Moisture and heat flow in unsaturated soil

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Moisture and heat flow in unsaturated soil

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Abstract

Many countries in the tropical and sub-tropical regions experience mainly two seasons: dry and wet. In general, this climatic condition is not conducive to the weathering of rock formation into residual soils. The shear strength of these residual soils is high when unsaturated. However, numerous slope failures in residual soils have occurred during heavy rainfalls. This is associated with a reduction in shear strength of the soil because of the decrease of the matric suction. Another aspect of climatic condition which has influence on slope stability is evaporation especially during the dry season. This process increases matric suction in the residual soil and hence the factor of safety of the slope will be increased. These two processes do not occur separately. Therefore by incorporating these two phenomena, the matric suction distribution in slope, and in turn the stability of slope, can be assessed more realistically.

Studying the moisture and heat flow in unsaturated soil under infiltration and evaporation processes has attracted the attention of many researchers. A numbers of approaches were proposed in the literature. In this research, a mathematical model was presented as a modification from the well-known Philip and de Vries (1957) model. The Philip and de Vries model was based on mass and energy conservation laws and appeared to be radically different from the Wilson et al. (1994) model which was established from the constitutive relationship for unsaturated soil. A detailed comparison of the two models was made and it was shown rigorously that the two models are similar to a large extent.

There is little experimental data for evaporation from soil column tests and none has been carried out for a full cycle of evaporation/infiltration. In this study, a series of three soil column tests under the processes of evaporation/infiltration continuously were carried out for sand and Singapore residual soil. It was found that the matric suction change in the top 10 to 15 cm of the soil, in the soil columns, 500 mm in height, depending on the drainage conditions and the temperature applied to the column.

The modified model established from this research was implemented in a Microsoft Excel program supplemented by Visual Basic Application programming. The program was evaluated using data from the literature. Three cases of infiltration and two cases of evaporation were used. The evaluation confirmed that the modified model was able to simulate both infiltration and evaporation processes well. Furthermore, the results indicate that the soil-water characteristic curve and the permeability function have important roles in numerical modelling.

A parametric study was conducted to evaluate the effects of the soil-water characteristic curve and the permeability function on numerical results from the modified model and the Wilson et al. (1994) model. The parametric study revealed that the differences in the numerical results are generally small except when the soil-water characteristic curve approaches that of a clay soil.

The modified model was also used to simulate the laboratory soil column tests. The results showed that the modified model is able to simulate the experimental data well. It is recommended that this modified equation, when extended to two-dimensional and three-dimensional spaces, can be used to assess slope stability for various soil types. However, as the effects of capillary phenomenon were not clear in the Sand 2 over Sand 1 column due to their saturated permeabilities that were not significantly different, more experimental data would be useful to test the rigour of the modified equation.

To my beloved family

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List of Symbols

A	: Cross section area of soil column perpendicular to the direction of flow	$[L^2]$
A_s	: Suction vector evaluated from the relationship for the void ratio obtained from the state surface	
A_T	: Thermal vector	
$[B]$	: gradient matrix	
C_w	: Interactive constant associated with the water phase	
C_{vl}^w	: Coefficient of consolidation for the liquid water phase	
C_{vv}^w	: Coefficient of consolidation for the water vapour phase	
C_h	: Thermal capacity of soil	
C_i	: Water capacity	
C_a	: Interactive constant associated with the air phase	
C_v^a	: Coefficient of consolidation for the air phase	
C_T	: Interactive thermal constant associated with the air phase	
$C(\psi)$	: Correction function in SWCC equation from Fredlund and Xing (1994)	
D	: Effective molecular diffusivity	$[L^2T^{-1}]$
D^*	: Diffusion coefficient of water vapour through the soil	
D_g	: Coefficient of transmission of gas phase through soil	
D_{vap}	: molecular diffusivity of water vapour in air	$[L^2T^{-1}]$
D_θ	: Isothermal water diffusivity	$[L^2T^{-1}]$
$D_{\theta w}$	: Diffusivity function	
$D_{\theta l}$	: Isothermal liquid water diffusivity	$[L^2T^{-1}]$
$D_{\theta v}$	: Isothermal watervapour diffusivity	$[L^2T^{-1}]$
D_T	: Thermal water diffusivity	$[L^2T^{-1}]$
D_{Tl}	: Thermal liquid water diffusivity	$[L^2T^{-1}]$
D_{Tv}	: Thermal water vapourdiffusivity	$[L^2T^{-1}]$
D_{Ta}	: Transport coefficient for absorbed liquid flow due to	$[L^2T^{-1}]$

	thermal gradient	
E	: Evaporative mass flux	
E_p	: Potential evaporation rate	$[LT^{-1}]$
E_a	: Actual evaporation rate	$[LT^{-1}]$
E_h	: Total bulk volumetric heat content of a porous medium	
G	: general symbol for source or sink, associated with a finite domain	
H	: The height of soil column in numerical modelling	$[L]$
H_s	: Sensible heat flux,	
H_l	: Latent heat flux of evaporation	
K_{im}	: Mean coefficient permeability of soil between node i and $i-1$	$[LT^{-1}]$
K_{ip}	: Mean coefficient permeability of soil between node i and $i+1$	$[LT^{-1}]$
L_v	: Latent heat of vaporization	
M_c	: Fluid mass capacity	
M_v	: Mass flux of liquid vapour,	
M_{ve}	: Mass flux of liquid vapour entering the soil element	
M_{ve}	: Mass flux of liquid vapour leaving the soil element	
M_{lwl}	: mass flux of liquid water leaving the soil element	
M_{lwe}	: mass flux of liquid water entering the soil element	
$\langle N \rangle$	: Vector of interpolation function	
P_{vs}^w	: Saturation vapour pressure of water at the water surface	
P_v^a	: Vapour pressure of the air above the water surface, and	
P_v	: Partial pressure due to water vapour in the air phase	
R	: Gas constant	
R^*	: Universal gas constant	
R_h	: Relative humidity	
R_N	: Net radiation of all wavelengths measured with a net-	

	radiometer input to the stand of vegetation per unit area	
S	: Degree of saturation	
T	: Temperature or absolute temperature	
V _o	: Initial overall volume of an element	[L ³]
V _w	: Volume of water in an element	[L ³]
W	: Differential heat of wetting,	
W _{wa}	: Molecular weight of water vapour	
a	: A soil parameter in SWCC equation from Fredlund and Xing (1994)	[ML ⁻¹ T ⁻²]
c'	: Effective cohesion	[ML ⁻¹ T ⁻²]
c _l	: Specific heat of liquid water	
c _s	: Specific heat of soil solid	
c _a	: Specific heat of air	
e	: Natural number, 2.71828.....	
g	: Gravitation acceleration	[LT ⁻²]
h _c	: Capillary height	[L]
h _p	: Pressure head	[L]
h _w	: Hydraulic or total head of water	[L]
i	: Hydraulic gradient	
I	: Infiltration rate	[LT ⁻¹]
k _s	: Saturated permeability coefficient of soil	[LT ⁻¹]
k _w	: Coefficient permeability of soil	[LT ⁻¹]
l	: Length of flow path	[L]
m	: A soil parameter in SWCC equation from Fredlund and Xing (1994)	
m _{lw}	: Mass of liquid water in the soil element	
m ₁ ^w	: Coefficient of volume change with respect to a change in net normal stress	
m ₂ ^w	: Coefficient of volume change with respect to a change in net matric suction	
n	: A soil parameter in SWCC equation from Fredlund and Xing (1994)	

f_v	: Transmission function depends on the mixing characteristic of the air in the atmosphere above evaporation surface	
p	: A soil parameter in permeability equation from Leong and Rahardjo (1997)	
q_m	: Total water flux density	$[ML^{-2}T^{-1}]$
q_l	: Liquid water flux density	$[ML^{-2}T^{-1}]$
q_v	: Water vapour flux density	$[ML^{-2}T^{-1}]$
t	: Elapsed time	$[T]$
u_a	: Pore-air pressure	$[ML^{-1}T^{-2}]$
u_w	: Pore-water pressure	$[ML^{-1}T^{-2}]$
v_w	: Velocity of water flow in soil	$[LT^{-1}]$
z	: Elevation from datum	$[L]$
α_T	: Thermal expansion coefficient	
β	: Tortuosity factor	
θ_r	: Residual volumetric water content	
θ_s	: Saturated volumetric water content	$[ML^{-2}T^{-2}]$
θ_w	: Volumetric water content	$[ML^{-2}T^{-2}]$
θ_a	: Volumetric air content	
λ	: Thermal conductivity of soil	
ψ	: Soil suction or matric suction	$[ML^{-1}T^{-2}]$
ψ_a	: Air-entry value	$[ML^{-1}T^{-2}]$
ψ_w	: Water-entry value	$[ML^{-1}T^{-2}]$
τ_f	: Average shear strength of soil	$[ML^{-1}T^{-2}]$
v	: Mass flow factor	
ρ_w	: Density of water	$[ML^{-3}]$
ρ_v	: Density of water vapour	$[ML^{-3}]$
$\rho_{vs T_0}$	: Saturation vapour density at T_0	$[ML^{-3}]$
Θ	: Dimensionless form of the SWCC	
ϕ'	: Effective angle of internal friction	

ϕ^b	: Angle indicating the rate of increasing in shear strength related to matric suction	
π	: Osmotic suction	$[ML^{-1}T^{-2}]$
σ	: Total stress	$[ML^{-1}T^{-2}]$
σ_T	: Pure water-air interfacial tension	
(u_a-u_w)	: Matric suction	$[ML^{-1}T^{-2}]$
$(\sigma-u_a)$	: Net normal stress	$[ML^{-1}T^{-2}]$
Δt	: Time step size	$[T]$
Δz	: Elevation step size	$[L]$
∇	: Gradient operator	
Γ	: surface bounding a finite sub region	
$\int_A$	: Integral over the area	
$\int_L$	: Integral over the length from corner node to corner node.	

Chapter 1 Introduction

1.1. BACKGROUND

Rapid population growth and economic development have fuelled the need for infrastructure and civil construction not only in deltaic but also in mountainous areas. As a result, the problem of slope stability is an important consideration in many countries. Many of the soils in tropical and sub-tropical regions are unsaturated. Therefore many residual soil slopes in these regions are in an unsaturated condition as the ground water table is usually deep relative to the ground surface. Slope stability problems in residual soils are receiving increasing attention due to the frequent occurrence of landslides in regions experiencing high seasonal rainfalls (Lim et al. 1996). The stability of a slope is governed by the distribution of pore water pressure within the slope. The pore-water pressure u_w or matric suction when referenced to the pore-air pressure u_a (i.e., $(u_w - u_a)$), has been found to play a significant role in the problem of slope stability (Lim et al., 1996). During rainfall, infiltration of rain water causes the pore-water pressure to increase or matric suction to be lost. This is associated with a reduction in shear strength of the soil and may cause a slope to fail. However, during dry periods, the pore-water pressure becomes more negative (or matric suction increases). This favourable condition increases the shear strength of the soils which in turn results in increasing the factor of safety of the slope against shear failure.

The relationship of matric suction change in unsaturated soil associated with the climatic condition change has attracted many researchers. There are a number of theories to model moisture and heat flow in unsaturated soil. The theories are presented for either infiltration or evaporation separately or coupled infiltration/evaporation situations. In general, the theory for water infiltration into unsaturated soil has been well researched. However, the theory for evaporation from an unsaturated soil is less well researched. Recently, some mathematical models based on different framework were presented for coupled moisture and heat flow in unsaturated soil. These models have been applied for different purposes and the differences between the models have not been investigated fully. In addition,

infiltration/evaporation experimental data for soil are scarce in the literature. Therefore, by introducing a comprehensive laboratory test program, the processes of infiltration/evaporation can be better understood.

1.2. OBJECTIVES AND SCOPE OF THE RESEARCH

In tropical regions many unsaturated soil slopes fail during the wet season. The stability of the unsaturated soil slope depends mostly on the matric suction profile within the slope. The matric suction profile changes continuously due to drying and wetting processes occurring daily. The present theories do not give a simple mathematical model for simulating infiltration and evaporation. Furthermore, experimental data on drying and wetting processes in soil are lacking or incomplete. Therefore, the objectives of this research are to review existing moisture and heat transfer models in unsaturated soil in order to derive a simple practical model for modelling drying and wetting processes in soil. The model should be verified with existing experimental data as well as with more complete experimental data.

Specifically the scope of the research will include the following:

- Review and evaluation of existing mathematical models for moisture and heat transfer in soil.
- Developing a simple practical 1-D mathematical model for predicting matric suction profile changes during evaporation and infiltration processes.
- Evaluation of the mathematical model using experimental data in the literature.
- Carrying out laboratory tests to determine the soil parameters needed in the mathematical model.
- Carrying out one dimensional (1-D) soil sand column tests for verifying the mathematical model.

1.3. OUTLINE OF THE THESIS

The thesis comprises of eight chapters, Chapter 1 gives the background, objectives and scope of the research, and an outline of the thesis.

Chapter 2 presents a brief review of unsaturated soil mechanics, a review of the past works on infiltration and evaporation processes and a review of coupled moisture and heat movement models.

Chapter 3 presents a modified model based on the framework of Philip and de Vries (1957). Subsequently an intensive comparison of the modified model with the Wilson et al. (1994) model is carried out,

Chapter 4 evaluates the modified model using data from published literature. A parametric study is also carried out to evaluate the differences between the Wilson et al. (1994) model and the modified model.

Chapter 5 describes the soil column set-up in the laboratory for this study. The equipment as well as calibrations of the measuring devices are presented in this chapter. The properties of soil used are also presented in this chapter.

Chapter 6 presents the soil column test programme and the test results.

Chapter 7 presents the numerical modelling of the soil column test and the comparison between the numerical results using the modified model and soil column test results described in Chapter 6.

Chapter 8 summarises the conclusions obtained in this research. Recommendations for future research are suggested at the end of this chapter.

Chapter 2 Literature Review

2.1. INTRODUCTION

There are two principal processes governing the exchange of water between the soil surface and the atmosphere (Wilson, 1990). Water molecules enter the soil surface almost exclusively in liquid form through infiltration. On the other hand, water molecules leave the ground surface exclusively in vapour form through evaporation. Numerous theories which describe these processes are available. However, the practical application sometimes entails the incorporated analysis of both infiltration and evaporation together. The infiltration process in porous media is a function of mechanical energy (total head) and porous media's properties (permeability and storage capabilities). Consequently, energy balance and vapour transfer within the soil as well as above the ground surface are ignored. On the contrary, the theory of evaporation is based on aerodynamic vapour transfer and energy balance at and above the ground surface.

In this chapter, theories related to infiltration and evaporation in unsaturated soil are presented. The first part discusses unsaturated soil mechanics. The theory of water flow in soils is presented next and the concept of the mass balance conservation law is described. Reviews of one-dimensional infiltration numerical and experimental modelling are then presented. The principles of the evaporation process and reviews of numerical modelling as well as experimental modelling of evaporation are presented subsequently. Finally concluding remarks are presented at the end of the chapter.

2.2. UNSATURATED SOIL MECHANICS

2.2.1. Background

Unsaturated soil is commonly defined as having three phases, namely: solid, water, and air. However this concept will be more correct if a fourth phase is included which is the air-water interface or contractile skin (Fisher 1926, Fredlund and Mogenstern 1977, Lu and Likos 2004). The broad spectrum of unsaturated soil

behaviour is associated to the wide range of degree of saturation, S . Figure 2-1 describes the changes in the degree of saturation, S , for a saturated – unsaturated soil system. In general, soil above the groundwater table, called vadose zone or unsaturated zone, can be readily divided into three parts. The first part, immediately above the groundwater table is called the capillary fringe with S near 100%. The second and third parts, above the capillary fringe, are unsaturated with degree of saturation, S , ranging from less than 100% with continuous water phase to near 0% where the water phase is discontinuous.

Figure 2-1 Description of saturated-unsaturated soil system (from Fredlund 2000)

There are three possible profiles of negative pore-water pressure distribution in the unsaturated zone as shown in Figure 2-2. In the case where there is no flux across the ground surface (e.g., infiltration or evaporation as well as transpiration), the pore-water pressure profile is in equilibrium at the hydrostatic condition (line 1 in Figure 2-2). If water goes out of the soil at the surface (e.g., evaporation or transpiration), the negative pore-water pressure profile will move in the direction such that pore-water pressure is more negative (curve 2 in Figure 2-2). On the contrary, if water enters the ground surface (e.g., infiltration from rainwater), the pore-water pressure profile will move to the right side of line 1, meaning that the pore-water pressure becomes less negative (curve 3 in Figure 2-2) in comparison with the hydrostatic condition.

Figure 2-2 The possible matric suction profiles in unsaturated soil
(from Fredlund and Rahardjo 1993a)

The theories of unsaturated soil mechanics can be viewed from the concept of classic constitutive relationships which are established in saturated soil mechanics, namely: seepage, shear strength, and volume changes (Figure 2-3). In Figure 2-3, the independent stress state variables, $(\sigma - u_a)$ and $(u_a - u_w)$ are shown. The term $(\sigma - u_a)$ is called net normal stress and the term $(u_a - u_w)$ is called matric suction where σ is normal stress, u_a is pore-air pressure and u_w is pore-water pressure. These variables were established in order to describe the behaviour of unsaturated soil (Fredlund and Morgenstern 1977).

2.2.2. Soil Suction

Soil suction or total suction consists of osmotic suction and matric suction as commonly accepted in unsaturated soil mechanics. Osmotic suction is referred to as osmotic potential due to the attraction that salts have for water through the phenomenon of osmotic. Matric suction is referred to as potential energy of water attracted to soil solids which resulted from capillarity and adhesion force (Saskatchewan Interactive 2007). In other words, soil suction can be understood as the free energy state of soil water (Edlefsen and Anderson 1943) or water being held by the soil particles in term of energy. The free energy of soil water can be measured in term of the partial vapor pressure of soil (Richard 1965). The thermodynamic relationship between soil suction and partial vapor pressure of soil water can be presented as follows:

$$\psi = -\rho_w \frac{R^*T}{W_{wa}} \ln \left(\frac{P_v}{P_0} \right) \quad (2.1)$$

where: ψ = soil suction or total suction,

R = universal gas constant,

T = absolute temperature,

ρ_w = water density at temperature = 20°C,

W_{wa} = molecular mass of water vapor, and

P_v, P_0 = partial, saturated vapor pressure.

Soil suction has two components, namely: matric and osmotic suctions. In equation form, this relation can be expressed as:

$$\psi = (u_a - u_w) + \pi \quad (2.2)$$

where: ψ = soil suction or total suction,

$(u_a - u_w)$ = matric suction, and

π = osmotic suction.

Matric suction can be calculated from the hydrostatic equilibrium for a capillary tube as shown in Figure 2-4. In this figure, pressure head is equal to the negative value of the elevation head (i.e., $u_w = -\rho_w gh_c$) at point C and the air pressure is atmospheric (i.e., $u_a = 0$). As a result, matric suction at point C, $(u_a - u_w)_c$ can be written as:

$$\psi_c = (u_a - u_w)_c = \rho_w gh_c > 0. \quad (2.3)$$

Pore-water in soil commonly contains dissolved salts. An increase in the concentration of dissolved salts will lead to a decrease in relative humidity and therefore pore-water pressure (Fredlund and Rahardjo 1993a). This causes osmotic suction. However the changes in osmotic suction are less significant than matric suction changes because most engineering problems in unsaturated soil are the result of environmental changes which caused the changes in matric suction (Fredlund and Rahardjo 1993a).

Figure 2-3 Categorization of unsaturated soil mechanics based on the type of engineering problem (from Fredlund and Rahardjo 1993b)

Figure 2-4 Capillary model for matric suction in soil (from Fredlund and Rahardjo 1993a)

Figure 2-5 Role of matric suction in soil (from Krahn and Fredlund 1972)

Figure 2-5 illustrates that osmotic suction is relatively constant compared with matric suction and the change in total suction is essentially due to the change in matric suction.

2.2.3. Soil - Water Characteristic Curve (SWCC)

The soil-water characteristic curve (SWCC) for a soil is usually defined as the relationship between the amount of water in the soil and the soil suction, ψ . The amount of water in the soil can be quantified in terms of gravimetric water content, w , volumetric water content, θ_w , or degree of saturation, S . In soil science, volumetric water content, θ_w , is most commonly used and SWCC is normally referred to as soil-water retention curve. The typical shape of the SWCC is sigmoid or S-shape if matric suction is plotted in a log scale. The water content of the soil decreases when its matric suction increases following the drying curve (desorption curve) and in the reverse cycle the water content will increase when matric suction decreases following the wetting curve (adsorption curve). Normally, the drying curve and the wetting curve do not coincide due to some of the air bubbles entrapped in soil and this phenomenon is known as hysteresis.

Figure 2-6 Illustration of terms for a typical SWCC
(modified from Fredlund and Xing 1994)

In Figure 2-6, parameters of the SWCC are presented: saturated volumetric water content, θ_s , the air entry value of the soil, ψ_r , the residual volumetric water content, θ_r , the water entry value ψ_w , and the residual air content, θ_a . Typical SWCC for different soil types are shown in Figure 2-7.

The air entry value of the soil, ψ_a , is defined as the matric suction required to remove water from the largest pores of the soil in the drying cycle (Brooks and Corey 1969). As the matric suction in soil increases from zero to air entry value, ψ_a , the volumetric water content, θ_w , remains constant. Beyond ψ_a , θ_w decreases steadily to the residual volumetric water content, θ_r . The residual volumetric water content is the water content at which further increases in matric suction do not change significantly the water content of the soil, and the corresponding matric suction is called residual matric suction.

Contrary to the above if water is added steadily to dry soil, matric suction decreases gradually until it reaches the water entry value, ψ_w (Hong et al 2004a, 2004b). The water entry value can be defined as the matric suction when water first enters the pores of dry soil in the wetting cycle. If water is continuously added to soil, the

water content will increase until it reaches the residual air content, θ_a . the residual air content, θ_a , is the air content that is trapped in the soil in an occluded form and does not permit replacement by water.

Figure 2-7 Typical drying SWCCs for a sandy soil, a silty soil and a clayey soil
(from Fredlund and Xing 1994)

There are a number of empirical equations proposed for the SWCC. Leong and Rahardjo (1997a) have assessed a number of these equations and have recommended that the Fredlund and Xing (1994) equation with parameter $C(\psi)$ equal to 1 be used to represent the SWCC. Thus, in this research, this equation will be used to describe the SWCC. The Fredlund and Xing equation is given as follows:

$$\theta_w = \theta_s C(\psi) \left\{ \frac{1}{\left[\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right]^m} \right\} \quad (2.4)$$

where: θ_w = volumetric water content,

θ_s = saturated volumetric water content,

$$C(\psi) = \left[1 - \frac{\ln \left[1 + \frac{\psi}{\psi_r} \right]}{\ln \left[1 + \frac{10^6}{\psi_r} \right]} \right] = 1 \text{ (assumed),}$$

a = soil parameter related to the inflection point of SWCC, ψ_a (kPa),

n = soil parameter related to the slope at the inflection point (where air-entry value occurs) in the SWCC,

m = soil parameter related to the residual water content, θ_r ,

e = natural number (i.e., 2.71828...), and

ψ_r = residual suction corresponding to the residual water content, θ_r (kPa).

2.2.4. Permeability Functions

Coefficient of water permeability, k_w , of an unsaturated soil is a function of pore space and pore-water pressure or water content. Therefore, the permeability coefficient, k_w , can generally be expressed in terms of volumetric water content, θ_w , which in turn depends on the matric suction. As a result if there is a further increase in matric suction of the soil, it will lead to further decrease in pore volume occupied by water, meaning that k_w will decrease. In other words the permeability coefficient with respect to water phase in this case is not a constant value. The plot of water coefficient of permeability, k_w , versus matric suction ($u_a - u_w$), can be described using the following equation suggested by Leong and Rahardjo (1997b):

$$k_w = \frac{k_s}{\ln \left[e + \left(\frac{\psi}{sa} \right)^{sn} \right]^{sm}} \quad (2.5)$$

where: sa , sm , sn = curve fitting constants.

The water coefficient of permeability, k_w , can be measured either in the laboratory or in the field. However, the procedure to determine the coefficient of permeability for unsaturated soil is a time – consuming process (Leong and Rahardjo 1997b). Thus, a number of permeability functions have been proposed to solve this problem.

They can be divided into three main categories namely: empirical, macroscopic, and statistical model. Leong and Rahardjo (1997b) have suggested a simple empirical equation to obtain permeability function for unsaturated soil for which a database is available. They suggested that rather than performing a complex integration procedure on the SWCC in order to yield the permeability function, the permeability function can be obtained as a product of the saturated permeability and the normalized SWCC raised to the power p . This permeability function is expressed as follows:

$$k_w = k_s \Theta^p \quad (2.6)$$

where: k_w = unsaturated coefficient of permeability,

k_s = saturated coefficient of permeability,

p = fitting parameter corresponding to the slope of permeability function,
and

$$\Theta = \frac{\theta_w}{\theta_s} = \text{normalized volumetric water content.}$$

Fredlund et al. (2001) evaluated Equation (2.6) with 323 sets of permeability data using the soil database in the SoilVision program (Fredlund 1997). The statistical analysis showed that the mean value of p is 3.29 and p could vary over a considerable range of 0.64 to 15.03. Generally p is greater than 1.0 and that there is a trend towards a larger p value for soil with higher plasticity.

2.2.5. Hysteresis

As mentioned in Section 2.2.3, all SWCCs show hysteresis. Hysteresis is a common feature in unsaturated soil which is caused by differences in sizes between the primary pores and interconnecting pore throats during the wetting and drying cycle. This phenomenon can be explained by using the “ink bottle” effect as shown in Figure 2-8.

When an empty thin tube is placed in a water tank, the water inside the tube will rise up until it reaches the maximum capillary height or the equilibrium state. In the SWCC's cycles in Figure 2-6, the wetting SWCC is analogous to what happen in

the case of the empty tube. The water inside the tube gradually rises until it reaches the equilibrium condition. In the drying SWCC, the reverse is the case.

Figure 2-8 Ink bottle effect in capillary tube (from Taylor 1948)

If in the above two cases, the radius of the tube, r , is constant, the equilibrium conditions in the wetting and drying cycles are the same (Figure 2-8a). Nevertheless, if the tubes have various radii, r , the equilibrium condition reached will be different as shown in Figure 2-8(b) and Figure 2-8(c). The wetting cycle in the expanded tube (Figure 2-8b) corresponds to the “bulb” portion being empty while in the drying cycle in Figure 2-8(c), the “bulb” space is being filled with water.

2.2.6. Moisture Movement in Unsaturated Soil

In this section, some theories relating to water flow in unsaturated soil are presented. The physical flow law in soil is briefly described followed by Darcy’s law for unsaturated soils. Following the presentation of the mass conservation concept is the derivation of Richards’ equation for infiltration. The theory of infiltration into soil and methods for solving flow equation are then discussed in detail.

2.2.6.1. Physical Flow Law in Soil

In saturated soil, water flow refers to the flow of water through soil voids only. But in unsaturated soil, water flow must include the flow of air. Flow of air only occurs when the air phase exists in a continuous form. In the understanding of the flow mechanism in both saturated and unsaturated soil the knowledge on the driving potential is required. In general, the hydraulic gradient is the driving potential of water flow in saturated and unsaturated soil. The flow of water in unsaturated soil can be defined more appropriately in terms of hydraulic head gradient, which is the combination of pressure and elevation head gradients (Fredlund and Rahardjo 1993a). This relationship is expressed as follows:

$$h_w = y + \frac{u_w}{\gamma_w} \quad (2.7)$$

where: h_w = hydraulic head or total head [L],

y = elevation from the datum point [L],

u_w = pore water-pressure [$ML^{-1}T^{-2}$], and

γ_w = unit weight of water [$ML^{-2}T^{-2}$].

The hydraulic gradient, i , is defined as the ratio of hydraulic head, h_w , to the length of the flow path, l , given as:

$$i = \frac{h_w}{l} \quad (2.8)$$

Generally, water will flow from a point with high total head to a point with lower total head. Meanwhile, the magnitude of the water flow in soil is largely dependent on the magnitude of hydraulic gradient.

2.2.6.2. Darcy's Law

Darcy formulated the flow equation while studying water infiltration into a soil column in 1856. Darcy's law states that, flow rate or discharge velocity of water through a porous media (e.g., soil) is proportional to hydraulic head gradient and

coefficient of permeability. This equation is usually written as:

$$v_w = -k_w \frac{\partial h_w}{\partial y} = k_w i \quad (2.9)$$

where: v_w = discharge velocity [LT^{-1}],

k_w = water coefficient of permeability [LT^{-1}], and

$\partial h_w / \partial y$ = hydraulic gradient, i , in y direction.

In lieu of using an expression of flow rate, v_w , the above equation can sometimes be presented in term of volumetric water flow, Q_w , as follows:

$$Q_w = -k_w \frac{\partial h_w}{\partial y} At = k_w i At \quad (2.10)$$

where: A = cross section of porous media perpendicular to the flow direction [L^2],
and

t = time for a certain flow quantity, Q_w , [T].

Darcy's law was initially established for flow in saturated soil, but later, some researchers (Richards 1931, Childs and Collis – George 1950) showed that it can also be used for describing the flow through unsaturated soil. However, the utilization of Darcy's law for unsaturated soil should take into account the basic concept of unsaturated soil flow condition in which the water coefficient of permeability, k_w , is no longer a constant but varies with changes in volumetric water content, θ_w , or matric suction, ψ , in the unsaturated soil (Fredlund and Rahardjo 1993a). Therefore, Equation (2.9) should be in the following form:

$$v_w = -k_w(\theta_w) \frac{\partial h_w}{\partial y} = k_w(\theta_w) i \quad (2.11)$$

where: $k_w(\theta_w)$ = permeability as a function of θ_w .

2.2.6.3. Fick's Law

It is an acceptable assumption that the rate of flow of an atmospheric entity is directly related to the gradient of that entity. For example, consider the transfer of

water vapour in one dimension (the z axis is chosen), the transfer expressed as a mass flux per unit area, J_a , is proportional to the concentration gradient. The above explanation is well known as Fick's law (1855). The law can be applied to unsaturated soil as it is consistent with the permeability concept for the water phase. Fick's law can be expressed as follows:

$$J_a = -D_a \frac{\partial C}{\partial y} \quad (2.12)$$

where: J_a = mass rate of air flow through a unit area of the soil (kgm^{-3}),

D_a = molecular diffusion coefficient (m^2s^{-1}),

C = concentration of the air expressed in term of the mass of air per unit volume of soil, and

$\partial C / \partial y$ = concentration gradient in the z direction.

A modified form of Fick's law is derived from Equation (2.12) by adding a coefficient of transmission for air flow through soil, D_a^* (Fredlund and Rahardjo 1993b). The coefficient of transmission is a function of the volume-mass of the soil (i.e., S and n) and the air density, ρ_a . The equation is written as follows:

$$J_a = -D_a^* \frac{\partial C}{\partial u_a} \frac{\partial u_a}{\partial y} = -D_a^* \frac{\partial [\rho_a (1-S)n]}{\partial u_a} \frac{\partial u_a}{\partial y} \quad (2.13)$$

where: u_a = pore air pressure (kPa), and

$\partial u_a / \partial y$ = pore air pressure gradient in the y direction.

The coefficient of transmission, D_a^* , can be related to the air coefficient of permeability, k_a . A steady-state air flow can be established through an unsaturated soil specimen with respect to average matric suction or an average degree of saturation. Consequently, within the unsaturated soil, the air coefficient is assumed to be a constant value. By applying a constant air density, ρ_{ma} , the mass rate of air flow through the soil specimen that is measured at the exit point can be written as follows:

$$v_a \frac{\partial V_a}{\partial t} = -D_a^* \frac{1}{\rho_{ma}} \frac{\partial u_a}{\partial y} \quad (2.14)$$

where: v_a = pore air pressure,

ρ_{ma} = constant air density corresponding to the pressure used in the measurement, and

$\partial V_a / \partial t$ = volume rate of the air flow across a unit area.

The pore-air pressure in Equation (2.14) can be expressed through the pore-air pressure head, h_a , within the soil specimen as follows:

$$v_a = -D_a^* g \frac{\partial h_a}{\partial y} \quad (2.15)$$

where: g = acceleration of gravity, and

$\partial h_a / \partial y$ = pore air pressure head gradient in the y direction.

As Equation (2.14) has a similar form with Darcy's law (i.e. Equation (2.10)), Equation (2.14) can be rewritten as follows:

$$v_a = -k_a \frac{\partial h_a}{\partial y} \quad (2.16)$$

where: the relationship between the air coefficient of transmission, D_a^* , and the air coefficient of permeability, k_a , is defined as follows:

$$k_a = D_a^* g \quad (2.17)$$

The hydraulic head gradient in Equation (2.14) consists of the pore-air pressure head gradient therefore Equation (2.15) has been used to calculate the air coefficient of permeability.

2.3. INFILTRATION PROCESS

Infiltration is a basic physical phenomenon of great importance to the hydrologist concerned with modelling the rainfall process (Bloomfield 1979). In general infiltration can be defined as a loss from rainfall which influences the peak and total flow within a rainfall event. The theoretical background to infiltration behaviour is drawn from studies of water movement in unsaturated porous media. In this section a literature review of the modelling of one-dimensional infiltration process into soil is presented.

2.3.1. Analytical Solution for One-dimensional Infiltration

The one dimensional flow in unsaturated soil (i.e., Richards' equation) formulated using Darcy's law (i.e., Equation (2.10)) is given as follows:

$$\frac{\partial \theta_w}{\partial t} = \frac{\partial}{\partial y} \left[k_w \frac{\partial (h_p + y)}{\partial y} \right] \quad (2.18)$$

where: θ_w = volumetric water content,

h_p = pressure head,

y = the vertical coordinate, positive upwards, and

k_w = permeability function with respect to water in y direction.

