Study of propagation-invariant beams and its application in optical tweezers for micromanipulation

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Study of Propagation-Invariant Beams and Its Application in Optical Tweezers for Micromanipulation

Balpreet Singh Ahluwalia

School of Electrical and Electronic Engineering

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SUMMARY

SUMMARY

Optical tweezers is a laser enabled non-invasive technique for micromanipulation of the microscopic sized particles. The momentum transferred by a highly focused laser beam is used to trap the micro-particles in three-dimensions. Laser micro-beam shaping directly enhances the trapping capability of the optical tweezers system. In this quest, researchers introduced the propagation-invariant beams in optical tweezers system. Such propagation-invariant beams are also referred as non-diffracting or Bessel beams and possess the distinct property of propagation-invariance of its core for a finite distance. Another interesting property of Bessel beams is the self-reconstruction of an obstructed or distorted Bessel beam along the propagation axis. These unique properties of the non-diffracting beams have been successfully exploited for optical manipulations in the optical tweezers system. In this thesis, various novel propagation-invariant fields are studied and employed for diversified optical trapping applications.

The fact that the laser beam can embed three-dimension intensity-null point, generated considerable interest due to its potential application in laser cooled atoms and optical trapping system. Such beams embedding three-dimensional intensity-null point were referred as optical bottle beam. A novel non-diffracting field termed as self-imaged optical bottle beams is reported in the thesis. A self-imaged beam possess an array of three-dimensional intensity-null points or bottles along the propagation axis and was generated by interfering two zero-order Bessel beams with varying radial wavevectors. Properties of the proposed self-imaged bottle beam are studied in depth and it is reported that such beams inherit the self-reconstruction property of non-diffracting beams. The

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minimum self-reconstruction distance of a distorted self-imaged bottle beam is also determined.

Rotating non-diffracting fields or helicon beams is another interesting propagation-invariant field. Numerical study of different regimes of the helicon beams is reported. Helicon beams were generated by interfering two Bessel beams with varying topological charges and varying radial wavevectors.

A self-imaged bottle beam based optical tweezers system was built and various trapping applications of low and high indices microparticles are reported. The transverse intensity profile of a self-imaged bottle beam oscillates along the propagation axis thus providing an opportunity to image different points of a bottle beam for different trapping applications. The bottle beam based optical tweezers system was further used to transfer on-axis 'pure' spin angular momentum to an absorptive particle. The absorptive particles are conventionally trapped at the central dark spot of a singular beam. Singular beams due to its helical wavefront possess orbital angular momentum. Thus on employing a singular beam to transfer spin angular momentum to an absorptive particle there would be a default transfer of an orbital angular momentum. This decoupling of spin and orbital angular momentum can be achieved by employing a self-imaged bottle beam system, which does not possess orbital angular momentum.

The optical element approach for the generation of different propagation-invariant beams is reported. Laser micro-beam shaping employing optical element approach is much more efficient in term of power conversion efficiency as compared to the conventional computer generated holograms. Furthermore, optical elements can be fabricated in micron size through electron beam lithography techniques, making them

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Design of a novel optical element for the direct generation of the proposed self-imaged bottle beam is reported. Conventional axicon structure was shaped with double-gradient on its surface, each generating Bessel beams with varying radial wavevector, interfering in the free-space propagation to generate a self-imaged bottle beam. The proposed optical element is coined as double-axicon due to its geometrical appearance. The proposed double-axicon can be considered as a dedicated optical element for the direct generation of self-imaged bottle beam, similar to an axicon for the generation of Bessel beam. The integration of a micro-double-axicon with the optical tweezers system for various trapping applications is also reported.

Kinoform micro-optical elements for the generation of propagation-invariant beams are also explored. Kinoform technique enables to fabricate the surface-relief micron structures with the resist thickness equivalent to 2π thickness, thus easing the fabrication procedure. Kinoform-micro-axicon and kinoform-micro-double-axicon are designed and used to generate a Bessel beam and a self-imaged bottle beam, respectively. Kinoform structures were experimentally found to generate high-quality non-diffracting beams. Power conversion efficiency of kinoform-micro-optical elements was experimentally found be higher than the conventional computer generated holograms.

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Investigation of the kinetics of the microscopic particles in an array of Bessel beams is reported. Composite hologram approach was employed to generate interference-free array of Bessel beams with long non-diffracting distance. A composite hologram consists of various holograms, each of which can be individually controlled. Composite hologram approach was employed to generate an identical array of Bessel beam, which was further introduced with inverted optical tweezers systems. The rod-like core of a Bessel beam in an array possessing non-diffracting and self-reconstruction properties was exploited for the parallel longitudinal guiding of microscopic particles. The capability to guide microparticles by the static potential landscape generated by an array of Bessel beams is an added advantage over dynamic techniques which are conventionally used to guide microscopic particles.

Another niche area of research in modern optics is the study of singular optics in non-diffracting beams. Study of singularity embedded in the propagation-invariant beams is imperative, as the evolution of vortex is influenced by the conical wavevector distribution of propagation-invariant beams. The evolution of an off-axis vortex or vortices embedded in a propagation-invariant beam is explored. The influence of various parameters on the recovery distance (distance required by an off-axis vortex to reach the center of a Bessel beam) of an off-axis vortex is reported. The parameters that influence the recovery distance of an off-axis vortex embedded in a Bessel beam are; radial wavevector of a Bessel beam, topological charge and offset-distance of the vortex. Furthermore, the evolution of the composite off-axis vortices embedded in a Bessel beam is also studied. It was observed that the composite off-axis vortices recovered and added their topological charges at the center of the Bessel beam. The evolution of the composite

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CHAPTER ONE

INTRODUCTION

"My goal is simple. It is complete understanding of the universe, why it is as it is and why it exists as all."

Stephen Hawking

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1.1 MOTIVATIONS

In the 20th century optics and photonics have played a phenomenal role in the development of science and has eventually resulted in many research breakthroughs. Photonics and optics have been the backbone in the fields like semiconductor industries, entertainment industries, bio-medical applications, security, defense, communication, laser therapy and the list keeps on growing. There are myriads of discoveries yet to be explored in optics and on using optics as a research tool in different areas of science.

The advent of laser technology has revolutionized the way research was done by the scientific community. Optical tweezers or optical trapping [1, 2] is one of the areas where lasers are used effectively. Optical tweezers is a non-invasive technique using laser beam for the micromanipulations of the microscopic particles. The fact that laser can be focused to a tight spot is being exploit to generate the desired force to trap and manipulate the microscopic particles. Optical tweezers provide a microscopic robotic-hand to pick and place the microparticle. The magnitude of force which optical tweezers technique can impart and the size of particles it can influence make it an imperative tool for numerous applications in colloidal physics, bio-medical applications, micro-fluids, atom trapping and etc. Physicists and biologists are interested in measuring the force with which two objects bond, have used optical tweezers techniques to pry the objects apart. The force used to break the bond is then equal to the force of the bond itself and current state of science enables to precisely measure this force. Optical tweezers also promise to serve as a great tool for industry, as they can be used to sort and select microscopic particles non-invasively.

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Optical tweezers has been used most significantly in bio-medical applications, which includes chromosomes manipulation and dissection, microsurgery and manipulation of cells in-vivo, in-vitro fertilization, DNA injection and analysis of DNA elasticity, force estimation for biological motors and other molecular motors, building and studying on enzyme structures and mechanical studies of bacteria flagella. The ability to remotely and non-invasively control the matter with tailorable optical potential landscape has had a major impact in physics and biology. The time is not far when researchers reach a stage where they can construct new types of material.

Enabling optical tweezers with novel trapping beams bonds to aggrandize the usefulness of an optical tweezers system in newer application [3-16]. Generation of the optical potential landscapes in optical tweezers can greatly influence the kinetics of microparticles for example optical sorting, binding and guiding. Researchers have recently explored laser micro-beam shaping to enhance optical trapping applications. Laser micro-beam shaping directly modulates the amplitude and the phase of the beam, which influence the trapping capability of the beam. Thus the study of laser micro-beam shaping and its application in optical tweezers is imperative.

In the quest of improvising novel trapping beams, researchers have incorporated propagation-invariant beams in the optical tweezers systems. Propagation-invariant beams are also commonly referred to as non-diffracting beams or Bessel beams. As the name suggest, propagation-invariant beam possess the unique property where the transverse intensity profile of the beam is non-diffracting for a finite distance. Moreover such beams possess the unique property of self-healing or self-reconstruction, whereby the distorted non-diffracting beams regains its original intensity profile during the free

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space propagation. The integration of propagation-invariant beams with the optical tweezers system has enabled researches to exploit the core of a Bessel beam for optical guiding applications. However, different types of propagation-invariant beams have not been exhaustively studied in optical tweezers application and thus make a challenging area of research. Moreover, the generation of novel propagation-invariant fields will also be beneficial in other fields of optics due their unique properties of self-reconstruction and non-diffraction.

Propagation-invariant beams are also successfully used in various applications due to its long depth of focus. Non-diffracting beams found application in imaging, meteorology for scanning optical systems and optical interconnects. Moreover, recently study of singular optics in non-diffracting beams has opened a niche area of research. Singular beam possessing helical wavefront would interact with the conical wavevectors distribution of the propagation invariant beams and is believed to reap newer phenomena and discoveries. For instance the evolution of off-axis optical vortices and evolution of composite off-axis optical vortices embedded in propagation-invariant beams are further unexplored and would be interesting to explore. Hence the study of singularity in propagation-invariant beams is important.

Furthermore, higher-order Bessel beams are successfully employed for the transfer of on-axis and off-axis orbital angular momentum in optical trapping set-up. Furthermore, optical vortices embedded in propagation-invariant beams also enabled the transfer of orbital angular momentum to different kinds of particles (high-index and low-index micro-particles, absorptive and birefringent micro-particles. Thus optical vortices

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embedded in propagation-invariant beams are also important for optical trapping applications.

1.2 OBJECTIVES

Optical tweezers since its discovery has been employed in multifarious applications. Laser micro-beam shaping further enhances the trapping applications as it modulates the intensity and thus directly influences the trapping capability of the laser beam. The main objective of this thesis is to study and explore propagation-invariant beams for optical trapping applications. The work is focused to generate novel propagation-invariant fields for the trapping applications as well as to study the properties of propagation-invariant beams, hence the broadly classified objectives of the thesis are stated below: -

- a) To study the properties of the propagation-invariant beams and to generate novel propagation-invariant fields. To generate propagation-invariant field, which can provide localized array of optical potential well for trapping of both high and low indices micro-particles. To incorporate novel propagation-invariant beams with the conventional optical trapping systems to aggrandize the optical micromanipulations capabilities of the optical tweezers system.
- b) To design high power-efficient micro-optical elements for the generation of non-diffracting beams. To study the properties of propagation-invariant beams generated by employing micro-optical element approach. To integrate such micro-optical elements with the optical tweezers systems for optical micromanipulations.

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- c) To generate an array of propagation-invariant beams with long non-diffracting distance and with minimum interference. To study the kinetics of microscopic particles in the optical landscape (optical intensity profile generated by the outer rings and the central core of Bessel beams) generated by an array of Bessel beam in an optical tweezers system. To exploit non-diffracting and self-construction properties of the core of Bessel beam array for parallel longitudinal guiding of microparticles for extended distance. To utilize the static optical intensity profile generated by an array of Bessel beams for such parallel longitudinal guiding.
- d) To study singular optics in non-diffracting beams by embedding optical vortex in propagation-invariant beams and studying its evolution. Furthermore, to explore the evolution of composite off-axis vortices embedded in a propagation-invariant beam. To determine the influence of propagation-invariant beam (for example radial wavevector) on the evolution of off-axis optical vortices.

1.3 MAJOR CONTRIBUTIONS OF THE THESIS

The major contributions of this thesis are on the study of propagation-invariant beams and their application for micromanipulation in an optical tweezers system. A novel propagation-invariant field (self-imaged bottle beam) is reported in the thesis. Self-imaged bottle beams possess an array of three-dimensional intensity-null points or bottles along the propagation axis. A self-imaged bottle beam was obtained by interference of two zero-order Bessel beams with varying radial wavevectors. The properties of the proposed self-imaged bottle beam have been studied in depth. The number of bottles in a self-imaged bottle beam and its periodicity was determined based on the parameters of

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interfering Bessel beams. Moreover, a self-imaged bottle beam was numerically and experimentally shown to inherit the unique property of self-reconstruction or self-healing of the non-diffracting beams.

Different regimes of rotating non-diffracting beams, which are also commonly known as helicon beams, was numerically studied and reported in the thesis. The term 'helicon beam' refers to a family of beams whose transverse intensity rotates along the propagation-axis. Interfering two Bessel beams with varying radial wavevectors and topological charges generates a helicon beam. The potential optical trapping applications of each regime of the helicon beams are also discussed.

Proposed self-imaged bottle beam was introduced in an optical tweezers system for the optical manipulation of high and low indices microparticles. The transverse intensity profile of a self-imaged bottle beam oscillates along the propagation axis. Thus imaging different points of the beam, bottle beam enabled optical tweezers system was used to trap both high and low indices microparticles.

A self-imaged bottle beam based optical tweezers system was further employed to transfer 'pure' on-axis spin angular momentum of a circular polarized light to an absorptive particle trapped in the central dark spot of the bottle beam. Singular beams are conventionally used to trap absorptive particles. Singular beams due to their helical wavefront carries an orbital angular momentum and thus both spin and orbital angular momentum of light are transferred to the trapped absorptive microparticle. A self-imaged bottle beam does not possess default orbital angular momentum, thus the transfer of 'pure' spin angular momentum to the trapped absorptive particle was accomplished.

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Design of micro-optical elements for the generation of propagation-invariant beams is reported in the thesis. The fabricated microscopic optical elements were further integrated with the conventional optical tweezers systems. The micro-optical elements approach for the generation of propagation-invariant beam is beneficial due to its high power-conversion efficiency contrary to the conventionally used computer-generated holograms. Generation of a propagation-invariant beam by micro-optical elements is also imperative for the miniaturized optical traps in lab-on-chip application. Design of micro-axicon for the generation of a Bessel beam with long non-diffracting distance is reported. The fabricated micro-axicon was integrated with an optical tweezers system for optical trapping and guiding applications of high-index microparticles.

Design of a novel micro-optical element for the direct generation of the proposed self-imaged bottle beam is reported in the thesis. The element is termed as double-axicon due to its geometrical appearance. The conventional conical surface of an axicon was tailored to possess double gradient on its surface to generate two superimposed Bessel beams, interfering to generate a self-imaged bottle beam. The generation of a self-imaged bottle beam by the proposed optical element was verified both in simulation and experiment. Double-axicon can be considered as a dedicated optical element for the generation of a self-imaged bottle beam. The proposed high power-efficient double-axicon was further integrated with the optical tweezers system for various trapping applications.

Design of kinoform enabled micro-axicon and micro-double-axicon for the generation of a Bessel beam and a self-imaged bottle beam is reported in the thesis. Kinoform allows fabricating micro-optical elements of dimensions equivalent to

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wavelength or 2π phase thickness. This is beneficial in fabrication as it becomes challenging for electron-beam lithography to fabricate optical elements of greater height. Kinoform-axicon and kinoform-double-axicon optical elements were experimentally found to possess high power-conversion efficiency.

The generation of an array of propagation-invariant beams (Bessel beams) employing composite hologram approach is reported. Experimental generation of an array of Bessel beams with long non-diffracting distance was achieved employing composite hologram technique. Furthermore, the core of a Bessel beam array was exploited in optical tweezers system for multiple-channel longitudinal optical guiding of microparticles. The core of a Bessel beam array is experimentally found to possess the longitudinal guiding capability even with few interfering outer rings.

Evolution of an off-axis vortex embedded in a propagation invariant beam is studied and reported in the thesis. An off-axis vortex embedded in a Bessel beam recover to the center of Bessel beam along the free space propagation contrary to an off-axis vortex embedded in a Gaussian beam which move towards infinity. In the thesis, study of recovery distance (the distance required by an off-axis vortex embedded in Bessel beam to reach to the center of the beam) of the optical vortex is reported. Recovery distance of an off-axis optical vortex embedded in a Bessel beam was found to be influenced by the offset distance of the vortex, the radial wavevector of the Bessel beam and the topological charge of the vortex. Furthermore, the study of composite off-axis optical vortices embedded in a Bessel beam is reported. The off-axis composite optical vortices add their respective topological charges at the center of the Bessel beam. Moreover, the

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evolution of an off-axis vortex embedded in a Bessel beam was found to be immune to the presence of other vortices.

The fabrication of various holograms and the micro-optical elements used in the thesis is not under the scope of this thesis. Candidate did not fabricate the holograms used in Chapter Three and Chapter Six and micro-optical elements reported in Chapter Five. However, the candidate was involved for choosing the desired parameters for the holograms, exploring free-space property of the beam generated from the holograms and integrating them with an optical tweezers system in Chapter Three and Chapter Six. Similarly in Chapter Five, besides the fabrication of micro-optical elements; the design of the optical elements, the study of the free-space properties of the generated beams and the integration of the micro-optical elements with an optical tweezers system were carried out by the candidate.

1.4 ORGANIZATION OF THESIS

The thesis is organized into eight chapters and to navigate the flow copied below is the organization of the thesis: -

- Chapter One is the introduction covering the motivation, objective, major contribution and organization of the thesis.
- Chapter Two provides an introduction to the optical tweezers technique and propagation-invariant beams.
- Chapter Three reports the generation of a novel propagation-invariant field, self-imaged bottle beam. Detailed study on the properties of the proposed beam is

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discussed in this Chapter. Numerical study of a rotating non-diffracting field (helicon beams) is also reported.

- Chapter Four reports the integration of a self-imaged bottle beam with the inverted optical tweezers system. Self-imaged bottle beam based optical tweezers system was employed for the trapping applications of high and low indices microparticle and for the transfer of on-axis 'pure' spin angular momentum to an absorptive microparticle.
- Chapter Five reports the micro-optical elements approach for the generation of propagation-invariant beams. Design of a micro-axicon for the generation of a Bessel beam is reported. Novel optical element (double-axicon) is reported for the direct generation of a self-imaged bottle beam. The integration of micro-optical elements with the optical tweezers system is discussed. Kinoform-axicon and kinoform-double-axicon are also studied and reported.
- Chapter Six shows the generation of an array of Bessel beam employing composite hologram approach and its usage for parallel longitudinal optical guiding of microparticles in an optical tweezers system.
- Chapter Seven reports the study of singular optics in non-diffracting optics. Evolution of a single and composite off-axis optical vortex embedded in a Bessel beam is studied.
- Chapter Eight consists of conclusions and recommendations for future work.

CHAPTER TWO

INTRODUCTION TO OPTICAL TWEEZERS AND PROPAGATION-INVARIANT BEAMS

*"When you are courting a nice girl an hour seems like a second. When you sit
on a red-hot cinder a second seems like an hour. That's relativity."*

Einstein, Albert

CHAPTER TWO

2.1 A GLIMPSE IN HISTORY:-OPTICS AND MANKIND

Light, the source of energy from the sun has always enchanted human race. Knowledge and keen interest in optics is well represented in the history of human mankind. Around 300 BC Euclid (Alexandria) in his *Optica* noted that light travels in straight lines and described the law of reflection. He assumed that vision involved the rays going from the eyes to the object and pursued the study to determine the relationship between the apparent sizes of the objects and the angles that they subtend at the eye. Ibn-al-Haitham commonly known as Alhazen, (965-1020) is believed to know about spherical aberration as he used spherical and parabolic mirrors!!

A major leap in optics application in ancient time was around 1303 when Bernard of Gordon (France) mentioned the use of spectacles as a way of correcting long-sightedness. Since then newer discoveries in optics keep on pouring and the hunger to see too small and too far lead the invention of microscopes and telescopes. It all started when in 1590 Zacharius Jensen (Netherlands) and in 1608 Hans Lippershey (Netherlands) constructed a compound microscope and telescope respectively, with a converging objective lens and a diverging lens. However it was in 1609 Galileo Galilei (Italy) who constructed his own version of Lippershey's telescope and started using it for astronomical observations, and reported that Jupiter has four moons. In 1611 Johannes Kepler (Germany) in his *Dioptrice*, presented an explanation of the principles involved in the convergent/divergent lens microscopes and telescopes. The phenomenon of total internal reflection was known to him, but he was unable to find a satisfactory explanation to it.

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In 1666 Isaac Newton (England) add one more dimension to the light when he reported the splitting up of white light into its component colors on passing it through a prism. In 1676 Olaf Römer (Denmark) almost calculated the speed of light from his observations of the eclipses of the moons of Jupiter and the value was as close as $2 \times 10^8 \text{ m.s}^{-1}$.

The dual nature of light was soon realized as more and more phenomenon were discovered which couldn't be explained based on rectilinear propagation of light. In 1678 Christiaan Huygens (Netherlands) reported his wave theory of light (*Traite de Lumiere* 1690). His understanding was based on his arguments that light is transmitted through an all-pervading aether that is made up of small elastic particles, each of which can act as a secondary source of wavelets. However, it took more than a decade when in 1801 Thomas Young (England) provided experimental support for the wave theory by demonstrating the interference of light. In 1816 Augustin Jean Fresnel (France) reported detailed explanation on diffraction and interference phenomena, emphasizing that they can be explained in terms of a wave theory of light. In 1819 Joseph Fraunhofer (Germany) described his investigations of the diffraction of light by gratings. ***The unique point is that he made such grating by winding fine wires around parallel screws!!*** Finally he published his theory of diffraction in 1823.

In 1828 William Nicol (Scotland), discovered a polarizing prism (commonly known as Nicol prism) made from two calcite components. Michael Faraday (England) proposed his famous Faraday Effect in 1845 and described the rotation of the plane of the polarized light when passed through glass in a magnetic field. However the most imperative Maxwell equations came to existence in 1865 when James Clerk Maxwell

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(Scotland) reported it and concluded that light is a form of electromagnetic wave, which is known as **Electromagnetic Theory of Light**.

2.2 INTRODUCTION TO OPTICAL TWEEZERS

English Mathematician and Physicist Isaac Newton, reported that ‘Every action has equal and opposite reaction’, which is commonly known as Newton’s third law of motion. Scottish mathematician and physicist James Maxwell, suggested photon carries momentum and energy. Combining these two principles, Bell Lab scientist Arthur Ashkin, led the basic arguments for the optical tweezers or optical trapping technique [1]. When a focused laser beam incident on microscopic particles, the light is refracted, reflected, scattered or absorbed (depending on the type of microparticle and the medium where it is placed) and in doing so there is a change in the momentum of light propagation and hence equal and opposite momentum should be experienced by the micro-particle. Choosing appropriate experimental set-up, Ashkin showed that it is possible to trap the particles exploiting the momentum of a focused laser beam.

Optical trapping or optical tweezers (as it is known more commonly) is a laser-enabled non-invasive, no-contact technique for micromanipulation of micrometer-sized objects. The trapped particle can be manipulated by moving the laser beam or sample stage; thus the laser acts as a “tweezer” to pick up and place the microparticle. The technique has its roots in the early seventies when Ashkin [1] found that it was possible to accelerate the micro-particles using the radiation pressure of laser beam. Till then the significance of the radiation pressure of light was felt only in astronomy for example comet tails, which is due to the radiation pressure of sun which pushes the dust particles

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and other particles of comet in the opposite direction of its motion. Soon, Ashkin realized that it is feasible to trap micro-particles three-dimensionally by employing two focused counter propagation laser beams. This was first optical trapping report. However, it was only in mid-eighties [2] when Ashkin realized single-beam gradient-force optical tweezers system employing highly focused laser beam. The present day optical trapping is pretty much same to the one reported by Ashkin in 1986.

Researchers have added more dimensions to optical tweezers technique since its inception. The research in this field is primarily focused on two aspects, first is on developing novel ways to trap, which includes employing novel laser beam [3-16] and employing novel optical trapping set-ups [17-38]. Second is employing optical tweezers as a scientific tool for the diversified applications [39-64].

Over the time researchers has proposed and validated the useful aspects of laser micro-beam shaping for optical tweezers. Novel laser beams like Laguerre-Gaussian (LG) beams [3-7], propagation-invariant beams [8-12] and employing optical potential landscapes [13-16] (different intensity patterns for example optical thermal ratchet and optical fractionation), have aggrandized the trapping application of an optical tweezers system. Each of above-mentioned beams or optical potential landscape has its distinct advantage, which is constructively being exploited in the optical tweezers system. For example singular beams like LG and higher-order Bessel beams having helical ramp in their phase possess orbital angular momentum [3, 4], which can be used to rotate microparticles [6, 7]. Similarly propagation-invariant beams possessing unique properties of non-diffraction and self-reconstruction [8-10] are used for the extended distance guiding application [11-12]. Employing interference pattern researchers reported optical

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binding of microparticles [13] and also used same technique for optical sorting and guiding purposes [14-16]. Moreover, researchers have also proposed different set-ups for optical trapping which includes optical fiber [17-20] and waveguide based optical trapping system [21, 22], evanescent wave trapping [23-25], holographic optical tweezers system [26-30] and laser-scanning techniques (time sharing optical trapping technique). Employing optical fiber based optical tweezers system researchers has recently reported optical stretchers, which has huge impact in bio-medical diagnosis applications [19, 20]. Similarly employing an evanescent beam and waveguides researchers reported optical guiding of microparticles for extended distance [21-25]. In time-sharing optical trapping set-up [31, 32], the laser beam is scanned at a very fast rate to trap multiple particles using a single beam. Similarly spatial light modulators (SLM) are used to dynamically shape the input beam into desired output beam; moreover SLM is also used to trap multiple particles using multiple beams generated by SLM [26-30]. Researchers also reported micropumps employing holographic optical tweezers technique [30]. Using phase contrast techniques and SLM researchers from Risø laboratory, Denmark, reported direct conversion of phase modulation to intensity modulation to enhance the trapping capability of the SLM [33-38].

As mentioned above, researchers also employed optical tweezers as a scientific tool and have dexterously explored it for myriads of applications. Optical tweezers has influenced multifarious fields, to name a few; microchemistry [39], colloidal physics [40-44], bio-medical applications [45-62], and many inter-disciplinary areas [63, 64] are being explored using optical tweezers technique. The ability to apply and to measure accurately the pico-Newton forces to micron-sized particles and at the same time

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dynamically measure the displacement in nanometer precision [65, 66] has further improvised the technique. Researchers are also reported seminal work on nanometric optical tweezers relying on nano-probes [67].

Optical trapping techniques have been used effectively for the field of microchemistry, studying the spectroscopy and chemistry of small micron-sized domains. Optical tweezers coupled with photochemistry and electrochemistry was performed for more insights on chemical reactions [39]. Optical tweezers has also been explored in colloidal physics [40–44] whereby contrasting discoveries were made for example liked-charged particles in colloids showed considerable attraction force (measured by optical tweezers) contrary to conventional theory [40].

Optical tweezers has greatly influenced the biomedical applications, which dates back to Ashkin experiment [45, 46] on virus and bacteria. Since then optical tweezers operating mainly on infrared wavelength has been exploited for various applications. Optical tweezers enabled molecule level motors [47-49] and cellular mechanism [50-52] has been reported in literature. Steve Chu [53] reported the stretching of DNA molecules by attaching microspheres at the ends of a DNA molecule, this lead to series of experiments to unwind the DNA molecules and overstretching of DNA molecules for newer phenomenon [54]. Optical tweezers has been used for the study of biological cell [55] measuring rigidity [56], mobility [57] and elasticity [58] of intra-cellular structures and cell membranes. Manipulations of chromosomes are another active area of research, which is influenced by optical tweezers [59-61]. Optical scissors integrated with optical tweezers have been explored for dissection of chromosomes [62].

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Besides above-mentioned areas optical tweezers have also been employed for various other studies. Ghislain *et al* [63] have proposed a scanning force microscope based on optical tweezers technique, by trapping a stylus particle it was shown that it is feasible to use probe with lower spring constant than mechanical cantilever used conventionally. Optical tweezers have also been explored in optical scanned probe microscope [64].

2.3 IMPACT OF OPTICAL TWEEZERS

As detailed above optical tweezers technique have numerous applications and many potential applications in physics, biology, medicine and microchemistry. Physicists and biologists interested in measuring the force with which two objects bond have used optical tweezers to pry the objects apart. The force needed to break the bond is equal to the force of the bond, which can be accurately measured in optical tweezers system. Optical tweezers on coupled with laser scissors have been used for cells surgery in various bio-medical applications.

The ability to remotely control matter with lasers has had a major impact in physics and biology, and has now reached the point where researchers can construct new types of material. The basic idea is if we can pick and place the atoms and molecules precisely then researchers can construct novel material. Although the state of science has not reached this level of precision but optical tweezers is one of the techniques, which can fulfill this scientific fiction.

Employing basic principles of optical tweezers researchers pioneered the dream to trap atoms. Optical tweezers provided the vital tool that Steven Chu later used to trap and cool atoms [68], which finally won him the Nobel Prize in Physics in the year 1997. The

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impact of optical tweezers can be estimated as Dr. Arthur Ashkin related to his discovery as ***“It led to Steve's Nobel Prize and, I believe, it will lead to two more Nobel Prizes.”***

The impact of this technology can be estimated as in March'02, the bi-monthly OpticsReport has identifies *“three additional multi-million and billion dollar industries that laser tweezers will impact. Single molecule manipulation is proving useful in the \$3 billion dollar fertility market, the \$100 million dollar electrophoresis (molecular separation) market, and the micro-machine market expected to reach \$100 billion in the next decade”* (OpticsReport is published by Breault Research Organization. Founded in 1979, Breault is an optics firm with over a dozen optical engineers and close ties to both the Arizona Optics Association and the University of Arizona's renowned Optical Sciences Centre).

2.4 ASHKIN'S EXPERIMENT

A. Ashkin [1] in 1970 pioneered the optical tweezers technique. Ashkin discovery of the optical tweezers were based on simple calculations whereby he determined the acceleration of the microparticle by the momentum transferred by a laser beam. In his calculations he used a laser of power $P=1$ W of wavelength (λ) $0.5145 \mu\text{m}$ on a dielectric sphere of radius equal to the wavelength and the density of the particle was assumed to be 1 gm/cc . He assumed sphere as lossless and the fraction of light reflected back (q) of order ~ 0.1 . The radiation pressure $F_{\text{rad}}=2qP/c$ was found to be 6.6×10^{-5} dynes, where c =speed of light, however the striking parameter was the acceleration of particle and it was calculated to be $1.2 \times 10^8 \text{ cm/sec}^2$ or $\sim 10^5$ times the acceleration of gravity. This led the basic argument for him to build a simple experimental set-up as shown in Figure

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2.1(a) to validate his calculations. He employed TEM_{00} laser mode of Argon ion laser beam and shined it horizontally to the chamber containing transparent latex spheres of size 0.59, 1.31 and 2.68 μm . It was observed that when off-centered light incident on the particle, it was sucked to the center of light and accelerated in the light propagation direction as shown in Figure 2.1(a). This acceleration was due to the radiation pressure of the light. Moreover Ashkin showed that this acceleration was not due to thermal effects based on his arguments presented in Ref. 1. In the extension of this idea, he used two opposing and equal Gaussian beams with same beam waist size as shown in Figure 2.1(b). The three-dimensional trapping of a microparticle was observed and the particle was in the equilibrium position. On blocking one of the beams particles accelerated in the direction of light propagation and on shining the other beam again, the particle regain its equilibrium position.

Figure 2.1 First experimental set-up used for optical trapping. Frame (a) shows the acceleration of the microparticle along the direction of light due to radiation force exerted by laser. Frame (b) shows first optical trap using two equal and counter-propagating Gaussian beams. Figures taken from Ref. [1].

