measure the Mueller matrix, 4 linearly independent input SOPs are required as mentioned in section 2.3.2.1. There are three methods to measure the PDL of this kind of system.

The deterministic Mueller matrix method is one choice. The measurement principle [26], system calibration, and uncertainty analysis [30] are given by R. M. Craig. This method

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applies four deterministic SOPs, vertical linear polarization, horizontal linear polarization, 45° (bisector) linear polarization and right circular polarization as the inputs. The optical powers for the four inputs are I_a , I_b , I_c and I_d and the corresponding output powers are I_1 , I_2 , I_3 and I_4 . With these powers, the first row of the Mueller matrix can be obtained [26]

$$\begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left[\frac{I_1}{I_a} + \frac{I_2}{I_b} \right] \\ \frac{1}{2} \left[\frac{I_1}{I_a} - \frac{I_2}{I_b} \right] \\ -\frac{1}{2} \left[\frac{I_1}{I_a} + \frac{I_2}{I_b} \right] + \frac{I_3}{I_c} \\ -\frac{1}{2} \left[\frac{I_1}{I_a} + \frac{I_2}{I_b} \right] + \frac{I_4}{I_d} \end{bmatrix} \quad (3.2)$$

The information about the global transmission extrema are contained in the first-row matrix elements. Applying the method of Lagrange multiplier, the transmitted power extrema can be written as [13, 26]

$$\begin{aligned} T_{\max} &= m_{11} + \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \\ T_{\min} &= m_{11} - \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \end{aligned} \quad (3.3)$$

By substituting Eq. (3.3) into Eq. (3.1), PDL can be calculated.

Actually Eq. (3.3) is a property of diattenuator Mueller matrix. It has been proven that the Mueller matrix can be decomposed into the product of three matrices corresponding to the three polarization effects by using polar decomposition, consequently, the transmission extrema are given by the first row of Mueller matrix [13]. This is the basis for the Mueller matrix method. Four specific SOPs, not 4 arbitrary SOPs, are used in [26], which gives the optimum accuracy.

The fundamental methods, deterministic all-states and pseudorandom all-states, are based on the definition of the PDL. These two methods expose the device under test (DUT) to

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all the states of polarization in a repeatable or pseudorandom way [26]. With the measurement of the corresponding output Stokes parameter component s_0 or optical power, maximum and minimum transmission can be obtained directly.

3.1.2 PDL measurement for a system which has no depolarization

When the system has no depolarization, which is the case for most of the optical fiber systems, the Mueller matrix has only 7 degrees of freedom. This kind of Mueller matrices can be determined by applying 3 linearly independent input SOPs. For such systems, there are basically 2 PDL measurement methods: Jones matrix method [29] and the method through measuring non-depolarizing Mueller matrix.

Jones matrix method measures PDL through determining the transmission Jones Matrix $\mathbf{J}$ with three input SOPs using Eq. (2.9). The extrema transmission can be given by the squares of the singular values of $\mathbf{J}$ or equivalently the eigenvalues of $\mathbf{J}^+ \mathbf{J}$ [14, 29]. Here, superscript “+” denotes the matrix conjugate transpose.

Actually, the Jones matrix is equivalent to the non-depolarizing Mueller matrix. So, Jones matrix method is essentially the same as measuring PDL through measuring the non-depolarizing Mueller matrix and extracting PDL information from the first row of the matrix.

From section 2.3, we know that for a system with PDL, the Mueller matrix is of size 4x4. If the system has depolarization effect, we have to apply 4 linearly independent input SOPs to get the transmission matrix. If the system does not have depolarization effect, only 3 input SOPs are required. Now, we come to a question: since we can measure PDL using 4 or 3 input SOPs, is it possible to use even less input SOPs? The answer is yes. These are the new methods proposed by us: two-states method [28] and unpolarized light method [31].

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3.2 Proposed Methods

3.2.1 Two-states method (TSM)

Since the measurement of the non-depolarization Mueller matrix requires three input SOPs, it is impossible to get the Mueller matrix with two input SOPs. However, if only PDL information is desired, it is not necessary to measure the whole Mueller matrix.

3.2.1.1 Theoretical analysis

Based on the property that the Mueller matrix governing the transmission property of any linear, time-invariant optical system with birefringence and PDL satisfies Lorentz transformation, we have [20]

$$\mathbf{M}^T \mathbf{G} \mathbf{M} = \sqrt{\det \mathbf{M}} \mathbf{G} \quad (3.4)$$

Here $\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ is called Minkowski metric and $\det \mathbf{M}$ is the determinant of

matrix $\mathbf{M}$. Please note that, Eq. (3.4) is actually equivalent to Eq. (2.17) which is for complex Mueller matrix and Stokes parameters. Based on Eq. (3.4) we can obtain the inverse matrix of $\mathbf{M}$, under the condition that $\det \mathbf{M} \neq 0$, as follows

$$\mathbf{M}^{-1} = \frac{1}{\sqrt{\det \mathbf{M}}} \mathbf{G} \mathbf{M}^T \mathbf{G} = \begin{pmatrix} m_{11} & -m_{21} & -m_{31} & -m_{41} \\ -m_{12} & m_{22} & m_{32} & m_{42} \\ -m_{13} & m_{23} & m_{33} & m_{43} \\ -m_{14} & m_{24} & m_{34} & m_{44} \end{pmatrix} / \sqrt{\det \mathbf{M}} \quad (3.5)$$

where m_{ij} are the elements of the Mueller matrix.

Thus, we have a bilinear equation from $\mathbf{M} \mathbf{M}^{-1} = \mathbf{I}$ ($\mathbf{I}$ is a 4×4 identity matrix) as [13]

$$m_{11}^2 - m_{21}^2 - m_{31}^2 - m_{41}^2 = m_{11}^2 - m_{12}^2 - m_{13}^2 - m_{14}^2 \quad (3.6)$$

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Combining with Eq. (3.3), we can easily get

$$\begin{cases} T_{\max} = m_{11} + \sqrt{m_{21}^2 + m_{31}^2 + m_{41}^2} \\ T_{\min} = m_{11} - \sqrt{m_{21}^2 + m_{31}^2 + m_{41}^2} \end{cases} \quad (3.7)$$

This means that the transmission extrema can be extracted not only from the first row of the Mueller matrix but also from the first column of non-depolarization Mueller matrix.

In order to measure m_{11}, m_{21}, m_{31} and m_{41} , two orthogonal input SOPs are required. We write these as $s_{in0a} (1 \ s_{in1} \ s_{in2} \ s_{in3})^T$ and $s_{in0b} (1 \ -s_{in1} \ -s_{in2} \ -s_{in3})^T$ where s_{in0a} and s_{in0b} denote the corresponding input optical powers. From Eq. (2.11), assuming that the corresponding output SOPs are $s_{out0a} (1 \ s_{out1a} \ s_{out2a} \ s_{out3a})^T$ and $s_{out0b} (1 \ s_{out1b} \ s_{out2b} \ s_{out3b})^T$, we obtain

$$\begin{aligned} s_{out0a} \begin{bmatrix} 1 \\ s_{out1a} \\ s_{out2a} \\ s_{out3a} \end{bmatrix} &= s_{in0a} \begin{bmatrix} m_{11} + s_{in1}m_{12} + s_{in2}m_{13} + s_{in3}m_{14} \\ m_{21} + s_{in1}m_{22} + s_{in2}m_{23} + s_{in3}m_{24} \\ m_{31} + s_{in1}m_{32} + s_{in2}m_{33} + s_{in3}m_{34} \\ m_{41} + s_{in1}m_{42} + s_{in2}m_{43} + s_{in3}m_{44} \end{bmatrix} \\ s_{out0b} \begin{bmatrix} 1 \\ s_{out1b} \\ s_{out2b} \\ s_{out3b} \end{bmatrix} &= s_{in0b} \begin{bmatrix} m_{11} - s_{in1}m_{12} - s_{in2}m_{13} - s_{in3}m_{14} \\ m_{21} - s_{in1}m_{22} - s_{in2}m_{23} - s_{in3}m_{24} \\ m_{31} - s_{in1}m_{32} - s_{in2}m_{33} - s_{in3}m_{34} \\ m_{41} - s_{in1}m_{42} - s_{in2}m_{43} - s_{in3}m_{44} \end{bmatrix} \end{aligned} \quad (3.8)$$

By solving the equations in (3.8), we can get the first column of Mueller matrix

$$\begin{bmatrix} m_{11} \\ m_{21} \\ m_{31} \\ m_{41} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left(\frac{s_{out0a}}{s_{in0a}} + \frac{s_{out0b}}{s_{in0b}} \right) \\ \frac{1}{2} \left(\frac{s_{out1a}s_{out0a}}{s_{in0a}} + \frac{s_{out1b}s_{out0b}}{s_{in0b}} \right) \\ \frac{1}{2} \left(\frac{s_{out2a}s_{out0a}}{s_{in0a}} + \frac{s_{out2b}s_{out0b}}{s_{in0b}} \right) \\ \frac{1}{2} \left(\frac{s_{out3a}s_{out0a}}{s_{in0a}} + \frac{s_{out3b}s_{out0b}}{s_{in0b}} \right) \end{bmatrix} \quad (3.9)$$

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Thus, using Eqs. (3.9), (3.7) and (3.1), PDL can be derived explicitly using two arbitrary input SOPs, as long as they are orthogonal, as shown below

$$\text{PDL} = 10 \log \left(\frac{R_a + R_b + \sqrt{R_a^2 + R_b^2 + 2R_a R_b \chi}}{R_a + R_b - \sqrt{R_a^2 + R_b^2 + 2R_a R_b \chi}} \right) \quad (3.10)$$

where $R = \frac{S_{out0}}{S_{in0}}$ is the power ratio between output and input; and $\chi = \sum_{i=1}^3 s_{outia} s_{outib}$ is the dot product between the two normalized SOP vectors which is the cosine of the angle between the two output SOPs in Stokes space.

3.2.1.2 Experimental verification

We have performed experiments to validate the above proposed PDL measurement methods. The DUTs are one spool of polarization maintaining fiber (PMF) with bending-induced PDL [32] and a side polished single-mode fiber (SMF).

The experiment setup is shown in Fig. 3.3. In our setup, the in-line polarimeter 1 (polarization analyzer) is used to monitor the input SOPs and input optical powers, and polarimeter 2 measures the corresponding output Stokes parameters. In order to generate two orthogonal input SOPs, a polarization controller (PC) is employed. The first SOP is generated arbitrarily and is measured by polarimeter 1. For the second SOP, one tunes the PC until polarimeter 1 indicates that the SOP is orthogonal to the first one. The tunable distributed feedback (DFB) laser is used as the fully-polarized light source. The details of the measurement system and control software are given in Appendix 3.

Fig. 3.3 Experiment setup for PDL measurement

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3.2.1.3 Measurement results

We compare the measurement results obtained using our method with the Jones and Mueller matrix methods. All the methods are implemented with the same setup. The 20-time measurement results for the two DUTs, measured at 1550nm wavelength, are shown in Fig. 3.4. To make the comparison between the three methods clearer, the mean values and standard deviations of the results are listed in Table 3.1.

Fig. 3.4 PDL measurement results for two DUTs: (a) side-polished fiber and (b) PMF with bending-induced PDL.

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Table 3.1 Mean value and standard deviation of PDL measurement results for two samples using three different methods.

Method	Side-polished fiber (dB)	Spooled PMF (dB)
Two-states method	0.268±0.009	7.105±0.043
Jones matrix method	0.395±0.088	7.128±0.083
Mueller matrix method	0.231±0.003	7.016±0.030

The measurement results in Table 3.1 show that our method is in close agreement with the other two methods.

Fig. 3.5 presents the measured PDL values for the same DUTs over a wavelength range 1510 - 1580 nm. Once again, results based on all the three methods are included. A close match is observed between the results from these methods. For both Figs. 3.4 and 3.5, the measured results of our method lie in between those for the other two methods. This further confirms the validity of our proposed two-states method. A detailed comparison of these 3 methods is given in section 3.3.

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Fig. 3.5 PDL vs. wavelength for two DUTs (a) side-polished fiber and (b) PMF with bending-induced PDL.

3.2.2 Unpolarized light method (ULM)

We can see that from all-states methods to deterministic fixed-states methods, the number of required input SOPs can be reduced from all states to 4, 3 and 2. Except for the all-states methods, the fixed-states methods measure the PDL through full or part measurement of the system transmission matrix using fully polarized input light. Later, we will show that by using unpolarized light, the PDL of non-depolarizing system can be measured with only one input.

3.2.2.1 Theoretical analysis

A non-depolarizing optical system with polarization dependent loss (PDL) has no depolarization effect on a fully polarized light. However, such a system can change the degree of polarization (DOP) of a partially polarized or unpolarized light [21]. In other words, a partially polarized or unpolarized light can be depolarized or polarized by a PDL component.

