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NONLINEAR HYPOELASTIC ANALYSIS OF REINFORCED CONCRETE FLAT PLATE FLOORS

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ABSTRACT

A three dimensional finite element model has been developed for nonlinear analysis of reinforced concrete flat plate structures in this study.

A 3-D hypo-elastic orthotropic material model based on equivalent uniaxial strain concept for modeling the material properties of uncracked concrete and cracked reinforced concrete was proposed. Material property of uncracked concrete under multiaxial stress state was simulated by uniaxial stress-equivalent uniaxial strain relationships. The five-parameter strength surface proposed by Willam and Warnke (1975) was employed to determine the ultimate peak stress and peak strain. A new model describing the material properties of cracked reinforced concrete with multiple non-orthogonal cracks has been developed. The material properties of the multi-directional cracked reinforced concrete are represented by the material properties of the intact solid concrete and a number of uniaxially cracked concrete with their coupling solids. Cracking effects due to multiple non-orthogonal cracks are traced in each uniaxially cracked concrete. Tension softening, aggregate interlock occurred at crack interface as well as tension stiffening and compression softening initiated in concrete between cracks due to multiple non-orthogonal cracks are all incorporated explicitly.

Tension softening effect of cracked plain concrete is in terms of stress-equivalent uniaxial strain relationship. In case of the reinforcement does not normal to crack plane, the tension stiffening effect is considered in crack normal direction by nonlinear interpolation between tension softening and tension stiffening. Compression softening effect of cracked reinforced concrete due to principal tensile strain, principal tensile stress normal to crack as well as shear stress in crack interface is considered. The shear friction effect of aggregate interlock and dowel action of uniaxially cracked concrete is modeled as a variable fraction of shear stiffness of uncracked concrete. Reinforcement is modeled as elasto-plastic material with possible strain-hardening having uniaxial stiffness property. Perfect bond is assumed between reinforcement and concrete.

A degenerate isoparametric serendipity shell element is employed in this study. A new layering scheme for shear correction was formulated directly in shell element formulations allowing the variation of shear strain distribution in the depth of the section. A numerical nonlinear analysis procedure was developed to solve the nonlinear equations.

The capability and performance of the finite element model have been evaluated by analyzing the response of deliberately selected reinforced concrete specimens under various loading conditions. The proposed material model has been proved to be capable of simulating the material properties of cracked reinforced concrete with good accuracy. The developed finite element model was proved to be an effective and efficient numerical tool for simulating the nonlinear behavior of RC flat plate structures. The deformational response, failure modes, and ultimate shear strength of the structure can be captured well. The developed finite element model is also applicable for nonlinear analysis of RC flat slabs with drop panels.

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LIST OF SYMBOLS

$\mathbf{C}$	=	Tangent compliance of cracked concrete in element coordinates
$\mathbf{C}_0$	=	Tangent compliance of the intact concrete in element coordinates
$\mathbf{C}_{iu}$	=	Tangent compliance of the i -th uniaxially cracked concrete in element coordinates
$\mathbf{C}_{i0}$	=	Tangent compliance of the coupling solid of the i -th uniaxially cracked concrete in element coordinates
$\mathbf{C}'_{icr}$	=	Tangent compliance of the i -th crack in its local crack coordinates
$\mathbf{C}'_{iu}$	=	Tangent compliance of the i -th uniaxially cracked concrete in the local crack coordinates
$\mathbf{C}'_{i0}$	=	Tangent compliance of the coupling solid of the i -th uniaxially cracked concrete in the local crack coordinates
$d\tau_{12}, d\tau_{13}$	=	Shear stress increment in crack interface pointing to σ_2 and σ_3 direction respectively
$d\gamma_{12cr}, d\gamma_{13cr}$	=	Shear crack strain increment in crack interface pointing to σ_2 and σ_3 direction respectively
$d\gamma_{12}, d\gamma_{13}$	=	Shear strain increment in crack interface of uniaxially cracked concrete pointing to σ_2 and σ_3 direction respectively
$d\gamma_{120}, d\gamma_{130}$	=	Shear strain increment of the coupling solid pointing to σ_2 and σ_3 direction respectively
$\mathbf{d}\boldsymbol{\varepsilon}$	=	Vector of strain increments
$\mathbf{d}\boldsymbol{\varepsilon}_0$	=	Strain increment of intact concrete in element coordinates
$\mathbf{d}\boldsymbol{\varepsilon}_{icr}$	=	Crack strain increment of the i -th crack in

element coordinates

- $\mathbf{d}\sigma$ = Vector of stress increments
- $\mathbf{d}\sigma'$ = Stress incremental vector in crack local coordinates
- $\mathbf{D}_L$ = Incremental concrete material matrix relative to the local orthotropic axes
- E_c = The initial modulus of elasticity of concrete
- E_i = Tangent modulus of elasticity in i direction of orthotropy ($i=1, 2, 3$)
- E_s = Modulus of elasticity of steel
- f'_{bc} = The equal biaxial compressive strength
- f'_c = The uniaxial compressive strength of concrete
- f'_t = The uniaxial tensile strength of concrete
- f_y = Yielding stress of steel
- G_{ij} = Shear modulus in plane i - j ($i, j=1, 2, 3$)
- β = Shear retention factor
- ε_1 = Tensile strain of uniaxial cracked concrete normal to crack plane
- ε_{10} = Tensile strain of the coupling solid in the direction normal to crack plane
- ε_{bc} = The equal biaxial compressive peak strain of concrete
- ε_c = The uniaxial compressive peak strain of concrete
- ε_{ic} = The equivalent uniaxial compressive peak strain
- ε_t = The uniaxial tensile peak strain of concrete
- ε_y = Strain at yielding of steel
- ϕ = Shear retention parameter
- η = Compression softening parameter

- v_{ij} = Transverse strain ratio for strain in the i-direction caused by stress in the j-direction (i, j=1, 2, 3)
- σ_{ic} = The maximum stress, associated with direction i, that occurs for the current particular stress ratio
- $\sigma_{ij}, \varepsilon_{ij}$ = The coordinates of some point on the descending branch of the stress-equivalent strain curve
- Δl = Arc-length
- $\Delta \lambda_j^i$ = The incremental load changing factor for jth iteration of ith incremental step
- ΔU_j^i = The incremental displacement vector for jth iteration of ith incremental step

CHAPTER 1 INTRODUCTION

1.1 BACKGROUND

1.1.1 Development of R. C. Flat Plate Structure

Reinforced concrete floor system spanning in more than one direction are classified into two categories by most building codes: the flat slab; and the two-way system with supports on all sides, usually called the two-way slabs. The flat plate floor system is a direct development from the earlier flat slab system, and is characterized by the removal of capitals at the top of the columns or drop panels surrounding each column as shown in Fig. 1.1. The development of this floor system is strongly related to that of flat slab floor system. The first flat slab building was the C. A. Bovey building in 1906. Following that, flat slabs was an instant commercial success, and more than 1000 structures were built around the world within 7 year after the first one (Sozen and Siess, 1963). Although there had been great progress since the flat slab was first introduced, there was no standard procedure for design at that time. The flat slab design was a subject of controversy until Westergaard and Slater published their comprehensive paper on slabs in 1921 (Westergaard and Slater 1921).

Compared to other floor systems particularly to the conventional beam-column-slab systems, the flat plate system has several advantages. Firstly, it reduces the building height and overall weight. It also offers flexibility in arranging room layout. From the construction point of view, a flat plate floor system will facilitate builders to increase site productivity by its simpler form-works. The other advantage is the ease of installation of Mechanical and Electrical facilities (M&E). Since the underside of the floor is kept free of beams, all M&E facilities can be mounted directly on the underside of the slab instead of bending them to avoid the position of beams as normally experienced in conventional beam-column slab. Given the above

advantages, flat plate has been widely used either in industry or in civil public buildings such as office buildings, warehouses or residential buildings.

Fig. 1.1 Types of Two-way Slab

1.1.2 General Behavior of R. C. Flat-Plate Structure

Despite its simple appearance, flat plate has a complex load bearing behavior in flexure and in shear. The slab-column connection part is the discontinuity region where high moments occur and the three-dimensional state of stresses is complicated. The general behavior of flat plate structures is dominated by the performance of this slab-column connection (D region). At initial loading this connection part remains nearly elastic. The first flexural crack usually occurs in the radial direction on the top surface near a column at 10 to 25 percent of the ultimate load. With increasing load, more radial cracks are formed and extend to the edges of

the slab. After very densely distributed radial cracks are formed, the tangential cracks appear near the column connection in circumferential directions. Thereafter, new radial and tangential cracks are formed between previously existing cracks until a web-like crack pattern is formed. The radial cracks are nearly vertical and can reach as deep as the compression zone of the slab. Tangential cracks are vertical in the tension surface and become inclined when approaching the middle of the section. The initial intact concrete in the tension surface and the web of the section is cut by these cracking planes and become multi-directional cracked concrete. The flexural and shear behaviors of the slab connection are significantly dictated by the material properties of these cracked concrete regions. Concrete in the compression zone experiences biaxial or triaxial compression. At the ultimate load, one of two failure modes will occur: flexural failure or punching shear failure. The flexural failure is associated with the yielding of tensile reinforcements and the compressive crushing of concrete. Initial yielding of steel generally occurred near the column and spread gradually to the edges. When punching shear failure happens, an inclined crack forms around the column and finally the column with a punching cone separates from the slab as shown in Fig. 1.2. It is a brittle type of failure where flexural reinforcement may not yield. This failure mode may even appear when a flat plate is subjected to substantial lateral loads due to earthquake or wind which cause high unbalanced moments and transverse shear forces to be transferred between slab and column.

A successful design of flat plate structures would inevitably need a better understanding for the rational analysis of its load effects and the accurate evaluation of its resistance capacities both in flexure and in shear. Therefore, much theoretical and experimental research has been conducted on slab-column connections, on the analysis of its flexural and shear behaviors, failure mechanisms, and design provisions.

Fig. 1.2 Shear Failure Mode of Flat Plate Slabs

1.1.3 Theoretical Studies on Flat-Plate Structures

A variety of models are available to estimate the ultimate strength of connections of RC flat plate structures. The control surface approach is the oldest and most widely used approach for punching. The approach was based on equilibrium of shear stress with the concrete strength on an assumed cylindrical control surface around a loaded area. The method has nothing to do with the physical reality of the punching phenomenon, but it is simple and when properly calibrated it leads to reasonable and consistent predictions. The method forms the basis for punching shear rules in most building codes (CEB-FIP; ACI; DIN). The main differences involve the definition of the control surface and the corresponding choice of concrete strength parameters (Regan and Braestrup 1985).

The flexural punching approach is to treat punching shear by means of ordinary flexural theory considering the fact that in many punching tests the ultimate load does not differ significantly from flexural capacity. The method ignores the effect of punching shear and assumes the strength is not much different from the flexural strength computed by the Yield Line Theory (Hognestad 1953). The models developed by Moe (1961) and Yitzhaki (1966) can be categorized into this group.

Based on assumed load transfer and failure mechanism enlightened from test observations or numerical simulations, various mechanical models have been developed to describe punching shear strength of slabs more rationally. Kinnunen and Nylander (1960) seem to have developed the first mechanical model for the

punching shear, which not only yielded good agreement with test results but also visualized the flow of forces. A rigid sector element cut by the radial cracks and tangential cracks is isolated. Equilibrium of forces acting on the sector is created. The failure is expressed in terms of a limitation of the inclined radial compressive stress and a limitation of the tangential compressive strain at the shear crack. In the plasticity based model proposed by Braestrup et al. (1976), a failure mechanism is assumed in which the deformation is concentrated in a rotationally symmetric surface and the concrete punching is assumed to be perpendicular to the slab. The equilibrium is expressed by equating the rate of external work done by applied load with the rate of internal work dissipated by the failure. The obtained punching load is an upper bound value. Based on the results obtained with numerical simulation of the failure phenomenon, Menétrey (1996) developed an analytical model which incorporates the shear transfer at the ultimate punching crack surface. The resulting punching load is computed by integrating the vertical components of the concrete tensile stresses around the ultimate punching crack. The influence of the flexural capacity is linked to the punching capacity with the punching crack inclination (Menétrey 1998). Another classical mechanical model is the truss model proposed by Alexander and Simmonds (1987). The model consists of a three-dimensional space truss composed of concrete compression struts and steel tension ties.

The analytical models reviewed above are based on the equilibrium condition at the ultimate limit state and are applied to estimate the ultimate shear strength. If more information is required, numerical tools such as finite element method can be resorted to in addition to expensive experimental methods. The degradation of stiffness due to progressive cracking, the initiation of diagonal shear cracks before the occurrence of punching shear failure as well as the post-failure structural responses can all be simulated by the finite element method.

1.1.4 Development of Nonlinear Finite Element Analysis (NLFEA) for Reinforced Concrete Flat Plate Structure

In the last twenty years, the nonlinear finite element analysis (NLFEA) approach has been developed as a versatile numerical tool to simulate the nonlinear behavior

of reinforced concrete flat plate structure. It is capable of providing significant information such as crack pattern, failure mode, ultimate strength, with respect to the local responses of slab column connection. The NLFEA approach can also be applied for global analysis to evaluate the structural performance of the whole structure within a complete loading history. A brief review of the available finite element models for reinforced concrete flat plate structure is presented as follows. The employed finite elements and material models are highlighted. Merits and demerits of the models reviewed are identified.

The importance of the local responses of the slab-column connection in relation to the structural performance of RC flat plate structures has been recognized. Considerable research efforts were devoted to develop finite element models simulating the localized behavior of slab-column connections. The reproduction of punching shear failure is the critical step in a successful simulation of punching shear behavior of RC flat plates. If such a failure mode were not simulated correctly, an undesirable overestimated ultimate strength of the structure may be predicted. Accurate representation of external three-dimensional stress state as well as on internal material response is thus necessitated.

In the study by de Borst and Nauta's (1985), an axisymmetric element in conjunction with an elastoplastic multiple non-orthogonal cracked concrete model was employed. The non-orthogonal crack approach proved to be able to handle the radial and tangential cracks that occurred in R. C. flat plates. The diagonal shear crack was captured by the smeared crack approach by depicting only the main cracks carrying no normal tensile stress. To study the failure mechanism and the evolution of punching shear failure of R. C. flat plate, Menétrey et al. (1997) developed a numerical model. A four-node quadrilateral axisymmetric element was used to generate the triaxial stress state. The incremental flow theory of plasticity as well as a fictitious crack model developed by Hillerborg et al. (1976) was employed to model the concrete material property. The localized punching shear failure, its initiation, and propagation were successfully generated. The size effect observed experimentally was also reproduced numerically by the proposed model. Parametric analyses were carried out to interpret the failure mechanisms of punching shear.

Studies by Gonzalez-Vidoso et al. (1988) and Hallgren (1996) also employed the axisymmetric element. It was shown that the axisymmetric element in conjunction with an appropriate triaxial material model for reinforced concrete is suitable for modeling the localized response of slab-column connection with axisymmetrical arrangements. However the element can not be used for important practical applications like orthogonal reinforcement or unsymmetrical punching.

In the study by Ozbolt et al. (1999), an eight-node solid finite element as well as the complicated microplane material model of concrete was used. The model was able to realistically predict the load bearing behavior, the deformations and the failure mechanism of slab-column connection. Even the shape of the punching cone as observed in the experiment was successfully predicted. Three dimensional elements were also used in studies by Dyngeland et al. (1994), Marzouk and Jiang (1996), Megally and Ghali (2000), Xiao and O'Flaherty (2000). The 3-D-element offers high flexibility and accuracy in modeling the localized punching shear behavior of slab-column connections of flat plate structures and generally leads to the most realistic results. Nevertheless, the analysis of structural performance of relatively large RC flat plate structures using 3-D elements becomes inefficient.

A shell element is probably a better choice to model RC flat plates when the structural responses of the structure is of more interest than localized responses of the slab-column connection. A layered shell element with a 3-D material model has gained essential success in modeling the flexural and punching shear behavior of flat plate structures.

Hand, Pecknold, and Schnobrich (1973) proposed a layering scheme for a shallow shell element to study the flexural behavior of slabs. The scheme allowed a discretized variation of material properties through the thickness as loading progresses yet retains the limited degrees of freedom. Different layers may have different material properties. Within the thickness of each layer, constant material properties were assumed. In studies by Polak and Vecchio (1993) and Polak (1998), a 3-D constitutive model derived from the modified compression field theory was incorporated into the shell element formulations by a special condensation scheme

for the material matrix. The smeared rotating orthogonal crack approach was used. Formulation was developed to obtain the transverse strain for incorporating the transverse shear reinforcement. The model was able to predict the capacity of the slabs and the type of failure. In the study by Loo and Guan (1997), a 3-D elastoplastic model based on smeared rotating crack approach was implemented into shell element with special layering scheme to describe the stiffness contribution of spandrel beams. Flat plates with two panels and spandrel beams were analyzed. Good predictions were obtained for punching shear strengths, collapse loads, and deformational responses. A shell element was also used by Marzouk and Chen (1993) to study the punching shear behavior of high-strength concrete slabs.

Finite element models employing shell elements along with the implementation of 3-D material models have shown good capability in modeling the structural performance of RC flat plate structures. However, such models have limitations in modeling localized behavior of slab-column connections. This is partly because the transverse shear stresses and shear strains within the cross section are not properly computed. A constant shear strain distribution across the depth of the section will be obtained because variation in the transverse displacement and cross-sectional rotation are not permitted through the thickness of the element.

Various procedures to improve the representation of transverse shear deformation along the cross section of laminated plates and shells had been developed. Figueras and Owen (1983) suggested some improvements by the implicit layering approach. They proposed the use of energy-based shear correction factors in the transverse shear stiffness and shear computations, which resulted in an improved representation of the shear stresses through the cross section. The capability of the implicit layering approach in modeling transverse shear deformation in reinforced concrete structures has been evaluated by Ramaswamy et al. (1995). A reinforced concrete beam under concentrated load and a slab under pure torsional moment were analyzed. The implicit layering approach was found to be suitable for cases where overall structural performance is concerned.

Mawnya and Davies (1974) presented a general quadratic multilayer plate element for the analysis of composite laminates. The formulation incorporated the effect of transverse shear deformation in each layer where two independent rotational degrees of freedom were added. Compatibility conditions between adjacent layers are so prescribed that no relative motion is allowed in the interface. In the plate formulations, the in-plane-membrane extension and the out-of-plane bending were dealt with separately. No coupling effects were considered. Each node of the plate element has the same DOFs. Based on the original layering concept proposed by Mawnya and Davies (1974), an explicit layering procedure was developed by Barzegar (1988) to incorporate the transverse shear deformation in reinforced concrete structures. The RC cross section is modeled using a stack of finite elements. Compatibility conditions are prescribed at the interface of two adjacent elements. The in-plane translations of slaved elements are condensed out by use of the constraint equations. The explicit layering has been applied to model a slab with anisotropic reinforcements under bending moment (Barzegar 1988) and RC beams under shear loads (Barzegar 1988; Ramaswamy et al. 1995). The explicit layering approach has shown superiority over the implicit layering procedure in representing the transverse deformational response. The highly localized cracking/crushing, the redistribution of stresses within the loading zone can be captured.

The explicit layering procedures proposed by Mawnya and Davies (1974) and Barzegar (1988) along with proper 3-D material models may also be promising in modeling the localized punching shear behavior of RC slab-column connections. However, the introduction of independent rotational DOFs in each layer makes the application of these explicit procedures much more expensive than the implicit layering scheme. In RC flat plate structures, as we know, the concentration of stresses diminishes very rapidly. Significant transverse shear stress only exists in small regions of a slab-column connection. Efficient finite element analysis of RC flat plates requires a correction of transverse shear deformation only on regions surrounding the columns. The explicit layering procedures do not provide the flexibility for such applications and they are not realistic to be employed for nonlinear analysis of relatively large RC flat plate structures.

The nonlinear finite element global analysis for structural responses of RC flat plates is directly relevant for design practice. Predicted information such as crack pattern, deflection, ultimate strength and ductility obtained from a global analysis is essential for evaluating the performance of the designed project or the rationality of the design codes used. Literature review indicated that finite element models with the application of shell elements are appropriate for global analysis of RC flat plate structures. However, the capability of such a model in simulating the localized punching shear behavior of slab-column connection needs to be enhanced by a proper layering scheme along with an accurate 3-D material model for reinforced concrete.

1.2 OBJECTIVES, SCOPE, AND LIMITATIONS

1.2.1 Research Objective

The primary purpose of the present study is to develop a finite element model which is suitable for nonlinear global analysis of reinforced concrete flat plate structures and is also capable of reproducing localized punching shear behavior of slab-column connections. A degenerate isoparametric shell element developed by Ahmad et al. (1970) is used with some simplifications. A new elaborate 3-D material model for reinforced concrete and a new layering scheme for incorporation of transverse shear deformation have been developed to enhance the capability of the shell element in modeling the localized punching shear behavior of slab-column connections. Computational efficiency required by global analysis has been considered in the development of material models, element formulations, and layering schemes.

1.2.2 Research Scope

- ✓ Develop a 3-D hypoelastic material model for reinforced concrete suitable for the implementation into shell element formulations. The behavior of concrete under triaxial stress states, crushing and post-crushing is considered. The

material properties of the multi-directional cracked reinforced concrete (MR-CRC) are described. The local interface cracking effects of tension softening and aggregate interlock in the crack interface are incorporated. The tension stiffening effect of cracked reinforced concrete (CRC) with multiple non-orthogonal cracks and reinforced in multiple non-orthogonal directions is modeled. The compression softening effect of CRC due to multiple non-orthogonal cracks is accounted for explicitly. Compression softening effect due to principal tensile strain, principal tensile stress normal to crack plane as well as shear stress in crack interface is considered.

- ✓ Develop a new layering scheme, flexible to be applied for incorporation of transverse shear deformation in local regions of slab-column connection. The layering scheme is formulated in shell element formulations.
- ✓ Develop a suitable nonlinear analysis procedure to simulate the nonlinear behavior of reinforced concrete structures due to material nonlinearities.
- ✓ Develop a comprehensive program based on the proposed material model, layered shell element formulations and the numerical analysis procedure. Necessary pre- and post- processing procedures are also developed to assist the preparation of input data and the interpretation of analysis results.
- ✓ Verify the developed constitutive material model on the element level by analyzing a range of deliberately selected specimens (panels under in-plane loading, beam under moderately shear, slab under torsional moment).
- ✓ Evaluate the capability of the proposed finite element model in modeling the localized punching shear behavior of slab-column connections by simulating biaxial orthogonally reinforced slabs under pure torsional moment, concentrated punching load, or unbalanced moment. Study the influence of transverse shear stress distribution on the predicted cracked pattern, stiffness, failure mode and ultimate strength.

- ✓ Evaluate the capability of the proposed finite element model in modeling the punching behavior of reinforced concrete flat slabs with drop panels.
- ✓ Evaluate the performance and the capability of the developed finite element model in predicting the structural responses of relatively large reinforced concrete flat plate structures by analyzing a multi-paneled flat plate with irregular layout and supported on 7 columns.
- ✓ Perform a parametric study using the developed finite element model to study the influences of a number of selected material parameters on the predicted behaviors.

1.2.3 Limitations

The present model is so far limited to a monotonic and proportional loading condition. Time dependent effects such as creep and shrinkage and thermal effects have not been included. Shear locking of the shell element is not investigated in the present study. The finite element model is limited to small displacement problems.

1.3 RESEARCH SIGNIFICANCE

In this study, a new finite element model is developed for nonlinear finite element analysis of RC flat plate structures. The numerical model is suitable for nonlinear global analysis of structural performance of flat plate structures and is also capable of simulating the localized punching shear behavior of slab-column connections. The developed finite element model is also applicable for modeling the nonlinear behavior of RC flat slabs with drop panels.

1.4 ORGANIZATION

The contents of the thesis are organized as follows.

- ✓ Chapter 2 presents a brief review of the different constitutive models for concrete employed in various analytical studies. The characteristics of some recently developed 3-D material models for concrete are discussed. The constitutive model for uncracked concrete and approaches for modeling cracking of concrete has been selected on this basis.
- ✓ In chapter 3, a 3-D hypo-elastic model developed in this study for uncracked concrete is presented. The model is based on slight modifications to the hypoelastic models proposed by Elwi and Murray (1979) and Balan et al. (1997). Elastic material properties are assumed for concrete in tension before cracking. The implementation of the material model for uncracked concrete under multiaxial stress state is then rectified by comparison of computed and experimental results.
- ✓ Chapter 4 presents the crack formulations developed in this thesis for simulating the material properties of CRC. Procedures explicitly incorporating the tension stiffening effect and compression softening effect due to multiple non-orthogonal cracks are described. A constitutive model for reinforcement is also provided in this chapter.
- ✓ The degenerated isoparametric serendipity shell element developed by Ahmad (1970) is simplified for a flat shape with constant thickness in Chapter 5. A layering scheme incorporating the transverse shear deformation is included in element formulations.
- ✓ In Chapter 6, a numerical nonlinear analysis procedure based on the arc-length method proposed by Fafard and Massicotte (1993) is described.
- ✓ The validity of the proposed constitutive material models and the capability of the developed finite element model have been examined by a series of numerical studies on a number of deliberately selected RC specimens in Chapter 7. Numerical results in comparison with experimental results are presented and discussed.

- ✓ Finally, the conclusions drawn from this study are presented in Chapter 8. Possible extensions of the present study in the future are also discussed.

CHAPTER 2 LITERATURE REVIEW OF MATERIAL MODELS

2.1 INTRODUCTION

The nonlinear finite element modeling of reinforced concrete structures involves the treatment of various nonlinearities exhibited by the two materials of reinforcing steels and concrete under various loading conditions. The nonlinearities of concrete include concrete response under compression up to crushing, post-crushing strain softening behavior, cracking of concrete, tension softening effect, aggregate interlock and dowel action in the crack interface, tension stiffening and compression softening effect for concrete between cracks. The nonlinearities of reinforcement include steel yielding and post yielding hardening responses. In this chapter, a brief overview of some of the developed analytical models for simulating different response behavior of concrete and reinforcement to various loads is reviewed. These include the various constitutive models employed for simulating steel and uncracked concrete responses, approaches simulating the occurrence of cracking and the treatment of cracks as well as the post cracking nonlinearities considered in different studies.

2.2 REINFORCEMENT MODELING

To model reinforcement in a finite element model, at least three alternative approaches have been developed: smeared, embedded, and discrete. In a smeared model, the reinforcement is assumed to be distributed over the concrete element with a particular orientation. Perfect bond is assumed between concrete and steel. In an embedded representation, the reinforcing bar is considered as an axial member built into the element such that its displacements are consistent with those of the element. In the discrete approach, the reinforcing bar is modeled by a truss or beam element. A significant advantage of the discrete approach is the capability of the

model to account for the possible relative displacement of the reinforcement with respect to the surrounding concrete. The application of the discrete model has to ensure that nodes in proper positions are available to incorporate the steel bars.

In general, a uniaxial stress-strain relationship is used for steel up to yielding. The post-yield behavior is idealized as either perfectly plastic or hardening plastic. For higher accuracy, a complete stress-strain curve may be used. An identical property is assumed in tension and compression. In cyclic loading conditions, the Bauschinger effect has been included through the use of kinematic hardening. Belarbi and Hsu (1994) showed that the yield point of the steel bars in cracked concrete is lower than that of bare bars. A simple constitutive model is proposed by the author to describe this behavior.

2.3 FAILURE CRITERIA FOR CONCRETE

The strength of concrete under multiaxial stresses is a function of the state of stress. Many failure criteria have been proposed in the past. The extensively used models are the four-parameter models of Ottosen (1977) and Hsieh et al. (1979) and the five-parameter model of Willam-Warnke (1975). Chen (1982) summarized that all these refined models gave a close estimate of relevant experimental data, contained all the stress invariants, reflected all the required characteristics concerning smoothness, convexity, symmetry, curved meridians, etc, and include most of the one- and two-parameter models as special cases. For example, the Willam-Warnke five-parameter model was used by Barzegar and Maddipudi (1997), Elwi and Murray (1979), and Balan et al. (1997). The four-parameter models of Hsieh et al. (1979), and Ottosen (1977) were employed in the study by Selby (1993) and Ramaswamy (1992) respectively. Excellent early surveys of such models were given in ASCE (1982, 1992).

2.4 UNCRACKED CONCRETE

2.4.1 Elastic Models

The linear elastic-fracture model was the first material model applied in finite element method for nonlinear analysis of reinforced concrete structures. A linear elastic isotropic material was assumed for concrete in this model. Cracking was described as a geometrical discontinuity, and bond was modeled by a link element between reinforcement and concrete. Despite its obvious shortcomings for modeling the nonlinearity of concrete due to compression, the model shown useful in many practical applications (Ngo and Scordelis 1967; Nilson, 1968; Phillips and Zienkiewicz 1976). Quite accurate results are generally obtained for reinforced concrete structures where the cracking of concrete dominates the nonlinear response of the structure.

2.4.2 Nonlinear Elastic Models

Nonlinear elastic models were developed from elastic models intended to improve the modeling of nonlinearity exhibited by concrete under multiaxial compressive stresses. The models are generally grouped as hyperelastic or hypoelastic models. The advantages of nonlinear elastic models are their simple form and their basis on curve fitting to a large number of experimental data. Some classical nonlinear elastic models that have been extensively used are briefly reviewed below.

2.4.2.1 Hyperelastic Models

Hyperelastic models are in the form of secant (total) formulations. The behavior of these models is both irreversible and path independent, and thus their applications are restricted primarily to monotonic or proportional loading regimes. In general, most of the isotropic secant constitutive models have been formulated basically as a simple extension of the isotropic linear elastic stress-strain relations. The two constant elastic moduli in linear models are replaced by secant moduli.

Cedolin et al. (1977) developed an isotropic secant constitutive model for concrete under compressive triaxial stress conditions. Secant bulk and shear moduli are defined as functions of octahedral normal and shear strains respectively. The closed form expressions were obtained by analyzing a large number of various triaxial experimental results. The coupling effect between deviatoric and volumetric components of response was not considered. The model can only provide a realistic simulation of response up to about seventy percent of the ultimate strength.

Experimental results showed that volumetric and deviatoric responses are coupled at high stress levels. To model this coupling effect, Kotsovos and Newman (1978) proposed a hyperelastic model in terms of bulk and shear moduli which were taken as functions of the octahedral normal and shear stress respectively. The function forms were determined by curve fitting of experimental results. The coupling effect was considered by including the volumetric strain caused by deviatoric stress.

Ottosen (1979) developed a nonlinear elastic isotropic model which is capable of coping with the dilation response of concrete under compressive stresses close to failure and the softening behavior in the post failure regime. Constitutive relations were in terms of modulus of elasticity and Poisson's ratio. These moduli are derived according to the concept of nonlinearity index, which is defined as a measure for actual loading in relation to the failure surface.

Bazant and Tsubaki (1980) developed a complex total stress-total strain constitutive model with the purpose of extending the total strain formulations to nonproportional loading and providing the path dependent behavior. The nonlinear triaxial behavior of plain concrete is described in the form of an algebraic relation between total strains and stresses. Certain path-dependent tensorial terms were added and thus the principal directions of stress and strain cease to coincide for nonproportional loading. The model has reproduced various characteristics such as dilatancy, compaction, and softening behavior. However, explicitness appears not possible in the formulations and the formulations must be differentiated before being applied in structural analysis.

2.4.2.2 Hypoelastic Models

Models of the hypoelastic type are formulated in terms of incremental stress and incremental strain. The state of stress depends in general on the current state of strain as well as on the stress path followed to reach the current state. Hypoelastic models are thus path dependent.

The hypoelastic model proposed by Liu et al. (1972) was an incremental orthotropic model. The model was formulated directly based on the isotropic linear elastic model modified by replacing the Young's modulus with the variable tangential modulus. The constitutive relations were expressed in form of principal stresses to principal strains on the principal stress axes. The ratios of the principal stress components were included in the constitutive relations and principal stress and principal strain axes were assumed to coincide.

Darwin and Pecknold (1976) proposed a constitutive model in the form of tangent formulations. In the model, concrete was assumed to have orthotropic material properties which were formulated on the principal stress axes. Principal stress axes and principal strain axes were assumed to coincide. An equivalent uniaxial strain concept was used to eliminate Poisson's ratio effect. Constitutive relations for a biaxial stress state were thus simplified and represented by stress-equivalent uniaxial strain relations. The equivalent uniaxial strain increments were accumulated in the principal stress directions. No rotation of principal axes was considered for this accumulation of equivalent uniaxial strain. The strength in each equivalent uniaxial stress-strain relationship was obtained from the biaxial strength envelope according to the current principal stress ratio. Elwi and Murray (1979) developed a nonlinear elastic three-dimensional (axisymmetric) model for concrete based on the same equivalent uniaxial strain concept. Nonlinear orthotropic material property was assumed in the model and defined in orthogonal principal stress directions. Coincidence of principal stress and strain axes was required. The same form of formulation was suggested for description of both equivalent uniaxial tension and compression constitutive relations. A five-parameter failure surface was

employed to determine the triaxial ultimate strengths and peak strains used in equivalent uniaxial stress-strain relationships.

Gerstle (1981) presented a nonlinear isotropic model using incremental octahedral stress-strain relationships. Constitutive relations were in form of tangent bulk and shear moduli which were assumed to decrease linearly from the initial values. No coupling between normal and shear response was considered and the model was limited to proportional loading. Stankowski and Gerstle (1985) developed a refined model of this type. Full coupling between hydrostatic and deviatoric effects were incorporated. Two coupling moduli were suggested to describe the volumetric variation due to deviator stress and the distortion due to hydrostatic stress. The model has features which are analogous to strain-hardening plasticity theory. The model appeared capable of predicting strains in concrete under arbitrary principal stress histories.

2.4.3 Plastic Models

Plastic models are characterized by the consideration of plastic deformation under a certain stress state. Plastic models can be classified into elastic perfectly plastic models and work hardening models (Chen and Chen 1975). The difference between the two models is that the first one has a constant yield surface while work hardening implies a changing (strengthening) yield surface. It is justified that material properties are uniquely determined by a stress state which requires yield the surface to be convex and the plastic strain flow normal to the plastic-potential functional surface (Non-associated flow rule) or normal to the yield surface (Associated flow rule). This normality property can be directly used to derive the constitutive relationships between incremental plastic strains and the current incremental stresses. In application, the total strain is divided into a solid elastic strain, a crack strain, and a plastic strain. Excessive dilatancy can be reduced by introducing a cap for the yield surface. Plastic models guarantee a stable and unique description of the material law when normality and convexity requirements are satisfied. Loading and unloading behavior can be simulated by these models.

Difficulties exist in these models to account for stiffness degradation and to simulate the post-peak strain softening behavior.

The plastic-fracture theory (Bažant and Kim 1979; Han and Chen 1986) combines plastic stress decrements with fracturing stress decrements, which reflect microcracking, and accounts for internal friction, pressure sensitivity, inelastic dilatancy, strain softening, and degradation of elastic moduli due to micro cracking.

Imran and Pantazopoulou (2001) proposed a plasticity model for concrete under triaxial compression. The model was reported to be capable of modeling the degradation of elastic moduli and the post-peak strain softening response. Volumetric strain was taken as a measure of mechanically induced damage. Failure was assumed to be a continuous process driven by the history rather than a marked distinct one. All yielding envelopes have the functional form, but the shape is expanding (prepeak response) or contracting (postpeak response) depending on the state of damage. A nonassociated flow rule is used to describe the directions of the plastic deformation.

2.4.4 Endochronic Models

Bažant and Bhat (1976) extended endochronic theory for metals to model concrete material. The model is characterized by the central concept of intrinsic time whose increment is a function of strain increments. Different from plasticity theory, the endochronic model is free from the yielding function and the normality rule. The model can be used to reproduce a wide range of concrete responses such as unloading, strain softening, volume dilation, hydrostatic pressure sensitivity. It was pointed out by Sandler (1978) that endochronic models without internal barriers are not suitable for the numerical solution of mechanical problems.

2.4.5 Damage Mechanics Models

Damage mechanicsl models (Benallal et al. 1986; Simo and Ju 1987; Lubliner et al. 1989; Oller et al. 1990; Lemaitre 1991; Krajcinovic and Vujosevic 1998; Scotta at

al. 2001) developed for concrete seem more comprehensive than all the previous models reviewed. The models were able to reproduce the nonlinear behavior of concrete under both tension and compression states. Elastic and plastic stiffness degradation effects due to internal damage can be incorporated. The simplest damage model is the isotropic damage model (Oliver et al. 1990), which has the form

$$\sigma = (1 - d) D_o \varepsilon \quad (1-1)$$

where D_o is the undamaged constitutive tensor and d is a scalar measure of damage that varies according to the prevailing strain conditions. Material damage that occurs in some anisotropic fashion can be represented by a damage tensor (Karihaloo and Fu 1990).

2.4.6 Micro-Plane Models

The micro-plane model is a microscopic non tensorial model (Bažant and Gambarova 1984; Bažant and Prat 1988; Bažant and Ožbolt 1990). The model has conceptual simplicity and was intended to overcome the difficulty and complexity of the macroscopic tensorial models in simulating the fracture, damage, and triaxial response of concrete. In the model, material properties are characterized separately on planes of various orientations within the material. On these microplanes there are only a few uniaxial stress and strain components in normal and shear directions. The macroscopic stiffness and the stress tensor are calculated for the actual strains on the microplanes by integrating the stress components on the microplanes over all directions. The integration is carried out numerically based on the method of virtual work. It has been shown that the model is able to generate most of the nonlinearity of concrete such as triaxial behavior, uni- or multi- directional cracking, and tensile fracture. Despite simplicity of the constitutive relations, the model still remains time consuming for practical structural analysis.

2.5 CRACKED CONCRETE

Cracking is the major contributor to the nonlinear behavior of reinforced concrete structures. The tensile cracking appears in different forms in plain concrete and reinforced concrete. In plain concrete, relatively few cracks are formed in comparison with their densely distributed appearance in reinforced concrete. Different views on this geometry discontinuity lead to different methods in describing this phenomenon.

2.5.1 Modeling Cracking in Concrete

2.5.1.1 Discrete Crack Models

The discrete crack model was first introduced by Ngo and Scordelis (1967), and later employed in the finite element analysis by Nilson (1968). In the model, a crack was thought of a geometrical discontinuity, which divided the initial continuous concrete into sub-solids which was assumed to be elastic. Cracking was modeled by progressively separating the element edges. The generally used discrete crack model is the fictitious crack model by Hillerborg et al. (1976). The model is based on fracture mechanics and defined a tension softening fracture process zone through a fictitious crack ahead of the pre-existing cracks whose faces are acted upon by certain closing stresses such that there is no stress concentration at the tip of the extended crack. The fictitious crack model assumes that the fracture process zone is of negligible thickness. Hence the cracks are modeled in a discrete fashion. When the normal tensile stress attains the level of tensile strength, the corresponding inter-element boundary nodes are released and the element is subjected to the assumed tension softening stress-displacement relation. A special linkage element may also be added between separated nodes to consider interface effects of cracking. The discrete crack model is appropriate for problems involving a few dominant cracks, such as the diagonal-tension cracks in reinforced concrete beams and punching shear cracks in reinforced concrete flat plates (Menétrey et al. 1997). However, the

requirements of the discrete crack models for complex re-meshing technology limit its application in practical structural analysis of large structures.

