Performance analysis and enhancement of multicarrier transmission systems

Zhang, Keli

2005

Zhang, K. L. (2005). Performance analysis and enhancement of multicarrier transmission systems. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/3977>

<https://doi.org/10.32657/10356/3977>

Nanyang Technological University

Downloaded on 26 Apr 2023 22:34:54 SGT

a 356 905

Performance Analysis and Enhancement of Multicarrier Transmission Systems

Zhang Keli

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2005

TK
5103.7
Z63
2005

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original re-
search and has not been submitted for a higher degree to any other University or
Institution.

28 Feb , 2005
.....

Date

.....

Zhang Keli

Acknowledgements

I would like to express my sincere gratitude and appreciation to my supervisor, **Assistant Professor Guan Yong Liang**, who has given me continuous encouragement and shown me the various aspects of research. Also from him, I learned how to think systematically, deeply, and carefully. I have also learned how to write and present my work clearly. As a supervisor, he demonstrates knowledge and patience that I need as a research student. From him, I started to know the world of research.

My appreciation also goes to **Joseph** and **Hannah**, for their assistance in providing the administration and technical support during my stay in Communication Laboratory IV. I am grateful to **Nanyang Technological University, Singapore** for offering me the research scholarship and the opportunity to carry out my research work.

I would also like to thank all the **friends in Communication Laboratory IV** and all my **flatmates**, who have added to the joys and tears of my days. Without them, my research life could never be as fulfilling and colorful.

Last but not least, I thank **my parents, my husband, my sister and my brother**, for all the love, support and understanding that they have given me to pursue my interest in research.

To all of them, I dedicate this thesis...

Summary

To support high transmission rates with sufficient robustness to radio channel impairments such as frequency-selective multipath fading, multicarrier modulation (MCM) has emerged as an attractive transmission scheme. Two important realizations of MCM in use or in research today are orthogonal frequency division multiplexing (OFDM) and multicarrier code division multiple access (MC-CDMA). Their performance can be shown to be closely dependent on the frequency-domain fading characteristics of the radio transmission channel. In this project, we consider generalized fading channels that contain a mixture of Rayleigh and/or Rician paths with or without inter-path correlations, and derive the first- and second-order fading statistics for every subcarrier in the received OFDM signal. It enables us to carry out exact bit error rate (BER) analysis of MCM systems down to the subcarrier level. The model also reveals that some multipath channels, such as those containing more than one independent Rician paths or those containing correlated paths, have non-uniform fading distributions among the MCM subcarriers. For such channels, we propose a novel “statistical OFDM adaptation” scheme to improve the system performance. It is shown that the proposed statistical OFDM adaptation can give considerable power saving and spectral efficiency improvement over non-adaptive OFDM systems.

We also look into the current state of the art in BER analysis for MC-CDMA with various equalization and combining schemes. For uplink MC-CDMA with

maximal ratio combining (MRC), we extend the BER analysis to include considerations of correlated channel paths with possibly different fading distributions (i.e., Rayleigh, Rician and Nakagami), fading power and fading severity. Closed-form BER expression that does not require integration is obtained for correlated Nakagami fading channels.

For MC-CDMA systems with orthogonality restoring combining (ORC), we have also achieved simpler BER formulations. A lower bound and a tight approximation to the BER expressions are developed. Through this study, it is found that the ORC performance is dominated by the weakest instantaneous subcarrier. Hence we propose a “Truncated ORC” scheme to omit the weakest subcarrier in the combining operation of ORC. Though with such simple modification, the proposed scheme is demonstrated to perform drastically better than the conventional ORC scheme, and impressively close to the optimal minimum mean square error combining (MMSEC) scheme.

Theoretically, the MMSEC scheme performs best among various equalization and combining schemes for downlink MC-CDMA. However, it is also most computationally complex to realize as it involves complex matrix inversion operation of a large matrix. In this thesis, a new method is proposed to calculate the equalizer coefficient for MMSEC based on reduced-size matrices.

Finally, in our study of the OFDM and MC-CDMA performance in correlated Nakagami fading channels, we found that computer simulation algorithm that can generate correlated Nakagami signals with arbitrary correlation and fading parameters is not available in the literature. Hence we propose two methods, namely Decomposition Method and Sim’s Method, to fill this research gap. Simulations show that both methods produce outputs that match well with the specifications.

List of Abbreviations and Symbols

Abbreviations

MCM	multicarrier modulation
OFDM	orthogonal frequency division multiplexing
MC-CDMA	multicarrier code division multiple access
CIR	channel impulse response
MRC	maximal ratio combining
ORC	orthogonality restoring combining
EGC	equal gain combining
MMSEC	minimum mean square error combining
ISI	inter-symbol interference
ICI	inter-carrier interference
DFT	discrete Fourier transform
IDFT	inverses discrete Fourier transform
FFT	fast Fourier transform
IFFT	inverses fast Fourier transform
DS-CDMA	direct sequence code division multiple access
DAB	digital audio broadcasting
DVB	digital video broadcasting

ADSL	asymmetric digital subscriber line
MUI	multiuser interference
BER	bit error rate
pdf	probability density function
jdf	joint density function
SNR	signal-to-noise ratio
FDM	frequency division multiplexing
QAM	quadrature amplitude modulation
AWGN	additive white Gaussian noise
RV	random variable
CLT	central limit theorem
LLN	law of large numbers

Symbols

N_c	number of subcarriers
n	subcarrier index, $1 \leq n \leq N_c$
N_p	number of paths
l	path index, $1 \leq l \leq N_p$
N_u	number of users
k	user index, $1 \leq k \leq N_u$
Ω	fading power
K	Rician K factor
m	Nakagami m parameter
$\mathbf{I}_a$	identity matrix with size $a \times a$
$\mathbf{C}_{\mathbf{x}}$	covariance matrix of vector $\mathbf{x}$

$s(t)$	transmitted signal
$r(t)$	received signal
$b_k(i)$	the i th data bit of the k th user
E_b	bit energy
N_0	one-sided noise density
T_s	symbol duration for OFDM (or MC-CDMA) systems
f_n	frequency of the n th subcarrier
f_0	frequency offset for all subcarriers
Δf	frequency spacing between subcarriers
M_n	QAM size chosen for the n th subcarrier
v_n	power control factor for the n th subcarrier
$c_{k,n}$	spreading code
$\alpha_{k,n}$	equalizer coefficient
U	decision variable
$\mathcal{MUI}$	multiuser interference (MUI)
$\mathcal{I}$	MUI from the same subcarrier of other users
$\mathcal{J}$	MUI from the other subcarriers of other users
η	noise
$P_{e,n}$	BER for the n th subcarrier
P_b	BER in AWGN channel
P_e	BER
$\hat{P}_e$	target BER for adaptive OFDM systems
T	matrix transpose
H	matrix Hermitian transpose
$E[\cdot]$	expectation operator
$Q(\cdot)$	Gaussian Q function

$\Gamma(\cdot)$	Euler Gamma function
${}_2F_1(\cdot, \cdot; \cdot, \cdot)$	Hypergeometric function
$u_{T_s}(\cdot)$	rectangular pulse with duration T_s
$p(x)$	pdf of RV x
$\text{var}(x)$	variance of RV x
$ x $	absolute value of x
$\mathcal{M}_x(\cdot)$	moment generating function (MGF) of RV x
$\Psi_x(\cdot)$	characteristic function (CF) of RV x
$\mathcal{O}(\cdot)$	complexity level

Contents

Acknowledgements	iii
Summary	iv
List of Abbreviations and Symbols	vi
List of Figures	xv
List of Tables	xix
1 Introduction	1
1.1 Motivation	1
1.2 Objectives	4
1.3 Major Contributions of the Thesis	5
1.4 Organization of the Thesis	7
2 Multicarrier Modulation	9
2.1 Wireless Propagation Channel	9
2.1.1 Flat Fading vs. Frequency-Selective Fading	10
2.1.2 Inter-Symbol Interference (ISI)	11
2.2 Measures to Mitigate ISI	12
2.2.1 Equalization for Single-Carrier Transmission	12
2.2.2 Multicarrier Modulation (MCM)	13
2.3 Fundamentals of OFDM	15
2.3.1 Transmission Principle	15

2.3.2	FFT-Based Implementation	18
2.3.3	Cyclic Prefix	19
2.4	MC-CDMA – Combining OFDM with CDMA	20
3	Frequency-Domain (Subcarrier) Fading Characteristics of OFDM Signals	23
3.1	Introduction	23
3.2	Subcarrier Fading Distributions	25
3.2.1	Channel Impulse Response of Frequency-Selective Fading Channels	25
3.2.2	Frequency-Domain (Subcarrier) Fading Statistics	27
3.2.3	Case Studies and Discussions	28
3.2.3.1	Channels with Independent Paths	28
3.2.3.2	Channels with Correlated Paths	33
3.3	Subcarrier Fading Correlations	35
3.4	Applications of the Subcarrier Fading Model	38
3.4.1	Subcarrier BER Evaluation	38
3.4.2	Statistical OFDM Adaptation	41
3.4.2.1	System Model	41
3.4.2.2	Statistical Power Control	43
3.4.2.3	Statistical Successive Bit Loading and Power Control	44
3.4.2.4	Performance Evaluation	47
3.5	Summary	49
4	BER Analysis and Performance of MC-CDMA Systems	52
4.1	Introduction	52
4.2	BER Formulations	53

4.2.1	Downlink Signal Model	53
4.2.2	Uplink Signal Model	57
4.2.3	Equalization and Combining Schemes	60
4.2.3.1	Orthogonality Restoring Combining (ORC)	60
4.2.3.2	Maximal Ratio Combining (MRC)	61
4.2.3.3	Equal Gain Combining (EGC)	61
4.2.3.4	Minimum Mean Square Error Combining (MMSEC)	62
4.3	Existing Works on BER Analysis of MC-CDMA and Monte Carlo Integration	66
4.4	Performance Comparison of Different Equalization and Combining Schemes	68
4.5	Summary	70
5	BER Analysis of Uplink MC-CDMA with MRC in Generalized Frequency-Selective Fading Channels	74
5.1	Introduction	74
5.2	Simplification of BER Formulas	75
5.3	Time- and Frequency-Domain Equivalence of the MRC Output	78
5.4	BER Analysis	80
5.4.1	Channels with Independent Nakagami Fading Paths	80
5.4.1.1	Existing Works	80
5.4.1.2	Determination of m Parameter through Moment- Matching	81
5.4.1.3	Verification of the Approximated Gamma RV γ	85
5.4.1.4	BER Expressions and Results	86
5.4.2	Channels with Correlated Nakagami Fading Paths	90

5.4.2.1	Existing Works	90
5.4.2.2	Modelling Correlated RVs as Weighted Sums of Uncorrelated RVs	91
5.4.2.3	Determination of Model Parameters through Moment- Matching	93
5.4.2.4	Model Verification	94
5.4.2.5	BER Expressions and Results	97
5.4.3	Channels with Rician/Rayleigh Fading Paths	98
5.4.3.1	Existing Works	98
5.4.3.2	Characteristic Function of the MRC Output	99
5.4.3.3	BER Expressions and Results	100
5.5	Generation of Correlated Nakagami Fading Channels	105
5.5.1	Parameter-Matching between Nakagami and Gamma RVs	106
5.5.1.1	Same m_l among Diversity Branches	107
5.5.1.2	Different m_l among Diversity Branches	107
5.5.2	Decomposition Method	109
5.5.3	Sim's Method	110
5.5.4	Comparison of Decomposition and Sim's Methods	112
5.5.5	Verification	113
5.6	Summary	117
6	Downlink MC-CDMA with ORC and MMSEC - BER Analysis and Enhancement	118
6.1	Introduction	118
6.2	MC-CDMA with ORC	119
6.2.1	BER Simplification	119

6.2.1.1	BER Lower Bound	120
6.2.1.2	Tight BER Approximation	121
6.2.1.3	Results and Discussions	122
6.2.2	Truncated ORC	126
6.2.2.1	Principle	126
6.2.2.2	Lower Bound of Truncated ORC	128
6.2.2.3	Complexity Comparison	130
6.2.2.4	Omitting More Subcarriers	131
6.2.2.5	Truncated ORC in Rician Fading Channels	133
6.3	Complexity Reduction for MMSEC	134
6.4	Summary	141
7	Conclusions and Recommendations	143
7.1	Conclusions	143
7.2	Recommendations for Further Research	147
	Author's Publications	149
	Bibliography	152
	A Investigation of Approximation Error in Higher Order Moments	166
	B Reduction of (5.32) to (5.20)	168

List of Figures

2.1	Bandwidth occupancy for (a) conventional FDM, and (b) MCM with spectrum overlapping.	14
2.2	Principle of OFDM transmission.	15
2.3	A generic OFDM transceiver model.	16
2.4	Cyclic prefix insertion of OFDM symbols.	19
2.5	Block diagram for a typical MC-CDMA system.	21
2.6	Principle of MC-CDMA transmission.	22
3.1	Block diagram and equivalent model of an OFDM system.	24
3.2	Channel impulse response and frequency response of a frequency-selective fading channel.	25
3.3	Simulated and predicted subcarrier power (Ω_n^f) and subcarrier K factor (K_n^f) for the channel specified in Table 3.1.	32
3.4	Simulated and theoretical pdf of the 1st and 29th subcarriers for the OFDM system specified for Fig. 3.3.	32
3.5	Simulated and predicted subcarrier power (Ω_n^f) and subcarrier K factor (K_n^f) for the channel specified in Table 3.2 with path correlations.	34
3.6	Subcarrier correlations for a 3-path or 8-path channel with i.i.d. Rayleigh fading.	37

3.7 Simulated and predicted subcarrier BERs of a 64-subcarrier 16-QAM OFDM system at $E_b/N_0 = 15$ dB. Fading channel conditions are specified in Table 3.1 with path correlations given in (3.21). 40

3.8 OFDM modulator with bit loading and power control. 42

3.9 Numerical example to assign 8 bits to 4 subcarriers, using the proposed successive bit loading and power control scheme. 46

3.10 Performance of statistical OFDM adaptations compared to non-adaptive OFDM and instantaneous OFDM adaptation at target BER of $\hat{P}_e = 10^{-3}$. The channel conditions are specified in Table 3.1. 51

3.11 Performance of statistical OFDM adaptations compared to non-adaptive OFDM and instantaneous OFDM adaptation at target BER of $\hat{P}_e = 10^{-6}$. The channel conditions are specified in Table 3.1. 51

4.1 Block diagram of a downlink MC-CDMA system. 54

4.2 An MC-CDMA transmitter model for user k 54

4.3 MC-CDMA receiver model for user k 56

4.4 Block diagram for an uplink MC-CDMA channel. 57

4.5 BER performance of downlink MC-CDMA (64 subcarriers and 1 user) with different equalization and combining schemes. 71

4.6 BER performance of downlink MC-CDMA (64 subcarriers and 32 user) with different equalization and combining schemes. 72

4.7 BER performance of downlink MC-CDMA (64 subcarriers and 64 user) with different equalization and combining schemes. 72

4.8 BER performance of uplink MC-CDMA (64 subcarriers and 32 user) with different equalization and combining schemes. 73

5.1 Value of $\sum_{i=1, i \neq n}^{N_c} (i - n)^{-2}$ as a function of the subcarrier index n ($N_c = 128$). 78

5.2	Comparison of approximated MRC pdf (5.18) with the exact MRC pdf (5.26). Branch parameters: (a) $\{m_l\} = \{2, 1\}$, $\{\Omega_l\} = \{3, 1\}$; (b) $\{m_l\} = \{10, 5, 1\}$, $\{\Omega_l\} = \{5, 3, 1\}$; (c) $\{m_l\} = \{1, 1, 1, 1\}$, $\{\Omega_l\} = \{2.1, 0.4, 2.6, 1.6\}$	86
5.3	Performance of uplink MC-CDMA with MRC in i.i.d. Rayleigh fading channels for various number of paths (for comparison with Fig. 4 in [24]).	89
5.4	Performance of uplink MC-CDMA with MRC for i.i.d. Rayleigh fading subcarriers with various number of users (for comparison with Fig. 1 in [27]).	89
5.5	Performance of uplink MC-CDMA with MRC in a channel with (a) 4 independent Nakagami fading paths, $\{m_l\} = [9, 5, 4, 2]$, $\{\Omega_l\} = [0.45, 0.25, 0.2, 0.1]$; (b) 3 independent Nakagami fading paths, $\{m_l\} = [4, 3, 1]$, $\{\Omega_l\} = [0.46, 0.38, 0.15]$	90
5.6	Comparison of the approximated MRC pdf (5.18) with the exact MRC pdf (5.26) for a dual-branch channel with branch m parameter $\{m_l\} = \{5, 5\}$, branch power $\{\Omega_l\} = \{2, 1\}$. Branch correlations: (a) $\rho = 0.1$; (b) $\rho = 0.5$; (c) $\rho = 0.9$	96
5.7	Comparison of the approximated MRC pdf (5.18) with the exact simulated MRC pdf for a 4-branch channel with branch m parameter $\{m_l\} = [4, 2.3, 3.1, 1.5]$, branch power $\{\Omega_l\} = [0.4779, 0.2353, 0.1985, 0.0882]$. Branch correlations: (a) independent branches; (b) correlated branches with normalized covariance matrix = $[1 \ 0.25 \ 0.40 \ 0.20; 0.25 \ 1 \ 0.22 \ 0.1; 0.4 \ 0.22 \ 1 \ 0.3; 0.2 \ 0.1 \ 0.3 \ 1]$	96

5.8 Performance of uplink MC-CDMA with MRC in a channel with 3 Nakagami fading paths, $\{m_l\} = [9, 7, 3]$, $\{\Omega_l\} = [0.5, 0.3, 0.2]$. Branch correlations: (a) independent paths; (b) correlated paths $\mathbf{C}_\gamma = [1 \ 0.4 \ 0.3; 0.4 \ 1 \ 0.7; 0.3 \ 0.7 \ 1]$ 98

5.9 Performance of uplink MC-CDMA with MRC in a channel with 3 correlated Rician fading paths, $\{K_l\} = [5, 3, 2]$, $\{\Omega_l\} = [0.4, 0.35, 0.25]$, channel (a) contains independent paths with $\mathbf{C}_\mathbf{x} = \mathbf{I}_6$; channel (b) contains correlated paths with $\mathbf{C}_\mathbf{x} = [1 \ 0.1 \ 0.2 \ 0 \ 0 \ 0; 0.1 \ 1 \ 0.5 \ 0 \ 0 \ 0; 0.2 \ 0.5 \ 1 \ 0 \ 0 \ 0; 0 \ 0 \ 0 \ 1 \ 0.1 \ 0.2; 0 \ 0 \ 0 \ 0.1 \ 1 \ 0.5; 0 \ 0 \ 0 \ 0.2 \ 0.5 \ 1]$. . . 102

5.10 Error floor values vs. number of users for uplink MC-CDMA with MRC. Number of subcarriers = 128. 103

5.11 User capacity of uplink MC-CDMA with MRC in channels with i.i.d. Rayleigh fading paths. 104

5.12 Normalized covariance of Gamma and corresponding Nakagami RVs. 108

5.13 Pdfs generated using Decomposition Method vs. theoretical pdfs for Nakagami RVs with equal and non-integer m -parameter values. 115

5.14 Pdfs generated using Decomposition and Sim’s Method vs. theoretical pdf for 3 Nakagami RVs with different m -parameter values. . . 116

6.1 BER of downlink ORC MC-CDMA with 32 subcarriers under independent/correlated Rayleigh subcarrier fading. 124

6.2 BER of downlink ORC MC-CDMA with 4/32/128/1024 i.i.d. Rayleigh fading subcarriers. 124

6.3 BER of downlink ORC MC-CDMA with 32 i.i.d. Rayleigh/Rician fading subcarriers. 125

6.4 BER of downlink ORC MC-CDMA with 64 i.i.d. Rayleigh/Nakagami fading subcarriers. 125

6.5 BER of downlink MC-CDMA (high system loading: 20 users) with different equalization and combining schemes in 4-path Rayleigh fading channel. 127

6.6 BER of downlink MC-CDMA (full system loading: 32 users) with different equalization and combining schemes in 4-path Rayleigh fading channel. 127

6.7 Theoretical BER lower bound for the Truncated ORC scheme of MC-CDMA with different numbers of users. 130

6.8 Required E_b/N_0 to achieve $BER = 10^{-3}$ for downlink MC-CDMA (32 subcarriers) with different Truncated ORC schemes in a channel with 4 i.i.d. Rayleigh fading paths. 132

6.9 Additional E_b/N_0 required by the Truncated ORC compared with MMSEC schemes, to achieve $BER = 10^{-3}$ in a channel with 4 i.i.d. Rician fading paths. 133

6.10 Reduced complexity order vs. system loading for MC-CDMA system with 32 subcarriers. 140

6.11 Required E_b/N_0 to achieve $BER=10^{-3}$ for MC-CDMA system with MMSEC in channels of 4 i.i.d. Rayleigh fading paths or of i.i.d. Rayleigh fading subcarriers. 140

List of Tables

3.1	ATDMA Picocell channel consisting of 5 Rician and 1 Rayleigh paths with decreasing power profile.	32
3.2	A channel with 3 Rayleigh fading paths of equal power.	34
3.3	Correspondence between channel impulse response (CIR) conditions and subcarrier fading conditions.	35
3.4	Correction factor for the QAM BER in Rician fading channels ($0 \leq K \leq 30$)	43
3.5	Required power for subcarriers to achieve $BER=10^{-3}$ at different constellation levels.	45
5.1	Comparison of correlation coefficients.	114

Chapter 1

Introduction

1.1 Motivation

With the increasing demand for high-speed wireless services, future wireless communication system will demand higher data transmission rates. However, when the transmission rate increases, the wireless signal propagation channel tends to suffer from frequency-selective multipath fading. This means that the received signal consists of several delayed and randomly weighted replicas of the transmitted signal. This gives rise to a form of self-interference, commonly referred to as inter-symbol interference (ISI). Multicarrier modulation (MCM) can combat this problem [1–6].

MCM refers to the generic technique of transmitting a high-rate data stream by dividing it into several parallel lower-rate streams and modulating them onto separate frequency carriers for concurrent transmission. By converting a high-rate serial data stream to several parallel low-rate data streams, each sub-stream has longer symbol durations, thus the relative amount of signal dispersion caused by the multipath delay spread is decreased. To eliminate ISI completely, a guard time is introduced into the MCM waveform before transmission. The effect of ISI can be totally eliminated if the length of guard time is longer than the maximum delay spread of the ISI channel and if the guard time is removed before demodulation.

CHAPTER 1. INTRODUCTION

Another important feature of MCM technique is that the subcarriers can be designed to overlap in the spectrum but remain orthogonal to each other. As long as the channel fading is quasi-static, i.e., the channel fading does not change much within one MCM waveform duration, the subcarrier orthogonality ensures that the receiver can fully separate the subcarriers while high spectral efficiency is achieved by spectral overlapping among the subcarriers.

Orthogonal frequency division multiplexing (OFDM) is a common form of MCM, in which the subcarriers are modulated using discrete Fourier transform (DFT) by fast Fourier transform (FFT). The use of fast algorithms such as FFT greatly enhances the implementation efficiency of OFDM, and hence its viability for realtime application. To enable multiple access, multicarrier code division multiple access (MC-CDMA), which combines the principles of direct sequence code division multiple access (DS-CDMA) with OFDM was introduced [7–9]. With MC-CDMA, every data symbol is spread and transmitted on multiple narrowband subcarriers. Multiple access is possible if data of different users are spread with a set of orthogonal or near-orthogonal codes. Since all subcarriers are unlikely to undergo deep fade simultaneously, frequency diversity is achieved by proper combining of the subcarriers at the receiver side.

For its implementation efficiency, multipath fading mitigation capability and high spectral efficiency, OFDM has been adopted for use in digital audio broadcasting (DAB), digital video broadcasting (DVB), asymmetric digital subscriber line (ADSL) as well as the next-generation wireless LAN standards, namely IEEE 802.11a and European high performance LAN (HIPERLAN/2). A recent trend is to use OFDM modulation in ultra wideband (UWB) systems [10, 11], while MC-CDMA is considered as one of the candidates for the future 4th generation wireless communications systems [12–16].

CHAPTER 1. INTRODUCTION

The multipath ISI effect, when viewed in frequency domain, corresponds to the frequency-selective fading effect. Since the data are carried by different subcarriers in the frequency domain, the MCM system performance is largely determined by the frequency response of the multipath fading channel. In the current literature, however, most practical channel characterization models, such as the COST model, ATDMA model, 3GPP model [17, 18], are presented as time-domain models in the form of channel impulse response (CIR) or power delay profile. The frequency-domain (subcarrier) fading models of these practical channels in MCM systems are not available. Hence most existing studies of MCM system performance are either based on Monte Carlo simulation [6, 19–22] or some simplified path/subcarrier fading assumptions [7, 23–28]. This motivates us to derive the subcarrier fading characteristics from the path fading information, so as to facilitate the analysis of OFDM system performance based on well-established time-domain channel models.

Although knowledge of subcarrier fading characteristics can be applied to analyze the performance of OFDM systems, the performance analysis of MC-CDMA systems is considerably more complicated because the receiver operations involve different ways of equalization and combining of a large number of correlated subcarriers, and the received signal is subject to multiuser interference (MUI), in addition to fading. In the literature, computer simulations appear to be the most commonly used approach to study the performance of MC-CDMA systems [19–22]. Although analytical performance formulations have also been reported [24, 26–29], they are either rather mathematically complex, hence computationally intensive to evaluate or based on simplified path/subcarrier fading. This motivates us to obtain more simplified and tractable performance analysis methodologies for MC-CDMA systems with different equalization and combining schemes.

CHAPTER 1. INTRODUCTION

Among the various equalization and combining schemes proposed for MC-CDMA, the minimum mean square error combining (MMSEC) scheme performs best, but it is also most computationally complex to realize as it involves complex matrix inversion of large size matrices[2, 30]. But orthogonality restoring combining (ORC) is the second best scheme for downlink MC-CDMA, but its performance gap from the optimal MMSEC is quite large especially at high signal-to-noise ratio (SNR) or low bit error rate (BER) regions. Therefore, improvements/enhancements of these equalization and combining schemes for MC-CDMA systems are desired.

1.2 Objectives

The major objectives of the project are to derive the frequency-domain fading properties of MCM channels, conduct in-depth performance analysis of MCM systems in generalized frequency-selective fading channels, achieve performance enhancement of MCM systems by improved modulation or detection schemes, and reduce complexity of optimal MCM detection schemes. The MCM systems considered are OFDM systems and MC-CDMA systems with different equalization and combining schemes. We aim to develop tractable BER formulas as well as to enhance the system performance based on the understanding/insights obtained from the performance analysis. Specifically, we aim to

- (i) Derive the frequency-domain fading models for MCM systems, i.e., the sub-carrier fading characteristics, in terms of their first-order fading statistic (i.e., probability density function (pdf)) and second-order statistic (i.e., fading correlations between subcarriers) based on the time-domain CIR information (i.e., path fading and correlation characteristics) of the wireless transmission channel.

CHAPTER 1. INTRODUCTION

- (ii) Analyze the BER performance of OFDM and MC-CDMA systems in frequency-selective fading channels with generalized path/subcarrier fading pdf and correlations. For MC-CDMA, we aim to achieve efficient analytical BER formulations for several important equalization and combining schemes, such as maximal ratio combining (MRC) and ORC.
- (iii) Enhance OFDM and MC-CDMA systems. The enhancement can come in two forms: to improve the system performance or to reduce the system complexity. Specially, we aim to propose new adaptive OFDM schemes based on knowledge of the subcarrier fading statistics. For MC-CDMA systems, we aim to improve the existing ORC and MMSEC schemes, which are important equalization and combining schemes for downlink MC-CDMA.

1.3 Major Contributions of the Thesis

The major contributions of the thesis include the following:

- (i) We derive the fading pdfs and fading correlations of the subcarriers in an MCM system which operates in multipath fading channels with arbitrary combination of uncorrelated or correlated Rician paths. This knowledge allows us to predict the exact BER performance at the subcarrier level for all possible practical OFDM channel conditions.
- (ii) We show that many practical MCM channels have non-uniform subcarrier fading distributions. For such channels, we propose a novel “statistical OFDM adaptation” scheme to achieve transmission power savings and spectral efficiency improvement at equalized subcarrier BERs based on knowledge of the subcarrier pdfs. Compared to the existing instantaneous OFDM adaptation schemes, our statistical adaptation scheme is computationally less

CHAPTER 1. INTRODUCTION

- complex and less costly to implement as it does not require the instantaneous channel state information to be fed back to the transmitter at short intervals.
- (iii) We simplify the analytical BER expressions of asynchronous uplink MC-CDMA with MRC. The results consider the effects of subcarrier correlations in generalized frequency-selective fading channels with correlated Nakagami or Rician fading paths. A new closed-form BER formula is derived for correlated Nakagami fading channels. This formula does not involve integration; hence it is much easier to evaluate. For channels with correlated Rician paths, the BER formula involves only one-dimensional integration, which can be easily evaluated numerically.
 - (iv) We develop two simplified BER formulas for downlink MC-CDMA with ORC. Specifically, a BER lower bound and a very tight BER approximation expression are obtained, both involving at most one-dimensional integration. Through this study, it is found that the ORC performance is dominated by the weakest instantaneous subcarrier. Hence we propose a novel “Truncated ORC” scheme by omitting the weakest subcarrier in the combining operation of conventional ORC. Though with such simple modification and hence with virtually no increase in complexity, the proposed scheme is demonstrated to perform impressively close to the optimal MMSEC scheme. The closed-form BER lower bound for the Truncated ORC scheme is also developed.
 - (v) The complexity of the optimal MMSEC of MC-CDMA systems is much higher than that of MRC and ORC because it involves matrix inversion. In this project, we propose a way to reduce the matrix inversion dimension for calculating the MMSEC equalizer coefficients.

- (vi) Two new fading generators are proposed to generate correlated Nakagami fading signals with arbitrary fading parameters and correlations. These simulators are useful in the computer simulations of MCM channels with correlated paths or subcarriers.

1.4 Organization of the Thesis

The thesis is organized as follows. Chapter 2 provides an overview of the MCM systems. Firstly, the ISI problem arising from frequency-selective fading channels is introduced. Several techniques to combat this ISI problem are discussed, among which MCM is shown to be efficient for solving this problem. Then the principles of the two MCM systems, OFDM and MC-CDMA, are presented.

In Chapter 3, the subcarrier fading distributions and the subcarrier fading correlations for the MCM systems are derived from the time-domain statistical CIR information. In this analysis, frequency-selective channels with arbitrary combinations of correlated or uncorrelated Rician/Rayleigh fading paths are considered. The derived subcarrier fading statistics are used to predict the subcarrier BERs of OFDM systems. For MCM channels which exhibit non-uniform subcarrier fading, a statistical adaptive OFDM scheme, which adapts the transmission parameters based on the subcarrier fading statistics instead of the instantaneous channel state information, is proposed and its performance is studied.

The signal models for downlink and uplink MC-CDMA channels are introduced in Chapter 4. A general approach to reduce the integration dimension of the BER formulas is discussed. Simulation results are given to verify our approach. Two special cases for which further simplification of the analytical BER expressions can be achieved, i.e., the MRC uplink channel and the ORC downlink channel, are discussed in Chapter 5 and Chapter 6 respectively.

CHAPTER 1. INTRODUCTION

In Chapter 5, we conduct a detailed BER analysis for the uplink MC-CDMA with MRC in channels with Rayleigh, Rician or Nakagami paths with arbitrary fading parameters and correlations. A simplified signal model for the MRC output in uplink channel is first developed. Specific BER formulations and their corresponding verification results for various fading channels are then given. In this chapter, we also proposed two methods to generate correlated Nakagami fading signals.

In Chapter 6, we analyze the performance of ORC and propose a Truncated ORC scheme which omits the weakest instantaneous subcarrier in the combining operation of ORC in order to improve system performance. Following that, a complexity reduction technique for MMSEC is presented and discussed.

Finally, conclusions are drawn in Chapter 7. Possible future extensions of the current works are also discussed.

Chapter 2

Multicarrier Modulation

2.1 Wireless Propagation Channel

The performance of wireless communication systems depends on the statistical properties of the wireless transmission channel. Hence the study of the multipath fading characteristics is a prerequisite to the understanding of wireless system performance. With good knowledge of the channel behavior, we can improve the system performance using techniques that mitigate the channel impairment effects.

While the propagation of radio waves through the wireless channels is a complicated mechanism characterized by various effects, such as free space attenuation, shadowing, reflections, diffractions and scatterings, accurate statistical channel models have been developed. In general, the random signal strength variation in a mobile radio channel can be decomposed into a large-scale path loss component, a slow-varying component due to shadowing and a small-scale fast-varying component due to multipath fading [31].

In this thesis, we focus on the effect of small-scale fading on broadband wireless communication. Small-scale fading refers to the rapid fluctuation of signal amplitude and phase over a short time interval or travel distance [31]. In the wireless propagation channel, the receiver receives several versions of the transmitted signal which have been reflected and diffracted by buildings, mountains and other

CHAPTER 2. MULTICARRIER MODULATION

obstructions and which hence suffer different propagation delays, attenuations and doppler shifts. These “multipath” signals may add constructively or destructively at the receiver, hence resulting in rapid fluctuations on the received signal strength. In practice, this phenomenon is referred to as “multipath fading”.

