Layered space-time architectures for wireless communications in flat fading channels

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Layered Space-Time Architectures for Wireless Communications in Flat Fading Channels

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School of Electrical & Electronic Engineering

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in fulfillment of the requirements for the degree of
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Summary

The increasing demand for higher data rates and better quality has motivated the deployment of multiple antennas at both the transmitter and the receiver sides in wireless communications. The problem discussed in our research is the development of fundamental space-time signal processing techniques to achieve different gains provided by multiple antennas. We focus on a point to point wireless link, where the radio channel is modeled as frequency flat and slow fading. We also consider different propagation conditions for the channel, particularly, one with rich scattering and the other with antenna correlation.

The first part of this thesis is concerned with spatial multiplexing (SM) systems, in which we discuss the detection schemes with emphasis on V-BLAST algorithm. The original iterative nulling and cancellation algorithm for V-BLAST has high computational complexity and requires repeated matrix pseudo-inverse which may lead to numerical instability. By studying the existing low complexity algorithms, we propose to replace the optimal detection order with a sub-optimal order to further reduce the complexity. The series of matrix pseudo-inverses are replaced by Gram-Schmidt orthogonalization process to improve numerical stability.

Secondly, linear precoding with maximum likelihood (ML) detection is proposed for SM systems over spatially correlated fading channels. Given the long term statistical

channel information at the transmitter, the precoder performs constellation multiplexing (CM) on the input symbols before transmitting them along the eigenvectors of the channel covariance matrix. The number of signaling branches and the modulation level are dictated by the correlation level and chosen to minimize the union bound on the symbol error probability of ML detection. Power allocation is also considered with a reduced rank linear receiver of successive interference cancellation (SIC) detection to improve system performance in highly correlated channels.

Next, we propose a combined SM and space-time block coding (STBC) system, namely, layered space-time block coding (LA-STBC) system that allows a tradeoff between spectral efficiency and diversity gain. Detection schemes for the combined system are proposed and exact error rate performance for the SIC based detection scheme is presented. In addition, an intensive study on the LA-STBC system is conducted. Techniques including optimal ordering, iterative decoding, and optimal transmit power allocation are proposed to improve the error rate performance when channel state information (CSI) is not available at the transmitter.

Finally, we consider linear precoding designs for the LA-STBC system with fading correlation. Given the channel covariance matrix, the precoder first allocates different power to each layer based on equating the error probability at each detection stage of SIC detection scheme. Then, the precoder is optimized for STBC at each layer given the power allocation. By taking advantages of the power allocation scheme for the SM system and the transmission scheme for the STBC system, the combined precoding strategy achieves further performance enhancement compared with either of the existing precoding schemes.

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Abbreviations

2D	two dimensional
3G	third-generation
AOA	angle-of-arrival
AWGN	additive white Gaussian noise
BER	bit error rate
BLAST	Bell lab layered space-time
BPSK	binary phase-shift keying
BS	base station
CDMA	code division multiple access
CM	constellation multiplexing
CSI	channel state information
DOA	direction-of-arrival
GSO	Gram Schmidt orthogonalization
I-CSI	instantaneous channel state information
i.i.d.	independent identically distributed
ISI	inter symbol interference
LA-STBC	layered space time block coding
LOS	line-of-sight
LT-CSI	long term channel state information

MIMO	multiple-input multiple-output
ML	maximum likelihood
MMSE	minimum mean square error
MRC	maximal ratio combining
MS	mobile station
MSE	mean square error
PAM	pulse amplitude modulation
pdf	probability density function
PEP	pairwise error probability
PSK	phase shift keying
PSNR	post processing signal-to-noise ratio
QAM	quadrature amplitude modulation
QoS	quality of service
QPSK	quadrature phase-shift keying
RR	reduced rank
SDMA	space division multiple access
SER	symbol error rate
SIC	successive interference cancellation
SINR	signal-to-interference-plus-noise ratio
SIR	signal-to-interference ratio
SISO	single-input single-output
SM	spatial multiplexing
SNR	signal-to-noise ratio
ST	space-time
STBC	space-time block coding
STBCs	space-time block codes
STC	space-time coding

STCs	space-time codes
STTCs	space-time trellis codes
SU	subscriber unit
SVD	singular value decomposition
TCM	trellis-coded modulation
ULAs	uniform linear arrays
V-BLAST	vertical Bell Lab layered space time
W-CDMA	wideband code division multiple access
WLAN	wireless local-area networks
ZF	zero-forcing

Notations

Throughout this thesis scalars are given by lowercase letters a , vectors by boldface lowercase letters $\mathbf{a}$, and matrices by boldface uppercase letters $\mathbf{A}$.

$ a $	absolute value of the complex scalar a
$a_{i,j}$	(i, j) -th element of matrix $\mathbf{A}$
$\ \mathbf{a}\ $	norm of the vector $\mathbf{a}$
$ \mathbf{A} $	determinant of the matrix $\mathbf{A}$
$\ \mathbf{A}\ _F$	Frobenius norm of the matrix $\mathbf{A}$
$\text{diag}(a_1, a_2, \dots, a_M)$	a diagonal matrix with diagonal elements $a_1, a_2, \dots, a_M$
$(\cdot)^*$	complex conjugate operation
$(\cdot)^T$	transpose operation
$(\cdot)^H$	conjugate transpose operation
$\mathbf{A}^+$	pseudo-inverse of the matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of the matrix $\mathbf{A}$
$\text{vec}(\mathbf{A})$	an operation that stacks the columns of the matrix $\mathbf{A}$ into a tall vector
$\text{rank}(\mathbf{A})$	rank of the matrix $\mathbf{A}$
$\mathbf{A}(:, i)$	the i -th column of the matrix $\mathbf{A}$
$\mathbf{A}(i, :)$	the i -th row of the matrix $\mathbf{A}$

$\mathbf{A}(:, i : j)$	the i th to the j th column of the matrix $\mathbf{A}$
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$
$\mathbf{0}$	a zero matrix
$\mathbf{I}_M$	an $M \times M$ identity matrix
$E\{a\}$	expectation of the random variable a
j	$\sqrt[2]{-1}$
$\mathcal{Q}(\cdot)$	quantization operation
$\langle \mathbf{a}, \mathbf{b} \rangle$	inner product of the vectors $\mathbf{a}$ and $\mathbf{b}$

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Chapter 1

Introduction

1.1 Motivation

Wireless communications have permeated every aspect of our lives and are making it possible for people to communicate reliably in any form at anytime anywhere. It has a wide range of connections from satellite that provides low bit rates but global coverage and cellular systems with continental coverage to high data rate local area networks and personal area networks with a maximum range of a few to a hundred meters. The introduction of these wireless services has increased the mobility and service area for many existing applications and numerous unthought-of applications. Wireless is a growing area of public networks and it plays an equally important role in private and dedicated communications systems.

The rapid growth of wireless communications is following the quest for higher data rate and improved quality of services (QoS) at lower costs. In conventional single transmit single receive antenna system, increasing data rate can be realized either by increasing the transmission bandwidth or the transmission power. However, as the frequency spectrum is a valuable resource and in some cases restricted, increasing the transmission bandwidth is not favorable. Increasing the transmission power is also undesirable as it requires an expensive radio frequency amplifier, reduces the battery life of the mobile unit and increases interference to other devices.

The use of multiple antennas at transmitter and receiver emerges as an alternative that does not require any extra bandwidth and no extra transmission power and promises huge performance gains over conventional single antenna systems [1]. Multiple antenna systems, also known as multiple-input multiple-output (MIMO) systems, create multiple paths, where each path from a transmit antenna to a receive antenna can be viewed as one signaling branch. The MIMO definition is the most general and contains the single-input multiple-output (SIMO) and multiple-input single-output (MISO) systems as special cases. MIMO systems provide different types of gains, including array gain, diversity gain, multiplexing gain, and interference reduction gain. Among them, two major attractive gains over conventional single antenna systems are:

Diversity gain: Spatial diversity is available if the paths fading between each pair of transmit and receive antennas are more or less independent. By transmitting the same signal over multiple independent branches, multiple independently faded replicas

of the data symbol can be obtained at the receiver [2]. The probability that all paths are degraded at the same time is significantly smaller than the probability that a single transmission branch is in a deep fade. Hence, fading is effectively mitigated and the link reliability is greatly improved.

Multiplexing gain(spectral efficiency gain): Transmit or receive diversity is a means to combat fading. A different line of thought is to utilize fading to increase the degrees of freedom. Essentially, if the path gains between all transmit and receive antennas fade independently, multiple parallel spatial channels are created. By transmitting independent information streams in parallel through the spatial channels, the data rate can be increased. The seminal work of [3, 4] suggested that provided that the multiple paths between all transmit and receive antennas are statistically independent, the maximum data rate at which error-free transmission over a fading channel is proportional to the minimum number of the transmit and receive antennas when channel information is known at the receiver.

Most of current research focuses on designing schemes to extract either maximal diversity gain or maximal spatial multiplexing gain. The objectives of realizing the above gains are often mutually conflicting. For example, it was observed in [5] that the coding structure from the orthogonal designs, while achieving the full diversity gain, reduces the achievable spatial multiplexing gain. In fact, each of the two design goals addresses only one aspect of the problem.

In an ideal environment of a large number of scatterers and antenna spacing larger

than half wavelength, the information theoretical capacity of a MIMO system grows linearly with the minimum number of transmit and receive antennas [1]. However, various measurements show that realistic MIMO channels show a significantly lower capacity growth [6–8]. This reduction in capacity is due to the spatial correlation of the MIMO channel coefficients.

The motivation of this thesis is to develop transmission strategies and signal processing techniques that achieve the most important gains: spatial multiplexing gain and diversity gain for flat fading channels in an independent fading environment and a more realistic spatially correlated environment.

1.2 Research Overview and Contributions

Since multi-antenna communications allows the transmission of information at very high rates, a major issue of concern is to keep the computational complexity of the decoding algorithm at the receiver within reasonable bounds. Our first focus is on the complexity issue of the Vertical Bell Labs Layered Space-Time (V-BLAST) detection algorithm [3, 9]. The existing iterative nulling and cancellation for V-BLAST has high computational complexity and requires repeated matrix pseudo-inverse calculations which may lead to numerical instability. A square-root algorithm and a modified decorrelating decision feedback detection algorithm have been proposed by other researchers to reduce complexity and improve numerical stability. In this thesis, we propose methods to further reduce the computational complexity of the existing low

complexity algorithms. The reduction by the proposed method is gained mainly from two ways. First, the optimal decoding order is replaced by a sub-optimal decoding order. Second, the series of calculations of pseudo-inverses are replaced by Gram-Schmidt Orthogonalization (GSO) process applied to the channel matrix for finding the weight vectors after the sub-optimal decoding order is determined.

Our second goal is to address the problem of MIMO spatial multiplexing (SM) systems in the presence of antenna fading correlation. Existing SM (V-BLAST and related) schemes rely on the linear independence of transmit antenna channel responses for substream separation and suffer considerably from high levels of fading correlation. As a result such algorithms simply fail to extract the capacity present in highly correlated spatial channels. We make the simple but key observation that fewer eigenmodes of the transmit covariance matrix are needed to send the independent substreams if those substreams are appropriately superposed to form a high order modulation. We build on this idea to present a new transmission scheme based on a precoder adjusting the order of constellation multiplexing as a function of correlation level. This yields a rate preserving MIMO system that can operate at any degree of correlation. The precoding scheme is analyzed assuming ML detection. In addition to this, we also consider a power allocation method combined with a reduced rank linear receiver for the successive interference cancellation (SIC) decoding scheme.

In the following, we conduct a study on the layered space-time block coding (LA-STBC) structure that achieves both multiplexing gain and diversity gain. The frame-

work of this structure is to partition all the transmit antennas into groups each applying a space-time block code independently. For a fixed number of transmit antennas, a flexible tradeoff between multiplexing gain and diversity gain is achieved by varying the codeword of each layer and combining method. We investigate several key aspects of the layering structure analogous to V-BLAST. We first propose different detection schemes for LA-STBC and exact bit error rate (BER) analysis is presented for SIC based decoding schemes. In order to enhance the error rate performance, several approaches are addressed. Optimal decoding ordering for each layer is given to improve the system performance and transmit power allocation that minimizes the overall BER is calculated at the receiver and fed back to the transmitter. Another method proposed is a computationally efficient iterative decoding algorithm that fully exploits the diversity of the system.

Finally, we look into the important aspects of the LA-STBC system when it is considered in spatially correlated channels. Owing to inherent orthogonality of the STBC structure, the LA-STBC can be formulated as an equivalent SM system. Compared with the well defined SM system, the LA-STBC is more robust to the antenna correlation and with guaranteed diversity even when the antennas are highly correlated. Existing linear precoding schemes are designed for SM systems or STBC systems based on the knowledge of the channel covariance at the transmitter. However, the problem of precoding designs has not been addressed elsewhere to improve the LA-STBC system performance in spatially correlated channels. In our research, we propose a precoder that incorporates the power allocation to each layer with the well known waterfilling so-

lution. As a result, the combined precoding scheme outperforms either the pure power allocation for SM systems or the pure waterfilling solution for STBC systems.

1.3 Organization of the Thesis

Following the introduction in this chapter, we review some background knowledge regarding MIMO systems, including channel model, MIMO transmission schemes and space-time processing techniques in Chapter 2.

Chapter 3 begins with the optimal ordering and iterative nulling and cancellation decoding algorithm of V-BLAST in Section 3.2. Existing low complexity decoding algorithms are presented in Section 3.3. The proposed low complexity decoding method is described in Section 3.4, followed by simulation results in Section 3.5.

In Chapter 4, we present the second part of our research of precoding designs for the SM system with long term channel statistical information available at the transmitter. The precoding techniques developed for ML detection and SIC based detection schemes are detailed in Sections 4.2 and 4.3, respectively.

The third part of our research on the spectral efficiency and diversity achieved architecture LA-STBC is given in Chapter 5. Detection schemes for the LA-STBC system are presented in Section 5.2 and performance analysis of the successive interference cancellation based decoding algorithm for the LA-STBC structure is given in Section 5.3.

An intensive study on LA-STBC is provided in Chapter 6. In Section 6.1, we present the basic framework and the group ZF-SIC and the group MMSE-SIC decoding algorithms. The derivation of the optimal decoding order for LA-STBC is given by Section 6.2. An iterative decoding method to improve receive diversity is given in Section 6.3. The optimal power allocation is derived in Section 6.4, followed by numerical results in Section 6.5.

The last part of our research about the precoding designs for LA-STBC over spatially correlated channels is presented in Chapter 7. We first analyze the performance of LA-STBC over spatially correlated channels in Section 7.2. Precoding schemes aiming at difference optimization targets are presented in Section 7.3 and compared by simulations in Section 7.4.

Finally, we provide the concluding remarks and suggestions for future research in Chapter 8.

Chapter 2

MIMO Wireless Systems and Space-Time Processing

In this chapter, we start with an overview of the wireless communication systems with multiple antennas, including channel models, transmission schemes and the prevalent precoding schemes.

2.1 System Model

2.1.1 Fading Model

The characteristic of wireless communication channels between the transmitter and receiver determines the performance of wireless systems. The transmission from trans-

mitter to receiver experiences a lot of limitations, including thermal noise, interference from other wireless channels, propagation loss, and self-interference. The most distinct difficulty in wireless system designs is the multiple paths of the same signal arriving at different times, which results in constructive or destructive combination of the arriving signals randomly [10]. The phenomenon is called multipath fading, and is the major and most challenging problem that needs to be dealt with in wireless communications.

Many physical factors in the radio propagation channel influence fading. These include the following: multipath propagation caused by the presence of reflecting objects and scatterers in the channel, relative motion between the transmitter and receiver, the speed of surrounding objects, and the transmission bandwidth of the signal. Depending on the relationship between the signal parameters and the channel parameters, different transmitted signals will undergo different types of fading. If the mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal, then the received signal undergoes flat fading. In the time domain, flat fading corresponds to a channel delay much less than the symbol duration. Hence, the channel induces no inter symbol interference (ISI). This type of fading is historically the most common type of fading described in the technical literature and is assumed throughout the thesis. When the channel possesses a constant gain and linear phase response over a bandwidth that is smaller than the bandwidth of the transmitted signal, the channel creates frequency selective fading. In this type of fading channel, equalization techniques is needed to compensate for ISI induced by time dispersion of the transmitted signals. Throughout this thesis, slow fading is also

assumed. That is, the channel impulse response changes at a rate much slower than the transmitted baseband signal. We further assume that the fading is quasi-static, i.e. constant during a burst or transmission frame and will change to another realization independently. Another slow fading model is block fading. It applies when several adjacent symbols are subject to the same fading. A detailed description on the types of fading channel can be found in [11].

The statistical nature of multipath fading can be modeled with various distributions. When there is a large number of scatterers present in the wireless channel, the signal at the receiver is the sum of several multipath components. It is shown in [11, 12] that for practical cases and over short distances, when no line of sight (LOS) exists between the transmitter and the receiver, the amplitude of the received signal follows a Rayleigh distribution; when there is a LOS signal, the amplitude of the received signal is Rician distributed. This assumption is valid when there are a large number of scatterers around the transmitter and the receiver. For both cases, the phase is uniformly distributed between 0 to 2π . In this thesis, we employ the commonly used Rayleigh distribution, which can accurately model indoor or urban channels.

2.1.2 MIMO Channel Model

A MIMO system is simply a wireless communication system in which the transmitter side and receiver side are equipped with multiple antennas. Consider a MIMO system with a transmit array of M_t antennas and a receive array of M_r antennas. The block

2.1. System Model

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diagram of such a system is shown in Fig. 2.1.

Figure 2.1: Block diagram of a MIMO system

As can be seen, the underlying nature of using multiple antennas as compared to the single-antenna systems is to create multiple signaling paths, each from a transmit antenna to a receive antenna. By collecting all branches, the MIMO channel can be fully described by a $M_r \times M_t$ matrix $\mathbf{H}$ as

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,M_t} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,M_t} \\ \vdots & \vdots & \vdots & \vdots \\ h_{M_r,1} & h_{M_r,2} & \cdots & h_{M_r,M_t} \end{bmatrix} \quad (2.1)$$

where the component $h_{i,j}$ is the fading coefficient from the j -th transmit antenna to the i -th receive antenna. In a rich scattering environment without antenna elements coupling, it is usually valid to assume that they are independent identically distributed (i.i.d.). Based on the flat Rayleigh fading channel model, the entries of the channel matrix can be modeled as samples of independent complex Gaussian random process

with zero mean and unit variance, i.e. $E\{|h_{i,j}|^2\} = 1$. Therefore, $|h_{i,j}|^2$ is a chi-squared distributed random variable with degrees of freedom of 2, i.e. χ_2^2 .

Let $\mathbf{s}_t$ denote the $M_t \times 1$ baseband transmitted signal vector at discrete time slot t with each element drawn from a unit power constellation, and $\mathbf{y}_t$ denote the $M_r \times 1$ baseband received signal vector. The received signal is a noisy superposition of the M_t transmitted signals corrupted by fading coefficients. The input and output relationship of the MIMO channel is modeled as:

$$\mathbf{y}_t = \mathbf{H}\mathbf{s}_t + \mathbf{n}_t \quad (2.2)$$

where $\mathbf{n}_t$ is the additive white Gaussian noise (AWGN) vector, whose components are statistically independent complex Gaussian random variables with zero mean and variance σ_n^2 .

As reported in [1, 3, 4], a MIMO system is capable of providing an enormous capacity increase compared to a traditional single-antenna system only if the channels between transmit antennas and receive antennas follow independent fading paths. In practice, however, the channel fading between different transmit-receive antenna pairs may be correlated due to, for example, insufficient antenna spacing. The capacity of MIMO systems can be significantly degraded if there is severe correlation present at the transmitter and/or receiver side [13, 14]. At worst, the capacity falls back to that of a SIMO/MISO with, potentially, additional array gain. In general, the correlation between all the entries in the channel matrix depends on the physical separation of the

antenna elements at both sides, the antenna polarization pattern, the wavelength, and the location of the surrounding scatterers. The propagation conditions determine the channel capacity that can be expected for a MIMO system. It is of great interest to characterize and model the MIMO channel for realistic capacity assessment, as well as for designing space-time (ST) modulation and coding techniques that are matched to channel statistics.

MIMO channel models are categorized into two main groups, i.e. non-physical (or statistical) and physical models [15]:

- Non-physical MIMO channel model:

1. Kronecker model: It is the state-of-the-art stochastic model that describes the spatial correlation properties of the MIMO channel as independent contribution of correlation from the transmitter and from the receiver, neglecting the statistical inter-dependence of both link ends [7, 8, 14, 16, 17]. In this case, the channel correlation matrix is well approximated by the Kronecker product of the correlation matrices from both ends, i.e.,

$$\mathbf{R}_H = \mathbf{R}_t^T \otimes \mathbf{R}_r, \quad (2.3)$$

where $\otimes$ stands for Kronecker product, $\mathbf{R}_H$ is the channel correlation matrix, $\mathbf{R}_t$ and $\mathbf{R}_r$ are the correlation matrices at the transmitter and receiver, respectively. Assuming that the channel coefficients are zero mean com-

plex Gaussian, the first and second order moments of the MIMO channel are enough to characterize the propagation matrix. Therefore, the channel response can be written mathematically as:

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2}, \quad (2.4)$$

where $\mathbf{H}_w$ is a $M_r \times M_t$ matrix with elements being zero mean, unit variance i.i.d complex Gaussian random variables.

2. Non-Kronecker model: The authors of [6,18] have shown that realistic indoor MIMO channels cannot be modeled adequately by Kronecker model and have proposed a stochastic channel model with joint correlation at both link ends.
- Physical MIMO channel model:
 1. One ring model: Fig. 2.2 shows the model and it is appropriate in the fixed wireless communication context, where the base station (BS) is usually elevated and unobstructed by local scatterers, and the subscriber unit (SU) is surrounded by local scatterers. For notational convenience, the BS and the SU assume the roles of transmitter and receiver, respectively. The distance between BS and SU is D and the radius of the scattering ring is R . Other parameters include the angle of arrival Θ at the BS, and the angle spread Δ . Since D and R are typically large compared to antenna spacing, $\Delta = \arcsin(R/D)$. Some assumptions are generally made in the “one ring” model:

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- The effective scatterers are assumed to be distributed uniformly in θ .
The effective scatterer located at angle θ is denoted by $S(\theta)$. A phase $\phi(\theta)$ is associated with $S(\theta)$, which represents the dielectric properties and the radial displacement from the scatterer ring of the actual scatterer that $S(\theta)$ represents [13]. Therefore, rays that are reflected by $S(\theta)$ are subject to the same $\phi(\theta)$.
- Only rays that are reflected by the effective scatterers exactly once are considered.
- All rays that reach the receiving antennas are equal in power.

Figure 2.2: Illustration of the one ring model

Suppose there are K effective scatterers, the normalized channel coefficient between transmit antenna TA_p and receive antenna RA_l is

$$h_{l,p} = \frac{1}{\sqrt{2\pi}} \cdot \int_0^{2\pi} \frac{1}{\sqrt{K}} \sum_{k=1}^K \delta(\theta - \theta_k) \exp \left\{ -j \frac{2\pi}{\lambda} (D_{TA_p \rightarrow S(\theta)} + D_{S(\theta) \rightarrow RA_l}) + j\phi(\theta) \right\} d\theta, \quad (2.5)$$

where $D_{X \rightarrow Y}$ is the distance between X and Y , and λ is the wavelength.

By the central limit theorem, in the limit of infinitely many scatterers, $h_{l,p}$ is complex Gaussian distributed with zero mean and unit variance. The covariance between two channel coefficients $h_{l,p}$ and $h_{m,q}$ is

$$E\{h_{l,p}h_{m,q}^*\} = \frac{1}{2\pi} \int_0^{2\pi} \exp\left\{-\frac{2\pi j}{\lambda} (D_{TA_p \rightarrow S(\theta)} - D_{TA_q \rightarrow S(\theta)} + D_{S(\theta) \rightarrow RA_l} - D_{S(\theta) \rightarrow RA_m})\right\} d\theta. \quad (2.6)$$

In general, numerical evaluation of (2.6) is difficult. But when Δ is small, which is often the case in fixed wireless channels, a useful approximation exists:

$$E\{h_{l,p}h_{m,q}^*\} \approx \frac{1}{2\pi} \int_0^{2\pi} \exp\left\{-\frac{2\pi j}{\lambda} \left[d_x^t(p, q) \left(1 - \frac{\Delta^2}{4} + \frac{\Delta^2 \cos 2\theta}{4}\right) + \Delta d_y^t(p, q) \sin \theta + d_x^r(l, m) \sin \theta + d_y^r(l, m) \cos \theta \right]\right\} d\theta \quad (2.7)$$

2. Two ring model: The model assumes that both BS and SU are surrounded by scatterers. This can be the case for indoor wireless channels. An illustration of the two ring model is shown in Fig. 2.3. The difficulty in this model is that each ray is reflected twice, and the signals reflected by the scatterers at the receive side are possibly not independent. Even if the number of scatterers goes to infinity, the channel coefficient is still not zero mean complex Gaussian. Therefore, the channel covariance matrix cannot completely describe the MIMO channel.

Figure 2.3: Illustration of the two ring model

Correlated MIMO channels are analyzed mainly in the context of the so-called Kronecker model [13] and the virtual representation model for uniform linear arrays (ULAs) [19]. The Kronecker model assumes separability in channel statistics induced by the transmit and receive antenna arrays and its validity has been supported by [20]. The virtual representation model does not assume separability of transmit and receive correlation but it is more applicable for ULAs.

The effect of correlated fading on the capacity of MIMO systems is shown in [13,21]. Its impact on the performance of different types of detection schemes for ST system is studied in [22] by simulation results. Previous studies on performance of space-time codes (STCs) over correlated channels, quantifying the loss of diversity and coding gain, include [23–25]. Given channel statistical knowledge at the transmitter, transmit antenna selection is considered for spatial multiplexing (SM) in [26,27] and for space-time coding (STC) in [28]. Precoding is also proposed as a method to improve the performance of ST system in correlated fading channels and precoding schemes with channel correlation knowledge will be reviewed in Section 2.3.2.

2.1.3 Transmission Over MIMO Channels

Fig. 2.1 shows a typical MIMO wireless system with M_t transmit antennas and M_r receive antennas. A digital source in the form of a bit stream is sent to the transmitter block that encompasses ST coding and mapping to complex modulation symbols (PSK, QAM, etc.). The ST coding block adds parity bits for protection against noise and also captures diversity from the space and possibly frequency or time dimensions in a fading channel. After coding, the bits are interleaved across space, time and frequency and mapped to data symbols to generate M_t outputs. After upward frequency conversion, filtering and amplification, the symbol streams are radiated from M_t antennas. These signals pass through the radio channel where they are attenuated and undergo fading in multiple dimensions before they arrive at the M_r receive antennas. Additive thermal noise at the receiver corrupts the received signal. These streams at the receiver are ST deinterleaved and ST decoded to recover the transmitted signals.

