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Yu, Zhuliang

2006

Yu, Z. (2006). New techniques for robust beamformer design. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/3944>

<https://doi.org/10.32657/10356/3944>

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New Techniques For Robust Beamformer Design

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2006

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Yu Zhuliang

Acknowledgments

I would like to express my heartfelt gratitude and appreciation to my advisor, Prof. Er Meng Hwa, deputy president, Nanyang Technological University. He had been very involved, providing advice, suggestions and encouragement throughout the project. Without his guidance and support, this thesis would not have been possible.

I would also like to thank the Assoc. Prof. Ser Wee, director of Center for Signal Processing, and all the colleagues in Center for Signal Processing, for their kind support and help in the past five years. I also wish to express my gratitude to my employer, Center for Signal Processing, Nanyang Technological University, for granting me the permission to take up this PhD programme on a part-time basis.

Finally, I wish to acknowledge the love, encouragement and support of my family, especially my wife, Dr. Gu Zhenghui.

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Summary

This research focuses on new techniques and schemes to improve the performance of adaptive beamformer. In this thesis, we first extend the conventional generalized sidelobe canceller (GSC) to deal with arbitrary transfer function relating signal source and sensors instead of the pure delay channel model used in conventional beamformers. The implementations in time and frequency domains are derived. Moreover, we also show that the extended GSC can be implemented as narrowband GSC at each frequency bin with the aid of the discrete Fourier transform (DFT) if the length of the DFT is long enough.

When an adaptive array has imperfections including steering direction error, array geometry error and channel phase error etc., we derive new robust adaptive beamformers by maximizing the array output power with respect to the so called, time-delay error or generalized phase error. A robust wideband beamformer in time domain is designed to treat general time-delay error. Another robust narrowband beamformer is also derived to deal with more kinds of array imperfections, such as array phase error. Those derived robust beamformers have advantage that they do not suffer from loss in degree-of-freedom (DOF) for interference suppression.

To make adaptive beamformer robust to arbitrary array imperfections, the idea of output power maximization is extended to derive robust beamformer against arbitrary array steering vector error. To prevent the robust beamformer from mistracking of the target signal, an uncertainty constraint on the array steering vector is introduced in optimization. To further improve system performance, a more compact uncertainty constraint is derived by utilizing the information of signal-plus-interference subspace. The proposed beamformers belong to the class of diagonal loading approach. Performance analysis of these beamformers is also carried out in this thesis.

The above approaches assume that partial information of the array steering vector is known. However, in some applications, the target source is moving or the array geometry is changing, no prior information of the array steering vector is known. In such case, to derive a robust beamformer, the true array steering vector must be estimated. Considering the application in speech enhancement against stationary interference and noise, we derive a robust beamformer using the on-line estimated array steering vector. The proposed method does not require prior information on the array steering vector.

List of Abbreviations and Symbols

Abbreviations

ASV	array steering vector
BCI	blind channel identification
BF	beamforming
DFT	discrete Fourier transform
DOA	direction of arrival
DOF	degree of freedom
DSB	delay-and-sum beamformer
ESB	eigenspace-based beamformer
EVD	eigenvalue decomposition
FFT	fast Fourier transform
GPE	generalized phase error
GSC	generalized sidelobe canceller
IFFT	inverse fast Fourier transform
IR	impulse response
LCMV	linearly constrained minimum variance

LMS	least mean square
LS	least squares
MCA	minor component analysis
MF	matched filter
MF-GSC	matched-filter array based generalized sidelobe canceller
MISO	multiple input single output
MSE	mean square error
MVDR	minimum variance distortionless response
NLMS	normalized least mean square
P/S	parallel to serial transform
PCA	principal component analysis
PRCB	proposed robust Capon beamformer
RCB	robust Capon beamformer
RLS	recursive least squares
RSA	robust subspace analysis
RSA-GSC	robust subspace analysis based generalized sidelobe canceller
SA	subspace analysis
SA-GSC	subspace analysis based generalized sidelobe canceller
SCB	standard Capon beamformer
SIR	signal-to-interference ratio
SINR	signal-to-interference-plus-noise ratio
SNR	signal-to-noise ratio
SOI	source of interest

S/P	serial to parallel transform
s.t.	subject to
TD	time delay
TD-GSC	time-delay estimated based generalized sidelobe canceller
TDOA	time difference of arrival
TF	transfer function
TFR	transfer function ratio
TFRV	transfer function ratio vector
TFV	transfer function vector
UCA	uniform circular array
ULA	uniform spaced linear array

Symbols

a	scalar
$\mathbf{a}$	vector in time domain
$\mathbf{A}$	matrix in time domain
$\mathbf{a}$	vector in frequency domain
$\mathcal{A}$	matrix in frequency domain
$\mathbb{C}$	set of complex numbers
$(\cdot)^*$	complex conjugate of a vector or matrix
$(\cdot)^H$	complex conjugate transpose of a vector or matrix
$(\cdot)^T$	transpose of a vector or matrix
$(\cdot)^{[n]}$	n th iteration
$\ \cdot\ _2$	Euclidean norm
$\ \cdot\ _F$	Frobenius norm
$\ \cdot\ _{\mathbf{R}}$	extended vector norm ($\mathbf{R}$ is positive matrix)
$\approx$	approximate to
$\star$	linear convolution
∇	gradient operator
$diag\{\mathbf{x}\}$	diagonal matrix whose i th diagonal element is the i th element of vector $\mathbf{x}$
$E\{\cdot\}$	expectation operator
$tr(\cdot)$	trace operator
$Re\{\cdot\}$	real operator
a_k	complex amplitude of the signal measured at the reference point

A_k	amplitude of narrowband signal
β	leakage parameter
$\mathbf{C}$	constrain matrix
$\mathbf{c}_j$	the j th column of the constrain matrix
d	inter-element distance of uniform linear array
Δ_m	delay difference
Δ_α	generalized phase error
ϵ	uncertainty level
Σ	the eigenvalue matrix
$\mathbf{F}_L$	DFT matrix
$h_m(t)$	impulse response function in continuous time
$h_m(n)$	impulse response function in discrete time
$\mathbf{h}_m$	filter coefficient vector
$\mathbf{H}$	filter coefficient matrix
$\mathbf{I}$	identity matrix
j	imaginary unit, i.e., $\sqrt{-1}$
K	number of signals
l_i	Lagrange multiplier
M	number of sensors in the array
N_s	number of snapshots to estimate correlation matrix
$n(t)$	noise component
$\mathbf{n}(t)$	array received noise vector
p	output power of array processor

$\hat{p}$	optimal output power of array processor
ρ	stepsize
$\mathbf{R}$	covariance matrix
$\Delta\mathbf{R}$	difference covariance matrix
$\hat{\mathbf{R}}$	the sample average covariance matrix
$\mathbf{R}_s$	covariance matrix of impinging sources
$\mathbf{R}_i$	covariance matrix of interferences
$\mathbf{R}_n$	covariance matrix of noise
$\mathbf{f}$	frequency response vector
$\mathbf{g}$	the weight vector of fixed beamformer of GSC
$\mathbf{B}$	the blocking matrix of GSC
$\mathbf{w}'$	the weight vector of GSC
ω_0	operating frequency of narrowband signal
t	time index
T_{60}	reverberation time
n	discrete time index
θ_k	incident azimuth angle of the k th source
ϕ_k	incident elevation angle of the k th source
$s(t)$	array incident signal
$\mathbf{s}(\omega_0, \theta, \phi)$	array steering vector
$\vec{\mathbf{u}}_k$	unit vector in the incident direction of the k th source
$\mathbf{u}_m$	the m th eigenvector
λ_m	the m th eigenvalue

$\mathbf{r}_m$	position vector of m th sensor
τ_k	time delay of arrival
α_k	phase shift of the signal
$x_m(t)$	array received signal at the m th sensor
$\mathbf{x}(t)$	array received signal snapshot
Γ	eigenvalue matrix
$\mathbf{U}$	the eigenvector matrix
$\mathbf{U}_n$	basis of noise subspace
$\mathbf{U}_s$	basis of signal subspace
v	propagation speed of signal
$\mathbf{W}_{L \times N}$	windowing matrix
$\mathcal{W}_{L \times N}$	transformed windowing matrix
$\mathbf{w}$	array weight vector
$\hat{\mathbf{w}}$	estimated optimal array weight vector
σ_k^2	the signal power of the k th signal
σ_n^2	the noise power

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Chapter 1

Introduction

1.1 Background and Motivation

Array signal processing has been studied for some decades as an attractive method for signal detection and estimation in hash environment. An array of sensors can be flexibly configured to exploit the spatial and temporal characteristics of the signal and noise, thus has many advantages over single sensor. It has found many applications in radar, radio astronomy, sonar, wireless communication, seismology, speech acquisition, medical diagnosis and treatment [1, 2, 3], etc.

There are two kinds of array beamformers, one is fixed beamformer and the other is adaptive beamformer. Adaptive beamformer automatically adjusts its weight according to some criteria. It significantly outperforms fixed beamformer in noise and interference suppression. The typical array beamformers are the linear constrained minimum variance (LCMV) beamformers [4, 5, 6, 7, 8]. In narrowband applications, a

famous representative of the LCMV is the Capon beamformer [9], and in broadband applications, the well studied LCMV beamformer is the Frost beamformer [10]. In ideal cases, the Frost and Capon beamformers have high performance in interference and noise suppression. However, the ideal assumptions of the array processors do not hold in practical applications [11,12,13,14,15]. The performance of adaptive array processors highly degrades when there are array imperfections such as steering direction error, time delay error, phase errors of the array sensors, multipath propagation effects, wavefront distortions, etc. This is known as the target signal cancellation problem of the adaptive beamformer. Tremendous work has been done to improve the robustness of adaptive beamformer [16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32].

To overcome the problem of target signal cancellation caused by steering direction error, multiple-point constraints [5,16] were introduced in adaptive array processor. The idea of this approach is intuitive. Multiple gain constraints at different directions in the vicinity of the assumed one are introduced. With these constraints, the array processor becomes robust in the region where constraints are imposed. The available number of constraints is limited because the constraints consume the degree-of-freedom (DOF) of array processor.

Another class of solution is to introduce derivative constraints into array processor [5,17,18,19,20,21,22,23]. With derivative constraints, the response of the array processor is almost flat in the vicinity of the steering direction. The array processor has widened beamwidth in the steering direction. With small steering error, the processor does not cancel the target signal. The widened beamwidth is achieved at the

cost of reduced capability in interference suppression because the additional derivative constraints consume the DOF of processor. In [18], the author pointed out that the performance of the derivative constrained array processor would change with different selection of reference coordinate point. This drawback arises because the derivative constraints obtained in [17] are not necessary and sufficient condition. In [20], a new technique was proposed for deriving a new set of constraints to control the spatial derivatives of the beampattern. The new set of constraints ensures that the array beampattern is independent of the coordinate origin. In [21,22], the authors proposed method to determine necessary and sufficient conditions for second-order spatial derivative constraints. The obtained constraints are generally in nonlinear format. Derivative constraints can be used to obtain not only a flat response of array processor, but also a flat null in the assumed signal direction in blocking matrix design [33].

A new set of constraints for robust array processor against steering error was also proposed in [24,25]. The idea is to minimize the weighted mean square deviation between the desired array response and the response of the processor over the variations in parameters, such as the steering error, the phase errors and array geometry error, etc. Although the constraints derived by this approach are quadratic [34], in [24], a set of linear constraints was derived approximately. In the approximation of quadratic constraints to linear constraints, a problem arises as to how many constraints should be selected. In [24], the author proposed a method to determine the number of necessary linear constraints and to select the constraints. For a single broadband source, propagating in a pure delay environment and being observed with an isotropic array, an accurate indication of the model dimension is derived [35]. It is independent of the

array configuration, being a function of the observed source time-bandwidth product only. This gives the theoretical number of necessary linear constraints.

Techniques restraining excess coefficients growth were also proposed in array processor to achieve robust performance. When array processor cancels target signal, the norm of the filter coefficients grows to a large value beyond the normal value to suppress noise and interference. In [26], an inequality constraint is imposed on the coefficient norm of adaptive beamformer to limit the growth of tap coefficients. The excess coefficient growth problem can also be solved using noise injection method [12]. Artificially generated noise is added to reference signals of adaptive filters. Although the artificial noise causes estimation errors in the beamformer coefficients, it prevents tap coefficients from growing excessively, resulting in robustness to array imperfections. A similar approach called leaky least mean square (LMS) algorithm can also be used [36].

Recently, robust methods with clear theoretical background have been proposed [27, 28, 29, 30]. An uncertainty set of array steering vector (ASV) is used. The true ASV is assumed to be in an ellipsoid centered at the nominal ASV. The designed beamformers are robust to arbitrary variation of the true ASV if it is in an assumed uncertainty set. These beamformers [27, 28, 29, 30] are equivalent and belong to the diagonal loading approach. The diagonal loading factor can be calculated from the constraint equation.

Other robust beamforming methods include calibration based approaches [37]. The calibration can generally eliminate the inherent error of the array processor, such as

geometry error, sensor response error, etc. However, it cannot eliminate the dynamic errors, such as steering error when the source is moving in the vicinity of assumed direction. Target tracking methods [31, 32] were introduced in array processor so that the look direction is steered to the continuously estimated direction-of-arrival (DOA). One problem is that this method may mistrack to interference in the absence of target signal unless some other methods are used to limit the tracking region.

The robust beamforming methods discussed above solve part of robust beamforming problems. More research works still need to be carried out, especially in real applications. As a very typical application in acoustics, e.g., sonar, acoustic field measurement, speech acquisition, microphone array has attracted attention for some decades. However, in acoustic applications, array usually has more complex imperfections. For examples, acoustic enclosure shows strong multipath or reverberation effects. These imperfections violate the assumptions of conventional robust beamforming methods, and make the research on robust microphone array a rather challenging topic.

Consequently, in this thesis, we study and propose some robust beamformers to combat more general array imperfections. The designed robust beamformers can be applied in more generic areas of beamforming, for example, antenna array processing, microphone array processing, etc.

1.2 Objectives

This thesis has three aims. The first aim is to extend the conventional generalized side-lobe canceller (GSC) to work with arbitrary transfer function relating the target source and the sensors. The implementations in time domain and frequency domain are also presented. Secondly, this thesis details the study on robust beamformers against various array imperfections, including modelled and arbitrary ones. The performance of these proposed robust beamformers is also discussed. Finally, we use robust subspace tracking method and improved ASV estimation method to estimate the true ASV of target speech signal in the presence of stationary interference/noise for speech enhancement. For specific application, suitable robust beamforming method can be selected from those proposed in this thesis according to the knowledge of array imperfections and signal properties.

In this thesis, we focus on the applications in acoustics although it is not necessary limited. The signals in acoustic applications are of narrowband or broadband. Broadband beamforming can be implemented in time domain directly. Likewise, broadband beamforming can be obtained by transforming signals into frequency domain using the fast Fourier transform (FFT), applying robust beamforming in each frequency bin, and transforming the result back to time domain again using the inverse fast Fourier transform (IFFT). Therefore, some robust beamformers discussed in frequency domain using narrowband form are not limited to narrowband applications.

1.3 Major Contributions of Thesis

The main contributions of this thesis are as following:

1. An extended GSC which adopts arbitrary transfer functions relating target source and sensors is proposed. Time and frequency domain implementations of the extended GSC are presented. We also show that the implementation of the extended GSC in frequency domain can be further simplified as independently applying beamforming in each frequency bin when the length of the FFT is long enough. Although this is not a new discovery, we show its applicability in analytic way.
2. Adaptive beamformers are proposed to achieve robustness by maximizing the output power of the conventional adaptive beamformer with respect to the modelled array imperfections. Two types of modelled errors are considered. One is the time delay error, which is a general modelled error including steering direction error, array geometry error and quantization error of the presteering delayer. The other is the generalized phase error, which also includes channel phase error, etc. The resulting beamformers do not suffer from loss in the DOF for interference and noise suppression. These methods are especially suitable for array with small number of sensors.
3. Robust adaptive beamformers against arbitrary ASV error are proposed. In practical applications, the array imperfection may not be described by simple model. These imperfections may cause arbitrary error in the ASV. To make the adaptive

beamformer robust to those arbitrary ASV errors, we propose robust beamformers by further extending the output power maximization method. Uncertainty constraint on the ASV is introduced. We also analyze the trade-off between the robustness and the output signal-to-interference-plus-noise ratio (SINR) of beamformer. An upper bound of the output SINR is derived. The study on the output SINR shows that the proposed methods can still be improved.

4. Projected nominal ASV onto signal-plus-interference subspace is proposed to replace the nominal ASV used in the proposed robust beamformers against arbitrary ASV error, to further improve the output SINR. The error between the nominal and true ASVs gives rise to low output SINR of the proposed array beamformer. With the available information on the signal-plus-interference subspace, we can estimate more accurate nominal ASV for robust beamforming. The optimal nominal ASV is derived as the projected one of the nominal ASV onto the signal-plus-interference subspace. A theoretical study is also carried out to prove that the proposed beamformer has higher output SINR.
5. Robust adaptive beamformer for speech enhancement against strong stationary interferences is proposed. If the ASV corresponding to target signal can be estimated, we can use this information in adaptive beamformer to avoid target signal cancellation. However, in many applications, the estimate of true ASV in presence of strong interferences is either impossible or quite difficult. In application of speech enhancement with stationary interference/noise, the nonstationarity of speech signal can be exploited to estimate the true ASV of the target signal. We propose a robust subspace tracking method for the ASV estimation. The adap-

tive beamformer with the estimated ASV demonstrates high performance in real applications.

1.4 Organization of Thesis

The remainder of the thesis is organized as follows. Chapter 2 provides the background materials of adaptive beamforming techniques. The basic principle and structure of adaptive beamformer are explained. The definition of the signal and noise model and formulations of the array processors are presented. The conventional array geometries and the well studied LCMV beamformer for narrowband and broadband applications are discussed. A key robustness problem, known as target signal cancellation, of adaptive beamformer due to array imperfections is then discussed.

The array imperfections in acoustic applications are much more complicated than those in antenna array applications. In Chapter 3, to extend conventional beamforming methods working with much complex array imperfections, we extend the conventional GSC to work with arbitrary transfer function relating the signal source and sensor. The extended GSC is implemented in time and frequency domains. We also simplify the frequency domain implementation of the extended GSC when the length of the FFT is long enough. The analysis shows that the extended GSC can be implemented at each frequency bin independently. This result reveals that wideband beamforming can be implemented as narrowband beamforming with the help of the FFT technique.

Some of array imperfections, such as steering direction error, array geometry error,

quantization error of presteering delayer, can be modelled as time delay error. To make the adaptive beamformer robust to time delay error, a robust wideband beamforming method is proposed in Chapter 4. To make the beamformer robust to more general array imperfections, e.g., sensor phase error and the time delay error discussed in Chapter 4, we further model all these array imperfections as generalized phase error (GPE). A robust beamformer against the GPE is proposed in Chapter 5.

The array imperfections discussed in Chapters 4 and 5 can be considered as model based errors. In many applications, there exist array imperfections which results in arbitrary error in the ASV. To make beamformer robust to the arbitrary error in the ASV, in Chapter 6, we propose robust beamformer by maximizing the output power of adaptive beamformer subject to some constraints on the uncertainty of the nominal ASV. Two beamformers are proposed in Chapter 6. Although these two beamformers have different constraints in optimization, they share similar mathematical solution as the diagonal loading method. The upper bounds of output SINR of these two beamformers are also derived. Although these beamformers are robust to the arbitrary error in the ASV, the error in the ASV affects the output SINR. A method is also proposed to use an estimated ASV, the projection of the nominal ASV onto the signal-plus-interference subspace, to replace the nominal ASV in adaptive beamforming. With the estimated nominal ASV and the uncertainty constraint, the proposed robust beamformer outperforms the former ones. Theoretical proof is also given.

Although the method proposed in Chapter 6 improves the output SINR of adaptive beamformer, the true ASV is most appreciated in adaptive beamforming. In many

applications, the estimation of the true ASV corresponding to the target source is either difficult or impossible. In Chapter 7, we propose a robust subspace tracking method to estimate the true ASV in application of speech enhancement with stationary noise and interference. The nonstationarity of speech signal is exploited for ASV estimation. The resulting beamformer can be used in the case when there are no priori information of ASV of the target signal is available. For example, the target source is moving.

Finally, Chapter 8 provides some concluding remarks and future work for possible extension.

Chapter 2

An Overview of Spatio-Temporal Array Processing: Models and Structures

2.1 Introduction

In this chapter, the framework for the design of robust adaptive beamformer in both narrowband and broadband applications will be presented. The chapter begins with an introduction to the basic principle and concepts in array processing. This is followed by the mathematical notations, signal and noise models that are relevant to the major part of this thesis. An overview of array beamformers is also given. Array geometries of various designs, which are frequently used in the subsequent chapters for simulations and performance evaluations, are also discussed in this chapter.

The basic structure and improved methods of the minimum variance beamformer [9,10] are reviewed and briefly discussed. The famous representative beamformers, the Capon beamformer and the Frost beamformer, as well as their alternative implementation, the GSC [38], are discussed. Various methods [5, 16, 5, 17, 18, 19, 20, 21, 22, 23, 26, 12, 36] to improve the robustness of the beamformers due to array imperfections are briefly reviewed. Their advantages and disadvantages are also briefly discussed.

The chapter is organized as follows. In Section 2.2, some basic concepts of the array processing are discussed. In Section 2.3, the signal model used in array processing for narrowband and broadband array applications are presented. The basic structures of the array beamformers are also discussed. It is followed by a brief introductions on the well used array geometries in Section 2.4. Three typical beamformers, the Capon beamformer, the Frost beamformer and the GSC are introduced in Section 2.5. In Section 2.6, we discuss the proposed robust beamformers in a uniform mathematical expression. A brief summary is given in Section 2.7.

2.2 Principle of Adaptive Beamformer

Array processing is a specialized branch of signal processing that focuses on signals conveyed by propagating waves. The received signals are obtained by means of an array of sensors located at different points in space in the field of interest. The aim of array processing is to extract useful characteristics of the received signal field (e.g., its signature, direction, speech of propagation). The collected signals at sensors are combined cleverly so as to enhance the signal-to-noise ratio (SNR) of target signal, to

characterize the signal wave field, and to track the signal sources as they move in space.

With distributed sensors, the array processing exploits not only the temporal information, but also the spatial information of signals. The temporal-spatial processing greatly extends the capability in information extraction. This technique receives great interest in past decades. In this thesis, we focus on the array processor in beamforming applications for signal detection and enhancement. Unless it is specially mentioned, the array processing refers to adaptive beamforming in this thesis.

The interest in using adaptive beamforming began in 1950s to solve the new problems in satellite communications. In [39], the theoretical basis for the operation of adaptive beamformer was given. In Applebaum's view, an adaptive beamformer is a system which selectively receives a signal when confronted with noise and interference. The performance index is a generalized SNR at the output of beamformer, and the system acts to maximizing the output SNR. In [40], Widrow et al. proposed another performance index, the similarity of the received signal with the reference signal in a LMS sense. Starting from these two reference works, the performance indexes for adaptive control law used in adaptive beamformers were considerably diversified. The usual performance indexes are: minimum mean square error (MSE), maximum SNR, maximum likelihood, minimum noise variance, minimum output power (variance)¹, maximum array gain, [41,5,6,7,8,42] etc. In these criteria, the minimum output power is widely applied in adaptive beamforming for its simplicity in practice. In this thesis, the adaptive beamformers discussed are all derived from this criterion.

¹With assumption of zero mean signal, the output power is equivalent to output variance.

Most of the pioneering works on adaptive arrays focus on antenna array for communication and radar applications. Another important application of adaptive array is the acoustic application, e.g., sonar, which works with low frequency signals. Although the signals as well as systems are different in those applications, the adaptive array techniques developed in communication and radar can be used in sonar too. The differences are rather in technologies than in principles. For specific applications, there are special problems of adaptive array. Nevertheless, they have similar principle.

An important concept in adaptive array is the classification of the array depending on the frequency operating bandwidth. According to such classification, adaptive arrays may be of narrowband or broadband type. A bandwidth exceeding 15% deviation from the central frequency is usually accepted as a broadband criterion. Although in principle no distinctions exist between the two situations, in practice, the technical means used to implement the array weights are different. Since broadband array can be transformed to narrowband array with the aid of the FFT technique, these two types of beamformers are not explicitly distinguished in this thesis.

2.3 Signal Representation in Array Processing

In most of the array processing examples, it is assumed that each signal consists of a point source which is located at a far distance away from the array, i.e., the array is located in the far field of the point sources. Thus each directional signal impinging on the array is considered as a plane wave. This is the typical scenario of conventional array processing.

Figure 2.1: Array with arbitrary geometry.

Consider an arbitrary array consisting of M isotropic elements as shown in Fig. 2.1. With different operating frequency band of the signals, i.e., broadband signal or narrowband signal, the signal models are discussed in Section 2.3.1 and Section 2.3.2, respectively.

2.3.1 Narrowband Signal Representation

As the array system shown in Fig. 2.1, it is assumed that the signal environment consists of K plane waves, each arriving at the array from a distinct direction, and

that all the plane waves are narrowband, with the same frequency ω_0 . The noiseless signal produced at the m th sensor of the array due to the k th plane wave can be expressed as follows:

$$s(m, k, t) = A_k e^{j\omega_0(t - \tau_{mk}) + \alpha_k}, \quad (2.1)$$

where t is the time, A_k is the amplitude of the signal produced, α_k is the phase displacement, τ_{mk} is the time delay at the m th sensor.

The expression for τ_{mk} is given by

$$\tau_{mk} = \frac{\vec{\mathbf{u}}_k \cdot \mathbf{r}_m}{v}, \quad (2.2)$$

where $\cdot$ denotes the dot product, $\vec{\mathbf{u}}_k$ is the unit vector in the incident direction (θ_k, ϕ_k) of the k th source, $\mathbf{r}_m$ is the position vector of the m th sensor, and v is the speed of the propagating wave. Clearly, $\vec{\mathbf{u}}_k$ and $\mathbf{r}_m$ can be expressed as

$$\vec{\mathbf{u}}_k = \begin{bmatrix} \cos \phi_k \cos \theta_k \\ \cos \phi_k \sin \theta_k \\ \sin \phi_k \end{bmatrix} \quad (2.3)$$

and

$$\mathbf{r}_m = \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}, \quad (2.4)$$

respectively.

Substituting (2.3) and (2.4) into (2.2), τ_{mk} can be expressed as

$$\tau_{mk} = \frac{1}{v} [(x_m \cos \theta_k + y_m \sin \theta_k) \cos \phi_k + z_m \sin \phi_k]. \quad (2.5)$$

The coordinate of each array element, $\mathbf{r}_m$, is assigned by taking the center of gravity of the array as the origin, such that

$$\sum_{i=0}^M \mathbf{r}_i = \mathbf{0}. \quad (2.6)$$

The selection of the center of gravity of the array as the coordinate origin will ensure that the array is equipped with the rotational invariance property [34]. This property is important as one would expect the array to have the same characteristic irrespective of whether the source is rotated in θ and/or ϕ or the array rigidly rotated by the corresponding $-\theta$ and/or $-\phi$ with a fixed source.

The sensor received signal in (2.1) can be represented by its complex amplitude, $s(m, k)$, defined by

$$\begin{aligned} s(m, k) &= A_k e^{-j\omega_0 \tau_{mk} + \alpha_k} \\ &= (A_k e^{j\alpha_k}) e^{-j\omega_0 \tau_{mk}} \\ &\triangleq a_k e^{-j\omega_0 \tau_{mk}}. \end{aligned} \quad (2.7)$$

The signal in (2.7) describes the noiseless signal output at the m th sensor. In practice, the array received signal also contains noise. In many array processing examples, the noise is assumed to be a white, ergodic random process. The output from each

element in the array is customarily filtered to the same narrowband frequency occupied by the actual received signal.

The observed signal at the m th sensor, $x_m(t)$, can be expressed as the sum of the noiseless signals produced by all the plane waves and the white noise as follows:

$$x_m(t) = \sum_{k=1}^K a_k e^{-j\omega_0 \tau_{mk}} + n_m(t), \quad (2.8)$$

where $n_m(t)$ is the complex random noise. Let $\mathbf{x}(t)$ be the vector of the observed signals derived at the output of the sensor elements, i.e.

$$\mathbf{x}(t) = [x_1(t) \ x_2(t) \ \cdots \ x_M(t)]^T, \quad (2.9)$$

where the superscript T denotes vector transpose.

Substitute (2.8) into (2.9), it follows that

$$\mathbf{x}(t) = \sum_{k=1}^K a_k \mathbf{s}_k(\omega_0, \theta_k, \phi_k) + \mathbf{n}(t), \quad (2.10)$$

where $\mathbf{s}_k(\omega_0, \theta_k, \phi_k)$ is the array steering vector of the k th source given by

$$\mathbf{s}_k(\omega_0, \theta_k, \phi_k) = \begin{bmatrix} e^{-j\omega_0 \tau_{k1}} \\ e^{-j\omega_0 \tau_{k2}} \\ \vdots \\ e^{-j\omega_0 \tau_{kM}} \end{bmatrix}, \quad (2.11)$$

and $\mathbf{n}(t)$ is the received noise vector given by

$$\mathbf{n}(t) = [n_1(t) \ n_2(t) \ \cdots \ n_M(t)]^T. \quad (2.12)$$

The structure of a narrowband beamformer comprising of M sensors is shown in Fig. 2.2. The sensor-collected signals, $x_m(t)$, $m = 1, \dots, M$, are weighted by w_m , $m = 1, \dots, M$ and summed up to form the output signal $y(t)$,

$$y(t) = \sum_{m=1}^M w_m^* x_m(t) = \mathbf{w}^H \mathbf{x}(t), \quad (2.13)$$

where $\mathbf{w}$ denotes the complex weight vector and $(\cdot)^H$ denotes the Hermitian transpose.

$$\mathbf{w} = [w_1 \ \cdots \ w_m \ \cdots \ w_M]^T. \quad (2.14)$$

For the sources that can be modelled by stationary stochastic process, the mean output power from the array system is given by

$$p(\mathbf{w}) = E\{|y(t)|^2\} = \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.15)$$

where $E\{\cdot\}$ denotes the expectation operator, and $\mathbf{R}$ is the $M \times M$ dimensional array covariance matrix defined by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}. \quad (2.16)$$

Figure 2.2: Narrowband beamformer with M sensors.

2.3.1.1 The Covariance Matrix of Narrowband Beamformer

The covariance matrix $\mathbf{R}$ in (2.16) is the statistical second-order property of the impinging signals. In real applications, $\mathbf{R}$ is estimated using the received array snapshots.

The estimated array covariance matrix, $\hat{\mathbf{R}}$, is given by

$$\hat{\mathbf{R}} = \frac{1}{N_s} \sum_{k=1}^{N_s} \mathbf{x}(k) \mathbf{x}^H(k), \quad (2.17)$$

where N_s is the number of snapshots, and $\mathbf{x}(k)$ is a column vector consisting of the k th sampled data of all the M sensors. When N_s approaches infinity, the estimated $\hat{\mathbf{R}}$ asymptotically approaches the true one.

When all the impinging sources and the noise are mutually uncorrelated or non-coherent, the array correlation matrix $\mathbf{R}$ is a non-negative matrix, and is given by

$$\begin{aligned}\mathbf{R} &= \sum_{k=1}^K \sigma_k^2 \mathbf{s}(\omega_0, \theta_k, \phi_k) \mathbf{s}^H(\omega_0, \theta_k, \phi_k) + \mathbf{R}_n \\ &\triangleq \mathbf{R}_s + \mathbf{R}_n,\end{aligned}\tag{2.18}$$

where σ_k^2 is the power of the k th source,

$$\sigma_k^2 = E\{|a_k|^2\}\tag{2.19}$$

and $\mathbf{R}_n$ is the covariance matrix of the noise, $\mathbf{R}_s$ is covariance matrix of the impinging directional sources. When the noise is white,

$$\mathbf{R}_n = \sigma_n^2 \mathbf{I},\tag{2.20}$$

where σ_n^2 is the power of noise and the matrix $\mathbf{I}$ is an identity matrix.

The eigen-decomposition (EVD) of the covariance matrix plays an important role in array processing algorithms and thus its structure will be explored further. It can be decomposed and expressed in the following form:

$$\begin{aligned}\mathbf{R} &= \mathbf{U} \mathbf{\Sigma} \mathbf{U}^H \\ &= \sum_{m=1}^M \lambda_m \mathbf{u}_m \mathbf{u}_m^H,\end{aligned}\tag{2.21}$$

where $\mathbf{\Sigma} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$, $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$ is the eigenvalue matrix and

$\mathbf{U} = [\mathbf{u}_1 \ \mathbf{u}_2 \ \cdots \ \mathbf{u}_M]$ is the corresponding eigenvector matrix, such that

$$\mathbf{R}\mathbf{u}_m = \lambda_m \mathbf{u}_m, \quad m = 1, \cdots, M. \quad (2.22)$$

An important property of the EVD of the matrix $\mathbf{R}$ is that, the eigenvalues λ_i of $\mathbf{R}$ is

$$\lambda_m = \begin{cases} \gamma_m + \sigma_n^2, & 1 \leq m \leq K \\ \sigma_n^2, & K + 1 \leq m \leq M \end{cases}, \quad (2.23)$$

where γ_m is the m th eigenvalue of the matrix $\mathbf{R}_s$.

Splitting the eigenvector matrix into

$$\mathbf{U} = [\mathbf{U}_s \ \mathbf{U}_n], \quad (2.24)$$

where

$$\begin{aligned} \mathbf{U}_s &= [\mathbf{u}_1 \ \mathbf{u}_2 \ \cdots \ \mathbf{u}_K], \\ \mathbf{U}_n &= [\mathbf{u}_{K+1} \ \cdots \ \mathbf{u}_M]. \end{aligned} \quad (2.25)$$

The matrix $\mathbf{U}_s$ is the eigen-basis of the so-called signal subspace, and the matrix $\mathbf{U}_n$ is the eigen-basis of the noise subspace. The ASV of each signal is a linear combination of the column vector of $\mathbf{U}_s$, i.e.,

$$\mathbf{s}(\omega_0, \theta_k, \phi_k) \in \text{span}(\mathbf{U}_s). \quad (2.26)$$

2.3.2 Broadband Signal Representation

For the broadband signal, the frequency of the signal covers a band, i.e., $\omega \in [\omega_1, \omega_2]$, where ω_1 and ω_2 are the lower and upper frequencies of the signal, respectively.

