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Channel Equalizer Design for ADSL Transceivers

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Master of Engineering

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Summary

G.DMT and G.Lite Asymmetric Digital Subscriber Line (ADSL) modems and Very-high Speed Digital Subscriber Line (VDSL) modems rely on discrete multitone modulation (DMT). Discrete multitone modulation is a multicarrier technique which makes efficient use of the channel, maximizing the throughput by sending different numbers of bits on different subchannels. In an ADSL discrete multitone receiver, a time-domain equalizer (TEQ) eliminates intersymbol interference (ISI) and intercarrier interference (ICI) by shortening the effective length of the channel impulse response. TEQ design in this thesis focuses on ADSL downstream (higher data rate) transmission. Many TEQ design methodologies have been proposed. One such TEQ design method is the minimum mean-squared error (MMSE). Another FIR TEQ, Maximum shortening SNR TEQ (MSSNR) attempts to replace the conventional MMSE TEQ in the ADSL modem design with the advantage of having a much lower complexity and higher output bit rate performance. In this thesis, we review the state-of-art TEQ designs (MSSNR and min-ISI), which optimize a measure of channel capacity at the output of the TEQ. The Minimum ISI (min-ISI) method outperforms all other finite impulse response (FIR) filter TEQ schemes in achievable bit rate to that of min-ISI with large number of TEQ taps. The objective of this research work is to analyze TEQ design schemes and propose certain modification to the MSSNR

design scheme in order to enhance the ADSL system performance in terms of the achievable bit rate and reduce the implementation complexity. A new efficient matrix computation design for Maximum Shortening SNR (MSSNR) TEQ is proposed and further studied through theoretical analysis. A comparison of arithmetic operations shows that our new implementation method outperforms other methods in computation complexity in the ADSL system. A new TEQ design using an iterative power method approach that requires fewer computations and has an arbitrary TEQ length is proposed. Our proposed power method presents a generally higher achievable bit rate and much stable performance than inverse power method. Bell system's carrier serving area (CSA) loop model has been used in the simulation to test the validity and effectiveness of the proposed design algorithm and iterative power method. The simulation results indicate that the power method TEQ yields generally higher bit rate than the optimum SSNR method and similar bit rate with the inverse power method.

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List of Abbreviations

ADSL	Asymmetrical Digital Subscriber Lines
AMI	Alternate Mark Inversion
ATM	Asynchronous Transfer Mode
AWGN	Additive White Gaussian Noise
CP	Cyclic Prefix
CAP	Carrierless Amplitude/Phase
CMA	Constant Modulus Algorithm
CSA	Carrier Serving Area
DAVIC	Digital Audio-Visual Council
DLC	Digital Loop Carrier
DMT	Discrete Multitone
DOCSIS	Data Over Cable Service Interface Specification
DSL	Digital Subscriber Lines
DVB	Digital Video Broadcasting
FEQ	Frequency Domain Equalizer
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FSK	Frequency Shift Keying
GSNR	Geometric Signal to Noise Ratio

HDSL	High-bit-rate DSL
ICI	Intercarrier Interference
IEEE	Institute of Electrical and Electronic Engineers
ISI	Intersymbol Interference
ISDN	Integrated Services Digital Network
ITU	International Telecommunication Union
LMS	Least Mean Square
MAC	Multiply-Accumulate Operation
MBR	Maximum Bit Rate
MCM	Multicarrier Modulation
MFB	Matched Filter Bound
MGSNR	Maximum Geometric SNR
Min-ISI	Minimum Intersymbol Interference
MMSE	Minimum Mean Squared Error
MPEG	Moving Picture Experts Group
MSSNR	Maximum Shortening SNR
NSP	Network Service Providers
OFDM	Orthogonal Frequency Division Multiplexing
PCM	Pulse Code Modulation
POTS	Plain Old Telephone System
PSK	Phase Shift Keying
PSTN	Public Switched Telephone Network
PTE	Per-tone Equalizer
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying

RADSL	Rate-adaptive ADSL
RAS	Remote Access Service
RF	Radio Frequency
SIR	Shortened Impulse Response
TCM	Time Compression Multiplexing
TEQ	Time-domain Equalizer
TIR	Target Impulse Response
UEC	Unit Energy Constraint
UTC	Unit Tap Constraint
VDSL	Very-high Speed Digital Subscriber Lines
VoIP	Voice over Internet Protocol

