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Internet Traffic Control with Applications in Multimedia Systems

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Doctor of Philosophy

2008

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

2008-04-11

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Date

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Cheng Chen

To My Families

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Summary

Multimedia applications with various Quality of Service (QoS) requirements have posed great challenges to the “flat” service of the Internet. In this thesis, we shall present several results to enhance the source adaptation and control of multimedia traffic. Specifically, a source adaptation framework and two pricing systems are designed. Our schemes do not require significant modifications to the architecture of the Internet. Thus, they can be readily deployed over the current Internet.

We begin by addressing the resource allocation problem through the introduction of globally and locally weighted Max-Min fairnesses. Through the study of their properties, a novel fairness index of fairness and the relevant theorems of convergence are proposed. These results are then used in the design of Internet traffic control schemes.

In source adaptation, we study the use of switch control in TCP congestion control algorithms and a new framework for designing responsive functions is proposed. By introducing a weight to each source, our proposed Monotonic Responsive Function (MRF) framework is able to achieve weighted fairness. With a proper setting of the weights, the framework is able to achieve both TCP-compatibility and Multimedia-compatibility. Thus, the framework can be used on the current

Internet without interrupting existing applications. Furthermore, we study a general class of monotonic responsive functions in terms of their suitability for source adaptations.

To better control the Internet traffic to meet diverse service requirements, we have adopted the price based mechanism to ensure better utilization of available resources and QoS assurance. We propose two novel pricing systems: The SumNet scheme aims to optimize the user's utility while the MinNet scheme aims to achieve weighted Max-Min fairness. Both schemes are based on the novel concept of the generalized Nash equilibrium. With the generalized Nash equilibrium, the proposed pricing systems have the feature that when the utility of the link capacity is low, i.e. QoS is better, users are charged a higher price. This avoids the problem of unfair profit for bottleneck resources which is the inevitable solutions under the conventional microeconomic pricing system. Hence with the proposed pricing systems, Internet Service Providers (ISPs) would have incentives to enlarge their capacities and invest in new technologies for these applications. This will be helpful for the evolution of the Internet.

Finally, we combine the proposed switch control scheme and the pricing system to construct a simple, harmonious network.

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Chapter 1

Introduction

1.1 The Evolution of the Internet: From Best Effort to QoS

The amazing success and continuing evolution of the Internet was unanticipated. The Internet began with a work sponsored by the U.S. Defense Advanced Research Projects Agency (DARPA) in the early 1970's, which was to develop a nonproprietary networking standard that would support computer-based command and control so as to avoid being constrained to a single vendor's equipment and networking technology. The original network was designed to serve a dozen nodes, meeting the needs of hundreds of scientific experts for application-sharing and data transmission. (The original design virtually limited the network to 256 nodes!) Since its first publication in 1974, the Internet was essentially deployed in the academic and military communities and grew at a modest rate until commercial services were available for the common population. The first Internet Service Provider (ISP) companies were formed during the late 1980s. Then in the mid-90's, along with

the success of World Wide Web (WWW), the Internet fuelled the so called *dot-com-boom* of Internet-related businesses. Then, by April 2000, the great dot-bomb imploded, fortunes were lost and the market had come to its senses.

Despite the various dramas during the evolution of the Internet, the Internet has gained a consistent growth in the number of users and the number of networks and machines connected to it. Figure 1.1 illustrates the increase in the number of the hosts connected to the Internet¹. The study shows that the Internet has maintained a steady growth of 50% – 80% per year since 1988. Nowadays, over a billion people² have accessed the Internet to exchange emails, join in discussions, look for information, share files and so on³. The Internet has substantially altered the way many people work, entertain and do business. This has brought up enormous market capitalization and opportunities.

Figure 1.1: The growth of Internet hosts.

¹From Hobbes' Internet Timeline. Online: <http://www.zakon.org/robert/internet/timeline>

²1,022,863,307, 15.7% of world population until March 2006. Online: <http://www.internetworldstats.com/stats.htm>

³The comprehensive listing of things now available for people to do online. Online: <http://internetization.com/index.html>

1.1 The Evolution of the Internet: From Best Effort to QoS

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On the other hand, great opportunities come with great challenges. It seems that today's Internet cannot be scaled to meet the number and nature of demands already placed on it, much less a new generation of more complex interactions. Technically, the Internet has its roots in the ARPANET, the experimental data network funded by DARPA in the early 1960s. An important goal then was to build a robust network that could survive active military attacks such as bombing. To achieve this, the ARPANET was built on the datagram model, where each individual packet is forwarded independently to its destination. The datagram network has the strength of simplicity and the ability to adapt automatically to changes in the network topology. This made the fast growth of the Internet possible. However, the datagram model has few resource management capabilities inside the network and therefore cannot provide resource guarantees to users. From the view-point of resource allocation, the Internet was initially conceived and implemented as a centrally supported shared source. Generally speaking, the growth in demand is met by increasing available resources. In that environment, a single service class, often called "best effort", is implemented and everyone has "equal right" to the available resource. However, the requirement of time-sensitive applications, coupled with an explosion in user population size and the increasing requirement in large multimedia content, have made it an impractical solution by simply adding more bandwidth.

The lack of resource management raises two important issues: one is performance assurance. When you try to reach a web site or to make an Internet phone call, some parts of the network may be so busy that your packets cannot get through at all. Most real-time applications such as video-conferencing also require some minimal level of resource to operate effectively.

Another issue is service differentiation. Different applications have diverse requirements. Interactive applications such as Internet telephony are sensitive to latency and packet losses. When the latency or the loss rate exceeds certain levels, these applications become literally unusable. In contrast, a file transfer can tolerate a fair amount of delay and losses without much degradation of perceived performance. Customer requirements may also vary depending on what the Internet is used for. For example, organizations that use the Internet for bank transactions or for control of industrial equipments are probably willing to pay more in order to receive preferential treatment for their traffic. For many service providers, providing multiple levels of services to meet different customer requirements is vital for the success of their business. The capability to provide resource assurance and service differentiation in a network is often referred to as quality of service (QoS).

Implementing these QoS capabilities in the Internet has been one of the toughest challenges in its evolution, touching on almost all aspects of Internet technologies and requiring changes to the basic architecture of the Internet. For more than a decade the Internet community has made continuous efforts to address the issue and has developed a number of new technologies for enhancing the Internet with QoS capabilities [2–9].

1.2 Motivation

More and more video applications or multimedia applications with video content are now becoming available on-line. This is a typical example of the network driving applications. The development of the network has made high quality services possible. On the other hand, applications are also driving the network technology for better support.

Since they are more sensitive to delay than data loss, video applications are likely to be implemented with the User Datagram Protocol (UDP) rather than upon Transmission Control Protocol (TCP), or high-layer protocols such as Real-time Transport Protocol (RTP) [10, 11], along with its partner protocols such as Real Time Streaming Protocol (RTSP), or Real Time Control Protocol (RTCP). These protocols define how the related data information, session information and feedback information are exchanged between users. However, they do not define any congestion control mechanism or traffic control mechanism. Thus, no QoS guarantee is presented and it must be provided by some other mechanisms. For the video technology itself, continuous efforts have been made to reduce the size of coded data, make video stream robust to missing information, adapting itself to network status and so on. However, to improve the final visual quality, the research on the improvement of network service is as important, if not more, as improvement in the coding technology.

Compared with traditional off-line applications, on-line multimedia applications, which normally involve a large amount of multimedia content consisting of text, audio, graphic, animation and video, have the following specific requirements from the service of the underlying network:

- (i). **Throughput:** A minimum throughput value is usually required. Unlike most traditional Internet applications over TCP traffic, which can continue to deliver data with perhaps degraded service, many multimedia applications require a firm minimum throughput.
- (ii). **Smoothness:** An on-line video application usually requires the underlying traffic to be smooth. With modern coding technology, CODEC (for COder-DECoder) is able to work under various coding rates. However, with large fluctuations in the coding rate, the quality may be very bad.

- (iii). **Delay:** A typical example of a delay-sensitive application is video-conference, which contains human interaction.
- (iv). **Delay variation (jitter):** The larger the allowable delay, the longer the real delay in delivering the data and the greater the size of the delay buffer required at receivers. This is critical especially when applications are run on a terminal with limited computational resources, such as PDA, intelligent mobile phone etc.
- (v). **Packet loss:** Real-time applications vary in the amount of packet loss, if any, that they can sustain.

Although some of the requirements are almost impossible to meet without advances in network devices, source adaptation is a potential method to improve the network service. Source adaptation pushes most of the complexity to the edge of network. It is highly scalable and is able to achieve rational upgrading on service. Moreover, it has the advantage of a better utilization of the available time varying network resources [12]. The requirements of source adaptation are relatively simple, thus it avoids the expensive upgrading of network devices. This reduces the business risk of Internet Service Providers (ISPs). Hence, source adaptation is attractive from both the technology and business points of view.

The evolution of the Internet obeys a spiral mode, i.e., old system is replaced by new system with new service while the preliminary services have remained. Thus, compatibility with the existing mechanisms is the key to the success of a new technology. On the current Internet, TCP traffic is still the dominant traffic. As of 2006, 75% of Internet traffic is TCP traffic followed by UDP traffic at 20%.

1.2 Motivation

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Thus, it is important for the new protocol to be compatible with TCP (i.e. TCP-compatible, also called TCP-friendly⁴).

With TCP, traffic on individual connections uses an Additive-Increase Multiplicative-Decrease (AIMD) algorithm to adapt its traffic to the network status. As the most popular congestion control scheme used in the Internet, numerous research efforts have proceeded to understand the behavior of TCP under various network situations. A representative but by no means exhaustive list would include the results in [7, 13–17]. To achieve *fairness*, TCP-based resource allocation requires all applications to use the same congestion control scheme. However, for on-line multimedia applications, AIMD has too much fluctuations and the requirement of TCP-compatibility conflicts with the real-time applications. TCP-compatibility requires that the sending rate of flow is “*generally within a factor of two of the sending rate of a TCP flow under the same conditions*” [7, 18]. The throughput that a TCP-compatible application gets is a fair share among all the competing flows. However, as we have mentioned earlier, a real-time application often has a minimal requirement for throughput (and schemes which are able to satisfy this requirement are called multimedia-friendly schemes). Therefore, in the case where the fair share is smaller than the minimal requirement, as is frequently the case, real-time applications crash. Hence, a new throughput allocation scheme is needed to satisfy both TCP-friendliness and multimedia-friendliness.

Another issue arose along with service differentiation of the Internet is the economic model. Since everyone and everything do not have to be treated equally, it is finally possible to adopt a “want more/pay more/get more” approach. This issue is as important as providing service differentiation. In the absence of such

⁴In this thesis, these two terms have different meanings: when competing over a link, a TCP-friendly scheme means it always achieves a factorized throughput of TCP while a TCP-compatible scheme always has the same throughput of TCP. The details of the definitions will be discussed later.

a model, the users may mark all their traffic as the highest priority and thus the system will degenerate into the best effort service. And without such an economic model to ensure that the service providers are compensated for providing enhanced service quality, no rational network service provider will do so.

Figure 1.2: The structure of the proposed source adaptation and pricing system.

1.3 Objectives

The main objectives of the research are as follows:

- (i). To study the fairness problem in resources allocation, develop methodology for its measurement and convergence in order to develop appropriate traffic control algorithms.
- (ii). To develop source adaptation systems that are able to provide bandwidth differentiation service, so that multimedia-friendly and TCP-friendly traffic control could be established based on the systems.
- (iii). To develop suitable pricing systems, which could be used by ISPs to implement a “want more/pay more/get more” approach and overcome the unfair tendency of the pricing system based on the traditional demand versus supply economic model.

- (iv). To develop “simple” solutions, which should be compatible with the architecture of the current Internet, thus they could be deployed easily and do not require a significant modification of the network devices.

1.4 Contributions of the Thesis

In this thesis, source adaptation schemes as illustrated in Figure 1.2, will be developed. In the proposed schemes, packets travel over the network from a source to a destination and collect the local information of links to perform the necessary adjustment. At each hop, the router may update the information carried on a fixed area of the packets. The information is fed back by the destination without any changes (We will also study schemes under the common TCP structure. In such a case, it is the destination which checks for packet loss and generates congestion signal if ECN mark is not enable.). The sources will then adjust their sending rates accordingly. Our algorithms do not require per-flow state for simplicity of router’s function.

The major contributions of the thesis include:

- (i). We examined Max-Min fairness and proposed the notion of weighted Max-Min fairness for resource allocation. A novel fairness index is defined under the condition of one resource bound, which is then used to develop our new schemes.
- (ii). A new Monotonic Responsive Function (MRF) framework is developed, which uses switch control to control the traffic of sources and extends TCP’s *fair* throughput allocation into a weighted fair throughput allocation. As a result, by choosing a proper weighting factor, traffic could be TCP-compatible and also multimedia-friendly. As an application example of the proposed results,

we also developed a simple structure that can provide a related differentiated throughput service.

- (iii). The responsive functions of the MRF framework are further analyzed and a new framework for the classification of responsive functions, which cover most of existing algorithms such as AIMD, GAIMD, BCC, CYRF etc., is established. We gave convergent conditions and the conditions for these responsive functions to be TCP-compatible.
- (iv). The new concept of generalized Nash equilibrium is used to develop a harmonious pricing system for network pricing and traffic control by solving an optimal problem. The so called SumNet is harmonious in the sense that it is able to avoid the unfair profits of bottlenecked links and the pricing system is QoS rational. We established its convergence conditions and studied its higher order form.
- (v). A MinNet scheme is developed under the structure of the SumNet. It inherits the advantages of the SumNet pricing system and is able to achieve weighted Max-Min fairness amongst the users of networks. Thus it is harmonious and globally fair.
- (vi). The idea of switch control in MRF framework is used to control the traffic in the MinNet so as to ease the requirements for computation in equation based traffic control.

1.5 Organization of the Thesis

In the next chapter, we present a review of key issues in Internet communication in terms of resource allocation and pricing mechanism, then the rest of the thesis

is divided into two parts. The first part, which includes Chapters 3-5, focuses on fairness index and the TCP congestion control problems. The second part, which includes Chapters 6-8, presents pricing systems that can achieve global optimal solutions and desired fairness. The main contents of each chapter are as follows:

In Chapter 2, the structure and the service model of the current Internet is first introduced. The integrated service and differentiated service as attempts to enhance QoS of the Internet are then discussed. We also present brief reviews of source adaptation algorithms and the concept of generalized Nash Equilibrium which will be used throughout this thesis.

In Chapter 3, we introduce the definition of weighted Max-Min fairness in throughput allocation and its features. In addition to global fairness, a local weighted fairness is studied for the case of a single bottleneck link and a novel fairness index is proposed which is used to establish the convergent conditions for local fairness. The weighted fairness will be used in the study of TCP congestion control algorithms in Chapters 4 and 5 while the global fairness will be pursued later in the development of appropriate pricing system.

In Chapter 4, we introduced the notion of Monotonic Responsive Function (MRF) for source adaptation. With this framework, a weighted fairness can be readily achieved by source adaptation and no change is required on the routers. The scheme can be TCP-compatible as well as multimedia-friendly.

In Chapter 5, we extend the category of the responsive functions of TCP congestion control under the concept of equal fairness. The switch control functions are classified into four categories. The convergent conditions for each category are proved and verified by simulations.

In Chapter 6, a dynamic pricing system, named SumNet, is provided by solving the optimal problem of users' utilities functions with a new Lagrangian factor.

It achieves the global optimal solution under the concept of Generalized Nash equilibrium. Hence it overcomes the unfair tendency of the pricing system based on the traditional demand versus supply economic model.

In Chapter 7, we study the weighted Max-Min fair allocation problem with a new pricing scheme. The proposed MinNet scheme is stable, weighted Max-Min fair and inherits the advantages of the SumNet's pricing system. The result is proved and demonstrated by simulations.

In Chapter 8, the structure in Chapter 7 and the algorithm in Chapter 4 are combined to provide a simple (the algorithm), harmonious (the economic model) scheme for traffic control and pricing.

Chapter 9 contains the conclusions and several suggestions for future research.

Chapter 2

Internet Traffic Control, Source Adaptation and GNE

2.1 The Current Internet

2.1.1 The Architecture

General speaking, Internet communication involves three components: applications, computers, and networks. The applications of concern here are distributed applications that involve the exchange of data, such as file transfer, email exchange, etc, between two computer systems. Applications are executed on computers, which can often support multiple simultaneous applications. Computers are connected to networks, as shown in Figure 2.1(a), and the data to be exchanged are transferred by the network from one computer to another.

2.1.2 The Protocol

The communication function is carried out by a *computer networking protocol suite*. A protocol suite can be viewed as a set of layers, each layer solves a set of problems

Figure 2.1: The Internet architecture and protocol.

involving the transmission of data, and provides a well-defined service to the upper layer protocols by using the services from some lower layers. The upper layers are logically closer to the user and deal with more abstract data, relying on the lower layer protocols to translate data into forms that can eventually be physically transmitted. A protocol suite itself is a set of individual protocols. A protocol implements the function of one or more layers of a protocol suite and is the set of standard rules used for data representation, signaling, authentication, and error detection required to send information over a communications channel. It is the basic mode or language of communication between devices on a network.

The most successful protocol suite is the *internet protocol suite* which underlies

the Internet. The internet protocol suite is sometimes called the *TCP/IP protocol suite*, after its two most important protocols: the Transmission Control Protocol (TCP) [21] and the Internet Protocol (IP) [22].

The TCP/IP protocol suite contains five relatively independent layers [23] and its deployment over the internet is shown in Figure 2.1(b):

- Applications layer
- Transport layer
- Internet layer
- Network access layer
- Physical layer

Remark 2.1.1. *The five layers model is a DoD (DoD stands for Department of Deference) reference model, created in the 1970s by DARPA for developing the Internet's protocols. Another popular reference model for Internet protocols is the seven layer Open System Interconnection (OSI) by the International Organization for Standardization (ISO) [24–26]. We prefer to use the DOD reference model because of its conciseness. The schemes throughout this thesis will be applied on the transport layer and the application layer.*

The physical layer covers the physical interface between a data transmission device (e.g., workstation, computer) and a transmission medium or network. This layer is concerned with specifying the characteristics of the transmission medium, the nature of the signals and the data rate, and related matters.

The network access layer is concerned with the exchange of data between an end system (server, workstation, etc.) and the network to which it is attached. The sending computer must provide the network with the address of the destination

computer, so that the network may route the data to the appropriate destination. The specific software used at this layer depends on the type of network to be used; and different standards have been developed for circuit switching, packet switching (e.g., frame relay), LAN (e.g., Ethernet), and others. Thus, it makes sense to separate those functions having to do with network access into a separate layer. By doing this, the remainder of the communication software, above the network access layer, need not be concerned about the specific of the network to be used. The same high-layer software should function properly regardless of the particular network to which the computer is attached to.

The network access layer is concerned with access to and routing data across a network for two end systems attached to the same network. In those cases where two devices are attached to different networks, procedures are needed to allow data to traverse multiple interconnected networks. This is the function of the internet layer. The Internet Protocol (IP) is used at this layer to provide the routing function across multiple networks. This protocol is implemented not only in the end systems but also in the routers. A router is a processor that connects two networks and whose primary function is to relay data from one network to the other on its route from the source to the destination end system.

The transport layer's responsibilities include end-to-end message transfer capabilities independent of the underlying network, along with error control, fragmentation and flow control. TCP and UDP are two typical transport layer protocols. TCP is a connection-oriented protocol which provides a "reliable" delivery by retransmitting the missed packets, while UDP is a connectionless datagram protocol which is an "unreliable" protocol, i.e., it does not guarantee the delivery of messages. From the traffic control's point of view, TCP has a mechanism to adjust its' traffic rate while UDP itself does not have one.

Finally, the application layer contains the logic needed to support the various user applications. For each different type of applications, a separate module is needed that is peculiar to that application, such as TELNET, File Transfer Protocol (FTP), Post Office Protocol version 3 (POP3), Simple Mail Transfer Protocol (SMTP), BitTorrent, Simple Network Management Protocol (SNMP), and Hypertext Transfer Protocol (HTTP) [27].

Unlike older communication systems, the internet protocol suite was deliberately designed to be independent of the underlying physical medium. Any communication network, wired or wireless, that can carry two-way digital data can carry Internet traffic. Thus, Internet packets flow through wired networks like copper wire, coaxial cable, and fibre optic, and through wireless networks like Wi-Fi. Together, all these networks, sharing the same protocols, form the Internet.

The advantage of the Internet architecture is its simplicity and efficiency. Consequently, it can only provide a simple connection service. There are other networking techniques that are capable of providing high level QoS, such as ATM networking [28]. However, the cost of building a new network is so high that no ISP would like to do that. Thus, in order to reduce the cost of the updating and make full use of the existing equipments and resources, a key issue of the Internet QoS is to update the Internet service without much modification of the current architecture.

2.2 Service Models

2.2.1 Best Effort Services

The current Internet does not support any form of active resource allocation [19]. There is no admission control so that users can inject packets into the network

as fast as possible. Under the best efforts service scheme, the Internet treats all packets equally. As the volume of traffic on the network grows and congestions occur, the packet delivery is slowed down. If congestion becomes severe, some packets are dropped more or less at random to ease the congestion. No distinction is made in terms of the relative importance or the timeliness requirements of any traffic.

The Internet currently relies on the TCP protocol in hosts to detect congestion and reduce the transmission rates accordingly [23]. TCP employs a window-based scheme for congestion control. The window corresponds to the amount of data in transit between the source and destination. If a lost of packet is detected, the window size is reduced by half. Otherwise, it is increased gradually. TCP-based resource allocation requires all applications to use the same congestion control scheme. However, many UDP-based applications do not support TCP-like congestion control. For example, real-time multimedia applications typically cannot cope with large fluctuations in the transmission rate [29, 30]. So to meet the requirement of such application, either the Internet is scaled to support QoS or adaptive methods should be found to reduce conflicts.

2.2.2 Integrated Services

The integrated service model was the first attempt to enhance the Internet with QoS capabilities [3]. The integrated service architecture is based on per-flow resource reservation. Before an application transmits data, it must make a resource reservation along the route. The Resource Reservation Setup Protocol (RSVP) [31] was standardized for signaling a source's requirements to the network and for setting up resource reservation along the route.

The integrated service is able to provide two new service models that a user

can select: the guaranteed service and the controlled load service. The former provides deterministic worst-case delay bound through strict admission control and fair queuing scheduling. This service was designed for applications that require absolute guarantee on delay. The latter one provides a less firm guarantee, it provides a service that is close to a lightly loaded best efforts network.

Although the resource allocation architecture, new service models, and RSVP protocol were standardized in the late 1990s, the deployment of integrated service architecture in internet service providers' backbones has been rather slow. One of the reasons may be due to its complexity that each node in a network has to implement per-flow classification and scheduling. The research on the integrated services began in the early 1990s. At that time, the World Wide Web (WWW) did not even exist. The integrated service is designed for long-lasting real-time applications such as video conferencing. The WWW, however, has significantly changed the Internet. Web-based applications now dominate the Internet, and much of Web traffic is short-lived transactions. Although per-flow reservation makes sense for long-lasting sessions, it is not appropriate for Web traffic. These mechanisms may not be able to cope with a very large number of flows at high speed.

2.2.3 Differentiated Services

Instead of making per-flow reservations, a differentiated service architecture uses a combination of edge policing, provisioning, and traffic prioritization to provide service differentiation [32].

In the differentiated service architecture, users' traffic is divided into a small number of forwarding classes. For each forwarding class, the amount of traffic that the users can inject into the network is limited at the edge of the network

and at the interior routers, a so-called Per-Hop Behavior (PHB) is applied to each forwarding class [6, 33]. By changing the total amount of traffic allowed in the network, service providers can adjust the level of resource provisioning and hence control the degree of resource assurance to users.

The edge of a differentiated service network is responsible for mapping packets to their appropriate forwarding classes. This packet classification is typically done based on the Service Level Agreement (SLA) between a user and his/her service provider. The nodes at the edge of the network also perform traffic policing to protect the network from misbehaving traffic sources. Nonconforming traffic may be dropped, delayed, or marked with a different forwarding class.

Differentiated services do not require resource reservation setup. The allocation of forwarding classes is typically specified as part of the SLA between the customer and its service provider, and the forwarding classes apply to traffic aggregates rather than to individual flows. The complex process of classification is needed only at the edge of network, where traffic rates are typically much lower.

The differentiated service approach relies on the provision to provide resource assurance. The quality of the assurance depends on how provision is carried out, and how the resources are managed in the network [9, 34–38]. Because of the dynamic nature of traffic flows, precise provision is usually difficult. Thus, it generally provides a less deterministic QoS guarantee.

Instead of an absolute service, it provides a relative differentiated service. Compared with an integrated service, although the service of a differentiated service is rather granulated, yet due to the simplicity of its architecture, it is believed that it will dominate the next generation Internet. In this thesis, we are concerned with the provision of differentiated services by source adaptation schemes.

Figure 2.2: The structure of source adaptation.

2.3 Source Adaptation

The structure of a source adaptation scheme is illustrated in Figure 2.2. It can be divided into two parts: responsive function and signal system. The signal system is normally embedded in each link and the signal is used to feedback the local state at a link to the sources. After receiving the signal, the responsive function will adjust each source's behavior accordingly. In the following, we shall use two examples to illustrate the above process.

2.3.1 TCP's Congestion Control Algorithms

TCP is a connection-concerned transport protocol [21,39]. It first builds a soft-state connection between the agents of source and destination before the transmission of a user's data. Once the connection is established, source and destination exchange information by the TCP-head carried by each data packet and acknowledge packet (ACK packet).

TCP's signal system does not require link's local state¹. The source assigns each packet a sequence number, and by the sequence number, the destination is able to detect the missing of incoming packets. Once packet loss is detected, the

¹Another way of congestion detection is by Explicit Congestion Notification (ECN) in the TCP head [40]. In that case, congested router will mark the ECN to identify a congestion. The mechanism requires support from the network device which limited its deployment.

destination treats it as a signal of network congestion and informs the source by duplicate ACK packets, which are packets with the same ACK sequence number. The number of duplicated ACK packets is different for the various implementations of TCP.

At the source side, there is a variable (called windows size) to identify the maximal number of packets a source can send without acknowledgement. Therefore, if one ignores the impact of Round Trip Time (RTT), the window size will represent the traffic rate. According to the received ACK packets, the source adjusts the windows size by the I (for increasing) policy or D (for decreasing) policy, for example, by the Additive-Increase Multiplicative-Decrease (AIMD) algorithm [41].

AIMD is the most popular algorithm and is widely deployed over the Internet. In the scheme, the window size is increased by 1 when the correct number of packets is acknowledged, and decreased to half its size when a loss is detected. The responsive function is therefore given as

$$I: \quad x_s(k+1) \leftarrow x_s(k) + 1, \quad (2.1)$$

$$D: \quad x_s(k+1) \leftarrow x_s(k) - \frac{x_s(k)}{2}, \quad (2.2)$$

where x_s denotes the window size of source s . It was shown in [42] that the AIMD scheme converges to fairness and efficiency when all flows in the network get the same feedback simultaneously.

Clearly, Equations (2.1) and (2.2) represent a switching strategy. In [43], a switch controller is defined as

Definition 2.3.1. Consider a plant P whose state variable is denoted by $X \in \mathbb{R}^r$.

2.3 Source Adaptation

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Suppose that we have a collection of state feedback controllers:

$$\tilde{U}(t) = K_{m(t)}(X(t)); \quad m(t) \in \{1, 2, \dots, n\}, \quad (2.3)$$

where $K_i (i = 1, 2, \dots, n) : \mathbb{R}^r \rightarrow \mathbb{R}^p$ are given continuous functions. The controllers in Equation (2.3) are called sub-controllers. A switched control is the law for switching one sub-controller to another one and is defined as

$$m(t) = I(X(t)), \quad (2.4)$$

where $I : \mathbb{R}^r \rightarrow \{1, 2, \dots, n\}$.

Hence, TCP's congestion control is a switch control system. It has two sub-controllers which are given by the responsive functions and the switching law is determined by the feedback signal.

TCP is the most successful congestion control scheme. One of the reasons may be that by AIMD, it is able to provide a sort of fairness to the users as well as guaranteeing the efficiency of the bandwidth usage [42]. Another reason may be its simplicity. It does not require the support from the routers and no complicated computation is required at the source and the destination. This means that it can be easily deployed.

2.3.1.1 Existing Responsive Functions of TCP Congestion Control

Besides AIMD, there are many other algorithms which have been studied to provide more choices [14, 15, 44–48]. The Generalized Additive-Increase Multiplicative-Decrease (GAIMD) scheme extends AIMD with the following increase and decrease

policies [44]:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + \alpha ; \alpha > 0, \quad (2.5)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \beta x_s(k) ; 0 < \beta < 1. \quad (2.6)$$

To be TCP-compatible, α and β must satisfy [47]

$$\alpha = \frac{3\beta}{2 - \beta}. \quad (2.7)$$

Clearly, the DECbit [49] scheme with

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + 1, \quad (2.8)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \frac{1}{8}x_s(k) \quad (2.9)$$

is not TCP-compatible.