The m th array sensor received signal $x_m(t)$ due to the k th source $s_k(t)$ cannot be expressed in the form like that in (2.8). It should be expressed as

$$x_m(t) = s_k(t) \star h_{mk}(t) + n_m(t) = \int h_{mk}(t - \tau) s_k(\tau) d\tau + n_m(t), \quad (2.27)$$

where $\star$ denotes linear convolution, $n_m(t)$ is the environment noise, $h_{mk}(t)$ is a linear transfer function that represents propagation effects between the k th source and m th sensor and any signal distortion in the sensor itself. In the case of ideal (non-dispersive) propagation and distortion-free omni-direction elements, $h_{mk}(t)$ corresponds to a pure time delay. In fact, this is the most often used assumption in array processing. In such case, the received signal becomes

$$x_m(t) = s_k(t + \tau_{mk}(\theta_k, \phi_k)) + n_m(t), \quad (2.28)$$

where $\tau_{mk}(\theta_k, \phi_k)$ is similar to the one in (2.2).

With K impinging sources presented, the array received signal at m th sensor is expressed as

$$x_m(t) = \sum_{k=1}^K s_k(t + \tau_{mk}(\theta_k, \phi_k)) + n_m(t). \quad (2.29)$$

To handle the broadband signals effectively, the array processor behind the sensors

Figure 2.3: Broadband presteeered beamformer with M sensors and J taps per sensor.

must be able to provide a phase shift that also varies with frequencies [4, 5, 7]. This is accomplished by the use of tapped delay lines as shown in Fig. 2.3.

A broadband beamformer in time domain is shown in Fig. 2.3. Compared with the narrowband beamformer shown in Fig. 2.2, the broadband beamformer uses tapped delay line filters to replace the single weight in narrowband beamformer. Generally, the weights in narrowband beamformer are complex numbers and in broadband beamformer are real numbers. The beamformer in Fig. 2.3 consists of M sensors and J taps per channel. The array received signal is first delayed by a set of pure time delays τ_m , $m = 1, \dots, M$. With these time delays, the main beam of the beamformer can be

steered to the desired direction. Its output is expressed as

$$y(t) = \mathbf{w}^T \mathbf{x}(t), \quad (2.30)$$

where the signal vector $\mathbf{x}(t)$ is defined as

$$\mathbf{x}(t) = [x_1(t) \ \cdots \ x_M(t) \ x_1(t - T_s) \ \cdots \ x_M(t - T_s) \ \cdots \ x_1(t - (J - 1)T_s) \ \cdots \ x_M(t - (J - 1)T_s)]^T \quad (2.31)$$

and the weight vector $\mathbf{w}$ is defined as

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_{JM} \end{bmatrix}^T. \quad (2.32)$$

For the sources that can be modelled by stationary stochastic processes, the mean output power from the array system is given by

$$p(\mathbf{w}) = E\{|y(t)|^2\} = \mathbf{w}^T \mathbf{R} \mathbf{w}, \quad (2.33)$$

where $\mathbf{R}$ is the $MJ \times MJ$ dimensional array covariance matrix defined by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^T(t)\}. \quad (2.34)$$

For the sampled signal, the array received vector can be expressed as

$$\mathbf{x}(n) = [x_1(n) \cdots x_M(n) \ x_1(n-1) \cdots x_M(n-1) \cdots x_1(n-J+1) \cdots x_M(n-J+1)]^T \quad (2.35)$$

and in practice, the matrix $\mathbf{R}$ is estimated using sample averaging,

$$\hat{\mathbf{R}} = \frac{1}{N_s} \sum_{k=1}^{N_s} \mathbf{x}(k)\mathbf{x}^T(k). \quad (2.36)$$

2.4 Array Geometry

The inter-element time delay in (2.2) is defined for an arbitrary array system. In practice, the geometry of the array system is well defined and hence the inter-element time delay can be expressed in a simplified form. Two commonly used array geometries, namely linear and circular arrays are examined in this section.

2.4.1 Uniform Linear Array

In many applications, such as sonar, radar and seismology, it is necessary to use antenna arrays with a very directive characteristic to meet the demand of long distance communication. The uniform linear array (ULA) is very simple in design and is found to be suitable for these applications as it has very high directivity.

A ULA consists of a number of sensor elements, which are equally spaced in a straight line. For an M -element ULA, with all the elements placed along the x-axis

Figure 2.4: Uniform linear array with M sensors.

as shown in Fig. 2.4, the inter-element time delay at the m th element can be derived from (2.2) by setting $y_m = z_m = 0$ and $\phi_m = 0^\circ$, i.e.,

$$\tau_m = \frac{x_m \sin \theta}{v}, \quad (2.37)$$

where θ is the incident angle of the directional source measured from the y-axis (broad side), and x_m is the x-coordinate of the m th element.

Taking the center of the array as the reference, it follows that

$$\tau_m = \left[(m-1) - \frac{N-1}{2} \right] \frac{d \sin \theta}{v}, \quad (2.38)$$

where d is the inter-element spacing.

2.4.2 Circular Array

As shown in Fig. 2.5, a circular array consists of a number of sensors arranged in a circular ring, with radius r in the azimuth plane. Assuming equal angular spacing

Figure 2.5: Circular array with M sensors.

among elements, the inter-element time delay is given by

$$\tau_m = \frac{1}{v} [x_m \sin \theta + y_m \cos \theta], \quad (2.39)$$

where

$$\begin{aligned} x_m &= r \sin \left(\frac{2(m-1)\pi}{M} + \phi_0 \right), \\ y_m &= r \cos \left(\frac{2(m-1)\pi}{M} + \phi_0 \right) \end{aligned} \quad (2.40)$$

for $m = 1, 2, \dots, M$, and ϕ_0 is the angular displacement of the 1st element with respect to the y-axis. For the purpose of coordinate symmetry, ϕ_0 is normally set to zero for odd number of elements. If M is even, ϕ_0 is set at $\frac{\pi}{M}$.

2.5 Linearly Constrained Minimum Variance Array Beamformers

Most of the well studied array beamformers are LCMV beamformers. The optimal weight vector of the LCMV beamformer is obtained by minimizing the output variance subject to a set of linear constraints. In this section, we briefly introduce the typical narrowband and broadband beamformers used in array processing. All of these beamformers can be transformed to GSC as discussed [38, 34]. The beamformers proposed in this thesis are derived from these typical beamformers, i.e., the Capon beamformer, the Frost beamformer as well as their GSC implementations.

2.5.1 Narrowband Linear Constrained Minimum Variance Beamformer

The structure of a narrowband beamformer comprising of M sensors is shown in Fig. 2.2. The adaptive beamformer adjusts its weight vector $\mathbf{w}$ automatically according to specific criterion without the requirement on the information of the stochastic property of the signals. A famous representative of the LCMV beamformer, the Capon beamformer [9], is designed to minimize its mean output power subject to a unity gain constraint in the look direction as an indirect way of rejecting noise and interferences incident on the array. The optimal weight vector $\hat{\mathbf{w}}$ is the solution of the following

constrained optimization problem

$$\begin{cases} \min_{\mathbf{w}} & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} & \mathbf{s}^H(\theta_0, \phi_0) \mathbf{w} = 1 \end{cases}, \quad (2.41)$$

where $\mathbf{s}(\theta_0, \phi_0)$ is the nominal ASV corresponding to the assumed target direction θ_0 and ϕ_0 . Using the Lagrange multipliers methodology [43], the optimal weight vector $\hat{\mathbf{w}}$ is given as [5]

$$\hat{\mathbf{w}} = \frac{\mathbf{R}^{-1} \mathbf{s}(\theta_0, \phi_0)}{\mathbf{s}^H(\theta_0, \phi_0) \mathbf{R}^{-1} \mathbf{s}(\theta_0, \phi_0)}. \quad (2.42)$$

The optimal output power of the array beamformer, i.e., the power of the target signal, is

$$\hat{p} = \frac{1}{\mathbf{s}^H(\theta_0, \phi_0) \mathbf{R}^{-1} \mathbf{s}(\theta_0, \phi_0)}, \quad (2.43)$$

where $\hat{p}$ is the estimated power of the target signal.

2.5.2 Broadband Linear Constrained Minimum Variance Beamformer

A typical broadband LCMV beamformer, also named Frost beamformer [10], is shown in Fig. 2.3. The optimal weight vector $\hat{\mathbf{w}}$ is obtained by minimizing the array output power while maintaining a certain frequency response in the look direction. In

mathematics, it is expressed as

$$\begin{cases} \min_{\mathbf{w}} \mathbf{w}^T \mathbf{R} \mathbf{w} \\ \text{s.t. } \mathbf{C}_0^T \mathbf{w} = \mathbf{f} \end{cases}, \quad (2.44)$$

where $\mathbf{C}_0$ is the constraint matrix. The most commonly used one, which is known as look direction constraint matrix, is defined as

$$\mathbf{C}_0 = \begin{bmatrix} \mathbf{c}_1 & \dots & \mathbf{c}_j & \dots & \mathbf{c}_J \end{bmatrix}, \quad j = 1, 2, \dots, J, \quad (2.45)$$

where

$$\mathbf{c}_j = \begin{bmatrix} \underbrace{0 \dots 0}_{(j-1)M \text{ 0s}} & \underbrace{1 \dots 1}_{M \text{ 1s}} & \underbrace{0 \dots 0}_{(J-j)M \text{ 0s}} \end{bmatrix}^T, \quad j = 1, \dots, J, \quad (2.46)$$

$\mathbf{f}$ is a vector of size $J \times 1$, which determines the frequency response in the look direction.

With the Lagrange multipliers methodology [43], the optimal weight vector $\hat{\mathbf{w}}$ to the problem of (2.44) is given by [10],

$$\hat{\mathbf{w}} = \mathbf{R}^{-1} \mathbf{C}_0 (\mathbf{C}_0^T \mathbf{R}^{-1} \mathbf{C}_0)^{-1} \mathbf{f}, \quad (2.47)$$

the optimal beamformer output power $\hat{p}$ is given by

$$\hat{p} = \mathbf{f}^T (\mathbf{C}_0^T \mathbf{R}^{-1} \mathbf{C}_0)^{-1} \mathbf{f}. \quad (2.48)$$

2.5.3 The Generalized Sidelobe Canceller

Although the Frost beamformer and the Capon beamformer have different implementations, they have similar mathematical expression. Both of them can be considered as a quadratic optimization problem subject to linear constraints. In [38], Griffiths et al. proposed an alternative implementation method of the LCMV processor by splitting the processor into three parts: the first one is a fixed beamformer which can be designed according to the requirements on the quiescent response of array processor; the second part is a block matrix which blocks target signal but pass noise and interference as much as possible; the third part is a reduced dimension multichannel adaptive filters. This is the well known GSC implementation. The diagram of a GSC is shown in Fig. 2.6. The fixed beamformer $\mathbf{g}$ in Fig. 2.6 can be selected as

$$\mathbf{g} = \mathbf{C}_0(\mathbf{C}_0^T \mathbf{C}_0)^{-1} \mathbf{f}, \quad (2.49)$$

and matrix $\mathbf{B}$, whose columns form a basis for the null space of $\mathbf{C}_0$, can be used as blocking matrix. The adaptive filter $\mathbf{w}'$ is obtained as

$$\min_{\mathbf{w}'} (\mathbf{g} - \mathbf{B}\mathbf{w}')^T \mathbf{R} (\mathbf{g} - \mathbf{B}\mathbf{w}'). \quad (2.50)$$

The solution $\hat{\mathbf{w}}'$ is

$$\hat{\mathbf{w}}' = (\mathbf{B}^T \mathbf{R} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{R} \mathbf{g}. \quad (2.51)$$

The relationship between $\hat{\mathbf{w}}$ and $\hat{\mathbf{w}}'$ is

$$\hat{\mathbf{w}} = \mathbf{g} - \mathbf{B}\hat{\mathbf{w}}'. \quad (2.52)$$

All of the LCMV beamformers can be implemented as GSC [34]. The GSC implementation has some advantages: GSC uses unconstrained optimization, which results in simple implementation and fast convergence in adaptation of filter weights; the fixed beamformer and blocking matrix can be flexibly chosen if they satisfy the necessary conditions. Consequently, GSC is widely used in practice.

The narrowband GSC implementation of the Capon beamformer is similar to the one shown in Fig. 2.6. The difference is that the multichannel adaptive filter of narrowband GSC has only one taps for each channel, while in broadband GSC, tapped delay line filters are used.

2.6 Robust Adaptive Array Beamformer

In ideal cases, the Frost and Capon beamformers have high performance in interference and noise suppression. However, the ideal assumptions of array processor do not hold in practical applications [11, 12, 13, 14, 15]. The performance of array processor highly degrades when there are array imperfections such as steering direction error, time delay error, phase errors of the array sensors, multipath propagation effects, wavefront distortions, etc. All of the implementations of adaptive beamformers, including the LCMV, GSC and others [19, 34], suffer from target signal cancellation when there are

Figure 2.6: GSC implementation of LCMV beamformer.

array imperfections.

To overcome the target signal cancellation problem caused by steering direction error, multiple-point constraints [5,16], derivative constraints [5,17,18,19,20,21,22,23] and a new set of constraints [24,25,35,34] are introduced in robust adaptive beamforming. Moreover, techniques restraining excess coefficients growth were also proposed in array processor to achieve robust performance [26], similar robust beamformer can be obtained by using the noise injection method [12] and the leaky LMS method [36].

Most of the above approaches can be expressed in a uniform optimization problem

as

$$\begin{cases} \min_{\mathbf{w}} \mathbf{w}^T \mathbf{R} \mathbf{w} \\ \text{s.t. } \mathbf{C}_0^T \mathbf{w} = \mathbf{f} , \\ f(\mathbf{w}) \leq 0 \end{cases} \quad (2.53)$$

where the linear constraints set $\mathbf{C}_0^T \mathbf{w} = \mathbf{f}$ contains the standard steering constraints in (2.41) and (2.52). It can also includes the linear constraints derived for robust beamformer, e.g., multiple-point constraints [5, 16] and derivative constraints [5, 17, 18, 19, 20], etc. Another constraint set $f(\mathbf{w}) \leq 0$ contains nonlinear inequality constraints for robust beamformer. The common nonlinear constraints derived so far for robust beamforming are of quadratic form [24, 25, 12, 26].

The solution of the universal optimization problem (2.53) can be refer to [43]. It is briefly introduced in this chapter. Ignore the inequality constraint in (2.53), the solution $\hat{\mathbf{w}}$ of the optimization problem (2.53) is given in (2.47), i.e.,

$$\hat{\mathbf{w}} = \mathbf{R}^{-1} \mathbf{C}_0 (\mathbf{C}_0^T \mathbf{R}^{-1} \mathbf{C}_0)^{-1} \mathbf{f}. \quad (2.54)$$

If $f(\hat{\mathbf{w}}) \leq 0$, the inequality constraint is inactive in the optimization, i.e., $\hat{\mathbf{w}}$ is the solution of (2.53). Otherwise, the optimal solution of (2.53) is obtained on the boundary of the constraints. The optimization problem can be expressed as

$$\left\{ \begin{array}{ll} \min_{\mathbf{w}} & \mathbf{w}^T \mathbf{R} \mathbf{w} \\ \text{s.t.} & \mathbf{C}_0^T \mathbf{w} = \mathbf{f} \\ & f(\mathbf{w}) = 0 \end{array} \right. \quad (2.55)$$

This quadratic programming problem with equality constraints can be solved using the Lagrange multiplier methodology [43]. Define a function as

$$f(\mathbf{w}, \mathbf{l}_1, \mathbf{l}_2) = \mathbf{w}^T \mathbf{R} \mathbf{w} + \mathbf{l}_1^T (\mathbf{C}_0^T \mathbf{w} - \mathbf{f}) + \mathbf{l}_2^T f(\mathbf{w}), \quad (2.56)$$

where $\mathbf{l}_1$ and $\mathbf{l}_2$ are the Lagrange multipliers. The optimal solution $\hat{\mathbf{w}}$ is obtained by solving

$$\frac{\partial f(\hat{\mathbf{w}}, \mathbf{l}_1, \mathbf{l}_2)}{\partial \mathbf{w}} = 0, \quad (2.57)$$

and $\mathbf{l}_1$ and $\mathbf{l}_2$ are solved from

$$\begin{cases} \mathbf{C}_0^T \hat{\mathbf{w}} = \mathbf{f}, \\ f(\hat{\mathbf{w}}) = 0. \end{cases} \quad (2.58)$$

Most of the above methods introduce additional constraints to the array processor, causing array processor suffering from loss in DOF, which results in degraded performance in noise and interference cancellation. In this thesis, we propose some robust methods which do not consume the DOF of array beamformer.

2.7 Summary

In this chapter, the principle of adaptive beamforming, signal model and notations, beamformer structures, robust beamforming methods are introduced. Most of the robust beamforming approaches have a uniform mathematical expression as the quadratic optimization problem with linear constraints and nonlinear inequality constraints. The general solution of such optimization problem is also briefly introduced.

Chapter 3

An Extended Generalized Sidelobe Canceller

3.1 Introduction

In most of the array processor, there is an assumption that the channel effect between the target source and each sensor is just a pure time delay. However, in real applications such as speech acquisition in adverse acoustic environments, source signal does not propagate along pure delay channels. There are multipath and reverberant effects in acoustic environment, implying that the incidence of the target signal to array can be from any directions. In such case, the conventional beamforming methods based on pure delay channel suffer from target signal cancellation. It is especially serious for microphone array in strong reverberant environments. The performance degradation is mainly due to the arbitrary transfer functions (TFs) relating source signal and sensors.

These TFs violate the assumption of pure delay channel model adopted by conventional beamforming methods [10, 17, 18, 33, 24].

To combat reverberant and multipath effects, matched-filter (MF) based beamformers [44, 45, 46, 47, 48] were proposed instead of delay-and-sum beamformer (DSB). MF beamformer significantly improves the quality of signal captured in reverberant environment [44, 45, 46, 47, 48]. However, to have high interference suppression performance, large number of sensors are required for MF beamformer. This drawback can be alleviated by combining MF beamformer with GSC. Consequently, fewer sensors are required to maintain high interference suppression performance. Another method to combat reverberant and multipath effects is the exact inverse filter [49]. This method has high dereverberation performance but no noise suppression capability. The GSC can also be combined with exact inverse filter to achieve high performance both in noise suppression and signal dereverberation.

In this chapter, we propose an extended GSC for wideband signal beamforming. It adopts arbitrary TFs relating signal source and sensors. The fixed beamformer and blocking matrix of GSC are extended to general spatial-spectral filters instead of spatial filter in conventional GSC. The implementation of the extended GSC in time and frequency domain is presented. Furthermore, an approximation of the extended GSC in frequency domain reveals that the GSC can be simplified as applying narrowband GSC at each frequency bin independently when the length of FFT is long enough.

This chapter is organized as follows. In Section 3.2, the extended channel model is formulated. Some of its special cases are also discussed. The extended GSC im-

plemented in time domain is formulated in Section 3.3. It is followed by the implementation of the extended GSC in frequency domain in Section 3.4. An simplified frequency domain implementation method is shown in Section 3.5, which indicates that the broadband beamformer can be implemented in frequency domain as narrow-band beamformer with the aid of FFT technique. Brief summary is given in Section 3.6.

3.2 System Model

3.2.1 Extended Data Model for Array Processing

The extended data model of array processor is shown in Fig. 3.1. There are M sensors. Each sensor picks up the additive environment noise/interference $n_m(t)$ and the filtered signal of the target signal $s(t)$ by the m th system impulse response (IR) $h_m(t)$, $m = 1, 2, \dots, M$. The sensor received signal $x_m(t)$ of the m th channel is designated as

$$x_m(t) = h_m(t) \star s(t) + n_m(t), \quad m = 1, 2, \dots, M. \quad (3.1)$$

The digitalized signal of the m th channel is

$$x_m(n) = s(nT_s) \star h_m(nT_s) + n_m(nT_s) \triangleq s(n) \star h_m(n) + n_m(n), \quad m = 1, 2, \dots, M, \quad (3.2)$$

where T_s represents the sampling period.

Figure 3.1: The extended data model of array processor.

In most applications, $h_m(n)$ can be approximated as FIR filter with length L though it is not necessary. The filter coefficient vector is defined as

$$\mathbf{h}_m = [h_m(0) \ h_m(1) \ \cdots \ h_m(L-1)]^T. \quad (3.3)$$

Arranging the received signals in vector form, the received sensor signal vector can be expressed as

$$\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{n}(n), \quad (3.4)$$

where

$$\begin{aligned}
 \mathbf{x}(n) &= [x_1(n) \ x_2(n) \ \cdots \ x_M(n)]^T, \\
 \mathbf{n}(n) &= [n_1(n) \ n_2(n) \ \cdots \ n_M(n)]^T, \\
 \mathbf{s}(n) &= [s(n) \ s(n-1) \ \cdots \ s(n-L+1)]^T, \\
 \mathbf{H} &= [\mathbf{h}_1 \ \mathbf{h}_2 \ \cdots \ \mathbf{h}_M]^T.
 \end{aligned} \tag{3.5}$$

3.2.2 Discussion on Data Model

In conventional array processing, the IR of each channel is assumed to be a pure delay function, i.e.,

$$h_m(t) = \delta(t - \tau_m), \tag{3.6}$$

where τ_m is the delay of the m th channel. When the array works in narrowband mode, the effect of channel impulse response is equivalent to a phase shift of target signal. In many array processing methods, the delay of each channel is pre-compensated before adaptive beamforming. However, with digitalized signal, τ_m may not just be the multiple of T_s . In such case, conventional digital delayer has time-delay error which results in target signal cancellation in adaptive beamforming.

When the delay τ_m is not multiple of T_s , the IR of each channel can be modelled as time-shift filter $h_m(k)$

$$h_m(n) = \frac{\sin((n - \tau_m/T_s)\pi)}{\pi(n - \tau_m/T_s)}, \quad n = \cdots, -1, 0, 1, \cdots \tag{3.7}$$

Although the taps of $h_m(n)$ should be infinity, the amplitude of $h_m(n)$ diminishes in same speed following $1/|n|$. Therefore we can use $2D + 1$ taps of $h_k(n)$ to approximate the ideal time-shift filter, where D is an integer.

The channel model is more complicated in real applications when there are various array imperfections including channel mismatches, sensor location error, sensor gain/phase error, wavefront distortion, multipath and reverberation effects in acoustic applications, etc. All of these effects can be modelled in $h_m(n)$. Therefore, the IR $h_m(n)$ cannot be expressed as a simple delay function. A more complicated filter (IIR or FIR) should be used to convey all the array imperfections.

3.3 Implementation of Extended GSC in Time Domain

The extended GSC in time domain is shown in Fig. 3.2. It is well known that GSC has three major parts, fixed beamformer, blocking filter¹ and multichannel adaptive filter. Compared with the conventional GSC in [38], the proposed one has some modifications in the fixed beamformer and blocking filter. The fixed beamformer is modified to be a multiple input single output (MISO) filter, which can be designed to achieve some criteria, such as coherently sum up all the multipath signals to achieve maximum SNR and dereverberate the reverberant target signal [44, 45, 46, 49]. The task of the blocking filter is to suppress target signal but pass interference and noise signal as much

¹It is also called blocking matrix.

Figure 3.2: The schematic of the extended GSC in time domain.

as possible. The conventional blocking filter is designed based on pure delay channel model. The resulting blocking filter only utilizes the spatial information of the incoming signals. With arbitrary TFs, signal components with different frequency have different response characteristics. The conventional blocking matrix can not block the target signal successfully. It must be changed to also introduce the temporal information of input signal to suppress all the components of target source. Otherwise, the target signal leaking into the multichannel adaptive filter results in unwanted target signal cancellation.

In the following discussion, we assume that the lengths of the FIR filter $g_m(n)$ and

the blocking filter $\mathbf{B}(n)$ are both L . The length of $w_m(n)$ is L_w . In Fig. 3.2, the output $y(n)$ of fixed beamformer is designated as

$$y(n) = \sum_{m=1}^M x_m(n) \star g_m(n) = \sum_{m=1}^M \sum_{j=0}^{L-1} x_m(n-j) g_m(j). \quad (3.8)$$

An example of designing $g_m(n)$ is to coherently sum up target signal propagating through channel $h_m(n)$, $m = 1, \dots, M$ and maximize the output SNR. This is known as MF array [44, 45, 46]. The filter coefficient $g_m(n)$ can be expressed as

$$g_m(n) = h_m(n_0 - n) \quad m = 1, 2, \dots, M, \quad (3.9)$$

where n_0 is a preselected delay.

The blocking filter $\mathbf{B}(n)$ is a matrix of size $N \times M$ ($N \leq M$),

$$\mathbf{B}(n) = \begin{bmatrix} b_{11}(n) & b_{12}(n) & \cdots & b_{1M}(n) \\ b_{21}(n) & b_{22}(n) & \cdots & b_{2M}(n) \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1}(n) & b_{N2}(n) & \cdots & b_{NM}(n) \end{bmatrix}. \quad (3.10)$$

The output vector $\mathbf{z}(n)$ of the blocking filter is expressed as

$$\mathbf{z}(n) \triangleq [z_1(n) \ z_2(n) \ \cdots \ z_N(n)]^T = \sum_{l=0}^{L-1} \mathbf{B}(n-l) \mathbf{x}(l), \quad (3.11)$$

where the the j th output signal $z_j(n)$ of the blocking filter is

$$z_j(n) = \sum_{m=1}^M \sum_{l=0}^{L-1} x_m(l) b_{jm}(n-l). \quad (3.12)$$

The design of the blocking filter $\mathbf{B}(n)$ has many choices. The key requirements of blocking filter are:

- (i) It can block all the target signal components including direct path component and multipath components.
- (ii) The blocking filter can pass the noise and interference as much as possible.
- (iii) Simple structure and low computational load required, however, it is unnecessary.
- (iv) Robustness of the blocking filter to system imperfections.

In this chapter, we propose a blocking filter design method, which is simple and easy to be implemented. Since

$$x_i(n) \star h_j(n) - x_j(n) \star h_i(n) = 0 \quad (3.13)$$

$$i, j = 1, 2, \dots, M, i \neq j,$$

in noiseless case, the target signal is blocked for any $h_i(n)$. The blocking filter can be

constructed as

$$\mathbf{B}(n) = \begin{bmatrix} h_2(n) & -h_1(n) & 0 & \cdots & 0 \\ 0 & h_3(n) & -h_2(n) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & h_M(n) & -h_{M-1}(n) \end{bmatrix}, \quad (3.14)$$

or

$$\mathbf{B}(n) = \begin{bmatrix} h_2(n) & -h_1(n) & 0 & \cdots & 0 \\ h_3(n) & 0 & -h_1(n) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_M(n) & \cdots & 0 & 0 & -h_1(n) \end{bmatrix}. \quad (3.15)$$

Other blocking filters can also be designed to achieve high performance and robustness. The multichannel adaptive filter is the same one as in conventional GSC. Its output $v(n)$ is

$$v(n) = \sum_{i=1}^N z_i(n) \star w_i(n) = \sum_{i=1}^N \sum_{j=0}^{L_w-1} z_i(j) w_i(n-j). \quad (3.16)$$

The error signal $e(n)$ is

$$e(n) = y(n-D) - v(n), \quad (3.17)$$

where D is the delay introduced. The filter weights vector $\mathbf{w}_i(n)$ of the multichannel adaptive filter is updated using the normalized least mean square (NLMS) method [42, 41],

$$\mathbf{w}_i(n+1) = \mathbf{w}_i(n) - \rho \frac{\mathbf{z}_i(n)}{\sum_{i=1}^N \|\mathbf{z}_i(n)\|^2} e(n), \quad i = 1, \dots, M, \quad (3.18)$$

where ρ is the stepsize of the NLMS algorithm, and $\mathbf{z}_i(n)$ is the input signal vector of the i th channel of multichannel adaptive filter.

$$\mathbf{z}_i(n) = [z_i(n) \ z_i(n-1) \ \cdots \ z_i(n-L_w+1)]^T. \quad (3.19)$$

The leaky LMS algorithm can also be used to prevent target signal from cancellation [12,36] when the target signal leaks into the multichannel adaptive filter. The updating formula of leaky LMS is

$$\mathbf{w}_i(n+1) = \beta \mathbf{w}_i(n) - \rho \frac{\mathbf{z}_i(n)}{\sum_{i=1}^M \|\mathbf{z}_i(n)\|^2} e(n), \quad i = 1, \dots, N, \quad (3.20)$$

where β ($0 < \beta \leq 1$) is the leakage parameter.

The implementation of the extended GSC in time domain is simple. However, when the length of IR is long, the computational load of the extended GSC is heavy. If the system delay is noncritical in some applications, the computational load can be reduced by implementing the extended GSC in frequency domain.

3.4 Implementation of Extended GSC in Frequency Domain

One of the advantages of frequency domain filtering is that the FFT can be used to reduce the computational complexity. From the property of discrete Fourier transform (DFT), we know that the circular convolution of two finite length sequences can be ob-

tained by transforming both sequences to their respective frequency domain(using the FFT), performing an element-by-element multiplication on the transformed samples, and transforming the result back to the time domain (using the inverse FFT). Another advantage of the frequency domain filtering is that the FFT performs an orthogonal transform on the input signal. With the power normalization method, it is possible to derive fast convergence algorithms for adaptive updating.

In this chapter, the extended GSC algorithm in frequency domain is derived based on the overlap-save technique [50]. Before the derivation of the extended GSC algorithm, some matrices used in the derivation are defined first.

Definition 3.1. *The DFT matrix $\mathbf{F}_L$ is a $L \times L$ matrix, whose (p, q) th element $(\mathbf{F}_L)_{p,q}$ is*

$$(\mathbf{F}_L)_{p,q} = e^{\frac{-j2\pi pq}{L}}, \quad p, q = 0, 1, \dots, L-1.$$

Lemma 3.1. *The inverse matrix of $\mathbf{F}_L$ is $\mathbf{F}_L^{-1} = \frac{1}{L}\mathbf{F}_L^H$.*

Proof. It is straightforward that

$$\frac{1}{L}\mathbf{F}_L^H\mathbf{F}_L = \mathbf{I}.$$

Therefore, $\mathbf{F}_L^{-1} = \frac{1}{L}\mathbf{F}_L^H$. □

Definition 3.2. *The windowing matrix $\mathbf{W}_{L \times N'}^{10}, \mathbf{W}_{N \times N'}^{01}, \mathbf{W}_{N' \times L}^{10}, \mathbf{W}_{N' \times N}^{01}, \mathbf{W}_{N' \times N'}^{10}$ and*

$\mathbf{W}_{N' \times N'}^{01}$ are defined as

$$\begin{aligned} \mathbf{W}_{L \times N'}^{10} &= [\mathbf{I}_{L \times L} \ \mathbf{0}_{L \times (N-1)}], \quad \mathbf{W}_{N' \times L}^{10} = (\mathbf{W}_{L \times N'}^{10})^T, \\ \mathbf{W}_{N \times N'}^{01} &= [\mathbf{0}_{N \times (L-1)} \ \mathbf{I}_{N \times N}], \quad \mathbf{W}_{N' \times N}^{01} = (\mathbf{W}_{N \times N'}^{01})^T, \\ \mathbf{W}_{N' \times N'}^{01} &= \begin{bmatrix} \mathbf{0}_{(L-1) \times (L-1)} & \mathbf{0}_{(L-1) \times N} \\ \mathbf{0}_{N \times (L-1)} & \mathbf{I}_{N \times N} \end{bmatrix}, \quad \mathbf{W}_{N' \times N'}^{10} = \begin{bmatrix} \mathbf{I}_{L \times L} & \mathbf{0}_{L \times (N-1)} \\ \mathbf{0}_{(N-1) \times L} & \mathbf{0}_{(N-1) \times (N-1)} \end{bmatrix}, \end{aligned}$$

where $N' = L + N - 1$.