Chapter 1

Introduction

1.1 Motivation

Nowadays customers can choose a suitable broadband access system from a number of technologies available. Traditional Telecommunication Companies utilize the telephone copper twisted pair network, which is based on the existing public switched telephone network (PSTN) infrastructure. It offers access to the Internet through the voiceband dial-up modems at rates up to 56 kbps and the telephone voice service. In addition, the telephone network offers high-speed broadband access in the form of digital subscriber line (DSL) technologies, such as the high-speed DSL version 2 at 1.544 Mbps to and from the service provider and asymmetric DSL (ADSL) at up to 8Mbps downstream and 1 Mbps upstream transferring rate. A higher speed version of DSL family, known as very-high speed DSL, runs over much shorter distances and offers rates up to 23 Mbps from the service provider. ADSL adopts discrete multitone (DMT) modulation as its modulation scheme. Being a multicarrier modulation method, DMT partitions the physical transmission channel into many virtually independent, flat narrowband subchannels. At the receiver, the transmitted symbols

may overlap with the adjacent ones because of the non-ideal physical channel spread. Time domain equalizer (TEQ) together with a cyclic prefix (CP) is used to combat and counteract the intersymbol interference (ISI) at the expense of introducing some redundancy. TEQ is also a crucial element in the ADSL modem design to achieve higher bit rate performance. The real time implementations of some ADSL TEQ training methods are complicated due to the redundant matrix computations in optimizing the objective function. This thesis focuses on the new TEQ algorithm design to enhance the output bit rate and to reduce the number of Multiply-Accumulate (MAC) operations required.

1.2 Objectives

Many TEQ design schemes are applicable for improving the ADSL transceiver communication performance. This study focuses on the equalization and modulation aspects with the following objectives:

- To propose a new matrix computation algorithm in the process of searching for an optimum equalizer design to reduce the implementation complexity in Chapter 5;
- To propose a new iterative TEQ scheme to solve eigenvalue and eigenvector and obtain higher and stable output data rate with arbitrary TEQ length and lower complexity in Chapter 6;
- To evaluate and optimize the performance of the proposed scheme through simulation and comparative analysis.

In essence, the goal of the project is to propose a new matrix manipulation technique and a new TEQ algorithm that requires lower computation and achieves higher data rates for ADSL or VDSL DMT modulated systems.

1.3 Major Contribution of the thesis

The major contributions of the thesis are:

- An extensive analysis of many previously reported TEQ design methodology for the ADSL system.
- The ADSL transceiver system with different TEQ design schemes is simulated and the simulation results are analyzed, with a focus on the output data rate performance.
- The proposed matrix computation scheme in the MSSNR TEQ design is analyzed theoretically and is found to outperform others in terms of the number of Multiplication and Addition calculations required.
- A new TEQ scheme using an iterative power method is proposed to obtain an arbitrary size of the TEQ taps (TEQ length) and stable performance.

1.4 Organization of the Thesis

This thesis is organized as follows. Chapter 2 introduces prevailing broadband access methods, and focuses on ADSL system and channel equalization and multicarrier modulation techniques. Chapter 3 discusses existing time domain equalizer design methods and the issue of intersymbol and intercarrier interference removal in multicarrier modulation systems. Chapter 4 details the simulation environment and the parameters used in the transmission rate performance analysis of all of the TEQ

design methods studied. Chapter 5 presents a new computationally efficient algorithm for the calculation of the matrices in the TEQ design that uses the Maximum Shortening SNR method. Chapter 6 proposes a new method adopting power iteration in solving maximum eigenproblem for an arbitrary length of TEQ. The proposed method suggests a TEQ design with an arbitrary equalizer length and more stable performance. Chapter 7 concludes this thesis with a summary of the project contributions and future work recommendations.

Chapter 2

Broadband Access Systems

2.1 High Speed Wireline Communication Systems

In the 1970s, telecommunications virtually meant plain old telephone service only. Technology primarily comprised conventional copper wires and electromechanical switches. Residential Internet usage has reached over 50 percent of the American population in 2002 with the majority using a dial-up connection [1]. However, broadband connections doubled between August 2000 and September 2001, totaling about 11 percent of all Internet connections [1]. As more residential users become accustomed to Internet access and users become reliant on the World Wide Web, the demand for high-speed, reliable and constant connection has also risen. Over the past 6 years, this demand has caused the development of higher speed residential connections such as DSL and cable modem. The growth in residential subscriptions to Internet also motivated multimedia applications' advance and prosperity. Sophisticated audio and video coding methods have reduced the bit rate requirements for audio and video transmission. This in turn motivated the evolution of modern communication systems to achieve these requirements.