Analytical solutions to the Richards' equation for unsaturated water flow, Equation (2.18), are generally very difficult because of the nonlinearity in soil permeability parameters. The problem is amplified in multi-layered soil with sharply contrasting hydraulic properties since water movement is generally transient (Choo and Yanful 2000). In order to deal with the above problem, some empirical exponential permeability functions for k_w have been suggested such that Richards' equation can be linearized whereby an analytical solution can be obtained. Philip (1972) proposed the following exponential permeability function for the mathematical study of certain unsaturated heterogeneous soils, as follows:

$$k_w = k_s e^{\alpha(h_p + \beta y)}, \quad \alpha > 0 \text{ and } k_s > 0. \quad (2.19)$$

where: α = parameter characterizing the dynamic of gravity and capillarity for soil-water movement in the particular soil, and

β = fitting parameter.

When α approaches infinity, the flow due to gravity dominates the soil-water movement. When α approaches zero, the flow can be either a saturated or a purely capillary-driven unsaturated flow (Parlange and Hogarth, 1985). Equation (2.19) was also recognized as a simple, adaptable and mathematically convenient equation for analytical solution of the infiltration problem (Pullan, 1990). Later, Philip

(1974) proposed a more general equation for unsaturated heterogeneous soil which can be written as:

$$k_w = k_s e^{\alpha[h_p + \beta(\lambda x + \mu y + \nu z)]} \quad (2.20)$$

where: λ, μ, ν = cosines of the normal to the planes of uniform k_w such that $(\lambda^2 + \mu^2 + \nu^2) = 1$.

Jaynes and Tyler (1984) proposed the following empirical equation for k_w , recognizing it as a modification of that proposed by Gardner (1958):

$$k_w = k_s e^{(b\sqrt{|h_p|})} \quad (2.21)$$

where: b = empirical constant.

Almost all of the analytical solutions for Richards' equation are based on the exponential hydraulic parameter model or the modification of Gardner (1958) equation by other researchers. The exponential model allows linearizing the governing flow equation and facilitates the process of deriving the analytical solution of the governing equation. This approach is widely known as quasi-linear approximation.

In this review, the work of Srivastava and Yeh (1991) which also used quasi-linear approximation is examined more closely. Srivastava and Yeh assumed the following exponential constitutive relations for k_w and θ_w :

$$k_w = k_s e^{\alpha^* h_p} \quad (2.22)$$

$$\theta_w = \theta_r + (\theta_s - \theta_r) e^{\alpha^* h_p} \quad (2.23)$$

where: α^* = soil pore size distribution parameter.

Substituting Equations (2.22) and (2.23) into Equation (2.18) to yield a linearized Richards' equation, Equation (2.18) can be expressed as follows:

$$\frac{\alpha(\theta_s - \theta_r) \partial \theta_w}{\partial t} = \frac{\partial^2 k_w}{\partial y^2} + \alpha \frac{\partial k_w}{\partial y} \quad (2.24)$$

Using the Laplace transformation technique and residue theorem (Ozisick 1980), Srivastava and Yeh (1991) obtained the analytical solution for Equation (2.24). The solution for Equation (2.24) is expressed in terms of the dimensionless coefficient of permeability, k' , and the outflow at the water table, q_{out} , as follows:

$$k' = q'_B - (q'_B - e^{\alpha h_{p0}}) e^{-z} - 4(q'_B - q'_A) e^{(L-y)/2} e^{-t/4} \sum_{n=1}^{\infty} \frac{\sin(\lambda_n y') \sin(\lambda_n L') e^{-\lambda_n^2 t'}}{1 + (L'/2) + 2\lambda_n^2 L'} \quad (2.25)$$

and

$$q_{out} = k_s \left[\frac{\partial k'}{\partial y'} + k' \right]_{z=0} = k_s q'_B - 4k_s (q'_B - q'_A) e^{(L-z)/2} e^{-t/4} \sum_{n=1}^{\infty} \frac{\lambda_n \sin(\lambda_n L') e^{-\lambda_n^2 t'}}{1 + (L'/2) + 2\lambda_n^2 L'} \quad (2.26)$$

In Equation (2.25) and (2.26) all the symbols with a prime (“ ’ ”) denote dimensionless parameters and are defined as follows:

$$\begin{aligned} q'_A &= \frac{q_A}{k_s} & q'_B &= \frac{q_B}{k_s} \\ y' &= \alpha z & L' &= \alpha L \\ k' &= \frac{k_w}{k_s} & t' &= \frac{\alpha k_s t}{\theta_s - \theta_r} \end{aligned} \quad (2.27)$$

where: q_A = initial boundary flux at the soil, which was used to determine the initial pressure distribution in the soil,

q_B = boundary flux at the soil surface for times greater than zero,

h_{p0} = initial pressure head, and

L = the depth to the water table, so $z = 0$ at the water table and $z = L$ at the surface of the soil column.

Srivastava and Yeh (1991) solved the above equations for a homogeneous soil column and a two layered soil column to obtain the pressure head profiles and moisture profiles for various outflow. The procedures to solve these equations are quite tedious. Srivastava and Yeh found that the constitutive equations, Equations (2.22) and Equation (2.23), do not accurately reproduce the soil behaviour near saturation. Further details can be found in Srivastava and Yeh (1991). In Chapter 4 the analytical results reported by Srivastava and Yeh (1991) will be used to verify the model proposed in this study.

2.3.2. Numerical Solutions for One-dimensional Infiltration

A review of numerical solutions for one-dimensional infiltration is presented in this section. The models are divided into three categories. In the first category, the water flow equation in unsaturated porous media in the form of Richards' equation and finite difference method was used to handle the non-linearities of material properties. In the second category, the saturated-unsaturated flow in deformable soil skeleton was solved with a combination of integral finite difference algorithm and explicit-implicit finite difference scheme. Lastly in the third category, the finite element method was used in a computer programme called SEEP/W from Geoslope International Limited for unsaturated flow. The experimental application of the models is also presented in this section.

2.3.2.1. Rubin and Steinhardt (1963)

Rubin and Steinhardt (1963) used the finite difference technique to study the problem of infiltration into an extremely dry soil column. In this model, they used the diffusivity form of Richards' equation to deal with the early absorption of infiltration into a vertical unsaturated soil column. The governing equation in the form of diffusivity can be written as follows:

$$\frac{\partial}{\partial y} \left[D(\theta_w) \frac{\partial \theta_w}{\partial y} \right] = \frac{\partial \theta_w}{\partial t} \quad (2.28)$$

where: $D(\theta_w)$ = diffusivity coefficient as a function of θ_w .

In considering soil-water relations during rain infiltration, Rubin and Steinhardt assumed that ponding of water on soil surface can occur only if rainfall intensity exceeds the saturated soil permeability. Otherwise the soil will tend to attain the uniform saturated condition at which rainfall intensity exactly matches saturated soil permeability. To demonstrate this concept, the form of empirical functions for the relations between water coefficient of permeability, k_w , volumetric water content, θ_w , and matric suction, $(u_a - u_w)$, were formulated. These equations were used to fit the experimental data for a semi-infinite column of Rehovot sand (that was initially very dry). The equations can be expressed as follows:

$$(u_a - u_w) = 11.3 + \left(\frac{3.19}{\theta_w} \right) - 0.05e^{1.5} + e^{-575\theta_w} + 16.3 \quad (2.29)$$

$$k_w = \frac{8400}{(u_a - u_w) + 14.45^5} \quad (2.30)$$

Computations based on Equations (2.28) and (2.29) showed that the results of moisture content and pressure head match well with the assumption and the experimental data. Although, the diffusivity formulation is useful in solving the aforementioned highly nonlinear problem, it has been rightly pointed out that the diffusivity equation cannot be used when the porous medium is heterogeneous (Narasimhan and Witherspoon 1978). This problem occurs at the interfaces of the materials. At these interfaces the distribution of volumetric water content, θ_w , is discontinuous which is termed the “jump” phenomenon. So it is desirable to be able to solve the infiltration problem with $(u_a - u_w)$ as the dependent variable.

2.3.2.2. Bloomfield (1979)

Bloomfield (1979) stated that most catchment models are basically lumped one-dimensional infiltration models. To continuously simulate the water status of catchments, several other catchment processes are generally added to the simple infiltration model. In his catchment model, Bloomfield introduced an infiltration model based on the finite difference method and some assumptions on pressure head, h_p , and hydraulic conductivity, k_w , relationships. The basic governing

equation in the infiltration model in soil is the continuity equation. This equation is the well known Richards' one-dimensional infiltration equation and is given as follows:

$$C(i) \frac{\partial h_p}{\partial t} = \frac{\partial}{\partial y} \left(k_w \frac{\partial h_p}{\partial y} \right) + \frac{\partial k_w}{\partial y} \quad (2.31)$$

where: $C(i) = d\theta_w/dh_p$,

An alternative form of the above equation can be expressed as:

$$\frac{\partial \theta_w}{\partial t} = \frac{\partial}{\partial y} \left[D(\theta_w) \frac{\partial \theta_w}{\partial y} \right] + \frac{\partial k_w}{\partial y} \quad (2.32)$$

where: $D(\theta_w) = \text{diffusivity function} = k_w \cdot (d\theta_w/dh_p)$.

Naturally, to solve Equation (2.31), the $C(i)$ and k_w functions must be specified, whilst to solve Equation (2.32), the $D(\theta_w)$ and k_w functions are needed. However in this one-dimensional infiltration model, Bloomfield used a simple algebraic expression to specify the relationships of soil-water characteristic, h_p , and hydraulic conductivity, k_w . In the expression, some idealized assumptions were made in order to describe the typical SWCC based on observed h_p and k_w relations. As part of this idealization, hysteresis has been neglected in the h_p relation. The SWCC was specified by three straight line segments joined by a parabolic curve. This h_p curve in the infiltration model is considered to be a wetting or drying boundary curve, or an "average position" of h_p relation. The general form used to specify the k_w relation is:

$$k_w = k_s \left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r} \right)^p \quad (2.33)$$

where: $p = \text{constant, assumed to be 4.}$

Schematic representations for the relations of h_p and k_w are shown in Figure 2-9 and Figure 2-10, respectively.

Figure 2-9. Soil-water characteristic curve

Figure 2-10. Hydraulic permeability function

Bloomfield used the semi-infinite lower boundary condition in his study because it allows simple specification of the initial condition within the soil as uniform head. Hence for the examination of infiltration models, uniform water content is again a simplifying assumption. Due to the general requirements of catchment models, the

infiltration curve was presented as hourly volumes. For infiltration study a shorter time interval ($\ll 1h$) is needed, particularly during periods of intense rainfall.

2.3.2.3. Narasimhan et al. (1977)

Narasimhan et al. (1977), Narasimhan and Witherspoon (1977), Narasimhan and Witherspoon (1978) developed a numerical model to simulate saturated-unsaturated groundwater flow in which the deformation of soil skeleton is handled according to Terzaghi's (1925) one-dimensional consolidation theory. The soil deformation may be non-elastic, and compressibility as well as the permeability characteristics of the saturated soil may be non-linear functions of effective stress. In addition, the permeability as well as the moisture characteristics show hysteresis phenomenon. The assumption that the air phase in the zone of partial saturation is continuous and is everywhere at atmospheric pressure was made in this model.

The governing equation for transient movement of groundwater in variably saturated deformable porous media was formulated from the fundamental equation of mass conservation, fluid mass capacity, and material properties. The permeability function assumed to be an exponential function of void ratio of the soil is as follows:

$$k_w = k_s \exp \left[\frac{2.303(e - e_0)}{C_k} \right] \quad (2.34)$$

where: C_k = slope of the best-fit straight line for the relationship of e versus $\log k_w$.

The governing equation can be written as follows:

$$G + \int_{\Gamma} \rho_w \frac{k_w \rho_w g}{\mu} \nabla (y + h_p) \cdot n dT = M_c \frac{dh_p}{dt} \quad (2.35)$$

where: G = general symbol for source or sink, associated with a finite domain,

ρ_w = water density,

V = volume of soil,

n = void ratio,

M_c = fluid mass capacity,

Γ = surface bounding a finite sub region, and

h_p = pressure head or pore-water pressure expressed in equivalent height of soil column.

An integrated finite difference method was used for numerically solving the governing equation of saturated-unsaturated flow in deformable porous media. The use of explicit stability scheme is recognized as a local phenomenon in an explicit-implicit procedure that helps the scheme in marching the time domain for the program. In this scheme the explicit change in potential are first computed for all elements in the system, after which implicit corrections are made only for those for which the stable time step is less than the time step being used. Time step sizes are automatically controlled in order to optimize the number of iteration, to control the maximum change in potential during a time step, and to obtain desired outputs. With these advantageous conditions, the combination of the two above-mentioned algorithms was reported to be an efficient tool for analyzing linear and nonlinear flow problems in porous media. By using this combination, a computer program named TRUST was developed for studying transient seepage in unsaturated deformable porous media.

2.3.2.4. Finite element model

Similar to other partial differential equations in science and engineering, the partial differential equation for water flow in unsaturated soil can also be analyzed with the finite element method (FEM). One commercially available finite element program, SEEP/W (Geo-slope/W 1998), can be used for solving the water phase partial differential equation in unsaturated soil. The basic procedures of FEM in this case are:

- Discretization of the problem domain by dividing it into elements.
- Determination of the property matrix for each element (i.e., permeability matrix, gradient matrix, etc.).
- Implementation of the boundary conditions.

- Solving the linear equations based on the above matrices.
- Determination of other necessary unknown parameters, such as total head, water content, storage etc.

The finite element formulation of the governing equation is formed by using Equation (2.18) that can be derived by applying the Galerkin method of weighted residual. The derived finite element equation for one-dimensional water flow is as follows:

$$t_e \int_A ([B]^T [k_w] [B]) dA \{h_{wn}\} = q t_e \int_L (\langle N \rangle^T) dL. \quad (2.36)$$

where: $[B]$ = gradient matrix,

$[k_w]$ = element coefficient of permeability matrix,

$\langle N \rangle^T \langle N \rangle = [M]$ = mass matrix,

$\langle N \rangle$ = vector of interpolation function,

$\{h_{wn}\}$ = vector of hydraulic heads at the nodal points,

t_e = thickness of the element,

q = unit flux across the side of an element,

$\int_A$ = integral over the area, and

$\int_L$ = integral over the length from corner node to corner node.

The transient and steady state seepage analyzed in SEEP/W (Geo-slope/W 1998) are based on Equation (2.36). Choo and Yanful (2000) used SEEP/W for investigating the dynamics of water flow in multi-layered soil columns in the laboratory. In their laboratory column study, a two-layered or three-layered soil was packed into a 1.0m high perspex cylinder.

For the drainage case in a two-layered soil column without evaporation, the finite element program produced reasonable results in comparison with laboratory measured pore-water pressure profiles up to 7.5 hrs. However, the predicted pore-water pressure profiles were not as good for elapsed times of 3 days and above. The reason was attributed to the discontinuity in the water phase of the soil column as water pockets were “locked inside”. The phenomenon was also recognised by Barbour and Yanful (1994). To date, there is no available method used for

estimating unsaturated permeability function that takes this phenomenon into account.

In the case of drainage in a three-layered soil column with evaporation, the finite element results showed good agreement in the coarse sand. The pressure head results in the sand-bentonite and fine sand layer were not as good as the ones from coarse sand layer. The reason for this problem is because the finite element model only modelled liquid water flow and was therefore not accurate in simulating flow where vapour transport was significant. Furthermore, the presence of discontinuous water pockets in the drainage sand was not captured by the method used for estimating the permeability function of the soil. The results in this case emphasize the need for accurate knowledge of unsaturated hydraulic properties in unsaturated flow modelling. The experiment set up, soil properties and modelling procedure can be found in details in Chapter 4.

2.4. EVAPORATION PROCESS

Evaporation of soil moisture into the atmosphere is a complex process that can be strongly tied to the distribution of temperature in the soil. When the soil surface is not sufficiently wet to keep the air in contact with it fully saturated with water vapour, evaporation becomes controlled by the rate at which water can flow upward to the dry surface (Milly 1984b). This soil moisture can occur in either the liquid or the gaseous phase and may be driven by gradient of either hydraulic potential (i.e., pressure head) or temperature.

In general, evaporation can be divided into two terms of vapour flow mechanism because evaporation process takes place over soil surface and vegetation cover. Firstly, the term evaporation usually refers to free water and bare soil surfaces. Secondly, the process in which water vapour transfers through vegetation cover from water is termed transpiration. The effect of transpiration on slope stability is much more complicated and is not within the scope of the present research. Hence in this section, the theoretical consideration of evaporation from the ground surface is briefly described.

2.4.1. Energy Balance

Evaporation occurs when liquid water is converted into water vapour and transferred into the atmosphere in this form. The process can only occur naturally if there is an input of energy either from the sun or from the atmosphere itself, and is controlled by the rate at which the energy, in the form of water vapour can diffuse away from the ground surface (Wilson 1990). Therefore, soil evaporation is an integral part of a complex hydrologic cycle. In this complex cycle, the fundamental principle of energy balance is reviewed first. The determination of actual evaporation is subsequently presented in the next section.

The process of natural evaporation is the change of phase from liquid water to water vapour which occurs within a limited soil zone near the interface between the atmosphere and the earth surface in response to the energy input from the sun. In water vapour, the water molecules are generally ten or more molecular diameters apart depending on vapour pressure (Shuttleworth 1976). In fact, at this separation the intermolecular force is very small. From liquid water it is necessary to increase the separation between all the water molecules to change to water vapour. To do this, it is necessary to supply energy to overcome the force for holding the water molecules together. The amount of energy required is directly related to the number of water molecules, which is in turn directly proportional to the mass of water involved. The amount of energy per unit mass of liquid water required to transform liquid water to water vapour is termed the latent heat of vaporization of water.

Thermal conduction occurs over limited distances very close to the soil surface at which the radiant energy is intercepted. Above the soil surface the primary exchange mechanism is thermal convection and thermal convection occurs only when mass motion of the air results in the transport and mixing of the heat content of the air between different levels in the atmosphere. The flow of energy between the surface and the atmosphere by the combined process of thermal conduction and thermal convection is considered as sensible heat (Shuttleworth 1976). The flux of sensible heat is normally away from the surface in the day time conditions when the temperature of the ground is greater than that of the atmosphere. On the contrary

this process is usually towards the surface at night when it supports the outward flux of radiation.

Figure 2-11 illustrates schematically the radiation balance at the earth's surface. In this diagram, S_T is the total incoming solar radiation in both direct and diffuse forms, S_R is that solar radiation immediately reflected at the surface, while L_D and L_U are the downward long wave radiation to the surface and upward long wave radiation from the surface, respectively.

In general the region that the incoming energy is divided extends over a finite height range between soil surface and the level at or above the vegetation at which radiation is measured. Within this region, temperature changes occur in both atmosphere and vegetation, and changes the absolute humidity of the air. Such changes represent a loss or gain in the energy available for partition into latent and sensible heat for any particular period, the magnitude being proportional to the rate of change of temperature and humidity with time (Shuttleworth and Calder 1979). In this situation, the energy lost or gained is termed storage S . Within this region, the energy absorbed or released by the biochemical processes of photosynthesis and respiration from the plants is called biochemical storage P .

Figure 2-11 Schematic diagram of the radiation balance at earth's surface
(from Shuttleworth 1979)

A component of the energy usually of more importance is the energy lost by thermal conduction into the ground. This soil heat flux, G , represents a loss of

energy during the day and again at night time. It is very significant at particular times of day for short vegetation and bare soil. The integrated value over a day is normally fairly small, but the long term integrated value can still have some influence on seasonal evaporation rates (Sinclair et al. 1975).

Although the majority of energy as sensible and latent heat leaves normal to the earth's surface, horizontal gradient of temperature and humidity observed across the ground necessarily imply that some of this energy is leaving parallel to the surface advection. The horizontal flux divergence, D , is generally neglected in energy balance studies but short range advection can be an important coefficient in the medium and large scale advection (Shuttleworth 1979).

The available energy, A , for a complete stand of vegetation is formulated by gathering the above terms and the formulation is expressed as follows:

$$A = R_N - D - G - S - P \quad (2.37)$$

or:

$$R_N = (H_s + H_l) + D + G + S + P \quad (2.38)$$

where: R_N = the net radiation of all measured wavelengths, and

H_s = sensible heat flux, H_l = latent heat flux of evaporation.

In the numerical models of vegetation/atmosphere interaction involving energy partition at several different levels in the canopy, it is necessary to take into account the energy available at each height (Shuttleworth and Calder 1979).

Black and Goldstein (1977) presented the components of heat flux over a stand of mixed orchard and rye grass that was irrigated (Figure 2-12). It might be quickly realized that in Figure 2-12, the latent heat flux, A , is the primary energy flux. In humid regions with freely available water at the surface, the latent heat becomes very large and dominates the energy budget. In fact, its magnitude may approach or equal the net radiation of all wavelengths or total net radiation, R_N . On the contrary, the opposite is true for arid environments where very little water is maintained at

the soil surface for evaporation. Consequently, in this case the sensible heat, H , becomes the dominating coefficient.

The most important concept that can be withdrawn from the above determination is the latent heat flux of evaporation, H_l , that specifies the evaporative mass flux of water across the ground surface, E , as follows:

$$E = \frac{H_l}{L_v} \quad (2.39)$$

where: L_v = latent heat of evaporation.

As presented before, soil water balance consists of two main basic inputs, namely: atmospheric water balance and surface radiation balance. Blight (1997) stated that the soil water balance controls the state of moisture in the unsaturated soil. Blight also proposed a simplified model of soil water balance as follows:

$$\text{Water input} = \text{water output} + \text{water stored in soil} \quad (2.40)$$

Figure 2-12 Energy balance over an irrigated mixed
(from Black and Goldstein 1977)

Each term in Equation (2.40) can be detailed in order to have a clear overview of this model. The details are as follows:

$$\text{Water input} = \text{infiltration} = \text{precipitation} - (\text{interception} + \text{runoff}) = P - (I + R_{\text{off}}) \quad (2.41)$$

$$\begin{aligned} \text{Water output} &= \text{water lost by evapotranspiration} + \text{water recharge to water table} \\ &= ET + R_{\text{ech}} \end{aligned} \quad (2.42)$$

$$\text{Water stored} = \text{change in total water stored in soil} = \Delta S \quad (2.43)$$

Substituting and regrouping the above three equations give the final equation illustrating Blight's model:

$$P - (I + R_{\text{off}}) = ET + R_{\text{ech}} + \Delta S \quad (2.44)$$

Blight's model is illustrated in Figure 2-13.

Figure 2-13 Components of the soil water balance (from Blight 1997).

2.4.2. Evaporative Flux

The evaporative flux can be divided into two types namely potential and actual evaporative flux. The potential evaporative flux refers to the evaporation from free water surface (WMO, 1974), while the actual evaporative flux refers to the evaporation from soil surface (Wilson, 1990). The determination of potential evaporative flux can be carried out by theoretical calculation as well as direct measurement.

In 1802, Dalton proposed a mass transfer method to specify the potential evaporative flux as follows:

$$E_p = f_v (P_{vs}^w - P_v^a) \quad (2.45)$$

where: E_p = potential evaporation flux,

f_v = transmission function depends on the mixing characteristic of the air in the atmosphere above evaporation surface = $f_v = (0.44 + 0.188u)$,

P_{vs}^w = saturation vapour pressure of water at the water surface,

P_v^a = vapour pressure of the air above the water surface, and

u = wind speed.

The mass transfer method was only recommended to approximate the potential evaporative flux because of difficulties in specify the surface temperature (after Wilson, 1990). Penman (1948) combined the mass transfer method (i.e., Equation (2.45)) and energy balance in soil surface (i.e., Equation (2.38)) to establish a more reliable formulation for evaporation from open water surface:

$$E_p = \frac{\Gamma R_N + v f_v^* (P_{vs}^w - P_v^a)}{\Gamma + v} \quad (2.46)$$

where: Γ = slope of the saturation vapour pressure versus temperature curve at the mean temperature of the air,

v = psychometric constant,

f_v^* = transmission function dependent on the mixing characteristic of the air in the atmosphere above evaporation surface = $0.4(1 + 0.17u_2)$, and

u_2 = wind speed at 2m above the water surface.

Rosenberg et al. (1983) concluded that the Penman formulation is most widely accepted for estimating the potential evaporation especially for evapo-transpiration. However, Granger (1989) observed that the Penman method gives reasonable evaluation of potential evaporation albeit typically overestimating its value. In general, different methods gave different values of potential evaporation due to the method's purpose and climatic influences (Wilson 1990).

Instead of a theoretical approach, the direct measurement (i.e., pan evaporation) can also be used in estimating evaporation. Pan evaporation is one of the most popular

methods due to its simplicity and believable application (Brutsaert, 1982). Besides, evapo-transpiration can also be estimated as 60 to 90% of the pan evaporation.

Wilson (1990) modified Penman (1948) model to estimate actual evaporative flux from unsaturated soil surface. The equation is presented as follows:

$$E_a = \frac{\Gamma R_N + v f_v^* P_v^a (B - A)}{(\Gamma + vA)} \quad (2.47)$$

where: E_a = actual evaporative flux from unsaturated soil surface,

$B = 1/\text{relative humidity of the air, and}$

$A = 1/\text{relative humidity of soil surface.}$

Equation (2.47) specifies the evaporation from soil surface based on climatic data input therefore it is more amendable for practical use. When A equal unity (i.e., soil surface at saturation state), Equation (2.47) will revert to the well known Penman equation (i.e., Equation (2.46)).

Wilson et al. (1997) presented a new concept to evaluate the actual evaporative flux from potential evaporation (i.e., E_a/E_p) which can be either determined theoretically or experimentally (i.e., pan evaporation). Assumption was made that under the same climatic influences (i.e., laboratory condition) evaporation from the water and soil surface is controlled primarily by atmospheric condition and surface temperature. Therefore, the evaporative flux from soil surface can be calculated following the same fashion as from water surface. This implies that the transmission function, f_v , is the same for the soil and water surfaces. The equation representing the concept of E_a/E_p is given as follows:

$$\frac{E_a}{E_p} = \frac{(P_v^s - P_v^a)}{(P_{vs}^w - P_v^a)} \quad (2.48)$$

where: P_v^s = actual vapour pressure at soil surface.

Equation (2.48) can be used for all types of soil (Wilson and Fredlund, 2000). The concept of E_a/E_p gives a clear relationship of actual soil evaporation with matric

suction in the soil. Using Equation (2.48) Wilson (1990) showed that the actual evaporative flux from soil is equal to potential evaporative flux until the matric suction at the soil surface exceeds 3000 kPa.

2.5. COUPLED MOISTURE AND HEAT MOVEMENT MODELS

In this section, the review concentrates on two approaches to deal with moisture content and heat distribution in soil during evaporation. The first approach refers to conservation laws of mass and energy in soil (Philip and de Vries, 1957 type model) whereas the second approach is based on constitutive relations (Dakshanamurthy and Fredlund, 1981). In the Philip and de Vries (1957) model, moisture and heat flow were driven by the combined effects of moisture and temperature gradients. However the widely accepted concept in geotechnical engineering is that the moisture in soil flows due to the total head gradient. Therefore, the model has been modified several times in order to satisfy the requirement of moisture flow as well as for specific problems of evaporation in deformable soil. On the other hand, Dakshanamurthy and Fredlund (1981) considered the combined transient flow of liquid water and water vapour in deformable unsaturated soil. The framework has been used by Wilson (1990) to model non-deformable unsaturated soil. These works will be reviewed in the following section.

2.5.1. Philip and de Vries (1957)

The general form of physically based model for water and heat transport in soil was first developed by Philip and de Vries (1957) based on the conservation of mass and energy laws. This work consolidated most of the knowledge on the influence of temperature gradient on moisture movement for a better understanding of coupled moisture and heat movement in soil (Socphocleuos, 1979, Camillo et al. 1983, Zhou et al. 1998). The (θ_w-T) based moisture and heat flow relationship and definition of transport coefficients from Philip and de Vries (1957) are summarized as follow:

$$\text{Moisture flow:} \quad \partial\theta_w/\partial t = \nabla(D_\theta \nabla\theta_w) + \nabla(D_T \nabla T) + \nabla(k_w) \quad (2.49)$$

$$\text{Heat flow:} \quad C(\partial T/\partial t) = \nabla(\lambda \nabla T) + \rho_l L_v \nabla(D_{\theta v} \nabla\theta_w) \quad (2.50)$$

Transport coefficients:

$$\text{Vapour moisture diffusivity, } D_{\theta v} = (\beta \theta_a) v D_{\text{vap}} \left[W_{\text{wa}} / (R^* T) \right] P_{\text{vs}} (\partial R_h / \partial T) \quad (2.51)$$

$$\text{Thermal vapour diffusivity, } D_{Tv} = \zeta (\beta \theta_a) v D_a v \left[W_{\text{wa}} / (R^* T) \right] R_h (dP_{\text{vs}} / dT) \quad (2.52)$$

$$\text{Liquid moisture diffusivity, } D_{\theta l} = k (\partial h_p / \partial \theta)_T \quad (2.53)$$

$$\text{Thermal liquid diffusivity, } D_{Tl} = \gamma h_p k \quad (2.54)$$

$$\text{Isothermal moisture diffusivity, } D_{\theta} = D_{\theta v} + D_{\theta l} \quad (2.55)$$

$$\text{Thermal moisture diffusivity, } D_T = D_{Tv} + D_{Tl} \quad (2.56)$$

where: ρ_l = liquid water density,

λ = thermal conductivity,

∇ = gradient operator,

D_{vap} = molecular diffusivity of water vapour in air

$$= 0.229 \times 10^{-4} \left(1 + \frac{T}{273} \right)^{1.75} (\text{m}^2 \text{s}^{-1}),$$

R_h = soil relative humidity (dimensionless),

h_p = pressure head (m),

θ_a = volumetric air content = $(n - \theta_w)$,

v = mass flow factor (dimensionless), and

β = tortuosity factor = $(n - \theta_w)$.

Cassel et al. (1969) reviewed the theoretical models proposed for predicting the moisture movement in soil in response to an imposed temperature gradient and concluded that the Philip and de Vries (1957) model showed good agreement of predicted and observed water movement. However, the $(\theta_w - T)$ system used in Philip and de Vries (1957) has prevented the possibility to incorporate moisture retention hysteresis and soil in-homogeneities into the model. Therefore, Socphocleuos (1979) attempted to modify the model into a $(h_p - T)$ system instead of $(\theta_w - T)$ system. The $(h_p - T)$ based system has since been used in many researches.

2.5.2. Milly (1982)

Milly (1982) used the Philip and de Vries, (1957) and de Vries, (1958) theories as the basis for the formulation of a more general mathematical model of moisture and heat flow in porous media. As described in the previous section, the Philip and de Vries (1957) theory is for non-isothermal moisture flow in the liquid and vapour phase. Their development of (θ_w-T) based formulation for moisture and heat flow in porous media required the assumption of moisture retention hysteresis and soil spatial variability to be ignored. However, given the importance of hysteresis in isothermal flow process, any theoretical treatment of non-isothermal process that neglects hysteresis is incomplete (Rubin, 1967). Therefore, specifically pressure head was used as the dependent variable in order to incorporate moisture retention hysteresis and soil inhomogeneities into this model. The moisture and heat flow equations are as follow:

Moisture flow equation:

$$\left[\left(1 - \frac{\rho_v}{\rho_l} \right) \frac{\partial \theta_w}{\partial h_p} + \frac{\theta_a}{\rho_l} \frac{\partial \rho_v}{\partial h_p} \right] \frac{\partial h_p}{\partial t} + \left[\left(1 - \frac{\rho_v}{\rho_l} \right) \frac{\partial \theta_w}{\partial T} + \frac{\theta_a}{\rho_l} \frac{\partial \rho_v}{\partial T} \right] \frac{\partial h_p}{\partial t} = \nabla \left[(k_w + D_{\psi v}) \nabla h_p + (D_{Tv} + D_{Ta}) \nabla T \right] + \frac{\partial k_w}{\partial z} \quad (2.57)$$

Heat flow equation:

$$\left[C + L\theta_a \frac{\partial \rho_v}{\partial T} - (\rho_l W + \rho_v L_v) \frac{\partial \theta}{\partial T} \right] \frac{\partial T}{\partial t} + \left[L\theta_a \frac{\partial \rho_v}{\partial \psi} - (\rho_l W + \rho_v L_v) \frac{\partial \theta}{\partial \psi} \right] \frac{\partial \psi}{\partial t} = \nabla \left[\lambda \nabla T + \rho_l (LD_{\psi v} + gTD_{Ta}) \nabla \psi \right] - c_l q_m \nabla T \quad (2.58)$$

where: q_m = total mass flux of water substance = $q_l + q_v = -\rho_l k_w \nabla (h_p + z) - D \nabla \rho_v$,

q_l, q_v = liquid and vapour flux, respectively,

ρ_l, ρ_v = density of liquid water and density of water vapour in the air-filled portion of pore space, respectively,

D, k_w = hydraulic conductivity and effective molecular diffusivity, respectively,

C = volumetric specific heat of soil = $C_d + c_l \rho_l \theta + c_p \rho_p \theta_a$,

c_l = specific heat of liquid water,

c_p = specific heat of liquid water vapour at constant pressure,

C_d = volumetric heat capacity of the porous medium,

W = differential heat of wetting,

L_0 = value of L at the temperature T_0 , and

D_{Ta} = transport coefficient for absorbed liquid flow due to thermal gradient.