In general, the difference between a MIMO communication system and a conventional SISO channel comes from the use of multiple antennas. By introducing the new dimension of space, the conventional signal processing techniques are developed to MIMO-related ST processing techniques including ST encoding/interleaving, ST decoding/deinterleaving and so on. The selection of ST transmission techniques varies a lot depending on whether the CSI is available at one or both sides of a wireless communication link. It is usually difficult for the transmitter to obtain the instantaneous CSI, hence, we assume the unavailability of instantaneous CSI (I-CSI) at the transmitter in

the thesis. While it is fairly common to assume that the CSI is available at the receiver, which can be obtained by a separate pilot channel or sending a training sequence.

2.2 MIMO Transmission Schemes

MIMO systems mainly provide two types of gains: SM gain and diversity gain. SM gain reflects the spectral efficiency and diversity gain reflects the link quality. To date, many transmission schemes for MIMO systems have been proposed in the literature. These schemes may coarsely be classified into rate-oriented, diversity-oriented, and rate-diversity-oriented techniques.

2.2.1 Rate Oriented Scheme: Spatial Multiplexing

SM schemes employ multiple antennas at both the transmitter and receiver, and launch independent data streams over the individual transmit antennas simultaneously. The vector encoding process is simply a demultiplexing operation followed by independent bit-to-symbol mapping of each substream. No inter-substream coding is required, although conventional coding of the individual substreams may be applied. All the substreams are transmitted in the same frequency band, so spectrum is used very efficiently. Since the user's data is sent in parallel over multiple antennas, the effective transmission rate is increased in roughly in proportion to the number of transmit antennas used. The total radiated power is constant and independent of M_t . At the receiver,

an antenna array picks up the superimposed data streams. If the multipath scattering is sufficient, the multiple substreams are all scattered slightly differently since they originate from different transmit antennas that are located at different points in space. By sophisticated signal processing, these slight differences in scattering allow the substreams to be identified and recovered by a linear/nonlinear receiver. A number of decoding techniques have been proposed to separate the multiplexed signals for SM systems [3, 9, 29–31]. Linear receivers such as the zero forcing (ZF) and the minimum mean square error (MMSE) receivers offer a significant computational reduction compared to nonlinear receivers and are more practical in systems with a large number of transmit and receive antennas. Nonlinear methods include successive interference cancelation (SIC) based V-BLAST and maximum likelihood (ML) detectors. Among the nonlinear techniques, ML is optimum in terms of minimizing the overall error probability, but its decoding complexity grows exponentially with the number of transmit antennas. V-BLAST compromises the performance for decoding complexity reduction by introducing layered space-time processing, where at each stage of detection, a selected layer is detected by a linear receiver such as ZF and MMSE algorithms and its contribution is cancelled from the received signals. Wherever linear receivers are deployed, the number of receive antennas should be no less than the number of transmit antennas [9].

- ZF receiver: The ZF decoding technique is described as the pseudo-inverse of the channel matrix, $\mathbf{W}_{zf} = \mathbf{H}^+$, that nulls the interference of the other substreams completely at the expense of enhanced noise power at the receiver.

- MMSE receiver: The MMSE decoding technique reduces the noise power by introducing inter-stream interference to minimize the mean squared error between the estimated symbols and the original transmitted symbols, and is given by
$$\mathbf{W}_{mmse} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_r})^+$$
- V-BLAST receiver: A ZF or MMSE receiver is used to decode one element of the transmitted signal vector at a time. By subtracting the interference of the decoded elements from the received signal vector, the detection of the other elements of the transmitted vector can be improved. This technique is called successive interference cancellation (SIC). The V-BLAST is one of the nonlinear techniques that first extract the strongest substream from the morass, then proceed with the remaining weaker signals, which are easier to recover once the stronger signals have been removed as a source of interference.
- Maximum likelihood decoding: The only decoding method that is not based on matrix inversion is the maximum likelihood (ML) decoding algorithm. ML compares the received signal with all possible transmitted vectors modified by $\mathbf{H}$ and choose the one with minimum Euclidean distance to the received vector. This decoding algorithm requires an exhaustive search through all possible transmitted vectors and has a computational complexity of M^{M_t} , where M represents the number of constellation points. For the ML decoding algorithm, the requirement of $M_t \leq M_r$ can be relaxed.

2.2.2 Diversity Oriented Scheme: Space-Time Coding

As opposed to V-BLAST schemes, STC techniques exploit spatial diversity, which yields an additional diversity and/or coding gain. Spatial diversity is available if the individual transmission paths from the transmit antennas to the receive antennas fade more or less independently. In this case, the probability that all paths are degraded at the same time is significantly smaller than the probability that a single transmission path is in a deep fade. By means of STC techniques, improved bit error rate performance is accomplished. This may in turn be translated to higher throughput if an adaptive modulation scheme/channel coding is applied. Based on code structure, STC techniques are divided into space-time trellis codes (STTCs) and space-time block codes (STBCs).

STTCs first introduced by Tarokh [2], are a generalization of trellis coded modulation (TCM) to multiple transmit antennas. They can simultaneously offer a substantial coding gain, spectral efficiency, and diversity gain on flat fading channels. At each time t , depending on the state of the ST encoder and the input bits, a transition branch is chosen. If the label of this branch is $s_t^1 s_t^2 \cdots s_t^{M_t}$, then constellation symbol s_t^m is transmitted by the m -th antenna. All the M_t symbols are transmitted simultaneously at time t . At the receiver, the decoder employs the Viterbi algorithm to perform maximum likelihood decoding. Assuming that perfect CSI is available at the receiver, the branch metric is computed as the squared Euclidean distance between the hypothesized

received symbols and the actual received signals as

$$\sum_{j=1}^{M_r} \left| y_t^j - \sum_{i=1}^{M_t} h_{j,i} s_t^i \right|^2 \quad (2.8)$$

The Viterbi algorithm selects the path with the minimum path metric as the decoded sequence. In STTCs, to achieve full transmit diversity, the minimum required number of trellis states grows exponentially with the number of transmit antennas and the spectral efficiency (in bits/s/Hz), resulting in an exponential increase in the decoding complexity [2].

STBCs, on the other hand, essentially perform a linear mapping of modulated symbols onto multiple antennas by means of a space-time (ST) mapping matrix. The key feature of the STBCs is that they are capable of achieving full diversity gain with a simple maximum likelihood decoding algorithm. Alamouti's scheme is historically the first STBC to provide full transmit diversity and full rate for two transmit antennas [32]. In Alamouti's ST encoder, a block of two modulated symbols s_1 and s_2 are mapped to the transmit antennas according to the code matrix given by

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2^* \\ s_2 & -s_1^* \end{bmatrix}. \quad (2.9)$$

That is, in the first symbol period, s_1 is transmitted from the first antenna, and s_2 is transmitted from the second antenna. In the next symbol period, s_2^* is transmitted by antenna 1, and $-s_1^*$ is transmitted by antenna 2. It is clear that the encoding is done

in both the space and time domains.

Assume that the channel remains constant during the two consecutive symbol periods. The two symbols s_1 and s_2 can be decoupled and decoded individually by simple linear combining followed by maximum likelihood decoding. Each of the symbol achieves a full transmit diversity of $M_t = 2$ due to the orthogonal structure of the codeword matrix (2.9). The very simple structure and linear processing of Alamouti's code has made it appealing to W-CDMA and CDMA2000. Later, Tarokh etc. proposed several different types of STBCs based on orthogonal design for more than two transmit antennas [5, 33]. We define the code rate as the ratio of the number of symbols in a codeword to the number of symbol periods used to transmit the codeword. Alamouti's STBC has two symbols transmitted over two consecutive symbols periods. Therefore, its code rate is 1. It is shown that for real signal modulation, e.g. PAM, orthogonal STBC with rate 1 can be constructed for arbitrary number of transmit antennas; while for the complex modulation such as M-QAM and M-PSK, a rate 1 orthogonal STBC does not exist for $M_t > 2$. There is a tradeoff among the orthogonality, the rate of the codeword, and the order of transmit diversity. The design of quasi-orthogonal codes can be found in [34–36], where the orthogonality is relaxed to provide higher symbol transmission rate. With the quasi-orthogonal structure, the ML decoding at the receiver can be done by searching pairs of symbols. However, these codes do not achieve the full diversity.

2.2.3 Rate-Diversity-Oriented Schemes

There are two types of rate-diversity-oriented space-time techniques. One achieves both diversity gain and multiplexing gain, and ensures a balance between the two. Examples of this type of techniques can be found in [37, 38] as a compromise between the rate-oriented V-BLAST and diversity-oriented STTCs. Another example achieving the tradeoff is the combined SM and STBC system considered in this thesis. The other without rate-diversity tradeoff endeavors to achieve full diversity gain and multiplexing gain simultaneously. Techniques of this category include the linear dispersion codes [39, 40], and the algebraic space-time codes [41]. These codes apply precoding at the transmitter and use a sphere decoder [42] at the receiver to do near ML decoding. The weakness of these codes compared with the tradeoff type techniques is the expansion of transmitted signal points due to the use of complex field precoding. In our research, only the rate-oriented SM and diversity-oriented STBC are considered.

2.3 Precoding Strategies for MIMO System

2.3.1 Transmitter with Instantaneous CSI

Precoding strategies in practical systems depend on the possibility to have channel state information (CSI) at the transmitter side. For duplexing schemes, some information about the channel can be fed back to the transmitter. Linear precoders and decoders can

then be designed through channel eigen-decomposition. Several criteria can be used for MIMO system optimization and lead to different solutions characterized by the power allocation strategy of the transmitter. The most popular criterion is the well known waterfilling solution, which aims to maximize the capacity [4]. Other designs using the weighted MMSE criterion and the maximization of the minimum singular value of the channel matrix have been proposed in [43] and [44]. These optimized schemes transform the MIMO channel into parallel and independent data streams, which lead to an eigenmode representation. The solutions only differ by the power allocation strategy in this representation. The maximum signal to noise ratio (SNR) solution [45] consists of transmitting the whole information on the most favored subchannel and results in good BER performance. Another interesting criterion to improve MIMO system performance is the minimum Euclidean distance between signal points at the receiver side, especially when a ML detector is used. This criterion has been applied to design coding and modulation schemes in [46]. This criterion is also used to choose between transmit diversity and SM system [47] or to perform antenna selection [48]. Recently, a precoding scheme based on maximizing the same criterion was proposed in [49].

2.3.2 Transmitter with Long Term CSI

Another field of study on precoding schemes is conducted under the constraints that CSI is limited to the second order statistics of channel fading and noise. This assumption

is of crucial relevance for systems in which the fading channel varies fast enough to make the condition of instantaneous CSI at the transmitter unrealistic. The long term CSI can be acquired from measurements of the opposite link or by feedback from the receiver. Based on channel covariance information, optimal transmitter design has been pursued in [50] based on a capacity criterion. Optimal transmission with eigenbeamforming that consists of transmission on the eigenmodes of the correlation matrix has been designed in [51–53], where the bit and power allocation for each eigenvector is determined by statistical waterfilling. To minimize the BER in the presence of transmit correlation, diagonal precoding in the form of power allocation and/or relative phase adjustment of the transmitted signals has been proposed in [54, 55] for the case of a 2×2 MIMO system. The precoding matrix is determined by numerical search to minimize the error probability of ML detection. Unitary and non-unitary precoding are also proposed in [56] for SM systems. Precoding by adjusting the phase and power of the input constellation for SM has been presented by [57] in a closed form. Coding and diversity gain tradeoff in space-time codes for correlated channel via precoding is discussed in [58] based on the virtual channel representation for correlated channels.

Chapter 3

Low Complexity Detection of V-BLAST

In Chapter 2 we reviewed the concept and the key designs of space-time system. It is known that for spatial multiplexing (SM) systems, V-BLAST is a preferred detection algorithm that realizes a good balance between the performance and decoding complexity [59]. V-BLAST alleviates the problem of exhaustive searching posed for ML detection by employing an iterative nulling and cancellation scheme: at each time instant, rather than jointly decoding the signals from all the transmit antennas. V-BLAST first decodes the “strongest” signal, then cancels the effect of this strongest signal from each of the received signals, and then proceeds to decode the “strongest” of the remaining transmit signals, and so on. The main computation in using the iterative nulling and cancellation algorithm for V-BLAST detection is the step where the optimal

ordering for the sequential estimation, detection of the transmitted signals, as well as the corresponding so-called nulling vector, or weight vector, is determined. These steps have computational complexity of order $\mathcal{O}(M_t^4)$, where M_t is the number of transmit antennas. When the number of transmit antennas and receive antennas is large, the repeated use of pseudo-inverse to calculate the nulling vectors may lead to numerical instability that hinders the implementation of the V-BLAST architecture [60].

A square-root algorithm based on QR decomposition of the channel matrix and unitary transformation is used in [60] to avoid the repeated computation of the nulling vectors. Instead, the QR decomposition is computed only once. Not only is the computational complexity is reduced, but also the numerical robustness is improved by the square-root algorithm. The computational complexity of the algorithm in [60] is further reduced in [61] by using a modified decorrelating decision feedback multiuser detection algorithm originally proposed for code division multiple access (CDMA) systems [62].

In this chapter, we propose methods to further reduce the total number of arithmetic operations required for calculating the weight vectors so that implementation in hardware for a large number of transmit antennas becomes possible. We propose to replace the optimal decoding order by a sub-optimal decoding order. From the simulation results, this sub-optimal decoding order causes insignificant degradation in the overall performance. The series of calculations of pseudo-inverses are replaced by a Gram-Schmidt Orthogonalization (GSO) process applied to the channel matrix for finding the weight vectors after the sub-optimal decoding order is determined. The remainder

of this chapter is organized as follows. We start by introducing the system model in Section 3.1. In the following, we detail the original V-BLAST detection algorithm in Section 3.2 and introduce the existing low complexity algorithms in Section 3.3, followed by our proposed reduced complexity algorithm in Section 3.4. The simulation results of the proposed algorithm and comparison of the linear and nonlinear detection schemes for SM system are provided in Section 3.5. Finally, Section 3.6 summarizes this chapter.

3.1 System Model

Consider a MIMO system equipped with M_t transmit antennas and M_r receivers as shown in Fig. 2.1. It is assumed that the channel is quasi-static flat Rayleigh fading and the channel coefficients are perfectly known to the receiver. For simplicity, we take a discrete-time base-band view of the detection process for a single transmitted vector symbol and signaling over a fixed linear matrix channel with additive white Gaussian noise (AWGN).

Let $\mathbf{s} = [s_1, s_2, \dots, s_{M_t}]^T$ denote the vector of transmitted symbols and the corresponding received signal vector is $\mathbf{y} = [y_1, y_2, \dots, y_{M_r}]^T$. Suppose the symbols are drawn from unit power constellation, i.e. $E\{|s_i|^2\} = 1$. The relationship between transmit and received signals may be written as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad (3.1)$$

where $\mathbf{H}$ is the channel matrix and $\mathbf{n}$ is the noise vector with each component being a complex white Gaussian noise process with variance σ_n^2 . The assumption that the channel is rich scattering means that the elements of the channel matrix (when viewed as random variables) are independent of one another.

3.2 V-BLAST Detection Algorithm

3.2.1 Iterative Nulling and Cancellation

Let the ordered set $\mathcal{S} = \{k_1, k_2, \dots, k_{M_t}\}$ be a permutation of the integers $\{1, 2, \dots, M_t\}$ specifying the order in which components of the transmitted symbol vector $\mathbf{s}$ are detected. The detection process proceeds generally as follows [9]:

- Step1: using a nulling vector $\mathbf{w}_{k_1}$ to form a decision statistic: $r_{k_1} = \mathbf{w}_{k_1}^T \mathbf{y}$
- Step2: slice r_{k_1} to obtain $\hat{s}_{k_1}$: $\mathcal{Q}(r_{k_1}) = \hat{s}_{k_1}$, where $\mathcal{Q}(\cdot)$ denotes the quantization operation appropriate to the constellation in use.
- Step3: assuming that $\hat{s}_{k_1} = s_{k_1}$, cancel contribution of s_{k_1} from the received vector $\mathbf{y}$, resulting in modified received vector $\mathbf{y}^{(2)}$: $\mathbf{y}^{(2)} = \mathbf{y} - \hat{s}_{k_1} \mathbf{H}(:, k_1)$, where $\mathbf{H}(:, k_1)$ denotes the k_1 -th column of $\mathbf{H}$.

Steps 1 – 3 are then performed for components $s_{k_2}, \dots, s_{k_{M_t}}$ by operating in turn on the progression of modified received vectors $\mathbf{y}^{(2)}, \dots, \mathbf{y}^{(M_t)}$. Fig. 3.1 shows the first

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Figure 3.1: Block diagram of the first detection stage of V-BLAST

Figure 3.2: Block diagram of all detection stages of V-BLAST

stage of detection in V-BLAST, where nulling vector $\mathbf{w}_{k_1}$ is used to detect the first stream. Fig. 3.2 is the block diagram that shows all M_t stages of detection.

The specifics of the detection process depend on the criterion chosen to compute the nulling vectors $\mathbf{w}_{k_i}$, the most common of these being MMSE or ZF. The detection process is described here with respect to the ZF criterion since it is somewhat simpler to state. The i -th ZF-nulling vector is defined as the unique minimum norm vector satisfying

$$\mathbf{w}_{k_i}^T \mathbf{H}(:, k_j) = 0, \text{ for } j > i \quad (3.2)$$

Thus, $\mathbf{w}_{k_i}$ is orthogonal to the subspace spanned by the contributions to $\mathbf{y}^{(i)}$ due to those symbols not yet estimated and cancelled. It is not difficult to show that the

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unique vector satisfying (3.2) is just the k_i -th row vector of $\mathbf{H}_{\bar{k}_i}^+$, where $\mathbf{H}_{\bar{k}_i}$ is defined to be the channel matrix obtained by zeroing columns $k_1, \dots, k_{i-1}$ of $\mathbf{H}$, and $+$ denotes the Moore-Penrose pseudo-inverse [9]. The post-detection SNR for the k_i -th detected component of $\mathbf{s}$ is easily obtained by

$$PSNR_{k_i} = \frac{1}{\sigma_n^2 \|\mathbf{w}_{k_i}\|^2}. \quad (3.3)$$

3.2.2 Optimal Ordering of Detection

When symbol cancellation is employed during the detection process, the system performance is heavily affected by the order in which the components of $\mathbf{s}$ are detected. Recall that all components of $\mathbf{s}$ are assumed to utilize the same constellation. The component with the smallest $PSNR_{k_i}$ will dominate the error rate performance of the system. Therefore, choosing the best $PSNR_{k_i}$ at each stage of the detection process will lead to the globally optimum ordering. The full ZF V-BLAST detection algorithm can now be described as a recursive procedure, which includes the determination of the optimal ordering. This is briefly described as follows:

initialization:

$$i \leftarrow 1 \quad (3.4)$$

$$\mathbf{G}_1 = \mathbf{H}^+ \quad (3.5)$$

$$k_1 = \arg \min_j \|\mathbf{G}_1(j, :)\|^2 \quad (3.6)$$

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$$\mathbf{y}^{(1)} = \mathbf{y} \quad (3.7)$$

recursion:

$$\mathbf{w}_{k_i} = \mathbf{G}_1(k_i, :) \quad (3.8)$$

$$r_{k_i} = \mathbf{w}_{k_i}^T \mathbf{y}^{(i)} \quad (3.9)$$

$$\hat{s}_i = \mathcal{Q}(r_{k_i}) \quad (3.10)$$

$$\mathbf{G}_{i+1} = \mathbf{H}_{k_i}^+ \quad (3.11)$$

$$k_{i+1} = \arg \min_j \|\mathbf{G}_{i+1}(j, :)\|^2 \quad (3.12)$$

$$\mathbf{y}^{(i+1)} = \mathbf{y}^{(i)} - \hat{s}_i \mathbf{H}(:, k_i) \quad (3.13)$$

$$i \leftarrow i + 1 \quad (3.14)$$

in the above algorithm, $\mathbf{G}_i(j, :)$ denotes the j -th row of $\mathbf{G}_i$. Thus, (3.6) and (3.12) determine the elements of the optimal ordering. Equations (3.8)-(3.10) compute, respectively, the ZF-nulling vector, the decision statistic, and the estimated component of $\mathbf{s}$. Finally, (3.13) performs the cancellation of the detected component from the received vector, and (3.11) computes the new pseudo-inverse for the next iteration. This new pseudo-inverse is based on a “deflated” version of $\mathbf{H}$, in which the columns $k_1, \dots, k_i$ have been zeroed.

We have described the iterative nulling and cancellation method for symbol detection in V-BLAST. It has been proven in [63] that choosing the signal with the largest SNR at

each step for nulling and cancellation achieves the global optimization that minimizes the probability of symbol errors. Since at each step of the original algorithm, a pseudo-inverse of the deflated channel matrix is computed which is of order $\mathcal{O}(M_t^4)$, the total complexity of the algorithm is $\frac{27}{4}M_t^4$ for the case of $M_r = M_t$ [60].

3.3 Low Complexity Detection Algorithms for V-BLAST

3.3.1 Square-Root Algorithm

An efficient square-root algorithm based on QR decomposition and unitary transformation has been adopted in [60] to avoid the repeated computation of nulling vectors via matrix inversion. Let the QR decomposition of channel matrix be $\mathbf{H} = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is orthonormal and $\mathbf{R}$ is upper triangular. The optimal ordering and the nulling vectors are all computed by unitary transformation on the QR decomposed matrices. The square-root algorithm uses both the matrix $\mathbf{Q}$ for nulling vector calculation and the inverse matrix $\mathbf{R}^{-1}$ for optimal ordering. The square-root algorithm performs repeated triangulization on the inverse matrix $\mathbf{R}^{-1}$. Even with back-substitution, this may lead to instability so [60] adopted a computationally complex series of transformation to avoid computing directly by inverting. Its computational complexity amounts to $\frac{2}{3}M_t^3 + 7M_t^2M_r + 2M_tM_r^2$. In particular, when $M_t = M_r$, the complexity is $\frac{29}{3}M_t^3$.

3.3.2 Decorrelating Decision Feedback Algorithm

The decorrelating decision-feedback detection is applied to the V-BLAST system by making the connection between a V-BLAST system and a synchronous CDMA system.

The received signal vector $\mathbf{y}$ is correlated with the conjugate transpose of the channel matrix as

$$\mathbf{H}^H \mathbf{y} = \mathbf{H}^H \mathbf{H} \mathbf{s} + \mathbf{H}^H \mathbf{n} \quad (3.15)$$

This correlation is analogous to the matched filter bank front-end of a CDMA multiuser receiver. In the above equation, the cross-correlation matrix $\mathbf{H}^H \mathbf{H}$ can be Cholesky decomposed as $\mathbf{L} \mathbf{L}^H$, where $\mathbf{L}$ is a lower triangular matrix. $\mathbf{L}^{-1}$ is applied to the correlator output in (3.15) to whiten the noise, and the resulting received signal is transformed to:

$$\hat{\mathbf{y}} = \mathbf{L}^H \mathbf{s} + \mathbf{n}. \quad (3.16)$$

Since $\mathbf{L}^H$ is upper triangular, the k -th component of $\hat{\mathbf{y}}$ can be expressed as:

$$\hat{y}_k = l_{k,k}^* s_k + \sum_{i=k+1}^{M_t} l_{k,i}^* s_i + n_k, \quad (3.17)$$

which contains interference from only $M_t - k$ signals. The last component $\hat{y}_{M_t}$ contains no interference, so a decision for this transmitted signal can be made first. The next signal can be detected by subtracting the interference contribution from the M_t -th signal using the previous decision. This procedure is repeated until all signals are detected.

The above decorrelating decision feedback method first cancels the interference using

the feedback of previous decisions, and then makes a decision on the current signal. The detection and decision-feedback are performed in decreasing order of received signal energies in the original decorrelating decision-feedback CDMA multiuser detector in [62]. The Cholesky decomposition is calculated only once, so repeated calculation of the pseudo-inverse is avoided. But the decreasing energy ordering for decorrelating decision-feedback is not the same as the decreasing SNR ordering in V-BLAST. In the V-BLAST system, the received power corresponds to the column norms of the channel matrix $\mathbf{H}$. In a CDMA system, the cross-correlation between the different user codes can be designed to be equal, so decreasing energy ordering is the same as decreasing SNR ordering. However, in the V-BLAST system, the channel gains corresponding to the spreading codes in CDMA system are random and generally do not have equal cross-correlations. So a modification to the original decorrelating decision-feedback method is needed to obtain the optimal ordering.

3.3.3 Modified Decorrelating Decision Feedback Algorithm

The modified decorrelating decision-feedback algorithm considers the optimal detection order in [61]. The Cholesky decomposition matrices $\mathbf{L}$ and $\mathbf{L}^H$ are reordered so that the detected signal has the largest SNR at every step. Define the cross correlation matrix as $\mathbf{A} = \mathbf{L}\mathbf{L}^H$, the signal to be detected with the largest SNR corresponds to the signal with the smallest diagonal entry of $\mathbf{A}^{-1}$. Since the diagonal entries of $\mathbf{A}^{-1}$ are equal to the column norm of $\mathbf{L}^{-1}$, calculation of $\mathbf{A}^{-1}$ can be omitted.

The smallest column norm of $\mathbf{L}^{-1}$ is first found and the column with the smallest norm is interchanged with the M_t -th column. The rows of $\mathbf{L}$, corresponding to the columns of $\mathbf{L}^{-1}$ are interchanged in the same way. Interchanging two columns of a matrix can be performed by post-multiplication by a unitary permutation matrix $\mathbf{U}$, and interchanging two rows of a matrix can be performed by pre-multiplication by a unitary permutation matrix $\mathbf{U}^H$.

This algorithm also utilizes QR decomposition, but it is different from the square-root algorithm in [60]. It utilizes symmetry properties to perform repeated triangularization on the conjugate transpose matrix $\mathbf{L}^H$, while the inverse matrix $\mathbf{L}^{-1}$ is triangularized by symmetry by the same transformation. The dominant computation of the modified decorrelating decision-feedback receiver is in the QR decomposition, the matrix inversion and the reordering and triangularization of matrix $\mathbf{L}^H$ and $\mathbf{L}^{-1}$. Thus the total complexity for the algorithm is $\frac{1}{2}M_t^3 + 2M_t^2M_r$. When $M_t = M_r$, it is $\frac{5}{2}M_t^3$ [61].

