Intersymbol blind multiuser detection for CDMA systems

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Intersymbol Blind Multiuser Detection for CDMA Systems

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2005

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Zhang Gaonan

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Summary

The telecommunications industry is one of the largest industries worldwide. One of the most active areas in this industry is wireless communication, which has covered a very wide array of applications. Most of the wireless communication systems today rely on a multiple-access technique, which is a mechanism to divide the common transmission medium among different users. Code-Division Multiple-Access (CDMA) is a multiple-access technique that has received considerable attention in recent years. In a CDMA system, each user spreads its information-bearing signal into a wideband signal by using specific code information. All users then transmit their wideband signals within the same frequency channel at the same time. This thesis deals with the development of the blind multiuser detectors for various CDMA systems by using the intersymbol information between the consecutive received signals, which has been considered as a novel development for the detection of wireless communication systems.

Multiuser detectors are widely developed for CDMA systems in recent literature. These detectors take into account the full structure of the multiuser interference (MUI) including the multiple-access interference (MAI) and the intersymbol inter-

ference (ISI) in CDMA systems to improve the performance of the detection. In this thesis, we design various intersymbol blind multiuser detectors for both asynchronous direct-sequence CDMA (DS-CDMA) systems and multicarrier CDMA (MC-CDMA) systems.

We first develop the intersymbol blind decorrelating detector for asynchronous DS-CDMA systems. The basic idea of the detector is the construction of a cross correlation matrix based on the consecutive received signals. With the constructed cross correlation matrix, the decorrelating detector can be developed without channel estimation.

Next, we develop two intersymbol group-blind multiuser detectors for asynchronous DS-CDMA systems. These detectors utilize the intersymbol information to construct a group constraint. Hence, the detectors can be derived in term of various constraint optimization criteria. It is shown that compared with conventional detectors without group constraint, both intersymbol group-blind multiuser detectors can improve the performance of the detection correspondingly in light of the channel feature of the signal model.

Finally, we propose a blind multiuser detector for asynchronous MC-CDMA systems. The derivation of the detector is referring to the proposed intersymbol blind decorrelating detector. For the blind adaptive implementation of the proposed detector, we develop a new subspace tracking algorithm. It is shown that the proposed algorithm can track the subspace of any Hamiltonian matrix, while the previous adaptive subspace track method can only estimate the subspace of the

positive Hamiltonian matrix. It is also noted that the proposed adaptive algorithm can be applied to the proposed intersymbol detectors for DS-CDMA systems in the same way.

Contents

Acknowledgements	i
Summary	ii
Contents	v
List of Figures	x
List of Tables	xii
List of Abbreviations	xiii
List of Symbols	xv
1 Introduction	1
1.1 Motivation	1
1.2 Objectives	2
1.3 Major Contributions of the Thesis	2
1.4 Multiple-Access Principles	3
1.5 Signal Model of CDMA Systems	6

1.5.1	Synchronous DS-CDMA Signal Model	8
1.5.2	Asynchronous DS-CDMA Signal Model in Multipath Channels	10
1.6	Basic Detection for CDMA systems	17
1.6.1	Canonical Form of Linear Detector	17
1.6.2	Performance Measures	18
1.6.3	Matched Filter	20
1.7	Organization of the Thesis	24
2	Conventional Linear Blind Multiuser Detectors	26
2.1	Introduction	28
2.2	Linear Detectors for Synchronous CDMA	30
2.2.1	Optimum Detector	30
2.2.2	Linear Decorrelating Detector	32
2.2.3	Linear MMSE Detector	34
2.3	LMS Blind Linear MMSE Detector for Synchronous CDMA Systems	35
2.4	RLS Blind Linear MMSE Detector for Synchronous CDMA Systems .	38
2.5	Subspace Blind Linear Detectors for Synchronous CDMA Systems . .	40
2.5.1	Subspace Decorrelating Detector	41
2.5.2	Subspace MMSE Detector	42
2.5.3	Blind Implementation	43
2.6	Subspace Detectors for Asynchronous CDMA Systems with Multipath.	45
2.7	Performance Analysis of the Decorrelating Detector and MMSE De- tector	50

2.7.1	SINR	50
2.7.2	Bit-error-rate (BER)	52
2.7.3	Asymptotic Multiuser Efficiency (AME) and Near-far Resistance	53
2.8	Conclusion	54
3	Intersymbol Decorrelating Detector for Asynchronous DS-CDMA	
	Systems	55
3.1	Introduction	55
3.2	Signal Model and Conventional Methods	57
3.2.1	Signal Model	57
3.2.2	Conventional Methods	57
3.3	Proposed Intersymbol Decorrelating Detector	58
3.4	Proposed Blind Implementation of the Intersymbol Decorrelating De- tector	64
3.5	Simulation Results	67
3.6	Conclusions	72
4	Intersymbol Group-blind Multiuser Detectors for Asynchronous	
	DS-CDMA Systems	73
4.1	Signal Model	74
4.2	Proposed Intersymbol Group Constraint	77
4.3	Proposed Intersymbol Blind Group Linear Detectors	87

4.3.1	Proposed Intersymbol Blind Group Linear Decorrelating De-	
	tector	87
4.3.2	Proposed Intersymbol Blind Group Linear Hybrid Detector . .	90
4.4	Simulation Results	93
4.4.1	Performance of the Proposed Detectors for Asynchronous CDMA	
	Systems	94
4.4.2	Performance of the Proposed Detectors for Synchronous CDMA	
	Systems	97
4.5	Conclusions	101
5	Intersymbol Blind Multiuser Detection for MC-CDMA Systems	103
5.1	Introduction	103
5.2	Signal Model	105
5.3	Proposed Blind Intersymbol Decorrelating Detector	109
5.3.1	Conventional Linear Detectors	110
5.3.2	Proposed Blind Intersymbol Decorrelating Detector	112
5.3.3	Proposed Intersymbol Multiuser Detection in Correlated Noise	115
5.4	Proposed Blind Adaptive Implementation	117
5.4.1	Proposed Scalar Function of I-PASTD algorithm	117
5.4.2	Proposed Subspace Tracking of I-PASTD Algorithm	119
5.4.3	Estimation of Ambiguous Scaling Factor	123
5.4.4	Blind Adaptive Implementation	127
5.5	Simulation Results	129

5.6	Conclusions	137
6	Conclusions and Recommendation	139
6.1	Conclusions	139
6.2	Recommendations for Further Research	143
	Author's Publication List	146
	Bibliography	149

List of Figures

1.1	Basic concept of FDMA, TDMA and CDMA	4
1.2	Signal model in a CDMA system with multipath distortion.	11
1.3	Bank of single-user matched filters	21
3.1	Bit-error-rate comparison of various linear detectors in an asynchro- nous CDMA system. $M = 3, G = 5, K = 8, d_k \geq 8$ ($k = 2, \dots, 8$). . . .	68
3.2	Bit-error-rate comparison of various linear detectors in an asynchro- nous CDMA system. $M = 5, G = 5, K = 8, d_2 = 3, d_3 = 2, d_k \geq 8$ ($k = 4, \dots, 8$).	70
3.3	Bit-error-rate comparison of various linear detectors in an asynchro- nous CDMA system. $M = 7, G = 5, K = 8, d_2 = 3, d_3 = 2, d_k \geq 8$ ($k = 4, \dots, 8$).	71
4.1	Bit-error-rate comparison of different detectors for asynchronous CDMA ($P = 500, K = 7$).	94
4.2	Bit-error-rate comparison of different detectors for asynchronous CDMA ($P = 500, K = 10$).	96

4.3	Bit-error-rate comparison of different detectors for asynchronous CDMA ($P = 500, K = 15$).	98
4.4	Bit-error-rate comparison of different detectors for synchronous CDMA ($P = 500, K = 10$).	99
4.5	Bit-error-rate comparison of different detectors for synchronous CDMA ($P = 500, K = 15$).	100
5.1	Transmitter block diagram for the k th user of the asynchronous MC- CDMA system.	107
5.2	Receiver block diagram of the asynchronous MC-CDMA system. . . .	107
5.3	Similarity between the proposed detector and the optimal decorrelat- ing detector.	130
5.4	Comparison of the SINRs of the proposed detector and other detectors for asynchronous MC-CDMA system ($K = 7$ users, SNR=2dB). . . .	132
5.5	Comparison of the SINRs of the proposed detector and other detectors for asynchronous MC-CDMA system ($K = 7$ users, SNR=6 dB). . . .	133
5.6	SINR of the proposed detector in a dynamic multiple-access channel (SNR=8dB).	134
5.7	Bit-error-rate comparison of the proposed detector and various no- adaptive detectors (7 users).	135
5.8	Bit-error-rate comparison of the proposed detector and various adap- tive detectors (9 users).	136

List of Tables

2.1	Algorithm of Subspace Blind Linear Decorrelating and Linear MMSE Detectors for synchronous CDMA Systems	44
2.2	Subspace Blind Linear Decorrelating and Linear MMSE Detectors for Asynchronous CDMA Systems	49
3.1	Blind Intersymbol Decorrelating Detector	66
4.1	Algorithms of Intersymbol Blind Group Linear Decorrelating detector and Blind Group Linear Hybrid Detector	93
5.1	Algorithm of I-PASTD	122
5.2	Blind Implementation of the Intersymbol Decorrelating Detector . . .	127

List of Abbreviations

ABBREVIATIONS	FULL EXPRESSIONS
AME	Asymptotic Multiuser Efficiency
AWGN	Additive White Gaussian Noise
BER	Bit-error-rate
BPSK	Binary Phase Shift Keying
CDMA	Code-division Multiple-access
D-RAKE	Decorrelating RAKE
DS-CDMA	Direct-sequence CDMA
DS-SS	Direct-sequence Spread-spectrum
EGC	Equal Gain Combining
FDMA	Frequency-division Multiple Access
GSM	Global System for Mobile
I-PASTD	Improved PASTD
ISI	Intersymbol Interference

LMS	Least Mean Squares
MAI	Multiple Access Interference
MC-CDMA	Multicarrier CDMA
MIMO	Multi-input and Multi-output
MMSE	Minimum Mean-square-error
MOE	Minimum Output Energy
MRC	Maximum Ratio Combining
MUI	Multiuser Interference
NMT	Nordic Mobile Telephone
OFDM	Orthogonal Frequency Division Multiplexing
PASTD	Projection Approximation Subspace Tracking Deflation
RLS	Recursive Least Square
SINR	Signal-to-interference and Noise Ratio
TDMA	Time-division Multiple Access
W-CDMA	Wideband CDMA

List of Symbols

$\mathbf{a}$	vector $\mathbf{a}$
$\mathbf{A}$	matrix $\mathbf{A}$
$\mathbf{A}^T$	transpose of matrix $\mathbf{A}$
$\mathbf{A}^H$	Hermitian (complex conjugate transpose) of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of matrix $\mathbf{A}$
$\mathbf{A}^\dagger$	pseudoinverse of matrix $\mathbf{A}$
$\text{tr}\{\mathbf{A}\}$	trace of matrix $\mathbf{A}$
$\text{diag}\{\mathbf{a}\}$	square diagonal matrix with $\mathbf{a}$ as diagonal
$\mathbf{A}_{k,l}$	element on the k th row and l th column of matrix $\mathbf{A}$
$\mathbf{A}_{N \times M}$	$N \times M$ matrix $\mathbf{A}$
$\mathbf{0}$	zero vector or zero matrix
$\mathbf{1}_n$	unit vector with a one in the n th entry
$\mathbf{I}$	identity matrix
$\mathbf{I}_{N \times N}$	$N \times N$ identity matrix

$\Re^N$	N dimension space
$\Re\{\mathbf{a}\}$	real part of $\mathbf{a}$
$E\{\mathbf{a}\}$	expectation of random variable $\mathbf{a}$
$ \mathbf{a} $	norm of $\mathbf{a}$
$\mathcal{N}(\mathbf{a}, \mathbf{B})$	Gaussian distribution with mean $\mathbf{a}$ and variance $\mathbf{B}$
$\odot$	convolutional integration

Chapter 1

Introduction

1.1 Motivation

Wireless communication is undergoing a revolution with the development of signal processing techniques and computer science during the last twenty years. A key aspect of a wireless communication system is the multiple access technique to share the common transmission medium between various users. The basic multiple access techniques typically include time-division multiple access (TDMA), frequency-division multiple access (FDMA) and code-division multiple access (CDMA). Among them, CDMA has gained popularity recently, and has been selected as a prime candidate for third-generation wide-band wireless systems, due to its improved bandwidth efficiency and flexibility. However, the communication of the CDMA systems suffers more interference compared with TDMA and FDMA. A large amount of research has been carried out in order to improve the detection of the signal in CDMA sys-

tems. This thesis deals with the development of the blind multiuser detectors for various CDMA systems.

1.2 Objectives

The main objective of the research work is to develop new blind linear multiuser detectors for various CDMA systems. The proposed detectors take into account the full structure of the multiuser interference (MUI) including multiple-access interference (MAI) and intersymbol interference (ISI), and then to improve the quality of the detection of the CDMA systems.

1.3 Major Contributions of the Thesis

In this thesis, we design various intersymbol blind multiuser detectors for both asynchronous direct sequence CDMA (DS-CDMA) systems and Multicarrier CDMA (MC-CDMA) systems.

We first develop the blind decorrelating detectors for asynchronous DS-CDMA systems. The basic idea of the detectors is the construction of a cross correlation matrix based on the consecutive received signal. Then, the decorrelating detector can be developed without channel estimation.

Next, we develop two intersymbol group-blind multiuser detectors for asynchronous DS-CDMA systems. These detector utilize the intersymbol information to construct a group constraint. Hence, the detectors can be derived in term of various

constraint optimization criteria. It is shown that compared with conventional detector without group constraint, both intersymbol group-blind multiuser detectors can improve the performance of the detection correspondingly in light of the channel feature of the signal model.

Finally, we propose the blind multiuser detector for asynchronous MC-CDMA systems. The derivation of the detector is referred to the proposed intersymbol blind decorrelating detector. For the blind adaptive implementation of the proposed detector, we develop a new subspace tracking algorithm. It is shown that our algorithm can track the subspace of any Hamiltonian matrix, while the previous adaptive subspace track method can only estimate the subspace of the positive Hamiltonian matrix. It is also noted that proposed adaptive algorithm can be applied to the proposed intersymbol detectors for DS-CDMA systems in the similar way.

1.4 Multiple-Access Principles

Wireless communication systems are usually organized in a cellular fashion. The wireless operator divides the covered area into different cells and places a base station in the center of each cell. There are three basic ways in which radio resources can be shared among multiple users in a cell. These can be viewed as ways of allocating regions in frequency and time to different users, as shown in Figure 1.1.

One of the basic multiple-access technique in Figure 1.1 is FDMA, in which the frequency band available for a given service is divided into subbands that are allocated to individual users who wish to use the service. FDMA is the principal

Figure 1.1: Basic concept of FDMA, TDMA and CDMA

multiplexing method used in the first-generation cellular telephone systems, such as the Nordic Mobile Telephone (NMT) [1]. FDMA is also used in some form in all other current cellular systems, in tandem with other multiple-access techniques that are used for further allocating the subchannels to multiple users [2].

Another multiple-access technique is TDMA, in which time is partitioned into equal-length slots assigned to each incoming digital stream. This method of channel sharing is widely used in the second-generation cellular systems, including the universal Global System for Mobile (GSM) systems [3, 4] and in the wireless personal network such as bluetooth network.

In FDMA or TDMA systems, each user within the same cell operates in a unique frequency or time channel. FDMA allows each user to use part of the spectrum all the time, while TDMA allows each user to use the entire available spectrum within a part of the time. These channels are said to be orthogonal because there is no

intra-cell interference between users. In a CDMA system, on the other hand, each user within the same cell operates in the same frequency band at the same time. CDMA assigns channels in a way that allows all users to use all of the available time and frequency resources simultaneously, through the assignment of a pattern or code to each user that specifies the way in which these resources will be used by a particular user. Typically, CDMA is implemented via spread-spectrum modulation, in which the pattern is the pseudorandom spreading code that determines the spreading sequence in the case of direct sequence, or the hopping pattern in the case of frequency hopping. In fact, FDMA and TDMA can be considered as special cases of CDMA where the spreading codes are orthogonal and nonoverlapping in the frequency domain and the time domain, respectively. CDMA is the basis for the principal standards being developed and deployed for 3G cellular telephony [5, 6, 7].

It is important to point out that the orthogonality of the spreading code is not imperative for CDMA systems. This leads to the multiple access interference which does not occur in FDMA and CDMA systems in theory. However, CDMA has a number of desirable advantages over FDMA or TDMA, whatever the receiver we use. In practical wireless communication systems, the transmitted signal will be received from a number of different paths. Since in a CDMA system each user is usually assigned a larger bandwidth than in FDMA or TDMA system, the different paths are better isolated. Hence, in a CDMA system the multipath interference can be suppressed more efficiently than in FDMA or TDMA systems. Another important advantage of CDMA technique is channel efficiency. In FDMA or TDMA systems,

each user is assigned a unique frequency or time channel, which causes a great loss of capacity. Consequently, only the channel of the silent user can be reused by another user. This requires that the silent period be long enough. In a CDMA system, this problem does not occur because each user is assigned the same frequency and time channel.

1.5 Signal Model of CDMA Systems

In order to discuss advanced signal processing methods used for CDMA receivers, it is useful to specify a general model first for the signal received by a wireless detector. We first consider the transmitter that transmits the sequence of different user's channel symbols over a CDMA channel. In this thesis, we consider only the binary (i.e. ± 1) and linear modulation systems, in which the symbols are transmitted into the channel by being modulated linearly onto a signaling waveform.

In a CDMA system, each user is assigned a unique signature waveform or code, which spreads data symbol sequence with rate $1/T$ (the data symbol rate) into a so-called chip sequence with rate N/T (the chip rate), where $N \geq 1$ is called the spreading factor. The way of the spreading operation is carried out depends on the type of CDMA systems. All users then transmit their chip sequence within the same frequency and time channel. The receiver finally tries to recover the desired user's data symbol sequence, based on the knowledge of the desired user's code information. Among various CDMA technologies, direct-sequence spread-spectrum (DS-SS) modulation gains most popularity as a multiple-access technology for per-

sonal, cellular, and satellite communication services. DS-CDMA is currently used in IS-95 system [8], and it is also the multiple-access technique that is employed in the third generation system proposal [9], which is referred to as wideband CDMA (W-CDMA) proposals. The IS-95 system has a bandwidth of 1.25 MHz, while the W-CDMA proposals aim at bandwidths of 5 MHz or more.

In a DS-CDMA system, each user spreads its data symbol sequence into a chip sequence by exploiting the direct-sequence signature waveform, which is characterized as follows:

- Spread the data symbol sequence by a factor N using a specific code sequence, which repeats each data symbol N times. The factor N is also called spreading gain.
- The normalized signature waveform of the spreading sequence for specific user k is given by

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} c_k[j] \psi(t - jT_c), \quad T \geq t \geq 0 \quad (1.1)$$

where $1/T$ is the symbol rate, $c_k[j]$ ($N-1 \geq j \geq 0$) is the pseudorandom spreading sequence allocated to the k th user (typically, $c_k[j] \in \{+1, -1\}$), and $\psi(t)$ is the normalized chip waveform with a chip interval $T_c = T/N$. In (1.1), the normalized signature waveform $s_k(t)$ has unit energy

$$\|s_k(t)\|^2 = \int_0^T s_k(t)^2 dt = 1, \quad (1.2)$$

and is assumed to be zero outside the duration $[0, T]$.

The signal models of two different DS-CDMA systems are introduced in the following Sections.

1.5.1 Synchronous DS-CDMA Signal Model

In DS-CDMA systems, each user is assigned a unique signature waveform, and the transmitted signals are the superposition of multiple signals. In particular, for a system of K users, we can write a transmitted signal for each user as

$$y_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k[i] s_k(t - iT), \quad k = 1, 2, \dots, K \quad (1.3)$$

where A_k and $b_k[i] \in \{+1, -1\}$ are, respectively, the amplitude and the transmitted symbol stream of the k th user. Then, the total transmitted signal is given by

$$\begin{aligned} y(t) &= \sum_{k=1}^K y_k(t) \\ &= \sum_{k=1}^K A_k \sum_{i=-\infty}^{\infty} b_k[i] s_k(t - iT), \end{aligned} \quad (1.4)$$

where $y(t)$ includes the signals of all users at the transmitter. Thus, the total received signal at the receiver is given by:

$$r(t) = \sum_{k=1}^K A_k \sum_{i=-\infty}^{\infty} b_k[i] s_k(t - iT) + v(t), \quad (1.5)$$

where $v(t)$ is the baseband complex Gaussian ambient noise with independent real and imaginary components and power spectral density σ^2 . The signal in (1.5) presents the most basic CDMA signal model, i.e., a baseband K -user time-invariant synchronous system with additive white Gaussian noise (AWGN), employing periodic spreading sequences and operating with a coherent binary phase shift keying

(BPSK) modulation format. Synchronous CDMA is normally considered in the CDMA downlink communication, i.e., base station to mobile users communication. In this scenario, all the active users are transmitted from base station simultaneously and within the same channel.

At the receiver, the received signal $r(t)$ is filtered by a chip-matched filter and then sampled at the chip rate. The sample corresponding to the j th chip of the i th symbol is thus given by

$$r_j[i] \triangleq \int_{iT+jT_c}^{iT+(j+1)T_c} r(t)\psi(t-iT-jT_c)dt, \quad j \in [0, N-1], i \in [-\infty, \infty]. \quad (1.6)$$

The resulting discrete-time signal corresponding to the i th symbol of all users is then given by

$$\mathbf{r}[i] = \sum_{k=1}^K A_k b_k[i] \mathbf{s}_k + \mathbf{v}[i], \quad (1.7)$$

where

$$\mathbf{r}[i] \triangleq \begin{bmatrix} r_0[i] \\ r_1[i] \\ \vdots \\ r_{N-1}[i] \end{bmatrix}, \quad \mathbf{s}_k \triangleq \frac{1}{\sqrt{N}} \begin{bmatrix} c_k[0] \\ c_k[1] \\ \vdots \\ c_k[N-1] \end{bmatrix}, \quad \mathbf{v}[i] \triangleq \begin{bmatrix} v_0[i] \\ v_1[i] \\ \vdots \\ v_{N-1}[i] \end{bmatrix}, \quad (1.8)$$

and

$$v_j[i] = \int_{iT+jT_c}^{iT+(j+1)T_c} v(t)\psi(t-iT-jT_c)dt, \quad j \in [0, N-1] \quad (1.9)$$

is a complex Gaussian random variable with independent real and imaginary components. According to the definition of the noise $v(t)$ in (1.5), the vector $\mathbf{v}(i)$ has a Gaussian distribution of $\mathbf{v}(i) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$.

By denoting

$$\mathbf{S} \triangleq [\mathbf{s}_1, \dots, \mathbf{s}_K],$$

$$\mathbf{A} \triangleq \mathbf{diag}[A_1, \dots, A_K],$$

$$\mathbf{b}[i] \triangleq [b_1[i], \dots, b_K[i]]^T,$$

the discrete-time signal model in (1.7) can also be expressed as

$$\mathbf{r}[i] = \mathbf{S}\mathbf{A}\mathbf{b}[i] + \mathbf{v}[i], \quad (1.10)$$

which is a more straightforward expression of the synchronous CDMA systems. The signal model of (1.10) can also be considered as a special case of MIMO systems while the input is single and the output is multiple [10].

1.5.2 Asynchronous DS-CDMA Signal Model in Multipath Channels

In the basic synchronous CDMA model of (1.10), the signals of the active users are assumed to be transmitted simultaneously and all of the transmitted signals suffered by white ambient noise only, while the influence of multipath is not considered. This is not the case in a practical wireless CDMA system, however, where the user's signals are asynchronously transmitted especially for CDMA uplink communication, i.e., mobile users to base station communication. Moreover, the physical channel exhibits dispersion due to multipath effects that further distort the signals. Multipath refers to the phenomenon by which multiple copies of a transmitted signal are received at the receiver, due to the presence of multiple radio paths between the

transmitter and receiver. These multiple paths arise due to reflections from objects in the path of the radio channel. A related phenomenon arises when the difference in path lengths is that the time delay of different arrivals is significant relative to a symbol interval. This results in dispersion of the retransmitted signal, and then causes ISI. Figure 1.2 illustrated a more general multiple-access signal model where the channel exhibits multipath distortion effects for each transmitted user.

Figure 1.2: Signal model in a CDMA system with multipath distortion.

In this figure, $y_k(t)$ is the transmitted signal for user k . Unlike the expression of the previous synchronous model in (1.3), $y_k(t)$ has the form of

$$y_k(t) = A_k \sum_{i=-\infty}^{\infty} b_k[i] s_k(t - iT - d_k), \quad k = 1, 2, \dots, K \quad (1.11)$$

where A_k and $b_k[i] \in \{+1, -1\}$ are, respectively, the amplitude and the transmitted symbol stream of the k th user, and $T \geq d_k \geq 0$ is the delay of the k th user's signal which specifies the difference between the asynchronous CDMA model and the synchronous CDMA model. Parameter $g_k(t)$ in Figure 1.2 denotes the impulse response of linear filter representing the channel between the transmitter and the

receiver of the k th user, and $v(t)$ represents the white ambient noise as we mentioned previously. All of the quantities $g_k(t)$ and $v(t)$ are, in general, random processes. Normally, the multipath channel impulse response of the k th user can be modeled as

$$g_k(t) = \sum_{m=1}^M \alpha_{m,k} \delta(t - \tau_{m,k}), \quad (1.12)$$

where M is the total number of paths in the channel, and $\alpha_{m,k}$ and $\tau_{m,k}$ are the complex path gain and the delay of the m th path for user k , respectively. For the convenience and without loss of generality, we assume $\tau_{1,k} < \tau_{2,k} \dots < \tau_{M,k}$. Throughout this thesis, it is assumed that the channel is slowly time varying, such that the path gains and delays remain constant over the duration of certain period.

By considering the asynchronism of each transmitted user, the received signal component of the k th user is given by [11]

$$\begin{aligned} r_k(t) &= y_k(t) \odot g_k(t) \\ &= \sum_{i=-\infty}^{\infty} b_k[i] \underbrace{A_k s_k(t - iT - d_k)}_{h_k(t - iT)} \odot g_k(t). \end{aligned} \quad (1.13)$$

We next denote the composite signature waveform of the k th user $h_k(t)$ as

$$\begin{aligned} h_k(t) &\triangleq A_k s_k(t - d_k) \odot g_k(t) \\ &= \sum_{j=0}^{N-1} c_k[j] \underbrace{\sum_{m=1}^M \frac{A_k}{\sqrt{N}} \alpha_{m,k} \psi(t - jT_c - d_k - \tau_{m,k})}_{\bar{g}_k(t - jT_c)}, \end{aligned} \quad (1.14)$$

where $\bar{g}_k(t)$ is the composite channel response, taking into account the effects of transmitter power, chip pulse waveform and the multipath channel, given by

$$\bar{g}_k(t) \triangleq \frac{A_k}{\sqrt{N}} \sum_{m=1}^M \alpha_{m,k} \psi(t - d_k - \tau_{m,k}). \quad (1.15)$$

In (1.15), $\bar{g}_k(t)$ is zero outside the time duration $[d_k + \tau_{1,k}, T_c + d_k + \tau_{M,k}]$ because $\psi(t)$ is zero outside the interval $[0, T_c]$. Therefore, the composite signature waveform $h_k(t)$ is zero outside the interval $[d_k + \tau_{1,k}, T + d_k + \tau_{M,k}]$.