A physical conceptual model of hysteresis is established in order that it may be applied to non-isothermal problems. The one-dimensional forms of the governing partial equations were solved numerically using the Galerkin finite element method. A computer program was developed to verify the accuracy of the theory. The numerical algorithm in the computer program was tested with different problems in order to illustrate particular features of the model. Three types of problems were chosen to test the validity of model. Firstly, an infiltration problem was solved in which the vapour and thermal effects were neglected. Secondly, a problem involving the isothermal redistribution of moisture in soil with consideration of hysteresis was tested for infiltration into very dry sand. Thirdly, vapour-dominated systems with strong coupling between moisture and heat were used. Although the numerical results showed good agreements in the tested problems, the model shows some significant limitations. The numerical results at about one day duration may be insufficient to test the validity of heat transfer in soil. At longer times, vapour transfer in soil is more dominant (Wilson et al. 1994, Choo and Yanful 2000). In the third problem, the transport of sensible heat by water vapour was assumed to be negligible. Every theory or numerical model should be tested against the field data. In this study the lack of experimental data to support the model is the main weakness of Milly's study.

2.5.3. Cahill and Parlange (1998)

Cahill and Parlange (1998) modified Philip and de Vries (1957) equations to examine the transport of moisture in soil with no vegetative cover. The modifications were made in both the moisture and heat flow equations. In the moisture flow equation, the moisture flow term due to thermal gradient was modified. In the heat flow equation, a thermal vapour diffusivity term was introduced into the latent heat flow and a sensible heat flow term was added into the equation. The impact of the magnitude of water vapour flux on mass/energy balance was evaluated. Since the measurements were made in term of soil moisture and not matric suction (i.e., pressure head), the (θ_w-T) system of Philip and de Vries (1957) was used. In addition the volumetric water content, θ_w , and permeability, k_w , functions are only dependent on matric suction. The modified formulations are as follows:

Moisture flow:

$$\partial\theta_w/\partial t = \nabla(D_\theta \nabla\theta_w) + \nabla(D_T^* \nabla T) + \nabla(k_w) \quad (2.59)$$

Heat flow:

$$C(\partial T/\partial t) = \nabla[(\lambda \nabla T) + \rho_l L_v \nabla(D_{\theta v} \nabla\theta_w)] + \nabla[\rho_l L_v \nabla(D_{Tv}^* \nabla T) + c_l \rho_l (T - T_0)(D_\theta \nabla\theta_w + D_T^* \nabla T + k_w)] \quad (2.60)$$

$$\text{Thermal vapour diffusivity: } D_{Tv}^* = \eta(\beta\theta_a) v D_a v (d\rho_{vs}/dT) \quad (2.61)$$

$$\text{Thermal moisture diffusivity: } D_T^* = D_{Tv}^* + D_{TI} \quad (2.62)$$

where: η = ratio of temperature gradient in the air versus overall average temperature gradient $= (\nabla T)_a / (\nabla T)$.

Calculations were carried out independent of Equations (2.59) and (2.60) for the mass of vapour flux term by using the measured values of temperature and moisture content in soil. Cahill and Parlange (1998) aimed to verify the modified terms, D_{Tv}^* and D_T^* , by comparing these two mass of vapour fluxes meaning that if the

modified terms are correct the two mass of vapour flux should be the same. Good agreement in magnitude and direction of vapour flux was reported. In other words, the Philip and de Vries (1957) model provided a good basis for predicting moisture and temperature in soil during evaporation. However, the theory for diffusive vapour flow in Philip and de Vries (1957) seems to underestimate the vapour flux and consequently overestimated the overall moisture content in the soil.

2.5.4. Thomas and He (1995)

Thomas and He (1995) proposed a theoretical model, extended from the work of Philip and de Vries (1957), for the analysis of coupled heat, moisture and air transfer. The model aimed to deal with the problem in deformable unsaturated soil. The proposed model contained four equations namely moisture, heat, dry air transfer and stress equilibrium equation, as follows:

For moisture flow:

$$\begin{aligned} n(\rho_l - \rho_v) \frac{\partial S_l}{\partial t} + (S\rho_l + S_g\rho_v) \frac{\partial n}{\partial t} + nS_v \frac{\partial \rho_v}{\partial t} - \rho_l \nabla \left(\frac{k_w}{\gamma_w} \nabla u_w \right) \\ - \rho_l \nabla (D_v \nabla \rho_v) - \nabla (\rho_g D_g \nabla u_g) - \nabla (\rho_l k_w \nabla z) = 0 \end{aligned} \quad (2.63)$$

For air flow:

$$\begin{aligned} n\rho_a (H-1) \frac{\partial S_l}{\partial t} + \rho_a (S_g + HS_l) \frac{\partial n}{\partial t} + n(S_g + HS_l) \frac{\partial \rho_a}{\partial t} \\ - \nabla \left[\rho_a \left(D_g \nabla u_g + H \frac{k_w}{\gamma_w} \nabla u_w \right) \right] - H \nabla (\rho_a k_w \nabla z) = 0 \end{aligned} \quad (2.64)$$

For heat flow:

$$\begin{aligned} C \frac{\partial T}{\partial t} + (T - T_0) \frac{\partial C}{\partial t} + nL_v \rho_v \frac{\partial S_g}{\partial t} + nL_v S_g \frac{\partial \rho_v}{\partial t} + \rho_v L_v S_g \frac{\partial n}{\partial t} - \nabla (\lambda \nabla T) \\ + c_l \rho_l \nabla [V_l (T - T_0)] + c_v \rho_l \nabla [V_v (T - T_0)] + c_v \nabla [\rho_v V_g (T - T_0)] \\ + c_a \nabla [\rho_a V_g (T - T_0)] + \rho_l L_v \nabla (V_v) + L_v \nabla (\rho_v V_g) = 0 \end{aligned} \quad (2.65)$$

Stress equilibrium:

$$\frac{\partial [B\Delta\epsilon_{ij} - BA_s\Delta(u_g - u_w) - \alpha_T Bm\Delta T]}{\partial y_j} + \frac{\partial (m\Delta u_g)}{\partial y_j} + \Delta b_i = 0 \quad (2.66)$$

where: S = degree of saturation, the subscript l , v , g present for liquid water, vapour and gas phase,

D_g = coefficient of transmission of gas phase through soil,

u_g = pore gas pressure,

ρ_g = gas density,

H = Henry's coefficient of solubility of dry air in water,

B = elastic matrix,

A_s = suction vector evaluated from the relationship for the void ratio obtained from the state surface,

A_T = thermal vector = $\alpha_T m$,

α_T = thermal expansion coefficient, and

$m^T = [1, 1, 0]$

The first two equations were extensions of Philip and de Vries (1957) model, the third equation was based on the Fick's law and the last equation was a combination of stress equilibrium and constitutive relationship for the state surface. The constitutive relationship for state surface could be linearized for a wide range of stress changes using the logarithmic form of stress state variable proposed by Fredlund (1979). The advantage of the state surface approach is that it allows the effects of suction and stress level to be introduced in terms of both void ratio and the degree of saturation of soil.

In order to solve the four differential equations above, a new algorithm was developed based on the use of a complex combination of finite element method for spatial discretization and finite difference method for the time variation of the variables. Thomas and He verified the numerical results with analytical solutions as well as experimental data. Very good agreement between numerical and analytical solutions were obtained that implied the accuracy of model's algorithm. The experimental results in term of temperature, degree of saturation and void ratio at

the steady state was compared with the numerical simulation. Similar trend of temperature distribution between experiment and numerical simulation was obtained. However, the temperature recorded at the point nearest to the heat source was approximately 10°C lower than the numerical simulation. The temperature at the other eight locations closely matched the numerical results. Overall behavioural pattern in term of a reduction in degree of saturation and an increase in void ratios near the heated zone was reflected in the numerical simulation. The differences in these two comparisons were in the zone far from the heat source during the cooling phase where temperature influence was not high. In general, the numerical results agreed with experimental data. In spite of the good agreements obtained between numerical results and experimental data, the drawbacks of the Thomas and He (1995) model are the needs to assume some soil parameters such as permeability function of the gas phase (i.e., dry air mixes with vapour), k_g , specific heat capacity of gas phase, C_g , thermal expansion coefficient of soil, α_T , and is highly computational intensive to solve. Therefore, in this research a numerical model is developed where soil parameters can be determined experimentally to produce reasonable results for practical use and which is less computationally intensive.

2.5.5. Dakshanamurthy and Fredlund (1981)

Dakshanamurthy and Fredlund (1981) presented a mathematical model for air, vapour and heat flow in non-frozen unsaturated swelling soil. The model comprises three differential equations namely: water, air and heat flow equations. The three equations were solved simultaneously for changes in the combined temperature and hydraulic boundary conditions. By solving the equation for heat flow, the change in temperature can be converted into change in pore-air pressure in the air flow equation. The change in pore-air pressure results in change in the pore-water pressure in the water flow equation. In order to form the equation for moisture flow, Darcy's law for unsaturated soil and modified Fick's law were used. Dakshanamurthy and Fredlund (1981) modified Fick's law to establish an expression of mass flux of liquid vapour in soil under a constant total pressure:

$$M_v = -(\alpha)(\theta_a) \left[D_{\text{vap}} \frac{W_{\text{wa}}}{(R^*)T} \right] \left(\frac{\partial P_v}{\partial y} \right) \quad (2.67)$$

or

$$M_v = -D^* \left(\frac{\partial P_v}{\partial y} \right) \quad (2.68)$$

where: M_v = mass flux of liquid vapour,

R^* = universal gas constant,

D^* = diffusion coefficient of water vapour through the soil, and

P_v = partial pressure due to water vapour in the air phase.

From Equation (2.68), the net flux of vapour through the soil element can be represented as follows:

$$\frac{\partial m_{\text{air}}}{\partial t} = - \left(D^* \frac{\partial^2 P_v}{\partial^2 y} \right) \quad (2.69)$$

Similarly, from Darcy's law the net flux of liquid water through the soil element is given as follows:

$$\frac{\partial m_{\text{water}}}{\partial t} = - \left(\frac{k_w}{\gamma_w} \frac{\partial^2 u_w}{\partial^2 y} \right) \quad (2.70)$$

Fredlund and Morgenstern (1976) established a constitutive relationship for the change in volume of water in an element as follows:

$$\frac{\Delta V_w}{V} = \left[m_1^w d(\sigma_y - u_a) + m_2^w d(u_a - u_w) \right] \quad (2.71)$$

where: u_a = pressure in air phase,

u_w = pressure in liquid water phase,

m_1^w = slope of the $d(\sigma_y - u_a)$ versus volumetric water content plot for $d(u_a - u_w)$ equals to zero, and

m_2^w = slope of the $d(u_a - u_w)$ versus volumetric water content plot for $d(\sigma_y - u_a)$ equals to zero.

The differential equation for moisture flow is a combination of Equations (2.69) and (2.70) and equalled the time differential of the constitutive relationship for moisture phase (i.e., Equation (2.71)). Therefore the moisture flow equation is given as follows:

$$m_1^w \frac{\partial(\sigma_y - u_a)}{\partial t} + m_2^w \frac{\partial(u_a - u_w)}{\partial t} = -\frac{k_w}{\gamma_w} \frac{\partial^2(u_w)}{\partial y^2} - D \frac{\partial^2(P_v)}{\partial y^2} \quad (2.72)$$

Assuming that the change in vertical stress σ_y with time equals to zero, Equation (2.72) can be presented as follows:

$$\frac{\partial(u_w)}{\partial t} = C_w \frac{\partial(u_a)}{\partial t} + C_{v1}^w \frac{\partial^2(u_w)}{\partial y^2} + C_{vv}^w \frac{\partial^2(P_v)}{\partial y^2} \quad (2.73)$$

where: C_w = interactive constant associated with the water phase,

C_{v1}^w = coefficient of consolidation for the liquid water phase, and

C_{vv}^w = coefficient of consolidation for the water vapour phase.

Similarly for the air phase, the air flow partial differential equation can be expressed as follows:

$$\frac{\partial(u_a)}{\partial t} = C_a \frac{\partial(u_w)}{\partial t} + C_v^a \frac{\partial^2(u_a)}{\partial y^2} + C_T \frac{\partial(T)}{\partial t} \quad (2.74)$$

where: C_a = interactive constant associated with the air phase,

C_T = interactive thermal constant associated with the air phase, and

C_v^a = coefficient of consolidation for the air phase.

The differential equation for heat flow was adopted from Aldrich (1956) as follows:

$$\frac{\partial T}{\partial t} = \left(\frac{\lambda}{C \rho_s} \right) \frac{\partial^2 T}{\partial y^2} \quad (2.75)$$

where : C = heat capacity of soil.

In Equation (2.75), Dakshanamurthy and Fredlund assumed that the heat flow in soil was only due to conduction. This assumption is only valid for isothermal or extremely low heat flow condition (Wilson 1990). Dakshanamurthy and Fredlund (1981) simulated a one dimensional soil column of 10 cm height for isothermal, non-isothermal and swelling problems. Because the laboratory data was not available, the simulation results were then compared with field observations from Regina clay of 2m in thickness. The simulation results seem to predict accurately the relationship of volume change versus time.

2.5.6. Wilson et al. (1994)

Wilson et al. (1994) extended the work of Dakshanamurthy and Fredlund (1981) to establish a model for predicting moisture and heat flow in non-deformable unsaturated soil. The moisture flow equation in the model assumed that the time derivative of the constitutive relationship for moisture phase equals the moisture flow in soil. Wilson et al. (1994) applied the expression of mass flux of liquid vapour in soil under a constant total pressure (i.e., Equation (2.67)) for a soil element. The amount of liquid vapour entering and leaving the soil element due to vapour diffusion is given by:

$$M_{ve} = -D \left(\frac{\partial P_v}{\partial y} \right) dx dz \quad (2.76)$$

$$M_{vl} = - \left[D \left(\frac{\partial P_v}{\partial y} \right) + \frac{\partial}{\partial y} \left(D \frac{\partial P_v}{\partial y} \right) dy \right] dx dz \quad (2.77)$$

where: M_{ve} = mass flux of liquid vapour entering the soil element (kg.s^{-1})

M_{vl} = mass flux of liquid vapour leaving the soil element (kg.s^{-1})

dx, dy, dz = dimensions of the unit element

Therefore the net change in mass flux of water vapour within the soil element is given as follows:

$$\frac{\partial m_v}{\partial t} = M_{vl} - M_{ve} = - \left[\frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) dy \right] dx dz = - \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) \quad (2.78)$$

where: m_v = mass of water vapour in the soil element (kg)

A correction factor $\left(-\frac{P_v}{P_a} \right) \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right)$ was introduced by Wilson (1990) into Equation (2.78) in order to compensate the flow of vapour due to advection such that:

$$\frac{\partial m_v}{\partial t} = - \left(\frac{P_a + P_v}{P_a} \right) \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) \quad (2.79)$$

Darcy's law was used to calculate the net change in volume of liquid water within the soil element. The resultant equation for liquid flow is given by the well known Richards' equation for the one-dimensional case:

$$\begin{aligned} \frac{\partial m_{lw}}{\partial t} &= M_{lwl} - M_{lwe} = - \left[k_w \left(\frac{\partial h_w}{\partial y} \right) + k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) dy \right] dx dz - \left[k_w \left(\frac{\partial h_w}{\partial y} \right) dx dz \right] \\ &= -k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) \end{aligned} \quad (2.80)$$

where: m_{lw} = mass of liquid water in the soil element (kg),

M_{lwl} = mass flux of liquid water leaving the soil element (kg s^{-1}), and

M_{lwe} = mass flux of liquid water entering the soil element (kg s^{-1}).

By assuming that the time derivative of constitutive relation given by Equation (2.71) is equal to the vapour pressure gradient and hydraulic gradient in the soil (i.e., $\sigma_y, u_a = 0$), an equation can be formed from Equations (2.79), (2.80), and (2.71) as follows:

$$\frac{\partial \left(\frac{V_w}{V} \right)}{\partial t} = \frac{\partial (\theta_w)}{\partial t} = \frac{\partial m_{lw}}{\partial t} + \frac{1}{\rho_l} \frac{\partial m_v}{\partial t} = -k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) - \frac{1}{\rho_l} \left(\frac{P_a + P_v}{P_a} \right) \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) \quad (2.81)$$

$$\text{or: } \frac{\partial h_p}{\partial t} = \frac{\frac{\partial}{\partial y} \left(k_w \frac{\partial h_p}{\partial y} \right)}{\rho_l g m_2^w} + \left(\frac{P_a + P_v}{P_a} \right) \frac{1}{\rho_l} \frac{\partial}{\partial y} \left(\alpha \theta_a D_{\text{vap}} \frac{W_{wa}}{R^* T} \frac{\partial P_v}{\partial y} \right) + \frac{\frac{\partial}{\partial y} (k_w)}{\rho_l g m_2^w} \quad (2.82)$$

In order to solve Equation (2.81), an equation to describe heat distribution in soil is required. Wilson et al. (1994) modified the equation for heat flow proposed by Jame and Norum (1980) in a freezing unsaturated porous medium:

$$C_h \frac{\partial T}{\partial t} = \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) - L_v \frac{\partial m_v}{\partial t} = \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) - L_v \left(\frac{P_a + P_v}{P_a} \right) \frac{\partial}{\partial y} \left(D^* \frac{\partial P_v}{\partial y} \right) \quad (2.83)$$

$$\text{or } \frac{\partial T}{\partial t} = \frac{\frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right)}{(C_h)} - \frac{L_v \left(\frac{P_a + P_v}{P_a} \right) \frac{\partial}{\partial y} \left[\left(\alpha \theta_a D_{\text{vap}} \frac{W_{wa} g}{\rho_l R^* T} \right) \frac{\partial P_v}{\partial y} \right]}{(C_h)} \quad (2.84)$$

A numerical model was established based on the two differential equations above (i.e, Equations (2.83) and (2.84)). Wilson et al. (1994) compared the experimental observation of moisture content and heat movement in a Beaver Creek sand column during evaporation with the numerical model results and found that they agreed well. The experimental data from Wilson et al. (1994) were employed to verify the modified model in this research. The details will be presented in Chapter 4.

Yanful et al. (2003) used the Wilson et al. (1994) model which was implemented in a commercial software, SoilCover (GeoAnalysis Ltd, 2000), for numerical modelling of four different one-dimensional homogenous soil columns of 0.25m in height and 0.115m in diameter. Column tests were carried out for coarse sand, fine sand, silt and clayey till individually to determine the suitable material (soil) for soil cover application in mine wastes and land fills. Generally numerical results and experimental data match well except for the fine sand and clayey till. The discrepancies for clayey till were attributed to the settlement characteristic during evaporation whereas measurement error was attributed for fine sand. Yanful et al. (2003) proposed silt or fine sand overlying clayey till and coarse sand suitable materials for soil cover application. Their numerical results also suggested that the clayey till could be an effective oxygen barrier in sulfide-bearing mine waste cover while the fine sand would be suitable evaporation and drainage barriers for the till.

The experimental data of clayey till from Yanful et al. (2003) will be used for comparison with the modified model in this research. The details will be presented in Chapter 4.

In general, the Wilson et al. (1994) model can handle the problem of moisture and heat flow in non-deformable soil. However, the Philip and de Vries (1957) framework also promises a simpler model for non-deformable soil. Therefore, an intensive comparison for a good understanding on the two models is needed and is carried out and presented in the next chapter.

2.6. SUMMARY

An overview of unsaturated soil mechanics was presented in this chapter. Past research works on infiltration and evaporation were reviewed. Models for predicting moisture and heat transfer in soil based on the Philip and de Vries (1957) and Dakshanamurthy and Fredlund (1981) frameworks were also reviewed. The Philip and de Vries framework seems to be suitable for moisture and heat flow in unsaturated soil under thermal and moisture gradients (Zhou et al., 1998). However, the adoption of these models in geotechnical engineering practice is not easy as described in Section 2.5.1.

The differences between Philip and de Vries type models and Dakshanamurthy and Fredlund (1981) type models have not been examined. Currently, the two types of model are still being used by different researchers. However, there is no research to report if they are really different or if they are different, their degree of differences. Therefore an in-depth study of the two approaches will be presented in the next chapter. The presentation of a modified equation in order to bridge between the two types of model is needed. As illustrated in this chapter, experimental verification of numerical results is important. Hence in this research a number of laboratory tests was also conducted to provide the verification.

Chapter 3 Theoretical Consideration

3.1. INTRODUCTION

The review on coupled moisture and heat movement models in Chapter 2 showed that a number of models are a modification of Philip and de Vries (1957) model. The Dakshanamurthy and Fredlund (1981) model which was further extended by Wilson (1990) and Wilson et al. (1994) used constitutive relationship from unsaturated soil mechanics. There appears to be a large difference between the models though they are based on similar principles. In this chapter, the Philip and de Vries (1957) type models are examined in detail. A modified model with easily quantifiable parameters is then proposed to model coupled moisture and heat movement in unsaturated soil. It is then shown that the modified model can be transformed into the Wilson et al. (1994) model with some simplifying assumptions as a means of reconciling the Philip and de Vries (1957) type model and Wilson et al. (1994) model. The modified model is recast into finite difference form and implemented in a spreadsheet program for solving 1-D infiltration and evaporation problem.

3.2. PHILIP AND DE VRIES (1957) TYPE FORMULATIONS

The general form of physically based model for water and heat transport in soil was first developed by Philip and de Vries (1957). In Philip and de Vries (1957) equations, moisture and heat flow were driven by the combined effects of moisture and temperature gradients. The original Philip and de Vries (1957) formulations have been modified a number of times in order to account for more characteristics of the soil during the evaporation process. Henceforth, the most complete formulation based on Philip and de Vries (1957) is derived and summarised in this section.

3.2.1. Mass conservation

Philip and de Vries (1957) extended the isothermal principles of liquid water flow by incorporating vapour flow and temperature gradients for moisture flow in porous

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media (Milly 1980). In Philip and de Vries (1957) formulation, volumetric water content, θ_w , was used (see Equation (2.49)). However, in geotechnical engineering moisture flow is due to total head gradient ($h_p + z$). Therefore, the general form of Philip and de Vries (1957) moisture flow equation in soil can also be expressed in term of pressure head, h_p , (Socphocleuos 1979; Milly 1980, 1982, 1984a; Camillo et al. 1983, Schieldge et al., 1982, de Vries, 1987) as follows:

$$q_m = q_l + q_v = -\rho_l k_w \nabla (h_p + z) - D \nabla \rho_v \quad (3.1)$$

- where: q_m = total water flux density ($\text{kgm}^{-2}\text{s}^{-1}$),
 q_l, q_v = liquid water and vapour components of q_m , respectively ($\text{kgm}^{-2}\text{s}^{-1}$),
 ρ_l = liquid water density (kgm^{-3}),
 k_w = unsaturated hydraulic permeability (ms^{-1}),
 ∇ = gradient operator,
 h_p = pressure head (m),
 D = effective molecular diffusivity = $D_{\text{vap}} v \alpha \theta_a$, (m^2s^{-1}),
 ρ_v = vapour density of water (kgm^{-3}),
 D_{vap} = $0.229 \times 10^{-4} \left(1 + \frac{T}{273}\right)^{1.75}$ molecular diffusivity of water vapour in air, (m^2s^{-1}),
 α = tortuosity of the air filled = $(\theta_a)^{2/3}$, and
 v = mass flow factor = 1.

To account for liquid flow under thermal gradient (Carry, 1966), Equation (3.1) can be rewritten as follows:

$$q_m = q_l + q_v = -\rho_l k_w \nabla (h_p + z) - D \nabla \rho_v - \rho_l D_{\text{TI}} \nabla T \quad (3.2)$$

- where: $\rho_l D_{\text{TI}} \nabla T$ = liquid flow under a thermal gradient ($\text{kgm}^2\text{s}^{-1}$),
 D_{TI} = liquid thermal diffusion coefficient ($\text{m}^2\text{s}^{-1}\text{K}^{-1}$), and
 T = absolute temperature (K).

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The general principle of conservation law for water vapour at macroscopic scale used to derive the governing equation for water mass flow in the porous medium (Philip and de Vries, 1957) is given as follows:

$$\frac{\partial \sum E}{\partial t} = -\nabla q \quad (3.3)$$

where: E = amount of substance or energy leaving or entering a unit of porous medium (XL^{-3}),

q = local average flux of energy or substance ($XL^{-2}T^{-1}$)

X = unit of substance or energy

The total mass of a unit of the porous medium is given by:

$$\sum_1^m E = \rho_l \theta_w + \rho_v \theta_a \quad (3.4)$$

where: θ_a = volumetric air content = $(n - \theta_w)$,

n = porosity of the porous media,

θ_w = volumetric water content,

$\sum_1^m E$ = total liquid mass flux per unit of porous medium (kgm^{-3}), and

m = total number of components.

Applying the principle of mass conservation for water and water vapour, the following equation can be obtained:

$$\frac{\partial}{\partial t} (\rho_l \theta_w + \rho_v \theta_a) = -\nabla q_m \quad (3.5)$$

Equation (3.5) can be expanded as follows:

$$\begin{aligned} & \frac{\partial \theta_w}{\partial h_p} \frac{\partial h_p}{\partial t} + \frac{(n - \theta_w)}{\rho_l} \frac{\partial \rho_v}{\partial h_p} \frac{\partial h_p}{\partial t} + \frac{\rho_v}{\rho_l} \frac{\partial (n - \theta_w)}{\partial h_p} \frac{\partial h_p}{\partial t} + \\ & + \frac{\partial \theta_w}{\partial T} \frac{\partial T}{\partial t} + \frac{(n - \theta_w)}{\rho_l} \frac{\partial \rho_v}{\partial T} \frac{\partial T}{\partial t} + \frac{\rho_v}{\rho_l} \frac{\partial (n - \theta_w)}{\partial T} \frac{\partial T}{\partial t} = -\frac{\nabla q_m}{\rho_l} \end{aligned} \quad (3.6)$$

Substituting Equation (3.2) into Equation (3.6), gives:

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$$\left[\frac{\partial \theta_w}{\partial h_p} + \frac{(n - \theta_w)}{\rho_l} \frac{\partial \rho_v}{\partial h_p} + \frac{\rho_v}{\rho_l} \frac{\partial (n - \theta_w)}{\partial h_p} \right] \frac{\partial h_p}{\partial t} + \left[\frac{\partial \theta_w}{\partial T} + \frac{(n - \theta_w)}{\rho_l} \frac{\partial \rho_v}{\partial T} + \frac{\rho_v}{\rho_l} \frac{\partial (n - \theta_w)}{\partial T} \right] \frac{\partial T}{\partial t} = -\nabla \frac{1}{\rho_l} \left[-\rho_l k_w \nabla (h_p + z) - D \nabla \rho_v - \rho_l D_{Tl} \nabla T \right] \quad (3.7)$$

Philip and de Vries (1957) defined a thermal liquid diffusivity, D_{Tl} , to calculate the amount of this liquid water movement:

$$D_{Tl} = \left(\frac{\partial h_p}{\partial T} \right) k_w \quad (3.8)$$

In Equation (3.8) the term $\partial h_p / \partial T$ implies that pressure head, h_p , is not only a function of volumetric water content, θ_w , but also temperature, T . In the same fashion, unsaturated hydraulic permeability, k_w , is a function of volumetric water content, θ_w , and temperature, T . In other words, h_p and k_w are as follows:

$$h_p = f(T, \theta_w) \quad (3.9)$$

$$k_w = f(T, \theta_w) \quad (3.10)$$

Unfortunately, there were no experimental data to support Equations (3.9) and (3.10). In order to overcome this problem, Philip and de Vries (1957) proposed a surface tension model for the calculation of, D_{Tl} (Equation (2.54)):

$$D_{Tl} = \gamma k_w h_p = \left(\frac{\partial \sigma_T}{\partial T} \right) k_w h_p \quad (3.11)$$

where: σ_T = pure water-air interfacial tension (dyne.cm⁻¹)

Cassel et al. (1969) concluded that the Philip and de Vries (1957) model showed good agreement between predicted and observed liquid water movement under a thermal gradient. However, the coefficient for dependence of pressure head on temperature seems greater than the actual coefficient of surface tension of pure water-air considered in Philip and de Vries (1957) (Jury and Miller, 1974, Kimball et al., 1976).

Socphocleuos (1979), proposed another model to overcome this difficulty by assuming that relative humidity, R_h , is only a function of pressure head, h_p . Using thermodynamic principle, when a local equilibrium exists between the liquid and vapour phase their free energies are equal as shown by the following equation (Edlefson and Anderson, 1943):

$$\rho_v = \rho_{vs|T_0} R_h = \rho_{vs|T_0} e^{\frac{h_p g}{RT}} \quad (3.12)$$

where: $\rho_{vs|T_0}$ = saturation vapour density at T_0 (g/m^3),

ψ = soil suction = h_p (m), and

R = gas constant = $461.5 \text{ J kg}^{-1} \text{ K}^{-1}$.

Differentiating the relative humidity, R_h , with respect to temperature gives:

$$0 = \frac{\partial R_h}{\partial T} = R_h \left(\frac{g}{RT} \frac{\partial h_p}{\partial T} - \frac{gh_p}{RT^2} \right) \quad (3.13)$$

Equation (3.13) can be simplified as follows:

$$\frac{\partial h_p}{\partial T} = \frac{h_p}{T} \quad (3.14)$$

Substituting Equation (3.14) into Equation (3.8) gives:

$$D_{T1} = k_w \left(\frac{h_p}{T} \right) \quad (3.15)$$

Socphocleuos (1979) attempted to evaluate the diurnal moisture flow mechanism which influences the subsurface flow using experimental data from field test. Equation (3.15) is therefore incorporated in the pressure head based equations of moisture and heat flow. A satisfactory agreement of volumetric water content profile between numerical results and field test data was reported. However, again experimental data to prove the significant improvement of Equation (3.15) towards the overall numerical results were not available.

Milly (1982) pointed out that Equation (3.15) contradicted Equation (3.9) where pressure head, h_p , is a function of temperature, T and volumetric water content, θ_w .

Furthermore, no experimental data were provided by Socphocleuos (1979) to support the validity of Equation (3.15). Although Milly (1982) agreed that the term for liquid flow under temperature gradient, $\rho_l D_{Tl} \nabla T$, should be incorporated in the moisture flow equation, the theory for its calculation was not clear and therefore in numerical modelling the term was an approximation.

Camillo et al. (1983) modified the work of Philip and de Vries (1957) to develop a model for problems in hydrology. Modifications were made to the equations of moisture flow and boundary conditions. The study emphasized on the sensitivities of environmental parameters and calibration analysis for estimation of daily evaporation. The two components for estimating the daily evaporation (i.e., temperature and moisture content at the soil surface) were found to fit well with the measurements. Camillo et al. (1983) used the definition of thermal liquid diffusion coefficient to calculate the amount of liquid water flow under thermal gradient (i.e., Equation (3.8)). The modified model did not give any specific relationship of pressure head, h_p with temperature, T and no experimental data was provided to show the contribution of liquid water flow under thermal gradient in the model.

Cahill and Parlange (1998) modified Philip and de Vries (1957) model to examine the behaviour of water transport in bare soil. The modifications were made in both the moisture and heat flow equations. In the equation of moisture flow, the liquid flow term due to thermal gradient was Equation (3.11). In the equation of heat flow, two modifications were made: thermal vapour diffusivity (i.e., Equation (2.61)) was introduced and a sensible heat flow term was added. Their study advanced the understanding of mass balance of moisture in soil under a thermal gradient and establishing the relationship between vapour flux and the moisture that remains in the soil. They showed experimentally that the energy and mass balance yielded good agreement in magnitude and direction of vapour flux. However, these modified equations are not meant to predict the change of pressure head in unsaturated soil which is an important variable in geotechnical engineering. Furthermore, there was insufficient experimental evidence to justify the use of liquid thermal diffusion coefficient, D_{Tl} . Besides, the governing equations for moisture flow are fairly complicated and contain two dependent relationships of

pressure head, h_p , and permeability function, k_w , on temperature, T , that difficult to determine experimentally. To date, there has been no agreement on how to define the liquid flow term under thermal gradient (i.e., Equation (3.8)) and its inclusion in the mathematical model is speculative.

The term $-D\nabla\rho_v$ in Equation (3.7) derived by Philip and de Vries (1957) and later used by others, ignored the dependency of relative humidity, R_h on temperature, T (Nakano and Miyazaki, 1979). The full form of the term $-D\nabla\rho_v$ considering these dependencies is as follows:

$$-D\nabla\rho_v = -D\left(\frac{g\rho_v}{RT}\right)\nabla h_p - D\left[e^{\frac{(h_p)g}{RT}}\frac{d\rho_0}{dT} - \frac{g\rho_v(h_p)}{RT^2}\right]\nabla T \quad (3.16)$$

Equation (3.7) can therefore be rewritten as follows:

$$\begin{aligned} & \left[\frac{\partial\theta_w}{\partial h_p} + \frac{(n-\theta_w)}{\rho_l} \frac{\partial\rho_v}{\partial h_p} + \frac{\rho_v}{\rho_l} \frac{\partial(n-\theta_w)}{\partial h_p} \right] \frac{\partial h_p}{\partial t} + \left[\frac{\partial\theta_w}{\partial T} + \frac{(n-\theta_w)}{\rho_l} \frac{\partial\rho_v}{\partial T} + \frac{\rho_v}{\rho_l} \frac{\partial(n-\theta_w)}{\partial T} \right] \frac{\partial T}{\partial t} \\ & = \frac{1}{\rho_l} \nabla \left[\left(\rho_l k_w + \frac{gD\rho_v}{RT} \right) \nabla h_p + D \left(e^{\frac{h_p g}{RT}} \frac{d\rho_0}{dT} - \frac{g\rho_v h_p}{RT^2} \right) \nabla T + \rho_l D_{\pi} \nabla T + \rho_l k_w \right] \end{aligned} \quad (3.17)$$

The final governing equation for one dimensional water and vapour flow is given as follows:

$$\begin{aligned} & \left[\frac{\partial\theta_w}{\partial h_p} + \frac{(n-\theta_w)}{\rho_l} \frac{\partial\rho_v}{\partial h_p} + \frac{\rho_v}{\rho_l} \frac{\partial(n-\theta_w)}{\partial h_p} \right] \frac{\partial(h_p)}{\partial t} + \left[\frac{\partial\theta_w}{\partial T} + \frac{(n-\theta_w)}{\rho_l} \frac{\partial\rho_v}{\partial T} + \frac{\rho_v}{\rho_l} \frac{\partial(n-\theta_w)}{\partial T} \right] \frac{\partial T}{\partial t} \\ & = \frac{\partial}{\partial y} \left[\left(k_w + \frac{gD\rho_v}{\rho_l RT} \right) \frac{\partial(h_p)}{\partial y} \right] + \frac{\partial}{\partial y} \left\{ \left[\frac{D}{\rho_l} \left(e^{\frac{(h_p)g}{RT}} \frac{d\rho_0}{dT} - \frac{g\rho_v(h_p)}{RT^2} \right) + D_{\pi} \right] \frac{\partial T}{\partial y} \right\} + \frac{\partial}{\partial y} (k_w) \end{aligned} \quad (3.18)$$

3.2.2. Energy Conservation

The solution for Equation (3.18) requires the temperature distribution in the porous medium. Hence, an equation describing heat transfer (i.e., heat distribution) is

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necessary. The amount of heat flow in a unit porous medium is given as (Philip and de Vries, 1957):

$$q_h = -\lambda \nabla T - \rho_l L_v D_{\theta v} \nabla (\theta_w) \quad (3.19)$$

where: λ = thermal conductivity $\lambda(y, T, t)$ ($\text{Wm}^{-1}\text{C}^{-1}$),

L_v = latent heat of vaporisation for liquid water = 2.43×10^6 (Jkg^{-1}),

$D_{\theta v}$ = vapour moisture diffusivity (m^2s^{-1}) = $D_{\text{vap}} \alpha \theta_a v \left(\frac{g}{RT} \right) \rho_v \frac{\partial(h_p)}{\partial \theta_w}$.