2.5.1.2 Smeared Crack Models

The smeared crack model developed by Rashid (1968) has been the most widely used crack approach. In the model, crack openings were assumed to be averaged or smeared within a certain domain. The cracked concrete was imagined to be a continuum whose properties were described in terms of stress-strain relations. The smeared crack model has shown its appropriateness in modeling CRC with evenly distributed cracks. However, the application of this smeared model in simulating fracture localization problems (problems in which behavior is dominated by a number of major cracks) runs into severe difficulty. Because of the strain softening nature of concrete the results of a smeared analysis may depend significantly on mesh size. A reduction of the element size to zero may cause the structure to fail with zero energy dissipation. Therefore, an appropriate localization limiter must be introduced which guarantees a localization of the damage in a mesh-independent area (volume). The simplest way to avoid this strain localization problem and the associated mesh sensitivity is to impose a lower limit on the element size, as introduced in the crack band model (Bažant and Oh 1983). A more general and fundamental way to overcome spurious mesh sensitivity is to adopt a nonlocal continuum approach (Pijaudier-Cabot and Bažant 1987; Bažant and Lin 1988; Bažant and Ožbolt 1990). The idea of a nonlocal continuum is to consider the stress to be a function of the mean of the strain from a certain representative volume of the material centered at that point. The fundamental parameter of this method is the so-called characteristic length over which the strains are averaged. Another available model is the nonlocal gradient approach developed by de Borst (1991) based on Cosserat theory. No characteristic material length factor is required in their model.

2.5.1.3 Various Smeared Crack Approaches

Before cracking, concrete can be seen as an isotropic material where the principal stress axes and principal strain axes coincide. When cracking criterion is violated, a

crack plane is formed usually perpendicular to the principal tensile stress direction. The subsequent loading may cause the principal axes to rotate from the initial crack axes to the current orientations. In addition, the principal stress axes may not coincide with principal strain axes. In this situation, significant difficulties appear in relation to how to properly describe the material properties of the current cracked concrete for the existence of three sets of axes—crack axes, principal stress axes, or principal strain axes. This anisotropy phenomenon leads to the development of three different crack approaches, the fixed-angle crack approach, the rotating-crack approach, and the multiple crack approach.

In the fixed-angle crack approach, it is assumed that the crack axes are fixed at their initial orientations and will not rotate with further loading. Material properties of cracked concrete are defined at the crack axes. Principal stresses can be built up and may exceed the crack stress. As new cracks are not allowed at the same point, numerical solutions with this approach can be far too stiff and can lead to collapse loads that are significantly too high. The anisotropy described earlier can not be avoided in this approach. However, the fixed-angle crack approach has the advantage of including the aggregate interlock effect and was thus applied in many material models for cracked concrete (Rashid 1968; ASCE 1982; Pang and Hsu 1996; Hsu and Zhang 1997; Zhang and Hsu 1998).

The rotating-angle crack approach is the simplest way of handling the anisotropy among the material axes. It is assumed in this approach that the crack axes rotate and align to the current principal strain axes with subsequent loading regardless of the previous crack direction (Cope et al. 1980; Milford and Schnobrich 1984; Gupta and Akber 1984; Crisfield and Wills 1987; Balakrishnan and Murray 1988; Balarbi and Hsu 1994; Pang and Hsu 1995). If proper shear modulus is used, the principal stress axes and the principal strain axes can be guaranteed to coincide. The material properties of cracked concrete are defined in the current principal strain axes. The stress-locking problem inherent with the fixed-angle crack approach is overcome by this approach. One feature of this approach is that no previously generated cracks

are memorized. This approach has been criticized by Bažant (1981, 1983) for being unphysical, and the formulation violates the tensorial invariance properties.

The multiple crack approach was proposed by de Borst and Nauta (1985) as an alternative to the rotating-angle crack approach to solve the stress locking problem of the fixed-angle crack approach. A secondary crack is allowed to occur after the change in principal stress directions has exceeded a certain threshold angle. The newly produced crack is not necessarily orthogonal to the previous cracks. The cracked concrete was considered to be composed of intact concrete and crack openings, idealized as springs in series (Rots and de Borst 1987; Rots 1988; Ramaswamy et al. 1995; Barzegar and Maddipudi 1997). The crack interface effects such as tension softening and aggregate interlock due to each crack were monitored by the corresponding crack strain. The basic idea of this approach is to remove the crack openings from the cracked concrete and leave the solid concrete to consider other nonlinearities such as plasticity.

2.5.2 Cracking Effects

Cracking in concrete initiates two kinds of effects: 1) local interface cracking effects occur within the crack interface including tension softening, aggregate interlock and dowel action, and 2) smeared cracking effects occurred in solid concrete between two adjacent cracks including tension stiffening and compression softening. For plain concrete, only the local interface cracking effects are necessary to consider while in reinforced concrete both kinds of effects have to be incorporated.

2.5.2.1 Local Interface Cracking Effects

Tension Softening Effect

Fig. 2.1 Plain Concrete under Tension

Concrete is a type of softening material which exhibits some softening effect when subjected to tension (Fig. 2.1). After cracking, a large deformation can occur at the crack interface where stress transfer is still possible. The softening takes place in a crack (or crack band) and is therefore a discrete phenomenon (Reinhardt *et al.* 1986). To model this localized tension softening effect using the smeared crack concept, the energy based criteria developed from fracture mechanics have been advanced in an attempt to remove the unobjectivity problem caused by the strength criteria (Hillerborg *et al.* 1976; Bažant and Cedolin 1979). The smeared crack models with strain decomposition are appropriate for incorporating such energy criteria. As the crack strain is equivalent to the local discrete crack opening, it is employed as a monitoring parameter screening the tension softening effect in the crack opening (de Borst and Nauta 1985; Rots and de Borst 1987; Rots 1988; Ramaswamy *et al.* 1994; Barzegar and Maddipudi 1997). The softening effect is simulated by the descending branch of the tensile stress-strain curve. The controlled parameters of these tension softening models are the tensile strength, energy consumed to form a unit area of a crack, the shape of the tensile softening diagram, and the crack band width (representative length). Proper procedures to define the representative length are necessary to incorporate these tensile softening models in a nonlinear finite element analysis algorithm (Bažant and Oh 1983; Rots 1988). Even more sophisticated procedures may have to be employed when dealing with inclined cracks in 3-D elements (Oliver 1989; Barzegar and Maddipudi 1997; Hauke and Maekawa 1998).

Aggregate Interlock Effect and Dowel Action

The transmission of shear stress at the crack interface involves mainly the two mechanisms of aggregate interlock and dowel action. The aggregate interlock effect of cracked plain concrete has been modeled by a constant shear retention stiffness obtained as a small fraction of the shear stiffness of uncracked concrete (ASCE 1982; Milford and Schnobrich 1984; Barzegar 1989; Hu and Schnobrich 1990). Some studies have considered varying the shear stiffness of cracked concrete as a function of the crack normal tensile strain or crack width (Cervenka 1985; Barzegar 1989; Balakrishnan and Murray 1988; Bathe et al. 1989). Other studies defined the shear modulus of cracked concrete by elastic normal or shear modulus, or the stresses and strains state in directions normal and parallel to the crack plane (Darwin and Pecknold 1976; Vecchio 1989; Pang and Hsu 1996; Elmorsi, Kianoush, and Tso 1998; Crisfield and Wills 1989). The confining effects of the normal compressive stress on the aggregate interlock effect were experimentally investigated by some researchers (Walraven 1981; Millard and Johnson 1985; Tassios and Vintzeleou 1987) and incorporated in some investigations (Bažant and Gambarova 1980; Vecchio and Collins 1986; Ramaswamy and Barzegar 1995). The quantification of dowel action requires treating cracks in a local discrete manner involving the local information such as crack spacing, crack width, bar diameter, bar spacing. It is hence extremely difficult to include dowel action in finite element methods, especially in 3-D cases. The very few developed models for dowel action are only applicable for a limited scope of problems (Bhide 1986; Pang and Hsu 1996).

Removing-Concept Based Smeared Crack Models

The modeling of local interface cracking effects of tension softening and aggregate interlock in the smeared crack approach usually employs a removing concept. The basic idea is to remove the discontinuity of cracks from the cracked concrete and keep the remaining solid to be continuous. The idea is realized by decomposing the total strain into a crack strain and a solid strain (de Borst and Nauta 1985; Rots

1988). The cracking effects are modeled by a crack material matrix and are monitored by the crack strain which is equivalent to the crack opening.

2.5.2.2 Smeared Cracking Effects in Reinforced Concrete

Tension Stiffening Effect

Fig. 2.2 Reinforced Concrete under Tension

The behavior of reinforced concrete under tension is quite different from that of plain concrete. In reinforced concrete, after cracking, the crack interface experiences the same tension softening as in plain concrete, while the reinforcement imparts some loads to the concrete between cracks via bond actions. Subsequent loading will result in the formation of more cracks between the previous adjacent cracks. It is obvious that both the tension softening effect and the bond action of concrete between adjacent cracks contribute to the total tensile stresses in the reinforced concrete tension member. This mechanism of CRC under tension is commonly termed as tension stiffening effect which plays a significant role in reducing the post-cracking deformations of reinforced concrete structures. A variety of models have been developed to account for this phenomenon. Tension stiffening can be calculated as a combination of tension softening and bond action with clear contributions defined separately (Balakrishnan and Murray 1988; Mohammadi 2002). Rots (1988) observed that the more distributed cracks in reinforced concrete consume more energy than the rare cracks in plain concrete and hence he treated the stiffness contribution as part of concrete and artificially increased the fracture energy of plain concrete. Ramaswamy and Barzegar (1995) accounted for the bond contribution as a part of concrete stiffness in reinforcement directions by a proposed tension stiffening model. In other models (Rots 1988; Barzegar and Maddipudi

1997), the contributions of the tension softening and the bond action were treated separately. Explicit bond action models were employed to describe the interaction between the concrete and the reinforcements. In models by scientists with structural engineering background, the two contributions were treated by a much simpler average concept and the mechanism was accounted for by a descending average stress versus average strain curve of concrete calibrated by experimental results (Vecchio and Collins 1986; Belarbi and Hsu 1994). No separation of tension softening and bond action is needed. The average stress-average strain models for tension stiffening effect are conceptually simple and are reliable and stable to be applied in finite element models as they were formulated directly from a large number of experimental data of large size reinforced concrete panels. It should be noted that the experimental investigations and the models only deal with the situations where reinforcement is normal to cracks.

Compression Softening Effect

CRC under compression in crack direction exhibits softening behavior in comparison with companion cylinder. This softening phenomenon has received much attention since it was first discovered in the web crushing beam tests by Robinson (1961). A number of experimental programs have been carried out to systematically investigate the parameters influencing this effect (Vecchio and Collins 1982; Shirai and Noguchi 1989; Belarbi and Hsu 1995). It was found that the principal tensile strain normal to the crack is the primary variable. Vecchio and Collins (1986), based on the test results of 30 panels, presented a compression softening model. The softening effect on the compressive peak stress is considered by a function of principal tensile strain normal to the crack. Belarbi and Hsu (1995) showed that the compressive peak stress was softened more in a proportional load path than in a sequential load path. In addition the compressive peak strain was also softened in the proportional load path. This may be explained by the existence of tensile stress which initiates its own softening effect on transverse compression in a proportional load path. It should be clear that all the available data and softening models are about uniaxially cracked concrete subjected to compression in the crack direction. The compression softening is monitored by the principal tensile strain

normal to the crack plane. Many studies for modeling the behavior of reinforced concrete have included the compression softening effect (Cervenka 1985; Polak 1993; Selby 1993; Ramaswamy and Barzegar 1995).

Averaging-concept based smeared crack models

The smeared cracking effects of tension stiffening and compression softening are the most crucial features of CRC and should be incorporated effectively and rationally in the material models. The smeared crack models based on averaging-concept are particularly suitable for modeling such smeared behavior. In these models, the crack opening discontinuity is imagined to be smeared or, to say, averaged in a certain domain rather than to be removed from the cracked concrete as in the removing-concept based smeared models. The smeared cracking effects are conveniently monitored by the principal tensile strain normal to the crack plane. Representative models within this category include the modified compression field theory (Vecchio and Collins 1986) and the softening truss models (Belarbi and Hsu 1994, 1995; Pang and Hsu 1995; Pang and Hsu 1996; Hsu and Zhang 1997; Zhang and Hsu 1998). Despite its simple and effective feature in modeling the smeared cracking property of uniaxially CRC, the averaging-concept based smeared crack model, when employed to model the CRC with multiple-nonorthogonal cracks, becomes extremely difficult.

2.6 SUMMARIZATION OF RECENTLY DEVELOPED 3-D MODELS FOR CRACKED REINFORCED CONCRETE

In the last fifteen years, several three-dimensional material models have been developed for MR-CRC.

Menétrey et al. (1997) developed a 3-D material model for cracked concrete. The fictitious crack model (Hillerborg et al. 1976) was used in a smeared manner to model concrete cracking. Total strain was decomposed into crack strain, elastic and

plastic strain. A non-associated flow plastic model was used to model the solid concrete. No tension stiffening and compression softening effect were included.

In the multiple non-orthogonal smeared crack model by de Borst and Nauta (1985), a removing-concept was used. The basic idea is to remove the discontinuity of cracks from the cracked concrete and keep the remaining solid to be continuous. The removing idea was realized by decomposing the total strain into crack strain of each cracks and solid strain. The crack strain was used to screen the local cracking effects of tension softening and aggregate interlock which were removed from the cracked concrete. The left solid concrete was modeled by a plastic model. Tension stiffening effect and compression softening effect were not incorporated in this model. The salient feature of their model is its capability of handling multiple non-orthogonal cracks. Tension stiffening and compression softening in CRC were not considered.

Employing the multiple nonorthogonal smeared crack approach, de Borst and Nauta (1985) and Ramaswamy et al. (1994) developed a 3-D hyper-elastic model for CRC respectively. The local cracking effects of plain concrete were removed from cracked concrete. The left solid concrete was modeled by a hyper-elastic model proposed by Ottosen (1979). Tension stiffening effect was considered as a part of concrete in reinforcement direction. Equilibrium conditions need to check in the crack planes and reinforcement was restricted to be in orthogonal directions. In addition, the effects of the multiple cracks on tension stiffening were not considered. Compression softening effect due to multiple cracks was roughly monitored by the principal tensile strain rather than by the corresponding tensile strain normal to each crack plane.

Barzegar and Maddipudi (1997) proposed a 3-D hypoelastic model for MR-CRC. In the model, the local cracking effects of plain concrete were removed by the same procedure proposed by de Borst and Nauta (1985). The solid concrete was modeled by the hypo-elastic model developed by Stankowski and Gerstle (1985). A bond-slip model was incorporated to model the interaction between the embedded reinforcement and the concrete. Tension stiffening effect was thus indirectly

modeled. However, the effect of multiple non-orthogonal cracks on the bond-slip behavior was not incorporated. No compression softening effect was considered in this model.

A three-dimensional constitutive model for analysis of reinforced concrete solids was presented by Selby and Vecchio (1997) by extrapolating the plane stress constitutive relationships of the modified compression field theory to three dimensions. The cracked concrete is modeled as an orthotropic nonlinear elastic material by the averaging-concept based smeared rotating orthogonal crack approach. Constitutive formulations were in secant form. Tension stiffening and compression softening effects were easily incorporated in this model. However, the compression softening effect was only accounted for by the principal tensile strain and no effect is considered when the second principal tensile strain is larger than the cracking strain.

Another recently developed 3-D material model for plain concrete was the hypo-plastic orthotropic model proposed by Balan et al. (1997). The smeared model was based on averaging-concept. No clear separation of crack strain, elastic strain and plastic strain was performed. The triaxial constitutive relationships were accounted for with the generalized uniaxial stress-equivalent uniaxial strain relations (Darwin and Pecknold 1977; Elwi and Murray 1979). The model employed the smeared rotating orthogonal crack approach. The equivalent uniaxial strain was taken as the monitoring parameter calculating the cracking effects. As the model was developed to describe the behavior of plain concrete, the tension stiffening and compression softening effects were not included.

All the 3-D removing-concept based multiple-nonorthogonal smeared crack models have the merits of handling the multiple nonorthogonal cracks which generally appear in reinforced concrete structures. The local cracking effects of tension softening and aggregate interlock were easily removed from cracked concrete. However, the smeared cracking effects of tension stiffening and compression softening due to the multiple non-orthogonal cracks, which are the essential feature of CRC, were not incorporated sufficiently or explicitly. All the 3-D averaging-

concept based smeared crack models have less difficulty in modeling smeared cracking effects of reinforced concrete. The obvious demerit of these models is that cracks formed at a certain point have to be restricted in orthogonal directions.

In this study, it is intended to develop explicit procedures to incorporate the tension stiffening and compression softening effects of CRC due to multiple non-orthogonal cracks.

2.7 SUMMARY

In reinforced concrete flat plate structures, steel bars usually are evenly distributed. Bond failure rarely occurs under this kind of arrangement. Perfect bond between reinforcement and concrete can be assumed. In this study, the smeared layer approach is adopted for modeling the reinforcements. An elastic-hardening plastic model is employed as the constitutive model of steel.

Among the models reviewed above, the incremental orthotropic models based on the equivalent uniaxial strain concept are conceptually the simplest models which represent concrete behavior under proportional loading quite well for both biaxial and triaxial loading conditions. Less difficulty exists among these hypoelastic models when the cyclic behavior of concrete is to be modeled. From both computational and practical viewpoints, these models offer an attractive approach for the nonlinear analysis of reinforced concrete structures, especially planar type structures such as beams, panels, slabs and shells. These models have been widely used in the finite element analysis of practical concrete structures, and the predicted structural responses compared reasonably well with experimental results for many cases investigated. In addition, the incremental orthotropic models seem to be a natural choice to model the cracked concrete for which an orthotropic material is commonly assumed. The formulations of these models are very convenient in modeling both the local cracking effects of plain concrete and the smeared cracking effects of reinforced concrete. Considering all these advantages inherent with the incremental orthotropic models based on the equivalent uniaxial strain concept,

these models were selected as the basis for a proposed material model to simulate the nonlinearity of reinforced concrete.

A 3-D hypoelastic model for uncracked concrete is developed in this study directly from the incremental orthotropic models based on the equivalent uniaxial strain concept. Elastic material behavior is assumed for concrete in tension before cracking. The five-parameter surface proposed by Willam and Warnke (1975) is employed to determine the ultimate strength of concrete under multiaxial stress states.

In finite element modeling of reinforced concrete flat plate structures, the treatment of cracked concrete has to involve the handling of the distributed flexural cracks in radial and tangential directions, diagonal shear cracks and the discrete punching shear crack. Since the smeared crack approach has an advantage in modeling distributed cracks and has the capability of representing the localization of cracks with a reasonable level of accuracy, it is thus chosen to accomplish this job. The material behavior of the cracked concrete is going to be modeled by a multiple non-orthogonal crack approach considering the characteristics of crack patterns in flat plate structures.

In this study, considerable efforts are directed to model the material properties of CRC to supplement the capability of the original incremental orthotropic models (Elwi and Murray 1979; Balan et al. 1997) in this aspect. The strain localization problem is considered by employing an energy based tension softening model. The anisotropy of tension stiffening, in cases where reinforcements are not normal to crack plane, is considered. The tension stiffening effect due to multiple non-orthogonal cracks in CRC is incorporated. Compression softening due to principal tensile strain, principal tensile stress normal to the crack plane as well as shear stress in crack interface is considered. The compression softening due to multiple non-orthogonal cracks is accounted explicitly. Aggregate interlock is simulated by a simple formulation.

CHAPTER 3 CONSTITUTIVE MODELING

3.1 INTRODUCTION

This chapter presents the development of a 3-D incremental hypoelastic orthotropic material model for uncracked concrete. The proposed model is based on a modification to the axisymmetric model proposed by Elwi and Murray (1979) and the 3D cyclic concrete model proposed by Balan et al. (1997). The general multiaxial relations are based on the equivalent uniaxial strain concept by Darwin and Pecknold (1977). Elastic material property is assumed for concrete in tension before cracking. The five-parameter surface proposed by Willam and Warnke (1975) is employed to determine the ultimate strength of concrete under multiaxial stress state.

3.2 TRIAXIAL STRESS-STRAIN RELATIONS

It is well established that concrete can be treated as an initially isotropic material that exhibits deformational anisotropy during loading (Darwin and Pecknold 1976; Elwi and Murray 1979; Chen 1982). Consequently, in structures where the stress state at every point is defined by three principal stresses, concrete can be characterized as a nonlinear orthotropic medium with directions of orthotropy being coincident with the principal stress directions (Bathe and Ramaswamy 1979; Elwi and Murray 1979). In this approach, the incremental constitutive relations of concrete in reference to the orthotropic axes (1, 2, and 3) are

$$\begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_3 \\ d\gamma_{12} \\ d\gamma_{23} \\ d\gamma_{31} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_2} & -\frac{\nu_{13}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{21}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{31}}{E_1} & -\frac{\nu_{32}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}} \end{bmatrix} \begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \\ d\tau_{12} \\ d\tau_{23} \\ d\tau_{31} \end{Bmatrix} \quad (3-1)$$

or in the following form

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \\ d\tau_{12} \\ d\tau_{23} \\ d\tau_{31} \end{Bmatrix} = \frac{1}{\Omega} \begin{bmatrix} E_1(1-\nu_{23}\nu_{32}) & E_1(\nu_{12}+\nu_{13}\nu_{32}) & E_1(\nu_{13}+\nu_{12}\nu_{23}) & 0 & 0 & 0 \\ E_2(\nu_{21}+\nu_{23}\nu_{31}) & E_2(1-\nu_{13}\nu_{31}) & E_2(\nu_{23}+\nu_{21}\nu_{13}) & 0 & 0 & 0 \\ E_3(\nu_{31}+\nu_{32}\nu_{21}) & E_3(\nu_{32}+\nu_{31}\nu_{12}) & E_3(1-\nu_{12}\nu_{21}) & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{12}\Omega & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{23}\Omega & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{31}\Omega \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_3 \\ d\gamma_{12} \\ d\gamma_{23} \\ d\gamma_{31} \end{Bmatrix} \quad (3-2)$$

where

$$\begin{aligned} d\sigma_i (i = 1, 2, 3) &= \text{Normal stress increments in } i \text{ direction} \\ d\tau_{ij} (i, j = 1, 2, 3) &= \text{Shear stress increments in plane } i-j \\ \nu_{ij} (i, j = 1, 2, 3) &= \text{Transverse strain ratio for strain in the } i\text{-direction caused by stress in the } j\text{-direction} \\ E_i (i = 1, 2, 3) &= \text{Tangent modulus of elasticity in } i \text{ direction of orthotropy} \\ d\varepsilon_i (i = 1, 2, 3) &= \text{Normal strain increments in } i \text{ direction} \\ d\gamma_{ij} (i, j = 1, 2, 3) &= \text{Shear strain increments in plane } i-j \\ G_{ij} (i, j = 1, 2, 3) &= \text{Shear modulus in plane } i-j \\ \Omega &= 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - \nu_{12}\nu_{23}\nu_{31} - \nu_{13}\nu_{32}\nu_{21} \end{aligned}$$

Equation (3-2) can be written in matrix form as

$$d\sigma = D_L d\varepsilon \quad (3-3)$$

where

$$\begin{aligned} \mathbf{d}\boldsymbol{\sigma} &= \text{Vector of stress increments} \\ \mathbf{d}\boldsymbol{\varepsilon} &= \text{Vector of strain increments} \\ \mathbf{D}_L &= \text{Incremental concrete material matrix relative to} \\ &\quad \text{the local orthotropic axes (1, 2, 3)} \end{aligned}$$

The three shear moduli are given as

$$G_{ij} = \frac{\bar{E}_i \bar{E}_j}{\bar{E}_i (1 + \nu_{ij}) + \bar{E}_j (1 + \nu_{ji})} \quad (3-4)$$

where $\bar{E}_i, \bar{E}_j$ are secant modulus of elasticity. Equation (3-4) can be derived from the secant form of equation (3-1) with a pure shear condition and an assumed isotropic shear property of concrete.

Since material matrix $\mathbf{D}_L$ in (3-3) is defined with reference to the directions of orthotropy, it must be transformed to the local element coordinate system. This requires the following transformation

$$\mathbf{D}_G = \mathbf{T}^T \mathbf{D}_L \mathbf{T} \quad (3-5)$$

Where matrix $\mathbf{T}$ takes the following form according

$$\mathbf{T} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1 m_1 & m_1 n_1 & n_1 l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\ l_3^2 & m_3^2 & n_3^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\ 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & (l_1 m_2 + l_2 m_1) & (m_1 n_2 + m_2 n_1) & (n_1 l_2 + n_2 l_1) \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & (l_2 m_3 + l_3 m_2) & (m_2 n_3 + m_3 n_2) & (n_2 l_3 + n_3 l_2) \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & (l_3 m_1 + l_1 m_3) & (m_3 n_1 + m_1 n_3) & (n_3 l_1 + n_1 l_3) \end{bmatrix} \quad (3-6)$$

where l_i, m_i, n_i are the direction cosines of the i th axis in the principal stress coordinate system with respect to the local element coordinate system (Table 3-1).

3.3 EQUIVALENT UNIAXIAL STRAIN

For the incremental constitutive relations in (3-1) it is necessary to describe now the determination of the nine incremental moduli. To this end the concept of equivalent

uniaxial strain, as proposed by Darwin and Pecknold (1976) is used in the material model. The method is briefly summarized as follows.

Let equation (3-2) be written as

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \\ d\tau_{12} \\ d\tau_{23} \\ d\tau_{31} \end{Bmatrix} = \begin{bmatrix} E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{23} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{31} \end{bmatrix} \begin{Bmatrix} d\varepsilon_{1u} \\ d\varepsilon_{2u} \\ d\varepsilon_{3u} \\ d\gamma_{12} \\ d\gamma_{23} \\ d\gamma_{31} \end{Bmatrix} \quad (3-7)$$

Comparing equation (3-2) with the above equation, the equivalent uniaxial strain can be expressed as

$$\begin{Bmatrix} d\varepsilon_{1u} \\ d\varepsilon_{2u} \\ d\varepsilon_{3u} \end{Bmatrix} = \frac{1}{\Omega} \begin{bmatrix} (1-\nu_{23}\nu_{32}) & (\nu_{12}+\nu_{13}\nu_{32}) & (\nu_{13}+\nu_{12}\nu_{23}) \\ (\nu_{21}+\nu_{23}\nu_{31}) & (1-\nu_{13}\nu_{31}) & (\nu_{23}+\nu_{21}\nu_{13}) \\ (\nu_{31}+\nu_{32}\nu_{21}) & (\nu_{32}+\nu_{31}\nu_{12}) & (1-\nu_{12}\nu_{21}) \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_3 \end{Bmatrix} \quad (3-8)$$

The variables $d\varepsilon_{iu}$ ($i = 1, 2, 3$) in equation (3-8) represent the equivalent uniaxial strain increments expressed in terms of the actual incremental strains $d\varepsilon_i$ ($i = 1, 2, 3$). The equivalent uniaxial strain increments can be determined from equation (3-7)

$$d\varepsilon_{iu} = \frac{d\sigma_i}{E_i} \quad (i = 1, 2, 3) \quad (3-9)$$

Equation (3-9) yields the total equivalent uniaxial strain by integration over the load path, i.e.

$$\varepsilon_{iu} = \int d\varepsilon_{iu} = \int \frac{d\sigma_i}{E_i} \quad (i = 1, 2, 3) \quad (3-10)$$

Equation (3-10) shows that the equivalent uniaxial strain increment represents the strain increment in the i -direction that the material would exhibit under a uniaxial stress increment with other stresses kept equal to zero. In reality, $d\varepsilon_{iu}$ depends on the 3D stress state so that ε_{iu} and $d\varepsilon_{iu}$ are not collinear with the corresponding stress along the load path. Both are, therefore, fictitious quantities, except for the uniaxial case, and are only significant as a measure for variation of material parameters.

Equation (3-8) is defined with respect to the material axes of orthotropy. Since these coincide with the current directions of principal stress, it follows that $d\epsilon_{iu}$ must be defined with respect to current axes of orthotropy. The last statement implies equivalence between the equivalent strain parameters of elastoplastic theory and the equivalent uniaxial strains in equation (3-8).

3.4 STRESS-EQUIVALENT UNIAXIAL STRAIN RELATIONSHIP FOR COMPRESSION

In the original models proposed by Elwi and Murray (1979) and by Balan et al. (1997), the response of plain concrete under tension was described by the same form as that for compression. The strain localization problem of smeared crack model was not considered. In the present study, an energy consumption based tension softening model is employed to avoid such problems. Before cracking, a linear elastic material is assumed for plain concrete under tension (Chapter 4, section 4.2.1).

For plain concrete under uniaxial compression, the constitutive relationship follows that of Darwin and Pecknold (1977) and assumes the total stresses to be functions of the current equivalent uniaxial strains. This assumption permits the evaluation of the incremental elastic moduli of equation (3-9) in the manner described in the following. For the purpose of this work, the uniaxial compressive stress-equivalent uniaxial compressive strain relationship suggested by Saenz (1964) is generalized and to describe the response of compression. Writing the Saenz relation in terms of equivalent strain yields (Elwi and Murray 1979)

$$\sigma_i = \frac{E_c \epsilon_{iu}}{1 + (R + R_E - 2) \frac{\epsilon_{iu}}{\epsilon_{ic}} - (2R - 1) \left(\frac{\epsilon_{iu}}{\epsilon_{ic}} \right)^2 + R \left(\frac{\epsilon_{iu}}{\epsilon_{ic}} \right)^3} \quad (3-11)$$

in which the following definitions apply

$$R_E = \frac{E_c}{E_0} \quad (3-12a)$$

$$E_0 = \frac{\sigma_{ic}}{\varepsilon_{ic}} \quad (3-12b)$$

$$R_\sigma = \frac{\sigma_{ic}}{\sigma_{if}} \quad (3-12c)$$

$$R_\varepsilon = \frac{\varepsilon_{if}}{\varepsilon_{ic}} \quad (3-12d)$$

$$R = R_E \frac{R_\sigma - 1}{(R_\varepsilon - 1)^2} - \frac{1}{R_\varepsilon} \quad (3-12e)$$

The type of curve described by equation (3-11) is shown in Fig. 3.1, in which variables appearing in the definitions of equations (3-11) and (3-12) are presented. More specifically,

- σ_{ic} = The maximum stress, associated with direction i , that occurs for the current particular stress ratio
- ε_{ic} = The equivalent uniaxial compressive strain corresponding to σ_{ic}
- E_c = The initial modulus of elasticity
- $\sigma_{if}, \varepsilon_{if}$ = The coordinates of some point on the descending branch of the stress-equivalent strain curve

Differentiating equation (3-11) with respect to ε_{iu} yields the tangent modulus E_t as follows

$$E_t = \frac{\partial \sigma_i}{\partial \varepsilon_{iu}} = E_c \frac{1 + (2R - 1) \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^2 - 2R \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^3}{\left[1 + (R + R_E - 2) \frac{\varepsilon_{iu}}{\varepsilon_{ic}} - (2R - 1) \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^2 + R \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^3 \right]^2} \quad (3-13)$$

Fig. 3.1 Compressive Stress-Equivalent Uniaxial Strain Relationship

3.5 TRANSVERSE STRAIN RATIOS

The following expression of Balan (1986) defines these ratios

$$\nu_{ij} = \sqrt{\nu_{iu} \nu_{ju} \frac{E_j}{E_i}} \quad (3-14)$$

ν_{iu} can be calculated by the equation proposed by Elwi and Murray (1979)

$$\nu_{iu} = \nu \left[1.0 + 1.3763 \frac{\varepsilon_{iu}}{\varepsilon_{ic}} - 5.3600 \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^2 + 8.586 \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^3 \right] \quad (3-15)$$

It follows that equation (3-14) satisfies the following conditions of symmetry

$$E_1 \nu_{12} = E_2 \nu_{21}; \quad E_1 \nu_{13} = E_3 \nu_{31}; \quad E_2 \nu_{23} = E_3 \nu_{32} \quad (3-16)$$

Variable Ω in equation (3-2) should be always positive. Thus, the limiting value of ν_{iu} in equation (3-15) is 0.5. This value corresponds to a limit of zero incremental volume change. As noted by Kotsovos and Newman (1977), the point at which this limit is reached corresponds to the onset of unstable microcrack propagation, which gives rise to the experimentally observed dilatancy phenomenon in the vicinity of the concrete material strength.

3.6 PRINCIPAL STRESSES AND PRINCIPAL DIRECTIONS

Principal Stresses

For a given stress state $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx})$, its principal stress state is determined by the following equation (Chen 1982)

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} \sigma_m \\ \sigma_m \\ \sigma_m \end{bmatrix} + \frac{2}{\sqrt{3}} \sqrt{J_2} \begin{bmatrix} \cos \theta_0 \\ \cos \left(\theta_0 - \frac{2}{3} \pi \right) \\ \cos \left(\theta_0 + \frac{2}{3} \pi \right) \end{bmatrix} \quad (3-17)$$

where

$$\theta_0 = \frac{1}{3} \cos^{-1} \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} \quad 0^\circ \leq \theta_0 \leq 60^\circ \quad (3-18)$$

and

$$\sigma_m = \frac{1}{3} I_1 = \frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) \quad (3-19)$$

$$J_2 = \frac{1}{6} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \quad (3-20)$$

$$J_3 = \begin{vmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_m \end{vmatrix} \quad (3-21)$$

Principal Directions

Let (x, y, z) and (X, Y, Z) denote the local element coordinate system and the principal stress coordinate system respectively. The cosines of the angles between the coordinate axes (x, y, z) and the coordinate axes (X, Y, Z) are listed in Table 3-1.

The angles are measured from the (x, y, z) axes to (X, Y, Z) axes. The principal direction $(l_i, m_i, n_i, i = 1, 2, 3)$ of any principal stress component $\sigma_i (i = 1, 2, 3)$ will have to satisfy the following equations:

$$\left. \begin{aligned} l_i(\sigma_x - \sigma_m) + m_i\tau_{xy} + n_i\tau_{zx} &= 0 \\ l_i\tau_{xy} + m_i(\sigma_y - \sigma_m) + n_i\tau_{yz} &= 0 \\ l_i\tau_{zx} + m_i\tau_{yz} + n_i(\sigma_z - \sigma_m) &= 0 \\ l_i^2 + m_i^2 + n_i^2 &= 1 \end{aligned} \right\} \quad (3-22)$$

Table 3-1 Direction Cosines

	x	y	z
X	l_1	m_1	n_1
Y	l_2	m_2	n_2
Z	l_3	m_3	n_3

Only two of the first three equations of (3-22) are independent. Suppose the two independent equations are

$$a_1l_i + a_2m_i + a_3n_i = 0 \quad (3-23)$$

$$b_1l_i + b_2m_i + b_3n_i = 0 \quad (3-24)$$

plus the last equation of (3-22), the required l_i, m_i, n_i can be resolved as (Arthur and Richard 2003):

$$l_i = \frac{a_3b_2 - a_2b_3}{\sqrt{[(a_1b_3 - a_3b_1)^2 + (a_2b_1 - a_1b_2)^2 + (a_3b_2 - a_2b_3)^2]}} \quad (3-25)$$

$$m_i = \frac{a_1b_3 - a_3b_1}{\sqrt{[(a_1b_3 - a_3b_1)^2 + (a_2b_1 - a_1b_2)^2 + (a_3b_2 - a_2b_3)^2]}} \quad (3-26)$$

$$n_i = \frac{a_2 b_1 - a_1 b_2}{\sqrt{[(a_1 b_3 - a_3 b_1)^2 + (a_2 b_1 - a_1 b_2)^2 + (a_3 b_2 - a_2 b_3)^2]}} \quad (3-27)$$

It is obvious that the coefficients of the two correlated equations will result in a minimum value of $(a_1 b_3 - a_3 b_1)^2 + (a_2 b_1 - a_1 b_2)^2 + (a_3 b_2 - a_2 b_3)^2$.

3.7 THE FIVE-PARAMETER ULTIMATE STRENGTH SURFACE

The evaluation of the variable moduli defined by equation (3-13) requires the specification of the value of peak stress σ_{ic} , and peak strain ε_{ic} . Since these parameters vary with each ratio of stresses, this is most conveniently done by specifying a stress surface in stress space to define the three values of σ_{ic} for each stress ratio, and a surface in equivalent uniaxial strain space to define the three values of ε_{ic} that corresponds to these σ_{ic} values.

A surface in stress space that defines the ultimate strengths σ_{ic} for any stress ratio is usually called “failure surface.” Because this name is somewhat misleading in the context of a material with strain softening behavior, it is proposed (Elwi and Murray 1979) to call such a surface an “ultimate strength surface.” The strength of concrete under a multiaxial stress state is determined by the “ultimate strength surface”. The five-parameter surface proposed by Willam and Warnke (1975) is employed in this study since it reproduces the principal features of the triaxial failure surface of concrete. It consists of a conical-shape surface with curved meridians and the elliptical type of noncircular cross section in the deviatoric plane (Fig. 3.2a). The surface is expressed in terms of the *average stress components* σ_m , τ_m and the angle of similarity θ :

$$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (3-28)$$

$$\tau_m = \frac{1}{\sqrt{15}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (3-29)$$

$$\cos \theta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{\sqrt{2[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}} \quad (3-30)$$

The tensile and compressive meridians are given by following quadratic equations (Fig. 3.2b):

$$\frac{\tau_{mt}}{f'_c} = \frac{r_t}{\sqrt{5}f'_c} = a_0 + a_1 \frac{\sigma_m}{f'_c} + a_2 \left(\frac{\sigma_m}{f'_c} \right)^2 \quad \text{at } \theta = 0 \quad (3-31a)$$

$$\frac{\tau_{mc}}{f'_c} = \frac{r_c}{\sqrt{5}f'_c} = b_0 + b_1 \frac{\sigma_m}{f'_c} + b_2 \left(\frac{\sigma_m}{f'_c} \right)^2 \quad \text{at } \theta = 60 \quad (3-31b)$$

The two curved meridians are then connected by an ellipsoidal surface. The surface is defined as a function of r_t , r_c , θ (Fig. 3.2c)

$$r(\sigma_m, \theta) = \frac{2r_c(r_c^2 - r_t^2)\cos\theta + r_c(2r_t - r_c)\sqrt{4(r_c^2 - r_t^2)\cos^2\theta + 5r_t^2 - 4r_tr_c}}{4(r_c^2 - r_t^2)\cos^2\theta + (r_c - 2r_t)^2} \quad (3-32)$$

The five parameters are determined from three simple tests and two strength points in the high-compression regime.