It has been shown that the non-depolarizing Mueller matrix satisfies Lorentz transformation and fulfills Eq. (3.4). Together with Eq. (2.11), we can easily obtain

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$$s_{out0}^2 - s_{out1}^2 - s_{out2}^2 - s_{out3}^2 = \sqrt{\det \mathbf{M}} (s_{in0}^2 - s_{in1}^2 - s_{in2}^2 - s_{in3}^2) \quad (3.11)$$

From the DOP definition Eq. (2.6), we can get the output and input DOPs as $\text{DOP}_{out} = \sqrt{s_{out1}^2 + s_{out2}^2 + s_{out3}^2} / s_{out0}$ and $\text{DOP}_{in} = \sqrt{s_{in1}^2 + s_{in2}^2 + s_{in3}^2} / s_{in0}$. Thus Eq. (3.11) becomes

$$\text{DOP}_{out} = \sqrt{1 - \sqrt{\det \mathbf{M}} (1 - \text{DOP}_{in}^2) s_{in0}^2 / s_{out0}^2} \quad (3.12)$$

Here $s_{in0} / s_{out0} = s_{in0} / (m_{11}s_{in0} + m_{12}s_{in1} + m_{13}s_{in2} + m_{14}s_{in3})$ and it has been shown that [18]

$$\sqrt{\det \mathbf{M}} = m_{11}^2 - m_{12}^2 - m_{13}^2 - m_{14}^2 \quad (3.13)$$

So, finally we have

$$\text{DOP}_{out} = \sqrt{1 - \sqrt{\det \mathbf{M}} (1 - \text{DOP}_{in}^2) / (1 + \bar{D} \cdot \bar{S}_{in})^2} \quad (3.14)$$

Here $\bar{S}_{in} = (s_{in1} \ s_{in2} \ s_{in3}) / s_{in0}$ is the input SOP and $\bar{D} = (m_{12} \ m_{13} \ m_{14}) / m_{11}$ is defined as the PDL vector of an optical component and we may easily find that $\text{PDL} = 10 \log \left[\frac{1 + |\bar{D}|}{1 - |\bar{D}|} \right]$ [13]. From Eq. (3.14), we can find that if $\text{DOP}_{in} = 0$, $\text{DOP}_{out} = |\bar{D}|$. This means that

$$\text{PDL} = 10 \log_{10} \left(\frac{1 + \text{DOP}_{out}}{1 - \text{DOP}_{out}} \right) \quad (3.15)$$

Eq. (3.15) suggests that with only one input SOP (unpolarized light), PDL can be determined by measuring the output DOP.

Next, the problem is to realize unpolarized light. The laser source we are using is a DFB laser diode (LD) with a 50 MHz linewidth. However, the traditional Lyot depolarizer cannot depolarize quasi-monochromatic light; the Mach-Zehnder interferometer-based depolarizer cannot effectively depolarize light with linewidth less than 100 MHz [33],

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and the dual fiber-ring depolarizer is also used for wide linewidth (10GHz) laser source [34]. Thus, we need to find another way to depolarize our laser source.

Although the traditional Lyot-based or Mach-Zehnder interferometer-based depolarizers cannot effectively depolarize quasi-monochromatic fully polarized light, they can make it partially polarized. From Eq. (3.14), we may find that the output DOP of a PDL device can be lower than that of the input if the input DOP is less than 1. This suggests the possibility of depolarizing further a partially polarized quasi-monochromatic light, which can be obtained by using a traditional wideband depolarizer with a variable PDL device. For completely depolarizing the input light, namely, $\text{DOP}_{\text{out}} = 0$, we may deduce from Eq. (3.14) that the conditions $|\bar{D}| = \text{DOP}_{\text{in}}$ and $\theta = \pi$ must be fulfilled.

3.2.2.2 Experiments and results

The experimental setup is shown in Fig. 3.6. The variable PDL component can be produced based on different principles and structures. Here we use bending-induced PDL in a PMF section [32]. The PDL value is adjusted by changing the bent fiber length (or number of coils) of PMF. The linewidth of the DFB LD is 50 MHz at 1550 nm. The DOP at the output port of a traditional depolarizer is about 30% measured using a polarimeter shown in Fig. 3.7(a). By adjusting the polarization controller and variable PDL component, the output DOP can be made to be as small as 0.16% as shown in Fig. 3.7(b). Next, the unpolarized light generated from the left part in Fig. 3.6 is launched into the DUT to perform PDL measurement.

According to Eq. (3.14), for completely depolarizing the light with DOP 29.8%, the PDL value of the variable PDL component should be 2.6694 dB, which is very close to our measured value 2.6712 dB. Because some attenuation of optical power is induced in this technique, we have to consider the relation between the PDL value of variable PDL component and the input DOP, as shown in Fig. 3.8. It can be observed that the PDL required at first increases linearly with input DOP, and it grows almost exponentially when input DOP is larger than 80%. If 10 dB attenuation is permitted, we can fully

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depolarize light with DOP up to 82%. In this way, we have realized a quasi-monochromatic depolarizer.

Fig. 3.6 Experimental configuration for quasi-monochromatic light depolarizer and PDL measurement.

Fig. 3.7 DOPs measured (a) at the output port of traditional depolarizer and (b) after the variable PDL component.

Fig. 3.8 The relation between the input DOP and the required PDL value for fully depolarizing the input.

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Fig. 3.9 Output DOPs for an unpolarized light through (a) an isolator and (b) a side-polished SMF.

The quasi-monochromatic unpolarized light is used to perform PDL measurement on two samples: one is an optical isolator, and the other is a section of side-polished single-mode fiber. The DOPs of light after passing through the two samples are shown in Fig. 3.9. The 20-time measurement results of the two samples are plotted in Fig. 3.10. To verify the proposed method, Jones matrix method [29] and Mueller matrix method [26] are also used to measure the same samples, whose results are also shown in Fig. 3.10. The mean values and standard deviations of measurement data of the three methods are summarized in Table.3.2. It is found that the result of our method agrees quite well with those of the other two methods.

Table 3.2 PDL measurement results of two samples

	Jones matrix method	Mueller matrix method	Unpolarized method
Isolator (dB)	0.2590 ± 0.0068	0.2382 ± 0.0013	0.2392 ± 0.0014
Side-polished SMF (dB)	1.1046 ± 0.0080	1.0341 ± 0.0021	1.0276 ± 0.0042

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(a)

(b)

Fig. 3.10 PDL measurement results of (a) optical isolator and (b) side-polished SMF.

Compared with the Jones matrix method, our method is less sensitive to birefringence perturbation because birefringence perturbation does not affect DOP [21]. Compared with Mueller matrix method, it can avoid the SOP-adjustment-induced error. And our proposed method apparently has the fastest measurement speed owing to its single input. It should be pointed out that Eq. (3.15) has been presented in some papers in a connotative mode [14] or in a simpler case [35]. But to the best of our knowledge, no experimental work has been reported, which may be due to the difficulty in obtaining high-quality unpolarized quasi-monochromatic light.

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3.3 Comparison between different methods

The previous sections in this chapter have discussed the applicability of different measurement methods to systems of different types. This section will focus on the measurement accuracy and speed.

A comparison of different methods is given in Table 3.3. The all-states methods and Mueller matrix method can be applied to the systems with all the three polarization effects. The deterministic all-states and pseudorandom all-states methods try to apply all states of polarization as input which is a very time consuming process spending tens of seconds to several minutes [36]. Thus, the results have some uncertainty. Also, setting the suitable SOP scan rate with respect to the power measurement time is critical. If the polarization scan rate is faster than the averaging time of the power measurement, results are erroneous [37]. When the measurement settings are correct, the all-states method has an accuracy which is almost the same as that of the Mueller matrix method [37].

The Mueller matrix method measures PDL through the measurement of the first column of Mueller matrix. Thus, 4 input SOPs are required and only s_0 or optical power is used. Since the SOPs may be affected by the change of birefringence induced by environment variation during the measurement, while the power does not get affected, the Mueller matrix method has the best accuracy among the fixed-states techniques. We can see from Table 3.1 that the Mueller matrix method has the minimum standard deviation. However, requirement of 4 specific input SOPs makes this method less time efficient than the other fixed-states methods.

When the system is not a depolarization system, the PDL can be extracted through the measurement of Jones matrix or non-depolarization Mueller matrix. The Jones matrix method needs 3 SOPs as input. By using the Lorentz transformation property of the Mueller matrix, the PDL information can be obtained without measuring the whole transmission Mueller matrix. So the required number of inputs can be further reduced. The two-states method utilizes two input SOPs and unpolarized light method only uses one input SOP. The Jones matrix method and the two-states method use the SOPs to calculate PDL. The results show that the Jones matrix method has much larger standard

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deviation. This is because the Jones matrix method needs three specific input SOPs while the two-states method only needs two arbitrary orthogonal SOPs. As a general rule of thumb, fewer inputs entail smaller standard deviation. Therefore, with only two input SOPs, the standard deviation of the two-states method is just slightly larger than the Mueller matrix method, although the two-states method uses SOPs to calculate PDL and SOPs which is more vulnerable to the environment perturbation. As for the measurement time, the adjustment time of two-states method is the shortest among all the fixed-states methods since the two-states method needs to adjust the input SOPs only once. The first SOP can be arbitrary and the second input SOP is adjusted until it is orthogonal to the first one. The unpolarized light method is the fastest one compared with the other methods because only one input state of polarization is required. However, the measurement accuracy is dependent on the DOP of input light and it is difficult to depolarize a narrow line-width laser source.

Table 3.3 Comparison between different PDL measurement methods

	System with depolarization	System without depolarization	Measure system transmission matrix	No. of inputs SOPs
Deterministic and Pseudorandom All-states	Yes	Yes	No	All-states
Mueller Matrix Method	Yes	Yes	Yes	4
Jones Matrix Method	No	Yes	Yes	3
Two-states Method	No	Yes	No	2
Unpolarized light Method	No	Yes	No	1

3.4 Conclusion

The definition of PDL is introduced and the existing measurement methods are reviewed with respect to their advantages and shortcomings. The existing methods use 4 or 3 input SOPs to measure the system transmission Matrix and then extract the PDL information or measure PDL directly according to the definition. However, if not all the polarization

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effects appeared in the system, especially no depolarization, it is not necessary to measure the whole transmission matrix because PDL can be obtained with only part of the matrix. Thus, the input SOPs can be reduced to two, which is our first new proposed method, two-states method. Till now, all the methods use fully polarized light as input. If the input is unpolarized light, only one input is enough for measuring PDL, which is our second new proposed method, unpolarized light method.

All the existing and proposed methods are compared with each other in terms of the polarization effects that appear in the system and the number of required SOPs. They all have advantages and limitations. Suggestions for choosing a suitable measurement method are given.

Chapter 4 POLARIZATION MODE DISPERSION MEASUREMENT

4.1 Introduction

The study of the polarization mode dispersion (PMD) originates from the practice of coherent communications [38]. Before the advent of the fiber-based erbium-doped optical amplifier (EDFA) (1990), the long-distance communications used coherent communications in order to extract the signal out of the noise. In 1970s, birefringence of the single-mode fiber was recognized and the output polarization state drift was found with the perturbation of the surrounding environment. The polarization tracking and control were under intense investigation because the coherent communications needs to align the signal polarization. In 1986, C. D Poole and R. E. Wagner proposed a phenomenological model of principal states of polarization (PSPs) to describe the polarization in optical fibers [39] which brought the research from the classical treatment of birefringence to the global treatment - modern development of PMD.

The description of the PMD can be started from the birefringence of the optical fiber. In reality, fibers have some amount of geometrical asymmetry due to imperfections in the manufacturing process or mechanical stress on the fiber after manufacture. The asymmetry breaks the degeneracy of the orthogonally polarized HE_{11} core modes, results in birefringence and causes the phase and group velocity difference between the two modes. These are the two polarization eigenmodes in the classical treatment of birefringence, such as the two modes polarized along the two axes of polarization maintaining fiber. For long fiber lengths, e.g. more than 1 km, the mode coupling in SMF makes identification of the eigenmodes difficult. Furthermore, when several fiber pieces are concatenated together, the random orientation of the birefringent axes of different pieces makes the application of such a model even impractical.

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In order to overcome this problem, the concept of the principal states of polarization was introduced [39]. C. D. Poole and R. E. Wagner found that for any linear optical transmission medium without PDL, there exist two orthogonal input SOPs for which the corresponding output SOPs are also orthogonal and show no dependence on wavelength to first order. The maximum differential delay exists between signals launched along these two input SOPs. They named these SOPs as principal states of polarization. Thus, the PMD can be expressed by a vector, which has a direction aligned to the slow PSP and the length of the PMD vector is the differential group delay (DGD) between the two PSPs. The PSPs have the following properties. The light of a finite spectral width (e.g. pulses) that is aligned with either of the two PSPs at the input will remain polarized to first order after propagating through the medium without depolarization. The input and output PSPs can be used as basis vectors for describing PMD in single-mode fibers of arbitrary length and configuration. As for the direction of the PSPs, they are not necessarily correlated with any local fiber birefringence, thus, the SOPs of the PSPs will vary during propagation. In the absence of polarization mode coupling, the PSPs are same as the eigen-modes of the fiber. If the differential delay becomes comparable to the coherence time of the light source, depolarization can be observed. In the presence of polarization mode coupling, pulse broadening occurs even for waves launched in a principal state because of the frequency dependence of the DGD and the frequency dependence of the output principal state. This is higher order PMD. Fig. 4.1 shows the trajectories of output SOPs with the change of frequency. When the system has only first order PMD, the output SOPs rotate around a fixed axis with a rotation angle which is proportional to the frequency change and the output SOP trajectories for different input SOPs are all circular and parallel as shown in Fig. 4.1 (a). When the higher order PMD is present, the trajectories of the output SOPs are complex curves. Fig. 4.1 (b) shows this condition. The PMD vector also changes with frequency.