The first term on the right hand side of Equation (3.19) represents the heat flow due to conduction while the second term represents the transfer of latent heat of vaporization. Socphocleuos (1979) introduced another term named sensible heat due to convection by the moisture movement and this term was used in Milly (1980, 1982) and Cahill and Parlange (1998). Equation (3.19) then can be rewritten as follows:

$$q_h = -\lambda \nabla T - \rho_l L_v D_{\theta v} \nabla (\theta_w) + c_l \rho_l q_m \nabla T \quad (3.20)$$

where: c_l = Specific heat of liquid water ($\text{cal kg}^{-1} \text{C}^{-1}$).

However, the sensible heat term was ignored in Camillo et al. (1983), Witono and Bruckler (1989), Wilson et al. (1994) and Lee et al. (2003) as its influence is very small. In Dakshanamurthy and Fredlund (1981), the right hand side of the heat flow equation, Equation (3.19), consisted of only the conductive term.

De Vries (1963) suggested the following equation to determine total bulk volumetric heat content of a porous medium, E_h :

$$E_h = (C_s \theta_s + C_w \theta_w + C_a \theta_a) (T - T_0) \quad (3.21)$$

$$\text{or} \quad E_h = C_h \cdot (T - T_0) \quad (3.22)$$

where: C_s, C_w, C_a = specific volumetric heat capacity of the soil solid, liquid water phase and air phase, respectively ($\text{Jm}^{-3}\text{C}^{-1}$),

C_h = specific heat capacity of soil element, and

$\theta_s, \theta_w, \theta_a$ = volumetric fraction of soil solid, water and air, respectively.

Milly (1980, 1982) introduced a term reflecting the initial consumption of latent heat, $L_0\rho_v\theta_a$ into Equation (3.22) to give:

$$E_h = C_h \cdot (T - T_0) + L_0\rho_v\theta_a \quad (3.23)$$

where L_0 = latent heat of vaporisation of liquid water at temperature T_0 (Jkg^{-1}).

Combining Equations (3.3), (3.20) and (3.22), the general equation for heat transport is written as follows:

$$\frac{\partial}{\partial t} [C_h \cdot (T - T_0) + L_0\rho_v\theta_a] = -\nabla [-\lambda \nabla T - \rho_l L_v D_{\theta w} \nabla (\theta_w)] - c_l \rho_l q_m \nabla T \quad (3.24)$$

$$\left[C_h - L_0\theta_a\rho_{vs} e^{\frac{(h_p)g}{RT}} \frac{h_p g}{R} \frac{1}{(T)^2} \right] \frac{\partial T}{\partial t} = -\nabla [-\lambda \nabla T - \rho_l L_v D_{\theta w} \nabla (\theta_w)] - c_l \rho_l q_m \nabla T \quad (3.25)$$

Substituting the definition of $D_{\theta w}$ given earlier into Equation (3.25), the equation for 1-D heat flow is given as follows:

$$\begin{aligned} & \frac{\partial T}{\partial t} \left[C_h - L_0\theta_a\rho_{vs} e^{\frac{(h_p)g}{RT}} \frac{h_p g}{R(T)^2} \right] \\ &= \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial y} \left\{ \left[\rho_l L_v \alpha D_{\text{vap}} \theta_a v \left(\frac{g}{RT} \right) \rho_v \right] \left(\frac{\partial h_p}{\partial y} \right) \right\} - c_l \rho_l q_m \frac{\partial T}{\partial y} \end{aligned} \quad (3.26)$$

3.3. PROPOSED MODIFIED MODEL

From the mathematical viewpoint, the Philip and de Vries (1957) formulation started from very basic soil principles (Socphocleuos, 1979, Milly, 1980, Thomas and He, 1995) therefore, it should be reasonably suitable for predicting heat and moisture profiles in unsaturated soil. Equations (3.18) and (3.26) are fairly complicated to solve analytically as well as numerically. Therefore, assumptions must be made in order to obtain a computational ammenable coupled heat and liquid flow model.

Firstly, although volume change induces change in the air phase, vapour flow is negligible. Volume change is negligible for most soils because they are drying

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under normal atmospheric pressure (Wilson 1990, Wilson et al., 1994). In this study the porosity of the soil is assumed to be constant meaning that the soil is considered to be non-deformable. Secondly, several formulations were presented in the previous section for the liquid flow term under thermal gradient, $\rho_l D_{Tl} \nabla T$. However to date, it has not been adequately proven experimentally for its existence or if it does exist, it is significant enough to affect the experiment results. Thirdly, pressure head, h_p , and permeability function, k_w , are only functions of volumetric water content, θ_w . Wilson (1990) measured soil – water characteristic curve for Beaver Creek sand at different temperatures (Figure 3-1) and found that temperature does not have a significant affect on the soil – water characteristic curve. To date, there is no experimental evidence to suggest that h_p and k_w are dependent on temperature. Fourthly, in Equation (3.26), sensible heat due to convection by moisture movement is assumed to be negligible. From the four assumptions above, the coupled heat and liquid flow equations, Equations (3.18) and (3.26), respectively, are rewritten as follows:

For liquid flow:

$$\left[\rho_l \frac{\partial \theta_w}{\partial t} + \theta_a \frac{\partial \rho_v}{\partial t} + \rho_v \frac{\partial (n - \theta_w)}{\partial t} \right] = -\nabla \left[-\rho_l k_w \nabla (h_p + z) - D \nabla \rho_v \right] \quad (3.27)$$

$$\begin{aligned} \frac{\partial \left[(\rho_l g)(h_p) \right]}{\partial t} \left[\rho_l \frac{\partial \theta}{\partial (u_a - u_w)} + \theta_a \frac{\partial \rho_v}{\partial (u_a - u_w)} - \rho_v \frac{\partial \theta_w}{\partial (u_a - u_w)} \right] = \\ \frac{\partial}{\partial y} \left\{ \rho_l k_w \frac{\partial}{\partial y} (h_p + z) + D \left(\frac{g \rho_v}{RT} \right) \frac{\partial h_p}{\partial y} + \left\{ D \left[e^{\frac{(h_p)g}{RT}} \frac{d\rho_0}{dT} - \frac{g \rho_v (h_p)}{RT^2} \right] \right\} \frac{\partial T}{\partial y} \right\} \end{aligned} \quad (3.28)$$

or:

$$\frac{\partial \left[(h_p) \right]}{\partial t} = \frac{1}{A^*} \frac{\partial}{\partial y} \left\{ \rho_l k_w \frac{\partial}{\partial y} (h_p + z) + B^* \frac{\partial h_p}{\partial y} + C^* \frac{\partial T}{\partial y} \right\} \quad (3.29)$$

$$\text{where: } A^* = (\rho_l g) \left[\rho_l \frac{\partial \theta}{\partial (u_a - u_w)} + \theta_a \frac{\partial \rho_v}{\partial (u_a - u_w)} - \rho_v \frac{\partial \theta_w}{\partial (u_a - u_w)} \right],$$

$$B^* = D \left(\frac{g \rho_v}{RT} \right), \text{ and}$$

$$C^* = \left\{ D \left[e^{\frac{(h_p)g}{RT}} \frac{d\rho_0}{dT} - \frac{g \rho_v (h_p)}{RT^2} \right] \right\}.$$

For heat flow:

$$\frac{\partial T}{\partial t} = \frac{\frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right)}{\left[C_h - L_0 \theta_a \rho_{vs} e^{\frac{h_{pg}}{RT}} \frac{h_p g}{RT^2} \right]} + \frac{\frac{\partial}{\partial y} \left[\rho_l L_v \theta_a D_{vap} v \left(\frac{g}{RT} \right) \rho_v \left(\frac{\partial h_p}{\partial y} \right) \right]}{\left[C_h - L_0 \theta_a \rho_{vs} e^{\frac{h_{pg}}{RT}} \frac{h_p g}{R (T)^2} \right]} \quad (3.30)$$

or

$$\frac{\partial T}{\partial t} = \frac{1}{K} \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} + G \frac{\partial h_p}{\partial y} \right) \quad (3.31)$$

where: $K = \left[C_h - L_0 \theta_a \rho_{vs} e^{\frac{h_{pg}}{RT}} \frac{h_p g}{RT^2} \right].$

Figure 3-1 Effect of temperature on moisture retention curve of the Beaver Creek sand (from Wilson, 1990)

The modified heat and liquid flow formulations (i.e., Equations (3.29) and (3.31)) are simpler than the original formulations of Philip and de Vries (1957) type. Furthermore, all parameters in the two equations can be determined experimentally.

3.3.1. Comparison of Modified Equations with Wilson et al. (1994) Equations

Wilson et al. (1994) formulation for moisture and heat flow in non-deformable soil was based on the works of Wilson (1990) for deformable soil which inherited the idea of Dakshanamurthy and Fredlund (1981). The Philip and de Vries (1957) type formulations and Wilson et al. (1994) formulations appeared to be very different. In this section an attempt is made to resolve the differences between the two formulations. In order to compare the two formulations, a brief review of Wilson (1990) heat and moisture flow equations in deformable soil is presented. The Wilson et al. (1994) equations are then compared term by term with the modified equations (i.e., Equations (3.29) and (3.30)) to evaluate the differences. The modified and Wilson (1994) formulations are summarised in the Table 3-1 and Table 3-2 for moisture and heat flow, respectively.

3.3.1.1. Moisture flow equation

Wilson (1990) considered a soil element as shown in Figure 3-2 during the evaporation process. The amount of liquid vapour entering (M_{ve}) and leaving (M_{vl}) the soil element due to vapour diffusion are given as follows:

$$M_{ve} = -D^* \left(\frac{\partial P_v}{\partial y} \right) dx dz \quad (3.32)$$

$$M_{vl} = - \left[D^* \left(\frac{\partial P_v}{\partial y} \right) + \frac{\partial}{\partial y} \left(D^* \frac{\partial P_v}{\partial y} \right) dy \right] dx dz \quad (3.33)$$

where:

$$D^* = D \frac{W_{wa}}{R^* T} = \alpha \theta_a D_{vap} \frac{W_{wa}}{R^* T} = D \frac{1}{RT} \quad (3.34)$$

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Therefore the net change in mass flux of water vapour within the soil element due to vapour diffusion, $m_{v(\text{diff})}$, is given as follows:

$$\frac{\partial m_{v(\text{diff})}}{\partial t} = M_{v_l} - M_{v_e} = - \left[\frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) dy \right] dx dz = - \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) \quad (3.35)$$

where:

$$\frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) = \frac{\partial}{\partial y} \left\{ D \left(\frac{g \rho_v}{RT} \right) \frac{\partial h_p}{\partial y} + D \left[e^{\frac{(h_p)_g}{RT}} \frac{dp_0}{dT} - \frac{g \rho_v (h_p)}{RT^2} \right] \frac{\partial T}{\partial y} \right\} \quad (3.36)$$

Figure 3-2 Soil element during evaporation process (vapour flow)

The dry air of bulk air phase (dry air and vapour) will move whenever there is a pore-air pressure gradient in the soil. The air flow in unsaturated soil can be calculated using Fick's law (Dakshanamurthy and Fredlund 1981, Fredlund and Dakshanamurthy, 1982, Fredlund and Rahardjo 1993a). Similar to Equation (3.35), the net change in mass flux of air (m_a) within the soil element is given by:

$$\frac{\partial m_a}{\partial t} = M_{al} - M_{ac} = - \left[\frac{\partial}{\partial y} \left(D_{atm} \frac{\partial P}{\partial y} \right) dy \right] dx dz = - \frac{\partial}{\partial y} \left(D_{air} \frac{\partial P}{\partial y} \right) \quad (3.37)$$

When the dry air moves, it also carries the water molecules (i.e., vapour molecules) along and the phenomenon is called advection. The amount of this water vapour, $m_{(adv)}$, can be estimated as a fraction of the net change of dry air (m_a), i.e.,

$$\frac{\partial m_{v(adv)}}{\partial t} = - \frac{\rho_v}{\rho_a} \frac{\partial}{\partial y} \left(D_{air} \frac{\partial P}{\partial y} \right) \quad (3.38)$$

where:

$$\frac{\rho_v}{\rho_a} = \frac{W_{wa} P_v}{W_a P_a} \quad (3.39)$$

It was assumed that atmospheric pressure remains constant (i.e., $\partial P = \partial u_a$) and therefore Equation (3.37) can be rewritten as follows:

$$\frac{\partial m_{v(adv)}}{\partial t} = - \frac{P_v}{P_a} \frac{\partial}{\partial y} \left(D \cdot \frac{\partial u_a}{\partial y} \right) \quad (3.40)$$

The combination of diffusion (i.e., Equation (3.35)) and advection (i.e., Equation(3.40)) gives the total change in the mass of water vapour in the soil element, $m_{v(diff+adv)}$, as:

$$\frac{\partial m_{v(diff+adv)}}{\partial t} = - \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) - \frac{P_v}{P_a} \frac{\partial}{\partial y} \left(D \cdot \frac{\partial u_a}{\partial y} \right) \quad (3.41)$$

In the case where deformation of soil is neglected then $\partial P_v = \partial u_a$ and Equation (3.41) can be rewritten as follows:

$$\frac{\partial m_{v(diff+adv)}}{\partial t} = - \frac{P_v + P_a}{P_a} \frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) \quad (3.42)$$

As vapour pressure and air volume do not remain constant with time, the change of total mass flux, m_{Tv} , should also be accounted for in the change of mass of vapour in soil element due to the change of vapour density and air void. The total inter-phase flux, $m_{v(inter-phase)}$, is given as follows:

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$$\frac{\partial m_{v(\text{inter-phase})}}{\partial t} = -\frac{\partial m_{v(\text{diff+adv})}}{\partial t} + \frac{\partial m_{Tv}}{\partial t} \quad (3.43)$$

where:

$$\frac{\partial m_{Tv}}{\partial t} = \rho_v \frac{\partial(\theta_a)}{\partial t} + \theta_a \frac{\partial(\rho_v)}{\partial t} \quad (3.44)$$

Therefore, the total mass of vapour change in the soil element can be represented as follows:

$$\frac{\partial m_{v(\text{inter-phase})}}{\partial t} = \frac{\partial}{\partial y} \left(D^* \frac{\partial P_v}{\partial y} \right) + \frac{P_v}{P_a} \frac{\partial}{\partial y} \left(D^* \frac{\partial u_a}{\partial y} \right) + \rho_v \frac{\partial(\theta_a)}{\partial t} + \theta_a \frac{\partial(\rho_v)}{\partial t} \quad (3.45)$$

The change in liquid water mass can be determined in the same way as for water vapour in the soil element as shown in Figure 3-3.

Figure 3-3 Soil element during water liquid flow

Darcy's law for unsaturated soil was used to calculate the net change in volume of liquid water within the soil element. The resultant equation for liquid flow is given by the well known Richard's equation:

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$$\begin{aligned}\frac{\partial m_{lw}}{\partial t} &= M_{lwl} - M_{lwe} = - \left[k_w \left(\frac{\partial h_w}{\partial y} \right) + k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) dy \right] dx dz - \left[k_w \left(\frac{\partial h_w}{\partial y} \right) dx dz \right] \\ &= -k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right)\end{aligned}\quad (3.46)$$

The equation describing the change in volumetric water content due to the combination of liquid and vapour phase is presented as follows:

$$\frac{\partial \frac{V_w}{V}}{\partial t} = \frac{\partial m_{lw}}{\partial t} - \frac{1}{\rho_w} \frac{\partial m_{(vtotal)final}}{\partial t} \quad (3.47)$$

or

$$\begin{aligned}\frac{\partial \frac{V_w}{V}}{\partial t} &= -k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) \\ &- \frac{1}{\rho_w} \left[\frac{\partial}{\partial y} \left(D \cdot \frac{\partial P_v}{\partial y} \right) + \frac{P_v}{P_a} \frac{\partial}{\partial y} \left(D \cdot \frac{\partial u_a}{\partial y} \right) + \rho_v \frac{\partial (\theta_a)}{\partial t} + \theta_a \frac{\partial (\rho_v)}{\partial t} \right]\end{aligned}\quad (3.48)$$

or

$$\begin{aligned}m_1^w \frac{\partial (\sigma_y - u_a)}{\partial t} + m_2^w \frac{\partial (u_a - u_w)}{\partial t} - \left[\rho_v \frac{\partial (\theta_a)}{\partial t} + \theta_a \frac{\partial (\rho_v)}{\partial t} \right] &= -k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) \\ - \frac{1}{\rho_w} \left\{ \frac{\partial}{\partial y} \left[D \left(\frac{g \rho_v}{RT} \right) \frac{\partial h_p}{\partial y} + D \left[e^{\frac{(h_p)g}{RT}} \frac{d\rho_0}{dT} - \frac{g \rho_v (h_p)}{RT^2} \right] \frac{\partial T}{\partial y} \right] + \frac{P_v}{P_a} \frac{\partial}{\partial y} \left(D \cdot \frac{\partial u_a}{\partial y} \right) \right\}\end{aligned}\quad (3.49)$$

Equation (3.49) describes the moisture content change in deformable unsaturated soil. For the case where soil deformation is considered negligible, Equation (3.49) becomes:

$$\begin{aligned}\frac{\partial (h_p)}{\partial t} &= \frac{1}{m_2^w} k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) \\ - \frac{1}{\rho_w m_2^w} \frac{P_v + P_a}{P_a} \frac{\partial}{\partial y} \left\{ D \left(\frac{g \rho_v}{RT} \right) \frac{\partial h_p}{\partial y} + D \left[e^{\frac{(h_p)g}{RT}} \frac{d\rho_0}{dT} - \frac{g \rho_v (h_p)}{RT^2} \right] \frac{\partial T}{\partial y} \right\}\end{aligned}\quad (3.50)$$

The modified equation (i.e., Equation (3.28)) and Wilson et al. (1994) equation (i.e., Equation (3.50)) have similar form, however they have some differences. The differences can be found by comparing term by term between the two formulations (Table 3-1). Firstly, on the left hand side of the modified equation there is an extra term to account for mass of moisture vapour (i.e., Equation (3.4)) whereas in Wilson et al. (1994) this term was not present as the constitutive relationship for soil was used. Wilson et al. (1994) assumed that the time derivative of constitutive relation given in Equation (2.70) can be equated to the vapour pressure and hydraulic gradients in the soil. Using this assumption the Wilson et al. (1994) equation of moisture flow was established, i.e.,

$$\frac{\partial \left(\frac{V_w}{V} \right)}{\partial t} = \frac{\partial (\theta_w)}{\partial t} = \frac{\partial m_{lw}}{\partial t} + \frac{1}{\rho_l} \frac{\partial m_v}{\partial t} = -k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) - \frac{1}{\rho_l} \left(\frac{P + P_v}{P} \right) \frac{\partial}{\partial y} \left(D^* \frac{\partial P_v}{\partial y} \right) \quad (3.51)$$

Actually in Wilson (1990) moisture flow equation (i.e., Equation (3.49)) there is a similar term as the extra term in the modified equation except for their signs. In Equation (3.50) for non-deformable soil, this term was eliminated by assuming that inter-phase flux is negligible (Wilson 1990). The inter-phase flux in Wilson (1990) was to account for part of the liquid phase changing into vapour phase during liquid flow. In Philip and de Vries (1957) (i.e., Equation (3.5)) it was considered that water vapour fills all the voids of the soil and the extra term accounts for mass of vapour, $\rho_v \theta_a$, whereas Wilson (1990) considered the inter-phase flux as a product of changing phase (i.e., liquid to vapour) during liquid flow. Therefore, the inter-phase flux in Wilson (1990) increases with time while liquid water decreases with time (flowing out of soil element). This explained the positive sign of inter-phase flux in Equation (3.43). Yang et al. (1998) using Philip and de Vries (1957) approach to develop a model for heat and moisture flow in deformable soil showed that in general the interaction between the phases in soil is too complicated to define the inter-phase flux reliably. If the mass of vapour in Equation (3.28) and inter-phase flux in Equation (3.5), is eliminated then the left hand side of Equations (3.28) and (3.50) are the same.

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Secondly, the two equations also have difference on the right hand side. In Equation (3.50), there is a correction factor, $(P_a + P_v)/P_a$, which accounted for the flow due to a combination of diffusion and advection. Wilson (1990) accounted for advection in the evaporation process. However, the advection amount was determined as a fraction of the vapour flow by diffusion (i.e., Equation (3.38)). This approximation constitutes the correction factor. Philip and de Vries (1957) had ignored the advection process and therefore this term is absent in Equation (3.28). The influence of this correction factor will be investigated in the parametric study described later in Chapter 4.

Table 3-1 Summary of modified and Wilson et al. (1994) formulations for moisture flow

Modified equation for moisture flow	
Left hand side	Right hand side
$\frac{\partial(h_w)(g)}{\partial t}(\rho_l m_2^w)$ $+\frac{\partial(h_w)(g)}{\partial t}\left[\theta_a \frac{\partial p_v}{\partial(u_a-u_w)}\right]$ $-\frac{\partial(h_w)(g)}{\partial t}(\rho_v m_2^w)$	$\frac{\partial}{\partial y}\left(k_w \frac{\partial h_p}{\partial y}\right)+\frac{\partial}{\partial y}(k_w)$ $+\frac{\partial}{\partial y}\left(D \frac{g \rho_v}{RT} \frac{\partial h_p}{\partial y}\right)$ $+\frac{\partial}{\partial y}\left\langle D\left[e^{\frac{(h_p)g}{RT}} \frac{dp_0}{dT}-\frac{g \rho_v(h_p)}{RT^2}\right]\right\rangle \frac{\partial T}{\partial y}$
Wilson et al. (1994) equation for moisture flow	
$\frac{\partial h_w}{\partial t}(\rho_l g m_2^w)$	$\frac{\partial}{\partial y}\left(k_w \frac{\partial h_p}{\partial y}\right)+\frac{\partial}{\partial y}(k_w)$ $+\left(\frac{P_a+P_v}{P_a}\right) \frac{1}{\rho_l} \frac{\partial}{\partial y}\left(\alpha \theta_a D_{\text {vap }} \frac{W_{\text {wa }}}{R^* T} \frac{\partial P_v}{\partial y}\right)$ $\Leftrightarrow$ $\frac{\partial}{\partial y}\left(k_w \frac{\partial h_p}{\partial y}\right)+\frac{\partial}{\partial y}(k_w)$ $+\left(\frac{P_a+P_v}{P_a}\right) \frac{1}{\rho_l} \frac{\partial}{\partial y}\left(\alpha \theta_a D_{\text {vap }} \frac{W_{\text {wa }}}{R^* T} P_v \frac{W_{\text {wa }} g}{R^* T} \frac{\partial h_p}{\partial y}\right)$ <p>where: $W_{\text {wa }}=\text {Molecular weight of water}=18.06$</p>

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	$R^* = \text{universal gas constant} = 8.314 \text{ J.kmole}^{-1}\text{K}^{-1}$ <p>Have: $\frac{W_{wa}}{R^*} = 0.0021722 \text{ (m}^{-1}\text{K}^{-1}) \sim \frac{1}{R}$</p> <p>Following the ideal gas law, have:</p> $P_v V = n R^* T \Leftrightarrow P_v = \frac{n}{V} R^* T$ $\Leftrightarrow P_v = \frac{M_{wa}}{W_{wa}} \frac{1}{V} R^* T \Leftrightarrow P_v = \rho_v \frac{R^* T}{W_{wa}}$ <p>therefore, the equation above can be represented as follows:</p> $k_w \frac{\partial}{\partial y} \left(\frac{\partial h_w}{\partial y} \right) + \frac{1}{\rho_w} \frac{P_v + P_a}{P_a} \frac{\partial}{\partial y} \left\{ D \left(\frac{g \rho_v}{RT} \right) \frac{\partial h_p}{\partial y} + D \left[e^{\frac{(h_p)_g}{RT}} \frac{d\rho_0}{dT} - \frac{g \rho_v (h_p)}{RT^2} \right] \frac{\partial T}{\partial y} \right\}$
--	--

3.3.1.2. Heat flow equation

The equation for heat flow of both modified model and Wilson et al. (1994) model are summarised in Table 3-2. For the heat flow equations (i.e., Equations (3.30) and (2.80)), there are also two differences although the form of the equations is similar. On the right hand side, all the terms are the same except for the sign of the heat flow term due to the latent heat of vaporization. Following the energy conservation law (i.e., Equation (3.20)), the heat flow terms due to conduction and latent heat of vaporization are consumptive and therefore, the sign of these terms should be the same (i.e., positive) to balance with the heat source on the left hand side of the heat flow equation. Wilson et al. (1994) equation for heat flow is a modification of Jame and Norum (1980) which is proposed to deal with heat flow in a freezing unsaturated porous medium. The negative sign of the latent heat of vaporisation in Wilson et al. (1994) can be understood that during the liquid to vapour phase change energy from the soil system is supplied for the process. The different physical meaning appears plausible. However in using a unit element formulation,

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the interaction of the unit element and its surrounding condition should be the only consideration. The effect of this difference toward the overall performance of the equation needs to be evaluated. Generally, the sign of latent heat of vaporisation should be positive as can be seen in the recent work of Prunty (2002). The differences between the two equations will be further elaborated though the parametric study presented later in Chapter 4.

Table 3-2 Summary of modified and Wilson et al. (1994) formulations for heat flow

Modified equation for heat flow		
Left hand side	Right hand side	
	Conductive	Latent heat of vaporisation
$\frac{\partial T}{\partial t} \left[C_h - L_0 \theta_a \rho_{vs} e^{\frac{h_p g}{RT}} \frac{h_p g}{R} \frac{1}{(T)^2} \right]$	$\frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right)$	$+L_v \frac{\partial}{\partial y} \left[\rho_l D_v \left(\frac{g}{RT} \right) \rho_v \left(\frac{\partial h_p}{\partial y} \right) \right]$
Wilson et al. (1994) equation for heat flow		
$\frac{\partial T}{\partial t} (C_h)$	$\frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right)$ or $\frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right)$	$-L_v \left(\frac{P + P_v}{P} \right) \frac{\partial}{\partial y} \left[\left(D \frac{1}{RT} \right) \frac{g P_v}{RT} \frac{\partial h_p}{\partial y} \right]$ or $-L_v \left(\frac{P + P_v}{P} \right) \frac{\partial}{\partial y} \left[D \left(\frac{g}{RT} \right) \rho_v \frac{\partial h_p}{\partial y} \right]$

3.4. NUMERICAL FORMULATION AND BOUNDARY CONDITIONS FOR MODIFIED MODEL

3.4.1. Finite Difference Method

In recent years, with ever growing availability of high speed, large capacity computers, the interest in the use of numerical methods, such as the finite difference and finite element methods for solving problems governed by differential equation has increased significantly. In general, finite difference methods are simple to formulate and are easy to apply. However, with the irregularity of the domain, finite

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difference method will suffer some difficulties in dividing the grid mesh (Ozisick, 1980). In the one-dimensional infiltration soil column, the difficulty of domain irregularity is not present. The concept of finite difference method is approximate in the sense that derivative at a point is approximated by difference quotients over a small interval, i.e., $\partial\phi/\partial x$ is replaced by $\delta\phi/\delta x$ where δx is small and δy is constant, but the solutions are not approximate in the sense of crude estimates (Desai and Christian, 1977). Basically, there are three types of partial differential equation, namely: elliptic, parabolic, and hyperbolic (Mitchell and Griffiths 1980). In the solution of partial differential equation with finite differences, the choice of a finite difference scheme depends mostly on the type of the partial differential equation. Consider the general form of partial differential equation as follows:

$$O_1 \frac{\partial^2 \phi}{\partial^2 x} + O_2 \frac{\partial^2 \phi}{\partial x \partial y} + O_3 \frac{\partial \phi}{\partial x} + O_4 \frac{\partial \phi}{\partial y} + O_5 (\phi) + O_6 (x, y) = 0 \quad (3.52)$$

In the above equation, assume that the coefficients $O_1, O_2, O_3, O_4, O_5, O_6$ are functions of two independent variables, x and y but not of the independent variable ϕ . The partial differential equation (i.e., Equation (3.52)) at a point (x_0, y_0) is called:

(1) Elliptic	if $(O_2)^2 - 4(O_1 O_3) < 0$,
(2) Parabolic	if $(O_2)^2 - 4(O_1 O_3) = 0$,
(3) Hyperbolic	if $(O_2)^2 - 4(O_1 O_3) > 0$.

Two basic approaches commonly used to discretize the derivatives in partial differential equation include: (1) the use of Taylor series expansion and (2) the control volume approach.

The idea of finite difference representation of a derivative can be introduced by recalling the definition of the derivative of the function $f(x, y)$ at $x = x_0, y = y_0$,

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} \quad (3.53)$$

With a function f where its derivatives are single-valued, finite and continuous functions of x , then by Taylor's theorem:

$$f(x + \Delta x) = f(x) + \Delta x f'(x) + \frac{1}{2}(\Delta x)^2 f''(x) + \frac{1}{6}(\Delta x)^3 f'''(x) + \dots$$

and

$$(3.54)$$

$$f(x - \Delta x) = f(x) - \Delta x f'(x) + \frac{1}{2}(\Delta x)^2 f''(x) - \frac{1}{6}(\Delta x)^3 f'''(x) + \dots$$

The above two expressions form the basis for developing finite difference approximations for the first derivative, df/dx , about x . Rearranging Equation (3.54) the forward and backward finite difference approximations, respectively, become:

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} + O(\Delta x) \quad \text{-- forward difference}$$

and

$$(3.55)$$

$$f'(x) = \frac{f(x) - f(x - \Delta x)}{\Delta x} + O(\Delta x) \quad \text{-- backward difference}$$

Subtracting forward difference from backward difference in Equation (3.55), the central difference is determined as:

$$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} + O(\Delta x)^2 \quad (3.56)$$

Figure 3-4. Concept of approximation in finite difference method

In Equations (3.55) and (3.56), $O(\Delta x)$ and $O(\Delta x)^2$ are the truncation error associated with the finite difference approximation. The approximation concept of finite difference can be illustrated as shown in Figure 3-4.

Clearly Equation (3.56) approximates the slope of tangent at P by the slope of the chord AB, and is called central difference. Equations (3.55) also approximates the slope of the tangent at P by either the slope of the chord PB or AP and are called forward difference and backward difference, respectively.

3.4.2. Finite Difference Formulation for Infiltration Problem

For infiltration, liquid water flow is predominant. In other words, the term for vapour flow term, $\rho_v \theta_a$, in the left hand side of Equation (3.4) or the two terms $(\theta_a \frac{\partial \rho_v}{\partial (u_a - u_w)})$ and $(-\rho_v \frac{\partial \theta_w}{\partial (u_a - u_w)})$ in Equation (3.28) are negligible. The term which accounts for mass of vapour leaving the porous medium, $-D\nabla \rho_v$, is also not present. Therefore, neglecting the vapour phase, Equation (3.28) can be rewritten as follows:

$$\frac{\partial \theta_w}{\partial h_p} \frac{\partial h_p}{\partial t} = \frac{\partial}{\partial y} \left(k_w \frac{\partial}{\partial y} h_p \right) + \frac{\partial}{\partial y} k_w \quad (3.57)$$

which is the well known Richards' equation.

Equation (3.57) is nonlinear because the permeability function, k_w , and the source term, θ_w , depend on water content. This equation can be discretized using finite difference scheme as follows:

$$\begin{aligned} c_i \frac{(h_p)_i^{j+1} - (h_p)_i^j}{(\Delta t)} = & \left(\frac{1}{\Delta y} \right) K_{ip} \left\{ (1 - \xi) \left[\frac{(h_p)_{i+1}^j - (h_p)_i^j}{(\Delta y)} \right] + \xi \frac{(h_p)_{i+1}^{j+1} - (h_p)_i^{j+1}}{(\Delta y)} \right\} \\ & - \left(\frac{1}{\Delta y} \right) K_{im} \left\{ (1 - \xi) \left[\frac{(h_p)_i^j - (h_p)_{i-1}^j}{(\Delta y)} \right] + \xi \frac{(h_p)_i^{j+1} - (h_p)_{i-1}^{j+1}}{(\Delta y)} \right\} + \frac{K_{ip} - K_{im}}{\Delta y} \end{aligned} \quad (3.58)$$

where: $c_i = \partial \theta / \partial h_p$ is the water capacity,

i = element node number,

- j = time step number,
 γ_w = unit weight of water,
 Δt = time step (s),
 Δy = elevation step or element height (m),
 K_{im} = $(K_{i-1} + K_i)/2$, arithmetic mean of permeability between node i and $i-1$
 K_{ip} = $(K_i + K_{i+1})/2$, arithmetic mean of permeability between node i and $i+1$
 ξ = 0, 1 or 0.5 for forward, backward or central difference scheme.

