Performance analyses of fast frequency-hopped communication systems against multitone jamming and multiple-access interference

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Performance Analyses of Fast Frequency-Hopped Communication Systems against Multitone Jamming and Multiple-Access Interference

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in FULFILMENT of the requirement for the degree of
Doctor of Philosophy

2007

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research done by me and has not been submitted for a higher degree to any other University or Institute.

30/01/2007

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Summary

Spread-spectrum (SS) is a modulation technique that can be used to mitigate the deleterious effects of intentional or unintentional interference and multi-path reception. Fast frequency-hopped (FFH) SS communication system is one of the very important techniques commonly used for SS communication systems. FFH/SS communication systems have attracted considerable interests in both military and commercial applications over the past few decades. The performance of an FFH/SS system may be vulnerable to intentional or unintentional interference, such as multitone jamming (MTJ) and multiple-access interference (MAI). In addition, the fading effect is also an important facet to be considered. In this dissertation, some problems regarding FFH/SS systems with diversity-combining methods against MTJ, MAI, fading effect and additive white Gaussian noise (AWGN) are addressed.

Analytical expressions for bit-error rate (BER) performance are firstly derived for an FFH M -ary frequency-shift-keying (FFH/MFSK) communication system with the worst-case MTJ and AWGN over nonfading channels. FFH/MFSK systems employing the linear-combining receiver, the product-combining receiver, the clipper receiver, and the maximum-likelihood (ML) receiver are investigated. For the fading case, two suboptimum ML receivers for FFH/MFSK communication systems are derived. These two receiver structures attempt to countermeasure the effects of the worst-case MTJ and AWGN over Rayleigh and Rician fading channels, respectively. In addition, an algorithm is presented to analyze the performance of the proposed receivers. Performance comparisons among various receivers show that the proposed

Summary

suboptimum receivers significantly outperform the other existing receivers over fading channels. The proposed suboptimum ML receivers with optimum diversity levels can effectively remove the effect of MTJ even under very low signal-to-jamming ratio (SJR) conditions.

Furthermore, we also present the analytical BER expressions for synchronous FFH multiple-access (FFH/MA) systems over Rician fading channels. Several diversity-combining methods are first studied for the perfect power control case. The numerical results show that both the clipper and product-combining receivers significantly outperform the linear-combining receiver. The clipper receiver with an optimum threshold level slightly outperforms the product-combining receiver. Based on the Gauss quadrature rules, analytical BER expressions of an asynchronous FFH/MA system over fading channels are presented in this thesis as well. The performance comparisons among different diversity-combining schemes show that the product-combining receiver outperforms the linear-combining receiver and the hard-limited diversity-combining receiver when the MAI is significant. In addition, the ML receiver structure and the corresponding analytical BER expressions for an FFH/MA system over Rayleigh fading channels are derived. The BER results of the proposed ML receiver can be regarded as the lower bound among the FFH/MA systems. When the interfering users have unequal power levels, the MAI of the system is modeled as alpha-stable noise. Based on this modeling, we derive the analytical BER expressions of the linear-combining, product-combining and clipper receivers over both Rician fading and Rayleigh fading channels by making use of the Fourier-Bessel series. The numerical results show that the two non-linear diversity-combining methods

Summary

significantly outperform the linear-combining receiver.

List of Abbreviations

List of Abbreviations

AWGN	additive white Gaussian noise
BFSK	binary frequency-shift-keying
BER	bit-error-rate
BPF	bandpass filter
CDMA	code-division multiple-access
DS	direct-sequence
FFH	fast frequency-hopped
GSM	global system for mobile communication
HDMV	hard-decision-majority-vote
IC	integrated circuit
LEO	low-earth-orbiting
LOS	line-of-sight
MA	multiple-access
MAI	multiple-access interference
MAP	maximum a posteriori probability
MFSK	M -ary frequency-shift-keying
ML	maximum-likelihood
MTJ	multitone jamming
PBNJ	partial-band noise jamming
PCS	personal communications systems
PDF	probability density function

List of Abbreviations

PSD	power spectral density
RF	radio-frequency
SFH	slow frequency-hopped
SJR	signal-to-jamming ratio
SNR	signal-to-noise ratio
SS	spread-spectrum

List of Symbols

List of Symbols

λ_b	arrive rate of Poisson distribution for BFSK case
σ_n^2	variance of Gaussian noise
p_b	average bit-error rate
p_s	average symbol-error rate
q_{ml}	jamming state of the m th branch and l th hop
$r_{m,l}$	envelope detector output of the m th branch and l th hop
z_m	final decision variable of m th branch
E_b	bit energy
E_h	hop energy
E_s	symbol energy
F	Rician fading factor
L	diversity level
M	modulation dimension
N	total number of non-overlapping FH bands
N_0	one-sided noise power spectral density
P_{J_T}	total jamming power of the MTJ

List of Symbols

Q	number of multitone jamming tones
R_b	bit rate
R_h	hop rate
$R_{m,l}$	square-law detector output of the m th branch and l th hop
T	clipping threshold of the clipper receiver
T_b	bit duration
T_h	hop duration
T_s	symbol duration
W_{ss}	total spread-spectrum bandwidth

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Chapter 1

Introduction

1.1 Motivation

Spread-spectrum (SS) is a signal transmission technique that was introduced more than half a century ago. SS signals are distinguished by the characteristic that the transmission bandwidth W is much greater than the information rate R in bits/s [1]. The bandwidth expansion is achieved by a second modulation that is independent of the information message. Over the past few decades, SS communication systems have attracted considerable interest in both military and commercial applications. Today, SS systems are widely used in the areas of military, commercial, consumer communications, data transmission, message privacy, signal hiding, etc. SS is also an essential technique for the next generation of mobile, cellular, and other personal communications systems (PCS) [2]. Applications and potential advantages of SS systems include the followings [3]:

1. Improved interference rejection
2. Code division multiple-access (CDMA)
3. Low-density power spectra for signal hiding

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4. High-resolution ranging
5. Antijam capability
6. Increased capacity and spectral efficiency in some mobile-cellular PCS applications
7. Graceful degradation of performance as the number of simultaneous users of an radio-frequency (RF) channel increases
8. Lower cost of implementation
9. Readily available integrated circuit (IC) components

Two most commonly used SS techniques are direct-sequence SS (DS/SS) and frequency-hopped SS (FH/SS). For the DS/SS system, the bandwidth spreading is achieved by direct modulation of a data-modulated carrier with a wideband spreading signal or code. For the FH/SS system, the transmitted radio frequency is constant within one hop duration but is changed from hop to hop. The transmitted hopped frequencies are generated by a digital frequency synthesizer which is controlled by the output of a frequency-hopping code sequence generator. FH/SS systems can be categorized into slow frequency-hopped SS (SFH/SS) and fast frequency-hopped SS (FFH/SS) systems. In a SFH/SS system, the hop rate R_h is less than the baseband message bit rate R_b . Thus more than one baseband bit is transmitted at the same frequency-hopped band before hopping to the next frequency-hopped band. In an FFH/SS system, the frequency hopping rate R_h is greater than the baseband message bit rate R_b . In this case one message bit is transmitted by more than one hop. FFH/SS is particularly beneficial in a jamming environment or when the transmission channel causes rapid signal fading as in microwave mobile telephony applications.

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The issues of interference-rejection in SS systems have become more and more important [4]. The growth in wireless communications makes it necessary to utilize the spectrum more efficiently. The increased sharing of spectrum translates into a higher likelihood of users interfering with one another. Interference-rejection techniques can increase cellular capacity which is inherently limited by co-channel interference and adjacent-channel interference. Furthermore, interference-rejection techniques are important in helping to facilitate compatibility during transitions between the old and new communication technologies. The CDMA overlay or coexistence with the Global system for mobile communication (GSM) results in interference - the key problem in making this new digital cellular format viable. In a satellite-based PCS, geostationary satellites can interfere with each other as well as with low-earth-orbiting (LEO) satellites, which limit capacity. Global positioning system (GPS) applications will potentially experience a mixture of both narrow-band and wideband interference.

In military applications, the most obvious application of interference-rejection is in mitigating the effects of intentional jamming. The mitigation of self-interference from harmonics produced by operating transmitters and receivers in close proximity to each other is another obvious application. In reconnaissance applications, a stand-off receiver covering a wide geographical region is subject to interference from nonintelligence-bearing signals operating in the same band. The above description shows that it is important to study the performance of SS systems for various types of intentional jamming and unintentional interference.

In practice, there are several sources of intentional and unintentional interference that are commonly encountered in SS communication systems, including partial-band noise

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jamming (PBNJ), wideband noise jamming, pulse-noise jamming, multitone jamming (MTJ), multiple-access interference (MAI), etc [5], [6]. In this thesis, we will mainly study the effects of MTJ and MAI.

In a practical wireless communication system, channel fading is a major impairment [1]. It has significant impact on performance of communication systems. Hence, it is important to study the channel fading effects on FFH communication systems. In practical system, Rician fading is a commonly used fading model and Rayleigh fading is a special case of Rician fading. In this thesis, we will focus on both Rayleigh fading and Rician fading channels.

1.2 Objectives and Major Contributions

The objective of this thesis is to develop theoretical analyses of the bit-error rate (BER) performance of various FFH M -ary frequency-shift-keying (MFSK) receivers with the worst-case MTJ and additive white Gaussian noise (AWGN) over both nonfading and fading channels. The performance study of FFH/MFSK systems with the existence of MAI is also carried out. In addition, we will also derive the optimum maximum-likelihood (ML) receiver structures for FFH/MFSK systems and develop the corresponding system performance. The system performance of the ML receiver can be regarded as the lower bound of the FFH/MFSK system. In addition, the design of suboptimum ML receivers with simpler structure is also an important aspect in this thesis. Furthermore, we will also study the effects of various system parameters on the BER performance.

The major contributions of this thesis can be summarized as followings. For the

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systems with MTJ, we derive the analytical BER expressions of the product-combining receiver and the clipper receiver against MTJ for an FFH/MFSK system. The optimum ML receiver structure and a suboptimum receiver which requires only the probability of jamming are derived for FFH/MFSK systems. In addition, two suboptimum receivers, namely, the ML-A receiver and the ML-B receiver, are derived to countermeasure the effects of the worst-case MTJ over Rayleigh and Rician fading channels, respectively. An algorithm is also presented to analyze the system performance of the two proposed suboptimum receivers. For the system with MAI, we present the analytical BER expressions for synchronous FFH multiple-access (FFH/MA) systems over fading channels. Furthermore, analytical BER expressions of an asynchronous FFH/MA system over fading channels are presented as well based on the Gauss quadrature rules. In addition, we also study the asynchronous FFH/MA system performance with MAI modeled as alpha-stable process. With the alpha-stable noise model, we can study the performance against the MAI of unequal power levels. The ML receiver structure for an FFH/MA system over Rayleigh fading channels is also derived in this thesis. Lastly, we also analyze the performance of the proposed ML receiver.

1.3 Organization of the Thesis

This thesis is organized as follows. In Chapter 2, the general FFH/MFSK system will be introduced and the literature review will be presented. We will introduce the transmitter, various diversity-combining receivers, AWGN, channel fading, MTJ, and

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MAI. In this chapter, we will describe the system mathematically and present the definitions of the system parameters.

In Chapter 3, the BER performance analyses of the product-combining receiver and the clipper receiver against MTJ over nonfading channels are presented. The ML receiver and a suboptimum receiver are derived as well. Following that, the performance comparisons among various diversity-combining receivers are presented.

In Chapter 4, we will derive two suboptimum ML receivers for FFH/MFSK communication systems. These two receiver structures attempt to countermeasure the effects of the worst-case MTJ and AWGN over Rayleigh and Rician fading channels, respectively. In addition, analytical BER expressions for the two proposed suboptimum receiver structures are derived and validated by simulation results. Performance comparisons among various receivers show that the proposed suboptimum receivers significantly outperform the other existing receivers over fading channels.

In Chapter 5, we present the analytical BER expressions for synchronous FFH/MA systems over fading channels. Fourier-Bessel series are employed in the analyses. Several diversity-combining methods including the linear-combining, product-combining and clipper receivers are studied in this chapter. Theoretical BER results, validated by simulation results for different diversity-combining methods, are presented for comparisons.

In Chapter 6, the ML receiver structure and the corresponding analytical BER expressions for asynchronous FFH/MA systems over Rayleigh fading channels are first derived. Following that, analytical BER expressions of asynchronous FFH/MA systems over both Rician fading and Rayleigh fading channels are presented. The power levels

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of all interfering users are assumed to be same. Theoretical BER expressions based on the Gauss quadrature rules are presented. An upper bound which can be computed easily is presented as well. In addition, a method is introduced to reduce the computational complexity of the multi-dimension Gauss quadrature rules. Lastly, we focus on the system where the interfering users have unequal power levels. We present analytical BER expressions of the system with MAI modeled as alpha-stable noise. Based on this modeling, we derive the analytical BER expressions of the linear-combining, product-combining and clipper receivers over both Rician fading and Rayleigh fading channels by making use of the Fourier-Bessel series.

In Chapter 7, we give the final conclusions of this thesis and some recommendations for future work.

Chapter 2

FFH/MFSK System Model and Literature Survey

In this chapter, we will describe the general FFH/MFSK system model that will be used in the analyses of the subsequent chapters. In particular, we will introduce the transmitter and receiver models, fading channels, and interference. For each part, we will also present the literature survey.

2.1 Transmitter Block Diagram

The transmitter block diagram of an FFH/MFSK communication system is shown in Fig. 2.1. The incoming binary data stream $\{d_b\}$ with bit energy E_b and bit duration T_b is fed into the binary-to- M -ary converter to produce the M -ary data stream $\{d_s\}$ with symbol energy $E_s = c \times E_b$ and symbol duration $T_s = c \times T_b$, where each M -ary symbol consists of $c = \log_2 M$ binary bits. The sequence is then fed into an MFSK modulator,

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which selects one of the M frequency slots based on the input symbol. The output of the MFSK modulator is mixed with a frequency synthesizer to hop to one of the total N non-overlapping FH bands with the hopping energy $E_h = E_s / L$ and the hopping rate $R_h = 1 / T_h$, where $T_h = T_s / L$ is the hopping duration and L is the number of hops per symbol (diversity level). In particular, we choose $f_m = m / T_h$, $m = 1, 2, \dots, M$, for the orthogonal MFSK modulator. The frequency synthesizer is driven by a frequency-hopping code generator. In this thesis, the hopping process is assumed to be done in a random manner unless otherwise specified. Following that, the signal is passed through a bandpass filter (BPF), whose pass-band is equal to the total SS bandwidth W_{ss} . The output of the BPF is up-converted to radio frequency before transmission.

Fig. 2.1 Block diagram of an FFH/MFSK transmitter

In order to have a fair comparison, we assume that the following parameters are constant throughout the subsequent parts:

- Total SS bandwidth W_{ss} ,
- Information bit duration T_b ,

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- One-sided power spectral density (PSD) of the noise N_0 .

With these assumptions, the number of total non-overlapping FH bands is given by

$$N = \left\lfloor \frac{W_{SS} \cdot T_b \cdot \log_2 M}{M \cdot L} \right\rfloor \quad (2.1)$$

Where $\lfloor \cdot \rfloor$ denotes taking the integer part. The symbol duration T_s is given by

$$T_s = T_b \cdot \log_2 M. \quad (2.2)$$

2.2 Receiver Block Diagram

Fig. 2.2 shows the block diagram of a general receiver of an FFH/MFSK communication system. The received signal is down-converted by an RF down-converter. After that, it is sent through a BPF with bandwidth equal to the total SS bandwidth W_{SS} . The frequency-hopping sequence generated by the frequency-hopping code generator is fed into the frequency synthesizer to dehop the received signal. Following that, the dehopped signal $r(t)$ is detected by the square-law detector. The square-law detector structure of the m th branch is shown in Fig. 2.3 and the output of the square-law detector in the l th hop can be expressed as

$$R_{m,l} = \left(\int_{(l-1)T_h}^{lT_h} r(t) \cos(2\pi f_m t) dt \right)^2 + \left(\int_{(l-1)T_h}^{lT_h} r(t) \sin(2\pi f_m t) dt \right)^2. \quad (2.3)$$

The corresponding envelope $r_{m,l}$ is given by $r_{m,l} = \sqrt{R_{m,l}}$.

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Fig. 2.2 Block diagram of an FFH/MFSK receiver

Fig. 2.3 Square-law detector structure

The square-law detector outputs $R_{m,l}$ ($m = 1, 2, \dots, M$, $l = 1, 2, \dots, L$) are then processed by a diversity-combining method denoted as the function $F(\cdot)$. For some diversity-combining methods, the information from other branches (denoted as H_l in Fig. 2.2) is required. The resultant outputs $y_{m,l}$ ($m = 1, 2, \dots, M$, $l = 1, 2, \dots, L$) are summed together to obtain the final decision variables z_m , which are then fed into a decision device

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where the branch with the largest decision variable among all the M branches is selected.

For an FFH/SS system, different diversity-combining techniques are used to combine the received hops corresponding to the same symbol. The suitable application of diversity-combining schemes can improve the system performance significantly. Gulliver et. al. [7] classified the combining techniques into “bin-processing” and “hop-processing” diversity-combining methods, and presented several conventional diversity-combining algorithms such as square-law linear-combining, product-combining, hard-decision-majority-vote (HDMV) diversity-combining, self-normalizing diversity-combining, clipped diversity-combining, gain-combining receiver, etc. In [8], the structure of the optimum ML receiver for an FFH binary frequency-shift-keying (BFSK) system with MTJ and AWGN conditions was presented. Some typical diversity-combining receivers are introduced in the following subsections.

2.2.1 Linear-Combining Receiver

For the linear-combining receiver, the received signals of L hops are directly summed together to obtain the decision variable of each branch. Hence the function $F(\cdot)$ of the linear-combining receiver can be expressed as

$$y_{m,l} = F(R_{m,l}) = R_{m,l}. \quad (2.4)$$

The performance of an FFH/BFSK system with a linear-combining receiver against PBNJ has been presented in [9]. In [10], the authors derived the performance expressions of an FFH/BFSK linear-combining receiver against MTJ based on Taylor-series expansion in central differences. This method allows the authors to efficiently

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analyze the performance of the linear-combining receiver with higher diversity levels. The performance analyses of the FFH/BFSK linear-combining and clipper receivers against MTJ over fading channels were given in [11] which show that the clipper receiver is superior to the linear-combining receiver when the jamming power is strong. A uniform quantizer was used for convenience of analyses in [11]. The performance analyses of an FFH/MFSK system with linear-combining receiver against MTJ over fading channels were presented in [12] where the characteristic functions were used to derive the analytical expressions. Only single-tone per-band MTJ was discussed in [10]-[12]. Upper bounds on the performance of chip synchronized FHMA/MFSK systems with the linear-combining receiver over a Rayleigh fading environment were presented in [13]. With the assumption that at most one hit occurs, closed-form expressions of the BER performance of an FFH/MA system with a linear-combining receiver over Rayleigh fading channels were presented in [14].

2.2.2 Product-Combining Receiver

For the product-combining receiver, the final decision variables are obtained by multiplying the received signals of L hops which can be expressed as

$$z'_m = \prod_{l=1}^L R_{m,l}, \quad m = 1, 2, \dots, M. \quad (2.5)$$

In our analyses, we can use the following variables to replace (2.5)

$$z_m = \ln \left(\prod_{l=1}^L R_{m,l} \right) = \sum_{l=1}^L \ln(R_{m,l}), \quad m = 1, 2, \dots, M. \quad (2.6)$$

Since (2.6) has the same performance as (2.5), so the equivalent function $F(\cdot)$ for the product-combining receiver is given by

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$$y_{m,l} = F(R_{m,l}) = \ln(R_{m,l}), \quad m = 1, 2, \dots, M. \quad (2.7)$$

The performance of an FFH/BFSK system with a product-combining receiver against PBNJ has been studied in [15]-[17]. The performance analysis of the same receiver against the single-tone per-band MTJ was shown in [18] where the compact characteristic functions of the natural logarithm of the product-combiner outputs were obtained based on the Taylor-series expansion. Furthermore, the performance of the FFH/BFSK product-combining receiver against both the single-tone per-band MTJ and the independent MTJ over fading channels was studied in [19].

2.2.3 Clipper Receiver

For the clipper receiver, the received signals are first limited by a clipping level and then summed together to obtain the decision variables. In this case, the function $F(\cdot)$ can be shown as

$$y_{m,l} = F(R_{m,l}) = \begin{cases} R_{m,l}, & R_{m,l} < T, \\ T, & R_{m,l} \geq T, \end{cases} \quad m = 1, 2, \dots, M \quad (2.8)$$

where T is the clipping threshold. The system performance with the clipper receiver is dependent on the clipping level T . Side information is required to obtain the optimum clipping level.

The analyses of an FFH/BFSK clipper receiver against PBNJ have been presented in [20] where the authors used L -fold convolution to derive the analytical expressions. The performance analyses of various FFH/BFSK receivers against MTJ over both nonfading and fading channels were shown in [21] and [22], respectively, where the authors used the Taylor-series expansion and the quantization approach to obtain the

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BER expression for higher diversity levels without much computational complexity. As mentioned earlier, the clipper receiver is shown to perform better than the linear-combining receiver in an FFH/BFSK system against MTJ over fading channels.

2.2.4 Other Diversity-Combining Techniques

In this section, we will introduce some other diversity-combining techniques which will not be analyzed in this thesis.

A. Self-Normalizing Receiver

For this case, the received signals of each hop are normalized by the sum of the M branches first. Unlike the receivers mentioned previously, the function $F(\cdot)$ of the self-normalizing receiver requires the information of other branches which is shown as H_l in Fig. 2.2, where

$$H_l = \sum_{m=1}^M R_{m,l}, \quad l = 1, 2, \dots, L. \quad (2.9)$$

The function $F(\cdot)$ for the self-normalizing receiver is given by

$$y_{m,l} = F(R_{m,l}, H_l) = \frac{R_{m,l}}{H_l}, \quad m = 1, 2, \dots, M, \quad l = 1, 2, \dots, L. \quad (2.10)$$

Probability of error analyses of an FFH/BFSK self-normalizing receiver against PBNJ have been presented in [23]. The performance of an FFH/BFSK self-normalizing receiver with PBNJ over fading channels was studied in [24]. In [25], the upper bound on the performance of an FFH/MFSK system against PBNJ over a Rician fading channel was also presented. The approach can avoid searching for the joint probability density function (PDF) of all the M branches. The BER performance of the FFH/BFSK self-normalizing receiver against MTJ has been presented in [26] based on Taylor-

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series expansion. In [26], it has been shown that single-tone per-band MTJ is more harmful than independent MTJ. An analytical study for the performance of an FFH/MFSK self-normalizing receiver in the presence of MTJ was presented in [27] where the authors gave an upper bound on BER using a similar method as in [25].

B. Hard-Decision-Majority-Vote Receiver

In a hard-decision-majority-vote (HDMV) diversity-combining receiver, a hard decision is first made on each of L hops. The branch that contains the most counts (votes) in L hops is declared the one containing the signal. In this case, the function $F(\cdot)$ can be shown as

$$y_{m,l} = F(R_{m,l}) = \begin{cases} 1, & R_{m,l} = \max_{n=1,2,\dots,M} \{R_{n,l}\}, \\ 0, & R_{m,l} \neq \max_{n=1,2,\dots,M} \{R_{n,l}\}, \end{cases} \quad m = 1, 2, \dots, M \quad (2.11)$$

In [28], the performance of HDMV diversity-combining receiver was measured in the presence of severe noise and multitone jamming plus system noise. The HDMV receiver reduced the BER reasonably well for noise jamming but gave little reduction for large multitone jamming. The authors in [29] analyzed the performance analyses of coherent FH system with the presence of PBNJ and AWGN. The locus of the optimum diversity level for the coherent HDMV receiver was also presented in [29], which allows one to obtain the optimum diversity level for the coherent HDMV receiver under different levels of signal-to-jamming ratio.

C. Gain Combining Receiver

For the gain-combining receiver, the function $F(\cdot)$ is given by

$$y_{m,l} = F(R_{m,l}) = a_{m,l} R_{m,l} \quad (2.12)$$

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where $a_{m,l}$ ($m=1,2,\dots, M$ and $l=1,2,\dots,L$) is the gain of the corresponding m th branch and l th hop. The most common used gain-combining technique is equal-gain combining, i.e. $a_{m,l}=1$ ($m=1,2,\dots, M$ and $l=1,2,\dots,L$). An equal-gain combining receiver is essentially a linear-combining receiver which has been introduced in previous section.

2.2.5 Maximum-Likelihood Receiver

From Fig. 2.2, it can be observed that the final decision over L hops is dependent on the vector $\mathbf{R}=[R_{1,1}, R_{2,1}, \dots, R_{M,1}, R_{1,2}, \dots, R_{M,L}]$ which contains all the relevant information in the received signal waveform. The maximum a posteriori probability (MAP) criterion is based on selecting the signal corresponding to maximizing the set of posterior probabilities $\{P(f_m | \mathbf{R})\}$, where f_m is the transmitted signal. In [1], it has been shown that this criterion maximizes the probability of a correct decision and, hence, minimizes the probability of error. Using the Bayes' rule [1] and assuming that all the M signals are equally probable, i.e., $P(f_m)=1/M$ for all M , the decision rule based on finding the signal that maximizes $P(f_m | \mathbf{R})$ is equivalent to finding the signal that maximizes $P(\mathbf{R} | f_m)$. The decision criterion based on the maximum of $P(\mathbf{R} | f_m)$ over the M signals is called the ML criterion. The receiver structure based on the ML criterion is called the ML receiver. It has been proved in [1] that the ML criterion maximizes the probability of a correct decision and, hence minimizes the probability of error. Thus the ML receiver is also called as the optimum receiver.

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The ML receiver for an FFH/MFSK system with PBNJ was first investigated in [30]. In [31] the authors presented the ML receiver structure for FFH/BFSK systems with MTJ and gave the corresponding simulation results. Furthermore, the authors also derived and analyzed two suboptimum receivers for large and small signal-to-noise ratio (SNR), respectively. The ML receiver structures for both synchronous and asynchronous FH/MA systems were presented and studied in [32] and [33], respectively. The signals are assumed to experience Rayleigh fading and Rician fading in [32] and [33], respectively. The optimum ML receiver structure proposed in [33] is not in closed form. Instead, a suboptimum ML receiver called the soft-limiter receiver was proposed and analyzed in [33].

2.3 Fading Channel Models

Transmission in wireless communication systems is carried out in a radio propagation environment, which places fundamental limitations to the performance of the wireless communication system. As introduced earlier, a typical received signal contains multiple components reflected off buildings or other large objects that arrive with different time delays and phases. This causes the multipath fading effect. A detailed study on the characterization of fading channels has been presented in [34]-[35]. In this thesis, we assume that the smallest spacing between frequency hopped slots is larger than the coherence bandwidth of the channel [1]. Consequently, each hop of a symbol experiences independent fading. In an FFH system, it is possible that the smallest spacing between adjacent frequency slots will not be greater than the coherence bandwidth of the channel, but it is not likely that the diversity transmissions for the

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same symbol will be sent over adjacent frequency slots. In fact, the hopping pattern can be easily designed to avoid the above scenario. Hence, the assumption of independent fading on each diversity reception is justifiable. In addition, the channel for each hop of a symbol is modeled as a frequency-nonselective slowly fading process. This implies that the signal bandwidth is much smaller than the coherence bandwidth of the channel and that the hop duration is much smaller than the coherence time of the channel [1]. The latter assumption is equivalent to requiring the hop rate to be large compared to the Doppler spread of the channel. As a result, the signal amplitude can be modeled as a Rician or Rayleigh random variable that remains fixed at least for one hop duration. These assumptions on the fading process are reasonable for FFH systems [36]. In the following, we will introduce the Rayleigh fading and Rician fading models.

2.3.1 Rayleigh Fading Channel

When the received signal is made up of multiple reflective rays with no significant line-of-sight (LOS) component, the signal amplitude due to small-scale fading has a Rayleigh PDF distribution and is referred to as Rayleigh fading. A Rayleigh fading model is suitable for urban areas without LOS and when the coverage area is small. The PDF of the received signal envelope r over Rayleigh fading channels is given by [1]

$$P(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \quad r \geq 0 \quad (2.13)$$

where σ^2 is the time-average power of the received signal. Fig. 2.4 shows the PDF of a Rayleigh faded random variable r with different values of σ .

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Fig. 2.4 PDF of Rayleigh fading channel with different values of σ

Rayleigh-fading channels have been widely considered in [13]-[15], [34]-[35], [37]-[38], and the references therein. A detailed survey on the characterization of Rayleigh fading channels for mobile digital communication systems was given in [34] and [35]. Furthermore, two practical techniques have been presented to mitigate the effects of Rayleigh fading channels. The BER performance of an FFH/BFSK system with product-combining receiver with PBNJ over Rayleigh fading channels has been studied in [15]. Gulliver et. al. focused on the noncoherent FFH/MFSK system and examined the BER performance with several diversity-combining methods over the Rayleigh fading channel [37]. The characteristic functions of the square-law detector outputs have been used to obtain a computationally efficient BER expression for different diversity levels. The BER performance of an FFH/BFSK linear-combining receiver

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against MTJ and AWGN over Rayleigh fading channels was presented in [38]. A computationally efficient BER expression for the proposed system was derived based on the Taylor-series expansion and was validated by simulation results. Upper bounds on the performance of FH/MA systems with hard-limited and linear-combining receivers in a Rayleigh fading environment with chip synchronism were presented in [13]. In [14], closed-form expressions for the BER performance of an FFH/MA system with the linear-combining receiver over Rayleigh fading channels were presented with the assumption that at most one hit occurs.