2.1.1 Flat Fading vs. Frequency-Selective Fading

An important characterization of multipath fading channel is (frequency-)flat fading vs. frequency-selective fading. If the wireless channel has almost constant gain and linear phase response over the bandwidth of the transmitted signal, the received signal will undergo flat fading, i.e., the channel effect is modelled as an amplitude scaling and a phase change effect. The bandwidth over which the channel has almost constant gain and linear phase response is called the coherence bandwidth of the channel. But if the coherence bandwidth is smaller than the signal bandwidth, the channel will exert different attenuation and phase shifts on different frequency components of the transmitted signal. The received signal will then undergo frequency-selective fading.

The coherence bandwidth, denoted as B_c , is related to the delay spread of the multipath fading channel. In some references [31, 32], B_c is approximated as inversely related to the maximum delay spread $\tau_{\max}$ of the channel, i.e.,

$$B_c \simeq \frac{1}{\tau_{\max}}. \quad (2.1)$$

For broadband wireless communication, data rates of at least a few mega bits per seconds (Mbps) may be required. For example, the data rate for the IEEE 802.11a and the European HIPERLAN/2 standard for broadband wireless LAN is up to 54 Mbps. Correspondingly, the symbol duration for such systems would be less than 1 micro second (μs). But wireless channels may introduce multipath

CHAPTER 2. MULTICARRIER MODULATION

delay spread of as much as $20\mu\text{s}$ [31]. Therefore the broadband wireless systems are commonly subject to frequency-selective fading as the signal bandwidth is much larger than channel coherence bandwidth B_c .

2.1.2 Inter-Symbol Interference (ISI)

As discussed above, the symbol duration for broadband wireless systems may be much less than the delay spread of the multipath channel. This implies that the received signal may be composed of many delayed replicas of the transmitted signal. These delayed signals may cause severe interference between adjacent symbols, known as inter-symbol interference (ISI). This effect, when viewed in frequency domain, corresponds to the frequency-selective fading response of the wireless channel.

Since ISI is a form of self-interference, it cannot be eliminated by simply increasing the transmitting power, as doing so merely leads to irreducible error floor. Without ISI compensation, we can only transmit data up to a certain data rate limit to ensure acceptable performance. This is not desirable. Therefore, it is important to design high speed wireless communication systems with ISI compensation in mind.

Many techniques have been developed to combat ISI. In the next section, we will outline some.

2.2 Measures to Mitigate ISI

2.2.1 Equalization for Single-Carrier Transmission

In conventional single-carrier systems, a matched filter is used in the receiver to maximize the signal to noise ratio (SNR) of the received signal and an equalizer is used to remove the ISI introduced by the multipath fading channel. The equalizer is commonly implemented as a multi-tap adaptive filter. Many equalizer structures and algorithms, such as linear equalizer (LE), decision feedback equalizer (DFE), fractionally spaced equalizer (FSE), blind equalizer (BE), etc., have been proposed for different channel characteristics. However, systems with very high data rates or long channel delay spread would increase the number of equalizer taps required and complexity of the equalizer too. Hence the system complexity for single-carrier broadband transmission is usually dominated by equalization [6].

CHAPTER 2. MULTICARRIER MODULATION

2.2.2 Multicarrier Modulation (MCM)

In recent years, multicarrier modulation (MCM) has attracted great interest in wireless communications as a promising and practical modulation technique to mitigate ISI. In MCM systems, a serial high-speed data stream is divided into many parallel low-rate data sub-streams and transmitted simultaneously onto a bank of carriers (each one is called a subcarrier). The transmission speed on every subcarrier is lower than the original data rate and the duration of the subcarrier data is enlarged to be several times the channel delay. Thus, the effect of ISI is reduced. To eliminate ISI completely, a guard time is introduced into the MCM waveform. The length of guard time is chosen to be longer than the maximal channel delay spread, such that the multipath component from one symbol cannot interfere with the next symbol. Furthermore, MCM ensures that every subcarrier undergo flat fading. This enables a simple one-tap equalization to be performed in the receiver at the subcarrier level.

Moreover, MCM is a spectrum efficient modulation scheme. In conventional frequency division multiplexing (FDM) system, the system bandwidth is divided into several non-overlapping subcarrier channels in order to eliminate inter-carrier interference. However this leads to spectral inefficiency, as shown in Fig. 2.1(a). In contrast, spectral overlapping is allowed in MCM systems, as shown in Fig. 2.1(b). The interference between subcarriers is eliminated by properly designing the subcarrier spacing so that they remain orthogonal to each other despite the spectral overlapping.

In fact, the concept of using parallel data transmission and overlapping subcarriers were proposed in mid-1960s. However, employing large number of modulators, oscillators and filters at the transmitter and receiver results in high implementation

CHAPTER 2. MULTICARRIER MODULATION

Figure 2.1: Bandwidth occupancy for (a) conventional FDM, and (b) MCM with spectrum overlapping.

complexity. Then, in 1970s, this limitation is greatly reduced by the use of discrete Fourier Transform (DFT) and fast Fourier Transform (FFT) to implement the MCM and demodulation. Meanwhile, the development of complementary metal oxide semiconductor (CMOS) technology has made great advancement. This helps made digital signal processing (DSP) very efficient for the calculation of FFT in real time. Hence, MCM is a very effective and efficient way to combat ISI. This is why MCM is very popular now and has been adopted in many broadband wireless standards such as DAB, DVB, ADSL, IEEE 802.11a, HIPERLAN/2, etc.

Two types of MCM systems are considered in this PhD project, namely orthogonal frequency division multiplexing (OFDM) and multicarrier code division multiple access (MC-CDMA). They will be discussed next.

2.3 Fundamentals of OFDM

2.3.1 Transmission Principle

The basic principle of OFDM is to split a high-rate data stream into a number of lower-rate data sub-streams which are transmitted simultaneously over separate Fourier subcarriers, as shown in Fig. 2.2. In other words, separate data symbols are transmitted on different frequency subcarriers in parallel. After going through frequency-selective multipath fading channel, the subcarriers suffer only flat fading because the subcarrier channels are ensured to be sufficiently narrowband [6]. As a result, only a simple one-tap frequency-domain equalization is needed to restore the amplitude and phase of the subcarrier data.

Figure 2.2: Principle of OFDM transmission.

CHAPTER 2. MULTICARRIER MODULATION

Figure 2.3: A generic OFDM transceiver model.

Fig. 2.3 shows the transceiver model for a generic OFDM system. Firstly the serial binary data stream goes through digital modulation which maps the binary data into symbols according to the modulation constellation, e.g., quadrature amplitude modulation (QAM) with Gray mapping [33, 34]. The resultant symbols are grouped into blocks of N_c symbols, where N_c is the number of subcarriers. After the serial to parallel (S/P) conversion, the n th modulated symbol, denoted as a_n , is transmitted onto the n th subcarrier. All the N_c subcarriers are then summed together and transmitted into the channel. Denoting the transmitted signal as $s(t)$, it has the form (for simplicity, we consider one OFDM symbol only)

$$s(t) = \sum_{n=1}^{N_c} a_n e^{j2\pi f_n t}, \quad 0 \leq t \leq T_s, \quad (2.2)$$

CHAPTER 2. MULTICARRIER MODULATION

where T_s is the OFDM symbol duration and f_n is the frequency for n th subcarrier. In OFDM, the frequency for the n th subcarrier is

$$f_n = f_0 + (n - 1) \Delta f, \quad 1 \leq n \leq N_c, \quad (2.3)$$

where f_0 is the first subcarrier frequency and the subcarrier spacing is chosen to satisfy

$$\Delta f = \frac{1}{T_s}. \quad (2.4)$$

After going through the multipath channel, every subcarrier suffers an amplitude scaling h_n and a phase distortion φ_n . These channel fading effects on the n th subcarrier can be modelled as $\tilde{h}_n = h_n \exp(j\varphi_n)$. Hence the received signal is

$$r(t) = \sum_{n=1}^{N_c} \tilde{h}_n a_n e^{j2\pi f_n t} + \eta(t), \quad 0 \leq t \leq T_s, \quad (2.5)$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN).

At the receiver side, as shown in Fig. 2.3, we perform OFDM demodulation to recover the symbol a_n on the n th subcarrier. If we do not consider AWGN for simplicity, a_n can be recovered by multiplying the received signal $r(t)$ with $e^{-j2\pi f_n t}$ and integrating over the OFDM symbol duration T_s , i.e.,

$$\begin{aligned} \hat{a}_n &= \frac{1}{T_s} \int_0^{T_s} r(t) e^{-j2\pi f_n t} dt \\ &= \frac{1}{T_s} \sum_{i=1}^{N_c} \tilde{h}_i a_i \int_0^{T_s} e^{-j2\pi(n-i)\Delta f t} dt \\ &= \frac{1}{T_s} \sum_{i=1}^{N_c} \tilde{h}_i a_i \int_0^{T_s} e^{-j2\pi \frac{(n-i)t}{T_s}} dt \\ &= \tilde{h}_n a_n \end{aligned} \quad (2.6)$$

where

$$\int_0^{T_s} e^{-j2\pi \frac{(n-i)t}{T_s}} dt = \begin{cases} 0, & n \neq i. \\ T_s, & n = i. \end{cases} \quad (2.7)$$

CHAPTER 2. MULTICARRIER MODULATION

The equations above indicate that by properly choosing the subcarrier frequency spacing according to (2.4) and integrating over T_s , the subcarriers are orthogonal to each other and there is no inter-carrier interference. The recovered data $\hat{a}_n$ on the n th subcarrier is only scaled by its fading parameter $\tilde{h}_n$. Hence, as shown in Fig. 2.3, the channel effect on the n th subcarrier can be easily compensated by the one-tap frequency-domain equalizer coefficient α_n , e.g., $\alpha_n = 1/\tilde{h}_n$. Therefore, although spectral overlapping among subcarriers occurs due to frequency spacing $\Delta f = 1/T_s$, the subcarriers remain orthogonal to each other and the data can be successfully recovered with simple frequency-domain equalization.

2.3.2 FFT-Based Implementation

As discussed in Section 2.2.2, it is the fast algorithms like FFT that makes OFDM and MCM transmission feasible today. In this section, we will explain why OFDM can be implemented using FFT.

Without loss of generality, let $f_0 = 0$ in (2.3). Then the transmitted signal $s(t)$ in (2.2) can be rewritten as

$$s(t) = \sum_{n=1}^{N_c} a_n e^{j2\pi(n-1)\Delta f t} = \sum_{n=1}^{N_c} a_n e^{j2\pi(n-1)t/T_s}. \quad (2.8)$$

If we sample $s(t)$ at the interval T_s/N_c , the i th ($1 \leq i \leq N_c$) sample is

$$s_i = \sum_{n=1}^{N_c} a_n e^{j2\pi(n-1)\frac{(i-1)T_s}{N_c} \frac{1}{T_s}} = \sum_{n=1}^{N_c} a_n e^{j2\pi(n-1)(i-1)/N_c}. \quad (2.9)$$

Note that (2.9) is exactly the same expression as inverse discrete Fourier transform (IDFT) of the symbol sequence $\{a_n\}$ [35]. Similarly it can also be proven that the demodulation process at the receiver side of OFDM can be implemented using DFT.

In practice, the number of subcarriers N_c is selected to be a power of 2. This is to enable fast algorithms, such as inverse fast Fourier transform (IFFT) and FFT,

CHAPTER 2. MULTICARRIER MODULATION

to be used in place of IDFT and DFT respectively [35]. Note that IFFT/FFT are just fast implementation algorithms of IDFT/DFT. Hence hereafter we will use the terms IDFT/DFT and IFFT/FFT interchangeably.

2.3.3 Cyclic Prefix

Another important feature of OFDM is the usage of cyclic prefix as a guard interval to eliminate ISI and inter-carrier interference (ICI). The principle of cyclic prefix is shown in Fig.2.4. Suppose the OFDM symbol duration is T_s , the signal of the last T_g interval is copied and appended to the start of the OFDM symbol. At the receiver side, this cyclic prefix is discarded and only the signal in the last T_s interval is used for demodulation. The length of cyclic prefix is chosen to be larger than the maximum expected channel delay spread, so that the multipath components of one OFDM symbol cannot interfere with the next OFDM symbol, hence ISI is removed. Besides eliminating ISI, cyclic prefix also prevents ICI. It ensures that the delayed replicas of the signal always have an integer number of cycles within the FFT integration interval. As a result, multipath signals with delays smaller than the cyclic prefix will not cause either ICI or ISI in an OFDM system.

Figure 2.4: Cyclic prefix insertion of OFDM symbols.

2.4 MC-CDMA – Combining OFDM with CDMA

OFDM is just a modulation technique. To enable multiple access, OFDM and code division multiple access (CDMA) can be combined. This leads to several hybrid multiple access techniques, such as MC-CDMA, multicarrier direct sequence CDMA (MC-DS-CDMA) and multitone CDMA (MT-CDMA) [20, 36–40].

In [20], it is shown that MC-CDMA outperforms the conventional DS-CDMA and two other forms of CDMA with OFDM modulation: MC-DS-CDMA and multitone CDMA in the downlink transmission. For uplink transmission, the results in [29] show that MC-CDMA also outperforms DS-CDMA and MC-DS-CDMA. Compared to OFDM, MC-CDMA is less sensitive to frequency offset [41] and has lower peak-to-average power ratio [12]. As a result, MC-CDMA is considered as one of the candidate technologies for the 4th generation wireless communication systems [12–16].

The block diagram of an MC-CDMA system is shown in Fig. 2.5. Each user's data stream is first spread with an unique code sequence, then the spread data will be OFDM modulated. To eliminate ISI, a guard interval in the form of a cyclic prefix is added before transmission, and removed immediately after reception. Similarly, the other users' data go through the same modulation process. The MC-CDMA signals from different users will then be added together in the transmitter (downlink signal) or channel (uplink signal). At the receiver side, the reverse processes are performed and despreading using the unique code sequence of the desired user can be used to extract the data of the desired user.

CHAPTER 2. MULTICARRIER MODULATION

Figure 2.5: Block diagram for a typical MC-CDMA system.

Another important block in MC-CDMA systems is the equalization and combining block, as shown in Fig. 2.5. Its functions are to compensate for the frequency-selective fading channel effect, to achieve diversity combining, as well as to minimize multiuser interference (MUI) of the MC-CDMA signal. Several equalization and combining schemes for MC-CDMA have been proposed in the literature, such as orthogonality restoring combining (ORC), equal gain combining (EGC), maximal ratio combining (MRC) and minimum mean square error combining (MM-SEC) [2, 19, 20]. They will be discussed in more details later in Chapters 4 to 6 of this thesis.

CHAPTER 2. MULTICARRIER MODULATION

Although MC-CDMA resembles OFDM in transmitting information in the frequency domain using Fourier subcarriers, the manner in which the subcarriers are utilized in both systems is very different. A more detailed graphical representation of the information carried by the subcarriers in MC-CDMA systems is shown in Fig. 2.6. While OFDM systems transmit one modulated symbol on one subcarrier, MC-CDMA transmit one data symbol on multiple subcarriers, as shown in Fig. 2.6(a). For OFDM systems, if a subcarrier is in deep fade, the data transmitted on the subcarrier may be lost. However, since MC-CDMA transmit the data over multiple subcarriers and it is unlikely for all subcarriers to fade simultaneously, data recovery will be more robust as frequency diversity is achieved.

(a) MC-CDMA transmission for one user

(b) MC-CDMA transmission for multiple users

Figure 2.6: Principle of MC-CDMA transmission.

Chapter 3

Frequency-Domain (Subcarrier) Fading Characteristics of OFDM Signals

3.1 Introduction

The block diagram of an OFDM system with frequency-domain equalization is shown in Fig. 3.1(a). As discussed in Section 2.3, the information data stream is first serial-to-parallel converted. The multicarrier modulation process can then be accomplished by performing inverse fast Fourier transform (IFFT) to generate the modulated subcarriers. After inserting the cyclic prefix, the signal is transmitted in the multipath fading channel. The receiver performs the reverse processes as the transmitter, i.e., FFT, parallel-to-serial conversion, etc.

In a quasi-static frequency-selective fading channel, as long as the cyclic prefix has the same or larger length compared to the maximum channel delay spread, it can be shown that the OFDM receiver output is equivalent to the input data frame multiplied by the DFT of the channel impulse response (CIR) [24, 42, 43], followed by corruption by additive white Gaussian noise (AWGN), as shown in Fig. 3.1(b). A detailed proof of this equivalent model of OFDM and the proof that the frequency-domain (subcarrier) channel fading is given by the DFT of CIR

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

can be found in [42]. Therefore, the subcarrier channel fading characteristics are crucial factors that affect the performance of OFDM systems.

In this chapter, we will focus on the derivation of the subcarrier fading characteristics from the path fading information (i.e., statistical CIR), and applications of the derived subcarrier fading model. In particular, we will derive both the first-order subcarrier fading statistic, which is the subcarrier fading distributions, and also the second-order subcarrier fading statistic, which is the fading correlations between subcarriers. We assume that the channel paths suffer Rician/Rayleigh fading.

Figure 3.1: Block diagram and equivalent model of an OFDM system.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

3.2 Subcarrier Fading Distributions

3.2.1 Channel Impulse Response of Frequency-Selective Fading Channels

Consider the statistical CIR of a typical frequency-selective fading channel with N_p paths, as shown in Fig. 3.2(a). The CIR is commonly represented by the tapped delay line model:

$$g(t) = \sum_{l=1}^{N_p} g_l \exp(j\psi_l) \delta(t - \tau_l) = \sum_{l=1}^{N_p} \tilde{g}_l \delta(t - \tau_l) \quad (3.1)$$

where g_l , ψ_l and τ_l are the fading envelope, phase and propagation delay for the l th path respectively, $\delta(\cdot)$ is the Dirac delta function. Without loss of generality, we set $\tau_1 = 0$. Convolution of the transmitted signal with the CIR gives the received signal.

Figure 3.2: Channel impulse response and frequency response of a frequency-selective fading channel.

In a Rician fading environment, the complex fading path gain $\tilde{g}_l$ can be modelled as a random variable (RV) which consists of a fixed complex component d_l^t and a random diffused component r_l^t with zero-mean complex Gaussian distribu-

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

tion [33, 44], i.e.,

$$\tilde{g}_l = d_l^t + r_l^t, \quad d_l^t = |d_l^t|e^{j\vartheta_l}, \quad (3.2)$$

where ϑ_l is uniformly distributed within $[0, 2\pi)$, representing the phase of the fixed component. The superscript t denotes the *time*-domain nature of the path fading parameters. The fading envelope $g_l = |\tilde{g}_l|$ follows the Rician probability density function (pdf), which is given as

$$p(g_l) = \frac{2(K_l^t + 1)g_l}{\Omega_l^t} \exp\left(-K_l^t - \frac{(K_l^t + 1)g_l^2}{\Omega_l^t}\right) I_0\left(2\sqrt{\frac{K_l^t(K_l^t + 1)}{\Omega_l^t}} g_l\right) \quad (3.3)$$

where $\Omega_l^t = E[g_l^2]$, $E[\cdot]$ denotes statistical expectation, $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind, and K_l^t is the Rician K factor defined as the power ratio between the fixed component d_l^t and the diffused component r_l^t , i.e.,

$$K_l^t = \frac{\text{fixed component power}}{\text{diffused component power}} = \frac{|d_l^t|^2}{\text{var}(r_l^t)} \quad (3.4)$$

where $\text{var}(\cdot)$ denotes the variance. The fading power of the l th Rician path is given by

$$\Omega_l^t = E[|g_l|^2] = |d_l^t|^2 + \text{var}(r_l^t). \quad (3.5)$$

When $K_l^t = 0$, the Rician distribution reduces to the Rayleigh distribution

$$p(g_l) = \frac{2g_l}{\Omega_l^t} \exp\left(-\frac{g_l^2}{\Omega_l^t}\right). \quad (3.6)$$

To consider the statistical correlation between different fading paths, we denote $\mathbf{C}_g$ as the path covariance matrix, which is defined as

$$\mathbf{C}_g = E[\tilde{\mathbf{g}}\tilde{\mathbf{g}}^H] - E[\tilde{\mathbf{g}}]E[\tilde{\mathbf{g}}^H] \quad (3.7)$$

where the superscript H indicates Hermitian transpose, and $\tilde{\mathbf{g}} = [\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_{N_p}]^T$. Note that $\mathbf{C}_g$ will be reduced to a diagonal matrix when the path fadings are independent.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

3.2.2 Frequency-Domain (Subcarrier) Fading Statistics

As shown in Fig. 3.2(b), taking DFT on the CIR gives the fading gains of the OFDM subcarriers, which constitute the discrete frequency response of the transmission channel. Provided that τ_l is an integer multiple of T_s/N_c , the complex fading gain $\tilde{h}_n = h_n \exp(j\varphi_n)$ of the n th subcarrier ($1 \leq n \leq N_c$, N_c is the FFT size) is [24, 43, 45, 46]

$$\begin{aligned}\tilde{h}_n &= \text{DFT}_n(\text{CIR}) \\ &= \sum_{l=1}^{N_p} d_l^t e^{-j2\pi(n-1)\frac{\tau_l}{T_s}} + \sum_{l=1}^{N_p} r_l^t e^{-j2\pi(n-1)\frac{\tau_l}{T_s}} \\ &= \text{DFT}_n(\mathbf{d}^t) + \text{DFT}_n(\mathbf{r}^t) \\ &= d_n^f + r_n^f\end{aligned}\tag{3.8}$$

where T_s is the OFDM symbol duration (excluding the cyclic prefix), $\mathbf{d}^t$ and $\mathbf{r}^t$ are the vector forms of $\{d_l^t\}$ and $\{r_l^t\}$ respectively, i.e., $\mathbf{d}^t = [d_1^t \ d_2^t \ \cdots \ d_{N_p}^t]$ and $\mathbf{r}^t = [r_1^t \ r_2^t \ \cdots \ r_{N_p}^t]$. The superscript f in the last line of (3.8) is to indicate the frequency-domain nature of the subcarrier fading parameters.

Note that in (3.8), d_n^f and r_n^f are a fixed complex component and a zero-mean complex Gaussian RV respectively, by virtue of the statistical nature of their respective time-domain counterparts (d_l^t and r_l^t). Hence the subcarrier fading envelope h_n can be modelled as a Rician RV, too. To determine the Rician parameters for h_n , we need to find out the corresponding fixed and diffused power as in (3.4). From (3.8), we have

$$|d_n^f|^2 = |\text{DFT}_n(\mathbf{d}^t)|^2 = \left| \sum_{l=1}^{N_p} \sqrt{\frac{K_l^t \Omega_l^t}{1 + K_l^t}} e^{-j(2\pi(n-1)\frac{\tau_l}{T_s} - \vartheta_l)} \right|^2.\tag{3.9}$$

From (3.8), the subcarrier fading relates to the path fading as $\tilde{h}_n = \mathbf{w}_n^T \tilde{\mathbf{g}}$, where $\tilde{\mathbf{g}}$ is the path fading vector as $\tilde{\mathbf{g}} = [\tilde{g}_1, \tilde{g}_2, \cdots, \tilde{g}_{N_p}]^T$ and $\mathbf{w}_n = [1 \ e^{-j2\pi(n-1)\tau_2/T_s}$

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

$\dots e^{-j2\pi(n-1)\tau_{N_p}/T_s}]^T$. The superscript T indicates transpose. Hence the diffused power of the n th subcarrier is generally given by [47, 48]

$$\text{var}(r_n^f) = \mathbf{w}_n^H \mathbf{C}_g \mathbf{w}_n \quad (3.10)$$

where $\mathbf{C}_g$ is the path covariance matrix defined in (3.7).

With (3.9) and (3.10), the K factor and fading power of the n th subcarrier can be obtained compactly as

$$K_n^f = \frac{|\text{DFT}_n(\mathbf{d}^t)|^2}{\mathbf{w}_n^H \mathbf{C}_g \mathbf{w}_n}, \quad (3.11)$$

$$\Omega_n^f = |\text{DFT}_n(\mathbf{d}^t)|^2 + \mathbf{w}_n^H \mathbf{C}_g \mathbf{w}_n. \quad (3.12)$$

In summary, we have shown for fading channels with Rician paths that OFDM subcarriers also have Rician fading gains, and their fading parameters can be determined from the time-domain CIR information using (3.11) and (3.12).

3.2.3 Case Studies and Discussions

In order to find out how the subcarrier power and K factor vary from subcarrier to subcarrier in typical wireless communication environment, various types of fading channels will next be investigated.

3.2.3.1 Channels with Independent Paths

When the fading channel consists only of independent paths, the channel covariance matrix $\mathbf{C}_g$ is a diagonal matrix whose l th element is the variance of r_l^t , hence the variance $\text{var}(r_n^f)$ of the diffused component of the subcarrier fading coefficient can be reduced to

$$\text{var}(r_n^f) = \mathbf{w}_n^H \mathbf{C}_g \mathbf{w}_n = \sum_{l=1}^{N_p} \text{var}(r_l^t) = \sum_{l=1}^{N_p} \frac{\Omega_l^t}{1 + K_l^t}. \quad (3.13)$$

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

As the power of the fixed component $|d_n^f|^2$ is given by (3.9), the subcarrier K factor and power are given respectively by:

$$K_n^f = \frac{\left| \sum_{l=1}^{N_p} \sqrt{\frac{K_l^t \Omega_l^t}{1+K_l^t}} e^{-j(2\pi n \frac{\tau_l}{T_s} - \vartheta_l)} \right|^2}{\sum_{l=1}^{N_p} \frac{\Omega_l^t}{1+K_l^t}}, \quad (3.14)$$

$$\Omega_n^f = \left| \sum_{l=1}^{N_p} \sqrt{\frac{K_l^t \Omega_l^t}{1+K_l^t}} e^{-j(2\pi n \frac{\tau_l}{T_s} - \vartheta_l)} \right|^2 + \sum_{l=1}^{N_p} \frac{\Omega_l^t}{1+K_l^t}. \quad (3.15)$$

Notice that $\text{var}(r_n^f)$ is the same for all subcarriers in this case, i.e., $\sum_{l=1}^{N_p} \frac{\Omega_l^t}{1+K_l^t}$ is a constant for all values of n in (3.13) to (3.15). This implies that the subcarrier power Ω_n^f and the subcarrier K factor K_n^f will have the same spectral profile. This will be illustrated later in Fig. 3.3.

We shall next study a few variations of channels with independent paths.

Channels with All Rayleigh Paths

When all the channel paths are Rayleigh fading, all the subcarriers will have zero K values because $K_l^t = 0$ for all l in (3.14), hence $K_n^f = 0$ and $\Omega_n^f = \text{var}(r_n^f) = \sum_{l=1}^{N_p} \text{var}(r_l^t)$. At a result, all the subcarriers will also have the same average power irrespective of the CIR profile of the transmission channel. In short, all the subcarriers will have Rayleigh pdfs with the same power.

Channels with One Rician Path and Multiple Rayleigh Paths

We now consider fading channel with several paths but only one of them is Rician, i.e., the i th path is Rician with K factor K_i^t and power Ω_i^t , while the other paths are Rayleigh. Practical examples of this type of channel can be found in [18, 49, 50]. etc. In this case, normally the first arriving path is Rician while the rest are Rayleigh.

From (3.9), the fixed component power for the n th subcarrier is

$$|d_n^f|^2 = \left| \sum_{l=1}^{N_p} \sqrt{\frac{K_l^t \Omega_l^t}{1+K_l^t}} e^{-j(2\pi n \frac{\tau_l}{T_s} - \vartheta_l)} \right|^2 = \frac{K_i^t \Omega_i^t}{1+K_i^t}, \quad \forall n \quad (3.16)$$

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

which indicates that d_n^f is the same for all subcarriers. Consequently, Ω_n^f and K_n^f will be the same for all subcarriers, too. This means that all the subcarriers will have exactly the same Rician pdf. Furthermore, as the subcarrier variance consists of contributions from both the Rician path and Rayleigh paths in the time domain, i.e., $\text{var}(r_n^f) = \sum_{l=1}^{N_p} \text{var}(r_l^t) > \text{var}(r_i^t)$, the subcarrier K factor is always lower than the K factor of the Rician path, i.e.,

$$K_n^f = \frac{\frac{K_i^t \Omega_i^t}{1+K_i^t}}{\frac{\Omega_i^t}{1+K_i^t} + \sum_{l=1, l \neq i}^{N_p} \frac{\Omega_l^t}{1+K_l^t}} < K_i^t, \quad \forall n. \quad (3.17)$$

Effectively, the Rayleigh paths in the channel paths reduces the K factor of the subcarriers. The more Rayleigh paths, the lower the K_n^f will be.

Channels with Multiple Rician and Rayleigh Paths

Examples of channels with more than one Rician paths can be found in [17, 18]. Having more than one Rician paths leads to a multi-point $\mathbf{d}^t$ profile in (3.9), e.g., $\mathbf{d}^t$ consists of two non-zero elements if there are two Rician paths. As a result, the corresponding frequency-domain $d_n^f = \text{DFT}_n(\mathbf{d}^t)$ will vary as a function of n , i.e., different subcarriers will have different d_n^f . Hence, different subcarriers will have different K factor K_n^f and power Ω_n^f .

To illustrate this case, let's consider an OFDM system implemented over the ATDMA picocell channel model in [17]. The channel is specified in Table 3.1. There are five Rician paths and one Rayleigh path in this channel model. The first Rician path has the largest power and K factor. Assuming that QAM symbols are input to the OFDM system, and the path delays are multiples of QAM symbol duration. Assuming that this OFDM system has 64 subcarriers, the simulated and predicated fading parameters K_n^f and Ω_n^f of its subcarriers are shown in Fig. 3.3. The simulation results for Ω_n^f and K_n^f are obtained by generating random CIR samples and computing the statistics of their FFT outputs. Specifically 100,000

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

samples of complex subcarrier gains are collected for every subcarrier. Ω_n^f is evaluated as the power for the n th subcarrier, d_n^f as the mean magnitude of the n th subcarrier, and $\text{var}(r_n^f)$ as its variance. Finally, the Rician K factor K_n^f for the n th subcarrier can be obtained by dividing $|d_n^f|^2$ by $\text{var}(r_n^f)$ based on (3.4). Fig. 3.4 shows the simulated pdf of the 1st and 29th subcarriers and their expected Rician pdf with power and K factors ($\Omega_1^f = 2.67$, $K_1^f = 14.96$ and $\Omega_{29}^f = 0.18$, $K_{29}^f = 0.05$) derived from (3.15) and (3.14), respectively.

The close match between the simulated results and the predicted results in Fig. 3.3 verifies the K_n^f and Ω_n^f formulas in (3.14) and (3.15). The close agreement of the simulated and expected pdf curves in Fig. 3.4 further testifies the validity of the derived frequency-domain subcarrier fading model.

From Fig. 3.3 and Fig. 3.4, it is obvious that the OFDM subcarriers in this ATDMA channel are not identically distributed. Some subcarriers, such as the 1st subcarrier, have high K factor and power, while others, such as the 29th subcarrier, have near zero K factor and low power.

Also we notice that the K factor and mean power for the subcarriers follow exactly the same spectral profile, as predicted earlier in Section 3.2.3.1 for channel with independent paths. Since a large K factor implies less fading and large mean power implies larger SNR under uniform AWGN spectral density, we expect that some subcarriers, e.g., subcarrier 1 to 10, will have better quality than others, e.g., subcarrier 30 to 40, in these channels. This is an example of a multicarrier channel in which the subcarriers have non-identical fading distributions. In this thesis, we will use the terms “non-uniform subcarrier fading distributions” or simply “non-uniform subcarrier fading” to indicate that the fading distributions of different subcarriers have different K factors and/or powers.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Table 3.1: ATDMA Picocell channel consisting of 5 Rician and 1 Rayleigh paths with decreasing power profile.

	path1	path2	path3	path4	path5	path6
$\Omega_l^t(\text{dB})$	0	-3.3	-9.3	-14.3	-20.3	-26.8
K_l^t	10	2.51	2.51	2.51	2.51	0
τ_l	0	1	4	6	8	10
$\vartheta_l(\text{deg})$	0	10	30	20	40	—

Figure 3.3: Simulated and predicted subcarrier power (Ω_n^f) and subcarrier K factor (K_n^f) for the channel specified in Table 3.1.

Figure 3.4: Simulated and theoretical pdf of the 1st and 29th subcarriers for the OFDM system specified for Fig. 3.3.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

3.2.3.2 Channels with Correlated Paths

There are practical cases where the path fadings are correlated [51–53]. For example, the situation will arise when the antenna spacing is small. Hence we shall now consider frequency-selective fading channel with correlated paths.

For channels with correlated paths, the channel covariance matrix $\mathbf{C}_g$ is no longer a diagonal matrix but the subcarrier K factor and power can still be computed from (3.11) and (3.12), respectively. In this case, the $\mathbf{w}_n^H \mathbf{C}_g \mathbf{w}_n$ term in (3.11) and (3.12) will generally be different for different subcarriers. This will result in different K factor and power values for different subcarriers. Hence we can conclude that the OFDM subcarriers in channels with correlated paths will always experience non-uniform fading among its subcarriers.

Consider a channel with 3 Rayleigh paths as shown in Table 3.2. If the path fadings are independent, all the subcarriers will experience uniform Rayleigh fading with equal powers, as discussed earlier in Section 3.2.3.1. However, if there are path correlations, e.g.,

$$\mathbf{C}_g = \begin{pmatrix} 1 & 0.9 & 0.5 \\ 0.9 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix},$$

the subcarriers will be subject to non-uniform fading. Fig. 3.5 presents the subcarrier powers Ω_n^f and K factors K_n^f of the OFDM system operating in this channel. In this case, all the subcarriers have zero K values (Rayleigh fading) but different powers, which is resulted from the path correlations. This clearly corresponds to non-uniform subcarrier fading, where the subcarriers have different fading powers.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Table 3.2: A channel with 3 Rayleigh fading paths of equal power.

	path1	path2	path3
$\Omega_l^t(\text{dB})$	0	0	0
K_l^t	0	0	0
τ_l	0	1	2

Figure 3.5: Simulated and predicted subcarrier power (Ω_n^f) and subcarrier K factor (K_n^f) for the channel specified in Table 3.2 with path correlations.