3.4.3. Finite Difference Formulation for Infiltration and Evaporation Problem

Equation (3.29) for moisture flow and Equation (3.30) for heat flow are nonlinear parabolic partial differential equations and can be discretized using finite difference scheme as follows:

For liquid and vapour flow:

$$\begin{aligned}
 \frac{(h_p)_i^{j+1} - (h_p)_i^j}{\Delta t} = & \frac{1}{G(\Delta y)} \left\langle \rho_w K_{ip} \left\{ (1-\xi) \left[\frac{(h_p)_{i+1}^j - (h_p)_i^j}{\Delta y} \right] + \xi \left[\frac{(h_p)_{i+1}^{j+1} - (h_p)_i^{j+1}}{\Delta y} \right] \right\} - \rho_w K_{im} \left\{ (1-\xi) \left[\frac{(h_p)_i^j - (h_p)_{i-1}^j}{\Delta y} \right] + \xi \left[\frac{(h_p)_i^{j+1} - (h_p)_{i-1}^{j+1}}{\Delta y} \right] \right\} \right\rangle \\
 & + \frac{1}{A(\Delta y)} \left\langle (H)_p \left\{ (1-\xi) \left[\frac{(h_p)_{i+1}^j - (h_p)_i^j}{\Delta y} \right] + \xi \left[\frac{(h_p)_{i+1}^{j+1} - (h_p)_i^{j+1}}{\Delta y} \right] \right\} - (H)_{im} \left\{ (1-\xi) \left[\frac{(h_p)_i^j - (h_p)_{i-1}^j}{\Delta y} \right] + \xi \left[\frac{(h_p)_i^{j+1} - (h_p)_{i-1}^{j+1}}{\Delta y} \right] \right\} \right\rangle \\
 & + \frac{1}{A(\Delta y)} \left\langle (I)_p \left\{ (1-\xi) \left[\frac{(T)_{i+1}^j - (T)_i^j}{\Delta y} \right] + \xi \left[\frac{(T)_{i+1}^{j+1} - (T)_i^{j+1}}{\Delta y} \right] \right\} - (I)_{im} \left\{ (1-\xi) \left[\frac{(T)_i^j - (T)_{i-1}^j}{\Delta y} \right] + \xi \left[\frac{(T)_i^{j+1} - (T)_{i-1}^{j+1}}{\Delta y} \right] \right\} \right\rangle \\
 & + \frac{\rho_w}{A(\Delta y)} (K_{ip} - K_{im})
 \end{aligned} \tag{3.59}$$

For heat flow:

$$\begin{aligned}
 \frac{(T_i^{j+1} - T_i^j)}{\Delta t} = & \left(\frac{1}{\Delta y M} \right) \left\langle \lambda_{ip} \left[(1-\xi) \left(\frac{T_{i+1}^j - T_i^j}{\Delta y} \right) + \xi \left(\frac{T_{i+1}^{j+1} - T_i^{j+1}}{\Delta y} \right) \right] - \lambda_{im} \left[(1-\xi) \left(\frac{T_i^j - T_{i-1}^j}{\Delta y} \right) + \xi \left(\frac{T_i^{j+1} - T_{i-1}^{j+1}}{\Delta y} \right) \right] \right\rangle + \\
 & \left(\frac{1}{\Delta y K} \right) \left\langle (G)_p \left\{ (1-\xi) \left[\frac{(h_p)_{i+1}^j - (h_p)_i^j}{\Delta y} \right] + \xi \left[\frac{(h_p)_{i+1}^{j+1} - (h_p)_i^{j+1}}{\Delta y} \right] \right\} - (G)_{im} \left\{ (1-\xi) \left[\frac{(h_p)_i^j - (h_p)_{i-1}^j}{\Delta y} \right] + \xi \left[\frac{(h_p)_i^{j+1} - (h_p)_{i-1}^{j+1}}{\Delta y} \right] \right\} \right\rangle
 \end{aligned} \tag{3.60}$$

where: γ_w = unit weight of water,

Δt = time step (s),

Δy = elevation step or element height (m),

K_{im} = $(K_{i-1} + K_i)/2$, arithmetic mean of permeability between node i and $i-1$

K_{ip} = $(K_i + K_{i+1})/2$, arithmetic mean of permeability between node i and $i+1$

λ_{ip} = $(\lambda_{i-1} + \lambda_i)/2$, arithmetic mean between node i and $i-1$

λ_{ip} = $(\lambda_i + \lambda_{i+1})/2$, arithmetic mean between node i and $i+1$

G_{ip} = $(\lambda_{i-1} + \lambda_i)/2$, arithmetic mean between node i and $i-1$

G_{ip} = $(\lambda_i + \lambda_{i+1})/2$, arithmetic mean between node i and $i+1$

3.4.4. Boundary Condition

To solve the governing differential equation, it is necessary to specify boundary conditions and its initial condition (pressure head or water content profiles). There are two types of boundary conditions for a 1-D model's top and bottom boundary conditions. The first type of boundary condition represents flux due to rainfall, evaporation rate at the top boundary or the liquid water flowing out from the bottom boundary. The second type of boundary condition represents a fixed value for pressure head (i.e., matric suction), temperature or relative humidity for both top and bottom boundary conditions. The boundary conditions for infiltration and evaporation problems in this study are presented in the following sections.

3.4.4.1. Boundary Condition for Infiltration Problem

Rubin (1966) presented three important phenomena related to the applied top boundary condition for the numerical model. These phenomena relate to the pre-ponding interval at the top of the soil. At the beginning, all the water enters the soil until ponding occurs and infiltration rate decreases. The phenomena can be briefly described as follows:

- If constant water flux is maintained at the surface, the pressure head at the surface must be constantly increasing until it reaches the limiting value, h_{lm} .

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- Ponding will only occur under constant rainfall intensity if, and only if, the rainfall intensity exceeds the saturated permeability of the soil.
- During non-ponding infiltration of rain with constant intensity R , the surface pressure must be continually approaching the limiting value of pressure head, h_{lm} . Rubin and Steinhardt (1963) also examined this condition experimentally.

Consider the top boundary condition where the surface flux is equal to the rainfall rate. This boundary condition can be applied in finite difference schemes by using mass conservation law and Darcy's law. The finite difference formulation for this boundary condition is as follows:

$$\frac{\partial \theta_w}{\partial t} = -\frac{\partial q}{\partial z} \quad (3.61)$$

The right hand side of Equation (3.61) can be recast in finite difference form as follows:

$$-\frac{\partial q}{\partial z} = \frac{-q_{i+1/2} + q_{i-1/2}}{\Delta z} = \frac{-q_{inf}}{\Delta z} + \frac{(-kj)}{\Delta z} = \frac{-q_{inf}}{\Delta z} - \frac{k}{\Delta z} \left(\frac{\partial h_w}{\partial z} \right) \quad (3.62)$$

or:

$$-\frac{\partial q}{\partial z} = \frac{-q_{inf}}{\Delta z} - \frac{K_{im}}{\Delta z} \left[1 + \frac{(h_p)_{ip}^{j+1} - (h_p)_{ip-2}^{j+1}}{\Delta z} \right] \quad (3.63)$$

where: q_{inf} = applied flux surface (m/s),

$(h_p)_{ip-1}^{j+1} = (h_p)_{ip-(\Delta z/2)}^{j+1}$, matric suction at the soil surface by shifting the coordinate $\Delta z/2$ up, and

$(h_p)_{ip-2}^{j+1} = (h_p)_{ip-(\Delta z)}^{j+1}$, matric suction at the node (i+1).

The coordinate system for the top boundary condition in the numerical modelling of the soil column is presented in the Figure 3-5.

Figure 3-5 Sketch of numerical coordinate of top boundary condition for infiltration problem

Substituting Equation (3.63) into Equation (3.58) the solution for the top boundary condition is obtained as follows:

$$(h_p)_i^{j+1} = (h_p)_i^j + \frac{\Delta t}{c_i} \left(\frac{1}{\Delta z} \right) \left\{ -q_{inf} - K_{im} \left[1 + \frac{(h_p)_{ip}^{j+1} - (h_p)_{ip-2}^{j+1}}{\Delta z} \right] \right\} \quad (3.64)$$

The second type of top boundary condition occurs when the capillary head at the surface equals to zero, meaning that ponding has occurred and there is no evaporation. As a result, the boundary condition must be changed. When it is desired to specify surface ponding at the start of the infiltration process, the surface pressure head is set to the depth of ponding for the entire ponded modelling.

For the bottom boundary condition a unit pressure head gradient (i.e., liquid flux going out) or constant pressure head can be applied. The unit pressure gradient means that there is a constant flow through the lower boundary. This boundary condition is applicable as long as the wetting front does not reach the bottom

boundary of the model. The finite difference equation for bottom flux is derived as follows:

$$(h_p)_n^{j+1} = (h_p)_n^j + \frac{\Delta t}{c_i} \left(\frac{1}{\Delta z} \right) \left\{ q_{out} + K_{nm} \left[1 + \frac{(h_p)_{np}^{j+1} - (h_p)_{np-2}^{j+1}}{\Delta z} \right] \right\} \quad (3.65)$$

where: q_{out} = liquid water going out from the bottom (m/s),

$(h_p)_{np-1}^{j+1} = (h_p)_{np-(\Delta z/2)}^{j+1}$, matric suction at the soil surface by shifting the coordinate $\Delta z/2$ up, and

$(h_p)_{np-2}^{j+1} = (h_p)_{np-(\Delta z)}^{j+1}$, matric suction at the node $[i+(n-1)]$.

The coordinate for bottom boundary condition for numerical modelling of soil column is presented in Figure 3-6:

Figure 3-6 Sketch of numerical coordinate of bottom boundary condition for infiltration problem

The constant head bottom boundary condition is applicable when the water table is present and does not move during the numerical modelling. This boundary

condition can be implemented by applying a constant pressure head at the bottom of the infiltration model. This implies that the water from the soil column can flow out freely at the bottom.

3.4.4.2. Boundary condition for Evaporation Problem

For evaporation problem, the top boundary condition is where the volumetric water content changes with relative humidity or temperature at the surface of the porous medium. Fredlund and Rahardjo (1993b) suggested that modified Fick's law may be used to describe the vapour flow through unsaturated soil. Therefore, this boundary condition is a combination of the mass conservation law with the modified Fick's law. The general boundary condition equation is given as follows:

$$\frac{\partial \theta_w}{\partial t} = -\frac{\partial (M_E)}{\partial z} \quad (3.66)$$

where: M_E = amount of vapour density leaving per unit of porous medium.

The finite difference scheme for Equation (3.66) is as follows:

$$\frac{\partial \theta_w}{\partial z} = -\frac{\partial (M_E)}{\partial z} = \frac{-(M_E)_{i+1/2} + (M_E)_{i-1/2}}{\Delta z} \quad (3.67)$$

Applying Equation (3.12) and definition of vapour diffusivity to the right hand side of Equation (3.67), Equation (3.67) can be rewritten as follows:

$$\frac{\partial \theta_w}{\partial z} = -\frac{\partial (M_E)}{\partial z} = -\frac{\rho_{vs} R_h}{\Delta z} + \left[-\frac{D_{\theta v}}{\Delta z} \left(\frac{\partial (\rho_v)}{\partial z} \right) \right]_{i-1/2} \quad (3.68)$$

or:

$$\frac{\partial \theta_w}{\partial z} = -\frac{\partial (M_E)}{\partial z} = -\frac{\rho_{vs} R_h}{\Delta z} - \frac{(D_{\theta v})_{im}}{\Delta z} \left(\frac{(\rho_v)_{np}^{j+1} - (\rho_v)_{np-2}^{j+1}}{\Delta z} \right) \quad (3.69)$$

Substituting Equation (3.69) into Equation (3.59) the solution for the top boundary condition is obtained as follows:

$$(h_p)_i^{j+1} = (h_p)_i^j + \frac{\Delta t}{c_i} \left(\frac{1}{\Delta z} \right) \left[\rho_{vs} R_h + (D_{\theta v})_{im} \left(\frac{(\rho_v)_{np}^{j+1} - (\rho_v)_{np-2}^{j+1}}{\Delta z} \right) \right] \quad (3.70)$$

The top boundary condition for the moisture and heat flow requires the top temperature for Equation (3.60). However, the change in temperature at the top node for the finite difference model in the case of heat flow is very fast in reaching the applied temperature. Therefore, for this case the applied temperature can be fixed at the top node of the finite difference model. In other words, by combining this boundary condition and the top boundary condition for vapour flow (i.e., Equation (3.70)) the finite difference model for the liquid and heat flow is solved.

Similar to the procedure for defining the bottom boundary condition the condition for the bottom node in the finite difference will be affected by the water table as well as the temperature applied. However, these conditions are not as complicated as the top boundary conditions. These conditions can be implemented by fixing the pressure head and temperature at the node.

In the special case when simultaneous infiltration and evaporation occur, there is a minor change in the top boundary condition:

$$\begin{aligned} (h_p)_i^{j+1} = (h_p)_i^j + \frac{\Delta t}{c_i} \left(\frac{1}{\Delta z} \right) & \left\{ -q_{inf} - K_{im} \left[-1 + \frac{(h_p)_{np-1}^{j+1} - (h_p)_{np-2}^{j+1}}{\gamma_w \Delta z} \right] \right\} \\ + \frac{\Delta t}{c_i} \left(\frac{1}{\Delta z} \right) & \left[\rho_{vs} R_h + (D_{\theta v})_{im} \left(\frac{(\rho_v)_{np-1}^{j+1} - (\rho_v)_{np-2}^{j+1}}{\Delta z} \right) \right] \end{aligned} \quad (3.71)$$

3.5. SPREADSHEET PROGRAM IMPLEMENTATION

In this study, a program was developed using Microsoft Excel spreadsheet and Microsoft Visual Basic to deal with moisture and heat flow in unsaturated soil. The spreadsheet is for computing Equations (3.59) and (3.60) for evaporation problem and Equation (3.58) for infiltration problem while the interaction process is handled by codes written in Microsoft Visual Basic. The program is explained in detail in the following sections.

3.5.1. Microsoft Excel Spreadsheet

The computational spreadsheet consists of two computing worksheets. One is for computing the moisture flow equation and the other is for computing the heat flow equation. The two worksheets were named moisture_cal and heat_cal, respectively. The moisture_cal worksheet has 24 columns representing the term in Equation (3.59) (Figure 3-7). The descriptions of the column are follows:

Column 1: Elevation height of the element,

Column 2: Temperature profile for time step j ,

Column 3: Matric suction profile for time step j ,

Columns 4 and 9: Effective molecular diffusivity calculation,

Columns 5 and 6: To calculate Equation (3.12),

Column 7: Volumetric air content,

Column 8: Coefficient of permeability with respect to water phase, k_w , computed using Equation (2.5),

Column 10: Calculation of Equation (3.58) for time step $j+1$, whereas the Equation (3.63) and (3.65) were employed for the top and the bottom cell, respectively,

Columns 11 and 12: Calculation of Equation (3.59) for time step $j+1$, whereas Equation (3.70) was employed for the top cell of column 11,

Column 13: Water capacity, c_i , in Equation (3.58),

Columns 14 and 15: Term A in Equation (3.59),

Columns 16 and 17: Permeability function, k_w , at the centre of the two consecutive elements, ($K_{im} = (K_{i-1} + K_i)/2$, $K_{ip} = (K_i + K_{i+1})/2$),

Columns 18 and 19: Effective molecular diffusivity at the centre of the two consecutive elements, ($D_{im} = (D_{i-1} + D_i)/2$, $D_{ip} = (D_i + D_{i+1})/2$)

Columns 20, 21 and 22: Mass balance moisture for time steps j and $j+1$, respectively,

Columns 23, and 24: Initial condition for matric suction and temperature, respectively.

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The heat_cal worksheet has 20 columns representing the term in Equation (3.60) (Figure 3-8). The descriptions of the columns are as follows:

Column 1: Elevation height of the element,

Column 2: Temperature profile for step j,

Column 3: Matric suction profile for step j,

Columns 4 and 13: Effective molecular diffusivity calculation,

Column 8: Thermal conductivity of soil element,

Columns 9, 10 and 11: Term M in Equation (3.31),

Column 14: Calculation of Equation (3.60). Known applied temperature was employed for the top cell,

Columns 15 and 16: Calculation of thermal conductivity at the centre of the two consecutive elements ($\lambda_{ip} = (\lambda_{i-1} + \lambda_i)/2$, $\lambda_{ip} = (\lambda_i + \lambda_{i+1})/2$).

Columns 17 and 18: Effective molecular diffusivity at the centre of the two consecutive elements, ($D_{im} = (D_{i-1} + D_i)/2$, $D_{ip} = (D_i + D_{i+1})/2$)

Columns 23, and 24: Initial condition for matric suction and temperature respectively.

3.5.2. Flow Chart of the Program

The program is marched in time using forward finite difference method. A Microsoft Visual Basic code was created in order to link the two equations for moisture and heat flow in the spreadsheet. The flow chart for the program is shown in Figure 3-9. The Microsoft Visual Basic code handles the implementation of the finite difference forward scheme and mass balance check. When the mass balance check is not satisfied (5% discrepancy), the program will halt and consequently the time step size needs to be reduced. The initial temperature and pressure head (matric suction) profile, input data, and time size step must be defined at the beginning. The implementation of the program is a process of a continuous loop where the result of the current step (j) will be used as an initial data for the next time step (j+1).

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1	2	3	4	5	6	7	8	9	10	11	12
Time Step j											
Time Step j+1											
z	T	(u _a -u _w)	D _{vap}	ρ _{va}	ρ _v	θ _a	k _w	D	(u _a -u _w) _{int}	(ρ _v) _{eva}	(u _a -u _w) _{int&eva}
0.5	24	14.84001301	8.30485E-05	0.021437431	0.02143511	0.543608193	4.91014E-11	4.70261E-11	14.83998401	0.00002	14.84000
0.49	24	10.0598101	8.30485E-05	0.021437431	0.021435858	0.520019297	3.50745E-10	4.36761E-11	10.05978859	0.00002	10.05980
0.48	24	7.740395897	8.30485E-05	0.021437431	0.02143622	0.49596123	1.93664E-09	4.03613E-11	7.740387801	0.00000	7.74039
0.47	24	6.694669241	8.30485E-05	0.021437431	0.021436384	0.47851219	5.79207E-09	3.80228E-11	6.694665605	0.00000	6.69467
0.46	24	6.107565793	8.30485E-05	0.021437431	0.021436476	0.465734767	1.2089E-08	3.63459E-11	6.10756369	0.00000	6.10756
0.45	24	5.713299273	8.30485E-05	0.021437431	0.021436537	0.45561109	2.08662E-08	3.50388E-11	5.713297865	0.00000	5.71330
i	i	i	i	i	i	i	i	i	i	i	i
i	i	i	i	i	i	i	i	i	i	i	i

13	14	15	16	17	18	19	20	21	22	23	24
Mass balance check											
Initial condition											
ci	ci(1)	A	kip/2	kim/2	Dip/2	Dim/2	Mass (1)	Mass (2)	MASS	T	(u _a -u _w)
0.032263936	-0.00069073	0.031573205		1.99923E-10		4.53511E-11	8.18E-06	8.18E-06	7.80E-12		34 12.47047355
0.076129213	-0.00163108	0.074498131	1.99923E-10	1.14369E-09	4.53511E-11	4.20187E-11	1.00E-05	1.00E-05	3.22E-12		26 8.578729903
0.140895375	-0.00301949	0.137875887	1.14369E-09	3.86436E-09	4.20187E-11	3.9192E-11	1.19E-05	1.19E-05	4.34E-12		26 6.506703685
0.196823045	-0.00421843	0.192604619	3.86436E-09	8.94053E-09	3.9192E-11	3.71843E-11	1.32E-05	1.32E-05	4.00E-12		26 5.710109605
0.240007179	-0.00514418	0.234863	8.94053E-09	1.64776E-08	3.71843E-11	3.56923E-11	1.42E-05	1.42E-05	3.27E-12		26 5.357497857
0.274198678	-0.00587716	0.26832152	1.64776E-08	2.65732E-08	3.56923E-11	3.44945E-11	1.50E-05	1.50E-05	2.67E-12		26 5.131537133
i	i	i	i	i	i	i	i	i	i	i	i
i	i	i	i	i	i	i	i	i	i	i	i
sum									2.53E-11		

Figure 3-7 The moisture_cal worksheet

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1	2	3	4	5	6	7	8	9	10	11	12
Time Step j											
M											
z	T	(u _a -u _w)	D _{vap}	ρ _{vs}	ρ _v	θ _a	λ	C _h	L _v	L _o *θ _a *ρ _v	C _h + L _o *θ _a *ρ _v
0.5	24	14.84000018	8.30485E-05	0.021437431	0.02143511	0.543608151	3.76E-01	1713800	795760	-0.00141876	1713799.999
0.49	24	10.0598046	8.30485E-05	0.021437431	0.021435858	0.520019255	3.77E-01	1713800	795760	-0.00092005	1713799.999
0.48	24	7.740391897	8.30485E-05	0.021437431	0.02143622	0.495961174	3.78E-01	1713800	795760	-0.00067518	1713799.999
0.47	24	6.6946666	8.30485E-05	0.021437431	0.021436384	0.478512138	3.79E-01	1713800	795760	-0.00056342	1713799.999
0.46	24	6.107564021	8.30485E-05	0.021437431	0.021436476	0.465734724	3.80E-01	1713800	795760	-0.00050029	1713799.999
0.45	24	5.713298008	8.30485E-05	0.021437431	0.021436537	0.455611056	3.80E-01	1713800	795760	-0.00045782	1713800
i	i	i	i	i	i	i	i	i	i	i	i
i	i	i	i	i	i	i	i	i	i	i	i

13	14	15	16	17	18	19	20	21	22	23	24
Time step j+1											
Initial condition											
D	T _{cal}	λ _{ip} /2	λ _{im} /2	D _{ip} /2	Dim/2					T	(u _a -u _w)
3.74215E-05	36.00		0.376556891		3.60886E-05					34	12.47047355
3.47557E-05	24.00	0.376556891	0.377696175	3.60886E-05	3.34368E-05					26	8.578729903
3.21179E-05	24.00	0.377696175	0.378688394	3.34368E-05	3.11874E-05					26	6.506703685
3.0257E-05	24.00	0.378688394	0.379410783	3.11874E-05	2.95898E-05					26	5.710109605
2.89226E-05	24.00	0.379410783	0.379958007	2.95898E-05	2.84025E-05					26	5.357497857
2.78824E-05	24.00	0.379958007	0.380404094	2.84025E-05	2.74493E-05					26	5.131537133
i	i	i	i	i	i					i	i
i	i	i	i	i	i					i	i

Figure 3-8 The heat_cal worksheet

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For the infiltration process where the temperature effect is not significant, the calculation process is performed in the moisture_cal worksheet. The calculation is carried out by copying results from column 10 to column 3 continuously for a desired number of time steps and at each time step the mass balance is checked. For the evaporation process, the interaction between temperature and pressure head (matric suction) profiles are solved by the computations of heat_cal and moisture_cal worksheets. Firstly, the computation starts in the heat_cal worksheet by copying results from column 14 to column 2 in the heat_cal worksheet and column 2 in moisture_cal worksheet. Secondly, the results from column 12 in moisture_cal worksheet is copied to column 3 in moisture_cal worksheet and column 3 in heat_cal worksheet. These two steps conclude the computation for time step j. This updating process will be carried out for a desired number of time steps.

The program was tested for accuracy against published analytical solutions and experimental data for infiltration and evaporation problems. The detail of the verification is presented later in Chapter 4.

3.6. SUMMARY

The modified model for moisture and heat flow in non-deformable unsaturated soil was established based on the work of Philip and de Vries (1957). Through an intensive comparison between the modified model and Wilson et al. (1994) model, the differences and similarities between the two models were highlighted and explained in detail. The forward finite difference formulations of the modified model were implemented using Microsoft Excel spreadsheet. The validations of modified model will be presented in the following chapters by comparing with analytical solutions and experimental results.

Figure 3-9 Flow chart for the program

Chapter 4 Evaluation of Modified Model

4.1. INTRODUCTION

In this chapter the Microsoft Excel spreadsheet program developed for the modified model described in Chapter 3 is evaluated. Firstly, the performance of the Microsoft Excel spreadsheet program is evaluated against analytical solutions as well as published experimental data for infiltration problem. Secondly, the program is verified with experimental data for evaporation problem. Thirdly, a parametric study is presented in order to compare the differences between Wilson et al. (1994) model and the modified model. A summary of this chapter is presented at the end.

4.2. VERIFICATION OF MODIFIED MODEL

4.2.1. Infiltration Problem

Three cases are used in the verification. Case 1 is from Srivastava and Yeh (1991), Case 2 is from Liakopoulos (1965) and Case 3 is from Choo and Yanful (2000). These three cases are presented in the following sections.

(i) Case 1: Srivastava and Yeh (1991)

Srivastava and Yeh (1991) presented closed form solution of the infiltration in a homogenous soil column based on Richards' equation. Review of Srivastava and Yeh (1991) in detail was presented in Section (2.3.1). The SWCC equation for the soil in this case is given by Equation (2.23) while the permeability function equation is given by Equation (2.22). The parameters of θ_s , θ_r , and α are 0.45, 0.2, and 0.01, respectively. The saturated permeability of the soil is 2.78×10^{-6} m/s. The closed form solution was based on Equation (2.25) and Equation (2.26).

In the Microsoft Excel spreadsheet program, the height of each soil element was set at 0.01m. The time step was set at 0.01s. The element height and time step size were chosen to be the same as Srivastava and Yeh (1991) in their numerical modelling. The initial pressure profile was the steady state infiltration profile at infiltration rate equal to 2.78×10^{-7} m/s (Srivastava and Yeh, 1991). After that, the infiltration rate was fixed at 2.50×10^{-6} m/s. The water table was fixed at the bottom of the column meaning that the matric suction at the bottom boundary was fixed at zero.

The closed form solution from Srivastava and Yeh, (1991) and the solution from the Microsoft Excel spreadsheet program are compared in Figure 4-1. The comparison shows that with the same input data, the program gives the same results as the closed form solution.

Figure 4-1 Matric suction distributions for the Case 1

(ii) Case 2: Liakopoulos (1965)

In Liakopoulos (1965) experiment, a 1-m high perspex column was packed with Del Monte sand and installed with a number of tensiometers to measure pore-water pressure continuously within the column. The initial condition of the 1-D soil column at $t < 0$ was set with vertical gradient in potential equal to unity (i. e. $\partial\phi/\partial z = 1$). On obtaining this condition, water was continuously added from the top of the soil column and water was allowed to flow out freely through a filter paper at the bottom. For $t > 0$, the test was started and the flow rates at the bottom of the soil column and the tensiometers readings along the column were recorded periodically. The physical properties of Del Monte sand were independently measured by Liakopoulos. The sand has a porosity of 29.75%. In this research, the SWCC and permeability functions given in Figure 4-2 and Figure 4-3 were fitted using Equation (2.4) and Equation (2.6), respectively. The parameters a , b , c and p were 90 kPa, 3.1, 2000 and 0.325, respectively. In the numerical model, the element height was set at 0.01m and time step size was set at 0.0001s.

Figure 4-2 SWCC of Del Monte sand in the first run

Figure 4-3 Permeability function of Del Monte sand in the first run

Narasimhan and Witherspoon (1978) modelled Liakopolus experiment using a program called TRUST based on Equation (2.35). The sand was assumed to be nondeformable and rigid, as is usually done in dealing with unsaturated flow. The permeability function was given by Equation (2.31). TRUST results match reasonably well with the experimental data. The modified model used the fitted SWCC and permeability function shown in Figure 4-2 and Figure 4-3, respectively. The numerical results using the modified model are plotted in Figure 4-4 and compared with the results from TRUST and Liakopoulos experimental data.

The numerical results using the modified model shown in Figure 4-4 are not as good as the results from TRUST, the reason is attributed to the SWCC and permeability function fitted curves. As can be seen in Figure 4-2 and Figure 4-3, the SWCC and permeability function fitted curves cannot fit the last part of the experimental curves. As can be seen in Figure 4-4, the results show reasonable agreements at 10, 20 and 30 minutes when the matric suction ranges from 4 - 7.5 (kPa). Within this matric suction range the SWCC and permeability fitted curves are close to the experimental curves. But above this matric suction range the SWCC and permeability function fitted curves do not match the experimental curves, so the results at 60 and 120 minutes from the modified model deviate from the experimental data.

Figure 4-4 Comparison between modified model and experimental data from Liakopoulos (1965) – first run.

The effect of the SWCC and the permeability functions fitted curves was investigated by changing the parameters of the SWCC and the permeability function to: $a = 85 \text{ kPa}$, $m = 3.2$, $n = 2500$, and $p = 0.45$, and respectively. The fitted SWCC and permeability function are shown in Figure 4-5 and Figure 4-6, respectively.

Figure 4-5 SWCC of Del Monte sand in the second run

Figure 4-6 Permeability function of Del Monte sand in the second run

The results using the modified model denoted as “second run” are given in Figure 4-7. As can be seen in Figure 4-7, the results from the modified model are

better than those obtained from the first run (i.e., Figure 4-4). But the pressure profile at 120 minutes is still less than those from TRUST and experimental data. This illustrates that accuracy of the SWCC and permeability function for numerical model is very important in numerical modelling.

Figure 4-7 Comparison between modified model and experimental data from Liakopoulos, 1965—second run.

(iii) Case 3: Choo and Yanful (2000)

As mentioned in Section 2.3.2.4, Choo and Yanful (2000) used SEEP/W to simulate two laboratory multi-layered soil columns subjected to downward drainage with and

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without evaporation. In this section downward drainage experimental data from one of the column tests are used to compare with those obtained from the modified model.

The 1m soil column consists of two soil layers, namely, till and fine sand. The thickness of each layer is 0.2m and 0.8m, respectively. The SWCC and permeability functions are presented in Figure 4-8 to Figure 4-11. In the numerical modelling Choo and Yanful used Green and Corey (1971) method for predicting permeability function. As illustrated in Figure 4-11, the fitted permeability curve for fine sand does not fit well in the high matric suction range Choo and Yanful used curve 1 for simulating the flow in the soil up to 7.5 hrs and curve 2 for 3 days and above. Curve 2 is a modification of curve 1 where the permeability function is decreased by one order of magnitude when matric suction exceeds 4.3 kPa. Figure 4-11 shows details of this modification.

For the modified model, the SWCC and permeability function used the formulation from Fredlund and Xing (1994) and Leong and Rahardjo (1977b), respectively. The fitted SWCC and permeability function are considered good. For till, the SWCC curve fitted parameters a , n and m were 24.32 kPa, 1.5 and 0.5, respectively and permeability fitted curve parameters s_a , s_n and s_m were 1.643 kPa, 1.5 and 0.976, respectively. For fine sand, the SWCC curve fitted parameters a , n and m were 1.29 kPa, 5.5 and 1.7, respectively and permeability fitted curve parameters s_a , s_n and s_m were 1.6 kPa, 3.1 and 9.04, respectively, for for matric suction up to 4.3 kPa and were 2.31 kPa, 2.26 and 17.20, respectively for matric suction from 4.3 kPa and above. When matric suction was greater than 4.3 kPa, the residual permeability was set at a constant value of 1×10^{-8} m/s. When matric suction was greater than 6.05 kPa, the residual permeability was changed to 1×10^{-9} m/s.

Initial soil condition was hydrostatic with the water table at the top of the column (i.e., 1m in the chosen coordinate system). The water table was lowered to -0.5m within 0.5 hrs to simulate the drop in the water table level. In the modified model, the element height was set at 0.02m whilst the time step size was set at 0.001s. After 2 hrs, the time step size was increased to 0.01s.

The comparison of the experimental data, SEEP/W and the modified model results are shown in Figure 4-12. The pressure head distribution up to 7.5 hrs obtained

from the modified model gave the same trend as SEEP/W. There are some small differences between the results and the differences are attributed to the difference in permeability functions used in the modified model. Nevertheless, the results agree well with each other.

Figure 4-8 SWCC for till used in the modified model

Figure 4-9 Permeability function for till used in the modified model

Figure 4-10 SWCC for fine sand used in the modified model

Figure 4-11 Permeability function for fine sand used in the modified model

Figure 4-12 Comparison of modified model results, SEEP/W results, and experimental data from Choo and Yanful (2000)

4.2.2. Evaporation Problem

The performance of the modified model was also evaluated with published evaporation experimental data. In this section, comparison with two cases of published experimental data were carried out.

Case 1 was a drying (i.e., evaporation) test on a homogenous Beaver Creek sand column (Wilson et al., 1994). The second case was an evaporation test on a clayey

till column (Yanful et al., 2003). The heights of the columns in case 1 and case 2 were 0.3 m and 0.25 m, respectively. The diameters of the two columns were 0.17 m and 0.115 m, respectively. The results reported in the two cases were in terms of gravimetric water content and volumetric water content versus depth of the columns, respectively. For consistency, the results in this section are presented in term of matric suction (kPa) versus depth (m).

(i) Case 1: Wilson et al. (1994)

Wilson et al. (1994) conducted a drying test on a Beaver Creek sand column. The sand was packed in a plexiglass column with 12 thermocouples to measure temperature distribution along the sand column. The sand column was then put into a constant temperature chamber. The gravimetric water content was determined from sand samples taken directly from the column through sampling ports. Liquid water was allowed to flow out through a drainage port at the bottom of the column. The column was exposed to a temperature of 38°C and a relative humidity of 10%. The potential evaporation rate at the top soil surface was 8 mm/day.