At the receiver end, the total received signal is the superposition of the signal of all users, plus additive Gaussian noise, which is given by

$$r(t) = \sum_{k=1}^K r_k(t) + v(t). \quad (1.16)$$

As mentioned before, $v(t)$ is a zero mean complex Gaussian noise process. After being sampled at the chip rate, the received signal $r(t)$ becomes a discrete-time sequence. The n th received signal sample during the i th symbol is then given by

$$\begin{aligned} r_n[i] &\triangleq r(iT + nT_c) \\ &= \sum_{k=1}^K \underbrace{r_k(iT + nT_c)}_{r_{k,n}[i]} + \underbrace{v(iT + nT_c)}_{v_n[i]}. \end{aligned} \quad (1.17)$$

For the convenience of analysis and without loss of generality, we assume throughout this thesis that $2T \geq T_c + d_k + \tau_{M,k}$, which means the delay caused by the multipath and the asynchronous transmission is within a two-symbol period. Thus, by using (1.13), we have

$$\begin{aligned} r_{k,n}[i] &\triangleq r_k(iT + nT_c) \\ &= \sum_{j=-\infty}^{\infty} b_k[j] h_k(iT + nT_c - jT) \\ &= b_k[i] \underbrace{h_k(nT_c)}_{h_k[n]} + b_k[i-1] \underbrace{h_k(T + nT_c)}_{h_k[N+n]}, \end{aligned} \quad (1.18)$$

where the third equation follows from the above assumption and the fact that $h_k(t)$

is zero outside the interval $[d_k + \tau_{1,k}, T + d_k + \tau_{M,k}]$. From (1.14) and (1.18), we have

$$\begin{aligned} h_k[n] &\triangleq h_k(nT_c) \\ &= \sum_{j=0}^{N-1} c_k[j] \underbrace{\bar{g}_k(nT_c - jT_c)}_{\bar{g}_k[n-j]}, \end{aligned} \quad (1.19)$$

where the sequence $\{\bar{g}_k[n]\}$ is obtained by sampling the composite channel response $\bar{g}_k(t)$ at chip rate, which is given by

$$\begin{aligned} \bar{g}_k[n] &\triangleq \bar{g}_k(nT_c) \\ &= \frac{A_k}{\sqrt{N}} \sum_{m=1}^M \alpha_{m,k} \psi(nT_c - d_k - \tau_{m,k}). \end{aligned} \quad (1.20)$$

Recalling that $\bar{g}_k(t)$ is nonzero only within the interval $[d_k + \tau_{1,k}, T + d_k + \tau_{M,k}]$, we denote $d_{k,s} \triangleq \lceil d_k + \tau_{1,k} \rceil$ and $d_{k,e} \triangleq \lceil d_k + \tau_{M,k} + T \rceil$, respectively. Then it is clear that $h_k[n]$ is nonzero within the interval $n \in [d_{k,s}, d_{k,e}]$. From (1.20) and denoting

$$\mathbf{h}_{k,all} \triangleq \begin{bmatrix} h_k[d_{k,s}] \\ h_k[d_{k,s} + 1] \\ \vdots \\ h_k[d_{k,e}] \end{bmatrix}, \quad \bar{\mathbf{g}}_k \triangleq \begin{bmatrix} \bar{g}_k[d_{k,s}] \\ \bar{g}_k[d_{k,s} + 1] \\ \vdots \\ \bar{g}_k[d_{k,e}] \end{bmatrix}, \quad (1.21)$$

and

$$\mathbf{C}_k \triangleq \begin{bmatrix} c_k[0] & & & \\ c_k[1] & \ddots & & \\ \vdots & \ddots & c_k[0] & \\ \vdots & & c_k[1] & \\ c_k[N-1] & \ddots & \vdots & \\ & \ddots & \vdots & \\ & & c_k[N-1] & \end{bmatrix}, \quad (1.22)$$

where $\mathbf{C}_k$ is an $(N + d_{k,e} - d_{k,s}) \times (d_{k,e} - d_{k,s} + 1)$ matrix formed from the spreading code of the k th user, we have

$$\mathbf{h}_{k,all} = \mathbf{C}_k \bar{\mathbf{g}}_k. \quad (1.23)$$

According to the received signal for the k th user in (1.18), we stack N successive sampled signals as following:

$$\mathbf{r}_k[i] \triangleq \begin{bmatrix} r_{k,0}[i] \\ r_{k,1}[i] \\ \vdots \\ r_{k,N-1}[i] \end{bmatrix}, \mathbf{h}_k \triangleq \begin{bmatrix} \mathbf{0} \\ h_k[d_{k,s}] \\ h_k[d_{k,s} + 1] \\ \vdots \\ h_k[N] \end{bmatrix}, \bar{\mathbf{h}}_k \triangleq \begin{bmatrix} h_k[N+1] \\ h_k[N+2] \\ \vdots \\ h_k[d_{k,e}] \\ \mathbf{0} \end{bmatrix}. \quad (1.24)$$

It is easy to obtain

$$\mathbf{r}_k[i] = \mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1]. \quad (1.25)$$

Finally, by denoting

$$\mathbf{r}[i] \triangleq \begin{bmatrix} r_0[i] \\ r_1[i] \\ \vdots \\ r_{N-1}[i] \end{bmatrix}, \mathbf{v}[i] \triangleq \begin{bmatrix} v_0[i] \\ v_1[i] \\ \vdots \\ v_{N-1}[i] \end{bmatrix}, \mathbf{b}[i] \triangleq \begin{bmatrix} b_1[i] \\ b_2[i] \\ \vdots \\ b_K[i] \end{bmatrix}, \quad (1.26)$$

the received composite signal vector from all active users and the background noise is given by

$$\begin{aligned} \mathbf{r}[i] &= \sum_{k=1}^K \mathbf{r}_k[i] + \mathbf{v}[i] \\ &= \sum_{k=1}^K (\mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1]) + \mathbf{v}[i], \end{aligned} \quad (1.27)$$

where $\mathbf{v}[i]$ is the background vector defined in (1.26) with a distribution of $\mathbf{v}(i) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_N)$.

Equation (1.27) presents the asynchronous CDMA signal model. Unlike the simple synchronous CDMA model in (1.10), the model in (1.27) considers the delay and the multipath channel of each user. For the desired user (we choose user 1 as the interested user without loss of the generality), (1.27) can be written as

$$\mathbf{r}[i] = \mathbf{h}_1 b_1[i] + \underbrace{\bar{\mathbf{h}}_1 b_1[i-1]}_{\text{ISI}} + \underbrace{\sum_{k \neq 1}^K (\mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1])}_{\text{MAI}} + \mathbf{v}[i]. \quad (1.28)$$

In (1.28), the first term contains the i th bit of the desired user 1; the second term contains the ISI from the previous bit of the first user; the third term contains the multiple access interference (MAI) from other users. The signal model of (1.28) can also be written as

$$\mathbf{r}[i] = \mathbf{H}_0 \mathbf{b}[i] + \mathbf{H}_1 \mathbf{b}[i-1] + \mathbf{v}[i], \quad (1.29)$$

where

$\mathbf{H}_0 = [\mathbf{h}_1, \dots, \mathbf{h}_K]$: signature matrix of the current symbols of all users including MAI;

$\mathbf{H}_1 = [\bar{\mathbf{h}}_1, \dots, \bar{\mathbf{h}}_K]$: signature matrix of the previous symbols of all users including ISI and MAI;

$\mathbf{b}[i] = [b_1[i], \dots, b_K[i]]^T$: current symbols of all users at the i th received signal vector;

$\mathbf{b}[i-1] = [b_1[i-1], \dots, b_K[i-1]]^T$: previous symbols of all users at the i th received signal vector.

1.6 Basic Detection for CDMA systems

This thesis is concerned with the design of advanced signal processing methods for the detection of CDMA systems. Before discussing these methods, however, it is of interest to briefly review some basic elements of signal processing for the above models [12, 13]. In this Section, the simplest strategy to demodulate CDMA signals, i.e., the single-user matched filter is analyzed as a basis for the following multiuser detection techniques. The basic performance measures for the detection efficiency checking of CDMA systems are also introduced.

1.6.1 Canonical Form of Linear Detector

According to the discrete-time signal models of (1.10) and (1.29), the canonical form of a linear detector for the desired user k can be represented by a correlating vector

$\mathbf{w}_k \in \mathbb{R}^N$ followed by the received signal $\mathbf{r}[i]$, such that the detection of user k is

$$\hat{b}_k[i] = \text{sgn}\{\Re(\mathbf{w}_k^H \mathbf{r}[i])\}. \quad (1.30)$$

For various linear detectors, the selection of $\mathbf{w}_k$ is based on the specific optimization criteria.

1.6.2 Performance Measures

Multisuser detection deals with the demodulation of mutually interfering digital streams of information. The main performance measures of multisuser detection include:

- Bit-error-rate (BER) $\mathbf{P}_k(\sigma)$: Bit-error-rate is defined as the detection error rate of the k th user. $\mathbf{P}_k(\sigma)$ is usually expressed by the form of Q-function, a complementary Gaussian cumulative distribution function which is defined as:

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \quad (1.31)$$

Q-function is a monotonically decreasing function. Its analytical properties have been extensively studied [14]. BER is the most important criteria for multisuser detection. It also provides the foundation for the following measures.

- Signal-to-interference and noise ratio (SINR): SINR is the ratio of powers due to the desired user and all other components at a soft-decision variable.
- Effective energy of the k th user $e_k(\sigma)$: Effective energy is defined as the energy that user k would require to achieve bit-error-rate equal to $\mathbf{P}_k(\sigma)$ in a single-

user Gaussian channel with the same background noise level, that is :

$$\mathbf{P}_k(\sigma) = Q\left(\frac{\sqrt{e_k(\sigma)}}{\sigma}\right) \quad (1.32)$$

- Asymptotic multiuser efficiency (AME) η_k : Asymptotic multiuser efficiency is another commonly used performance measure for a multiuser detector [15]. Asymptotic multiuser efficiency measures the slope with which $\mathbf{P}_k(\sigma)$ goes to zero in the high signal-to-noise ration region, that is:

$$\eta_k = \lim_{\sigma \rightarrow 0} \frac{e_k(\sigma)}{A_k^2} \quad (1.33)$$

Sometimes, it is easier to calculate the asymptotic multiuser efficiency than the bit-error-rate, which allows intuitive depictions of the effects of unequal received energies.

- Near-far resistance: Based on the AME, a related performance measure, the near-far resistance is denoted as the multiuser asymptotic efficiency minimized over the received energies of all the other users, that is:

$$\bar{\eta}_k = \inf_{A_k > 0, j \neq k} \eta_k \quad (1.34)$$

Near-far resistance depends on the signature waveforms and the type of demodulator. It is another important measure for multiuser detection besides BER. It is also noted that when the BER is given as the weighted sum of Q -functions, it is very convenient to compute near-far resistance.

Among the above criteria, the main performance measure of interest in digital communications in general, and in multiuser detection in particular, is the BER. The

performance measures SINR and near-far resistance are popular in the multiuser detection as well. The SINR is useful to assess the quality of multiuser detectors, especially when used in conjunction with error-control codes. Since the received user's energies do not remain constant all the time, the near-far problem is also common in CDMA systems with mobile transmitters.

1.6.3 Matched Filter

In the literature of multiuser detection, the matched filter is frequently referred to as the conventional detector or RAKE receiver. It consists of a bank of correlators for the desired user, where each correlator despreads the received sequence matched to a different channel tap, using the desired user's spread code, followed by a linear combiner, which linearly combines the different correlator outputs in order to generate an estimate of the desired user's symbol sequence. The matched filter was originally proposed by North [16]. The single-user matched filter receiver has been used for CDMA demodulation since the inception of direct-sequence spread-spectrum for multiuser applications [17].

Among all the detection techniques for CDMA systems, matched filter is the simplest approach and is a single user detection technique. It provides the foundation of multiuser detection techniques and is still adopted in many practical communication systems due to its simplicity.

A. Construction of Filter

According to the discrete-time model of (1.10) and (1.29), the bit symbol of each

Figure 1.3: Bank of single-user matched filters

user can be obtained by its own matched filter (see Fig 1.3), where each filter is synchronized to the corresponding user. For both synchronous and asynchronous models, the matched filter for user k has the same canonical form of (1.30) with the value $\mathbf{w}_k = \mathbf{s}_k$, where $\mathbf{s}_k$ is the spreading code of user k as defined in (1.8). In the following, we analyze the performance of the matched filter. Since the analysis methods for both synchronous and asynchronous CDMA are similar, we take the synchronous model as an example.

Based on the synchronous model of (1.7) and (1.10), the estimated k th user's

data bit at i th symbol is given by:

$$\begin{aligned}
 y_k[i] &= \mathbf{w}_k^T \mathbf{r}[i] \\
 &= \mathbf{s}_k^T \mathbf{r}[i] \\
 &= A_k b_k[i] + \sum_{j \neq k} A_j b_j[i] \rho_{jk} + v_{sk}[i],
 \end{aligned} \tag{1.35}$$

and

$$\hat{b}_k[i] = \text{sgn}(y_k[i]), \tag{1.36}$$

where

$$v_{sk}[i] = \mathbf{s}_k^T \mathbf{v}[i] \tag{1.37}$$

is the correlation between the spreading code of the k th user and the back ground noise, and ρ_{jk} is the correlation of the spreading codes between the j th and k th users.

B. Bit-error-rate

The computation of the BER of the matched filter is straightforward since the background noise vector $\mathbf{v}[i]$ of (1.7) is of Gaussian distribution. Then, the BER of the k th user can be written as:

$$\begin{aligned}
 \mathbf{P}_k(\sigma) &= P\{b_k[i] = +1\}P\{y_k < 0 | b_k[i] = +1\} + P\{b_k[i] = -1\}P\{y_k > 0 | b_k[i] = -1\} \\
 &= \frac{1}{2}P \left[v_{sk}[i] > A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] + \frac{1}{2}P \left[v_{sk}[i] < -A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] \\
 &= P \left[v_{sk}[i] > A_k - \sum_{j \neq k} A_j b_j \rho_{jk} \right] \\
 &= \frac{1}{2^{K-1}} \sum_{e_1 \in [-1, 1]} \dots \sum_{e_1 \in [-1, 1], j \neq k} \dots \sum_{e_K \in [-1, 1]} Q \left(\frac{A_k}{\sigma} + \sum_{j \neq k} e_j \frac{A_j}{\sigma} \rho_{jk} \right),
 \end{aligned} \tag{1.38}$$

where Q -function is defined as in (1.31). By considering the worse case of (1.38) where the overall interference is the sum of the cross correlation between the desired user and each interference user, the upper bound of the BER $\mathbf{P}_k$ can be given by:

$$\mathbf{P}_k \preceq Q \left(\frac{A_k}{\sigma} - \sum_{j \neq k} \frac{A_j}{\sigma} |\rho_{jk}| \right) \quad (1.39)$$

C. Signal-to-interference and Noise Ratio

According to the definition of SINR, the SINR of the matched filter for the k th user is straightforward, which is given by

$$SINR_k = \frac{A_k^2}{\sigma^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2}. \quad (1.40)$$

It is seen that the SINR of the matched filter is seriously influenced by the correlation between the desired user and the interfering users.

D. Asymptotic Multiuser Efficiency and Near Far Resistance.

According to the BER in (1.39), the asymptotic multiuser efficiency of the matched filter for user k is obtained by :

$$\eta_k = \max \left\{ 0, 1 - \sum_{j \neq k} \frac{A_j}{A_k} |\rho_{jk}| \right\}. \quad (1.41)$$

Based on the AME in (1.41), the near-far resistance is selected as the minimum asymptotic efficiency of all users. Clearly, the matched filter can not entirely avoid near far problem ($\eta_k = 0$), i.e., the influence among different users may generate errors even though the noise is close to zero.

As a simple and convenient detector for CDMA systems, the matched filter does not consider the influence of the channel. Its performance is very limited compared

with the following introduced multiuser detectors. Especially, it suffers the near-far problem. The simulation results of the matched filter will be presented in the following chapter as a comparison with other detectors.

1.7 Organization of the Thesis

The rest of the thesis is organized as follows.

Chapter 2 introduces the batch algorithms and the adaptive algorithms for the basic approaches of the linear blind multiuser detection. We first treat simple synchronous CDMA model and present the principal techniques for blind multiuser detection. We then generalize these methods into the more general asynchronous CDMA channels with multipath fading.

Chapter 3 develops a new intersymbol decorrelating detector for asynchronous DS-CDMA systems based on the signal subspace spanned by the neighboring received signals. By using the cross correlation matrix between adjacent symbols, the proposed detector can be easily implemented without the estimations of the noise level, the multipath channel and the multipath length.

In Chapter 4, a series of intersymbol group-blind multiuser detectors for asynchronous CDMA systems are developed. The new techniques make use of an important cross correlation matrix between the adjacent symbols as a constraint to suppress the influence from the interfering users. The proposed detectors provide a substantial performance gain over the conventional blind detectors for both downlink CDMA and uplink CDMA.

In Chapter 5, we present the intersymbol blind multiuser detection for asynchronous MC-CDMA systems with Rayleigh fading channels. In order to implement the proposed detector blind adaptively, a new recursive subspace tracking algorithm for any Hamiltonian matrices is developed in this chapter, while the previously reported subspace tracking methods can only estimate the subspace of positive Hamiltonian matrix.

Conclusions and recommendation for further research work are presented in Chapter 6.

Chapter 2

Conventional Linear Blind Multiuser Detectors

As mentioned in Chapter 1, CDMA is gaining popularity as a multiple-access technology for personal, cellular, and satellite communication services. Considerable research in the field of multiuser detection for CDMA systems has been developed. Multiuser detection deals with the demodulation of digitally modulated signals in the presence of multiaccess interference. Such a technology finds its major applications in CDMA receiver design. The first comprehensive introduction of multiuser detection was found in [18], where the key concepts and a fairly complete coverage of the design and analysis of receivers for CDMA systems were presented extensively. Multiuser detection techniques can substantially increase the capacity of CDMA systems. Over the past decade, a significant amount of research has been focused on the linear multiuser detection. For example, methods for adaptive linear decorrelat-

CHAPTER 2. CONVENTIONAL LINEAR BLIND MULTIUSER DETECTORS 25

ing detector that require the transmission of training sequences have been proposed in [19, 20]. An alternative linear detector, the linear Minimum Mean-square-error (MMSE) detector, however, can be adopted either through the use of training sequence [21, 22] or in the blind mode [23, 24]. Timing-free blind multiuser detection has been considered recently in [25], while noncooperative multiuser detection, in which none of the signature waveforms is known, is treated in [26].

Recently, a large amount of reported research have been focused on the blind multiuser detection for CDMA systems, where only the information related to the interested user is available. Blind adaptation schemes are especially attractive for the downlink of CDMA systems, since in a dynamic environment it is very difficult for a mobile user to obtain accurate information about other active users in the channel, such as their signature waveforms, and the frequent use of training sequence will waste channel bandwidth.

In this chapter, we will present the batch algorithms and the adaptive algorithms for the basic approaches of linear blind multiuser detection. For the sake of exposition, we first treat simple synchronous CDMA model and present the principal techniques for blind multiuser detection. We then generalize these methods into the more general asynchronous CDMA channels with multipath fading.

The definition of linear blind multiuser detection is given in Section 2.1. In Section 2.2, the basic linear multiuser detectors for synchronous CDMA signal model are introduced. In Sections 2.3-2.5, various adaptive approaches for the implementation of blind multiuser detection are discussed, respectively. All these approaches

are based on the linear MMSE detector. In Section 2.6, the blind adaptive multi-user detection for general asynchronous CDMA systems with multipath channels is discussed in detail. The adaptive implementation of the detectors in this section is derived from subspace method via subspace tracking. Finally, Section 2.7 presents the analytical performance assessments.

The proposed algorithms in this chapter include:

- LMS blind linear MMSE detector for synchronous CDMA systems.
- RLS blind linear MMSE detector for synchronous CDMA systems.
- Subspace blind linear decorrelating detector and linear MMSE detector for synchronous CDMA systems.
- Subspace blind linear decorrelating detector and linear MMSE detector for asynchronous CDMA systems with multipath.

2.1 Introduction

In this section, we present the definition of blind linear multiuser detector. Since the canonical form of a linear detector has been given in (1.30), we only focus on the explanation of blind multiuser detection.

In multiuser detection, the detector for the desired user at the receiver end may assume to have the knowledge of

- (a). The signature waveform of the desired user.
- (b). The signature waveforms of the interfering users.

- (c). The timing (bit-epoch and carrier phase) of the desired user.
- (d). The timing (bit-epoch and carrier phase) of each of the interfering users.
- (e). The received amplitudes of interfering users (relative to that of the desired user).

For the blind multiuser detection, the detector does not need the information of the signature waveforms, timing and amplitudes of the interfering users, and only the signature waveform and timing of the desired user is assumed to be known. It should be pointed out that for any linear detectors, the number of active users must be estimated. One good subspace method for the estimation of the number of active users is given by [24]. Blind multiuser detection is extremely useful in the CDMA downlink communication, i.e., transmission of data from base station to individual mobile station. In such cases, it is impossible for the desired user's detector to obtain the information of interfering users. Even in systems where the knowledges of (b), (d) and (e) are available (as in the case of a centralized base station which demodulates every active user), it is usually computationally intensive to incorporate all the knowledge into the receiver parameters. Therefore, blind multiuser detection, especially the computationally efficient blind adaptive multiuser detection gains much attention during the development of multiuser detection.

It should be noted that the conventional matched filter introduced in the previous chapter does not require the knowledge of (b), (d) and (e) either. However, it is never considered as a multiuser detector since its performance is severely limited by the near-far problem, and even in the presence of perfect power control, its bit-

error-rate is in orders of magnitude far from optimal. On the contrary, the detector discussed in the following is able to use different optimization methods to overcome the near-far problem and achieve better performance.

2.2 Linear Detectors for Synchronous CDMA

In this section, we introduce the basic linear multiuser detectors for synchronous CDMA systems. Firstly, the optimum detector is presented and analyzed. In order to avoid the unbearable complexity of the optimum detector, two of the most popular suboptimal linear detectors, i.e., the linear decorrelating and the linear MMSE detectors, are next introduced. Finally, the blind adaptive implementation of the linear decorrelating and the linear MMSE detectors will be given in the following sections.

2.2.1 Optimum Detector

According to the model of K -user basic synchronous CDMA system in (1.5) and (1.10), the linear optimum detector tries to find the most likely $\mathbf{b}[i]$ with the minimum probability of error. Base on the independence among all users and the feature of the white background noise, the minimum probability of error of $\mathbf{b}[i]$ is equivalent to the maximization of

$$\exp \left(-\frac{1}{2\sigma^2} \int_{(i-1)T}^{iT} [r(t) - \sum_{k=1}^K b_k[i] A_k s_k(t - iT)]^2 dt \right), \quad (2.1)$$

or, equivalent maximizations of

$$\begin{aligned}\Omega(\hat{\mathbf{b}}[i]) &= 2 \int_{(i-1)T}^{iT} \left[\sum_{k=1}^K A_k b_k[i] s_k(t - iT) \right] r(t) dt - \int_{(i-1)T}^{iT} \left[\sum_{k=1}^K A_k b_k[i] s_k(t - it) \right]^2 dt \\ &= 2\mathbf{b}^T[i] \mathbf{A} \mathbf{r}[i] - \mathbf{b}^T[i] \mathbf{S}^H \mathbf{A}^2 \mathbf{S} \mathbf{b}[i],\end{aligned}\tag{2.2}$$

where $\hat{\mathbf{b}}[i]$ is the estimated symbol vector; the definition of $r(t)$, A_k , $b_k[i]$, $s_k(t - iT)$, $\mathbf{S}$, $\mathbf{A}$, $\mathbf{r}[i]$ and $\mathbf{b}[i]$ can be found in (1.5) and (1.10). The maximization of (2.2) is a combinatorial optimization problem. Such combinatorial optimization problem can always be solved by exhaustive search method, namely, computation of the function for every possible argument and selection of the one that maximizes the function. Since the exhaustive search method requires the computational complexity of $O(2^K)$, the optimum detector is not used in practical CDMA systems because of its unbearable computational complexity especially when the number of users is large.

Remark 2.1:

The optimum multiuser detector achieves the minimum probability of BER for all users, and optimum asymptotic multiuser efficiency, as well as optimum near-far resistance in CDMA systems [27].

Remark 2.2:

The optimum multiuser detector requires not only the signature waveform and timing of every active user, but also the received amplitudes of all users and the noise level. Therefore, it cannot be considered as a blind multiuser detector.

Remark 2.3:

Although the optimum synchronous multiuser detection was shown to be solvable in polynomial time for nonpositive cross correlation in Sankaran and Ephremides [28], the computing complexity for the implementation of the optimum detector is unbearable for practical detection.

Altogether, the optimum detector yields the achievable lower-bound of BER in theory. It serves as a baseline of comparison for other suboptimum multiuser detectors.

2.2.2 Linear Decorrelating Detector

The huge gaps in performance and complexity between the conventional matched filter and the optimum detector motivate extensive research for other suboptimal methods which present good performance and complexity tradeoffs. One of the most popular suboptimal detectors is the decorrelating detector.

The linear decorrelating detector is designed to completely eliminate the MAI caused by other interfering users, at the expense of enhancing the ambient noise [29]. According to the basic K -user synchronous CDMA model of (1.10), which is given by

$$\begin{aligned}\mathbf{r}[i] &= \sum_{k=1}^K A_k b_k[i] \mathbf{s}_k + \mathbf{v}[i] \\ &= \mathbf{S} \mathbf{A} \mathbf{b}[i] + \mathbf{v}[i],\end{aligned}\tag{2.3}$$

the decorrelating detector for the desired k th user has the canonical form of (1.30)

with the weight vector $\mathbf{d}_k = \mathbf{w}_k \in \mathbf{S}$, satisfying

$$\begin{aligned}\mathbf{d}_k^H \mathbf{s}_k &= 1 \\ \mathbf{d}_k^H \mathbf{s}_m &= 0 \quad m = 1, 2, \dots, K, \text{ and } m \neq k.\end{aligned}\tag{2.4}$$

Assuming that the user signature sequences are linearly independent, i.e., the matrix $\mathbf{S}$ in (2.3) has full column rank of K , the expression for the linear decorrelating detector can be given by

$$\mathbf{d}_k = \mathbf{S}(\mathbf{S}^H \mathbf{S})^{-1} \mathbf{e}_k,\tag{2.5}$$

where $\mathbf{e}_k$ is an N -dimension column vector with all zeros except the k th factor equaling to one. More detail of the definition and the derivation of the decorrelating detector in (2.5) can be referred to [18]. The decorrelating detector is also called zero-forcing detector [30]. From the perspective of implementation, the decorrelating detector has two desirable features, i.e.,

- (a). The knowledge of the received amplitudes is not required.
- (b). The detector can be decentralized conveniently in the sense that the demodulation of individual user can be realized independently.

In contrast to the derivation of the optimum detector, the decorrelating detector is simple and of a natural strategy which supposes to know neither the received amplitudes nor any prior distributions for itself. It is then natural to consider joint maximum likelihood estimation of amplitude and transmitted bits. The following parts present the blind adaptive implementations and the performance analysis of the decorrelating detector. It is proven that the decorrelating detector is optimal

in least-squares, near-far resistance, and maximum-likelihood when the received amplitudes are not available [31].

2.2.3 Linear MMSE Detector

The linear minimum mean square error detector is a more fruitful suboptimum detector for multiuser detection. Unlike the linear decorrelating detector which is designed to eliminate the MAI completely at the expense of enhancing the ambient noise, the linear MMSE detector is designed to minimize the total effect of the MAI and the ambient noise at the detector output [32].

According to the synchronous CDMA model of (2.3), the linear MMSE detector for the desired User k has the canonical form of (1.30) with the weight vector $\mathbf{m}_k = \mathbf{w}_k$, obtained by calculating the minimum output mean-squared error, i.e.,

$$\mathbf{m}_k \triangleq \arg \min_{\mathbf{m}_k} E\{|A_k b_k[i] - \mathbf{m}_k^H \mathbf{r}[i]|^2\}. \quad (2.6)$$

Assuming that the user signature sequences are linearly independent, the expression for the linear MMSE detector can be given by [18]

$$\mathbf{m}_k = \mathbf{S}(\mathbf{S}^H \mathbf{S} + \sigma^2 |\mathbf{A}|^{-2})^{-1} e_k. \quad (2.7)$$

Exploiting the result of (2.7), we have

$$\begin{aligned} \mathbf{m}_k^H \mathbf{s}_k &= [(\mathbf{S}^H \mathbf{S} + \sigma^2 |\mathbf{A}|^{-2})^{-1} \mathbf{S}^H \mathbf{S}]_{1,k} \\ \|\mathbf{m}_k\|^2 &= [(\mathbf{S}^H \mathbf{S} + \sigma^2 |\mathbf{A}|^{-2})^{-1} \mathbf{S}^H \mathbf{S} (\mathbf{S}^H \mathbf{S} + \sigma^2 |\mathbf{A}|^{-2})^{-1}]_{k,k}. \end{aligned} \quad (2.8)$$

Compared with the decorrelating detector in (2.4), it is clear that the linear MMSE detector output can not remove all MAI. However, it is also obvious that

$\|\mathbf{m}_k\|^2 < \|\mathbf{d}_k\|^2$, i.e., the effects of ambient noise are reduced by linear MMSE detector. In fact, the linear MMSE detector is easier to be implemented compared with the decorrelating detector, and the MMSE detector maximizes the SINR ratio at the output [33].

Another expression of the linear MMSE detector can be developed by considering the optimization problem of (2.6) directly, i.e.,

$$\begin{aligned}\mathbf{m}_k &\triangleq \arg \min_{\mathbf{m}_k} [\mathbf{m}_k^H \mathbf{R} \mathbf{m}_k - 2\mathbf{m}_k^H \Re\{A_k E(\mathbf{r}[i]b_k[i])\}] \\ &= |A_1|^2 \mathbf{R}^{-1} \mathbf{s}_1,\end{aligned}\tag{2.9}$$

where $\mathbf{R}$ is the autocorrelation matrix of the received signal, which is denoted as

$$\mathbf{R} \triangleq E\{\mathbf{r}[i]\mathbf{r}^H[i]\}.\tag{2.10}$$

The matrix $\mathbf{R}$ can be estimated from the received signals by the corresponding sample autocorrelation. The detector in (2.9) can be derived directly from the definition of the MMSE detector in (2.6).

2.3 LMS Blind Linear MMSE Detector for Synchronous CDMA Systems

In this section, we present a blind adaptive algorithm, i.e., least-mean-squares (LMS) algorithm to resolve the linear MMSE detector in (2.9). The LMS method is derived from an alternative description of the linear MMSE detector. In order to explain this, we consider the following constrained minimum-output-energy (MOE) optimization

problem:

$$\mathbf{m}_k \triangleq \arg \min_{\mathbf{m}_k} E\{\|\mathbf{m}_k^H \mathbf{r}[i]\|^2\}, \quad \text{subject to: } \mathbf{m}_k^H \mathbf{s}_k = 1. \quad (2.11)$$

The MOE method in (2.11) was first proposed in [23]. It has been proven that the solution of (2.11) is equivalent to the solution of the MMSE detector in (2.9). To show this, we consider the Lagrangian:

$$\begin{aligned} \zeta(\mathbf{m}_k) &\triangleq E\{\|\mathbf{m}_k^H \mathbf{r}[i]\|^2\} - 2\lambda[(\mathbf{m}_k^H \mathbf{s}_k) - 1] \\ &= \mathbf{m}_k^H \mathbf{R} \mathbf{m}_k - 2\lambda \mathbf{m}_k^H \mathbf{s}_k + 2\lambda \end{aligned} \quad (2.12)$$

By applying differential operator into (2.12), the solution is given by

$$\begin{aligned} \mathbf{m}_k &= \lambda \mathbf{R}^{-1} \mathbf{s}_k \\ \lambda &= \mathbf{s}_k^H \mathbf{R}^{-1} \mathbf{s}_k. \end{aligned} \quad (2.13)$$

Since λ is always a real positive factor, it is seen that the MMSE detector in (2.9) and the solution of (2.13) differ only by a positive scaling constant. Such a scaling constant does not affect the linear decision rules of the above solution, i.e., the linear MOE detector is equivalent to the linear MMSE detector.