2.3.2 Rician Fading Channel

The model for Rician fading is similar to that for Rayleigh fading, except that in the Rician fading channel, a strong dominant component is present. This dominant component is due to the LOS path. The Rician fading model is used when a strong component exists in addition to the scattered components. The PDF of the received signal envelope r over Rician fading channels can be shown as [1]

$$P(r) = \frac{r}{\sigma^2} e^{-(r^2 + \alpha^2)/2\sigma^2} I_0\left(\frac{r\alpha}{\sigma^2}\right), \quad r \geq 0 \quad (2.14)$$

where α is the peak amplitude of the dominant signal, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, and σ^2 is the time-average power of the scattered components. The Rician fading channel is often described in terms of a parameter F which is defined as the ratio between the dominant signal power and the variance of the multipath. It is given by $F = \alpha^2 / (2\sigma^2)$. F is known as the Rician fading factor and completely specifies the Rician distribution. As $F \rightarrow 0$, i.e., $\alpha \rightarrow 0$,

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the Rician distribution approaches a Rayleigh distribution. For $F \gg 1$, the Rician PDF will converge to that of Gaussian distribution. The PDF curves of a Rician faded random variable r with $\sigma = 0.5$ and different values of F are shown in Fig. 2.5.

Fig 2.5 PDF of Rician fading channel with $\sigma = 0.5$ and various F

The performance over a Rician fading channel has been widely studied in the literature. The BER of an FFH/BFSK product-combining receiver with PBNJ over Rician fading channels was presented in [16]. Robertson and Sheltry [39] presented a method to examine the BER performance of the FH/MFSK receiver with MTJ and AWGN over Rician fading channels. In [39], Marcum's Q -function was used to derive the BER with no diversity-combining. Besides the method proposed in [39], Teh et. al. [40] presented the uniform quantization approach to investigate the BER performance for an FFH/BFSK self-normalizing receiver over a Rician fading channel with the

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worst-case MTJ and AWGN. As an extension, the quantization approach was applied to examine the BER performance of the FFH/BFSK system with the worst-case MTJ and additive white Gaussian noise (AWGN) over Rician fading channels for the clipper and linear-combining receivers [11] as well as the product-combining receiver [41]. Annamalai et. al. [42] presented a general method for calculating the average error rates and outage performance of a broad class of coherent, differentially coherent and noncoherent communication systems with diversity reception in various fading environments. Wang and Jiang [12] presented their analytical study of an FFH/MFSK linear-combining receiver with MTJ and AWGN over Rician fading channels. They used the characteristic function approach to analyze the uncoded system. In [43], the authors presented the BER of an SFH/MA system over a Rician fading channel.

2.4 Interference Models

Since the performance of an FFH/SS system may be vulnerable to intentional or unintentional interference, the issues of interference suppression in FFH/SS systems have become more and more important [44]. In the following, some commonly encountered interference models in FFH/SS systems are introduced.

2.4.1 Additive White Gaussian Noise

AWGN is the most commonly encountered noise for a communication channel [1]. As shown in Fig. 2.6, AWGN is the background noise spreads over the whole frequency spectrum, which is introduced primarily by electronic components and amplifiers at the receiver. As AWGN is commonly encountered in physical communication channels

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and because of its mathematical tractability, this is the predominant channel model used in many communication system analyses and designs.

Fig. 2.6 Additive white Gaussian noise in FH system

The AWGN denoted as $n(t)$ can be expressed as a white Gaussian process with power spectral density (PSD) $\Phi_{nn}(f) = \frac{1}{2}N_0$. The mean of the $n(t)$ is zero and the auto-correlation function of $n(t)$ is given by

$$E(n(t_1)n(t_2)) = \frac{N_0}{2} \delta(t_1 - t_2) \quad (2.15)$$

where $\delta(\bullet)$ denotes the delta function.

2.4.2 Multitone Jamming

As mentioned in Chapter 1, there are two types of jamming waveforms commonly studied, namely, PBNJ and MTJ. The PBNJ is modeled as a zero-mean Gaussian random process with a flat power spectral density over a fraction of the total SS bandwidth W_{ss} and zero elsewhere. Many results for the performance of FFH/SS with PBNJ have been presented in [9], [16], [23]-[25], [45], and the references therein.

Fig. 2.7 Independent multitone jamming

Fig. 2.8 Single-tone per-band multitone jamming

MTJ is more harmful against FH/SS systems and we will focus on this jamming model in this thesis. For an FH/SS system with MTJ, the total jamming power is denoted as P_{J_T} , which is equally divided among Q jamming tones. As indicated in [5], the jammer is assumed to have the complete knowledge of the transmission system except that it does not know the actual hopping pattern being used. The task of the MTJ is to choose the number of jamming tones Q and the tone spacing such that the received BER is maximized. MTJ can be categorized into independent MTJ and single-tone per-band MTJ, as shown in Figs. 2.7 and 2.8, respectively. For independent MTJ, the

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jamming tones can be present at any of the frequency slots within the total SS bandwidth. For the single-tone per-band MTJ, a particular FH band is jammed with at most one jamming tone at one of the M adjacent frequency slots for MFSK systems. The MTJ tone can be expressed as [10]

$$J(t) = \sqrt{2}a_j \cos(2\pi f_j t + \phi_j) \quad (2.16)$$

where $\sqrt{2}a_j$ is the amplitude of the jamming tone, f_j is the jamming frequency which is assumed to coincide exactly with one of the M frequency slots within a particular FH band, and ϕ_j is the random phase of the jamming tone. It has been shown in [46] that the single-tone per-band MTJ gives the worst-case BER for a HDMV diversity-combining technique. It was also concluded that the MTJ is more harmful than the corresponding PBNJ under noise-free conditions since the MTJ can focus its power on only one channel of a MFSK receiver. Hence, in this thesis we focus on the system performance against the worst-case single-tone per-band MTJ. The distribution of signal tones and single-tone per-band MTJ tones with 4-ary FSK modulation is shown in Fig. 2.9. In this case, the transmitter performs L hops/symbol with $L = 3$ being the diversity level and the output of the modulator is pseudo-randomly hopped into one of the N non-overlapping FH bands.

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Fig. 2.9 Transmitted signals of an FFH/SS system with single-tone per-band MTJ

The performance analyses of FFH/SS systems with MTJ under various channel conditions have been widely studied. In [10], the performance of an FFH/BFSK linear-combining receiver against MTJ has been studied based on Taylor-series expansion in central differences. The authors obtained the analytical expressions through the characteristic functions of decision statistics. The performance analyses of the FFH/BFSK linear-combining receiver and the clipper receiver against MTJ over fading channels were given in [47] which show that the clipper receiver is superior to the linear-combining receiver when the jamming power is strong. In [47], a uniform quantizer was used for analyses. The performance of an FFH/MFSK system with the linear-combining receiver against MTJ over fading channels was studied in [12] where the characteristic functions were used to derive the analytical expressions. The BER performance of the product-combining receiver against the single-tone per-band MTJ was shown in [18] where the compact characteristic functions of the natural logarithm of the product-combiner outputs were obtained based on the Taylor-series expansion. Furthermore, the performance analyses of the FFH/BFSK product-combining receiver

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against both the single-tone per-band MTJ and the independent MTJ over fading channels were presented in [19]. The FFH/BFSK self-normalizing receiver against MTJ was investigated in [26] based on the Taylor-series expansion. In [27], the authors gave an upper bound on the BER performance of an FFH/MFSK self-normalizing receiver in the presence of MTJ using a similar method as in [25]. In [31], the authors presented the ML receiver structure for FFH/BFSK systems with MTJ and gave simulation results for the proposed ML receiver.

2.4.3 Multiple-Access Interference

Multiple-access (MA) techniques are nowadays widely used, as they allow many users to share a wireless communication channel. Due to the rapid development of technology and increasing demand for personal wireless communications, MA techniques have become increasingly important. One of the important types of MA techniques is FH/MA. FH/MA has been used in military applications due to its significant anti-jamming properties, low probability of intercept and low probability of detection in the presence of unintended listeners [48]. The main advantage of using FH/MA is that it does not require contiguous bandwidth, power control and sophisticated synchronization at the receiver [48]-[49]. When two users transmit their signals in the same frequency hop, a collision can occur and the receiver can fail to distinguish between the two users. Thus, the communication of other users can interfere with the desired transmitted signal and affect its detection. This kind of interference is called multiple-access interference (MAI). The MAI in an FH/MA system can be categorized into two types: synchronous MAI and asynchronous MAI. When a collision occurs, the MAI in one hop can be expressed as

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$$I(t) = \sum_{c=1}^C \sqrt{2}a_c \cos(2\pi f_c(t - \tau_c') + \phi_c') \quad (2.17)$$

where C is the number of the interfering users, $\sqrt{2}a_c$ is the amplitude of the c th interfering user in the hop, f_c is the modulation frequency, ϕ_c' is a random phase uniformly distributed over $[-\pi, \pi]$, and τ_c' is the time delay between the c th interfering user and the reference user. The delays τ_c' ($c = 1, 2, \dots, C$) are zero for synchronous systems and are assumed to be uniformly distributed over $[-T_h, T_h]$ for asynchronous systems.

The effects of MAI in FH/MA systems have been studied by many researchers. Upper bounds on the error probability of SFH/MA systems have been presented in [50]. In [51], a Gaussian approximation technique was proposed for evaluating the symbol error probability of noncoherent hybrid DS/SFH SSMA systems. Furthermore, a detailed analysis has been presented in [52] for an FH/CDMA system with unequal power levels among various users over a nonfading channel. In [53], BER expressions for an asynchronous FH/MA system were presented. The proposed BER expressions are exact for only one interfering user case. When there is more than one interfering users, the corresponding upper bounds were presented. For an FFH system, diversity-combining techniques can be employed to mitigate the MAI and fading effects. Upper bounds on the BER performance of chip synchronized FHMA/MFSK systems with hard-limited and linear-combining receivers over a Rayleigh fading environment were presented in [13]. In [14], closed-form expressions for the BER performance of an FFH/MA system with a linear-combining receiver over Rayleigh fading channels were presented with the assumption that at most one hit occurs. In [54], Gaussian

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approximations for the MAI were developed and used to analyze the performance of the proposed system for synchronous and asynchronous hopping under nonfading and Rayleigh fading channels. To study the MAI more efficiently, the MAI was characterized by the alpha-stable random variable model in [55] and [56]. In [57] and [58], signal detection in alpha-stable noise has been investigated and the Cauchy receiver was found to offer the performance close to that of the corresponding optimum receivers. Under the alpha-stable model assumption for MAI, the performance of the conventional receivers in FH systems with MFSK modulation was studied in [59].

Chapter 3

Performance Analyses of FFH

Systems with MTJ over Nonfading

Channels

In this chapter, analytical BER expressions are derived for FFH/MFSK communication systems with the worst-case MTJ and AWGN. FFH/MFSK systems employing the product-combining, clipper, and ML receivers are investigated. Our analyses validated by simulation results show that the optimum ML receiver provides a lower bound on the BER performance. The product-combining receiver and the clipper receiver give nearly the same performance which is better than that of the linear-combining receiver. The effects of the diversity level on BER performance are also studied in this chapter.

3.1 Introduction

Many researchers have studied the effects of intentional or unintentional interference on the BER performance of various FFH diversity-combining techniques. The authors in [10] derived the performance of an FFH/BFSK linear-combining receiver against MTJ based on Taylor-series expansion in central differences. This approach enables the authors to efficiently analyze the performance of the linear-combining receiver with higher diversity levels. The performance of the product-combining receiver against single-tone per-band MTJ was shown in [18] where the authors obtained the compact characteristic functions of the product-combiner outputs based on the Taylor-series expansion. The analyses of an FFH/BFSK clipper receiver against PBNJ have been presented in [20] where the authors used L -fold convolutions to derive the analytical expressions. The performance analyses of various FFH/BFSK receivers against MTJ over nonfading were shown in [21], where the authors used the Taylor-series expansion and the quantization approach to obtain the BER expression for higher diversity levels without much extra computational complexity. The performance of an FFH/MFSK self-normalizing receiver against PBNJ over a Rician fading channel was also studied in [25] where the authors used the upper bound of BER to approach the system performance. The theoretical analyses of the FFH/BFSK self-normalizing receiver against MTJ have been presented in [26] where the authors showed that the single-tone per-band MTJ is more harmful than the independent MTJ. An analytical study for the performance of an FFH/MFSK self-normalizing receiver in the presence of MTJ was presented in [27] where the authors gave the upper bound of BER using the similar method as in [25]. In [31], the authors presented the ML receiver structure for FFH/BFSK systems with MTJ

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and gave the simulation results of the proposed ML receiver. Furthermore, the authors also derived and analyzed two suboptimum receivers in [31] for large and small SNR, respectively.

From the above literature survey, we conclude that the performance analyses of the product-combining, clipper, and ML receivers in FFH/MFSK systems with MTJ and AWGN have not been investigated. Hence, we focus on the BER performance of an FFH/MFSK system with diversity-combining against the single-tone per-band MTJ and AWGN. The single-tone per-band MTJ is chosen in our subsequent analyses because it results in the poorest system performance as compared with other types of MTJ as well as PBNJ under nonfading and low SJR conditions [12], [17], [26]. Furthermore, a suboptimum ML receiver structure which requires only information on the probability of jamming is derived in this chapter for FFH/MFSK systems.

3.2 Performance Analyses of Various FFH/MFSK Systems

The transmitter and the receiver structures were shown in Figs. 2.1 and 2.2, respectively. The dehopped signal $r(t)$ in Fig. 2.2 can be expressed as

$$r(t) = \sqrt{2}a_s \cos(2\pi f_m t + \phi_s) + J(t) + n(t), \quad m = 1, 2, \dots, M \quad (3.1)$$

where $\sqrt{2}a_s$ is the amplitude of the desired signal, f_m is the signal frequency depending on the incoming data, ϕ_s is the random phase uniformly distributed over $[-\pi, \pi]$, $J(t)$ is the MTJ, and $n(t)$ is the Gaussian noise process with zero mean and variance $\sigma_n^2 = N_0 B$, where N_0 is the one-sided PSD of the noise and $B = 1/T_h$. The

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MTJ tone can be expressed as

$$J(t) = \sqrt{2}a_J \cos(2\pi f_J t + \phi_J) \quad (3.2)$$

where $\sqrt{2}a_J$ is the amplitude of the jamming tone, f_J is the jamming frequency which is assumed to coincide exactly with one of the M frequency slots within a particular FH band, and ϕ_J is the random phase of the jamming tone.

The SNR of the system is defined as $SNR = E_b / N_0$ where E_b is the bit energy of the original binary data at the transmitter before it is converted into M -ary data. The SJR of the system is defined as $SJR = E_b / N_J$, where $N_J = P_{J_T} / W_{SS}$ is the equivalent PSD of the MTJ, and P_{J_T} denotes the total power of the MTJ. It is assumed that the total power of the MTJ is distributed over Q equal-power jamming tones, each with power $P_J = P_{J_T} / Q$.

Without loss of generality, we assume that f_1 was transmitted in the subsequent analyses. The square-law detector output of the l th hop is denoted as $R_{m,l,q_{ml}}$, where $m = 1, 2, \dots, M$ denotes one of the M frequency slots, $q_{ml} = 1$ or 0 denotes whether the l th hop of the m th branch is jammed or not jammed, respectively, and $l = 1, 2, \dots, L$. The PDF of $R_{m,l,q_{ml}}$ is denoted as $P_{R_{m,l,q_{ml}}}(R_{m,l})$, which can be obtained using the Taylor-series approximation [10]

$$P_{R_{1,l,1}}(R_{1,l}) \approx \sum_{b=-1}^1 \frac{1}{6\sigma_n^2} \exp\left(-\frac{R_{1,l} + 2(a_S^2 + a_J^2 + b\sqrt{3}a_S a_J)}{2\sigma_n^2}\right) \times I_0\left(\frac{\sqrt{2R_{1,l}(a_S^2 + a_J^2 + b\sqrt{3}a_S a_J)}}{\sigma_n^2}\right) U(R_{1,l}), \quad (3.3)$$

$$P_{R_{1,l,0}}(R_{1,l}) = \frac{1}{2\sigma_n^2} \exp\left(-\frac{R_{1,l} + 2a_S^2}{2\sigma_n^2}\right) \times I_0\left(\frac{\sqrt{2R_{1,l}a_S^2}}{\sigma_n^2}\right) U(R_{1,l}), \quad (3.4)$$

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$$P_{R_{m,l}}(R_{m,l}) = \frac{1}{2\sigma_n^2} \exp\left(-\frac{R_{m,l} + 2a_J^2}{2\sigma_n^2}\right) \times I_0\left(\frac{\sqrt{2R_{m,l}a_J^2}}{\sigma_n^2}\right) U(R_{m,l}), \quad m = 2, 3, \dots, M, \quad (3.5)$$

and

$$P_{R_{m,l,0}}(R_{m,l}) = \frac{1}{2\sigma_n^2} \exp\left(-\frac{R_{m,l}}{2\sigma_n^2}\right) U(R_{m,l}), \quad m = 2, 3, \dots, M \quad (3.6)$$

where $U(\cdot)$ is the unit step function. The characteristic function of $R_{m,l,q_{ml}}$ is defined as

$$\Phi_{R_{m,l,q_{ml}}}(iv) = \int_{-\infty}^{\infty} P_{R_{m,l,q_{ml}}}(R_{m,l}) e^{iR_{m,l}v} dR_{m,l}. \quad (3.7)$$

It has also been shown that [10]

$$\Phi_{R_{1,l}}(iv) = \sum_{b=-1}^1 \frac{1}{3(1-2iv\sigma_n^2)} \cdot \exp\left(\frac{2iv(a_S^2 + a_J^2 + b\sqrt{3}a_S a_J)}{1-2iv\sigma_n^2}\right), \quad (3.8)$$

$$\Phi_{R_{1,l,0}}(iv) = \frac{1}{1-2iv\sigma_n^2} \cdot \exp\left(\frac{2iv a_S^2}{1-2iv\sigma_n^2}\right), \quad (3.9)$$

$$\Phi_{R_{m,l}}(iv) = \frac{1}{1-2iv\sigma_n^2} \cdot \exp\left(\frac{2iv a_J^2}{1-2iv\sigma_n^2}\right), \quad m = 2, 3, \dots, M, \quad (3.10)$$

and

$$\Phi_{R_{m,l,0}}(iv) = \frac{1}{1-2iv\sigma_n^2}, \quad m = 2, 3, \dots, M. \quad (3.11)$$

3.2.1 Analysis of the Product-Combining Receiver

For the product-combining receiver, the function $F(\cdot)$ is given in (2.7) with $y_{m,l}$

replaced by $y_{m,l,q_{ml}}$. The corresponding PDF of $y_{m,l,q_{ml}}$ can be expressed as

$$P_{y_{m,l,q_{ml}}}(y_{m,l}) = P_{R_{m,l,q_{ml}}}(R_{m,l}) \cdot \frac{dR_{m,l}}{dy_{m,l}} = P_{R_{m,l,q_{ml}}}(e^{y_{m,l}}) \cdot e^{y_{m,l}}, \quad m = 1, 2, \dots, M, \quad l = 1, 2, \dots, L \quad (3.12)$$

where $P_{R_{m,l,q_{ml}}}(R_{m,l})$ can be obtained from (3.3) to (3.6). As shown in Fig. 2.2, the final

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decision variables are defined as $z_m = \sum_{l=1}^L y_{m,l,q_{ml}}$, $m = 1, 2, \dots, M$, and the PDF of z_m is given by

$$P_{z_m}(z_m) = P_{y_{m,1,q_{m1}}}(y_{m,1}) \otimes P_{y_{m,2,q_{m2}}}(y_{m,2}) \otimes \dots \otimes P_{y_{m,L,q_{mL}}}(y_{m,L}), \quad m = 1, 2, \dots, M \quad (3.13)$$

where $\otimes$ denotes convolution.

Since all L hops play the same role in the PDF of $P_{z_m}(z_m)$ and the only difference is whether they are jammed or not jammed, we can define

$$P_{y_{m,1}}(y_m) = P_{y_{m,l,q_{ml}}}(y_{m,l}), \quad \text{for } q_{ml} = 1 \quad (3.14)$$

and

$$P_{y_{m,0}}(y_m) = P_{y_{m,l,q_{ml}}}(y_{m,l}), \quad \text{for } q_{ml} = 0. \quad (3.15)$$

We also define

$$P_{z_m}^{[j_m]}(x_m) = [P_{y_{m,1}}(y_m)]^{\otimes j_m} \otimes [P_{y_{m,0}}(y_m)]^{\otimes (L-j_m)} \quad (3.16)$$

where $j_m = \sum_{l=1}^L q_{ml}$ denotes the number of jammed hops in the m th frequency slot out of

L hops per symbol and the term $(\otimes j_m)$ denotes $(j_m - 1)$ -fold convolution.

Since f_1 is assumed to be transmitted, an error occurs if $z_m > z_1$, for any of the $m = 2, 3, \dots, M$. Therefore, the conditional probability of symbol error for the sum of all L hops given that j_m hops are jammed is

$$\begin{aligned} P_{J,j}(j_1, j_2, \dots, j_M) = 1 - \int_0^\infty \left[\int_0^{z_1} P_{z_2}^{[j_2]}(z_2) dz_2 \times \int_0^{z_1} P_{z_3}^{[j_3]}(z_3) dz_3 \times \dots \right. \\ \left. \times \int_0^{z_1} P_{z_M}^{[j_M]}(z_M) dz_M \right] P_{z_1}^{[j_1]}(z_1) dz_1 \end{aligned} \quad (3.17)$$

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where $j = \sum_{m=1}^M j_m$ denotes the total number of jammed hops for all M frequency slots.

Then the conditional probability of symbol error for the sum of all L hops given that j hops have been jammed can be expressed as

$$p_j = \sum_{j_1=0}^j \sum_{j_2=0}^{j-j_1} \sum_{j_3=0}^{j-j_1-j_2} \dots \sum_{j_{M-1}=0}^{j-j_1-j_2-j_3-\dots-j_{M-2}} \binom{j}{j_1} \times \binom{j-j_1}{j_2} \times \binom{j-j_1-j_2}{j_3} \times \dots \times \binom{j-j_1-j_2-\dots-j_{M-2}}{j_{M-1}} \times \left(\frac{1}{M}\right)^j \times p_{J,j}(j_1, j_2, \dots, j_M). \quad (3.18)$$

Last, the probability of the M -ary symbol error can be shown as

$$p_s = \sum_{j=0}^L \binom{L}{j} \left(\frac{Q}{N}\right)^j \left(1 - \frac{Q}{N}\right)^{L-j} p_j \quad (3.19)$$

where N is given by (2.1). Assuming all symbols are equal-probable, the average BER is then given by [page 260, 1]

$$p_b = \frac{M/2}{M-1} p_s. \quad (3.20)$$

3.2.2 Analysis of the Clipper Receiver

For the clipper receiver, the function $F(\cdot)$ is given in (2.8) with $y_{m,l}$ replaced by $y_{m,l,q_{ml}}$. The PDF of $y_{m,l,q_{ml}}$ is given by

$$P_{y_{m,l,q_{ml}}}(y_{m,l}) = P_{R_{m,l,q_{ml}}}(R_{m,l}) \cdot U(T - R_{m,l}) + w_{m,l} \cdot \delta(y_{m,l} - T) \quad (3.21)$$

where T is the clipping level and $w_{m,l} = \int_T^\infty P_{R_{m,l,q_{ml}}}(R_{m,l}) dR_{m,l}$. When the l th hop is jammed, $w_{m,l}$ can be expressed as [21]

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$$w_{1,l} = \frac{1}{3} \sum_{k=-1}^1 Q \left(\frac{\sqrt{2(a_s^2 + a_j^2 + k\sqrt{3}a_s a_j)}}{\sigma_n}, \sqrt{\frac{C}{\sigma_n^2}} \right) \quad (3.22)$$

where $Q(a,b)$ is Marcum's Q -function [1]. Similarly, the values of $w_{m,l}$ for the other 3 possible combinations can be obtained from (3.22) by setting $a_j = 0$, $a_s = 0$, and $a_s = a_j = 0$, respectively. Once we obtain the PDF of $y_{m,l,q_{ml}}$, the subsequent analyses are the same as the product-combining receiver which are given by (3.13) to (3.20).

3.2.3 Analysis of the Maximum-Likelihood Receiver

The structure of the ML receiver against MTJ over nonfading channels is given by [8]

$$z_m = \sum_{l=1}^L y_{m,l,q_{ml}} = \sum_{l=1}^L \left[\alpha_{m,l,q_{ml}} + \ln I_0 \left(\beta_{m,l,q_{ml}} \sqrt{R_{m,l,q_{ml}}} \right) - \ln I_0 \left(\gamma_{m,l,q_{ml}} \sqrt{R_{m,l,q_{ml}}} \right) \right] \quad (3.23)$$

where

$$\alpha_{m,l,q_{ml}} = \frac{2q_{ml}a_j^2 - A_{m,l}^2}{\sigma_n^2}, \quad \beta_{m,l,q_{ml}} = \frac{A_{m,l}}{\sigma_n^2}, \quad \text{and} \quad \gamma_{m,l,q_{ml}} = \frac{\sqrt{2q_{ml}a_j^2}}{\sigma_n^2} \quad (3.24)$$

with

$$A_{m,l} = \sqrt{2(a_s^2 + q_{ml}(a_j^2 + 2a_s a_j \cos \phi_{m,l}))}. \quad (3.25)$$

Note that $\phi_{m,l}$ is the phase difference between the signal tone and the MTJ tone in the m th branch and l th hop.

In this case, the quantization approach is used to obtain the analytical expressions. Since we only study single-tone per-band MTJ, we can replace $\phi_{m,l}$ by ϕ_l , where ϕ_l is the phase difference between the signal tone and the MTJ tone of the l th hop. We define a vector $\phi = [\phi_1, \phi_2, \dots, \phi_L]$ and let $p_b(\phi)$ be the probability of bit error conditioned on ϕ .

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Thus the average BER is

$$p_b = E_{\phi}(p_b(\phi)) \quad (3.26)$$

where $E_{\phi}(\cdot)$ denotes taking the expectation with respect to ϕ .

Since the PDF of $y_{m,l,1}$ depends on ϕ_l , in order to avoid the L -dimensional integration in (3.26), we use the following Taylor-Series approximation [10, eq. (6)] to obtain the unconditioned PDF of $y_{m,l,1}$

$$P_{y_{m,l,1}}(y_{m,l}) \approx \sum_{b=-1}^1 \frac{1}{3} P_{y_{m,l,1}}^{[b]}(y_{m,l}) \quad (3.27)$$

where the superscript $[b]$ represents the condition $\cos \phi_l = b \cdot \frac{\sqrt{3}}{2}$, for $b=-1, 0$ and 1 .

The accuracy of this approximation have been discussed in [60] and verified in [10].

Let $\hat{y}_{m,l,q_{ml}}$ be the G -level uniform quantizer output of $y_{m,l,q_{ml}}$, the decision statistics are then expressed as

$$\hat{z}_m = \sum_{l=1}^L \hat{y}_{m,l,q_{ml}}. \quad (3.28)$$

The PDFs of the discrete random variables $\hat{y}_{m,l,q_{ml}}$ are given by

$$P_{\hat{y}_{m,l,1}}(\hat{y}_{m,l}) = \sum_{g=-G/2}^{G/2} v_{m,l,1}(g) \delta(\hat{y}_{m,l} - c_g) \quad (3.29)$$

and

$$P_{\hat{y}_{m,l,0}}(\hat{y}_{m,l}) = \sum_{g=-G/2}^{G/2} v_{m,l,0}(g) \delta(\hat{y}_{m,l} - c_g) \quad (3.30)$$

where $c_g = 2gC/G$ for $g = -G/2, -G/2+1, \dots, G/2$, and C is set at a substantially higher level such that the probability of $\hat{y}_{m,l,q_{ml}}$ exceeding this value is negligible.

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Following that, we use the following algorithm to derive $v_{m,l,1}(g)$ and $v_{m,l,0}(g)$.

Step 1: Set $v_{m,l,1}(g) = 0$, $v_{m,l,0}(g) = 0$, for $g = -G/2, -G/2 + 1, \dots, G/2$ and $m = 1, 2, \dots, M$.