The Binomial Congestion Control (BCC) scheme further extends the GAIMD scheme with the following I and D policies [50]:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + \frac{\alpha}{x_s^k(k)} ; \alpha > 0, \quad (2.10)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \beta x_s^l(k) ; 0 < \beta < 1. \quad (2.11)$$

The SQRT ($k = l = 0.5$) and IIAD ($k = 1, l = 0$) schemes are two special cases of BCC [50]. Note that $k + l = 1$ assures TCP-friendliness of BCC.

Sastry and Lam [51] proposed an interesting framework, called Choose Your

2.3 Source Adaptation

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Response Function(CYRF) for memoryless window-based source adaptation protocols:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + \frac{x_s(k)}{\tilde{f}(x_s(k))}, \quad (2.12)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - x_s(k)\tilde{g}(x_s(k)), \quad (2.13)$$

where $\tilde{f}(x_s)$ and $\tilde{g}(x_s)$ are two monotonic nondecreasing functions, satisfying $\tilde{f}(x_s) > 0$, $1 \geq \tilde{g}(x_s) > 0$ for all $x_s \geq 1$. One example of the CYRF scheme is the SIGMOID scheme [51] which used a sigmoid function as its D policy as follows:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + 1, \quad (2.14)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \frac{\gamma x_s(k)}{(1 + e^{-(\alpha x_s(k) - \beta)})} \quad (2.15)$$

where γ , β and α are positive constants which decide the maximum, the knee point and the smoothness of the knee of the sigmoid function.

Sastry and Lam [51] showed that a set of CYRF flows step-wisely converges to fairness and efficiency. Further, CYRF can be easily made TCP-friendly/TCP-compatible. CYRF covers schemes such as AIMD and GAIMD. However, the requirement of CYRF is still a little conservative for it excludes some of BCC. Another example is a responsive function proposed in [51], named LOG, which is defined as

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + \frac{\log(x_s(k))}{x_s(k)}, \quad (2.16)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \frac{\log(x_s(k))}{2}, \quad (2.17)$$

and it is also not CYRF. Thus the concept of 1-CYRF is introduced in [51] to cover the case. It is a function with a smooth increase policy whose $\tilde{f}(x_s)$ and $\tilde{f}(x_s)\tilde{g}(x_s)$

are monotonically nondecreasing (one of them must be strictly increasing) and $\tilde{f}(x_s)\tilde{g}(x_s) > 1$.

2.3.2 Price Based Source Adaptation

To introduce service differentiation into the Internet, an important issue is how to make users obey the rules and mark their traffic by the proper priority when such services are provided. Otherwise, all users may mark their traffic with the highest priority and the priority model would degenerate into the existing best effort service.

The price based marking scheme is a possible solution to this problem [52–59]. In such schemes, the network computes a price for the resource it provides and the users are charged accordingly. Through either static or dynamic pricing mechanisms, the “want more/pay more/get more” structure may control the abuse of resources. An additional advantage is that such a pricing system is also helpful for the profit allocation among the service providers in the case where a connection may be built over several networks [60,61].

In [1], Low and Lapsley proposed a source adaptation scheme to maximize the users’ utilities. It contains a pricing system at the link side. The optimal problem is defined as

$$\begin{aligned} & \max \left\{ \sum U_s(x_s) \right\}, \\ & \text{subject to } \sum x_s \leq c_l \text{ for all links,} \end{aligned}$$

where U_s is the utility function of source s , c_l is the capacity of link l .

The problem was solved by using the gradient projection method [62,63] and accordingly a distributed algorithm was proposed.

In their pricing system, a *price* is computed at each link as

$$p_l(k+1) = p_l(k) + \gamma_l(\bar{x}_l(k) - c_l),$$

where p_l is the price, $\bar{x}_l$ is the total traffic in the link and γ_l is a constant. The feedback signal is the sum of the links' prices along a source's path, i.e. $\sum p_l$, and this value is then used by the source to compute its traffic rate. The responsive function is

$$x_s = \max\{\min\{x_s^+, \dot{U}_s^{-1}(\sum p_l)\}, x_s^-\},$$

where x_s^+ and x_s^- are two bounds of source's traffic, and $\dot{U}_s^{-1}$ is the inverse function of the first derivative of the utility function of source s .

The scheme in [1] has the advantages of stability and optimality. However, with his pricing system, the network works as a conventional microeconomic system [64,65]. Under the conventional microeconomic system, the price of a link goes up when the link is over-loaded (i.e., when the demand for bandwidth at a link exceeds the capacity); otherwise, it goes down. With this pricing scheme, the bottleneck links have an unfair advantage in getting more profit than under-loaded links, and hence service providers have limited or even no incentive to enlarge their capacities.

2.3.2.1 Existing Pricing Systems

There are already a number of dynamic pricing systems when all the traffic has the same priority. One interesting scheme is the model proposed by Kelly in [52, 56] for elastic traffic. In this model, a source selects the charge per unit time that it is willing to pay, thereafter the source's rate is determined by the network according to a proportional fairness criterion applied to the rate per unit time.

Another one is based on the Nash bargaining solution from cooperative game theory [1, 53–55, 57, 58, 60, 61]. Most available schemes consider the issue of maximizing the benefit of all sources, which is the sum of the source utilities. In general, the primal system problem is formulated as a centralized optimal rate allocation problem, and centralized coordination is required among the sources to derive at the optimal solution. Much communication overhead will be needed when all sources are located in a network distributed over a vast geographical area. It is thus desirable to find a decentralized solution at the source level in which the source tries to optimize his/her own performance [58]. Such the solution was obtained in [1, 58] by using the gradient projection method to solve the dual problem. It is worth mentioning that He and Walrand [60] had proposed a fair revenue sharing policy based on the weighted proportional fairness criterion. The idea of [60] is similar to that of [1, 58] and the overall price is also determined by the most bottlenecked link. The only difference is that He and Walrand studied the problem from the link's point of view.

However, with the pricing systems in [1, 53–55, 57, 58, 60, 61], the network works as a conventional microeconomic system [64, 65]. That is, the price goes up when demand increases. However, network services are fundamentally different from goods in microeconomic systems. The quality of each goods is always the same regardless of the number of customers while QoS can only be guaranteed when the total demand on a link is less than its capacity. The quality will decrease when the total temporal demand on a link is greater than its capacity. With the pricing systems in [1, 53–55, 57, 58, 60, 61], the price goes up (down) when QoS decreases (increases). Clearly, this is not reasonable from the QoS point of view. Furthermore, a consequence of these pricing systems is that the price of under-loaded links will be minimal when the network is in equilibrium. This is unfair

to the under-loaded links which provide the best QoS. As a result, for the pricing systems in [1, 53–55, 57, 58, 60, 61], there is no incentive for these links to enlarge their capacities. Meanwhile, all the sources will increase their transmission rates when the price falls and decrease their transmission rates when the price rises. They fail to utilize better QoS and the available bandwidth. Moreover, only one priority class was studied in [1, 53–55, 57, 58, 60, 61].

Another problem is that by focusing on the case of a single priority class, their results cannot be extended in a straightforward manner to the case of multiple priority classes. This is because when there are multiple priority classes, these pricing systems are not consistent. While a higher priority class can still obtain a better QoS, and is charged a higher price, however, within each class, it is charged a lower price when the network is under-loaded even though the QoS is relatively better. Marbach [66] proposed an interesting static pricing system for priority services based on the assumption that changes in the traffic allocation of an individual source have a negligible effect on the overall network. With the static pricing system in [66], ISPs also have limited or no motivation to increase their capacities.

Hence, it is necessary to design a new pricing system that would enable service providers to make fair profit by charging more for providing better QoS. The issues that should be addressed when the pricing system is designed include: 1) pricing the bandwidth in such a way that links have incentives to improve their capacities to support new applications and sources have incentives to mark their traffic by the proper priority; 2) differentiating the different features of service from that of the conventional goods; and 3) the ease of implementing the pricing system in a decentralized and simple manner.

2.4 The concept of Generalized Nash Equilibrium

In this section, we briefly review the concept of generalized Nash equilibrium [67]. The normal form is adopted to describe a class of 2-player games [65]. For simplicity, it is assumed that there are two basic strategies $\hat{U}_s$ and $\hat{U}_c$ where $\hat{U}_s$ corresponds to “behave selfishly” and $\hat{U}_c$ means “contribute to the common good”. The strategies of players 1 and 2, $U_1(k)$ and $U_2(k)$, are in the set $\{\hat{U}_s, \hat{U}_c\}$. The utility payoff functions $f_i (i = 1, 2)$ have the following properties:

$$f_1(\hat{U}_c, \hat{U}_s) \leq f_1(\hat{U}_s, \hat{U}_s) \leq f_1(\hat{U}_c, \hat{U}_c) \leq f_1(\hat{U}_s, \hat{U}_c), \quad (2.18)$$

$$f_2(\hat{U}_s, \hat{U}_c) \leq f_2(\hat{U}_s, \hat{U}_s) \leq f_2(\hat{U}_c, \hat{U}_c) \leq f_2(\hat{U}_c, \hat{U}_s). \quad (2.19)$$

An example of a lunch partner is used to illustrate the above model.

Example 2.4.1. *Two players work in the same company, and they have agreed to always split the bill evenly. There is a restaurant that offers a luxurious \$20 meal and a simpler \$10 meal. Suppose each player, if he/she had to pay only his/her own bill, would prefer the \$10 meal. $\hat{U}_s$ and $\hat{U}_c$ correspond to “choose \$20 meal” and “choose \$10 meal”, respectively. The utility payoff matrix is given in Table 2.1.*

Table 2.1: Two players' utility payoffs of lunch partner.

	(player 1, \$10)	(player 1, \$20)
(player 2, \$10)	(4,4)	(5,0)
(player 2, \$20)	(0,5)	(3,3)

The concept of generalized Nash equilibrium is then defined as follow [67]:

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Definition 2.4.1. Suppose that each player maintains the same incentive, a strategy pair (U_1, U_2) is a generalized Nash equilibrium with respect to the incentive if player 1 (or 2) has no incentive to deviate from U_1 (or U_2) provided that the opponent maintains U_2 (or U_1).

It can be shown from Definition 2.4.1 that a type of *generalized Nash equilibrium* is associated with and determined by a corresponding type of incentive. There are two types of incentives as listed in Table 2.2.

Table 2.2: Two types of incentives.

Type	Incentive
1	maximize his/her own utility payoff
2	maximize the opponent's utilization payoff

The first type of incentive is the conventional one that is widely used in the existing game theory [68]. It can be shown from (2.18) and (2.19) that

$$f_1(U_1, \hat{U}_s) \leq f_1(\hat{U}_s, \hat{U}_s) ; \forall U_1$$

$$f_2(\hat{U}_s, U_2) \leq f_2(\hat{U}_s, \hat{U}_s) ; \forall U_2.$$

In other words, each player has no incentive to deviate from $\hat{U}_s$ provided that the opponent maintains $\hat{U}_s$. Thus, $(\hat{U}_s, \hat{U}_s)$ is the first type of generalized Nash equilibrium [27] that is based on the assumption that each player is *rational*. However, this will yield a suboptimal solution for the scenario in the Example 2.4.1 as highlighted below.

Example 2.4.2. Consider Example 2.4.1, and suppose that each player's incentive is to maximize his/her own utilization payoff. The dominant strategy is to "choose \$20 meal", and the first type of generalized Nash equilibrium will be obtained if

both players choose the \$20 meal. However, this equilibrium leads to a lower utility payoff for each player, i.e. $(3,3)$, than that of (“choosing \$10 meal”, “choosing \$10 meal”), which is $(4,4)$.

On the other hand, it can be shown that

$$\begin{aligned} f_2(\hat{U}_c, \hat{U}_c) &\geq f_2(U_1, \hat{U}_c) ; \forall U_1, \\ f_1(\hat{U}_c, \hat{U}_c) &\geq f_1(\hat{U}_c, U_2) ; \forall U_2. \end{aligned}$$

Thus, if each player’s incentive belongs to the second type, each player has no incentive to deviate from $\hat{U}_c$ provided that the opponent maintains $\hat{U}_c$. $(\hat{U}_c, \hat{U}_c)$ is thus the second type of generalized Nash equilibrium.

The second type of generalized Nash equilibrium is based on the assumption that each player is *wise*. In other words, each player does not assume that he/she is smarter than the opponent, and fully understands the law of action and reaction. He/She knows that if he/she takes the opponent’s strategy as given and chooses his/her own strategy to *maximize* his/her expected utility payoff, the opponent will also take his/her strategy as given and choose proper strategies to *maximize* his/her expected utility payoff. As a result, he/she cannot *eventually maximize* his/her utility payoff. On the other hand, when each player chooses to *maximize* the opponent’s utility payoff, he/she can *eventually maximize* his/her utility payoff at the equilibrium, provided the opponent also adopt the same incentive strategy.

Since

$$f_1(\hat{U}_s, \hat{U}_s) + f_2(\hat{U}_s, \hat{U}_s) \leq f_1(\hat{U}_c, \hat{U}_c) + f_2(\hat{U}_c, \hat{U}_c), \quad (2.20)$$

the second type of generalized Nash equilibrium is better than the first type of

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generalized Nash equilibrium when both players are regarded as a whole. Thus, a perfect cooperation is resulted when each player tries to maximize the opponent's utility payoff. Meanwhile, it can also be verified that $f_i(\hat{U}_s, \hat{U}_s) \leq f_i(\hat{U}_c, \hat{U}_c) (\forall i)$. Therefore, the objective of the second type of incentive is to eventually maximize each player's own utilization payoff at the equilibrium. This will be especially the case when inequalities (2.18) and (2.19) are satisfied. The essential ideas are illustrated through the examples given below.

Example 2.4.3. Consider Example 2.4.1 again, and suppose that each player's incentive is to maximize the opponent's utilization payoff. The dominant strategy is to "choose \$10 meal", and the second type of generalized Nash equilibrium will be obtained if both players choose the \$10 meal. Obviously, this equilibrium leads to a higher utility payoff for each player than that of ("choose \$20 meal", "choose \$20 meal").

Example 2.4.4. Consider an example given in [61]. Two players A and B have identical strategy space with elements "high" and "low". The corresponding utility payoff matrix is given in Table 2.3.

Table 2.3: Utilization payoff matrix.

$A \downarrow B \rightarrow$	low	high
low	(5,5)	(0,10)
high	(10,0)	(0, 0)

Consider the case that each player's incentive is to maximize the opponent's utilization payoff, the generalized Nash equilibrium is ("low", "low") and the corresponding utilization payoff is (5,5). Clearly, this is a Pareto optimal point [27].

We can also consider the case where inequalities (2.18) and (2.19) are not satisfied.

Example 2.4.5. *There are two players, Bob and Susan. Their strategies are A and B , and a and b , respectively. The corresponding utility payoff matrix is given in Table 2.4.*

Table 2.4: Utilization payoff matrix of Bob and Susan.

Bob ↓ \ Susan →	a	b
A	(1,2)	(3,1)
B	(0,-2)	(2, -3)

Assume that each player's incentive is to maximize the opponent's utilization payoff, the generalized Nash equilibrium is (A, b) and the corresponding utilization payoff is $(3,1)$. Obviously, this is a Pareto optimal point [27].

If all players' incentives belong to the second type, they will share more and more resources including information, knowledge, technologies, and so on. Their utilization payoffs will be increased accordingly

The second type of incentive can be employed in the development of Internet communication in terms of QoS and resource control. The following examples further illustrate the points.

Example 2.4.6 (TCP Congestion Control). *The cooperation among the sources of TCP congestion control is an interesting application of the generalized Nash equilibrium concept. When the sources detect a congestion, their reactions and corresponding throughput are shown in Table 2.5. The AIMD algorithm of congestion control is clearly of the second type of incentive. It can avoid network crash due to the congestion.*

Example 2.4.7 (ISPs' incentive). *Consider a route providing connection service between two destinations. Each link of the route belongs to different ISPs and the*

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Table 2.5: Sources' reactions to congestion.

$1 \downarrow \backslash 2 \rightarrow$	Reduce traffic rate	Ignore
Reduce traffic rate	(fair share, fair share)	(none, all)
Ignore	(all, none)	(none, none)

Table 2.6: The impact on price due to ISPs' reactions to the lack of resources under normal microeconomic mode.

ISP 1 $\downarrow \backslash$ ISP 2 $\rightarrow$	Enlarge service	Ignore
Enlarge service	(fair price, fair price)	(low, high)
Ignore	(high, low)	(high, high)

ISPs charge the users with a dynamic pricing system, i.e. the price fluctuates with the demands. Consider the case when the bandwidth is limited, the service providers may choose to enlarge the bandwidth or just ignore it. With the pricing systems in [1, 53–55, 57, 58, 60, 61], the network works as a conventional microeconomic system [64, 65]. The corresponding price due to ISPs reactions is shown in Table 2.6. No rational ISPs will choose to enlarge their services which would result in a reduction of their prices. This increases the cost of users even though the QoS is not increased. Consequently, this will lead to a stagnant in the industry and finally nobody wins. Hence, appropriate solutions are needed to overcome this.

Chapter 3

Fairness Indices for Resource Allocation

Fundamentally, many problems we encounter in the Internet all boil down to the issue of resource allocation. Packets get dropped or delayed because the resources in the network cannot meet all the traffic demands. A network, in its simplest form, consists of shared resources such as bandwidth and buffers, has to serve traffic from multiple competing users. A network that supports QoS must therefore take an active role in the resource allocation process and decides who should get the resources and how much. We shall focus on the issue of throughput allocation.

In this chapter, we shall first introduce a network model plus the accompanying notations that will be used throughout this thesis. The definition of fairness is then provided. After that, the case when there is only one resource bound is considered and a novel fairness index for a numerical measure of the fairness is proposed. The sufficient condition for convergence is also derived.

3.1 Network Model

In this thesis, we consider a network that has L unidirectional links and S sources. Each source uses a part of the network and does not change its route. The following

denotations are used throughout the thesis:

Links:

- L : The set of links, $L = \{1, 2, \dots, L\}$.
- L_s : The set of links that source s uses, $L_s \subseteq L$.
- c_l : The capacity of link l .
- $\bar{x}_l$: The total demand on link l .

Sources:

- Ω : The set of sources, $\Omega = \{1, 2, \dots, S\}$.
- Ω_l : The set of sources that passes through link l , $\Omega_l \subseteq \Omega$.
- x_s : The total transmission rate of source s . Under the TCP structure, x_s represents the window size of source s as we assume that the round trip time is fixed. x_s is bounded by the lower and upper bounds x_s^- and x_s^+ , respectively, i.e. $x_s^- \leq x_s \leq x_s^+$.

Delays:

- $\tau_{s,l}^f$: The forward delay from source s to link l .
- $\tau_{l,s}^b$: The backward delay from link l to source s .
- τ_s : The round trip delay time of source s , i.e. $\tau_s = \tau_{s,l}^f + \tau_{l,s}^b$.

Throughout the thesis, k will represent the discrete time instance. In the case of continuous time problems, t represents the time variable.

We shall study both the problems of a single service priority of the current Internet and the differentiated services with multiple priority classes. In the case

of the services with multiple priority classes, the traffic is divided into two priority classes. The network will give a better service to traffic of a higher priority class. A higher priority class of traffic is the one with a larger class index, i.e. a priority 2 traffic will get a better service than priority 1 traffic. Within each priority class, the traffic from different sources will be treated the same way. From this point of view, the service with a single service priority is just a special case of the service with multiple priority classes.

3.2 Fairness of Throughput Assignment

3.2.1 Feasible Solution

Consider a network with only one level of service priority. Each source s will transmit at the rate x_s that satisfies $x_s^- \leq x_s \leq x_s^+$. The throughput assignment problem is to determine a rate vector $\mathbf{x} = [x_1, x_2, \dots, x_S]^T$ for all the sources over the network. A feasible solution of the problem is defined as

Definition 3.2.1 (Feasible Solution). *A flow rate vector $\mathbf{x} = [x_1, x_2, \dots, x_S]^T$ is a feasible solution to the throughput assignment problem if and only if $\mathbf{x} \in \mathbb{R}^S$ satisfies*

$$\begin{aligned} x_s^- &\leq x_s \leq x_s^+; & \forall s \in \Omega, \\ \sum_{s \in \Omega_l} x_s &\leq c_l; & \forall l \in L, \end{aligned}$$

where $\Omega_l \subset \Omega$ is the set of sources which uses link l .

The set of all the feasible solutions is denoted by $\mathbf{X}_f$. Throughout this thesis, we shall make the following assumption on x_s^- to guarantee that $\mathbf{X}_f$ is not empty.

Assumption 3.2.1.

$$\sum_{s \in \Omega_l} x_s^- \leq c_l; \quad \forall l \in L.$$

3.2.2 Max-Min Fairness and Weighted Max-Min Fairness

Fairness is an important performance criterion when estimating a feasible solution. However, there exist different notions of the concept of fairness. For different purposes, different notions of fairness were defined [52, 78, 79]. Amongst all these notions of fairness, the concept of Max-Min fairness is very important for the traffic control of the Internet [80, 81] as it can be used to maximize the service of sources that may receive the poorest service.

3.2.2.1 Max-Min Fairness

Figure 3.1: The water-filling analogy.

A perfect interpretation (or the origin) of Max-Min fairness is the water-filling

3.2 Fairness of Throughput Assignment

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analogy as shown in Figure 3.1¹. The system resource is represented as a fixed amount of water. Each user is represented as a container and the capacity of the container stands for the requirement of users from the resource. To fairly allocate the water to the users, the system infuses the containers *synchronously* with the *same speed* until the containers are full or the water is exhausted. Consequently, the allocation is Max-Min fair. With Max-Min fairness, if a user is unsatisfied, he/she will find that all the other unsatisfied users have an equal share of the resource as him/her. The precise mathematical definition of Max-Min fairness is given as follows:

Definition 3.2.2 (Max-Min Fairness). *A feasible flow rate vector $\mathbf{x}^*$ is Max-Min fair if and only if for every feasible rate vector $\mathbf{x}$ with $x_i > x_i^*$ for some source i , there exists another source j such that $x_j^* \leq x_i^*$ and $x_j < x_j^*$.*

In other words, a feasible flow rate vector is Max-Min fair if any rate x_i cannot be increased without decreasing some x_j which is smaller than or equal to x_i . The following two examples can be used to demonstrate the characteristics of Max-Min fairness.

Example 3.2.1 (One Bottleneck Link). *Three sources share one link with capacity 24. The x_s^- and x_s^+ of sources are listed as below:*

$$\begin{aligned} x_1^- &= 1; & x_1^+ &= 6, \\ x_2^- &= 3; & x_2^+ &= 10, \\ x_3^- &= 0; & x_3^+ &= 12. \end{aligned}$$

The corresponding Max-Min fair assignment $\mathbf{x}^$ is $[6, 9, 9]^T$. In this case, the system*

¹The example shows the case where there is only one resource bound.

first satisfies source 1 with the smallest requirements and splits all the rest equally to the other two sources.

Figure 3.2: The topology of the example in the case of multiple bottleneck links.

Example 3.2.2 (Multiple Bottleneck Links). Consider the case whose topology is as shown in Figure 3.2. Suppose that the capacity of each link is 1 and x_s^- and x_s^+ of each source is 0 and ∞ , i.e. sources are unbounded. The Max-Min fair allocation is

$$\mathbf{x}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \\ x_4^* \\ x_5^* \\ x_6^* \\ x_7^* \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{3}{8} \\ \frac{3}{8} \end{bmatrix}.$$

Sources 2, 3, 4 and 5 are bottlenecked by link 3 and each gets an equal share of the throughput. Sources 6 and 7 are bottlenecked by link 4. Since source 3 has gotten

its maximal throughput, which is under the bound of link 3, sources 6 and 7 got to split the rest of the throughput of link 4, while source 1 takes all of the rest of link 1.

Clearly, Max-Min fairness is based on the assumption that all sources have the same priority and should be given equal treatment. However, with the evolution of the Internet and the ever increasing on-line applications, this is not necessary as it will limit the efficiency of the networks. For example, an FTP application can tolerate a relative small throughput at a cost of extending the total transmission time. However, for real-time video, if the throughput does not satisfy a minimal amount, the application may work poorly or even unable to work at all. Thus, it makes sense to reduce the share of the throughput of FTP applications so as to support more critical real-time video applications.

In consideration of this possibility, we generalize the concept of Max-Min Fairness by assigning each source s a predefined weighting factor w_s ²:

Definition 3.2.3 (Weighted Max-Min Fairness). A rate vector $\mathbf{x}^* \in \mathbf{X}_f$ is Max-Min fair with respect to $[w_1, w_2, \dots, w_S]^T$ if, for every feasible rate vector $\mathbf{x}$ with $x_i > x_i^*$ for some source i , there exists another source j such that $\frac{x_j^*}{w_j} \leq \frac{x_i^*}{w_i}$ and $x_j < x_j^*$.

The weights can be interpreted as the competitive ability of the sources to request for the resource. In a “want more/pay more/get more” service approach, it may represent the users’ “willingness-to-pay”.

In the water-filling analogy for the Weighted Max-Min Fairness, the system infuses water into containers with a speed which is linearly proportional to the weight of each user, instead of with the same speed. Thus, the original Max-Min

²A similar definition of weighted Max-Min fairness is also proposed in [78, 82]

fairness can be seen as a special case of Weighted Max-Min fairness where all users have equal weights.

From the analogy, it is easy to derive the following propositions:

Proposition 3.2.1. *The Max-Min fair rate vector $\mathbf{x}^*$ w.r.t. $[w_1, w_2, \dots, w_S]^T$ always exists if $\mathbf{X}_f$ is not empty.*

Proposition 3.2.2. *The Max-Min fair rate vector $\mathbf{x}^*$ w.r.t. $[w_1, w_2, \dots, w_S]^T$ is unique.*

The Examples 3.2.1 and 3.2.2 are extended to the weighted case as follows:

Example 3.2.3 (One Bottleneck Link). *We extend Example 3.2.1 by giving each source a weight:*

$$w_1 = 1; w_2 = 3; w_3 = 2.$$

The corresponding Max-Min fair assignment $\mathbf{x}^$ is $[14/3, 10, 7/3]^T$. In this case, source 2 first achieves its upper bound and sources 1 and 3 partition the rest of the throughput according to their weights.*

Example 3.2.4 (Multiple Bottleneck Links). *We assign each source in Example 3.2.2 a weight as follows:*

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \\ 2 \\ 4 \\ 4 \\ 6 \end{bmatrix}.$$

The corresponding weighted Max-Min fair allocation and x_s^*/w_s are

$$\mathbf{x}^* = \begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \\ x_4^* \\ x_5^* \\ x_6^* \\ x_7^* \end{bmatrix} = \begin{bmatrix} \frac{35}{44} \\ \frac{5}{44} \\ \frac{1}{11} \\ \frac{35}{132} \\ \frac{35}{66} \\ \frac{4}{11} \\ \frac{6}{11} \end{bmatrix}, \text{ and } \begin{bmatrix} \frac{x_1^*}{w_1} \\ \frac{x_2^*}{w_2} \\ \frac{x_3^*}{w_3} \\ \frac{x_4^*}{w_4} \\ \frac{x_5^*}{w_5} \\ \frac{x_6^*}{w_6} \\ \frac{x_7^*}{w_7} \end{bmatrix} = \begin{bmatrix} \frac{5}{44} \\ \frac{5}{44} \\ \frac{1}{11} \\ \frac{35}{264} \\ \frac{35}{264} \\ \frac{1}{11} \\ \frac{1}{11} \end{bmatrix}.$$

In this case, sources 3, 6 and 7 are bottlenecked by link 4 and they split the overall throughput of the link according to their weights; Sources 1 and 2 are bottlenecked by link 1 and they split the rest of throughput of link 1, which is the total throughput of link 1 subtracting the throughput of source 3; A similar situation can be observed for link 3. Note that link 2 is not a bottleneck link of any source.

Remark 3.2.1. in [83], a mathematical definition of Max-Min fairness is given as: Suppose that $f_s(x_s)$ is an increasing and continuous function of x_s . A rate vector $\mathbf{x}^* \in \mathbf{X}_f$ is Max-Min fair with respect to $[f_1, f_2, \dots, f_S]^T$ if, for every feasible rate vector $\mathbf{x}$ with $x_i > x_i^*$ for some source i , there exists another source j such that $f_j(x_j^*) \leq f_i(x_i^*)$ and $x_j < x_j^*$. Thus, our Weighted Max-Min Fairness is the case when $f_s(x_s) = x_s/w_s$.

3.2.3 Properties of Weighted Max-Min Fairness

It is interesting to note from Example 3.2.4 that with the weighted Max-Min fairness, the network has the feature that if the link l is the most congested link for all sources $s \in \Omega_l$, the capacity is shared among all sources $s \in \Omega_l$ with respect to

their weights w_s (in case of Max-Min fairness, it is shared equally). Hence we have the following definition and key result on weighted Max-Min fairness.

Definition 3.2.4. For a given $\mathbf{x} \in \mathbf{X}_f$, link l is a bottleneck link for source $i \in \Omega_l$ if it is the most congested link in the path from source i to its destination, i.e. $\sum_{j \in \Omega_l} x_j = c_l$ and $\frac{x_j}{w_j} \leq \frac{x_i}{w_i}$, $\forall j \in \Omega_l$.

Proposition 3.2.3. A rate vector $\mathbf{x}^* \in \mathbf{X}_f$ is Max-Min fair w.r.t. $[w_1, w_2, \dots, w_S]^T$ if and only if for each source s , it is bottlenecked by a link/some links or $x_s^* = x_s^+$.