Definition 3.3. The transformed windowing matrix $\mathcal{W}_{L \times N'}^{10}$, $\mathcal{W}_{N \times N'}^{01}$, $\mathcal{W}_{N' \times L}^{10}$, $\mathcal{W}_{N' \times N}^{01}$,

$\mathcal{W}_{N' \times N'}^{10}$ and $\mathcal{W}_{N' \times N'}^{01}$ are defined as

$$\begin{aligned} \mathcal{W}_{L \times N'}^{10} &= \mathbf{F}_L \mathbf{W}_{L \times N'}^{10} \mathbf{F}_{N'}^{-1}, \quad \mathcal{W}_{N \times N'}^{01} = \mathbf{F}_N \mathbf{W}_{N \times N'}^{01} \mathbf{F}_{N'}^{-1}, \\ \mathcal{W}_{N' \times L}^{10} &= \mathbf{F}_{N'} \mathbf{W}_{N' \times L}^{10} \mathbf{F}_L^{-1}, \quad \mathcal{W}_{N' \times N}^{01} = \mathbf{F}_{N' \times 2L} \mathbf{W}_{N' \times N}^{01} \mathbf{F}_N^{-1}, \\ \mathcal{W}_{N' \times N'}^{10} &= \mathbf{F}_{N'} \mathbf{W}_{N' \times N'}^{10} \mathbf{F}_{N'}^{-1}, \quad \mathcal{W}_{N' \times N'}^{01} = \mathbf{F}_{N'} \mathbf{W}_{N' \times N'}^{01} \mathbf{F}_{N'}^{-1}. \end{aligned}$$

Lemma 3.2. The transformed windowing matrices have the following properties

$$\mathcal{W}_{L \times N'}^{10} = \frac{L}{N'} (\mathcal{W}_{N' \times L}^{10})^H, \quad \mathcal{W}_{N' \times N}^{01} = \frac{N'}{N} (\mathcal{W}_{N \times N'}^{01})^H.$$

Proof. Refer to Appendix A.1. □

Lemma 3.3.

$$\begin{aligned}\mathbf{w}_{N' \times N'}^{01} &\approx \frac{N}{N'} \mathbf{I}_{N' \times N'}, \\ \mathbf{w}_{N' \times N'}^{10} &\approx \frac{L}{N'} \mathbf{I}_{N' \times N'}.\end{aligned}$$

Proof. Refer to Appendix A.2. □

The filtering of $x_j(n)$ by an FIR filter $g_j(n)$ of length L involves computation of a linear convolution

$$y_j(n) = \sum_{i=0}^{L-1} g_j(i) x_j(n-i). \quad (3.21)$$

This process can be efficiently implemented in frequency domain using circular convolution when L is large. The computation can be greatly reduced when L is large by using the FFT technique. In matrix notation, the vector $\tilde{\mathbf{y}}_j(m)$ of length $N' = N + L - 1$ that results from the circular convolution of signal $x_j(n)$ and its associated filter $g_j(n)$ is given by

$$\tilde{\mathbf{y}}_j(m) = \check{\mathbf{X}}_j(m) \tilde{\mathbf{g}}_j, \quad (3.22)$$

where m stands for the m th data block, the matrix $\check{\mathbf{X}}_j(m)$ is a circular matrix

$$\check{\mathbf{X}}_j(m) = \begin{bmatrix} x_j(mN - L + 1) & x_j(mN + N - 1) & \cdots & x_j(mN - L + 2) \\ x_j(mN - L + 2) & x_j(mN - L + 1) & \cdots & x_j(mN - L + 3) \\ \vdots & \vdots & \ddots & \vdots \\ x_j(mN - 1) & x_j(mN - 2) & \cdots & x_j(mN) \\ x_j(mN) & x_j(mN - 1) & \cdots & x_j(mN + 1) \\ x_j(mN + 1) & x_j(mN) & \cdots & x_j(mN + 2) \\ \vdots & \vdots & \ddots & \vdots \\ x_j(mN + N - 1) & x_j(mN + N - 2) & \cdots & x_j(mN - L + 1) \end{bmatrix}, \quad (3.23)$$

and the vectors $\tilde{\mathbf{y}}_j(m)$ and $\tilde{\mathbf{g}}_j$ are expressed as

$$\begin{aligned} \tilde{\mathbf{y}}_j(m) &= [\tilde{y}_j(mN - L + 1) \ \tilde{y}_j(mN - L + 2) \ \cdots \ \tilde{y}_j(mN) \ \cdots \ \tilde{y}_j(mN + N - 1)]^T, \\ \tilde{\mathbf{g}}_j &= [\mathbf{g}_j^T \ \mathbf{0}_{1 \times (N-1)}]^T = \mathbf{W}_{N' \times L}^{10} \mathbf{g}_j, \\ \mathbf{g}_j &= [g_j(0) \ \cdots \ g_j(L - 1)]^T. \end{aligned} \quad (3.24)$$

The results of a linear convolution $\mathbf{y}_j(m)$ is identical to the last N samples of $\tilde{\mathbf{y}}_j(m)$,

$$\begin{aligned} \mathbf{y}_j(m) &= \mathbf{W}_{N \times N'}^{01} \tilde{\mathbf{y}}_j(m) \\ &= \mathbf{W}_{N \times N'}^{01} \check{\mathbf{X}}_j(m) \tilde{\mathbf{g}}_j \\ &= \mathbf{W}_{N \times N'}^{01} \check{\mathbf{X}}_j(m) \mathbf{W}_{N' \times L}^{10} \mathbf{g}_j \\ &= \mathbf{W}_{N \times N'}^{01} \mathbf{F}_{N'}^{-1} \mathcal{D}_{\check{\mathbf{X}}_j}(m) \mathcal{W}_{N' \times L}^{10} \mathbf{g}_j, \end{aligned} \quad (3.25)$$

where

$$\begin{aligned}\mathcal{D}_{\check{\mathbf{X}}_j}(m) &= \mathbf{F}_{N'} \check{\mathbf{X}}_j(m) \mathbf{F}_{N'}^{-1}, \\ \mathbf{g}_j &= \mathbf{F}_L \mathbf{g}_j.\end{aligned}\tag{3.26}$$

Lemma 3.4. *The matrix $\mathcal{D}_{x_j}(m)$ in (3.26) is a diagonal matrix whose diagonal elements are given by the DFT of the first column of $\check{\mathbf{X}}_j(m)$.*

Proof. Refer to Appendix A.3. □

With the above results, the extended GSC can be easily derived in frequency domain. The output $\mathbf{y}(m)$ of the fixed beamformer is

$$\begin{aligned}\mathbf{y}(m) &= \sum_{i=1}^M \mathbf{y}_i(m) \\ &= \mathbf{W}_{N \times N'}^{01} \sum_{i=1}^M \check{\mathbf{X}}_i(m) \mathbf{W}_{N' \times L}^{10} \mathbf{g}_i \\ &= \mathbf{W}_{N \times N'}^{01} \mathbf{F}_{N'}^{-1} \sum_{i=1}^M \mathcal{D}_{\check{\mathbf{X}}_i}(m) \mathbf{W}_{N' \times L}^{10} \mathbf{g}_i.\end{aligned}\tag{3.27}$$

The output signal vector of i th blocking filter $\mathbf{z}_i(m)$ is expressed as

$$\begin{aligned}\mathbf{z}_i(m) &= [z_i(mN) \cdots z_i(mN + N - 1)]^T \\ &= \mathbf{W}_{N \times N'}^{01} \mathbf{F}_{N'}^{-1} \sum_{j=1}^M \mathcal{D}_{\check{\mathbf{X}}_j}(m) \mathbf{W}_{N' \times L}^{10} \mathbf{b}_{ij},\end{aligned}\tag{3.28}$$

where

$$\begin{aligned} \mathbf{b}_{ij} &= \mathbf{F}_L \mathbf{b}_{ij}, \\ \mathbf{b}_{ij} &= [b_{ij}(0) \ \cdots \ b_{ij}(L-1)]^T. \end{aligned} \quad (3.29)$$

The output signal vector $\mathbf{v}(m)$ of the multichannel adaptive filter is

$$\begin{aligned} \mathbf{v}(m) &= [v(mN) \ \cdots \ v(mN + N - 1)]^T \\ &= \mathbf{W}_{N \times N''}^{01} \mathbf{F}_{N''}^{-1} \sum_{i=1}^M \mathcal{D}_{\check{\mathbf{z}}_i}(m) \mathcal{W}_{N'' \times L_w}^{10} \mathbf{w}_i, \end{aligned} \quad (3.30)$$

where $N'' = L_w + N - 1$ and $\check{\mathbf{Z}}_i(m)$ is a circular matrix

$$\check{\mathbf{Z}}_i(m) = \begin{bmatrix} z_i(mN - L_w + 1) & z_i(mN + N - 1) & \cdots & z_i(mN - L_w + 2) \\ z_i(mN - L_w + 2) & z_i(mN - L_w + 1) & \cdots & z_i(mN - L_w + 3) \\ \vdots & \vdots & \ddots & \vdots \\ z_i(mN - 1) & z_i(mN - 2) & \cdots & z_i(mN) \\ z_i(mN) & z_i(mN - 1) & \cdots & z_i(mN + 1) \\ z_i(mN + 1) & z_i(mN) & \cdots & z_i(mN + 2) \\ \vdots & \vdots & \ddots & \vdots \\ z_i(mN + N - 1) & z_i(mN + N - 2) & \cdots & z_i(mN - L_w + 1) \end{bmatrix}, \quad (3.31)$$

and

$$\begin{aligned}\mathcal{D}_{\check{\mathbf{Z}}_i}(m) &= \mathbf{F}_{N''} \check{\mathbf{Z}}_i(m) \mathbf{F}_{N''}^{-1}, \\ \mathbf{w}_i &= \mathbf{F}_{L_w} \mathbf{w}_i, \\ \mathbf{w}_i &= [w_i(0) \cdots w_i(L_w - 1)]^T.\end{aligned}\tag{3.32}$$

Define the error vector $\mathbf{e}(m)$ in frequency domain as

$$\mathbf{e}(m) = \mathbf{F}_N(\mathbf{y}(m, D) - \mathbf{v}(m)),\tag{3.33}$$

where $\mathbf{y}(m, D)$ is the delayed signal vector of $\mathbf{y}(m)$,

$$\mathbf{y}(m, D) = [\tilde{y}_j(mN - D) \cdots \tilde{y}_j(mN + N - D - 1)]^T.\tag{3.34}$$

The cost function $C(m)$ for the derivation of multichannel adaptive filter in frequency domain is define as

$$C(m) = \mathbf{e}^H(m) \mathbf{e}(m).\tag{3.35}$$

Lemma 3.5.

$$\begin{aligned}\nabla C_k(m) &= \frac{\partial C(m)}{\partial \mathbf{w}_k^*} = -\mathcal{W}_{L_w \times N''}^{10} \mathcal{D}_{\check{\mathbf{Z}}_k}^H(m) \mathcal{W}_{N'' \times N}^{01} \mathbf{e}(m), \\ E\{\nabla^2 C_k(m)\} &= E\left\{\frac{\partial \nabla C_k(m)}{\partial \mathbf{w}_k^T}\right\} \approx \frac{N}{N''} \mathcal{W}_{L_w \times N''}^{10} \hat{\mathcal{R}}_{z_k}(m) \mathcal{W}_{N'' \times L_w}^{10},\end{aligned}$$

where

$$\hat{\mathcal{R}}_{z_k}(m) \approx \alpha \hat{\mathcal{R}}_{z_k}(m - 1) + (1 - \alpha) \mathcal{D}_{\check{\mathbf{Z}}_k}^H(m) \mathcal{D}_{\check{\mathbf{Z}}_k}(m),$$

and α is a forgetting factor.

Proof. Refer to Appendix A.4. □

Lemma 3.6. *If $\hat{\mathcal{R}}_{z_k}(m)$ is invertible, we have*

$$\mathcal{W}_{N'' \times L_w}^{10} E\{\nabla^2 C(m)\}^{-1} \mathcal{W}_{L_w \times N''}^{10} = \frac{N''}{N} \mathcal{W}_{N'' \times N''}^{10} \hat{\mathcal{R}}_{z_k}^{-1}(m).$$

Proof. Refer to Appendix A.5. □

With the above Lemmas, it is straightforward that LMS type algorithm for the extended GSC is given by

Theorem 3.1. *The constrained Newton-LMS type algorithm is*

$$\hat{\mathbf{w}}_k^{10}(m) = \hat{\mathbf{w}}_k^{10}(m-1) + \rho \mathcal{W}_{N'' \times N''}^{10} \hat{\mathcal{R}}_{z_k}^{-1}(m) \mathcal{D}_{\check{Z}_k}(m) \mathcal{W}_{N'' \times N}^{01} \mathbf{e}(m),$$

and the unconstrained Newton-LMS algorithm is

$$\hat{\mathbf{w}}_k^{10}(m) = \hat{\mathbf{w}}_k^{10}(m-1) + \rho \hat{\mathcal{R}}_{z_k}^{-1}(m) \mathcal{D}_{\check{Z}_k}^H(m) \mathcal{W}_{N'' \times N}^{01} \mathbf{e}(m),$$

where ρ is the stepsize.

Proof. Refer to Appendix A.6. □

The algorithms in Theorem 3.1 have low computational load because the matrices $\mathcal{D}_{\tilde{z}_k}^H(m)$ and $\mathcal{R}_{z_k}(m)$ are all diagonal matrices. The matrix inverse and multiplication operations are simplified to elements inverse and multiplication.

3.5 Simplified Frequency Domain Implementation of Extended GSC

The extended GSC with long filter length implemented in frequency domain is computational effective compared with the one in time domain. From the derivations in Section 3.4, we find that the calculation of $\mathbf{v}(m)$ need to transform $x_i(n)$ into frequency domain, then filter it by $B_{M \times N}(n)$ in frequency domain and transform the result back to time domain to get $z_i(n)$. Then the calculated $z_i(n)$ are transformed to frequency domain again and filtered by $w_i(n)$. The result is transformed back to time domain to get $v(n)$. The reason for such operations can be explained from (3.28). Only part of the output vector of blocking filter is identical to the output signal of linear convolution. To construct input signal of multichannel adaptive filter, the former data block of the output signal of the blocking filter must be borrowed as shown in (3.30)-(3.31).

A question arises whether it is possible to pass over the intermediate FFT and IFFT to further reduce computational load. The answer is YES when the length of FFT is far greater than that of the filter, i.e., $N' \gg L$ and $N'' \gg L_w$.

Theoretically, the filter response from the the received signal $x_i(n)$ to $v(n)$ can be derived.

Lemma 3.7. *The signal $v(n)$ in Fig. 3.2 can be expressed as*

$$v(n) = \sum_{i=1}^M \sum_{l=0}^{L+L_w-2} x_i(l) d_i(n-l),$$

where

$$d_i(n) = \sum_{n=1}^M \sum_{j=0}^{L+L_w-2} b_{ni}(j) w_n(n-j).$$

Proof. The proof is straightforward. □

Remark 3.1. *The combined IR $d_i(n)$ is of length $L' = L + L_w - 1$. We can consider that the output signal $v(n)$ is the sum of filtered signals of $x_i(n)$ by filter $d_i(n)$. The combined IR $d_i(n)$ is the sum of linear convolution of $b_{ni}(n)$ and $w_n(n)$. Therefore, the filtering method in frequency domain can also be applied.*

The signal vector $\mathbf{v}(m)$ of length N can be calculated as

$$\begin{aligned} \mathbf{v}(m) &= \mathbf{W}_{N \times N_3}^{01} \sum_{i=1}^M \check{\mathbf{X}}_i(m) \mathbf{W}_{N_3 \times L'}^{10} \mathbf{d}_i \\ &= \mathbf{W}_{N \times N_3}^{01} F_{N_3}^{-1} \sum_{i=1}^M \mathcal{D}_{\check{\mathbf{X}}_i}(m) \mathcal{W}_{N_3 \times L'}^{10} \mathbf{d}_i \\ &= \mathbf{W}_{N \times N_3}^{01} F_{N_3}^{-1} \sum_{i=1}^M \mathcal{D}_{\check{\mathbf{X}}_i}(m) \tilde{\mathbf{d}}_i, \end{aligned} \tag{3.36}$$

where $N_3 = N + L + L_w - 2$ and the matrix $\check{\mathbf{X}}_j(m)$ is a circular matrix

$$\check{\mathbf{X}}_j(m) = \begin{bmatrix} x_j(mN - L' + 1) & x_j(mN + N - 1) & \cdots & x_j(mN - L' + 2) \\ x_j(mN - L' + 2) & x_j(mN - L' + 1) & \cdots & x_j(mN - L' + 3) \\ \vdots & \vdots & \ddots & \vdots \\ x_j(mN - 1) & x_j(mN - 2) & \cdots & x_j(mN) \\ x_j(mN) & x_j(mN - 1) & \cdots & x_j(mN + 1) \\ x_j(mN + 1) & x_j(mN) & \cdots & x_j(mN + 2) \\ \vdots & \vdots & \ddots & \vdots \\ x_j(mN + N - 1) & x_j(mN + N - 2) & \cdots & x_j(mN - L' + 1) \end{bmatrix}, \quad (3.37)$$

and

$$\begin{aligned} \mathbf{d}_i &= \mathbf{F}_{L'} \mathbf{d}_i, \\ \mathbf{d}_i &= [d_i(0) \ \cdots \ d_i(L' - 1)]^T, \\ \tilde{\mathbf{d}}_i &= \mathcal{W}_{N_3 \times L'}^{10} \mathbf{d}_i, \\ \mathcal{D}_{\check{\mathbf{X}}_i}(m) &= \mathbf{F}_{N_3} \check{\mathbf{X}}_i(m) \mathbf{F}_{N_3}^{-1}, \end{aligned} \quad (3.38)$$

respectively.

Lemma 3.8. *The vector $\tilde{\mathbf{d}}_i$ in (3.38) can be expressed as*

$$\tilde{\mathbf{d}}_i = \sum_{n=1}^M \mathcal{B}_{ni} \tilde{\mathbf{w}}_n,$$

where

$$\begin{aligned}\mathbf{B}_{ni} &= \text{diag}\{\tilde{\mathbf{b}}_{ni}\}, \\ \tilde{\mathbf{b}}_{nj} &= \mathbf{F}_{N_3} \mathbf{W}_{N_3 \times L}^{10} \mathbf{b}_{nj}, \\ \tilde{\mathbf{w}}_n &= \mathbf{F}_{N_3} \mathbf{W}_{N_3 \times L_w}^{10} \mathbf{w}_n.\end{aligned}$$

Define the error as same in (3.33) and cost function in (3.35), we have

Lemma 3.9.

$$\begin{aligned}\nabla C_k(m) &= \frac{\partial C(m)}{\partial \mathbf{w}_k^*} = -\mathbf{W}_{L_w \times N_3}^{10} \sum_{i=1}^M \mathbf{B}_{ki}^H \mathcal{D}_{\check{X}_i}^H(m) \mathbf{W}_{N_3 \times N}^{10} \mathbf{e}(m), \\ E\{\nabla^2 C_k(m)\} &= E\left\{\frac{\partial \nabla C_k(m)}{\partial \mathbf{w}_k^T}\right\} \approx \frac{N''}{N} \mathbf{W}_{L_w \times N_3}^{10} \hat{\mathcal{R}}_k(m) \mathbf{W}_{N_3 \times L_w}^{01},\end{aligned}$$

where

$$\hat{\mathcal{R}}_k(m) \approx \alpha \hat{\mathcal{R}}_k(m-1) + (1-\alpha) \left(\sum_{i=1}^M \mathbf{B}_{ki}^H \mathcal{D}_{\check{X}_i}^H(m) \right)^H \left(\sum_{i=1}^M \mathbf{B}_{ki}^H \mathcal{D}_{\check{X}_i}^H(m) \right),$$

and α is a forgetting factor.

Proof. Similar to Lemma 3.5. □

The LMS-type algorithm can be derived as

Theorem 3.2. *The constrained Newton-LMS type algorithm is*

$$\hat{\mathbf{w}}_k^{10}(m) = \hat{\mathbf{w}}_k^{10}(m-1) + \rho \mathbf{W}_{N'' \times N''}^{10} \hat{\mathcal{R}}_k(m)^{-1} \left(\sum_{i=1}^M \mathbf{B}_{ki}^H \mathcal{D}_{\check{X}_i}^H(m) \right) \mathbf{W}_{N'' \times N}^{01} \mathbf{e}(m),$$

and the unconstrained Newton-LMS algorithm is

$$\hat{\mathbf{w}}_k^{10}(m) = \hat{\mathbf{w}}_k^{10}(m-1) + \rho \hat{\mathcal{R}}_k^{-1}(m) \left(\sum_{i=1}^M \mathcal{B}_{ki}^H \mathcal{D}_{\check{X}_i}^H(m) \right) \mathbf{w}_{N'' \times N}^{01} \mathbf{e}(m),$$

where ρ is the stepsize.

Proof. Proof is similar to Theorem 3.1. □

The extended GSC implementation in Theorem 3.2 does not need twice FFT /IFFT operations as the implementation in Theorem 3.1. Since $\hat{\mathcal{R}}_k(m)$, $\mathcal{B}_{ki}^H$ and $\mathcal{D}_{\check{X}_i}^H(m)$ are all diagonal matrices. Therefore, the matrix inverse, multiplication can be simplified as element operations.

To further reveal the frequency implementation of the extended GSC, without loss of generality, we assume that the FFT length N_3 is far greater than the filter length L , L_w and $L + L_w - 1$. In such case, $\mathbf{w}_{N'' \times N}^{01}$ can be approximated as an identity matrix. Therefore, the adaptive filter updating in (3.2) can be approximated as independent updating at each frequency bin. This means that the extended GSC can be implemented in simple manner when the FFT length is far greater than the filter length. This simple implementation shown in Fig. 3.3 can be described as:

Step 1 Arrange the input data in blocks.

Step 2 Apply FFT of the input data block of each channel.

Step 3 Apply narrow band beamforming at each frequency bin independently.

Figure 3.3: The schematic of simplified extended GSC in frequency domain.

Step 4 Apply IFFT of the output signals of narrow band beamformers and construct the output signal using overlap-save method [50].

3.6 Summary

An extended GSC adopts a general model of channel relating the signal source and each sensor is proposed. Arbitrary channel transfer function can be used in the extended GSC while in conventional GSC, pure delay channel is assumed. Implementation of

the extended GSC in time and frequency domain is discussed. The frequency domain implementation saves the computational load when the filter length is long. The discussion on the implementation in frequency domain also reveals that the GSC can be implemented by transforming data into frequency domain and applying beamforming at each frequency bin independently, after that, transforming the signal back to time domain. Although this is not a new discovery, our derivation provides another way to understand the applicability of such simplified approach. Since the conventional GSC is a special case of the proposed one, the methods derived in this chapter can also be applied in conventional GSCs.

Chapter 4

Adaptive Presteering Filter for Robust Beamforming

4.1 Introduction

As discussed in Chapter 2, the Frost beamformer [10,38,18,51] can be used to enhance wideband signal, such as speech signal impinging on a microphone array, against interference and noise [52,53]. This beamformer consists of a presteering frontend that is composed of variable time delayers, with which the main lobe of its beam pattern can be steered to the look direction. The back end is a signal processor consisting of multichannel tapped delay lines whose weights are iteratively adjusted to minimize the output power subject to some linear constraints, which are used to maintain a certain frequency response in the look direction.

Using the conventional Frost beamformer, target signal cancellation occurs when

there are array imperfections, including steering direction errors, array geometry errors, etc. The target signal cancellation phenomena is more severe when the signal-to-noise ratio (SNR) is high [5, 12]. Many methods, including multiple constraints [16, 54, 55], derivative constraints [17, 19], improved spatial filter [36, 56] and soft quadratic response constraints [57, 12], have been proposed to solve this problem. By applying these techniques in adaptive beamformer, target signal cancellation phenomena due to steering direction error is solved to some extent. However, these beamformers suffer from loss in DOF for interference and noise suppression. For array with a small number of sensors, the loss in DOF is unbearable because it reduces the performance in interference and noise suppression. Furthermore, some of these approaches have the common assumption that the error is a pure steering direction error so that the steering vector can be modelled as a vector function of steering direction error. In real applications, there are multiple imperfections, such as steering direction error, array geometry error, quantization errors in the presteering delays. All of these imperfections usually exist simultaneously. In our knowledge, few methods have been proposed to ensure processor's robustness in face of multiple imperfections mentioned above.

Another type of robust beamformer proposed in [32] employs the beamformer output power as an objective function to correct steering direction error. The estimate of steering direction error is obtained by maximizing the output power of Capon beamformer [9]. This method does not suffer from loss in DOF for interference rejection. By extending the idea in [32] for wideband signal beamforming in time domain, a new method is proposed in this chapter to overcome multiple array imperfections mentioned above.

In this chapter, the array imperfection including steering direction error, array geometry error and quantization error in presteering delayers are all modelled as time-delay errors for array channels. An interpolation FIR filter [50] is introduced to replace the presteering delays. The coefficients of interpolation FIR filter is obtained by maximizing the output power of the LCMV beamformer. An efficient steepest descent method is derived to iteratively update the coefficients of the interpolation filters. Computer simulation results show that the proposed method is convergent when the time-delay errors are small.

This chapter is organized as follows. The basic problem is formulated in Section 4.2, the time-delay error is defined. In Section 4.3, the proposed robust beamforming method is derived. Two implementations of the robust beamformer are discussed in Section 4.4. Simulation results are shown in Section 4.5 to demonstrate the performance of the proposed method. Followed by a brief summary in Section 4.6.

4.2 Problem Formulation

Consider a presteered wideband beamformer with M sensors and J taps per sensor as shown in Fig. 2.3. As discussed in Section 2.3.2, the continuous time signal received by the m th sensor is designated by $x_m(t)$, $m = 1, 2, \dots, M$, which can be written as

$$x_m(t) = s(t - \tau_m) + n_m(t), \quad m = 1, 2, \dots, M, \quad (4.1)$$

where $s(t)$ is the target signal, τ_m denotes the propagation delay difference of the target signal at the output of the m th sensor, and $n_m(t)$ represents the totality of interference and noise observed at the output of the m th sensor. The frontend of a presteered linearly constrained adaptive beamformer is either digital or analog time delay placed immediately after each sensor, and its function is to steer the array response to the direction of interest. These steering time delays aim to align the target signal component $s(t - \tau_m)$ at the output of each sensor exactly in phase. So it is required that the time delay added to the output of each sensor is able to completely compensate the actual delay difference τ_m . Those additive time delays are usually calculated based on the knowledge of the DOA of the target signal and the array geometry. However, due to practical imperfections such as the steering direction error and array geometry errors, etc., the real delay differences τ_m are impossible to be perfectly compensated. The misalignment of the target signal component at the output of each sensor will cause the desired signal to be regarded as interference and then cancelled by the adaptive beamformer. After the steering delay compensation, the signal of each sensor output becomes

$$x_m(t) = s(t - \Delta_m) + n_k(t + \tau_m - \Delta_m), \quad m = 1, 2, \dots, M, \quad (4.2)$$

where Δ_m represents the residual delay difference of the target signal. It is clear that the target signal components are perfectly aligned only when Δ_m , $m = 1, 2, \dots, M$, are all equal. The digitalized signal of the output at the m th sensor is

$$x_m(n) = s(nT_s - \Delta_m) + n_m(nT_s + \tau_m - \Delta_m), \quad (4.3)$$

where T_s represents the sampling period. Similar to the signal defined in (2.35), the vector of the array signal at time n for the adaptive beamformer is defined as

$$\mathbf{x}(n) = [x_1(n) \cdots x_M(n) \ x_1(n-1) \cdots x_M(n-1) \cdots x_1(n-J+1) \cdots x_M(n-J+1)]^T. \quad (4.4)$$

Following the time delays in Fig. 2.3 is a multichannel adaptive beamformer whose weights are iteratively adjusted to minimize the output while maintaining a certain frequency response in the look direction. The output signal $y(n)$ of beamformer is

$$y(n) = \mathbf{w}^T \mathbf{x}(n), \quad (4.5)$$

where $\mathbf{w}$ is defined in (2.32). The discussion of the output power of the LCMV beamformer as well as its optimal solution can be refer to Section 2.3.2 and 2.5.2. In this chapter, we denote the covariance matrix of array signal as $\mathbf{R}_x$ instead of $\mathbf{R}$.

$$\mathbf{R}_x = E\{\mathbf{x}(n)\mathbf{x}(n)^T\}. \quad (4.6)$$

Moreover, the optimal output power $\hat{p}$ of the LCMV beamformer is also shown in this section for easy referencing.

$$\hat{p} = \mathbf{f}^T (\mathbf{C}_0^T \mathbf{R}_x^{-1} \mathbf{C}_0)^{-1} \mathbf{f}. \quad (4.7)$$

As we have discussed, when there are time delay errors, i.e., Δ_m not all equal, the

constraint matrix $\mathbf{C}_0$ does not match the true array scenario and can cause serious target signal cancellation. In the next section, the proposed algorithm aims to use a slightly adjustable constraint matrix $\mathbf{C}$ instead of the fixed $\mathbf{C}_0$ so that target signal cancellation does not occur in the presence of small time delay errors.

4.3 Proposed Algorithm for Robust Beamformer

Assume that $\mathbf{x}'(n)$ is a signal vector received by the sensor array when there exist time delay errors. The desired signal components in $\mathbf{x}'(n)$ are not perfectly aligned in phase, i.e., Δ_m are not all equal in $\mathbf{x}'(n)$. Assume also that $\mathbf{x}(n)$ is the signal vector related to $\mathbf{x}'(n)$ by an accurate time-shift operation that is used to correct the misalignment in Δ_m , i.e., $\mathbf{x}(n)$ is the corresponding signal vector of $\mathbf{x}'(n)$ except that the desired signal components in $\mathbf{x}'(n)$ are ideally aligned. Then, the optimal weight vector $\hat{\mathbf{w}}$ in (2.47) with $\mathbf{C}_0$ given by (2.45) and (2.46) corresponds only to $\mathbf{x}(n)$ but not to $\mathbf{x}'(n)$. The time-shift operation between $\mathbf{x}(n)$ and $\mathbf{x}'(n)$ can be approximately realized through a linear transformation matrix $\mathbf{H}$,

$$\mathbf{x}'(n) = \mathbf{H}\mathbf{x}(n). \quad (4.8)$$

Here, assuming for the moment that $\Delta_m, m = 1, 2, \dots, M$ are known, the matrix $\mathbf{H}$ of size $MJ \times MJ$ is defined as

$$\mathbf{H} = [\mathbf{h}_{1,1}^T \cdots \mathbf{h}_{M,1}^T \mathbf{h}_{1,2}^T \cdots \mathbf{h}_{M,2}^T \cdots \mathbf{h}_{1,J}^T \cdots \mathbf{h}_{M,J}^T]^T, \quad (4.9)$$

where $\mathbf{h}_{m,j}$ is the $((j-1)M + m)$ th row of $\mathbf{H}$. Each $\mathbf{h}_{m,j}$ is a $1 \times MJ$ row vector consisting of subrow vectors $\mathbf{g}_1, \dots, \mathbf{g}_J$ as

$$\mathbf{h}_{m,j} = [\mathbf{g}_1 \ \cdots \ \mathbf{g}_i \ \cdots \ \mathbf{g}_J], \quad (4.10)$$

where $\mathbf{g}_i$ is a $1 \times M$ row vector given by

$$\mathbf{g}_i = [0 \ \cdots \ 0 \ h_m(i-j) \ 0 \ \cdots \ 0], \quad (4.11)$$

in which the only possibly nonzero element of $\mathbf{g}_i$ is at its m th position and denoted by $h_m(i-j)$. Here, $h_m(n)$ are the interpolation filter coefficients that shift a signal along the time axis by Δ_m , i.e.,

$$h_m(n) = \frac{\sin(\pi(n - \Delta_m/T_s))}{\pi(n - \Delta_m/T_s)}, \quad n = \dots, -1, 0, 1, \dots \quad (4.12)$$

Since $h_m(n)$ has infinite length, to compensate the time delay error Δ_m completely would require J to be infinite. Nevertheless, (4.9) is still a good approximation in practice if J is large enough, because the magnitude of $h_m(n)$ diminishes as fast as $1/|n|$.

Minimizing the beamformer output power for $\mathbf{x}'(n)$ gives

$$\min_{\mathbf{w}'} p' = \min_{\mathbf{w}'} \mathbf{w}'^T \mathbf{R}_{x'} \mathbf{w}', \quad (4.13)$$

where $\mathbf{w}'$ and p' represent the weight vector and the output power for $\mathbf{x}'(n)$, respec-

tively. Using (4.6) and (4.8), (4.13) becomes

$$\min_{\mathbf{w}'} p' = \min_{\mathbf{w}'} \mathbf{w}'^T \mathbf{H} \mathbf{R}_x \mathbf{H}^T \mathbf{w}'. \quad (4.14)$$

Comparing (4.14) with (2.44), the correct constraint matrix for $\mathbf{w}'$ is no longer $\mathbf{C}_0$; instead it is given by

$$\mathbf{C} = \mathbf{H} \mathbf{C}_0. \quad (4.15)$$

Therefore, the new optimization problem is

$$\begin{cases} \min_{\mathbf{w}'} \mathbf{w}'^T \mathbf{R}_{x'} \mathbf{w}' \\ \text{s.t. } \mathbf{C}^T \mathbf{w}' = \mathbf{f} \end{cases}. \quad (4.16)$$

The optimal weight vector $\hat{\mathbf{w}}'$ to the problem of (4.16) is given by

$$\hat{\mathbf{w}}' = \mathbf{R}_{x'}^{-1} \mathbf{C} (\mathbf{C}^T \mathbf{R}_{x'}^{-1} \mathbf{C})^{-1} \mathbf{f}, \quad (4.17)$$

the beamformer output power $\hat{p}'$ is given by

$$\hat{p}' = \mathbf{f}^T (\mathbf{C}^T \mathbf{R}_{x'}^{-1} \mathbf{C})^{-1} \mathbf{f}. \quad (4.18)$$

In practice, however, since Δ_m is unknown, $\mathbf{H}$ cannot be directly computed using (4.9) -(4.12). It follows that $\mathbf{C}$ cannot be directly solved using (4.15). To overcome this difficulty, we assume that Δ_m is small so that the norm of $\mathbf{C} - \mathbf{C}_0$, denoted by $\|\mathbf{C} - \mathbf{C}_0\|_2$, is also small, implying that $\mathbf{C}$ lies in the neighborhood of $\mathbf{C}_0$. Furthermore,

when the selected $\mathbf{C}$ satisfies the real scenario of array signals with time delay errors, the beamformer output retains the desired signal with minimal distortion. Otherwise, the desired signal is cancelled more or less as interference. So, it is clear that the optimal output power p' should have a local maximum with respect to different selections of $\mathbf{C}$. Therefore, a method to find the desired $\mathbf{C}$ is to perform a local search in the vicinity of $\mathbf{C}_0$ to maximize the optimal beamformer output $\hat{p}'$. For example, if the target signal cancellation is caused by steering direction error, the search for $\mathbf{C}$ is analogous to a small but important readjustment to the steering direction of the presteered beamformer so that this steering direction lies in the true DOA of the target signal [32]. Thus, the criterion for this local search algorithm is given by

$$\begin{cases} \max_{\mathbf{C}} \min_{\mathbf{w}'} \mathbf{w}'^T \mathbf{R}_{x'} \mathbf{w}' \\ \text{s. t. } \mathbf{C}^T \mathbf{w}' = \mathbf{f} \\ \|\mathbf{C} - \mathbf{C}_0\|_2^2 \leq \delta \end{cases}, \quad (4.19)$$

it can be simplified as

$$\begin{cases} \max_{\mathbf{C}} \hat{p}' \\ \text{s.t. } \|\mathbf{C} - \mathbf{C}_0\|_2^2 \leq \delta \end{cases}, \quad (4.20)$$

where δ is a small positive real number used to control the size of the feasible region around $\mathbf{C}_0$. The value of δ is experimentally set based on the norm of time delay errors.