1. The uniaxial compressive strength f'_c
2. The uniaxial tensile strength. f'_t
3. The equal biaxial compressive strength f'_{bc}
4. The high-compressive-stress point $(\sigma_m / f'_c, \tau_m / f'_c) = (-\bar{\xi}_1, \bar{r}_1)$ on the tensile meridian ($\theta = 0^\circ$, $\bar{\xi}_1 > 0$)
5. The high-compressive-stress point $(\sigma_m / f'_c, \tau_m / f'_c) = (-\bar{\xi}_2, \bar{r}_2)$ on the compressive meridian ($\theta = 60^\circ$, $\bar{\xi}_2 > 0$)

Moreover, the two parabolas have to pass through a common apex σ_{m0} at the hydrostatic axis; this imposes the condition

$$r_t = r_c = 0 \quad \text{for } \rho = \frac{\sigma_{m0}}{f'_c} > 0 \quad (3-33)$$

By the above 5 test results and the constraint condition of equation (3-33), the parameters in equation (3-31) can be determined as follows

$$\left. \begin{aligned} a_0 &= \frac{2}{3} \bar{f}_{bc}' a_1 - \frac{4}{9} \bar{f}_{bc}'^2 a_2 + \sqrt{\frac{2}{15}} \bar{f}_{bc}' \\ a_1 &= \frac{1}{3} (2 \bar{f}_{bc}' - \bar{f}_i') a_2 + \left(\frac{6}{5} \right)^{1/2} \frac{\bar{f}_i' - \bar{f}_{bc}'}{2 \bar{f}_{bc}' + \bar{f}_i'} \\ a_2 &= \frac{\sqrt{\frac{6}{5}} \bar{\xi}_1 (\bar{f}_i' - \bar{f}_{bc}') - \sqrt{\frac{6}{5}} \bar{f}_i' \bar{f}_{bc}' + \bar{r}_1 (2 \bar{f}_{bc}' + \bar{f}_i')}{(2 \bar{f}_{bc}' + \bar{f}_i') \left(\bar{\xi}_1^2 - \frac{2}{3} \bar{f}_{bc}' \bar{\xi}_1 + \frac{1}{3} \bar{f}_i' \bar{\xi}_1 - \frac{2}{9} \bar{f}_i' \bar{f}_{bc}' \right)} \end{aligned} \right\} \quad (3-34)$$

$$\rho = \frac{-a_1 - \sqrt{a_1^2 - 4a_0 a_2}}{2a_2} \quad (3-35)$$

$$\left. \begin{aligned} b_0 &= -\rho b_1 - \rho^2 b_2 \\ b_1 &= \left(\bar{\xi}_2 + \frac{1}{3} \right) b_2 + \frac{\sqrt{\frac{6}{5}} - 3\bar{r}_2}{3\bar{\xi}_2 - 1} \\ b_2 &= \frac{\bar{r}_2 \left(\rho + \frac{1}{3} \right) - \sqrt{\frac{2}{15}} (\rho + \bar{\xi}_2)}{(\bar{\xi}_2 + \rho) \left(\bar{\xi}_2 - \frac{1}{3} \right) \left(\rho + \frac{1}{3} \right)} \end{aligned} \right\} \quad (3-36)$$

(a) Perspective view

(b) Hydrostatic section

(c) Deviatoric section

Fig. 3.2 Five Parameter Failure Surface (Willam-Warnke 1975)

The ultimate strength surface is therefore completely defined and serves to evaluate the three σ_{ic} values required in equation (3-13) for any ratio of stresses.

3.8 THE ULTIMATE STRAIN SURFACE

Equation (3-13) also requires the evaluation of the equivalent uniaxial strains, ε_{ic} , associated with the ultimate strength points, σ_{ic} . For this purpose, it was postulated that there is a surface in the equivalent uniaxial strain space that has the same form as the ultimate strength surface (Elwi and Murray 1979). The surface is defined by replacing σ_m , τ_m in Eqns. (3-31) and (3-32) with the following equivalent quantities:

$$\varepsilon_m = \frac{1}{3}(\varepsilon_{1u} + \varepsilon_{2u} + \varepsilon_{3u}) \quad (3-37)$$

and

$$\gamma_m = \frac{2}{\sqrt{15}} \sqrt{(\varepsilon_{1u} - \varepsilon_{2u})^2 + (\varepsilon_{2u} - \varepsilon_{3u})^2 + (\varepsilon_{3u} - \varepsilon_{1u})^2} \quad (3-38)$$

The remaining parameters required by equation (3-12) are values of σ_{if} , ε_{if} on the descending branch of the stress-equivalent uniaxial strain curve (Fig. 3.1). The definition of these points is not possible on any rigorous experimental basis since the descending branch of the stress-strain curve is highly test-dependent. The following assumptions are adopted (Elwi and Murray 1979)

$$\varepsilon_{if} = 4 \varepsilon_{ic} \quad (3-39a)$$

$$\sigma_{if} = \frac{1}{4} \sigma_{ic} \quad (3-39b)$$

The description of the determination of all the material variables required for the implementation of the constitutive relationship is now complete.

3.9 CURRENT MATERIAL STRENGTH

The determination of the three current concrete strength values σ_{ic} ($i = 1, 2, 3$) is based on the ultimate strength surface in equation (3-31) as follows. If a point $(\sigma_1, \sigma_2, \sigma_3)$ on the loading trajectory in principal stress space represents the current stress state of the material, then a line that extends from the origin through the current stress point penetrates the ultimate strength surface at point $(\sigma_{1c}, \sigma_{2c}, \sigma_{3c})$, where σ_{ic} is the required current concrete strength of concrete in the i -direction.

In similar manner, the ultimate strain surface in the equivalent uniaxial strain space is used to determine the current ultimate equivalent uniaxial strains ε_{ic} ($i = 1, 2, 3$). The same line extended from the origin through the current stress state point $(\sigma_1, \sigma_2, \sigma_3)$ in stress space is used to penetrate the ultimate strain surface to obtain the current ultimate equivalent uniaxial strains ε_{ic} in the i -direction. The use of the current stress state rather than the current equivalent uniaxial strain state can be easily understood in a proportional loading case where the ratio between each stress keeps constant, however, the resulting ratio between equivalent uniaxial strains will vary in the loading history due to the nonlinearity of concrete in compression. This treatment guarantees the constant peak equivalent uniaxial strains to be obtained for the whole loading history.

3.10 COMPRESSIVE STRENGTH OF PLAIN CONCRETE UNDER TENSION-COMPRESSION

To determine the compressive strength of plain concrete under biaxial tension compression from the above ultimate strength surface, another important fact, the failure mode, has to be considered. For concrete under proportional tension-compression loading, either tensile splitting or compression crushing will occur. If the concrete fails in crushing or a combination of crushing and splitting, no more residual strength can remain both for tension and compression. However, if the concrete fails in tension splitting, a large value of compressive strength remains

parallel to the splitting direction although no tensile strength remains in the tension direction. Therefore, the tensile strength obtained from the ultimate strength surface can be used for analysis directly. The failure mode has to be considered to determine the compressive strength in addition to the use of the ultimate strength surface. According to test data provided by Kupfer (1969), compressive crushing or combined crushing and splitting controlled failure mode occurs when the absolute stress ratio of σ_1/σ_3 is less than 0.052. Tensile splitting controlled failure mode occurs when the absolute stress ratio of σ_1/σ_3 is larger than 0.103. In this study, for the absolute stress ratio of σ_1/σ_3 less than 0.052, the compressive strength σ_{3c} is directly determined by the current stress ratio σ_1/σ_3 from the ultimate strength surface. For the absolute stress ratio of σ_1/σ_3 larger than 0.052, the compressive strength σ_{3c} takes the value of $1/0.052$ times the tensile strength σ_{1c} and is limited to not more than the uniaxial compressive strength f'_c of concrete. The triaxial tension-compression case is treated similarly.

3.11 IMPLEMENTATION

A flow chart describing the implementation of the constitutive relationship as suggested by Elwi and Murray (1979) is shown in Fig. 3.3. The strain input parameters, $\varepsilon_t, \varepsilon_c, \varepsilon_{bc}, \bar{\eta}, \bar{\lambda}_1, \bar{\lambda}_2$, appearing in Fig. 3.3 are analogous to the stress input parameters, $f'_t, f'_c, f'_{bc}, \bar{\xi}, \bar{r}_1, \bar{r}_2$, defined in the preceding section.

Fig. 3.3 Flow Chart for Implementation of Constitutive Equations (Elwi and Murray 1979)

Fig. 3.4 Flow Chart for Constitutive Relation to Follow Specified Stress Path (Elwi and Murray 1979)

Although Fig. 3.3 is the normal type of application, it is useful in comparing with experimental results to be able to determine the strains associated with a predefined stress path. A flow chart as suggested by Elwi and Murray (1979) for this type of application, which requires iteration, is shown in Fig. 3.4.

3.12 VERIFICATION OF THE MODEL

A comparison of predicted strains with selected sets of measurements by Kupfer, Hilsorf, and Rüschi (1969), hereinafter referred to as the KHR data, in a series of two-dimensional tests is shown in Fig. 3.5. The response of concrete under uniaxial tension is modeled by the tension softening model in the next section. The softening parameter λ takes the value of 750 (Chapter 4, section 4.2.1).

The strength parameters used by Elwi and Murray (1979) are applied:

$$f'_c = 32.06 \text{ MPa}; f'_t = 0.091 f'_c; f'_{bc} = 1.15 f'_c; \bar{r}_1 = 0.048835 - 0.513326(\bar{\epsilon}) - 0.038292(\bar{\epsilon})^2; \\ \bar{r}_2 = 0.087963 - 0.909151(\bar{\epsilon}) - 0.232785(\bar{\epsilon})^2.$$

The deformation parameters used in this study are:

$$\epsilon_c = -0.00215; \epsilon_t = 0.000075; E_c = 32407 \text{ MPa}; \epsilon_{bc} = 1.75 \epsilon_c; \nu = 0.195; \\ \bar{\lambda}_1 = 0.025115 - 0.538802(\bar{\eta}) - 0.010786(\bar{\eta})^2; \bar{\lambda}_2 = 0.048340 - 1.027311(\bar{\eta}) - 0.230689(\bar{\eta})^2.$$

(a) Stress Ratios -1:0

(b) Stress Ratios -1:-1

Fig. 3.5 Comparison of Predicted Strains with Measurements of KHR Data (to be continued)

(c) Stress Ratios -1:-0.52

(d) Stress Ratios -1:0.103

Fig. 3.5 Comparison of Predicted Strains with Measurements of KHR Data

Fig. 3.5 (a) shows the uniaxial compression comparison, Fig. 3.5 (b) shows the equal biaxial compression comparison. Since the peak stresses and corresponding

strains are control points on the surfaces, exact correlation is obtained at these points. Fig. 3.5 (c) shows the comparison for biaxial compression in the ratio of -1:-0.52. Since the five parameter failure surface can not reproduce the exact strength, there is a discrepancy in this instance. A comparison of typical response in the tension-compression zone is shown in Fig. 3.5 (d).

The preceding comparisons are those of the three-dimensional theory with plane stress tests. A comparison of the theory with some of the three-dimensional tests of Schickert and Winkler (1977), hereinafter referred to as the SW data, is shown in Fig. 3.6. These tests were carried out by loading along a hydrostatic stress path, and then incrementing the three principal stresses in such a way that the hydrostatic stress remained constant, while the deviatoric stress varied.

The strength parameters used in this study are:

$$f'_c = 30.6 \text{ MPa} ; f'_t = 0.06 f'_c ; f'_{bc} = 1.21 f'_c ; \bar{r}_1 = 0.053627 - 0.512079(\bar{\xi}) - 0.038226(\bar{\xi})^2 ; \\ \bar{r}_2 = 0.095248 - 0.891175(\bar{\xi}) - 0.244420(\bar{\xi})^2 .$$

The deformation parameters used in this study are:

$$\varepsilon_c = -0.00283 ; \varepsilon_t = 0.000075 ; E_c = 26201 \text{ MPa} ; \varepsilon_{bc} = 1.3 \varepsilon_c ; \nu = 0.18 ; \\ \bar{\lambda}_1 = 0.025077 - 0.539616(\bar{\eta}) - 0.024032(\bar{\eta})^2 ; \bar{\lambda}_2 = 0.045268 - 0.974103(\bar{\eta}) - 0.043383(\bar{\eta})^2 .$$

(a) Stress Ratios: -1:0.5:0.5

(b) Stress Ratios: -1:0:1

Fig. 3.6 Comparison of Predicted Strains with Measurements of SW Data (to be continued)

(c) Stress Ratios: -1:-1:2

Fig. 3.6 Comparison of Predicted Strains with Measurements of SW Data

It can be seen that the model gives a reasonable simulation of the SW experimental stress-strain response along each of stress paths.

CHAPTER 4 MODELING CRACKED CONCRETE AND REINFORCEMENT

4.1 INTRODUCTION

Cracking is the major contributor to the nonlinear behavior of reinforced concrete structures. Cracking in concrete initiates two kinds of effects: 1) Local interface cracking effects occur within the crack interface including tension softening, aggregate interlock and dowel action, and 2) Smeared cracking effects occur in solid concrete between two adjacent cracks including tension stiffening and compression softening. For plain concrete, only the local interface cracking effects are necessary to consider while in reinforced concrete both kinds of effects have to be incorporated.

This chapter presents the development of cracking formulations to model the material properties of cracked plain and reinforced concrete. An energy based tension softening model is developed to avoid the strain localization problem for plain concrete under tension. An average concept is employed to describe the stress-strain relationship to retain computation efficiency dealing with strain localization in planar-type structures. A tension stiffening model considering the anisotropy between reinforcement and crack plane is proposed. The compression softening effect due to principal tensile strain, principal tensile stress normal to crack plane as well as shear stress in the crack interface is incorporated. Aggregate interlock is simulated by a variable fraction of stiffness of uncracked concrete. A new model describing the material properties of CRC with multiple non-orthogonal cracks is introduced. The local interface cracking effects of tension softening, aggregate interlock as well as the smeared cracking effects of tension stiffening and compression softening due to multiple non-orthogonal cracks are all incorporated explicitly. A constitutive model for reinforcement is also provided in this chapter.

4.2 CRACKING EFFECTS

4.2.1 Tension Softening Effect

To avoid the strain localization problem and the associated mesh sensitivity due to the tension softening effect of cracked plain concrete in the smeared crack approach, the characteristic material length needs to be defined properly (Bažant and Oh 1983; Rots 1988). When dealing with inclined cracks in 3-D elements, however, this becomes a sophisticated process which involves the element geometry, stress and strain state of a certain domain (Oliver 1989; Barzegar and Maddipudi 1997; Hauke and Maekawa 1998). In addition, it is difficult to formulate the crack localization in a layered shell element because, when a transverse shear crack is considered, the crack localization must be considered across several layers in an element. Since the main advantage of the layered shell formulation is that the stiffness calculation of each layer is independent from the adjacent layers, the crack localization treatment will render the finite element formulation significantly more complicated. To overcome these difficulties and make it possible the implementation of a tension softening model in finite element shell analysis, Yamamoto and Vecchio (2001) proposed a softening model based on the average tensile stress and average tensile strain concept for thick shell element. The representative length of material was taken as one half thickness of the thick shell element. This average concept is employed in this study to incorporate the tension softening for cracked plain concrete effectively and efficiently. The proposed formulations for the tension softening stress is as follows

$$\sigma_1 = \begin{cases} E_{1c} \varepsilon_{1u} & \varepsilon_{1u} < \varepsilon_{1c} \\ \sigma_{1c} e^{-\lambda(\varepsilon_{1u} - \varepsilon_{1c})} & \varepsilon_{1c} < \varepsilon_{1u} < \varepsilon_{ch} \\ 0 & \varepsilon_{1u} > \varepsilon_{ch} \end{cases} \quad (4-1)$$

where

$$E_{1c} = \frac{\sigma_{1c}}{\varepsilon_{1c}} \quad (4-2)$$

where σ_{1c} and ε_{1c} are the tensile peak stress and tensile peak strain under multiaxial stress state just prior to cracking and are kept constant thereafter. σ_{1c} and ε_{1c} can be calculated by the method discussed in Section 3.8. ε_{ch} is taken as the yield strain of steel considering the fact that few of new cracks form after the yielding of steel in reinforced concrete specimens under uniaxial tension tests (Fields 1998). The tension softening coefficient λ representing the descending shape is thus determined from the uniaxial tension by the following condition,

$$\int \sigma_1 d\varepsilon_{1u} = \frac{G_f}{L_r} \quad (4-3)$$

G_f is the energy consumed within the representative material length L_r which is assumed to be the thickness of thin and moderate thick shell elements

$$L_r = d \quad (4-4)$$

The proposed tension softening model is shown in Fig. 4.1.

Fig. 4.1 Proposed Tension Softening Model

4.2.2 Tension Stiffening Effect

The same form of softening shape is used in the proposed tension stiffening model. The softening coefficient λ takes a constant value of 750 as suggested by Fields (1998). After the yielding of steel, the tensile stress decreases linearly to zero at

strain ε_{te} . The tensile stress contributed by the tension stiffening effect is accounted by the following equation

$$\sigma_1 = \begin{cases} E_{1c} \varepsilon_{1u} & \varepsilon_{1u} < \varepsilon_{1c} \\ \sigma_{1c} e^{-750(\varepsilon_{1u} - \varepsilon_{1c})} & \varepsilon_{1c} \leq \varepsilon_{1u} < \varepsilon_{ch} \\ \sigma_{ch} \frac{\varepsilon_{te} - \varepsilon_{1u}}{\varepsilon_{te} - \varepsilon_{ch}} & \varepsilon_{ch} \leq \varepsilon_{1u} < \varepsilon_{te} \\ 0 & \varepsilon_{1u} \geq \varepsilon_{te} \end{cases} \quad (4-5)$$

where σ_{1c} , ε_{1c} , ε_{ch} and E_{1c} are defined in equation (4-1). ε_{te} takes 0.05 in this study. Fig. 4.2 is the comparison between the proposed model and other tension stiffening models (Vecchio and Collins 1986; Belarbi and Hsu 1994).

Fig. 4.2 Comparison between the Proposed Model and Other Tension Stiffening Models

4.2.3 Anisotropy in Tension Stiffening Effect

For an arbitrary crack orientation at a characteristic point, the reinforcements may not be normal to the crack plane but have a skew angle with the crack (Fig. 4.3). This phenomenon is termed here as anisotropy of tension stiffening effect. It is necessary to compute the tension stiffening effect while accounting for this skew angle. Existing models handling this anisotropy simulate the tension stiffening effect either in the reinforcement direction (Gilbert and Warner 1977; Ramaswamy 1992; Ramaswamy et al. 1994; Ramaswamy et al. 1995) or in the crack normal

direction (Hauke and Maekawa 1998; Rots 1988; Mohammadi 2002). Equilibrium conditions need to be examined for the models oriented in the reinforcement direction. The energy interpolation approach is usually used for models oriented in crack normal direction. The energy consumed in the crack normal direction is approximated by either linear (Rots 1988) or nonlinear (Hauke and Maekawa 1998) interpolation between tension softening and tension stiffening. In the model developed by Mohammadi (2002), named the transient tension softening-stiffening model, the tension stiffening is considered to be a combination of the bridging tension softening stress as well as bond stress. The anisotropy is considered by a reduction parameter on the bond part. This model has the advantages of simplicity. As the tension softening model proposed in this study has been smeared in the thickness of the element, it is possible to separate the bond contribution from the tension stiffening effect. The basic concepts of separation and combination in the model by Mohammadi (2002) are incorporated in the proposed procedure for handling the anisotropy of the tension stiffening effect. The tensile stress σ_1 normal to the crack plane is expressed as follows

$$\sigma_1 = \sigma_{1f} + \sum_{i=1}^n \cos^2 \theta_i (\sigma_{1t} - \sigma_{1f}) \quad (4-6)$$

and

$$\sum_{i=1}^n \cos^2 \theta_i \leq 1 \quad (4-7)$$

where σ_{1f} is tension softening stress and can be calculated by equation (4-1); σ_{1t} is tension stiffening stress which is accounted by equation (4-5); θ_i is the angle between the i -th reinforced direction and the crack normal direction (Fig. 4.3); n is the number of reinforced directions. Fig. 4.4 illustrates the resulting tension stiffening curve by the proposed model when the skew angle between crack norm and the reinforcement is 60 degrees. The tensile strength σ_{1c} in the above tension models should be determined by the failure surface of concrete under multiaxial stress state.

Fig. 4.3 Anisotropy of Tension Stiffening Effect

Fig. 4.4 The Proposed Model for Anisotropy of Tension Stiffening Effect

4.2.4 Aggregate Interlock and Dowel Action

In the smeared rotating angle crack approach, the aggregate interlock effect in cracked plain concrete has been commonly modeled by a shear modulus defined either by the secant modulus of elasticity on principal axes or by principal stresses and strains (Vecchio 1989; Crisfield and Wills 1989; Polak and Vecchio 1993; Pang and Hsu 1996; Selby and Vecchio 1997; Balan et al. 1997). These models tend to provide lower predictions of shear strength since only the contribution of plain concrete can be incorporated. In the smeared fixed angle crack approach, a constant or variable fraction of the shear stiffness of uncracked concrete is often used to account for the aggregate interlock effect in cracked plain concrete (ASCE 1982;

Milford and Schnobrich 1984; Barzegar 1989). Analysis experiences (Ramaswamy et al. 1995) showed that a value as small as 1% of the shear stiffness of uncracked concrete can be used to model cracked plain concrete. One of the advantages of these models is convenience to indirectly incorporate the dowel action contribution of reinforcements across the crack interface. Barzegar (1989) suggested that any value greater than 20% of shear modulus of uncracked concrete may be used to model the shear capacity of CRC.

In the study by Barzegar (1989), the variable shear retention factor proposed by Al-Mahaidi (1979) was used. It took the form as follows

$$\beta = \delta \left(\frac{\varepsilon_t}{\varepsilon_{1u}} \right)^\phi \quad (4-8)$$

where

- δ = 0.4, instantaneous reduction factor just after cracking
- ε_{1u} = Equivalent uniaxial tensile strain normal to crack
- ϕ = 1.0, shear retention parameter
- ε_t = Crack strain of concrete under uniaxial tension

Equation (4-8) is used in this study to define the shear retention factor. The instantaneous reduction is not considered and thus δ equals to 1.0. The shear parameter ϕ takes a value of 1.2 for cracked plain concrete. The model is shown in Fig. 4.5. The proper choice of shear parameter ϕ and the performance of the proposed model in modeling the shear behavior of CRC are investigated by analyzing a number of reinforced concrete panels under pure shear and tension-shear actions.

Fig. 4.5 Proposed Shear Retention Factor

4.2.5 Compression Softening Effect

CRC under compression in the crack plane exhibits softening behavior in comparison with a companion cylinder. The severity of cracking is obviously a major variable governing the softening of concrete. Since it is contributed primarily by the crack widths, the principal tensile strain normal to crack plane represents the severity of cracking and is often used as a measure of the “smeared cracks” to quantify the compression softening effect (Vecchio and Collins 1986; Belarbi and Hsu 1995). It has been found that the principal tensile strain has only softening effects on peak compressive stress and no obvious softening effects on peak compressive strain. Belarbi (1991) showed that the presence of tensile stress in concrete causes the build-up of the stress concentration at the crack tips and facilitates the propagation of cracks and the crushing of concrete due to compressive stresses. Therefore, the tensile stress was recognized as the second major variable for compression softening effect. It was found (Belarbi 1991) that the tensile stress not only further softens the peak compressive strength but also softens the peak compressive strain.

The softening effects due to both principal tensile strain and tensile stress were reflected in the model proposed by Belarbi and Hsu (1995). The softening coefficient of peak compressive strength was evaluated as a function of principal tensile strain and tensile stress represented by different loading path as

$$\zeta_{\sigma} = \frac{0.9}{\sqrt{1 + k_{\sigma} \varepsilon_{1u}}} \quad (4-9)$$

where k_{σ} takes the value of 250 for sequential loading path where the tensile stress releases before the application of compressive stress, and 400 for proportional loading path where no tensile stress is released when compression is applied. The softening coefficient of peak compressive strain due to the tensile stress in concrete in proportional loading path is accounted for by the following formulation

$$\zeta_{\varepsilon} = \frac{1}{\sqrt{1 + k_{\varepsilon} \varepsilon_{1u}}} \quad (4-10)$$

where k_{ε} takes the value of 500. ε_{1c}

In this study, the cracked concrete is treated as an assembly of intact concrete, uniaxial cracked concrete and its coupling solid connected in series (Section 4.3.1). Multiaxial stresses are applied on the intact concrete. The principal tensile stress will reduce the compressive peak stress and peak strain of the intact concrete obtained from the ultimate failure surface through the stress ratio. This situation has to be considered before the development of compression softening model for the uniaxial cracked concrete. If equation (4-9) and equation (4-10) are directly used to model the compression softening due to principal tensile stress and principal tensile strain for uniaxial cracked concrete, the total compression softening on the cracked concrete (Fig. 4.9) will be overestimated. In this study, adjustments have been made on equation (4-9) and equation (4-10) to model the compression softening of the uniaxial cracked concrete (Fig. 4.9). The softening parameter k_{σ} in equation (4-9) takes the same value of 200 for both sequential loading and proportional loading. The softening parameter k_{ε} for in equation (4-10) also takes the value of 200 for proportional loading.

In addition to the severity of cracking measured by the tensile strain and the tensile stress normal to a crack plane, the intensity of shear stress in the crack interface has been found to be another primary factor in the softening of the compressive strength

of concrete parallel to crack plane (Pang 1991). In his study, an empirical equation is suggested to take into account the softening due to shear. The softening parameter k_σ in equation (4-9) is related to the parameter η

$$k_\sigma = \frac{400}{\eta} \quad (4-11)$$

Where

$$\eta = \frac{\rho_t f_{ty} - \sigma_t}{\rho_l f_{ly} - \sigma_l} \quad (4-12)$$

and

- ρ_l, ρ_t = Reinforcement ratio in the l and t directions, respectively
- f_{ly}, f_{ty} = Yield strengths of steels in the l and t directions, respectively
- σ_l, σ_t = Stresses of steels in the l and t directions, respectively

It is obvious that equation (4-11) is not a direct reflection of intensity of shear and it is not convenient to be applied in general cases. Therefore, in this study, a simple equation is proposed to relate the softening coefficient k_σ directly to the shear stress at the crack interface to account for the intensity of shear

$$\eta = \begin{cases} e^{-1.6 \frac{|\tau_{1j}|}{f_t'}} & |\tau_{1j}| < f_t' \\ 0.2 & |\tau_{1j}| \geq f_t' \end{cases} \quad (4-13)$$

where $j = 2, 3$ are the principal directions to consider compression softening effect.

Equation (4-13) is illustrated in Fig. 4.6.

Fig. 4.6 Relationship of Softening Parameter η with Respect to Shear Stress

From the above discussion, the proposed compression softening model for the uniaxial cracked concrete (illustrated in Fig. 4.9) is summarized below. The softening coefficient of compressive peak stress in sequential or proportional loading is expressed

$$\zeta_{\sigma} = \frac{0.9}{\sqrt{1 + \frac{200}{\eta} \varepsilon_1}} \quad (4-14)$$

The softening coefficient of compressive peak strain is accounted by the following formulation

$$\zeta_{\varepsilon} = \begin{cases} \frac{1}{\sqrt{1 + \frac{200}{\eta} \varepsilon_1}} & \text{Proportional loading} \\ 1.0 & \text{Sequential loading} \end{cases} \quad (4-15)$$

Where η is given in equation (4-13).

It should be noted that all the available data and softening models are about uniaxially cracked concrete which is subjected to compression in the crack plane. The compression softening is monitored by the principal tensile strain normal to the crack plane. In case of three dimensional structures such as R. C. flat plate structure in this study, cracking can occur in two principal directions with the third principal direction in compression. Insufficient experimental data exist on how the compressive behavior changes in the presence of biaxial cracking. In the study by Polak (1992), the effective tensile strain approach suggested by Kirschner and Collins (1986) is used. If ε_1 and ε_2 are tensile strains, then the effective tensile strain ε_1^e to be used is defined as

$$\varepsilon_1^e = \sqrt{\varepsilon_1^2 + \varepsilon_2^2} \quad (4-16)$$

In the presence of multiple non-orthogonal cracks, the quantification of compression softening effect becomes more complex. In the study by Ramaswamy (1992), the compression softening effects due to each crack is simplified by the employment of principal tensile strain of the cracked concrete rather than the respective tensile strain normal to each crack plane.

In this study, the compression softening effects of multi-directionally cracked concrete is accounted for in a more explicit manner. The softening effect due to each crack is considered separately and is monitored by the tensile strain normal to its crack plane. The detailed procedure is explained in the following section.

4.3 PROPOSED MODEL FOR CRC WITH MULTIPLE NON-ORTHOGONAL CRACKS

Multidirectional cracking is a general phenomenon in reinforced concrete and the crack orientations may not be orthogonal to each other (Fig. 4.7). In each crack interface, tension softening, aggregate interlock, and dowel action of reinforcement are initiated. In concrete between cracks, tension stiffening and compression softening are mobilized. All these cracking effects due to each crack are affected by

those of the other cracks. Since little experimental data is available on these coupling effects of cracking, the modeling of material properties of CRC with multiple non-orthogonal cracks becomes extremely difficult. This situation turns even worse when the cracked concrete is reinforced in multiple non-orthogonal orientations as illustrated in Fig. 4.7. The effect due to the skew angle between each crack plane and each reinforcing direction on the tension stiffening has to be considered in addition to the incorporation of the coupling effects among the cracks.

**Fig. 4.7 Multi-Directionally Cracked Concrete
Reinforced in Multiple Orientations**

The local interface cracking effects of tension softening and aggregate interlock due to multiple non-orthogonal cracks can be handled by the smeared crack model proposed by de Borst and Nauta (1985). In their model, a removing-concept was used. The basic idea is to remove the discontinuity of cracks from the cracked concrete and keep the remaining solid to be continuous. The removing idea was realized by decomposing the total strain increment of cracked plain concrete into solid strain and crack strain increments corresponding to the intact concrete and each crack as follows

$$d\epsilon = d\epsilon_0 + d\epsilon_{1cr} + \dots + d\epsilon_{icr} + \dots + d\epsilon_{ncr} \quad (4-17)$$

$d\epsilon$	=	Total strain increment of cracked plain concrete in element coordinates
$d\epsilon_{icr}$	=	Crack strain increment of the i -th crack in element coordinates
$d\epsilon_0$	=	Solid strain increment of intact concrete in element coordinates

In their model, the cracked concrete with multiple non-orthogonal cracks, in essence, was represented as an assembly of intact concrete and crack openings connected in series. This is illustrated in Fig. 4.8 for the case of biaxial cracked concrete. The crack strain was used to monitor the properties of tension softening and aggregate interlock occurred in local crack interface. Tension stiffening and compression softening in CRC were not considered in that model.

Fig. 4.8 Rheological Representation of Biaxial-Cracked Concrete by the Crack Model by de Borst and Nauta (1985)

In this study, a new material model has been developed to simulate the material properties of CRC with multiple non-orthogonal cracks. The localized interface properties of tension softening and aggregate interlock as well as the smeared properties of tension stiffening and compression softening in concrete between cracks are all incorporated explicitly. The model developed by the author is based on the concept of springs in series describing the coupling effect of cracks by de Borst and Nauta (1985). Before the development of the model, the underlying assumption has to be clarified. Multiple cracks are assumed to take their coupling effect on the tension stiffening and compression softening of concrete between cracks by the same mechanism as that for the interface tension softening and aggregate interlock.

4.3.1 Stiffness Matrix of CRC

In the author's proposed model, the crack strain $d\epsilon_{icr}$ of the i -th crack in equation (4-17) is further decomposed into a strain increment $d\epsilon_{iu}$ corresponding to a uniaxial-cracked concrete and a strain increment $d\epsilon_{i0}$ corresponding to the coupling solid of the uniaxial-cracked concrete

$$\mathbf{d}\boldsymbol{\varepsilon}_{icr} = \mathbf{d}\boldsymbol{\varepsilon}_{iu} - \mathbf{d}\boldsymbol{\varepsilon}_{i0} \quad (4-18)$$

The cracks illustrated in Fig. 4.8 are thus replaced by a uniaxial cracked concrete minus its coupling solid as shown in Fig. 4.9. The material properties of MR-CRC are thus represented by the material properties of the intact concrete and each uniaxially cracked concrete with its coupling solid.

Through strain decomposition and accumulation onto each uniaxial cracked concrete, its strain state can be conveniently obtained. It is obvious that the reproduction of uniaxial cracked concrete in this model significantly facilitates tracing the smeared cracking effects of tension stiffening and compression softening due to each crack.

Fig. 4.9 Rheological Representation of Biaxial-Cracked Concrete by the Crack Model Proposed in This Study

Input equation (4-18) into equation (4-17) we obtain

$$\mathbf{d}\boldsymbol{\varepsilon} = \mathbf{d}\boldsymbol{\varepsilon}_0 + \sum_{i=1}^n (\mathbf{d}\boldsymbol{\varepsilon}_{iu} - \mathbf{d}\boldsymbol{\varepsilon}_{i0}) \quad (4-19)$$

It is equivalent to the following equation

$$\mathbf{C}\mathbf{d}\boldsymbol{\sigma} = \mathbf{C}_0\mathbf{d}\boldsymbol{\sigma} + \sum_{i=1}^n (\mathbf{C}_{iu}\mathbf{d}\boldsymbol{\sigma} - \mathbf{C}_{i0}\mathbf{d}\boldsymbol{\sigma}) = \left[\mathbf{C}_0 + \sum_{i=1}^n (\mathbf{C}_{iu} - \mathbf{C}_{i0}) \right] \mathbf{d}\boldsymbol{\sigma} \quad (4-20)$$

where

- $\mathbf{d}\boldsymbol{\sigma}$ = Stress increment vector in element coordinates
- $\mathbf{C}$ = Tangent compliance of cracked concrete in element coordinates
- $\mathbf{C}_{iu}$ = Tangent compliance of the i -th uniaxially cracked concrete in element coordinates

- $\mathbf{C}_{i0}$ = Tangent compliance of the coupling solid of the i -th uniaxially cracked concrete in element coordinates
- $\mathbf{C}_0$ = Tangent compliance of the intact concrete in element coordinates

From equation (4-20), the compliance of the CRC can be expressed as

$$\mathbf{C} = \left[\mathbf{C}_0 + \sum_{i=1}^n (\mathbf{C}_{iu} - \mathbf{C}_{i0}) \right] = \left[\mathbf{C}_0 + \sum_{i=1}^n \mathbf{C}_{icr} \right] \quad (4-21)$$

where $\mathbf{C}_{icr} = \mathbf{C}_{iu} - \mathbf{C}_{i0}$ is the material compliance of the i -th crack in the element coordinates and it can be obtained by transformation from its local crack coordinates

$$\mathbf{C}_{icr} = \mathbf{N}_i^T \mathbf{C}'_{icr} \mathbf{N}_i = \mathbf{N}_i^T (\mathbf{C}'_{iu} - \mathbf{C}'_{i0}) \mathbf{N}_i \quad (4-22)$$

where

- $\mathbf{C}'_{icr}$ = Tangent compliance of the i -th crack in its local crack coordinates
- $\mathbf{C}'_{iu}$ = Tangent compliance of the i -th uniaxially cracked concrete in the local crack coordinates
- $\mathbf{C}'_{i0}$ = Tangent compliance of the coupling solid of the i -th uniaxially cracked concrete in the local crack coordinates

$\mathbf{N}_i$ is transformation matrix of the i -th uniaxially cracked concrete from the local crack coordinates to the element coordinates

$$\mathbf{N}_i = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & 2l_1m_1 & 2m_1n_1 & 2n_1l_1 \\ l_2^2 & m_2^2 & n_2^2 & 2l_2m_2 & 2m_2n_2 & 2n_2l_2 \\ l_3^2 & m_3^2 & n_3^2 & 2l_3m_3 & 2m_3n_3 & 2n_3l_3 \\ l_1l_2 & m_1m_2 & n_1n_2 & (l_1m_2 + l_2m_1) & (m_1n_2 + m_2n_1) & (n_1l_2 + n_2l_1) \\ l_2l_3 & m_2m_3 & n_2n_3 & (l_2m_3 + l_3m_2) & (m_2n_3 + m_3n_2) & (n_2l_3 + n_3l_2) \\ l_3l_1 & m_3m_1 & n_3n_1 & (l_3m_1 + l_1m_3) & (m_3n_1 + m_1n_3) & (n_3l_1 + n_1l_3) \end{bmatrix} \quad (4-23)$$

The material compliance of the intact concrete can also be obtained by transformation from the principal coordinate as follows

$$\mathbf{C}_0 = \mathbf{N}_0^T \mathbf{C}'_0 \mathbf{N}_0 \quad (4-24)$$

where $\mathbf{N}_0$ is the transformation matrix of the intact concrete from the principal coordinates to the element coordinates and can be calculated from equation (4-23).

Input equations (4-22) and (4-24) into equation (4-21), we can get

$$\mathbf{C} = \left[\mathbf{N}_0^T \mathbf{C}'_0 \mathbf{N}_0 + \sum_{i=1}^n \mathbf{N}_i^T (\mathbf{C}'_{iu} - \mathbf{C}'_{i0}) \mathbf{N}_i \right] = \left[\mathbf{N}_0^T \mathbf{C}'_0 \mathbf{N}_0 + \sum_{i=1}^n \mathbf{N}_i^T \mathbf{C}'_{icr} \mathbf{N}_i \right] \quad (4-25)$$

Inverting equation (4-25), we get the material stiffness matrix $\mathbf{D}$ of CRC with multiple non-orthogonal cracks as follows

$$\mathbf{D} = \mathbf{C}^{-1} = \left[\mathbf{N}_0^T \mathbf{C}'_0 \mathbf{N}_0 + \sum_{i=1}^n \mathbf{N}_i^T \mathbf{C}'_{icr} \mathbf{N}_i \right]^{-1} \quad (4-26)$$

4.3.2 Material Compliance Matrix $\mathbf{C}'_0$ of the Intact Concrete

Multiaxial stresses are applied on the intact concrete. It is modeled by the material model developed for uncracked concrete in Chapter 3. Triaxial principal stresses and equivalent uniaxial strains are used to define its material property. The material compliance $\mathbf{C}'_0$ of the intact concrete (Fig. 4.9) in equation (4-26) can be calculated by equation (3-1) in Chapter 3.

4.3.3 Material Compliance Matrix $\mathbf{C}'_{icr}$ of Cracks

All the cracking effects of tension softening and aggregate interlock and tension stiffening and compression softening will be accounted for by the material compliance $\mathbf{C}'_{icr}$ of each crack in equation (4-26). The calculation of $\mathbf{C}'_{icr}$ is through the tensile, shear, and compressive crack strains caused by cracking. These crack strains, which are equivalent to a crack opening, consist a crack strain vector $\mathbf{d}\boldsymbol{\varepsilon}_{cr}$ in equation (4-18). Procedures are developed below to calculate the material compliance $\mathbf{C}'_{icr}$ of each crack.

(a) Uniaxial cracked concrete

(b) Coupling solid of the uniaxial cracked concrete

Fig. 4.10 Coordinates of Uniaxial Cracked Concrete and its Coupling Solid

Fig. 4.10 shows the local crack coordinates of the uniaxial cracked concrete and its coupling solid. The local crack axes of the uniaxial cracked concrete are fixed at principal axes at the initiation of that crack. The same local crack coordinates are used for the coupling solid. Uniaxial material property is assumed along each principal axis of the uniaxial cracked concrete and the coupling solid. If the crack is in opening state, tension softening and tension stiffening have to be considered in σ_1 direction by use of tensile crack strain. Aggregate interlock in the crack interface needs to be accounted through shear crack strain. If σ_2 , or σ_3 , or both directions are under compression, compression softening effect has also to be taken into account through a newly developed concept of compression crack strain.

4.3.3.1 Tensile crack strain and shear crack strain versus cracking effects of tension softening, tension stiffening, and aggregate interlock

In σ_1 direction of the uniaxial cracked concrete as shown in Fig. 4.10 (a), tension softening and tension stiffening due to cracking has to be considered. If the cracked concrete is not affected by any reinforcement, only tension softening is included. The tension softening model expressed in equation (4-1) and illustrated in Fig. 4.1 is used. If the crack concrete is reinforced as shown in Fig. 4.3, the tension stiffening model considering anisotropy of reinforcement developed in equation (4-6) and illustrated in Fig. 4.4 is used. Such a tension softening or tension stiffening curve used in σ_1 direction of the uniaxial cracked concrete is shown in Fig. 4.11. In the same direction of the coupling solid, linear elastic material is assumed and can be modeled by the linear ascending portion of the tension curve of the uniaxially cracked concrete (Fig. 4.11).

