Research and development of optical filters for wavelength division multiplexing systems

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Research and Development of Optical Filters for Wavelength Division Multiplexing Systems

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2005

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Date

Zheng Rui Tao

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Abstract

Due to the advantages of low insertion loss, low polarization sensitivity, compactness, low cost, all-fiber geometry and ease of fabrication, fiber Bragg gratings (FBGs) have evolved into critically important optical filters for a wide variety of applications in wavelength division multiplexed (WDM) systems. This thesis investigates the theoretical issues involved in the synthesis or design of optical filters based on FBGs as well as the experimental issues involved in the fabrication of optical filters based on chirped fiber Bragg gratings (CFBGs).

In many system applications, it is critically important to find a grating with a desired complex reflection response (i.e. reflection spectrum and phase response). As such, this thesis studies various optimization methods for the development of a powerful and versatile methodology for the synthesis or design of FBG-based filters with complicated characteristics. As a result, a new staged continuous tabu search (SCTS) optimization algorithm is developed for solving global optimization problems. A novel synthesis method based on the proposed SCTS algorithm is developed for the design of optical bandpass filters and linear phase optical filters based on FBGs. As a further improvement on this SCTS-based synthesis method, a novel two-stage hybrid optimization method is proposed for the synthesis of FBG-based filters with more complicated characteristics. Using this hybrid method, an optical bandpass filter

is designed, fabricated and tested. To further demonstrate the effectiveness of the hybrid method, three linear phase optical filters with different grating lengths are designed using this method.

Compared with a uniform FBG, a CFBG is a grating with its Bragg wavelength varying linearly or nonlinearly along the grating length. CFBGs have been widely used for compensation of fiber dispersion, for increasing the data rate, and for pulse compression. The stretching-and-writing method is investigated in great detail for tailoring the profiles of chirped FBGs with stepped-chirped profiles, and the experimental results are in good agreement with the theoretical predictions. In addition, the method is further improved to enable the fabrication of chirped FBGs with continuously chirped profiles, and the fabricated FBGs have nonlinear group delay characteristics and the experimental results agree well with the theoretical predictions.

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List of Abbreviations

AFBG	Apodized fiber Bragg grating
ASA	Active simulated annealing
AWG	Arrayed waveguide grating
CFBG	Chirped fiber Bragg grating
CTS	Continuous tabu search
EDFA	Erbium-doped fiber amplifier
FBG	Fiber Bragg grating
GA	Genetic algorithm
GLM	Gel'fand-Levitan-Marchenko
IGA	Improved genetic algorithm
NCFBG	Nonlinearly chirped fiber Bragg grating
SA	Simulated annealing
SCTS	Staged continuous tabu search
TFF	Thin film filter
TMM	Transfer matrix method
TS	Tabu search
UV	Ultra violet
VSF	Vestigial single sideband
WDM	Wavelength division multiplexing

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1 Introduction

1.1 Background

Optical filters are now key elements in optical communication systems especially in those systems using the wavelength division multiplexing (WDM) technique [1–2]. They have thus attracted considerable attention in the last few years and filters with different characteristics have been designed and fabricated to meet the system requirements.

A fiber Bragg grating (FBG) is essentially a filter written into the core of a segment of optical fiber via the interference of two ultraviolet (UV) beams from a UV laser (as shown in Fig. 1-1).

Figure 1-1 Schematic diagram of a typical interferometric system used for the fabrication of fiber Bragg gratings.

As shown in Fig. 1-1, the interference pattern forms a periodic refractive index change (or index perturbation) longitudinally along the fiber. Each refractive index change or jump acts as a series of reflectors, reflecting back a small amount of light with wavelength that corresponds to the Bragg wavelength. The Bragg wavelength, λ_B , of an FBG is the wavelength that fulfills the Bragg condition, that is,

$$\lambda_B = 2\bar{n}_{eff}\Lambda \quad (1.1)$$

where $\bar{n}_{eff}$ is the average effective index over the grating and Λ is the perturbation period or grating period. Because there are typically tens of thousands of these perturbation periods of index changes or reflectors in a row, an FBG-based filter generally has excellent characteristics such as an almost-

squared reflective spectrum. Compared with two other main types of optical filters, namely, thin film filters (TFFs) and arrayed waveguide gratings (AWGs), FBG-based filters have many advantages such as small size, low loss, low polarization sensitivity, all-fiber geometry, easy fabrication, and low cost [3].

The formation of permanent gratings by photosensitivity in an optical fiber was first demonstrated by Hill *et al.* in 1978 [4]. Photosensitivity means that exposure of a doped fiber with UV light will increase the refractive index of the doped fiber. Typical values for the index change are between 10^{-6} to 10^{-3} , depending on the intensity of the UV exposure and the types of dopants in the fiber. Using techniques such as hydrogen loading, an index change of as high as 10^{-2} can be achieved [5].

Many efforts have been put into the fabrication of different types of FBGs, such as the interferometric technique, point-by-point method, and phase-mask techniques. Meltz and co-workers were the first to demonstrate the interferometric fabrication technique [6]. They utilized an interferometer to split the incoming UV light into two beams and subsequently recombine the two beams to form an interference pattern to side-expose a photosensitive fiber. Because only the core was doped, a permanent refractive index modulation can be induced in the core [7]. The point-by-point technique for fabricating FBGs is accomplished

by inducing a change in the index of refraction corresponding to a grating plane one step at a time along the core of the fiber [8]. However, the phase-mask technique is one of the most effective methods because it employs a simple diffractive optical element (or a phase mask) to spatially modulate the UV beam [9]. To write gratings with an arbitrary index modulation profile, the scheme suggested by Asseh *et al.* [10] may be used.

1.2 Motivation

An FBG can be designed and fabricated with a complex spectral response and it thus has a variety of applications in WDM systems [7], such as wavelength selective devices [11], dispersion compensation [12], pulse compression [13–14], and pulse multiplication [15–16]. In addition, any change in the fiber properties, such as strain or temperature will change the modal index or grating pitch, and thus will change the Bragg wavelength of the grating. Thus, FBGs can also be used as sensing devices with applications ranging from structural monitoring to chemical sensing [17].

To design fiber gratings for a wide variety of system applications, it is crucial to have sound mathematical tools for the analysis, synthesis and characterization of

fiber gratings. It is well known that the coupled-mode theory [18] can be used to analyze the wave propagation in a grating when the structure of the grating (such as index modulation profile of the grating) is given. However, many practical applications will require the synthesis or design of FBGs with prescribed characteristics.

The synthesis of FBGs is to find a grating structure from a specified or prescribed complex reflective spectrum (i.e. reflective spectrum and phase response) [19–25]. The simplest approach is to use the approximate Fourier relation between the reflective coefficient and the coupling coefficient of a grating. This method is only suitable for the design of weak gratings. For strong gratings (i.e. high-reflectivity gratings), one can determine the coupling coefficient using classical inverse scattering techniques. Song and Peral *et al.* [19–20] have shown how one can design corrugated gratings by solving two coupled integral equations which are called the Gel'fand-Levitan-Marchenko (GLM) equations. However, this algorithm is quite complicated, and the results are not always accurate for highly reflecting gratings. Another approach for solving the inverse scattering problem is the differential inverse scattering method, which is also referred to as the layer-peeling method. This method has been applied to the design of several types of FBGs [21–23], but their designed profiles (e.g. index modulation profiles) typically have long grating lengths, making practical realization difficult. Moreover, when specified or prescribed ideal filter

characteristics are given, it is desirable to have a weighting mechanism to weight the different target requirements of the desired filter responses. For example, when designing an optical bandpass filter, one may need to weight the linear phase more than the sharp spectral peak because the filter dispersion is a more critical parameter. Neither the GLM method nor the layer-peeling algorithm can support such a mechanism in a satisfactory way.

To overcome these drawbacks, optimization techniques have been employed [24–25]. Compared with the design methods as described earlier, optimization techniques can facilitate the task of weighting the different requirements of the complex spectrum of the filter [24]. Another advantage is that the filter designed by the optimization methods can be more practically realized by imposing additional constraints to suit the fabrication conditions. Skaar and Bae *et al.* [24–25] employed the standard genetic algorithm (GA) and simulated annealing algorithm (SA) directly to design several types of optical filters; however these algorithms are not powerful enough for the design of optical filters with more complex characteristics. In addition, they used an FBG model based on piecewise uniform grating sections, which always involves a large number of variables for the global optimization problems. It is known that a large number of variables always make the global optimization problem complicated. Thus it is still difficult to use these methods for solving more complicated problems such as the design of a linear phase optical filter, which requires both a target reflective

spectrum and a target dispersion response to be designed with high accuracy.

Chirped fiber Bragg gratings (CFBGs) are gratings with their Bragg wavelengths varying linearly or nonlinearly along the grating length. CFBGs have been widely used for dispersion compensation, pulse multiplication and pulse compression [12], [13], [15], [26], [27]. However, due to the high cost of either a linearly chirped or a nonlinearly chirped phase mask, various techniques have been proposed to fabricate CFBGs using a uniform phase mask, which include temperature gradient or strain gradient method [28], dual-scanning technique [29], and shifting the Bragg wavelength by adding a converging lens before the mask [30]. However, these methods are generally not stable enough to enable fabrication of high quality CFBGs. Byron *et al.* [31] proposed a stretching-and-writing method for the fabrication of CFBGs. Compared with other approaches, the method proposed by Byron *et al.* [31] is comparably simple; only one additional motorized stage is required to be inserted into the standard UV exposure system for inscribing CFBGs. However, the method has not been investigated in great detail, and tailoring of CFBGs with an arbitrary grating profile is still a challenge.

1.3 Objectives

The main objective of this thesis is on the design and development of optical fiber based filters for application in WDM systems. One aim of the thesis is to develop an efficient synthesis method of FBGs based on some powerful optimization techniques. First, a novel global optimization algorithm, namely, a staged continuous tabu search (SCTS) algorithm has been developed as described in Chapter 3 [32]. The efficiency of the SCTS algorithm is demonstrated by testing a set of benchmark functions. Using the SCTS method and a model of cascaded apodized FBGs, the designs of an FBG-based bandpass filter and an FBG-based linear phase filter are presented [33]. A hybrid method combining the SCTS algorithm with a local optimization method, namely, the Quasi-Newton algorithm, is also proposed to increase the optimization efficiency [34].

Another aim of the thesis is to investigate in great detail the stretching-and-writing technique [31] for the fabrication of CFBGs with arbitrary chirped profiles for different system applications. Stepped-chirp fiber Bragg gratings on a pre-stretched fiber with arbitrary group delay responses are designed and fabricated. As an extension of this method, a novel method is proposed [35] for the inscription of asymmetric chirped gratings with nonlinear group delay responses using a uniform phase mask.

1.4 Major Contributions

The original contributions of this thesis are summarized below.

- A new SCTS algorithm is developed for global optimization problems (see Chapter 3).
- A new SCTS synthesis method of FBGs is demonstrated by designing a 25-GHz optical bandpass filter and a 50-GHz linear phase optical filter. The optical bandpass filter is based on a model of piecewise uniform FBGs and the linear phase optical filter is based on a model of cascaded apodized FBGs (see Chapter 4).
- A new two-stage hybrid method is proposed for the optimization of FBGs. An optical bandpass filter is designed and fabricated to demonstrate the effectiveness of the method. Three linear phase optical filters with different grating lengths are also designed (see Chapter 5).
- The stretching-and-writing method is studied in great depth. Using the method, a linearly-chirped FBG and a nonlinearly-chirped FBG are designed and fabricated using a uniform phase mask (see Chapter 6).
- A new method is proposed for the writing of nonlinearly-chirped gratings in fibers under pre-stretched conditions. The fabricated grating has a continuous chirped profile and an asymmetric spectrum (see Chapter 7).

1.5 Outline

The introduction of FBGs is given in Chapter 2. First, the coupled-mode theory is reviewed and the connection between the mathematical model and the physical quantities is described. Section 2.2 presents the well-known methods for solving the coupled-mode equations numerically. Both the numerical integration method (Runge-Kutta) and the commonly used transfer matrix method (TMM) are described in Section 2.3. Using the TMM, various types of FBGs introduced in Section 2.1 are solved numerically as presented in Section 2.4. Section 2.5 presents techniques for the apodization of FBGs.

Physically, the properties of an FBG are determined by the index modulation and the grating period. To tailor the FBG's properties, one can thus either modulate the refractive index or vary the grating period, or both. Chapters 3 to 5 focus on the concept of index modulation while Chapters 6 and 7 discuss the variation of the grating period.

Chapter 3 presents the proposed SCTS algorithm. First, a standard tabu search algorithm and a continuous tabu search algorithm are introduced in Section 3.1 and Section 3.2, respectively. The detailed description of the SCTS algorithm is presented in Section 3.3. The efficiency of the SCTS algorithm is tested by using

a set of benchmark functions which are listed in the Appendix.

Chapter 4 describes the application of the SCTS algorithm to the synthesis of FBGs. Using an FBG model of piecewise uniform sections, an optical bandpass filter is designed as presented in Section 4.3, and a linear phase optical filter based on a model of apodized FBGs in cascade is also designed as described in Section 4.4.

Chapter 5 presents a hybrid optimization concept, and the proposed hybrid method is employed to solve the synthesis problem of FBGs. Section 5.2 describes the flowchart of the method. Section 5.3 presents the design and fabrication of an optical bandpass filter, and Section 5.4 presents the design of three linear optical phase filters with different grating lengths.

Chapter 6 presents a method for the fabrication of chirped gratings with a uniform phase mask (i.e. the stretching-and-writing method). Section 6.2 presents an experimental setup used for the fabrication of stepped-chirp FBGs using this method. The relationship between the grating parameters and the fabrication conditions is analyzed. Section 6.3 describes the fabrication of a linearly-chirped FBG and a nonlinearly-chirped FBG using the fabrication method presented in Section 6.2.

Chapter 7 presents a new method for the fabrication of a nonlinearly chirped grating in a pre-stretched fiber. Section 7.2 describes the experimental setup of the method. Section 7.3 analyzes the method, and a grating fabricated using the method is presented in Section 7.4.

Finally, Chapter 8 presents a summary of the thesis as well as the recommendations for future work.

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2 Theory of Fiber Bragg Grating

In this chapter, the mathematical models used in simulating the behavior of fiber Bragg gratings (FBGs) are investigated. The coupled-mode theory, which has been widely used for the analysis of the field propagation in corrugated structures, is briefly reviewed. The coupled-mode theory and the commonly used numerical techniques are used for computing the reflection and transmission spectra of fiber gratings, and some results are presented.

First, the physical structures and types of FBGs are introduced in Section 2.1. Section 2.2 describes the coupled-mode theory. Section 2.3 derives the analytical solution of a uniform FBG with a constant refractive index modulation and period and presents two simple and fast techniques for analyzing more complex grating structures. As examples to calculate the reflective and transmissive spectra of the grating, Section 2.4 uses the transfer matrix method (TMM) to determine the spectral responses of the different types of FBGs as described in Section 2.1. Section 2.5 introduces apodization techniques of FBGs and

compares the spectra of apodized gratings with four types of apodization profiles.

2.1 Introduction of FBGs

As described in Chapter 1, an FBG is formed by a periodic refractive index change (or index perturbation) longitudinally along the fiber core. Figure 2-1 shows the schematic diagram of a grating, where the grating period is represented by Λ and the average effective refractive index is given by $\bar{n}_{eff} = n_0 + \Delta n_{dc}$, where n_0 is the effective index without UV exposure and Δn_{dc} is the “dc” index change spatially averaged over the grating period.

Figure 2-1 Example of an FBG and its refractive index profile [1].

From Fig. 2-1, the effective refractive index $n_{eff}(z)$ has a profile that can be assumed as

$$n_{eff}(z) = \bar{n}_{eff}(z) + \Delta n_{ac}(z) \cos \left[\frac{2\pi z}{\Lambda} + \phi(z) \right] \quad (2.1)$$

where $\Delta n_{ac}(z)$ is the “ac” index change (i.e. index modulation), $\phi(z)$ describes the grating chirp and z ($0 \leq z \leq L_g$) is the longitudinal coordinate along the grating length with L_g being the grating length.

Thus the optical properties of a fiber grating are essentially determined by the variation of the induced index change (including the “dc” and “ac” index changes) along the fiber axis z . Depending on the index variations, there are different types of FBGs. Uniform FBGs are gratings that have a uniform pitch and constant index modulation. Currently, non-uniform FBGs are preferred such as apodized FBGs which have high side-lobe suppression. Some common types of fiber gratings can be classified according to the variation of the induced index change along the fiber axis. For example, such types of gratings are listed below.

- a. Uniform FBG
- b. Apodized FBG with variable-dc index change
- c. Apodized FBG with zero-dc index change (or constant-dc index change)
- d. Chirped FBG

- e. Phase-shift FBG
- f. Super-structured FBG (or sampled FBG).

The index modulation profiles of these FBGs are shown in Fig. 2-2 below.

Figure 2-2 Index modulation profiles of some common types of fiber gratings [2]. (a) Uniform FBG. (b) Apodized FBG with variable-dc index change. (c) Apodized FBG with zero-dc index change (or constant-dc index change). (d) Chirped FBG. (e) Phase-shift (e.g. π phase shift) FBG. (f) Super-structured FBG.

The size of the grating period relative to the grating length has been greatly enlarged for the purpose of illustration. The reflective spectra of the gratings with the index modulation profiles illustrated in Fig. 2-2 will be discussed in Section 2.4.

2.2 Coupled-Mode Theory

The relation between the spectral response of a fiber grating and the corresponding grating structure is usually described by the coupled-mode theory [2–5]. While other techniques are available, only the coupled-mode theory is considered here because it is straightforward and intuitive. Furthermore, it accurately models the optical properties of most fiber gratings of interests. The coupled-mode theory has been described in detail in a number of texts [6–9]. Thus the details of the coupled-mode theory are presented in Appendix A. The notations in Appendix A follow closely with those of [9].

For a uniform fiber grating, the z -dependence of the index perturbation $\delta n_{eff}(z)$ is approximately quasi-sinusoidal in the sense as shown in Eq. (2.1), and it can be written as

$$\delta n_{eff}(z) = n_{eff}(z) - n_0 = \Delta n_{dc} + \Delta n_{ac} \cos\left[\frac{2\pi z}{\Lambda} + \phi\right] \quad (2.2)$$

where Δn_{dc} , Δn_{ac} and ϕ are constant values for uniform gratings.

The fiber is assumed to be lossless and single mode in the wavelength range of

interest. Thus only one forward and one backward propagating mode are considered. By substituting Eq. (2.2) into Eqs. (A2)–(A4), the well-known coupled-mode equation can be derived. In the coupled-mode equation, the amplitude of the forward mode and the backward mode can be simplified as follows [2]

$$\frac{dE_f}{dz} = i\hat{\sigma}E_f(z) + i\kappa E_b(z) \quad (2.3)$$

$$\frac{dE_b}{dz} = -i\hat{\sigma}E_b(z) - i\kappa^* E_f(z) \quad (2.4)$$

where the forward mode is represented by $E_f(z) \equiv A(z) \exp[i\delta z - \phi(z)/2]$ and the backward mode is represented by $E_b(z) \equiv B(z) \exp[i\delta z - \phi(z)/2]$. $A(z)$ and $B(z)$ are the slowly varying amplitudes of the forward mode and backward mode respectively, κ is the ‘ac’ coupling coefficient, κ^* is the complex conjugate of κ . And $\hat{\sigma}$ is the ‘dc’ coupling coefficient, which can be defined for a single-mode Bragg grating as

$$\kappa = i \frac{\pi}{\lambda} \Delta n_{ac} \quad (2.5)$$

$$\hat{\sigma} = \delta + \sigma \quad (2.6)$$

where $\delta = \beta - \frac{\pi}{\Lambda} = 2\pi\bar{n}_{eff}\left(\frac{1}{\lambda} - \frac{1}{\lambda_B}\right)$ is the detuning parameter at the wavelength λ with β being the propagation constant and λ_B being the Bragg wavelength which has been defined in Eq. (1.1). The coefficient σ is defined in Eq. (A5), and it can be defined for a single-mode FBG as

$$\sigma = \frac{2\pi}{\lambda} \Delta n_{dc} \quad (2.7)$$

If the grating phase ϕ is not a constant but a function of grating length z , the “dc” coupling coefficient $\hat{\sigma}$ in Eq. (2.6) becomes

$$\hat{\sigma} = \delta + \sigma - \frac{1}{2} \cdot \frac{d\phi}{dz} \quad (2.8)$$

The phase term $\frac{1}{2} \cdot \frac{d\phi}{dz}$ describes the possible chirp of the grating period, where $\phi(z)$ is defined in Eqs. (2.1) and (2.2). For a chirped grating, the phase term $\frac{1}{2} \cdot \frac{d\phi}{dz}$ is given by [2]

$$\frac{1}{2} \frac{d\phi}{dz} = -\frac{4\pi\bar{n}_{eff}}{\lambda_B^2} \frac{d\lambda_{eff}}{dz} \quad (2.9)$$

where λ_{eff} is the “effective” Bragg wavelength and the “chirp” $\frac{d\lambda_{eff}}{dz}$ is a measure of the rate of change of the “effective” Bragg wavelength with the position of the grating, and is usually given in units of nm/cm.