The multipath fading channels which exhibit uniform/non-uniform subcarrier fading distributions are summarized in Table 3.3. For channels with no or only one Rician path and the paths experience independent fading, all the subcarriers will have uniform subcarrier fading. On the other hand, (3.11) and (3.12) imply that a channel will have non-uniform subcarrier fading distributions if either $|\text{DFT}(\mathbf{d}^t)|^2$ or $\mathbf{w}_n^H \mathbf{C}_{\mathbf{g}} \mathbf{w}_n$ have different values for different subcarriers. These situations will arise if there are 2 or more paths in the channel and at least 2 of them are Rician fading (such that $\mathbf{d}^t$ will consist of 2 or more non-zero elements), or if the channel paths are correlated (such that $\mathbf{C}_{\mathbf{g}}$ is non-diagonal).

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Table 3.3: Correspondence between channel impulse response (CIR) conditions and subcarrier fading conditions.

CIR Conditions \ Subcarrier Fading		Uniform Subcarrier Fading	Non-uniform Subcarrier Fading
Independent paths	All Rayleigh paths	✓	
	One Rician path	✓	
	More than one Rician path		✓
Correlated paths			✓

3.3 Subcarrier Fading Correlations

The subcarrier fading distributions were derived earlier in Section 3.2.2. In this section, we will derive the subcarrier fading correlations from the time-domain path fading information.

Define the path fading vector as $\tilde{\mathbf{g}} = [\tilde{g}_1, \tilde{g}_2, \dots, \tilde{g}_{N_p}]^T$, and the subcarrier fading vector as $\tilde{\mathbf{h}} = [\tilde{h}_1, \tilde{h}_2, \dots, \tilde{h}_{N_c}]^T$. From (3.8), their relationship can be expressed compactly as

$$\tilde{\mathbf{h}} = \mathbf{W}\tilde{\mathbf{g}}, \quad (3.18)$$

where

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j2\pi\tau_1/T_s} & e^{-j2\pi\tau_2/T_s} & \dots & e^{-j2\pi\tau_{N_p}/T_s} \\ e^{-j4\pi\tau_1/T_s} & e^{-j4\pi\tau_2/T_s} & \dots & e^{-j4\pi\tau_{N_p}/T_s} \\ \dots & \dots & \dots & \dots \\ e^{-j(N_c-1)2\pi\tau_1/T_s} & e^{-j(N_c-1)2\pi\tau_2/T_s} & \dots & e^{-j(N_c-1)2\pi\tau_{N_p}/T_s} \end{bmatrix}$$

Define the subcarrier covariance matrix $\mathbf{C}_h$ as

$$\mathbf{C}_h = E[\tilde{\mathbf{h}}\tilde{\mathbf{h}}^H] - E[\tilde{\mathbf{h}}] E[\tilde{\mathbf{h}}^H]. \quad (3.19)$$

Since $\tilde{\mathbf{h}}$ and $\tilde{\mathbf{g}}$ are linearly related through (3.18), the subcarrier covariance matrix $\mathbf{C}_h$ is related to the path covariance matrix $\mathbf{C}_g$ by [47]

$$\mathbf{C}_h = \mathbf{W}\mathbf{C}_g\mathbf{W}^H. \quad (3.20)$$

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Note that even if the channel paths are independent and $\mathbf{C}_g$ is a diagonal matrix, (3.20) predicts that the subcarrier fading may still be correlated. This is visualized in Fig. 3.6 which shows the normalized correlations between any subcarrier and its neighboring subcarriers for an OFDM channel with 3 or 8 independent and identically distributed (i.i.d.) Rayleigh paths. The path delay is assumed to be one symbol duration. For example, for the 3-path channel in Fig. 3.6, the correlation between subcarrier n and subcarrier $(n + 1)$ is 0.98, the correlation between subcarrier n and subcarrier $(n + 2)$ is 0.95, and so on. It indicates that even although the fading paths are independent, the neighboring subcarriers are highly correlated.

Fig. 3.6 also shows that channels with different number of paths will lead to different amount of subcarrier correlations. For the 3-path channel, the correlations between 5 neighboring subcarriers are above 0.8. Only subcarriers that are far apart have low correlations. But when the number of independent paths in the channel increases from 3 to 8, the correlations between neighboring subcarriers drop rapidly, e.g., the correlation between subcarrier n and subcarrier $(n + 5)$ drops quickly to 0.1 and the correlations between most of the subcarriers are below 0.2. In general, the more the number of channel paths, the less correlated will be the subcarriers. This is clearly a result of the well-known property that the coherence bandwidth of a fading channel decreases with increasing channel delay spread.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Figure 3.6: Subcarrier correlations for a 3-path or 8-path channel with i.i.d. Rayleigh fading.

3.4 Applications of the Subcarrier Fading Model

3.4.1 Subcarrier BER Evaluation

We have demonstrated that the fading statistics of individual subcarriers in an OFDM system can be deduced from the fading statistics of channel paths. Such knowledge is expected to be useful in the design and analysis of OFDM systems under arbitrary channel and system conditions. As an illustration, we consider the performance of an OFDM system with 64 subcarriers modulated with 16-QAM (square QAM with Gray mapping [34]) and the bit energy to noise density ratio E_b/N_0 is equal to 15 dB for all subcarriers. The channel is assumed to be that specified in Table 3.1. To further generalize the channel condition, we also introduce random correlations among the channel paths by assuming the following path correlation matrix

$$\mathbf{C}_g = \begin{pmatrix} 1 & 0.82 & 0.50 & 0.36 & 0.11 & 0 \\ 0.82 & 1 & 0.73 & 0.41 & 0.13 & 0.05 \\ 0.50 & 0.73 & 1 & 0.81 & 0.39 & 0.14 \\ 0.36 & 0.41 & 0.81 & 1 & 0.76 & 0.38 \\ 0.11 & 0.13 & 0.39 & 0.76 & 1 & 0.75 \\ 0 & 0.05 & 0.14 & 0.38 & 0.75 & 1 \end{pmatrix}. \quad (3.21)$$

For this channel, the subcarrier fadings are non-uniform, and the subcarrier fadings are correlated (as discussed in Section 3.3). To evaluate the BER performance of such an OFDM system, currently the only known approach is by Monte Carlo simulation. With our subcarrier fading model, we shall show that the BER can now be reliably predicted, even down to the subcarrier level.

Denoting $P_{e,n}$ as the BER of the n th subcarrier under a fading pdf $p(h_n)$, we have

$$P_{e,n} = \int_0^\infty P_b \left(h_n, M_n, \frac{E_b}{N_0} \right) p(h_n) dh_n, \quad (3.22)$$

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

where $P_b(h_n, M_n, E_b/N_0)$ is the BER for a subcarrier modulated with a QAM constellation of size M_n , conditioned on the fading h_n . For simplicity in notation, we denote $P_b(h_n, M_n, E_b/N_0)$ as simply P_b . The exact closed-form expression of P_b for square QAM with Gray mapping was derived in [34] as

$$P_b = \frac{1}{\log_2 \sqrt{M}} \sum_{i=1}^{\log_2 \sqrt{M}} P_b(i), \quad (3.23)$$

where

$$P_b(i) = \frac{1}{\sqrt{M}} \sum_{k=1}^{(1-2^{-i})\sqrt{M}-1} \left\{ (-1)^{\lfloor \frac{2^{i-1}k}{\sqrt{M}} \rfloor} \cdot \left(2^{i-1} - \left\lfloor \frac{2^{i-1}k}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \cdot \operatorname{erfc} \left((2k+1) \sqrt{\frac{3 \log_2 M h_n^2 E_b/N_0}{2(M-1)}} \right) \right\},$$

$\lfloor \cdot \rfloor$ denotes the next smaller integer.

As discussed in Section 3.2, h_n follows the Rician pdf $p(h_n)$ given by (3.3). The subcarrier K factor and power required in (3.3) are given by (3.11) and (3.12), respectively. Then substituting (3.3) and (3.23) into (3.22), we can obtain the subcarrier BER $P_{e,n}$.

The subcarrier BERs computed from (3.22) is shown in Fig. 3.7, together with the simulated BER. The close match between the simulated and predicted subcarrier BER values proves that our subcarrier fading model can greatly facilitate the analysis of OFDM system performance under generalized fading channels. Furthermore, we can see from Fig. 3.7 that the subcarrier BERs vary over a large range (almost 3 order of magnitudes). This is due to the large difference in the subcarrier fading powers Ω_n^f and the subcarrier K factors K_n^f associated with this channel.

As the subcarriers with high BER will dominate the overall performance, the average BER is only 0.0141 even though most of the subcarriers have much lower

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

BER values than the average. Therefore adaptive OFDM schemes should be developed so that the high-quality subcarriers are made to carry more bits or use less power than the low-quality subcarriers.

Figure 3.7: Simulated and predicted subcarrier BERs of a 64-subcarrier 16-QAM OFDM system at $E_b/N_0 = 15$ dB. Fading channel conditions are specified in Table 3.1 with path correlations given in (3.21).

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

3.4.2 Statistical OFDM Adaptation

3.4.2.1 System Model

In the literature, many algorithms have been proposed to adapt the transmission parameters of the OFDM system based on the instantaneous subcarrier fading gains [54–60]. For such systems, the channel variation must be slow and the receiver must be able to accurately measure and feedback the instantaneous subcarrier fading gains reliably to the transmitter. For channels with non-uniform subcarrier fading distributions, we propose a different kind of adaptive OFDM schemes, called statistical adaptive OFDM, which performs the adaptation based on the subcarrier fading parameters, instead of the instantaneous subcarrier fading gains. Compared to the instantaneous OFDM adaptation schemes, our proposed statistical adaptation schemes are considerably simpler and less costly to implement as they do not need to feedback instantaneous channel state information to the transmitter at short intervals. Instead, the adaptation only needs to be updated when the subcarrier fading statistics change substantially, e.g., when the terminal goes from indoor to outdoor, or from rural to urban area.

The adaptive OFDM transmitter is shown in Fig. 3.8. We have N_c parallel subcarriers in the OFDM modulator and two sets of variables $\{M_n\}$ and $\{v_n\}$ to adapt. M_n is the adaptive QAM size for the n th subcarrier (bit loading) and v_n ($v_n \geq 0$) is the power scaling factor for the n th subcarrier (power control). The objective of the adaptive OFDM scheme is to minimize the transmitting power, while equalize all the subcarrier BER at a target BER level $\hat{P}_e$ and maintain the overall transmission rate. The transmission rate used here refers to the number of bits transmitted per OFDM symbol.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Figure 3.8: OFDM modulator with bit loading and power control.

Since subcarriers with high BER values are going to dominate the overall system BER in OFDM systems with non-uniform subcarrier fading, the optimum system design is that all subcarriers have the same error rates [61]. Hence, in our proposed adaptive OFDM scheme, the BER $P_{e,n}$ of every subcarrier are maintained to be the same, i.e.,

$$P_{e,n} = \hat{P}_e. \quad (3.24)$$

Note that the subcarrier BER $P_{e,n}$ given in (3.22) is a one dimensional integral formula. Recently, a closed-form BER approximation formula for QAM in Rician fading channel is given in [62], i.e.,

$$P_{e,n} \approx 0.2\varepsilon\Delta_n \exp [K_n^f(\Delta_n - 1)] , \quad (3.25)$$

where

$$\Delta_n = \left[\frac{1.6\Omega_n^f E_b/N_0}{(1 + K_n^f)(M_n - 1)} + 1 \right]^{-1}.$$

The values of ε are provided in Table 3.4 for commonly used QAM sizes. Note that ε is also a function of the Rician K factor. The Rician K factor considered is

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

within the range of $0 \leq K \leq 30$, which is wide enough to cover the practical range of K for multipath fading channels [63]. In [62], it is confirmed that (3.25) is a tight BER approximation formula. It will hence be used for our proposed adaptive OFDM scheme.

Table 3.4: Correction factor for the QAM BER in Rician fading channels ($0 \leq K \leq 30$)

	4 QAM	16 QAM	64 QAM	256 QAM	1024 QAM
ε	$1.335 - 0.00875K$	$1.15 - 0.0125K$	$0.9 - 0.0125K$	$0.725 - 0.01125K$	$0.56 - 0.008K$

3.4.2.2 Statistical Power Control

A simple power control scheme is considered first. In this scheme, M_n is kept the same for all subcarriers and only v_n is varied so as to achieve the target BER $\hat{P}_e$ for all subcarriers. Introducing the power control parameter v_n to (3.25) simply replaces E_b/N_0 in (3.25) with $v_n E_b/N_0$. And solving it for v_n with $P_{e,n} = \hat{P}_e$ gives

$$v_n = \frac{(1 + K_n^f)(M_n - 1)}{1.6\Omega_n^f E_b/N_0} [f(K_n^f) - 1] , \quad (3.26)$$

where

$$f(K_n^f) = \begin{cases} (5\hat{P}_e)^{-1}, & K_n^f = 0. \\ K_n^f [W_0(5\hat{P}_e K_n^f e^{K_n^f})]^{-1}, & K_n^f > 0. \end{cases} \quad (3.27)$$

The function $W_0(z)$ in (3.27) is the principal value of the Lambert's W -function, i.e., it is the root of the equation [64]

$$\ln W(z) = \ln z - W(z), \quad z > 0. \quad (3.28)$$

With these formulas available, the adaptation can be done following the procedures below:

- 1) Measure the subcarrier fading parameters K_n^f and Ω_n^f or obtain them from the CIR using (3.11) and (3.12);
- 2) Based on the target BER level $\hat{P}_e$, calculate v_n using (3.26);
- 3) Repeat Step 2 for all subcarriers.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

3.4.2.3 Statistical Successive Bit Loading and Power Control

Next, we consider a statistical OFDM adaptation scheme which simultaneously adapts the subcarrier power and the subcarrier QAM-size based on knowledge of the subcarrier fading parameters. We call this scheme “statistical successive bit loading and power control”. This scheme attempts to load bits successively to subcarriers with the objective of using minimum power to achieve certain transmission rate at a target BER level $\hat{P}_e$. The adaptation problem can be formulated as

$$\{M_n, v_n\} = \arg \left\{ \min \sum_{n=1}^{N_c} v_n \right\}. \quad (3.29)$$

The above optimization must be constrained by maintaining the subcarrier BER $P_{e,n}$ at the target BER level of $\hat{P}_e$, i.e.,

$$P_{e,n} |_{\{K_n^f, \Omega_n^f, M_n, v_n\}} = \hat{P}_e, \quad \forall n \quad (3.30)$$

and by maintaining the transmission rate $\sum_{n=1}^{N_c} \log_2 M_n$ to be the same before and after adaptation.

Denote $\mathbf{Q}_M$ as the set of available QAM sizes. For square QAM, we have $\mathbf{Q}_M = \{4, 4^2, 4^3, \dots\}$. In the successive bit loading and power control scheme, the bits are loaded bit by bit to the subcarrier which requires minimum power to achieve minimum transmitting power [65, 66]. Since the constellation size for square QAM is in the form of 4^q (q is an integer number), we need to load two bits at a time to the subcarriers. The procedures to perform the optimization task are summarized below:

- 1) Measure the subcarrier fading parameters K_n^f and Ω_n^f or obtain them from the CIR using (3.11) and (3.12);
- 2) Using the v_n formula (3.26), calculate the power needed by each subcarrier for all the available QAM sizes $\mathbf{Q}_M$ to meet the target BER $\hat{P}_e$;

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

- 3) Starting with the first two bits, load them onto the subcarrier (denoted as the n th subcarrier) which requires the least power calculated in Step 2;
- 4) Similarly for the next two bits, load them to the subcarrier which requires least power. But note that when comparing the power between subcarriers, for the n th subcarrier which has just been loaded with 2 bits, the additional power to increase the QAM size by one level is used for comparison;
- 5) Repeat Step 4 until all the bits are loaded.

Next we will use an example to illustrate how the above successive bit loading and power control scheme works. Considering an OFDM system with 4 subcarriers, we are going to load 8 bits to the subcarriers using the algorithm described above. The QAM sizes available for the system are assumed to be $\mathbf{Q}_M = \{4, 16, 64\}$. The required powers for the subcarriers to achieve the target BER at different QAM sizes are assumed to be given in Table 3.5 (note that their values are imaginary but ensured to be increasing with larger QAM size).

Table 3.5: Required power for subcarriers to achieve $\text{BER}=10^{-3}$ at different constellation levels.

Subcarrier Index	Constellations	QPSK	16-QAM	64 QAM
1		1000	2200	4900
2		200	400	900
3		1000	2200	4900
4		100	200	500

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Figure 3.9: Numerical example to assign 8 bits to 4 subcarriers, using the proposed successive bit loading and power control scheme.

As shown in Fig. 3.9, we load 2 bits every time to the subcarriers which requires least power. For example, for the first 2 bits, since the 4th subcarrier requires the least power with QPSK constellation, we load them to the 4th subcarrier. For the next 2 bits, the 4th subcarrier needs only additional 100 unit of power to carry 2 more bits, which is still the least. Hence we load the second 2 bits to the 4th subcarrier again. Similarly the third 2 bits are loaded to the second subcarrier and the last 2 bits are loaded to the second subcarrier again as the additional power it needs is the least. As a result, the first 4 bits are loaded to the 4th subcarrier and the remaining 4 bits are loaded to the 2nd subcarrier in this example. And the least power that we are able to achieve to load 8 bits are 200 power units (for the 4th subcarrier to carry 4 bits) plus 400 power units (for the 2nd subcarrier to carry 4 bits), which is 600 units of power in total.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

3.4.2.4 Performance Evaluation

In this section, the performance of the proposed statistical adaptive OFDM schemes is evaluated over the channel model specified in Table 3.1. The detailed procedures to perform the proposed statistical adaptive scheme are given in Section 3.4.2.3. As discussed earlier, this channel have non-uniform subcarrier fading in the frequency domain, hence our proposed statistical OFDM adaptation scheme is applicable to it. The OFDM system operating over this channel employs 32 subcarriers and the available QAM size is assumed to be $\mathbf{Q}_M = \{4, 16, 64, 256, 1024\}$. Fig. 3.10 shows the performance of various non-adaptive/adaptive OFDM schemes. The x axis is the average transmit power ($\frac{\sum_{n=1}^{N_c} v_n E_b/N_0}{N_c}$) required to achieve the target BER of $\hat{P}_e = 10^{-3}$ and the y axis is the number of bits transmitted per subcarrier. For example, if QAM size is 4 for all subcarriers in the non-adaptive OFDM scheme, the number of bits per subcarrier is 2 and $E_b/N_0 = 23.8$ dB is required to achieve $\hat{P}_e = 10^{-3}$. Note that for the adaptive OFDM schemes, the E_b/N_0 required for different subcarriers and the bits carried by different subcarrier may be different among subcarriers, hence the E_b/N_0 values and the bits transmitted per subcarrier shown in the figure are the average values.

From Fig. 3.10, we can see that the “statistical power control” scheme performs almost the same as the non-adaptive scheme. This is because we need to give large amount of power to the poor subcarriers to ensure that it achieves BER of 10^{-3} . However, with power control, we can achieve the same BER for all the subcarriers. This is not possible for the non-adaptive system.

On the other hand, with adaptive bit loading, we can turn off bad subcarriers which otherwise would require large amount of power to meet the BER target. Hence, the “statistical successive bit loading and power control” scheme (refers as

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

“statistical successive adaptation” in the figures), can give considerable power saving of up to 12 dB compared to the non-adaptive OFDM scheme, while maintaining the same spectral efficiency.

For the sake of completeness, we also compare our statistical OFDM adaptation schemes with the instantaneous OFDM adaptation [56] in Fig. 3.10. As seen from the figure, the instantaneous adaptation scheme performs better than our statistical adaptation schemes. This is not surprising since it optimizes the transmission parameters for every OFDM symbol sent, but it also has higher implementation complexity than our proposed statistical adaptation system. Nonetheless, the performance of the proposed statistical successive adaptation is only 3.5 dB away from the instantaneous adaptation scheme at a spectral efficiency of 2 bits per subcarrier, as shown in Fig. 3.10. This amount is not significant compared to the gain achieved by the statistical successive adaptation over the non-adaptive scheme, which is 12 dB. Hence, the proposed statistical adaptation system is competitive to the instantaneous adaptation system in terms of complexity performance trade-off.

The performance of the adaptive OFDM schemes tends to saturate when the bits transmitted per subcarrier reaches the highest QAM size, which is 10 bits per subcarrier in Fig. 3.10. This is because the largest QAM size assumed is 1024, which corresponds to 10 bits, hence the system has no choice but to let all subcarriers carry 10 bits in order to meet the transmission rate requirement.

Different applications may have different BER requirement to ensure the quality of service. Therefore the effect of different target BER $\hat{P}_e$ on the adaptive OFDM system performance is also investigated in this section. Fig. 3.11 shows the performance gains obtained from both the instantaneous and statistical OFDM adaptation schemes over the non-adaptive OFDM scheme for a target BER of

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

$\hat{P}_e = 10^{-6}$. The system parameters and the channel conditions are kept the same as those for Fig. 3.10.

Comparing Fig. 3.11 with Fig. 3.10, both the instantaneous and statistical successive OFDM adaptation schemes are seen to give more performance gain when $\hat{P}_e$ is lowered from 10^{-3} to 10^{-6} . For example, the statistical successive adaptation scheme gives about 12 dB gain for $\hat{P}_e = 10^{-3}$ and this increases to 20.5 dB for $\hat{P}_e = 10^{-6}$ at the average spectral efficiency of 2 bits per subcarrier. The “power control only” scheme still performs no better than the non-adaptive scheme. The reason is the same as that given earlier for Fig. 3.10.

In addition, practical wireless systems may have certain constraint on the total transmit power. This corresponds to certain maximum power value shown on the x-axes of Fig. 3.10 and 3.11. This maximum power value will imply a maximum supported bit rate shown on the y-axes of Fig. 3.10 and 3.11. For example, for Fig. 3.10 (target BER $\hat{P}_e = 10^{-3}$), if the max power constraint is 20 dB, then it implies that a bit loading level not higher than 5 bits per subcarrier (on the average) can be supported by the successive bit loading scheme.

3.5 Summary

In this chapter, the subcarrier fading distributions of MCM systems is derived based on the path fading distribution and path correlation information. Frequency-selective channels with arbitrary Rician/Rayleigh path fading and inter-path correlations are considered.

It is discovered that certain MCM channels experience different fading severity among the subcarriers, i.e., non-uniform subcarrier fading. A channel will have non-uniform subcarrier fading distributions if there are 2 or more resolvable paths in the channel and at least 2 of them are Rician, or if the path fadings are correlated.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

For channels with non-uniform subcarrier fading, a novel bit loading and power control scheme that requires less power to achieve certain target BER is proposed and studied. The scheme adapts the subcarrier modulation parameters based on the long-term subcarrier fading statistics, instead of instantaneous channel state information. Hence the proposed adaptive OFDM scheme has considerable complexity and cost advantage. In addition, considerable performance gains can also be achieved by the proposed statistical adaptation scheme over the non-adaptive OFDM scheme.

CHAPTER 3. FREQUENCY-DOMAIN (SUBCARRIER) FADING CHARACTERISTICS OF OFDM SIGNALS

Figure 3.10: Performance of statistical OFDM adaptations compared to non-adaptive OFDM and instantaneous OFDM adaptation at target BER of $\hat{P}_e = 10^{-3}$. The channel conditions are specified in Table 3.1.

Figure 3.11: Performance of statistical OFDM adaptations compared to non-adaptive OFDM and instantaneous OFDM adaptation at target BER of $\hat{P}_e = 10^{-6}$. The channel conditions are specified in Table 3.1.

Chapter 4

BER Analysis and Performance of MC-CDMA Systems

4.1 Introduction

Multicarrier code division multiple access (MC-CDMA) is a multiple access technique that combines direct sequence (DS) CDMA with OFDM. In MC-CDMA systems, the signal is first spread with a high-rate code sequence, then it is converted into several parallel data streams for transmission over multiple subcarriers using OFDM modulation. Therefore, the MC-CDMA transmitter is basically an OFDM modulator preceded with DS spreading, while the MC-CDMA receiver is an OFDM demodulator followed by DS despreading and combining. In this chapter, we will first present the signal models of MC-CDMA systems, and then propose some means to simplify its BER analysis.

Most wireless networks make use of two different channels to provide 2-way communication: downlink and uplink channels. In the downlink channel, the base station transmits while the mobile user receives; the reverse is true for the uplink channel. There are important distinctions between the downlink and uplink channels. In downlink transmission, all the user signals can be time aligned by the base station before transmission, hence the different user signals are synchronous

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

and subjected to the same channel effects. In the uplink transmission, different user signals are generated and transmitted independently by different users, so the uplink signals from different users tend to be asynchronous and have different channel impairments.

Due to the different nature of the uplink and downlink channels in MC-CDMA system, we will describe their signal models separately.

4.2 BER Formulations

4.2.1 Downlink Signal Model

The block diagram of a downlink MC-CDMA system is shown in Fig. 4.1. Suppose there are N_u simultaneous active users, their data are separately spread using different spreading codes, permitting multiple access based on the CDMA principle. Then the spread signals are time aligned, added together and modulated using OFDM, which is achieved by efficient IFFT operation as explained earlier in Section 2.3.2. After inserting the cyclic prefix, the combined MC-CDMA signal of all users is sent to the channel. At the receiver, the reverse processes are performed. The cyclic prefix is first removed to eliminate multipath ISI. The FFT demodulation converts the OFDM waveform back to CDMA signals. After despreading using the appropriate spreading code sequence, a frequency-domain equalization and combining block is used to reduce the channel impairment and achieve diversity gain.

An equivalent model for the spreading and OFDM modulation operations in the MC-CDMA transmitter is shown in Fig. 4.2. Fig. 4.2 considers the k th user. The user data is first spread using the code $\{c_{k,n}\}$, then the n th spread chip is modulated onto frequency w_n . Consider N_c subcarriers with binary phase shift

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Figure 4.1: Block diagram of a downlink MC-CDMA system.

Figure 4.2: An MC-CDMA transmitter model for user k .

keying (BPSK) modulation. The transmitted signal corresponding to the k th user can be expressed as

$$s_k(t) = \sum_{i=-\infty}^{\infty} \sqrt{\frac{2E_b}{N_c T_s}} \sum_{n=1}^{N_c} b_k(i) c_{k,n} u_{T_s}(t - iT_s) \cos(w_n t), \quad (4.1)$$

where E_b and T_s are the bit energy and symbol duration respectively, $u_{T_s}(t)$ represents a rectangular waveform with amplitude 1 and pulse duration T_s , $b_k(i)$ is the i th transmitted data bit of user k , $c_{k,n}$ is the spreading code, $w_n = 2\pi f_0 + 2\pi(n - 1)\Delta f$ is the radian frequency of the n th subcarrier, and the frequency spacing is $\Delta f = 1/T_s$.

For downlink transmission, all user signals are synchronously combined before transmission, hence they experience the same channel fading. Similar to an OFDM

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

system, as long as the length of the cyclic prefix is equal to or larger than the maximum delay spread of the channel, ISI can be eliminated at the receiver by discarding the cyclic prefix. Hence the effects of the channel on the received signal include the subcarrier (flat) fading gain h_n , the subcarrier fading phase φ_n , and the AWGN $\eta(t)$. Therefore, the received signal $r(t)$ after cyclic prefix removal is

$$r(t) = \eta(t) + \sum_{i=-\infty}^{\infty} \sqrt{\frac{2E_b}{N_c T_s}} \sum_{k=1}^{N_u} \sum_{n=1}^{N_c} h_n b_k(i) c_{k,n} u_{T_s}(t - iT_s) \cos(w_n t + \varphi_n), \quad (4.2)$$

where $\eta(t)$ is zero mean with single-sided power spectral density N_0 .

Fig. 4.3 shows the receiver model of the k th MC-CDMA user. The received signal $r(t)$ after cyclic prefix removal is firstly OFDM demodulated and then despread by multiplying with the spreading code $c_{k,n}$ of the k th user.

Since ISI is already eliminated, the use of time-domain equalization is not necessary in MC-CDMA systems, and only frequency-domain equalization is necessary. Frequency-domain equalizer aims to compensate for the phase rotation and the amplitude fading effects of the frequency-selective multipath channel on the subcarriers, as well as to minimize multiuser interference (MUI) arising from loss of orthogonality among the different spreading codes due to frequency-selective fading. It performs adjustments of subcarrier gains and phases. For simplicity, we assume that the channel phase φ_n of the subcarriers is perfectly known to the equalizers, and can be perfectly corrected by the equalizers. Hence we only consider the amplitude correction for the performance analysis of MC-CDMA systems. As shown in Fig. 4.3, the amplitude correction is accomplished by α_n ($n = 1, \dots, N_c$), which is also called equalizer coefficient. The α_n values for various types of equalization and combining schemes, including MRC, EGC, ORC and MMSEC, will be discussed in detail in Section 4.2.3.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Figure 4.3: MC-CDMA receiver model for user k .

Without loss of generality, we take user 1 as the reference user, then the decision variable U of user 1 is

$$U = \int_0^{T_s} r(t) \sum_{n=1}^{N_c} c_{1,n} \cos(w_n t + \varphi_n) \alpha_n dt = D + \mathcal{MUI} + \eta, \quad (4.3)$$

where $r(t)$ is given by (4.2), D , $\mathcal{MUI}$ and η are the desired signal component, MUI component and additive noise component, respectively. It can be shown that

$$D = \sqrt{\frac{E_b T_s}{2N_c}} b_1 \sum_{n=1}^{N_c} h_n \alpha_n, \quad (4.4)$$

$$\mathcal{MUI} = \sqrt{\frac{E_b T_s}{2N_c}} \sum_{k=2}^{N_u} b_k \sum_{n=1}^{N_c} c_{k,n} c_{1,n} h_n \alpha_n, \quad (4.5)$$

and the noise η has zero mean and variance

$$\text{var}(\eta) = \frac{N_0 T_s}{4} \sum_{n=1}^{N_c} \alpha_n^2. \quad (4.6)$$

The noise is Gaussian distributed, hence the BER for user 1 conditioned on the set of subcarrier fading coefficients $\{h_n\}$ and the set of user data $\{b_k\}$ is (assuming, without loss of generality, that user 1 transmits one)

$$P_e|\{\{h_n\}, \{b_k\}\} = Q\left(\frac{\sum_{n=1}^{N_c} h_n \alpha_n + \sum_{k=2}^{N_u} b_k \sum_{n=1}^{N_c} c_{k,n} c_{1,n} h_n \alpha_n}{\sqrt{\frac{N_0 N_c}{2E_b} \sum_{n=1}^{N_c} \alpha_n^2}}\right), \quad (4.7)$$

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

where $Q(\cdot)$ is the Gaussian Q function.

By averaging (4.7) over the joint distribution function (jdf) of all h_n , we obtain the BER conditioned on $\{b_k\}$

$$P_e|\{b_k\} = \int_0^\infty \cdots \int_0^\infty (P_e|\{\{h_n\}, \{b_k\}\}) jdf(\{h_n\}) dh_1 \cdots dh_{N_c}. \quad (4.8)$$

Assuming b_k takes values of ± 1 with equal probability, we have the final BER as

$$P_e = \frac{1}{2^{N_u-1}} \sum_{b_2 \in \{\pm 1\}} \sum_{b_3 \in \{\pm 1\}} \cdots \sum_{b_{N_u} \in \{\pm 1\}} P_e|\{b_k\}. \quad (4.9)$$

4.2.2 Uplink Signal Model

The block diagram of the uplink MC-CDMA channel is shown in Fig. 4.4. For uplink transmission, the base station receives signals from different users through different channels. This leads to different channel amplitudes, phases and delays for different users. More importantly, synchronization between users is now not possible, hence the misalignment in the signal arrival time among different users need to be considered in the performance analysis.

Figure 4.4: Block diagram for an uplink MC-CDMA channel.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Taking into the considerations of all the factors mentioned earlier, the transmitted uplink signal of the k th user in an MC-CDMA system with BPSK-modulated subcarriers is given by

$$s_k(t) = \sum_{i=-\infty}^{\infty} \sqrt{\frac{2E_b}{N_c T_s}} \sum_{n=1}^{N_c} b_k(i) c_{k,n} u_{T_s}(t - iT_s) \cos(w_n t + \theta_{k,n}). \quad (4.10)$$

Compared to the transmitted signal in the downlink case given in (4.1), the additional parameter $\theta_{k,n}$ in (4.10) is to account for the random phase of the k th user's transmitter.