When PDL is present, even for a system with only first order PMD, the trajectories of the output SOPs are also complex curves. This time, the two PSPs are not orthogonal and the PMD vector becomes a complex vector.

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In the following parts of this chapter PMD measurement methods are divided into two categories: methods for systems without PDL and with PDL. For each category, we begin with a literature review. Then, through the analysis of the disadvantages of the existing methods, new methods are introduced.

Fig. 4.1 Trajectories of output SOPs when (a) the system has 1st order PMD and (b) high order PMD.

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4.2 Measurement methods for the system without PDL

The PMD vector measurement methods for systems without depolarization and PDL are mainly of three types: geometric method, differential method and the method based on the system transmission matrix.

4.2.1 Literature review

When the optical fiber system has only birefringence, the two PSPs are orthogonal and the output SOP rotates around one of the PSP when frequency changes. This is the basis of differential methods.

4.2.1.1 Geometric method – Poincaré sphere technique (PST)

The Poincaré sphere technique is used to measure the first order PMD for any linear optical transmission medium without PDL [40]. Using the Stokes-parameters formalism, one has

$$\frac{d\vec{S}(\omega)}{d\omega} = \vec{\Omega} \times \vec{S}(\omega) \quad (4.1)$$

where $\vec{S}(\omega)$ is the Stokes vector representing the output SOP and $\vec{\Omega}$ represents the PMD vector. This equation implies that a frequency change $\Delta\omega$ causes a rigid sphere rotation γ around the axis $\vec{\Omega}$ and the axis of rotation is assumed constant under the first order approximation as shown in Fig. 4.1 (a). Thus, we have [40]

$$\gamma = |\vec{\Omega}| \Delta\omega \quad (4.2)$$

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Using the above equation, the amplitude of PMD vector or DGD can be calculated with two adjacent SOPs. Since Eq. (4.2) is only a simple transform of the differential equation Eq. (4.1), we consider PST as one of the differential method.

4.2.1.2 Differential method - C. D. Poole's method

The basis of C. D. Poole's method is also Eq. (4.1). With the help of the vector identity $\vec{a} \times \vec{b} \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{a})\vec{c}$, PMD vector can be derived [41]

$$\vec{\Omega} = \frac{\frac{d\vec{S}_i}{d\omega} \times \frac{d\vec{S}_j}{d\omega}}{\frac{d\vec{S}_i}{d\omega} \cdot \vec{S}_j} \quad (4.3)$$

where $\vec{S}_i$ and $\vec{S}_j$ are two output SOPs corresponding to two different input SOPs.

When the input SOP is close to one of the PSPs or the cross product in Eq. (4.3) is small, the output will not show much dependence on frequency. So during the measurement, 3 input states (linear polarizations at 0° and 45° and left hand circular polarization) are used. The error can be minimized by choosing the pairs of SOPs satisfying the criteria

$$\frac{\left| \frac{d\vec{S}_i}{d\omega} \times \frac{d\vec{S}_j}{d\omega} \right|}{\left| \frac{d\vec{S}_i}{d\omega} \right| \left| \frac{d\vec{S}_j}{d\omega} \right|} > 0.707 \quad \text{and} \quad \left| \left(\vec{P}_{b-} \right)_j \times \vec{S}_i \right| > 0.707 \quad i \neq j \quad (4.4)$$

where $\vec{P}_{b-} = -\vec{P}_{b+} = \frac{\vec{\Omega}}{|\vec{\Omega}|}$ is one of the principal states. It was found that at every wavelength at least one pair of scans would satisfy both of these conditions.

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4.2.1.3 The method based on the transmission matrix - Mueller matrix method

The above mentioned methods have the advantage that the input SOPs are not required. However, the authors did not give the methods for measuring higher order PMD. Actually, the PMD vector aligns with the rotation axis in PST, but we have to change the algorithm of PST by using 3 SOPs at different frequencies to determine the pointing direction of rotation axis. With the PMD vector, we can calculate higher order PMD through differentiation with respect to frequency. C. D. Poole's method has given the first order PMD, so we can calculate higher order PMD through the differentiation too. However, there are three experimental difficulties: 1) when the input SOP is close to one of the PSPs, the output SOP will rotate little with the frequency change. This will result in erroneous PMD calculation. So several different input SOPs are required. 2) C. D. Poole's method needs the frequency difference to be as small as possible. Thus the creation and measurement of small frequency differences become the issue especially for large PMD values for which the required frequency difference could be even smaller. 3) Small frequency differences cause small rotation of the output SOP. Thus the inaccuracies of the polarimeter become more severe. This will result in low experimental signal-to-noise ratios (SNRs). These could be the reasons why the authors of PST and C. D. Poole's method did not introduce the algorithm to calculate higher order PMD.

The method based on the system transmission matrix, Mueller matrix method (MMM), can overcome these problems.

The Mueller matrix method was developed by Jopson et al. in 1999 [22, 42]. It uses the Mueller matrix as the tool to calculate the PMD vector. As mentioned in section 2.3.2.2, for a fiber transmission system without depolarization and PDL, the transmission rotates the polarization of light in Stokes space. This rotation is described by the 3×3 Mueller matrix with 3 DOFs. Two independent input SOPs are required for measuring such a Mueller matrix.

The calculation of the MMM is based on the calculation of the difference rotation $\mathbf{R}_\Delta$ derived from Eq. (4.1) which relates the output Stokes vectors at the two frequencies

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$$\bar{S}(\omega + \Delta\omega) = \mathbf{R}_\Delta \bar{S}(\omega) \quad (4.5)$$

and

$$\mathbf{R}_\Delta = \mathbf{R}(\omega + \Delta\omega) \mathbf{R}^T(\omega) \quad (4.6)$$

where $\mathbf{R}(\omega)$ is the transmission Mueller matrix at frequency ω and superscript ‘T’ denotes the matrix transpose. This difference rotation can be expressed in terms of its rotation angle ϕ and rotation axis $\mathbf{r}$ in the form

$$\mathbf{R}_\Delta = \cos \phi \cdot \mathbf{I} + (1 - \cos \phi) \mathbf{r} \mathbf{r} - (\sin \phi) \mathbf{r} \times \quad (4.7)$$

where $\mathbf{I}$ is the 3×3 unit matrix, $\mathbf{r} \mathbf{r}$ is a dyadic, and $\mathbf{r} \times$ is the cross-product operator. Combining Eq. (4.6) and (4.7), the rotation angle and axis can be determined through the measured matrices $\mathbf{R}(\omega + \Delta\omega)$ and $\mathbf{R}(\omega)$

$$\cos \phi = \frac{1}{2} (\text{Tr} \mathbf{R}_\Delta - 1) \quad (4.8)$$

and

$$\begin{aligned} r_1 \sin \phi &= \frac{1}{2} (R_{23} - R_{32}) \\ r_2 \sin \phi &= \frac{1}{2} (R_{31} - R_{13}) \\ r_3 \sin \phi &= \frac{1}{2} (R_{12} - R_{21}) \end{aligned} \quad (4.9)$$

where the R_{ij} are elements of $\mathbf{R}_\Delta$. Thus, the PMD vector is determined using

$$\bar{\Omega} = \Delta\tau \cdot \mathbf{r} \quad \text{and} \quad \Delta\tau = \frac{\phi}{\Delta\omega} \quad (4.10)$$

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Same as JME, the condition $\Delta\tau\Delta\omega < \pi$ needs to be satisfied in order to avoid the ambiguities of the multiple-valued argument function. The authors also gave the calculation of second order PMD vector through $\bar{\Omega}_\omega = d\bar{\Omega}/d\omega$.

4.2.2 Experiments and comparison between different methods

For PMD measurement, the same experimental setup as the one used to measure PDL in section 3.2.1.2 is adopted. We have measured a lot of DUTs. Here, we only give the results for a piece of 3m PMF because from the results we can see the difference between different methods clearly. During the experiments, three input SOPs were launched at each frequency or wavelength. They are $[1 \ 0 \ 0]^T$, $[-1 \ 0 \ 0]^T$ and $[0 \ 1 \ 0]^T$. The wavelength sweeping range is from 1530 nm to 1535 nm with a wavelength step size of 0.1 nm. The SOP measurement results are shown in Fig. 4.2. The results show that the output SOP trajectories are circles and parallel to each other.

During the calculation, PST can use any input SOP as long as the input is not close to the PSPs; C. D. Poole's method chooses two of them under the condition of Eq. (4.4); and MMM method chooses the first and third input SOPs.

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Fig. 4.2 Input SOPs and output SOP trajectories for PMD measurement: (a) $[1\ 0\ 0]^T$, (b) $[-1\ 0\ 0]^T$ and (c) $[0\ 1\ 0]^T$. (d) shows that the output trajectories are parallel.

The measurement results of PST are shown in Fig. 4.3 and Table 4.1. Here we just give the DGD values. From the results we can see that the increase of the trajectory radius results in an increase of measurement accuracy. The trajectory of the output c has the biggest radius and thus has much smallest measurement standard deviation. This is caused by the polarimeter. If the rotation angle between two adjacent output SOPs is bigger, then the relative SOP measurement error will be smaller.

Fig. 4.3 DGD calculation results using PST for 3 output SOP trajectories with wavelength step 0.1 nm.

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Table 4.1 the mean and standard deviation values measured using PST for three output SOP trajectories.

	SOP _a	SOP _b	SOP _c
1 st order DGD (ps)	3.9498±1.5608	4.2436±1.8878	4.0551±0.4893

In order to reduce the standard deviation, we tried to increase the wavelength step size for calculation. We found that with the increase of the step size, the standard deviation keeps on decreasing; however, the mean DGD value is also decreasing which is shown in Fig. 4.4 and table 4.2. This makes it difficult to choose a suitable wavelength step for calculation.

Fig. 4.4 DGD calculation results using PST for SOP_c with wavelength step ranging from 0.1 nm to 0.4 nm

Table 4.2 DGD calculation results using PST for SOP_c with different step wavelength.

	0.1 nm step	0.2 nm step	0.3 nm step	0.4 nm step
1 st order DGD (ps)	4.0551±0.4893	3.7698±0.2044	3.5676±0.0832	3.3251±0.0897

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The measurement results using C. D. Poole’s method is shown in Fig. 4.5 and Table 4.3. The different pairs of SOPs give different results.

Fig. 4.5 DGD calculation results using C. D. Poole’s method with (a) SOP_a and SOP_b, (b) SOP_a and SOP_c, and (c) SOP_b and SOP_c

Table 4.3 DGD values calculated by C. D. Poole’s method using different pairs of SOPs.

	SOP _a and SOP _b	SOP _a and SOP _c	SOP _b and SOP _c
1 st order DGD (ps)	47.323±129.61	3.8526±0.38720	4.0375±0.4533

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Fig. 4.6 DGD results calculated using C. D. Poole’s method with different wavelength step.

Table 4.4 DGD calculation results for SOP_b SOP_c using C.D. Poole’s method with different step wavelength.

	0.1 nm step	0.2 nm step	0.3 nm step	0.4 nm step
1 st order DGD (ps)	4.0375±0.4533	4.1650±0.3057	4.3672±0.2055	4.7895±0.3739

The results in Fig. 4.6 and Table 4.4 show that the increase of the wavelength step size leads to erroneous results. This is because the C. D. Poole’s method needs to calculate the derivative of SOP with respect to frequency. Thus, a smaller wavelength step size gives better accuracy.

The results for MMM are shown in Fig. 4.7 and Table 4.5. With the increase of the wavelength step size, the standard deviation decreases dramatically and the mean DGD value almost remains the same. Thus, the MMM has much better accuracy than the PST and C. D. Poole’s method. However, wavelength step size cannot increase without limit. The condition $\Delta\tau\Delta\omega < \pi$ has to be fulfilled to avoid the ambiguities of the multiple-valued argument function. In our calculation, we found that the optimum results are achieved at $\Delta\tau\Delta\omega = \pi/2$ which corresponds to 0.5 nm step wavelength.

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Fig. 4.7 DGD calculation results using MMM with different step wavelength.

Table 4.5 Calculation results using MMM with different step wavelength

	0.1 nm step	0.3 nm step	0.5 nm step	0.7 nm step
1 st order DGD (ps)	3.9592±0.3928	3.9541±0.1127	3.9570±0.0742	3.9594±0.1199

From the experiment results and the algorithms of these methods, we can conclude that the PST and C. D. Poole’s method only need the output SOPs to perform calculation. However, these two methods have the difficulty to choose suitable input SOPs and require small wavelength step to increase the measurement accuracy. On the contrary, the matrix method, MMM, has a much smaller standard deviation. This is because this method measures the system transmission matrices for different wavelengths to perform calculation; small wavelength step is not required. However, the matrix method has its shortcomings, too. This method needs to measure both input and output SOPs and the number of input SOPs required is more than the geometric and differential methods. So the matrix method needs relatively longer measurement time.