By solving the coupled-mode equations (2.3) and (2.4), one can find the complex reflection coefficient of the grating $\rho(\lambda) = \frac{E_b(0; \lambda)}{E_f(0; \lambda)}$, where $E_b(0; \lambda)$ and $E_f(0; \lambda)$ are the backward and forward electric fields at position $z = 0$. Then the reflective spectrum can be calculated by $R = |\rho|^2$, and the phase response is $\theta = \arg(\rho)$.

2.3 Numerical Solution Methods

The solutions of the coupled-mode equations as defined in Eqs. (2.3) and (2.4) must satisfy two appropriate boundary conditions, i.e. $E_f(0) = 1$ and $E_b(L_g) = 0$.

2.3.1 Analytical Solutions of Uniform FBGs

A uniform grating has a constant coupling coefficient κ and a constant value of σ over the grating length L_g . In this situation, the coupled-mode equations can be solved analytically.

By differentiation of Eqs. (2.3) and (2.4), one can obtain $\frac{d^2 E_f}{dz^2} = (\kappa^2 - \hat{\sigma}^2)E_f$

and $\frac{d^2 E_b}{dz^2} = (\kappa^2 - \hat{\sigma}^2)E_b$. By solving these simple equations, one can obtain

expressions for the reflective coefficient ρ and the transmission coefficient t_x as follows

$$\rho = \frac{-i\kappa \sinh(SL_g)}{S \cosh(SL_g) - i\hat{\sigma} \sinh(SL_g)} \quad (2.10)$$

$$t_x = \frac{S}{S \cosh(SL_g) - i\hat{\sigma} \sinh(SL_g)} \quad (2.11)$$

where the parameter S is defined as

$$S^2 = \kappa^2 - \hat{\sigma}^2 \quad (2.12)$$

Thus, the reflective spectrum and the transmissive spectrum can be obtained by

$$R = |\rho|^2 \quad \text{and} \quad T = |t_x|^2, \text{ respectively.}$$

2.3.2 Direct Numerical Integration Method for Non-Uniform FBGs

As described above, the complex reflection coefficient can be further defined as

$$\rho(z; \lambda) = \frac{E_b(z; \lambda)}{E_f(z; \lambda)} \quad (2.13)$$

where $0 \leq z \leq L_g$. By differentiating both sides of Eq. (2.13) with respect to z and substituting the result into the coupled-mode equations (2.3) and (2.4), the following Riccati equation [10] can be obtained

$$\frac{d\rho(z; \lambda)}{dz} = -2i\hat{\sigma}\rho - \kappa(z)\rho^2 + \kappa^*(z) \quad (2.14)$$

By applying the boundary condition $\rho(L_g; \lambda) = 0$, one can start at the end of the

grating and use the Runge-Kutta methods [11] to work the equation backwards to $z = 0$. The reflection coefficient of the grating $\rho(0; \lambda)$ can then be obtained.

Although the Runge-Kutta method is simple, it requires a large number of steps to ensure convergence of the solution. Therefore, in some cases this method could be slow to yield the solution compared to the transfer matrix method which is to be described next. In addition, compared to the transfer matrix method, this method cannot be applied here to solve the Riccati equation because the phase shifts cannot be modeled or incorporated into the Riccati equation.

2.3.3 Transfer Matrix Method for Non-Uniform FBGs

In the transfer matrix method (TMM), a non-uniform FBG is divided into a number of serially-connected uniform sub-gratings or sections (as shown in Fig. 2-3). Every uniform section has an analytic transfer matrix. The transfer matrix for the entire structure can be obtained by multiplying the individual transfer matrices.

Figure 2-3 Schematic diagram of piecewise-uniform FBGs for modeling a non-uniform FBG.

In Fig. 2-3, $E_f(j;\lambda)$ and $E_b(j;\lambda)$ are the complex electric fields of the forward and backward propagation waves, respectively, describing the j^{th} section.

In the figure, δl_j , Λ_j , $\Delta n_{ac,j}$ and $\bar{n}_{eff,j}$ are the length, grating period, amplitude of index modulation and average effective index of the j^{th} section, respectively. L_g is the length of the whole grating. The electric fields at the input and output ports of the FBG are given by

$$\begin{bmatrix} E_f(0;\lambda) \\ E_b(0;\lambda) \end{bmatrix} = T_1 \cdot T_2 \cdots T_N \begin{bmatrix} E_f(N;\lambda) \\ E_b(N;\lambda) \end{bmatrix} \quad (2.15)$$

where $T_j = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$ is the 2×2 transfer matrix of the j^{th} section of the FBG.

The elements of the transfer matrix are defined as

$$T_{11} = \cosh(S_j \delta l_j) - \frac{i \hat{\sigma}_j}{S_j} \sinh(S_j \delta l_j) \quad (2.16)$$

$$T_{22} = \cosh(S_j \delta l_j) + \frac{i\hat{\sigma}_j}{S_j} \sinh(S_j \delta l_j) \quad (2.17)$$

$$T_{12} = -\frac{i\kappa_j}{S_j} \sinh(S_j \delta l_j) \quad (2.18)$$

$$T_{21} = \frac{i\kappa_j}{S_j} \sinh(S_j \delta l_j) \quad (2.19)$$

where κ_j , $\hat{\sigma}_j$ and S_j can be obtained from Eqs. (2.5), (2.6) and (2.12), respectively.

Replacing the N multiplied 2×2 matrices in Eq. (2.15) by a single 2×2 matrix, the transfer function of the whole grating can be simply expressed as

$$\begin{bmatrix} E_f(0; \lambda) \\ E_b(0; \lambda) \end{bmatrix} = T \cdot \begin{bmatrix} E_f(N; \lambda) \\ E_b(N; \lambda) \end{bmatrix} \quad (2.20)$$

where the matrix T is

$$T = \prod_{j=1}^N T_j \quad (2.21)$$

Applying the boundary condition $E_b(N; \lambda) = 0$ (i.e. there is no input to the right side of the FBG), the reflection coefficient ρ and the transmission coefficient t_x , which are functions of wavelength, are given by

$$\rho(\lambda) = \frac{E_b(0; \lambda)}{E_f(0; \lambda)} \quad (2.22)$$

$$t_x(\lambda) = \frac{E_f(N; \lambda)}{E_f(0; \lambda)} \quad (2.23)$$

The reflective spectrum and the transmissive spectrum can then be obtained by $R = |\rho|^2$ and $T = |t_x|^2$. The delay time τ_ρ of light reflected off a grating corresponds to the phase change of ρ relative to the wavelength λ , and is given by [2]

$$\tau_\rho = -\frac{\lambda^2}{2\pi c} \cdot \frac{d\theta_\rho}{d\lambda} \quad (2.24)$$

where θ_ρ is the cumulative phase of ρ and c is the speed of light in vacuum.

The dispersion of the grating d_ρ is therefore given by [2]

$$d_\rho = \frac{d\tau_\rho}{d\lambda} = -\frac{\lambda^2}{2\pi c} \left(\frac{d^2\theta_\rho}{d\lambda^2} + \frac{2}{\lambda} \cdot \frac{d\theta_\rho}{d\lambda} \right) \quad (2.25)$$

To apply the TMM for apodized and chirped gratings, one simply needs to assign constant values of σ , κ , and $\frac{1}{2} \cdot \frac{d\phi}{dz}$ to each uniform section, where these might be the z -dependent values at the center of each section. For phase-shifted and sampled gratings, a phase-shift matrix $T_{phase, j}$ must be inserted between the factors T_j and T_{j+1} in the product in Eq. (2.21). The phase-shift matrix can be calculated as [3]

$$T_{phase, j} = \begin{bmatrix} \exp\left(\frac{-i\phi_j}{2}\right) & 0 \\ 0 & \exp\left(\frac{i\phi_j}{2}\right) \end{bmatrix} \quad (2.26)$$

where ϕ_j is the shift in the phase of the grating itself for discrete phase shifts (see Fig. 2-2(e)). For sampled gratings (see Fig. 2-2(f)), ϕ_j can be defined as [3]

$$\frac{\phi_j}{2} = \frac{2\pi n_{eff}}{\lambda} \Delta z_0 \quad (2.27)$$

where Δz_0 is the separation between two grating sections.

2.4 Application of TMM to Various Types of FBGs

The TMM has been discussed above in details. In this section, the method is used to calculate the spectra of different types of FBGs as listed in Fig. 2-2.

Figure 2-4 shows the reflective spectrum and group delay response of a 20-mm long uniform FBG. The index modulation (“ac” index change) is 1×10^{-4} . The index profile of this grating is illustrated in Fig. 2-2(a), which has a constant index modulation and a constant grating period. Typically, relatively high side-lobe levels (more than -10 dB) are found on both sides of the center reflectivity peak.

Figure 2-4 Reflective spectrum and group delay response of a uniform FBG with $L_g = 20$ mm and $\Delta n_{ac} = 1 \times 10^{-4}$.

Fiber gratings are not infinite in length so they have a beginning and an end; thus, they begin abruptly and end abruptly. The Fourier transform of such a grating with a “rectangular” index function immediately yields the well-known sinc function, which is associated with the side-lobe structure in the reflective spectrum (see Fig. 2-4). Conversely, a grating with a similar index modulation profile (e.g. Gaussian index profile) will have its side lobes diminished substantially. The suppression of the side lobes in the reflective spectrum by gradually increasing the coupling coefficient with penetration into, as well as gradually decreasing on exiting from, the grating is called apodization. However, simply changing the index modulation profile also causes the “dc” index change

along the grating length (see Fig. 2-2(b)). As a result, the local Bragg wavelength also changes, and a distributed Fabry–Perot interferometer pattern is formed [12], which causes the side lobes to appear on the short wavelength side of the reflective spectrum of the grating. Figure 2-5 shows the reflective spectrum and group delay response of such a Gaussian-apodized FBG with variable-dc index change. The grating length is 20 mm and the maximum index modulation is 1×10^{-4} . To prevent such a side lobe from occurring, the key is to maintain an unchanging average refractive index throughout the length of the grating while gradually altering the index modulation profile (see Fig. 2-2(c)).

Figure 2-5 Reflective spectrum and group delay response of a Gaussian-apodized FBG with variable-dc index change. $L_g = 20$ mm and the maximum index modulation is 1×10^{-4} .

Figure 2-6 shows the reflective spectrum and group delay response of a

Gaussian-apodized FBG with zero-dc index change (or constant-dc index change). The index profile of the grating is illustrated in Fig. 2-2(c), which has a Gaussian-function index modulation profile. The grating length is 20 mm and the maximum index modulation is 1×10^{-4} . The grating is observed with a high side-lobe suppression level (less than -60 dB) in the reflective spectrum. However, the in-band group delay response shows significant ripples, which will give rise to in-band dispersion.

Figure 2-6 Reflective spectrum and group delay response of a Gaussian-apodized FBG with zero-dc index change (or constant-dc index change). $L_g = 20$ mm and the maximum index modulation is 1×10^{-4} .

Figure 2-7 shows the reflective spectrum and group delay response of a linearly-chirped FBG. The corresponding index modulation profile of the grating is illustrated in Fig. 2-2(d), in which the grating period is linearly

decreased. The grating length is 20 mm and the maximum index modulation is 2×10^{-4} . The grating is observed with a linear group delay response and a wide band reflective spectrum.

Figure 2-7 Reflective spectrum and group delay response of a linearly-chirped FBG. $L_g = 20$ mm and index modulation $\Delta n_{ac} = 2 \times 10^{-4}$. The chirp rate is 1.4 nm/cm.

Figure 2-8 shows the reflective spectrum and group delay response of a 20-mm long phase-shifted FBG. The index modulation (“ac” index change) is 1×10^{-4} . The index modulation profile of the grating is illustrated in Fig. 2-2(e), which has a π phase shift inserted in the center position of the grating. It is found that the π phase shift of the grating at the center of the grating opens a narrow transmission resonance (or a notch depth) at the designed wavelength.

Figure 2-8 Reflective spectrum of an FBG with insertion of a π phase shift at the center of the grating. $L_g = 20$ mm and $\Delta n_{ac} = 1 \times 10^{-4}$.

Figure 2-9 shows the reflective spectrum of a 100-mm long periodic super-structured FBG [2]. The index modulation profile is shown in Fig. 2-2(f), in which a uniform grating except for regions where the “ac” index change is set to zero. The “ac” index change in the non-zero regions is 2.5×10^{-4} , and there are 50 sections of such non-zero regions. It is found that a multi-channel filter can be designed using this index modulation profile as shown in Fig. 2-9.

Figure 2-9 Reflective spectrum of a uniform grating with periodic superstructure, where the 100-mm long grating has 50 200- μm long grating sections spaced 1800 μm apart and $\Delta n_{ac} = 2.5 \times 10^{-4}$ [2].

2.5 Apodization of FBG

For the numerical analysis commonly used for the computation of the spectral response and also for defining the functions for apodization and chirp, it is convenient to define the index perturbation function as

$$\delta n(z) = \bar{\Delta n}_{ac} \cdot f(z) \cdot \cos\left[\frac{2\pi}{\Lambda} + \phi(z)\right] + \Delta n_{dc} \quad (2.28)$$

where $\bar{\Delta n}_{ac}$ is the peak amplitude (maximum) of the “ac” effective index change over the grating, Λ is the grating period, $f(z)$ is the normalized apodization shading function, $\phi(z)$ describes the grating chirp, and Δn_{dc} should be a constant value to avoid the presence of the side lobe in the short wavelength region of the reflective spectrum (see Fig. 2-5). The general idea of using TMM to solve such an apodized FBG is that the grating structure can be divided into a number of uniform grating sections, and each of these sections is described by an analytic transfer matrix. The transfer matrix for the entire structure can be obtained by multiplying the individual transfer matrices. Consequently, the final reflective spectrum and the phase response can be obtained by applying the initial values and boundary conditions to the grating.

Some commonly used functions are as follows [13],

1. Raised cosine: $f(z) = \frac{1}{2} \left\{ 1 + \cos \left[\frac{\pi(z - L_g/2)}{L_g} \right] \right\}$
2. Gaussian: $f(z) = \exp \left[-\frac{(4 \ln 2)(z - L_g/2)^2}{(L_g/3)^2} \right]$
3. Sine: $f(z) = \sin \left(\frac{\pi z}{L_g} \right)$
4. Quadratic sine: $f(z) = \sin^2 \left(\frac{\pi z}{L_g} \right)$

These apodization profiles are plotted in Fig. 2-10 for comparison. Using the TMM as described above, the corresponding reflective spectra of the different types of apodized FBGs are computed as shown in Fig. 2-11. The Gaussian-apodized FBG is found to have the highest side-lobe suppression ratio, in which the side lobes are less than -65 dB, while the uniform FBG has more than -10 dB. In addition, the suppression of the side lobes also has an effect on the reflectivity value at the Bragg wavelength. The Gaussian-apodized FBG has the largest insertion loss and the reflectivity value at the Bragg wavelength is around -0.1 dB.

Figure 2-10 Some commonly used apodization profiles [13].

Figure 2-11 Reflective spectra of various apodized FBGs ($L_g = 20$ mm and the maximum of “ac” index change is 1×10^{-4}).

2.6 Summary

In this chapter, several types of FBGs have been introduced. Two popular modeling methods for solving the coupled-mode equations for FBGs have been discussed. The method of the Riccati equation solved by the Runge-Kutta algorithm is simpler and quicker but it cannot solve FBGs with insertion of the phase shifts. Comparably, the TMM is more flexible and has been widely used. As examples, the TMM has been applied to calculate the spectra of different types of FBGs, including the apodized FBGs with different apodization profiles.

In this thesis, the calculations of the spectra of FBGs are all based on the TMM.

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3

A Staged Continuous Tabu Search Algorithm for Global Optimization

In this chapter, a novel staged continuous tabu search (SCTS) algorithm is developed for solving global optimization problems of multi-minima functions with multi-variables. This method comprises three stages that are based on the continuous tabu search (CTS) algorithm with different neighbor-search strategies, with each devoting to a particular task. The method searches for the global optimum thoroughly and efficiently over the space of solutions compared to a single process of CTS. The effectiveness of the proposed SCTS algorithm is evaluated using a set of benchmark multimodal functions whose global and local minima are known. The numerical test results obtained indicate that the proposed method is more efficient than the improved genetic algorithm that has been published previously.

Section 3.1 introduces the strategy of a standard tabu search (TS) algorithm. Section 3.2 gives a brief review on the CTS algorithm. Based on the CTS algorithm, a new SCTS algorithm is developed and the detailed description of the

algorithm is described in Section 3.3. Section 3.4 presents experimental results of a set of benchmark functions obtained from the new algorithm and compares the method with the CTS algorithm and with an improved genetic algorithm.

3.1 Introduction

Optimization is a scientific branch using both scientific methods and technological approaches to satisfy technical, economical and social requirements in an ideal way. Usually, optimization problems in engineering can be formulated as nonlinear programming problems. Due to the multi-modal and ill-conditioned character of the objective functions, it is difficult to solve these engineering problems with traditional methods. Hence the study of global optimization methods has become one of the most important topics for engineering designers.

Tabu search (TS) is an iterative search method originally developed by Glover and Laguna [1], which has been successfully applied to a variety of combinatorial global optimization problems [2-4]. A good analogy is mountain climbing (see Fig. 3-1), where the climber must selectively remember key elements of the path traveled (using adaptive memory) and must be able to

strategize choices along the way (using responsive exploration).

Figure 3-1 A good analogy of the TS algorithm is the problem of hill climbing. In the problem, the climber will selectively remember key elements of the path traveled [1].

A rudimentary form of this algorithm may be roughly summarized as follows. It starts from an initial solution s that is randomly selected. From this current solution s , a set of neighbors, called s' , is generated by pre-defining such a set of “moves” or perturbation of current solution (see details in Section 3.2). To avoid the endless reiterative cycle, the neighbors of the current solution, which belong to a subsequently defined “tabu list”, are systematically eliminated. The objective function to be minimized is then evaluated for each generated solution

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s' , and the best neighborhood of s becomes the new current solution even if it is worse than s . The “move” that generates the new selected current solution will also be stored in the “tabu list”, which is circular. When it is full, it is updated by eliminating the previous estimated solution. Then a new “iteration” is performed; the previous procedure is repeated by starting from the new current point until a pre-defined stopping condition is satisfied. Usually, the algorithm will stop after a given number of iterations have occurred without any improvement on the value of the objective function.

The more general form of the method uses more advanced recency and frequency memory than that embodied in the simple tabu list, together with associated intensification and diversification strategies that exploit these memory structures (Glover and Laguna [1]). However, simpler forms of TS are sometimes used for conducting prototype studies, and in some instances such methods perform remarkably well without resorting to more powerful forms of TS.

Compared with analytical methods, even simple versions of the TS algorithm have a smaller probability of becoming trapped in a local optimum. The method is also organized to take advantage of problem-specific information, which is in contrast to the classical forms of some other methods such as genetic algorithm

(GA) and simulated annealing (SA) approaches¹. Because of this focus, the method demonstrates a highly attractive convergence velocity as well as a high level of reliability.

Tabu search also includes *candidate list strategies* for generating and sampling neighbors (see, e.g., Chapter 3 of [1]). These strategies are extremely important, since often only a relatively small subset of neighbors is generated at any given iteration, especially when a large number of neighborhoods are used, as in the case of multi-variable problems whose neighbors are generated in a multi-dimensional space. Siarry and Berthiau have proposed a continuous tabu search (CTS) approach [5] for optimization of nonlinear functions, and it employs a special candidate list strategy to generate neighbors. In this method, the solution space is divided into several regions. Neighbors are generated in these regions and the remainder of the method consists of an elementary form of TS that uses only the simple tabu list construction as previously mentioned. The authors have reported impressive results for optimizing functions of two or three variables. But when the number of variables increases, the efficiency of the CTS algorithm is not satisfactory and it must be improved, especially for those problems with high dimensions [5].