The base station receives signals from all the asynchronous users to obtain a received signal:

$$r(t) = \eta(t) + \sum_{i=-\infty}^{\infty} \sqrt{\frac{2E_b}{N_c T_s}} \sum_{k=1}^{N_u} \sum_{n=1}^{N_c} h_{k,n} b_k(i) c_{k,n} u_{T_s}(t - iT_s - \zeta_k) \cos(w_n t + \phi_{k,n}) \quad (4.11)$$

where $\phi_{k,n} = \theta_{k,n} + \varphi_{k,n} - w_n \zeta_k$, $h_{k,n}$ and $\varphi_{k,n}$ are the fading amplitude and phase for the n th subcarrier of user k , and ζ_k is the time misalignment of user k with respect to user 1, the reference user. Then the decision variable U of user 1 is [29]

$$U = \int_0^{T_s} r(t) \sum_{n=1}^{N_c} c_{1,n} \cos(w_n t + \phi_{1,n}) \alpha_{1,n} dt = D + I + J + \eta, \quad (4.12)$$

where D and η are the desired signal and noise components respectively, and $\alpha_{1,n}$ is the equalizer coefficient for the n th subcarrier. Representing $h_{1,n}$ as h_n and $\alpha_{1,n}$ as α_n , the desired signal component D takes the same form as (4.4), and the AWGN noise component η has zero mean and identical variance as (4.6). Hence the desired signal component D and the noise component η of the MC-CDMA decision variable have the same expressions in both the uplink and the downlink. However, the MUI component is much more complicated in the uplink than in the downlink due to the asynchronous nature of the uplink. For simplicity in analysis,

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

the uplink MUI is divided into two parts: I (MUI from the same subcarrier of other users) and J (MUI from the other subcarriers of other users) [29], i.e.,

$$I = \sqrt{\frac{E_b T_s}{2N_c}} \sum_{k=2}^{N_u} \sum_{n=1}^{N_c} h_{k,n} \alpha_n \cos(\phi_{k,n} - \phi_{1,n}) [b_k(-1) c_{k,n} c_{1,n} \zeta_k + b_k(0) c_{k,n} c_{1,n} (T_s - \zeta_k)], \quad (4.13)$$

$$J = \sqrt{\frac{E_b T_s}{2N_c}} \sum_{k=2}^{N_u} \sum_{n=1}^{N_c} \sum_{q=1, q \neq n}^{N_c} h_{k,n} \alpha_n \left(\int_0^{\zeta_k} b_k(-1) c_{k,n} c_{1,n} \cos[(2\pi f_n - 2\pi f_q)t + \phi_{k,n} - \phi_{1,n}] dt + \int_{\zeta_k}^{T_s} b_k(0) c_{k,n} c_{1,n} \cos[(2\pi f_n - 2\pi f_q)t + \phi_{k,n} - \phi_{1,n}] dt \right), \quad (4.14)$$

where $b_k(0)$ and $b_k(-1)$ represent the current and previous data bit of the k th user, respectively. Since the user data and fading parameters of different users are uncorrelated, the summands in (4.13) and (4.14) are uncorrelated too. Even though the subcarrier fading gain $h_{k,n}$ of the same user may be correlated to some extent, the summands in (4.13) and (4.14) in this case are still uncorrelated due to the presence of other uncorrelated variables such as the subcarrier phase $\phi_{k,n}$ in the equations. Moreover, since the number of summands in (4.13) and (4.14) are very large (e.g., N_c can be at least 64 and N_u can be as large as N_c), both I and J are the summation of large number of uncorrelated terms. Hence, Central Limit Theorem (CLT) can be applied to approximate I and J as Gaussian random variables (RVs) [67], with zero mean and variance given by [29]

$$\text{var}(I) = \frac{E_b T_s (N_u - 1) \sigma^2}{3N_c} \sum_{n=1}^{N_c} \alpha_n^2, \quad (4.15)$$

$$\text{var}(J) = \frac{E_b T_s (N_u - 1) \sigma^2}{4N_c \pi^2} \sum_{n=1}^{N_c} \alpha_n^2 \sum_{i=1, i \neq n}^{N_c} \frac{1}{(i - n)^2}, \quad (4.16)$$

where σ^2 is the subcarrier fading power of the interfering users and it is assumed to be equal for all the interfering users. Therefore, the BER of an uplink MC-CDMA

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

system conditioned on the set of subcarrier fading amplitudes $\{h_n\}$ is

$$P_e|\{h_n\} = Q \left(\frac{\sum_{n=1}^{N_c} h_n \alpha_n}{\sqrt{\left(\frac{2(N_u-1)\sigma^2}{3} + \frac{N_c}{2E_b/N_0} \right) \sum_{n=1}^{N_c} \alpha_n^2 + \frac{(N_u-1)\sigma^2}{2\pi^2} \sum_{n=1}^{N_c} \sum_{i=1, i \neq n}^{N_c} \frac{1}{(i-n)^2} \alpha_n^2}} \right). \quad (4.17)$$

The final BER P_e can then be obtained by averaging (4.17) over the jdf of $\{h_n\}$, i.e.,

$$P_e = \int_0^\infty \cdots \int_0^\infty (P_e|\{h_n\}) \text{jdf}(\{h_n\}) dh_1 \cdots dh_{N_c}. \quad (4.18)$$

4.2.3 Equalization and Combining Schemes

As indicated in (4.7) and (4.17), the BERs of MC-CDMA downlink and uplink channels are dependent on the choice of the equalizer coefficient α_n of the receiver. The purpose of α_n is mainly to reduce the effect of channel impairments and MUI. Several equalization and combining schemes have been developed for MC-CDMA, namely orthogonality restoring combining (ORC), equal gain combining (EGC), maximal ratio combining (MRC) and minimum mean square error combining (MMSEC).

4.2.3.1 Orthogonality Restoring Combining (ORC)

ORC aims to restore the orthogonality of the frequency-domain spreading codes after they suffer different amplitude of subcarrier fading in a frequency-selective fading channel, hence the equalizer coefficient is set to be:

$$\alpha_n = \frac{1}{h_n}. \quad (4.19)$$

If orthogonal spreading codes with zero cross correlation such as the Walsh-Hadamard codes are used, MUI vanishes with ORC. However, the noise in the

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

received signal may be amplified by a large α_n value due to a small h_n value, i.e., deep fade at the n th subcarrier causes noise amplification on the subcarrier. This is not desirable.

4.2.3.2 Maximal Ratio Combining (MRC)

MRC is a classical diversity combining scheme that offers maximum SNR improvement in the presence of spectrally white Gaussian noise [68, 69]. For this scheme, the equalizer coefficient is

$$\alpha_n = h_n. \quad (4.20)$$

The major motivation behind MRC is that received subcarriers with large fading amplitude have high signal to noise ratio, hence they should be given more weight in the combining. However, MRC amplifies the difference among the subcarrier amplitudes arising from frequency-selective fading, hence it worsens the loss of orthogonality between the spreading codes of different users, giving rise to higher MUI.

4.2.3.3 Equal Gain Combining (EGC)

With EGC, the equalizer coefficient is chosen to be

$$\alpha_n = 1. \quad (4.21)$$

Hence EGC only tries to compensate for the subcarrier phase offset caused by fading while leaving the subcarrier amplitude unchanged. As a result, it will not amplify the noise nor the MUI, and its implementation is simpler than the other equalization and combining schemes.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

4.2.3.4 Minimum Mean Square Error Combining (MMSEC)

The equalizer coefficient expressions for the previous three combining schemes (ORC, MRC and EGC) apply for both the downlink and uplink channels. For MMSEC, we consider the downlink channels first. In the literature, different equalizer coefficient expressions for MMSEC have been proposed for MC-CDMA systems [19, 30, 70]. For completeness, we will derive the equalizer coefficient expressions for two representative forms of MMSEC schemes, namely “MMSEC per subcarrier” and “MMSEC per user”.

MMSEC per subcarrier The “MMSEC per subcarrier” scheme aims to find α_n that minimize the mean square error between the decision variable U_n for every subcarrier and the transmitted data bit. For easier presentation of the derivation, we use α'_n to represent $\alpha_n \times c_{1,n}$, where $c_{1,n}$ is the n th spreading code for the desired user (user 1). As shown in Fig. 4.3, the received signal will go through an OFDM demodulation process, a despreading process and an equalization process. After OFDM demodulation but before despreading and equalization, the signal for the n th subcarrier is

$$y_n = \int_0^{T_s} r(t) \cos(w_n t + \phi_{1,n}(0)) dt = D_n + \mathcal{MUI}_n + \eta_n \quad (4.22)$$

where

$$D_n = \sqrt{\frac{E_b T_s}{2N_c}} b_1 c_{1,n} h_n, \quad (4.23)$$

$$\mathcal{MUI}_n = \sqrt{\frac{E_b T_s}{2N_c}} h_n \sum_{k=2}^{N_u} b_k c_{k,n}. \quad (4.24)$$

The noise η_n is of zero mean and variance $N_0 T_s / 4$. After equalization and despreading, the decision variable for the n th subcarrier is $U_n = y_n \times \alpha'_n$ for the n th subcarrier.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Applying MMSE criteria at the subcarrier level gives

$$\alpha'_n = \arg \min_{\alpha'_n} |b_1 - U_n|^2. \quad (4.25)$$

Solution of the equation above is [71, 72]

$$\alpha'_n = R_{y_n y_n}^{-1} R_{b_1 y_n}, \quad (4.26)$$

where $R_{\Delta\Lambda}$ represents the correlation between Δ and Λ . For the “MMSE per subcarrier” scheme, y_n is a scalar, hence $R_{b_1 y_n}$ and $R_{y_n y_n}$ are both scalars given respectively by

$$R_{b_1 y_n} = E[b_1 \times (D_n + \mathcal{MUI}_n + \eta_n)] = \sqrt{\frac{E_b T_s}{2N_c}} c_{1,n} h_n, \quad (4.27)$$

$$R_{y_n y_n} = E[(D_n + \mathcal{MUI}_n + \eta_n)^2] = \frac{N_u E_b T_s}{2N_c} h_n^2 + \frac{N_0 T_s}{4}, \quad (4.28)$$

where $E[\cdot]$ denotes statistical expectation. Since $R_{y_n y_n}$ is scalar, $R_{y_n y_n}^{-1}$ is just the reciprocal $R_{y_n y_n}$. Hence we have

$$\alpha'_n = \alpha_n \times c_{1,n} = \frac{h_n c_{1,n}}{h_n^2 + \frac{1}{2} \frac{N_c}{N_u} \frac{1}{E_b/N_0}}, \quad (4.29)$$

and

$$\alpha_n = \frac{h_n}{h_n^2 + \frac{1}{2} \frac{N_c}{N_u} \frac{1}{E_b/N_0}}. \quad (4.30)$$

After equalization and combining, the final decision variable $U = \sum_{n=1}^{N_c} U_n = \sum_{n=1}^{N_c} y_n c_{1,n} \alpha_n$.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

MMSEC per user Denoting the vectors $\boldsymbol{\alpha}' = [\alpha'_1, \alpha'_2, \dots, \alpha'_{N_c}]$, $\mathbf{y} = [y_1, y_2, \dots, y_{N_c}]^T$, we have $U = \mathbf{y}^T \boldsymbol{\alpha}'$. The “MMSEC per user” scheme aims to find $\boldsymbol{\alpha}'$ that minimize the mean square error between U and b_1 , i.e.,

$$\boldsymbol{\alpha}' = \arg \min_{\boldsymbol{\alpha}'} |b_1 - U|^2. \quad (4.31)$$

The solution of the equation above is

$$\boldsymbol{\alpha}' = \mathbf{R}_{\mathbf{y}\mathbf{y}}^{-1} \mathbf{R}_{b_1\mathbf{y}}. \quad (4.32)$$

Let $\mathbf{b} = [b_1, b_2, \dots, b_{N_u}]^T$, $\boldsymbol{\eta} = [\eta_1, \eta_2, \dots, \eta_{N_c}]^T$, $\mathbf{C}_d$ is a $N_c \times N_u$ matrix with k th column being the spreading code for k th user, and $\mathbf{H}$ is a diagonal matrix with n th diagonal element equals to h_n . We have

$$\mathbf{y} = \sqrt{\frac{E_b T_s}{2N_c}} \mathbf{H} \mathbf{C}_d \mathbf{b} + \boldsymbol{\eta}. \quad (4.33)$$

Then the matrix $\mathbf{R}_{b_1\mathbf{y}}$ is

$$\begin{aligned} \mathbf{R}_{b_1\mathbf{y}} &= E \left[b_1 \times \left(\sqrt{\frac{E_b T_s}{2N_c}} \mathbf{H} \mathbf{C}_d \mathbf{b} + \boldsymbol{\eta} \right) \right] \\ &= \sqrt{\frac{E_b T_s}{2N_c}} E[b_1 \mathbf{H} \mathbf{C}_d \mathbf{b}] \\ &= \sqrt{\frac{E_b T_s}{2N_c}} \mathbf{H} \mathbf{c}_1 \end{aligned} \quad (4.34)$$

where $\mathbf{c}_1$ is the first column of $\mathbf{C}_d$.

The matrix $\mathbf{R}_{\mathbf{y}\mathbf{y}}$ is

$$\begin{aligned} \mathbf{R}_{\mathbf{y}\mathbf{y}} &= E \left[\left(\sqrt{\frac{E_b T_s}{2N_c}} \mathbf{H} \mathbf{C}_d \mathbf{b} + \boldsymbol{\eta} \right) \times \left(\sqrt{\frac{E_b T_s}{2N_c}} \mathbf{H} \mathbf{C}_d \mathbf{b} + \boldsymbol{\eta} \right)^T \right] \\ &= \frac{E_b T_s}{2N_c} E [\mathbf{H} \mathbf{C}_d \mathbf{b} \mathbf{b}^T \mathbf{C}_d^T \mathbf{H}^T] + E [\boldsymbol{\eta} \times \boldsymbol{\eta}^T] \\ &= \frac{E_b T_s}{2N_c} \mathbf{H} \mathbf{C}_d \mathbf{C}_d^T \mathbf{H} + \frac{N_0 T_s}{4} \mathbf{I}_{N_c} \end{aligned} \quad (4.35)$$

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

where $\mathbf{I}_{N_c}$ is an identity matrix with size $N_c \times N_c$. Substituting (4.34) and (4.35) into (4.32), and taking note that $\mathbf{c}_1$ corresponds to the despreading process, we obtain the set of equalizer coefficients for “MMSEC per user” scheme for downlink MC-CDMA as

$$\boldsymbol{\alpha} = \sqrt{\frac{2N_c}{E_b T_s}} \left(\mathbf{H} \mathbf{C}_d \mathbf{C}_d^T \mathbf{H} + \frac{N_c}{2E_b/N_0} \mathbf{I}_{N_c} \right)^{-1} \cdot \mathbf{h} \quad (4.36)$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_{N_c}]^T$ and $\mathbf{h} = [h_1, h_2, \dots, h_{N_c}]^T$.

The MMSEC scheme for uplink MC-CDMA is similar to that for downlink channels, except that the MUI power is larger in uplink channels. In uplink channels, the MUI for the n th subcarrier consists of two terms I_n (MUI from the same subcarrier of other users) and J_n (MUI from the other subcarriers of other users). The variance of I_n and J_n can be deduced from (4.15) and (4.16) respectively to be

$$\text{var}(I_n) = \frac{E_b T_s (N_u - 1) \sigma^2}{3N_c}, \quad (4.37)$$

$$\text{var}(J_n) = \frac{E_b T_s (N_u - 1) \sigma^2}{4N_c \pi^2} \sum_{i=1, i \neq n}^{N_c} \frac{1}{(i - n)^2}, \quad (4.38)$$

where σ^2 is the subcarrier fading power of other interfering users. Replacing $\mathcal{MUI}_n$ in (4.22) with $(I_n + J_n)$, and substituting it into (4.27) and (4.28), we have

$$R_{b_1 y_n} = E[b_1 \times (D_n + I_n + J_n + \eta_n)] = \sqrt{\frac{E_b T_s}{2N_c}} c_{1,n} h_n, \quad (4.39)$$

$$\begin{aligned} R_{y_n y_n} &= E[(D_n + I_n + J_n + \eta_n)^2] \\ &= \frac{E_b T_s}{2N_c} h_n^2 + \frac{E_b T_s (N_u - 1) \sigma^2}{3N_c} + \frac{E_b T_s (N_u - 1) \sigma^2}{4N_c \pi^2} \sum_{i=1, i \neq n}^{N_c} \frac{1}{(i - n)^2} + \frac{N_0 T_s}{4}. \end{aligned} \quad (4.40)$$

Hence the “MMSEC per user” equalizer coefficient for uplink MC-CDMA is

$$\alpha_n = \frac{h_n}{h_n^2 + \frac{2}{3}(N_u - 1)\sigma^2 + \frac{N_u - 1}{2\pi^2}\sigma^2 \sum_{i=1, i \neq n}^{N_c} \frac{1}{(i - n)^2} + \frac{N_c}{2} \frac{1}{E_b/N_0}}. \quad (4.41)$$

4.3 Existing Works on BER Analysis of MC-CDMA and Monte Carlo Integration

So far, we have obtained the expressions for the equalizer coefficients $\{\alpha_n\}$ of ORC, MRC, EGC and MMSEC. By substituting these $\{\alpha_n\}$ expressions into (4.7) or (4.17), we can obtain the theoretical BER expressions of downlink or uplink MC-CDMA respectively.

However, the BER performance of MC-CDMA systems is not easy to analyze as the receiver operations involve coherent combining of a large number of possibly correlated fading subcarriers. Signal analysis is further complicated by the presence of MUI which may also contain fading parameters, as shown in Section 4.2.1 to 4.2.3.

One obvious way to compute the BER of MC-CDMA systems is through numerical simulations [19–22]. However, it is very time consuming and when the channel conditions change, we need to re-do the simulation again. In order to avoid time consuming simulations and obtain in-depth understanding on the systems, theoretical performance analysis is always desirable. Central limit theorem (CLT) or the law of large numbers (LLN) have been exploited by some researchers to simplify the BER analysis, by assuming independent fading among large number of subcarriers [7, 23, 25].

In [29], Monte Carlo integration is used to evaluate the uplink BER formulas of MC-CDMA systems with MRC and EGC. These BER formulas involve nested integrations of dimension equal to the number of subcarriers. Simplified BER formulation in the form of closed-form expressions [20, 26, 27] or one-dimensional integral formulas [24, 27, 28] have also been achieved in the literature. However, these results are either lower bound formulas, or they are constrained by certain assumptions. For example, the BER provided in [20] is a BER lower bound for

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

downlink MC-CDMA with MRC. And the BER lower bound given in [26] assumes that the channel is noiseless. In addition, assumptions of i.i.d. fading among the channel paths [23, 24] or independent fading among the subcarriers [7, 25–28] are also commonplace.

For BER analysis without the constraints of the above-mentioned conditions, we have to revert to the generalized multi-dimensional integration formula in (4.8) for the downlink BER, or (4.18). Both expressions are exact but involve multi-dimensional integration which is cumbersome to evaluate.

In this chapter, we use Monte Carlo integration to calculate the multi-dimensional integration in (4.8) and (4.18). Monte Carlo integration (not Monte Carlo simulation) is a method to evaluate multi-dimensional integration and it was employed in [29] to evaluate the BER performance for uplink MC-CDMA systems. In the theory of Monte Carlo integration, an integration involving pdf or jdf is viewed as a statistical averaging process. By generating a set of RVs with pertinent distributions and correlations, the multi-dimensional integration is evaluated by taking the expected mean value of the RVs [73].

The most challenging task in the implementation of Monte Carlo integration is usually the generation of RVs of specific distributions and correlations. The integration dimension in (4.8) and (4.18) is equal to the number of subcarriers N_c , which could be as large as 2048 in digital video broadcasting (DVB) systems. Also, since the subcarrier fading is generally correlated (as discussed in Section 3.3), we need to generate correlated RVs, which further complicates the implementation.

In the next section, we will compare the performance difference between various equalization techniques. We also show in the next section that we can actually make use of the time and frequency equivalence of multicarrier signals to simplify the implementation of frequency-domain Monte Carlo integration.

4.4 Performance Comparison of Different Equalization and Combining Schemes

As shown in (4.7) and (4.17), the BER conditioned on the subcarrier fading process, $P_e|\{h_n\}$, is basically some function of $\{h_n\}$, i.e., $f(\{h_n\})$. The overall P_e is obtained by averaging $f(\{h_n\})$ over the jdf of $\{h_n\}$, i.e.,

$$P_e = \int_0^\infty \cdots \int_0^\infty f(\{h_n\}) jdf(\{h_n\}) dh_1 \cdots dh_{N_c}. \quad (4.42)$$

According to (3.8) and provided that τ_l is an integer multiple of T_s/N_c , the subcarrier fading gain $\tilde{h}_n = h_n \exp(j\varphi_n)$ is related to the path fading gain $\tilde{g}_l = g_l \exp(j\psi_l)$ by the DFT operation, i.e.,

$$\tilde{h}_n = \sum_{l=1}^{N_p} \tilde{g}_l \exp \left[-j2\pi(n-1)\frac{\tau_l}{T_s} \right]. \quad (4.43)$$

Hence, the fading envelope $\{h_n\}$ is related to $\{g_l\}$ and $\{\psi_l\}$ by the following function Λ_n :

$$\begin{aligned} h_n &= \sqrt{\left(\sum_{l=1}^{N_p} g_l \cos(\psi_l - j2\pi(n-1)\tau_l/T_s) \right)^2 + \left(\sum_{l=1}^{N_p} g_l \sin(\psi_l - j2\pi(n-1)\tau_l/T_s) \right)^2} \\ &\equiv \Lambda_n(\{g_l\}, \{\psi_l\}). \end{aligned} \quad (4.44)$$

For simplicity in representation, we write $\Lambda_n(\{g_l\}, \{\psi_l\})$ as Λ_n . Equation (4.44) implies that the subcarrier fading process in the frequency domain is uniquely determined by the channel path fading process in the time domain. Hence the frequency-domain integration over the subcarrier fading processes $\{h_n\}$ in (4.42) can be transformed into a time-domain integration over the channel path fading processes $\{g_l\}$ and $\{\psi_l\}$, as shown below:

$$P_e = \int_0^\infty \cdots \int_0^\infty f(\Lambda_1, \dots, \Lambda_{N_c}) jdf(\{g_l\}, \{\psi_l\}) dg_1 \cdots dg_{N_p} d\psi_1 \cdots d\psi_{N_p}. \quad (4.45)$$

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Equation (4.45) implies that in order to obtain the system BER using Monte Carlo integration, we need only generate N_p RVs for the fading amplitudes and N_p RVs for the phases of channel paths (the channel fading phases are usually i.i.d. over $[0, 2\pi)$ [33]).

Next we will compare the performance for different equalization techniques.

After substituting the equalizer coefficients $\{\alpha_n\}$ in (4.19), (4.20), (4.21), (4.30) or (4.36) into (4.7), we can obtain the BER for the downlink MC-CDMA system with different equalization and combining schemes accordingly. Similarly, substituting them into (4.17) gives the uplink BER.

Let's consider the indoor channel model specified in Table 3.2 which consists of 3 independent Rayleigh fading paths. For this channel, the BER of a 64-subcarrier MC-CDMA system with various equalization and combining schemes are shown in Fig. 4.5-4.8. Fig. 4.5-4.7 are for the downlink channels with 1 user, 32 users and 64 users (full load condition), respectively. Fig. 4.8 is for an uplink channel with 32 users. Note that although there are 64 subcarriers, to obtain the BER, we only need to generate 3 independent Rayleigh RVs for the path amplitudes and 3 i.i.d. RVs uniformly distributed over $[0, 2\pi)$ for the phase of the channel paths, or equivalently 6 i.i.d. Gaussian RVs.

The spreading codes chosen for the downlink is the orthogonal Walsh-Hadamard codes. As seen from Fig. 4.5-4.7, the BER performance of ORC does not degrade with increasing number of users since it restores the orthogonality between the users and hence eliminates MUI. MRC performs best under the single-user condition, but it degrades quickly with increasing number of users since it destroys the orthogonality between users. EGC performs not as good as MRC for the single-user condition, but its performance does not degrade as quickly when the number of users in the system increases. As the MMSEC scheme considers all the channel effects including MUI, fading and noise in its design, it performs best all the

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

time. The “MMSEC per user” scheme outperforms the “MMSEC per subcarrier” scheme, but the former requires knowledge of the code information for all users and is computationally more complex in its equalizer coefficient expression.

Fig. 4.8 shows that the uplink BER performance is much worse than the downlink BER performance because the asynchronous transmission in the uplink channel results in much higher MUI. The uplink performance also exhibits an error floor at high E_b/N_0 level due to the MUI power predominating over the AWGN power. Moreover, MRC is the best scheme for uplink channels and MMSEC performs almost the same as MRC. This is because the MUI is so large that the first term h_n^2 in the denominator of (4.41) can almost be ignored, hence the equalizer coefficient for MMSEC (4.41) becomes the same as that for MRC (4.20).

Note that the BER performance curves in Fig. 4.5-4.8 are actually obtained using the proposed time-domain Monte Carlo integration. The results match very well with that obtained using pure Monte Carlo simulations, despite the fact that the computation time has been greatly reduced.

4.5 Summary

In this chapter, we present the signal model for both downlink and uplink MC-CDMA channels. The theoretical BER formulations are in the form of multi-dimensional integration, which can be evaluated using Monte Carlo integration. The most challenging task in the implementation of Monte Carlo integration is usually the generation of RVs. Hence in this chapter, we map the frequency-domain integration to a time-domain integration over the jdf of all channel paths. Comparing (4.42) and (4.45), the dimension of integration is reduced from N_c to $2N_p$, as the number of channel paths N_p is normally much less than the number

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

of subcarriers N_c . Therefore, the computational complexity of BER evaluation is significantly reduced.

Figure 4.5: BER performance of downlink MC-CDMA (64 subcarriers and 1 user) with different equalization and combining schemes.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Figure 4.6: BER performance of downlink MC-CDMA (64 subcarriers and 32 user) with different equalization and combining schemes.

Figure 4.7: BER performance of downlink MC-CDMA (64 subcarriers and 64 user) with different equalization and combining schemes.

CHAPTER 4. BER ANALYSIS AND PERFORMANCE OF MC-CDMA SYSTEMS

Figure 4.8: BER performance of uplink MC-CDMA (64 subcarriers and 32 user) with different equalization and combining schemes.

Chapter 5

BER Analysis of Uplink MC-CDMA with MRC in Generalized Frequency-Selective Fading Channels

5.1 Introduction

In the last chapter, we have shown that the dimension of Monte Carlo integration for the computation of BER of MC-CDMA system can be reduced substantially by mapping the BER expressions from the frequency domain to the time domain. Next, we shall develop further simplification for the BER expressions of MC-CDMA with MRC and ORC. The case of MRC will be discussed in this chapter, while ORC will be discussed in the next chapter.

MRC offers maximum improvement in the presence of spectrally white Gaussian noise [68, 69]. In [29], it is shown that MRC achieves better performance than EGC for the uplink MC-CDMA, and the resultant system has lower error floor than DS-CDMA and MC-DS-CDMA. The BER plots in Fig. 4.8 in the previous chapter also show that MRC is the best combining scheme for the uplink MC-CDMA.

In this chapter, we will focus on the BER analysis for uplink MC-CDMA with MRC in generalized frequency-selective fading channels. To our knowledge, the

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

most general performance analysis of uplink MC-CDMA with MRC is given in [29], where Monte Carlo integration is used to evaluate the BER expressions. Although simplified BER formulations can be found in [24, 27, 28], they are based on the assumptions of independent and identically distributed (i.i.d.) fading among the channel paths [24], or independent fading among the subcarriers [27, 28]. Furthermore, all the works reported in [20, 24, 26–29] for the BER analysis for MC-CDMA with MRC only consider Rayleigh fading channels. Other fading pdfs are not considered.

In this chapter, we will generalize and extend the performance analysis of uplink MC-CDMA with MRC to Rayleigh, Rician and Nakagami fading channels with arbitrary fading parameters and correlations.

5.2 Simplification of BER Formulas

The signal model for uplink MC-CDMA and the corresponding BER expressions have been derived in Section 4.2.2. In (4.12), the decision variable U for uplink MC-CDMA systems is shown to consist of four components, namely the desired signal D , the noise η , the MUI component I from the same subcarrier of other users and the MUI component J from the other subcarriers of other users. For MRC, the equalizer coefficient $\alpha_n = h_n$. Substituting it to (4.4), we can obtain the desired signal D

$$D = \sqrt{\frac{E_b T_s}{2N_c}} \sum_{n=1}^{N_c} h_n^2. \quad (5.1)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

The variance for the interference I , J and the noise η , as deduced from (4.15), (4.16) and (4.6) respectively, are

$$\text{var}(I) = \frac{E_b T_s (N_u - 1) \sigma^2}{3 N_c} \sum_{n=1}^{N_c} h_n^2, \quad (5.2)$$

$$\text{var}(J) = \frac{E_b T_s (N_u - 1) \sigma^2}{4 N_c \pi^2} \sum_{n=1}^{N_c} h_n^2 \sum_{i=1, i \neq n}^{N_c} (i - n)^{-2}, \quad (5.3)$$

$$\text{var}(\eta) = \frac{N_0 T_s}{4} \sum_{n=1}^{N_c} h_n^2, \quad (5.4)$$

where N_u is the number of users, N_c is the number of subcarriers, E_b is the bit energy, N_0 is the noise power density, T_s is MC-CDMA symbol duration and σ^2 is the subcarrier fading power of the interfering users.

Thus the BER expression for the uplink MC-CDMA signal with MRC, conditioned on the set of subcarrier fading amplitudes $\{h_n\}$, is

$$P_e | \{h_n\} = Q \left(\frac{\sum_{n=1}^{N_c} h_n^2}{\sqrt{\left(\frac{2(N_u-1)\sigma^2}{3} + \frac{N_c}{2E_b/N_0} \right) \sum_{n=1}^{N_c} h_n^2 + \frac{(N_u-1)\sigma^2}{2\pi^2} \sum_{n=1}^{N_c} \sum_{i=1, i \neq n}^{N_c} (i - n)^{-2} h_n^2}} \right). \quad (5.5)$$

The average BER P_e can then be obtained by averaging (5.5) over the joint distribution function (jdf) of $\{h_n\}$, i.e.,

$$P_e = \int_0^\infty \cdots \int_0^\infty (P_e | \{h_n\}) \text{jdf}(\{h_n\}) dh_1 \cdots dh_{N_c}. \quad (5.6)$$

Note that the presence of $\sum_{i=1, i \neq n}^{N_c} (i - n)^{-2}$ in (5.5) makes it difficult to further simplify the $P_e | \{h_n\}$ expression. However, we noticed that its value does not vary much for different values of n . As an example, Fig. 5.1 shows the value of $\sum_{i=1, i \neq n}^{N_c} (i - n)^{-2}$ as a function of the subcarrier index n , assuming 128 subcarriers in total. As seen from the figure, the values of $\sum_{i=1, i \neq n}^{N_c} (i - n)^{-2}$ are very similar

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

for most values of n . Hence we approximate it as a constant a whose value only depends on N_c , i.e.,

$$\sum_{i=1, i \neq n}^{N_c} \frac{1}{(i-n)^2} \simeq \frac{1}{N_c} \sum_{n=1}^{N_c} \sum_{i=1, i \neq n}^{N_c} \frac{1}{(i-n)^2} = a. \quad (5.7)$$

Later we shall show using simulation results that the effect of the approximation made in (5.7) on the final BER value is negligible.

With this approximation, the term $\sum_{n=1}^{N_c} h_n^2$ becomes a common factor in the denominator of the conditional uplink BER expression in (5.5), which can then be simplified to

$$P_e|\{h_n\} = Q \left(\sqrt{\frac{\sum_{n=1}^{N_c} h_n^2}{\frac{2(N_u-1)\sigma^2}{3} + \frac{(N_u-1)\sigma^2 a}{2\pi^2} + \frac{N_c}{2E_b/N_0}}} \right). \quad (5.8)$$

Let's define a combined subcarrier fading variable

$$\chi = \sum_{n=1}^{N_c} h_n^2, \quad (5.9)$$

the average BER can be obtained by averaging (5.8) over the probability density function (pdf) of χ , i.e.,

$$P_e = \int_0^\infty Q(\sqrt{\nu\chi}) p(\chi) d\chi, \quad (5.10)$$

where ν is a constant given by

$$\nu = \left[\frac{2(N_u-1)\sigma^2}{3} + \frac{(N_u-1)\sigma^2 a}{2\pi^2} + \frac{N_c N_0}{2 E_b} \right]^{-1}. \quad (5.11)$$

To evaluate (5.10), the pdf of χ must first be obtained. However, in general it is not easy to find the pdf of χ for large number of subcarriers whose fading gains $\{h_n\}$ may be correlated, or may even have different fading characteristics as explained in Chapter 3. In this thesis, we circumvent this problem by transforming the subcarrier-domain integration in (5.10) into a path-domain integration. This will be elaborated in the next section.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.1: Value of $\sum_{i=1, i \neq n}^{N_c} (i-n)^{-2}$ as a function of the subcarrier index n ($N_c = 128$).