As illustrated in Fig. 4.11, the tensile crack strain increment $d\epsilon_{1cr}$ normal to crack plane can be expressed as follows

$$d\epsilon_{1cr} = d\epsilon_{1u} - d\epsilon_{10} \quad (4-27)$$

where

$d\epsilon_{1u}$ = Tensile strain increment of uniaxial cracked concrete normal to crack plane

$d\epsilon_{10}$ = Tensile strain increment of the coupling solid in the direction normal to crack plane

From the constitutive relations shown in Fig. 4.4, the above equation is equivalent to

$$d\epsilon_{1cr} = \frac{1}{E_1} d\sigma_1 - \frac{1}{E_{1c}} d\sigma_1 = \left(\frac{1}{E_1} - \frac{1}{E_{1c}} \right) d\sigma_1 \quad (4-28)$$

where

- E_1 = Tangent modulus of elasticity on the descending branch of the tension softening or stiffening curve corresponding to current tensile strain ε_{1u} , which is greater than cracking strain ε_{1c}
- E_{1c} = Tensile modulus of elasticity of concrete
- $d\sigma_1$ = Normal stress increment in crack normal direction

Fig. 4.11 Tensile Crack Strain

Similarly, the shear crack strain increments can be evaluated as

$$d\gamma_{12cr} = d\gamma_{12} - d\gamma_{120} = \frac{1}{\beta G_0} d\tau_{12} - \frac{1}{G_0} d\tau_{12} = \left(\frac{1}{\beta G_0} - \frac{1}{G_0} \right) d\tau_{12} \quad (4-29a)$$

$$d\gamma_{13cr} = d\gamma_{13} - d\gamma_{130} = \frac{1}{\beta G_0} d\tau_{13} - \frac{1}{G_0} d\tau_{13} = \left(\frac{1}{\beta G_0} - \frac{1}{G_0} \right) d\tau_{13} \quad (4-29b)$$

where

- β = Shear retention factor of the uniaxially cracked concrete
- G_0 = Initial shear modulus of uncracked concrete
- $d\tau_{12}, d\tau_{13}$ = Shear stress increment in crack interface pointing to σ_2 and σ_3 direction respectively
- $d\gamma_{12cr}, d\gamma_{13cr}$ = Shear crack strain increment in crack interface pointing to

σ_2 and σ_3 direction respectively

$d\gamma_{12}, d\gamma_{13}$ = Shear strain increment in crack interface of uniaxially cracked concrete pointing to σ_2 and σ_3 direction respectively

$d\gamma_{120}, d\gamma_{130}$ = Shear strain increment of the coupling solid pointing to σ_2 and σ_3 direction respectively

From equations (4-28) and (4-29), the constitutive relationships of the crack can be written in matrix form as

$$\mathbf{d}\boldsymbol{\varepsilon}'_{cr} = \mathbf{C}'_{cr} \mathbf{d}\boldsymbol{\sigma}' \quad (4-30)$$

where $\mathbf{d}\boldsymbol{\varepsilon}'_{cr}$ is the incremental crack strain vector in its local crack coordinates

$$\mathbf{d}\boldsymbol{\varepsilon}'_{cr} = \begin{Bmatrix} d\varepsilon_{1cr} \\ d\gamma_{12cr} \\ d\gamma_{13cr} \end{Bmatrix} \quad (4-31)$$

$\mathbf{d}\boldsymbol{\sigma}'$ is the incremental stress vector in crack local coordinates

$$\mathbf{d}\boldsymbol{\sigma}' = \begin{Bmatrix} d\sigma_1 \\ d\tau_{12} \\ d\tau_{13} \end{Bmatrix} \quad (4-32)$$

$\mathbf{C}'_{cr}$ is the tangent material compliance of the crack in its local crack coordinates and

$$\mathbf{C}'_{cr} = \begin{bmatrix} \left(\frac{1}{E_1} - \frac{1}{E_{1c}} \right) & 0 & 0 \\ 0 & \left(\frac{1}{\beta G_0} - \frac{1}{G_0} \right) & 0 \\ 0 & 0 & \left(\frac{1}{\beta G_0} - \frac{1}{G_0} \right) \end{bmatrix} \quad (4-33)$$

From equations (4-28) and (4-29), the constitutive relationships of the uniaxially cracked concrete (Fig.4.10) can also be written in matrix form as

$$\mathbf{d}\boldsymbol{\varepsilon}'_u = \mathbf{C}'_u \mathbf{d}\boldsymbol{\sigma}' \quad (4-34)$$

where $\mathbf{d}\boldsymbol{\varepsilon}'_u$ is the strain incremental vector of uniaxially cracked concrete in its local crack coordinates

$$d\mathbf{\epsilon}'_u = \begin{Bmatrix} d\epsilon_{1u} \\ d\gamma_{12} \\ d\gamma_{13} \end{Bmatrix} \quad (4-35)$$

$\mathbf{C}'_u$ is the tangent material compliance of the uniaxially cracked concrete in its local crack coordinates and

$$\mathbf{C}'_u = \begin{bmatrix} \frac{1}{E_1} & 0 & 0 \\ 0 & \frac{1}{\beta G_0} & 0 \\ 0 & 0 & \frac{1}{\beta G_0} \end{bmatrix} \quad (4-36)$$

4.3.3.2 Compressive crack strain versus compression softening effect

If σ_i ($i = 2, 3$) direction of the uniaxial cracked concrete as shown in Fig. 4.3 (a) is in compression and the crack is under opening state, the compression softening effect due to cracking is considered. A softened compressive uniaxial stress-equivalent uniaxial strain curve expressed by equation (3-11) is used for σ_i direction (Fig. 4.12 curve b). The peak stress and peak strain of the constitutive curve is determined by equations (4-14) and (4-15). In the same direction of the coupling solid (Fig. 4.3 (b)), a uniaxial compressive stress-strain curve is used (Fig. 4.12 curve a).

As shown in Fig. 4.12, the compression softening in the uniaxial cracked concrete introduces additional compressive strain, termed here as compressive crack strain, compared to the response of the coupling solid. Within an incremental stress $d\sigma_i$ from σ_i^j to σ_i^{j+1} , due to progressive increase of tensile strain ϵ_{1u} in 1-direction, the compressive stress-strain curve gradually changes from curve b^j to curve b^{j+1} (Fig. 4.12). For simplicity, the progressive change from curve b^j to curve b^{j+1} is treated as a sudden change. It is assumed that firstly the compressive curve b^j does not change within the load increment. At this stage, the state point P^j progresses to the

interim state point P^{jm} along the tangent stiffness line. One part of the compressive crack strain $d\varepsilon_{icra}$ is produced and can be evaluated as

$$d\varepsilon_{icra} = d\varepsilon_{iu} - d\varepsilon_{i0} = \frac{1}{E_i} d\sigma_i - \frac{1}{E_{i0}} d\sigma_i = \left(\frac{1}{E_i} - \frac{1}{E_{i0}} \right) d\sigma_i \quad (i = 2, 3) \quad (4-37)$$

where

- E_i = Tangent modulus of elasticity at P^j , $i = 2, 3$
- E_{i0} = Tangent modulus of elasticity on the compressive curve of the coupling solid with the same compressive stress at P^j
- $d\sigma_i$ = Compressive stress increment in i -direction, $i = 2, 3$
- $d\varepsilon_{iu}$ = Compressive strain increment of the uniaxial cracked concrete in i -direction due to $d\sigma_i$, $i = 2, 3$
- $d\varepsilon_{i0}$ = Compressive strain increment of the coupling solid in i -direction due to $d\sigma_i$, $i = 2, 3$

Fig. 4.12 Compressive Crack Strain due to Compression Softening Effect

After the interim state point P^{jm} , a sudden change is assumed from P^{jm} to P^{j+1} and another part of compressive crack strain $d\epsilon_{icrb}$ is generated. $d\epsilon_{icrb}$ can be calculated directly from the constitutive curves b^j and b^{j+1} given the same stress state of $\sigma_i + d\sigma_i$.

The compressive crack strain is thus obtained as

$$d\epsilon_{icr} = d\epsilon_{icra} + d\epsilon_{icrb} \quad (i = 2, 3) \quad (4-38)$$

Similar to the treatment of normal tensile crack strain and shear crack strain, the compressive crack strain is treated as a kind of crack strain. The total compressive crack strain of all the cracks is assumed to be contributed by each crack and evaluated by a simple summation of the contribution of each crack. These compressive crack strains can be included into the crack strains in equation (4-17).

As the part of compressive crack strain $d\epsilon_{icrb}$ can be evaluated directly from constitutive curves, it is treated separately as stress-free strain and is added to equation (4-17) directly. To evaluate the free nodal displacement corresponding to this stress-free strain $d\epsilon_{icrb}$, a fictitious stress is first calculated as

$$d\sigma_{icrb} = E_i^{j+1} d\epsilon_{icrb} \quad (4-39)$$

where E_i^{j+1} is the tangent modulus of elasticity at P^{j+1} on curve b^{j+1} shown in Fig. 4. 12. The equivalent nodal force vector due to stress-free strain $d\epsilon_{icrb}$ is obtained by integrating the stress $d\sigma_{icrb}$ over the element domain. Eventually, the equivalent nodal force vector is added to the externally applied load vector.

Therefore, only the compressive crack strain $d\epsilon_{icra}$ is taken into account to evaluate the material compliance of cracks. The constitutive relationship expressed in equation (4-37) for compressive crack strains can be included in matrix form for tensile and shear strains in equation (4-30). The crack strain vector is thus written as

$$\mathbf{d}\boldsymbol{\varepsilon}'_{cr} = \begin{Bmatrix} d\varepsilon_{1cr} \\ d\varepsilon_{2cr} \\ d\varepsilon_{3cr} \\ d\gamma_{12cr} \\ d\gamma_{13cr} \end{Bmatrix} \quad (4-40)$$

$\mathbf{d}\boldsymbol{\sigma}'$ in equation (4-32) is expanded as

$$\mathbf{d}\boldsymbol{\sigma}' = \begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \\ d\tau_{12} \\ d\tau_{13} \end{Bmatrix} \quad (4-41)$$

The tangent material compliance $\mathbf{C}'_{cr}$ of the crack in equation (4-33) is expanded as

$$\mathbf{C}'_{cr} = \begin{bmatrix} \left(\frac{1}{E_1} - \frac{1}{E_{1c}}\right) & 0 & 0 & 0 & 0 \\ 0 & \left(\frac{1}{E_2} - \frac{1}{E_{20}}\right) & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{E_3} - \frac{1}{E_{30}}\right) & 0 & 0 \\ 0 & 0 & 0 & \left(\frac{1}{\beta G_0} - \frac{1}{G_0}\right) & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{1}{\beta G_0} - \frac{1}{G_0}\right) \end{bmatrix} \quad (4-42)$$

Equation (4-42) is for the case where all the directions of 2 and 3 in the crack coordinates are in compression. If the direction is in tension, the corresponding term in equation (4-42) is set to zero. By use of equation (4-42), all the cracking effects of tension softening, tension stiffening, aggregate interlock and compression softening are accounted for.

The strain incremental vector $\mathbf{d}\boldsymbol{\varepsilon}'_u$ of uniaxially cracked concrete in equation (4-35) is rewritten as

$$\mathbf{d}\boldsymbol{\varepsilon}'_u = \begin{Bmatrix} d\varepsilon_{1u} \\ d\varepsilon_{2u} \\ d\varepsilon_{3u} \\ d\gamma_{12} \\ d\gamma_{13} \end{Bmatrix} \quad (4-43)$$

The tangent material compliance $\mathbf{C}'_u$ of the uniaxially cracked concrete in equation (4-36) is transferred to

$$\mathbf{C}'_u = \begin{bmatrix} \frac{1}{E_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{E_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\beta G_0} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\beta G_0} \end{bmatrix} \quad (4-44)$$

4.3.4 Tracing Strain State of the Intact Concrete and Each Uniaxial Cracked Concrete

To evaluate the stiffness matrix by equation (4-26) requires tracing the strain state of the intact concrete and each uniaxial cracked concrete. The strain increment of the intact concrete is evaluated by the decomposition of the total strain increment

$$\mathbf{d}\boldsymbol{\varepsilon}'_0 = \mathbf{C}'_0 \mathbf{d}\boldsymbol{\sigma}' = \mathbf{C}'_0 \mathbf{N}_0 \mathbf{d}\boldsymbol{\sigma} = \mathbf{C}'_0 \mathbf{N}_0 \mathbf{D} \mathbf{d}\boldsymbol{\varepsilon} \quad (4-45)$$

The equivalent uniaxial strain increment $\mathbf{d}\boldsymbol{\varepsilon}'_{0u}$ can then be obtained from equation (3-8). The current equivalent strain state is obtained by accumulation of the strain increment of the above

$$\boldsymbol{\varepsilon}'_{0u} = \int \mathbf{d}\boldsymbol{\varepsilon}'_{0u} \quad (4-46)$$

The initial strain state of each uniaxial cracked concrete is defined as the strain state of the intact concrete just when the crack occurs. The subsequent strain increment is calculated by the decomposition of the total strain increment

$$d\boldsymbol{\varepsilon}'_{iu} = \mathbf{C}'_{iu} d\boldsymbol{\sigma}' = \mathbf{C}'_{iu} \mathbf{N}_i d\boldsymbol{\sigma} = \mathbf{C}'_{iu} \mathbf{N}_i \mathbf{D} d\boldsymbol{\varepsilon} \quad (4-47)$$

where i refers to the i -th uniaxial cracked concrete and $\mathbf{C}'_{iu}$ is defined in equation (4-44). The current strain state of each uniaxial cracked concrete is obtained by accumulation of the strain increment as

$$\boldsymbol{\varepsilon}'_{iu} = \int d\boldsymbol{\varepsilon}'_{iu} \quad (4-48)$$

From equations (4-45) to (4-48) the strain state of the intact concrete and each uniaxial cracked concrete can be traced and the modulus of elasticity corresponding to current strain state can be obtained from constitutive relationships. The material property of CRC can thus be defined completely by the above developed model.

4.3.5 Definition of Concrete Crushing in Multidirectional-Cracked Concrete and Treatment of Post Crushing

When the absolute value of the principal compressive strain is larger than the minimum absolute value of the peak strains of the uniaxial cracked concrete, compressive crushing is assumed for the multidirectional-cracked concrete. It can be expressed as follows

$$|\varepsilon_3| > \text{Min}(|\varepsilon_{ic}|_{i=1, \dots, n}) \quad (4-49)$$

where ε_{ic} is the compressive peak strain of the i -th uniaxial cracked concrete. The stress and equivalent uniaxial strain on the compressive constitutive curve at crushing are taken as the peak stress and peak strain of compressive curve modeling the post crushing response of each uniaxial cracked concrete (Fig. 4.13).

Fig. 4.13 Compressive Curve for Fictitious Uniaxial Cracked Concrete

4.3.6 Numerical Experiments on Compression Softening Effect of MR-CRC

Since there is currently a lack of proper data to verify the developed material model in modeling the compression softening effect of MR-CRC on the element level, two numerical experiments have been designed and analyzed. The performance of the proposed model has been demonstrated in this numerical example. A biaxial orthogonally cracked concrete with constant normal tensile strains, a special case of multidirectional non-orthogonally cracked concrete, is under compression in direction parallel to crack planes.

The first experiment is under equal biaxial tension of tensile strain of 0.004. In the second experiment, the constant tensile strains in the two orthogonal directions are 0.004 and 0.012 respectively. The compressive strength of concrete is 25 MPa and the compressive peak strain is 0.002. The predicted compressive stress-strain responses of the biaxial cracked concrete are shown in Fig. 4.14 along with the uniaxial compressive curve as well as the softened compressive curve of each

(a) The specimen with equal biaxial tensile strains

(b) The specimen with unequal biaxial tensile strains

Fig. 4.14 Predicted Compressive Response of the Numerical Experiments

uniaxial cracked concrete. The predicted strength and stiffness before crushing in both numerical experiments are found to be lower than those of any uniaxial cracked concrete. The predicted strength of the equal biaxial tension experiment is 13.9 MPa which is 17.3% lower than the strength (16.8 MPa) of the uniaxial cracked concrete with tensile strain 0.004 and is 9.7% lower than the predicted strength (15.4 MPa) by the effective tensile strain approach ($\varepsilon_1^e = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}$, Kirschner and Collins 1986). The predicted strength of the unequal biaxial tension experiment is 11.2 MPa which is 8.2% lower than the strength (12.2 MPa) of the uniaxial cracked concrete with tensile strain 0.012 and is 6.7% lower than the predicted strength (12.0 MPa) by the effective tensile strain approach ($\varepsilon_1^e = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}$, Kirschner and Collins 1986). The developed approach in this study predicted apparent lower compressive strength and stiffness for MR-CRC than the effective tensile strain approach did ($\varepsilon_1^e = \sqrt{\varepsilon_1^2 + \varepsilon_2^2}$, Kirschner and Collins 1986).

4.3.7 Cracking Criteria for Secondary Cracks

Allowing cracks to form every time the stresses rotate slightly and violate the tensile strength in the new principal direction leads to excessive formation of new cracks and closing of existing cracks. A threshold angle scheme had been proposed by de Borst and Nauta (1985) to restrict new cracks to form at an integration point only when the angle between the principal stress direction and the normal to the existing crack has exceeded the threshold angle. Typically the threshold angle used ranges from 15 to 30 degrees. However, it has been observed that for some problems this range is not sufficient and angles as large as 60 degree have been employed (de Borst and Nauta 1985). In this study, the strength criterion is augmented by a minimum threshold angle criterion, whereby a new crack is initiated only if the strength criterion is violated and the crack normal of a new crack is directed at a skew angle larger than the threshold angle with respect to the normals of any previously existing cracks. Because such a criterion may cause the tensile stresses to build up to levels greater than the cracking strength of concrete

under uniaxial tension, a tension cutoff procedure is employed limiting the tensile stresses at a sampling point to the uniaxial cracking strength of concrete.

4.3.8 Closing and Opening of Cracks

In smeared crack analyses of concrete members there is a likelihood of the occurrence of spurious cracking. This means there are quite a number of sampling points which crack, but only show small crack strain. The sampling points with small crack strains pose a problem as a number of them show unloading, even close and sometimes open again in the later stage of the loading process. Unloading of cracks occurs for instance when new cracks at the same integration point arise. For unloading response, a secant approach as suggested by de Borst and Nauta (1985) is adopted in this study. The tangent moduli appearing in equation (4-26) are replaced by secant moduli counterparts. This assumption is too simple as in reality some residual strain upon closing of a crack is expected, but this procedure is numerically stable (de Borst and Nauta 1985). When a crack fully closes, i.e. when the normal stress in the crack changes from tension into compression, the full elastic stiffness is inserted. Although upon closing of a crack, the normal stress and normal strain both vanish, this needs not to be true for the shear stress and shear strain.

4.4 LOADING, UNLOADING AND RELOADING CRITERIA

The state of material loading or unloading affects the evaluation of stiffness matrix. The loading, unloading, and reloading conditions developed in this study are defined by a loading function

$$f = \sqrt{\sum_{i=1}^3 \left(\frac{\varepsilon_i}{\varepsilon_{ic}} \right)^2} \quad (4-50)$$

Where ε_i is the i -th principal strain and

$$\varepsilon_{ic} = \begin{cases} \varepsilon_i & \varepsilon_i \geq 0 \\ \varepsilon_c & \varepsilon_i < 0 \end{cases} \quad (4-51)$$

In which

- ε_t = Crack strain of concrete under uniaxial tension
- ε_c = Compressive peak strain of concrete under uniaxial compression

The condition of material loading is $f_i > f_{i-1}$ and $f_i > f_{\max}$, where f_i is the loading function in current loading step and f_{i-1} is the loading function for previous loading step. $f_{\max}$ is the maximum value of the loading function in (4-50) during the previous loading history. The unloading condition is $f_i < f_{i-1}$. Reloading is assumed when $f_i > f_{i-1}$ and $f_i < f_{\max}$. Under loading conditions, the tangent modulus of elasticity is used to evaluate the tangent stiffness matrix in equation (4-26). Under unloading and reloading conditions, the secant modulus is used to replace the tangent modulus in equation (4-26).

4.5 MODELING REINFORCEMENT

Fig. 4.15 Reinforcements Description

A typical arrangement of reinforcement in a shell element is shown in Fig. 4.15. The in-plane and out-of-plane reinforcement are treated differently in this study.

4.5.1 In-Plane Reinforcement

Fig. 4.16 Equivalent Thickness of In-Plane Reinforcement

In-plane reinforcement is the primary reinforcement situated in the plane of the shell element (Fig. 4.15). The in-plane reinforcement can be arranged at an arbitrary orientation in the element plane. It is treated as an equivalent uniaxial layered material placed at the depth of the centerline of the bars and smeared out over the spacing of bars. The equivalent thickness of the reinforcement layer is evaluated as follows (Fig. 4.16)

$$\delta_s = \frac{A_s}{s} \quad (4-52)$$

where

$$\begin{aligned} A_s &= \text{Area of cross section of bar} \\ s &= \text{Spacing of bars} \end{aligned}$$

As many layers are used as there are layers of bars in the cross section. This in-plane reinforcement layer is modeled separately independent from concrete layers. The stiffness contribution by the in-plane reinforcement is added to the element stiffness matrix. The bond between steel and concrete is assumed to remain perfect. The in-plane reinforcement lies in element plane xy . Therefore, only inplane strains are needed to calculate the strain in the reinforcement direction. The material matrix for a reinforcement component, in the direction of the reinforcement, is

$$\mathbf{D}_{sp'} = \begin{bmatrix} E_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4-53)$$

The matrix is then transformed to element xyz local coordinate system

$$\mathbf{D}_{sp} = \mathbf{T}_{sp}^T \mathbf{D}_{sp'} \mathbf{T}_{sp} \quad (4-54)$$

where $\mathbf{T}_{sp}$ is the transformation matrix for in-plane reinforcement. E_s is the tangent modulus of steel and can be evaluated by the constitutive relationship for in-plane reinforcement. $\mathbf{D}_{sp}$ can then be applied to calculate stiffness contributed by this in-plane reinforcement layer.

Belarbi and Hsu (1994) showed that when reinforcing bars are surrounded by concrete as in membrane elements, the yield stress of steel is lower than a bare steel bar. The determination of the lowered yield stress involves the detailed tension stiffening model. This phenomenon is accounted and the stress-strain relationship used for in-plane reinforcement in tension is modeled by a tri-linear curve as shown in Fig. 4.17. The steel stress is calculated by the follow equation

$$\sigma_s = \begin{cases} E_s \varepsilon_s & \varepsilon_s < \varphi \varepsilon_y \\ \varphi f_y + E_z (\varepsilon_s - \varphi \varepsilon_y) & \varphi \varepsilon_y \leq \varepsilon_s \leq k \varepsilon_y \\ \varphi f_y + E_z (k - \varphi) \varepsilon_y + E_p (\varepsilon_s - k \varepsilon_y) & \varepsilon_s > k \varepsilon_y \end{cases} \quad (4-55)$$

where

$$E_z = \zeta E_s, \quad E_p = \alpha E_s \quad (4-56)$$

E_s = Modulus of elasticity of steel

ζ = 7.5% in this study

α = 2.5%, strain hardening parameter

ε_y = Strain at yielding of steel

f_y = Yield stress of steel

Fig. 4.17 Stress-Strain Relationship of In-Plane Reinforcement

To model membrane elements in this study, ϕ takes the value of 0.8~0.9 and k takes the value 1.5.

The stress-strain relationship used for in-plane reinforcement in compression is modeled by a bi-linear curve as shown in Fig. 4.17. The stress of steel is expressed as

$$\sigma_s = \begin{cases} E_s \varepsilon_s & |\varepsilon_s| < |\varepsilon_y| \\ -f_y & |\varepsilon_s| \geq |\varepsilon_y| \end{cases} \quad (4-57)$$

4.5.2 Out-Of-Plane Reinforcement

The out-of-plane reinforcement refers to the reinforcement not situated in the element plane xy (Fig. 4.15). It can be directed at an arbitrary angle with respect to the element surface normal direction z . The out-of-plane reinforcement is specified

as a property of concrete and is treated along with the concrete layers. It contributes to the stiffness only in reinforcement direction. The constitutive relationship for out-of-plane steel is taken to be linear elastic perfectly plastic. The stress-strain relationship used is the same as that for in-plane reinforcement under compression as shown in Fig. 4.18.

Fig. 4.18 Stress-Strain Relationship for In-Plane Reinforcement in Compression and Out-Of-Plane Reinforcement

The mechanism how the out-of-plane reinforcement participates in the transverse shear resistance has been illustrated in the study by Polak (1992). This study follows the procedure proposed by Polak (1992) to incorporate the stiffness contribution by out-of-plane reinforcement. This can be done by first finding the strain in the direction of reinforcement. The stress is then found by use of the constitutive relationship for out-of-plane steel. The material matrix for reinforcement, $\mathbf{D}_{st'}$, is formulated as

$$\mathbf{D}_{st'} = \begin{bmatrix} \rho E_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-58}$$

where ρ is the reinforcement ratio.

The matrix is then transformed to local element coordinate system x, y, z

$$\mathbf{D}_{st} = \mathbf{T}_{st}^T \mathbf{D}_{st} \mathbf{T}_{st} \quad (4-59)$$

where $\mathbf{T}_{st}$ is the transformation matrix between the out-of-plane reinforcement and the local element coordinate system. For a concrete layer with out-of-plane reinforcement, the total material matrix is found by the sum of the concrete and reinforcement contributions

$$\mathbf{D}^* = \mathbf{D}_c + \mathbf{D}_{st} \quad (4-60)$$

where $\mathbf{D}_c$ is the contribution by concrete which can be evaluated by equation (4-26).

The combined material stiffness matrix is a 6×6 matrix as follows

$$\mathbf{D}^* = \begin{bmatrix} \mathbf{D}_{11}^* & \mathbf{D}_{12}^* & \mathbf{D}_{13}^* & \mathbf{D}_{14}^* & \mathbf{D}_{15}^* & \mathbf{D}_{16}^* \\ \mathbf{D}_{21}^* & \mathbf{D}_{22}^* & \mathbf{D}_{23}^* & \mathbf{D}_{24}^* & \mathbf{D}_{25}^* & \mathbf{D}_{26}^* \\ \mathbf{D}_{31}^* & \mathbf{D}_{32}^* & \mathbf{D}_{33}^* & \mathbf{D}_{34}^* & \mathbf{D}_{35}^* & \mathbf{D}_{36}^* \\ \mathbf{D}_{41}^* & \mathbf{D}_{42}^* & \mathbf{D}_{43}^* & \mathbf{D}_{44}^* & \mathbf{D}_{45}^* & \mathbf{D}_{46}^* \\ \mathbf{D}_{51}^* & \mathbf{D}_{52}^* & \mathbf{D}_{53}^* & \mathbf{D}_{54}^* & \mathbf{D}_{55}^* & \mathbf{D}_{56}^* \\ \mathbf{D}_{61}^* & \mathbf{D}_{62}^* & \mathbf{D}_{63}^* & \mathbf{D}_{64}^* & \mathbf{D}_{65}^* & \mathbf{D}_{66}^* \end{bmatrix} \quad (4-61)$$

The $\mathbf{D}^*$ matrix must then be modified to enforce the zero normal stress conditions.

For σ_z to equal zero, the following must hold

$$\varepsilon_z = \frac{-(\mathbf{D}_{31}^* \varepsilon_x + \mathbf{D}_{32}^* \varepsilon_y + \mathbf{D}_{34}^* \gamma_{xy} + \mathbf{D}_{35}^* \gamma_{yz} + \mathbf{D}_{36}^* \gamma_{zx})}{\mathbf{D}_{33}^*} \quad (4-62)$$

Modification of the $\mathbf{D}^*$ requires that

- i) row 3 and column 3 be removed

ii) the entries in the new $\mathbf{D}_{cs}$ matrix be:

$$\mathbf{D}_{csij} = \mathbf{D}_{ij}^* - \frac{\mathbf{D}_{i3}^* \mathbf{D}_{3j}^*}{\mathbf{D}_{33}^*} \quad (4-63)$$

matrix $\mathbf{D}_{cs}$ is a 5×5 matrix

In this study, the incremental form of equations (4-62) and (4-63) is used.

CHAPTER 5 SHELL ELEMENT AND LAYERING SCHEME

5.1 INTRODUCTION

The degenerated isoparametric shell element developed by Ahmad et al. (1970) is adopted in this study to model RC flat plates. The shell formulations are simplified considering the flat shape with constant thickness of flat plate floors to improve the computational efficiency. A layering scheme for correction on transverse shear deformation is formulated directly in element formulations. Each node of the layered shell element is allowed to have different degrees of freedom.

5.2 SHELL ELEMENT

Reinforced concrete slabs can be seen as a special type of shells with a flat geometry and constant thickness. For finite element modeling of such reinforced concrete slabs, a three-dimensional element is a natural choice. However, certain difficulties will be encountered. In the first place the retention of three degrees of freedom at each node leads to large stiffness coefficients for relative through-thickness displacements. This presents numerical problems and inevitably leads to ill-conditioned equations when the shell thickness becomes small compared with the other dimensions of the element (Ahmad et al. 1970). The second factor is that of computational economy. The use of several nodes across the shell thickness ignores the well-known fact that even for thick shells the “normals” to the middle surface remain practically straight after deformation. Thus an unnecessarily high number of degrees of freedom has to be carried, involving penalties of computer time (Ahmad et al. 1970). All these difficulties can be avoided by using a thick shell element in which the constraint of straight “normals” is introduced to overcome the second problem and the strain energy corresponding to stresses perpendicular to the middle surface is ignored. It should be noted that the two

constraints introduced correspond only to a part of the usual assumptions of shell theory. Thus, the statement that after deformation the normals remain normal to the deformed middle surface has been deliberately omitted. This omission permits the shell to experience shear deformations. Benefiting from all these advantages of this shell element over other three dimensional and two dimensional elements, it has been extensively used for finite element modeling of reinforced concrete slabs (Ramaswamy et al. 1995; Loo and Guan 1997; Polak 1998).

The serendipity shell element developed by Ahmad et al. (1970) was the first degenerated shell element formulated. It has been the most widely used element of degenerated type. The quadratic element has eight nodes, each with three translations and two rotations as degree of freedom. The displacement field and geometry of the element are defined using quadratic serendipity shape functions. The rank of the stiffness matrix remains correct when evaluated using numerical integration. The serendipity element performs not well for thin shells although reduced integration improves the performance somewhat.

In design practice, the thickness of reinforced concrete flat plate structures is commonly not very thin according to building code specifications that limit the allowable deflection of the slab. The shear locking problem associated with thin plates is not considered in the present study. Thereafter, the formulation of serendipity degenerate shell element is employed in conjunction with full integration scheme.

It should be noted that the shell element is formulated primarily for modeling the curved shape of shells with variable thickness. Transformations necessary are somewhat involved and indeed sophisticated programming is needed. If the standard formulation of the shell element is employed to model reinforced concrete slabs characterized by flat geometry and constant thickness, additional unnecessary computational efforts will be introduced. In this study, the shell formulations are thus directly simplified. Fig. 5.1 demonstrates the shell element simplified from the degenerated isoparametric shell element developed by Ahmad et al. (1970). The element formulations are presented as follows.

Fig. 5.1 Shell Element with Constant Thickness and Flat Shape

5.2.1 Coordinate Systems

Four systems are used in this study to describe the geometry, displacements, strains and stresses.

Global coordinate system— $X Y Z$

This system is usually chosen for convenience. The nodal displacements, global stiffness, and load vector are referred to this system (see Fig. 5.2).

Fig. 5.2 Global Coordinate System and Local Element Coordinate System

Local element coordinate system— $x\ y\ z$

The local element coordinate system is defined in the middle surface of an element (Fig. 5.1).

Natural coordinate system— $\xi\ \eta\ \zeta$

This system is a normalized coordinate system (see Fig. 5.1). ξ, η vary from -1 to $+1$ between edges of an element. ζ directly is set as z . This enables a consistent definition of shape functions, element geometry and stress resultants for all elements.

Principal axis coordinate system— $x'\ y'\ z'$

This system is defined at sampling points consistent with the orientation of the principal strain axes or principal stress axes. The sampling points in this study are the Gauss Points.

5.2.2 Shape Functions

The element mid-surface is defined by only two coordinates ξ, η and a considerable saving in computer effort is obtained. The same shape functions are used in geometric and functional interpolation. The coordinates of each node are as follows:

$$\left. \begin{array}{ll} (\xi_1, \eta_1) = (-1, -1), & (\xi_2, \eta_2) = (1, -1), \\ (\xi_3, \eta_3) = (1, 1), & (\xi_4, \eta_4) = (-1, 1), \\ (\xi_5, \eta_5) = (0, -1), & (\xi_6, \eta_6) = (1, 0), \\ (\xi_7, \eta_7) = (0, 1), & (\xi_8, \eta_8) = (-1, 0) \end{array} \right\} \quad (5-1)$$

and the shape functions for each node are:

$$N_i(\xi, \eta) = \begin{cases} \frac{1}{4}(1 + \xi_i \xi)(1 + \eta_i \eta)(\xi_i \xi + \eta_i \eta - 1), & (i = 1, 2, 3, 4) \\ \frac{1}{2}(1 - \xi^2)(1 + \eta_i \eta), & (i = 5, 7) \\ \frac{1}{2}(1 - \eta^2)(1 + \xi_i \xi), & (i = 6, 8) \end{cases} \quad (5-2)$$

The geometry of an element is described by the coordinates in its local element coordinate system as:

$$\begin{cases} x = \sum_{i=1}^8 N_i(\xi, \eta) x_i \\ y = \sum_{i=1}^8 N_i(\xi, \eta) y_i \end{cases} \quad (5-3)$$

where x_i, y_i are the coordinates of nodes.

5.2.3 Displacement Field and Strain Definition

Displacement field

From the formulation for a curved shell element, the displacement field can be directly written as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^8 N_i(\xi_i, \eta_i) \begin{Bmatrix} u_i + z\theta_{iy} \\ v_i - z\theta_{ix} \\ w_i \end{Bmatrix} \quad (5-4)$$

where u, v, w are displacements in the directions of the local element x, y, z axes.

u_i, v_i, w_i are three translations of the i -th nodal point and θ_{ix}, θ_{iy} are rotations of the i -th nodal point around x and y axes respectively.

Strain definition

There are only five independent strains because it is assumed that the stress in the z direction is zero. The strains are:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{Bmatrix} \quad (5-5)$$

Substitute (5-4) into (5-5),

$$= \sum_{i=1}^8 \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & z \frac{\partial N_i}{\partial x} \\ 0 & \frac{\partial N_i}{\partial y} & 0 & -z \frac{\partial N_i}{\partial y} & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & -z \frac{\partial N_i}{\partial x} & z \frac{\partial N_i}{\partial y} \\ 0 & 0 & \frac{\partial N_i}{\partial y} & -N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{ix} \\ \theta_{iy} \end{Bmatrix} \quad (5-6)$$

Let

$$\boldsymbol{\delta}_i^e = \{u_i \quad v_i \quad w_i \quad \theta_{ix} \quad \theta_{iy}\}^T \quad (5-7)$$

$$\boldsymbol{\delta}^e = \{\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \dots, \boldsymbol{\delta}_8\}^T \quad (5-8)$$

where $\boldsymbol{\delta}^e$ is called element nodal displacement vector.

$$\mathbf{B}_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & z \frac{\partial N_i}{\partial x} \\ 0 & \frac{\partial N_i}{\partial y} & 0 & -z \frac{\partial N_i}{\partial y} & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & -z \frac{\partial N_i}{\partial x} & z \frac{\partial N_i}{\partial y} \\ 0 & 0 & \frac{\partial N_i}{\partial y} & -N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & N_i \end{bmatrix} \quad (i = 1, 2, \dots, 8) \quad (5-9)$$

$$\mathbf{B} = \{\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_8\} \quad (5-10)$$

Where $\mathbf{B}$ is called *strain matrix*. Thus, the strain vector can be rewritten as:

$$\boldsymbol{\varepsilon} = \sum_{i=1}^8 \mathbf{B}_i \delta_i^e = \mathbf{B} \boldsymbol{\delta}^e \quad (5-11)$$

5.2.4 Element Stiffness and Numerical Integretion

Jacobian matrix

To evaluate the stiffness matrix of an element, it is necessary to calculate the strain matrix $\mathbf{B}$. The element strains are obtained in terms of derivatives of element displacements with respect to the local element coordinates. Because the element displacements are defined in the natural coordinate system using (5-4), it is needed to relate the x, y derivatives to the ξ, η derivatives. By inverting Eqn. (5-3), we can get

$$\left. \begin{aligned} \xi &= \xi(x, y) \\ \eta &= \eta(x, y) \end{aligned} \right\} \quad (5-12)$$

We require the derivatives $\partial/\partial x, \partial/\partial y$, and it is natural to use the chain rule in the following form:

$$\left. \begin{aligned} \frac{\partial}{\partial \xi} &= \frac{\partial}{\partial x} \frac{\partial x}{\partial \xi} + \frac{\partial}{\partial y} \frac{\partial y}{\partial \xi}, \\ \frac{\partial}{\partial \eta} &= \frac{\partial}{\partial x} \frac{\partial x}{\partial \eta} + \frac{\partial}{\partial y} \frac{\partial y}{\partial \eta}. \end{aligned} \right\} \quad (5-13)$$

or in the matrix form:

$$\begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \quad (5-14)$$

If we define the *Jacobian operator* as:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (5-15)$$

then, Eqn. (5-14) can be rewritten as

$$\begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \quad (5-16)$$

and, its inverse relationship is

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = \mathbf{J}^{-1} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} \quad (5-17)$$

Substituting Eqn. (5-3) into (5-15), we can get

$$\mathbf{J} = \sum_{i=1}^8 \begin{bmatrix} \frac{\partial N_i}{\partial \xi} x_i & \frac{\partial N_i}{\partial \xi} y_i \\ \frac{\partial N_i}{\partial \eta} x_i & \frac{\partial N_i}{\partial \eta} y_i \end{bmatrix} \quad (5-18)$$

The element stiffness matrix $\mathbf{K}^e$ can be evaluated by the following equation

$$\mathbf{K}^e = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-1}^1 \int_{-1}^1 \mathbf{B}^T \mathbf{D} \mathbf{B} |\mathbf{J}| d\xi d\eta dz \quad (5-19)$$

where $\mathbf{D}$ is the material stiffness matrix and h is the thickness of the element.

The internal force vector $\mathbf{F}^e$ is defined as

$$\mathbf{F}^e = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{-1}^1 \int_{-1}^1 \mathbf{B}^T \boldsymbol{\sigma} |\mathbf{J}| d\xi d\eta dz \quad (5-20)$$

In this study, A 3×3 full Gauss numerical scheme is employed to calculate the the above element stiffness.

5.3 PROPOSED LAYERING SCHEME FOR INCORPORATION OF TRANSVERSE SHEAR DEFORMATION

The shell element formulations developed in section 5.1 along with layering procedures can be applied to model the nonlinear material properties of concrete and reinforcement through the cross section. However, it still has limitations in that the transverse shear stresses and shear strains within the cross section are not properly computed. A constant shear strain distribution across the depth of the section will be obtained because variation in the transverse displacement and cross-sectional rotation are not permitted through the thickness of the element. For nonlinear analysis of reinforced concrete flat plate structures where transverse shear stress is significant, this misrepresentation of shear strain and shear stress distribution will affect the prediction of cracking patterns of the structure, the ultimate state response and hence particularly the failure mode of the structure. In RC flat plate structures, the concentration of stresses diminishes very rapidly.