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¹ In recent years, “hybrid variants” of GA and SA methods have emerged that seek to incorporate problem-specific information in a better manner than the classical versions. Some of the more effective instances of these hybrid approaches make use of TS strategies.

In this chapter, a new multi-level candidate list method is introduced to give a more effective approach for optimization of nonlinear functions, namely, a staged continuous tabu search (SCTS) algorithm. The algorithm comprises three stages that are based on the CTS algorithm. Each stage focuses on one task with a special neighborhood definition, and the three combined stages are for global optimization. The algorithm is described in details in the following sections.

3.2 Continuous Tabu Search Algorithm

Before the SCTS algorithm is presented, the CTS algorithm developed by Siarry and Berthiau [5] is briefly reviewed.

For the following optimization problem:

$$\min_{s \in \Psi^k} [\Phi(s)], \quad (3.1)$$

where $\Phi(s)$ is the objective function to be minimized, and $s = [x_1, x_2, \dots, x_k]^T$

is defined as

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$$s \in \Psi^k \text{ and } \Psi^k = \{s \mid a_j \leq x_j \leq b_j\}, j = 1, 2, \dots, k. \quad (3.2)$$

where a_j and b_j are the boundary values of the j^{th} element of s , and k represents the dimension of the problem or the number of variables. The basic process of the CTS method, which is organized around a simple version of the tabu search, can be summarized as follows:

1. Generate a random point s that belongs to the space Ψ^k as the current solution.
2. A set of neighbors, $s' \in \Psi^k$, is then generated by applying s with a series of perturbations or “moves”. Generation of neighbors is defined by the following method: the neighborhood space Ψ^k of the current solution s is deemed as a ball $B(s, r)$ centered on s with a radius r . Considering a set of concentric balls with radii $h_0, h_1, \dots, h_n$, the space is partitioned into n concentric ‘crowns’. Hence n neighbors of s are obtained by selecting one point randomly inside each crown and eliminating those neighbors that belong to the “tabu list”. Figure 3-2 shows an example to generate neighbors for a problem with two variables ($k = 2$). The space is partitioned into four concentric “crowns”, and four neighbors are produced randomly in their own crown areas.

Figure 3-2 Partition of the current solution neighborhood (two variables and $n = 4$). The neighborhood s_j ($j = 1, 2, 3, 4$) is selected randomly in its own crown area [5].

3. Evaluate these neighbors with the objective function, choose the best neighbor s^* and replace the starting point s even if it is worse than the current solution. Then update the “tabu list”.
4. Clear the “tabu list”: in particular, some solution belonging to the “tabu list” can release their tabu status if their “aspiration levels” are sufficiently high.
5. Check the stopping condition and return to step (2) if the condition is not met. Otherwise, stop the iteration procedure and report the results.

Figure 3-3 shows the flow chart of this algorithm, where the main stages include the initial solution, generation of neighbors, selection of the solution and tabu list clearance.

Figure 3-3 General flow chart of a standard TS algorithm.

From the results reported in [5], the strategy of generating neighbors in CTS is more efficient than a naive candidate-list strategy based solely on random sampling, and usually produces neighbors distributed over the whole solution space.² However, the method generally encounters difficulties in finding global optima for high-dimension problems.

² Other ways of applying tabu search to continuous nonlinear optimizations are described in [1] but were not tested in [5].

3.3 Staged Continuous Tabu Search Algorithm

A new staged continuous Tabu search (SCTS) algorithm is developed here to improve on the CTS algorithm. The SCTS algorithm likewise employs the same rudimentary form of tabu search embodies in CTS, but it provides an enhanced candidate list strategy that subdivides the CTS approach into three independent processes that generate candidate neighbors in a different way. The first stage attempts to survey the whole solution space to localize a “prospective point”, which is a solution likely to produce a global optimum. The objective of the second stage is to find a point close to the global optimum. The third stage starts from the solution found in the second stage, and eventually converges to the global optimum point. The proposed SCTS algorithm is described in following sections.

3.3.1 Generation of Neighborhoods

As described in the CTS method in Section 3.2, the neighborhoods are generated in a ball $B(s, r)$ centered on s with a radius r . All neighbors s' meet the condition: $|s' - s| \leq r$.

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In the first stage, the radius r_1 is defined so that the ball $B_1(s, r_1)$ contains the whole k -dimension space Ψ^k . With radii $r_1^1, r_1^2, \dots, r_1^{n_1}$, the ball is partitioned into k concentric “crowns” centered on the current solution. One neighbor is produced in each crown. Thus the j^{th} neighbor s_j' is generated with the condition:

$$r_1^{j-1} \leq |s_j' - s| \leq r_1^j, (r_1^0 = 0). \quad (3.3)$$

As the ball $B_1(s, r_1)$ includes the whole space Ψ^k , it should be possible for all solutions within it to become the neighbors of the current solution s so that the process can investigate the whole solution space. We define the “moves” to generate neighbors such that some elements of the current solution are randomly replaced. The number of replaced elements depends on different crowns. For example, the j^{th} neighbor s_j' is generated by replacing any j elements of the current solution.

The radius r_2 for the generation of neighbors in the second stage is defined as the minimum radius of radii $r_1^1, r_1^2, \dots, r_1^{n_1}$ defined in the first stage. Followed with another partition process with a set of radii $r_2^1, r_2^2, \dots, r_2^{n_2}$, the ball $B_2(s, r_2)$ is divided into n_2 sections. The j^{th} neighbor s_j' is generated with a condition given by

$$r_2^{j-1} \leq |s_j' - s| \leq r_2^j, (r_2^0 = 0). \quad (3.4)$$

As described above, the minimum radius defined in the first stage is propagated in only one dimension of the current solution. Considering the condition defined in Eq. (3.2), the boundary can be proportionally divided for every dimension into n_2 partitions. The neighbors can then be generated by replacing the j^{th} element of the current solution (x_j') with a number computed by:

$$x_j' = a_j + (j_2 + \mu) \cdot \frac{(b_j - a_j)}{n_2}, \text{ where } j = 1, 2, \dots, k; \quad j_2 = 1, 2, \dots, n_2. \quad (3.5)$$

where μ is a random value between 0 and 1. It can be seen that the number of neighbors in this stage is $k \times n_2$.

The minimum of radii $r_2^1, r_2^2, \dots, r_2^{n_2}$ is set as the radius r_3 to generate neighbors in the third stage. Instead of partitioning to generate neighbors, the radius of the ball $B_3(s, r_3)$ decreases with an increase in the iteration number. The generation of the j^{th} neighbor is defined as

$$x_j' = x_j + \mu_j \cdot \frac{b_j - a_j}{n_2} \cdot \left(\frac{M_3 - m}{M_3} \right) \quad (3.6)$$

where x_i and x_i' are the j^{th} elements of the current solution s and the neighbor s' produced, respectively, m is the iteration number without any improvement on the current solution, and M_3 is the maximum allowable number of iterations without any improvement on the current solution.

3.3.2 Tabu List

In the underlying TS algorithm, a tabu list stores some solutions that have recently been selected. It is used to qualify the algorithm to select solutions that have not been selected before so as to escape from being recycled.

Because the three stages in the SCTS algorithm are independent of each other, the tabu lists in these stages are thus independent of each other. The list obtained in the first stage will store those ‘prospective solutions’ found in recent iterations.

In the second and third stages, the lists will store the attributes of ‘moves’ or perturbations that generate the best neighbors in recent iterations. The tabu list in each stage is always reset at the beginning of each stage.

3.3.3 Stopping Conditions

The stopping conditions for the three processes are defined below.

1. The program will stop after a given number of iterations without any improvement on the value of the objective function. The number of iterations varies in different stages.
2. The results satisfy the successful conditions. Such successful conditions are specially applied to those problems with known global optima (such as those benchmark test functions). For example, the results are close to the global optimum in a satisfactory level.
3. The search procedure will stop after a pre-defined maximum number of iterations.

In the SCTS algorithm, all stages will be terminated if any one of the stopping conditions is satisfied. That is, if the process is in the first stage or second stage, it will move into the next stage. However, if the process is in the third stage, the algorithm will stop and report the result.

3.3.4 Description of the Algorithm

Figure 3-4 shows a flowchart that summarizes the steps of the proposed SCTS algorithm. The following notations are used.

- Ψ^k : Space of feasible solutions (k dimensions).
- s_0 : Current solution.
- n_1 : Length of neighbors generated in the first stage, which is equal to k here.
- n_2 : The section number divided within the boundary of every element of s_0 .
- n_3 : Length of neighbors generated in the third stage.
- s' : The neighborhood of s_0 .
- s^* : The best solution in s' .
- s_{opt} : Current best solution found.
- $Mv(j)$: Maximum number of iterations without improvement of s_{opt} in the j^{th} stage.

As pointed out previously, the neighbors in the first stage are generated in the largest range so as to explore most of the space. While in the last stage, only a reduced space is used so that the solution will eventually converge to the global optimum.

Figure 3-4 Algorithmic description of the proposed SCTS algorithm.

The sensitivity of some main parameters for a single CTS algorithm has been discussed in [5]. Usually, these parameters should be adjusted empirically according to the nature of the problem so as to achieve an efficient optimization. As inherited from the CTS algorithm, it is found that the properties of some parameters in the SCTS algorithm are similar to the CTS algorithm. These parameters are not analyzed individually, and a set of empirical values is applied in the experiments for testing the benchmark functions. These empirical values are listed in Table 3-1.

Table 3-1 Parameters of the SCTS algorithm used for benchmark test functions.

List of parameters used in the SCTS algorithm	Parameters used for benchmark functions
Number of neighbors in the first stage (n_1)	Number of variables
Number of sections in the second stage (n_2)	5
Number of neighbors in the third stage (n_3)	Number of variables
Number of neighbors in the second stage	5×Number of variables
Maximum number of iterations without any improvement of the objective function value (Mv)	{20, 8, 5}
Maximum number of iterations of SCTS algorithm	8000

3.4 Experimental Results

To demonstrate the effectiveness of the SCTS algorithm, the important parameters to be studied are convergence, speed and robustness.

Convergence means the search for the global optimum of a function to be evaluated. The test for convergence employed here is the relative error between the optimum obtained by the algorithm, X_{opt} , and a theoretical value of the optimum, X_{theo} , of each function. The relative error, E_{relative} , is defined as [6]

$$E_{\text{relative}} = \frac{|X_{\text{opt}} - X_{\text{theo}}|}{X_{\text{theo}}} \quad (3.7)$$

If the theoretical value of the optimum is zero, the relative error in Eq. (3.7) is defined as

$$E_{\text{relative}} = |X_{\text{opt}} - X_{\text{theo}}| \quad (3.8)$$

The criterion of speed means the time taken by the algorithm to find the global optimum of the objective function. However, the computation time also depends on the computation speed of the computer. Thus, one can define the speed

criterion by determining the number of evaluations of the objective function required till a global optimum is found. Robustness means that the algorithm is versatile and can be applied to solving a variety of functions. A set of commonly used benchmark functions whose global optima are known (as listed in the Appendix B) is chosen to test the algorithm. These test functions represent various practical problems in engineering. To obtain a statistical comparison of the optimization results, every test has been performed 100 times (starting from various randomly selected points) to ensure that the results obtained are reliable.

Table 3-2 Experimental data of the SCTS and CTS algorithms.

Function	Success rate (%)		Number of evaluation functions	
	CTS	SCTS	CTS	SCTS
Goldprice	100	100	1636	696
Hartmann34	100	100	528	691
Branin	100	100	668	491
Shubert	100	100	1123	521

Table 3-2 shows the results obtained from both the CTS and the proposed SCTS algorithms for the four test functions, namely, Goldprice, Hartmann34, Branin and Shubert. The criterion of success is the percentage of trials (out of the 100 tests for each function) that can reach the global optimum with a relative error of less than 1%. Experimental data obtained from the CTS algorithm [5] is also shown in the table. From the table, it can be seen that both the algorithms can successfully find the global optima of all the four test functions. Compared with

the CTS algorithm, the SCTS algorithm reduces the number of evaluations of the Goldprice test function from 1636 to 696 and the Shubert test function from 1123 to 521. This means that the SCTS algorithm has a faster computation rate for these two particular functions. However, for the Hartmann34 and Branin functions, the SCTS algorithm does not show much improvement over the CTS algorithm, showing that the algorithms are equally good for these two particular functions.

Table 3-3 shows a comparison of the experimental data of various test functions obtained by the SCTS algorithm with an improved genetic algorithm (IGA). It is noted that the IGA algorithm can potentially yield a complete set of optima when dealing with multimodal problems [6]. These test functions have variables from 1 to 20 as given in the Appendix. In the table, the minimum found (Max) is the maximum value of the optimum found over 100 tests and the minimum found (Min) is the minimum value of the optimum found over 100 tests.

From Table 3-3, the SCTS algorithm outperforms the IGA algorithm in two ways. One advantage is that the SCTS algorithm can find the global optima (see, for example, the Brown1, Brown 3 and F10n functions) that the IGA algorithm fails to find. In these cases, the SCTS algorithm can successfully find the global optima for those functions that the IGA algorithm cannot obtain a successful rate

of 100%. Moreover, the relative errors obtained by the SCTS algorithm for these functions can reach a satisfactory level of close to zero. The other advantage is that the SCTS algorithm greatly reduces the computation time as indicated by the smaller number of evaluations of the test functions. In addition, the SCTS algorithm reduces the relative error for those functions that the IGA algorithm also yields a successful rate of 100%. These test functions are F1, F3, Branin, Goldprice, Shubert1, Shubert2, Shubert, Hartmann34, F5n and F15n.

3.5 Conclusion

An effective global optimization algorithm, namely, the SCTS algorithm, has been proposed. The method combines three stages of the CTS algorithm with three strategies of neighborhood generation. From the test results of a number of benchmark test functions, it has been found that the SCTS algorithm is more efficient than the original CTS algorithm and the IGA algorithm in terms of the success rate and computation efficiency. From the experimental results based on these test functions, the SCTS algorithm might be considered as a powerful tool for solving a variety of engineering problems such as the optimization of the fiber Bragg grating (FBG) designs. In the next chapter, the SCTS algorithm will be used to solve the synthesis problem of FBG.

Table 3-3 Experimental data of the test functions obtained by the proposed SCTS algorithm and the improved genetic algorithm (IGA).

Function	Number of variables	Theoretical minimum	Minimum found (Min)		Minimum found (Max)		Average of relative errors over 100 tests (%)		Number of evaluation of test functions		Success rate (%)	
			IGA	SCTS	IGA	SCTS	IGA	SCTS	IGA	SCTS	IGA	SCTS
F1	1	-1.1232	-1.1232	-1.1232	-1.1139	-1.1223	0.03	0	784	134	100	100
F3	1	-12.0312	-12.0312	-12.0312	-11.9270	-12.0203	0.12	0	744	181	100	100
Branin	2	0.3979	0.3979	0.3979	0.4018	0.3983	0.48	0	2040	492	100	100
Goldprice	2	3	3.003	3.002	3.0296	3.0029	0.43	0	4632	696	100	100
Shubert1	2	-186.7309	-186.6857	-186.7304	-184.9554	-186.3406	0.53	0.014	8853	2194	100	100
Shubert2	2	-186.7309	-186.7047	-186.7302	-184.9295	-185.9505	0.53	0.024	4116	2053	100	100
Shubert	2	-186.7309	-186.7280	-186.7269	-184.8753	-186.5490	0.49	0.0003	2364	521	100	100
Hartmann34	3	-3.8628	-3.8611	-3.8621	-3.8246	-3.8591	0.51	0.018	1680	560	100	100
Brown1	20	2	8.5516	2.0018	111.2914	2.0020	2692.67	0.05	128644	111430	0	100
Brown3	20	0	0.6746	0.0006	5.9122	0.0010	2.324	0.0009	106859	15142	5	100
F5n	20	0	0.0022	0.0001	0.5906	0.0010	0.067	0.0004	99945	17443	100	100
F10n	20	0	0.0496	0.0001	4.0660	0.0010	1.197	0.0008	113929	19931	49	100
F15n	20	0	0.0034	0.0003	0.7361	0.0009	0.075	0.0008	102413	19660	100	100

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4

Synthesis of Fiber Bragg Gratings with a Staged Continuous Tabu Search Algorithm

In this chapter, a design method is developed for solving the synthesis problem of fiber Bragg gratings (FBGs) by using the staged continuous tabu search (SCTS) algorithm developed in Chapter 3. The method involves the use of a transfer matrix method (TMM) for calculating the reflective spectrum and phase response of an FBG (which is modeled using either piecewise uniform sections or cascaded apodized sections), and the SCTS optimization algorithm for obtaining an optimal fiber grating design. The proposed synthesis method enables dynamic adjustment of different requirements to be made to both the reflective spectrum and phase response of the filter.

Two aspects are important in the synthesis of FBG using optimization techniques, which are a powerful and efficient global optimization algorithm and a practical FBG model. In Ref. [1] and [2], genetic algorithm (GA) and simulated annealing (SA) were used as the global optimization algorithms, and a model of piecewise uniform FBGs was employed. However, they used the standard GA and SA,

which would not be efficient enough for complex optimization problems such as the synthesis of FBGs. The model of piecewise uniform FBGs introduces a large number of variables into the optimization process. It is known that the global optimum is always difficult to find for problems with a large number of variables, especially for a complex synthesis problem that requires both the target spectrum and the target phase response to be obtained.

Because the SCTS method has been demonstrated to be powerful and efficient by experimenting on a set of benchmark functions (see Chapter 3), the SCTS algorithm could be one of the best choices for the synthesis of FBGs. Section 4.1 reviews some synthesis methods of FBGs. Section 4.2 describes the synthesis of FBG using the SCTS algorithm. Then in Section 4.3, an optical bandpass filter is designed using the same model of piecewise uniform FBGs as in [1] and [2]. The model of piecewise uniform FBGs always involves a large number of variables. It is known that global optimization problems with a large number of variables are always difficult to deal with and are time consuming to yield the solutions. To reduce the number of variables of the global optimization problems, a model of cascading apodized FBGs (AFBGs) is proposed, and a linear phase optical filter with small in-band dispersion is designed based on this model (see Section 4.4).

4.1 Introduction

The synthesis problem of a fiber grating is to find an index modulation profile that gives a complex reflection response which accurately approximates a desired complex reflection response (i.e. reflective spectrum and phase response). The simplest approach is to use the approximate Fourier relation between the reflective coefficient and the coupling coefficient of a grating. This method is only suitable for weak gratings. For strong gratings (i.e. high-reflectivity gratings), one can find the coupling coefficient using classical inverse scattering techniques. Song *et al.* [5] report on how one can design corrugated gratings by solving two coupled integral equations, which are called the Gel'fand-Levitan-Marchenko (GLM) equations. Peral *et al.* [6] have further proposed an iterative, numerical method for solving the GLM equations in the design of fiber gratings. This algorithm converges relatively fast and gives satisfactory results, but it is complex and the results are always not accurate for highly reflecting gratings [1]. Another approach for solving the inverse scattering problem is the differential inverse scattering method, which is also referred to as the layer-peeling method. This method has been applied to the design of several types of FBGs [7–9], but their designed profiles (e.g. index modulation profile) always have long grating lengths, making practical realization difficult. Moreover, when specifying ideal filter characteristics, it is desirable to have a weighting mechanism to weight the different target requirements of the filter responses. For example, when designing

an optical bandpass filter, one may be interested in weighting the linear phase more than the sharp spectral peaks because the filter dispersion is a more critical parameter. Neither the GLM method nor the layer-peeling algorithm supports such a weighting mechanism in a satisfactory way.

To overcome these difficulties, optimization techniques are one of the solutions. When the optimization techniques were applied to the synthesis of FBGs [1–2], the synthesis problems were formulated as nonlinear objective functions. An optimal solution of the grating design may be obtained by using an optimization algorithm to find the global optimum of the objective function. Comparing with the synthesis methods described earlier, optimization techniques can facilitate the task of weighting the different requirements to the filter spectrum [1]. Another advantage is that the results obtained by the optimization method are more practical by imposing additional constraints to suit the fabrication conditions. Two aspects are important for this kind of filter synthesis method. On one hand, the formulated objective function should be based on an easy-to-fabricate model so that the designed FBGs can be practically realized. On the other hand, due to the multi-modal and ill-conditioned character of those objective functions formulated by different synthesis problems of FBGs, it is difficult to solve these problems with traditional optimization algorithms. Hence, a powerful and efficient global optimization algorithm is important for synthesis of FBGs. Refs. [1] and [2] have used the standard GA and SA algorithms directly. However, they

have not demonstrated that the methods they were using are powerful ones. And the models used in [1] and [2] are the model of piecewise uniform sections, which always takes a large number of variables for the global optimization problems. Thus the methods presented in [1–2] are still not good enough for solving a more complex problem such as the design of a linear phase optical filter which requires both the target reflective spectrum and the target dispersion response to be obtained.