5.3 Time- and Frequency-Domain Equivalence of the MRC Output

The relationship between the subcarrier fading gain $\tilde{h}_n = h_n \exp(j\varphi_n)$ and the path fading gain $\tilde{g}_l = g_l \exp(j\psi_l)$ was given in Section 3.2.2. Referring to (3.8), the complex subcarrier fading gain is related to the time-domain CIR by

$$\tilde{h}_n = \sum_{l=1}^{N_p} \tilde{g}_l \exp \left[-j2\pi(n-1)\frac{\tau_l}{T_s} \right]. \quad (5.12)$$

In matrix form, (5.12) can be re-written as

$$\tilde{\mathbf{h}} = \mathbf{W}\tilde{\mathbf{g}}, \quad (5.13)$$

where

$$\tilde{\mathbf{h}} = [\tilde{h}_1, \dots, \tilde{h}_n, \dots, \tilde{h}_{N_c}]^T,$$

$$\tilde{\mathbf{g}} = [\tilde{g}_1, \dots, \tilde{g}_l, \dots, \tilde{g}_{N_p}]^T,$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ e^{-j2\pi\tau_1/T_s} & e^{-j2\pi\tau_2/T_s} & \dots & e^{-j2\pi\tau_{N_p}/T_s} \\ e^{-j4\pi\tau_1/T_s} & e^{-j4\pi\tau_2/T_s} & \dots & e^{-j4\pi\tau_{N_p}/T_s} \\ \dots & \dots & \dots & \dots \\ e^{-j(N_c-1)2\pi\tau_1/T_s} & e^{-j(N_c-1)2\pi\tau_2/T_s} & \dots & e^{-j(N_c-1)2\pi\tau_{N_p}/T_s} \end{bmatrix}.$$

The $N_c \times N_p$ matrix $\mathbf{W}$ has the property that $\mathbf{W}^H \mathbf{W} = N_c \mathbf{I}_{N_p}$ and $\mathbf{I}_{N_p}$ is a $N_p \times N_p$ identity matrix. The superscripts T and H denote matrix transpose and matrix Hermitian transpose respectively.

The term $\sum_{n=1}^{N_c} h_n^2$ can now be represented as

$$\sum_{n=1}^{N_c} h_n^2 = \tilde{\mathbf{h}}^H \tilde{\mathbf{h}} = \tilde{\mathbf{g}}^H \mathbf{W}^H \mathbf{W} \tilde{\mathbf{g}} = N_c \tilde{\mathbf{g}}^H \tilde{\mathbf{g}} = N_c \sum_{l=1}^{N_p} g_l^2. \quad (5.14)$$

This signifies that the amount of frequency diversity available to an uplink MC-CDMA signal with MRC is exactly equal to the amount of path diversity available in the frequency-selective fading channel in which the MC-CDMA system operates, regardless of the path number and path delays.

Let's define the combined path fading variable

$$\gamma = \sum_{l=1}^{N_p} g_l^2. \quad (5.15)$$

The BER expression in (5.10) can now be written as

$$P_e = \int_0^\infty Q\left(\sqrt{\nu N_c \gamma}\right) p(\gamma) d\gamma. \quad (5.16)$$

Compared to (5.10), (5.16) is much easier to compute because the pdf of γ is generally easier to analyze than the pdf of χ . This is because the number of significant channel paths N_p is normally much less than the number of subcarriers N_c . For example, N_c can be 64 for the IEEE 802.11a wireless LAN standard, or as large as 2048 for the DVB standard, while N_p in practical wireless communication systems is normally less than 10 [17, 18]. Furthermore, fading among the subcarriers is normally correlated (even for channels with independent fading paths as

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

discussed in Section 3.3). This increases the complexity in determining the pdf of χ . Therefore, we shall use (5.16) and the pdf of γ to derive the expression of P_e in the next section.

5.4 BER Analysis

Notice that (5.16) is equivalent to the BER expression for a generic MRC system, so the problem now is to find the pdf of γ , which is the MRC output of multiple fading paths $\{g_i\}$. We will derive the pdf of γ with arbitrarily correlated paths with Rayleigh, Rician or Nakagami fading (Rayleigh fading is discussed as a special case of Nakagami or Rician fading). To our knowledge, such generalized problem has not been fully solved in the existing literature.

5.4.1 Channels with Independent Nakagami Fading Paths

5.4.1.1 Existing Works

Nakagami fading distribution, also known as m -distribution, is widely adopted for modelling fading channels because of its good fit to empirical measurements [74–76], as well as the tractability it renders to BER evaluation [77]. A variety of fading effects can be modelled as Nakagami fading with different m -parameters, including Rayleigh and one-sided Gaussian fading as special cases when m equals to 1 and 1/2, respectively. As a result, the performance analysis of MRC reception in Nakagami fading channels is of widespread interest.

Some pdf formulas for the combined output of MRC systems of independent Nakagami signals are given in [52, 69, 77–81]. The results in [79] and [80] are complicated, hence difficult to use. The results in [69] are limited to integer values of m parameter for the diversity branches, while in [77], the author only considers the case of equal m parameter values for all the branches. Approximations is

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

involved in deriving the closed-form result in [81] (with enough accuracy though) and correlated fading is not considered in [81].

In this thesis, we follow the approach in [78] because it can also be used for the correlated fading in the next section. In [78], Nakagami obtained the exact pdf expression for MRC output, subject to the assumption that all diversity branches have the same m_l/Ω_l ratio where Ω_l is the branch power. If the diversity branches have different m_l/Ω_l ratios, the MRC output is approximated as another Gamma distribution. This approximation is useful as it greatly simplifies the evaluation of MRC system performance in Nakagami fading channels.

It is worth taking note that the moment generating function (MGF) approach in [81, 82] is widely known in the literature for directly calculating the BER, without the need to calculate the pdf of the MRC output. However, the MGF of the MRC output available in the literature is only for independent Nakagami channels or correlated channels with the same m parameters. Hence we still follow the pdf approach in this thesis, aiming to analyze the MRC systems with arbitrary fading parameters and fading correlations.

5.4.1.2 Determination of m Parameter through Moment-Matching

Assuming that the path gains $\{g_l\}$ have independent Nakagami fading. The Nakagami- m distribution is given by [78]

$$p(g_l) = \frac{2}{\Gamma(m_l)} \left(\frac{m_l}{\Omega_l} \right)^{m_l} g_l^{2m_l-1} \exp \left[- \left(\frac{m_l}{\Omega_l} \right) g_l^2 \right] \quad (5.17)$$

where $\Omega_l = E[g_l^2]$ and $m_l = \frac{E^2[g_l^2]}{E[(g_l^2 - E[g_l^2])^2]}$. $\Gamma(\cdot)$ is the Euler Gamma function and $E[\cdot]$ denotes statistical expectation. The square of Nakagami RV $\gamma_l = g_l^2$ is widely known to follow the Gamma distribution [69, 77, 78]

$$p(\gamma_l) = \frac{(m_l/\Omega_l)^{m_l} e^{-(m_l/\Omega_l)\gamma_l} \gamma_l^{m_l-1}}{\Gamma(m_l)}. \quad (5.18)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

Hence the MRC signal

$$\gamma = \gamma_1 + \gamma_2 + \dots + \gamma_{N_p} \quad (5.19)$$

can be viewed as the summation of N_p number of independent Gamma RVs.

If the set of Gamma RVs $\{\gamma_l\}$ are independent with identical m_l/Ω_l ratio for all values of l , it was shown in [69] that γ follows exactly a Gamma distribution with m and Ω given by

$$m = \sum_{l=1}^{N_p} m_l, \quad (5.20)$$

$$\Omega = \sum_{l=1}^{N_p} \Omega_l. \quad (5.21)$$

However, if the set of Gamma RVs $\{\gamma_l\}$ has different m_l/Ω_l ratios, the exact pdf of γ becomes much more complicated to derive.

Our approach is to approximate such MRC output as a Gamma RV with new m parameter and power Ω . To derive the formulas of m and Ω in terms of the branch parameters m_l and Ω_l , we match the moments of the exact MRC output with the moments of the approximated Gamma RV. This analysis is novel as it caters for arbitrary number of branches with arbitrary values of m_l , Ω_l and m_l/Ω_l ratios.

According to [67], an RV can be characterized by its moment generating function (MGF), which is defined as

$$\mathcal{M}_x(t) = \int_{-\infty}^{\infty} p(x) e^{tx} dx, \quad (5.22)$$

where $p(x)$ is the pdf of x . Denoting μ_q as the q th order moment of the RV x , we have [67]

$$\mathcal{M}_x(t) = \sum_{q=0}^{\infty} \frac{\mu_q}{q!} t^q, \quad (5.23)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

where $q!$ means the factorial of q . As MGF uniquely defines an RV [67], (5.23) shows that an RV is fully described by all its moments. Since we approximate γ as a Gamma RV, we can determine the m parameter and power Ω of γ by matching the first two moments of the approximated Gamma RV to the true MRC output defined in (5.19). In this case, it is straightforward to see that Ω still follows (5.21). Hence what we are interested in now is the formula for m .

The MGF of a Gamma RV has a simple form given by

$$\mathcal{M}_\gamma(t) = \int_{-\infty}^{\infty} \frac{(m/\Omega)^m e^{-(m/p)\gamma} \gamma^m}{\Gamma(m)} e^{t\gamma} d\gamma = \left(1 - \frac{\Omega}{m}t\right)^{-m}. \quad (5.24)$$

For an MRC system with N_p independent branches, the MGF for γ is

$$\mathcal{M}_\gamma(t) = \prod_{l=1}^{N_p} \mathcal{M}_{\gamma_l}(t) = \prod_{l=1}^{N_p} \left(1 - \frac{\Omega_l}{m_l}t\right)^{-m_l}. \quad (5.25)$$

Case 1: If m_l/Ω_l is the same, for all branches, the MGF for γ is

$$\mathcal{M}_\gamma(t) = \left(1 - \frac{\sum_{l=1}^{N_p} \Omega_l}{\sum_{l=1}^{N_p} m_l}t\right)^{-\sum_{l=1}^{N_p} m_l}. \quad (5.26)$$

Since the MGF for γ has the same form as (5.24), γ is a Gamma RV with m parameter given by (5.20) and power given by (5.21). This conclusion is consistent with that of [69].

Case 2: If m_l/Ω_l are different for different branches, we will determine m of the MRC output using the definition of the m parameter of a Gamma RV [78]

$$m = \frac{E^2[\gamma]}{E[\gamma^2] - E^2[\gamma]}. \quad (5.27)$$

The moments of γ can be obtained from

$$E[\gamma^q] = \frac{d^q}{dt^q} \mathcal{M}_\gamma(t)|_{t=0}. \quad (5.28)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Given $\mathcal{M}_\gamma(t)$ in (5.25), using Taylor series expansion [83], and as t approach 0, we have

$$\begin{aligned}\mathcal{M}_\gamma(t) &\simeq \prod_{l=1}^{N_p} \left(1 + t \Omega_l + \frac{t^2 (m_l + 1) \Omega_l^2}{2 m_l} \right) \\ &= 1 + t \sum_{l=1}^{N_p} \Omega_l + \frac{t^2}{2} \left(\sum_{l=1}^{N_p} \frac{m_l + 1}{m_l} \Omega_l^2 + \sum_{l=1}^{N_p} \sum_{i=1, i \neq l}^{N_p} \Omega_l \Omega_i \right).\end{aligned}\quad (5.29)$$

Hence

$$E[\gamma] = \sum_{l=1}^{N_p} \Omega_l, \quad (5.30)$$

and

$$\begin{aligned}E[\gamma^2] &= \sum_{l=1}^{N_p} \frac{m_l + 1}{m_l} \Omega_l^2 + \sum_{l=1}^{N_p} \sum_{i=1, i \neq l}^{N_p} \Omega_l \Omega_i \\ &= \left(\sum_{l=1}^{N_p} \Omega_l \right)^2 + \sum_{l=1}^{N_p} \frac{\Omega_l^2}{m_l}.\end{aligned}\quad (5.31)$$

Substituting the above two equations into (5.27), we have the m parameter for a N_p -branch MRC output:

$$m = \frac{(\sum_{l=1}^{N_p} \Omega_l)^2}{\sum_{l=1}^{N_p} \frac{\Omega_l^2}{m_l}}. \quad (5.32)$$

Note that (5.32) is reduced to (5.20) when the m_l/Ω_l ratios are equal among all branches. The proof is given in Appendix B of the thesis. Therefore, (5.32) is the generalized expression as it can handle arbitrary m_l and Ω_l combinations.

In summary, the MRC output of Nakagami fading branches with unequal m_l/Ω_l ratios is approximated as a Gamma RV and its m parameter is determined by matching its first two moments to the true MRC output.. As we match only the first two moments in our MRC model, there are approximation errors which are contributed by the higher order moments. The most significant approximation error component caused by the 3rd order moment is presented in Appendix A. The

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

results shown that larger difference in m_l/Ω_l ratios gives rise to larger approximation error, while zero difference in m_l/Ω_l ratios results in an exact Gamma model for the MRC output.

5.4.1.3 Verification of the Approximated Gamma RV γ

Next we will use an example to verify the accuracy of approximating the MRC output γ of Nakagami fading diversity branches with different m_l/Ω_l ratios as a Gamma RV with parameters m and Ω given by (5.32) and (5.21) respectively. For Nakagami fading channel with integer m_l values and distinct m_l/Ω_l ratios, an exact pdf for the MRC output γ has actually been derived in [69] as

$$p(\gamma) = \left[\prod_{l=1}^{N_p} \left(\frac{m_l}{\Omega_l} \right)^{m_l} \right] \times \left\{ \sum_{i=1}^{N_p} \sum_{q=1}^{m_i} \frac{\frac{d^{m_i-q}}{ds^{m_i-q}} \left[\prod_{l=1, l \neq i}^{N_p} \left(\frac{m_l}{\Omega_l} + s \right)^{-m_l} \right] \Big|_{s=-m_i/\Omega_i}}{(m_i - q)!} \gamma^{q-1} \frac{\exp(-\gamma m_i/\Omega_i)}{(q-1)!} \right\}. \quad (5.33)$$

It is clear that our approximate pdf has a much simpler form than (5.33). Furthermore, our approximate pdf is good for any value of m_l , while (5.33) can only handle integer m_l values.

In Fig. 5.2, the exact pdf (5.33) is compared with our approximated Gamma pdf for three different MRC channels. The curves (a) in Fig. 5.2 are for a 2-branch Nakagami fading channel, the curves (b) are for a 3-branch Nakagami fading channel and the curves (c) are for a 4-branch Rayleigh fading channel. The m_l and Ω_l values for these channels are shown in the figure caption. Fig. 5.2 shows that the approximated MRC pdf match very well with the exact MRC pdf given in (5.33) for all cases.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.2: Comparison of approximated MRC pdf (5.18) with the exact MRC pdf (5.26). Branch parameters: (a) $\{m_l\} = \{2, 1\}$, $\{\Omega_l\} = \{3, 1\}$; (b) $\{m_l\} = \{10, 5, 1\}$, $\{\Omega_l\} = \{5, 3, 1\}$; (c) $\{m_l\} = \{1, 1, 1, 1\}$, $\{\Omega_l\} = \{2.1, 0.4, 2.6, 1.6\}$.

5.4.1.4 BER Expressions and Results

The BER of uplink MC-CDMA with MRC was given in (5.16). Substituting the Gamma pdf (5.18) into it, we have

$$P_e = \int_0^\infty Q\left(\sqrt{\nu N_c \gamma}\right) \frac{(m/\Omega)^m e^{-(m/\Omega)\gamma} \gamma^{m-1}}{\Gamma(m)} d\gamma \quad (5.34)$$

$$= \frac{\left(\frac{m}{\Omega}\right)^m \Gamma\left(\frac{1}{2} + m\right) {}_2F_1\left(m, \frac{1}{2} + m; 1 + m, -\frac{m}{\Omega}\right)}{2\sqrt{\pi} \Gamma(1 + m)},$$

where ${}_2F_1(\cdot)$ is the hypergeometric function and the constant ν is given in (5.11), the m is parameter given by (5.32) and the power Ω is given by (5.21).

With $m_l = 1$ for the fading paths, (5.34) becomes applicable to channels with Rayleigh fading paths with different path powers. Furthermore, for i.i.d. Rayleigh

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

fading branches, (5.34) is reduced to the following closed-form formula

$$P_e = \left[\frac{1}{2}(1 - \varsigma) \right]^{N_p} \sum_{q=0}^{N_p-1} \binom{N_p-1+q}{q} \left[\frac{1}{2}(1 + \varsigma) \right]^q, \quad (5.35)$$

where

$$\varsigma = \sqrt{\frac{\nu N_c \Omega / N_p}{1 + \nu N_c \Omega / N_p}}. \quad (5.36)$$

Next, we will verify the derived BER expressions (5.34) and (5.35) for uplink MC-CDMA with MRC in channels with independent Nakagami-Rayleigh fading paths. First of all, we will compare the BER results obtained from (5.35) with those available in [24] (for channels with i.i.d. Rayleigh fading paths) and [27] (for channels with i.i.d. Rayleigh fading subcarriers). Using Laplace transform and residual method, the authors of [24] prove that the MRC output in frequency domain $\chi = \sum_{n=1}^{N_c} h_n^2$ is a Chi-square distributed RV of order $2N_p$ and the resultant BER expression is in one-dimensional integration form. The authors of [27] derive the characteristic function of the MRC decision variable and propose three methods to compute the resultant BER. One of the methods employs numerical calculation, the other two methods involve either Hypergeometric function or residue calculation. In addition, the results in [24, 27] are also based on the simplifying assumption that the uplink is a synchronous channel. In contrast, our analysis has shown that, if the channel paths are i.i.d. Rayleigh fading, the BER of MC-CDMA with MRC has a simple closed-form expression (5.35) which has no constraint of synchronous transmission.

The BER results computed using our formula (5.35) are shown in Fig. 5.3 and Fig. 5.4 with system parameters corresponding to Fig. 4 in [24] (which plots the BER for different number of paths) and Fig. 1 in [27] (which plots the BER for different number of users), respectively. Clearly, our BER results in both

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

figures match exactly with those in [24] and [27]. This verify our simplified BER expression (5.35).

In Fig. 5.5, we use computer simulations of asynchronous uplink MC-CDMA with 128 subcarriers and 10 active users to verify our BER expression in (5.34). There are three types of BER plots in Fig. 5.5: one simulation plot and two analytical plots (obtained with or without the approximation made in (5.7)). The two channels considered are a 4-path Nakagami channels with identical m_l/Ω_l ratio and a 3-path Nakagami channels with non-identical m_l/Ω_l ratios.

The “analytical with approximation (5.7)” plots in Fig. 5.5 are obtained by using (5.34), while the “analytical without approximation (5.7)” plots are obtained by Monte Carlo Integration of the conditional BER given in (5.5). It can be seen that the analytical BER values with or without approximation (5.7) are indistinguishable, hence the effect of the approximation made in (5.7) on the system BER values is negligible.

For Nakagami fading channels, if the fadings for different channel paths are independent and have identical m_l/Ω_l ratio, the MRC output γ follows an exact Gamma distribution. Hence, as seen for curves (a) in Fig. 5.5, both the simulated and analytical BER plots match very well. On the other hand, when the fading in different channel paths have different m_l/Ω_l ratios, γ is only approximately Gamma-distributed. Hence, some slight mismatch can be observed between the simulated and analytical BER values for curves (b) in Fig. 5.5, but the mismatch is very small and hence acceptable.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.3: Performance of uplink MC-CDMA with MRC in i.i.d. Rayleigh fading channels for various number of paths (for comparison with Fig. 4 in [24]).

Figure 5.4: Performance of uplink MC-CDMA with MRC for i.i.d. Rayleigh fading subcarriers with various number of users (for comparison with Fig. 1 in [27]).

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.5: Performance of uplink MC-CDMA with MRC in a channel with (a) 4 independent Nakagami fading paths, $\{m_l\} = [9, 5, 4, 2]$, $\{\Omega_l\} = [0.45, 0.25, 0.2, 0.1]$; (b) 3 independent Nakagami fading paths, $\{m_l\} = [4, 3, 1]$, $\{\Omega_l\} = [0.46, 0.38, 0.15]$.

5.4.2 Channels with Correlated Nakagami Fading Paths

5.4.2.1 Existing Works

Although statistical independence among the diversity branches is desired in MRC systems, there are cases where this assumption is not valid [51, 52]. In this section, we will analyze the BER performance of uplink MC-CDMA with MRC in Nakagami fading channels with not just arbitrary fading parameters, but also arbitrary fading correlations.

Dual-branch MRC system with correlated Nakagami fading branches is discussed in [52, 69, 78, 84]. The study is further generalized to arbitrary number of diversity branches in [51, 77, 79, 82, 85–89]. However, the results for correlated Nakagami fading in [51, 77, 79, 82, 85–88] are subject to the constraint of identical branch statistical parameters m_l and power Ω_l . And the results in [87–89] are

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

limited to the case of m_l must be integer values or at least half-integer (multiples of $1/2$).

In this section, we propose a way to obtain the statistics of MRC systems in correlated Nakagami fading channel without any constraints on the fading parameters and correlations of the diversity branches. The general principle of our proposed approach is as follows. Cholesky decomposition [90] is used to transform the correlated Nakagami fading branches into independent ones by matching their first and second moments. Then the previous derived formulas in Section 5.4.1 for independent Nakagami fading branches are used to obtain the final MRC output statistics.

5.4.2.2 Modelling Correlated RVs as Weighted Sums of Uncorrelated RVs

First of all, denote

$$\boldsymbol{\gamma} = [\gamma_1 \ \gamma_2 \ \dots \ \gamma_l \ \dots \ \gamma_{N_p}]^T \quad (5.37)$$

where γ_l is the square of branch fading envelope, i.e., $\gamma_l = g_l^2$. As g_l is a Nakagami RV, γ_l is a Gamma RV following the pdf given in (5.18). The covariance matrix $\mathbf{C}_\gamma$ is defined as

$$\mathbf{C}_\gamma = E[(\boldsymbol{\gamma} - \mathbf{M}_\gamma)(\boldsymbol{\gamma} - \mathbf{M}_\gamma)^T] \quad (5.38)$$

where

$$\mathbf{M}_\gamma = E[\boldsymbol{\gamma}] . \quad (5.39)$$

For positive definite matrix $\mathbf{C}_\gamma$, it is always possible to perform Cholesky decomposition to obtain

$$\mathbf{C}_\gamma = \mathbf{L}\mathbf{L}^T \quad (5.40)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

where $\mathbf{L}$ is a lower triangular matrix, i.e.,

$$\mathbf{L} = \begin{bmatrix} \zeta_{11} & 0 & 0 & \cdots & 0 \\ \zeta_{21} & \zeta_{22} & 0 & \cdots & 0 \\ \vdots & \cdots & \ddots & 0 & 0 \\ \zeta_{(N_p-1)1} & \cdots & \cdots & \ddots & 0 \\ \zeta_{N_p1} & \cdots & \cdots & \cdots & \zeta_{N_p N_p} \end{bmatrix}. \quad (5.41)$$

Denote $\mathbf{w}$ as a vector which consists of independent Gamma RVs with identity covariance matrix, i.e.,

$$\mathbf{w} = [w_1 \ w_2 \ \cdots \ w_{N_p-1} \ w_{N_p}]^T \quad (5.42)$$

$$\mathbf{M}_{\mathbf{w}} = E[\mathbf{w}] \quad (5.43)$$

$$\mathbf{C}_{\mathbf{w}} = E[(\mathbf{w} - \mathbf{M}_{\mathbf{w}})(\mathbf{w} - \mathbf{M}_{\mathbf{w}})^T] = \mathbf{I}_{N_p} \quad (5.44)$$

where $\mathbf{I}_{N_p}$ denotes the identity matrix with size $N_p \times N_p$. It can be shown that

$$E[\mathbf{L}(\mathbf{w} - \mathbf{M}_{\mathbf{w}})(\mathbf{w} - \mathbf{M}_{\mathbf{w}})^T \mathbf{L}^T] = \mathbf{L} \mathbf{L}^T = \mathbf{C}_{\gamma}. \quad (5.45)$$

Hence we can deduce that

$$\gamma = \mathbf{L} \mathbf{w} \quad (5.46)$$

or equivalently

$$\begin{cases} \gamma_1 = \zeta_{11} w_1 \\ \gamma_2 = \zeta_{21} w_1 + \zeta_{22} w_2 \\ \vdots \\ \gamma_l = \sum_{i=1}^l \zeta_{li} w_i \\ \vdots \\ \gamma_{N_p} = \sum_{i=1}^{N_p} \zeta_{N_p i} w_i \end{cases}. \quad (5.47)$$

Therefore, correlated diversity branches are transformed to different weighted sums of independent branches.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

5.4.2.3 Determination of Model Parameters through Moment-Matching

First of all, we will find the m parameters and powers of independent Gamma RVs $\{w_i\}$ ($1 \leq i \leq N_p$) by matching the first and second moments of both sides of (5.47). To match the first moment, we have

$$\Omega_l = \sum_{i=1}^l \zeta_{li} \Omega_{w,i} \quad (5.48)$$

where Ω_l is the mean of γ_l and $\Omega_{w,i}$ is the mean of w_i . For $\{w_i\}$ that has identity covariance matrix, the variance of w_i is 1, i.e.,

$$\text{var}(w_i) = \frac{\Omega_{w,i}^2}{m_{w,i}} = 1 \quad \forall i \quad (5.49)$$

Substituting (5.49) into (5.48), we have

$$\Omega_l = \sum_{i=1}^{l-1} \zeta_{li} \sqrt{m_{w,i}} + \zeta_{ll} \sqrt{m_{w,l}} \quad (5.50)$$

then $m_{w,i}$ is given by

$$m_{w,i} = \zeta_{ii}^{-2} \left(\sum_{q=1}^{i-1} \zeta_{iq} \sqrt{m_{w,q}} - \Omega_i \right)^2. \quad (5.51)$$

For $i = 1$, it is obvious from the first equation in (5.47) that

$$m_{w,1} = m_1. \quad (5.52)$$

With this, all the other $m_{w,i}$ for $i \geq 2$ can be obtained from (5.51) and $\Omega_{w,i}$ can be derived from (5.49) as

$$\Omega_{w,i} = \sqrt{m_{w,i}}. \quad (5.53)$$

Referring to (5.47), summing up both sides of all the equations in (5.47), we get

$$\begin{aligned} \gamma &= w_1 \sum_{i=1}^{N_p} \zeta_{i1} + w_2 \sum_{i=2}^{N_p} \zeta_{i2} + \dots + w_{N_p} \zeta_{N_p N_p} \\ &= \sum_{i=1}^{N_p} \left(w_i \sum_{q=i}^{N_p} \zeta_{qi} \right) \end{aligned} \quad (5.54)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Hence for correlated Nakagami fading channels, the MRC output γ is transformed into the sum of weighted independent Gamma RVs $\{w_i\}$ with m parameters and powers of $\{w_i\}$ given by (5.51) and (5.53) respectively. Invoking the results derived in the Section 5.4.1 for independent Nakagami fading signal, γ can be approximated as a new Gamma RV with power Ω given by (5.21) and m parameter given by (5.32), i.e.,

$$m = \Omega^2 \left[\sum_{i=1}^{N_p} m_{w,i}^{-1} \left(\Omega_{w,i} \sum_{q=i}^{N_p} \zeta_{qi} \right)^2 \right]^{-1} \quad (5.55)$$

5.4.2.4 Model Verification

First, we compare the pdf of the approximated Gamma RV with the pdf formulas derived in the literature. The author in [78] derived the pdf of $g = \sqrt{\gamma}$ for dual branch Nakagami fading channel with equal m parameter to be

$$p(g) = \frac{2g\sqrt{\pi}}{\{\sigma_1\sigma_2(1-\rho)\}^{m_l} \Gamma(m_l)} \times \exp\left(-\frac{(\sigma_1+\sigma_2)g^2}{2\sigma_1\sigma_2(1-\rho)}\right) \times \left(\frac{g^2}{2\iota}\right)^{m_l-\frac{1}{2}} \times I_{m_l-0.5}(\iota g^2) \quad (5.56)$$

where $I_{m_l-0.5}$ is the modified Bessel of the first kind and $(m_l - 0.5)$ th order, m_l is the common m parameter of both diversity branches, $\sigma_1 = \frac{\Omega_1}{m_l}$, $\sigma_2 = \frac{\Omega_2}{m_l}$, $\iota = \sqrt{\frac{(\sigma_1-\sigma_2)^2+4\sigma_1\sigma_2\rho}{4\sigma_1^2\sigma_2^2(1-\rho)^2}}$ and ρ is the normalized correlation coefficient of the two branches.

In our model, γ is approximated as a Gamma RV. Correspondingly $g = \sqrt{\gamma}$ is a Nakagami RV with power Ω and m parameters given in (5.21) and (5.55) respectively. Fig. 5.6 shows the pdf of g based on our proposed model compared to (5.56) for different values of ρ . Since (5.56) can only handle the case of equal m_l values, we chose $m_1 = m_2 = 5$. The Ω_l parameters are randomly chosen. As seen from Fig. 5.6, the approximated pdf of g obtained from our proposed model matches well with the exact pdf in (5.56) for all values of ρ .

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

In Fig. 5.6, it is also shown that the fading severity after MRC combining varies with the amount of correlation between the diversity branches. For low correlation ($\rho = 0.1$), two branches of $m_l = 5$ each result in a final $m = 8.3$ after MRC, signifying a diversity gain of almost two. When the branches are highly correlated ($\rho = 0.9$), the diversity gain drops to only $m = 5.2$ for the combined signal, which is almost the same as the m parameter without combining.

Equation (5.56) is applicable to MRC with dual branch Nakagami fading channels with equal m_l . For MRC with more number of paths, arbitrary fading parameters and arbitrary fading correlations, our literature survey shows that no exact pdf of g is available. Therefore our proposed MRC model has to be verified by simulations. Fig. 5.7 shows the simulated pdf of g compared that of approximating g as a Nakagami RV with Ω and m given in (5.21) and (5.55), respectively. The channel for Fig. 5.7 consists of 4 Nakagami fading paths with randomly chosen m_l ($m_1 = 4, m_2 = 2.3, m_3 = 3.1, m_4 = 1.5$) and Ω_l ($\Omega_1 = 0.4779, \Omega_2 = 0.2353, \Omega_3 = 0.1985, \Omega_4 = 0.0882$). Furthermore, we assume that there is no path correlations for the curves (a) in Fig. 5.7 while for curves (b), the path correlations are characterized by the following normalized covariance matrix

$$\mathbf{C}_g = \begin{bmatrix} 1 & 0.25 & 0.40 & 0.20 \\ 0.25 & 1 & 0.22 & 0.10 \\ 0.40 & 0.22 & 1 & 0.30 \\ 0.20 & 0.10 & 0.30 & 1 \end{bmatrix}.$$

As seen from Fig. 5.7, the true pdfs of g obtained from the simulations are well modelled by our approximated proposed MRC model for all cases.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.6: Comparison of the approximated MRC pdf (5.18) with the exact MRC pdf (5.26) for a dual-branch channel with branch m parameter $\{m_l\} = \{5, 5\}$, branch power $\{\Omega_l\} = \{2, 1\}$. Branch correlations: (a) $\rho = 0.1$; (b) $\rho = 0.5$; (c) $\rho = 0.9$.

Figure 5.7: Comparison of the approximated MRC pdf (5.18) with the exact simulated MRC pdf for a 4-branch channel with branch m parameter $\{m_l\} = [4, 2.3, 3.1, 1.5]$, branch power $\{\Omega_l\} = [0.4779, 0.2353, 0.1985, 0.0882]$. Branch correlations: (a) independent branches; (b) correlated branches with normalized covariance matrix = $\begin{bmatrix} 1 & 0.25 & 0.40 & 0.20 \\ 0.25 & 1 & 0.22 & 0.1 \\ 0.4 & 0.22 & 1 & 0.3 \\ 0.2 & 0.1 & 0.3 & 1 \end{bmatrix}$.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

5.4.2.5 BER Expressions and Results

After obtaining the pdf of γ , we may substitute it into (5.16) to calculate the uplink MC-CDMA BER with MRC. The resultant BER expression has the same form as (5.34), with m given by (5.55) and power Ω given by (5.21). Note that the m parameter given in (5.55) is the most general one, as it can cater for both independent and correlated paths with any m_l/Ω_l ratios.

Fig. 5.8 shows the BER results of uplink MC-CDMA channels with either independent or correlated Nakagami fading paths. There are 128 subcarriers and 10 active users. Similar to Fig. 5.5, the three types of BER plots in Fig. 5.8 are: one simulation plot and two analytical plots obtained with or without the approximation made in (5.7). Since the analytical BER values with or without approximation (5.7) are indistinguishable in Fig. 5.8, the effect of the approximation made in (5.7) on the system BER values is verified to be negligible for correlated Nakagami fading channels.

As shown earlier in Fig. 5.6 and 5.7, when the fading in different channel paths have different m_l/Ω_l ratios or are correlated, the MRC output γ is approximately Gamma-distributed. Hence, some mismatch can be observed between the analytical and simulated BER plots in Fig. 5.8.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.8: Performance of uplink MC-CDMA with MRC in a channel with 3 Nakagami fading paths, $\{m_l\} = [9, 7, 3]$, $\{\Omega_l\} = [0.5, 0.3, 0.2]$. Branch correlations: (a) independent paths; (b) correlated paths $C_\gamma = [1 \ 0.4 \ 0.3; 0.4 \ 1 \ 0.7; 0.3 \ 0.7 \ 1]$.

Note that moment generating function approach is often used to derive the BER [81, 82, 85]. It can be applied to independent channels, or correlated channels with identical branches. However it cannot be applied to the most general case as correlated channels with arbitrary fading parameters (integer and non-integer m values, non-identical m parameter for various branches), which is addressed in this thesis.

5.4.3 Channels with Rician/Rayleigh Fading Paths

5.4.3.1 Existing Works

The BER expression of an MRC system with independent Rayleigh diversity branches is available in [33, 91]. For channels with Rician fading paths or correlated paths, the BER expressions are given in [91–95]. To our knowledge, there

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

is no closed-form BER formula of MRC in arbitrarily correlated Rician fading channel with arbitrary fading parameters for the paths. In this section, based on the characteristic function of the MRC output γ derived in [95] and the approach to calculate BER in [94], a one-dimensional integral BER expression will be developed for generalized Rician/Rayleigh fading channels with arbitrary fading parameters and fading correlations.