We now consider the LMS algorithm to achieve an adaptive implementation of $\mathbf{m}_k$ based on (2.11). We first decompose $\mathbf{m}_k$ into two orthogonal parts, given by

$$\mathbf{m}_k = \mathbf{s}_k + \mathbf{x}_k, \quad (2.14)$$

where $\mathbf{x}_k$ is the orthogonal counterpart of $\mathbf{s}_k$ in $\mathbf{m}_k$. Denote

$$\mathbf{P}_k = \mathbf{I}_N - \mathbf{s}_k (\mathbf{s}_k^H \mathbf{s}_k)^{-1} \mathbf{s}_k^H = \mathbf{I}_N - \mathbf{s}_k \mathbf{s}_k^H. \quad (2.15)$$

It is easy to obtain

$$\mathbf{x}_k = \mathbf{P}_k \mathbf{m}_k = \mathbf{P}_k \mathbf{x}_k, \quad (2.16)$$

which is derived with the assumption that the spreading code of the desired user has be normalized, i.e.,

$$\mathbf{s}_k^H \mathbf{s}_k = 1 \quad (2.17)$$

Hence, the constrained optimization problem in (2.11) can be rewritten as a unconstrained optimization problem:

$$\mathbf{x}_k = \arg \min_{\mathbf{x}_k} E\{\|(\mathbf{s}_k + \mathbf{P}_k \mathbf{x}_k)^H \mathbf{r}[i]\|^2\}. \quad (2.18)$$

Applying the LMS algorithm into the cost function of (2.18), the adaptive implementation of $\mathbf{x}_k$ is given by

$$\begin{aligned} \mathbf{x}_k[i+1] &= \mathbf{x}_k[i] - \mu \frac{d}{d\mathbf{x}_k} \|(\mathbf{s}_k + \mathbf{P}_k \mathbf{x}_k)^H \mathbf{r}[i]\|_{\mathbf{x}_k=\mathbf{x}_k[i]}^2 \\ &= \mathbf{x}_k[i] - 2\mu \left[(\mathbf{s}_k + \mathbf{P}_k \mathbf{x}_k[i])_k^H \mathbf{r}[i] \right]^* \mathbf{P}_k \mathbf{r}[i] \\ &= \mathbf{x}_k[i] - 2\mu \left[(\mathbf{s}_k + \mathbf{P}_k \mathbf{x}_k[i])_k^H \mathbf{r}[i] \right]^* (\mathbf{I}_N - \mathbf{s}_k \mathbf{s}_k^H) \mathbf{r}[i] \\ &= \mathbf{x}_k[i] - 2\mu \left[(\mathbf{s}_k + \mathbf{P}_k \mathbf{x}_k[i])_k^H \mathbf{r}[i] \right]^* [\mathbf{r}[i] - (\mathbf{s}_k^H \mathbf{r}[i]) \mathbf{s}_k], \end{aligned} \quad (2.19)$$

where μ is the step size of the iteration. The optimization of μ can improve the convergence rate of LMS algorithm. The convergence analysis of the LMS algorithm for MMSE detector is presented in [23]. It is noted that the increase of the step size μ will accelerated the convergence speed of the LMS algorithm. However, the errors in the stable state are also enlarged as μ increases. An alternative stochastic gradient algorithm for blind adaptive multiuser detection is developed in [34]. In

[35], the LMS algorithm for the blind adaptive implementation of the decorrelating detector is proposed. The performance loss incurred by LMS algorithm compared with training based MMSE detector is analyzed in [36, 37]. In [38], a two-stage adaptive method is proposed, where the adaptive method is used to predict the output during the first stage, and then the obtained output is used to train the second stage to achieve the improved performance. Since the LMS MMSE detector only requires the knowledge of the signature form of the desired user, it is considered as blind multiuser detector.

2.4 RLS Blind Linear MMSE Detector for Synchronous CDMA Systems

The LMS MMSE detector in the previous section is easy to be implemented with a very low computational complexity in the order of $O(N)$ per update where N is the spreading factor of the CDMA system. Unfortunately, the convergence rate of the LMS algorithms is usually slow. In this section, the recursive least square (RLS) algorithm, which has a much faster convergence compared with LMS algorithm, is introduced for the blind adaptive implementation of the linear MMSE detector.

Unlike the LMS algorithm, the RLS algorithm tries to minimize the sum of exponentially weighted constrained optimization problem in (2.11), given by

$$\mathbf{m}_k[i] \triangleq \arg \min_{\mathbf{m}_k} \sum_{n=0}^i \lambda^{i-n} E\{\|\mathbf{m}_k^H \mathbf{r}[n]\|^2\}, \quad \text{subject to: } \mathbf{m}_k^H \mathbf{s}_k = 1, \quad (2.20)$$

where $0 < \lambda < 1$ is the forgetting factor. The use of forgetting factor in the

RLS algorithm is intended to ensure that data in the distant past are forgotten in order to afford the possibility of following the statistical variations of the observable data when the filter operates in a nonstationary environment. The influence of the forgetting factor on the RLS algorithm is analyzed in [39] and [40]. Normally, the forgetting factor is chosen as $1 - \lambda \ll 1$.

Denoting

$$\begin{aligned}\mathbf{R}[i] &\triangleq \sum_{n=0}^i \lambda^{i-n} \mathbf{r}[n] \mathbf{r}^H[n] \\ &= \lambda \mathbf{R}[i-1] + \mathbf{r}[i] \mathbf{r}^H[i],\end{aligned}\tag{2.21}$$

the solution to the optimized problem in (2.20) can be given by

$$\mathbf{m}_k[i] = \mathbf{R}^{-1}[i] \mathbf{s}_k (\mathbf{s}_k^H \mathbf{R}^{-1}[i] \mathbf{s}_k)^{-1}.\tag{2.22}$$

Since $(\mathbf{s}_k^H \mathbf{R}^{-1}[i] \mathbf{s}_k)^{-1}$ is only a positive real parameter which has no influence on the performance of the detector, the most work of the algorithm falls on the resolving of the inverse matrix of $\mathbf{R}[i]$. According to (2.21) and the matrix inversion lemma [41], and by denoting $\Phi[i] \triangleq \mathbf{R}^{-1}[i]$, we have

$$\Phi[i] = \lambda^{-1} \Phi[i-1] - \frac{\lambda^{-2} \Phi[i-1] \mathbf{r}[i] \mathbf{r}^H[i] \Phi[i-1]}{1 + \lambda^{-1} \mathbf{r}^H[i] \Phi[i-1] \mathbf{r}[i]}.\tag{2.23}$$

Hence, the RLS algorithm for the blind adaptive implementation of the linear MMSE detector can be summarized as

$$\begin{aligned}k[i] &\triangleq \frac{\lambda^{-2} \Phi[i-1] \mathbf{r}[i]}{1 + \lambda^{-1} \mathbf{r}^H[i] \Phi[i-1] \mathbf{r}[i]} \\ \Phi[i] &= \lambda^{-1} \Phi[i-1] - k[i] \mathbf{r}^H[i] \Phi[i-1] \\ \mathbf{m}_k[i] &= \Phi[i] \mathbf{s}_k.\end{aligned}\tag{2.24}$$

The algorithm in (2.24) is based on the matrix inversion lemma which requires the complexity of $O(N^2)$ for the recursive updating of $\Phi[i]$. However, the RLS MMSE detector presents satisfying convergence properties compared with LMS MMSE detector [36]. In [42], a well-known improved rotation-based RLS algorithm, i.e., QR-RLS algorithm [43], is proposed to halve and enable more accurate computation than that with the RLS algorithm that operate directly on $\mathbf{R}^{-1}[i]$. Another advantage of the rotation-based QR approach is that the computation can be easily be mapped to systolic array structures for parallel implementations.

2.5 Subspace Blind Linear Detectors for Synchronous CDMA Systems

Subspace-based methods were first proposed in CDMA systems for delay and channel estimation [44, 45] and [46]. Subspace blind multiuser detection was first proposed in [24], where the blind decorrelating and MMSE detector were developed based on the estimation of the signal subspace spanned by the user's signature waveforms.

According to the synchronous model in the previous chapter, we reconsider the autocorrelation matrix of the received signal:

$$\begin{aligned}\mathbf{R} &= E\{\mathbf{r}[i]\mathbf{r}^H[i]\} \\ &= \mathbf{S}\mathbf{A}\mathbf{S}^H + \sigma^2\mathbf{I}_N.\end{aligned}\tag{2.25}$$

It should be pointed out that the matrix $\mathbf{R}$ plays a very important role for the subspace based multiuser detection. By performing the eigendecomposition of $\mathbf{R}$,

we obtain

$$\begin{aligned}\mathbf{R} &= [\mathbf{U}_s, \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \Lambda_n \end{bmatrix} [\mathbf{U}_s, \mathbf{U}_n]^H \\ &= \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H,\end{aligned}\tag{2.26}$$

where $\Lambda_s = \text{diag}(\lambda_1, \dots, \lambda_K)$ contains K largest eigenvalues of the signal subspace in $\mathbf{R}$, and $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors. Both $\Lambda_n = \sigma^2 \mathbf{I}_{N-K}$ and $\mathbf{U}_n$ are, respectively, eigenvalues and orthonormal eigenvectors of the noise subspace. It is noted that the eigendecomposition of (2.26) is based on the assumption that the background noise vector in $\mathbf{r}[i]$ is white with the variance of $\sigma^2 \mathbf{I}_N$. According to (2.25) and (2.26), it is easy to obtain

$$\mathbf{SAS}^H = \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_K) \mathbf{U}_s^H.\tag{2.27}$$

Utilizing the eigendecomposition of $\mathbf{R}$, we next discuss the subspace based expression for linear decorrelating detector and linear MMSE detector.

2.5.1 Subspace Decorrelating Detector

Recalling that the linear decorrelating detector must be in the range of user subspace and orthogonal to all interfering users except the desired user, the decorrelating detector can be expressed by the following constrained optimization problem, i.e., the decorrelating detector for the k th user is the unique signal $\mathbf{d} \in \text{range}(\mathbf{U}_s)$, which satisfies

$$\mathbf{d} \triangleq \arg \min_{\mathbf{d}} E\{\|\mathbf{d}^H \mathbf{SAb}[i]\|^2\}, \quad \text{subject to: } \mathbf{d}^H \mathbf{s}_k = 1.\tag{2.28}$$

where User k is the desired user. To show this, we consider

$$\begin{aligned}
\mathbf{d} &\triangleq \arg \min_{\mathbf{d}} E\{\|\mathbf{d}^H \mathbf{S} \mathbf{A} \mathbf{b}[i]\|^2\}, \\
&\text{subject to: } \mathbf{d}^H \mathbf{s}_k = 1 \\
&= \arg \min_{\mathbf{d}} \mathbf{d}^H \mathbf{S}^H \mathbf{A}^2 \mathbf{S} \mathbf{d} \\
&\text{subject to: } \mathbf{d}^H \mathbf{s}_k = 1 \\
&= \arg \min_{\mathbf{d}} A_k^2 (\mathbf{d}^H \mathbf{s}_k)^2 + \sum_{i=1, i \neq k}^K A_i^2 (\mathbf{d}^H \mathbf{s}_i)^2 \\
&\text{subject to: } \mathbf{d}^H \mathbf{s}_k = 1 \\
&= \arg \min_{\mathbf{d}} A_k^2 + \sum_{i=1, i \neq k}^K A_i^2 (\mathbf{d}^H \mathbf{s}_i)^2.
\end{aligned} \tag{2.29}$$

The solution of $\mathbf{d}$ in (2.29) is achieved if and only if $\mathbf{d}^H \mathbf{s}_i = 0$, for $i = 1, 2, \dots, K$ and $i \neq k$. Since $\mathbf{d}$ is assumed to be in the signal subspace and the solution for (2.29) is unique under the assumption that the signature waver form $\mathbf{S}$ is tall with full rank of K , it is clear that $\mathbf{d}$ is an equivalent expression of the decorrelating detector for User k . It has been proven in [24] that the solution of the decorrelating detector in (2.29) can be expressed by subspace form as following

$$\mathbf{d}_k = \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_K) \mathbf{U}_s^H \mathbf{s}_k. \tag{2.30}$$

2.5.2 Subspace MMSE Detector

According to the definition of the linear MMSE detector in (2.13) and the eigen-decomposition of (2.26), the subspace expression of the linear MMSE detector is

straightforward, given by

$$\begin{aligned}
 \mathbf{m}_k &= \mathbf{R}^{-1} \mathbf{s}_k \\
 &= (\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H + \frac{1}{\sigma^2} \mathbf{U}_n \mathbf{U}_n^H) \mathbf{s}_k \\
 &= (\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H) \mathbf{s}_k,
 \end{aligned} \tag{2.31}$$

where the third equation is based on the fact that $\mathbf{U}_n$ is the orthogonal subspace of the signal subspace.

2.5.3 Blind Implementation

By observing the detectors in (2.30) and (2.31), it is easy to know that the main work for the blind implementation of the subspace decorrelating detector and subspace MMSE detector is the subspace tracking of the signal subspace. In (2.30) and (2.31), the signal subspace components ($\mathbf{U}_s$, Λ_s , and σ^2) can be blindly estimated from the received signal [47]. Various recursive subspace tracking methods have been proposed for the blind adaptive implementation of the subspace decorrelating and MMSE detectors [48, 49, 50].

Table 2.1 summarize the algorithms of subspace blind linear MMSE and linear decorrelating detectors for synchronous CDMA systems.

In this table, step 3 is only required by the decorrelating detector. At the last step, the signal is detected by the differential detection in order to remove the influence of the complex phase factor incurred by subspace estimation [37]. In theory, the subspace decorrelating detector and the subspace MMSE detector achieve similar performance. However, the subspace MMSE detector considerably outperforms the

Table 2.1: Algorithm of Subspace Blind Linear Decorrelating and Linear MMSE Detectors for synchronous CDMA Systems

Signal Frame: $\mathbf{r}[0], \mathbf{r}[1], \dots, \mathbf{r}[P]$

STEP 1. Compute autocorrelation

$$\mathbf{R} = \frac{1}{P+1} \sum_{n=0}^P \mathbf{r}[n] \mathbf{r}[n]^H$$

STEP 2. Compute eigendecomposition of $\mathbf{R}$

$$\mathbf{R} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H$$

STEP 3. Parameter estimation

$$\sigma^2 = \frac{1}{N-2K} (\text{Sum of the mini } N-2K \text{ eigenvalues of } \mathbf{R})$$

STEP 4. Form decorrelating and MMSE detectors

$$\mathbf{d}_k = \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_{N-2K})^{-1} \mathbf{U}_s^H \mathbf{s}_k$$

$$\mathbf{m}_k = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k$$

STEP 5. Perform differential detection

$$z_k[i] = (\mathbf{d}_k \text{ or } \mathbf{m}_k)^H \mathbf{r}[i] \quad \hat{\beta}_k[i] = \text{sign} (\Re\{z_k[i] z_k[i-1]^*\})$$

subspace decorrelating detector in practical situation as it is more robust against the channel estimation error.

2.6 Subspace Detectors for Asynchronous CDMA Systems with Multipath.

The subspace based blind multiuser detection for CDMA systems is widely discussed and developed in recent years. In this section, we present the basic subspace methods for the detection of asynchronous CDMA systems. According to the definition of (1.11)-(1.29), the asynchronous signal model of CDMA systems with multipath can be rewritten as

$$\begin{aligned}
 \mathbf{r}[i] &= \sum_{k=1}^K \mathbf{r}_k[i] + \mathbf{v}[i] \\
 &= \sum_{k=1}^K (\mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1]) + \mathbf{v}[i] \\
 &= \mathbf{H}_0 \mathbf{b}[i] + \mathbf{H}_1 \mathbf{b}[i-1] + \mathbf{v}[i],
 \end{aligned} \tag{2.32}$$

where all the symbols in (2.32) can be referred to the definition in (1.11)-(1.29). It is seen that the autocorrelation matrix for the received signal in (2.32) can be given by

$$\begin{aligned}
 \mathbf{R} &= E\{\mathbf{r}[i] \mathbf{r}^H[i]\} \\
 &= [\mathbf{H}_0, \mathbf{H}_1][\mathbf{H}_0, \mathbf{H}_1]^H + \sigma^2 \mathbf{I}_N,
 \end{aligned} \tag{2.33}$$

where $\mathbf{H}_0$ and $\mathbf{H}_1$ are the signature matrices of the current symbols and the previous symbols, respectively, as defined in (1.29). For the convenience of analysis, both $\mathbf{H}_0$ and $\mathbf{H}_1$ are assumed to be “tall” with full column rank. By performing the

2.6 Subspace Detectors for Asynchronous CDMA Systems with Multipath. 46

eigendecomposition of the matrix $\mathbf{R}$, we obtain

$$\begin{aligned}\mathbf{R} &= [\mathbf{U}_s, \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \\ & \Lambda_n \end{bmatrix} [\mathbf{U}_s, \mathbf{U}_n]^H \\ &= \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \sigma^2 \mathbf{I}_N\end{aligned}\quad (2.34)$$

where $\Lambda_s = \text{diag}(\lambda_1, \dots, \lambda_{2K})$ contains $2K$ largest eigenvalues of $\mathbf{R}$ and $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors of the signal subspace spanned by $[\mathbf{H}_0, \mathbf{H}_1]$. Both $\Lambda_n = \sigma^2 \mathbf{I}_{N-2K}$ and $\mathbf{U}_n$ are, respectively, the eigenvalues and the orthonormal eigenvectors of the background noise subspace. It should be pointed out that the eigendecomposition of the autocorrelation matrix $\mathbf{R}$ in (2.34) is for asynchronous CDMA system, while the eigendecomposition of the autocorrelation matrix $\mathbf{R}$ in (2.26) is for synchronous CDMA system. In other words, matrix $\mathbf{U}_s$ and $\mathbf{U}_n$ in (2.34) and (2.26) have different meaning, respectively, although they have the same form. Normally, the signal subspace of the asynchronous CDMA systems has more ranks compared with the signal subspace of the synchronous CDMA systems as the existence of the MAI and ISI.

Based on the eigendecomposition of (2.34) and referring to the definition of the decorrelating detector and the MMSE detector in (2.4) and (2.6), it is easy to obtain the subspace forms of the decorrelating detector and the MMSE detector for asynchronous CDMA systems with multipath by using the similar methods in the previous section, i.e.,

$$\begin{aligned}\mathbf{d}_k &= \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_{2K}) \mathbf{U}_s^H \mathbf{h}_k \\ \mathbf{m}_k &= (\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H) \mathbf{h}_k\end{aligned}\quad (2.35)$$

2.6 Subspace Detectors for Asynchronous CDMA Systems with Multipath. 47

where $\mathbf{d}_k$ is the decorrelating detector for User k , $\mathbf{m}_k$ is the MMSE detector for User k and $\mathbf{h}_k$ is composite channel response vector of User k denoted by (1.24).

It is seen from (2.35) that unlike the subspace methods for synchronous systems, where the linear detectors can be resolved as long as the signal subspace components are identified, the subspace detectors for asynchronous model require the information of the composite channel response vector of the desired user, i.e., $\mathbf{h}_k$. Such vector can be viewed as the channel-distorted original spreading sequence $\mathbf{s}_k$.

The channel estimation of $\mathbf{h}_k$ can be achieved by a training sequence [51, 52, 53]. For the blind multiuser detection, the channel estimation can be completed by exploiting the spreading sequence of the desired user and the orthogonality between the signal subspace and the noise subspace [54, 55, 56].

We now present the principle for the channel estimation of $\mathbf{h}_k$. By assuming that the k th desired user has been synchronized, and according to the definition of the composite channel response vector in (1.22) and (1.23), the channel $\mathbf{h}_k$ can be expressed as

$$\mathbf{h}_k = \bar{\mathbf{C}}_{k,up} \bar{\mathbf{g}}_k, \quad (2.36)$$

2.6 Subspace Detectors for Asynchronous CDMA Systems with Multipath. 48

where

$$\mathbf{C}_{k,up} \triangleq \begin{bmatrix} c_k[0] & & & & \\ c_k[1] & \ddots & & & \\ \vdots & \ddots & & c_k[0] & \\ \vdots & & & c_k[1] & \\ \vdots & \ddots & & \vdots & \\ c_k[N-1] & \cdots & c_k[N-M-1] & & \end{bmatrix}, \quad (2.37)$$

and $\bar{\mathbf{g}}_k$ is the channel response denoted by (1.21). Since matrix $\mathbf{C}_{k,up}$ only contains the spreading sequence of the desired user, which is preknown by the detector, the blind channel estimation of $\mathbf{h}_k$ falls on the estimation of channel response $\bar{\mathbf{g}}_k$.

Recalling that when the ambient noise is white, the signal subspace and the noise subspace can be identified through the eigendecomposition on the autocorrelation matrix of the received signal in (2.34). Hence, the channel response $\bar{\mathbf{g}}_k$ can be estimated by exploiting the orthogonality between the signal subspace and the noise subspace. Since the noise subspace in (2.34) is orthogonal to the column space $[\mathbf{H}_0, \mathbf{H}_1]$ and $\mathbf{h}_k$ is one of the column vectors in $\mathbf{H}_0$, we have

$$\mathbf{U}_n^H \mathbf{h}_k = \mathbf{U}_n^H \bar{\mathbf{C}}_k \bar{\mathbf{g}}_k = \mathbf{0}. \quad (2.38)$$

According to the above relationship, the estimation of the channel response $\bar{\mathbf{g}}_k$ can be determined by calculating the minimum eigenvector of the matrix $\bar{\mathbf{C}}_k^H \mathbf{U}_n \mathbf{U}_n^H \bar{\mathbf{C}}_k$, i.e.,

$$\bar{\mathbf{g}}_k = \text{Min-eigenvector } (\bar{\mathbf{C}}_k^H \mathbf{U}_n \mathbf{U}_n^H \bar{\mathbf{C}}_k). \quad (2.39)$$

2.6 Subspace Detectors for Asynchronous CDMA Systems with Multipath. 49

It should be pointed that the condition for such channel estimation to be unique is that matrix $\mathbf{U}_n^H \bar{\mathbf{C}}_k$ has full column rank.

Table 2.2 summarizes the subspace blind linear MMSE and decorrelating detectors for asynchronous CDMA systems.

Table 2.2: Subspace Blind Linear Decorrelating and Linear MMSE Detectors for Asynchronous CDMA Systems

Signal Frame: $\mathbf{r}[0], \mathbf{r}[1], \dots, \mathbf{r}[P]$

STEP 1. Compute autocorrelation

$$\mathbf{R} = \frac{1}{P+1} \sum_{n=0}^P \mathbf{r}[n] \mathbf{r}[n]^H$$

STEP 2. Compute eigendecomposition of $\mathbf{R}$

$$\mathbf{R} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H$$

STEP 3. Parameter estimation

$$\sigma^2 = \frac{1}{N-2K} (\text{Sum of the mini } N-2K \text{ eigenvalues of } \mathbf{R})$$

$$\bar{\mathbf{g}}_k = \text{Min-eigenvector } (\bar{\mathbf{C}}_k^H \mathbf{U}_n \mathbf{U}_n^H \bar{\mathbf{C}}_k)$$

$$\mathbf{h}_k = \bar{\mathbf{C}}_k \bar{\mathbf{g}}_k$$

STEP 4. Form decorrelating and MMSE detectors

$$\mathbf{d}_k = \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_{N-2K})^{-1} \mathbf{U}_s^H \mathbf{h}_k$$

$$\mathbf{m}_k = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_k$$

STEP 5. Perform differential detection

$$z_k[i] = (\mathbf{d}_k \text{ or } \mathbf{m}_k)^H \mathbf{r}[i] \quad \hat{\beta}_k[i] = \text{sign} (\Re\{z_k[i] z_k[i-1]^*\})$$

It is seen from this table that unlike the algorithms of synchronous case, the

2.7 Performance Analysis of the Decorrelating Detector and MMSE Detector 50

eigendecomposition of the signal subspace has $2K$ eigenvectors as the existence of ISI. In addition, special composite channel response vector of the desired user is required to be estimated for asynchronous case, while the detectors in Table 2.1 do not have such a requirement. The channel estimation of the desired user in Table 2.2 is based on the assumption that the background noise is white. In [57], the blind channel estimation in correlated background noise is proposed by exploiting two independent antennas.

2.7 Performance Analysis of the Decorrelating Detector and MMSE Detector

After the discussion of the above linear multiuser detectors, we now focus on the performance analysis of the decorrelating detector and MMSE detector. For the convenience of the analysis and without loss of the generality, the detectors for the synchronous CDMA systems are considered in this section.

2.7.1 SINR

When a linear detector $\mathbf{w}_k$ for the desired User k is applied to the received signal $\mathbf{r}[i]$ in (1.10), the signal-to-interference and noise ratio at the output of the detector

2.7 Performance Analysis of the Decorrelating Detector and MMSE Detector 51

is given by

$$\begin{aligned} \text{SINR}(\mathbf{w}_k) &= \frac{E\{\mathbf{w}_k^H \mathbf{r}[i] b_k[i]\}^2}{E\{\text{Var}\{\mathbf{w}_k^H \mathbf{r}[i] b_k[i]\}\}} \\ &= \frac{A_k^2 (\mathbf{w}_k^H \mathbf{s}_k)^2}{\sum_{i \neq k}^K A_i^2 (\mathbf{w}_k^H \mathbf{s}_i)^2 + \sigma^2 \mathbf{w}_k^H \mathbf{w}_k}, \end{aligned} \quad (2.40)$$

where the second equation is based on the assumption that the user bit streams are independent and the noise is independent of the user bits. According to the definition of (2.40), the SINR of the decorrelating for synchronous CDMA systems is straightforward, which is given by

$$\begin{aligned} \text{SINR}(\mathbf{d}_k) &= \frac{A_k^2 (\mathbf{d}_k^H \mathbf{s}_k)^2}{\sum_{i \neq k}^K A_i^2 (\mathbf{d}_k^H \mathbf{s}_i)^2 + \sigma^2 \mathbf{d}_k^H \mathbf{d}_k} \\ &= \frac{A_k^2 (\mathbf{d}_k^H \mathbf{s}_k)^2}{\sigma^2 \mathbf{d}_k^H \mathbf{d}_k}, \end{aligned} \quad (2.41)$$

where the second equality is obtained with the property of the decorrelating detector. It has been proved in [18] that SINR of the decorrelating detector in (2.41) is equivalent to

$$\text{SINR}(\mathbf{d}_k) = \frac{A_k^2}{\sigma^2 [\mathbf{R}^{-1}]_{kk}}, \quad (2.42)$$

where $\mathbf{R}$ is the autocorrelation matrix of the received signal denoted by (2.10).

By applying the MMSE detector in (2.31) into (2.41), the SINR of the MMSE

2.7 Performance Analysis of the Decorrelating Detector and MMSE Detector 52

detector is given by

$$\begin{aligned}
 \text{SINR}(\mathbf{m}_k) &= \frac{A_k^2(\mathbf{m}_k^H \mathbf{s}_k)^2}{\sum_{i \neq k}^K A_i^2(\mathbf{m}_k^H \mathbf{s}_i)^2 + \sigma^2 \mathbf{m}_k^H \mathbf{m}_k} \\
 &= \frac{A_k^2[(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k)^H \mathbf{s}_k]^2}{(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k)^H \mathbf{R}(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k) - A_k^2[(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k)^H \mathbf{s}_k]^2} \\
 &= \frac{A_k^2[\mathbf{s}_k^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k]^2}{(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k)^H \mathbf{R}(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k) - A_k^2[(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k)^H \mathbf{s}_k]^2} \quad (2.43) \\
 &= \frac{A_k^2[\mathbf{s}_k^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k]^2}{\mathbf{s}_k^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k - A_k^2[(\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k)^H \mathbf{s}_k]^2} \\
 &= \frac{A_k^2 \mathbf{s}_k^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k}{1 - A_k^2 \mathbf{s}_k^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{s}_k}.
 \end{aligned}$$

2.7.2 Bit-error-rate (BER)

Since the decorrelating detector is the projection of the signal of the desired user on the orthogonal space spanned by the interfering signal, its BER is invariant to the amplitudes of the interfering users. The BER analysis of the decorrelating detector is the simplest among all multiuser detectors. It is noted that the output of the received signal after the detection of the decorrelating detector contains two parts only. One part is the signal due to the desired user, and the other part is the interference due to the background noise which is assumed to be white Gaussian distribution with zero mean and variance σ^2 . Therefore, in light of (2.42), the BER of the decorrelating detector for the desired k th user is easily given by

$$P_e(\mathbf{d}_k) = Q\left(\frac{A_k}{\sigma \sqrt{\mathbf{R}^{-1}_{kk}}}\right). \quad (2.44)$$

Clearly, the BER of the decorrelating is independent of the interfering amplitudes.

Since the linear MMSE detector does not totally remove the interfering users, the analysis of its BER is not so straightforward in contrast to the decorrelating

2.7 Performance Analysis of the Decorrelating Detector and MMSE Detector 53

detector. By taking the first user as the desired user for example, the general expression of the BER of the linear MMSE detector can be given by [58]:

$$\begin{aligned} P_e(\mathbf{m}_1) &= P(\hat{b}_1[i] \neq b_1[i]) \\ &= \frac{1}{2^{K-1}} \sum_{[b_2, \dots, b_K] \in \{-1, +1\}^{K-1}} Q\left(\frac{A_1 \mathbf{m}_1^H \mathbf{s}_1 + \sum_{k=2}^K A_k b_k \mathbf{m}_1^H \mathbf{s}_k}{\|\mathbf{m}_1\| \sigma}\right). \end{aligned} \quad (2.45)$$

It has been proven that for sufficiently low interferer power, the liner MMSE has smaller BER than that of the liner decorrelating detector; while for relatively high-power interferers, the MMSE detector performs quite similarly to the decorrelating detector [18].

2.7.3 Asymptotic Multiuser Efficiency (AME) and Near-far Resistance

The AME measures the exponential decay rate of the error probability as the background noise approaches to zero. Since the linear decorrelating detector and the linear MMSE detector converge to the same result as the background noise is infinite small, the two detectors possess the same AME and near-far resistance. Hence, it is suffice to find the AME and the near-far resistance of the decorrelating detector. According to the definition of AME in (1.33) and the BER of the decorrelating detector in (2.44), it is easy to obtain the AME of the decorrelating detector, which is given by

$$\eta_k = \frac{1}{[\mathbf{R}^{-1}]_{kk}}. \quad (2.46)$$

Clearly, the AME of the decorrelating is independent of the interfering users. Therefore, the near-far resistance of the decorrelating detector and the MMSE detector can be given by

$$\bar{\eta}_k = \eta_k = \frac{1}{[\mathbf{R}^{-1}]_{kk}}. \quad (2.47)$$

It is easy to infer in term of (2.47) that both the decorrelating detector and the linear MMSE detector achieve the optimal near-far resistance, which is always larger than zero no matter what the power of the interfering users is. In [24], the AME of the subspace MMSE and decorrelating detectors under the signature waveform mismatch is analyzed.

2.8 Conclusion

In this chapter, the conventional blind linear multiuser detectors are introduced. We first present the definition of blind multiuser detection. In terms of this definition, various conventional blind linear multiuser detectors are proposed. As the two of the most popular blind multiuser detectors, blind linear MMSE detector and blind linear decorrelating detector are specially addressed. Meanwhile, the adaptive implementation and the performance analysis of blind MMSE detector and blind decorrelating detector are explained in detail. These two detectors will be adopted as a reference for the comparison of the proposed new methods in the later chapters.

