Optical solitons in passively mode-locked fiber ring lasers

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Optical Solitons in Passively Mode-locked Fiber Ring Lasers

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

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SUMMARY

Optical solitons have attracted especial attention due to their theoretical importance and potential practical applications in optical communications. Self-starting, passively mode-locked erbium-doped fiber lasers are promising sources of optical solitons for laboratory, telecommunication and signal-processing applications because of their simplicity, tunability and ultrashort pulse operation. Although this type of lasers have been intensively studied, many mechanisms are still not very clear. In this dissertation, the author presents a series of theoretical and experimental investigations on the soliton property, evolution, and interaction in a passively mode-locked erbium-doped fiber laser.

First of all, a passively mode-locked erbium-doped fiber soliton laser exploiting the nonlinear polarization rotation (NPR) technique was set up and the soliton characteristics of the laser were extensively investigated. Self-started mode locking was obtained in the laser simply by increasing the pump power and adjusting the polarization state of the light. Upon mode locking, stable soliton operations of the laser could be achieved. Comparing with the solitons formed in a single mode fiber, the solitons formed in the laser are of many new characteristics: sideband generation, pump power hysteresis, energy quantization, various modes of multiple soliton operation and so on. It was also found that depending on the specific polarization settings, three operation regimes: traditional solitons (single-pulse solitons), twin-pulse solitons, and noise-like mode-locked pulses, could be obtained in the laser.

In particular, twin-pulse solitons, as a novel form of solitons, were observed in the laser. The intensity profile of a twin-pulse soliton has two peaks with close and fixed peak separation, which is different to the traditional soliton whose intensity profile has only one peak. Intensive investigations on the properties of the twin-pulse solitons revealed that they have exactly all the same properties as those of the traditional single-pulse solitons. Energy quantization, pump power hysteresis,

harmonic mode locking, and soliton interaction of the twin-pulse solitons, have been observed in the laser.

To have a better understanding on the soliton properties of the laser, numerical simulations on the fiber laser have been carried out. A numerical model based on the coupled Ginzburg-Landau equations (CGLE), which takes into account not only the group velocity dispersion (GVD) and nonlinear effects but also the loss, gain, gain dispersion, and birefringence of fibers, was built up to simulate the operation of the laser. Particularly, to closely simulate the condition of a fiber laser, a pulse tracing technique was used in the calculation, in which both the pulse propagation in the fiber and the periodic discrete actions of the cavity components on the pulse are included. Using this model, all the operation characteristics of the fiber laser have been numerically demonstrated. In particular, the numerical simulations confirmed the existence of the twin-pulse solitons in the laser.

Soliton interaction in the fiber laser has also been systemically studied based on the experimental observations. Multiple solitons can coexist in the laser cavity and form various distribution modes such as the bound solitons, soliton bunch, and harmonic mode locking. Through carefully analyzing the experimental results, the existence of three types of soliton interaction, which play dominating roles on the soliton dynamics in a laser, have been identified: a global type of soliton interaction caused by the existence of unstable continuous wave (CW) emission; a type of long-range soliton interaction caused by the resonant sidebands; and the direct soliton interaction. Since different types of the soliton interaction have different interaction ranges and strengths, depending on the concrete laser operation conditions, they can exist either individually or concurrently, and consequently cause different modes of soliton operation.

Suggested by the average soliton theory, it was generally believed that the output of the laser is uniform soliton pulses. However, experimental studies on the laser output revealed that it is not always the case. Two distinct types of soliton pulse amplitude modulation have been observed in the laser under different polarization

Summary

settings: a low-frequency modulation caused by the NPR technique itself, and a high-frequency modulation introduced by the unstable dispersive waves. Both experimental observation and numerical analysis have shown that the soliton pulse nonuniformity is an intrinsic feature of passively mode-locked laser using the NPR technique. Stable uniform soliton pulses can only be obtained when the pump power is carefully controlled.

In addition, an interesting phenomenon, the Fermi-Pasta-Ulam (FPU) recurrence, which is characterized as the reversible behavior of the modulation instability (MI), was observed in the laser. With an appropriate laser parameter setting, CW lasing could coexist with the solitons in the laser. It is shown that when the strength of the CW component is strong, it becomes unstable due to MI and exhibits modulation sidebands in its spectrum. Benefiting from the ring cavity configuration of the laser, it was observed that the intensity of the MI sidebands changes periodically even with the fixed experimental conditions, demonstrating the FPU recurrence nature of the phenomenon.

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Chapter 1

Introduction

Solitons, as a special kind of nonlinear waves that can propagate undistorted on their pulse shapes, have been found in a wide range of nonlinear physical systems from fluids and plasma physics to optics, biological and atmospheric phenomena. Optical solitons have attracted especial attention in the last two decades due to their theoretical importance and potential practical applications in the optical communication and signal processing systems. In the context of optical fibers, solitons are very narrow, high-intensity optical pulses that can retain their shape through the balancing interaction between the anomalous fiber group velocity dispersion and the nonlinear Kerr effect. One of the most promising applications of the soliton theory is in the field of optic fiber communications [1-7]. Hasegawa and Tappert firstly predicted the existence of solitons in optical fibers and proposed the use of solitons for optic communications in 1973 [1]. Seven years later, Mollenauer et al. at the Bell Laboratories experimentally demonstrated the propagation of solitons in optical fibers [2], which opened up the intensive studies on optical fiber solitons that is still lasting now. Both the theory and experimental investigations on optical solitons have progressed rapidly. And various applications of optical solitons in optic communication and signal processing have been suggested and investigated [3-7].

A short pulse laser source is necessary for the generation of optical solitons. Mode locking is widely used in lasers to generate ultrashort pulses [8, 9]. In the case of a mode-locked fiber laser, apart from the cavity components that are necessary for achieving mode locking, the cavity is mainly made up of optical fibers. It is natural to expect that under certain conditions optical solitons can also be formed in the lasers. Indeed, soliton operation has been obtained in various ultrashort pulse mode-locked fiber lasers [10-14].

This chapter is intended to provide a basic overview on soliton theory and mode-locked fiber soliton lasers. Section 1.1 gives a brief review on the development of mode-locked fiber soliton lasers. The basic theories of mode locking and soliton generation are also provided. Section 1.2 presents the motivation and objectives of the research. Section 1.3 lists out the main contributions of the research. An overview of the dissertation is provided in section 1.4.

1.1 Development of mode-locked fiber soliton lasers

Since the low-loss silica fiber was available in 1970s, fiber lasers have attracted more and more attention due to their flexible, easy established structure, and various applications. Many different rare-earth ions, such as erbium (Er^{3+}), neodymium (Nd^{3+}), and ytterbium (Yb^{3+}) have been used to dope the normal fibers. These rare-earth-doped fibers then can be used as the gain medium in fiber lasers or amplifiers depending on the required operating wavelength range.

With the fast development of optical communications in the last two decades, the request for stable ultrashort pulse laser sources has become the focus of research. Mode-locked fiber lasers are capable of producing pulses with widths in a very wide range of from tens of fs to 1 ns and a wide repetition rate range of from less than 1MHz to 1THz. The versatility, as well as the flexible and compact size of optical fibers, is quite unique in laser technology, and thus open up mode-locked fiber lasers to a large range of applications. From a practical standpoint, mode-locked, erbium-doped fiber lasers (EDFLs) stand out since such lasers are capable of generating ultrashort optical pulses in the spectral range of 1.5 μm , which coincides with the low-loss region of silica fibers used for optical communications and fiber-based signal processing.

1.1.1 Mode locking and ultrashort pulse generation

The word “mode locking” firstly appeared in the paper of Hargroves et al. in 1964 [15]. It means the locking of multiple longitude modes in a laser cavity. By introducing a relatively weak modulation synchronous with the round trip time of the laser, the coherence between the phases of different modes could be realized, and pulsed radiation could be produced. The history of laser mode locking is a progression of new and better ways to generate shorter and shorter pulses, and of improvements in the understanding of mode-locking processes. Over the last four decades, mode locking has been used in all kinds of lasers to generate ultrashort pulses as short as less than 100 fs [9].

The theory of mode locking is very complicated. Here the author only gives the basic physics of mode locking. A large number of longitude modes can be stimulated simultaneously in the gain bandwidth of a laser provided that the pump is strong. The frequency spacing among the modes is given by

$$\Delta\nu = c/L_{opt} \quad (1.1.1)$$

where L_{opt} is the optical length during one round trip inside the cavity. Therefore, the total optical field can be written as:

$$E(t) = \sum_{m=-M}^M E_m \exp[i(\phi_m - \omega_m t)] \quad (1.1.2)$$

where E_m , ϕ_m and ω_m are the amplitude, phase and frequency of the specific modes among $2M+1$ modes permitted by the laser gain bandwidth. If all these modes operate independently of each other without definite phase relationship between them, the interference terms in the total intensity $|E(t)|^2$ averages out to zero. Under this situation, the laser works in a multimode CW state.

Mode locking occurs when the phases of various longitude modes are synchronized such that the phase difference between any two neighboring modes is locked to a constant value ϕ , viz. $\phi_m = \phi_0 + m\phi$. Since the mode frequency $\omega_m = \omega_0 + 2m\pi\Delta\nu$, if we assume for simplicity that all modes have the same amplitude E_0 , the total intensity can be analytically calculated and presented as:

$$|E(t)|^2 = \frac{\sin^2[(2M+1)\pi\Delta\nu t + \phi/2]}{\sin^2(\pi\Delta\nu t + \phi/2)} E_0^2 \quad (1.1.3)$$

The intensity shows as a periodic function with period $\tau_r = 1/\Delta\nu$, which is just the cavity round trip time. Under mode locking, the laser output is in the form of a pulse train with a repetition rate equals to $\Delta\nu$.

Under the modulation in the cavity, the initiated pulse is shortened every time it passes through the resonator. This shortening process continues until the pulse becomes so short and its spectrum so wide that the pulse lengthening or spectrum narrowing mechanisms, such as the finite bandwidth of the gain, spring to action. The pulse width is estimated from Eq. (1.1.3) as $\tau_p = [(2M+1)\Delta\nu]^{-1}$. Since the $[(2M+1)\Delta\nu]$ represents the total bandwidth of all mode-locked modes, the pulse width is inversely related to the spectral bandwidth over which phases of various modes can be synchronized. In practice, the exact relationship between the pulse width and the gain bandwidth depends on the nature of the gain broadening. In rare-earth doped fibers, the fiber characters such as the birefringence also affect the pulse width.

In general, there are two sorts of mode locking: active mode locking, and passive mode locking. Both methods have been used in fiber lasers to achieve ultrashort optical pulses.

1.1.2 Soliton generation in mode-locked fiber lasers

Soliton operation is an intrinsic feature of mode-locked fiber lasers with anomalous cavity dispersion and has been intensively studied [10-14].

In general, a pulse propagating in optical fibers is affected by both the group velocity dispersion (GVD) and the nonlinear optical Kerr effect. The GVD broadens the pulse in time domain, while the nonlinear self-phase modulation (SPM) broadens the pulse in frequency domain, which corresponds to narrowing the pulse in time domain. If these two effects on a pulse totally compensate with each other, it will maintain its pulse shape and pulse width during the propagation in the fiber i.e. form a soliton. The formation of solitons in optical fibers is described by the nonlinear Schrödinger Equation (NLSE) [16,17].

In a fiber laser, after mode locking, a mode-locked pulse is formed in the laser. If the peak power of the mode-locked pulse is strong, due to the nonlinear optical Kerr effect in the cavity, SPM occurs, which narrows the pulse width. If the strength of the SPM is strong enough that it can balance the pulse width broadening caused by the cavity dispersion, the pulse will propagate in the fiber without changing its pulse width. In fact, since the SPM is the pulse intensity dependent, as a pulse propagates in anomalous-dispersion fiber, it can adjust its intensity so that the effects induced by the optical Kerr effect and the anomalous GVD are automatically balanced. Namely, optical solitons is actually a generic property of mode-locked fiber lasers with anomalous cavity dispersion. Due to the existence of gain and loss, the soliton formation in a fiber laser is governed by the Ginzburg-Landau equation (GLE) [17,18].

Soliton operation can be achieved in both actively and passively mode-locked fiber lasers through the soliton shaping of the mode-locked pulses. Once the soliton operation is achieved in a laser, the pulse shape and pulse width are no longer determined by the mode locking mechanism, but by the soliton shaping. Comparing

with a conventional mode-locked pulse, solitons have narrower pulse width and therefore higher peak power.

1.1.3 Actively mode-locked fiber soliton lasers

Active mode locking is an efficient way to achieve laser mode locking. A typical arrangement for an integrated actively mode-locked fiber laser is shown in Fig. 1.1. An active modulator is necessary in actively mode-locked lasers to modulate either the amplitude or the phase of the intracavity optical field at a frequency that equals to integer multiples of the cavity longitude mode spacing [19-22]. As far as the modulation frequency is matched to the cavity length, active mode locking of fiber lasers can be easily obtained. With this mode locking technique, high repetition rate pulses with good noise performance can be generated. For instance, optical pulses of less than 6 ps duration generated at repetition rate to 40 GHz have been reported in an actively mode-locked erbium-doped fiber ring laser [23].

Figure 1.1 Typical experimental setup of an actively mode-locked fiber laser

Without soliton shaping an actively mode-locked fiber laser produces mode-locked pulses of Gaussian pulse profile, whose width is determined by the laser gain

bandwidth, and the modulation frequency and strength [19]. For the EDFLs this is normally in the range of several tens of picoseconds. If the peak power of the pulses is strong enough, soliton shaping happens and shortens the mode-locked pulse as well as changes the pulse profile from a Gaussian shape to a Sech shape [24,25]. However, comparing with the passively mode-locked fiber lasers, soliton operation cannot be easily achieved in actively mode-locked fiber lasers. The main reason for the difficulty is that harmonic mode locking is normally implemented. In a mode locked state too many mode-locked pulses are generated in the laser cavity, and they share the laser cavity energy. The energy of each mode locked pulse is therefore weak. And determined by the mode-locking technique, the mode-locked pulse width is initially broad, which leads to that the peak power of the pulse achievable is also low. Therefore, only very weak nonlinear SPM could be actually generated by the pulses, which limits the strength of their soliton shaping. To obtain ultra narrow soliton pulses in the laser either strong pumping and/or low repetition rate operation are necessary. Actively mode-locked fiber lasers also suffer from the pulse drop-out problem [26], which could also be traced back to the gain competition between the pulses.

1.1.4 Passively mode-locked fiber soliton lasers

Passive mode locking is an alternative to produce ultrashort optical pulses in lasers. It makes use of a nonlinear device whose response to the entering optical pulse is intensity dependant so that the optical pulse is shortened every time passing through it. A semiconductor saturable absorber [27-31] or a Sagnac loop mirror [35-43] is often used for this purpose. Comparing with actively mode-locked fiber lasers, passively mode-locked fiber lasers can produce much shorter and more intense optical pulses as well as keep a relatively low component count simultaneously. As a result, soliton operation can be easily obtained after mode locking in a passively mode-locked fiber laser. Passively mode-locked fiber soliton lasers (PMLFSLs) have some special features, e.g. the sideband generation, pump power hysteresis, energy quantization [40], multiple soliton operation [41] and passive harmonic mode locking [42,43].

Type 1. Semiconductor saturable absorber mode-locked fiber laser

Saturable absorbers are widely used for passively mode locking lasers. The mode locking technique bases on the mutual interaction of light with the laser gain medium and the saturable absorber. Therefore, properties of the saturable absorber such as the recovery time, saturable absorption strength play an important role on the quality of the mode locking. For mode-locked fiber lasers using this technique, the most suitable saturable absorbers are the specially designed semiconductor structures. A single layer of InGaAsP grown on an InP substrate [27], a multiple-quantum-well semiconductor amplifier [28], or a saturable Bragg reflector based on InGaAs/GaAs multi-quantum well [29-31] has been used to passively mode locking the EDFLs. Figure 1.2 shows a typical laser configuration of an erbium-doped fiber soliton laser mode locked by using a semiconductor saturable Bragg reflector.

Figure 1.2 Schematic configuration of a fiber laser mode locked using a semiconductor saturable absorber

Soliton operation of such lasers has been observed by several authors [30,31]. As the relaxation times of semiconductor saturable absorbers are generally quite long, limited by the saturable absorber recovery time, the conventional mode-locked pulses have broad pulse width. However, with the soliton shaping, pulse width is no longer determined by the recovery time of the absorber, but by the soliton effect in the laser. In this case the pulse width significantly narrower than the absorber recovery time becomes possible. After a soliton is formed in the cavity, the function of the saturable absorber turns to stabilize the soliton through suppressing the

background noise. Like the soliton operation of other fiber lasers, here again the saturable absorber initiates the mode locking in the lasers.

Apart from mode locking, semiconductor saturable absorbers were also used for the timing stabilization of harmonically mode-locked fiber lasers [31]. It was suggested that the phase effects in semiconductor saturable absorber could lead to pulse repulsion, which provides self-organization of the pulse repetition rate. However, for the application the recovery time and saturation energy density of the saturable absorbers must be carefully designed.

Type 2. Additive pulse mode-locked fiber laser

In the case of passive mode locking, a certain intensity dependent loss or dispersion mechanism must be used to favor the pulsed operation over continuous radiation. A simple method to compress a pulse is to subtract some energy from the pulse wings and transfer it to the pulse center. Additive pulse mode locking (APM) is an implementation of this idea, where the output pulse is re-injected into the laser after a phase modulation such that the pulse center and leading edge add in phase with the intra-cavity pulse, and the pulse tail out of phase [32-34].

In the past few years, fiber lasers using the APM have been widely investigated as potentially compact sources for ultrashort optical pulses. A successful integrated source using APM is the figure-of-eight laser [35-43], in which the passive mode locking is achieved based on the nonlinear amplifying loop mirror (NALM). Another attractive technique for fiber APM sources is the nonlinear polarization rotation (NPR) [44-56]. Low-birefringence fibers are always required in the laser systems using APM.

- **Figure-of-eight laser**

The first successful integrated source of ultrashort soliton pulses using APM is the figure-of-eight laser, first reported by Duling [35] and Richardson in 1991 [36].

Figure 1.3 shows a typical configuration of the laser. Because of its appearance, this configuration is referred to as figure-of-eight laser.

Figure 1.3 Schematic configuration of a figure-of-eight laser

The ring cavity on the left acts as a nonlinear amplifying loop mirror (NALM), whose transmission is intensity dependant and can shorten the optical pulses just as a saturable absorber does. [35-37]. Self-started mode locking is initiated in the lasers through the nonlinear switching of the NALM. When light comes to the 3dB coupler, it is split into two equal counter-propagating parts. As the doped fiber providing amplification is placed close to the 3dB coupler, one part is amplified at the entrance to the loop while the other part experiences the amplification just before leaving the loop. As the two parts have different intensity as they propagate in the loop, therefore, they acquire different nonlinear phase shifts at completing the propagation in the NALM loop. The two parts meet again at the 3dB coupler and interfere with each other. Depending on the nonlinear phase difference light will be reflected or pass through the NALM. If the phase difference is close to π , the light passes through the NALM. Otherwise partial of the light is reflected. For a pulse, the phase difference is not a constant but varies along the pulse profile. If the NALM is adjusted so that the phase shift for the central intense part of pulse is around π , this part will be transmitted, while the low intensity pulse wings will get

reflected due to smaller nonlinear phase shifts. With this mechanism an artificial saturable absorber effect is achieved and mode locking can be realized.

If the mode-locked pulses have high peak power, soliton shaping occurs, and soliton operation of the lasers is then obtained. Depending on the cavity design, soliton pulses with different pulse widths such as 290 fs [38], 320 fs [36], and 1.3 ps [39] have been reported using this configuration of lasers. Multiple soliton operation was also found to be a generic property of the lasers. Under the multiple soliton operation solitons either form soliton bunches, or scatter randomly in the whole cavity with fixed soliton separations [40,41]. Passive harmonic mode locking of the soliton pulses was also observed in such lasers [42,43].

- **Passively mode-locked fiber ring laser using nonlinear polarization rotation**

Another attractive technique for APM in fiber lasers is the nonlinear polarization rotation (NPR). Nonlinear birefringence of optical fibers has been previously used for the polarization switching and optical pulse shaping [44,45]. Hoffer et al firstly exploited NPR as a possible self-sustaining mechanism for passive mode locking [46]. But owing to the short interaction lengths and the relatively high-birefringence fibers used, at the very beginning, mode locking could only be initiated by the use of an active modulator. With the development of fiber technology, the first successful self-started passively mode-locked fiber laser using this technique was demonstrated by Matsas et al in 1992 [47]. Figure 1.4 provides a schematic design of a fiber ring laser passively mode locked by using this NPR technique.

The principle of the technique is quite simple. When a linearly polarized light is incident to a piece of weakly birefringent fiber, the polarization of the light will generally become elliptically polarized in the fiber. The orientation and ellipticity of the final light polarization is fully determined by the fiber length and its birefringence. However, if the intensity of the light is strong, the nonlinear optical Kerr effect in the fiber must be considered, which introduces extra changes to the

light polarization [48, 49]. As the polarization change introduced by the optical Kerr effect depends on the light intensity, if a polarizer is put behind the fiber, the light intensity transmission through the polarizer will become light intensity dependent. Through appropriately selecting the orientation of the polarizer or the length of the fiber, an artificial saturable absorber effect with ultra-fast response could then be achieved in such a system, where light of higher intensity experiences less absorption loss on the polarizer. The NPR technique makes use of this artificial saturable absorption to achieve the passive mode locking in a fiber laser. Once a mode-locked pulse is formed, the nonlinearity of the fiber further shapes the pulse into an optical soliton and consequently the ultrashort soliton operation is obtained in the laser. Soliton operation is almost a generic feature of the fiber lasers mode-locked by this technique and has been intensively investigated [47,50-52].

Figure 1.4 Schematic configuration of a passively mode-locked fiber ring laser exploiting the nonlinear polarization rotation.

Considerable efforts have been made to understand and improve fiber lasers making use of NPR for self-starting passive mode locking. In 1993, Tamura et al obtained 77 fs pulses with a high energy of 90 pJ at the repetition rate of 45 MHz in a NPR mode-locked erbium-doped fiber ring laser by using a section of large positive dispersion fiber in the cavity [53]. The laser is referred to as stretched-pulse or dispersion-managed fiber laser since pulses circulating inside the cavity stretch significantly in the piece of fiber with normal GVD. As the dispersion management

technique could significantly reduce the output pulse width, this kind of fiber lasers are intensively investigated [54-56].

1.2 Motivation and objectives

Due to their intrinsic stability in propagation, optical solitons have been proposed to use as information carrier for the long-distance optic fiber communications. Self-starting passively mode-locked erbium-doped fiber lasers, as kind of attractive sources for ultrashort soliton pulses for their simplicity, tunability and ultrashort pulse operation, have been intensively studied. Although the basic principles of the operation of a PMLFSL are generally understood, many features of the laser still remain to be investigated.

Soliton operation is a natural consequence of mode-locked fiber lasers in the anomalous cavity dispersion regime under strong pumping. Due to the balance effect between the GVD and SPM, soliton operation is ready to be obtained after the mode locking of the fiber laser. However, as in a fiber laser, besides the optical fibers, there also exist other components e.g. the gain medium and the output coupler, whose existence affects the detailed dynamics of the formed solitons. Therefore, compared with the solitons formed in single-mode fibers, the solitons obtained in a fiber laser are of new characteristics, which need to be further investigated.

Generally, the soliton formation in optical fibers is governed by the NLSE, and the soliton pulses have a single-peak Sech-form intensity profile. Recently, a special bound soliton state with fixed and very close separation is observed in a fiber laser, which caused a lot of interest [57]. However, the property and formation mechanism of this state need to be further investigated.

A major limitation for the soliton applications is the soliton interaction. To understand the nature and consequence of soliton interaction, extensive theoretical

and experimental investigations have been carried out [58-65]. In the framework of the solitons described by the NLSE, two sorts of soliton interaction have been revealed: the direct soliton interaction and the long-range dispersive waves mediated soliton interaction. Gordon has firstly studied the direct soliton interaction in optical fibers [58]. He found that solitons exert forces on their neighbors if they are closely spaced. Either attractive or repulsive interaction can be resulted in depending on the relative phase difference between the solitons. However, the strength of the force decreases exponentially with the soliton separation. Direct soliton interaction can be greatly reduced in transmission links by the use of band-pass filters or fast saturable absorbers [59,60]. Apart from the direct soliton interaction, Smith and Mollenauer reported the observation of a long-range soliton interaction caused by the existence of dispersive waves [61]. Compared with the direct soliton interaction, this long-range soliton interaction is soliton phase independent, but it exists even when solitons are separated far apart.

However, a real long-distance soliton fiber optic communication system also involves in the soliton losses and periodic amplifications. Therefore, the dynamics of solitons in the system is not described by the NLSE, which is integral and describes only the conservative systems, but by the Ginzburg-Landau equation (GLE), where soliton is formed not only as a result of the balanced interaction between the fiber Kerr nonlinearity and dispersion, but also as a result of the balance between fiber losses and gain generated by the fiber amplifiers. As a soliton propagating in the system periodically experiences loss and amplification, dispersive waves with discrete spectra are generated, which resonantly draw energy from the soliton. Socci and Romagnoli have analyzed the soliton interaction induced by these resonant dispersive waves [62]. They have shown that the resonant dispersive waves could result in a long-range soliton interaction as well as enhance the direct soliton interaction. Either bound states of solitons or random wiggling of the soliton positions could be caused by the resonant dispersive wave mediated soliton interaction. Afanasjev et al [63,64], Malomed [65-67] have also theoretically investigated interactions between the solitons described by the GLE. They have found that bound states of solitons with fixed, discrete soliton separations could be

formed under either the direct soliton interaction or the dispersive waves mediated soliton interaction.

A fiber laser is also periodical gain-and-loss system, in which, an optical pulse periodically experience the amplification of the fiber amplifier and the output loss. In this sense, a passively mode-locked soliton fiber ring laser can be regarded as a miniature of a soliton optic fiber communication system. Study on the soliton propagation and interaction in the laser cavity therefore gives a direct insight into the soliton interactions in the long-haul optic communication systems. So far soliton operation in passively mode-locked fiber lasers has been extensively investigated. Various modes of soliton operation such as soliton bunching, stable randomly spaced soliton distribution, harmonic mode locking, and transient stochastic soliton evolutions have been experimentally observed. Theoretical studies on the possible physical mechanisms for the formation of the various soliton operation modes have also been carried out by some authors, e.g. Kutz et al have theoretically investigated the effect of laser gain depletion and recovery on the soliton distribution in the lasers [68], Pilipetskii et al studied the fiber acoustic effect on the soliton interaction in a fiber soliton laser [69]. There is no doubt that these effects always exist in a fiber soliton laser and affect the soliton dynamics. However, how strong are their influences? Do they play a dominating role on the soliton dynamics in a laser? Does any other soliton interaction exist, which plays an even stronger role on the soliton operation of a laser? And how are these different soliton operation-modes formed in a laser? All these questions have so far not been clearly addressed. To clarify them is not only important for understanding the soliton operation of the passively mode-locked fiber soliton lasers, but also potentially useful for the future application of solitons in the ultra-high-bit-rate optical communication systems.