5.4.3.2 Characteristic Function of the MRC Output

For frequency-selective fading channels with Rician paths, the path gain g_l follows the Rician distribution given in (3.3). For easy reference, the Rician pdf in (3.3) is rewritten here

$$p(g_l) = \frac{2(K_l + 1)g_l}{\Omega_l} \exp\left(-K_l - \frac{(K_l + 1)g_l^2}{\Omega_l}\right) I_0\left(2\sqrt{\frac{K_l(K_l + 1)}{\Omega_l}}g_l\right) \quad (5.57)$$

where $\Omega_l = E[g_l^2]$, $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind, and the Rician K factor K_l is defined as the power ratio between the line of sight component and the diffused component. With $K_l = 0$, g_l becomes Rayleigh faded with pdf given in (3.6).

It is well-known that the complex fading gains of a Rician fading path can be modelled as complex Gaussian processes, i.e.,

$$\tilde{\mathbf{g}} = [g_1 \exp(j\psi_1), \dots, g_{N_p} \exp(j\psi_{N_p})]^T = \mathbf{X}_c + j\mathbf{X}_s \quad (5.58)$$

where ψ_l is the phase of the l th fading paths, both $\mathbf{X}_c$ and $\mathbf{X}_s$ are $N_p \times 1$ real Gaussian random vectors. Denote $\mathbf{X} = [\mathbf{X}_c; \mathbf{X}_s]$, $\boldsymbol{\mu} = E[\mathbf{X}]$ as the mean vector, and $\mathbf{C}_x$ as the covariance matrix of $\mathbf{X}$. By making use of the approach in [96] for the jdf of Gaussian RVs in $\mathbf{X}_c$ and $\mathbf{X}_s$, the authors of [95] derive the characteristic

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

function of $\gamma = \sum_{l=1}^{N_p} g_l^2$ as

$$\begin{aligned}\Psi_\gamma(j\omega) &= \int_0^\infty \exp(jw\gamma)p(\gamma)d\gamma \\ &= \frac{\exp\{j\omega \sum_{i=1}^{2N_p} \frac{\epsilon_i^2}{(1-2j\omega\vartheta_i)}\}}{\prod_{i=1}^{2N_p} (1-2j\omega\vartheta_i)^{\frac{1}{2}}}\end{aligned}\quad (5.59)$$

where ϑ_i are the eigenvalues of $\mathbf{C}_x$, ϵ_i is given by $\boldsymbol{\epsilon} = [\epsilon_1, \epsilon_2, \dots, \epsilon_{2N_p}]^T = \mathbf{V}^T \boldsymbol{\mu}$, $\mathbf{C}_x = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^T$ and $\boldsymbol{\Lambda} = \text{diag}(\vartheta_1, \vartheta_2, \dots, \vartheta_{2N_p})$.

5.4.3.3 BER Expressions and Results

From (5.16), the BER of uplink MC-CDMA with MRC is

$$P_e = \int_0^\infty Q\left(\sqrt{\nu N_c \gamma}\right) p(\gamma) d\gamma \quad (5.60)$$

where ν is a constant given in (5.11), N_c is the number of subcarriers, and $p(\gamma)$ is the pdf of γ .

Utilizing an alternative expression for the Gaussian Q function given in [94, 97]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2 \sin^2 \phi}\right) d\phi \quad (5.61)$$

and substituting (5.61) into (5.60), we have

$$P_e = \frac{1}{\pi} \int_0^{\pi/2} \int_0^\infty \exp\left(-\frac{\nu N_c \gamma}{2 \sin^2 \phi}\right) p(\gamma) d\gamma d\phi. \quad (5.62)$$

Comparing the inner integration of (5.62) with the CF of γ defined in (5.59), they are found to be equivalent, i.e.,

$$\int_0^\infty \exp\left(-\frac{\nu N_c \gamma}{2 \sin^2 \phi}\right) p(\gamma) d\gamma \equiv \Psi_\gamma\left(-\frac{\nu N_c}{2 \sin^2 \phi}\right). \quad (5.63)$$

Therefore, with the closed-form CF expression of γ given in (5.59), the BER of uplink MC-CDMA with MRC in correlated Rician fading channel can now be

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

obtained by the equation shown below

$$\begin{aligned}
 P_e &= \frac{1}{\pi} \int_0^{\pi/2} \Psi_\gamma \left(-\frac{\nu N_c}{2 \sin^2 \phi} \right) d\phi \\
 &= \frac{1}{\pi} \int_0^{\pi/2} \frac{\exp \left[\left(-\frac{\nu N_c}{2 \sin^2 \phi} \right) \sum_{i=1}^{2N_p} \frac{\epsilon_i^2}{\left(1 + \frac{\nu N_c \vartheta_i}{\sin^2 \phi} \right)} \right]}{\prod_{i=1}^{2N_p} \left(1 + \frac{\nu N_c \vartheta_i}{\sin^2 \phi} \right)^{\frac{1}{2}}} d\phi.
 \end{aligned} \tag{5.64}$$

Since only single-dimensional integration is involved and the integration limits are finite, (5.64) can be easily evaluated numerically. Also, (5.64) is able to cover the case of independent paths with the covariance matrix $\mathbf{C}_x$ being a diagonal matrix, and the Rayleigh fading case with the mean vector $\boldsymbol{\mu} = \mathbf{0}$.

To verify our BER expression (5.64) for Rician fading channels, Fig. 5.9 compares the BER obtained using (5.64) with simulations. There are 128 subcarriers and 10 active users for the MC-CDMA system. The K_l , Ω_l and correlations for the fading paths are randomly selected. In Fig. 5.9, the “analytical with approximation (5.7)” plots are obtained by using (5.64), while the “analytical without approximation (5.7)” plots are obtained by Monte Carlo Integration of the conditional BER given in (5.5).

All Fig. 5.5, Fig. 5.8 and Fig. 5.9 show that the analytical BER values with or without approximation (5.7) are indistinguishable, hence the effect of the approximation made in (5.7) on the system BER values is negligible for Rayleigh, Rician as well as Nakagami channels. Although not shown in this thesis, the same verification has also been carried out for other values of subcarrier number N_c and fading parameters, e.g., N_c from 16 to 256, Rician K factor from 0 to 20, and Nakagami m parameter from 1 to 20. The same conclusion that the effect of the approximation made in (5.7) on the system BER is insignificant can be reached.

For Rician fading, since (5.64) is mathematically exact, its BER results match very well with those computed using full Monte Carlo integration, as shown in Fig. 5.9.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.9: Performance of uplink MC-CDMA with MRC in a channel with 3 correlated Rician fading paths, $\{K_l\} = [5, 3, 2]$, $\{\Omega_l\} = [0.4, 0.35, 0.25]$, channel (a) contains independent paths with $\mathbf{C}_x = \mathbf{I}_6$; channel (b) contains correlated paths with $\mathbf{C}_x = [1 \ 0.1 \ 0.2 \ 0 \ 0 \ 0; 0.1 \ 1 \ 0.5 \ 0 \ 0 \ 0; 0.2 \ 0.5 \ 1 \ 0 \ 0 \ 0; 0 \ 0 \ 0 \ 1 \ 0.1 \ 0.2; 0 \ 0 \ 0 \ 0.1 \ 1 \ 0.5; 0 \ 0 \ 0 \ 0.2 \ 0.5 \ 1]$.

As seen from Fig. 5.3 - Fig. 5.5, Fig. 5.8 and Fig. 5.9, the uplink MC-CDMA exhibits the familiar error floor at high E_b/N_0 level due to the MUI power predominating over the AWGN power. With our simple uplink BER expressions of (5.64) and (5.34), the error floor can be easily evaluated by setting E_b/N_0 in (5.11) to infinity. Fig. 5.10 shows the dependence of the error floor values on the number of users in channels with correlated Rician and Nakagami paths. The Nakagami channel model is the same as the channel (b) in Fig. 5.8, while the Rician channel model has the same parameters as the channel (b) in Fig. 5.9. The Rician channel suffers higher error floor because its channel paths are subject to more severe fading than the Nakagami channel.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Furthermore, with our closed-form or one-dimensional integral BER formulas, the effect of various system or channel parameters on the system performance can also be obtained with ease. As an example, Fig. 5.11 shows how many users the MC-CDMA system can support in order to meet a target BER of 10^{-3} in channels with 4 or 8 i.i.d. Rayleigh paths. Such user capacity results can be easily obtained from (5.34) by simple numerical root finding. Besides, our earlier analysis predicts that the larger the number of channel paths, the more diversity the MC-CDMA system can achieve. This explains why the channel with more paths in Fig. 5.11 can accommodate more users.

Figure 5.10: Error floor values vs. number of users for uplink MC-CDMA with MRC. Number of subcarriers = 128.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.11: User capacity of uplink MC-CDMA with MRC in channels with i.i.d. Rayleigh fading paths.

5.5 Generation of Correlated Nakagami Fading Channels

In Section 5.4.2, we have studied the performance of MC-CDMA systems in correlated Nakagami fading channels. The computer simulation of such systems will require the generation of correlated Nakagami signals. In the literature, algorithms for generating correlated Nakagami signals with the same m -parameter can be found in [98–100]. The case of $m < 1$ is considered in [98]. The cases of integer and non-integer m parameter values are considered in [99], but the simulated Nakagami RVs are still constrained to have the same fading parameters. As pointed out in [89] for the cases of angle, polarization, Rake and macro diversity systems, there exist many practical scenarios in which different diversity branches may have different fading parameters.

In this section, techniques for generating correlated Nakagami signals with arbitrary fading parameters are proposed. Our approach is to generate correlated Gamma RVs and take their square roots to obtain the desired correlated Nakagami RVs. Two methods are proposed to generate correlated Gamma RVs: Cholesky decomposition method (referred to as Decomposition Method) and Sim's Method. The former is more versatile, although it is not mathematically exact for certain combinations of fading parameters. The latter is mathematically exact, but only when the fading channel parameters fulfill certain constraints. Before elaborating on these two methods, let's look at the relationship between the covariance parameters of the desired Nakagami RVs and their corresponding Gamma RVs.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

5.5.1 Parameter-Matching between Nakagami and Gamma RVs

For a N_p -branch diversity system with Nakagami fading branches, the fading envelope g_l of the l th ($1 \leq l \leq N_p$) diversity branch follows the Nakagami- m distribution expressed in (5.17) with m_l as the m -parameter and Ω_l as the mean fading power.

Denoting vector $\mathbf{g} = [g_1 \ g_2 \ \dots \ g_{N_p}]^T$, the covariance matrix of $\mathbf{g}$ is defined as

$$\mathbf{C}_{\mathbf{g}} = E [(\mathbf{g} - E[\mathbf{g}])(\mathbf{g} - E[\mathbf{g}])^T] . \quad (5.65)$$

The normalized covariance (or correlation coefficients) between g_l and g_i is defined as

$$\rho_g(l, i) = \frac{\text{cov}(g_l, g_i)}{\sqrt{\text{var}(g_l) \times \text{var}(g_i)}} = \frac{E(g_l g_i) - E(g_l)E(g_i)}{\sqrt{\text{var}(g_l) \times \text{var}(g_i)}} \quad (5.66)$$

where $\text{cov}(g_l, g_i)$ is the (l, i) th element of $\mathbf{C}_{\mathbf{g}}$.

The square of a Nakagami RV, $\gamma_l = g_l^2$, follows the Gamma distribution whose pdf follows (5.18) with m parameter equals to m_l and the Ω_l parameter in (5.18) is $\Omega_l = E[g_l^2] = E[\gamma_l]$. For $\boldsymbol{\gamma} = [\gamma_1 \ \gamma_2 \ \dots \ \gamma_{N_p}]^T$, its covariance matrix $\mathbf{C}_{\boldsymbol{\gamma}}$ and correlation coefficient $\rho_{\boldsymbol{\gamma}}$ are similarly defined as (5.65) and (5.66) respectively.

Here, the notations $g_l \sim \mathcal{N}(m_l, \Omega_l)$ and $\gamma_l \sim \mathcal{G}(m_l, \Omega_l)$ are used to indicate that g_l and γ_l follow the Nakagami and Gamma distribution, respectively. Our approach is to generate correlated Nakagami RVs $g_l \sim \mathcal{N}(m_l, \Omega_l)$ with covariance matrix $\mathbf{C}_{\mathbf{g}}$, from correlated Gamma RVs $\gamma_l \sim \mathcal{G}(m_l, \Omega_l)$ with corresponding covariance matrix $\mathbf{C}_{\boldsymbol{\gamma}}$. Therefore, given the $\mathbf{C}_{\mathbf{g}}$ or ρ_g of the desired Nakagami RVs, we need to determine the $\mathbf{C}_{\boldsymbol{\gamma}}$ or $\rho_{\boldsymbol{\gamma}}$ for the corresponding Gamma RVs. We consider two cases separately: same m_l among diversity branches and different m_l among diversity branches.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

5.5.1.1 Same m_l among Diversity Branches

When all the diversity branches have the same m -parameter, i.e., $m_l = m$ for all l , the relationship between the Gamma correlation coefficient ρ_γ and the corresponding Nakagami correlation coefficient ρ_g is given by the expression (139) in [78], i.e.,

$$\rho_g = \frac{\Gamma^2(m + 0.5)}{\Gamma(m) \times \Gamma(m + 1) - \Gamma^2(m + 0.5)} \times [{}_2F_1(-0.5, -0.5; m, \rho_\gamma) - 1] \quad (5.67)$$

where ${}_2F_1(\cdot)$ is the hypergeometric function. It is also pointed out in [78] that (5.67) gives $\rho_\gamma \approx \rho_g$ numerically. Hence, we can either directly equate ρ_γ with ρ_g , or calculate the exact ρ_γ in (5.67) numerically, or by using the methodology described in [99]. Fig. 5.12 (a) shows the value of ρ_γ vs. ρ_g as computed from (5.67) for $m = 1, 10$ and 20 . It clearly shows that ρ_γ is almost identical to ρ_g for all values of ρ between 0 and 1 for all values of m .

5.5.1.2 Different m_l among Diversity Branches

For different m_l among diversity branches, we do not know of any formula in the literature that gives the relationship between ρ_γ and ρ_g . Hence simulations are used to ascertain it. The simulation results are plotted in Fig. 5.12 (b) for different combinations of m_l values. It is again found that $\rho_\gamma \approx \rho_g$. Therefore we conclude that it is sufficiently accurate to equate the correlation coefficients of Gamma RVs with their corresponding Nakagami RVs.

After knowing ρ_γ , $\mathbf{C}_\gamma$ can be obtained from (5.66). We are now ready to generate correlated Gamma RVs in the next section. Two methods for generating correlated Gamma RVs with arbitrary m -parameter values are presented. The first method, called Decomposition Method, is based on the linear summations of weighted independent Gamma RVs with the weighting coefficients determined

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

by the Cholesky decomposition of $\mathbf{C}_\gamma$. The second method, called Sim's Method, was developed by Sim [101] who generates correlated Gamma RVs from a set of independent Gamma- and Beta-distributed RVs.

(a) Same m -parameters (ρ_g computed from (5.67)).

(b) Different m -parameters (ρ_g and ρ_γ obtained from simulation).

Figure 5.12: Normalized covariance of Gamma and corresponding Nakagami RVs.

5.5.2 Decomposition Method

Let's start with the Decomposition Method. Cholesky decomposition, which is commonly used for generating correlated Gaussian processes [102, 103], is used here to transform correlated Gamma RVs into linear combination of independent Gamma RVs by matching their first and second moments. As discussed in Section 5.4.2.2, after performing Cholesky decomposition on the covariance matrix $\mathbf{C}_\gamma$, a lower triangular matrix $\mathbf{L}$ can be uniquely obtained from the covariance matrix $\mathbf{C}_\gamma$ such that

$$\mathbf{C}_\gamma = \mathbf{L}\mathbf{L}^T. \quad (5.68)$$

Then the correlated Gamma RVs $\{\gamma_l\}$ can be modelled as weighted sums of independent Gamma RVs $\{w_i\}$ ($1 \leq i \leq N_p$), as shown in (5.46), i.e.,

$$\gamma = \mathbf{L}\mathbf{w} \quad \text{or} \quad \gamma_l = \sum_{i=1}^l \zeta_{li} w_i \quad (5.69)$$

where ζ_{li} is the (l, i) th entry of $\mathbf{L}$. The statistical parameters of $\{w_i\}$ were derived in Section 5.4.2.3 by matching the first and second moments of both sides of (5.69), i.e., $m_{w,i}$ is given by (5.51) and $\Omega_{w,i}$ is given by (5.53). For easy reference, they are reproduced here

$$m_{w,i} = \zeta_{ii}^{-2} \left(\sum_{q=1}^{i-1} \zeta_{iq} \sqrt{m_{w,q}} - \Omega_i \right)^2, \quad (5.70)$$

$$\Omega_{w,i} = \sqrt{m_{w,i}}. \quad (5.71)$$

Therefore, based on (5.69) with parameters given by (5.70) and (5.71), our Decomposition Method proposes to generate correlated Gamma RVs $\gamma_l \sim \mathcal{G}(m_l, \Omega_l)$ from linear combinations of independent Gamma RVs $w_i \sim \mathcal{G}(m_{w,i}, \Omega_{w,i})$.

Generation of independent Gamma RVs with integer m -parameter is straightforward by summing up squared Gaussian RVs [67, 104]. However, (5.70) does not

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

usually produce $m_{w,i}$ with integer values. A commonly used method for the generating of Gamma RVs with non-integer m values is the probability switch method, which however is approximate and only valid for $m \geq 1$. We adopt the Jöhnk's algorithm in [105], which is able to generate exact Gamma RVs with arbitrary non-integer m -parameter values.

5.5.3 Sim's Method

The algorithm given in [101] provides an alternative way to generate correlated Gamma RVs. In this algorithm, the desired correlated Gamma vector γ is obtained by

$$\gamma = \mathbf{A}\mathbf{z} \quad (5.72)$$

where $\mathbf{z} = [z_1 \ z_2 \ \dots \ z_{N_p}]^T$ is a Gamma-distributed random vector with diagonal covariance matrix, i.e., $\{z_i\}$ are independent. Denoting $z_i \sim \mathcal{G}(m_{z,i}, \Omega_{z,i})$, the parameters $\Omega_{z,i}$ and $m_{z,i}$ must satisfy

$$\frac{m_{z,i}}{\Omega_{z,i}} = \frac{m_i}{\Omega_i} = \lambda_i. \quad (5.73)$$

The matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ \beta_{21}b_{21} & 1 & 0 & \dots & 0 \\ \vdots & \dots & \ddots & 0 & \vdots \\ \beta_{(N_p-1)1}b_{(N_p-1)1} & \dots & \dots & 1 & 0 \\ \beta_{N_p1}b_{N_p1} & \dots & \dots & \beta_{N_p(N_p-1)}b_{N_p(N_p-1)} & 1 \end{bmatrix}. \quad (5.74)$$

β_{ij} ($1 \leq j < i \leq N_p$) for (5.74) is calculated from

$$\beta_{ij} = \frac{\lambda_j}{\lambda_i} \quad (5.75)$$

and b_{ij} are independent Beta-distributed RVs that follow the pdf of

$$p(b_{ij}) = \frac{\Gamma(t_1 + t_2)b_{ij}^{t_1-1}(1-b_{ij})^{t_2-1}}{\Gamma(t_1)\Gamma(t_2)} \quad (5.76)$$

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

where t_1 and t_2 are the shape parameters with values $t_1 = a_{ij} > 0$ and $t_2 = m_{z,j} - a_{ij} > 0$. We denote the Beta RV b_{ij} with corresponding shape parameters as $b_{ij} \sim \mathcal{B}(a_{ij}, m_{z,j} - a_{ij})$.

The values of a_{ij} and $m_{z,i}$ are obtained from the iterative formulas below:

$$m_{z,1} = m_1 \quad (5.77a)$$

$$a_{i1} = \lambda_1 \lambda_i \mathbf{C}_\gamma(1, i) \quad (5.77b)$$

$$a_{ij} = \lambda_i \lambda_j \mathbf{C}_\gamma(i, j) - \sum_{q=1}^{j-1} \left(\frac{a_{iq} a_{jq}}{m_{g,q}} \right) \quad (5.77c)$$

$$m_{z,i} = m_i - \sum_{j=1}^{i-1} a_{ij} \quad (5.77d)$$

where $\mathbf{C}_\gamma(i, j)$ is the (i, j) th entry of $\mathbf{C}_\gamma$.

Finally, based on (5.72) and (5.74), the desired RV γ_i will be given by

$$\gamma_1 = z_1 \quad \text{and} \quad \gamma_i = z_i + \sum_{j=1}^{i-1} \beta_{ij} b_{ij} z_j \quad (5.78)$$

Notice that when a Gamma RV $z_j \sim \mathcal{G}(m_{z,j}, m_{z,j}/\lambda_j)$ is multiplied by a Beta RV $b_{ij} \sim \mathcal{B}(a_{ij}, m_{z,j} - a_{ij})$, we obtain another Gamma RV $b_{ij} z_j \sim \mathcal{G}(a_{ij}, a_{ij}/\lambda_j)$. With a scaling factor β_{ij} defined by (5.75), the RV $\beta_{ij} b_{ij} z_j$ will be characterized by $\beta_{ij} b_{ij} z_j \sim \mathcal{G}(a_{ij}, a_{ij}/\lambda_i)$. As a result, γ_i in (5.78) is formed by a summation of several Gamma RVs with equal λ_i values, or m_i/Ω_i ratios. According to [78], this ensures that γ_i has an exact Gamma distribution.

The independent Beta RV b_{ij} in (5.74) or (5.78) can be generated using the following well-known relationship: if y_1 and y_2 are independent Gamma RVs with m -parameters $m_{y_1} = \Omega_{y_1}$ and $m_{y_2} = \Omega_{y_2}$ respectively, then $Y = \frac{y_1}{y_1 + y_2}$ is a Beta RV, i.e., $Y \sim \mathcal{B}(m_{y_1}, m_{y_2})$.

Note that, similar to the matrix $\mathbf{L}$ used in the Decomposition Method, the matrix $\mathbf{A}$ used in the Sim's Method is a lower triangular matrix. However, $\mathbf{L}$ is a fixed matrix while $\mathbf{A}$ is not due to the random nature of b_{ij} in $\mathbf{A}$.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

In summary, after determining the parameters $m_{z,i}$ from (5.77a) - (5.77d) and $\Omega_{z,i}$ from (5.73), the Jöhnk's algorithm discussed earlier is used to generate the independent Gamma random vector $\mathbf{z}$. Next, $\mathbf{A}$ can be obtained from (5.74) to (5.77d). Finally, as per (5.72), every new random realization of $\mathbf{z}$ will be multiplied with a new random realization of $\mathbf{A}$ to generate the desired correlated Gamma vector $\boldsymbol{\gamma}$.

5.5.4 Comparison of Decomposition and Sim's Methods

The advantage of our proposed Decomposition Method is its capability of generating correlated Gamma RVs for any fading parameters and cross-correlation. It is also simpler to implement as it requires only the generation of independent Gamma RVs, whereas the Sim's Method also requires the generation of independent Beta RVs. The drawback of the Decomposition Method is that its kernel, i.e., expression (5.69), may not be an exact pdf model if the $\frac{m_{w,i}}{\zeta_{li}\Omega_{w,i}}$ ratio is not the same for all w_i weighted by ζ_{li} associated with the generation of γ_l in (5.69) [78], so minor statistical inaccuracy can be expected in the generated outputs. On the other hand, Sim's Method is exact by ensuring the same λ_i value (i.e., $\frac{m_i}{\Omega_i}$ ratio) for all the summands in (5.78). However, it carries some constraints on the specified statistical parameters [101], as shown below:

$$m_i > \sum_{q=1}^{i-1} a_{iq} \quad (5.79)$$

$$\lambda_i \lambda_j \mathbf{C}_{\boldsymbol{\gamma}}(i, j) \in \left(\sum_{q=1}^{j-1} \frac{a_{iq} a_{jq}}{m_{z,q}}, \quad m_{z,j} + \sum_{q=1}^{j-1} \frac{a_{iq} a_{jq}}{m_{z,q}} \right) \quad (5.80)$$

where $x \in (x_1, x_2)$ means that x must lie in the range between x_1 and x_2 .

Hence our recommendation is that if high accuracy is required and the constraint conditions are fulfilled, use the Sim's Method, otherwise use the Decomposition Method.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

In our experiments, it was also found that the Decomposition Method can give better results if the m -parameters in the vector γ in (5.69) are arranged in ascending order, i.e., $m_1 \leq m_2 \leq \dots \leq m_{N_p}$. This can be easily achieved, without loss of generality, by appropriately adjusting the numbering order of the desired RVs prior to their computer generation. By doing so, we observed that the variation of the $\frac{m_{w,i}}{\zeta_{li}\Omega_{w,i}}$ ratio for all w_i weighted by ζ_{li} associated with the generation of γ_l in (5.69) is reduced. Hence, the weighted summation of independent Gamma RVs w_i approximates better to a net Gamma RV. In the next section, we shall show that the statistical inaccuracy in the Decomposition Method is fairly small for most practical purposes.

5.5.5 Verification

As an illustration for correlated Nakagami RVs with equal non-integer m -parameter, we generate four Nakagami RVs whose fading parameters and covariance values are exactly the same as those in [99]. The m -parameters for all four Nakagami RVs are set equally to be 2.18, while their powers are set to $\{\Omega_l\} = [20.1, 14.8, 30.9, 25.9]$. The normalized covariance (correlation coefficients) matrix is specified in [99]:

$$\rho_g = \begin{pmatrix} 1 & 0.795 & 0.604 & 0.372 \\ 0.795 & 1 & 0.795 & 0.604 \\ 0.604 & 0.795 & 1 & 0.795 \\ 0.372 & 0.604 & 0.795 & 1 \end{pmatrix}$$

For this example, we use the Decomposition Method to generate the correlated Nakagami RVs. The pdfs of the generated RVs for all four RVs match well with the expected pdfs given by (5.17). For brevity, only the pdfs of the 1st and 3rd RVs are shown in Fig. 5.13. The correlation coefficients obtained from our results are compared in Table 5.1 with those obtained from [99]. The results show that our Decomposition Method is able to produce fairly accurate statistical correlation.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

Table 5.1: Comparison of correlation coefficients.

Correlation coefficients	By Decomposition Method	By [99]	Specified
$\rho_g(1, 2)$	0.820	0.802	0.795
$\rho_g(1, 3)$	0.629	0.612	0.604
$\rho_g(1, 4)$	0.396	0.369	0.372

We next look at the more general case of correlated Gamma RVs with different m -parameters. Using both the Decomposition Method and Sim's Method, we generate three Nakagami fading RVs with parameters: $\{m_l\} = [1, 2, 3]$, $\{\Omega_l\} = [0.300, 0.667, 1.000]$, and correlation coefficients

$$\rho_g = \begin{pmatrix} 1 & 0.7 & 0.3 \\ 0.7 & 1 & 0.4 \\ 0.3 & 0.4 & 1 \end{pmatrix}$$

Fig. 5.14 shows that the pdfs of both generated RVs match very well with the theoretical pdfs given by (5.17). The correlation coefficient matrices produced by the Sim's Method and the proposed Decomposition Method are

$$\begin{pmatrix} 1 & 0.695 & 0.294 \\ 0.695 & 1 & 0.394 \\ 0.294 & 0.394 & 1 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 0.686 & 0.308 \\ 0.686 & 1 & 0.414 \\ 0.308 & 0.414 & 1 \end{pmatrix}$$

respectively. Both matrices match the ρ_g specification very well.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED FREQUENCY-SELECTIVE FADING CHANNELS

(a) Generated Nakagami RV 1.

(b) Generated Nakagami RV 3.

Figure 5.13: Pdfs generated using Decomposition Method vs. theoretical pdfs for Nakagami RVs with equal and non-integer m -parameter values.

CHAPTER 5. BER ANALYSIS OF UPLINK MC-CDMA WITH MRC IN GENERALIZED
FREQUENCY-SELECTIVE FADING CHANNELS

Figure 5.14: Pdfs generated using Decomposition and Sim's Method vs. theoretical pdf for 3 Nakagami RVs with different m -parameter values.

5.6 Summary

In this chapter, we present a way to evaluate the BER of uplink MC-CDMA with MRC in generalized frequency-selective fading channels consisting of Rayleigh, Rician or Nakagami fading paths with arbitrary path correlations and fading parameters. For channels with correlated Nakagami fading paths, we derive a simple closed-form BER formula (5.34). The formula is exact if the channel paths have independent Nakagami fading with identical m_l/Ω_l ratio, otherwise the formula is approximated, but nonetheless adequately accurate. For channels with correlated Rician fading paths, the BER formula is simplified to a one-dimensional integration form (5.64), which is fairly easy to compute and exact. The derived BER formulas (5.34) and (5.64) are also useful in terms of computing other system performance measures such as error floor and user capacity. Therefore, in this chapter, we have obtained tractable or simplified analytical BER formulas for asynchronous uplink MC-CDMA with MRC in generalized frequency-selective fading channels with correlated paths.

To facilitate computer simulations, we also proposed two methods, namely Decomposition Method and Sim's Method, to generate Nakagami fading signals with arbitrary correlation and fading parameters. Simulations show that both methods produce outputs that match well with the specifications.

Chapter 6

Downlink MC-CDMA with ORC and MMSEC - BER Analysis and Enhancement

6.1 Introduction

For the downlink MC-CDMA channels under high load conditions, orthogonality restoring combining (ORC) outperforms maximal ratio combining (MRC) over all values of SNR and equal gain combining (EGC) for high SNR values [19, 21]. Hence ORC is an important equalization and combining scheme for downlink MC-CDMA. However, most of the existing works on ORC are based on computer simulations [19, 21]. In this chapter, we address the problem of BER performance evaluation and enhancement for MC-CDMA with ORC. A lower bound and a tight approximation to the analytical BER of MC-CDMA with ORC will be developed. Both the lower bound and approximate BER expressions involve at most one-dimensional integration, hence they can be evaluated much more efficiently than the existing techniques found in the literature. Based on the findings of this analysis, we propose a novel “Truncated ORC” scheme to enhance the ORC performance without increasing the computational complexity.

Theoretically, the equalization and combining scheme which performs best in

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

downlink MC-CDMA is the minimum mean square error combining (MMSEC) scheme. However, it is also most computationally complex to realize as it involves complex matrix inversion operation of a large matrix. Therefore, in this chapter, we also propose a new method to calculate the equalizer coefficient for MMSEC based on reduced-size matrices.

6.2 MC-CDMA with ORC

6.2.1 BER Simplification

The equalizer coefficient for ORC is $\alpha_n = h_n^{-1}$. It aims to restore the amplitude orthogonality of the frequency-domain spreading codes used in MC-CDMA transmission which suffer from different subcarrier fading in a frequency-selective fading channel. For downlink transmission, the MUI model was developed in (4.5) to be $\sqrt{\frac{E_b T_s}{2N_c}} \sum_{k=2}^{N_u} b_k \sum_{n=1}^{N_c} c_{k,n} c_{1,n} h_n \alpha_n$. Hence if spreading codes with zero cross correlation, such as the Walsh-Hadamard codes, are used, the MUI vanishes with ORC. Therefore, the downlink BER for ORC, conditioned on the set of subcarrier fading amplitudes $\{h_n\}$, can be deduced from the general downlink BER in (4.7) to be:

$$P_e|\{h_n\} = Q\left(\frac{D}{\sqrt{\text{var}(\eta)}}\right) = Q\left(\sqrt{\frac{2N_c E_b/N_0}{\sum_{n=1}^{N_c} h_n^{-2}}}\right) \quad (6.1)$$

where D denotes the desired signal component and η denotes the noise component.

The average downlink BER P_e can then be obtained by averaging (6.1) over the joint distribution function (jdf) of $\{h_n\}$, i.e.,

$$P_e = \int_0^\infty \cdots \int_0^\infty Q\left(\sqrt{\frac{2N_c E_b/N_0}{\sum_{n=1}^{N_c} h_n^{-2}}}\right) \text{jdf}(\{h_n\}) dh_1 \cdots dh_{N_c}. \quad (6.2)$$

The multi-dimensional integration in (6.2) is clearly not easy to evaluate because the subcarrier fading gains $\{h_n\}$ are correlated. In the next section, we will

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

propose two ways to simplify its evaluation: BER lower bound and BER approximation.

6.2.1.1 BER Lower Bound

Let $h_{\min}$ be the minimum value of the set of instantaneous subcarrier fading gains $\{h_n\}$. By exploiting the fact that the sum of squared reciprocals of a set of random numbers is always larger than the squared reciprocal of the smallest number in the set, i.e., $\sum_{n=1}^{N_c} h_n^{-2} > h_{\min}^{-2}$, we have a BER lower bound of MC-CDMA with ORC:

$$P_e > \int_0^\infty Q\left(\sqrt{2N_c E_b/N_0} h_{\min}\right) p(h_{\min}) dh_{\min} \quad (6.3)$$

where $p(h_{\min})$ denotes the pdf of $h_{\min}$.