4.2 Synthesis of FBGs using the SCTS Algorithm

In this chapter, the SCTS algorithm developed in Chapter 3 is applied to solve the synthesis problem of FBGs. Figure 4-1 shows a general diagram of the synthesis method using the SCTS algorithm.

Figure 4-1 Block diagram of the synthesis method using the SCTS algorithm.

As shown in Fig. 4-1, the synthesis problem of FBG is formulated as an objective function which measures the error between the complex reflection response of FBG and the target one. The whole synthesis method can be deemed as a “black box”. The synthesized FBG can be obtained by the following steps.

1. When a target complex reflective spectrum (i.e. the reflective spectrum and the phase response) is input into the synthesis method, the FBG model is formulated as an objective function to be minimized.
2. The SCTS algorithm will produce a set of solutions. These solutions represent different grating profiles of the FBG.
3. These solutions are sent to the objective function built in step (1). The values of the objective function are calculated and sent back to the algorithm.
4. The SCTS algorithm then checks the calculated values of the objective function. If the pre-defined stopping conditions are not reached, the process will enter the next iteration and will be recycled from step (2). Otherwise, the algorithm will output the best solution found (i.e. the optimized structural parameters of the gratings).

To demonstrate the efficiency of the synthesis method, an optical bandpass filter and a linear phase optical filter are designed using different FBG models in the sections followed.

4.3 Design of an Optical Bandpass Filter

As described in Chapter 2, a non-uniform FBG can be divided into N piecewise uniform grating sections so that the complex reflection response can be calculated using the transfer matrix method (TMM). Thus the reflective spectrum (Eq. (4.1)) and the phase response (Eq. (4.2)) of a grating can be written as a function of the index modulation profile, $\overrightarrow{\Delta n_{ac}}$, as

$$R = \mathbf{R} \left(\overrightarrow{\Delta n_{ac}} \right) \quad (4.1)$$

$$\theta = \boldsymbol{\theta} \left(\overrightarrow{\Delta n_{ac}} \right) \quad (4.2)$$

where $\overrightarrow{\Delta n_{ac}} = [\Delta n_{ac,1}, \Delta n_{ac,2}, \dots, \Delta n_{ac,N}]$, $\Delta n_{ac,j}$ is the index modulation depth of the j^{th} grating section, and $j = 1, 2, \dots, N$.

An interesting application of the synthesis of FBG is to synthesize a fiber-optic bandpass filter. A target optical bandpass filter with 25 GHz (i.e. 0.2 nm) bandwidth is characterized by

$$R_{\text{target},\lambda} = \begin{cases} 1; & 1549.9 \text{ nm} \leq \lambda \leq 1550.1 \text{ nm} \\ 0; & \lambda < 1549.9 \text{ nm and } \lambda > 1550.1 \text{ nm} \end{cases} \quad (4.3)$$

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A step size of the wavelength (i.e. resolution) of 0.01 nm was used in the simulation. Thus an objective function can be built to measure the error between the calculated spectrum and the target one as follows

$$error(\overrightarrow{\Delta n_{ac}}) = \sum_{j \in \text{window}} W_j^R \times \sqrt{|\mathbf{R}_j(\overrightarrow{\Delta n_{ac}}) - R_{\text{target},j}|} \quad (4.4)$$

In Eq. (4.4), $\mathbf{R}_j(\overrightarrow{\Delta n_{ac}})$ is the calculated reflectivity at the j^{th} wavelength (the details involved in calculating the reflective spectrum can be found in Section 2.3.3), and W_j^R is the weight parameter of the j^{th} wavelength for the reflectivity. $R_{\text{target},j}$ is the target reflectivity at the j^{th} wavelength, which is defined in Eq. (4.3). Using the objective function as defined in Eq. (4.4), the synthesis problem of FBG can be formulated as a global optimization problem, and the aim is to find the global optimum of the objective function. Additional constraints for each element of $\overrightarrow{\Delta n_{ac}}$ can also be imposed to tailor for different fabrication conditions (see Eq. (4.5)) to ensure that the filter design can be practically realized

$$\Delta n_{j,a} \leq \Delta n_{ac,j} \leq \Delta n_{j,b}, \quad (j = 1, 2, \dots, N) \quad (4.5)$$

where N is the number of grating sections, $\Delta n_{j,a}$ and $\Delta n_{j,b}$ are the boundaries set

for the j^{th} element of $\overrightarrow{\Delta n_{ac}}$.

In this design, the number of sections is chosen as $N = 40$, and the boundary of the j^{th} element of $\overrightarrow{\Delta n_{ac}}$ (i.e. $\Delta n_{ac,j}$) is set as $[0, 0.0002]$, and the parameter values of the SCTS algorithm are listed in Table 3-1 of Chapter 3. The weight parameter W_{λ}^R at a particular wavelength λ of the reflective spectrum is defined as

$$W_{\lambda}^R = \begin{cases} 10; & \lambda \in \text{Stopband} \\ 1; & \lambda \in \text{Passband} \end{cases} \quad (4.6)$$

As seen in Eq. (4.6), the weight values in stopband are chosen as 10 times than that in the passband so as to suppress the side lobe effectively. Figure 4-2 shows the optimized index modulation profile of a 10.5-mm long FBG-based bandpass filter designed using the SCTS algorithm. The corresponding reflective spectrum is illustrated in Fig. 4-3. It can be seen that the spectrum of the uniform FBG (without optimization) has undesirable secondary maxima or side lobes of up to $\sim 30\%$ on both sides of the main reflection peak. The side lobes could create serious crosstalk or interference in the dense-WDM applications. These side lobes are greatly suppressed by the optimized FBG using the SCTS algorithm.

Figure 4-2 Optimized index modulation profile of an FBG-based bandpass filter using the SCTS algorithm.

Figure 4-3 Reflective spectrum of an optimized FBG-based bandpass filter using the SCTS algorithm. Solid line is the reflective spectrum of the optimized FBG. Dotted line is the desired spectrum (target spectrum). Dashed line is the reflective spectrum of a uniform FBG (non-optimized).

Figure 4-4 Representation of Figure 4-3 in dB unit. Solid line is the reflective spectrum of the optimized FBG. Dashed line is the reflective spectrum of a uniform FBG (non-optimized).

To show the side-lobe level more clearly, Fig. 4-4 shows the reflective spectra of Fig. 4-3 in dB unit. The side-lobe level of the optimized FBG is found to be less than -20 dB while that of the uniform FBG (non-optimized) with the same length is more than -7 dB.

To verify that the optimum solution of the FBG design obtained by the SCTS algorithm is indeed the global one, two other algorithms, namely, the GA and the adaptive SA (ASA) algorithm presented in references [3–4] are employed here for comparison. Table 4-1 shows the optimum values of the objective function (as defined in Eq. (4.4)) obtained by the three algorithms. It is clear that the SCTS

algorithm provides the best result with the smallest optimum value.

Table 4-1 The minimum value of the objective function obtained by the three algorithms.

Algorithm applied	GA	ASA	SCTS
Minimum found	4.9583	5.3451	4.9370

It is noted that the softwares of the algorithms in references [3–4] are used directly. However the softwares did not give the exact number of evaluations of the objective function which can indicate the speed of optimization. To compare the computation speed, the three software programs were run on the same computer, and it was found that the GA algorithm had the longest computation time while the computation time of the ASA algorithm is similar to that of the SCTS algorithm.

As described in Section 2.5 of Chapter 2, bandpass filters can also be designed using traditional apodization techniques. Different apodized functions have different side-lobe suppression ratios. By substituting the different apodized profiles into the objective function as defined in Eq. (4.4), different values of the objective function can be obtained. Table 4-2 shows the calculated values of the objective function. The values obtained depend very much on the definition of the weight parameters. With the weight parameter defined in Eq. (4.6), the

Gaussian profile has a relatively high value because of the comparably high insertion loss in the passband in spite of the highest side-lobe suppression ratio as presented in Section 2.5. The sine profile has the smallest value of 9.253. However, all the apodized profiles have higher values of the objective functions than that of the optimal profile. It can be concluded that optimization techniques are generally superior to the traditional apodization techniques in the design of optical bandpass filters.

Table 4-2 The values of the objective function using different apodized functions.

Apodized functions	Uniform	Gaussian	Sine	Raised-cosine	Quadratic-sine
Values of the objective function	48.9262	30.2651	9.253	14.1170	15.8763

In this design, the number of sub-grating sections N is a constant value rather than a variable in the optimization process. It is known that the larger the number of variables used in the optimization process the more complex the optimization problem will become, and a larger computation time would be required. On the other hand, it is generally difficult to obtain satisfactory results (e.g. smaller side-lobe suppression level) when a smaller number of variables is employed in the optimization process. In this design, the number of sub-grating sections used is 40 (the total grating length is about 10 mm), which corresponds to the number of variables used in the optimization process. And it was found

that the design yielded a side-lobe suppression level of -20 dB which is quite satisfactory for a grating of only 10 mm long.

4.4 Design of Linear Phase Optical Filter

Optimization of FBGs based on the model of piecewise uniform grating sections has been applied to the design of an optical bandpass filter (see Section 4.3) using the SCTS algorithm. The results show that the SCTS algorithm is more efficient than the improved GA and the ASA. However, the problem of the model of piecewise uniform sections is that the optimization process always has a large number of variables (for example, 40 in the design of the optical bandpass filter in Section 4.3). It is known that a global optimization problem with a large number of variables is always complicated to deal with; thus it is difficult to obtain satisfied optimal design of the gratings using the model of piecewise uniform FBGs. In particular, the problem will become more complex when both the target reflective spectrum and the desired phase response are required to be designed at the same time.

In Chapter 2, the FBGs with apodized index modulation profiles have been introduced. In this section, a model based on a cascaded apodized FBGs (AFBGs)

is presented. By applying the SCTS algorithm together with the model of cascaded AFBGs (instead of uniform FBG sections), the number of variables in the optimization problem is reduced because the number of AFBGs used is always relatively small. Thus the method presented in this section is comparably simpler than the method using the model of piecewise uniform sections. The effectiveness of the method has been verified through the design of a linear phase optical filter, which is a problem with requirements in both the reflective spectrum (i.e. bandpass filter) and the phase response (i.e. zero in-band dispersion).

4.4.1 Modeling of Apodized FBGs in Cascade

Similar to Fig. 2-3 as presented in Chapter 2, the model of cascaded AFBGs is shown in Fig. 4-5.

Figure 4-5 Schematic diagram of a model based on N cascaded AFBGs.

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In Fig. 4-5, $\Delta n_{ac,j}^{\max}$, δl_j and f_j are the maximum amplitude of the ‘ac’ index change, and the length and apodization function of the j^{th} AFBG, respectively. As described in Chapter 2, an AFBG can be characterized by specifying the maximum index modulation amplitude and the apodization function. Thus each AFBG section can be represented by a 2×2 matrix using the transfer matrix method (TMM). When N AFBGs are cascaded, the electrical field amplitudes between the input port and the output port are related as

$$\begin{bmatrix} E_f(0) \\ E_b(0) \end{bmatrix} = T_1^{ap} \cdot T_2^{ap} \cdots T_N^{ap} \begin{bmatrix} E_f(N) \\ E_b(N) \end{bmatrix} \quad (4.7)$$

where T_j^{ap} is the transfer matrix of the j^{th} apodized grating and $j = 1, 2, \dots, N$.

Applying the boundary condition, $E_b(N) = 0$, the complex reflection coefficient can be expressed as

$$\rho = \frac{E_b(0)}{E_f(0)} \quad (4.8)$$

Thus the corresponding delay time and dispersion can be calculated using Eq. (2.24) and Eq. (2.25).

If N AFBGs are cascaded and the apodization functions (i.e. f_j in Fig. 4-5) are pre-defined, the index profile can be represented by a vector $\overrightarrow{Var}$ which is given by

$$\overrightarrow{Var} = \{\Delta n_{ac,1}^{\max}, \dots, \Delta n_{ac,N}^{\max}, \delta l_1, \dots, \delta l_N\} \quad (4.9)$$

It is noted that the vector $\overrightarrow{Var}$ consists of the elements of the index modulations and the grating lengths of the N AFBGs. The grating period, Λ , is kept fixed for ease of fabrication for all the AFBGs. If a vector $\overrightarrow{Var}$ is given, the reflection spectrum and the dispersion response can be computed using Eq. (4-8) and Eq. (2-25). In the synthesis problem of N cascaded AFBGs, the aim is to find such a vector $\overrightarrow{Var}$ that produces a reflection spectrum and dispersion response as close as possible to the target filter characteristics. An error function, which measures the discrepancy between the calculated complex reflection spectrum and the target complex spectrum, is defined as

$$error(\overrightarrow{Var}) = \sum_{j \in \text{window}} W_j^R \times \sqrt{|R_j(\overrightarrow{Var}) - R_{\text{target}}|} + b \times \sum_{k \in \text{passband}} |D_k(\overrightarrow{Var}) - D_{\text{target}}| \quad (4.10)$$

In Eq. (4.10), $R_j(\overrightarrow{Var})$ and $D_k(\overrightarrow{Var})$ are the reflectivity of the j^{th} wavelength and the dispersion of the k^{th} wavelength, respectively, W_j^R is the weight parameter of the j^{th} wavelength for the reflectivity, and R_{target} and D_{target} are the

target reflectivity spectrum and the target dispersion response, respectively. The weighting between the reflective spectrum and the dispersion response can be adjusted by changing the value of b . Using the objective function as defined in Eq. (4-10), one can formulate the synthesis problem of FBGs as a global optimization problem, and the aim is to find the global optimum of the objective function. Additional constraints for $\overrightarrow{Var}$ can also be imposed to tailor for different fabrication conditions to ensure that the filter design can be practically realized.

4.4.2 FBG-Based Linear Phase Filter

In some applications, especially for high bit-rate add/drop multiplexing systems (10 Gbit/s and above), FBGs with a near-squared spectral response and in-band linear phase response (or zero dispersion) are required to minimize the crosstalk between the different wavelength channels [10]. Different kinds of AFBGs have been proposed to produce near-squared spectral responses (see Section 2.5), but the dispersion caused by their non-linear phase characteristics limits their applications (see Fig. 2-6). Using the proposed synthesis method, one can design a linear phase optical filter with 50 GHz (0.4 nm) bandwidth by cascading the AFBGs.

In this design of a linear phase filter, the length of $\overrightarrow{Var}$ for N cascaded AFBGs is $2 \times N$ so the synthesis problem is an optimization problem with $2 \times N$ variables. To avoid long computation time, the value of N should not be too large. Here $N = 5$ is used in the design. As described in Section 2.5, the type of apodization profile will determine the value of the side-lobe suppression in the reflective spectrum. Compared with other apodized profiles such as Gaussian, raised-cosine and sine profiles, the quadratic-sine apodization profile provides a larger side-lobe suppression ratio and a comparably high reflectivity in the passband. As an example, the quadratic-sine apodization profile is employed here in the linear phase filter design.

With an in-band linear phase response (or $D_{\text{target}} = 0$), the target reflective spectrum of a linear phase filter can be expressed as

$$R_{\text{target}}^{\lambda} = \begin{cases} 1; & \lambda \in \text{Passband} \\ 0; & \lambda \in \text{Stopband} \end{cases} \quad (4.11)$$

To obtain a 0.4-nm bandwidth, the passband is defined from 1549.8 nm to 1550.2 nm. In the stopband, suppression of the side lobes in the wavelengths close to the center wavelength is always more critical than in other wavelength regions. To increase the optimization efficiency, the weight parameter W_j^R of the reflective

spectrum in the objective function (as defined in Eq. (4.10)) is given as

$$W_j^R = \begin{cases} \frac{\lambda_0}{|\lambda_0 - j|}; & j \in \text{stopband} \\ \varepsilon; & j \in \text{passband} \end{cases} \quad (4.12)$$

where λ_0 is the center wavelength and ε is a numerical value. The value of ε was chosen by means of trials and errors during the course of carrying out many experimental simulations. And it was found that $\varepsilon = 1$ gave promising results that meet the specifications of the filter design as described below. However, it should be noted that it is still possible that other values of ε might yield better results than those of this design with $\varepsilon = 1$. In Eq. (4.10), b is a varying parameter which can be used to dynamically balance the error between the reflective spectrum and the dispersion response.

Figure 4-6 (the solid line) shows an effective index modulation profile of a designed linear phase filter using the proposed synthesis method incorporating the SCTS algorithm.

Figure 4-6 The effective index modulation profiles of an optimal linear phase filter designed using the proposed synthesis method incorporating the SCTS algorithm (solid line) and a quadratic sine-apodized filter (dashed line).

From Fig. 4-6, it can be seen that the length of the filter is about 16.3 mm. The profile consists of five quadratic-sine AFBGs whose lengths are 3.33 mm, 3.07 mm, 5.6 mm, 2.1 mm, and 2.2 mm. And the corresponding index modulation amplitudes of these gratings are 8×10^{-5} , 1.1×10^{-5} , 5.59×10^{-4} , 4.2×10^{-5} , and 1.76×10^{-5} , respectively. Such a profile can be realized or written into the fiber using existing methods such as the recently proposed polarization control method [11]. For the purpose of comparison, an AFBG with a quadratic-sine apodization profile and of the same grating length (16.3 mm) is also shown in Fig. 4-6 (the dotted line). Because the optimized filter was designed with a quadratic

sine-apodized function, the optimized filter was compared with a quadratic sine-apodized filter to show that the linear phase filter design has been optimized for a smooth group-delay response.

Figure 4-7 shows the reflection spectrum as calculated from the index-modulation profile of the designed linear phase filter (see the solid line of Fig. 4.6). It is found that the reflection spectrum of the designed filter has side lobes of less than -28 dB, which are an acceptable level for many practical applications.

Figure 4-7 Calculated reflective spectrum of an optimal linear phase filter using the proposed synthesis method incorporating the SCTS algorithm.

Figures 4-8 and 4-9 show, respectively, the in-band group delay response and the in-band dispersion response as calculated from the index modulation profile of

the designed linear phase filter (see the solid line of Fig. 4-6). It can be seen that the group delay response of the designed filter is much more flattened than that of the single-apodized FBG. The ripple of the in-band group delay response of the designed filter is less than 1 ps while it is more than 80 ps for the single-apodized FBG. The worst of the in-band dispersion of the optimal FBG is less than 100 ps/nm while the in-band dispersion ripple of the single-apodized FBG is just slightly more than 2000 ps/nm (see Fig. 4-9). These findings have shown that the synthesized filter has been optimized for zero dispersion as required of a linear phase optical filter.

Figure 4-8 Group delay response of an optimal linear phase filter using the proposed synthesis method incorporating the SCTS algorithm (solid line) and the quadratic sine-apodized filter (dashed line).

Figure 4-9 Dispersion response of an optimal linear phase filter using the proposed synthesis method incorporating the SCTS algorithm (solid line) and the quadratic sine-apodized filter (dashed line).

The reflective spectrum of a quadratic sine-apodized FBG-based filter has a near-squared shape. In Fig. 4-8, it can be seen that the group delay response of the quadratic sine-apodized filter (the dashed line) has two peaks outside the slowly-varying valley. These peaks are arising from the sharp roll-offs on both sides (i.e. at the edges of the passband) of the near-squared spectrum. An FBG filter can be deemed as a physical passive device which is characterized by stability and causality conditions, and thus its spectrum and phase response can be explained using the Hilbert transform theory [12]. According to this theory, there is always a sudden change in the phase response at the wavelength where the spectrum has a sharp roll-off. Using the group delay response as defined in Eq.

(2.24), the group delay response will thus have a peak at the wavelength where the spectrum has a sharp roll-off. However, it can be seen from Fig. 4-8 (the solid line) that the SCTS optimized filter has a smoother in-band group delay response, and thus its group delay response has smaller peaks than those of the apodized filter. It is noted that, in Fig. 4-8, these smaller peaks in the group delay response of the optimized filter actually appear (although not fully shown) at the edges of the window. The filter dispersion response is simply the derivative of the group delay response with respect to wavelength and can be calculated using Eq. (2.25). As such, the peaks (or valleys) will appear in the dispersion responses as expected (as shown in Fig. 4-9).