As $\hat{p}' = \mathbf{f}^T (\mathbf{C}^T \mathbf{R}_{x'}^{-1} \mathbf{C})^{-1} \mathbf{f}$ in view of (4.7), (4.20) is equivalent to

$$\begin{cases} \min_{\mathbf{C}} \frac{1}{\mathbf{f}^T (\mathbf{C}^T \mathbf{R}_{x'}^{-1} \mathbf{C})^{-1} \mathbf{f}} \\ \text{s.t. } \|\mathbf{C} - \mathbf{C}_0\|_2^2 \leq \delta \end{cases}. \quad (4.21)$$

We choose to minimize the reciprocal of the beamformer output power instead of maximizing the output power itself, because the former method is numerically more stable than the latter when using the gradient method. Since $\mathbf{C} = \mathbf{H}\mathbf{C}_0$ and $\mathbf{H}$ is a function matrix of Δ_m , (4.21) can be rewritten as

$$\begin{cases} \min_{\Delta_m} \frac{1}{\hat{p}'} = \min_{\Delta_m} \frac{1}{\mathbf{f}^T (\mathbf{C}_0^T \mathbf{H}^T \mathbf{R}_{x'}^{-1} \mathbf{H} \mathbf{C}_0)^{-1} \mathbf{f}}, \\ \text{s. t. } \|\mathbf{H}\mathbf{C}_0 - \mathbf{C}_0\|_2^2 \leq \delta \end{cases} \quad (4.22)$$

The problem of (4.22) is a multidimensional nonlinear optimization problem. In this chapter, the gradient method is used to find the optimal Δ_m . Let $\mathbf{Q} = \mathbf{C}_0^T \mathbf{H}^T \mathbf{R}_{x'}^{-1} \mathbf{H} \mathbf{C}_0$, the partial derivative of $1/\hat{p}'$ in (4.22) with respect to Δ_m is given by

$$\frac{\partial}{\partial \Delta_m} \left(\frac{1}{\hat{p}'} \right) = -\frac{1}{\hat{p}'^2} \frac{\partial \hat{p}'}{\partial \Delta_m}, \quad (4.23)$$

where

$$\frac{\partial \hat{p}'}{\partial \Delta_m} = -\mathbf{f}^T \mathbf{Q}^{-1} \frac{\partial \mathbf{Q}}{\partial \Delta_m} \mathbf{Q}^{-1} \mathbf{f}, \quad (4.24)$$

and

$$\frac{\partial \mathbf{Q}}{\partial \Delta_m} = \mathbf{C}_0^T \frac{\partial \mathbf{H}^T}{\partial \Delta_m} \mathbf{R}_{x'}^{-1} \mathbf{H} \mathbf{C}_0 + \mathbf{C}_0^T \mathbf{H}^T \mathbf{R}_{x'}^{-1} \frac{\partial \mathbf{H}}{\partial \Delta_m} \mathbf{C}_0. \quad (4.25)$$

The term $\frac{\partial \mathbf{H}}{\partial \Delta_m}$ can be calculated using (4.9)-(4.12), and its computation is essentially that of $\frac{\partial h_m(n)}{\partial \Delta_m}$, which is given by

$$\frac{\partial h_m(n)}{\partial \Delta_m} = \begin{cases} 0, & n - \Delta_m/T_s = 0, \\ -\frac{\cos[\pi(n - \Delta_m/T_s)]}{T_s(n - \Delta_m/T_s)} + \frac{\sin[\pi(n - \Delta_m/T_s)]}{T_s \pi(n - \Delta_m/T_s)^2}, & n - \Delta_m/T_s \neq 0. \end{cases} \quad (4.26)$$

As the time delay differences Δ_m are all relative quantities compared to each other, it is required to fix one of them, say $\Delta_1 = 0$, and adjust other to find the local maximum of $\hat{p}'$. The use of the gradient search method requires that the iterative process is convergent. Extensive simulations show that the proposed optimization process always converges when steering vector errors are small, although a theoretical proof is not yet available.

The proposed algorithm is summarized as follows.

Step 1: When new signal data $\mathbf{x}'(n)$ is received, the correlation matrix $\mathbf{R}_{x'}$ is updated by

$$\mathbf{R}_{x'}^{[n]} = \alpha \mathbf{R}_{x'}^{[n-1]} + (1 - \alpha) \mathbf{x}'(n) \mathbf{x}'^T(n), \quad (4.27)$$

where $(\cdot)^{[n]}$ denotes the n th iteration. The scale α is the update coefficient and close to one.

Step 2: The partial derivatives of $1/\hat{p}'$ with respect to Δ_m is calculated using (4.23)-(4.26), and then $\Delta_m, m = 2, \dots, M$, are updated as

$$\Delta_m^{[n]} = \Delta_m^{[n-1]} - \mu \frac{\partial}{\partial \Delta_m} \left(\frac{1}{\hat{p}'} \right), \quad n = 1, 2, \dots, \quad (4.28)$$

where μ is a small positive number used to control the convergence speed. $\Delta_1^{[n]}$ is fixed as zero. The initial values $\Delta_m^{[0]}$ are set to be zeros.

Step 3: The constraint matrix is updated by

$$\mathbf{C}^{[n]} = \mathbf{H}^{[n]} \mathbf{C}_0, \quad (4.29)$$

where $\mathbf{H}^{[n]}$ is computed according to (4.9)-(4.12) with the new $\Delta_m^{[n]}$.

Step 4: Check whether $\|\mathbf{C}^{[n]} - \mathbf{C}_0\|_2^2 \leq \delta$. If this condition is violated, it is known that the incoming signals are all interferences and noises and, hence, $\mathbf{C}^{[n]}$ needs to be reset to $\mathbf{C}_0$. Otherwise, Steps 2-4 are repeated until $\mathbf{C}^{[n]}$ converges to the desired constraint matrix $\hat{\mathbf{C}}$.

Step 5: The traditional iterative algorithm is employed to find the optimal $\hat{\mathbf{w}}'$ based on $\hat{\mathbf{C}}$ as shown in (4.17).

4.4 Implementation of Proposed Robust Beamformer

In the discussion above, the estimate of optimal time-delay errors is derived. The error compensated constraint matrix is then used in robust adaptive beamformer. In some applications, if the system imperfections do not change with time, we only need to estimate those time delay errors when the system begins to run. Then, the estimated values are used to compensate the array processor. It can be considered as an on-line calibration.

In this section, we suggest two implementations of the proposed beamformer. The first one is the same as the derivation in above section. The constraint matrix $\mathbf{C}$ is reconstructed according to the estimated delays. The new constructed constraint matrix $\mathbf{C}$ is used to substitute the nominal constraint matrix $\mathbf{C}_0$. The other processing procedures for LCMV beamformer are unchanged. The advantage of such implementation is that there is no extra computational load for the time delay error compensation. How-

Figure 4.1: New array processor with presteering filter.

ever, when there are more complicated constraints used in LCMV, such as derivative constraint, it is difficult to reconstruct these constraints according to the estimated time delay errors.

The other implementation is shown in Fig. 4.1. Before the adaptive array processor, each channel signal is processed by an interpolation FIR filter. The coefficients of this FIR filter can be calculated by (4.12) using the estimated optimal time delay error. This method has the advantage that it does not affect the following array processor. All the algorithms proposed for Frost beamformer can work without any change. The disadvantage is that such presteering filters need extra computational load.

4.5 Numerical Experiments

In this section, the proposed algorithm is evaluated by computer simulations. The conventional Frost beamformer and fixed beamformer are also implemented for the purpose of performance comparison.

A four-element uniform linear microphone array is used in this simulation. The spacing between microphones is 0.04m, and the sampling rate is 8kHz. Bandlimited Gaussian signals (0.3–3.4)kHz are used as source signals, and the assumed direction of the target signal is 0° . The number of taps for each sensor is $J = 16$. The look direction filter $\mathbf{f}$ in (4.16) is designed to be a lowpass filter with passband (0.0–3.0)kHz. In the simulations, the step size μ is 10^{-8} , and the allowed maximal iteration times is 200.

The first set of simulations shows the capability of the proposed algorithm in widening the acceptance angle when there has steering direction error. In these simulations, noise with -30dB power are added. The spatial power responses of the proposed beamformer, the conventional Frost beamformer, and the fixed beamformer are plotted in Fig. 4.2. It is shown that the acceptance angle of the conventional Frost algorithm is quite narrow, which is significantly widened by the proposed algorithm. Note that the parameter in (4.21) is set to control the range of acceptance angle. Here, $\delta = 5.8$ is used so that the acceptance angles are in the range of $[-10^\circ, 10^\circ]$. The fixed beamformer has a wider main lobe, but its sidelobe level is much higher. In Fig. 4.3 and 4.4, different δ values are used to show that the widened acceptance angle can be adjusted by using different δ , which are determined by experiment.

Figure 4.2: Power response of adaptive beamformer with -30dB noise ($\delta = 5.8$).

Figure 4.3: Power response of adaptive beamformer with -30dB noise ($\delta = 2.8$).

Figure 4.4: Power response of adaptive beamformer with -30dB noise ($\delta = 0.7$).

To show the effect of noise on the widened acceptance angle, in the next set of simulations, noise with -20dB power are added. From the results shown in Fig. 4.5-4.7, the proposed method significantly improve the robustness in adaptive beamformer. Compared with results shown in Fig. 4.2-4.4, with the stronger noise, the robustness of the Frost beamformer is improved, i.e., the power response of the Frost beamformer is not as sharp as the one shown in Fig. 4.2-4.4. This verifies the conclusion that the target signal cancellation is more serious when the input SINR is high.

The next set of simulations demonstrates the performance of the proposed beamformer with array geometry error. The actual positions of array sensors slightly differ from the nominal ones. Assume that the four sensors are placed randomly away from their presumed locations, i.e., $\mathbf{r}(n) = (k-1)d\mathbf{a}_x + \mathbf{r}_n(k)$, $k = 1, \dots, 4$, $d = 0.04m$, where

Figure 4.5: Power response of adaptive beamformer with -20dB noise ($\delta = 5.8$).

Figure 4.6: Power response of adaptive beamformer with -20dB noise ($\delta = 2.8$).

Figure 4.7: Power response of adaptive beamformer with -20dB noise ($\delta = 0.7$).

$\mathbf{a}_x$ is the unit vector along the x axis, and $\mathbf{r}_n(k)$ is the positional error vector of the k th sensor. In this simulation, $\mathbf{r}_n(k)$ are generated as two-dimensional random Gaussian noises with variance $0.04d^2$. Fig. 4.8 shows the output power of the proposed algorithm in this case. The assumed sensor locations are at $(0.00m, 0.00m)$, $(0.04m, 0.00m)$, $(0.08m, 0.00m)$, $(0.12m, 0.00m)$. The actual sensor position are $(0.0045m, -0.0004m)$, $(0.0367m, -0.0103m)$, $(0.0827m, 0.0087m)$, $(0.1235m, -0.0058m)$. It is shown that the proposed algorithm is able to avoid target signal cancellation compared with the conventional Frost algorithm. The proposed algorithm still maintains a certain range of acceptance angle around 0° , although it is narrower than that in Fig. 4.2. This is because we still use $\delta = 5.8$, which must now tolerate both steering direction error and array geometry error. We could also increase δ to further widen the acceptance

angle if required. With different choices of δ , the power responses of the proposed method are shown in Fig. 4.8-4.10. The results indicate that the widened acceptance of the proposed method can be adjusted by using different value of δ . Generally, when the array with arbitrary geometry error, large value of δ is required to maintain wide acceptance angle.

The next experiment is to show the performance of the proposed method with the existence of directional interference. In Fig. 4.11 and 4.12, the simulations are carried out with $-30dB$ background noise and $2dB$ interference at 70° . The acceptance of angle of the proposed algorithm is significantly widened even with the existence of interference. The simulation results also shows that fixed beamformer outputs signal with higher power than the proposed method does because the output signal contains the interference signal.

The last set of experiments shows the array response in 3-D, i.e., gain versus angle and frequency. Assume that the array has 5° steering direction error. With the optimal weight estimated by the proposed method, the array response is calculated by varying direction and frequency. As shown in Fig. 4.13, the beamformer does not have signal cancellation problem. The peak response is adjusted to the true DOA of target signal automatically. We also carried out experiments to calculate the frequency response of the array by varying the DOA of target signal. The allowed DOA error is assumed to be 10° . From Fig. 4.14, we find that the proposed array has almost flat response in the range $[-10^\circ, 10^\circ]$. By adding an interference at -70° , similar flat response is obtained and shown in Fig. 4.15. Compared with the result in Fig. 4.14, we find that

Figure 4.8: Power response of adaptive beamformer in the presence of array positional error ($\delta = 5.8$).

Figure 4.9: Power response of adaptive beamformer in the presence of array positional error ($\delta = 8.8$).

Figure 4.10: Power response of adaptive beamformer in the presence of array positional error ($\delta = 10.0$).

Figure 4.11: Power response of adaptive beamformer with -30dB noise and 3dB interference ($\delta = 5.8$).

Figure 4.12: Power response of adaptive beamformer with -30dB noise and 3dB interference ($\delta = 2.8$).

the array response to negative angle is lower than that to positive angle. From these results, we also find that the array has flat response to the low frequency signal and sharp response to the high frequency signal. This coincides with the array processing theory [6].

4.6 Summary

A new method has been proposed to improve the robustness of LCMV beamformer against general array imperfections such as steering direction error and array geometry error, etc. The novelty of this method is modelling of array imperfection as time delay error and replacing the constraint matrix by time shifted one of conventional

Figure 4.13: The array response versus angle and frequency (Four-sensor array, steering error 5°).

Figure 4.14: The frequency response of the array versus angle (Four-sensor array).

Figure 4.15: The frequency response of the array versus angle (Four-sensor array with a $0dB$ interference at -70°).

constraint matrix. The estimation of the new constraint matrix based on output power maximization is also proposed. As this model does not require any knowledge of array manifold functions, it is robust against various types of errors that cause the conventional Frost algorithm to fail. Two types of implementations of the proposed method are proposed. Simulation results have shown that the sensitivity of the adaptive beamformer to steering vector errors can be significantly lowered by the proposed algorithm.

Chapter 5

Robust Beamformer Against Generalized Phase Errors

5.1 Introduction

In Chapter 4, a robust wideband beamformer in time domain is proposed. The time-delay error can be efficiently compensated in adaptive beamforming. If the adaptive beamformer is implemented in frequency domain, it is possible to make the adaptive beamformer robust to more kinds of imperfections.

As discussed in Chapter 2, the Capon beamformer has high performance in interference suppression if the ASV corresponding to the source of interest (SOI) is known accurately. However, some of the underlying assumptions on environment, source and sensor array can be violated when adaptive arrays are used in practical applications. This may give rise to mismatch between nominal and actual ASVs, which results in tar-

get signal cancellation in adaptive beamformer. Target signal cancellation problem can be caused by common array imperfections, including steering direction error [58, 59], imperfect array calibration error [12], near-far field problem [60], multipath and reverberation effects of environment [31], etc. In this chapter, we focus on improving the robustness of adaptive beamformer to steering direction error, array geometry error and array sensor phase errors. In narrowband array processing, all of these errors can be modelled as GPEs. Since adaptive beamformer is very sensitive to phase error, designing an array processor robust to phase errors has become an important research topic.

Many robust methods [11, 61, 17, 20, 62, 34] have been proposed to avoid performance degradation due to array imperfections. To combat steering direction error, the multiple-point constraints [11, 61], the derivative constraints [5, 17, 20, 23], the weight norm constraint [5, 62, 34] and the artificial noise injection technique [42, 12] were proposed. Most of these methods introduce additional constraints which consume the DOF of array processor. Consequently, they suffer from performance loss in interference and noise suppression. Moreover, some methods are only robust to steering direction error, and some constraints are derived with the assumption of ideal array. In practice, there are multiple array imperfections including array geometry error, steering direction error, sensor phase error. It is important to make the beamformer robust to all these imperfections.

In [32], a new approach was proposed for robust beamforming in the presence of steer direction error. It iteratively searches for the optimal ASV by maximizing the

mean output power of Capon beamformer using first-order Taylor series approximation in terms of steering direction error. This method does not suffer from performance loss in interference/noise suppression. However, its performance degrades when there exist multiple errors, such as steering direction error, array geometry error and array sensor phase error, because the ASV in [32] is assumed to be a vector function of steering direction. When multiple imperfections exist, the assumed model of ASV is violated.

In this chapter, a new model of the ASV is adopted. All of these array imperfections are modelled as GPEs. The GPEs are broad-sense errors which simplify the mathematical expression of the ASV in terms of array imperfections. Therefore, the ASV can be described as a vector function of GPEs. The output power of Capon beamformer can also be expressed as a function of GPEs. A new approach is derived based on this model to search for the optimal estimates of GPEs by maximizing the output power of Capon beamformer. The estimated GPEs can be used to compensate the errors in the actual ASV. Since there is no additional constraints introduced in adaptive beamforming, this new method does not suffer from performance loss in interference/noise suppression.

This chapter is organized as follows. In Section 5.2, the GPE is defined and the problem is formulated. An approximated LS solution is given in Section 5.3. This solution only works when the GPEs are small. In practice, we use the gradient based method which is proposed in Section 5.4. Some numerical simulation results are shown in Section 5.5. Followed by a brief summary in Section 5.6.

5.2 Generalized Phase Error Model and Associated Robust Beamformer

As discussed in Chapter 2, the optimal output power $\hat{p}$ in (2.43) of the Capon beamformer shown in Fig. 2.2 is given by

$$\hat{p} = \frac{1}{\mathbf{s}^H(\theta_0, \phi_0) \mathbf{R}^{-1} \mathbf{s}(\theta_0, \phi_0)}. \quad (5.1)$$

In this chapter, we discuss one-dimensional linear array, therefore, the elevation angel ϕ_0 in the array steering vector is omitted in the following context. Therefore, the output power is expressed as

$$\hat{p} = \frac{1}{\mathbf{s}^H(\theta_0) \mathbf{R}^{-1} \mathbf{s}(\theta_0)}. \quad (5.2)$$

The output power in (5.2) has an ambiguity problem when we select the nominal ASV $\mathbf{s}(\theta_0)$ with different norm. For example, if we use another nominal ASV $\mathbf{s}(\theta_0)' = \alpha \mathbf{s}(\theta_0)$ in (5.2), where α is a nonzero scale. Although the output SINR of the Capon beamformer does not change, the optimal output power $\hat{p}'$ is changed to

$$\hat{p}' = \frac{\hat{p}}{|\alpha|^2}. \quad (5.3)$$

The scale $|\alpha|^2$ in (5.3) affects the output power maximization with respect to nominal ASV. In some beamforming methods, the robustness of beamformer is improved by

replacing the nominal ASV with a corrected one [32]. When the corrected nominal ASV has different norm compared to the original one, the output power maximization method may lead to a wrong solution. In this chapter, we will show that the proposed method avoids such problem because the corrected ASV has constant norm whatever the GPEs are.

If the nominal ASV is accurately known, the Capon beamformer can output desired signal with high SINR. Nevertheless, in practical applications, the true ASV is unknown or known but with some errors. In such case, target signal cancellation occurs, and the output power in (5.2) decreases. In [32], a method is proposed to solve target signal cancellation problem by maximizing the output power in (5.2) with respect to steering direction error. An optimal steering direction is estimated to construct the nominal ASV for robust beamforming. This approach works well with the only existence of steering direction error. In the following section, a new method is proposed for robust beamforming in the case of multiple imperfections.

As discussed in Chapter 2, for an ideal sensor array with known geometry, the ASV $\mathbf{s}(\theta)$ is defined as an $M \times 1$ complex vector

$$\mathbf{s}(\theta) = [e^{j\omega_0\tau_1(\theta)} \ \dots \ e^{j\omega_0\tau_m(\theta)} \ \dots \ e^{j\omega_0\tau_M(\theta)}]^T. \quad (5.4)$$

Assume that the direction of target signal is θ_0 , while the true direction is $\theta_s = \theta_0 + \Delta$, where Δ is the steering direction error, the time delay $\tau_m(\theta_s)$ may deviate from the nominal one $\tau_m(\theta_0)$. Moreover, with the existence of array geometry error, the time delay $\tau_m(\theta_0)$ has more complicated errors because it is calculated using the assumed

geometry information. However, both types of errors can be represented by phase error in each channel. In addition, the array sensor phase error affects the ASV in (5.4) directly. Therefore, the effects of the multiple errors can be expressed by GPEs in array processing. Based on this model, the actual ASV can be express as

$$\mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha}) = [e^{j\omega_0\tau_1^0(\theta_0)+j\Delta\alpha_1}, \dots, e^{j\omega_0\tau_M^0(\theta_0)+j\Delta\alpha_M}]^T, \quad (5.5)$$

where $\tau_m^0(\theta_0), m = 1, \dots, M$, denotes the assumed steering delay corresponding to the assumed target direction θ_0 . $\Delta\alpha_m$ is the unknown GPE of the m th channel. $\Delta\boldsymbol{\alpha}$ is an $M \times 1$ real vector defined as

$$\Delta\boldsymbol{\alpha} = [\Delta\alpha_1 \ \dots \ \Delta\alpha_m \ \dots \ \Delta\alpha_M]^T. \quad (5.6)$$

Notice that the norm of $\mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha})$ is constant whatever the phase error $\Delta\boldsymbol{\alpha}$ is, i.e., $\|\mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha})\|_2^2 = M$. This property is useful for the proposed method because the compensation of phase errors in ASV does not change the scale of output power.

In the simple case when there is only steering error, $\Delta\alpha_m$ can be expressed as

$$\Delta\alpha_m = \omega_0(\tau_m(\theta_s) - \tau_m(\theta_0)), \quad (5.7)$$

where θ_s denotes the actual direction of the target signal. On the other hand, pure sensor phase error results in constant additive component in $\Delta\alpha_m$. If there is only array geometry error, $\Delta\alpha_m$ can be expressed as a function of sensor location error. In practice, sensor array simultaneously has these imperfections. Therefore, $\Delta\alpha_m$ is

considered to be the composition of all these errors.

The existence of $\Delta\alpha$ causes the array processor to wrongly update its adaptive weights, which finally results in target signal cancellation. A solution to this problem is to replace the nominal ASV in (5.1) by an estimate ASV in (5.5). Hence, we estimate $\Delta\alpha$ to construct an estimate of true ASV according to (5.5). Since the error $\Delta\alpha$ causes target signal cancellation in Capon beamformer, it can be estimated by maximizing the output power of Capon beamformer with respect to $\Delta\alpha$. The maximization of the output power (5.1) is equivalent to minimizing $\mathbf{s}^H(\theta_0, \Delta\alpha)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \Delta\alpha)$ because $\mathbf{s}^H(\theta_0, \Delta\alpha)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \Delta\alpha)$ is a scale. The estimate of $\Delta\alpha$ can be formulated as

$$\begin{aligned}\Delta\hat{\alpha} &= \arg \max_{\Delta\alpha} \hat{p} \\ &= \arg \min_{\Delta\alpha} \mathbf{s}^H(\theta_0, \Delta\alpha)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \Delta\alpha).\end{aligned}\tag{5.8}$$

This is a nonlinear multi-dimensional optimization problem. To search for the optimal $\Delta\hat{\alpha}$, the well known nonlinear multi-dimensional optimization techniques [43] can be applied. With the estimated $\Delta\hat{\alpha}$, the robust beamformer can be expressed as

$$\begin{cases} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad \mathbf{s}^H(\theta_0, \Delta\hat{\alpha}) \mathbf{w} = 1 \end{cases}.\tag{5.9}$$

5.3 Approximated Least Square Solution

The term $\mathbf{s}^H(\theta_0, \Delta\alpha)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \Delta\alpha)$ in (5.8) can be approximated as a quadratic function of $\Delta\alpha$ when the elements $\{\Delta\alpha_m\}$ are small. To simplify the optimization problem

in (5.8), with small $\Delta\alpha_m$, the ASV can be approximated using first-order Taylor series in terms of $\Delta\alpha$,

$$\mathbf{s}(\theta_0, \Delta\alpha) \approx \mathbf{s}(\theta_0, 0) + \mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\alpha, \quad (5.10)$$

where $\mathbf{S}_{\Delta\alpha}(\theta_0)$ denotes the first order derivative matrix defined as

$$\mathbf{S}_{\Delta\alpha}(\theta_0) = \left. \frac{\partial \mathbf{s}(\theta_0, \Delta\alpha)}{\partial \Delta\alpha^T} \right|_{\Delta\alpha=0} = j \cdot \text{diag}\{\mathbf{s}(\theta_0, \mathbf{0})\}. \quad (5.11)$$

Substituting (5.10) into $\mathbf{s}^H(\theta_0, \Delta\alpha)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \Delta\alpha)$, it yields

$$\begin{aligned} q &\triangleq \mathbf{s}^H(\theta_0, \Delta\alpha)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \Delta\alpha) \\ &\approx (\mathbf{s}(\theta_0, 0) + \mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\alpha)^H \mathbf{R}^{-1} (\mathbf{s}(\theta_0, 0) + \mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\alpha) \\ &= \mathbf{s}^H(\theta_0, \mathbf{0})\mathbf{R}^{-1}\mathbf{s}(\theta_0, \mathbf{0}) + 2\text{Re}\{\mathbf{s}^H(\theta_0, 0)\mathbf{R}^{-1}\mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\alpha\} \\ &\quad + \Delta\alpha^T \mathbf{S}_{\Delta\alpha}^H(\theta_0)\mathbf{R}^{-1}\mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\alpha. \end{aligned} \quad (5.12)$$

The optimal $\Delta\hat{\alpha}$ of the robust beamformer can be obtained when

$$\frac{\partial q}{\partial \Delta\alpha} = 2\text{Re}\{\mathbf{S}_{\Delta\alpha}^H(\theta_0)\mathbf{R}^{-1}\mathbf{s}(\theta_0, 0)\} + 2\mathbf{S}_{\Delta\alpha}^H(\theta_0)\mathbf{R}^{-1}\mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\alpha = \mathbf{0}, \quad (5.13)$$

hence,

$$\Delta\hat{\alpha} = -\{\mathbf{S}_{\Delta\alpha}^H(\theta_0)\mathbf{R}^{-1}\mathbf{S}_{\Delta\alpha}(\theta_0)\}^{-1}\text{Re}\{\mathbf{S}_{\Delta\alpha}^H(\theta_0)\mathbf{R}^{-1}\mathbf{s}(\theta_0, \mathbf{0})\}. \quad (5.14)$$

The optimal ASV is given by

$$\mathbf{s}(\theta_0, \Delta\hat{\alpha}) \approx \mathbf{s}(\theta_0, \mathbf{0}) + \mathbf{S}_{\Delta\alpha}(\theta_0)\Delta\hat{\alpha}. \quad (5.15)$$

If the phase error is small, then the ASV found in (5.15) is a near optimal estimate of the actual ASV. However, when the phase error is relatively large, the first-order Taylor series approximation given by (5.10) cannot guarantee that (5.15) is close to the actual ASV. To avoid this problem, we propose an iterative steepest descent method for GPE estimation in the next section.

5.4 Iterative Method Based on Steepest Descent Technique

Since $\Delta\hat{\alpha}$ is the parameter vector to minimize q in (5.12), according to the optimization theory [43, 63], it can be estimated using steepest descent technique. However, the feasible regions of the solution should be considered. In case of the target signal is not the dominant signal, i.e., at least one interference signal has larger power than target signal do, the output power maximization methods may yield wrong solution to output interference and suppress the target signal. Therefore, additional constraints must be used on the feasible region of the estimated GPEs. In this chapter, we introduce a constraint that the error compensated ASV must in a small ball around the nominal ASV, i.e.,

$$\|\hat{\mathbf{s}} - \mathbf{s}(\theta_0)\|_2^2 \leq \xi, \quad (5.16)$$

where $\hat{\mathbf{s}}$ and $\mathbf{s}(\theta_0)$ are the error compensated ASV and the nominal ASV, respectively. ξ is the allowable distance between $\hat{\mathbf{s}}$ and $\mathbf{s}(\theta_0)$. With the feasible region constraints, simulation results in Section 5.5 will show that steepest descent based method converges

to correct GPEs.

The gradient of q in (5.12) to $\Delta\boldsymbol{\alpha}$ can be obtained as

$$\begin{aligned}\frac{\partial q}{\partial \Delta\boldsymbol{\alpha}} &= \frac{\partial \mathbf{s}^H(\theta_0, \Delta\boldsymbol{\alpha})}{\partial \Delta\boldsymbol{\alpha}} \mathbf{R}^{-1} \mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha}) + \frac{\partial \mathbf{s}^T(\theta_0, \Delta\boldsymbol{\alpha})}{\partial \Delta\boldsymbol{\alpha}} (\mathbf{R}^{-1} \mathbf{s}^H(\theta_0, \Delta\boldsymbol{\alpha}))^T \\ &= 2\text{Re}\{\mathbf{S}_{\Delta\boldsymbol{\alpha}}^H(\theta_0, \Delta\boldsymbol{\alpha}) \mathbf{R}^{-1} \mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha})\},\end{aligned}\quad (5.17)$$

where

$$\mathbf{S}_{\Delta\boldsymbol{\alpha}}(\theta_0, \Delta\boldsymbol{\alpha}) = \frac{\partial \mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}^T} = j \cdot \text{diag}\{\mathbf{s}(\theta_0, \Delta\boldsymbol{\alpha})\}. \quad (5.18)$$

Since the phase errors are all relative quantities compared to each other, it is required to fix one of them, say $\Delta\alpha_1 = 0$, and adjust the others to find the maximum of output power. In the proposed approach, we assume that $\Delta\alpha_1 \equiv 0$. The iterative steepest descent algorithm can be described as following:

Step 1 Initialize $\Delta\boldsymbol{\alpha}^{[0]} = 0$ and $\hat{\mathbf{s}}^{[0]} = \mathbf{s}(\theta_0, 0)$, where $(\cdot)^{[n]}$ denotes the n th iteration.

Step 2 Calculate the gradient vector $\left. \frac{\partial q}{\partial \Delta\boldsymbol{\alpha}} \right|_{\Delta\boldsymbol{\alpha}=\Delta\boldsymbol{\alpha}^{[n-1]}}$ by (5.17).

Step 3 Update the phase error vector $\Delta\boldsymbol{\alpha}^{[n]}$ by

$$\Delta\boldsymbol{\alpha}^{[n]} = \Delta\boldsymbol{\alpha}^{[n-1]} + \mu \left. \frac{\partial q}{\partial \Delta\boldsymbol{\alpha}} \right|_{\Delta\boldsymbol{\alpha}=\Delta\boldsymbol{\alpha}^{[n-1]}}, \quad (5.19)$$

where μ is the stepsize.

Step 4 Use $\Delta\boldsymbol{\alpha}^{[n]}$ to update the steering vector by

$$\hat{\mathbf{s}}^{[n]} = \text{diag}\{e^{j\Delta\psi_1^{[n]}}, \dots, e^{j\Delta\psi_N^{[n]}}\} \hat{\mathbf{s}}^{[0]}. \quad (5.20)$$

Step 5 Check the convergence of $\Delta\boldsymbol{\alpha}^{[n]}$ by validating $\|\Delta\boldsymbol{\alpha}^{[n]} - \Delta\boldsymbol{\alpha}^{[n-1]}\|_2^2 \geq \epsilon$ and

limit the search region by validating $\|\hat{\mathbf{s}}^{[n]} - \mathbf{s}(\theta_0, \mathbf{0})\|_2^2 \leq \xi$. If both of them are true, repeat Step 1-4. Here ϵ is the error tolerance and ξ is a real number to limit the search region to prevent algorithm from mistracking.

Step 6 Replace the ASV $\mathbf{s}^H(\theta_0, \Delta\hat{\boldsymbol{\alpha}})$ by the estimated $\hat{\mathbf{s}}^{[n]}$ in (5.9) if the algorithm converges within the feasible search region. Otherwise, $\mathbf{s}(\theta_0, \mathbf{0})$ is used to replace $\mathbf{s}^H(\theta_0, \Delta\hat{\boldsymbol{\alpha}})$ in (5.9).

5.5 Numerical Experiments

In this section, some numerical experiments were carried out to illustrate the effectiveness of the proposed approach. In the following simulations, a uniform linear array comprising eight isotropic sensors with half-wavelength inter-element spacing is used. The nondirectional background noise is spatial white noise with unit variance. The power of target signal is $10dB$ and its assumed DOA is 0° . Two strong interferences with power and DOA ($20dB, 60^\circ$) and ($20dB, 80^\circ$) are considered in the simulations. All the simulation results are the average of 200 Monte-Carlo experiment results except that the beampattern is calculated by one experiment. For the purpose of performance comparison, the standard Capon beamformer (SCB) and delay-and-sum beamformer (DSB) are also included in the simulations.

In the first experiment, it is assumed that the GPE is caused by steering direction error. The parameters used in simulation are $\epsilon = 0.005$, $\xi = 10$ and $\mu = 0.1$. We assume

Figure 5.1: Spatial power responses of the array processors with steering direction error only.

that the steering direction error is in the range of $[-5^\circ, 5^\circ]$. In Fig. 5.1, the spatial power response of the array processor is shown. Compared with the DSB and the SCB, the proposed RCB can achieve remarkably flat response in the region around the assume direction. Moreover, the maximal steering error can be controlled by selecting suitable ξ . An experiment on controlling the allowable steering error is shown in Fig. 5.2, where ξ is choose to be 3, 7 and 13. The maximal allowable steering direction errors are 4, 8 and 12 degrees, respectively.

As we know, the array sample covariance matrix approaches the true one when the number of snapshots approaches infinity. However, the covariance matrix is always

Figure 5.2: Maximal allowable steering error control of the proposed RCB.

estimated with limited number of snapshots. Hence, the estimation error in array covariance matrix is unavoidable. In this section, we also carried out some experiments to show the robustness of the proposed method when the number of snapshots is limited. Fig. 5.3 shows the output SINR of each array processor versus the number of snapshots. We can find out that the proposed robust Capon beamformer (RCB) is robust to small number of snapshots, and its output SINR increases with the number of snapshots. For the SCB, its output SINR decreases with the number of snapshots. Although the performance of the DSB is independent to the number of snapshots, its output SINR is far below that of the RCB. The beampattern of the array processors are also shown in Fig. 5.4 and 5.5. In these two figures, the proposed RCB always has high response to the target signal, while the SCB has serious signal cancellation effect. The DSB also has high response to the target signal. However, it does not null out the interferences.

In the next experiment, we study the performance of the proposed beamformer under random GPEs. One realization of random GPEs may be the phase errors caused by the array geometry error or sensor phase error. The largest absolute value of random GPEs has unit value. The parameters used in simulation are $\epsilon = 0.005$, $\xi = 4$ and $\mu = 0.1$. In Fig. 5.6, spatial power response of the array processors under random GPEs is shown. Clearly, the proposed RCB is robust to the random GPEs and it has almost flat response around the DOA of the target signal. The SCB shows serious target signal cancellation even around the true DOA of the target signal. In Fig. 5.7, the output SINR versus the number of snapshots is also plotted. The proposed RCB has higher performance than the others do even with small number of snapshots.

Figure 5.3: Output SINR of the array processors versus the number of snapshots (The steering direction error is 5°).

Figure 5.4: Beampatterns of the array processors with steering error only (The covariance matrix is estimated with 100 snapshots. The vertical lines stand for the directions of incident signals.).

Figure 5.5: Beampatterns of the array processors with steering error only (The covariance matrix is estimated with 2000 snapshots. The vertical lines stand for the directions of incident signals.).

Figure 5.6: Spatial power response of the array processors under random phase error.

Figure 5.7: Output SINR of the array processors under random phase error.

In the next experiment, we study the performance of the array processors under both random GPEs and steering direction error. The random GPE is similar to the above one. The steering direction error is 3° . The parameters used in simulation are $\epsilon = 0.005$, $\xi = 8$ and $\mu = 0.1$. From the results shown in Fig. 5.8 and Fig. 5.9, similar conclusion can be obtained as the above experiments. The proposed RCB has the best performance among all the beamformers.