The SWCC in Wilson et al. (1994) was presented in term of gravimetric water content. Therefore, in this study the SWCC was transformed and presented in term of volumetric water content as shown in Figure 4-13. The SWCC parameters a , n and m used in the Microsoft Excel spreadsheet program were 4.03 kPa, 41.88 and 0.31, respectively. The permeability function in Wilson et al. (1994) was generated using the method proposed by Laliberte et al. (1968) while the one used in the Microsoft Excel spreadsheet program was from Leong and Rahardjo (1997b) (Equation (2.5)). The parameters s_a , s_n and s_m of the permeability function were 4.05 kPa, 19.45 and 0.435, respectively (Figure 4-14). The thermal conductivity of Beaver Creek sand is presented in Figure 4-15. The determination of thermal conductivity of soil was described in detail in Chapter 5. The time step size and element height used in the modelling were set at 0.1 s and 0.005 m, respectively.

The results from experiment, Wilson et al. (1994) model and modified model are plotted in Figure 4-16 for elapsed time of 1, 5 and 21 days. As can be seen in Figure 4-16, the modified model results match well with the experimental data as well as Wilson et al. (1994) model results. However, there are some small discrepancies

between the two models. The discrepancies can be explained by the difference between the two models which was presented in Chapter 3.

Figure 4-13 SWCC of Beaver Creek sand

Figure 4-14 Permeability of Beaver Creek sand

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Figure 4-15 Thermal conductivity of Beaver Creek sand from Wilson et al. (1994)

Figure 4-16 Comparison with experimental data from Wilson et al. (1994)

(ii) Case 2: Yanful et al. (2003)

Yanful et al. (2003) examined the use of soil covers for protecting the environment from atmospheric oxygenation through rainfall infiltration by isolating sulphide minerals. Soil cover with low hydraulic permeability will maintain the cover at high degree of saturation longer. Subsequently the evaporation process will lower the soil cover's water content. In this section a comparison of the modified model with Yanful et al. (2003) of Halton clayey till column experimental data is made. The soil column has a height of 0.25 m and a diameter of 0.115 m. Yanful et al. (2003) used a number of time-domain reflectometry (TDR) probes and thermocouples for measuring the volumetric water content and temperature distribution in the soil column, respectively. The water table was set at the bottom of the column. The test was performed under a range of temperature from 18-27°C and a relative humidity of around 32%. The test experiment was carried out for a duration of 12 days, long enough to achieve maximum evaporation under the laboratory condition (Choo and Yanful, 2000).

The SWCC and permeability functions of Halton clayey till are shown in Figure 4-17 and Figure 4-18, respectively. The SWCC parameters of a , n and m (Equation (2.4)) used in the modified model were 8000 kPa, 0.6 and 11, respectively. The permeability function used in the modified model was obtained from ACUPIM/W then fitted with Equation (2.5). The fitted parameters s_a , s_n and s_m for the permeability function were 1000 kPa, 1.05 and 3.5, respectively. Yanful et al. (2003) used a commercial software named SoilCover (Geo-Analysis Ltd, 2000) which used Wilson et al. (1994) model to simulate the problem.

The minimum and maximum time step sizes used in the modified model modelling were 1s and 1000s, respectively. A small time step size was necessary when the soil condition changed from saturated to unsaturated to maintain numerical stability. When the soil became unsaturated, the time step size was increased accordingly. The initial condition was set at hydrostatic as the soil column was saturated before starting the test. The initial time step applied was 1s while the element height was set at 0.001m. The evaporative flux applied was 6.7×10^{-8} m/s and the top temperature applied was 27°C.

Results from the modified model, SoilCover and experimental data are compared in Figure 4-20. As can be seen in Figure 4-20, the numerical results from SoilCover and modified model are not very close to the experimental data but the modified model gives a better agreement. The reason for this discrepancy may be attributed to cracks in the clayey till soil column formed during the evaporation process (Yanful et al., 2003) which created more vapour flow than the numerical models.

Figure 4-17 SWCC of Halton clayey till

Figure 4-18 Permeability function of Halton clayey till predicted using ACUPIM/W

Figure 4-19 Thermal conductivity for Halton clayey till

Figure 4-20 Comparison with experimental data from Yanful et al. (2003)

4.3. PARAMETRIC STUDY TO EVALUATE THE DIFFERENCES BETWEEN WILSON ET AL. (1994) MODEL AND MODIFIED MODEL

As described in Chapter 3, the modified model and Wilson et al. (1994) model have differences in some terms. Therefore a parametric study was performed to evaluate the differences between the two models numerically. The parametric study consisted of seven cases. In each case, different SWCCs and permeability functions were used. The SWCC parameters were chosen based on the typical shapes of SWCC. Leong and Rahardjo (1997a) illustrated the effect of curve fitting on SWCC using parameters a , n and m varying from 10 to 10000 kPa, 0.5 to 4 and 0.5 to 4, respectively. When parameter a equals 1000 kPa, the air entry value of the soil, ψ_a , is around 9000 kPa which is much higher than all the soils considered in this study therefore a range from 50 to 450 kPa is selected for parameter a . The parameter n does not effect the shape of the SWCC much when n changes from 2 to 4. Similarly, the parameter m does not effect the shape of the SWCC much when it varies from 3 to 4. Hence, the ranges of parameters n and m used in this parametric study are from 0.5 to 1.5 and 1 to 3, respectively. The permeability functions were then generated from the SWCC using ACUPIM/W. The SWCC and permeability function parameters are summarised in Table 4-1 and shown in Figure 4-21 and Figure 4-22, respectively, for cases 1 to 7.

The soil column used in this parametric study is 0.5m in height and is homogeneous. Figure 4-23 to Figure 4-25 show the Wilson et al. (1994) and modified models results for the homogeneous soil column at elapsed times of 4, 8 and 12 days. The results presented are in term of matric suction versus height of the soil column. The curves in Figure 4-23 to Figure 4-25 are annotated with “M” and “W” where “M” stands for modified model results and “W” stands for Wilson et al. (1994) model results.

Figure 4-23 compared cases 1, 2 and 3 where a and n were constant and m was varied from 1 to 3. The results show a close agreement between the two models.

Chapter 4 Evaluation of Modified Model

The SWCCs and permeability functions for these cases show that the slopes of SWCCs (which in turn define the shape of permeability functions) are quite steep and comparable with sandy soils' SWCCs. The results show that there is negligible differences and therefore parameter m is not significant

Figure 4-24 presented the comparison for cases 2, 4 and 5 where a and m were constant and n was varied from 0.5 to 1.5. The greatest difference appears in case 4 whereas the other two cases only show a small discrepancy. The SWCC for case 4 is gentler compared with the other two. The shape of the SWCC in case 4 is similar to clayey soil's SWCC. Therefore the parameter n affects the numerical results and will cause the modified model and Wilson et al. (1994) model results to be different.

Figure 4-25 showed the results for cases 2, 6 and 7 where n and m were constant and a was varied from 50 to 450 kPa. The discrepancies between the two models are very small. Therefore the effect of parameter a on the modified model and Wilson et al. (1994) model results is negligible.

In order to further clarify the differences between the two models, two more cases were modelled where SWCCs were also gentle (i.e., lower and upper bound of case 4). The SWCCs and permeability functions for these two cases are presented in Figure 4-27 and Figure 4-28, respectively. The parameters of SWCC and permeability functions are tabulated in Table 4-2.

The results for cases 8, 4 and 9 are shown in Figure 4-26. The results show that as the SWCC becomes gentle there are more differences between the two models' results. Therefore the differences between the modified model and Wilson et al. (1994) model results become more pronounced as the SWCC become gentler, i.e., it is expected that numerical modelling of clay soil will give different results depending if the modified model and Wilson et al. (1994) model is based.

Table 4-1 SWCC and permeability function parameters for the parametric study

case	SWCC			Permeability function		
	a (kPa)	n	m	sa (kPa)	sn	sm
1	200	1	1	153	0.7	12
2	200	1	2	116	0.8	11
3	200	1	3	70	0.8	11
4	200	0.5	2	20	0.6	12.5
5	200	1.5	2	100	0.5	7
6	50	1	2	27	0.8	11
7	450	1	2	300	1	10

Table 4-2 SWCC and permeability function parameters for the additional two cases

case	SWCC			Permeability function		
	a (kPa)	n	m	sa (kPa)	sn	sm
8	50	0.5	2	10	0.6	13
9	450	0.5	2	10	0.5	13

Figure 4-21 SWCCs used in the parametric study

Figure 4-22 Permeability functions used in the parametric study

Figure 4-23 Effect of parameter “m”

Figure 4-24 Effect of parameter “n”

Figure 4-25 Effect of parameter “a”

Figure 4-26 Effect of parameter a with $n = 0.5$ and $m = 2$

Figure 4-27 SWCCs of the additional two cases

Figure 4-28 Permeability functions of the additional two cases

4.4. SUMMARY

In this chapter the modified model was evaluated for three cases of infiltration and two cases of evaporation. A parametric study comparing the results from the modified model and Wilson et al. (1994) model for nine sets of SWCC and permeability function was carried out.

For the infiltration cases, generally all the results agreed well with each other. The close agreement was attributed to the correct element and time step sizes. The infiltration cases also emphasized the importance of “correct” SWCC and permeability function in the numerical modelling. Therefore it is important to obtain reliable estimation of SWCC and permeability function.

For the evaporation cases, generally good agreement between numerical results and experimental data was obtained. However, the agreement was not as good as that for the infiltration cases. Furthermore, the experimental data from the literature only showed the suction profiles. Temperature profiles were not available for comparison.

A parametric study of nine cases was carried out in order to clarify the differences between the Wilson et al. (1994) and the modified models. The results showed that in general the two models agreed well with each other. Differences were detected if the shape of SWCC approaches that of clay soil. However, the differences are quite small.

From the evaluation and parametric study in this chapter, it is clear that the modified model was able to model the infiltration and evaporation processes in soil. As mentioned in Chapter 2, there were not many experimental data of soil column under evaporation in the literature to fully validate the modified model experimentally. Therefore, soil column tests were carried out in this study to provide more complete cases for the evaluation of the modified model. The soil column test, test results, and modelling test results are presented in Chapters 5, 6, and 7, respectively.

Chapter 5 One-Dimensional Soil Column Setup and Soil Properties

5.1. INTRODUCTION

The one-dimensional flux movement of fluid (water or water vapour) within soil is commonly studied using a soil column. The flux of fluid can be upward for evaporation or downward for infiltration. Laboratory tests are important tools to validate a model. Although the modified model was evaluated with published data in Chapter 4, the evaluation was hampered at times by incomplete soil and test information. Therefore in this chapter, soil column test to provide more complete evaluation of the modified model are described. The soil column test programme and results are presented in the next chapter.

5.2. SOIL COLUMN SETUP

A schematic of the soil column set-up is shown in Figure 5-2. The column is made of a perspex cylinder of height 500 mm, internal diameter 140 mm and a wall thickness of 3 mm. Along the perspex cylinder are sensor ports spaced at intervals of 50 mm where sensors can be installed into the soil column. The column sits on a stainless steel base where a coarse porous stone is placed. The stainless steel base has an outlet where a constant head boundary condition can be imposed. Soil is filled layer by layer with thickness equal to the distance between two sensor ports. After placement of each soil layer, sensors (ceramic tip of small-tip tensiometer and temperature sensor) are installed and the sensor port is sealed using a connector. The connector is designed to prevent water leak but allows the tube of the small-tip ceramic tensiometer or temperature sensor cable to pass through.

For the infiltration tests, an electronic weighing balance is placed at the outlet of the soil column steel base. For evaporation test, the electronic weighing balance is used to measure the potential evaporation and a sensor is used to measure the relative humidity and temperature above the soil column. The constant water level tank is mounted above the soil column to provide a constant intensity infiltration. The

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intensities are controlled by a valve that allows a minimum intensity of 1×10^{-9} m/s to be applied to the soil column.

All the sensors were connected to a MicroDac[®] data acquisition system (Grayhill Inc. 1998). The MicroDac[®] data acquisition system consists of a 32-I/O module mounting rack and a controller connected via a 50 pin ribbon cable (Figure 5-1). Depending on the instruments to be monitored, different types of I/O modules can be plugged into the mounting rack. Computer software for the soil column tests was developed by Leong (2002) for data acquisition by a computer via the MicroDac[®] controller. The software allows setting different sampling rates for one test.

Figure 5-1 Data acquisition system

5.3. INSTRUMENTS FOR SOIL COLUMN

The measuring devices used in the soil column test are: small-tip tensiometers for measuring changes in pore-water pressure in the soil column; pressure transducers for enabling the tensiometers to be read automatically by a data acquisition system; temperature sensors for measuring the temperature distribution within the soil column; electronic weighing balance to measure the outflow during infiltration test and potential evaporation during evaporation test; relative humidity and temperature transmitter for measuring relative humidity and temperature above the soil surface of the soil column during evaporation test. The measuring devices are described in detail in the following sections.

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Figure 5-2 Schematic of the soil column test

5.3.1. Tensiometer - Transducer System

Tensiometers are widely used for negative pore-water pressure measurement in soil. The small-tip tensiometer consists of three main parts namely, tensiometer shaft with jet-fill cup, pressure transducer and ceramic tip. Figure 5-3 presents the schematic drawing of a jet-fill cup. De-aired water is used to fill the jet-fill cup to remove air bubbles in the system in order to avoid error in measurement.

Figure 5-3 Jet-fill cup (from Soilmoisture Equipment Corporation 1986)

Good measurement of the negative pore-water pressure during the test depends on the contact between the small-tip tensiometer's ceramic tip and the soil (Figure 5-4). When the ceramic tip is saturated only water can flow through the wall of the porous ceramic cup as long as the matric suction does not exceed the air-entry value of the ceramic tip (100 kPa). When the tip is in contact with soil, water flows due to the difference of the pressure head gradient between the inner side and outer side (i.e., pore water pressure in the soil) of the tip. The flow of water will stop when there is equilibrium of pressure between both sides of the ceramic tip. The negative pore-water pressure exerts a tension in the de-aired water in the tensiometer's shaft and registered by the pressure transducer.

The tensiometer – transducer system must be serviced properly prior to its use in order to obtain reliable and accurate results. This procedure consists of removal of air that accumulates in the tensiometer and filling the tensiometer with de-aired water. Air can accumulate inside the small tip tensiometer due to the diffusion of air through the ceramic cup and also due to cavitation of water inside the tensiometer when the pore-water pressure becomes less than -80 kPa (Stannard 1992). As the tensiometers' ceramic tips are installed at different elevations in comparison with

their pressure transducers, corrections are required to account for the difference in the elevation heads between the ceramic tip and the point of pressure measurement by the pressure transducer.

Figure 5-4 Small-tip tensiometer with flexible coaxial tubing
(from Soilmoisture Equipment Corporation 1986)

5.3.2. Temperature Sensor

The temperature distribution in the soil column is measured using an integrated circuit temperature sensor, IC 590kH (Figure 5-5). The size of the IC 590kH is approximately a 5mm cube. The IC 590kH is functionally a two terminal IC which produces an output current proportional to the absolute temperature. The technical specifications of the IC 590kH are: operating voltage range from 4 to 30 Vdc; nominal current output is $289.2\mu\text{A}$ at 25°C ; nominal temperature coefficient is $1\mu\text{A}/^{\circ}\text{C}$; operating temperature range for IC 590kH is from -55°C to 150°C .

(a) Before epoxy

(b) After epoxy

Figure 5-5 Temperature sensor

The temperature sensors must be installed into the soil column and therefore the temperature sensors must be waterproofed. For waterproofing the temperature sensor is placed in a short copper tube and sealed with epoxy (Figure 5-5 b). The temperature sensor within the copper tube is then calibrated. Details of the temperature sensors calibration curves are presented in the Appendix.

5.3.3. Relative Humidity and Temperature Transmitter

Relative humidity and temperature are monitored at the surface of the soil column during the evaporation process. Accurate measurement data of relative humidity and temperature are required as this give the top boundary condition for the numerical modelling. In this study the HUMICAP[®]H-sensor model HMP 133Y presented in Figure 5-6 is used to measure relative humidity and temperature.

The specifications of the HMP 133Y are as follows:

Range of relative humidity measurement is from 0 to 100%RH. The accuracies (including nonlinearity and repeatability) achievable when calibrated against high quality and certificated humidity standard are $\pm 1\%$ RH (0 to 100%RH) and $\pm 2\%$ RH (90 to 100%RH). The range for temperature measurement is from -20 to 80°C. The accuracy achievable at 20°C is $\pm 0.2^\circ\text{C}$. Operating voltage is 11 to 30 Vdc. Two output types are available for this device namely, selectable and scalable at range of 0-20mA/0-1V and 4-20mA/0-5V. Based on the requirement of the MicroDac[®] data acquisition, the 0-5V output range of the device was used. The calibrations of the relative humidity and transmitter sensor are also given in the Appendix.

Figure 5-6 Relative humidity and temperature transmitter model HMP 133Y.

5.3.4. Electronic Weighing Balance

The amount of bottom outflow during infiltration test and potential evaporation during evaporation test are measured using an electronic weighing balance. The electronic weighing balance used is SHIMADZU model EB 3200HU (Figure 5-7) with a stainless steel weighing platform with a maximum range of 2kg and an accuracy of $\pm 0.01\text{g}$. The digital output signal that appeared on the LCD screen of the balance was directly captured by the personal computer using an IFB-102A interface through the balance's data I/O connector.

Figure 5-7 Electronic weight balance

5.4. SOIL USED IN THE COLUMN TEST

Three types of soils were selected for the soil column tests namely Sand 1, Sand 2 and Residual soil. The two sands have high saturated permeability and low air entry value so it is expected that test durations are reasonable for both infiltration and evaporation tests. The Residual soil has much lower saturated permeability than the two sands and test duration is expected to be longer. Residual soil is used as it covers two-third of Singapore land area. All the three soils used have negligible deformation behaviour during drying and wetting. Soil classification tests, drying and wetting soil-water characteristic curves determinations and saturated and unsaturated permeability tests were performed for the three soils and results are presented in the following sections.

5.4.1. Basic Properties of the Soils

The soils were classified according to the Unified Soil Classification System (USCS) (ASTM D 2487-93). The grain-size analysis, specific gravity, Atterberg limits, relative density (minimum and maximum density) and compaction tests were conducted in accordance to ASTM D 422-63 (1990), ASTM D 854-92, ASTM D 4318-95a, ASTM D 4254-91 and ASTM D4253-93, respectively. The test results are summarised in Table 5-1, Figure 5-8 and Figure 5-9.

5.4.2. Determinations of Soil Water Characteristic Curves (SWCC)

To model both evaporation and infiltration processes, both drying and wetting SWCCs are required. In this study, two methods were employed for SWCC determination namely Tempe pressure cell and pressure plate extractor. Both equipments used are manufactured by Soilmoisture Equipment Corporation CA, USA (1999). Tempe pressure cell can be used to determine the drying SWCC up to 100kPa while pressure plate extractor can be used to determine both drying and wetting SWCCs of soils up to 200kPa. In general, the two equipments work under a similar principle described in ASTM D 2325-68 (1994).

Figure 5-8 Grain size distributions

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Table 5-1 Test results and test standards used

No	Description	Sand 1	Sand 2	Residual soil	Standard/Reference
1	Unified Soil Classification System	SM	SP-SM	SC	ASTM: D 2487 - 93
2	Specific gravity, G_s	2.63	2.58	2.51	ASTM: D 854 - 92
3	Grain size analysis (dry sieving)	*Y	Y	Y	ASTM: D 422 - 63
4	Atterberg limits				
	Liquid limit	*N	N	55	ASTM: D 4318 - 95a
	Plastic limit	N	N	30	ASTM: D 4318 - 95a
	Plasticity index	N	N	25	ASTM: D 4318 - 95a
5	Relative density				
	Minimum dry density (Mg/m^3)	1.44	1.51	N	ASTM: D 4254 - 91
	Maximum dry density (Mg/m^3)	1.87	1.99	N	ASTM: D 4253 - 93
	Maximum void ratio, e_{max}	0.82	0.75	N	
	Minimum void ratio, e_{min}	0.41	0.3	N	
6	Compaction test				
	Maximum dry density (Mg/m^3)	N	N	1.72	ASTM: D 698 - 91
	Optimum water content (%)	N	N	15.5	ASTM: D 698 - 91
7	Saturated permeability				
	Contant head test (3 different void ratios)	Y	Y	Y	ASTM: D 2343 - 68

Note: *N: no test was carried out

*Y : test was carried out.

Figure 5-9 Compaction tests for Residual soil

5.4.2.1. Tempe Pressure Cell Tests

Several types of Tempe pressure cell were introduced by Soilmoisture Corporation. Although they have differences in diameter and height of the cell body, the working principle is the same. In this study, the Tempe pressure cell with dimensions of 56mm in diameter and 30mm in height was used. A cross-sectional view of the Tempe pressure cell is shown in Figure 5-10.

The most important component in the Tempe pressure cell is the 1 bar high air entry ceramic disk. The disk's saturated permeability value is around 8.6×10^{-8} m/s (Soilmoisture Equipment Corp., 1999) sufficient to maintain a stable condition for water flowing out during the test. Water flows out of the soil specimen through a water outlet in the base of the Tempe cell. Therefore, pore-water pressure during the test is zero while the matric suction in soil equals the applied air pressure. In order to prevent air leak during the test, all components of the Tempe pressure cell are sealed with rubber O-rings and vacuum grease.

In all SWCC tests, the void ratio of the soil was controlled. Soils were compacted in the Tempe cell bodies until the desired void ratios were obtained. The Tempe pressure cells were assembled and connected to a water filled burette for saturation by allowing the water to percolate upwards till air bubble are no longer visible at the air inlet. When saturation of soil specimens had been completed, the Tempe pressure cells were weighed to determine the saturated volumetric water content of the soils. This was checked against the void ratio of the soil. Air pressure corresponding to the desired matric suction was applied next. During the test, the water level at the outlet was maintained using a water tank.

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(a) Disassembled components of a Tempe pressure cell

(b) Assembled Tempe pressure cell with soil specimen

Figure 5-10 Cross-sectional view of Tempe pressure cell used
(from Soilmoisture Equipment Corp., 1999)

The volumetric water content of the soils corresponding to the applied matric suctions was calculated from the weight of the Tempe cell after equilibrium condition had been established. Time to reach equilibrium condition is shown with

plot of cells' weight versus elapsed time in the Appendix. Relationship between volumetric water content and matric suction gave the SWCC. The SWCC test data of soils are fitted with Equation (2.3) from Fredlund and Xing (1994) with $C(\psi) = 1$ as recommended by Leong and Rahardjo (1997a). The fitted SWCCs of Sand 1, Sand 2 and Residual soil obtained from Tempe pressure cell are shown in Figure 5-11, Figure 5-12 and Figure 5-13, respectively, and the SWCCs parameters are summarised in Table 5-2.

Table 5-2 SWCC parameters a , n and m in Equation (2.3) for Sand 1, Sand 2 and Residual soil

Material \ Parameter	a (kPa)	n	m
Sand 1	2.41	7.6	0.38
Sand 2	3.51	3.62	0.6
Residual soil	3.9	1.85	0.71

Figure 5-11 Soil-water characteristic curve of Sand 1

Figure 5-12 Soil-water characteristic curve of Sand 2

Figure 5-13 Soil-water characteristic curve of Residual soil

5.4.2.2. Drying and Wetting SWCCs using Volumetric Pressure Plate Extractor

Volumetric pressure plate extractor with hysteresis attachment manufactured by Soilmoisture Corporation (1999) was used to measure drying and wetting SWCCs.

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The equipment has dimensions of 128.3 mm in diameter and 120mm in height. The cutaway view of the volumetric pressure plate extractor is shown in Figure 5-14. The schematic diagram of the volumetric pressure plate extractor test set-up is shown in Figure 5-15.

The height of all the soil specimens in SWCCs tests is about 40mm, up to the pressure inlet of the pressure plate extractor. The extractor incorporates a 2-bar ceramic disk which permits a matric suction of up to 200 kPa to be applied. The ceramic disk was placed submerged in water in a vacuum desiccator with a small vacuum applied to saturate the ceramic disk. Afterwards, the ceramic disk was assembled into the bottom pedestal of the volumetric pressure extractor. Similar to the Tempe cell test, soils were compacted in the volumetric pressure plate extractor to control the void ratio.

Figure 5-14 Cut away view of volumetric pressure plate
(from Soilmoisture Equipment Corp., 1999)

The soil specimen was saturated by filling water into the burette that was connected to the outlet hole at the base of the pressure plate extractor. Due to the low permeability of the 2-bar ceramic disk (6.3×10^{-9} m/s), the saturation process took about 14 hrs for Sand 1 and Sand 2, and about 48 hrs for Residual soil to complete.

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Saturation was deemed completed when water flowing into the bottom of the volumetric pressure extractor through the soil balances the water flowing out from the air pressure inlet. The volumetric pressure extractor was then assembled and the water level in the ballast tube was adjusted with the aid of the air trap to a reference mark.

Air pressure was then applied through the vapour saturator. As water leaves the soil specimen into the bottom water compartment, the water level in the ballast tube will move. Using both the air trap and burette, the water level in the ballast tube can be adjusted to the reference mark and the volume of the water outflow from the soil specimen can be measured. The water outflow with time was recorded periodically until the equilibrium condition is reached, i.e., when water outflow ceases.

Figure 5-15 Schematic diagram of pressure plate extractor
(from Soilmoisture Equipment Corp., 1999)

For the wetting SWCC, applied air pressure was reduced incrementally. During the test, water will be drawn into the soil specimen. Both the air trap and burette can be manipulated to move the water level in the ballast tube to the reference mark and the volume of water inflow into the soil specimen can be measured. The water inflow with time was recorded periodically until equilibrium condition is obtained,

i.e., when the water inflow ceases. At the equilibrium condition, the air pressure is decreased to the next matric suction level and the test procedure is repeated.

In this study, drying and wetting SWCCs were obtained for Sand 1, Sand 2 and Residual soil as shown in Figure 5-16, Figure 5-17 and Figure 5-18, respectively. The fitted SWCCs using Fredlund and Xing (1994) equations are shown as lines in these figures. The parameters a , n , and m for the fitted curves are summarised in Table 5-3.

Table 5-3 Parameters a , n and m in Eq. (2.3) for Sand 1, Sand 2 and Residual soil

Parameter Material	a (kPa)		n		m	
	Drying	Wetting	Drying	Wetting	Drying	Wetting
Sand 1	2.81	3.01	2.97	3.49	0.69	0.73
Sand 2	3.5	2.76	3.86	3.5	0.65	0.75
Residual soil	3.4	1.98	1.5	4.5	0.71	0.7

Figure 5-16 Drying and wetting soil-water characteristic curves of Sand 1

Figure 5-17 Drying and wetting soil-water characteristic curves of Sand 2

Figure 5-18 Drying and wetting soil-water characteristic curves of Residual soil

5.4.3. Saturated and Unsaturated Permeability of Soils

The determination of unsaturated permeability function, k_w , from SWCC using statistical model requires the value of the saturated coefficient of permeability. Therefore accurate measurement of the saturated coefficient of permeability is very

important (Leong and Rahardjo, 1997b). The saturated coefficient of permeability can be obtained by carrying out laboratory permeability tests namely, rigid-wall and flexible-wall permeability tests. Results of the saturated coefficient of permeability are presented in the following section. The determination of permeability function from SWCC was compared with results from direct measurement of permeability function using a triaxial permeameter for unsaturated soil described in Section 5.4.3.2. Comparisons of the direct and indirect measured unsaturated coefficient of permeability are presented in Section 5.4.3.3.

5.4.3.1. Saturated Coefficient of Permeability

The procedure for the rigid-wall permeability test is listed in ASTM D2434-68 (1994). The constant head rigid-wall permeameter used in this study comprises a perspex cylinder with top cap and bottom pedestal, a water tank to maintain a constant pressure head, electronic weighing scale for measuring water flowing out and a stop watch. A schematic diagram of the constant head rigid-wall permeameter setup is shown in Figure 5-19. The perspex cylinder has dimensions of 30 cm in height and 7 cm in diameter. Soil specimens were compacted into the cylinder at different void ratios (three different void ratios for each type of soil were used). A filter paper was placed on the bottom pedestal in order to prevent soil from falling through the outlet. Depending on soil types, different pressure heads were used. In this study, pressure head of 0.5 m was applied for Sand 1 and Sand 2 while pressure head of 1.0 m was applied for Residual soil. After the compaction, the soil specimen was saturated by allowing water to percolate from the bottom upwards with the top outlet opened. Saturation was deemed completed when water flowing out from the top outlet has no air bubbles. Once saturation was completed, the pressure head was applied and the amount of water flowing out of the permeameter with time was measured using an electronic weighing scale. Saturated coefficient of permeability of the soil was calculated using Darcy's law given in the equation below:

$$Q_w = h_w k_s A t \quad (4.1)$$

Results of the saturated coefficient of permeability determined using rigid-wall permeameter are summarised in Table 5-4.

Figure 5-19 Schematic diagram of constant-head rigid-wall permeameter setup

Table 5-4 Results of saturated coefficient of permeability obtained from constant head rigid-wall permeameter test

Soil type	Saturated coefficient of permeability, k_s (m/s)		
Sand 1	$e = 0.42$	$e = 0.57$	$e = 0.75$
	8.3×10^{-6}	1.5×10^{-5}	5.2×10^{-5}
Sand 2	$e = 0.4$	$e = 0.56$	$e = 0.742$
	2.9×10^{-7}	9.4×10^{-7}	8×10^{-6}
Residual soil	$e = 0.47$	$e = 0.624$	$e = 0.87$
	1.05×10^{-11}	4.5×10^{-9}	9×10^{-8}

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A disadvantage of the rigid-wall permeameter is the zero confining pressure and therefore the soil particles may rearrange during the test. To overcome this problem, flexible-wall permeameter tests were also conducted to supplement the rigid-wall permeameter tests at different cell pressures. The flexible-wall permeameter set up used is shown in Figure 5-20. Rubber membrane of 50 mm in diameter is used to form the soil specimen. High permeability porous stone ($k_s = 9.2 \times 10^{-5}$ m/s) and filter paper were placed at the bottom and top of the soil specimen. Similar to the rigid-wall permeameter, soil specimens in this test were also prepared for three different void ratios. The soil specimen's dimensions are 70 mm in height and 50 mm in diameter. Results from flexible-wall permeameter tests at different cell pressures are summarised in Figure 5-21, Figure 5-22, and Figure 5-23, respectively.

Figure 5-20 Schematic diagram of flexible-wall permeameter set up

Figure 5-21 Results of saturated coefficient of permeability at different cell pressure and different void ratio for Sand 1

Figure 5-22 Results of saturated coefficient of permeability at different cell pressure and different void ratio for Sand 2

Figure 5-23 Results of saturated coefficient of permeability at different cell pressure and different void ratio for Residual soil

5.4.3.2. Determination of Coefficient of Permeability Function from SWCC

The SWCC obtained from either Tempe pressure cell or volumetric pressure plate extractor can be used to determine the permeability function. The permeability function, k_w , can be obtained from the SWCC and saturated coefficient of permeability using the statistical method described in Fredlund and Rahardjo (1993a). The statistical method for calculation of the permeability function involves integration of the SWCC:

$$k_w(\theta_w) = \frac{k_s}{k_{sc}} A_d \sum_{j=1}^m \left[(2j+1-2i)(u_a - u_w)_j^{-2} \right] \tag{4.2}$$

- where: m = total number of interval between the saturated volumetric water content, θ_s , and the lowest volumetric water content on the experiment SWCC, θ_L ,
- i = midpoint of segment is numbered,
- j = a counter from i to m,

$$k_{sc} = A_d \sum_{j=1}^m \left[(2j+1-2i)(u_a - u_w)_j^{-2} \right], \quad \text{saturated coefficient of permeability,}$$

$$A_d = \frac{T_s^2 \rho_l g}{2\mu_w} \frac{\theta_s^p}{N^2} \quad \text{adjusting constant,}$$

μ_w = absolute viscosity of water,

p = a constant which accounts for the interaction of pores of various sizes, assumed to be 2,

N = total number of interval computed between the saturated volumetric water content, θ_s , and zero volumetric water content.

The procedure for calculation of the permeability function using the statistical method was coded into a computer program, ACUPIM/W, by Leong (1994). The permeability functions generated by ACUPIM/W for the three soils are presented in Figure 5-24, Figure 5-25, and Figure 5-26, respectively. The permeability functions generated by ACUPIM/W were fitted with the Equation (2.4). The values of s_a , s_n , s_m for Sand 1, Sand 2 and Residual soil are tabulated in Table 5-5.

Table 5-5 Summarised parameters of permeability function

Soil type		Parameters		
		sa (kPa)	sn	sm
Sand 1	Drying	3.62	2.21	5.5
	Wetting	3.15	2.20	6.05
Sand 2	Drying	4.21	4.02	4.51
	Wetting	3.25	3.56	4.30
Residual soil	Drying	4.11	1.21	6.79
	Wetting	3.55	2.05	4.50

Figure 5-24 Permeability function of Sand 1 from ACUPIM/W

Figure 5-25 Permeability function of Sand 2 from ACUPIM/W

Figure 5-26 Permeability functions of Residual soil from ACUPIM/W.

5.4.3.3. Direct Measurement of Unsaturated Permeability

Direct measurement of permeability is more desirable. Direct measurement of permeability of unsaturated soils has the disadvantages of higher equipment cost, longer test duration and involves more complicated procedures. However, direct measurements are more reliable and can provide the basis for prediction of permeability functions using the indirect method (Agus et al. 2003). A constant-head triaxial permeability flexible wall test was set up at NTU (Agus 2000). The permeameter consists of a stainless steel cylindrical cell, a top cap and a bottom pedestal. The top cap and bottom pedestal are designed specially with a grooved water compartment where 5-bar ceramic disks are fixed using epoxy. The saturated permeabilities of the two high air entry ceramic disks were 6.65×10^{-10} m/s and 3.11×10^{-10} m/s. The schematic drawing of the NTU triaxial permeameter is shown in Figure 5-27. Test procedure is generally divided into four phases namely, soil specimen preparation, soil specimen saturation, unsaturated consolidation, and water permeability test.