The necessity of Proposition 3.2.3 can be easily derived from the water-filling analogy. The proof for sufficiency will be deferred to later, after we have introduced some other relevant results. To prove its sufficiency, we first group all sources into N groups, $\tilde{\Omega}_r (1 \leq r \leq N \leq S)$ according to their throughput, arranged in increasing order:

- Two sources i, j are in the same group $\tilde{\Omega}_r$ if their rates satisfy

$$\frac{x_i}{w_i} = \frac{x_j}{w_j}. \quad (3.1)$$

- For any 2 sources $i \in \tilde{\Omega}_{r_1}$, $j \in \tilde{\Omega}_{r_2}$, with $r_1 < r_2$, then

$$\frac{x_i}{w_i} < \frac{x_j}{w_j}. \quad (3.2)$$

This is illustrated by the following example.

Example 3.2.5. Consider the network in Example 3.2.4. We have $x_3^*/w_3 = x_6^*/w_6 = x_7^*/w_7 = 1/11$, $x_1^*/w_{11} = x_2^*/w_2 = 5/44$, and $x_4^*/w_4 = x_5^*/w_5 = 35/264$. The sources are therefore grouped into the following groups:

$$\tilde{\Omega}_1 = \{3, 6, 7\},$$

3.2 Fairness of Throughput Assignment

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$$\tilde{\Omega}_2 = \{1, 2\},$$

$$\tilde{\Omega}_3 = \{4, 5\}.$$

Through this grouping rule, we have the following proposition.

Proposition 3.2.4. *The sources which are bottlenecked by the same link are in the same group.*

Proof. This can be easily derived from Definition 3.2.4 and the grouping rule. $\square$

Link l is said to be associated with $\tilde{\Omega}_r$ if link l is the bottleneck link of source s in $\tilde{\Omega}_r$. We also group all associated links into N groups, $\tilde{L}_r (1 \leq r \leq N)$ accordingly, as illustrated by the following example.

Example 3.2.6. *In Example 3.2.5, links 1, 3 and 4 are associated with $\tilde{\Omega}_r (1 \leq r \leq 3)$: Link 4 is the bottleneck link of sources 3, 6 and 7; link 1 is the bottleneck link of both sources 1 and 2; and link 3 is the bottleneck link of both sources 4 and 5. Note that link 1 is not a bottleneck link for source 4 because $x_4^* = 35/132 < 1$. According to $\tilde{\Omega}_r$ in the example, links 1, 3 and 4 are grouped into the following groups:*

$$\tilde{L}_1 = \{4\},$$

$$\tilde{L}_2 = \{1\},$$

$$\tilde{L}_3 = \{3\}.$$

Extending the above example, we have the following propositions:

Proposition 3.2.5. $\cup_{r=1}^N \tilde{L}_r \subseteq L$.

Proposition 3.2.6. *For source s which belongs to $\tilde{\Omega}_i$ (accordingly, its bottleneck link l belongs to $\tilde{L}_i$) and link j which belongs to $\tilde{L}_k$. If $k < i$, source s will not pass through link j , i.e. $j \notin L_s$.*

Proof. We prove by contradiction. Suppose there exists a link j , $j \in \tilde{L}_k$, $k < i$ and $j \in L_s$. Hence there must exist at least one source, s_1 say, which is bottlenecked by link j , i.e., x_{s_1}/w_{s_1} is greater than all the other sources which pass through link j . On the other hand, according to Equation 3.2, $x_{s_1}/w_{s_1} < x_s/w_s$. It contradicts with the earlier assertion. This proves the proposition. $\square$

We are now ready to prove the sufficiency of Proposition 3.2.3.

Proof of the sufficiency of Proposition 3.2.3. We shall use induction.

We first consider link $l \in \tilde{L}_1$. For any source $s \in \Omega_l$, either $s \in \tilde{\Omega}_1$ or $x_s = x_s^+$. For any two sources i and j , where $\{i, j\} \subseteq \tilde{\Omega}_1$ and $\{i, j\} \subseteq \Omega_l$, $x_i/w_i = x_j/w_j$. For any source i , where $x_i = x_i^+$, and source j , where $j \in \tilde{\Omega}_1$ and $\{i, j\} \subseteq \Omega_l$, $x_i/w_i < x_j/w_j$. We denote the set of such sources as $\hat{\Omega}_l$.

Consequently, for any source $s \in \Omega_l$, while link $l \in \tilde{L}_1$, either $x_s = x_s^+$ or

$$x_s = \frac{w_s(c_l - \sum_{i \in \hat{\Omega}_l} x_i)}{\sum_{i \in \Omega_l \cap \hat{\Omega}_l} w_i}. \quad (3.3)$$

Recall the intuition of weighted Max-Min fairness, thus such an allocation is weighted Max-Min fair.

Now we suppose that for group m , the allocation is weighted Max-Min fair, we consider group $m + 1$. Consider link $l \in \tilde{L}_{m+1}$. For any source $s \in \Omega_l$, there are three possible cases: $x_s = x_s^+$, $s \in \tilde{\Omega}_k$ where $k \leq m$ or $s \in \tilde{\Omega}_{m+1}$ (according to Proposition 3.2.6). For the first two cases, source s already gets the maximum throughput it can get. For the remaining case, Equation 3.3 can be easily derived. Hence, it is also weighted Max-Min fair.

By induction, we can know that it is sufficient for N groups.

For any other sources, which satisfy $x_s = x_s^+$ while the links they pass do not belong to any link groups, they already get the maximum throughput. Thus, for all sources, the allocation is weighted Max-Min fair. $\square$

3.3 Fairness over Single Bottleneck Link

3.3.1 Weighted Fairness

Max-Min fairness and weighted Max-Min fairness deal with the fairness problem under multiple bounds whence they correspond to the fairness problem over a network. On the other hand, as Proposition 3.2.3 implied, when the allocation is weighted Max-Min fair, the corresponding sources are bottlenecked by *one* link. Furthermore, to achieve such a fairness, it depends on not only the protocol, which is used to exchange information by sources and links, but also the traffic control algorithm employed by sources. However, it is difficult to study the characteristics of such algorithms directly in the case of multiple bottleneck links. Thus it is necessary to consider the fairness with only one resource bound, i.e. one bottleneck link [15, 42, 46].

Definition 3.3.1 (Weighted Fairness). *A rate vector $\mathbf{x}^*$ is fair with respect to $[w_1, w_2, \dots, w_S]^T$, if $\frac{x_j^*}{w_j} = \frac{x_i^*}{w_i}$ for all i and j .*

Therefore, the weighted fairness is a type of locally weighted fairness while the weighted Max-Min fairness is a type of globally weighted fairness.

3.3.2 A Novel Fairness Index

A weighted fairness index for the quantitative measurement of weighted fairness is defined as [43]

$$F = \sum_{s=1}^S \frac{x_s^2}{w_s^2} - \frac{1}{S} \left(\sum_{s=1}^S \frac{x_s}{w_s} \right)^2, \quad (3.4)$$

where x_s and w_s are the traffic rate and the weight of source s and S is the total number of sources.

Figure 3.3: The numerical illustration of F in the case of two users with $w_1 = 2$ and $w_2 = 2$. From (d), it is easy to know that the fairness indices of dots which are located on the parallel of $x_1 = 2x_2$ are equal.

3.3 Fairness over Single Bottleneck Link

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Figure 3.3 illustrates the value of F in a two-user case with $w_1 = 2$ and $w_2 = 1$. It can be shown from Equation (3.4) that

$$F = \sum_{1 \leq i, j \leq S, i \neq j} \left(\frac{x_i}{w_i} - \frac{x_j}{w_j} \right)^2.$$

F has the following features:

- (i). $F \geq 0, \forall x_1, \dots, x_S$.
- (ii). $F = 0$ when $\frac{x_1}{w_1} = \dots = \frac{x_S}{w_S}$.

Thus, the fairness index is a kind of Lyapunov function [43].

As a comparison, one may use a widely used fairness index which is proposed by Jain et al. in [84]

$$\tilde{F} = \frac{(\sum_{s=1}^S x_s)^2}{S \sum_{s=1}^S x_s^2}. \quad (3.5)$$

$\tilde{F}$ always lies between 0 and 1. When x_s are equal, $\tilde{F}$ is 1 and refers to a fair state. Obviously, $\tilde{F}$ only considered the “equality” of the traffic rate, i.e. fairness in the case that the weights of all sources are equal while the proposed F has considered a more common case. The physical meaning of F in (3.4) with equal weights and $\tilde{F}$ in (3.5) for $S = 2$ can be explained by using Figure 3.4.

It can be easily derived that

$$\begin{aligned} F &= \sum_{s=1}^2 x_s^2 - \frac{(\sum_{s=1}^2 x_s)^2}{2} \\ &= \sum_{s=1}^2 \left(x_s - \frac{\sum_{i=1}^2 x_i}{2} \right)^2 \\ &= \rho(P, L). \end{aligned}$$

In other words, F represents the distance from point $P(x_1, x_2)$ to line $L(x_1 = x_2)$.

Figure 3.4: Illustration of two types of fairness.

On the other hand, we have

$$\begin{aligned}
 \tilde{F} &= \frac{(\sum_{s=1}^2 x_s)^2}{2 \sum_{s=1}^2 x_s^2} \\
 &= \frac{2(\frac{\sum_{s=1}^2 x_s}{2})^2}{\sum_{s=1}^2 x_s^2} \\
 &= \cos^2(\theta).
 \end{aligned}$$

It represents the angle between line $(0,0)-(x_1, x_2)$ and line $x_1 = x_2$.

3.4 Sufficient Condition for the Convergence of Weighted Fairness

Since the fairness index represents the fairness state, we have the following theorem for the change of fairness:

Theorem 3.4.1. *Let Δx_s be a small change to x_s . F in (3.4) approaches 0 (i.e. fairness) if*

$$\sum_{1 \leq i, j \leq S, i \neq j} \left(\frac{x_i}{w_i} - \frac{x_j}{w_j} \right) \left(\frac{\Delta x_i}{w_i} - \frac{\Delta x_j}{w_j} \right) < 0 \quad (3.6)$$

3.4 Sufficient Condition for the Convergence of Weighted Fairness

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is satisfied. □

Proof. Let ΔF be the change in F corresponding to small changes Δx_i in x_i . It is easily derived that

$$\begin{aligned}
 \Delta F &= \sum_{i=1}^S \frac{\partial F}{\partial x_i} \Delta x_i \\
 &= \sum_{i=1}^S \left[\frac{2x_i}{w_i^2} - \frac{2}{S w_i} \left(\sum_{s=1}^S \frac{x_s}{w_s} \right) \right] \Delta x_i \\
 &= \frac{2}{S} \sum_{i=1}^S \left(S \frac{x_i}{w_i} - \sum_{j=1}^S \frac{x_j}{w_j} \right) \frac{\Delta x_i}{w_i} \\
 &= \frac{2}{S} \sum_{1 \leq i, j \leq S, i \neq j} \left(\frac{x_i}{w_i} - \frac{x_j}{w_j} \right) \left(\frac{\Delta x_i}{w_i} - \frac{\Delta x_j}{w_j} \right).
 \end{aligned}$$

Thus if Equation (3.6) is satisfied, ΔF is negative and F reduces. Therefore, F approaches 0. □

Similarly, we have the following two results.

Theorem 3.4.2. *Let Δx_s be a small change to x_s . If*

$$\sum_{1 \leq i, j \leq S, i \neq j} \left(\frac{x_i}{w_i} - \frac{x_j}{w_j} \right) \left(\frac{\Delta x_i}{w_i} - \frac{\Delta x_j}{w_j} \right) = 0, \quad (3.7)$$

the fairness of system does not change. □

Theorem 3.4.3. *Let Δx_s be a small change to x_s . If*

$$\sum_{1 \leq i, j \leq S, i \neq j} \left(\frac{x_i}{w_i} - \frac{x_j}{w_j} \right) \left(\frac{\Delta x_i}{w_i} - \frac{\Delta x_j}{w_j} \right) > 0, \quad (3.8)$$

the fairness of system will become worse. □

Chapter 4

Responsive Functions for Source Adaptation

4.1 Introduction

Many new applications are being widely deployed in the Internet, such as voice over IP, video conference, video streaming, audio streaming, mission-critical financial data, and so on. Many of these applications employ the user datagram protocol (UDP) and have different requirements on the bandwidth from those of existing applications using transmission control protocol (TCP). For example, multimedia applications have the following two distinguished requirements [12, 29, 89, 90]:

- (i). The sending rate should be adjusted smoothly;
- (ii). The sending rate should be greater than a threshold, i.e. multimedia applications usually consume high bandwidth.

TCP-friendly source adaptation schemes do not work well for multimedia applications [29, 30, 90, 91]. It seems that TCP-friendliness and multimedia-friendliness “contradict” each other [12]. Thus, these new applications present new problems

for the Internet. There are two possible approaches to address these problems. One is to enhance the Internet with resource reservation, admission control [92] and so on. The other is to adopt source adaptation schemes such that the sending rate of an application is adjusted according to the current network condition. Compared to the first approach, the second one has the advantage of better utilization of the available time-varying network resources [12].

One important type of source adaptation is equation-based source adaptation [7, 47, 93]. The advantage of this type of source adaptation, as compared to the switch control law of TCP, is a smoother sending rate. It can also be used to maintain a relatively steady sending rate. However, it relies heavily on the accurate measurement of the round trip time, steady-state loss event rate and the TCP retransmit time out value, especially the steady-state loss event rate that is very difficult to be measured. Furthermore, it has another two disadvantages: 1) It is more moderately responsive to transient changes in congestion, including a slower response to an abrupt change in the available bandwidth. As a result, it cannot aggressively find and use the available bandwidth [47, 94]; 2) The second requirement of multimedia applications cannot be satisfied by equation-based source adaptation when a large number of sources share a link.

In this chapter, instead of equation-based source adaptation, we provide a new framework for source adaptation based on switch control. The proposed framework is composed of an Increase (*I*) policy and a Decrease (*D*) policy. The *I* and *D* policies are:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + w_s f\left(\frac{x_s(k)}{w_s}\right), \quad (4.1)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - w_s g\left(\frac{x_s(k)}{w_s}\right), \quad (4.2)$$

where w_s is the weighting factor of source s , $f(x)$ is a monotonically non-increasing function, $g(x)$ is a monotonically non-decreasing function with $f(x) > 0$, $x \geq g(x) > 0$ and at least one of $f(x)$ and $g(x)$ should be strictly monotonic. A set of sources that uses the proposed increase-decrease policy will step-wisely converge to weighted fairness and efficiency. Since both $f(x)$ and $g(x)$ are monotonic functions, we call our scheme the Monotonic Responsive Function (MRF) scheme.

We derived our main theoretical results from the weighted fairness index defined in Equation (3.4). MRF is based on the concept of switched control [43], so the network with the proposed MRF is a switched system. The performance index (3.4) can be regarded as a common Lyapunov function of the switched system. The MRF is thus an application of switched control [43].

By adopting the proposed MRF as the source adaptation scheme, differentiated bandwidth services can be provided with necessary feedback signals from the destination. With the proposed MRF as the source adaptation scheme, a source can find and use the available bandwidth and also adjust its sending rate smoothly without interrupting the existing TCP traffic. Hence, with the proposed framework, more and varied applications over the Internet can be supported.

4.2 Requirements on Responsive Functions

4.2.1 Fairness

It has been shown that TCP with the AIMD algorithm has a feature that the traffic rate is inversely proportional to the square root of the loss rate [13]. This feature is recommended as TCP-friendliness for congestion control. Consequently, a TCP-friendly traffic should obtain a throughput *generally within a factor of two of the sending rate of a TCP flow under the same conditions* [7]. Obviously, because of

the existence of the weighting factor, the TCP-friendly traffic and TCP traffic are not equally fair when compared to the case of all TCP traffic. On the other hand, the factor implies a *weighted fairness*. Thus in view of throughput allocation, we have the following definition for TCP-friendliness:

Definition 4.2.1 (TCP-friendliness). *A scheme is TCP-friendly if the throughput allocation is weighted fair, when it is competing with TCP flow.*

We distinguish TCP-compatibility from TCP-friendliness as follow:

Definition 4.2.2 (TCP-compatibility). *A scheme is TCP-compatible if the throughput allocation is equal fair, when it is competing with TCP flow.*

Thus, TCP-friendliness is TCP-compatible with respect to unknown weights. Similarly, the weighting factor is applied to the concept of TCP-friendliness and TCP-compatibility. Hence, a source with traffic rate x_s is said to be

- TCP-friendly with respect to w_s if the corresponding traffic flow with traffic rate x_s/w_s is TCP-friendly;
- TCP-compatible with respect to w_s if the corresponding traffic flow with traffic rate x_s/w_s is TCP-compatible.

Intuitively speaking, TCP-friendly with respect to w_s will maintain the traffic rate of the source to be inversely proportional to the square root of the loss rate. Thus, it does not interrupt other competing TCP traffic [7]. TCP-compatible with respect to w_s guarantees that the traffic rate of the source is proportional to w_s when competing with TCP traffic. These concepts are very useful for multimedia applications over the Internet in the sense that by controlling the weights, it can change the throughput allocation without interrupting TCP traffic.

4.2.2 1-Responsiveness

Let Δ_I and Δ_D be the increase and decrease in the window size resulting from a single application of I and D , respectively. An algorithm is said to be 1-responsive if

$$|\Delta_I| \leq |\Delta_D|. \quad (4.3)$$

In other words, the decrease in the window size from a single application of D must at least wipe out the previous increase resulting from the last application of I .

It can be easily shown that AIMD, GAIMD, BCC, LOG and SIGMOID are 1-responsive. Thus, unless otherwise stated, the protocols are supposed to be 1-responsive.

4.2.3 Smoothness and Efficiency

Smoothness is an important property when video, audio or speech is transmitted over the network [30]. Same as in [51], a window increase (decrease) policy is said to be smooth if the window size increase (decrease) from a single I/D policy is at least an order of magnitude smaller than the current window size, i.e.

$$|\Delta x| \ll x. \quad (4.4)$$

Meanwhile, the policies are required to move the total bottleneck link utilization closer to the link capacity. This can be achieved by the principle of negative feedback [42], i.e., each source increases its window size when the bottleneck link is under-utilized and decreases its window size when it is overloaded.

4.3 The MRF Framework

4.3.1 Network Model

We only consider the classical memoryless model of source adaptation. Generally, it is very difficult to analyze the fairness of a network protocol. Similar to [42, 51], the following assumptions are imposed on the network:

Assumption 4.3.1 (Synchronization Assumption). *It is assumed that all the sources in the network get the same feedback and get the feedback simultaneously.*

Remark 4.3.1. *This can normally be viewed as sources are bottlenecked by a single link.*

Assumption 4.3.2 (Feedback Signal). *The feedback is binary and limited to a single bit indicating whether the network is overloaded, i.e. a '1', or if the bandwidth is not fully utilized, i.e. a '0' [40].*

Assumption 4.3.3 (Responsive Functions). *Assume that all sources use the same responsive functions. When an ACK is received and the feedback is 1, the next window size is computed by the policy D . If the feedback is 0, the policy I is used to calculate the next window size.*

The capacity of network is treated as a constant, i.e. when the sum of window sizes exceeds the constant, an overloaded signal will be sent out [42, 44, 50].

It is important to note that these assumptions are used to simplify the analysis. As it can be seen later in the simulation studies that our results hold even with the normal Internet setting, which does not satisfy the ideal condition.

4.3.2 Convergence of MRF

Let $f(x)$ be a monotonically non-increasing function and $g(x)$ a monotonically non-decreasing function, with $f(x) > 0$, $x \geq g(x) > 0$, and that the sending rates of sources are adjusted by increase policy (4.1) and decrease policy (4.2).

We shall first consider the case of $S = 2$. Suppose two sources share a bottleneck link. The result is as given below.

Theorem 4.3.1. *(2-flow Fairness for the MRF) Let $-f(x)$ and $g(x)$ be two monotonically non-decreasing functions (at least one of them strictly increasing) with $f(x) > 0$, $x \geq g(x) > 0$. Then policies I and D given in Equations (4.1) and (4.2) guarantee convergence to the weighted fairness and efficiency for two sources sharing a bottleneck link.* $\square$

Proof. For policy I (4.1), $\Delta x_s = w_s f(\frac{x_s}{w_s})$ and $(\frac{x_1}{w_1} - \frac{x_2}{w_2})(\frac{\Delta x_1}{w_1} - \frac{\Delta x_2}{w_2}) = (\frac{x_1}{w_1} - \frac{x_2}{w_2})(f(\frac{x_1}{w_1}) - f(\frac{x_2}{w_2})) \leq 0$. While for policy D (4.2), $\Delta x_s = -w_s g(\frac{x_s}{w_s})$ and $(\frac{x_1}{w_1} - \frac{x_2}{w_2})(\frac{\Delta x_1}{w_1} - \frac{\Delta x_2}{w_2}) = (\frac{x_1}{w_1} - \frac{x_2}{w_2})(g(\frac{x_2}{w_2}) - g(\frac{x_1}{w_1})) \leq 0$.

Thus, by Theorems 3.4.1 and 3.4.2 in case of $S = 2$, we need at least one of D and I to ensure $\Delta F < 0$ so that F decreases over each congestion epoch and eventually becomes 0. Meanwhile, once $x_1/w_1 = x_2/w_2$, this equality is maintained under synchronous feedback and the values of the window sizes will increase or decrease in lockstep with each other.

Note that the principle of negative feedback is applied in the proposed MRF, with the window size increased by policy I (4.1), and decreased by policy D (4.2). Meanwhile, the window size is always non negative by the proposed policies I (4.1) and D (4.2). Therefore, the MRF is efficient. $\square$

Theorem 4.3.2. *Let $-f(x)$ and $g(x)$ be two monotonically non-decreasing functions (at least one of them strictly increasing) with $f(x) > 0$, $x \geq g(x) > 0$. Then*

policies I and D given in Equations (4.1) and (4.2) guarantee convergence to the weighted fairness and efficiency for $S (> 2)$ sources sharing a bottleneck link. $\square$

Proof. We shall use the induction method to show that the proposed MRF satisfies the condition of Theorems 3.4.1 and 3.4.2. From Theorem 4.3.1, this is true for $S = 2$. Suppose that it is true for $S = k$, i.e.

$$\sum_{1 \leq i, j \leq k, i \neq j} \left(\frac{x_j}{w_j} - \frac{x_i}{w_i} \right) \left(\frac{\Delta x_j}{w_j} - \frac{\Delta x_i}{w_i} \right) \leq 0. \quad (4.5)$$

Consider the case of $S = k + 1$. Note that both $f(x)$ and $-g(x)$ are non-increasing functions. This implies that Δx is non-increasing for both policy D and policy I . Thus, for any x_{k+1} , we have

$$\left(\frac{x_{k+1}}{w_{k+1}} - \frac{x_i}{w_i} \right) \left(\frac{\Delta x_{k+1}}{w_{k+1}} - \frac{\Delta x_i}{w_i} \right) \leq 0; \forall i.$$

It follows that

$$\sum_{i=1}^k \left(\frac{x_{k+1}}{w_{k+1}} - \frac{x_i}{w_i} \right) \left(\frac{\Delta x_{k+1}}{w_{k+1}} - \frac{\Delta x_i}{w_i} \right) \leq 0. \quad (4.6)$$

From (4.5) and (4.6), we obtain

$$\sum_{1 \leq i, j \leq (k+1), i \neq j} \left(\frac{x_j}{w_j} - \frac{x_i}{w_i} \right) \left(\frac{\Delta x_j}{w_j} - \frac{\Delta x_i}{w_i} \right) \leq 0.$$

Conditions for Theorems 3.4.1 and 3.4.2 are therefore satisfied.

Following a similar argument as in the proof of Theorem 4.3.1, we obtain the results of Theorem 4.3.2. $\square$

4.3.2.1 Convergence of Existing Responsive Functions

The convergence to fairness of AIMD is first shown in [42] by Jain and Chiu. Sastry and Lam extended the proof under the same conditions [51]. However, the Choose-Your-Responsive-Function (CYRF) they provided in [51] failed to cover all the existing alternative algorithms and they have to introduce the notion of 1-CYRF.

The convergence of existing responsive functions such as AIMD, GAIMD, BCC, LOG, and SIGMOID can be derived directly by Theorem 4.3.2 and assuming equal weighting factor. Furthermore, it is much easier to use the proposed results to show the convergence of the BCC than the result from Jain's fairness index [84], which is shown in [51]. Moreover, unlike [51], no approximation is used in our proof.

4.3.3 TCP-friendly MRF

To guarantee the display quality of video at the receiver side, smooth source adaptation schemes are desirable [30]. This type of source adaptation schemes are also required to be TCP-friendly w.r.t. predefined weighting factors. In this section, we shall derive a sufficient condition for the proposed MRF with a smooth increase policy to be TCP-friendly w.r.t. the predefined weighting factor. We require 1-responsiveness since the analysis is highly related to the activation of the increase/decrease policies in steady state. 1-responsiveness assures that the policy is invoked with the order $DII \cdots IDII \cdots$.

Similar to [51], the behavior of the proposed MRF in the steady state is considered. Suppose x is the maximum window size in the epoch just before the application of policy D . The window size just after the application of policy D , i.e. $(x - g(x))$, is also the initial window size of the epoch due to the steady state condition. Assume that the first time instant when policy I is used in an epoch

is t . Using the first order Taylor's expansion, the window sizes in the epoch are approximated as

$$\begin{aligned}
 \frac{x_s(k)}{w_s} &\approx \frac{x_s}{w_s} - g\left(\frac{x_s}{w_s}\right), \\
 \frac{x_s(k+1)}{w_s} &\approx \frac{x_s}{w_s} + f\left(\frac{x_s}{w_s}\right) - (1 + f'\left(\frac{x_s}{w_s}\right))g\left(\frac{x_s}{w_s}\right), \\
 \frac{x_s(k+2)}{w_s} &\approx \frac{x_s}{w_s} + 2(1 + f'\left(\frac{x_s}{w_s}\right))f\left(\frac{x_s}{w_s}\right) \\
 &\quad - (1 + f'\left(\frac{x_s}{w_s}\right))^2 g\left(\frac{x_s}{w_s}\right), \\
 &\vdots \\
 \frac{x_s(k+m)}{w_s} &\approx \frac{x_s}{w_s} + m(1 + f'\left(\frac{x_s}{w_s}\right))^{m-1} f\left(\frac{x_s}{w_s}\right) \\
 &\quad - (1 + f'\left(\frac{x_s}{w_s}\right))^m g\left(\frac{x_s}{w_s}\right),
 \end{aligned}$$

where m is the number of successive applications of policy I .

Since the network is in steady state, $x_s(k+m) = x_s$, the number of applications of policy I is

$$m = \frac{g\left(\frac{x_s}{w_s}\right)(1 + f'\left(\frac{x_s}{w_s}\right))}{f\left(\frac{x_s}{w_s}\right)}, \quad (4.7)$$

and numbers of packets sent during the epoch is

$$\begin{aligned}
 S_{m+1} &= \sum_{j=0}^m x_s(k+j) \\
 &= w_s \sum_{j=0}^m \frac{x_s(k+j)}{w_s} \\
 &= \frac{m+1}{2} x_s \left(2 - \frac{(1 + f'\left(\frac{x_s}{w_s}\right))^m g\left(\frac{x_s}{w_s}\right)}{\frac{x_s}{w_s}} \right). \quad (4.8)
 \end{aligned}$$

The following result gives a characterization of TCP-friendliness w.r.t. w_s in the steady state and it can be easily proved by using the definition of TCP-friendliness

4.3 The MRF Framework

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w.r.t. w_s and the corresponding result in [51].

Lemma 4.3.1. *TCP-friendliness: A 1-responsive flow is TCP-friendly w.r.t. w_s in the deterministic steady state if and only if*

$$\frac{S_{m+1}}{w_s} \propto (m+1)^2. \quad (4.9)$$

□

Substituting Equations (4.7) and (4.8) into Equation (4.9), we have

Theorem 4.3.3. *A smooth 1-responsive flow is TCP-friendly w.r.t. w_s in the steady state if and only if*

$$\frac{(1 + f'(\frac{x_s}{w_s}))g(\frac{x_s}{w_s})}{f(\frac{x_s}{w_s})} \propto \frac{x_s}{w_s}. \quad (4.10)$$

□

Similarly, we have the following result on TCP-compatibility w.r.t. w_s .

Lemma 4.3.2. *TCP-compatibility: A 1-responsive flow is TCP-compatible w.r.t. w_s in the deterministic steady state if and only if*

$$\frac{S_{m+1}}{w_s} = \frac{3(m+1)^2}{2}. \quad (4.11)$$

□

Substituting Equations (4.7) and (4.8) into Equation (4.11), we have

Theorem 4.3.4. *A smooth 1-responsive flow is TCP-compatible w.r.t. w_s in the steady state if and only if*

$$f(\frac{x_s}{w_s}) = \frac{3(1 + f'(\frac{x_s}{w_s}))g(\frac{x_s}{w_s})}{\frac{2x_s}{w_s} - (1 + f'(\frac{x_s}{w_s}))^m g(\frac{x_s}{w_s})}. \quad (4.12)$$

□

With the theorem, the results for GAIMD in [47], i.e. Equation (2.7), could be derived directly.

4.3.4 Applications of the MRF

In this subsection, we shall apply the MRF to design extended source adaptation schemes.