All of these simulation results show that the proposed beamformer can achieve remarkably flat response in the acceptance region. That means the proposed beamformer is robust when there are multiple array imperfections. Moreover, the proposed beamformer has highest output SINR.

Figure 5.8: Spatial power response of the array processors under random phase error and steering direction error.

Figure 5.9: Output SINR of the array processors under random phase error.

5.6 Summary

A new approach is proposed for robust beamforming against various array errors including steering direction error, array geometry error and sensor phase error. All of these array imperfections are modelled as GPEs in the ASV. An optimization method is proposed to obtain the optimal estimate of GPEs. The estimated GPEs are used to compensate the errors in nominal ASV. The resulting beamformer demonstrates high robustness to those array imperfections. Moreover, the proposed beamformer does not suffer from performance loss in interference rejection. It can be applied in arrays with small number of sensors.

Chapter 6

Beamformer Robust to Arbitrary Error in Array Steering Vector

6.1 Introduction

If the ASV can be modelled as a vector function of some parameters, like steering direction error [32], time-delay error [64], and general phase errors [65], robust beamformer can be constructed by maximizing the output power of the SCB to these parameters in their feasible ranges. Efficient gradient descent method [43] can be derived to search for the optimal parameters. With these estimated optimal parameters, the error in the ASV can be compensated. The signal cancellation effect in the array output is then reduced. Detail discussion of these methods can refer to Chapter 4 and 5.

In practice, the nominal ASV corresponding to the target source cannot be expressed in a compact mathematical form in term of array imperfections. In such case,

new robust methods should be developed. In this chapter, we further extend the idea used in [32, 65, 64] to design an adaptive beamformer robust to arbitrary ASV error. Since the output power of the SCB is a function of the assumed nominal ASV, we can maximize the output power of the SCB with respect to all feasible ASVs instead of the modelled parameters of the ASV in [32, 65, 64]. Although nonzero scaling of ASV does not change the output SINR of the SCB, it introduces an arbitrary scale in the output power. To eliminate the ambiguity of the output power, we assume that the ASV has unit norm. If there is no other constraint on ASV, the design of the array processor can be simplified to a principal component analysis (PCA) or minor component analysis (MCA) problem [66]. Nevertheless, when the target signal is not the dominant one, such array beamformer may wrongly suppress the target signal and retrieve interference as the output signal. To solve this problem, we introduce an additional uncertainty constraint on the ASV. This uncertainty constraint of the ASV is also used in some robust methods [27, 30, 29, 67]. It assumes that the feasible ASV is in an ellipsoid whose center is the nominal ASV. With this uncertainty constraint, the designed Capon beamformer is robust to arbitrary ASV error even with the existence of strong interferences.

Two types of RCBs are derived with and without using of the additional constraint on the norm of the array ASV in optimization. Theoretical analysis shows that the proposed RCBs can be generalized as the diagonal loading approach. The optimal output SINRs of the proposed RCBs are also derived. Unfortunately, the RCBs cannot achieve the optimal output SINR. Although the proposed RCBs cannot achieve the optimal output SINR, numerical experiments show that the RCBs demonstrate

outstanding robustness to the ASV error and have relatively high output SINR.

Our studies also indicate that the robust beamformer has degraded output SINR when the uncertainty level is large. This is because the robustness of the beamformers [27, 28, 29, 30] is obtained at the cost of reduced capability in noise and interference suppression. The performance degradation is serious with large uncertainty level. Hence, compact uncertainty set is needed to guarantee the performance on noise and interference suppression. In this chapter, we also propose a new compact uncertainty set which has lower uncertainty level than those [29, 30] do. The compact uncertainty constraint is obtained by replacing the nominal ASV in conventional uncertainty set [29, 30] with the projection of the nominal ASV onto the signal-plus-interference subspace. Using the new compact constraint with the beamformers proposed in Section 6.2, higher output SINR is achievable. Theoretical analysis and simulation results show the SINR improvement.

This chapter is organized as follows. In Section 6.2, two kinds of RCBs are derived based on output power maximization. The RCB proposed in Section 6.2.1 is derived based on Capon beamformer and uncertainty constraint on the nominal ASV. The RCB proposed in Section 6.2.1 has one more constraint on the norm of the ASV. The discussion in Section 6.2.3 show the difference between these two RCBs. In Section 6.2.4, the optimal output SINR of the proposed RCBs is derived. Analysis shows that the proposed RCBs cannot achieve the highest output SINR. To further improve output SINR of the proposed RCBs, a new RCB with a compact uncertainty set is proposed in Section 6.3. Theoretical proof of its high output SINR is also given. In Section

6.4, various simulation results are carried out to show the performance of the proposed RCBs. Brief summary is given in Section 6.5.

6.2 Beamformers Robust to Arbitrary ASV Error

As discussed in Chapter 2, the Capon beamformer has serious target signal cancellation problem when the nominal ASV has error. For time-delay error and GPE, the robust beamforming methods are proposed in Chapter 4 and 5. In this section, we formulate RCB robust to arbitrary ASV error. For convenience, in the following context in this chapter, we ignore the angle index in the ASV.

6.2.1 Robust Beamformer with Uncertainty Constraint

In practical applications, the true ASV $\mathbf{s}_0$ is always unknown or known but with some errors. If $\mathbf{s}_0$ deviates from the true one, target signal cancellation occurs. The output power in (2.43) decreases. A method to overcome target signal cancellation is to search for an optimal ASV $\mathbf{s}$, which results in maximal output power $\hat{p}$. We assume that the true ASV $\mathbf{s}_0$ belongs to the following uncertainty ellipsoid [27, 30, 29]:

$$\mathbf{s}_0 \in \left\{ \mathbf{s} \mid (\mathbf{s} - \bar{\mathbf{s}}_0)^H \mathbf{D}^{-1} (\mathbf{s} - \bar{\mathbf{s}}_0) \leq 1 \right\}, \quad (6.1)$$

where $\bar{\mathbf{s}}_0$ is the nominal ASV and $\mathbf{D}$ is a positive definite matrix. If $\mathbf{D} = \frac{1}{\epsilon} \mathbf{I}$, we have

$$\mathbf{s}_0 \in \left\{ \mathbf{s} \mid \|\mathbf{s} - \bar{\mathbf{s}}_0\|_2^2 \leq \epsilon \right\}, \quad (6.2)$$

where ϵ is the uncertainty level. With the uncertainty constraint (6.2), robust beam-former can be formulated as

$$\begin{cases} \max_{\mathbf{s}} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad \mathbf{s}^H \mathbf{w} = 1 \\ \|\mathbf{s} - \bar{\mathbf{s}}_0\|_2^2 \leq \epsilon \end{cases} \quad (6.3)$$

This optimization problem can be solved in two steps. First, we fix $\mathbf{s}$ and search the minimal output power. Then we search for the maximal value of the minimal output power to all the feasible $\mathbf{s}$. For any given $\mathbf{s}$, the output power of the SCB is given in (2.43). Since $\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}$ is a scalar, maximizing $\frac{1}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}}$ is equivalent to minimizing $\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}$. The optimization problem in (6.3) is simplified to

$$\begin{cases} \min_{\mathbf{s}} \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \\ \text{s.t.} \quad \|\mathbf{s} - \bar{\mathbf{s}}_0\|_2^2 \leq \epsilon \end{cases} \quad (6.4)$$

The optimization problem (6.4) has similar mathematical form as [30, 29]. It can be solved using the Lagrange multiplier methodology [43]. The optimal solution is obtained on the boundary of the constraint set. Therefore (6.4) can be reformulated as

$$\begin{cases} \min_{\mathbf{s}} \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \\ \text{s.t.} \quad \|\mathbf{s} - \bar{\mathbf{s}}_0\|_2^2 = \epsilon \end{cases} \quad (6.5)$$

To exclude the trivial solution $\mathbf{s} = \mathbf{0}$ to (6.5), we assume that

$$\|\bar{\mathbf{s}}_0\|_2^2 > \epsilon. \quad (6.6)$$

Define a function

$$f = \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} + g(\|\mathbf{s} - \bar{\mathbf{s}}_0\|_2^2 - \epsilon), \quad (6.7)$$

where $g \geq 0$ is the Lagrange multiplier. The optimal vector $\hat{\mathbf{s}}$ is obtained by setting the differentiation of (6.7) with respect to $\mathbf{s}^*$ to zero,

$$\frac{df}{d\mathbf{s}^*} = \mathbf{R}^{-1} \hat{\mathbf{s}} + g(\hat{\mathbf{s}} - \bar{\mathbf{s}}_0) = \mathbf{0}. \quad (6.8)$$

The above equation yields

$$\hat{\mathbf{s}} = (g^{-1} \mathbf{R}^{-1} + \mathbf{I})^{-1} \bar{\mathbf{s}}_0. \quad (6.9)$$

The Lagrange multiplier g is the root of the constraint equation

$$\|\hat{\mathbf{s}} - \bar{\mathbf{s}}_0\|_2^2 = \|(\mathbf{I} + g\mathbf{R})^{-1} \bar{\mathbf{s}}_0\|_2^2 = \epsilon. \quad (6.10)$$

As shown in (2.21), the eigenvalue decomposition of $\mathbf{R}$ is

$$\mathbf{R} = \mathbf{U} \Sigma \mathbf{U}^H. \quad (6.11)$$

Substituting (6.11) into (6.10) and denoting $\mathbf{y} = \mathbf{U}^H \bar{\mathbf{s}}_0$, it yields

$$h(g) = \sum_{i=1}^M \frac{|y_i|^2}{(1 + \lambda_i g)^2} = \epsilon, \quad (6.12)$$

where y_i is the i th element of vector $\mathbf{y}$.

Consider the derivative of $h(g)$ to g ,

$$\frac{dh(g)}{dg} = -2 \sum_{i=1}^M \frac{|y_i|^2 \lambda_i}{(1 + \lambda_i g)^3} < 0, \quad (6.13)$$

when $g \geq 0$. The function $h(g)$ is a monotonically decreasing function with respect to g . Moreover, $\lim_{g \rightarrow \infty} h(g) = 0$ and $h(0) = \|\mathbf{U}^H \bar{\mathbf{s}}_0\|_2^2 = \|\bar{\mathbf{s}}_0\|_2^2 > \epsilon$. The equation (6.12) has a unique solution $\hat{g}$. Substituting $\hat{g}$ into (6.9), it yields

$$\hat{\mathbf{s}} = (\hat{g}^{-1} \mathbf{R}^{-1} + \mathbf{I})^{-1} \bar{\mathbf{s}}_0 = \bar{\mathbf{s}}_0 - (\mathbf{I} + \hat{g} \mathbf{R})^{-1} \bar{\mathbf{s}}_0. \quad (6.14)$$

The corresponding optimal weight of the beamformer is

$$\hat{\mathbf{w}}_0 = \frac{\mathbf{R}^{-1} \hat{\mathbf{s}}}{\hat{\mathbf{s}}^H \mathbf{R}^{-1} \hat{\mathbf{s}}} \quad (6.15)$$

and the output power and SINR are

$$\hat{p} = \frac{\|\hat{\mathbf{s}}\|_2^2}{\hat{\mathbf{s}}^H \mathbf{R}^{-1} \hat{\mathbf{s}}}, \quad \rho = \frac{\mathbf{w}_0^H \mathbf{R}_s \mathbf{w}_0}{\mathbf{w}_0^H \mathbf{R}_n \mathbf{w}_0}, \quad (6.16)$$

where $\mathbf{R}_s$ and $\mathbf{R}_n$ are the sample covariance matrices of the target signal and noise,

respectively. The numerator $||\hat{\mathbf{s}}||_2^2$ in $\hat{p}$ is used to eliminate the effect of norm of the ASV on the estimated power.

6.2.2 Robust Beamformer with Multiple Constraints On Uncertainty of Steering Vector

In Section 6.2.1, the optimal beamformer is derived ignoring the condition that the ASV has unit norm. The optimal ASV estimate $\hat{\mathbf{s}}$ in (6.14) may not have unit norm. Its effect on output power estimate is eliminated by the normalization shown in (6.16). In this Section, the unit norm constraint of the ASV is considered in the derivation of robust beamformer. From the new derivation following, a clear relationship between the proposed approach and PCA/MCA based beamformer is shown.

In practice, the ASV $\mathbf{s}_0$ is always unknown or known but with some error. If $\mathbf{s}_0$ deviates from the true one, target signal cancellation is inevitable. This results in decreasing of the output power in (2.43). A solution to this problem is to search for an optimal ASV $\mathbf{s}$, which results in maximal output power $\hat{\sigma}_s^2$. We also assume that the ASV has unit norm. Therefore, the robust beamformer can be formulated as

$$\left\{ \begin{array}{l} \max_{\mathbf{s}} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad \mathbf{s}^H \mathbf{w} = 1 \\ \quad \quad ||\mathbf{s}||_2^2 = 1. \end{array} \right. , \quad (6.17)$$

This is equivalent to

$$\begin{cases} \min_{\mathbf{s}} \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \\ \text{s.t.} \quad \|\mathbf{s}\|_2^2 = 1 \end{cases}, \quad (6.18)$$

which becomes a PCA/MCA problem [66]. The optimal $\hat{\mathbf{s}}$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}$.

However, if the target signal is not the dominant one, this method leads to a wrong solution. Therefore, additional constraint must be incorporated in the optimization problem in (6.17). We assume the true ASV $\mathbf{s}_0$ belongs to an uncertainty set as shown in (6.1). With this uncertainty constraint on ASV, the robust beamformer is constructed by maximizing the output power of the SCB when an imprecise knowledge of its steering vector $\mathbf{s}_0$ is available.

$$\begin{cases} \max_{\mathbf{s}} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} \quad \mathbf{s}^H \mathbf{w} = 1 \\ \|\mathbf{s}\|_2^2 = 1 \\ \|\mathbf{s} - \bar{\mathbf{s}}_0\|_2^2 \leq \epsilon \end{cases}. \quad (6.19)$$

This is equivalent to

$$\begin{cases} \min_{\mathbf{s}} \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \\ \text{s.t.} \quad \|\mathbf{s}\|_2^2 = 1 \\ \mathbf{s}^H \bar{\mathbf{s}}_0 + \bar{\mathbf{s}}_0^H \mathbf{s} \geq 2 - \epsilon \end{cases}. \quad (6.20)$$

The optimization problem (6.20) can be solved by the Lagrange multiplier methodology [43]. According to the optimization theory [43], inequality constraint may be

inactive during optimization. Otherwise, the optimal solution is on the boundary of the constraints. Hence, we define a phase shifted vector $\tilde{\mathbf{u}}_1$ of $\mathbf{u}_1$ as $\tilde{\mathbf{u}}_1 = e^{j\phi}\mathbf{u}_1$, where $\mathbf{u}_1$ is the eigenvector of $\mathbf{R}$ corresponding to the largest eigenvalue. If

$$\max_{\phi}(\tilde{\mathbf{u}}_1^H \bar{\mathbf{s}}_0 + \bar{\mathbf{s}}_0^H \tilde{\mathbf{u}}_1) = 2\mathbf{u}_1^H \bar{\mathbf{s}}_0 e^{j\phi_{opt}} \geq 2 - \epsilon, \quad (6.21)$$

$$\phi_{opt} = -\arg(\mathbf{u}_1^H \bar{\mathbf{s}}_0),$$

the uncertainty constraint is inactive during optimization. The vector $\tilde{\mathbf{u}}_1$ is the optimal solution to (6.20). Otherwise, the optimal solution is obtained on the boundary of the constraints. Define

$$f(\mathbf{s}, g_1, g_2) = \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} + g_1(\|\mathbf{s}\|_2^2 - 1) + g_2(\mathbf{s}^H \bar{\mathbf{s}}_0 + \bar{\mathbf{s}}_0^H \mathbf{s} - 2 + \epsilon). \quad (6.22)$$

The minimization of (6.22) to $\mathbf{s}$ is obtained when

$$\frac{\partial f(\mathbf{s}, g_1, g_2)}{\partial \mathbf{s}^*} = \mathbf{R}^{-1} \mathbf{s} + g_1 \mathbf{s} + g_2 \bar{\mathbf{s}}_0 = 0. \quad (6.23)$$

It yields,

$$\mathbf{s} = -g_2(\mathbf{R}^{-1} + g_1 \mathbf{I})^{-1} \bar{\mathbf{s}}_0. \quad (6.24)$$

Substituting (6.24) into constraint $\mathbf{s}^H \bar{\mathbf{s}}_0 + \bar{\mathbf{s}}_0^H \mathbf{s} = 2 - \epsilon$, we obtain

$$\hat{g}_2 = \frac{2 - \epsilon}{-2\bar{\mathbf{s}}_0^H (\mathbf{R}^{-1} + g_1 \mathbf{I})^{-1} \bar{\mathbf{s}}_0} = \frac{2 - \epsilon}{-2 \sum_{i=1}^M |\tilde{s}_i|^2 \frac{\lambda_i}{\lambda_i g_1 + 1}}, \quad (6.25)$$

where $\tilde{s}_i$ is the i th element of vector $\tilde{\mathbf{s}}_0 = \mathbf{U}^H \bar{\mathbf{s}}_0$. The calculation of $\hat{g}_2$ can be ignored

when the beamformer is not used for power estimation because the scale $\hat{g}_2$ does not influence the output SINR.

Substitute (6.24) into the constraint $\|\mathbf{s}\|_2^2 = 1$, the optimal estimate $\hat{g}_1$ of g_1 can be calculated from the following equation.

$$\psi(g_1) = \frac{\bar{\mathbf{s}}_0^H (\mathbf{R}^{-1} + g_1 \mathbf{I})^{-2} \bar{\mathbf{s}}_0}{(\bar{\mathbf{s}}_0^H (\mathbf{R}^{-1} + g_1 \mathbf{I})^{-1} \bar{\mathbf{s}}_0)^2} = \frac{\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)^2}{\left(\sum_{i=1}^M |\tilde{s}_i|^2 \frac{\lambda_i}{\lambda_i g_1 + 1}\right)^2} = \frac{4}{(2 - \epsilon)^2}. \quad (6.26)$$

It is proved in Appendix B.1 that the above equation has unique solution in the range $(-1/\lambda_1, +\infty)$.

With the estimates $\hat{g}_1, \hat{g}_2$, the optimal solution $\hat{\mathbf{s}}$ is given by

$$\hat{\mathbf{s}} = -\hat{g}_2 (\mathbf{R}^{-1} + \hat{g}_1 \mathbf{I})^{-1} \bar{\mathbf{s}}_0. \quad (6.27)$$

The corresponding optimal weight of the beamformer is given by

$$\hat{\mathbf{w}}_0 = \frac{\hat{\mathbf{R}}^{-1} \hat{\mathbf{s}}}{\hat{\mathbf{s}}^H \hat{\mathbf{R}}^{-1} \hat{\mathbf{s}}} \quad (6.28)$$

and the estimates of the signal power $\hat{p}$ and output SINR ρ are given by

$$\hat{p} = \frac{1}{\hat{\mathbf{s}}^H \mathbf{R}^{-1} \hat{\mathbf{s}}} \quad (6.29)$$

and

$$\rho = \frac{\mathbf{w}_0^H \mathbf{R}_s \mathbf{w}_0}{\mathbf{w}_0^H (\mathbf{R}_i + \mathbf{R}_n) \mathbf{w}_0}, \quad (6.30)$$

Figure 6.1: Geometrical illustration of the proposed RCBs.

where $\mathbf{R}_s$, $\mathbf{R}_i$ and $\mathbf{R}_n$ are the covariance matrices of the target signal, interference and nondirectional noise, respectively.

6.2.3 Discussion on Proposed Methods

In this section, we show the difference between the proposed RCBs in Section 6.2.1 and Section 6.2.2. For notation convenience, we denote the RCB in Section 6.2.1 as RCB1 and the RCB in Section 6.2.2 as RCB2. The uncertainty constraint (6.2) of the ASV is shown as the circle E in Fig. 6.1. Without loss of generality, the geometrical illustrations of the proposed RCBs are shown in 2-Dimension. Moreover, we assume that the target signal and interferences are well separated.

The RCB1 in (6.4) is obtained by maximizing the output power subject to a con-

straint that the feasible ASV is in the uncertainty constraint E , which is a circle centered at $\bar{\mathbf{s}}_0$ as shown in Fig. 6.1. The true ASV $\mathbf{s}_0$ with unit norm locates inside the uncertainty set E . It is obvious that we prefer to select the nominal ASV as a scaled version of $\mathbf{s}_0$. The target signal cancellation can be avoided and the output SINR of beamformer is high. If the nominal ASV is selected as the one which deviates from $\mathbf{s}_0$. Target signal cancellation occurs. The amount depends on the degree of deviation. As a consequence, the output power decreases.

As discussed in (5.3), if we select the nominal ASV with smaller norm, the output power has larger value. It is also known that the solution of the RCB1 is obtained on the boundary of constraints. Therefore, the solution of (6.5) may be $\mathbf{s}_1$ which deviates from $\mathbf{s}_0$ but with small norm as shown in Fig. 6.1. The effect in output power reducing can be compensated by the small norm of the nominal ASV. Clearly, the output SINR of RCB1 may not be the optimal one because target signal cancellation exists due to the deviation of the nominal ASV. This effect is much more serious if the uncertainty set is constructed as an ellipsoid as shown in Fig. 6.2. When the uncertainty level ϵ in Fig. 6.1 is small, the SINR degradation of RCB1 can be ignored.

The RCB2 in (6.19) has one additional constraint in optimization that the nominal ASV has fixed norm $\|\mathbf{s}\|_2^2 = 1$. With this additional constraint, the solution of (6.19) must always be on the circle B with unit radius. The solution $\mathbf{s}_2$ should be close to $\mathbf{s}_0$ to prevent target signal cancellation, otherwise, the output power will decrease provided the interferences are far away from the SOI.

The geometrical illustrations in Fig. 6.1 and 6.2 show the difference between the

Figure 6.2: Geometrical illustration of the proposed RCBs with ellipsoid constraint set.

proposed RCBs. From these illustrations, it is easy to conclude that the RCB2 should have higher output SINR than RCB1 does. When the uncertainty level ϵ is small, the differences should be slight, as verified in simulation.

Although the proposed RCBs have different constraints in optimization, their solutions have similar mathematical forms as shown in (6.9) and (6.24).

$$\begin{aligned}
 RCB1 : \quad \hat{\mathbf{s}} &= (\hat{g}^{-1} \hat{\mathbf{R}}^{-1} + \mathbf{I})^{-1} \bar{\mathbf{s}}_0, \\
 RCB2 : \quad \hat{\mathbf{s}} &= -\hat{g}_2 (\hat{\mathbf{R}}^{-1} + \hat{g}_1 \mathbf{I})^{-1} \bar{\mathbf{s}}_0 \\
 &= -\frac{\hat{g}_2}{\hat{g}_1} (\hat{g}_1^{-1} \hat{\mathbf{R}}^{-1} + \mathbf{I})^{-1} \bar{\mathbf{s}}_0.
 \end{aligned} \tag{6.31}$$

Both of them belong to the diagonal loading approach. The differences between these two RCBs are the diagonal loading factors and the scale factors of $\hat{\mathbf{s}}$. The scale factor

of $\hat{\mathbf{s}}$ does not affect the output SINR of a beamformer. It can be ignored for SINR analysis. In the performance analysis carried out in the next section, we treat these two RCBs in uniform mathematical form similar to RCB1.

6.2.4 Performance Analysis of the Proposed RCBs

In this section, the optimal output SINR of the proposed beamformer is derived. A complete performance analysis of output SINR under general array imperfections represents a formidable analytical task. In this chapter, we assume that the array processor only has steering vector error. The theoretical covariance matrix is used in analysis. In such case, the performance degradation of the Capon beamformer is caused by the error in the nominal ASV. This assumption simplifies the performance analysis.

Before we derive the optimal output SINR of the proposed RCB, we first give the output SINR of the Capon beamformer in Lemma 6.1.

Lemma 6.1. *Assume that the covariance matrices of the SOI and interference/noise are $\mathbf{R}_s$ and $\mathbf{R}_n$, respectively. The covariance matrix of array snapshot is $\mathbf{R} = \mathbf{R}_s + \mathbf{R}_n$. When the nominal ASV is given as $\mathbf{s}$, and the true ASV is $\mathbf{s}_0$, the output SINR ρ of the Capon beamformer is given by*

$$\rho = \frac{\rho_o \cos^2(\theta)}{1 + \sin^2(\theta)\rho_o(\rho_o + 2)},$$

where θ is the angle between the vector $\mathbf{s}$ and $\mathbf{s}_0$, and ρ_o is the output SINR of the

Capon beamformer when $\mathbf{s}_0$ is known, and

$$\cos^2(\theta) = \frac{|\mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\mathbf{s}\|_{\mathbf{R}}^2},$$

$$\rho_o = \sigma_s^2 \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0 = \sigma_s^2 \|\mathbf{s}_0\|_{\mathbf{R}}^2,$$

where $\|\mathbf{x}\|_{\mathbf{R}}^2 \triangleq \mathbf{x}^H \mathbf{R}_n^{-1} \mathbf{x}$ is the extended vector norm ($\mathbf{R}_n$ is a positive matrix); σ_s^2 is the power of the SOI. If $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$, the extended vector norm $\|\cdot\|_{\mathbf{R}}$ can be replaced by the Euclidian norm, and

$$\cos^2(\theta) = \frac{|\mathbf{s}_0^H \mathbf{s}|^2}{\|\mathbf{s}_0\|_2^2 \|\mathbf{s}\|_2^2},$$

$$\rho_{opt} = \frac{\sigma_s^2}{\sigma_n^2} \|\mathbf{s}_0\|_2^2.$$

Proof. Refer to Appendix B.2. □

Lemma 6.1 indicates that the output SINR of the Capon beamformer is determined by the angle between the nominal ASV and true ASV. Moreover, it is easy to find that the output SINR ρ is a monotonically increasing function of $\cos^2(\theta)$. From (6.27) and (6.28), we find that the proposed RCBs have similar mathematical form as the Capon beamformer except that the nominal vector $\bar{\mathbf{s}}_0$ is replaced by the estimated one, $\hat{\mathbf{s}}$. Therefore, the performance of the proposed RCBs can be analyzed via the angle between $\hat{\mathbf{s}}$ and $\mathbf{s}_0$. Different diagonal loading factors result in different $\hat{\mathbf{s}}$. The optimal diagonal loading factor should be selected to obtain the highest output SINR. In Lemma 6.2, we derive the optimal diagonal load factor and the output SINR of the

RCB.

Lemma 6.2. *Assume that the covariance matrix of the interference/noise is $\mathbf{R}_n$ and its eigen-decomposition is*

$$\mathbf{R}_n = [\mathbf{U}_i \ \mathbf{U}_n] \begin{bmatrix} \Sigma_i & \mathbf{0} \\ \mathbf{0} & \Sigma_n \end{bmatrix} [\mathbf{U}_i \ \mathbf{U}_n],$$

where $\mathbf{U}_i$ and $\mathbf{U}_n$ are the eigenvector matrices which span the interference and noise subspaces respectively. The diagonal matrices $\Sigma_i = \text{diag}\{\lambda_1, \dots, \lambda_K\}$ and $\Sigma_n = \sigma_n^2 \mathbf{I}$ are the corresponding eigenvalue matrices. The nominal ASV is

$$\hat{\mathbf{s}} = (g_1^{-1} \hat{\mathbf{R}}^{-1} + \mathbf{I})^{-1} \bar{\mathbf{s}}_0,$$

If $\lambda_i \gg \sigma_n^2, i = 1, \dots, K$, the optimal output SINR ρ_u of the output SINR is

$$\rho_u = \frac{\sigma_s^2 \|P_{\mathbf{U}_n} \mathbf{s}_0\|_2^2}{\sigma_n^2},$$

which is achieved when

$$g_1 = \frac{-1}{\sigma_n^2 + \sigma_s^2 \|P_{\mathbf{U}_n} \mathbf{s}_0\|_2^2},$$

provided that $\mathbf{s}_0^H P_{\mathbf{U}_n} \bar{\mathbf{s}}_0 \neq 0$. The matrix $P_{\mathbf{U}_n} = \mathbf{U}_n \mathbf{U}_n^H$ is the projection matrix to the subspace spanned by $\mathbf{U}_n$. The power and ASV of the SOI are σ_s^2 and $\mathbf{s}_0$, respectively.

Proof. Refer to Appendix B.3. □

Lemma 6.2 indicates that the optimal output SINR of the proposed RCBs are

achievable with negative diagonal loading factor. Since $\lambda_1 \geq \sigma_n^2 + \sigma_s^2 \|\mathbf{s}_0\|_2^2 \geq \sigma_n^2 + \sigma_s^2 \|P_{\mathbf{U}_n} \mathbf{s}_0\|_2^2$, the optimal value of g_1 is not in $(-1/\lambda_1, \infty)$, which is the solution range of the proposed RCBs. The proposed RCBs cannot achieve the highest output SINR. Nevertheless, the simulation results in the next section will show that the proposed RCBs still have high output SINRs.

6.3 Proposed Robust Beamformer with New Uncertainty Constraint

As shown in (6.9) and (6.24), the optimal ASVs $\hat{\mathbf{s}}$ used for the RCB1 and the RCB2 are

$$\begin{aligned} RCB1 : \quad \hat{\mathbf{s}} &= (g^{-1} \hat{\mathbf{R}}^{-1} + \mathbf{I})^{-1} \bar{\mathbf{s}}_0, \\ RCB2 : \quad \hat{\mathbf{s}} &= -\hat{g}_2 (\hat{\mathbf{R}}^{-1} + \hat{g}_1 \mathbf{I})^{-1} \bar{\mathbf{s}}_0, \end{aligned} \tag{6.32}$$

where g is calculated from (6.12), $\hat{g}_1$ and $\hat{g}_2$ are calculated from (6.26) and (6.25). From our studies, we find that the output SINR of the RCBs degrade with large uncertainty set because the robustness of the RCBs are obtained at the cost of reduced capability in noise suppression. In this section, we derive a new method to improve the output SINR of the RCB while maintaining the robustness to ASV error.

Figure 6.3: Geometry model of the uncertainty set.

6.3.1 Derivation of New Compact Uncertainty Constraint

In this Section, we derive a new compact uncertainty constraint which is used to replace the constraint in (6.5). The eigenvalue decomposition of the covariance matrix is

$$\mathbf{R} = \mathbf{U}\mathbf{\Gamma}\mathbf{U}^H. \quad (6.33)$$

Refer to (2.24), the orthogonal bases $\mathbf{U}_s$ of the steering vectors $\mathbf{s}_k$, $k = 0, \dots, K-1$, are obtained by extracting the eigenvectors corresponding to the largest K eigenvalues.

$\mathbf{U}_s$ spans a linear space H ,

$$H = \{\mathbf{s} | \mathbf{s} = \mathbf{U}_s \mathbf{c}, \mathbf{c} \in \mathbb{C}^K\}, \quad (6.34)$$

where $\mathbb{C}^K$ is K -dimensional complex vector space. Assume that there are two signals. Without loss of generality, H is illustrated in 3-dimension as shown in Fig. 6.3. Herein, the nominal ASV $\bar{\mathbf{s}}_0$ does not coincide with the actual one $\mathbf{s}_0$. The conventional RCBs

can be regarded as searching for an optimal ASV in the uncertainty set B to maximize the output power. In this chapter, we derive a more compact constraint set instead.

Although the actual ASV $\mathbf{s}_0$ is unknown, it locates in the space H . With this property, a new nominal ASV $\hat{\mathbf{s}}_0$ in H can be estimated to form a compact uncertainty set with smaller uncertainty level. This new ASV $\hat{\mathbf{s}}_0$ is designed as a vector in H and nearest to $\bar{\mathbf{s}}_0$. It can be expressed as

$$\hat{\mathbf{s}}_0 = \mathbf{U}_s \hat{\mathbf{c}}, \quad (6.35)$$

where $\hat{\mathbf{c}}$ is the solution to the following optimization problem

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \|\mathbf{U}_s \mathbf{c} - \bar{\mathbf{s}}_0\|_2^2. \quad (6.36)$$

The optimal solution of $\hat{\mathbf{s}}_0$ is

$$\hat{\mathbf{s}}_0 = \mathbf{U}_s \mathbf{U}_s^H \bar{\mathbf{s}}_0. \quad (6.37)$$

Since $\hat{\mathbf{s}}_0$ is the projection of $\bar{\mathbf{s}}_0$ onto the signal-plus-interferences subspace H , it is straightforward that the distance between the estimated ASV $\hat{\mathbf{s}}_0$ and the actual one $\mathbf{s}_0$ is shorter than that between $\bar{\mathbf{s}}_0$ and $\mathbf{s}_0$, i.e.,

$$\|\mathbf{s}_0 - \hat{\mathbf{s}}_0\|_2^2 \leq \|\mathbf{s}_0 - \bar{\mathbf{s}}_0\|_2^2. \quad (6.38)$$

The new uncertainty constraint can be formulated as

$$\|\mathbf{s}_0 - \hat{\mathbf{s}}_0\|_2^2 \leq \epsilon', \quad (6.39)$$

where ϵ' is the new uncertainty level.

6.3.2 Proposed Robust Capon Beamformer with New Uncertainty Constraint

In this section, we discuss the application of the new compact uncertainty set in RCB1. Similar conclusion can be obtained with RCB2. With the new uncertainty constraint (6.39), the proposed RCB is formulated as

$$\begin{cases} \min_{\mathbf{s}} \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \\ \text{s.t.} \quad \|\mathbf{s} - \hat{\mathbf{s}}_0\|_2^2 \leq \epsilon' \end{cases} \quad (6.40)$$

This optimization problem (6.40) can be solved using the Lagrange multiplier methodology [43]. The optimal solution of (6.40) is obtained on the boundary of the constraint. Therefore, (6.40) can be reformulated as

$$\begin{cases} \min_{\mathbf{s}} \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \\ \text{s. t.} \quad \|\mathbf{s} - \hat{\mathbf{s}}_0\|_2^2 = \epsilon' \end{cases} \quad (6.41)$$

Similar to the derivation in Section 6.2.1, the optimal solution of the above problem is

$$\hat{\mathbf{s}} = (g^{-1} \mathbf{R}^{-1} + \mathbf{I})^{-1} \hat{\mathbf{s}}_0, \quad (6.42)$$

where the Lagrange multiplier g is the root of the constraint equation

$$\|\hat{\mathbf{s}} - \hat{\mathbf{s}}_0\|_2^2 = \|(\mathbf{I} + g\mathbf{R})^{-1}\hat{\mathbf{s}}_0\|_2^2 = \epsilon'. \quad (6.43)$$

Substituting (6.33) into (6.43) and denoting $\mathbf{y} = \mathbf{U}^H \hat{\mathbf{s}}_0$, it yields

$$h(g) = \sum_{i=1}^M \frac{|y_i|^2}{(1 + \lambda_i g)^2} = \epsilon', \quad (6.44)$$

where y_i is the i th element of vector $\mathbf{y}$.