Table 2.1 High-speed data communication standards with service data rate and brief description

Standard	Data Rates	Description
V.32 (voiceband modem)	9.6 kbps	full-duplex using PSK
V.34 (voiceband modem)	33.6 kbps	channel precoding
V.90 (voiceband modem)	56 kbps down 33.6 kbps up	pulse code modulation
ISDN (Integrated Service Digital Network)	144 kbps	two 64kbps and one 16kbps channel
HDSL(Hight-bit-rate DSL)	1.544 Mbps US / 2.048 Mbps Europe	two wire pairs, reach of 3658m
HDSL2 (High-bit-rate DSL)	1.544 Mbps US / 2.048 Mbps Europe	one wire pair, reach of 3658m
ADSL (Asymmetric DSL)	1.5 Mbps – 8 Mbps down 16 kbps – 1 Mbps up	one wire pair, reach of 5486m requires splitters
RADSL (Rate-adaptive ADSL)	1.5 Mbps – 8 Mbps down 16 kbps – 640 kbps up	adaptive rates, one wire pair, requires splitters
G.lite (splitterless ADSL)	up to 1.5 Mbps down up to 512 kbps up	no splitter
VDSL (Very high rate DSL)	6 Mbps – 22 Mbps down 3 Mbps – 13 Mbps up	one wire pair, reach of 1371 m
DOCSIS (Data over cable)	27 Mbps or 36 Mbps down 320 kbps – 10.24 Mbps up	cable TV infrastructure
IEEE 802.14 (Cable modem)	27 Mbps or 36 Mbps down 320 kbps – 20.48 Mbps up	cable TV infrastructure

Many technologies have been developed for high-speed communications as shown in Table 2.1. Logically voiceband modems is the first solution making use of the original telephone network infrastructure. Due to the bandwidth constraint of traditional telephone voice lines, the data rate of voiceband modems cannot satisfy the requirements of most of the residential and business applications. Integrated Service Digital Network (ISDN) instead provides higher bit rates than voiceband modems. Initially T1/E1 lines offered only business solution for having higher bit rates than ISDN. However the cost of installation and maintenance of T1/E1 lines are relatively high, which motivated the service providers to search for cheaper alternatives. DSL technologies offered cheaper options to T1/E1 lines and higher transfer rates than ISDN. The first DSL technology was High-bit-rate DSL (HDSL), which was targeted for business applications. HDSL is the most mature xDSL. It has been fully standardized by ANSI (American National Standards Institute) and ETSI (European Technical Standards Institute). HDSL requires two twisted pair lines to carry data at a rate of 1.544 Mbps both upstream and downstream. Another alternative of HDSL technology uses three twisted pair copper lines to carry 2.048 Mbps. The first DSL technology dedicated for residential customers was Asymmetric DSL (ADSL). Initially planned for video-on-demand applications, ADSL has evolved into a major Internet access technology. The emerging Very-high-rate DSL (VDSL) standard is intended to be a bridge between fiber and copper communication technology. Asymmetric VDSL is targeted to deliver 13/22 Mbps downstream and 3 Mbps upstream. For symmetric VDSL, the targeted data rate is 6, 9 or 13 Mbps. The main broadband access market competitor for DSL technology is cable modem. Coaxial cable technologies enable cable TV service provider aggregate TV interactive service,

access to the Internet, voice-over Internet Protocol (VoIP) and always-on service at connection speeds ranging from 0.5-1.5 Mbps.

2.1.1 Voiceband Modem

Voiceband modems are the most common type of modem, and by far the most widespread: over 100 million are in daily use, and 95% of Internet access was via voiceband modems in 2003 [56]. Voiceband modems will remain the primary method of Internet access for many years to come. Voiceband modems connect directly to the central office (CO) voice switch, can dial any number and hence work on any plain old telephone service (POTS) line. However, voiceband modems cannot exceed 56kbps, they tie up one phone line (costing the subscriber more), and they tie up a port on the CO voice switch. Moreover, users often stay online for hours rather than minutes (voice calls typically last only a few minutes). Thus they cost the CO significantly more than voice calls, they degrade overall performance, and they force the phone companies to upgrade their existing expensive voice switches.

Voiceband modems have existed for almost 30 years, the earliest being Bell 103 modem transferring data at 300 bps using the frequency shift keying (FSK) modulation scheme and capable of supporting duplex operations. Recently the V.34 modem supports 33.6kbps. A typical dial-up connection via a voiceband modem is shown in Figure 2.1. It's worth noting that 56kbps is the fastest speed achievable over a dial-up POTS line. A telephone channel utilizes the frequency band from about 200 Hz to 3400Hz. This bandwidth is enough to transmit human voice.

Bell 202 was the first modem to achieve 1200bps using half duplex FSK modulation scheme. Vadic Inc. introduced the VA 3400 modem in 1973, which was the first modem using phase shift keying (PSK) and featured full duplex 1200bps

transmission [2]. V.22b was doubled the bit rate to 2400 bps later. The V.32 was the first modem to use trellis coding and featured echo cancellation, which enabled transmission of data in both directions at the same time in the same frequency bands. V.32 modems achieved a bit rate of 9600 bps. Constellation shaping, bandwidth optimization and channel-dependent coding enabled the V.34 modem to achieve 28.8 kbps, which increased to 33.6 kbps in 1995.