Step 2: Divide $[0, R_m]$ into K small intervals as $R_{m,l}(k) = (k-1/2) \cdot R_m / K$, $k = 1, 2, \dots, K$, where R_m satisfies the conditions

$$1 - \int_0^{R_m} P_{R_{m,l,1}}(R_{m,l}) dR_{m,l} < \varepsilon, \quad m = 1, 2, \dots, M \quad (3.31)$$

and

$$1 - \int_0^{R_m} P_{R_{m,l,0}}(R_{m,l}) dR_{m,l} < \varepsilon, \quad m = 1, 2, \dots, M \quad (3.32)$$

where ε is an error limit and should be small enough to ensure good accuracy, K should also be large enough to ensure good accuracy. From (3.23) we can derive

$$y_{1,l,1}^{[b]}(k) = \alpha_{1,l,1}^{[b]} + \ln I_0 \left(\beta_{1,l,1}^{[b]} \sqrt{R_{1,l}(k)} \right) - \ln I_0 \left(\gamma_{1,l,1}^{[b]} \sqrt{R_{1,l}(k)} \right), \quad b = -1, 0, 1, \quad (3.33)$$

$$y_{1,l,0}(k) = \alpha_{1,l,0} + \ln I_0 \left(\beta_{1,l,0} \sqrt{R_{1,l}(k)} \right) - \ln I_0 \left(\gamma_{1,l,0} \sqrt{R_{1,l}(k)} \right), \quad (3.34)$$

$$y_{m,l,1}^{[b]}(k) = \alpha_{m,l,1}^{[b]} + \ln I_0 \left(\beta_{m,l,1}^{[b]} \sqrt{R_{m,l}(k)} \right) - \ln I_0 \left(\gamma_{m,l,1}^{[b]} \sqrt{R_{m,l}(k)} \right), \quad m = 2, 3, \dots, M, \quad (3.35)$$

and

$$y_{m,l,0}(k) = \alpha_{m,l,0} + \ln I_0 \left(\beta_{m,l,0} \sqrt{R_{m,l}(k)} \right) - \ln I_0 \left(\gamma_{m,l,0} \sqrt{R_{m,l}(k)} \right), \quad m = 2, 3, \dots, M \quad (3.36)$$

where $\alpha_{m,l,q_{ml}}$, $\beta_{m,l,q_{ml}}$, and $\gamma_{m,l,q_{ml}}$ ($m = 1, 2, \dots, M$) are defined in (3.24), the superscript

$[b]$ represents the condition $\cos \phi_l = b \cdot \frac{\sqrt{3}}{2}$ as mentioned earlier, for $b = -1, 0$ and 1 .

Step 3: For every value of k , by using (3.27) and with the assumption that f_1 was

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transmitted, we obtain

$$v_{1,l,1}(g_{1,l,1}^{[b]}) = v_{1,l,1}(g_{1,l,1}^{[b]}) + \frac{1}{3} P_{R_{1,l}}(R_{1,l}(k)) \cdot \frac{R_1}{K}, \quad b = -1, 0, 1, \quad (3.37)$$

$$v_{1,l,0}(g_{1,l,0}) = v_{1,l,0}(g_{1,l,0}) + P_{R_{1,0}}(R_{1,l}(k)) \cdot \frac{R_1}{K}, \quad (3.38)$$

$$v_{m,l,1}(g_{m,l,1}^{[b]}) = v_{m,l,1}(g_{m,l,1}^{[b]}) + \frac{1}{3} P_{R_{m,l}}(R_{m,l}(k)) \cdot \frac{R_m}{K}, \quad m = 2, 3, \dots, M, \quad (3.39)$$

and

$$v_{m,l,0}(g_{m,l,0}) = v_{m,l,0}(g_{m,l,0}) + P_{R_{m,0}}(R_{m,l}(k)) \cdot \frac{R_m}{K}, \quad m = 2, 3, \dots, M. \quad (3.40)$$

Note that the PDF of $R_{m,l,q_{ml}}$ ($m = 1, 2, \dots, M$) is given by (3.2) to (3.6), and $g_{1,l,1}^{[b]}$, $g_{1,l,0}$, $g_{m,l,1}^{[b]}$, and $g_{m,l,0}$ satisfy the following conditions

$$c'_{g_{1,l,1}^{[b]}-1} + C/G < y_{1,l,1}^{[b]}(k) \leq c'_{g_{1,l,1}^{[b]}} + C/G, \quad b = -1, 0, 1, \quad (3.41)$$

$$c'_{g_{1,l,0}-1} + C/G < y_{1,l,0}(k) \leq c'_{g_{1,l,0}} + C/G, \quad (3.42)$$

$$c'_{g_{m,l,1}^{[b]}-1} + C/G < y_{m,l,1}^{[b]}(k) \leq c'_{g_{m,l,1}^{[b]}} + C/G, \quad m = 2, 3, \dots, M, \quad (3.43)$$

and

$$c'_{g_{m,l,0}-1} + C/G < y_{m,l,0}(k) \leq c'_{g_{m,l,0}} + C/G, \quad m = 2, 3, \dots, M \quad (3.44)$$

where

$$c'_{-G/2-1} \rightarrow -\infty, \quad c'_{G/2} \rightarrow \infty, \quad \text{and} \quad c'_g = c_g \text{ for } g = -G/2, -G/2+1, \dots, G/2-1.$$

The PDF of the quantized output $\hat{y}_{m,l,q_{ml}}$ can be obtained from (3.29) and (3.30), and similar to the product-combining receiver the average BER of the ML receiver can be obtained from (3.13) to (3.20). The differences are to replace $y_{m,l,q_{ml}}$ and z_m with the corresponding quantized outputs $\hat{y}_{m,l,q_{ml}}$ and $\hat{z}_m$, respectively. In addition, the

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convolution operation in (3.16) is replaced by discrete convolution and the integration in (3.17) is replaced by summation.

3.3 Suboptimum Maximum-Likelihood Receiver

In [8], the authors derived the ML receiver and two suboptimum receivers against MTJ over nonfading channels. However, these optimum and suboptimum receivers require information on the jamming location which is difficult to obtain in practice. In this section, a new suboptimum ML receiver structure for FFH/MFSK communication systems in the presence of MTJ and AWGN is derived.

As shown in [1], the ML receiver selects the largest $P(\mathbf{R} | f_m)$ for $m = 1, 2, \dots, M$. In this system, $\mathbf{R} = [R_{1,1}, R_{2,1}, \dots, R_{M,1}, R_{1,2}, \dots, R_{M,L}]$. By defining $\hat{\mathbf{R}}_l = (R_{1,l}, R_{2,l}, \dots, R_{M,l})$, $P(\mathbf{R} | f_m)$ can be written as

$$P(\mathbf{R} | f_m) = \prod_{l=1}^L P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m). \quad (3.45)$$

Because all M branches are independent of each other, $P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m)$ is given by

$$\begin{aligned} P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m) &= \frac{Q}{N} P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m, J) + \left(1 - \frac{Q}{N}\right) P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m, NJ) \\ &= \frac{Q}{MN} \sum_{i=1}^M \left[\prod_{j=1, j \neq i}^M P_{R_{j,l}}(R_{j,l} | f_m, NJ) \cdot P_{R_{i,l}}(R_{i,l} | f_m, J) \right] + \left(1 - \frac{Q}{N}\right) \prod_{i=1}^M P_{R_{i,l}}(R_{i,l} | f_m, NJ) \\ &= \prod_{i=1}^M P_{R_{i,l}}(R_{i,l} | f_m, NJ) \left[1 - \frac{Q}{N} + \frac{Q}{MN} \sum_{j=1}^M \frac{P_{R_{j,l}}(R_{j,l} | f_m, J)}{P_{R_{j,l}}(R_{j,l} | f_m, NJ)} \right] \end{aligned} \quad (3.46)$$

where $P_{R_{i,l}}(R_{i,l} | f_m, J)$ and $P_{R_{i,l}}(R_{i,l} | f_m, NJ)$ denote the PDF of the l th hop and the i th branch being jammed and not jammed, respectively, for $i = 1, 2, \dots, M$.

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Next we define

$$P_{R_{m,j}}(R_{m,l} | S, J) = P_{R_{m,j}}(R_{m,l} | f_m, J), \quad (3.47)$$

$$P_{R_{m,j}}(R_{m,l} | NS, J) = P_{R_{j,l}}(R_{j,l} | f_m, J), \quad i = 1, 2, \dots, M \text{ and } i \neq m, \quad (3.48)$$

$$P_{R_{m,j}}(R_{m,l} | S, NJ) = P_{R_{m,j}}(R_{m,l} | f_m, NJ), \quad (3.49)$$

and

$$P_{R_{m,j}}(R_{m,l} | NS, NJ) = P_{R_{j,l}}(R_{j,l} | f_m, NJ), \quad i = 1, 2, \dots, M \text{ and } i \neq m \quad (3.50)$$

where $P_{R_{m,j}}(R_{m,l} | S, J)$, $P_{R_{m,j}}(R_{m,l} | NS, J)$, $P_{R_{m,j}}(R_{m,l} | S, NJ)$, and $P_{R_{m,j}}(R_{m,l} | NS, NJ)$

denote the PDF of the signal hop with jamming, the non-signal hop with jamming, the signal hop without jamming, and the non-signal hop without jamming, respectively.

Then, we can rewrite (3.46) as

$$P_{\hat{R}_l}(\hat{R}_l | f_m) = \prod_{i=1, i \neq m}^M P_{R_{i,j}}(R_{i,l} | NS, NJ) \cdot P_{R_{m,j}}(R_{m,l} | S, NJ) \\ \times \left[1 - \frac{Q}{N} + \frac{Q}{MN} \sum_{j=1, j \neq m}^M \frac{P_{R_{j,j}}(R_{j,l} | NS, J)}{P_{R_{j,j}}(R_{j,l} | NS, NJ)} + \frac{Q}{MN} \frac{P_{R_{m,j}}(R_{m,l} | S, J)}{P_{R_{m,j}}(R_{m,l} | S, NJ)} \right]. \quad (3.51)$$

Dividing $P_{\hat{R}_l}(\hat{R}_l | f_m)$ by $\prod_{i=1}^M P_{R_{i,j}}(R_{i,l} | NS, NJ)$, we obtain the new decision variables

$$x_m = \prod_{l=1}^L \left\{ \frac{P_{R_{m,j}}(R_{m,l} | S, NJ)}{P_{R_{m,j}}(R_{m,l} | NS, NJ)} \left[1 - \frac{Q}{N} + \frac{Q}{MN} \sum_{j=1, j \neq m}^M \frac{P_{R_{j,j}}(R_{j,l} | NS, J)}{P_{R_{j,j}}(R_{j,l} | NS, NJ)} + \frac{Q}{MN} \frac{P_{R_{m,j}}(R_{m,l} | S, J)}{P_{R_{m,j}}(R_{m,l} | S, NJ)} \right] \right\}. \quad (3.52)$$

Since taking the natural logarithm of x_m does not affect the final decision, the final

decision statistics of the proposed suboptimum receiver are given by

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$$z_m = \ln(x_m) = \sum_{l=1}^L y_{m,l} \quad (3.53)$$

where

$$y_{m,l} = \ln \left\{ \frac{P_{R_{m,l}}(R_{m,l} | S, NJ)}{P_{R_{m,l}}(R_{m,l} | NS, NJ)} \left[1 - \frac{Q}{N} + \frac{Q}{MN} \sum_{j=1, j \neq m}^M \frac{P_{R_{j,l}}(R_{j,l} | NS, J)}{P_{R_{j,l}}(R_{j,l} | NS, NJ)} + \frac{Q}{MN} \frac{P_{R_{m,l}}(R_{m,l} | S, J)}{P_{R_{m,l}}(R_{m,l} | S, NJ)} \right] \right\}. \quad (3.54)$$

By substituting (3.3) to (3.6) into (3.54) and after some mathematical manipulations, we obtain

$$y_{m,l} = -\frac{a_s^2}{\sigma_n^2} + \ln I_0 \left(\frac{\sqrt{2a_s^2 R_{m,l}}}{\sigma_n^2} \right) + \ln \left[1 - \frac{Q}{N} + \frac{Q}{MN} \sum_{i=1, i \neq m}^M \exp \left(-\frac{a_j^2}{\sigma_n^2} \right) I_0 \left(\frac{\sqrt{2a_j^2 R_{i,l}}}{\sigma_n^2} \right) \right. \\ \left. + \frac{Q}{3MN} \frac{\sum_{b=-1}^1 \exp \left(\frac{-a_j^2 - b\sqrt{3}a_s a_j}{\sigma_n^2} \right) \cdot I_0 \left(\frac{\sqrt{2(a_s^2 + a_j^2 + b\sqrt{3}a_s a_j) R_{m,l}}}{\sigma_n^2} \right)}{I_0 \left(\frac{\sqrt{2a_s^2 R_{m,l}}}{\sigma_n^2} \right)} \right]. \quad (3.55)$$

In this thesis, we call this receiver the ML-III receiver. From (3.55) it can be seen that the proposed ML-III receiver only requires information on Q/N , SNR and SJR. Since the decision statistics depend on $R_{m,l}$ and $R_{i,l}$, $i=1,2,\dots,m-1,m+1,\dots,M$, the final decision variables z_m of the M branches are dependent on each other. Consequently it is difficult to derive a closed-form BER expression. Hence in this chapter we only present simulation results for this proposed receiver.

3.4 Numerical Results and Discussions

In this section, numerical BER results for various noncoherent FFH/MFSK receivers

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with the single-tone per-band MTJ and AWGN over non-fading channels are presented. The worst-case band MTJ is considered and the number of interfering tones Q is chosen to maximize the BER for each SJR level. The worst-case BER results are plotted against SJR. The BER results for the linear-combining receiver and the self-normalizing receiver are obtained from [12] and [27], respectively. The BER results for the product-combining receiver, the clipper receiver, and the ML receiver are obtained based on the analytical expressions derived in Section 3.3. The BER performance of the two suboptimum receivers proposed in [31], namely, the ML-I receiver and the ML-II receiver, are also presented for comparison. A comparison of analytical and simulation results are also provided to validate the BER expressions derived in Section 3.3.

In Figs. 3.1 to 3.3, the worst-case analytical and simulation BER results of the product-combining receiver, the clipper receiver, and the ML receiver are presented, respectively, with SNR=6dB, $M=32$ and $L=1, 3$, and 5. The analytical BER results are obtained based on the BER expressions derived in Section 3.3. The close match between both the analytical and simulation results validates our analyses presented in Section 3.3. Fig. 3.3 also justifies the accuracy of using the quantization approach to analyze the performance of the ML receiver. The above observation is valid for other modulation dimension and SNR levels though only some typical results with $M=32$ and SNR=6dB are provided.

Although higher diversity levels can mitigate the MTJ effects, it will cause more noncoherent diversity-combining loss as introduced in [9], [20] and [23]. Figs. 3.1 and 3.2 show that when SJR is less than 28dB, a diversity order of $L=3$ provides the best

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performance for both the product-combining receiver and the clipper receiver. This is because $L=3$ provides sufficient mitigation of the MTJ effects and outweighs the corresponding noncoherent diversity-combining loss incurred by the system. The improvement in system performance due to diversity is only applicable to a limited range of SJR which is less than 28dB in this case because SNR is not large enough to outweigh the disadvantages of increased noncoherent diversity-combining loss introduced by higher diversity levels. In other words, when SJR is greater than a certain value, the optimum diversity level will decrease as SJR increases.

In Fig. 3.3, it can be observed that higher diversity does not improve the system performance of the ML receiver throughout the SJR range, while the product-combining receiver and the clipper receiver can benefit from it. The reason is that the ML receiver makes use of information from other branches and can reduce the MTJ effects significantly. Consequently, the higher diversity level of the ML receiver counteracts the MTJ effects less than the product-combining receiver and the clipper receiver while introducing the same noncoherent diversity-combining loss.

In Figs. 3.4 and 3.5, the performance for various receivers, namely, linear-combining receiver [12], product-combining receiver, clipper receiver, ML receiver, and ML-I receiver [8] against the worst-case band MTJ and AWGN are shown with $M=4$ and $M=32$, respectively. The SNR is fixed at 13.35dB and the diversity level is fixed at 3. The BER results show that the optimum ML receiver provides a lower bound on the BER performance for the FFH/MFSK system in the presence of the worst-case band MTJ and AWGN, while the suboptimum ML-I receiver has the second best performance. Among the other receivers, it can be observed that the product-combining

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receiver and the clipper receiver give nearly the same performance which is better than the linear-combining receiver. It should be noted that these receivers are easier to implement as they do not require any side information. Comparing Figs 3.4 with 3.5, it can be observed that the BER decreases as the modulation level M increases. Note that it has been shown in [1] that larger M will improve the system performance over an AWGN channel only.

A performance comparison of the proposed ML-III receiver with some existing receivers is shown in Fig. 3.6 with SNR=13.35dB and $L=3$. The performance of the ML-III receiver was obtained via simulation. Only the BER results with $M=2$ are presented, but for the proposed ML-III receiver can be easily extended for $M>2$. The BER results in Fig. 3.6 show that the ML-III receiver performs slightly better than the product-combining receiver, the clipper receiver, and the linear-combining receiver. Although the BER results of the ML receiver and ML-I receiver are better than that of the ML-III receiver, the ML-III receiver does not require the information of jamming location which is difficult to obtain in practice.

In Fig. 3.7, performance comparisons of various receivers with optimum diversity levels are presented for SNR=13.35dB, $M=32$ and $10\text{dB} \leq \text{SJR} \leq 30\text{dB}$. The optimum diversity levels for various receivers under different conditions are shown in Table 3.1. From Fig. 3.7, we can observe that the ML receiver and the ML-I receiver still perform the best and the second best, the clipper receiver performs better than the product-combining receiver, and the linear-combining receiver gives the worst performance. From Fig. 3.7, it can also be observed that the effects of SJR on the performance of the ML receiver and the ML-I receiver are less as compared to the other receivers. The

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reason is that the ML receiver and the ML-I receiver have the information of the MTJ which helps the receivers to mitigate the effects of MTJ. Thus performance of the ML receiver and the ML-I receiver is less sensitive to the SJR as compared to the other receivers. From Fig. 3.7 we also notice that the receivers which have more complicated structure or require more side information have better performance. The above observations are applicable to other SNR levels and modulation dimensions.

In Table 3.1, the optimum diversity levels for various receivers with different SNR and M values are shown. From Table 3.1, the following conclusions can be drawn. Firstly, the optimum diversity levels of the ML receiver and the suboptimum ML-I receiver are much less than those of the product-combining receiver, the clipper receiver and the self-normalizing receiver. This is because the ML receiver and the ML-I receiver have side information on the MTJ and can reduce the MTJ effects significantly. Thus there is no need to use higher diversity levels to countermeasure the MTJ effects. Secondly, the optimum diversity level for the linear-combining receiver is always 1. The reason is that the linear-combining receiver does not limit the MTJ before diversity combining and it is unable to mitigate the MTJ effects effectively while it incurs more noncoherent diversity-combining loss as the diversity level increases. Thirdly, the optimum diversity level increases as M increases except for the linear-combining receiver. When M increases, the number of non-overlapping FH bands decreases, and thus the signal tone is more likely to be jammed. Therefore a higher order of diversity is required to mitigate more probable MTJ effects when M increases. Lastly, the optimum diversity level decreases as SNR decreases due to the increasing noncoherent diversity-combining loss.

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Fig. 3.1 Worst-case analytical and simulation BER results of the product-combining receiver with SNR=6dB and $M=32$

Fig. 3.2 Worst-case analytical and simulation BER results of the clipper receiver with SNR=6dB and $M=32$

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Fig. 3.3 Worst-case analytical and simulation BER results of the ML receiver with $\text{SNR}=6\text{dB}$ and $M=32$

Fig. 3.4 BER performance comparisons of various receivers with $\text{SNR}=13.35\text{dB}$, $M=4$, and $L=3$

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Fig. 3.5 BER performance comparisons of various receivers with SNR=13.35dB, $M=32$, and $L=3$

Fig. 3.6 BER performance comparisons among ML-III receiver and other existing receivers with SNR=13.35dB, $M=2$, and $L=3$

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Fig. 3.7 BER performance comparisons of various receivers with optimum diversity levels for SNR=13.35dB and $M=32$

Receiver	SNR=6dB M=32	SNR=13.35dB M=32	SNR=13.35dB M=4
Linear-combining receiver	1	1	1
Self-normalizing receiver	1	17	3
Product-combining receiver	3	9	3
Clipper receiver	3	9	3
ML-I receiver	1	5	1
ML receiver	1	5	1

Table 3.1 Optimal diversity levels of various receivers

3.5 Conclusions

In this chapter, we have derived new analytical BER expressions for the product-

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combining and clipper receivers against MTJ for FFH/MFSK systems over nonfading channels. In addition, an algorithm was presented to obtain the analytical BER performance of the FFH/MFSK ML receiver. Numerical results have shown that the ML receiver provides a lower bound on the BER performance. In addition, there is an optimum diversity level for various diversity-combining schemes and the optimum diversity level of the ML receiver is less than that of the product-combining receiver and the clipper receiver. Lastly, we derived a new suboptimum ML receiver which does not require information on the jamming location. Simulation results have shown that this receiver has better BER performance than other receivers.

Chapter 4

Suboptimum ML Receiver Structures for FFH Systems with MTJ over Fading Channels

In this chapter, we derive two suboptimum ML receivers for FFH/MFSK communication systems with the worst-case MTJ and AWGN over Rayleigh and Rician fading channels, respectively. In addition, analytical BER expressions for the two proposed suboptimum structures are derived and validated by simulation results. Performance comparisons among various receivers show that the proposed suboptimum receivers significantly outperform other existing receivers over fading channels. The optimum diversity level of the suboptimum ML receiver for the Rayleigh fading case is found to be higher than that of the Rician fading case. In addition, the proposed suboptimum ML receivers with optimum diversity levels can effectively remove the effects of MTJ even under very low SJR conditions.

4.1 Introduction

The effects of the MTJ on FFH/MFSK systems over nonfading channels have been introduced in Chapter 3. In this chapter, we will focus on the fading channel cases. Robertson and Sheltry [39] presented a method to examine the BER performance of the FH/MFSK receiver with MTJ and AWGN over Rician fading channels. In [39], the Marcum's Q -function was used to derive the BER expression, and no diversity-combining scheme was considered. The performance analyses of the FFH/BFSK linear-combining and clipper receivers against MTJ over fading channels were given in [11]. It has been shown in [11] that the clipper receiver is superior to the linear-combining receiver when the jamming power is strong. A uniform quantizer is used to simplify the analyses in [11]. The performance study of an FFH/MFSK system with linear-combining receiver against MTJ over fading channels was presented in [12] where the characteristic functions are used to derive the analytical expressions. The performance analysis of an FFH/BFSK product-combining receiver against both single-tone per-band MTJ and independent MTJ over fading channels was presented in [19]. The BER results of the FFH/BFSK self-normalizing receiver against MTJ have been presented in [26] based on Taylor-series expansion. In [26] the authors showed that single-tone per-band MTJ is more harmful than independent MTJ. An analytical study of the performance of an FFH/MFSK self-normalizing receiver in the presence of MTJ was presented in [27]. The ML diversity-combining structure for an FFH/MFSK system with PBNJ was first investigated in [30], and the ML receiver structure for FFH/BFSK systems with MTJ was presented in [31]. Furthermore, the authors also derived two suboptimum receivers in [31]

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for large and small SNR, respectively. In this chapter, we will derive two suboptimum receiver structures for FFH/MFSK systems against MTJ over Rayleigh and Rician fading channels, respectively. In addition, we will analyze the performance of these two suboptimum ML receivers in the presence of single-tone per-band MTJ.

4.2 Proposed Suboptimum ML Receivers over Fading Channels

4.2.1 System Description

The transmitter and receiver structures have been shown in Figs. 2.1 and 2.2, respectively.

The dehopped signal $r(t)$ in Fig. 2.2 can be expressed as

$$r(t) = \sqrt{2}a_s \cos(2\pi f_m t + \phi_s) + J(t) + n(t), \quad m = 1, 2, \dots, M \quad (4.1)$$

where $n(t)$ is the Gaussian noise process with zero mean and variance $\sigma_n^2 = N_0 B$, and $J(t)$ is the MTJ and can be expressed as

$$J(t) = \sqrt{2}a_j \cos(2\pi f_j t + \phi_j). \quad (4.2)$$

All parameters in (4.1) and (4.2) have the same definitions as those in Chapter 3. Since it has been shown in [27] and [61] that the system performance is not sensitive to different types of MTJ fading conditions, we assume that the MTJ does not experience fading in our subsequent analyses. The desired signal amplitude is modeled as a frequency-nonselective, slowly Rician fading random variable with PDF given by [1]

$$P_{a_s}(a_s) = \frac{a_s}{\sigma_s^2} e^{-(a_s^2 + \alpha_s^2)/2\sigma_s^2} I_0\left(\frac{a_s \alpha_s}{\sigma_s^2}\right) U(a_s). \quad (4.3)$$

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The parameters α_s^2 and $2\sigma_s^2$ are the power levels of the direct and diffused components of the desired signal, respectively. The total average power of the desired signal is assumed to remain constant from hop to hop. The Rician fading factor defined as $F_s = \alpha_s^2 / 2\sigma_s^2$ determines the level of fading, i.e., essentially no fading ($F_s = 1000$), Rician fading ($F_s = 10$), and Rayleigh fading ($F_s = 0$). In this chapter, we mainly focus on the Rician fading ($F_s = 10$) and Rayleigh fading ($F_s = 0$) cases.

The SNR of the system is defined as $\text{SNR} = E_b / N_0$ where E_b is the bit energy of the original binary data at the transmitter before it is converted into M -ary data. The SJR of the system is defined as $\text{SJR} = E_b / N_J$, where $N_J = P_{J_T} / W_{SS}$ is the equivalent PSD of the MTJ, and P_{J_T} denotes the total power of the MTJ. It is also assumed that the total power of the MTJ is distributed over Q equal-power jamming tones, each with power $P_J = P_{J_T} / Q$.

4.2.2 Proposed Suboptimum ML Receiver Structures

In this section, we will derive two suboptimum receiver structures based on the ML criterion for Rayleigh fading and Rician fading channels, respectively. As presented in Section 3.3 and [1], the ML receiver is to select the largest $P(\mathbf{R} | f_m)$ with $\mathbf{R} = [R_{1,1}, R_{2,1}, \dots, R_{M,1}, R_{1,2}, \dots, R_{M,L}]$ for $m = 1, 2, \dots, M$. Based on the same definition of $\hat{\mathbf{R}}_l = (R_{1,l}, R_{2,l}, \dots, R_{M,l})$ as in Chapter 3, $P(\mathbf{R} | f_m)$ can be written as

$$P(\mathbf{R} | f_m) = \prod_{l=1}^L P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m) \quad (4.4)$$

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where $P_{\hat{\mathbf{R}}_l}(\cdot)$ denotes the PDF of $\hat{\mathbf{R}}_l$. Similarly, since all the M slots within one hop are independent of each other, we have

$$P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m) = \prod_{i=1}^M P_{\mathbf{R}_{i,l}}(\mathbf{R}_{i,l} | f_m) \quad (4.5)$$

where $P_{\mathbf{R}_{i,l}}(\cdot)$ is the PDF of the received signal of the l th hop and the i th slot. We define

$P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)$ and $P_{R_{m,l}}(R_{m,l} | q_{ml}, NS)$ as

$$P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S) = P_{R_{m,l}}(R_{m,l} | f_m) \quad (4.6)$$

and

$$P_{R_{m,l}}(R_{m,l} | q_{ml}, NS) = P_{R_{m,l}}(R_{m,l} | f_i), \quad i=1,2,\dots,M, \text{ and } i \neq m \quad (4.7)$$

where $\phi_{m,l}$ represents the phase difference between the desired signal tone and the jamming tone, S denotes a signal hop and NS denotes a non-signal hop. Following that, equation (4.5) can be rewritten as

$$\begin{aligned} P_{\hat{\mathbf{R}}_l}(\hat{\mathbf{R}}_l | f_m) &= \prod_{i=1}^M P_{\mathbf{R}_{i,l}}(\mathbf{R}_{i,l} | f_m) = P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S) \cdot \prod_{i=1, i \neq m}^M P_{R_{i,l}}(R_{i,l} | q_{il}, NS) \\ &= \frac{P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)}{P_{R_{m,l}}(R_{m,l} | q_{ml}, NS)} \prod_{i=1}^M P_{R_{i,l}}(R_{i,l} | q_{il}, NS). \end{aligned} \quad (4.8)$$

Since the final decision is to select the largest $P(\mathbf{R} | f_m)$, for $m=1,2,\dots,M$, it does not affect the final decision if (4.8) is divided by the factor $\prod_{i=1}^L P_{R_{i,l}}(R_{i,l} | q_{il}, NS)$. Hence the equivalent decision statistics is given by

$$z'_m = \prod_{l=1}^L \frac{P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)}{P_{R_{m,l}}(R_{m,l} | q_{ml}, NS)}. \quad (4.9)$$

Taking the natural logarithm on z'_m also does not affect the final decision, thus we can

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obtain decision statistics of the ML receiver structure as

$$y_{m,l} = \ln \left(\frac{P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)}{P_{R_{m,l}}(R_{m,l} | q_{ml}, NS)} \right) \quad (4.10)$$

and

$$z_m = \sum_{l=1}^L \ln \left(\frac{P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)}{P_{R_{m,l}}(R_{m,l} | q_{ml}, NS)} \right). \quad (4.11)$$

Case I: Rayleigh Fading

When the signal undergoes Rayleigh fading, the conditional PDF of $R_{m,l}$ is given by

$$P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S) = \frac{1}{\beta_S} \exp\left(-\frac{R_{m,l} + 2q_{ml}a_J^2}{\beta_S}\right) \times I_0\left(\frac{2q_{ml}\sqrt{2R_{m,l}a_J^2}}{\beta_S}\right) U(R_{m,l}) \quad (4.12)$$

and

$$P_{R_{m,l}}(R_{m,l} | q_{ml}, NS) = \frac{1}{2\sigma_n^2} \exp\left(-\frac{R_{m,l} + 2q_{ml}a_J^2}{2\sigma_n^2}\right) \times I_0\left(\frac{2q_{ml}\sqrt{2R_{m,l}a_J^2}}{2\sigma_n^2}\right) U(R_{m,l}) \quad (4.13)$$

where $\beta_S = 2(\sigma_n^2 + 2\sigma_S^2)$. Using (4.10), (4.12), and (4.13) we can obtain the suboptimum

ML receiver for Rayleigh fading channels as

$$y_{m,l} = \frac{2\sigma_S^2}{\beta_S \sigma_n^2} (R_{m,l} + 2q_{ml}a_J^2) + \ln I_0(2q_{ml}\sqrt{2a_J^2 R_{m,l}} / \beta_S) - \ln I_0(q_{ml}\sqrt{2a_J^2 R_{m,l}} / \sigma_n^2). \quad (4.14)$$

This receiver structure is called the ML-A receiver in the subsequent analyses.