The corresponding optimal weight of the robust beamformer and its output SINR are given by

$$\mathbf{w}_0 = \frac{\mathbf{R}^{-1}\hat{\mathbf{s}}}{\hat{\mathbf{s}}^H \mathbf{R}^{-1}\hat{\mathbf{s}}}, \quad \rho = \frac{\mathbf{w}_0^H \mathbf{R}_s \mathbf{w}_0}{\mathbf{w}_0^H \mathbf{R}_n \mathbf{w}_0}. \quad (6.45)$$

6.3.3 Analysis of Output SINR

In this section, an analysis of the output SINR of the proposed RCB (PRCB) and the conventional RCB is carried out. Since a complete analysis of SINR performance under general array imperfections represents a formidable analytical task, in this chapter, a simplified problem is discussed. We assume that only steering vector error exists in the array processor and the theoretical covariance matrix is used. In such case, the performance degradation of the Capon beamformer is caused by the error in the nominal ASV.

When there is only ASV error, a general conclusion on the output SINR of Capon beamformer is given in Lemma 6.1. Lemma 6.1 indicates that the output SINR of

Capon beamformer is determined by the angle between the nominal and true ASVs. Moreover, it is easy to show that the output SINR ρ is a monotonically increasing function of $\cos^2(\theta)$. The PRCB and the RCB have similar mathematical form as Capon beamformer except that the nominal vector $\mathbf{s}$ is replaced by the estimated one $\hat{\mathbf{s}}$ or $\bar{\mathbf{s}}$. Therefore, the output SINR of the PRCB (RCB) can be analyzed via the angle between $\hat{\mathbf{s}}$ ($\bar{\mathbf{s}}$) and $\mathbf{s}_0$.

Lemma 6.3. *The ASVs used in calculation of array optimal weight for the conventional RCBs in Section 6.2 and the PRCB are $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. According to (6.42), we have*

$$\mathbf{s}_1 = (g_1^{-1}\mathbf{R}^{-1} + \mathbf{I})^{-1}\bar{\mathbf{s}}_0,$$

$$\mathbf{s}_2 = (g_2^{-1}\mathbf{R}^{-1} + \mathbf{I})^{-1}\hat{\mathbf{s}}_0,$$

where the scales g_1 and g_2 are the optimal diagonal loading factors. Denoting

$$\begin{aligned}\cos^2(\theta_1) &= \frac{\|\mathbf{s}_0^H \mathbf{s}_1\|_{\mathbf{R}}^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\mathbf{s}_1\|_{\mathbf{R}}^2}, \\ \cos^2(\theta_2) &= \frac{\|\mathbf{s}_0^H \mathbf{s}_2\|_{\mathbf{R}}^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\mathbf{s}_2\|_{\mathbf{R}}^2},\end{aligned}$$

we have

$$\cos^2(\theta_1) \leq \cos^2(\theta_2).$$

Proof. Refer to Appendix B.4. □

According to Lemma 6.1 and 6.3, it can be concluded that the output SINR ρ_2 of

the PRCB is higher than that of the conventional RCB ρ_1 , i.e.,

$$\rho_2 \geq \rho_1. \quad (6.46)$$

6.4 Numerical Experiments

6.4.1 Performance Evaluation of RCBs without Projected ASV

In this section, some numerical simulations were carried out to evaluate the performance of the proposed RCBs. A uniform linear array containing eight sensors with half-wavelength spacing is used to enhance the SOI in the presence of strong interferences as well as uncertainty in ASV. There are two kinds of uncertainty under consideration. One is the well studied steering direction error, the other is random ASV error.

In the simulations, the estimate of signal power and SINR were the average of 200 Monte-Carlo experiments. The beampatterns were obtained from one Monte-Carlo simulation. The nondirectional noise was a spatially white Gaussian noise whose power is $-10dB$. The power of the SOI is $\sigma_0^2 = 10dB$, and the powers of the two interferences are $\sigma_1^2 = \sigma_2^2 = 20dB$. The DOA of the SOI is $\theta_0 = 0^\circ$. The DOAs of the two interferences are $\theta_1 = 60^\circ$ and $\theta_2 = 80^\circ$, respectively. The performance of the SCB is also demonstrated for the purpose of comparison.

In the first simulation, the array was assumed to have steering direction error $\Delta = 3^\circ$. The covariance matrix was estimated with different number of snapshots. It is well known that the covariance matrix estimated using sample averaging method

Figure 6.4: Output power and SINR versus the number of snapshots with steering error $\Delta = 3^\circ$, $\epsilon = 0.12$.

asymptotically approaches the true one. In the case only small number of snapshots available, the estimated error in covariance matrix also affects the performance of beamformer. The result shown in Fig. 6.4 indicates that the output powers of the RCBs are close to the true one even the number of snapshots is small. With increasing number of snapshots, the output SINRs are improved for the proposed RCBs. However, for the SCB, due to steering direction error, the target signal is cancelled. Its output SINR remains a low level. From the beampattern shown in Fig. 6.6, the SCB has serious target signal cancellation problem.

In Fig. 6.4, the RCB1 shows higher output SINR than RCB2 does. The explanation can refer to Section 6.2.3. Another experiment is done with small uncertainty level, from the result shown in Fig. 6.5, we find that the RCB1 and RCB2 have similar output SINRs. This verifies the conclusion made from geometrical illustration in Section 6.2.3.

Figure 6.5: Output power and SINR versus the number of snapshots with steering error $\Delta = 3^\circ$, $\epsilon = 0.01$.

Figure 6.6: Comparison of beampattern of RCB and SCB (The vertical dot lines indicate the direction of incident signals. $\Delta = 3^\circ$, $\epsilon = 0.15$).

Figure 6.7: Comparison of power estimation of RCB and SCB versus steering direction (The vertical dot lines and the diamonds indicate the direction and the true power of each signal. $\epsilon = 0.1$).

In second experiment, the output powers of signals at different directions are estimated when the array has arbitrary ASV error. The ASV error is simulated by a random complex vector with norm 0.3. The covariance matrix is estimated from 100 snapshots. The directions and powers of the five sources are $(-55^\circ, 10dB)$, $(-25^\circ, 20dB)$, $(0^\circ, 10dB)$, $(20^\circ, 20dB)$ and $(50^\circ, 20dB)$, respectively. With the existence of random ASV error, serious target signal cancellation effect exists in the SCB. It gives rise to large error in the estimated output power. On the other hand, the proposed RCBs do not suffer from the target signal cancellation. The simulation results in Fig. 6.7 show that the proposed RCBs give estimation results with significantly higher accuracy than SCB does.

The last experiment is carried out to compare the output SINR of the proposed

Figure 6.8: Comparison of output SINR.

beamformers with the upper bound. The output SINR of the SCB with known ASV is also evaluated. The experiment using similar parameters as the first experiment except that the steering direction error changes from 1° to 10° . The results in Fig. 6.8 show that the bounds of the RCBs are lower than the SINR of the SCB with known ASV. The output SINRs of RCBs are close to the bound when the steering error is small. Although the output SINRs of the RCBs are lower than their bound, they are robust to steering vector error as shown in all the experiments.

6.4.2 Performance Evaluation of RCBs Using Projected ASV

In this section, some numerical experiments were carried out to evaluate the performance of the proposed RCB with new compact constraint set. An ULA containing ten sensors with half-wavelength spacing is used to enhance the SOI in the presence of strong interferences as well as the uncertainty in ASV. Two kinds of uncertainty in

ASV are considered in this section. One is the well studied steering direction error, the other is random ASV error. The performance of the SCB [9], the RCB in Section 6.2.1 and [29], and the eigenspace-based beamformer (ESB) [68] are also included for the purpose of performance comparison.

6.4.2.1 Performance Evaluation Considering Steering Direction Error

We assume that the array has steering direction error. The assumed DOA of the SOI is $\theta_0 = 0^\circ$ in this section. However, the actual one is Δ . In the first experiment, the DOA and the power of the SOI are $(6^\circ, 10dB)$. The DOAs and the powers of two interferences are $(60^\circ, 20dB)$ and $(80^\circ, 20dB)$, respectively. The output SINR of each beamformer versus the number of snapshots is shown to illustrate the beamformer performance because the covariance matrix is always estimated with limited number of snapshots in practical applications.

From the results shown in Fig. 6.9(a), it can be found that PRCB, RCB and ESB all perform well at small number of snapshots N . Since the projected ASV has high accuracy in this experiment, PRCB and ESB have similar output SINR (The curves of PRCB and ESB in Fig. 6.9 overlap). The SINR of RCB is lower than that of PRCB and ESB, and SCB completely fails. In Fig. 6.9(b), it is found that PRCB and ESB have similar beampatterns. Although the response of the RCB to the SOI is similar to that of PRCB and ESB, its response peaks at $\theta = 0^\circ$. This causes larger noise gain of RCB. Moreover, Fig. 6.9(b) implies that the interference suppression performance of RCB is lower than that of PRCB and ESB. All of these factors result in lower output

SINR of RCB.

In the next experiment, the interferences are moved close to the SOI, e.g., the DOAs and the powers of the two interferences are changed to $(10^\circ, 20dB)$ and $(20^\circ, 20dB)$, respectively. The output SINR versus the number of snapshots N is shown in Fig. 6.10(a), where PRCB shows the highest output SINR. Since the DOAs of the interferences are close to that of the SOI in this experiment, the error in the projected ASV increases. It results in performance degradation of ESB. On the other hand, PRCB can tolerate ASV error to some extent.

It is known that the performance of ESB degrades when the dimension of signal-plus-interference subspace is high. In this experiment, we evaluate the performance of beamformers with large number of interferences, i.e., the dimension of signal-plus-interference subspace is high. Considering six interferences in the system, whose DOAs and powers are $(60^\circ, 10dB)$, $(80^\circ, 20dB)$, $(-30^\circ, 20dB)$, $(-50^\circ, 10dB)$, $(-70^\circ, 20dB)$ and $(-85^\circ, 20dB)$, respectively. The results shown in Fig. 6.11(a) clearly indicate that ESB has poor performance because of large error in the projected ASV due to the high dimension of signal-plus-interference subspace. Meanwhile, PRCB outperforms the other beamformers in output SINR.

It is also known that the performance of ESB strongly depends on the accurate knowledge of the dimension of signal-plus-interference subspace. The following three experiments evaluate the performance of the beamformers when error exists in the estimation of the dimension of signal-plus-interference subspace. In these experiments, the DOA and the power of the SOI is $(6^\circ, 10dB)$. The DOAs and the powers of the two

(a) Comparison of output SINR of beamformers versus number of snapshots

(b) Comparison of the beampatterns ($N = 100$)

Figure 6.9: Performance comparison of beamformers with 3 impinging sources (vertical dot lines stand for the directions of incident signals, $\Delta = 6^\circ$, $\epsilon = 8.9$, $\epsilon' = 0.1$).

(a) Comparison of output SINR of beamformers versus number of snapshots

(b) Comparison of the beampatterns ($N = 100$)

Figure 6.10: Performance comparison of beamformers with 3 impinging sources (vertical dot lines stand for the directions of incident signals, $\Delta = 6^\circ$, $\epsilon = 5.0$, $\epsilon' = 4.0$).

(a) Comparison of output SINR of beamformers versus number of snapshots

(b) Comparison of the beampatterns ($N = 100$)

Figure 6.11: Performance comparison of beamformers with 6 impinging sources (vertical dot lines stand for the directions of incident signals, $\Delta = 6^\circ$, $\epsilon = 7.0$, $\epsilon' = 3.1$).

Figure 6.12: Comparison of output SINR of beamformers versus number of snapshots with 3 impinging sources (The dimension of signal-plus-interference subspace is overestimated as 4. $\epsilon = 8.9, \epsilon' = 4.0$).

interferences are $(60^\circ, 20dB)$ and $(80^\circ, 20dB)$, respectively. The results in Fig. 6.12 are obtained when the dimension of the subspace is overestimated as four. With this overestimated dimension parameter, the performance of ESB seriously degrades, while the performance of PRCB is still better than those of the others. Two more experiments are carried out to evaluate the performances of these beamformers when the dimension parameter is underestimated as two. When the SOI is not the dominant signal, the results in Fig. 6.13 show that both PRCB and ESB fail. On the other hand, if the SOI becomes the dominant signal with power $\sigma_0^2 = 30dB$, these two beamformers still work, as can be found in Fig. 6.14. From these experiments, we suggest that overestimation of the dimension of signal-plus-interference subspace is needed for the PRCB to guarantee its robustness.

Figure 6.13: Comparison of output SINR of beamformers versus number of snapshots with 3 impinging sources (The dimension of signal-plus-interference subspace is underestimated as 2. SOI is not the dominant signal. $\epsilon = 9.0, \epsilon' = 0.1$).

Figure 6.14: Comparison of output SINR of beamformers versus number of snapshots with 3 impinging sources (The dimension of signal-plus-interference subspace is underestimated as 2. SOI is the dominant signal. $\epsilon = 9.0, \epsilon' = 8.0$).

6.4.2.2 Performance Evaluation Considering Low Input SNR

It is well known that the performance of ESB depends on the accurate knowledge of the signal-plus-interference subspace. The estimate of the dimension of the signal-plus-interference subspace is not reliable in the case of low input SNR. In this section, we show that in the case of low input SNR, the PRCB still demonstrates high robustness and has higher output SINR than the ESB does provided overestimate of the signal-plus-interference subspace is adopted.

In the first simulation, we assume that there are 7 signals. The power and DOA of the target signal are $(-5dB, 6^\circ)$. The DOAs and powers of the other six interferences are $(60^\circ, 10dB)$, $(80^\circ, 20dB)$, $(-30^\circ, 20dB)$, $(-50^\circ, 10dB)$, $(-70^\circ, 20dB)$ and $(-85^\circ, 20dB)$, respectively. The eigenvalues estimated from two covariance matrices, which are estimated using 900 snapshots and 50 snapshots, are shown in Figure 6.15. From Figure 6.15, it is quite difficult to reliably determine the dimension of the signal-plus-interference subspace, i.e., 7 signals. If the subspace dimension is correctly estimated (e.g., 7), in Figure 6.16, it is shown that both the PRCB and the ESB work but the PRCB has higher performance than the ESB does. If the subspace dimension is overestimated (e.g., 8), in Figure 6.17, it is shown that the PRCB is still robust and has higher performance than the ESB does. It should be noted that overestimate of subspace is easy to be realized. Therefore, we suggest using this skill to guarantee the robustness of the PRCB.

If we change the power of the target source to be $-10dB$, all the other parameters remain unchanged. From Figure 6.18-6.20, similar conclusions can be obtained.

(a) The eigenvalues of the estimated covariance matrix (900 snapshots)

(b) The eigenvalues of the estimated covariance matrix (50 snapshots)

Figure 6.15: The eigenvalues of the estimated covariance matrix (-5dB SNR).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.16: Performance comparison of the PRCB, RCB, ESP, SCB at -5dB input SNR (subspace correctly estimated to be 7).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.17: Performance comparison of the PRCB, RCB, ESP, SCB at -5dB input SNR (subspace overestimated to be 8).

(a) The eigenvalues of the estimated covariance matrix (900 snapshots)

(b) The eigenvalues of the estimated covariance matrix (50 snapshots)

Figure 6.18: The eigenvalues of the estimated covariance matrix (-10dB SNR).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.19: Performance comparison of the PRCB, RCB, ESP, SCB at -10dB input SNR (subspace correctly estimated to be 7).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.20: Performance comparison of the PRCB, RCB, ESP, SCB at -10dB input SNR (subspace overestimated to be 8).

In the above simulations, high dimension of the signal-plus-interference subspace is used. In the next simulation, we use low dimension of signal-plus-interference subspace. There are 3 impinging signals. The power and DOA of the target signal are $(-5dB, 6^\circ)$. The power and DOA of the interferences are $(20dB, 60^\circ)$ and $(20dB, 80^\circ)$. From the eigenvalues shown in Figure 6.21, which are estimated from two covariance matrices estimated using 900 snapshots and 50 snapshots, it seems that the estimate of subspace dimension is not reliable when the covariance matrix estimated using small number of snapshots.

If the dimension of signal-plus-interference is underestimated, e.g., to be 2. The results shown in Figure 6.22 indicates that both the PRCB and the ESB fail to work. This is not surprising because the projection based method is sensitive to correct knowledge of signal-plus-interference subspace, especially when the target signal is weaker than the interferences. To prevent this robustness problem, we still use the skill, overestimate of the signal-plus-interference subspace. When the dimension of the signal-plus-interference subspace is overestimated to be 4, the result shown in Figure 6.23 indicates that the PRCB has higher performance than the ESB does.

If we change the power of the target signal to be $-10dB$, similar conclusions can be obtained from the results shown in Figure 6.24 - 6.26.

(a) The eigenvalues of the estimated covariance matrix (900 snapshots)

(b) The eigenvalues of the estimated covariance matrix (50 snapshots)

Figure 6.21: The eigenvalues of the estimated covariance matrix (-5dB SNR).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.22: Performance comparison of the PRCB, RCB, ESP, SCB at -5dB input SNR (subspace underestimated to be 2).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.23: Performance comparison of the PRCB, RCB, ESP, SCB at -5dB input SNR (subspace overestimated to be 4).

(a) The eigenvalues of the estimated covariance matrix (900 snapshots)

(b) The eigenvalues of the estimated covariance matrix (50 snapshots)

Figure 6.24: The eigenvalues of the estimated covariance matrix (-10dB SNR).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beampattern

Figure 6.25: Performance comparison of the PRCB, RCB, ESP, SCB at -10dB input SNR (subspace underestimated to be 2).

(a) Comparison of output SINR of the PRCB, RCB, ESP, SCB under different number of snapshots

(b) Comparison of beam pattern

Figure 6.26: Performance comparison of the PRCB, RCB, ESP, SCB at -10dB input SNR (subspace overestimated to be 4).

6.4.2.3 Performance Evaluation Considering Random Error in Array Steering Vector

In Section 6.4.2.1, it is assumed that the array has steering direction error only. In this section, random error in ASV is considered and the performance of beamformers is evaluated. The ASV error in the simulations are generated as random complex vector whose norm is 40% of the norm of the true ASV.

Fig. 6.27 shows that PRCB and ESB have similar performance. The results in Fig. 6.28 are obtained using 6 signals similar to those described in Section 6.4.2.1 except that the DOA of the SOI is changed to 0° . The results in Fig. 6.29 and Fig. 6.30 are obtained when error exists in the estimate of the dimension of signal-plus-interference subspace. Comparing the corresponding results shown in Section 6.4.2.1, we find that the PRCB is also robust to arbitrary ASV error. Moreover, it has high output SINR regardless of the type of ASV error.

6.5 Summary

Robust Capon beamformers are proposed based on output power maximization subject to uncertainty constraint on the nominal ASV. Two types of RCBs are discussed. Although they are derived with different set of constraints, they share similar mathematical form. Performance analysis is also carried out to study the optimal output SINR of the proposed RCBs. Although the RCBs cannot achieve the optimal output SINR, simulation shows they have high robustness to arbitrary ASV error. To im-

Figure 6.27: Comparison of output SINR of beamformers with arbitrary ASV error (The parameters of target source and two interferences are $(0^\circ, 10\text{dB})$, $(60^\circ, 20\text{dB})$ and $(80^\circ, 20\text{dB})$. $\epsilon = 4.0$, $\epsilon' = 0.1$).

Figure 6.28: Comparison of output SINR of beamformers with arbitrary ASV error with 7 impinging sources ($\epsilon = 4.0$, $\epsilon' = 2.5$).

Figure 6.29: Comparison of output SINR of beamformers versus number of snapshots with 3 impinging sources (The dimension of signal-plus-interference subspace is overestimated as 4. $\epsilon = 6.0, \epsilon' = 0.3$).

prove the performance of the RCBs with large uncertainty set, a new constraint on the uncertainty of ASV is proposed. The new constraint is formed as replacing the nominal ASV in the original constraint as the projected one into signal-plus-interference subspace. It is proved that the improved RCB has higher performance than other RCBs. Simulations on various cases were carried out to show the effectiveness of the improved RCB. In the cases when the dimension of signal-plus-interferences subspace cannot be obtained accurately, the improved RCB still has high SINR improvement if overestimate of subspace dimension is adopted.

(a) Comparison of output SINR of beamformers versus the number of snapshots (The target signal is not the dominant signal. $\epsilon = 4.0, \epsilon' = 0.1$)

(b) Comparison of output SINR of beamformers versus the number of snapshots (The target signal is the dominant signal. $\epsilon = 9.0, \epsilon' = 9.0$)

Figure 6.30: Comparison of output SINR of beamformers versus the number of snapshots with 3 impinging sources (The dimension of signal-plus-interference subspace is underestimated as 2).

Chapter 7

Robust Subspace Analysis and Its Application for Speech Enhancement

7.1 Introduction

High quality speech acquisition in adverse environment has received increasing interest in applications such as speech recognition and hand-free telephone communications. Conventional single channel speech enhancement methods based on temporal information processing, such as spectral subtraction [69], do not provide sufficient improvement on speech quality, especially when noise is strong. In the past few decades, microphone array, which incorporates both signal spatial and temporal information, has been proposed as a promising technique for speech enhancement and noise suppression.

To improve their robustness of adaptive beamformer, numerous methods [17, 18, 19, 33, 24, 35, 26] have been proposed to combat array imperfections, including steering direction error, sensor location error, array channel mismatch, etc. These methods assume that target signal propagates through known direct path and the geometry of the array is also available. However, in applications such as speech acquisition in adverse acoustic environments, the source signal propagates not merely along direct path. There are also unknown multipath and reverberation effects. In such cases, target signal is often cancelled to some extent with conventional adaptive beamforming approaches.

Concerning the existence of arbitrary array imperfections and reverberation effects, the channel relating target source and each sensor is more precisely modelled as linear filter. Compared with the relatively simple channel model used in conventional adaptive beamforming, more parameters are incorporated in the new model. In Chapter 4 and 5, robust beamformer against generalized time-delay error or phase error are discussed. Moreover, in Chapter 6, beamformers robust to arbitrary error in ASV is also proposed. All of these beamformer require the prior information on the ASV of the target signal, e.g., the DOA of target signal, array geometry information etc. In some applications, these information are unknown prior. In some cases, the position of the target signal is changing so that the ASV is not fixed. Moreover, we have shown that the robustness of array beamformer is obtained at cost of reduced performance in noise interference suppression. Uncertainty constraint with large uncertainty level is not appropriate in practice. In such cases, the conventional robust beamformers may not work.

An alternative method is to use estimated transfer function (TF) vector in adaptive beamforming as discussed in Chapter 3. A straightforward solution for TF identification with training signal [46] is easy to understand and implement. However, this off-line measurement method has limited applications due to two reasons. The first one is the unavailability of training signal in some cases. The other one is that, training signal has to be transmitted frequently when the environment changes or signal source moves. In many applications, on-line identification of TFs is appreciated. One potential solution is the blind channel identification (BCI) technique ([70,71] and references therein). However, BCI technique in acoustic applications did not prove itself to be as successful as in wireless communications because the acoustic channel impulse response is long and complex.

In [31], subspace tracking method [72] with the prior measured mean energy of response vector in frequency domain was used to identify the channel TFs. It was pointed out that the orthogonal basis for signal subspace does not correspond to the TF vector [72]. However, at high SNR with rank-one signal subspace, the TF vector corresponds to signal subspace basis vector up to a multiplicative scale. Therefore, in [31], the author proposed to apply subspace tracking during the period with active speech so that high input SNR could be guaranteed. Nevertheless, when noise is strong, the speech activity detection becomes unreliable. Moreover, the identified subspace basis still has large estimation error when point noise exists.

In many speech enhancement applications, noise and interference can be considered as stationary or slowly varying signal. With this assumption, we propose a robust signal

subspace analysis method for transfer function identification in frequency domain. It estimates the signal subspace using differential signal covariance matrix. The proposed method does not need a speech activity detector. Moreover, it has low estimation error because differential covariance matrix weakens the effect of noise and interference in subspace estimation. To simplify the TF identification, we estimate transfer function ratios (TFRs) instead of TFs in this chapter. With the estimated TFR vector, an suboptimal matched-filter array [46] based GSC (MF-GSC), which is a special case of the extended GSC discussed in Chapter 3, is formed to suppress noise and interference.

This chapter is organized as follows. The system model is discussed in Section 7.2. The MF-GSC is briefly introduced. In Section 7.3, the robust signal subspace analysis method for transfer function ratio vector (TFRV) estimation exploiting the nonstationarity of speech signal as well as the upper bound of the estimation error are derived. Some numerical results using simulated signal and actually recorded signal are shown in Section 7.4 to evaluate the performance of the proposed method. In Section 7.5, a brief conclusion is given.

7.2 System Model and Matched-Filter Array Based GSC

A microphone array system with M sensors is studied in this chapter. The target speech signal $s(k)$ propagates through the i th channel with impulse response (IR) $h_i(k)$, $i = 1, 2, \dots, M$, and is corrupted by uncorrelated additive noise $n_i(k)$. The

noise $n_i(k)$ may include environment noise, sensor noise and interferences if there is any. The sensor received signal $x_i(k)$ of the i th channel is expressed as

$$x_i(k) = h_i(k) \star s(k) + n_i(k), \quad i = 1, 2, \dots, M. \quad (7.1)$$

Multiplying both sides of (7.1) by a rectangular window function over the m th analysis frame and applying the DFT operator, it yields

$$\mathbf{x}(m, \omega) \approx s(m, \omega) \mathbf{h}(\omega) + \mathbf{n}(m, \omega), \quad (7.2)$$

where

$$\begin{aligned} \mathbf{x}(m, \omega) &= [x_1(m, \omega) \ x_2(m, \omega) \ \cdots \ x_M(m, \omega)]^T, \\ \mathbf{n}(m, \omega) &= [n_1(m, \omega) \ n_2(m, \omega) \ \cdots \ n_M(m, \omega)]^T, \\ \mathbf{h}(\omega) &= [h_1(\omega) \ h_2(\omega) \ \cdots \ h_M(\omega)]^T, \end{aligned} \quad (7.3)$$

where $\omega = 0, 1, \dots, N - 1$ denotes the frequency index. The approximation in (7.2) is justified for frame length N sufficiently large. The transformed signals $x_i(m, \omega)$, $n_i(m, \omega)$, $s(m, \omega)$ and the channel transfer function $h_i(\omega)$ are the short time Fourier transform of $x_i(k)$, $n_i(k)$, $s(k)$ and $h_i(k)$, respectively.

When $\mathbf{h}(\omega)$ is known, an extended GSC (See Chapter 3) in frequency domain can be constructed as shown in Fig. 7.1. However, in many applications, true transfer function vector (TFV) $\mathbf{h}(\omega)$ is difficult to obtain, while as shown in Section 7.3, its scaled version $\tilde{\mathbf{h}}(\omega) = \alpha(\omega) \mathbf{h}(\omega)$ is easier to estimate, where $\alpha(\omega)$ is a nonzero complex scalar. To

Figure 7.1: Schematic of MF-GSC in frequency domain.

remove the ambiguity of $\alpha(\omega)$, without loss of generality, the TFV is normalized to its first element. The resulting new vector is given by

$$\tilde{\mathbf{h}}(\omega) = \left[1 \quad \frac{h_2(\omega)}{h_1(\omega)} \quad \dots \quad \frac{h_M(\omega)}{h_1(\omega)} \right]^T. \quad (7.4)$$

It is clear that the elements in the vector $\tilde{\mathbf{h}}(\omega)$ are the TFRs to the first channel. We call this vector $\tilde{\mathbf{h}}(\omega)$ as TFRV.

If TFRV $\tilde{\mathbf{h}}(\omega)$ is obtained, an MF-GSC is formed as

$$\begin{aligned}
 \mathbf{g}(\omega) &= \frac{\tilde{\mathbf{h}}(\omega)}{\|\tilde{\mathbf{h}}(\omega)\|^2}, \\
 y(m, \omega) &= \mathbf{g}^H(\omega) \mathbf{x}(m, \omega), \\
 \mathbf{B}(\omega) &= \begin{bmatrix} -\tilde{h}_2(\omega) & \cdots & -\tilde{h}_M(\omega) \\ 1 & \vdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}, \\
 \mathbf{z}(m, \omega) &= [z_1(m, \omega) \cdots z_{M-1}(m, \omega)]^T = \mathbf{B}(\omega)^T \mathbf{x}(m, \omega), \\
 v(m, \omega) &= \mathbf{w}^H(m, \omega) \mathbf{z}(m, \omega), \\
 e(m, \omega) &= y(m, \omega) - v(m, \omega), \\
 \mathbf{w}(m+1, \omega) &= \mathbf{w}(m, \omega) + \rho_f \frac{e^*(m, \omega) \mathbf{z}(m, \omega)}{P_{\mathbf{x}}(m, \omega)}, \\
 P_{\mathbf{x}}(m, \omega) &= \alpha P_{\mathbf{x}}(m-1, \omega) + (1-\alpha) \|\mathbf{x}(m, \omega)\|^2,
 \end{aligned} \tag{7.5}$$

where $y(m, \omega)$, $\mathbf{z}(m, \omega)$, $v(m, \omega)$, $e(m, \omega)$ are the output signals of the fixed beam-former, the blocking matrix, the multichannel noise canceller and the MF-GSC, respectively. $\mathbf{B}(\omega)$ is the blocking matrix. α ($0 < \alpha < 1$) is the forgetting factor and ρ_f ($0 < \rho_f < 2$) is the stepsize of the NLMS method [73, 41]. Since the adaptive weight $\mathbf{w}(m, \omega)$ of noise canceller should be updated only when there is no target signal, the power of signal $\mathbf{x}(m, \omega)$ is used instead of the power of $\mathbf{z}(m, \omega)$ to normalize the step-size. With this modification, the multichannel noise canceller can always on because the adaptation term $\rho_f \frac{e^*(m, \omega) \mathbf{z}(m, \omega)}{P_{\mathbf{x}}(m, \omega)}$ is very small when target speech signal exists.

The speech component $\hat{s}(m, \omega)$ in the output signal of fixed beamformer is

$$\hat{s}(m, \omega) = h_1(\omega)s(m, \omega). \quad (7.6)$$

It indicates that, when $\tilde{\mathbf{h}}(\omega)$ is used to form MF-GSC, the output speech signal is the filtered original signal by the first channel. This MF-GSC does not have dereverberation effect. If dereverberation is required, like [31], the prior measured mean energy of response vector can be used in case of small variation of source position. In this chapter, only the SINR improvement of target speech signal is considered.

7.3 Robust Signal Subspace Analysis Method for Transfer Function Ratio Vector Estimation

Practical applications usually require the TFRV $\tilde{\mathbf{h}}(\omega)$ to be estimated online. In this section, we first formulate the estimate of $\tilde{\mathbf{h}}(\omega)$ as rank one signal subspace estimation problem. Then, we discuss how noise affects the signal subspace. The upper bound of estimation error and its relationship to SNR are also derived. Finally, a new robust signal subspace analysis method is proposed for TFRV estimation.

The covariance matrix $\mathbf{R}_{\mathbf{x}}(m, \omega)$ of $\mathbf{x}(m, \omega)$ is

$$\mathbf{R}_{\mathbf{x}}(m, \omega) \triangleq E\{\mathbf{x}(m, \omega)\mathbf{x}^H(m, \omega)\} = \mathbf{R}_s(m, \omega) + \mathbf{R}_n(m, \omega), \quad (7.7)$$

where $\mathbf{R}_s(m, \omega) \triangleq E\{|s(m, \omega)|^2 \mathbf{h}(\omega)\mathbf{h}^H(\omega)\}$ and $\mathbf{R}_n(m, \omega) \triangleq E\{\mathbf{n}(m, \omega)\mathbf{n}^H(m, \omega)\}$

are the covariance matrices of target signal component and noise, respectively. It is apparent that, if $\mathbf{R}_n(m, \omega) = \mathbf{0}$, $\mathbf{h}(\omega)$ is in the signal space of $\mathbf{R}_x(m, \omega)$, i.e.,

$$\mathbf{h}(\omega) \in \text{span}(\mathbf{u}_1) = \phi \mathbf{u}_1, \quad (7.8)$$

where $\mathbf{u}_1$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_x$, and ϕ is a nonzero complex scale. When $\mathbf{u}_1$ is estimated, the TFRV $\tilde{\mathbf{h}}(\omega)$ can be easily constructed through dividing $\mathbf{u}_1$ by its first element.

If $\mathbf{R}_n(m, \omega) \neq \mathbf{0}$, the relationship in (7.8) may not hold. Accordingly, the estimated TFRV deviate from the true one. In Theorem 7.1, we show that certain types of noise do not introduce perturbation on signal subspace.

Theorem 7.1. *If the noise covariance matrix $\mathbf{R}_n$ and signal covariance matrix $\mathbf{R}_s$ are commute, i.e., $\mathbf{R}_n \mathbf{R}_s = \mathbf{R}_s \mathbf{R}_n$, such noise does not introduce perturbation on signal subspace.*

Proof. Refer to C.1. □

The white noise, whose covariance matrix is $\sigma_n^2 \mathbf{I}$, is a special case of Theorem 7.1. However, most of the realistic noises, especially point noises, do not satisfy Theorem 7.1. If such noise exists, the accuracy of estimated TFRV can not be guaranteed. In fact, the estimated signal subspace is still applicable if $SNR \gg 1$. To support this straightforward conclusion, the upper bound of estimation error of signal subspace is given in Theorem 7.2.