Another interesting topic is the soliton pulse evolution. Although in a fiber laser due to the influence of the laser gain and output, a pulse circulating in the cavity periodically changes its intensity, it was theoretically demonstrated that at a fixed position in the cavity the pulse has a fixed pulse profile and pulse intensity, which is invariant of the time, namely, a soliton. This property of solitons formed in the fiber

laser is known as the average soliton [70]. Suggested by the average soliton theory of the laser, it was generally believed that the output of the lasers is uniform soliton pulses. Until very recently, Kim and Spaulding et al theoretically analyzed the pulse dynamics of the lasers passively mode-locked by using the NPR technique [71, 72]. They found that due to the interaction between the nonlinear polarization rotation and the passive polarizer inside the cavity, there is an intrinsic intensity fluctuation of the output soliton pulses unless the polarizer is aligned with either the slow or the fast axis of the fiber. To our knowledge, so far this theoretical prediction has not been experimentally confirmed. In addition, it is noticed that for the sake of simplicity, in their analysis Kim et al. have ignored the effect of resonant sideband generation of solitons in the laser. But in practice this effect always exists and significantly affects the properties of the solitons. So in what extent their theoretical prediction describes the real soliton pulse dynamics in the lasers needs to be further studied.

The main objective of this research is to investigate the properties of optical solitons in passively mode-locked fiber lasers. In experiment, a passively mode-locked erbium-doped fiber ring laser will be set up for experimental investigation. In theory, the pulse propagation and soliton formation in a fiber laser will be investigated. Numerical simulations on the laser system will also be carried out to get a better understanding on the operation of the laser. In particular, some important topics on optical solitons such as the multi-pulse solitons, soliton interaction, and soliton pulse nonuniformity will be emphasized in the study based on the experimental and numerical investigations.

1.3 Main contributions of the research

First of all, a passively mode-locked fiber laser using the NPR technique has been built up; various operation features of the laser have been examined. Three operation regimes of the laser have been obtained and investigated. Soliton

operation, as an intrinsic feature of the laser have been achieved and detailedly studied.

A numerical model based on the coupled GLEs has also been built up to simulate the laser system. Different to the models used by other groups, the so-called master-equation simplification was not made in this model. Instead, a pulse tracing technique is used in the numerical simulations. With the help of the numerical model, the features of the laser operation have been precisely reproduced.

A novel form of solitons, twin-pulse solitons were observed and investigated in the fiber laser. The properties of this soliton state were intensively studied for the first time. It was found that except that their intensity profiles have two peaks, the twin-pulse solitons have all the same properties with those of the traditional single-pulse solitons. Numerical calculation reproduced the existence of the twin-pulse solitons as another intrinsic operation state of the fiber laser.

Soliton interaction has been systemically studied in the fiber laser for the first time. Based on the experimental observations, three types of soliton interaction with different impacts have been identified and detailedly investigated. It is demonstrated that the formation of the various multiple soliton modes in a fiber laser could be well explained as the results of different types of soliton interactions. Numerical simulations on soliton interaction were also processed. Bound solitons, soliton bunch, soliton collision, and soliton oscillation have been numerically demonstrated.

Periodical amplitude variations on the output soliton pulses have been firstly observed in the fiber laser. Unlike the condition anticipated by the average soliton theory, experimental study on the practical laser output revealed that the soliton pulses of the laser are not always uniform. It was found that only under appropriate pump level, stable uniform soliton output could be obtained. While, for other cases, the soliton peak power is slowly modulated. Two types of soliton pulse modulations, one caused by the NPR technique itself, and the other introduced by

the unstable dispersive waves, have been observed. Numerical calculation reproduced the experimental observations. Both the experimental and the numerical results show that the nonuniformity of soliton pulses is actually an intrinsic phenomenon for NPR mode-locked fiber lasers.

Finally, as a byproduct, the Fermi-Pasta-Ulam (FPU) recurrence has been firstly observed in the fiber laser. It is shown that when the strength of the CW component is strong, it becomes unstable due to MI and exhibits modulation sidebands in its spectrum. Benefiting from the ring cavity configuration of the laser, it is observed that the intensity of the MI sidebands changes periodically even with the fixed experimental conditions, demonstrating the FPU recurrence nature of the phenomenon.

1.4 Overview of the dissertation

The dissertation is organized as follows:

Chapter 1 brings out as an introduction of the research.

Chapter 2 presents the theoretical background of the pulse propagation in optical fibers. Major fiber characteristics that affect the pulse evolution in an optical fiber including the GVD, fiber nonlinearity, and modal birefringence are discussed. As an important factor, the gain and gain dispersion in erbium-doped fibers are also introduced. Equations that describe the pulse evolution in a real gain and loss fiber system, e.g. a fiber laser, are derived out step by step in this chapter.

Chapter3 demonstrates the experimental investigations on a passively mode-locked fiber soliton laser using the NPR technique. The mechanism of the NPR technique and its implementations in fiber lasers is discussed in detail. Three operation regimes of the laser are obtained, and their respective characteristics are studied.

Chapter 4 demonstrates the numerical investigations on the passively mode-locked fiber soliton laser. A numerical model based on the coupled complex GLEs is built up to simulate the operation of the laser.

Chapter 5 presents the experimental observation on the Fermi-Pasta-Ulam (FPU) recurrence of the MI of the CW component in the laser cavity.

Chapter 6 addresses the study on soliton interaction. Three types of solitons interactions are identified and discussed in the laser. Different modes of multiple soliton operation are interpreted as the effects of various soliton interactions.

Chapter 7 provides the observation of amplitude modulations on the soliton pulses of the laser. Two distinct types of soliton pulse modulation are observed and investigated. The mechanism of the soliton pulse nonuniformity is also studied.

Chapter 8 concludes the dissertation, summarizes the achievements of the research and gives recommendations for future work.

Chapter 2

Theory of pulse propagation in optical fibers

The envelope of a light wave in a single-mode optical fiber is deformed by the dispersive (variation of the group velocity as a function of the frequency) and nonlinear (variation of the phase velocity as a function of the wave intensity) properties of the fiber. The dispersive property of the light wave envelope is decided by the group velocity dispersion (GVD). And the nonlinear properties of the light wave envelope are determined by a combination of Kerr effect (an effect of the increase in the refractive index n in proportion to the light intensity) and stimulated Brillouin and Raman scattering. In a fiber laser, the gain and gain dispersion introduced by the erbium-doped fiber amplifier will also complicate the pulse propagation. In addition, the fiber birefringence induced polarization evolution is another important aspect that must be taken into account when analyzing the pulse propagation in practical fiber systems. For a good understanding of the soliton operation in fiber lasers, it is necessary to first consider the theory of wave propagation in fibers in the presence of these effects.

The objective of this chapter is to derive the basic equations that govern the nonlinear pulse propagation in single-mode fibers. Section 2.1 introduces the group velocity dispersion in fibers and its effect on the pulse propagation. Section 2.2 provides the theory of nonlinear pulse propagation in optical fibers. In this section, the Nonlinear Schrödinger Equation is derived and the fundamental soliton solution of this equation is given. Section 2.3 discusses the gain property introduced by the erbium-doped fibers. By include the gain and gain dispersion, the pulse propagation is then described by the Ginzburg-Landau equation. Finally, the polarization characters introduced by the fiber birefringence are discussed in Section 2.4.

2.1 Linear pulse propagation in optical fibers

2.1.1 Group velocity dispersion in optical fibers

When an optical pulse propagates within an optical fiber, it becomes distorted due to intramodal (i.e. chromatic) dispersion and intermodal delay effects [16, 17]. The distortion effects can be explained by analyzing the group velocities of the guided modes. Group velocity is the speed at which energy in a particular mode travels along the fiber. For an optical signal that normally contains a finite number of frequency modes, a phenomenon known as the group velocity dispersion (GVD) occurs. GVD, also referred to as the chromatic dispersion, exhibit as pulse spreading that occurs within a single-mode fiber (SMF), which arises due to the finite spectral width of the optical signal. This phenomenon is called GVD since the dispersion is a result of the group velocity being a function of the wavelength.

Since the refractive index of the medium is frequency dependent, which can be written as $n(\omega)$, chromatic dispersion always exists with the propagation of light in fibers. For CW light or pulse with big pulsewidth (usually $T_0 > 100$ ps), this dispersion can be ignored. But in the case of short pulses, especially for the ultrashort pulses as those presented in this dissertation, fiber dispersion plays a critical role because different spectral components associated with the pulse travel at different speeds given by $c/n(\omega)$. Because of dispersion, a pulse will broaden with the propagation in fibers, which can be detrimental for optical communication systems.

On a fundamental level, the origin of dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons. The refractive index can be well approximated by the Sellmeier equation at frequencies far from the material resonances:

$$n^2(\omega) = 1 + \sum_{j=1}^m \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2}, \quad (2.1.1)$$

where ω_j is the resonance frequency and B_j is the strength of j th resonance. The sum in Eq. (2.1.1) extends over all material resonances that contribute to the frequency range of interest. For silica, a three-term Sellmeier equation is typically used, accounting for the resonances in the ultraviolet and infrared.

$$n^2(\omega) = 1 + \frac{B_1 \omega_1^2}{\omega_1^2 - \omega^2} + \frac{B_2 \omega_2^2}{\omega_2^2 - \omega^2} + \frac{B_3 \omega_3^2}{\omega_3^2 - \omega^2} \quad (2.1.2)$$

Mathematically, the effects of fiber dispersion are accounted for by expanding the mode-propagation constant β in a Taylor series about the frequency ω_0 at which the pulse spectrum is centered:

$$\beta(\omega) = n(\omega) \frac{\omega}{c} = \beta_0 + \beta_1 (\omega - \omega_0) + \frac{1}{2} \beta_2 (\omega - \omega_0)^2 + \dots \quad (2.1.3)$$

where

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m} \right)_{\omega=\omega_0} \quad (m = 0, 1, 2, \dots) \quad (2.1.4)$$

The parameters β_1 and β_2 are related to the refractive index n and its derivatives through the relations

$$\beta_1 = \frac{1}{v_g} = \frac{n_g}{c} = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right), \quad (2.1.5)$$

$$\beta_2 = \frac{1}{c} \left(2 \frac{dn}{d\omega} + \omega \frac{d^2 n}{d\omega^2} \right), \quad (2.1.6)$$

where n_g is the group index and v_g is the group velocity. Physically speaking, the envelope of an optical pulse moves at the group velocity while parameter β_2 represents dispersion of the group velocity and is responsible for pulse broadening. β_2 is referred to as the group velocity dispersion (GVD) parameter.

Figure 2.1 shows how β_2 vary with wavelength λ in fused silica [16].

Figure 2.1 Group velocity dispersion β_2 for fused silica as a function of wavelength (After Ref. [16]).

In the figure, β_2 vanishes at the wavelength of about $1.27\mu\text{m}$, which is referred to as the zero-dispersion wavelength and is denoted as λ_D . For wavelengths such that $\lambda < \lambda_D$, the fiber is said to exhibit normal dispersion as $\beta_2 > 0$. In the normal dispersion regime, the higher frequency (blue shifted) components of an optical pulse travel slower than lower frequency (red-shifted) components. By contrast, the fiber shows anomalous dispersion as $\beta_2 < 0$ when the light wavelength exceeds the zero-dispersion wavelength ($\lambda > \lambda_D$). And in this regime, the blue-shifted components of an optical pulse travel faster than red-shifted components. The anomalous-dispersion regime is of considerable interest for the study of solitons. Because it is

in this regime that optical fibers support solitons through a balance between the dispersive and nonlinear effects.

2.1.2 Pulse propagation in dispersive media

Basically, like all electromagnetic phenomena, the propagation of optical fields in the media is governed by Maxwell's equations. Then deriving from the Maxwell's equations, one can get the fundamental wave equation

$$\nabla \times \nabla \times \bar{E} = -\frac{1}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \bar{P}}{\partial t^2}, \quad (2.1.7)$$

where $\bar{E}$ and $\bar{P}$ are electric field vector and the induced electric polarization, respectively. c is the speed of light in vacuum and has a relation $1/c^2 = \mu_0 \epsilon_0$, where μ_0 is the vacuum permeability and ϵ_0 is the vacuum permittivity.

Before solving the wave equation, two simplifications are made. First, by just considering linear polarized light propagation in the media, the vectors in Eq. (2.1.7) can be replaced by scalars. Second, by ignoring the nonlinear effects temporarily in this section, Eq. (2.1.7) is linear and can be easily converted to the frequency domain.

Using the two simplifications and also adopting the operation

$$\nabla \times \nabla \times \bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}, \quad (2.1.8)$$

one can easily get the wave propagating equation in frequency domain:

$$\nabla^2 E(z, \omega) + n^2(\omega) \frac{\omega^2}{c^2} E(z, \omega) = 0 \quad (2.1.9)$$

where $E(z, \omega)$ is the Fourier transform of $E(z, t)$ defined as:

$$E(z, \omega) = \int_{-\infty}^{\infty} E(z, t) \exp(i\omega t) dt \quad (2.1.10)$$

In addition, the paraxial approximation is also implied in Eq. (2.1.9), which is applicable and reasonable in laser optics by replace $\bar{r}$ with z along which the light propagates.

Next, another function $F(z, \omega)$ is brought in to further simplify the wave equation. $F(z, \omega)$ is assumed to be a slowly varying function of z .

$$F(z, \omega) = E(z, \omega) \exp[-i\beta(\omega)z] \quad (2.1.11)$$

Therefore, the wave equation (2.1.9) can be further simplified as:

$$\frac{\partial F(z, \omega)}{\partial z} = -i\beta(\omega)F(z, \omega) , \quad (2.1.12)$$

where $\beta(\omega) = n(\omega)\frac{\omega}{c}$ is the propagation constant varying with frequency ω . In obtaining Eq. (2.1.12), the second derivative $\partial^2 F / \partial z^2$ is neglected since $F(z, \omega)$ is assumed to be a slowly varying function of z .

Now let's go on to study the propagation equation of an optical pulse. The electric field of an optical pulse can be generally written as:

$$f(z, t) = E(z, t) \exp\{i[\omega_0 t - \beta(\omega_0)z]\} , \quad (2.1.13)$$

where $E(z, t)$ is the pulse envelop, which contains both the amplitude and phase information. ω_0 is the central frequency of the pulse and $\beta(\omega_0)$ is the propagation

constant at the frequency of ω_0 . Correspondingly, in the frequency domain, an optical pulse can be described by the Fourier transform of $f(z, t)$ and written as:

$$F(z, \omega) = E(z, \omega - \omega_0) \exp[i\beta(\omega_0)z] \quad (2.1.14)$$

The function of $F(z, \omega)$ obeys the conventional transmission relation of Eq. (2.1.12)

If the pulse width is big enough so that the dispersion effect can be ignored, each spectral component of the pulse at ω can be well approximated to propagate with the same phase constant $\beta(\omega_0)$. But when the width of an optical pulse τ_p is so short that the spectral bandwidth $\delta\omega \propto \frac{1}{\tau_p}$ is a significant fraction of ω_0 , one must account for the fact that each spectral component of $F(z, \omega)$ has its own phase constant $\beta(\omega)$. In this condition, by substituting Eq. (2.1.14) and Eq. (2.1.3) into Eq. (2.1.12), one can finally obtain an equation describing the scalar envelop function $E(z, \omega - \omega_0)$:

$$\frac{\partial E(z, \omega - \omega_0)}{\partial z} = -i\delta\beta E - i\beta_1 E - i\frac{\beta_2}{2}(\omega - \omega_0)^2 E \quad (2.1.15)$$

Since a perturbation induced by the amplitude of the field itself usually exists (i.e. $n=n(\omega)+\Delta n$), the factor $\delta\beta$ is imported to denote the perturbation it induced on β (i.e. $\beta=\beta(\omega)+\delta\beta$).

At this point, one can go back to the time domain by taking inverse Fourier transform of Eq. (2.1.15), and then obtain the propagation equation for $E(z, t)$:

$$\frac{\partial E(z, t)}{\partial z} + \beta_1 \frac{\partial E(z, t)}{\partial t} = i\frac{\beta_2}{2} \frac{\partial^2 E(z, t)}{\partial t^2} - i\delta\beta E(z, t) \quad (2.1.16)$$

In practice, a synchronous coordinate system moving at the group velocity v_g is always used to make Eq. (2.1.16) even simpler in form. The transformation between the original and synchronous coordinate systems is as:

$$\begin{cases} \tau = t - \frac{z}{v_g} \\ z' = z \end{cases} \quad (2.1.17)$$

Finally, the propagation equation of an optical pulse can be reformed as:

$$\frac{\partial E(z', \tau)}{\partial z'} = i \frac{\beta_2}{2} \frac{\partial^2 E(z', \tau)}{\partial \tau^2} - i \delta \beta E(z', \tau) \quad (2.1.18)$$

Eq. (2.1.18) describes the propagation of a short pulse in dispersive media. It will be further used to acquire pulse propagation in single-mode fibers in the next section of this chapter.

Based on Eq. (2.1.18), one can get some features of pulse propagation in dispersive media. If the GVD parameter $\beta_2 = 0$, i.e. there is no group velocity dispersion, then

$$\frac{\partial E}{\partial z'} = 0 \quad (2.1.19)$$

Therefore, any envelope function can propagate any distance without distortion. While unfortunately, the only medium where $\beta_2 = 0$ is a vacuum.

As $\beta_2 \neq 0$, then the pulse shapes become distorted as they propagate in the media. In practice, GVD always plays an important role in pulse propagation, especially in ultrashort pulse propagation.

2.2 Nonlinear pulse propagation in optical fibers

2.2.1 Nonlinear effects in optical fibers

Besides the frequency dependence, the refractive index of the optical media is always intensity dependent. Hence when the peak power of a pulse is very strong, an interesting manifestation of that occurs through self-phase modulation (SPM) arising from the Kerr nonlinearity. This phenomenon leads to the spectral broadening of an optical pulse [16,17].

On a fundamental level, the origin of nonlinearity is related to the nonlinear response of the electric dipolar polarization $\bar{P}$ under the influence of an applied electric field $\bar{E}$:

$$\bar{P} = \epsilon_0 \left(\chi^{(1)} \cdot \bar{E} + \chi^{(2)} : \bar{E}\bar{E} + \chi^{(3)} : \bar{E}\bar{E}\bar{E} + \dots \right), \quad (2.2.1)$$

where ϵ_0 is the vacuum permittivity, and $\chi^{(j)}$ ($j=1,2,\dots$) is the j th order susceptibility. The linear susceptibility $\chi^{(1)}$ represents the dominant contribution to $\bar{P}$, whose effects are included through the refractive index n . The second-order susceptibility $\chi^{(2)}$ is caused by the asymmetry in molecular structure, which is responsible for nonlinear effects such as the second-harmonic generation (SHG) and sum-frequency generation (SFG). Since SiO_2 has a symmetric molecular structure, $\chi^{(2)}$ vanishes for silica glasses. Therefore, in the case of a silica fiber, the lowest-order nonlinear effects originate from the third-order susceptibility $\chi^{(3)}$. These third-order nonlinear effects include the third-order harmonic generation, four-wave mixing, and nonlinear refraction. However, as the third-harmonic generation and the four-wave mixing both need strict phase matching condition, they are not efficient in single-mode fibers. Most of the nonlinear effects in optical fibers such as the optical Kerr effect, stimulated Raman and Brillouin scattering, originate from nonlinear refraction, which can automatically occur provided that the light intensity reach

some thresholds. In its simplest form, the nonlinear refractive index can be written as:

$$n(\omega, |E|^2) = n(\omega) + n_2 |E|^2, \quad (2.2.2)$$

where $n(\omega)$ is the linear part given by Eq. (2.1.1), $|E|^2$ is the optical intensity inside the fiber, and n_2 is the nonlinear-index coefficient determined by $\chi^{(3)}$. Normally, n_2 of optical fibers has a very small value, which is in the range of $2.2 \sim 3.4 \times 10^{-20} \text{ m}^2/\text{W}$ [73]. However, because the core of SMFs is very small, which is normally $< 10 \text{ }\mu\text{m}$ in diameter, and the light is confined to propagate in the fiber core with almost ignorable loss, a very long distance of light-matter interaction is thus possible, which in consequence makes the efficiency of the nonlinear optical process very high in optical fibers.

The intensity dependence of the refractive index leads to a large number of interesting nonlinear effects. The two most widely studied are self-phase modulation (SPM) and cross-phase modulation (XPM).

Self-phase modulation refers to the self-induced phase shift experienced by an optical field during its propagation in optical fibers. Its magnitude can be obtained by noting that the phase of an optical field changes by

$$\phi = nk_0 L = \left(n(\omega) + n_2 |E|^2 \right) k_0 L, \quad (2.2.3)$$

where $k_0 = \frac{2\pi}{\lambda}$, and L is the fiber length. The intensity-dependent nonlinear phase shift due to SPM is $\phi_{NL} = n_2 |E|^2 k_0 L$. SPM is responsible for the spectral broadening of ultrashort pulses and the formation of optical solitons in the anomalous-dispersion regime of fibers.

Cross-phase modulation refers to the nonlinear phase shift of an optical field induced by another field having a different wavelength, direction, or polarization state. Its origin can be understood by noting for example that the total electrical field E is given by

$$E = \frac{1}{2} \bar{x} [E_1 \exp(-i\omega_1 t) + E_2 \exp(-i\omega_2 t) + c.c.], \quad (2.2.4)$$

when two optical fields at frequencies ω_1 and ω_2 , polarized along the same axis, propagate simultaneously inside the fiber. The nonlinear phase shift introduced by the field E_2 to the field E_1 is $\phi_{nl} = n_2 k_0 L \cdot 2|E_2|^2$. Therefore in this case the total nonlinear phase shift for the field E_1 would be

$$\phi_{nl} = n_2 k_0 L (|E_1|^2 + 2|E_2|^2), \quad (2.2.5)$$

where the two terms on the right side are due to SPM and XPM, respectively. Among other things, XPM is responsible for the asymmetric spectral broadening of the co-propagating optical pulses. XPM also gives an anticipation of soliton interaction and the existence of paired solitons in fiber soliton lasers.

2.2.2 Nonlinear Schrödinger equation

By including the nonlinear part of refractive index into $\delta\beta$ of Eq. (2.1.16) using

$\beta(\omega) = n(\omega) \frac{\omega}{c}$, one can get:

$$\frac{\partial E}{\partial z} + \beta_1 \frac{\partial E}{\partial t} = i \frac{\beta_2}{2} \frac{\partial^2 E}{\partial t^2} - i\gamma |E|^2 E, \quad (2.2.5)$$

where

$$\gamma = \frac{\pi n_2}{\lambda}. \quad (2.2.6)$$

When introducing the normalized time τ , distance ξ and wave envelope U by:

$$\begin{aligned}\tau &= \frac{t - z/v_g}{\tau_p} \\ \xi &= \frac{1}{\tau_p^2} |\beta_2| z \\ U &= \tau_p \left(\frac{\alpha}{\beta_2} \right)^{1/2} E\end{aligned}\tag{2.2.7}$$

where τ_p is the pulse width, Eq. (2.2.5) is then reduced to the standard nonlinear Schrödinger equation (NLSE):

$$i \frac{\partial U}{\partial \xi} + \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} + |U|^2 U = 0\tag{2.2.8}$$

Eq. (2.2.8) gives a simple description over the propagation of an optical pulse in nonlinear dispersive media, which is affected by both the GVD and nonlinear effects denoted respectively by the second and third term in the equation.

2.2.3 Fundamental Soliton solution

From Eq. (2.2.5) or (2.2.8), it's easy to understand that the propagation of an optical pulse is the combinative effect of both GVD and nonlinear effects. The nonlinear optical Kerr effect always generates a positive frequency chirp in the center part of a pulse, while the frequency chirp induced by the GVD can be either positive or negative depending on the sign of the GVD parameter. Since in anomalous dispersion regimes ($\beta_2 < 0$), the fiber generates a negative linear frequency chirp, under the simultaneous action of the nonlinear Kerr effect, the frequency chirp on a pulse may be eliminated and no pulse distortion will happen as it propagating in the fiber. In fact, as the chirp induced by the nonlinear Kerr effect is the pulse intensity dependent, as a pulse with strong intensity propagates in anomalous-dispersion

fiber, it can adjust its intensity so that the chirp induced by the two effects is automatically balanced. Namely, optical solitons is actually a generic property of fibers in the anomalous dispersion regime.

The fundamental soliton solution can be derived theoretically from NLSE by assuming the balance between the GVD and nonlinear terms. By solving the NSLE using the inverse scattering method [17], one can get the solution of fundamental solitons written as:

$$E = \left(\frac{\beta_2}{\alpha} \right)^{1/2} \frac{1}{\tau_p} \operatorname{sech} \left(\frac{t - z/v_g}{\tau_p} \right) \exp \left(-i \frac{\pi z}{4z_0} \right) \quad (2.2.9)$$

where α is the fiber loss. z_0 represents the soliton period that is defined as:

$$z_0 = \frac{\pi}{2} \cdot \frac{\tau_p^2}{|\beta_2|}. \quad (2.2.10)$$

The wave package in Eq. (2.2.9) is referred to as the fundamental soliton because its shape does not change with propagation.

2.3 Pulse propagation in erbium-doped fiber

2.3.1 Gain profile of erbium-doped fibers

Rare-earth doped fibers manufactured by doping the rare-earth ions in SMF core have been widely used for amplifiers in optic fiber communication systems. Various doping ions are chosen to provide gain for light in different wavelength regions. Among them, erbium-doped fiber amplifiers (EDFAs) have attracted especial

attention, as they have a broadband gain in the 1.55 μm optical fiber communication window.

By pumping the ions from their ground state to the excited states, population inversion could be achieved between the two energy levels of the ions. Then the stimulated emission happens and amplifies the light incident to the fiber whose frequency is within the gain profile. The optical spectrum of a fiber amplifier is mainly determined by the property of the dopants in the fiber. Based on the rate equations, the gain coefficient of an optical amplifier can be calculated by $g = \sigma(N_2 - N_1)$, where σ is the transition cross-section, and N_1 and N_2 are the atomic densities in the two energy levels of the ions. In practice, there're many factors that affect the gain coefficient of doped fiber amplifiers. Different rare-earth doped fiber amplifier or even the fiber amplifiers doped with the same rare-earth ion but with different doping densities and fiber properties could have different gain profiles [74].

The pumping scheme of the Er^{3+} ions can be regarded as a three-level system, which is schematically illustrated as Fig. 2.2. Efficient pumping is possible using semiconductor lasers operating near 980 nm and 1480 nm wavelengths.

Figure 2.2 Energy levels of Er^{3+} ions

Generally, an amplifier is characterized by its small signal gain, gain bandwidth, gain saturation power and the noise figure. For a homogeneously broadened gain medium as Er^{3+} ions, the gain coefficient can be written as [75]

$$g(\omega) = \frac{g_0}{1 + (\omega - \omega_0)^2 T_2^2 + P/P_s} \quad (2.3.1)$$

where g_0 is the small signal gain, ω is the frequency of the incident light, ω_0 is the resonant frequency of the dopants, T_2 is the dipole relaxation time of the dopants, P is the power of the incident light, and P_s is the saturation power of the gain medium. The saturation power depends on the stimulated emission cross-section and the upper energy level lifetime of the dopants. In the case of three-level pumping scheme, it is also the pump strength dependent. In practice, the gain saturation can be negligible for most doped fiber amplifier for single pulse amplification, i.e. $P/P_s \ll 1$. By neglecting the P/P_s term, the gain coefficient becomes:

$$g(\omega) = \frac{g_0}{1 + (\omega - \omega_0)^2 T_2^2} \approx g_0 \left[1 - \frac{(\omega - \omega_0)^2}{\Omega_g^2} \right] \quad (2.3.2)$$

This equation shows that the gain is governed by a Lorentzian profile with the maximum value happening when the signal frequency coincides with the atomic transition frequency ω_0 . The gain bandwidth Ω_g is defined as the full width at half maximum (FWHM) of the gain spectrum $g(\omega)$. For a Lorentzian profile, Ω_g is inversely related to the dipole relaxation time T_2 of the dopants.