4.2.3 Our method: Round trip method (RTM)

From the above literature review and method comparison, we know that except for the PST, other methods need to employ two or three input SOPs at different wavelengths.

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This will incur some drawbacks. There are two ways to perform the wavelength sweeping during the measurement. The first way is to fix one SOP, sweep the wavelength, and repeat this process for other SOPs. The wavelength repeatability of the laser source becomes a challenge. The drift of the wavelength will cause the output SOP to rotate and lead to erroneous results especially for large PMD condition. The second way is to set one wavelength and sweep SOPs. This method also has its weakness; since the calculations of all these methods need to use the output SOPs at different wavelengths with the same input SOP, the accuracy for achieving the desired input SOPs at different wavelength becomes an issue. In order to overcome the above mentioned difficulties, we propose a new method, called the round trip method (RTM). This method only employs one input SOP, so the wavelength sweeping is required once and the requirement of wavelength repeatability is avoided. Also, this algorithm does not require the input SOP information, so the input SOP can be arbitrary. Thus, the problem of achieving a desired input SOPs is also avoided.

The round trip method measures the SOPs at both ends of the DUT as shown in Fig. 4.8. First, a fully polarized light is launched from one end of the DUT and one measures the SOP $\bar{S}$ at the other end. Because of the Fresnel reflection, part of the light is reflected back to the launching end. In order to measure the reflected SOP $\bar{T}$, an optical circulator is applied. Here we assume that the PMD of the circulator is zero.

The frequency dependence of the Stokes parameters obeys Eq. (4.1). If we name the transmission Mueller matrix from laser source output port to the splicing point of circulator 1 and DUT as $\mathbf{M}_1$ and the Mueller matrix of the DUT as $\mathbf{M}$, we can get the equation for forward direction [22]

$$\bar{\Omega}_\times = \frac{\partial(\mathbf{M}\mathbf{M}_1)}{\partial\omega} \cdot (\mathbf{M}\mathbf{M}_1)^{-1} = \left(\frac{\partial\mathbf{M}}{\partial\omega} \mathbf{M}_1 + \mathbf{M} \frac{\partial\mathbf{M}_1}{\partial\omega} \right) (\mathbf{M}_1^{-1} \mathbf{M}^{-1}) \quad (4.11)$$

where superscript ‘-1’ denotes the matrix inverse.

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Since the PMD of the circulator is neglected, we have $\frac{\partial \mathbf{M}_1}{\partial \omega} = 0$, then the above equation becomes

$$\tilde{\Omega}_\times = \frac{\partial \mathbf{M}}{\partial \omega} \mathbf{M}^{-1} \quad (4.12)$$

Fig. 4.8 Experimental setup for RTM

So, for the forward direction, there is no effect of $\mathbf{M}_1$. By using the matrix expression of

$$\tilde{\Omega}_\times = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \text{ and Eq. (4.1), we get a group of equations}$$

$$s_1' = 0 - \Omega_3 s_2 + \Omega_2 s_3 \quad (4.13)$$

$$s_2' = \Omega_3 s_1 + 0 - \Omega_1 s_3 \quad (4.14)$$

$$s_3' = -\Omega_2 s_1 + \Omega_1 s_2 + 0 \quad (4.15)$$

where $\Omega_i (i = 1, 2, 3)$ are the three components of $\tilde{\Omega}$, and prime denotes differentiation with respect to frequency.

Since only two equations are independent among Eqs. (4.13-15), we need another equation to determine all the three components of $\tilde{\Omega}$.

If we define the Mueller matrix from splicing point to polarimeter 2 as $\mathbf{M}_2$, the propagation of the reflected light can be expressed as

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$$\frac{\partial \bar{T}}{\partial \omega} = \mathbf{M}_2 \mathbf{R} \mathbf{M}^{-1} \mathbf{M} \frac{\partial \mathbf{M}^T}{\partial \omega} \mathbf{R} \bar{S} + \mathbf{M}_2 \mathbf{R} \mathbf{M}^T \mathbf{R} \frac{\partial \bar{S}}{\partial \omega} \quad (4.16)$$

where $\mathbf{R} = \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}$ is the reflection matrix. Under the condition that the system does not contain polarization dependent loss (PDL), the system transmission Mueller matrix is orthogonal matrix, $\mathbf{M}^T = \mathbf{M}^{-1}$. Eq. (4.16) becomes

$$\frac{\partial \bar{T}}{\partial \omega} = \mathbf{M}_2 \mathbf{R} \mathbf{M}^T (-\bar{\Omega} \times \mathbf{R} \bar{S} + \mathbf{R} \bar{\Omega} \times \bar{S}) = \mathbf{M}_2 \mathbf{R} \mathbf{M}^T \begin{bmatrix} 2\Omega_2 s_3 \\ -2\Omega_1 s_3 \\ 2\Omega_2 s_1 - 2\Omega_1 s_2 \end{bmatrix} \quad (4.17)$$

Since $\mathbf{M}_2$, $\mathbf{R}$ and $\mathbf{M}^T$ are also orthogonal matrices, by using the fact that the determinant of the orthogonal matrix is 1, we have

$$\left| \frac{\partial \bar{T}}{\partial \omega} \right| = \det \left(\mathbf{M}_2 \mathbf{R} \mathbf{M}^T \begin{bmatrix} 2\Omega_2 s_3 \\ -2\Omega_1 s_3 \\ 2\Omega_2 s_1 - 2\Omega_1 s_2 \end{bmatrix} \right) = \begin{vmatrix} 2\Omega_2 s_3 \\ -2\Omega_1 s_3 \\ 2\Omega_2 s_1 - 2\Omega_1 s_2 \end{vmatrix}$$

And

$$|T|^2 = 4(\Omega_2^2 + \Omega_1^2) s_3'^2 + 4s_3'^2 \quad (4.18)$$

Substituting Eq. (4.13-15) in Eq. (4.18), we can get the three components of $\bar{\Omega}$

$$\Omega_1 = \frac{s_2 s_3' \pm \sqrt{s_1^2 [(s_1^2 + s_2^2) a - s_3'^2]}}{s_1^2 + s_2^2} \quad (4.19)$$

$$\Omega_2 = \frac{\Omega_1 s_2 - s_3'}{s_1} \quad (4.20)$$

$$\Omega_3 = \frac{s_2' + \Omega_1 s_3}{s_1} \quad (4.21)$$

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where $\Omega_2^2 + \Omega_1^2 = \frac{|T|^2 - 4s_3'^2}{4s_3^2} \equiv a$ in Eq. (4.19).

The term under the square root in Eq. (4.19) is equal to or greater than zero which is proved in Appendix 4.

The Eqs. (4.19-4.21) are valid when s_1 and s_3 do not equal to zero. If s_1 or s_3 or both of them equal to zero, $\bar{\Omega}$ cannot be calculated. However, because the continuity of the physical system, we can get $\bar{\Omega}$ at the corresponding frequencies through some numerical methods such as curve fitting or interpolation.

Eqs. (4.19-4.21) give us two sets of $\bar{\Omega}$. In order to remove the false one, we need to use the output SOPs again. Since the output SOP rotates around the PMD vector with the frequency change, the vector determined by 3 adjacent SOPs is parallel to the PMD vector. Through the comparison of the vector with the two solutions of Eq. (4.19), the correct solution can be found.

During the experiments, two pieces of concatenated PMF was adopted as a sample having higher order PMD. The total length of the pieces is 3 m. The wavelength is swept from 1544 to 1549 nm. Fig. 4.9(a) shows the trajectories of the forward and backward SOPs. We can see that the backward trajectory is almost two times longer than the forward trajectory. This is because the backward SOP passes through the PMD component two times. In order to increase the SNR, curve fitting has been done for the Stokes parameters as shown in Fig. 4.9(b) and (c). The SOPs on the fitted curve are used for PMD calculation. The PMD results measured using the round trip method and the Mueller matrix method are plotted in Fig. 4.9(d). The results of round trip method agree with the MMM and the accuracy is comparable.

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(c)

(d)

Fig. 4.9 Round trip method measurement results of (a) SOP trajectories for forward and backward direction, curve fitting of (b) forward SOPs and (c) backward SOPs, and (d) DGD values calculated using RTM and MMM.

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4.3 Measurement methods for the systems with PDL

In an optical fiber system with both birefringence and polarization dependent loss or gain, the two PSPs are not orthogonal. In such a case, PMD is fully described by a complex PMD vector $\vec{W} = \vec{\Omega} + i\vec{\Lambda}$, which includes two real vectors $\vec{\Omega}$ and $\vec{\Lambda}$. The PMD equation may becomes [43]

$$d\vec{S}/d\omega = \vec{\Omega} \times \vec{S} - (\vec{\Lambda} \times \vec{S}) \times \vec{S} \quad (4.22)$$

Since PSPs are frequency independent to first order, $d\vec{S}_p/d\omega = 0$, Eq. (4.22) becomes

$$\vec{\Omega} - \vec{\Lambda} \times \vec{S}_p = \lambda \vec{S}_p \quad (4.23)$$

where λ is a real eigenvalue to be determined.

4.3.1 Literature review

The currently available methods for the measurement of the complex PMD vector in frequency domain are divided into two categories. The first category is based on the differential equations, which involves derivatives of polarization states with respect to optical frequency. Such methods include the complex plane method [44] and the generalized Poincaré sphere method (GPST) [45]. The second category is based on analysis of the transmission matrices, such as the Jones matrix eigenanalysis (JME) method [46], generalized Mueller matrix method (GMMM) [47] and virtual generalized Mueller matrix method (VGMMM) [48]. In principle, all methods suitable for the case with PDL can be used to measure an optical fiber system without PDL.

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4.3.1.1 Complex plane method

The complex plane method was proposed by A. Eyal and M. Tur in 1997 [44]. For a linear medium without ideal polarizer, an input optical field $\vec{E}_i$ will produce an output field which depends on the optical frequency through $\vec{E}_o(\omega) = \mathbf{T}(\omega)\vec{E}_i$, where $\mathbf{T}(\omega)$ is a frequency dependent complex Jones matrix [46]. Taking the derivative of $\vec{E}_o(\omega)$ with respect to ω gives [46]

$$\vec{E}_o'(\omega) = [\mathbf{T}'\mathbf{T}(\omega)^{-1}] \vec{E}_o \equiv \mathbf{N}(\omega)\vec{E}_o \quad (4.24)$$

The PSPs of the system are the eigenvectors of $\mathbf{N}$, and their propagation characteristics can be derived from the respective eigenvalues. By using the complex plane representation, the optical field is separated into a unit intensity polarization vector, $\hat{\varepsilon}$, and a complex amplitude $A(\omega)$

$$\vec{E} = \begin{bmatrix} E_u \\ E_v \end{bmatrix} = E_u \begin{bmatrix} 1 \\ \chi \end{bmatrix} = A(\omega)\hat{\varepsilon} \quad \text{with} \quad A(\omega) = E_u(1 + |\chi|^2)^{1/2} \quad \text{and} \quad \hat{\varepsilon} = (1 + |\chi|^2)^{-(1/2)} \begin{bmatrix} 1 \\ \chi \end{bmatrix} \quad (4.25)$$

The new form of differential equation (4.24) in terms of new descriptors χ and $A(\omega)$ can be derived as

$$\chi_0' = -n_{12}\chi_o^2 + (n_{22} - n_{11})\chi_o + n_{21} \quad (4.26a)$$

$$A_o'(\omega) = \left[n_{11} + n_{12}\chi_o + \left[\ln(1 + |\chi_o|^2)^{1/2} \right]' \right] A_o(\omega) \quad (4.26b)$$

where n_{ij} are the elements of $\mathbf{N}$. Since the principal states of polarization do not change with frequency to the first order, which is equivalent to $\chi_o' = 0$, their complex plane representations can be easily obtained from Eq. (4.26a)

$$\chi_{o\pm} = \frac{1}{2n_{12}} \left[(n_{22} - n_{11}) \pm \sqrt{(n_{22} - n_{11})^2 + 4n_{12}n_{21}} \right] \quad (4.27)$$

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The group delays of the PSPs are contained in Eq. (4.26b). By using the substitution $A_o(\omega) = \sigma_o(\omega)\exp[j\phi_o(\omega)]$, Eq. (4.26b) reduces to

$$[\ln \sigma_{o\pm}(\omega) + j\phi_{o\pm}(\omega)] = n_{11} + n_{12}\chi_{o\pm} \quad (4.28)$$

Thus, the difference between the propagation characteristics of the two PSPs are given by

$$\Delta\rho = \Delta(\ln \sigma)' + jDGD = \sqrt{(n_{22} - n_{11})^2 + 4n_{12}n_{21}} \quad (4.29)$$

where $DGD = \phi'_{o+} - \phi'_{o-}$ is the differential group delay and $\Delta(\ln \sigma)' = \Delta(\sigma' / \sigma)$ is the frequency derivative of the attenuation/amplification difference (differential attenuation slope).