Theorem 7.2. *Assume that $\mathbf{R}_s, \mathbf{R}_n$, and $\mathbf{R}_x = \mathbf{R}_s + \mathbf{R}_n \in \mathbb{C}^{M \times M}$ are non-negative*

Hermitian matrices, where $\mathbf{R}_s$ is of rank one with eigenvalue λ_s , $\mathbf{R}_n$ has full rank with eigenvalues $\lambda_i \in [\sigma_{n,min}^2, \sigma_{n,max}^2] (\sigma_{n,max}^2 < \lambda_s)$. $\mathbf{u}_1$ and $\tilde{\mathbf{u}}_1$ are the eigenvectors corresponding to the largest eigenvalues of $\mathbf{R}_s$ and $\mathbf{R}_x$, respectively. We have,

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|_2^2 \leq 2 \left(1 - \sqrt{1 - \left(\frac{1}{\rho - k} \right)^2} \right),$$

where the SNR ρ and the scale k are defined as $\rho = \frac{\lambda_s}{\|\mathbf{R}_n\|_F}$ and $k = \frac{\sigma_{n,max}^2}{\|\mathbf{R}_n\|_F}$. When SNR is high, approximately,

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|_2^2 \leq \frac{1}{\rho^2}.$$

Proof. Refer to C.2. □

Remark 7.1. Theorem 7.2 reveals that signal subspace can be used for TFRV estimation if SNR is high enough. Nevertheless, when SNR is low, the accuracy of the estimated signal subspace can not be guaranteed.

Assume that the noise is stationary, i.e. $\mathbf{R}_n(m, \omega) = \mathbf{R}_n(\omega)$. Taking the difference between two covariance matrix estimates, we have

$$\begin{aligned} \Delta \mathbf{R}_x(m, \omega) &\triangleq \mathbf{R}_x(m, \omega) - \mathbf{R}_x(m - l, \omega) \\ &= (\sigma_s^2(m, \omega) - \sigma_s^2(m - l, \omega)) \mathbf{h}(\omega) \mathbf{h}^H(\omega) \\ &\triangleq \Delta \sigma_s^2(m, \omega) \mathbf{h}(\omega) \mathbf{h}^H(\omega). \end{aligned} \tag{7.9}$$

If $\Delta \sigma_s^2(m, \omega)$ is nonzero, the subspace of $\Delta \mathbf{R}_x(m, \omega)$ produces estimate of signal sub-

space with high accuracy. With this property, we propose a method which is robust to stationary or slowly varying noise.

In real applications, the noise is stationary or slowly varying. Moreover, signal covariance matrix is estimated using time average

$$\begin{aligned}\hat{\mathbf{R}}_{\mathbf{x}}(m, \omega) &= \frac{1}{L} \sum_{i=m-L+1}^m \mathbf{x}(i, \omega) \mathbf{x}^H(i, \omega) \\ &= \mathbf{R}_{\mathbf{s}}(m, \omega) + \mathbf{R}_{\mathbf{n}}(\omega) + \mathbf{E}_{\mathbf{x}}(m, \omega),\end{aligned}\tag{7.10}$$

where $\mathbf{E}_{\mathbf{x}}(m, \omega)$ is an estimation error matrix. It includes the time varying part of the noise covariance matrix and other estimation errors such as time average error. Taking the difference between two estimates of $\hat{\mathbf{R}}_{\mathbf{x}}(m, \omega)$, we have

$$\begin{aligned}\Delta \hat{\mathbf{R}}_{\mathbf{x}}(m, \omega) &\triangleq \hat{\mathbf{R}}_{\mathbf{x}}(m, \omega) - \hat{\mathbf{R}}_{\mathbf{x}}(m-l, \omega) \\ &\triangleq \Delta \sigma_s^2(m, \omega) \mathbf{h}(\omega) \mathbf{h}^H(\omega) + \Delta \mathbf{E}_{\mathbf{x}}(m, \omega),\end{aligned}\tag{7.11}$$

where $\Delta \mathbf{E}_{\mathbf{x}}(m, \omega) \triangleq \mathbf{E}_{\mathbf{x}}(m, \omega) - \mathbf{E}_{\mathbf{x}}(m-l, \omega)$ is a differential estimation error matrix. If $\Delta \mathbf{E}_{\mathbf{x}}(m, \omega)$ is a non-negative Hermitian matrix, it is proved in Theorem 7.2 that the estimate of TFRV has high accuracy if SNR is high enough. When $\Delta \mathbf{E}_{\mathbf{x}}(m, \omega)$ is an arbitrary matrix, the error bound of TFRV estimation is given in Theorem 7.3.

Theorem 7.3. *Assume that $\mathbf{R}_s$ is non-negative Hermitian matrix, $\mathbf{R}_x = \mathbf{R}_s + \mathbf{R}_n$, where $\mathbf{R}_n \in \mathbb{C}^{M \times M}$, and the Euclidean norm of $\mathbf{R}_n$ is $\|\mathbf{R}_n\|_2^2 = \varepsilon$. $\mathbf{R}_s$ is of rank one with largest eigenvalue λ_s ($\lambda_s > 2\varepsilon$). $\mathbf{u}_1$ and $\tilde{\mathbf{u}}_1$ are the eigenvectors corresponding to*

the largest eigenvalues of $\mathbf{R}_s$ and $\mathbf{R}_x$, respectively. We have

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|_2^2 \leq 2 \left(1 - \sqrt{1 - \left(\frac{2}{\rho - 2} \right)^2} \right),$$

where ρ is the SNR defined as $\rho \triangleq \frac{\lambda_s}{\varepsilon}$. When SNR ρ is high, approximately,

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|_2^2 \leq \frac{4}{\rho^2}.$$

Proof. Refer to C.3. □

Remark 7.2. Theorem 7.3 indicates that the estimated subspace has sufficient accuracy if SNR is high enough regardless of the type of error matrix $\Delta \mathbf{E}_x(m, \omega)$ in (7.11).

Consider the differential matrix in (7.11), the alternative SNR is guaranteed high in the case either $\Delta \sigma_s^2(m, \omega)$ is large or $\|\Delta \mathbf{E}_x(m, \omega)\|_2$ is small. However, $\Delta \sigma_s^2(m, \omega)$ is not always positive, nor does it always have large value. In order to get a reliable estimate, a new weighted differential covariance matrix $\Delta \bar{\mathbf{R}}(m, \omega)$ is proposed for subspace analysis

$$\Delta \bar{\mathbf{R}}(m, \omega) = \frac{1}{K} \sum_{i=m-K+1}^m w_i \Delta \hat{\mathbf{R}}_x(i, \omega), \quad (7.12)$$

where the weight w_i is used to accumulate the power of $\Delta \sigma_s^2(i, \omega)$ and suppress $\Delta \mathbf{E}_x(i, \omega)$.

In this chapter, we adopt a simple and efficient weight design method. First, the trace

of matrix $\Delta\hat{\mathbf{R}}_{\mathbf{x}}(i, \omega)$ is calculated by

$$\begin{aligned} \text{tr}\{\Delta\hat{\mathbf{R}}_{\mathbf{x}}(i, \omega)\} &= \text{tr}\{\Delta\sigma_s^2(i, \omega)\mathbf{h}(\omega)\mathbf{h}^H(\omega)\} + \text{tr}\{\Delta\mathbf{E}_{\mathbf{x}}(i, \omega)\} \\ &\approx \Delta\sigma_s^2(i, \omega)\|\mathbf{h}(\omega)\|_2^2, \end{aligned} \quad (7.13)$$

which indicates that the trace of matrix $\Delta\hat{\mathbf{R}}_{\mathbf{x}}(i, \omega)$ has the same sign as $\Delta\sigma_s^2(i, \omega)$.

Hence w_i is given by

$$w_i = \begin{cases} 1, & \text{tr}\{\Delta\hat{\mathbf{R}}_{\mathbf{x}}(i, \omega)\} > \eta; \\ 0, & |\text{tr}\{\Delta\hat{\mathbf{R}}_{\mathbf{x}}(i, \omega)\}| \leq \eta; \\ -1, & \text{tr}\{\Delta\hat{\mathbf{R}}_{\mathbf{x}}(i, \omega)\} < -\eta. \end{cases} \quad (7.14)$$

where $\eta > 0$ is a threshold. With the estimated $\Delta\bar{\mathbf{R}}(m, \omega)$, signal subspace can be obtained by eigenvalue decomposition (EVD), or using recursive principal component estimate method (See [74] and reference therein).

7.4 Numerical Experiments

In this section, we evaluate the performance of the proposed method using a microphone array for speech enhancement and noise suppression. The linear microphone array, as shown in Fig. 7.2, consists of 5 elements with inter-element distance $0.04m, 0.06m, 0.10m$ and $0.16m$. It is placed in the center of a conference room with dimension $(5.8m \times 4.0m \times 3.2m)$. The position of each signal source is also shown in Fig. 7.2. The signal sources are both omni-directional. The room IR relating the speech

source and each microphone is obtained through image method [75] with a sampling rate $8kHz$ and a different reverberation time T_{60} , which can be expressed as a function of the average absorption coefficient γ of the walls, according to Eyring's formula,

$$T_{60} = \frac{0.163V}{-S \log(1 - \gamma)}, \quad (7.15)$$

with V the volume of the room and S the total surface of the room.

Figure 7.2: Configuration of the microphone array system.

In all of the following simulations, a male speech counting from one to ten is used as target signal. The noise signal is recorded in a conference room with lowpass spectrum. Since the target speech is nonstationary, herein, average SNR ς is used for performance evaluation. For a given signal $x(i)$, average SNR is defined as

$$\varsigma \triangleq \frac{\langle |x(i)|^2 \rangle_{T_s} - \langle |x(i)|^2 \rangle_{T_n}}{\langle |x(i)|^2 \rangle_{T_n}}, \quad (7.16)$$

where $\langle \cdot \rangle_T$ denotes the average in time interval T . T_s and T_n denote the time intervals in which speech signal is active and inactive, respectively. In Fig. 7.3, the

clear speech signal and the microphone received signal with -5dB SNR are shown together with the average SNR in frequency domain.

Figure 7.3: Signal waveform and its SNR at each frequency bin (average SNR: -5dB).

Three different MF-GSC processors are used in simulation, RSA-GSC, SA-GSC and TD-GSC (They are of same structure, but with TFRVs estimated from different methods, namely, the proposed robust subspace analysis (RSA), conventional subspace analysis (SA) and time delay (TD) estimate method). A robust speech detector [76] is used for SA-GSC and TD-GSC.

K	20	25	30	35	40	45
SINR(dB)	17.01	18.5	19.1	19.3	19.5	19.6

Table 7.1: Output SINR of RSA-GSC with different average factor K .

7.4.1 Performance Evaluation of RSA and SA for TFRV Estimation

In this simulation, reverberation time is $T_{60} = 50ms$. FFT length N is selected as 64 to cover the strong early reflections. $L = 50$ blocks of signal are used to obtain an estimate of covariance matrix of received array signal. The performance of the proposed method depends on the number of averaging blocks K . Since it is difficult to derive a theoretical optimal value of K , in this paper, K is determined by experiment. An experiment was carried out with different K under $T_{60} = 150ms$ and input SINR 0dB. As shown in Table 7.1, large K is appreciated to have high output SINR. However, the output SINR changes little if K is greater than 30. Therefore, in the following simulations, K is selected as 30. Since the estimated TFRV $\tilde{\mathbf{h}}(\omega)$ is a scaled version of true TFV $\mathbf{h}(\omega)$, in order to eliminate the effect of arbitrary scale, a projection estimation error E_p , which is the error between the true TFV and the projection misalignment vector [77] of the estimated TFRV $\tilde{\mathbf{h}}(\omega)$, is defined as

$$E_p(\omega) \triangleq \min_g \|\mathbf{h}(\omega) - g\tilde{\mathbf{h}}(\omega)\|_2^2 = \|\mathbf{h}(\omega) - \frac{\tilde{\mathbf{h}}^H(\omega)\mathbf{h}(\omega)}{\|\tilde{\mathbf{h}}(\omega)\|_2^2}\tilde{\mathbf{h}}(\omega)\|_2^2. \quad (7.17)$$

In Fig. 7.4, a comparison of estimation error between RSA and SA under different SNR is shown. The RSA method outperforms SA, especially when SNR is low. The relative inferior performance of SA method is caused by the unreliability of speech

detector at low SNR or the perturbation error on signal subspace due to noise.

For high input SNR, at some frequency bins, the performance of RSA is inferior to that of SA. This can be explained that the noise reduction effect brought by difference operation is not as significant as the signal power reduction effects. However, with high input SNR, all the estimation errors of RSA and SA methods are so small that they do not affect the performance of GSC in a significant manner.

Figure 7.4: Estimation error at different input SNR.

From subfigure (c) in Fig. 7.3, for a signal with -5dB average SNR, the SNR of the signal component above 2500Hz is higher than -5dB . This results in the lower estimation error at high frequency bins above 2500Hz in Fig. 7.4.

7.4.2 SNR Improvement of MF-GSC Using Simulated Data

In this section, we compare the SNR improvement performance of the proposed RSA-GSC for speech enhancement with that of SA-GSC and TD-GSC. The parameters used in (7.5) are $\rho_f = 0.01$ and $\alpha = 0$. The noise cancellation filter in GSC is always on in the simulation.

Figure 7.5: Comparison of SNR improvements among RSA-GSC, SA-GSC and TD-GSC under different reverberation time T_{60} .

Fig. 7.5 shows the SNR improvements of various methods under different room reverberation times. It is found that RSA-GSC outperforms SA-GSC, especially at low input SNR. For high input SNR, RSA-GSC and SA-GSC have similar performance because the TFRVs estimated by RSA and SA are both very close to the true values. Moreover, both RSA-GSC and SA-GSC outperform TD-GSC, more obviously at low input SNR.

It is observed that the performance of TD-GSC does not have smooth curve in Fig. 7.5. It has very poor output SNR when the input SNR is low, because in this case the estimated time delay has large error. With higher input SNR, the accurate time delay estimation causes the TD-GSC to have high output SNR.

The simulation results also show that the SNR improvement of all the GSC processors degrade with increasing reverberation time. The situation is especially serious for TD-GSC because the true channel impulse response is quite different from the pure delay method even if accurate time delay is estimated. Such channel impulse response mismatch causes severe signal cancellation in the output signal of TD-GSC.

7.4.3 Performance of Speech Enhancement Using Real Recorded Data

The simulation of each method using real recorded data is carried out to evaluate the performance in realistic environment. The data are recorded from a microphone array in a real conference room with reverberation time $T_{60} \approx 200ms$. The speech and the noise signal are played by loudspeakers placed at different locations. The average input signal SNR is around $0dB$. This recorded data are processed off-line using the three different GSCs. It is shown in (7.6) that the output speech signal is the filtered original signal by the reference channel. Since the impulse response of the reference channel is unknown, in this chapter, we evaluate the performance through the waveform and sonogram of the output signal of each GSC. The waveforms and their sonogram of each signal are shown in Fig. 7.6. The output signal of RSA-GSC shown in Fig. 7.6(c) has

good quality compared with the original speech signal in Fig. 7.6(a), and the noise corrupted speech signal in Fig. 7.6(b) is greatly enhanced with low distortion. The output signal of SA-GSC is also enhanced as shown in Fig. 7.6(d). However, the residue noise is still high and some target signal cancellation occurs. In Fig. 7.6(e), we find that the signal cancellation effect of TD-GSC is severe. Hence, the output signal has low SNR and large distortion.

7.5 Summary

A microphone array for speech enhancement and noise suppression using TFRV estimated by robust signal subspace analysis method is proposed in this chapter. The proposed method outperforms conventional SA-GSC and TD-GSC, especially when input SNR is low. Moreover, the proposed RSA-GSC does not require speech activity detector which is not reliable at low input SNR. Experiments using simulated and recorded data show that the proposed method performs well even in adverse environment.

(a) Clear speech signal and its sonogram,

(b) Noise corrupted speech signal and its sonogram,

(c) Enhanced speech signal using RSA-GSC and its sonogram,

(d) Enhanced speech signal using SA-GSC and its sonogram,

(e) Enhanced speech signal using TD-GSC and its sonogram,

Figure 7.6: The waveforms and their sonogram of the speech signal.

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

In this thesis, we have focused on improving the robustness and output SINR of adaptive beamformer. Several approaches were proposed to make the adaptive beamformer robust to various array imperfections, including generalized time-delay and phase error, arbitrary array steering vector error. Meanwhile, some of our approaches also improve the output SINR of the adaptive beamformers.

Our work has been carried out in four aspects. Firstly, we derived an extended GSC which adopts arbitrary transfer function relating signal source and each sensor. The implementation of the extended GSC in time domain is straightforward. However, it has high computational load with long channel impulse response length. The frequency domain implementation reduces computational load by exploiting the FFT technique. When the length of FFT is long enough, we derived another simplified implementation

of the extended GSC approximately in frequency domain. It indicates that the GSC can be independently implemented at each frequency bin. Therefore, robust wideband beamformer in time domain can also be studied as narrowband beamformer in frequency domain using FFT/IFFT techniques. Although this is not a new discovery, our derivation shows the applicability of this simplified implementation for the extended GSC. Since the conventional GSC is a special case of the extended GSC when the pure delay channel is used, the derived methods can also be applied to conventional GSCs.

Secondly, some beamformers robust to various array imperfections were proposed in this thesis. Most of the conventional robust beamformers only deal with array steering direction error. We proposed robust beamformer against generalized time-delay error, which includes more broad class of array imperfections, such as steering direction error, array geometry error and quantization time delay error. We also proposed beamformer robust to generalized phase error. Compared with time-delay error, the generalized phase error includes more types of array imperfections such as sensor phase error. Since array imperfections result in target signal cancellation, the robust beamformers are derived to maximize the output power of adaptive beamformer with respect to time delay error or generalized phase error. In the optimization procedure, the feasible range of the time or phase errors can be constrained to avoid of mistracking target signal. Both robust beamformers have advantage that they do not suffer from performance loss in interference suppression. In the acceptance region, these beamformers have almost flat response. Moreover, the proposed beamformers are also robust to the errors in array covariance matrix estimation due to limited available snapshots, as verified by simulations.

Thirdly, we extend the beamformer robust to arbitrary array steering vector error. The time-delay error and generalized phase error can be considered as modelled array imperfections. Many imperfections in practice cannot be expressed in simple model. They cause arbitrary error in array steering vector. Although these errors are arbitrary, we can assume that the true array steering vector locates in an ellipsoid centered at the nominal array steering vector. With this assumption, we also proposed robust beamformers which is derived by maximizing the output power of adaptive beamformer subject to constraint that the true array steering vector locates in an uncertainty set. Using the Lagrange multiplier methodology, the resulting robust beamformer belongs to diagonal loading approach. Its diagonal loading factor is calculated from the constraint equation.

According to optimization theory, if the inequality constraint is active, it is treated as equality constraint. The optimal solution is obtained at the boundary of constraints. To eliminate this effect on the final output SINR, we also proposed robust beamformer with additional constraint that the norm of array steering vector is constant. The resulting robust beamformer also belongs to diagonal loading approach. It has higher output SINR than the former one does. Theoretical analysis of the upper bound of their output SINRs was also carried out in this thesis. The result show that the SINR of the proposed methods do not approach the upper bound. There still has room for improving the output performance of the proposed robust beamformer. The derivations also reveal the relationship between the proposed robust methods and the robust beamformer based on principal component analysis. The applicability of these beamformers was also discussed.

A new uncertainty set of array steering vector was proposed in this thesis. We use the known knowledge on the signal-plus-interference subspace to construct a more compact uncertainty constraint set of array steering vector. With the new uncertainty constraint, the output SINR of the robust beamformer is higher than that of the one which uses conventional uncertainty constraint set. Theoretical proof on the SINR improvement was also carried out.

Lastly, according to specific application, we proposed beamformers for speech enhancement in presence of strong stationary noise. The estimate of true channel transfer functions relating signal source and sensors are sometime difficult. If some reverberation effects in speech are allowed, the estimation of TFs can be simplified as estimation of scaled ASV at each frequency bins. Since the speech signal is nonstationary, this property can be exploited to estimate the ASV corresponding to the target signal, robust matched-filter based GSC can be constructed using the estimated ASV. A robust subspace analysis method is proposed to estimate the true ASV. The ASV is estimated as the eigenvector of the differential array covariance matrix. Theoretical analysis on the upper bound of estimation error is provided. Experiments using simulation data and real recorded data show that the proposed methods have high performance even in adverse environments.

8.2 Recommendations for Further Research

Based on the techniques and schemes developed in this thesis, the scope of the current work may be further extended.

In this thesis, we have chosen the ellipsoid to model the uncertainty in array steering vector. Since large uncertainty level of uncertainty set results in lower output SINR, compact uncertainty set of array steering vector is appreciated. In practice, there are multiple array imperfections. Each array imperfection has different effect on the array steering vector. Therefore, method to model an uncertainty ellipsoid efficiently using the prior information on array imperfections is important in practice. It can be further studied.

In speech enhancement, we have studied how to estimate ASV when stationary noise exists. However, some of the noise are nonstationary, e.g., speech interference, music signal, etc. To estimate the ASVs in the presence of nonstationary signals is an interest topic in real applications. The well studied second order information, e.g., array covariance matrix, cannot be used to estimate the ASVs. In [72], it is pointed out that the eigenvectors of array covariance matrix are not corresponding to the ASVs of incident signals. In some speech enhancement applications, we can assume that the positions of noises are fixed relative to the microphone array. In such case, it is possible to calibrate the subspace of noises. Although the noises are nonstationary, their subspace basis are unchanged. With this assumption, we can use the calibrated noise subspace to estimate the ASV of target signal using the nonstationary property of signals. An optimization problem as following can be formed to estimate the ASV $\mathbf{s}$. When there are K observations of array covariance matrices $\hat{\mathbf{R}}(k)$, $k = 1, \dots, K$ available, $\mathbf{s}$ can be obtained as the solution of

$$[\sigma_{opt}(k), \Sigma_{opt}(k), \mathbf{s}_{opt}] = \min_{\sigma(k), \Sigma(k), \mathbf{s}} J(\sigma(k), \Sigma(k), \mathbf{s}), \quad (8.1)$$

where the cost function is defined as

$$J(\sigma(k), \Sigma(k), \mathbf{s}) = \sum_{k=1}^K \|\hat{\mathbf{R}}(k) - [\mathbf{s} \ \mathbf{U}] \begin{bmatrix} \sigma(k) & \mathbf{0}^T \\ \mathbf{0} & \Sigma(k) \end{bmatrix} [\mathbf{s} \ \mathbf{U}]^H\|_2^2, \quad (8.2)$$

where $\sigma(k)$ is the varying power of target signal. The matrix $\Sigma(k)$ is the matrix obtained as $\Sigma(k) = \mathbf{U}^H \mathbf{R}_n(k) \mathbf{U}$, $\mathbf{R}_n(k)$ is the covariance matrix of noise. Matrix $\mathbf{U}$ is the calibrated subspace basis of noise. The estimated $\mathbf{s}_{opt}$ can then be used as the nominal ASV in MF-GSC for speech enhancement and noise suppression.

Another technique, i.e., blind channel identification (BCI) ([70, 71] and references therein) can also be applied in speech enhancement using microphone array. BCI can be used to estimate the system transfer functions up to a multiplicative scale. If the transfer functions are estimated, the extended GSC can not only suppress interferences but also remove the reverberation effects in output signal. However, conventional BCI techniques can work well when the length of channel is short. In acoustic applications, the length of channel is long up to thousands taps. This results in poor performance of BCI. Nevertheless, some property of acoustic enclosure can be exploited. For example, in a room, the dominant pole of channel is almost not changing [78]. Therefore, we can first measure the dominant pole of the room, then using the Laguerre filter [79, 80] or Kautz filter [81] instead of conventional FIR filter in BCI. Since Laguerre or Kautz filter can efficiently shorten the number of coefficient to approximate an impulse response, the number of adaptive coefficients can be reduced. Consequently, the Laguerre or Kautz filter based BCI have better performance for channel identification in acoustic application.

Author's Publications

Journal Papers

1. Z. L. Yu and M. H. Er, "Robust subspace analysis and its application in microphone array for speech enhancement," *IEICE Trans. Fundamentals.*, vol. E88-A, no. 7, pp. 1708–1715, July 2005.
2. Z. L. Yu and M. H. Er, "An improved array steering vector estimation method and its application in speech enhancement," *EURASIP Journal on Applied Signal Processing*, vol. 2005, no. 18, pp. 2930–2937, 2005.
3. Q. Zou, Z. L. Yu, and Z. Lin, "A robust algorithm for linearly constrained adaptive beamforming," *IEEE Signal Processing Letters*, vol. 11, no. 1, pp. 26–29, Jan 2004.
4. Z. L. Yu and M. H. Er, "A robust minimum variance beamformer with new constraint on uncertainty of steering vector," to appear in *EURASIP Signal Processing*.
5. Z. L. Yu and M. H. Er, "A robust capon beamformer against uncertainty of

nominal steering vector,” to appear in EURASIP Journal on Applied Signal Processing.

6. Z. L. Yu and M. H. Er, “Speech enhancement with nonstationary noise exploiting calibrated noise subspace,” submitted to IEEE Trans. Acoust. Speech and Signal Processing.
7. Z. L. Yu and M. H. Er, “Robust beamformer against generalized phase errors,” submitted to Proc. IEE.

Conference Papers

1. Z. L. Yu and M. H. Er, “A robust capon beamformer with new uncertainty constraint on steering vector,” in Proc. ICASSP, May 2006, pp. 1029-1032.
2. Z. L. Yu and M. H. Er, “An extended GSC with arbitrary transfer functions in time and frequency domain,” in *Proc. ISCAS*, vol. 3, May 2004, pp. 629-632.
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Appendix A

Appendix of Chapter 3

A.1 Proof of Lemma 3.2

Proof. Since

$$\mathbf{W}_{N' \times L}^{10} = \mathbf{F}_{N'} \mathbf{W}_{N' \times L}^{10} \mathbf{F}_L^{-1},$$

we have,

$$\begin{aligned} (\mathbf{W}_{N' \times L}^{10})^H &= (\mathbf{F}_L^{-1})^H \mathbf{W}_{L \times N'}^{10} \mathbf{F}_{N'}^H \\ &= \frac{N'}{L} \mathbf{F}_L \mathbf{W}_{L \times N'}^{10} \mathbf{F}_{N'}^{-1} \\ &= \frac{N'}{L} \mathbf{W}_{L \times N'}^{10}. \end{aligned}$$

□

A.2 Proof of Lemma 3.3

Proof. Since $\mathbf{W}_{N' \times N'}^{10} = \mathbf{F}_{N'} \mathbf{W}_{N' \times N'}^{10} \mathbf{F}_{N'}^{-1}$, we have

$$\mathbf{W}_{N' \times N'}^{10} = \mathbf{F}_{N'}^{-1} \mathbf{W}_{N' \times N'}^{10} \mathbf{F}_{N'},$$

where $\mathbf{W}_{N' \times N'}^{10}$ is a diagonal matrix, and $\mathbf{W}_{N' \times N'}^{10}$ is a circulant matrix. Inverse transform of the diagonal element of $\mathbf{W}_{N' \times N'}^{10}$ gives the first column of $\mathbf{W}_{N' \times N'}^{10}$, which is expressed as

$$\mathbf{w} = [w_0 \ w_1 \ \cdots \ w_{N'}]^T = \mathbf{F}_{N'}^{-1} [0 \ \cdots \ 0 \ 1 \ \cdots \ 1]^T.$$

The element of vector $\mathbf{w}$ can be expressed explicitly as

$$w_k = \frac{(-1)^k}{N'} \sum_{l=L}^{N'-1} e^{-jl\pi k/N'}.$$

The first element w_0 of $\mathbf{w}$ is $w_0 = \frac{N}{N'}$. Moreover, we have

$$\|\mathbf{w}\|_2^2 = \mathbf{w}^H \mathbf{w} = \frac{N}{N'}.$$

When N is far greater than L , i.e., N is close to N' , w_0 is the dominant element. And

$$\mathbf{W}_{N' \times N'}^{10} \approx \frac{N}{N'} \mathbf{I}_{N' \times N'}.$$

□

A.3 Proof of Lemma 3.4

Proof. Denote the p th row vector of $\mathbf{F}_{N'}\check{\mathbf{X}}_j(m)$ as $[x_{p,0} \ x_{p,1} \ \cdots \ x_{p,N'-1}]$, where $x_{p,0}$ is the p th element of the DFT sequence of the first column of $\check{\mathbf{X}}_j(m)$. Due to the property of DFT and circular shift sequence [82], we have

$$x_{p,k} = x_{p,0} e^{\frac{-j2\pi pk}{N'}}.$$

Therefore, the row vector can be expressed as

$$[x_{p,0} \ x_{p,1} \ \cdots \ x_{p,N'-1}] = x_{p,0} \left[1 \ e^{\frac{-j2\pi p}{N'}} \ \cdots \ e^{\frac{-j2\pi p(N'-1)}{N'}} \right].$$

Refer to Lemma 3.1, we conclude that the p th row vector of $\mathbf{F}_{N'}\check{\mathbf{X}}_j(m)\mathbf{F}_{N'}^{-1}$ is

$$\left[\underbrace{0 \ \cdots \ 0}_{p-1 \text{ elements}} \ x_{p,0} \ 0 \ \cdots \ 0 \right].$$

Therefore, $\mathcal{D}_{\check{\mathbf{X}}_j}(m)$ is a diagonal matrix whose diagonal elements are given by the DFT of the first column of $\check{\mathbf{X}}_j(m)$.

□

A.4 Proof of Lemma 3.5

Proof. Refer to (3.30) and (3.33),

$$\mathbf{e}(m) = \mathbf{F}_N(\mathbf{y}(m, D) - \mathbf{v}(m)) = \mathbf{F}_N \left(\mathbf{y}(m, D) - \mathbf{W}_{N \times N''}^{01} \mathbf{F}_{N''}^{-1} \sum_{i=1}^M \mathcal{D}_{\check{\mathbf{z}}_i}(m) \mathcal{W}_{N'' \times L_w}^{10} \mathbf{w}_i \right). \quad (\text{A.1})$$

Therefore,

$$\begin{aligned} \nabla C_k(m) &= \frac{\partial C(m)}{\partial \mathbf{w}_k^*} \\ &= \frac{\partial \mathbf{e}^H(m)}{\partial \mathbf{w}_k^*} \mathbf{e}(m) \\ &= -\mathcal{W}_{L_w \times N''}^{10} \mathcal{D}_{\check{\mathbf{z}}_k}^H(m) \mathcal{W}_{N'' \times N}^{01} \mathbf{e}(m), \end{aligned}$$

and

$$\begin{aligned} E\{\nabla^2 C_k(m)\} &= E\left\{\frac{\partial \nabla C_k(m)}{\partial \mathbf{w}_k^T}\right\} \\ &= E\left\{-\mathcal{W}_{L_w \times N''}^{10} \mathcal{D}_{\check{\mathbf{z}}_k}^H(m) \mathcal{W}_{N'' \times N}^{01} \frac{\partial \nabla C_k(m)}{\partial \mathbf{w}_k^T}\right\} \\ &= E\left\{\mathcal{W}_{L_w \times N''}^{10} \mathcal{D}_{\check{\mathbf{z}}_k}^H(m) \mathcal{W}_{N'' \times N}^{01} \mathcal{W}_{N \times N''}^{01} \mathcal{D}_{\check{\mathbf{z}}_k}(m) \mathcal{W}_{N'' \times L_w}^{10}\right\}. \end{aligned}$$

Since

$$\mathcal{W}_{N'' \times N''}^{01} = \mathcal{W}_{N'' \times N}^{01} \mathcal{W}_{N \times N''}^{01} = \mathcal{W}_{N'' \times N''}^{01} \approx \frac{N}{N''} \mathbf{I}_{N'' \times N''},$$

we have

$$\begin{aligned}
 E\{\nabla^2 C_k(m)\} &= E\{\mathbf{W}_{L_w \times N''}^{10} \mathbf{D}_{\check{Z}_k}^H(m) \mathbf{W}_{N'' \times N}^{01} \mathbf{W}_{N \times N''}^{01} \mathbf{D}_{\check{Z}_k}(m) \mathbf{W}_{N'' \times L_w}^{10}\} \\
 &\approx \frac{N}{N''} \mathbf{W}_{L_w \times N''}^{10} E\{\mathbf{D}_{\check{Z}_k}^H(m) \mathbf{D}_{\check{Z}_k}(m)\} \mathbf{W}_{N'' \times L_w}^{10} \\
 &\approx \frac{N}{N''} \mathbf{W}_{L_w \times N''}^{10} \hat{\mathbf{R}}_{z_k}(m) \mathbf{W}_{N'' \times L_w}^{01},
 \end{aligned}$$

where

$$\hat{\mathbf{R}}_{z_k}(m) = \alpha \hat{\mathbf{R}}_{z_k}(m-1) + (1-\alpha) \mathbf{D}_{\check{Z}_k}^H(m) \mathbf{D}_{\check{Z}_k}(m),$$

and α is the forgetting factor. □

A.5 Proof of Lemma 3.6

Proof. This lemma is justified by post-multiply both sides of (A.2) by $\hat{\mathbf{R}}_{z_k}(m) \mathbf{W}_{N'' \times L_w}^{10}$,

$$\mathbf{W}_{N'' \times L_w}^{10} E\{\nabla^2 C(m)\}^{-1} \mathbf{W}_{L_w \times N''}^{10} = \frac{N''}{N} \mathbf{W}_{N'' \times N''}^{10} \hat{\mathbf{R}}_{z_k}(m)^{-1}. \quad (\text{A.2})$$

The left side is

$$\mathbf{W}_{N'' \times L_w}^{10} E\{\nabla^2 C(m)\}^{-1} \mathbf{W}_{L_w \times N''}^{10} \hat{\mathbf{R}}_{z_k}(m) \mathbf{W}_{N'' \times L_w}^{10} = \frac{N''}{N} \mathbf{W}_{N'' \times L_w}^{10}.$$

The right side is

$$\begin{aligned}
 \frac{N''}{N} \mathbf{W}_{N'' \times N''}^{10} \hat{\mathbf{R}}_{z_k}(m)^{-1} \hat{\mathbf{R}}_{z_k}(m) \mathbf{W}_{N'' \times L_w}^{10} &= \frac{N''}{N} \mathbf{W}_{N'' \times N''}^{10} \mathbf{W}_{N'' \times L_w}^{10} \\
 &= \frac{N''}{N} \mathbf{W}_{N'' \times L_w}^{10}.
 \end{aligned}$$

The equality holds. □

A.6 Proof of Theorem 3.1

Proof. The Newton-LMS algorithm is given as

$$\begin{aligned}\hat{\mathbf{w}}_k(m) &= \hat{\mathbf{w}}_k(m-1) - \rho' E\{\nabla^2 C(m)\}^{-1} \nabla C(m) \\ &= \hat{\mathbf{w}}_k(m-1) + \rho \left(\mathbf{W}_{L_w \times N''}^{10} \hat{\mathcal{R}}_{z_k}(m) \mathbf{W}_{N'' \times L_w}^{10} \right)^{-1} \mathbf{W}_{L_w \times N''}^{10} \mathcal{D}_{\check{Z}_k}^H(m) \mathbf{W}_{N'' \times N}^{01} \mathbf{e}(m).\end{aligned}\tag{A.3}$$

Pre-Multiplying both side of (A.3) by $\mathbf{W}_{N'' \times L_w}^{10}$, it yields

$$\begin{aligned}\hat{\mathbf{w}}_k^{10}(m) &= \hat{\mathbf{w}}_k^{10}(m-1) + \rho \mathbf{W}_{N'' \times L_w}^{10} \left(\mathbf{W}_{L_w \times N''}^{10} \hat{\mathcal{R}}_{z_k}(m) \mathbf{W}_{N'' \times L_w}^{10} \right)^{-1} \\ &\quad \mathbf{W}_{L_w \times N''}^{10} \mathcal{D}_{\check{Z}_k}^H(m) \mathbf{W}_{N'' \times N}^{01} \mathbf{e}(m) \\ &= \hat{\mathbf{w}}_k^{10}(m-1) + \rho \mathbf{W}_{N'' \times N''}^{10} \hat{\mathcal{R}}_{z_k}^{-1}(m) \mathcal{D}_{\check{Z}_k}^H(m) \mathbf{W}_{N'' \times N}^{01} \mathbf{e}(m),\end{aligned}\tag{A.4}$$

where

$$\hat{\mathbf{w}}_k^{10}(m) = \mathbf{W}_{N'' \times L_w}^{10} \hat{\mathbf{w}}_k(m),\tag{A.5}$$

and Lemma A.5 is used.