Ashkin provided series of experiments [69-72] and studied the radiation-pressure of laser beam for levitation [69], other potential applications [71], alternating-beams

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radiation-pressure trapping of microspheres and also studied it for an Earnshaw theorem [72]. Figure 2.2 shows the Ashkin experiment to levitate microparticles in the air employing the radiation pressure of the laser beam. In this experiment laser beam (mode TEM_{00}) was directed vertically-up and when laser power was around 100-500 mW the levitation of a microparticle in the air was achieved. The restoring force on the levitated particle was exerted by the second beam (incident horizontally) as shown in the Figure 2.2. It was found that the radiation pressure of a laser beam was less than the Vander Waals attraction between the particle and the base. This problem was eliminated acoustically by employing piezo-electric vibration to the stage. The equilibrium position of the levitated microparticle was at a height where the gravitational forces balance the radiation pressure.

Figure 2.2 Optical levitation of the microparticle in the air using the radiation pressure of the laser beam. Equilibrium position is reached when the radiation pressure balances the gravitational forces. Figure taken from Ref. [69].

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The present day optical trapping realizes on the gradient force rather than radiation pressure, which was again reported by Ashkin in 1986 [2]. It can be constructed by tightly focusing a laser beam using a high numerical aperture microscope objective lens. The beam focused to a very small spot size, creates a strong intensity maximum at the focus, which can be used to trap high-index transparent particles. In such case highly focused beam generates a gradient force pulling high-index particles (refractive index of target particle greater than the medium) towards the highest intensity point (focus). Similarly it was observed that a focused laser beam would push or repel the low-index particle (refractive index of target particle lower than the medium) away from the highest intensity spot and such low-index particles are trapped in dark regions. Figure 2.3 shows a simple arrangement to trap microparticles by a focused laser beam. Spatially filtered argon laser is incident on water immersion objective lens with high numerical aperture (NA 1.25) to a sample stage containing microparticles. Optical trapping of microparticles was achieved using single beam realizing on the strong gradient force, which is proportional to the intensity gradient.

Figure 2.3 Schematic diagram of the experimental set-up for single-beam gradient force optical trapping. Figure taken from Ref. [2].

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2.5 OPTICAL FORCE INVOLVED WITH OPTICAL TRAPPING

Optical tweezers is a technique whereby microscopic particles can be trapped in the focus point of a strongly converging laser beam. The explanation of optical trapping is based on Newton's Third law of motion, which states that every action has an equal and opposite reaction. It is widely known that each photon possesses energy (E) and momentum (p), which is given as: -

$$E = h\nu \quad (2.1)$$

$$p = \frac{h}{\lambda} \quad (2.2)$$

where h is the Planck's constant, ν is the frequency of light. Hence, when light incident on a particle, it cause change in the momentum of the light propagation, therefore equal and opposite force should be experienced by the microparticle. Employing such optical forces it is feasible to impart force big enough to trap microparticle in three-dimensions against gravitational forces and other forces incurred due to Brownian motions.

Optical trapping however works in two regimes based on the size of the microparticles being trapped. In the Mie regime the overall diameter of a microparticle is much larger as compared with the wavelength of the laser and in the Rayleigh regime where the diameter of a microparticle is much smaller compared with the wavelength. Therefore the two regimes of operations are: -

Rayleigh regime (diameter of particle $\ll \lambda$)

Mie regime (diameter of particle $\gg \lambda$)

In the Rayleigh regime, the trapped sphere is much smaller than the wavelength of the trapping beam. Rayleigh particles in general require lesser power to get trapped as compared to the Mie particles. To calculate the trapping force in such regime the dipole

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model and the conservation of momentum are used. Under this model, the scattering and gradient force components are readily separated. The scattering force is mainly due to the reflection and the absorption of the light and can be determined for a known particle for example for a microsphere of radius a , this force is given as [73]: -

$$F_{scat} = \frac{I_o \sigma n_m}{c} \quad (2.3)$$

$$\sigma = \frac{128\pi^5 a^6}{3\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 \quad (2.4)$$

where I_0 is the intensity of the incident light, σ is the scattering cross section of the sphere, n_m is the index of refraction of the medium, c is the speed of light in vacuum, m is the ratio of the index of refraction of the microparticle to the index of refraction of the medium n_p/n_m , and λ is the wavelength of the trapping laser. The scattering force is in the direction of light propagation and is proportional to intensity.

When light interacts with the particles, polarization occurs. This polarization causes the microparticle to experiences a gradient force. This gradient force is proportional to the gradient intensity and points towards the direction of intensity gradient. The time-averaged gradient force which is proportional to the polarizability of the sphere (α) can be given as [73]: -

$$F_{grad} = \frac{2\pi^5 \alpha}{cn_m^2} \Delta I_o \quad (2.5)$$

$$\alpha = n_m^2 a^3 \left(\frac{m^2 - 1}{m^2 + 2} \right) \quad (2.6)$$

In Mie regime where the diameter of the particle is bigger than the wavelength, ray optics explanations are used for force calculations. In Mie regime, the conservation of

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momentum is used and the trapping forces exerted onto the sphere are mainly due to the refraction of the light rays. The total force can be calculated by summing all the force acting onto the microsphere, which can be classified as scattering and gradient force. When the focused light is incident on the object the radiation pressure is imparted to the particle, which generates two primary optical forces: scattering and the gradient forces. Scattering force is primarily due to the reflection of light from the dielectric transparent particles. When the incident light is reflected from the surface of a particle, a force (scattering force) is imparted to the particle. The scattering force however can also be imparted to the particle on absorption of light by the particle. This force works against any type of trapping, as it pushes the particle along the propagation of light. It is the gradient force, which is responsible for the stable trapping. Therefore in order to stably trap the particle, gradient force should be of greater magnitude than the scattering force. Gradient force arises due to the refraction of the light from the microscopic sphere. For a high-index microparticle the gradient force is towards the intensity gradient and is in reverse direction for the low-index microparticles. The gradient force traps the microparticle (high index) in the maximum intensity region.

Figure 2.4 shows the schematic diagram to illustrate the trapping principles. In Figure 2.4(a) the Mie particle (high-index particle) is off-center to the incident beam. The thick line in Figure 2.4(a) shows more intense region as compared to thin line. The net change of momentum of light due to the particle will be more by intense regions therefore the particle pushes the light to the left side. From Newton's third law it is evident that light would push the particle to the right side towards more intense region or focus point of the converging beams. In Figure 2.4(b) the Mie particle is on the center of

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beam axis but is placed just beneath the focus point of converging beam. In this case as shown in Figure 2.4(b) the particle pushes the light downwards and experience equal and opposite force by light pulling the particle upwards towards the focus point. Therefore based on Figure 2.4(a, b) it is clear that the highly converging beam traps the microparticle at the focal point of a highly converging beam. Figure 2.4(c) shows Rayleigh particle placed in the converging laser beam. When the polarizable particle is placed in the electric field the net field decreases, and for the system to be stable or possess minimum energy the microparticle has to move to a point with maximum intensity [71] and thus the particle is trapped at the focus point.

Figure 2.4 Schematic diagrams to illustrate the trapping principles of Mie and Rayleigh particles. Frame (a) shows a Mie particles off-axis to the incident beam. Frame (b) shows the on-axis Mie particle but lying just beneath the focus point of converging beam. Frame (c) shows a Rayleigh particle in the converging beam. Figure taken from Ref. [71]

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2.6 INTRODUCTION TO PROPAGATION-INVARIANT BEAMS

The rectilinearly propagation of light is observed usually suggest that light is a stream of particles. However, few optical phenomena indicate that the law of rectilinear propagation of light does not hold and such phenomenon can be satisfactorily explained only on the assumption that light is a wave. This lead researcher to envisage the dual nature of light. Historically, the diffraction effects are associated with violation of the rectilinear propagation of light on encountering an obstacle. Any field of wavelength λ confined to a finite area of radius r (spot size) in a transverse plane will be subject to diffractive spreading in free space propagation. The characteristic distance beyond which the spreading becomes evident is r^2/λ , the Rayleigh range. With the advent of laser the diffraction effects are not only connected with light transmission through apertures and obstacles, it is related with the laser beam divergence and this affect is more prominent if the beam transverse dimension is comparable with the wavelength.

There exist families of non-diffracting beams or propagation-invariant beams [74] (also referred as Bessel beams) with a characteristic propagation distance over which they do not diffract. The non-diffracting beams are defined as the monochromatic optical field, whose transverse intensity profile remains unchanged during free-space propagation for a finite distance. The propagation-invariant beams have a conical wavevector distribution and its core is immune to diffraction within the non-diffraction distance. J. Durnin *et al* [8, 9] reported the experimental generation of a non-diffracting beam. Since then propagation-invariant beams have been widely studied and generalized [74-76]. Durnin initially reported scalar theory for non-diffracting beams, which was further, studied as vectorial electromagnetic beams based on Maxwell equations [77-80]. Researchers also

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explored partial coherent propagation-invariant beams [81, 82] and reported non-diffracting beams with controlled spatial coherence [83].

Study of singular optics in non-diffracting optics also opened a niche area of research [84-89]. Free-space propagation of the propagation-invariant beams carrying composite optical vortices was reported and the approach was based on superposition of two Bessel beams carrying different vorticity. Such superposition of propagation invariant-beam possessing different topological charges caused intensity modulation and other interesting effects like braided nodal lines [86-88]. An array of propagation-invariant beams carrying optical vortices is recently reported [89].

Propagation-invariant beams or Bessel beams due to its unique property of non-diffracting core for a finite distance have been explored for various applications. Bessel beams are also explored in non-linear regime [90, 91]. Bessel beam based interferometry [92] to accurately characterize the curvature of a reflecting surface was reported recently. Such set-up holds potential usage for optical testing of the non-stationary surfaces and could be used as a general method for real-time non-contact sensing. Advantage of using such propagation-invariant beams in laser Compton Scattering system was also reported [93], which have an edge to increase the flux of scattered photons through maintaining an efficient interaction of electron beam and laser beam for a longer distance. Due to its longer depth of focus propagation-invariant beams found application in optical interconnects [94] and metrology for scanning optical systems [95]. Imaging applications of the nondiffracting beams have been realized for an extremely long focal depth. The application of a nondiffracting beam to the third-harmonic generation [96] is also reported. Researchers studied conical emission of light in Raman scattering stimulated by

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a Bessel beam [97]. The Bessel beam pump found its usage in the design of the distributed-feedback laser [98]. Recently propagation-invariant beams have found tremendous application in an optical tweezers system [11, 99-102] where it provides a tool to play with microscopic particles. Higher-order Bessel beam has been used to transfer orbital angular momentum to both high and low indices microparticles [100, 101]. The non-diffracting core of a Bessel beam provides optical potential landscape for optical guiding of microparticles for an extended distance.

2.7 THEORY

Durnin provided the underlying theory of propagation-invariant beams by solving the Helmholtz equation (Whitaker's solution of Helmholtz equation), which can be obtained in the cylindrical co-ordinate under the restriction that their complex amplitude is separable as the product of the functions, which is given as [8, 9]: -

$$U(r, \phi, z) = R(r) * \phi(\phi) * Z(z) \quad (2.7)$$

Putting above equation in Helmholtz equation and solving [74, 75]: -

$$U(r, \phi, z) = J_o(k_r r) * \exp(il\phi) * \exp(-ik_z z) \quad (2.8)$$

$$k_z^2 = k^2 - k_r^2 \quad (2.9)$$

where k is the propagation constant given by $2\pi/\lambda$ and λ is the wavelength of the input beam. J_o is the first-order Bessel function. The intensity profile of a propagation-invariant beam contains a Bessel function as shown in Eq. (2.8), therefore such beams are also commonly known as Bessel beam.

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Non-diffracting beams can also be determined employing angular spectrum approach. The diffraction of a wave is usually studied in an angular spectrum approach where the angular spectrum of a plane wave is multiplied with the propagation transmission function of the respective beam. The transmission equation of a wave can be given as [103]: -

$$H(\rho; \Delta z) = \exp \left[i \Delta z \sqrt{\left(\frac{2\pi}{\lambda} \right)^2 - \rho^2} \right] \quad (2.9)$$

where ρ is the radial spatial frequency, λ is wavelength of interest and Δz is the propagation distance. For the wave to be propagation-invariant there should be no spread of its angular spectrum or in other words angular spectrum must be unchanged during propagation. This will be fulfilled if the angular spectrum of the wave possesses only a single radial frequency. This gives the condition on the angular spectrum of propagation-invariant waves in cylindrical coordinates, which are given as [103]: -

$$A(\rho; \phi, z) = \delta(\rho - k_r) \phi(\phi) \exp \left[i \Delta z \sqrt{\left(\frac{2\pi}{\lambda} \right)^2 - \rho^2} \right] \quad (2.10)$$

$$\phi(\phi) = \exp(il\phi) \quad (2.11)$$

where δ is the Dirac delta function and k_r is the single radial wavevector. From Eq. (2.10) the general solution for the amplitude of a propagation-invariant wave can be obtained as: -

$$U(r, \theta, z) = \frac{1}{2\pi} \int_{\rho=0}^{\infty} \int_{\phi=0}^{2\pi} \delta(\rho - k_r) \exp(il\phi) \exp \left[i \Delta z \sqrt{\left(\frac{2\pi}{\lambda} \right)^2 - \rho^2} \right] \exp[ir \cos(\phi - \theta)] \rho d\rho d\phi \quad (2.12)$$

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Integrating Equation (2.12) over the entire limits of radial frequency (ρ) and azimuthal angle (ϕ) the equation can be further simplified to: -

$$U(r, \theta, z) = \frac{1}{2\pi} k_r \exp \left[i\Delta z \sqrt{\left(\frac{2\pi}{\lambda} \right)^2 - \rho^2} \right] \int_{\phi=0}^{2\pi} \exp(il\phi) \exp[ik_r \cos(\phi - \theta)] d\phi \quad (2.13)$$

$$U(r, \theta, z) = k_r (i)^l J_l(k_r r) \exp(il\theta) \exp \left[iz \sqrt{\left(\frac{2\pi}{\lambda} \right)^2 - \rho^2} \right] \quad (2.14)$$

$$k_z = \sqrt{\left(\frac{2\pi}{\lambda} \right)^2 - \rho^2} \quad (2.15)$$

J_l denotes the l^{th} order Bessel beam. Eq. (2.14) is similar to Eq. (2.8), which can be achieved by substituting Eq. (2.15) in Eq. (2.14): -

$$U(r, \theta, z) = k_r (i)^l J_l(k_r r) \exp(il\theta) \exp[izk_z] \quad (2.16)$$

The transverse intensity profile of propagation-invariant beams does not change with distance for a finite distance and thus can be given as: -

$$I(x, y, z \geq 0) = I(x, y) \quad (2.17)$$

The transverse intensity of an ideal propagation-invariant beam consists of infinite concentric rings, each carrying equal energy and thus carries infinite energy. This is a reason, why an ideal nondiffracting beam cannot be experimentally realized and hence why the diffraction cannot be overcome practically. In experiments, only their approximations usually referred as the pseudo-nondiffracting (P-N) beams can be obtained.

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Durnin showed for the first time the experiment generation of a propagation-invariant beam. He employed an expanded laser beam illuminating the annular ring mask placed at the front focal plane of the lens as shown in Figure 2.5. The collimated laser beam incident the annular slit with mean diameter d and the width of the ring is Δd . The annular slit serves as a secondary source generating spherical waves. Every point in the annular aperture acts like a point source of spherical waves as shown in Figure 2.6(a). The annular ring is placed at the front focal point of a Fourier lens of focal length f . Each spherical wavelets leaves the lens as a plane wave propagating at an angle ϕ given as $\phi = \tan^{-1}(d/2f)$. Each of them is transformed by the Fourier lens to the beam of parallel rays propagating with the angle θ with respect to the optical axis as shown in Figure 2.6(a) [105]. The angle θ is given by [8, 9]: -

$$\theta = \sin^{-1}(v_0 \lambda) \quad (2.18)$$

$$\theta = \tan^{-1} \left(\frac{k_r}{k_z} \right) \quad (2.19)$$

where v_0 is the radial frequency given by $k_r/2\pi$ and k_r is the radial propagation constant. The interference of wavevector behind the lens has transverse intensity profile, which is immune to diffraction for a finite distance and can be approximate to a zero-order Bessel beam. The transverse profile of a zeroth-order Bessel beam possess central bright core surrounded by concentric rings. As shown in Figure 2.6(b) the Bessel beam can be thought of as light waves that are all arranged on a cone. The bright center of the beam that lies along the axis of the cone is created from light that arrives from an angle on this

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cone. The longitudinal distance Z_{max} where the beam is non-diffracting depends on the angle θ or radial wavevector k_r and is given as (8, 9): -

$$Z_{max} = \frac{R}{\tan \theta} \propto R \left(\frac{k_z}{k_r} \right) \quad (2.20)$$

where R is the radius of lens as shown in Figure 2.5. Full width half maximum ($FWHM$) of the central spot and the spot size of a Bessel beam (r_o) are given as: -

$$FWHM = \frac{2.253}{k_r} \quad (2.21)$$

$$r_o = \frac{2.405}{k_r} \quad (2.22)$$

Figure 2.5 Experimental setup for the generation of first order Bessel beam as used by Durnin. The collimated light beam incident the annular slit with mean diameter d and width Δd is the width of the ring. The focal length of the lens is f . Z_{max} represent the maximum non-diffracting distance of the Bessel beam. Figure taken from Ref [8].

Figure 2.6 Schematic diagrams to explain the generation of Bessel beam from the annular ring placed at the focal length of the lens. Z_{max} represent the maximum non-diffracting distance of a Bessel beam. (a) Each point in the annular aperture forms a parallel beam, which interferes to generate Bessel beam with conical wavevector distribution as shown in (b). Figure taken from Ref [105].

2.8 PHYSICS BEHIND NON-DIFFRACTING BEAMS

The transverse intensity profile of the propagation-invariant beams (Bessel beams) does not change within a finite distance during free space propagation. Propagation-invariant beams are generally expressed in Fourier domain as the diffraction of beam can be easily analyzed employing angular spectrum approach based on the following facts: -

- (a) Diffraction of a beam can be explained by the spread of the angular spectrum in the frequency domain.
- (b) The spread of angular spectrum is linked with the relative phase change of the Fourier component.
- (c) Ideally non-diffracting beam should have no diffraction along the propagation direction; so it should have no spread in its angular spectrum.

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Based on the above criteria the necessary and sufficient condition on the angular spectrum of a beam to possess the properties of propagation-invariant beams is given as [104]: -

$$F(\nu, \phi) = A(\phi) * \delta(\nu - \nu_o) \quad (2.23)$$

$F(\nu, \phi)$ is the angular spectrum of the beam in the cylindrical coordinate (r, ϕ, z) and ν and ϕ are the corresponding Fourier integral operator for r and ϕ respectively. $A(\phi)$ is the input field; $\delta(\nu - \nu_o)$ is the delta function and ν_o is the radial frequency. Angular spectrum of the non-diffracting beams contains only single radial frequency so that the relative phases of the plane wave components remain unchanged under propagation. This explains the non-diffracting property of the Bessel beams. The radial frequency in the frequency domain is directly linked with the radial wavevector (k_r) as: -

$$\nu_o = \frac{k_r}{2\pi} \quad (2.24)$$

When any beam satisfies the above conditions then its angular spectrum is superposition of the plane waves whose wavevectors (propagation vectors) cover a conical surface as shown in Figure 2.6(b). The conical angle (θ) formed is linked with the radial frequency ν_o as stated in Eq. (2.18). The angular spectrum of the Bessel beam possesses a single ring in the frequency domain as it contains single radial frequency. Any beam showing non-diffracting property should have a single radial frequency in the angular spectrum and thus a single radial and longitudinal wavevectors (k_r and k_z , respectively). Therefore, Bessel beams can be fully characterized by its radial wavevector. As discussed above Durnin showed experimental generation of a non-diffracting beams by employing an annular aperture at the focal length of a lens, which in principle is selecting a single radial

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frequency in the angular spectrum and hence fulfilling the condition represented by Eq. (2.23).

2.9 CONTROVERSY OVER NON-DIFFRACTING BESSEL BEAM

The inception paper by Durnin [8, 9] on non-diffracting beams generated considerable interest as well as controversies. A comment to Durnin first paper was given by D. Debeer *et al* [106] whereby they compared the ‘non-diffracting beams’ as a line image and equivalent to a Poisson’s spot. Durnin [107] replied to this by emphasizing that indeed the non-diffracting beams can be viewed as line image as it falls under the category of any beam possessing central maxima along the propagation axis and draws comparison to an axicon employed by McLeod [108, 109] to produce a line image field. However Durnin mainly emphasized on the fact that there exist several techniques to generate such line images but it does not mean all can generate non-diffracting beams as they might not necessary inherit the other properties of non-diffracting beams. In 1991 P. Sprangle *et al* [110] commented on the non-diffracting property of a Bessel beam core as compared to a Gaussian beam of dimension equivalent to size of a Bessel beam. To this Durnin replied [111] clarifying that the non-diffracting property of a Bessel beam core should be compared with the Gaussian beams of size equivalent to the central spot (core) of a Bessel beam and not to the Gaussian beam whose diameter is equal to full Bessel beam itself. After the initial controversies the imperative non-diffracting beams with many unique properties have been widely studied and employed for multifarious applications over two decade.

2.10 PROPERTIES OF PROPAGATION-INVARIANT BEAMS

Propagation-invariant beams possess myriads of unique and imperative properties, which have been explored in diversified applications. Propagation-invariance is however the most versatile property of such beams. The rod-alike core of a Bessel beam, which can be considered, as line-focus does not diffract over a finite distance, and the transverse intensity profile of such beams are constant for the same distance. Figure 2.7(a) shows the transverse intensity profile of a Bessel beam, having concentric outer rings. The central core of the Bessel beam shown in Figure 2.7(a) can be calculated employing Eq. (2.22). Figure 2.7(b) shows the three-dimensional intensity profile of a Bessel beam. Figure 2.8 shows the transverse intensity distribution of a Bessel beam shown in Figure 2.7(a). Intensity profile of a non-diffracting beam possesses Bessel function. This is the reason why intensity distribution obtained in Figure 2.7 and Figure 2.8 resembles that to a Bessel function. Energy in each ring is constant however as the area of each ring is different (minimum for the central core and increase outward) therefore the intensity is maximum in the central core and decrease outwards. Bessel beam is thus like an interference pattern whereby the outer rings compensate the spreading of central core. Figure 2.9 shows the longitudinal beam formation of a Bessel beam. The conical wavevector distribution and the non-diffracting property of a Bessel beam is evident from the Figure 2.9. Figure 2.7 to Figure 2.9 are generated using an angular spectrum approach based simulation in MATLAB programming.

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Figure 2.7(a) Transverse intensity of a Bessel beam. Each vertical and horizontal unit is 0.01 mm. Frame (b) shows the three-dimensional intensity profile of a Bessel beam. Each unit of X and Y axis is 0.01mm and the unit of Z axis (representing the intensity) is normalized.

Figure 2.8 Radial intensity distribution of a Bessel beam shown in Figure 2.7(a). The vertical unit is arbitrary and each horizontal unit is 0.01 mm. The vertical and horizontal axis represents the normalized intensity and the transverse axis of a Bessel beam, respectively.

Figure 2.9 Simulated longitudinal intensity profile of a Bessel beam generated using an axicon. Each unit of vertical axis is 0.04mm and the horizontal axis representing the beam propagation axis is normalized with respect to Zmax (maximum non-diffracting distance of a Bessel beam).

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Another important property of a nondiffracting beam is its resistance against amplitude and phase distortions referred as self-healing or self-reconstruction property [10]. The way Bessel beams are formed; it is able to reconstruct itself after being distorted by a non-transparent obstacle. The transverse intensity profile of a Bessel beam obstructed by an opaque obstacle can reconstruct itself during free-space propagation. In geometrical interpretation a Bessel beam represents a coherent superposition of the plane waves with the wavevectors covering the conical surface. The presence of the non-transparent obstacle along the propagation path of a Bessel beam blocks the superposition of the plane waves in the shadow region behind it, deforming its transverse intensity distribution for a finite distance. However, a Bessel beam can be thought of as a light waves that are all arranged on a cone, hence the central core of the Bessel beam is created from light that arrives from an angle on this cone. Thus if the small portion of the core is distorted at the center it can reconstruct its transverse intensity profile as the light waves are still coming along the outside of the shadow caused by the obstacle. The beam thus reforms or "self-heals" itself after passing beyond the obstruction. Minimum distance (Z_{min}) [10] required by a distorted Bessel beam to reconstruct its transverse intensity distribution depends on the radial dimension (r_a) of the obstacle and is given as [10]: -

$$Z_{min} = r_a \left(\frac{k}{k_r} \right) \quad (2.25)$$

The condition $Z_{max} > Z_{min}$ should be satisfied for an obstructed Bessel beam to reconstruct its transverse intensity profile completely. Self-reconstruction property of a distorted Bessel beam is shown in Figure 2.10. An obstacle in Figure 2.10(a) distorts the transverse intensity of the Bessel beam. During the free space propagation the Bessel beam self-heals its intensity profile as shown in Figure 2.10(b-d). Figure 2.10(d) shows

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the reconstructed Bessel beam after obstruction. This self-healing property of a Bessel beam is very effectively used in optical tweezers for the manipulation of microparticles for the extended distance.

Figure 2.10 Self-reconstruction property of a Bessel beam. (a) A non-transparent obstacle blocks the central part of the core at $z=0$. During the free-space propagation the beam self-heals, (b) $z=Z_{min}/2$, (c) $z=Z_{min}$ and (d) $z=4Z_{min}$. The figure is taken from Ref [10].

2.11 EXPERIMENTAL GENERATION OF BESSEL BEAM

An ideal Bessel beam (nondiffracting beam) consists of an infinite number of rings and hence carries infinite energy. In practice, however, only pseudo-nondiffracting beams can be obtained as an approximation to the ideal Bessel beam. Non-diffracting beams can be experimentally generated by using an annular ring placed at the focal length of the Fourier lens [8] as reported by Durnin. However, there exist few other techniques to approximate such beams, which include computer generated holograms [112-114] or an axicon [108, 109] and Fabry-Perot etalon [115, 116].

Bessel beams are widely generated by employing holographic approach and the phase transmission function to generate such beams is given as [113-114]: -

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$$T_l(r, \theta) = \exp(il\theta) * \exp\left(\frac{-i2\pi r}{r_o}\right) \quad [2.26]$$

$$k_r = \frac{2\pi}{r_o} \quad [2.27]$$

where l represents the order of a Bessel beam. The phase given in above equation is recorded in computer-generated holograms and on incident a Gaussian beam or a plane wave the Bessel beam can be reconstructed. Spatial light modulators (SLM) have been used to generate propagation-invariant beams using the phase given in Eq. (2.26). Generation of a Bessel beams using holographic approach suffers the disadvantage of being less power efficient. Also considerable amount of power is being wasted in higher orders of diffraction in case of spatial light modulators.

Fabry-Perot technique has also been explored for the generation of propagation-invariant beams [115, 116]. A spatially filtered Gaussian beam illuminates Fabry-Perot and the output is then imaged to obtain series of concentric rings by the lens. The width of these rings is tailorable and is determined by the finesse of the cavity. An annular aperture is used to filter only one of the rings of this interference pattern and is placed at a focal distance of a Fourier lens similar to the lens used in annular aperture technique.

However most frequently technique to generate Bessel beams is through an axicon [108-109, 117], which is a conical optical element, with a cone angle given as [108, 117]: -

$$\Phi \approx (n-1)\gamma \quad [2.28]$$

where n is the refractive index of an axicon and γ is the wedge angle of an axicon. An axicon converts the input plane wave into beam with a conical wavevectors distribution

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approximating to a Bessel beam near its optics axis. The wedge angle of the axicon determines the radial wavevector of a Bessel beam generated and is given as [108, 117]: -

$$k_r = k(n-1)\gamma \quad [2.29]$$

$$k_r = k.\Phi \quad [2.30]$$

Bessel-like fields generated by above-mentioned techniques have slightly different intensity distributions, however they all have a common property of diffraction-invariance in propagation within a finite distance. The on-axis intensity of the Bessel beam generated by an annular aperture oscillates around its initial value before dropping abruptly to zero after Z_{max} . However using the Fabry-Perot etalon method such an intensity oscillation can be reduced but the average intensity value is increased along the propagation axis and it is reduced sharply to zero at the maximum nondiffracting distance (Z_{max}) [116]. Axicon induced Bessel beam shows a linear increase of on-axis intensity with noticeably oscillations, however, by apodising the incident beam it is possible to reduce such oscillations significantly. This can be achieved by using Gaussian beam as an input to an axicon. Hence the axicon has been widely accepted for the generation of Bessel beam in many applications.

The maximum nondiffracting distance (Z_{max}) of the Bessel beam generated by various techniques differs and depends on different parameters [117]. In the annular slit approach a thin annular aperture of radius R_o and width Δ placed at the focal length f of a Fourier lens generates a nondiffracting field with a maximum nondiffracting distance of $Z_{max} \sim (f)^2.\lambda / R_o.\Delta$. Similarly the Fabry-Perot etalon with a mirror spacing d and a finesse F , followed by an annular spatial filter, generates a Bessel beam with a maximum nondiffracting distance of $Z_{max} \sim 2dF$. An axicon of a radius R_a , index of refraction n and

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cone angle (Φ) generates an approximate Bessel beam near optical axis with a maximum nondiffracting distance of $Z_{max} \sim R_d/\Phi$; for small cone angles, $\Phi \sim (n-1)\gamma$, where γ is wedge angle.

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LASER MICRO-BEAM SHAPING

*"The important thing in science is not so much to obtain new facts as to
discover new ways of thinking about them."*

William Lawrence Bragg

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3.1 INTRODUCTION

Since its discovery, laser has been applied to diversified pool of applications. Optical tweezers is one such area where laser forms the backbone of the technique. Shaping the intensity and the phase profile of a laser beam is desirable in many dedicated applications and thus laser micro-beam shaping gained considerable interest. Especially in optical tweezers, laser micro-beam shaping has been employed in a great deal as it directly influences the trapping capability of the beam. Integration of a spatial light modulator with the optical tweezers system provided an opportunity to study the kinetics of microparticles in various optical potential landscapes and novel laser modes in an optical tweezers system. Holographic optical tweezers (HOT) enabled by a spatial light modulator [26-30], were also explored to shape an incident beam into an array of Gaussian or LG beams for trapping applications. Optical elements approach to shape the incident beam was also explored; such techniques are more efficient in terms of power conversion, which is desirable in optical tweezers system [11].

Laser micro-beam shaping of an incident Gaussian mode to Laguerre-Gaussian mode [3] and to propagation-invariant fields (Bessel beam) [8] was reported. Furthermore, such beams were employed for various trapping applications [6, 11]. Interference pattern of two beams were explored in depth for optical tweezers applications [13]. Interference of two counter-propagating plane waves was employed for optical binding applications [13] and interference of LG beams and Bessel beams were explored for optical micromanipulations [118, 119]. Researchers also reported various novel families of beams, which have potential applications in optical tweezers. Mathieu beams (Bessel beam in elliptical co-ordinates) [120, 121], Parabolic Bessel beams [122],

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and optical bottle beam [123] were reported and each of them possesses distinct advantage if employed in an optical tweezers system. However, optical bottle beams embedding three-dimensional intensity null point generated considerable interest due to its potential application in atom cooling and optical tweezers.

3.2 OPTICAL BOTTLE BEAMS

Laser beams embedding three-dimensional intensity null points or voids (generally known as bottle) generated considerable interest due to its potential application in optical confinement of cold atoms, optical dipole traps for laser-cooled atoms and in optical micromanipulations employing optical tweezers system. Recently bottle beams were extensively studied and myriads of techniques to generate such beams were reported [123-132]. Bottle beam embedding three-dimensional intensity voids holds potentially application in blue-detuned optical traps where neutral atoms are trapped in dark region, hence providing long lifetimes and long atomic coherence times [133] which are essential for ultrasensitive spectroscopy [134].