3.4 Proposed Low Complexity Detection Algorithm

The standard V-BLAST algorithm and the existing low complexity algorithms consist of M_t iterations and for each iteration, the strongest sub-stream signal among the remaining sub-stream signals is found and the corresponding nulling vector is obtained. This means that the processes for finding the decoding order and finding the nulling vectors are mixed together. If we can separate the two processes, a significant reduction in the amount of arithmetic operations required can be obtained. In this section, a sub-

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optimal order based on QR decomposition that comes close to the error performance of the optimal order is introduced. In order to obtain the sub-optimal order, we first review the modified Gram-Schmidt algorithm without ordering.

Denote $\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{u}^H \mathbf{v}$ as the inner product of the vectors $\mathbf{u}$ and $\mathbf{v}$. Partition the channel matrix $\mathbf{H}$ into its columns corresponding to the M_t transmitted signals as

$$\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2 \ \cdots \ \mathbf{h}_{M_t}]. \quad (3.18)$$

The Gram-Schmidt algorithm and the calculation of the weight vectors can be summarized following the order $1, \dots, M_t$ as:

initialization:

$$i \leftarrow 1 \quad (3.19)$$

$$r_{11} = \|\mathbf{h}_1\| \quad (3.20)$$

$$\mathbf{q}_1 = \mathbf{h}_1 / r_{11} \quad (3.21)$$

recursion:

$$i \leftarrow i + 1 \quad (3.22)$$

$$\mathbf{u}_i = \mathbf{h}_i - \sum_{j=1}^{i-1} \langle \mathbf{h}_i, \mathbf{q}_j \rangle \mathbf{q}_j \quad (3.23)$$

$$r_{1:i-1,i} = \langle \mathbf{h}_i, \mathbf{q}_{1:i-1} \rangle \quad (3.24)$$

$$r_{ii} = \|\mathbf{u}_i\| \quad (3.25)$$

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$$\mathbf{q}_i = \mathbf{u}_i / r_{ii} \quad (3.26)$$

In every step i the vectors $\mathbf{q}_1, \dots, \mathbf{q}_i$ form an orthonormal basis of the vector space spanned by $\mathbf{h}_1, \dots, \mathbf{h}_i$ and the vectors $\mathbf{q}_j$ ($i < j \leq M_t$) contain the components of the corresponding $\mathbf{h}_j$ orthogonal to this vector space. The diagonal element r_{ii} denotes the length of $\mathbf{h}_i$ orthogonal to $\mathbf{q}_1, \dots, \mathbf{q}_{i-1}$. Furthermore, the coefficients r_{ij} specify the component of $\mathbf{h}_j$ ($i < j \leq M_t$) in the direction of $\mathbf{q}_i$.

The resulting QR decomposition of channel matrix has the form:

$$[\mathbf{h}_1 \cdots \mathbf{h}_{M_t}] = [\mathbf{q}_1 \cdots \mathbf{q}_{M_t}] \begin{bmatrix} r_{11} & \cdots & r_{1,M_t} \\ 0 & \ddots & \vdots \\ 0 & \cdots & r_{M_t,M_t} \end{bmatrix}. \quad (3.27)$$

Since the computation of $\mathbf{R}^{-1}$ requires much less arithmetic operations than that of $\mathbf{H}^{-1}$, the sub-optimal order can be obtained by sorting the row norms of $\mathbf{R}^{-1}$ in ascending order. Once the decoding order is obtained, the next step is to find the corresponding nulling vectors. Let the decoding order be $S_{sub} = \{k'_1, k'_2, \dots, k'_{M_t}\}$. We permute the columns of $\mathbf{R}$ according to the inverse of sub-optimal decoding order as

$$\mathbf{R}_1 = [\mathbf{r}_{k'_{M_t}} \mathbf{r}_{k'_{M_t-1}} \cdots \mathbf{r}_{k'_1}]. \quad (3.28)$$

This is because all the sub-stream signals will be decoded from an upper triangular matrix by back-substitution. The strongest signal corresponding to the M_t -th row of the upper triangular matrix will be decoded first and the weakest sub-stream signal

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corresponding to the first row will be decoded last. Interchanging the columns of a matrix is done by post-multiplication by a unitary permutation matrix $\mathbf{P}$ as we described in modified decorrelating decision feedback method. Now that $\mathbf{R}_1$ is not upper triangular, there exists a unitary matrix Θ that transforms $\mathbf{R}_1$ into an upper triangular form [64], which is denoted by $\mathbf{R}_2$. Similarly, $\mathbf{Q}$ has to be updated to $\mathbf{Q}_1 = \mathbf{Q}\Theta^H$ while the permuted $\mathbf{R}_2 = \Theta\mathbf{R}_1$. Therefore, each row of $\mathbf{Q}_1^H$ can be seen as a nulling vector. After $\mathbf{Q}_1^H$ is applied to the received signals, the output at the receiver is:

$$\mathbf{x} = \mathbf{Q}_1^H \mathbf{y} = \mathbf{R}_2 \mathbf{P}^H \mathbf{s} + \mathbf{Q}_1^H \mathbf{n} \quad (3.29)$$

where $\mathbf{P}^H \mathbf{s}$ is the transmitted signal with reordered components according to the sub-optimal order and defined to be $\mathbf{s}'$. Since $\mathbf{R}_2$ is upper triangular, the i -th component of $\mathbf{r}$ can be expressed as:

$$x_i = r_{i,i} s'_i + \sum_{k=l+1}^{M_t} r_{i,k} s'_k + n'_i \quad (3.30)$$

where the noise variance $\sigma_{n'_i}^2 = \sigma_n^2$ as the unitary matrix does not change the noise distribution.

The last component x_{M_t} contains no interference, so a decision for this transmitted signal can be made first. The next signal can be detected by subtracting the interference contribution from the M_t -th signal using the previous decision. This procedure is repeated until all signal are detected. When decoding the weakest sub-stream signal, which is the last one, all other sub-stream signals have already been cancelled.

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The above decision-feedback method first cancels the interference using the feedback of previous decisions, and then makes a decision on the current signal. The detection and decision-feedback are performed in the proposed sub-optimal order. The QR decomposition is calculated only once, so repeated calculation of the pseudo-inverse is avoided.

The performance of the iterative nulling and cancellation detection, the square-root algorithm and the modified decorrelating decision feedback detection with optimal ordering are the same [61]. Although in the latter two algorithms, repeated inverse of the deflated channel matrix is avoided, the QR decomposition matrices in the square-root algorithm and the Cholesky decomposition matrices in the modified decorrelating algorithm have to be reordered and triangularized at every step. While by adopting the low complexity algorithm proposed in our work, the sub-optimal detection order is decided before detection so the repeated reordering and triangularization operation becomes unnecessary. Thus further reduces the computational complexity. On the whole, the repeated inversion of channel matrix, reordering and triangularization are all avoided.

In a practical implementation, directly inverting the channel matrix $\mathbf{H}$ may lead to instability, so we use $\mathbf{R}^{-1}$ to obtain the sub-optimal detection order. We first perform Gram-Schmidt algorithm decomposition to get the upper triangular matrix $\mathbf{R}$, since the row norm of $\mathbf{R}^{-1}$ is equal to the row norm of $\mathbf{H}^+$. Then we calculate each row norm of $\mathbf{R}^{-1}$ and sort them to get the detection order. The rows of $\mathbf{R}$ are reordered

3.4. Proposed Low Complexity Detection Algorithm

Figure 3.3: Uncoded SER of 4×6 V-BLAST with optimal detection order and sub-optimal detection order

according to the sub-optimal order and triangularized. The dominant computation of the proposed low complexity method is in the QR decomposition via the Gram-Schmidt algorithm, the matrix inversion and reordering and triangularization on $\mathbf{R}^{-1}$ only once. The complexity of QR decomposition is $2M_t^2(M_r - \frac{1}{3}M_t)$. The complexity of calculating by back-substitution is $\frac{1}{3}M_t^3$. Finding the sub-optimal detection order by computing the row norm of $\mathbf{R}^{-1}$ takes $\frac{1}{2}M_t^2$ operations and triangularization of $\mathbf{R}$ takes M_t^2 operations. So the total complexity for the proposed algorithm is $2M_t^2M_r - \frac{1}{3}M_t^3$. When $M_t = M_r$, it is $\frac{5}{3}M_t^3$. This complexity is less than that of the modified decorrelating decision

Figure 3.4: Uncoded SER of 8×12 V-BLAST with optimal detection order and sub-optimal detection order

feedback algorithm [61] and the square-root algorithm [60].

3.5 Simulation Results

In this sub-section, we employ the proposed method to detect each substream in the V-BLAST system and compare the performance with the initial results, which utilize the original iterative nulling and cancellation detection algorithm. The simulation is conducted under the same condition as in the V-BLAST prototype in [63] with 16-

Figure 3.5: Performances of the detection schemes for SM in 4×4 MIMO

QAM and QPSK modulation schemes, respectively. We use the notation $M_t \times M_r$ to represent a MIMO configuration with M_t transmit antennas and M_r receive antennas. Fig. 3.3 shows the performance comparison of two detection orders for a 4×6 MIMO system, where the two curves are indistinguishable for both modulation schemes. For 8×12 MIMO system, the performance degradation caused by the sub-optimal ordering is insignificant as can be seen in Fig. 3.4.

We also consider the other linear and nonlinear detection schemes for SM introduced in Section 2.2.1. We use BPSK modulation in our simulation to compare their

performances in terms of uncoded BER. The simulation results are shown in Fig. 3.5.

3.6 Conclusions

A sub-optimal decoding order that is determined before detection starts and the GSO algorithm for QR decomposition to reduce the computational complexity for V-BLAST is studied. The repeated computation of triangularization is avoided by the sub-optimal ordering. It is shown that one can decrease the detection complexity of an uncoded MIMO system without significant loss in performance by using a sub-optimal ordering. Among the detection schemes for SM systems, the diversity of ML detection equals to the number of receive antennas, whereas, the diversity order of the ZF algorithm and the MMSE algorithm equals the number of receive antennas minus the number of transmit antennas plus one.

Chapter 4

Spatial Multiplexing in Spatially Correlated Fading Channels

In the previous chapter, we addressed the problem of a low complexity detection algorithm for one of spatial multiplexing (SM) structures, V-BLAST, under the assumption of i.i.d. Rayleigh multipath fading. It was shown that the MIMO systems achieve linear growth in multiplexing gain with the number of antennas. However, the i.i.d. assumption is idealistic and most realistic channels exhibit clustered scattering and spatially correlated links. This is motivated by the physics of the underlying scattering environment [13, 19]. In such realistic conditions, the capacity of MIMO channels can be substantially lower depending on the level of correlation. Both linear and nonlinear receiver structures work well when the channel matrix has independent entries, however, their error rate performance may be severely degraded when the channel is

ill-conditioned [28, 65]. Designing appropriate transmission techniques is therefore an important and practical issue for successful deployment of MIMO system. In this chapter, we investigate the problem of precoding strategies for SM systems at a given data rate.

Precoding techniques have been investigated by several researchers in related context. These include rotating the signal constellation for SM in a polarized channel [55] and in the presence of spatial correlation for the case of two transmit and two receive antennas [54]. Both above schemes adopt diagonal precoding with known channel correlation matrix at the transmitter. Linear precoding is considered in [51, 52] for space-time coded systems with knowledge of fading correlation. Different from the above schemes, we consider approaches in improving performances of two decoding methods: one case is with ML detection and the other is based on a reduced rank SIC receiver. The key advantage of our precoding scheme for ML detection is that the simplified pairwise error probability (PEP) analysis offers a direct insight for matching signaling schemes to channel characteristics. Reducing rank of the linear receiver at each stage of SIC demonstrates another yet effective way to enhance system performance in a highly correlated channel.

We base our analysis on the assumption that the transmitter is aware of the second order statistics of the channel, i.e. the channel correlation, and not the instantaneous CSI (I-CSI). This is because the second order statistics of the channel are a function of the local scattering environment [66] and thus vary much slower than the I-CSI so that

they can be sent by the receiver at regular intervals using a low rate feedback logical channel [67]. We first design the precoding strategy for ML decoding schemes based on constellation multiplexing and select the optimal precoder from all the candidates that minimizes the union bound on the symbol error rate (SER) for different correlation scenarios. Motivated by combining precoding and the BLAST receiver with known channel correlation at the transmitter in [53,68], we propose to reduce the rank of the ZF receiver based on singular value decomposition (SVD) at the early stages of SIC detection. The reduced rank receiver is obtained by deleting the ranks of a ZF receiver that carry the least channel information. Different from the full rank ZF receiver, the reduced rank receiver allows a small amount of interference from other streams in the interest of suppressing noise power so as to increase the post processing SNR of the desired stream.

The rest of the chapter is organized as follows. The system model is described in Section 4.1. The precoding techniques with maximum likelihood detection and SIC based detection schemes are detailed in Sections 4.2 and 4.3. The performance of the system is evaluated under different correlation environments, which is followed by conclusions in Section 4.4. The results in this chapter are published in [Zhao6, Zhao7, Zhao8].

4.1 System Model

The system model considered in this chapter is the same as that in Chapter 3 but we assume the presence of local scatterers around both the transmitter and the receiver. Therefore, the entries of the channel matrix $\mathbf{H}$ are zero-mean circularly symmetric complex Gaussian with spatial correlation function [13]

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H}_w \mathbf{R}_t^{1/2} \quad (4.1)$$

where $\mathbf{R}_t$ is the transmit covariance matrix, $\mathbf{R}_r$ is the receive covariance matrix and $\mathbf{H}_w$ is a matrix with each element being a zero mean, i.i.d complex Gaussian random variable. Let $\mathbf{R}_\mathbf{H}$ be the general $M_t M_r \times M_t M_r$ positive definite autocorrelation matrix that describes the correlation between all channel elements as

$$\mathbf{R}_\mathbf{H} = E\{\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^H\}, \quad (4.2)$$

where the operator $\text{vec}(\cdot)$ stacks the columns of a matrix into a tall vector. We use $\mathbf{A} \otimes \mathbf{B}$ to denote Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$. Using the property of Kronecker product, $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B})$, the channel correlation matrix is given by

$$\mathbf{R}_\mathbf{H} = \mathbf{R}_t^T \otimes \mathbf{R}_r. \quad (4.3)$$

It is assumed that the I-CSI is perfectly estimated at the receiver. The long term channel state information (LT-CSI) $\mathbf{R}_t$ and $\mathbf{R}_r$ is sent to the transmitter by the receiver. We further assume that the signal bandwidth is much smaller than the coherence frequency of the channel and thus it can be modeled as frequency flat. The received signal can be written as:

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{n} \quad (4.4)$$

where $\mathbf{F}$ is the precoding matrix and the total radiated power is $E\{\|\mathbf{F}\mathbf{s}\|^2\} = \text{tr}(\mathbf{F}\mathbf{F}^H) = M_t$. When the transmitted signals are not precoded before launched into the channel, the precoder can be seen as an identity matrix, i.e. $\mathbf{F} = \mathbf{I}_{M_t}$.

4.2 Precoding with ML Detection

4.2.1 ML Detection and PEP

In this section, ML detection is considered at the receiver. When there is no precoding matrix applied to the transmitted symbol vector, the ML detector computes the metric over the set of all possible transmitted symbol vectors $\mathcal{S}_s$ and decides in favor of the codeword that minimizes the squared Frobenius norm as:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s}_i \in \mathcal{S}_s} \|\mathbf{y} - \mathbf{H}\mathbf{s}_i\|_F^2, \quad (4.5)$$

where $\|\cdot\|_F$ stands for Frobenius norm. For random channels the metric of interest is typically the average error probability. Exact calculation of the symbol or bit error probability is difficult. We evaluate the pairwise error probability (PEP) which determines the probability of erroneous decoding of $\mathbf{s}_m$ when $\mathbf{s}_n$ is transmitted. Then we upper bound all the possible error events in the vector constellation using the union bound for PEP.

Let the error vector be denoted by $\mathbf{e}_{n,m} = \mathbf{s}_n - \mathbf{s}_m$. The PEP for a given $\mathbf{H}$ is denoted by $P(s^n \rightarrow s^m | \mathbf{H})$ and given by [10]

$$P(\mathbf{s}_n \rightarrow \mathbf{s}_m | \mathbf{H}) = Q \left(\sqrt{\frac{\|\mathbf{H}\mathbf{e}_{n,m}\|^2}{2\sigma_n^2}} \right) \quad (4.6)$$

Using the Chernoff bound $Q(x) \leq e^{-x^2/2}$ to upper bound the PEP and the fact that $\text{vec}(\mathbf{H})$ is complex normal distributed with zero mean and covariance $\mathbf{R}_{\mathbf{H}}$, it can be shown that

$$\begin{aligned} P(\mathbf{s}_n \rightarrow \mathbf{s}_m) &= E_{\mathbf{H}} \{P(\mathbf{s}_n \rightarrow \mathbf{s}_m | \mathbf{H})\} \\ &\leq \left| \mathbf{I}_{M_t M_r} + \frac{1}{4\sigma_n^2} \mathbf{R}_{\mathbf{H}} [(\mathbf{e}_{n,m} \mathbf{e}_{n,m}^H)^* \otimes \mathbf{I}_{M_r}] \right|^{-1}, \end{aligned} \quad (4.7)$$

where $E_{\mathbf{H}}\{\cdot\}$ is the expectation taken over all the random channel matrices $\mathbf{H}$. This is strictly true when $\mathbf{R}_{\mathbf{H}}$ is nonsingular. We rewrite $\mathbf{R}_{\mathbf{H}}$ as

$$\mathbf{R}_{\mathbf{H}} = (\mathbf{U}_t \Sigma_t \mathbf{U}_t^H)^T \otimes (\mathbf{U}_r \Sigma_r \mathbf{U}_r^H), \quad (4.8)$$

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where $\mathbf{R}_t = \mathbf{U}_t \Sigma_t \mathbf{U}_t^H$ and $\mathbf{R}_r = \mathbf{U}_r \Sigma_r \mathbf{U}_r^H$ are the eigenvalue decompositions of the transmit correlation matrix and the receive correlation matrix. Following the property of determinant $|\mathbf{I} + \mathbf{A}\mathbf{B}| = |\mathbf{I} + \mathbf{B}\mathbf{A}|$ and the distributive property of Kronecker product, we have

$$P(\mathbf{s}_n \rightarrow \mathbf{s}_m) \leq \left| \mathbf{I}_{M_t M_r} + \frac{1}{4\sigma_n^2} (\Sigma_t \otimes \Sigma_r) [\mathbf{U}_t^T (\mathbf{e}_{n,m} \mathbf{e}_{n,m}^H)^* \mathbf{U}_t^* \otimes \mathbf{I}_{M_r}] \right|^{-1}. \quad (4.9)$$

For two matrices $\mathbf{A}$ and $\mathbf{B}$, the eigenvalues of $\mathbf{A} \otimes \mathbf{B}$ are given by $\lambda_{A_i} \lambda_{B_j}$, where λ_{A_i} ($i = 1, \dots, r_A$) are eigenvalues of $\mathbf{A}$ and λ_{B_j} ($j = 1, \dots, r_B$) are eigenvalues of $\mathbf{B}$ with $r_A = \text{rank}(\mathbf{A})$, $r_B = \text{rank}(\mathbf{B})$. Using this property, it is easy to show that at high SNRs,

$$P(\mathbf{s}_n \rightarrow \mathbf{s}_m) \leq \left(\frac{1}{4\sigma_n^2} \right)^{-M_r} \left(\lambda_s \left[\prod_{i=1}^{M_r} \lambda_{r_i} \right]^{1/M_r} \right)^{-M_r}, \quad (4.10)$$

where λ_s is the only non-zero eigenvalue of $\Sigma_t \mathbf{U}_t^T (\mathbf{e}_{n,m} \mathbf{e}_{n,m}^H)^* \mathbf{U}_t^*$ and λ_{r_i} ($i = 1, \dots, M_r$) are the eigenvalues of Σ_r . From the above inequality, it is seen that the diversity gain is M_r and the coding gain is $\lambda_s \left[\prod_{i=1}^{M_r} \lambda_{r_i} \right]^{1/M_r}$. We note that both the diversity gain and coding gain of ML detection are dependent on the error codeword properties, as well as the channel correlation characteristics.

Figure 4.1: Illustration of four superposed BPSK constellation yielding a 16-QAM constellation on the left, and two superposed BPSK constellation yielding a 4-QAM constellation on the right

4.2.2 Constellation Multiplexing and Precoder Selection

Motivated by (4.9), we define the unitary transform of the codeword error correlation matrix as

$$\begin{aligned} \mathbf{d}_{n,m} \mathbf{d}_{n,m}^H &= \mathbf{U}_t^T (\mathbf{e}_{n,m} \mathbf{e}_{n,m}^H)^* \mathbf{U}_t^* \\ &= \mathbf{U}_t^T \mathbf{e}_{n,m}^* (\mathbf{U}_t^T \mathbf{e}_{n,m}^*)^H, \end{aligned} \quad (4.11)$$

where $\mathbf{d}_{n,m} = \mathbf{U}_t^T \mathbf{e}_{n,m}^*$ is the unitary transform of the original error vector. It is natural to transmit the signal vector along the eigenmodes of the transmit correlation matrix. That is, we consider the transmitted signals of the form $\mathbf{s} = \mathbf{U}_t \mathbf{s}_{F_1}$. In other words, the transmitted signals are precoded by the eigenvectors of the transmit correlation matrix before launched onto the channel. Thus, $\mathbf{d}_{n,m}^* \mathbf{d}_{n,m}^T$ becomes the codeword error correlation matrix corresponding to transmitted vector signals $\mathbf{s}_{F_1}$, and the precoder is given by $\mathbf{F}_1 = \mathbf{U}_t$. For correlated MIMO channels, the correlation matrix is always singular or near singular due to clustered scattering. To avoid transmission over those weak

eigenmodes corresponding to very small eigenvalues, data should be multiplexed on the relatively strong eigenmodes that corresponds to large eigenvalues. In the following, we will apply a precoding transformation to the transmitted vectors and launch them along the strong eigenmodes. The precoding process is to transform a low level modulated symbol vector into a high level modulated symbol vector, and is given below:

$$\mathbf{s}_{F_1} = \mathbf{F}_2 \mathbf{s}_{F_2} \quad (4.12)$$

where the elements of $\mathbf{s}_{F_2}$ are low level modulated, $\mathbf{F}_2$ functions as the precoder, the output vector of the precoder $\mathbf{F}_2$ has high level modulated symbols. To illustrate the precoder, we first consider the simplest case: BPSK modulation with $M_t = 4$ data streams. Transforming the 4×1 BPSK modulated signal vector into a 2×1 vector with 4-QAM modulated symbols is realized by setting $\mathbf{F}_2$ as:

$$\mathbf{F}_2 = \begin{bmatrix} 1 & i & 0 & 0 \\ 0 & 0 & 1 & i \end{bmatrix}. \quad (4.13)$$

the output vector of this precoder has two 4-QAM symbols as its elements. Hence, $\mathbf{F}_1$ should be revised accordingly to consist of the first two eigenvectors corresponding to the two largest eigenvalues, and thus is given by $\mathbf{U}_t(:, 1 : 2)$. Similarly, encoding a four-BPSK-symbol vector into one 16-QAM symbol is realized by letting $\mathbf{F}_2$ be:

$$\mathbf{F}_2 = \begin{bmatrix} \sqrt{1.6} & \sqrt{1.6}i & \sqrt{0.4} & \sqrt{0.4}i \end{bmatrix}, \quad (4.14)$$

Figure 4.2: Block diagram of the signal model with precoders

Figure 4.3: Equivalence of the signal model with precoders

and the superposed four BPSK symbols is equivalent to a 16-QAM symbol. Thus $\mathbf{F}_1$ should be composed of the first eigenvector corresponding to the largest eigenvalues, that is, $\mathbf{U}_t(:, 1)$. The superposed BPSK constellation yields a constellation equivalent to 4-QAM and 16-QAM as illustrated in Fig. 4.1. If transmission is along the first three eigenmodes, the precoder will need to transform the 4×1 vector into a 3×1 vector, which includes one 4-QAM symbol and two BPSK symbols. Hence, $\mathbf{F}_2$ can be modified as

$$\mathbf{F}_2 = \begin{bmatrix} 1 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (4.15)$$

with $\mathbf{F}_1 = \mathbf{U}_t(:, 1 : 3)$. The signaling process via precoders $\mathbf{F}_1$ and $\mathbf{F}_2$ is illustrated in Fig. 4.2. Thus, the combined precoder is given by $\mathbf{F} = \mathbf{F}_1 \mathbf{F}_2$, which satisfies the power constraints.

When channel correlation is high, not all the eigenmodes of the transmit correlation

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matrix should be utilized, resulting in different precoder selections. The number of eigenmodes used for transmission is determined by the channel correlation level, and is numerically evaluated by the union bound on the SER. The Union bound on the SER can be obtained by evaluating all the possible pairwise error events. Let M_s be the size of the constellation. Therefore, the size of $\mathcal{S}_s$ is $|\mathcal{S}_s| = M_s^{M_t}$. The number of all possible pairwise error vectors is then $M_s^{M_t}(M_s^{M_t} - 1)$. Hence, the union bound on the SER is given by:

$$P_s \leq M_s^{-M_t} \sum_n \sum_m P(\mathbf{s}_n \rightarrow \mathbf{s}_m) \quad (4.16)$$

At high SNRs, substituting (4.10) into the above inequality, we have

$$P_s \leq M_s^{-M_t} \left(\frac{1}{4\sigma_n^2} \right)^{-M_r} \left[\prod_{i=1}^{M_r} \lambda_{r_i} \right]^{-1} \sum_n \sum_m \lambda_s^{-M_r} \quad (4.17)$$

where $\lambda_s = \mathbf{e}_{n,m}^H \mathbf{U}_t \Sigma_t \mathbf{U}_t^H \mathbf{e}_{n,m}$, with $\mathbf{e}_{n,m} = \mathbf{F}_1 \mathbf{F}_2 \mathbf{e}_{F_2,n,m}$, and $\mathbf{e}_{F_2,n,m} = \mathbf{s}_{F_2,n} - \mathbf{s}_{F_2,m}$.

Different selections of the precoder results in different union bounds. We will choose the precoder that minimize $\sum_n \sum_m \lambda_s^{-M_r}$.