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Figure 5-27 Schematic drawing of NTU triaxial permeameter (from Agus, 2000)

The soil specimen preparation followed the same procedure as the saturated permeability test. The specimen's dimensions are 50 mm in height and 70 mm in diameter. The high air entry ceramic disks were saturated by applying cell pressure of 300 kPa. Subsequently, water was allowed to flow through the ceramic disks for half a day by opening both the top and bottom flushing lines.

The bottom water pressure was set at 5-10 kPa higher than the top water pressure and 10 kPa lower than the cell pressure. This process is to remove air from the soil specimen with water flowing from bottom to top. The saturation of the soil specimen can be checked using Skempton's pore-water pressure coefficient B . The B value was measured by increasing the cell pressure and then monitoring the corresponding change in pore-water pressure. The ratio between the change of pore-water pressure developed and the change in applied cell pressure is the pore-water pressure coefficient B . The soil specimen is saturated when B is nearly 1. The B values of all three soils used in this research were 0.98, therefore, the soil specimens were considered saturated.

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After saturation, the soil specimen was isotropically consolidated to the desired net normal stress and matric suction value. Double drainage was adopted in order to shorten the consolidation process. The water drained out (from bottom and top of soil specimen) during consolidation process was monitored by using two auto volumetric change indicators. The net confining pressure was maintained at a constant value since only the permeability coefficients as a function of matric suction was examined. For the drying cycle, reducing the top water pressure results in a higher outflow rate compared with inflow rate as water tends to flow out of specimen prior to permeability measurement. Conversely, for the wetting cycle, water tends to flow into the specimen. The consolidation process was considered to be completed when there was negligible amount of water flowing into or out of the soil specimen meaning that at this state the soil specimen is in equilibrium to the applied stress state condition.

After the soil specimen has equilibrated at a specified matric suction, the water permeability test was performed. The test was carried out by applying a hydraulic gradient between the bottom and the top of the specimen. The hydraulic gradients used for Sand 1, Sand 2 and Residual soil were 20, 20 and 30 respectively. The water inflow and outflow were monitored until steady-state flow condition was established i.e. where the inflow and outflow rates were the same.

The water coefficient of permeability of the soil, k_w was determined by treating the disk-soil-disk arrangement as a three-layered system as illustrated in Figure 5-28 (Agus et al. 2003). The permeability of the disk-soil-disk system, k , can be determined using Darcy's law as follows:

$$k = \frac{Q}{iat} \quad (4.3)$$

where: Q = Volumetric water flow (m^3),

k = Coefficient of permeability of the disk-soil-disk system (m/s),

i = Hydraulic gradient, and

a = Cross section of soil specimen (m^2).

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The permeability of the disk-soil-disk system can be expressed as a function of the permeability of the individual components as follows:

$$k = \frac{(H_t + H_s + H_b)}{\left(\frac{H_t}{k_t} + \frac{H_s}{k_w} + \frac{H_b}{k_b} \right)} \quad (4.4)$$

where: H_t, H_b, H_s = Height of top, bottom ceramic disk and soil specimen, respectively (m)

k_t, k_b, k_w = Permeability of top, bottom ceramic disk and soil, respectively (m)

Substituting Equation (4.3) into Equation (4.4), the unsaturated permeability of the soil, k_w . The equation is given as follows (Agus, 2000):

$$k_w = \frac{H_s}{\left(\frac{H}{\left(\frac{Q}{iat} \right)} - \left(\frac{H_t}{k_t} + \frac{H_b}{k_b} \right) \right)} \quad (4.5)$$

where: H = Total height of disk-soil-disk system (m)

Figure 5-28 Illustration of disk-soil-disk arrangement system

The unsaturated permeability test results for Sand 1, Sand 2, and Residual soil are shown in Figure 5-29, Figure 5-30 and Figure 5-31, respectively. Also shown in the figures are the permeability functions from ACUPIM/W. The comparisons showed that the unsaturated permeability determined from direct measurements were slightly higher than the unsaturated permeability estimated by ACUPIM/W. However, the discrepancies are less than one order in magnitude which is within the accuracy of experimental error. Therefore in the numerical modelling the permeability function used is that generated by ACUPIM/W.

Figure 5-29 Permeability function, k_w , for Sand 1

Figure 5-30 Permeability function, k_w , for Sand 2

Figure 5-31 Permeability function, k_w , for Residual soil

5.4.3.4. Potential Evaporation Test

The actual evaporative flux from soil surface is an important boundary condition for numerical modelling of the evaporation process. As mentioned in Chapter 2 the actual evaporative flux can be determined theoretically (i.e., Equation 2.43) or experimentally through the potential evaporation (i.e., Equation 2.44). In this study, the second method was chosen. The potential evaporation test was carried out at four different applied temperatures to the water surface: 24°C (normal condition), 49°C (temperature applied to Sand 1 column), 40°C (temperature applied to Residual soil column) and 33°C (temperature applied to layered sand column). A container having the same diameter as the soil column was set on top of an electronic weighing. The light bulb and relative humidity and temperature transmitter were placed above the water surface. The results of potential evaporative flux tests are presented in the Figure 5-32 to Figure 5-35.

Figure 5-32(a) to Figure 5-35(a) show that the temperature fluctuation is less than $\pm 1^\circ\text{C}$ and can therefore be treated as almost constant. Similarly, Figure 5-32(b) to Figure 5-35(b) show that potential evaporative flux fluctuation is less than one order of magnitude and therefore the potential evaporative flux can be considered as almost constant.

(a) Temperature

(b) Evaporative flux

Figure 5-32 Potential evaporation test at temperature of about 49°C

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(a) Temperature

(b) Evaporative flux

Figure 5-33 Potential evaporation test at temperature of about 40°C

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Figure 5-34 Potential evaporation test at temperature of about 33oC

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(a) Temperature

(b) Evaporative flux

Figure 5-35 Potential evaporation test at temperature of about 24°C

5.4.3.5. Specific Heat Capacity and Thermal Conductivity of Soils

In order to deal with heat transfer in soil, specific heat capacity of soil, C_h , and thermal conductivity of soil, λ are important properties. These properties depend on the grain size distribution, mineralogical content and volumetric water content which in turn depends on matric suction. Both specific heat capacity and thermal conductivity of soil can be determined theoretically as suggested by de Vries (1963). It was found that the calculated and measured values agreed well (Milly, 1980, Camillo et al., 1983, and Wilson, 1990). In this study, C_h and λ variables were also determined using de Vries (1963) approach.

The equation for calculating specific heat capacity of soil was given in Equation (3.20). The specific volumetric heat capacity of the liquid water phase, C_w , at 10°C and 40°C are 4.154×10^6 and $4.18 \times 10^6 \text{ Jm}^{-3}\text{C}^{-1}$, respectively. The specific volumetric heat capacity of the soil solid, C_s for Sand 1 and Sand 2 can be estimated from data of fine sand whereas C_s for Residual soil where quartz is dominant mineral with approximately 5% kaolinite (Rahardjo et al., 2000) can be estimated from data of quartz material. The C_s value of fine sand at temperature 35°C is $2.27 \times 10^6 \text{ Jm}^{-3}\text{C}^{-1}$, while the C_s value of quartz material at 60°C is $2.438 \times 10^6 \text{ Jm}^{-3}\text{C}^{-1}$ (de Vries, 1963). The volumetric soil solid content, θ_{solid} is constant during the evaporation test. The specific volumetric heat capacity of the air portion, C_a , at 10°C is $0.0125 \times 10^6 \text{ Jm}^{-3}\text{C}^{-1}$.

The thermal conductivity of the soil, λ , can be calculated as follows (de Vries, 1963):

$$\lambda = \frac{\sum_{i=0}^N k_i \theta_i \lambda_i}{\sum_{i=0}^N k_i \theta_i} \quad (4.6)$$

where: k_i = weighting factor for the types of phases (i.e., soil solid, water and air) = $1/3 \left\{ \sum_{i=1}^3 \left[1 + (\lambda_i / \lambda_w - 1) g_i \right] \right\}^{-1}$,

g_i = depolarization factor of the ellipsoid in the direction of the i-axis,

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λ_w = thermal conductivity of water phase, and

N = number of type of phases.

Jame (1977) showed that the thermal conductivity of the air phase can be calculated from the thermal conductivities of dry air ($\lambda_{\text{dry_air}}$), and vapour phases (λ_{vapour}) as follows:

$$\lambda_a = \lambda_{\text{dry_air}} + \lambda_{\text{vapour}} \quad (4.7)$$

The value of (λ_{vapour}) varies linearly from 0 to 0.0736 W/m °C for gravimetric water content from 0% to 12.1% (Jame, 1977). The value of ($\lambda_{\text{dry_air}}$) is widely accepted to be 0.025 W/m °C (Wilson, 1990)

The value of g_i depends on the shape of the soil particle, more specifically the ratio of the axes not on their absolute value, meaning that it only depends on the shape of the ellipsoid describing the soil particle. The sum of the three orthogonal g_i values equals unity. For spherical soil particle, $g_1 = g_2 = g_3 = 1/3$. Jame (1977) assumed that the depolarisation factors g_1 and g_2 of the air phase decreased linearly from 0.333 to 0.105 when gravimetric water content decreases from 23.6% to 12.1%, and from 0.105 to 0.015 when gravimetric water content decreases from 12.1% to 0%.

The thermal conductivities of the three soils calculated using the above discussions are presented in the Figure 5-36, Figure 5-37, and Figure 5-38, respectively. As can be seen in these figures the values of soils' thermal conductivities are non-linear. Therefore for computational convenience in the spreadsheet program, a third-order polynomial was fitted to the curves as shown in the figures. The polynomial function was then employed in the spreadsheet program.

Figure 5-36 Thermal conductivity of Sand 1

Figure 5-37 Thermal conductivity of Sand 2

Figure 5-38 Thermal conductivity of Residual soil

5.5. SUMMARY

The one-dimensional soil column setup and soil properties were presented in this chapter. All soils' properties required as input data for the modified model were determined experimentally. The SWCCs of the soils were determined for a full drying and wetting cycle using pressure plate extractor whereas the drying parts were also carried out using Tempe pressure cell. The unsaturated permeability functions were determined from SWCCs using ACUPIM/W. Direct measurements of unsaturated permeability at confining pressure of 200 kPa was compared with results from ACUPIM/W with saturated permeability at the confining pressure cell of 200 kPa. The comparison showed that the two sets of results are in good agreement with a difference of less than one order of magnitude.

For the thermal conductivity determination, the method proposed by Jame (1977) and later used by Wilson (1990) was employed. The relationship of thermal conductivity versus volumetric water content was determined for Sand 1, Sand 2 and Residual soil.

Chapter 6 Soil Column Test Programme and Results

6.1. INTRODUCTION

Soil column tests are commonly performed to study the infiltration process (Choo and Yanful, 2000). Limited soil column tests have been performed to study the evaporation process (Yanful et al. 2003). When these tests are reported, not all soil properties are reported and it is difficult to use these data without making some assumptions. For this research, three soil column tests using Sand 1, Sand 2 and Residual soil were performed to provide more complete sets of data for verification of the numerical models. Both infiltration and evaporation tests were performed on each soil column. The first soil column consists of a 0.5m height of Sand 1. The second soil column consists of a 0.25m height of Sand 2 over 0.25 m height of Sand 1. The third soil column consists of 0.5m height of Residual soil. The preparation of the soil column has been described in Chapter 5. The short column height of 0.5m was selected as small tip tensiometers were used to measure the pore-water pressure which have a limitation of -85 kPa (Stannard 1990). It was expected that during the evaporation process, soil near the top of the soil column may experience matric suction of greater than 85 kPa.

6.2. TEST PROGRAMME

Each soil column was subjected to seven different stages:

- (1) Saturation – where water was allowed to percolate slowly from the base of the soil column to the top surface and overflow to saturate the soil column and to check that the small tip tensiometers were working properly (Figure 6-1a),
- (2) Drawdown – where water was allowed to drain by gravity flow through the base of the soil column where the pressure head was maintained at zero to reach the hydrostatic condition (Figure 6-1b),

(3) Ambient evaporation – the soil column was allowed to dry under the ambient condition for a long duration. Potential evaporation was also measured at the same time using a container of water placed on an electronic weighing scale (Figure 6-1c),

(4) Temperature evaporation - After ambient evaporation, a light bulb was positioned above the soil column to simulate evaporation under a heat source. Potential evaporation was also measured at the same time using a container of water placed under an identical heat source on a electronic weighing scale (Figure 6-1d),

(5) Cooling down after temperature evaporation – the light bulb was turned off and the soil column was allowed to cool down under ambient condition (Figure 6-1e),

(6) Infiltration – water at a flow rate of $q_{inf} < k_s$ was introduced at the top of the soil column. A piece of filter paper was placed on the top surface of the soil column so that the water can be uniformly spread over the top surface and to minimize disturbance to the top surface of the soil column. The flow rate of $q_{inf} < k_s$ was selected so that the soil column test was conducted under the no-ponding condition. Water flowing out from the base of the soil column was collected in a beaker placed on an electronic weighing scale (Figure 6-1f), and

(7) Redistribution of pressure head after infiltration – the water flow was stopped and the water in the soil column was allowed to drain (redistribute) under gravity flow and ambient condition (Figure 6-1g). The third soil column was subjected to two additional stages: ambient evaporation and temperature evaporation. The details of the three soil column tests are summarized in Table 6-1.

Figure 6-1 Soil column test stages

Table 6-1 Test programme

Material	No	Test stage	Elapsed time	Evaporation/Infiltration rate (column cross section of 0.015394 m ²)		Boundary condition			
						Pressure head (m)		Temperature (°C)	
						Top (z = 0.5m)	Bottom (z = 0m)	Top (z = 0.5m)	Bottom (z = 0m)
Sand 1	1	Saturation process	2 days	-	-	$h_p = 0$	$h_p = 0.5$	-	-
	2	Drawdown	8 hrs	-	-	-	$h_p = 0$	-	-
	3	Ambient evaporation	12 hrs	1.71×10^{-8}	1.47744	Q_{eva}	$h_p = 0$	24	23.5
	4	Temperature evaporation	17 hrs	8.25×10^{-8}	7.128	Q_{eva}	$h_p = 0$	49.5	23.5
	5	Cooling down after temperature evaporation	12 hrs	3.15×10^{-8}	2.7216	Q_{eva}	$h_p = 0$	23.5	23.5
	6	Infiltration	8 hrs	7.21×10^{-7}	62.2944	$q_{infil} < k_s$	$h_p = 0$	-	-
	7	Redistribution of pressure head after infiltration	4 hrs	-	-	$q_{infil} = 0$	$h_p = 0$	-	-
Sand 2 over Sand 1	1	Saturation process	4 days	-	-	$h_p = 0$	$h_p = 0.5$	-	-
	2	Drawdown	12 hrs	-	-	-	$h_p = 0$	-	-
	3	Ambient evaporation	48 hrs	1.78×10^{-8}	2.592	Q_{eva}	$h_p = 0$	23.5	23
	4	Temperature evaporation	12 hrs	7.35×10^{-8}	6.3504	Q_{eva}	$h_p = 0$	34	24
	5	Cooling down after temperature evaporation	12 hrs	3.25×10^{-8}	2.808	Q_{eva}	$h_p = 0$	23.5	23.5
	6	Infiltration	8 hrs	1.08×10^{-6}	93.312	$q_{infil} < k_s$	$h_p = 0$	-	-
	7	Redistribution of pressure head after infiltration	10 hrs	-	-	$q_{infil} = 0$	$h_p = 0$	-	-

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Material	No	Test stage	Elapsed time	Evaporation/Infiltration rate (column cross section of 0.015394 m ²)		Boundary condition			
				m/s	mm/day	Pressure head (m)		Temperature (°C)	
						Top (z = 0.5m)	Bottom (z = 0m)	Top (z = 0.5m)	Bottom (z = 0m)
Singapore residual soil	1	Saturation process	8 days	-	-	$h_p = 0$	$h_p = 0.5$	-	-
	2	Drawdown	20 hrs	-	-	$q_{infil} = 0$	$h_p = 0$	-	-
	3	Ambient evaporation	72 hrs	1.65×10^{-8}	1.4256	Q_{eva}	$h_p = 0$	24	23.5
	4	Temperature evaporation	24 hrs	7.95×10^{-8}	6.8688	Q_{eva}	$h_p = 0$	40	23.5
	5	Cooling down after temperature evaporation	8 hrs	3.15×10^{-8}	2.7216	Q_{eva}	$h_p = 0$	25	24
	6	Infiltration	25 hrs	1.05×10^{-8}	0.9072	$q_{infil} < k_s$	$h_p = 0$	-	-
	7	Redistribution of pressure head after infiltration	40 hrs	3.25×10^{-8}	2.808	$q_{infil} = 0$	$h_p = 0$	-	-
	8	Ambient evaporation	24 hrs	1.65×10^{-8}	1.4256	Q_{eva}	$h_p = 0$	24	23.5
	9	Temperature evaporation	10 hrs	7.9×10^{-8}	6.8256	Q_{eva}	$h_p = 0$	39	24

6.3. TEST RESULTS

In this section, the results of the three soil columns during rapid drawdown of water table, evaporation under atmospheric condition, evaporation under high temperature condition, and infiltration are presented. Pressure head profiles and temperature profiles were plotted for various elapsed times. The pressure head-time history was plotted at various heights of the soil column: 0.45m, 0.35m, 0.3m and 0.15m. Similarly temperature-time history was also plotted at various heights: 0.5m, 0.45m, 0.35m and 0.15m. All four plots are shown for each test for a better view of the changes in pressure head and temperature in the soil column.

6.3.1. Homogenous Soil Column of Sand 1

Drawdown test was started first with a saturated soil column at hydrostatic condition. The water table was fixed at the bottom of the soil column (i.e., $z = 0\text{m}$) and the soil surface was covered with a plastic plate to minimise evaporation. The test results are shown in Figure 6-2. In general the development of pressure head in the Sand 1 column was relatively fast (i.e., within 1 hour). This could be attributed to the fact that the saturated permeability of Sand 1 is high. As can be seen in Figure 6-2(a), the pressure head profile developed toward the unsaturated hydrostatic line water drained out of the column. As shown in the pressure head-time history in Figure 6-2(c), after elapsed time of 1 hour of experiment the pressure head hardly changed until the end of the test (8 hrs). The reason can be explained using the SWCC presented in Figure 5-16 and unsaturated permeability, k_w , presented in Figure 5-24. The Sand 1 was able to retain water up to the air-entry value of 2.5 kPa and the value of k_w becomes smaller and smaller when the matric suction increases. The lower value of k_w will prevent the water from flowing out of the column easily. Therefore, the hydrostatic condition may never be reached. A similar phenomenon was also reported by Choo and Yanful (2000) and Yang (2002). The temperature-time history was also presented in Figure 6-2(d). There was a temperature fluctuation of about $\pm 0.5^\circ\text{C}$ within the accuracy of the sensors as shown in Figure 6-2(d). Overall, there is very little difference in the temperature along the column as

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(a) Pressure head profile (b) Temperature profile

(c) Pressure head -time history

(d) Temperature -time history

Figure 6-2 Drawdown test on Sand 1 column

the difference is less than $\pm 0.5^{\circ}\text{C}$ and the temperature can be considered as uniform with depth.

The pressure head profile during the ambient evaporation test is shown in Figure 6-3(a). The pressure head profile tends to move to the left (i.e., getting more negative) in the zone of 15-20 cm from the top of the column. The development of pressure head could be due to the difference in vapour density which is dependent on the temperature and relative humidity in the soil and in the air. The saturated vapour density in the air corresponding to temperature of 24°C and relative humidity of 75% was 0.0162 kg/m^3 whereas the vapour density at the soil surface was 0.0239 kg/m^3 . Therefore, a vapour density gradient exists between the soil and the air and cause an outflow of moisture content in the form of vapour from the soil. The temperature profile was relatively uniformly at around 24°C along the column as shown in Figure 6-3(b) and Figure 6-3 (d).

For the temperature evaporation test, the temperature applied at the top of the soil column was 49.5°C . The pressure head profiles and temperature profiles during the temperature evaporation test are presented in Figure 6-4(a) and Figure 6-4(b), respectively. As can be seen in these two figures, as the temperature increases the pressure head becomes more negative. However, the high pore-water pressure zone was formed at the top 10 cm of the soil column faster than the lower part of the soil column. The reason could be explained by comparing the temperature gradients in the upper and lower parts of the soil column which in turn affects the vapour density gradient of these parts. From Figure 6-4(b), the temperature gradients were estimated to be 2°C/cm and 0.2°C/cm in the upper and lower parts of the soil column, respectively. The pressure head development rate in this test was also much faster than that in the ambient evaporation test (1m in 10 hrs compared with 0.2m in 12 hrs). As the measurement range of the small-tip tensiometer range is small (i.e., 85 kPa) then the continuous evaporation in the top of the soil column may lead to desaturation of ceramic tip in the upper part of the soil column. Therefore in this test the temperature evaporation test was stopped when the small-tip reading near the top of the soil column was approximately 30 kPa.

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Figure 6-3 Ambient evaporation test on Sand 1 column

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Figure 6-4 Temperature evaporation test on Sand 1 column

After the temperature evaporation test, the column was cooled down by exposing the top soil surface to ambient condition before the next test on infiltration. The test results are presented in Figure 6-5. In general the pressure head in soil column had a tendency of becoming less negative as can be seen in Figure 6-5(a). From Figure 6-5(d) it can be observed that the top temperature decreases rapidly to the temperature in the lower part of the soil column. As the top soil temperature reduces to the ambient condition the pressure head reading increases. This could be attributed to the fact that the top part of soil column has a high pressure head (i.e., -3m) whereas the pressure head in the lower part was much lower (i.e., -0.5m) therefore causing moisture content to move up to reduce the high pressure head gradient (i.e., increasing moisture content). The test was terminated after the temperature profile became stable after 12 hrs.

The infiltration test was started with an infiltration rate of 6.02×10^{-7} m/s and the water table set at $z = 0$ m. The test results are presented in Figure 6-6. A plastic cover was placed at the top of the soil column to minimise the possible evaporation. The infiltration rate was selected so that the pressure head can develop slowly. As more water infiltrates into the soil column, the pressure head profile increases towards the right to almost 0 m throughout the soil column (Figure 6-6 a). The test was stopped after 4 hrs.

In the final Sand 1 column test, the column was left under ambient condition for redistribution of the pressure head. The results are presented in Figure 6-7. As can be seen in Figure 6-7(c) the pressure head profile of soil column decreased continually (from -0.12m down to -0.38m at the height of 0.45m) causing the pressure head profile to move back to the left progressively. The decrease in pressure head profile in this test tends toward the hydrostatic condition as in the drawdown test. As the whole column test duration was very long (approximate 65 hrs continuously), the small tip tensiometer-transducer at the height of 0.3m malfunctioned during this test and no data was recorded as shown in Figure 6-7(a).

(a) Pressure head profile (b) Temperature profile

(c) Pressure head -time history

(d) Temperature -time history

Figure 6-5 Cooling down process on Sand 1 column

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(a) Pressure head profile (b) Temperature profile

(c) Pressure head -time history

(d) Temperature -time history

Figure 6-6 Infiltration test on Sand 1 column

Figure 6-7 Redistribution after infiltration test on Sand 1 column

6.3.2. Layered Soil Column of Sand 1 and Sand 2

In this section, the test results from the layered soil column of 0.25 m Sand 2 overlying 0.25m Sand 1 are presented. The purpose of this series of layered soil column tests was to examine whether different types of soil can create a significant change in pressure head and temperature profile under processes of infiltration and evaporation.

Drawdown test was started first with a saturated soil column at hydrostatic condition. The water table was lowered to the bottom of soil column during this test and the soil surface was covered with a plastic plate to minimise evaporation. The results are shown in Figure 6-8. In general the development of pressure head in the layered sand column was not as fast as Sand 1 column test. In this test it took nearly 3 hrs to fully achieve a stable pressure head profile. The test showed that the pressure head was continuous across the Sand 2 – Sand 1 interface. As can be seen in Figure 6-8(a), the pressure head profile was developing towards the hydrostatic condition as water drained out of the soil column. It was also expected that the pressure head profile will come to the hydrostatic line however the final hydrostatic profile was not reached. As shown in Figure 6-8(c) the pressure head-time history after 4 hrs was nearly constant till the end of the test. The reason can be explained using the SWCCs presented in Figure 5-16 and Figure 5-17 and unsaturated permeability, k_w , presented in Figure 5-29 and Figure 5-30 for Sand 1 and Sand 2, respectively. Both sands were able to retain water up to the air-entry value of about 2.5 kPa and the values of unsaturated permeability, k_w , are smaller when the matric suction increases. The lower unsaturated permeability will prevent the water from flowing out of the column easily. Therefore, the hydrostatic condition may be obtained at much longer test duration or may never be reached. The same phenomenon was also observed in the Sand 1 column test. The temperature-time history was also presented in Figure 6-8(d). There was a fluctuation of about $\pm 0.5^\circ\text{C}$ along the soil column during the test. Therefore, the temperature can be considered as uniform with depth during the test.

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(a) Pressure head profile

(b) Temperature profile

(c) Pressure head -time history

(d) Temperature -time history

Figure 6-8 Drawdown test on layered sand column

The pressure head profile during the ambient evaporation test is shown in Figure 6-9(a). The pressure head profile tends to move left (i.e., getting more negative) in the zone of 10-15 cm from the top of the column. The development of pressure head could be due to the difference in vapour density which is dependent on temperature and relative humidity in the soil and in the air. The saturated vapour density in the air corresponding to temperature of 23°C and relative humidity of 78% was 0.0162 kg/m³ whereas the vapour density at the soil surface was 0.0214 kg/m³. Therefore, a vapour density gradient existed between the soil and the air and caused an outflow of moisture content in the form of water vapour from the soil. The temperature profile was relatively uniform at around 23.5°C along the column during the first 3 hrs as shown in Figure 6-9(b) and Figure 6-9(d). The temperature decreased gradually to about 22.75°C at the end of the experiment.

For the temperature evaporation test, the temperature applied at the top of the soil column was 34°C. The pressure head profiles and temperature profiles during the temperature evaporation test are presented in Figure 6-10(a) and Figure 6-10(b), respectively. As can be seen in these two figures, as the temperature increases the pressure head becomes more negative. However, the pore-water pressure change only occurred in the top 10-15 cm of the column. The pore-water pressure was less negative than in the Sand 1 column test. The reason could be explained by comparing the temperature gradients in Figure 6-4(d) and Figure 6-10(d). The temperature gradient in the upper part of the layered sand column did not vary much after 5 hrs. Therefore the pore-water pressure development in the upper part of the layered sand column was slower than that in the Sand 1 column. The temperature profile presented in Figure 6-10(b) does not show any change of temperature across the Sand 2 – Sand 1 interface.

Figure 6-9 Ambient evaporation test on layered sand column

After the temperature evaporation test, the column was cooled down by exposing the top soil surface to ambient condition before the next test on infiltration. The test results are presented in Figure 6-11. In general the pressure head profile in the soil column do not change as can be seen in Figure 6-11(a). From Figure 6-11(d) it can be seen that the top temperature decreases rapidly to the temperature in the lower part of the soil column. The test was stopped after the temperature distribution in soil became stable after 12 hrs.

The infiltration test was started with an infiltration rate of 6.0×10^{-7} m/s and the water table set at $z = 0$ m. The test results are presented in Figure 6-12. A plastic cover was placed at the top of the soil column to minimise possible evaporation. The infiltration rate was selected so that the pressure head can develop slowly. As more water infiltrates into the soil column, the pressure head profile in the top 15 cm of the soil column increases towards the right (Figure 6-12a). The test was stopped after 8 hrs. The temperature profile was relatively uniform throughout the test.

In the final layered sand column test, the column was left under ambient condition for redistribution of the pressure head. The results are presented in Figure 6-13. As can be seen in Figure 6-13(c) the pressure head of soil decreased continually (from -0.18 m down to -0.4 m in the height of 0.45 m) causing the pressure head profile to move back to the left progressively. The decrease in pressure head profile in this test tends towards the hydrostatic condition as in the drawdown test. Due to the long duration of infiltration – evaporation test the small-tip tensiometer transducer at the heights of 0.25 m and 0.2 m malfunctioned during the test. A similar problem also occurred in Sand 1 column test. The temperature profile was relatively uniform throughout the test.

Figure 6-10 Pressure head profile of temperature evaporation test on layered sand column

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Figure 6-11 Cooling down process after temperature evaporation test on layered sand column

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Figure 6-12 Pressure head profile of infiltration test on layered sand column

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Figure 6-13 Redistribution after infiltration test on layered sand column

6.3.3. Homogenous Soil Column of Residual Soil

In this section, the test results from the residual soil column are presented. The purpose of this series of soil column test was to examine the behaviour of local residual soil under processes of infiltration and evaporation

Drawdown test was started first with a saturated soil column at hydrostatic condition. The water table was fixed at the bottom of soil column during this test and the soil surface was covered with a plastic plate to minimise evaporation. The results are shown in Figure 6-14. In general the development of pressure head in the residual soil column is relatively slow compared with the other two columns. In this test it took nearly 20 hrs to fully achieve a stable pressure head profile. This could be attributed to the fact that the saturated permeability of residual soil is very low. As can be seen in Figure 6-14(a), the pressure head profile was developing toward the hydrostatic condition as water drained out of the soil column. It was expected that the pressure head profile will come to the hydrostatic line however the final hydrostatic profile was not reached. As shown in Figure 6-14(c), the pressure head after 6 hrs of experiment was nearly constant till the end of the test. The reason can be explained using the SWCC presented in Figure 5-18 and unsaturated permeability, k_w , presented in Figure 5-26. The residual soil was able to retain water up to the air-entry value of about 2.5 kPa and the value of unsaturated permeability becomes smaller when the matric suction increases. The lower unsaturated permeability will prevent the water from flowing out of the column easily. Therefore, the hydrostatic condition may be obtained at longer test duration or may never be reached, similar to the cases of Sand 1 and layered sand column. The temperature-time history is also presented in Figure 6-14(d). There was a fluctuation of about $\pm 0.5^\circ\text{C}$ along the soil column during the test therefore the temperature can be considered as uniform with depth.

The pressure head profile during the ambient pressure evaporation test is shown in Figure 6-15(a). The pressure head profile tends to move left (i.e., getting more negative) in the zone of 10-20 cm from the top of the column. The development of pressure head could be due to the difference in vapour density which is dependent on temperature and relative humidity in the soil and in the air. Similar trends of results were also observed found in Sand 1 column and layered sand column. The saturated vapour density in the air corresponding to temperature of 23.5°C and relative humidity of 75% was 0.016 kg/m^3 whereas the vapour density at the soil surface was 0.0213 kg/m^3 . Therefore, a vapour density gradient existed between the soil and the air and caused an outflow of moisture content in the form of water

vapour from the soil. However, in this test the test duration was much longer than the previous two columns but the change in pressure head was smaller (i.e., -0.05m compared to -1m and -0.2m for sand 1 and layered soil column, respectively). The reason could be due to the lower saturated permeability, k_s , and vapour diffusivity coefficient of the residual soil. Quantitatively, vapour diffusivity coefficient of soil surface in this test was $1.977 \times 10^{-11} \text{ m}^2/\text{s}$ whereas the values for Sand 1 column and layered sand column were 3.42×10^{-11} and $2.91 \times 10^{-11} \text{ m}^2/\text{s}$, respectively. The temperature profile was distributed uniformly, about 23°C along the column as presented in Figure 6-15(b) and Figure 6-15(d).

For the temperature evaporation test, the temperature applied at the top of the soil column was 40°C . The pressure head profiles and temperature profiles during the temperature evaporation test are presented in Figure 6-16(a) and Figure 6-16(b), respectively. As can be seen in these two figures, as the temperature increases the pressure head becomes more negative. In this test the desaturated zone was also confined to the top 10 cm of the column similar to the Sand 1 column test. As presented in Figure 6-16(a), the maximum pressure head observed in residual soil column was approximately half of that in Sand 1 column even though the test duration was doubled. This was attributed to both the lower saturated permeability, k_s which in turn affects unsaturated permeability, k_w , and temperature gradient applied at the top of the residual soil column. The temperature gradient in the upper part of the residual soil column did not vary much after 12 hrs. Therefore the pore-water pressure development in the upper part of the residual soil column was slower than that in the Sand 1 column. Therefore from these two column tests, it is clear that the higher temperature applied to the top of the soil column the more negative the pressure head in the soil.

After the temperature evaporation test, the column was cooled down by exposing the top soil surface to ambient condition before the next test on infiltration. The test results are presented in Figure 6-17. In general the pressure head profile in the soil column did not change much as can be seen in Figure 6-17(a). From Figure 6-17 (d) it can be seen that the temperature at the top of the column decreased rapidly to the temperature as in the lower part of the soil column. After temperature distribution in the soil column was similar to the ambient temperature and the small tip tensiometer-transducer readings were stable, test was stopped after 8 hrs.

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Figure 6-14 Drawdown test on residual soil column

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Figure 6-15 Ambient evaporation test on residual soil column

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The infiltration test was started with an infiltration rate of 9.05×10^{-8} (m/s) and the water table was set at $z = 0$ m. The test results are presented in Figure 6-18. A plastic cover was placed at the top of the soil column to minimise possible evaporation. The infiltration rate was selected so that the pressure head can develop slowly. As more water infiltrated into the soil column, the pressure head profile in the top 15 cm of the soil column increased towards the right (Figure 6-18a). The temperature distribution along the soil column was nearly uniform as shown in Figure 6-18(b). The temperature profile was relatively uniform throughout the test. The test was stopped after 25 hrs.