Figure 2.1 Local access with voiceband modem and central office

33.6 kbps was considered the highest possible data rate that could be achieved on a telephone voice channel until 56 kbps modems were introduced in 1996. The V.90 standard for these modems did not appear until 1998. The V.90 modems, which are also called Pulse Code Modulation (PCM) modems, are asymmetric. It supports a downstream bit rate of 56.6 kbps and an upstream bit rate of 33.6 kbps. A V.90 modem operates at 8000 symbols/s. An eight-bit code word is carried by every symbol. However, only 7 bits carry information, and the eighth bit is reserved for control purpose. In practice, PCM modems rarely achieve 56.6 kbps and are generally

limited to 50 kbps [2]. The fastest voiceband modems now available use one of two technologies: K56flex or X2. K56flex, developed by Lucent Technologies and Rockwell, is the most widely supported 56kbps modem protocol today, supported by over 700 leading PC, modem, and remote access server manufacturers and Internet Service Providers (ISPs) worldwide. The next voiceband modem technology standard will be V.PCM, which is a version of 56kbps modems being standardized by the Telecommunications Industry Association (TIA). V.PCM will provide better interoperability between 56K modems from different vendors, meaning your connection will be faster more often.

2.1.2 Integrated Services Digital Network (ISDN)

Integrated Services Digital Network (ISDN) is a digital network providing voice and data communications at rates of up to 1.544 (T1 rates in the US) or 2.048 Mbps (E1 rates in Europe) for Primary Rate Interface (PRI) service (actually slightly less since one 64kbps B channel is used for signaling). It was introduced by CCITT (now ITU) in 1976. Initially ISDN systems were designed on time compression multiplexing (TCM) or alternate mark inversion (AMI). However, 2B1Q (2 binary 1 quaternary) and 4B3T (4 binary 3 ternary) transmission offered greater loop reach and therefore were adopted in the standard. Many corporate users rely on T1/E1/PRI service for Internet connectivity to link their distant enterprise locations. Individuals in many areas can order Basic Rate Interface (BRI) ISDN, which provides two 64 kbps B channels (for data or voice) and one 16kbps D channel (for signaling: who's calling, number dialed, and so on). BRI modems use the existing POTS lines to the phone company, but the CO must have ISDN line cards installed, instead of typical POTS line cards, a substantial expense for the phone company. Two B channels can be

combined into one 128 Kbps data channel via the BOUNDING technique (defined by the Bandwidth ON Demand Interoperability Group), yielding a high-speed data pipe for select customers. BRI can operate only up to 3 miles from the CO, limiting deployment in a service area. Basic rate ISDN uses one of the four available signal levels to represent two bits, hence 2B1Q. It also supports full duplex communication using echo cancellation. The bandwidth used is about 80 kHz. Provided that the loops are unloaded, basic rate ISDN modems can handle bridge taps by using an adaptive equalizer.

It's certain that an analog telephone network can be easily connected to one or more data networks – like X.21, X.25, X400. But all these data networks have their proper infrastructure, are separated from one another and thus are not integrated. In contrast to them, the advantage of ISDN is that it can integrate more services into a single network. Figure 2.2 illustrates the idea of services integration in ISDN system.

Figure 2.2 Services integration in ISDN. TA stands for terminal adaptors. S is the reference point between user terminals and the NT2. U is the reference point between NT1 devices and line-termination equipment in the carrier network.

Two interface techniques are used at the subscribers' location: the Universal (U) and the Subscriber/Termination (S/T). The U interface plugs directly to the phone

line. The S/T interface requires a separate Network Termination (NT-1) unit to connect to the U interface at the phone line, but it allows up to eight BRI devices to share the line. Setting up a BRI channel can be complex, since there are so many options to select and configure. BRI is in many cases the best data access available at moderate cost. Although BRI has been available for over a decade, there are fewer than one million subscribers in the U.S. so far. Even with the demand for Internet bandwidth, phone companies have been slow to deploy BRI since it ties up their voice switches with lengthy data calls. Nonetheless, as the cost comes down, and as setup becomes more standardized, ISDN will gain more users.

Three methods are used to increase the transmission rate of ISDN – basic rate ISDN transmission extension (BRITE), mid-span repeater, and extended-range basic rate ISDN. BRITE uses time division multiplexed channels and digital loop carrier to extend the reach of basic rate ISDN. A mid-span repeater can extend the loop reach by a factor of two. Extended range ISDN uses more advanced digital communication methods to extend the range of basic rate ISDN; e.g. trellis coding enables transmission of 160 kbps for up to 8.5km without a repeater.