To illustrate the precoder selection process, we first consider the simple case of $M_t = 4$ and $\mathbf{s}_{F_2}$ with BPSK modulation. For BPSK modulation, the symbols of $\mathbf{s}_{F_2}$ belong to the set $\{1, -1\}$. Suppose the error vector given by the difference between the transmitted vector and the decoded vector is $\mathbf{e}_{F_2,n,m} = [e_1, \dots, e_4]^T$. The number of all possible error vectors is 240. The elements of the error vector $e_i, i = 1, \dots, 4$, belong to the set $\{0, 2, -2\}$ and there are altogether 80 distinct error vectors since some error vectors are equal due to the symmetry of the constellation. If $\mathbf{F}_2$ is given by (4.13) for

example, then

$$\begin{aligned} \lambda_s &= [e_1 \ e_2 \ e_3 \ e_4] \begin{bmatrix} 1 & 0 \\ -i & 0 \\ 0 & 1 \\ 0 & -i \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} 1 & i & 0 & 0 \\ 0 & 0 & 1 & i \end{bmatrix} [e_1 \ e_2 \ e_3 \ e_4]^T \\ &= \lambda_1(e_1^2 + e_2^2) + \lambda_2(e_3^2 + e_4^2). \end{aligned} \quad (4.18)$$

Similarly, when $\mathbf{F}_2$ is given by (4.14), λ_s changes to

$$\lambda_s = [1.6(e_1^2 + e_2^2 + e_1e_3 + e_2e_4) + 0.4(e_3^2 + e_4^2)]\lambda_1. \quad (4.19)$$

For a given precoder, $\sum_n \sum_m \lambda_s^{-M_r}$ is computed. The precoder which minimizes the union bound on the SER is selected.

4.2.3 Combined ML and SIC Decoding

The computational complexity of the ML detection can be reduced by introducing the SIC method [59]. We regard the output of precoder $\mathbf{F}_2$ as the reduced number of input substreams with different modulation level. For example, the output of (4.13) can be seen as two 4-QAM modulated substreams instead of four BPSK modulated substreams. Since the performance of SIC is limited by the first detected substream due to error propagation, we separate the substream that transmits along the eigenmode corresponding to the largest eigenvalue and perform ML detection on this substream. The

Figure 4.4: Performance of detection schemes for 4×4 SM system with fading correlation, $\rho = 0.8$

contribution of the estimates from the first detected substreams is cancelled out from the received signals. The procedure is repeated to decode the remaining substreams. The reduced number of substreams are separated by linear receivers such as ZF and MMSE at each stage of SIC, followed by ML detection. The successive decoding of each substream proceeds according to their indexes since they coincide with the descending order in which the eigenvalues of the transmit correlation matrix are arranged.

Figure 4.5: Comparison of different precoders for 4×4 SM system with ML detection, $\rho = 0.8$

4.2.4 Simulation Results

This section illustrates the effects of spatial correlation on the performance of a 4×4 SM system by Monte-Carlo simulation. We use BPSK to modulate the four transmitted symbols. The proposed precoder is employed to improve the error rate performance of the ML detector for different channel correlation levels. We assume that the correlated channel follows the Kronecker model and correlation exists only at the transmitter as many other literatures did [52,54]. This case is well suited to downlink situation where an elevated access point transmits to a subscriber in a rich scattering environment. The

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Figure 4.6: Comparison of different precoders for 4×4 SM system with ML detection, $\rho = 0.3$

correlation matrix adopted in the simulation has an exponential form:

$$\begin{bmatrix} 1 & \rho & \cdots & \cdots & \rho^{M_t-1} \\ \rho^* & 1 & \rho & \cdots & \rho^{M_t-2} \\ \vdots & & \cdots & & \vdots \\ \rho^{M_t-1*} & \rho^{M_t-2*} & \cdots & \rho^* & 1 \end{bmatrix}. \quad (4.20)$$

The performance of SM system is first evaluated in terms of uncoded BER with both linear and nonlinear receivers when the correlation level is $\rho = 0.8$ and compared

Figure 4.7: Performance of combined ML and SIC detection for 4×4 SM system, $\rho = 0.8$

with that of the ML detection scheme with $\rho = 0.3$ and $\rho = 0$. Fig. 4.4 shows the performance degradation with different detection schemes. Though ML detection is optimal among all the linear and nonlinear detection schemes, it is sensitive to high correlation of the channel coefficients.

Then, we compare the proposed precoding schemes with the ML detection scheme. Fig. 4.5 demonstrates the performance of different precoders versus input SNR when the correlation level is $\rho = 0.8$. Under this correlation scenario, the eigenvalues of the transmit correlation matrix is $\lambda_1 = 3.1032$, $\lambda_2 = 0.5593$, $\lambda_3 = 0.2088$, and $\lambda_4 =$

0.1287. Given the eigenvalues, the union bound on the SER with each precoder can be computed. We compare these union bounds and find that the optimal transmission strategy is to precode the four BPSK symbols into a 16-QAM symbol via (4.14) and transmit it over the eigenvector corresponding to the first eigenvalue, which is also the largest eigenvalue. This result agrees well with the simulation results shown in the figure.

When the correlation level is $\rho = 0.3$, the eigenvalues of the transmit correlation matrix is given by $\lambda_1 = 1.5767$, $\lambda_2 = 1.087$, $\lambda_3 = 0.7503$, and $\lambda_4 = 0.856$. Different from the previous case, the distribution of eigenvalues of this correlated scenario is less spread out. We calculate the union bounds on the SER of all the precoders given the eigenvalues. It is found that the best precoder for this correlation environment is to transmit the two 4-QAM symbols precoded by (4.13) along the first two eigenvectors corresponding to λ_1 and λ_2 , which is validated in Fig. 4.6. It is also noted that under low correlated scenario, precoding four BPSK symbols into a 16-QAM symbol and transmitting it over the first eigenvector performs worst among all the precoders.

We study the performance of the combined ML and SIC detection under the correlation level $\rho = 0.8$. Fig. 4.7 displays the comparison between the combined ML and SIC detection and ML detection with different precoders employed. It is seen that at a given data rate, when the number of output substreams of precoder $\mathbf{F}_2$ becomes larger, which means more substreams are to be separated by the linear receiver, the performance difference between the combined detector and ML detection becomes more

distinct.

4.3 Precoding with RR-SIC Detection

4.3.1 RR-SIC Algorithm

In this section, we consider linear receivers to separate the transmitted data streams when precoding is not applied. Let the row vector $\mathbf{w}_k$ be an arbitrary linear receiver for the k -th stream with dimension $1 \times M_r$. Then the estimate of the k -th stream $\hat{s}_k$ is given by:

$$\begin{aligned}\hat{s}_k &= \mathbf{w}_k \mathbf{y} \\ &= \mathbf{w}_k \mathbf{H} \mathbf{s} + \mathbf{w}_k \mathbf{n}\end{aligned}\tag{4.21}$$

The post processing SNR of the k -th multiplexed stream is [48]

$$PSNR_k = \frac{|\mathbf{w}_k \mathbf{h}_k|^2}{N_o \|\mathbf{w}_k\|^2 + \sum_{j \neq k} |\mathbf{w}_k \mathbf{h}_j|^2}\tag{4.22}$$

where $\mathbf{h}_k$ is the k -th column of $\mathbf{H}$.

For a standard ZF receiver, $\mathbf{w}_k = \mathbf{H}^\dagger(k, :)$, where $\mathbf{H}^\dagger(k, :)$ is the k -th row of pseudo-inverse of $\mathbf{H}$. The post processing SNR of the k -th decoded data stream is given as:

$$PSNR_k = \frac{1}{N_o [(\mathbf{H}^H \mathbf{H})^{-1}]_{kk}}\tag{4.23}$$

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The ZF receiver results in significant performance degradation when channel correlation is high. When $\mathbf{H}$ is ill-conditioned, $\mathbf{s}$ will be poorly determined by its full rank receiver [64]. In this situation, the ill-conditioned channel can be well approximated by a lower rank channel. We assume that the channel matrix has rank $r = \min(M_t, M_r)$. The channel matrix $\mathbf{H}$ can be expressed via the singular value decomposition (SVD) as follows:

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \quad (4.24)$$

where $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_{M_r}]$ and $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_{M_t}]$ and $\mathbf{\Sigma} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r)$. If we reduce the rank of the ZF receiver to q ($q < r$), the linear receiver for the k -th stream is given by

$$\mathbf{w}_k^R = \mathbf{e}_k^T \mathbf{V}_q \mathbf{\Sigma}_q^{-1} \mathbf{U}_q^H \quad (4.25)$$

where $\mathbf{U}_q = [\mathbf{u}_1, \dots, \mathbf{u}_q]$, $\mathbf{V}_q = [\mathbf{v}_1, \dots, \mathbf{v}_q]$, $\mathbf{\Sigma}_q = \text{diag}(\sigma_1, \dots, \sigma_q)$ and $\mathbf{e}_k$ denotes a vector with 1 at the k -th entry and zero elsewhere. The combined response of the reduced rank receiver $\mathbf{w}_k^R$ is given by:

$$\begin{aligned} \mathbf{w}_k^R \mathbf{H} &= \mathbf{e}_k^T \mathbf{V}_q \mathbf{V}_q^H \\ &= [a_1, \dots, a_k, \dots, a_{M_t}] \end{aligned} \quad (4.26)$$

where a_i ($i \neq k$) are the residual responses of the k -th receiver and constitute the k -th row of $\mathbf{V}_q \mathbf{V}_q^H$. The output of the receiver, $\hat{s}_k$ is a weighted sum of the desired symbol

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and multi-stream interference:

$$\hat{s}_k = a_1 s_1 + \cdots + a_{M_t} s_{M_t} + \mathbf{w}_k^R \mathbf{n} \quad (4.27)$$

After some simplification, the post processing SNR for the k -th stream can be written as:

$$PSNR_k = \frac{[\mathbf{V}_q \mathbf{V}_q^H]_{kk}^2}{N_o [\mathbf{V}_q \Sigma_q^{-2} \mathbf{V}_q^H]_{kk} + \sum_{j \neq k} |[\mathbf{V}_q \mathbf{V}_q^H]_{kj}|^2} \quad (4.28)$$

A reduced rank receiver allows a small amount of interference from other streams in the interest of suppressing noise power so as to increase the post processing SNR of the k -th stream. When the receiver rank is reduced to a small number, the post processing SNR is quickly saturated, which causes an error floor to occur at median input SNRs.

We now present the algorithm of RR-SIC receiver. We use (M, N) to denote the algorithm mode where the reduced rank receivers are used at the first M stages of detection in SIC and for each of the M stages the rank of receiver is N less than the full rank of that stage. For the remaining $M_t - M$ stages, full rank ZF receivers are used. As the detection proceeds, the rank of a deflated channel matrix becomes so small that further reducing the rank of the receiver may lead to early saturation of the post processing SNR and cause a high error floor for RR-SIC receiver. For each of the first M stages, if N is selected to be a large value, the performance will also have an error floor that lies above the required error rate. So in the RR-SIC algorithm, both M and N are usually chosen to have small values in order that the RR-SIC receiver

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outperforms the ZF-SIC receiver even at high input SNRs.

We assume a fixed detection order (e.g., detecting according to the order of antenna indexes) and the algorithm can be described as follows:

initialization:

$$\mathbf{H}_1 = \mathbf{H}, \mathbf{y}_1 = \mathbf{y}$$

recursion:

$$\text{for } i = 1 \text{ to } M_t$$

- Step1: compute SVD of the channel matrix $\mathbf{H}_i = \mathbf{U}_{r_i} \Sigma_{r_i} \mathbf{V}_{r_i}^H$, and $\text{rank}(\mathbf{H}_i)$ is r_i
- Step2: if $i \leq M$, reduce the rank of the channel matrix to $q_i = r_i - N$, the reduced rank receiver is given by $\mathbf{w}_i^R = \mathbf{e}_i^T \mathbf{V}_{q_i} \Sigma_{q_i}^{-1} \mathbf{U}_{q_i}^H$; otherwise, apply the full rank receiver $\mathbf{w}_i = \mathbf{e}_i^T \mathbf{V}_{r_i} \Sigma_{r_i}^{-1} \mathbf{U}_{r_i}^H$
- Step3: estimate i -th stream, $\hat{s}_i = \mathbf{w}_i \mathbf{y}_i$ and subtract the contribution of the i -th stream, $\mathbf{y}_{i+1} = \mathbf{y}_i - \mathbf{H}(:, i) \sqrt{p_i} \hat{s}_i$
- Step4: null the i -th column of $\mathbf{H}_i$ to obtain $\mathbf{H}_{i+1}$

For each stage of detection, SVD is computed for $\mathbf{H}$, whose columns corresponding to the previously detected streams are zeroed. For the first M stages (including the M -th stage), the reduced rank receiver that has N rank less than the full rank receiver

is applied at each detection stage. After the M -th stage, the receiver falls back to the full rank ZF receiver at each stage of detection.

4.3.2 Power Allocation

We consider two transmission modes for RR-SIC: one is transmission from eigenvectors of the correlation matrix with power allocation to each eigenmode; the other is to allocate power to each transmitted signal without phase adjustment.

It is assumed that the LT-CSI is available at the transmitter in the form of the matrix $\mathbf{R}_{\mathbf{H}\mathbf{H}} = E\{\mathbf{H}^H\mathbf{H}\}$. The LT-CSI $\mathbf{R}_{\mathbf{H}\mathbf{H}}$ measures the transmit correlation and has the following expression [53]:

$$\mathbf{R}_{\mathbf{H}\mathbf{H}} = \mathbf{R}_t \cdot \text{tr}(\mathbf{R}_r). \quad (4.29)$$

The linear precoding matrix that maximizes the capacity, minimizes the mean square error (MSE) between the transmitted signal and the detected signals has the general form [53]:

$$\mathbf{F} = \mathbf{U}_F \cdot \Phi, \quad (4.30)$$

where the columns of $\mathbf{U}_F$ are eigenvectors of $\mathbf{R}_{\mathbf{H}\mathbf{H}}$, i.e., $\mathbf{R}_{\mathbf{H}\mathbf{H}} = \mathbf{U}_F \Lambda \mathbf{U}_F^H$ with $\Lambda = \text{diag}(\lambda_{11}, \dots, \lambda_{M_t M_t})$. Φ is a diagonal matrix $\Phi = \text{diag}(\phi_{11}, \dots, \phi_{M_t M_t})$ that defines the power allocated to different eigenvectors. In this transmission mode, we consider two power allocation schemes: MMSE waterfilling and uniform allocation.

1. MMSE waterfilling: the power allocation matrix that minimizes the MSE is given by [44]:

$$|\phi_{ii}|^2 = \left(\frac{M_t + \sum_{n=1}^{\bar{N}} \lambda_{nn}^{-1} \lambda_{ii}^{-1/2}}{\sum_{n=1}^{\bar{N}} \lambda_{nn}^{-1/2}} - \frac{1}{\lambda_{ii}} \right)^+. \quad (4.31)$$

2. Uniform power allocation: equal power is transmitted from each eigenvector by setting $\phi_{ii} = 1$.

In the latter mode, equal power transmission and unequal power transmission strategies are considered. Unequal power allocation is designed to compensate for the lack of diversity of the reduced rank receiver. The precoding matrix only defines the power allocation over the transmit antennas that has the form: $\mathbf{F} = \text{diag}(\sqrt{P_1}, \dots, \sqrt{P_{M_t}})$.

1. Equal power allocation: we simply let $\mathbf{F} = \mathbf{I}_{M_t}$.
2. Unequal power allocation: as shown in (4.27), the reduced rank receiver cannot completely eliminate the multi-stream interference. Therefore, we may be able to increase the post processing SNR of the desired stream by assigning higher input power to it. We treat the first M streams detected by reduced rank receivers as the first group and the remaining $M_t - M$ streams detected using full rank receiver as the second group. We assign higher input power to the first group than the second group. Among each group equal power is allocated over the inputs of the group.

$$\mathbf{F} = \begin{bmatrix} \sqrt{\frac{P_1}{M}} \mathbf{I}_M & 0 \\ 0 & \sqrt{\frac{P_2}{M_t - M}} \mathbf{I}_{M_t - M} \end{bmatrix} \quad (4.32)$$

where P_1 and P_2 are the power allocated to the first group and the second group of inputs respectively and $P_1 + P_2 = P_o$.

4.3.3 Simulation Results

In this section we evaluate the error rate performance of the proposed RR-SIC algorithm with index-order detection in the case of MIMO channels with severe correlation. We assume that the transmit antennas placed at a high base station exhibit correlation while the receive antennas are in an uncorrelated rich-scattering environment. We consider a MIMO system with four transmit antennas and four receive antennas.

The correlated channel model adopted in our simulation is described in [13]. We assume the multi-antenna array is in linear arrangement at the transmitter. The relative spacing between neighboring antennas is $\delta = \frac{d}{\lambda} = 4$, where d is the absolute spacing and λ is the wavelength of the signal. We consider the “broadside” array configuration as defined in [13] and the incoming waves are uniformly distributed in angle spread Δ [13].

In a 4×4 MIMO system, four streams are to be detected so that there are four detection stages altogether. When $\Delta = 2^\circ$, we adopt the mode $(M, N) = (1, 1)$. The reduced rank receiver is applied only to the first detection stage of SIC, i.e., a rank-3 receiver is used to detect the first stream. Sequentially, full rank receivers are used at the second and third stages followed by maximal ratio combining at the fourth stage of detection. When $\Delta = 3^\circ$, we also consider the $(1, 1)$ mode, i.e., a rank-3 receiver is

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Figure 4.8: Performance comparison of ZF-SIC and RR-SIC for 4×4 system, RR-SIC mode (1,1)

used at the first stage of detection, and full rank receivers are utilized for the remaining detection stages as in the case of $\Delta = 2^\circ$. The performance of RR-SIC is measured in terms of symbol error rate (SER) as a function of average input SNR of all substreams. Fig. 4.8 illustrates performance comparison between an ordered ZF-SIC (ZF-BLAST) receiver, where full rank ZF receivers are used at each stage of detection, and a RR-SIC receiver. Equal power is allocated over all the transmit antennas for both receivers. Fig. 4.8 shows that the RR-SIC receiver outperforms the ZF-BLAST in the scenarios of $\Delta = 2^\circ$, and $\Delta = 3^\circ$. We observe the appearance of an error floor for RR-SIC receiver of mode (1,1) begins at about 30 dB in the case of $\Delta = 3^\circ$. If a SER of approximately

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Figure 4.9: Effect of unequal power allocation on performance of RR-SIC for 4×4 system with angle spread $\Delta = 3^\circ$, RR-SIC mode (1, 1)

10^{-4} is required for an appropriate service, the RR-SIC receiver may not meet the SER requirement in this case. Therefore, we consider other power allocation strategies for RR-SIC of (1, 1) mode when $\Delta = 3^\circ$. The simulation result is shown in Fig. 4.9. For the case of transmission not from the eigenvector of the channel correlation matrix, the input power assigned to the first stream has a ratio of 0 dB, 3 dB, and 5 dB, respectively, above the other streams, where equal power is assigned. The unequal power allocation scheme helps reduce the interference power of other streams when detecting the stream at the first stage. The error rate performance has a lower error floor than in the case of equal power allocation. But this effect reverses for larger input power ratio between

the first and the other streams because reducing power of the other streams may affect their post processing SNR in the following stages of detection. We also evaluate the impact of transmission from the eigenmode of the channel correlation matrix and using the well known MMSE waterfilling and uniform power allocation schemes. It is shown that the two schemes perform worse than the unequal power allocation scheme.

4.4 Conclusions

This chapter presents a precoding scheme which combines constellation multiplexing and transmission over the eigenvectors of the transmit correlation matrix given long term channel statistics at the transmitter. Given the eigenvalues of the transmit correlation matrix, the optimal precoder that minimizes the union bound on the SER is selected from the precoder candidates. It is noted that the signal should be transmitted over the eigenvector corresponding to the largest eigenvalue when channel correlation is high; while in the low correlated channel, where eigenvalues are evenly distributed, more eigenmodes should be utilized for transmission. For the RR-SIC algorithm where a reduced rank receiver is applied to the first few stages of detection for SIC, both the number of stages where reduced rank receivers are used for detection and the rank reduced at each of the stages must be limited to a small value. The constraints are proposed to avoid the phenomenon of an error floor to occur at low SNRs. The advantage of the RR-SIC algorithm is more significant in the case of unequal power allocation since the scheme can reduce the error floor for the RR-SIC receiver.

Chapter 5

Detection for Layered Space-Time Block Coding System

In the previous two chapters, we discussed the problems of the spatial multiplexing (SM) transmission in an i.i.d. channel and a spatially correlated channel. It has been stated in Chapter 2 that SM and space-time block coding (STBC) are developed to exploit the data rate advantage and the diversity advantage of multiple antennas, respectively. The tradeoffs between the two schemes are considered in [69]. Given a MIMO system, both diversity gain and multiplexing gain can be obtained simultaneously [69]. In this chapter, we consider combined SM and STBC system, namely, layered space-time block coding (LA-STBC) system, which can provide a flexible tradeoff between error probability and transmission efficiency with low complexity.

The basic framework of LA-STBC is to partition all the available transmit anten-

nas into several groups, each transmitting space-time coded blocks independently at the same time. At the receiver, each filtered version of a codeword is extracted independently or serially similar to the SM detection algorithms. The outputs of group receivers are sent to a space-time decoder at each layer, which performs maximum likelihood (ML) decoding to decode the transmitted signals embedded in each codeword with low complexity. On one hand, the signals transmitted within each antenna group are space-time coded, hence higher transmit diversity can be obtained compared with pure SM systems. On the other hand, the overall data rate is higher than pure STBC systems due to the independence of the signals transmitted by different antenna groups. Similar ideas have been adopted in [37, 38] and [70], where space-time trellis codes (STTCs) and space-time block codes (STBCs) are combined with BLAST, respectively. Here we focus on the structure where STBCs are used as component codes at each layer and propose several detection schemes for the LA-STBC system. The detection schemes include linear group receivers and serial processing that combines group interference suppression and group interference cancellation. In addition, performance of detection based on successive interference cancellation (SIC) with imperfect decision feedback is analyzed, which has never been discussed before as far as we know. For the sake of brevity, the STBC considered here is Alamouti's code [32] and the number of transmit antennas is assumed to be even. We notice that there is another transmission scheme similar to the LA-STBC system called quasi-orthogonal STBC [34, 35], which is structured to provide data rate 1 for complex signals when transmit antennas is four. The quasi-orthogonal STBC spreads independent Alamouti's codes in both space and time

Figure 5.1: Main layout of the combined system

domains so that the data rate of the code cannot be more than one; while in LA-STBC systems the data rate is the sum of code rate of the individual component codes.

This chapter is organized as follows. In Section 5.1, we introduce the symbol model of LA-STBC. The detection schemes are presented in Section 5.2, followed by performance analysis in Section 5.3. Finally, a summary is provided in Section 5.4. The results in this chapter are published in [Zhao1, Zhao2, Zhao5].

5.1 System Model

We first lay out the basic architecture of the LA-STBC system and the layered signal processing in Fig. 5.1. Consider a MIMO communication link consisting of M_t transmit antennas and M_r receive antennas ($M_r \geq M_t$). We use $\mathbf{H}$ to denote a $M_r \times M_t$ channel matrix. The channel propagation coefficients $h_{i,j}$ between each pair of transmit antenna and receive antenna are assumed to be i.i.d. complex Gaussian with zero mean and unit variance. The channel is assumed quasi-static fading (i.e., the fading coefficients are

constant during a frame, and change from one frame to another frame independently) [71]. It is assumed that perfect channel state information (CSI) is only known at the receiver. A sequence of n information bearing symbols $\{b_l\}_{l=1}^n$ drawn from a unit energy constellation is sent to a serial-to-parallel converter and multiplexed into q sub-groups, i.e., $\{b_l\}_{l=1}^{k_1}$, $\{b_l\}_{l=k_1+1}^{k_2}$, $\dots$, and $\{b_l\}_{l=k_{q-1}+1}^n$. Each group is sent to a space-time encoder, and the codeword is defined as a layer of the layered space-time coding system. The i -th layer is denoted by $\mathbf{S}_i$, which is associated with M_i transmit antennas and modulated over p symbol intervals. Therefore, $\mathbf{S}_i$ has dimension $M_i \times p$ and $M_1 + \dots + M_q = M_t$. The q space-time coded blocks are transmitted in parallel from all the M_t transmit antennas and the channel matrix can be seen as a collection of $M_r \times M_i$ matrix group, $\mathbf{H}_i$, $i \in [1, q]$:

$$\mathbf{H} = [\mathbf{H}_1 \quad \mathbf{H}_2 \quad \dots \quad \mathbf{H}_q] \quad (5.1)$$

We assume the channel is flat fading, so the baseband equivalent of the received signal is given by

$$\begin{aligned} \mathbf{Y} &= \mathbf{H}\mathbf{S} + \mathbf{N} \\ &= \sum_{i=1}^q \mathbf{H}_i \mathbf{S}_i + \mathbf{N}, \end{aligned} \quad (5.2)$$

where $\mathbf{S} = [\mathbf{S}_1^T, \dots, \mathbf{S}_q^T]^T$, and $\mathbf{N}$ denotes the noise matrix.

5.2 Detection for LA-STBC

The decoding schemes for LA-STBC are similar to the SM detection schemes. We first introduce the group receiver that separates the filtered version of codeword of each

layer.

5.2.1 MMSE Group Receiver

The group receiver is defined as a $M_r \times M_r$ matrix. In this case, the group receiver minimizes squared error between the output of the group receiver and the filtered version of the desired space-time coded block. For the k -th block, the group receiver $\mathbf{W}_k^{mmse}$ is given by the following cost function:

$$\begin{aligned} \mathbf{W}_k^{mmse} &= \operatorname{argmin}_{\mathbf{W}} E \{ \|\mathbf{W}\mathbf{Y} - \mathbf{H}_k\mathbf{S}_k\|^2 \} \\ &= E \{ \mathbf{Y}(\mathbf{H}_k\mathbf{S}_k)^H \} [E \{ \mathbf{Y}\mathbf{Y}^H \}]^{-1} \\ &= \mathbf{H}_k\mathbf{H}_k^H (\mathbf{H}\mathbf{H}^H + \sigma_n^2 \mathbf{I}_{M_r})^{-1} \end{aligned} \quad (5.3)$$

Assuming that the residual inter block interference is negligible in the output of the group receiver, then the output of the k -th group receiver only contains the desired block and additive Gaussian noise, i.e.,

$$\begin{aligned} \tilde{\mathbf{Y}}_k &= \mathbf{W}_k^{mmse} \mathbf{Y} \\ &= \mathbf{W}_k^{mmse} \mathbf{H}_k \mathbf{S}_k + \tilde{\mathbf{N}}_k, \quad 1 \leq k \leq M_t/2. \end{aligned} \quad (5.4)$$

where we define $\mathbf{H}_k^{vir} = \mathbf{H}_k$ to be the virtual channel matrix and $\mathbf{H}_k^{eq} = \mathbf{W}_k^{mmse} \mathbf{H}_k$ to be the equivalent channel matrix for space-time coded block $\mathbf{S}_k$, and $E \{ \tilde{\mathbf{N}}_k \tilde{\mathbf{N}}_k^H \} = p\sigma_n^2 \mathbf{W}_k^{mmse} \mathbf{W}_k^{mmseH}$. Hence, $\mathbf{S}_k$ can be detected by linear processing with ML detection.