Figure 2.3 shows a typical absorption and emission spectrum of erbium-doped fiber amplifiers [76]. The shapes of the absorption and emission spectrum of erbium-doped fiber amplifiers can be associated with the energy-level transitions of Er^{3+} ions shown in Fig. 2.2. Details of the spectrum also depend on the fiber composition and the co-dopants. Worthy of mentioning here, the gain of isolated Er^{3+} ions is homogeneously broadened, which exhibit a Lorentzian profile. However, as Er^{3+} ions are doped into silica fiber, due to the interaction of the ions with the silica host and

other co-dopants such as germania and alumina within the fiber core, their energy levels are no longer discrete, but consist of a number of closely spaced levels. As a result, the gain and absorption spectrum is broadened with a double peak structure as shown in Fig. 2.3. For erbium-doped fiber, the gain bandwidth could reach approximately 40 nm, which can correspondingly support as short to 60 fs pulse. But in practice, not the whole part of the gain can be efficiently used due to some effects such as the nonuniform gain profile, the fiber birefringence, and also the cavity effect in fiber lasers.

Figure 2.3 Typical absorption and emission spectra of the erbium-doped fibers (After Ref. [76]).

2.3.2 Ginzburg-Landau equation

When an optical pulse propagates in the erbium-doped fibers, the effect of the light amplification must be considered. Generally, the interaction of the doped ions with the light is governed by the Maxwell-Bloch equations [18]. However, in the case of that the pulse widths is larger comparing with the dipole relaxation time T_2 of the Er^{3+} ions ($T_2 < 0.1$ ps), the rate equation approximation can be made and consequently the gain term can be simply added into the NLSE.

As the gain dispersion is to reduce the gain for spectral components away from the gain peak ω_0 , therefore, the frequency dependence of the gain can be approximated by:

$$g(\omega) = g(\omega_0) + \left. \frac{\partial g}{\partial \omega} \right|_{\omega=\omega_0} (\omega - \omega_0) + \frac{1}{2} \left. \frac{\partial^2 g}{\partial \omega^2} \right|_{\omega=\omega_0} (\omega - \omega_0)^2 + \dots \quad (2.3.3)$$

Without loss of the generality, assuming that the pulse carrier frequency coincides with the gain peak, the equation describing optical pulse propagation in the doped fiber becomes:

$$\frac{\partial E}{\partial z} = -\beta_1 \frac{\partial E}{\partial t} - \frac{i}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} + i\gamma |E|^2 E + \frac{g}{2} E + \frac{g}{2\Omega_g^2} \frac{\partial^2 E}{\partial t^2} \quad (2.3.4)$$

where g is the peak gain coefficient, Ω_g is the gain bandwidth. The time dependence of the peak gain coefficient varies with time due to the gain saturation and is determined by [77]

$$\frac{\partial g}{\partial t} = \frac{g_0 - g}{T_1} - \frac{g|E|^2}{E_s} \quad , \quad (2.3.5)$$

where g_0 is the small signal gain, T_1 is the population decay time, and E_s is the saturation energy. As for the erbium doped ions T_1 is in the time scale of 10 ms, if the pulse width is far narrower than it, which is normally the case, the T_1 term is negligible during pulse amplification, and $g(t)$ becomes:

$$g(t) = g_0 \exp \left[-\frac{1}{E_s} \int_{-\infty}^t |E|^2 dt \right] \quad . \quad (2.3.6)$$

Typical values of E_s for erbium-doped fibers are about 10 μJ . As the pulse energies are normally much smaller than the saturation energy E_s , gain saturation is

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negligible over the duration of a single pulse. However, in the case of a pulsed fiber laser, the pulse circulating in the laser cavity, the average power of the light may still saturate the gain and determines the saturated gain value.

In reality, one may also need to take into account the fiber loss (~ 0.2 dB/km) for long distance light propagation in fibers.

$$\frac{\partial E}{\partial z} = -\beta_1 \frac{\partial E}{\partial t} - \frac{i}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} + i\gamma |E|^2 E + \frac{g - \alpha_f}{2} E + \frac{g}{2\Omega_s^2} \frac{\partial^2 E}{\partial t^2}, \quad (2.3.7)$$

where α_f represents the fiber loss. When $\alpha_f = 0$ and $g = 0$, the equation reduces back to NLSE.

Eq. (2.3.7) can be also written in a dimensionless form by making the transformation of Eq. (2.2.7):

$$i \frac{\partial U}{\partial \xi} + \frac{1}{2} (1 - id) \frac{\partial^2 U}{\partial \tau^2} + N^2 |U|^2 U = \frac{i}{2} \mu U, \quad (2.3.8)$$

where

$$\begin{aligned} d &= \frac{g T_2^2}{n_0 |\beta_2|}, \\ \mu &= \left(\frac{g}{n_0} - \alpha_f \right) L_D, \end{aligned} \quad (2.3.9)$$

and

$$N = \left(\frac{\mathcal{P}_0 T_0^2}{|\beta_2|} \right)^{1/2} \quad (2.3.10)$$

is the soliton order. Eq. (2.3.8) is known as the Ginzburg-Landau equation (GLE).

2.3.3 Soliton solution

Different to the NLSE, which is integrable and has an exact soliton solution, the GLE is non-integrable. However, it is found that in the case of anomalous fiber dispersion, the equation also has solitary wave solutions in the sense of optical pulses whose shape does not change during propagation. When $N=1$, fundamental soliton solution can be obtained in the case of a single eigen-value by using a trial-and-error method [18]:

$$U(\xi, \tau) = \text{sech}(p\tau) \exp[iq \ln \cosh(p\tau) + i\Gamma \xi] , \quad (2.3.11)$$

where p , q and Γ are constants depending on d which was already defined in Eq. (2.3.9). Comparing with the case in undoped fiber, due to the influence of the fiber gain, the solitons in the doped fibers become frequency chirped. And for a SMF of GVD β_2 , the peak power required to form a fundamental soliton is

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \approx \frac{3.11|\beta_2|}{\gamma T_{FWHM}^2} \quad (2.3.12)$$

where T_{FWHM} is the FWHM (full width half maximum) soliton pulse width.

$N > 1$ corresponds to high-order solitons. However, although theoretically predicted, by now no high-order soliton has been ever experimentally observed to our knowledge due to its intrinsic instability.

2.4 Pulse propagation in linearly birefringent fibers

2.4.1 Fiber birefringence

In all the above processes, the author has ignored the polarization characteristic of light in the fiber. Actually, even a SMF is not truly single mode because it can

support two degenerate modes that polarized in two orthogonal directions. Under ideal conditions, the two polarization states would not couple to each other. But in practice, all fibers exhibit some modal birefringence because of small departures from cylindrical symmetry due to random variations in core shape and/or stress-induced anisotropy along the fiber length. Mathematically, the strength of modal birefringence is defined as:

$$B = \delta\beta / k_0 = |n_x - n_y|, \quad (2.4.1)$$

where $\delta\beta = |\beta_x - \beta_y|$ is the difference in the propagation constant, and n_x and n_y are the respective modal refractive indices for the two orthogonally polarized components. Here it is assumed the fiber is linearly birefringent, i.e. the fiber has two principle axes along which it is capable of maintaining the state of linear polarization of the incident light in the absence of nonlinear effects. The axis along which the mode index is smaller, namely the group velocity is larger for light propagating in this direction, is called the fast axis. And similarly, the axis with the larger mode index is called the slow axis. This assumption is ideally the case for polarization-maintaining fibers (PMF), where the built-in birefringence is made much larger than random changes occurring due to stress and core-shape variations.

For a given value of B , after propagating a length of L , the two polarization components of the incident light will gain a phase difference of

$$\Delta\phi = \frac{2\pi(n_x - n_y)L}{\lambda} = \frac{2\pi BL}{\lambda} = 2\pi \cdot \frac{L}{L_B}, \quad (2.4.2)$$

where $L_B = 2\pi / |\beta_x - \beta_y| = \lambda / B$ is called the beat length.

From Eq. (2.4.2), it can be seen that the light varies its polarization periodically along the fiber. At a multiple distance of L_B , a multiple phase difference of 2π will be generated between the two polarization components, which means that the light

returns to its initial polarization state. From a viewpoint of physics, the two polarization modes exchange their powers as propagating in the fiber with a period of the beat length L_B . Different single-mode fibers could have very different beat lengths. For a standard SMF, L_B is in the range of several meters, while for the PMF, as a strong birefringence is deliberately built in the fibers, they could have a beat length of several millimeters.

When the nonlinear effects in optical fibers become dominant, a sufficiently intense optical field can induce nonlinear birefringence whose magnitude is intensity dependent. Consequently, the refractive index is described as

$$n_j = n_j^L + \Delta n_j \quad (j = x, y), \quad (2.4.3)$$

where n_j^L is the linear part of the refractive index. And the nonlinear contributions Δn_x and Δn_y are given by

$$\Delta n_x = n_2 \left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right), \quad \Delta n_y = n_2 \left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right), \quad (2.4.4)$$

where n_2 is the nonlinear parameter as in Sec. 2.2. Normally one takes the value of n_2 the same for the two polarizations, which is also practically just the case. In Eq. (2.4.4), the first term is responsible for SPM. The second term results in XPM since the nonlinear phase shift acquired by one polarization component dependent on the intensity of the other polarization component. The presence of this term induces a nonlinear coupling between the two polarization eigen-modes E_x and E_y .

2.4.2 Coupled nonlinear Schrödinger equations

By taking into account the nonlinear fiber birefringence, the nonlinear Schrödinger equation is transformed into two coupled equations [78,79]:

$$\begin{aligned}
i\left(\frac{\partial U}{\partial z} + k' \frac{\partial U}{\partial t}\right) - \frac{1}{2}k'' \frac{\partial^2 U}{\partial t^2} + \gamma\left(|U|^2 + \frac{2}{3}|V|^2\right)U + \frac{\gamma}{3}V^2U^* \exp[-2i(k_0 - l_0)z] &= 0 \\
i\left(\frac{\partial V}{\partial z} + l' \frac{\partial V}{\partial t}\right) - \frac{1}{2}l'' \frac{\partial^2 V}{\partial t^2} + \gamma\left(|V|^2 + \frac{2}{3}|U|^2\right)V + \frac{\gamma}{3}U^2V^* \exp[2i(k_0 - l_0)z] &= 0
\end{aligned}
\tag{2.4.5}$$

where U and V are the slowly varying amplitudes of the two orthogonally polarized eigen-modes, U^* and V^* are their complex conjugates, respectively. In the above equations, k and l are used to represent the propagation constants of the two modes, where

$$\begin{aligned}
k(\omega) &= k_0\omega_0 + k'(\omega - \omega_0) + \frac{1}{2}k''(\omega - \omega_0)^2 + \dots \\
l(\omega) &= l_0\omega_0 + l'(\omega - \omega_0) + \frac{1}{2}l''(\omega - \omega_0)^2 + \dots
\end{aligned}
\tag{2.4.6}$$

Eqs. (2.4.6) are the Taylor series of $k(\omega)$ and $l(\omega)$ about the center frequency ω_0 , where k_0 and l_0 , k' and l' , k'' and l'' are their respective zero to second order Taylor expansion parameters. Note that the modal birefringence leads to different group velocities for the two polarization components, i.e. $k' \neq l'$ in general. But in contrast, the GVD is always regarded the same for both polarization components having the same frequency, i.e. $k'' = l''$. It should also be pointed out that the last terms in Eqs. (2.4.5) results from the four-wave mixing, which can often be neglected in highly birefringent fibers ($L_B \sim 1$ cm typically).

For simplification, the author firstly transforms Eqs. (2.4.5) into the synchronous coordinate system by using a similar transformation as that in Sec. 2.2.

$$\begin{cases} T = t - \frac{z}{\bar{v}_g} = t - \bar{\beta}z, \\ Z = z \end{cases}
\tag{2.4.7}$$

where

$$\bar{\beta} = \frac{k' + l'}{2}. \quad (2.4.8)$$

Applying the transformation and also substituting $\bar{\beta}$ and $k'' = l''$ into the equations, Eqs. (2.4.5) then take the form:

$$\begin{aligned} \frac{\partial U}{\partial Z} &= \frac{l' - k'}{2} \frac{\partial U}{\partial T} - \frac{i}{2} k'' \frac{\partial^2 U}{\partial T^2} + i\gamma \left(|U|^2 + \frac{2}{3} |V|^2 \right) U + \frac{i}{3} \gamma U^2 V^* \exp[-2i(k_0 - l_0)Z] \\ \frac{\partial V}{\partial Z} &= \frac{k' - l'}{2} \frac{\partial V}{\partial T} - \frac{i}{2} k'' \frac{\partial^2 V}{\partial T^2} + i\gamma \left(|V|^2 + \frac{2}{3} |U|^2 \right) V + \frac{i}{3} \gamma U^2 V^* \exp[2i(k_0 - l_0)Z] \end{aligned} \quad (2.4.9)$$

Then by defining $\delta = \frac{k' - l'}{2}$, $\beta = \frac{k_0 - l_0}{2}$, $U = u \exp(-i\beta z)$ and $V = v \exp(i\beta z)$, then substituting these definitions into Eqs. (2.4.9), one can get a simpler form of the coupled nonlinear Schrödinger equations as:

$$\begin{aligned} \frac{\partial u}{\partial Z} &= i\beta u - \delta \frac{\partial u}{\partial T} - \frac{i}{2} k'' \frac{\partial^2 u}{\partial T^2} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* \\ \frac{\partial v}{\partial Z} &= -i\beta v + \delta \frac{\partial v}{\partial T} - \frac{i}{2} k'' \frac{\partial^2 v}{\partial T^2} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + \frac{i\gamma}{3} u^2 v^* \end{aligned} \quad (2.4.10)$$

Since the gain medium exists in the laser cavity, its gain and gain dispersion effect must be also involved in the pulse propagation equations. Therefore, the coupled NLSEs in (2.4.10) are written as two coupled GLEs:

$$\begin{aligned} \frac{\partial u}{\partial z} &= i\beta v - \delta \frac{\partial v}{\partial t} - \frac{i}{2} k'' \frac{\partial^2 u}{\partial t^2} + i\gamma \left(|u|^2 + 2|v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial v}{\partial z} &= -i\beta u + \delta \frac{\partial u}{\partial t} - \frac{i}{2} k'' \frac{\partial^2 v}{\partial t^2} + i\gamma \left(2|u|^2 + |v|^2 \right) v + \frac{i\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2} \end{aligned} \quad (2.4.11)$$

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where g is the gain coefficient, and Ω_g is the gain bandwidth. In this case, the gain saturation results from the light along both polarizations so the saturated gain coefficient is calculated by

$$g = g_0 / \left[1 + \frac{\int (|u|^2 + |v|^2) dt}{E_s} \right]. \quad (2.4.12)$$

Chapter 3

Experimental investigations

Passively mode-locked fiber lasers are attractive sources of ultrashort soliton pulses for laboratory, telecommunication and signal-processing applications because of their simplicity, tunability and ultrashort pulse operation. In this research, a passively mode-locked fiber laser using the nonlinear polarization rotation (NPR) technique is set up for experimental investigations. By making up the laser ring cavity with anomalous-dispersion single-mode fibers, soliton operation of the laser is expected. It is of interests to find out the operation characteristics of such a fiber laser, in particular, what properties optical solitons in the fiber laser have.

In this chapter, section 3.1 introduces the basic mechanism of the NPR technique and its implementation in fiber lasers. Section 3.2 describes the experimental setup and measurement systems. Based on the laser setup, by appropriately adjusting the settings of the polarization controllers, three distinct mode-locked operation regimes i.e. the traditional soliton operation, twin-pulse soliton operation, and noise-like pulse operation, have been observed. Their respective properties have also been investigated, which are sequentially presented in Sect. 3.3 to 3.5. Especially, as a novel form of solitons in fiber lasers, twin-pulse solitons are investigated for the first time in Sec. 3.4.

3.1 Nonlinear polarization rotation mode locking

3.1.1 Nonlinear polarization rotation technique

Nonlinear polarization rotation (NPR) mode locking technique exploits the nonlinear birefringence of the single-mode fibers to realize an artificial saturable

absorber effect in the laser cavity for a self-started mode locking. To illustrate the principle of the technique, a configuration as shown in Fig. 3.1 is used. A piece of birefringent fiber is placed between two linear polarizers (To distinguish, the one after the fiber is referred to as an analyzer.). Suppose the fiber is linearly birefringent, the author selects its two polarization eigenmodes as the x (vertical, slow) and y (horizontal, fast) axis. The polarizer and analyzer are at such positions that their polarizations have angles of θ and φ , respectively, to the vertical polarization axis.

Figure 3.1 Schematic diagram of the laser cavity equivalence.

At this configuration, any incident light will first be transformed into linearly polarized light by the polarizer, whose polarization orientation is determined by the bias of the polarizer. Then because the fiber is birefringent, the linearly polarized light will be transformed into elliptically polarized light after propagating through the fiber. The elliptically polarized light will become linearly polarized light again after the analyzer. The evolution of the light polarization is also schematically shown in Fig. 3.1.

If the light intensity is weak, after propagating through the fiber, the ellipticity and azimuth of the light polarization are fully determined by the linear fiber birefringence and the polarizer. But if the light intensity is strong, the nonlinear effects of the fiber must be taken into account. Then because of the fiber nonlinearity, the ellipticity and azimuth of the light polarization depend on the pulse intensity, which in turn causes that the cavity transmission depends on the light intensity. Through appropriately selecting the orientation of the analyzer, a situation could be achieved with the setup that the stronger the light is, the larger the light transmission through the analyzer is. Physically, such an effect is equivalent to that generated by a saturable absorber, which is widely used for mode locking in lasers [50-52]. Here in this setup, the artificial saturable absorber effect let the light of higher intensity experience less absorption loss on the analyzer, which shortens the pulse round by round and furthermore leads to passive mode locking. As this artificial saturable absorber effect is generated based on the nonlinear Kerr effect, which has a recovery time in the order of several femtosecond, from the laser mode-locking point of view, ultrashort pulses could be generated by this technique.

3.1.2 Cavity transmission analysis

To explain the mode locking mechanism mathematically, the author analyzes the light intensity transmission T of the setup in Fig. 3.1. The transmission function T is defined as

$$T = \frac{I_{out}}{I_{in}} = \frac{\bar{E}_{out}^* \cdot \bar{E}_{out}}{\bar{E}_{in}^* \cdot \bar{E}_{in}} \quad , \quad (3.1.1)$$

where I_{in} , I_{out} are the input and output light intensity; $\bar{E}_{in}$, $\bar{E}_{out}$ is the respective field vector of the input and output light; and $\bar{E}_{in}^*$, $\bar{E}_{out}^*$ are their complex conjugates.

Eq. (3.1.1) can be calculated by using the Jones Matrix method. Through simple matrix operation (Appendix A), the linear intensity transmission of light through the setup in Fig. 3.1 can be denoted by

$$T_l = \sin^2 \theta \sin^2 \varphi + \cos^2 \theta \cos^2 \varphi + \frac{1}{2} \sin 2\theta \sin 2\varphi \cos(\Delta\phi_l) \quad (3.1.2)$$

where $\Delta\phi_l = \Delta\beta L$ is the linear phase delay between the two polarization components caused by the linear birefringence of the fiber. However, if the light intensity is strong, the nonlinear effect must be considered. By considering the nonlinear phase induced by the fiber nonlinearity, the light intensity transmission is written as

$$T = \sin^2 \theta \sin^2 \varphi + \cos^2 \theta \cos^2 \varphi + \frac{1}{2} \sin 2\theta \sin 2\varphi \cos(\Delta\phi_l + \Delta\phi_{nl}), \quad (3.1.3)$$

where $\Delta\phi_{nl}$ is the nonlinear phase delay caused by the fiber nonlinearity. For simplification, the extra phase delay introduced by the nonlinear fiber birefringence is assumed to be small. Then the light intensity transmission can be written as:

$$T = T_l - \frac{1}{2} \sin(2\theta) \sin(2\varphi) \sin(\Delta\phi_l) \Delta\phi_{nl}. \quad (3.1.4)$$

The fiber nonlinearity introduces a transmission change on the linear transmission T_l . And this transmission change not only depends on the light intensity, introduced by the nonlinear phase delay $\Delta\phi_{nl}$, but also depends on the orientations of the polarizer θ and the analyzer φ , as well as the linear phase delay $\Delta\phi_l$. As a result, to achieve an artificial absorber effect in the cavity, it is necessary to properly select the combination of all the parameters.

Once the orientations of the polarizer and analyzer with respect to the fiber polarization principal axes are fixed, the linear transmission of the setup is a

sinusoidal function of the linear phase delay between the two polarization components. Although the relative orientation between the polarizer and the analyzer could be arbitrarily selected, in order to possibly achieve a stable mode locking operation, it is preferred to choose the relative orientation between the polarizer and the analyzer 90-degree, as under such a selection the linear transmission could be set to zero.

Considering that the polarizer and the analyzer are set orthogonal, the linear and nonlinear light transmission can be simplified as:

$$T_l = \sin^2 2\theta \frac{[1 - \cos(\Delta\phi_l)]}{2}, \quad (3.1.5)$$

$$T = T_l + \frac{1}{2} \sin^2(2\theta) \sin(\Delta\phi_l) \Delta\phi_{nl}, \quad (3.1.6)$$

Obviously, the maximum linear intensity transmission is limited by $\sin^2 2\theta$. From Eq. (3.1.6), one can get that with the increase of the light intensity, either an increase or a decrease in the transmission could be generated depending on the linear phase delay selection. Firstly, the orientation of polarizer determines the projection of light on the two polarization axes of the fiber, therefore, determines whether a positive $\Delta\phi_{nl}$ or a negative $\Delta\phi_{nl}$ would be generated. The magnitude of $\Delta\phi_{nl}$ is proportional to the intensity of light. Then further depending on the linear phase delay, either an increase or decrease in the intensity transmission could be caused by the nonlinear polarization rotation.

Since $\Delta\phi_{nl}$ is proportional to the light intensity, i.e. $\Delta\phi_{nl} \propto |E_{in}|^2$, the nonlinear cavity transmission can be written as

$$T = T_l + \kappa |E_{in}|^2. \quad (3.1.7)$$

Here κ is a parameter related with the θ , $\Delta\phi_l$, and also the nonlinear birefringence of the fiber. As far as $\kappa > 0$, a saturable absorber effect can be obtained [80].

Strictly speaking, the nonlinear phase delay between the two polarization components of an optical pulse traversing a piece of birefringent fiber can only be determined by numerically solving the coupled NLSEs (2.4.11). However, for standard single-mode fiber, in the case of CW with power P_0 is launched to the fiber at an angle θ from the slow axis, the relative nonlinear phase shift between the two polarization components is [16]

$$\Delta\phi_{nl} = \gamma L(1-B)(P_x - P_y) = \frac{\gamma P_0 L \cos(2\theta)}{3} \quad (3.1.8)$$

P_x and P_y are the respective power of the two polarization components. Then the expression of κ is:

$$\kappa = -\frac{\gamma L}{12} \sin(4\theta) \sin(2\theta) \sin(\Delta\phi_l) \quad (3.1.9)$$

To get an artificial saturable absorber effect, $\kappa > 0$ requires that the linear phase shift $\Delta\phi_l \in (\pi, 2\pi)$. Using Eq. (3.1.9), one can estimate the optimum selection for the polarizer orientation θ where the strongest saturable absorber effect could be achieved:

$$\begin{aligned} \frac{d\kappa}{d\theta} &= \frac{d}{d\theta} \left(\frac{\gamma L}{12} \sin(4\theta) \sin(2\theta) \sin(\Delta\phi_l) \right) = 0 \\ \Rightarrow \frac{\gamma L}{3} \sin(\Delta\phi_l) \sin 2\theta (3\cos^2 2\theta - 1) &= 0 \\ \Rightarrow \cos 2\theta &= \pm \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{n\pi}{2} \pm 27.4^\circ \end{aligned} \quad (3.1.10)$$

Therefore in this case, the optimum θ happens at around 27.4 degree.

In practice, it is difficult to set the linear phase delay just within the range where the saturable absorption effect is achieved. Therefore, a polarization controller is normally inserted in the setup to efficiently control the value of the linear phase

delay $\Delta\phi$. Mathematically, this is equivalent to add a variable linear phase delay term i.e.

$$\Delta\phi_l = \Delta\phi_{pc} + \Delta\phi_f . \quad (3.1.11)$$

where $\Delta\phi_{pc}$ represents the phase delay introduced by the polarization controllers, which is tunable in experiment. While, $\Delta\phi_f$ is the linear phase delay introduced by the fiber itself, which is a constant for fixed fiber length at certain wavelength.

From the cavity transmission function T, it can be deduced that saturable absorber effect can only be obtained within a certain range of linear phase bias. In order to get mode locking, one needs to rotate the polarization controllers to set $\Delta\phi_{pc}$ to the position that the saturable absorber effect can be obtained in the laser.

3.2 Experimental setup

3.2.1 Laser setup

To achieve the self-started mode locking using the NPR technique, a ring cavity is normally used [81]. In this research, a passively mode-locked fiber ring laser using the NPR technique schematically shown in Fig. 3.2 is built up.

The fiber laser has a ring cavity of about 5.5 meters long, which comprises of a 3.5 meter long 2000 ppm erbium-doped fiber with a GVD of about 10 ps/nm·km (i.e. $\beta_2 \sim -12.75$ ps²/km), a piece of 1 meter long single-mode dispersion shifted fiber, whose GVD is 2 ps/nm·km (i.e. $\beta_2 \sim -2.55$ ps²/km), and a piece of 1 meter long standard single-mode telecomm fiber (SMF 28), whose GVD is 18 ps/nm·km (i.e. $\beta_2 \sim -22.9$ ps²/km). The erbium-doped fiber acts as the gain medium. The nonlinear polarization rotation technique is used to achieve the self-started mode locking in the laser. For this purpose, a polarization dependent isolator together with two polarization controllers, one consists of two quarter-wave plates and a half-wave

plate, the other consists of two quarter-wave plates, is used to adjust the polarization of light in the cavity. The polarization dependent isolator and the polarization controllers are mounted on a 7 cm long fiber bench to achieve accurate polarization adjustments. The laser is pumped by a pigtailed InGaAsP semiconductor diode of wavelength 1480 nm, whose light is coupled into the cavity through a 1480/1550 nm WDM coupler. The output of the laser is taken via a 10 % fiber coupler.

Figure 3.2 Schematic of the fiber soliton laser setup. $\frac{\lambda}{4}$: Quarter-wave plate; $\frac{\lambda}{2}$: Half-wave plate;

PI: Polarization dependent isolator; WDM: Wavelength-division-multiplex coupler.

In the laser cavity, the fiber bench is the key to realize passive mode locking. The function of each component on the fiber bench is as follows:

- Polarization dependent isolator (PI)

The PI (polarization dependent isolator) has both the functions of a polarizer and a polarization analyzer. Light passing through the PI becomes linearly polarized. After one round trip of propagation, the light will pass the PI again. This time it then acts as a polarization analyzer. The polarization dependent isolator can also block the light in the opposite direction so that the cavity is a unidirectional ring.