Significant transverse shear stress only exists in small regions of a slab-column connection. Efficient finite element analysis of RC flat plate structures requires that shell elements with shear correction being applied in regions surrounding the columns and normal shell elements without shear correction to being used outside the slab-connection regions.

The explicit layering procedures developed by Mawnya and Davies (1974) and Barzegar (1988) are suitable to be applied in regions of slab-column connections. The introduction of two independent rotational DOFs at each layer highly improves the accuracy in representing the transverse shear deformation. However, the explicit layering procedures do not provide the flexibility to be applied along with normal shell element without shear correction. This is because, in their procedures, all the nodes of one element have the two independent rotational DOFs at each layer.

In this study, a more flexible layering scheme for shear correction is formulated in element formulations. Each node of the formulated element can be specified as a shear correction node with two independent rotational DOFs at each layer or normal node without shear correction.

5.3.1 Layering Scheme and Nodal Displacement

In the author's proposed layering scheme, the shell element shown in Fig. 5.2 is divided into n layers with m ($m < n$) layers below and $n-m$ layers above the reference plane (Fig. 5.3 (a)). The layers are numbered from bottom to top. Different layers may have different thickness. The reference plane may not be the middle surface of the element. Each node of the shell element can be specified as normal node or node with shear correction.

The normal node shown as Node i in Fig. 5.3 (a) has in-plane displacements u_i and v_i of the reference plane, the lateral displacement w_i , and normal rotations θ_{ix} and θ_{iy} . The displacement vector of this normal node i can be expressed by equation (5-7) as $\delta_i^e = \{u_i \quad v_i \quad w_i \quad \theta_{ix} \quad \theta_{iy}\}^T$. At this node, each layer has the same rotation

(a) Layering Scheme

(b) Assumed deformation of the layered element

Fig. 5.3 Proposed Layering Scheme for Shear Correction

θ_{ix} around x-axis and θ_{iy} around y-axis. The rotation vector around x-axis of all the layers at node i is written as

$$\boldsymbol{\theta}_{ix} = \{\theta_{ix1} \quad \theta_{ix2} \quad \dots \quad \theta_{ixn}\}^T = \{\theta_{ix} \quad \theta_{ix} \quad \dots \quad \theta_{ix}\}^T \quad (5-21a)$$

and that around y-axis is

$$\boldsymbol{\theta}_{iy} = \{\theta_{iy1} \quad \theta_{iy2} \quad \dots \quad \theta_{iyn}\}^T = \{\theta_{iy} \quad \theta_{iy} \quad \dots \quad \theta_{iy}\}^T \quad (5-21b)$$

Shown in Fig. 5.3 (a), the node k is specified as a node with shear correction. The displacement components of the node k consists of the in-plane displacements u_k and v_k of the reference plane, the lateral displacement w_k , and independent normal rotations θ_{kxj} and θ_{kyj} in each layer. These may be listed by the vector

$$\boldsymbol{\delta}_k^e = \{u_k \quad v_k \quad w_k \quad \theta_{kx1} \quad \theta_{kx2} \quad \dots \quad \theta_{kxn} \quad \theta_{ky1} \quad \theta_{ky2} \quad \dots \quad \theta_{kyn}\}^T \quad (5-22)$$

Let

$$\boldsymbol{\theta}_{kx} = \{\theta_{kx1} \quad \theta_{kx2} \quad \dots \quad \theta_{kxn}\}^T \quad (5-23a)$$

and

$$\boldsymbol{\theta}_{ky} = \{\theta_{ky1} \quad \theta_{ky2} \quad \dots \quad \theta_{kyn}\}^T \quad (5-23b)$$

Equation (5-22) is rewritten in the following simpler form as

$$\boldsymbol{\delta}_k^e = \{u_k \quad v_k \quad w_k \quad \boldsymbol{\theta}_{kx}^T \quad \boldsymbol{\theta}_{ky}^T\}^T \quad (5-24)$$

5.3.2 Displacement Field and Strain Definition

Displacement field

Fig 5.3 (b) represents the cross section of the layered shell in which the reference xy-plane is defined as the plane $z=0$. The deformation of the shell element is described by the inplane displacements of the reference plane u_o and v_o , the transverse displacement w , which is taken as constant through the thickness, and two normal rotations θ_{xj} and θ_{yj} of each layer of the shell element. The normal rotations, though assumed uniform for any particular layer, vary from layer to layer.

By considering continuity of the in-plane displacements at the interface of the layers, the in-plane displacements of any point $P(x, y, z)$ in the j^{th} layer can be obtained

$$\begin{cases} u(x, y, z) = u_o + \mathbf{H}\boldsymbol{\theta}_y \\ v(x, y, z) = v_o - \mathbf{H}\boldsymbol{\theta}_x \end{cases} \quad (5-25)$$

where

$$\boldsymbol{\theta}_x = \{\theta_{x1} \quad \theta_{x2} \quad \dots \quad \theta_{xn}\}^T \quad (5-26a)$$

$$\boldsymbol{\theta}_y = \{\theta_{y1} \quad \theta_{y2} \quad \dots \quad \theta_{yn}\}^T \quad (5-26b)$$

$$\mathbf{H} = \begin{cases} \left\{ \begin{array}{c|c|c|c|c|c} \mathbf{0} & z + \sum_{k=j+1}^m t_k & -t_{j+1} & \dots & -t_m & \mathbf{0} \end{array} \right\} & z < 0 \\ \left\{ \begin{array}{c|c|c|c|c|c} \mathbf{0} & t_{m+1} & \dots & t_{j-1} & z - \sum_{k=m+1}^{j-1} t_k & \mathbf{0} \end{array} \right\} & z > 0 \end{cases} \quad (5-27)$$

The displacements u_o , v_o , θ_x , and θ_y can be interpolated from the nodal displacement of each node

$$u_o = \sum_{i=1}^8 N_i u_i \quad v_o = \sum_{i=1}^8 N_i v_i \quad \theta_x = \sum_{i=1}^8 N_i \theta_{ix} \quad \theta_y = \sum_{i=1}^8 N_i \theta_{iy} \quad (5-28)$$

where θ_{ix} and θ_{iy} are defined in equation (5-21) and (5-23) for a normal node without shear correction and a node with shear correction respectively.

Input equation (5-28) into (5-25), we obtain

$$\begin{cases} u(x, y, z) = \sum_{i=1}^8 N_i u_i + \mathbf{H} \sum_{i=1}^8 N_i \theta_{iy} = \sum_{i=1}^8 N_i (u_i + \mathbf{H} \theta_{iy}) \\ v(x, y, z) = \sum_{i=1}^8 N_i v_i - \mathbf{H} \sum_{i=1}^8 N_i \theta_{ix} = \sum_{i=1}^8 N_i (v_i - \mathbf{H} \theta_{ix}) \end{cases} \quad (5-29)$$

From equations (5-21) and (5-27), for a normal node without shear correction, the following relationship can be obtained

$$\begin{cases} \mathbf{H} \theta_{iy} = z \theta_{iy} \\ \mathbf{H} \theta_{ix} = z \theta_{ix} \end{cases} \quad (5-30)$$

Equation (5-29) is thus rewritten as

$$\begin{cases} u(x, y, z) = \sum_{i=1}^8 N_i \begin{cases} (u_i + \mathbf{H} \theta_{iy}) & \text{for shear correction node} \\ (u_i + z \theta_{iy}) & \text{for normal node} \end{cases} \\ v(x, y, z) = \sum_{i=1}^8 N_i \begin{cases} (v_i - \mathbf{H} \theta_{ix}) & \text{for shear correction node} \\ (v_i - z \theta_{ix}) & \text{for normal node} \end{cases} \end{cases} \quad (5-31)$$

Transverse displacement w is obtained directly from interpolation of nodal displacements as

$$w = \sum_{i=1}^8 N_i w_i \quad (5-32)$$

Substitute (5-31) and (5-32) into (5-5), we can get the strain vector expressed in equation (5-11)

$$\boldsymbol{\varepsilon} = \sum_{i=1}^8 \mathbf{B}_i \boldsymbol{\delta}_i^e = \mathbf{B} \boldsymbol{\delta}^e$$

where, for a normal node, $\boldsymbol{\delta}_i^e$ and $\mathbf{B}_i$ are defined by equations (5-7) and (5-9) respectively. For a shear correction node, $\boldsymbol{\delta}_i^e$ is defined by equation (5-32) and $\mathbf{B}_i$ is expressed as

$$\mathbf{B}_i = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & 0 & \mathbf{H} \frac{\partial N_i}{\partial x} \\ 0 & \frac{\partial N_i}{\partial y} & 0 & -\mathbf{H} \frac{\partial N_i}{\partial y} & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & -\mathbf{H} \frac{\partial N_i}{\partial x} & \mathbf{H} \frac{\partial N_i}{\partial y} \\ 0 & 0 & \frac{\partial N_i}{\partial y} & -\frac{\partial \mathbf{H}}{\partial z} N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial x} & 0 & \frac{\partial \mathbf{H}}{\partial z} N_i \end{bmatrix} \quad (5-33)$$

where $\mathbf{H}$ is $1 \times n$ and $\mathbf{B}_i$ is $5 \times (2n+3)$.

The nodal displacement vector $\boldsymbol{\delta}^e$ of the element is formed by equation (5-8). The strain matrix $\mathbf{B}$ of the element is formed by inputting $\mathbf{B}_i$ of each node into equation (5-10).

5.3.3 Element Stiffness

The element stiffness matrix $\mathbf{K}^e$ of the layered shell element is evaluated by

$$\mathbf{K}^e = \sum_{i=1}^n \int_{-1}^1 \int_{-1}^1 \mathbf{B}_i^T \mathbf{D}_i \mathbf{B}_i |\mathbf{J}| d\xi d\eta t_i \quad (5-34)$$

where $\mathbf{D}$ is the material stiffness matrix and n is the number of layers. The internal force vector $\mathbf{F}^e$ is calculated by

$$\mathbf{F}^e = \sum_{i=1}^n \int_{-1}^1 \int_{-1}^1 \mathbf{B}_i^T \boldsymbol{\sigma} |\mathbf{J}| d\xi d\eta t_i \quad (5-35)$$

5.3.4 Discussion Regarding the Layering Scheme

1. Relationships of the proposed layering scheme for shear correction to the shell element derived in section 5.2 and to the plate element developed by Mawenya and Davies (1975).

If all the nodes of the element are normal nodes without shear correction, the layered shell formulation developed in this section is exactly the same as the shell element formulation developed in section 5.2. If all the nodes are specified as shear correction nodes, the layered shell element above will have no difference with the plate element developed by Mawenya and Davies (1975) for elastic problems.

2. Advantages of the proposed layering scheme

The proposed layering scheme for incorporation of transverse shear deformation has such advantages that it can be conveniently employed to specify shear correction nodes where needed. Shell elements with all the nodes specified as shear correction nodes can be applied in slab-column connection regions. Shell elements with all the nodes being normal nodes will be applied in regions far from the connections. Shell elements with part of the nodes specified with shear correction may be applied in transitional regions of a RC flat plate. Computational efficiency and accuracy can thus reach an elegant compromise when a relative large RC flat plate is to be analyzed.

3. Incorporation of components protruding the slab surface

In the proposed layering scheme, the reference plane can be defined at any position through the section. Different number of layers with different thickness of each layer is allowed above and below the reference plane. Each layer can have independent rotational DOFs around x and y axes. This layering scheme can thus be conveniently applied to include the column stub and drop panels protruding from the surface of the slab when necessary. Fig. 5.4 illustrates such an example. The reference plane is defined at the mid-surface of a slab rather than the mid-surface of the element. Four layers of equal thickness are used above and below the reference plane for the slab. Additional four layers with equal thickness of h_1 , are added onto the top of the slab to model the components protruding from the slab surface. Each node of such a layered shell element has 2 additional independent rotational degrees of freedom at the mid-surface of each layer.

Fig. 5.4 Layering Scheme Incorporating Components Protruding Slab Surface

5.3.5 Modeling Concrete and Reinforcement by the Proposed Layering Scheme

All the layers divided through the thickness of the shell element as shown in Fig. 5.3 (a) are used for concrete. Out-of-plane reinforcement is specified as the material property of concrete layers (see section 4.5.2). Reinforcement layers are used to model the in-plane reinforcement. They are superimposed into concrete layers at their depths considering their relative thin equivalent thickness. Uniaxial material properties are assumed for the reinforcement layers. Since perfect bond is assumed between reinforcement and concrete in this study, the displacement field of reinforcement layers can be determined from concrete layers. Therefore, reinforcement layers, which may locate within any layer of concrete at their depths, are not plotted in Fig. 5.3 (a).

Within each layer, uniform properties of material and stress state are assumed and take the values of the middle surface of that layer. Different layers have different material properties and stress state. Each layer has one Gaussian integration point on its mid-surface. Material properties, stresses and strains are evaluated at such Gaussian integration point of each layer. It is thus that Z in equation (5-27) takes the value of the mid-surface of the layer to be treated. Before the treatment of the reinforcement layer, the concrete layer (j^{th} layer in equation (5-27)) containing the reinforcement layer needs to be identified.

Each of concrete layers is specified with a parameter indicating the consideration of tension stiffening effect or not as shown in Fig. 5.5. According to past CEB guidelines (CEB 1990), it is assumed that the extent of influence of reinforcement is limited within 7.5 bar diameters from the reinforcement. Outside this zone, only tension softening effect is considered.

The element stiffness contributed by in-plane reinforcement and that contributed by concrete and out-of-plane reinforcement are evaluated independently by equation (5-34). The total element stiffness $\mathbf{K}^e$ of the layered element is obtained by a combination of such two contributions as

$$\mathbf{K}^e = \mathbf{K}_c^e + \mathbf{K}_s^e \tag{5-36}$$

where $\mathbf{K}_c^e$ is the element stiffness contribution by concrete and out-of-plane reinforcement and $\mathbf{K}_s^e$ is the element stiffness contribution by in-plane reinforcement.

Fig. 5.5 Definition of Tension Stiffening Zone

CHAPTER 6 NONLINEAR ANALYSIS METHOD

6.1 INTRODUCTION

The responses of reinforced concrete structures are highly nonlinear due to material properties of concrete and reinforcement. Several critical points on the load-deformation response curve generally appear upon cracking of concrete, yielding of steel, and formation of ultimate failure. The analysis of complete nonlinear responses of RC structures requires the incorporation of powerful nonlinear numerical methods capable of passing through these critical points. Such capability is clearly beneficial and is often essential if the ultimate collapse is to be properly defined, to reveal the failure mode as well as the maximum load, and throw light on the ductility or deformation capacity of the structure.

The available numerical methods able to snap through the critical points on the load-deformation response curve are the displacement control method (Batoz and Dhatt 1979) and the arc-length method. In the displacement control method, a prescribed component of displacement vector is used to control the increments. The approach suffers from a lack of generality since only a limited number of engineering problems permit modeling via prescribed displacements (Rots 1985). Furthermore, the direct displacement control is incapable of tracing the snap back responses. The arc-length method is much more versatile as the increment is controlled by all the components of the displacement vector. Both the snap through and snap back behavior of the structure can be traced. The arc-length method was originally proposed by Riks (1972) and Wempner (1971) and later modified by Crisfield (1981) and Ramm (1981). Initially it was presented to deal with geometric nonlinearities encountered in aeronautics, involving tracing post-buckling response, snapping through limit or bifurcation points. Later, it was introduced to trace material nonlinearities in reinforced concrete structures by some authors (Crisfield 1982; May and Ganaba 1988; Lam and Morley 1992). Recently, the arc-length

method has been used along with more sophisticated material models for reinforced concrete structures (Park et al. 1995; Gomes and Awruch 2001).

In this study, an incremental/iterative nonlinear analysis procedure is developed based on the arc-length method proposed by Fafard and Massicotte (1993). Increments are controlled by a variable incremental work, while iterations within each increment are controlled by a constant arc-length of displacement components and load components. The following presents the detailed strategies of the developed procedure.

6.2 ITERATIVE STRATEGIES

The incremental form of the nonlinear equilibrium equations of a structure for the j th iteration of the i th incremental step can be written in the following general form:

$$\mathbf{K}_{j-1}^i \delta \mathbf{U}_j^i = \mathbf{F}_j^i - \mathbf{P}_{j-1}^i \quad (6-1)$$

where $\mathbf{F}_j^i$ denotes the external load vector applied on the structure at j th iteration; $\mathbf{P}_{j-1}^i$ is the internal nodal forces summed at each node of the structure up to the previous iteration; $\delta \mathbf{U}_j^i$ denotes the displacement variation for the j th iteration; $\mathbf{K}_{j-1}^i$ is the tangent stiffness at the $j-1$ th iteration. The external loads $\mathbf{F}_j^i$ can be decomposed as follows:

$$\mathbf{F}_j^i = \mathbf{F}_{j-1}^i + \delta \mathbf{F}_j^i \quad (6-2)$$

while $\delta \mathbf{F}_j^i$ can be written in the form of

$$\delta \mathbf{F}_j^i = \delta \lambda_j^i \hat{\mathbf{F}} \quad (6-3)$$

where $\delta \lambda_j^i$ is the load changing factor for the j th iteration of the i th incremental step, and $\hat{\mathbf{F}}$ is a reference load vector. By letting $\mathbf{R}_{j-1}^i$ denote the unbalanced forces

$$\mathbf{R}_{j-1}^i = \mathbf{F}_{j-1}^i - \mathbf{P}_{j-1}^i \quad (6-4)$$

equation (6-1) can be rewritten as:

$$\mathbf{K}_{j-1}^i \delta \mathbf{U}_j^i = \mathbf{F}_{j-1}^i + \delta \mathbf{F}_j^i - \mathbf{P}_{j-1}^i = \delta \mathbf{F}_j^i + (\mathbf{F}_{j-1}^i - \mathbf{P}_{j-1}^i) = \delta \lambda_j^i \hat{\mathbf{F}} + \mathbf{R}_{j-1}^i \quad (6-5)$$

For convenience, Eqn. (6-5) is replaced by the following two equations:

$$\mathbf{K}_{j-1}^i \delta^F \mathbf{U}_j^i = \hat{\mathbf{F}} \quad (6-6)$$

$$\mathbf{K}_{j-1}^i \delta^R \mathbf{U}_j^i = \mathbf{R}_{j-1}^i \quad (6-7)$$

Thus, $\delta \mathbf{U}_j^i$ is broken into two parts accordingly:

$$\delta \mathbf{U}_j^i = \delta \lambda_j^i \delta^F \mathbf{U}_j^i + \delta^R \mathbf{U}_j^i \quad (6-8)$$

To solve the equilibrium equation (6-5), a loading constraint condition is needed to get the load changing factor $\delta \lambda_j^i$. Different methods in determining $\delta \lambda_j^i$ give rise to different nonlinear analysis methods. Newton-Raphson method uses constant load increment as load constraint condition while displacement control method uses constant displacement increment (Batoz and Dhatt 1979; May et al. 1988). The arc-length method uses a combination of the above two loading constraint conditions. The load constraint condition is defined based on both the load changing factor $\delta \lambda_j^i$ and the n displacement incremental components of the structure. It usually can be expressed as the following function:

$$f(\delta \mathbf{U}_j^i, \delta \lambda_j^i) = 0 \quad (6-9)$$

The loading constraint condition of Riks' arc-length method (Riks 1979) can be expressed as:

$$(\Delta \mathbf{U}_j^i)^T \Delta \mathbf{U}_j^i + (\Delta \lambda_j^i)^2 (\hat{\mathbf{F}})^T (\hat{\mathbf{F}}) = \Delta l^2 \quad (6-10)$$

where

Δl	=	Arc-length
$\Delta \mathbf{U}_j^i$	=	The incremental displacement vector for j th iteration of i th incremental step
$\Delta \lambda_j^i$	=	The incremental load changing factor for j th iteration of i th incremental step

Crisfield (1981) reduced the constraint equation (6-10), excluding the load term, to the following equation:

$$(\Delta \mathbf{U}_j^i)^T \Delta \mathbf{U}_j^i = \Delta l^2 \quad (6-11)$$

Crisfield's arc-length method has the advantage of limiting the length of the displacement increment, for all iterations within a step, to a constant value. However it has the disadvantage that there exist two roots or even complex roots. Therefore, some complicated procedures are usually needed (Lam and Morley 1992; Carrera 1994). In Ramm's method (Ramm 1981), similar to Crisfield's method only the first displacement increment $\Delta \mathbf{U}_1^i$ must fall on the hypersphere of radius Δl . The solution along the curve is found by following a hyper-plane orthogonal to $\Delta \mathbf{U}_1^i$ expressed by

$$(\Delta \mathbf{U}_1^i)^T \delta \mathbf{U}_j^i = 0 \quad (6-12)$$

One advantage of Ramm's method is that there is only one value for $\delta \lambda_j^i$. Convergence is always possible as long as it exists physically. Whereas the arc-length is imposed only at the first iteration, this may lead in rare occasions where there is an abrupt change in the response to difficulties in tracing the load-displacement response of a structure (Fafard and Massicotte 1993).

To avoid both disadvantages of the above arc-length methods, Fafard and Massicotte (Fafard and Massicotte 1993) proposed a nonlinear analysis approach called *the modified Crisfield-Ramm arc-length method*. The method had three basic ideas.

First, the displacement variation vector $\delta \mathbf{U}_j^i$ of each iteration is in the orthogonal direction to the incremental displacement vector $\Delta \mathbf{U}_{j-1}^i$ of the previous iteration,

$$(\Delta \mathbf{U}_{j-1}^i)^T (\delta \mathbf{U}_j^i) = 0 \quad (6-13)$$

Substituting equation (6-8) into (6-13) leads to the load changing factor $\delta \lambda_j^i$ as follows

$$\delta \lambda_j^i = - \frac{(\Delta \mathbf{U}_{j-1}^i)^T \delta^R \mathbf{U}_j^i}{(\Delta \mathbf{U}_{j-1}^i)^T \delta^F \mathbf{U}_j^i} \quad (6-14)$$

Second, the actual desired incremental displacement vector $\Delta \mathbf{U}_j^i$ is reduced by a factor of α

$$\Delta \mathbf{U}_j^i = \alpha \Delta \mathbf{U}_{j-1}^i + \alpha \delta \mathbf{U}_j^i \quad (6-15)$$

as illustrated in Fig. 6.1 for a two-dimensional problem. The value of α can be evaluated by

$$\alpha = \frac{\Delta l_{iteration}^i}{\|\Delta \bar{\mathbf{U}}_j^i\|} \quad (6-16)$$

where

$$\Delta \bar{\mathbf{U}}_j^i = \Delta \mathbf{U}_{j-1}^i + \delta \mathbf{U}_j^i \quad (6-17)$$

Fig. 6.1 Modified Crisfield-Ramm Arc-Length Method
(Fafard and Massicotte 1993)

Third, the arc-length $\Delta l_{iteration}^i$ was as defined by Crisfield arc-length method as in equation (6-11). Since the arc-length is defined only in the displacement space without consideration of the load vector, the *modified Crisfield-Ramm arc-length method* may not be able to completely treat the abrupt changes in response. There may still exist some possibilities that the variation of displacements is confined to be very small and constant while the load variation can be surprisingly large due to the sharp variation of stiffness. Consequently, in the proposed iterative strategies, the author maintained the first two main ideas of the above *modified Crisfield-Ramm arc-length method*, but redefined the arc-length in equation (6-11) by adopting Riks arc-length definition (Riks 1979) as follows:

$$(\Delta \mathbf{U}_j^i)^T \Delta \mathbf{U}_j^i + A_0 (\Delta \lambda_j^i)^2 (\hat{\mathbf{F}})^T (\hat{\mathbf{F}}) = (\Delta l_{iteration}^i)^2 \quad (6-18)$$

where A_0 is a scalar that governs the relative contributions of the displacement and load increments. Eqn. (6-16) becomes

$$\alpha = \Delta l_{iteration}^i / \sqrt{(\Delta \bar{\mathbf{U}}_j^i)^T (\Delta \bar{\mathbf{U}}_j^i) + A_0 (\Delta \lambda_j^i)^2 (\hat{\mathbf{F}})^T (\hat{\mathbf{F}})} \quad (6-19)$$

6.3 INCREMENTAL STRATEGIES

In the proposed strategies, the incremental loading or unloading is controlled by a “length” of incremental work $\Delta l_{increment}^i$. As the load reference vector is defined arbitrarily without consideration of proper magnitude, the value of incremental work used to control loading should be properly selected.

6.3.1 Initial Increment

In the first incremental step, the value $\Delta l_{increment}^i$ is initially calculated by the following equation,

$$\Delta l_{increment}^1 = \sqrt{(\hat{\mathbf{F}})^T (\delta^F \mathbf{U}_1^1)} \quad (6-20)$$

At the second incremental step, it is adjusted by multiplying it by a factor κ .

$$\Delta l_{increment}^2 = \kappa \Delta l_{increment}^1 \quad (6-21)$$

Factor κ is used to ensure $\Delta l_{increment}^2$ fall in a rational scope which may lead to the desired loading steps of 20-100 in the thesis. κ is determined such that the maximum compressive principal stress of concrete at the end of the first increment equals to 2.5% of the uniaxial compressive strength of concrete:

$$\kappa = 0.025 (-f'_c) / \sigma_{3min}^1 \quad (6-22)$$

where,

$$\sigma_{3min}^1 = \text{Minimum compressive principal stress of concrete at the end of the first increment}$$

$$f_c' = \text{Cylinder compressive strength of concrete}$$

The value of incremental work $\Delta I_{\text{increment}}^i$ is variable at any incremental step. When the number of iterations happens to be larger than expected, the incremental work can increase or decrease by a factor of γ ,

$$\Delta I_{\text{increment}}^i = \gamma \Delta I_{\text{increment}}^{i-1} \quad (6-23)$$

which is used to adjust the incremental length of work, ensuring that the solution will definitely converge to a point on the solution path. γ varies within the range of 0.05 to 1.0.

6.3.2 Determination of Incremental Sign

The sign of the load changing factor $\delta \lambda_1^i$ of each initial incremental step must be determined according to some correction method. Incorrect choice of the sign will inevitably lead to the so called “track back”, where an iterative procedure may converge to the previously computed points. The main available criteria were summarized by Feng et al (1996) as: change of current stiffness parameter, change of incremental work, eigenvalue analysis, sign of the determinant of the current stiffness matrix. In this study, the sign of the loading parameter $\delta \lambda_1^i$ of each initial incremental step is determined by a criterion in work form, which is based on the concept that once the tangent stiffness changes from positive to negative, the displacement variation corresponding to the reference load vector will change directions; the resulting work is negative. Thus, the sign is determined by the following equation:

$$\text{sign} = \left| \left(\hat{\mathbf{F}} \right)^T \left(\delta^F \mathbf{U}_1^i \right) \right| / \left(\left(\hat{\mathbf{F}} \right)^T \left(\delta^F \mathbf{U}_1^i \right) \right) \quad (6-24)$$

Fig. 6.2 Flow Chart of the Proposed Incremental/Iterative Methods

6.4 CONVERGENCE CRITERION

A convergence criterion can take the forms of load, displacement, or energy (Bathe 1996). In this study, convergence is checked by comparing the displacement variation of each iteration to the accumulated displacement increment within that incremental step. The criterion takes the following form:

$$\sqrt{\frac{(\delta \mathbf{U}_j')^T (\delta \mathbf{U}_j')}{(\Delta \mathbf{U}_j')^T (\Delta \mathbf{U}_j')}} \leq \text{ETOL} \quad (6-25)$$

ETOL is a displacement convergence tolerance and it takes a value of 5% in this study.

6.5 FLOW CHART FOR THE INCREMENTAL/ITERATIVE METHOD DEVELOPED

Fig. 6.2 shows the flow chart for the proposed incremental/iterative procedure in solving the equilibrium equation (6-1).

CHAPTER 7 NUMERICAL STUDIES AND DISCUSSION OF RESULTS

7.1 INTRODUCTION

Based on the three dimensional material model, the layered shell element formulations, and arc-length numerical analysis procedure developed in this study, a finite element analysis program named USTAR has been developed using C++ language. The necessary pre-process and post-process procedures have also been compiled to facilitate the creation of input data files and interpretation of analyzed results. The validity and the performance of the proposed material model and the numerical procedures have been examined by a series of numerical studies in this chapter. The details of these studies are reported in the following sections.

The 3-D constitutive material model developed herein for CRC was verified by numerical simulation of a number of RC specimens. The validity and capability of the proposed tension stiffening, compression softening, and aggregate interlock models have been examined by analyzing reinforced concrete panels under uniform tension-compression stresses and pure shear stresses. The performance of the proposed crack formulations for MR-CRC has been investigated by simulating four anisotropically reinforced concrete panels under uniform tension-shear stresses and a reinforced concrete beam without web shear reinforcement under moderate shear. The simulation of the RC beam has also been used to test the validity of the developed layered shell element formulations.

The capability of the proposed finite element model in modeling the punching shear behavior of slab-column connections has been evaluated by simulating biaxial orthogonal reinforced slabs under pure torsional moment, concentrated punching load, or unbalanced moment. The flexibility and performance of the layered shell element for incorporating the transverse shear deformation and the influence of transverse shear stress distribution on the predicted cracked pattern, stiffness, failure

mode and ultimate strength of RC flat plate structure has also been investigated. A reinforced concrete flat slab with drop panel was studied to evaluate the performance of the developed layered shell element in modeling such slabs. The suitability and performance of the developed finite element model for global analysis of structural responses of relatively large reinforced concrete flat plate structures was studied by analyzing a multi-paneled flat plate with irregular layout and supported on 7 columns. At last, some parametric studies of the influence of several factors on punching shear behavior of R. C. flat plates have been carried out to further prove the validity of the proposed finite element model.

In the analyses, if no direct experimental data were available, the properties of concrete were assumed as: Young's modulus $E_c = 4733\sqrt{f'_c}$ (MPa), Poisson's ratio $\nu = 0.2$, tensile strength $f'_t = 0.33\sqrt{f'_c}$ (MPa), peak tensile strain $\varepsilon_t = f'_t / E_c$, shear retention parameter in equation (4-8) $\beta = 0.8$, and fracture energy $G_f = 0.06$ N/mm, equal biaxial compressive strength $f'_{bc} = 1.15f'_c$, equal biaxial compressive peak strain $\varepsilon_{bc} = 1.75\varepsilon_c$. Unless otherwise stated, a threshold angle of 10° has been used for the initiation of secondary cracks with respect to previously existing cracks in these analyses.

7.2. VERIFICATION OF THE PROPOSED 3-D MATERIAL MODEL FOR CRC

7.2.1 Specimens under Uniaxial Tension and Biaxial Tension-Compression

The capability of the proposed tension stiffening and compression softening models in simulating the material properties of the uni-directional-cracked concrete has been verified by numerical examples presented in this section.

To verify the capability of the proposed model in modeling the material properties of uni-directionally CRC under uniaxial tension and biaxial tension-compression stresses, the experimental specimens PB13, PB24, PB25, PB27 tested by Bhide and Collins (1986), and E2-1, E4-1, E10-1A, F2, F3, F4 of Belarbi and Hsu (1991) have been analyzed. The geometries, material properties of concrete, reinforcement detailing, and loading conditions are presented in Table 7.1. PB13 is under uniaxial tension. PB24, PB 25, PB27, E2-1, E4-1, E10-1A are under biaxial tension compression stresses. These specimens were first subjected to tension. After attaining a desired average tensile strain, the tensile stresses were kept constant and compressive stresses were gradually applied in the transverse direction until failure. F2, F3, F4 are under proportional biaxial tension-compression stresses. In the analyses, an 8×8 mesh was used. The loading condition of the specimens under sequential biaxial tension compression was realized by alternatively applying tensile and compressive stress increments in the two directions respectively after the desired average tensile strain was attained.

Table 7.1 Specimen dimensions, material properties, and loading details of panels under tension and biaxial tension-compression

Label	Loading type	ε_i	Dimension L×W×T (mm)	Concrete		Reinforcement			
				f'_c (MPa)	ε_c	ρ_x (%)	f_x (MPa)	ρ_y (%)	f_y (MPa)
PB13	UT	—	890×890×70	23.4	0.001813	1.085	414	0	0
PB24	T-C (s)	0.0055	890×890×70	20.4	0.001975	1.097	407	0	0
PB25	T-C (s)	0.0065	890×890×70	20.6	0.001975	2.195	414	0	0
PB27	T-C (s)	0.0100	890×890×70	37.9	0.003150	2.023	502	0	0
E2-1	T-C (s)	0.0020	1397×1397×178	47.7	0.002250	1.200	448	0.540	544
E4-1	T-C (s)	0.0040	1397×1397×178	40.6	0.002050	1.200	495	0.540	498
E10-1A	T-C (s)	0.0100	1397×1397×178	36.89	0.002150	1.169	462	0.540	498
F2	T-C (p)	—	1397×1397×178	40.1	0.002000	1.270	447	0.540	498
F3	T-C (p)	—	1397×1397×178	42.6	0.002000	1.270	447	0.540	498
F4	T-C (p)	—	1397×1397×178	41.9	0.002200	1.270	447	0.540	498

Note:

- ε_i = Tensile strain at start of compression phase
- UT = Uniaxial tension
- T-C = Biaxial tension compression
- s = Sequential loading (tension followed by compression)
- p = Proportional loading
- f'_c = Uniaxial compressive strength of concrete
- ε_c = Uniaxial compressive peak strain of concrete
- ρ_x = Reinforcement ratio in x direction
- ρ_y = Reinforcement ratio in y direction
- f_x = Yield stress of reinforcement in x direction
- f_y = Yield stress of reinforcement in y direction

Table 7.2 Experimental and analytical cracking loads of specimens under tension-compression stresses

Label	Experiment (MPa)	Analysis (MPa)	Analysis /Experiment
PB13	1.48	1.77	1.20
PB24	1.71	1.63	0.95
PB25	2.79	1.83	0.66
PB27	4.00	2.31	0.58
E2-1	1.96	2.41	1.23
E4-1	1.99	2.23	1.12
E10-1A	1.93	2.21	1.15
			Mean 0.98
			COV 25.0%

The experimental and analytical cracking loads are compared in Table 7.2. It is shown that the assumption of $f_t = 0.33\sqrt{f_c}$ (MPa) resulted in a lower prediction of cracking load for PB24, PB25, and PB27 with compressive concrete strength less than 25 MPa from Bhide’s test and a higher prediction for E2-1, E4-1, and E10-1A with compressive concrete strength more than 35 MPa from Belarbi’s test. This is consistent with the predictions from both studies by Bhide (1986) and by Belarbi (1991). The predicted tensile strengths of the panels are presented in Table 7.3. The predicted tensile and compressive strengths of the panels over experimental results have an average value of 1.02 and a coefficient of variation of 6.51%. It is demonstrated that the predicted ultimate tensile strengths are adequately accurate. The ratios of the predicted softened compressive strengths due to different levels of tensile strain over experimental results have an average value of 1.01 and a coefficient of variation of 16.6% (Table 7.3). Fig. 7.1 illustrates the tensile response of PB13. Axial load versus axial average tensile strain response of the panel and average tensile stress to average tensile strain of the concrete are all shown in the figure. The predicted tensile responses follow closely to experimental results except for a higher prediction of cracking stress. The proposed tension stiffening model proved to be capable of predicting the stiffness and strength contributed by CRC.

Fig.7.2 presents the tensile responses of other specimens except PB13. All the predicted tension responses are in good agreement with the experimental results. There is strong evidence that the proposed tension stiffening model along with the proposed model for reinforcement can capture tensile behavior of uniaxially CRC with good accuracy.

Table 7.3 Experimental and analytical ultimate loads of specimens under tension-compression

Label	Tension			Compression		
	Experiment (MPa)	Analysis (MPa)	Analysis/ Experiment	Experiment (MPa)	Analysis (MPa)	Analysis / Experiment
PB13	4.71	4.74	1.01	—	—	—
PB24	4.82	4.73	0.98	-11.07	-9.35	0.84
PB25	9.99	9.35	0.94	-13.33	-9.35	0.70
PB27	11.25	10.72	0.95	-15.38	-14.79	0.96
E2-1	4.86	5.55	1.14	-23.41	-29.80	1.27
E4-1	5.58	5.68	1.02	-20.78	-21.18	1.02
E10-1A	7.76	8.31	1.07	-12.91	-15.78	1.22
F2	—	—	—	-11.85	-12.81	1.08
F3	—	—	—	-18.74	-17.56	0.94
F4	—	—	—	-20.85	-22.32	1.07
			Mean 1.02			Mean 1.01
			COV 6.51%			COV 16.6%

(a) Tensile stress-tensile strain

(b) Axial load-tensile strain

Fig. 7.1 Tensile Response of PB13

(a) Axial load-tensile strain of PB24

(c) Axial load-tensile strain of PB25

Fig. 7.2 Tensile Response of Specimens under Sequential Tension and Compression Loading (to be continued)

(c) Axial load-tensile strain of PB27

(d) Axial load-tensile strain of E2-1

Fig. 7.2 Tensile Response of Specimens under Sequential Tension and Compression Loading (to be continued)

(e) Axial load-tensile strain of E4-1

(f) Axial load-tensile strain of E10-1A

Fig. 7.2 Tensile Response of Specimens under Sequential Tension and Compression Loading

(a) Compressive stress-compressive strain of E2-1

(b) Compressive stress-compressive strain of E4-1

Fig. 7.3 Compressive Response of Specimens E2-1, E4-1 and E10-A under Sequential Tension and Compression Loading (to be continued)

(c) Compressive stress-compressive strain of E10-1A

Fig.7.3 Compressive Response of Specimens E2-1, E4-1 and E10-1A under sequential tension and compression loading

Fig. 7.3 presents the compressive responses of specimens with sequential loading. The predicted responses are found to be very close to the experimental results except that predicted peak strains were relatively high in all these specimens. This is because no reduction of peak strain due to principal tensile strain is considered for sequential loading. Fig. 7.4 shows the compressive response of specimens under proportional loading. It can be observed that the peak stress and peak strain in compression were predicted with reasonable accuracy. The predicted compressive response follows the experimental results up to failure. The proposed compression softening model has demonstrated good capability in modeling the compression softening effect due to tensile stress and the severity of cracking which is monitored by average tensile strain.

The failure modes of all the specimens have been predicted correctly. PB13 was predicted to fail in reinforcement yielding while both reinforcement yielding and concrete crushing were captured in all the other specimens.

(a) Compressive stress-compressive strain of F2

(b) Compressive stress-tensile strain of F2

Fig. 7.4 Compressive Response of Specimens F2, F3, and F4 under Proportional Tension-Compression Loading (to be continued)

(c) Compressive stress-compressive strain of F3

(d) Compressive stress-tensile strain of F3

Fig. 7.4 Compressive Response of Specimens F2, F3, and F4 under Proportional Tension-Compression Loading (to be continued)

(e) Compressive stress-compressive strain of F4

(f) Compressive stress-tensile strain of F4

Fig. 7.4 Compressive Response of Specimens F2, F3, and F4 under Proportional Tension-Compression Loading

7.2.2 Specimens under Pure Shear

Fig. 7.5 Dimensions of Panels under Pure Shear Loading
(Unit: mm)

Six RC panels B1, B2, B3, B4, B5, B6 tested by Pang (1991) were analyzed to examine the performance of the proposed model in simulating the material properties of uniaxial CRC under pure shear stresses. The dimensions of each panel were 1397×1397×178 (mm) as shown in Fig. 7.5. The reinforcement was arranged at 45 degrees with respect to the principal external loading directions, 1-axis and 2-axis. The steel percentages in the longitudinal direction (*l*-axis in Fig. 5.5) were greater than those in the transverse direction (*t*-axis in Fig. 7.5). The test variable of these panels was the ratio of longitudinal steel tension force to the transverse steel tension force (or η -ratio). Material details of concrete and reinforcement are given in Table 7.4. Proportional tension and compression loads of equal magnitude (pure shear) were applied along the 1-axis and 2-axis respectively. This was equivalent to applying a shear stress in *l*-*t* direction.