4.5 Conclusion

By applying the SCTS algorithm to the optimization of FBG, a novel synthesis method has been developed. An optical bandpass filter with 25 GHz bandwidth has been designed based on the FBG model of piecewise uniform grating sections. The results show that the SCTS algorithm is superior to both the adaptive SA algorithm and the improved GA algorithm. A linear phase optical filter with 50 GHz bandwidth has been designed based on the FBG model of cascaded apodized sections. The simulation results show that the proposed

SCTS-based synthesis method can be further developed into a powerful toolbox for a variety of fiber grating designs.

However, the linear phase filter design obtained from the SCTS method as presented in Section 4.4 is still not that satisfactory in the sense that the in-band group-delay ripples must be smoothed out more. Chapter 5 will present a design method which is better than the SCTS method resulting in better filter designs.

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5 Two-Stage Hybrid Optimization of Fiber Bragg Gratings

Based on the optimization methods presented in Chapter 3 and Chapter 4, this chapter further presents a hybrid optimization method for the synthesis of fiber Bragg gratings (FBGs) with complex characteristics. The hybrid optimization method is a two-tier search which employs a global optimization algorithm (i.e. the staged continuous tabu search (SCTS) algorithm presented in Chapter 3) and a local optimization method, namely, the Quasi-Newton method. First, the SCTS global optimization algorithm is used to find a “promising” FBG structure that has a complex spectral response (i.e. reflective spectrum and phase response) as close as possible to the targeted one (i.e. the same process as the method presented in Chapter 4). Then a local optimization method (i.e. the Quasi-Newton method) is applied to further optimize the “promising” FBG structure obtained from the SCTS algorithm to obtain a targeted complex spectral response. A dynamic mechanism for weighting of different requirements of the complex spectral response is employed to enhance the optimization efficiency. To demonstrate the effectiveness of the hybrid optimization method, the

synthesis of three linear phase optical filters based on FBGs with different grating lengths is presented.

This chapter is organized as follows. Section 5.1 gives a literature review. The proposed hybrid optimization algorithm is presented in detail in Section 5.2. Section 5.3 presents the three designs of the linear phase optical filters with different grating lengths by using the hybrid optimization algorithm.

5.1 Introduction

Fiber Bragg gratings (FBGs) have been widely used as optical filters in wavelength division multiplexed (WDM) systems due to a number of advantages, which include low insertion loss, low polarization sensitivity, compactness, low cost, all-fiber geometry, and easy fabrication. Section 4.1 presents a review of some recently published synthesis methods, which can be used to determine the index modulation profile of an FBG corresponding to a given desired spectral response [1–6]. Among them, synthesis of FBGs with some optimization techniques is used in this work because it can be facilitated with a weighting mechanism. Moreover, the synthesis method can produce results that can be practically realized by imposing additional constraints to suit the fabrication

conditions. In Section 4.2 and Section 4.3, the method is improved in two ways. First, an efficient global optimization algorithm is used (i.e. the proposed SCTS algorithm as presented in Chapter 3). Second, the FBG model of either piecewise uniform sections or cascaded apodized sections is used for different design targets. If the design objective requires only the spectrum to be optimized, the optimization for this problem can be solved by using the model of piecewise uniform FBGs. Section 4.3 presents the synthesis of an optical bandpass filter using this model. When both the desired phase response and the spectrum of the filter are required, for example, the synthesis problem of linear phase optical filters, the optimization problems are always complex and difficult to solve. To solve this problem, the model of cascaded apodized FBGs (AFBGs) can be used (see Section 4.4). As an example, Section 4.4 presents the design of a linear phase filter based on the model of cascaded AFBGs.

When a synthesis problem of FBG is to be solved with the optimization techniques, the synthesis problem is formulated as a nonlinear function (or objective function) to be minimized. An optimized index modulation profile can be obtained by searching for the minimum of the objective function with some optimization algorithms. The formulated objective functions are generally complicated, thus traditional methods (i.e. local optimization algorithms) are not efficient enough to solve these problems because these methods always require a “promising” or initial solution (which is generally unknown) that is close to the

global optimum. Some global optimization methods can be used instead, such as the genetic algorithm (GA) and the simulated annealing (SA) algorithm (see [5], [6] and Chapter 4).

A global optimization method is good at the global search, and it can search for the global optimum starting from any randomly selected solution. However, such global search processes are always poor at convergence. The computational efficiency generally reduces with an increase in the number of variables. Local optimization algorithms are usually good at fine tuning. However, they are poor at finding global optima if they do not begin from a “promising” solution (i.e. a solution close to the global optimum). Thus, a hybrid concept has been recently proposed to increase the searching efficiency by combining a global optimization algorithm and a local optimization method [7–8].

Based on the work in the former two chapters, a new method using a hybrid optimization algorithm is proposed to further improve on the design of FBG-based filters in this chapter. The method consists of two steps. In the first step, the SCTS algorithm (as presented in Chapter 4) is used to obtain a “promising” FBG structure that has a complex spectral response (i.e. reflective spectrum and phase response) as close as possible to the targeted one. The optimization process can be based on either the model of piecewise uniform

sections or the model of cascaded AFBGs. In the second step of the proposed hybrid optimization algorithm, a local optimization method, namely, the Quasi-Newton method is used to further optimize the FBG structure obtained from the SCTS optimization algorithm to achieve the desired complex spectral response. In this step, the variables to be optimized are the index modulation profiles. As the “promising” initial solution, which is obtained from the first step of the hybrid algorithm (i.e. the SCTS algorithm), is known, the second step involving the use of the Quasi-Newton algorithm ensures that the global optimum is found efficiently.

5.2 Proposed Hybrid Optimization Algorithm

Figure 5-1 shows the flow chart of the proposed hybrid optimization algorithm used for the synthesis of FBGs. It can be seen from Fig. 5-1 that the first step involves the use of the global optimization technique (the SCTS algorithm) to optimize the cascaded uniform FBGs or AFBGs. Then the second step makes use of a local optimization algorithm (the Quasi-Newton method) to further optimize the index modulation profile obtained from the first step based on the cascading of the uniform FBGs. When both processes employ the same model of FBGs, the hybrid concept is only a combination of a global optimization algorithm and a

local optimization algorithm [7–8]. When the global search process and the local search process use different models of FBGs, it should be noted that the method makes use of the hybrid concept in two aspects. First, a global optimization algorithm is combined with a local optimization algorithm. Second, the use of the models of FBGs is also hybrid, i.e. the two search processes use different FBG models.

Figure 5-1 Schematic diagram of the proposed hybrid optimization method.

The SCTS algorithm is used as the global search algorithm in this work, which has been presented in detail in Chapter 3. And the global search process has been presented in Chapter 4 and is based on the model of piecewise uniform FBGs or

cascaded AFBGs. Thus only the local search process is discussed in this chapter.

After the “promising” FBG structure is obtained from the SCTS process (the global optimization process), a local optimization algorithm is used to further optimize the FBG structure starting from this “promising” index modulation profile. In this second step, the length of the “promising” index modulation profile is divided into M points placed at equal intervals from one another, that is, the index modulation function, $\Delta n_{ac}(z)$, is sampled at the discrete point z_j , where $j = 1, 2, \dots, M$. For every interval, the index modulation can be assumed to be constant. Thus the transfer matrix method (TMM) can be applied to calculate the complex spectral response of the whole grating if the grating profile is known. The FBG model employed in this second step is the piecewise uniform grating sections which has been described in Section 4.3 of Chapter 4. Thus the error function for the index modulation, $\overrightarrow{\Delta n_{ac}} = [\Delta n_{ac,1}, \Delta n_{ac,2}, \dots, \Delta n_{ac,M}]$, can be defined as

$$error(\overrightarrow{\Delta n_{ac}}) = \sum_{i \in \text{window}} W_j^R \times \sqrt{|R_j(\overrightarrow{\Delta n_{ac}}) - R_j^{\text{target}}|} + b \times \sum_{k \in \text{passband}} |D_k(\overrightarrow{\Delta n_{ac}}) - D_k^{\text{target}}| \quad (5.1)$$

where $R_j(\overrightarrow{\Delta n_{ac}})$ and $D_k(\overrightarrow{\Delta n_{ac}})$ are the calculated reflectivity of the j^{th} wavelength and the calculated dispersion response of the k^{th} wavelength,

respectively (the details involved in calculating the reflectivity and dispersion response can be found in Section 2.3.3), W_j^R is the weight parameter of the j^{th} wavelength for the reflectivity, and R_j^{target} and D_k^{target} are the target reflectivity of the j^{th} wavelength and the target dispersion response of the k^{th} wavelength, respectively. The weighting between the reflective spectrum and the dispersion response can be adjusted by changing the value of b . Then the Quasi-Newton method is employed to find the optimum of Eq. (5.1). The Quasi-Newton method, showing high efficiency in solving multimode nonlinear optimization problems, has become a standard local optimization method, and its detailed description can be found in reference [9].

5.3 Design of Bandpass Optical Filter

In this section, the hybrid optimization method is applied to the design of a bandpass optical filter. As mentioned above, the SCTS algorithm is used in the first step of the hybrid optimization process. Because Section 4.3 has presented the design of a bandpass optical filter using the SCTS algorithm based on the piecewise uniform FBGs, this design can be considered as a “promising” structure, and the Quasi-Newton method is further applied to optimize the “promising” structure in the design of a bandpass optical filter here.

The desired reflective spectrum of a bandpass optical filter with 0.2 nm (or 25 GHz) bandwidth is given by

$$R_{\text{target}, \lambda} = \begin{cases} 1; & 1549.9 \text{ nm} \leq \lambda \leq 1550.1 \text{ nm} \\ 0; & \lambda < 1549.9 \text{ nm and } \lambda > 1550.1 \text{ nm} \end{cases} \quad (5.2)$$

In this bandpass filter design, the resolution of wavelength used was 0.01 nm and the phase response of the filter is not considered here. Unlike the bandpass filter designed in Chapter 4, here the total length of the grating used was 20 mm, and the total number of the piecewise uniform FBGs was 40. The boundary of each element of $\overrightarrow{\Delta n_{ac}}$ was set as [0, 0.0002]. The designed filter presented in Chapter 4 has a total grating length of 10.5 mm and a total number of sub-grating sections of 40, which corresponds to a sub-grating length of about 0.25 mm. Because the beam size of the UV exposure system was about 0.5 mm, which corresponds to a minimum allowable sub-grating length in order to produce the FBG with an index profile that closely matches the desired one. Thus, the designed filter as presented in Chapter 4 cannot be fabricated using this fabrication system. In this chapter, an FBG-based bandpass filter was designed using the proposed two-stage hybrid method and the designed filter was also fabricated to verify the effectiveness of the hybrid method. As such, the hybrid optimized filter was chosen to have a total grating length of 20 mm and a total number of sub-grating sections of 40, which corresponds to a sub-grating length of 0.5 mm that meets

the fabrication requirement of this system.

Figure 5-2(a) shows the hybrid-optimized FBG index modulation profile of the FBG-based bandpass filter obtained by the hybrid optimization method. For comparison, the index modulation profile obtained in the first stage of the hybrid method (i.e. the SCTS process) is illustrated in Fig. 5-2(b). As mentioned above, this profile is used as the “promising” structure in the second stage of the hybrid method. As a comparison, the profile of a sine-apodized FBG with a grating length of 20 mm is also shown in Fig. 5-2(b). The corresponding reflective spectra of the three FBG-based bandpass filters illustrated in Figs. 5-2(a) and 5-2(b) are shown in Fig. 5-3.

Figure 5-2(a) The optimized FBG index modulation profile of an optical bandpass filter designed by the hybrid optimization method.

Figure 5-2(b) The solid line is the “promising” index modulation profile of an optical bandpass filter obtained by the first stage of hybrid method (i.e. the SCTS process), and the dashed line is the index modulation profile of a sine-apodized FBG (divided into 40 sections).

Figure 5-3 Reflective spectra corresponding to the three index modulation profiles shown in Figs 5-2(a) and 5-2(b). The solid line is the reflective spectrum of a hybrid-optimized FBG-based bandpass filter. The dashed line is the reflective spectrum of a SCTS-optimized FBG-based bandpass filter (i.e. the first stage of the hybrid method). The dotted line is the reflective spectrum of a sine-apodized FBG-based bandpass filter with the same grating length as those of the other two optimized filters.

As shown in Fig. 5-3, it is noted that the SCTS-optimized FBG (i.e. the FBG obtained by the SCTS algorithm) is superior to a standard sine-apodized FBG because the spectrum of the SCTS-optimized FBG (the dashed line in Fig. 5-3) is steeper than the spectrum of the standard sine-apodized FBG (the dotted line in Fig. 5-3), and their side lobes are nearly of the same level (around -25 dB). The hybrid-optimized FBG shows the best performance in the reflective spectrum. It is found that the spectrum of the hybrid-optimized FBG (the solid line in Fig. 5-3) is the steepest compared to the other two, and the side lobes are suppressed to as

low as -30 dB. It can be concluded that the two-stage hybrid algorithm is more efficient than the single-stage SCTS process.

Asseh *et al.* [10] have proposed a technique for the fabrication of long gratings with complex profiles, in which a large number of small sub-gratings were exposed in sequence by UV pulses. Each sub-grating has a few hundred periods. Thus, the depth of the index modulation of each sub-grating can be tuned by adjusting the offset of the fiber dithering away from the phase mask. That is, if the offset of the fiber dithering is half of the grating period, the index modulation will be completely averaged out (i.e. no index modulation). Now if there is no offset of the fiber dithering from the phase mask, the index modulation will be maximum. Using this method, one should be able to fabricate an FBG with an optimized profile as shown in Fig. 5-2(a). The measured reflective spectrum is illustrated in Fig. 5-4. In Fig. 5-4, the solid line is the measured spectrum of a fabricated 20-mm long FBG with the optimized profile obtained using the hybrid method (see Fig. 5-2(a)). As a comparison, a 20-mm uniform FBG was also fabricated and measured and its reflective spectrum is shown as the dashed line in the figure. Compared with the spectrum of the uniform FBG, the optimized FBG has a steeper spectrum with side lobes less than -20 dB. The side-lobe suppression level is higher than the theoretical value of -30 dB (see Fig. 5-3) due probably to the fabrication errors. The possible fabrication errors are due to the positioning error of the translation stage, fluctuation of the UV laser power and

some possible dirty spots on the phase mask. The procedures that could be taken to enhance the fabrication accuracy are listed below.

- (1) Use more precise positioning stage,
- (2) Improve the stability of the UV laser power,
- (3) Use a cleaner phase mask.

Figure 5-4 Measured reflective spectrum of a 20-mm long hybrid-optimized FBG-based bandpass filter (solid line) and measured reflective spectrum of a uniform FBG-based bandpass filter with the same length (dashed line).

5.4 Design of Linear Phase Optical Filters

Section 4.4 has presented the SCTS algorithm for the design of a linear phase

optical filter based on the FBG model of cascaded AFBGs. In this section, to demonstrate the effectiveness of the proposed hybrid optimization method, three linear phase optical filters with different grating lengths are designed with each having a 50 GHz bandwidth (i.e. 0.4 nm in the 1550-nm wavelength window).

In the first step of the hybrid optimization process, the optimization variable is a vector $\vec{Var}$ as defined in Eq. (4.9). The length of $\vec{Var}$ for N cascaded AFBGs is $2 \times N$ so the problem becomes an optimization problem with $2 \times N$ variables. Here $N = 5$ is chosen. As described in Section 2.5, the type of the apodization profile used will determine the extent of the side-lobe suppression in the reflective spectrum. Compared with other apodization profiles such as Gaussian, raised-cosine and sine, the quadratic-sine apodization profile can provide a large side-lobe suppression and a relatively high reflectivity in the passband. Thus the quadratic-sine apodization profile is chosen in the filter design here. As defined in Eq. (4.9) of Chapter 4, the elements from $(N + 1)th$ to $(2 \times N)th$ of the vector $\vec{Var}$ describe the grating lengths of the cascaded AFBGs. By setting different boundary values for these elements, one can control the total grating lengths in a practical range. After a “promising” index modulation profile is obtained in the first step of the hybrid process, it is set as the initial solution for the second step of the hybrid process. In the second step of the hybrid method, the index modulation profile is divided into M sections, thus the number of variables in the second step of optimization is M .

The objective function for the first step of the hybrid method is defined in Eq. (4.10) of Chapter 4. And the objective function for the second step of the hybrid method is defined in Eq. (5.1). With an in-band linear phase response (or $D_{\text{target}} = 0$), the target reflective spectrum of a linear phase filter can be expressed as Eq. (4.11) of Chapter 4. The weight parameter W_j^R in the objective functions of the two stages (i.e. Eq. (4.10) and Eq. (5.1)) is given as Eq. (4.12) of Chapter 4.

5.4.1 Design of an FBG-Based Linear Phase Filter with a Grating Length of 17.1 mm

Figure 5-5(a) shows the index modulation profile of a designed linear phase filter with a grating length of 17.1 mm. As shown in Fig. 5-5(a), the dashed line is the “promising” index modulation profile obtained from the first step of the hybrid optimization algorithm, and this profile consists of the cascade of five AFBGs. The solid line is the index modulation profile obtained from the second step of the hybrid optimization algorithm. As a comparison, the index modulation profile of a single quadratic-sine apodized FBG with a 17.1 mm length is also plotted (the dotted line).

Figure 5-5(a) The index modulation profiles of a 17.1 mm-long optimized linear phase filter. The dashed line corresponds to the profile obtained from the first step (i.e. using the SCTS algorithm) of the hybrid optimization algorithm. The solid line corresponds to the profile obtained from the second step (i.e. using the Quasi-Newton method) of the hybrid optimization algorithm. The dotted line is the profile of a single quadratic-sine apodized FBG.

The corresponding reflective spectra and group delay responses are shown in Fig. 5-5(b) and Fig. 5-5(c), respectively. It is noted that the wavelength resolution used to calculate the reflective spectra and group delay responses is 0.01 nm. In Fig. 5-5(b), the side-lobe suppression of the profile obtained from the first step is -28 dB (the dashed line) while the side-lobe suppression of the profile obtained from the second step is more than -35 dB (the solid line). Furthermore, the profile obtained from the second step gives a sharper reflective spectrum than that obtained from the first step. It is found that the reflective spectra of the

optimized FBGs (obtained by the first stage of the hybrid method or the second stage of the hybrid method) are not as good as that of the single apodized FBG (AFBG) which has the lowest side lobes of -45 dB (the dotted line). However, the side lobes of the optimized FBGs have already been suppressed to an acceptable level of less than -20 dB.

Figure 5-5(b) The corresponding reflective spectra calculated from the index modulation profiles shown in Fig. 5-5(a). The dashed line corresponds to the reflective spectrum obtained from the first step (i.e. using the SCTS process) of the hybrid algorithm. The solid line corresponds to the reflective spectrum obtained from the second step (i.e. using the Quasi-Newton method) of the hybrid algorithm. The dotted line is the reflective spectrum of a single quadratic-sine apodized FBG with a length of 17.1 mm.

Figure 5-5 (c) The corresponding group delay responses calculated from the index modulation profiles shown in Fig. 5-5(a). The dashed line corresponds to the group delay response obtained from the first step (i.e. using the SCTS process) of the hybrid algorithm. The solid line corresponds to the group delay response obtained from the second step (i.e. using the Quasi-Newton method) of the hybrid algorithm. The dotted line corresponds to the group delay response of a single-apodized FBG with a length of 17.1 mm.

In Fig. 5-5(c), compared with the single apodized FBG with the same grating length (the dotted line), the ripples of the in-band group delay responses of the profiles obtained from the first step (the dashed line) and the second step (the solid line) are greatly reduced. Thus it is found that the grating optimized by the hybrid optimization algorithm has linear phase characteristics and near-ideal squared reflective spectrum.

To distinguish the two flattened group delay lines in Fig. 5-5(c), Fig. 5-5(d) shows a zoomed-in figure of Fig. 5-5(c) in the passband. It is found that the in-band group delay ripple of the profile obtained from the first step is 1 ps (the dashed line) and it is less than 0.1 ps for the profile obtained from the second step (the solid line). It can be concluded that the two-stage hybrid algorithm has greatly improved the performance of the optimized grating in both the reflective spectrum and the in-band group delay response compared with the first stage of the hybrid method (i.e. the SCTS process).

Figure 5-5(d) The zoomed-in figure of Fig. 5-5(c).

5.4.2 Design of an FBG-Based Linear Phase Filter with a Grating Length of 25.8 mm

Figure 5-6(a) shows the index modulation profile of an optimized FBG with a length of 25.8 mm obtained from the hybrid optimization algorithm.