Weighted AIMD (WAIMD) is an extension of the most popular source adaptation scheme, AIMD, introduced in [41]. The window size is increased by w_s and decreased by half when a loss is detected, i.e.

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + w_s, \quad (4.13)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \frac{x_s(k)}{2}. \quad (4.14)$$

It was shown in [42] that the AIMD converges to fairness and efficiency when all flows in the network get the same feedback simultaneously.

The generalized WAIMD (GWAIMD) extends the WAIMD with the following increase and decrease policies:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + \alpha w_s ; \alpha > 0, \quad (4.15)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \beta x_s(k) ; 0 < \beta < 1. \quad (4.16)$$

The weighted BCC (WBCC) further extends the GWAIMD with the following I and D policies:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + \frac{\alpha w_s^{k+1}}{x_s^k(k)} ; \alpha > 0, \quad (4.17)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \beta w_s^{1-l} x_s^l(k); 0 < \beta < 1. \quad (4.18)$$

SQRT ($k = l = 0.5, w_s = 1$) and IIAD ($k = 1, l = 0, w_s = 1$) are two special cases of WBCC [50].

4.4 TCP-Friendliness and Multimedia-Friendliness

4.4.1 TCP-Friendliness and Multimedia-Friendliness: a Twin-Pair

In this section, GAIMD and GWAIMD are taken as an example to show the relationship between TCP-friendliness (compatibility) and multimedia-friendliness (compatibility).

For GWAIMD, $f(x) = \alpha$ and $g(x) = \beta x$. Clearly, $f'(x) = 0$. It can be shown from Equation (4.12) that when α and β satisfy Equation (2.7), GWAIMD is TCP-compatible with respect to w_s .

Multimedia applications are usually very sensitive to the abrupt changes of the sending rates. To avoid this, the value of β should be very small. For example, it can be $1/8$ and the scheme is [49]

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + 1, \quad (4.19)$$

$$\text{D: } x_s(k+1) \leftarrow x_s(k) - \frac{x_s(k)}{8}. \quad (4.20)$$

Note that

$$\alpha = 1 > \frac{3\beta}{2-\beta} = \frac{3 \times 1/8}{2-1/8} = \frac{1}{5}. \quad (4.21)$$

Clearly, it is not TCP-compatible [47] but it is TCP-compatible with respect to

5. This is as shown below.

It can be shown from Equation (4.21) that the following scheme:

$$\text{I: } \tilde{x}_s(k+1) \leftarrow \tilde{x}_s(k) + \frac{1}{5}, \quad (4.22)$$

$$\text{D: } \tilde{x}_s(k+1) \leftarrow \tilde{x}_s(k) - \frac{\tilde{x}_s(k)}{8} \quad (4.23)$$

is TCP-compatible [47].

Let $\tilde{x}_s(k) = \frac{x_s(k)}{5}$. Equations (4.19) and (4.20) can be derived from Equations (4.1), (4.2), (4.22) and (4.23). In other words, the scheme that is described by Equations (4.19) and (4.20) is TCP-compatible with respect to 5.

Therefore, multimedia-friendliness (compatibility) is actually TCP-friendliness (compatibility) with respect to a weighting factor, so multimedia-friendliness (compatibility) and TCP-friendliness (compatibility) are actually a twin-pair. The extension of TCP-friendliness (compatibility) is very useful for multimedia applications over the Internet.

4.4.2 Source Adaptation for Multimedia Applications

As shown in the previous subsection, w_s decides the throughput that an MRF traffic gets over a TCP flow. α and β are the adjusting steps of the MRF, thus they are related to the smoothness of the traffic. To be TCP friendly, α and β must satisfy Equation (2.7). By this relationship, we can easily design a source adaptation scheme to support a multimedia application which has requirements on sending rates. The values of α and β can be predefined according to Equation (2.7) and the smoothness requirement of multimedia applications. Since it is impossible to know the number of active sources in advance, the value of w_s cannot be predefined, and a method is provided to determine it in this subsection.

Suppose that there are an upper bound x_s^+ and a lower bound x_s^- for the sending rate of a multimedia application (also source) s . Let x_l be the link capacity of the bottleneck link. Same as [1], we require the following assumption.

Assumption 4.4.1. *The lower bounds x_s^- of all multimedia applications satisfy*

$$\sum_{\text{all SA sources on bottleneck link}} x_s^- < x_l. \quad (4.24)$$

For the MRF source s with a fixed weighting factor w_s , the sending rate converges to a throughput which is given as

$$r_s = \frac{x_l w_s}{\sum_{\text{all competitive source } i} w_i}. \quad (4.25)$$

Since it is impossible for a source to know the weighting factors of other sources in advance and the network is usually time varying, it is very difficult for a source to properly choose its weighting factor such that the actual throughput is within a predefined interval. To address this issue, a simple source adaptation (SA) scheme is required.

Figure 4.1: The structure of the source adaptation scheme.

Figure 4.1 illustrates the overall structure of the scheme. There is an SA model and an MRF at the source side. The MRF works as follows: it sends out packets, receives feedback information from the corresponding destination and adjusts its window size accordingly.

The SA model keeps on monitoring the sending rate of the MRF algorithm during each time period T_s and compares it with the lower bound x_s^- and the upper bound x_s^+ . If the sending rate is within the interval, the weighting factor is not adjusted. Otherwise, the SA model adjusts the weighting factor of the MRF accordingly.

Specifically, the SA model records the bytes of data sent out during the period and then computes an average value by the following sliding window method:

$$\bar{r}_s(k+1) = r_s(k) \times a_s + \bar{r}_s(k) \times (1 - a_s), \quad (4.26)$$

where r_s is the sending rate of source s , $\bar{r}_s$ is the average sending rate and $a_s \leq 1$ is a positive factor.

$\bar{r}_s$ is then compared with the bounds. If $\bar{r}_s > x_s^+$ or $\bar{r}_s < x_s^-$, then the weight w_s is adjusted by:

$$w_s(k+1) = w_s(k) + \gamma_s \left(\frac{\frac{x_s^+ + x_s^-}{2} - \bar{r}_s(k+1)}{\frac{x_s^+ + x_s^-}{2}} \right) \quad (4.27)$$

$$= w_s(k) + \gamma_s \left(1 - \frac{2\bar{r}_s(k+1)}{x_s^+ + x_s^-} \right), \quad (4.28)$$

where γ_s is a positive factor. Otherwise, w_s does not change. In the next section, the proposed scheme will be verified by experimental results.

It should be mentioned here that the service we provide is not a strict one. Although the user defines an upper bound and a lower bound, there is still a chance that the service can go beyond the bounds. However, the possibility is very small and there is usually a buffer associated with a multimedia application to smoothen out the effect of this event with small possibility. Moreover, compared to other structures which provide different bandwidth sharing services, the proposed scheme is much simpler and does not require any change to the routers. Also,

the proposed scheme can work harmoniously with networks that are dominated by TCP traffic, such as the current Internet.

4.5 Simulation Results

Figure 4.2: The topology of simulation.

We have simulated the proposed scheme in Ns-2 simulator [95]. A new source adaptation agent is employed instead of using the conventional TCP congestion avoidance mechanism to reduce the disturbance from other mechanisms such as slow-start or time-out etc. [44, 47]. Most of the experiments use the standard “dumbbell” topology as illustrated in Figure 4.2. It has a single bottleneck link with a default bandwidth of 16MB and a delay of 30ms. Each source s_i sends traffic to the corresponding destination d_i . The algorithm of RED is deployed at routers r_1 and r_2 to drop packets in case of congestion [96]. Several TCP-Reno flows (with different RTT) were introduced as the background traffic. The packet size is fixed at 500 bytes and all sources start at random times. We use GWAIMD as the example to illustrate our results.

4.5.1 Fairness

We first consider the fairness of two competing WAIMD sources. Two groups of w_1 and w_2 are tested.

Group 1: $w_1 = 1$, i.e. it is just a normal AIMD source, and $w_2 = 4$. The corresponding fairness index is computed as

$$F(k) = \frac{(x_1(k) - \frac{x_2(k)}{4})^2}{2}.$$

Group 2: $w_1 = w_2 = 1$, i.e. both of them are normal AIMD sources. The corresponding fairness index is computed as

$$\tilde{F}(k) = \frac{(x_1(k) - x_2(k))^2}{2}.$$

The dynamics of $F(k)$ and $\tilde{F}(k)$ are shown in Figure 4.3. Clearly, the weighted fairness is achieved by using the WAIMD and the dynamic of the fairness index of the weighted case is as good as, if not better than, the equal weight case.

Figure 4.3: Weighted fairness indices of two competing WAIMD flows.

4.5.2 Weighted Throughput

In this subsection, we run simulations with different weighting factors. There are two sources and the weighting factors are

$$w_1 = 1 ; w_2 \in [1.5, 30].$$

In other words, one is the normal AIMD flow and the other works with a weighting factor in the range $[1.5, 30]$. The simulation runs for 200s each time. The monitored traffic and background traffic start at random times during the first 10 sec. The throughput is averaged from 20 sec to 180 sec for each traffic when the simulation runs in steady-state. Figure 4.4 summarizes the throughput ratio. It can be seen that the throughput of each traffic is proportional to the weighting factor.

Figure 4.4: The throughput ratio with respect to $\frac{w_2}{w_1}$.

4.5.3 Smoothness

In this section, we test the smoothness of traffic under the control of WAIMD and GWAIMD. Three experiments are carried out as described below.

- (i). Both sources are under the control of WAIMD and their weighting factors are

$$w_1 = 5 ; w_2 = 1.$$

In other words, a WAIMD flow with a weighting factor of 5 is competing with an AIMD flow. It can be seen in Figure 4.5(a) that the fluctuation in the window size of the WAIMD flow is much larger. Therefore, there is a cost of larger fluctuation when the WAIMD is used to provide bandwidth differentiation service. This should be avoided in certain situations, and multimedia application is such an example.

- (ii). The scenario is similar to the above except that the GWAIMD in Equations (4.19) and (4.20) replaces the WAIMD with weighting factor 5. The windows size is shown in Figure 4.5(b). Clearly, bandwidth differentiation service can also be provided by GWAIMD, and the fluctuation of GWAIMD is significantly smaller than WAIMD in the above scenario. This is very attractive for both TCP and UDP applications.

- (iii). In this scenario, we show the competition of three traffic flows: an AIMD, a GAIMD by Equations (4.22) and (4.23) and a GWAIMD by Equations (4.19) and (4.20). The following conclusions can be drawn from Figure 4.5(c).

- The GAIMD by Equations (4.22) and (4.23) is compatible to AIMD.
- The sending rate is adjusted more smoothly by GWAIMD and GWAIMD is multimedia friendly. Meanwhile, it is also TCP-friendly with respect to 5. Thus, multimedia friendliness is actually TCP-friendliness with respect to certain weighting factor.

4.5 Simulation Results

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Figure 4.5: The comparison of smoothness between WAIMD and GWAIMD.

4.5.4 Source Adaptation for Multimedia Applications

In this subsection, we will verify the source adaptation scheme for multimedia applications. The bottleneck link has a capacity of 1Mbps and employs RED to provide random dropping. There are 10 applications sending data from s_i to d_i where $1 \leq i \leq 10$: 9 of them are normal FTP which generate TCP-Reno traffic and the other is the proposed source adaptation scheme. The Round Trip Time (RTT) is 0.01 sec. The proposed source adaptation scheme employed the following GWAIMD scheme:

$$\text{I: } x_s(k+1) \leftarrow x_s(k) + 0.007519w_s(k); \quad (4.29)$$

$$D: \quad x_s(k+1) \leftarrow x_s(k) - 0.005x_s(k). \quad (4.30)$$

The adjust time period, i.e. T_s is set at 5 sec. The upper bound, i.e. x_s^+ is 250Kbps and the lower bound, x_s^- is 200Kbps. The adjusting step, γ_s , is selected as 0.1 and the value of a_s is set at 0.2. The simulation is run for 1200s.

4.5.4.1 The Traffic Control

We first tested the scenario that all the sources start at random time in the first 20 sec. Figures 4.6(a) and 4.6(b) show the traffic rate and the weighting factor of the source adaptation scheme under the typical setting. It can be shown that the scheme can adjust the sending rate to the desired interval. Furthermore, compared to TCP traffic, our scheme yields a much smoother traffic.

Figure 4.6: The traffic rate and corresponding weight of the source adaption scheme.

Then the scenario that there is an abrupt change in the network load is tested. We examined the scenario where only 5 FTP applications are opened in the beginning and the other 4 FTP applications are started at around the time of 400

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sec. It can be illustrated from Figure 4.7(a) that the sending rate of the source adaptation application decreases at about 400 sec. The weighting factor is thus increased by our scheme to yield the desired throughput.

Figure 4.7: The traffic rate and corresponding weight of the source adaption scheme in the case of an abrupt change.

We also tested the scenario that there are two SA sources. The settings are the same except that one of them now has the upper and lower bounds as 350Kbps and 300Kbps, respectively. Figures 4.8(a) and 4.8(b) show the traffic rates and the weighting factors. Again, the scheme adjusts the traffic rates to the desired intervals.

4.5.4.2 Effect of Different Parameter Choices

In this section, we will examine the effect of some parameters on the source adaptation.

We first tested a different choice of β . Experimental results are shown in Figures 4.9(a) and 4.9(b) when Equations (4.29) and (4.30) are replaced by

$$I: \quad x_s(k+1) \leftarrow x_s(k) + 0.076923w_s(k); \quad (4.31)$$

Figure 4.8: The traffic rate and corresponding weight in the case of two source adaption schemes.

$$D: \quad x_s(k+1) \leftarrow x_s(k) - 0.05x_s(k) . \quad (4.32)$$

Comparing Figures 4.6(a) and 4.9(a), it is clear that the fluctuation of the sending rate increases with a large value of β .

Figure 4.9: The traffic rate and corresponding weight of the source adaption schemes with different underlying algorithm.

We also tested the performance with a different choice of γ_s . We consider the case that γ_s is set at 0.5 and GWAIMD is given by Equations (4.29) and (4.30).

Experimental results are illustrated in Figures 4.10(a) and 4.10(b). Obviously, with a bigger γ_s , both the rate of change and the oscillation of the weighting factor increase.

Figure 4.10: The traffic rate and corresponding weight of the source adaption schemes with a bigger γ_s .

4.6 Conclusion

A simple MRF framework has been proposed in this chapter for the analysis and design of memoryless window-based congestion control protocols. The MRF can be used to provide weighted fair bandwidth allocation without any change to the router of the existing Internet. The framework can be used to design both TCP-friendly and multimedia-friendly schemes or provide bandwidth differentiation service by source adaptation. A necessary and sufficient condition is derived for stepwise convergence of the MRF to the fairness. Many existing protocols, like AIMD, GAIMD, BCC, can be constructed by using the MRF. The requirements of the proposed increase-decrease policies are less conservative than that of the CYRF. With so many types of applications in the Internet, the simple framework

proposed in this chapter should be useful for the evolution of the Internet.

Chapter 5

Analysis of a General Class of Responsive Functions

5.1 Introduction

In the previous chapter, the MRF defined in Equations (4.1) and (4.2) requires that $f(x)$ is a monotonically non-increasing function and $g(x)$ a monotonically non-decreasing function with at least one of them strictly monotonic. In this chapter, we shall study more general cases of the responsive functions.

As in the previous chapter, we still require that $f(x) > 0$, $x \geq g(x) > 0$. This is to guarantee that the I policy always increases the window size and the D policy decreases the window size while bounded away from 0 (i.e. there is no negative window size after a D policy). Another condition is 1-responsiveness, i.e. $g(x) > f(x)$, thus each D policy will cover the effect of the I policy.

Instead of the specified decrease / increase functions, we consider the general case where $f(x)$ and $g(x)$ are only required to be monotonic functions, and at least one of $f(x)$ and $g(x)$ is strictly monotonic. All the responsive functions are classified into 4 types according to the monotony of their responsive functions. Our

main theoretical result is that the type-1 scheme, which is the MRF proposed in Chapter 4, i.e. $f(x)$ is a monotonic non-increasing function and $g(x)$ a monotonic non-decreasing function, converges to fairness and efficiency. We shall also study the convergent conditions of types-2, 3 and 4 schemes. We shall show that our results include the CYRF and extend the 1-CYRF schemes. And similar to [51], Theorems 4.3.3 and 4.3.4 are used to make 1-responsive functions TCP-friendly and TCP-compatible (in this case, with respect to weight 1). The proposed scheme is also used to design multimedia-friendly congestion control protocols, which can be helpful when extending the network service for different applications.

5.2 Four Types of Responsive Functions

We follow the network model and conditions such as 1-responsiveness and smoothness as defined in Chapter 4.

We first classify the responsive functions, i.e. $f(x)$ and $g(x)$, according to their monotony under the assumptions that $f(x)$ and $g(x)$ are monotonic functions. The monotony of I/D is divided into the following groups:

- *Group I_1* : $f(x) > 0$ and it is a non-increasing function.
- *Group I_2* : $f(x) > 0$ and it is an increasing function.
- *Group D_1* : $x \geq g(x) > 0$ and it is a non-decreasing function.
- *Group D_2* : $x \geq g(x) > 0$ and it is a decreasing function.

The schemes are then classified into 4 types according to the groups that I and D belong to. These are defined as in Table 5.1.

5.2 Four Types of Responsive Functions

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Table 5.1: The classification of responsive functions.

$D \downarrow \backslash I \rightarrow$	$Group I_1$	$Group I_2$
$Group D_1$	type-1 (MRF)	type-3
$Group D_2$	type-2	type-4

Remark 5.2.1. For simplicity of classification of type-1 functions in Table 5.1, we have included cases where both $f(x)$ and $g(x)$ are not strictly monotonic. However, in subsequent analysis, it is required at least one of $f(x)$ and $g(x)$ is strictly monotonic. Thus, the actual type-1 functions must satisfy the additional condition that at least one of $f(x)$ and $g(x)$ is strictly monotonic.

Figure 5.1: The groups of I/D policy.

We now give an explanation of the proposed classification in the case that two sources have the same weight. On the Chiu's phase plot [42, 44], the equally fair line in the case of two users is a line which is parallel to the line $x_1 = x_2$, where x_i stands for the window size of source i . Assume that the windows of the two sources are (x_1, x_2) with $x_2 > x_1$, and an I policy is carried out. The I policy is represented as the adjustment vector $(f(x_1), f(x_2))$. Intuitively, if $f(x_2)/f(x_1) \leq 1$ like the vector shown in Figure 5.1, after the application of I policy, the system will be closer to the desired fairness. Otherwise, the system will step away from

the desired fairness (in comparison with the fairness of the current point). Similar argument can be used to describe the D policy. Hence, the monotony property is actually determined by the direction where the adjustment vector points to. When the sources have different weights, similar discussions can be derived with respect to x_s/w_s .

Type-1 responsive functions are convergent since each of the policies will contribute to fairness (thus it is a *step-wise convergent* function). Type-4 responsive functions are not convergent since none of the policies will contribute to improved fairness. These will be addressed in Section 5.3. The convergence of types-2 and 3 policies will depend on the given policies since one of the policies contributes to fairness while the other does not. If such a responsive function converges eventually, it is called an *epoch-wise convergent* function. We shall give a detail proof to the hypothesis and derive the conditions for convergence of types-2 and 3 functions in Section 5.4.

5.3 Convergence of Type-1 and Type-4 Responsive Functions

5.3.1 Convergent Result of Type-1 Responsive Function

A type-1 responsive function is MRF which is defined in Chapter 4. Thus, the convergence is derived directly from Theorem 4.3.2.

Clearly, AIMD, GAIMD, BBC, LOG and SIGMOID are special cases of type-1 responsive functions. The corresponding $f(x)$ and $g(x)$ are

$$\text{AIMD: } f(x) = 1 ; g(x) = \frac{x}{2}, \quad (5.1)$$

5.3 Convergence of Type-1 and Type-4 Responsive Functions

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$$\text{GAIMD: } f(x) = \alpha ; g(x) = \beta x ; \alpha > 0; \beta > 0, \quad (5.2)$$

$$\text{BCC: } f(x) = \frac{\alpha}{x^k} ; g(x) = \beta x^l ; k \geq 0; l \geq 0, \quad (5.3)$$

$$\text{LOG: } f(x) = \frac{\log(x)}{x} ; g(x) = \frac{\log(x)}{2}, \quad (5.4)$$

$$\text{SIGMOID: } f(x) = 1 ;$$

$$g(x) = \frac{\gamma x}{(1 + e^{-(\alpha x - \beta)})}. \quad (5.5)$$

Note that $\tilde{f}(x)$ and $\tilde{g}(x)$ of AIMD, GAIMD, BBC, LOG and SIGMOID in the CYRF framework of (2.12) and (2.13) are

$$\text{AIMD: } \tilde{f}(x) = x ; \tilde{g}(x) = \frac{1}{2}, \quad (5.6)$$

$$\text{GAIMD: } \tilde{f}(x) = \frac{x}{\alpha} ; \tilde{g}(x) = \beta ; \alpha > 0; \beta > 0, \quad (5.7)$$

$$\text{BCC: } \tilde{f}(x) = \frac{x^{k+1}}{\alpha} ; \tilde{g}(x) = \beta x^{l-1} ; k \geq 0; l \geq 0, \quad (5.8)$$

$$\text{LOG: } \tilde{f}(x) = \frac{x^2}{\log(x)} ; \tilde{g}(x) = \frac{\log(x)}{2x}, \quad (5.9)$$

$$\text{SIGMOID: } \tilde{f}(x) = x ;$$

$$\tilde{g}(x) = \frac{\gamma}{(1 + e^{-(\alpha x - \beta)})}. \quad (5.10)$$

Clearly, $\tilde{g}(x)$ of BCC is not a monotonic non-decreasing function when $0 < l < 1$ and $\tilde{g}(x)$ of LOG is not a monotonic non-decreasing function. Thus, BCC with $0 < l < 1$ and LOG are not CYRF. This implies that the requirements of the MRF are less conservative than those of CYRF [51].

We shall now derive condition for CYRF to be type-1 responsive functions and the result is as given below.

Theorem 5.3.1. *Suppose that $\tilde{f}(x) > 0$, $1 \geq \tilde{g}(x) > 0$ for all $x \geq 1$, $\tilde{f}(x) \leq \tilde{f}'(x)x$, and $\tilde{g}(x) + x\tilde{g}'(x) \geq 0$. The CYRF framework of (2.12) and (2.13) is a type-1 responsive functions.* $\square$

Proof. Note that

$$\begin{aligned} f(x) &= \frac{x}{\tilde{f}(x)}, \\ g(x) &= x\tilde{g}(x). \end{aligned}$$

It can be easily shown that

$$\begin{aligned} f'(x) &\leq 0, \\ g'(x) &\geq 0, \\ x \geq g(x) &\geq 0, \\ f(x) &> 0. \end{aligned}$$

Thus, the framework of (2.12) and (2.13) is a type-1 responsive functions. $\square$

5.3.2 Convergence of Type-4 Responsive Function

It is easy to see that the policies satisfy the requirement of Theorem 3.4.3, and each application of I and D decreases the fairness. Hence type-4 responsive function is not convergent.

5.4 Convergence of Type-2 and Type-3 Responsive Functions

We now consider the convergence of types-2 and 3 responsive functions. Note that it is difficult to analyze such schemes since one of the policies will reduce fairness. Thus, we shall assume the 1-responsive property. Under the assumption, the policies will be activated with an order $DII \cdots DII \cdots$. We treat the interval between

5.4 Convergence of Type-2 and Type-3 Responsive Functions

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two D 's as an *epoch*. Thus if at the end of each epoch, the fairness increases in comparison with that at the beginning of the epoch, the system will approach fairness eventually. We shall study the procedure and derive the convergent conditions of types-2 and 3 responsive functions.

For convergence of type-2 and type-3 responsive functions, it is further required that both the $f(x)$ and $g(x)$ are strictly monotonic. Otherwise, they are not convergent.

In this section, we shall analyze a procedure which contains two sources. Suppose that two sources start from an arbitrary position $[x_2(0), x_1(0)]$, $(x_2(0)/w_2 > x_1(0)/w_1)$. If after a series of D/I policies, the fairness index approaches 0, i.e. the window sizes of the two sources approach the same value, then the corresponding responsive function is convergent. Our conclusion can be extended to the case of S sources case.

5.4.1 f-e Transformation

We shall consider the convergence analysis via the following transformation:

$$\mathbf{f} = \frac{x_1}{w_1} - \frac{x_2}{w_2}, \quad (5.11)$$

$$\mathbf{e} = x_1 + x_2. \quad (5.12)$$

The two transformations are linearly independent and thus the system's adjustment policies can be written as:

$$[\mathbf{f}(k+1), \mathbf{e}(k+1)] = [\mathbf{f}(k), \mathbf{e}(k)] + [\Delta_f^f(k), \Delta_f^e(k)], \quad (5.13)$$

$$[\mathbf{f}(k+1), \mathbf{e}(k+1)] = [\mathbf{f}(k), \mathbf{e}(k)] + [\Delta_g^f(k), \Delta_g^e(k)] \quad (5.14)$$

where $[\Delta_f^f(k), \Delta_f^e(k)]$ and $[\Delta_g^f(k), \Delta_g^e(k)]$ are given as

$$[\Delta_f^f(k), \Delta_f^e(k)] = \left[f\left(\frac{x_1(k)}{w_1}\right) - f\left(\frac{x_2(k)}{w_2}\right), \right. \\ \left. w_1 f\left(\frac{x_1(k)}{w_1}\right) + w_2 f\left(\frac{x_2(k)}{w_2}\right) \right], \quad (5.15)$$

$$[\Delta_g^f(k), \Delta_g^e(k)] = \left[-g\left(\frac{x_1(k)}{w_1}\right) - g\left(\frac{x_2(k)}{w_2}\right), \right. \\ \left. w_1 g\left(\frac{x_1(k)}{w_1}\right) + w_2 g\left(\frac{x_2(k)}{w_2}\right) \right]. \quad (5.16)$$

The following propositions can be derived easily.

Proposition 5.4.1. *The system approaches fairness when $\mathbf{f}$ approaches 0.*

Proposition 5.4.2. *The D policy is activated when $\mathbf{e}$ is bigger than a constant.*

Recall that the window increase or decrease policy is smooth, i.e. $|\Delta x| \ll x$. Suppose that the increase policy of a smooth protocol is successively applied two times: $x(k+1) = x(k) + \Delta x(k)$, $x(k+2) = x(k+1) + \Delta x(k+1)$. Since $|\Delta x(k)| \ll x(k)$ and $|\Delta x(k+1)| \ll x(k+1)$, we can write $x(k+2) \approx x(k) + 2\Delta x(k)$ or in general

$$x(k+m) \approx x(k) + m\Delta(x(k)),$$

i.e.

$$\Delta(x(k+m)) \approx m\Delta(x(k))$$

after m successive applications of I . Specifically, in the $\mathbf{f}$ - $\mathbf{e}$ system, we have the following lemma.

5.4 Convergence of Type-2 and Type-3 Responsive Functions

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Lemma 5.4.1. *After m successive applications of smooth I , we have*

$$\begin{aligned}\Delta_f^f(k+m) &\approx m\Delta_f^f(k), \\ \Delta_f^e(k+m) &\approx m\Delta_f^e(k).\end{aligned}$$

□

5.4.2 Convergent Conditions for Type-2 and Type-3 Responsive Functions

We shall first derive the convergent condition for type-2 responsive functions. Because of the 1-responsiveness property, the procedure of an epoch should be $DI \cdots ID$. We then have

$$[\mathbf{f}(k+1), \mathbf{e}(k+1)] = [\mathbf{f}(k), \mathbf{e}(k)] + [\Delta_g^f(k), \Delta_g^e(k)], \quad (5.17)$$

and according to Lemma 5.4.1, we have

$$\begin{aligned}[\mathbf{f}(k+2), \mathbf{e}(k+2)] &= [\mathbf{f}(k+1), \mathbf{e}(k+1)] + [\Delta_f^f(k+1), \Delta_f^e(k+1)], \\ &\approx [\mathbf{f}(k+1), \mathbf{e}(k+1)] + [\Delta_f^f(k), \Delta_f^e(k)].\end{aligned}$$

Specifically, after m successive applications of I policy, we have

$$\begin{aligned}[\mathbf{f}(k+m), \mathbf{e}(k+m)] &\approx [\mathbf{f}(k+1), \mathbf{e}(k+1)] + [m\Delta_f^f(k), m\Delta_f^e(k)].\end{aligned}$$

Since in the current epoch, D is the first and the last action, according to

Proposition 5.4.2 and smoothness of the responsive functions, we have

$$m\Delta_f^e(k) \approx -\Delta_g^e(k).$$

Thus

$$m = \frac{-\Delta_g^e(k)}{\Delta_f^e(k)}.$$

To guarantee that the system approaches fairness after each epoch, it requires

$$m\Delta_f^f(k) > -\Delta_g^f(k).$$

Thus we have the following theorem:

Theorem 5.4.1. *For a type-2 responsive function, if it satisfies*

$$\frac{-\Delta_g^e(k)}{\Delta_f^e(k)} > \frac{-\Delta_g^f(k)}{\Delta_f^f(k)}, \quad (5.18)$$

where Δ is as defined in Equations (5.15) and (5.16), then the responsive function approaches fairness by epoch. $\square$

By a similar procedure we have the following convergent condition for type-3 responsive functions:

Theorem 5.4.2. *For a type-3 responsive function, if it satisfies*

$$\frac{-\Delta_g^e(k)}{\Delta_f^e(k)} < \frac{-\Delta_g^f(k)}{\Delta_f^f(k)},$$

where Δ is as defined in Equations (5.15) and (5.16), then the responsive function approaches fairness by epoch. $\square$

5.5 Simulation Results

It should be noted that the convergence of type-2 and type-3 responsive functions is related with the actual w_s . In this section, we will study the schemes with $w_s = 1$. The normal schemes such as AIMD, GAIMD, BCC (IIAD, SQRT) and LOG are type-1 and hence their convergent results are well understood. In this section, we will study the responsive function given by

$$f(x) = \frac{\alpha}{x^k \log(x)}, \quad (5.19)$$

$$g(x) = \frac{\beta x^l}{\log(x)}. \quad (5.20)$$

We name our scheme R-LOG (for Reverse-LOG). Two cases are studied with (k, l) set as $(0, 1)$ and $(1, 0)$ as the examples of type-1 and type-2 responsive functions, respectively.