Next we will find $p(h_{\min})$. Let's arrange the set of $\{h_n\}$ in increasing order to obtain a set of ordered statistics $h_{(r)}$, i.e., $h_{\min} \leq h_{(2)} \leq \dots h_{(r)} \dots \leq h_{(N_c-1)} \leq h_{\max}$. Denote the pdf of h_n as $p_{h_n}(\cdot)$ and cumulative distribution function (cdf) of h_n as $F_{h_n}(\cdot)$. If the subcarrier fading gains $\{h_n\}$ are i.i.d., the pdf of the r th ordered statistic $h_{(r)}$ is given by [106]

$$p(h_{(r)}) = \frac{N_c!}{(r-1)!(N_c-r)!} [F_{h_n}(h_{(r)})]^{r-1} [1 - F_{h_n}(h_{(r)})]^{N_c-r} p_{h_n}(h_{(r)}) . \quad (6.4)$$

Considering Rayleigh fading with normalized power, i.e.,

$$p_{h_n}(x) = 2x \exp(-x^2) . \quad (6.5)$$

Setting $r = 1$ in (6.4), the pdf of $h_{\min}$ is

$$p(h_{\min}) = 2N_c h_{\min} \exp(-N_c h_{\min}^2) . \quad (6.6)$$

Substituting (6.6) into (6.3), we have a simple closed-form expression for the BER lower bound of MC-CDMA with ORC:

$$P_e > \frac{1}{2} \left(1 - \sqrt{\frac{E_b/N_0}{E_b/N_0 + 1}} \right) . \quad (6.7)$$

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Notice that the pdf of $h_{\min}$ in (6.6) is a Rayleigh pdf, and (6.7) is equivalent to the BER formula for single-carrier BPSK/QSPK modulation in flat Rayleigh fading channel. This means that although ORC restores the orthogonality between users and eliminates MUI in the downlink, it achieves no diversity gain due to the accompanying noise amplification effect.

6.2.1.2 Tight BER Approximation

Equation (6.3) is essentially derived based on the inequality $\sum_{n=1}^{N_c} h_n^{-2} > h_{\min}^{-2}$. This inequality relationship can be tightened by means of a correction factor ε as shown below:

$$\sum_{n=1}^{N_c} h_n^{-2} = h_{\min}^{-2} + h_{(2)}^{-2} + \cdots + h_{\max}^{-2} \approx h_{\min}^{-2} + \varepsilon. \quad (6.8)$$

The correction factor ε can be approximated by matching the first moments of $\sum_{n=1}^{N_c} h_n^{-2}$ and $h_{\min}^{-2}$, i.e.,

$$\varepsilon = E\left(\sum_{n=1}^{N_c} h_n^{-2}\right) - E(h_{\min}^{-2}). \quad (6.9)$$

Then P_e from (6.2) can be approximated by

$$P_e \approx \int_0^\infty Q\left(\sqrt{\frac{2N_c E_b/N_0}{h_{\min}^{-2} + \varepsilon}}\right) p(h_{\min}) dh_{\min}, \quad (6.10)$$

which is a simple one-dimensional integral formula. For subcarriers with i.i.d. Rayleigh fading, the pdf of $h_{\min}$ is given in (6.6). In the next subsection, it will be shown that (6.10) gives a very tight approximation to the true BER of (6.2).

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

6.2.1.3 Results and Discussions

The downlink BER simulation results, the lower bound values and the approximate BER values of an MC-CDMA system with 32 subcarriers and ORC are shown in Fig. 6.1. The number of users will not affect the BER because we use orthogonal codes, e.g., Walsh-Hadamard codes, which causes the MUI to vanish with ORC. The “BER lower bound” plot is obtained by (6.7) and the “BER approximation” plot is obtained by (6.10) with $p(h_{\min})$ given in (6.6). From the figure, we can see that the “BER approximation” plot is very close to the true BER for all values of E_b/N_0 . The “BER lower bound” plot is also very tight for high E_b/N_0 values. These suggest that the noise amplification effect is dominant in ORC at high E_b/N_0 and $h_{\min}$ is the subcarrier which amplifies the noise most, hence the performance of ORC is mainly dependent on $h_{\min}$. At low E_b/N_0 , the BER approximation method also accounts for the effects of other subcarriers by means of the correction factor ε , so it is close to the true BER for all E_b/N_0 values.

The discussion so far has assumed independent subcarrier fading. In practice, this is often not true, so we should also consider the cases where subcarrier fading are correlated. Let’s consider an MC-CDMA system with 32 subcarriers in a channel with 4 i.i.d. Rayleigh fading paths. This channel will have correlated Rayleigh fading subcarriers, as discussed in Section 3.3. The triangular markers in Fig. 6.1 are the BER simulation results in such a channel with correlated subcarrier fading. It can be seen that the performance under correlated subcarrier fading is very similar to that under independent subcarrier fading obtained earlier. Therefore, the BER lower bound and the approximate BER formulas in (6.3) and (6.10) respectively can also be applied for correlated subcarrier fading. Next we will explain the reason for this phenomenon.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

In the synchronous downlink, the performance of MC-CDMA with orthogonal spreading and ORC is independent of the number of users (up to the number of subcarriers) because MUI is eliminated by ORC. The BER lower bound (6.7) also suggests that the system performance is independent of the number of subcarriers. This is verified in Fig. 6.2, which shows that the BER of an ORC MC-CDMA system with 4, 32, 128 or 1024 i.i.d. Rayleigh fading subcarriers are practically invariant at $\text{BER} < 10^{-2}$. This indicates that the ORC performance hardly changes as we increase the number of independent subcarriers beyond 4. For a frequency-selective fading channel, subcarriers with large frequency spacing experience relatively independent fading. As long as there are 4 independent subcarriers among the 32 correlated subcarriers in the MC-CDMA, Fig. 6.2 predicts that the ORC MC-CDMA will not perform much differently from system with independent subcarriers. This explains why in Fig. 6.1, the performance for independent subcarriers or correlated subcarriers are almost the same.

In addition to Rayleigh fading, the lower bound and the approximate BER formulas have also been tested under other subcarrier fading conditions such as independent/corrected Rician/Nakagami fading. As an illustration, Fig. 6.3 and Fig. 6.4 show the BER results for i.i.d. Rician and Nakagami fading subcarriers, respectively. Again the BER lower bound turns out to be tight for high E_b/N_0 values and the approximate BER formula is tight for all values of E_b/N_0 . Therefore, it can be concluded that the BER lower bound (6.3) and the BER approximation (6.10), both of which require only simple one-dimensional integration, can reliably predict the BER of downlink MC-CDMA with ORC in various fading conditions.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Figure 6.1: BER of downlink ORC MC-CDMA with 32 subcarriers under independent/correlated Rayleigh subcarrier fading.

Figure 6.2: BER of downlink ORC MC-CDMA with 4/32/128/1024 i.i.d. Rayleigh fading subcarriers.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Figure 6.3: BER of downlink ORC MC-CDMA with 32 i.i.d. Rayleigh/Rician fading subcarriers.

Figure 6.4: BER of downlink ORC MC-CDMA with 64 i.i.d. Rayleigh/Nakagami fading subcarriers.

6.2.2 Truncated ORC

6.2.2.1 Principle

In the previous section, we have shown that the performance of ORC MC-CDMA with orthogonal spreading is limited by the weakest instantaneous subcarrier. Exploiting this insight, we propose a “Truncated ORC” scheme, which omits the weakest instantaneous subcarrier in the combining operation of ORC. Fig. 6.5 and Fig. 6.6 compare the BER results of the proposed Truncated ORC scheme with the known equalization and combining schemes for MC-CDMA, i.e., EGC, MRC, ORC and MMSEC. Note that, there are two different forms of MMSEC schemes for MC-CDMA, namely “MMSEC per subcarrier” and “MMSEC per user”, as discussed in Section 4.2.3. Since “MMSEC per user” is the most optimal scheme, the “MMSEC” used in this chapter refers to this scheme. In the simulations, 32 subcarriers are used and the channel is assumed to consist of 4 i.i.d. Rayleigh fading paths. Fig. 6.5 is for 20 users and Fig. 6.6 is for 32 users (full system loading condition).

The results in Fig. 6.5 and 6.6 show that, MRC and EGC have unacceptable error floors. ORC does not have error floor and outperforms MRC and EGC at high E_b/N_0 values. The proposed Truncated ORC scheme can achieve significant performance gain over the conventional ORC. Comparing Fig. 6.5 and Fig. 6.6, it is noted the performance difference between the proposed Truncated ORC scheme and the optimal MMSEC scheme gets smaller when the MC-CDMA system has more users. At full system loading, the Truncated ORC scheme performs no more than 3 dB worse than the optimal MMSEC scheme.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Figure 6.5: BER of downlink MC-CDMA (high system loading: 20 users) with different equalization and combining schemes in 4-path Rayleigh fading channel.

Figure 6.6: BER of downlink MC-CDMA (full system loading: 32 users) with different equalization and combining schemes in 4-path Rayleigh fading channel.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

6.2.2.2 Lower Bound of Truncated ORC

A BER lower bound will next be derived for the proposed Truncated ORC scheme. Assuming that orthogonal Walsh-Hadamard codes are employed, the MUI for downlink MC-CDMA with ORC, as deduced from (4.5), is

$$\mathcal{MUI} = \sqrt{\frac{E_b T_s}{2N_c}} \sum_{k=2}^{N_u} b_k \sum_{n=1}^{N_c} c_{k,n} c_{1,n} \quad (6.11)$$

where E_b is the bit energy, T_s is MC-CDMA symbol duration, b_k is the data for the k th user, N_u is the number of users and N_c is the number of subcarriers. Since we omit one subcarrier in the combining operation of the Truncated ORC scheme, its MUI will not be zero. Instead, we have $\sum_{n, n \neq i}^{N_c} c_{k,i} c_{1,n} = \pm 1$ where i denotes the subcarrier index for $h_{\min}$. Hence the MUI power is

$$\text{var}(\mathcal{MUI}) = \frac{E_b T_s}{2N_c} (N_u - 1). \quad (6.12)$$

For the desired signal component D in (4.4), we have

$$D = \sqrt{\frac{E_b T_s}{2N_c}} \sum_{n, n \neq i}^{N_c} h_n \alpha_n = \sqrt{\frac{E_b T_s}{2N_c}} (N_c - 1). \quad (6.13)$$

Hence the conditional BER for downlink MC-CDMA with the proposed Truncated ORC scheme is

$$\begin{aligned} P_e | \{h_n\} &= Q \left(\frac{D}{\sqrt{\text{var}(\mathcal{MUI}) + \text{var}(\eta)}} \right) \\ &= Q \left(\frac{N_c - 1}{\sqrt{N_u - 1 + \frac{N_c N_0}{2E_b} \left(\sum_{n=1}^{N_c} h_n^{-2} - h_{\min}^{-2} \right)}} \right). \end{aligned} \quad (6.14)$$

Similar to ORC, we obtain the lower bound for the Truncated ORC scheme by using the fact that $\sum_{n=1}^{N_c} h_n^{-2} - h_{\min}^{-2} > h_{(2)}^{-2}$, where $h_{(2)}$ denotes the second smallest

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

value of the instantaneous subcarrier fading gains. Hence the lower bound for the Truncated ORC scheme conditioned on $h_{(2)}$ is

$$P_e|h_{(2)} = Q \left(\frac{N_c - 1}{\sqrt{N_u - 1 + \frac{N_c N_0}{2E_b h_{(2)}^2}}} \right). \quad (6.15)$$

For i.i.d. Rayleigh faded subcarriers, the pdf of $h_{(2)}$ can be obtained by setting $r = 2$ in (6.4), i.e.,

$$p(h_{(2)}) = 2N_c(N_c - 1)h_{(2)} \exp[-(N_c - 1)h_{(2)}^2] - 2(N_c - 1)N_c h_{(2)} \exp(-N_c h_{(2)}^2). \quad (6.16)$$

Then the lower bound for the Truncated ORC scheme with single user can be derived to be

$$P_e > \frac{1}{2} \left[1 - N_c \sqrt{\frac{\frac{N_c - 1}{N_c} \frac{E_b}{N_0}}{1 + \frac{N_c - 1}{N_c} \frac{E_b}{N_0}}} + (N_c - 1) \sqrt{\frac{\left(\frac{N_c - 1}{N_c}\right)^2 \frac{E_b}{N_0}}{1 + \left(\frac{N_c - 1}{N_c}\right)^2 \frac{E_b}{N_0}}} \right]. \quad (6.17)$$

Fig. 6.7 shows the simulated BER results and the BER lower bound (6.17) for an MC-CDMA system with the proposed Truncated ORC scheme. There are 32 i.i.d. Rayleigh fading subcarriers and the system loading condition varies from low system loading (single user), half system loading (16-users) to full system loading (32 users). Fig. 6.7 shows that the lower bound (6.17) is tight for $\text{BER} < 10^{-6}$, and the MC-CDMA downlink performances for 1, 16 and 32 users are not distinguishable although MUI is not zero for the Truncated ORC scheme. This is because the MUI power is very small compared to the amplified noise power in (6.14).

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Figure 6.7: Theoretical BER lower bound for the Truncated ORC scheme of MC-CDMA with different numbers of users.

6.2.2.3 Complexity Comparison

Although the performance of the proposed Truncated ORC scheme for downlink MC-CDMA is not as good as the MMSEC scheme, the MMSEC scheme is computationally much more complex to implement than the Truncated ORC scheme. In this section, we will compare the computational complexity of conventional ORC, Truncated ORC, and MMSEC, assuming that, the subcarrier fading gains are known to all combining schemes. Computational complexity of a signal processing algorithm is commonly measured in terms of the number of additions and multiplications required. For ORC, every subcarrier is divided by its own fading coefficient, so there are N_c arithmetic operations as there are N_c subcarriers. For the proposed Truncated ORC scheme, we need to identify the instantaneous minimum subcarrier fading coefficient $h_{\min}$ first, which need $N_c - 1$ comparison operations. After suppressing this subcarrier and combining the rest, the total number of opera-

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

tions needed for the Truncated ORC scheme is $N_c - 1$ comparisons and $N_c - 1$ additions. Hence both ORC and Truncated ORC has complexity in the order of N_c , i.e., $\mathcal{O}(N_c)$, while the Truncated ORC scheme can achieve much better BER performance than the ORC scheme, as shown in the last section.

The computationally complexity of the MMSEC scheme is much higher than the ORC and Truncated ORC schemes. The equalizer coefficient vector α for MMSEC is given in (4.36). In (4.36), there is a need to perform matrix inversion for a matrix of size $N_c \times N_c$, hence MMSEC has minimal complexity order of $\mathcal{O}(N_c^3)$. Although the performance of the proposed Truncated ORC scheme is slightly worse than MMSEC, the former has much lower computational complexity, hence it can serve as a competitive equalization and combining scheme for downlink MC-CDMA.

6.2.2.4 Omitting More Subcarriers

In this section, we investigate the effect of truncating more than the weakest subcarriers in the Truncated ORC scheme. Fig. 6.8 shows the required E_b/N_0 to achieve $P_e = 10^{-3}$ when the weakest few subcarriers are omitted in ORC combining. For example, the “ORC-2” scheme cuts off the weakest two subcarriers and the “ORC-3” scheme cuts off the weakest three subcarriers. Since ORC is designed for high load conditions, the number of users considered in Fig. 6.8 ranges from 20 to 32 for a 32-subcarrier MC-CDMA system. For up to 24 users, the ORC performance with different number of Truncated subcarriers are similar. But when the number of users is increased further, the performance of “ORC-2” scheme and “ORC-3” scheme becomes worse than the proposed “ORC-1” scheme. This shows that as more and more subcarriers are omitted in the ORC combining, although the noise amplification effect is reduced, the MUI increases significantly due to

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

loss of orthogonality between the Truncated Walsh-Hadamard spreading codes. As shown earlier, if we cut off only $h_{\min}$, the resultant MUI contribution per interfering user is $\sqrt{\frac{E_b T_s}{2N_c}} \sum_{n, n \neq i}^{N_c} c_{k,n} c_{1,n} = \sqrt{\frac{E_b T_s}{2N_c}} \times (\pm 1)$ and the corresponding total MUI power is $\frac{E_b T_s}{2N_c} (N_u - 1)$. However, if we cut off three subcarriers, the MUI component $\sqrt{\frac{E_b T_s}{2N_c}} \sum_{n, n \neq i}^{N_c} c_{k,n} c_{1,n} = \sqrt{\frac{E_b T_s}{2N_c}} \times (\pm 3)$ or $\sqrt{\frac{E_b T_s}{2N_c}} \times (\pm 1)$, hence the MUI power could become 9 times larger. Therefore, as more subcarriers are Truncated before ORC combining, the MUI increases and dominates over the noise reduction, resulting in performance degradation at high load conditions. From this study, it is evident that the proposed Truncated ORC scheme which omits only the weakest subcarrier achieves the optimum trade off between noise reduction and loss of orthogonality.

Figure 6.8: Required E_b/N_0 to achieve $\text{BER} = 10^{-3}$ for downlink MC-CDMA (32 subcarriers) with different Truncated ORC schemes in a channel with 4 i.i.d. Rayleigh fading paths.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

6.2.2.5 Truncated ORC in Rician Fading Channels

So far, only Rayleigh fading channel has been considered for the proposed Truncated ORC scheme. In this section, its performance is investigated under Rician fading environment. Fig. 6.9 shows the additional E_b/N_0 required by the Truncated ORC compared to the MMSEC schemes to achieve $\text{BER}=10^{-3}$ in a channel with 4 i.i.d. Rician fading paths. For Rayleigh fading channels ($K=0$), the performance difference in E_b/N_0 to achieve $\text{BER}=10^{-3}$ between the Truncated ORC and MMSEC schemes is 3 dB, while for Rician fading channels ($K > 0$), the performance difference between Truncated ORC and MMSEC is smaller, e.g., only around 1.5 dB for $K \geq 2$.

Figure 6.9: Additional E_b/N_0 required by the Truncated ORC compared with MMSEC schemes, to achieve $\text{BER} = 10^{-3}$ in a channel with 4 i.i.d. Rician fading paths.

6.3 Complexity Reduction for MMSEC

In Section 4.4, we have shown that MMSEC is the optimal equalization and combining scheme in downlink MC-CDMA. However, it is also the most computationally complex to realize as it needs to perform real-time matrix inversion of a large matrix. In this section, we propose a way to reduce the matrix inversion dimension for calculating the MMSEC equalizer coefficient for downlink MC-CDMA. Note that in the literature, many complexity reduction methods are proposed for MMSE channel estimator [46, 107] and DS-CDMA detector [108, 109]. Our proposed scheme, however, is specific to the MMSE combiner/equalizer for MC-CDMA downlink with Walsh-Hadamard spreading codes.

The MMSEC equalizer coefficient vector α was derived in (4.36) to be:

$$\alpha = \sqrt{\frac{2N_c}{E_b T_s}} \left(\mathcal{H} \mathcal{C}_d \mathcal{C}_d^T \mathcal{H} + \frac{N_c}{2E_b/N_0} \mathbf{I}_{N_c} \right)^{-1} \cdot \mathbf{h} \quad (6.18)$$

where $\mathbf{I}_{N_c}$ is an identity matrix with size $N_c \times N_c$, $\mathcal{H}$ is a diagonal matrix as shown below

$$\mathcal{H} = \begin{bmatrix} h_1 & 0 & \cdots & 0 \\ 0 & h_2 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & h_{N_c} \end{bmatrix}, \quad (6.19)$$

$\mathbf{h} = [h_1, h_2, \cdots, h_{N_c}]^T$, $\mathcal{C}_d$ is a size $N_c \times N_u$ matrix whose k th column is the spreading code for the k th user, i.e.,

$$\mathcal{C}_d = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{N_u 1} \\ c_{12} & c_{22} & \cdots & c_{N_u 2} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ c_{1N_c} & c_{2N_c} & \cdots & c_{N_u N_c} \end{bmatrix}, \quad (6.20)$$

N_c and N_u are the number of subcarriers and the number of users respectively. We use the orthogonal Walsh-Hadamard codes for downlink spreading as they have zero cross-correlation.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Note that the matrix to be inverted in (6.18) is

$$\mathbf{R} = \mathcal{H}\mathcal{C}_d\mathcal{C}_d^T\mathcal{H} + \frac{N_c}{2E_b/N_0}\mathbf{I}_{N_c}$$

$$= \begin{bmatrix} h_1^2x_{11} + \frac{N_c}{2E_b/N_0} & h_1h_2x_{12} & \cdots & h_1h_{N_c}x_{1N_c} \\ h_2h_1x_{21} & h_2^2x_{22} + \frac{N_c}{2E_b/N_0} & \cdots & h_{N_c}h_2x_{2N_c} \\ \vdots & \vdots & \cdots & \vdots \\ h_{N_c}h_1x_{N_c1} & h_{N_c}h_2x_{N_c2} & \cdots & h_{N_c}h_{N_c}x_{N_cN_c} + \frac{N_c}{2E_b/N_0} \end{bmatrix} \quad (6.21)$$

where $x_{n,i}$ is the (n,i) th element of $\mathcal{C}_d\mathcal{C}_d^T$, i.e.,

$$x_{n,i} = \sum_{k=1}^{N_u} c_{kn}c_{ki}. \quad (6.22)$$

When $n = i$, we have $x_{n,n} = N_u$. This implies that the diagonal elements of $\mathcal{C}_d\mathcal{C}_d^T$ in $\mathbf{R}$ is always equals to N_u . If orthogonal spreading codes such as Walsh-Hadamard codes are used, the characteristics of $x_{n,i}$ will enable us to simplify the computation of $\mathbf{R}^{-1}$, and hence the corresponding MMSEC equalizer coefficient α .

We now consider the simplification of $\mathbf{R}^{-1}$ for different values of N_u .

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Case 1: $N_u = N_c$ (full system loading condition)

In this case, due to the code orthogonality of Walsh-Hadamard codes, we have

$$x_{n,i} = 0, \quad \text{if } n \neq i. \quad (6.23)$$

Hence $\mathbf{R}$ becomes a diagonal matrix, and the MMSEC equalizer coefficient in (6.18) can be reduced to

$$\boldsymbol{\alpha} = \left[\frac{h_1}{h_1^2 + \frac{1}{2} \frac{N_c}{N_u} \frac{1}{E_b/N_0}}, \quad \frac{h_2}{h_2^2 + \frac{1}{2} \frac{N_c}{N_u} \frac{1}{E_b/N_0}}, \quad \dots, \quad \frac{h_{N_c}}{h_{N_c}^2 + \frac{1}{2} \frac{N_c}{N_u} \frac{1}{E_b/N_0}} \right]^T. \quad (6.24)$$

This implies that the n th equalizer coefficient only depends on the fading gain h_n of the n th subcarrier. Hence there is no need for matrix inversion to find $\mathbf{R}^{-1}$.

Note also that the ‘MMSEC per user’ equalizer coefficients in (6.24) is exactly the same as the ‘MMSEC per subcarrier’ equalizer coefficients in (4.30). This explains why the ‘MMSEC per subcarrier’ scheme is observed in Fig. 4.7 to perform the same as the optimum ‘MMSEC per user’ scheme under full system load.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Case 2: $N_u = \frac{N_c}{2}$ (half system loading condition)

In this case, for Walsh-Hadamard codes, we have

$$x_{n,i} = \begin{cases} N_u, & \text{if } n = i \text{ or } |n - i| = N_c/2. \\ 0, & \text{otherwise.} \end{cases} \quad (6.25)$$

Hence $\mathbf{R}$ has the form

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{R}_3 & \mathbf{R}_4 \end{bmatrix} \quad (6.26)$$

where $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3$ and $\mathbf{R}_4$ are all diagonal matrices. For example,

$$\mathbf{R}_1 = \begin{bmatrix} N_u h_1^2 + \frac{N_c}{2E_b/N_0} & 0 & \cdots & 0 \\ 0 & N_u h_2^2 + \frac{N_c}{2E_b/N_0} & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & \cdots & N_u h_{N_c/2}^2 + \frac{N_c}{2E_b/N_0} \end{bmatrix}. \quad (6.27)$$

$\mathbf{R}_2, \mathbf{R}_3$ and $\mathbf{R}_4$ have similar forms. In this case, it can be shown that the inverse of $\mathbf{R}$ is also composed of four diagonal matrices, i.e.,

$$\mathbf{R}^{-1} = \begin{bmatrix} \mathbf{R}'_1 & \mathbf{R}'_2 \\ \mathbf{R}'_3 & \mathbf{R}'_4 \end{bmatrix} \quad (6.28)$$

where $\mathbf{R}'_1, \mathbf{R}'_2, \mathbf{R}'_3$ and $\mathbf{R}'_4$ are all diagonal matrices. Denoting the n th diagonal element of $\mathbf{R}_1$ as $r_{1,n}$, and the n th diagonal element of $\mathbf{R}'_1$ as $r'_{1,n}$, and so on, the elements of $\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \mathbf{R}_4$ and the elements of $\mathbf{R}'_1, \mathbf{R}'_2, \mathbf{R}'_3, \mathbf{R}'_4$ are related by the following 2×2 matrix inversion

$$\begin{bmatrix} r'_{1,n} & r'_{2,n} \\ r'_{3,n} & r'_{4,n} \end{bmatrix} = \begin{bmatrix} r_{1,n} & r_{2,n} \\ r_{3,n} & r_{4,n} \end{bmatrix}^{-1}. \quad (6.29)$$

Therefore, we do not need to invert matrix $\mathbf{R}$ with size $N_c \times N_c$, which has complexity order $\mathcal{O}(N_c^3)$, as shown in Section 6.2.2.3. With (6.29), we can obtain $\mathbf{R}^{-1}$ by simply inverting 2 by 2 matrices for $N_c/2$ times. Hence the complexity order is $\frac{N_c}{2} \mathcal{O}(2^3)$, which is much lower than $\mathcal{O}(N_c^3)$.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Case 3: $N_u = \frac{N_c}{4}$ or $N_u = \frac{3}{4}N_c$

In this case, it can be shown that the matrix $\mathbf{R}$ consists of 16 diagonal matrices, i.e.,

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2 & \mathbf{R}_3 & \mathbf{R}_4 \\ \mathbf{R}_5 & \mathbf{R}_6 & \mathbf{R}_7 & \mathbf{R}_8 \\ \mathbf{R}_9 & \mathbf{R}_{10} & \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{13} & \mathbf{R}_{14} & \mathbf{R}_{15} & \mathbf{R}_{16} \end{bmatrix}. \quad (6.30)$$

$\mathbf{R}^{-1}$ is also composed of 16 diagonal matrices. Similar to the half system loading case, the elements of these diagonal matrices are related through

$$\begin{bmatrix} r'_{1,n} & r'_{2,n} & r'_{3,n} & r'_{4,n} \\ r'_{5,n} & r'_{6,n} & r'_{7,n} & r'_{8,n} \\ r'_{9,n} & r'_{10,n} & r'_{11,n} & r'_{12,n} \\ r'_{13,n} & r'_{14,n} & r'_{15,n} & r'_{16,n} \end{bmatrix} = \begin{bmatrix} r_{1,n} & r_{2,n} & r_{3,n} & r_{4,n} \\ r_{5,n} & r_{6,n} & r_{7,n} & r_{8,n} \\ r_{9,n} & r_{10,n} & r_{11,n} & r_{12,n} \\ r_{13,n} & r_{14,n} & r_{15,n} & r_{16,n} \end{bmatrix}^{-1}. \quad (6.31)$$

Therefore, for system loading at $N_u = \frac{N_c}{4}$ or $N_u = \frac{3}{4}N_c$, the elements of $\mathbf{R}^{-1}$ can be computed by the inversion of 4×4 matrices. As a result, the complexity of MMSEC is reduced to $\frac{N_c}{4}\mathcal{O}(4^3)$.

Similarly we can reduce the matrix for other system loading conditions. As a summary, Fig. 6.10 shows the relationship between the reduced complexity level and the system loading of MMSEC for an MC-CDMA system with 32 subcarriers. The horizontal axis represents the system loading ratio N_u/N_c while the vertical axis represents the complexity level in the form of $\mathcal{O}(y^3)$ (y is a dummy variable). Note that the complexity of the original MMSEC is $\mathcal{O}(N_c^3)$ or $\mathcal{O}(32^3)$ in this case. From Fig. 6.10, we can see that as long as the number of users is even, we can reduce the complexity to at least $\mathcal{O}((N_c/2)^3)$. If N_u/N_c is some special values, such as $1/2$ or $1/4$, we can reduce the complexity even more. Since the proposed technique of matrix inversion size reduction is derived mathematically, the reduced-complexity MMSEC is expected to have exactly the same performance as the full-complexity MMSEC scheme. In addition, it is also worth taking note that the reduction of matrix size is independent of channel fading conditions.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Case 4: N_u is an odd number

If there are odd number of users in the system, however, we cannot reduce the matrix inversion size as before. But this problem can be circumvented easily. For example, by simply transmitting the signal of a dummy user, we can change N_u to an even number and make use of the method proposed earlier to reduce the matrix inversion size. As shown in Fig. 6.10, the complexity order of MMSEC can be reduced to at least $\mathcal{O}((N_c/2)^3)$ when N_u is even.

With the dummy user, the MMSEC complexity can be reduced, but the performance may degrade slightly since one more interferer is now present in the system. Fig. 6.11 shows the required E_b/N_0 for a 32-subcarrier MC-CDMA system with MMSEC to achieve $\text{BER}=10^{-3}$ as N_u varies from 1 to 32 (full system loading condition). The channels considered are: a channel with 4 i.i.d. Rayleigh fading paths, and a channel with i.i.d. Rayleigh fading subcarriers. As seen from Fig. 6.11, around 0.2 dB more of E_b/N_0 is required to add one more user to the system with 4 i.i.d. Rayleigh fading paths while maintaining system performance at $\text{BER}=10^{-3}$. Similarly the additional required E_b/N_0 to support one more user in the channel with i.i.d. Rayleigh fading subcarriers is also not much. Hence adding one dummy user does not degrade the system performance much, but the complexity may be reduced a lot by taking advantage of our proposed matrix inversion size reduction technique.

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

Figure 6.10: Reduced complexity order vs. system loading for MC-CDMA system with 32 subcarriers.

Figure 6.11: Required E_b/N_0 to achieve $BER=10^{-3}$ for MC-CDMA system with MMSEC in channels of 4 i.i.d. Rayleigh fading paths or of i.i.d. Rayleigh fading subcarriers.

6.4 Summary

In this chapter, a BER lower bound and an approximate BER formula are proposed for downlink MC-CDMA with ORC in channels with independent or correlated subcarrier fading. For Rayleigh fading, the lower bound can be reduced to a closed-form expression. Numerical results show that under many fading conditions, the approximate BER formula is very tight for all BER ranges and the lower bound is tight for BER less than 10^{-3} . Hence we have achieved simple but useful BER formulations for MC-CDMA systems with ORC.

Through the study of ORC performance, the noise amplification effect of the weakest subcarrier is found to be very dominant, hence a novel Truncated ORC scheme which omits the weakest instantaneous subcarrier in the combining operation is proposed. It can achieve significant performance gain over the conventional ORC and it is only less than 3 dB worse than the optimum MMSEC scheme at full system load. The closed-form BER lower bound for the Truncated ORC scheme is also developed, which is found to be tight for BER smaller than 10^{-6} . The Truncated ORC scheme has complexity $\mathcal{O}(N_c)$ where N_c is the number of subcarriers used, so it can serve as a competitive alternative to the optimal MMSEC scheme which has complexity $\mathcal{O}(N_c^3)$ for downlink MC-CDMA.

In this chapter, we also propose a method to reduce the computational complexity of the MMSEC scheme, which needs to perform matrix inversion of size $N_c \times N_c$. When the number of users N_u in the MC-CDMA system is even, we can reduce the matrix inversion size to at least $N_c/2$. When N_u/N_c has some special values, the complexity order can be reduced even more. For example, the complexity order is reduced to $\mathcal{O}(2^3)$ at $N_u/N_c = 1/2$ (half system loading condition). This technique can be directly used for even number of N_u . If N_u is an odd number, by simply transmitting a dummy user, the system performance does not degrade

CHAPTER 6. DOWNLINK MC-CDMA WITH ORC AND MMSEC - BER ANALYSIS AND ENHANCEMENT

much, while we can apply the proposed technique to reduce the matrix inversion size.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

In this thesis, we have analyzed the subcarrier fading characteristics of multicarrier modulation (MCM) schemes such as orthogonal frequency division multiplexing (OFDM) and multicarrier code division multiple access (MC-CDMA), under generalized frequency-selective fading conditions. We have also developed several approaches to simplify the analytical expressions of bit error rate (BER) performance, as well as to enhance the performance of OFDM and MC-CDMA systems with various equalization and combining schemes.

For the subcarrier fading analysis study, we have shown that for multipath fading channels with arbitrary combinations of correlated Rician and Rayleigh paths, the subcarrier fading can be modelled as Rician too. The K factors and mean fading powers of individual subcarriers can be precisely derived from the K factors and powers of the time-domain channel impulse response (CIR), which is widely used to characterize practical wideband mobile radio channels.

A wide variety of frequency-selective fading channels containing different combinations of independent/correlated Rayleigh and Rician fading paths are then

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

studied. In this study, we have discovered that many practical channels are subject to non-uniform subcarrier fading, which means that the K factors and/or powers of individual subcarriers are different. Results show that, for channels with independent paths, as long as there are more than one Rician path in the channel, the K factors and powers across the subcarriers will be different (i.e. non-uniform subcarrier fading), but high- K subcarriers will have high power, and vice versa. For channels with only one Rician path and the remaining path(s) are Rayleigh, all the subcarriers will be Rician distributed with the same K factor (i.e. uniform subcarrier fading), but this K factor will be smaller than the K factor of the Rician path. On the other hand, channels with correlated paths are shown to always yield non-uniform fading across the subcarriers.