Chapter 3

Intersymbol Decorrelating Detector for Asynchronous DS-CDMA Systems

3.1 Introduction

In the previous chapters, we introduced the signal models of the synchronous and asynchronous DS-CDMA systems. In addition, we also discussed the conventional blind linear multiuser detectors for these systems. In this chapter, we develop a new intersymbol decorrelating detector for asynchronous DS-CDMA systems based on the signal subspace spanned by the neighboring received signals.

It is noted that the previously mentioned multiuser detection techniques for the asynchronous CDMA systems require the channel estimation of the composite

channel response vector of the desired user, including the delay spread and the parameters of the multipath channel of the desired user. In this chapter, we attempt to avoid channel estimation based on a new multiuser detection scheme that exploits the intersymbol information which is not considered in the previously mentioned detectors. The proposed detector is attractive for its simplicity because only the synchronization of the desired user is assumed to be known. Such an assumption is required by most blind multiuser detectors [59, 60, 61].

A brief review of the signal model of asynchronous DS-CDMA system, and the conventional subspace decorrelating and subspace MMSE detectors are given in section 3.1. In section 3.2, the intersymbol decorrelating detector is introduced. The basic idea of the proposed detector is to construct an intersymbol cross correlation matrix of the consecutive symbols, and then to derive the decorrelating detector for the desired user. The proposed detector can be implemented conveniently without the estimation of background noise, delay spread and multipath channel of the desired user. Section 3.3 presents the blind implementation of the proposed decorrelating detector. To demonstrate the performance of the proposed detector, simulation examples are given in section 3.4 to compare with the performance of the conventional linear detectors. The chapter is ended with some conclusions in section 3.5.

3.2 Signal Model and Conventional Methods

3.2.1 Signal Model

We reconsider an asynchronous CDMA signal model where the delay and the multipath channel of each user are presented. Based on the asynchronous model of (1.27)-(1.29) for the first chapter, a K user asynchronous DS-CDMA system is given by

$$\begin{aligned}\mathbf{r}[i] &= \sum_{k=1}^K (\mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1]) + \mathbf{v}[i] \\ &= \mathbf{H}_0 \mathbf{b}[i] + \mathbf{H}_1 \mathbf{b}[i-1] + \mathbf{v}[i],\end{aligned}\tag{3.1}$$

where $\mathbf{h}_k$, $\bar{\mathbf{h}}_k$, $\mathbf{H}_0$, $\mathbf{H}_1$, and $\mathbf{v}[i]$ as denoted in the first chapter, are the signature vector of User k for the current bit, the signature vector of User k for the previous bit, the signature matrix of the current bits of all users, the signature matrix of the previous bits of all users and the background noise, respectively.

3.2.2 Conventional Methods

For the convenience of the analysis and without loss of generality, let us assume that User 1 is the desired user, which has been synchronized without delay. We then consider the conventional linear decorrelating detector and linear MMSE detector of the desired user. According to the method in (2.35), the subspace based decorrelating detector and MMSE detector for asynchronous CDMA systems can be

given by

$$\begin{aligned}\mathbf{d}_1 &= \mathbf{U}_s(\Lambda_s - \sigma^2 \mathbf{I}_{2K})\mathbf{U}_s^H \mathbf{h}_1 \\ \mathbf{m}_1 &= (\mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H) \mathbf{h}_1.\end{aligned}\tag{3.2}$$

where $\mathbf{U}_s$ and Λ_s are denoted in (2.34), respectively. As mentioned in Chapter 2, the signal subspace components ($\mathbf{U}_s, \Lambda_s$, and σ^2) can be estimated from the received signal, and the channel parameter $\mathbf{h}_1$ can be estimated with the prior knowledge of the spreading sequence of the desired user. The implementation algorithm of the above detectors is given in Table 2.2.

3.3 Proposed Intersymbol Decorrelating Detector

We now consider a new intersymbol method for the detection of the asynchronous CDMA systems. Based on the assumption that the receiver is synchronized to User 1, i.e. $d_1 = 0$, the signal in (3.1) can be written as

$$\mathbf{r}[i] = \mathbf{h}_1 b_1[i] + \bar{\mathbf{h}}_1 b_1[i-1] + \tilde{\mathbf{H}} \tilde{\mathbf{b}}[i] + \mathbf{v}[i],\tag{3.3}$$

where $\tilde{\mathbf{H}} = [\mathbf{h}_2, \dots, \mathbf{h}_K, \bar{\mathbf{h}}_2, \dots, \bar{\mathbf{h}}_K]$ and $\tilde{\mathbf{b}}[i] = [b_2[i], \dots, b_K[i], b_2[i-1], \dots, b_K[i-1]]^T$ are, the signature matrix and the received bits of the interfering users, respectively. It is noted from (2.37) that only the first M entries of $\bar{\mathbf{h}}_1$ are non-zero, where M is the total number of paths in the channel. Thus, the ISI caused by $\bar{\mathbf{h}}_1$ can be removed by truncating the first $G \geq M$ entries of $\mathbf{r}[i]$. For convenience and without

loss of generality, we choose $G = M$ to obtain

$$\underline{\mathbf{r}}[i] = \underline{\mathbf{h}}_1 b_1[i] + \underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i] + \underline{\mathbf{v}}[i], \quad (3.4)$$

where $\underline{\mathbf{r}}[i]$ is an $(N - G) \times 1$ vector which consists of the last $N - G$ entries of $\mathbf{r}[i]$. Similarly, $\underline{\mathbf{h}}_1$, $\underline{\tilde{\mathbf{H}}}$ and $\underline{\mathbf{v}}[i]$ are composed of the last $N - G$ rows of $\mathbf{h}_1$, $\tilde{\mathbf{H}}$ and $\mathbf{v}[i]$, respectively. The signal in (3.4) is a truncation of the signal in (3.3) which indicates that the ISI of the desired user is completely eliminated. It should be pointed out that the ISI of the interfering users in (3.4) still exists as their non-zero delays. This is an advantageous property for the following proposed intersymbol cross correlation matrix.

We now develop a linear decorrelating detector for User 1 by utilizing the intersymbol information of (3.4). We first construct the autocorrelation matrix of the received signal $\underline{\mathbf{r}}[i]$, which is given by

$$\underline{\mathbf{R}} = E\{\underline{\mathbf{r}}[i]\underline{\mathbf{r}}[i]^H\} = \underline{\mathbf{h}}_1 \underline{\mathbf{h}}_1^H + \underline{\tilde{\mathbf{H}}} \underline{\tilde{\mathbf{H}}}^H + \sigma^2 \mathbf{I}_{N-G}. \quad (3.5)$$

We next construct a useful cross correlatin matrix of the received signal $\underline{\mathbf{r}}[i]$ to obtain the intersymbol information, i.e.,

$$\begin{aligned} \bar{\underline{\mathbf{R}}} &= E\{\underline{\mathbf{r}}[i-1]\underline{\mathbf{r}}[i]^H\} + E\{\underline{\mathbf{r}}[i]\underline{\mathbf{r}}[i-1]^H\} \\ &= E\{(\underline{\mathbf{h}}_1 b_1[i-1] + \underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i-1] + \underline{\mathbf{v}}[i-1])(\underline{\mathbf{h}}_1 b_1[i] + \underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i] + \underline{\mathbf{v}}[i])^H\} \\ &\quad + E\{(\underline{\mathbf{h}}_1 b_1[i] + \underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i] + \underline{\mathbf{v}}[i])(\underline{\mathbf{h}}_1 b_1[i-1] + \underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i-1] + \underline{\mathbf{v}}[i-1])^H\} \\ &= E\{(\underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i-1])(\underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i])^H\} + E\{(\underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i])(\underline{\tilde{\mathbf{H}}} \tilde{\mathbf{b}}[i-1])^H\} \\ &= \underline{\tilde{\mathbf{H}}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \underline{\tilde{\mathbf{H}}}^H, \end{aligned} \quad (3.6)$$

where the third equation is based on the assumption that the back ground noise is white. In (3.7) $\underline{\mathbf{R}}$ is made up of the cross correlation of the consecutively received signals. It is noted that $\underline{\mathbf{R}}$ only contains the signature matrix of all interfering users excepted for the desired user since the ISI of the desired user has been truncated in (3.4). It is also observed that the subspace spanned by $\underline{\mathbf{R}}$ does not include the interference of the white background noise. By performing the eignedeecompositions of the matrix $\underline{\mathbf{R}}$ and $\bar{\underline{\mathbf{R}}}$, we obtain

$$\begin{aligned}\underline{\mathbf{R}} &= [\underline{\mathbf{U}}_s, \underline{\mathbf{U}}_n] \begin{bmatrix} \underline{\Lambda}_s \\ \underline{\Lambda}_n \end{bmatrix} [\underline{\mathbf{U}}_s, \underline{\mathbf{U}}_n]^H \\ &= \underline{\mathbf{U}}_s \underline{\Lambda}_s \underline{\mathbf{U}}_s^H + \underline{\mathbf{U}}_n \underline{\Lambda}_n \underline{\mathbf{U}}_n^H\end{aligned}\quad (3.7)$$

$$\begin{aligned}\bar{\underline{\mathbf{R}}} &= [\bar{\underline{\mathbf{U}}}_s, \bar{\underline{\mathbf{U}}}_n] \begin{bmatrix} \bar{\underline{\Lambda}}_s \\ \mathbf{0} \end{bmatrix} [\bar{\underline{\mathbf{U}}}_s, \bar{\underline{\mathbf{U}}}_n]^H \\ &= \bar{\underline{\mathbf{U}}}_s \bar{\underline{\Lambda}}_s \bar{\underline{\mathbf{U}}}_s^H,\end{aligned}\quad (3.8)$$

where $\underline{\Lambda}_s = \text{diag}(\lambda_1, \dots, \lambda_{2K-1})$ contains $(2K - 1)$ largest eigenvalues of the signal subspace in $\underline{\mathbf{R}}$ and $\underline{\mathbf{U}}_s$ contains the corresponding orthonormal eigenvectors. Both $\underline{\Lambda}_n = \sigma^2 \mathbf{I}_{N-G-2K+1}$ and $\underline{\mathbf{U}}_n$ are, respectively, eigenvalues and orthonormal eigenvectors of the noise subspace. Similarly, $\bar{\underline{\Lambda}}_s = \text{diag}(\bar{\lambda}_1, \dots, \bar{\lambda}_{2K-2})$ contains $2K - 2$ non-zero eigenvalues and $\bar{\underline{\mathbf{U}}}_s$ contains the corresponding eigenvectors (Here we assume that $\tilde{\underline{\mathbf{H}}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\underline{\mathbf{H}}}^H$ has full rank of $2K - 2$). Obviously, $\bar{\underline{\mathbf{R}}}$ only contains ISI and MAI of all interfering users.

The construction of $\bar{\underline{\mathbf{R}}}$ plays a key role for the following derived decorrelating detector. Since $\bar{\underline{\mathbf{R}}}$ is formed from the cross correlation matrix of the consecutively

received signals, the proposed detector is hence termed as intersymbol decorrelating detector.

We now consider the relationship between the subspace spanned by $\bar{\mathbf{R}}$ and the subspace spanned by $\tilde{\mathbf{H}}$.

Proposition 3.1: The null space of $\bar{\mathbf{U}}_s$ is equal to the null space of $\tilde{\mathbf{H}}$, i.e.,

$$\text{null}(\tilde{\mathbf{H}}) = \text{null}(\bar{\mathbf{U}}_s) = \text{range}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \quad (3.9)$$

Proof:

Denote an $(N - G) \times 1$ vector $\mathbf{x}$. The proof of the proposition is equivalent to the proof of the following items.

- i. if $\mathbf{x}^H \tilde{\mathbf{H}} = \mathbf{0} \implies \mathbf{x}^H \bar{\mathbf{U}}_s = \mathbf{0}$.
- ii. if $\mathbf{x}^H \bar{\mathbf{U}}_s = \mathbf{0} \implies \mathbf{x}^H \tilde{\mathbf{H}} = \mathbf{0}$.

We now consider $\bar{\mathbf{R}}\bar{\mathbf{R}}^H$, which is given by

$$\begin{aligned} \bar{\mathbf{R}}\bar{\mathbf{R}}^H &= \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H \\ &= \tilde{\mathbf{H}} \mathbf{W} \tilde{\mathbf{H}}^H \\ &= \bar{\mathbf{U}}_s \bar{\mathbf{\Lambda}}_s \bar{\mathbf{U}}_s^H \bar{\mathbf{U}}_s \bar{\mathbf{\Lambda}}_s \bar{\mathbf{U}}_s^H \\ &= \bar{\mathbf{U}}_s \bar{\mathbf{\Lambda}}_s^2 \bar{\mathbf{U}}_s^H, \end{aligned} \quad (3.10)$$

where

$$\mathbf{W} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \quad (3.11)$$

and the third equality follows from the definition of (3.8). Obviously, $\bar{\mathbf{R}}\bar{\mathbf{R}}^H$ is positive and has the same eigenvectors as that of $\bar{\mathbf{R}}$. To prove the first item, we

have

$$\begin{aligned}
\mathbf{x}^H \tilde{\mathbf{H}} &= \mathbf{0} \implies \mathbf{x}^H \tilde{\mathbf{H}} \mathbf{W} \tilde{\mathbf{H}}^H \mathbf{x} = 0 \\
&\implies \mathbf{x}^H \bar{\mathbf{U}}_s \bar{\Lambda}_s^2 \bar{\mathbf{U}}_s^H \mathbf{x} = 0 \\
&\implies \mathbf{x}^H \bar{\mathbf{U}}_s = 0
\end{aligned} \tag{3.12}$$

where the last step follows from the fact that $\bar{\Lambda}_s^2$ is a positive diagonal matrix. For the second item, the proof is as follows:

$$\begin{aligned}
\mathbf{x}^H \bar{\mathbf{U}}_s &= \mathbf{0} \implies \mathbf{x}^H \bar{\mathbf{U}}_s \bar{\Lambda}_s^2 \bar{\mathbf{U}}_s^H \mathbf{x} = 0 \\
&\implies \mathbf{x}^H \tilde{\mathbf{H}} \mathbf{W} \tilde{\mathbf{H}}^H \mathbf{x} = 0 \\
&\implies \mathbf{x}^H \tilde{\mathbf{H}} = \mathbf{0}.
\end{aligned} \tag{3.13}$$

Similarly, the last step in (3.13) is derived because $\mathbf{W}$ is also positive and of full rank. By using items i and ii, we have $\text{null}(\bar{\mathbf{U}}_s) = \text{null}(\tilde{\mathbf{H}})$. In addition, $\text{null}(\bar{\mathbf{U}}_s)$ can be easily expressed by its projection matrix, i.e., $\text{null}(\bar{\mathbf{U}}_s) = \text{range}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)$, where $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ projects any signal onto the null space of $\tilde{\mathbf{H}}$.

Proposition 3.1 shows that $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ projects any signal onto the null space of $\tilde{\mathbf{H}}$. Since $\tilde{\mathbf{H}}$ only contains the signature vectors of the interfering users, it is natural to explore the optimal linear detector by removing the influence generated by $\tilde{\mathbf{H}}$. According to [24], the decorrelating detector for User 1 can be defined as the unique signal $\mathbf{d} \in \text{range}(\mathbf{U}_s)$, such that $\mathbf{d}^H \mathbf{h}_1 > 0$, and $\mathbf{d}^H \tilde{\mathbf{H}} = \mathbf{0}$, where $\mathbf{U}_s$ is given in (3.7). Clearly, the decorrelating detector for User 1 is orthonormal to $\tilde{\mathbf{H}}$. By considering the property of Proposition 3.1, we then propose another form of the decorrelating detector for the desired user, which is given by the following proposition.

Proposition 3.2: In terms of the matrix $\bar{\mathbf{R}}$ in (3.8), the decorrelating detector for User 1 in (3.2) can be given by

$$\mathbf{d}_1 = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \quad (3.14)$$

Proof:

Based on (2.4) and the eigendecomposition of (3.8), the decorrelating detector for User 1 must and only satisfies the follow conditions:

$$\mathbf{d}_1 = (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \in \text{range}(\mathbf{U}_s), \text{ and } \begin{cases} \mathbf{d}_1^H \mathbf{h}_1 > 0 \\ \mathbf{d}_1^H \tilde{\mathbf{H}} = 0 \end{cases} \quad (3.15)$$

Since $\text{range}(\mathbf{U}_s) = \text{null}(\mathbf{U}_n)$, the proof of the first condition is equivalent to the proof of $\mathbf{U}_n^H \mathbf{d}_1 = \mathbf{0}$. Then, it is easy to obtain

$$\begin{aligned} \mathbf{U}_n^H \mathbf{d}_1 &= \mathbf{U}_n^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \\ &= \mathbf{U}_n^H \mathbf{h}_1 - \mathbf{U}_n^H \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H \mathbf{h}_1 \\ &= \mathbf{0} \end{aligned} \quad (3.16)$$

where the third equality follows from the fact that $\mathbf{h}_1 \in \text{range}(\mathbf{U}_s)$ is orthogonal to $\mathbf{U}_n$, and $\text{range}(\mathbf{U}_n) = \text{null}(\tilde{\mathbf{H}}, \mathbf{h}_1)$ belongs to the subspace of $\text{null}(\tilde{\mathbf{H}}) = \text{null}(\bar{\mathbf{U}}_s)$, i.e., $\mathbf{U}_n^H \bar{\mathbf{U}}_s = \mathbf{0}$. We now consider the second condition. With (3.8) and Proposition 3.1, it is easy to obtain

$$\begin{aligned} \mathbf{d}_1^H \mathbf{h}_1 &= \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 = \mathbf{h}_1^H \bar{\mathbf{U}}_n \bar{\mathbf{U}}_n^H \mathbf{h}_1 > 0 \\ \mathbf{d}_1^H \tilde{\mathbf{H}} &= \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \tilde{\mathbf{H}} = \mathbf{0} \end{aligned} \quad (3.17)$$

Therefore, $\mathbf{d}_1$ is the decorrelating detector of User 1.

3.4 Proposed Blind Implementation of the Intersymbol Decorrelating Detector

Proposition 3.2 shows that $\mathbf{d}_1$ is equivalent to the decorrelating detector for the desired user. We now consider the blind implementation of $\mathbf{d}_1$.

Utilizing the property of Proposition 3.1, we project the received signal in (3.4) onto $\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ directly to obtain

$$\begin{aligned} \tilde{\mathbf{r}}[i] &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{r}[i] \\ &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) [\mathbf{h}_1 b_1[i] + \tilde{\mathbf{H}} \tilde{\mathbf{b}}[i] + \mathbf{v}[i]] \\ &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) [\mathbf{h}_1 b_1[i] + \mathbf{v}[i]] \end{aligned} \quad (3.18)$$

where the third equality follows from Proposition 3.1, i.e., $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \tilde{\mathbf{H}} = \mathbf{0}$.

It is obvious from (3.18) that the MAI and ISI of all interfering users are removed from $\tilde{\mathbf{r}}[i]$. The autocorrelation matrix of $\tilde{\mathbf{r}}[i]$ can be given by

$$\begin{aligned} \tilde{\mathbf{R}} &= E\{\tilde{\mathbf{r}}[i] \tilde{\mathbf{r}}^H[i]\} \\ &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) (\mathbf{h}_1 \mathbf{h}_1^H + \sigma^2 \mathbf{I}_{N-G}) (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \\ &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) + \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H). \end{aligned} \quad (3.19)$$

3.4 Proposed Blind Implementation of the Intersymbol Decorrelating Detector 65

We next premultiply $\mathbf{d}_1$ by $\tilde{\mathbf{R}}$ to obtain

$$\begin{aligned}
 \tilde{\mathbf{R}}\mathbf{d}_1 &= \tilde{\mathbf{R}}(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \\
 &= [(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \\
 &\quad + \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)] (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \\
 &= [\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \sigma^2] (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \\
 &= [\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \sigma^2] \mathbf{d}_1.
 \end{aligned} \tag{3.20}$$

Clearly, the decorrelating detector of the desired user $\mathbf{d}_1$, i.e., $(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1$ is an eigenvector of $\tilde{\mathbf{R}}$ with the corresponding eigenvalue $\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \sigma^2$. In fact, $\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \sigma^2$ is the largest eigenvalue of $\tilde{\mathbf{R}}$. In order to prove this, we consider the rest eigenvectors of $\tilde{\mathbf{R}}$. Denoting $\mathbf{u}$ as an eigenvector of $\tilde{\mathbf{R}}$ which is orthogonal to $\mathbf{d}_1$, i.e., $\mathbf{u}^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 = 0$, we have

$$\begin{aligned}
 \tilde{\mathbf{R}}\mathbf{u} &= [(\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) + \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)] \mathbf{u} \\
 &= \sigma^2 (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{u} \\
 &\preceq \sigma^2 \mathbf{u}
 \end{aligned} \tag{3.21}$$

where the second equality follows from $\mathbf{u}^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 = 0$. It is clear from (3.21) that $\mathbf{h}_1^H (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \sigma^2$ is the largest eigenvalue of $\tilde{\mathbf{R}}$, and all of the rest eigenvalues of $\tilde{\mathbf{R}}$ are less than σ^2 . Therefore, the proposed detector $\mathbf{d}_1$ can be implemented by calculating the principle eigenvector of $\tilde{\mathbf{R}}$, i.e.,

$$\begin{aligned}
 \mathbf{d}_1 &= (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 \\
 &= \arg \max \tilde{\mathbf{R}} \\
 &= \arg \max (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \tilde{\mathbf{R}} (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H).
 \end{aligned} \tag{3.22}$$

3.4 Proposed Blind Implementation of the Intersymbol Decorrelating Detector 66

Table 3.1: Blind Intersymbol Decorrelating Detector
Sampling Frame: $\mathbf{r}[0], \mathbf{r}[1], \dots, \mathbf{r}[P]$
STEP 1. Compute autocorrelation and cross correlation
$\mathbf{R} = \frac{1}{P+1} \sum_{i=0}^P \mathbf{r}[i] \mathbf{r}[i]^H$ $\bar{\mathbf{R}} = \frac{1}{P} (\sum_{i=0}^{P-1} \mathbf{r}[i] \mathbf{r}[i+1]^H + \sum_{i=1}^P \mathbf{r}[i-1] \mathbf{r}[i]^H)$
STEP 2. Compute eigendecomposition of $\bar{\mathbf{R}}$
$\bar{\mathbf{R}} = [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n] \begin{bmatrix} \bar{\mathbf{\Lambda}}_s \\ \mathbf{0} \end{bmatrix} [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]^H = \bar{\mathbf{U}}_s \bar{\mathbf{\Lambda}}_s \bar{\mathbf{U}}_s^H$
STEP 3. Form decorrelating detector
$\mathbf{d}_1 = \text{Max-eigenvector } (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \bar{\mathbf{R}} (\mathbf{I}_{N-G} - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)$
STEP 4. Perform differential detection
$z_1[i] = \mathbf{d}_1^H \mathbf{r}[i] \quad \hat{\beta}_1(n) = \text{sign} (\Re\{z_1[i] z_1[i-1]^*\})$

Table 3.1 summarizes the implementation of the proposed intersymbol decorrelating detector. It can be observed from this table that the realization of the detector is very simple compared with the subspace methods in Table 2.2. We do not require any information on background noise level, the delay spread and the parameters of the multipath channel. The only requirement is the synchronization of the desired user. In contrast, some of these parameters must be known or estimated in the detectors of Table 2.2. It is noted from this table that the detector in step 3 is equal to the decorrelating detector up to an ambiguous phase factor which will deteriorate the performance of the detector. In order to overcome this problem, the differential

detection is adopted in step 4 to remove the ambiguity of the phase.

It should be pointed out that the basic idea of the proposed detector is to utilize the eigendecomposition of the cross-correlation matrix $\bar{\mathbf{R}}$ to null out the ISI and MAI of all interfering users. According to (3.4), $\bar{\mathbf{R}}$ has $2(K - 1)$ eigenvectors spanned by $\mathbf{h}_k \bar{\mathbf{h}}_k^H + \bar{\mathbf{h}}_k \mathbf{h}_k^H$ for $k = 2, \dots, K$. However, if the k th interfering user has similar delays to that of the desired user, i.e., $d_k \approx 0$, the eigenvalues corresponding to the k th user in $\bar{\mathbf{R}}$ will be small since only the first d_k entries of $\bar{\mathbf{h}}_k$ are non-zero. Because it is usually difficult to estimate the small eigenvalue's subspace accurately, more errors will occur on the estimation of the k th user's eigenvectors at STEP 2 of Table 3.1 for small d_k . In this case, the estimated $\bar{\mathbf{U}}_s$ may lose a part of the k th user's information, and the proposed detector can not totally null out the interference of the k th user. Then, the performance of the proposed detector will be deteriorated.

3.5 Simulation Results

In this section, experimental results are provided to illustrate the performance of the proposed intersymbol decorrelating detector. We test the proposed method in an asynchronous CDMA system with $K=8$ users and spreading gain $N = 31$. The spreading sequences for all users are generated by Gold codes with the crosscorrelation of $1/31$ for any two users. In all cases, we simulate a severe near-far situation that each interfering user is at least with a power of 10 dB more than that of the desired user. Meanwhile, User 1 is assumed to be synchronized and all interfering users have the delay less than one bit symbol. The multipath gains in each user's

channel are randomly chosen and kept fixed, which have been normalized with equal power.

We compare the performance of five detectors, namely: the true decorrelating detector, the proposed decorrelating detector, the blind zero-forcing (decorrelating) detector in [62], the constraint optimized detector in [63] and the conventional matched-filter. The proposed detector is implemented by the algorithm given in Table 3.1, where the length of signal frame is $P = 500$. The truncating window of the proposed detector is $G = 5$, i.e., the first five entries of the received signal vector are truncated to remove the ISI of the desired user. Figure 3.1 illustrates the BERs

Figure 3.1: Bit-error-rate comparison of various linear detectors in an asynchronous CDMA system. $M = 3$, $G = 5$, $K = 8$, $d_k \geq 8$ ($k = 2, \dots, 8$).

of these detectors with multipath length $M = 3$. In this example, the initial delay

d_k of each interfering user is assumed to be randomly distributed between 8 and 31 in terms of the chip cycle T_c . It is seen from this figure that the matched filter performs poorly compared with the other four linear detectors because it is near-far limited. It is also observed that the performance of the proposed decorrelating detector is close to the performance of the zero-forcing (decorrelating) detector in [62], and is a little better than that of the constraint optimized detector in [63]. In addition, there is no significant performance difference between the proposed detector and the true decorrelating detector. Clearly, our method performs well in this case because the signal channel is ideal for the proposed detector, i.e., the ISI of the desired user is totally removed because $M < G$, and all interfering users have significant intersymbol interference as the delay $d_k \geq 8$.

We next consider a dispersive channel with multipath length $M = 5$. In this channel, two users are assumed to have similar delays to that of use 1, i.e., $d_2 = 3$ and $d_3 = 2$, while the delays of the other five interfering users are still longer than $8T_c$. The performance of various detectors is shown in Figure 3.2. It is seen that all detectors except the matched filter have certain performance deterioration as the length of multipath increases. However, although the ISI of the desired user is also removed in this case, i.e., $M=G$, the proposed detector suffers more performance loss and its BER is a little higher than that of the constraint optimized detector in [63]. As explained in the previous section, this is because User 2 and User 3 have small delays relative to User 1 and the corresponding eigenvalues in $\bar{\mathbf{R}}$ are close to zero. Thus, more errors will occur on the estimation of their eigenvectors within the

Figure 3.2: Bit-error-rate comparison of various linear detectors in an asynchronous CDMA system. $M = 5$, $G = 5$, $K = 8$, $d_2 = 3$, $d_3 = 2$, $d_k \geq 8$ ($k = 4, \dots, 8$).

limited signal frame, which leads to the deterioration of the proposed detector.

In Figure 3.3, the multipath length increases to $M = 7$ and the delays of the interfering users are the same as the delays in Figure 3.2. It is seen that the performance deterioration of the proposed detector is more apparent compared with Figure 3.2, and the proposed detector is obviously inferior to the decorrelating detector in [62] and the constraint optimized detector in [63] although it is still significantly superior to the matched filter. The reason for such deterioration is that the signal in (3.4) cannot totally remove the ISI of the desired user since $M > G$. Hence, the remanent ISI of the desired user in $\bar{\mathbf{R}}$ also influences the accuracy of the proposed detector.

Figure 3.3: Bit-error-rate comparison of various linear detectors in an asynchronous CDMA system. $M = 7$, $G = 5$, $K = 8$, $d_2 = 3$, $d_3 = 2$, $d_k \geq 8$ ($k = 4, \dots, 8$).

In general, our method is more sensitive to the multipath length and the delays of the interfering users compared with the decorrelating detector in [62] and the constraint optimized detector in [63]. However, our method is very simple which only requires to know the delay of the desired user, while the decorrelating detector in [62] requires the knowledge of the delay and the signature sequence of the desired user as well as the estimation of the noise level σ^2 and the desired user's multipath channel impulse response $\mathbf{g}_1$. The detector in [63] also requires information of the delay, the signature sequence and the channel estimation of the desired user.

3.6 Conclusions

In this chapter, a new intersymbol decorrelating detector is developed for asynchronous CDMA systems with multipath. The detector makes use of an important cross correlation matrix between adjacent symbols to mitigate the influence of the interfering users. It is shown that the proposed detector can be easily implemented without the estimations of the noise level, the multipath channel and the multipath length, while these estimations are required for the conventional blind MMSE detector and blind decorrelating detector.

It is observed from the simulation results that the proposed achieve a satisfying performance as the signal channel is good. However, the proposed method suffers more deterioration when the delays of the interfering users are closed to the multipath length. In this case, the performance of the proposed detector is inferior to the performance of the conventional MMSE and decorrelating methods. In order to increase the performance of the proposed detector, an improved group detection method is proposed in the next chapter.