Figure 5-6(a) The solid line is the index modulation profile of a 25.8-mm long optimized linear phase filter obtained from the hybrid algorithm, and the dotted line is the index modulation profile of a 25.8-mm long single quadratic-sine apodized FBG.

The corresponding reflective spectrum and group delay response of the optimized FBG are shown in Fig. 5-6(b) and Fig. 5-6(c) (the solid line), respectively. It can be seen from Fig. 5-6(b) that the almost-squared reflective spectrum of the optimized FBG has good side-lobe suppression values of less

than -37 dB. Although the reflective spectrum of the optimized FBG is not as good as the reflective spectrum of a single apodized FBG (the dotted line in Fig. 5-6(b)), which has a side-lobe suppression value of -50 dB, the side lobes of the optimized FBG have been suppressed to an acceptably low level.

Figure 5-6 (b) The corresponding reflective spectra calculated from the index modulation profiles shown in Fig. 5-6(a). The solid line corresponds to the reflective spectrum obtained from the hybrid algorithm. The dotted line is the reflective spectrum of a 25.8-mm long single apodized FBG.

It can be seen from Fig. 5-6(c) that the in-band group delay ripple of the optimized FBG is less than 0.2 ps, which is a lot flatter than that of the single apodized FBG. Thus it is found that the grating optimized by the hybrid optimization algorithm has linear phase characteristics and near-ideal squared reflective spectrum.

Figure 5-6(c) The corresponding group delay responses calculated from the index modulation profiles shown in Fig. 5-6(a). The solid line corresponds to the group delay response obtained from the hybrid algorithm. The dotted line corresponds to the group delay response obtained from a 25.8-mm long single apodized FBG.

5.4.3 Design of an FBG-Based Linear Phase Filter with a Grating Length of 31.1 mm

Similarly, Figure 5-7(a) shows the index modulation profile of an optimized FBG with a length of 31.1 mm obtained from the hybrid optimization algorithm.

Figure 5-7(a) The solid line is the index modulation profile of a 31.1-mm long optimized linear phase filter obtained from the hybrid algorithm. The dashed line is the index modulation profile of a 31.1-mm long single quadratic-sine apodized FBG.

The corresponding reflective spectrum and group delay response of the optimized FBG are shown in Fig. 5-7(b) and Fig. 5-7(c) (the solid line), respectively. It can be seen from Fig. 5-7(b) that the almost-squared reflective spectrum of the optimized FBG has good side-lobe suppression values of less than -45 dB. Although the reflective spectrum of the optimized FBG is not as good as the reflective spectrum of a single apodized FBG (the dotted line in Fig. 5-6(b)), which has a side-lobe suppression value of -55 dB, the side lobes of the optimized FBG have been suppressed to an acceptably low level.

Figure 5-7(b) The corresponding reflective spectra calculated from the index modulation profiles shown in Fig. 5-7(a). The solid line corresponds to the reflective spectrum obtained from the hybrid algorithm. The dotted line is the reflective spectrum of a 31.1-mm long single apodized FBG.

It can be seen from Fig. 5-7(c) that the in-band group delay ripple of the optimized FBG is less than 0.3 ps, which is a lot flatter than that of the single apodized FBG. Thus it is found that the grating optimized by the hybrid optimization algorithm has linear phase characteristics and near-ideal square reflective spectrum.

Figure 5-7(c) The corresponding group delay responses calculated from the index modulation profiles shown in Fig. 5-7(a). The solid line corresponds to the group delay response obtained from the hybrid algorithm. The dotted line corresponds to the group delay response obtained from a 31.1-mm long single apodized FBG.

5.4.4 Discussion

Table 5-1 shows the performance comparison of the three linear phase optical filters with different grating lengths as designed by the hybrid optimization algorithm. It is noted that the designed filter with a longer grating length has a higher side-lobe suppression value. For example, the designed filter with the longest grating length of 31.1 mm has a side lobe of as low as -45 dB while the

designed filter with the shortest grating length of 17.1 mm has a side lobe of only -35 dB. However, it can also be seen from the table that the group delay ripple increases with the grating length of the designed filter. The designed filter with the longest grating length of 31.1 mm has the in-band group delay ripple of 0.3 ps while the designed filter with the shortest grating length of 17.1 mm has the group delay ripple of as low as 0.1 ps. It is found that a long grating length will yield a high side-lobe suppression at the expense of a large group delay ripple (see Table 5-1). Thus, one can conclude from this finding that there is a trade-off between a high side-lobe suppression value and a large group delay ripple for a long grating length.

Table 5-1 Comparison of the performances of the three linear phase optical filters with different grating lengths as designed by the hybrid optimization algorithm.

Grating length (mm)	Maximum side lobe value (dB)	Maximum in-band group delay ripple (ps)
17.1	-35	0.1
25.8	-37	0.2
31.1	-45	0.3

It should be noted that all the three optimized FBGs have arbitrary index modulation profiles. The two main issues involved in the fabrication of the

optimized index modulation (i.e. the “ac” index change) profiles are described as follows. First, the fabrication process must be able to realize such a complex index modulation profile along the grating length while maintaining an unperturbed “dc” index change along the grating length. The fabrication process to realize these index modulation profiles is similar to that used in the fabrication of apodized index modulation profiles and the profile shown in Fig. 5-2. This is because the fabrication of these two types of profiles is to produce some desired index modulation values corresponding to the grating positions while maintaining an unperturbed “dc” index change. To meet this first requirement, a well-developed technique of fiber dithering (which involves dithering of the fiber during the UV exposure) can be used [10], [12]. In the fiber dithering method, the first step is to divide the grating into a number of sub-gratings. Each sub-grating inscribed by the fiber dithering will cause the index modulation depth to be averaged out. That is, if the offset of the fiber dithering is half of the grating period, the index modulation will be completely averaged out (i.e. no index modulation). Now if there is no offset of the fiber dithering from the phase mask, the index modulation will be maximum. It is noted that, to maintain an unperturbed ‘dc’ index change along the grating length, all the sub-gratings must be exposed with the same total number of UV pulses (i.e. the exposure energy levels of all the sub-gratings are the same)._

Second, the fabrication process must be able to introduce some π phase shifts (i.e.

negative index modulation values) along the grating length (see Figs. 5-5, 5-6 and 5-7). To meet this second requirement, the fiber should be moved relatively to the phase mask (which is also a well-developed method), and this would enable the insertion of π phase shifts into the gratings during the fabrication process [4], [10–12]. However, it is still difficult to precisely control such π phase shifts during fabrication; thus the fabrication of complex FBGs (see Figs. 5-5, 5-6 and 5-7) is still a challenge using the method in [10–12].

5.5 Conclusion

A novel synthesis method of FBGs using a hybrid optimization algorithm has been presented. The effectiveness of the proposed method has been demonstrated by synthesizing three linear phase optical filters with 50 GHz bandwidth. The designed filters have reasonably flat group delay responses and near-squared spectra, and their grating lengths can be practically realized. The proposed hybrid synthesis method of FBGs can be further developed into a powerful toolbox for a variety of fiber grating designs.

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6

Stepped-Chirp Fiber Bragg Grating on a Pre-Stretched Fiber

This chapter presents the design, fabrication and testing of piece-wise stepped-chirp fiber Bragg gratings (FBGs) with arbitrary group delay responses using a uniform phase mask and a bare fiber section under varying pre-stretching conditions. The technique involves writing a series of sub-gratings on a pre-stretched fiber whose length is varied during the UV exposure. Two motorized translation stages are employed to control the length of the fiber to produce varying grating pitches at each writing step. Piece-wise stepped chirping of the FBGs can thus be obtained using this method. Furthermore, as the fiber is moved relatively to the phase mask, an apodized index modulation profile of the FBG can be produced by means of fiber dithering. A linearly chirped unapodized FBG and a quadratically chirped apodized FBG have been fabricated. The measured group delay responses of these two types of FBGs agree well with analytical predictions.

Section 6.1 reviews various methods used for the fabrication of chirped gratings.

Section 6.2 introduces the experimental setup and presents the analysis of the grating profile and the corresponding fabrication parameters. Section 6.3 presents the fabrication of two types of chirped gratings using the proposed method.

6.1 Introduction

Chirped fiber Bragg gratings (CFBG) are gratings with their Bragg wavelengths varying linearly or nonlinearly along the grating length. CFBGs have been widely used for dispersion compensation, pulse multiplication and pulse compression [1-3]. Several techniques have been developed for fabrication of CFBGs for different applications. One method uses a chirped phase mask which is expensive and the written gratings always exhibit a fixed group delay characteristics. To overcome this drawback, considerable attempts have been made to fabricate chirped gratings using less expensive uniform phase masks. These methods are dual-scanning technique [4], shifting the Bragg wavelength by inserting a converging lens before the mask [5], and moving fiber/phase mask-scanning beam technique [6]. Asseh *et al.* [7] has proposed a technique for the fabrication of long gratings with complex profiles, in which a large number of small partially overlapping sub-gratings were exposed in sequence by UV pulses.

Each sub-grating contained a few hundred periods. Thus, some complex properties of the FBG such as chirp, phase shifts, and apodization can be obtained by adjusting the phase offset between two partially overlapping sub-gratings. Petermann *et al.* [8] has improved on this method by using a UV CW source and a sawtooth movement of the interference pattern; hence the grating period can be varied with such an interferometric scanning setup. However, this method requires highly accurate positioning and an in-situ interferometric system for monitoring the position of the translation stage.

The design of stepped-chirp FBGs has been discussed by Kashyap [9]. Using a stepped-chirp phase mask, a stepped-chirp FBG can be written [10]. Another technique has been proposed by Ref. [11] which employed a stretching-and-writing method to fabricate stepped-chirp FBGs using a uniform phase mask. In this method, the Bragg period along the grating length can be adjusted by controlling the in-fiber strain of the fiber (and hence the pulling force) during the UV exposure. Because the interferometric pattern used to inscribe the grating is formed by a uniform phase mask, the exposure process [11] is quite stable compared with that presented in Ref. [8]. Furthermore, the moving distance of the stage which stretches the fiber is very short (i.e. a change in the grating period is very small). Thus the interferometric system that is needed to monitor the fabrication process in [8] is not required in this method [11]. However, Ref. [11] has not investigated the relationship between the group delay response of an FBG

and its fabrication parameters, and hence tailoring of the chirped gratings to obtain arbitrary group delay responses is still a challenge.

In this chapter, an improved method for inscribing the step-chirped FBGs is presented to generate arbitrary group delay responses using a uniform phase mask together with stretching both ends of the fiber (and thus the term pre-stretching). Two motorized stages are employed to apply compressive forces to adjust the tension of the pre-stretched fiber during the UV exposure. Thus a grating structure with a desired group delay response can be inscribed. In order to control the continuity of the phase of the grating, the two motorized stages are dynamically translated during the writing process. The fiber can be moved relatively to the phase mask during the fabrication process, and thus the fiber can be dithered by the two motorized stages to realize an apodized grating profile [7]. To obtain a constant distribution of strain along the pre-stretched fiber, the coating of the fiber is stripped off.

The main improvement of the proposed method over the method of [11], as discussed above, is that the two motorized stages are simultaneously stretching the fiber, and thus the phase errors due to gaps or overlaps between neighbored subgratings can be minimized. Furthermore, the relationship between the fabrication parameters and the properties of the produced chirped FBGs is

analyzed in this work. Linearly-chirped and nonlinearly-chirped FBGs have been fabricated using the proposed pre-stretching method. The measured reflective spectra and the group delay responses of these two types of FBGs are consistent with the analytical predictions.

6.2 Fabrication System and Analysis

Figure 6-1 shows the schematic diagram of the setup used for fabricating the FBGs with arbitrary group delay responses. The UV beam from a frequency-doubled Argon laser is folded by a mirror mounted on a motorized translation stage and focused using a cylindrical lens onto the pre-stretched fiber through the phase mask. The UV beam is scanned along the pre-stretched fiber and the scanning velocity is controlled by a motorized translation stage. The fiber is clamped by two motorized stages (i.e. Stage A and Stage B). The mounted fiber is then pre-stretched before the UV exposure so that the fiber length can either be increased (by stretching the fiber) or decreased (by releasing the fiber). The plastic coating of the fiber might reduce the exposure efficiency and hence reducing the grating depth. The gratings used for dispersion compensation usually require high reflectivities so they are often inscribed with the fiber coatings stripped off. To obtain a constant distribution of the in-fiber strain along

the fiber length, the plastic coating of a segment of the fiber between the two holders is removed including the segment without the inscribed grating. The coating in the segment in the holders is not stripped off to avoid slippage.

Figure 6-1 Schematic diagram of the system setup for fabricating the chirped FBGs with arbitrary group delay responses.

The photosensitivity of the fiber could be changed with different in-fiber strain. However, the change of the in-fiber strain is quite small, and thus the effect of the photosensitivity due to the in-fiber strain can be ignored in the experiment. The original distance between the two stages is L_1 during the UV scanning of the first sub-grating.

The index modulation of the grating will be averaged out when the UV beam illuminates the fiber while the fiber is moving. To avoid this, the UV beam must be shielded when the fiber is stretched. Thus the UV beam will be shielded, after every sub-grating is written. It is noted that after the UV beam is shielded both

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ends of the fiber (with length L_1) are stretched and the fiber length now becomes $L_2 = L_1 + x_1$, where x_1 is the amount of stretching on both fiber ends, and $L_2 > L_1$ for $x_1 > 0$ (i.e. stretching the fiber) or $L_2 < L_1$ for $x_1 < 0$ (i.e. releasing the fiber). That is, the stretched length $x_1 = x_A^1 + x_B^1$ is the amount of movement of Stage A and Stage B, where x_A^1 and x_B^1 are the distances moved by Stage A and Stage B, respectively (see Fig. 6-1). And l_1 is the distance between Stage A and the position between the first sub-grating and the second sub-grating. The fiber will move relatively to the phase mask once the fiber is stretched by the two stages. This change in the relative position between the fiber and the phase mask could produce gaps or overlaps between two neighboring sub-gratings. To minimize such errors, the writing position of the UV beam on the fiber should be kept stationary relative to the phase mask. Thus, the stretching distances of Stage A and Stage B should follow the following relationship:

$$x_A^1 = \frac{x_1 l_1}{L_1} \quad \text{and} \quad x_B^1 = x_1 - x_A^1 = \frac{x_1 (L_1 - l_1)}{L_1} \quad (6.1)$$

Then the shutter is then re-opened to write the next sub-grating section. After the fiber is stretched, the grating period of the first sub-grating is elongated (i.e. $L_2 > L_1$). Thus the difference between the first sub-grating period and the second sub-grating period can be described by

$$\Delta\Lambda_1 = \frac{x_1}{2L_1} \Lambda_p \quad (6.2)$$

where Λ_p is the period of the phase mask. Thus a chirped grating fabricated with N steps can be formed after fabricating a number of consecutive sub-gratings with a series of stretched lengths between any two neighboring sub-gratings as described by $\vec{X} = [x_1, x_2, \dots, x_{N-1}]$. Variation of the fiber length during stretching is given by $\vec{L} = [L_1, L_2, \dots, L_N]$, where L_j is the fiber length when inscribing the j^{th} sub-grating. Using the fact that $L_j \gg x_j$, the following relationship holds $L_1 \approx L_2 \approx \dots \approx L_N$. The longitudinal position of the j^{th} sub-grating along the fiber length is $z_j = j \times \Delta z$, where $j = 1, 2, \dots, N$, is the section number and Δz is the length of each sub-grating. Thus the Bragg wavelength at position z_j can be approximately given as

$$\lambda(z_j) = \lambda_1 + 2n_{\text{eff}} \sum_{m=1}^j \Delta\Lambda_m = \lambda_1 + \frac{n_{\text{eff}} \Lambda_p}{L_1} \sum_{m=1}^{\frac{z_j}{\Delta z}} \vec{X}(m) \quad (6.3)$$

where n_{eff} is the effective index of the grating, and λ_1 is the Bragg wavelength of the first sub-grating which can be determined by the initial strain applied to the fiber. Thus the fabricated grating will follow a stepped-chirped structure whose period is similar to those presented in [9], [10]. Figure 6-2 shows the schematic diagram of such a stepped-chirp grating.

Figure 6-2 Schematic diagram of a step-chirped grating with N equal sections. Each grating section has a length of δl and a period of Λ_j .

The total length of the grating is denoted as L_g [9], [10].

The time delay at the wavelength λ_i can be given by

$$\tau(\lambda_i) = 2n_{eff}z/c \quad (6.4)$$

It should be noted that the distance between Stage A and the exposure position, that is, $\vec{l} = [l_1, l_2, \dots, l_{N-1}]$ (where $l_{N-1} > l_2 > l_1$), becomes longer during the exposure process. The stretched lengths $\vec{x}_A = [x_A^1, x_A^2, \dots, x_A^{N-1}]$ and $\vec{x}_B = [x_B^1, x_B^2, \dots, x_B^{N-1}]$ must be dynamically set according to Eq. (6.1) to obtain a stepped-chirp grating with a continuous phase.

6.3 Experimental Results and Discussion

In the experiment, the grating was inscribed into a germanium-doped fiber (Stocker Yale PMS 50). The period of the phase mask is 1070 nm. The power of the UV laser is 50 mW.

First, a linearly chirped grating was fabricated. To obtain a linearly chirped grating, the different stretched lengths were set to be the same (i.e. $x_1 = x_2 = \dots = x_{N-1} = x_c$, where x_c is a constant value). For a linearly chirped FBG, the corresponding set of Bragg wavelengths of the sub-gratings along the stretched fiber can be derived from Eq. (6.3) as

$$\lambda(z_j) = \lambda_1 + \frac{n_{eff}\Lambda_p x_c}{L_1} \times \left(\frac{z_j}{\Delta z} - 1\right) \quad (6.5)$$

Thus the group delay $\tau(\lambda)$ of a linearly chirped FBG relates to this set of Bragg wavelengths as

$$\tau(\lambda) = \frac{2\Delta z L_1}{\Lambda_p x_c c} (\lambda - \lambda_1) + \frac{2n_{eff}\Delta z}{c} \quad (6.6)$$

The dispersion of a linearly chirped FBG can then be derived from Eq. (6.6) as

$$D = \frac{d\tau}{d\lambda} = \frac{2\Delta z L_1}{\Lambda_p x_c c} \quad (6.7)$$

The velocity of the translation stage is 0.01 mm/s. The initial length of the pre-stretched fiber, L_1 , between Stage *A* and Stage *B* is $L_1 = 193$ mm. The number of sub-gratings is 50 and the total length of the FBG is 25 mm. Thus the length of each sub-grating is $\Delta z = 25 \text{ mm}/50 = 0.5 \text{ mm}$. The stretched length profile was set as $\vec{X} = \{5, 5, \dots, 5\} \text{ } \mu\text{m}$ and thus $x_c = 5 \text{ } \mu\text{m}$. By substituting these parameter values into Eq. (6.7), the calculated dispersion of the FBG is 120.25 ps/nm.

Figure 6-3 shows the measured reflective spectrum and group delay response of a fabricated linearly chirped FBG. The in-band dispersion is ~ 123.9 ps/nm and is quite close to the theoretical value of 120.25 ps/nm, and Fig. 6-4 shows the deviation of the linear portion of the group delay response. Some ripples can be observed in the reflective spectrum as shown in Fig. 6-3, and the largest ripple is ~ 10 dB at a wavelength of 1552.7 nm.

Figure 6-3 Measured reflective spectrum and group delay response of a linearly chirped FBG.

Figure 6-4 Deviation of the linear time delay of the measured in-band group delay response, and the estimated dispersion factor is 123.9 ps/nm.