- Polarization controller ($\frac{\lambda}{4} + \frac{\lambda}{4}$)

Two quarter-wave plates together functions as an arbitrary polarization rotator and phase shifter, which is usually referred to as a polarization controller (PC). Rotatable retardation plates have been widely used in optics to adjust the polarization of the light. Quarter-wave plate (QWP) and half-wave plate (HWP) are two most commonly used in practical experiments. Different combination of QWP and HWP, e.g. a QWP+HWP, a QWP+QWP, or a QWP+HWP+QWP could be used to transform any input polarization into any required arbitrary polarization, i.e. function as a polarization controller [82]. Using wave-plates as PC has both the advantage of accurate controlling the phase shift with any value between $0-2\pi$, and of introducing no effects on other parts of the cavity. In the cavity, there are two PCs altogether, which locate before and after the PI respectively. The PCs are used to adjust the linear phase bias so that artificial saturable absorber effect can be achieved in the laser. By rotating the wave-plates, their relative orientation of polarization axis changes which changes the relative phase delay between two polarizations and consequently the final polarization state.

- Polarization controller ($\frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{4}$)

As the author mentioned above, a QWP+HWP+QWP system acts as a PC. In the cavity, the second set of PC is to bias the system so that the highest-intensity light can has the maximum transmission through the PI. This second set of PC is not needed in principle, but is required in practice. It can be used to compensate for the uncertain polarization transformation in the fiber due to bending, pressure etc.

Now let's have a look at the operation procedure of the laser. Any incident light will be transformed into a linearly polarized light by the polarizer (the PI). Then the first PC transforms the linearly polarized light into an elliptically polarized light.

Generally, an elliptically polarized light can be treated as a superposition of a left-hand and a right-hand circularly polarized mode of different intensity. The two polarizations will experience different linear and nonlinear phase shifts through the fiber. Therefore, the polarization ellipse is rotated after propagating through the fiber. And because of the nonlinearity of the fiber, the ellipticity and azimuth of the light polarization depend on the pulse intensity, which in turn causes that the cavity transmission depends on the light intensity. The elliptically polarized light will then become linearly polarized light again after the analyzer (the PI). By carefully choosing the linear phase delay of the PC, one can achieve an artificial saturable-absorber effect in the laser cavity, which furthermore leads to the passively mode locking.

Mode locking is started in passively mode-locked fiber lasers by the spike formed by sudden beating or fluctuation of the cavity mode under certain pump power. So it is regarded as self-starting. Since the higher-intensity spike will experience smaller loss, it will be amplified and shortened every time it passes the polarizer until a mode-locked pulse is formed. Furthermore, when an appropriate polarization controller position is set, which makes the GVD and Kerr nonlinearity totally compensate with each other, a stable soliton pulse can be formed immediately after the mode locking. In the laser, depending on the settings of the wave-plates, either the single-pulse soliton or a twin-pulse soliton operation can be obtained. Furthermore, apart from the two types of soliton operations, the laser can also operate in a noise-like mode-locked pulse regime. Altogether, three operation regimes will be discussed for the laser in the following sections.

3.2.2 Measurement system

In experiment, the laser output is analyzed with an optical spectrum analyzer (AQ-6317B) and a commercial optical autocorrelator (APE Pulsescope 50). Just as its name implies, an optical spectrum analyzer (OSA) is used to get the optical spectrum of an input light signal. As for the one the author uses in experiment (AQ9317B), its wavelength response range is from 350 to 1750 nm, which well

includes the range of the author's interest (~ 1540 to 1620 nm). And the highest resolution it supports is 0.01 nm. The autocorrelator is used to measure the pulse width of ultrashort pulses. The one the author uses (Pulsescope APE) has a scan range from 0.5 to 50 ps. And the resolution is optional depending on the respective scan range. At the condition of using 0.5 ps scan range, the resolution could reach 2 fs, while as the scan range is 50 ps, the highest resolution is only 2 ps. With the OSA and the autocorrelator, the pulse profile of the laser output in both frequency and time domain can be obtained.

In addition, to study the evolution of the pulses in the cavity, a 50 GHz wide bandwidth sampling oscilloscope (Agilent 86100A), together with a 25 GHz high-speed photo-detector (New Focus 1414FC) is used to monitor the laser output. Referring to the resolution of the oscilloscope and the photodetector, the smallest separation their combined system can resolve is around 40 ps. Therefore, for the ultrashort single-pulse solitons (~ 340 fs) and the twin-pulse solitons (separation ~ 1 ps), it can only give a pulse image depending on the respective intensity, but not the real pulse profile. However, as described above, an autocorrelator whose scan range can cover from 0.5 to 50 ps, is also used to monitor the laser output. Obviously, the autocorrelator and the oscilloscope system have an overlap in their application ranges. With this overlap of their measurement capabilities, one can well observe the dynamics of the laser output at any time scale of interest.

An IR detector (Anritsu AM9331A) and a power meter (Anritsu MT9810A) were also used to measure the output power of the laser. The system has a wavelength range of from 940 to 1640 nm, and an optical power range from -50 to $+35$ dBm. Moreover, for some specific problems such as the study on the soliton interaction and soliton pulse nonuniformity in Chap.5 and Chap.6, some other equipments are used including a rf spectrum analyzer (HP4396B), a DC to 2 GHz photodetector, and a low speed digital oscilloscope (Fluke PM3390B). The rf spectrum analyzer together with a 7 GHz/ 25 GHz photodetector was used to measure the rf intensity spectrum of the laser output. And the low-speed oscilloscope together with the 2

GHz photodetector was used to monitor the low frequency variations of the laser output.

With these precise equipments, the author can carry out the experimental investigations efficiently and reliably.

3.3 Single-pulse soliton operation of the laser

3.3.1 Single-pulse soliton emission

As mentioned above, in experiment, if the pump power has been increased well beyond the mode-locking threshold, soliton operation can be obtained by appropriately adjusting the orientations of the wave-plates. In experiment, soliton operation of the lasers is confirmed by the sideband generation on the mode-locked pulse spectrum or the significant narrowing of the mode locked pulse widths. The soliton pulse obtained in the laser is similar with those reported previously in passively mode-locked fiber lasers [36, 50]. Here it is called single-pulse soliton, which indicates the soliton pulses have a single-peak intensity profile, to distinguish from the twin-pulse soliton, a novel form of solitary waves that will be discussed later in the next section. Figure 3.3 presents the optical spectrum, and its corresponding autocorrelation trace of a single-pulse soliton of the laser. The pulse width is around 340 fs (FWHM) from the autocorrelation trace. The spectral bandwidth is around 9.6 nm from the optical spectrum. Thus a time-bandwidth product ($\Delta\nu\Delta\tau$) of around 0.39 is obtained.

Using a high-speed oscilloscope and a photo-detector, the author also monitored the pulse dynamics in the cavity. Figure 3.4 shows the oscilloscope trace of the laser output in the same case as that in Fig. 3.3. From the figure, one can get that there is only one soliton pulse circulating with the fundamental cavity round trip time $T \sim 26$ ns in the laser.

Figure 3.3 A single-pulse soliton operation state of the laser. (a): Optical spectrum (Log). The inset is the linear scaled spectrum; (b): Autocorrelation trace.

Figure 3.4 Oscilloscope trace of the laser output as only one single-pulse soliton exists in the cavity. The cavity round trip time is around 26 ns.

Actually, single-pulse soliton operation can be achieved in the laser in a wide range of operation parameters such as different wave-plate settings and pump power. However it is to note that the solitons obtained under different laser operation conditions could have different properties in terms of pulse energy, pulse width and center wavelengths. Figure 3.5 shows that the soliton center wavelength does change as the polarization settings are adjusted. In the laser, it is observed that by changing the wave-plate orientations, the center wavelength of solitons could be tuned in a range from around 1551 nm to 1558 nm. In the meantime, the spectral bandwidth, the pulse duration, and pulse energy also slightly changed. The author has measured the soliton pulse parameters under different polarization settings as presented in Table 3.1. Here the lowest output power that can support the soliton operation, P_{out} , is measured to estimate the corresponding soliton energy, which is reasonable due to soliton energy quantization effect. It could be seen that the shorter the center wavelength is, the broader the spectral bandwidth is, and correspondingly the shorter the pulse width is and the higher the pulse energy is.

Figure 3.5 Soliton spectra under different polarization settings.

Table 3.1 Soliton parameters under different polarization settings.

Setting	λ_c (nm)	$\Delta\lambda$ (nm)	P_{out} (dBm)	$E_{soliton}$ (pJ)
1	1551.8	12	-18.12	43.2
2	1553.2	10.9	-18.19	42.5
3	1554.6	10.1	-18.33	41.1
4	1556.6	8.2	-18.59	38.8

* λ_c : soliton center wavelength; $\Delta\lambda$: 3 dB spectral bandwidth; P_{out} : output power of the laser as only one soliton exist in the cavity; $E_{soliton}$: soliton energy.

Fixing the settings of the wave-plates and only changing the pump power, the pulse width also slightly changed as shown in Fig. 3.6. The soliton bandwidth increases with the pump power, from ~ 9 nm at $P=18$ mW, ~ 9.9 nm at $P=21$ mW, to ~ 10.4 nm at $P=24$ mW, indicating that the pulse becomes narrower. The center wavelength of the soliton only slightly changed.

Figure 3.6 Soliton spectra under different pump power.

It should be noted that the adjustment of the pump power is limited by the energy quantization of the solitons. Further increasing the pump power, a new soliton will be formed and the soliton parameters change back.

3.3.2 Sideband generation

As can be seen from Fig. 3.3a, Fig. 3.5 or Fig. 3.6, apart from the broadband continuous soliton spectrum, there are also several sharp discrete spectral lines superimposed on the soliton spectrum. The generation of these spectral lines on the soliton spectrum is known as the sideband emission of the soliton lasers. In fact the sidebands appear in pairs on the soliton spectrum. Each pair locates almost symmetrically on each side of the soliton spectral peak, however, with unsymmetrical spectral intensities.

The mechanism for the sideband generation has been intensively studied and already well understood by now [83-92]. As a soliton circulates in the cavity, it periodically endures the amplification of the erbium-doped fiber and cavity loss introduced by the discrete cavity components such as the output coupler, and the polarizer. As a result, the soliton reshapes itself periodically in the cavity with the cavity round trip time and sheds the excess energy into the linear dispersive waves. The radiation of the dispersive waves is a generic feature of soliton operation in the presence of periodical perturbations, e.g. the soliton propagation in an amplified transmission link or a fiber laser. The emitted dispersive waves have a continuous spectrum like the soliton spectrum. However, if the perturbations are periodic, then the dispersive waves emitted by a soliton at each perturbation can interference with one another. If at certain wavelengths the constructive interference condition is fulfilled, dispersive waves at the wavelengths will be resonantly enhanced, which form the discrete sidebands in the soliton spectrum.

Under the assumption that the soliton is a NLSE soliton, the sideband positions on the soliton spectrum can be calculated as [84]

$$\Delta\omega_m = \pm \frac{1}{T_0} \sqrt{m \frac{8Z_0}{Z_p} - 1} \quad , \quad (3.3.1)$$

where Z_0 and Z_p are the soliton period and the laser cavity length, m is integers representing the order of the sidebands. However, solitons formed in a laser is normally frequency chirped. Therefore, the actual sideband positions also slightly shift with the pump power variations [89]. The strength of the experimentally observed sidebands is normally asymmetric, which is a result of the asymmetric linear cavity loss of the laser with wavelength [92]. In addition, since the soliton period Z_0 is a function of the GVD and soliton pulse width T_0 as

$$Z_0 = \frac{\pi}{2} \cdot \frac{T_0^2}{|\beta_2|} \quad , \quad (3.3.2)$$

The position of sidebands is often used to estimate the average dispersion of the laser cavity.

Physically, sideband emission of a soliton resonantly draws energy away from the soliton. It was theoretically shown that the energy exchange rate is proportional to the spectral strength of the soliton at the location of the sideband [90]. In order for a soliton to be stable in the cavity, the cavity gain must not only compensate the linear cavity losses caused by the cavity output and cavity component absorptions, but also balance the loss caused by the resonant dispersive wave emission. In the case of laser soliton operation the soliton pulse width narrows with the increase of pump power. A narrower soliton pulse means a broader soliton spectrum, as the sideband positions only slightly shift with the pump power, which further means a stronger energy exchange rate from soliton to the dispersive waves. Depending on the concrete laser parameters, experimentally increasing the pump power could significantly increase the sideband strength, which can be also seen from Fig. 3.6. In addition, altering the linear phase delay by adjusting the polarization controllers will also change the pulse parameter of the solitons and consequently change the strength of the sidebands as shown in Fig. 3.5. Dispersive waves are linear waves. They are intrinsically unstable against the modulation instability (MI). When their strengths grow beyond the MI threshold, they generate their own sidebands through the MI. A phenomenon known as sub-sideband generation caused by the MI of the

sidebands then will also appear in the soliton spectrum [93]. Generally, as far as the sidebands are stable, they have no influence on the soliton stability. This is because sidebands are radiative wave in nature. They have no interaction with the solitons emitting them. However, when they are unstable, they affect the soliton evolution in the cavity. The modulation instability of the sidebands can be easily suppressed experimentally through reducing the pump power. Nevertheless, in the practice one cannot reduce the sideband strength to too low value, as when they are too weak, the soliton itself will also become weakly stable against the environmental perturbations.

3.3.3 Pump power hysteresis

A special feature of soliton operation in passively mode-locked fiber soliton lasers (PMLFSLs) is the pump power hysteresis [38,95]. To obtain soliton operation in the laser from a CW state, the pump power must be beyond a certain pump threshold (~ 80 mW for single-pulse soliton operation). However, once soliton operation is achieved, the pump power can be reduced to a very low value (as low as 18 mW) while still maintaining the soliton operation. This property of soliton operation in fiber lasers is known as the pump power hysteresis. The physical mechanism of pump-power hysteresis for soliton operation is now well understood. It is turned out that the pump hysteresis is related to the simultaneously generation of multiple solitons in the cavity. In order for a soliton pulse to be formed in a laser cavity, the laser must first become mode-locked. The threshold one observed for the self-starting of soliton operation of the laser is in fact its mode-locking threshold. Immediately after the mode locking, multiple soliton pulses will be simultaneously generated due to the soliton energy quantization. Once multiple solitons are formed in the cavity, decreasing the pump power will only reduce the number of solitons in the cavity, while the soliton operation itself is not destroyed. The soliton operation remains until the last soliton in the cavity is destroyed. And because supporting one soliton in the cavity requires less pump power than obtaining the mode locking, the soliton operation of a laser therefore displays pump power hysteresis.

Not only the soliton operation but also the formation and annihilation of an individual soliton in the lasers exhibit pump hysteresis. Figure 3.7 shows an experimentally measured relationship between the laser output power /pump power and the number of solitons in a similar fiber laser cavity [94]. Through carefully controlling the pump power, solitons can be generated or destroyed one by one in the lasers. With solitons already existing in cavity, increasing the pump power to a certain value, a new soliton will be generated. The pump power increase required for a new soliton generation is significantly less than that required for the same laser to self-start mode locking. And depending on the number of solitons already existing in the cavity, the pump power increase required for the generation of a new soliton varies. The more of the solitons in the cavity, the larger is the pump power increase for the next soliton generation. This can be understood by considering the existence of the dispersive waves. It is to emphasize that the output power shown in Fig. 3.7 consists of the power of both the soliton and the dispersive waves. When more soliton are in the cavity, the power of dispersive waves also increases. Therefore, the total pump power required for a new soliton generation increases. However, also from Fig. 3.7, it is observed that independent of the number of solitons in cavity, the pump power decrease between two adjacent soliton annihilations is almost always the same. This could be used to estimate the energy of an individual soliton.

Obviously, the mechanism of soliton formation in a laser with or without solitons in the cavity is different. In the case that there is no soliton in the cavity, in order for solitons to be formed in the cavity, the laser must first become mode-locked. Mode locking in a laser always requires that the pump strength be beyond the mode-locking threshold. Mode locking generates an optical pulse in the laser cavity. If the pulse is strong enough, soliton shaping then occurs, which converts the pulse into a soliton. While if there are solitons in cavity, mode locking of a laser is already realized. New solitons will then be formed either through the pulse splitting [95] or the soliton shaping of the dispersive waves [96] or unstable CW depending on the concrete laser operation conditions.

Figure 3.7 Output power versus number of solitons in cavity. The generation and annihilation of each individual soliton exhibit clear pump hysteresis. (After Ref. [94])

3.3.4 Energy quantization and multiple soliton operation

Soliton operation of the laser is characterized by simultaneous generation of multiple soliton pulses in the cavity. When they are far apart separated from each other, all the soliton pulses have the same properties, i.e. the same pulse energy and pulse duration. This property of soliton operation is known as the energy quantization effect.

Energy quantization is a well-known feature of solitons in passively mode-locked fiber lasers. Grudinin et al. have firstly reported an experimental observation of energy quantization of single-pulse solitons in a figure-of-eight fiber laser [40]. Since in a fiber laser, in addition to the propagation in the fiber, an optical pulse in the cavity also endures losses introduced by other cavity components such as the polarizer and the output coupler, in order for a soliton to survive in a laser cavity,

the gain it experienced must compensate these losses. This internal energy balance of a soliton pulse then results in that all the fundamental solitons in a laser cavity have exactly the same peak power, and therefore the same pulse width. More concretely in a fiber laser using the NPR technique, a peak power clamp mechanism effect exists and limits the peak power of the solitons. Under this mechanism, multiple soliton pulses with the same pulse properties are generated with strong pump. This mechanism of the soliton energy quantization will be further discussed in Chap.4.

Experimentally, soliton energy quantization can be easily identified through monitoring the laser output using a high-speed oscilloscope. To demonstrate this, the author has shown in Fig. 3.8 the oscilloscope trace of a typical multiple single-pulse soliton operation state of the laser. 5 single-pulse solitons coexist in the laser cavity. The soliton nature of the pulses is confirmed by their optical spectrum, which shows no profile difference with that in Fig. 3.3a except for an intensity increase due to the coexistence of multiple solitons. As can be seen in the Fig. 3.8, all the solitons have exactly the same pulse height in the oscilloscope trace. Since the single-pulse soliton in the laser has a pulse width of only about 340 fs, obviously the electronic detection system used in the experiment cannot display the exact soliton pulse form. However, as the oscilloscope pulse height is directly proportional to the actual soliton pulse energy, the equal oscilloscope pulse height demonstrates clearly that all the solitons in the cavity have the same pulse energy.

The energy quantization of the solitons can be used to accurately determine the energy of a single soliton. The soliton energies provided in Table 3.1 are measured this way. Frequently soliton energy of a fiber laser is estimated based on the laser output power and the number of solitons inside the laser cavity as shown. But apart from the soliton pulses there also exist the dispersive waves in a laser, which contribute to the laser output power as well. Therefore, this method generally overestimates the energy of a soliton pulse. Due to the energy quantization of the soliton pulses, it is however possible to eliminate the influence of the dispersive waves through measuring the laser output power change associated with the

annihilation of a soliton. As shown in Fig. 3.7, when the pump power is continually reduced so that the solitons are eliminated one by one, the output power of the laser indeed exhibits equal amount of energy jumps. It is to note that generally when one soliton is destroyed, the output power of a laser will not make an energy jump of the single-pulse soliton energy. This is because after a soliton is destroyed the extra energy stored in the gain medium is converted into the dispersive waves, which adds up with the dispersive waves already existing and make the total laser output power almost unchanged. To determine the soliton energy, every time after a soliton is destroyed, one needs to reduce the dispersive wave strength to the same level as that before a soliton was destroyed. Only under this condition the output power difference measured will give the energy of a soliton. Nevertheless, with the aid of a high-speed oscilloscope and an optical spectrum analyzer, one could easily determine the number of solitons and the relative strength of dispersive waves in a cavity.

Figure 3.8 The oscilloscope trace of a multiple single-pulse soliton operation state of the laser.

As mentioned above, multiple soliton operation is a generic feature of soliton operation in PMLFSLs. Multiple soliton pulses coexist in the cavity as a result of the energy quantization effect of the soliton operation and can form various distribution modes under the effect of the soliton interaction. Irregularly distributed soliton trains with random but fixed temporal separations, soliton bunches, harmonic mode locking and bound solitons, have all been reported before in

PMLFSLs [41-43,97]. All these operation modes have also been observed in the present laser. Figure 3.8 already presented a stable irregularly distributed multiple soliton state. As shown in the figure, there are 5 pulses coexisting in the cavity. Since the separations between solitons are very large, no mutual interaction exists between each other, and consequently, they exhibit the same height in the oscilloscope trace, which shows the effect of the energy quantization. Figure 3.9a shows a case of soliton bunching. From the oscilloscope trace, one can see that soliton pulses are bunched together and move together in the cavity. Figure 3.9b shows the oscilloscope trace of a harmonic mode locking state. In this case, 8 solitons distribute evenly in the cavity. As a result, the repetition rate of the laser output is enhanced 8 times higher. Experimentally it is found that all these specific multiple soliton operation modes are formed under the effect of various soliton interactions, which will be expanded for discussion in Chap. 5.

Figure 3.9 Oscilloscope traces of multiple single-pulse soliton operation of the laser. (a): Soliton bunch; (b): Harmonic mode locking.

3.4 Twin-pulse soliton operation of the laser

3.4.1 Twin-pulse soliton emission

Multi-hump soliton generation and its dynamics have recently attracted considerable attention [98-101]. Multi-hump spatial solitons, a kind of self-guided beams that consist of more than one transverse mode therefore have more than one peak in their intensity profiles, were recently observed in experiments [98-100]. Although multi-hump temporal solitons have been theoretically predicted [101], they were not experimentally observed.

Apart from the traditional single-pulse solitons, by appropriately setting the orientations of the wave-plates, the laser can also emit a twin-pulse form of soliton pulses [57, 102-106]. Compared with the single-pulse solitons, the twin-pulse solitons are characterized as that, their soliton profiles have double intensity peaks that have discrete and fixed separations. Figure 3.10 shows a typical experimentally measured twin-pulse soliton spectrum and its corresponding autocorrelation trace. Because the soliton has a profile of twin-pulse form, its optical spectrum becomes strongly modulated as in Fig. 3.10a. Clearly both from the autocorrelation trace and the optical spectrum, it is to see that the two pulses in a twin-pulse soliton have very close pulse separation. The almost symmetric soliton spectrum with a deep dip in the center further suggests that the optical phase difference between the two pulses is about π . The spectral modulation period gives an indirect measure of the twin-pulse peak separation in the time domain. As can be determined either from the autocorrelation trace or the spectral modulation, the pulse peak separation between the twin pulses is about 930 fs, and each of the two pulses in the soliton has a pulse width roughly equal to that of the single-pulse solitons in the laser. Figure 3.11 shows the oscilloscope trace as only one twin-pulse soliton exists in the cavity.

Figure 3.10 A twin-pulse soliton operation state of the laser. (a): Optical spectrum (Log). The inset is the linear scaled spectrum; (b): Autocorrelation trace.

Figure 3.11 Oscilloscope trace of the laser output as only one twin-pulse soliton exists in the cavity. The cavity round trip time is around 26 ns.

At the first beginning, this operation was regarded as a state of bound solitons formed under direct soliton-soliton interaction in passively mode-locked fiber ring lasers [102, 103]. However, further detailed experimental studies revealed that the two pulses in a bound-soliton always behave as a unit. They are in fact not separable. This state is very stable. Once obtained, even further varying the laser operation condition such as carefully changing the pump power and /or the cavity linear phase delay will not destroy the state or alter the soliton separation. And no coexistence of unbound single-pulse solitons or other type of pulses was found with the bound solitons. Their properties show that they are not just binding states of single-pulse solitons due to direct soliton interaction, but rather a novel form of multi-pulse solitons in the fiber laser, which is then called twin-pulse solitons [104-106]. Extended experimental investigations on the twin-pulse solitons have revealed properties of the solitons such as energy quantization, pump power hysteresis [105], quasi-harmonic and harmonic mode-locking [106], which are exactly the same as those of the conventional single-pulse solitons in the laser.

3.4.2 Sideband generation and pump power hysteresis

Like the case of the single-pulse solitons, the spectral sidebands caused by the dispersive wave emission can also be seen from the optical spectrum of twin-pulse solitons in Fig. 3.10a. The mechanism for the sideband generation is the same with that of single-pulse solitons. However, one may have noted that the strength of the sidebands greatly weakens comparing with those of the single-pulse solitons. This can be understood as considering that the dispersive waves are linear waves and experience the linear cavity transmission in the cavity. Since the twin-pulse solitons are obtained at different linear phase bias with single-pulse solitons, the corresponding linear loss is also different, which in consequence cause the difference in their sideband strengths.

The twin-pulse soliton operation has a pump threshold of about 130 mW in the laser, whose value may vary over a range of tens of milli-watts depending on the setting

of the polarization controllers. While, the same as the case of single-pulse solitons, once the twin-pulse soliton operation is obtained, it can remain there even when the pump power is reduced to as low as 32 mW, exhibiting a feature of pump power hysteresis.

3.4.3 Energy quantization and multiple twin-pulse soliton operation

Multiple twin-pulse solitons can also coexist in the laser cavity just as single-pulse solitons, and in particular, as the multiple solitons are far apart from each other to be exempt from the mutual interactions, all of them have exactly the same pulse energy and twin-pulse separation. It is believed that the same mechanisms as described in Sec. 3.3 for single-pulse solitons are also responsible for the pump power hysteresis and energy quantization feature of the twin-pulse soliton operation of the laser. Using the same method described previously for measuring the single-pulse soliton energy the author also experimentally measured the energy of individual twin-pulse soliton in the laser. The result is shown in Fig. 3.12. Through carefully decreasing the pump power, the number of twin-pulse solitons in the laser cavity can be reduced one by one. By keeping the strength of dispersive waves always at the same level, the output power of the laser then exhibited discrete power drops corresponding to the annihilation of twin-pulse solitons. Within the experimental measurement error range, all the jumps do exhibit equal amount of output power changes, which shows that the energy of all twin-pulse solitons in cavity is the same.

Figure 3.12 Experimentally measured output power versus number of twin-pulse solitons in the laser cavity.

Figure 3.13a shows again the oscilloscope trace of a multiple twin-pulse soliton operation state of the laser. Figure 3.13b shows its corresponding optical spectrum. Like the case of the single-pulse soliton operation, multiple twin-pulse solitons can coexist in the laser cavity and exhibit a static distribution pattern. As can be seen from Fig. 3.13a, all the soliton pulses in the cavity are far apart from each other and so have exactly the same pulse energy. The twin-pulse nature of each spike is identified by their optical spectrum and autocorrelation trace, which were simultaneously monitored in the experiment. Comparing the soliton spectra of the laser under the multiple twin-pulse soliton operation to that measured when only one twin-pulse soliton was in the cavity as shown in Fig. 3.10a, except that the soliton spectral strength is increased, there is no other visible spectral profile changes. Figure 3.13b also shows the soliton spectrum after one twin-pulse soliton in the cavity is destroyed. Annihilation of one soliton generally causes the solitons to rearrange their relative positions in the cavity, forming a new static oscilloscope pulse pattern. However, in the optical spectrum the only change observable is a small drop on the spectral strength as shown in Fig. 3.13b. The same property was also observed on their autocorrelation traces. When multiple pulses are in the cavity, except that the strength of the autocorrelation trace is increased, there is no

measurable difference to that when only one twin-pulse soliton is in the cavity. All these experimental results clearly demonstrated that each of the twin-pulse solitons in the cavity has not only the same pulse energy, but also the same twin-pulse separation and pulse width, namely they are identical. Although in the laser both the single-pulse solitons and the twin-pulse solitons are observed respectively, no coexistence of both was observed in the cavity in all the experiments, which can also be seen from Fig. 3.8 and Fig. 3.13.