Chapter 4

Intersymbol Group-blind

Multiuser Detectors for

Asynchronous DS-CDMA Systems

In this chapter, a series of intersymbol group-blind multiuser detectors for asynchronous CDMA systems are developed. Group-blind multiuser detectors were first proposed by [57] for CDMA uplink communication. The group detectors in [57] make use of the spreading sequence and the estimated multipath channels of all known users to construct a group constraint to suppress the intracell interference, while blindly suppressing the intercell interference, and then to enhance the performance of the detectors. Unlike the methods in [57], the proposed intersymbol group-blind multiuser detectors utilize the correlation information between consecutively received signals to generate the corresponding group constraint. It is shown

that by incorporating this group constraint, the proposed detectors can provide different performance gains in both uplink and downlink environments. In addition, the proposed detectors only require to estimate the multipath channel of the desired user and do not require the channel estimation of other users.

The asynchronous CDMA model in the first chapter is reviewed and analyzed briefly in Section 4.1. In Section 4.2, the intersymbol group constraint is discussed in detail. Unlike the group detectors in [57] and [64], the proposed intersymbol group constraint only requires the channel estimation of the interested user. The implementation of various intersymbol group-blind multiuser detectors based on the intersymbol constraint is developed in Section 4.3. In Section 4.4, simulation examples are provided to demonstrate the performance of the proposed detectors. Section 4.5 contains the conclusion.

4.1 Signal Model

Similar to Chapter 3, the signal model of the proposed intersymbol group-blind multiuser detectors is also based on the asynchronous CDMA systems presented in Chapter 1. According to (1.21)-(1.29), the signal model of asynchronous CDMA systems can be given by

$$\begin{aligned}
 \mathbf{r}[i] &= \sum_{k=1}^K (\mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1]) + \mathbf{v}[i] \\
 &= \mathbf{H}_0 \mathbf{b}[i] + \mathbf{H}_1 \mathbf{b}[i-1] + \mathbf{v}[i] \\
 &= \mathbf{H} \bar{\mathbf{b}}[i] + \mathbf{v}[i],
 \end{aligned} \tag{4.1}$$

where $\mathbf{H}_0 = [\mathbf{h}_1, \dots, \mathbf{h}_K]$, $\mathbf{H}_1 = [\bar{\mathbf{h}}_1, \dots, \bar{\mathbf{h}}_K]$, $\mathbf{H} = [\mathbf{H}_0, \mathbf{H}_1]$, $\mathbf{b}[i] = [b_1[i], \dots, b_K[i]]^T$, $\mathbf{b}[i-1] = [b_1[i-1], \dots, b_K[i-1]]^T$ and $\bar{\mathbf{b}}[i] = [\mathbf{b}^T[i], \mathbf{b}^T[i-1]]^T$. The signature vectors of the k th user are given by

$$\mathbf{h}_k = \begin{bmatrix} \mathbf{0} \\ h_k[d_{k,s}] \\ h_k[d_{k,s} + 1] \\ \vdots \\ h_k[N] \end{bmatrix}, \quad \bar{\mathbf{h}}_k = \begin{bmatrix} h_k[N+1] \\ h_k[N+2] \\ \vdots \\ h_k[d_{k,e}] \\ \mathbf{0} \end{bmatrix}, \quad (4.2)$$

where

$$\mathbf{h}_{k,all} = \mathbf{C}_k \bar{\mathbf{g}}_k = \begin{bmatrix} h_k[d_{k,s}] \\ h_k[d_{k,s} + 1] \\ \vdots \\ h_k[d_{k,e}] \end{bmatrix} \quad (4.3)$$

is given by

$$\mathbf{C}_k = \begin{bmatrix} c_k[0] & & & & \\ c_k[1] & \ddots & & & \\ \vdots & \ddots & c_k[0] & & \\ \vdots & & c_k[1] & & \\ c_k[N-1] & \ddots & \vdots & & \\ & \ddots & \vdots & & \\ & & c_k[N-1] & & \end{bmatrix}, \quad \bar{\mathbf{g}}_k = \begin{bmatrix} \bar{g}_k[d_{k,s}] \\ \bar{g}_k[d_{k,s} + 1] \\ \vdots \\ \bar{g}_k[d_{k,e}] \end{bmatrix}. \quad (4.4)$$

All of the above definitions can be referred to Chapter 1 for more details. It is noted that the signal model in (4.1) is a general expression of the asynchronous CDMA

systems. The synchronous CDMA system can be considered as a special case of (4.1) where all active users have the same delay.

In practical CDMA systems, there exists two kinds of communications, i.e., uplink communication and downlink communication. During the uplink CDMA communication, the users in the cell transmitted their signals asynchronously to the base station. Meanwhile, the base station also receives the asynchronous signals from the interfering out-cell users. In this case, it is often assumed that $\mathbf{H}_0$ and $\mathbf{H}_1$ are mutually independent, and $\mathbf{H}$ is a tall matrix with full column rank of $2K$ as the existence of the delays of different users [63]. However, during the downlink communication, the signals to the in-cell users are transmitted synchronously from the base station plus the signal of asynchronous out-cell users as well. In this case, the full-rank condition on $\mathbf{H}$ is hardly satisfied as argued in [65]. It can be explained that in synchronous CDMA communication ($d_k = 0$ for $k = 1, \dots, K$), the non-zero elements in $\mathbf{H}_1$ is limited by the number of active Users K and the multipath length M , i.e., only an $M \times K$ sub-matrix in $\mathbf{H}_1$ is non-zero. Hence, the signature matrix $\mathbf{H}$ will not be of full column rank unless K is smaller than M .

Based on the signal model in (4.1), the autocorrelation matrix of the received signal is given by

$$\begin{aligned} \mathbf{R} &= \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_N \\ &= \mathbf{U}_s\Lambda_s\mathbf{U}_s^H + \mathbf{U}_n\Lambda_n\mathbf{U}_n^H. \end{aligned} \tag{4.5}$$

where $\Lambda_s = \text{diag}(\lambda_1, \dots, \lambda_L)$ contains L largest eigenvalues of $\mathbf{R}$ and $\mathbf{U}_s$ contains the corresponding orthonormal eigenvectors of the signal subspace spanned by $\mathbf{H}\mathbf{H}^H$.

Both $\Lambda_n = \sigma^2 \mathbf{I}_{N-L}$ and $\mathbf{U}_n$ are, respectively, the eigenvalues and the orthonormal eigenvectors of the noise subspace.

Referring to (2.37), the k th desired user's channel $\mathbf{h}_k$ can be estimated in terms of the received signal and $\mathbf{C}_k$ when the desired user is synchronized, i.e.,

$$\mathbf{g}_k = \text{Max-eigenvector } (\mathbf{C}_{k,up}^H \mathbf{U}_s \mathbf{U}_s^H \mathbf{C}_{k,up}) \quad (4.6)$$

$$\mathbf{h}_k = \mathbf{C}_{k,up} \mathbf{g}_k \quad (4.7)$$

where $\mathbf{C}_{k,up}$ is composed of the first N rows of the matrix $\mathbf{C}_k$ as denoted in (2.37) of Chapter 2.

4.2 Proposed Intersymbol Group Constraint

The conventional blind multiuser detectors discussed in the previous chapters only have the knowledge of the spreading sequences of the desired user. However, the base station has the knowledge of the spreading sequences of all in-cell users. Hence, some performance gains may be obtained if the detector has the knowledge of the spreading sequences of a group of users. Such expectation has been achieved in [57], where various group-blind linear multiuser detectors were presented. These detectors first exploit the spreading sequences of the known users to estimate their signature vectors based on the methods in (4.6) and (4.7). By using the estimated signature vectors, several group-blind detectors can be developed with the following group constraint

$$\mathbf{w}_k^H \tilde{\mathbf{H}} = \mathbf{1}_{P,k}^T \quad (4.8)$$

where P is the number of columns in $\tilde{\mathbf{H}}$, $\mathbf{w}_k$ is the detector for User k , $\tilde{\mathbf{H}}$ contains the estimated signature vectors of a group of known users, which is a subset of $\mathbf{H}$, and $\mathbf{1}_{P,k}$ is denoted as a P dimension column vector with all zero entries except for the k -th entry. The constraint in (4.8) is the key idea of the group-blind zero-forcing detector and the group-blind hybrid detector in [57]. In order to give a clearer explanation of the group constraint, we can rewrite Equation (4.8) as the following constraints:

$$\begin{cases} \mathbf{w}_k^H \mathbf{h}_k = 1 \\ \mathbf{w}_k^H \hat{\mathbf{H}} = 0 \end{cases} \quad (4.9)$$

where $\hat{\mathbf{H}}$ contains the remaining column vectors of $\tilde{\mathbf{H}}$ except for $\mathbf{h}_k$. In (4.9), the first constraint is to assure the existence of the desired user's signal in the detector, and the second constraint is used to zero-force the interference of the known user's signals. Hence, the essence of the group-blind multiuser detection is to construct a group constraint, which exactly removes the influence of some interfering users, thereby achieving performance gains. Since the group constraint in [57] requires the known user's spreading sequences and the estimations of their multipath channels and delays, such techniques are only fit for the CDMA uplink, i.e., only the base station knows the spreading sequences of all intracell users.

In this section, we develop two improved blind linear multiuser detectors. The basic idea for the proposed detectors is to construct a new group constraint, which is similar to the group constraint in (4.9), by exploiting the intersymbol information of the received signals. By using this method, we only need the channel estimation

of the desired user and the spreading sequences of other users are not required to be known by the detector. It is shown that the proposed detectors provide performance gain over the blind linear de correlating detector and the blind linear MME detector in both downlink and uplink cases.

Let us assume without loss of generality that User 1 is the desired user, and the receiver is synchronized to User 1, i.e. $d_1 = 0$. Then, the signal in (4.1) can be rewritten as

$$\mathbf{r}[i] = \mathbf{h}_1 b_1[i] + \bar{\mathbf{h}}_1 b_1[i-1] + \check{\mathbf{H}} \check{\mathbf{b}}[i] + \mathbf{v}[i] \quad (4.10)$$

where $\check{\mathbf{H}} = [\mathbf{h}_2, \dots, \mathbf{h}_K, \bar{\mathbf{h}}_2, \dots, \bar{\mathbf{h}}_K]$ and $\check{\mathbf{b}}[i] = [b_2[i], \dots, b_K[i], b_2[i-1], \dots, b_K[i-1]]^T$ are, respectively, the signature matrix and received symbols of the interfering users. We next consider the channel estimation of $\mathbf{h}_1$, which is the signature vector of the desired signal $b_1[i]$. Meanwhile, we will also estimate $\bar{\mathbf{h}}_1$ since it is required by the following derived detectors. With the assumption that User 1 is synchronized, we denote

$$\bar{\mathbf{C}}_{1,up} = \begin{bmatrix} \bar{\mathbf{c}}_1[0] & & \mathbf{0} \\ \vdots & \ddots & \bar{\mathbf{c}}_1[0] \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{c}}_1[N-1] & \dots & \bar{\mathbf{c}}_1[N-M-1] \end{bmatrix} \quad (4.11)$$

$$\bar{\mathbf{C}}_{1,dn} = \begin{bmatrix} 0 & \bar{\mathbf{c}}_1[N-1] & \dots & \bar{\mathbf{c}}_1[N-M] \\ \vdots & & \ddots & \vdots \\ \vdots & \mathbf{0} & & \bar{\mathbf{c}}_1[N-1] \\ 0 & \dots & \dots & 0 \end{bmatrix} \quad (4.12)$$

where both $\bar{\mathbf{C}}_{1,up}$ and $\bar{\mathbf{C}}_{1,dn}$ are $N \times (M+1)$ matrices. $\bar{\mathbf{C}}_{1,up}$ is constructed by the first N rows of $\bar{\mathbf{C}}_1$ defined in (4.4), and $\bar{\mathbf{C}}_{1,dn}$ is formed by the remaining M rows of $\bar{\mathbf{C}}_1$ plus an $(N-M) \times (M+1)$ zero matrix. According to the definitions of $\mathbf{h}_1$ and $\bar{\mathbf{h}}_1$ and the denotation in (4.11) and (4.12), it is easy to obtain

$$\mathbf{h}_1 = \bar{\mathbf{C}}_{1,up} \mathbf{g}_1 \quad (4.13)$$

$$\bar{\mathbf{h}}_1 = \bar{\mathbf{C}}_{1,dn} \mathbf{g}_1. \quad (4.14)$$

where $\mathbf{g}_1$ is the multipath channel of User 1. Since $\bar{\mathbf{C}}_{1,up}$ and $\bar{\mathbf{C}}_{1,dn}$ are easy to construct with the prior knowledge of the spreading sequences of the desired user, we only need to consider the estimation of $\mathbf{g}_1$. The multipath channel vector $\mathbf{g}_1$ of the desired user can be estimated by exploiting the orthogonality between the signal subspace and the interfering noise subspace. Since $\mathbf{h}_1$ belongs to the subspace of $\mathbf{U}_s$, and the signal subspace of $\mathbf{U}_s$ is orthogonal to $\mathbf{U}_n$, we have

$$\mathbf{U}_n^H \mathbf{h}_1 = \mathbf{U}_n^H \bar{\mathbf{C}}_{1,up} \mathbf{g}_1 = \mathbf{0}. \quad (4.15)$$

Hence, the estimation of $\mathbf{g}_1$ can be uniquely determined by computing the principle eigenvector of matrix $\bar{\mathbf{C}}_{1,up}^H \mathbf{U}_s \mathbf{U}_s^H \bar{\mathbf{C}}_{1,up}$ [66, 67], i.e.,

$$\hat{\mathbf{g}}_1 = \text{Max-eigenvector} (\bar{\mathbf{C}}_{1,up}^H \mathbf{U}_s \mathbf{U}_s^H \bar{\mathbf{C}}_{1,up}). \quad (4.16)$$

Clearly, the estimation of $\mathbf{g}_1$ is very similar to the way in Chapter 2. Based on (4.16), the signature vectors $\mathbf{h}_1$ and $\bar{\mathbf{h}}_1$ can be determined according to (4.13) and (4.14). After the channel estimation, we construct the cross correlation matrix $\bar{\mathbf{R}}$ again, which is given by

$$\begin{aligned}\bar{\mathbf{R}} &= E\{\mathbf{r}[i]\mathbf{r}[i-1]^H\} + E\{\mathbf{r}[i-1]\mathbf{r}[i]^H\} \\ &= \mathbf{H} \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} \mathbf{H}^H\end{aligned}\tag{4.17}$$

where $\bar{\mathbf{R}}$ is generated by the cross correlation of the received signal. It is seen that $\bar{\mathbf{R}}$ is an $N \times N$ matrix and is irrelevant to the background noise, i.e., $\bar{\mathbf{R}}$ only contains the signal subspace spanned by the active users. This matrix plays an important role in this chapter since it will be exploited to develop the proposed detectors.

Recalling that the basic idea of the group-blind multiuser detection is to suppress the interference of some known users based on the group constraint, we naturally expect that the more subspace of the interfering users the constraint removes, the better performance improvement the detector achieves. In this section, we utilize $\bar{\mathbf{R}}$ to develop a constraint similar to the group constraint in (4.8) for the desired user, which is given by

$$\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H.\tag{4.18}$$

Let us now consider the properties of the above constraint with the following propositions to analytically demonstrate the performance improvement brought by the proposed constraint.

Proposition 4.1: Based on the signal model in (4.1) and the definition of $\mathbf{R}$ in (4.5), the constraint $\mathbf{w}_1^H \mathbf{H} = \mathbf{1}_{2K,1}^T$ implies the constraint $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$, i.e.,

$$\mathbf{w}_1^H \mathbf{H} = \mathbf{1}_{2K,1}^T \implies \mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H \quad (4.19)$$

Proof: The proof of Proposition 4.1 is straightforward by directly substituting the constraint $\mathbf{w}_1^H \mathbf{H} = \mathbf{1}_{2K,1}^T$ into $\mathbf{w}_1^H \bar{\mathbf{R}}$, i.e.,

$$\begin{aligned} \mathbf{w}_1^H \bar{\mathbf{R}} &= \mathbf{w}_1^H \mathbf{H} \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} \mathbf{H}^H \\ &= \mathbf{1}_{2K,1}^T \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} \mathbf{H}^H \\ &= \mathbf{1}_{2K,K+1}^T \mathbf{H}^H \\ &= \bar{\mathbf{h}}_1^H \end{aligned} \quad (4.20)$$

where the last equality follows from the definition of $\mathbf{H}$ in (4.1), i.e., $\mathbf{H}[:, K+1] = \bar{\mathbf{h}}_1$. Hence, Proposition 4.1 is proven.

In the above proposition, the constraint $\mathbf{w}_1^H \mathbf{H} = \mathbf{1}_{2K,1}^T$ is the optimal group constraint for group-blind multiuser detection since it ensures that the detector $\mathbf{w}_1$ is orthogonal to the subspace spanned by all interfering users except the desired user. The result of Proposition 4.1 shows that the proposed constraint is the necessary condition of the optimal group constraint. It means that the proposed constraint $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ will not remove the subspace spanned by the desired user and it is possible to achieve the optimal group constraint based on this constraint. Therefore, we are interested in the feature of the proposed constraint, and want to know if this

constraint implies the optimal group constraint or in what extend it removes the subspace spanned by the interfering users. Propositions 4.2 to 4.4 summarize the proposed constraint's ability to resist the interference brought by the interfering users in different cases.

Proposition 4.2: The constraint $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ implies the constraint $\mathbf{w}_1^H \mathbf{H} = \mathbf{1}_{2K,1}^T$ if $\mathbf{H}$ has full column rank of $2K$.

Proof: We proceed with a proof by contradiction. Assuming that $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ and $\mathbf{w}_1^H \mathbf{H} \neq \mathbf{1}_{2K,1}^T$, then, $\mathbf{w}_1^H \mathbf{H}$ can be expressed as

$$\mathbf{w}_1^H \mathbf{H} = \mathbf{1}_{2K,1}^T + \mathbf{f}^H \quad (4.21)$$

where $\mathbf{f} = [\mathbf{f}_{up}^T, \mathbf{f}_{dn}^T]^T$ is a non-zero $2K$ dimensional column vector, and $\mathbf{f}_{up}$ and $\mathbf{f}_{dn}$ are respectively the first K entries and the last K entries of $\mathbf{f}$. By denoting $\check{\mathbf{f}} = [\mathbf{f}_{dn}^T, \mathbf{f}_{up}^T]^T$ and substituting (4.21) into $\mathbf{w}_1^H \bar{\mathbf{R}}$, we have

$$\begin{aligned} \mathbf{w}_1^H \bar{\mathbf{R}} &= \bar{\mathbf{h}}_1^H \\ \implies \mathbf{w}_1^H \mathbf{H} \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} \mathbf{H}^H &= \bar{\mathbf{h}}_1^H \\ \implies (\mathbf{1}_{2K,1}^T + \mathbf{f}^H) \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} \mathbf{H}^H &= \bar{\mathbf{h}}_1^H \\ \implies (\mathbf{1}_{2K,K+1}^T + \check{\mathbf{f}}^H) \mathbf{H}^H &= \bar{\mathbf{h}}_1^H \\ \implies \bar{\mathbf{h}}_1^H + \check{\mathbf{f}}^H \mathbf{H}^H &= \bar{\mathbf{h}}_1^H \\ \implies \check{\mathbf{f}} &= \mathbf{0} \\ \implies \mathbf{f} &= \mathbf{0} \end{aligned} \quad (4.22)$$

where the third right arrow follows from the fact that

$$\mathbf{f}^H \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} = \check{\mathbf{f}}^H,$$

and the fifth right arrow is based on the fact that $\mathbf{H}$ is of full column rank. Hence, (4.22) contradicts (4.21).

Proposition 4.2 shows that the proposed constraint is equivalent to the optimal group constraint which removes all MAI and ISI on condition that $\mathbf{H}$ is of full column rank. Recalling that the signature matrix $\mathbf{H}$ in asynchronous CDMA model, which is given in (4.1), is assumed to be of full-rank, we naturally expect that the proposed constraint provides the greatest performance gain in the CDMA uplink communication.

As mentioned before, the full-rank condition on $\mathbf{H}$ is hardly satisfied in the CDMA downlink communication, where the intracell users are transmitted synchronously and the intercell users are transmitted asynchronously. In this case, we divide $\mathbf{H}$ into $\mathbf{H}_0$ and $\mathbf{H}_1$ according to (4.1), where $\mathbf{H}_0$ contains the subspace spanned by the desired user and MAI, and $\mathbf{H}_1$ contains the subspace spanned by ISI. In the CDMA downlink, most treatments assume that $\mathbf{H}_0$ is full-rank and independent of $\mathbf{H}_1$, while $\mathbf{H}_1$ is not full-rank when the number of active users K exceeds the maximum length of the multipath channel M . We next analyze the property of the proposed constraint under different conditions of $\mathbf{H}_1$ and $\mathbf{H}_0$.

Proposition 4.3: For non-full-rank $\mathbf{H}$, the constraint $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ implies $\mathbf{w}_1^H \mathbf{H}_1 = \mathbf{0}$ if $\mathbf{H}_0$ is independent of $\mathbf{H}_1$ with full column rank.

Proof: The proof is similar to the proof of Proposition 4.2, given by

$$\begin{aligned}
\mathbf{w}_1^H \bar{\mathbf{R}} &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H \mathbf{H} \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} \mathbf{H}^H &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H [\mathbf{H}_0 & \mathbf{H}_1] \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} [\mathbf{H}_0 & \mathbf{H}_1]^H &= \bar{\mathbf{h}}_1^H \quad (4.23) \\
\Rightarrow \mathbf{w}_1^H \mathbf{H}_0 \mathbf{H}_1^H + \mathbf{w}_1^H \mathbf{H}_1 \mathbf{H}_0^H &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H \mathbf{H}_1 \mathbf{H}_0^H &= \mathbf{0} \\
\Rightarrow \mathbf{w}_1^H \mathbf{H}_1 &= \mathbf{0}
\end{aligned}$$

where the fourth arrow follows from the fact that $\bar{\mathbf{h}}_1 \in \text{range}(\mathbf{H}_1)$ and $\mathbf{H}_0$ is independent of $\mathbf{H}_1$; the last arrow is based on the fact that $\mathbf{H}_0$ is of full column rank.

Proposition 4.3 shows that the proposed constraint can totally remove the ISI of the received signal even with non-full-rank $\mathbf{H}$ for the CDMA downlink, i.e., the interference incurred by $\mathbf{H}_0$ can be entirely cleared by the proposed constraint as long as $\mathbf{H}_0$ is independent of $\mathbf{H}_1$ with full column rank. In particular, if we further assumed that $\bar{\mathbf{h}}_1$ is independent of the other vectors in $\mathbf{H}_1$, better result can be achieved by the proposed constraint. To show this, we have the following proposition:

Proposition 4.4: For non-full-rank $\mathbf{H}$, the constraint $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ implies $\mathbf{w}_1^H \mathbf{H}_0 = \mathbf{1}_{K,1}^T$ if $\mathbf{H}_0$ is independent of $\mathbf{H}_1$ and $\bar{\mathbf{h}}_1$ is independent of all the remaining vectors in $\mathbf{H}_1$.

Proof: Similar to the proof of Proposition 4.3, we consider

$$\begin{aligned}
\mathbf{w}_1^H \bar{\mathbf{R}} &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H [\mathbf{H}_0 \ \mathbf{H}_1] \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix} [\mathbf{H}_0 \ \mathbf{H}_1]^H &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H \mathbf{H}_0 \mathbf{H}_1^H + \mathbf{w}_1^H \mathbf{H}_1 \mathbf{H}_0^H &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H \mathbf{H}_0 \mathbf{H}_1^H &= \bar{\mathbf{h}}_1^H \\
\Rightarrow \mathbf{w}_1^H \mathbf{H}_0 &= \mathbf{1}_{K,1}^T
\end{aligned} \tag{4.24}$$

where the third arrow is derived from the independence assumption between $\mathbf{H}_0$ and $\mathbf{H}_1$, and the last arrow is based on the fact that $\bar{\mathbf{h}}_1$ can not be constructed by the linear combination of the other vectors in $\mathbf{H}_1$. Proposition 4.4 shows that the proposed constraint can totally remove the MAI of interfering users and keep the signal of the desired user if the signature vector of the desired user in ISI $\mathbf{H}_1$ is independent of the other signature vectors in ISI. It is noted that the result of (4.24) is very closed to the expression of the group constraint in (4.8) where $\tilde{\mathbf{H}}$ is replaced by $\mathbf{H}_0$. In this case, we do not require the full-rank of $\mathbf{H}_0$. In fact, if we still assumed that $\mathbf{H}_0$ is of full column rank, the constraint in Proposition 4.4 can eliminate both MAI and ISI according to Proposition 4.3 and Proposition 4.4, which means our proposed constraint may achieve the optimal group constraint on certain condition even without a full-rank $\mathbf{H}$.

The results of Propositions 4.2-4.4 show that the proposed constraint $\mathbf{w}_1^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ is able to remove the MAI and ISI to certain extent according to the properties of

the signature matrix $\mathbf{H}$. We then develop two improved blind linear detectors by considering this constraint.

4.3 Proposed Intersymbol Blind Group Linear Detectors

In this section, we develop two intersymbol blind group linear detectors based on the proposed group constraint.

4.3.1 Proposed Intersymbol Blind Group Linear Decorrelating Detector

The intersymbol blind group linear decorrelating detector considers the minimization of the following constraint cost function

$$\begin{aligned} \mathbf{d}_{opt} = \arg \min_{\mathbf{d}} \|\mathbf{d}^H \mathbf{H}\|^2, \quad \mathbf{d} \in \text{range}(\mathbf{H}) \\ \text{subject to: } \begin{cases} \mathbf{d}^H \mathbf{h}_1 = 1 \\ \mathbf{d}^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H \end{cases} \end{aligned} \quad (4.25)$$

where $\mathbf{d}_{opt}$ is the proposed detector. The first constraint in (4.25) is used to keep the information of the desired user, and the second constraint is used to suppress the MAI and ISI on certain degree based on the property of $\mathbf{H}$ as shown in Propositions 4.2-4.4. The two constraints compose an equivalent group-blind constraint to (4.9). It has been shown in [57] that the improved blind linear decorrelating detector in

(4.25) is equivalent to the decorrelating detector in theory, i.e., $\mathbf{d}_{opt} = \mathbf{d}_1$. Proposition 4.5 presents the resolution of the proposed blind group linear decorrelating detector.

Proposition 4.5: Based on the eigendecomposition in (4.5), the solution of the intersymbol blind group linear decorrelating detector in (4.25) is given by

$$\mathbf{d}_{opt} = \mathbf{U}_s(\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H (\lambda_1 \mathbf{h}_1 + \bar{\mathbf{R}} \lambda_2) \quad (4.26)$$

where

$$\lambda_2 = [\alpha \bar{\mathbf{R}}^H \mathbf{U}_s(\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} - \theta \theta^H]^\dagger [\alpha \bar{\mathbf{h}}_1 - \theta]$$

$$\lambda_1 = \frac{1}{\alpha} [1 - \theta^H \lambda_2]$$

$$\theta = \bar{\mathbf{R}}^H \mathbf{U}_s(\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H \mathbf{h}_1$$

$$\alpha = \mathbf{h}_1^H \mathbf{U}_s(\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H \mathbf{h}_1.$$

Proof: Referring to the method in [57], we use Lagrange multiplier to resolve the problem in Proposition 4.5. For the cost function in (4.25), we have

$$\begin{aligned} \mathbf{d}_{opt} &= \arg \min_{\mathbf{d}} |\mathbf{d}^H \mathbf{H}|^2 + \Re\{\lambda_1^* (\mathbf{h}_1^H \mathbf{d} - 1) + \lambda_2^H (\bar{\mathbf{R}}^H \mathbf{d} - \bar{\mathbf{h}}_1)\} \\ &= (\mathbf{H} \mathbf{H}^H)^\dagger (\lambda_1 \mathbf{h}_1 + \bar{\mathbf{R}} \lambda_2). \end{aligned} \quad (4.27)$$

Combining the constraint $\mathbf{h}_1^H \mathbf{d} = 1$ with (4.27), we obtain

$$\mathbf{h}_1^H (\mathbf{H} \mathbf{H}^H)^\dagger (\lambda_1 \mathbf{h}_1 + \bar{\mathbf{R}} \lambda_2) = 1. \quad (4.28)$$

Hence, it is easy to obtain

$$\lambda_1 = \frac{1}{\mathbf{h}_1^H (\mathbf{H} \mathbf{H}^H)^\dagger \mathbf{h}_1} [1 - \mathbf{h}_1^H (\mathbf{H} \mathbf{H}^H)^\dagger \bar{\mathbf{R}} \lambda_2]. \quad (4.29)$$

Substituting (4.29) into (4.27), we have

$$\mathbf{d}_{opt} = \frac{(\mathbf{H}\mathbf{H}^H)^\dagger}{\mathbf{h}_1^H(\mathbf{H}\mathbf{H}^H)^\dagger\mathbf{h}_1} [\mathbf{h}_1 - \mathbf{h}_1\mathbf{h}_1^H(\mathbf{H}\mathbf{H}^H)^\dagger\bar{\mathbf{R}}\lambda_2 + \mathbf{h}_1^H(\mathbf{H}\mathbf{H}^H)^\dagger\mathbf{h}_1\bar{\mathbf{R}}\lambda_2]. \quad (4.30)$$

Combining the constraint $\mathbf{d}_{opt}^H\bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H$ with (4.30) and denoting $\alpha \triangleq \mathbf{h}_1^H(\mathbf{H}\mathbf{H}^H)^\dagger\mathbf{h}_1$,

we obtain

$$\lambda_2 = [\alpha\bar{\mathbf{R}}^H(\mathbf{H}\mathbf{H}^H)^\dagger\bar{\mathbf{R}} - \bar{\mathbf{R}}^H(\mathbf{H}\mathbf{H}^H)^\dagger\mathbf{h}_1\mathbf{h}_1^H(\mathbf{H}\mathbf{H}^H)^\dagger\bar{\mathbf{R}}]^\dagger[\alpha\bar{\mathbf{h}}_1 - \bar{\mathbf{R}}^H(\mathbf{H}\mathbf{H}^H)^\dagger\mathbf{h}_1]. \quad (4.31)$$

According to the eigendecomposition in (4.5), the matrix $(\mathbf{H}\mathbf{H}^H)^\dagger$ can be replaced by $\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H$. We then rewrite α and λ_2 as

$$\begin{aligned} \alpha &= \mathbf{h}_1^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\mathbf{h}_1 \\ \lambda_2 &= [\alpha\bar{\mathbf{R}}^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\bar{\mathbf{R}} \\ &\quad - \bar{\mathbf{R}}^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\mathbf{h}_1\mathbf{h}_1^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\bar{\mathbf{R}}]^\dagger \\ &\quad \cdot [\alpha\bar{\mathbf{h}}_1 - \bar{\mathbf{R}}^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\mathbf{h}_1] \\ &= [\alpha\bar{\mathbf{R}}^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\bar{\mathbf{R}} - \theta\theta^H]^\dagger[\alpha\bar{\mathbf{h}}_1 - \theta], \end{aligned} \quad (4.32)$$

where θ is denoted as $\theta \triangleq \bar{\mathbf{R}}^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\mathbf{h}_1$. Substituting α and θ into (4.29), it is easy to obtain

$$\begin{aligned} \lambda_1 &= \frac{1}{\alpha}[1 - \mathbf{h}_1^H\mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H\bar{\mathbf{R}}\lambda_2] \\ &= \frac{1}{\alpha}[1 - \theta^H\lambda_2]. \end{aligned} \quad (4.33)$$

Finally, the proposed detector is given by

$$\mathbf{d}_{opt} = \mathbf{U}_s(\Lambda_s - \sigma^2\mathbf{I}_L)^{-1}\mathbf{U}_s^H(\lambda_1\mathbf{h}_1 + \bar{\mathbf{R}}\lambda_2). \quad (4.34)$$

4.3.2 Proposed Intersymbol Blind Group Linear Hybrid Detector

The intersymbol blind group linear hybrid detector considers the following constrained optimization problem

$$\begin{aligned} \mathbf{m}_{opt} = & \arg \min_{\mathbf{m}} E\{|b_1(n) - \mathbf{m}^H \mathbf{r}_1(n)|^2\}, \quad \mathbf{m} \in \text{range}(\mathbf{H}) \\ \text{subject to: } & \begin{cases} \mathbf{m}^H \mathbf{h}_1 = 1 \\ \mathbf{m}^H \bar{\mathbf{R}} = \bar{\mathbf{h}}_1^H \end{cases} \end{aligned} \quad (4.35)$$

where $\mathbf{m}_{opt}$ is the proposed detector. Proposition 4.6 gives the implementation for the proposed blind linear hybrid detector.