Case II: Rician Fading

When the signal undergoes Rician fading, the characteristic function of the conditional

PDF $P_{R_{m,l}}(R_{m,l} | q_{ml}, S)$ is given by (please refer to Appendix A for detailed derivation)

$$\Phi_{R_{m,l}}(iv | q_{ml}, S) = \frac{1}{\pi} \int_0^\pi \frac{1}{1 - iv\beta_S} \exp\left(\frac{2iv\{\alpha_S^2 + q_{ml}(a_J^2 + 2\alpha_S a_J \cos \phi_{m,l})\}}{1 - iv\beta_S}\right) d\phi_{m,l}. \quad (4.15)$$

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The characteristic function of the conditional PDF $P_{R_{m,l}}(R_{m,l} | q_{ml}, S)$ can also be given by

$$\Phi_{R_{m,l}}(iv | q_{ml}, S) = \frac{1}{\pi} \int_0^\pi \Phi_{R_{m,l}}(iv | q_{ml}, \phi_{m,l}, S) d\phi_{m,l}. \quad (4.16)$$

By comparing (4.15) and (4.16), we can approximate the characteristic function of $P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)$ as

$$\Phi_{R_{m,l}}(iv | q_{ml}, \phi_{m,l}, S) \approx \frac{1}{1-iv\beta_S} \exp\left(\frac{2iv[\alpha_S^2 + q_{ml}(a_J^2 + 2\alpha_S a_J \cos \phi_{m,l})]}{1-iv\beta_S}\right). \quad (4.17)$$

By performing the inverse transformation of (4.17), the PDF of $P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S)$ can be derived as

$$P_{R_{m,l}}(R_{m,l} | q_{ml}, \phi_{m,l}, S) \approx \frac{1}{\beta_S} \exp\left(-\frac{R_{m,l} + 2[\alpha_S^2 + q_{ml}(a_J^2 + 2\alpha_S a_J \cos \phi_{m,l})]}{\beta_S}\right) \\ \times I_0\left(\frac{2\sqrt{2R_{m,l}[\alpha_S^2 + q_{ml}(a_J^2 + 2\alpha_S a_J \cos \phi_{m,l})]}}{\beta_S}\right) U(R_{m,l}). \quad (4.18)$$

The PDF of $P_{R_{m,l}}(R_{m,l} | q_{ml}, NS)$ is the same as (4.13), hence, by using (4.10), (4.13), and (4.18) we can obtain the suboptimum ML receiver for Rician fading channels as

$$y_{m,l} = \frac{2\sigma_S^2}{\beta_S \sigma_n^2} R_{m,l} + \frac{q_{ml} a_J^2}{\sigma_n^2} - 2 \frac{A_{m,l}^2}{\beta_S} + \ln I_0(2\sqrt{2A_{m,l}^2 R_{m,l}} / \beta_S) - \ln I_0(q_{ml} \sqrt{2a_J^2 R_{m,l}} / \sigma_n^2) \quad (4.19)$$

where

$$A_{m,l} = \sqrt{2(\alpha_S^2 + q_{ml}(a_J^2 + 2\alpha_S a_J \cos \phi_{m,l}))}. \quad (4.20)$$

This receiver structure is called the ML-B receiver in the subsequent analyses.

From (4.14) and (4.20), we observe that the proposed suboptimum ML receivers require information on the noise variance, the signal and the jamming powers. In the followings, we provide some suggestions on how the side information may be obtained. The noise variance can be estimated from a “noise-only” channel for noise power

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measurement [20], and the energy of the signal and jamming can be extracted from energy detector [70]. In the subsequent analyses, we assume that side information can be obtained perfectly which is consistent with our aim of deriving the optimum receiver. Comparing with the conventional diversity-combining receivers, the proposed suboptimum receivers are required to perform Bessel and logarithm functions. Thus, the proposed suboptimum receivers are more complex to implement.

4.3 Probability of Bit Error Analyses

Without loss of generality, we assume that f_1 was transmitted in the subsequent analyses. The PDF of the final decision variables z_m , which are the sum of $y_{m,l}$ over L hops, are given by

$$P_{z_m}(z_m) = P_{y_{m,1}}(y_{m,1}) \otimes P_{y_{m,2}}(y_{m,2}) \otimes \dots \otimes P_{y_{m,L}}(y_{m,L}), \quad m=1,2,\dots,M \quad (4.21)$$

where $P_{y_{m,l}}(\cdot)$ is the PDF of $y_{m,l}$. Following the same notation used in Section 3.2.1, we define

$$P_{y_{m,1}}(y_m) = P_{y_{m,l}}(y_{m,l} | q_{ml} = 1), \quad (4.22)$$

$$P_{y_{m,0}}(y_m) = P_{y_{m,l}}(y_{m,l} | q_{ml} = 0), \quad (4.23)$$

and

$$P_{z_m}^{[j_m]}(z_m) = [P_{y_{m,1}}(y_m)]^{\otimes j_m} \otimes [P_{y_{m,0}}(y_m)]^{\otimes (L-j_m)} \quad (4.24)$$

where $j_m = \sum_{l=1}^L q_{ml}$ denotes the number of jammed hops in the m th frequency slot out of L

hops per symbol. Then, we use the quantization approach to obtain the analytical BER

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expressions. Let $\hat{y}_{m,l}$ be the G -level uniform quantizer output of $y_{m,l}$, the decision statistics can then be expressed as

$$\hat{z}_m = \sum_{l=1}^L \hat{y}_{m,l}. \quad (4.25)$$

Using the same definition as in (4.22) and (4.23), the PDFs of the discrete random variables $\hat{y}_{m,l}$ are given by

$$P_{\hat{y}_{m,l}}(\hat{y}_m) = \sum_{g=-G/2}^{G/2} v_{m,l}(g) \delta(\hat{y}_m - c_g) \quad (4.26)$$

and

$$P_{\hat{y}_{m,0}}(\hat{y}_m) = \sum_{g=-G/2}^{G/2} v_{m,0}(g) \delta(\hat{y}_m - c_g) \quad (4.27)$$

where $c_g = 2gC/G$ for $g = -G/2, -G/2+1, \dots, G/2$, and C is set at a substantially high level such that the probability of $\hat{y}_m$ exceeding this value is negligible. In the following two subsections, we show the steps involved to derive the values of $v_{m,l}(g)$ and $v_{m,0}(g)$ for Rayleigh fading and Rician fading channels, respectively.

Case I: Rayleigh Fading

Step 1: Set $v_{m,l}(g) = 0$, $v_{m,0}(g) = 0$, for $g = -G/2, -G/2+1, \dots, G/2$ and $m=1, 2, \dots, M$.

Step 2: Divide $[0, R_m]$ into K small intervals as $R_m(k) = (k-1/2) \cdot R_m / K$, $k=1, 2, \dots, K$,

where R_m and K should be set high enough to ensure good accuracy. From (4.14) we can derive

$$y_{m,l}(k) = \frac{2\sigma_s^2}{\beta_s \sigma_n^2} (R_m(k) + 2a_j^2) + \ln I_0(2\sqrt{2a_j^2 R_m(k)} / \beta_s) - \ln I_0(\sqrt{2a_j^2 R_m(k)} / \sigma_n^2) \quad (4.28)$$

and

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$$y_{m,0}(k) = \frac{2\sigma_s^2}{\beta_s \sigma_n^2} R_m(k) \quad (4.29)$$

where $y_{m,1}(k)$ and $y_{m,0}(k)$ represent the corresponding output of function $F(\cdot)$ for $R_m(k)$, with $q_{ml} = 1$ and 0, respectively.

Step 3: For each value of k , with the assumption that f_i was transmitted, $v_{m,1}(g)$ and $v_{m,0}(g)$ can be rewritten as

$$v_{m,1}(g_{m,1}(k)) = v_{m,1}(g_{m,1}(k)) + P_{R_{m,1}}(R_m(k)) \cdot \frac{R_m}{K} \quad (4.30)$$

and

$$v_{m,0}(g_{m,0}(k)) = v_{m,0}(g_{m,0}(k)) + P_{R_{m,0}}(R_m(k)) \cdot \frac{R_m}{K}. \quad (4.31)$$

Note that $g_{m,1}(k)$ and $g_{m,0}(k)$ denote the location of $y_{m,1}(k)$ and $y_{m,0}(k)$ in the G -level quantizer, respectively, which satisfy the following conditions

$$c'_{g_{m,1}(k)-1} + C/G < y_{m,1}(k) \leq c'_{g_{m,1}(k)} + C/G \quad (4.32)$$

and

$$c'_{g_{m,0}(k)-1} + C/G < y_{m,0}(k) \leq c'_{g_{m,0}(k)} + C/G \quad (4.33)$$

where $c'_{-G/2-1} \rightarrow -\infty$, $c'_{G/2} \rightarrow \infty$, $c'_g = c_g$ for $g = -G/2, -G/2+1, \dots, G/2-1$,

$P_{R_{m,0}}(R_m) = P_{R_{m,j}}(R_{m,l} | q_{ml} = 0)$ and $P_{R_{m,1}}(R_m) = P_{R_{m,j}}(R_{m,l} | q_{ml} = 1)$. Note that $P_{R_{1,1}}(R_1)$ and $P_{R_{1,0}}(R_1)$ can be obtained from (4.12) by setting $q_{ml} = 1$ and 0, respectively, and $P_{R_{m,1}}(R_m)$

and $P_{R_{m,0}}(R_m)$ for $m=2,3,\dots,M$, can be obtained from (4.13) by setting $q_{ml} = 1$ and 0, respectively.

Case II: Rician Fading

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The performance of the proposed suboptimum ML receiver for Rician fading channels depends on the phase difference $\phi_{m,l}$. We replace $\phi_{m,l}$ with ϕ_l where ϕ_l denotes the difference between the signal tone and the MTJ tone of the l th hop. We define $\phi = [\phi_1, \phi_2, \dots, \phi_L]$ and let $p_b(\phi)$ be the probability of bit error conditioned on ϕ . Thus the average probability of bit error is

$$p_b = E_{\phi}(p_b(\phi)) \quad (4.34)$$

where $E_{\phi}(\cdot)$ denotes taking the expectation over ϕ . In order to avoid the L -dimensional integration in obtaining the PDF of z_m , we use the following approximation based on the Taylor-series expansion to obtain the unconditional PDF $P_{y_{m,l}}(y_{m,l} | q_{ml} = 1)$ [18]

$$P_{y_{m,l}}(y_{m,l} | q_{ml} = 1) \approx \sum_{b=-1}^1 \frac{1}{3} P_{y_{m,l}} \left(y_{m,l} | q_{ml} = 1, \cos \phi_l = b \cdot \frac{\sqrt{3}}{2} \right). \quad (4.35)$$

Thus the steps involved are given as follows:

Step 1: Set $v_{m,1}(g) = 0$, $v_{m,0}(g) = 0$, for $g = -G/2, -G/2 + 1, \dots, G/2$ and $m = 1, 2, \dots, M$.

Step 2: Divide $[0 \ R_m]$ into K small intervals as $R_m(k) = (k - 1/2) \cdot R_m / K$, $k = 1, 2, \dots, K$,

where R_m and K should be set high enough to ensure good accuracy. From (4.19) we can derive

$$\begin{aligned} y_{m,1}^{[b]}(k) &= \frac{2\sigma_s^2}{\beta_s \sigma_n^2} R_m(k) + \frac{a_j^2}{\sigma_n^2} - \frac{4(\alpha_s^2 + a_j^2 + b\alpha_s a_j \sqrt{3})}{\beta_s} \\ &+ \ln I_0(2\sqrt{4(\alpha_s^2 + a_j^2 + b\alpha_s a_j \sqrt{3})} R_m(k) / \beta_s) - \ln I_0(\sqrt{2a_j^2 R_m(k)} / \sigma_n^2) \end{aligned} \quad (4.36)$$

and

$$y_{m,0}(k) = \frac{2\sigma_s^2}{\beta_s \sigma_n^2} R_m(k) - \frac{4\alpha_s^2}{\beta_s} + \ln I_0(2\sqrt{4\alpha_s^2 R_m(k)} / \beta_s) \quad (4.37)$$

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where the superscript $[b]$ represents the condition $\cos \phi_l = b \cdot \frac{\sqrt{3}}{2}$ as mentioned in (4.35)

for $b=-1, 0$ and 1 .

Step 3: For each value of k , assuming that f_l was transmitted, $v_{m,1}(g)$ and $v_{m,0}(g)$ are given by

$$v_{m,1}(g_{m,1}^{[b]}(k)) = v_{m,1}(g_{m,1}^{[b]}(k)) + \frac{1}{3} P_{R_{m,1}}^{[b]}(R_m(k)) \cdot \frac{R_m}{K}, \quad b = -1, 0, 1 \quad (4.38)$$

and

$$v_{m,0}(g_{m,0}(k)) = v_{m,0}(g_{m,0}(k)) + P_{R_{m,0}}(R_m(k)) \cdot \frac{R_m}{K} \quad (4.39)$$

where $g_{m,1}^{[b]}(k)$ and $g_{m,0}(k)$ denote the location of $y_{m,1}^{[b]}(k)$ and $y_{m,0}(k)$ in the G -level quantizer, respectively, which satisfy the following conditions

$$c'_{g_{m,1}^{[b]}(k)-1} + C/G < y_{m,1}^{[b]}(k) \leq c'_{g_{m,1}^{[b]}(k)} + C/G, \quad b = -1, 0, 1 \quad (4.40)$$

and

$$c'_{g_{m,0}(k)-1} + C/G < y_{m,0}(k) \leq c'_{g_{m,0}(k)} + C/G. \quad (4.41)$$

Note that $c'_{-G/2-1} \rightarrow -\infty$, $c'_{G/2} \rightarrow \infty$, $c'_g = c_g$ for $g = -G/2, -G/2+1, \dots, G/2-1$,

$P_{R_{m,1}}^{[b]}(R_m) = P_{R_{m,1}}(R_{m,1} | q_{ml} = 1, \cos \phi_l = b\sqrt{3})$, and $P_{R_{m,0}}(R_m) = P_{R_{m,1}}(R_{m,1} | q_{ml} = 0)$. Here

$P_{R_{1,1}}^{[b]}(R_1)$ and $P_{R_{1,0}}(R_1)$ can be obtained from (4.18) by setting $\cos \phi_l = b\sqrt{3}$ (with $b=-1, 0$,

and 1) for $q_{ml} = 1$ and 0 , respectively. Note that $P_{R_{m,1}}^{[b]}(R_m)$ and $P_{R_{m,0}}(R_m)$ for $m=2, 3, \dots, M$, can be obtained from (4.13) by setting $q_{ml} = 1$ and 0 , respectively.

After obtaining the PDF of the discrete random variable $\hat{y}_{m,l}$ for both cases as described above, we then use discrete convolution to obtain the discrete PDF of the final

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decision statistics given by

$$P_{\hat{z}_m}^{[j_m]}(\hat{z}_m) = [P_{\hat{z}_m,1}(\hat{z}_m)]^{\otimes j_m} \otimes [P_{\hat{z}_m,0}(\hat{z}_m)]^{\otimes (L-j_m)}. \quad (4.42)$$

Since f_1 is assumed to be transmitted, an error occurs if $\hat{z}_m > \hat{z}_1$, for any of the $m = 2, 3, \dots, M$. Therefore, the conditional probability of symbol error for the sum of all L hops given that j_m hops are jammed is

$$p_{J,j}(j_1, j_2, \dots, j_M) = 1 - \sum_{g_1=-GL/2}^{GL/2} \sum_{g_2=-GL/2}^{g_1} \dots \sum_{g_M=-GL/2}^{g_{M-1}} P_{\hat{z}_1}^{[j_1]}(2g_1 C/G) \\ \times P_{\hat{z}_2}^{[j_2]}(2g_2 C/G) \dots \times P_{\hat{z}_M}^{[j_M]}(2g_M C/G) \quad (4.43)$$

where $j = \sum_{m=1}^M j_m$ denotes the total number of jammed hops for all M frequency slots.

Following that, the conditional probability of symbol error for the sum of all L hops given that j hops have been jammed can be expressed as

$$p_j = \sum_{j_1=0}^j \sum_{j_2=0}^{j-j_1} \sum_{j_3=0}^{j-j_1-j_2} \dots \sum_{j_{M-1}=0}^{j-j_1-j_2-j_3-\dots-j_{M-2}} \binom{j}{j_1} \times \binom{j-j_1}{j_2} \times \binom{j-j_1-j_2}{j_3} \times \dots \\ \times \binom{j-j_1-j_2-\dots-j_{M-2}}{j_{M-1}} \times \left(\frac{1}{M}\right)^j \times p_{J,j}(j_1, j_2, \dots, j_M). \quad (4.44)$$

Lastly, the probability of an M -ary symbol error can be shown as

$$p_s = \sum_{j=0}^L \binom{L}{j} \left(\frac{Q}{N}\right)^j \left(1 - \frac{Q}{N}\right)^{L-j} p_j \quad (4.45)$$

where N is the total number of FH bands available given by (2.1) and Q is the total number of jamming tones. The average probability of bit error is then given by [page 260, 1]

$$p_b = \frac{M/2}{M-1} p_s. \quad (4.46)$$

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In the above analysis, the computational complexity mainly depends on the diversity level L . Although the PDF calculation of the quantizer outputs are independent with the diversity level, the computational complexity of the discrete convolution in (4.42) increases with the diversity level L .

4.4 Numerical Results and Discussions

In this section, numerical BER results of the proposed suboptimum ML receivers with the single-tone per-band MTJ and fading channels are presented. These two suboptimum receivers, named ML-A and ML-B receivers, are derived for Rayleigh fading and Rician fading channels, respectively. The worst-case band MTJ is considered and the number of multiple interfering tones Q is chosen to maximize the BER for each SJR level. The corresponding worst-case BER results are plotted against SJR. The SNR is fixed at $E_b / N_0 = 13.35\text{dB}$, corresponding to a BER of 10^{-5} for a BFSK system under no fading and jammer-free conditions. For the subsequent numerical results, the BER for the ML-A receiver and the ML-B receiver are only presented for the Rayleigh fading channel and the Rician fading channel, respectively. The BER for the linear-combining receiver, the self-normalizing receiver and the product-combining receiver obtained from [12], [27] and [61], respectively, are also presented for performance comparisons. Both analytical and simulation results of the proposed suboptimum ML receivers are also provided to validate the BER expressions derived in Section 4.3.

In Figs. 4.1 and 4.2, the worst-case analytical and simulation BER results of the ML-A and ML-B receivers are presented, respectively, with $M=32$ and various diversity levels L . The desired signal experiences Rayleigh fading in Fig. 4.1 and Rician fading with the

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Rician fading factor $F_s = 10$ in Fig. 4.2. The analytical BER results are obtained based on the BER expressions derived in Section 4.3. Due to the simulation time constraint, the simulation results are only provided when the BER is greater than 10^{-5} in Figs. 4.1 and 4.2. The close match in both analytical and simulation results justifies the accuracy of using the quantization approach to analyze the performance of these two suboptimum ML receivers and validates our analyses given in Section 4.3. Note that the performance analyses of the ML-B receiver are based on the PDF approximation of (4.17) and (4.18) and the simulation results are independent of that approximation. Thus the close match in Fig. 4.2 also verifies the accuracy of the PDF approximation for the Rician fading channels and we can conclude that it is reasonable to derive the suboptimum ML receiver structure over the Rician fading channels by using the PDF approximation in (4.17) and (4.18). From Figs. 4.1 and 4.2, it can be seen that a larger L improves the system performance significantly and there exists an optimum diversity level for both ML-A and ML-B receivers. For the Rayleigh fading case, the optimum diversity level for the ML-A receiver with SJR=25dB is 37. On the other hand, for Rician fading case, the optimum diversity level for the ML-B receiver is 15 with the same SJR. Comparing Figs. 4.1 and 4.2, we observe that the optimum diversity level of the ML-A receiver is much greater than the optimum diversity level of the ML-B receiver. The reason is that the ML-A receiver in Fig. 4.1 undergoes more serious fading than the ML-B receiver in Fig. 4.2 and a higher order of diversity is required to countermeasure the effects of more serious fading.

In Fig. 4.3, the performance comparisons for various receivers, namely, the linear-combining receiver [12], the self-normalizing receiver [27], the product-combining

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receiver [61], and the ML-A receiver against the worst-case band MTJ and AWGN are shown with $M=32$ over Rayleigh fading channels. The diversity level is fixed at $L=5$. Similar performance comparisons for the Rician fading case are shown in Fig. 4.4 with the ML-A receiver replaced by the ML-B receiver. The BER results show that the two proposed suboptimum receivers significantly outperform the other existing receivers for the FFH/MFSK system in the presence of the worst-case band MTJ and AWGN under the respective fading environments. This is because the suboptimum ML receivers can countermeasure the MTJ and fading effects effectively by using the ML criterion. In Figs 4.3 and 4.4, it can also be seen that the BER improvement between the two proposed suboptimum receivers and other existing receivers with small SJR is much greater than that with large SJR. It is to say that the receivers derived by the ML criterion can reduce the MTJ effects more effectively than other existing receivers.

It is well known that for an FFH system, higher diversity levels can counteract the MTJ and fading effects more while introducing more noncoherent diversity-combining loss. Hence there exists an optimum diversity level for the system. The optimum diversity levels for various receivers with both Rayleigh and Rician fading channels are shown in Table 4.1. From Table 4.1, it can be observed that the optimum diversity levels for the two proposed suboptimum receivers are higher than those of other receivers, especially for the ML-A receiver over Rayleigh fading channels. The reason is that the two proposed suboptimum receivers can mitigate the MTJ and fading effects more effectively than other receivers by using the ML criterion. Thus higher diversity levels for the two proposed suboptimum receivers can introduce more benefits than the disadvantages caused by noncoherent diversity-combining loss and higher optimum diversity levels can

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be achieved. From Table 4.1, we can also observe that the optimum diversity levels over Rayleigh fading channels are greater than those over Rician fading channels. The reason is that a higher order of diversity is required to outweigh the degradation caused by more serious fading.

The performance comparisons of various receivers with the optimum diversity level for $M=32$ are presented in Figs. 4.5 and 4.6 for the Rayleigh fading and Rician fading channels, respectively. From Figs. 4.5 and 4.6, we can observe that under the optimum diversity level condition, the suboptimum receivers significantly outperform the other existing receivers. It can also be observed that the effects of SJR on the performance of the suboptimum ML receivers are less as compared to other receivers. In fact, the BER results for the proposed suboptimum receivers are almost independent of SJR for the range of SJR under consideration. This is because the suboptimum ML receivers have the information of the MTJ which helps the receivers to mitigate the effects of the MTJ effectively, even under strong jamming condition. Thus the performance of the suboptimum ML receivers is less sensitive to the SJR than the other receivers. For strong-to-moderate jamming condition, the proposed suboptimum ML receivers are better than all the other existing receivers by at least a few orders of magnitude in BER performance. However, the suboptimum ML receivers require more side information. This side information may be obtained a “noise-only” channel and energy detector as described in Section 4.2. In addition, the proposed suboptimum receivers are required to perform the Bessel logarithm functions. Thus, there is a tradeoff between the performance and the complexity of various receiver structures. The above observations are applicable to other SNR levels and modulation dimensions as well.

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The performance comparisons of the proposed suboptimum receivers with different modulation dimensions M and the corresponding optimum diversity levels are presented in Figs. 4.7 and 4.8 for the Rayleigh fading and Rician fading channels, respectively. It can be observed that the optimum diversity level increases and the BER performance improves as the modulation level M increases. It can be seen from (2.1) that when M increases, the bandwidth of each FH band increases and thus the number of total non-overlapping FH bands N decreases. Consequently when M increases, a higher diversity level is required to offset the reduction of the available frequency-hopped bands and the optimum diversity level increases. A higher optimum diversity level can counteract the MTJ and fading effects more effectively although it introduces more noncoherent diversity-combining loss. From the numerical results, we can conclude that the advantages introduced by the increase of M are much greater than the disadvantages.

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Fig. 4.1 Worst-case analytical and simulation BER results of the ML-A receiver over Rayleigh fading channels with $\text{SNR}=13.35\text{dB}$ and $M=32$

Fig. 4.2 Worst-case analytical and simulation BER results of the ML-B receiver over Rician fading channels ($F_s = 10$) with $\text{SNR}=13.35\text{dB}$ and $M=32$

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Fig. 4.3 BER performance comparisons of various receivers over Rayleigh fading channels with SNR=13.35dB, $M=32$, and $L=5$

Fig. 4.4 BER performance comparisons of various receivers over Rician fading channels ($F_s = 10$) with SNR=13.35dB, $M=32$, and $L=5$

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Fig. 4.5 BER performance comparisons of various receivers over Rayleigh fading channels with SNR=13.35dB, $M=32$, and optimum diversity levels

Fig. 4.6 BER performance comparisons of various receivers over Rician fading channels ($F_s = 10$) with SNR=13.35dB, $M=32$, and optimum diversity levels

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Fig. 4.7 BER performance of the ML-A receiver over Rayleigh fading channels with SNR=13.35dB and different M

Fig. 4.8 BER performance of the ML-B receiver over Rician fading channels ($F_s = 10$) with SNR=13.35dB and different M

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<div>Fading</div> <div>Receiver</div>	Rayleigh Fading	Rician Fading
Linear-combining receiver	9	7
Self-normalizing receiver	17	13
Product-combining receiver	13	11
ML-A receiver	37	N.A.
ML-B receiver	N.A.	15

Table 4.1 Optimum diversity level of various receivers under both Rayleigh and Rician fading channels with $M=32$, $\text{SNR}=13.35\text{dB}$, and $\text{SJR}=25\text{dB}$

4.5 Conclusions

In this chapter, we have proposed two suboptimum ML receiver structures for FFH/MFSK communication systems with the worst-case single-tone per-band MTJ and AWGN over both Rayleigh fading and Rician fading channels, respectively. In addition, the analytical BER of these two proposed receivers were derived and validated by simulation results. Numerical results have shown that there exists an optimum diversity level for the two proposed suboptimum ML receivers and the optimum diversity level is higher for the Rayleigh fading case as compared to the Rician fading case. Furthermore, these two suboptimum ML receivers can significantly improve the system performance as compared to the existing diversity-combining receivers. In particular, it has been observed that the effects of SJR on the performance of the suboptimum ML receivers are less as compared to the other existing receivers. The trade-off of these two suboptimum receivers is that they require side information and are slightly more complicated to be implemented than the other receivers.

Chapter 5

Performance Study of Synchronous FFH/MA Systems over Fading Channels

In this chapter, we present analytical BER expressions for synchronous FFH/MA systems over fading channels. Several diversity-combining methods including linear-combining, product-combining and clipper receivers are studied in this chapter. Theoretical BER results, validated by simulation results for various diversity-combining methods, are presented for comparisons. In addition, the effects of some parameters such as modulation dimension, diversity level and fading factor are also discussed in this chapter.

5.1 Introduction

The effects of MTJ on FFH/MFSK systems over both nonfading and fading channels have been presented in Chapters 3 and 4, respectively. In this chapter, we will focus on the effects of MAI. For commercial applications, multiple access capability has been an important facet, thus various FFH/MA systems using noncoherent MFSK modulation have been proposed in [13], [14], [62]-[65], and the references therein. Upper bounds on the performance of FH/MA MFSK systems with hard-limited and linear-combining receivers in a Rayleigh fading environment with chip synchronism were presented in [13]. In [14], closed-form expressions for the BER performance of an FFH/SSMA system with linear combining over Rayleigh fading channels were presented with the assumption that at most one hit occurs. In [62], a calculation method based upon the Gauss quadrature rules was presented for assessing the performance of an asynchronous FH/MA system. Both nonfading and frequency non-selective Rayleigh fading channels were considered in [62]. Furthermore, the effects of frequency and time errors over nonfading and Rayleigh fading channels have been studied in [63] and [64], respectively. The performance improvement due to equal-gain space diversity for FFH/SSMA systems over Rayleigh fading channels was presented in [65]. In the performance analyses of [14], [64], and [65], the authors assumed that the in-phase and quadrature-phase terms of the detector have independent Gaussian distributions. For the Rician fading cases, since the mean of the in-phase and quadrature-phase terms of the detector are not zero, the performance analyses will become very complicated if the approaches proposed in [64] and [65] are employed.