Since there are three input SOP independent variables n_{12} , n_{21} and $(n_{12} - n_{21})$ for the determination of PSPs and their differential propagation characteristics, a series of $K(\geq 3)$ different input polarizations are used. $\chi'_o(\omega_o)$ is approximated by $(\chi_o(\omega_o + \Delta\omega) - \chi_o(\omega_o - \Delta\omega)) / (2\Delta\omega)$. By solving this set of K linear equations, n_{12} , n_{21} and $(n_{12} - n_{21})$ can be found through Eq. (4.26a). Thus, PSPs and DGD are calculated from Eqs. (4.27) and (4.29).

4.3.1.2 Generalized Poincaré sphere method

In the year 2004, Liang Chen et al. proposed the generalized Poincaré sphere technique (GPST) for measuring the complex PMD vector [45]. The solutions for Eq.(4.23) are

$$\begin{cases} \bar{S}_p^\pm = \pm \left[\frac{\lambda}{\lambda^2 + \Lambda^2} \bar{\Omega} + \frac{(\bar{\Omega} \cdot \bar{\Lambda})}{\lambda(\lambda^2 + \Lambda^2)} \bar{\Lambda} \right] + \frac{1}{\lambda^2 + \Lambda^2} \bar{\Omega} \times \bar{\Lambda} \\ \lambda = \sqrt{\frac{1}{2} \left[\Omega^2 - \Lambda^2 + \sqrt{(\Omega^2 - \Lambda^2)^2 + 4(\bar{\Omega} \cdot \bar{\Lambda})^2} \right]} \end{cases} \quad (4.30)$$

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From Eq. (4.30), we note that the fast and slow output state Stokes vectors $\bar{S}_p^+$ and $\bar{S}_p^-$ are not orthogonal to each other in general unless $\bar{\Omega} \times \bar{\Lambda} = 0$ or $\bar{\Lambda} = 0$ (no PDL). The real and imaginary parts of complex PMD vector, DGD and differential attenuation slope (DAS), can be expressed as

$$\tau_{\text{DGD}} = \text{Re}(\sqrt{\bar{W} \cdot \bar{W}}) = \lambda; \quad \eta = \text{Im}(\sqrt{\bar{W} \cdot \bar{W}}) = \frac{\bar{\Omega} \cdot \bar{\Lambda}}{\tau_{\text{DGD}}} \quad (4.31)$$

It is important to note that DAS parameter η is related to PDL, but it is not equal to the PDL of the system.

In order to use the dynamic PMD equation to measure the PMD vector, three pairs of output Stokes vectors are used: $\bar{S}_1(\omega + \Delta\omega)$, $\bar{S}_1(\omega)$; $\bar{S}_2(\omega + \Delta\omega)$, $\bar{S}_2(\omega)$; and $\bar{S}_3(\omega + \Delta\omega)$, $\bar{S}_3(\omega)$. Using Eq. (4.22), we can get the following coupled equations at an angular frequency $\omega + \Delta\omega/2$

$$\begin{cases} \frac{d\bar{S}_1}{d\omega} \equiv \bar{\Omega} \times \bar{S}_1 - (\bar{\Lambda} \times \bar{S}_1) \times \bar{S}_1 \\ \frac{d\bar{S}_2}{d\omega} \equiv \bar{\Omega} \times \bar{S}_2 - (\bar{\Lambda} \times \bar{S}_2) \times \bar{S}_2 \\ \frac{d\bar{S}_3}{d\omega} \equiv \bar{\Omega} \times \bar{S}_3 - (\bar{\Lambda} \times \bar{S}_3) \times \bar{S}_3 \end{cases} \quad (4.32)$$

where $\bar{S}_i \equiv \bar{S}_i(\omega + \Delta\omega/2) \approx \frac{\bar{S}_i(\omega + \Delta\omega) + \bar{S}_i(\omega)}{|\bar{S}_i(\omega + \Delta\omega) + \bar{S}_i(\omega)|}$ and

$$\frac{d\bar{S}_i}{d\omega} \equiv \frac{d\bar{S}_i(\omega + \Delta\omega/2)}{d\omega} \approx \frac{\bar{S}_i(\omega + \Delta\omega) - \bar{S}_i(\omega)}{\Delta\omega}, \quad i = 1, 2, 3$$

Eq. (4.32) can be further simplified for the real part of the PMD vector

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$$\begin{cases} \bar{\Omega} \cdot (\bar{S}_1 + \bar{S}_2) = \frac{1}{1 - \bar{S}_1 \cdot \bar{S}_2} \left(\frac{d\bar{S}_2}{d\omega} - \frac{d\bar{S}_1}{d\omega} \right) \cdot (\bar{S}_1 \times \bar{S}_2) \\ \bar{\Omega} \cdot (\bar{S}_2 + \bar{S}_3) = \frac{1}{1 - \bar{S}_2 \cdot \bar{S}_3} \left(\frac{d\bar{S}_3}{d\omega} - \frac{d\bar{S}_2}{d\omega} \right) \cdot (\bar{S}_2 \times \bar{S}_3) \\ \bar{\Omega} \cdot (\bar{S}_1 + \bar{S}_3) = \frac{1}{1 - \bar{S}_1 \cdot \bar{S}_3} \left(\frac{d\bar{S}_3}{d\omega} - \frac{d\bar{S}_1}{d\omega} \right) \cdot (\bar{S}_1 \times \bar{S}_3) \end{cases} \quad (4.33)$$

and for the imaginary part of the PMD vector

$$\begin{cases} \bar{\Lambda} \cdot (\bar{S}_1 + \bar{S}_2) = \frac{1}{1 - \bar{S}_1 \cdot \bar{S}_2} \left(\bar{S}_1 \cdot \frac{d\bar{S}_2}{d\omega} + \bar{S}_2 \cdot \frac{d\bar{S}_1}{d\omega} \right) \\ \bar{\Lambda} \cdot (\bar{S}_2 + \bar{S}_3) = \frac{1}{1 - \bar{S}_2 \cdot \bar{S}_3} \left(\bar{S}_2 \cdot \frac{d\bar{S}_3}{d\omega} + \bar{S}_3 \cdot \frac{d\bar{S}_2}{d\omega} \right) \\ \bar{\Lambda} \cdot (\bar{S}_1 + \bar{S}_3) = \frac{1}{1 - \bar{S}_1 \cdot \bar{S}_3} \left(\bar{S}_1 \cdot \frac{d\bar{S}_3}{d\omega} + \bar{S}_3 \cdot \frac{d\bar{S}_1}{d\omega} \right) \end{cases} \quad (4.34)$$

By solving the Eqs. (4.33) and (4.34), $\bar{\Omega}$ and $\bar{\Lambda}$ can be obtained.

4.3.1.3 Jones matrix eigenanalysis method

The measurement methods in the first category (based on differential equations) do not require the knowledge of input states of polarization, thus only a polarization controller (PC) is needed for changing the input polarization states [44]. However, these measurement methods suffer from high noise due to high-precision demand in setting up the frequencies with a very small step size as required by differentiation [22]. The methods in the second category are based on the analysis of system transmission matrices, so the requirement for small step size is avoided which makes these methods achieve high SNR.

Jones matrix eigenanalysis method (JME) was developed by B. L. Heffner in 1992 [46]. This method measures PMD through the measurement of Jones matrix. Thus, as introduced in section 2.2.1, three specific input SOPs (linear polarization at 0°, 90° and

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45°) are required. With the frequency sweeping, the corresponding output SOP trajectories are measured. The Jones matrix $\mathbf{J}$ can be calculated at each wavelength using Eq. (2.9) and we can obtain the eigenvalue relation [46]

$$[\mathbf{J}(\omega + \Delta\omega)\mathbf{J}^{-1}(\omega) - (1 + i\tau_g \Delta\omega)\mathbf{I}]y = 0 \quad (4.35)$$

where τ_g is the group delay, y is the output PSP at frequency ω , $\mathbf{I}$ is a 2×2 unit matrix and the superscript '-1' denotes the matrix inversion.

The two PSPs are the eigenvalues of the above equation $\rho_k = 1 + i\tau_{g,k} \Delta\omega$, $k = 1, 2$ and the DGD can be expressed as

$$\Delta\tau = |\tau_{g,1} - \tau_{g,2}| = \left| \frac{\text{Arg}(\rho_1 / \rho_2)}{\Delta\omega} \right| \quad (4.36)$$

where Arg denotes the argument function, e.g., $\text{Arg}(ae^{i\theta}) = \theta$.

During the measurement, the condition $\Delta\tau \Delta\omega < \pi$ needs to be satisfied in order to avoid the ambiguities of the multiple-valued argument function.

With the PMD calculated above, Heffner proposed the method for calculation of the differential group delay dispersion (DGDD), the wavelength derivative of DGD [49]

$$D_n = \frac{2(\tau_n - \tau_{n-1})}{\lambda_n - \lambda_{n-2}} \quad (4.37)$$

where τ_n is the DGD at n-th wavelength λ_n calculated using Eq (4.6). Actually, the definition of second order DGD is the frequency derivative of DGD which is slightly different from DGDD. However, the calculation principle is the same. The only thing that needs to be changed in Eq. (4.37) is to replace the wavelengths with the frequencies.

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4.3.1.4 Generalized Mueller matrix method (GMMM)

The existing JME method is considered to be competent for measuring the PMD in a system with PDL/G. However, it only presents DGD information of the complex PMD vector [46]. As shown in section 4.2.1.3 and 4.2.2, the Mueller matrix method has been demonstrated to be a better technique to attain the low-noise high-resolution PMD data in an optical fiber system without PDL. The governing equation between the Mueller matrix and the complex PMD vector has been derived. Thus, Hui Dong et al. developed a generalized Mueller matrix method for PMD measurement in an optical fiber system with PDL [47] in 2006.

For a system with both birefringence and PDL, the 4×4 Mueller matrix $\mathbf{M}$ has been demonstrated to be corresponding to a Lorentz transformation and only 7 elements are independent. Therefore, three non-normalized input SOPs (including the optical power) are sufficient for the determination of Mueller matrix. For ease of calculation, $(s_{in0} \ s_{in0} \ 0 \ 0)^T$, $(s_{in0} \ -s_{in0} \ 0 \ 0)^T$ and $(s_{in0} \ 0 \ s_{in0} \ 0)^T$ are selected. Here, s_{in0} is the input optical power. If their corresponding output SOPs are measured using a polarimeter, the 12 elements in the first three columns of Mueller matrix can be determined directly. By using the Lorentz transformation property of Mueller matrix, $\mathbf{M}^T \mathbf{G} \mathbf{M} = \sqrt{\det \mathbf{M}} \mathbf{G}$, the last column of the Mueller matrix can be obtained

$$m_{14} = \det \begin{pmatrix} m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{pmatrix} / \sqrt{\det \mathbf{M}}, \quad m_{24} = \det \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{43} \end{pmatrix} / \sqrt{\det \mathbf{M}},$$

$$m_{34} = -\det \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{41} & m_{42} & m_{43} \end{pmatrix} / \sqrt{\det \mathbf{M}} \quad \text{and} \quad m_{44} = \det \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} / \sqrt{\det \mathbf{M}} \quad (4.38)$$

Since $\sqrt{\det \mathbf{M}}$ can be determined using $\sqrt{\det \mathbf{M}} = m_{11}^2 - m_{21}^2 - m_{31}^2 - m_{41}^2$ [20], these 4 elements can be calculated.

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When the input polarization state is fixed, the output polarization states at two consecutive optical frequencies can also be related by [22]

$$\bar{S}(\omega + \Delta\omega) = \mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega)\bar{S}(\omega) = \mathbf{M}_\Delta\bar{S}(\omega) \quad (4.39)$$

where $\mathbf{M}_\Delta = \mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega)$.

Actually, Eq. (4.39) has been used in section 4.2.1.3. The differences are that in section 4.2.1.3, the Mueller matrix has 3 degrees of freedom with size of 3×3 and $\bar{S}$ is a 3-dimensional Stokes vector under the condition that the system has no PDL, while here, Mueller matrix has 7 degrees of freedom with a size of 4×4 and $\bar{S}$ is a 4-dimensional Stokes vector.