5.2.2 ZF Group Receiver

In order to completely eliminate the interference among the space-time coded blocks, ZF group receivers are proposed, but the ZF group receivers will enhance the noise and result in performance degradation. The ZF group receiver is described as a matrix of dimension $M_t \times M_r$, that makes the channel matrix block diagonal, i.e.,

$$\mathbf{W}^{zf} \mathbf{H} = \begin{bmatrix} \mathbf{H}_1^{vir} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2^{vir} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_q^{vir} \end{bmatrix}, \quad (5.5)$$

where each block on the main diagonal of the above matrix is the virtual channel of the corresponding space-time coded block. If $\mathbf{W}^{zf} = \mathbf{H}^{-1}$, then $\mathbf{H}_i^{vir} = \mathbf{I}_{M_i}$. The virtual channel matrix of the space-time codeword is not unique and depends on the decomposition of the original channel matrix. For example, a 4×4 channel matrix can be decomposed into four matrices as

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}, \quad (5.6)$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are all 2×2 matrices. In fact, the dimension that denotes the number of receive antennas can be indefinite in the four submatrices, as long as the dimension indicating the number of transmit antennas remains 2. In order to make the

channel matrix block diagonal, the ZF group receiver takes the following form:

$$\mathbf{W}^{zf} = \begin{bmatrix} \mathbf{B}^{-1} & -\mathbf{D}^{-1} \\ \mathbf{A}^{-1} & -\mathbf{C}^{-1} \end{bmatrix}, \quad (5.7)$$

so that

$$\mathbf{W}^{zf}\mathbf{H} = \begin{bmatrix} \mathbf{B}^{-1}\mathbf{A} - \mathbf{D}^{-1}\mathbf{C} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{A}^{-1}\mathbf{B} - \mathbf{C}^{-1}\mathbf{D} \end{bmatrix}. \quad (5.8)$$

In this way, the space-time coded blocks are separated. $\mathbf{B}^{-1}\mathbf{A} - \mathbf{D}^{-1}\mathbf{C}$ is the virtual channel matrix for $\mathbf{S}_1$ and $\mathbf{A}^{-1}\mathbf{B} - \mathbf{C}^{-1}\mathbf{D}$ is the virtual channel matrix for $\mathbf{S}_2$. That is,

$$\tilde{\mathbf{Y}}_1 = (\mathbf{B}^{-1}\mathbf{A} - \mathbf{D}^{-1}\mathbf{C})\mathbf{S}_1 + \tilde{\mathbf{N}}_1 \quad (5.9)$$

$$\tilde{\mathbf{Y}}_2 = (\mathbf{A}^{-1}\mathbf{B} - \mathbf{C}^{-1}\mathbf{D})\mathbf{S}_2 + \tilde{\mathbf{N}}_2 \quad (5.10)$$

where the covariance matrix of $\tilde{\mathbf{N}}_1$ and $\tilde{\mathbf{N}}_2$ are $E\{\tilde{\mathbf{N}}_1\tilde{\mathbf{n}}_1^H\} = 2\sigma_n^2(\mathbf{B}^{-1}\mathbf{A} - \mathbf{D}^{-1}\mathbf{C})(\mathbf{B}^{-1}\mathbf{A} - \mathbf{D}^{-1}\mathbf{C})^H$ and $E\{\tilde{\mathbf{N}}_2\tilde{\mathbf{n}}_2^H\} = 2\sigma_n^2(\mathbf{A}^{-1}\mathbf{B} - \mathbf{C}^{-1}\mathbf{D})(\mathbf{A}^{-1}\mathbf{B} - \mathbf{C}^{-1}\mathbf{D})^H$ respectively.

5.2.3 QR Group Receiver and SIC

To avoid noise enhancement brought due to a ZF group receiver, we propose a unitary transformation based group receiver followed by a space-time decoder, which operates block by block in an SIC fashion.

We first perform a QR decomposition [64] on the channel matrix $\mathbf{H}$, i.e., $\mathbf{H} = \mathbf{QR}$,

where $\mathbf{Q}$ is a unitary matrix and $\mathbf{R}$ is an upper triangular matrix. Then the group receiver and the received signal after the group receiver are given by

$$\mathbf{W}^{qr} = \mathbf{Q}^H \quad (5.11)$$

$$\tilde{\mathbf{Y}} = \mathbf{R} \mathbf{S} + \tilde{\mathbf{N}} \quad (5.12)$$

Since $\mathbf{R}$ is upper triangular, it can be written as:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \cdots & \mathbf{R}_{1,q} \\ \mathbf{0} & \mathbf{R}_{22} & \cdots & \mathbf{R}_{2,q} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{R}_{q,q} \end{bmatrix} \quad (5.13)$$

and $\tilde{\mathbf{Y}}_k$ can be expressed as:

$$\tilde{\mathbf{Y}}_k = \mathbf{R}_{k,k} \mathbf{S}_k + \sum_{i=k+1}^q \mathbf{R}_{k,i} \mathbf{S}_i + \tilde{\mathbf{N}}_k \quad (5.14)$$

which contains only interference from $(q - k)$ blocks.

The last block $\tilde{\mathbf{Y}}_q$ contains no interference, and $\mathbf{R}_{q,q}$ is the virtual channel of block $\mathbf{S}_q$. A space-time decoder is applied to estimate this transmitted block first. The next coded block can be detected by subtracting the interference contribution from the q -th block $\mathbf{S}_q$ using the previous decision. This procedure is repeated until all the blocks are detected. The virtual channel matrix of $\mathbf{S}_k$ is $\mathbf{R}_{k,k}$ and $E\{\tilde{\mathbf{N}}_k \tilde{\mathbf{N}}_k^H\} = p\sigma_n^2$ for

Figure 5.2: BER performance comparison between the proposed detection schemes for 4×4 combined system and SM(2,4) system that randomly selects two transmit antennas with MMSE receiver, $r_s = 2$, $\rho = 0$.

$$k = 1, \dots, q.$$

5.2.4 Simulations

We consider a MIMO system with four transmit antennas and four receive antennas. The performance of the proposed detection schemes in terms of bit error rate (BER) is presented for the combined system. We compare the performances with a SM system that has the same spatial code rate as the combined system. We define the spatial code rate $r_s = N/T$, where N independent symbols are transmitted over T symbol periods,

Figure 5.3: BER performance of the proposed detection schemes for 4×4 combined system and SM(2,4) system as a function of correlation level ρ , SNR=15 dB

as the average number of independent symbols transmitted from all the antennas over unit symbol periods [71]. Throughout, BPSK modulated Alamouti's coding scheme over a quasi-static Rayleigh channel is used for the combined system. We assume that only transmit antennas exhibit correlation while receive antennas are in an uncorrelated environment. We investigate the impact of spatial correlation on the error rate performance of the combined system. The spatial correlation matrix is assumed to have an exponential form as (4.20).

Fig. 5.2 displays the BER performance of a 4×4 LA-STBC system using different detection schemes over an i.i.d. Rayleigh fading channel. We compare the performances

with that of a 4×4 SM system employing an MMSE receiver under the same channel condition. The data rate of the combined system is $r_s = 2$. For a fair comparison, the SM system randomly selects two out of four antennas to transmit independent signals for each channel realization, which is denoted as SM (2,4). Among all the detection schemes, the ZF group receiver has the worst performance with diversity order 3, which is the same as SM (2,4). The MMSE group receiver has slightly larger diversity order than the ZF group receiver. SIC brings more diversity order to the MMSE group receiver in the MMSE-SIC scheme. The QR group receiver with the SIC scheme has the largest diversity order of 4, which is the same as Alamouti's scheme. But the data rate of Alamouti's scheme is $r_s = 1$. According to the diversity-multiplexing tradeoff theory in [69], for the LA-STBC system, the maximum diversity gain is higher than a pure SM; while the maximum multiplexing gain is higher than a pure STBC system. Fig. 5.3 demonstrates the performance of the LA-STBC system and SM (2,4) as a function of the transmit correlation level ρ at the average SNR equal to 15 dB.

5.3 Performance Analysis

In this section, we give the exact performance analysis of the layered space-time decoding algorithm. Since the decisions fed back by the layers with lower diversity are not completely correct, the higher diversity promised for the following layers cannot be realized due to non-negligible interference. The residual interference caused by error propagation from the previous layers will be modeled as Gaussian variables in our

performance analysis as will be explained later. For simplicity, we assume that BPSK modulation is used and each group of bits is encoded by Alamouti's code [32] so that $p = M_i = 2$ ($1 \leq i \leq q$), and the codeword of the i -th layer may be expressed by:

$$\mathbf{S}_i = \begin{bmatrix} b_{2i-1} & b_{2i}^* \\ b_{2i} & -b_{2i-1}^* \end{bmatrix}. \quad (5.15)$$

Following the detection order, we calculate the BER from the first layer to the q -th layer. Suppose the virtual channel matrix for the first layer is $\mathbf{H}_1^{vir}$ with dimension $(M_r - M_t + M_1) \times M_1$. Suppose the entries of $\mathbf{H}_1^{vir}$ are $\{h_{i,j}^v\}_{i=1, j=1}^{M_1, M_r - M_t + M_1}$. The receiver applies a matched filter and the output of the matched filter for each bit b_l is:

$$r_l = \left(\sqrt{\sum_{i=1}^{M_1} \sum_{j=1}^{M_r - M_t + M_1} |h_{i,j}^v|^2} \right) b_l + \tilde{n}_l, \quad l = 1, 2 \quad (5.16)$$

where $\tilde{n}_l$ is complex Gaussian with zero mean and variance σ_n^2 . The instantaneous SNR per received bit is given by

$$\gamma = \left(\sum_{i=1}^{M_1} \sum_{j=1}^{M_r - M_t + M_1} |h_{i,j}^v|^2 \right) \rho \quad (5.17)$$

where $\rho = E_b/\sigma_n^2$ is the average SNR per bit and E_b is the energy of each bit. The error probability for deciding $b_{1,2}$ given a channel realization is $Q(\sqrt{2\gamma})$. The BER is obtained by averaging the bit error probability over the probability density function

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(pdf) of γ . The pdf of γ is a function of M_1 and $M_r - M_t + M_1$ and given by:

$$p(\gamma) = \left(\frac{\gamma}{\rho}\right)^{K-1} \cdot e^{-\frac{\gamma}{\rho}} \cdot \frac{1}{\Gamma(K)\rho} \quad (5.18)$$

with $K = M_1(M_r - M_t + M_1)$. It is a central χ^2 distribution with $2K$ degree of freedom.

Following the method of [72], the BER of b_l ($l = 1, 2$) is given by

$$\begin{aligned} P_{e,1} &= \int_0^\infty Q(\sqrt{2\gamma})p(\gamma)d\gamma \\ &= \frac{1}{2\sqrt{\pi}(1 + \rho g_{BPSK})^K} \frac{\Gamma(K + \frac{1}{2})}{\Gamma(K + 1)} \mathcal{H}\left(K, \frac{1}{2}; K + 1; \frac{1}{1 + \rho g_{BPSK}}\right), \end{aligned} \quad (5.19)$$

where $\mathcal{H}(a, b; c; x)$ is the Gauss hypergeometric function, $\Gamma(\cdot)$ is the gamma function, and $g_{BPSK} = \sin^2(\frac{\pi}{2}) = 1$. Since the bits in one layer have the same BER, let $P_{e,k}$ denote the BER of the k -th layer. Given $P_{e,1}$, we can calculate $P_{e,k}$ successively for $2 \leq k \leq q$ as will be elaborated in the following.

For a given k ($k > 1$), the interference matrix is the second term of (6.3). Suppose the estimate of b_l ($l = 2i - 1, 2i$) is $\hat{b}_l$ for the i -th layer. Then

$$\mathbf{S}_i - \hat{\mathbf{S}}_i = \begin{bmatrix} b_{2i-1} - \hat{b}_{2i-1} & b_{2i}^* - \hat{b}_{2i}^* \\ b_{2i} - \hat{b}_{2i} & -b_{2i-1}^* + \hat{b}_{2i-1}^* \end{bmatrix}. \quad (5.20)$$

In order to quantify the effect of interference, we assume equal probability of $\sqrt{E_b}$ and $-\sqrt{E_b}$. For any $l = 2i - 1, 2i$ ($1 \leq i \leq q - 1$), it is easy to verify that

$$E\{b_l - \hat{b}_l\} = 0 \quad (5.21)$$

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and

$$E\{|b_l - \hat{b}_l|^2\} = 4E_b, \quad \text{when } b_l \neq \hat{b}_l, \quad (5.22)$$

or

$$E\{|b_l - \hat{b}_l|^2\} = 0, \quad \text{when } b_l = \hat{b}_l. \quad (5.23)$$

The value of $E\{|b_l - \hat{b}_l|^2\}$ has a Bernoulli distribution with probability functions given by

$$\begin{aligned} Pr(E\{|b_l - \hat{b}_l|^2\} = 4E_b) &= P_{e,i}, \\ Pr(E\{|b_l - \hat{b}_l|^2\} = 0) &= 1 - P_{e,i}. \end{aligned} \quad (5.24)$$

Suppose the entries of $\mathbf{H}_i(\mathbf{S}_i - \hat{\mathbf{S}}_i)$ are $\{w_{s,t}^i\}_{s=1,t=1}^{M_r,2}$. Since $\mathbf{H}_i$ is independent of $\mathbf{S}_i - \hat{\mathbf{S}}_i$ and the elements of $\mathbf{H}_i$ are i.i.d. complex Gaussian variables with zero mean and variance 1, the entries of the interference matrix $\{w_{s,t}^i\}_{s=1,t=1}^{M_r,2}$ are Gaussian distributed with the mean and the variance given by

$$E\{w_{s,t}^i\} = 0, \quad (5.25)$$

$$E\{|w_{s,t}^i|^2\} = E\{|b_{2i-1} - \hat{b}_{2i-1}|^2\} + E\{|b_{2i} - \hat{b}_{2i}|^2\}. \quad (5.26)$$

Therefore, the total interference and noise power of the k -th layer is

$$E_{total}^k = \sum_{i=1}^{k-1} (E\{|b_{2i-1} - \hat{b}_{2i-1}|^2\} + E\{|b_{2i} - \hat{b}_{2i}|^2\}) + \sigma_n^2. \quad (5.27)$$

The average SNR per bit of the k -th layer is $\rho_k = \frac{E_b}{E_{total}^k}$.

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Let Ω be the set of values that ρ_k will take, so that

$$\Omega = \left\{ \frac{E_b}{\sigma_n^2}, \frac{E_b}{4E_b + \sigma_n^2}, \dots, \frac{E_b}{\sum_{i=1}^{k-1} k_i \cdot 4E_b + \sigma_n^2} \right\}. \quad (5.28)$$

Let $Pr(\rho_k = \omega)$ be the probability that ρ_k takes the value $\omega \in \Omega$, so that

$$\begin{aligned} & Pr\left(\rho_k = \frac{E_b}{j \cdot 4E_b + \sigma_n^2}\right) \\ &= \sum_{j_1 + \dots + j_{k-1} = j, 0 \leq j_i \leq k_i} \prod_{i=1}^{k-1} \binom{k_i}{j_i} (P_{e,i})^{j_i} (1 - P_{e,i})^{k_i - j_i} \end{aligned} \quad (5.29)$$

for $j = 0, \dots, \sum_{i=1}^{k-1} k_i$. The conditional error probability of the k -th layer at a given value of ρ_k is given by $P_{e,k}(b_l \neq \hat{b}_l | \rho_k)$, which is computed in the same way as for the first layer with $K = M_k(M_r - M_t + M_1 + \dots + M_k)$. The BER of the k -th layer is

$$P_{e,k} = \sum_{\omega \in \Omega} P_{e,k}(b_l \neq \hat{b}_l | \rho_k) Pr(\rho_k = \omega), \quad l = 2k - 1, 2k. \quad (5.30)$$

We calculate the BER of each layer from $P_{e,1}$ to $P_{e,q}$ successively, and therefore, the overall BER P_e is obtained by averaging across all the layers, $P_e = \frac{1}{q} \sum_{k=1}^q P_{e,k}$.

We consider a MIMO system with six transmit antennas and six receive antennas, denoted by 6×6 , where information bits are modulated using BPSK and each layer is encoded by Alamouti's code. The channel is assumed to be i.i.d. quasi-static Rayleigh fading. The BER performance of all three layers based on the SIC detection scheme are plotted in Fig. 5.4. We also plot the corresponding analytical performance curves for the

Figure 5.4: Exact BER performance of each layer for 6×6 combined system

three layers. It is shown that the theoretical results agree with the simulation results. For comparison, the simulation curves for all the layers assuming perfect interference cancellation are also provided, where the diversity order for the three layers are 4, 8 and 12, respectively.

5.4 Conclusions

We have presented detection schemes for the LA-STBC system that employ Alamouti's STBC as the component codeword at each layer. Given the same data rate, the LA-

STBC system achieves larger diversity order than the SM system. Among the detection schemes, the QR-SIC scheme achieves the same diversity order as the STBC system, but has higher data data rate than the STBC system. The advantages of LA-STBC have made it a promising space-time architecture for future applications. We observe from the performance analysis when imperfect decisions are fed back, the performance of the later detected layers of large diversity order are severely degraded. The overall system performance is limited by the first layer, which has the lowest diversity order. The significant gap between ideal performance assuming perfect interference cancellation and the actual performance with error propagation indicates the room for improvement.

Chapter 6

Improved Techniques for Layered Space-Time Block Coding

In the last chapter, we introduced the basic framework of the LA-STBC system that can achieve both the spectral efficiency gain and the diversity gain, and at the same time keep the computational complexity low. It has been seen that in the LA-STBC system, the diversity order of an earlier detection stage is less than that of a later one. Considering this characteristic and the effect of error propagation, it is apparent that the overall system performance is limited by the earlier detection stages.

Motivated by the similarities between the LA-STBC system and V-BLAST in terms of layered structure and detection algorithms, in this chapter, we make several attempts to improve the original LA-STBC system based on successive interference cancellation (SIC) decoding schemes. These approaches include optimal detection ordering, trans-

mit power allocation and iterative extension of the SIC decoding algorithm. We first consider the detection order by computing the post processing SNR of each layer. Although detection ordering can mitigate this problem, it may not sufficiently compensate for the diversity loss. Hence, transmit power allocation for LA-STBC is considered to further improve the performance of early detection stages and enhance the overall performance. Transmit power allocation has been investigated for V-BLAST in [73, 74]. In [38], a power allocation scheme is presented independent of channel information using curve fitting techniques for the layered space-time trellis coding system. In our research, the transmit power is determined at the receiver for a given channel realization and transferred to the transmitter. It is adapted according to the channel condition, which necessitates a feedback mechanism. However, the feedback overhead needed for transmit power allocation is relatively small, compared to the system with instantaneous channel state information (I-CSI) feedback. We also observe that the decisions of the last decoded layer have the highest reliability. Utilizing the estimates with higher quality to decode the layer with lower quality is another way to enhance the overall performance.

The remainder of this chapter is organized as follows. System model and decoding schemes based on SIC are described in Section 6.1. We determine the optimal detection order for each channel realization in Section 6.2, followed by iterative decoding and transmit power allocation scheme in Section 6.3 and Section 6.4. Numerical results and conclusions are provided in Section 6.5 and Section 6.6. The results in this chapter are published in [Zhao3].

6.1 System Model and Decoding Schemes

6.1.1 System Model

The system model has been introduced in Chapter 5. In this chapter, we consider the power allocation. The baseband equivalent of the received signal is modified to

$$\mathbf{Y} = \mathbf{H} \mathbf{F} \mathbf{S} + \mathbf{N}, \quad (6.1)$$

where the transmit power allocation is denoted by a diagonal matrix

$$\mathbf{F} = \text{diag}(\sqrt{P_1/M_1} \mathbf{I}_{M_1}, \sqrt{P_2/M_2} \mathbf{I}_{M_2}, \dots, \sqrt{P_q/M_q} \mathbf{I}_{M_q}), \quad (6.2)$$

and the total transmit power is constrained by $\sum_{i=1}^q P_i = M_t$.

The decoding scheme for layered space-time coding system is similar to the V-BLAST decoding algorithm that detects each layer based on SIC. The difference is that each layer is a space-time codeword and we introduce group interference nulling and group interference cancellation to detect the codeword of each layer as in Fig. 6.1.

In the layered space-time decoding scheme, we incorporate detection ordering and transmit power allocation to the initial decoding scheme. Suppose the processing for each layer follows the order $\{l_1, l_2, \dots, l_q\}$, which is a permutation of $\{1, 2, \dots, q\}$. For a given k , we assume that the previous $k - 1$ layers have been decoded. Let $\hat{\mathbf{S}}_{l_i}$ be the

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Figure 6.1: Decoding procedure for the layered space-time coding system

estimate of $\mathbf{S}_{l_i}$ where $1 \leq i \leq k-1$. We reconstruct the signal due to $\mathbf{S}_{l_i}$ and subtract it from the received signal $\mathbf{Y}$ of (6.1) as

$$\begin{aligned} \mathbf{Y}^{(k)} &= \mathbf{Y} - \sum_{i=1}^{k-1} \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} \hat{\mathbf{S}}_{l_i} \\ &= \sum_{i=k}^q \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} \mathbf{S}_{l_i} + \sum_{i=1}^{k-1} \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} (\mathbf{S}_{l_i} - \hat{\mathbf{S}}_{l_i}) + \mathbf{N} \end{aligned} \quad (6.3)$$

where $\mathbf{Y}^{(k)}$ is the updated received signal after group interference cancellation from the decoded $k-1$ layers. We also define $\mathbf{H}^{(k)}$ to be the updated channel matrix at the k -th detection stage as

$$\mathbf{H}^{(k)} = [\mathbf{H}_{l_k} \quad \mathbf{H}_{l_{k+1}} \quad \cdots \quad \mathbf{H}_{l_q}] \quad (6.4)$$

for $1 \leq k \leq q-1$. Two group receivers, namely, a group ZF receiver and a group MMSE receiver, are applied to suppress the interference from the undetected layers.

6.1.2 Group ZF-SIC Decoding Scheme

From (6.3), we see that $\mathbf{S}_{l_k}$ is in the direction of $\mathbf{H}_{l_k}$ and the interference towards $\mathbf{S}_{l_k}$ lies in the linear space spanned by $\mathbf{H}_{l_{k+1}}, \dots, \mathbf{H}_{l_q}$. A group interference nulling filter is needed to eliminate the interference from block $\mathbf{S}_{l_{k+1}}, \dots, \mathbf{S}_{l_q}$. If we define $\mathbf{H}_{\bar{l}_k}^{(k)}$ as

$$\mathbf{H}_{\bar{l}_k}^{(k)} = [\mathbf{H}_{l_{k+1}} \quad \mathbf{H}_{l_{k+2}} \quad \dots \quad \mathbf{H}_{l_q}] \quad (6.5)$$

for $1 \leq k \leq q-1$, the ZF group receiver for $\mathbf{S}_{l_k}$ should lie in the null space of $\mathbf{H}_{\bar{l}_k}^{(k)}$. It is satisfied when $\text{rank}(\mathbf{H}_{\bar{l}_k}^{(k)}) < M_r$. This defines the dimension condition necessary to guarantee that the filtered version of $\mathbf{S}_{l_k}$ can be separated under the zero-interference constraints, i.e., $M_r > M_t - M_1$. Thus, it is theoretically possible to support some situations where $M_t > M_r$. Assuming the dimension condition is satisfied and $\bar{L}_{l_k} = \text{rank}(\mathbf{H}_{\bar{l}_k}^{(k)})$, the singular value decomposition for $\mathbf{H}_{\bar{l}_k}^{(k)}$ is

$$\mathbf{H}_{\bar{l}_k}^{(k)} = [\bar{\mathbf{U}}_{l_k} \quad \bar{\mathbf{U}}_{l_k}^{(0)}] \bar{\Sigma}_{l_k} \bar{\mathbf{V}}_{l_k}^H \quad (6.6)$$

where $\bar{\mathbf{U}}_{l_k}^{(0)}$ is composed of the last $(M_r - \bar{L}_{l_k})$ left singular vectors that forms an orthogonal basis for the null space of $\mathbf{H}_{\bar{l}_k}^{(k)}$. Therefore, $[\bar{\mathbf{U}}_{l_k}^{(0)}]^H \mathbf{H}_{\bar{l}_k}^{(k)} = \mathbf{0}$. The columns of $\bar{\mathbf{U}}_{l_k}^{(0)}$ can be used as the ZF group receiver for the l_k -th layer as

$$\mathbf{W}_{l_k}^{zf} = [\bar{\mathbf{U}}_{l_k}^{(0)}]^H, \quad k = 1, \dots, q-1. \quad (6.7)$$

The output of the ZF group receiver is given by

$$\begin{aligned} \mathbf{Y}_{l_k} &= \mathbf{W}_{l_k}^{zf} \mathbf{Y}^{(k)} \\ &= \sqrt{\frac{P_{l_k}}{M_{l_k}}} \mathbf{W}_{l_k}^{zf} \mathbf{H}_{l_k} \mathbf{S}_{l_k} + \mathbf{W}_{l_k}^{zf} \sum_{i=1}^{k-1} \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} (\mathbf{S}_{l_i} - \hat{\mathbf{S}}_{l_i}) + \mathbf{W}_{l_k}^{zf} \mathbf{n}, \end{aligned} \quad (6.8)$$

$$k = 1, 2, \dots, q-1.$$

The filtered version of codeword $\mathbf{S}_{l_k}$ is extracted and its equivalent channel matrix is given by

$$\mathbf{H}_{l_k}^{eq} = \mathbf{W}_{l_k}^{zf} \mathbf{H}_{l_k}, \quad k = 1, \dots, q-1. \quad (6.9)$$

When $k = q$, the group ZF receiver and equivalent channel matrix of the l_q -th layer are $\mathbf{W}_{l_q}^{zf} = \mathbf{I}_{M_r}$ and $\mathbf{H}_{l_q}^{eq} = \mathbf{H}_{l_q}$, respectively. The second term of (6.8) is the interference matrix due to imperfect interference cancellation. The covariance matrix of the equivalent noise matrix is $E\{\mathbf{W}_{l_k}^{zf} \mathbf{N} \mathbf{N}^H \mathbf{W}_{l_k}^{zfH}\} = p \cdot \sigma_n^2 \mathbf{I}_{M_r - \bar{L}_{l_k}}$. It is seen that each entry of the equivalent noise matrix is i.i.d. complex Gaussian variables with zero mean and variance σ_n^2 .