Figure 3.13 Multiple twin-pulse soliton operation of the laser. (a): Oscilloscope trace of the state; (b): Optical spectra of the state. The lower spectral strength trace corresponds to the spectrum after one twin-pulse soliton is destroyed.

As described above, multiple twin-pulse solitons can be simultaneously generated in the laser cavity as a result of the energy quantization effect, and form various distribution modes under the effect of their mutual interactions such as stable irregular distribution mode, twin-pulse soliton bunches, quasi- and harmonic mode locking of twin-pulse solitons just as the single-pulse solitons do. Figure 3.13a is a stable irregular distributed multiple twin-pulse soliton state. 5 twin-pulse solitons are scattered in the cavity with large, irregular, and fixed separations. Figure 3.14a and b presents both a bunch state and a harmonically mode-locked state of twin-pulse solitons. Soliton interaction will be specifically discussed in Chap. 6.

Figure 3.14 Oscilloscope traces of multiple twin-pulse soliton operation of the laser. (a): A bunch state of twin-pulse solitons; (b): A harmonically mode-locked twin-pulse soliton state.

3.4.4 Comparisons between single-pulse and twin-pulse solitons

Comparisons between the two types of soliton operations of the laser based on the experimental observations are provided as follows:

- I. Pump power. The mode-locking threshold of a laser depends on the concrete experimental condition, such as the orientations of the wave-plates in the laser, which is difficult to be compared between the cases of single-pulse soliton and the twin-pulse soliton operation. However, the minimum pump power required for maintaining a single soliton pulse in a laser cavity could be directly compared. In the experiment it is found out while a minimum pump power of around 32 mW is required for supporting the twin-pulse soliton operation of the laser, only around 18 mW of pump power is required to support the single-pulse soliton operation.
- II. Polarization settings. In experiment, the two types of soliton operation are switchable simply by adjusting the polarization orientation of the polarization controllers. It is observed that the single-pulse soliton operation can exist in a large polarization range, while the twin-pulse soliton operation can only be observed in a very narrow range. And their operation regimes are just adjacent to each other. Since they operate under different polarization settings, which correspond to different linear phase delays, their sideband strengths are also different.
- III. Soliton properties. As described above, except that they have very different pulse profiles, both the two forms of solitons exhibit exactly the same properties. Energy quantization, pump power hysteresis, various multiple soliton operation modes, for both the single-pulse and twin-pulse solitons have observed in the experiments. This strongly suggested that the twin-pulse solitons is an intrinsic soliton operation state of the fiber laser just as the

single-pulse solitons. The close similarity in properties between the twin-pulse soliton and the single pulse soliton could also be illustrated by comparing their oscilloscope traces in Fig. 3.4 and Fig. 3.11. For the benefit of comparison, the author has recorded the two traces using exactly the same detection system and scales. Compared to the condition of single pulse soliton, expect that the bound solitons have higher pulse height; the oscilloscope traces show almost no difference. Although due to the limitation in resolution of the detection system, the oscilloscope traces are quite similar, except that the twin-pulse solitons shows higher intensities, in experiment, from their quite different optical spectra, one can easily distinguish the two forms of soliton operations of the laser.

3.5 Noise-like pulse of the laser

Besides the soliton operations demonstrated above, the passively mode-locked fiber lasers can also emit a noise-like mode-locked pulse. Noise-like pulses are often observed in high power passively mode-locked fiber lasers based on the nonlinear polarization rotation [107-111]. Because the temporal behavior resembles noise burst feature, it is thus named noise-like mode. This operation mode has also been obtained in the laser just by properly adjusting the polarization controllers. Figure 3.15 presents the optical spectrum, and the autocorrelation trace of such an operation state. As shown in Fig. 3.15a, the noise-like pulse has a broad spectrum that is quite different with those of the solitons. The spectral bandwidth is as broad as around 11 nm, showing that the laser is still mode-locked. Its autocorrelation exhibits as a very short spike with wide shoulders, which is very typical for the noise-like pulses. Since the autocorrelation measurement is non-collinear and therefore was affected only by the light intensity, for a noise-like mode it cannot give the real profile in time domain. Thus to observe the temporal behavior, the 50 GHz oscilloscope and the 25 GHz photo-detector are used to monitor the output of the laser in such a state. Figure 3.16 presents a typical oscilloscope trace of noise-like operation state at the same pump level with Fig. 3.15. Similar to those observed

in other fiber lasers [107-111], there is only one intense pulse envelope present in the cavity per round trip.

Figure 3.15 Noise-like mode of the laser. Pump = 196 mW. (a): Optical spectrum (Log); (b): The corresponding autocorrelation trace.

With an expanded window as shown in Fig. 3.17, it is found that the noise-like pulse actually has an envelope of up to several tens of picoseconds in time domain. And here the noise-like nature of the mode can also be seen. Furthermore, the author observed that the width of the output pulse envelope varies depending on the total energy in the cavity. Decreasing the pump power decreases the width of the output pulse envelope. The pulse envelopes for different pump power are recorded as shown in Fig. 3.17. However, it is found that the optical spectrum of the laser

output remains the same as decreasing the pump power except an in-whole reduction on the spectral intensity as shown in Fig. 3.18. This shows that the spectrum of the mode is not defined by the entire noise-like pulse envelope, but by the fine structure in the pulse.

Figure 3.16 Oscilloscope trace of noise-like pulse operation of the laser recorded with multi-round cavity trips. The cavity round trip time is around 26 ns.

The mechanism for the generation of the noise-like pulses has been previously discussed [107,111]. However, what is the exact mechanism for noise-like pulse generation has not been clearly presented. The investigations show that this noise-like pulse is formed because the accumulation of a large amount of nonlinear chirping as the pulse intensity becomes so high under low cavity loss. Experimentally, this noise-like regime of operation occurs between the single-pulse soliton regime and the non-stationary regime. Detailed investigations on mechanism of the noise-like pulse formation will be presented in the next chapter.

Figure 3.17 Oscilloscope traces of noise-like pulses under different pump power. From (a) to (c), the pump power is 120 mW, 141mW, and 196 mW, respectively.

Figure 3.18 Optical spectra (Lin) under different pump power. (a): Pump= 165 mW; (b) Pump= 196 mW. The spectrum shows no difference except an overall intensity change.

Chapter 4

Numerical simulations

As introduced in Chap. 2, the pulse propagation in a fiber laser is governed by the coupled Ginzburg-Landau equations (CGLE). GLE/NSLE is a nonlinear partial differential equation that cannot be solved analytically except for some special cases. Therefore for a better understanding on the operation of the fiber soliton laser, especially to get an insight on the observed experimental phenomena, it is necessary to carry out the numerical simulations.

A numerical model based on the CGLE is built up to simulate the dynamics of the fiber laser. This chapter aims to elaborate the numerical simulations performed on the basis of the model. Section 4.1 introduces the methodology used in the numerical simulation. The simulation process and parameters are presented in Sec. 4.2. The three operation regimes and all their detailed features obtained in experiment have been precisely reproduced in the numerical simulations and presented in Sec. 4.3. Direct comparison between the experiments and numerical simulations are also provided in this section. Furthermore, based on both the experimental and numerical results, the operation characteristics of the laser are studied in Sec. 4.4 at a more comprehensive level. Particularly, the formation of different operation regimes and the properties of solitons such as the energy quantization have been further understood through the numerical simulations.

4.1 Numerical methodology

4.1.1 Mathematical preparations

As the author discussed in Chap. 2, nonlinear pulse propagation in optical fibers are described by the coupled NLSEs (2.4.10):

$$\begin{aligned}\frac{\partial u}{\partial Z} &= i\beta u - \delta \frac{\partial u}{\partial T} - \frac{i}{2} k'' \frac{\partial^2 u}{\partial T^2} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i\gamma}{3} v^2 u^* \\ \frac{\partial v}{\partial Z} &= -i\beta v + \delta \frac{\partial v}{\partial T} - \frac{i}{2} k'' \frac{\partial^2 v}{\partial T^2} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + \frac{i\gamma}{3} u^2 v^*\end{aligned}\quad (4.1.1)$$

In numerical simulation, there is a need to transform the equations into a dimensionless form. For this purpose, the following transformations are defined:

$$\tau = \frac{T}{t_o}, \quad \xi = \frac{z}{L_D}, \quad L_D = \frac{t_o^2}{|k''|}, \quad \text{and} \quad L_{NL} = \frac{1}{\gamma P_o}$$

where L_D and L_{NL} are called the dispersion length and the nonlinear length, respectively. In addition, the two wave functions endure the following transformation:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \left[\frac{L_D}{L_{NL} P_o} \right]^{\frac{1}{2}} \begin{pmatrix} u \\ v \end{pmatrix} \quad (4.1.2)$$

Substituting the above definitions into Eqs. (4.1.1), one can get:

$$\begin{aligned}\frac{\partial X}{\partial \xi} &= i\beta' X - \delta' \frac{\partial X}{\partial \tau} + \frac{i}{2} \frac{\partial^2 X}{\partial \tau^2} + i \left(|X|^2 + \frac{2}{3} |Y|^2 \right) X + \frac{i}{3} Y^2 X^* \\ \frac{\partial Y}{\partial \xi} &= i\beta' Y - \delta' \frac{\partial Y}{\partial \tau} + \frac{i}{2} \frac{\partial^2 Y}{\partial \tau^2} + i \left(|Y|^2 + \frac{2}{3} |X|^2 \right) Y + \frac{i}{3} X^2 Y^*\end{aligned}\quad (4.1.3)$$

where $\beta' = \beta L_D$, and $\delta' = \delta L_D / t_o$.

Letting $X=u$, $Y=v$, $\beta' = \beta$, and $\delta' = \delta$, then one can obtain the *dimension-less* coupled NLSEs as follows:

$$\begin{aligned}\frac{\partial u}{\partial \xi} &= i\beta u - \delta \frac{\partial u}{\partial \tau} + \frac{i}{2} \frac{\partial^2 u}{\partial \tau^2} + i \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i}{3} v^2 u^* \\ \frac{\partial v}{\partial \xi} &= i\beta u - \delta \frac{\partial v}{\partial \tau} + \frac{i}{2} \frac{\partial^2 v}{\partial \tau^2} + i \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + \frac{i}{3} u^2 v^*\end{aligned}\quad (4.1.4)$$

where the magnitude of δ and β are given by:

$$\delta = \frac{(k' - l')t_o}{2|k'|} = \frac{(k_o - l_o)t_o}{2|k'| \omega_o} = \frac{\frac{\Delta n \cdot \omega_o}{c} t_o}{2 \omega_o \left(\frac{\lambda_o}{2\pi c^2} D(\lambda_o) \right)} = \frac{\pi c \Delta n t_o}{\lambda_o D(\lambda_o)}, \quad (4.1.5)$$

$$\beta = \frac{(k_o - l_o)t_o^2}{2|k'|} = \frac{\frac{\Delta n \cdot \omega_o}{c} t_o^2}{2 \left(-\frac{\lambda_o}{2\pi c^2} D(\lambda_o) \right)} = \frac{-2(\pi c)^2 \Delta n t_o^2}{D(\lambda_o) \lambda_o^2} = \delta t_o \omega_o. \quad (4.1.6)$$

Here Δn represents the refractive index difference between the two polarization modes, λ_o is the (center) wavelength of the propagating signal, and $D(\lambda_o)$ is the dispersion parameter at λ_o . Considering the gain and gain dispersion in an EDFL, the coupled NLSEs are written into CGLE:

$$\begin{aligned}\frac{\partial u}{\partial \xi} &= i\beta u - \delta \frac{\partial u}{\partial \tau} + \frac{i}{2} \frac{\partial^2 u}{\partial \tau^2} + i \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + \frac{i}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial u}{\partial \tau} \\ \frac{\partial v}{\partial \xi} &= i\beta u - \delta \frac{\partial v}{\partial \tau} + \frac{i}{2} \frac{\partial^2 v}{\partial \tau^2} + i \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + \frac{i}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial v}{\partial \tau}\end{aligned}\quad (4.1.7)$$

4.1.2 Split-step Fourier method

The NLSEs /GLEs are nonlinear partial differential equations. Thus, it is difficult to get analytical solutions except for some unique cases in which the inverse scattering

approach can be used. In view of that, to understand the nonlinear pulse propagation in optical fibers, it is often necessary to employ a numerical approach.

The split-step Fourier (SSF) method has been extensively used to solve the pulse propagation problems in nonlinear dispersive media [16]. It is relatively quicker compared to most finite difference methods due to the use of the fast Fourier transform (FFT) algorithm. To understand the philosophy behind the SSF method, it is useful to write the NLSE in the form of:

$$\frac{\partial A(z,T)}{\partial z} = (\hat{D} + \hat{N})A(z,T) \quad (4.1.8)$$

where $A(z,T)$ is the pulse envelope, $\hat{D}$ is a propagation operator that accounts for dispersion and absorption in a linear medium, and $\hat{N}$ is a nonlinear operator that governs the effect of fiber nonlinearities on pulse propagation.

In general, dispersion and nonlinearity act together along the length of the fiber. The SSF method obtains an approximate solution by assuming that in propagation the optical field over a small distance of h , the dispersive and nonlinear effects can be assumed to act independently of each other. More specifically, propagation from z to $z+h$ is carried out in two steps. In the first step, the nonlinearity $\hat{N}$ acts alone and dispersion $\hat{D} = 0$. In the second step, dispersion $\hat{D}$ acts alone and the nonlinearity $\hat{N}$ is set to zero.

Mathematically,

$$A(z+h,T) \approx \exp(h\hat{D})\exp(h\hat{N})A(z,T) \quad (4.1.9)$$

In order to estimate the accuracy of the SSF method, a formal exact solution of Eq. (4.1.8) is given by:

$$A(z+h, T) = \exp[h(\hat{D} + \hat{N})]A(z, T) \quad (4.1.10)$$

Comparing Eq. (4.1.9) with (4.1.10), it can be seen that the SSF method has ignored the non-commutating nature of the operators. Since for two noncommuting operators $\hat{a}$ and $\hat{b}$,

$$\exp(\hat{a})\exp(\hat{b}) = \exp\left(\hat{a} + \hat{b} + \frac{1}{2}[\hat{a}, \hat{b}] + \frac{1}{12}[\hat{a} - \hat{b}, [\hat{a}, \hat{b}]] + \dots\right), \quad (4.1.11)$$

considering $\hat{a} = h\hat{D}$ and $\hat{b} = h\hat{N}$, the dominant error result from the single commutator $\frac{h^2}{2}[\hat{D}, \hat{N}]$. Thus, the SSF method is accurate to second order of the step size h .

In order to improve the accuracy of the SSF method, a different approach was adopted to propagate the optical pulse over one segment from z to $z+h$. The main difference is that the effect of nonlinearity is included in the middle of the segment rather than at the segment boundary. As a result, the given equation below will replace Eq. (4.1.9) in the earlier discussion.

$$A(z+h, T) \approx \exp\left(\frac{h}{2}\hat{D}\right)\exp(h\hat{N})\exp\left(\frac{h}{2}\hat{D}\right)A(z, T) \quad (4.1.12)$$

This method is known as the symmetrized SSF method because of the symmetric form of the exponential operators in the equation. Comparing with Eq. (4.1.9), the most advantage of using Eq. (4.1.12) is that the dominant error is caused by the double commutator in Eq. (4.1.11) and is of the third order of the step size h .

The implementation of the symmetrized SSF method is relatively straightforward. As shown in Fig. 4.1, the fiber length is divided into a large number of segments that may not be equally spaced. The optical pulse is propagated from segment to segment using the prescription of Eq. (4.1.12). The use of the FFT algorithm makes

the numerical evaluation relatively faster and this results in the SSF method being faster than most finite difference methods by up to two orders of magnitude.

Figure 4.1 Schematic illustration of the symmetrized split-step Fourier method (After Ref. [16]).

4.1.3 Procedure of numerically solving the equations

To use the SSF method, one must transform the coupled NLSEs/CGLE to an appropriate format, which can be further translated into a computer program. The transforming procedure can be summarized as follows:

Firstly, the energy exchange terms in the CGLE are eliminated by defining:

$u' = \frac{(u + iv)}{\sqrt{2}}$ and $v' = \frac{(u - iv)}{\sqrt{2}}$. Physically, this transformation has the same effect as

changing the linearly polarized beam into a circularly polarized beam. In this case, the energy exchange term due to the four-wave mixing effect disappears. The energy exchange is now contained in the u' and v' components, since either u' or v' contains both u and v as in the earlier definitions. The CGLE is then transformed into:

$$\begin{aligned}
\frac{\partial u'}{\partial z} &= i\beta v' - \delta \frac{\partial v'}{\partial t} + \frac{i}{2} k'' \frac{\partial^2 u'}{\partial t^2} + \frac{2i\gamma}{3} (|u'|^2 + 2|v'|^2) u' + \frac{g}{2} u' + \frac{g}{2\Omega_g^2} \frac{\partial^2 u'}{\partial t^2} \\
\frac{\partial v'}{\partial z} &= i\beta u' - \delta \frac{\partial u'}{\partial t} + \frac{i}{2} k'' \frac{\partial^2 v'}{\partial t^2} + \frac{2i\gamma}{3} (2|u'|^2 + |v'|^2) v' + \frac{g}{2} v' + \frac{g}{2\Omega_g^2} \frac{\partial^2 v'}{\partial t^2}
\end{aligned} \quad (4.1.13)$$

Then the equations are solved using the SSF method. In the program, the propagation terms are solved in the frequency domain. For simplicity of the calculation, $x = \frac{(u' + v')}{\sqrt{2}}$ and $y = \frac{(u' - v')}{\sqrt{2}}$ are defined. Then the two equations take the form:

$$\begin{aligned}
\frac{\partial x}{\partial z} &= i\beta x - \delta \frac{\partial x}{\partial t} + \frac{i}{2} k'' \frac{\partial^2 x}{\partial t^2} + \frac{g}{2\Omega_g^2} \frac{\partial^2 x}{\partial t^2} \\
\frac{\partial y}{\partial z} &= -i\beta y + \delta \frac{\partial y}{\partial t} + \frac{i}{2} k'' \frac{\partial^2 y}{\partial t^2} + \frac{g}{2\Omega_g^2} \frac{\partial^2 y}{\partial t^2}
\end{aligned} \quad (4.1.14)$$

Solving the two differential equations will give x and y . The results of x and y are then transferred back into u' and v' using: $u' = \frac{x + y}{\sqrt{2}}$ and $v' = \frac{x - y}{\sqrt{2}}$.

And the nonlinear terms are solved in the time domain:

$$\begin{aligned}
\frac{\partial u'}{\partial z} &= \frac{2i\gamma}{3} (|u'|^2 + 2|v'|^2) u' + \frac{g}{2} u' \\
\frac{\partial v'}{\partial z} &= \frac{2i\gamma}{3} (2|u'|^2 + |v'|^2) v' + \frac{g}{2} v'
\end{aligned} \quad (4.1.15)$$

4.2 Numerical simulations

4.2.1 Simulation process

Soliton operation of lasers was traditionally modeled by the master equation [34,56, 112], which takes into account the cavity dispersion, nonlinear SPM, gain and saturable absorption in a laser. An advantage of the model is that it can give an explicit analytical solution of the mode-locked pulse of a laser, which therefore provides a clear understanding on the soliton nature of the pulse and its dependence on various laser operation parameters. However, since the analytical solution is only possible under the assumption that the pulse change within one cavity round trip is small, and the model also ignores the discrete actions of the cavity components, this master-equation model have limited application in simulating a real fiber soliton laser.

A fundamental difference of the numerical model used in this thesis work to those used by other groups is that the author didn't make the so-called master-equation simplification. Therefore, there is no limitation on the action of cavity components to the soliton pulse when it circulates in the cavity. To closely simulate the laser operation, in particular, considering that the experimental laser cavity comprises segments of optical fibers with different dispersion, and also that the polarization controllers and the polarization-dependent isolator have discrete actions on the light beam circulating in the cavity, the numerical calculation is performed in the following way. The numerical calculation was started with a weak noise light beam incident to the laser system. According to the laser cavity configuration, within one round trip the noise light beam will first propagate in the three pieces of fibers with different group velocity dispersions, where the light propagation is described by the Eq. (4.1.7). Then it will pass through the polarization controllers and the polarization-dependent isolator. The actions of the polarization controllers and the isolator are modeled by their corresponding transformation matrixes. After one round trip of propagation in the cavity, the final state of the light is then used as the

starting state for the next round of calculation. This process repeats until a steady state of light pulse is obtained.

With this pulse tracing method, the effect of nonlinear coupling between the two orthogonal polarization components of the light in a birefringent fiber has been explicitly taken into account. The frequency-dependent linear cavity loss and the saturable absorber effect introduced by the combined effect of the nonlinear light polarization rotation and the passive polarizer are also automatically included in the calculation. In addition, since every next round of numerical calculation starts with the calculation result of the previous round as the initial condition, effects of periodic gain and loss perturbation on the soliton pulses are also included. Furthermore, as in the numerical simulations apart from the optical pulse the dispersive waves within a calculation window are also traced, the effect of the resonant sideband emission by the soliton is then taken into account.

The round-trip model has several advantages in comparison with the master equation model. The calculation is made following the pulse propagation in the cavity; therefore, pulse evolution within one round trip can be studied. Within each step of calculation the pulse's variation is always small, even if the change of a pulse within one round trip could be big. As there is no limitation on the pulse change within one round trip, dynamical process of soliton evolution, such as the process of a new soliton generation in the cavity, soliton interaction etc. can be investigated. In addition, the effect of discrete cavity components on the soliton, the influence of the dispersive waves and the different order of the cavity components on the soliton property are automatically included in the calculation.

4.2.2 Simulation parameters

In order to easily compare with the experimental results, whenever possible the real experimental values is chosen for the simulation. Referring to the experimental configuration of the laser, the parameters used in the simulation are listed as follows:

- Nonlinear coefficient of the fiber $\gamma = 3 \text{ W}^{-1}\text{km}^{-1}$;
- GVD parameter $\kappa'' = 2 \text{ ps}^2/\text{nm km}$ (for the dispersion-shifted fiber);
- GVD parameter $\kappa'' = 10 \text{ ps}^2/\text{nm km}$ (for the erbium-doped fiber);
- GVD parameter $\kappa'' = 18 \text{ ps}^2/\text{nm km}$ (for standard single-mode fiber);
- Gain bandwidth of the laser $\Omega_g = 2\pi \times 10 \text{ THz}$;
- Gain saturation energy $E_s = 120$;
- Cavity Length $L = 6 \text{ m}$;
- Beat Length $L_b = L/4$;
- Angles between the polarizer and the analyzer orientations to the fast axis of the birefringent fiber $\theta = \pi/8$ and $\varphi = \pi/2 + \pi/8$.

In all of the simulations, unless it is explicitly specified, the same simulation parameters as shown above are used. The two variables that can be adjusted depending on specific conditions in the simulations are:

- Small signal gain of erbium-doped fiber $g_0 = \text{varying}$. This parameter corresponding to the pump power in practical experiments.
- Linear phase delay $\Delta\phi = \text{varying}$. In practical experiments, this parameter can be altered by adjusting the polarization controllers.

4.3 Simulation results

By varying the linear phase delay and the gain parameter, the three operation regimes observed in experiment could all be numerically calculated.

4.3.1 Single-pulse solitons

The process of soliton generation is simulated as shown in Fig. 4.2. Analog with the experimental observations, the laser first becomes mode-locked from a fluctuating spike under the artificial saturable absorber effect. After mode locking, soliton

shaping occurs, and shapes the optical pulse into a stable soliton after hundreds of round trips. The formation of a soliton pulse is characterized by the appearance of the spectral sidebands.

Figure 4.2 Numerically simulated soliton formation process of the laser.

Figure 4.3 shows a numerically calculated steady-state single-pulse soliton operation of the laser. From Fig. 4.3a, one can get the soliton pulse width is around 300 fs. The spectral sidebands can also be seen in the calculated spectrum shown in Fig. 4.3b, which is well consistent with the experimental observations. For a more direct comparison, Fig.4.4 shows a numerical calculated soliton spectrum together with an experimental observed one in one figure. Both the soliton shape and the sideband positions could be precisely reproduced by the simulation.

Figure 4.3 Numerically calculated soliton operation of the laser. $g_0=285$, $\Delta\Phi_{pc}=1.3\pi$. (a) Soliton pulse output. (b) Soliton spectra.

Figure 4.4 Comparison of soliton spectra obtained in simulation and experiment.

Numerical simulations show that the soliton operation is in fact a generic feature of the laser, which can be obtained in a wide range of laser parameters such as different laser cavity lengths, cavity losses, fiber birefringence, and laser gain strength. Exactly like the experimental observations, the soliton obtained under different operation conditions have different pulse parameters. Figure 4.5 compares the optical spectra of the laser output under different gain parameters. The same trends as discussed in experimental observations are observed from the figure. As the gain is increased, the optical spectrum of the pulse becomes broader, and correspondingly the pulse width becomes shorter. Under weak gain, the laser operates in a mode-locked state. Once the pulse becomes a soliton, sidebands appear on the spectrum. After the soliton operation is obtained, increasing the gain parameter will also increase the strength of the sidebands.

Figure 4.5 Comparison of the numerically calculated optical spectra of a mode locked fiber laser under different gain strength. $\Delta\Phi_{pc}=1.2\pi$.

4.3.2 Twin-pulse solitons

Figure 4.6 shows a state of twin-pulse soliton operation of the laser numerically calculated in the simulation. It is seen that two solitons with exactly the same pulse width and height bind tightly together with a fixed separation of 1.125 ps, which is

less than 5 times of the pulse width. Both the pulse shape and the pulse separation never changed within thousands of round trips in the cavity, which shows that the twin-pulse soliton operation is very stable. Figure 4.7 compares the numerically calculated soliton spectrum with the experimentally observed one. The corresponding optical spectra of the twin-pulse solitons also exhibit the same properties as those observed in the experiments. In particular, the spectra have dips in the center, which suggested that the phase difference between the two solitons is about π . [102-104]

Figure 4.6 A numerically calculated state of twin-pulse soliton operation. $g_0=270$, $\Delta\Phi_{pc}=1.2\pi$. (a) Twin-pulse soliton profile. (b) Twin-pulse soliton spectrum.

Figure 4.7 Twin-pulse soliton spectra obtained in simulation and experiment.

4.3.3 Noise-like pulse

Noise-like pulse operation has also been reproduced in the numerical calculation as shown in Fig. 4.8. The noise burst nature of the state can be observed from the temporal evolution shown in Fig. 4.8a, which consists of a lot of very short and intense pulses arbitrarily changing on successive round trips. And the overall spectrum shown in Fig. 4.8b is very broad. These phenomena agree well with the experimental observations. Also the same with the experimental results, if the gain parameter is increased in the calculation, the fine pulse number in the noise-like pulse envelope increases and correspondingly the duration of the whole envelope is broadened. However, it is noted that the calculated optical spectra of the state are also noise-like, which could not be experimentally observed because of the average effect of the equipment.

Figure 4.8 Noise like pulse operation calculated for the laser. $g_0=700$, $\Delta\Phi_{pc}=1.45\pi$. (a): Noise-like pulse on successive round trips (b): Corresponding optical spectrum

4.4 Discussions

4.4.1 Average solitons

The mechanism of soliton formation in a single-mode fiber is well understood as the result of the continuous balance between anomalous GVD and the nonlinear SPM. However, in a fiber laser, as a pulse circulating in the cavity, it not only propagates in the fibers, but also passes through various cavity components such as the polarizer and the output coupler. These components will discretely change the parameter of the pulse, and consequently the pulse energy and polarization vary periodically along the laser cavity. Therefore, strictly speaking, the soliton pulse obtained in a fiber laser is different with that formed in a single-mode fiber.