The unconstrained Newton-LMS algorithm is obtained by removing $\mathbf{W}_{N'' \times N''}^{10}$ from (A.4). □

Appendix B

Appendix of Chapter 6

B.1 Analysis of the Solution of (6.26)

The diagonal loading factor g_1 is obtained by solving

$$\psi(g_1) = \frac{4}{(2 - \epsilon)^2}. \quad (\text{B.1})$$

The derivative of $\psi(g_1)$ to g_1 is given by

$$\frac{d\psi(g_1)}{dg_1} = \frac{\left(\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)^2\right)^2 - \left(\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)^3\right) \left(\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)\right)}{\left(\sum_{i=1}^M |\tilde{s}_i|^2 \frac{\lambda_i}{\lambda_i g_1 + 1}\right)^4}. \quad (\text{B.2})$$

When $g_1 > -1/\lambda_1$, the item $\frac{\lambda_i}{\lambda_i g_1 + 1} > 0$. Using Schwartz inequality [43], we have

$$\left(\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)^2\right)^2 - \left(\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)^3\right) \left(\sum_{i=1}^M |\tilde{s}_i|^2 \left(\frac{\lambda_i}{\lambda_i g_1 + 1}\right)\right) \leq 0. \quad (\text{B.3})$$

Hence $\frac{d\psi(g_1)}{dg_1} \leq 0$, $\psi(g_1)$ is a monotonically decreasing function of g_1 . Additionally,

$$\begin{aligned} \lim_{g_1 \rightarrow \infty} \psi(g_1) &= 1, \\ \lim_{g_1 \rightarrow \frac{-1}{\lambda_i}} \psi(g_1) &= 1/|\tilde{s}_i|^2 \geq 1/|\mathbf{u}_1^H \bar{\mathbf{s}}_0|^2 \geq \frac{4}{(2-\epsilon)^2}. \end{aligned} \tag{B.4}$$

There is a unique root of (6.26) in the range $(\frac{-1}{\lambda_1}, \infty)$.

B.2 Proof of Lemma 6.1

The optimal weight vector $\mathbf{w}$ of adaptive filter is

$$\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{s}}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}} = \gamma \mathbf{R}^{-1} \mathbf{s}. \tag{B.5}$$

The output power p_s of the target signal and the output power p_n of the noise/interference are

$$p_s = \sigma_s^2 |\mathbf{w}^H \mathbf{s}_0|^2, \quad p_n = \mathbf{w}^H \mathbf{R}_n \mathbf{w}. \tag{B.6}$$

Since

$$\mathbf{R} = \sigma_s^2 \mathbf{s}_0 \mathbf{s}_0^H + \mathbf{R}_n, \tag{B.7}$$

we have

$$\mathbf{R}^{-1} = \mathbf{R}_n^{-1} - \frac{\sigma_s^2 \mathbf{R}_n^{-1} \mathbf{s}_0 \mathbf{s}_0^H \mathbf{R}_n^{-1}}{1 + \sigma_s^2 \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0} = \mathbf{R}_n^{-1} - \frac{\sigma_s^2 \mathbf{R}_n^{-1} \mathbf{s}_0 \mathbf{s}_0^H \mathbf{R}_n^{-1}}{1 + \rho_o}. \tag{B.8}$$

The optimal weight can also be given by

$$\mathbf{w} = \frac{\gamma}{1 + \rho_o} \left[(1 + \rho_o) \mathbf{R}_n^{-1} - \sigma_s^2 \mathbf{R}_n^{-1} \mathbf{s}_0 \mathbf{s}_0^H \mathbf{R}_n^{-1} \right] \mathbf{s}. \quad (\text{B.9})$$

Hence,

$$\begin{aligned} |\mathbf{w}^H \mathbf{s}_0|^2 &= \frac{\gamma^2 \left| (1 + \rho_o) \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}_0 - (\sigma_s^2 \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0) (\mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}_0) \right|^2}{(1 + \rho_o)^2} \\ &= \frac{\gamma^2 |\mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s}_0|^2}{(1 + \rho_o)^2} \\ &= \frac{\gamma^2 \|\mathbf{s}\|_{\mathbf{R}}^2 \|\mathbf{s}_0\|_{\mathbf{R}}^2 \cos^2(\theta)}{(1 + \rho_o)^2}, \end{aligned} \quad (\text{B.10})$$

where

$$\cos^2(\theta) = \frac{|\mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\mathbf{s}\|_{\mathbf{R}}^2} \quad (\text{B.11})$$

and

$$\begin{aligned} p_n &= \frac{\gamma^2}{(1 + \rho_o)^2} \left[(1 + \rho_o) \mathbf{R}_n^{-1} \mathbf{s} - \sigma_s^2 \mathbf{R}_n^{-1} \mathbf{s}_0 \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s} \right]^H \mathbf{R}_n \left[(1 + \rho_o) \mathbf{R}_n^{-1} \mathbf{s} - \sigma_s^2 \mathbf{R}_n^{-1} \mathbf{s}_0 \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s} \right] \\ &= \frac{\gamma^2}{(1 + \rho_o)^2} \left[(1 + \rho_o)^2 \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s} - 2(1 + \rho_o) \sigma_s^2 |\mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}|^2 + \sigma_s^2 \rho_o |\mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}|^2 \right] \\ &= \frac{\gamma^2}{(1 + \rho_o)^2} \left[(1 + \rho_o)^2 \|\mathbf{s}\|_{\mathbf{R}}^2 - (2 + \rho_o) \sigma_s^2 \|\mathbf{s}\|_{\mathbf{R}}^2 \|\mathbf{s}_0\|_{\mathbf{R}}^2 \cos^2(\theta) \right] \\ &= \frac{\gamma^2 \|\mathbf{s}\|_{\mathbf{R}}^2}{(1 + \rho_o)^2} \left[(1 + \rho_o)^2 - (2 + \rho_o) \rho_o \cos^2(\theta) \right] \\ &= \frac{\gamma^2 \|\mathbf{s}\|_{\mathbf{R}}^2}{(1 + \rho_o)^2} \left[1 + 2\rho_o \sin^2(\theta) + \rho_o^2 \sin^2(\theta) \right]. \end{aligned} \quad (\text{B.12})$$

The output SINR is

$$\rho = \frac{p_s}{p_n} = \frac{\rho_o \cos^2(\theta)}{1 + 2\rho_o \sin^2(\theta) + \rho_o^2 \sin^2(\theta)}.$$

If $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$, the weighted norm $\|\cdot\|_{\mathbf{R}}$ is equivalent to $\|\cdot\|_2$. The conclusion in Lemma 6.1 is straightforward.

B.3 Proof of Lemma 6.2

The proposed RCB uses the ASV $\hat{\mathbf{s}}_0$ given in (6.27) instead of the nominal ASV $\bar{\mathbf{s}}_0$ in the calculation of optimal weight vector (6.28). Refer to Lemma 6.1, the bound of output SINR of the proposed RCB can be obtained by studying the angle between the ASV $\hat{\mathbf{s}}_0$ and the true one $\mathbf{s}_0$.

The array covariance matrix can be expressed as

$$\mathbf{R} = \sigma_s^2 \mathbf{s}_0 \mathbf{s}_0^H + \mathbf{R}_n, \quad (\text{B.13})$$

Using matrix inversion lemma, we have

$$\mathbf{R}^{-1} = \mathbf{R}_n^{-1} - \frac{\sigma_s^2}{1 + \xi} (\mathbf{R}_n^{-1} \mathbf{s}_0) (\mathbf{R}_n^{-1} \mathbf{s}_0)^H, \quad (\text{B.14})$$

where $\xi = \sigma_s^2 \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0$.

Using matrix inversion lemma again,

$$(\mathbf{R}^{-1} + g\mathbf{I})^{-1} = (\mathbf{R}_n^{-1} + g\mathbf{I})^{-1} + \frac{k(\mathbf{I} + g\mathbf{R}_n)^{-1}\mathbf{s}_0\mathbf{s}_0^H(\mathbf{I} + g\mathbf{R}_n)^{-1}}{1 - k\mathbf{s}_0^H(\mathbf{R}_n + g\mathbf{R}_n^2)^{-1}\mathbf{s}_0}, \quad (\text{B.15})$$

where $k = \sigma_s^2/(1 + \xi)$.

Substituting (B.15) into (6.28), it yields

$$\begin{aligned} \hat{\mathbf{s}}_0 &= (\mathbf{R}^{-1} + g_1\mathbf{I})^{-1}\bar{\mathbf{s}}_0 \\ &= (\mathbf{R}_n^{-1} + g_1\mathbf{I})^{-1}\bar{\mathbf{s}}_0 + d(\mathbf{I} + g_1\mathbf{R}_n)^{-1}\mathbf{s}_0, \end{aligned} \quad (\text{B.16})$$

where $d = k\mathbf{s}_0^H(\mathbf{I} + g_1\mathbf{R}_n)^{-1}\bar{\mathbf{s}}_0/(1 - k\mathbf{s}_0^H(\mathbf{R}_n + g_1\mathbf{R}_n^2)^{-1}\mathbf{s}_0)$.

Assume that the angle between $\hat{\mathbf{s}}_0$ and $\mathbf{s}_0$ is θ , we have

$$\cos^2(\theta) = \frac{|\hat{\mathbf{s}}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\hat{\mathbf{s}}_0\|_{\mathbf{R}}^2}. \quad (\text{B.17})$$

The items in (B.17) can be calculated as

$$\begin{aligned} \hat{\mathbf{s}}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0 &= \bar{\mathbf{s}}_0^H (\mathbf{I} + g_1\mathbf{R}_n)^{-1} \mathbf{s}_0 + d^* \mathbf{s}_0^H (\mathbf{R}_n + g_1\mathbf{R}_n^2)^{-1} \mathbf{s}_0, \\ \|\hat{\mathbf{s}}_0\|_{\mathbf{R}}^2 &= \hat{\mathbf{s}}_0^H \mathbf{R}_n^{-1} \hat{\mathbf{s}}_0 \\ &= \bar{\mathbf{s}}_0^H (\mathbf{I} + g_1\mathbf{R}_n)^{-2} \mathbf{R}_n \bar{\mathbf{s}}_0 + 2\text{Re}\{d\bar{\mathbf{s}}_0^H (\mathbf{I} + g_1\mathbf{R}_n)^{-2} \mathbf{s}_0\} + |d|^2 \mathbf{s}_0^H (\mathbf{I} + g_1\mathbf{R}_n)^{-2} \mathbf{s}_0, \end{aligned} \quad (\text{B.18})$$

where $\text{Re}\{\cdot\}$ is the real operator.

If we assume that the eigenvalues of interference in Σ_i are far greater than the vari-

ance of noise σ_n^2 . Using the eigen-decomposition in (2.21), (B.18) can be approximated as

$$\begin{aligned}\hat{\mathbf{s}}_0^H \mathbf{R}_n^{-1} \mathbf{s}_0 &= \bar{\mathbf{s}}_0^H (\mathbf{I} + g_1 \mathbf{R}_n)^{-1} \mathbf{s}_0 + d^* \mathbf{s}_0^H (\mathbf{R}_n + g_1 \mathbf{R}_n^2)^{-1} \mathbf{s}_0 \\ &\approx \frac{1}{1 + \sigma_n^2 g_1} \bar{\mathbf{s}}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{s}_0 + \frac{d^*}{\sigma_n^2 (1 + \sigma_n^2 g_1)} \mathbf{s}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{s}_0 \\ &= \frac{\psi_c}{1 + \sigma_n^2 g_1} + \frac{d^* \psi_0}{\sigma_n^2 (1 + \sigma_n^2 g_1)}\end{aligned}\quad (\text{B.19})$$

and

$$\begin{aligned}\|\hat{\mathbf{s}}_0\|_{\mathbf{R}}^2 &= \bar{\mathbf{s}}_0^H (\mathbf{I} + g_1 \mathbf{R}_n)^{-2} \mathbf{R}_n \bar{\mathbf{s}}_0 + 2 \text{Re}\{d \bar{\mathbf{s}}_0^H (\mathbf{I} + g_1 \mathbf{R}_n)^{-2} \mathbf{s}_0\} + |d|^2 \mathbf{s}_0^H (\mathbf{I} + g_1 \mathbf{R}_n)^{-2} \mathbf{s}_0 \\ &\approx \frac{\sigma_n^2 \bar{\mathbf{s}}_0^H \mathbf{U}_n \mathbf{U}_n^H \bar{\mathbf{s}}_0}{(1 + g_1 \sigma_n^2)^2} + 2 \text{Re}\left\{\frac{d \bar{\mathbf{s}}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{s}_0}{(1 + g_1 \sigma_n^2)^2}\right\} + \frac{|d|^2 \mathbf{s}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{s}_0}{(1 + g_1 \sigma_n^2)^2} \\ &= \frac{\sigma_n^2 \psi_b}{(1 + g_1 \sigma_n^2)^2} + 2 \text{Re}\left\{\frac{d \psi_c}{(1 + g_1 \sigma_n^2)^2}\right\} + \frac{|d|^2 \psi_0}{(1 + g_1 \sigma_n^2)^2},\end{aligned}\quad (\text{B.20})$$

where

$$\begin{aligned}\psi_c &= \bar{\mathbf{s}}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{s}_0, \\ \psi_0 &= \mathbf{s}_0^H \mathbf{U}_n \mathbf{U}_n^H \mathbf{s}_0, \\ \psi_b &= \bar{\mathbf{s}}_0^H \mathbf{U}_n \mathbf{U}_n^H \bar{\mathbf{s}}_0.\end{aligned}\quad (\text{B.21})$$

If the angle between $\hat{\mathbf{s}}_0$ and $\mathbf{s}_0$ is θ , we have

$$\begin{aligned}f = \cos^2(\theta) &= \frac{|\hat{\mathbf{s}}^H \mathbf{R}_n^{-1} \mathbf{s}_0|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\hat{\mathbf{s}}\|_{\mathbf{R}}^2} \\ &= \frac{|\psi_c + \frac{d^* \psi_0}{\sigma_n^2}|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 (\sigma_n^2 \psi_b + 2 \text{Re}\{d \psi_c\} + \frac{|d|^2 \psi_0}{\sigma_n^2})}.\end{aligned}\quad (\text{B.22})$$

Substituting

$$d = \frac{k\mathbf{s}_0^H(\mathbf{I} + g_1\mathbf{R}_n)^{-1}\bar{\mathbf{s}}_0}{1 - k\mathbf{s}_0^H(\mathbf{R}_n + g_1\mathbf{R}_n^2)^{-1}\mathbf{s}_0} \approx \frac{k\sigma_n^2\psi_c^*}{\sigma_n^2(1 + g_1\sigma_n^2) - k\psi_0} = \frac{k\sigma_n^2\psi_c^*}{\beta}, \quad (\text{B.23})$$

where $\beta = \sigma_n^2(1 + g_1\sigma_n^2) - k\psi_0$, into (B.22), we have

$$\begin{aligned} f(\beta) = \cos^2(\theta) &= \frac{|\psi_c + \frac{d^*\psi_0}{\sigma_n^2}|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2(\sigma_n^2\psi_b + 2\text{Re}\{d\psi_c\} + \frac{|d|^2\psi_0}{\sigma_n^2})} \\ &= \frac{|\psi_c|^2(\beta + k\psi_0)^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2(\sigma_n^2\psi_b\beta^2 + 2k\sigma_n^2|\psi_c|^2\beta + k^2\sigma_n^2\psi_0|\psi_c|^2)}. \end{aligned} \quad (\text{B.24})$$

It is obvious that if $|\psi_c|^2 = 0$, then $\cos^2(\theta) \equiv 0$. In such case, the beamformer does not work. The maximum value of $\cos^2(\theta)$ is achieved when $df(\beta)/d\beta = 0$. After some straightforward algebraic manipulations, it yields

$$\beta = 0. \quad (\text{B.25})$$

Hence,

$$\sigma_n^2(1 + g_1\sigma_n^2) - k\psi_0 = 0. \quad (\text{B.26})$$

Therefore, the upper bound of the output SINR is achieved when the value of g_1 satisfies

$$g_1 = \frac{-1}{\sigma_n^2 + \sigma_s^2\psi_0} = \frac{-1}{\sigma_n^2 + \sigma_s^2\|P_{\mathbf{U}_n}\mathbf{s}_0\|_2^2} \quad (\text{B.27})$$

and the corresponding output SINR is

$$\rho_o = \frac{\sigma_s^2\|P_{\mathbf{U}_n}\mathbf{s}_0\|_2^2}{\sigma_n^2}. \quad (\text{B.28})$$

B.4 Proof of Lemma 6.3

Proof. Using the eigen-decomposition of $\mathbf{R}$, we have

$$\begin{aligned}
 \mathbf{s}_1 &= (g_1^{-1}\mathbf{R}^{-1} + \mathbf{I})^{-1}\bar{\mathbf{s}}_0 \\
 &= \mathbf{U} \begin{bmatrix} \frac{g_1\lambda_1}{1+g_1\lambda_1} & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & \frac{g_1\lambda_M}{1+g_1\lambda_M} \end{bmatrix} \mathbf{U}^H \bar{\mathbf{s}}_0 \\
 &\triangleq \mathbf{U}\mathbf{D}_1\mathbf{U}^H \bar{\mathbf{s}}_0
 \end{aligned} \tag{B.29}$$

and

$$\begin{aligned}
 \mathbf{s}_2 &= (g_2^{-1}\mathbf{R}^{-1} + \mathbf{I})^{-1}\hat{\bar{\mathbf{s}}}_0 \\
 &= \mathbf{U} \begin{bmatrix} \frac{g_2\lambda_1}{1+g_2\lambda_1} & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & \frac{g_2\lambda_M}{1+g_2\lambda_M} \end{bmatrix} \mathbf{U}^H \mathbf{U}_s \mathbf{U}_s^H \bar{\mathbf{s}}_0 \\
 &= \mathbf{U} \begin{bmatrix} \frac{g_2\lambda_1}{1+g_2\lambda_1} & \cdots & 0 & \mathbf{0} \\ 0 & \ddots & 0 & \vdots \\ 0 & \cdots & \frac{g_2\lambda_K}{1+g_2\lambda_K} & \mathbf{0} \\ 0 & \cdots & 0 & \mathbf{0} \end{bmatrix} \mathbf{U}^H \bar{\mathbf{s}}_0 \\
 &\triangleq \mathbf{U}\mathbf{D}_2\mathbf{U}^H \bar{\mathbf{s}}_0.
 \end{aligned} \tag{B.30}$$

The true ASV $\mathbf{s}_0$ is spanned by $\mathbf{U}_s$,

$$\mathbf{s}_0 = \mathbf{U}_s \mathbf{c}_0, \quad (\text{B.31})$$

where $\mathbf{c}_0$ is the coefficient vector. Assume that g_1 is the optimal diagonal loading factor for the RCB. Choosing $g_2 = g_1$, it yields

$$\begin{aligned} \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_1 &= \mathbf{c}_0^H \mathbf{U}_s^H \mathbf{R}_n^{-1} \mathbf{U} \mathbf{D}_1 \mathbf{U}^H \bar{\mathbf{s}}_0 \\ &= \mathbf{c}_0^H \mathbf{U}_s^H \mathbf{R}_n^{-1} [\mathbf{U}_s \ \mathbf{U}_n] \mathbf{D}_1 \mathbf{U}^H \bar{\mathbf{s}}_0 \\ &= \mathbf{c}_0^H [\mathbf{K} \ \mathbf{0}] \mathbf{D}_1 \mathbf{U}^H \bar{\mathbf{s}}_0 \\ &= \mathbf{c}_0^H [\mathbf{K} \ \mathbf{0}] \mathbf{D}_2 \mathbf{U}^H \bar{\mathbf{s}}_0 \\ &= \mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_2, \end{aligned} \quad (\text{B.32})$$

where $\mathbf{K} = \mathbf{U}_s^H \mathbf{R}_n^{-1} \mathbf{U}_s$.

We have

$$\begin{aligned} \|\mathbf{s}_1\|_{\mathbf{R}}^2 &= \bar{\mathbf{s}}_0^H \mathbf{U} \mathbf{D}_1 \mathbf{U}^H \mathbf{R}_n^{-1} \mathbf{U} \mathbf{D}_1 \mathbf{U}^H \bar{\mathbf{s}}_0 = \mathbf{v}^H \mathbf{D}_1 \mathbf{E} \mathbf{D}_1 \mathbf{v}, \\ \|\mathbf{s}_2\|_{\mathbf{R}}^2 &= \bar{\mathbf{s}}_0^H \mathbf{U} \mathbf{D}_2 \mathbf{U}^H \mathbf{R}_n^{-1} \mathbf{U} \mathbf{D}_2 \mathbf{U}^H \bar{\mathbf{s}}_0 = \mathbf{v}^H \mathbf{D}_2 \mathbf{E} \mathbf{D}_2 \mathbf{v}, \end{aligned} \quad (\text{B.33})$$

where

$$\begin{aligned} \mathbf{E} &= \mathbf{U}^H \mathbf{R}_n^{-1} \mathbf{U}, \\ \mathbf{v} &= \mathbf{U}^H \bar{\mathbf{s}}_0. \end{aligned} \quad (\text{B.34})$$

The covariance matrix $\mathbf{R}_n$ consists of two parts, $\mathbf{R}_i$ and $\sigma_n^2 \mathbf{I}$, which are the covari-

ance matrices of the interferences and background noise, respectively. Since the signal space of the interferences is a subspace of $\mathbf{U}_s$, $\mathbf{R}_n$ can be expressed as

$$\mathbf{R}_n = \mathbf{R}_i + \sigma_n^2 \mathbf{I} = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \mathbf{D}_i & \mathbf{0} \\ \mathbf{0} & \sigma_n^2 \mathbf{I} \end{bmatrix} [\mathbf{U}_s \ \mathbf{U}_n]^H, \quad (\text{B.35})$$

where the matrix $\mathbf{D}_i$ of size $K \times K$ is not necessary a diagonal matrix. The inverse matrix of $\mathbf{R}_n^{-1}$ can be expressed as

$$\mathbf{R}_n^{-1} = [\mathbf{U}_s \ \mathbf{U}_n] \begin{bmatrix} \mathbf{D}_i^{-1} & \mathbf{0} \\ \mathbf{0} & \sigma_n^{-2} \mathbf{I} \end{bmatrix} [\mathbf{U}_s \ \mathbf{U}_n]^H. \quad (\text{B.36})$$

Therefore,

$$\mathbf{E} = \mathbf{U}^H \mathbf{R}_n^{-1} \mathbf{U} = \begin{bmatrix} \mathbf{D}_i^{-1} & \mathbf{0} \\ \mathbf{0} & \sigma_n^{-2} \mathbf{I} \end{bmatrix}. \quad (\text{B.37})$$

With the derived $\mathbf{E}$ in (B.37) and new definition of matrix $\mathbf{D}_3$,

$$\mathbf{D}_3 = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & 0 & \cdots & 0 \\ 0 & \cdots & \frac{g_1 \lambda_{K+1}}{1+g_1 \lambda_{K+1}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \frac{g_1 \lambda_M}{1+g_1 \lambda_M} \end{bmatrix}, \quad (\text{B.38})$$

we have,

$$\begin{aligned}
\|\mathbf{s}_1\|_{\mathbf{R}}^2 &= \mathbf{v}^H \mathbf{D}_1 \mathbf{E} \mathbf{D}_1 \mathbf{v} \\
&= \mathbf{v}^H (\mathbf{D}_2 + \mathbf{D}_3) \mathbf{E} (\mathbf{D}_2 + \mathbf{D}_3) \mathbf{v} \\
&= \mathbf{v}^H \mathbf{D}_2 \mathbf{E} \mathbf{D}_2 \mathbf{v} + \mathbf{v}^H \mathbf{D}_3 \mathbf{E} \mathbf{D}_3 \mathbf{v} \\
&= \|\mathbf{s}_2\|_{\mathbf{R}}^2 + \mathbf{v}^H \mathbf{D}_3 \mathbf{E} \mathbf{D}_3 \mathbf{v}.
\end{aligned} \tag{B.39}$$

Since $\mathbf{D}_3 \mathbf{E} \mathbf{D}_3$ is a non-negative Hermitian matrix, we have

$$\|\mathbf{s}_1\|_{\mathbf{R}}^2 \geq \|\mathbf{s}_2\|_{\mathbf{R}}^2. \tag{B.40}$$

Denoting the angles between $\mathbf{s}_0$ and $\mathbf{s}_1$ as θ_1 and the angle between $\mathbf{s}_0$ and $\mathbf{s}_2$ as θ_2 ,

we have

$$\cos^2(\theta_1) = \frac{|\mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_1|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\mathbf{s}_1\|_{\mathbf{R}}^2} \leq \frac{|\mathbf{s}_0^H \mathbf{R}_n^{-1} \mathbf{s}_1|^2}{\|\mathbf{s}_0\|_{\mathbf{R}}^2 \|\mathbf{s}_2\|_{\mathbf{R}}^2} = \cos^2(\theta_2). \tag{B.41}$$

□

The optimal factor g_2 should be selected to maximize $\cos^2(\theta_2)$, therefore, the corresponding $\cos^2(\theta_2)$ must be greater than or equal to $\cos^2(\theta_1)$.

Appendix C

Appendix of Chapter 7

C.1 Proof of Theorem 7.1

Proof. Since $\mathbf{R}_n$ and $\mathbf{R}_s$ are Hermitian and commute matrices, according to Theorem 2.5.5 in [83], there is a single unitary matrix $\mathbf{U}$ that transforms both $\mathbf{R}_n$ and $\mathbf{R}_s$ into diagonal matrices.

$$\mathbf{R}_n = [\mathbf{u}_1 \ \mathbf{U}_2] \text{diag}\{\sigma_{n,1}^2, \dots, \sigma_{n,M}^2\} [\mathbf{u}_1 \ \mathbf{U}_2]^H,$$

$$\mathbf{R}_s = [\mathbf{u}_1 \ \mathbf{U}_2] \text{diag}\{\sigma_s^2, 0, \dots, 0\} [\mathbf{u}_1 \ \mathbf{U}_2]^H.$$

Therefore,

$$\mathbf{R} = \mathbf{R}_s + \mathbf{R}_n = [\mathbf{u}_1 \ \mathbf{U}_2] \text{diag}\{\sigma_{n,1}^2 + \sigma_s^2, \dots, \sigma_{n,M}^2\} [\mathbf{u}_1 \ \mathbf{U}_2]^H.$$

□

C.2 Proof of Theorem 7.2

Lemma C.1. *Let $\mathbf{R}_x = \mathbf{R}_s + \mathbf{R}_n$, where $\mathbf{R}_s$ is a rank one non-negative Hermitian matrix with its non-zero eigenvalue λ_s , and $\mathbf{R}_n$ is a full rank non-negative Hermitian matrix with its eigenvalues $\lambda_i \in [\sigma_{n,min}^2, \sigma_{n,max}^2]$. If $\lambda_s > \sigma_{n,max}^2$, matrix $\mathbf{R}_x$ has the maximum eigenvalue σ_{max}^2 and the other eigenvalues $\{\lambda_i\}$ as*

$$\sigma_{max}^2 \in [\lambda_s + \sigma_{n,min}^2, \lambda_s + \sigma_{n,max}^2],$$

$$\lambda_i \in [\sigma_{n,min}^2, \sigma_{n,max}^2].$$

Proof. Refer to Weyl Theorem [84]. □

We give the proof of Theorem 7.2 in the following.

Proof. Let $\mathbf{D} = \mathbf{R}_x \mathbf{u}_1 - \sigma_s^2 \mathbf{u}_1$. Then

$$\|\mathbf{D}\|_F = \|\mathbf{R}_s \mathbf{u}_1 + \mathbf{R}_n \mathbf{u}_1 - \sigma_s^2 \mathbf{u}_1\|_F = \|\mathbf{R}_n \mathbf{u}_1\|_F \leq \|\mathbf{R}_n\|_F.$$

Define unitary matrices $\mathbf{U} = [\mathbf{u}_1 \ \mathbf{U}_2]$ and $\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1 \ \tilde{\mathbf{U}}_2]$ as the eigenvector matrices of $\mathbf{R}_s$ and $\mathbf{R}_x$, respectively. We have

$$\mathbf{U}^H \mathbf{R}_s \mathbf{U} = \begin{bmatrix} \lambda_s & \mathbf{0}^H \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{U}}^H \mathbf{R}_x \tilde{\mathbf{U}} = \begin{bmatrix} \sigma^2 & \mathbf{0}^H \\ \mathbf{0} & \mathbf{M} \end{bmatrix}.$$

Let $\delta = \min |L(\lambda_s) - L(\mathbf{M})|$, where $L(\cdot)$ stands for the set of eigenvalues. According to

Lemma C.1 and the assumption $\sigma_{n,max}^2 < \lambda_s$, δ is obtained as

$$\delta = \lambda_s - \sigma_{n,max}^2.$$

Using the first $\sin \Theta$ theorem [85,84], we obtain

$$\|\sin \Theta[\mathbf{u}_1, \tilde{\mathbf{u}}_1]\|_F \leq \frac{\|\mathbf{D}\|_F}{\delta} \leq \frac{\|\mathbf{R}_n\|_F}{\delta} = \frac{1}{\rho - k},$$

where $\Theta[\mathbf{u}_1, \mathbf{u}_2]$ stands for the angle between $\mathbf{u}_1$ and $\mathbf{u}_2$. Since

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 = \|\mathbf{u}_1\|^2 + \|\tilde{\mathbf{u}}_1\|^2 - 2\mathbf{u}_1^H \tilde{\mathbf{u}}_1 = 2(1 - \cos \Theta[\mathbf{u}_1, \tilde{\mathbf{u}}_1]),$$

therefore,

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 \leq 2 \left(1 - \sqrt{1 - \left(\frac{1}{\rho - k} \right)^2} \right).$$

With high SNR ρ , using approximation $(1 - x)^{1/2} \approx 1 - \frac{1}{2}x$, when $x \ll 1$, we have

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 \leq \left(\frac{1}{\rho - k} \right)^2.$$

Since $k \leq 1$, we have

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 \leq \left(\frac{1}{\rho - k} \right)^2 \approx \frac{1}{\rho^2}.$$

□

C.3 Proof of Theorem 7.3

Proof. Define unitary matrix $\mathbf{U} = [\mathbf{u}_1 \ \mathbf{U}_2]$ as the eigenvector matrix of $\mathbf{R}_s$, where $\mathbf{u}_1$ is the eigenvector of $\mathbf{R}_s$ corresponding to eigenvalue λ_s . We have

$$\mathbf{U}^H \mathbf{R}_s \mathbf{U} = \begin{bmatrix} \lambda_s & \mathbf{0}^H \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{U}^H \mathbf{R}_n \mathbf{U} = \begin{bmatrix} e & \mathbf{f}^H \\ \mathbf{g} & \mathbf{E} \end{bmatrix}.$$

If $\lambda_s > 2\|\mathbf{E}\|_2 = 2\varepsilon$ and $\frac{\varepsilon\|\mathbf{g}\|_2}{(\lambda_s - \varepsilon)^2} < \frac{1}{4}$, according to [86], for vector $\mathbf{p} \in \mathbb{C}^{M-1}$ and $\|\mathbf{p}\|_2 \leq \frac{2\|\mathbf{g}\|_2}{\lambda_s - 2\varepsilon}$, a new vector $\tilde{\mathbf{u}} = \mathbf{u} + \mathbf{U}_2 \mathbf{p}$ is the eigenvector of $\mathbf{R}_x$ corresponding to its largest eigenvalue. Moreover, we have

$$\|\sin \Theta[\mathbf{u}_1, \tilde{\mathbf{u}}_1]\|_2 \leq \frac{2\|\mathbf{g}\|_2}{\sqrt{(\lambda_s - 2\|\mathbf{R}_n\|_2)^2 + 4\|\mathbf{g}\|_2^2}}. \quad (\text{C.1})$$

Since $\|\mathbf{g}\|_2 \leq \|\mathbf{R}_n\|_2 = \varepsilon$, we have

$$\|\sin \Theta[\mathbf{u}_1, \tilde{\mathbf{u}}_1]\|_2 \leq \frac{2\varepsilon}{\sqrt{(\lambda_s - 2\varepsilon)^2 + 4\varepsilon^2}} \leq \frac{2\varepsilon}{\lambda_s - 2\varepsilon}. \quad (\text{C.2})$$

Since

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 = \|\mathbf{u}_1\|^2 + \|\tilde{\mathbf{u}}_1\|^2 - 2\mathbf{u}_1^H \tilde{\mathbf{u}}_1 = 2(1 - \cos \Theta[\mathbf{u}_1, \tilde{\mathbf{u}}_1]),$$

therefore,

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 \leq 2 \left(1 - \sqrt{1 - \left(\frac{2\varepsilon}{\lambda_s - 2\varepsilon} \right)^2} \right).$$

With high SNR ρ , using approximation $(1 - x)^{1/2} \approx 1 - \frac{1}{2}x$, when $x \ll 1$, we have

$$\|\mathbf{u}_1 - \tilde{\mathbf{u}}_1\|^2 \leq \left(\frac{2\varepsilon}{\lambda_s - 2\varepsilon} \right)^2 \approx \frac{4}{\rho^2}.$$

□