2.1.3 Digital Subscriber Line

The latest and most promising digital technology, Digital Subscriber Line (DSL) modems, provide a quantum leap over ISDN or voiceband modem data bandwidth. Asymmetric DSL (ADSL), the leading contender, delivers up to 8 megabits-per-second (Mbps) downstream (to the subscriber) and up to 1 Mbps upstream (to the network). ADSL modems save the phone companies money because they combine voice and broadband data on each existing phone line. Within five years from 2003, a

significant portion of U.S. households will have high-speed data connections to the Internet via ADSL modems. Following are the important features of ADSL:

- A single local loop (the wires that transmit to the local CO) can provide both voice and data. Lifeline voice is retained—the phone will work in a power outage, but the data connection will be down. Thus only one local loop per subscriber is required.
- Data is modulated at higher frequencies (up to 1.1 MHz) than voice signals, and can be passed over 18,000 foot local loops to the subscriber. Typical ADSL will be "rate adaptive", meaning that the fastest connections depend on the quality and length of the local loop.
- A splitter at both ends separates the voice (below 4 kHz) from the higher frequency data: voice and data to and from the user pass over the single POTS line to the CO, where voice is sent to the CO voice switch and data is sent to the network via ATM. Thus, phone companies do not need to install more local loops, do not have to buy more voice switches, and will not have degraded service due to data traffic through their voice switches. Subscribers get better data service, even if they are online all the time.

There are several versions of xDSL technologies where "x" stands for a different technology. The first member of DSL family is HDSL. It was developed with the data rate of 1.544 kbps using 2B1Q transmission, which supported repeaterless transmission up to 3500m using the same signal processing techniques as ISDN. Of all xDSL forms, ADSL is the most significant for mass deployment because voice and data coexist on the same local loop. Ironically, ADSL was first developed for digital video to the home, but video alone did not drive up demand as the Internet

has. However, when ADSL is available, there will be virtually unlimited video sources available to the subscriber.

Unlike video-on-demand applications, Internet browsing does not have a fixed bit rate requirement that needs to be supported. Rate-adaptive ADSL automatically determines the highest rate it can provide over a given loop. Rate-adaptive ADSL (RADSL) supports downstream rates up to 7-10 Mbps and upstream rates up to 512-900 kbps. To separate the data band from the Plain Old Telephone System (POTS) band, a splitter has to be installed at the customer side. This expensive process motivated a low rate ADSL standard that can avoid the installation of the splitter. Splitterless ADSL or G.lite targets a data rate of 1.5 Mbps downstream primarily for Internet applications. ADSL forum has suggested a system reference model as shown in Figure 2.3

Figure 2.3 ADSL system Model where ATU-C refers to the ADSL transmission Unit at the network end and ATU-R refers to ADSL transmission Unit at the customer premises end. ATU-R is the ADSL modem where TEQ is located.

ADSL was the first DSL standard to adopt multicarrier modulation. Very High Rate DSL (VDSL) has been standardized to use multicarrier modulation for its sole

line code. VDSL is essentially a higher speed version of ADSL. And it is proposed as a way to connect a consumer to a fiber optical communication network in the consumer's neighborhood [3].

2.1.4 Cable Modem

The demand in the high-speed communication market has pushed the deregulation of the telecommunications and cable companies and increased competition between network service providers, which has caused all Network Service Providers (NSP) to find new revenue streams. Cable companies, in an effort to remain competitive, have exploited their existing cable network and have developed technology that allows for the delivery of IP traffic to residential homes. By using standard radio frequencies (RF), which cable companies understand well, cable providers can offer extremely fast Internet access through the use of cable modems. Initially, cable TV system could not support two-way communication for duplex application. One ad-hoc solution uses POTS voice band modems for the uploading data and the cable channel for the downstream. In 1994, the IEEE proposed the 802.14 cable TV Media Access Control and Physical Protocol Working Group to formulate standards [4]. Their standard was finalized in 1997 and an international cable modem standard was developed in the following year. Digital Video Broadcast (DVB)/Digital Audio-Visual Council (DAVIC) technology is the incumbent European standard for digital set-top boxes and is now starting to be employed for cable modems. As the name suggests, DVB focuses on digitized video delivery and requires MPEG II framing.

Cablelabs formed their own group to propose their own standard for cable modems. They released the Data Over Cable System Interface Specification (DOCSIS) in early 1996. Both the DOCSIS and IEEE standards are similar in the

physical layer but have differences in the media access control layer. The IEEE standard is based on Asynchronous Transfer Mode (ATM) whereas the DOCSIS standard supports variable length packets for the delivery of Internet Protocol (IP) traffic. Although better in some aspects, the IEEE ATM approach has a higher implementation cost than DOCSIS. Therefore, nearly all of the major cable modem vendors use the DOCSIS standard [2].