Based on the derived subcarrier fading statistics, we have demonstrated that the BER performance of an OFDM system operating in generalized fading channels with a mixture of independent/correlated Rayleigh and Rician paths is accurately analyzed and predicted down to subcarrier level. Furthermore, OFDM systems operating in channels with non-uniform subcarrier fading will experience unequal BER performance across the subcarriers if they are identically modulated, and the poor subcarriers will degrade the overall system performance a lot. For such channels, we have proposed a statistical OFDM adaptation scheme to improve the system performance. The proposed scheme effectively performs adaptive bit loading and power control on the OFDM subcarriers based on the long-term subcarrier fading statistics, instead of instantaneous subcarrier fading gains, hence it has lower implementation complexity compared to the existing adaptive OFDM schemes found in the literature. Through simulation, it is demonstrated that it can achieve considerable power saving and spectral efficiency improvement over non-adaptive OFDM systems.

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

For the generic BER analysis of MC-CDMA systems, we have mapped the BER integration domain from the frequency (subcarrier) domain to the time (channel path) domain so that Monte Carlo integration can be carried out more efficiently to compute the BER value. With this, the performance of various equalization and combining schemes including orthogonality restoring combining (ORC), equal gain combining (EGC), maximal ratio combining (MRC) and minimum mean square error combining (MMSEC) can be obtained more efficiently.

For the BER analysis of uplink MC-CDMA with MRC, we have considered frequency-selective fading channels with correlated Rayleigh, Rician or Nakagami paths. New closed-form BER formula is derived for correlated Nakagami fading channels. This closed-form BER formula no longer involves integration. For correlated Rayleigh or Rician fading channels, the full analytical BER formula is simplified to contain only one-dimensional integration, hence it can be easily integrated numerically.

We have also managed to simplify the analytical BER expression of downlink MC-CDMA with ORC. A BER lower bound is first obtained. It is then refined to give a very tight BER approximation. Both the BER lower bound and the approximate BER expressions involve only one-dimensional integration, hence they are much easier to evaluate than the available multi-dimensional integration expressions. For Rayleigh fading channels, the lower bound can be reduced to closed-form expression. Numerical results show that our approximate BER formulation is very tight under many fading channel conditions.

Through the study of ORC performance, it is found that the BER performance of MC-CDMA with ORC is dominated by the weakest instantaneous subcarrier, which suffers from the worst noise amplification effect. Hence we have proposed a “Truncated ORC” scheme which omits the weakest instantaneous subcarrier in

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

the combining operation of ORC in an attempt to enhance the ORC performance. Though with such simple modification, the proposed scheme is demonstrated to perform much better than the conventional ORC scheme and impressively close to the optimal MMSEC scheme, while the computational complexity of MMSEC is high as it involves matrix inversion operation. The closed-form BER lower bound for the Truncated ORC scheme is also developed.

For the optimal MMSEC scheme of MC-CDMA systems, its complexity is much higher compared to the MRC and ORC schemes because it involves matrix inversion operation of matrices size $N_c \times N_c$. In this project, one way to reduce the matrix inversion size for calculating the MMSEC equalizer coefficient is proposed. As long as the number of users is even, the matrix inversion size is reduced to at least $N_c/2$. When N_u is an odd number, by simply transmitting a dummy user (it is shown that the system performance does not degrade much), the complexity can also be reduced by our proposed matrix inversion size reduction technique.

In this thesis, two methods, namely Decomposition Method and Sim's Method, are proposed to generate Nakagami fading signals with arbitrary fading parameters and correlations. The Decomposition Method is more flexible and efficient. The Sim's Method is mathematically exact but carries constraints on the permissible simulation parameters. Simulations show that both methods produce outputs that match well with the specifications.

I believe that the achievements stated above have contributed towards better understanding, more efficient evaluations and enhancements of the performance of MCM systems operating in generalized frequency-selective fading channels.

7.2 Recommendations for Further Research

The scope of the current work may be further extended in the following areas:

- The proposed statistical OFDM adaptation algorithm in this thesis aims to minimize the transmission power while maintaining equalized subcarrier BERs and constant transmission rate for OFDM channels with non-uniform subcarrier fading. This adaptation objective can be extended to include maximization of transmission rate with equalized subcarrier BERs and constant transmission power, and minimization of overall system BER subject to constant transmission rate and power, as different practical applications may have different requirements. Moreover, the statistical OFDM adaptation scheme can also be combined with instantaneous adaptation schemes to form hybrid schemes that can achieve even better trade-off between performance and complexity.
- This thesis has focused mainly on the BER performance analysis of uncoded MCM systems. It is instructive to extend the system performance analysis to coded MCM systems (i.e., with channel coding). In this analysis, the subcarrier fading correlations need to be taken into consideration. At the same time, the effect of time/frequency domain interleaving on the system performance should also be investigated.
- This thesis has proposed a statistical OFDM adaptation scheme in this thesis, which gives considerable power saving over the non-adaptive OFDM systems for MCM channels with non-uniform subcarrier fading distributions. This idea of statistical adaptation can similarly be applied to MC-CDMA systems to achieve system adaptability and performance enhancement.

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

- Our current Nakagami fading generator generates Nakagami signal samples with arbitrary m parameters and inter-path correlations. However, the samples are memoryless, i.e., they have flat Doppler spectrum. A more general design capable of generating Nakagami waveforms with specified time correlation or Doppler spectrum (e.g., Clark's spectrum) would be very useful.
- Last but not the least, the performance analysis conducted in this thesis may also be extended to include new features, such as space-time coded modulation or multi-user detection. Future MCM systems are likely to be incorporated with these new features, hence their performance analysis are also highly desirable.

Author's Publications

Journal Papers

1. K. L. Zhang, Z. F. Song and Y. L. Guan, "Simulation of Nakagami fading channels with arbitrary cross-correlation and fading parameters," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1463-1468, Sept. 2004.
2. K. L. Zhang, Z. F. Song and Y. L. Guan, "Multicarrier channels with non-identical subcarrier fading distributions - analysis and adaptation," *IEEE Commun. Lett.*, vol. 8, no. 6, pp. 368-370, Jun. 2004.
3. K. L. Zhang and Y. L. Guan, "Performance of Asynchronous MC-CDMA Systems with Maximal Ratio Combining in Frequency-Selective Fading Channels," *EURASIP Journal on Applied Signal Processing: Special Issue on Multicarrier Communications and Signal Processing*, no. 10, pp. 1595-1603, Aug. 2004.

Conference Papers

1. K. L. Zhang and Y. L. Guan, "Simplified BER formulations for MC-CDMA systems with orthogonality restoring combining," in *Proc. 4th Int. Conf. on Information, Communications and Signal Processing 4th IEEE Pacific-Rim Conf. on Multimedia (ICICS & PCM '03)*, Singapore, Dec. 2003.

AUTHOR'S PUBLICATIONS

2. K. L. Zhang and Y. L. Guan, "Two low-complexity enhancements to multi-carrier modulation systems," in *Proc. First Singapore-Japan International Workshop on Info-Communications Technologies for the Ubiquitous Networked Society*, Singapore, Dec. 2003.
3. K. L. Zhang, Z. F. Song and Y. L. Guan, "Cholesky decomposition model for correlated MRC diversity systems in Nakagami fading channels," in *Proc. IEEE Veh. Technol. Conf. (VTC'02)*, Vancouver, Sep. 2002, vol. 3, pp. 1515 -1519.
4. K. L. Zhang, Z. F. Song and Y. L. Guan, "A generalized model for the combined output statistics of MRC diversity systems in Nakagami fading channels," in *Proc. International Symposium on Communication Systems, Networks and Digital Signal Processing (CSNDSP'02)*, Stafford, United Kingdom, Jul. 2002, pp.422-425.
5. Z. F. Song, K. L. Zhang and Y. L. Guan, "Statistical adaptive modulation for QAM-OFDM systems," in *Proc. IEEE Global Commun. Conf. (GLOBECOM '02)*, Taipei, Nov. 2002.
6. Z. F. Song, K. L. Zhang and Y. L. Guan, "Joint bit-loading and power-allocation for OFDM systems based on statistical frequency-domain fading model," in *Proc. IEEE 56th Veh. Technol. Conf. (VTC'02)*, Vancouver, Sep. 2002, vol. 2, pp. 724 -728.
7. Z. F. Song, K. L. Zhang and Y. L. Guan, "Generating correlated Nakagami fading signals with arbitrary correlation and fading parameters," in *Proc. IEEE Int. Conf. Commun. (ICC'02)*, New York, Apr. 2002, vol. 3, pp. 1363 -1367.

AUTHOR'S PUBLICATIONS

8. Y. L. Guan, Z. F. Song, K. L. Zhang, H. Meng and Y. Wang, "Statistical bit-loading and power-control for OFDM systems with unequal subcarrier fading distributions," in *Proc. International Symposium on Communication Systems, Networks and Digital Signal Processing (CSNDSP'02)*, United Kingdom, Stafford, United Kingdom, Jul. 2002, pp. 232-235.
9. Y. L. Guan, Z. F. Song and K. L. Zhang, "Modelling of frequency-domain fading characteristics for OFDM systems," in *Proc. 3rd Int. Conf. Information, Communications and Signal Processing (ICICS'01)*, Singapore, Oct. 2001.

Bibliography

- [1] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, pp. 5–14, May 1990.
- [2] S. Hara and R. Prasad, "Design and performance of multi-carrier DS-CDMA system in multi-path fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, no. 5, pp. 1584–1595, Sept. 1999.
- [3] A. R. S. Bahai and B. R. Saltzberg, *Multi-carrier digital communications: theory and applications of OFDM*, Kluwer Academic, New York, 1999.
- [4] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," *IEEE Signal Processing Mag.*, pp. 29–48, May 2000.
- [5] T. Keller and L. Hanzo, "Adaptive multicarrier modulation: a convenient framework for time-frequency processing in wireless communications," *Proceedings of the IEEE*, vol. 88, no. 5, pp. 611–640, May 2000.
- [6] R. V. Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House publisher, Boston, London, 2000.
- [7] N. Yee, J. P. Linnartz, and G. Fettweiss, "Multi-carrier CDMA in indoor wireless radio networks," in *Proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'93)*, 1993, pp. 468–472.

BIBLIOGRAPHY

- [8] K. Fazel and L. Papke, "Performance of Convolutional-Coded CDMA/OFDM for Mobile Communication System," in *Proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'93)*, Sept. 1993, pp. 468–472.
- [9] A. Chouly, A. Brajal, and S. Jourdan, "Orthogonal multicarrier techniques applied to direct sequence spread spectrum CDMA systems," in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM'93)*, Nov. 1993, pp. 1723–1728.
- [10] S. Stroh, "Ultra-wideband: multimedia unplugged," *IEEE Spectrum*, vol. 40, no. 9, pp. 23–27, Sept. 2003.
- [11] H. J. Park, M. J. Kim, Y. J. So, Y. H. You, and H. K. Song, "UWB communication system for home entertainment network," *IEEE Trans. Consumer Electronics*, vol. 49, no. 2, pp. 302–311, May 2003.
- [12] A. C. McCormick and E. A. Al-Susa, "Multicarrier CDMA for future generation mobile communication," *Electronics & Communication Engineering Journal*, vol. 14, no. 2, pp. 52–60, Apr. 2002.
- [13] S. Sigdel, K. M. Ahmed, and R. M. A. P. Rajatheva, "Performance evaluation of multicarrier CDMA uplink system with antenna array and multiuser detection over correlated multipath channel," in *IEEE Vehicular Technology Conference (VTC'02-Fall)*, 2002, pp. 1958–1962.
- [14] V. Nangia and K. L. Baum, "Experimental broadband OFDM system: field results for OFDM and OFDM with frequency domain spreading," in *IEEE Vehicular Technology Conference (VTC'02-Fall)*, 2002, pp. 223–227.

BIBLIOGRAPHY

- [15] W. Zhou, X. Lu, and J. Zhu, "M-ary MC-CDMA system for 4G," in *IEEE Vehicular Technology Conference (VTC'01-Fall)*, 2001, pp. 2234–2238.
- [16] J. M. Pereira, "Fourth generation: now, it is personal!," in *Proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'00)*, 2000, pp. 1009–1016.
- [17] T. Ojanpera and R. Prasad, *Wideband CDMA for Third Generation Mobile Communications*, Artech House publisher, Boston, 1998.
- [18] M. Patzold, *Mobile Fading Channels*, Wiley, 1981.
- [19] S. Kaiser, "On the performance of different detection techniques for OFDM-CDMA in fading channels," in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM'95)*, 1995, pp. 2059–2063.
- [20] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, pp. 126–133, Dec. 1997.
- [21] R. Le Gouable and M. Helard, "Performance of single and multi-user detection techniques for a MC-CDMA system over channel model used for HIPERLAN2," in *Proc. IEEE Int. Symposium on Spread Spectrum Techniques and Applications (ISSSTA'00)*, 2000, pp. 718–722.
- [22] I. Siaud, R. Le Gouable, and M. Helard, "On multicarrier transmission techniques over recorded indoor propagation channel models for future broadband RLANs at 60 GHz," in *Proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'01)*, 2001, pp. F-53 – F-58.

BIBLIOGRAPHY

- [23] N. Yee and J. P. Linnartz, "BER for multi-carrier CDMA in indoor Rician fading channel," in *Proc. Asilomar Conference on Signals, Systems and Computers*, 1993, pp. 426–430.
- [24] J. Park, J. Kim, S. Choi, N. Cho, and D. Hong, "Performance of MC-CDMA systems in non-independent Rayleigh fading," in *Proc. IEEE Int. Conf. Communications (ICC'99)*, 1999, pp. 506–510.
- [25] Y. Kim, S. Choi, C. You, and D. Hong, "BER computation of an MC-CDMA system with carrier frequency offset," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP'99)*, 1999, vol. 5, pp. 2555–2558.
- [26] Q. Shi and M. Latva-aho, "An exact error floor for downlink MC-CDMA in correlated Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 6, no. 5, pp. 196–198, May 2002.
- [27] Q. Shi and M. Latva-aho, "Exact bit error rate calculations for synchronous MC-CDMA over a Rayleigh fading channel," *IEEE Commun. Lett.*, vol. 6, no. 7, pp. 276–278, July 2002.
- [28] Z. Li and M. Latva-aho, "Simple analysis of MRC receivers for MC-CDMA systems in fading channels," in *Proc. Int. Conf. on Info-tech and Info-net (ICII 2001)*, 2001, pp. 560–565.
- [29] X. Gui and T. S. Ng, "Performance of asynchronous orthogonal multicarrier CDMA system in frequency selective fading channel," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1084–1091, July 1999.
- [30] D. Mottier, D. Castelain, J. F. Herlard, and J. Y. Baudais, "Optimum and sub-optimum linear MMSE multi-user detection for multi-carrier CDMA

BIBLIOGRAPHY

- transmission systems,” in *IEEE Vehicular Technology Conference (VTC'01 FALL)*, 2001.
- [31] T. S. Rappaport, *Wireless Communications*, Prentice Hall, New York, 1996.
- [32] G. L. Stüber, *Principle of Mobile Communication*, Kluwer Academic Publishers, 2nd edition, 2001.
- [33] J. G. Proakis, *Digital Communications*, McGraw-Hill, 4th edition, 2001.
- [34] K. Cho and D. Yoon, “On the general BER expression of one- and two-dimensional amplitude modulations,” *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1074–1080, July 2002.
- [35] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications*, Prentice Hall, New York, 1996.
- [36] W. Nabhane and H. V. Poor, “Blind joint equalization and multiuser detection in dispersive MC-CDMA/MC-DS-CDMA/MT-CDMA channels,” in *IEEE Military Communications Conference Proceedings (MILCOM'02)*, 2002, vol. 2, pp. 814–819.
- [37] W. Xu and L. B. Milstein, “On the performance of multicarrier RAKE systems,” *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1812–1823, Oct. 2001.
- [38] S. Kondo and L. B. Milstein, “Performance of multicarrier DS CDMA systems,” *IEEE Trans. Commun.*, vol. 44, no. 2, pp. 238–246, Feb. 1996.
- [39] H. Matsutani and M. Nakagawa, “Multi-carrier DS-CDMA using frequency spread coding,” *IEICE Trans. Fundamentals*, vol. E82-A, no. 12, pp. 2634–2642, Dec. 1999.

BIBLIOGRAPHY

- [40] Q. M. Rahman and A. B. Sesay, "Performance analysis of MT-CDMA system with diversity combining," in *IEEE Military Communications Conference Proceedings (MILCOM'01)*, 2001, vol. 2, pp. 1360–1364.
- [41] Q. Shi and M. Latva-aho, "Effect of frequency offset on the performance of asynchronous MC-CDMA systems in a correlated Rayleigh fading channel," in *Proc. Int. Conf. on Info-tech and Info-net (ICII 2001)*, 2001, pp. 448–452.
- [42] O. Edfors, M. Sandel, J. J. van de Beek, D. Landström, and F. Sjöberg, *An Introduction to Orthogonal Frequency-Division Multiplexing*, Lulea Tekniska University, Research report 1996:16, Sept. 1996.
- [43] B. Yang, K. B. Letaief, R. S. Cheng, and Z. Cao, "Channel Estimation for OFDM Transmission in Multipath Fading Channels Based on Parametric Channel Modelling," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 467–479, Mar. 2001.
- [44] L. J. Greenstein, D. G. Michelson, and V. Erceg, "Moment Method Estimation of the Ricean K-factor," *IEEE Commun. Lett.*, vol. 3, no. 6, pp. 175–176, June 1999.
- [45] Y. Li, L. J. Cimini, and N. R. Sollenberger, "Robust channel estimation for OFDM systems with rapid dispersive fading channels," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 902–914, July 1998.
- [46] O. Edfors, M. Sandel, J. J. van de Beek, S. K. Wilson, and P. O. Brjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, no. 7, pp. 931–939, July 1998.
- [47] R. A. Johnson and D. W. Wichern, *Applied Multivariate Statistical Analysis*, Prentice Hall, 1992.

BIBLIOGRAPHY

- [48] M. S. Srivastava and E. M. Carter, *An introduction to applied multivariate statistics*, Academic Press, North Holland, 1983.
- [49] J. Kivinen, X. Zhao, and P. Vainikainen, "Empirical characterization of wide-band indoor radio channel at 5.3 GHz," *IEEE Trans. Antennas Propagat.*, vol. 49, no. 8, pp. 1192–1203, Aug. 2001.
- [50] W. Mohr, "Modeling of wideband mobile radio channels based on propagation measurements," in *Proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'95)*, 1995, pp. 397–401.
- [51] V. A. Aalo, "Performance of maximal-ratio diversity systems in a correlated Nakagami-fading environment," *IEEE Trans. Commun.*, vol. 43, no. 8, pp. 2360–2369, Aug. 1995.
- [52] G. Fedele, L. Izzo, and M. Tanda, "Dual diversity reception of M-ary DPSK signals over Nakagami fading channels," in *IEEE Sixth Int. Symp. Personal, Indoor and Mobile Radio Communication (PIMRC'95)*, Toronto, Canada, Sept. 1995, pp. 1195–1201.
- [53] J. Salz and J. H. Winters, "Effect of fading correlation on adaptive arrays in digital wireless communications," *IEEE Trans. Veh. Technol.*, vol. 43, no. 4, pp. 1049–1057, Nov. 1994.
- [54] J. K. Cavers, "Variable-rate transmission for Rayleigh fading channels," *IEEE Trans. Commun.*, vol. COM-20, pp. 15–22, Feb. 1972.
- [55] W. T. Webb and R. Steele, "Variable rate QAM for mobile radio," *IEEE Trans. Commun.*, pp. 2223–2230, July 1995.

BIBLIOGRAPHY

- [56] S. T. Chung and A. J. Goldsmith, "Adaptive multicarrier modulation for wireless systems," in *Proc. IEEE Asilomar Conf. on Signals, Systems and Computers*, Pacific Grove, CA, USA, Nov. 2000, pp. 1603–1607.
- [57] A. J. Goldsmith and S. G. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1218–1230, Oct. 1997.
- [58] Y. W. Cheong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Select. Areas Commun.*, vol. 17, no. 10, pp. 1747–1758, Oct. 1999.
- [59] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication," *IEEE Trans. Commun.*, vol. 48, no. 1, pp. 23–27, Jan. 2000.
- [60] S. T. Chung and A. J. Goldsmith, "Degree of freedom in adaptive modulation: a unified view," *IEEE Trans. Commun.*, vol. 49, no. 9, pp. 1561–1571, Sept. 2001.
- [61] R. F. H. Fischer and J. B. Huber, "A new loading algorithm for discrete multitone transmission," in *Proc. IEEE Global Telecommunications Conf. (GLOBECOM'96)*, Nov. 1996, pp. 724–728.
- [62] Z. Song, K. Zhang, and Y. L. Guan, "Statistical adaptive modulation for QAM-OFDM systems," in *Proc. IEEE Global Commun. Conf. (GLOBECOM '02)*, Taipei, Nov. 2002.
- [63] J. D. Parsons, *The Mobile Radio Propagation Channel*, New York: John Wiley, 2000.

BIBLIOGRAPHY

- [64] R. M. Corless, G. H. Gonnet, D. E. G. Hare, and D. J. Jeffrey, "On the Lambert W function," *Adv. Comput. Math.*, vol. 5, pp. 329–359, 1996.
- [65] S. K. Lai, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Adaptive trellis coded MQAM and power optimization for OFDM transmission," in *IEEE Vehicular Technology Conference (VTC'99)*, May 1999, pp. 290–294.
- [66] K. K. Wong, S. K. Lai, R. S.-K. Cheng, K. B. Letaief, and R. D. Murch, "Adaptive spatial-subcarrier trellis coded MQAM and power optimization for OFDM transmissions," in *IEEE Vehicular Technology Conference (VTC'00)*, Tokyo, May 2000, pp. 2049–2053.
- [67] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, 4th edition, 2002.
- [68] W. C. Jakes, *Microwave Mobile Communication*, Wiley, New York, 1974.
- [69] E. K. Al-Hussaini and A. M. Al-Bassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Trans. Commun.*, vol. 33, pp. 1315–1319, Dec. 1985.
- [70] D. N. Kalofonos, M. Stojanovic, and J. G. Proakis, "On the performance of adaptive MMSE detectors for a MC-CDMA system in fast fading Rayleigh channels," in *IEEE Ninth Int. Symp. Personal, Indoor and Mobile Radio Communication (PIMRC'98)*, Sept. 1998, pp. 1309–1313.
- [71] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, 4th edition, 2002.
- [72] S. Verdu, *Multiuser detection*, Cambridge University Press, 1998.
- [73] R. Y. Rubinstein, *Simulation and the Monte Carlo Method*, Wiley, New York, 1981.

BIBLIOGRAPHY

- [74] G. L. Turin, "Simulation of urban vehicle monitoring systems," *IEEE Trans. Veh. Technol.*, pp. 9–16, Feb. 1972.
- [75] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. Commun.*, vol. COM-25, no. 7, pp. 673–680, 1977.
- [76] A. Wojnar, "Rayleigh, Rice and Nakagami – In search of efficient models of fading radio channels," in *Int. Wroclaw Symp. Electromagnetic Compatibility*, Wroclaw, Poland, 1988, pp. 797–802.
- [77] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1141–1150, July 1999.
- [78] M. Nakagami, "The m -distribution—A general formula of intensity distribution of rapid fading," *Statistical Methods in Radio Wave Propagation*, 1960.
- [79] M. S. Alouini, A. Abdi, and M. Kaveh, "Sum of gamma variates and performance of wireless communication systems over Nakagami-fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, pp. 1471–1480, Nov. 2001.
- [80] G. P. Efthymoglou, V. A. Aalo, and H. Helmken, "Performance analysis of coherent DS-CDMA systems in a Nakagami fading channel with arbitrary parameters," *IEEE Trans. Veh. Technol.*, vol. 46, no. 2, pp. 289–297, May 1997.
- [81] A. Annamalai, C. Tellambura, and V. K. Bhargava, "Efficient computation of MRC diversity performance in Nakagami fading channel with arbitrary parameters," *Electronics Letters*, vol. 34, pp. 1189–1190, June 1998.

BIBLIOGRAPHY

- [82] A. Annamalai and C. Tellambura, "Error rates for Nakagami-m fading multichannel reception of binary and M-ary signals," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 58–68, Jan. 2001.
- [83] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, 9th printing, New York: Dover, 1972.
- [84] J. Reig and N. Cardona, "Bivariate Nakagami-m distribution with arbitrary fading parameters," *IEE Electronic Lett.*, vol. 38, no. 25, pp. 1715–1717, Dec. 2002.
- [85] A. Annamalai, C. Tellambura, and V. K. Bhargava, "Exact evaluation of maximal-ratio and equal-gain diversity receivers for M-ary QAM on Nakagami fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1335–1344, Sept. 1999.
- [86] C. Mun, C. H. Kang, and H. K. Park, "Approximation of SNR statistics for MRC diversity systems in arbitrarily correlated Nakagami fading channels," *IEE Electronic Lett.*, vol. 35, no. 4, pp. 266–267, Feb. 1999.
- [87] P. Lombardo, G. Fedele, and M. M. Rao, "MRC performance for binary signals in Nakagami fading with general branch correlation," *IEEE Trans. Commun.*, vol. 47, no. 1, pp. 44–52, Jan. 1999.
- [88] J. Luo, J. R. Zeidler, and S. McLaughlin, "Performance analysis of compact antenna arrays with MRC in correlated Nakagami fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 1, pp. 267–277, Jan. 2001.

BIBLIOGRAPHY

- [89] M. Z. Win, G. Chrisikos, and J. H. Winters, "MRC performance for M-ary modulation in arbitrarily correlated Nakagami fading channels," *IEEE Commun. Lett.*, vol. 4, no. 10, pp. 301–303, Oct. 2000.
- [90] E. Kreyszig, *Advanced engineering mathematics*, Wiley, New York, 8th edition, 1998.
- [91] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*, Wiley, New York, 2000.
- [92] K. W. Yip and T. S. Ng, "Matched filter bound for multipath Rician-fading channels," *IEEE Trans. Commun.*, vol. 46, no. 4, pp. 441–445, Apr. 1998.
- [93] D. D. N. Bevan, V. T. Ermolayev, and A. G. Flaksman, "Coherent multi-channel reception of binary modulated signals with dependent Rician fading," *IEE Proc. Commun.*, vol. 148, no. 2, pp. 105–111, Apr. 2001.
- [94] Y. Ma and T. J. Lim, "Unified analysis of error probability for MRC in correlated fading channels," *IEE Electronic Lett.*, vol. 35, no. 16, pp. 1314–1315, Aug. 1999.
- [95] R. K. Mallik and M. Z. Win, "Error probability of binary NFDK and DPSK with postdetection combining over correlated Rician channels," *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 1975–1978, Dec. 2000.
- [96] G. L. Turin, "The characteristic function of Hermitian quadratic forms in complex normal variables," *Biometrika*, vol. 47, pp. 199–201, 1960.
- [97] M. S. Alouini and A. J. Goldsmith, "A unified approach for calculating error of linearly modulated signals over generalized fading channels," *IEEE Trans. Commun.*, vol. 47, no. 9, pp. 1324–1334, Sept. 1999.

BIBLIOGRAPHY

- [98] K. W. Yip and T. S. Ng, "A simulation model for Nakagami- m fading channels, $m < 1$," *IEEE Trans. Commun.*, vol. 48, no. 2, pp. 214–221, Feb. 2000.
- [99] Q. T. Zhang, "A decomposition technique for efficient generation of correlated Nakagami fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, no. 11, pp. 2385–2392, Nov. 2000.
- [100] J. Luo and J. R. Zeidler, "A statistical simulation model for correlated Nakagami fading channels," in *Proc. Int. Conf. Commun. Technol.*, 2000, vol. 2, pp. 1680–1684.
- [101] C. H. Sim, "Generation of poisson and Gamma random vectors with given marginal and covariance matrix," *Journal of Statistical Computation and Simulation*, vol. 47, no. 1&2, pp. 1–10, 1993.
- [102] B. Natarajan, C. R. Nassar, and V. Chandrasekhar, "Generation of correlated Rayleigh fading envelopes for spread spectrum applications," *IEEE Commun. Lett.*, vol. 4, no. 1, pp. 9–11, Jan. 2000.
- [103] S. D. Gordon and J. A. Ritcey, "Generating correlated Nakagami fading channels," in *Proc. Asilomar Conf. on Signals, Systems and Computers*, 1996, pp. 684–688.
- [104] M. D. Yacoub, J. E. Vargas, and L. G. R. Guedes, "On higher order statistics of the Nakagami- m distribution," *IEEE Trans. Veh. Technol.*, vol. 48, no. 3, pp. 790–794, May 1999.
- [105] N. D. Wallace, "Computer generation of gamma random variates with non-integral shape parameters," *Commun. ACM*, vol. 17, no. 12, Dec. 1974.

BIBLIOGRAPHY

- [106] S. S. Wilks, *Mathematical Statistics*, Wiley, New York, 1962.
- [107] M. H. Hsieh and C. H. Wei, "Channel Estimation for OFDM Systems based on Comb-Type Pilot Arrangement in Frequency Selective Fading Channels," *IEEE Trans. Consumer Electronics*, vol. 44, no. 1, pp. 217–225, Feb. 1998.
- [108] L. Li, M. Tulino, and S. Verdu, "Design of reduced-rank MMSE multiuser detectors using random matrix methods," *IEEE Trans. Info. Theo.*, vol. 50, no. 6, pp. 986–1008, June 2004.
- [109] E. Strom and S. L. Miller, "A reduced complexity adaptive near-far resistant receiver for DS-CDMA," in *Proc. IEEE Global Commun. Conf. (GLOBE-COM '93)*, Nov. 1993, pp. 1734–1738.

Appendix A

Investigation of Approximation Error in Higher Order Moments

As shown in Section 5.4.1.2, in the case of Nakagami fading diversity branches with unequal m_i/Ω_i ratios, the MRC output amplitude can be approximated as a Gamma RV whose m parameter can be obtained by matching the first and second moments of the approximated Gamma RV to the true MRC output defined in (5.19). Due to this approximation, some errors can be expected. In this appendix, we investigate the most significant approximation error computed in our proposed MRC model: the 3rd order moments.

The 3rd moment for a Gamma RV with parameter m and power Ω is given by

$$E[\gamma^3] = \frac{d^3}{dt^3} \mathcal{M}_\gamma(t)|_{t=0} = \Omega^3 \left(1 + \frac{3}{m} + \frac{2}{m^2} \right). \quad (\text{A.1})$$

If we approximate the MRC output γ as a Gamma RV with parameters m and Ω given by (5.32) and (5.21) respectively, the 3rd order moment of γ , as deduced from (A.1), is

$$E[\gamma^3]_{\text{appro}} = \left(\sum_{l=1}^{N_p} \Omega_l \right)^3 + 3 \sum_{l=1}^{N_p} \Omega_l \cdot \sum_{l=1}^{N_p} \frac{\Omega_l^2}{m_l} + 2 \left[\sum_{l=1}^{N_p} \frac{\Omega_l^2}{m_l} \right]^2 \left(\sum_{l=1}^{N_p} \Omega_l \right)^{-1}. \quad (\text{A.2})$$

The exact 3rd order moment for γ can be obtained from the MGF of γ in (5.25),

APPENDIX A. INVESTIGATION OF APPROXIMATION ERROR IN HIGHER ORDER MOMENTS

i.e.,

$$E[\gamma^3]_{\text{exact}} = \frac{d^3}{dt^3} \left[\prod_{l=1}^{N_p} \left(1 - \frac{\Omega_l}{m_l} t \right)^{-m_l} \right] \Big|_{t=0}. \quad (\text{A.3})$$

Hence the error or mismatch between these two 3rd order moments can be shown to be

$$\text{err}(E[\gamma^3]) = |E[\gamma^3]_{\text{exact}} - E[\gamma^3]_{\text{appro}}| = \frac{2}{\sum_{l=1}^{N_p} \Omega_l} \sum_{l=1}^{N_p} \sum_{i=1, i \neq l}^{N_p} \left[\Omega_l \Omega_i \left(\frac{\Omega_l}{m_l} - \frac{\Omega_i}{m_i} \right)^2 \right]. \quad (\text{A.4})$$

Equation (A.4) implies that the approximation error due to the 3rd order moment depends on the squared difference between the m_l/Ω_l ratios. Similar behavior can be observed for the approximation error due to higher order moments. Therefore large difference in m_l/Ω_l ratios gives rise to larger approximation error, while zero difference in m_l/Ω_l results in an exact Gamma model for the MRC output.

Appendix B

Reduction of (5.32) to (5.20)

As stated in Section 5.4.1.2, (5.32) is reduced to (5.20) when the m_l/Ω_l ratios are equal among all branches. The detailed proof is shown in this appendix.

Denoting

$$\frac{\Omega_1}{m_1} = \frac{\Omega_2}{m_2} = \dots = \frac{\Omega_{N_p}}{m_{N_p}} = a, \quad (\text{B.1})$$

Substituting $\Omega_l = a \times m_l$ into (5.32), we have

$$\begin{aligned} m &= \frac{(\sum_{l=1}^{N_p} a \times m_l)^2}{\sum_{l=1}^{N_p} \frac{(a \times m_l)^2}{m_l}} \\ &= \frac{a^2 \times (\sum_{l=1}^{N_p} m_l)^2}{a^2 \times \sum_{l=1}^{N_p} m_l} \\ &= \sum_{l=1}^{N_p} m_l. \end{aligned} \quad (\text{B.2})$$

Therefore, (5.32) is reduced to (5.20) when the m_l/Ω_l ratios are equal among all branches.