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Second, the pre-stretching method is applied to the fabrication of apodized quadratically chirped FBGs. A quadratically chirped FBG will have a quadratic Bragg wavelength distribution along the grating length. According to Equation (6.3), when the stretched lengths are set in the vector form as $\vec{X}(j) = x_1 - k(j-1)$, where $j = 1, 2, \dots, N-1$, and k is a constant value, the Bragg wavelength distribution along the grating length will be a quadratic function. From Eq. (6.3) and Eq. (6.4), the group delay as a function of the Bragg wavelength of a quadratically chirped FBG can be derived as

$$\tau(\lambda) = -\frac{2n_{eff}\Delta z(x_1 + k/2)}{kc} \left[1 - \frac{2kL_1}{n_{eff}\Lambda_p(x_1 + k/2)^2} (\lambda - \lambda_1) \right]^{1/2} + C \quad (6.8)$$

where C is a constant. The Taylor series approximation of Eq. (6.8) is given by

$$\begin{aligned} \tau(\lambda) = \tau(\lambda_1) + \frac{2\Delta z L_1}{c\Lambda_p(x_1 + k/2)} (\lambda - \lambda_1) \\ + \frac{1}{2} \left(\frac{\Delta z k L_1^2}{2n_{eff}\Lambda_p^2 c(x_1 + k/2)^3} \right) (\lambda - \lambda_1)^2 + \dots \end{aligned} \quad (6.9)$$

The following parameter values are used in the experiment: the velocity of the translation stage is 0.005 mm/s, the initial length of the pre-stretched fiber, L_1 , between Stage A and Stage B is $L_1 = 193$ mm, the number of sub-gratings is 100,

the total length of the FBG is 50 mm (hence $\Delta z = 50 \text{ mm}/100 = 0.5 \text{ mm}$), and the stretched length of the j^{th} section was set as $x(j) = x_1 - 0.029(j-1) \text{ } \mu\text{m}$ where $x_1 = 2.5 \text{ } \mu\text{m}$ and $j = 1, 2, \dots, N-1$. By substituting these parameter values into Eq. (6.9), the linear coefficient of the time delay is calculated as 164.1 ps/nm and the quadratic coefficient (or dispersion slope) is calculated as 21.3 ps/nm^2 .

Figure 6-5 shows the measured reflective spectrum and group delay response of a fabricated quadratically chirped FBG. Figure 6-6 shows the quadratic part of the measured in-band group delay response (i.e. linear portion of the group delay is eliminated), and the estimated linear coefficient and quadratic coefficient are 158.9 ps/nm and 19.7 ps/nm^2 , respectively.

Figure 6-5 Measured reflective spectrum and group delay response of a quadratically chirped FBG.

Figure 6-6 Deviation of the linear time delay of the measured in-band group delay response of a quadratically chirped FBG. The estimated dispersion is 158.9 ps/nm and the estimated dispersion slope is 19.7 ps/nm².

As described above, the large ripples in the reflective spectra and group delay responses of both the linearly chirped FBG and the quadratically chirped FBG could possibly due to the positioning errors of the stages used for stretching the fiber (i.e. errors in controlling $\bar{x}_A$ and $\bar{x}_B$ as shown in Eq. (6.1)); hence some possible gaps (or overlaps) between neighboring sub-gratings were produced in the two fabricated FBGs. This problem could be overcome by using more precise controllers such as PZT controllers instead of the motorized stages (i.e. Stage A and Stage B) as presented here.

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It is noted that there is a limit to the maximum achievable bandwidth of the fabricated grating with this method due to the physical failure feature of the silica fiber (which is an inherent property of the fiber material) when a significant amount of strain is applied to the fiber. If the force (or strain) applied to the fiber is below the ‘ultimate force’ value, the fiber will retain its elastic property. That is, the stretched fiber will go back to its original length when the applied strain is removed from the fiber. Assuming that the ‘ultimate force’ of the fiber is F_u and the elastic coefficient of the fiber is k_f , the maximum allowable stretched length ΔL of the fiber under the ‘ultimate force’ is given by

$$\Delta L_{\max} = F_u / k_f \quad (6.12)$$

It is known that

$$\frac{\Delta L_{\max}}{L_1} = \frac{2\Delta\Lambda_{\max}}{\Lambda_p} = \frac{\Delta\lambda_{\max}}{\lambda_1} \quad (6.13)$$

where $\Delta\Lambda_{\max}$ is the maximum allowable change of the grating period and $\Delta\lambda_{\max}$ corresponds to the maximum allowable bandwidth of the spectrum. The maximum allowable bandwidth of the spectrum is thus given by

$$\Delta\lambda_{\max} = \frac{F_u}{L_1 k_f} \lambda_1 = \frac{F_u \bar{n}_{eff} \Lambda_p}{L_1 k_f} \quad (6.14)$$

6.4 Conclusion

This chapter has presented the design and fabrication of the piece-wise stepped-chirp fiber Bragg gratings (FBGs), which were written using a uniform phase mask while a section of the bare fiber was under pre-stretching condition. FBGs with arbitrary group delay responses have been demonstrated. The group delay response can be arbitrarily generated by setting the stretched length of the fiber during the UV exposure to introduce phase shifts into the FBG to realize the apodized index modulation profile. A linearly chirped FBG and a quadratically chirped FBG have been developed using this method. The measured group delay responses of these two types of FBGs show a good agreement with the analytical predictions.

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7

Nonlinearly Chirped Grating Written in a Pre-Stretched Fiber

In this chapter, a novel fabrication method is reported on the generation of continuous chirp in a fiber Bragg grating (FBG) using a uniform phase mask. The fiber is stretched before the grating is written. The stretched length of the fiber is varied while the UV beam is scanned. The grating fabricated with this method has a nonlinear group delay response and an asymmetric passband reflective spectrum. The analytical results agree with those obtained from the experiments.

Section 7.1 reviews several methods on the fabrication of chirped FBGs using a uniform phase mask. Section 7.2 introduces the experimental setup of the proposed fabrication method. Theoretical analysis of the proposed fabrication process is presented in Section 7.3, and a nonlinearly-chirped FBG was fabricated using the proposed method and was measured as presented in Section 7.4.

7.1 Introduction

The accumulated dispersion and nonlinearities of the optical fiber are two of the most troublesome issues in fiber transmission operating at speed higher than 10 Gbit/s. Furthermore, in optical communication systems operating at 40 Gbit/s, advanced modulation formats such as the vestigial single sideband (VSB) will require optical filters that could reject half of the unwanted band of the signal with a high rejection ratio on one side and tolerable roll-off band on the other [1]. In addition, an optical filter can be considered as an important filtering component at the output of an optical transmitter if it can function as a dispersion-compensating device as well as a half-band filter.

Nonlinearly chirped fiber Bragg gratings (NCFBGs) have been recognized as one of the most attractive solutions to the above-mentioned problem [2–3]. However, due to the high cost of a nonlinearly chirped phase mask, various techniques have been proposed on the fabrication of NCFBGs using a uniform phase mask, which include temperature gradient or strain gradient method [4], dual-scanning technique [5], and shifting the Bragg wavelength by adding a converging lens before the mask [6].

The Bragg wavelength can be tuned by continuously changing the phase of the

grating (see Eqs. (2.8) and (2.9)). The phase can be directly inserted into the grating by adjusting the relative position of the fiber to the phase mask. Thus a chirped profile can be produced by continuously changing the phase of the grating [7–8]. Using such a moving fiber/phase mask technique, a nonlinearly chirped grating with a high-order group delay response can be realized by producing a complex phase profile. However, producing such a complex phase profile usually requires complicated control of the velocity of the fiber-mounting platform. Precise control of a complex velocity profile of the fiber is not easy, thus making fabrication of the nonlinearly chirped gratings difficult, and hence large fabrication errors are expected using this method.

Tuning the Bragg wavelength of a grating using the phase-mask technique can also be achieved by applying strain to the fiber during the UV illumination stage [9]. The method has been demonstrated on the fabrication of stepped chirped gratings [10] and it has also been discussed in Chapter 6. In this chapter, a new technique is presented on the inscription of a continuous and nonlinearly chirped grating in a fiber under pre-stretched condition. With this method, a self-apodized, continuous and nonlinearly chirped grating can be produced. The main parameters that determine the properties of the chirped grating are analyzed. Furthermore, an asymmetric roll-off half-band feature in the reflective spectrum of the fabricated NCFBG is also found. The analytical and experimental results agree reasonably well with each other. The ripple of the in-band group delay

response is found to be less than 20 ps.

7.2 Experimental Setup

Figure 7-1 shows the schematic diagram of the setup used in the fabrication of the nonlinearly chirped FBGs. The UV beam from a frequency-doubled Argon laser is folded by a mirror mounted on a motorized translation stage and focused using a cylindrical lens onto the fiber through the phase mask. The UV beam is scanned along the fiber and its velocity is controlled by the translation stage. A fixed tower clamps one end of the fiber while a movable stage holds the other fiber end. The key improvement of the proposed method over the previously reported technique [9] is that, in the proposed method, the stretched length of the fiber is adjusted while the UV beam is scanned, and hence continuously chirped grating pitches can be produced. However, the technique presented in [9] can only be used to fabricate stepped-chirp gratings. The plastic coating of the fiber might reduce the UV exposure efficiency resulting in a decrease of the depth of the index modulation of the grating. However, the gratings required for dispersion compensation are generally strong so the gratings must be inscribed with the fiber coatings stripped off. To obtain a constant distribution of the in-fiber strain along the fiber length, the plastic coating of the segment of the

fiber (including the fiber segment without the grating being inscribed) between the two holders is removed. The coating of the fiber segment within the holders is not stripped off to avoid slippage.

Figure 7-1 Proposed experimental setup for fabricating nonlinearly chirped fiber Bragg gratings.

Because the fiber moves relatively to the phase mask (due to the adjustment of the stretched fiber length) during the scanning of the UV beam, a gradual phase shift will be added to the fiber grating being written [7]. However, the proposed method is different from the moving fiber/phase mask scanning beam technique [8] in two ways. First, the velocity of the fiber is not equal to that of the stage that holds the fiber. Second, the fiber is stretched and hence the grating period is chirped during the fabrication process. With reference to the operating principle of the proposed method as described above, the resultant index modulation profile, $\delta n_{eff}(z)$, along the grating length z can be described by [11]

$$\delta n_{eff}(z) = \Delta n_{ac}(z) \cos \left[\frac{2\pi z}{\Lambda(z)} + \phi_f(z) \right] + \Delta n_{dc} \quad (7.1)$$

where $\phi_f(z)$ is the grating phase due to the movement of the fiber, $\Lambda(z)$ is the grating period due to the stretching of the fiber during the scanning of the UV beam, $\Delta n_{ac}(z)$ is the “ac” index change, and Δn_{dc} is the “dc” index change which can be approximated to be a constant here. In the following section, the characteristics of $\phi_f(z)$, $\Lambda(z)$ and $\Delta n_{ac}(z)$ are analyzed.

7.3 Analysis of the Grating Profiles

The operating principle of the proposed method has been described above. This section presents the analysis of the phase of the grating, the period of the grating and the index modulation of the grating produced by this method.

In Fig. 7-1, l is defined as the distance between the initial position of the UV beam (before starting to inscribe the grating) and the fixed tower which clamps the fiber, and L is defined as the fiber length between the fixed tower and the other end which is moved by a motorized stage. When the stage moves, the fiber length will change from L into L' . With reference to Fig. 7-1, the velocity of the stage

holding the fiber is v_f and v_s is the scanning velocity of the UV beam. Usually $v_f \ll v_s$ [8], thus one can assume that $L \cong L'$ (i.e. the fiber length can be approximated to be a constant value during the fabrication process). Considering that the plastic coating of the fiber is removed, the velocity profile of the fiber at the writing position x (i.e. at a position where the UV beam is illuminated on the fiber) can be approximately given by

$$v_f^x(x) \approx \frac{v_f}{L} x \quad (7.2)$$

where $x = l + z$. The velocity profile $v_f^x(x)$ along the fiber length x is schematically shown in Fig. 7-2.

Figure 7-2 Schematic diagram of the velocity profile of the fiber when one end of the fiber is moved with velocity v_f by a motorized stage.

As shown in Fig. 7-2, the fiber velocity $v_f^0(0)$ is zero at $x = 0$ (i.e. at position O where the fiber is clamped by the fixed tower, see Fig. 7-1), and the fiber velocity $v_f^L(L)$ is v_f at $x = L$ (i.e. at the position where the motorized stage moves the fiber, see Fig. 7-1). As described above, the fiber length L can be assumed to be constant and thus the approximation sign in Eq. (7.2) can be used.

Figure 7-3(a) illustrates the index modulation profile of an FBG fabricated by a conventional method (i.e. the fiber is not moved during the scanning of the UV beam), and the phase of the grating can be continuous (i.e. no phase shift is inserted into the grating). In the proposed setup (see Fig. 7-1), a gradual phase shift can be inserted into the grating as the fiber moves relatively to the phase mask during the scanning of the UV beam. Figure 7-3(b) shows this situation where a phase shift is inserted between two adjacent grating sections when the fiber moves relatively to the phase mask during the UV exposure.

If the fiber has moved a distance of Δx_f , the corresponding phase shift inserted into the grating, $\Delta\phi_f$, can be described by

$$\Delta\phi_f(x) = \Delta x_f \frac{2\pi}{\Lambda_p / 2} = \frac{4\pi}{\Lambda_p} \Delta x_f \quad (7.3)$$

where Λ_p is the period of the phase mask. When the fiber moves a distance of Δx_f ,

the UV beam scans a distance of Δx , which is given by

$$\Delta x = \frac{\Delta x_f}{v_f^x(x)} [v_s + v_f^x(x)] \quad (7.4)$$

Figure 7-3 (a) Schematic diagram of the index modulation profile of the grating when the fiber is not moved (i.e. it has a continuous phase property). (b) Schematic diagram of the index modulation profile of the grating when the fiber is moved a distance of Δx (i.e. a phase shift is inserted into a small grating section).

After the UV beam has scanned a distance of Δx , the corresponding phase shift inserted into the grating, $\Delta\phi_f$, which is a function of the fiber length x , can be obtained by substituting Eq. (7.4) into Eq. (7.3), and it is given by

$$\Delta\phi_f(x) = \frac{4\pi}{\Lambda_p} \left(\frac{\Delta x}{v_s + v_f^x(x)} \right) v_f^x(x) \quad (7.5)$$

The phase at a particular position z of the grating (where $x = l + z$) can be obtained by performing an integration of the phase shift inserted into the grating as defined in Eq. (7.5) to yield

$$\phi_f(z) = \int_l^{l+z} \frac{\Delta\phi_f(x)}{\Delta x} dx \quad (7.6)$$

By putting Eq. (7.5) into Eq. (7.6), the phase of the grating produced by moving the fiber along the grating length z can be shown to be

$$\phi_f(z) = -\frac{2\pi}{\Lambda_p} \alpha (2l + z)z \quad (7.7)$$

where $\alpha = \frac{v_f}{v_s L}$, and the phase at the starting position $z = 0$ is presumed to be zero.

At the same time, the length of the stretched fiber is changing. If the UV beam scans a distance of Δx , the change in the fiber length, ΔL , can be estimated as

$$\Delta L = \frac{v_f}{v_s} \Delta x \quad (7.8)$$

and the change in the grating period, $\Delta\Lambda$, can be assumed to be

$$\frac{\Delta\Lambda}{\Lambda_p/2} = \frac{\Delta L}{L} \quad (7.9)$$

Thus the profile of the grating period, $\Lambda(z)$, can be obtained by integrating $\Delta\Lambda$ (as defined in Eq. (7.9)) to yield

$$\Lambda(z) = \Lambda_0 + \int_l^{l+z} \frac{\Delta\Lambda}{\Delta x} dx \quad (7.10)$$

where Λ_0 is the grating period at the starting position $z = 0$ of the grating.

Using Eqs. (7.8)–(7.10), the chirped period $\Lambda(z)$ along the grating length z can be shown to be

$$\Lambda(z) = \Lambda_0 + \left(\frac{\alpha\Lambda_p}{2} \right) z \quad (7.11)$$

By substituting Eq. (7.7) and Eq. (7.11) into Eq. (7.1), a general function of the index modulation profile can be shown to be

$$\delta n_{eff}(z) = \Delta n_{ac}(z) \cos \left[\frac{2\pi z}{\Lambda_p / 2} + \phi_{eff}(z) \right] + \Delta n_{dc} \quad (7.12)$$

where the “effective phase” of the grating, ϕ_{eff} , is given by

$$\phi_{eff}(z) = \phi_f(z) - \frac{4\pi v_f z^2}{\Lambda_p (v_s L + v_f z)} \quad (7.13)$$

In terms of more readily understandable parameters, the “effective phase” term can take a form which is similar to that given by Eq. (2.9) (see Section 2.2 of Chapter 2) as follows

$$\frac{1}{2} \cdot \frac{d\phi_{eff}}{dz} = - \left(\frac{4\pi \bar{n}_{eff} z}{\lambda_D^2} \right) \frac{d\lambda_{eff}}{dz} \quad (7.14)$$

where $\lambda_D = \bar{n}_{eff} \Lambda_p$ is the designed Bragg wavelength, $\bar{n}_{eff}$ is the effective

index of the fiber, λ_{eff} is the “effective” Bragg wavelength, and $\frac{d\lambda_{eff}}{dz}$ is the

“effective” chirp parameter that measures the rate of change of the designed “effective” Bragg wavelength with the grating position. By substituting Eq. (7.13) into Eq. (7.14), the “effective” chirp parameter in Eq. (7.14) can be rewritten as

$$\frac{d\lambda_{eff}}{dz} = \bar{n}_{eff} \Lambda_p \alpha \left(3 + \frac{l}{z+l} \right) \quad (7.15)$$

By performing a Taylor series approximation on Eq. (7.15), Eq. (7.15) becomes

$$\frac{d\lambda_{eff}}{dz} = \bar{n}_{eff} \Lambda_p \alpha \left(4 - \frac{z}{l} \right) \quad (7.16)$$

It should be noted that the higher-order terms of the Taylor series expansion have been ignored in Eq. (7.16) because they are very small compared to the term in Eq. (7.16). By integrating Eq. (7.16), the “effective” Bragg wavelength along the grating length z can be shown to be

$$\lambda_{eff}(z) = \lambda_0 + \left(4\bar{n}_{eff} \Lambda_p \alpha \right) z - \left(\frac{\bar{n}_{eff} \Lambda_p \alpha}{2l} \right) z^2 \quad (7.17)$$

where $\lambda_0 = \lambda_{eff}(0)$ is the “effective” Bragg wavelength at position $z = 0$. It is noted

from Eq. (7.17) that the “effective” Bragg wavelength is nonlinearly distributed along the grating length because it is of a second-order polynomial in position z .

The group delay experienced by an optical signal reflecting from a particular position z of the grating is given by $\tau = 2z\bar{n}_{eff} / c$ (see Eq. (6.4)), where c is the speed of light in vacuum. By substituting z in this definition of the group delay τ into Eq. (7.17), the group delay of the grating as a function of the “effective” Bragg wavelength can be shown to be

$$\tau(\lambda_{eff}) = \frac{1}{2c\alpha\Lambda_p}(\lambda_{eff} - \lambda_0) + \frac{1}{2} \left(\frac{3}{32c\alpha^2\bar{n}_{eff}\Lambda_p^2 l} \right) (\lambda_{eff} - \lambda_0)^2 \quad (7.18)$$

It can be seen from Eq. (7.18) that the grating has a nonlinear (or second-order) group delay characteristics. Furthermore, the linear coefficient (i.e. the coefficient of the first term of Eq. (7.18)) and the quadratic coefficient (i.e. the coefficient of the second term of Eq. (7.18)) of the group delay characteristics are both positive. These coefficients can be determined by using the parameter values used in the fabrication.

As shown in Eq. (7.2) and Fig. 7-2, the velocity of the fiber increases with the scanning distance of the UV beam along the fiber. As discussed in [8], the

refractive index modulation will become weaker and weaker with an increase in the fiber velocity until it is completely averaged out when the fiber moves too quickly across the interference pattern formed by the phase mask. Thus the fabricated grating will have a semi-apodized index modulation profile.

7.4 Results and Discussion

Section 7.3 has presented the analysis of the phase profile, the grating period profile and the index modulation profile of the grating fabricated using the proposed method. This section presents the fabrication of a nonlinearly chirped grating with this method.

The values of the fabrication parameters are given as follows. The velocity of the scanning beam is 0.01 mm/s and the velocity of the stage that moves the fiber is 0.01 $\mu\text{m/s}$. The period of the phase mask is 1070 nm. The length of the fiber is 143 mm. The grating was inscribed in a germanium-doped photosensitive fiber (PMS-50 from Stock Yale). The group delay was measured using Agilent's optical dispersion analyzer (model 86038A). The wavelength resolution is 3 pm and the modulation frequency of the analyzer is 500 MHz. Figure 7-4 shows the measured reflective spectrum of a fabricated nonlinearly chirped grating which

has a 3-dB bandwidth of 1.2 nm (the dark line). By putting the parameter values used in the fabrication as given above into Eq. (7.17), the chirp profile of the grating can be estimated theoretically, and this will allow the reflective spectrum and the group delay response to be calculated using the transfer matrix method as described in Section 2.3.3 of Chapter 2. The calculated reflective spectrum of the nonlinearly chirped grating (the gray line) is also shown in Fig. 7-4 for comparison with the experimental result. It can be seen that the two results agree quite well with each other, and the small discrepancy between the measured and theoretical spectra might be due to fabrication errors.