In the final Residual soil column test, the column was left under ambient condition for redistribution of the pressure head. The results are presented in Figure 6-19. As can be seen in Figure 6-19(c) the pressure head decreased continually (from -0.12m down to -0.38m at the height of 0.45m) causing the pressure head profile to move back to the left progressively. The decrease in pressure head profile tends towards the hydrostatic condition as in the drawdown test. The small-tip tensiometer at the height of 0.25m malfunctioned during the test. The temperature profile was relatively uniform throughout the test. It was also exhibiting a diurnal change with the ambient temperature of the laboratory.

The column test for the ambient evaporation test was carried out after the temperature evaporation test. The test results are presented in Figure 6-20 and Figure 6-21. Compared with the results from ambient evaporation test presented in Figure 6-3 (Sand 1 column), Figure 6-9 (layered sand column) and Figure 6-15 (1st cycle of residual soil), the development rate of pressure head profile in Figure 6-20(c) was extremely fast although the temperature profile remain at around 23°C with a diurnal fluctuation as shown in Figure 6-20(d). The results for next temperature evaporation test is presented in Figure 6-21. The change in pressure head profile was quite gradual (Figure 6-21a) whereas the change in temperature in the top 10 cm of the soil column was quite rapid up to 2 hrs. The temperature profile did not vary much after 10 hrs. Both the ambient evaporation test and temperature evaporation test showed missing data points in the pressure head profile and temperature profile due to malfunction of tensiometers and temperature sensors.

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Figure 6-16 Temperature evaporation test on residual soil column

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Figure 6-17 Cooling down process after temperature evaporation test on residual soil column

Figure 6-18 Infiltration test on residual soil column

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Figure 6-19 Redistribution after infiltration test on residual soil column

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Figure 6-20 Ambient evaporation test (2nd cycle) on residual soil column

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Figure 6-21 Temperature evaporation test (2nd cycle) on residual soil column

6.4. SUMMARY

The results of three series of soil column tests were plotted and presented in this chapter. In each series of test, a full cycle of drying and wetting process were simulated. In general, the data showed that there was a limited zone of the soil column that was affected most by evaporation and infiltration. The development rate of matric suction profile was strongly related to the drainage boundary, temperature applied and soil type. For the residual soil column another cycle of drying and wetting tests were performed. However the long duration of the test caused more sensors to malfunction. The experimental results presented in this chapter will be used to validate the formulations presented in Chapter 3 in the next chapter.

Chapter 7 Numerical Modelling of Soil Column Tests Described in Chapter 6

7.1. INTRODUCTION

In this chapter, the Microsoft Excel spreadsheets `moisture_cal` and `heat_cal` developed in Chapter 3 are employed for the numerical modelling of the soil column tests described in Chapter 6. The input data was presented in Chapter 5. The details of the experimental setup and the test details can be found in Chapter 6. The numerical modelling is divided into the seven experimental stages of the soil column tests for Sand 1, layered sand, and residual soil columns. Each series of modelling reproduced a full cycle of infiltration and evaporation.

7.2. MODELLING FOR SAND 1 COLUMN

For Sand 1 column, the SWCCs and permeability functions were presented in Figure 5-16 and Figure 5-24, respectively. The parameters of the SWCCs and permeability functions were summarised in Table 5-3 and Table 5-5, respectively. The thermal conductivity of Sand 1 was calculated based on de Vries (1963) theory and shown in Figure 5-36. All the parameters required for the spreadsheet program were described in Section 3.5.

7.2.1. Modelling for Drawdown Test

The water table was fixed at the bottom of the column i.e., $h_p = 0\text{m}$ at $z = 0\text{m}$ was for the bottom node during the modelling of the drawdown test of Sand 1 column. The temperature profile presented in Figure 6-2(b) was quite uniform as the fluctuation of temperature along the column was less than $\pm 1^\circ\text{C}$. Therefore, in the numerical model, the effect of temperature was not considered and the Microsoft Excel spreadsheet `heat_cal` was not used. The numerical results are compared with the experimental data in Figure 7-1(a). In this simulation, the initial time step size and element height used were 0.00001s and 0.001m , respectively. When the soil column was fully unsaturated i.e., pressure head profile is all negative the time step was increased to 0.1s . The drying SWCC and the corresponding permeability

function presented in Figure 5-16 and Figure 5-24, respectively, were employed for the numerical modelling.

The drawdown test was simulated for duration of 2 hrs. As can be seen in Figure 7-1(a) the results from the numerical solution showed reasonable agreement with the pressure head profile observed at the end of the test where both showed a trend towards hydrostatic condition.

7.2.2. Modelling for Ambient Evaporation Test

After drawdown, the simulation for ambient evaporation test was carried out. The initial pressure head profile for this modelling was obtained from the last numerical results of the drawdown test. As the temperature profile distribution in the soil column presented in Figure 6-3(b) was relatively constant therefore the initial temperature profile was set at 23.5 °C along the column. The applied evaporative flux at the top of Sand 1 column was 1.7×10^{-8} m/s which was calculated based on Equation (2.44) and Figure 5-35. The temperature boundary conditions at top and bottom of the column were fixed at 23.5°C. The time step size and element height were set at 0.1s and 0.001m, respectively. The drying SWCC and the corresponding permeability function presented in Figure 5-16 and Figure 5-24, respectively, were employed for the numerical modelling. The numerical results for pressure head and temperature are compared with the experimental data in Figure 7-2(a) and Figure 7-2(b), respectively. As can be seen in Figure 7-2, the numerical results gave the same trend and value as the experimental observation.

7.2.3. Modelling for Temperature Evaporation Test

The pressure head profile at 12 hrs of the ambient evaporation test simulation and a uniform temperature profile of 23.5°C were employed as the initial conditions for this simulation as shown in Figure 7-3. The evaporative flux applied at the top of the soil column was 8.25×10^{-8} m/s, calculated based on Equation (2.44) and Figure 5-32. The temperature at the soil surface recorded by the temperature sensor was applied as the temperature at the top of the soil column. The pressure head at the bottom boundary was set at zero (i.e., $h_p = 0$ m at $z = 0$ m) and the temperature at the bottom boundary was fixed at 23.5°C. In the simulation, time step size and element

height of 0.01s and 0.001m, respectively, were employed. The drying SWCC and the corresponding permeability function presented in Figure 5-16 and Figure 5-24, respectively, were employed for the numerical modelling. The pressure head profiles at various times are shown in Figure 7-3(a). Figure 7-3(a) showed good agreement between numerical results and experimental data. The temperature profiles at various times are presented in Figure 7-3(b). The temperature profiles showed reasonable agreement at 5 and 10 hrs but the numerical results for 17 hrs overestimated the experimental temperature profile. The overestimation could be attributed to the specific heat capacity and the thermal conductivity of the soil which were not determined experimentally. However, the temperature profiles obtained from the numerical solution, in general, reflected the trend of the experimental observations.

7.2.4. Modelling for Cooling Down after Temperature Evaporation Test

Prior to the infiltration test, the soil column was again exposed to ambient laboratory condition to obtain a stable pressure head profile. The evaporation flux applied at the top of the soil column was reduced to 1.7×10^{-8} m/s as the top temperature was around 24°C. During this process the pressure head profile reduced as the soil's vapour density was much lower than that in the air. The numerical simulation was carried out for 12 hrs. In the simulation, time step size and element height of 0.01s and 0.001m, respectively, were employed. The wetting SWCC and the corresponding permeability function presented in Figure 5-16 and Figure 5-24, respectively, were employed for the numerical modelling. The pressure head profiles showed close agreement between the numerical and experimental data as presented in Figure 7-4(a). As shown in Figure 7-4(b), the numerical results for temperature overestimated the experimental temperature profile. This could be attributed to the specific heat capacity and the thermal conductivity of the soil which were not determined experimentally.

7.2.5. Modelling for Infiltration Test

The simulation for infiltration test was carried out right after the simulation for the cooling down after temperature evaporation test with an infiltration flux of 7.21×10^{-7} m/s. As the infiltration rate was small, evaporation could affect the results. However during the infiltration test, a plastic plate was used to cover the top of the soil column so evaporation can be assumed to be negligible. Furthermore, as shown in Figure 7-5(b) the temperature distribution along the column was reasonably uniform therefore heat movement in the column was not modelled. The numerical simulation ended after 4 hrs. In the simulation, time step size and element height of 0.1s and 0.001m, respectively, were employed. The wetting SWCC and the corresponding permeability function presented in Figure 5-16 and Figure 5-24, respectively, were employed for the numerical modelling. The pressure head profiles from the numerical modelling are presented in Figure 7-5(a). The pressure head near the soil surface increased after rainfall was applied and the rate of increase was higher in the upper part of the soil column. In general, the numerical results reflected the trend of the experimental pressure head profile movements well.

7.2.6. Modelling for Redistribution of Pressure Head after Infiltration

After the infiltration simulation, the redistribution of pressure head was simulated with the results at the end of the infiltration simulation as the initial condition. In the simulation, time step size and element height of 0.01s and 0.001m, respectively, were employed. The drying SWCC and the corresponding permeability function presented in Figure 5-16 and Figure 5-24, respectively, were employed for the numerical modelling. The results of the simulation are presented in Figure 7-6(a). After 4 hrs the pressure head profile tends to move to the hydrostatic condition. Similarly, the redistribution test was carried out with a plastic cover at the top of the soil column meaning that negligible evaporation occurred during the test. As shown in Figure 7-6(b), the temperature profile along the soil column was relatively uniform and therefore, heat movement was not modelled.

(a) Pressure head profile

(b) Temperature Profile

Figure 7-1 Numerical results for drawdown test on Sand 1 column

(a) Pressure head profile
(b) Temperature Profile
Figure 7-2 Numerical results for ambient evaporation test on Sand 1 column

Figure 7-3 Numerical results for temperature evaporation test on Sand 1 column

(a) Pressure head profile
(b) Temperature Profile
Figure 7-4 Numerical results for cooling down after temperature evaporation test on Sand 1 column

Figure 7-5 Numerical results for infiltration test on Sand 1 column

Figure 7-6 Numerical results redistribution of pressure head after infiltration test on Sand 1 column

7.3. MODELLING FOR LAYERED SAND COLUMN

For Sand 2 over Sand 1 column, the SWCCs and permeability functions of Sand 2 were presented in Figure 5-17 and Figure 5-25, respectively. The SWCCs and permeability functions of Sand 1 were presented in Figure 5-16 and Figure 5-24, respectively. The parameters of the SWCCs and permeability functions of Sand 1 and Sand 2 were summarised in Table 5-3 and Table 5-5, respectively. The thermal conductivity of Sand 2 and Sand 1 were calculated and shown in Figure 5-37 and Figure 5-36, respectively. All the parameters required for the spreadsheet program were described in Section 3.5.

7.3.1. Modelling for drawdown test

The water table was fixed at the bottom of the column i.e., $h_p = 0\text{m}$ at $z = 0\text{m}$ for the bottom node during the modelling of the drawdown test of the layered sand column. The temperature profile presented in Figure 7-7(b) was quite uniform as the fluctuation of temperature along the column was $\pm 1^\circ\text{C}$. Therefore, in the numerical model, the effect of temperature was not considered and the Microsoft Excel spreadsheet `heat_cal` was not used. The numerical results are compared with the experimental data in Figure 7-7(a). In this simulation, the initial time step size and element height used were 0.0000001s and 0.001m , respectively. When the soil column was fully unsaturated i.e., pressure head profile is all negative the time step was increased to 0.1s . The drying SWCCs and the corresponding permeability functions presented in Figure 5-17, Figure 5-16, Figure 5-25 and Figure 5-24 were employed for the numerical modelling.

The drawdown test was simulated for duration of 3 hrs. The results show a reasonable movement of pressure head profile especially at the end of the test (i.e., 3 hrs). As can be seen in Figure 7-7(a), the numerical results showed reasonable agreement with the pressure head profile observed at the end of the test. The experimental data did not show a kink at the interface between the two sand layers. As mentioned in Section 6.3.2 the differences in saturated permeability, k_s , and unsaturated permeability, k_w , were not much between the two sands, therefore the capillary barrier phenomenon was not well observed. However, as can be seen in

Figure 7-7(a), the numerical results showed the effect of the small permeability difference. The numerical results showed large discrepancies with experimental data during the first few hours where the top pressure head values tend to decrease faster.

7.3.2. Modelling for Ambient Evaporation Test

After drawdown, the simulation for ambient evaporation was carried out. The initial pressure head profile for this modelling was obtained from the last numerical results of the drawdown test. As the temperature profile distribution in the soil column presented in Figure 7-8(b) were relatively constant, the initial temperature profile was set at 24°C along the column. The actual evaporative flux for layered sand column was calculated to be 1.7×10^{-8} m/s based on Equation (2.44) and Figure 5-35. The temperature boundary conditions at top and bottom of the column were fixed at 24°C. The time step size and element height were set at 0.01s and 0.001m, respectively. The drying SWCCs and the corresponding permeability functions presented in Figure 5-17, Figure 5-16, Figure 5-25 and Figure 5-24 were employed for the numerical modelling. The numerical results for pressure head and temperature are compared with the experimental data in Figure 7-8(a) and Figure 7-8(b), respectively. As can be seen in Figure 7-8(a), the numerical results showed the same trend as the experimental data even though there was a discrepancy between the two in the lower part of the column. This is due to the difference in the initial condition. As mentioned earlier, the initial condition of this test was the last numerical result of the drawdown test. Therefore if the simulation was carried out with the experimental data as the initial condition, better agreement would be expected. The experimental data of pressure head profiles did not change significantly after 48 hrs, therefore the numerical simulation was stopped at 48 hrs.

7.3.3. Modelling for Temperature Evaporation Test

The numerical pressure head profile at 48 hrs of the ambient evaporation test and a uniform temperature profile of 24°C were employed as the initial condition for this simulation as shown in Figure 7-9. The evaporative flux applied at the top of the soil column was 8.25×10^{-8} m/s, calculated based on Equation (2.44) and Figure 5-

32. The temperature at the soil surface recorded by the temperature sensor was applied as the temperature at the top of the soil column. The pressure head at the bottom boundary was set at zero (i.e., $h_p = 0\text{m}$ at $z = 0\text{m}$) and the temperature at the bottom boundary was fixed at 24°C . In the simulation, time step size and element height of 0.01s and 0.001m , respectively, were employed. The drying SWCCs and the corresponding permeability functions presented in Figure 5-17, Figure 5-16, Figure 5-25 and Figure 5-24 were employed for the numerical modelling. The pressure head profiles at the various times are shown in Figure 7-9(a). Figure 7-9(a) showed good agreement between numerical results and experimental data. The temperature profile during temperature evaporation test is presented in Figure 7-9(b). The comparison for temperature profiles was reasonably good at 5 hrs but the numerical results at 12 hrs slightly overestimated the experimental temperature profile. The overestimation was attributed to the specific heat capacity and the thermal conductivity of the soil which were not determined experimentally.

7.3.4. Modelling for Cooling Down after Temperature Evaporation Test

Prior to the infiltration test, the soil column was again exposed to ambient laboratory condition to obtain a stable pressure head profile. The evaporation flux applied at the top of the soil column was reduced to $1.7 \times 10^{-8} \text{ m/s}$ as the top temperature was around 24°C . During this process the pressure head profile reduced as the soil's vapour density was much lower than that in the air. The simulation ended at 12 hrs. In the simulation, time step size and element height of 0.01s and 0.001m were employed. The wetting SWCCs and the corresponding permeability functions presented in Figure 5-17, Figure 5-16, Figure 5-25 and Figure 5-24 were employed for the numerical modelling. The pressure head profiles showed close agreement between the numerical and experimental data as presented in Figure 7-10(a). Figure 7-10(b) showed that the modelling results for the temperature profile overestimated the experimental results. The overestimation was attributed to the specific heat capacity and the thermal conductivity of the soil which were not determined experimentally.

7.3.5. Modelling for Infiltration Test

The simulation for infiltration test was carried out right after the simulation of the cooling down after temperature evaporation test with an infiltration flux of 5×10^{-6} m/s. The infiltration test was carried out with a plastic plate covering the top of the soil column. Therefore, evaporation was negligible and not modelled. Furthermore, as shown in Figure 7-11(b) the temperature distribution along the column was reasonably uniform throughout the test. In the simulation, time step size and element height of 0.1s and 0.001m, respectively, were employed. The wetting SWCCs and the corresponding permeability functions presented in Figure 5-17, Figure 5-16, Figure 5-25 and Figure 5-24 were employed for the numerical modelling. The numerical pressure head profiles presented in Figure 7-11(a) demonstrated the entire transient effect of a rainfall on pressure head profile. The pressure head near the soil surface increased after rainfall was applied and the rate of increase was higher in the upper part of the soil column. In general, the numerical results reflected the trend of the experimental pressure head profile movements.

7.3.6. Modelling for Redistribution of Pressure Head after Infiltration

After the infiltration test simulation, the redistribution of pressure head was simulated using the results at the end of the infiltration test simulation as the initial condition. In the simulation, time step size and element height of 0.01s and 0.001m, respectively, were employed. The drying SWCCs and permeability functions presented in Figure 5-17, Figure 5-16, Figure 5-25 and Figure 5-24 were employed for the numerical modelling. The results of the simulation are presented in Figure 7-12(a). The redistribution test was carried out with a plastic cover at the top of the soil column meaning that evaporation was negligible during the test. As shown in Figure 7-12(b), the temperature profile along the soil column was relatively uniform therefore effect of temperature was not simulated. After 10 hrs the pressure profile tended to move to the hydrostatic condition. Reasonably good agreement was obtained between the numerical and experimental pressure head profiles.

(a) Pressure head profile

(b) Temperature Profile

Figure 7-7 Numerical results for drawdown test on layered sand column

Figure 7-8 Numerical results for ambient evaporation test on layered sand column

(a) Pressure head profile

(b) Temperature Profile

Figure 7-9 Numerical results for temperature evaporation test on layered sand column

(a) Pressure head profile
(b) Temperature Profile

Figure 7-10 Numerical results for redistribution of pressure head after temperature evaporation test on layered sand column

Figure 7-11 Numerical results for infiltration test on layered sand column

(a) Pressure head profile
(b) Temperature Profile

Figure 7-12 Numerical results for redistribution of pressure head after infiltration test on layered sand column

7.4. MODELLING FOR RESIDUAL SOIL COLUMN

For residual soil column, the SWCCs and permeability functions were presented in Figure 5-18 and Figure 5-26, respectively. The SWCCs and permeability functions parameters were presented in Table 5-3 and Table 5-5, respectively. The thermal conductivity of residual soil was calculated and shown in Figure 5-37. All the parameters required for the spreadsheet program were described in Section 3.5. The modelling was only performed for the first seven stages of the column test as the sensors started malfunctioning in stages 8 and 9.

7.4.1. Modelling for drawdown test

The water table was fixed at the bottom of the column i.e., $h_p = 0\text{m}$ at $z = 0\text{m}$ for the bottom node during the modelling of the drawdown test of the Residual soil column. The temperature profile presented in Figure 7-13(b) was relatively uniform as the fluctuation of temperature along the column was less than $\pm 1^\circ\text{C}$. Therefore, in the numerical model, the effect of temperature was not considered and the Microsoft Excel spreadsheet *heat_cal* was not used. In this simulation, the initial time step size and element height used were 0.01s and 0.001m, respectively. When the soil column was fully unsaturated i.e., pressure head profile is all negative, the time step was increased to 0.1s. The drying SWCC and the corresponding permeability function presented in Figure 5-18 and Figure 5-26, respectively, were employed for the numerical modelling. The numerical results are compared with the experimental data in Figure 7-13(a).

The drawdown test was simulated for a duration of 20 hrs. The results showed a reasonable agreement of pressure head profile with the experimental data. The duration of drawdown test for Residual soil column was much longer than that for Sand 1 and layered sand column as the saturated permeability, k_s , of the Residual soil was more than three orders lower in magnitude than the Sand 1 and Sand 2.

7.4.2. Modelling for Ambient Evaporation Test

After drawdown, the simulation for ambient evaporation test was carried out. The initial pressure head condition for this modelling was obtained from the last

numerical results of the drawdown test. As the temperature profile in the soil column presented in Figure 7-14(b) were relatively uniform, the initial temperature profile was set at 23.5 °C along the column. The applied evaporative flux at the top of residual soil column was 1.68×10^{-8} m/s which was calculated based on Equation (2.44) and Figure 5-35. The temperature boundary conditions at top and bottom of the column were fixed at 23.5°C. The time step size and element height were set at 0.01s and 0.001m, respectively. The drying SWCC and the corresponding permeability function presented in Figure 5-18 and Figure 5-26, respectively, were employed for the numerical modelling. The numerical and experimental pressure head profiles and temperature profiles are compared in Figure 7-14(a) and Figure 7-14(b), respectively. As can be seen in Figure 7-14, the numerical results gave the same trend and values as the experimental observation. The experimental pressure head profiles did not change significantly after 3 days, therefore simulation was carried out for duration of 3 days only.

7.4.3. Modelling for Temperature Evaporation Test

The pressure head profile at 3 days of the ambient evaporation test simulation and a uniform temperature profile of 23.5°C were employed as the initial conditions for this simulation as shown in Figure 7-15. The evaporative flux applied at the top of the soil column was 6.43×10^{-8} m/s, calculated based on Equation (2.44) and Figure 5-32. The temperature at the soil surface recorded by the temperature sensor was applied as the temperature at the top of the soil column. The pressure head at the bottom boundary was set at zero (i.e., $h_p = 0$ m at $z = 0$ m) and the temperature at the bottom boundary was fixed at 23.5°C. In the simulation, time step size and element height of 0.01s and 0.001m, respectively, were employed. The drying SWCC and the corresponding permeability function presented in Figure 5-18 and Figure 5-26, respectively, were employed for the numerical modelling. The pressure head profiles at various times presented in Figure 7-15(a) showed good agreement with the experimental data. The temperature profiles at various times presented in Figure 7-15(b) showed reasonable agreement up to 12 hrs but the numerical results for 24 hrs overestimated the experimental data. The overestimation was attributed to the specific heat capacity and the thermal conductivity of the soil which were not determined experimentally.

7.4.4. Modelling for Cooling Down after Temperature Evaporation Test

Prior to the infiltration test, the soil column was again exposed to ambient laboratory condition to obtain a stable pressure head profile. The evaporation flux applied at the top of the soil column was reduced to 1.7×10^{-8} m/s as the top temperature was around 24°C. During this process the pressure head profile reduced as the soil's vapour density was much lower than that in the air. The numerical simulation was carried out for 8 hrs. In the simulation, time step size and element height of 0.01s and 0.001m, respectively, were employed. The wetting SWCC and the corresponding permeability function presented in Figure 5-18 and Figure 5-26, respectively, were employed for the numerical modelling. The pressure head profiles showed close agreement between the numerical results and experimental data as presented in Figure 7-16(a). The numerical results for the temperature profile overestimated the experimental results. The overestimation was attributed to the specific heat capacity and the thermal conductivity of the soil which were not determined experimentally.

7.4.5. Modelling for Infiltration Test

The simulations for infiltration test was carried out right after the simulation of the cooling down after temperature evaporation test. An infiltration flux of 9.1×10^{-8} m/s was applied on the Residual soil column. The infiltration test was carried out with a plastic plate covering the top of the soil column. Therefore evaporation was negligible and temperature effect was not modelled. Furthermore as shown in Figure 7-17(b), the temperature distribution along the column was reasonably uniform. The numerical simulation was carried out for 24 hrs. In the simulation, time step size and element height of 0.1s and 0.001m, respectively, were employed. The wetting SWCC and the corresponding permeability function presented in Figure 5-18 and Figure 5-26, respectively, were employed for the numerical modelling. The numerical results of pressure head profiles presented in Figure 7-17(a) demonstrated the entire transient process of the effect of rainfall on pressure head profile. The pressure head near the soil surface increased rapidly after rainfall was applied. In general, the numerical results reflected well the trend of experimental pressure head profile movements.

7.4.6. Modelling for Redistribution of Pressure Head after Infiltration

After the infiltration simulation, the redistribution of pressure head test was simulated with the results at the end of the infiltration simulation as initial condition. In the simulation, time step size and element height of 0.01s and 0.001m, respectively, were employed. The drying SWCC and the corresponding permeability function presented in Figure 5-18 and Figure 5-26, respectively, were employed for the numerical modelling. The redistribution test was carried out with a plastic cover at the top of the soil column meaning that evaporation was negligible during the test. Therefore temperature effect was not simulated. Furthermore, the temperature profile along the soil column was relatively uniform. Generally, the comparison of numerical and experimental pressure head profiles shown in Figure 7-18 is not as good as the corresponding comparison for the Sand 1 and layered sand columns. Although the trend in the upper part of the Residual soil column was reasonable, the trend in the lower part was poor.

7.5. SUMMARY

In this chapter, numerical modelling of the soil column test conducted for this research was presented. In general, the numerical results agree reasonably well with the experimental data. The soil properties, SWCCs and permeability functions which were determined independently and used as input data are reasonable and reliable as shown by the good agreement of the numerical results and experimental data. However, the heat capacity and the thermal conductivity of soils may need to be measured experimentally as shown by the lesser agreement between numerical results and experimental data. In the numerical modelling, very small step size was used to maintain numerical stability as the forward finite difference scheme was used. Due to time constrain, the criterion for numerical stability was not investigated. Furthermore, the use of other finite difference schemes such as central difference and backward difference schemes which could have shortened the computation time was not explored.

Figure 7-13 Numerical results for drawdown test on layered Residual soil column

(a) Pressure head profile
(b) Temperature profile

Figure 7-14 Numerical results for ambient condition evaporation test on Residual soil column

Figure 7-15 Numerical results for temperature evaporation test on Residual soil column

Figure 7-16 Numerical results for cooling down after temperature test on Residual soil column

Figure 7-17 Numerical results for infiltration test on Residual soil column

(a) Pressure head profile
(b) Temperature profile

Figure 7-18 Numerical results for redistribution of pressure head test on Residual soil column

Chapter 8 Conclusion and Recommendation

8.1. INTRODUCTION

The objective of this research is to review existing moisture and heat transfer models in unsaturated soil in order to derive a simple practical model for modelling drying and wetting processes in soil. The model has been verified with existing experimental data as well as with more complete experimental data. Through the works presented in Chapters 2 to 7, the objective of this research has been largely met. The main conclusions obtained in this research are presented in this chapter. Subsequently, recommendations for future research are presented at the end of the chapter.

8.2. CONCLUSION

The major findings of this research are summarised as follows:

1. The mathematical models for evaporation process in the literature were reviewed. It was found that the Philip and de Vries type models seem to be suitable for moisture and heat flow in unsaturated soil under thermal and moisture gradients. However, the proliferation of models based on the Philip and de Vries (1957) has not improved the understanding of moisture and heat flow in unsaturated soil. Some of the terms in these models have no theoretical basis and have parameters that cannot be determined experimentally.
2. In Chapter 3, a rationalizing of the Philip and de Vries type models was performed and a modified model was proposed. The model relied on mass and energy conservations for moisture and heat flows in non-deformable unsaturated soil. With assumptions, this model can be successfully transformed into Wilson et al. (1994) model which was based on the constitutive relationship for unsaturated soil. However differences between some terms in the Wilson et al. (1994) and modified models were identified. Through a parametric study presented in Chapter 4, it was shown that in general the two models gave comparable results except for soil with SWCC

similar to those of clay soil. Even for clay soil, the differences in pressure head distribution in the soil are small and possibly negligible.

3. The modified model was implemented in the Microsoft Excel environment with Visual Basic Application programming. The forward finite difference scheme was used. The Microsoft Excel spreadsheet program includes two spreadsheets, namely, `inf_cal` and `heat_cal` for modelling pressure head and temperature, respectively. The inputs for the program are basic properties of soil, SWCC, permeability function, thermal conductivity and heat capacity of soil. The modified model was evaluated against published data. For the infiltration cases the modified model gave numerical results that agreed well with published experimental data and related numerical solutions. For the evaporation cases, the numerical results from the modified model also gave close agreement with published data. However, the degree of agreement was less than that in the infiltration problem as some assumptions had to be made on soils properties and the soils were assumed to be non-deformable. So far there was no published data where infiltration and evaporation occurred concurrently or consecutively.
4. Soil column tests, involving three different soils namely Residual soil, Sand 1 and Sand 2 were carried out. For Sand 1 and layered sand columns, a total of seven stages of test involving drawdown of water table, ambient evaporation, temperature evaporation, cooling down after temperature evaporation, infiltration and redistribution of pressure head after infiltration were performed. For the Residual soil column two additional stages (2nd ambient evaporation and 2nd temperature evaporation) were carried out. The soils used in these soil columns were characterised for their basic soil properties, drying and wetting SWCCs and permeability functions. The permeability functions were also estimated from the SWCC using a computer program, ACUPIM/W. The comparison showed that the discrepancies between the ACUPIM/W results and direct measurements obtained from triaxial permeameters were less than one order in magnitude, which was within the acceptable experimental accuracy for permeability determination.

5. In general, the data showed that there was a limited zone within the soil column that was severely affected by evaporation and infiltration (i.e., 15 to 20 cm from the surface of the soil columns). The development rate of matric suction profile was strongly related to the drainage boundary, temperature applied and soil type. The experimental observation on the three soil columns were presented and found to be consistent in trend. Generally the durations of tests on soil columns involving Sand 1 and the layered sand were shorter than the tests on the Residual soil column. This is because the saturated permeability of sands were three orders higher than that of Residual soil. For the layered sand column, the effect of capillary barrier in the soil interface was not clearly observed as the saturated permeability of the two sands did not differ much.
6. Reasonably good agreements were observed between numerical results obtained from the modified model and experimental data. The numerical results showed that the modified model was able to replicate infiltration and evaporation processes in unsaturated soil well.

8.3. RECOMMENDATION FOR FUTURE RESEARCH

Through the course of this research it was discovered that the phenomenon of moisture and heat flow in soil was highly complex. It is thought that much more research is needed to fully understand the processes and to review if the processes can be completely modelled using a mathematical model. The shortcomings in this thesis can only be addressed with further research. Some of the improvements that can be made to the current research include:

1. The properties of the soil used in the soil column test should be fully determined to include the thermal characteristics. Heat capacity and thermal conductivity of the soil were not determined in this research. It is thought that better agreement of temperature profile would be possible using independently measured heat capacity and thermal conductivity of the soil.
2. The implementation of soil column test could involve more robust instrumentation so that longer duration test as those seen for the Residual

Chapter 8 Conclusion and Recommendation

soil column test will not suffer from lack of data due to malfunctioning of the instruments.

3. Small changes in pressure head and temperature within the soil column could be detected using more sensitive and accurate instrumentation.
4. The rigour of the modified model could be tested by means of more comprehensive soil column tests. In the present research the soil column test shows a small range of pressure head and temperature changes except at the top of the soil column.
5. The computational efficiency of the modified model could be improved by adopting the backward difference scheme or other numerical scheme so that larger time-step size can be used.
6. Field observations could be carried out for the verification of the modified model.
7. The modified model could be extended to 2-D or even 3-D so that it can be incorporated in a slope stability analysis program.

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APPENDIX

Calibration of pressure transducer:

Channel	Transducer serial number	Calibration constants ($y=mx + c$)		R^2
		m	c	
1	2334483	-0.0067	7.8353	0.9997
2	2334492	-0.0068	7.7553	0.9996
3	2351742	-0.0067	7.842	0.9998
4	2334494	-0.0068	7.8557	0.9999
5	2334493	-0.0072	7.8756	0.9998
6	2351743	-0.0071	7.7765	0.9999
7	2334490	-0.0068	7.7906	0.9999
8	2334486	-0.0068	7.9305	0.9989
9	2334491	-0.0067	7.8065	0.9999

Calibration of temperature sensor:

Channel	Calibration constants (y=mx + c)		R ²
	m	c	
1	0.0619	0.6582	0.9967
2	0.061	1.3397	0.9964
3	0.0686	-4.1739	0.9828
4	0.0679	-3.5241	0.9902
5	0.0626	-0.1892	0.9913
6	0.0623	0.0088	0.9932
7	0.0636	-0.4039	0.9944
8	0.0648	-0.5726	0.9935
9	0.0645	0.0632	0.9919
10	0.0656	-1.8625	0.9917

**Calibration of temperature and, relative humidity transmitter
(serial no 99TCS40058):**

Parameter	Calibration constants (y=mx + c)		R ²
	m	c	
Temperature	0.0264	-23.255	0.9979
Relative humidity	0.1146	-57.573	0.9944

Elapsed time-weight history of Tempe pressure cell for drying
SWCC determination:

Volume of water extracted/absorbed versus matric suction applied from Volumetric Pressure Plate for drying and wetting SWCC determination:

Sand 1			
Drying		Wetting	
Extracted water (ml)	Matric suction (kPa)	Absorbed water (ml)	Matric suction (kPa)
0	0.1	0	200
7	2	0	150
15	3	1	100
10	4	2	50
30	5	2	30
15	6	3	20
10	10	4	15
5	20	8	13
5	30	12	10
5	40	14	8
5	80	15	5
4	100	25	4
3	150	15	3
2	200	5	2
		10	0.1

Appendix

Sand 2			
Drying		Wetting	
Extracted water (ml)	Matric suction (kPa)	Absorbed water (ml)	Matric suction (kPa)
5	0.1	0	200
10	2	0.2	100
15	3	0.5	50
25	4	0.7	30
30	6	2	20
30	8	3	15
20	10	4	13
15	15	5	10
15	30	10	8
10	50	15	6
10	100	20	5
5	200	22	4
		25	3
		30	2
		38	0.1

Residual soil			
Drying		Wetting	
Extracted water (ml)	Matricsuction (kPa)	Absorbed water (ml)	Matricsuction (kPa)
0	0.1	0	200
8	2	0.2	100
9	2.5	0.5	50
7.5	3	0.7	30
15.5	5	2	20
18.5	8	3	15
25.5	12	4	13
25	20	5	10
29	50	10	8
15	100	15	6
10	150	20	5
5	200	22	4
		25	3
		30	2
		25	0.1