In this chapter, we will present another method to analyze the system performance over Rician fading channels. We consider a synchronous FFH/MA system employing MFSK

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modulation with noncoherent square-law detection over Rician fading channels. The hopping patterns are assumed to be random. We derive BER expressions for the linear-combining, product-combining, and clipper receivers based on the Fourier-Bessel series [66]. These BER expressions are validated by the corresponding simulation results.

5.2 System Description

In the following analyses, we consider the presence of K simultaneous users operating synchronously, which will be the case for the base-to-mobile transmission. Each of these users is frequency hopped to a set of N frequency-hopped bands. We assume that the hopping patterns are independent and identically distributed (i.i.d.). We further assume that the reference user and the $K-1$ interfering users experience mutually independent Rician fading. Given that there are N frequency-hopped bands available and that all hopping patterns are independent, the probability of the reference user being jammed by another user is given by

$$P_h = \frac{1}{N}. \quad (5.1)$$

We define the vector $\mathbf{K}_{ml} = (k_{1,ml}, k_{2,ml}, \dots, k_{C_{ml},ml})$ to represent the index of the active users for the received signal in the m th branch and the l th hop after dehopping, where C_{ml} represents the corresponding number of active users among all K users in the system. The active user in the m th branch and the l th hop means that this user collides with the signal in this branch and hop. For convenience, the elements $k_{c,ml}$, for $c=1, 2, \dots$, and C_{ml} , are arranged in ascending order. For illustration, assuming that users 1, 4, and 6 are present in the second branch and the third hop after dehopping, then in this case $m=2$, $l=3$, the

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number of active users C_{23} is equal to 3, and thus the corresponding vector $\mathbf{K}_{23}$ is given by (1,4,6).

The transmitter and the general receiver structures of an FFH/MA communication system have been shown in Chapter 2. The received signal consists of the desired signal corrupted by AWGN and MAI. The received signal over each hop duration T_h is down-converted, filtered and dehopped. Then, the dehopped signal is detected by the square-law detector. The square-law detector outputs denoted as $R_{m,l} = r_{m,l}^2$ (for $m = 1, 2, \dots, M$, $l = 1, 2, \dots, L$), where $r_{m,l}$ is the envelope of the m th branch and the l th hop, are processed by different diversity-combining methods denoted by the function $F(\cdot)$. The resultant outputs $y_{m,l}$ are summed together over L diversity receptions before making the final decision.

The dehopped signal of the l th hop $r_l(t)$ can be expressed as

$$r_l(t) = \sum_{m=1}^M \sum_{c=1}^{C_{ml}} \sqrt{2} a_{k_{c,ml}} \cos(2\pi f_m t + \phi_{k_{c,ml}}) + n(t) \quad (5.2)$$

where $a_{k_{c,ml}}$ is the signal amplitude of the user $k_{c,ml}$, f_m is the modulation frequency with $m=1, 2, \dots$, or M , $\phi_{k_{c,ml}}$ is the random phase uniformly distributed over $[-\pi, \pi]$, and $n(t)$ is Gaussian noise with zero mean and variance $\sigma_n^2 = N_0 B$, where N_0 is the one-sided PSD of the noise and $B = 1/T_h$. In the following analyses, we assume that user 1 is the reference user.

The k th user's signal amplitude is modeled as a frequency-nonselective, slowly Rician fading random variable with PDF given by [1]

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$$P_{a_k}(a_k) = \frac{a_k}{\sigma_k^2} e^{-(a_k^2 + \alpha_k^2)/2\sigma_k^2} I_0\left(\frac{a_k \alpha_k}{\sigma_k^2}\right) \cdot U(a_k). \quad (5.3)$$

The parameters α_k^2 and $2\sigma_k^2$ are the power levels of the direct and diffused components of the desired signal, respectively. The total average power of the desired signal is assumed to remain constant from hop to hop. The Rician fading factor defined as $F_k = \alpha_k^2 / 2\sigma_k^2$ determines levels of fading, e.g., essentially no fading ($F_k = 1000$), Rician fading ($F_k = 10$), and Rayleigh fading ($F_k = 0$).

5.3 Probability of Bit Error Analyses

The conditional PDF of the envelope $r_{m,l}$ given that C_{ml} users with index of $\mathbf{K}_{ml} = (k_{1,ml}, k_{2,ml}, \dots, k_{C_{ml},ml})$ transmitting in the m th branch and the l th hop can be expressed using the Fourier-Bessel series as [66]

$$P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml}) = \frac{2r_{m,l}}{R^2} \sum_{n=1}^{\infty} \frac{G_n}{J_1^2(\lambda_n)} J_0(\lambda_n \frac{r_{m,l}}{R}), \quad \text{when } r_{m,l} < R \quad (5.4)$$

where λ_n is the n th zero of $J_0(\cdot)$ and G_n is the Fourier-Bessel coefficient which can be replaced by $\Phi_{r_{m,l}}^{[h]}(\lambda_n / R | \mathbf{K}_{ml})$. Note that in (5.4), the value of R should be sufficiently large so that the probability of $r_{m,l}$ being larger than R is negligible. The Hankel characteristic function $\Phi_{r_{m,l}}^{[h]}(\rho | \mathbf{K}_{ml})$ of $r_{m,l}$ is defined as [67]

$$\Phi_{r_{m,l}}^{[h]}(\rho | \mathbf{K}_{ml}) = \int_0^{\infty} P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml}) J_0(r_{m,l} \rho) dr_{m,l}. \quad (5.5)$$

With the above definition, the Hankel characteristic function for the k th user is given by

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$$\begin{aligned}\Phi_k^{[h]}(\rho) &= \int_0^\infty \frac{a_k}{\sigma_k^2} e^{-(a_k^2 + \alpha_k^2)/2\sigma_k^2} I_0\left(\frac{a_k \alpha_k}{\sigma_k^2}\right) J_0(a_k \rho) da_k \\ &= \exp\left(-\frac{\rho^2 \sigma_k^2}{2}\right) J_0(\alpha_k \rho).\end{aligned}\quad (5.6)$$

Therefore the Hankel characteristic function for C_{ml} independent users for one hop in the same branch is given by

$$\Phi_U^{[h]}(\rho | \mathbf{K}_{ml}) = \prod_{c=1}^{C_{ml}} \exp\left(-\frac{\rho^2 \sigma_{k_{c,ml}}^2}{2}\right) J_0(\alpha_{k_{c,ml}} \rho). \quad (5.7)$$

The Hankel characteristic function for Gaussian noise is given by [66]

$$\Phi_G^{[h]}(\rho) = \exp\left(-\frac{\rho^2 \sigma_n^2}{2}\right). \quad (5.8)$$

Multiplying the Hankel characteristic functions in (5.7) and (5.8), we obtain the Hankel characteristic function of $r_{m,l}$ as

$$\Phi_{r_{m,l}}^{[h]}(\rho | \mathbf{K}_{ml}) = \exp\left(-\frac{\rho^2 \sigma_n^2}{2}\right) \cdot \prod_{c=1}^{C_{ml}} \exp\left(-\frac{\rho^2 \sigma_{k_{c,ml}}^2}{2}\right) J_0(\alpha_{k_{c,ml}} \rho). \quad (5.9)$$

Using (5.4) and (5.9) and replacing G_n with $\Phi_{r_{m,l}}^{[h]}(\lambda_n / R | \mathbf{K}_{ml})$ in (5.4), we can obtain the PDF of $r_{m,l}$ denoted as $P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml})$. Based on the PDF $P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml})$, in the following, we will derive the exponential characteristic function expressions for the variables $y_{m,l}$ of the product-combining, linear-combining, and clipper receivers, respectively.

Case I: Product-combining receiver

For the product-combining receiver, the final decision statistics can be expressed as

$$z_m = \ln\left(\prod_{l=1}^L r_{m,l}^2\right) = \sum_{l=1}^L \ln(r_{m,l}^2) = \sum_{l=1}^L y_{m,l} \quad (5.10)$$

where $y_{m,l} = 2 \ln(r_{m,l})$. In this case, the exponential characteristic function $\Phi_{y_{m,l}}(i\nu | \mathbf{K}_{ml})$

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is given by

$$\Phi_{y_{m,l}}(i\nu | \mathbf{K}_{ml}) = \int_0^\infty P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml}) r_{m,l}^{2i\nu} dr_{m,l}. \quad (5.11)$$

Substituting (5.4) into (5.11) and after some manipulations, the exponential characteristic function $\Phi_{y_{m,l}}(i\nu | \mathbf{K}_{ml})$ is obtained as

$$\Phi_{y_{m,l}}(i\nu | \mathbf{K}_{ml}) = \sum_{n=1}^{\infty} \frac{2\Phi_{r_{m,l}}^{[h]}(\lambda_n / R | \mathbf{K}_{ml}) \lambda_n^{-2-2i\nu} R^{2i\nu}}{J_1^2(\lambda_n)} \left[2^{1+2i\nu} \frac{\Gamma(1+i\nu)}{\Gamma(-i\nu)} - \lambda_n J_{-1}(\lambda_n) S_{1+2i\nu,0}(\lambda_n) \right] \quad (5.12)$$

where $\Gamma(\cdot)$ represents the Gamma function and the $S_{a,b}(\cdot)$ represents the Lommel function [68].

Case II: Linear-combining receiver

For the linear-combining receiver, the final decision statistics are given by

$$z_m = \sum_{l=1}^L R_{m,l} = \sum_{l=1}^L r_{m,l}^2 = \sum_{l=1}^L y_{m,l} \quad (5.13)$$

where $y_{m,l} = R_{m,l} = r_{m,l}^2$. The corresponding exponential characteristic function

$\Phi_{y_{m,l}}(i\nu | \mathbf{K}_{ml})$ is

$$\Phi_{y_{m,l}}(i\nu | \mathbf{K}_{ml}) = \sum_{n=1}^{\infty} \sum_{p=0}^{\infty} \frac{\Phi_{r_{m,l}}^{[h]}(\lambda_n / R | \mathbf{K}_{ml}) \lambda_n^{2p} (-1)^p}{J_1^2(\lambda_n) 2^{2p} (p!)^2 (-i\nu R^2)^{p+1}} \gamma(p+1, -i\nu R^2) \quad (5.14)$$

where $\gamma(a, x)$ represents the incomplete gamma function [68].

Case III: Clipper receiver

For the clipper receiver, the final decision statistics are given by

$$z_m = \sum_{l=1}^L y_{m,l}, \text{ where } y_{m,l} = \begin{cases} r_{m,l}^2, & r_{m,l} \leq T \\ T^2, & r_{m,l} > T \end{cases} \quad (5.15)$$

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where T is the clipper threshold. Following that, the exponential characteristic function

$\Phi_{y_{m,l}}(iv | \mathbf{K}_{ml})$ can be obtained as

$$\Phi_{y_{m,l}}(iv | \mathbf{K}_{ml}) = \int_0^T P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml}) e^{ivr_{m,l}^2} dr_{m,l} + e^{ivT^2} \int_T^\infty P_{r_{m,l}}(r_{m,l} | \mathbf{K}_{ml}) dr_{m,l}. \quad (5.16)$$

Substituting (5.4) into (5.16) and assuming R is larger than the clipper threshold T , we can obtain

$$\begin{aligned} \Phi_{y_{m,l}}(iv | \mathbf{K}_{ml}) &= \sum_{n=1}^{\infty} \sum_{p=0}^{\infty} \frac{\Phi_{r_{m,l}}^{[h]}(\lambda_n / R | \mathbf{K}_{ml}) \lambda_n^{2p} (-1)^p}{J_1^2(\lambda_n) 2^{2p} (p!)^2 (-ivR^2)^{p+1}} \gamma(p+1, -ivT^2) \\ &+ \sum_{n=1}^{\infty} \frac{2\Phi_{r_{m,l}}^{[h]}(\lambda_n / R | \mathbf{K}_{ml}) e^{ivT^2}}{J_1^2(\lambda_n)} \left[\frac{T}{R} \lambda_n^{-1} J_{-1}\left(\frac{T}{R} \lambda_n\right) S_{1,0}\left(\frac{T}{R} \lambda_n\right) - \lambda_n^{-1} J_{-1}(\lambda_n) S_{1,0}(\lambda_n) \right]. \end{aligned} \quad (5.17)$$

From the above, we have obtained the exponential characteristic functions $\Phi_{y_{m,l}}(iv | \mathbf{K}_{ml})$ for the product-combining, linear-combining, and clipper receivers as shown in (5.12), (5.14), and (5.17), respectively. Following that, we can obtain the exponential characteristic functions of the final decision statistics z_m which are given by

$$\Phi_{z_m}(iv | \mathbf{K}) = \prod_{l=1}^L \Phi_{y_{m,l}}(iv | \mathbf{K}_{ml}), \quad (5.18)$$

where the interference pattern matrix $\mathbf{K}$ is defined as

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_{11} & \cdots & \mathbf{K}_{1l} & \cdots & \mathbf{K}_{1L} \\ \vdots & & & & \vdots \\ \mathbf{K}_{m1} & & \ddots & & \mathbf{K}_{mL} \\ \vdots & & & & \vdots \\ \mathbf{K}_{M1} & \cdots & \mathbf{K}_{Ml} & \cdots & \mathbf{K}_{ML} \end{pmatrix}. \quad (5.19)$$

By performing the inverse transformation of the exponential characteristic functions, we can obtain the PDF expressions of the final decision variables as

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$$P_{z_m}(z_m | \mathbf{K}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{z_m}(i\nu | \mathbf{K}) e^{-iz_m \nu} d\nu. \quad (5.20)$$

In the subsequent analyses we assume that f_1 was transmitted by the reference user 1. Thus the vector $\mathbf{K}_{1l}$ can be restricted by $\mathbf{K}_{1l} = (1, k_{2,1l}, \dots, k_{C_{1l},1l})$ over all L hops. Then the conditional probability of symbol error for the sum of all L hops given interference pattern $\mathbf{K}$ can be expressed as

$$p_s(\mathbf{K}) = 1 - \int_0^{\infty} P_{z_1}(z_1 | \mathbf{K}) \int_0^{z_1} P_{z_2}(z_2 | \mathbf{K}) \dots \int_0^{z_{L-1}} P_{z_L}(z_L | \mathbf{K}) dz_1 dz_2 \dots dz_{L-1}. \quad (5.21)$$

By averaging over all the possible combinations of the interference patterns, the average symbol error can be obtained as

$$p_s = \sum_{\text{all possible } \mathbf{K}} p_s(\mathbf{K}) \cdot \prod_{l=1}^L \left\{ \left(\frac{1}{MN} \right)^{C_{1l}-1} \cdot \prod_{m=2}^M \left(\frac{1}{MN} \right)^{C_{ml}} \right\} \cdot \left(\frac{N-1}{N} \right)^{LK - \sum_{l=1}^L \sum_{m=1}^M C_{ml}}, \quad (5.22)$$

where $\mathbf{K}$ is given by (5.19). Lastly, the average probability of bit error is given by [1]

$$p_b = \frac{M/2}{M-1} p_s. \quad (5.23)$$

5.4 Numerical Results and Discussions

In this section, the BER results for the linear-combining, product-combining, and clipper receivers are obtained based on the analytical expressions derived in Section 5.3. For the clipper receiver, we assumed that the optimum threshold level was used. From the analyses in Section 5.3, we observe that the theoretical BER expressions for all three receivers are applicable for nonfading, Rician fading and Rayleigh fading channels with the Rician fading factor F_k equals to 1000, 10 and 0, respectively. In this chapter, we

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focus on Rayleigh fading and Rician fading channels which are more commonly encountered in practical systems. In the subsequent discussion, we assume that all users have equal power levels and experience i.i.d. fading. Thus we denote F as the Rician fading factor of all users. In order to have a fair comparison, the total SS bandwidth W_{ss} is assumed to be fixed and the number of the available frequency-hopped bands N for the BFSK modulation case is set to 1000.

In Fig. 5.1, the analytical and simulation BER results of the linear-combining, product-combining, and clipper receivers over Rician fading channels are presented versus SNR, with $M=4$, $L=3$, and $K=100$. The close match in both analytical and simulation BER results validates our analyses presented in Section 5.3. From Fig. 5.1, we can also observe that under the assumption that the optimum threshold level is used, the clipper receiver performs better than the product-combining receiver which in turn outperforms the linear-combining receiver. Thus, the non-linear diversity-combining schemes can significantly improve the system performance as compared to that of the linear-combining receiver. For instance, when SNR=15dB, the BER results of both the product-combining and clipper receivers are more than 10 times better than that of the linear-combining receiver. The improvement in BER performance is even more significant as SNR increases. Note that although the clipper receiver performs better than the product-combining receiver, the appropriate setting of the clipping threshold requires side information on the desired signal level which is assumed to be known *a priori*. In practice, the desired signal level can be estimated by using some energy detectors which will make the receiver structure more complex. On the other hand, the product-combining receiver is completely nonparametric. Thus, the product-combining receiver is a preferred

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diversity-combining method to countermeasure the MAI and fading effects effectively and practically if no side information is available. If the side information about the desired signal level is available, then the clipper receiver will be preferred.

The theoretical BER results of the linear-combining, product-combining, and clipper receivers over Rician fading channels with $M=4$ and $K=100$ are shown in Figs. 5.2, 5.3, and 5.4, respectively, with different diversity levels L . From Fig. 5.2, we observe that the linear-combining receiver performs the best when there is no diversity, i.e., $L=1$. This is because the diversity-combining improvement against the MAI and the fading effects can not outweigh the noncoherent diversity-combining loss of the system. For the product-combining receiver, we observe from Fig. 5.3 that there is an optimum diversity level for the system. For instance, when $\text{SNR} > 15\text{dB}$, $L=4$ gives the best performance, which is significantly better than the no diversity case. This is because the product-combining receiver with a higher order of diversity can counteract the MAI and fading effects more effectively and offset the noncoherent diversity-combining loss of the system. Similar observations can be applied to the clipper receiver as shown in Fig. 5.4 except that the corresponding optimum diversity level is higher. It can also be observed from Table 5.1 that for the case of Rayleigh fading channels, the corresponding optimum diversity levels are higher than that of the Rician fading channels.

The BER results with $L=3$, $K=100$, and different modulation levels M are presented for the product-combining receiver over both Rayleigh and Rician fading channels in Figs. 5.5 and 5.6, respectively. We observe from Figs. 5.5 and 5.6 that a higher modulation level M performs better over both Rayleigh and Rician fading channels, and the incremental improvement in the BER performance becomes smaller as M gets larger.

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This is consistent with the observation in [1] where it has been shown that a larger modulation level M can give rise to better BER performance over AWGN channels. The above observations for the product-combining receiver in Figs. 5.5 and 5.6 are also applicable to the linear-combining receiver and the clipper receiver as shown in Figs. 5.7 and 5.8, respectively.

In Fig. 5.9, some typical BER results for the three receivers over both Rayleigh fading and Rician fading channels are presented versus the number of users K with SNR=15dB, $M=4$, and $L=3$. From Fig. 5.9, we can observe that in general, the BER increases as K increases due to the presence of more MAI in the system. In addition, we can also observe from Fig. 5.9 that for the Rician fading channel, the performance of the product-combining receiver is only slightly worse than that of the clipper receiver but is significantly better than that of the linear-combining receiver. A similar observation is also applicable for the Rayleigh fading channel. Thus, these BER results demonstrate the effectiveness of employing non-linear diversity-combining techniques in combating MAI.

It is well known that for a noncoherent FFH/SSMA system which employs various diversity-combining schemes, higher diversity level can counteract the MAI and fading effects more effectively while more noncoherent diversity-combining loss is introduced. Hence there exists an optimum diversity level for the system. Some typical optimum diversity levels for various receivers with both Rayleigh and Rician fading channels are shown in Table 5.1 for $M=4$ and SNR=10dB. From Table 5.1, it can be observed that the optimum diversity level for Rayleigh fading channels is in general greater than that for Rician fading channels. This is because a higher diversity order is needed to counteract more serious fading conditions. We can also observe from Table 5.1 that the optimum

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diversity level of the clipper receiver is higher than that of the linear-combining and clipper receivers while the number of user is 10. When the number of user is large, i.e. $K=100$, the optimum diversity levels of the clipper and product-combining receivers are similar whereas the optimum diversity level of the linear-combining receiver is lighter lower.

Fig. 5.1 Analytical and simulation BER results of the linear-combining, product-combining, and clipper receivers over Rician fading channels ($F = 10$) with $L=3$, $M=4$, and $K=100$

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Fig. 5.2 BER performance of the linear-combining receiver over Rician fading channels

($F = 10$) with $M=4$ and $K=100$

Fig. 5.3 BER performance of the product-combining receiver over Rician fading channels

($F = 10$) with $M=4$ and $K=100$

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Fig. 5.4 BER performance of the clipper receiver over Rician fading channels ($F = 10$)
with $M=4$ and $K=100$

Fig. 5.5 BER performance of the product-combining receiver over Rayleigh fading
($F = 0$) channels with $L=3$, $K=100$, and various M

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Fig. 5.6 BER performance of the product-combining receiver over Rician fading

($F = 10$) channels with $L=3$, $K=100$, and various M

Fig. 5.7 BER performance of the linear-combining receiver over Rician fading ($F = 10$)

channels with $L=3$, $K=100$, and various M

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Fig. 5.8 BER performance of the clipper receiver over Rician fading ($F = 10$) channels with $L=3$, $K=100$, and various M

Fig. 5.9 BER performance comparisons of various receivers over Rayleigh fading ($F = 0$) and Rician fading channels ($F = 10$) with $\text{SNR}=15\text{dB}$, $M=4$, and $L=3$

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Fading \ Receiver	Rayleigh Fading ($F=0$)			Rician Fading ($F=10$)		
	$K=10$	$K=50$	$K=100$	$K=10$	$K=50$	$K=100$
Linear-combining receiver	4	2	2	2	1	1
Product-combining receiver	5	3	2	2	2	2
Clipper receiver	6	3	2	3	2	2

Table 5.1 Optimum diversity levels of various receivers under both Rayleigh and Rician fading channels with $M=4$ and $\text{SNR}=10\text{dB}$

5.5 Conclusions

In this chapter, we have derived new analytical BER expressions for the linear-combining, product-combining, and clipper receivers for synchronous FFH/SSMA systems over both Rician and Rayleigh fading channels. These analytical BER expressions are validated via simulation results. From the numerical results, we conclude that the product-combining receiver and the clipper receiver outperform the linear-combining receiver significantly over both Rayleigh fading and Rician fading channels. The clipper receiver with optimum threshold level slightly outperforms the product-combining receiver. However, the product-combining receiver is easier to implement as compared to the clipper receiver since it does not require any side information on the signal level. In addition, a higher modulation dimension can improve the system performance with a fixed total SS bandwidth although the incremental improvement in the BER performance will diminish as M increases. Lastly, the optimum diversity levels for diversity-combining receivers over Rayleigh fading channels are higher than over Rician fading channels.

Chapter 6

Bit-Error Rate Analyses of

Asynchronous FFH/MA Systems

over Fading Channels

In this chapter, we will study the performance of asynchronous FFH/MA systems over fading channels. The ML receiver structure and the corresponding analytical BER expressions for asynchronous FFH/MA system over Rayleigh fading channels are first derived. Numerical results are also presented to illustrate the superiority of the proposed ML receiver in terms of BER. Then, analytical BER expressions for asynchronous FFH/MA systems over both Rician fading and Rayleigh fading channels are presented. The power levels of all interfering users are assumed to be the same. The linear-combining and product-combining receivers are employed in the system. Theoretical BER expressions based on the Gauss quadrature rules are presented. An upper bound on the BER which can be computed easily is also presented. In addition, a method is introduced to reduce the computational complexity of the multi-dimension Gauss

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quadrature rules. The effects of some parameters such as diversity level, fading factor, and number of users are also discussed.

Lastly, we consider the system where the interfering users have unequal power levels. We present analytical BER expressions of the system with MAI modeled as alpha-stable noise. Based on this modeling, we derive analytical BER expressions for the linear-combining, product-combining and clipper receivers over both Rician fading and Rayleigh fading channels by making use of the Fourier-Bessel series. The accuracy of the analyses is confirmed by simulation. The analytical results in environments varying from urban settings to office buildings are also presented in this chapter.

6.1 Maximum-Likelihood Receiver with Side Information over Rayleigh Fading Channels

6.1.1 Introduction

Performance studies on the conventional receivers and the ML receiver for FH/FSK communication systems with MAI and AWGN can be found in [13]-[14], [32]-[33], [50] and the references therein. Upper bounds on the error probability have been presented in [50] for an SFH/MA system. For an FFH system, upper bounds on the performance of hard-limited and linear-combining receivers over a Rayleigh fading channel were presented in [13]. Closed-form expressions for the BER performance of an FFH/MA system with a linear-combining receiver over Rayleigh fading channels were presented in [14] with the assumption that at most one hit occurs. The ML receiver structures for both synchronous and asynchronous systems were presented and studied in [32] and [33],

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respectively. The signals are assumed to experience Rayleigh fading and Rician fading in [32] and [33], respectively. In [33], the PDF of the square-law detector output is obtained by using the Hankel characteristic function and the ML receiver structure is derived based on that PDF expression. Since a two-dimensional numerical integration is required for obtaining the PDF, the optimum ML receiver structure proposed in [33] is not in closed-form and is thus difficult to implement in practice. Furthermore, no numerical results were presented for the exact ML receiver in [33]. Instead, a suboptimum ML receiver called the soft-limiter receiver was proposed and analyzed in [33].

In this section, we will derive the ML receiver structure for asynchronous FFH/MA systems over Rayleigh fading channels based on the ML criterion. The proposed ML receiver requires information on the noise variance and the delay and frequency of the interfering users. It can outperform the hard-limited receiver [13], the linear-combining receiver [13] and the soft-limiter receiver [33]. Comparing with the ML receiver derived in [33], our proposed ML receiver is in closed-form and is easy to implement. BER performance of the proposed ML receiver is also analyzed. Furthermore, numerical results are presented to demonstrate the superiority of the proposed ML receiver.

6.1.2 System Description

In an FFH/MA communication system, K simultaneous users are assumed to operate asynchronously and the total available SS bandwidth is divided into a set of N frequency-hopped bands. It is assumed that all the users employ the independent Markov hopping pattern and experience mutually i.i.d. Rayleigh fading with variance σ^2 . Under this assumption, an interfering user will just collide with the signal partially in a hop. The

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timing misalignment of the k th user is τ_k which is uniformly distributed over $[-T_h, T_h]$.

We assume that user 1 is the reference user and the other $K-1$ users are the interfering users. Assuming that there are C_l interfering users in the l th hop, the l th de-hopped signal over one hop duration T_h can be expressed as

$$r_l(t) = \sqrt{2}a_{s,l} \cos(2\pi f_s t + \phi_{s,l}) + \sum_{c=1}^{C_l} I_c(t) + n(t) \quad (6.1)$$

where $a_{s,l}$ is the Rayleigh faded signal amplitude of the reference user, f_s is the modulation frequency of the reference user, $\phi_{s,l}$ is the random phase which is uniformly distributed over $[-\pi, \pi]$, $I_c(t)$ denotes the MAI and $n(t)$ is the Gaussian noise process with one-sided PSD level equals to N_0 .