Since the inverse of a Lorentz transformation and the product of two Lorentz transformations are still Lorentz transformations, $\mathbf{M}_\Delta$ should also be a Lorentz transformation. Its polar decomposition has the following form [13]

$$\mathbf{M}_\Delta = t_0 \begin{pmatrix} 1 & \bar{0}^T \\ \bar{0} & \mathbf{m}_R \end{pmatrix} \begin{pmatrix} 1 & \bar{D}^T \\ \bar{D} & \mathbf{m}_D \end{pmatrix} = t_0 \begin{pmatrix} 1 & \bar{D}^T \\ \mathbf{m}_R \bar{D} & \mathbf{m}_R \mathbf{m}_D \end{pmatrix} \quad (4.40)$$

where $\bar{0}^T = (0 \ 0 \ 0)$, t_0 is the transmittance for unpolarized light in frequency domain. And

$$\mathbf{m}_R = \begin{pmatrix} \cos\phi + r_1^2(1 - \cos\phi) & r_1 r_2(1 - \cos\phi) + r_3 \sin\phi & r_1 r_3(1 - \cos\phi) - r_2 \sin\phi \\ r_1 r_2(1 - \cos\phi) - r_3 \sin\phi & \cos\phi + r_2^2(1 - \cos\phi) & r_2 r_3(1 - \cos\phi) + r_1 \sin\phi \\ r_1 r_3(1 - \cos\phi) + r_2 \sin\phi & r_2 r_3(1 - \cos\phi) - r_1 \sin\phi & \cos\phi + r_3^2(1 - \cos\phi) \end{pmatrix} \quad (4.41)$$

$$\mathbf{m}_D = \begin{pmatrix} \sqrt{1-D^2} + (1-\sqrt{1-D^2})d_1^2 & (1-\sqrt{1-D^2})d_1 d_2 & (1-\sqrt{1-D^2})d_1 d_3 \\ (1-\sqrt{1-D^2})d_1 d_2 & \sqrt{1-D^2} + (1-\sqrt{1-D^2})d_2^2 & (1-\sqrt{1-D^2})d_2 d_3 \\ (1-\sqrt{1-D^2})d_1 d_3 & (1-\sqrt{1-D^2})d_2 d_3 & \sqrt{1-D^2} + (1-\sqrt{1-D^2})d_3^2 \end{pmatrix} \quad (4.42)$$

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where $\hat{r} = (r_1 \ r_2 \ r_3)^T$ is a unit vector, ϕ is the rotation angle around $\hat{r}$ in Stokes space, and $\bar{D} = D(d_1 \ d_2 \ d_3)^T$ is a vector representing PDL/G in frequency domain. We know that [18]

$$\frac{d\mathbf{M}}{d\omega}\mathbf{M}^{-1} = \lim_{\Delta\omega \rightarrow 0} \frac{\mathbf{M}_\Delta - \mathbf{I}}{\Delta\omega} = \begin{pmatrix} \eta_\omega & \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Lambda_1 & \eta_\omega & -\Omega_3 & \Omega_2 \\ \Lambda_2 & \Omega_3 & \eta_\omega & -\Omega_1 \\ \Lambda_3 & -\Omega_2 & \Omega_1 & \eta_\omega \end{pmatrix} \quad (4.43)$$

where $\mathbf{I}$ is an identity matrix, $\eta_\omega = \frac{d \ln \sqrt{\det \mathbf{M}}}{d\omega}$, $\bar{\Omega} = (\Omega_1 \ \Omega_2 \ \Omega_3)$ and $\bar{\Lambda} = (\Lambda_1 \ \Lambda_2 \ \Lambda_3)$ are the real and imaginary parts of the complex PMD vector $\bar{W}$. Comparing Eq. (4.40) and Eq. (4.43), assuming that the higher-order PMD effects between the two frequencies are negligible, we have

$$\bar{\Lambda} = t_0 \bar{D} / \Delta\omega, \quad \bar{\Omega} = \phi \hat{r} / \Delta\omega \quad (4.44)$$

Here, ϕ and $\hat{r}$ can be extracted from $\mathbf{m}_R$ based on Eq. (4.8) and Eq. (4.9). To avoid measurement ambiguities, the condition $|\bar{\Omega}| \Delta\omega < \pi$ should be satisfied. Finally, DGD and DAS can be calculated using Eqs. (4.30) and (4.31).

4.3.1.5 Virtual generalized Mueller matrix method (VGMMM)

Compared with the methods in the first category, the above JME and GMMM methods also have drawbacks. Since the knowledge of the input polarization states is required, not only that the system configuration is more complex, but also setup of the predefined input polarization states tends to introduce errors. In order to solve these problems, a virtual GMMM (VGMMM) was proposed [48] by Hui Dong et al. in 2007. Compared to previous methods, this technique measures the complex PMD vector using a relatively large frequency step without the knowledge of three input polarization states. So, it combines the advantages of both matrix-based methods and differentiation-based methods and overcomes their shortcomings. Measurement accuracy can be greatly

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improved by optimizing the frequency step using a theoretical relation and system setup is simplified as the knowledge of input states is not required.

For a general optical fiber system with PDL/G, if the input polarization state $\vec{S}_{in} = (s_{in0} \ s_{in1} \ s_{in2} \ s_{in3})^T$ is fixed and the optical frequency is swept, at the output end of the system, the output polarization states at two adjacent frequencies are related by Eq. (4.39). The matrix $\mathbf{M}_\Delta(\omega) = \mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega)$ contains the information of the complex PMD vector. Based on the polar decomposition of $\mathbf{M}_\Delta(\omega)$, the complex PMD vector can be determined easily with a relatively large $\Delta\omega$ [47]. However, in order to obtain $\mathbf{M}_\Delta(\omega)$, we have to measure the Mueller matrices at two adjacent frequencies. This is the step that requires knowledge of input states of polarization [47]. In order to overcome this problem, we can consider it from another point of view. Based on Eq. (4.39), $\mathbf{M}_\Delta(\omega)$ can be determined by $\vec{S}_{out}(\omega)$ and $\vec{S}_{out}(\omega + \Delta\omega)$ directly as long as $\vec{S}_{in}$ is fixed, thus the knowledge of $\vec{S}_{in}$ is not required at all.

Since $\mathbf{M}_\Delta(\omega)$ corresponds to a Lorentz transformation [47], it can be determined using three inputs. For three fixed but unknown input polarization states $\vec{S}_{in}$, $\vec{T}_{in}$ and $\vec{U}_{in}$, their corresponding outputs are measured at two adjacent frequencies as $\vec{S}_{out}(\omega)$, $\vec{S}_{out}(\omega + \Delta\omega)$, $\vec{T}_{out}(\omega)$, $\vec{T}_{out}(\omega + \Delta\omega)$, $\vec{U}_{out}(\omega)$ and $\vec{U}_{out}(\omega + \Delta\omega)$. With these 3 pairs of outputs at two frequencies, $\mathbf{M}_\Delta(\omega)$ can be calculated using the method presented in the first part of section 2.3.2.2. And the optimum accuracy is achieved when the three input SOP are coplanar and the angles between them are 120° in Stokes space [e.g., $(1 \ 0 \ 0)^T$, $(-1/2 \ \sqrt{3}/2 \ 0)^T$, and $(-1/2 \ -\sqrt{3}/2 \ 0)^T$].

The complex PMD vector can be extracted from $\mathbf{M}_\Delta(\omega)$ according to Eqs. (4.40-42) and (4.44); the differential group delay and differential attenuation slope can be obtained from Eq. (4.31). GMMM works fine if the higher order PMD effects are negligible at two adjacent frequencies, which means the product $\text{DGD} \cdot \Delta\omega$ should be small enough. But

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too small a $\Delta\omega$ may cause inaccuracies because of the equipment limitation. So there is a tradeoff for the choice of a suitable $\Delta\omega$.

4.3.2 Method proposed: Flexible input Mueller matrix method (FIMMM)

From the above description, we can see that for PMD measurement, we have to sweep input optical frequencies and change the SOPs under each frequency. There are two ways to perform the measurement process.

The first is to set one SOP then sweep wavelength and repeat this process for other SOPs. This process can get good SOP accuracy; however, the repeatability of the laser source becomes a problem. The drift of the wavelength will cause the output SOP to rotate and lead to erroneous results especially for large PMD condition. In table 4.6, the absolute wavelength accuracy and the wavelength repeatability for some commonly used tunable laser sources are given. The absolute wavelength accuracy means the maximum difference between the actual wavelength and the displayed wavelength of the TLS. It will result in the inaccuracy of the wavelength step size and affect the calculated PMD values. By applying wavelength meter during the measurement, this problem can be solved. Wavelength repeatability is the random uncertainty in reproducing a wavelength after detuning and re-setting the wavelength. Most of the PMD vector measurement methods require multiple input SOPs under same wavelength, but the wavelength repeatability problem makes this requirement impossible to meet. Because of the randomness of the wavelength repeatability, it is difficult to overcome this problem. Besides the wavelength repeatability, the tuning speed of laser sources is not fast (hundreds of milliseconds to several seconds). Thus, the measurement speed for this sweep method is slow.

Table 4.6 Absolute wavelength accuracy and wavelength repeatability for some tunable laser sources.

Tunable Laser Source	Absolute Wavelength Accuracy	Wavelength Repeatability
----------------------	------------------------------	--------------------------

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Anritsu MG9637A; MG9638A	$< \pm 0.1 \text{ nm}$	$\pm 0.035 \text{ nm}$ (80 nm range); $\pm 0.007 \text{ nm}$ (at $\pm 3 \text{ nm}$ after calibration)
Anritsu MG9541	$\pm 0.055 \text{ nm}$ (Valid for 10 h after wavelength calibration and at constant temperature)	$\pm 0.035 \text{ nm}$ (constant temperature)
Ando AQ8201-13/- 13B ECL; AQ8201-13A/-13D ECL.	$< \pm 0.2 \text{ nm}$	$\pm 0.050 \text{ nm}$
Agilent 81480A; 81680A; 81682A.	$\pm 0.01 \text{ nm}$	$\pm 0.001 \text{ nm}$, typ. $\pm 0.0005 \text{ nm}$
Agilent 81640A; 81642A.	$\pm 0.015 \text{ nm}$	$\pm 0.001 \text{ nm}$, typ. $\pm 0.0005 \text{ nm}$
Agilent 81689A.	$\pm 0.3 \text{ nm}$	$\pm 0.05 \text{ nm}$

The second way of conducting measurements is to set one wavelength then sweep SOPs. Since the calculations of all these methods need to use the output SOPs at different wavelength with the same input SOP, we need to use either a manual or computer-controlled PC to tune the input SOP and use an in-line polarimeter to monitor the input SOP until it reaches the desired SOP. Because of the SOP measurement accuracy of the inline polarimeter, the input SOPs at different wavelengths may not be the same. This will also cause erroneous PMD measurement results. Table 4.7 gives the SOP measurement accuracy of some in-line polarimeters.

Table 4.7 SOP accuracy of some polarimeters

Polarization Analyzer (Polarimeter)	SOP Measurement Accuracy
Thorlabs TXP IPM5300 Fast Inline Polarimeter.	$\pm 0.25 \text{ deg.}$ on Poincaré Sphere
General Photonics PolaDetect™ POD-101A in-line polarimeter.	1% max. of Agilent 8509C

In order to overcome the problems in the sweep process, we propose a new method, called the flexible input Mueller matrix method (FIMMM). From the above analysis, we

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can see that the problems come from the TLS repeatability for the first method and input SOP setting accuracy for the second method. From Eqs. (4.39 - 4.41), we already know that the PMD vector is fully described by the matrix $\mathbf{M}_\Delta = \mathbf{M}(\omega + \Delta\omega)\mathbf{M}^{-1}(\omega)$ in Eq. (4.39) ($\mathbf{M}(\omega + \Delta\omega)$, $\mathbf{M}(\omega)$ are the system Mueller matrices under frequency $\omega + \Delta\omega$ and ω). So, if we can measure the system transmission matrices under the corresponding frequencies, the PMD vector can be derived accordingly. In section 2.3.2.2 we have proven that for the system without depolarization, the system Mueller matrix can be measured with 3 pairs of input and out SOPs. This implies that the matrix $\mathbf{M}_\Delta$ can be measured with flexible input SOPs under different frequencies as long as they are not linearly dependent. Thus, the measurement process becomes: sweep the wavelength once with 3 input SOPs at each wavelength and the 3 input SOPs need not be the same for different wavelengths. So the TLS repeatability problem and input SOP accuracy problem are solved. Since wavelength is only required to be swept once and the input SOPs under different wavelengths are flexible, the measurement speed of FIMMM is also improved.

The experimental setup is shown in Fig. 4.10. The laser source is Anritsu MG9638A and the two polarimeters are both Thorlabs TXP IPM5300 Fast Inline Polarimeter. Here, the inline polarimeter before the FUT is just used to measure the input SOP which is different from the GMMM. After the second inline polarimeter, an OSA or wavelength meter could be used to measure the wavelength. Since, we only want to see the effects of laser source repeatability and polarimeter accuracy, the OSA is not connected. Computer-controlled PC tunes input SOP through 3 control voltages. Before the measurement, The PC is calibrated to achieve 3 input coplanar SOPs with 120° angles in between in Stokes space [e.g., $(1 \ 0 \ 0)^T$, $(-1/2 \ \sqrt{3}/2 \ 0)^T$, and $(-1/2 \ -\sqrt{3}/2 \ 0)^T$], in the middle of the PMD measurement wavelength range. During the measurement, the PC is set according to the calibration. Since the PDL and PMD of the fiber link from the laser source to inline polarimeter (1.5 m SMF and PC) is quite small, the input SOPs under different wavelengths still remain angles close to 120° although the Stokes parameters may be different. Thus, the optimum accuracy is achieved.

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Fig. 4.10 Experimental configuration for PMD measurement in an optical fiber system.

In our experiment, the fiber system under test (FUT) comprises four PMF sections (3m-long length and 4ps DGD for each) interleaved by two single-mode fiber sections (1m-long for each) and 1 m side-polished PMF as a PDL component. In order to simulate distributed PDL, the second PMF section is spooled on a drum with a 1.5cm diameter, so bending-induced PDL is induced [32].. The measurement results using FIMMM and VGMMM (first sweeping process) with 0.2 nm wavelength step size are shown in Fig. 4.11. The agreement is very good. From Table 4.6, we know that the wavelength repeatability for Anritsu MG9638A is ± 0.035 nm. In order to see the effect of wavelength repeatability, we set wavelength step size to 0.1 nm so that the repeatability is comparable with the step size. This time, as shown in Fig. 4.12, the SNR of VGMMM is worse than FIMMM.