However, under the steady state, at any fixed point in the cavity, the pulse has fixed pulse shape and peak power, which is invariant with time. This feature shows that within one round trip in the cavity, the average dispersion experienced by the pulse is balanced with the average nonlinear phase modulation. Therefore, instead of the continuous balance between the dispersion and nonlinearity in single-mode fibers, the soliton formation in a laser should be understood in the sense that the average property of the pulse within one round trip of cavity behaves as a soliton. This property of the laser solitons is referred to as average soliton [72].

4.4.2 Sideband asymmetry and soliton wavelength tuning

Comparing the operation states under different simulation parameters, one could get a better understanding of the operation mechanisms of a passively mode-locked fiber ring laser.

In Sec. 3.1, the nonlinear polarization rotation (NPR) mode locking technique has been discussed. It was shown that the linear cavity transmission is a sinusoidal function of the linear cavity phase delay. While, for the simplicity of the discussion, the dispersion property of the fiber have been skipped. However, an optical fiber is intrinsically dispersive. If the dispersion of the optical fibers is further considered, the linear intensity transmission should also be a sinusoidal function of the wavelength.

A direct consequence of the sinusoidal form of linear cavity transmission with wavelength is the asymmetrical sideband strength in the soliton spectrum. It is well known that sidebands are formed as a result of the periodic discrete perturbations on the laser solitons. Early numerical simulations on soliton sideband generation have ignored the laser cavity effect, which showed symmetrical sidebands strength [84,85]. However, all experimental results have exhibited sidebands with asymmetric strengths on the soliton spectrum [86-88]. This discrepancy to the experimental results was caused because the effect of the linear cavity loss dispersion of a real laser was not taken into account in the simulations [92]. After incorporating the linear cavity loss dispersion effect in the simulations, the numerically calculated soliton spectrum reproduces details of those of the experimental results. As shown in Fig. 4.3b, the sidebands asymmetry is obvious. Depending on the laser parameter settings, the sidebands could become more or less asymmetric. In the fiber soliton lasers mode locked by the nonlinear polarization rotation technique, mode locking can only occur in the linear cavity phase delay range where an artificial saturable absorber effect can be generated. This restriction on the linear cavity phase delay also limits the wavelength range of solitons that can be formed in the lasers. As the linear cavity loss of the lasers varies in a sinusoidal form with wavelength, therefore, independent of the central wavelength of a soliton formed, over the range of the soliton spectrum the linear cavity loss is always asymmetric. Sidebands are linear waves, so they experience this linear cavity loss asymmetric and consequently exhibit different spectral strengths. Like that observed experimentally, increasing the gain parameter, the sidebands strength will grow up. When the sidebands become too strong, due to modulation instability, they become unstable and generate their own sub-sidebands [93]. This phenomenon was also numerically reproduced and shown in Fig. 4.9.

Once the linear cavity phase delay is fixed, a soliton is always formed in the lasers at the wavelength where the cavity has the minimum loss. As the linear cavity loss of the fiber lasers is a sinusoidal function of the linear phase delay, through cavity dispersion it further becomes a sinusoidal function of the wavelength. Therefore, shifting the linear cavity phase delay eventually changes the central wavelength of

the solitons formed in the lasers. The larger the cavity dispersion, the bigger is the continuous soliton wavelength tuning range [92,113].

Figure 4.9 Sub-sideband generation of the lasers as a result of the dispersive wave modulation instability. Cavity length $L=12$ m, fiber beat length $L_b=L/5$, $g_0=325$, $\Delta\Phi_{pc}=1.4\pi$.

While, although the cavity transmission of the lasers is a periodic function of the linear cavity phase delay with a period of 2π , only within half of the period the artificial saturable absorber effect can be achieved. Despite shifting the linear cavity phase delay can continuously tune the central wavelength of the solitons, the tuning range is limited by the mode locking range of the linear cavity phase delay.

4.4.3 Regimes of laser operation

In the previous discussion (Sec. 3.1.2), it was assumed that the magnitude of the nonlinear polarization rotation caused by the Kerr effect is small so that an explicit mathematical formula could be derived, which has clearly shown that the generation of an artificial saturable absorber effect in the system. However, as the nonlinear polarization rotation is both the intensity and the fiber length dependent, intrinsic to the setup, it can also cause the polarization switching [44,46]. Above the polarization switching point, the setup then generates an artificial negative feedback

effect on the optical pulse, which then limits the maximum pulse peak height. This power clamping mechanism severely determines the operation regimes of the laser.

Numerical simulations show that depending on the selection of the linear cavity phase delay the fiber soliton lasers can exhibit various regimes of operation, each with distinctively different features. Figure 4.10 shows a case of the laser operation when the linear cavity phase delay is selected close to the polarization switching point. In this case multiple mode-locked pulses with weak peak power are obtained. The pulses fill in the whole simulation window. As the pulse peak power is weak, gain competition and pulse peak clamping result in that their peaks vary constantly. It is difficult to obtain a steady state where all pulses have exactly the same pulse height as that under the soliton operation of the lasers. Increasing pump power only increases the number of pulses but not their peak power as the peak is clamped by the polarization switching power. On the contrary, if the linear cavity phase delay is set far away from the polarization switching point, a mode-locked pulse first is slowly built up to a very high peak power and very narrow pulse width and then quickly collapses as shown in Fig. 4.11. This process repeats continuously. When the pump power is strong, several of such pulses generate and disappear randomly in the cavity forming a state of so-called noise-like pulse operation as shown in Fig. 4.8.

With some other linear cavity phase delay selections between the two states, soliton operations of the laser could be obtained. However, at different linear cavity phase delays settings the obtained solitons exhibit different behaviors. With the linear cavity phase delay selected close to the polarization switching point, the power required for the nonlinear polarization switching is weak, the generated soliton thus have lower peak power. Consequently, their pulse widths are broad. In this case less dispersive waves would be emitted by the solitons. As a result, the solitons in the cavity experience less dispersive wave mediated long-range soliton interaction. Solitons are stable and tend to stay with fixed separations in the cavity. The twin-pulse soliton operation is obtained in the vicinity.

Figure 4.10 A state of laser operation when the linear cavity phase delay is selected close to the polarization switching point. $g_0=240$, $\Delta\Phi_{pc}=1.08\pi$.

Figure 4.11 A state of laser operation when the linear cavity phase delay is selected far away from the polarization switching point. $g_0=405$, $\Delta\Phi_{pc}=1.5\pi$.

Both numerical calculations and experimental observations show that only when the linear phase bias is selected close to the polarization switching point, a twin-pulse soliton operation is possible to be obtained. The formation of the twin-pulse solitons still need to be further investigated. A plausible explanation for the formation of the state is that it is formed through the cross phase coupling. With a linear phase bias near the polarization switching point, the switching power is small, therefore the peak power of the emitted pulses are very weak. As the peak power of each individual pulse is not strong enough to generate enough SPM to compensate the GVD, it cannot form a soliton just by itself. Whereas, if two such pulses are very close to each other, they can work together to form a twin-pulse soliton with the help of the cross phase modulation between them.

As the linear cavity phase delay setting moves away from the nonlinear polarization switching point, soliton peak power increases and pulse width decreases. Consequently, the dispersive wave emission becomes stronger and the dispersive wave mediated soliton interaction becomes stronger as well. The dispersive wave mediated soliton interaction is essentially a random kind of interaction. Under existence of the interaction solitons move around in the cavity. Carefully controlling the pump power, the strength of dispersive waves could be reduced. Therefore, in the steady state individual soliton scattering randomly in the cavity is frequently observed under the linear cavity phase delay setting. The numerically simulated regimes of laser operation are well in agreement with the experimental observations.

4.4.4 Soliton energy quantization and multiple soliton generation

Experimental studies in Chap. 3 have shown that the soliton energy quantization is a generic feature of the passively mode locked fiber soliton lasers. Energy quantization of the solitons could be explained by the internal energy balance of the laser. It is well known that for an optical soliton to form in a single-mode optical

fiber, the fundamental requirement is that the dispersion of the fiber is balanced by the nonlinear phase modulation of the pulse. If the fiber loss could be ignored, this requirement can actually be satisfied by pulses of various energies and pulse width. Therefore, for pulse propagation even in the same single-mode fiber solitons of different energies can form and coexist. However, in the case of solitons in a laser, as a laser is a dissipative system, in addition to the above fundamental soliton formation requirement, a soliton must also satisfy the internal energy balance. Namely, when a soliton circulates in the laser cavity, it experiences the cavity gain and loss, in order to survive the gain it experienced must balance the loss. This energy balance requirement ultimately determines the energy of a soliton. Therefore, with fixed laser parameters, solitons formed in a laser have exactly the same soliton parameters.

As the energy of solitons formed in a laser is determined by the internal energy balance, and the gain experienced by a soliton is the pump strength dependent, increasing pump power results in that a soliton has even higher peak power and even narrower pulse width. At a certain value of the laser gain, the soliton pulse width will become so narrow that its spectral bandwidth is larger than that of the effective gain bandwidth of the laser, and then the effective gain bandwidth will impose an extra loss to the soliton and limit its further narrowing. As in this case a narrow soliton pulse experiences more loss than two solitons of broad pulse width, the soliton is no longer stable and will split into two [95]. A multiple soliton operation of a laser is then obtained. Multiple soliton operation is a natural consequence of soliton lasers under strong pumping. After a new soliton is generated, as more solitons will share the same laser gain, the effective gain experienced by each soliton will become less, the laser will eventually stabilized at a new steady state where all soliton have exactly the same pulse width and peak power. Further increasing pump power the above process repeats. Nevertheless, it is to see that even in a fiber soliton laser through carefully controlling the laser parameters stable gain bandwidth limited soliton pulses could still be obtained.

Figure 4.12 shows a numerically calculated multiple soliton operation state of the laser. Within the calculation window, multiple soliton pulses coexist and all the solitons have exactly the same pulse width and peak power. Numerically it was confirmed that the number of solitons in the cavity is determined by the laser gain. The larger the laser gain, the more solitons can be generated. In particular, in the steady state all solitons in the cavity have exactly the same height. The numerically calculated multiple soliton operation states consist with those obtained experimentally.

Figure 4.12 A numerically calculated state of multiple soliton operation. $g_0=268$, $\Delta\Phi_{pc}=1.15\pi$.

However, careful numerical studies on the multiple soliton formation, especially on the relation of soliton peak power with the cavity linear phase delay turns out that the multiple soliton formation of the soliton fiber lasers is with a different scenario to that described above. It is actually caused by a pulse peak power clamp effect of the laser cavity. The laser is mode locked by using the NPR technique. Intrinsic to the mode-locking technique there is also a nonlinear polarization switching effect. Depending on the linear cavity phase delay, the effect sets a maximum peak power on the pulses in the cavity. If the peak power of a pulse formed in the cavity is

lower than the value, the nonlinear polarization rotation generates a positive feedback on it. While if the peak power of a pulse is larger than the value, the nonlinear polarization rotation will then generate a negative intensity feedback on it, which suppresses the further increase of the peak of the pulse. This effect leads to the consequence that if the effective gain of the laser is high, instead of generating a single pulse with gain bandwidth limited pulse width, multiple pulses will be generated in the laser cavity. As all the pulse equally share the total laser gain, in a steady state all pulses will have exactly the same pulse height. The soliton nature of each pulse then determines the pulse width.

With fixed cavity length and fiber nonlinearity, the polarization switching power is determined by the linear cavity phase delay. Setting the linear cavity phase delay closer to the polarization switching point leads to the formation of solitons with lower peak power and larger pulse width. The nonlinear polarization switching power is also the cavity length and birefringence dependent. In order to obtain narrower soliton pulse in the laser, it is always preferred to use short laser cavity and low-birefringence fibers to construct the laser cavity.

Chapter 5

Observation of FPU recurrence

A characteristic of passively mode-locked fiber soliton lasers is that CW lasing can coexist with the soliton operation under certain laser parameter settings. And due to soliton energy quantization, with a fixed polarization setting, the strength of the CW component can be controlled by the pump power. As the CW component is a linear wave and all linear waves are intrinsically unstable in the laser due to the modulation instability, when the strength of the CW component becomes strong, it eventually becomes unstable with symmetric spectral sidebands appearing in the spectrum.

Modulation instability (MI) is a universal effect existing in many nonlinear systems as a result of the interplay between the nonlinear and dispersive effects. It has been widely studied in diverse fields such as fluid dynamics, plasma physics and nonlinear optics since 1960s [114]. In the context of optical fibers, MI is referred to as a process in which due to the interplay between the self-phase modulation and the anomalous dispersion, weak perturbations grow exponentially into pulse trains from a steady CW state. Tai et al reported the first experimental observation of MI in optical single-mode fibers in 1985 [115]. They found that the MI could be interpreted as a special case of the four-wave mixing whose phase-matching condition is self-generated by the nonlinear change of the refractive index and the anomalous dispersion. Agrawal studied the MI in an erbium-doped fiber amplifier (EDFA) [116]. He found that, comparing with the case of the undoped fibers, the threshold for MI could be considerably reduced by the introduction of the internal gain. Tang et al have reported the observation of MI of the dispersive waves in a fiber ring laser [117]. It was found that when the linear dispersive waves in the lasers are strong enough, they become unstable as a result of MI. MI has even been used for generating pulse trains with high repetition rates from the CW states [118].

However, MI is a process that would not lead to a thermalization of energy. Instead, theoretical studies have shown that after a certain length of propagation, the energy that spreads to different frequencies due to MI eventually returns to the initial mode, and consequently the initial waveform is reconstructed or almost reconstructed. This reversible behavior of MI is now well known as the Fermi-Pasta-Ulam (FPU) recurrence [119,120]. FPU recurrence has been predicted for a large number of conservative nonlinear systems, including those described by the nonlinear Schrödinger equation [121,122]. As also predicted by theory, MI could appear to be of the limit cycle type, simply because the recurrence time is much longer than the time scale of interest. For this reason, although a lot of theoretical study has been done on FPU recurrence, there are rare experimental observations in optics. Only until very recently, Simaey et al experimentally demonstrated the FPU recurrence in optical fibers using a specially designed laser source [123,124].

This chapter presented the experimental observation on the FPU recurrence of the MI of CW component in the passively mode-locked fiber soliton ring laser. Section 5.1 presents the basic theory of MI. Section 5.2 reports the CW emission and its MI in the present laser. The observation of the FPU recurrence is presented in section 5.3. And Section 5.4 is the discussion.

5.1 Theory of modulation instability

Mathematically, if ignoring the fiber loss, the propagation of CW light inside an optical fiber can be described by the NLSE:

$$i \frac{\partial E}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 E}{\partial T^2} - \gamma |E|^2 E, \quad (5.1.1)$$

where E is the electric field envelope, z is the propagation distance, T is the time in a reference frame traveling at the group velocity of the light, β_2 is the GVD of the fiber, and γ is the nonlinear coefficient of the fiber.

Considering a perturbed CW of the form of

$$E(z, T) = \left[\sqrt{P_0} + a(z, T) \right] \exp(i\phi_{NL}) \quad (|a|^2 \ll P_0), \quad (5.1.2)$$

where P_0 is the incident power, $a(z, T)$ is the perturbation function, and $\phi_{NL} = \gamma P_0 z$ is the nonlinear phase shift induced by the SPM. Assuming that the perturbation has a wave number of κ and frequency of Ω , then the function of the perturbation have the form of

$$a(z, T) = a_0 \exp[i(\kappa z - \Omega T)] \quad (5.1.3)$$

Substituting Eq. (5.1.2) and (5.1.3) into Eq.(5.1.1), and then to make the equation have a nontrivial solution, the following dispersion relation is obtained:

$$\kappa = \pm \frac{1}{2} |\beta_2 \Omega| \left[\Omega^2 + \text{sgn}(\beta_2) \Omega_c^2 \right]^{1/2}, \quad (5.1.4)$$

where

$$\Omega_c = \left(\frac{4\gamma P_0}{|\beta_2|} \right)^{1/2} = \left(\frac{4}{|\beta_2| L_{NL}} \right)^{1/2}. \quad (5.1.5)$$

From Eq. (5.1.4), it can be seen that, as $\beta_2 < 0$, κ is imaginary for $|\Omega| < \Omega_c$, and the perturbation $a(z, t)$ grows exponentially with z . As a result, CW is inherently unstable in anomalous dispersion regime with frequency sidebands generated. This instability is referred to as modulation instability (MI) because it leads to a spontaneous temporal modulation of the CW beam and transform it into pulse train. On the contrary, as $\beta_2 > 0$, κ is real for all Ω , and the steady state is stable against small perturbations.

The gain spectrum of MI shows as two symmetric bands with respect to the signal frequency:

$$g(\Omega) = |\beta_2 \Omega| (\Omega_c^2 - \Omega^2)^{1/2}. \quad (5.1.6)$$

And the maximum gain is obtained as

$$\Omega_{\max} = \pm \frac{\Omega_c}{2} = \pm \left(\frac{2\gamma P_0}{|\beta_2|} \right)^{1/2}, \quad (5.1.7)$$

with a peak value of

$$g_{\max} = g(\Omega_{\max}) = 2\gamma P_0. \quad (5.1.8)$$

It is to see that the modulation frequency and the peak gain are both power dependent.

5.2 CW lasing and its MI in the laser

Coexistence of CW lasing and soliton operation was frequently observed in passively mode-locked fiber soliton lasers [125]. This effect is especially pronounced in the fiber soliton lasers mode locked by using the NPR technique. When extra gain exists in the laser, some modes that satisfy the resonance condition of the laser will become lasing as a residual CW light background in the cavity. As a linear wave, the strength of CW component is determined by the linear transmission of the cavity. Although ideally due to the existence of the saturable absorber effect in the laser, under an optimized soliton operation all CW components would experience negative effective gain and therefore be suppressed, in practice if the minimum linear cavity loss position is not appropriately set and/or the pump power is strong, CW lasing can still build up and coexist with the solitons. Here the author presents in Fig. 5.1 a typical optical spectrum of the laser output when the CW lasing coexists with the solitons in the cavity. The position of the CW

component on the soliton spectrum is determined by the linear cavity phase bias and laser gain profile. Because of the soliton energy quantization, the strength of the CW component can be experimentally controlled by the pump power. Simply increasing the pump power, the CW strength can be significantly increased.

Figure 5.1 A typical optical spectrum of the laser output when the CW lasing coexists with the single-pulse solitons in the cavity.

As the laser is operating in the anomalous dispersion region, a CW component is intrinsically unstable due to the modulation instability. It is observed that when the strength of the CW component was strong, it became unstable as characterized by the appearance of the MI sidebands in the spectrum. Figure 5.2 shows the enlarged views of the spectral image of the CW component under the same polarization setting of Fig. 5.1. In Fig. 5.2a, the CW component is a very clear single mode wave with a wavelength of 1557.74 nm, in which the MI sidebands are too weak to be seen. Fixing all the other parameters, and increasing the pump power from 100 mW in Fig. 5.2a to 157 mW in Fig. 5.2b, the MI of the CW becomes so strong that the modulation sidebands are clearly to see in symmetric positions to the original mode as shown in Fig. 5.2b.

Figure 5.2 Enlarged views of the CW component. (a): Pump =100mW, (b): Pump =157 mW.

The laser output was also simultaneously monitored with a 25 GHz Photo-detector and a 50 GHz high-speed sampling oscilloscope. It is observed that whenever the spectral sidebands appear on the optical spectrum, the high-speed oscilloscope traces exhibit a noisy background [106], indicating that the CW has become unstable.

5.3 Observation of the FPU recurrence

From Fig. 5.2b, one can get from the spectral pattern that the modulation frequency of the first-order sidebands is about 0.01 THz, which corresponds to the maximum-gain frequency $f_{\max}$ of the MI. Theoretically, as shown in Eq. (5.1.7), in the anomalous-dispersion regime of optical fibers the maximum gain of the MI is described by

$$\Omega_{\max} = 2\pi f_{\max} = \left(\frac{2\gamma P_0}{|\beta_2|} \right)^{1/2}, \quad (5.3.1)$$

In the present laser, the average dispersion $\beta_2 \approx -5.15 \text{ ps}^2\text{km}^{-1}$ as estimated from the soliton sidebands, and the nonlinear coefficient $\gamma \approx 3 \text{ W}^{-1}\text{km}^{-1}$. Thus we can get that the peak power of the CW component P_0 is about 3.5 mW. By using Simaey's analysis result ($\gamma P_0 z \approx 8$ [123]) to estimate the FPU recurrent period in the condition of Fig. 5.2b, it is obtained the recurrence distance is around 780 km, which corresponds to a period of around 2.6 ms. In experiment, the OSA has a minimum recovery time of about 1 s, which is much bigger than the recurrent period. For this reason one cannot measure the actual recurrent period directly from the OSA.

However, by using a DC coupled photodetector and a low-speed oscilloscope, the actual recurrence period of the MI of the CW component under the same condition of Fig. 5.2b in our laser has been measured. Figure 5.3 shows the corresponding oscilloscope trace. In obtaining Fig. 5.3, the author used a self-written fiber Bragg grating ($\lambda_0 \approx 1557.74 \text{ nm}$, $\Delta\lambda \leq 0.08 \text{ nm}$, $r \geq 30\text{dB}$) to suppress the center CW component after the output, so that the observed intensity change would be totally due to the appearance of the spectral sidebands. A light intensity variation with a period of around 7 ms can be clearly seen on the oscilloscope trace, which confirms that the intensity of the spectral sidebands varies periodically. This result is very

close to the 2.6 ms that was theoretically estimated above. It is to note that the high frequency components of the output such as the mode-locked pulses and the high frequency modulations of MI cannot be displayed on this oscilloscope trace. Obviously, this experimental observation agrees well with the analytical estimation.

Figure 5.3 The low-speed oscilloscope trace of the output under the same operation condition as Fig. 5.2b. A recurrence period of around 7 ms is clearly observed.

5.4 Discussions

The experimental results clearly show that the MI of the CW component of the laser does have a reversible behavior, which has been theoretically predicted in nonlinear systems as the FPU recurrence. As that described above, FPU recurrence is a general feature of MI in conservative nonlinear systems. When a CW propagates in the nonlinear media, initially all the energy is concentrated in the central mode. But since the CW is intrinsically unstable against MI, over time, it becomes modulated which is characterized as the appearance of the symmetrical spectral sidebands as shown in Fig. 5.2b. Thus the energy flows from the central mode into the sidebands, and as a result, the energy in the central mode decreases. This process continues

until a certain point is reached at which the energy starts to flow back from the sidebands to the central component. And after some time, the energy is totally back to the central mode, and the CW is reconstructed. This reciprocal energy exchange between the central mode and the sidebands is the FPU recurrence. As a result, in experiment one can observe the intensity of the MI sidebands vary periodically with time as shown in Fig. 5.3.

The experimental observation of the FPU recurrence benefits from two aspects of the present fiber soliton laser, one is the existence of erbium-doped fiber in the cavity, and the other is the ring structure of the laser. The existence of the erbium-doped fiber greatly lowers the threshold for MI, which makes the MI easier to be observed. When the gain is equal to the loss in the cavity, the laser ring can be approximately regarded as a conservation system, and then the light can travel in the cavity round after round. The long propagation path and periodical insertion of amplifiers needed to observe the FPU recurrence in transmission systems are therefore avoided in the present experiment, which makes it much easier to observe the phenomenon.

Chapter 6

Soliton interaction

As presented in Chap. 3, multiple soliton pulses can be simultaneously generated and form various distribution modes such as stable randomly spaced soliton distribution, soliton bunching, and harmonic mode locking in the laser cavity. It is of significance to find out the physical mechanisms for the formation of the various modes of multiple soliton operation, in particular, how they are related to the soliton interaction in the laser. For this purpose, various experiments are designed to identify different soliton interactions and their impacts on soliton dynamics in the fiber laser.

This chapter presents detailed investigations on soliton interaction. Based on the experimental observations, three sorts of soliton interactions, which are identified as a global type of soliton interaction caused by the existence of unstable CW emission, a type of long-range soliton interaction caused by the resonant sidebands, and the direct soliton interaction, are discussed from section 6.1 to 6.3. Numerical simulations on the soliton interactions are processed in section 6.4.

6.1 Global soliton interaction

6.1.1 Observation of global soliton interaction

As discussed in Chap. 5, CW emission is often observed to coexist with the soliton operation in the fiber laser. Generally the existence of a weak CW wave will not affect the property of the solitons in the cavity. However, when the strength of the CW component becomes strong, due to the MI, it eventually becomes unstable as discussed in 5.1.2. Experimentally it is observed that whenever the CW component

becomes unstable as characterized by the appearance of sidebands on the CW spectral component, a noisy background is observed in the oscilloscope trace. Under the influence of the noise background all solitons in the cavity start to move, suggesting that the unstable CW components could introduce a kind of global soliton interaction mechanism to the solitons.

Figure 6.1 shows a series of the experimentally recorded oscilloscope traces that demonstrate the existence of the global type of soliton interaction in the laser. The oscilloscope window is scaled to show exactly one cavity round trip of ~ 26 ns. In the state shown in Fig. 6.1, there are five solitons coexisting in the cavity. Two solitons have a very small soliton separation (~ 5 ps) and are bound together. Details about the properties and binding mechanism of this bound soliton state will be discussed later. As the detection system cannot resolve these two solitons, it appears as a large pulse on the oscilloscope trace. Here the bound-soliton is used as the trigger signal for the measurements (it therefore has a fixed position in the oscilloscope traces), and the relative movement of the other three solitons in the cavity is experimentally investigated. Figure 6.1a and b show the relative position changes of the three soliton-pulses under fixed laser parameters. It is observed that the three solitons ceaselessly moved with respect to each other with an average speed of about half division/second. Their movements were purely random. The solitons could come close or move apart from each other. Occasionally, when they moved very close to each other, they bind temporally together and form a weakly stable bound solitons. However, it could be eventually destroyed by the environmental perturbations. In such a state of laser operation, the movement of one soliton affects all the other solitons in the cavity no matter how far apart they are. It is therefore believed that there is a global type of soliton interaction between them.

Figure 6.1 Oscilloscope traces demonstrating unstable CW lasing induced global soliton interaction. (a) and (b): Two consecutive instants under the existence of noisy background. Solitons change their positions in the cavity; (c): Noisy background is suppressed. Solitons form a stable irregular pattern.

Experimentally the physical mechanism of this global soliton interaction has been further investigated and it is found out that the cause is the existence of an unstable CW component in the laser. To demonstrate this, the experimentally measured soliton spectrum of the state is shown in Fig. 6.2. It is to see that there is actually CW lasing coexisting with the soliton operation. Through adjusting the pump power, the strength of the CW lasing can be controlled. As the laser is operating in the anomalous dispersion region, a CW component is intrinsically unstable due to the MI. It has been demonstrated in Chap. 5 that when the strength of the CW component was strong, it became unstable as characterized by the appearance of the modulation instability sidebands in the spectrum [126]. In the oscilloscope trace, instability of the CW component is also represented by the appearance of background noise, as can be clearly seen in Fig. 6.1a and b. Reducing the strength of the CW component, its instability becomes less obvious, and consequently the noisy background disappears from the oscilloscope trace as shown in Fig. 6.1c. Without the noisy background all solitons then stay stable in the cavity, forming a stable irregular distribution pattern.

Figure 6.2 Optical spectrum of the laser output under the same condition of Fig. 6.1a and b. The strong line in the spectrum is the CW emission.