We implement the proposed scheme in ns-2 simulator [95] by replacing the responsive function of TCP-Reno, thus it inherits other mechanisms such as slow-start and timeout. In our experiments, the standard “dumbbell” topology (see Figure 4.2) is used: a single bottleneck link with RED queue. Several TCP-reno flows run as the background traffic. All the flows start at random time.

5.5.1 An Example of Type-1 Responsive Functions

The specific R-LOG(0, 1) responsive function is

$$f(x) = \frac{\alpha}{\log(x)}, \quad (5.21)$$

$$g(x) = \frac{\beta x}{\log(x)}. \quad (5.22)$$

R-LOG $(0, 1)$ belongs to type-1, hence according to Theorem 4.3.2 it is convergent. Figure 5.2 shows the window sizes of two R-LOG $(0, 1)$ flows running over a bottleneck link. We can see that the two flows share the link equally and thus it is convergent.

Figure 5.2: The convergence of two R-LOG $(0, 1)$.

We also study the features of R-LOG $(0, 1)$ when competing with TCP-Reno traffic. Figure 5.3 shows the window sizes of the competing TCP-Reno and the proposed scheme, it is obvious that the proposed scheme has a much smoother window size variation than the TCP-Reno scheme. This is an important advantage since many applications, such as multimedia, prefer smooth traffic.

5.5.1.1 TCP-Friendliness

TCP-friendliness requires that the throughput of a scheme is inversely proportional to the square root of the dropping rate. According to the condition for a 1-responsiveness scheme in Theorem 4.3.3 with $w_s = 1$, we have

$$\dot{f}(x) = -\frac{\alpha}{x \log^2(x)}.$$

Figure 5.3: The window sizes of TCP-reno and R-LOG (0, 1).

Hence

$$\begin{aligned} \frac{(1 + \dot{f}(x))g(x)}{f(x)} &= \frac{\left(1 + \frac{\alpha}{x \log^2(x)}\right) \frac{\beta x}{\log(x)}}{\frac{\alpha}{\log(x)}} \\ &\approx \frac{\beta}{\alpha} x \\ &\propto x. \end{aligned}$$

Thus, R-LOG(0, 1) is TCP-friendly. This can be seen in Figure 5.4, which shows the dropping rate and the throughput of a TCP-reno and the proposed scheme under the same environment.

5.5.1.2 TCP-Compatibility

In this subsection, we will illustrate the choices of α and β to get TCP-compatibility. From Equations (4.12), (5.21) and (5.22),

$$\frac{\alpha}{\log(x)} = \frac{3(1 + \dot{f}(x))g(x)}{2x - (1 + \dot{f}(x))^m g(x)}$$

Figure 5.4: The throughput of R-LOG (0,1) and TCP-reno with respect to loss rate.

$$\begin{aligned}
 &= \frac{3(1 + \frac{\alpha}{x \log^2(x)}) \frac{\beta x}{\log(x)}}{2x - (1 + \frac{\alpha}{x \log^2(x)})^m \frac{\beta x}{\log(x)}} \\
 \frac{\alpha}{\beta} &= \frac{3}{2} \frac{1}{\frac{1}{1 + \frac{\alpha}{x \log^2(x)}} - \frac{\beta(1 + \frac{\alpha}{x \log^2(x)})^{m-1}}{2 \log(x)}}
 \end{aligned}$$

i.e. when $\alpha/\beta \approx 3/2$, the proposed scheme is TCP-compatible.

Figure 5.5 shows the *normalized throughput differentiation* with different α/β ratio. The normalized throughput differentiation is defined as

$$d = \left(\frac{2(T_1 - T_2)}{T_1 + T_2} \right)^2,$$

where T_1 and T_2 are the throughput of the 2 flows. We can see that the difference of the two flows is the smallest at about $\alpha/\beta = 3/2$. This verifies the proposed theoretical results.

Figure 5.5: The throughput differentiation with respect to α - β ratio.

5.5.2 An Example of Type-2 Responsive Functions

The specific R-LOG (1, 0) responsive function is

$$f(x) = \frac{\alpha}{x \log(x)}, \quad (5.23)$$

$$g(x) = \frac{\beta}{\log(x)}. \quad (5.24)$$

It is easy to show that R-LOG (1, 0) is a type-2 responsive function, i.e. its D policy is a decreasing function instead of an increasing function. And it is 1-responsive.

To prove its convergence, we have

$$\begin{aligned} [\Delta_f^f(k), \Delta_f^e(k)] &= [\alpha(\frac{1}{x_1 \log(x_1)} - \frac{1}{x_2 \log(x_2)}), \\ &\quad \alpha(\frac{1}{x_1 \log(x_1)} + \frac{1}{x_2 \log(x_2)})], \\ [\Delta_g^f(k), \Delta_g^e(k)] &= [-\beta(\frac{1}{\log(x_1)} - \frac{1}{\log(x_2)}), \\ &\quad -\beta(\frac{1}{\log(x_1)} + \frac{1}{\log(x_2)})]. \end{aligned}$$

We use m and m' to denote the two sides of Equation 5.18 and they are computed as

$$\begin{aligned} m &= \frac{-\Delta_g^e(k)}{\Delta_f^e(k)} \\ &= \frac{\log(x_1 x_2)}{\log(x_1) \log(x_2)} / \frac{\log(x_1^{x_1} x_2^{x_2})}{x_1 x_2 \log(x_1) \log(x_2)} \\ &= \frac{x_1 x_2 \log(x_1 x_2)}{\log(x_1^{x_1} x_2^{x_2})} \end{aligned}$$

and

$$\begin{aligned} m' &= \frac{-\Delta_g^f(k)}{\Delta_f^f(k)} \\ &= \frac{\log(x_2/x_1)}{\log(x_1) \log(x_2)} / \frac{\log(x_2^{x_2}/x_1^{x_1})}{x_1 x_2 \log(x_1) \log(x_2)} \\ &= \frac{x_1 x_2 \log(x_2/x_1)}{\log(x_2^{x_2}/x_1^{x_1})} \end{aligned}$$

Hence,

$$\begin{aligned} m - m' &= \frac{x_1 x_2}{\log(x_1^{x_1} x_2^{x_2}) \log(x_2^{x_2}/x_1^{x_1})} \\ &\quad \times (\log(x_1 x_2) \log(x_2^{x_2}/x_1^{x_1}) - \log(x_2/x_1) \log(x_1^{x_1} x_2^{x_2})) \\ &= \frac{x_1 x_2}{\log(x_1^{x_1} x_2^{x_2}) \log(x_2^{x_2}/x_1^{x_1})} (2 \log(x_1) \log(x_2)) (x_2 - x_1) \\ &> 0. \end{aligned}$$

Thus the condition of Theorem 5.4.1 is satisfied, and so R-Log $(1, 0)$ is convergent. The experimental result of its convergence is illustrated in Figure 5.6. And similar to the case of R-Log $(0, 1)$, we show the comparison of R-Log $(1, 0)$ with TCP-Reno in Figure 5.7.

Figure 5.6: The convergence of two R-LOG (1, 0).

5.5.2.1 TCP-Friendliness

Similar to the above experiment, for TCP-friendliness, we have

$$f'(x) = -\frac{\alpha(1 + \log(x))}{x^2 \log^2(x)}.$$

Hence

$$\begin{aligned} \frac{(1 + f'(x))g(x)}{f(x)} &= \frac{(1 - \frac{\alpha(1 + \log(x))}{x^2 \log^2(x)}) \frac{\beta}{\log(x)}}{\frac{\alpha}{x \log(x)}} \\ &\approx \frac{\beta}{\alpha} x \\ &\propto x. \end{aligned}$$

Thus, R-LOG(1, 0) is TCP-friendly. This can be seen in Figure 5.8, which shows the dropping rate and the throughput of a TCP-reno and the proposed scheme under the same environment.

Figure 5.7: The window sizes of TCP-reno and R-LOG (1, 0).

5.5.2.2 TCP-Compatibility

To be TCP-compatible, from Equations (4.12), (5.23) and (5.24), we have

$$\begin{aligned} \frac{\alpha}{x \log(x)} &= \frac{3(1 + f'(x))g(x)}{2x - (1 + f'(x))^k g(x)} \\ &= \frac{3(1 - \frac{\alpha(1+\log(x))}{x^2 \log^2(x)}) \frac{\beta}{\log(x)}}{2x - (1 - \frac{\alpha(1+\log(x))}{x^2 \log^2(x)})^k \frac{\beta}{\log(x)}} \end{aligned}$$

$$\frac{\alpha}{\beta} = \frac{3}{2} \frac{1}{\frac{1}{(1 - \frac{\alpha(1+\log(x))}{x^2 \log^2(x)})} - \frac{\beta(1 - \frac{\alpha(1+\log(x))}{x^2 \log^2(x)})^{k-1}}{2x \log(x)}}$$

i.e. when $\alpha/\beta \approx 3/2$, the proposed scheme is TCP-compatible.

The normalized throughput differentiation with respect to different α/β ratio is illustrated Figure 5.9. Notice that the least different point is a little larger than that in Figure 5.5, this can be derived by differentiating the left-hand side of α/β .

Figure 5.8: The throughput of R-LOG (1,0) and TCP-reno with respect to loss rate.

5.6 Conclusion

In this chapter, the convergence of a general class of monotonic responsive functions for congestion control has been analyzed. We have classified all the responsive functions into 4 types according to the monotony of their responsive functions. The condition for type-1 responsive functions is in fact the MRF considered in Chapter 4 and it can be readily used to construct the desirable increase-decrease policies. Many existing protocols, like AIMD, GAIMD and BCC can be constructed by using our framework. We have also derived the convergent conditions for types-2 and 3 responsive functions. The proposed framework also includes a simple method to provide TCP-friendliness and TCP-compatibility for these types of responsive functions. We verified our results via a case study where a possible multimedia-friendly congestion control protocol can be designed.

Figure 5.9: The throughput differentiation with respect to α - β ratio.

Chapter 6

SumNet: A Fully Cooperative Network

6.1 Introduction

In the previous chapters, we have considered the various issues on source adaptation. The source adaptation schemes are based on the binary feedback of TCP congestion control. Thus, the simplification of the feedback information limits its performance over networks with complex topology. From this chapter on, we will consider deploying a pricing mechanism in links and use the price as the feedback information to provide traffic control. We start with an optimal problem and then provide another scheme with weighted Max-Min fairness in the next chapter. The switched control source adaptation studied in Chapters 4 and 5 will be combined with the pricing system to an MRF based harmonious networks in Chapter 8.

There are several issues that should be addressed when a source adaptation scheme is designed. They are: 1) efficient bandwidth allocation to different sources, taking into account their different requirements; 2) the utilization of the available bandwidth; and 3) the ease of implementing the source adaptation scheme in a distributed and simple manner with minimal communication overheads [1].

Besides the requirement of differentiated bandwidth service, the various applications may also have different requirements on such services in terms of delay, jitter and loss rate. Hence it is also necessary to classify Internet traffic into different priority classes such that the services can be provided in a classified manner [2].

In this chapter, we consider the design of a price based framework by using generalized Nash equilibrium [67]. The SumNet consists of a novel pricing scheme at links' side and a corresponding source adaptation scheme at sources' side. Unlike the static pricing system in [66], the prices associated with the different priority classes in our system are adjusted according to the status of network: Within each class, each link will increase its price when the load of the respective priority class is below a predefined bound, and decrease its price when it is above the bound. Our pricing system is based on the observation that service of network is fundamentally different from goods in the conventional microeconomic pricing mechanism in the sense that the quality of each goods is always the same regardless of the number of customers while the QoS will change according to the overall load condition. Same as [66], in our pricing system, the higher priority class is charged a higher price. Our pricing system will outperform those in [1, 52–58, 60, 61] due to the following three features of the proposed system:

- (i). The proposed system is consistent in the sense that the price is higher when QoS is better;
- (ii). Each link has an incentive to increase its capacity for new real time applications because it can earn more profit;
- (iii). All links are independent agents and they levy charges for the use of their bandwidth.

Meanwhile, for better QoS, the sending rate of each source is set to be a monotonic non-decreasing function of the price, i.e., each source will increase its transmission rate when the price does not fall and decrease the transmission rate when the price falls. With our scheme, the whole network is fully cooperative in the sense that besides all sources are cooperating with each other, sources and links are also cooperative with each other. This distinguishes our scheme from existing cooperative schemes that are based on the conventional microeconomic system [1] [58].

6.2 Network Model and Notations

We shall consider a network that has L unidirectional links and S sources. All the traffic is divided into two different priority classes and all sources are allowed to split their traffic between the two classes. Class 2 traffic has a higher priority than class 1 traffic. The various parameters and notations of links and sources and delay quantities are defined below. This is an expanded list of notations as compared to the one defined in Chapter 3.

Links:

- L : The set of links, $L = \{1, 2, \dots, L\}$.
- L_s : The set of links that source s uses, $L_s \subseteq L$.
- c_l : The capacity of link l .
- $c_{0l} = [c_{0l,1}, c_{0l,2}]^T$: The 2 upper bounds of traffic in link l with $c_{0l,2} \leq c_{0l,1} < c_l$. $c_{0l,1}$, which is the target capacity of link l , is set to be slightly smaller than c_l as recommended in [99, 100] to ensure better performance.
- $\mathbf{c} = [c_{01,1}, \dots, c_{0L,1}, c_{01,2}, \dots, c_{0L,2}]^T$: The upper bound vector.

- $p_l = [p_{l,1}, p_{l,2}]^T$: The bandwidth price of link l . $p_{l,1}$ is the bandwidth price of class 1 traffic, and $(p_{l,1} + p_{l,2})$ is the bandwidth price of class 2 traffic. p_l is bounded component-wise by the lower bound $p_l^- = [p_{l,1}^-, p_{l,2}^-]^T$ and the upper bound $p_l^+ = [p_{l,1}^+, p_{l,2}^+]^T$, i.e., $p_{l,1}^- \leq p_{l,1} \leq p_{l,1}^+$, and $p_{l,2}^- \leq p_{l,2} \leq p_{l,2}^+$.
- $\mathbf{p} = [p_{1,1}, \dots, p_{L,1}, p_{1,2}, \dots, p_{L,2}]^T$: The price link vector.
- $\bar{x}_l = \bar{x}_{l,1} + \bar{x}_{l,2}$: The total demand on link l . $\bar{x}_{l,1}$ is the total demand of class 1 traffic, and $\bar{x}_{l,2}$ is the total demand of class 2 traffic.
- γ_l : The step size on the adjustment of price of link l .

Sources:

- Ω : The set of sources, $\Omega = \{1, 2, \dots, S\}$.
- Ω_l : The set of sources that uses link l , $\Omega_l \subseteq \Omega$.
- $x_s = x_{s,1} + x_{s,2}$: The total transmission rate of source s . $x_{s,1}$ is the transmission rate of priority 1 traffic, and $x_{s,2}$ is the transmission rate of priority 2 traffic. $x_{s,1}$ is bounded by $x_{s,1}^-$ (the lower bound) and $x_{s,1}^+$ (the upper bound), i.e. $x_{s,1}^- \leq x_{s,1} \leq x_{s,1}^+$. Similarly, $x_{s,2}^- \leq x_{s,2} \leq x_{s,2}^+$.
- $\mathbf{x} = [x_{1,1}, \dots, x_{S,1}, x_{1,2}, \dots, x_{S,2}]^T$: The rate vector.
- $U_s = [U_{s,1}, U_{s,2}]^T$: The utility functions of source s . Both $U_{s,1}$ and $U_{s,2}$ are strictly concave functions with continuous second derivatives.
- $D_s = [D_{s,1}, D_{s,2}]^T$: The demand functions of source s . $D_{s,1}(\cdot) = \dot{U}_{s,1}^{-1}(\cdot)$, and $D_{s,2}(\cdot) = \dot{U}_{s,2}^{-1}(\cdot)$.
- $\bar{p}_s = [\bar{p}_{s,1}, \bar{p}_{s,2}]^T$: The price vector that source s has to pay to use the necessary links where $\bar{p}_{s,1} = \sum_{l \in L_s} p_{l,1}$, and $\bar{p}_{s,2} = \sum_{l \in L_s} (p_{l,1} + p_{l,2})$. Let

$\bar{p}_{s,1}^+ = \sum_{l \in \mathbf{L}_s} p_{l,1}^+$, $\bar{p}_{s,1}^- = \sum_{l \in \mathbf{L}_s} p_{l,1}^-$, $\bar{p}_{s,2}^+ = \sum_{l \in \mathbf{L}_s} (p_{l,1}^+ + p_{l,2}^+)$, and $\bar{p}_{s,2}^- = \sum_{l \in \mathbf{L}_s} (p_{l,1}^- + p_{l,2}^-)$. Clearly, $\bar{p}_{s,1}^- \leq \bar{p}_{s,1} \leq \bar{p}_{s,1}^+$, and $\bar{p}_{s,2}^- \leq \bar{p}_{s,2} \leq \bar{p}_{s,2}^+$.

Delays:

- $\tau_{s,l}^f$: The forward delay from source s to link l .
- $\tau_{l,s}^b$: The backward delay from link l to source s .
- τ_s : The round trip delay time of source s . $\tau_s = \tau_{s,l}^f + \tau_{l,s}^b$.

In the above notations, the second subscript is used to describe the priority of the traffic. In the case where the system has only one priority, we will omit the subscript for conciseness of the notations. For example, p_l will be used to indicate the only link price in the case of one priority and x_s will represent the transmission rate of source s , instead of $p_{l,1}$ and $x_{s,1}$.

Similar to [1], we assume that the network satisfies

Assumption 6.2.1. $\sum_{s \in \Omega_l} x_{s,2}^- \leq c_{0l,2}$ and $\sum_{s \in \Omega_l} [x_{s,1}^- + x_{s,2}^-] \leq c_{0l,1}$ for all $l \in L$.

Assumption 6.2.2. The demand functions $D_{s,1}$ and $D_{s,2}$ satisfy

$$D_{s,1}(\bar{p}_{s,1}^+ - \bar{p}_{s,1}^-) \geq x_{s,1}^-; \forall s,$$

$$D_{s,1}(0) \geq x_{s,1}^+; \forall s,$$

$$D_{s,2}(\bar{p}_{s,2}^+ - \bar{p}_{s,2}^-) \geq x_{s,2}^-; \forall s,$$

$$D_{s,2}(0) \geq x_{s,2}^+; \forall s.$$

Remark 6.2.1. The values of $p_{l,1}^+$, $p_{l,1}^-$, $p_{l,2}^+$, and $p_{l,2}^-$ can be determined by using the demand functions in the conventional microeconomic system. To provide the desired QoS and fully utilize the available resources at any time, their values can be set according to the statistics of traffic flows.

We define two bound projections. Let $g(y)$ be a function of y with $y^- \leq y \leq y^+$.

The lower and upper bound projections are defined as:

$$(g(y))_{y,y^-}^+ = \begin{cases} g(y), & \text{if } (y > y^-), \text{ or } (y = y^- \text{ and } g(y) \geq 0) \\ 0, & \text{if } y = y^- \text{ and } g(y) < 0 \end{cases}, \quad (6.1)$$

$$(g(y))_{y,y^+}^- = \begin{cases} g(y), & \text{if } (y < y^+), \text{ or } (y = y^+ \text{ and } g(y) \leq 0) \\ 0, & \text{if } y = y^+ \text{ and } g(y) > 0 \end{cases}. \quad (6.2)$$

When the first condition of each projection holds, we say that the projection is inactive; otherwise, the projection is active. If y and $g(y)$ are vectors, the projections are to be interpreted component-wise.

6.3 SumNet with One Priority

We begin with the simple case where the network has only one priority in which no service differentiation is provided.

6.3.1 Mathematical Formulation

Same as [58] and [1], we begin from the following optimization problem:

$$\max_{\mathbf{x}} \left\{ \sum_{s=1}^S U_s(x_s) \right\}, \quad \text{subject to } \mathbf{x} \in \mathbf{X}_f. \quad (6.3)$$

In other words, all sources intends to choose an optimal source rate vector $\mathbf{x}^*$ to maximize their utilization payoffs. Clearly, links' utilization payoffs should also be maximized by the optimal vector $\mathbf{x}^*$. Otherwise, all links will also set their price vector $\mathbf{p}$ to maximize their own utilization payoffs and as a result, both sources and links cannot maximize their own utilization payoffs when in equilibrium. To

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avoid this, we introduce a new type of Lagrangian factor $(p_l^+ - p_l)$, so that the corresponding Lagrangian function $L(\mathbf{x}, \mathbf{p})$ is

$$\begin{aligned} L(\mathbf{x}, \mathbf{p}) &= \sum_{s=1}^S U_s(x_s) - \sum_{l=1}^L (p_l^+ - p_l)(\bar{x}_l - c_{0l}) \\ &= \sum_{s=1}^S f_s(x_s, \mathbf{p}) + \sum_{l=1}^L (p_l^+ - p_l)c_{0l}, \end{aligned} \quad (6.4)$$

where function $f_s(x_s, \mathbf{p})$ is given by

$$f_s(x_s, \mathbf{p}) = U_s(x_s) - (\bar{p}_s^+ - \bar{p}_s)x_s. \quad (6.5)$$

Let $\mathbf{D}(\mathbf{p})$ denote the maximization of Equation (6.4), i.e.

$$\begin{aligned} \mathbf{D}(\mathbf{p}) &= \max_{\mathbf{x}} L(\mathbf{x}, \mathbf{p}) \\ &= \sum_{s=1}^S \max_{\mathbf{x}} \{f_s(x_s, \mathbf{p})\} + \sum_{l=1}^L (p_l^+ - p_l)c_{0l} \\ &= \sum_{s=1}^S f_s(x_s^*, \mathbf{p}) + \sum_{l=1}^L (p_l^+ - p_l)c_{0l} \\ &= \sum_{s=1}^S U_s(x_s^*) + \sum_{l=1}^L (p_l^+ - p_l)(c_{0l} - \bar{x}_l^*), \end{aligned} \quad (6.6)$$

where $[x_1^*, \dots, x_S^*] \in \mathbf{X}_f$ is the unique solution of the problem and $\bar{x}_l^*$ is the link demand in case of the solution.

It can be easily shown that $\mathbf{D}(\mathbf{p})$ has a continuous first derivative. The dual problem is

$$\nabla_{\Omega, \mathbf{L}} = \min_{\mathbf{p}} \{\mathbf{D}(\mathbf{p})\}. \quad (6.7)$$

6.3.2 Cooperative Links

The proposed system is set up by solving the dual problem (6.7) via the gradient projection method [62,63]. To obtain an optimum solution, the gradients of $\mathbf{D}(\mathbf{p})$ is used in the proposed pricing system, and computed as

$$\frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_l} = \bar{x}_l - c_{0l}. \quad (6.8)$$

The price of link l is adjusted in the same direction as the gradient of $\mathbf{D}(\mathbf{p})$ as follows:

$$\dot{p}_l = \gamma_l ((c_{0l} - \bar{x}_l)_{p_l, p_l^+}^-)_{p_l, p_l^-}^+. \quad (6.9)$$

From Equations (6.8) and (6.9), in discrete time, we obtain

$$p_l(k+1) = \max\{p_l^-, \min\{p_l^+, p_l(k) + \gamma_l(c_{0l} - \bar{x}_l(k))\}\}, \quad (6.10)$$

where the initial value of $p_l(0)$ is set as $p_l(0) = (p_l^- + p_l^+)/2$.

6.3.3 Feedback Signal

Source s is feedback the scalar price of $\bar{p}_s(k)$ given by

$$\bar{p}_s(k) = \sum_{l \in \mathbf{L}_s} p_l(k - \tau_{l,s}^b). \quad (6.11)$$

Since the feedback signal is the SUM of the utilized links' prices, the pricing framework is called SumNet.

The signal has the following two functions:

- (i). It serves as the charging signal, i.e., it is used to determine the total amount of money that each source should pay and the distribution of profit among

links;

- (ii). It serves as the controlling signal. i.e., it is used by each source to adjust its sending rate.

Remark 6.3.1. *It should be pointed here that $(\bar{p}_s^+ - \bar{p}_s(k))$ is actually used in the traffic control of the source adaptation algorithm. Since $\bar{p}_s^+$ is fixed, we suppose that $\bar{p}_s^+$ is known when the connection is set up. This also applies to the case of multiple priority classes.*

6.3.4 Cooperative Sources

To provide a distributed source adaptation scheme that will converge to the optimal solution, we need to solve the following two optimization problems [62, 70]:

$$\max_{x_s} \{f_s(x_s, \mathbf{p})\}. \quad (6.12)$$

Let $x_s^*(\bar{p}_s)$ be the unique optimal solution to problem (6.12). Since $f_s(x_s, \mathbf{p})$ are concave functions, by the Kuhn-Tucker theorem [101], $x_s^*(\bar{p}_s)$ is

$$x_s^*(\bar{p}_s) = D_s((\bar{p}_s^+ - \bar{p}_s(k))). \quad (6.13)$$

The solution $x_s^*(\bar{p}_s)$ is different from the conventional demand function in microeconomic system and the solutions in [1] and [58] where the sending rate is a monotonically decreasing function of the price.

To avoid abrupt changes of the sending rates, $x_s(k)$ is computed by a sliding window method as

$$x_s(k+1) = \lambda_s x_s(k) + (1 - \lambda_s) x_s^*(\bar{p}_s), \quad (6.14)$$

$$x_s(0) = x_s^*(\bar{p}_s(0)); 0 \leq \lambda_s < 1. \quad (6.15)$$

6.3.5 Incentives of Sources and Links

In this section, we shall show that the incentives of both sources and links belong to the second type of the generalized Nash equilibrium discussed in Section 2.4.

Since $\gamma_l > 0$, Equation (6.10) implies that the price is a nondecreasing function of QoS, i.e.,

- (i). When the system is under-loaded, i.e. the QoS can be guaranteed, the link can then increase the price;
- (ii). On the other hand, when the system is over-loaded, the QoS cannot be guaranteed, so the link decreases the price.

This implies that the proposed pricing system has the feature that links reduce the price when the sending rates of sources are relatively large, and increase the price when the sending rate of sources are relatively small. Thus, the links also try to maximize the utility payoffs of sources.

Since $D_s(\cdot)$ is a decreasing function, Equation (6.13) implies that

- (i). Source s knows that the system is under-loaded when the price does not fall, it thus increases its sending rate accordingly to improve the utilization of the available bandwidth;
- (ii). On the other hand, when the system is over-loaded as the price falls, source s decreases its sending rate accordingly to guarantee the desired QoS of all sources.

In other words, sources increase the sending rates when price is high, and reduce the sending rates when price is low. Therefore, the sources try to maximize the

utility payoffs of links.

Therefore, the incentives of both the links and the sources belong to the second type. With the proposed solution, sources and links are fully cooperative, and achieve the second type of generalized Nash equilibrium.

6.3.6 Stability of the Pricing System

In this section, we shall establish the global asymptotical stability of the proposed pricing system by using Lyapunov functions.

Let $\bar{p}_s^*$ be the price of source s at equilibrium and x_s^* be the corresponding sending rate of source s i.e., $x_s^* = D_s(\bar{p}_s^+ - \bar{p}_s^*)$. We define

$$M_s = \int_{\bar{p}_s^*}^{\bar{p}_s^+} (D_s(\bar{p}_s^+ - \delta) - x_s^*) d\delta. \quad (6.16)$$

and its derivative along the solution is

$$\dot{M}_s = (D_s(\bar{p}_s^+ - \bar{p}_s) - x_s^*) \sum_l \dot{p}_l.$$

We define the following function for link l :

$$W_l = (c_{0l} - \bar{x}_l^*)(p_l^+ - p_l). \quad (6.17)$$

Its derivative along the trajectory is

$$\dot{W}_l = (\bar{x}_l - c_{0l})\gamma_l((c_{0l} - \bar{x}_l)_{p_l, p_l^+}^-)_{p_l, p_l^-}^+ - (\bar{x}_l - \bar{x}_l^*)\dot{p}_l$$

The overall Lyapunov function is defined as

$$V = \sum_{s=1}^S M_s + \sum_{l=1}^L W_l. \quad (6.18)$$

V is nonnegative, and

$$\dot{V} = \sum_{l=1}^L [(\bar{x}_l - c_{0l}) \gamma_l ((c_{0l} - \bar{x}_l)_{p_l, p_l^+}^-)^l]_{p_l, p_l^-}.$$

Clearly, $\dot{V}$ is non-positive and vanishes on the set

$$\{\mathbf{p} : \text{for all } l = 1, \dots, L, \text{ either } (c_{0l} = \bar{x}_l), \text{ or } (c_{0l} > \bar{x}_l \text{ and } p_l = p_l^+); \}.$$

Following similar arguments as in [102] and according to Lyapunov's stability theorem, it can be shown that the only trajectory that can remain identically in the above set is the unique equilibrium. Thus, the equilibrium is globally asymptotically stable.