6.1.3 The Proposed ML Receiver

Since the amplitudes and phase angles of all users' signals are assumed to be Rayleigh distributed and uniformly distributed, respectively, the de-hopped signal of the k th user can be decomposed into the in-phase part $\xi_{k,l}$ and the quadrature-phase part $\eta_{k,l}$, where $\xi_{k,l}$ and $\eta_{k,l}$ are independent Gaussian random variables with mean zero and variance σ^2 . Consequently, the in-phase and quadrature-phase outputs of the m th matched filter are given by

$$\begin{aligned} r_{m,l,c} = & \frac{\xi_{1,l} T_h}{2} \delta(f_s - f_m) + \sum_{c=1}^{C_l} \frac{\xi_{k_c^{(l)},l} |\tau_{k_c^{(l)}}|}{2} \text{sinc}(\pi(f_{k_c^{(l)}} - f_m)\tau_{k_c^{(l)}}) \cos(\pi(f_{k_c^{(l)}} - f_m)\tau_{k_c^{(l)}}) \\ & + \sum_{c=1}^{C_l} \frac{\eta_{k_c^{(l)},l} |\tau_{k_c^{(l)}}|}{2} \text{sinc}(\pi(f_{k_c^{(l)}} - f_m)\tau_{k_c^{(l)}}) \sin(\pi(f_{k_c^{(l)}} - f_m)\tau_{k_c^{(l)}}) + n_{m,l,c} \end{aligned} \quad (6.2)$$

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and

$$r_{m,l,s} = \frac{\eta_{1,l} T_h}{2} \delta(f_s - f_m) + \sum_{c=1}^{C_l} \frac{\eta_{k_c^{(l)},l} |\tau_{k_c^{(l)}}|}{2} \text{sinc}(\pi(f_{k_c^{(l)}} - f_m) \tau_{k_c^{(l)}}) \cos(\pi(f_{k_c^{(l)}} - f_m) \tau_{k_c^{(l)}}) \\ - \sum_{c=1}^{C_l} \frac{\xi_{k_c^{(l)},l} |\tau_{k_c^{(l)}}|}{2} \text{sinc}(\pi(f_{k_c^{(l)}} - f_m) \tau_{k_c^{(l)}}) \sin(\pi(f_{k_c^{(l)}} - f_m) \tau_{k_c^{(l)}}) + n_{m,l,s} \quad (6.3)$$

respectively, where $\delta(\cdot)$ denotes the delta function, $n_{m,l,c}$ and $n_{m,l,s}$ are independent Gaussian noise components with zero mean and variance $N_0 T_h / 4$ and the square-law detector output $R_{m,l}$ is given by $R_{m,l} = r_{m,l,c}^2 + r_{m,l,s}^2$. In (6.2) and (6.3), the subscript $k_c^{(l)}$ represents the c th interfering user's number in the l th hop. Assuming that the information about the frequency $f_{k_c^{(l)}}$ and the delay $\tau_{k_c^{(l)}}$ of all interfering users is known at the receiver, the in-phase output $r_{m,l,c}$ and the quadrature-phase output $r_{m,l,s}$ are independent of each other [13]. Since $\xi_{k,l}$ and $\eta_{k,l}$ ($k=1, k_1^{(l)}, k_2^{(l)}, \dots, k_{C_l}^{(l)}$) are i.i.d. zero mean Gaussian random variables with variance σ^2 , $r_{m,l,c}$ and $r_{m,l,s}$ are independent Gaussian variables with zero mean and variance $\sigma_{m,l}^2$ given by

$$\sigma_{m,l}^2 = \frac{T_h^2}{4} \delta(f_s - f_m) \sigma^2 + \sum_{c=1}^{C_l} \frac{\tau_{k_c^{(l)}}^2}{4} \text{sinc}^2(\pi(f_{k_c^{(l)}} - f_m) \tau_{k_c^{(l)}}) \sigma^2 + \frac{1}{4} N_0 T_h. \quad (6.4)$$

Consequently the PDF of the square-law detector output $R_{m,l}$ is given by

$$P_{R_{m,l}}(R_{m,l}) = \frac{1}{2\sigma_{m,l}^2} e^{-R_{m,l}/2\sigma_{m,l}^2} U(R_{m,l}). \quad (6.5)$$

From [32] and [33], the decision variable $y_{m,l}$ for the ML receiver is given by

$$y_{m,l} = \ln \left(\frac{P_{R_{m,l}}(R_{m,l} | f_m = f_s)}{P_{R_{m,l}}(R_{m,l} | f_m \neq f_s)} \right) \quad (6.6)$$

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where $P_{R_{m,l}}(R_{m,l} | f_m = f_s)$ and $P_{R_{m,l}}(R_{m,l} | f_m \neq f_s)$ denote the PDF of $R_{m,l}$ in (6.5) for the signal branch and non-signal branch, respectively. Substituting (6.4) and (6.5) into (6.6), we obtain

$$y_{m,l} = A_{m,l} + B_{m,l}R_{m,l} \quad (6.7)$$

where

$$A_{m,l} = \ln \left(\frac{\sigma_{m,l,NS}^2}{\sigma_{m,l,S}^2} \right), \quad B_{m,l} = \frac{1}{2\sigma_{m,l,NS}^2} - \frac{1}{2\sigma_{m,l,S}^2},$$

$$\sigma_{m,l,NS}^2 = \sum_{c=1}^{C_l} \frac{\tau_{k_c^{(l)}}^2}{4} \text{sinc}^2(\pi(f_{k_c^{(l)}} - f_m)\tau_{k_c^{(l)}})\sigma^2 + \frac{1}{4}N_0T_h,$$

and

$$\sigma_{m,l,S}^2 = \frac{T_h^2}{4}\sigma^2 + \sum_{c=1}^{C_l} \frac{\tau_{k_c^{(l)}}^2}{4} \text{sinc}^2(\pi(f_{k_c^{(l)}} - f_m)\tau_{k_c^{(l)}})\sigma^2 + \frac{1}{4}N_0T_h.$$

The variables $y_{m,l}$ are then summed together over L diversity receptions to produce the final decision variable z_m of the proposed ML receiver. From (6.7), we observe that the proposed ML receiver requires the information on the noise PSD N_0 , the Rayleigh fading variance σ^2 , the frequency $f_{k_c^{(l)}}$ and the delay $\tau_{k_c^{(l)}}$ of all the interfering users. Once the coefficients $A_{m,l}$ and $B_{m,l}$ have been obtained, the proposed ML receiver can be implemented easily as shown in (6.7). In comparison with the optimum ML receiver as presented in [31, equation (25)], the proposed ML receiver in (6.7) is much easier to implement. The disadvantage of the proposed ML receiver presented in (6.7) is that it requires certain side information. In the followings, we provide some suggestions on how the side information may be obtained with the expense of extra complexity. The noise

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PSD can be estimated from a “noise-only” channel for noise power measurement [69], and the Rayleigh fading variance of the signal, the delay and frequency of the interfering users can be extracted from energy detectors [5], [70]. In subsequent analyses, we assume that the side information can be obtained perfectly which is consistent with our aim of deriving the optimum receiver.

6.1.4 Performance Analysis of the Proposed ML Receiver

To evaluate the performance of the proposed ML receiver, we first denote $\mathbf{f}_l^{(l)} = [f_{k_1^{(l)}}, f_{k_2^{(l)}}, \dots, f_{k_{C_l}^{(l)}}]$ and $\boldsymbol{\tau}_l^{(l)} = [\tau_{k_1^{(l)}}, \tau_{k_2^{(l)}}, \dots, \tau_{k_{C_l}^{(l)}}]$ to represent the information of frequencies and time delays of the interfering users over the l th hop duration. Based on (6.5) and assuming that $\mathbf{f}_l^{(l)}$, $\boldsymbol{\tau}_l^{(l)}$ and f_s are known, the characteristic function of $R_{m,l}$ can be obtained as

$$\Phi_{R_{m,l}}(iv | \mathbf{f}_l^{(l)}, \boldsymbol{\tau}_l^{(l)}, f_s) = \frac{1}{1 - 2iv\sigma_{m,l}^2}. \quad (6.8)$$

Thus the characteristic function of $y_{m,l}$ in (6.7) is given by

$$\Phi_{y_{m,l}}(iv | \mathbf{f}_l^{(l)}, \boldsymbol{\tau}_l^{(l)}, f_s) = \left(\frac{\sigma_{m,l,NS}^2}{\sigma_{m,l,S}^2} \right)^{iv} \Phi_{R_{m,l}} \left(\left(\frac{1}{2\sigma_{m,l,NS}^2} - \frac{1}{2\sigma_{m,l,S}^2} \right) iv \right). \quad (6.9)$$

The average characteristic function $\Phi_{y_{m,l}}(iv | f_s)$ can be computed numerically by taking the expectation of $\Phi_{y_{m,l}}(iv | \mathbf{f}_l^{(l)}, \boldsymbol{\tau}_l^{(l)}, f_s)$ over $\mathbf{f}_l^{(l)}$ and $\boldsymbol{\tau}_l^{(l)}$. Since all the L hops are independent of each other, the subscript l can be ignored and the characteristic function of the final decision statistics z_m can be obtained as

$$\Phi_{z_m}(iv | f_s) = [\Phi_{y_m}(iv | f_s)]^L. \quad (6.10)$$

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By performing the inverse transformation of the characteristic function, we can obtain the PDF of the final decision variable z_m , denoted as $P_{z_m}(z_m | f_S)$. The conditional symbol error probability is given by

$$p_s(f_S) = 1 - \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} P_{z_1}(z_1 | f_S) dz_1 \times \cdots \int_{-\infty}^{\infty} P_{z_{S-1}}(z_{S-1} | f_S) dz_{S-1} \times \int_{-\infty}^{\infty} P_{z_{S+1}}(z_{S+1} | f_S) dz_{S+1} \cdots \right. \\ \left. \times \int_{-\infty}^{\infty} P_{z_M}(z_M | f_S) dz_M \right] P_{z_S}(z_S | f_S) dz_S. \quad (6.11)$$

We can use quantization and summation to replace the multi-dimensional integration to evaluate (6.11). Lastly, the average BER is given by

$$p_b = \frac{1}{2(M-1)} \sum_{s=1}^M p_s(f_S). \quad (6.12)$$

6.1.5 Numerical Results and Discussions

In this section, numerical BER results for the proposed ML receiver are obtained based on the analytical expressions derived in Section 6.1.4. In order to have a fair comparison, the total SS bandwidth W_{SS} for the MFSK system is assumed to be fixed and the corresponding number of the available frequency-hopped bands N for BFSK modulation case without diversity is fixed at $N=3000$.

In Figs. 6.1 and 6.2, the analytical and simulated BER results of the proposed ML receiver are presented versus the number of users and SNR, respectively. The close match in both analytical and simulation results validates our analyses presented in Section 6.1.4. From Figs. 6.1 and 6.2, we also observe that for the proposed ML receiver, a higher order of diversity can improve the system performance significantly. This is because a higher order of diversity can counteract the fading effects and the MAI more effectively with

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side information on the interference. It can also be observed from Fig. 6.2 that diversity-combining of the proposed ML receiver can mitigate the MAI and fading effects significantly when SNR is large.

The analytical BER results of the proposed ML receiver are presented in Fig. 6.3 with $L=5$, $\text{SNR}=15\text{dB}$, and different modulation dimensions. From Fig. 6.3, we can observe that a higher modulation level M performs better. This is consistent with the observation in [1] where it has been shown that a larger modulation level M can give rise to better BER performance over AWGN channels only.

The analytical BER results of the hard-limited receiver [13], the linear-combining receiver [13], the soft-limiter (clipper) receiver [33], and the proposed ML receiver are presented in Fig. 6.4 with $M=4$, $\text{SNR}=15\text{dB}$, and $L=5$. From Fig. 6.4, it can be observed that the ML receiver significantly outperforms the other three conventional receivers. It can also be observed that the improvement between the proposed ML receiver and the conventional receivers increases as the number of users increases. This is because the ML receiver has perfect MAI information and can counteract the MAI more efficiently.

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Fig. 6.1 Analytical and simulated BER results of the proposed ML receiver with $M=4$, $SNR=15dB$, and different diversity levels

Fig. 6.2 Analytical and simulated BER results of the proposed ML receiver with $M=4$, $K=100$, and different diversity levels

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Fig. 6.3 Analytical BER results of the proposed ML receiver with $L=5$, $\text{SNR}=15\text{dB}$, and different modulation dimensions

Fig. 6.4 BER performance of the hard-limited, linear-combining, soft-limiter, and the proposed ML receivers with $M=4$, $\text{SNR}=15\text{dB}$, and $L=5$

6.2 Error Probability Analyses of Various FFH/MA Systems over Rician Fading Channels

6.2.1 Introduction

Various FH/MA systems have been proposed in many previous studies. Upper bounds on the error probability of SFH/MA systems have been presented in [50]. In [51], a Gaussian approximation technique was proposed to evaluate the symbol error probability of noncoherent hybrid SFH/DS systems. Furthermore, a detailed analysis has been presented in [52] for an FH/CDMA system with unequal power levels among various users over a nonfading channel. In [53], BER expressions for an asynchronous FH/MA system were presented. The proposed BER expressions are exact for only the one interfering user case. When there is more than one interfering user, the corresponding upper bounds were presented.

For an FFH system, diversity-combining techniques can be employed to mitigate the MAI and the fading effects. Upper bounds on the performance of FH/MA systems with hard-limited and linear-combining receivers over a Rayleigh fading environment with chip synchronism were presented in [13]. In [14], closed-form expressions for the BER performance of an FFH/MA system with a linear-combining receiver over Rayleigh fading channels were presented with the assumption that at most one hit occurs. In [54], Gaussian approximations for the MAI were developed and used to analyze the performance of the proposed system for synchronous and asynchronous hopping under nonfading and Rayleigh fading channels. The performance improvement due to equal-gain space diversity for an FFH/MA system over Rayleigh fading channels was presented

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in [65]. In [71], the authors presented the symbol error probabilities of synchronous and asynchronous FH/MA systems through semi-analytic Monte-Carlo simulations.

From the above literature review, we observed that in the performance analyses of [52] and [53], the authors extended the results of [66] to obtain the system performance assuming that the received signals from different branches are independent of each other. However this assumption can not be extended to the asynchronous case. Furthermore in the performance analyses of [13], [14], and [65], only Rayleigh fading channels were considered. In a wireless communication system, asynchronous system is practical for mobile to station cases. Rician fading is a commonly encountered channel fading model and Rayleigh fading can be regarded as a special case of Rician fading. Consequently, we consider an asynchronous FFH/MA communication system with linear-combining and product-combining receivers over Rician fading channels in this section.

6.2.2 System Description

We consider the presence of K simultaneous users operating asynchronously, which will be the case for the mobile-to-base transmission. The total available SS bandwidth is divided into a set of N frequency-hopped bands. All users hop over these FH bands under the control of hop sequences. It is assumed that the users employ independent Markov hopping patterns. We further assume that all users experience mutually i.i.d. Rician fading or Rayleigh fading. Given that there are N frequency-hopped bands available and that all hopping patterns are independent, the probability of the reference user being interfered by another user is given by $\left(\frac{2}{N} - \frac{1}{N^2}\right)$ [50].

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At baseband, the MFSK frequencies assigned to the M symbols are equal to $\frac{m}{T_h}$ Hz,

where $m=1,2,\dots, M$, and T_h is the hopping duration. We assume that user 1 is the reference user and the other $K-1$ users are the interfering users. The dehopped signal of the l th hop interval for the reference user in Fig. 2.2 can be expressed as

$$r_l(t) = \sqrt{2}a_{s,l} \cos(2\pi f_s t + \phi_{s,l}) + \sum_{c=1}^{C_l} I_c(t) + n(t) \quad (6.13)$$

where $a_{s,l}$ is the Rician faded signal amplitude of the reference user, f_s is the modulation frequency of the reference user, $\phi_{s,l}$ is the random phase uniformly distributed over $[-\pi, \pi]$, C_l is the number of the interfering users, and $n(t)$ is the Gaussian noise process with one-sided PSD level equals to N_0 . In (6.13), $I_c(t)$ denotes the MAI due to the $k_{c,l}$ th user ($c=1,2,\dots,C_l$) in the l th hop interval and can be expressed as

$$I_c(t) = \begin{cases} \sqrt{2}a_{k_{c,l}} \cos(2\pi f_{k_{c,l}}(t - \tau'_{k_{c,l}}) + \phi'_{k_{c,l}}) \cdot U(t - \tau'_{k_{c,l}} - lT_h), & \tau'_{k_{c,l}} \geq 0 \\ \sqrt{2}a_{k_{c,l}} \cos(2\pi f_{k_{c,l}}(t - \tau'_{k_{c,l}}) + \phi'_{k_{c,l}}) \cdot U((l+1)T_h + \tau'_{k_{c,l}} - t), & \tau'_{k_{c,l}} < 0 \end{cases} \quad (6.14)$$

where $a_{k_{c,l}}$ is the amplitude of the $k_{c,l}$ th user, $f_{k_{c,l}}$ is the modulation frequency of the $k_{c,l}$ th user, $\phi'_{k_{c,l}}$ is the random phase uniformly distributed over $[-\pi, \pi]$, and $\tau'_{k_{c,l}}$ is the random time delay between the $k_{c,l}$ th user and the reference user uniformly distributed over $[-T_h, T_h]$.

The k th user's signal amplitude is modeled as a frequency-nonselective, slowly Rician fading random variable with PDF given by [1]

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$$P_{a_k}(a_k) = \frac{a_k}{\sigma_k^2} e^{-(a_k^2 + \alpha_k^2)/2\sigma_k^2} I_0\left(\frac{a_k \alpha_k}{\sigma_k^2}\right) \cdot U(a_k). \quad (6.15)$$

The parameters α_k^2 and $2\sigma_k^2$ are the power levels of the direct and diffused components of the k th user's signal, respectively. The total average power is assumed to remain constant from hop to hop. The Rician fading factor defined as $F_k = \alpha_k^2 / 2\sigma_k^2$ determines level of fading condition, e.g., essentially no fading ($F_k = 1000$), Rician fading ($F_k = 10$), and Rayleigh fading ($F_k = 0$). In this section, we assume all interfering users have the same power levels and experience i.i.d. fading. Consequently, the parameters α_k , σ_k , and F_k can be simplified to α , σ , and F , respectively.

6.2.3 Probability of Bit Error Analyses

A. Performance analyses of the system

Since all L hops are assumed to be independent and symmetric, we can ignore the subscript l in the subsequent analyses. In this section, we also assume that all users have the same power level and experience i.i.d. fading. With the assumption that user 1 is the reference user and f_S ($S=1,2,\dots,M$) is transmitted by the reference user, the average joint PDF over C active users ($C=0,1,2,\dots,K-1$) of the processed outputs

$P_{y_1, y_2, \dots, y_M}(y_1, y_2, \dots, y_M | f_S)$ in one hop can be expressed as

$$P_{y_1, y_2, \dots, y_M}(y_1, y_2, \dots, y_M | f_S) = \sum_{C=0}^{K-1} P_{y_1, y_2, \dots, y_M}(y_1, y_2, \dots, y_M | f_S, C) \binom{K-1}{C} (p_h)^C (1-p_h)^{K-1-C} \quad (6.16)$$

where $P_{y_1, y_2, \dots, y_M}(y_1, y_2, \dots, y_M | f_S, C)$ is the joint PDF when there are C interfering users in

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the hop and $p_h = \left(\frac{2}{N} - \frac{1}{N^2} \right)$. The corresponding characteristic function is given by

$$\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S) = \sum_{C=0}^{K-1} \binom{K-1}{C} (p_h)^C (1-p_h)^{K-1-C} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C) \quad (6.17)$$

where $\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C)$ is defined as

$$\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C) = \int \dots \int P_{y_1, y_2, \dots, y_M}(y_1, y_2, \dots, y_M | f_S, C) e^{-i(v_1 y_1 + v_2 y_2 + \dots + v_M y_M)} dy_1 dy_2 \dots dy_M. \quad (6.18)$$

Since all the hops are independent, the joint characteristic function of the final decision variable z_m can be obtained as

$$\Phi_{z_1, z_2, \dots, z_M}(iv_1, iv_2, \dots, iv_M | f_S) = \left[\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S) \right]^L. \quad (6.19)$$

After performing the inverse transformation of the $\Phi_{z_1, z_2, \dots, z_M}(iv_1, iv_2, \dots, iv_M | f_S)$, we can obtain the joint PDF $P_{z_1, z_2, \dots, z_M}(z_1, z_2, \dots, z_M | f_S)$ and the symbol error rate is given by

$$p_s(f_S) = 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} P_{z_1, z_2, \dots, z_M}(z_1, z_2, \dots, z_M | f_S) dz_1 \dots dz_{S-1} dz_{S+1} \dots dz_M dz_S \quad (6.20)$$

where $S=1, 2, \dots$, or M denotes the corresponding MFSK frequency branch of the reference user, i.e., $f_S = \frac{S}{T_h}$. After taking expectation with respect to f_S , the average

symbol error rate is given by

$$p_s = \frac{1}{M} \sum_{S=1}^M p_s \left(\frac{S}{T_h} \right) \quad (6.21)$$

and the average BER is given by [1]

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$$p_b = \frac{M/2}{M-1} p_s. \quad (6.22)$$

From the above analyses, we can compute the average BER using (6.17)-(6.22) once the characteristic function $\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_s, C)$ in (6.17) has been obtained.

The envelope of the m th branch r_m can be decomposed into in-phase and quadrature-phase terms, i.e., $r_m = \sqrt{(r_{m,in})^2 + (r_{m,qu})^2}$, where the in-phase and quadrature-phase outputs of the matched filter are given by

$$r_{m,in} = \int_0^{T_h} r(t) \cos\left(2\pi \frac{m}{T_h} t\right) dt \quad (6.23)$$

and

$$r_{m,qu} = \int_0^{T_h} r(t) \sin\left(2\pi \frac{m}{T_h} t\right) dt, \quad (6.24)$$

respectively. Substituting (6.13) and (6.14) into (6.23) and (6.24), the in-phase and quadrature-phase outputs with C interfering users can be expressed as

$$r_{m,in} = \frac{a_s T_h}{\sqrt{2}} \cos(\phi_s) \delta(f_s - f_m) + \sum_{c=1}^C \frac{a_{k_c} |\tau_{k_c}|}{\sqrt{2}} \left[\cos((f_{k_c} - f_m) \tau_{k_c} + \phi_{k_c}) \text{sinc}(\pi(f_{k_c} - f_m) \tau_{k_c}) \right] + n_{m,in} \quad (6.25)$$

and

$$r_{m,qu} = -\frac{a_s T_h}{\sqrt{2}} \sin(\phi_s) \delta(f_s - f_m) - \sum_{c=1}^C \frac{a_{k_c} |\tau_{k_c}|}{\sqrt{2}} \left[\sin((f_{k_c} - f_m) \tau_{k_c} + \phi_{k_c}) \text{sinc}(\pi(f_{k_c} - f_m) \tau_{k_c}) \right] + n_{m,qu} \quad (6.26)$$

where $\tau_{k_c} = T_h - \tau'_{k_c}$, $\phi_{k_c} = \phi'_{k_c} - 2\pi f_{k_c} \tau'_{k_c}$, and $n_{m,in}$ and $n_{m,qu}$ are independent Gaussian noise components with zero mean and variance $N_0 T_h / 4$. The variables τ_{k_c} and ϕ_{k_c} introduced in (6.25) and (6.26) have the same distributions as τ'_{k_c} and ϕ'_{k_c} , respectively.

We define the vectors $\mathbf{a}_C$, $\boldsymbol{\phi}_C$, $\boldsymbol{\tau}_C$ and $\mathbf{f}_C$ to represent the amplitude, phase, delay

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and frequency of C interfering users in the particular hop duration. For instance, $\mathbf{a}_C$ can be expressed as $\mathbf{a}_C = [a_{k_1}, a_{k_2}, \dots, a_{k_C}]$. We also define a_S and ϕ_S to represent the amplitude and phase of the reference user, respectively. With the above definitions, the envelope r_m conditioned on all the parameters $\mathbf{a}_C, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, \mathbf{f}_C, a_S, \phi_S$ and f_S has the Rician distribution given as

$$P_{r_m}(r_m | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, \mathbf{f}_C, a_S, \phi_S, f_S) = \frac{r_m}{\sigma_n^2} \exp\left(-\frac{r_m^2 + \rho_m^2}{2\sigma_n^2}\right) I_0\left(\frac{r_m \rho_m}{\sigma_n^2}\right) U(r_m) \quad (6.27)$$

where $\sigma_n^2 = N_0 T_h / 4$ is the Gaussian noise variance and

$\rho_m = \sqrt{(r_{m,in} - n_{m,in})^2 + (r_{m,qu} - n_{m,qu})^2}$. With the probability distribution in (6.27), the PDF

and the corresponding characteristic function of y_m for the product-combining and linear-combining receivers are presented below.

Case I: Product-combining receiver

For the product-combining receiver, the final decision statistics can be expressed as

$$z_m = \ln\left(\prod_{l=1}^L r_{m,l}^2\right) = \sum_{l=1}^L \ln(r_{m,l}^2) = \sum_{l=1}^L y_{m,l} \quad (6.28)$$

where $y_{m,l} = 2 \ln(r_{m,l})$. By ignoring the subscript l , the conditional PDF of y_m can be expressed as

$$P_{y_m}(y_m | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, \mathbf{f}_C, a_S, \phi_S, f_S) = \frac{e^{2y_m}}{\sigma_n^2} \exp\left(-\frac{e^{2y_m} + \rho_m^2}{2\sigma_n^2}\right) I_0\left(\frac{e^{y_m} \rho_m}{\sigma_n^2}\right) \quad (6.29)$$

and the corresponding characteristic function is given by [68]

$$\Phi_{y_m}(i\nu | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, \mathbf{f}_C, a_S, \phi_S, f_S) = \int_0^\infty p_{y_m}(y_m | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, \mathbf{f}_C, a_S, \phi_S, f_S) e^{iy_m \nu} dy_m$$

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$$= -\frac{\gamma(1+i\nu)(2\sigma_n^2)^{\frac{1}{2}+i\nu}}{\rho_m} \exp\left(-\frac{\rho_m^2}{4\sigma_n^2}\right) M_{\frac{1}{2}+i\nu,0}\left(-\frac{\rho_m^2}{2\sigma_n^2}\right) \quad (6.30)$$

where $\gamma(\cdot)$ is the Gamma function and $M_{\lambda,\mu}(\cdot)$ is the Whittaker function [68].

Case II: Linear-combining receiver

For the linear-combining receiver, the corresponding y_m are the square-law detector outputs R_m and the conditional PDF is given by

$$P_{y_m}(y_m | a_K, \phi_C, \tau_C, f_C, a_S, \phi_S, f_S) = \frac{1}{2\sigma_n^2} \exp\left(-\frac{y_m + \rho_m^2}{2\sigma_n^2}\right) I_0\left(\frac{\sqrt{y_m} \rho_m}{\sigma_n^2}\right) U(y_m) \quad (6.31)$$

and the corresponding characteristic function is given by [68]

$$\begin{aligned} \Phi_{y_m}(i\nu | a_K, \phi_C, \tau_C, f_C, a_S, \phi_S, f_S) &= \int_0^\infty P_{y_m}(y_m | a_K, \phi_C, \tau_C, f_C, a_S, \phi_S, f_S) e^{i\nu y_m} dy_m \\ &= \frac{1}{1-2i\nu\sigma_n^2} \exp\left(\frac{2i\nu\rho_m^2}{1-2i\nu\sigma_n^2}\right). \end{aligned} \quad (6.32)$$

When all the parameters $a_C, \phi_C, \tau_C, f_C, a_S, \phi_S$ and f_S are given, the y_m outputs of all M branches are independent of each other. Thus the joint PDF of the M branches can be given as

$$\Phi_{y_1, y_2, \dots, y_M}(i\nu_1, i\nu_2, \dots, i\nu_M | a_K, \phi_C, \tau_C, f_C, a_S, \phi_S, f_S) = \prod_{m=1}^M \Phi_{y_m}(i\nu_m | a_K, \phi_C, \tau_C, f_C, a_S, \phi_S, f_S). \quad (6.33)$$

To obtain the average characteristic function $\Phi_{y_1, y_2, \dots, y_M}(i\nu_1, i\nu_2, \dots, i\nu_M | f_S, C)$ as shown in (6.17), we first take the expectation over f_C given by

$$\Phi_{y_1, y_2, \dots, y_M}(i\nu_1, i\nu_2, \dots, i\nu_M | a_K, \phi_C, \tau_C, a_S, \phi_S, f_S) =$$

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$$\frac{1}{C^M} \underbrace{\sum_{f_{h_1}=1/T_h}^{M/T_h} \cdots \sum_{f_{h_C}=1/T_h}^{M/T_h} \cdots \sum_{f_{h_C}=1/T_h}^{M/T_h}}_{C+1} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | a_K, \phi_C, \tau_C, f_C, a_S, \phi_S, f_S). \quad (6.34)$$

Then, we have

$$\begin{aligned} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C) &= E\left(\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C)\right) \\ &= \iint_{\mathbb{R}} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | a_K, \phi_C, \tau_C, a_S, \phi_S, f_S) \\ &\quad \times P_{a_S}(a_S) P_{\phi_S}(\phi_S) P_{a_C}(a_C) P_{\phi_C}(\phi_C) P_{\tau_C}(\tau_C) da_S d\phi_S da_C d\phi_C d\tau_C \end{aligned} \quad (6.35)$$

where $E(\cdot)$ denotes taking the expectation. Note that $P_{a_S}(a_S)$ and $P_{a_C}(a_C)$ are Rician distributed PDFs while $P_{\phi_S}(\phi_S)$, $P_{\phi_C}(\phi_C)$, and $P_{\tau_C}(\tau_C)$ are PDFs with uniform distribution.

In part C, we will make use of Gauss quadrature rules to obtain $\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C)$ in order to avoid the complicated multiple integrations in (6.35).

B. A BER upper bound of the system

For the analyses in part A, an M -dimensional inverse transformation is required to obtain the joint PDF $P_{z_1, z_2, \dots, z_m}(z_1, z_2, \dots, z_m | f_S)$ after obtaining the characteristic function $\Phi_{z_1, z_2, \dots, z_m}(iv_1, iv_2, \dots, iv_M | f_S)$ in (6.19). Although we can use quantization to perform the inverse transformation, it is complicated and time consuming. In this subsection, we will give an upper bound on the BER which does not require the M -dimensional inverse transformation. The upper bound on the BER is given by

$$P_s = \frac{1}{M} \sum_{S=1}^M P_s(f_S) \leq \frac{1}{M} \sum_{S=1}^M \sum_{m=1, m \neq S}^M P_s\left(z_S - z_m < 0 | f_S = \frac{S}{T_h}\right). \quad (6.36)$$

We define $z_{S,m} = z_S - z_m$ and $y_{S,m,l} = y_{S,l} - y_{m,l}$ where $m \neq S$. By ignoring the subscript l ,

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the characteristic function of $y_{S,m}$ is given by

$$\begin{aligned}\Phi_{y_{S,m}}(i\nu | f_S) &= E\left(\Phi_{y_{S,m}}(i\nu | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, f_C, a_S, \phi_S, f_S)\right) \\ &= E\left(\Phi_{y_S}(i\nu | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, f_C, a_S, \phi_S, f_S) \Phi_{y_m}(-i\nu | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, f_C, a_S, \phi_S, f_S)\right) \quad (6.37)\end{aligned}$$

where $\Phi_{y_S}(i\nu | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, f_C, a_S, \phi_S, f_S)$ and $\Phi_{y_m}(-i\nu | \mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, f_C, a_S, \phi_S, f_S)$ for the product-combining receiver and the linear-combining receiver can be obtained from (6.30) and (6.32), respectively. The expectations with respect to $\mathbf{a}_K, \boldsymbol{\phi}_C, \boldsymbol{\tau}_C, f_C, a_S, \phi_S$, and f_S in (6.37) are the same as the expectations in (6.34) and (6.35) and can be taken based on the Gauss quadrature rules which will be described in part C. Since the final decision variable $z_{S,m}$ is the summation of $y_{S,m}$ over L independent hops, the characteristic function of $z_{S,m}$ is given by

$$\Phi_{z_{S,m}}(i\nu | f_S) = \left[\Phi_{y_{S,m}}(i\nu | f_S) \right]^L. \quad (6.38)$$

With a one-dimensional inverse transformation, we can obtain the PDF $P_{z_{S,m}}(z_{S,m} | f_S)$ and the upper bound of symbol error rate is given by

$$P_{s-bound} = \frac{1}{M} \sum_{S=1}^M \sum_{m=1, m \neq S}^M \int_{-\infty}^0 P_{z_{S,m}}(z_{S,m} | f_S) dz_{S,m}. \quad (6.39)$$

The corresponding BER upper bound is $p_{b-bound} = \frac{M/2}{M-1} P_{s-bound}$.