Interaction between solitons in the presence of a frequency-shifted CW wave was also investigated by Loh et al [127]. It was found that the existence of a weak

frequency-shifted CW causes the soliton central frequency to shift. So far, how the solitons interact with an unstable CW in the laser cavity is not fully understood. However, considering that both the CW component and the solitons coexist in the laser cavity and share the same laser gain, the existence of mutual influence between them seems understandable. Hence, the experimental observations may be explained in the following way. When a stable CW wave coexists in the cavity, all the solitons would experience the same central frequency shift, overall the relative soliton velocity will not change. While if the CW component becomes unstable, each soliton would experience different local perturbations, therefore, they would have different central frequency shifts. Due to the different center frequency shifts, the solitons begin to move randomly with respect to each other in the cavity.

6.1.2 Passive harmonic mode locking

Passive harmonic mode locking (HML) is a well-known phenomenon in passively mode-locked fiber lasers and has been intensively investigated previously [42,43]. Grudinin et al firstly reported the experimental observation of this phenomenon [42]. They showed that under certain conditions the multiple solitons in a passively mode-locked fiber laser could arrange themselves automatically to form a stable state where all the soliton pulses are equally spaced in the cavity. As in such a state the solitons in cavity has a similar distribution as that of optical pulses of a harmonically mode-locked laser, they called the state the passive harmonic mode locking of a fiber soliton laser. The multiple soliton formation in a passively mode-locked fiber laser is a result of the soliton energy quantization, which is known as a typical characteristic of soliton operation in the laser cavity. However, what mechanisms drive the multiple solitons equally distributing in the laser cavity has so far not been clearly explained. Philipetskii et al have numerically studied the effect of acoustically induced long-term soliton interaction in a fiber laser and found that the effect could cause the multiple solitons in a laser cavity either to bunch or form HML [69]. Kutz et al simulated the effect of laser gain depletion and recovery on the soliton distribution in the cavity. They demonstrated that competition for gain between the solitons could results in equally spaced soliton distributions [68].

Obviously, these two effects always exist in a passively mode-locked fiber laser, but the passive HML does not always appear. It would be interesting to know under what conditions will the influence of the two effects on the solitons become significant, or if there are other mechanisms which have played a even stronger role on the formation of the HML of the lasers.

In 6.1.1, the author has experimentally demonstrated that the unstable CW lasing could introduce a global type of soliton interaction in the laser. Worthy of mentioning, obviously in a state like Fig. 6.1a and b, the soliton interactions induced by the gain depletion and recovery, the fiber acoustic effect should also exist. Nevertheless, their effects seem very weak to be measured. Experimentally it is observed that once the noise background was suppressed, the solitons would stay in the cavity at the positions where they were just before the background was suppressed. They never changed their positions except that the unstable component was created again in the cavity. With this method, one could control the soliton distribution in the cavity and obtain various stable soliton distribution patterns.

It is found that the unstable CW lasing mediated global soliton interaction plays an essential role on the formation of the harmonically mode-locked states. Fig. 6.3a shows the oscilloscope trace of one stable harmonically mode-locked single-pulse soliton operation state. The cavity round trip time of the laser is about 26 ns. Six single-pulse solitons coexist in the cavity, equally spaced, and as a result, the repetition rate of the laser emission becomes 6 times of its fundamental value (37.8 MHz). Fig. 6.3b gives the power spectrum of the laser output in the harmonically mode-locked state. From the RF spectrum, it is seen that under the HML the fundamental repetition frequency is suppressed more than 40 dB, and the output repetition rate is 266.8 MHz. Harmonically mode-locked states formed with various numbers of single-pulse solitons have also been observed e.g. Fig. 3.9b.

Figure 6.3 A harmonic mode locking state of single-pulse solitons. (a): Oscilloscope trace; 6 solitons separate evenly in the cavity. (b): Corresponding power spectrum.

Harmonically mode-locked twin-pulse soliton states have also been obtained. In Chap.3, the author already presented a twin-pulse soliton HML state as Fig. 3.14b. Here another experimentally observed such state is shown in Fig. 6.4. In Fig. 6.4 fifteen twin-pulse solitons coexist in the cavity and are equally spaced. Consequently, the pulse repetition rate becomes 15 times of the laser fundamental repetition rate. As the author has presented, HML of various numbers of single-pulse solitons (Fig. 3.9b) or twin-pulse solitons (Fig. 3.14b) have been observed in

the experiments. Empirically it was found that the more solitons in the cavity, the higher possibility they form HML.

Figure 6.4 A harmonic mode locking state of twin-pulse solitons. 15 twin-pulse solitons coexist and distribute evenly in the cavity

The author investigated the formation of the harmonically mode-locked states in the laser. It comes to her attention that whenever a harmonic mode-locking state occurs in the laser, the oscilloscope trace displays a strong noise background, which spreads over the whole laser cavity as can be clearly seen in Fig. 6.3a, 3.9b, 3.14b and 6.4, which indicates the existence of the unstable CW lasing. While in the other states of the soliton operation, there is no such noise background e.g. the oscilloscope traces shown in Fig. 3.8, 3.9a, 3.13a and 3.14a. It has been revealed that the unstable CW lasing could induce a kind of global soliton interaction in the laser. Intuitively it is to imagine that under the global soliton interaction, solitons will gradually adjust their positions. The only stable state they could eventually reach would be a harmonically mode-locked state as only in such a state all the solitons subject to the same influence caused by an unstable CW. Actually all of the harmonically mode-locked states are obtained this way.

6.2 Long-range soliton interaction

6.2.1 Dispersive wave mediated soliton interaction

The soliton interaction in the laser becomes more complicated as considering the existence of the radiative dispersive waves. It is well known that a soliton circulating in the laser cavity endures periodical gain and loss perturbations, and consequently periodically reshapes itself and sheds energy into linear dispersive waves. The dispersive waves form a low-intensity pedestal accompanying the soliton. Simth and Mollenauer firstly discovered that the dispersive waves could introduce a kind of long-range interaction between solitons. They analytically showed that this long-range soliton interaction is an oscillation function depending on the initial separation of solitons and is only effective in a range determined by the duration of the dispersive-wave pedestal [61]. Hence, intensive study has been carried out on the long-range soliton interaction. In particular, bound solitons with fixed separations are thereby anticipated [62, 63].

Experimentally it is found that the dispersive waves can form a modulated quasi-cw pedestal around the solitons with an extent of about 500 ps. It is observed that when the separation between two solitons is less than 500 ps, they will interact with each other mediated through their dispersive waves. They may oscillate with each other. Under certain conditions they can be bound together through the dispersive waves. On the contrary, when the separation is larger than 500 ps, there is no sign for the existence of mutual interaction between solitons if the CW has been well suppressed to avoid the global soliton interaction discussed in last section.

The experimental observations are well in agreement with previous anticipations. Specifically, not only the bound states of solitons are formed as a result of this long-range soliton interaction, it is believed that the formation of the soliton bunching observed in fiber soliton lasers could also be attributed to a result of this soliton interaction.

6.2.2 Bound solitons and soliton bunch

Experimentally, bound states of single-pulse solitons were already observed in passively mode-locked fiber soliton lasers. For instance, Seong and Kim reported the bound states of single-pulse solitons in a figure-of-eight laser [130]. In the present laser, bound single-pulse solitons have also been frequently observed. In section 6.2.1, the author already mentioned the existence of a bound soliton state in Fig. 6.1. Here detailed discussion on this issue is presented. In Fig. 6.1 there are two solitons binding together, forming a state of bound solitons. The oscilloscope trace is not uniform any more as other multiple soliton states such as Fig. 3.8 and Fig. 3.9a. This is because that the oscilloscope cannot resolve the short separation between the bound two solitons. Thus, the bound solitons exhibits as an extra higher spike. In the experiment as the author increased or decreased the background noise, neither the state of bound solitons was destroyed nor the pulse separation was changed, indicating that the binding force between the two solitons is stronger than the force of the global soliton interaction induced by the unstable CW. Figure 6.5 shows a similar state, where only two bound solitons exist in the cavity. From the autocorrelation trace one can find that the two solitons have a fixed pulse separation of about 5 ps. The optical spectrum shows modulation with a period of 1.8 nm, which exactly corresponds to the 5 ps separation in time domain. Bound states of solitons with various soliton separations and soliton numbers are frequently obtained in the laser. Coexistence of bound solitons with different pulse separations and bound states of more than two solitons with equal separations has also been observed. Figure 6.6 shows for example two other bound-soliton states observed in the laser: Fig. 6.6a is a state where 5 solitons bind together with equal separations in the cavity, and Fig. 6.6b is a two soliton bound state with a large pulse separation of around 100 ps.

Figure 6.5 The optical spectrum and autocorrelation trace (inset) of a state of bound single-pulse solitons. Soliton separation is about 5 ps, which is ~ 15 times of the soliton pulse width.

Experimentally it is found that the larger the pulse separation, the weaker is the binding energy between the bound two solitons, therefore, the easier is the state to be destroyed by the perturbations. A common characteristic of the bound solitons is that their soliton separations are fairly large. Previous studies on the solitons interaction have shown that the direct soliton interaction could be neglected if the soliton separation is larger than 5 pulse-widths [127]. As in the laser the soliton only has a pulse width of about 340 fs, the separations of the bound solitons are then far bigger than that value. Therefore, it is unlikely that the observed bound solitons are formed due to the direct soliton interaction. Considering the properties of the bound solitons, it is believed that they are formed under long-range soliton interaction mediated through the dispersive waves.

Figure 6.6 Two other states of bound single-pulse solitons observed in the laser. (a): The autocorrelation trace of 5 solitons binding together with an equal separation around 6 ps. (b) Oscilloscope trace of 2 solitons binding together with a separation of around 100 ps.

In addition, it was found that even the state of soliton bunching, which was frequently observed in the fiber soliton lasers, could also be a result of the dispersive wave mediated long-range soliton interaction. It is a special case of the bound solitons. To illustrate this, the author shows a typical case of soliton bunching observed in the laser in Fig. 6.7. Figure 6.7a shows that a bunch of solitons moves stably in the laser cavity with the fundamental cavity repetition rate. Figure 6.7b is the expansion of the soliton bunch. 17 solitons coexist in the bunch with fixed relative soliton separations. Note that all the soliton separations in the

bunch are less than 500 ps. Under fixed experimental conditions a bunch structure of solitons can also stay for a long time in the cavity, like the case of bound solitons. Nevertheless, it is far less stable than the states of bound solitons shown above. The author believes this is because solitons in a bunch normally have very large separations, consequently, very weak binding energy.

Figure 6.7 A state of soliton bunching in the cavity. (a) Overview of the state; (b): Detailed structure of the bunch.

6.2.3 Bound twin-pulse solitons

Just like the single-pulse solitons, when the separation between two twin-pulse solitons is small, they interact with each other. Under certain conditions, stable bound states of twin-pulse solitons are observed in the laser. For the convenience of comparison, the optical spectrum and autocorrelation trace of the twin-pulse solitons, and a typical irregularly distributed multiple twin-pulse soliton state are shown again in Fig. 6.8.

Figure 6.8 Twin-pulse soliton operation of the laser. (a): The optical spectrum and autocorrelation trace (inset). (b): A typical multiple twin-pulse soliton operation with irregular distribution.

Figure 6.9 shows the optical spectrum and autocorrelation trace of one of the bound twin-pulse soliton states observed in the laser. From the autocorrelation trace it is to see that two twin-pulse solitons are binding together with a separation of about 6.8 ps. The bound state is also evidenced by the measured optical spectrum. A stable spectral modulation with a period of about 1.2 nm, which corresponds exactly to the measured soliton separation of 6.8 ps, is clearly visible on the twin-pulse soliton spectrum.

Figure 6.9 The optical spectrum and autocorrelation trace (inset) of a bound state of two twin-pulse solitons. The separation between the bound two twin-pulse solitons is around 6.8 ps.

Other bound states of twin-pulse solitons with different pulse separations are also observed in the research. Figure 6.10 shows a bound state of twin-pulse solitons with soliton separation of about 12.8 ps. This separation corresponds to an optical spectral modulation of about 0.63 nm. It is to note that with the use of a large scan range of the autocorrelator (APE PulseScope 50), the detailed twin-pulse structure of each soliton cannot be clearly resolved on the autocorrelation trace. But as one decreases the scan range to show only the center peak, the same autocorrelation trace as shown in Fig. 6.8 is obtained, confirming that each individual solitons are actually twin-pulse solitons. Figure 6.11 shows the corresponding oscilloscope trace of Fig. 6.10. Unlike that in Fig. 6.8b, the pulse height in Fig. 6.11 is no longer uniform. There is a pulse whose height is almost twice of that of the others, which

represents the bound twin-pulse solitons. Here again since the photodetector and the sampling oscilloscope are not fast enough to resolve such a short separation, the bound state of the twin-pulse solitons can only be displayed as a pulse of extra height in the oscilloscope trace. In Fig. 6.11, altogether 5 twin-pulse solitons coexist with one bound twin-pulse soliton pair in the cavity.

Figure 6.10 The optical spectrum and corresponding autocorrelation trace (inset) of another bound state of two twin-pulse solitons. The separation between the bound two twin-pulse solitons is around 12.8 ps.

Figure 6.11 The corresponding oscilloscope trace under the same condition of Fig. 6.10.

Coexistence of bound twin-pulse solitons with different soliton separations has also been observed in the laser. Figure 6.12 shows the optical spectrum and autocorrelation trace of such a state. From the autocorrelation trace there are two bound states of twin-pulse solitons coexist in the laser cavity, one with a pulse separation of about 12.8 ps and the other of about 19 ps. Due to the coexistence of the two different bound solitons, the corresponding optical spectrum no longer exhibits clear periodic modulation as those observed in Fig. 6.9 and Fig. 6.10. Apart from the bound states of two twin-pulse solitons, bound states of three twin-pulse solitons have also been experimentally obtained in the laser. Figure 6.13 shows the spectrum and autocorrelation trace of such a case. The optical spectrum exhibits a similar modulation as those shown in Fig. 6.9, while the autocorrelation trace exhibits five peaks with equal peak-to-peak separation of about 6.8ps, suggesting that three twin-pulse solitons are binding together with equal soliton-to-soliton separations. To confirm it, the author also simultaneously checked the oscilloscope trace, and found that there is only one bound twin-pulse soliton triplet in the cavity.

Figure 6.12 Coexistence of two bound twin-pulse solitons with different pulse separations. The inserted autocorrelation trace shows there are two bound states, one with a pulse separation of 12.8ps, and another 19ps.

Figure 6.13 A three twin-pulse soliton bound state. Three twin-pulse solitons bind together with a separation of 6.8 ps.

Experimentally it is found that the appearance of bound states of solitons depends strongly on the initial condition of the laser operation. All the bound states of soliton are stable in the sense that if the laser parameters are fixed, they can remain there for several hours. However, if any of the laser parameters is changed, e.g. the pump power or the orientations of the wave-plates is slightly changed, the bound states may be destroyed. And the same with those of single-pulse solitons, the larger the separation between the bound solitons is, the weaker the binding energy is, and the more easily they could be destroyed.

6.3 Direct soliton-soliton interaction

6.3.1 Introduction on direct soliton interaction

When two solitons are close enough (the separation in between is less than 5 pulse widths), another kind of soliton interaction, direct soliton interaction or soliton interference, becomes significant. Direct soliton interaction is kind of interaction which is most intensively studied in previous analytical and numerical studies [58-

60]. Gordon has derived expressions describing the case of two-soliton interaction in the framework of the NLSE based on the perturbation theory. He found that the motion of two neighboring solitons with nearly equal amplitudes and frequencies could be described in terms of interaction forces that decrease exponentially with their distance of separation and vary sinusoidally with their relative phase [58]. With a very short separation, two fundamental solitons with the same amplitudes and frequencies may attract or repel depending on their initial relative phase Ψ_0 . In the case of two in-phase solitons ($\Psi_0=0$), the relative separation between adjacent solitons changes with propagation periodically as [131]

$$q(\xi) = q_0 + \ln|\cos[2\xi \exp(-q_0)]|, \quad (5.4.1)$$

where q_0 is the initial normalized soliton separation, and ξ is the normalized space axes. Under this condition, the two solitons collide with each other periodically. While in the case of two out-of-phase solitons ($\Psi_0=\pi$),

$$q(\xi) = q_0 + \ln|\cosh[2\xi \exp(-q_0)]|, \quad (5.4.2)$$

which shows that the relative separation increases monotonically. In all, it is to see that the direct interaction between two NLSE solitons could result in soliton collision, or soliton repulsion depending on their relative phase difference.

Since a fiber laser is a periodical gain-and-loss system, the dynamics of solitons in the system is not described by the NLSE, but by the Ginzburg-Landau equation (GLE). Some researchers have theoretically investigated interactions between the solitons described by the GLE and found that bound states of solitons with fixed, discrete soliton separations could be formed under either the direct soliton interaction or the dispersive waves mediated soliton interaction [63-67].

Although extensive theoretical studies have been done on direct soliton interaction, in particular, in the case of solitons described by the GLE, formation of bound solitons caused by direct soliton interaction was predicted; nevertheless, no

experimental studies on the direct soliton interaction in passively mode-locked fiber lasers were reported. This is partially because direct soliton interaction depends sensitively on the relative soliton phase difference, while in practice because of the unavoidable environmental perturbations, the relative soliton phase always varies, it is difficult to obtain a situation where solitons directly interact with constant phase difference. Even if the environmental perturbations could be suppressed, the existence of other forms of soliton interaction, such as the long-range soliton interaction mediated through the resonant dispersive waves, which always exists when the separation of two solitons comes into the range of < 500 ps, complicates the dynamics of the direct soliton interaction in a laser.

6.3.2 Formation of twin-pulse solitons

Although in the experiment one could frequently observe that solitons moved across or collide one another during a transient process mostly when a laser parameter is suddenly and significantly changed, and it doubtlessly shows the existence of direct soliton interaction in the system, this process normally occurs very quick, and it is difficult to experimentally investigate their interaction dynamics. However, it is noticed that the separation between the two pulses in a twin-pulse solitons is merely 930 fs as shown in Fig. 3.10 and Fig. 6.8a. Obviously the direct soliton interaction exists between them. The optical spectrum also shows that the two solitons in a twin-pulse solitons is out of phase. The twin-pulse solitons can be regarded as a special case of bound solitons formed in the presence of the direct soliton interaction.

Comparing with the bound solitons formed by the resonant dispersive waves mediated soliton interaction, this special bound twin-pulse soliton state exhibited a number of unique properties. In particular, they are super stable. Independent of the concrete laser operation conditions, e.g. the laser pump power, and the exact orientations of the wave plates, whenever such a state of bound solitons is obtained, it has the same pulse separation and phase difference. Increasing pump power, the number of the bound-soliton increases, while decreasing pump power carefully, the

bound-soliton as an entity is destroyed one by one in the cavity. Multiple bound-solitons of exactly the same property can coexist in the cavity. Like the multiple soliton operation of the laser described above, the multiple bound-solitons can form the various modes of operation as described above, such as bunching and passive HML of the bound-solitons. They can also interact with each other and form bound twin-pulse solitons through dispersive waves mediated soliton interaction as illustrated in 6.2.3. All these features suggest that the bound-solitons formed under the direct soliton interaction have very strong binding energy. It is virtually a new form of soliton pulse.

Worthy of clarifying, the twin-pulse soliton formed in the laser is not a second-order soliton defined by the NLSE or GLE. Although they resemble each other in the way that they both comprise two pulses, their formation and properties are quite different. Both NLSE and GLE have higher-order soliton solutions (as $N > 1$ in Eq. 2.3.8). Higher-order solitons are not as stable as the fundamental solitons, quite another, their shape and amplitude change periodically along the propagation, which can be interpreted as an oscillatory mutual interaction between the GVD and SPM. However, the laser solitons are different with those solutions of the NLSE or GLE, since the formation of laser solitons is not only defined by the propagating properties in fibers, but also affected by the laser components. Namely, the laser solitons are first of all mode-locked pulses. A peak power clamping mechanism (see Sec. 4.4.3) exists, which together with the gain bandwidth limitation makes the formation of higher-order soliton not possible in the laser. However, the formation of twin-pulse solitons is a special result of strong direct soliton interaction. Although the formation mechanism of twin-pulse solitons need to be further investigated, they are still obtained under the balanced affect between the average cavity dispersion and the nonlinear effect. Moreover, different with higher-order solitons, twin-pulse solitons are quite stable: their shapes and twin-pulse separations never change. They behave as stable units and have exactly same properties as fundamental solitons, from which they are regarded as an intrinsic soliton state of the laser.

6.4 Numerical evidences

To have a better understanding on the soliton interaction in the laser, in particular, to find out the influence of the resonant dispersive waves on the soliton interaction, the author has also numerically simulated the soliton interaction of the laser. Again the simulations are based on the pulse tracing technique discussed in Chap. 4.

6.4.1 Bound solitons and soliton bunch

Through increasing the pump power, which in the model corresponds to increasing the small signal gain and gain saturation energy, multiple soliton pulses can be automatically generated in the model. Depending on the laser operation parameter settings, new solitons can be formed one by one through either the soliton-shaping of dispersive waves or the unstable CW components, or the pulse splitting. Here we will focus on the soliton interaction without considering the generation or elimination of solitons. Figure 6.14 shows a case of two solitons bound with a large pulse separation in the cavity. It is to see that in the steady state, the two solitons propagate in the laser cavity stably with a fixed soliton separation. Dispersive waves between the two solitons are clearly to see, which suggests strongly that the two solitons could be bound through the dispersive waves mediated soliton interaction. Numerical studies show that such a state of bound solitons is just marginally stable. When laser parameters e.g. the gain, are slightly changed, the separation of the solitons varies. Figure 6.14b shows the corresponding optical spectrum of the state. From the interference patterns, the phase difference between the two bound solitons varies slowly from round to round. If the gain parameter is significantly reduced so that the laser gain can no longer support two solitons in the cavity, one of the solitons gradually disappears, while the other still remains as a soliton but with increased pulse intensity exactly as that observed experimentally. Bound solitons could also be formed in the presence of the direct soliton interaction. For comparison, Fig. 6.15 shows the case where two solitons have close pulse separation (the twin-pulse soliton) so that the direct soliton interaction between

them cannot be ignored. Like the experimental observations, in the steady state the two solitons propagate in the cavity with fixed soliton separation and fixed phase difference. The state of bound solitons is significantly more stable than that shown in Fig. 6.14. Even with about 8 % gain or saturation energy changes, it remains the exactly same pulse separation, despite that the soliton intensity also varies with the gain change. If a big gain reduction is made, intensities of both solitons decrease and eventually both of the bound two solitons are destroyed together.

Figure 6.14 A numerically calculated state of bound solitons, where two solitons are bound through the resonant dispersive wave mediated long-range interaction. Cavity linear phase delay bias $\phi=1.5\pi$. Gain parameter $g=420$, gain saturation energy $E_s=300$. (a): Time evolutions of the state. (b): Optical spectral variations.

Figure 6.15 A numerically calculated state of bound solitons, where the direct soliton interaction between the two solitons exists. Cavity linear phase delay bias $\varphi=1.3\pi$. Gain parameter $g=620$, gain saturation energy $E_s=300$. (a): Intensity profile in time domain. (b): Optical spectrum.

Soliton bunches have also been obtained in the numerical simulations as shown in Fig. 6.16. Soliton bunching seems a very general feature of the laser. Multiple solitons bind through the resonant dispersive waves in the cavity and move together with the same velocity. In the simulations, limited by the computer speed, a calculation window of 128 ps has been selected, which is a very small window compared with the actual cavity round trip time of the laser. A direct consequence of the limited simulation window size is that the solitons are crowded in the window. In the case of large number of solitons, artificial direct soliton interaction

effects may generate. However, considering that the fundamental soliton pulse width of the laser is only about 340 fs, it can still accommodate many solitons with enough large soliton separations in such a window. These numerical results at least provide some useful insight into the soliton interaction in the laser.

Figure 6.16 A state of soliton bunching numerically calculated. Cavity linear phase delay bias $\varphi=1.1\pi$. Gain parameter $g=415$, gain saturation energy $E_s=200$.

6.4.2 Soliton oscillation and soliton collision

Collision of solitons and oscillation of soliton separation, as a result of strong dispersive waves mediated soliton interaction, have also been numerically reproduced. Figure 6.17 shows a case of the numerically obtained relative soliton separation oscillation. Different to the case of bound solitons, where two solitons keep a fixed soliton separation between them, the separation between two solitons oscillates randomly as they propagate in the cavity. Because of the strong coupling between the solitons through dispersive waves, the intensities of the two solitons also vary stochastically.

Figure 6.17 Relative soliton separation oscillation caused by the resonant dispersive wave mediated soliton interaction. $g_0=352$, $\Delta\Phi_{pc}=1.4\pi$.

Figure 6.18 shows a numerically calculated soliton collision process. Two solitons move with different velocities eventually meet in the cavity and make a collision. The detailed collision process varies case by case. Sometimes, they just simply cross each other with slight velocity changes, sometimes one soliton is first destroyed, but after several round trips of complicated variation, it is eventually rebuilt up and the final state still with two solitons in the cavity.

Figure 6.18 A state of soliton collision numerically calculated. Cavity linear phase delay bias $\varphi=1.4\pi$. Gain parameter $g=420$, gain saturation energy $E_s=300$.

6.4.3 Discussions

The numerical simulations on soliton interaction show the following general characteristics of soliton interaction in the laser: In the case of weak dispersive waves in the cavity, the multiple solitons generated in the laser have no tendency of movement, i.e. the solitons do not spread out in the laser cavity but fix at their initial positions with irregular soliton separations. They behave just like a tightly soliton bunch. In such a state, solitons could have random but fixed separations during their propagation in the cavity, and both direct and dispersive wave mediated soliton interaction could coexist. In the case of strong resonant dispersive wave emission, the multiple solitons have the tendency of moving away from each other. Occasionally, bound state of solitons can be formed; however, it is not stable. Changing the simulation parameters can easily destroy it.

It is also proved that both the dispersive wave mediated long-range soliton interaction and the direct soliton interaction could result in the formations of bound solitons. However, the bound solitons formed under the two conditions are quite different. The bound solitons formed through the dispersive waves are marginally stable and easy to be destroyed by perturbations. While, the bound solitons formed in the presence of the direct soliton interaction exhibit super stability.

6.5 Summary on soliton interaction

In summary, based on the experimental and numerical investigations, comparisons of the three types of soliton interaction are listed in Table. 6.1. Worthy of mentioning here, the author used the previous predictions to say that the effective range of the direct soliton interaction is around 5 pulse widths. However, since the solitons in a fiber laser are average solitons, which are not strictly the same with those used in theoretical predictions, to what extent this restriction is applicable still needs further studying. One of the key concepts is that the solitons in a fiber laser is actually not strictly Sech form, on the contrary, they could have long tails

depending on the setting of the parameters. In the numerical simulation, it is found that, as adjusting the parameters so that the soliton tail is very long, the direct soliton interaction could exist to more than 10 pulse widths. Further investigations on this issue are expected in the future, which is beyond the scope of this dissertation.

Table 6.1 Comparisons of the three types of soliton interaction.