Ozeri *et al* generated three-dimensional intensity null point using holographic phase plate technique [124] and J. Arlt *et al* reported bottling effect by interfering two Laguerre-Gaussian beams with different Gouy phases [123]. Interfering Bessel beams were also shown to generate an array of bottles embedded in the beam, with additional properties of self-reconstruction and propagation-invariance of a Bessel beam inherited [125]. Combination of an axicon and a lens was used to generate a bottle beam [127, 128] and its conversion to hollow beam by changing the relative distance between them was also reported [127]. Bottle beams can also be generated by interfering two Gaussian

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beams with different radius of curvature [129]. Generation of bottle beams directly from laser cavity [130, 131] and the generation of partially coherent bottle beam using partially coherent laser source focused by a lens [132] were also explored recently. The three-dimensional intensity-null point of a bottle beam is different from LG beam and higher-order Bessel beam where center null point is only in two-dimension.

Optical bottle beams [123] or high-order p -mode Bessel beams [125] contains intensity-null point or points along the propagation direction surrounded by region of higher intensity. Optical bottle beam was initially generated by interfering Laguerre-Gaussian (LG) beams with different radial mode index (p) [123]. LG beams with $l=0$; $p=0$ and $l=0$; $p=2$ were interfered destructively at the common focus in such a manner that on-axis intensity at the focus is zero, where l and p are azimuthal and radial indices of LG beam, respectively. The LG beam in cylindrical co-ordinate is represented by the following equation [118]: -

$$E(LG_p^l) \propto \left[\frac{r\sqrt{2}}{w(z)} \right]^{|l|} L_p^{|l|} \left[\frac{2r^2}{w^2(z)} \right] \cdot e^{\left(\frac{-r^2}{w^2(z)} \right)} \cdot e^{\left(\frac{-ikr^2z}{2(z^2+z_R^2)} \right)} \cdot \exp(-il\phi) \cdot \exp \left[-i(2p+|l|+1) \tan^{-1} \frac{z}{z_R} \right] \quad (3.1)$$

where $L_p^{|l|}$ is a generalized Laguerre polynomial, $w(z)$ is the radius of the beam at the position z , z_R is the Raleigh range. Gouy phase shift given as φ [118]: -

$$\varphi = \exp\{-i(2p+l+1)\} * \tan^{-1} \left(\frac{z}{z_r} \right) \quad (3.2)$$

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It was shown that relative Gouy phases of two interfering LG beam can be adjusted such that on-axis intensity at the common focus is zero. Moreover, on the both sides of the focus, the different Gouy phases interfere constructively giving intensity maxima hence encompassing an optical bottle or intensity null point in three dimensions as shown in Figure 3.1.

Figure 3.1 Intensity void or bottle generated by interfering LG beam with different radial index (p). Figure taken from Ref [123].

The optimized array of three-dimensional bottle beams with additional non-diffracting property was subsequently reported [125]. Such an array of bottle beams was generated by passing a Laguerre-Gaussian beam (LG) with $p > 0$ and $l = 0$ through an axicon. Such an array has p null-intensity points or bottles where p is the radial index of the LG beam used to generate optical bottle beam. Experimentally it was generated by passing a LG beam with $p > 0$ through an axicon. Radial mode index p of LG beam represents the number of concentric rings (p concentric rings when $l = 0$ and $p + 1$ rings when $l > 0$) as shown in Figure 3.2. When LG beam with $p > 0$ and $l = 0$ is focused through an axicon; each ring of LG beam is individually transformed to a Bessel beam. These

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Bessel beams interfere destructively to generate p bottles or null-intensity points in the beam as show in Figure 3.3 and Figure 3.4. Figure 3.3(a) shows the longitudinal intensity distribution of a beam embedding a single bottle on passing LG beam with $l=0$ and $p=1$ through an axicon. Similarly, Figure 3.3(b) shows on-axis intensity distribution of Figure 3.3(a) making the presence of the bottle clear. Figure 3.4(a) shows a beam embedding five bottles, which was generated by passing LG beam with $l=0$ and $p=5$ through an axicon and similarly Figure 3.4(b) plots the intensity profile distribution of Figure 3.4(a) along the longitudinal axis. Thus it was reported that such technique could generate a bottle beam where the number of bottles generated would be equal to the radial index (p) of the LG beam used to generate such beams.

Figure 3.2 LG beam with $l=1$ and (a) $p=0$, (b) $p=1$; and (c) $p=2$. Figure taken from Ref. [125].

Figure 3.3(a) Generation of optical bottle beam by passing LG beam of $l=0$ and $p=1$ through an axicon; (b) shows on-axis intensity of the generated beam. Figure taken from Ref. [125].

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Figure 3.4(a) Generation of optical bottle beam by passing LG beam of $l=0$ and $p=5$ through an axicon; (b) shows on-axis intensity of the generated beam. Figure taken from Ref. [125].

3.3 PROPOSED SELF-IMAGED OPTICAL BOTTLE BEAM

Based on reported literature it is known that to increase the number of bottles or intensity-null points in the beam we have to increase the value of radial index p of LG beam. This is not a convenient way to generate a bottle beam with large number of bottles or null intensity points. Firstly, to generate higher p -modes of LG beam (say $p > 10$) is experimentally difficult and then the alignment of LG beam (with high radial index, $p > 10$) with an axicon is even more critical. Secondly, focusing of an LG beam through an axicon is an approximate method to generate Bessel beam close to the optical axis [135]. In Ref. [135], the LG beam with $p = 0$ is focused through an axicon but as we increase the radial mode index of the LG beam to a large value (say $p = 10$ or higher) then the Bessel beam generated from the outer concentric rings of the LG beam are not accurate and consequently the interference of such Bessel beams generating bottles will not be identical, which can also be observed in Figure 3.4. Therefore an alternative approach to generate a beam embedding large number of bottles exploiting the self-

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imaging phenomena is explored in the thesis. Such beams are referred as self-imaged bottle beam for the terminology sake.

As mentioned-above a beam embedding an arrays of bottles was generated by passing a LG beam with $p>0$ through an axicon. When LG beam with $p>0$ is focused through an axicon; each ring of LG beam is individually transformed to a Bessel beam. These Bessel beams interfere destructively to generate p bottles or null intensity points in the beam. The extension of this idea is to interfere Bessel beams with different radial frequencies or radial propagation constants (k_r). It was foreseen that such an arrangement will generate an interference pattern of two or more Bessel beams which will be similar to incident LG beam with $p>0$ on an axicon. Radial wavevector of a Bessel beam can be spatially filtered by using an annular ring at the focal length of Fourier lens as used by Durnin to generate a Bessel beam. Axicon which are conventionally used to generate a Bessel beam selects single radial frequency and thus a single radial propagation constant (k_r) which can determined by its wedge $k_r=k(n-1)\gamma$, where k is propagation constant and n is the refractive index of the axicon. The double annular rings approach or computer-generated holographic techniques can be applied to interfere Bessel beams with different radial wavevectors. In double annular ring approach, each ring would select different radial wavevector, which depends on its radius. Thus double annular ring will generate an interference pattern of two Bessel beams with varying radial wavevector. However, computer-generated holograms approach was used in this case, due to the availability of spatial light modulator (SLM). Numerically study of such beams is also explored employing beam propagation method.

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This understanding was exploited by dexterously choosing the appropriate values of k_{r1} and k_{r2} (radial propagation constant of two interfering Bessel beams) to generate a beam with embedded array of bottles. The Bessel beams can be interfered based on the azimuthal index (l) with the same radial propagation constant [119] or based on the radial frequency with different radial propagation constant or wavevectors (k_r) [136, 137]. In the present case two zero-order Bessel beams are interfered with varying radial wavevectors. The intensity of two interfering Bessel beams is given as: -

$$I(r, \phi, z) = J_0(k_{r1} \cdot r)^2 + J_0(k_{r2} \cdot r)^2 + 2A' J_0(k_{r1} \cdot r) * J_0(k_{r2} \cdot r) * \cos[(k_{z1} - k_{z2}) + \Delta] \quad (3.3)$$

where the two zero-order interfering Bessel beams can be given as: -

$$E_{01} = J_0(k_{r1} \cdot r) \exp(ik_{z1} \cdot z) \quad (3.4)$$

$$E_{02} = J_0(k_{r2} \cdot r) \exp(ik_{z2} \cdot z) \quad (3.5)$$

The Bessel beams interfered in the above equation E_{01} and E_{02} are of different longitudinal and radial propagation constants k_{r1} , k_{r2} and k_{z1} , k_{z2} . A' is an amplitude factor, generally taken as 1 and Δ is an arbitrary additional phase change in the optical system. Due to the presence of the cosine term and its dependency on z-axis in Eq. (3.3), it is evident that the longitudinal intensity will oscillate along the z-axis and will generate bottles where two Bessel beam interfere destructively.

3.3.1 Simulation

Generation of the proposed self-imaged optical bottle beam was initially obtained numerically by interfering two Bessel beams with varying radial propagation constant, $k_{r2} = 2k_{r1}$, where k_{r1} is 0.2/pixel and each pixel is given by 15 μm . Beam of spot size (w_0)

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1.92 mm and wavelength 633 nm was used in simulation. The azimuthal indices (l_1 and l_2) of the interfering beams were set to zero. Angular spectrum approach was employed for the beam generation and propagation. The beam propagation in free-space can be described by the Fourier transform of plane waves given as: -

$$A(x, y, z) = FT^{-1} \{ FT[A(x, y, z - \Delta z)] \exp(-ik_z \Delta z) \} \quad (3.6)$$

where FT and FT^{-1} denote the Fourier transform and inverse Fourier transform respectively, and k_z and k_r are the longitudinal and radial components of the free-space wavevector respectively. Δz is the propagation distance and $A(x, y, z)$ is the field distribution of the beam at a distance of z .

The self-imaged beam showing periodical variation of its transverse intensity with embedded bottles or intensity null point within each period is obtained as shown in Figure 3.5. Figure 3.5 shows the longitudinal intensity field distribution of the proposed beam. Three periods of oscillations with embedded bottles is shown in Figure 3.5(a, b). Figure 3.5(b) shows the contour map of the intensity distribution of a self-imaged bottle beam obtained in Figure 3.5(a). Thus it is evident that two interfering Bessel beams with varying radial wavevectors generates a self-imaged beam with bottle or intensity voids embedded within each period.

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Figure 3.5 Longitudinal intensity profile of a self-imaged optical bottle beam embedding three bottles generated by interfering Bessel beams with $k_{r2}=2k_{r1}$, k_{rl} is 0.2/pixel and each pixel is given by 15 μm . Frame (a) shows the intensity of the self-imaged optical bottle beam along the propagation direction. Frame (b) shows the contour map of frame (a). Each vertical and horizontal unit of frames (a, b) is in pixel and 1 cm, respectively.

Figure 3.6(a-l) shows transverse intensity snapshots of a single period of a self-imaged bottle beam along the propagation direction. Distance between the frames is 25 mm. It is clear from Figure 3.6 that a self-imaged optical bottle beam varies its transverse intensity along the propagation axis and contains embedded bottle along the propagation axis. Figure 3.6(f, g) represents a bottle embedded in the beam, where Bessel beams interfere destructively and Figure 3.6(d) represents intensity maxima, where Bessel beams interfere constructively. Figure 3.7(a-l) plots the transverse beam profile and the intensity distribution of a self-imaged bottle beam along the propagation axis. Figure 3.7 shows the intensity cancellation near the bottling point in a self-imaged bottle beam. The distance between the frames is 10 mm. A bottle is obtained in Figure 3.7(e), which is also clear in Figure 3.7(f), where there is a sharp dip of the intensity at the core of the beam. The embedded bottle is surrounded by higher intensity region, which are shown in Figure 3.7(b, l).

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Figure 3.6 Transverse intensity profile of a self-imaged optical bottle beam (one complete cycle). Distance between the frames is 20 mm. The bottling effect is observed in frames (f, g). The vertical and horizontal axis represents a dimension of around 3.8 mm.

Figure 3.7 Transverse intensity profile of a self-imaged optical bottle beam (one complete cycle). Distance between the frames is 10 mm. The bottling effect is observed in frames (f, g). Each vertical and horizontal unit of frames (a, b) is in pixel and each pixel is given by 15 μm .

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3.3.2 Study of bottles embedded in the proposed self-imaged bottle beams

Properties of the proposed self-imaged bottle beam were further studied. The proposed bottle beam holds an edge over earlier reported bottles beams [123, 125] as it is self-imaged so it can have large numbers of bottles embedded in the beam without necessary increasing the radial index of LG beam [125]. Each bottle embedded in the beam was further found identical irrespective to earlier reported works. By just varying the parameters (radial wavevectors) of interfering Bessel beams the location and the number of bottles can also be pre-determined. Figure 3.8 shows the feasibility to vary the position of a bottle in the beam by tailoring the radial wave vector of the interfering Bessel beams. Figure 3.8(a-b) shows the self-imaged bottle beams obtained by interfering Bessel beams with radial wavevectors 0.2/pixel and 0.25/pixel and 0.2/pixel and 0.3/pixel, respectively. Figure 3.9 shows the feasibility to choose the number of bottles in a self-imaged bottle beam; again this can be achieved by carefully altering the radial wavevectors of the interfering Bessel beams. Figure 3.9(a-b) shows the self-imaged bottle beams obtained by interfering Bessel beams with radial wavevectors 0.2/pixel and 0.35/pixel and 0.2/pixel and 0.45/pixel, respectively. It is also observed that the bottle obtained by superimposing two Bessel beams with varying radial wavevectors generate an array of identical bottles contrary to earlier reported work. Figure 3.10 shows the longitudinal intensity distribution of the self-imaged bottle beams shown in Figure 3.9. It can be noticed from Figure 3.10 that each bottle in the beam is identical, contrary to Figure 3.4.

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Figure 3.8 Bottle locations in the beam can be pre-determined by tailoring the radial wavevector of interfering Bessel beam. The radial wavevector of interfering Bessel beams in frame (a) is 0.2/pixel and 0.25/pixel and for frame (b) its 0.2/pixel and 0.3/pixel, respectively. Each vertical and horizontal unit of frames (a, b) is in pixel and 1 cm respectively.

Figure 3.9 Number of bottles in a self-imaged optical bottle beam can be pre-determined by tailoring the radial wavevector of interfering Bessel beam. The radial wavevector of interfering Bessel beams in frame (a) is 0.2/pixel and 0.35/pixel and for frame (b) its 0.2/pixel and 0.45/pixel, respectively. Each vertical and horizontal unit of frames (a, b) is in pixel and 1 cm, respectively.

Figure 3.10 Longitudinal intensity distribution of the self-imaged bottle beams obtained in Figure 3.9. Each bottle is identical in a beam. Each vertical and horizontal unit of frames (a, b) is in pixel and 1 cm, respectively.

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The behaviors of bottles embedded in the beam were further studied and generalized. It was noticed that each bottle embedded in the beam is highly influenced by the two radial propagation constant (k_{r1} and k_{r2}) chosen in the simulation. The bigger the difference between k_{r1} and k_{r2} the smaller and higher the will be the numbers of bottles embedded in the beam. The number of bottles (N) embedded in the beam was also determined, which is given as: -

$$N = \frac{Z_{\max OBB}}{P} \quad (3.7)$$

where $Z_{\max OBB}$ is defined as the maximum distance of a beam where bottling effect is observed and P is the oscillation period of the self-imaged beam and is given by: -

$$P = \frac{2\pi}{(k_{z1} - k_{z2})} \quad (3.8)$$

Each Bessel beam has its maximum distance of non-diffracting propagation ($Z_{\max}$) given as $Z_{\max 1, 2} = w_o \cdot k / k_{r1, 2}$ (w_o is the beam spot size). Bottling effect will appear in a self-imaged optical bottle beam within a distance where both Bessel beams are non-diffracting. If either of the interfering Bessel beams crosses the $Z_{\max}$ distance the bottling effect will ceases therefore $Z_{\max OBB}$ is given as: -

$$Z_{\max OBB} = \min(Z_{\max 1}, Z_{\max 2}) \quad (3.9)$$

Eq. (3.7) was used to analyze and verify the number of bottles for Figure 3.5. The parameters chosen in Figure 3.5 generated two Bessel beams with maximum non-diffracting distance ($Z_{\max 1, 2}$) of 142 cm and 70 cm, respectively. Therefore $Z_{\max OBB}$ of a self-imaged optical bottle beam is 70cm from Eq. (3.9). Periodicity (P) of the self-imaged beam calculated from Eq. (3.8) is 23.36 cm. Hence the chosen parameters can only

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possess three bottles in the self-imaged optical bottle beam based on Eq. (3.7). This is consistent with the simulation result shown in Figure 3.5 where a self-imaged bottle beam possesses three bottles.

When either of the interfering Bessel beams surpasses its Z_{max} the bottling effect ceases, however the other Bessel beam is still non-diffracting and is obtained beyond the Z_{maxOBB} distance. This is clear when the difference between the radial wavevectors of interfering Bessel beams is high. For example parameters chosen in Figure 3.11 (radial wavevectors k_{r1} and k_{r2} given as 0.3/pixel and 0.9/pixel) generate two Bessel beams where the Z_{max} of first Bessel beam is almost three times of the second Bessel beam, therefore after the Z_{maxOBB} , the first Bessel beam is obtained, as shown in Figure 3.11. It is evident from the Figure 3.11 that a self-imaged bottle beam is obtained within a distance where both the Bessel beam are non-diffracting, beyond that the bottling effect ceases and the other Bessel beam will be obtained. However, the other Bessel beam interferes with the diffracting Bessel beam, causing noticeable interference in the outer ring, which can also be observed in Figure 3.11.

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Figure 3.11 Longitudinal propagation of a self-imaged optical bottle beam and Bessel beam. Beyond Z_{maxOBB} the bottling effect ceases and the Bessel beam is obtained. Each vertical and horizontal unit of frames (a, b) is in pixel (each pixel is $15\ \mu\text{m}$) and 1 cm, respectively

3.3.3 Experimental generation of beam embedding an array of bottles

Experimental generation of a self-imaged optical bottle beam is reported employing different techniques. Different approaches can be used to interfere Bessel beams with varying radial wavevector to obtain a beam with embedded bottles. Spatial light modulators (SLM), holograms and novel optical element double-axicon are explored to generate such beams. However, the details of (double-axicon) will be discussed in Chapter Five.

In the SLM approach, Boulder Nonlinear Systems spatial light modulator is used. The SLM is a reflection type with at least 32 phase levels or quantization levels representing phase increment (from 0 to 2π) controlled by a PC. The hologram information is loaded in the computer to control the phase distribution of the SLM, thus

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the SLM acts as the loaded hologram to diffract the incident laser beam. SLM allows dynamic control on the size and other dimensions of each bottle in the beam by sending series of information from computer. In the experiment, the hologram that generates a self-imaged bottle beam is obtained from the superposition equation of two Bessel beams with different radial propagation constant ($k_{r2}=2k_{r1}$) as reported in Eq. (3.3). The transmission function to generate such bottle beams can be obtained by linear addition of transmission function of the two Bessel beams and is given as: -

$$T(r, \theta) = \exp(il_1\theta)\exp(-ik_{r1}r) + \exp(il_2\theta)\exp(-ik_{r2}r) \quad (3.10)$$

where l_1 and l_2 are the orders of the Bessel functions, k_{r1} and k_{r2} are the radial wave vectors of the respective Bessel beams. In the present case $l_1 = l_2 = 0$, as the interference of two zero-order Bessel beams is desired. The above equation contains both amplitude and phase information but only the phase information was used for the beam reconstruction. On incident a Gaussian beam to the SLM, the self-imaged bottle beam was obtained. The intensity of the self-imaged optical bottle beam oscillates along the propagation axis with a bottle embedded in each oscillation similar to simulations results. Figure 3.12 shows the transverse intensity distribution of a self-imaged bottle beam obtained experimentally and captured at a close proximity to one of the bottles embedded in the beam. Figure 3.12(e, f) clearly shows the presence of a bottle embedded in the beam and surrounded by higher intensity shown in Figure 3.12(a, h). The other bottles obtained experimentally vary similarly to the bottle shown in Figure 3.12. The self-imaging phenomena and the presence of bottles both ceased after the maximum non-diffracting distance of the self-imaged bottle beam (Z_{maxOBB}), which is congruous to the simulation result, as discussed above.

Figure 3.12 Experimental generation of a self-imaged optical bottle beam. The images are captured at 20 cm from SLM. The distances between the frames are 2 cm. Frame (e, f) shows one of the bottle embedded in the beam. Frame (a, h) shows the constructive interference of the beam.

The spatial light modulator approach to generate a self-imaged bottle beam is useful as it allows the dynamic control on the parameters of the beam. However, in spatial light modulator considerable amount of power is wasted in higher orders of diffraction. Employing high power-efficient technique for the laser-beam shaping is crucial in many applications including optical tweezers. Therefore a self-imaged bottle beam was generated using phase-only type holograms for higher power conversion efficiency. The phase distribution required for a self-imaged bottle beam is taken from Eq. (3.10). After the quantization of the phase values into binary, e-beam lithography was employed to generate a mask followed by fabrication using UV exposure. Q-2001CT mask aligner (Quintel Corporation) with wavelength 365nm and irradiance of 15mw/cm^2 to expose the photoresist film (AZ5214) was used for the fabrication. Since the speed and duration of the spin coating, the exposure time and the development time determine the

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profile heights of the photoresist film; the calibration was done to get a precise phase distribution on the film. The adopted parameters in the fabrication process were exposure duration of 15s, spin speed of 3500 rpm, development time of 15s. The hologram made of photoresist is a phase-only type, which would have higher diffraction efficiency than its transmission counterpart. The parameters used for the hologram was $k_{r1}=1/\text{pixel}$ and $k_{r2}=0.2/\text{pixel}$ and the size of each pixel is $15\text{ }\mu\text{m}$. Free-space propagation of a self-imaged bottle beam generated from the hologram is shown in Figure 3.13. The self-imaged bottle beam varies its transverse intensity profile periodically with alternative bright and dark central spot. One full period of the self-imaged bottle beam is shown in Figure 3.13(a-f). The bottle is obtained in Figure 3.13(d) and the bright spot is shown in Figure 3.13(a, f). The distance between each frame is 5 cm and first frame is at 15 cm from the hologram. The beam generated by the holographic technique also possesses serious of embedded bottle along the propagation axis, congruous to Figure 3.5.

Figure 3.13 Free space propagation of the experimental generated self-imaged bottle beam using a hologram. One full period of the self-imaged bottle beam is shown above.

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3.4 SELF-HEALING PROPERTY OF SELF-IMAGED BOTTLE BEAM

A self-imaged bottle beams belong to the family of propagation-invariant beams and hence the study of its self-reconstruction property will be beneficial. Moreover, self-healing property of such beams will also be useful in optical tweezers applications. A self-imaged bottle beam was obtained by superimposing two or more Bessel beams hence the knowledge of its minimum self-reconstruction distance is essential. The robustness of a self-imaged bottle beam towards a non-transparent obstacle is studied. Self-imaged bottle beam was found to inherit the self-healing property of the propagation-invariant beams. The periodicity of the self-imaged bottle beam was also found to be immune to the obstacle and regains its original periodicity. This self-healing property of the bottle beam together with the self-imaging property would commence an opportunity to use such beams for optical stacking and trapping of micro-particles in the optical tweezers system for an extended distance.

Initially the self-healing property of a self-imaged optical bottle beam was numerically studied. The self-imaged bottle beam was numerically simulated (as explained in the above section) and then its self-reconstruction property was studied by blocking the bottle beam by a non-transparent obstacle. The radial wavevectors of the Bessel beams interfered in the simulation were $k_{r2}=5k_{r1}$, where k_{r1} is 0.2/pixel and each pixel is given by 15 μm . A beam with a spot size (w_0) of 1.92 mm and a wavelength of 633 nm was used in the simulation. The azimuthal indices (l) of the interfering Bessel beams were set to zero, as the interference of two zero-order Bessel beams was required. The radial dimension of the non-transparent obstacle used in the simulation was 0.232 mm. The obstacle was placed such that it distorts some portion of the central spot as well

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as the first outer ring of the beam as highlighted by an arrow in Figure 3.14(a). The obstacle distorts both the transverse intensity distribution and the periodicity of a self-imaged bottle beam. The self-imaged bottle beam self-heals and reconstructs its transverse intensity distribution during free-space propagation behind the obstacle as shown in Figure 3.14. The transverse intensity distribution of the self-imaged bottle beam recovers fully after a distance of 17.5 cm as shown in Figure 3.14(e), although in Figure 3.14 it represents a distance of 20 cm (distance between Figure 3.14(a) and Figure 3.14(e)) this is to maintain the homogeneous distance between frames. After the healing period Figure 3.14(a-e), the self-imaged beam propagates similarly to the unobstructed self-imaged beam.

Figure 3.14(a-f) Self-reconstruction of an obstructed self-imaged optical bottle beam. A non-transparent obstacle is placed in frame (a), distorting the transverse intensity distribution of beam in frames (a-e). The reconstructed bottle beam is shown in frames (e-f). Distance between consecutive frames is 5 cm. Frames (c, f) shows the bottling points of the beam. Each vertical and horizontal unit of frames (a, b) is in pixel and each pixel is given by 15 μm .

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The longitudinal intensity distribution of the self-imaged bottle beam was studied to determine the influence of a non-transparent obstacle on the periodicity of a self-imaged bottle beam. Figure 3.15(a) shows the longitudinal intensity distribution of an unobstructed self-imaged bottle beam, conversely Figure 3.15(b) show the longitudinal intensity distribution of an obstructed self-imaged bottle beam. The obstacle was placed on the second bottle in Figure 3.15(b). The location of an obstacle is highlighted by an arrow in Figure 3.15(b). The obstacle deforms the intensity profile and the periodicity of the beam for a finite distance, which can be observed from Figure 3.15(b); however along the propagation axis the beam self-reconstruct its original periodicity as observed by comparing the third bottle of Figure 3.15(b) to Figure 3.15(a).

Figure 3.15(a, b) Longitudinal intensity distribution of unobstructed and obstructed self-imaged bottle beam, respectively. In frame (b) a non-transparent obstacle is placed as highlighted by an arrow. An obstructed self-imaged bottle beam regains its original periodicity along the propagation direction. Each vertical and horizontal unit of frames (a, b) is in pixel (each pixel is $15\ \mu\text{m}$) and $1\ \text{cm}$, respectively. The horizontal axis represents the propagation axis and the vertical axis represents the side view of the beam.

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The minimum distance required by a distorted self-imaged bottle beam to reconstruct its intensity distribution was further determined. In the above simulation the minimum self-reconstruction distance Z_{minSI} required by a distorted self-imaged bottle beam was 17.5 cm when the radial size of the obstacle was 0.232 mm. It is known from literature [10] that for the individual Bessel beam the presence of the non-transparent obstacle blocks the conical distribution of the plane waves in the shadow region behind it and deforms its transverse intensity distribution for a finite distance $Z_{min} = r_a \cdot (k/k_r)$ where r_a is the radial dimension of an obstacle.

The schematic diagram shown in Figure 3.16 explains the minimum self-reconstruction distance (Z_{minSI}) of the obstructed self-imaged bottle beam. A self-imaged bottle beam is the superposition of two Bessel beams each generated from different annular rings with different radial wavevectors as shown in Figure 3.16. In geometrical interpretation a Bessel beam represents the coherent superposition of the plane waves with the wavevectors covering the conical surface. The presence of the non-transparent obstacle blocks the superposition of the plane waves in the respective shadow region for the individual Bessel beams deforming the transverse intensity distribution for a finite distance (Z_{min}). In the similar manner the presence of the obstacle in Figure 3.14(a) or Figure 3.15 is blocking the converging superimposed plane waves that are interfering to form a self-imaged bottle beam as schematically shown in Figure 3.16. The shadow or the distortion regions obtained by the obstacle in Figure 3.16 is different for each interfering Bessel beam. Hence the transverse intensity distribution of the obstructed self-imaged beam will be restored if both the interfering Bessel beams reconstruct individually. Each Bessel beam if obstructed individually by the same obstacle will

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reconstruct after a distance Z_{min1} and Z_{min2} , which is given by $Z_{min1,2} = r_a(k/k_{r1,2})$. Therefore the minimum self-reconstruction distance required for a distorted self-imaged bottle beam should be given as: -

$$Z_{minSI} = \max(Z_{min1}, Z_{min2}) \quad (3.11)$$

In the case discussed in Figure 3.14, the parameters used in simulation gives $Z_{min1} > Z_{min2}$, which were calculated to be 17.3 cm and 3.5 cm, respectively. Hence theoretically Z_{minSI} is 17.3 cm, which matches closely with the simulation result of 17.5 cm obtained in Figure 3.14. Thus Eq. (3.11) can be used to determine the minimum self-reconstruction distance of an obstructed self-imaged bottle beam.

Figure 3.16 Schematic diagram for the explanation of self-reconstruction period of a distorted self-imaged bottle beam by a non-transparent obstacle.

Thus a novel propagation-invariant field, termed as a self-imaged optical bottle beam, was reported. The unique property of such beams is that it possesses an array of three-dimensional intensity-null points or bottles. The transverse intensity profile of such beams oscillates along the propagation axis. Moreover, the self-imaged bottle beams

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inherit the interesting property of self-reconstruction of the propagation-invariant beams. Thus a self-imaged bottle beam provides an opportunity to integrate such beams with an optical tweezers system. Contrary to the Bessel beam core, the self-imaged bottle beam provides oscillating intensity distribution and thus creating three-dimensional trapping potential. Self-imaged bottle beam possessed few distinct advantages in an optical tweezers application, which is reported in the next Chapter.

3.5 HELICON BEAMS

3.5.1 Introduction

Propagation-invariant beams and its interference gained considerable interest due to many potential applications [86, 103, 118, 136-140]. Initially researchers reported the interference of two zero-order Bessel beams ($l_1=l_2=0$) with varying radial wavevector ($k_{r1} \neq k_{r2}$) in order to generate smaller central core of the resultant beam [138]. However, later Bouchal *et al* and S. C. Cerda *et al* reported the self-imaging phenomena by interfering two zero-order Bessel beams ($l_1=l_2=0$) with varying radial wavevectors ($k_{r1} \neq k_{r2}$) [139, 136-137]. The same understanding was exploited in the above section by dexterously choosing the radial wavevector of superimposing zero-order Bessel beams to obtain self-imaged optical bottle beams. Interest on interfering higher-order Bessel beams ($l_1, l_2 > 0$) or singular Bessel beams with same radial wavevector ($k_{r1} = k_{r2}$) was reported to generate much richer vortex content during the free space propagation [86]. Interference of higher-order Bessel beams with varying radial wavevector ($k_{r1} \neq k_{r2}$) and varying topological charges ($l_1 \neq l_2$) generated a novel rotating propagation-invariant field coined as **helicon beams** [103, 140]. Even substantial research has been reported on interfering Bessel

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beams, still lot of interesting phenomena can be observed on tailoring the parameters of interfering Bessel beams.

Bessel beams can be interfered based on azimuthal indices (l) and radial wavevectors ($k_{r,l}$). Interfering zero-order Bessel beams ($l_1=l_2=0$) with different radial wavevectors ($k_{r,1} \neq k_{r,2}$) generates a self-imaged bottle beam as reported above. However, interfering higher-order Bessel beams with different radial wavevectors ($k_{r,1} \neq k_{r,2}$) and azimuthal indices ($l_1 \neq l_2$) generates a special rotating and self-imaged non-diffracting field termed as helicon beams. As long as the interfering Bessel beams possess different radial wavevectors ($k_{r,1} \neq k_{r,2}$), the transverse intensity profile of the resultant beam will possess self-imaging effect whereby the transverse intensity profile will oscillate along the propagation axis. The following cases can be classified for the interference of two Bessel beams based on k_r and l : -

- Different k_r and $l=0$ (Self-imaged optical bottle beam)
- Different k_r and different l (Rotating nondiffracting field or Helicon beams)
- Different k_r and same $l>0$ (Self-imaged non-diffracting beam)
- Same k_r and different l (Non-diffracting beam with intensity modulation)

Most of the above regimes of interfering Bessel beam are covered [86, 103, 118, 136-140] however only inception work on a helicon wave was reported [103]. Therefore, detailed numerical study of the different regimes for the helicon waves is reported in the thesis. Moreover, the potential applications of the helicon waves in the trapping application are also discussed.