6.4 SumNet with Multiple Priorities

6.4.1 Relative Differentiated Delay Services

We shall now consider a network that can provide a relative differentiated delay service. We assume that all the traffic is divided into two different priority classes [2, 34], and a strict priority scheduler is used. That is, for differentiation, each link will transmit class 1 traffic only if all class 2 traffic has been served.

The throughput of each class is bounded by a Russian Doll Model (RDM) [99]. Let $\mathbf{X}_f$ be a feasible set of sending rates, then according to the RDM, $\mathbf{X}_f$ is defined

as

$$\begin{aligned} \mathbf{X}_f &= \{ \mathbf{x} \in \mathbb{R}_+^{2S} | x_{s,1}^- \leq x_{s,1} \leq x_{s,1}^+, \\ &\quad x_{s,2}^- \leq x_{s,2} \leq x_{s,2}^+ ; \text{ for } \forall s \in \Omega, \\ &\quad \bar{x}_{l,2} = \sum_{s \in \Omega_l} x_{s,2} \leq c_{0l,2}, \\ &\quad \bar{x}_{l,1} + \bar{x}_{l,2} = \sum_{s \in \Omega_l} (x_{s,1} + x_{s,2}) \leq c_{0l,1}, \forall l \in \mathbb{L} \}. \end{aligned}$$

$\mathbf{X}_f$ is a nonempty, convex, and compact set under Assumption 6.2.1.

It should be pointed out that other scheduling schemes [103–111] can also be adopted to determine the service order of the two priority classes. Since there is neither resource reservation nor admission control, class 2 traffic does not get absolute delay assurance but a smaller delay than class 1 traffic.

Sources are free to choose the priority of their traffic, but are charged based on their traffic allocation. In particular, link l charges the prices of $p_{l,1}$ and $(p_{l,1} + p_{l,2})$ respectively for each unit of class 1 and class 2 traffic. Normally, class 2 traffic is more expensive than the class 1 one, i.e.

$$p_{l,2} > 0.$$

The values of $p_{l,1}$ and $p_{l,2}$ are determined by link l according to the total demand $\bar{x}_l$ (i.e. $\bar{x}_{l,1} + \bar{x}_{l,2}$) and its upper bound $c_{0l,1}$, and the total demand on priority 2, $\bar{x}_{l,2}$ and its upper bound $c_{0l,2}$, respectively.

Each source is allowed to split its traffic between the two priorities. Source s needs to determine the values of $x_{s,1}$ and $x_{s,2}$, i.e.,

- (i). Its sending rate, i.e., the amount of traffic submitted per unit time, and
- (ii). Its traffic allocation, i.e., the priority of the submitted traffic.

6.4.2 Mathematical Formulation

Similar to the one priority case, we also begin from the following optimization problem:

$$\begin{aligned} \max_{\mathbf{x}} \{ & \sum_{s=1}^S [U_{s,1}(x_{s,1}) + U_{s,2}(x_{s,2})] \}, \\ \text{subject to } & \mathbf{x} \in \mathbf{X}_f. \end{aligned} \quad (6.19)$$

By introducing a new type of Lagrangian factors as $(p_{l,1}^+ - p_{l,1})$ and $(p_{l,2}^+ - p_{l,2})$, the corresponding Lagrangian function $L(\mathbf{x}, \mathbf{p})$ is

$$\begin{aligned} L(\mathbf{x}, \mathbf{p}) &= \sum_{s=1}^S [U_{s,1}(x_{s,1}) + U_{s,2}(x_{s,2})] \\ &\quad - \sum_{l=1}^L (p_{l,1}^+ - p_{l,1})(\bar{x}_{l,1} + \bar{x}_{l,2} - c_{0l,1}) \\ &\quad - \sum_{l=1}^L (p_{l,2}^+ - p_{l,2})(\bar{x}_{l,2} - c_{0l,2}) \\ &= \sum_{l=1}^L [(p_{l,1}^+ - p_{l,1})c_{0l,1} + (p_{l,2}^+ - p_{l,2})c_{0l,2}] \\ &\quad + \sum_{s=1}^S [f_{s,1}(x_{s,1}, \mathbf{p}) + f_{s,2}(x_{s,2}, \mathbf{p})], \end{aligned} \quad (6.20)$$

where functions $f_{s,1}(x_{s,1}, \mathbf{p})$ and $f_{s,2}(x_{s,2}, \mathbf{p})$ are given by

$$\begin{aligned} f_{s,1}(x_{s,1}, \mathbf{p}) &= U_{s,1}(x_{s,1}) - (\bar{p}_{s,1}^+ - \bar{p}_{s,1})x_{s,1}, \\ f_{s,2}(x_{s,2}, \mathbf{p}) &= U_{s,2}(x_{s,2}) - (\bar{p}_{s,2}^+ - \bar{p}_{s,2})x_{s,2}. \end{aligned}$$

Let $\mathbf{D}(\mathbf{p})$ denote the maximization of Equation (6.20), i.e.

$$\mathbf{D}(\mathbf{p}) = \max_{\mathbf{x}} L(\mathbf{x}, \mathbf{p})$$

$$\begin{aligned}
&= \sum_{s=1}^S [\max_{x_{s,1}} \{f_{s,1}(x_{s,1}, \mathbf{p})\} + \max_{x_{s,2}} \{f_{s,2}(x_{s,2}, \mathbf{p})\}] \\
&\quad + \sum_{l=1}^L [(p_{l,1}^+ - p_{l,1})c_{0l,1} + (p_{l,2}^+ - p_{l,2})c_{0l,2}] \\
&= \sum_{s=1}^S [f_{s,1}(x_{s,1}^*, \mathbf{p}) + f_{s,2}(x_{s,2}^*, \mathbf{p})] \\
&\quad + \sum_{l=1}^L [(p_{l,1}^+ - p_{l,1})c_{0l,1} + (p_{l,2}^+ - p_{l,2})c_{0l,2}] \\
&= \sum_{s=1}^S [U_{s,1}(x_{s,1}^*) + U_{s,2}(x_{s,2}^*)] \\
&\quad + \sum_{l=1}^L (p_{l,1}^+ - p_{l,1})(c_{0l,1} - \bar{x}_{l,1}^* - \bar{x}_{l,2}^*) \\
&\quad + \sum_{l=1}^L (p_{l,2}^+ - p_{l,2})(c_{0l,2} - \bar{x}_{l,2}^*), \tag{6.21}
\end{aligned}$$

where $[x_{s,1}^*, x_{s,2}^*, \dots, x_{S,1}^*, x_{S,2}^*] \in \mathbf{X}_f$ is the unique solution of the problem, and $\bar{x}_{l,1}^*$ and $\bar{x}_{l,2}^*$ are the link demands of priorities 1 and 2 in case of the solution, and $\bar{x}_l^* = \bar{x}_{l,1}^* + \bar{x}_{l,2}^*$.

It can be easily shown that $\mathbf{D}(\mathbf{p})$ has a continuous first derivative. The dual problem is

$$\nabla_{\Omega, \mathbf{L}} = \min_{\mathbf{p}} \{\mathbf{D}(\mathbf{p})\}. \tag{6.22}$$

6.4.3 The First Order System

6.4.3.1 The First Order Pricing System

Same as the one priority case, the dual problem (6.22) is solved via the gradient projection method. The gradients of $\mathbf{D}(\mathbf{p})$ in Equation (6.22) are used in the

proposed pricing system, and are computed as

$$\begin{aligned}\frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_{l,1}} &= \bar{x}_{l,1} + \bar{x}_{l,2} - c_{0l,1} \\ &= \bar{x}_{l,-} - c_{0l,1},\end{aligned}\tag{6.23}$$

$$\frac{\partial \mathbf{D}(\mathbf{p})}{\partial p_{l,2}} = \bar{x}_{l,2} - c_{0l,2}.\tag{6.24}$$

The prices are adjusted in the same direction as the gradients of $\mathbf{D}(\mathbf{p})$ and bounded by the upper and lower bounds. The first order pricing system of link l is given as

$$\dot{p}_{l,1} = \gamma_l((c_{0l,1} - \bar{x}_l)_{p_{l,1}, p_{l,1}^+}^-)_{p_{l,1}, p_{l,1}^-}^+, \tag{6.25}$$

$$\dot{p}_{l,2} = \gamma_l((c_{0l,2} - \bar{x}_l)_{p_{l,2}, p_{l,2}^+}^-)_{p_{l,2}, p_{l,2}^-}^+. \tag{6.26}$$

Consequently, in discrete time implementation, we obtain

$$p_{l,1}(k+1) = p_{l,1}(k) + \gamma_l(c_{0l,1} - \bar{x}_l(k)), \tag{6.27}$$

$$p_{l,2}(k+1) = p_{l,2}(k) + \gamma_l(c_{0l,2} - \bar{x}_l(k)). \tag{6.28}$$

where the initial values of $p_{l,1}(0)$ and $p_{l,2}(0)$ are set as $(p_{l,1}^- + p_{l,1}^+)/2$ and $(p_{l,2}^- + p_{l,2}^+)/2$, respectively.

With the values of $p_{l,1}$ and $p_{l,2}$ determined, the prices of classes 1 and 2 are $p_{l,1}(k+1)$ and $(p_{l,1}(k+1) + p_{l,2}(k+1))$, respectively. The price of class 2 traffic is higher than that of class 1. This is reasonable because class 2 traffic receives better QoS. Within each class, the price is higher when the QoS is better. Therefore, the proposed pricing system is consistent in the sense that the traffic is charged a higher price when it receives a better QoS. Meanwhile, with the proposed pricing system, all links are independent agents and they charge a price for the use of their

bandwidth according to Equations (6.27) and (6.28).

6.4.3.2 Feedback Signal

Source s is fed back two scalar prices of $\bar{p}_{s,1}(k)$ and $\bar{p}_{s,2}(k)$ that are given by

$$\bar{p}_{s,1}(k) = \sum_{l \in \mathbf{L}_s} p_{l,1}(k - \tau_{l,s}^b), \quad (6.29)$$

$$\bar{p}_{s,2}(k) = \sum_{l \in \mathbf{L}_s} [p_{l,1}(k - \tau_{l,s}^b) + p_{l,2}(k - \tau_{l,s}^b)]. \quad (6.30)$$

6.4.3.3 The Source Adaptation Scheme

To provide a distributed source adaptation scheme, we need to solve the following two optimization problems:

$$\max_{x_{s,1}} \{f_{s,1}(x_{s,1}, \mathbf{p})\}, \quad (6.31)$$

$$\max_{x_{s,2}} \{f_{s,2}(x_{s,2}, \mathbf{p})\}. \quad (6.32)$$

Let $x_{s,1}^*(\bar{p}_{s,1})$ and $x_{s,2}^*(\bar{p}_{s,2})$ be the unique optimal solutions to problems (6.31) and (6.32), respectively. Since both $f_{s,1}(x_{s,1}, \mathbf{p})$ and $f_{s,2}(x_{s,2}, \mathbf{p})$ are concave functions, by the Kuhn-Tucker theorem, $x_{s,1}^*(\bar{p}_{s,1})$ and $x_{s,2}^*(\bar{p}_{s,2})$ are

$$x_{s,1}^*(\bar{p}_{s,1}) = D_{s,1}(\bar{p}_{s,1}^+ - \bar{p}_{s,1}), \quad (6.33)$$

$$x_{s,2}^*(\bar{p}_{s,2}) = D_{s,2}(\bar{p}_{s,2}^+ - \bar{p}_{s,2}). \quad (6.34)$$

To avoid abrupt changes of the sending rates, the same sliding window method

in Equation (6.14) is used to compute $x_{s,1}(k)$ and $x_{s,2}(k)$ as

$$x_{s,1}(k) = \lambda_s x_{s,1}(k-1) + (1 - \lambda_s) x_{s,1}^*(\bar{p}_{s,1}), \quad (6.35)$$

$$x_{s,1}(0) = x_{s,1}^*(\bar{p}_{s,1}(0)), \quad (6.36)$$

$$x_{s,2}(k) = \lambda_s x_{s,2}(k-1) + (1 - \lambda_s) x_{s,2}^*(\bar{p}_{s,2}), \quad (6.37)$$

$$x_{s,2}(0) = x_{s,2}^*(\bar{p}_{s,2}(0)); 0 \leq \lambda_s < 1. \quad (6.38)$$

Same as the one priority case, it is easy to see that our algorithm has the second type of incentive.

6.4.4 Stability of the First Order Pricing System

In this section, we shall establish the global asymptotical stability of the proposed first-order pricing system by using Lyapunov functions.

Let $(\bar{p}_{s,1}^*, \bar{p}_{s,2}^*)$ be the price of source s at equilibrium and $(x_{s,1}^*, x_{s,2}^*)$ be the corresponding sending rate of source s i.e., $x_{s,1}^* = D_{s,1}(\bar{p}_{s,1}^+ - \bar{p}_{s,1}^*)$, $x_{s,2}^* = D_{s,2}(\bar{p}_{s,2}^+ - \bar{p}_{s,2}^*)$. We define two functions as

$$M_{s,1} = \int_{\bar{p}_{s,1}^*}^{\bar{p}_{s,1}^+} (D_{s,1}(\bar{p}_{s,1}^+ - \delta) - x_{s,1}^*) d\delta, \quad (6.39)$$

$$M_{s,2} = \int_{\bar{p}_{s,1}^*}^{\bar{p}_{s,2}^+} (D_{s,2}(\bar{p}_{s,2}^+ - \delta) - x_{s,2}^*) d\delta. \quad (6.40)$$

We then have the following function for source s :

$$M_s = M_{s,1} + M_{s,2}. \quad (6.41)$$

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Its derivative along the solution is

$$\begin{aligned}\dot{M}_s &= (D_{s,1}(\bar{p}_{s,1}^+ - \bar{p}_{s,1}) - x_{s,1}^*) \sum_l \dot{p}_{l,1} \\ &\quad + (D_{s,2}(\bar{p}_{s,2}^+ - \bar{p}_{s,2}) - x_{s,2}^*) \sum_l \dot{p}_{l,1} \\ &\quad + (D_{s,2}(\bar{p}_{s,2}^+ - \bar{p}_{s,2}) - x_{s,2}^*) \sum_l \dot{p}_{l,2}.\end{aligned}$$

We also define the following function for link l :

$$W_l = W_{l,1} + W_{l,2}, \quad (6.42)$$

where

$$W_{l,1} = (c_{0l,1} - \bar{x}_l^*)(p_{l,1}^+ - p_{l,1}), \quad (6.43)$$

$$W_{l,2} = (c_{0l,2} - \bar{x}_{l,2}^*)(p_{l,2}^+ - p_{l,2}). \quad (6.44)$$

The derivative of W_l along the trajectory is

$$\begin{aligned}\dot{W}_l &= (\bar{x}_l - c_{0l,1})\gamma_l((c_{0l,1} - \bar{x}_l)_{p_{l,1}, p_{l,1}^+}^-)_{p_{l,1}, p_{l,1}^-}^+ \\ &\quad + (\bar{x}_{l,2} - c_{0l,2})\gamma_l((c_{0l,2} - \bar{x}_{l,2})_{p_{l,2}, p_{l,2}^+}^-)_{p_{l,2}, p_{l,2}^-}^+ \\ &\quad - (\bar{x}_l - \bar{x}_l^*)\dot{p}_{l,1} - (\bar{x}_{l,2} - \bar{x}_{l,2}^*)\dot{p}_{l,2}.\end{aligned}$$

The overall Lyapunov function is defined as

$$V = \sum_{s=1}^S M_s + \sum_{l=1}^L W_l. \quad (6.45)$$

V is nonnegative, and

$$\begin{aligned} \dot{V} = & \sum_{l=1}^L [(\bar{x}_l - c_{0l,1})\gamma_l((c_{0l,1} - \bar{x}_l)_{p_{l,1}, p_{l,1}^+}^-)^+_{p_{l,1}, p_{l,1}^-} \\ & + (\bar{x}_l - c_{0l,2})\gamma_l((c_{0l,2} - \bar{x}_l)_{p_{l,2}, p_{l,2}^+}^-)^+_{p_{l,2}, p_{l,2}^-}]. \end{aligned}$$

Clearly, $\dot{V}$ is non-positive and vanishes on the set

$$\begin{aligned} \{\mathbf{p} : & \text{for all } l = 1, \dots, L, \text{ either } (c_{0l,1} = \bar{x}_l), \\ & \text{or } (c_{0l,1} > \bar{x}_l \text{ and } p_{l,1} = p_{l,1}^+); \\ & \text{either } (c_{0l,2} = \bar{x}_l), \text{ or } (c_{0l,2} > \bar{x}_l \text{ and } p_{l,2} = p_{l,2}^+).\}. \end{aligned}$$

According to Lyapunov's stability theorem, it can be shown that the only trajectory that can remain identically in the above set is the unique equilibrium. Thus, the equilibrium is globally asymptotically stable.

6.4.5 High Order Pricing System

We can extend the first order pricing system of (6.25) and (6.26) to a higher order one as follows:

$$\dot{\eta}_{l,1} = (G_{l,1}\eta_{l,1} + B_{l,1}(\bar{x}_l - c_{0l,1}))_{\eta_{l,1},0}^+, \quad (6.46)$$

$$\dot{p}_{l,1} = (H_{l,1}\eta_{l,1} + d_{l,1}(c_{0l,1} - \bar{x}_l))_{p_{l,1}, p_{l,1}^+}^-, \quad (6.47)$$

$$\dot{\eta}_{l,2} = (G_{l,2}\eta_{l,2} + B_{l,2}(\bar{x}_l - c_{0l,2}))_{\eta_{l,2},0}^+, \quad (6.48)$$

$$\dot{p}_{l,2} = (H_{l,2}\eta_{l,2} + d_{l,2}(c_{0l,2} - \bar{x}_{l,2}))_{p_{l,2}, p_{l,2}^+}^-, \quad (6.49)$$

where $\eta_{l,1} = [\eta_{l,1,1}, \dots, \eta_{l,1,m_{l,1}}]^T$ and $\eta_{l,2} = [\eta_{l,2,1}, \dots, \eta_{l,2,m_{l,2}}]^T$ are two vectors with dimensionalities $m_{l,1}$ and $m_{l,2}$, respectively; $d_{l,1}$ and $d_{l,2}$ are two constants; and matrices $G_{l,1}$, $B_{l,1}$, $H_{l,1}$, $G_{l,2}$, $B_{l,2}$, and $H_{l,2}$ are given by

$$\begin{aligned} G_{l,1} &= \begin{bmatrix} -g_{l,1,1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -g_{l,1,m_{l,1}} \end{bmatrix}, \\ B_{l,1} &= \begin{bmatrix} b_{l,1,1} \\ \vdots \\ b_{l,1,m_{l,1}} \end{bmatrix}, \\ H_{l,1} &= \begin{bmatrix} -h_{l,1,1} & \cdots & -h_{l,1,m_{l,1}} \end{bmatrix}, \\ G_{l,2} &= \begin{bmatrix} -g_{l,2,1} & 0 & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -g_{l,2,m_{l,2}} \end{bmatrix}, \\ B_{l,2} &= \begin{bmatrix} b_{l,2,1} \\ \vdots \\ b_{l,2,m_{l,2}} \end{bmatrix}, \\ H_{l,2} &= \begin{bmatrix} -h_{l,2,1} & \cdots & -h_{l,2,m_{l,2}} \end{bmatrix}, \end{aligned}$$

with $g_{l,1,i}$ and $g_{l,2,i}$ being nonnegative constants, and $b_{l,1,i}$, $b_{l,2,i}$, $h_{l,1,i}$ and $h_{l,2,i}$ being positive constants. Note that in Equations (6.46) and (6.48), the projections are defined component-wise and that the lower bound for both vectors $\eta_{l,1}$ and $\eta_{l,2}$ is $\mathbf{0}$ (the zero vector).

Remark 6.4.1. When $m_{l,1} = m_{l,2} = 1$, $g_{l,1,1} = g_{l,2,1} = 0$, $b_{l,1,1} = b_{l,2,1} = 1$, $h_{l,1,1} = h_{l,2,1} = \lambda_l \gamma_l$, and $d_{l,1} = d_{l,2} = \gamma_l$, where λ_l is a non-negative gain factor, then Equations (6.46) - (6.49) are reduced to the following second-order pricing system:

$$\dot{\eta}_{l,1} = (\bar{x}_l - c_{0l,1})_{\eta_{l,1},0}^+, \quad (6.50)$$

$$\dot{p}_{l,1} = \gamma_l (c_{0l,1} - \lambda_l \eta_{l,1} - \bar{x}_l)_{p_{l,1}, p_{l,1}^+}^-, \quad (6.51)$$

$$\dot{\eta}_{l,2} = (\bar{x}_{l,2} - c_{0l,2})_{\eta_{l,2},0}^+, \quad (6.52)$$

$$\dot{p}_{l,2} = \gamma_l (c_{0l,2} - \lambda_l \eta_{l,2} - \bar{x}_{l,2})_{p_{l,2}, p_{l,2}^+}^-. \quad (6.53)$$

We shall now establish the global asymptotic stability of the high-order pricing system. The overall Lyapunov function is chosen as Equation (6.45), with function M_s of source s given by Equations (6.39)-(6.41), and function W_l of link l given by Equation (6.42) but $W_{l,1}$ and $W_{l,2}$ are replaced by

$$W_{l,1} = \sum_{i=1}^{m_{l,1}} \frac{h_{l,1,i}}{2b_{l,1,i}} \eta_{l,1,i}^2 + (c_{0l,1} - \bar{x}_l^*)(p_{l,1}^+ - p_{l,1}), \quad (6.54)$$

$$W_{l,2} = \sum_{i=1}^{m_{l,2}} \frac{h_{l,2,i}}{2b_{l,2,i}} \eta_{l,2,i}^2 + (c_{0l,2} - \bar{x}_{l,2}^*)(p_{l,2}^+ - p_{l,2}). \quad (6.55)$$

The derivative of W_l along the solution is given by

$$\begin{aligned} \dot{W}_l = & \sum_{i=1}^{m_{l,1}} \frac{h_{l,1,i}}{b_{l,1,i}} \eta_{l,1,i} (b_{l,1,i}(\bar{x}_l - c_{0l,1}) - g_{l,1,i} \eta_{l,1,i})_{\eta_{l,1,i},0}^+ \\ & + (\bar{x}_l - c_{0l,1}) (d_{l,1} (c_{0l,1} - \bar{x}_l) - \sum_{i=1}^{m_{l,1}} h_{l,1,i} \eta_{l,1,i})_{p_{l,1}, p_{l,1}^+}^- \\ & - (\bar{x}_l - \bar{x}_l^*) \dot{p}_{l,1} \\ & + \sum_{i=1}^{m_{l,2}} \frac{h_{l,2,i}}{b_{l,2,i}} \eta_{l,2,i} (b_{l,2,i}(\bar{x}_{l,2} - c_{0l,2}) - g_{l,2,i} \eta_{l,2,i})_{\eta_{l,2,i},0}^+ \end{aligned}$$

$$\begin{aligned}
& +(\bar{x}_{l,2} - c_{0l,2})(d_{l,2}(c_{0l,2} - \bar{x}_{l,2}) - \sum_{i=1}^{m_{l,2}} h_{l,2,i} \eta_{l,2,i})_{p_{l,2}, p_{l,2}^+}^- \\
& -(\bar{x}_{l,2} - \bar{x}_{l,2}^*) \dot{p}_{l,2} \\
= & - \sum_{i=1}^{m_{l,1}} \frac{g_{l,1,i} h_{l,1,i}}{b_{l,1,i}} \eta_{l,1,i}^2 - (\bar{x}_l - \bar{x}_l^*) \dot{p}_{l,1} \\
& + \sum_{i=1}^{m_{l,1}} h_{l,1,i} \eta_{l,1,i} (\bar{x}_l - c_{0l,1}) \\
& + (\bar{x}_l - c_{0l,1})(d_{l,1}(c_{0l,1} - \bar{x}_l) - \sum_{i=1}^{m_{l,1}} h_{l,1,i} \eta_{l,1,i})_{p_{l,1}, p_{l,1}^+}^- \\
& - \sum_{i=1}^{m_{l,2}} \frac{g_{l,2,i} h_{l,2,i}}{b_{l,2,i}} \eta_{l,2,i}^2 - (\bar{x}_{l,2} - \bar{x}_{l,2}^*) \dot{p}_{l,2} \\
& + \sum_{i=1}^{m_{l,2}} h_{l,2,i} \eta_{l,2,i} (\bar{x}_{l,2} - c_{0l,2}) \\
& + (\bar{x}_{l,2} - c_{0l,2})(d_{l,2}(c_{0l,2} - \bar{x}_{l,2}) - \sum_{i=1}^{m_{l,2}} h_{l,2,i} \eta_{l,2,i})_{p_{l,2}, p_{l,2}^+}^-.
\end{aligned}$$

It follows that

$$\begin{aligned}
\dot{V} \leq & \sum_{l=1}^L \left[\sum_{i=1}^{m_{l,1}} h_{l,1,i} \eta_{l,1,i} (\bar{x}_l - c_{0l,1}) \right. \\
& + (\bar{x}_l - c_{0l,1})(d_{l,1}(c_{0l,1} - \bar{x}_l) - \sum_{i=1}^{m_{l,1}} h_{l,1,i} \eta_{l,1,i})_{p_{l,1}, p_{l,1}^+}^- \\
& + \sum_{i=1}^{m_{l,2}} h_{l,2,i} \eta_{l,2,i} (\bar{x}_{l,2} - c_{0l,2}) \\
& \left. + (\bar{x}_{l,2} - c_{0l,2})(d_{l,2}(c_{0l,2} - \bar{x}_{l,2}) - \sum_{i=1}^{m_{l,2}} h_{l,2,i} \eta_{l,2,i})_{p_{l,2}, p_{l,2}^+}^- \right].
\end{aligned}$$

Clearly, $\dot{V}$ is non-positive and vanishes on the set

$$\{(\eta, \mathbf{p}) : \text{for all } l = 1, \dots, L, \text{ either } (c_{0l,1} = \bar{x}_l),$$

$$\begin{aligned} &\text{or } (c_{0l,1} > \bar{x}_l \text{ and } p_{l,1}^+ = p_{l,1}, \eta_{l,1} = \mathbf{0}); \\ &\text{either } (c_{0l,2} = \bar{x}_{l,2}), \text{ or } (c_{0l,2} > \bar{x}_{l,2} \text{ and } p_{l,2}^+ = p_{l,2}, \eta_{l,2} = \mathbf{0}). \} \end{aligned}$$

Similar to the first order pricing system, the only trajectory that can remain identically in the above set is the unique equilibrium. Thus, the equilibrium is globally asymptotically stable.

6.5 Simulation Results

6.5.1 Utility functions

There are many possible choices of functions $U_{s,1}(\cdot)$ and $U_{s,2}(\cdot)$, and two suitable ones are

$$U_{s,1}(\cdot) = \alpha_1 \log(\cdot); \alpha_1 > 0, \quad (6.56)$$

$$U_{s,2}(\cdot) = \alpha_2 \log(\cdot); \alpha_2 > 0. \quad (6.57)$$

It can be shown from Equations (6.33) and (6.34) that the corresponding $x_{s,1}^*(\bar{p}_{s,1})$ and $x_{s,2}^*(\bar{p}_{s,2})$ are

$$\begin{aligned} x_{s,1}^*(\bar{p}_{s,1}) &= \frac{\alpha_1}{\bar{p}_{s,1}^+ - \bar{p}_{s,1}(k)}, \\ x_{s,2}^*(\bar{p}_{s,2}) &= \frac{\alpha_2}{\bar{p}_{s,2}^+ - \bar{p}_{s,2}(k)}. \end{aligned}$$

We test our scheme by Ns-2 [95]. The network topology is shown in Figure 6.1. There are three sources S_i ($i = 1, 2, 3$) that send out UDP packets.

Figure 6.1: Network topology.

In all simulations, each source transmits data for a total period of 120s with their starting times staggered by an interval of 40s, i.e. sources 1, 2, and 3 start transmitting at times 0s, 40s, and 80s, respectively. We set the minimal transmitting rate of each priority for all sources as 1×10^6 bps while the maximal transmitting rate as 1.6×10^7 bps. The utility functions of all sources are set to $10^6 \log(x_s)$ and the smooth factor, λ_s , is set as 0, i.e. the case with the most fluctuating traffic is studied.

The values of both $p_{l,1}^-$ and $p_{l,2}^-$ are 0.013, and those of both $p_{l,1}^+$ and $p_{l,2}^+$ are set as 1. The target value, $c_{0l,1}$, is set to 1.9×10^7 bps which is slightly smaller than the actual link capacity as recommended in [100], and $c_{0l,2}$, at $c_{0l,1}/4$. The stepsize γ_l used by the router to adjust its link price is set as 5×10^{-9} .