According to the DOCSIS standard, the downstream channel uses 64 or 256 Quadrature Amplitude Modulation (QAM) on a carrier of 6 MHz. The data rate is either 27 or 36 Mbps. The upstream modulation method is Quadrature Phase Shift Keying (QPSK) and 16 QAM on a variable carrier between 200 kHz and 3.2 MHz. Data rate is between 320 kbps and 10 Mbps [5]. All cable users on the same local area cable network share this data rate.

The major advantage for cable modem technology is its use of the Ethernet protocol at the network layer to move IP traffic. This allows NSPs to rely on the customer's PC to handle network traffic at the application layer without any significant investment by the cable company [6]. Additionally, it makes cable modems extremely easy to connect to existing PCs. Cable networks also have an excellent advantage in their ability to scale the network. Finally, cable networks have the added benefit of the "always-on" network connection. Although cable networks use modems and modem technology, they are in a connectionless state similar to a LAN environment. The most apparent disadvantage to current cable networks are the way in which bandwidth is shared across users within a service node. Although the bandwidth associated with a cable network can reach speeds of 30 Mbps downstream and 10 Mbps upstream, this bandwidth is shared over the number of users within the node [7].

Figure 2.4 shows how the cable modem fits into the home cable setup. The cable modem is located between the radio frequency (RF) module and the Ethernet card so that Internet and other data communications traffic can be managed separately from the video signals.

Figure 2.4 Cable Modem setup with cable splitter separating the two different applications of cable TV and internet services

2.2 Channel Equalization

The all-pass assumption made in the non-dispersive AWGN channel is rarely practical. Due to the scarcity of the frequency spectrum, we usually filter the transmitted signal to limit its bandwidth so that efficient sharing of the frequency resource can be achieved. Moreover, many practical channels are bandpass and, in fact, they often respond differently to inputs with different frequency components, i.e., they are dispersive. We have to refine the simple AWGN (or non-dispersive) model to accurately represent this type of practical channels. One such commonly employed refinement is the dispersive channel model

$$r(t) = u(t) * h_c(t) + n(t)$$

where $u(t)$ is the transmitted signal, $h_c(t)$ is the impulse response of the channel, and $n(t)$ is AWGN with a two sided power spectral density $N_0/2$. In essence, we model the dispersive characteristic of the channel by a linear time-invariant filter with impulse response $h_c(t)$. The simplest dispersive channel is the bandlimited channel of which the channel impulse response $h_c(t)$ is that of an ideal lowpass filter. This lowpass filtering smears the transmitted signal in time causing the effect of a symbol to spread to adjacent symbols when a sequence of symbols is transmitted. The resulting interference, intersymbol interference (ISI), degrades the bit error rate performance of the communication system. There are two major ways to mitigate the detrimental effects of ISI. The first method is to design bandlimited transmission pulses, which minimize the effect of ISI. We will describe such a design for the simple case of bandlimited channels. The ISI-free pulses obtained are called the Nyquist pulses. The second method is to filter the received signal to cancel the ISI introduced by the channel impulse response. This approach is generally known as equalization.

Telephone lines, ADSL lines and cable lines all cause some distortion in transmitted signals because of nonideal channel impulse response. The channel's frequency response $C(f)$ can be expressed as [8]

$$C(f) = A(f)e^{j\theta(f)} \quad (2.1)$$

where $A(f)$ and $\theta(f)$ are the amplitude response and the phase response, respectively. An ideal channel that does not distort the transmitted signal is obtained when $A(f)$ is a constant and $\theta(f)$ is a linear function of frequency, f .

Figure 2.5 Intersymbol interference caused by dispersion. The left two graphs show that if the transmitted symbols are placed far enough, there will be no overlaps at the output. Whereas the right two graphs illustrates the case of overlapping at the output caused by the short transmitting interval between successive input symbols.

When signals are transmitted through a non-ideal channel, the channel may disperse the signal in such a way that a pulse interferes with adjacent pulses at the sampling instants, causing intersymbol interference (ISI), as shown in Figure 2.5. In Figure 2.5, if the transmitted signal is “well placed”, there will not be any ISI at the output. But most of the time, in the pursuit of higher output data rate we want to “place” the input pulses closer to one another. And this causes the problem of intersymbol interference. The received bits are different from the transmitted bits. At the receiver, we may employ a linear filter with adjustable coefficients to compensate for the channel distortion. The filter coefficients from measurements made on the channel characteristics. These adjustable filters are called channel equalizers.