Before concrete cracked, the whole panel acted like an elastic homogenous material and little deformation was generated. When the tensile stress reached the tensile strength of concrete, the concrete cracked. Uniformly distributed cracks formed normal to the 1-direction. Due to different reinforcement ratios in *l*-*t* direction,

shear stress and shear deformation were generated soon after cracking. Subsequently, the uniformly CRC was under tension, compression and shear stresses. The shear stiffness was reduced apparently after cracking. After the steel yielded in the t -direction, a rapid increase of deformation was produced with a very small increase of loads. The maximum applied load was reached when the concrete began crushing.

It was obvious that these specimens are very good examples of uniaxially CRC under combined tension, compression, and shear stresses. The modeling of such specimens has to deal with the anisotropy of tension stiffening effect, aggregate interlock and dowel action, yielding of steel, and compression softening effect due to tensile stress, tensile strain, and the intensity of shear. To model these panels, an 8×8 mesh scheme was used. The yield stresses of steel used in analysis were approximated by the equation $\phi f_y = 0.89(1 - 1/1000\rho)f_y$ as suggested by Pang (1991). Three series of analyses were conducted. In the first analysis, the shear retention parameter ϕ in equation (4-8) took the value of 0.8 to consider both aggregate interlock of concrete and dowel action of reinforcement, and the compression softening effect due to shear stress was considered ($\eta < 1.0$). In the second analysis, the shear retention parameter ϕ equaled to 0.9, while the compression softening effect due to shear stress was considered ($\eta < 1.0$). In the third analysis, the shear retention parameter ϕ equaled to 0.8, and the compression softening effect due to shear stress was not considered ($\eta = 1.0$). The analyses were an attempt to study the sensitivity of the predicted behavior to the choice of shear retention factor and the consideration of compression softening effect due the shear stress in crack interface. The predicted shear responses of all analyses are compared with the experimental results in Fig. 7.6.

The effect of different values of shear retention parameters is obvious in these panels. The use of a lower shear retention factor ($\phi=0.9$) did not have a pronounced effect untill the yielding of steel. After yielding of steel, the differences of deformational responses between the analyses using different shear retention factors became apparent. The predicted strength and stiffness for $\phi=0.9$ were lower than

those for $\phi=0.8$ which corresponds to a higher shear retention factor. This may be explained with the observation from experiment. After yielding of steel in t direction, the strain of steel in that direction increased much faster than in the l direction. The resistance of the panels to the subsequent loading might be performed more through shear deformation.

According to Fig. 7.6, the ultimate failure load and the ductility were predicted with good accuracy by the analysis that considered the compression softening effect due to shear. The analysis without such consideration resulted in a much higher prediction of ductility. The consideration of compression softening effect due to shear stress in the crack interface by the developed procedure has made it possible to capture compressive crushing of concrete at a time close to the observed value.

Despite the apparent larger cracking loads, the responses predicted by the analysis with $\phi=0.8$ and consideration of the compression softening effect by shear stress followed closely the experimental curves. The deformations before cracking, after cracking, and after yielding of steel were all predicted fairly well. Compression crushing failure, as observed in the experiments, was captured successfully in all the panels. The crushing failure has been identified to be caused by the gradual softening effect on compressive strength due to the tensile stress, the tensile strain normal to crack plane, and the development of shear in the crack interface. Table 7.5 presents the predicted shear strengths of these specimens from analysis with $\phi=0.8$ and $\eta < 1.0$. The ratios of predicted shear strengths of the panels over experimental results have an average value of 1.02 and a coefficient of variation of 3.96%. This numerical example has identified a proper choice of $\phi=0.8$ if strong dowel action in uniaxially CRC is present, and it also highlighted the importance of considering the compression softening effect due to shear stress to accurately capture the compressive crushing failure of cracked concrete. The proposed model has good performance and good capability in modeling the shear behavior and the compression softening behavior of uni-directionally CRC.

In the above numerical studies, the developed tension stiffening model, compression softening model as well as aggregate interlock models have shown a

good capability in modeling the material properties of uniaxially CRC under tension, compression, and shear stresses.

Table 7.4 Material properties of panels under pure shear loading

Label	$\eta_0 = \frac{\rho_l f_{ty}}{\rho_l f_{ly}}$	Concrete		Reinforcement				
		f'_c (MPa)	ϵ_c	ρ_x (%)	f_x (MPa)	ρ_y (%)	f_y (MPa)	f_o/f_y
B1	0.480	45.2	0.002150	1.193	463	0.596	445	0.75
B2	0.690	44.1	0.002350	1.789	446	1.193	462	0.80
B3	0.332	44.9	0.002150	1.789	446	0.596	445	0.75
B4	0.189	44.8	0.002050	2.982	470	0.596	445	0.75
B5	0.349	42.8	0.002200	2.982	470	1.193	462	0.80
B6	0.570	43.0	0.002200	2.982	470	1.789	445	0.85

Note:

- f'_c
- =
- Uniaxial compressive strength of concrete
- ϵ_c
- =
- Uniaxial compressive peak strain of concrete
- ρ_x
- =
- Reinforcement ratio in x direction
- ρ_y
- =
- Reinforcement ratio in y direction
- f_x
- =
- Yield stress of reinforcement in x direction
- f_y
- =
- Yield stress of reinforcement in y direction

Table 7.5 Experimental and predicted ultimate loads of panels under pure shear loading

Label	Experiment (MPa)	Analysis (MPa)	Analysis / Experiment
B1	3.96	3.94	0.99
B2	6.13	6.49	1.06
B3	4.36	4.65	1.07
B4	5.00	4.93	0.99
B5	7.16	7.51	1.05
B6	9.15	8.79	0.96
			Mean 1.02
			COV 3.96%

(a) Shear stress-shear strain of B1

(b) Shear stress-shear strain of B2

Fig. 7.6 Shear Stress τ_{lt} -Shear Strain γ_{lt} of Panels B1, B2, B3, B4, B5, and B6 under Pure Shear Loading (to be continued)

(c) Shear stress-shear strain of B3

(d) Shear stress-shear strain of B4

Fig. 7.6 Shear Stress τ_{lt} -Shear Strain γ_{lt} of Panels B1, B2, B3, B4, B5, and B6 under Pure Shear Loading (to be continued)

(e) Shear stress-shear strain of B5

(f) Shear stress-shear strain of B6

Fig. 7.6 Shear Stress τ_u -Shear Strain γ_u of Panels B1, B2, B3, B4, B5, and B6 under Pure Shear Loading

7.2.2 Modeling Material Properties of MR-CRC

7.2.2.1 Delft Beam

The first example testing the suitability of the proposed model to handle multi-directionally cracked concrete was the delft beam tested by Walraven (1978). The performance and the sensitivity of the proposed model for modeling the material properties of multidirectional cracked plain concrete and lightly reinforced concrete have been examined in this example. The validity and capability of the proposed layered shell element in modeling transverse shear deformation was also scrutinized. The beam was simply supported at the ends and subjected to concentrated loads at points 500 mm on either side of the center (Fig. 7.7a). In the experiment it was observed that vertical cracks initially formed in the pure moment zone of the beam. As the loads were increased, new inclined cracks formed between the supports and the loading points. It was reported that the final failure of the beam was due to the penetration of these inclined cracks deep into the compression zone resulting in a sudden failure. The beam had a depth of 450 mm and a thickness of 200mm. No web shear reinforcement was provided. The properties of concrete were $f'_c = 35$ (MPa), Poisson's ratio 0.2, uniaxial compressive peak strain $\epsilon_c = 0.002$. The reinforcement at the bottom of the beam had a Young's modulus $E_s = 210,000$ MPa and a yield stress $f_y = 440$ MPa. The threshold angle used in this example was 10° . The shear parameter β for the shear retention factor used was 1.2. The geometry and finite element mesh and layering scheme are shown in Fig. 7.7. Two meshes have been used to model the beam geometry. In mesh A, shell elements were used vertically and load was applied in the plane of the elements. The longitudinal reinforcements at the bottom of the beam were smeared into the shaded elements as shown in Fig. 7.7 (a). Tension stiffening effect was considered only in the shaded and stared elements. In mesh B, shell elements were used horizontally and load was applied transversely to the elements (Fig. 7.7 (b)). Eight layers of equal thickness were used for concrete and one layer was used for reinforcement (Fig. 7.7 (c)). Tension stiffening effect was considered in the bottom two layers. All the nodes of the shell elements were specified with shear correction.

The characteristic material length L_r was taken as either the whole thickness (d) or one-half of the thickness ($d/2$) of the beam in the analysis with mesh A. Fig. 7.8 (a) shows the predicted load versus mid-span deflection responses. The predicted deformational response curve by analysis $L_r=d$ follows closely the observed curve and the predicted strength is 73 kN compared to the observed 69 kN in the experiment. Diagonal tension shear cracks pointing to the load point were reproduced as shown in Fig. 7.9 (b). The failure of the beam was predicted to be due to the penetration of the diagonal shear crack deep into the compression zone near the load point. No crushing of concrete and yielding of steel were observed in the analysis. The analysis with $L_r=d/2$ predicted a higher stiffness and strength (82 kN) than the analysis with $L_r=d$ did (Fig. 7.8 (a)). The predicted diagonal shear crack for $L_r=d/2$ extended more directly to the supporting point (Fig. 7.9 (c)). A similar trend was observed for computations on this beam using a higher threshold angle (de Borst and Nauta 1985). At failure of the beam, concrete crushing and yielding of steel were predicted. The beam was predicted to fail in shear compression.

The predicted load-center deflection response curve using mesh B with $L_r=d$ is presented in Fig. 7.8 (b) in comparison with the predicted response curve by mesh A with $L_r=d$ and the observed experimental result. The analysis provided accurate prediction of stiffness and strength (72.3 kN). The localized diagonal shear crack was captured (Fig. 7.9 (d)) and it is in good agreement with that obtained by use of mesh A (Fig. 7.9 (b)). At failure, no crushing of concrete and yielding of steel were predicted. The shear stress distribution predicted by mesh B (Fig. 7.10 (a)) was found to fairly close to the result predicted by mesh A (Fig. 7.10 (b)).

Layering scheme of mesh B (A-A)

Fig. 7.7 Geometry and Finite Element Mesh of Delft Beam (Unit: mm)

(a) Load versus deflection curves for analyses by Mesh A

(b) Load versus deflection curves for analyses by Mesh A and Mesh B

Fig. 7.8 Load-Deflection Responses of Delft Beam

(a) Observed crack pattern

(b) Predicted cracks transferring no normal stress at impending failure
by mesh A and $Lr=d$

(c) Predicted cracks transferring no normal stress at impending failure
by mesh A and $Lr=d/2$

**Fig. 7.9 Crack Pattern of the Delft-Beam at Impending Failure
(to be continued)**

(d) Analyzed cracks transferring no normal stress at impending failure
by mesh B

Fig. 7.9 Crack Pattern of the Delft-Beam at Impending Failure

(a) Shear stress distribution predicted by use of mesh B at load of 7.3 kN

(b) Shear stress distribution predicted by use of mesh A at load of 7.3 kN

Fig. 7.10 Predicted Shear Stress Distribution of Delft Beam

This numerical analysis example indicated the importance of the tension softening model in reproducing the diagonal shear failure in cracked plain concrete. The use of the whole thickness of the element as characteristic material length which results in a brittle property of concrete is preferable for reproducing the diagonal shear failure mode in cracked plain concrete or lightly reinforced concrete. The shear tension parameter can take the value of 1.2 in this case. The developed layering scheme has shown in this example to have good capability to model the transverse shear behavior of RC thick shell elements. The material model for plain concrete is not sensitive to the choice of mesh scheme.

7.2.2.2 Reinforced Concrete Panels under Tension-Shear Loading Tested by Bhide (1986)

Fig. 7.11 Reinforcement Details of Panels by Bhide Subjected to Combined Tension and Shear

Four reinforced concrete panels subjected to pure shear or combined tension and shear have been analyzed to examine the capability and performance of the proposed model in simulating the material properties of CRC with multiple non-orthogonal cracks. All the panels had the dimension of $890\text{mm} \times 890\text{mm} \times 70\text{mm}$ and were reinforced only in the tension direction as shown in Fig. 7.11. The material properties of concrete and reinforcements as well as the ratios of loading are presented in Table. 7.7. The first crack was formed when the principal stress exceeded the strength criterion. Due to the lack of reinforcement in one direction, significant shear stress was generated in the crack interface after cracking. With loading, the principal stress direction rotated gradually and secondary cracks were formed towards the reinforcement direction. The failure was characterized by rapid widening of the newly developed secondary cracks. All these panels exhibited a ductile failure in the experiments by Bhide (1986).

Table 7.7 Material properties and loading details of panels by Bhide (1986)

Label	Loading ratio $v : f_x$	Concrete			Reinforcement	
		f'_c (MPa)	f'_t (MPa)	ϵ_c	ρ_x (%)	f_y (MPa)
PB19	1:1.0	20.0	1.80	0.0019	2.195	411
PB20	1:2.0	21.7	1.80	0.0019	2.195	424
PB21	1:3.1	21.8	2.00	0.0018	2.195	402
PB22	1:6.1	17.6	2.00	0.0012	2.195	433

Note:

- f'_c = Uniaxial compressive strength of concrete
- f'_t = Uniaxial tensile strength of concrete
- ϵ_c = Uniaxial compressive peak strain of concrete
- ρ_x = Reinforcement ratio in x direction
- v = Applied shear stress
- f_x = Applied normal stress
- f_y = Yielding stress of reinforcement

One element was used for the analyses of all the panels. The shear parameter ϕ had the value of 1.2. The predicted strengths are presented in Table 7.8. The predicted shear strengths of the panels were in good agreement with the experimental results. The predicted deformational responses of these panels are shown in Fig. 7.12. PB19 under combined tension and the largest proportion of shear was predicted to fail soon after the first cracking. As shown in Fig. 7.12 and Table 7.8, it is clear that the increase of the proportion of tension from PB20 to PB22 increased the difference between the predicted cracking load and ultimate failure load. These predictions were reasonable since the larger proportion of shear caused the formation of cracking towards more to the unreinforced direction and thus led to earlier failure. The predicted post-failure softening response in strength was not observed in the experiment. This is because shear stiffening contributed by the dowel action of reinforcement was not incorporated in the present study. Studies (Bhide and Collins

1987) indicated that, for large crack widths, the dowel action of the reinforcing bars cross the cracks becomes important in transmitting the required interface shear stresses. While not significantly influencing the failure load, this dowel action made the specimens considerably more ductile. Despite this predicted earlier drop in load, the predicted response curves of all these panels followed reasonable well the experimental results until the predicted ultimate failure.

Table 7.8 Results of panels by Bhide (1986)

Label	Experiment			Analysis			v_u (Analysis) / v_u (Experiment)
	v_{cr} (MPa)	v_u (MPa)	$\frac{v_u}{v_{cr}}$	v_{cr} (MPa)	v_u (MPa)	$\frac{v_u}{v_{cr}}$	
PB19	1.23	1.28	1.04	1.19	1.19	1.00	0.93
PB20	0.94	1.42	1.51	0.92	1.16	1.26	0.82
PB21	0.73	1.42	1.95	0.75	1.16	1.55	0.82
PB22	0.44	1.03	2.34	0.36	0.97	2.69	0.94

(a) Shear stress versus shear strain of PB19

(b) Shear stress versus shear strain of PB20

Fig. 7.12 Deformational Response of Panels by Bhide (to be continued)

(c) Shear stress versus shear strain of PB21

(d) Shear stress versus shear strain of PB22

Fig. 7.12 Deformational Response of Panels by Bhide

(a) Observed crack pattern at the initiation of the secondary crack

(b) Observed crack pattern at the ultimate load

Fig. 7.13 Observed and Predicted Crack Pattern of PB21 (to be continued)

(c) Predicted crack pattern

Fig. 7.13 Observed and Predicted Crack Pattern of PB21**PB21**

Detailed predicted information for panel PB21 will be provided in the following part. The first crack was predicted at an angle of 71° with respect to the reinforcement (Experiment 66°). Four cracks were formed at each Gaussian point with the last crack in an inclination of 39° to the reinforcement (Experiment 31°). As shown in Fig. 7.13, the predicted crack pattern matched the observed result reasonably well.

Fig. 7.14 (a) shows the predicted principal tensile stress versus principal strain relationship. The predicted response follows reasonably the observed response. Also, the predicted shear stress versus principal tensile strain agreed well with the experimental result until failure (Fig. 7.14 (b)). As seen from Fig. 7.14 (c), the predicted strain response in the longitudinal and transverse direction agrees well with the observed results. The strain response pattern was reasonably well predicted.

At lower load, the longitudinal strain increased faster than the transverse strain while at high load the reverse was true. This corresponded well with the crack pattern. The initial deformation was mostly contributed by the initial cracks close to the transverse direction while the deformations after the formation of secondary cracks were contributed significantly by the widening of these secondary cracks approaching the longitudinal direction. The predicted rotation of the principal stress was in good agreement with the observed result (Fig. 7.14 (d)). Fig. 7.14 (e) shows the rotation of the principal strain direction. After the formation of the first crack at loading of 0.75MPa, the predicted rotation of principal strain direction followed reasonably with the experimental result.

(a) Principal tensile stress versus principal tensile strain

Fig. 7.14 Deformational Response of PB21 (to be continued)

(b) Shear stress versus principal tensile strain of PB21

(c) Shear stress versus longitudinal and transverse strains

Fig. 7.14 Deformational Response of PB21 (to be continued)

■

(d) Principal stress direction versus shear stress

(e) Principal strain versus shear stress

Fig. 7.14 Deformational Response of PB21

PB22

As seen in Fig. 7.15 (c), the predicted shear stress versus shear strain response curve for PB22 followed closely the observed curve. The analyzed principal stress-principal strain curve agrees well with the experimental result (Fig. 7.15 (a)). Good correlation has been observed between the predicted and observed variation of principal stress direction (Fig. 7.15 (b)). Omitting the tension stiffening effect in modeling this panel resulted in a significant drop in stiffness, strength and ductility (Fig. 7.15 (c)). The developed tension stiffening model has shown a good capability in modeling the stiffness and ductility of MR-CRC. To study the sensitivity of the proposed material model to mesh size, the panel was reanalyzed with an 8×8 mesh. It was found that the predicted response curves by mesh 8×8 and mesh 1×1 were almost identical, that is no mesh size effect was observed in the analysis of this panel.

(a) Principal tensile stress versus principal tensile strain

Fig. 7.15 Deformational Response of PB22 (to be continued)

(b) Principal stress direction versus shear stress

(c) Shear stress-shear strain response

Fig. 7.15 Deformational Response of PB22

The good performance in predicting the deformational responses of the above panels up to ultimate failure proved the good capability of the proposed model to handle the tension stiffening effect of cracked reinforced concrete with multiple non-orthogonal cracks. Finite element modeling of the above reinforced concrete panels under tension and shear stresses showed that the developed material model is able to simulate the behavior characteristics of CRC with multiple non-orthogonal cracks reasonably well. The crack pattern, deformational response, and the ultimate failure load and failure mode were all captured well. The mobilization of principal directions, the propagation of cracking, and mode of ultimate failure were all reproduced successfully. It seemed, however, the lack of representation of dowel action resulted in a lower prediction of shear strength and ductility of CRC with multiple non-orthogonal cracks. Furthermore, the post-failure responses captured in this simulation has proved the validity and indicated the good performance of the arc-length numerical analysis procedure developed in this study.

The good capability of the proposed 3-D hypo-elastic material model in simulating the material properties of cracked plain and CRC had made it possible to apply it to nonlinear finite element analysis of reinforced concrete flat plates. The performance and the suitability of the developed material model with the application of the developed layered formulation for shell elements in modeling the punching shear behavior of RC flat plates will be demonstrated systematically in the following section.

7.3 MODELING THE PUNCHING SHEAR BEHAVIOR OF REINFORCED CONCRETE FLAT PLATE STRUCTURES

7.3.1 RC Slab under Pure Torsion by Marti et al. (1987)

An experimental study (Marti et al. 1987) and theoretical investigation (Marti and Kong 1987) have been conducted on nine orthogonally reinforced concrete slabs subjected to pure torsional moments in the reinforcement direction. The studies have indicated that the compression softening effect due to transverse tensile strain has a significant influence on both stiffness and strength and that a yield line approach may considerably overestimate the ultimate torsional resistance. In this study, one specimen (ML6) of the nine slabs was selected and analyzed to test the capability of the developed material model in simulating the behavior of such slabs.

Fig. 7.16 Reinforcement of ML6 (Unit: mm)

The slab had dimensions of 200mm×1700mm×1700mm and was loaded by couples of equal and opposite forces at the corners of the specimen. Fig. 7.16 shows the details of the reinforcement of 1/4 of the slab. Steel 10M had a yield strength of 551 MPa and that of Steel 15M was 481 MPa. The compressive strength of concrete f'_c was 23.3 MPa and peak compressive strain was 0.00248. In the analysis 1/4 of the slab as shown in Fig. 7.17 was analyzed due to the symmetry. 25 shell elements were used to model the slab. The concentrated upward corner load was uniformly distributed in the shaded element. Eight layers of equal thickness were used for

concrete and four layers were used for reinforcements. Two analyses were conducted with the first one considering the compression softening effect of cracks and the second one without such consideration.

Fig. 7.17 Finite Element Mesh for ML6 (Unit:mm)

The predicted twisting moment versus corner deflection is portrayed in Fig. 7.18. The deformational curve predicted by the first analysis with compression softening effect agreed closely the observed curve. Uniaxial cracks were predicted in the bottom and top three layers of concrete at approximately 45° and 135° with respect to the x-axis, respectively. Biaxial cracks were predicted in the middle two layers. It was indicated that concrete within the outer six layers were under tension normal to cracks and under compression parallel to cracks and concrete in the middle two layers was under biaxial tension. The failure of the slab was predicted to be caused by the progressive crushing of concrete on the bottom and top layers. No yielding of steel was predicted in any direction. The predicted crack pattern and failure mode agreed fairly well with the observed results. The predicted ultimate torsional moment by the first analysis was 58.6 kN which was 7.9% lower than the observed value (63.6 kN). According to Fig. 7.18, the omission of the compression softening effect in the second analysis has resulted in an apparent increase of stiffness and an unconservative predicted strength of 69.2 kN which was 8.8% higher than the observed strength. It was reported by Marti et al. (1987) that conservative strengths were predicted for this slab by the ACI code (34.1 kN), Canadian code (45.2 kN)

and Lower bound analysis (52.4 kN). The yield line approach without consideration of compression softening resulted in very unconservative strength of 113kN. The need to account for compression softening effect of cracking has been demonstrated in this numerical example. The tension stiffening model and compression softening model in this study have shown good capability in simulating the torsional behavior of RC slabs under significant twisting moment. The progressive degradation of stiffness and strength of cracked concrete were simulated well and the compressive crushing failure in the torsional slab due to tensile strain, tensile stress and shear stress was captured with good accuracy.

Fig. 7.18 Averaging Twisting Moment versus Corner Deflection Curve of ML6

7.3.2 Slabs under Punching Load Tested by Elstner and Hognestad (1956)

The capability of the proposed finite element model in simulating the punching shear behavior of a reinforced concrete flat plate under concentrated load was investigated by analyzing a group of slabs tested by Elstner and Hognestad (1956). All the slabs were square (1829mm×1829mm) with a thickness of 152mm, supported at the edges and loaded through a centrally located column stub. The material properties of the slabs selected for analysis are presented in Table 7.9 and Table 7.10. Compression steel was provided in the A-series but was omitted in the B-series. In slabs B10, B13, B15, inclined transverse shear reinforcements were provided. Slab A-5 has a column stub size of 356mm in a comparison with 254mm for the column stub of slab A-7b to study the column size effect. Slabs A-9 and A-10, compared to slabs A-7b and A-5 respectively, had 50 percent of the tension reinforcement concentrated directly under the column.

Slab A-2c

The significant step in modeling the punching shear behavior of a reinforced concrete flat plate is the reproduction of diagonal shear failure mode. Such capability of the proposed material model in conjunction with the developed layered shell element was investigated by analyzing slab A-2c. Five analyses named A-2ca, A-2cb, A-2cc, A-2cd, A-2ce, were conducted. For the symmetry of the slab, 1/4 was modeled by use of 25 layered shell elements. The mesh used in all the five analyses is shown in Fig. 7.19. Only the normal shell element was employed in analysis A-2ca; in A-2cb, all of nodes were specified as shear correction nodes; In A-2cc, only the nodes of the starred elements shown in Fig. 7.19 were specified as nodes with shear correction; as a comparison with A-2cd, in analysis A-2ce, compression softening effect was omitted in contrast to analysis A-2cd where it was considered. Eight layers of equal thickness were used for concrete in the previous four analyses introduced (Fig. 7.19 (b)). In A-2cd, additional four layers, of 75mm thickness on top of the eight layers for the slab, were employed to incorporate the stiffness of the column stub (Fig. 7.19 (c)). The stirrups of the column stub were smeared into the layers for the column. Both the shaded and starred elements in A-

2cd were specified with shear correction. The shear parameter used in these analyses was $\beta = 0.8$.

Fig. 7.20 shows the predicted load-center deflection responses and failure loads along with the experimental results. A-2ca without shear correction on shell elements predicted the maximum failure load of 550 kN which was 18.8% higher than the observed value (463kN). The deformational responses predicted by A-2cb with shear correction only on elements adjacent to the column stub and by A-2cc with shear correction on all elements were nearly identical. The failure loads predicted by A-2cb and by A-2cc were 423kN and 424 kN, which were around 9% lower than the observed failure load. Moreover, both analyses of A-2cb and A-2cc with shear correction predicted a lower stiffness and strength than A-2Ca without shear correction. Compared to A-2cb and A-2cc, it was clear that the incorporation of the column stub in analysis A-2cd increased both the predicted stiffness and failure load. The predicted failure load by A-2cd was 461kN which was around 10% higher than the values predicted by A-2cb and A-2cc. As shown in Fig. 7.20, the omission of the compression softening effect in A-2ce resulted in an apparent increase in ductility and an 8.9% increase of the ultimate shear strength.

Similar crack patterns on the bottom surface of the slab were predicted in all the five analyses. Fig. 7.21 portrayed the crack pattern on the bottom surface predicted by A-2cd. Both the radial and circumferential cracks typical in a slab under punching shear were all captured. At some Gauss points, three cracks in different directions were reproduced. Fig. 7.22 shows the cracks on section A carrying no normal stress predicted by A-2ca, A-2cb, A-2cc, and A-2cd. No localized diagonal crack was formed in the middle of the section in analysis A-2ca with the application of normal shell element. In analyses A-2cb, A-2cc, A-2cd, obvious diagonal crack was reproduced. It is of interest to observe that the diagonal crack predicted by A-2cb and A-2cc extended into the slab element under column stub. The inclusion of the column stub in A-2cd seemed to have pushed the diagonal crack outside the slab under the column stub. The characteristic of the predicted crack patterns can be explained by the transverse shear stress distribution predicted by each analysis as shown in Fig. 7.23. In analysis A-2ca with no shear correction, a constant transverse

shear stress distribution resulted. No shear stress concentration occurred along the section. No pronounced diagonal shear crack could be generated and the deformation of the slab connection was mostly due to flexure. In analysis A-2cb and A-2cc, concentration of transverse shear stress was generated in the middle of the section at the column slab connection and it caused a diagonal shear crack which would diagonally extend to the top and bottom surface of the slab. Since most of the shear stresses were transferred at the column slab connection, the constant shear distribution outside the connection elements in A-2cc would not have an obvious effect on crack pattern. The inclusion of the slab-column stub in A-2cd significantly reduced the concentration of shear stress in the cross section just under the column face. However, a heavy shear stress concentration was produced in the column root near the side face. As a consequence, the main diagonal shear crack in the slab cross section was predicted away from the column face.

Yielding of longitudinal bottom reinforcement was captured in all the analyses. However, the yielding of steel was concentrated in elements under or adjacent to the column stub and did not spread to the edge of the slab. A typical picture of the stress distribution in the bottom steel in the x direction obtained from A-2cd is shown in Fig. 7.24. Concrete crushing was reproduced by analysis A-2ca on the top surface around the column stub and spread towards the corners. In analyses A-2cb and A-2cc, concrete crushing was only found on the top surface of the slab element just under the column stub. Analysis A-2cd, showed that concrete crushing occurred in uniaxial cracked concrete in the top layer and in multidirectional cracked concrete in the second and third layers from the top surface of the slab around the column stub as shown in Fig. 7.25. There was no doubt flexural failure was predicted by A-2ca while punching shear failure was reproduced by all the analyses with shear correction on shell elements.

The importance of shear correction on shell elements in reproducing the punching shear failure mode has been highlighted in this example. Results from analyses A-2cb and A-2cc showed no much difference in terms of the predicted stiffness, strength, crack pattern and failure mode. The proposed layering scheme for shear correction has shown good capability in modeling the nonlinear behavior of slabs

under significant transverse shear stresses. The specification of shear correction only on elements in the slab column connection rather than globally on all elements was proved to be effective and efficient. The layering formulations developed in this study provided the availability of such a choice. The inclusion of the column stub improved the representation of transverse shear stress and predicted more reasonable crack patterns although no improvement could be made on the representation of axial stress of the column stub. In analysis A-2cd, concrete crushing in multidirectional cracked concrete was captured in top layers of the slab. The consideration of compression softening in multidirectional cracked concrete led to a conservative prediction of ultimate shear strength of the slab.

Table 7.9 Material properties of A-series slabs by Elstner and Hognestad (1956)

Slab No.	Column Size (mm)	d , (mm)	f'_c (MPa)	f_y (MPa)	Longitudinal reinforcement							
					Tension mat				Compression mat			
					Dia. (mm)	Spacing		ρ (%)	Dia. (mm)	Spacing		ρ (%)
						Bot., mm	Top, mm			Bot., mm	Top, mm	
A-1b	254	118	25.2	332	19.1	229	191	1.15	12.7	178	203	0.56
A-1d	254	118	36.8	332	19.1	229	191	1.15	12.7	178	203	0.56
A-2b	254	114	19.5	321	25.4	203	159	2.47	19.1	191	229	1.15
A-2c	254	114	37.7	321	25.4	203	159	2.47	19.1	191	229	1.15
A-7b	254	114	27.9	321	25.4	203	159	2.47	19.1	191	229	1.15
A-9	254	114	29.9	321	25.4	64 305	64 229	7.02 1.64	19.1	191	229	1.15
A-5	356	114	27.8	321	25.4	203	159	2.47	19.1	191	229	1.15
A-10	356	114	29.7	321	25.4	64 305	64 229	6.28 1.48	19.1	191	229	1.15
A-3b	254	114	22.6	321	25.4	133	108	3.70	19.1	191	229	1.15
A-3d	254	114	34.5	321	25.4	133	108	3.70	19.1	191	229	1.15

Note:

- f'_c = Uniaxial compressive strength of concrete
- ρ = Reinforcement ratio
- d = Effective depth of slab
- f_y = Yield stress of reinforcement

Table 7.10 Material properties of B-series slabs by Elstner and Hognestad (1956)

Slab No.	Column Size (mm)	d , (mm)	f'_c (MPa)	f_y (MPa)	Longitudinal reinforcement				Shear Reinforcement		
					Tension mat				Reinforce -ment (Nums.- Dia)	f_{yv} (MPa)	θ (deg.)
					Dia. (mm)	Spacing		ρ (%)			
						Bot., mm	Top, mm				
B2	254	114	47.6	321	12.7	238	213	0.50			
B4	254	114	47.7	303	15.9	187	162	0.99			
B9	254	114	43.9	341	22.2	187	152	2.00			
B14	254	114	50.5	325	25.4	165	133	3.00			
B10	254	114	46.4	335	22.2	187	152	3.00	4 -12.7	331	45
B13	254	114	48.0	328	25.4	165	133	3.00	10-15.9	373	20
B15	254	114	48.9	332	25.4	165	133	3.00	6 -12.7	331	45

Note:

- f'_c = Uniaxial compressive strength of concrete
- ρ = Reinforcement ratio
- d = Effective depth of slab
- f_y = Yield stress of reinforcement
- f_{yv} = Yield stress of shear reinforcement
- θ = Angle between shear reinforcement and slab surface

(a) Finite element mesh

(b) Section A-A for A-2ca, A-2cb, A-2cc, A-2ce

(c) Section A-A for A-2cd

Fig. 7.19 Finite Element Mesh, and Layering Details for Slab A-2c

Fig. 7.20 Load-Deflection Curve of A-2c
(Numbers in bracket are predicted failure loads)

Fig. 7.21 Predicted Crack Pattern on Bottom Surface by A-2cd

(a) Predicted crack pattern by A-2ca

(b) Predicted crack pattern by A-2cb

(c) Predicted crack pattern by A-2cc

(d) Predicted crack pattern by A-2cd

Fig. 7.22 Predicted Crack Pattern on Section A of Slab A-2c at Failure

(a) Predicted by A-2ca at load of 41.2kN

(b) Predicted by A-2cb at load of 40.6kN

(c) Predicted by A-2cc at load of 41.0kN

(d) Predicted by A-2cd at load of 41.0kN

**Fig. 7.23 Distribution of Predicted Transverse Shear Stress
on Section A of Slab A-2c**

Fig. 7.24 Predicted Stress Distribution of Longitudinal Bottom Steel in X Direction at Failure by A-2cd

(a) Top layer predicted by A-2ca

(b) Top layer predicted by A-2cd

Fig. 7.25 Distribution of Crushing Points and Principal Compressive Stress for Slab A-2c (to be continued)

(c) Second layer from the top by A-2cd

(d) Third layer from the top predicted by A-2cd

Fig. 7.25 Distribution of Crushing Points and Principal Compressive Stress for Slab A-2c

Other slabs

A combination of shear parameter of 0.8, shell element with shear correction for slab column connection, and inclusion of column stub, was used to analyze the other slabs listed in Table 7.9. All the slabs except B2, and B4 were predicted to fail in punching shear. This was consistent with the experimental results. The predicted ultimate strengths of all the slabs by use of the above combination are presented in Table 7.11. The ratios of the predicted ultimate shear strengths of the slabs over the experimental results have an average value of 0.92 and a coefficient of variation of 12.0%. Good accuracy was obtained on predicting the shear strength of the slabs. Slabs A-1b, A-2b, A-3b, A-7b whose concrete had a compressive strength less than 30 MPa were predicted to have shear strengths lower than 90% of the observed shear strengths. The strengths of slabs A-3b and A-3d whose reinforcement ratio were larger than 3% were predicted to be more than 15% lower than the observed value. The ultimate strengths of slabs B2 and B4 that failed in flexure, slab A-5 with enlarged column, and slabs A-9 and A-10 with concentrated reinforcement were predicted with good accuracy. The lower predicted shear strength of slab B13 implied the difficulty of the developed finite element model to predict the shear strength of slabs with relatively heavy transverse shear reinforcement.

The predicted load versus center deflection responses of these slabs are plotted in Fig. 7.26 against the experimental results. All the predicted response curves agreed reasonably well with the observed curves.

Table 7.11 Ultimate loads of slabs by Elstner and Hognestad (1956)

Slab No.	Experiment (kN)	Analysis (kN)	Analysis/ Experiment
A-1b	364	309	0.85
A-1d	351	387	1.10
A-2b	400	320	0.80
A-2c	467	461	0.99
A-7b	511	405	0.79
A-9	445	419	0.94
A-5	533	487	0.91
A-10	489	480	0.98
A-3b	445	368	0.83
A-3d	547	450	0.82
B2	200	220	1.10
B4	333	347	1.04
B9	505	433	0.86
B14	578	556	0.96
B10	533	514	0.96
B13	778	601	0.77
B15	689	615	0.89
			Mean 0.92
			COV 12.0%

Fig. 7.26 Load-Deflection Response of Slabs by Elstner and Hognestad (1956) (to be continued)

Fig. 7.26 Load-Deflection Response of Slabs by Elstner and Hognestad (1956) (to be continued)

Fig. 7.26 Load-Deflection Response of Slabs by Elstner and Hognestad (1956)

7.3.3 Slabs under Unbalanced Moment Tested by Moe (1961)

Finite element analyses were conducted on slabs under eccentric column load that had been tested by Moe (1961) to investigate the capability and performance of the developed finite element model in simulating punching shear behavior of slabs under unbalanced moment. Ten slabs from the M-series were selected for analysis. All the slabs were 1829mm square with an overall thickness of 152mm, and were simply supported along four edges with the corner free to lift. The data of the specimens are given in Table 7.12. Because of symmetry, 1/2 of the slabs were modeled. The mesh used for slabs M1A, M2, M3, M4, and M5 is shown in Fig. 7.27. Fifty shell elements were used. Nodes of the stated elements were specified as nodes with shear correction. Eight layers of equal thickness of 19 mm were used for the slab and four additional layers of equal thickness of 89 mm on top of the eight layers for the slab were employed to incorporate the column stub (Fig. 7.27 (b)). The unbalanced moment was applied by two uniform vertical line loads as shown in Fig. 7.27 (a). The finite element mesh and layering scheme for slabs M6, M7, M8, M9, and M10 are shown in Fig. 7.28, with 45 elements. Eight layers of equal thickness of 19 mm were used for the slab and additional four layers of equal thickness of 75 mm on top of the eight layers for the slab were employed to incorporate the column stub (Fig. 7.28 (b)).

(a) Finite element mesh

(b) Section A-A

Fig. 7.27 Finite Element Mesh and Layering Scheme for Slab M1A, M2, M3, M4, M5

(a) Finite element mesh

(b) Section A-A

**Fig. 7.28 Finite Element Mesh and Layering Scheme
for Slabs M6, M7, M8, M9, M10**

Table 7.12 Data of slabs of M-series tested by Moe (1961)

Slab No.	Column Size (mm)	f_c' (MPa)	f_y (MPa)	Tension mat				Compression mat In both directions Dia. @ Spacing	Eccentricity (mm)
				Dia. (mm)	Spacing		ρ (%)		
					Bot., mm	Top, mm			
M1A	305	20.8	481	19.1	178	152	1.50		0
M2	305	25.7	481	19.1	178	152	1.50		191
M3	305	22.7	481	19.1	178	152	1.50		330
M4	305	24.6	481	19.1	178	152	1.50		432
M5	305	27.0	481	19.1	178	152	1.50		610
M6	254	26.5	328	15.9	140	121	1.34		165
M7	254	25.0	328	15.9	140	121	1.34		64
M8	254	24.6	328	15.9	140	121	1.34	15.9@305mm	432
M9	254	23.2	328	15.9	140	121	1.34		127
M10	254	21.1	328	15.9	140	121	1.34	15.9@305mm	305

Table 7.13 presents the predicted failure loads and failure modes in comparison with the experimental results. The ratio of the predicted failure loads over the observed value had a mean value of 0.96, and a coefficient variation of 13.6%. The developed finite element model has shown good capability in predicting the shear strength of slabs under unbalance moment. Slabs M4, and M5 with large eccentricity and without reinforcements at the top were predicted to fail in negative bending. The failure was caused by vertical flexural cracking on the top surface of the slab near the column. No diagonal shear crack was formed in the cross section of the slab column connection. Unlike slabs M4 and M5, slabs M8, and M10 with large eccentricity and reinforcement at the top were predicted to fail in punching shear. Shear failure was also captured in M1A under concentrated load and in M2, M3, M6, M7, and M9 under less unbalanced moment. Fig. 7.29 shows the typical predicted crack pattern on section A-A of slabs M3, M8, and M10. All the failure modes predicted for the slabs were found to agree with the observed results. It has been noticed from Fig. 7.29 that a large amount of diagonal shear cracks carrying no normal stress were predicted in the slab under the column stub in M3, M8, and M10 with large eccentricity. It may be that these shear cracks resulted in 10-20% lower predicted shear strengths than the observed values (Table 7.13). For more

accurate simulation, it seems that the inclusion of axial compressive stress from the column stub is necessary.