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3.5.2 Interfering Bessel beams with varying topological charge and radial wavevector (Helicon waves)

Researchers reported interesting rotating propagation-invariant field whose transverse intensity profile is non-diffracting except the rotation of intensity around its optical axis. Such unique fields were obtained by interfering higher-order Bessel beams with varying azimuthal indices (l_1, l_2) and varying radial wavevectors (k_{r1}, k_{r2}) [103]. Such rotating beam can also be obtained by interfering two Laguerre-Gaussian beams; however the cases discussed here are limited to non-diffracting rotating beams generated by interference of two or more propagation-invariant beams. The intensity of the interfering Bessel beams can be given as: -

$$I_1(r, \theta, z) = J_{l_1}(k_{r1}r) \exp(il_1\theta) \exp[izk_{z1}] \quad (3.12)$$

$$I_2(r, \theta, z) = J_{l_2}(k_{r2}r) \exp(il_2\theta) \exp[izk_{z2}] \quad (3.13)$$

As reported in Chapter Two, an angular spectrum approach is usually employed to visualize non-diffracting fields; therefore such rotating beams or helicon beams can also be studied in frequency domain. The angular spectrum of a helicon field generated by the superposition of two non-diffracting beams (with parameters k_{r1}, k_{z1}, l_1 and k_{r2}, k_{z2} and l_2) can be given by the linear addition of the angular spectrum of individual Bessel beams as reported in Eq. (2.10) and is given as [103]: -

$$A(\rho, \phi, z) = \delta(\rho - k_{r1}) \exp[i(k_{z1}.z + l_1\phi)] + \delta(\rho - k_{r2}) \exp[i(k_{z2}.z + l_2\phi)] \quad (3.14)$$

The amplitude of the helicon beam is therefore can be obtained as [103]: -

$$U(r, \theta, z) = k_{r1}(i)^{l_1} J_{l_1}(k_{r1}r) \exp(il_1\theta + k_{z1}.z) + k_{r2}(i)^{l_2} J_{l_2}(k_{r2}r) \exp(il_2\theta + k_{z2}.z) \quad (3.15)$$

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Substituting the value of θ' as given below in Eq. (3.15) it can be further simplified as

[103]: -

$$\theta' = \theta + z \left(\frac{k_{z2} - k_{z1}}{l_2 - l_1} \right) \quad (3.16)$$

$$U(r, \theta, z) = \exp \left[iz \left(\frac{(l_2 z_1 - l_1 z_2)}{l_2 - l_1} \right) \right] U(r, \theta', 0) \quad (3.17)$$

The intensity distribution of the helicon wave can be determined by squaring the amplitude given in Eq. (3.17) and is given as [103]: -

$$I(r, \theta, z) = k_{r1}^2 J_{l1}(k_{r1}r)^2 + k_{r2}^2 J_{l2}(k_{r2}r)^2 + 2k_{r1}k_{r2}J_{l1}(k_{r1}r)J_{l2}(k_{r2}r) \cos \left[(l_2 - l_1) \left(\theta' + \frac{\pi}{2} \right) \right] \quad (3.18)$$

$$I(r, \theta, z) = I(r, \theta', 0) = I(r, \theta + z \left(\frac{k_{z1} - k_{z2}}{l_2 - l_1} \right), 0) \quad (3.19)$$

Hence just like a Bessel beam it can be noticed that the transverse intensity of the helicon beam is propagation-invariant in the free-space except the rotation of the intensity profile around its optical axis, as observed from Eq. (3.19). The rate of rotation of the helicon beams are proportional to the difference of the squared radial wavevectors of the interfering Bessel beams and is inversely proportional to the difference of the topological charge of interfering Bessel beams [103]: -

$$\frac{\partial \theta}{\partial z} = - \left(\frac{k_{z2} - k_{z1}}{l_2 - l_1} \right) \approx \frac{1}{2\pi} \left(\frac{k_{r2}^2 - k_{r1}^2}{l_2 - l_1} \right) \quad (3.20)$$

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The longitudinal distance required by such rotating fields for complete 2π rotation can be given as [103]: -

$$\Delta z_{2\pi} = \frac{2\pi}{\partial\theta/\partial z} = 4\pi k \frac{(l_2 - l_1)}{k_{r2}^2 - k_{r1}^2} \quad (3.21)$$

Therefore it can be observed from Eq. (3.20) and Eq. (3.21) that to generate helicon beams it is necessary to interfere Bessel beams with varying radial wavevectors and topological charge. Numerical simulations of the helicon beams are carried and various cases are discussed below.

3.5.3 Study of various helicon fields

Numerical simulations of helicon waves were obtained by interfering Bessel beams with varying radial wavevectors and topological charges and the results were found consistent with Ref. [103]. However, contrary to Ref. [86, 103] different regimes of the helicon waves are discussed. The parameters chosen for the simulations were 500 nm wavelength and beam waist size of 1.92 mm. The rotating non-diffracting field or the helicon waves were generated by interfering two Bessel beams with radial wavevector ($k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and each pixel is given as 15 μm) and varying topological charge $l_1=4$ and $l_2=2$. Figure 3.17(a-f) shows the transverse intensity profile of a helicon wave obtained numerically. The transverse intensity profile of the helicon wave rotates anti-clockwise direction along the propagation direction in Figure 3.17. The orientation of the arrows in Figure 3.17 highlights the rotation of the helicon beam in anti-clockwise direction in each quadrant. Rotation of the helicon field is in clockwise direction if $(l_2-l_1) > 0$ and is in anti-clockwise direction or $(l_2-l_1) < 0$. Figure 3.18 shows the longitudinal

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intensity distribution of the helicon field generated in Figure 3.17. Figure 3.18 shows a longitudinal distance of $\min(Z_{max1}, Z_{max2})$, where Z_{max1} and Z_{max2} are the maximum non-diffracting distance (Z_{max}) of the interfering Bessel beams. It was noticed that helicon beams are obtained within a distance where both the interfering Bessel beams are non-diffracting. If either of the interfering Bessel beams crosses Z_{max} distance, the rotation was ceased. In all the cases discussed here $k_{r2} > k_{r1}$ if converse is true, than the rotation direction and the distribution of topological charges at the center and outer rings is reversed.

Figure 3.17(a-f) Transverse intensity of the rotating helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=4$ $l_2=2$. Each pixel is $15\mu\text{m}$. The transverse intensity profile rotates in anti-clockwise direction. Each unit of y-axis and x-axis is $62.5\mu\text{m}$.

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Figure 3.18 Longitudinal intensity distribution of the helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=4$, $l_2=2$. Each pixel is $15\mu\text{m}$. The units of y-axis is arbitrary and the x-axis represents the longitudinal axis and each unit is $\min(Z_{\max1}, Z_{\max2})/50$, where $Z_{\max1}$ and $Z_{\max2}$ are the maximum non-diffracting distance of interfering Bessel beams.

As reported in Ref. 86 when the condition on the topological charge $l_1 > |l_2|$ of the interfering Bessel beam, is satisfied, there is l_2 charge at the center of beam and $(l_2 - l_1)$ charge in the outer rings, with alternating signs in the alternating outer rings. For the case shown in Figure 3.17 the condition $l_1 \geq |l_2| \geq 0$ was fulfilled and thus there is l_2 charge at the center of beam. Therefore under such conditions there is an intensity minimum at the center of the beam and the first ring of the resultant beam is closed enclosing charge l_2 as observed in Figure 3.17 and Figure 3.18. Such intensity profile can be used to trap low-index microparticles in the center within the first ring and the transfer of orbital angular momentum can be accomplished. As discussed in Chapter Two the low-indices microparticle are repelled from the higher intensity region and are trapped in the doughnut-structured beams.

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However, in the literature reported in Ref. 86 the case discussed are when interfering Bessel beams satisfy the following conditions $k_{r1}=k_{r2}$ and $l_1 \geq |l_2| \geq 0$. However, here generalized cases with all the possible conditions and resultant intensity profiles of the helicon waves are discussed.

Other possibility is when $l_1 < |l_2|$ but $|l_1| > |l_2|$ for example, topological charges $l_1=-3$ and $l_2=-1$ of the interfering Bessel beams. The resultant intensity profile of the helicon beams under such condition is found consistent with condition $l_1 > |l_2|$, and a charge of l_2 at the center of the helicon beam and $(l_2 - l_1)$ charge in the outer rings were obtained. The result of the numerical simulation of a helicon beam under such conditions, with $k_{r1}=2k_{r2}$ and topological charge $l_1=-3$ and $l_2=-1$ is shown in Figure 3.19 and Figure 3.20. Figure 3.19(a-c) shows the transverse intensity profile of the resultant helicon waves and Figure 3.20 shows the longitudinal intensity distribution of the helicon wave obtained in Figure 3.19. The transverse and longitudinal intensity profile of Figure 3.19 and Figure 3.20 is similar to the one obtained in Figure 3.17 and Figure 3.18. However, the rotation in the present case is in clockwise direction as observed in Figure 3.19 contrary to anti-clockwise rotation obtained in Figure 3.17, which is due to the sign of $(l_2 - l_1)$ as explained above. The arrows in Figure 3.19 highlight the rotation of the helicon beam in clockwise direction in different quadrants. The central closed ring of the resultant helicon beam embedding topological charge of l_2 obtained under such conditions can also be used in an optical tweezers application for the trapping of a low index microparticle.

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Figure 3.19(a-f) Transverse intensity of the rotating helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=-3$ and $l_2=-1$. Each pixel is $15\mu\text{m}$. The transverse intensity profile rotates in clockwise direction. Each unit of y-axis and x-axis is $62.5\mu\text{m}$.

Figure 3.20 Longitudinal intensity distribution of the helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=-3$ and $l_2=-1$. Each pixel is $15\mu\text{m}$. The units of y-axis is arbitrary and the x-axis represents the longitudinal axis and each unit is $\min(Z_{max1}, Z_{max2})/50$, where Z_{max1} and Z_{max2} are the maximum non-diffracting distance of interfering Bessel beams.

Helicon beams obtained by interfering the Bessel beams under following conditions $l_1 \geq |l_2|$ and $l_2=0$, possess zero charge at the center of the helicon beam and the bright central core was obtained as shown in Figure 3.21. The radial wavevectors of the

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interfering Bessel beams chosen in the simulation were $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ (pixel=15 μm) and the topological charge of the interfering Bessel beams were $l_1=2$ and $l_2=0$. The intensity profile of the beam rotates in the anti-clockwise direction along the propagation axis, which can be observed from the outer rings of the helicon beam in Figure 3.21. The arrows in Figure 3.21 highlight the rotation of the helicon beam in anti-clockwise direction in different quadrants. Figure 3.22 shows the longitudinal profile of the helicon beam shown in Figure 3.21. The presence of the central bright core is clear from Figure 3.22. Such intensity pattern obtained by interfering Bessel beams can be used in trapping and guiding applications of the high-indices microparticles at the center of the beam.

Figure 3.21(a-c) Transverse intensity of the rotating helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=2$ and $l_2=0$. The intensity profile rotates in anti-clockwise direction. The units of y-axis and x-axis are in pixel and each pixel is given by 15 μm .

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Figure 3.22 Longitudinal intensity distribution of the helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=2$, $l_2=0$. Each pixel is $15\mu\text{m}$. The units of y-axis is arbitrary and the x-axis represents the longitudinal axis and each unit is $\min(Z_{max1}, Z_{max2})/50$, where Z_{max1} and Z_{max2} are the maximum non-diffracting distance of interfering Bessel beams.

Helicon beams obtained under the condition $l_1 < |l_2|$ of the interfering Bessel beams, possess (l_2-l_1) charge at the center and due to this the central ring of the helicon beam is broken as shown in Figure 3.23. The broken central ring is clear on comparing Figure 3.24 to Figure 3.18. Figure 3.23 shows transverse intensity profile of the rotating helicon field obtained by interfering Bessel beams with radial wavevector of $k_{r1}=2k_{r2}$ and $k_{r1}=3/\text{pixel}$ (pixel= $15\mu\text{m}$) and topological charges $l_1=2$ and $l_2=4$. The helicon beam obtained in Figure 3.23 rotates in clockwise direction. The arrows in Figure 3.23 highlight the rotation of the helicon beam in clockwise direction in different quadrants. Figure 3.24 shows the longitudinal intensity distribution of the helicon beam obtained in Figure 3.23. It is worth mentioning that under $l_1 < |l_2|$ condition the helicon beams obtained cannot be used to trap the low-indices particle at the center of the beam due to broken ring. Therefore as long as condition $l_1 < |l_2|$ of the interfering Bessel beams are

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satisfied the rotating beam obtained has broken central ring which was observed in all the possible combination under $l_1 < |l_2|$ condition.

Figure 3.23(a-f) Transverse intensity of the rotating helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=2$, $l_2=4$. The intensity profile rotates in clockwise direction. Each unit of y-axis and x-axis is $15 \mu\text{m}$.

Figure 3.24 Longitudinal intensity distribution of a helicon wave generated by interfering Bessel beams with $k_{r2}=2k_{r1}$ and $k_{r1}=3/\text{pixel}$ and $l_1=4$, $l_2=2$. Each pixel is $15 \mu\text{m}$. The units of y-axis is arbitrary and the x-axis represents the longitudinal axis and each unit is $\min(Z_{\max1}, Z_{\max2})/50$, where $Z_{\max1}$ and $Z_{\max2}$ are the maximum non-diffracting distance of interfering Bessel beams.

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Various regimes of the helicon waves were numerically simulated and discussed and their potential applications in optical tweezers system were also reported. A helicon beam similar to a self-imaged bottle beam was obtained by interfering two Bessel beams. In the next Chapter the proposed self-imaged bottle beam was integrated with an inverted optical tweezers system for various micromanipulations.

CHAPTER FOUR

OPTICAL MICROMANIPULATION USING PROPAGATION-INVARIANT BEAMS

“Music is the arithmetic of sounds as optics is the geometry of light.”

Claude Debussy

CHAPTER FOUR

4.1 INTRODUCTION

Optical trapping technique makes use of the optical forces generated by a focused laser beam to trap and manipulate microscopic particles [1, 2]. It is evident from previous Chapters that the micromanipulation of particles in optical trapping technique mainly depends on the input laser beam. Gaussian beam is normally used to trap high-index particles [2] and singular beams like LG beam [6] and higher-order Bessel beam [101] are used to trap both high and low-index particles and to transfer orbital angular momentum [100, 102] to the trapped particles. Bessel beams are used to trap and levitate or guide microparticles exploiting its non-diffracting and self-reconstruction properties [11, 99]. Laser beams employed in optical tweezers can broadly be classified (based on their trapping capability), as propagation-invariant beams or non-diffracting beams (like Bessel beam) and diffracting beams (like Gaussian beam and LG beam). Diffracting beams diverge away very rapidly after being tightly focused. Highly focused beams are usually employed in the optical tweezers; hence diffracting beams are limited to trap microparticles at or near the beam focus. Contrary to this, employing non-diffracting beams this problem can be solved by exploiting its non-diffracting property, which enables to trap microparticle for extended distance and even in different chambers [99]. As the core of a Bessel beam does not diffract rapidly due to its non-diffracting property so it can be used to trap microparticles all along the portion of the beam where it remains diffraction free. Also, non-diffracting beams have unique property of self-reconstruction, whereby the beam self-reconstruct itself around any obstacle it meets, i.e. it can be use to trap particles in different planes along the beam propagation length. However, a Bessel

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beam does not provide real three-dimension trapping, contrary to a Gaussian beam or a LG beam based optical tweezers system [11].

Figure 4.1 shows the schematic diagram comparing different beams used in optical trapping system and proposed bottle beam based optical tweezers. The main difference between a diffracting and non-diffracting beams used in optical tweezers is that the former one can be focused down to a tight spot and can trap particle in three-dimensions, as depicted in Figure 4.1(a, b). Figure 4.1(a) and Figure 4.1(b) shows a Gaussian beam and an LG beam trapping a particle in three-dimensions, respectively. While the core of Bessel beam, approximates like a rod of light provides only two-dimensional trapping potential, as shown in Figure 4.1(c, d). Figure 4.1(c) shows a particle transversely trapped in the core of a Bessel beam, which will finally get levitated all along in the core of the Bessel beam as shown in Figure 4.1(d). Thus a high-index particle trapped in Bessel beam core is loosely defined in the longitudinal axis. Employing the proposed self-imaged bottle beam can be useful for solving above-mentioned problems. Contrary to previously used beam in the optical tweezers the transverse intensity profile of the self-imaged bottle beam varies periodically (alternative dark and bright spots) along the propagation axis as shown in Figure 4.1(e, f) and hence generating a three-dimensional potential well. Thus the self-imaged bottle beam based optical tweezers system will provide an additional control to the trapped high-index particle in the longitudinal axis due to bottling effect in the core of the beam. Figure 4.1(e) shows a high-index particle trapped transversely in a constructive interference region of a self-imaged bottle beam. The particle cannot cross the potential well created by the bottling effect as shown in Figure 4.1(f). Moreover, the proposed self-imaged

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bottle beam belongs to the family of propagation-invariant beams and inherit the beneficial property of self-reconstruction as reported in Chapter Three.

Figure 4.1 Schematic diagram comparing various beam used in optical trapping. (a) Gaussian beam, (b) LG beam, (c, d) Bessel beam and (e, f) Proposed self-imaged bottle beam.

The transverse intensity profile of a self-imaged bottle beam oscillate along the propagation axis contrary to other conventional beams used in trapping (Gaussian, LG and Bessel beams) thus it becomes imperative to image appropriate point of a self-imaged bottle beam onto the sample stage for desired trapping application. Thus, just by imaging the appropriate points of the self-imaged bottle beam for example central dark or central bright spot; it can easily be modified for different trapping applications. The region where there is a constructive interference in a self-imaged bottle beam, it can be used as a Bessel beam based tweezers system for trapping and guiding application. Similarly, region where there is a bottling effect, the same set-up can be used to trap a low-index and an absorptive particle.

The other unique property of the proposed system is that it can stably trap a low-index and an absorptive particle at the central dark region without the default transfer of the orbital angular momentum (OAM) associated with the singular beams. Conventionally to trap the low-index and the absorptive particles singular beams are employed and thus the default transfer of the orbital angular momentum of such beam is

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transferred to the trapped particles. The default transfer of OAM carried by singular beams might not be desirable for the stable trapping in colloidal and bio-medical applications. This detrimental point can be solved employing self-imaged bottle beam based optical trapping system, as the central dark spot or null-intensity points in a self-imaged bottle beam is due to destructive interference of interfering Bessel beam contrary to singular beam where the central dark spot is due to singularity (vortex), carrying orbital angular momentum.

4.2 SELF-IMAGED BOTTLE BEAM BASED OPTICAL TWEEZERS SYSTEM

A self-imaged bottle beam based optical tweezers system is reported for various trapping applications. A self-imaged bottle beam was generated by employing hologram technique as reported in Chapter Two. Figure 4.2 shows experimentally generated self-imaged bottle beam in the free space and its corresponding images at the sample stage after employing ten times telescopic arrangement as shown in the experimental set-up in Figure 4.3. The unique intensity oscillation property of a self-imaged bottle beam enables to image different points of the beam for different trapping application. Figure 4.2(a-d) shows the two periods of a self-imaged bottle beam in the free space and Figure 4.2(e-h) shows the corresponding points of a beam imaged onto the trapping sample stage. Different points of the self-imaged bottle beam can be imaged onto the trapping stage by adjusting the distance between the hologram and lens f_3 (as shown in Figure 4.3). Figure 4.3 shows the experimental set-up of bottle beam based optical tweezers system. The light from an Nd:YVO₄ laser (1 W, 1064nm) was expanded five times by the lenses f_1 and f_2 (focal length of lenses f_1 and f_2 was 5 cm and 25 cm, respectively) before incident to

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the fabricated hologram. The generated self-imaged bottle beam was telescoped down by ten times by the lenses f_3 and f_4 (focal length of f_3 and f_4 was 25 cm and 2.5 cm, respectively) at the sample stage to reduce the central spot of diameter around 8~ 10 μm . X-Y translation was provided to the lens f_4 to manipulate the beam spot over the sample stage and x-y-z translation was provided to the sample stage. An objective lens of magnification 40X (Zeiss objective lens NA 0.65 CP-Achromat) was used to image the microparticles. IR-sensitive Hamamatsu C5332 camera was used to image both the particles and the beam. Dichoric mirror and lens used in the experiment was IR-coated for higher efficiency. Optical Fiber illumination (Thorlabs OSL1-EC) was employed for imaging purpose.

Point A and B in Figure 4.3 shows two distinct transverse intensity profile of the self-imaged bottle beam imaged at the sample stage, which could be used for different trapping applications. Hence, bottle beam based trapping system provides an opportunity to employ single beam for different trapping applications by appropriately opting for the different points of the bottle beam in the trapping applications. Point B corresponds to the bright spot of the self-imaged bottle beam, which is similar to the intensity profile of the Bessel beam and hence can be used to trap and levitate high-index microparticles. Point A represents the dark spot or the destructive interference point of the self-imaged bottle beam imaged onto the sample stage, which can be used to trap a low-index or an absorptive microparticle. Hence, just by imaging different points of the optical bottle beam, the set-up can be used as LG beam or Bessel beam based optical trapping set-up.

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Figure 4.2 Self-imaged bottle beam in the free space propagation (a-d) and imaged onto the sample stage (e-f) for trapping applications.

Figure 4.3 Schematic diagram of the experimental set-up for the self-imaged bottle beam based optical tweezers system.

Initially a bottle beam based optical tweezers system was used to trap and manipulate the low-indices microparticles. Point A of Figure 4.3 was imaged onto the sample stage for the optical manipulation of the low-index particles. Figure 4.4(a-d) shows the transverse trapping and manipulation of a low-index microparticle (hollow

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silica spheres) of dimensions around 8~10 μm . The transverse dimension of the bottle was made comparable to the dimension of the low-index particle, as shown in Figure 4.4(a). The circle in Figure 4.4(a) highlights the low-index particle chosen for trapping and the dotted arrow shows the predetermined direction for manipulation. The low-index microparticle is repelled from the region of higher intensity and is trapped in the dark spot or minimum intensity region. In this case low-index particle get trapped in the bottle or intensity void. Once the low-index particle was transversely trapped, it can be manipulated over the sample stage as depicted in Figure 4.4 (b-d). Arrows marked in the Figure 4.4 (b-d) shows the relative distance between the reference particle and the trapped particle.

The experiment was carried employing Nd:YVO₄ laser (1 W, 1064nm) with output power of 150 mW. The power conversion efficiency of the fabricated hologram was experimentally determined around ~40%, thus the generated self-imaged bottle beam possesses power of around 60 mW employed in experiment above. Other losses for example absorption and reflection from the optical elements and inefficient coupling of laser beam to back aperture of objective lens is approximated to cause less than 10% of power loss. The approximated power (P) at the sample stage in the above experiment was around ~50 mW, thus the force experienced (F) by the microparticles can be approximated as $F = Q.(n/c).P$ where Q is the quality factor (which can take maximum value of 0.3), P is the power at the sample stage, c is the speed of light and n ($n=1.33$ in this case) is the refractive index. Optical trapping system employed in the experiment using 100X oil immersion objective lens, water as the medium the Q -value is approximated as ~0.1. Thus the force experienced by the microparticles is approximated

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as ~ 22 pN. Details on required power level at the sample stage for optical trapping can further be found in Appendix A.

Figure 4.4 Optical trapping and manipulation of low-index particle (hollow silica sphere) in the bottle, employing self-imaged bottle beam based optical tweezers system.

Figure 4.4 shows the transverse or two-dimensional trapping of a low-index particle. However, as explained above the oscillating intensity profile of the bottle beam can be further used to localize the trapped low-index particles in three-dimensions. Figure 4.5 shows the schematic diagram to explain the technique to trap a low-index particle three-dimensionally using a self-imaged bottle beam. Shown schematically in Figure 4.5(a) is a low-index particle transversely trapped in one of the central dark spot of a self-imaged bottle beam. On imaging the bottle just above the sample stage by slowly changing the distance between the lens f_3 and hologram (Figure 4.3) the low-index particle can be levitated as shown schematically in Figure 4.5(b).

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Figure 4.5 Optical trapping and manipulation of low-index particle (hollow silica sphere) in the bottle, employing self-imaged bottle beam based optical tweezers system.

Employing the technique depicted in Figure 4.5 a low-index particle was localized in three-dimensions. Figure 4.6 shows the trapping and levitation of a single low-index microparticle (hollow silica sphere) by the bottle beam. The low-index particle of diameter around $8\ \mu\text{m}$ was initially trapped transversely as shown in Figure 4.6(a) at the central dark region of the bottle. The hollow circle marks one of the reference microparticle. The dotted arrow marked in Figure 4.6(a) signifies the predetermined direction of manipulation and the arrows marked in Figure 4.6(b-d) highlights the relative distance of the trapped particle and the trapped particle. By imaging the bottle just above the sample stage the low-index particle was levitated as explained in Figure 4.5(b). Figure 4.6(b) shows the levitation of a low-index particle and as the objective lens images the levitated particle in Figure 4.6(b) the reference particles highlighted by a hollow circle was imaged out-of-focus and thus blur. Once the particle was levitated it can be

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manipulated over the sample stage as marked by the arrow in Figure 4.6(a-c). In Figure 4.6(c) the focus of the objective lens is at the plane of the reference particles and hence the levitated low-index particle is out-of-focus and blur. Figure 4.6(d) shows the final position of a low-index particle when the laser beam was switched-off and the levitated low-index particles settled down at the bottom of the sample stage.

Figure 4.6 Optical trapping and manipulation of a low-index particle (hollow silica sphere) in the bottle, employing self-imaged bottle beam based optical tweezers system. The scale bar shows the dimension of $5\mu\text{m}$.

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On imaging point B of a self-imaged bottle beam the same tweezers system can easily change to Bessel beam based optical tweezers. Figure 4.7(a-f) shows the optical trapping and levitation of a high-index particle (silica sphere) using a self-imaged bottle beam based tweezers systems. Figure 4.7(a) shows the silica spheres of diameter $5\text{ }\mu\text{m}$ in the sample stage (input beam blocked), the hollow circle in Figure 4.7(a) highlights the microparticle opt for levitation and the arrows in Figure 4.7 mark one of the reference microparticles. Figure 4.7(b) shows the transverse trapping of a silica sphere in the central core of a self-imaged bottle beam (at a constructive interference point), when the input power is 30mW. On increasing the power of the laser to 150 mW, the particle was levitated along the longitudinal direction as shown in Figure 4.7(b-e). Longitudinal distance between Figure 4.7(a-e) is around $90\text{ }\mu\text{m}$, which is the dimension of the sample stage used in the experiment. It can be noticed from Figure 4.7(c) that the trapped sphere is just levitated and hence blur. In Figure 4.7(d) the objective lens images the levitated sphere and thus the reference microparticles are out-of-focus and blur. Finally the levitated microparticle reaches the top of sample stage as shown in Figure 4.7(e) and the reference particle (highlighted by the arrows) is out-of-focus and thus disappears. The objective lens images the reference plane in Figure 4.7(f) after blocking the input beam. The levitated sphere highlighted by the hollow circle is missing and all other reference spheres are present in the reference plane, with nominal transverse displacement due to the Brownian motion. Hence, a self-imaged bottle beam based optical tweezers system can be used to trap low-index particle at the central dark spot and at the same time can also be used to trap and guide high-index microparticle at the central bright spot.

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Figure 4.7 Optical trapping and levitation of a high-index particle (silica sphere) at the constructive interference of a self-imaged bottle beam. The scale bar shows the dimension of 10 μm .

4.3 TRANSFER OF ON-AXIS PURE SPIN ANGULAR MOMENTUM EMPLOYING BOTTLE BEAM BASED OPTICAL TWEEZERS SYSTEM

4.3.1 Introduction

Polarization depicts the orientation of the electric field vectors. When electric field oscillates in one plane with respect to the direction of propagation, the light is said to be linear polarized. When the electric field vector rotates with respect to the direction of the propagation, the beam is circular polarized. An elliptical polarized light with electric field vector E and angular frequency ω can be given as: -

$$E = E_o e^{(i\omega t)} \cos(\phi.x) + i E_o e^{(i\omega t)} \sin(\phi.y) \quad (4.1)$$

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where ϕ indicated the degree of ellipticity of the polarized light, when $\phi = 0$ or $\pi/2$ the light is said to be plane polarized light and when $\phi = \pi/4$, the light is circular polarized.

When light is circular polarized then each photon carries a spin angular momentum (SAM) given as $\sigma\hbar$, where σ is (+1 or -1) for right or left circular polarized light and zero for plane polarized light. In 1936, Beth [141] reported that the circular polarized light could exert a torque upon a birefringent wave plate which changes the polarization of the light beam and hence experience equal and opposite torque. Besides spin angular momentum light can carry orbital angular momentum (OAM), which arises due to the helical phase fronts of the beam [3, 6].

Optical tweezers acts as a versatile tool to study the spin angular momentum possessed by the polarized light. The particles that alter the polarization of the input light experience the equal and opposite momentum, (spin angular momentum). The trapped microparticle would experience the momentum, which is due to the change of polarization of the light beam after transmitting from the trapped microparticle. Absorptive particles (Teflon, metallic particles) and birefringent particles (calcite, liquid crystal) are commonly used to study spin angular momentum of light employing optical tweezers system.