6.1.3 Group MMSE-SIC Decoding Scheme

The MMSE group receiver is defined as an $M_r \times M_r$ matrix. Unlike the ZF group receiver that nulls the interference of the undetected layers completely with variable dimension of the equivalent channel matrix at each detection stage, the MMSE group receiver minimizes the mean square error between the output of the group receiver and the filtered version of the desired space-time codeword. The MMSE group receiver

6.1. System Model and Decoding Schemes

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does not change the dimension of the original channel matrix of each layer. The MMSE group receiver for the l_k -th layer is given by the following cost function:

$$\begin{aligned}\mathbf{W}_{l_k}^{mmse} &= \arg \min_{\mathbf{W}} E \left\{ \left\| \mathbf{W} \mathbf{Y}^{(k)} - \sqrt{\frac{P_{l_k}}{M_{l_k}}} \mathbf{H}_{l_k} \mathbf{S}_{l_k} \right\|_F^2 \right\} \\ &= \frac{P_{l_k}}{M_{l_k}} \mathbf{H}_{l_k} \mathbf{H}_{l_k}^H \left(\mathbf{H}^{(k)} \mathbf{H}^{(k)H} + \sigma_n^2 \mathbf{I}_{M_r} \right)^{-1}, \\ &k = 1, 2, \dots, q-1.\end{aligned}\quad (6.10)$$

The output of the MMSE group receiver for the l_k -th layer is given by

$$\begin{aligned}\mathbf{Y}_{l_k} &= \mathbf{W}_{l_k}^{mmse} \mathbf{Y}^{(k)} \\ &= \sqrt{\frac{P_{l_k}}{M_{l_k}}} \mathbf{W}_{l_k}^{mmse} \mathbf{H}_{l_k} \mathbf{S}_{l_k} + \mathbf{W}_{l_k}^{mmse} \sum_{i=k+1}^q \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} \mathbf{S}_{l_i} \\ &\quad + \mathbf{W}_{l_k}^{mmse} \sum_{i=1}^{k-1} \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} (\mathbf{S}_{l_i} - \hat{\mathbf{S}}_{l_i}) + \mathbf{W}_{l_k}^{mmse} \mathbf{N}, \\ &k = 1, 2, \dots, q-1.\end{aligned}\quad (6.11)$$

When $k = q$, the MMSE group receiver of the l_q -th layer is given by $\mathbf{W}_{l_q}^{mmse} = \mathbf{I}_{M_r}$. By definition of the MMSE group receiver $\mathbf{W}_{l_k}^{mmse}$, the equivalent channel matrix for codeword $\mathbf{S}_{l_k}$ is

$$\mathbf{H}_{l_k}^{eq} = \mathbf{W}_{l_k}^{mmse} \mathbf{H}_{l_k}, \quad k = 1, \dots, q. \quad (6.12)$$

It is seen from (6.11) that the output of the MMSE group receiver consists of the filtered version of the desired codeword, the inter-layer interference, error propagation caused by the imperfect decision feedback, and additive Gaussian noise. The covariance matrix of the equivalent noise matrix is $E\{\mathbf{W}_{l_k}^{mmse} \mathbf{N} \mathbf{N}^H \mathbf{W}_{l_k}^{mmseH}\} = p \cdot \sigma_n^2 \mathbf{W}_{l_k}^{mmse} \mathbf{W}_{l_k}^{mmseH}$.

Suppose the filtered version of the space-time codeword $\mathbf{S}_{l_k}$ is extracted and the

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previous $k - 1$ layers are perfectly detected. For the sake of exposition, we assume the space-time encoder of each layer uses Alamouti's scheme [32], so that $M_{l_k} = 2$ ($1 \leq k \leq q$), and the codeword of the l_k -th layer may be expressed by

$$\mathbf{S}_{l_k} = \begin{bmatrix} b_{2l_k-1} & b_{2l_k}^* \\ b_{2l_k} & -b_{2l_k-1}^* \end{bmatrix}. \quad (6.13)$$

The structure of the transmitted signal is such that $\mathbf{y}_{l_k}$ may be expressed as

$$\begin{bmatrix} \mathbf{Y}_{l_k}(:, 1) \\ \mathbf{Y}_{l_k}(:, 2)^* \end{bmatrix} = \sqrt{\frac{P_{l_k}}{2}} \begin{bmatrix} \mathbf{H}_{l_k}^{eq}(:, 1) & \mathbf{H}_{l_k}^{eq}(:, 2) \\ -\mathbf{H}_{l_k}^{eq}(:, 2)^* & \mathbf{H}_{l_k}^{eq}(:, 1)^* \end{bmatrix} \begin{bmatrix} b_{2l_k-1} \\ b_{2l_k} \end{bmatrix} + \begin{bmatrix} \mathbf{N}_{l_k}(:, 1) \\ \mathbf{N}_{l_k}(:, 2)^* \end{bmatrix} \quad (6.14)$$

where $\mathbf{N}_{l_k}$ is the total of noise and interference at $\mathbf{Y}_{l_k}$. We use notation $\mathbf{A}^{eff}$ to denote the effective channel matrix of channel $\mathbf{A}$, which is given by

$$\mathbf{A}^{eff} = \begin{bmatrix} \mathbf{A}(:, 1) & \mathbf{A}(:, 2) \\ -\mathbf{A}(:, 2)^* & \mathbf{A}(:, 1)^* \end{bmatrix}. \quad (6.15)$$

We note that each effective channel is rendered orthogonal regardless of the channel realization.

For group ZF-SIC decoding scheme, $\mathbf{N}_{l_k} = \mathbf{W}_{l_k}^{zf} \mathbf{N}$, and b_{2l_k-1} and b_{2l_k} can be detected by left multiplying (6.14) by $(\mathbf{H}_{l_k}^{eff})^H$ as [71]

$$\mathbf{r}_{l_k} = \sqrt{\frac{P_{l_k}}{2}} (\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq, eff} [b_{2l_k-1} \ b_{2l_k}]^T + \tilde{\mathbf{n}}_{l_k}. \quad (6.16)$$

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Since $\mathbf{H}_{l_k}$ and $\mathbf{H}_{l_k}^{eq}$ are identical in this case, we have $(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq,eff} = \|\mathbf{H}_{l_k}\|_F^2 \mathbf{I}_2$, and $E\{\tilde{\mathbf{n}}_{l_k} \tilde{\mathbf{n}}_{l_k}^H\} = \|\mathbf{H}_{l_k}\|_F^2 \sigma_n^2 \mathbf{I}_2$. The corresponding received SNR is given by

$$\gamma_{l_k} = \frac{P_{l_k} \|\mathbf{H}_{l_k}\|_F^2}{2E_{n,l_k}}, \quad (6.17)$$

where $E_{n,l_k} = \sigma_n^2$.

For group MMSE-SIC decoding scheme, $\mathbf{H}_{l_k}$ and $\mathbf{H}_{l_k}^{eq}$ are different, and (6.16) should be modified to

$$\begin{aligned} \mathbf{r}_{l_k} &= \sqrt{\frac{P_{l_k}}{2}} (\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq,eff} [b_{2l_k-1} \ b_{2l_k}]^T \\ &+ (\mathbf{H}_{l_k}^{eff})^H \sum_{i=k+1}^q \sqrt{\frac{P_{l_i}}{2}} \mathbf{H}_{l_i}^{eq,eff} [b_{2l_i-1} \ b_{2l_i}]^T + \tilde{\mathbf{n}}_{l_k}, \end{aligned} \quad (6.18)$$

where $\mathbf{H}_{l_i}^{eq} = \mathbf{W}_{l_k}^{mmse} \mathbf{H}_{l_i}$. Hence, the received SNR for the bits of the l_k -th layer is given by

$$\gamma_{l_k,1} = \frac{\frac{P_{l_k}}{2} \left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq,eff} \right]_{11} \right|^2}{E_{int,l_k,1} + E_{n,l_k,1}}, \quad (6.19)$$

$$\gamma_{l_k,2} = \frac{\frac{P_{l_k}}{2} \left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq,eff} \right]_{22} \right|^2}{E_{int,l_k,2} + E_{n,l_k,2}}, \quad (6.20)$$

where

$$\begin{aligned} E_{int,l_k,1} &= \\ \frac{P_{l_k}}{2} \left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq,eff} \right]_{12} \right|^2 &+ \sum_{i=k+1}^q \frac{P_{l_i}}{2} \left(\left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_i}^{eq,eff} \right]_{11} \right|^2 + \left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_i}^{eq,eff} \right]_{12} \right|^2 \right), \end{aligned} \quad (6.21)$$

$$E_{int,l_k,2} = \frac{P_{l_k}}{2} \left(\left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_k}^{eq,eff} \right]_{21} \right|^2 + \sum_{i=k+1}^q \frac{P_{l_i}}{2} \left(\left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_i}^{eq,eff} \right]_{21} \right|^2 + \left| \left[(\mathbf{H}_{l_k}^{eff})^H \mathbf{H}_{l_i}^{eq,eff} \right]_{22} \right|^2 \right) \right), \quad (6.22)$$

and

$$E_{n,l_k,1} = E_{n,l_k,2} = \sigma_n^2 \sum_{i=1}^2 \sum_{j=1}^{M_r} |\mathbf{H}_{l_k}^H(i, :) \mathbf{W}_{l_k}^{mmse}(:, j)|^2. \quad (6.23)$$

We can prove that $E_{int,l_k,1}$ and $E_{int,l_k,2}$ are equal. Therefore, the interference and noise power for the l_k -th layer can be denoted by E_{int,l_k} and E_{n,l_k} , and the received SNR for the l_k -th layer is denoted by γ_{l_k} .

6.2 Detection Order of Two Decoding Schemes

In the previous section, we described the group ZF-SIC and group MMSE-SIC decoding algorithms when the decoding order is predetermined. In the following, we will discuss how to determine the decoding order based on the channel realizations at the receiver. At the first detection stage, let $\mathbf{H}^{(1)} = \mathbf{H}$ denote the channel matrix and $S^{(1)} = \{1, 2, \dots, q\}$ be the set consisting of indexes of the undetected layers at the first detection stage. For a given k , the channel matrix at the k -th detection stage is updated as $\mathbf{H}^{(k)} = \mathbf{H}_{l_{k-1}}^{(k-1)}$ by eliminating $\mathbf{H}_{l_{k-1}}$ from $\mathbf{H}^{(k-1)}$. And $S^{(k-1)}$ is updated accordingly by deleting l_{k-1} from $S^{(k-1)}$, and is denoted by $S^{(k)} = S_{l_{k-1}}^{(k-1)}$.

As the post processing SNR defined on the layer is proportional to the squared Frobenius norm of the virtual matrix as shown in (6.17). The layer to be detected at

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the k -th stage should have the largest squared Frobenius norm of the virtual channel matrix and is given as

$$l_k = \arg \max_{j \in S^{(k)}} \|\mathbf{H}_j\|_F^2. \quad (6.24)$$

The modified algorithm of the group ZF-SIC decoding scheme can be given as follows.

initialization:

$$i \leftarrow 1 \quad (6.25)$$

$$\mathbf{Y}^{(1)} = \mathbf{Y} \quad (6.26)$$

$$\mathbf{H}^{(1)} = \mathbf{H} \quad (6.27)$$

$$S^{(1)} = \{1, \dots, q\} \quad (6.28)$$

$$l_1 = \arg \max_{j \in S^{(1)}} \|\mathbf{H}_j\|_F^2 \quad (6.29)$$

Recursion:

$$\mathbf{Y}_{k_i} = \mathbf{W}_{k_i}^{zf} \mathbf{Y}^{(i)} \quad (6.30)$$

$$\hat{\mathbf{S}}_{k_i} = D(\mathbf{Y}_{k_i}) \quad (6.31)$$

$$\mathbf{Y}^{(i+1)} = \mathbf{Y}^{(i)} - \sqrt{\frac{P_{l_i}}{M_{l_i}}} \mathbf{H}_{l_i} \hat{\mathbf{S}}_{l_i} \quad (6.32)$$

$$\mathbf{H}^{(i+1)} = \mathbf{H}_{\bar{k}_i}^{(i)} \quad (6.33)$$

$$S^{(i+1)} = S_{\bar{l}_i}^{(i)} \quad (6.34)$$

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$$l_{i+1} = \arg \max_{j \in S^{(i+1)}} \|\mathbf{H}_j\|_F^2 \quad (6.35)$$

$$i \leftarrow i + 1 \quad (6.36)$$

where $D(\cdot)$ is the space-time decoder associated with ML decoding. In the case of the MMSE group receiver, the post processing SNR on the defined layer is not proportional to the squared Frobenius norm of the virtual channel matrix and can be calculated by (6.23). For a given k , let γ_{l_j} represents the post processing SNR of the l_j -th layer where $j \in S^{(k)}$, then (6.24) should be changed to

$$l_k = \arg \max_{j \in S^{(k)}} \gamma_{l_j}, \quad (6.37)$$

where $\mathbf{W}_{l_j}^{mmse}$ is required to get γ_{l_j} . When calculating $\mathbf{W}_{l_j}^{mmse}$ using (6.10), $\mathbf{H}^{(k)}$ needs to be modified as

$$\mathbf{H}^{(k)} = [\mathbf{H}_{l_j} \quad \mathbf{H}_{l_j}^{(k)}], \quad (6.38)$$

and $\mathbf{H}_{l_j}$, $\mathbf{H}_{l_j}^{eq}$, and $\mathbf{H}_{l_i}^{eq} (i \in S^{(k)}, i \neq j)$ are obtained in the similar way as described in Section 6.1.3, where detection order is predetermined. The detection process for the group MMSE-SIC algorithm is the same as that for the group ZF-SIC algorithm in (6.25)–(6.36), except for (6.29) and (6.35) changed to (6.37) with $k = 1$ and $k = i + 1$ accordingly.

6.3 Iterative Decoding Schemes

The imperfect decision feedback in the successive decoding schemes, namely, group ZF-SIC and group MMSE-SIC, cause error propagation that becomes a bottleneck in overall performance. Dynamic ordering is an effective method to overcome the bottleneck. However, there exists another dimension that can be exploited to improve performance. Despite the diversity order is limited by that of the first detected layer, the last detected layer is relatively more reliable than any other layers. A simple way to improve system performance is to subtract the contribution of the last decoded layer from the received signal after an initial application of the original algorithm. This forms a feedback flow in the reversed order, which can be seen as an iteration of the SIC algorithm. Each iteration is composed of q levels, and each level is processed successively at the same order as in the original algorithm.

To evaluate the performance of the iterative decoding algorithm, we employ BPSK modulation and Alamouti's code as the component code at each layer. We use notation $M_t \times M_r$ to represent a MIMO configuration with M_t transmit antennas and M_r receive antennas. Fig. 6.2 and Fig. 6.3 demonstrate the performance improvement of the iterative decoding schemes with the group ZF-SIC and the group MMSE-SIC algorithms, respectively, for a 6×6 LA-STBC system. We vary the iteration depth from 1 to 2. The performances of the original algorithms without iterative decoding are included as references in both figures. As evident from the figures, large performance gains are achieved by using iterative decoding for both algorithms. The original algorithms with

Figure 6.2: Performance of iterative group ZF-SIC decoding scheme for 6×6 MIMO system

detection ordering are simulated as comparison. For the group ZF-SIC receiver, the iterative decoding outperforms the detection ordering, which means it can bring more gains than detection ordering in compensating for the loss of imperfect decision feedback. While for the group MMSE-SIC algorithm, the iterative decoding cannot offer as much benefit as the detection ordering.

Figure 6.3: Performance of iterative group MMSE-SIC decoding scheme for 6×6 MIMO system

6.4 Transmit Power Allocation

As the overall system performance is limited by the first detected layer, allocating more power to the earlier detected layers and less power to the later detected layers seems a straightforward way to improve the performance. The unequal power allocation strategy was first adopted by Tarokh in [37] for a layered space-time trellis coded system, where the power allocated to a layer is half of that to the layer right before it along the detection order, i.e., $P_{l_k} = 2P_{l_{k+1}}$. This scheme is novel but has not been optimized.

In this section, we will derive a transmit power allocation scheme for each channel realization that minimizes the BER, which is motivated by the idea of [73] for V-BLAST system. We first express the BER of each layer as a function of transmit power.

In group ZF-SIC and group MMSE-SIC decoding schemes, the residual interference caused by error propagation from the previous layers will be modeled as Gaussian variables. Following the detection order $\{l_1, l_2, \dots, l_q\}$, we calculate the bit error rate (BER) from the l_1 -th layer to the l_q -th layer. The exact BER expression of SIC based detection schemes with BPSK modulation for LA-STBC system has been proposed in Chapter 5.

In calculating the BER of the l_k -th layer, we let Ω be the set of values that σ_{n,l_k}^2 will take, so that

$$\Omega = \left\{ \sigma_n^2, 2P_{l_1} + \sigma_n^2, 2P_{l_2} + \sigma_n^2, \dots, \sum_{i=1}^{k-1} 4P_{l_i} + \sigma_n^2 \right\} \quad (6.39)$$

Let $Pr(\sigma_{n,l_k}^2 = \omega)$ be the probability that σ_{n,l_k}^2 takes the value $\omega \in \Omega$ and $P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = \omega)$ be the conditional bit error probability of the l_k -th layer at a given value of $\sigma_{n,l_k}^2 = \omega$.

If the cancellation of previously detected $k-1$ layers are perfect, the bit error probability of the l_k -th layer will be given as

$$P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = \sigma_n^2) Pr(\sigma_{n,l_k}^2 = \sigma_n^2) = P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = \sigma_n^2) \prod_{i=1}^{k-1} (1 - P_{e,i})^2 \quad (6.40)$$

When only one bit is erroneously detected for the previous $k-1$ detection stages, the

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bit error probability of the l_k -th layer is given by

$$\begin{aligned} & \sum_{i=1}^{k-1} P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = 2P_{l_i} + \sigma_n^2) Pr(\sigma_{n,l_k}^2 = 2P_{l_i} + \sigma_n^2) \\ &= \sum_{i=1}^{k-1} P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = 2P_{l_i} + \sigma_n^2) 2P_{e,i} (1 - P_{e,i}) \prod_{j \neq i, 1 \leq j \leq k-1} (1 - P_{e,j})^2 \end{aligned} \quad (6.41)$$

The probability when more than one bits are erroneously detected in the previous $k - 1$ detection stages results in higher order terms of $P_{e,i}$, and will not be taken into consideration for calculating BER of the k -th detection stage, therefore, the BER of the k -th stage is given by

$$\begin{aligned} P_{e,k} &= \sum_{\omega \in \Omega} P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = \omega) Pr(\sigma_{n,l_k}^2 = \omega) \\ &\approx P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = \sigma_n^2) + \sum_{i=1}^{k-1} P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = 2P_{l_i} + \sigma_n^2) 2P_{e,i} \end{aligned} \quad (6.42)$$

The approximation in (6.42) results from neglecting second and higher order terms of $P_{e,i}$. If we assume the error propagation from the $k - 1$ previous layers given one bit is erroneously detected has the same effect on the l_k -th layer, the conditional bit error probability of the k -th detection stage will be given as

$$P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = 2P_{l_i} + \sigma_n^2) = P_{e,k}(\gamma_{l_k} | \sigma_{n,l_k}^2 = 4 + \sigma_n^2), \quad i = 1, \dots, k-1, \quad (6.43)$$

where the equation exists when $P_{l_i} = M_{l_i} = 2$. The overall BER P_e is obtained by averaging across all the layers as

$$\begin{aligned} P_e &= \frac{1}{q} \sum_{i=1}^q P_{e,i} \\ &= \frac{1}{q} \sum_{i=1}^q \left(\prod_{j=i+1}^q \left(1 + 2P_{e,j}(\gamma_{l_j} | \sigma_{n,l_j}^2 = 4 + \sigma_n^2) \right) \right) \cdot P_{e,i}(\gamma_{l_i} | \sigma_{n,l_i}^2 = \sigma_n^2). \end{aligned} \quad (6.44)$$

In the following, we will derive the transmit power allocation scheme that minimizes the BER of group ZF-SIC and group MMSE-SIC decoding schemes. For simplicity, we assume the conditional bit error probability $P_{e,j}(\gamma_{l_j}|\sigma_{n,l_j}^2 = 4 + \sigma_n^2)$ does not depend on the transmit power and define it to be α_j .

We use the Lagrange multiplier to find the transmit power $\{E_{l_i}\}$ that minimizes the overall BER in (6.44) under the total transmit power constraint. The cost function may be expressed as

$$J(P_{l_1}, P_{l_2}, \dots, P_{l_q}) = P_e + \lambda \left(\sum_{i=1}^q P_{l_i} - M_t \right) \quad (6.45)$$

where λ is the Lagrange multiplier.

Letting $\partial J / \partial P_{l_i} = 0$, a set of equations is given by

$$\frac{dP_{e,i}(\gamma_{l_i}|\sigma_{n,l_i}^2 = \sigma_n^2)}{dP_{l_i}} = -\frac{q\lambda}{\prod_{j=i+1}^q (1 + 2\alpha_j)}, \quad i = 1, 2, \dots, q. \quad (6.46)$$

In the case of the group ZF-SIC decoding scheme, the total of noise and the interference due to the imperfect decision feedback is Gaussian distributed, and the bit error probability for a given γ_{l_i} is found as

$$P_{e,i}(\gamma_{l_i}) = Q(\sqrt{2\gamma_{l_i}}), \quad i = 1, 2, \dots, q, \quad (6.47)$$

and the BER expression of BPSK modulated symbols can be tightly approximated by

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an exponential function of γ_{l_i} as

$$P_{e,i}(\gamma_{l_i}) \approx 0.2 \exp(-1.04\gamma_{l_i}). \quad (6.48)$$

By curve fitting techniques, the bit error probability of the group MMSE-SIC decoding scheme for a given γ_{l_i} can also be approximated by an exponential function of γ_{l_i} as

$$P_{e,i}(\gamma_{l_i}) \approx 0.2 \exp(-0.56\gamma_{l_i}). \quad (6.49)$$

By the similarity between (6.48) and (6.49), we define the bit error probability of both decoding schemes as

$$P_{e,i}(\gamma_{l_i}) = c_1 \exp(-c_2 P_{l_i} \beta_{l_i}). \quad (6.50)$$

Therefore, (6.46) can be rewritten as

$$-c_1 c_2 \beta_{l_i} \exp(-c_2 P_{l_i} \beta_{l_i}) = -\frac{q\lambda}{\prod_{j=i+1}^q (1 + 2\alpha_j)}, \quad i = 1, 2, \dots, q. \quad (6.51)$$

By solving q simultaneous equations of (6.51) and the power constraint, we can find

the solution for the transmit power as

$$P_{l_i} = -\frac{1}{c_2\beta_{l_i}} \ln \left(\frac{q\lambda}{c_1c_2\beta_{l_i} \prod_{j=i+1}^q (1+2\alpha_j)} \right), \quad i = 1, 2, \dots, q,$$

$$\lambda = \exp \left(-\frac{M_t c_2 + \sum_{i=1}^q \frac{1}{\beta_{l_i}} \ln \frac{q}{c_1c_2\beta_{l_i} \prod_{j=i+1}^q (1+2\alpha_j)}}{\sum_{i=1}^q \frac{1}{\beta_{l_i}}} \right). \quad (6.52)$$

As shown in (6.52), β_{l_i} are required in calculating P_{l_i} , which entails computing the virtual channel matrix $\mathbf{H}_{l_i}$ ($i = 1, 2, \dots, q$) and the equivalent channel matrix $\mathbf{H}_{l_i}^{eq}$ ($i = 1, 2, \dots, q$). Note that the conditional probability α_j needs to be computed in order to obtain the transmit power. If we set $\alpha_j = 0$ ($j = 2, 3, \dots, q$) to ignore error propagations in the overall BER equation in (6.44), the transmit power allocation may be simplified to

$$P_{l_i} = -\frac{1}{c_2\beta_{l_i}} \ln \left(\frac{q\lambda}{c_1c_2\beta_{l_i}} \right), \quad i = 1, 2, \dots, q,$$

$$\lambda = \exp \left(-\frac{M_t c_2 + \sum_{i=1}^q \frac{1}{\beta_{l_i}} \ln \frac{q}{c_1c_2\beta_{l_i}}}{\sum_{i=1}^q \frac{1}{\beta_{l_i}}} \right). \quad (6.53)$$

6.5 Numerical Results

In this section, the performance of group ZF-SIC and group MMSE-SIC decoding schemes for LA-STBC system with the detection ordering and optimal transmit power allocation is evaluated, and compared with that of the decoding schemes without detection ordering and with equal power allocation.

Figure 6.4: Effects of transmit power calculated using (6.52) and (6.53) on uncoded BER performance of 6×6 MIMO system

Fig. 6.4 shows the effects of transmit power allocation on the uncoded BER performance of a 6×6 MIMO system. For group ZF-SIC and group MMSE-SIC schemes, the transmit power is calculated using (6.52) and (6.53) with their respective α_j and β_{i_i} . We observe that for each decoding scheme, two BER curves obtained by considering and neglecting error propagation are almost indistinguishable. This indicates that error propagation hardly affects the performance of transmit power allocation. Therefore, we use the simpler (6.53) rather than (6.52) in the subsequent results.

Fig. 6.5 shows the effects of the detection order and proposed transmit power al-

Figure 6.5: BER Performance of each decoding stage of group ZF-SIC decoding scheme for 6×6 MIMO system

location on the uncoded BER performance of each detection stage for a 6×6 MIMO system with group ZF-SIC decoding. The scheme without detection ordering and with equal transmit power allocation and the scheme with detection ordering and equal power transmit power allocation is also provided for comparison. Error propagation is ignored when calculating the transmit power coefficients by (6.53). It is shown that the detection ordering cannot increase the diversity but only increase the average SNR of each detection stage. When incorporating the proposed transmit power allocation, the diversity order of all detection stages are increased to the same level and the curves

Figure 6.6: BER Performance of each decoding stage of group MMSE-SIC decoding scheme for 6×6 MIMO system

are almost indistinguishable. Therefore, the overall BER performance of the system is improved. Furthermore, the effects of error propagation may be reduced by improving the BER performance of the early detection stages.

Similar performance comparison for group MMSE-SIC decoding scheme is shown in Fig. 6.6 under the same conditions as in Fig. 6.5. Different from the case of group ZF-SIC decoding scheme, in the absence of transmit power allocation, the detection ordering alone can bring diversity gain to all the detection stages and the curves of all detection stages are similar to one another. When employing both detection order and

Figure 6.7: Overall BER Performance of group ZF-SIC and group MMSE-SIC decoding schemes for 6×6 MIMO system

transmit power allocation, the diversity order of all the curves is further improved to the same level, and the curves of all the detection stages cannot be distinguished.