Table 7.13 Ultimate loads and failure modes of slabs tested by Moe (1961)

Slab No.	Predicted response			Observed failure mode	Ultimate load		
	Yielding of steel	Formation of shear crack	Predicted failure mode		Experiment (kN)	Analysis (kN)	Analysis/ Experiment
M1A	yes	yes	shear	shear	432	363	0.84
M2	yes	yes	shear	shear	292	281	0.96
M3	no	yes	shear	shear	207	187	0.90
M4	no	no	Negative bending	Negative bending	132	160	1.21
M5	no	no	Negative bending	Negative bending	101	119	1.18
M6	yes	yes	shear	shear	239	229	0.96
M7	yes	yes	shear	shear	311	295	0.95
M8	yes	yes	shear	shear	149	123	0.83
M9	yes	yes	shear	shear	267	253	0.95
M10	yes	yes	shear	shear	178	144	0.81
							Mean 0.96
							COV 13.6%

(a) M3

(b) M8

(c) M10

**Fig. 7.29 Predicted Crack Pattern (Cracks Carrying No Normal Stress)
On Section A-A of Slab M3, M8, and M10**

Fig. 7.30 shows the predicted load-center deflection responses of the slabs along with the observed results, which agreed reasonably well with the experimental response curves.

Fig. 7.30 Load-Deflection Response of Slabs by Moe (1961)
(to be continued)

Fig. 7.30 Load-Deflection Response of Slabs by Moe (1961)

Fig. 7.31 shows the deflection profile at section A-A of slab M10 at a load of 133kN. The predicted deflections agreed well with the observed deflections. The edge of the column applying the eccentric load was predicted to have the maximum deflection. Fig. 7.32 depicts the predicted crack pattern on the bottom surface of slab M6 at failure. A relatively larger area of circumferential cracks was predicted on the bottom surface at the side with loading than on the opposite side. Compared to the crack pattern of a slab under concentrated load, the center of the distribution of the circumferential cracks seems to have shifted from the center of the column to the edge of the column with loading. The predicted crack pattern on the bottom surface agreed considerably well with the observed one.

Fig. 7.31 Deflection Profile at Section A-A of Slab M10 at Load of 133kN

Fig. 7.33 shows the concrete strain on the top surface of the slab M8 close to the column corner along the loading direction. The predicted compressive strain at point B agreed well with the observed strain. The tensile strain at point A also agreed closely with the tested value up to 100 kN.

From the above results, the developed finite element model has shown good capability in predicting the ultimate shear strength, the failure modes, and deformational characteristics of slabs under unbalanced moment.

(a) Observed crack pattern

(b) Predicted crack pattern

Fig. 7.32 Crack Pattern on Bottom Surface of Slab M6 at Failure

Fig. 7.33 Concrete Strain on Top Surface of M8 (In Thousandths)

7.3.4 Slab with Drop Panel Tested by Megally (1998)

The flexibility of the developed layering formulations to perform shear correction of shell elements has been demonstrated in the above examples with column stubs. The possibility of simulating the punching shear behavior of flat slabs with drop panel was investigated by a numerical simulation of slab I-2 with drop panel tested by Megally (1998).

Fig. 7.34 Geometry of Slab I-2 with Drop Panel Tested by Megally (1998)

(Unit: mm)

The geometry of the slab I-2 is shown in Fig. 7.34. The specimen was simply supported along its four edges. At the center of the slab, a reinforced concrete column protruded on both slab sides. The column was square with side width of 250mm. The slab had a dimension of 1900mm×1900mm×150mm with a 75 mm thick drop panel extending 350 mm away from column faces. The reinforcement details are shown in Fig. 7.35. The cylinder compressive strength of concrete was 41.7 MPa. No. 10 bars had a diameter of 11.3 mm, a yield stress of 461 MPa, and a yield strain of 0.00256. No. 15 bars had a diameter of 16.0 mm, a yield stress of 445 MPa, and a yield strain of 0.0022.

(a) Top reinforcement material (No. 15 bars)

(b) Bottom reinforcement material (No. 10 bars)

(c) Section A-A (enlarged scale)

Fig. 7.35 Reinforcement Details of Slab I-2

The finite element mesh used in this example is shown in Fig. 7.36 (a). 25 shell elements were used to model the slab. Nodes of the started and shaded elements were specified with shear correction. The drop panel was modeled by adding four layers below the bottom of the slab as shown in Fig. 7.36 (b).

(a) Finite element mesh

(b) Section A-A

Fig. 7.36 Finite Element Mesh and Layering Scheme for Slab I-2 by Megally (1998)

Fig. 7.37 depicts the predicted crack pattern on the top surface of the slab. The first radial crack at the column face was predicted at a load of 150 kN in comparison with the observed value of 180 kN (Fig. 7.37 (a)). The first circumferential flexural crack was predicted to occur at the end of the drop panel at a load of 220 kN as shown in Fig. 7.37 (b). The experimental counterpart was observed at about 210 kN

at approximately 390 mm from the column face; i.e. slightly less than $d/2$ from the end of the drop panel. With loading more circumferential cracks were predicted on the slab surface inside the drop panel. With further loading more cracks were predicted to form outside the drop panel. Similar behavior was observed in the experiment. Fig. 7.37 (c) shows the predicted crack pattern on the top surface at failure. The distribution of transverse shear stress at a load of 30 kN is shown in Fig. 7.38 (a). At the end of the drop panel, a concentration of shear stress was generated at the corner of the drop panel. The analyzed crack pattern at failure along the cross section A-A was shown in Fig. 7.38 (b). The initiation of the diagonal shear crack was captured by the analysis. Additional shear cracks at the corner of the drop panel observed in the experiment were also reproduced.

(a) Predicted initial crack pattern at load 150kN

Fig. 7.37 Predicted Crack Pattern on Top Surface of Slab I-2
(to be continued)

(b) Predicted crack patter on the occurrence of circumferential cracks at 220kN

(c) Predicted crack pattern at failure

Fig. 7.37 Predicted Crack Pattern on Top Surface of Slab I-2

(a) Shear stress distribution along section A-A at a load of 30 kN

(b) Predicted cracks carrying no normal stress on section A-A

Fig. 7.38 Predicted Shear Stress and Crack Pattern on Section A-A of Slab I-2

The predicted ultimate shear strength of this slab was 680 kN which was 5.95% lower than the observed value of 723 kN. The analyzed load-deflection response curve at the slab center was plotted against the experimental curve as shown in Fig. 7.39. The predicted deformational response curve agreed reasonably well with the experimental curve. The accuracy of the finite element prediction can also be seen from Fig. 7.40 which shows the vertical slab deflection profiles along the centerline of the specimen at different values of applied shear force. The profiles shown in Fig. 7.40 are plotted for one slab half because of symmetry.

This example has confirmed the ability of the developed finite element model to simulate the punching shear behavior of reinforced concrete flat slabs with drop panels. Convenience and flexibility in such a numerical simulation have also been demonstrated.

Fig. 7.39 Load-Deflection Response of Slab I-2 at Slab Center

Fig. 7.40 Slab Centerline Deflection Profiles on Half of the Slab I-2

7.3.5 Slab with Irregular Layout Tested By Tan and Teng (2001)

An analysis of a multi-panel slab S7C with irregular layout tested by Tan and Teng (2001) was performed to evaluate the performance of the developed finite element model in simulating the behavior of a multi-panel slab with relative large size. The slab was support on 7 columns which were pinned at the bottom. The slab was subjected to uniformly distributed vertical load. The geometry and the reinforcement details are shown in Fig. 7.41 and Fig. 7.42 respectively. The slab had a thickness of 100 mm and the column sizes were 200 mm×400 mm. The clear concrete cover was 15 mm. The cylinder compressive strength of concrete was 39.75 MPa. The modulus of elasticity of concrete was 27 GPa. The yield of stress of T-10, R-10, and mesh steel was 518 MPa, 372 MPa, and 575 MPa respectively. The modulus of elasticity of the T-10, and R-10 was 177GPa, 196 GPa respectively. The finite element mesh used in this study is shown in Fig. 7.43. 175 shell elements were used to model the slab. Nodes of the stared elements were specified with shear correction. Each column was simulated by use of 4 shell elements. The mid-node of the bottom element was pinned as shown in Fig. 7.43 to simulate the boundary condition.

Layout (Unit mm)

Section A-A (Unit mm)

Fig. 7.41 Geometry and Layout of Seven Column Slab S7c Tested by Tan and Teng (2001)

Fig. 7.42 Reinforcement Details of S7C

(a) Plain view of the finite element mesh

(b) Perspective view of the finite element mesh

Fig. 7.43 Finite Element Mesh Used for S7C

Fig. 7.44 shows the comparison of the predicted deflection contours at 177kN (including self-weight) along with the observed results. The position where the maximum deflection occurred was predicted correctly. The predicted contour pattern was shown to conform reasonably well to the observed result. The deformed mesh at failure is shown in Fig. 7.45.

Fig. 7.44 Deflection Contour of Slab S7C at 177kN

Fig. 7.45 The Deformed Mesh of S7C

Fig. 7.46 shows the predicted crack pattern on the top surface of S7C along with the observed one. The predicted cracks along columns B, G, and E linked together and formed a clear column strip. Cracks from columns C and G approached each other.

Circumferential cracks were predicted around the columns. The predicted crack pattern on the top surface agreed fairly well with the observed one. Fig. 7.47 shows the predicted crack pattern on the bottom surface of slab S7C in comparison with the observed crack pattern. The predicted cracks formed crack streams following the middle strips of the slab. The positions and orientations of these crack streams agreed well with the observed crack pattern. Biaxial cracking was predicted in areas where maximum deflection occurred in each panel of the slab. This biaxial cracking could also be found in the observed crack pattern.

Fig. 7.48 shows the stress distribution of steel in the x-direction at failure. Yielding of steel was found both in the middle strip on the bottom surface and in the column strip on the top surface. Fig. 7.49 shows principal compressive stress distribution and crushing points on the bottom surface. Yield of steel and crushing of concrete were first observed at the slab connection around column F in the analysis. The localized diagonal shear crack at the slab-column connection at column F observed in the experiment was not captured clearly in sections of slab-column connection. A flexural failure which started at the connection around column F was predicted for this slab. In the experiment, the specimen basically failed in flexure, though most of the slab connections were tested individually for punching shear after the punching shear crack formed at column F at a total load of 558 kN. It has been noticed that both the experiment and numerical simulation disclosed the same weakest connection which was around column F. A few reasons may explain this. Column F was a corner support surrounded by a slab with irregular layout that was under biaxial bending and torsion. In addition, column F was weaker in the span FE direction which was the shortest span compared to others. Significant biaxial shear stresses can be generated along the section of the slab-column connection.

The predicted ultimate strength of the slab was 522 kN which was 6.5% lower than the observed value. Fig. 7.50 shows the analyzed load-deflection response of slab S7C at the maximum deflection point with the experimental results. The predicted response curve followed fairly well the observed response curve.

(a) Observed crack pattern

(b) Predicted crack pattern

Fig. 7.46 Crack Pattern on Top Surface of S7C

(a) Observed crack pattern

(b) Predicted crack pattern

Fig. 7.47 Crack Pattern on Bottom Surface of S7C

(a) Stress of steel in x-direction on bottom surface

(b) Stress of steel in x-direction on top surface

Fig. 7.48 Stress Distribution of Steel in X-Direction at Failure of S7C

Fig. 7.49 Distribution of Principal Compressive Stress and Crushing Points on the Bottom Surface of S7C

Fig. 7.50 Load-Deflection Response of S7C at the Maximum Deflection Point

From the predicted crack patterns, stress distributions of concrete and steel on the top and bottom surface of the slab, the column and middle strips can be distinguished clearly although the slab in this example has an irregular layout. The suitability of the finite element model developed in this study for global nonlinear analysis of relative large RC flat plate structures has been demonstrated in this

numerical example. In addition, the computation time of this slab was around 2 hours for a normal micro-computer. Computational stability and efficiency make it promising to be applied in design practice for reinforced concrete flat plate structures with irregular layout.

7.4 PARAMETRIC STUDY FOR STRENGTH OF R. C. SLABS UNDER PUNCHING SHEAR

The developed finite element model has shown good performance in simulating the behavior of reinforced concrete slabs under punching shear. It has been employed to study the influence of several factors on the behavior of R. C. slabs under concentrated punching shear forces. The factors investigated were slab thickness, concrete strength, and reinforcement ratio.

7.4.1 Influence of Slab Thickness

To investigate the influence of slab thickness on the shear behavior of slabs under concentrated punching shear force, six analyses were conducted. The reference slab was slab A-2c tested by Elstner and Hognestad (1956). The variable was the slab thickness as shown in Table. 7.14. Other parameters of the slabs analyzed were identical to those of slab A-2c. The predicted strengths of the slabs are presented in Table 7.15 and predicted deformational responses plotted in Fig. 7.51. The predicted shear strengths versus slab thickness are shown in Fig. 7.52. The results show that the gradual increase of slab thickness leads to a corresponding increase in ultimate shear strength and cracking load. All the slabs were predicted to fail in punching shear. The yielding of steel either did not occur or was very close to ultimate failure. No obvious effect of slab thickness has been shown on yield load, ductility and failure mode. The predicted strengths are also shown in Fig. 7.53 in terms of relative punching strengths plotted against effective depth. The relative strength is the ratio of the ultimate shear stress for the slab calculated as $P_u / d(\sum b + \pi d)$, to the ultimate stress for the slab with $h = 240$ mm ($d = 200$ mm), where $\sum b$ is the peripheral of the column section. The results show a continuous reduction of relative strength with increasing depth throughout the range covered ($62 \text{ mm} \leq d \leq 312 \text{ mm}$). The curve drawn in the figure indicates that the ultimate shear stress τ_u is proportional to $\sqrt[3]{1/d}$. The results presented here are consistent with the experimental findings by Kinnunen, Nylander (1960) and by Regan (1986).

Table 7.14 Predictions for slabs with different thickness

Thickness h (mm)	Effective Depth (mm)	Ultimate Load (kN)	Cracking Load (kN)	Cracking load/ ultimate load	Yield Load (kN)	Yield load/ ultimate load	Predicted failure mode
100	62	216	33	0.15	—	—	shear
150	112	432	70	0.16	417	0.97	shear
200	162	570	116	0.20	534	0.94	shear
240	202	668	145	0.22	—	—	shear
300	262	843	251	0.30	—	—	shear
350	312	1082	268	0.25	1068	0.99	shear

Fig. 7.51 Load-Center Deflection Response of Slabs with Different Thicknesses

Fig. 7.52 Predicted Shear Strength versus Slab Thickness

Fig. 7.53 Predicted Shear Strength versus Effective Depth of Slab

7.4.2 Influence of Concrete Strength

It has been generally accepted that it is the tensile strength rather than the compressive strength of concrete which controls the punching behavior of R. C. slabs. Two groups of analyses were performed to provide more evidence on this aspect. In the first group of analyses, the slab A-2b tested by Elstner and Hognestad (1956) was re-analyzed using different tensile strengths of concrete, $f_t' = 1.46$ MPa, 2.20 MPa, 3.00 MPa. Fig. 7.54 shows the predicted load versus center deflection responses. The increase of tensile strength of concrete is seen to increase stiffness and ultimate strength (320 kN, 368 kN, 449 kN). Punching failure was predicted for $f_t' = 1.46$ MPa, and 2.20 MPa, while flexural failure was obtained for $f_t' = 3.00$ MPa. In the second group of analyses, the slab A-2c tested by Elstner and Hognestad (1956) was re-analyzed using the same tensile strength ($f_t' = 2.02$ MPa) of concrete and different compressive strengths ($f_c' = 25$ MPa, 35 MPa, 45 MPa). The predicted load versus center deflection responses are shown in Fig. 7.55. It was found that the increase in compressive strength did not lead to the obvious increase of ultimate shear strength of the slab (409 kN, 427 kN, 424 kN) although a slight increase in stiffness was observed. All the predicted failure modes were punching shear failure. The analyzed result further proved that tensile strength is more important than compressive strength of concrete with regard to the punching shear behavior of R. C. slabs.

Fig. 7.54 Load-Center Deflection Responses of Slab Predicted with Different Tensile Strengths of Concrete

Fig. 7.55 Load-Center Deflection Responses Predicted with Different compressive strengths of concrete

However, since the tensile strength and the compressive strength of concrete are normally interrelated, their combined effect, in terms of influence of compressive strength of concrete, was investigated by the following analyses. Slab A-2c was recalculated with the use of different compressive strengths (20 MPa, 25 MPa, 35 MPa, 45 MPa, 55 MPa) and their corresponding tensile strengths according to $f_t = 0.33\sqrt{f_c}$. The predicted shear strengths are presented in Table 7.15 and deformational responses plotted in Fig. 7.56. The analysis result showed that the increase in compressive strength of concrete not only increased the stiffness and the ductility of the slab, but also the cracking load, yield load, and ultimate shear strength. No influence of compressive strength on the failure mode was observed. All the analyses predicted the same failure mode of shear with clear formation of diagonal shear crack on the section of slab near the column.

Table 7.15 Predictions for slabs with different compressive strength of concrete

f_c (MPa)	Ultimate Load (kN)	Cracking Load (kN)	Cracking load/ ultimate load	Yielding Load (kN)	Yielding load/ ultimate load	Predicted failure mode
20	318	45	0.14	—	—	shear
25	372	48	0.13	367	0.99	shear
35	463	66	0.14	411	0.89	shear
45	491	86	0.18	434	0.88	shear
55	551	103	0.19	444	0.81	shear

Fig. 7.56 Load-Center Deflection Response Curves Predicted with Different Compressive Strengths of Concrete

The shear strength versus compressive strength of concrete is shown in Fig. 7.57. The predicted strength P_u in the above analyses was fitted well by $P_u \propto \sqrt{f'_c}$, while $P_u \propto \sqrt[3]{f'_c}$ underestimated and $P_u \propto f_c'^{\frac{2}{3}}$ overestimated the shear strength for compressive strengths of concrete larger than 25 MPa. Fig. 7.58 shows the predicted strengths in the above analyses as well as the predicted strengths of slabs with reinforcement ratio between 2% and 3% tested by Elstner and Hognestad (1956) as graphs of nominal shear stresses τ_u calculated for the CEB perimeter plotted against compressive strength of concrete. The influence of compressive strength of concrete on the predicted ultimate shear stresses was found to be better represented by $\tau_u \propto \sqrt{f'_c}$ than $\tau_u \propto f_c'^{\frac{2}{3}}$ or $\tau_u \propto \sqrt[3]{f'_c}$. This result indicated that the shear strength of slab was proportional to $\sqrt{f'_c}$.

Fig. 7.57 Predicted Shear Strength versus Compressive Strength of Concrete

Fig. 7.58 Relative Strength versus Compressive Strength of Concrete Obtained for Slabs in this analysis and slabs by Elstner and Hognestad (1956)

7.4.3 Influence of Reinforcement Ratio

The influence of the reinforcement ratio was studied by simulating slabs with different percentages: $\rho = 0.50, 1.15, 2.00, 2.47, \text{ and } 3.70\%$. The compressive strength of concrete was $f'_c = 40 \text{ MPa}$ for all the analyses. The geometry, boundary and loading condition were identical to those of slabs tested by Elstner and Hognestad (1956). The predicted strengths and failure modes are presented in Table 7.16. The analyzed load-deflection response curves are shown in Fig. 7.59. The predicted shear strength is plotted against the reinforcement ratio in Fig. 7.60. The increase of percentage of reinforcement was found to increase the stiffness and failure load, decrease the ductility, and not affected the cracking load for slabs with reinforcement ratio less than 3%. Slabs with lower reinforcement ratio ($\rho \leq 1.15\%$) were predicted to fail in flexure while punching shear failure was predicted for those with higher reinforcement ratio ($\rho > 1.15\%$).

Table 7.16 Predictions for slabs with different reinforcement ratio

Reinforcement ratio ρ (%)	Ultimate Load (kN)	Cracking Load (kN)	Cracking load/ ultimate load	Yielding Load (kN)	Yielding load/ ultimate load	Predicted failure mode
0.50	256	59	0.23	171	0.67	flexure
1.15	409	59	0.14	257	0.63	flexure
2.00	417	67	0.16	383	0.92	shear
2.47	485	74	0.15	413	0.85	shear
3.00	544	74	0.14	510	0.94	shear
3.70	504	67	0.13	—	—	shear

Fig. 7.59 Load-Center Deflection Response Curves of Slabs with Different Reinforcement Ratios

Fig. 7.60 Predicted Shear Strength versus Reinforcement Ratio

Fig. 7.61 Predicted Relative Shear Strength versus Reinforcement Ratio

The predicted relative strengths in this analysis along with those predicted for slabs tested by Elstner and Hognestad (1956) that failed in punching shear are plotted against the reinforcement ratio in Fig. 7.61. The expressions of either $\tau_u \propto (1 + 50\rho)$ with $\rho \leq 2.00\%$ as in the CEB Complements or $\tau_u \propto \sqrt[3]{\rho}$ as in British codes can fit the predicted relative strength well.

Analysis results from the above parametric study confirm the general understanding of the influence that parameters have on the punching shear behavior of reinforced concrete slabs. This partly proved the validity and applicability of the developed finite element model in modeling punching shear behavior of reinforced concrete flat plate structures.

CHAPTER 8 SUMMARY AND CONCLUSIONS

8.1 SUMMARY

In this study, a three dimensional nonlinear finite element model for nonlinear analysis of reinforced concrete flat plate structures has been developed.

A 3-D hypo-elastic orthotropic material model based on the equivalent uniaxial strain concept for modeling the material properties of uncracked concrete and CRC was proposed. The material properties of uncracked concrete under multiaxial stress state were simulated by uniaxial stress-equivalent uniaxial strain relationships. Linear elastic material properties were assumed for uncracked concrete under tensile stress. The five-parameter strength surface proposed by Willam and Warnke (1975) was employed to determine the ultimate peak stress and peak strain. The stress-strain response of uncracked concrete has been verified in the present study by comparing the analytical results with the available data on uncracked concrete response to multiaxial states of stress.

A new model describing the material properties of CRC with multiple non-orthogonal cracks has been developed. The material properties of the MR-CRC are represented by the material properties of the intact concrete and a number of uniaxially cracked concrete with their coupling solids. Cracking effects due to multiple non-orthogonal cracks are traced in each uniaxially cracked concrete. Tension softening and aggregate interlock at crack interfaces as well as tension stiffening and compression softening initiated in concrete between cracks due to multiple non-orthogonal cracks are all incorporated explicitly in the proposed model.

An exponential descending curve was used to describe the tension softening effect of cracked plain concrete under tension in terms of a stress-equivalent uniaxial strain relationship. The thickness of shell element was taken as the representative material length of concrete. Similar softening curve with different softening parameter along with a linear descending branch after yielding of steel was used to

model the tension stiffening effect of CRC. In case the reinforcement is not normal to the crack plane, the tension stiffening effect is considered in crack normal direction by nonlinear interpolation between tension softening and tension stiffening. Compression softening of CRC due to principal tensile stress, principal tensile strain normal to crack as well as shear stress in crack interface is considered. The shear friction effect of aggregate interlock and dowel action of uniaxially cracked concrete is modeled as a variable fraction of shear stiffness of uncracked concrete.

The degenerate isoparametric serendipity shell element developed by Ahmad (1970) is employed and simplified for flat shapes with constant thickness. A layering scheme for shear correction was included directly in the element formulations allowing the variation of shear strain distribution over the depth of the section. A special formulation proposed by Polak (1992) was implemented to permit the arrangement of transverse shear reinforcements. A numerical nonlinear analysis procedure is developed based on the arc-length approach proposed by Fafard and Massicotte (1993) to solve the nonlinear equations. Increments are controlled by a variable "length" in form of work. Iterations within an increment are controlled by a constant arc-length of displacements and load.

The 3-D constitutive material model developed in this study for CRC has been verified by numerical simulation of a number of RC specimens. The capability and performance of the proposed finite element model in modeling the punching shear behavior of reinforced concrete flat plate structures has been evaluated by simulating RC flat plates or flat slabs under various loading.

8.2 CONCLUSIONS

The proposed tension stiffening model has shown good capability in predicting the deformational response and the ultimate tensile strength of CRC under tension loading (panels PB13, PB24, PB25, PB27 tested by Bhide and Collins 1986; and panels E2-1, E4-1, E10-1A, F2, F3, F4 tested by Belarbi and Hsu 1991).

The nonlinear responses of the delft beam have been successfully simulated in this study. The crack pattern and diagonal shear failure mode were reproduced. The ultimate shear strength was predicted with good accuracy. The simulation indicated the localized cracking effects of tension softening and aggregate interlock due to multiple non-orthogonal cracks were represented well by the proposed model. This analysis example also highlighted the importance of the tension softening model in reproducing the diagonal shear failure in cracked plain concrete. The use of the whole thickness as the characteristic material length which results in a brittle property of concrete is preferable for reproducing such a brittle shear failure. The shear tension parameter can take the value of 1.2 to simulate the aggregate interlock effect of cracked plain concrete and lightly reinforced concrete. The validity of the proposed layering scheme for incorporation of transverse shear deformation is also corroborated in this example.

Finite element modeling of reinforced concrete panels under combined tension and shear (PB19, PB20, PB21, PB22, Bhide 1986) showed the good capability of the developed material model to simulate the crack pattern, deformational response, the ultimate shear strength, and failure mode of these panels. The mobilization of principal directions, the propagation of cracking, and the ultimate failure were all reproduced. The analysis indicated that the anisotropy of tension stiffening effect was treated well by the proposed model. The tension stiffening effect due to multiple non-orthogonal cracks was simulated with good accuracy. It was shown the neglect of dowel action resulted in a premature failure and lower shear strength to be predicted for CRC with multiple non-orthogonal cracks.

The predicted load-deflection responses of slabs (Slab by Elstner and Hognestad 1956; slabs by Moe 1961; slab by Megally 1998; slab by Tan and Teng 2001) analyzed in this study agreed reasonably well with the observed response curves. Good performance of the developed model has been demonstrated in modeling the tension stiffening effect of multidirectional cracked concrete reinforced in multi-directions.

Analyzed results for panels (PB24, PB25, PB27 by Bhide and Collins 1986; E2-1, E4-1, E10-1A by Belarbi and Hsu 1991) under sequential biaxial tension compression loading and panels (F2, F3, F4 by Belarbi and Hsu 1991) under proportional biaxial tension compression loading indicated the good performance of the developed material model in simulating the compression softening behavior due to transverse principal tensile strain and transverse principal tensile stress. Analyses of panels (B1, B2, B3, B4, B5, B6 tested by Pang (1991)) under pure shear loading showed that the proposed compression softening model has the capability of handling the compression softening effect due to the shear stress in the crack interface. Compressive crushing failure was successfully captured in all the panels simulated and the failure was identified to be caused by the gradual softening effect on compressive strength due to the tensile stress, the tensile strain normal to crack plane, and the development of shear stress in the crack interface. The shear retention parameter β can take the value of 0.8 for uniaxially cracked concrete with strong dowel action effect.

The lack of proper experimental data make it difficult to verify the proposed model in modeling the compression softening effect due to multiple cracks at element level. A numerical experiment study was thus presented in section 4.3.6 to demonstrate the performance of the proposed model in simulating the compression softening effect of MR-CRC. Moreover, in this context, the model has also been examined in analyses of RC slabs at the structural level. The developed compression softening model for CRC has simulated the stiffness and strength of a RC flat plate under torsional moment reasonably well (ML6, Marti et al. 1987). Compressive crushing failure was captured in multidirectional cracked concrete before the occurrence of punching shear failure in some slabs (slabs A-2c, B14 by Elstner and Hognestad 1956). The inclusion of compression softening due to multiple cracks avoids overestimating the shear strength of slabs under punching shear.

The formulated layering scheme for shear correction has improved the representation of transverse shear strain distributions in shell element. The scheme has the flexibility to be applied at any node of the shell element where shear

correction is needed. The importance of shear correction on shell elements in reproducing punching shear failure has been highlighted in the analysis of A-2c. The inclusion of the column stub improved the representation of transverse shear stress and led to a more reasonable crack pattern to be predicted. The proposed layering scheme made it possible to include the column stub in the analysis with shell elements. The characteristics of punching shear behavior as well as the ultimate strength of RC flat plates under a concentrated load or unbalanced moment were captured well by the developed finite element model. The modeling of a RC flat slab with drop panel (Megally 1998) showed the good capability of the proposed layering scheme for incorporating a drop panel into the analysis. Provisions need to be taken to strengthen the corner of the drop panel as local shear failure was predicted in the analysis and observed in the experiment. The more precise simulation of a slab under high unbalanced moment may necessitate the inclusion of the axial compressive stress from the column stub to avoid premature shear failure in the slab element under the column stub. The simulation of the multi-panel RC slab with irregular layout (Tan and Teng 2001) demonstrated the suitability of the developed finite element model for global analysis of relatively large RC flat plate structures. The good performance may promise its application in assisting the design practice for reinforced concrete flat plate structures with irregular layout.

Results obtained from parametric studies provided confirmative information for the general understanding of the influence that some parameters have on the punching shear behavior of reinforced concrete slabs. The increase of slab thickness increases cracking load and ultimate shear strength but does not affect the ductility and failure mode. The parametric study confirmed that it is the tensile strength rather than the compressive strength of concrete that controls the punching behavior of R. C. flat plates. The increase in compressive strength of concrete not only increased the stiffness and the ductility of the slab, but also increased the cracking load, yielding load, and ultimate shear strength. No influence of compressive strength on the failure mode was observed. The increase of percentage of reinforcement was found to increase the stiffness and ultimate shear strength, however, decrease the ductility. No effect of reinforcement ratio was observed on the cracking load.

The developed arc-length nonlinear analysis procedure has shown good capabilities in tracing the material nonlinearities of reinforced concrete structures. The critical points of cracking of concrete, yielding of steel, formation of failure mechanism on the load-deformation response curve can be captured reasonably well and passed through successfully. The procedure also enabled tracing the post ultimate responses of RC structures. The complete response curves of the whole loading history obtained by the nonlinear analysis procedure have made it easy to determine the ultimate state in strength and in deformation, and to evaluate the nature of failure.

In general, the developed finite element model has been proved to be capable of simulating the localized punching shear behavior of reinforced concrete slab-column connection and also to be suitable for nonlinear global analysis of relatively large reinforced concrete flat plate structures. The application of the finite element model developed in this study so far has been restricted to static proportional loading. Care should be taken for any further extension to non-proportional loading.

8.3 RECOMMENDATIONS FOR FUTURE WORK

Based on the present study, some of areas where some future work needs to be done are identified below:

- ✓ Extend the material model to incorporate material properties of reinforced concrete under non-proportional or cyclic loading conditions.
- ✓ Develop procedures to include pre-stressing steels in concrete.
- ✓ Formulate a more precise model for aggregate interlock and dowel action effect in MR-CRC in slabs.
- ✓ Conduct experimental studies of compression softening effect due to multiple cracks to further verify the proposed material model for CRC at element level.

REFERENCES

Ahmad S., Irons B. M., and Zienkiewicz O. C., "Analysis of Thick and Thin Shell Structures by Curved Finite Elements", International Journal for Numerical Methods in Engineering, Vol. 2, 1970, pp. 419-451.

Al Mahaidi R. S. H., "Nonlinear Finite Element Analysis of Reinforced Concrete Deep Members", PhD Thesis, Cornell University, 1979.

Alexander S. B., and Simmonds S. H., "Tests of Column-Flat Plate Connections", ACI Structural Journal, V. 89, No. 5, Sept.-Oct., 1992, pp. 495-502.

Alexander S. B., and Simmonds S. H., "Ultimate Strength of Slab-Column Connections", ACI Structural Journal, V. 84, No. 3, May-June, 1987, pp. 255-261.

Arthur P. B., and Richard J. S., "Advanced Mechanics of Material", John Wiley & Sons, Inc, 2003

ASCE, Task Committee on Finite Element Analysis of Reinforced Concrete Structures, Nilson A. H., Editor, "State of the Art Report on Finite Element Analysis of Reinforced Concrete", ASCE, 1982.

ASCE, Task Committee on Finite Element Analysis of Reinforced Concrete Structures, Isenberg J., Editor, "Finite Element Analysis of Reinforced Concrete II", ASCE, 1992.

Balakrishnan S., Elwi A. A., and Murray A. W., "Effect of Modeling on NLFE Analysis Of Concrete Structures", Journal of Structural Engineering, ASCE, V. 114, No. 7, July, 1988, pp. 1467-1487.

Balakrishnan, S., and Murray, D. W., "Concrete Constitutive Model for NLFE Analysis of Structures", Journal of Structural Engineering, ASCE, V. 114, No. 7, July 1988, pp. 1449-1466.

Balan T. A., "A Constitutive Relationship for Structurally Non-uniform Materials in Complex Stressed State", Strength of Mat., V. 10, 1986, pp. 1351-1357.

Balan T. A., Filippou F. C., and Popov E. P., "Constitutive Model for 3D Cyclic Analysis of Concrete Structures", Journal of Engineering Mechanics, ASCE, V. 123, No. 2, Feb., 1997, pp. 143-153.

Barzegar F., "Layering of RC Membrane and Plate Elements in Nonlinear Analysis", Journal of Structural Engineering, ASCE, V. 114, No. 11, Nov., 1988, pp. 2474-2492.

Barzegar F., and Maddipudi S., "Three-Dimensional Modeling of Concrete Structures. I: Plain Concrete", Journal of Structural Engineering, V. 123, NO. 10, October, 1997, pp. 1339-1346.

Barzegar F., and Maddipudi S., "Three-Dimensional Modeling of Concrete Structures. II: Reinforced Concrete", Journal of Structural Engineering, V. 123, NO. 10, October, 1997, pp. 1347-1356.

Barzegar, F., "Analysis of RC Membrane Elements with Anisotropic Reinforcement", Journal of Structural Engineering, ASCE, V. 115, No. 3, Mar., 1989, pp. 647-665.

Bathe K. J., "Finite Element Procedures", Prentice Hall, New Jersey, 1996.

Bathe K. J., and Ramaswamy S., "On Three-Dimensional Nonlinear Analysis of Concrete Structures", Nuclear Engrg. Des., V. 52, No. 3, 1979, pp. 385-409.

Bathe K. J., Walczak J., Welch A. and Mistry N., "Nonlinear Analysis of Concrete Structures", Computers and Structures, V. 32, No. 3/4, 1989, pp. 563-590.

Bazant Z. P., "Comment on Orthotropic Models for Concrete and Geomaterials", Journal of Engineering Mechanics, V. 109, No. 6, 1983, pp. 849-865.

Bažant Z. P., “Discussion at IABSE Colloq. on Advanced Mechanics of Reinforced Concrete”, Final Report, Delft University Press, Delft, The Netherlands, 1981, pp. 490-491.

Bažant Z. P., and Bhat P. D., “Endochronic Theory of Inelasticity and Failure of Concrete”, Journal of Engineering Mechanics, V. 102, No. EM4, August, 1976, pp. 701-722.

Bažant Z. P., and Cedolin L., “Blunt Crack Band Propagation in Finite Element Analysis” Journal of Engineering Mechanics, V. 105, No. EM2, April, 1979, pp. 297-315.

Bažant Z. P., and Gambarova P. G., “Crack Shear in Concrete: Crack Band Microplane Model”, Journal of Structural Engineering, V. 110, No. 9, September, 1984, pp. 2015-2035.

Bažant Z. P., and Gambarova P., “Rough Cracks in Reinforced Concrete”, Journal of Structural Engineering, V. 106, No. ST 4, April, 1980, pp. 819-842.

Bažant Z. P., and Kim S. S., “Plastic-Fracturing Theory for Concrete”, Journal of Engineering Mechanics, V. 105, No. EM3, June, 1979, pp. 407-428.

Bažant Z. P., and Lin F. B., “Nonlocal Smeared Cracking Model for Concrete Fracture” , Journal of Structural Engineering, V. 114, No. 11, November, 1988, pp. 2493-2510.

Bažant Z. P., and Oh B. H., “Crack Band Theory for Fracture of Concrete”, Mater. Structures, RILEM 93, 1983, pp. 155-177.

Bažant Z. P., and Ožbolt J., “Nonlocal Microplane Model for Fracture, Damage, and Size Effect in Structures”, Journal of Engineering Mechanics, V. 116, No. 11, October, 1990, pp. 2485-2505.

Bažant Z. P., and Prat P. C., “Microplane Model for Brittle-Plastic Material: I. Theory”, Journal of Engineering Mechanics, V. 114, No. 10, October, 1988, pp. 1672-1688.

Bazant Z. P., and Prat P. C., "Microplane Model for Brittle-Plastic Material: II. Verification", Journal of Engineering Mechanics, V. 114, No. 10, October, 1988, pp. 1689-1702.

Bazant Z. P., and Tsubaki T. (1980) "Total Strain Theory and Path-Dependence of Concrete", Journal of Engineering Mechanics, ASCE, V. 106, EM. 6, Dec., 1980, pp. 1151-1173.

Belarbi A., "Stress-Strain Relationships of Reinforced Concrete in Biaxial Tension-Compression", PhD. Thesis, University of Houston, 1991.

Belarbi A., and Hsu T. T. C., "Constitutive Laws of Concrete in Tension and Reinforcing Bars Stiffened by Concrete", ACI Structural Journal, V. 91, No. 4, July-August, 1994, pp. 465-474.

Belarbi A., and Hsu T. T. C., "Constitutive Laws of Softened Concrete in Biaxial Tension -Compression", ACI Structural Journal, V. 92, No. 5, September-October, 1995, pp. 562-573.

Benallal A., Billardon R. and Lemaitre J., "Continuum Damage Mechanics and Local Approach to Fracture: Numerical Procedures", Computer Method in Applied Mechanics and Engineering, Vol.92, 1991, pp. 141-155.

Bhide S. B., "Reinforced Concrete Elements in Shear and Tension", Ph. D. Thesis, University of Toronto, Canada, 1986.

Braestrup M. W., Nielsen M. P., Jensen B. C., Bach F., "Axisymmetric Punching of Plain and Reinforced Concrete", Report R75, Struct. Research Laboratory, Technical Univ. of Denmark, Copenhagen, 1976.

Carrera E. "A Study on Arc-Length-Type Methods and their Operation Failures Illustrated By a Simple Model", Computers & Structures, Vol. 50, No. 2, 1994, pp. 217-229.

Cedolin L., Crutzen, Y. R. J., and Dei Poli S., "Triaxial Stress-strain Relationship for concrete", Journal of Engineering Mechanics, ASCE, V. 103, No. EM3, June, 1977, pp. 423-439.

Cervenka, V., "Constitutive Model for Cracked Reinforced Concrete", ACI Journal, Proceedings V. 82, No. 6, Nov.-Dec., 1985, pp. 877-882.

Chen A. C. T., and Chen W. F., "Constitutive Relations for Concrete", Journal of Engineering Mechanics, V. 101, No. EM4, August, 1975, pp. 465-481.

Chen W. F. "Plasticity in Reinforced Concrete", McGraw-Hill Book Co., New York, 1982.