	Origin	Effective range	Effective strength	Consequences
Global interaction	Unstable CW	Global whole cavity	Weak	Irregular scattering, Quasi- and HML
Long-range interaction	Dispersive waves	Long ~500 ps	Middle	Soliton bunch, Weakly bound solitons, Soliton oscillation
Direct interaction	Interference	Short ~5 τ_0	Strong	Twin-pulse solitons, Stable bound solitons, Soliton collision

Chapter 7

Soliton pulse nonuniformity

The solitons formed in a fiber laser is known as the average solitons [72]. Suggested by the average soliton theory of the laser, it was generally believed that the output of the lasers is uniform soliton pulses. Until very recently, Kim and Spaulding et al theoretically analyzed the pulse dynamics of the lasers passively mode-locked by using the NPR technique [70,71]. They found that due to the interaction between the nonlinear polarization rotation and the passive polarizer inside the cavity, there exists an intrinsic intensity fluctuation of the output soliton pulses unless the polarizer is aligned with either the slow or the fast axis of the fiber. So far this theoretical prediction has not been experimentally confirmed. In addition, it is noticed that for the sake of simplicity, in their analysis Kim et al. have ignored the effect of resonant sideband generation of solitons in the laser. But in practice this effect always exists and significantly affects the properties of the solitons [87-89]. So in what extent their theoretical prediction describes the real soliton pulse dynamics in the lasers needs to be further studied.

This chapter presents the observation of two distinct types of soliton pulse amplitude modulation. Section 7.1 presents the observation on a low-frequency modulation on the soliton intensity, which is found to be intrinsic in the fiber laser mode-locked using the NPR technique. And section 7.2 shows the observation of another kind of pulse nonuniformity introduced by the unstable dispersive waves in the fiber laser.

7.1 Intrinsic pulse nonuniformity

7.1.1 Experimental observations

The experiments were conducted on the laser setup presented in Chap.3 (Fig.3.2). Experimentally the uniformity of the soliton output of the laser could be easily investigated by simultaneously monitoring the optical spectra of the solitons with an OSA and the RF spectra of the laser output with a RF spectrum analyzer. It was found that independent of the orientation of the passive polarizer in the cavity and the linear light polarization rotation bias, either stable uniform or nonuniform soliton pulse output could be observed by changing the pump strength. For example Fig. 7.1 shows the respective soliton spectra of the laser as the pump power is slightly increased from 97 mw in (a) to 100 mw in (b), 105 mw in (c) and 115 mw in (d). Associated with the pump power increase, the spectral strength of the dispersive waves increases from (a) to (d). Figure 7.2 presents the RF spectra of the laser output corresponding to the four states in Fig. 7.1, respectively. In the RF spectra only the fundamental RF component is displayed, which corresponds to the cavity repetition rate of 37.8 MHz. Except the pump power, all the other experimental parameters were fixed in obtaining the figures.

Figure 7.1 Experimentally measured soliton spectra of the laser under different pump levels. From (a) to (d), the pump power is 97mW, 100mW, 105mW and 115mW, respectively.

Figure 7.2 The corresponding rf spectra of the laser under the same conditions of Fig. 7.1.

Figure 7.1 shows that as the pump power is increased, the sideband strength also increases slightly. When the pump power is too strong, even the CW emissions had also appeared in experiment as shown in Fig. 7.1c and d. However, the CW components did not affect the soliton property. From Fig. 7.2 it is to see that when the sidebands are weak, correspondingly, the soliton peak power is weak, there is only one peak located at 37.8 MHz in the RF spectrum of Fig. 7.2a, which indicates that the laser have very uniform soliton output under the condition. However, as the pump power is increased to a certain level that depends on the settings of the waveplates in the experiment, two symmetric frequency sidebands appeared beside the main fundamental peak in Fig. 7.2b, showing that the output soliton pulses are no longer uniform, but endure a periodic amplitude modulation. The modulation frequency is about 2.1 MHz, which is the frequency difference between the

sidebands and the 37.8 MHz main peak. This modulation frequency corresponds to a period of 18 round trips. Slightly increasing the pump power further, we found that the modulation strength increases, as shown in Fig. 7.2c, and even the harmonic sidebands appear as in Fig. 7.2d. Increasing pump power the modulation frequency increases slightly as well. The modulation frequency in Fig. 7.2c is 2.5 MHz, corresponding to a period of 15 round trips. And the modulation frequency in Fig. 7.2d is 2.7 MHz, corresponding to a period of 14 round trips.

The uniformities of solitons under different polarizer orientations have also been examined. Experimentally it is found that independent of the orientation of the polarizer, as long as the pump power is appropriately adjusted, one can always obtain stable uniform soliton output. On the contrary, with all the other conditions fixed when the pump power is increased, the output pulses always become nonuniform, exhibiting a periodic-kind pulse amplitude variation. The nonuniformity of the output soliton pulses is actually an intrinsic feature of the laser.

7.1.2 Numerical evidences

To get an insight into the physical mechanism of the observed soliton pulse non-uniformity, the author has further conducted numerical simulations to study the soliton evolution in the laser. Again the simulations are based on the numerical model described in Chap.4.

By properly setting the linear cavity phase bias, which corresponds in the experiment to appropriately select the orientations of the wave-plates so as to ensure the laser is operating in the positive feedback regime, soliton operation can be always obtained in the simulation. The exact soliton parameters, such as the pulse width and peak power, are determined by the laser parameter settings. However, in all the simulation it was found that when the value of the gain is so selected that the generated soliton pulse peak power is weaker than that of the polarization switching power of the laser, one can always obtain the ideally uniform soliton pulse train as

shown in Fig. 7.3a. Fig. 7.3d is the corresponding spectrum of Fig. 7.3a. In this case the soliton pulses are very stable and uniform from round to round. In the simulations the orientation of the passive polarizer is not aligned with the principal axes of the optical fiber. But obviously one could still obtain stable uniform soliton pulse output from the laser. This result is very different from Kim et al's theoretical predictions. The author has also checked laser output under different polarizer orientations, and observed the similar results. However, independent of the polarizer's orientation and the linear cavity light polarization rotation bias, if the gain is set so large that the soliton peak power becomes larger than the polarization switching power, periodic-like amplitude variations always appear on the output soliton pulses as shown in Fig. 7.3b. This can be more obviously observed on its corresponding spectrum shown in Fig. 7.3e. The strength of the soliton sidebands changes with a period of around 20 round trips, showing that the amplitudes of the output soliton pulses are no longer uniform, but vary periodically. If the gain is further increased, the strength of the pulse intensity variation increases, and the modulation period reduces as shown in Fig. 7.3c and its corresponding spectrum Fig. 7.3f. In this case, the amplitude modulation become stronger and more complicated, as indicated by the appearance of the harmonic sidebands in the experiment as shown in Fig. 7.2d. It is to note that only the gain parameter, which experimentally corresponds to the pump power, is increased in the calculation conditions of Fig. 7.3.

From the numerical simulation, one can get that as increasing the gain parameter, the strength of the sidebands increases and periodical intensity fluctuations are then observed, which is qualitatively consistent with the experimental results. However, it should be noted that due to the average effect of the experimentally recorded optical spectra, the periodical change of the sideband strength could not be observed on the optical spectra recorded in the experiment. One could only observe the slight increase of the sideband strength as the pump power is increased in the experiment as shown in Fig. 7.1. Obviously, the numerical simulations well reproduce details of the experimental observations, which demonstrated that the observed periodic

soliton pulse intensity fluctuation is indeed an intrinsic feature of the fiber soliton lasers.

Figure 7.3 Soliton pulses and the corresponding optical spectra calculated under different pump strength. $\Delta\Phi_{pc}=1.4\pi$. (a) and (d): $g=300$; (b) and (e): $g=360$; (c) and (f): fluctuation, $g=450$.

All the other parameters are fixed

7.1.3 Mechanism of the pulse nonuniformity

The physical origin of the observed soliton pulse nonuniformity is further investigated based on the above experimental results and numerical simulations. It turned out this feature of the laser is again related to the peak power clamp effect of the laser cavity, which could still be explained as a result of interaction between the NPR of the soliton pulse and the polarizer in the cavity. As we know the laser is mode-locked by using the NPR effect. Through the combined action of the nonlinear light polarization rotation and a passive polarizer in the cavity, an artificial saturable absorption effect is generated in the laser, which results in the mode locking of the laser and further stabilizes the solitons once they are formed. However, since the NPR is soliton peak power dependent, which also cause the polarization switching in the cavity. As the author has discussed in Chap. 4, the nonlinear polarization switching effect clamps the maximum peak power of the solitons formed in the cavity. In a laser once the linear cavity phase delay is selected, the nonlinear polarization switching power is also fixed. When the laser gain is weak so that the formed solitons have a peak power that is weaker than the polarization switching power, the soliton peak is determined by the cavity gain as a result of the soliton internal energy balance. In this case the nonlinear polarization switching plays no role and the soliton output of the laser is stable and uniform. However, if the gain of the laser is so strong that the solitons formed in the cavity have a peak power that is higher than the nonlinear polarization switching power, the cavity peak power clamp effect occurs, which generates a negative feedback on the pulse peak. In the laser cavity apart from the soliton pulses there are also resonant dispersive waves, which share with the solitons the laser gain. Once the peak power of the solitons become so high that the loss they experienced is larger than that experienced by the resonant dispersive waves, as a result of the gain competition, the strength of the dispersive waves will be amplified, consequently the effective gain experienced by the solitons is reduced and the soliton peak power decreases. However, as the soliton peak intensity decreases to a certain value where the loss they experienced will become again less than that experienced by the resonant dispersive waves, their intensity will start to increase. Therefore, as a result

of the nonlinear polarization switching and the gain competition with the resonant dispersive waves the soliton peak becomes periodically modulated. If the gain of the laser is not too strong, the competition between the soliton and the resonant dispersive waves is weak, a slow periodic-like modulation is caused. However, if the gain is high, this competition also becomes stronger, a quicker and complicated soliton peak modulation is then observed. When the cavity gain becomes so large that it can support an extra soliton pulse, a new soliton will then be generated. Consequently the effective gain experienced by each soliton in the cavity is reduced. A new stable state of multiple soliton operation will then be obtained. The soliton peak modulation is also reflected on the spectral intensity variation of the resonant sidebands. As the dispersive waves are adaptive to the soliton parameters and compete the laser gain with the soliton, therefore, the dispersive wave intensity variation is even more complicated.

Based on the mechanism discussed above, it is to see the soliton pulse peak non-uniformity is an intrinsic feature of the laser. In order to obtain stable soliton output the pump strength/gain parameter must be carefully controlled so that the peak power of the solitons is below the polarization switching threshold. On the other side, as the polarization switching threshold is related to the linear polarization bias in the cavity, with a fixed pump power, one can also suppress the pulse nonuniformity by appropriately adjusting the linear polarization bias to a position where the polarization switching power is higher. In this case as long as the pump power is increased, the pulse nonuniformity will always appear again. These two methods have been proved both feasible in the simulations. However, in practical experiment, since the accuracy of adjusting the linear polarization bias by rotating the wave-plates is limited, and also the cavity operation is much more sensitive to the polarization change than to the pump power, it is very difficult to suppress the pulse intensity fluctuation by adjusting the linear polarization bias.

7.2 Unstable dispersive wave induced nonuniformity

7.2.1 Experimental observations

Apart from the soliton pulse variation discussed above, it is found that the modulation instability (MI) of the dispersive waves could introduce another frequency of soliton pulse intensity modulation.

Since soliton pulses transfer their energy into the dispersive waves in the reshaping process, before a new soliton is created, increasing the pump power will significantly increase the energy of the dispersive waves rather than the energy of solitons. As dispersive waves are linear waves, they are intrinsically unstable against the MI. When their strength is strong enough, they become unstable due to MI and consequently produce new discrete spectral components in the soliton spectrum. A phenomenon so called sub-sideband generation then appears in the laser. In the laser, by increasing the pump power, provided that the polarization orientations have already been adjusted to a position where the linear loss is small, which indicates the linear dispersive waves experience less loss in the cavity and can then grow very strong, the MI induced sub-sidebands can be observed. Figure 7.4 gives the optical spectrum and the corresponding RF spectrum of the laser output of such a state that the MI sub-sidebands are apparent. In Fig. 7.4a, the sub-sidebands are clearly seen, as a manifestation of the strong MI of the dispersive waves.

An unusual phenomenon that attracts the author's attention in this state is that some slow frequency components appear in the RF spectrum as shown in Fig. 7.4d, which shows the appearance of a periodic modulation on the soliton pulses. The modulation frequency is about 9.4 MHz, corresponding to ~ 4 cavity round trips. Different with the pulse modulation presented in the last section, this modulation has a higher frequency. As the author continually reduced the pump power, the sub-sidebands on the soliton spectrum become weaker and weaker, and as an

consequence, the modulation frequency sidebands on the intensity spectrum becomes weaker as well, which shows that this amplitude modulation is connected with the MI of the dispersive waves. This process is recorded in Fig. 7.4.

Figure 7.4 The optical spectra and corresponding rf spectra as the sidebands become unstable. From (a) to (c), the pump power is slight reduced. (d)-(f) are the respective rf spectra of (a)-(c).

With proper polarization settings, the author has also observed the coexistence of the two modulation frequencies. Figure 7.5 presents the optical spectrum and corresponding RF spectrum of such a state.

Figure 7.5 Optical spectrum and corresponding rf spectrum of a state where the two amplitude modulations coexist.

In this case, the MI induced sub-sidebands are clearly seen on the optical spectrum as in Fig 7.5a. And in the RF spectrum, four frequency sidebands appear symmetrically to the main fundamental peak: the inner two sidebands have a frequency of around 3 MHz, and the outsider sidebands have a frequency of around 11.4 MHz.

7.2.2 Discussions

In Fig. 7.5 the inner sidebands are caused by the generic pulse nonuniformity in NPR mode-locked fiber lasers, which have been detailedly discussed in the last section [132]. But what mechanisms could be responsible for the other pair of sidebands with higher modulation frequency?

Experimentally it was found this modulation frequency only occurs when the MI of the sidebands appear and could be suppressed by reducing the pump power to eliminate the sub-sidebands. The author have tried different polarization orientations and found that only at the positions where the linear loss is small so that the MI induced sub-sidebands can be apparently observed, could this modulation frequency occurs, which strongly suggested that this modulation on the soliton intensity may be related to the unstable dispersive waves. Generally it is known that the existence of stable dispersive waves will not affect the property of the solitons in the cavity. They can even form a stationary pattern in the laser cavity, which further stabilizes the propagation of solitons [133]. However, when the dispersive waves become unstable due to the MI, they no longer form a stable pattern in the cavity. It is to image that under the MI the dispersive waves would form some movable background pattern relative to the solitons. Then the interference between the unstable dispersive waves and solitons would cause the soliton amplitude to temporally fluctuate. As in the laser the NPR technique is used to achieve the passive mode locking, the cavity transmission sensitively depends on the nonlinear phase shift of the pulses traveling in the cavity. Any temporal pulse amplitude fluctuation will affect the cavity transmission, and eventually introduces actual pulse energy variation.

Chapter 8

Conclusion

In this dissertation, the optical soliton operations of a passively mode-locked fiber laser have been extensively investigated. A great amount of information on soliton properties, soliton evolution and interaction in fiber lasers has been obtained. This chapter summarizes the accomplishments in the research and presents some suggestions for the future work.

8.1 Summary of accomplishments

➤ Operation regimes of a passively mode-locked fiber soliton laser

First of all, the operation characteristics of the fiber soliton laser have been experimentally studied. Three operation regimes, viz. single-pulse soliton operation, twin-pulse soliton operation, and noise-like pulse operation, have been demonstrated in the laser under different polarization settings. As an intrinsic feature of passively mode-locked fiber lasers, soliton operation can be obtained in the laser under the balanced effect between the fiber nonlinearity and anomalous cavity dispersion. Optical solitons in a fiber laser exhibit some special characteristics such as the sideband generation, pump power hysteresis, energy quantization, various modes of multiple soliton operation and so on, which have all been demonstrated in the laser.

➤ Numerical simulations on the fiber laser

To get a better understanding on the experimental results, a numerical model has been built up based on the coupled complex GLEs to numerically simulate the fiber laser system. Different with the models used by other groups, the so-called master-

equation simplification was not used in this model. By using a single pulse tracing technique, the model has explicitly taken into account the effect of nonlinear coupling between the two orthogonal polarization components of the light in a birefringent fiber, the frequency-dependent linear cavity loss, the nonlinear saturable absorber effect introduced by the combined effect of the nonlinear light polarization rotation and the passive polarizer. The dispersive wave radiation is also implicitly considered in the model. The numerical simulation results reproduced all the details of the experimental observations. Single-pulse soliton operation, twin-pulse soliton operation, and noise-like pulse generation have all been numerically reproduced for the laser. Especially, the properties of soliton operation such as the soliton energy quantization, sideband generation and soliton wavelength tuning have been well interpreted in the numerical simulations. The numerical model also helped to understand some specific phenomena observed in the experiment such as soliton interaction and soliton nonuniformity. Comparing those numerically calculated results with the experimental observations, one can get an insight of the operation of a fiber laser and optical solitons.

➤ **Twin-pulse soliton emission**

An important accomplishment in the present dissertation is the investigation on a novel twin-pulse soliton emission in the laser. They are referred to as twin-pulse solitons due to that their intensity profile has a twin-pulse structure. Since the twin-pulse solitons are characterized as two pulses binding together tightly with closed, fixed separation, at the first beginning, they were regarded as a bound soliton state formed under the direct soliton interaction. However, further investigation on the properties of this state revealed that the two pulses in a bound-soliton always behave as a unit. They are in fact not separable. Interaction of such solitons also revealed that they are not simply a state that two solitons bind together, but rather a form of multi-pulse (-hump) solitons in the laser. All the features of the single-pulse soliton in a fiber laser, such as energy quantization, pump power hysteresis, soliton bunching, quasi- and harmonic mode locking, have also been observed on the twin-pulse solitons. Numerical simulations also confirmed the existence of twin-pulse

solitons, which shows that the multi-hump solitons emission is another intrinsic feature of the passively mode-locked fiber laser.

➤ Soliton interaction

Various modes of multiple soliton operations have been experimentally obtained in the fiber laser under soliton interactions. Experimental studies revealed the existence of three dominating types of soliton interaction that affect the soliton dynamics in the laser. They are a global type of soliton interaction caused by unstable CW lasing, a local type of long-range soliton interaction caused by the resonant dispersive waves, and direct soliton-soliton interaction.

The impact of each soliton interaction on the formation of the various modes of soliton operation of the laser has also been investigated. The global type of soliton interaction has the largest interaction range but the weakest interaction strength among the three types of soliton interactions. However, this type of soliton interaction can be well suppressed by carefully controlling the pump strength. Experimentally it is found that the global soliton interaction plays a key role on the formation of the passive harmonic mode locking of the laser. The local type of dispersive wave mediated long-range soliton interaction has an interaction range of around 500 ps in the laser, which is determined by the spatial extension of the resonant dispersive waves. This type of soliton interaction can either make solitons oscillate with each other, or form soliton bunches and states of bound solitons depending on the concrete experimental conditions. The direct soliton interaction only occurs when the separation of two adjacent solitons is less than 2 ps (~ 5 times of the soliton pulse width) in the laser. This kind of soliton interaction has the strongest interaction strength. Bound solitons formed by the direct soliton interaction have very strong binding energy between the pulses. Therefore, they are very stable and practically behave like a new form of soliton. The numerical simulations reproduced both the bound solitons binding through the dispersive waves mediated soliton interaction and the direct soliton interaction. The numerical results clearly show that solitons binding through the direct soliton interaction have

stronger stability. The oscillation and collision of solitons, induced by the long-range and direct soliton interaction respectively, were also reproduced in the numerical calculation. Furthermore, the numerical simulations also demonstrated that multiple soliton operation is an inherent feature of the laser. With a limited simulation window, it has been shown that the multiple solitons tend to bunch together to form various soliton bunches under the existence of the dispersive wave mediated soliton interaction and the direct soliton interaction.

➤ **Amplitude variations of soliton pulses**

Unlike previous viewpoint that soliton pulses from a fiber laser are uniform, periodic amplitude variations of the soliton pulses from the fiber soliton laser have been experimentally observed. It is found out that pulse nonuniformity is intrinsic in NPR mode-locked lasers and can be eliminated by appropriately controlling the pump power. Numerical simulations on the laser operation confirmed the existence of this soliton pulse nonuniformity. Both the experimental results and the numerical simulations suggested that the soliton pulse nonuniformity is caused by a peak power clamp effect induced through the interaction of the nonlinear soliton pulse polarization rotation and the passive polarizer in the cavity. When the peak intensity of the soliton pulse is too high, the generated nonlinear polarization rotation could bias over the polarization switching point, and consequently drives the laser cavity from the positive feedback regime to the negative feedback regime, which consequently causes the amplitude of the soliton pulses to periodically vary.

In addition, experimental observations show that the MI of the dispersive waves could also affect the uniformity of the output pulses of the laser. Since the dispersive waves are linear waves, due to modulation instability, they are intrinsically unstable when their strength becomes strong. Then the interference between the soliton pulses and their unstable dispersive waves will make the amplitude of the solitons pulse temporally varying.

➤ **MI and FPU recurrence of the CW component**

Besides the optical solitons in the laser, CW component could be also observed in the laser cavity as the pump power is strong. It has been experimentally observed that the CW component in a fiber soliton ring laser becomes unstable due to the modulation instability (MI) when its strength is strong, and its MI manifests a reversible behavior-the FPU recurrence. The observation on the FPU recurrence attributes to the ring structure of the laser cavity, as well as the existence of erbium-doped fiber in the cavity. The existence of the erbium-doped fiber greatly lowers the threshold for MI, which makes the MI easier to be observed. And the ring structure of the laser cavity permits the light to travel in the cavity round after round. The long propagation path and periodical insertion of amplifiers needed to observe the FPU recurrence in transmission systems are therefore automatically satisfied in the experiment, which makes it much easier to observe the phenomenon.

8.2 Recommendations for future work

Despite that the discussions are mainly focused on the soliton operations in a passively mode-locked fiber soliton laser, the author believes that they could be commonly applied for the soliton operations in other systems, such as fiber communication links and other laser systems. In particular, study on optical solitons in the dispersion-managed fiber lasers, the double-cladding fiber lasers and solid-state lasers are expected. On what extent the conditions in other systems consist with those observed in the present fiber laser, and what other new properties solitons in these systems could have, need to be further investigated.

The soliton interaction has been discussed in Chap. 5. However, since the soliton pulses in a fiber laser are not exactly Sech form, whose properties are not only determined by their soliton nature but also by the laser parameters. Strictly speaking, they are not the same with those described by the NLSE or GLE. Therefore some theoretical predictions on soliton interactions in NLSE or GLE may be not accurate in this case. In the numerical simulations, some quantitative

disagreement with those predicted for ideal solitons have been observed. Further investigations on this issue are thereby suggested.

Moreover, it is noted that in all our results on bound solitons, the phase difference between the bound two solitons is always around π . So far what mechanisms caused this phase locking of the solitons is still a mystery to us. Will solitons with the same phase bind together stably? If yes, under what conditions will these bound states happen? Do the three-pulse solitons or four-pulse solitons exist stably as the twin-pulse solitons do? What conditions do they require to be obtained? All these questions are still waiting for answers in the further work.

Finally, a major drawback for the application of passively mode-locked fiber lasers is the low and unpredictable repetition rates of the laser output. Although passive harmonic mode locking has been observed in the laser, it is still difficult to accurately control the laser repetition rate. Therefore, how to increase and stabilize the repetition rate of the passively mode-locked fiber soliton laser is an important topic of the further work. In addition, for soliton application in practical communication systems, to compensate the transmission loss, amplification of solitons is usually necessary after propagation of certain distance. How to efficiently amplify soliton pulses still needs to be further investigated.

Appendix A

Calculation of the cavity transmission

Figure 3.1 Schematic diagram of the laser cavity equivalence.

Figure 3.1 is shown again for the convenience of the calculation. The coordinate system is chosen to coincide with the two polarization eigen-modes of the birefringent fiber. The polarizer and analyzer are at such positions that their polarizations have angles of θ and φ , respectively, to the x axis.

Light with any arbitrary polarization becomes linearly polarized after passing through the polarizer, which can be described by the field vector:

$$\vec{E}_{in} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = A \cdot \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

where $\vec{E}_{in}$ is the incident electrical field vector, E_x and E_y are its respective components along the two orthogonal polarization axes, and A is its amplitude. The intensity of the incident light is $I_{in} = \vec{E}_{in}^* \cdot \vec{E}_{in} = A^2$.

The light is incident to the fiber and experiences a phase retardation of $\Delta\phi_F$ between the two light polarization components due to the fiber birefringence. Referring to the Jones matrix of a phase shifter or a retarder, the transformation of the fiber is written as:

$$J_F = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\Delta\phi_F) \end{pmatrix}$$

Then the light will pass through the analyzer, whose transformation matrix is the same with that of a polarizer:

$$J_P = \begin{pmatrix} \cos^2 \varphi & \sin \varphi \cos \varphi \\ \sin^2 \varphi \cos \varphi & \sin^2 \varphi \end{pmatrix}$$

Therefore, the field vector of the output light after passing through the configuration is:

$$\begin{aligned} \bar{E}_{out} &= J_P \cdot J_F \cdot \bar{E}_{in} = A \cdot \begin{pmatrix} \cos^2 \varphi & \sin \varphi \cos \varphi \\ \sin^2 \varphi \cos \varphi & \sin^2 \varphi \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\Delta\phi_F) \end{pmatrix} \cdot \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \\ &= \begin{pmatrix} \cos \varphi \cdot [\cos \theta \cos \varphi + \sin \theta \sin \varphi \cdot \exp(i\Delta\phi_F)] \\ \sin \varphi \cdot [\cos \theta \cos \varphi + \sin \theta \sin \varphi \cdot \exp(i\Delta\phi_F)] \end{pmatrix} \end{aligned}$$

From the definition of the intensity transmission parameter, the transmission of light passing through the setup is

$$T = \frac{I_{out}}{I_{in}} = \frac{\bar{E}_{out}^* \cdot \bar{E}_{out}}{\bar{E}_{in}^* \cdot \bar{E}_{in}}$$

$$\begin{aligned}
&= [\cos \theta \cos \varphi + \sin \theta \sin \varphi \cdot \exp(i\Delta\phi_F)] \cdot [\cos \theta \cos \varphi + \sin \theta \sin \varphi \cdot \exp(-i\Delta\phi_F)] \cdot (\cos^2 \varphi + \sin^2 \varphi) \\
&= \cos^2 \theta \cos^2 \varphi + \sin^2 \theta \sin^2 \varphi + \sin \theta \sin \varphi \cos \theta \cos \varphi \cdot [\exp(i\Delta\phi_F) + \exp(-i\Delta\phi_F)] \\
&= \cos^2 \theta \cos^2 \varphi + \sin^2 \theta \sin^2 \varphi + \left(\frac{1}{4} \sin 2\theta \sin 2\varphi\right) \cdot (2 \cos \Delta\phi_F) \\
&= \cos^2 \theta \cos^2 \varphi + \sin^2 \theta \sin^2 \varphi + \frac{1}{2} \sin 2\theta \sin 2\varphi \cos \Delta\phi_F
\end{aligned}$$

As the nonlinear effect is weak, one just need consider the linear birefringence of the fiber, and the linear cavity transmission can be obtained by letting $\Delta\phi_F = \Delta\phi_l$:

$$T_l = \sin^2 \theta \sin^2 \varphi + \cos^2 \theta \cos^2 \varphi + \frac{1}{2} \sin 2\theta \sin 2\varphi \cos(\Delta\phi_l).$$

If taking the nonlinear birefringence into account, $\Delta\phi = \Delta\phi_l + \Delta\phi_{nl}$, then the cavity transmission becomes:

$$T = \sin^2 \theta \sin^2 \varphi + \cos^2 \theta \cos^2 \varphi + \frac{1}{2} \sin 2\theta \sin 2\varphi \cos(\Delta\phi_l + \Delta\phi_{nl}).$$

Appendix B

Publications of the author

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