Proposition 4.6: Based on the eigendecomposition in (4.5), the solution of the improved blind linear hybrid detector in (4.35) is given by

$$\mathbf{m}_{opt} = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H (\bar{\lambda}_1 \mathbf{h}_1 + \bar{\mathbf{R}} \bar{\lambda}_2) \quad (4.36)$$

where

$$\bar{\lambda}_2 = [\bar{\alpha} \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} - \bar{\theta} \bar{\theta}^H]^\dagger [\bar{\alpha} \bar{\mathbf{h}}_1 - \bar{\theta}]$$

$$\bar{\lambda}_1 = \frac{1}{\bar{\alpha}} [1 - \bar{\theta}^H \bar{\lambda}_2]$$

$$\bar{\theta} = \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1$$

$$\bar{\alpha} = \mathbf{h}_1^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1.$$

Proof: Similar to the proof of Proposition 4.5, we still use Lagrange multiplier

to resolve the problem in Proposition 4.6. For the cost function in (4.35), we have

$$\begin{aligned} \mathbf{m}_{opt} &= \arg \min_{\mathbf{m}} E\{|b_1(n) - \mathbf{m}^H \mathbf{r}_1(n)|^2\} + \Re\{\lambda_1^*(\mathbf{h}_1^H \mathbf{m} - 1) + \bar{\lambda}_2^H(\bar{\mathbf{R}}^H \mathbf{m} - \bar{\mathbf{h}}_1)\} \\ &= \arg \min_{\mathbf{m}} \mathbf{m}^H \mathbf{R} \mathbf{m} - 2\Re\{\mathbf{h}_1^H \mathbf{m}\} + \Re\{\lambda_1^*(\mathbf{h}_1^H \mathbf{m} - 1) + \bar{\lambda}_2^H(\bar{\mathbf{R}}^H \mathbf{m} - \bar{\mathbf{h}}_1)\}. \end{aligned} \quad (4.37)$$

Denoting $\bar{\lambda}_1 \triangleq \lambda_1 - 2$ and substituting $\bar{\lambda}_1$ into (4.37), we obtain

$$\begin{aligned} \mathbf{m}_{opt} &= \arg \min_{\mathbf{m}} \mathbf{m}^H \mathbf{R} \mathbf{m} + \Re\{\bar{\lambda}_1^*(\mathbf{h}_1^H \mathbf{m} - 1) + \bar{\lambda}_2^H(\bar{\mathbf{R}}^H \mathbf{m} - \bar{\mathbf{h}}_1)\} \\ &= \mathbf{R}^{-1}(\bar{\lambda}_1 \mathbf{h}_1 + \bar{\mathbf{R}} \bar{\lambda}_2). \end{aligned} \quad (4.38)$$

Referring to the methods in (4.29)-(4.31) and denoting $\bar{\alpha} \triangleq \mathbf{h}_1^H \mathbf{R}^{-1} \mathbf{h}_1$, we have

$$\begin{aligned} \bar{\lambda}_1 &= \frac{1}{\bar{\alpha}}[1 - \mathbf{h}_1^H \mathbf{R}^{-1} \bar{\mathbf{R}} \bar{\lambda}_2] \\ \bar{\lambda}_2 &= [\bar{\alpha} \bar{\mathbf{R}}^H \mathbf{R}^{-1} \bar{\mathbf{R}} - \bar{\mathbf{R}}^H \mathbf{R}^{-1} \mathbf{h}_1 \mathbf{h}_1^H \mathbf{R}^{-1} \bar{\mathbf{R}}]^\dagger [\bar{\alpha} \bar{\mathbf{h}}_1 - \bar{\mathbf{R}}^H \mathbf{R}^{-1} \mathbf{h}_1]. \end{aligned} \quad (4.39)$$

It is seen from (4.10) and (4.17) that $\mathbf{h}_1$ and $\bar{\mathbf{R}} \in \text{range}(\mathbf{U}_s)$, i.e., $\mathbf{h}_1^H \mathbf{U}_n = \mathbf{0}$ and $\bar{\mathbf{R}}^H \mathbf{U}_n = \mathbf{0}$. Thus, by denoting $\bar{\theta} \triangleq \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1$, we can rewrite $\bar{\alpha}$, $\bar{\lambda}_1$ and $\bar{\lambda}_2$ in the following subspace forms

$$\begin{aligned} \bar{\alpha} &= \mathbf{h}_1^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1 \\ \bar{\lambda}_2 &= [\bar{\alpha} \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} - \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1 \mathbf{h}_1^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \bar{\mathbf{R}}]^\dagger \\ &\quad \cdot [\bar{\alpha} \bar{\mathbf{h}}_1 - \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1] \\ &= [\bar{\alpha} \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} - \bar{\theta} \bar{\theta}^H]^\dagger [\bar{\alpha} \bar{\mathbf{h}}_1 - \bar{\theta}] \\ \bar{\lambda}_1 &= \frac{1}{\bar{\alpha}}[1 - \mathbf{h}_1^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} \bar{\lambda}_2] = \frac{1}{\bar{\alpha}}[1 - \bar{\theta}^H \bar{\lambda}_2]. \end{aligned} \quad (4.40)$$

Finally, the proposed detector can be given by

$$\mathbf{m}_{opt} = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H (\bar{\lambda}_1 \mathbf{h}_1 + \bar{\mathbf{R}} \bar{\lambda}_2). \quad (4.41)$$

In the above results, two intersymbol blind group linear detectors are expressed in subspace forms. In these detectors, the influence of the interfering users are mitigated by the proposed constraints, which are constructed from the cross correlation matrix $\bar{\mathbf{R}}$ and the channel estimation of the desired user. After suppressing the interfering signals, the signals of the desired user are identified by the transformation of the subspace based on the zero-forcing or the hybrid criteria. Unlike the group constraint in [57], our proposed constraints utilize the cross correlation matrix $\bar{\mathbf{R}}$ to improve the performance of the detectors and then avoid the channel estimation of other users except the desired user. Therefore, our proposed detectors can be implemented blindly for both downlink and uplink CDMA channels, while the conventional blind group methods are only suitable for uplink communication channels and require the channel estimation of all known intracell users. In addition, the performance gain provided by the proposed constraint depends on the property of the signature matrix $\mathbf{H}$. When $\mathbf{H}$ is of full column rank (uplink-CDMA), the proposed constraint can totally remove MAI and ISI to achieve the best performance gain. When $\mathbf{H}$ is not of full rank (downlink-CDMA), the proposed constraint can at least remove the influence of ISI. Table 4.1 summarizes the algorithms of the proposed detectors. Similar to the detectors in Chapter 3, the detectors in Table 4.1 still use differential detection to avoid the problem of phase ambiguity.

Table 4.1: Algorithms of Intersymbol Blind Group Linear Decorrelating detector and Blind Group Linear Hybrid Detector

Signal Frame: $\mathbf{r}[0], \mathbf{r}[1], \dots, \mathbf{r}[P]$
STEP 1. Compute autocorrelation and cross correlation $\mathbf{R} = \frac{1}{P+1} \sum_{i=0}^P \mathbf{r}[i] \mathbf{r}[i]^H$ $\bar{\mathbf{R}} = \frac{1}{P} (\sum_{i=1}^P \mathbf{r}[i] \mathbf{r}[i-1]^H + \sum_{i=1}^P \mathbf{r}[i-1] \mathbf{r}[i]^H)$
STEP 2. Compute eigendecomposition of $\mathbf{R}$ $\mathbf{R} = \mathbf{U}_s \Lambda_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H$
STEP 3. Parameter estimation $\sigma^2 = \frac{1}{N-L} (\text{Sum of the mini } N-L \text{ eigenvalues of } \mathbf{R})$ $\mathbf{g}_1 = \text{Max-eigenvector } (\bar{\mathbf{C}}_{1,up}^H \mathbf{U}_s \mathbf{U}_s^H \bar{\mathbf{C}}_{1,up})$ $\mathbf{h}_1 = \bar{\mathbf{C}}_{1,up} \mathbf{g}_1$ $\bar{\mathbf{h}}_1 = \bar{\mathbf{C}}_{1k,dn} \mathbf{g}_1$
STEP 4. Compute $\alpha, \bar{\alpha}, \theta, \bar{\theta}, \lambda_1, \bar{\lambda}_1, \lambda_2, \bar{\lambda}_2$ $\alpha = \mathbf{h}_1^H \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H \mathbf{h}_1; \bar{\alpha} = \mathbf{h}_1^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1$ $\theta = \bar{\mathbf{R}}^H \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H \mathbf{h}_1; \bar{\theta} = \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1$ $\lambda_2 = [\alpha \bar{\mathbf{R}}^H \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} - \theta \theta^H]^\dagger [\alpha \mathbf{h}_1 - \theta]$ $\bar{\lambda}_2 = [\bar{\alpha} \bar{\mathbf{R}}^H \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \bar{\mathbf{R}} - \bar{\theta} \bar{\theta}^H]^\dagger [\bar{\alpha} \bar{\mathbf{h}}_1 - \bar{\theta}]$ $\lambda_1 = \frac{1}{\alpha} [1 - \theta^H \lambda_2]; \bar{\lambda}_1 = \frac{1}{\bar{\alpha}} [1 - \bar{\theta}^H \bar{\lambda}_2]$
STEP 5. Form decorrelating detector and hybrid detector $\mathbf{d}_{opt} = \mathbf{U}_s (\Lambda_s - \sigma^2 \mathbf{I}_L)^{-1} \mathbf{U}_s^H (\lambda_1 \mathbf{h}_1 + \bar{\mathbf{R}} \lambda_2)$ $\mathbf{m}_{opt} = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H (\bar{\lambda}_1 \mathbf{h}_1 + \bar{\mathbf{R}} \bar{\lambda}_2)$

4.4 Simulation Results

In this section, simulation results are provided to demonstrate the performance of the proposed intersymbol blind group linear detectors. Similar to the previous chapter, we test the proposed methods in a CDMA system with spreading gain $N = 31$. The spreading sequences for all users are generated by Gold codes. Both the asynchronous and synchronous cases are considered. In each case, User 1 is assumed to be the desired user which is synchronized, and each user has $M = 5$ multipath delays. The multipath gains in each user's channel are randomly chosen and kept fixed, which have been normalized with equal power. In addition, we

simulate a severe near-far case in which the power of each interfering user is 10 dB more than that of the desired user. In each simulation, an eigendecomposition is performed on the autocorrelation matrix of the received signals. The length of the signal frame is $P = 500$. The algorithmic details of the matrix operations and parameter estimation involved in computing the various detectors can be seen from the Table 4.1.

Figure 4.1: Bit-error-rate comparison of different detectors for asynchronous CDMA ($P = 500$, $K = 7$).

4.4.1 Performance of the Proposed Detectors for Asynchronous CDMA Systems

We consider the signals of the CDMA uplink communication as the asynchronous systems, where the received signal at the base station is created by the asynchronous

intercell and intracell users. For an asynchronous CDMA system, we assume that the delay of each interfering user is randomly distributed within $[0, 26T_c]$ where T_c is the chip period. Hence, the maximum delay spread is limited within one symbol interval. Meanwhile, it is also assumed that the signature matrix $\mathbf{H}$ is of full column rank as the existence of the delays of interfering users. We compare the performance of six detectors, i.e., the conventional blind decorrelating and MMSE detectors implemented by the algorithms in Table 2.2, the group-blind zero-forcing and hybrid detectors in [57], and the proposed improved blind linear decorrelating and hybrid detectors implemented by the algorithm in Table 4.1.

Figure 4.1 compares the BERs of various detectors, which only require to estimate the channel of the desired user, in an asynchronous system with 7 users. It shows that the proposed blind linear decorrelating and hybrid detectors provide substantial BER improvement over the conventional decorrelating and MMSE detectors. It is also observed that there exists certain differences in performance between the proposed decorrelating detector and hybrid detector. This is because the two detectors use the same constraint to suppress the ISI and MAI. When $\mathbf{H}$ is of full rank, the proposed constraint totally removes the ISI and MAI. Hence, both proposed detectors can zeroforce all signals from interfering users to achieve similar performance.

We next compare the performance of the group-blind detectors in [57] with our proposed detectors. The results of the simulation are shown in Figure 4.2 and Figure 4.3. In Figure 4.2, a ten-user asynchronous system is considered. It is assumed

Figure 4.2: Bit-error-rate comparison of different detectors for asynchronous CDMA ($P = 500$, $K = 10$).

that there are eight known intracell users and two unknown intercell users for the group-blind detectors. It is seen that the group-blind zero-forcing detector is a little superior to the proposed decorrelating detector. The group-blind hybrid detector also achieves a better performance than the proposed hybrid detector. However, it should be pointed out that our proposed detectors only need the channel estimation of the desired user, while the group-blind detectors require the channel estimation of all intracell users.

Figure 4.3 considers a fifteen-user system with eight intracell known users and seven intercell unknown users. Such situation may occur at the edge of the cell, where the number of the intracell users is close to the number of the intercell users.

It is observed that both the group-blind detectors and the proposed detectors have a deterioration compared with Figure 4.2. In this case, the performance of the proposed decorrelating detector and the hybrid detector is close to that of the group-blind zero-forcing detector and the group-blind hybrid detector, respectively. It means that the group-blind detectors are more sensitive to the number of the unknown users. This is because the group-blind detectors use the channel estimation of all known users to construct the group constraint to improve the performance. Hence, the more users the detectors know, the better performance gain the detectors achieve. However, our proposed detectors utilize the cross correlation of the received symbols and the channel estimation of the desired user to construct the corresponding constraint. Such constraint, which can remove both MAI and ISI in asynchronous CDMA communication, is irrelevant to the number of unknown users.

4.4.2 Performance of the Proposed Detectors for Synchronous CDMA Systems

We next consider the performance of the proposed detectors in the CDMA downlink, i.e. synchronous CDMA. In downlink-CDMA communication, the signal received by the desired user is made up of the synchronous intracell users and some asynchronous intercell users. In this case, the signature matrix $\mathbf{H}_1$, and hence the signature matrix $\mathbf{H}$ may not be of full rank when the number of synchronous intracell users surpasses the multipath length M .

It is noted from the above simulation results that the performance of the con-

Figure 4.3: Bit-error-rate comparison of different detectors for asynchronous CDMA ($P = 500$, $K = 15$).

ventional blind MMSE detector and the proposed hybrid detector are close to the performance of the conventional blind decorrelating detector and the proposed decorrelating detector, respectively. However, the two decorrelating detectors require the estimate of the noise level. In addition, the group-blind detectors in [57] cannot be used for downlink-CDMA because they need to know the spreading sequences of the other intracell users. Therefore, we only compare three detectors for synchronous CDMA communication, i.e., the conventional MMSE detector, the proposed hybrid detector and the true linear MMSE detector, where all these detectors do not require to estimate the variance of the background noise σ^2 . The true linear MMSE

detector is given by

$$\mathbf{m}_{true} = \mathbf{R}^{-1}\mathbf{h}_1. \quad (4.42)$$

In order to guarantee the accuracy of the true MMSE detector, the correlation matrix $\mathbf{R}$ in (4.42) is created by the average of 10000 received samples at a high signal-to-noise ratio.

Figure 4.4: Bit-error-rate comparison of different detectors for synchronous CDMA ($P = 500$, $K = 10$).

Figure 4.4 compares the performance of the above three detectors. The system is assumed to have eight synchronous intracell users and two asynchronous intercell users. It is seen from this figure that the proposed hybrid detector still achieves a better performance than that of the conventional MMSE detector, although both

Figure 4.5: Bit-error-rate comparison of different detectors for synchronous CDMA ($P = 500$, $K = 15$).

are inferior to the true linear MMSE detector. However, the performance gain of the proposed hybrid detector over the convention MMSE detector is not as obvious as the case in Figure 4.1. This is because the constraint in the proposed hybrid detector can only remove the ISI for synchronous CDMA, while it can remove both MAI and ISI for asynchronous CDMA.

In Figure 4.5, the system has seven asynchronous intercell users and eight synchronous intracell users. In this case, the signature matrix $\mathbf{H}_1$ has more column rank since the number of the asynchronous users is increased. It implies that the proposed constraint can remove more MAI. It is observed as expected that the proposed hybrid detector provides more performance gain over the conventional MMSE de-

tector compared with the result of Figure 4.4, and is closer to the true linear MMSE detector.

Theoretically, both the conventional detectors and the proposed detectors converge to the true linear MMSE detector at high signal-to-noise ratio when the signal frame length P is infinite. However, for a finite frame length P , the proposed detectors significantly outperform the conventional detectors no matter in the CDMA uplink or in the CDMA downlink. The reason for such a performance improvement is that more multiuser environment information is incorporated in forming the proposed detectors.

4.5 Conclusions

In this chapter, new intersymbol blind group linear multiuser detection techniques are proposed for both downlink of the synchronous CDMA system and uplink of the asynchronous CDMA systems. The new techniques make use of an important cross correlation matrix between the adjacent symbols as a constraint to suppress the influence from the interfering users. Two intersymbol blind group linear multiuser detectors are developed based on different optimal criteria with the constraint constructed by the cross correlation matrix.

Similar to the detector in Chapter 3, the proposed detectors can be implemented blindly only with the channel estimation of the desired user, while the implementation of the proposed methods in this chapter is more complicate than that of Chapter 3 as the introduction of the proposed group constraint. Simulation results show that

the proposed improved blind linear detectors provide a substantial performance gain over the conventional blind detectors.

It is noted that the proposed methods in Chapter 3 and Chapter 4 are based on the DS-CDMA signal model. As a matter of fact, the intersymbol detection can be exploited for MC-CDMA systems as well. In the next chapter, the intersymbol blind multiuser detection for MC-CDMA systems is proposed. In addition, a new recursive subspace tracking algorithm for Hamiltonian matrix matrices is also presented in Chapter 5 for the blind adaptive implementation of the proposed detector.

Chapter 5

Intersymbol Blind Multiuser

Detection for MC-CDMA Systems

5.1 Introduction

In this chapter, we present the intersymbol blind multiuser detection for asynchronous multicarrier code-division multiple-access (MC-CDMA) systems with Rayleigh fading channels. Referring to the previous chapters, our method is still based on the utilization of the cross correlation matrix between the consecutively received signals. In order to implement the proposed detector blind adaptively, a new recursive subspace tracking algorithm for any Hamiltonian matrices is developed in this chapter, while the previously reported subspace tracking methods can only estimate the subspace of positive Hamiltonian matrix. Although our proposed subspace tracking algorithm is developed for the detection of the MC-CDMA systems, it can be easily

applied to the previously proposed intersymbol blind multiuser detectors in Chapter 3 and Chapter 4.

MC-CDMA systems are gaining more popularity recently [68, 69]. The MC-CDMA basically is the combination of orthogonal frequency division multiplexing (OFDM) and CDMA [70, 71, 72]. The main advantage of OFDM is that the chip duration can be significantly increased to reduce selective fading. In MC-CDMA systems, each user is transmitted simultaneously over multiple subcarriers without any spreading per subcarrier. With the narrow band of the subcarriers, the frequency selective fading can be reduced to the limit so that the signal associated with each subcarrier behaves like one with flat fading. The MC-CDMA systems have also been considered as a promising candidate for future wideband wireless communications, and various multiuser detections for MC-CDMA systems were proposed in the literature. For example, several decorrelating detectors for synchronous MC-CDMA systems were developed in [73, 74]. In [75], the receivers for asynchronous MC-CDMA systems using equal-gain combining (EGC) and maximum-ratio combining (MRC) were reported. In addition, the performance of the MRC and postdetection EGC algorithms over generalized gamma fading channels is analyzed in [76]. However, a perfect estimation of the timing and the fading channel of the desired user was required for the proposed detectors in [75] and [76].

In this chapter, Section 5.2 presents the signal model of the asynchronous MC-CDMA system. Section 5.3 develops the proposed decorrelating detector. The detector is analogous to the previous proposed intersymbol detector for asynchronous

DS-CDMA systems, which is based on the key idea of using a cross correlation matrix constructed by the consecutively received signals. It is shown that our method does not require channel estimation, which leads to a simpler implementation. In Section 5.4, a new subspace tracking algorithm is developed to obtain the blind adaptive implementation of the proposed detector. It will be shown that the main task of the proposed detector is the subspace tracking of the cross correlation matrix which is not positive definite. However, the existing subspace tracking methods require the estimated signal space to be positive [77, 78, 79, 80]. Enlightened from the PASTD algorithm in [81, 82], a new algorithm is developed in this chapter to track the subspace of any Hamiltonian matrices for the implementation of the proposed detector. It is shown that the corresponding implemented detector is equivalent to the decorrelating detector up to an ambiguous complex scaling factor. Since the phase of the scaling factor may substantially deteriorate the performance of the detector, an adaptive algorithm is developed to compensate the phase of the factor according to the property of the background noise. In Section 5.5, simulation results are provided to demonstrate the performance of the proposed detector. Section 5.6 contains the conclusion.

5.2 Signal Model

Let us consider an asynchronous MC-CDMA system with K simultaneous users and N subcarriers in a frequency selective Rayleigh fading channel. The system uses the Gold codes of length N followed by an OFDM modulator to construct the spreading

codes of different users. The transmitter block diagram of the asynchronous MC-CDMA system for User k is shown in Figure 5.1. In this figure, the transmitter spreads the i th data symbol of the k th user, i.e., $b_k[i] \in \{1, -1\}$, in the frequency domain by using the k th user's spreading code $\{c_{k,n}|_{n=0,1,\dots,N-1}\}$. The transmitted signal for the k th user can be written as

$$s_k(t) = a_k \sum_{i=-\infty}^{\infty} b_k[i] \sum_{n=0}^{N-1} c_{k,n} P(t - iT_s - \tau_k) e^{j2\pi n(t-\tau_k)/T_s} \quad (5.1)$$

where a_k is the amplitude of the k th user, τ_k , the delay of the k th user, is a random variable and uniformly distributed in $[0, T_s]$, $P(t)$ is the rectangular pulse waveform uniformly normalized in the interval $[0, T_s]$ and T_s is the symbol duration.

After passing through the frequency selective fading channel, $s_k(t)$ is affected by multipath fading and additive white Gaussian noise. By suitably choosing N and $P(t)$ [83], it can be assumed that each subcarrier undergoes independent frequency nonselective slow Rayleigh fading. Without loss of generality, the phase of channel response is assumed to be tracked by the receiver and hence becomes zero. In CDMA systems, phase tracking of channel response is required by each mobile user. The reference of the tracking is provided by the specific pilot signaling channel. Hence, it is easy for each receiver to obtain the phase of the channel response. After the phase tracking, the received signal of all active users is given by

$$y(t) = \sum_{k=1}^K a_k \sum_{i=-\infty}^{\infty} b_k[i] \sum_{n=0}^{N-1} c_{k,n} \beta_{k,n} P(t - iT_s - \tau_k) e^{j2\pi n(t-\tau_k)/T_s} + z(t) \quad (5.2)$$

where $z(t)$ is the complex AWGN process with zero mean and variance σ^2 , $\beta_{k,n}$ is the amplitude of multipath channel's fading coefficient with a Rayleigh distribution

for the k th user on the n th subcarrier.

Figure 5.1: Transmitter block diagram for the k th user of the asynchronous MC-CDMA system.

Figure 5.2: Receiver block diagram of the asynchronous MC-CDMA system.

At the receiver end, the received signal in (5.2) is correlated with each subcarrier frequency as shown in Figure 5.2. After sampling the output of each subcarrier

correlator at the symbol rate, the output of the n th correlator for the i th symbol is given by

$$\begin{aligned} r_n[i] &= \frac{1}{\sqrt{T_s}} \int_{iT_s}^{(i+1)T_s} y(t) e^{-j2\pi nt/T_s} dt \\ &= \sum_{k=1}^K [h_{k,n} b_k[i] + \bar{h}_{k,n} b_k[i-1]] + \omega_n[i] \end{aligned} \quad (5.3)$$

where

$$h_{k,n} = \frac{1}{\sqrt{T_s}} \sum_{l=0}^{N-1} a_k c_{k,l} \beta_{k,l} e^{-j2\pi l \tau_k / T_s} \int_{\tau_k}^{T_s} e^{j2\pi(l-n)t/T_s} dt \quad (5.4)$$

$$\bar{h}_{k,n} = \frac{1}{\sqrt{T_s}} \sum_{l=0}^{N-1} a_k c_{k,l} \beta_{k,l} e^{-j2\pi l \tau_k / T_s} \int_0^{\tau_k} e^{j2\pi(l-n)t/T_s} dt \quad (5.5)$$

$$\omega_n[i] = \frac{1}{\sqrt{T_s}} \int_{iT_s}^{(i+1)T_s} z(t) e^{-j2\pi nt/T_s} dt \quad (5.6)$$

for $n = 0, 1, \dots, N-1$.

Without loss of generality, it is still assumed that the desired user is User 1, which has been synchronized to the receiver, i.e., $\tau_1 = 0$. According to (5.5), it is easy to prove

$$\bar{h}_{1,n} = 0, \quad n = 0, 1, \dots, N-1 \quad (5.7)$$

which implies that the ISI of the desired user is completely removed by synchronization operation, while the ISI and MAI of other users still exist because of their time-shifted replicas. By using (5.4)-(5.6) and stacking N outputs of the subcarrier correlator into a vector as following

$$\mathbf{r}[i] \triangleq [r_0[i], r_1[i], \dots, r_{N-1}[i]]^T, \quad (5.8)$$

the final output of the receiver can be given by

$$\begin{aligned}\mathbf{r}[i] &= \mathbf{h}_1 b_1[i] + \sum_{k=2}^K [\mathbf{h}_k b_k[i] + \bar{\mathbf{h}}_k b_k[i-1]] + \boldsymbol{\omega}(i) \\ &= \mathbf{h}_1 b_1[i] + \mathbf{H}_I \mathbf{b}_I[i] + \boldsymbol{\omega}[i]\end{aligned}\tag{5.9}$$

where

$$\begin{aligned}\mathbf{h}_k &\triangleq [h_{k,0}, h_{k,1}, \dots, h_{k,N-1}]^T \\ \bar{\mathbf{h}}_k &\triangleq [\bar{h}_{k,0}, \bar{h}_{k,1}, \dots, \bar{h}_{k,N-1}]^T \\ \mathbf{H}_I &\triangleq [\mathbf{h}_2, \mathbf{h}_3, \dots, \mathbf{h}_K, \bar{\mathbf{h}}_2, \bar{\mathbf{h}}_3, \dots, \bar{\mathbf{h}}_K] \\ \mathbf{b}_I[i] &\triangleq [b_2[i], b_3[i], \dots, b_K[i], b_2[i-1], b_3[i-1], \dots, b_K[i-1]]^T \\ \boldsymbol{\omega}[i] &\triangleq [\omega_0[i], \omega_1[i], \dots, \omega_{N-1}[i]]^T.\end{aligned}$$

In (5.9), $\mathbf{h}_1$ and $b_1[i]$ contain the information of the desired user. The MAI and ISI of the interfering users are represented by $\mathbf{H}_I$ and $\mathbf{b}_I[i]$. It is easy to observe from (5.6) that $\boldsymbol{\omega}[i]$ is a complex white Gaussian noise vector with zero means and an $N \times N$ covariance matrix

$$\text{Var}_{\boldsymbol{\omega}[i]} = \begin{bmatrix} \frac{\sigma^2}{\sqrt{T_s}} & 0 & \cdots & 0 \\ 0 & \frac{\sigma^2}{\sqrt{T_s}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\sigma^2}{\sqrt{T_s}} \end{bmatrix}.\tag{5.10}$$

5.3 Proposed Blind Intersymbol Decorrelating Detector

As mentioned in the previous chapters, we consider the blind symbol-by-symbol detection for the desired user. The canonical form of a linear detector for the desired

user can be represented by a correlating vector $\mathbf{w}_1 \in \mathfrak{R}^N$, which is followed by the received signal $\mathbf{r}[i]$, such that the detection of User 1 is

$$\hat{b}_1[i] = \text{sgn}\{\Re(\mathbf{w}_1^H \mathbf{r}[i])\} \quad (5.11)$$

For the two special cases of $\mathbf{w}_1 = \mathbf{c}_1$, where $\mathbf{c}_1 = [c_{1,0}, c_{1,1}, \dots, c_{1,N-1}]^T$, and $\mathbf{w}_1 = [c_{1,0}\beta_{1,0}, c_{1,1}\beta_{1,1}, \dots, c_{1,N-1}\beta_{1,N-1}]^T$, the detector will become EGC and MRC, respectively.