The propagation delay of the links is set as 10ms. Sources as well as links adjust their transmitting rates/prices at every 50ms time intervals. The Round Trip Time of the sources is smaller than the adjustment period. Thus, it guarantees that in most cases, the scheme works in the synchronous case. In the following simulations, this setting is used unless otherwise stated.

6.5.2 The One Priority Case

To compare SumNet with those in [1] [58], we shall first consider the case where all sources only mark their traffic by priority 1.

6.5.2.1 Convergence

Figure 6.2(a) shows the received rate of each source at the destination in the synchronous case. Obviously, the sum of the trace is constant at about the target value, and the received rates vary in accordance with the dynamics of the link price in Figure 6.2(b). In case of congestion, sources can adjust their rates that are convergent to an equal share of bandwidth which gives a maximal overall utility and thus the proposed scheme provides an optimal and fair network service. As a comparison, we also tested the scheme proposed in [1] and the corresponding results are shown in Figures 6.2(c) and 6.2(d). Clearly, both algorithms can achieve similar results on source adaptation. However, it can be seen in Figure 6.2(d) that the price is only nonzero at bottleneck links when an equilibrium is reached [1], [58], [100]. Clearly, this is unfair to links that are under-loaded. It can be seen from Figure 6.2(b) that our proposed system has avoided this problem.

We shall also test an asynchronous setting where $\tau_{s,l}^f$ and $\tau_{l,s}^b$ may be different, network states may be probed by different sources at different rates, and different links may update their prices with different frequency. To justify the convergence in the asynchronous case, the propagation delay is set as 30ms. Although this delay has disturbed the convergence of the algorithm, the proposed algorithm can still converge by setting a relatively smaller γ_l . The experimental results are illustrated in Figures 6.3(a) and 6.3(b), for the case the propagation delay of each link is 30ms and the value of γ_l is 5×10^{-10} .

6.5 Simulation Results

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(a) The received rate at the destination by our proposed scheme. (b) The link price for each link by our proposed pricing system.

(c) The received rate at the destination by the scheme by Low et al. [1]. (d) The link price for each link by the pricing system by Low et al. [1].

Figure 6.2: The received rate and link price by our proposed scheme and the pricing system by Low et al. [1].

(a) The receive rate at the destination.

(b) The link price for each link.

Figure 6.3: The proposed scheme in the case of asynchronous algorithm.

6.5.2.2 Comparison of Pricing Systems

We shall now compare the values of $\mathbf{p}^*$ that are obtained by our proposed pricing system and those in [1, 58], which use the traditional Nash Equilibrium [112, 113]. The results are shown in Table 6.1. Obviously, the pricing systems in [1, 58] are unfair to the under-loaded links. There is no incentive for the under-loaded links to adjust their prices according to the schemes in [1, 58]. On the other hand, the bottleneck links have an unfair advantage in getting more profit than the under-loaded links, and hence they do not have any incentive to increase their capacities. This is shown in the following example.

Table 6.1: The price in equilibrium.

	The price of under-loaded link	The price of fully utilized link
Our proposed system	p_l^+	p_l^*
The systems in [1, 58]	p_l^-	p_l^*

The capacity of link 2 is now slightly enlarged with $c_{02,1}$ set at 2.0×10^7 bps. Obviously, link 1 is congested while link 2 is under-loaded during the time period

of 40s to 80s. It can be observed from Figures 6.2(d) and 6.4(a) that the pricing system in [1] has the following two features:

- The price of congested link is higher than that of under-loaded link.
- The price of link 2 decreases during the time period of 80s to 120s although the capacity is enlarged.

Thus, with the scheme in [1], there is no incentive for a service provider to enlarge the capacity. This is not helpful for the evolution of the Internet.

(a) The link price by the scheme by Low et al [1] with slightly enlarged link 2. (b) The link price by the proposed scheme with slightly enlarged link 2.

Figure 6.4: The comparison of link price with a slightly enlarged link.

On the other hand, it can be shown from Figures 6.2(b) and 6.4(b) that our proposed pricing system has the following two features:

- The price of under-loaded link is higher than that of congested link.
- The price of link 2 increases after its capacity is enlarged.

Therefore, with the proposed pricing system, each link has an incentive to increase its capacity because it can get more profit. This is very helpful for the evolution of the Internet.

6.5.3 The Two Priority Case

For the two priority case, we shall implement the second order pricing system defined by Equations (6.50) - (6.53) in discrete time as

$$\begin{aligned}\eta_{l,1}(k+1) &= \eta_{l,1}(k) + (\bar{x}_l(k) - c_{0l,1})_{\eta_{l,1}(k),0}^+, \\ p_{l,1}(k+1) &= p_{l,1}(k) + \gamma_l(c_{0l,1} - \lambda_l \eta_{l,1}(k+1) - \bar{x}_l(k)), \\ \eta_{l,2}(k+1) &= \eta_{l,2}(k) + (\bar{x}_{l,2} - c_{0l,2})_{\eta_{l,2},0}^+, \\ p_{l,2}(k+1) &= p_{l,2}(k) + \gamma_l(c_{0l,2} - \lambda_l \eta_{l,2} - \bar{x}_{l,2}(k)).\end{aligned}$$

Then $p_{l,1}(k)$ and $p_{l,2}(k)$ are bounded by their lower and upper bounds. When λ_l is set as 0, the system is degraded to the first order pricing system defined by Equations (6.27) and (6.28).

In this simulation, we set γ_l as 1×10^{-9} . Figures 6.5(a) and 6.5(b) show the traffic rates and link prices of the first order pricing system, i.e., $\lambda_l = 0$; while Figures 6.5(c) and 6.5(d) show those of a typical second order pricing system with $\lambda_l = 0.8$. Obviously, both systems can be used to achieve the desired stability. Moreover, it can be observed by comparing Figures 6.5(b) and 6.5(d) that the convergent speed of the second order system is faster than that of the first order system. To illustrate this feature clearly, we focus on the dynamics of $p_{2,2}$ during the time interval from 60s to 150s with different choices of λ_l . It can be seen from Figure 6.6(a) that

- (i). When there are new sources joining the network, the convergence tends to be faster with an increase in λ_l ;
- (ii). When there are sources leaving the network, the convergence speed is the same regardless of the choice of λ_l .

It can be observed from Figures 6.5(a) and 6.5(c) that during the time period of 80s to 120s, the dynamics of traffic are more stable with the second order pricing system. On the other hand, it can be seen from Figure 6.6(b) that the dynamics of $p_{2,1}$ have a bigger fluctuation with the increment of λ_l .

(a) The traffic rate with $\lambda_l = 0$. (b) The dynamics of link price with $\lambda_l = 0$. (c) The traffic rate with $\lambda_l = 0.8$. (d) The dynamics of link price with $\lambda_l = 0.8$.

Figure 6.5: The performance of the first and second order pricing systems.

Figure 6.7 illustrates the queueing delays of the two classes of traffic at each link. Since the class 2 traffic is always served first, its queueing delay is much smaller than that of class 1. Therefore, the proposed relative delay service can be achieved. Meanwhile, it is noticed that after 80 sec, the queueing delay of link 1 drops sharply. This is because after source 3 begins to send packets, it shares the

(a) The price dynamics of link 2 priority 2 traffic. (b) The price dynamics of link 2 priority 1 traffic.

Figure 6.6: The price dynamics of link 2 with different choices of λ_l .

Figure 6.7: The link delay of each priority.

bandwidth of link 2 with sources 1 and 2. This reduces the total traffic rate over link 1 and thus leads to a small queueing delay at link 1.

6.5.4 Impacts of Different Choices of Links' Adjustment Step Size

Finally, we test the effect of different choices of γ_l on the stability of the proposed high order pricing system. The experimental results are demonstrated in Figure 6.8 with different choices of γ_l with $\lambda_l = 0.8$. It can be shown that $p_{2,1}$ eventually converges to the equilibrium when γ_l is selected as 1×10^{-9} . However, when γ_l is selected as 1.5×10^{-9} or 2×10^{-9} , $p_{2,1}$ does not converge. Moreover, the fluctuation becomes larger with the increment of γ_l .

Figure 6.8: The dynamics of $p_{2,1}$ with different choices of γ_l .

6.6 Conclusion

In this chapter, a fully cooperative network was designed by using the new concept of generalized Nash Equilibrium. The SumNet works as follows: each source knows

that the network is under-loaded when the price does not fall, it thus increases the transmission rate accordingly to improve the utilization of the available bandwidth. Controversially, when each source knows that the network is over-loaded because the price falls, it decreases its transmission rate accordingly. On the other hand, each link increases its price when the system is under-loaded, and decreases its price when the system is over-loaded. In other words, the price goes down at the bottleneck links. Clearly, the price of under-loaded links can reach the maximal one when the network operates in equilibrium. This is fair to the under-loaded links as they provide better QoS and these links have the incentive to adjust the price according to the system's condition.

SumNet is reasonable from the Quality of Service (QoS) point of view, since the QoS of the network is fundamentally different from quality of goods in the conventional microeconomic system in the sense that the quality of each goods is always the same regardless of the number of customers while the QoS will change according to the overall load condition. Thus when the link is under-loaded and the QoS is better, the price should be high and vice versa. Clearly, our scheme is different from the conventional microeconomic system [68]. With SumNet, each link has the incentive to increase its capacity because it can get more profit. This would be very helpful for the evolution of the Internet.

However, the feedback information of SumNet is the sum of link prices, and it served as both the charging signal and the control signal. With the SumNet, a source that needs to pass through longer distances to its destination will get a smaller share of bandwidth at the bottleneck link. Obviously, this is unfair to these sources. Therefore, the whole network with the SumNet is still not very harmonious although sources and links are fully cooperative. In the following chapter, we will propose an alternative scheme, named MinNet, which is able to achieve weighted

6.6 Conclusion

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Max-Min fairness as well as retaining the advantages of SumNet's pricing system.

Chapter 7

MinNet: Harmonious and Stable Networks

7.1 Introduction

In this chapter, we shall provide an alternative pricing scheme that is called MinNet. The feedback information in the proposed MinNet includes a charging signal and a controlling signal. This is different from the MaxNet [81] and the eXplicit Control Protocol (XCP) [80] where only the control signal is feedback to each source. In our scheme, the sum of link prices will serve as the charging signal to determine the cost that each source should pay and the distribution of the profit among service providers. This is the same as the SumNet scheme discussed in Chapter 6, so that each link has the incentive to increase its capacity because it can get more profit. But we now have a new control signal. The value of the most congested link is used as the control signal to adjust the sending rate, and this is the same as MaxNet [81] and XCP [80]. Compared to the previous scheme based on the sum of prices [1], the MinNet can achieve Max-Min fairness in bandwidth share. Meanwhile, since different sources may have different willingness-to-pay, the concept of weighted

Max-Min fairness is also studied in the MinNet scheme. Our results show that the weighted Max-Min fairness can also be achieved when the sending rate is adjusted by using the price of the most congested link. This concept is very useful for multimedia applications over the Internet because most multimedia applications have the requirement of a minimal sending rate.

7.2 Network Model and Notations

In this chapter, we shall focus on the network with only one priority for conciseness of presentation. It can be extended to the multiple priority classes in the way similar to the SumNet scheme presented in Chapter 6, and we present the experimental results in the simulation parts.

The various parameters and notations of links and sources and delay quantities are the same as those defined in Section 6.2 except that they are now for the case of one priority. Because of the new control signal, additional notations are also required. For completeness, the complete list of notations are given as follows:

Links:

- $\mathbf{L}$: The set of links, $\mathbf{L} = \{1, 2, \dots, L\}$.
- $\mathbf{L}_s$: The set of links that source s uses, $\mathbf{L}_s \subseteq \mathbf{L}$.
- c_l : Capacity of link l .
- c_{0l} : Target capacity of link l . c_{0l} is set to be slightly smaller than c_l as recommended in [100] for better performance.
- p_l : Bandwidth price of link l . It is bounded by p_l^- (the lower bound) and p_l^+ (the upper bound), i.e. $p_l^- \leq p_l \leq p_l^+$. p_l is time-varying.

- Q_l : The auxiliary control signal of link l , $Q_l = p_l - p_l^+$. This is time-varying and is used by the sources to construct a control signal.
- $\bar{x}_l$: Total demand on link l .
- n_s : The most congested link on the path of source s , $n_s = \arg \min_{l \in \mathbf{L}_s} \{Q_l\}$. n_s is the *bottleneck link* or *controlling link* of source s .

Sources:

- Ω : The set of sources, $\Omega = \{1, 2, \dots, S\}$.
- Ω_l : The set of sources that uses link l , $\Omega_l \subseteq \Omega$.
- x_s : The transmission rate of source s . It is bounded by x_s^- (the lower bound) and x_s^+ (the upper bound), i.e. $x_s^- \leq x_s \leq x_s^+$.
- $\mathbf{x}$: The rate vector, $\mathbf{x} = [x_1, x_2, \dots, x_S]^T$.
- w_s : Weighting factor of source s .
- U_s : Utility functions of source s . $U_s : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a strictly concave function with a continuous second derivative. It is assumed that all the utility functions are the same with respect to the weighted transmission rate, i.e.

$$U_s(x_s) = U\left(\frac{x_s}{w_s}\right). \quad (7.1)$$

$U : \mathbb{R}_+ \rightarrow \mathbb{R}$ is also a strictly concave function with a continuous second derivative.

- q_s : the controlling signal for source s , $q_s = \min_{l \in \mathbf{L}_s} \{Q_l\}$.
- D_s : The demand function of source s , $D_s(\cdot) = w_s \dot{U}^{-1}(\cdot)$.

- $\bar{p}_s$: The sum of prices source s has to pay to use the necessary links.

Delays:

- $\tau_{s,l}^f$: The forward delay from source s to link l .
- $\tau_{l,s}^b$: The backward delay from link l to source s .
- τ_s : The round trip delay time of source s . $\tau_s = \tau_{s,l}^f + \tau_{l,s}^b$.

We assume that the network satisfies the following two assumptions.

Assumption 7.2.1. $\sum_{s \in \Omega_l} x_s^- < c_l$ for all $l \in L$, and

$$\frac{x_i^-}{w_i} = \frac{x_j^-}{w_j} ; \forall i, j \in \Omega. \quad (7.2)$$

Assumption 7.2.2. p_l^+ and p_l^- satisfy

$$w_s \dot{U}^{-1}(-\min_{l \in L_s} \{p_l^- - p_l^+\}) \geq x_s^- ; \forall s, \quad (7.3)$$

$$w_s \dot{U}^{-1}(0) \geq x_s^+ ; \forall s, \quad (7.4)$$

where $\dot{U}^{-1}(\cdot)$ is the inverse of $\dot{U}(\cdot)$, the first derivative of $U(\cdot)$.

7.3 The MinNet

For ease of exposition, we shall provide the MinNet scheme for the case of one service priority. It is easy to extend the MinNet to the multiple priority case, similar to that presented for SumNet in Chapter 6.

7.3.1 Cooperative Links

Same as the SumNet scheme, the price of link l is adjusted by

$$p_l(k+1) = \max\{p_l^-, \min\{p_l^+, p_l(k) + \gamma_l(c_{0l} - \bar{x}_l(k))\}\}. \quad (7.5)$$

As a result, the price is a non-decreasing function of QoS, i.e.,

- (i). When the system is under-loaded, i.e. $c_{0l} > \bar{x}_l(k)$, QoS can be guaranteed, the link can then increase the price;
- (ii). On the other hand, when the system is over-loaded, i.e. $c_{0l} < \bar{x}_l(k)$, QoS cannot be guaranteed, the link then decreases the price.

Thus, in the MinNet scheme, the links will cooperate with the sources and try to provide the desired QoS to all sources.

After the value of p_l is computed by Equation (7.5), it is used to compute the sum of the price $\bar{p}_s$ and to update the value of q_s , if necessary.

7.3.2 The Feedback Signals

Unlike the SumNet scheme, where the feedback signal is the sum of route's prices, two separate signals are now used in the MinNet scheme. The signal feedback from links to sources consists of two parts:

- (i). **Charging Signal:** This is the sum of the link prices.

At time t , the source s receives a charging signal of

$$\bar{p}_s(k) = \sum_{l \in \mathbf{L}_s} p_l(k - \tau_{l,s}^b), \quad (7.6)$$

Note that for simplicity, it is assumed that the price of link l is simply p_l and depends on the service providers. It can be set to be a more general function of p_l . Each packet has one space in the header to include this value. This will be used to determine the cost that each source shall pay and the distribution of the profit among the service providers.

- (ii). **Controlling Signal:** This is determined by the auxiliary controlling signal, and is used by each source to adjust its sending rate. That is, the controlling signal is

$$\begin{aligned} q_s(k) &= \min_{l \in \mathbf{L}_s} \{Q_l(k - \tau_{l,s}^b)\}, \\ &= \min_{l \in \mathbf{L}_s} \{p_l(k - \tau_{l,s}^b) - p_l^+\}. \end{aligned} \quad (7.7)$$

Each packet has another space in the header to include this value. It is initially set as 0 by source s . Once the packet is transmitted to link l , link l first computes the current $Q_l = (p_l(k) - p_l^+)$, then compares it to the corresponding value in the header. If the local value is smaller than the value carried by the packet, then the link updates the value in the header. Ultimately, the packet will contain the price information from the bottleneck link along the path. When the packet reaches the receiver, it is returned to source s (typically piggybacked on a transport protocol acknowledgement packet).

The most congested link for source s is the one with the lowest Q_l . That is, we define n_s of source s as $\operatorname{argmin}_{l \in \mathbf{L}_s} \{Q_l\}$, i.e. n_s is the most congested link in the path from source s to its destination. Same as XCP [80] and MaxNet [81], the sending rate of source s is determined by the price of link n_s .

7.3.3 Cooperative Sources

Source s prefers links that can provide the desired quality of service (QoS). On the other hand, source s should be able to pay for the cost. So instead of Equation (6.13), the sending rate is determined differently as described below.

The source first compute

$$\hat{x}_s(k) = D_s(-q_s(k)), \quad (7.8)$$

where the demand function of the source s is $D_s(\cdot) = w_s \dot{U}^{-1}(\cdot)$. The sending rate $x_s(k)$ is then adjusted as

$$\begin{aligned} x_s(k) &= \max\{\min\{x_s^+, \hat{x}_s(k)\}, x_s^-\} \\ &= \min\{x_s^+, \hat{x}_s(k)\} \end{aligned} \quad (7.9)$$

to meet the upper and lower bounds.

Two major differences between Equations (6.13) and (7.8) are identified as below.

- (i). The control signal in Equation (6.13) is the sum of prices while that in Equation (7.8) is the price of the bottlenecked link;
- (ii). A weighting factor of w_s is introduced in Equation (7.8).

Thus, the MinNet scheme is different from the SumNet scheme. We will show that the MinNet scheme is fair to all sources.

It can be shown from Equations (7.5) and (7.7) that q_s has the following features:

- If all link $l \in L_s$ are under-loaded, q_s increases.

- If there exists one link $l \in L_s$ that is congested, i.e. the source s is bottlenecked at the link, then q_s decreases.

Since $\ddot{U} < 0$, D_s is a decreasing function and hence Equation (7.8) implies that

- Source s knows that the system is under-loaded when q_s does not fall, it thus increases the sending rate accordingly to improve the utilization of the available bandwidth; and
- When source s knows that the system is over-loaded because q_s falls, it decreases the sending rate accordingly to guarantee the desired QoS of all sources.

Therefore, besides all sources are cooperative with each other, they also cooperate with the links to ensure the utilization of the available bandwidth.

7.3.3.1 The Relationship Between The Proposed Scheme and The MRF

The relationship between the proposed scheme (7.5), (7.7) and (7.8) and the MRF is given in the following proposition:

Proposition 7.3.1. *The proposed scheme (7.5), (7.7) and (7.8) is a type of MRF when the number of bottleneck links is 1.*

Proof. It can be illustrated from (7.8) that the increase policy and decrease policy are given by

$$\text{I: } x_s(k+1) = x_s(k) + (D_s(q_s(k+1)) - x_s(k)), \quad (7.10)$$

$$\text{D: } x_s(k+1) = x_s(k) - (x_s(k) - D_s(q_s(k+1))). \quad (7.11)$$

When that the number of bottleneck link is 1, it can be easily shown from (7.5) that

$$(x_s(k) - D_s(q_s(k+1))) \leq x_s(k), \quad (7.12)$$

$$\frac{\partial(x_s(k) - D_s(q_s(k+1)))}{\partial x_s(k)} = 1 - \gamma_l \dot{D}_s > 0. \quad (7.13)$$

Therefore, the proposed scheme is a type of MRF. $\square$

7.3.4 A Harmonious Network

The MinNet system is different from the conventional microeconomic system and the existing schemes in [1] [58]. With Equations (7.5) and (7.9), links and sources are harmonious, the QoS of all sources can be guaranteed, and the available resource is fully utilized.

The MinNet system has the following two features:

- (i). Sources increase the sending rates when the price is high, and reduce the sending rates when the price is low. Therefore, the sources try to maximize utilization payoffs of links.
- (ii). Links reduce the price when the sending rates of sources are relatively large, and increase the price when the sending rate of sources are relatively small. Thus, the links also try to maximize utilization payoffs of sources.

Therefore, the incentives of sources and links belong to the second type of generalized Nash equilibrium, and the network with the MinNet scheme is fully cooperative.

7.4 Weighted Max-Min Fairness in Steady State

In this section, we shall show that the MinNet system can achieve weighted Max-Min fairness by employing similar method as that used in the proof of Proposition 3.2.3. We first group all sources into N groups, $\tilde{\Omega}_r (1 \leq r \leq N \leq S)$ according to their sending rates in steady state and accordingly the links are all grouped into N groups, $\tilde{L}_r (1 \leq r \leq N)$.

The following features of $\tilde{L}_r$ can be easily derived from Equation (7.8):

- Two links i, j are in the same group $\tilde{L}_r$, if

$$Q_i = Q_j = \tilde{q}_r,$$

where $\tilde{q}_r$ is defined as the price of the group.

- For any 2 links $i \in \tilde{L}_{r_1}, j \in \tilde{L}_{r_2}$, with $r_1 < r_2$, then

$$Q_i < Q_j.$$

That is, $\tilde{q}_{r_1} < \tilde{q}_{r_2}$.

Equation (7.8) can thus be written as: for any source $s \in \tilde{\Omega}_r$,

$$\begin{aligned} x_s &= D_s(-\tilde{q}_r) \\ &= w_s * \dot{U}^{-1}(-\tilde{q}_r). \end{aligned} \tag{7.14}$$

We shall first study the dynamics of the price for each link l in group $\tilde{L}_1$, i.e. the group with the smallest $\tilde{q}$. Note that if $l \in \tilde{L}_1$, and $s \in \Omega_l$, then $s \in \tilde{\Omega}_1$. It can

7.4 Weighted Max-Min Fairness in Steady State

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be shown from Equation (7.5) that

$$p_l(k+1) = p_l(k) + \gamma_l(c_{0l} - \sum_{s \in \Omega_l \cap \tilde{\Omega}_1} D_s(-\tilde{q}_1)). \quad (7.15)$$

Since $p_l(k+1) = p_l(k)$ in steady state, it follows that for any source $i \in \Omega_l$,

$$x_i = \frac{w_i}{\sum_{s \in \Omega_l \cap \tilde{\Omega}_1} w_s} c_{0l}.$$

Clearly, source i will share the bandwidth according to its weight.

We then study another link in group $\tilde{L}_2$. Clearly, if $l \in \tilde{L}_2$ then $l \notin \tilde{L}_1$. Further, if $s \in \Omega_l$, then $s \in \tilde{\Omega}_1 \cup \tilde{\Omega}_2$. Similar to Equation (7.15), it can be derived that

$$\begin{aligned} p_l(k+1) &= p_l(k) + \gamma_l(c_{0l} - \sum_{s \in \Omega_l \cap \tilde{\Omega}_2} D_s(-\tilde{q}_2) \\ &\quad - \sum_{s \in \Omega_l \cap \tilde{\Omega}_1} D_s(-\tilde{q}_1)). \end{aligned}$$

Then for any source $i \in \Omega_l \cap \tilde{\Omega}_2$, we can obtain

$$x_i = \frac{w_i}{\sum_{s \in \Omega_l \cap \tilde{\Omega}_2} w_s} (c_{0l} - \sum_{s \in \Omega_l \cap \tilde{\Omega}_1} D_s(-\tilde{q}_1))$$

in steady state. Clearly, the sources which belong to $\tilde{\Omega}_1$ are not controlled nor bottlenecked at this link, and we have already shown that their sending rates are maximized. All other sources receive a weighted share of the remaining bandwidth, which is also based on a weighted Max-Min fair allocation. Thus the allocation of rates to sources passing this link l is weighted Max-Min fair.

Similar to Proposition 3.2.3, we now show that the MinNet is Max-Min fair

w.r.t. $[w_1, w_2, \dots, w_S]$ by using the induction method.

Assume that the capacity allocation to sources which pass through $\tilde{L}_{(n-1)}$, $\tilde{L}_{(n-2)}$, $\dots$, $\tilde{L}_1$ is weighted Max-Min fair. Then, we shall show that the capacity allocated to sources in the set $\tilde{\Omega}_n$ is also weighted Max-Min fair. In general, links in $\tilde{L}_n$ will have a set of sources traversing them which are controlled by other links whose price is lower and a set of sources which are controlled by the current link. Thus, we can get the generalized capacity allocation as

$$x_i = \frac{w_i}{\sum_{s \in \Omega_l \cap \tilde{\Omega}_n} w_s} (c_{0l} - \sum_{s \in \Omega_l \cap \tilde{\Omega}_1} D_s(-\tilde{q}_1) - \dots - \sum_{s \in \Omega_l \cap \tilde{\Omega}_{(n-1)}} D_s(-\tilde{q}_{n-1})).$$

Notice that the sources passing through the links with lower prices are not bottlenecked at the links. We have assumed these sources already have weighted Max-Min allocations at other links. The remaining capacity is allocated to the sources which are bottlenecked at the link with respect to their weights, thus satisfying the weighted Max-Min condition for such sources. Therefore the weighted Max-Min condition is satisfied by all sources traversing the link. By induction, all allocations are weighted Max-Min fair and global fairness is achieved.

7.5 Simulation Results

We implement the proposed scheme in Ns-2 with the utility function

$$U(x) = \alpha \log(x); \alpha > 0. \quad (7.16)$$

Thus, the demand function is

$$D_s(k) = \frac{-w_s \alpha}{\min_{l \in L_s} \{p_l(k) - p_l^+\}}. \quad (7.17)$$

The simulation uses the topology shown in Figure 7.1. Sources 1 and 2 run for 120 sec and start at 0 sec and 40 sec, respectively. Sources 3 and 4 start at 60 sec and 80 sec respectively and run until the end. The minimal transmitting rate of the sources is 1×10^6 bps while the maximal transmitting rate is 1.6×10^7 bps. The utility function of the source is set to $10^6 \log(x)$. The link capacity is $c_l = 2.0 \times 10^7$ bps and the target value, c_{0l} , is set to 1.9×10^7 bps which is slightly smaller than the actual link capacity as recommended in [100] for a better performance. The stepsize γ_l used by the link to adjust its link price is set to 1.0×10^{-8} . The minimum price and the maximum price are set at 0.013 and 1 respectively. These settings are used in the following simulations unless otherwise stated.

Figure 7.1: The topology of simulation.

7.5.1 Stability and Efficiency

Figures 7.2(a) and 7.2(b) show the traffic rates and the link prices of MinNet in the case that the weighting factors of all sources are 1's. It can be observed that during 0-60 (meanwhile the source 2 starts), the state of the two links are identical and the price is the same since source 1 and source 2 have same path. During

60-80, source 3 starts. Thus link 2 has 3 traffics while link 1 has 2. This makes link 2 the congested one of the two links. According to the MinNet, the link 1 should has a high price. Since 80 sec, source 4 starts and source 1 and source 2 stops, the state of link 1 and link 2 are always the same. Thus it leads to same link price and source rate.

As a comparison, Figure 7.2(c) shows the results with the SumNet scheme presented in Chapter 6. Clearly, both schemes can be used to *actually maximize* the profits of sources and links.

However, it can be seen from Figure 7.2(c) that during the 60-120 sec time period when all sources are running, sources 1 and 2 get a smaller traffic rate than those of sources 3 and 4. This is because sources 1 and 2 pass two congested links, they thus pay a higher price than sources 3 and 4 which pass only one congested link. As illustrated in Figure 7.2(a), the MinNet scheme has overcome the problem. And during the same period, all sources get equal share of the bandwidth, thus it achieves Max-Min fairness.

We shall test the case where different sources have different weighting factors. In this scenario, $w_s (1 \leq s \leq 4)$ are give by

$$w_1 = w_2 = 2, w_3 = 1, w_4 = 3, \quad (7.18)$$

and the values of $x_s^- (1 \leq s \leq 4)$ are

$$x_1^- = x_2^- = 2 \times 10^6, x_3^- = 1 \times 10^6, x_4^- = 3 \times 10^6. \quad (7.19)$$

From Figure 7.2(d), we observe that

- (i). Sources 1 and 2 always have the same traffic rate, since their weights are the

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(a) The traffic rates of sources under MinNet. (b) The link prices of links under MinNet.

(c) The traffic rates of sources under SumNet. (d) The traffic rates of sources with different weight under MinNet.