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(b)

Fig. 4.11 Measured complex PMD vector (a) $\tilde{\Omega}$ and (b) $\tilde{\Lambda}$ using FIMMM (solid lines) and VGMMM (dashed lines) for wavelength step size 0.2 nm.

(a)

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Fig. 4.12 Measured complex PMD vector (a) $\bar{\Omega}$ and (b) $\bar{\Lambda}$ have worse SNR using VGMMM (dashed lines) compared with FIMMM (solid lines) when the wavelength step size is 0.1 nm.

The effect of polarimeter measurement accuracy can be seen clearly when birefringence and PDL are small. Fig. 4.13 shows the results when the side-polished PMF and one section of PMF are removed. For smaller birefringence and PDL, the output SOP rotation is smaller for a fixed wavelength step. Thus, polarimeter measurement accuracy plays an important role during the measurement especially for the second sweeping process.

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(b)

Fig. 4.13 Measured complex PMD vector (a) $\tilde{\Omega}$ and (b) $\tilde{\Lambda}$ have worse SNR using VGMMM (dashed lines) compared with FIMM (solid lines) when using the second sweeping process.

4.4 Conclusion

The methods for the measurement of PMD vector have been studied in this chapter. Following the objective of improving the measurement efficiency and reducing the number of inputs, a new round trip method is developed for the PMD measurement when the system has no PDL/G. The measurement accuracy for the existing methods are also discussed.

In the presence of PDL/G, the PMD vector becomes a complex vector. The signal to noise ratio (SNR) for the existing differential methods is relatively low and the methods based on the system transmission matrix either cannot fully retrieve the complex PMD vector or have relative large equipment error induced measurement uncertainty. Thus a new method, flexible input Mueller matrix method, is proposed. The proposed method can achieve better SNR and measurement efficiency.

Chapter 5 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The basic polarization effects of optical fiber systems have been examined. For systems that have different polarization effects, the numbers of degrees of freedom are different. Thus the measurement of system transmission matrix requires different number of input states of polarization (SOPs). Based on the fact that the Mueller matrix of a non-depolarizing system satisfies the Lorentz transformation, we proved that three arbitrary inputs are sufficient to measure the Mueller matrix and no input parameters are redundant. Further, a convenient way to calculate the Mueller matrix with these 3 pairs of input and output SOPs is presented in order to avoid nonlinear equations that arise using traditional calculations. We also proved that when the three input SOPs are coplanar, the angles between each other are 120 degrees and their magnitudes are equal to 1, the measurement achieves the optimum accuracy.

Polarization dependent loss or gain (PDL/G) is one of the basic polarization effects in optical fiber system. The existing PDL/G measurement methods have been introduced and compared. In order to increase the measurement efficiency, two new PDL/G measurement methods, two-states method and unpolarized light method, are proposed for non-depolarizing system. Consequently, guidance for choosing a suitable PDL/G measurement method for different conditions is provided.

The polarization mode dispersion (PMD) measurement methods can be classified into two categories: methods for the system with and without PDL/G. In each category, the methods can be further divided into two kinds: methods based on the differential equations and methods based on the analysis of the system transmission matrices. For systems without PDL/G, there are many existing methods. In order to reduce the measurement time and the number of input SOPs, we proposed round trip method which uses only one input SOP. For systems with PDL/G, the differential methods do not require the information of input SOPs but the SNR is relative poor especially for large wavelength step size. For matrix based methods, Jones matrix eigenanalysis method (JME)

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cannot provide the differential attenuation slope information, generalized Mueller matrix method (GMMM) and virtually generalized Mueller matrix method (VGMMM) cannot avoid the measurement error by the equipments. So, the flexible input Mueller matrix method (FIMMM) is proposed. Actually, GMMM, VGMMM and FIMMM are three successive stages for complex PMD vector measurement. Firstly, generalized Mueller matrix method provides the theoretical method to retrieve complex PMD vector information from Mueller matrix. Next, virtual generalized Mueller matrix method combines the advantages of differential methods and matrix methods, and reduces the requirements for input SOPs. Finally, flexible input Mueller matrix method tries to reduce the effects of equipment measurement errors to the minimum.

5.2 Recommendations for further research

The proposed PMD vector measurement method FIMMM can calculate PMD using the input and output SOPs at two adjacent wavelengths during measurement. Thus, it has the potential to choose the wavelength step size dynamically. For all the current measurement methods, the wavelength step size is fixed over the entire measurement wavelength range. However, the rate of change of PMD with wavelength is not a constant. The wavelength step size should be smaller when PMD changes dramatically. In chapter 4, we demonstrated that the SNR of PMD measurement is closely related to the wavelength step size. This also requires choosing a suitable wavelength step size dynamically during the measurement. So, we can further work on the determination of wavelength step size criteria and combine it with FIMMM.

Most of the PDL/G and PMD measurement methods we discussed need to launch the light from one end of the fiber link and measure the output from another side. For components measurements, these methods are good. However, for real underground fiber links, which are usually tens to hundreds of kilometers long, it is not practical to measure the input and output SOPs at the same time. So, we need to develop single-end or round-trip measurement methods for PDL/G and complex PMD vector measurement. The so

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called polarimetric optical time domain reflectometry (POTDR) is one of the technologies that can achieve this aim.

APPENDIX 1: CONVERSION FROM JONES MATRIX TO MUELLER MATRIX

Suppose the Jones matrix that needs to be converted is

$$\mathbf{J} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \quad (\text{A1.1})$$

and its conjugate matrix is

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad (\text{A1.2})$$

Then the corresponding Mueller matrix is [50]

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \quad (\text{A1.3})$$

where

$$m_{11} = (G_{11}J_{11} + G_{21}J_{21} + G_{12}J_{12} + G_{22}J_{22})/2, \quad m_{12} = (G_{11}J_{11} + G_{21}J_{21} - G_{12}J_{12} - G_{22}J_{22})/2,$$

$$m_{13} = (G_{11}J_{12} + G_{21}J_{22} + G_{12}J_{11} + G_{22}J_{21})/2, \quad m_{14} = i(G_{11}J_{12} + G_{21}J_{22} - G_{12}J_{11} - G_{22}J_{21})/2,$$

$$m_{21} = (G_{11}J_{11} + G_{12}J_{12} - G_{21}J_{21} - G_{22}J_{22})/2, \quad m_{22} = (G_{11}J_{11} + G_{22}J_{22} - G_{21}J_{21} - G_{12}J_{12})/2,$$

$$m_{23} = (G_{12}J_{11} + G_{11}J_{12} - G_{22}J_{21} - G_{21}J_{22})/2, \quad m_{24} = i(G_{11}J_{12} + G_{22}J_{21} - G_{21}J_{22} - G_{12}J_{11})/2,$$

$$m_{31} = (G_{11}J_{21} + G_{21}J_{11} + G_{12}J_{22} + G_{22}J_{12})/2, \quad m_{32} = (G_{11}J_{21} + G_{21}J_{11} - G_{12}J_{22} - G_{22}J_{12})/2,$$

$$m_{33} = (G_{11}J_{22} + G_{21}J_{12} + G_{12}J_{21} + G_{22}J_{11})/2, \quad m_{34} = i(G_{11}J_{22} + G_{21}J_{12} - G_{12}J_{21} - G_{22}J_{11})/2,$$

Appendix 1

$$m_{41} = i(G_{21}J_{11} + G_{22}J_{12} - G_{11}J_{21} - G_{12}J_{22})/2, \quad m_{42} = i(G_{21}J_{11} + G_{12}J_{22} - G_{11}J_{21} - G_{22}J_{12})/2,$$

$$m_{43} = i(G_{21}J_{12} + G_{22}J_{11} - G_{11}J_{22} - G_{12}J_{21})/2, \quad m_{44} = (G_{22}J_{11} + G_{11}J_{22} - G_{12}J_{21} - G_{21}J_{12})/2,$$

And $i = \sqrt{-1}$.

APPENDIX 2: THE SOLVABLE CONDITION AND OPTIMUM ACCURACY FOR NON-DEPOLARIZATION MUELLER MATRIX MEASUREMENT

In this appendix, the solvable condition and the condition for the optimum accuracy of Eq. (2.27) are discussed [16].

Eq. (2.28) gives the direct condition that Eq. (2.27) can be solved, which is the determinant of the coefficient matrix cannot be equal to 0. And inversely we can consider the condition for the optimum accuracy is that the absolute value of this determinant can be up to its maximum. Because Eq. (2.28) only indicates the requirements of the output Stokes parameters, we have to find the requirement of the input Stokes parameters at first. From Eq. (2.27), we have

$$\begin{pmatrix} iS_{out0} & S_{out1} & S_{out2} & S_{out3} \\ it_{out0} & t_{out1} & t_{out2} & t_{out3} \\ iu_{out0} & u_{out1} & u_{out2} & u_{out3} \\ A_{out0} & A_{out1} & A_{out2} & A_{out3} \end{pmatrix} \begin{pmatrix} iv_{out0} \\ v_{out1} \\ v_{out2} \\ v_{out3} \end{pmatrix} = \sqrt{|\mathbf{M}|} \begin{pmatrix} iS_{in0} & S_{in1} & S_{in2} & S_{in3} \\ it_{in0} & t_{in1} & t_{in2} & t_{in3} \\ iu_{in0} & u_{in1} & u_{in2} & u_{in3} \\ \sqrt{|\mathbf{M}|}A_{in0} & \sqrt{|\mathbf{M}|}A_{in1} & \sqrt{|\mathbf{M}|}A_{in2} & \sqrt{|\mathbf{M}|}A_{in3} \end{pmatrix} \begin{pmatrix} iv_{in0} \\ v_{in1} \\ v_{in2} \\ v_{in3} \end{pmatrix} \quad (\text{A2.1})$$

Take the notations that $\mathbf{F}_{out} = \begin{pmatrix} iS_{out0} & S_{out1} & S_{out2} & S_{out3} \\ it_{out0} & t_{out1} & t_{out2} & t_{out3} \\ iu_{out0} & u_{out1} & u_{out2} & u_{out3} \\ A_{out0} & A_{out1} & A_{out2} & A_{out3} \end{pmatrix}$, $\mathbf{F}_{in} = \begin{pmatrix} iS_{in0} & S_{in1} & S_{in2} & S_{in3} \\ it_{in0} & t_{in1} & t_{in2} & t_{in3} \\ iu_{in0} & u_{in1} & u_{in2} & u_{in3} \\ A_{in0} & A_{in1} & A_{in2} & A_{in3} \end{pmatrix}$

and $\tilde{\mathbf{F}}_{in} = \begin{pmatrix} iS_{in0} & S_{in1} & S_{in2} & S_{in3} \\ it_{in0} & t_{in1} & t_{in2} & t_{in3} \\ iu_{in0} & u_{in1} & u_{in2} & u_{in3} \\ \sqrt{|\mathbf{M}|}A_{in0} & \sqrt{|\mathbf{M}|}A_{in1} & \sqrt{|\mathbf{M}|}A_{in2} & \sqrt{|\mathbf{M}|}A_{in3} \end{pmatrix}$, then $\mathbf{F}_{out} \vec{v}_{out} = \sqrt{|\mathbf{M}|} \tilde{\mathbf{F}}_{in} \vec{v}_{in}$. Due

to $\vec{v}_{out} = \mathbf{M} \vec{v}_{in}$, we have

$$\mathbf{F}_{out} \mathbf{M} \vec{v}_{in} = \sqrt{|\mathbf{M}|} \tilde{\mathbf{F}}_{in} \vec{v}_{in} \quad (\text{A2.2})$$

Appendix 2

Because $\vec{V}_{in}$ can be arbitrary, the square matrices on the left and right sides must be equal

$$\mathbf{F}_{out} \mathbf{M} = \sqrt{|\mathbf{M}|} \tilde{\mathbf{F}}_{in} \quad (\text{A2.3})$$

Finally we obtain a relation as

$$|\mathbf{F}_{out}| = |\mathbf{M}| |\tilde{\mathbf{F}}_{in}| = |\mathbf{M}|^{\frac{3}{2}} |\mathbf{F}_{in}| \quad (\text{A2.4})$$

Based on Eq. (A2.4), we can study the above-mentioned conditions using the inputs if we consider the matrix determinant as the criteria. By calculation, we can obtain

$$|\mathbf{F}_{in}| = s_{in0}^2 t_{in0}^2 u_{in0}^2 \left\{ \left[\hat{S}_{in} \cdot (\hat{T}_{in} \times \hat{U}_{in}) \right]^2 - \left| (\hat{T}_{in} - \hat{S}_{in}) \times (\hat{U}_{in} - \hat{S}_{in}) \right|^2 \right\} \quad (\text{A2.5})$$

Here “ $\wedge$ ” indicates a normalized 3-D Stokes vector. In Fig. A2.1, we show the geometrical meaning of Eq. (A2.5). The first item inside the bracket of Eq. (A2.5) stands for the square of the volume of a parallelepiped composed of $\hat{S}_{in}$, $\hat{T}_{in}$ and $\hat{U}_{in}$; the second item denotes the 4-time value of the square of the area of the triangle shadowed in the figure.