5.3.1 Conventional Linear Detectors

A. Linear Decorrelating Detector

Recalling that the decorrelating detector is designed to completely eliminate the MAI and ISI from all interfering users at the expense of enhancing the ambient noise, it has the form of (5.11) with the weight vector $\mathbf{d}_1 = \mathbf{w}_1$, which is the unique vector as follows:

$$\begin{aligned} \mathbf{d}_1^H \mathbf{h}_1 &= 1 \\ \mathbf{d}_1^H \mathbf{H}_I &= \mathbf{0}. \end{aligned} \quad (5.12)$$

We now consider the autocorrelation matrix of the received signal in (5.9), which is given by

$$\mathbf{R} = E\{\mathbf{r}[i]\mathbf{r}(i)^H\} = [\mathbf{h}_1, \mathbf{H}_I][\mathbf{h}_1, \mathbf{H}_I]^H + \frac{\sigma^2}{\sqrt{T_s}}\mathbf{I}_N. \quad (5.13)$$

By performing the eigendecomposition of $\mathbf{R}$, we obtain

$$\mathbf{R} = [\mathbf{U}_s, \mathbf{U}_n] \begin{bmatrix} \Lambda_s & \mathbf{0} \\ \mathbf{0} & \frac{\sigma^2}{\sqrt{T_s}}\mathbf{I}_{N-2K+1} \end{bmatrix} [\mathbf{U}_s, \mathbf{U}_n]^H \quad (5.14)$$

where $\mathbf{U}_s$ is an $N \times (2K - 1)$ dimensional orthonormal basis of the signal subspace spanned by $[\mathbf{h}_1, \mathbf{H}_I]$ and $\mathbf{U}_n$ is the basis for the white noise subspace orthogonal to $\mathbf{U}_s$. The diagonal matrix Λ_s contains the $2K - 1$ largest eigenvalues of $\mathbf{R}$ and the remaining minimum eigenvalues have the same value of $\frac{\sigma^2}{\sqrt{T_s}}$. As we have mentioned many times in the previous chapters, the canonical expression of the decorrelating detector in subspace form can be given by

$$\mathbf{d}_1 = \mathbf{U}_s (\Lambda_s - \frac{\sigma^2}{\sqrt{T_s}} \mathbf{I}_{2K-1})^{-1} \mathbf{U}_s^H \mathbf{h}_1, \quad (5.15)$$

where the decorrelating detector for MC-CDMA system is very close to the decorrelating detector for DS-CDMA system.

B. Linear MMSE Detector

Similarly, the linear MMSE detector has the form of (5.11) with the weight vector $\mathbf{m}_1 = \mathbf{w}_1$, obtained by calculating the minimum output mean-squared error, i.e.,

$$\mathbf{m}_1 \triangleq \arg \min_{\mathbf{m}_1} E\{|b_1[i] - \mathbf{m}_1^H \mathbf{r}[i]|^2\}. \quad (5.16)$$

In terms of the signal subspace components ($\mathbf{U}_s$ and Λ_s), the linear MMSE detector can be expressed as

$$\mathbf{m}_1 = \mathbf{U}_s \Lambda_s^{-1} \mathbf{U}_s^H \mathbf{h}_1. \quad (5.17)$$

It should be pointed out that for the implementation of the subspace based detectors in (5.15) and (5.17), the channel estimation of the desired user $\mathbf{h}_1$ is required.

C. Decorrelating RAKE Detector

The decorrelating RAKE (D-RAKE) detector is a two-stage blind adaptive detector with a comprising of a set of adaptive weight vectors at stage 1 and a coherent combiner at stage 2 [84, 85]. The D-RAKE detector for MC-CDMA requires the oversampling of the received signal. Assuming each subcarrier is sampled at L times of the symbol-rate, the received signal $\mathbf{r}[i]$ in (5.9) becomes an $NL \times 1$ vector, and $\mathbf{R}$ in (5.13) becomes an $NL \times NL$ matrix. Then, the D-RAKE detector has the form of

$$\mathbf{d}_{RAKE} = \mathbf{R}^{-1} \mathbf{C} (\mathbf{C} \mathbf{R}^{-1} \mathbf{C}^H)^{-1} \mathbf{p} \quad (5.18)$$

where

$$\begin{aligned} \mathbf{C} &= (\text{diag}[c_{1,0}, c_{1,1}, \dots, c_{1,N-1}]) \otimes \mathbf{1}_{L \times 1}, \\ \mathbf{p} &= [\frac{1}{\sqrt{T_s}} a_1 \beta_{1,0}, \frac{1}{\sqrt{T_s}} a_1 \beta_{1,1}, \dots, \frac{1}{\sqrt{T_s}} a_1 \beta_{1,N-1}]^T \end{aligned} \quad (5.19)$$

The D-RAKE detector has a lower complexity compared with the subspace decorrelating and MMSE detectors because it can be implemented via standard LMS algorithm without the channel estimation of the desired user. However, its performance is always inferior to the MMSE detector [85].

5.3.2 Proposed Blind Intersymbol Decorrelating Detector

The blind intersymbol decorrelating detector developed in this chapter is based on the intersymbol method presented in Chapter 3. We first construct a cross correlation matrix $\bar{\mathbf{R}}$, which is obtained from the cross correlation of the consecutively

received signals, i.e.,

$$\begin{aligned}\bar{\mathbf{R}} &= E\{\mathbf{r}[i]\mathbf{r}[i-1]^H\} + E\{\mathbf{r}[i-1]\mathbf{r}[i]^H\} \\ &= \mathbf{H}_I \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \mathbf{H}_I^H.\end{aligned}\quad (5.20)$$

where $\bar{\mathbf{R}}$ contains only the subspace spanned by the MAI and ISI of all interfering users and does not contain information of the desired user and the noise subspace.

By performing the eigendecomposition of $\bar{\mathbf{R}}$, we obtain

$$\begin{aligned}\bar{\mathbf{R}} &= [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n] \begin{bmatrix} \bar{\Lambda}_s & \\ & \mathbf{0} \end{bmatrix} [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]^H \\ &= \bar{\mathbf{U}}_s \bar{\Lambda}_s \bar{\mathbf{U}}_s^H,\end{aligned}\quad (5.21)$$

where $\bar{\Lambda}_s = \text{diag}(\bar{\lambda}_1, \dots, \bar{\lambda}_{2K-2})$ contains $2K - 2$ non-zero eigenvalues, $\bar{\mathbf{U}}_s$ are the corresponding eigenvectors and $\bar{\mathbf{U}}_n$ contains the remaining eigenvectors which are orthogonal to $\bar{\mathbf{U}}_s$. It should be noted that the eigenvalues of $\bar{\mathbf{R}}$ are not always positive according to the definition in (5.20). Referring to the lemmas in Chapter 3, we also have the following lemma:

Lemma 1: The null space of $\bar{\mathbf{U}}_s$ is equal to the null space of $\mathbf{H}_I$, i.e.,

$$\text{null}(\mathbf{H}_I) = \text{null}(\bar{\mathbf{U}}_s) = \text{range}(\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H). \quad (5.22)$$

By projecting the signal in (5.9) onto $\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H$ directly, we obtain

$$\begin{aligned}\tilde{\mathbf{r}}[i] &= (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{r}[i] \\ &= (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) [\mathbf{h}_1 b_1[i] + \mathbf{H}_I \mathbf{b}_I[i] + \boldsymbol{\omega}[i]] \\ &= (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) [\mathbf{h}_1 b_1[i] + \boldsymbol{\omega}[i]]\end{aligned}\quad (5.23)$$

where the third equality follows from lemma 1, i.e., $(\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{H}_I = \mathbf{0}$. We then consider the following optimization problem:

$$\mathbf{w}_{opt} \triangleq \arg \min_{\mathbf{w}} E\{|b_1[i] - \mathbf{w}^H \tilde{\mathbf{r}}[i]|^2\}. \quad (5.24)$$

Referring to the method in Chapter 3, it is easy to show that the optimized solution for (5.24) is given by

$$\mathbf{w}_{opt} = [\mathbf{h}_1^H (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \frac{\sigma^2}{\sqrt{T_s}}]^{-1} (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1. \quad (5.25)$$

Since $[\mathbf{h}_1^H (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1 + \frac{\sigma^2}{\sqrt{T_s}}]^{-1}$ is only a positive real scalar factor which is irrelevant to the performance of the detector, $\mathbf{w}_{opt}$ can be rewritten as

$$\mathbf{w}_{opt} = (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{h}_1. \quad (5.26)$$

It has been proven in Chapter 3 that $\mathbf{w}_{opt}$ is equivalent to the decorrelating detector for the desired user. Meanwhile, the detector $\mathbf{w}_{opt}$ is equivalent to the principal eigenvector of $E\{(\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{r}[i] \mathbf{r}^H[i] (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\}$ up to a complex phase factor α , i.e.,

$$\begin{aligned} \hat{\mathbf{w}}_{opt} &= \text{Principal-eigenvector } E\{(\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{r}[i] \mathbf{r}^H[i] (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H)\} \\ &= \alpha \mathbf{w}_{opt}. \end{aligned} \quad (5.27)$$

It is noted that although the intersymbol decorrelating detector proposed in this chapter has similar expression as that of the intersymbol decorrelating detector in Chapter 3, the difference between them should be addressed. The intersymbol decorrelating detector in Chapter 3 is based on the signal model of DS-CDMA systems where the received signal vector need to be truncated for the implementation

of the detector. While such a requirement is not required for the detector in this chapter based on the signal model of (5.9)

5.3.3 Proposed Intersymbol Multiuser Detection in Correlated Noise

Although the proposed intersymbol detector in this thesis primarily focuses on the channel with white ambient noise, the intersymbol information can be exploited as well when the noise is not white. We next briefly discuss the intersymbol multiuser detection in the presence of correlated ambient noise.

In order to utilize the intersymbol information, we consider two independent antennas at the receiver. The two augmented received signal vectors at the two antennas can be written respectively as

$$\begin{aligned}\mathbf{r}_1[i] &= \mathbf{h}_{1,1}b_1[i] + \mathbf{H}_{1,I}\mathbf{b}_I[i] + \mathbf{v}_1[i] \\ \mathbf{r}_2[i] &= \mathbf{h}_{2,1}b_1[i] + \mathbf{H}_{2,I}\mathbf{b}_I[i] + \mathbf{v}_2[i]\end{aligned}\tag{5.28}$$

where $[\mathbf{h}_{1,1}, \mathbf{H}_{1,I}]$ and $[\mathbf{h}_{2,1}, \mathbf{H}_{2,I}]$ contain the channel information corresponding to the respective antennas. It is assumed that the two antennas are well separated so that the ambient noise is spatially uncorrelated, i.e., $E\{\mathbf{v}_1[i]\mathbf{v}_2[i]^H\} = 0$. Without loss of generality, we consider the decorrelating detector for the first receiver, i.e., $\mathbf{r}_1[i]$. Denote further

$$\begin{aligned}\mathbf{R}_{12} &\triangleq E\{\mathbf{r}_1[i]\mathbf{r}_2[i]^H\} = [\mathbf{h}_{1,1}, \mathbf{H}_{1,I}][\mathbf{h}_{2,1}, \mathbf{H}_{2,I}]^H \\ \mathbf{R}_{12I} &\triangleq E\{\mathbf{r}_1[i]\mathbf{r}_2[i-1]^H\} + E\{\mathbf{r}_1[i-1]\mathbf{r}_2[i]^H\} = \mathbf{H}_{1,I} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \mathbf{H}_{2,I}^H.\end{aligned}\quad (5.29)$$

Here we assume that $[\mathbf{h}_{1,1}, \mathbf{H}_{1,I}]^H[\mathbf{h}_{1,1}, \mathbf{H}_{1,I}]$ and $[\mathbf{h}_{2,1}, \mathbf{H}_{2,I}]^H[\mathbf{h}_{2,1}, \mathbf{H}_{2,I}]$ are full rank. By performing the eigendecompositions of $\mathbf{R}_{12}\mathbf{R}_{12}^H$ and $\mathbf{R}_{12I}\mathbf{R}_{12I}^H$, we obtain

$$\begin{aligned}\mathbf{R}_{12}\mathbf{R}_{12}^H &= [\mathbf{h}_{1,1}, \mathbf{H}_{1,I}][\mathbf{h}_{2,1}, \mathbf{H}_{2,I}]^H[\mathbf{h}_{2,1}, \mathbf{H}_{2,I}][\mathbf{h}_{1,1}, \mathbf{H}_{1,I}]^H = \mathbf{U}_{1,12}\Lambda_{1,12}\mathbf{U}_{1,12}^H \\ \mathbf{R}_{12I}\mathbf{R}_{12I}^H &= \mathbf{H}_{1,I} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \mathbf{H}_{2,I}^H \mathbf{H}_{2,I} \begin{bmatrix} \mathbf{0} & \mathbf{I}_{K-1} \\ \mathbf{I}_{K-1} & \mathbf{0} \end{bmatrix} \mathbf{H}_{1,I}^H = \mathbf{U}_{1,12I}\Lambda_{1,12I}\mathbf{U}_{1,12I}^H\end{aligned}\quad (5.30)$$

where $\Lambda_{1,12}$ is a positive diagonal matrix containing $2K - 1$ eigenvalues of $\mathbf{R}_{12}\mathbf{R}_{12}^H$ and $\mathbf{U}_{1,12}$ are the corresponding eigenvectors, and $\Lambda_{1,12I}$ contains $2K - 2$ positive eigenvalues of $\mathbf{R}_{12I}\mathbf{R}_{12I}^H$ with the corresponding eigenvectors $\mathbf{U}_{1,12I}$. With (5.29), (5.30) and the full rank assumption of $[\mathbf{h}_{1,2}, \mathbf{H}_{2,I}]^H[\mathbf{h}_{1,2}, \mathbf{H}_{2,I}]$, it is easy to see that $\text{range}(\mathbf{U}_{1,12}) = \text{range}([\mathbf{h}_{1,1}, \mathbf{H}_{1,I}])$ contains the signal subspace at receiver 1, and $\text{range}(\mathbf{U}_{1,12I}) = \text{range}(\mathbf{H}_{1,I})$ only contains the subspace spanned by all interfering users except the desired user. Recalling that the decorrelating detector must be in the range of user subspace and orthogonal to all interfering users except the desired user, we can express the detector as $\mathbf{d}_{cor} = \mathbf{U}_{1,12}\mathbf{g}_1$, where $\mathbf{d}_{cor}$ must be orthogonal to the subspace spanned by interfering users, i.e.,

$$\mathbf{d}_{cor}^H \mathbf{U}_{1,12I} = \mathbf{g}_1^H \mathbf{U}_{1,12}^H \mathbf{U}_{1,12I} = 0 \quad (5.31)$$

Hence, an estimation of $\mathbf{g}_1$ can be obtained by computing the minimum eigenvector of the matrix $\mathbf{U}_{1,12}^H \mathbf{U}_{1,12} \mathbf{U}_{1,12}^H \mathbf{U}_{1,12}$, and the decorrelating detector can be derived accordingly.

5.4 Proposed Blind Adaptive Implementation

This section considers the blind adaptive implementation of the decorrelating detector in (5.27). Clearly, the main task of the realization of the detector focuses on the subspace tracking of the intersymbol cross correlation matrix $\bar{\mathbf{R}}$, i.e., the estimation of $\bar{\mathbf{U}}_s$. In the literatures, all previously proposed subspace tracking methods require the estimated signal space to be a positive Hamiltonian matrix [41]. Unfortunately, $\bar{\mathbf{R}}$ may not be a positive Hamiltonian matrix. To tackle this problem, a new iterative algorithm for the subspace tracking of $\bar{\mathbf{R}}$ is developed. Since our algorithm is derived from the projection approximation subspace tracking deflation (PASTD) algorithm in [81, 82], it is termed as improved PASTD (I-PASTD) algorithm. It should be noted that the proposed approach can track the subspace of any Hamiltonian matrix.

5.4.1 Proposed Scalar Function of I-PASTD algorithm

Similar to the PASTD algorithm in [81], let us consider the minimization of the scalar function

$$J(\mathbf{W}) = \text{tr}(\mathbf{C}) - 2\text{tr}(\mathbf{W}^H \mathbf{C} \mathbf{W}) + \text{tr}(\mathbf{W}^H \mathbf{C} \mathbf{W} \mathbf{W}^H \mathbf{W}) \quad (5.32)$$

with a matrix argument $\mathbf{W} \in \mathbb{R}^{N \times r}$ ($r < N$). It is shown in [81] that when $\mathbf{C}$ is of positive conjugate symmetry, the solution to the above scalar function will converge to a global minimum where the column span of $\mathbf{W}$ equals the principal subspace of $\mathbf{C}$. We now discuss the property of $J(\mathbf{W})$ when $\mathbf{C}$ is not positive definite.

Lemma 2: For any conjugate symmetric matrix $\mathbf{C} \in \mathbb{R}^{N \times N}$, $\mathbf{W}$ is a stationary point of $J(\mathbf{W})$ if and only if $\mathbf{W} = \mathbf{U}_r \mathbf{Q}$ where $\mathbf{U}_r \in \mathbb{R}^{N \times r}$ contains any r distinct eigenvectors of $\mathbf{C}$ and $\mathbf{Q} \in \mathbb{R}^{r \times r}$ is an arbitrary unitary matrix.

Proof: The gradient of $J(\mathbf{W})$ in (5.32) can be written as

$$\nabla_J = [-2\mathbf{C} + \mathbf{C}\mathbf{W}\mathbf{W}^H + \mathbf{W}\mathbf{W}^H\mathbf{C}]\mathbf{W} \quad (5.33)$$

If $\mathbf{W} = \mathbf{U}_r \mathbf{Q}$, where $\mathbf{Q}$ is orthogonal and $\mathbf{U}_r$ contains r distinct eigenvectors of $\mathbf{C}$, it is pellucid to demonstrate $\nabla_J = 0$.

On the other hand, if $\nabla_J = 0$, we obtain

$$\mathbf{W}^H \nabla_J = (\mathbf{W}^H \mathbf{C} \mathbf{W})(\mathbf{W}^H \mathbf{W} - \mathbf{I}) + (\mathbf{W}^H \mathbf{W} - \mathbf{I})(\mathbf{W}^H \mathbf{C} \mathbf{W}) = \mathbf{0} \quad (5.34)$$

where $\mathbf{W}^H \mathbf{C} \mathbf{W}$ is conjugate symmetric with full rank r and is not positive definite.

Denote $\mathbf{W}^H \mathbf{W} - \mathbf{I} = \mathbf{U}_A^H \Lambda_A \mathbf{U}_A$ as the eigendecomposition with eigenvalues λ_i^A , and let $\mathbf{B} = \mathbf{U}_A \mathbf{W}^H \mathbf{C} \mathbf{W} \mathbf{U}_A^H = \{b_{i,j}\}$. We can express (5.34) as

$$\mathbf{B} \Lambda_A + \Lambda_A \mathbf{B} = \mathbf{0} \quad (5.35)$$

where $\mathbf{B}$ is still conjugate symmetric with full rank r but not positive definite.

Equation (5.35) implies $(\lambda_i^A + \lambda_j^A)b_{i,j} = 0$ for $\forall i$ and j . Since $\mathbf{B}$ is of full rank, there are at least r nonzero entries in $\mathbf{B}$ where these entries belong to the different rows and

the different columns. According to these r entries and (5.35), it is straightforward to obtain at least r constrained equations $\lambda_i^A + \lambda_j^A = 0$, where the combinations of $(\lambda_i^A, \lambda_j^A)$ pairs in the r constraints are different, respectively. Consequently, the only solution to the above r equations is $\lambda_i^A = 0$ for $\forall i$, i.e.,

$$\mathbf{W}^H \mathbf{W} - \mathbf{I} = 0 \quad (5.36)$$

Accordingly, $\nabla_J = 0$ is equivalent to $\mathbf{C}\mathbf{W} = \mathbf{W}\mathbf{W}^H\mathbf{C}\mathbf{W}$. If we denote $\mathbf{W}^H\mathbf{C}\mathbf{W} = \mathbf{U}_B^H \Lambda_B \mathbf{U}_B$ as the eigendecomposition of $\mathbf{W}^H\mathbf{C}\mathbf{W}$ and $\mathbf{U}_r = \mathbf{W}\mathbf{U}_B^H$, it is easy to obtain $\mathbf{C}\mathbf{U}_r = \mathbf{U}_r \Lambda_B$. Obviously, $\mathbf{U}_r$ contains r distinct eigenvectors of $\mathbf{C}$ and $\mathbf{W}$ can be expressed as $\mathbf{W} = \mathbf{U}_r \mathbf{U}_B$, where $\mathbf{U}_B$ is a unitary matrix.

Hence, lemma 2 is proven.

Lemma 2 shows that $\mathbf{W}$ contains an orthonormal basis of the signal subspace of $\mathbf{C}$ when $J(\mathbf{W})$ reaches stationary points. It should be noted that $\mathbf{W}$ does not always contain the dominant subspace of $\mathbf{C}$ when $\mathbf{C}$ is not positive definite. For special case that $\mathbf{W}$ is a vector, the solution to the stationary point of $J(\mathbf{W})$ will be given by one normalized eigenvector of $\mathbf{C}$.

5.4.2 Proposed Subspace Tracking of I-PASTD Algorithm

Referring to (5.20), we denote $\mathbf{x}_1[i] \in \mathbb{R}^N$ and $\mathbf{x}_2[i] \in \mathbb{R}^N$ as the received N dimensional vectors at the i th snapshot, where $\mathbf{x}_1[i]$ and $\mathbf{x}_2[i]$ are correlated with correlation $E[\mathbf{x}_1[i]\mathbf{x}_2[i]^H] = \mathbf{C}_{half}$. We then denote the cross correlation matrix $\mathbf{C}$

as the estimated signal space, which is given by

$$\mathbf{C} \triangleq E[\mathbf{x}_1[i]\mathbf{x}_2[i]^H + \mathbf{x}_2[i]\mathbf{x}_1[i]^H] = \mathbf{C}_{half} + \mathbf{C}_{half}^H \quad (5.37)$$

where $\mathbf{C}$ is a conjugate symmetric matrix. Referring to the method in [86], the expectation in (5.32) is replaced with the exponentially weighted sum to yield

$$\begin{aligned} J(\mathbf{W}[i]) &= \sum_{m=1}^i \beta^{i-m} \{ \text{tr}[\mathbf{x}_1[m]\mathbf{x}_2[m]^H + \mathbf{x}_2[m]\mathbf{x}_1[m]^H] \\ &\quad - 2\text{tr}[\mathbf{W}^H[i](\mathbf{x}_1[m]\mathbf{x}_2[m]^H + \mathbf{x}_2[m]\mathbf{x}_1[m]^H)\mathbf{W}[i]] \\ &\quad + \text{tr}[\mathbf{W}^H[i](\mathbf{x}_1[m]\mathbf{x}_2[m]^H + \mathbf{x}_2[m]\mathbf{x}_1[m]^H)\mathbf{W}[i]\mathbf{W}^H[i]\mathbf{W}[i]] \} \\ &= \text{tr}[\mathbf{C}[i]] - 2\text{tr}[\mathbf{W}^H[i]\mathbf{C}[i]\mathbf{W}[i]] + \text{tr}[\mathbf{W}^H[i]\mathbf{C}[i]\mathbf{W}[i]\mathbf{W}^H[i]\mathbf{W}[i]] \end{aligned} \quad (5.38)$$

where $\mathbf{C}[i]$ is the exponentially weighted correlation matrix defined as

$$\begin{aligned} \mathbf{C}[i] &= \sum_{m=1}^i \beta^{i-m} [\mathbf{x}_1[m]\mathbf{x}_2[m]^H + \mathbf{x}_2[m]\mathbf{x}_1[m]^H] \\ &= \beta\mathbf{C}[i-1] + \mathbf{x}_1[i]\mathbf{x}_2[i]^H + \mathbf{x}_2[i]\mathbf{x}_1[i]^H. \end{aligned} \quad (5.39)$$

In (5.38), the forgetting factor $0 < \beta < 1$ is used to provide the tracking capability when the environment is not stationary. Noting that $J(\mathbf{W}[i])$ in (5.38) is a fourth-order function of $\mathbf{W}[i]$, we denote

$$\mathbf{y}_1[m] \triangleq \mathbf{W}^H[m-1]\mathbf{x}_1[m] \quad (5.40)$$

$$\mathbf{y}_2[m] \triangleq \mathbf{W}^H[m-1]\mathbf{x}_2[m] \quad (5.41)$$

to approximate $\mathbf{W}^H[i]\mathbf{x}_1[m]$ and $\mathbf{W}^H[i]\mathbf{x}_2[m]$, respectively. Thus, $J(\mathbf{W}[i])$ can be

rewritten as

$$\begin{aligned}
\bar{J}(\mathbf{W}[i]) &= \sum_{m=1}^i \beta^{i-m} \{ \text{tr}[\mathbf{x}_1[m]\mathbf{x}_2[m]^H + \mathbf{x}_2[m]\mathbf{x}_1[m]^H] \\
&\quad - 2\text{tr}[\mathbf{W}^H[i](\mathbf{x}_1[m]\mathbf{y}_2[m]^H + \mathbf{x}_2[m]\mathbf{y}_1[m]^H)] \\
&\quad + \text{tr}[(\mathbf{y}_1[m]\mathbf{y}_2[m]^H + \mathbf{y}_2[m]\mathbf{y}_1[m]^H)\mathbf{W}^H[i]\mathbf{W}[i]] \} \\
&= \text{tr}[\mathbf{C}[i]] - 2\text{tr}[\mathbf{W}^H[i]\mathbf{C}_{xy}[i]] + \text{tr}[\mathbf{C}_{yy}[i]\mathbf{W}^H[i]\mathbf{W}[i]]
\end{aligned} \tag{5.42}$$

where

$$\begin{aligned}
\mathbf{C}_{xy}[i] &= \sum_{m=1}^i \beta^{i-m} [\mathbf{x}_1[m]\mathbf{y}_2[m]^H + \mathbf{x}_2[m]\mathbf{y}_1[m]^H] \\
&= \beta\mathbf{C}_{xy}[i-1] + \mathbf{x}_1[i]\mathbf{y}_2[i]^H + \mathbf{x}_2[i]\mathbf{y}_1[i]^H
\end{aligned} \tag{5.43}$$

$$\begin{aligned}
\mathbf{C}_{yy}[i] &= \sum_{m=1}^i \beta^{i-m} [\mathbf{y}_1[m]\mathbf{y}_2[m]^H + \mathbf{y}_2[m]\mathbf{y}_1[m]^H] \\
&= \beta\mathbf{C}_{yy}[i-1] + \mathbf{y}_1[i]\mathbf{y}_2[i]^H + \mathbf{y}_2[i]\mathbf{y}_1[i]^H.
\end{aligned} \tag{5.44}$$

Clearly, $\bar{J}(\mathbf{W}[i])$ is a modified cost function which is quadratic in terms of $\mathbf{W}[i]$. It was shown in [86] that $\bar{J}(\mathbf{W}[i])$ is a good approximation to $J(\mathbf{W}[i])$ and the matrix $\mathbf{W}[i]$ minimizing $\bar{J}(\mathbf{W}[i])$ is a good estimate for the signal subspace of $\mathbf{C}[i]$. Applying the gradient-descent technique into (5.42), the optimal $\mathbf{W}[i]$ is given by

$$\mathbf{W}[i] = \mathbf{C}_{xy}[i]\mathbf{C}_{yy}^{-1}[i] \tag{5.45}$$

where $\mathbf{W}[i]$ contains r distinct eigenvectors of $\mathbf{C}[i]$ based on the result of Lemma 2.

It was proven in [87] that $\mathbf{C}[i]$ can be replaced by its expected value for large i , i.e.,

$$\lim_{i \rightarrow \infty} \mathbf{C}[i] \cong \lim_{i \rightarrow \infty} E\{\mathbf{C}[i]\} = \frac{1}{1-\beta}\mathbf{C} \tag{5.46}$$

which shows that $\mathbf{C}[i]$ will converge to $\mathbf{C}$ up to a real constant factor as long as i is large enough. Hence, it is naturally expected that $\mathbf{W}[i]$ will converge to r distinct orthogonal eigenvectors of $\mathbf{C}$.

Table 5.1: Algorithm of I-PASTD

Choosing: $\mathbf{u}_k[0]$ and $d_k[0]$,

For $i = 1, 2, \dots$ Do

$$\mathbf{C}_1[i] = \mathbf{x}_1[i]\mathbf{x}_2[i]^H + \mathbf{x}_2[i]\mathbf{x}_1[i]^H$$

For $k = 1 : r$ Do

$$y_k[i] = \mathbf{u}_k^H[i-1]\mathbf{C}_k[i]\mathbf{u}_k[i-1]$$

$$d_k[i] = \beta d_k[i-1] + y_k[i]$$

$$\mathbf{u}_k[i] = \mathbf{u}_k[i-1] + [\mathbf{C}_k[i] - y_k[i]\mathbf{I}_N]\mathbf{u}_k[i-1]/d_k[i]$$

$$\mathbf{M} = \mathbf{u}_k(i)\mathbf{u}_k^H[i-1]\mathbf{C}_k[i]$$

$$\mathbf{C}_{k+1}[i] = \mathbf{C}_k[i] - \mathbf{M} - \mathbf{M}^H + y_k[i]\mathbf{u}_k[i]\mathbf{u}_k^H[i]$$

End

Let us consider the computation of $\mathbf{W}[i]$ now.