Figure 7.2: The convergence and weighted Max-Min fairness of MinNet.

same and they pass through the same path.

- (ii). During the 60-80 sec time period, link 2 is congested, source 1 (and also source 2) and source 3 share the bandwidth with a ratio of 2 : 1.
- (iii). During the 80-120 sec time period, link 1 becomes congested. Source 1 (and also source 2) and source 4 share the bandwidth with a ratio of 2 : 3, while on the under-loaded link 2, source 3 takes all the bandwidth available.
- (iv). During the 120-160 sec time period when source 1 is turned off, link 1 is

congested. Sources 2 and 4 share the bandwidth with a ratio of 2 : 3, while on the under-loaded link 2, source 3 takes all the bandwidth available.

Thus, our scheme can be used to achieve a weighted Max-Min fairness.

7.5.2 Profit with Enlarged Capacity

We next consider the effect of an increment of capacity on the link price. The capacity of link 2 is slightly enlarged to 2.1×10^7 bps and the target value c_{0l} is set at 2.0×10^7 bps. The traffic rates and link price are illustrated in Figures 7.3(a) and 7.3(b) when the network reaches its equilibrium. Compared to those given in Figures 7.2(a) and 7.2(b), we can see that link 2 now has a higher price after its capacity is enlarged. As a comparison, we also give the prices of MaxNet [118], with the utility function $\alpha \log(x + 1)$, before and after an enlarged link capacity as illustrated in Figures 7.3(c) and 7.3(d). Same as the scenario tested in the MinNet, the capacity of link 2 is also enlarged in MaxNet. However, the price of link 2 dropped in MaxNet when the capacity is enlarged. This implies that the service provider has no incentive to increase the link capacity. Thus, the MinNet scheme could be helpful for the evolution of the Internet.

7.5.3 Stability with Propagation Delay

In the previous simulations, we assume that the adjustments are carried out synchronously. In this subsection, we set the propagation delay of each link at 0.03s. The other setting is unchanged except that the step size γ_l is set as 5.0×10^{-9} . Figure 7.4 illustrates the traffic rate of sources while the weighting factors are given in Equation (7.18). Clearly, MinNet is still convergent with an appropriate γ_l when there is propagation delay.

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(a) The traffic rates of sources under MinNet (b) The link price of links under MinNet with enlarged link 2.

(c) The link price of links under MaxNet (d) The link price of links under MaxNet without enlarged link 2.

Figure 7.3: The traffic rate and price with and without an enlarged link.

Figure 7.4: The traffic rates of sources with propagation delay.

7.5.4 The Extension to Multi-Priority High Order System

Similar as SumNet in the previous Chapter, MinNet can be easily extended to Multi-priority case.

We shall implement the link pricing system as the second order pricing system defined by Equations (6.50) - (6.53). In discrete time, they are

$$\begin{aligned}\eta_{l,1}(k+1) &= \eta_{l,1}(k) + (\bar{x}_l(k) - c_{0l,1})_{\eta_{l,1}(k),0}^+, \\ p_{l,1}(k+1) &= p_{l,1}(k) + \gamma_l(c_{0l,1} - \lambda_l \eta_{l,1}(k+1) - \bar{x}_l(k)), \\ \eta_{l,2}(k+1) &= \eta_{l,2}(k) + (\bar{x}_{l,2} - c_{0l,2})_{\eta_{l,2},0}^+, \\ p_{l,2}(k+1) &= p_{l,2}(k) + \gamma_l(c_{0l,2} - \lambda_l \eta_{l,2} - \bar{x}_{l,2}(k)).\end{aligned}$$

Then $p_{l,1}(k)$ and $p_{l,2}(k)$ are bounded by their lower and upper bounds. When λ_l is set as 0, the system is degraded to the first order pricing system defined by Equations (6.27) and (6.28).

The control signal is then given as

$$q_{s,1}(k) = \min_{l \in L_s} \{p_{l,1}(k - \tau_{l,s}^b) - p_{l,1}^+\}, \quad (7.20)$$

$$q_{s,2}(k) = \min_{l \in L_s} \{\min\{p_{l,1}(k - \tau_{l,s}^b) - p_{l,1}^+, p_{l,2}(k - \tau_{l,s}^b) - p_{l,2}^+\}\}. \quad (7.21)$$

At source side, the same utility function as Equation (7.16) is used. Thus, the demand function is

$$D_{s,1}(k) = \frac{-w_s \alpha}{q_{s,1}(k)}, \quad (7.22)$$

$$D_{s,2}(k) = \frac{-w_s \alpha}{q_{s,2}(k)}. \quad (7.23)$$

We test the high order system with the topology shown in Figure 7.1 and same

setting for links. The weights of source 1, 2, 3 and 4 are 1, 2, 2 and 1 respectively. α is set at 5×10^5 . The results is shown in Figure 7.5. Expect the convergence to the weighted Max-Min fairness, we can observe the same phenomenon of high order system as in high order SumNet system, i.e.

- (i). When there are new sources joining the network, the convergence tends to be faster with an increase in λ_l ;
- (ii). When there are sources leaving the network, the convergence speed is the same regardless of the choice of λ_l .

7.6 Conclusion

In this chapter, we have proposed the MinNet scheme. In this MinNet scheme, the price of the bottleneck link is sent to the sources so that they can adjust their sending rates, instead of the sum of the prices in the SumNet scheme. With proper setting, our scheme can be used to achieve Max-Min bandwidth sharing hence it avoids the unfair allocation in the SumNet scheme to the sources with shorter paths. Also, by introducing a weighting factor for each source, the scheme can achieve a weighted Max-Min bandwidth fairness, thus it is attractive to sources with different bandwidth requirements. And at the same time, the MinNet obeys the second type of generalized Nash Equilibrium. Thus, the MinNet is a cooperative and harmonious network.

(a) The traffic rates of priority 1 of sources under first order system.

(b) The traffic rates of priority 1 of sources under second order system where $\lambda_l = 0.8$.

(c) The traffic rates of priority 2 of sources under first order system.

(d) The traffic rates of priority 2 of sources under second order system where $\lambda_l = 0.8$.

(e) The link price in case of two priorities under first order system.

(f) The link price in case of two priorities under second order system where $\lambda_l = 0.8$.

Figure 7.5: The traffic rate and link price of MinNet in case of priorities.

Chapter 8

MRF Based MinNet

8.1 Introduction

From the results of Chapters 6 and 7, we note that the MinNet scheme inherits the advantage of the SumNet pricing system, and it avoids the unfair profit for the bottleneck resources. It gives service providers incentives to improve their services and hence it is helpful for the evolution of the Internet. Meanwhile, from the perspective of resource allocation, it achieves a weighted Max-Min fairness which overcomes the preference of SumNet for the short route users. Hence it is fair for the users of network. However, the adopted source adaptation strategy is a little bit complicated, which may not be preferred for an environment with limited computational capacity such as PDA, intelligent mobile phone etc.

In Chapter 4, we have proposed the framework of Monotonic Responsive Function (MRF) for designing both TCP-friendly and multimedia-friendly source adaptations schemes, which is given as

$$x_s(k+1) = \begin{cases} x_s(k) + w_s f(\frac{x_s(k)}{w_s}); & \text{if system is under-loaded} \\ x_s(k) - w_s g(\frac{x_s(k)}{w_s}); & \text{otherwise} \end{cases}, \quad (8.1)$$

where $f(x)$ is a monotonically non-increasing function and $g(x)$ is a monotonically non-decreasing function, with $f(x) > 0$, and $x \geq g(x) > 0$.

The MRF is simpler than the source adaptation schemes used in both SumNet and MinNet. In this chapter, we shall use MRF as the source adaptation under the MinNet structure and propose a switched algorithm to simplify the pricing system.

8.2 A Simple Framework for Relative Differentiated Services

We consider a network that is the same as MinNet in Chapter 7, which has L unidirectional links and S sources. We shall first provide a simplified version of the MinNet by integrating the MRF into it. The simplified MinNet is then extended to design a dynamic pricing system and a simple source adaptation scheme to provide both relative differentiated bandwidth service and relative differentiated delay service.

8.2.1 The Simplified MinNet

The MinNet has many attractive features, such as it can be applied to design a harmonious network with links willing to increase their capacities to support new applications, a relative differentiated bandwidth service can be provided, and so on. However, it is still a little complex to implement the MinNet directly into the current Internet. This can be overcome by combining the MRF and the MinNet to design a new framework as presented below:

- (i). Cooperative link pricing system.

Each link now computes the following two local signals which are different

from the MinNet scheme.

(a) Controlling signal.

A single bit is used in the MRF to represent the current network status, and this is different from the MinNet where the price of the bottlenecked link is used to describe the current network status. It is desired to inherit this advantage of the MRF so that the controlling signal is given by

$$Q_l(k) = \begin{cases} 1; & \text{if } c_{0l} > \bar{x}_l(k) \\ 0; & \text{otherwise} \end{cases} \quad (8.2)$$

Link l compares Q_l to q_s in the header of the packet from any source $s \in \Omega_l$. If Q_l is smaller than q_s carried by the packet, then the link updates the value of q_s . When the packet reaches the receiver, it is returned to source s (typically piggybacked on a transport protocol acknowledgement packet).

(b) Charging signal.

Although the continuous charging signal in MinNet can be used to achieve a better performance, the discrete charging signal is favored by sources due to its simplicity. The pricing system is thus simplified as

$$p_l(k) = \begin{cases} p_l^+; & \text{if } Q_l(k) = 1 \\ p_l^-; & \text{otherwise} \end{cases} \quad (8.3)$$

In other words, the price of link l is fixed at p_l^+ when the QoS is guaranteed and is fixed at p_l^- when the QoS cannot be guaranteed.

(ii). Feedback signals.

Same as the MinNet scheme, the feedback charging signal is the sum of the utilized links' local charging signal and the feedback control signal is the lowest local control signal of the utilized links, as defined in Equations (7.6) and (7.7).

(iii). Source adaptation scheme.

The source adaptation algorithm in MinNet is replaced by the MRF

$$x_s(k+1) = \begin{cases} x_s(k) + w_s f(\frac{x_s(k)}{w_s}); & \text{if } q_s(k) = 1 \\ x_s(k) - w_s g(\frac{x_s(k)}{w_s}); & \text{otherwise} \end{cases} \quad (8.4)$$

It can be shown from Equations (8.3) and (8.4) that the pricing system (8.3) and the source adaptation scheme (8.4) are loosely coupled by the control signal $Q_l(k)$. And same as XCP [80] and MaxNet [81], the sending rate of source s is determined by the status of link n_s .

With the above three simplifications, the simplified MinNet is sufficient for the non-real time services and can be readily implemented into the Internet. However, for new real time applications, it is also necessary to extend the simplified MinNet such that it can be used to provide relative differentiated delay services. This is studied in the next subsection.

8.2.2 The Extension of the Simplified MinNet

We now extend the simplified MinNet into the case of two priority classes like SumNet.

8.2.2.1 Cooperative Link Pricing System

Since there are two different priority classes, two digits are required to indicate the status of each link. Similar to Equation (8.2), the values of $Q_{l,1}(k)$ and $Q_{l,2}(k)$ are set according to the overall load on link l as

$$Q_{l,1}(k) = \begin{cases} 1; & \text{if } c_{0l,1} > \bar{x}_l(k) \\ 0; & \text{otherwise} \end{cases}, \quad (8.5)$$

$$Q_{l,2}(k) = \begin{cases} 1; & \text{if } c_{0l,2} > \bar{x}_{l,2}(k) \\ 0; & \text{otherwise} \end{cases}. \quad (8.6)$$

After the values of $Q_{l,1}(k)$ and $Q_{l,2}(k)$ are computed by Equations (8.5) and (8.6), they are used to update the values of $q_{s,1}(k)$ and $q_{s,2}(k)$ if necessary.

We also extend the simplified pricing system (8.3) to the case that there are two different priority classes. The pricing system is given as follows:

$$p_{l,1}(k) = \begin{cases} p_{l,1}^+; & \text{if } Q_{l,1}(k) = 1 \\ p_{l,1}^-; & \text{otherwise} \end{cases}, \quad (8.7)$$

$$p_{l,2}(k) = \begin{cases} p_{l,2}^+; & \text{if } Q_{l,2}(k) = 1 \\ p_{l,2}^-; & \text{otherwise} \end{cases}. \quad (8.8)$$

Clearly, the new prices $p_{l,1}(k)$ and $p_{l,2}(k)$ are also non-decreasing functions of QoS, i.e., the prices are high when the QoS is guaranteed, and are low when the QoS cannot be guaranteed.

Similarly, after the values of $p_{l,1}$ and $p_{l,2}$ are computed by Equations (8.7) and (8.8), they are used to compute the sum of price $\bar{p}_{s,1}$ and $\bar{p}_{s,2}$ to construct the feedback charging signal.

8.2.2.2 The Feedback Signals

Similar to the MinNet scheme, separate charging signal and controlling signal are sent from links to sources. In other words, the signal feedback from links to sources consists of two parts:

- (i). **Charging Signal:** This is the sum of the link prices.

Since there are two different priority classes, at time k , the source s could receive two charging signals as

$$\bar{p}_{s,1}(k) = \sum_{l \in L_s} p_{l,1}(k - \tau_{l,s}^b), \quad (8.9)$$

$$\bar{p}_{s,2}(k) = \sum_{l \in L_s} (p_{l,1}(k - \tau_{l,s}^b) + p_{l,2}(k - \tau_{l,s}^b)). \quad (8.10)$$

Since each packet can only be marked by one priority, each packet only needs one space in the header to include this value. This will be used to determine the cost that each source should pay and the distribution of the profit among the service providers. The total cost that source s needs to pay at time k is given by $x_{s,1}(k)\bar{p}_{s,1}(k) + x_{s,2}(k)\bar{p}_{s,2}(k)$.

- (ii). **Controlling Signal:** This is determined by the auxiliary controlling signals, and is used by each source to adjust its sending rate. That is, the controlling signals are

$$q_{s,1}(k) = \min_{l \in L_s} \{Q_{l,1}(k - \tau_{l,s}^b)\}, \quad (8.11)$$

$$q_{s,2}(k) = \min_{l \in L_s} \{Q_{l,2}(k - \tau_{l,s}^b)\}. \quad (8.12)$$

Each packet has another space in the header to include these values. Both of them are initially set as 1's by source s . Once the packet is transmitted to link l , link l compares the price carried by the packets with its current price and updates $q_{s,1}$ and $q_{s,2}$ if necessary.

It should be pointed out that two different schemes are provided by (7.21) and (8.12) to compute the controlling signal for priority 2 class. Fairness can be achieved among all traffic by (7.21) while it can be only achieved within each class by (8.12).

8.2.2.3 Cooperative Source Adaptation Scheme

Similar to the simplified MinNet scheme, MRF is applied to design source adaptation scheme.

The sending rates of $\hat{x}_{s,1}(k)$ and $\hat{x}_{s,2}(k)$ are first computed by

$$\hat{x}_{s,1}(k) = \begin{cases} x_{s,1}(k-1) + w_{s,1}f_{s,1}(\frac{x_{s,1}(k-1)}{w_{s,1}}); & \text{if } q_{s,1}(k) = 1 \\ x_{s,1}(k-1) - w_{s,1}g_{s,1}(\frac{x_{s,1}(k-1)}{w_{s,1}}); & \text{otherwise} \end{cases}, \quad (8.13)$$

$$\hat{x}_{s,2}(k) = \begin{cases} x_{s,2}(k-1) + w_{s,2}f_{s,2}(\frac{x_{s,2}(k-1)}{w_{s,2}}); & \text{if } q_{s,2}(k) = 1 \\ x_{s,2}(k-1) - w_{s,2}g_{s,2}(\frac{x_{s,2}(k-1)}{w_{s,2}}); & \text{otherwise} \end{cases}, \quad (8.14)$$

i.e. for each priority, sources employ the MRF algorithm to control the sending rate for the priority. Then it is bounded by the upper and lower bounds:

$$x_{s,1}(k) = \max\{\min\{\hat{x}_{s,1}(k), x_{s,1}^+\}, x_{s,1}^-\}, \quad (8.15)$$

$$x_{s,2}(k) = \max\{\min\{\hat{x}_{s,2}(k), x_{s,2}^+\}, x_{s,2}^-\}. \quad (8.16)$$

The users may choose to use only one priority. If users choose to use two priorities simultaneously, they can use weights to control the two MRF algorithms. The base algorithms of MRF for one source or two sources are not required to be identical.

8.3 Simulation Results

8.3.1 The Implementation of the Proposed Scheme

We have implemented the proposed scheme in Ns-2 simulator. The topology used in the simulation is shown in Figure 8.1. There are two links with capacity 20Mbps and propagation delay 10ms. The values of $p_{l,1}^-$ and $p_{l,2}^-$ are both 0.013, and those of $p_{l,1}^+$ and $p_{l,2}^+$ are 1. The target value, $c_{ol,1}$, is set to 19Mbps which is slightly smaller than the actual link capacity as recommended in [100], and $c_{ol,2}$, at $c_{ol,1}/4$.

The link pricing system and the source adaptation algorithm require periodic adjustments of the price, control signal, and the transmission rates. In our implementation, the adjustment of links and sources are carried out at every T_l and T_s time intervals. That is, at every T_l time interval, the links compute their local control signals and prices by Equations (8.5)-(8.8), while the sources adjust their transmission rates every T_s time interval by Equations (8.13)-(8.16) according to the feedback signals they received in the previous time period. Each time the sources finish the adjustments, they send out a “carrier” packet with high priority which contains their current rates. When links receive such a packet, they use the information to update the local database plus updating the charging signal and pricing signal of the packet. The packet is feedback to the source when it arrives at the destination. Sources do their adjustments according to the feedback information.

Figure 8.1: The topology of simulation.

In the simulation, all sources are using the following AIMD algorithm:

$$f_{s,1}(x_{s,1}) = \frac{x_{s,1}^-}{20} ; f_{s,2}(x_{s,2}) = \frac{x_{s,2}^-}{20} , \quad (8.17)$$

$$g_{s,1}(x_{s,1}) = \frac{x_{s,1}}{8} ; g_{s,2}(x_{s,2}) = \frac{x_{s,2}}{8} , \quad (8.18)$$

where the values of $x_{s,1}^-$ and $x_{s,2}^-$ are both 1 Mbps and those of $x_{s,1}^+$ and $x_{s,2}^+$ are both 16 Mbps.

Sources as well as links adjust their transmitting rates/prices every 50ms. The Round Trip Time of sources is smaller than the adjustment period. Thus, it guaranteed that in most cases, the proposed scheme works in the synchronous case. In the following simulation, this setting will be used unless otherwise stated.

8.3.2 The Overall Utilization

With the proposed control scheme, the average utilization of the proposed scheme should be smaller than the target value. Since the rate increase of AIMD is linear, we can easily estimate the utilization by computing the average of the maximal rate and the minimal rate. Obviously, the maximal rate is 19Mbps. The minimal rate is related to the ratio of T_l and T_s (assuming that the adjustment periods of all the links and sources are identical). If T_l and T_s are the same, it means that sources only decrease their rates once for the control signal of 0. The utilization is

$(19 + 19 \times (1 - 0.125))/2 = 17.81\text{Mbps}$. If T_l is twice that of T_s , sources decrease their rates twice for a control signal of 0, since during the T_l time period, the sources probe twice and the link state does not change. Hence, we have $(19 + 19 \times (1 - 0.125) \times (1 - 0.125))/2 = 16.77\text{Mbps}$. Figure 8.2 shows the total utilization of these two cases. Thus, with a careful design, we can achieve a good overall utilization of the system.

Figure 8.2: The total traffic rate with different ratio of T_l and T_s .

8.3.3 Relative Differentiated Bandwidth Services

Two cases are studied.

Case 1 Between two sources sharing the same path.

In the simulation, sources 1 and 2 start at 0 sec and 40 sec, respectively, and both run for 120 sec. The weighting factors for priorities 1 and 2 of source 1 are 1.5 and 1, and 1 and 0.5 for source 2. The experimental results are shown in Figure 8.3. Clearly, before the starting of source 2, source 1 takes all the available bandwidth. Since the throughput of priority 2 is guaranteed,

priority 1 can only take the remaining bandwidth of priority 2. After source 2 starts to send traffic, there are two competing traffic at each priority. We can see that the ratio between two sources at one priority is determined by the weights of sources at the priority. Therefore, a relative differentiated bandwidth service can be achieved when two sources share the same path.

Figure 8.3: Two sources with different weights for each priority.

Case 2 Among sources that have different paths.

For simplicity, all the weighting factors for both priorities of each source are set at 1's. All these 4 sources start at random time and source 1 stops at 120 sec. All the other sources run for 200 sec. We shall further study the following two sub-cases:

Case 2.1 The global synchronization case.

In this case, all sources and links carry out their adjustments at the same time and the information they used are identical, links may generate command 0 at the same time. Thus, sources 1 and 2 receive the same number of command 0 (i.e. decrease the sending rate) as sources 3 and

4. In such a case, the weighted Max-Min fairness is achieved and this is shown in Figure 8.4.

Figure 8.4: The throughput of sources under global synchronization.

Case 2.2 The asynchronous case.

The experimental results are shown in Figure 8.5. We note that sources 1 and 2 get smaller bandwidths than sources 3 and 4. In other words, the relative differentiated service cannot be achieved. The reason behind this is that links 1 and 2 may generate the control command of 0 at different time, sources 3 and 4 only response to that from one link while sources 1 and 2 response to both links.

8.3.4 Relative Differentiated Delay Services

Figure 8.6 shows the link delay of each priority in the previous simulation. Obviously, class 2 traffic gets a relatively smaller delay. Although the link delay of all packets is not related to the adjustment of the proposed scheme and vice versa, the pricing system is an efficient tool to guarantee that all packets are marked with the proper priority.

Figure 8.5: The throughput of sources with the same weights for each priority under the asynchronous case.

8.3.5 The Pricing System

The link price reflects the change of the link state and Figure 8.7 shows the average link price per second in the previous simulation. Since the link capacity can always guarantee the minimal requirement, the value of $p_{l,2}(k)$ is always 1. We can see that after 120 sec when the source 1 is turn off, the price of priority 1 is higher than that before 120 sec. This is because that, after source 1 is closed, the competition for bandwidth eases, thus the QoS for each priority increases. Accordingly, the price increases. This implies that the pricing system has the following feature: When the capacity is enlarged, the price can also be increased. The service provider then has more chance to earn more profit. This is very helpful for the evolution of the Internet.

Figure 8.6: The link delay of each priority.

8.4 Conclusion

A simple MinNet combined with the MRF source adaptation framework have been designed to provide both relative differentiated bandwidth service and relative differentiated delay service. In the service model, all traffic is divided into two priority classes, with each link transmits priority 1 traffic only if all priority 2 traffic has been served, and each source is allowed to split its traffic between the two priorities. Both priority classes are charged according to a differentiated pricing system. The pricing is piecewise, i.e., it is fixed at relatively larger values when the network is under-loaded and relatively smaller values when the network is over-loaded. The new pricing system could be favored by sources due to its simplicity. Meanwhile, the MRF framework is used as source adaptation algorithm which is both TCP-friendly and multimedia-friendly. The framework only requires slight changes to the current Internet infrastructure and therefore can be easily implemented within the current Internet infrastructure.

Similar to the SumNet and MinNet schemes, the proposed scheme is based on

Figure 8.7: The price of links.

the assumption that both sources and links intend to maximize their opponents' utilization payoffs. Each source adjusts its sending rate according to its requirement on the sending rate and the status of the current network to improve the overall utilization of the whole network. To guarantee the QoS of other sources, each source tries to use priority 1 and only uses priority 2 if necessary. Each link has the incentive to increase its capacity for new real time applications because it can earn more profit by charging more for providing better QoS.

Chapter 9

Conclusions and Recommendations For Future Research

9.1 Conclusions

The Internet is evolving into a multi-service network. The simplex “best effort” service is no longer sufficient to support the various requirements from the new applications and hence the next generation Internet should be capable of providing guaranteed QoS. In this thesis, we studied the behavior of networks by employing novel source adaptation and pricing schemes to enhance the differences of various services. Specifically our contributions are as follows:

- (i). In the resource allocation problems, the concept and the property of weighted Max-Min fairness were studied. By such a fairness measure, the priorities of sources are identified by weighting factors and the network maximizes the service of sources that may receive the poorest service. Besides global fairness, a local fairness which is bounded by a single resource bound was also studied and a novel fairness index is introduced as a numerical measure

and used for further analysis of source adaptation schemes.

- (ii). A source adaptation framework named Monotonic Responsive function (MRF) was proposed for TCP's congestion control. By introducing a weighting factor to each source, the MRF enhances the weighted fairness in relation to the TCP's equal fairness concept, i.e. the sources will share the capacity of the bottle-neck link according to predefined weighting factors. The conditions for MRF to be TCP-friendly and TCP-compatible were proposed. Hence, the MRF framework can be used to design TCP responsive functions which are TCP-friendly and multimedia-friendly.

As an example of the possible applications of the results, a simple scheme for bandwidth differentiation service is proposed which requires no changes from the routers and thus it is easy to be deployed over the current Internet.

We also examined more general responsive functions to extend the choice of TCP's congestion control. Specifically, TCP's responsive functions are divided into 4 types according to the monotony of their responsive functions: type-1 responsive functions, which include most of the existing protocols such as AIMD, GAIMD, BCC, CYRF etc. are convergent; type-4 responsive functions are not convergent; type-2 and type-3 responsive functions are conditionally convergent, and their convergent conditions are derived. The results can be used to enhance the service of both the current TCP or MRF source adaptations.

- (iii). Two harmonious pricing schemes based on the new concept of generalized Nash Equilibrium were proposed. The schemes are cooperative in the sense that besides all sources cooperate with each other, the sources and the links also cooperate with each other. The new pricing systems allow us to formulate

a reasonable pricing framework to charge the sources. With the scheme, each link has an incentive to increase its capacity because it can get more profit. This can be very helpful for the evolution of the Internet. Also, the system takes QoS into consideration and can be easily extended to differentiated services. This distinguishes our pricing schemes from existing cooperative schemes.

The two specific distributed schemes proposed are: the SumNet scheme which aims to optimize the utility by solving the optimization problem through a gradient projection method; and the MinNet scheme which is formulated to achieve a global weighted Max-Min fair throughput allocation. Both schemes are stable, efficient and require no per-flow state at the routers. Thus, it is simple and easy to be deployed over the Internet. Finally, a simplification of the MinNet scheme with the source adaptation determined by the MRF framework is also proposed.

9.2 Recommendations for Future Research

In this section, we present some research directions which can be pursued in the future.

- (i). In Chapters 4 and 5, we have analyzed TCP's responsive functions under the classical Chiu and Jain's synchronous structure [42, 44, 51, 125]. Although studies have shown that the results are still correct under more complex environment, the assumptions are too ideal and they limit the study of more complex responsive functions. There are already many proposal to study the behavior of TCP-like switch control under randomized environments [16, 45, 126]. However, most of them model the Internet channel as a random process,

which generates a series of dropping events, and studied the behavior of a single source's responses. This approach has two problems: 1) The accuracy of representing the random Internet dropping as a random process, and 2) It omits the impact of sources' behavior on the random process. Thus, the behavior of responsive functions under a source-coupled random environment may be one of our future work.

- (ii). Optimization of utilization and global fairness are both important criteria when designing a pricing system. However, the SumNet scheme can only achieve the optimization of users' utility while the MinNet scheme is only able to achieve global fairness. In the real world, the users may prefer one characteristic at the cost of a small decrease in another. Hence, a combined system of the SumNet and MinNet schemes, that can provide a well controlled trade-off between optimization of utility and global fairness would be more likely to meet the users' requirements.
- (iii). In Chapters 6 and 7, we have used the concept of Lyapunov stability and simple control theory to prove convergence and stability. However, the status of the Internet are always changing and time-varying. Hence, to achieve better performance, advanced control theory for switching and time-varying systems may be needed in order to analyze the dynamic behavior of the SumNet and MinNet schemes, especially the convergence speed of the schemes.
- (iv). In Chapter 8, the proposed simplified MinNet scheme is able to achieve weighted Max-Min fairness conditionally. Further research should be pursued to overcome this conditional fairness.

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- (a). Chen, C., Li, Z. G., and Soh, Y. C., "MRF: A framework for source and destination based bandwidth differentiation service," *IEEE/ACM Transactions on Networking*, vol. 15, no. 3, pp. 697–708, June 2007.
- (b). Chen, C., Li, Z. G., and Soh, Y. C., "Analysis of monotonic responsive functions for congestion control," in *The 13th International MultiMedia Modeling Conference (MMM2007)*, ser. 2, vol. 2, Singapore, Jan. 2007, pp. 383–392.
- (c). Chen, C., Li, Z. G., and Soh, Y. C., "TCP-Friendly source adaptation for multimedia applications over the internet," in *15th International Packet Video Workshop (PV2006)*, Hangzhou, China, Apr. 2006.
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- (g). Li, Z. G., Chen, C., and Soh, Y. C., "Cooperative Control Based Charging Scheme for Differentiated Service," *Submitted to IEEE Transactions on Automatic Control*, 2007.
- (h). Li, Z. G., Chen, C., and Soh, Y. C., "Pricing Based Differentiated Bandwidth and Delay Services With Weighted Max-Min Fairness," *Submitted to IEEE/ACM Transactions on Networking*, 2007.

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