C. Using Gauss quadrature rules to compute multiple-integrations

In this part we will apply the Gauss quadrature rules to compute the multiple level integrations in (6.35) and (6.37). We will first introduce the Gauss quadrature rules for one dimension, and then we will make use of these rules to perform the expectation operation in (6.35) and (6.37). Finally, we will introduce a method to reduce the

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computational complexity.

C.1. Using Gauss quadrature rules with one dimension

In general, the N_g point Gauss quadrature rules for the integral of a function $f(t)$ can be expressed as [73]

$$\int_a^b \omega(t) f(t) dt = \sum_{j=1}^{N_g} \omega_j f(t_j) \quad (6.40)$$

where $\omega(t)$ is called weighting function, ω_j is the weight and t_j is the corresponding abscissas. The N_g points quadrature rule $\{\omega_j, t_j\}_{j=1}^{N_g}$ corresponding to the weighting function $\omega(t)$ can be obtained by the following steps [73]:

Step 1: Generate the Gram matrix $\mathbf{G}$ as given by

$$\mathbf{G} = \begin{bmatrix} \mu_0 & \mu_1 & \cdots & \mu_{N_g} \\ \mu_1 & \mu_{1+1} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \mu_{(N_g-1)+N_g} \\ \mu_{N_g} & \mu_{N_g+1} & \cdots & \mu_{2N_g} \end{bmatrix} \quad (6.41)$$

where $\mu_p = \int_a^b \omega(t) t^p dt$, for $p = 0, 1, \dots, 2N_g$, is the moment of the weighting function $\omega(t)$.

Step 2: Decompose the Gram matrix $\mathbf{G}$ by Cholesky decomposition as $\mathbf{G} = \mathbf{R}^T \mathbf{R}$, where $\mathbf{R}$ can be written as

$$\mathbf{R} = \begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N_g+1} \\ 0 & r_{2,2} & \cdots & r_{2,N_g+1} \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & r_{N_g+1,N_g+1} \end{bmatrix}. \quad (6.42)$$

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Step 3: Obtain the three term recurrence relationship matrix $\mathbf{J}$ as

$$\mathbf{J} = \begin{bmatrix} \xi_1 & \zeta_1 & 0 & \cdots & 0 \\ \zeta_1 & \xi_2 & & \ddots & \vdots \\ 0 & & \ddots & \ddots & 0 \\ \vdots & \ddots & & \ddots & \zeta_{N_g-1} \\ 0 & \cdots & 0 & \zeta_{N_g-1} & \xi_{N_g} \end{bmatrix}. \quad (6.43)$$

The coefficients $\{\xi_j\}_{j=1}^{N_g}$ and $\{\zeta_j\}_{j=1}^{N_g-1}$ can be expressed as

$$\xi_j = \frac{r_{j,j+1}}{r_{j,j}} - \frac{r_{j-1,j}}{r_{j-1,j-1}}, \quad j=1,2,\dots,N_g \quad \text{and} \quad \zeta_j = \frac{r_{j+1,j+1}}{r_{j,j}}, \quad j=1,2,\dots,N_g-1 \quad (6.44)$$

with $r_{0,0} = 1$ and $r_{0,1} = 0$.

Step 4: Generate the Gauss quadrature rules. The abscissas t_j , for $j=1,2,\dots,N_g$, are the eigenvalues of the matrix $\mathbf{J}$, and we assume $\mathbf{q}_j = (q_{1,j}, q_{2,j}, \dots, q_{N_g,j})$, for $j=1,2,\dots,N_g$, as the corresponding eigenvectors. Then the weights can be obtained as $\omega_j = q_{1,j}^2 \times \mu_0$.

In the multiple level integrations of (6.35) and (6.37), there are two kinds of weighting functions, i.e., the first is the Rician distribution for the amplitude integration and the second is the uniform distribution for the phase and delay. For the Rician distribution case, the weighting function is given in (6.15), thus the set of $2N_g+1$ moments of the k th user can be obtained by [68]

$$\begin{aligned} \mu_p &= \int_0^\infty \frac{a_k}{\sigma_k^2} e^{-(a_k^2 + \alpha_k^2)/2\sigma_k^2} I_0\left(\frac{a_k \alpha_k}{\sigma_k^2}\right) t^p dt \\ &= \frac{-i(2\sigma_k^2)^{\frac{k+1}{2}} \gamma(\frac{1}{2}k+1)}{\alpha_k} e^{-\frac{\alpha_k^2}{4\sigma_k^2}} M_{\frac{1}{2}(k+1),0}\left(-\frac{\alpha_k^2}{2\sigma_k^2}\right), \quad p=0,1,\dots,2N_g. \end{aligned} \quad (6.45)$$

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For the uniform distribution case, we normalize the distribution of the phase and the delay to the interval $[-1, 1]$. The set of $2N_g + 1$ moments can be obtained by

$$\mu_p = \int_{-1}^1 1 \times t^p dt = \frac{1 - (-1)^{p+1}}{(p+1)}, \quad p = 0, 1, \dots, 2N_g. \quad (6.46)$$

Using (6.45) or (6.46) in the above four steps, we can obtain the Gauss quadrature rules for the amplitude integration and the phase (or the delay) integration in one dimension.

C.2. Using Gauss quadrature rules in multiple dimensions

Since the multiple level integrations in (6.35) and (6.37) are similar, we will discuss (6.35) only in the following analyses, as the same method can be applied in (6.37) easily. The integration region $\mathbb{R}$ in (6.35) is Euclidean $(3C+2)$ -space and we divide the region $\mathbb{R}$ into two regions $\mathbb{R}_1$ and $\mathbb{R}_2$. The region $\mathbb{R}_1$ is composed of all the amplitudes and has $(C+1)$ dimensions, and the region $\mathbb{R}_2$ is composed of all the phases and delays and has $(2C+1)$ dimensions. For convenience, we rewrite the parameters in (6.35) as

$$\mathbf{A} = [A_1 \ A_2 \ \dots \ A_{C+1}] = [\mathbf{a}_C \ a_S] \text{ and } \mathbf{B} = [B_1 \ B_2 \ \dots \ B_{2C+1}] = [\pi\phi_K \ \pi\phi_S \ T_h \mathbf{r}_K]. \quad (6.47)$$

With the definition in (6.47), (6.35) can be rewritten as

$$\begin{aligned} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C) &= \left(\frac{1}{2\pi}\right)^{C+1} \left(\frac{T_h}{2}\right)^C \\ &\times \int_{\mathbb{R}_1} \prod_{c=1}^{C+1} P_{A_c}(A_c) \int_{\mathbb{R}_2} \prod_{c=1}^{2C+1} P_{B_c}(B_c) \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, \mathbf{A}, \mathbf{B}) d\mathbf{B} d\mathbf{A} \end{aligned} \quad (6.48)$$

where $P_{A_c}(A_c)$ is the Rician PDF given in (6.15) and $P_{B_c}(B_c)$ is the uniform distribution.

We employ N_1 points Gauss quadrature rules $\{\omega_{A,j}, t_{A,j}\}_{j=1}^{N_1}$ for the Rician distribution

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and N_2 points Gauss quadrature rules $\{\omega_{B,j}, t_{B,j}\}_{j=1}^{N_2}$ for the normalized uniform distribution. Consequently, the average characteristic function is given by

$$\begin{aligned} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, C) &= \left(\frac{1}{2\pi}\right)^{C+L} \left(\frac{T_h}{2}\right)^C \\ &\times \sum_{j_1, j_2, \dots, j_{C+1}=1}^{N_1} \sum_{j_1, j_2, \dots, j_{2C+1}=1}^{N_2} \prod_{c_1=1}^{C+1} \omega_{A, j_{c_1}} \prod_{c_2=1}^{2C+1} \omega_{B, j_{c_2}} \Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, t_{A, j_1}, \dots, t_{A, j_{C+1}}, t_{B, j_1}, \dots, t_{B, j_{2C+1}}) \end{aligned} \quad (6.49)$$

where $\Phi_{y_1, y_2, \dots, y_M}(iv_1, iv_2, \dots, iv_M | f_S, t_{A, j_1}, \dots, t_{A, j_{C+1}}, t_{B, j_1}, \dots, t_{B, j_{2C+1}})$ can be obtained by (6.30) or (6.32).

C.3. Employing symmetric integration rules for hypercubes $\mathbb{R}_2$

In this part, we will introduce the method presented in [74]-[75] to reduce the computational points in (6.49). We define an n -dimensional basic rule for the integral function $f(t_1, t_2, \dots, t_n)$ as

$$\mathfrak{R}(\beta_1 \beta_2 \dots \beta_n) f = \frac{1}{n!} \sum_{\lambda_1=\pm 1} \sum_{\lambda_2=\pm 1} \dots \sum_{\lambda_n=\pm 1} \sum_{\mathbf{P}} f(\lambda_1 \beta_{P_1}, \lambda_2 \beta_{P_2}, \dots, \lambda_n \beta_{P_n}) \quad (6.50)$$

where $\mathbf{P} = (P_1 \ P_2 \ \dots \ P_n)$ is the $n!$ permutations of $(1 \ 2 \ \dots \ n)$. From [75], the convolution product (denoted by $*$) of two basic rules is given by

$$\mathfrak{R}(\beta_1 \beta_2 \dots \beta_r) * \mathfrak{R}(\gamma_1 \gamma_2 \dots \gamma_s) = \mathfrak{R}(\beta_1 \beta_2 \dots \beta_r \gamma_1 \gamma_2 \dots \gamma_s). \quad (6.51)$$

We also define an r -dimension rule extended from the s -dimension rule as

$$\begin{aligned} E'_s(\gamma) \mathfrak{R}(\beta_1 \beta_2 \dots \beta_s) &= D_0 \{\mathfrak{R}(\gamma)\}^r + D_1 \{\mathfrak{R}(\gamma)\}^{r-1} * \{\mathfrak{R}(\beta_1) + \mathfrak{R}(\beta_2) + \dots + \mathfrak{R}(\beta_s)\} \\ &+ D_2 \{\mathfrak{R}(\gamma)\}^{r-2} * \sum_{i=1}^s \sum_{j=i+1}^s \mathfrak{R}(\beta_i) * \mathfrak{R}(\beta_j) + \dots + D_s \{\mathfrak{R}(\gamma)\}^{r-s} * \prod_{j=1}^s \mathfrak{R}(\beta_j) \end{aligned} \quad (6.52)$$

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where $D_t = \frac{(-1)^{s-t} r! (r-s)}{(r-s)! s! (r-t)}$. With the above definitions, we present a rule $\bar{\mathbb{C}}_{N_2}^{(2C+1)}$ to

calculate the integration $\int_{\mathbb{R}_2}$. The rule $\bar{\mathbb{C}}_{N_2}^{(2C+1)}$ is given by

$$\bar{\mathbb{C}}_{N_2}^{(2C+1)} = E_{N_2-1}^{(2C+1)}(0) (\mathbb{C}_{N_2})^{N_2-1} + \frac{1}{3\beta_1^2} \left[E_{N_2-1}^{(2C+1)}(0) \{\Re(\beta_1)\}^{N_2-1} - E_{N_2-2}^{(2C+1)}(0) \{\Re(\beta_1)\}^{N_2-2} \right] \quad (6.53)$$

where $\mathbb{C}_{N_2}$ is the one-dimensional N_2 -point Gauss-Legendre quadrature rule and β_1 is the first abscissa of the rule $\mathbb{C}_{N_2}$. The rule $\bar{\mathbb{C}}_{N_2}^{(2C+1)}$ has the same degree with the product rule $(\mathbb{C}_{N_2})^{2C+1}$ presented in part C.2 and requires fewer computational points. The number of computational points required by $\bar{\mathbb{C}}_{N_2}^{(2C+1)}$ is

$$v(\bar{\mathbb{C}}_{N_2}^{(2C+1)}) = \begin{cases} 1 + (N_2 - 1) \binom{2C+1}{1} + \dots + (N_2 - 1)^{N_2-2} \binom{2C+1}{N_2-2} + 2^{N_2-1} \binom{2C+1}{N_2-1}, & N_2 \text{ is odd} \\ 1 + N_2 \binom{2C+1}{1} + \dots + (N_2)^{N_2-2} \binom{2C+1}{N_2-2} + 2^{N_2-1} \binom{2C+1}{N_2-1}, & N_2 \text{ is even} \end{cases} \quad (6.54)$$

and that required by the product rule $(\mathbb{C}_{N_2})^{2C+1}$ is $v((\mathbb{C}_{N_2})^{2C+1}) = (N_2)^{2C+1}$. For example, based on (6.54), $\bar{\mathbb{C}}_4^{(15)}$ requires 5381 computational points while $(\mathbb{C}_4)^{15}$ requires $4^{15} = 1.07 \times 10^9$ computational points.

6.2.4 Numerical Results and Discussions

In this section, numerical BER results for the product-combining receiver and the linear-combining receiver are obtained based on the analytical expressions derived in Section 6.2.3. We consider both Rician fading and Rayleigh fading channels with the Rician

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fading factor F_k equals to 10 and 0, respectively. In the subsequent numerical results, we assume that all users have equal power levels and experience i.i.d. fading. Thus we denote F as the Rician fading factor for all users. In order to have a fair comparison, the total SS bandwidth W_{ss} is assumed to be fixed and the number of available frequency-hopped bands N for the BFSK modulation case without diversity is set to 3000.

In Figs. 6.5 and 6.6, the analytical and simulation BER results of the product-combining receiver and the linear-combining receiver are presented respectively, for both Rayleigh fading and Rician fading channels, versus the number of users K , with $M=4$, $L=3$, and $\text{SNR}=13.35\text{dB}$. To study the efficiency of the proposed analytical expressions, the analytical results with different Gauss quadrature rule orders N_1 and N_2 are presented as well, where N_1 is the Gauss quadrature rule order for the amplitude and N_2 is the Gauss quadrature rule order for the phase and delay. From these two figures, we observe that the analytical results with $N_1 = 8$ and $N_2 = 3$ are nearly the same as those obtained using $N_1 = 4$ and $N_2 = 2$. These analytical results are also very close to the corresponding simulation results. The close match between both analytical and simulation BER results validates our analyses presented in Section 6.2.3. In Figs. 6.5 and 6.6, it can be observed that the system performance deteriorates as the number of users increases. In addition, we can observe that the system performance deteriorates more significantly for the Rician fading case as compared to the Rayleigh fading case when the number of users increases. This is because the MAI in the Rician fading channel is more dominant than in the Rayleigh fading channel.

The upper bounds obtained in Section 6.2.3 and simulation BER results of the product-combining and linear-combining receivers over Rician fading channels are

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presented in Fig. 6.7. From Fig. 6.7, we can observe that the upper bounds are close to the simulation results.

The theoretical BER results for the product-combining receiver over Rician fading channels with $M=4$ and $K=100$ are plotted versus SNR in Fig. 6.8 with different diversity levels L . From Fig. 6.8, we can observe that higher diversity order gives better BER performance when the SNR is large, which is significantly better than the no diversity case. This is because the product-combining receiver can counteract the MAI and the fading effects effectively. It is well known that for a noncoherent FFH system, higher diversity level can also introduce more noncoherent diversity-combining loss. Hence there exists an optimum diversity level for the system. From Fig. 6.8, we can observe that the optimum diversity level increases with increasing SNR.

In Fig. 6.9, the BER results with $M=4$, $K=100$, and different Rician fading factors F are plotted versus SNR for the product-combining receiver with $L=1, 2$, and 3 . In Fig. 6.9, the optimum diversity levels with SNR=9dB over Rayleigh fading, Rician fading, and non-fading channels are 3, 2, and 1, respectively. Thus we conclude that the optimum diversity level will increase as the Rician fading factor decreases. This is because a higher diversity order is required to mitigate more serious fading effects.

The analytical BER results of the product-combining, linear-combining, soft-limited (clipper) [33] and hard-limited [13] receivers are presented for comparison in Figs. 6.10 and 6.11 over Rayleigh fading and Rician fading channels, respectively, with $M=4$, $L=3$, and SNR=13.35dB. The performance of the soft-limited receiver and the hard-limited receiver are obtained from simulation with the optimum clipper level. From Figs. 6.10 and 6.11, it can be observed that the soft-limited receiver always gives the best

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performance over both Rayleigh fading and Rician fading channels and the hard-limited receiver performs the worst. When the MAI is strong, we can observe that the product-combining receiver performs better than the linear-combining receiver, especially in the Rician fading channel case. This is because the product-combining receiver can countermeasure the MAI and the fading effects more efficiently than the linear-combining receiver. Although the soft-limited receiver performs the best, it requires side information on signal amplitude to obtain the clipping threshold. Thus, there exists a tradeoff between system performance and the complexity of the receiver structure.

Fig. 6.5 Analytical and simulation BER results of the product-combining receiver over both Rayleigh fading and Rician fading channels with $M=4$, $L=3$, and $\text{SNR}=13.35\text{dB}$

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Fig. 6.6 Analytical and simulation BER results of the linear-combining receiver over both Rayleigh fading and Rician fading channels with $M=4$, $L=3$, and $\text{SNR}=13.35\text{dB}$

Fig. 6.7 Upper bounds and simulation BER results over both Rician fading channels with $M=4$, $L=3$, and $\text{SNR}=13.35\text{dB}$

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Fig. 6.8 BER performance of the product-combining receiver over a Rician fading channel ($F = 10$) with $M=4$ and $K=100$

Fig. 6.9 BER performance of the product-combining receiver with $M=4$ and $K=100$ over various fading channels

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Fig. 6.10 BER performance comparisons among various diversity-combining receivers with $M=4$, $L=3$, and $\text{SNR}=13.35\text{dB}$ over Rayleigh fading channel

Fig. 6.11 BER performance comparisons among various diversity-combining receivers with $M=4$, $L=3$, and $\text{SNR}=13.35\text{dB}$ over Rician fading channel

6.3 Performance Analyses with MAI Modeled as Alpha-Stable Process

6.3.1 Introduction

The performance analyses of asynchronous FFH/MA systems have been introduced in Section 6.2.1. The analyses in Section 6.2 are based on the assumption that all interfering users have the same power level. In this section, we will focus on the system where the interfering users have unequal power levels which are more practical in the real-life application. In [55] and [56], the MAI was characterized by the alpha-stable random variable model. In [57] and [58], signal detection in alpha-stable noise has been investigated and the Cauchy receiver is found to offer performance close to that of the corresponding optimum receiver. Under the alpha-stable model assumption for MAI, the performance study of the conventional receivers in FHSS systems with MFSK modulation was presented in [59]. In this section, we study the analytical BER performance for an asynchronous FFH/MA communication system with MAI modeled as alpha-stable noise over Rician fading channels. The BER expressions for the linear-combining, product-combining, and clipper receivers based on the Fourier-Bessel series [66] will be presented. These BER expressions are validated by the corresponding simulation results.

6.3.2 System Description

The system model is similar to that introduced in Section 6.2.2 and the only difference is the model of the MAI. In this MA communication system, a receiver is located at the

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center of a plane where there is a large number $(K+1)$ of transmitters and only one of them is communicating with the receiver. The distance between the desired transmitter and the receiver is denoted as d_0 and the distances between the desired receiver and interfering terminals are denoted as d_k for $k=1,2,\dots,K$, where $d_1 \leq d_2 \leq \dots \leq d_K$. The attenuation of signal amplitude over distance is given by [59]

$$a(d_k) = \frac{G}{d_k^p}, \quad k=0,1,2,\dots,K, \quad (6.55)$$

where G depends on the transmitter power and p denotes the attenuation factor. We assume that transmitting terminals form a Poisson point process with the expected number of terminals per unit area given by λ [76]. In addition, we assume that the expected number of terminals per unit area for binary FSK (BFSK) with no diversity is λ_b , so that the expected number of interfering terminals per unit area with MFSK and L -level diversity is given by $\lambda = \frac{ML}{N \log_2 M} \lambda_b$, where N is the number of available FH bands.

The dehopped signal in the l th hop interval for the reference user in Fig. 2.2 can be expressed as

$$r_l(t) = \sqrt{2}a(d_0)\Re_{s,l} \cos(2\pi f_s t + \phi_{s,l}) + \sum_{k=1}^K Z_k(t) + n(t) \quad (6.56)$$

where $\Re_{s,l}$ is a Rician fading random variable, f_s is the modulation frequency, $\phi_{s,l}$ is the random phase uniformly distributed over $[-\pi, \pi]$, and $n(t)$ is the Gaussian noise process with one-sided PSD equal to N_0 . In (6.56), $Z_k(t)$ denotes the k th user's interference at the particular hop and can be expressed as

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$$Z_k(t) = \sqrt{2}a(d_k)\mathfrak{R}_k \cos(2\pi f_k(t - \tau_k') + \phi_k') \quad (6.57)$$

where $a(d_k)$ is the amplitude of the k th interfering user and is given by (6.55), $\mathfrak{R}_k$ is a Rician fading random variable of the k th interfering user at a particular hop, f_k is the corresponding modulation frequency, ϕ_k' is the random phase uniformly distributed over $[-\pi, \pi]$, and τ_k' is the random time delay between the k th interfering user and the reference user uniformly distributed over $[-T_h, T_h]$.

All the users are assumed to experience mutually i.i.d. Rician fading and the distribution of Rician fading random variables $\mathfrak{R}_{s,l}$ and $\mathfrak{R}_k$ in (6.56) and (6.57) is given by [1]

$$P_{\mathfrak{R}}(r) = \frac{r}{\sigma_r^2} e^{-(r^2 + \alpha_r^2)/2\sigma_r^2} I_0\left(\frac{r\alpha_r}{\sigma_r^2}\right) \cdot U(r). \quad (6.58)$$

The parameters α_r^2 and $2\sigma_r^2$ are the normalized power levels of the direct and diffused components of the desired signal. Note that $\alpha_r^2 + 2\sigma_r^2 = 1$ and the Rician fading ratio defined as $F = \alpha_r^2 / 2\sigma_r^2$ determines the level of fading, e.g., essentially no fading ($F = 1000$), Rician fading ($F = 10$), and Rayleigh fading ($F = 0$).

6.3.3 Probability of Bit Error Analyses

We assume that all L hops are independent of each other, thus the subscript l can be ignored in the subsequent analyses. The envelope detector output of the m th branch r_m can be decomposed into in-phase and quadrature-phase terms, i.e., $r_m = \sqrt{r_{m,c}^2 + r_{m,s}^2}$, where $r_{m,c}$ and $r_{m,s}$ are the in-phase and quadrature-phase outputs of the matched filter,

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respectively, which can be expressed as

$$r_{m,c} = \frac{a(d_0)\Re_s T_h}{\sqrt{2}} \cos(\phi_s) \delta(f_s - f_m) + \sum_{k=1}^K \frac{a(d_k)\Re_k |\tau_k|}{\sqrt{2}} \cos((f_s - f_m)\tau_k + \phi_k) \text{sinc}(\pi(f_k - f_m)\tau_k) + n_{m,c} \quad (6.59)$$

and

$$r_{m,s} = \frac{a(d_0)\Re_s T_h}{\sqrt{2}} \sin(\phi_s) \delta(f_s - f_m) - \sum_{k=1}^K \frac{a(d_k)\Re_k |\tau_k|}{\sqrt{2}} \sin((f_s - f_m)\tau_k + \phi_k) \text{sinc}(\pi(f_k - f_m)\tau_k) + n_{m,s} \quad (6.60)$$

where $\tau_k = T_h - \tau'_k$, $\phi_k = \phi'_k - 2\pi f_k \tau'_k$, and $n_{m,c}$ and $n_{m,s}$ are independent Gaussian noise components with zero mean and variance $\sigma_n^2 = N_0 T_h / 4$. The variables τ_k and ϕ_k introduced in (6.59) and (6.60) have the same distribution with τ'_k and ϕ'_k , respectively, as shown in (6.57).

A. To obtain the conditional PDF $P_{r_m}(r_m | f_s)$

From (6.59) and (6.60), we observe that the three components of the envelope detector output, including the desired signal, the MAI and the Gaussian noise, are all spherical symmetric. Consequently, we can obtain the PDF of r_m by using the Fourier-Bessel series as [66]

$$P_{r_m}(r_m) = \frac{2r_m}{R^2} \sum_{q=1}^{\infty} \frac{G_q}{J_1^2(\beta_q)} J_0(\beta_q \frac{r_m}{R}), \quad \text{when } r_m < R \quad (6.61)$$

where β_q is the q th zero of $J_0(\cdot)$ and G_q is the Fourier-Bessel coefficient which can be replaced by $\Phi_{r_m}^{[h]}(\beta_q / R)$. Note that in (6.61), the value of R should be sufficiently large

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so that the probability of r_m larger than R is negligible. The Hankel characteristic function $\Phi_{r_m}^{[h]}(\rho)$ of r_m is defined as

$$\Phi_{r_m}^{[h]}(\rho) = \int_0^{\infty} P_{r_m}(r_m) J_0(r_m \rho) dr_m. \quad (6.62)$$

Furthermore, the PDF of r_m can be calculated by the Hankel characteristic function as

$$P_{r_m}(r_m) = r_m \int_0^{\infty} \rho J_0(r_m \rho) \Phi_{r_m}^{[h]}(\rho) d\rho. \quad (6.63)$$

According to the above definition, the conditional Hankel characteristic function for the desired signal given f_s can be obtained by

$$\begin{aligned} \Phi_s^{[h]}(\rho | f_s) &= \int_0^{\infty} \frac{r}{\sigma_s^2} e^{-(r^2 + \alpha_s^2)/2\sigma_s^2} I_0\left(\frac{r\alpha_s}{\sigma_s^2}\right) J_0(r\rho) \delta(f_s - f_m) dr \\ &= \exp\left(-\frac{\rho^2 \sigma_s^2}{2}\right) J_0(\alpha_s \rho) \delta(f_s - f_m), \end{aligned} \quad (6.64)$$

where $\alpha_s = \frac{a(d_0)T_h}{\sqrt{2}}\alpha_r$ and $\sigma_s = \frac{a(d_0)T_h}{\sqrt{2}}\sigma_r$. Similarly, the Hankel characteristic function for Gaussian noise is given by [66]

$$\Phi_G^{[h]}(\rho) = \exp\left(-\frac{\rho^2 \sigma_n^2}{2}\right). \quad (6.65)$$

Under the assumption that the distances between the desired receiver and the MAI are Poisson distributed, the MAI can be modeled as bivariate spherically symmetric alpha-stable noise when the number of the interferers K is large [59]. For practical systems with a finite number of users, it has been shown in [59] that the alpha-stable noise model can approximate quite well even for $K=10$ when $4/3 < p < 4$. Thus, in the subsequent analysis, we will make use of this alpha-stable noise model to approximate the MAI.

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Now we rewrite the MAI components in (6.59) and (6.60) as

$$\mathbf{Y}_m = \begin{bmatrix} Y_{m,c} \\ Y_{m,s} \end{bmatrix} = \sum_{k=1}^{\infty} \frac{G}{d_k^p} \begin{bmatrix} \cos((f_s - f_m)\tau_k + \phi_k) \\ \sin((f_s - f_m)\tau_k + \phi_k) \end{bmatrix} \left| \frac{\tau_k}{\sqrt{2}} \right| \text{sinc}(\pi(f_k - f_m)\tau_k) \Re_k, \quad (6.66)$$

where $\mathbf{Y}_m$ is the vector representation of the MAI components in the m th branch.