In (5.45), $\mathbf{W}[i]$ can be calculated from $\mathbf{C}_{xy}[i]$ and $\mathbf{C}_{yy}^{-1}[i]$. The iterative calculation of $\mathbf{C}_{xy}[i]$ is straightforward according to (5.43). However, the computation of $\mathbf{C}_{yy}^{-1}[i]$ needs the matrix inverse calculation, which is usually troublesome for adaptive implementation. Noting that $\mathbf{C}_{yy}^{-1}[i]$ turns into a scalar when $\mathbf{W}[i]$ is a vector, the deflation technique that is similar to the PASTD algorithm in [81] is adopted to avoid the matrix inverse computation. We first estimate one eigenvector of $\mathbf{C}[i]$ by

computing $\mathbf{C}_{xy}[i]$ and scalar $\mathbf{C}_{yy}^{-1}[i]$, and then remove the subspace spanned by the estimated eigenvector from $\mathbf{C}[i]$ to track the second eigenvector in the same way. The same method is repeated until all the desired eigenvectors are estimated sequentially. It should be pointed out that the estimated eigenvector may not be the principal eigenvector since $\mathbf{C}[i]$ is not positive definite, i.e., the estimated subspace may not be the principal subspace. Fortunately, we are interested in the whole space spanned by $\bar{\mathbf{R}}$ in (5.21). Hence, the sequence of the estimated eigenvectors will not influence the result of proposed detector. Table 5.1 summarizes the algorithm of I-PASTD. In this table, $d_k[i]$ is the counterpart of $\mathbf{C}_{yy}[i]$ and $\mathbf{u}_k[i]$ is the k th eigenvector. The last two steps describe the deflation procedure which removes the subspace spanned by the current eigenvector for the estimation of next eigenvector. This algorithm can track the subspace of any Hamiltonian matrix. By denoting $\mathbf{x}_1[i] = \mathbf{x}_2[i]$, the implementation of the I-PASTD algorithm is equivalent to the PASTD algorithm, and the estimated eigenvector in each loop is always the principal eigenvector.

5.4.3 Estimation of Ambiguous Scaling Factor

The above I-PASTD algorithm can be utilized to evaluate the subspace of the matrix $\bar{\mathbf{R}}$ in (5.21). For tracking the subspace of $\bar{\mathbf{R}}$, $\mathbf{x}_1[i]$ and $\mathbf{x}_2[i]$ in the I-PASTD algorithm are replaced by $\mathbf{r}[i]$ and $\mathbf{r}[i-1]$, respectively. Because $\bar{\mathbf{R}}$ has the rank of $2K-2$, only $2K-2$ eigenvectors are tracked to acquire an orthonormal basis of $\bar{\mathbf{R}}$. With the estimated basis $\bar{\mathbf{U}}_s$, $\hat{\mathbf{w}}_{opt}$ can also be resolved by calculating the principal eigenvector of (5.27) through the I-PASTD algorithm. In this case, $\mathbf{x}_1[i]$ and $\mathbf{x}_2[i]$ in I-PASTD

algorithm are given by

$$\mathbf{x}_1[i] = \mathbf{x}_2[i] = (\mathbf{I}_N - \bar{\mathbf{U}}_s \bar{\mathbf{U}}_s^H) \mathbf{r}[i] \quad (5.47)$$

As mentioned before, $\hat{\mathbf{w}}_{opt}$ is equal to the decorrelating detector up to an ambiguous scaling factor α . To avoid serious performance deterioration due to the phase of α , a differential detection could be adopted [67]. However, the differential detection itself creates certain deterioration on performance. In this section, a new approach is proposed to compensate the phase of α .

Let us express α as $\alpha = |\alpha|e^{j\phi}$, where $|\alpha|$ and ϕ are respectively the amplitude and phase of α . Then, the optimal detector can be rewritten as

$$\hat{\mathbf{w}}_{opt} = \alpha \mathbf{w}_{opt} = |\alpha|e^{j\phi} \mathbf{w}_{opt}. \quad (5.48)$$

Since the real positive amplitude does not influence the performance of the detector, only phase ϕ is required to be estimated. Recalling that the elements of the background noise $\boldsymbol{\omega}[i]$ in (5.9) are independent, zero mean and identically distributed complex variables, let us consider

$$E[\boldsymbol{\omega}[i]\boldsymbol{\omega}^T[i]] = \begin{bmatrix} E[\omega_0[i]\omega_0[i]] & 0 & \cdots & 0 \\ 0 & E[\omega_1[i]\omega_1[i]] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & E[\omega_{N-1}[i]\omega_{N-1}[i]] \end{bmatrix}. \quad (5.49)$$

The complex white Gaussian distributed variable $\omega_n[i]$ can be written as

$$\omega_n[i] = p_n[i] + jq_n[i], \quad n = 0, 1, \dots, N-1. \quad (5.50)$$

where $p_n[i]$ and $q_n[i]$ are the real and imaginary parts of $\omega_n[i]$, respectively, which are independent, zero mean and identically distributed real variables. Hence, it is easy to obtain

$$\begin{aligned} E[\omega_n[i]\omega_n[i]] &= E\{[p_n[i] + jq_n[i]][p_n[i] + jq_n[i]]\} \\ &= E[p_n[i]p_n[i]] - E[q_n[i]q_n[i]] \\ &= 0 \end{aligned} \quad (5.51)$$

where $n = 0, 1, \dots, N - 1$. Substituting (5.51) into (5.49), we obtain

$$E[\boldsymbol{\omega}[i]\boldsymbol{\omega}^T[i]] = \mathbf{0}. \quad (5.52)$$

For the estimation of the phase, we consider

$$\begin{aligned} &\hat{\mathbf{w}}_{opt}^H E[\mathbf{r}[i]\mathbf{r}^T[i]] (\hat{\mathbf{w}}_{opt}^H)^T \\ &= \hat{\mathbf{w}}_{opt}^H E\{[\mathbf{h}_1 b_1[i] + \mathbf{H}_I \mathbf{b}_I[i] + \boldsymbol{\omega}[i]][\mathbf{h}_1 b_1[i] + \mathbf{H}_I \mathbf{b}_I[i] + \boldsymbol{\omega}[i]]^T\} (\hat{\mathbf{w}}_{opt}^H)^T \\ &= (|\alpha| e^{j\phi} \mathbf{w}_{opt})^H (\mathbf{h}_1 \mathbf{h}_1^T + \mathbf{H}_I \mathbf{H}_I^T) [(|\alpha| e^{j\phi} \mathbf{w}_{opt})^H]^T \\ &= |\alpha|^2 e^{-j2\phi} \end{aligned} \quad (5.53)$$

where the second equality follows from (5.48) and (5.52), the last equality follows from the properties of the decorrelating detector in (5.12). For the adaptive iteration of ϕ , the expectation $E[\mathbf{r}[i]\mathbf{r}^T[i]]$ is replaced with the exponentially weighted sum to yield

$$\mathbf{R}_T[i] = \sum_{m=1}^i \beta^{i-m} \mathbf{r}[m]\mathbf{r}^T[m] = \beta \mathbf{R}_T[i-1] + \mathbf{r}[i]\mathbf{r}^T[i]. \quad (5.54)$$

Thus, the estimated phase ϕ at the i th step is given by

$$\hat{\phi} = -\frac{1}{2} \text{phase}\{\hat{\mathbf{w}}_{opt}^H[i] \mathbf{R}_T[i] (\hat{\mathbf{w}}_{opt}^H)^T[i]\}. \quad (5.55)$$

It is noted that the estimated phase $\hat{\phi}$ in (5.55) has an ambiguity of 180 degree because both $\hat{\phi}$ and $\hat{\phi} + \pi$ satisfy (5.53). In order to avoid this ambiguity, let us denote $\mathbf{c}_1 = [c_{1,0}, c_{1,0}, \dots, c_{1,N-1}]^T$ and consider

$$\begin{aligned}
 e^{-\hat{\phi}} \hat{\mathbf{w}}_{opt}^H[i] \mathbf{R}_T[i] \mathbf{c}_1 &= e^{\phi - \hat{\phi}} \mathbf{w}_{opt}^H[i] (\mathbf{h}_1 \mathbf{h}_1^T + \mathbf{H}_I \mathbf{H}_I^T) \mathbf{c}_1 \\
 &= e^{\phi - \hat{\phi}} \mathbf{h}_1^T \mathbf{c}_1 \\
 &= e^{\phi - \hat{\phi}} \frac{1}{\sqrt{T_s}} \sum_{n=0}^{N-1} a_1 c_{1,n}^2 \beta_{1,n} \begin{cases} > 0, & \hat{\phi} = \phi \\ < 0, & \hat{\phi} + \pi = \phi \end{cases} \tag{5.56}
 \end{aligned}$$

where the third equation follows from the fact that $h_{1,n} = \frac{1}{\sqrt{T_s}} a_1 c_{1,n} \beta_{1,n}$, and $\beta_{1,n}$, the amplitude of desired user's channel fading coefficient, is positive. Hence, the ambiguity of 180 degree is eliminated.

It should be pointed out that the method of (5.56) is based on the assumption that the phase of the channel response of the desired user is tracked, i.e., $\mathbf{h}_1$ is a real vector. When the phase tracking cannot be completed, the calculation in (5.56) does not make sense since $\mathbf{h}_1$ is a complex vector. In this case, the encoder tail of the CDMA frame can be used to solve the ambiguity of 180 degree. It is known that an 8-bit encoder tail, which is preknown by each mobile user, is allocated to each CDMA frame. Such encoder tail can be used as training bits in each frame to check the possible values of ϕ and to decide which one is the true value.

Table 5.2: Blind Implementation of the Intersymbol Decorrelating Detector

I-PASTD algorithm for subspace tracking of $\mathbf{R}$	
Choosing: $\mathbf{u}_k[0]$ and $d_k[0]$ suitably	
For $i = 1, 2, \dots$	Do
$\bar{\mathbf{R}}_1[i] = \mathbf{r}(i)\mathbf{r}[i-1]^H + \mathbf{r}[i-1]\mathbf{r}[i]^H$	
For $k = 1 : 2K - 2$	Do
$y_k[i] = \mathbf{u}_k^H[i-1]\bar{\mathbf{R}}_k[i]\mathbf{u}_k[i-1]$	
$d_k[i] = \beta d_k[i-1] + y_k[i]$	
$\mathbf{u}_k[i] = \mathbf{u}_k[i-1] + [\bar{\mathbf{R}}_k[i] - y_k[i]\mathbf{I}_N]\mathbf{u}_k[i-1]/d_k[i]$	
$\mathbf{M} = \mathbf{u}_k[i]\mathbf{u}_k^H[i-1]\bar{\mathbf{R}}_k[i]$	
$\bar{\mathbf{R}}_{k+1}[i] = \bar{\mathbf{R}}_k[i] - \mathbf{M} - \mathbf{M}^H + y_k[i]\mathbf{u}_k[i]\mathbf{u}_k^H[i]$	
End	
$\bar{\mathbf{U}}_s[i] = [\mathbf{u}_1[i], \dots, \mathbf{u}_{2K-2}[i]]$	
$\tilde{\mathbf{y}}[i] = (\mathbf{I}_N - \bar{\mathbf{U}}_s[i]\bar{\mathbf{U}}_s^H[i])\mathbf{r}[i]$	
I-PASTD algorithm for tracking the principal eigenvector of $\tilde{\mathbf{R}}$:	
Choosing $\hat{\mathbf{w}}_{opt}[0], \tilde{d}[0]$,	
$\tilde{\mathbf{R}}[i] = \tilde{\mathbf{y}}[i]\tilde{\mathbf{y}}^H[i]$	
$\tilde{y}[i] = \hat{\mathbf{w}}_{opt}^H[i-1]\tilde{\mathbf{R}}[i]\hat{\mathbf{w}}_{opt}[i-1]$	
$\tilde{d}[i] = \beta\tilde{d}[i-1] + \tilde{y}[i]$	
$\hat{\mathbf{w}}_{opt}[i] = \hat{\mathbf{w}}_{opt}[i-1] + [\tilde{\mathbf{R}}[i] - \tilde{y}[i]\mathbf{I}_N]\hat{\mathbf{w}}_{opt}[i-1]/\tilde{d}[i]$	
Estimate the phase $\phi[i]$ of $\hat{\mathbf{w}}_{opt}[i]$ and form the detector	
$\mathbf{R}_T[i] = \beta\mathbf{R}_T[i-1] + \mathbf{r}[i]\mathbf{r}^T[i]$, computing $\phi[i]$ by (5.55) and (5.56)	
$\mathbf{w}_{opt}[i] = e^{-j\phi[i]}\hat{\mathbf{w}}_{opt}[i]$	

5.4.4 Blind Adaptive Implementation

Based on the procedures previously presented, the blind adaptive implementation of the proposed intersymbol decorrelating detector includes the following steps:

- (i). Perform I-PASTD to estimate $\bar{\mathbf{U}}_s$ in (5.21);
- (ii). Perform I-PASTD to estimate $\hat{\mathbf{w}}_{opt}$ as the principal unit-norm eigenvector of $\tilde{\mathbf{R}}$;
- (iii). Use the method of (5.55) and (5.56) to estimate the phase of $\hat{\mathbf{w}}_{opt}$;
- (iv). Form the detector after phase compensation.

Table 5.2 summarizes the blind adaptive algorithm for the proposed decorrelating

detector. It is seen that the main task of the algorithm is the subspace tracking of $\bar{\mathbf{U}}_s$. The implementation is totally blind and simple since channel estimation of the desired user is not required.

5.5 Simulation Results

In this section, simulation results are presented to illustrate the performance of the proposed decorrelating detector implemented by the algorithm in Table 5.2 for asynchronous MC-CDMA system with Rayleigh fading channel. It is assumed that User 1 is the desired user that has been synchronized. We use a 31-subcarrier MC-CDMA system with Gold code of length $N = 31$ to simulate a severe near-far case in which the power of each interfering user is 10 dB more than that of the desired user. A normalized Rayleigh channel with amplitude $\beta_{k,n}$ is generated with the pdf defined as

$$p(\beta_{k,n}) = \frac{\beta_{k,n}}{\sigma^2} \exp\left(-\frac{\beta_{k,n}^2}{2\sigma^2}\right), \quad \beta_{k,n} > 0$$

where $k = 1, 2, \dots, K; n = 1, 2, \dots, N$ and $E[\beta_{k,n}^2] = 2\sigma^2 = 1$. The delay of each interfering user is randomly selected from 1 to 31.

Figure 5.3 compares the similarity between the proposed detector and its optimal value. The optimal decorrelating detector for User 1 is given by

$$\mathbf{d}_{opt} = \sum_{k=1}^{2K-1} ([\mathbf{h}_1, \mathbf{H}_I]^H [\mathbf{h}_1, \mathbf{H}_I])_{1,k}^{-1} [\mathbf{h}_1, \mathbf{H}_I]_k \quad (5.57)$$

where $([\mathbf{h}_1, \mathbf{H}_I]^H [\mathbf{h}_1, \mathbf{H}_I])_{i,j}^{-1}$ denotes the (i, j) th element of $([\mathbf{h}_1, \mathbf{H}_I]^H [\mathbf{h}_1, \mathbf{H}_I])^{-1}$, and $[\mathbf{h}_1, \mathbf{H}_I]_k$ denotes the k th column vector of the matrix $[\mathbf{h}_1, \mathbf{H}_I]$. The parameters $\mathbf{h}_1$ and $\mathbf{H}_I$ are presumed to be known for the optimal detector. The similarity between the two detectors is calculated by

$$\text{Similarity}[i] = \frac{|\mathbf{w}_{opt}[i]^H \mathbf{d}_{opt}|}{\sqrt{|\mathbf{d}_{opt}^H \mathbf{d}_{opt}| |\mathbf{w}_{opt}[i]^H \mathbf{w}_{opt}[i]|}} \quad (5.58)$$

Figure 5.3: Similarity between the proposed detector and the optimal decorrelating detector.

where $\mathbf{w}_{opt}[i]$ is the proposed detector at the i th step.

It is observed from Figure 5.3 (a) that the proposed detector with a large forgetting factor $\beta = 0.99$ is more stable and closer to its optimal value in stable state

compared with the detector with a much lower forgetting factor $\beta = 0.92$, while a faster transient process is obtained for the detector with lower forgetting factor. In Figure 5.3 (b), a heavy-loaded environment, i.e., $K = 10$, is considered. It is seen that the stable state of the detector with a large forgetting factor is still close to the true value though more steps are required for the convergency of the detector as the number of the interfering users increases. Similarly, the convergency speed is improved by introducing a smaller forgetting factor at the price of more error in stable state. The results in Figure 5.3 demonstrate the fast convergency and the strong robustness of the proposed detector under different loading conditions.

It is also observed that the use of small forgetting factor enhances the convergent speed of the estimated detector, but increases the sensitivity of the algorithm to the noise as well. This is because the forgetting factor is used to ensure that the data in the distant past are forgotten in order to afford possibility of following the statistical variations of the observable data. Small forgetting factors are forcing the next estimated parameter set to depend on few back in time, and then increase the sensitivity of the algorithm to the change of the environments. For a good compromise of the convergent speed and the stability of the algorithm, typical choices of β are between 0.98 to 0.995. In the following simulations, the forgetting factor is $\beta = 0.99$.

Figure 5.4 presents the average signal to interference and noise ratios of six detectors, i.e., the true MMSE detector in (5.17) which is presumed to know the channel information of all users, the blind subspace MMSE detector in [88] where the

Figure 5.4: Comparison of the SINRs of the proposed detector and other detectors for asynchronous MC-CDMA system ($K = 7$ users, $\text{SNR}=2\text{dB}$).

PASTD algorithm is used to track the signal subspace, the proposed decorrelating detector, the blind adaptive D-RAKE detector in [85] with an oversampling factor $L = 3$, the EGC detector and the MRC detector. The system is assumed to have 7 active users, and the performance measure SINR is defined as

$$\text{SINR}[i] = \frac{|\mathbf{d}^H[i]\mathbf{h}_1|^2}{|\mathbf{d}^H[i](\mathbf{r}[i] - \mathbf{h}_1 b_1[i])|^2} \quad (5.59)$$

where $\mathbf{d}[i]$, $\mathbf{r}[i]$ and $b_1[i]$ are the detector, the received signal and the symbol of desired user at the i th step, respectively. It should be noted that all the data plotted in Figure 5.4-5.6 are the average of 100 random runs. It is seen from Figure 5.4 that the proposed detector significantly outperforms the EGC and MRC detectors and

Figure 5.5: Comparison of the SINRs of the proposed detector and other detectors for asynchronous MC-CDMA system ($K = 7$ users, $\text{SNR}=6$ dB).

is better than D-RAKE detector as well. It is also observed that the true MMSE detector achieves the best performance, and the proposed method and the subspace MMSE detector have close SINRs. However, the subspace MMSE detector requires the channel estimation of the desired user which is not required for the proposed method. When SNR increases to 6dB, similar simulation results are shown in Figure 5.5. In this figure, all detectors except the EGC and MRC detectors present manifest performance increment as SNR increases. It can be explained that the performance of the EGC and MRC detectors is limited by the multiuser interference introduced by the successive symbols of the interfering users due to the timing offset between

different users, while the other detectors have more freedom than the EGC and MRC detectors to suppress such interference.

Figure 5.6: SINR of the proposed detector in a dynamic multiple-access channel (SNR=8dB).

Figure 5.6 depicts the SINR of the proposed detector in dynamic multiple-access channel where the number of the interfering users varies with time. The simulation starts with 6 interfering users in the channel. At the 1001th iteration, 2 new interfering users enter the channel, and at the 2001th iteration, 4 interfering users exit the channel. It is seen from this figure that the proposed detector can adapt to the dynamic channel traffic promptly and the SINR does not have significant variations as the number of interfering users changes.

Figure 5.7: Bit-error-rate comparison of the proposed detector and various no-adaptive detectors (7 users).

Figure 5.7 compares the BER performance of the optimal decorrelating detector, the proposed detector, the EGC detector and the MRC detector. The comparison is based on the same channel condition and system parameters given at the beginning of this section, and the number of the total users is fixed at 7. It is shown that the proposed detector outperforms both EGC and MRC detectors, and its performance is more close to that of the optimal decorrelating detector compared with EGC and MRC detectors, especially when the SNR is high. It reveals the fact that the proposed detector is more capable of rejecting the MAI and ISI from other users, compared with the EGC and MRC detectors.

Figure 5.8: Bit-error-rate comparison of the proposed detector and various adaptive detectors (9 users).

Finally, Figure 5.8 compares the BER performance of the proposed detector and other adaptive detectors for a system with 7 intracell users and 2 intercell users. The compared detectors are respectively the semi-blind MMSE detector in [89], which is presumed to know all incell user's signature sequences, the proposed detector, the D-RAKE and the blind subspace MMSE detectors. The D-RAKE detector requests the simplest implementation by using LMS algorithm. However, its performance is limited compared with other adaptive methods. The proposed detector and the subspace MMSE detector have the same complexity which is higher than the D-RAKE detector because both of them utilizing the subspace method to derive the

detector. It is observed that the proposed method outperforms the subspace MMSE detector. This is because that the former does not require the channel estimation of the desired user and then avoids any possible estimation errors. Another reason is that the subspace MMSE method has no phase compensation for the final detector. The semi-blind MMSE detector in this figure requires the channel estimation of all incell users. This detector is based on the group multiuser detection technology, which achieves the best performance with the highest complexity among all detectors compared.

5.6 Conclusions

This chapter presents a new decorrelating detector for asynchronous MC-CDMA systems over a frequency selective Rayleigh fading channel. Similar to the previous chapters, the intersymbol multiuser detection technology is used for the MC-CDMA systems. The main advantage of the proposed detector is its simplicity since no channel estimation is needed except the synchronization of the desired user.

In order to implement the proposed detector blind-adaptively, a new recursive algorithm is presented for the subspace tracking. The proposed subspace tracking is derived from the PASTD algorithm which is then termed as improved PASTD (IPASTD) algorithm. It is shown that the proposed algorithm can adaptively track the subspace of any conjugate symmetric matrix, while previously reported subspace tracking algorithms require the estimated signal space to be positive definite. Such algorithm can be used for any subspace tracking application without the limitation of

the positive definition of the estimated subspace.

By using the proposed subspace tracking method, the proposed detector can be implemented adaptively with only an ambiguous complex scaling factor. Since the phase of the scaling factor may substantially deteriorate the performance of the detector, a new adaptive algorithm is presented to estimate the phase factor of the proposed detector by exploiting the property of the white background noise. Simulation results show that the proposed method provides a good performance/complexity compromise compared with the conventional adaptive and non-adaptive detectors.

Chapter 6

Conclusions and Recommendation

This chapter consists of two sections. In Section 6.1, we summarize the main conclusions of this thesis. Section 6.2 gives some suggestions for further research.

6.1 Conclusions

Wireless communication today has become one of the most prosperous areas of technology development since telecommunication is one of the largest industries worldwide for the time being. Among various wireless communication technologies, CDMA techniques are widely considered as the preferred multiple access strategy in third generation mobile communications due to efficient use of available bandwidth. Owing to the multiple propagation paths, there always exist multiple access interference (MAI) and intersymbol interference (ISI) in a CDMA system. Both MAI and ISI increase the mutual interference of individual users's channel and also deteriorate the whole wireless channel. Therefore, the conventional single user de-

tection techniques, such as matched filter, suffer obvious performance deterioration especially when the traffic load is heavy.

In order to improve the detection of the CDMA systems, numerous multiuser detectors have been developed. Among these detectors, the optimum detector achieves the minimum BER, the optimum asymptotic multiuser efficiency and the optimum near-far resistance. However, the optimum detector assumes the availability of the channel information of each user as well as the distribution of the noise. Such assumption is not applicable in real communication environment. In addition, the unbearable computational complexity of the optimum detector makes it unacceptable for practical implementation. Because of this, some sub-optimum multiuser detectors have been developed. The linear decorrelating detector and the linear MMSE detector are two of the most popular sub-optimum multiuser detectors. These two detectors need a much simpler implementation compared with the optimum detector. The decorrelating detector achieves the optimum near far resistance when the received amplitudes are unavailable, while the MMSE detector achieves a better performance in lower SIR channel environment. Since these two detectors only require the signature waveform and the timing of the desired user, they can be implemented blindly. For the blind implementation, the channel estimation of the desired user is required for asynchronous CDMA systems. In addition to the decorrelating detector and the MMSE detector, the group-blind multiuser detectors are developed in order to improve the performance. The conventional group-blind detectors make use of the spreading sequence and the estimated multipath channels of all known users in

the group to construct a group constraint to suppress the intracell interference, while blindly suppressing the intercell interference, and then to enhance the performance of the detection. Since the conventional group-blind detectors require the information of all known users in the group, these detectors are only applicable for CDMA uplink and the implementation of these detectors are much more complicated than that of the decorrelating and MMSE detectors.

In this thesis, a number of intersymbol blind multiuser detectors for CDMA systems is developed based on the useful intersymbol information of the consecutive received signals. We first developed a blind intersymbol decorrelating detector for asynchronous DS-CDMA systems. It is noted that with the existence of ISI and MAI, the current received signal has certain correlation with its previous and following ones. The basic idea of the proposed detector is to exploit such correlation to construct a useful cross correlation matrix, and then to develop the decorrelating detector based on the constructed cross correlation matrix. The advantage of the proposed detector is its simplicity since the estimation of the composite channel response of the desired user is not required, while such estimation is necessary for the conventional blind decorrelating detector and MMSE detector. Since the detector is derived from the cross correlation matrix of the consecutive received signals, it is hence termed as the intersymbol decorrelating detector.

We propose two intersymbol group-blind multiuser detectors for asynchronous DS-CDMA systems. The proposed detectors still utilize the correlation information between consecutively received signals to generate a new group constraint. By

incorporating this group constraint, the proposed detectors can provide different performance gains for both uplink and downlink CDMA communications. Unlike the previously reported group-blind detectors, the proposed methods only need to estimate the multipath channel of the desired user and do not require the channel estimation of the other users. Simulation results show that the proposed intersymbol group-blind detectors has a better performance compared with the conventional group-blind detectors. Furthermore, our detectors require a much simpler implementation.

In addition, we introduce the intersymbol blind multiuser detection for MC-CDMA system. The MC-CDMA scheme is a promising technique for relaxing the capacity limitation problems of DS-CDMA systems [90]. The MC-CDMA scheme is a combination of CDMA and OFDM which can better utilize the spectrum and meet the demand of wireless mobile multimedia services. With the narrow band of the subcarriers in OFDM, the frequency selective fading can be reduced to the limit so that the signal associated with each subcarrier behaves like one with flat fading. However, the DS-CDMA systems need a complex RAKE receiver to equalize the channel fading. Therefore, MC-CDMA is also widely considered as an effective approach for future wireless multimedia communications [91]. In this thesis, we develop a decorrelating detector for asynchronous MC-CDMA systems over a frequency selective Rayleigh fading channel. The detector is analogous to the proposed intersymbol detector for asynchronous DS-CDMA systems, which is based on the key idea of using a cross correlation matrix constructed by the consecutively re-

ceived signals. It is shown that our method does not require the channel estimation of the desired user, which leads to a simpler implementation. We also develop a blind adaptive implementation for the proposed detector. Since the implementation of the detector requires the estimation of a signal subspace which is not positive definite, a new adaptive subspace tracking method is developed to track both positive and non-positive Hamiltonian matrix. It should also be noted that the blind adaptive algorithm can be applied to the intersymbol decorrelating detector and the intersymbol group-blind detectors conveniently with minor modification.

The main advantage of the proposed detectors in this thesis is simplicity. By introducing the intersymbol information into the proposed detectors, the implementations for both non-adaptive and adaptive detectors can be simplified significantly compared with the conventional detectors without using the intersymbol information. Another contribution of this thesis is the proposal of a new adaptive subspace tracking algorithm that provides promising performance. Unlike all of the previously proposed adaptive subspace tracking methods which require the estimated subspace to be positive, our method can track any Hamiltonian matrix, which is convenient to be implemented especially when the estimated subspace is not positive.

6.2 Recommendations for Further Research

Our focus has been on the situation in which the ambient noise is additive white Gaussian noise. However, as increasingly practical techniques for multiuser detection become available, the situation in which practical multiple-access will be ambient-

noise limited can be realistically envisioned. For example, in the urban and indoor radio channels [92, 93], and underwater acoustic channels [94], the ambient noise is known through experimental measurements to be non-Gaussian, due to the impulsive nature of the human-made electromagnetic interference and of a great deal of nature noise as well. In view of the lack of realism of an AWGN model for ambient noise arising in many scenarios in which multiuser detection techniques may be applied, natural questions arise concerning the applicability, robustness, and performance of multiuser detection techniques for non-Gaussian multiple-access channels.

Although we briefly analyze the detection for asynchronous MC-CDMA system in correlated noise in Chapter 5, it is far from complete since no performance analysis and simulation results are given. Meanwhile, the detection method in Chapter 5 is based on the assumption that the signal received in two antennas are independent of each other. In the future, research could be conducted on the various CDMA systems with the model of the non-Gaussian phenomena encountered in practice which are characterized by sharp spikes, occasional bursts, heavy outliers and so on. In addition, the signal models throughout this thesis are assumed to be under slow Doppler frequency shifting case, i.e., the parameters of the signal models are constant during the detection. It should be an interesting and promising research direction to step in the tracking ability of the proposed adaptive detectors for the Doppler frequency shifting.

Another research topic of great interest is the extension of the proposed intersymbol detection methods to the multi-input and multi-output (MIMO) CDMA

systems [94]. As the MIMO system is based on the antenna array, array signal processing technologies may be combined with the proposed intersymbol methods to improve the performance of detection.

Author's Publication List

Journal Publication

- [1] Gaonan Zhang, Guoan Bi and Liren Zhang, "Blind multiuser detection for asynchronous MC-CDMA system without channel estimation", *IEEE Trans. VT*, vol. 53, no. 4, Jul. 2004.
- [2] Gaonan Zhang, Guoan Bi and Liren Zhang "Improved blind linear detectors for CDMA in fading multipath channels", *IEEE Trans. Commun.*, Accepted by IEEE trans. Commun., 2004.
- [3] Gaonan Zhang, Guoan Bi and Qian Yu "Intersymbol decorrelating detector for asynchronous CDMA networks with multipath", Submitted to *EURASIP Journal on Wireless Communications and Networking*, Revised and submitted, May. 2004.
- [4] Gaonan Zhang, Guoan Bi and Qian Yu "Blind intersymbol decorrelating detector for asynchronous multicarrier CDMA System", Accepted by *Signal Processing*, 2005.

- [5] Gaonan Zhang, Guoan Bi and A. C. Kot, "Intersymbol decorrelating detector for asynchronous CDMA Systems", submitted to *JCN*, Dec. 2003.
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Conference Publication

- [7] Gaonan Zhang, Guoan Bi and Yuanxin Zhang, "New Group-Blind intersymbol detectors for downlink CDMA", *IEEE Conf. ICASSP*, May 2003.
- [8] Qian Yu, Gaonan Zhang and Guoan Bi, "Blind multipath estimation with Toeplitz displacement for long code DS-CDMA", *IEEE Conf. ICC*, no. 1, pp. 3292-3296, May 2003.
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