2.2.1 Telephone Lines

The structure of a feedforward equalizer used in a telephone line transmission system is shown in Figure 2.6. If the frequency response of an equalizer is $E(f)$, then $|E(f)|$

Figure 2.6 Structure of a feedforward equalizer

must compensate for the channel distortion. A possible solution would be to make the equalizer frequency response equal the inverse of the channel impulse response, i.e.,

$$E(f) = \frac{1}{C(f)} \tag{2.2}$$

Equation (2.2) implies that the equalizer is the inverse function of the channel response so that the linear channel distortion can be compensated. An ideal inverse channel filter can completely eliminate ISI caused by the channel. Since it forces the ISI to be zero at the sampling times, the equalizer based on (2.2) is called the zero-forcing equalizer. In the ADSL system, zero forcing equalizer is the frequency domain equalizer.

The disadvantage of the zero-forcing equalizer is that it ignores the presence of additive noise. So the use of a zero-forcing equalizer may amplify the noise significantly. If $C(f)$ in (2.2) is small in a frequency range, then the channel equalizer,

$E(f) = 1/C(f)$, gives a large gain in the same frequency range where noise presents.

Therefore, the noise in this frequency range is amplified as a result.

Another solution to compensate channel distortion is to use a minimum mean square error (MMSE) equalizer. Figure 2.7 shows a basic model for a communication system that contains an equalizer. A digital message m_k is transmitted through an analog channel. The received signal y_k is corrupted by additive noise represented by n_k . The receiver equalizes the distortion in the channel and then detects the symbols (sequence of bits) that were transmitted. If the output from the equalizer is $f(y(k))$ as shown in Figure 2.5 and the desired response at the output of an equalizer is the transmitted symbol $s(k)$ in Figure 2.6, the k th sample of the error signal between $f(y(k))$ and $s(k)$ is

Figure 2.7 A simple equalizer model

$$e(k) = f(y(k)) - s(k) \quad (2.3)$$

Then, the mean squared error (MSE) is defined as

$$MSE = E\{e^2(k)\} \quad (2.4)$$

where $E\{\cdot\}$ is the expectation operator. We can obtain the minimum MSE solution by differentiating the MSE in (2.4) with respect to the equalizer coefficients, setting the derivative to zero, and solving for the equalizer coefficients.

Both zero-forcing and MMSE equalizers are linear equalizers to combat ISI. When a channel has a spectral null (i.e., $C(f) = 0$), the linear equalizers may not compensate ISI sufficiently. An alternative is to use a nonlinear equalizer such as a

decision-feedback equalizer. Neural networks may also be used as the decision function in Figure 2.6.

The computation time needed to adapt the equalizer coefficients increases dramatically as the length of a channel increases. Therefore, shorter channels are desired, particularly for real-time transceiver design. Channel shortening will be discussed later.

2.2.2 DSL Lines

Figure 2.8 shows the typical channel characteristic using carrier-serving-area (CSA) digital subscriber loop 2. In an ideal case, the channel frequency response $A(f)$ is constant. In practice, the channel frequency response over the entire bandwidth range is not constant. Its frequency response, as shown in Figure 2.8 (b), is not flat. The CSA loop 2 impulse response contains 512 taps. To invert this 512-tap channel, the channel equalizer mentioned in Section 2.2.1 is an all-pole IIR filter obtained from (2.2) with 511 poles, which may not be stable since the poles could be outside the unit circle. The IIR filter might also amplify the channel noise. An FIR filter is used to approximate the IIR filter. The FIR filter will have to be long enough, and small changes in IIR coefficients may require large changes in FIR coefficients. To make the equalizer problem tractable for a broadband channel, discrete multitone (DMT) modulation is proposed. DMT modulation is the standard for data transmission in ADSL and VDSL system [9]. Using DMT modulation, the channel is divided into a large number of parallel, virtually independent and approximately flat subchannels. The non-ideal channel responses cause ISI between two adjacent subchannels.

(a) Impulse response of CSA loop 2

(b) Frequency response of CSA 2

Figure 2.8 Impulse and frequency response of carrier-serving-area loop 2

A guard interval, called the cyclic prefix (CP) is used to combat ISI and ICI. We assume the channel impulse response has a known length L . The prefix consists simply of copying the last $L-1$ values from each symbol and prepending them in the

same order to the front of the symbol. By having this buffer of essentially junk data in the front, the convolution of the impulse response with the signal at the end of a symbol does not affect any of the actual data at the beginning of the next symbol. In addition, by repeating the last elements at the beginning, the first real “data” elements of each symbol experience overlap with the “end” of the symbol, just as in cyclic convolution. This means the linear convolution of the channel impulse response with the concatenated symbol becomes concatenated cyclic convolutions of the impulse response with the individual symbols. Since cyclic convolution directly corresponds to multiplication in the frequency domain, this has great import with respect to equalization, as we will see later. After the time-domain signal passes through the channel, it is broken into the parallel symbols and the prefix is simply discarded. In the ADSL standard [9], the CP is a copy of the last 32 samples and the symbol length is 512 samples. Figure 2.9 shows the principle of cyclic prefix.