A birefringent particle (calcite, in this case) has two refractive indices n_o and n_e for the ordinary and the extraordinary rays. Due to this anisotropic nature of the calcite when the light beam pass through it, the ordinary and the extraordinary rays undergo different phase shifts, and if the light is linear or circular polarized it results in the change of the angular momentum carried by the light, which would be experienced by the

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microparticle. For a light beam with field vector $\mathbf{E}$ and angular frequency ω , the total angular momentum carried by the light is given as [142]: -

$$\mathbf{J} = \epsilon_0 \int d\mathbf{r}^3 \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) \quad (4.2)$$

This contains both the orbital and spin angular momentum, however the spin angular momentum is given as [142]: -

$$\mathbf{J}_{spin} = \frac{\epsilon_0}{2i\omega} \int d\mathbf{r}^3 (\mathbf{E}^* \times \mathbf{E}) \quad (4.3)$$

where $\mathbf{E}^*$ is the complex conjugate of the electric field vector and ϵ_0 is the permittivity. When the light passes through a birefringent particle the polarization of the light changes, which results, to the torque on the particle causing it to either rotate or align with the polarization axis depending on circular or linear polarization of the incident light respectively. The torque (τ) experienced by the microparticle can be determined by evaluating the change of the angular momentum of the incident ($\mathbf{j}_{before}$) and emerging light ($\mathbf{j}_{after}$) given as [142]: -

$$\boldsymbol{\tau} = \mathbf{j}_{before} - \mathbf{j}_{after} \quad (4.4)$$

$$\boldsymbol{\tau} = \frac{\epsilon_0}{2i\omega} \int d\mathbf{r}^3 (\mathbf{E}_{before}^* \times \mathbf{E}_{before} - \mathbf{E}_{after}^* \times \mathbf{E}_{after}) \quad (4.5)$$

Moreover, the incident elliptical polarized light can be written in terms of components parallel and perpendicular to the optics axis of the particle [142]: -

$$\mathbf{E} = E_o e^{i\omega t} (\cos \phi \cos \theta - i \sin \phi \sin \theta) \hat{\mathbf{i}} + E_o e^{i\omega t} (\cos \phi \sin \theta + i \sin \phi \cos \theta) \hat{\mathbf{j}} \quad (4.6)$$

The angle θ is between the fast axis of the quarter-wave plate (used to polarize light) and the optic axis of the birefringent particle (calcite). The birefringent particle has two

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refractive indices n_o and n_e for the ordinary and extraordinary rays, therefore the ray passing through the calcite particle of thickness d and refractive index n of the calcite would bring a phase change of $k.d.n$, where k is the free-space wavenumber. Therefore the emergent light field can be given as [142]: -

$$\mathbf{E} = E_o e^{i\omega t} e^{ikdn_e} (\cos\phi \cos\theta - i \sin\phi \sin\theta) \hat{i} + E_o e^{i\omega t} e^{ikdn_o} (\cos\phi \sin\theta + i \sin\phi \cos\theta) \hat{j} \quad (4.7)$$

Therefore the torque on the birefringent particle by the polarized light can be calculated by putting Eq. (4.7) in Eq. (4.5) and is given as [142]: -

$$\tau = -\frac{\epsilon}{2\omega} E_o^2 \sin[kd(n_o - n_e)] \cos 2\phi \sin 2\theta + \frac{\epsilon}{2\omega} E_o^2 [1 - \cos(kd(n_o - n_e))] \sin 2\phi \quad (4.8)$$

This is the torque experienced by the birefringent particle trapped in the polarized light. When the light is linearly polarized, that is $\phi = 0$ or $\pi/2$, the second term of equation is zero and the torque experience is proportional to $\sin 2\theta$. The torque experienced by a calcite in a linear polarized light can be given as: -

$$\tau \propto -\frac{\epsilon_o}{2\omega} E_o^2 \sin[kd(n_o - n_e)] \sin 2\theta \quad (4.9)$$

Thus a calcite in linear polarized light would be stable when $\sin 2\theta = 0$ or $\theta = 0$, hence the fast axis of the calcite will align with the plane of polarization of the linear polarized light at equilibrium. The torque will be maximum when the incident beam is circular polarized light $\phi = \pi/4$ and the trapped particle acts as a half-wave plate that is $k.d.(n_o - n_e) = \pi$. In this case the first term in equation becomes zero and the torque is only due to the second term. In such case particle experience the constant torque; this causes the angular rotation of the trapped particle at a constant angular velocity.

4.3.2 Spin angular momentum of light in optical tweezers system

Angular momentum possessed by a light beam can be classified as spin angular momentum [141] and orbital angular momentum [3] each due to the polarization states and the helical phase structure of the light, respectively. Researchers used optical tweezers systems to study the light-matter interaction, and to transfer the angular momentum of light to the trapped particles [142-145]. The spin angular momentum instigates the rotation of the trapped particle on its own axis and the orbital angular momentum causes the particle to orbit around the beam axis [102, 143]. The transfer of spin angular momentum by birefringence depends on the polarization change of the incident light after passing through a birefringent particle, which again depends on the thickness of the birefringent particle and wavelength of light.

Initially the transfer of a spin angular momentum due to light polarization was studied employing conventional upright optical tweezers system. The schematic diagram of the experimental set-up is shown in Figure 4.8. The linearly polarized Gaussian beam from HeNe laser with an output power of 30 mW at wavelength 632.8nm was incident to quarter and half wave-plates to alter the polarization of the incident beam. Pair of lenses and periscope mirrors was used to focus the beam into upright microscope (Axiostar from Carl Zeiss) and an objective lens of magnification 100X, oil immersion of numerical aperture 1.25 (Zeiss CP-Achromat) was used to reduce down the beam to a dimension of 5 μm at the sample stage. A filter is used to block the reflected laser light for the viewing application at the output port.

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Figure 4.8 Schematic diagram of upright optical trapping experimental set-up for the transfer to the calcite particle due to polarization of light.

Figure 4.9(a-c) shows the alignment of the calcite particle with the plane of polarization of incident beam by varying the orientation of a half-waveplate. As reported in Ref. [142] and depicted in Eq. (4.9) that a birefringent microparticle in a linear polarized light will align its fast axis with the plane of polarization of the light, thus by smoothly rotating the half-plate, the plane of polarization of light is altered causing the rotation of the calcite particle as shown in Figure 4.9. Similarly on inserting a quarter-waveplate the incident light is made circular polarized and the transfer of spin angular momentum to the calcite particle was obtained as shown in Figure 4.10[a-f]. The calcite rotates with a constant angular momentum due to a constant transfer of the torque by a spin angular momentum of light. The results shown in Figure 4.9 and Figure 4.10 are in congruous with earlier reported literature [142].

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Figure 4.9 Trapping and alignment of the calcite particle by a linear polarized Gaussian beam in an upright optical tweezers system. The arrow highlights the alignment of the calcite on changing the polarization of light. The scale bar shows the dimension of 5μm.

Figure 4.10 Transfer of spin angular momentum of the circular polarized light to a calcite particle by a Gaussian beam based upright optical tweezers system. The arrow highlights the rotation of the calcite on changing the polarization of light. The scale bar shows the dimension of 5μm.

4.3.3 Transfer of on-axis ‘pure’ spin angular momentum to an absorptive particle

The orbital angular momentum transferred to a trapped particle in optical tweezers can be intrinsic and extrinsic in nature contrary to the spin angular momentum, which is always intrinsic [143]. In optical tweezers the intrinsic and the extrinsic nature of the orbital angular momentum is defined based on the position of the trapped particle. If a particle is trapped at the center of the beam (on-axis) the orbital angular momentum transferred is referred as intrinsic and if the particle is trapped at off-axis, the transfer of orbital angular momentum is extrinsic [102, 143]. Hence it becomes significant to determine the position of a trapped particle as it affects the transfer of angular momentum of light.

The orbital angular momentum of light can be transferred through scattering [100] and absorption [7, 144] and the transfer of spin angular momentum is through absorption [144] and birefringence [142]. Absorption is a common and uniform mode of transfer for both orbital and spin angular momentum of light. When the particles are absorptive in nature and on-axis trapping is desired then both the spin and orbital angular momentum are intrinsic in nature and act in an interchangeable fashion and hence can be added or cancelled from the total angular momentum [144, 145]. Metallic particles are usually employed as an absorptive particle to transfer the spin and orbital angular momentums of the light. Trapping of a metallic particle having complex refractive index was initially achieved by scanning laser beam (TEM_{00} mode) in circular fashion [146]. Recently researcher proposed optical spanner [144] based on optical tweezers systems employing singular beam to trap absorptive particles.

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As an absorptive particle repels from the higher intensity region, therefore dark spots (surrounded by bright regions) are required to trap absorptive particles. The absorptive particles are trapped at central dark or minimum intensity region of the singular beams [146-149]. Using optical spanner different combinations of angular momentum was transferred [144] like orbital plus spin angular momentum, orbital minus spin angular momentum and pure orbital angular momentum. However the transfer of 'pure' on-axis spin angular momentum to the trapped absorptive particle is not possible employing conventional optical spanner, as the default orbital angular momentum possessed by the singular beam would always be transferred to the trapped particle. The transfer of pure orbital angular momentum (decoupling of orbital and spin angular momentum) to the metallic particle can be achieved, where the mode of transfer of angular momentum was scattering [147].

The optical tweezers system based on a self-imaged bottle beam can trap an absorptive particle at the bottling or the central dark spot and the transfer the 'pure' on-axis spin angular momentum can be accomplished. The self-imaged bottle beam [123, 125-126] has embedded optical bottles or intensity-null points along the propagation axis, which can be used to trap and to transfer the 'pure' spin angular momentum to the absorptive particles. This bottling effect of the self-imaged bottle beam is due to the destructive interference of the superimposed Bessel beam and not due to the singularity, hence there is no default orbital angular momentum carried by a self-imaged bottle beam.

The experimental set-up of the optical tweezers system using a self-imaged bottle beam was similar to Figure 4.3, and the quarter waveplate was inserted just before light

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incident the hologram to polarize the light. An IR-filter was used before CCD to observe clearly the rotation of the trapped particle.

Copper oxide (CuO) was used as the probing absorptive particle of diameter around 4~6 μm and little detergent was added to the sample to increase the mobility of the particles. The transverse trapping of an absorptive particle was achieved in the central dark spot or bottle of a self-image bottle beam. On introducing the quarter waveplate the spin angular momentum of circular polarized light was transferred to the CuO particle due to the absorption of light energy by the particle causing its rotation around its own axis as shown in Figure 4.11(a-f). The arrows in Figure 4.11 highlight the rotation of the CuO particles in different quadrants. The rotation of the CuO particle was due to the transfer of the 'pure' spin angular momentum of the light as a self-imaged bottle beam does not possess any singularity and hence no transfer of the default orbital angular momentum of the light. The torque on the absorptive particle is given as [145]: -

$$\tau = \frac{P_{abs} \sigma}{\omega} \quad (4.10)$$

where P_{abs} is the power absorbed by the particle and σ is +1 and -1 for right and left circularly polarized light, respectively and 0 for plane-polarized light and ω is the frequency of the light. The torque experienced by the particle allows it to overcome the drag force and a particle rotates with constant angular velocity.

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Figure 4.11 Transfer of 'pure' on-axis spin angular momentum of the circular polarized light to an absorptive particle (copper oxide) a self-imaged bottle beam based optical tweezers system. The scale bar shows the dimension of $5\mu\text{m}$.

4.4 DISCUSSION

Allen and Padgett [151] recently put across an important question where they reported the possibility where the matter (preferably an atom as a probing particle) may respond to the local value of spin angular momentum (in optical spanner based tweezers system), which could be varying in magnitude and sign at different positions of the wavefront when spin angular momentum is transferred using singular beams [151]. They validate this based on the fact that the spin angular momentum depends on the intensity gradient, which is influenced by the singularity of the host beam and depends on $\text{mod}(l)$, where l is the topological charge of singularity. Using the self-imaged bottle beam optical tweezers eliminates the default dependence of spin angular momentum on the singularity of the host beam. However, it could still be possible that an atom trapped in a self-imaged bottle beam could respond to the local spin angular momentum at various positions of the beam, as the bottling effect is obtained due to the destructive interference of two

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superimposed Bessel beams generating different intensity gradient at various locations of the wavefronts. Although as suggested in Ref. [151], for the particles of size in micrometers the observation of such local spin angular momentum is not substantial and hence, the self-imaged bottle beam based optical tweezers system can be used for the transfer of 'pure' on-axis spin angular momentum to the microscopic absorptive particles.

4.5 COMPARISONS OF DIFFERENT BEAMS EMPLOYED IN OPTICAL TRAPPING SYSTEM

Different laser beams employed with optical trapping system can be broadly classified as diffracting and non-diffracting beams. The main difference between a diffracting and non-diffracting beams used in optical tweezers is that the former one can be focused down to a tight spot trapping few microparticles at focal vicinity, (for example Gaussian beam and LG beam), contrary to non-diffracting beams. Different laser beams used in optical trapping system are Gaussian beam, Laguerre-Gaussian beam (LG beam), Bessel beam and self-imaged bottle beam or bottle beam. Each of them possesses some distinct advantage over others for specific application. However, self-imaged bottle beam adds most of the advantageous (which can be observed from Table 4.1 below) and thus could be preferred for various applications. Different beams employed in optical trapping systems are: -

- Gaussian beam: - The most important beneficial property of Gaussian beam (which no other beams have) is that it does not require any laser beam shaping and thus the output of any laser beam (possessing Gaussian distribution) can be directly employed for trapping applications. Furthermore, when focused all the power of the beam is localized in the central bright spot thus low-power laser

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(~30mW) can be employed for optical trapping. Gaussian beam when focused also provides three-dimensional trapping potential and thus can be used for 3-D trapping of high-index microparticles.

However the bottlenecks of Gaussian beams is that it can only be employed to trap high-index particle at the central bright spot and the low index and absorptive microparticles are repelled away from the central bright spot. Moreover, Gaussian beam is diffracting beam therefore it does not possess non-diffracting property and thus cannot be used for guiding/levitation application.

- Laguerre-Gaussian beam: - LG beam possessing doughnut-shaped intensity profile (central dark point with bright ring) is also a diffracting beam which can be focused tightly (thus the input power is localized into doughnut-shaped beam) to trap particles in 3-D. Due to its unique intensity profile (central hollow) it enables to trap all kinds of microparticle, for example low-index and absorptive microparticles at the central dark spot and high-index microparticle at the bright ring. However, just like Gaussian beam as LG beam does not belong to the family of non-diffracting beam it cannot be used for guiding and levitation applications. Moreover, similar to Gaussian beam, LG beam can also be use for trapping fewer numbers of particles at focal vicinity of strongly converging LG beam.
- Bessel beam: - Bessel beam belong to unique family of non-diffracting or propagation-invariant beams. Such beams as the name suggests does not diffract within a certain range and thus is exploit for trapping, guiding (levitation), trapping of multiple particles in different chambers (planes). Such beam possess unique property of self-reconstruction whereby the beam after trapping particles

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regains its original intensity distribution and thus can be re-employed for optical trapping. Thus Bessel beam can be employed for trapping multiple particles all along its non-diffracting region.

The big bottleneck with the Bessel beam is that it can be employed for 3-D trapping applications, as the central non-diffracting core of Bessel beam pushes the trapped particle along the propagation axis. Thus a high-index particle trapped in Bessel beam core is loosely defined in the longitudinal axis and is levitated due to radiation pressure imparted by the non-diffracting core of Bessel beam.

- Self-imaged bottle beam: - The proposed self-imaged bottle beam is the only beam whose transverse intensity profile varies along the propagation axis, and thus different regions of self-imaged bottle beam can be employed for diversified applications. The proposed self-imaged bottle beam belongs to the family of propagation-invariant beams and inherits the beneficial properties of self-reconstruction and non-diffraction of a Bessel beam. Similar to Bessel beam, self-imaged bottle beam can also be employed for trapping multiple particles in multiple planes. Most importantly self-imaged bottle beam add the beneficial property of Gaussian beam and LG beam of providing potential well for localizing micro-particles in 3-D. The transverse intensity profile of the self-imaged bottle beam varies periodically (alternative dark and bright spots) along the propagation and hence generating a three-dimensional potential well. Thus the self-imaged bottle beam based optical tweezers system will provide an additional control to the trapped particle (as compared to micro-particle trapped in the core of a Bessel beam) in the longitudinal axis as the particle cannot cross the potential

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well created by destructive and constructive interference effect in the core of self-imaged bottle beam. However, the bottleneck of employing self-imaged bottle beam is that similar to Bessel beam, power is shared in many outer rings, thus a laser with output power more than 100 mW is usually required.

Thus it is evident that self-imaged bottle beam can be widely applied for various applications, as it combines the beneficial property of non-diffracting and self-reconstruction properties of propagation-invariant beam (which possess the capability to trap multiple micro-particles) and that of diffracting beams like Gaussian or LG beam of providing three-dimensional potential well for optical trapping applications. So applications where researchers want to trap multiple microparticles of different refractive indices (high and low indices microparticles), the self-imaged bottle beam can be a good choice. However, for applications where the trapping and force analysis of a single microparticle is of interest (mostly in biological application), Gaussian beam or LG beam are better choice, as the intensity distribution of these beams are more uniform and thus the calibration and calculation of imparted force, torque and the observation of resultant phenomena is simple. Bessel beam can be useful when researchers predominately want to use levitation and guiding of high-index micro-particles as the main application.

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Table 4.1 Comparison of various laser beams employed in optical trapping system

	Gaussian Beam	Laguerre-Gaussian (LG) beam	Bessel beam	Bottle beam
High Index Micro-particle Trapping	Yes	Yes (At the Outer Ring)	Yes (At the central Bright Core)	Yes (At the constructive interference)
Low Index Micro-particle Trapping	No	Yes (At the central intensity null point)	Yes (Higher order Bessel beam)	Yes (At the destructive interference)
Absorptive Micro-particle Trapping	No	Yes (At the central intensity null point)	Yes (Higher order Bessel beam)	Yes (At the destructive interference)
2-D Trapping Potential	Yes	Yes	Yes	Yes
3-D Trapping Potential	Yes	Yes	No (The rod-alike core pushes the trapped particle)	Yes (Can localize low & high index particles in 3-D at destructive & constructive interference)
Levitation and guiding	No	No	Yes (Through out its non-diffracting distance)	Yes (Through out its constructive interference)
Power Localization at center	Yes	Yes	No (Power is wasted outer rings)	No (Power is wasted in outer rings)
Non-diffracting Property	No	No	Yes	Yes

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MICRO-OPTICAL ELEMENT APPROACH FOR LASER BEAM SHAPING IN OPTICAL TWEEZERS APPLICATIONS

“The scientist, by the very nature of his commitment, creates more and more questions, never fewer. Indeed the measure of our intellectual maturity, one philosopher suggests, is our capacity to feel less and less satisfied with our answers to better problems.”

G.W. Allport

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5.1 WHY OPTICAL ELEMENTS FOR LASER BEAM SHAPING?

It is evident from last two Chapters that the laser micro-beam shaping is necessary and important for various applications in optical trapping. In optical tweezers, laser micro-beam shaping is utmost important as it increase the trapping capability of a laser beam. Laser micro-beam shaping directly influences the phase and intensity of the trapping beam. Moreover, in optical tweezers system it is essential to employ power-efficient optical elements, as the trapping force is directly proportional to the laser power. It is known from literature that holographic techniques can be used for laser micro-beam shaping, where the phase of the desired beam is stored in the holograms and on incident of a reference beam, the desired beam can be reconstructed. Researchers proposed to use phase-only hologram to increase the power-efficiency of such techniques, still holograms seldom provides power-efficiency more than 50%-60%. Employing spatial light modulator provides freedom to dynamically change the parameters of the phase information of the desired beam and thus have been employed in various trapping application. However, in spatial light modulator approach considerable amount of power is wasted in higher diffraction orders, which can be undesirable in many applications.

High-power efficient techniques to generate Bessel beams are even more crucial, as power is shared equally in the outer rings [11]. Based on literature it is known that there exist a few techniques to generate propagation-invariant beams, however most commonly used techniques re annular aperture technique as proposed by Durnin [8, 9], holographic techniques and employing axicon approach (optical element approach). The first two techniques are less power efficient; in an annular aperture technique only a fraction of light passes through the aperture and rest is wasted. Similarly, holograms have

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the bottlenecks of being less power-efficient. Therefore, for beams like LG beams and Bessel beams, where it is possible to generate them through a single optical element (spiral phase plate and axicon, respectively) is preferred over other techniques. Optical elements approach is a power-efficient technique and many times power conversion efficiency reaches as high as 90%~95%. Besides that unlike spatial light modulator, such optical elements do not possess higher orders of diffraction. Therefore, the edge of such optical elements over other a holographic technique is clear. Moreover, the optical lithography technique enables to fabricate such optical elements in micrometer range making it further beneficial. Micro-optical elements are easier to integrate with the conventional optical tweezers system with minimal changes. Moreover, micro-optical elements hold potential applications for laser beam shaping in miniaturized optical traps. Researchers are eyeing on next generation of miniaturized optical traps for lab-on-a-chip technologies by integrating the laser on the chip [152]. One of the feasibilities for a miniaturized optical trap is to use micro-lensed diode lasers (MDL) [153]. Hence it can be foreseen that such micro-optical elements will be highly useful to shape the laser beam for lab-on-a-chip applications where all the components are in micrometer range. The light emitting from the micro-lasers in the chip is diverging and the integration of micro-axicon would allow availing the advantages of propagation-invariant beams for miniaturized optical traps.

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5.2 MICRO-AXICON FOR BESSEL BEAM GENERATION AND ITS APPLICATION IN OPTICAL TWEEZERS

5.2.1 Introduction to an axicon

Axicon is an optical element, which was first reported by McLeod [108] in 1954, to produce long and narrow focal lines along the optical axis. Axicon are also used to approximate Bessel beam near the optical axis [116]. Due to the unique property of possessing line focus, the axicons generated considerable interest both in its design innovation and its utilization in myriads of applications. Such design innovations and potential applications of an axicon can be found in several reported reviews [154-157]. An axicon finds its applications in alignment and metrology [158-161], due to its long and narrow focus; it provides better depth and resolution. Axicons were also explored for non-collinear harmonic generation [162-166] and have been used for optical pumping of plasma [167]. Axicons found its tremendous application in optical coherence tomography [168-170]. Axicons were explored for atom trapping and guiding applications [171-174] and various applications in interferometry [175-180], for example in radial shear interferometry. Recently, the axicon were used to approximate a Bessel beam, which found tremendous application in an optical tweezers system for micromanipulation of particles [11, 100-101].

An axicon is a conical optical element with a very small wedge angle (γ). The working principle of an axicon is illustrated in Figure 5.1. The incident parallel rays of light at any positions in the axicon are refracted at approximately the same angle. For example as shown schematically in Figure 5.1 both red and green light rays incident at different position to the axicon are refracted at the same angle (θ), thus producing a

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conical wave, which creates the Bessel beam. Bessel beams generated from an axicon can also be thought as interference pattern of two plane waves traveling in opposite direction. Bessel beam generated by an axicon possess a bright and narrow focus.

Figure 5.1 Schematic diagram to show the working principle of an axicon. Incident beam irrespective of its position in the axicon are refracted approximately at the same angle to generate a Bessel beam with conical wavevector distribution.

An ordinary lens creates a point focus and an axicon produces line focus along the optical axis. The length of the line focus can vary from a couple of millimeters [11] up to several kilometers [181] depending on the design of the axicon. Using the phase integral approach, Herman *et al* [116] reported the field generated by an axicon approximates to the Bessel beam (Eq. 13 of Ref. 116). Herman *et al* reported detailed study of the axicon fields and its optimization [116]. The phase of an axicon as a function of distance (r) from the center of the axicon can be given as: -

$$\phi(r) = r.(n-1).\cos(\gamma) \quad (5.1)$$

where n is the refractive index of the axicon material and γ is the wedge angle of the axicon.

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There are various documented techniques to fabricate axicons or axicon-like elements through the means of vapor deposition of dielectric layers through shadow masks, structural transfer into substrate by reactive-ion etching or using spherical aberration lens [182-183]. Regardless of the type, it is normally difficult to fabricate an axicon with a small wedge angle ($<0.5^\circ$), especially when the aforementioned fabrication techniques are employed for large-aperture (bigger diameter) axicons. In optical trapping application it is desirable to generate a Bessel beam with long non-diffracting distance so that it can be easily coupled with the optical trapping setup. The maximum non-diffracting distance of a Bessel beam is directly proportional to the aperture of the axicon and inversely proportional to the wedge angle of an axicon. Therefore, axicon required for optical trapping application should be of a smaller angle and the greater aperture (diameter). However, in all earlier reported literatures [182] the Bessel beams generated by the micro-axicon had very short nondiffracting distance and hence cannot be applied for optical tweezers systems.

5.2.2 Design of micron sized axicon for the Bessel beam generation

Micro-axicon for the Bessel beam generation possesses clear advantages over a computer-generated holograms and an annular aperture technique for optical trapping applications due to its high power-conversion efficiency. Design of a high power-efficient micro-axicon is realized, which is further fabricated employing electron-beam lithography (EBL). The main concern for the selection of the parameters was to design a micro-axicon such that the Bessel beam generated by it has long nondiffracting distance

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enabling its integration with an inverted optical tweezers system and at the same time it is easier to be fabricated using EBL techniques.

In optical trapping the parameters of a Bessel beam that are utilized for trapping applications are the maximum non-diffracting distance (Z_{max}) given by Eq. (2.20) and the dimension of the core of a Bessel beam (r_o), given by Eq. (2.22). Reverse engineering approach was employed to design the parameters of a micro-axicon. The wedge angle (γ) and the diameter (d) of a micro-axicon are the two parameters required for micro-axicon design and fabrication. Following steps were considered to determine the parameters of a micro-axicon for the generation of a Bessel beam and its application in optical tweezers: -

- Firstly the parameters of a Bessel beam required at the sample stage (after downsizing by the telescopic arrangement) were approximated. Keeping in mind the size of microparticles and desired application of the Bessel beam trapping being investigated here, which is levitation or guiding of microparticles by a Bessel beam.
- Then the parameters of a Bessel beam in the free space were calculated by multiplying the parameter of a Bessel beam at the sample stage by the approximated magnification of the telescopic arrangement that will be used in the experiment.
- Finally various possible design parameters of a micro-axicon were explored to generate a Bessel beam with the desired parameters, keeping the constraint of fabrication process.

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Based on above-mentioned arguments and the experience on Bessel beam trapping, the desirable size of the Bessel beam core (r_o) at the sample stage was assumed to be around 6~15 μm . This is due to the fact that the microparticles been used in most of the experiment are of diameter around 3~5 μm and the core of a Bessel beam should be around two or three times of the particle's diameter for a Bessel beam to have tighter trap and sufficient transverse space in the rod-alike core of a Bessel beam to levitate particles. Keeping this in mind the design of a micro-axicon was carried in such a manner that the core of a Bessel beam (r_o) obtained at the sample stage is within the range $6 \mu\text{m} < r_o < 15 \mu\text{m}$.

The other important parameter for the Bessel beam in optical tweezers is its maximum non-diffracting distance (Z_{max}), which is used for the levitation and guiding of microparticles in optical tweezers applications. In all the experiment discussed here, the longitudinal dimension of the sample stage where the microparticles were placed was around 100 μm , therefore the Bessel beam at the sample stage should possess the $Z_{max} > 100 \mu\text{m}$. However, $Z_{max} > 500 \mu\text{m}$ at the sample stage was chosen to ensure that besides the possible minor missalignment in the set-up, the non-diffracting distance of a Bessel beam should cover the entire sample stage.

The telescopic arrangement that will be used in the experiment was taken as constant while determining the design parameters of the micro-axicon. The telescopic arrangement is necessary in the trapping experiment to reduce the dimension of the Bessel beam onto the sample stage. On using $M\times$ telescopic arrangement the core of a Bessel beam will be downsized by M times (onto the sample stage) to that of core size of a Bessel beam in free space. Similarly the Z_{max} will be downsized to M^2 times (onto the

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sample stage) to that Z_{max} of the Bessel beam in free space on using $M\times$ telescopic arrangement. A $10\times$ telescopic arrangement was chosen for the design consideration of the micro-axicon. Keeping this parameter constant the parameters for the Bessel beam in the free space can be determined. The core of a Bessel beam (r_o) at the sample stage which was approximated earlier to be $6\text{ }\mu\text{m} < r_o < 15\text{ }\mu\text{m}$, is $60\text{ }\mu\text{m} < r_o < 150\text{ }\mu\text{m}$ in the free space employing $10\times$ telescopic arrangement. Similarly, the maximum non-diffracting distance of a Bessel beam at the sample calculated earlier to be $Z_{max} > 500\text{ }\mu\text{m}$, is $Z_{max} > 5\text{ cm}$ in the free space. However, higher the Z_{max} , easier it is to integrate a Bessel beam for guiding applications.

Once the range of parameters of the required Bessel beam to be generated by a micro-axicon is known, various possible parameters of micro-axicon can be determined. The core of a Bessel beam is inversely proportional to the wedge angle and is given as $r_o = 2.405/k(n-1)\gamma$. The maximum non-diffracting distance of a Bessel beam is inversely proportional to the wedge angle of the micro-axicon and directly proportional to the diameter (d) of the micro-axicon $Z_{max} = d/(n-1)\gamma$, assuming that the input beam waist (w_o) is matched to the diameter of micro-axicon ($d=w_o$). As the core of a Bessel beam (r_o) is not influenced by the micro-axicon diameter it was initially determined. The wavelength of interest in this case is 1064 nm and the refractive index of the micro-axicon material (SU-8 resist) is 1.59 (n). Table 1 shows various values of r_o with respect to the wedge angle of the micro-axicon. Looking up to different values of r_o , the range of angles which can provided the desired value of r_o is from $0.3^\circ \sim 0.5^\circ$ degrees. The wedge angle (γ) of a micro-axicon was opted as 0.5° and the calculation of micro-axicon diameter was done, keeping the desired Z_{max} range and fabrication constraint in consideration. Table 2 shows

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the various possible diameter of a micro-axicon and the corresponding non-diffracting distance of the generated Bessel beam, keeping wedge angle constant $\gamma=0.5^\circ$. Table 2 also shows the height of the micro-axicon on varying the diameter of axicon. The height of a micro-axicon becomes an important parameter on selection the resist material and the accelerating voltage in electron beam lithography techniques. Under the given experimental conditions (in the lab) for fabrication the accelerating voltage used in e-beam writing was 25~35 Kev and under such conditions the resist was calibrate for maximum thickness of around 8~11 μm . In general, the higher the resist thickness in e-beam writing more demanding is the process. Therefore, keep this height constraint as $< 8\mu\text{m}$, there is two choice of the diameter of a micro-axicon given as 2 mm and 1 mm as highlighted in Table 2, which can generate a Bessel beam with $Z_{max} > 5 \text{ cm}$. Therefore, the micro-axicon with wedge angle 0.5° and diameter 1mm or 2mm are capable to generate Bessel beam which can be effectively used in optical tweezers. The parameters of a micro-axicon opted for the fabrication was wedge angle of 0.5° and 1 mm diameter.

Table 5.1 Various possible wedge angles of micro-axicon (γ) and the corresponding diameter of the generated core of Bessel beam (r_o)

Wedge angle of micro- axicon (γ)	0.2 degrees	0.5 degrees	0.8 degrees	1 degrees	1.25 degrees	1.5 degrees	1.75 degrees
Core of Bessel beam (r_o)	195.6 μm	79.23 μm	48.79 μm	39.11 μm	31.69 μm	26.41 μm	22.638 μm

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Table 5.2 Various possible diameters (d) of micro-axicon and the corresponding maximum non-diffracting distance of the generated Bessel beam (Z_{max})

Wedge angle of micro-axicon (γ)	Diameter of micro-axicon (d)	Z_{max} of the generated Bessel beam	Height of the micro-axicon
0.5 degrees	3 mm	58.3 cm	11.8 μm
0.5 degrees	2 mm	38.38 cm	7.85 μm
0.5 degrees	1 mm	19.4 cm	3.94 μm
0.5 degrees	0.5 mm	9.7 cm	1.963 μm

The parameters chosen for the micro-axicon are shown in Figure 5.2. Micro-axicon with a diameter of 1 mm and a height of $\sim 4 \mu\text{m}$ giving a small wedge angle (γ) of $\sim 0.5^\circ$ was chosen for fabrication. The radial wavevector of a Bessel beam (k_r) generated using the chosen micro-axicon can be obtained from the wedge angle (γ) given as $k_r = k \cdot (n-1) \cdot \gamma$ where k is the propagation vector (λ in this case is 1064nm) and n is the refractive index of the axicon (~ 1.59 , refractive index of axicon material which is SU-8 resist in present case). The radial wavevector was determined to be 30373.66 m^{-1} . The maximum propagation-invariant distance and the size of central spot of the generated Bessel beam can be theoretically calculated as Z_{max} of $\sim 19.4 \text{ cm}$ and r_o of $\sim 80 \mu\text{m}$. The calculated parameters, Z_{max} and r_o can be exploited in optical tweezers applications and also the required diameter and the height of the micro-axicon; can easily be fabricated by electron beam lithography techniques.

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Figure 5.2 Schematic diagram of micron sized axicon for the generation of Bessel beam. (Not to scale).