Fig. A2.1. Geometrical meaning of Eq. (A2.5).

1. The solvable condition

The solvable condition of Eq. (2.27) is $|\mathbf{F}_{out}| \neq 0$, viz., $|\mathbf{F}_{in}| \neq 0$ which can be observed from Eq. (A2.4). From Eq. (A2.5), we know there are two cases that $|\mathbf{F}_{in}|$ is possible to be equal to 0, which are discussed as below.

Appendix 2

$$1) \hat{S}_{in} \cdot (\hat{T}_{in} \times \hat{U}_{in}) = 0 \text{ and } (\hat{T}_{in} - \hat{S}_{in}) \times (\hat{U}_{in} - \hat{S}_{in}) = 0$$

From the first equation, we know $\hat{S}_{in}$, $\hat{T}_{in}$ and $\hat{U}_{in}$ are coplanar in Stokes space. In this case, we know the second equation stands for the shadow area in Fig. A2.2. Then

$$|\mathbf{F}_{in}| = -4s_{in0}^2 t_{in0}^2 u_{in0}^2 (\text{Area of the shadow region})^2 \quad (\text{A2.6})$$

Fig. A2.2. Geometrical meaning of Eq. (A2.6).

So in this case, when the area of the shadow region is zero, Eq. (2.27) can not be solved. This will happen when $\hat{T}_{in} - \hat{S}_{in}$ is parallel to $\hat{U}_{in} - \hat{S}_{in}$. While the inputs are completely polarized lights, which are typically used in real measurements, the condition becomes the three inputs can not be linearly superposed.

$$2) |\hat{S}_{in} \cdot (\hat{T}_{in} \times \hat{U}_{in})| = |(\hat{T}_{in} - \hat{S}_{in}) \times (\hat{U}_{in} - \hat{S}_{in})|$$

We can easily derive $\hat{S}_{in} \cdot [(\hat{T}_{in} - \hat{S}_{in}) \times (\hat{U}_{in} - \hat{S}_{in})] = \hat{S}_{in} \cdot (\hat{T}_{in} \times \hat{U}_{in})$. This means two relations must be satisfied as: $|\hat{S}_{in}| = 1$ and $\hat{S}_{in}$ is parallel to $(\hat{T}_{in} - \hat{S}_{in}) \times (\hat{U}_{in} - \hat{S}_{in})$ (Viz., $\hat{S}_{in}$ is orthogonal to $\hat{T}_{in} - \hat{S}_{in}$ and $\hat{U}_{in} - \hat{S}_{in}$). In Fig. A2.3, it is easily observed that $|\hat{T}_{in}|$ and $|\hat{U}_{in}|$ will larger than 1 under such relations. But this is impossible. So $|\mathbf{F}_{in}|$ will not be 0 in this case.

Fig. A2.3. Geometrical relations under condition 2.

In conclusion, the solvable condition for Eq. (2.27) is the three inputs are not linearly superposed in Stokes space while they are completely polarized.

2. The condition for optimum accuracy

We consider the condition for optimum accuracy is that the absolute value of the determinant achieves its maximum. Based on Eq. (A2.4), $\text{abs}(|\mathbf{F}_{\text{out}}|)$ and $\text{abs}(|\mathbf{F}_{\text{in}}|)$ reach their maximum simultaneously.

From Eq. (A2.5), we easily know if one item inside the bracket is equal to 0, and another item is up to its maximum value at the same time, $\text{abs}(|\mathbf{F}_{\text{in}}|)$ reaches its maximum.

$$1) \hat{S}_{\text{in}} \cdot (\hat{T}_{\text{in}} \times \hat{U}_{\text{in}}) = 0$$

Now $\hat{S}_{\text{in}}$, $\hat{T}_{\text{in}}$ and $\hat{U}_{\text{in}}$ are coplanar, and then

$$\text{abs}(|\mathbf{F}_{\text{in}}|) = s_{\text{in}0}^2 t_{\text{in}0}^2 u_{\text{in}0}^2 |(\hat{T}_{\text{in}} - \hat{S}_{\text{in}}) \times (\hat{U}_{\text{in}} - \hat{S}_{\text{in}})|^2 \quad (\text{A2.7})$$

By calculation, we easily confirm $|(\hat{T}_{\text{in}} - \hat{S}_{\text{in}}) \times (\hat{U}_{\text{in}} - \hat{S}_{\text{in}})|^2$ will achieve its maximum value in this case. The requirements are that $\hat{S}_{\text{in}}$, $\hat{T}_{\text{in}}$ and $\hat{U}_{\text{in}}$ are with 120 degrees between them, and their magnitudes are equal to 1, as shown in the Fig. A2.4.

Fig. A2.4. Geometrical relations of three normalized inputs for optimum accuracy.

At present, we easily obtain that $\text{abs}(\mathbf{F}_{\text{in}}) = 6 \frac{3}{4} s_{\text{in}0}^2 t_{\text{in}0}^2 u_{\text{in}0}^2$. As a comparison, $\text{abs}(\mathbf{F}_{\text{in}}) = 4 s_{\text{in}0}^2 t_{\text{in}0}^2 u_{\text{in}0}^2$ when three inputs are $(1 \ 0 \ 0)^T$, $(-1 \ 0 \ 0)^T$ and $(0 \ 1 \ 0)^T$.

$$2) (\hat{T}_{\text{in}} - \hat{S}_{\text{in}}) \times (\hat{U}_{\text{in}} - \hat{S}_{\text{in}}) = 0$$

In this case, we already know $\hat{S}_{\text{in}} \cdot [(\hat{T}_{\text{in}} - \hat{S}_{\text{in}}) \times (\hat{U}_{\text{in}} - \hat{S}_{\text{in}})] = \hat{S}_{\text{in}} \cdot (\hat{T}_{\text{in}} \times \hat{U}_{\text{in}})$.

If $(\hat{T}_{\text{in}} - \hat{S}_{\text{in}}) \times (\hat{U}_{\text{in}} - \hat{S}_{\text{in}}) = 0$, then $\hat{S}_{\text{in}} \cdot (\hat{T}_{\text{in}} \times \hat{U}_{\text{in}}) = 0$. So

$$\text{abs}(\mathbf{F}_{\text{in}}) = s_{\text{in}0}^2 t_{\text{in}0}^2 u_{\text{in}0}^2 \text{abs}\left\{\left[\hat{S}_{\text{in}} \cdot (\hat{T}_{\text{in}} \times \hat{U}_{\text{in}})\right]^2\right\} = 0.$$

In conclusion, the maximum value will be achieved when $\hat{S}_{\text{in}}$, $\hat{T}_{\text{in}}$ and $\hat{U}_{\text{in}}$ are coplanar, their angles between each other are 120 degrees and their magnitudes are equal to 1.

APPENDIX 3: MEASUREMENT SYSTEM AND CONTROL SOFTWARE

The experimental setup of our measurement system consists of a tunable laser source (TLS), computer-controlled polarization controller (PC), three polarimeters, and an optical spectrum analyser (OSA) as shown in Fig. A3.1. The whole system is controlled and synchronized by a computer. The laser source is Anritsu MG9638A tunable laser source which communicates with the computer through NI-488.2 PC-GPIB interface. The PC is General Photonics PolaRite II polarization controller which is controlled by voltage signals generated by an ICP DAS PIO-DA16 multi-channel D/A board. Two of the three polarimeters are Thorlabs TXP Series fast inline polarimeter IPM5300. They are plugged into the Thorlabs TXP Series 4-channel test and measurement platform TXP5004. The communication between TXP5004 and the computer is through a USB cable. The third one is Thorlabs polarimeter model PA430. It communicates with computer through RS-232 serial port. Inline polarimeter measures the SOP through tapping a small amount of light power from the light path. Actually, the SOP measured by inline polarimeter is different from the SOPs at its input and output ports because of the fiber pigtails inside. But this does not affect the measurement because we can treat the short pigtails of the inline polarimeter as part of our fiber system under test. In some circumstance, the exact SOP at the fiber end needs to be measured, such as $\bar{S}$ in round trip PMD measurement. Thus, the ordinary polarimeter is required. This kind of polarimeter measures SOP using a photo detector with rotating waveplates. Since the coupling from the fiber end to the polarimeter is free space, the SOP is not changed. Some times, this kind of polarimeter is called ‘end polarimeter’ in order to differentiate with inline polarimeter. In section 4.3.2, the wavelength repeatability and absolute wavelength accuracy are introduced. So, an OSA, Ando AQ6317B, is used to measure the wavelength. It also communicates with the computer through 488.2 interface.

Fig A3.1 Experimental setup of our measurement system

Fig. A3.2 Flow chart of the control software

Appendix 3

The control software for our measurement system is written in MS Visual Basic 6.0 (VB). Since the driver for Thorlabs TXP5004 is only for C programming language, a dynamic link library (DLL) is written to wrap the original driver.

The software flow chart is shown in Fig. A3.2. All the hardware configurations and communication parameters are stored in an INI file which is loaded during control software initialization. Then, the equipments are initialized according to the settings. The use of INI and initialization process makes this software easy to incorporate new equipments. After initialization, the software gives a function list as shown in Fig. A3.3, from which the user can choose the desired one.

Fig. A3.3 Main functions of the control software

Here, the options of “Polarization Controller” and “Anritsu TLS Control Panel” will lead to the control panels for these two equipments. If PDL or PMD measurement options are selected, user can future select desired measurement methods as shown in Fig. A3.4. Some of the PMD measurement methods, such as Poincaré sphere technique, C. D. Poole’s method, and Generalized Poincaré sphere method, do not have corresponding options because they can use the measured data of JME or MMM.

(a)

(b)

Fig. A3.4 Choosing of (a) PDL and (b) PMD measurement methods.

In each measurement window, the instructions on how to connect fiber system or component under test with our measurement system are given for the convenience of users. Fig. A3.5 is an example.

Appendix 3

Fig. A3.5 Measurement window for FIMMM and input SOPs calibration window.

All the measurement methods and equipment settings can be executed through menu commands.

APPENDIX 4: THE PROOF OF REAL SOLUTION FOR ROUND TRIP PMD MEASUREMENT METHOD

We can prove that the results determined by Eq. (4.19) are real through the following steps.

If the items under square root in Eq. (4.19) are equal or greater than zero, then Ω_1 is real.

The items are

$$\begin{aligned}
 (s_1^2 + s_2^2)\alpha - s_3'^2 &= (s_1^2 + s_2^2) \frac{|T|^2 - 4s_3'^2}{4s_3^2} - 4s_3'^2 \\
 &= \frac{s_1^2 + s_2^2}{4s_3^2} \left(|T|^2 - 4s_3'^2 - \frac{4s_3^2 s_3'^2}{s_1^2 + s_2^2} \right) \\
 &= \frac{s_1^2 + s_2^2}{4s_3^2} \left(|T|^2 - 4 \frac{s_3'^2}{s_1^2 + s_2^2} \right) \\
 &= \frac{|T|^2 (s_1^2 + s_2^2) - 4s_3'^2}{4s_3^2} \tag{A4.1}
 \end{aligned}$$

The sign of Eq. (A4.1) is determined by its numerator. Using Eqs. (4.18), (4.21) and $s_1^2 + s_2^2 + s_3^2 = 1$, we have

$$\begin{aligned}
 |T|^2 (s_1^2 + s_2^2) - 4s_3'^2 &= \left[4(\Omega_2^2 + \Omega_1^2)s_3^2 + 4s_3'^2 \right] (s_1^2 + s_2^2) - 4s_3'^2 \\
 &= 4(\Omega_2^2 + \Omega_1^2)(s_1^2 + s_2^2)s_3^2 + 4s_3'^2 (s_1^2 + s_2^2) - 4s_3'^2 \\
 &= 4(\Omega_2^2 + \Omega_1^2)(s_1^2 + s_2^2)s_3^2 - 4s_3'^2 s_3^2 \\
 &= 4(\Omega_2^2 + \Omega_1^2)(s_1^2 + s_2^2)s_3^2 - 4s_3^2 (-\Omega_2 s_1 + \Omega_1 s_2)^2
 \end{aligned}$$

Appendix 4

$$\begin{aligned}
&= 4(\Omega_1^2 + \Omega_2^2)(s_1^2 + s_2^2)s_3^2 - 4s_3^2(\Omega_1^2 s_2^2 + \Omega_2^2 s_1^2 - 2\Omega_1\Omega_2 s_1 s_2) \\
&= 4s_3^2(\Omega_1^2 s_2^2 + \Omega_2^2 s_1^2 - 2\Omega_1\Omega_2 s_1 s_2) \\
&= 4s_3^2(\Omega_1 s_1 - \Omega_2 s_2)^2 \geq 0
\end{aligned}$$

Thus, the items under square root in Eq. (4.19) are also equal or greater than 0. So, the solutions of Eq. (4.19) are real. This satisfies the definition of PMD vector.

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Journal:

J. Q. Zhou, H. Dong, S. Aditya, Y. D. Gong, P. Shum, X. Guo, and L. Ma, "*Two-states method for polarization dependent loss measurement*," Optical Fiber Technology, vol. 13, pp. 139-142, 2007.

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