Figure 7-5 shows the measured and calculated group delay responses of the nonlinearly chirped grating. It can be seen that the measured group delay response agrees quite well with the theoretically calculated group delay response within the in-band region. By putting the experimental parameter values as given above into Eq. (7-18), the linear and quadratic coefficients of the group delay response can be theoretically estimated as 223.9 ps/nm and 74.9 ps/nm², respectively, and the measured linear and quadratic coefficients of the group delay response are 215.8 ps/nm and 73.6 ps/nm², respectively. These two sets of results are very close with each other.

Figure 7-4 Measured reflective spectrum (dark line) and calculated reflective spectrum (gray line) of the fabricated nonlinearly chirped grating.

Figure 7-6 shows the ripples of the group delay response. It can be seen that the ripples of the in-band group delay response are quite small due to the fact that the index modulation was apodized. The peak-to-peak ripple of the in-band group delay response is about 20 ps.

Figure 7-5 Measured group delay response (with markers) and calculated group delay response (solid line) of the nonlinearly chirped grating.

Figure 7-6 Measured in-band group delay ripple of the nonlinearly chirped grating.

It is noted that Fig. 7-4 shows that the asymmetric reflective spectrum has a sharp roll-off of more than 25 dB over a 0.1 nm interval on the right side of the spectrum but only over a 1.0 nm interval on the other side of the spectrum. Thus the nonlinearly chirped grating can be potentially used as a VSB filter [1].

In optical fiber communication systems, a pre-distortion compensating device can be used as a dispersion-compensating device as well as a half-band filter at the transmitter output before signal transmission through the first span of the optical fiber. Thus the nonlinearly chirped FBG proposed here can be potentially used for such a purpose in VSB-formatted optical communication systems. It should be noted that, using the proposed method, the dispersion of the nonlinearly chirped FBG could also be tailored to suit a particular

VSF-formatted optical communication system. Furthermore, the residual high-order dispersion coefficient at the end of a fiber link could either be positive or negative depending on the system design. Thus the nonlinearly chirped FBG can also be potentially used as a high-order dispersion compensator with a positive dispersion coefficient to compensate for the negative high-order dispersion coefficient at the end of an optical fiber communication system.

7.5 Comparison of the Method with the Stepped-Stretching Method Presented in Chapter 6

The stepped-stretching method for the fabrication of CFBGs requires devices with high positioning precision to ensure that there are no gaps or overlaps between two neighboring sub-gratings. The continuous-stretching method presented here requires continuous stretching of the fiber during the UV exposure. Thus the gratings fabricated by this method will have no gaps inserted into the grating, and therefore the reflection spectra as well as the group delay responses of the gratings will be smoother than those of the gratings fabricated by the method presented in Chapter 6.

It should be noted that the CFBGs fabricated by this method will usually have

higher-order dispersion properties; thus it is difficult to fabricate linearly CFBGs. Hence the method presented in Chapter 6 is more suitable for the fabrication of linearly CFBGs,

The maximum allowable bandwidth of the filter fabricated by the stepped-stretch method presented in Chapter 6 is related to the strain of the fiber (see Eq. (6.14)). It should be noted, in this chapter, that the fiber actually moves relatively to the phase mask with the UV exposure on the fiber and the grating will vanish if the fiber moves too fast. Thus the maximum allowable bandwidth of the filter fabricated with the method presented here is not related to the strain of the fiber.

7.6 Conclusion

A new method has been presented for the fabrication of the nonlinearly chirped fiber Bragg gratings (FBGs) using only a uniform phase mask. The fiber was pre-stretched and the stretched length was controlled by a motorized stage while the UV beam was scanned along the fiber. An asymmetric chirped grating with a nonlinear group delay response was fabricated to demonstrate the effectiveness of the proposed method. The ripple of the group delay response of the grating is relatively small. Experimental results have shown good agreement with those

obtained by theoretical predictions. The method enables the fabrication of complex chirped profiles with a relatively simple operation. The half-band filtering property of the nonlinearly chirped FBG can be potentially used as a vestigial single sideband (VSB) filter for advanced-format optical communications systems. Furthermore, the proposed method may be extended to the fabrication of FBGs with higher-order group delay characteristics for application in the next generation of high-speed optical communication systems.

References

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8

Conclusions and Recommendations for Future Work

In this chapter, the previous chapters will be concluded with a special emphasis on the results. Some future works will also be suggested.

8.1 Conclusions

In Chapter 2, the coupled-mode equations have been derived for the analysis of the coupling between the forward and backward propagating waves in a fiber grating. The connection between the mathematical model and the physical grating has been studied. Once the mathematical model is established, the reflection and transmission responses of a grating can be analyzed. When analyzing a non-uniform grating, numerical tools such as the well-known Runge-Kutta numerical integration method or the transfer-matrix method (TMM) must be used. Gratings with different types of grating profiles have been

simulated with the TMM. Using the TMM, some apodized gratings have been analyzed.

Chapter 3 was devoted to the development of an effective global optimization algorithm, namely, the staged continuous tabu search (SCTS) algorithm. The method combines three stages of the continuous tabu search (CTS) algorithm with three strategies for neighborhood generations. From the test results of a number of benchmark test functions as listed in the Appendix, it was found that the SCTS algorithm is more efficient than the original CTS and the improved genetic algorithm (IGA) in terms of the success rate and computational efficiency. From the experimental results of these test functions, the SCTS algorithm can be considered as a potentially powerful tool for solving a variety of engineering problems such as the optimization of the fiber Bragg grating (FBG) design.

Chapter 4 has presented the application of the SCTS optimization algorithm developed in Chapter 3 to the synthesis problem of FBGs. First, an optical bandpass filter was designed based on an FBG model of piecewise uniform sections. The simulation results showed that the SCTS algorithm was superior to the adaptive simulated annealing (ASA) algorithm and the genetic algorithm (GA) algorithm. In addition, a linear phase optical filter with 50 GHz bandwidth was designed based on an FBG model of cascaded apodized sections. The

promising simulation results showed that the proposed SCTS-based synthesis method can be further developed into a powerful toolbox for a variety of fiber grating designs.

Based on the works presented in Chapter 4, Chapter 5 has presented the proposal of a two-stage hybrid optimization algorithm for the design of FBG. This hybrid method employs a global optimization algorithm (i.e. the SCTS algorithm as presented in Chapter 3) to find a “promising” solution, and uses a local optimization algorithm (i.e. the Quasi-Newton method) to converge this “promising” solution to a global optimum. Using this hybrid method, an optical bandpass filter was designed and fabricated based on the same FBG model of piecewise uniform grating sections in the two stages of the hybrid method. The simulation results showed that the performance of the designed hybrid-optimized filter was better than the filter designed using the SCTS algorithm. The measured reflective spectrum of the fabricated hybrid-optimized filter also showed a large side-lobe suppression level (-20 dB). By using different FBG models in the two stages of the hybrid optimization process, three linear phase optical filters with different grating lengths have been designed. The simulation results showed that the hybrid optimization method greatly increases the computational efficiency compared with that of a single SCTS process, and it can be considered as a powerful synthesis method for the design of FBGs with complex profiles.

Chapter 6 and Chapter 7 were devoted to the development of fabrication methods of chirped FBGs with a uniform phase mask. Chapter 6 has presented the design and fabrication of a piecewise stepped-chirp FBG written using a uniform phase mask while a section of the bare fiber was under a pre-stretching condition. FBGs with arbitrary group delay responses were fabricated and tested. The group delay response can be arbitrarily generated by setting the stretched length of the fiber during the UV exposure. An unapodized linearly chirped FBG and an apodized quadratically chirped FBG were also fabricated using this method. The measured group delay responses of these two types of FBGs showed good agreement with their analytical predictions.

Chapter 7 has presented on the improvement of the method presented in Chapter 6 and has described the proposal of a new method for the fabrication of a nonlinearly chirped fiber Bragg grating using only a uniform phase mask. The fiber was pre-stretched and the stretched length was controlled by a motorized stage while the UV beam was scanned along the fiber. An asymmetric chirped grating with a nonlinear group delay response was fabricated to demonstrate the effectiveness of the proposed method. The ripple of the group delay response of the grating is relatively small. The experimental results showed good agreement with those obtained by theoretical predictions. The method enables the fabrication of complex chirped profiles with a relatively simple operation. The

half-band filtering property of the nonlinearly chirped FBG can be potentially used as a vestigial single sideband (VSB) filter for advanced-format optical communications systems. Furthermore, the proposed method may be extended to the fabrication of FBGs with high-order group delay characteristics for application in the next generation of high-speed optical communication systems.

8.2 Recommendations for Future work

The SCTS algorithm presented in Chapter 3 consists of three independent CTS stages, which have only used the simple construction of the tabu lists. One interesting issue is the use of more advanced operators in the tabu search algorithm such as recency and frequency memory rather than the normal operators embodied in the simple tabu list, together with associated intensification and diversification strategies that exploit these memory structures. Combination of some local search methods in the search process could also increase the computational efficiency.

To design an FBG-based filter with a target spectrum and a target phase response, the thesis proposes a hybrid optimization method to increase the search efficiency. One approach that can be considered is the use of some multi-objective

optimization methods that could make the optimization program more intelligent. For example, the number of serially-connected FBGs could also be optimized to achieve the target grating responses.

The synthesis of FBGs using optimization techniques has been developed in this dissertation. One could also consider the design of long period gratings using the optimization techniques developed in this thesis. Some potential applications of the proposed optimization methods are the design of long period gratings for use as an optical fiber sensor (with a special spectrum, for example, a linear spectrum) or a gain flattening filter in an erbium-doped fiber amplifier (EDFA).

The stretching-and-writing method has been demonstrated on the fabrication of stepped-chirp gratings in Chapter 6. To tailor chirped gratings with different chirp profiles, one could use more precise positioning devices such as PZT controllers rather than the motorized stages as reported here. Because the precise positioning devices could reduce the phase errors during the fabrication process, high quality fiber gratings with desired spectral responses could be obtained.

Chapter 7 has presented a new method for the fabrication of a chirped grating with a continuous chirped profile. With a constant stretching velocity on the fiber, a nonlinearly-chirped grating was fabricated. One could also set some more

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Chapter 8 Conclusions and Recommendations for Future Work

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complex velocity profiles to realize more complex grating profiles to realize, for an example, a high-order dispersion compensator.

Another interest issue is to design a VSB filter with a target spectrum and a target dispersion response using the hybrid optimization algorithm presented in Chapter 5. In the designed filter, the grating could have a nonlinear-chirp profile which can be fabricated using the method presented in Chapter 7.

Appendix A: Coupled-mode theory

The transverse component of the electric field can be written as a superposition of the ideal modes, which are labeled as j (i.e. the modes in an ideal waveguide with no grating perturbation), such that

$$\vec{E}_t(x, y, z, t) = \sum_j [A_j(z) \exp(i\beta_j z) + B_j(z) \exp(-i\beta_j z)] \vec{e}_{jt}(x, y) \exp(-i\omega t) \quad (\text{A1})$$

where $i = \sqrt{-1}$, $\exp(-i\omega t)$ is the time-dependent factor, ω is the angular optical frequency, and $\beta_j = (2\pi / \lambda) n_{eff}^j$ is the mode propagation constant of the j^{th} mode at the wavelength λ and n_{eff}^j is the effective index of the j^{th} mode. $A_j(z)$ and $B_j(z)$ are the slowly varying amplitudes of the j^{th} mode traveling in the $+z$ and $-z$ directions, respectively. Similarly, $\beta_k = (2\pi / \lambda) n_{eff}^k$ is the mode propagation constant of the k^{th} mode at the wavelength λ and n_{eff}^k is the effective index of the k^{th} mode. In Eqs. (A2) and (A3) below, $A_k(z)$ and $B_k(z)$ are the slowly varying amplitudes of the k^{th} mode traveling in the $+z$ and $-z$ directions, respectively. The transverse mode fields $\vec{e}_{jt}(x, y)$ and $\vec{e}_{kt}(x, y)$ might describe the radiation LP modes, or they might describe the cladding modes. When the waveguide has a grating perturbation, the modes will be coupled between the j^{th} mode and the k^{th} mode such that the amplitudes A_j and B_j of the j^{th} mode will

evolve along the z axis according to

$$\begin{aligned} \frac{dA_j}{dz} = & i \sum_k A_k(z) [K_{kj}^t(z) + K_{kj}^z(z)] \exp[i(\beta_k - \beta_j)z] \\ & + i \sum_k B_k(z) [K_{kj}^t(z) - K_{kj}^z(z)] \exp[-i(\beta_k + \beta_j)z] \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \frac{dB_j}{dz} = & i \sum_k A_k(z) [K_{kj}^t(z) - K_{kj}^z(z)] \exp[i(\beta_k + \beta_j)z] \\ & + i \sum_k B_k(z) [K_{kj}^t(z) + K_{kj}^z(z)] \exp[-i(\beta_k - \beta_j)z] \end{aligned} \quad (\text{A3})$$

where $K_{kj}^t(z)$ is the transverse coupling coefficient between modes j and k , and

it is defined as

$$K_{kj}^t(z) = \frac{\omega}{4} \iint_{\infty} \Delta \varepsilon(x, y, z) \cdot \vec{e}_{kt}(x, y) \cdot \vec{e}_{jt}^*(x, y) dx dy \quad (\text{A4})$$

where $\Delta \varepsilon \cong 2n_{co}\delta n_{co}$ is the perturbation to the permittivity with n_{co} being the index of the core and δn_{co} being the index perturbation of the core. When the fibers are weakly guided, it can be assumed that $n_{co} \cong n_{eff}$ and $\delta n_{co} \cong \Delta n_{dc}$, thus $\Delta \varepsilon \cong 2n_{eff}\delta n_{eff}$. And $\vec{e}_{jt}^*(x, y)$ is the complex conjugate of $\vec{e}_{jt}(x, y)$. Generally, the longitudinal coefficient $K_{kj}^z(z) \ll K_{kj}^t(z)$ for fiber modes, and thus this coefficient is usually neglected. For a waveguide without index perturbation (i.e. $\Delta \varepsilon = 0$), the modes will propagate without affecting each other

in the absence of the grating.

If two new coefficients are defined as

$$\sigma_{kj}(z) = \omega n_{eff}(z) \Delta n_{dc}(z) \iint_{\text{core}} \vec{e}_{kt}(x, y) \cdot \vec{e}_{jt}^*(x, y) dx dy \quad (\text{A5})$$

$$\kappa_{kj}(z) = \omega n_{eff}(z) \Delta n_{ac}(z) \iint_{\text{core}} \vec{e}_{kt}(x, y) \cdot \vec{e}_{jt}^*(x, y) dx dy \quad (\text{A6})$$

where σ_{kj} and κ_{kj} are the “dc” (period-averaged) coupling coefficient and the “ac” coupling coefficient, respectively, then the general transverse coupling coefficient can be written as

$$K_{kj}^t(z) = \sigma_{kj}(z) + \kappa_{kj}(z) \cos\left[\frac{2\pi}{\Lambda}z + \phi(z)\right] \quad (\text{A7})$$

Appendix B: List of Test Functions

- F1 (1 variable):

$$f(x) = 2(x - 0.75)^2 + \sin(5\pi x - 0.4\pi) - 0.125$$

where $0 \leq x \leq 1$.

- F3 (1 variable):

$$f(x) = -\sum_{i=1}^5 i \sin[(i+1)x + i]$$

where $-10 \leq x \leq 10$.

- Branin (2 variables):

$$f(x, y) = a(y - bx^2 + cx - d)^2 + h(1 - g)\cos(x) + h$$

where $a = 1, b = \frac{5.1}{4\pi^2}, c = \frac{5}{\pi}, d = 6, h = 10, g = \frac{1}{8\pi}$.

- Goldprice (2 variables):

$$f(x, y) = [1 + (x + y + 1)^2(19 - 14x + 3x^2 - 14y + 6xy + 3y^2)] \times \\ [30 + (2x - 3y)^2(18 - 32x + 12x^2 + 48y - 36xy + 27y^2)]$$

where $-2 \leq x \leq 2, -2 \leq y \leq 2$.

- Hartmann1 ($H_{3,4}$) (3 variables)

$$f(x) = -\sum_{i=1}^4 c_i \exp\left[-\sum_{j=1}^3 a_{ij}(x_j - p_{ij})^2\right]$$

Appendix B: List of Test Functions

where $0 < x_j < 1$ for $j = 1 - 3$.

i	a_{ij} ($j = 1, 2, 3$)			c_i ($j = 1$)	p_{ij} ($j = 1, 2, 3$)		
1	3.0	10.0	30.0	1.0	0.3689	0.1170	0.2673
2	0.1	10.0	35.0	1.2	0.4699	0.4387	0.7470
3	3.0	10.0	30.0	3.0	0.1091	0.8732	0.5547
4	0.1	10.0	35.0	3.2	0.0382	0.5743	0.8828

- Shubert1 and Shubert2 (2 variables):

$$f(x, y) = \left\{ \sum_{j=1}^5 j \cdot \cos[(j+1)x + j] \right\} \times \left\{ \sum_{j=1}^5 j \cdot \cos[(j+1)y + j] \right\} + \beta[(x+1.42513)^2 + (y+0.80032)^2]$$

where $-10 \leq x, y \leq 10$, $\beta = 0.5$ for Shubert1, and $\beta = 1$ for Shubert2.

- Shubert (2 variables):

$$f(x_1, x_2) = \left\{ \sum_{j=1}^5 j \cdot \cos[(j+1)x_1 + j] \right\} \times \left\{ \sum_{j=1}^5 j \cdot \cos[(j+1)x_2 + j] \right\}$$

where $-10 \leq x_i \leq 10$, $i=1,2$.

- Brown1 (20 variables):

$$f(x) = \left[\sum_{i \in J} (x_i - 3) \right]^2 + \sum_{i \in J} [10^{-3} (x_i - 3)^2 - (x_i - x_{i+1}) + e^{20(x_i - x_{i+1})}]$$

where $J = \{1, 3, \dots, 19\}$, $-1 \leq x_i \leq 4$ for $1 \leq i \leq 20$ and $x = [x_1, \dots, x_{20}]^T$.

Appendix B: List of Test Functions

- Brown3 (20 variables):

$$f(x) = \sum_{i=1}^{19} \left[(x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)} \right]$$

where $x = [x_1, \dots, x_{20}]^T$ and $-1 \leq x_i \leq 4$ for $1 \leq i \leq 20$

- F5n (20 variables):

$$f(x) = (\pi / 20) \times \left\{ 10 \sin^2(\pi y_1) + \sum [(y_i - 1)^2 \times (1 + 10 \sin^2(\pi y_{i+1}))] + (y_{20} - 1)^2 \right\}$$

where $y_i = 1 + 0.25(x_i - 1)$, $x = [x_1, \dots, x_{20}]^T$, $-10 \leq x_i \leq 10$.

- F10n (20 variables):

$$f(x) = \left(\frac{\pi}{20} \right) \times \left\{ 10 \sin^2(\pi x_1) + \sum_{i=1}^{19} [(x_i - 1)^2 \times (1 + 10 \sin^2(\pi x_{i+1}))] + (x_{20} - 1)^2 \right\}$$

where $x = [x_1, x_2, \dots, x_{20}]^T$ and $-10 \leq x_i \leq 10$.

- F15n (20 variables):

$$f(x) = (1/10) \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^{19} [(x_i - 1)^2 (1 + \sin^2(3\pi x_{i+1}))] + (1/10)(x_{20} - 1)^2 [1 + \sin^2(2\pi x_{20})] \right\}$$

where $x = [x_1, x_2, \dots, x_{20}]^T$ and $-10 \leq x_i \leq 10$.

AUTHOR'S PUBLICATIONS

These publications result from the works of this thesis

Journal Papers

- [1] **R. T. Zheng**, N. Q. Ngo, P. Shum, S. C. Tjin, and L. N. Binh, "A staged continuous tabu search algorithm for the global optimization and its applications to the design of fiber Bragg gratings," *Computational Optimization and Applications*, vol. 30, pp. 319–335, 2005.
- [2] N. Q. Ngo, **R. T. Zheng**, P. Shum, Y. H. Lee, S. C. Tjin, and S. Y. Li, "Tabu search synthesis of cascaded fiber Bragg gratings for linear phase filters," *Optics Communications*, vol. 241, pp. 79–85, 2004.
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