5.2.3 Study of Bessel beam generated by micron sized axicon

Micro-axicon structures were initially simulated in MATLAB to study the Bessel beam evolution. The three-dimensional profile of a simulated micro-axicon is shown in Figure 5.3. Figure 5.4 shows the longitudinal intensity distribution of a Bessel beam generated by the micro-axicon shown in Figure 5.2. In the simulation the input beam waist was matched to the diameter of the micro-axicon. The parameters of the simulated micro-axicon were same as shown in Figure 5.2. The wavelength used in simulation was 1064 nm and the refractive index of the axicon was chosen to be 1.59 to match the experimental conditions (refractive index of SU-8 resist). It was observed from Figure 5.4 that micro-axicon generates conical wave distribution field to approximate a Bessel beam near the optical axis. The Bessel beam generated employing a micro-axicon was found to be non-diffracting for a distance around 20 cm, which is congruent to the theoretical calculations. The transverse intensity distribution of the Bessel beam generated by a micro-axicon was further analyzed. Figure 5.5 shows the transverse beam profile and the intensity distribution at a distance of 15 cm from the element. It is evident from Figure

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5.4 that field generated by micro-axicon is close approximation to a Bessel beam having concentric rings and the intensity profile similar to a Bessel functions as observed from Figure 5.5.

Figure 5.3 Simulated micro-axicon with the parameters shown in Figure 5.1. The x-y axes are in pixel where each pixel is given as 2.5 μm , representing the dimension of micro-axicon and the z-axis is arbitrary.

Figure 5.4 Longitudinal beam distribution of micro-axicon. The y-axis is in pixel where each pixel is given as 2.5 μm , representing the dimension of micro-axicon. Each unit of x-axis is 3mm.

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Figure 5.5 Transverse beam distribution and intensity profile of Bessel beam generated by micro-axicon. The units are in pixel where each pixel is given as 2.5 μm .

The designed micro-axicon was fabricated in SU-8 resist using electron beam lithography techniques. The optical microscopic image of the fabricated micro-axicon is shown in Figure 5.6. The fabricated micro-axicon is a stepped structure however the step size is very small thus the surface profile looks almost continuous. The surface topology of the fabricated micro-axicon was studied by Deteck³ Surface Profiler and compared with the ideal parameters in Figure 5.7. The ideal and measured surface profiles of the fabricated axicon are plotted in Figure 5.7. The close proximity between the ideal and the fabricated profiles of the axicons, confirms the feasibility of EBL for the fabrication of a micro-axicon with high accuracy.

Figure 5.6 Microscopic image of the fabricated micro-axicon.

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Figure 5.7 Plot of ideal and measured physical profile of fabricated micro-axicon by Deteck³ Surface Profiler.

The fabricated micro-axicon was used to study experimentally the profile of the generated Bessel beam at various planes from the element using a diode detector mounted on an x–y scanning stage. The designed micro-axicon was found to generate a high-quality Bessel beam. Detailed illustration of the measured intensity profiles as a function of distance from the fabricated element is shown in Figure 5.8. Figure 5.8(a, b) shows the forming-up of a Bessel beam at a distance of 2 cm and 6 cm from the micro-axicon, respectively. At a propagating distance of ~8 cm, as shown in Figure 5.8(c), high quality Bessel beam has been formed-up. It was further observed that such a high quality beam had propagation invariant distance in excess of 20 cm and the measured center spot of the Bessel beam was around ~80 μm, which is in close harmony with the theoretically determined with the given parameters. Figure 5.9 shows the three-dimensional intensity profile of the experimentally generated Bessel beam at a distance of 15 cm from the micro-axicon measured through a commercial beam-profiler. Figure 5.9 shows the

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intensity profile of a generated Bessel beam, which matches to the Bessel function, verifying the generation of a high-quality Bessel beam by the fabricated micro-axicon. The slight discrepancy between the predicted and the measured spot size is mainly due to the point spread function of the detector. Furthermore, the equations for the optical case were derived within the paraxial limit and not the full electromagnetic vector field.

Figure 5.8 Intensity profile of Bessel beam generated by micro-axicon at various propagation distances from the element (a) 2cm, (b) 6 cm, (c) 8 cm, (d) 12 cm, (e) 15 cm and (f) 20 cm. The dimension of vertical and horizontal axis is 1.5 mm.

Figure 5.9 Experimentally measured three-dimensional intensity distribution of the Bessel beam generated by the fabricated micro-axicon, at a distance of 15 cm from the element.

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5.2.4 Integration of micro-axicon with inverted optical tweezers system

Micro-optical element offers a high accuracy, high power-conversion efficiency and reproducibility which provides an opportunity to integrate them with the optical tweezers system for laser beam shaping in contrast to other approaches like holograms which suffers a constraint of being less efficient in terms of power. The efficiency (in terms of power) of a micro-axicon was found to be around 85% to 90% for various fabricated micro-axicons. Such high conversion efficiency is essential for a Bessel beam to be effectively used in optical trapping applications, as each ring of the Bessel beam shares equal power [11]. The Bessel beam generated by a micro-axicon was used for various trapping application. The micro-axicon was integrated with an inverted optical tweezers and the experimental set-up is shown in Figure 5.10. The light from the Nd:YVO₄ infrared laser (1 W, 1064nm) incident to the fabricate element (micro-axicon) The Bessel beam generated from the micro-axicon was telescoped down by 10× telescopic arrangement (lenses 1 and 2) onto the sample stage to reduce the central spot of the Bessel beam (r_o) to about 8~10 μm and the non-diffracting distance (Z_{max}) of 2 mm at the sample stage. However, no telescopic arrangement was required to expand the laser beam before incident on a micro-axicon due to its micron-size. Owing to its micron size such optical element can easily be integrated in the conventional trapping system with minimum changes. An x-y translation was provided to lens 2 to manipulate the beam spot over the sample stage and the x-y-z translation was provided to the sample stage. Fibre based illumination was used and the image was captured employing 40× objective lens which couples the light to the CCD. The Bessel beam generated by the micro-axicon was imaged onto the sample stage and is shown in Figure 5.11.

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Figure 5.10 Schematic diagram of the experimental set-up employing fabricated micro-axicon with the inverted optical tweezers.

Figure 5.11 Micro-axicon generated Bessel beam imaged at the sample stage using 10 $\times$ telescopic arrangements.

The high quality Bessel beam generated by a micro-axicon was used for various trapping application as shown in Figure 5.12 and Figure 5.13. Bessel beam generated was tested for both the transverse trapping (Figure 5.12) and levitation (Figure 5.13) applications. In one of the experiments the core of a Bessel beam generated from a micro-axicon was employed for transversely trapping and manipulation of silica spheres of

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diameter $5\ \mu\text{m}$. Figure 5.12(a-f) shows the transverse trapping and manipulation of two silica spheres which are transversely trapped in the central core of a Bessel beam generated by the fabricated micro-axicon. The input power used in the experiment was around 80mW. Once the spheres are transversely trapped in the central core of a Bessel beam as shown in Figure 5.12(a) it was manipulated over the sample stage as depicted by the arrows. The dimension and the direction of the arrows highlight the distance and the direction of the trapped spheres with respect to other reference spheres, respectively. On increasing the laser power the silica spheres trapped in the core of a Bessel beam can be levitated due to the non-diffracting property of the Bessel beam core, which is reported below.

Figure 5.12 (a-f) Transverse trapping and manipulation of two silica spheres of diameter $5\ \mu\text{m}$ in the core of a Bessel beam.

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The non-diffracting property of a Bessel beam generated by the fabricated micro-axicon was experimentally studied in an optical tweezers system. The core of a Bessel beam is known to possess non-diffracting propagation, which has been very effectively utilized for longitudinal guiding of microparticles in the optical tweezers system [11]. To study the non-diffracting and self-reconstruction properties of a Bessel beam generated by the micro-axicon, a silica sphere of 3 μm in diameter was trapped and levitated as shown in Figure 5.13(a-f). Figure 5.13(a) shows reference plane where a silica sphere, marked by a hollow circle is opt for the levitation. Figure 5.13(b) shows the transverse trapping of the chosen silica sphere in the core of a Bessel beam. The particle trapped at the center of a Bessel beam levitates due to radiation pressure on increasing laser power ~ 150 mW. The levitated sphere reaches the top of the sample stage and thus it is missing in Figure 5.13(c) when objective lens images the reference plane, which is further highlighted by a hollow circle. The incident laser beam is blocked in Figure 5.13(c-f). Figure 5.13(d-f) shows the levitated sphere as it settles down at the reference plane after the laser beam was blocked. The arrows in Figure 5.13(d-f) highlight the levitated sphere as it settles down to the reference plane. The objective lens images the reference plane thus the levitated sphere is blur (being out-of-focus) in Figure 5.13(d, e). The levitated sphere settles down to the reference plane and thus clearly imaged by the objective lens in Figure 5.13(f). Hence, on comparing the Figure 5.13(a) and Figure 5.13(c), selective levitation of a sphere by the core of a Bessel beam is evident, suggesting high beam quality generated by the micro-axicon

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Figure 5.13(a-f) Selective levitation of a micro-sphere by a Bessel beam generated by the fabricated micro-axicon. The circle in frame (a) marks the sphere chosen for levitation and the arrows in frame (d-f) mark the levitated particle as it settles down after the laser beam is stopped.

5.3 MICRO-DOUBLE-AXICON FOR SELF-IMAGED BOTTLE BEAM GENERATION AND ITS APPLICATION IN OPTICAL TWEEZERS

5.3.1 Introduction

In Chapter Three, a novel self-imaged bottle beam was introduced and its application in optical tweezers was reported in Chapter Four. A self-imaged bottle beam embeds three-dimensional intensity-null points or bottles, which generated considerable interest due to its potential applications in optical confinement of cold atoms and optical manipulations techniques. Researchers reported myriads of techniques to generate bottle beam [123-132]. Holographic phase plate approach and the interference effect of two Laguerre-Gaussian beams with different Gouy phases were initially used to generate such beams. Interference of two Gaussian beams with different radius of curvature or interfering Bessel beams with different radial wavevectors (k_r) were also shown to

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generate a beam embedding bottle or bottles. However, most of the aforementioned techniques employed complex optical set-up with multiple optical elements and/or low power-efficient holographic elements. Thus, the unavailability of a reliable and a reproducible fabrication technique in the realization of a 'single' power-efficient optical element to directly generate bottle beams restricted its widespread usage. Hence, it is evident that a high power-efficient optical element dedicated to directly generate beam with a sequence of three-dimensional intensity voids or bottles will be useful. Such single optical element for the generation of a self-imaged bottle beam will be complementing to an axicon [108] for a Bessel beams and a spiral phase plate [185] for a Laguerre-Gaussian beam. Moreover, as discussed above optical element approach for laser beam shaping besides being high-power efficient, also provides an opportunity to fabricate such elements at the micron level, employing electron beam lithography techniques. A novel design of an optical element is proposed for the direct generation of a self-imaged bottle beam.

5.3.2 Design of micro-double-axicon for self-imaged bottle beam generation

A novel design of high power-efficient 'single' optical element to directly generate a self-imaged bottle beam is proposed. Such an optical element can be fabricated at submicron-sized lithographic resolution. The interference of two Bessel beams with different radial wavevectors generates a self-imaged beam with embedded sequence of three-dimensional intensity voids as reported in Chapter Three. Bessel beams are usually generated using a conical element called an axicon with a very small wedge angle (γ), which determines the radial wavevector (k_r) of a Bessel beam $k_r = k(n-1)\gamma$, where k is

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propagation constant and n is the refractive index of the axicon. The radial wavevector (k_r) of a Bessel beam is therefore determined by the wedge angle (γ), where other parameters remain constant. Conventional axicons have a single gradient (wedge angle), hence, it can be foreseen that the conical element with double-gradient structure (Figure 5.14(b)) will generate two superimposed Bessel beams with different radial wavevectors, interfering to generate a self-imaged beam with embedded intensity voids or bottles. For the sake of a simple terminology, the proposed micro-optical element is referred to as **double-axicon** following its geometrical appearance. Figure 5.14(a, b) shows the schematic diagram of a conventional micro-axicon and the proposed micro-double-axicon, respectively. The parameters shown in Figure 5.14(a, b) are chosen for the simulation. It is evident from Figure 5.14(b) that proposed optical element possess two wedge angles and thus can potential generate the approximation of two superimposed Bessel beams with different radial wavevector embedding a series of intensity voids or bottles. Such micro-optical element also holds potential application in the optical tweezers systems due to its high-power conversion efficiency.

To validate the design of the proposed optical element, an axicon and a double-axicon optical element were simulated using beam propagation method. The parameters of a micro-axicon and a micro-double-axicon opted for the simulation is shown in Figure 5.14. Figure 5.15(a, b) shows the beam propagation along the longitudinal direction by a conventional axicon and a double-axicon, respectively. Gaussian beam was chosen as an input beam for the simulation results shown in Figure 5.15. The input beam diameter was matched with the diameter of the simulated optical element. The axicon converges the incident Gaussian beam into a conical wavevector distribution approximating a Bessel

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beam. As reported above the working principle of an axicon is that rays incident at any position of the axicon can be considered to be refracted at the same angle forming a conical wavevector distribution, which can also be noticed from Figure 5.15(a). Hence it is understood that the rays being refracted from a double-axicon structure will be from two different angle-zones generating superimposed conical wavevector distribution, interfering to generate a self-imaged bottle beam as also obtained in the numerical simulation shown in Figure 5.15(b). The beam generated by the proposed double-axicon generated a series or an array of intensity-null points as also observed from Figure 5.15(b).

Figure 5.14 Schematic diagrams of a conventional axicon and the proposed double-axicon (not to scale). The double-axicon possesses two gradients whereby the lower and upper wedge angles are 0.1° (γ_1) and 0.08° (γ_2), respectively.

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Figure 5.15 Longitudinal beam formations of an axicon and a double-axicon (which are depicted in Figure 5.14), after incident a Gaussian beam. The unit of vertical axis is in pixel where each pixel is given as $15\ \mu\text{m}$ and each unit of horizontal axis is $3\ \text{mm}$.

Figure 5.16 shows the free-space propagation of a beam generated by micro-double-axicon and its transverse intensity profile. Noted that Figure 5.16(c) and Figure 5.16(g) depict the intensity void at a distance of $14\ \text{cm}$ and $25\ \text{cm}$ where two superimposed Bessel beams interfere destructively, whereas, Figure 5.16(d) and Figure 5.16(h) shows the corresponding transverse intensity profile of the previous figures. Conversely, Figure 5.16(a) and Figure 5.16(e) show the central bright spot at a distance of $9.6\ \text{cm}$ and $20.5\ \text{cm}$, and likewise, Figure 5.16(b) and Figure 5.16(f) show the transverse intensity profile of the corresponding figures respectively. The double-axicon

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structures with various combinations (different diameter and gradients) were simulated and the bottling effect was observed in all the cases. It is evident from Figure 5.15(b) and Figure 5.16 that the proposed micro-double-axicon generates two superimposed Bessel beams embedding an array of three-dimensional intensity null points or bottles concurrent to previous reported results in Chapter Three. Thus this novel structure can be used to directly shape the incident Gaussian beam to a self-imaged bottle beam.

Figure 5.16 Simulated free-space propagation of the bottle beam generated using proposed double-axicon (a, c, e, g) and corresponding transverse intensity profile (b, d, f, h). The intensity voids appear in frames (c, g). Distance of frames (a-d) from the element is 9.6 cm, 14 cm, 20.5 cm, and 25 cm, respectively. The unit of vertical and horizontal axis is in pixel where each pixel is given as 15 μm .

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5.3.3 Study of self-imaged bottle beam generated by micron sized double-axicon

The micro-double-axicon was fabricated in the negative-toned SU8-2025 resist with a JEOL-JSM6600 scanning electron microscope converted lithographic tool based on ELPHY PLUS 3.0 beam blanking system. Like other members of the SU8 resist family, SU8-2025 is a high sensitive ($\sim 2.5 \mu\text{C}/\text{cm}^2$) and a low contrast (~ 0.6) photo-resist. Such properties are very important for the fabrication of multi-level and continuous surface relief structure. As the desired optical element required two gradients of different wedge angles, intensive characterization of resist-thickness with-respect-to exposure dosage for the formation of a single grey-scale continuous surface relief structure is absolutely necessary. The exposure dosage has to be carefully controlled during the fabrication of a single conical micro-optical element with two different gradient slopes as the key challenge during this fabrication process depends on the cautiously control of dosage increment in the two wedge angle zone.

Fabrication of various double-axicon structures was done; the one used in the experiments is 4 mm in diameter with a lower and upper wedge angle of 0.23° (γ_1) and 0.39° (γ_2), respectively, which can be calculated from Figure 5.17. The surface profile of the fabricated optical element was determined using Dteck³ Surface Profiler as shown in Figure 5.17. In Figure 5.17 the horizontal axis is in micron and vertical axis in kilo-angstrom. The surface profile difference between the desired and the fabricated structures was found to be less than 10%. Arising from the close proximity between the desired and the fabricated profiles of the double-axicon, it confirms the feasibility of electron-beam lithography techniques for the fabrication of micro-double-axicon with high accuracy.

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Initially the fabricated micro-double-axicon was used to study the beam generation in free-space. Consistent to the simulation results, the beam generated using fabricated micro-double-axicon possessed a sequence of three-dimensional intensity voids or bottles along the propagation axis as depicted in Figure 5.18. Intensity voids or bottles are obtained both in the near field and in the far field from the element. Figure 5.18(a, b) shows the oscillating intensity property of a self-imaged bottle beam in the near field, where bright and dark center is obtained in Figure 5.18(a) and Figure 5.18(b), respectively. Figure 5.18(c-f) shows the beam propagation obtained by the fabricated double-axicon in the far field. The bottles were obtained at a distance of 12 cm and 25 cm from the fabricated element, where the Bessel beams interfere destructively, as shown in Figure 5.18(d, f). Series of such three-dimensional intensity null points were observed all along the propagation axis. Figure 5.18(c, e) shows the bright central spot of a self-imaged bottle beam where the Bessel beam interferes constructively in the far field at a distance of 8 cm and 18 cm from the element.

Thus, this 'single' micro-element (double-axicon) directly shapes the incident Gaussian beam to a self-imaged beam embedding three-dimensional intensity voids which is analogous to micron-size spiral phase plates [186] and axicon [187], shaping Gaussian beam into Laguerre-Gaussian and Bessel beams, respectively.

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Figure 5.17 Dektak³ Surface Profiler measurement of a fabricated micro-double-axicon. Horizontal axis is in micron and vertical axis in kilo-angstrom.

Figure 5.18 Experimental generation of a self-imaged bottle beam using fabricated micro-double-axicon at (a) 2 cm, (b) 3 cm, (c) 8 cm and (d) 12 cm, (e) 18 and (f) 25 cm. Noted that intensity voids appears in frames (b) and (d) and (f).

5.3.4 Integration of micro-double-axicon with inverted optical tweezers system

Most of the fabricated double-axicon was 80% to 90% power-efficient (which is desirable for optical tweezers and optical traps for laser-cooled atoms) and thus possessed this distinct advantage over low power-efficient holographic elements conventionally

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used to generate a bottle beam. Integration of such a high power-efficient micro-optical element with the conventional inverted optical tweezers systems for optical micromanipulations is reported. The experimental set-up was similar to Figure 5.10. Owing to its micron-size the double-axicon can easily be integrated in the present trapping system with minimal changes. As explained in Chapter Four, contrary to a Bessel and a Laguerre-Gaussian beams, the transverse intensity profile of such beams varies periodically along the propagation axis, hence provides an additional advantage to image different points of the beam (as shown in Figure 5.18(e) and Figure 5.18(f)) onto the sample stage to obtain preferred trapping results. For the present experiment central dark spot of the beam was imaged onto the stage (which is congruous to Figure 5.18(f) in the free-space) as shown in Figure 5.19. The light from the Nd:YVO₄ infrared laser (1 W, 1064 nm) was incident to the fabricated element (beam waist of the Gaussian beam was made equal to the diameter of double-axicon ~4 mm) and using the appropriate telescopic arrangement (ten times in present case) the central dark spot was made (8~10 μm) comparable with the dimensions of a low-index particle, as shown in Figure 5.20(a). Figure 5.20(b-f) illustrates the optical micromanipulation of a low-index particle (hollow silica sphere) using a double-axicon based optical tweezers systems. The low-index particles are repelled from the region of higher intensity and are trapped in the dark spot or minimum intensity region. The particle was transversely trapped in the intensity void and was manipulated over the sample stage in the predetermined direction highlighted by the dotted arrow in Figure 5.20(a). The solid arrows in Figure 5.20 highlight two reference particles. The direction of the solid arrow suggests the relative direction of the

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trapped particle with the reference particle and the length of the lines indicates the relative distance between the trapped and reference particles.

Figure 5.19 Central dark spot of self-imaged bottle beam (generated from micron-sized double-axicon) imaged onto the sample stage for optical manipulations.

Figure 5.20 Double-axicon-enabled optical tweezers system to transversely trap a low-index microparticle of diameter around 10 μm and its manipulation over the sample stage.

Thus, a double-axicon directly generates a self-imaged bottle beam and it can be easily integrated with the optical tweezers systems for trapping applications. As discussed above an optical tweezers system enabled by a double-axicon can be used for trapping

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both low and high indices microparticle, by imaging different points of oscillating intensity profile of a self-imaged bottle beam.

5.4 KINOFORM MICRO-OPTICAL ELEMENTS

5.4.1 Introduction

A kinoform can be described as a phase-only optical element where only the phase modulation of the desired wavefront is recorded as a surface-relief profile [188, 189]. Such structures differs from the conventional holograms as kinoform reconstructed field does not have unwanted diffraction orders. The kinoform technique realizes on the fact that the sections or thickness of the microelement that are one wavelength thick do not contribute to the change of phase, hence can be removed. Thus the kinoform based micro-optical elements (axicon or double-axicon) allow each wavelength multiple of the phase modulation value to be ‘wrapped’ back to its 2π equivalent surface thickness. This will allow the surface-relief profiles of the micro-optical elements with high aperture (diameter) and thus with high-height to be micro-fabricated with the same physical height as of 2π equivalent surface thickness, easing the fabrication process. Besides that kinoform structures are also more efficient in the use of spatial frequency and reconstruction energy or power-conversion efficiency.

As discussed in above sections that fabricating micron sized axicon and double-axicon with large diameter is beneficial for the trapping applications, as it has bigger non-diffracting distance. However, increasing the diameter of the optical element, increase the height and thus the resist thickness required to fabricate the element. Fabricating micro-optical elements with higher height becomes challenging and more tedious besides

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become prone to fabrication errors. Kinoform approach for fabricating such structures avoids such difficulties, as the thickness required to form any structures in kinoform is constant and is equivalent to surface relief profile of a single wavelength or 2π equivalent phase.

The power conversion efficiency of kinoform axicon and double-axicon structures was experimentally found to be more than the conventional holograms required to generate corresponding beams. As a matter of fact the power-conversion efficiency of the kinoform structures was found only little less than the optical element approach (axicon and double-axicon), detail comparison of power conversion of these elements are mentioned latter. Kinoform enables axicon and double-axicon to be fabricated with more accuracy and at the same time it allows fabricating such structures with large diameters or aperture.

5.4.2 Kinoform axicon

Kinoform technique enables to fabricate a kinoform axicon with large diameter and hence generating a Bessel beam with long non-diffracting distance, with the resist thickness or surface relief profile equivalent to a single wavelength or 2π equivalent phase. For example if an axicon is required to generate a Bessel beam with four times the maximum non-diffracting distance to the Bessel beam generated by an axicon reported in Figure 5.2, without changing the central core of Bessel beam significantly. In this case the diameter of the new axicon should be ~ 4 mm which is four times the diameter of an axicon shown in Figure 5.2. Keeping the angle of a micro-axicon close to 0.5° say $\sim 0.46^\circ$ (the reason to opt 0.46° , instead of 0.5° , is to make the equivalent phase of the element

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equal to integer multiple of 2π , as shown in Figure 5.21) and the diameter of 4 mm requires the resist thickness of $16.2\text{ }\mu\text{m}$, as shown in Figure 5.21(a). Employing electron beam lithography technique fabricating micro-structures with vertical dimensions of $16.8\text{ }\mu\text{m}$ is demanding, moreover the accelerating voltage of 25~35 KeV, used in all fabrication (available in the lab) was not enough to penetrate into the resist (SU8-2025) with such vertical heights. Thus the importance of kinoform enabled micro-optical elements was realized. The kinoform-axicon for the generation of an identical Bessel beam is shown in Figure 5.21(b). The diameter of the kinoform-axicon is still 4 mm and the resist thickness required this time is equivalent to 2π phase thickness ($1.8\text{ }\mu\text{m}$ only) as depicted in Figure 5.21(b).

Figure 5.21 (a) Axicon designed to generate a Bessel beam with four times the non-diffracting distance to an axicon depicted in Figure 5.2; (b) Design of an equivalent kinoform-axicon for the generation of an identical Bessel beam.

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The phase change (Δ) when light beam of wavelength (λ), enters a medium of refractive index (n) and travel a distance of t , is given by: -

$$\Delta = \frac{2\pi}{\lambda}(n-1)t \quad (5.2)$$

Thus the distance of surface relief structure (t') to generate 2π equivalent phase ($\Delta=2\pi$) difference is given as: -

$$t' = \frac{\lambda}{(n-1)} \quad (5.3)$$

In the present context the wavelength (λ) of interest is 1064 nm and the refractive index (n) of SU8-2025 resist is 1.59. Thus the value (t') is determined to be around 1.8 μm , as also shown in Figure 5.21(b). Thus usefulness of the kinoform is evident, as the kinoform-axicon structure requires a surface relief thickness of only 1.8 μm contrary to axicon structures, which requires thickness of 16.8 μm for the identical Bessel beam.

Kinoform approach was used to fabricate kinoform-axicon for the Bessel beam generation. Figure 5.22 shows the top-view of the fabricated micro-kinoform-axicon taken from optical microscope. Each concentric ring represents the 2π equivalent phase or 1.8 μm resist thickness. Similarly Figure 5.23 shows the surface profile of the fabricated element by commercial available Dteck³ Surface Profiler, which sketches the profile of the element. In Figure 5.23 the horizontal axis is in micron and the vertical axis is in kilo-angstrom. Each segment in Figure 5.23 represents the phase change of 2π , which is shown by concentric ring in Figure 5.22.

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Figure 5.22 Top view of a fabricated kinoform-axicon taken from optical microscope. The diameter of kinoform axicon is 4 mm as depicted in Figure 5.21(b).

Figure 5.23 DeTeck³ surface profiler measurement of a fabricated micron sized kinoform-axicon. Horizontal axis is in micron and vertical axis in kilo-angstrom.

Fabricated kinoform-axicon was studied for the experimental generation of a Bessel beam. Figure 5.24 shows the free-space beam propagation of a Bessel beam generated from the fabricated kinoform axicon. The core of a Bessel beam was experimentally found to possess non-diffracting property for a larger distance. Figure 5.24(a, b) shows the Bessel beam formation in the near field and Figure 5.24(c-f) shows a

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non-diffracting Bessel beam propagating to a larger distance which is excess with 50 cm. Figure 5.25(a) shows the transverse intensity and the corresponding intensity profile of a Bessel beam is shown in Figure 5.25(b) at a distance of 12 cm from the kinoform-axicon, measured using commercial beam profiler. The intensity distribution in Figure 5.25 (b) is along the horizontal and vertical lines shown in Figure 5.25(a). The Bessel function alike transverse intensity profiles of the generated beam as shown in Figure 5.25 from kinoform-axicon strengths its accuracy to generate a high-quality Bessel beam.

Figure 5.24 Bessel beam generation from fabricated kinoform-axicon with long non-diffracting distance, (a) 3 cm, (b) 6.5 cm, (c) 15.5 cm (d) 25.5 cm, (e) 39 cm and (f) 49 cm. The vertical and horizontal axis shows a dimension of 4 mm.

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Figure 5.25 (a) Transverse intensity profile of a Bessel beam generated from fabricated kinoform-axicon at a distance of 12 cm from the fabricated element and (b) the corresponding intensity distribution of a Bessel beam. Frame (b) sketches the intensity along the vertical and horizontal lines depicted in frame (a). The vertical and horizontal axis of frame (a) shows a dimension of 4 mm.

5.4.3 Kinoform double-axicon

The kinoform-double-axicon was further studied. The kinoform technique was explored to fabricate a kinoform-double-axicon for the generation of a self-imaged bottle beam. As proposed above the double-axicon possesses two different wedge angles in two different zones, contrary to axicon. Thus in the kinoform technique the angles for each 2π equivalent phase should correspond to the individual angle zones, as highlighted in Figure 5.26. Figure 5.26(a) shows a double-axicon with 4 mm diameter and two different wedge angles of around $\sim 0.21^\circ$ and $\sim 0.41^\circ$, similarly Figure 5.21(b) shows the corresponding kinoform double-axicon to generate an identical self-imaged bottle beam. As shown in Figure 5.26(b) the angle of each 2π equivalent phase is either 0.21° or 0.41° corresponding to two different angle zones, respectively. It is worth again mentioning that a double-axicon of surface relief thickness of $10.8 \mu\text{m}$ designed in Figure 5.26(a) can be fabricated with only $1.8 \mu\text{m}$ thickness which corresponds to 2π equivalent phase,

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employing kinoform techniques. Figure 5.27 shows the surface profile of the fabricated kinoform double-axicon sketched by Deteck³ Surface Profiler. It can be observed from Figure 5.27 that unlike Figure 5.23 all the segments of the Figure 5.27 are not identical. This is due to the fact that the kinoform-double-axicon possess two different angle zones unlike an axicon possessing a single angle, therefore two angles zones in Figure 5.27 represents two different segments. Figure 5.28 shows the optical microscope image of the fabricated element.

Figure 5.26 (a) Double-axicon designed to generate a self-imaged bottle beam (b) Design of an equivalent kinoform double-axicon for the generation of an identical self-imaged bottle beam.

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Figure 5.27 Dektak³ Surface Profiler measurement of a fabricated micron sized kinoform-double-axicon. Horizontal axis is in micron and vertical axis in kilo-angstrom.

Figure 5.28 Top view of a fabricated kinoform-double-axicon taken from optical microscope. The diameter of the fabricated kinoform double-axicon is 4 mm.

Similar to double-axicon, kinoform double-axicon generates a beam embedding an array of three-dimensional intensity null points. Figure 5.29 shows the beam generated on incident a Gaussian beam onto the fabricated kinoform double-axicon. Arrays of bottles or intensity null points were embedded all along the beam propagation axis. Figure 5.29(b, d) shows the intensity null point where the superimposed Bessel beam

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interferes destructively at a distance of 12.5 cm and 27 cm from the fabricated element. Similarly, Figure 5.29(a, c) shows the constructive interference of the superimposed Bessel beams at a distance of 6 cm and 20.5 cm from the fabricated element.

Thus kinoform enabled micro-optical elements were designed, fabricated and employed for laser micro-beam shaping. Kinoform technique enables easy fabrication due to reduced resist thickness and the beam generated by it was found to be of high quality. Kinoform axicon and kinoform double-axicon was fabricated to generate high quality Bessel beam and self-imaged bottle beam respectively.

Figure 5.29 Self-imaged bottle beam generation from a fabricated kinoform-double-axicon. Fames (a-d) are at following distance, (a) 6 cm, (b) 12.5 cm, (c) 20.5 cm and (d) 27 cm.

5.5 STUDY OF POWER CONVERSION EFFICIENCY OF DIFFERENT MICRO-OPTICAL ELEMENTS

As discussed in above sections employing power-efficient optical elements are crucial in the optical tweezers applications. Thus a detailed comparison of power

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conversion efficiency of various fabricated optical elements was done. Various fabricated axicons and double-axicon were tested at different input powers. Optical elements such as micron-sized axicon and double-axicon were found to possess average power efficiency of about 89% and 84% respectively, as shown in the tables below. Such high power conversion efficiency is imperative in many applications, especially in an optical tweezers system. The error margin of the commercial power meter used for the measurement was 5mW, thus the reading obtained possess maximum percentage error of 4%~5%.