According to [59], the characteristic function of $\mathbf{Y}_m$ can be given by

$$\Phi_{\mathbf{Y}_m}(i\mathbf{v}) = \exp\left(-\gamma_m \|\mathbf{v}\|^\alpha\right), \quad (6.67)$$

where $\alpha = 2/p$, $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, and $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$. The dispersion parameter γ_m when

$1 < p < 4$ can be obtained by

$$\gamma_m = \lambda \pi G^\alpha \frac{\Gamma(1-\alpha/2)}{2^\alpha \Gamma(1+\alpha/2)} E \left| \frac{\tau_k}{\sqrt{2}} \text{sinc}(\pi(f_k - f_m)\tau_k) \right|^\alpha E |\Re_k|^\alpha, \quad (6.68)$$

where $\Gamma(\cdot)$ is Gamma function. Note that in (6.68), $\Re_k$ is Rician distributed, τ_k is

uniformly distributed and that f_k is uniformly distributed over $\left[\frac{1}{T_h}, \frac{M}{T_h}\right]$. Thus we can

obtain the expressions for $E[|\Re_k|^\alpha]$ and $E \left| \frac{\tau_k}{\sqrt{2}} \text{sinc}(\pi(f_k - f_m)\tau_k) \right|^\alpha$ as given by [68]

$$E[|\Re_k|^\alpha] = \frac{(2\sigma_r^2)^{\frac{1+\alpha}{2}} \Gamma\left(1+\frac{\alpha}{2}\right)}{i\alpha_r} \exp\left(-\frac{\alpha_r^2}{4\sigma_r^2}\right) M_{\frac{1+\alpha}{2}, 0}\left(-\frac{\alpha_r^2}{2\sigma_r^2}\right) \quad (6.69)$$

and

$$E \left| \frac{\tau_k}{\sqrt{2}} \text{sinc}(\pi(f_k - f_m)\tau_k) \right|^\alpha = \frac{T_h^\alpha}{2^{\alpha/2}(\alpha+1)M} + \sum_{\substack{k=1 \\ k \neq m}}^M \frac{B\left(\frac{1+\alpha}{2}, \frac{1+\alpha}{2}\right)}{M\pi^{\alpha+1}|f_k - f_m|^\alpha}, \quad (6.70)$$

respectively, where $M_{u,v}(\cdot)$ is Whittaker's function and $B(x, y)$ is the Beta function

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[68]. Substituting (6.69) and (6.70) into (6.68), we can obtain the dispersion parameter of the m th branch γ_m . Then the Hankel characteristic function of the MAI components of the m th branch is obtained by [56]

$$\Phi_{MAI,m}^{[h]}(\rho) = \exp(-\gamma_m \rho^\alpha). \quad (6.71)$$

Combining (6.64), (6.65) and (6.71) together, the conditional Hankel characteristic function $\Phi_{r_m}^{[h]}(\rho | f_S)$ of r_m can be obtained as

$$\Phi_{r_m}^{[h]}(\rho | f_S) = \Phi_S^{[h]}(\rho | f_S) \Phi_{MAI,m}^{[h]}(\rho) \Phi_G^{[h]}(\rho). \quad (6.72)$$

After replacing G_q with $\Phi_{r_m}^{[h]}(\frac{\beta_q}{R} | f_S)$ in (6.61), we can obtain the conditional PDF

$$P_{r_m}(r_m | f_S).$$

B. To obtain the conditional PDF $P_{z_m}(z_m | f_S)$

In the following, we will derive the exponential characteristic functions of y_m and z_m in Fig. 2.2 for various diversity-combining methods including the product-combining, linear-combining, and clipper receivers. The exponential characteristic functions $\Phi_{z_m}(iv | f_S)$ for the final decision statistics z_m of these three receivers can be obtained by

$$\Phi_{z_m}(iv | f_S) = [\Phi_{y_m}(iv | f_S)]^L. \quad (6.73)$$

The corresponding conditional PDF expressions $P_{z_m}(z_m | f_S)$ can be obtained by performing the inverse transformation of (6.73). In the following, we will derive the exponential characteristic functions $\Phi_{y_m}(iv | f_S)$ for these three receivers.

Case I: Product-combining receiver

For the product-combining receiver, the final decision statistics can be expressed as

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$$z_m = \ln \left(\prod_{l=1}^L r_{m,l}^2 \right) = \sum_{l=1}^L \ln(r_{m,l})^2 = \sum_{l=1}^L y_{m,l} \quad (6.74)$$

where $y_{m,l} = 2 \ln(r_{m,l})$. By ignoring the subscript l , the conditional exponential characteristic function $\Phi_{y_m}(i\nu | f_s)$ is given by

$$\Phi_{y_m}(i\nu | f_s) = \int_0^\infty P_{r_m}(r_m | f_s) r_m^{2i\nu} dr_m. \quad (6.75)$$

Substituting (6.61) into (6.75) and after some manipulations, the exponential characteristic function $\Phi_{y_m}(i\nu | f_s)$ can be obtained as

$$\Phi_{y_m}(i\nu | f_s) = \sum_{q=1}^{\infty} \frac{2\Phi_{r_m}^{[h]}(\frac{\beta_q}{R} | f_s) \beta_q^{-2-2i\nu} R^{2i\nu}}{J_1^2(\beta_q)} \left[2^{1+2i\nu} \frac{\Gamma(1+i\nu)}{\Gamma(-i\nu)} - \beta_q J_{-1}(\beta_q) S_{1+2i\nu,0}(\beta_q) \right], \quad (6.76)$$

where $S_{a,b}(\bullet)$ denotes the Lommel function [68].

Case II: Linear-combining receiver

For the linear-combining receiver, the final decision statistics can be expressed as

$$z_m = \sum_{l=1}^L (r_{m,l})^2 = \sum_{l=1}^L y_{m,l}, \quad (6.77)$$

where $y_{m,l} = R_{m,l} = r_{m,l}^2$. The corresponding exponential characteristic function $\Phi_{y_m}(i\nu | f_s)$ can be shown as

$$\Phi_{y_m}(i\nu | f_s) = \sum_{q=1}^{\infty} \sum_{p=0}^{\infty} \frac{\Phi_{r_m}^{[h]}(\frac{\beta_q}{R} | f_s) \beta_q^{2p} (-1)^p}{J_1^2(\beta_q) 2^{2p} (p!)^2 (-i\nu R^2)^{p+1}} \gamma(p+1, -i\nu R^2), \quad (6.78)$$

where $\gamma(a, x)$ denotes the incomplete Gamma function [68].

Case III: Clipper receiver

For the clipper receiver, the final decision statistics can be expressed as

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$$z_m = \sum_{l=1}^L y_{m,l}, \text{ where } y_{m,l} = \begin{cases} r_{m,l}^2, & r_{m,l} \leq T \\ T^2, & r_{m,l} > T \end{cases} \quad (6.79)$$

where T is the clipper threshold. Following that, the exponential characteristic function $\Phi_{y_m}(i\nu | f_S)$ can be obtained as

$$\Phi_{y_m}(i\nu | f_S) = \int_0^T P_{r_m}(r_m | f_S) e^{i\nu r_m^2} dr_m + e^{i\nu T^2} \int_T^\infty P_{r_m}(r_m | f_S) dr_m. \quad (6.80)$$

Substituting (6.61) into (6.80) and assuming R is larger than the clipping threshold T , we can obtain

$$\begin{aligned} \Phi_{y_m}(i\nu | f_S) &= \sum_{q=1}^{\infty} \sum_{p=0}^{\infty} \frac{\Phi_{r_m}^{[h]} \left(\frac{\beta_q}{R} | f_S \right) \beta_q^{2p} (-1)^p}{J_1^2(\beta_q) 2^{2p} (p!)^2 (-i\nu R^2)^{p+1}} \gamma(p+1, -i\nu T^2) \\ &+ \sum_{q=1}^{\infty} \frac{2\Phi_{r_m}^{[h]} \left(\frac{\beta_q}{R} | f_S \right) e^{i\nu T^2}}{J_1^2(\beta_q)} \left[\frac{T}{R} \beta_q^{-1} J_{-1} \left(\frac{T}{R} \beta_q \right) S_{1,0} \left(\frac{T}{R} \beta_q \right) - \beta_q^{-1} J_{-1}(\beta_q) S_{1,0}(\beta_q) \right]. \quad (6.81) \end{aligned}$$

C. To obtain the average BER expressions

The exponential characteristic functions $\Phi_{y_m}(i\nu | f_S)$ for the product-combining, linear-combining, and clipper receivers have been derived in (6.76), (6.78), and (6.81), respectively. The corresponding $\Phi_{z_m}(i\nu | f_S)$ is then given by (6.73). Assuming that all M branches are independent of each other [53], the conditional probability of symbol error given that f_S is transmitted by the desired transmitter can be expressed as

$$p_s(f_S) = 1 -$$

$$\int_0^\infty P_{z_S}(z_S | f_S) \int_0^{z_S} P_{z_1}(z_1 | f_S) \dots \int_0^{z_S} P_{z_{S-1}}(z_{S-1} | f_S) \int_0^{z_S} P_{z_{S+1}}(z_{S+1} | f_S) \dots \int_0^{z_S} P_{z_M}(z_M | f_S) dz_1 dz_2 \dots dz_M. \quad (6.82)$$

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By averaging f_s over $(S = 1, 2, \dots, M)$, the average symbol error rate can be obtained as

$$p_s = \frac{1}{M} \sum_{s=1}^M p_s(f_s). \quad (6.83)$$

Lastly, the average probability of bit error is given by [1]

$$p_b = \frac{M/2}{M-1} p_s. \quad (6.84)$$

6.3.4 Numerical Results and Discussions

In this section, BER results for the linear-combining, product-combining, and clipper receivers are obtained based on the analytical expressions derived in Section 6.3.3. For the clipper receiver, we assume that the optimum threshold level was used. From the analyses in Section 6.3.3, we observe that the theoretical BER expressions for all three receivers are applicable for non-fading, Rician fading and Rayleigh fading channels with the Rician fading factor F equal to 1000, 10 and 0, respectively. In this section, we focus on the Rayleigh fading and Rician fading channels which are more commonly encountered in practical systems. In the subsequent discussion, the distance between the desired terminal and the receiver is assumed to be $d_0 = 1$ and the SNR is defined as

$$SNR = \frac{G^2}{N_0}.$$

In Fig. 6.12, the analytical and simulation BER results of the linear-combining, product-combining, and clipper receivers over Rician fading channels are presented, with $L=3$, $M=4$, $SNR=15\text{dB}$, and $p=2$. The performance is plotted against the expected number of terminals per unit area for BFSK and no diversity case λ_b . The close match in both analytical and simulation results validates our analyses presented in Section 6.3.3. From

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Fig. 6.12, we also observe that when the optimum threshold level is used, the clipper receiver performs the best and the product-combining receiver outperforms the linear-combining receiver. Thus, non-linear diversity-combining schemes can significantly improve the system performance as compared to the linear-combining receiver. Note that although the clipper receiver performs better than the product-combining receiver, the appropriate setting of the clipping threshold requires side information on the desired signal level which is assumed to be known *a priori*. In practice, the desired signal level can be estimated using energy detectors which will make the receiver structure more complex. On the other hand, the product-combining receiver is completely nonparametric, thus, the product-combining receiver is a preferred diversity-combining method to countermeasure the MAI and fading effects effectively and practically if no side information is available. It can also be observed that the system performance becomes worse when λ_b increases due to the presence of more MAI.

The theoretical BER results of the linear-combining, product-combining, and clipper receivers are shown in Figs. 6.13, 6.14, and 6.15, respectively, with $M=4$, $\text{SNR}=15\text{dB}$, $p=2$, and various diversity levels over both Rician fading ($F=10$) and Rayleigh fading ($F=0$) channels. From Fig. 6.13, we observe that there is no diversity gain for the linear-combining receiver. This is because for the linear-combining receiver, the diversity-combining improvement against the MAI and the fading can not outweigh the noncoherent diversity-combining loss of the system. For the product-combining and clipper receivers, we observe from Figs. 6.14 and 6.15 that higher diversity levels can improve the BER performance significantly over both Rician fading and Rayleigh fading channels, especially when the MAI is moderate. From Fig. 6.14, it can also be observed

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that there is an optimum diversity level for the system. For instance, $L=3$ gives the best performance when $\lambda_b > 0.025$ and $L=5$ gives the best performance when $\lambda_b < 0.025$ for the product-combining receiver over Rician fading channels, which are significantly better than the no diversity case. In fact, similar observations can be seen for the clipper receiver when L increases. This is because the product-combining and clipper receivers with a higher order of diversity can counteract the MAI and fading effects more effectively and offset the noncoherent diversity-combining loss of the system. Comparing Figs. 6.13 and 6.14, we can also conclude that the clipper receiver outperforms the product-combining receiver.

In Fig. 6.16, we study the system performance under various environments including Urban I ($p=1.1$), Ground-wave ($p=2$), and Indoor II ($p=3$) [57]. Only the numerical results for the product-combining receiver are illustrated in Fig. 6.16. From Fig. 6.16, we can observe that the system performance does not always improve when the attenuation parameter p increases. This is because the desired signal is also attenuated although the MAI is reduced when p increases. It can also be observed from Fig. 6.16 that the performance with small p deteriorates more than that of a larger p when λ_b increases. In fact, the MAI becomes dominant when λ_b increases. Thus the channel with larger attenuation factor p will attenuate MAI more than it attenuates the desired signal. The above observation is also applicable to other diversity-combining receivers.

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Fig. 6.12 Analytical and simulation BER results of the linear-combining, product-combining, and clipper receivers over Rician fading ($F = 10$) channels with $L=3$, $M=4$, SNR=15dB, and $p=2$

Fig. 6.13 Analytical BER results of the linear-combining receiver over both Rician fading ($F = 10$) and Rayleigh fading ($F = 0$) channels with $M=4$, SNR=15dB, and $p=2$

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Fig. 6.14 Analytical BER results of the product-combining receiver over both Rician fading ($F = 10$) and Rayleigh fading ($F = 0$) channels with $M=4$, $\text{SNR}=15\text{dB}$, and $p=2$

Fig. 6.15 Analytical BER results of the clipper receiver over both Rician fading ($F = 10$) and Rayleigh fading ($F = 0$) channels with $M=4$, $\text{SNR}=15\text{dB}$, and $p=2$

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Fig. 6.16 Analytical BER results of the product-combining receiver over both Rician fading ($F = 10$) and Rayleigh fading ($F = 0$) channels with $M=4$, $\text{SNR}=15\text{dB}$, $L=3$, and different attenuation factors p

6.4 Conclusions

In this chapter, the ML receiver structure was first derived for asynchronous FFH/MA systems over Rayleigh fading channels. The proposed ML receiver requires the perfect information of the noise variance, the delay and frequency of the interfering users. Numerical results have shown that the proposed ML receiver significantly outperforms the conventional receivers. In addition, we have derived the new analytical BER expressions of the product-combining receiver and the linear-combining receiver for asynchronous FFH/MA systems over both Rician and Rayleigh fading channels. The analytical BER results were presented with the assumption that all the interfering users have the same power level. Numerical results have shown that the optimum diversity

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level will increase as the Rician fading factor decreases and the performance improvement due to the diversity combining with large SNR is greater than that with low SNR. From the numerical results, it can also be observed that the product-combining receiver is effective for the system with strong MAI and the soft-limited receiver performs the best with the optimum clipping threshold. Lastly, new analytical BER expressions of the linear-combining, product-combining, and clipper receivers for asynchronous FFH/MA systems over both Rician and Rayleigh fading channels have been presented. The MAI in the system is modeled as the alpha-stable noise and the Fourier-Bessel series is employed in the analyses. From the numerical results, we can observe that the product-combining receiver and the clipper receiver outperform the linear-combining receiver significantly over both Rayleigh fading and Rician fading channels. Higher expected number of terminals per unit area λ_b will further deteriorate the system performance due to more MAI.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

In this thesis, we have studied the FFH/MFSK communication systems with AWGN and interference for both fading and nonfading channels. MTJ and MAI are the two kinds of interference considered in this thesis. For MTJ, we focused on the system performance against the worst-case single-tone per-band MTJ since it has been shown to result in worse BER than independent MTJ [12], [26]. For MAI, we considered both synchronous and asynchronous systems. The BER of systems with various diversity-combining methods, including the linear-combining, product-combining, clipper, and ML receivers were studied for both nonfading and fading channels. In addition, ML receivers and some suboptimum ML receivers were proposed for different channel conditions and the corresponding BER expressions were also derived. In the analyses, it was assumed that

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the total SS bandwidth and the bit duration of the original binary data stream were fixed.

New analytical BER expressions of the product-combining receiver and the clipper receiver against MTJ over non-fading channels were derived. In addition, a new algorithm was introduced to obtain the analytical BER performance of the FFH/MFSK ML receiver. Numerical results have shown that the ML receiver always provides a lower bound of the BER performance in the presence of worst-case band MTJ and AWGN. The performance of the product-combining receiver and the clipper receiver is better than the linear-combining receiver. We also observed that higher diversity in the clipper receiver and the product-combining receiver can reduce the effects of the interference while introducing more noncoherent diversity-combining loss. Hence there will be an optimum diversity level for these receivers. When the jamming power is high, we observed the optimum diversity levels of these receivers (except for the linear-combining receiver), increase as M increases.

We have also proposed two ML receiver structures for FFH/MFSK communication systems with single-tone per-band jamming and AWGN over both Rayleigh fading and Rician fading channels. A new PDF approximation for Rician fading channels was presented. This approximation can be used to design the suboptimum receiver and analyze the system performance. The analytical BER expressions of these two receivers were derived and validated by simulation. Numerical results show that there exists an optimum diversity level for these suboptimum ML receivers and the optimum diversity level is higher for the Rayleigh fading case as compared with the Rician fading case. Furthermore, these two suboptimum ML receivers can significantly improve the system performance as compared to existing receivers. In particular, it has been observed that the

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effects of SJR on the performance of the suboptimum ML receivers are less as compared to the other existing receivers. In fact, the BER results for the proposed suboptimum receivers with optimum diversity levels are almost independent of SJR. However, these two suboptimum receivers require more side information and are more complicated to implement than the other existing receivers.

In this thesis, we have also derived new analytical BER expressions for the linear-combining, product-combining, and clipper receivers for synchronous FFH/MA systems over both Rician and Rayleigh fading channels. Both Hankel characteristic and exponential characteristic functions were used in the analyses. The analytical BER results were derived based on the Fourier-Bessel series and validated by simulation results. From the numerical results, the following conclusions can be drawn. Firstly, the product-combining receiver and the clipper receiver significantly outperform the linear-combining receiver over both Rayleigh fading and Rician fading channels. The clipper receiver with an optimum threshold slightly outperforms the product-combining receiver. However, the product-combining receiver is easier to implement compared to the clipper receiver as it does not require any side information on the signal level. Secondly, higher modulation dimension can improve the system performance with a fixed total SS bandwidth although the incremental improvement in BER performance will diminish as M increases. Thirdly, the optimum diversity levels for various diversity-combining receivers over Rayleigh fading channels are higher than that for Rician fading channels. Lastly, the two non-linear diversity-combining techniques can effectively combat the effects of MAI as compared to the linear-combining receiver.

For an asynchronous FFH/MA system, we first derived the ML receiver structure over

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Rayleigh fading channels based on the ML criterion. The proposed ML receiver requires information on the noise variance, the delay and frequency of the interfering users. BER expressions for the proposed ML receiver were also presented and numerical BER results have shown that the proposed ML receiver significantly outperforms other existing receivers. In addition, we have also derived new analytical BER expressions for the product-combining receiver and the linear-combining receiver for asynchronous FFH/MA systems over both Rician and Rayleigh fading channels based on Gauss quadrature rules. In the analyses, we assumed that all interfering users have the same power level. Analytical BER results were presented and validated by simulation. From the numerical results, it can be observed that the optimum diversity level increases with increasing SNR and the performance improvement due to diversity-combining with large SNR is greater than that with low SNR. When the interfering users have unequal power levels, we have derived new analytical BER expressions for linear-combining, product-combining, and clipper receivers with MAI modeled as alpha-stable noise. Fourier-Bessel series were employed in the analyses. From the numerical results, we can observe that the product-combining receiver and the clipper receiver significantly outperform the linear-combining receiver over both Rayleigh fading and Rician fading channels. The clipper receiver with an optimum threshold slightly outperforms the product-combining receiver. Lastly, we observe that a higher arrival rate will further deteriorate the system performance due to more MAI.

7.2 Recommendations for Further Research

In this thesis, we have studied the system performance of the linear-combining, product-

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combining and clipper receivers against MAI. Since it has been shown that the self-normalizing receiver can counteract the effects of MTJ and PBNJ effectively [25]-[27], it might be interesting to study the performance of the self-normalizing receiver against MAI. Furthermore, we have only presented the ML receiver for asynchronous FFH/MA systems over Rayleigh fading channels. Since the ML receiver can provide the best performance and Rician fading is also commonly encountered, it is meaningful to derive and study the ML receiver over Rician fading channels. As we have discussed in this thesis, the ML receiver always requires some side information on the interference and the channel. Thus, it is also useful to derive and study more practical suboptimum ML receivers against MAI over both Rayleigh and Rician fading channels, which require less side information than the ML receiver.

In this thesis, we assumed that the channel for each hop of a symbol is modeled as a frequency-nonselective slowly fading process. Frequency-selective fast fading is also commonly encountered in wireless communication systems, especially in urban environments. Some equalization techniques have been employed to reduce the effects of frequency-selective fast fading. Consequently, it is useful to study the performance improvement by employing both diversity-combining and equalization techniques while the signal and the interference experience frequency-selective fast fading. In addition, it is also possible to extend the above study to other types of modulation, such as orthogonal-frequency-division-multiplexing (OFDM), etc. Lastly, we only consider an uncoded system in the thesis, the effects of various types of error-correction codes are also an interesting topic for further research.

Appendix A

Appendix A

In this appendix, we will derive the characteristic functions $\Phi_{R_{m,l}}(iv | q_{m,l}, S)$ over Rician fading channels as shown in (4.15). When the signal undergoes Rician fading, the conditional PDF of the $R_{m,l}$ is given by

$$P_{R_{m,l}}(R_{m,l} | q_{m,l}, S) = \frac{1}{\pi} \cdot \frac{1}{2\sigma_n^2} \int_0^\infty \int_0^\pi \exp\left(-\frac{R_{m,l} + 2(a_s^2 + q_{m,l}(a_j^2 + 2a_s a_j \cos \phi))}{2\sigma_n^2}\right) \\ \cdot I_0\left(\frac{\sqrt{2R_{m,l}(a_s^2 + q_{m,l}(a_j^2 + 2a_s a_j \cos \phi))}}{\sigma_n^2}\right) \cdot \frac{a_s}{\sigma_s^2} \exp\left(-\frac{a_s^2 + \alpha_s^2}{2\sigma_s^2}\right) I_0\left(\frac{a_s \alpha_s}{\sigma_s^2}\right) d\phi da_s. \quad (\text{A.1})$$

The characteristic functions $\Phi_{R_{m,l}}(iv | q_{m,l}, S)$ can be obtained as

$$\Phi_{R_{m,l}}(iv | q_{m,l}, S) = \frac{1}{\pi} \cdot \frac{1}{2\sigma_n^2} \cdot \frac{1}{\sigma_s^2} \int_0^\infty \int_0^\pi \exp\left(-\frac{a_s^2 + q_{m,l}(a_j^2 + 2a_s a_j \cos \phi)}{\sigma_n^2}\right) \\ \cdot \exp\left(-\frac{a_s^2 + \alpha_s^2}{2\sigma_s^2}\right) I_0\left(\frac{a_s \alpha_s}{\sigma_s^2}\right) \cdot a_s \cdot \int_0^\infty \exp\left[-\left(\frac{1}{2\sigma_n^2} - iv\right) R_{m,l}\right] \\ \cdot I_0\left(2\sqrt{\frac{a_s^2 + q_{m,l}(a_j^2 + 2a_s a_j \cos \phi)}{2\sigma_n^4}} \cdot \sqrt{R_{m,l}}\right) \cdot dR_{m,l} d\phi da_s. \quad (\text{A.2})$$

By applying the following formula [67, p. 709, equation (6.614-3)]

$$\int_0^\infty e^{-\alpha x} I_0(2\sqrt{\beta x}) dx = \frac{1}{\alpha} \exp\left(\frac{\beta}{\alpha}\right), \quad \text{Re}(\alpha) > 0, \quad (\text{A.3})$$

Appendix A

equation (A.2) can be simplified to

$$\begin{aligned}
 \Phi_{R_{mJ}}(iv|q_{ml}, S) &= \frac{1}{\pi} \cdot \frac{1}{2\sigma_n^2} \cdot \frac{1}{\sigma_s^2} \int_0^\infty \int_0^\pi \exp\left(-\frac{a_s^2 + q_{ml}(a_J^2 + 2a_s a_J \cos \phi)}{\sigma_n^2}\right) \\
 &\quad \cdot \exp\left(-\frac{a_s^2 + \alpha_s^2}{2\sigma_s^2}\right) I_0\left(\frac{a_s \alpha_s}{\sigma_s^2}\right) \cdot a_s \cdot \frac{2\sigma_n^2}{1 - i2v\sigma_n^2} \\
 &\quad \cdot \exp\left(\frac{a_s^2 + q_{ml}(a_J^2 + 2a_s a_J \cos \phi)}{2\sigma_n^4} \cdot \frac{2\sigma_n^2}{1 - i2v\sigma_n^2}\right) \cdot d\phi da_s \\
 &= \frac{1}{\sigma_s^2(1 - i2v\sigma_n^2)} \cdot \exp\left(-\frac{\alpha_s^2}{2\sigma_s^2}\right) \cdot \exp\left[\frac{q_{ml}a_J^2}{\sigma_n^2(1 - i2v\sigma_n^2)}\right] \\
 &\quad \cdot \int_0^\infty a_s \cdot \exp\left[-\left(\frac{1}{\sigma_n^2} + \frac{1}{2\sigma_s^2} - \frac{1}{\sigma_n^2(1 - i2v\sigma_n^2)}\right)a_s^2\right] \\
 &\quad \cdot I_0\left(\frac{\alpha_s}{\sigma_s^2} a_s\right) \cdot J_0\left(\frac{4q_{ml}va_J}{1 - i2v\sigma_n^2} a_s\right) \cdot da_s
 \end{aligned} \tag{A.4}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. Furthermore, by making use of the following expression [67, p. 718, equation (6.633-4)]

$$\int_0^\infty x e^{-\alpha x^2} I_0(\beta x) J_0(\gamma x) dx = \frac{1}{2\alpha} \exp\left(\frac{\beta^2 - \gamma^2}{4\alpha}\right) \cdot J_0\left(\frac{\beta\gamma}{2\alpha}\right), \quad \text{Re}(\alpha) > 0, \tag{A.5}$$

equation (A.4) can be simplified to

$$\begin{aligned}
 \Phi_{R_{mJ}}(iv|q_{ml}, S) &= \frac{1}{(1 - iv\beta_s)} \cdot \exp\left[\frac{\alpha_s^2(1 - i2v\sigma_n^2)}{2\sigma_s^2(1 - iv\beta_s)} - \frac{\alpha_s^2}{2\sigma_s^2}\right] \\
 &\quad \cdot \exp\left\{-\left[\frac{1}{\sigma_n^2} - \frac{1}{\sigma_n^2(1 - i2v\sigma_n^2)} + \frac{8v^2\sigma_s^2}{(1 - i2v\sigma_n^2)(1 - iv\beta_s)}\right] \cdot q_{ml}a_J^2\right\} \cdot J_0\left(\frac{4v\alpha_s}{1 - iv\beta_s} q_{ml}a_J\right) \\
 &= \frac{1}{1 - iv\beta_s} J_0\left(\frac{4q_{ml}v\alpha_s a_J}{1 - iv\beta_s}\right) \cdot \exp\left(\frac{2iv(\alpha_s^2 + q_{ml}a_J^2)}{1 - iv\beta_s}\right),
 \end{aligned} \tag{A.6}$$

Appendix A

where $\beta_s = 2(\sigma_\omega^2 + 2\sigma_s^2)$. Using $J_0(z) = \frac{1}{\pi} \int_0^\pi e^{iz \cos \phi} d\phi$ [68], (A.6) can be rewritten as

$$\begin{aligned} \Phi_{R_{mJ}}(iv | q_{mI}, S) &= \frac{1}{\pi} \int_0^\pi \frac{1}{1-iv\beta_s} \exp\left(\frac{4ivq_{mI}\alpha_s a_J \cos \phi_{mJ}}{1-iv\beta_s}\right) \cdot \exp\left(\frac{2iv(\alpha_s^2 + q_{mI}a_J^2)}{1-iv\beta_s}\right) d\phi_{mJ} \\ &= \frac{1}{\pi} \int_0^\pi \frac{1}{1-iv\beta_s} \exp\left(\frac{2iv[\alpha_s^2 + q_{mI}(a_J^2 + 2\alpha_s a_J \cos \phi_{mJ})]}{1-iv\beta_s}\right) d\phi_{mJ}. \end{aligned} \quad (\text{A.7})$$

Author's Publications

- [1] Y. Han and K. C. Teh, "Performance study of suboptimum maximum-likelihood receivers for FFH/MFSK systems with multitone jamming over fading channels," *IEEE Trans. Veh. Tech.*, vol. 54, pp. 82-90, Jan. 2005.
- [2] Y. Han and K. C. Teh, "Error probabilities and performance comparisons of various FFH/MFSK receivers with multitone jamming," *IEEE Trans. Commun.*, vol. 53, pp. 769-772, May 2005.
- [3] Y. Han and K. C. Teh, "Maximum-likelihood receiver with side information for asynchronous FFH-MA/MFSK systems over Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 10, pp. 435-437, Jun. 2006.
- [4] Y. Han and K. C. Teh, "FFH/MFSK multiple-access communications with various diversity-combining techniques over fading channels," accepted for publication in the *IEE Proc. Commun.*
- [5] Y. Han and K. C. Teh, "Performance study of asynchronous FFH/MFSK communications using various diversity-combining techniques with MAI modeled as alpha-stable process," accepted for publication in the *IEEE Trans. Wireless Commun.*

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