Fig. 6.7 shows the comparison of overall uncoded BER performance between the group ZF-SIC and group MMSE-SIC decoding schemes. For each decoding scheme, the performance with arbitrary detection order and equal transmit power allocation is compared with that employing detection order and optimal transmit power allocation scheme. The Tarokh's power allocation scheme meant for layered space-time trellis coded systems with detection order is also provided for comparison. It can be seen that

a system with detection ordering and optimal transmit power allocation significantly outperforms that under other conditions for both decoding schemes. It is shown that detection ordering combined with optimal power allocation achieves 4 – 5 dB SNR gain at an uncoded BER of 10^{-4} for both group MMSE-SIC and ZF-SIC decoding schemes. The group MMSE-SIC scheme has better BER performance but less diversity gain than the group ZF-SIC decoding scheme when decoding proceeds in arbitrary order and with equal power allocation. When detection ordering and optimal power allocation are applied to both decoding schemes, the group MMSE-SIC not only outperforms but also gains diversity order over the group ZF-SIC decoding scheme.

6.6 Conclusions

In this chapter, an extensive study on LA-STBC system is conducted, including dynamic detection ordering, iterative decoding and transmit power allocation. These techniques are aimed at mitigating error propagation due to the imperfect decision feedback inherent in SIC based algorithms. The improvement brought by the detection ordering is less than iterative decoding algorithm. The transmit power allocation method proposed to minimize the BER averaged over all detection stages performs best among all the techniques. Based on the numerical results, it has been found that combining optimal transmit power allocation with detection ordering can dramatically improve the overall uncoded BER performance.

Chapter 7

Layered Space-Time Block Coding with Spatial Correlation

In the previous two chapters, we have conducted an extensive study on the LA-STBC architecture. The analysis and simulations were presented assuming an idealized channel model with i.i.d. fading between different transmit-receive antenna pairs. However, most realistic MIMO channels exhibit spatially correlated fading due to clustered scattering and relatively small antenna spacing. The performance of STBC and SM systems may be degraded in the presence of channel correlation. The correlation at transmitter or receiver may reduce the degrees of freedom in SM that makes the separation of input streams rather difficult. Meanwhile, the reduced degrees of freedom affects not only the available diversity but also the coding gain in the diversity achieving space-time coding system. Appropriate transmission designs by exploiting the partial channel state

information (CSI) is therefore necessary to enhance the performance of the LA-STBC system.

In this chapter, we are concerned with the optimization of the transceiver structure for an LA-STBC system that operates over a frequency flat fading channel, under the assumption that only long term channel state information (LT-CSI) is available at the transmitter [53, 67, 68]. This assumption is of crucial relevance for systems in which the fading channel is sufficiently fast-varying to make the condition of instantaneous CSI (I-CSI) at the transmitter not realistic. The LT-CSI can be acquired by the transmitter either directly from measurement of the opposite link [75] or by feedback from the receiver. When Alamouti's STBC is employed as a codeword at each layer, the LA-STBC system can be transformed to an equivalent SM system. For the equivalent SM system, the transceiver comprises linear precoding and nonlinear receivers, such as ZF-SIC, MMSE-SIC, ML detection schemes, and linear precoding techniques known from the literature [44]. In addition, we propose a linear precoding design that incorporates the power allocation and the well known statistical waterfilling scheme. The proposed precoder first allocates transmission power among layers based on minimizing the distance to the decision boundary. Given the transmit power for each layer, the precoder then performs a waterfilling strategy to minimize the upper bound on the pair-wise error probability (PEP) of STBC at each layer.

The remainder of this chapter is organized as follows. System and correlation models are described in Section 7.1. We present the combined precoding scheme for the LA-

STBC system in Section 7.3 and the performance of the equivalent SM in Section 7.2, followed by simulation results and conclusions in Sections 7.4 and 7.5. The results in this chapter are published in [Zhao4].

7.1 System Model

The system model is the same as that in Chapter 5, but we assume the presence of local scatterers around the transmitter and/or the receiver. We introduce a linear precoding matrix $\mathbf{F}$ before transmitting signals. Therefore, the baseband received signal is written as

$$\mathbf{Y} = \mathbf{H}\mathbf{F}\mathbf{S} + \mathbf{N}. \quad (7.1)$$

In optimizing the transceiver structure, the power constraint $E\{\|\mathbf{F}\|_F^2\} = \text{tr}(\mathbf{F}\mathbf{F}^H) = M_t$ is imposed. This condition is clearly satisfied when no precoder is employed, i.e. $\mathbf{F} = \mathbf{I}$.

Consider a MIMO channel where the channel propagation coefficients are assumed to be correlated. The autocorrelation matrix that describes the correlation between all channel elements is given by (4.2) in Chapter 4. Suppose $\mathbf{H}_i$ to be the sub-channel corresponding to the i -th layer. For a given k , the correlation matrix of $\mathbf{H}_k$ is given by

$$\begin{aligned} \mathbf{R}_{\mathbf{H}_k} &= E\{\text{vec}(\mathbf{H}_k)\text{vec}(\mathbf{H}_k)^H\}, \\ &= \mathbf{R}_{\mathbf{H}}(\mathbf{k}, \mathbf{k}) \end{aligned} \quad (7.2)$$

where

$$\mathbf{k} = \sum_{i=1}^{k-1} M_r M_i + 1 : \sum_{i=1}^k M_r M_i. \quad (7.3)$$

If the channel has a separable correlation function, the correlation between the channel propagation coefficients h_{ij} and h_{lm} , i.e., $E\{h_{ij}h_{lm}^*\}$, is given by the product of the spatial correlation at the receiver and the spatial correlation. The channel correlation is modeled by the Kronecker product. The validity of the Kronecker correlation model has been supported by [20]. We assume spatial correlation only exists at the transmitter side as we did in Chapter 4.

Hence, the channel correlation is given by

$$\mathbf{R}_{\mathbf{H}} = \mathbf{R}_t^T \otimes \mathbf{I}_{M_r}, \quad (7.4)$$

and the correlation matrix of $\mathbf{H}_k$ is given by

$$\mathbf{R}_{\mathbf{H}_k} = \mathbf{R}_t^T(k_1 : k_2, k_1 : k_2) \otimes \mathbf{I}_{M_r}, \quad (7.5)$$

where $k_1 = \sum_{i=1}^{k-1} M_i + 1, k_2 = \sum_{i=1}^k M_i$.

7.2 Performance Analysis

In this section, we analyze the performance of the proposed transceiver algorithm in a 4×4 LA-STBC. We suppose the output codeword of the encoder at each layer is a Alamouti's code, and there are two layers altogether. Therefore,

$$\mathbf{S}_i = \begin{bmatrix} b_{2i-1} & b_{2i}^* \\ b_{2i} & -b_{2i-1}^* \end{bmatrix}, \quad i = 1, 2. \quad (7.6)$$

The LA-STBC system can be equivalently converted to a SM system, whose channel matrix is an aggregate channel matrix of the channel matrix in the LA-STBC system as

$$\mathbf{H}_{ag} = \begin{bmatrix} \mathbf{H}(:, 1) & \mathbf{H}(:, 2) & \mathbf{H}(:, 3) & \mathbf{H}(:, 4) \\ -\mathbf{H}(:, 2)^* & \mathbf{H}(:, 1)^* & -\mathbf{H}(:, 4)^* & \mathbf{H}(:, 3)^* \end{bmatrix}, \quad (7.7)$$

where $\mathbf{H}(:, i)$ is the i -th column of $\mathbf{H}$. The aggregate received signal vector before the array processing can be written as

$$\begin{bmatrix} \mathbf{Y}(:, 1) \\ \mathbf{Y}(:, 2)^* \end{bmatrix} = \mathbf{H}_{ag} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} + \begin{bmatrix} \mathbf{N}(:, 1) \\ \mathbf{N}(:, 2)^* \end{bmatrix}, \quad (7.8)$$

where $\mathbf{Y}(:, i)$ is the column of $\mathbf{Y}$ that represents the received signal at symbol period i .

The same definition also applies to $\mathbf{N}(:, i)$. We redefine (7.8) as

$$\mathbf{y}_{ag} = \mathbf{H}_{ag}\mathbf{b} + \mathbf{n}_{ag}. \quad (7.9)$$

The array processor then operates on the aggregated channel matrix and signal vector.

We observe that the aggregate channel matrix can be rewritten with transmit correlation as

$$\mathbf{H}_{ag} = \begin{bmatrix} \mathbf{H}_w & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_w^* \end{bmatrix} \begin{bmatrix} \mathbf{R}_t^{\frac{1}{2}}(:, 1) & \mathbf{R}_t^{\frac{1}{2}}(:, 2) & \mathbf{R}_t^{\frac{1}{2}}(:, 3) & \mathbf{R}_t^{\frac{1}{2}}(:, 4) \\ -\mathbf{R}_t^{\frac{1}{2}*}(:, 2) & \mathbf{R}_t^{\frac{1}{2}*}(:, 1) & -\mathbf{R}_t^{\frac{1}{2}*}(:, 4) & \mathbf{R}_t^{\frac{1}{2}*}(:, 3) \end{bmatrix}. \quad (7.10)$$

We remark that the correlation matrix of the aggregated channel related to the LT-CSI at the transmitter is

$$E \{ \mathbf{H}_{ag}^H \mathbf{H}_{ag} \} = \mathbf{R}_{ag} \cdot \text{tr}(\mathbf{I}_{M_r}) \quad (7.11)$$

with $\mathbf{R}_{ag}$ defined as $\mathbf{R}_{ag} = (\mathbf{R}_{t,ag}^{\frac{1}{2}})^H \mathbf{R}_{t,ag}^{\frac{1}{2}}$ and

$$\mathbf{R}_{t,ag}^{\frac{1}{2}} = \begin{bmatrix} \mathbf{R}_t^{\frac{1}{2}}(:, 1) & \mathbf{R}_t^{\frac{1}{2}}(:, 2) & \mathbf{R}_t^{\frac{1}{2}}(:, 3) & \mathbf{R}_t^{\frac{1}{2}}(:, 4) \\ -\mathbf{R}_t^{\frac{1}{2}*}(:, 2) & \mathbf{R}_t^{\frac{1}{2}*}(:, 1) & -\mathbf{R}_t^{\frac{1}{2}*}(:, 4) & \mathbf{R}_t^{\frac{1}{2}*}(:, 3) \end{bmatrix}. \quad (7.12)$$

It is easy to see that the expected value of the correlated channel matrix is not full rank when high correlation exists between antennas. One extreme case can be obtained by simply assuming the correlation matrix to be an all-one matrix. In this case, the channel matrix has rank equal to one; however, the aggregated channel has a minimum

rank equal to two which comes from the STBC no matter what the value of the channel coefficients are or if they are correlated. It is this guaranteed diversity order that keeps the performance of LA-STBC robust in correlated environments. Note that with the built-in transmit diversity, the proposed transceiver achieves at least the same diversity order as the STBC used at each layer.

After converting the LA-STBC system into an equivalent SM system, we employ a linear or nonlinear receiver on the aggregate channel matrix $\mathbf{H}_{ag}$ of (7.10) instead of performing group receivers on the original channel matrix $\mathbf{H}$ to decode the transmitted signals. The diversity order of the first detected layer is increased to $2M_r - M_t + 2$. Suppose in the 4×4 MIMO system, each layer is equipped with 2 transmit antennas, the diversity gain achieved by the first layer when using group ZF receiver is only 4; while in the equivalent SM channel, the first layer has diversity order of 6. Furthermore, the noise amplification caused by linear receivers is reduced in the aggregate channel matrix. This can be reflected by inspecting the transmit correlation matrix of the aggregate channel matrix $\mathbf{R}_{ag}$. Take the 4×4 channel as an example, if the correlation matrix follows the exponential structure as

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho^* & 1 & \rho & \rho^2 \\ \rho^{2*} & \rho^* & 1 & \rho \\ \rho^{3*} & \rho^{2*} & \rho^* & 1 \end{bmatrix}, \quad (7.13)$$

then $\mathbf{R}_{ag}$ is given by

$$\mathbf{R}_{ag} = \begin{bmatrix} 2 & 0 & \rho^2 + \rho^{2*} & \rho^3 - \rho^* \\ 0 & 2 & \rho - \rho^{3*} & \rho^2 + \rho^{2*} \\ \rho^2 + \rho^{2*} & \rho^* - \rho^3 & 2 & 0 \\ \rho^{3*} - \rho & \rho^2 + \rho^{2*} & 0 & 2 \end{bmatrix}. \quad (7.14)$$

The performance of the SM system using linear receiver is determined by the diagonal element of the inverse of the transmit correlation matrix. If the two transmit correlation matrix are full rank, the diagonal elements of $\mathbf{R}_t^{-1}$ and $\mathbf{R}_{ag}^{-1}$ are given by

$$\begin{aligned} \mathbf{R}_t^{-1}(1, 1) &= \mathbf{R}_t^{-1}(4, 4) = \frac{1}{1 - \rho\rho^*} \\ \mathbf{R}_t^{-1}(2, 2) &= \mathbf{R}_t^{-1}(3, 3) = \frac{1 + \rho\rho^*}{1 - \rho\rho^*} \end{aligned} \quad (7.15)$$

and

$$\begin{aligned} \mathbf{R}_{ag}^{-1}(i, i) &= \frac{8 - 2\rho\rho^* - 4\rho^2\rho^{2*} - 2\rho^3\rho^{3*}}{16 - 8\rho\rho^* - 15\rho^2\rho^{2*} - 4\rho^3\rho^{3*} + 6\rho^4\rho^{4*} + 4\rho^5\rho^{5*} + \rho^6\rho^{6*}}, \\ i &= 1, 2, 3, 4. \end{aligned} \quad (7.16)$$

It is easy to see that the diagonal element of $\mathbf{R}_{ag}^{-1}$ is smaller than that of $\mathbf{R}_t^{-1}$. According to [76], the mean received SNR of the i -th layer is inversely proportional to the i -th diagonal element of the inverse of the transmit correlation matrix. Hence, the noise power enhanced with $\mathbf{R}_{ag}$ is less than that enhanced by $\mathbf{R}_t$. Fig. 7.1 demonstrates the results when performing ZF-SIC, MMSE-SIC, and ML detection schemes on the

Figure 7.1: Detection schemes for a 4×4 layered space-time block coding system in correlated channel, $\rho = 0.8$

aggregated channel matrix for LA-STBC in correlated channel. The performances of the group ZF-SIC and the group MMSE-SIC receivers are also provided for comparison. It is shown that transforming a LA-STBC into an equivalent SM system and performing SIC based detection schemes, the diversity gain is enhanced as expected. The performances of the ZF-SIC and the MMSE-SIC detection schemes are very close, which can be explained as noise power enhancement by the inverse of aggregated channel matrix is negligible. The ML detection scheme is optimum and has a diversity order of 8.

7.3 Precoding for LA-STBC system

7.3.1 MMSE-Based Precoding

Given the equivalent SM system in (7.9), we first optimize the general transceiver scheme by minimizing the mean square error (MSE) between the output of a linear receiver and the transmitted signals. Suppose the linear receiver is $\mathbf{W}$ and it estimates the transmitted symbols as $\hat{\mathbf{b}} = \mathbf{W}\mathbf{y}_{ag}$. From (7.9), we have

$$\hat{\mathbf{b}} = \mathbf{W}\mathbf{y}_{ag} = \mathbf{W}\mathbf{H}_{ag}\mathbf{F}\mathbf{b} + \mathbf{W}\mathbf{n}_{ag}. \quad (7.17)$$

The MSE matrix is given by

$$E \left\{ (\hat{\mathbf{b}} - \mathbf{b})(\hat{\mathbf{b}} - \mathbf{b})^H \right\} = \text{MSE}(\mathbf{F}, \mathbf{W}) \quad (7.18)$$

where

$$\text{MSE}(\mathbf{F}, \mathbf{W}) = (\mathbf{W}\mathbf{H}_{ag}\mathbf{F})(\mathbf{W}\mathbf{H}_{ag}\mathbf{F})^H + \sigma_n^2 \mathbf{W}\mathbf{W}^H. \quad (7.19)$$

The cumulative MSE of the estimate of $\mathbf{b}$ is $E\{\|\hat{\mathbf{b}} - \mathbf{b}\|^2\} = \text{tr}(\text{MSE}(\mathbf{F}, \mathbf{W}))$. The optimal $\mathbf{W}$ that minimizes the whole $\text{MSE}(\mathbf{F}, \mathbf{W})$ matrix is the same as the MMSE receiver [77], which is known to minimize the $\text{tr}(\text{MSE}(\mathbf{F}, \mathbf{W}))$ and is given by

$$\mathbf{W}_{opt} = \mathbf{F}^H \mathbf{H}_{ag}^H (\mathbf{H}_{ag} \mathbf{F} \mathbf{F}^H \mathbf{H}_{ag}^H + \sigma_n^2 \mathbf{I})^{-1}. \quad (7.20)$$

Substituting (7.20) into the expression of $\text{MSE}(\mathbf{F}, \mathbf{W})$, we obtain

$$\text{MSE}(\mathbf{F}) = (\mathbf{I} + \sigma_n^{-2} \mathbf{F}^H \mathbf{H}_{ag}^H \mathbf{H}_{ag} \mathbf{F})^{-1}. \quad (7.21)$$

In the following, we will determine $\mathbf{F}_{opt}$ based on transmit power constraints stated in Section 7.1.

The joint transmitter and receiver design that minimizes $\text{tr}(\text{MSE}(\mathbf{F}, \mathbf{W}))$ can be obtained by minimizing $\text{tr}(\text{MSE}(\mathbf{F}))$ with respect to $\mathbf{F}$. Since the transmitter is given only the LT-CSI, we cannot minimize the trace of (7.21). Instead, it is reasonable to consider $E\{\text{tr}(\text{MSE}(\mathbf{F}))\}$ as the loss function. It can be shown that $E\{\text{tr}(\text{MSE}(\mathbf{F}))\} \geq \text{tr}(E\{\text{MSE}(\mathbf{F})\})$ where $E\{\text{MSE}(\mathbf{F})\} = (\mathbf{I} + \sigma_n^{-2} \mathbf{F}^H E\{\mathbf{H}_{ag}^H \mathbf{H}_{ag}\} \mathbf{F})^{-1}$. Therefore, we minimize the lower bound $\text{tr}(\overline{\text{MSE}}(\mathbf{F})) = \text{tr}(E\{\text{MSE}(\mathbf{F})\})$. The solution for $\mathbf{F}_{opt}$ is obtained by following the steps outlined in [44]. It is

$$\mathbf{F}_{opt} = \mathbf{U}_F \Phi \quad (7.22)$$

where $\mathbf{U}_F$ is obtained from the eigenvalue decomposition of the LT-CSI $E\{\mathbf{H}_{ag}^H \mathbf{H}_{ag}\} = \mathbf{U}_F \Lambda \mathbf{U}_F^H$ and Φ is a diagonal matrix with the following entry:

$$|\phi_{ii}|^2 = \left(\frac{M_t + \sum_{l=1}^{\bar{M}_t} \lambda_{ll}^{-1}}{\sum_{l=1}^{\bar{M}_t} \lambda_{ll}^{-1/2}} \lambda_{ii}^{-1/2} - \frac{1}{\lambda_{ii}} \right)^+ \quad (7.23)$$

where $(x)^+ = \max(x, 0)$ and $\bar{M}_t \leq M_t$ is such that $|\phi_{ii}|^2 > 0$ for $l \in [1, \bar{M}_t]$ and $|\phi_{ii}|^2 = 0$ for all other l .

Some remarks on the results of the optimization (7.20) and (7.22) are in order: 1) In case we relax the assumption of LT-CSI at the transmitter, i.e., we allow $\mathbf{F}$ to depend on the instantaneous CSI, the resulting $\mathbf{F}$ coincides with the results derived in [44]. In other words, if both sides of the link have access to the channel matrix, the setting that minimizes the MSE results in linear precoder and decoder. In this case, $\mathbf{F}$ is obtained from the singular value decomposition (SVD) of the aggregated channel matrix $\mathbf{H}_{ag}$. 2) Following the approach of [44], the linear precoder $\mathbf{F}$ can be obtained alternatively by minimizing $\text{tr}(\overline{\text{MSE}}(\mathbf{F}))$ subject to a power constraint or by maximizing the information rate, i.e., minimizing $|\overline{\text{MSE}}(\mathbf{F})|$, which results in a waterfilling solution.

7.3.2 Combined SM and STBC Precoding

A closed form precoder for a SM system in correlated MIMO channels has been presented in [57], wherein power allocation and phase shift are considered for each input symbol based on equating the symbol error rate (SER) of each detection stage when SIC is used as the receiver. It is shown that the original LA-STBC can be equivalently converted to an aggregated SM system when employing Alamouti's code at each layer and SIC based detection schemes is performed on the aggregated channel matrix to decode the input signals. We are therefore motivated to consider optimal power allocation for the input symbols given the aggregated transmit correlation $\mathbf{R}_{ag}$ available at the transmitter.

We first consider a 4×4 LA-STBC MIMO system. Each of the four symbols is

multiplied with a power coefficient as

$$\mathbf{b} = [\sqrt{P_1}b_1 \ \sqrt{P_2}b_2 \ \sqrt{P_3}b_3 \ \sqrt{P_4}b_4]^T, \quad (7.24)$$

where the positive power allocated to the i -th input signal b_i is selected to satisfy $\sum_{i=1}^4 P_i = M_t$. Following the idea of [57] in deriving the optimal power allocation, we multiply the inverse of an independent channel matrix to $\mathbf{y}_{ag}$ and obtain

$$\mathbf{z} = \begin{bmatrix} \mathbf{R}_t^{\frac{1}{2}}(:,1) & \mathbf{R}_t^{\frac{1}{2}}(:,2) & \mathbf{R}_t^{\frac{1}{2}}(:,3) & \mathbf{R}_t^{\frac{1}{2}}(:,4) \\ -\mathbf{R}_t^{\frac{1}{2}*}(:,2) & \mathbf{R}_t^{\frac{1}{2}*}(:,1) & -\mathbf{R}_t^{\frac{1}{2}*}(:,4) & \mathbf{R}_t^{\frac{1}{2}*}(:,3) \end{bmatrix} \begin{bmatrix} \sqrt{P_1}b_1 \\ \sqrt{P_2}b_2 \\ \sqrt{P_3}b_3 \\ \sqrt{P_4}b_4 \end{bmatrix} + \begin{bmatrix} \mathbf{H}_w & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_w^* \end{bmatrix}^+ \mathbf{n}_{ag}. \quad (7.25)$$

We can obtain the estimate of the input signals by applying maximal ratio combining (MRC) on $\mathbf{z}$ as $\hat{\mathbf{s}} = (\mathbf{R}_{t,ag}^{\frac{1}{2}})^H \mathbf{z}$. For arbitrary transmit correlation matrix, denote the (i,j) -th entry of $\mathbf{R}_t$ by r_{ij}^t . We have

$$\mathbf{R}_{ag} = \begin{bmatrix} 2 & 0 & r_{13}^t + r_{42}^t & r_{14}^t - r_{32}^t \\ 0 & 2 & r_{23}^t - r_{41}^t & r_{24}^t + r_{31}^t \\ r_{31}^t + r_{24}^t & r_{32}^t - r_{14}^t & 2 & 0 \\ r_{41}^t - r_{23}^t & r_{42}^t + r_{13}^t & 0 & 2 \end{bmatrix}. \quad (7.26)$$

Denoting the (i, j) -th element of $\mathbf{R}_{ag}$ by r_{ij} , the estimated signals can be expressed as

$$\begin{aligned}\hat{s}_1 &= 2\sqrt{P_1}b_1 + r_{13}\sqrt{P_3}b_3 + r_{14}\sqrt{P_4}b_4 + e_1, \\ \hat{s}_2 &= 2\sqrt{P_2}b_2 + r_{23}\sqrt{P_3}b_3 + r_{24}\sqrt{P_4}b_4 + e_2, \\ \hat{s}_3 &= 2\sqrt{P_3}b_3 + r_{31}\sqrt{P_1}b_1 + r_{32}\sqrt{P_2}b_2 + e_3, \\ \hat{s}_4 &= 2\sqrt{P_4}b_4 + r_{41}\sqrt{P_1}b_1 + r_{42}\sqrt{P_2}b_2 + e_4.\end{aligned}\tag{7.27}$$

The symbol error probability for s_i is governed by the variance of the additive noise term e_i and the minimum possible distance from $\hat{s}_i$ to its corresponding symbol region boundary. Since the noise variance for each estimated signal is the same, the other factor that makes the error probability different for each input signal is the minimum distance, which is governed by the power assigned to each input signal. Suppose we decode the above signals in a SIC fashion. It is shown that s_1 and s_2 are subject to the same interference variance and noise variance, and they are decoded at the first stage of SIC. The minimum distances for $\hat{s}_1$ and $\hat{s}_2$ from their respective decision boundary are given by [57]

$$\begin{aligned}\delta_1 &= 2\sqrt{P_1}d_{min} - |r_{13}|\sqrt{P_3}d_{max} - |r_{14}|\sqrt{P_4}d_{max}, \\ \delta_2 &= 2\sqrt{P_2}d_{min} - |r_{23}|\sqrt{P_3}d_{max} - |r_{24}|\sqrt{P_4}d_{max}\end{aligned}\tag{7.28}$$

where d_{min} and d_{max} are the minimum and maximum distances of a constellation point to the decision boundary. Assume the detection of b_1 and b_2 is perfect, and their contributions are cancelled out from the estimated signals $\hat{s}_3$ and $\hat{s}_4$. Therefore, the

minimum distances for s_3 and s_4 are given by

$$\delta_3 = 2\sqrt{P_3}d_{min}, \quad \delta_4 = 2\sqrt{P_4}d_{min} \quad (7.29)$$

According to the BER balancing criterion proposed in [57], $\delta_1 = \delta_2 = \delta_3 = \delta_4$. It is obvious that $P_1 = P_2$ and $P_3 = P_4$. For BPSK and 4-QAM constellations, $d_{min} = d_{max} = 1$, and with the power constraint $\sum_{i=1}^4 P_i = M_t = 4$, we are allowed to derive a simple closed-form solution for the precoder. Without taking into account the phase shift, the closed-form precoder is actually a power allocation scheme that is obtained as

$$\begin{aligned} P_{1,2} &= \frac{2 \cdot \left(1 + \frac{|r_{13}| + |r_{14}|}{2}\right)^2}{\left(\left(1 + \frac{|r_{13}| + |r_{14}|}{2}\right)^2 + 1\right)}, \\ P_{3,4} &= \frac{2}{\left(\left(1 + \frac{|r_{13}| + |r_{14}|}{2}\right)^2 + 1\right)}. \end{aligned} \quad (7.30)$$

Certain special cases for this precoder turn out to be of immediate interest in their interpretation. 1) Uncorrelated case: With no correlation $r_{ij} = 0 (i \neq j)$, which yields equal power transmission, justifying the standard equal power design. 2) Fully correlated case: With full correlation $r_{13} = 0, r_{14} = 2$, and $d_{max} = d_{min}$, we find $P_{1,2} = 1.6$ and $P_{3,4} = 0.4$. Interestingly, this corresponds to the power allocation for a regular 2D constellation. For instance a 16-QAM constellation can be seen as the superposition of two 4-QAM constellations with respective power ratio 4, the second 4-QAM points being centered about the constellation point of the first 4-QAM points.