Table 5.3 Power conversion efficiency of various fabricated axicon at different input powers. The maximum percentage error of each reading is less than 5%

Axicon	Input Power	Output Power	Power Efficiency	Average
1.	120 mW	105 mW	87.5%	
2.	120 mW	110 mW	91.66%	
3.	100 mW	90 mW	90%	89%
4.	100 mW	90mW	90%	
5.	100 mW	90mW	90%	

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Table 5.4 Power conversion efficiency of various fabricated double-axicon at different input powers. The maximum percentage error of each reading is less than 5%

Double-Axicon	Input Power	Output Power	Power Efficiency	Average
1.	120 mW	105 mW	87.5%	
2.	120 mW	100 mW	83.5%	
3.	100 mW	85 mW	85%	84.2%
4.	100 mW	80mW	80%	
5.	100 mW	85mW	85%	

Power conversion efficiency of the kinoform optical structures was also determined experimentally. It was found that the power conversion efficiency of kinoform axicon and kinoform double-axicon are only little less as compared to axicon and double-axicon. The average power conversion efficiency of a kinoform axicon and a kinoform double-axicon was experientially determined to be around 80% and 78%, respectively. Kinoform techniques thus provide reasonable power conversion efficiency and easiness of fabrication and allow fabricating optical elements with bigger aperture as reported above. Moreover, the power conversion efficiency of kinoform structures is higher as compared to computer-generated holograms, used conventionally for generating propagation-invariant beams. Furthermore, there are no higher diffraction orders in kinoform structures contrary to spatial light modulator technique to generate such beams.

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Table 5.5 Power conversion efficiency of various fabricated kinoform axicon at different input powers. The maximum percentage error of each reading is less than 5%

Kinoform Axicon	Input Power	Output Power	Power Efficiency	Average
1.	120 mW	100 mW	83.5%	
2.	120 mW	95 mW	79.16%	
3.	100 mW	80 mW	80%	79.5%
4.	100 mW	75mW	75%	
5.	100 mW	80mW	80%	

Table 5.6 Power conversion efficiency of various fabricated kinoform double-axicon at different input powers. The maximum percentage error of each reading is less than 5%

Kinoform Double-Axicon	Input Power	Output Power	Power Efficiency	Average
1.	120 mW	95 mW	79.16%	
2.	120 mW	95 mW	79.16%	
3.	100 mW	75 mW	75%	77.66%
4.	100 mW	75mW	75%	
5.	100 mW	80mW	80%	

Thus high-power efficient micro-optical elements for laser micro-beam shaping are beneficial in optical tweezers applications. The utility of such micro-optical elements were explored by integrating them with optical tweezers system for various trapping applications. High-power conversion efficiency and micro-size are added advantages of optical element technique for laser micro-beam shaping.

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BESSEL BEAM ARRAY FOR LONGITUDINAL OPTICAL GUIDING

*"An experiment is a question which science poses to Nature, and a
measurement is the recording of Nature's answer."*

Max Planck

6.1 INTRODUCTION

A single beam optical tweezers technique [2] has aggrandized the study of light-matter interaction and has been successfully applied in multifarious fields from colloidal science [14] and micro-fluidics [15] to bioscience. On the other hand, specially designed optical potential landscape has shown strong capability to precisely control the kinetics of microparticles for optical sorting [14], optical fractionation [15], optical binding [13] and optical guiding [11]. In three-dimensional optical sorting of microparticles, by size and refractive indices, researchers employed a three-dimensional optical lattice using a diffractive optical element. While in optical fractionation, a designed optical landscape, generated from a spatial light modulator, was used to deflect and sort the microparticles in the presence of external forces (fluid flow). Using the concept of dual optical traps (two counter-propagating beams) optical binding process was reported and the formation of microparticles in a periodic arrangement was achieved.

However, one of the earlier observed phenomenon of optical landscape transferring optical momentum to the microparticle that resulted in a strong particle kinetic motion is the transverse optical guiding by A. Ashkin *et al* [1]. The phenomenon of transverse optical guiding has since taken many forms. S. Kawata *et al* [190] reported the transverse guiding of microparticles in the presence of evanescent field and recently evanescent field has shown to be capable of guiding and sorting of microspheres using a Y-branched optical waveguide [191]. Using symmetrical optical potentials, coupled with the concept of optical thermal ratchet [192], has shown the ability to transversely guide microparticles both in forward and backward directions. Apart from transverse optical

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guiding, little work has been focused onto longitudinal optical guiding of microparticles and hence not being extensively studied.

Judging from previous literatures, a Bessel beam has been proven to offer a well defined longitudinal optical potential for longitudinal guiding [11]. J. Arlt *et al* [11] reported the levitation of microparticles using a single Bessel beam, whereby the particles get guided along a defined longitudinal direction in the core of a Bessel beam. However, to increase the throughput of the longitudinal guiding of microparticles there is a need to develop a technique where the core of a Bessel beam array can be used as multiple-channels for massive longitudinal guiding. The key advantage of using the Bessel beam is its long non-diffracting distance and the self-reconstructing capability that is deemed to be more suitable for multiple-channel longitudinal optical guiding.

6.2 GENERATION OF AN ARRAY OF BESSEL BEAM USING COMPOSITE HOLOGRAM

To integrate an array of Bessel beam in optical tweezers system the main concern is the generation of a Bessel beam array with minimum interference and maintaining a long non-diffracting propagation distance while conserving considerable power in each Bessel beam to optically guide. Furthermore, to have similar guiding capabilities in the longitudinal direction for each Bessel beam in an array necessitates the experimental generation of an array of identical Bessel beam. A novel and efficient way to generate an array of Bessel beams using composite holograms approach was employed [193]. In a composite hologram many holograms are arranged in a single hologram, each of which can store different phase information. When laser light is incident on a composite

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hologram each individual hologram reconstruct individually. This technique of generating an array whereby each beam in an array would be addressed by different sub-hologram holds a distinctive edge over other techniques as the phase information of each hologram can be easily altered without influencing the full array. Moreover, location of each hologram can be independently altered in an array to obtain customized array of a propagation-invariant beams. Figure 6.1 shows a composite hologram to generate a 3 X 3 array of Bessel beams. Each individual hologram in the composite holograms shown in Figure 6.1 is the hologram to generate a Bessel beam. Other beneficial property of composite hologram for the generation of Bessel beam array can found in the Ref. [193].

Figure 6.1 Composite hologram possessing 3 X 3 sub-holograms, each to generate individual Bessel beam. The vertical and horizontal axis shows a dimension of 2.1 mm. Figure modified from Ref [193].

Each ring of a Bessel beam shares equal power and hence the requirement to have power-efficient diffractive optical element for parallel optical guiding is crucial. An array of Bessel beam can be generated using a spatial light modulator (SLM) [193]; however in the present work this approach was not involved due to power constraint. An estimate of

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the input power required by the Bessel beam array generated by spatial light modulator for parallel guiding was determined to study the feasibility. An estimate of power required in the core of Bessel beam (where levitation takes place) to levitate silica particles of dimensions around 3~5 μm is around 5~10 mW at the trapping stage. It is known that power in Bessel beam is evenly divided among all the rings, thus if each Bessel beam in an array possess ten rings, the power required in each Bessel beam is around 50~100 mW at the sample stage. Thus a 3×3 array of Bessel beams at the sample stage should have 450~900mW of power. This estimated power is only for a single order of light pattern generated by a spatial light modulator and without taking consideration of power conversion efficiency of the spatial light modulator. So if spatial light modulator generates N order and has 40% of power conversion efficiency (usual SLM power conversion efficiency) the input power to the spatial light modulator will be $(1125\sim2250) \times N$ mW. So it was foreseen that spatial light modulator technique is not desirable for the generation of a Bessel beam array for the trapping application, mainly due to power constraint. Furthermore, G. Sinclair *et al* [194] has recently reported the disadvantages of using spatial light modulator where the lateral displacements of optical traps are limited by the optical efficiency of the spatial light modulator, while the longitudinal displacements are limited by the aberrations of the objective lens. Due to above-mentioned constraints the composite hologram was fabricated as phase-only, on-axis holograms, instead of using spatial light modulator.

UV lithography techniques were employed for the fabrication of various composite holograms. Close view of fabricated composite hologram is shown in Figure 6.2. Various arrays of identical Bessel beams were fabricated with each Bessel beam of

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diameter around 0.7 mm and the radial wavevector of 0.2/pixel, where each pixel was chosen as 10 μm and the wavelength of interest was 1064 nm. These parameters generated an array of Bessel beams with 21 cm of non-diffracting distance.

Figure 6.2 Fabricated composite hologram for the generation of Bessel beam array. The Bessel beam array was found to be propagation invariant for a distance of 20 cm. The vertical and horizontal axis shows a dimension of around 2 mm.

The array of Bessel beams generated using fabricated hologram was initially analyzed in free space propagation. Figure 6.3(a) shows the transverse intensity profile of 2×2 array of a Bessel beam at a distance of 10 cm generated from the fabricated composite hologram. Figure 6.3(b) shows the three dimensional intensity profile of a Bessel beam array using beam profiler. Figure 6.3 clearly illustrates that each peak (representing the core of a Bessel beam) is impervious to the presence of other beams in an array and has identical peak values. As the beam propagates further noticeable interference can be observed at the outer rings, however the core of each Bessel beam in an array remains interference-free as shown in Figure 6.4. The array of Bessel beams at a distance of 20 cm does show noticeable interference at the outer rings however the core of each Bessel beams is impervious to any interference as noticed in Figure 6.4. In optical

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tweezers the core of a Bessel beam will be used for the longitudinal guiding. Interference in the outer rings does not affect the guiding property of the core of Bessel beam array, which was also observed experimentally and reported latter. Figure 6.5 shows the intensity distribution of the Bessel beam array generated by the fabricated composite hologram along the vertical and horizontal lines shown in Figure 6.4. It is clear from Figure 6.5 that the core of each Bessel beam is not affected by the interference and also possesses identical peak value. The necessary condition of a Bessel beam to maintain the conical wavevector distribution is well preserved by the fabricated composite hologram as each Bessel beam in an identical array was found to have same non-diffracting distance, with identical core radius and peak value. Generating array of identical Bessel beams is important to enable the array to possess similar guiding ability, which was also experimentally observed and reported latter.

Figure 6.3 (a) Experimental generation of a Bessel beam array using a composite hologram at a distance of 10 cm from the element. Frame (b) shows the three-dimension intensity distribution of Bessel beam array measured using beam profiler. The vertical and horizontal axis shows a dimension of 4 mm.

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Figure 6.4 3 x 3 Bessel beam array at a distance of 20 cm from the composite hologram. Noticeable interference at the outer rings of the Bessel beam can be noticed however the core of Bessel beam is still impervious to interference. Vertical and horizontal lines map the intensity in Figure 6.5. The vertical and horizontal axis shows a dimension of 5 mm.

Figure 6.5 Intensity distributions of an array of Bessel beam along the vertical and horizontal lines as shown in Figure 6.4. It is clear that a composite hologram generates an array of Bessel beam with identical peak intensity and also the interference of outer rings does not affect the core of each Bessel beam.

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6.3 PARALLEL LONGITUDINAL GUIDING OF MICROPARTICLES USING AN ARRAY OF BESSEL BEAM

The array of Bessel beams was used for the parallel longitudinal optical guiding of microparticles. The core of each Bessel beam in the array has shown, experimentally, to have a high efficient longitudinal guiding efficiency. The composite hologram was integrated with the inverted optical trapping set-up as shown in Figure 6.6. The light from the Nd:YVO₄ infrared laser (1 W, 1064nm) was expanded five times before incident to the fabricated hologram. The array of Bessel beams was telescoped down by ten times onto the sample stage to reduce the central spot of the each Bessel beam to 10~15 μm and the non-diffracting distance at the sample stage was around 2 mm.

It was realized that different combination of objective lens were required to view full array and to comprehensively observe the guiding phenomena, as objective lens has a trade-off between its resolution and field-of-view. Figure 6.7(a) shows 4 $\times$ 3 array of Bessel beam imaged from the sample stage using a 10 $\times$ objective lens. However, the 10 $\times$ objective lens does not provide convincing images of 3~5 μm microparticles; hence 25 $\times$ and 40 $\times$ objective lenses, as schematically drawn by arrows in Figure 6.7 (b, c) was used to image 1 $\times$ 2 array and a single Bessel beam respectively. It can be observed from Figure 6.7(b) that there is slight interference in the outer rings of a Bessel beam from adjacent beams; however the core of each Bessel beam is unaffected. Due to self-reconstruction and propagation-invariant properties of Bessel beam core, it was experimentally found to possess guiding property even with a few interfering outer rings. Moreover it is worth mentioning that precise imaging of a Bessel beam array onto the sample stage is important to preserve guiding property of each Bessel beam. Slight miss-alignment in the

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set-up causes distortion or inclined incident of Bessel beam array onto the sample stage, ceasing its guiding capabilities.

Figure 6.6 Experimental set-up of optical tweezers system employing array of Bessel beams. Strict alignment is absolute necessary to preserve the conical wavevector distribution of Bessel beam.

Figure 6.7 (a) 4 × 3 array of Bessel beam imaged from the sample stage using 10× objective lens; (b) an array of 1 × 2 Bessel beams imaged from the sample stage using 25× objective, (c) a single Bessel beam imaged using 40× objective.

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Each Bessel beam in an array was found equally capable to guide the microparticles longitudinally. Initially the trapping and guiding capability of each Bessel beam in an array was experimentally studied individually at a time. Silica spheres of 3 μm in diameter were placed in a sample stage. After preparing the sample appreciable time (60~90 mins) was allowed so that most of the silica spheres settled at the bottom of the sample stage. Figure 6.8(a-d) shows the levitation of the silica spheres by a single Bessel beam in an array. The radiation pressure (along the direction of beam propagation) is responsible for such longitudinal optical guiding. Figure 6.8(a) shows three microparticles at the bottom of the sample stage transversely trapped in the core of a Bessel beam. Arrows in Figure 6.8(a) marks three microparticles trapped transversely in the core of a Bessel beam. The spheres trapped in the core of a Bessel beam levitates to the top of sample stage on increasing the power (around 200 mW in this experiment), as shown in Figure 6.8(b-d). In Figure 6.8(b) one of the trapped spheres was levitated and thus disappear as the objective lens images the reference plane. Two of the levitated spheres reached the top of sample stage as shown in Figure 6.8(c). Objective lens images the levitated sphere in Figure 6.8(c); hence all reference particles are defocused and disappeared. Moreover, in Figure 6.8(c) the third sphere is stacked just below the two levitated spheres (as highlighted by an arrow in Figure 6.8 (c)), which finally get levitated and imaged in Figure 6.8(d). Hence, all the three silica spheres transversely trapped in the core of Bessel beam in Figure 6.8(a) are guided to the top of sample stage as shown in Figure 6.8(d). The longitudinal distance between the Figure 6.8(a) and Figure 6.8(c) is 90 μm , as reported earlier the sample stage used in the experiment had vertical dimension of around 100 μm . This guiding is due to the non-diffracting nature of the Bessel beam core.

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Similar to this, other Bessel beams in the array were found to preserve their non-diffracting nature, which can be used for the parallel massive longitudinal guiding of microparticles.

Figure 6.8 (a-d) Selective levitation of three silica spheres (marked with arrows) of diameter 3 μm in the non-diffracting core of a single Bessel beam in an array. The longitudinal distance between the frame (a) and frame (c) is 90 μm .

Using a 25 $\times$ objective lens an array of 1 $\times$ 2 Bessel beams was imaged from the sample stage, corresponding to Figure 6.7(b). Figure 6.9 shows parallel longitudinal guiding of the silica spheres (3 μm in diameter) in the core of 1 $\times$ 2 array of Bessel beams. Figure 6.9(a) shows 1 $\times$ 2 array of Bessel beams at the reference plane (bottom of sample stage) transversely trapping three and two microparticles. Subsequently in Figure 6.9(b) two microparticles get levitated in the core of first Bessel beam to the top of sample

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stage. In the second Bessel beam in Figure 6.9(b) the microparticle is stacked just below the imaged plane, which finally gets levitated in Figure 6.9(c). In Figure 6.9(c) three microparticles that were initially trapped by the first Bessel beam at the bottom of the sample stage get levitated to the top of sample stage. Finally all the particles trapped in the core in Figure 6.9(a) (three and two particles, in first and second Bessel beam, respectively) are guided to the top of sample stage as shown in Figure 6.9(d). In Figure 6.9(b-d) the objective lens images the levitated particles and hence all reference particles are out-of-focus and thus disappeared. The close-up of all the microparticles trapped in the core of each Bessel beam is shown as a inset in Figure 6.9. The longitudinal distance (defocus level) between the frames (a) and (d) in Figure 6.9 is around 90 μm .

The concept of exploiting non-diffracting nature of a Bessel beam core in an array to provide multiple-channels to longitudinally guide microparticles is schematically explained in Figure 6.10. One other key advantage that a Bessel beam possesses is the capability to form potential washboard profile, such an optical potential will propel microparticles that is already trapped at the outer rings of Bessel beam to “hop” and accumulate at the central core of Bessel beam [195]. Therefore using such an array of Bessel beam; it would be possible to first collect the microparticles at close proximity to the core of a Bessel beam array and then by increasing the input power to guide them along the longitudinal direction massively. The non-diffracting cores of Bessel beam array act like a parallel longitudinal path for the massive guiding of microparticles using optical means without the need of any external forces (for example fluid flow). The radiation pressure of the non-diffracting Bessel beam coupled with its unique self-reconstruction property is solely responsible for such longitudinal guiding.

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Figure 6.9(a-d) Levitation of silica spheres of diameter $3\ \mu\text{m}$ in the non-diffracting cores of a 1×2 array of Bessel beam imaged using $25\times$ objectives. Frame (a) shows the transverse trapping of three and two microparticles in an array of Bessel beam, which subsequently get levitated to the top of sample stage in frames (b-d).

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Figure 6.10 Schematic representations of microparticles guiding in an array of Bessel beams. The core of Bessel beam array serves as parallel path to optically guide microparticles.

To quantitatively study the efficiency of longitudinal guiding in 3×2 array of Bessel beam, a graph showing levitated distance by the microparticles in the core of each Bessel beam was experimentally determined as shown in Figure 6.11. The estimate of power experienced by the microparticles was also determined. The laser power used in the experiment was 760 mW and silica spheres of diameter $3 \mu\text{m}$ were used. The composite hologram was found to be 40%~45% power efficient corresponding to 55~60 mW power of each Bessel beam in an identical 3×2 array and around 7~10 mW power at the central core of each Bessel beam where optical guiding occur. It can be seen from Figure 6.11 that each Bessel beam guides the microparticles to a similar distance. The vertical dimension of the sample stage used in all the experiments was $100 \mu\text{m}$; therefore maximum longitudinal distance particles can be levitated is $100 \mu\text{m}$ in the present experimental conditions.

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Figure 6.11 Graph showing levitated distance of 3 μm silica spheres in the core of 3×2 Bessel beam array. An array of Bessel beam was experimentally found to have similar guiding property.

Such a multiple-channel longitudinal optical guiding technique exploiting the unique advantage of diffraction-invariance of the Bessel beam core holds promise application in colloidal science and biology. Using composite hologram technique, the non-diffractive parameter can be tailored as required. With or without any external microfluidic flow, this technique can still very much be used to guide microparticles through static optical fields, thus this is another advantage over optical fractionation [15] or optical peristalsis [191] where external flow rate or dynamic optical fields is important to the eventual guiding efficiency.

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STUDY OF SINGULAR OPTICS IN NON-DIFFRACTING OPTICS

"Scientific principles and laws do not lie on the surface of nature. They are hidden, and must be wrested from nature by an active and elaborate technique of inquiry."

John Dewey

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7.1 INTRODUCTION

Non-diffracting and singular optics are the niche areas of modern optics with substantial research interest and potential applications [99, 196, 203-216]. At singularity the phase of the beam is undefined and the amplitude vanishes to zero. Such beams are characterized by having intensity null point at the center therefore singular beams is usually referred as doughnut beams. Singularity is also referred as optical vortex or screw dislocation. Properties of such dislocations or singularities have been studied in depth in the literature [3-5, 7, 185, 197-202, 203-211]. Historically, singularity has been studied and observed in laser speckle fields [197]. Singularity embedded in the laser beam mode has been explored [198-200], Laguerre-Gaussian (LG) mode gained more interest; therefore singular beams are also referred as LG beams. Singularity in non-linear medium and the generation of the optical vortex solitons are reported [201-202]. The singular beam possesses a unique helical wavefront distribution and the degree of circulation is characterized by a topological or azimuthal charge number l [4-5]. Due to its unique helical wavefront distribution, singular beams carries an orbital angular momentum [3, 7], which is different from the spin angular momentum of a circular polarized light [142-143].

The generation of such singular beams has been extensively covered and diversified techniques are reported. Hermite-Gaussian (HG) mode and a cylindrical lens were used initially to generate a singular beam [203]. However, computer generated holograms have been most commonly used to generate a singular beam. Such technique relies on encoding the desired helical phase information to the incident beam [204-205]. The event of spatial light modulators (SLM) has provided more freedom to the generation

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and study of singular beams, whereby the parameters of the singular beams can be dynamically altered [206]. Employing a single high-power efficient optical element (spiral phase plates) researchers directly modulate the phase of an incident beam to desired helical phase distribution [185, 207]. Nonspiral optical elements was also used to generate an optical vortex by employing a wedge to introduce a linear phase variation on one-half of the incident Gaussian beam, which transform to a singular beam along the propagation axis [208].

Substantial amount of research in the field of singular optics has been reported [209-211] whereby singularities or wavefront discontinuities are hosted in the diffracting beam (Gaussian beams) and its evolution in the free-space propagation were studied. Trajectories of an off-axis optical vortex embedded in a Gaussian beam was influenced by the parameters like the inter-vortex separation, vortex radius ($\tanh$ or r vortex), background beam amplitude and phase gradient whereby off-axis vortex moved towards infinity, rotates or gives complex intensity pattern during the free-space propagation [209-210].

Singular optics is rarely being studied in the propagation-invariant beams. Study of singularities embedded in the propagation-invariant beams has opened up a niche area of research. Bouchal [84] pointed out the connection of singular and non-diffracting optics, whereby he reported the new prospective effects that could be observed if singular optics is studied in the propagation-invariant beams. The interesting properties of the propagation-invariant beams like non-diffracting and self-reconstruction are believed to interact differently with the embedded singularities. Moreover, it was foreseen that conical wavevector distribution of a propagation-invariant beam might further influence

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the evolution or dynamics of the vortices embedded in it. Recently a striking difference in the evolution of an off-axis vortex embedded in a Bessel beam was reported. It was found that an off-axis optical vortex embedded in a Bessel beam moved towards the center of the Bessel beam [85] contrary to an off-axis optical vortex embedded in a Gaussian beam, which moved outward (towards infinity) as reported by Rozas *et al* [209]. However, different parameters influencing the evolution of an offset optical vortex embedded in the propagation-invariant beam was not studied in detail.

Composite off-axis optical vortices embedded in a single Gaussian beam as well as composite optical vortices generated by the superposition of the two beams carrying different vorticity have been convincingly covered in literature [86-88, 209-210, 212]. Composite optical vortices generated by the superimposing two noncollinear beams carrying different vorticity also generated lot of interesting phenomena where the composite vortices rotate, merge, annihilate or moved towards infinity depending on the relative phase and amplitude of the individual beams and inter-vortex separations [212]. Free-space propagation of the propagation-invariant beams carrying composite optical vortices was also reported, however the approach was on superimposing of two Bessel beams carrying different vorticity. Such superposition of propagation invariant-beam possessing different topological charges caused intensity modulation and other interesting effects like braided nodal lines [86-88]. However, composite off-axis optical vortices embedded in a single propagation-invariant beam have not been studied.

7.2 STUDY OF OFFSET VORTEX EMBEDDED IN PROPAGATION-INVARIANT BEAM

7.2.1 Introduction

Interest in the off-axis vortex generated when it was reported that an off-axis optical vortex (*tanh* vortex) hosted in the diffracting beam exhibit fluid-like motion [209]. Also an off-axis optical vortex was experimentally generated by utilizing the rotational Doppler Effect using a rotating prism [211]. Angular spectrum theory was employed to stimulate the evolution of an off-axis optical vortex embedded in the propagation-invariant beam. A Bessel beam can be generated by a hologram with a phase transmission function reported in Eq. (2.26). An optical vortex can be embedded off-centered to the Bessel beam by changing the co-ordinates of $\theta(x, y)$, to $\theta(x-x_n, y-y_n)$ in the Eq. (7.1) so that the vortex can be shifted to an arbitrary point (x_n, y_n) : -

$$T(r, \theta) = \exp(-i2\pi r/r_0) \exp[in\theta(x, y)] \quad (7.1)$$

Figure 7.1 shows simulation result for the evolution of an off-axis optical vortex embedded in the Bessel beam. The parameters used in the simulations were wavelength of 633 nm, the beam radius (w_0) was 1.92 mm, the topological charge of the optical vortex was one and the offset distance of the optical vortex was around thirty-eight pixels from the center of the beam (each pixel is given as 15 μm). As reported in Ref. 85 it was observed that an off-axis optical vortex embedded in a Bessel beam moved towards the center of the Bessel beam during the free-space propagation. The arrow in Figure 7.1(a) shows the initially offset of the optical vortex to an arbitrary distance from the center of Bessel beam. During the free space propagation the optical vortex (screw dislocation) interacts with the rings of Bessel beam, (which can be considered as non-generic edge

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dislocations [85]) and forms mixed edge-screw type dislocations, as highlighted by an arrow in Figure 7.1(c). Mixed edge-screw dislocation moved to the center of the Bessel beam as shown in Figure 1(c-e) and stable screw dislocation is formed at the center of Bessel beam as shown in Figure 7.1(f). Once the stable screw dislocation is formed at the center of a Bessel beam, the edge dislocation moved towards the infinity, as highlighted by the arrow in Figure 7.1(f). The propagation of such edge-screw dislocation is explained in detail in Ref. 85.

Figure 7.1 Free space propagation of an off-axis vortex embedded in a Bessel beam. An off-centered vortex is shown by an arrow in frame (a). The off-axis vortex moves towards the center of the Bessel beam as shown in frames (a-e). On reaching the center the screw dislocation remains there and the edge dislocation moves towards infinity (frame (f)). Frames (a-f) are at distance $3Z_r$, $5Z_r$, $7Z_r$, $9Z_r$, $11Z_r$ and $13Z_r$, respectively and Z_r is the Rayleigh distance.

To verify the screw dislocation or vortex at the center of Bessel beam, interference pattern was studied. It is known from literature that a vortex or a screw dislocation has a fork-type dislocation in its interference pattern. Figure 7.2 shows the

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interference of a Bessel beam with the inclined Gaussian beam. Figure 7.2(a-c) shows the interference pattern of a Bessel beam shown in Figure 7.1(a, c, f) respectively. An arrow in Figure 7.2(a) highlights the dislocation formed by an off-axis vortex in Figure 7.1(a). Similarly stable screw dislocation of Figure 7.1(f) is shown in Figure 7.2(c) verifying the presence of fork-type dislocation and hence the optical vortex at the center of the Bessel beam.

Figure 7.2 Interference pattern of a Bessel beam embedding an off-axis vortex. Frame (a-c) is the interference of a Bessel beam shown in Figure 7.1(a, c, f), respectively. The presence of fork type dislocation verifies the presence of vortex at the center of the Bessel beam in Figure 7.1(f).

Researchers [85] have also calculated the radial velocity of an off-axis optical vortex, which can be given $v_r = \Delta r / \Delta z$, where Δr is the distance covered in the radial direction (r) during the propagation of distance Δz in z direction. Therefore the radial velocity of an off-axis optical vortex can be given as follows [85]: -

$$v_r = \frac{\Delta r}{\Delta z} = \frac{k_r}{k_z} = \tan \gamma = \gamma \quad (7.2)$$

where k_r and k_z are radial and longitudinal wavevectors of a Bessel beam and the γ is the cone angle of the Bessel beam. Therefore the radial velocity is directly proportional to the

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cone angle of a Bessel beam. This recovery of the off-axis optical vortex to the center of the beam is influenced by the non-diffractive and self-reconstructive properties of the propagation-invariant beam, which is due to the conical wavevector distribution of the beam. Hence it becomes significant to determine the influence of the radial wavevector of a Bessel beam on the evolution of the off-axis optical vortex to the center of the beam.

7.2.2 Study of recovery distance of an off-axis vortex embedded in a propagation-invariant beam

An off-axis optical vortex is embedded in a Bessel beam and the influence of radial wavevector of the Bessel beam on the recovery distance of the off-axis optical vortex is initially determined. The terminology ‘recovery distance’ is used to state the distance required by an offset optical vortex embedded in a Bessel beam to move to the center of a Bessel beam. The parameters used in the simulations were wavelength of 633 nm, the beam radius (w_0) was 1.92 mm, the topological charge of the vortex was one and the offset distance of the vortex was thirty-eight pixels from the center of the beam (size of each pixel was 15 μm). Figure 7.3 shows the graph of the recovery distance of the off-axis vortex with the radial wavevector of the Bessel beam varying from 0.1/pixel to 0.6/pixel ($6.67 \times 10^3 \text{ m} < k_r < 40 \times 10^3 \text{ m}$). In Figure 7.3 the offset distance of the optical vortex was kept constant and the radial wavevector of a Bessel beam was altered. It was observed that the higher the radial wavevector of the Bessel beam, the faster the evolution of the off-axis optical vortex to the center of the beam or lesser is the recovery distance. However increasing the radial wavevector (k_r) of a Bessel beam causes shorter non-diffracting distance of a Bessel beam (Z_{max} is inversely proportional to k_r). Once the

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recovery distance of an off-axis optical vortex is more than the Z_{max} , the vortex cannot be recovered fully. This can be understood, as after Z_{max} the Bessel beam is diffracting and the off-axis optical vortex embedded in the diffracting beam would move outward with the increasing vortex radius as reported in Ref. 209.

Figure 7.3 Plot of the recovery distance of the off-axis vortex of topological charge one with the radial wavevector of the Bessel beam. The vortex is offset to a distance of thirty-eight pixels from the center of the beam.

It was further considered that the recovery distance of the off-axis optical vortex should also be influenced by the offset distance of the optical vortex from the center of the beam. Influence of the offset distance on the recovery distance of an off-axis optical vortex was studied. Figure 7.4 shows the relation between the recovery distance of an off-axis optical vortex and the offset distance of the optical vortex from the center of the beam. In Figure 7.4 the radial wavevector of a Bessel beam was kept constant and the offset distance of the optical vortex was altered. It was observed that the recovery distance of the off-axis optical vortex is almost linearly dependant on the offset distance

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of the optical vortex from the center of the beam. The higher the offset distance, the slower the recovery (higher recovery distance) of the off-axis optical vortex. For a given propagation-invariant beam there is a critical offset distance of the optical vortex beyond which the recovery distance surpasses the maximum non-diffracting distance of a Bessel beam ceasing the recovery of the off-axis optical vortex. For the parameters chosen in the simulation the maximum non-diffracting distance (Z_{max}) of the Bessel beam is around 57 cm ($Z_{max}=w_0 \cdot (k/k_r)$). In Figure 7.4 the critical offset distance is around fifty pixels (one pixel is 15 μm , which is around 0.75 mm) after which the recovery distance surpasses the Z_{max} .

Figure 7.4 Plot of the recovery distance of the off-axis vortex of topological charge one with the offset distance of the vortex from the center of the beam. The radial wavevector of the Bessel beam is 0.5/pixel.

It was further analyzed that besides the radial wavevector and the offset distance, the recovery distance of the off-axis optical vortex also depends on the topological charge

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of the optical vortex. Figure 7.5 shows the plot of the recovery distance of an off-axis optical vortex with the topological charge of the optical vortex. From the plot it is noticed that as the topological charge of the off-axis optical vortex increases the recovery distance of the off-axis optical vortex to the center of the beam also increase. For the parameters chosen in the simulation (radial wavevector of the Bessel beam being 0.5/pixel and the optical vortex offset distance of thirty-eight pixels) an off-axis optical vortex of the topological charge more than seven cannot be fully recovered as then the recovery distance surpasses the Z_{max} (which was around 57 cm for the chosen parameters). It was further observed that the off-axis optical vortices embedded in a Bessel beam tends to move towards the center of the Bessel beam with the same vortex core radius within the propagation-invariant distance (Z_{max}), which is contrasting to the vortex core size in Gaussian beam which becomes bigger along the propagation axis due to diffraction [209]. However, once the Bessel beam crosses Z_{max} the optical vortex starts behaving as it is embedded in the diffracting beams and increases its core size.