Cope R. J., et al., "Modeling of Reinforced Concrete Behavior for Finite Element Analysis of Bridge slabs", Numerical Methods for Nonlinear Problems, C. Taylor et al., eds., Vol. 1, Pineridge Press, Swansea, U.K., 1980, pp. 457-470.

Crifield, M. A., and Wills, J., "Analysis of R/C Panels Using Different Concrete Models", Journal of Engineering Mechanics, ASCE, V. 115, No. 3, 1989, pp. 578-597.

Crisfield M. A., "A Fast Incremental/Iterative Solution Procedure that Handles "Snap-Through"", Computers & Structures, Vol. 13, 1981, pp. 55-62.

Crisfield M. A., "Accelerated solution techniques and concrete cracking", Computer Methods in Applied Mechanics and Engineering, Vol. 33, 1982, pp. 585-607.

Darwin, D., and Pecknold, D. A., "Analysis of RC Shear Panels under Cyclic Loading", Journal of Structural Engineering, ASCE, V. 102, ST2, Feb., 1976, pp. 355-369.

de Borst R., "Continuum Models for Discontinuous Media", Proc. Of the International RILEM/ESIS Conference on "Fracture Processes in Concrete, Rock and Ceramics", Ed. by van Mier, Rots and Bakker, E & FN Spon/Chapman & Hall, Noordwijk, 1991, pp. 601-618.

de Borst R., and Nauta P., "Non-orthogonal Cracks in a Smeared Finite Element Model", Engineering Computations, V. 2, March, 1985, pp. 35-46.

Dyngeland T., Hoiseth K., Opheim E., Hole A. "Nonlinear Analysis of Reinforced Concrete Members Subjected to Punching shear", Computer Modeling of Concrete Structures. Proc. Of EURO-C 1994 Int. Conference, Innsbruck, 1994.

Elmorsi, M., Kianoush, M. R., and Tso, W. K., "Nonlinear Analysis of Cyclically Loaded Reinforced Concrete Structures", ACI Structural Journal, V. 95, No. 6, Nov.-Dec., 1998, pp. 725-739.

Elstner R. C., and Hognestad E., "Shear Strength of Reinforced Concrete Slabs", ACI Journal, Proceedings V. 28, No. 1, July, 1956, pp. 29-58.

Elwi A. A., and Murray A. W., "A 3D Hypoelastic Concrete Constitutive Relationship", Journal of Engineering Mechanics, ASCE, V. 105, EM4, August, 1979, pp. 623-641.

Fafard M., and Massicotte, B., "Geometrical Interpretation of the Arc-Length Method", Computers & Structures, Vol. 46, No. 4, 1993, pp. 603-615.

Feng Y. T., Perić † D., and Owen D. R. J., "A New Criterion for Determination of Initial Loading Parameter in Arc-Length Methods", Computers & Structures, Vol. 68, No. 3, 1996, pp. 479-486.

Fields K. L., "Tension Stiffening Response of High Strength Reinforced Concrete Tensile Members", Master Thesis, University of New Brunswick, 1998.

Figueras J. A., and Owen D. R. J., "Anisotropic Elasto-Plastic Finite Element Analysis of Thick And Thin Plates and Shells", International Journal for Numerical Methods in Engineering, Vol. 19, 1983, pp. 541-566.

Foster S., "An Application of the Arc Length Method Involving Concrete Cracking", International Journal for Numerical Methods in Engineering, Vol. 33, 1992, pp. 269-285.

Gerstle K. H., "Simple Formulation of Biaxial Concrete Behavior", Journal of the American Concrete Institute, V. 78, No. 1, 1981, pp. 62-68.

Gilbert R. I. And Warner R. F., "Tension Stiffening in Reinforced Concrete Slabs", Journal of Structural Division, Proceedings ASCE, Vol. 104, No. ST12, Dec., 1978, pp. 1885-1900.

Gomes H. M., and Awruch A. M., "Some aspects on three dimensional numerical modeling of reinforced concrete structures using the finite element method", Advances in Engineering Software, Vol. 32, 2001, pp. 257-277.

González-Vidoso F., Kotsovos M. D., and Pavlović M. N., "Symmetrical Punching of Reinforced Concrete Slabs: An Analytical Investigation Based on Nonlinear Finite Element Modeling", ACI Structural Journal, V. 85, May-June, 1988, pp. 241-250.

Gupta A. K., and Akbar H., "Cracking in Reinforced Concrete Analysis", Journal of Structural Engineering, ASCE, V. 110, No. 8, Aug., 1984, pp. 1735-1746.

Hallgren M., "Punching Shear Capacity of Reinforced High Strength Concrete Slabs", PhD thesis, KTH Stockholm, TRITA-BKN. Bull.23, 1996, Royal Institute of Technology, Stockholm, 1996, pp.206.

Han D. J., and Chen W. F., "Strain-Space Plasticity Formulation for Hardening-Softening Materials with Elastoplastic Coupling", International Journal of Solids and Structures, V. 22, No. 8, 1986, pp. 935-950.

Hand F. R., Pecknold D. A., and Schnobrich W. C., "Nonlinear Layered Analysis of RC Plates and Shells", Journal of Structural Division, Proceedings ASCE, Vol. 99, No. ST7, July, 1973, pp. 1491-1505.

Hauke B., and Maekawa K., "Three-Dimensional R/C-Model with Multi-directional Cracking", Computational Modeling of Concrete Structures, eds de Borst, etc, 1998 Balkema, Rotterdam, pp. 93-102.

Hillerborg A., Modeer M., and Petersson P. E., "Analysis of Crack Formation and Crack Growth in Concrete by Means of Fracture Mechanics and Finite Elements", Cement and Concrete Research, V.6, No. 6, 1976, pp. 773-782.

Hognestad E., "Yielding Line Theory for the Ultimate Strength of Reinforced Concrete Slabs", ACI Journal, Proceedings V. 24, No. 7, March, 1953, pp. 637-656.

Hsieh S. S., Ting E. C., and Chen W. F., "An Elastic Fracture Model for Concrete", Proceedings of 3rd Engineering Mechanics Division Specialty Conference, ASCE, Austin, TX, 1979, pp. 437-440.

Hsu T. T. C., and Zhang L. X., "Nonlinear Analysis of Membrane Elements by Fixed-Angle Softened-Truss Model," ACI Structural Journal, V. 94, No. 5, September-October, 1997, pp. 483-492.

Hu H. T. and Schnobrich W. C., "Nonlinear Analysis of Cracked Reinforced Concrete", ACI Structural Journal, V. 87, No. 2, March-April, 1990, pp. 199-207.

Imran I., and Pantazopoulou S. J., "Plasticity Model for Concrete under Triaxial Compression", Journal of Engineering Mechanics, V. 127, No. 3, March, 2001, pp. 281-290.

Karihaloo B. L., and Fu D., "An Anisotropic Damage Model for Plain Concrete", Engineering Fracture Mechanics, V. 35, No. 1/2/3, 1990, pp. 205-209.

Kinnunen S., Nylander H., "Punching of Concrete Slabs without Shear Reinforcement", Transactions No. 158, Royal Institute of Technology, Stockholm, 1960, pp. 112.

Kirschner U., and Collins M. P., "Investigating the Behavior of Reinforced Concrete Shell Elements", University of Toronto, Publication No. 86-09, 1986, pp. 82.

Kotsovos M. D., and Newman J. B., "Behavior of Concrete under Multiaxial stresses", ACI Structural Journal, V. 74, No. 9, 1977, pp. 443-446.

Kotsovos M. D., and Newman J. B., “Generalized Stress-Strain Relations for Concrete”, Journal of Engineering Mechanics, V. 104, EM4, August, 1978, pp. 845-856.

Krajcinovic D., and Vujosevic M., “Strain Localization—short to long correlation length transition”, International Journal of Solids and Structures, V. 35, 1998, pp. 4147-4166.

Kupfer H., Hilsdorf H. K. and Rusch H., “Behavior of Concrete under Biaxial Stresses”, ACI Journal, Proceedings V. 66, No. 8, August, 1969, pp. 656-666.

Lam W. F., and Morley C. T., “Arc-Length Method for Passing Limit Points in Structural Calculation”, Journal of Structural Engineering, ASCE, V. 118, No. 1, January, 1992, pp. 169-185.

Liu, T. C. Y., Nilson, A. H., and Slate, F. O., “Biaxial Stress-Strain Relations for Concrete”, Journal of Structural Engineering, ASCE, V. 98, ST5, 1972, pp. 1025-1034.

Loo Y. C., and Guan H., “Cracking and Punching Shear Failure Analysis of RC Flat Plates”, Journal of Structural Engineering, ASCE, V. 123, No. 10, Oct., 1997, pp. 1321-1330.

Lubliner, Oliver J., Oller S., Oñate E., “A Plastic Damage Model for Concrete”, International Journal of Solid and Structures, V. 25, No. 3, 1989, pp. 299-326.

Marti P., Leesti P. and Khalifa W. U., “Torsion Tests on Reinforced Concrete Slab Elements”, Journal of Structural Engineering, ASCE, V. 113, No. 5, May, 1987, pp. 994-1010.

Marti P., and Kong K., “Response of Reinforced Concrete Slab Elements to Torsion”, Journal of Structural Engineering, ASCE, V. 113, No. 5, May, 1987, pp. 976-993.

Mazars J., "A Description of Micro- and Macroscale Damage of Concrete Structures", Engineering Fracture Mechanics, Vo. 25, No. 5/6, 1986, pp. 729-737.

Marzouk and Chen Z. W., "Finite Element Analysis of High Strength Concrete Slabs", ACI Structural Journal, V. 90, No. 5, Sept.-Oct., 1993, pp. 505-513.

Marzouk H. and Jiang D., "Finite Element Evaluation of Shear Enhancement of High-Strength Concrete Plates", ACI Structural Journal, Nov.-Dec., 1996, pp. 667-673.

May I. M., Naji J. H. and Ganaba* T. H., "Displacement Control for the Non-Linear Analysis of Reinforced Concrete Structures", Eng. Comput. Vol. 5, December, 1988, pp. 266-273.

Megally S. and Ghali A., "Punching of Concrete Slabs Due to Column Moment Transfer", Journal of Structural Engineering, ASCE, V. 126, No. 2, Feb. 2000, pp. 180-189.

Megally S. H., "Punching Shear Resistance of Concrete Slabs to Gravity and Earthquake Forces", PhD. Thesis, The University of Calgary, 1998.

Menétrey P., "Analytical Computation of the Punching Strength of Reinforced Concrete", ACI Structural Journal, V. 93, No. 5, Sept.-Oct., 1996, pp. 503-511.

Menétrey P., Walther R., Zimmermann T., Willam K. J., and P.E. Regan, "Simulation of Punching Failure in Reinforced-Concrete Structures", Journal of Structural Engineering, V. 123, No. 5, May, 1997, pp. 652-659.

Menétrey P., "Relationships between Flexural and Punching Failure", ACI Structural Journal, V. 95, No. 4, July-August, 1998, pp. 412-419.

Milford R. V., and Schnobrich W.C., "Nonlinear Behavior of Reinforced Concrete Cooling Towers", Report ISSN: 0069-4274, University of Illinois, Urbana, Ill. 1984.

Millard S. G., and Johnson R. P., "Shear Transfer in Cracked Reinforced Concrete", Magazine of Concrete Research, Vol. 37, No. 130, 1985, pp. 3-15.

Moe J., "Shear Strength of Reinforced Concrete Slabs and Footings under Concentrated Loads", Portland Cement Association, Research and Development Laboratories, Bulletin D47, 1961.

Mohammadi M. S., "Micro Computational Approach to Post Cracking Constitutive Laws of Reinforced Concrete and Application to Nonlinear Finite Element Analysis", Ph.D Thesis, The University of Tokyo, 2002.

Ngo D. and Scordelis A. C., "Finite Element Analysis of Reinforced Concrete Beams", ACI Journal, *Proceedings* V. 64, March, 1967, pp. 152-163.

Nilson, A. H., "Nonlinear Analysis of Reinforced Concrete by Finite Element Method", ACI Journal, *Proceedings* V. 66, No. 9, Sep., 1968, pp. 757-766.

Oliver J., "A Consistent Characteristics Length for Smeared Cracking Model", Int. J. Numer. Methods in Engrg., V. 28, 1989, pp. 461-474.

Oller S., Oñate E., Oliver J., and Lubliner J., "Finite Element Nonlinear Analysis of Concrete Structures Using a Plastic-Damage Model", Engineering Fracture Mechanics, V. 35, No. 1/2/3, 1990, pp. 219-231.

Ottosen N. S., "A Failure Criterion for Concrete", Journal of Engineering Mechanics, ASCE, V. 103, No. EM4, August, 1977, pp. 527-535.

Ottosen N. S., "Constitutive Model for Short Time Loading of Concrete", Journal of Engineering Mechanics, ASCE, V. 105, EM. 1, 1979, pp. 127-141.

Ozbolt J., Mayer U., Vocke H., Eligehausen R., "Das FE-Programm MASA in Theorie und Anwendung", BuStb 94 (1999), H.10, 403-412.

Pang X. B., "Constitutive Laws of Reinforced Concrete in Shear", PhD. Thesis, University of Houston, 1991.

Pang X. B., and Hsu T. T. C., "Behavior of Reinforced Concrete Membrane Elements in Shear", ACI Structural Journal, V. 92, No. 6, November-December, 1995, pp. 1-15.

Park H., Klingner R. E., Wheat D. L., "Numerical Techniques for Predicting Brittle Failure Of Reinforced Concrete Planar Structures", Journal of Structural Engineering, ASCE, V. 121, No. 10, Oct., 1995, pp. 1507-1513.

Phillips D. V. and Zienkiewicz O. C., "Finite Element Nonlinear analysis of Concrete Structures", Proceedings of the Institution of Civil Engineers, Part 2, vol. 61, March, 1976, pp. 59-88.

Pijaudier-Cabot G., and Bažant Z. P., "Nonlocal Damage Theory", Journal of Engineering Mechanics, V. 113, No. 10 October, 1987, pp. 1512-1533.

Polak M. A., "Modeling Punching Shear of Reinforced Concrete Slabs Using Layered Finite Elements", ACI Structural Journal, V. 95, No. 1, Jan.-Feb., 1998, pp. 71-80.

Polak M. A., "Nonlinear Analysis of Reinforced Concrete Shells", Ph. D. Thesis, University of Toronto, Canada, 1992.

Polak M. A., and Vecchio F. J., "Nonlinear Analysis of Reinforced-Concrete Shells", Journal of Structural Engineering, ASCE, V. 119, No. 12, Dec., 1993, pp. 3439-3462.

Ramaswamy A., "Nonlinear Inelastic Finite Element Analysis of Reinforced Concrete Structures with Emphasis on Shear and Torsion", Ph. D. Thesis, The Louisiana State University and Agricultural and Mechanical Col., 1992.

Ramaswamy A., Barzegar F. and Voyiadjis G. Z., "Post Cracking Formulation for Analysis of RC Structures Based on Secant Stiffness", Journal of Engineering Mechanics ASCE, Vol. 120, No. 12, 1994, pp. 2621-2640.

Ramaswamy A., Barzegar F., and Voyiadjis G. Z., "Study of Layering Procedures in Finite Element Analysis of RC Flexural and Torsional Elements", Journal of Structural Engineering, V. 121, No. 12, December, 1995, pp. 1773-1783.

Ramm E. "Strategies for Tracing Non-Linear Responses near Limit Points", in Non-linear Finite Element Analysis in Structural Mechanics (Eds. W. Wunderlich, E. Stein and K. J. Bathe), Springer-Verlag, New York, 1981, pp. 68-89.

Rashid Y. R., "Ultimate Strength Analysis of Prestressed Concrete Pressure Vessels", Nuclear Engineering and Design, V. 7, No. 4, 1968, pp. 334-344.

Regan P. E., "Symmetric Punching of Reinforced Concrete Slabs", Mag. Concrete Res., V. 38, No. 136, 1986, pp. 115-128.

Regan P. E., and Braestrup M. W., "Punching Shear in Reinforced Concrete", A State of Art Report by CEB Bull.168, Lausanne, 1985.

Reinhardt H. W., Cornelissen H. A. W., and Hordijk D. A., "Tensile Tests and Failure Analysis of Concrete", Journal of Structural Engineering, V. 112, No. 11, Nov., 1986, pp. 2462-2477.

Riks E., "An Incremental Approach to the Solution of Snapping and Buckling Problems", Int. J. Solids Structures, Vol. 15, 1979, pp. 529-551.

Riks E., "The Application of Newton's Method to the Problem of Elastic Stability," J. Appl. Mech. Vol. 39, 1972, pp. 1060-1066.

Rots J. G., "Computational Modeling of Concrete Fracture", Ph. D Thesis, Delft University of Technology (NL), 1988.

Rots J. G., and de Borst R., "Analysis of Mixed-Mode Fracture in Concrete", Journal of Engineering Mechanics, V. 113, No. 11, November, 1987, pp. 1739-1758.

Saenz L. P., discussion of "Equation for the stress-strain curve of concrete", ACI Journal, Proceedings V. 61, No. 9, Sept., 1964, pp. 1229-1235.

Sandler I. S., "On the Uniqueness and Stability of Endochronic Theories of Material Behavior", Journal of Applied Mechanics ASME, V. 45, June, 1978, pp. 263-266.

Schickert G., and Winkler H., "Results of Test Concerning Strength and Strain of Concrete Subjected to Multiaxial Compressive Stresses", Deutscher Ausschuss für Stahlbeton, Heft 277, Berlin, Germany, 1977.

Scotta R., Vitaliani R., Saelta A., Oñate E., Hanganu A., "A Scalar Damage Model with a Shear Retention Factor for the Analysis of Reinforced Concrete Structures: Theory and Validation", Computers and Structures, V. 79, 2001, pp. 737-755.

Selby R. G., "Three-Dimensional Constitutive Relations Reinforced Concretes", Ph. D Thesis, University of Toronto, Canada, 1993.

Selby R. G., and Vecchio F. J., "A Constitutive Model for Analysis of Reinforced Solids", Canadian Journal of Civil Engineering, V. 24, 1997, pp. 460-470.

Simo J. C., and Ju J. W., "Strain- and Stress-Based Continuum Damage Models-II Computational Aspects", International Journal of Solid and Structures, V. 23, No. 7, 1987, pp. 841-869.

Sozen M. A., and Siess C. P., "Investigation of Multi-Panel Reinforced Concrete Floor Slabs", ACI Journal, Proceedings V. 60, No. 8, August, 1963, pp. 999-1027.

Stankowski T., and Gerstle K. H., "Simple Formulation of Concrete Behavior under Multiaxial Loading History", ACI Structural Journal, V. 82, March-April, 1985, pp. 213-221.

Tan H. N., and Teng S., "Concrete Flat-Plate Floors with Irregular Column Layout: Flexural Design and Test of a 7-Column Floor Specimen", Final Report, School of Civil and Environmental Engineering, Nanyang Technological University, 2001.

Tassios T. P., and Vintzeleou E. N., "Concrete-to-Concrete Friction", Journal of Structural Engineering, V. 113, NO. 4, 1987, pp. 832-849.

Vecchio F. J., "Nonlinear Finite Element of Reinforced Concrete Membranes", ACI Structural Journal, V. 86, No. 1, January-February, 1989, pp.26-35.

Vecchio F. J., and Collins M. P. "The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear", ACI Structural Journal, V. 83, No. 2, March-April, 1986, pp.219-231.

Walraven J. C., "Fundamental Analysis of Aggregate Interlock", Journal of Structural Engineering, V. 107, NO. 11, November, 1981, pp. 2245-2270.

Walraven J. C., "The Influence of Depth on the Shear Strength of Concrete Beams without Shear Reinforcement", Rep. 5-78-4, Stevens Lab., Delft Univ. of Technol., Delft, the Netherlands, 1978.

Wempner G. A., "Discrete Approximations Related to Non-Linear Theories of Solids", Int. J. Solids Struct. Vol. 7, 1971, pp. 1581-1599.

Westergaard H. M. and Slater W. A., "Moments and Stresses in Slabs." Proceedings of the American Concrete Institute, Vol.17, 1921, pp. 415-538.

Willam K. J. and Warnke E. P., "Constitutive Model for Triaxial Behavior of Concrete", Proceeding of IABSE, Vol. 19, 1975, pp. 1-30.

Xiao R. Y., and O'Flaherty T., "Finite-Element Analysis of Tested Concrete Connections", Computers and Structures, Vol. 78, 2000, pp. 247-255.

Yamamoto T., Vecchio F. J., "Analysis of Reinforced Concrete Shells For Transverse Shear And Torsion", *ACI Structural Journal*, V. 98, No. 2, March-April, 2001, pp.191-200.

Yitzhaki D., "Punching Strength of Reinforced Concrete Slabs", ACI Journal, Proceedings V. 63, No. 5, May, 1966, pp. 527-542.

Zhang L. X., and Hsu T. T. C., "Behavior and Analysis of 100Mpa Concrete Membrane Elements", Journal of Structural Engineering, ASCE, V. 124, No. 1, Jan., 1998, pp. 24-34.

APPENDIX A

USTAR USER'S MANUAL

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- A.2 Format of Input Data File
- A.3 Sample of Input Data File
- A.4 User's Manual for USTAR
- A.5 Output Data Files

Fig. A.1 Interface of USTAR

A.1 INTRODUCTION

Based on the finite element model of this thesis, computational software named USTAR has been developed using C++ language. USTAR is specifically for

nonlinear finite element analysis of reinforced concrete flat plate structures. It may also be applicable for other RC planner-type structures such as shear walls, beams and shell structures. USTAR runs in an independent environment (Main Window) where pre-process, computation progress monitor and control, post-process, and view operation functions are integrated (Fig. A.1).

Fig. A.2 Pre-Process Function of USTAR

Pre-process functions in USTAR are to facilitate the creation of input data file. For simple structures with rectangular layout, such a data file can be created by the pre-process functions automatically (Fig. A.2). Only a little necessary information regarding material properties, geometrical features, load and boundary conditions is required for the users to provide through user-computer communication interfaces.

To monitor the computational progress and quit the computation however save the analyzed result whenever necessary, computation progress monitor and control function is provided in USTAR (Fig. A.3). The loading steps and the load-displacement response curve are demonstrated dynamically on the functional

window with loading. The user can specify the load component and displacement component preferred to be monitored.

Fig. A.3 Computation Progress Monitor and Control Interface

Fig. A.4 Post-Process Function

Necessary post-process functions presented in USTAR are to assist the interpretation of analyzed results. As shown in Fig. A.4, such post-process functions

include demonstrations of deformations, stresses of reinforcement, stresses and strains of concrete, crushing points and cracks of concrete.

In addition to the pre- and post- process functions and the computation progress control function, view operation and zoom functions are also provided in USTAR. View operation functions include front, left, top, and perspective view operations. Zoom in, zoom out, zoom window, etc are integrated in zoom functions.

In the present version of USTAR, 8 layers of equal thickness are specified for concrete of slab and additional 4 layers of equal thickness are allowed for concrete of components protruding slab surface such as drop panel, column stub and spandrel beams. The maximum of 4 layers are restricted for in-plane reinforcements. The USTAR at present only permits nodal loads to be applied.

A.2 FORMAT OF INPUT DATA FILE

It should be noted that USTAR uses SI unit system. The user must input all variables that belong to the same type, in the same units; for example length in millimeter, stresses in MPa, forces in N, throughout the whole input file. The input data file of an analysis should provide the following information sequentially. The nodes and elements are numbered from 0.

1. General Information

The first line of the data file is of 12 variables regarding the general information of the analysis. The variables are sequenced as:

G1, G2, G3, G4, G5, G6, G7, G8, G9, G10, G11, G12

where

G1	=	Uniaxial tensile strength of concrete f_t'
G2	=	Uniaxial compressive strength of concrete f_c'

G3	=	Equal biaxial compressive strength of concrete f'_{bc}
G4	=	Uniaxial tensile peak strain of concrete ϵ_t
G5	=	Uniaxial compressive peak strain of concrete ϵ_c
G6	=	Equal biaxial compressive peak strain of concrete ϵ_{bc}
G7	=	Number of Nodes
G8	=	Number of Elements
G9	=	Number of nodes with the application of load
G10	=	Number of nodes with supports
G11	=	Young' modulus of concrete E_c
G12	=	Poisson's ratio of concrete ν

2. Coordinates of Each Node

The coordinates of the all the nodes in the analysis are presented in sequence following the first line. One line is for one node. Each line is taken as the following format:

$$x, y, z$$

x, y, z are the coordinates of x, y, z of each node.

3. Information of Elements

Information of elements directly follows the coordinates of nodes. One line is for one element. Information of each element is sequenced by series number of each element. Each line is taken as the following format:

$$E1\text{---}E2, E3, E4, E5, N1, N2, N3, N4, N5, N6, N7, N8$$

where

E1	=	Series number of element
E2	=	Element type:

0: No shear correction

1: All nodes of the element are specified with shear correction

2: Additional four layers are added to incorporate components protruding slab surface and all nodes are specified with shear correction

E3 = Sign of tension stiffening

0: No tension stiffening is considered

1: Tension stiffening is considered

E4 = Slab thickness

E5 = Thickness of components protruding slab surface. If no such components, E5 is set with zero.

N1-N8 = Series numbers of the eight nodes of the element presented in node sequence of the element (Fig. 5.2)

4. Information of In-plane Reinforcements

In the present version of USTAR, four layers of in-plane reinforcements as shown in Fig. 4.15 can be arranged for analysis. Two layers, named bottom_x and bottom_y, are next to bottom surface while the other two layers, named top_x and top_y, are next to top surface. Each layer of reinforcement can be in any in-plane direction. The information of the four reinforcement layers is so sequenced as bottom_x, bottom_y, top_x and top_y. For each layer, information of each element regarding that layer is presented in one line according to series number of element. The following is the general format of one line:

E1----R1, R2, R3, R4, R5, R6, R7, R8, R9, R10

where

E1 = Series number of element

R1 = Young's modulus of reinforcement

R2 = Yielding stress of reinforcement

R3 = ϕ in equation (4-55)

R4	=	k in equation (4-55)
R5	=	Diameter of reinforcing bar
R6	=	Spacing of reinforcing bars
R7	=	Cover of reinforcing bar, measured from bottom/top surface to the centroid of the reinforcing bar
R8	=	Angle between reinforcement and X axis in global coordinate system
R9	=	Angle between reinforcement and Y axis in global coordinate system
R10	=	Angle between reinforcement and Z axis in global coordinate system

If no reinforcement is present in the reinforcement layer of the element, the variables from R1-R10 are set to zero.

5. Information of Out-of-plane Reinforcements

Out-of-plane reinforcement can be arranged at any angle with respect to normal of slab surface. In the present version of USTAR, the out-plane-plane reinforcement has to be smeared across the whole thickness of the element. Each element can only have one type of out-of-plane reinforcement. One line is used for out-of-plane reinforcement information of one element. The following is the general format of one line:

E1----O1, O2, O3, O4, O5, O6

where

E1	=	Series number of element
O1	=	Young's modulus of reinforcement
O2	=	Yielding stress of reinforcement
O3	=	Reinforcement ratio
O4	=	Angle between reinforcement and X axis in global coordinate system

- O5 = Angle between reinforcement and Y axis in global coordinate system
- O6 = Angle between reinforcement and Z axis in global coordinate system

If no out-of-plane reinforcement is available in the element, the variables from O1-O6 are set to zero.

6. Information of Loading

Only nodal load is allowed in the present version of USTAR. The load information of each node with loading consists six components: L1, L2, L3, L4, L5, L6. Each component can be a very small value which will result in elastic responses of the analyzed structure. The information is arranged in one line as:

N1----L1, L2, L3, L4, L5, L6

where

- N1 = Series number of node with loading
- L1 = Force in X direction in global coordinate system
- L2 = Force in Y direction in global coordinate system
- L3 = Force in Z direction in global coordinate system
- L4 = Moment around X-axis in global coordinate system
- L5 = Moment around Y-axis in global coordinate system
- L6 = Moment around Z-axis in global coordinate system

7. Information of Support

In USTAR, each node has six DOFs, each of which can be set 0 for no restriction and 1 for being fixed. The support information of each node with restrictions consists six components: S1, S2, S3, S4, S5, and S6. The information is arranged in one line as:

N1----S1, S2, S3, S4, S5, S6

where

- N1 = Series number of node with loading
- S1 = Translation in X direction in global coordinate system
- S2 = Translation in Y direction in global coordinate system
- S3 = Translation in Z direction in global coordinate system
- S4 = Rotation around X-axis in global coordinate system
- S5 = Rotation around Y-axis in global coordinate system
- S6 = Rotation around Z-axis in global coordinate system

A.3 SAMPLE OF INPUT DATA FILE

The input data file of slab I-2 (Megally 1998) analyzed in section 7.3.4 of this thesis is attached as follows:

2.13000,41.7,47.96,0.00007,0.002000,0.003500,96,25,8,38,30834.00,0.200000
0.00,0.00,0.00
0.00,62.500000,0.00
0.00,125.00,0.00
0.00,212.00,0.00
0.00,300.00,0.00
0.00,387.00,0.00
0.00,475.00,0.00
0.00,580.00,0.00
0.00,687.00,0.00
0.00,793.00,0.00
0.00,900.00,0.00
62.500000,0.00,0.00
62.500000,125.00,0.00
62.500000,300.00,0.00
62.500000,475.00,0.00
62.500000,687.00,0.00
62.500000,900.00,0.00
125.00,0.00,0.00
125.00,62.500000,0.00
125.00,125.00,0.00
125.00,212.00,0.00
125.00,300.00,0.00
125.00,387.00,0.00
125.00,475.00,0.00
125.00,580.00,0.00
125.00,687.00,0.00
125.00,793.00,0.00
125.00,900.00,0.00
212.00,0.00,0.00
212.00,125.00,0.00
212.00,300.00,0.00
212.00,475.00,0.00
212.00,687.00,0.00
212.00,900.00,0.00
300.00,0.00,0.00

300.00,62.500000,0.00
300.00,125.00,0.00
300.00,212.00,0.00
300.00,300.00,0.00
300.00,387.00,0.00
300.00,475.00,0.00
300.00,580.00,0.00
300.00,687.00,0.00
300.00,793.00,0.00
300.00,900.00,0.00
387.00,0.00,0.00
387.00,125.00,0.00
387.00,300.00,0.00
387.00,475.00,0.00
387.00,687.00,0.00
387.00,900.00,0.00
475.00,0.00,0.00
475.00,62.500000,0.00
475.00,125.00,0.00
475.00,212.00,0.00
475.00,300.00,0.00
475.00,387.00,0.00
475.00,475.00,0.00
475.00,580.00,0.00
475.00,687.00,0.00
475.00,793.00,0.00
475.00,900.00,0.00
580.00,0.00,0.00
580.00,125.00,0.00
580.00,300.00,0.00
580.00,475.00,0.00
580.00,687.00,0.00
580.00,900.00,0.00
687.00,0.00,0.00
687.00,62.500000,0.00
687.00,125.00,0.00
687.00,212.00,0.00
687.00,300.00,0.00
687.00,387.00,0.00
687.00,475.00,0.00
687.00,580.00,0.00
687.00,687.00,0.00
687.00,793.00,0.00
687.00,900.00,0.00
793.00,0.00,0.00
793.00,125.00,0.00
793.00,300.00,0.00
793.00,475.00,0.00
793.00,687.00,0.00
793.00,900.00,0.00
900.00,0.00,0.00
900.00,62.500000,0.00
900.00,125.00,0.00
900.00,212.00,0.00
900.00,300.00,0.00
900.00,387.00,0.00
900.00,475.00,0.00
900.00,580.00,0.00
900.00,687.00,0.00
900.00,793.00,0.00
900.00,900.00,0.00
0---2,1,152.00,75.00,0,17,19,2,11,18,12,1
1---2,1,152.00,75.00,2,19,21,4,12,20,13,3
2---2,1,152.00,75.00,4,21,23,6,13,22,14,5
3---0,1,152.00,0.00,6,23,25,8,14,24,15,7
4---0,1,152.00,0.00,8,25,27,10,15,26,16,9
5---2,1,152.00,75.00,17,34,36,19,28,35,29,18
6---2,1,152.00,75.00,19,36,38,21,29,37,30,20
7---2,1,152.00,75.00,21,38,40,23,30,39,31,22
8---0,1,152.00,0.00,23,40,42,25,31,41,32,24
9---0,1,152.00,0.00,25,42,44,27,32,43,33,26
10---2,1,152.00,75.00,34,51,53,36,45,52,46,35
11---2,1,152.00,75.00,36,53,55,38,46,54,47,37
12---2,1,152.00,75.00,38,55,57,40,47,56,48,39


```

13----0,1,152.00,0.00,40,57,59,42,48,58,49,41
14----0,1,152.00,0.00,42,59,61,44,49,60,50,43
15----0,1,152.00,0.00,51,68,70,53,62,69,63,52
16----0,1,152.00,0.00,53,70,72,55,63,71,64,54
17----0,1,152.00,0.00,55,72,74,57,64,73,65,56
18----0,1,152.00,0.00,57,74,76,59,65,75,66,58
19----0,1,152.00,0.00,59,76,78,61,66,77,67,60
20----0,1,152.00,0.00,68,85,87,70,79,86,80,69
21----0,1,152.00,0.00,70,87,89,72,80,88,81,71
22----0,1,152.00,0.00,72,89,91,74,81,90,82,73
23----0,1,152.00,0.00,74,91,93,76,82,92,83,75
24----0,1,152.00,0.00,76,93,95,78,83,94,84,77
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7----200000.00,445.00,0.999999,1.50,16.00,130.00,27.00,0.00,90.00,90.00
8----200000.00,445.00,0.999999,1.50,16.00,130.00,27.00,0.00,90.00,90.00
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19----200000.00,445.00,0.999999,1.50,16.00,130.00,27.00,0.00,90.00,90.00
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19----200000.00,445.00,0.999999,1.50,16.00,115.00,43.00,90.00,0.00,90.00
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2----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
3----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
4----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
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6----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
7----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
8----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
9----180000.00,461.00,0.999999,1.50,11.300000,205.00,29.00,0.00,90.00,90.00
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```


28,0,1,0,1,0,0
34,0,1,0,1,0,0
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68,0,1,0,1,0,0
79,0,1,0,1,0,0
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10,1,0,1,0,1,0
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16,0,0,1,0,0,0
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89,0,0,1,0,0,0
90,0,0,1,0,0,0
91,0,0,1,0,0,0
92,0,0,1,0,0,0

A.4 USER’S MANUAL FOR USTAR

To analyze one RC structure by USTAR is usually started from creating new project. Click “Project” menu on USTAR interface and then after a pop menu jumps out click “New Project” menu. Add the directory and the name of the project to be created in the edit boxes of a small window jumped out and Click “Return” button after finished. The next step is to use the pre-process function to create an input data file.

A.4.1 Pre-process Functions

Click “Preprocessing” menu on USTAR interface.

A.4.1.1 General Information of Concrete

Then click “Concrete Element” and “Material Properties”. Fill in the material properties of concrete in the edit boxes on a popped out small window and click “Return” menu after finished.

A.4.1.2 Mesh

The present version of USTAR can auto mesh the rectangular layout to rectangular meshes. Then click “Concrete Element” and “Geometrical Properties”. Provide the height, width and rotation of a rectangular layout as well as the number of meshes in each direction in the edit boxes and click “Return” menu after finished. Then click “Concrete Element”, “Auto Mesh” and “Show Mesh” menus. The resulted mesh will appear on the screen. The user is allowed to use the “view operation” and “zoom” function to adjust the appearance of the mesh on the screen.

A.4.1.3 Reinforcement

Click “Reinforcement” menu. A window for collection of information of reinforcement will be popped out. Fill up each edit box and click “Window Select” button on the window to attach the information in the window onto elements with such properties.

Fig. A.5 Window for Collection of Information of Reinforcement

A.4.1.4 Load

Click “Loading Information” menu. Fill up each edit box for each load component on a popped out window. Click “Window Select” button to attach the information onto nodes being applied with such loads.

A.4.1.5 Support

Click “Support Information” menu. Fill up each edit box on a popped out window with “0” for free of restriction and “1” for being fixed. Click “Window Select” button to attach the information onto nodes with such support information.

A.4.1.6 Create input data file

Click “Forming Data File” menu. An input data file will be generated automatically.

A.4.2 Computation

Click “Analysis” menu on the interface of USTAR. Then click “Parameters” to input parameters regarding the computation progress monitor and control on a popped out window. The user can input 50 for “Number of step” and 0.001-0.01 for “Arc-length” for the first computation and adjust them according to the predicted response after a few of steps. Click “Return” button on the window after finished input of parameters. Click “Computing” menu. A computation progress monitor and control window will pop out and the computing progress will be demonstrated on the window. The user can click “Terminate” menu to stop the analysis. The current analyzed results will be saved and the program will be terminated.

A.4.3 Post-process Functions

USTAR provides necessary post-process function to visualize the analyzed results. Before operating any post-process function, the user needs to click “Post

Processing”, “Drawing Out” and “Show Mesh” to prepare data for post-processing. Click “Analysis” menu on the interface of USTAR.

A.4.3.1 Animation of Deformation

The deformation of the structure within the whole loading history can be demonstrated by animation. The user just clicks “Drawing Out” and “Animation” menus and inputs control parameters. The user may try 100 for scale for deformation and 0.1 for time delay of animation first.

A.4.3.2 Deformation Contour

USTAR provides function to draw deformation contour at any load step for selected elements. The user clicks “Load Step” and inputs the load step at which the deformation is to check. Then click “Element Selection” menu to select the elements to show. Click “Deformation” menu, then “Preparation” menu and select which deformation component to check. The deformation contour will be drawn on a new window.

A.4.3.3 Stresses of Reinforcement

The user clicks “Load Step” and inputs the load step and layer number. “1” is for bottom layers of reinforcements and “2” is for top layers. Then click “X on” or “Y on” to select the direction of reinforcement.

A.4.3.4 Stresses and Strains of Concrete

USTAR provides functions to visualize the stresses or strains of concrete at each layer or on any section of elements. The user clicks “Load Step” and inputs the load step and layer number (1-8). “1” is for bottom layer and “8” is for the top layer. Then click “Concrete Stress Out” or “Concrete Strain Out” to show stress or strain. The user needs to click the corresponding menu to show the stress or strain components interested. If the distribution of stress or strain on section of elements is

interested, the user should click “Section” menu and select the direction of the section and distance of the section from the center of the element which is taken -1 to 1.

A.4.3.5 Cracks

USTAR provides functions to visualize the predicted cracks on the bottom or top surface of each layer or on any section of elements. The user should enter load steps and select elements. Then click “Cracks”, “Static Show” and “Preparation” menus to prepare data. If the surface crack pattern is interested, then click “One layer”, “Surface View” and “Bottom/Top” menus to show the cracks. If the section crack pattern is interested, the user should click “Section” menu and enter parameters about the section. Then click “Multi-Layer”, “Surface View” and “Section_View” menus to show the cracks. The user can use the View Operation functions and Zoom function to adjust the appearance of drawing on the screen. Functions to visualize the cracks in the whole elements and to show the crack propagation dynamically are also available in USTAR.

A.4.3.6 Crushing of Concrete

The user should enter load steps and series number of concrete layer, and select elements first. Then click “Crush” and “On” menus to show.

A.5 Output Data Files

USTAR generates the following data files:

*.crk	=	Cracks of concrete
*.cru	=	Crushing of concrete
*.dis	=	Nodal displacement
*.ele	=	Stress of concrete
*.in	=	Input data file

- *.lad = Load versus displacement data generated from computation progress monitor and control function**
- *.pal = Strain of concrete**
- *.scl = Data file for view operation**
- *.str = Stress of steel**