Given $k (k = 1, 2)$, the total power allocated to the k -th layer is assumed to be

7.3. Precoding for LA-STBC system

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$P_{2k-1} + P_{2k}$, which is computed by the approach presented above. With the power constraints, the STBC precoder of the k -th layer is given by solving the following optimization problem [52]:

$$\begin{aligned} \mathbf{F}_k = \arg \max_{\mathbf{F}_k} & \left| \mathbf{I}_{M_k} + \frac{\beta_{min} \mathbf{R}_t^T(k_1 : k_2, k_1 : k_2) \mathbf{F}_k \mathbf{F}_k^H}{4\sigma_n^2} \right|, \\ s.t. \quad & \text{tr}(\mathbf{F}_k \mathbf{F}_k^H) \leq P_{2k-1} + P_{2k}, \end{aligned} \quad (7.31)$$

where β_{min} is the minimum Euclidean distance between two constellation points. The solution of the above optimization problem is the classical waterfilling power allocation along the eigenmode of the transmit correlation matrix of the k -th layer, and is given by

$$\mathbf{F}_k = \mathbf{U}_k \Phi_k \quad (7.32)$$

where $\mathbf{U}_k$ is defined by the following eigenvector decomposition of $\mathbf{R}_t^T(k_1 : k_2, k_1 : k_2)$ as:

$$\mathbf{R}_t^T(k_1 : k_2, k_1 : k_2) = \mathbf{U}_k \Lambda_k \mathbf{U}_k^H. \quad (7.33)$$

Suppose $\Lambda_k = \text{diag}(\lambda_{k,1}, \dots, \lambda_{k,M_k})$. Then Φ_k is found to be a diagonal matrix as $\Phi_k = \text{diag}(\phi_{k,1}, \dots, \phi_{k,M_k})$, in which each element is given by

$$\phi_{k,i}^2 = \left(\frac{P_{2k-1} + P_{2k}}{\bar{M}_k} + \frac{4\sigma_n^2}{\beta_{min}} \left(\frac{\sum_{l=1}^{\bar{M}_k} \lambda_{k,l}^{-1}}{\bar{M}_k} - \lambda_{k,i}^{-1} \right) \right)^+, \quad (7.34)$$

where $(x)^+ = \max(x, 0)$ and $\bar{M}_k \leq M_k$ is such that $\phi_{k,i}^2 > 0$ for $i \in [1, \bar{M}_k]$, and $\phi_{k,i}^2 = 0$ for all other i .

Figure 7.2: Effect of correlation on the LA-STBC system, input SNR=8 dB

7.4 Simulation Results

This section illustrates the effects of spatial correlation on the performance of a 4×4 LA-STBC system by Monte-Carlo simulation. Combined STBC and SM linear precoder is employed to improve the error rate performance. We assume the correlated channel follows the Kronecker model and correlation exists only at the transmitter. The correlation matrix adopted in the simulation has an exponential form. For simplicity, the output STBC of the encoder at each layer is Alamouti's STBC. BPSK modulation is adopted in our simulation. In the following, the BER is averaged over all transmitted

Figure 7.3: Precoding schemes on LA-STBC system with ZF-SIC decoder, $\rho = 0.8$ data streams.

The performance of the proposed precoder for the LA-STBC system is first evaluated in terms of uncoded BER as a function of correlation level ρ with ZF-SIC and MMSE-SIC decoding schemes performed on the aggregate channel matrix. In Fig. 7.2, we find that both with and without the proposed precoder, the MMSE-SIC decoding scheme on the aggregate channel is more robust to the spatial correlation than the ZF-SIC decoder. The performance degradation for MMSE-SIC in a correlated channel is very small even when correlation level as high as 0.9.

Figure 7.4: Precoding schemes on LA-STBC system with MMSE-SIC decoder, $\rho = 0.8$

Fig. 7.3 demonstrates the uncoded BER performance of different precoders versus input SNR when the correlation level is $\rho = 0.8$ using ZF-SIC decoder. The STBC precoder is defined when the waterfilling scheme is applied to each layer with same power constraints. On the other hand, the definition of the power allocation scheme is simply transmitting signals over transmit antennas with different power allocated to each layer. It is observed that the STBC precoder performs better than the power allocation scheme at low SNR while the latter takes the lead when SNR is high. The MMSE waterfilling precoding algorithm performs as well as the power allocation scheme

Figure 7.5: Precoding schemes on LA-STBC system with ML detection, $\rho = 0.8$

with slightly better performance at high SNR. While the proposed precoder always performs best regardless of the input SNR value. The proposed precoder achieves nearly 1.5 dB gain for ZF-SIC compared with system without precoding at BER of 10^{-3} .

The same features of the precoding schemes remains for the MMSE-SIC decoder as shown in Fig. 7.4. The gains achieved by the proposed precoder is more than 2 dB compared with no precoding scheme at BER of 10^{-3} for the MMSE-SIC decoder. It is noted that among the precoding schemes other than the proposed precoder, the

MMSE waterfilling scheme performs best and has higher diversity order than the proposed precoder. It is because the MMSE waterfilling precoder with the MMSE decoder is optimum in terms of minimizing the trace of minimum square error between the transmitted signals and the output signals of the decoder.

In Fig. 7.5, the precoding schemes are evaluated with ML detection at the receiver. Different from the previous two cases, the STBC precoder shows the lowest BER; while the proposed precoder has the same diversity order as the STBC precoder with a right shift in the curve. Since the proposed precoder is targeted for SIC based detection algorithms, its optimality is not guaranteed for ML detection. The optimal precoder for ML detection has been discussed in Chapter 4.

7.5 Conclusions

In this chapter, we have studied the performance of an LA-STBC system over spatially correlated channels and proposed a precoder under the assumption that the CSI available at the transmitter is limited to the second-order statistics of the channel. Different transceiver structures have been studied by simulation and it is shown that relevant benefits can be obtained by exploiting the LT-CSI at the transmitter. By converting an LA-STBC system into an equivalent SM system, the diversity order of the ZF-SIC and the MMSE-SIC decoding schemes is increased compared with the group ZF-SIC and the group MMSE-SIC decoding algorithms. Among the precoding schemes, the proposed precoder that combines a STBC precoder and power allocation schemes outperforms

either one of other precoding schemes but only for the SIC based detection algorithms.

Chapter 8

Conclusions and Future Work

In this chapter, we give some concluding remarks on this work and discuss several directions for future work.

8.1 Conclusions

A number of signal processing techniques have been investigated for multiple input multiple output (MIMO) antenna systems in flat fading Rayleigh channels. The transmission schemes considered are spatial multiplexing (SM) and the proposed layered space-time block coding (LA-STBC) systems. The receiver processing techniques include narrow-band ZF, MMSE, ML detection, V-BLAST for SM systems, and group ZF, group MMSE for LA-STBC systems. Besides, we have explored the precoding strategies for the SM and LA-STBC systems over spatially correlated fading channel to improve

their error rate performance. The main contributions of our work are summarized as follows.

To avoid the prohibitively large number of computational operations, we proposed in Chapter 3 a low complexity version of the V-BLAST architecture, which can reduce the complexity by an order of magnitude. The original V-BLAST algorithm and the existing two low complexity algorithms all adopt optimal ordering and have the same performance. Although in the two low complexity algorithms, repeated inverses of the channel matrix are avoided, optimal order still needs to be determined at each stage of V-BLAST. Therefore, reordering and repeated triangularization of the decomposed matrices are required at each stage of V-BLAST according to the optimal order. By adopting our proposed sub-optimal ordering, the repeated reordering and triangularization are both avoided. It is shown by simulations that our low complexity V-BLAST algorithm incurs no significant loss in SER performance compared with the original algorithm.

Notice that the performance of SM system will be degraded in the presence of spatial correlated fading. In Chapter 4, we propose a precoding technique that minimizes the union bound on the SER of ML detection for SM over correlated Rayleigh fading channels. If the correlation matrix follows the well known Kronecker structure, the receive correlation will have no impact on the precoding design. The precoder performs a constellation multiplexing (CM) process that superposes the original low order modulated symbols to form a high order modulation signaling along eigenvectors of

transmit correlation matrix with rate equivalent to the rate of all original streams. The constellation modulation and the number of signaling branches is adapted to the correlation level. When channel correlation is high, fewer eigenvectors corresponding to the largest eigenvalues should be used for transmission; while in the low correlated channel, where eigenvalues are evenly distributed, more eigenmodes should be utilized for transmission. The rate preserving precoding scheme can operate smoothly at any degree of correlation. It draws the bridge between pure SM and CM over spatial correlated channels.

We present a LA-STBC structure that provides a tradeoff between spectral efficiency and diversity order in Chapter 5. It works as a combination of traditional space-time block coding system and SM system. Detection schemes of the LA-STBC similar to those of traditional SM are presented. Performance of the detection based on successive interference cancellation (SIC) is analyzed. Its superiority over the pure SM in terms of diversity gain and the pure STBC system in terms of data rate is validated in our work.

In Chapter 6, we further study a few techniques of the LA-STBC structure to improve the performance over uncorrelated channels. A number of important aspects including decoding order, an iterative decoding scheme, and transmit power allocation are discussed. These techniques are aimed to compensate for the loss of diversity order due to the error propagation of SIC based detection schemes. The decoding order is given based on the post processing SNR criteria. The iterative decoding scheme

efficiently exploits the receive diversity to achieve more gains than the detection ordering technique. Transmit power allocation is the most effective in minimizing the BER averaged over all detection stages. Further improvement can be achieved by combining optimal transmit power allocation with detection ordering that the performance of each layer becomes almost the same.

We propose in Chapter 7 the precoding design for the LA-STBC system over spatially correlated channel. Due to the orthogonality inherent in STBC, LA-STBC system shows robustness to the channel correlation but it still suffers a certain level performance degradation. Based on partial channel knowledge at transmitter, we present a combined precoding design that incorporates the well known statistical waterfilling scheme for STBC system and a power allocation scheme for SM system with SIC detection. The proposed precoder takes advantage of both schemes and has better performance than either one of the two schemes.

8.2 Future Work

There are several possible directions that can follow the work done in this thesis.

1. High order modulation:

The performance analysis for SIC based decoding algorithms for LA-STBC system is presented assuming BPSK modulation, which is a relatively simple case. It will be more useful to extend the results to the more general M-PSK and M-QAM

modulation schemes.

2. Rate-diversity space-time codes:

Our previously studied layered space-time architecture is the simplest kind of rate-diversity space-time transmission schemes, which is a new research direction. As a matter of fact, the tradeoff between rate and diversity only exists when the modulation for the transmitted symbols is fixed. Recent efforts in [39–41, 78] revealed that, if linear precoding or constellation rotation in the complex field is applied, the tradeoff disappears. This is ensured by the information-theoretic study in [69]. Nevertheless, the techniques on this rate-diversity topic are far from mature and there is still a lot of research potential. For example, the peak-to-average power ratio, the resolution of complex numbers, and the joint design and decoding with outer channel coding are issues.

3. Precoding techniques:

Recent research has demonstrated the value of feeding back to the transmitter information of the channel state by the receiver. The type of feedback may vary largely, depending on its nature, e.g., required rate, instantaneous or statistical CSI, leading to various transmitter design schemes. The precoding designs considered in this work only exploit the low rate long term statistical CSI in the form of channel covariance matrix, especially under the widely used Kronecker correlation model. However, the Kronecker model may not be applicable to a lot of realistic scenarios. It is necessary to investigate the precoding for both SM and STBC systems in channels with more generalized correlation form.

Author's Publications

Journal Papers

1. **L. Zhao** and V. K. Dubey, "Detection schemes for space-time block code and spatial multiplexing combined system," *IEEE Commons. Lett.*, vol. 9, no. 1, Jan. 2005.
2. **L. Zhao** and V. K. Dubey, "Performance of successive interference cancellation based decoding scheme for layered space-time coded communication system," *Submitted to IEEE Commons. Lett.*, 2005.
3. **L. Zhao** and V. K. Dubey, "Transmit power allocation for layered space-time block coding system," *Submitted to EURASIP Journal on Applied Signal Processing*, 2005.
4. **L. Zhao** and V. K. Dubey, "Linear Precoding for layered space-time block coding with correlation feedback," *Submitted to IEEE Signal Processing Lett.*, 2005.

Conference Papers

5. **L. Zhao** and V. K. Dubey, "Transmit diversity and combining scheme for spatial multiplexing over correlated channels," in *Proc. IEEE 59th Vehicular Technology Conference*, May 2004, vol. 1, pp. 380-383.
6. **L. Zhao** and V. K. Dubey, "Reduced rank receiver for successive interference cancellation in correlated MIMO channel," in *Proc. The 9th International Conference on Communications Systems*, Sept. 2004, pp. 306-310.
7. **L. Zhao** and V. K. Dubey, "Maximum likelihood detection and constellation multiplexing in correlated MIMO Channel," in *Proc. IEEE International Symposium on Consumer Electronics*, June 2005.
8. **L. Zhao** and V. K. Dubey, "Precoding with maximum likelihood detection for spatial multiplexing in correlated channel," submitted to *IEEE GLOBECOM*, 2005.

Bibliography

- [1] G. J. Foschini and M. J. Gans, “On limits of wireless communications in fading environment when using multiple antennas,” *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.
- [2] V. Tarokh, N. Seshadri, and A. R. Calderbank, “Space-time codes for high data rate wireless communication: performance criterion and code construction,” *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
- [3] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas,” *Bell Labs Technical J.*, vol. 2, no. 1, pp. 41–59, 1996.
- [4] E. Telatar, “Capacity of multi-antenna Gaussian channels,” *European Trans. on Telecommunications*, vol. 10, no. 6, pp. 589–595, Nov. 1999.
- [5] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.
- [6] M. Herdin, H. Ozelik, H. Hofstetter, and E. Bonek, “Variation of measured indoor MIMO capacity with receive direction and position at 5.2 GHz,” *Electron. Lett.*, vol. 38, no. 21, pp. 1283–1283, Oct. 2002.

- [7] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE J. Select. Areas Commun.*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.
- [8] K. Yu, M. Bengtsson, B. Ottersten, D. McNamara, P. Karlsson, and M. Beach, "Second order statistics of NLOS indoor MIMO channels based on 5.2ghz measurements," in *Proc. IEEE GLOBECOM*, vol. 1, Nov. 2001, pp. 156–160.
- [9] G. D. Golden, C. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *Electron. Lett.*, vol. 35, no. 1, pp. 14–16, Jan. 1999.
- [10] J. G. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, 1995.
- [11] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Upper Saddle River, New Jersey: Prentice Hall PTR, 2002.
- [12] W. C. Jakes, *Microwave Mobile Communications*. John Wiley & Son, Inc., 1974.
- [13] D. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna system," *IEEE Trans. Commun.*, vol. 48, no. 3, pp. 502–513, Mar. 2000.
- [14] D. Gesbert, H. Bolcskei, D. Gore, and A. J. Paulraj, "Outdoor MIMO wireless channels: models and performance prediction," *IEEE Trans. Commun.*, vol. 50, no. 12, pp. 1926–1934, Dec. 2002.
- [15] K. Yu and B. Ottersten, "Models for MIMO propagation channels, a review," *Wiley Journal on Wireless Communications and Mobile Computing*, vol. 2, no. 7, pp. 653–666, Nov. 2002.

- [16] K. I. Pedersen, J. B. Andersen, J. P. Kermoal, and P. E. Mogensen, "A stochastic multiple-input-multiple-output radio channel model for evaluation of space-time coding algorithms," vol. 2, pp. 24–28, Sept. 2000.
- [17] M. T. Ivrlac, W. Utschick, and J. A. Nossek, "Fading correlations in wireless MIMO communication systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 5, pp. 819–828, June 2003.
- [18] H. Ozcelik, M. Herdin, W. Weichselberger, J. Wallace, and E. Bonek, "Deficiencies of 'Kronecker' MIMO radio channel model," *Electron. Lett.*, vol. 39, no. 16, pp. 1209–1210, Aug. 2003.
- [19] A. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2563–2579, Oct. 2002.
- [20] D. Chizhik, J. Ling, P. W. Wolniansky, R. A. Valenzuela, N. Costa, , and K. Huber, "Multiple input multiple output measurements and modeling in Manhattan," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 321–331, Apr. 2003.
- [21] D. Chizhik, F. Rashid-Farrokh, J. Ling, and A. Lozano, "Effect of antenna separation on the capacity of BLAST in correlated channels," in *Proc. IEEE-APS Conference on Antennas and Propagation for Wireless Communications*, Nov. 2000, pp. 183–185.
- [22] M. O. Damen, A. Abdi, and M. Kaveh, "On the effect of correlated fading on several space-time coding and detection schemes," in *Proc. 54th Vehicular Technology Conference, Fall*, vol. 1, 2001, pp. 13–16.
- [23] H. Bolcskei and A. J. Paulraj, "Performance of space-time codes in the presence of spatial fading correlation," in *Proc. Asilomar Conference*, Sept. 2000, pp. 687–693.

- [24] J. Wang, M. K. Simon, M. P. Fitz, and K. Yao, "On the performance of space-time codes over spatially correlated Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 52, no. 6, pp. 877–881, June 2004.
- [25] E. A. Jorswieck and A. Sezgin, "Impact of spatial correlation on the performance of orthogonal space-time block codes," *IEEE Commun. Lett.*, vol. 8, no. 1, pp. 21–23, Jan. 2004.
- [26] R. Narasimhan, "Spatial multiplexing with transmit antenna and constellation selection for correlated MIMO fading channels," *IEEE Trans. Signal Processing*, vol. 51, no. 11, pp. 2829–2838, Nov. 2003.
- [27] D. Gore, R. W. Heath, and A. Paulraj, "Statistical antenna selection for spatial multiplexing systems," in *Proc. IEEE International Conference on Communications*, vol. 1, Apr. 2002, pp. 450–454.
- [28] D. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2580–2588, Oct. 2002.
- [29] R. van Nee, A. van Zelst, and G. Awater, "Maximum likelihood decoding in a space division multiplexing system," in *Proc. IEEE 51st Vehicular Technology Conference, spring*, vol. 1, May 2000, pp. 6–10.
- [30] A. van Zelst, "Space division multiplexing algorithms," in *Proc. 10th Mediterranean Electrotechnical Conference*, vol. 3, May 2000, pp. 1218–1221.
- [31] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE J. Select. Areas Commun.*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

-
- [32] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.
 - [33] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 451–459, Mar. 1999.
 - [34] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 1–4, Jan. 2001.
 - [35] C. B. Papadias and G. J. Foschini, "Capacity-approaching space-time codes for systems employing four transmitter antennas," *IEEE Trans. Inform. Theory*, vol. 49, no. 3, pp. 726–733, Mar. 2003.
 - [36] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal nonorthogonality rate 1 space-time block code for 3+ Tx antennas," in *Proc. IEEE 6th Symp. Spread-Spectrum Tech. and Applications*, 2000, pp. 429–432.
 - [37] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Combined array processing and space-time coding," *IEEE Trans. Inform. Theory*, vol. 45, no. 4, pp. 1121–1128, May 1999.
 - [38] M. Tao and R. S. Cheng, "Generalized layered space-time codes for high data rate wireless communications," *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1067–1075, July 2004.
 - [39] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. Inform. Theory*, vol. 48, no. 7, pp. 1804–1824, July 2002.
 - [40] R. W. Heath and A. J. Paulraj, "Linear dispersion codes for MIMO systems based on frame theory," *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2429–2441, Oct. 2002.

-
- [41] H. E. Gamal and M. O. Damen, "Universal space-time coding," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1097–1119, May 2003.
- [42] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1639–1642, July 1999.
- [43] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted mmse criterion," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2198–2206, Dec. 2001.
- [44] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Processing*, vol. 50, no. 5, pp. 1051–1064, May 2002.
- [45] P. Stoica and G. A. Ganesan, "Maximum-SNR spatial-temporal formatting designs for MIMO channels," *IEEE Trans. Signal Processing*, vol. 50, no. 12, pp. 3036–3042, Dec. 2002.
- [46] J. Boutros, E. Viterbo, C. Rastello, and J. C. Belfiore, "Good lattice constellations for both Rayleigh fading and gaussian channels," *IEEE Trans. Inform. Theory*, vol. 42, no. 2, pp. 502–518, Mar. 1996.
- [47] J. R. W. Heath and A. Paulraj, "Switching between multiplexing and diversity based on constellation distance," in *Proc. Allerton Conf. Commun., Contr., and Comput.*, Oct. 2000.
- [48] ———, "Antenna selection for spatial multiplexing systems based on minimum error rate," in *Proc. IEEE International Conference on Communications*, vol. 7, pp. 2276–2280.

-
- [49] L. Collin, O. Berder, P. Rostaing, and G. Burel, "Optimal minimum distance-based precoders for MIMO spatial multiplexing systems," *IEEE Trans. Signal Processing*, vol. 52, no. 3, pp. 617–627, Mar. 2004.
- [50] S. A. Jafar, S. Vishwanath, and A. Goldsmith, "Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback," in *Proc. IEEE International Conference on Communications*, vol. 7, June 2001, pp. 2266–2270.
- [51] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel correlation," *IEEE Trans. Inform. Theory*, vol. 49, no. 7, pp. 1673–169, July 2003.
- [52] H. Sampath and A. Paulraj, "Linear precoding for space-time coded systems with known fading correlations," *IEEE Commun. Lett.*, vol. 6, no. 6, pp. 239–241, June 2002.
- [53] O. Simeone and U. Spagnolini, "Combined linear pre-equalization and BLAST equalization with channel correlation feedback," *IEEE Commun. Lett.*, vol. 7, no. 10, pp. 487–489, Oct. 2003.
- [54] R. U. Nabar, H. Bolcskei, and A. J. Paulraj, "Transmit optimization for spatial multiplexing in the presence of spatial fading correlation," in *Proc. IEEE GLOBE-COM*, vol. 1, Nov. 2001, pp. 131–135.
- [55] H. Bolcskei, R. U. Nabar, V. Erceg, D. Gesbert, and A. J. Paulraj, "Performance of spatial multiplexing in the presence of polarization diversity," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, May 2001, pp. 2437–2440.

- [56] Z. Hong, R. W. H. K. Liu, and A. M. Sayeed, "Spatial multiplexing in correlated fading via the virtual channel representation," *IEEE J. Select. Areas Commun.*, vol. 21, no. 5, pp. 856–866, June 2003.
- [57] J. Akhtar and D. Gesbert, "A closed-form precoder for spatial multiplexing over correlated MIMO channels," in *Proc. IEEE GLOBECOM*, vol. 4, Dec. 2003, pp. 1847–1852.
- [58] J. H. Kotecha, Z. Hong, and A. M. Sayeed, "Coding and diversity gain tradeoff in space-time codes for correlated MIMO channels," in *Proc. IEEE GLOBECOM*, Dec. 2003, pp. 646–650.
- [59] W. Choi, R. Negi, and J. Cioffi, "Combined ML and DFE decoding for the V-BLAST system," in *Proc. IEEE International Conference on Communications*, vol. 3, June 2000, pp. 1243–1248.
- [60] B. Hassibi, "An efficient square-root algorithm for BLAST," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, vol. 2, 2000, pp. 737–740.
- [61] W. Zha and S. D. Blostein, "Modified decorrelating decision-feedback detection of BLAST space-time system," in *Proc. IEEE International Conference on Communications*, vol. 1, May 2002, pp. 335–339.
- [62] A. Duel-Hallen, "Decorrelating decision-feedback multiuser detector for synchronous code-division multiple-access channel," *IEEE Trans. Commun.*, vol. 41, no. 2, pp. 285–290, Feb. 1993.
- [63] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless

- channel,” in *URSI International Symposium on Signals, Systems, and Electronics*, 1998, pp. 295–300.
- [64] G. H. Golub and C. F. V. Loan, *Matrix Computation*. Johns Hopkins University Press, 1996.
- [65] R. W. Heath, S. Sandhu, and A. J. Paulraj, “Antenna selection for spatial multiplexing systems with linear receivers,” *IEEE Commun. Lett.*, vol. 5, no. 4, pp. 142–144, Apr. 2001.
- [66] J. Salz and J. H. Winters, “Effect of fading correlation on adaptive arrays in digital mobile radio,” *IEEE Trans. Veh. Technol.*, vol. 43, no. 4, pp. 1049–1057, Nov. 1994.
- [67] M. T. Ivrlac, T. P. Kurpjuhn, C. Brunner, and W. Utschick, “Efficient use of fading correlations in MIMO systems,” in *Proc. 54th IEEE Vehicular Technology Conference, Fall*, vol. 4, pp. 2763–2767.
- [68] O. Simeone, Y. Bar-Ness, and U. Spagnolini, “Linear and nonlinear preequalization/equalization for MIMO systems with long-term channel state information at the transmitter,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 2, pp. 373–378, Mar. 2004.
- [69] L. Zheng and D. N. C. Tse, “Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels,” *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.
- [70] S. Baro, G. Bauch, A. Pavlic, and A. Semmler, “Improving BLAST performance using space-time block codes and turbo decoding,” in *Proc. IEEE GLOBECOM*, vol. 2, Nov. 2000, pp. 1067–1071.
- [71] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

-
- [72] H. Shin and J. H. Lee, "Exact symbol error probability of orthogonal space-time block codes," in *Proc. IEEE GLOBECOM*, vol. 2, Nov. 2002, pp. 1197–1201.
- [73] S. H. Nam, O. S. Shin, and K. B. Lee, "Transmit power allocation for a modified V-BLAST system," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1072–1079, 2004.
- [74] S. T. Chung, A. Lozano, and H. C. Howard, "Approaching eigenmode BLAST channel capacity using V-BLAST with rate and power feedback," in *Proc. IEEE 54th Vehicular Technology Conference, Fall*, vol. 2, Oct. 2001, pp. 915–919.
- [75] B. M. Hochwald and T. L. Marzetta, "Adapting a downlink array from uplink measurements," *IEEE Trans. Signal Processing*, vol. 49, no. 3, pp. 642–653, Mar. 2001.
- [76] X. Li and Z. Xie, "Performance losses in V-BLAST due to correlation," *IEEE Antennas Wireless Propagat. Lett.*, vol. 3, pp. 291–294, 2004.
- [77] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. New Jersey: Prentice Hall, PTR, 1993.
- [78] M. O. Damen, A. Tewfik, and J. C. Belfiore, "A construction of a space-time code based on number theory," *IEEE Trans. Inform. Theory*, vol. 48, no. 3, pp. 753–760, Mar. 2002.