Figure 7.5 Plot of the recovery distance of the off-axis vortex with the topological charge of the vortex. The radial wavevector of the Bessel beam is 0.5/pixel and the vortex is offset to a distance of thirty-eight pixels from the center of the beam.

7.3 STUDY OF COMPOSITE OFF-AXIS VORTICES EMBEDDED IN PROPAGATION-INVARIANT BEAM

The evolution of the composite optical vortices embedded in a propagation-invariant beam was explored. Free-space propagation of a Bessel beam embedding composite off-axis optical vortices was studied. Composite off-axis vortices can be embedded in the Bessel beam by changing the co-ordinates of $\theta(x, y)$, to $\theta(x-x_{l,2}, y-y_{l,2})$ in Eq. (7.2) so that the optical vortices can be shifted to the arbitrary points (x_l, y_l) and (x_2, y_2) and given as :-

$$T(r, \theta) = \exp(-i2\pi r/r_0) \prod_n^N \exp[in\theta(x_n, y_n)] \quad (7.3)$$

The parameters used in the simulation were wavelength of 633 nm; radial wavevector (k_r) of 0.5/pixel and the size of each pixel was 15 μm . Figure 7.6 shows the stimulated free-space propagation of a Bessel beam embedding two off-axis vortices. Two optical vortices, each of topological charge one, were offset to a distance of thirty-eight pixels from the center as highlighted by circles in Figure 7.6(a). Figure 7.6 and 7.7 show the intensity distribution and the interference pattern of a Bessel beam embedding composite vortices, respectively. The composite off-axis optical vortices moved towards the center of the beam and add their topological charges at the center of the beam during the free-space propagation to generate a second-order Bessel beam as shown in Figure 7.6(a-f). Figure 7.7 showing the fork-like interference pattern of Figure 7.6 with the tilting plane beam validates the presence of a second-order optical vortex in the center of Bessel beam as shown by circle in Figure 7.7(f). Congruent to Ref. 85 both the edge and mixed edge-screw dislocations each moving towards the infinity and center of the Bessel beam

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respectively, was observed. Marked with circles in Figure 7.6(c) are the edge dislocations, which are moving towards infinity, and marked with circles in Figure 7.6(e, f) are the edge dislocations generated from the mixed edge-screw dislocations after it passed through the center of the Bessel beam congruent to the results reported in Ref. 85. Each mixed edge-screw dislocation generates one stable screw dislocation at the center of Bessel beam, which adds the topological charges. After stably adding the topological charges the second-order Bessel beam maintains its non-diffracting propagation till it reaches Z_{max} .

Figure 7.6 Free-space propagation of a Bessel beam, possessing composite off-axis vortices of topological charge one. Frames (a-f) are at distance $3Z_r$, $8Z_r$, $13Z_r$, $18Z_r$, $23Z_r$ and $28Z_r$, respectively and Z_r is the Rayleigh distance.

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Figure 7.7 Interference pattern of a Bessel beam each corresponding to Figure 7.6(a-f). Frames (a-f) are at distance $3Z_r$, $8Z_r$, $13Z_r$, $18Z_r$, $23Z_r$ and $28Z_r$, respectively and Z_r is the Rayleigh distance.

Bessel beam embedding composite off-axis optical vortices of opposite signs was also simulated. Figure 7.8 shows the free-space propagation of a Bessel beam embedding composite off-axis optical vortices of opposite topological signs (topological charge being one). Figure 7.9(a-f) shows the interference pattern with a tilted plane wave, each corresponding to Figure 7.8(a-f). It was observed that the two off-axis optical vortices of opposite signs canceled their topological charges at the center of the beam to generate a zeroth-order Bessel beam. Congruence to Figure 7.6 and 7.7 the beam dynamics is same, each off-axis optical vortex interact with the outer ring of a Bessel beam to generate mixed edge-screw dislocation which travel towards the center of Bessel beam, where stable screw dislocation is formed at the center and edge dislocation moves towards infinity.

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Figure 7.8 Free-space propagation of a Bessel beam, possessing composite off-axis vortices of topological charge one (opposite sign). Frames (a-f) are at distance $3Z_r$, $8Z_r$, $13Z_r$, $18Z_r$, $23Z_r$ and $28Z_r$, respectively and Z_r is the Rayleigh distance.

Figure 7.9 Interference pattern of a Bessel beam each corresponding to Figure 7.8(a-f). Frames (a-f) are at distance $3Z_r$, $8Z_r$, $13Z_r$, $18Z_r$, $23Z_r$ and $28Z_r$, respectively and Z_r is the Rayleigh distance.

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It was further noticed that the evolution and recovery distance of each off-axis optical vortex embedded in a Bessel beam is independent and immune to the presence of the other optical vortices. The evolution of an off-axis optical vortex along the longitudinal axis was explored to study the inter-vortex influence. The parameters used in the simulation were wavelength of 633 nm; radial wavevector (k_r) of 0.4/pixel and the size of each pixel was 15 μm . Figure 7.10 compares the recovery distance of an off-axis optical vortex embedded in a Bessel beam in the presence of other optical vortex of equal and opposite topological signs. In Figure 7.10(a) a single off-axis optical vortex of topological charge one is offset to a distance of thirty-eight pixels, which is recovered to the center of the Bessel beam at a distance of fifty steps (each step is $Z_{\text{max}}/100$). On introducing the additional optical vortex of equal and opposite topological signs (with same offset distance from the center) in Figure 7.8(b) and Figure 7.8(c), respectively, it was noticed that the composite off-axis optical vortices still reached the center of the Bessel beam at the same recovery distance of about fifty steps, adding or canceling their topological charges to generate second or zeroth-order Bessel beam, respectively. The arrows in Figure 7.10 highlight the recovery distance of an off-axis optical vortex in all cases. Thus the recovery distance required by an offset optical vortex is immune to the presence of same or opposite sign vortex.

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Figure 7.10 Longitudinal intensity profile of a Bessel beam to study the recovery distance of the off-axis vortex in the presence of another vortex with same and different topological signs. The plot shows the evolution of an off-axis vortex or vortices for the following cases: - (a) a single off-axis vortex with a offset distance of thirty-eight pixel (b) composite off-axis vortices of topological charge one and same topological signs with a offset distance of thirty-eight pixels and (c) composite off-axis vortices of topological charge one and opposite topological signs with a offset distance of thirty-eight pixels. Each vertical axis is in pixel ($15\ \mu\text{m}$) and horizontal axis is $Z_{\text{max}}/100$.

Other cases were also studied by offsetting the optical vortices with different inter-vortex distances and with different-topological charges. The recovery distance of an off-axis optical vortex is also immune to different inter-vortex distance and to the higher topological charges. Figure 7.11 compares the recovery distance of an off-axis optical vortex embedded in a Bessel beam with different inter-vortex distance and with higher topological charges. Figure 7.11(a) shows the evolution of the composite off-axis optical vortices of same topological charges one (same sign) but with different offset distances of thirty-eight and sixty-eight pixels. Similar to Figure 7.10(a) the optical vortex, which is offset to a distance of thirty-eight pixels reached the center of the Bessel beam at a distance of fifty steps and other optical vortex (offset to the distance of sixty-eight pixels) recovered to the center at a distance of around eighty steps. The upright and the inverted arrows in Figure 7.11(a) highlights the recovery distance of off-axis vortices offset to a

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distance of thirty-eight and sixty eighty-pixel respectively. This is in consistent to Figure 7.2, which illustrates that the optical vortex with higher offset distance will have longer recovery distance. Figure 7.11(b) shows the evolution of the composite off-axis optical vortex with different topological charges one and two, respectively and an offset distance of thirty-eight pixels. The vortex with the topological charge one reached to the center of the Bessel beam at a distance of fifty steps similar to Figure 7.10(a) but the optical vortex with topological charge two reached at the center of the Bessel beam at a distance of around sixty steps. Again this is congruence to the results shown in Figure 7.3, whereby higher topological charges will take longer recovery distance. The upright and the inverted arrows in Figure 7.11(b) highlights the recovery distance of off-axis vortices with a topological charge one and two respectively.

Figure 7.11 Longitudinal intensity profile of a Bessel beam to study the recovery distance of the off-axis vortex with different inter-vortex distance and different topological charges. The plot shows the evolution of an off-axis vortex or vortices for the following cases: - (a) composite off-axis vortices of same topological charge (one) and sign but with different offset distances of thirty-eight and sixty-eighty pixels and (b) composite off-axis vortices with different topological charges one and two and an offset distance of thirty-eight pixels. Each vertical axis is in pixel (15 μm) and horizontal axis is $Z_{max}/100$.

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This property of the composite optical vortices whereby they are not influenced by the presence of other optical vortex and inter-vortex distance is also congruent with the evolution of composite r vortices in Gaussian beam (diffracting beam). Indebetouw [213] and Rozas [209] reported that each individual r vortex in a Gaussian beam is immune to the presence of other vortex in the beam and are not influenced by the inter-vortex distance. Rozas have proposed the detail mathematical model to explain such independence [209]. However vortex-vortex or inter-vortex distance dependence in a Gaussian beam was observed for the point vortex [209].

In general, optical vortex has been defined as point vortex, r vortex and $\tanh$ vortex based on the core function of the optical vortex [209-210]. The complex function describing an optical vortex in cylindrical coordinates can be given as [217] -

$$\psi(r, \phi) = c(r) \cdot \exp(il\phi) \quad (7.4)$$

$$\phi = \tan^{-1} \left[\frac{(y-b)}{(x-a)} \right] \quad (7.5)$$

where, $c(r)$ describes the core function of an optical vortex and l describes the topological charge of a vortex. The phase of an optical vortex at position $r_o=(a, b)$ is given by ϕ . Based on the value of $c(r)$ various types of optical vortices were studied in singular optics [217] -

$$c(r) = 1 \quad (\text{Point vortex}) \quad (7.6)$$

$$c(r) = (r - r_o) \quad (r \text{ vortex}) \quad (7.7)$$

$$c(r) = \tanh \left[\frac{(r - r_o)}{r_o} \right] \quad (\tanh \text{ vortex}) \quad (7.8)$$

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Here, r_0 is the vortex core radius. However, as the intensity of the outer rings of Bessel beam drops with increasing radius, thus the insertion of an r vortex [209] in a Bessel beam would cause bright rings of radially increasing height. This prohibits the experimental realization of an r vortex in a Bessel beam. Therefore in all the cases discussed above, point vortex has been used (the core function of vortex is taken as 1).

7.4 EXPERIMENTAL STUDY OF SINGULAR OPTICS IN PROPAGATION-INVARIANT BEAMS

Experimental realization of an off-axis optical vortex and vortices embedded in propagation-invariant beams is reported. For a single off-axis optical vortex embedded in Bessel beam, an axicon and spiral phase plate (SPP) [214-215] was employed. Axicon are used to approximate the Bessel beam near its optics axis and spiral phase plate embed 2π phase retardation on the input beam to generate helical singular beam. To embed a single off-axis optical vortex in a Bessel beam, initially an off-axis optical vortex was embedded in a Gaussian beam by intentional misalignment of SPP with the input Gaussian beam [216]. Gaussian beam embedding an off-axis optical vortex was then incident on the axicon, to realize off-axis optical vortex embedded in a Bessel beam.

The spiral phase plate imparts the helical wavefront ($e^{il\phi}$) to the input beam. Gaussian beam was used as the input beam incident to the spiral phase plate generating a singular beam, which can be expressed as [216]: -

$$E(r, \phi, z) = e^{\left(\frac{-r^2}{w^2(z)}\right)} \cdot e^{-i\left(\frac{kr^2z}{2(z^2 + z_R^2)} + \tan^{-1}\frac{z}{z_R}\right)} \cdot e^{il\phi} \quad (7.9)$$

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where ϕ is the phase given as $\tan^{-1}(y/x)$, l is the topological charge, k is the wave number and $w(z)$ is the radius of the beam at the position z , z_R is the Rayleigh range. By displacing the SPP or intentional misalignment to the input beam, 2π phase retardation can be generated at the desired portions of the input beam forming an off-axis optical vortex. The mathematical expression of an off-axis optical vortex beam embedded in a Gaussian beam can be given as [216]: -

$$E(r, \phi, z) = e^{\left(\frac{-r^2}{w^2(z)}\right)} \cdot e^{-i\left(\frac{kr^2z}{2(z^2+z_R^2)} + \tan^{-1}\frac{z}{z_R}\right)} \cdot e^{il\phi} \quad (7.10)$$

$$\phi^1 = \arctan\left(\frac{y \pm \Delta y}{x \pm \Delta x}\right) \quad (7.11)$$

where Δy and Δx are the displacement of the optical vortex away from the beam central axis. Hence an off-axis vortex was embedded in the Gaussian beam before shining it to an axicon.

The experimental set-up is shown in Figure 7.12, the light from Nd:YVO₄ infrared laser (1 W, 1064nm) was incident on a SPP, which was misalignment to the input beam, the output beam was expanded four times by the telescopic arrangement and incident on the axicon, as shown in Figure 7.12. However, there exists a limitation to such technique, whereby the off-axis optical vortex cannot be off-centered to a greater distance on outer rings of the Bessel beam. This is due to the limitation of off-axis vortex in Gaussian beam itself. For an off-axis vortex in a Gaussian beam the vortex should be fully enclosed in a Gaussian beam else full 2π phase retardation is not imparted to a Gaussian beam and thus to a Bessel beam (for more details consult Ref. 216). Care should be taken to fully embed the optical vortex in a Gaussian beam, so that the

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enclosed intensity null or singularity is well within the Gaussian beam. However, an off-axis optical vortex can be imparted to any outer ring of a Bessel beam employing computer-generated holograms (spatial light modulator), which is discussed later. When a Gaussian beam embedding an optical off-axis vortex is incident to the axicon, the Bessel beam embedding off-axis optical vortex in its core was obtained. Figure 7.13 shows the core of a Bessel beam embedding an off-axis optical vortex. Figure 7.13 (a-c) shows the formation of a Bessel beam embedding off-axis vortex in the near field. An off-axis vortex embedded in the core of a Bessel beam is clear in Figure 7.13(d) During the free-space propagation the off-axis optical vortex, recovers to the center of a Bessel beam as shown in Figure 7.13(f) consistent to the simulation results.

Figure 7.12 Schematic diagram of the experimental set-up to embed off-axis vortex in a Bessel beam. The blue line depicts the Gaussian mode from Nd:YVO₄ infrared laser (1 W, 1064nm), incident on off-centered SPP. Gaussian beam embedding off-axis vortex is shown by green light, which incident on axicon. Purple line depicts Bessel beam embedding off-axis vortex.

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Figure 7.13 Free-space propagation of a Bessel beam embedding off-axis vortex. The off-axis vortex moves towards the center of the Bessel beam during the free-space propagation (a-e) and forms higher-order Bessel beam (f). Frame (a) is at 2 cm from an axicon. Distance between each frame is 5 mm. The vertical and horizontal axis shows a dimension of 5 mm.

Experimental generation of composite off-axis optical vortices and the verification of the addition and cancellation of the topological charges at the center of the Bessel beam were obtained employing spatial light modulator (SLM). The SLM used in the experiment had 512 by 512 pixels and the size of each pixel is 15 μm by 15 μm . The hologram information was loaded in the computer to control the phase distribution of the

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SLM. The phase information can be obtained from Eq. 7.3. Figure 7.14 and Figure 7.15 show the experimental results of a Bessel beam embedding composite off-axis optical vortices of same and different topological signs respectively. Congruence to the simulation results the off-axis optical vortices of similar and opposite topological signs, add or cancel their topological charge at the center of the Bessel beam generating second or zero order Bessel beam respectively. Moreover it was also noted that after the addition or cancellation of the optical vortices, the Bessel beam maintains its non-diffracting property, which can be understood based on the fact that the non-diffracting property of the Bessel beam is due to the conical wavevector distribution of the beam, which is immune to the vortices migrations.

Figure 7.14 Experimental result of two off-axis vortices of topological charge one embedded in the Bessel beam. Congruent to the simulation the off-axis vortices add their topological charge at the center of the Bessel beam generating second-order Bessel beam.

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Figure 7.15: - Experimental result of two off-axis vortices of topological charge one with opposite signs embedded in the Bessel beam. Congruent to the simulation the off-axis vortices cancel their topological charge at the center of the Bessel beam generating zero-order Bessel beam.

As reported in the above-section propagation-invariant beam does provide a unique host beam to study singular optics. The optical vortex evolution in propagation-invariant beams is influenced by the conical wavevector distribution. In the above-section study of the evolution of a single off-axis optical vortex embedded in a Bessel beam are determined. Furthermore, the evolution of the composite optical vortices embedded in a Bessel beam is studied numerically and experimentally.

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CONCLUSION AND RECOMMENDATION

"I do not know what I may appear to the world, but to myself I seem to have been only a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

ISAAC NEWTON

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8.1 Conclusion

In this thesis investigation of the propagation-invariant field for the optical trapping applications is reported. A novel non-diffracting field, self-imaged optical bottle beams, by interfering two zero-order Bessel beams with varying radial wavevectors is obtained. A self-imaged bottle beam possesses an array of three-dimensional intensity-null points or bottles along the propagation axis. Further investigation of the properties of the proposed self-imaged bottle beam is reported. A self-imaged bottle beam was reported to inherit the self-reconstruction property of the non-diffracting beams. Experimental realization of a self-imaged bottle beam employing computer generate holograms is obtained.

Numerical study of various regimes of the rotating non-diffracting beams referred as helicon beams is also reported. Helicon beams were obtained by interfering two Bessel beams with varying topological charges and varying radial wavevectors.

The proposed self-imaged bottle beam was employed in the optical tweezers system for various trapping applications of low and high indices microparticles. The unique property of oscillating transverse intensity profile of a self-imaged bottle beam along the propagation axis was exploited to image various points of a bottle beam for different trapping applications. The transfer of 'pure' on-axis spin angular momentum of the circular polarized light to an absorptive microparticle by a bottle beam based optical tweezers system is also reported.

Micro-optical elements approach for the generation of propagation-invariant beams is also explored. Micro-optical element approach for laser micro-beam shaping is high-power efficient technique, which is desirable in optical trapping applications. The

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design of a micron-axicon for the generation of a Bessel beam with long non-diffracting distance is reported. Fabricated micro-axicon was integrated with an optical tweezers system for the trapping and guiding applications. A novel optical element for the direct generation of the proposed self-imaged bottle beam is designed and reported. Conventional axicon structure was shaped with double-gradient on its surface, each generating a Bessel beam with varying radial wavevector, interfering in the free-space propagation to generate a self-imaged bottle beam. Such novel optical element was termed as double-axicon due to its geometrical appearance. A self-imaged bottle beam generated by the fabricated micro-double-axicon was employed in the optical tweezers system for the trapping application.

Furthermore, the kinoform based micro-optical element for the generation of propagation-invariant beams is also reported. Kinoform technique enables to fabricate the surface-relief micron structures with the resist thickness equivalent to 2π thickness, thus easing the fabrication procedure and reducing the fabrication errors. Kinoform-axicon and kinoform-double-axicon was employed for the experimental generation of a Bessel beam and a self-imaged bottle beam, respectively.

The kinetics of the microscopic particles in an array of propagation-invariant or Bessel beams is investigated. Employing the concept of composite hologram approach an array of Bessel beam with minimum interference and long non-diffracting distance was obtained. The static potential landscape generated by the core of Bessel beam array possessing non-diffracting and self-reconstruction property was exploited for the parallel longitudinal guiding of microscopic particle.

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The studied of singular optics in non-diffracting beams is further explored. The evolution of an off-axis optical vortex embedded in a Bessel beam is thoroughly investigated. The influence of various parameters on the evolution of an off-axis optical vortex to the center of Bessel beam is reported. The evolution of the composite off-axis optical vortices embedded in a Bessel beam is also studied. Composite off-axis optical vortices recovered and added their topological charge at the center of the Bessel beam. Moreover, the evolution of the composite off-axis optical vortices embedded in the Bessel beam was found to be immune to the presence of other optical vortices.

8.2 Recommendations

The recommendation for future work based on the outcome of the research carried out in the thesis can be broadly classified into following fields: -

- (a) To study the application of a self-imaged bottle beam in laser cooled trapping.

Bottle beam embedding three-dimensional intensity voids holds potentially application in blue-detuned optical traps where neutral atoms are trapped in dark region; hence the study of trapping laser cooled atoms in an array in a self-imaged bottle beam can provide interesting results. The locations of the bottles embedded in the self-imaged bottle beam can be tailored and thus it will be interesting to study the interaction of closely trapped clouds of atom in the respective bottle of a self-imaged bottle beam. It will be interesting to observe how closely spaced array of cooled atoms interacts. An array of cooled atoms in the optical potential well generated by self-imaged bottle would have applications in quantum computing. Moreover recently researchers has stated modulational instability in arrays of

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Bose-Einstein condensates (BEC) [218], such studies can further be explored employing self-imaged optical bottle beams, which provide an array of trapping spots. Furthermore, an array of laser cooled atoms acts as an atomic tunnel array, which can potentially be used for macroscopic quantum interference.

- (b) Furthermore, the present techniques for the generation of BEC are all-optical techniques employing two crossed laser beams. This is contrary to earlier magnetic-optical technique, which relies on interaction with the magnetic dipole of atoms and magnetic confinement techniques, which work only with certain atoms in some of their energy states. However, now all-optical techniques allow dynamic, fast and much more efficient ways of cooling atoms. Thus it is easy to foresee that laser beam shaping might further be useful for exploring further findings in BEC. Applications like atomic lithography, coherent matter wave optics and coherent atomic interferometry, a technique to control atoms for investigating quantum entanglements for quantum logic is among the applications that would be of interest in BEC employing self-imaged optical bottle beam.
- (c) Another interesting potential application of such unique self-imaged bottle beam possessing periodic optical intensity profile (alternating bright and dark three-dimensional trapping potential well) is to employ these beams to fabricate photonics crystal. Self-imaged bottle beam enables to trap an array of microparticles of different (or alternating) refractive indices. An alternating array of high index and low index microparticles can be trapped in alternating constructive and destructive interference, respectively. Employing 'trap-and-lock' technique, where after trapping particles are locked in their respective by

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solidifying the medium (sol-gel for example), then it would be possible to generate such structures. Thus if the medium where trapping take place is sol-gel, then once the trapping is establish the medium can be exposed to UV light which will solidify micro-particles permanently. It will be thus interesting to explore the trapping of high and low indices particles in an array of self-imaged bottle beams which will would provide an opportunity to generate three-dimensional micro-structures in sol-gel or other preferable solidifying solution to act as photonics crystals.

- (d) Micro-optical elements are designed and employing them with optical tweezers applications tested its usefulness. However, the imperative application of such microscopic elements is for laser beam shaping in lab-on-chip application. It will be interesting to integrate such micro-axicon with the vertical cavity surface emitting laser (VCSEL) to shape the diverging beam into non-diffracting beam in miniaturized optical trap in lab-on-chip applications. For instance integrating composite array of Bessel beams on-chip could possibly be explored for on-chip optical sorting and parallel guiding of micro-particles. Recently there has been strong push to explore miniaturized optical trapping applications whereby all optical components required for trapping are incorporated on the chip itself. Thus for such applications, micron-sized optical elements would be useful when employed for on-chip laser micro-beam shaping and trapping applications. On-chip optical sorting of cells and guiding is an emerging area of research and micro-optical elements would be employed to shape laser beam on chip directly. Moreover incorporating all micro-optical elements (usually required for laser

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beam shaping in optical trapping) on chip itself will greatly miniaturize the set-up and would provide an opportunity of a solitary device to act truly as a lab-on-chip or trapping-on-chip.

- (e) Furthermore, there is scope to further study the design parameters and optimization of micro-optical elements like double-axicon for the generation of self-imaged optical bottle beam. Double-axicon designed can further be explored to have triple or multiple gradients on its surface profile and dedicated study on resultant bottles (null-intensity points) can be explored further. It will be interesting to study multiple refraction on beam in multiple-gradient double-axicon and analyze the resultant interferes to generate bottling effect. Systematic study to report the techniques to shape the bottle shape, dimensions and location on tailoring the gradient and aperture size of double-axicon would also be beneficial.
- (f) Another area, which could be explored, is to design micro-axicon in elliptical shape and to study if that could potentially generate elliptical shaped Bessel beam, which is commonly known as Mathieu beams. Presently Mathieu beams are experimentally generated only through holograms thus it will be beneficial to have a dedicated high-power efficient optical element for the generation of such beams just like an axicon for Bessel beams and a double-axicon for self-imaged bottle beams. Moreover, there is scope to study the limit till an axicon can be altered elliptically preserving the non-diffracting property of the generated beam. Normal axicon produces a line-focus along the longitudinal axis and possess circular geometry in transverse axis. However elliptical-axicon will not have

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circular geometry and outer rings of the elliptical-Bessel beam would be broken. Due to ellipticity there will be non-uniform intensity distribution at the central spot of the resultant elliptical-Bessel beam generating non-uniform gradient force experienced by the microparticles. Thus it will be interesting to explore the interaction of such intensity profile beam with microparticles in optical trapping system.

- (g) Study of singular optics in a non-diffracting beam is reported in the thesis. It has been shown that evolution of optical vortices in a Bessel beam is different from the evolution of optical vortex in a Gaussian beam. Thus it will be interesting to study the evolution of the optical vortices in the different propagation-invariant modes for example hosting vortices in Mathieu beam (elliptical Bessel beam) and parabolic Bessel beams and studying its evolution. It can be foreseen that the evolution of an off-axis optical vortex or vortices placed in propagation-invariant beams like Mathieu beams and parabolic Bessel beam (which does not have closed outer rings or circular geometry) will certainly be different from Bessel beam (which has symmetrical closed outer rings). It will be interesting to explore how does off-axis vortices behave when placed on different locations in Mathieu beams and parabolic Bessel beams. Similarly the study of evolution of composite off-axis vortices placed in such beams might generate some interesting results.

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APPENDIX A**Building Optical Trapping System: - Components and Cost**

Conventional optical trapping system can be easily build employing laser, optical microscope and few essential optical elements. However, before building a practical and economical optical trapping set-up, it is essential to first determine the types of particles (high index, low-index or absorptive) and application (trapping, sorting, guiding, rotating e.t.c.), desired to carry out in optical trapping system. This will determine the type of beam required (for example Gaussian beam, LG beam, Bessel beam or proposed self-imaged bottle beam, as discussed in Section 4.4) and the required laser power for optical trapping application. Usually the power around 5~20mW at the sample stage (trapping plane) is sufficient to trap a microparticle (high-index transparent silica or polystyrene microparticles) of dimension 1~10 μm (that covers most of the colloidal particles and bio-logical cells). However in general for practical optical trapping set-up a laser beam (of good quality and stability with $M^2 < 1.1$) with output power around 30~50 mW is sufficient to trap micro-particles of dimension 1~10 μm , when operating on optical trapping set-up with 100X oil-immersion objective lens with N.A. in range of 1.2~1.4. Bigger and heavier micro-particles would require laser with higher output power. Subsequently an array of micro-particle would require laser beam of higher power.

There are various losses of power in optical trapping set-up (losses due to optical elements, attenuation, poor coupling of laser beam into the back aperture of objective lens, reflection, absorption and attenuation within the medium). Thus after all losses, power of around 5~20 mW is required at the sample stage for generating enough gradient force to trap single microparticle and to overcome gravitational and other fluidic forces due to liquid medium. The major loss of laser power in various experiments carried in the thesis is due to laser micro-beam shaping. Various techniques employed like computer generate holograms (CGH), micro-fabricated optical elements (like spiral phase plates, axicon and double-axicon) cause loss of power during laser beam shaping. It was found that computer generated holograms used employed in Chapter 4 thesis for laser beam

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shaping provides around 40%~50% power conversion efficiency and various optical elements (micro-axicon and micro-double-axicon employed in Chapter 5) provides power conversion efficiency of around 80%~90%.

Another important part of optical trapping set-up is microscopic objective lens, which is employed to focus the trapping beam down to micrometer range. Three-dimensional optical trapping of a micro-particle depends on the gradient force, which depends on how strongly laser beam is focused at the focal vicinity. Thus an objective lens with higher magnification and higher numerical aperture (N.A.) is crucial for optical trapping. For conventional single particle optical trapping employing oil immersion 100X objective lens with N.A.>1.2 would require 30mW~50mW laser beam to trap microparticle of dimension 1~10 μm in comparison to 60X objective lens with smaller N.A. say ~0.7, which would require higher laser power of range 50mW ~100mW for trapping particles of similar dimensions 1~10 μm . A basic optical trapping set-up requires following items: -

- Laser: - laser beam with appropriate power is essential for optical trapping. In optical trapping it is essential to employ beam of good quality and also stability so that the intensity distribution is uniform (usually, $M^2 < 1.1$). He-Ne laser (cost around S\$5,000~S\$15,000 depending on different brands and specifications) with around output power of 50 mW and above can be employed when working on Gaussian beam or LG beam based optical trapping system employing 100X oil immersion objective lens with N.A. >1.2. However, most of the optical trapping experiments in this thesis required high power laser beam, as self-imaged bottle beam and Bessel beam where employed for optical trapping. Such beams share the power equally to the outer rings, thus limiting the power in the central core where trapping takes place. Therefore, high power Neodymium Doped Yttrium Orthvanadate laser (Nd:YVO₄) laser with maximum continuous output power capability of 1 W at 1064nm wavelength was employed (cost around S\$25,000~S\$30,000) in most of the experiment in the thesis. Other lasers like diode pumped solid-state lasers (DPSS) with maximum output laser power around 1~1.5 W can also be employed; such laser cost around S\$20,000.

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- Microscope Objective Lens: - Microscope objective lens is employed to tightly focus the laser beam for generating optical gradient force, which is responsible for optical trapping of microparticles. Tighter the focus, higher will be the gradient force. Most of the optical trapping system employs oil immersion objective lens with high numerical aperture (N. A. in range of 1.2 ~1.4). In the thesis optical upright microscope (Axiostar from Carl Zesis) with 100X oil-immersion objective lens of 1.4 N.A. (Carl Zesis) was employed (cost around ~S\$24,000).
- Dichoric Mirrors: - Dichoric mirrors are employed in most of the optical trapping set-up to reflect the beam of desired wavelength into the objective lens and transmit the beam with different wavelengths. Dichoric mirrors ensure highly reflectivity for chosen wavelength. The optical trapping set-up used in this thesis Dichoric mirror was coated for IR as 1064nm wavelength laser beam was employed (such dichoric mirrors cost around S\$500~3,000\$, depending on various brand and quality).
- CCD Camera and Computer/monitor: - Mostly all optical trapping set-ups require CCD camera and monitor or computer for imaging the real time trapping. CCD camera of decent quality cost around S\$2000.
- Optical Lens: - A pair of lens and periscope arrangement is usually is employed in optical trapping system. A pair of lens is employed to focus and image the beam output from laser to the sample stage through objective lens. Pair of lens allows additional freedom to adjust the focal plane of the laser beam to an appropriate plane at the sample stage. In many practical optical trapping systems the usage of periscope arrangement becomes necessary to guide the laser beam appropriately. Optical elements used with the optical trapping system used in this thesis were IR coated, as wavelength of interested was 1064nm. Cost of optical lens usually required with practical optical system cost around S\$1,000.
- Laser protective eyewear glasses: - It is essential to use to laser protective eyewear glasses special when working in infrared invisible range. The cost of such glass is around S\$ 200.