Fig 2.9 Cyclic prefix. The initial N samples of a symbol are appended at the back of the symbol.

2.2.3 Cable Lines

High-speed data transmission via coaxial cable also suffers from ISI introduced by the spread of the impulse response of the cable channel. In the IEEE 802.14 standard for cable modems, quadrature amplitude modulation (QAM) is recommended. An

adaptive equalizer in a cable modem for combating ISI consists of a feedforward equalizer combined with a decision feedback equalizer as shown in Figure 2.10. There are 16 complex taps; the feedforward equalizer and decision feedback equalizer each have eight taps. There are no training sequences required so that equalization for cable modems is blind [10], [11].

Figure 2.10 Equalizer for Cable modem

Single-carrier modulation is suggested by IEEE 802.14 standard since it is market-ready and the technology is well understood and developed. Theoretically, multicarrier modulation such as DMT has advantages in performance. IEEE 802.14 officially formed a research group to investigate multicarrier modulation for future application. Therefore, DMT modulation and equalization methods may be used in future cable modems [10].

2.3 Discrete Multitone Modulation

High-speed transmission technologies mentioned in the previous section require broadband channels. Inter-symbol interference (ISI) is a major drawback associated with the applications of broadband channels. This undesirable effect is caused by the spectral shaping of the channel. The variation of magnitude and phase responses of

the channel over frequency causes adjacent symbols to interfere with each other at the receiver. Two approaches or remedies to combat ISI are full channel equalization and multicarrier modulation (MCM).

Full channel equalization gets rid of the spectral shaping effect of a channel using an equalizer. Although linear equalizers are easy to implement, they enhance noise and thus degrade the performance of the system. Therefore, more complicated nonlinear equalizers, such as the decision feedback equalizer, have been proposed. One of the disadvantages of nonlinear equalizers is their computational complexity, especially under high sampling rates.

Multicarrier modulation is one possible solution for high-speed digital communications. In contrast to single carrier modulation, multicarrier modulation [12]:

- avoids full equalization of a channel
- uses available bandwidth efficiently by controlling the power and number of bits in each subchannel
- is robust against impulse noise and fast fading due to its long symbol duration
- avoids narrowband distortion by simply disabling one or more subchannels

Multicarrier modulation has been standardized for G.DMT and G.lite ADSL [9] as well as digital audio/video broadcasting [13, 14].

In multicarrier modulation, the channel is partitioned into a large number of small bandwidth subchannels. If a subchannel were narrow enough to assume to be represented by a complex constant, then no ISI would occur in this subchannel. Thus, information can be transmitted over these narrowband subchannels without ISI, and the total number of bits transmitted is the sum of the number of bits transmitted in each subchannel. If the available power were distributed over the subchannels using

the SNR of each subchannel, then high spectral efficiency could be achieved. This principle dates back to Shannon's 1948 paper [15] and has been applied in practical systems as early as the late 1950s [16].

Efficiently dividing the channel into subchannels became tractable only in the 1990s with the cost vs. performance provided by programmable fixed-point digital signal processors and the advancement in digital signal processing methods [16]. One of the most efficient ways to partition a channel into a large number of narrowband channels is the Fast Fourier Transform (FFT) [16]. Multicarrier modulation implemented via a FFT is called Discrete Multitone (DMT) modulation or Orthogonal Frequency Division Multiplexing (OFDM). DMT is more common in wireline applications, whereas OFDM is more common for wireless applications. And DMT is baseband and OFDM is bandpass transmission.

Interchannel Interference (ICI) comes from the fact that the carrier frequencies for DMT lose their orthogonality due to the non-ideal frequency response of the channel. If we looked at the FFT of a block at the input to the channel, then we will see a sinc function at each carrier frequency since the IFFT modulates each carrier with a rectangular pulse. The DFT basis is orthonormal, so that each of the basis functions (sinusoids) is orthogonal to all the rest; this means each of the sincs are orthogonal (zero inner product) as well. The frequency response of the channel has the effect of attenuating certain frequencies more than others, so that each of the sincs is changed by a different amount. Since the inner product is a measure of the similarity of two functions, two previously "completely dissimilar" sinc functions now have at least some degree of similarity; i.e., they are no longer orthogonal. Without orthogonal carriers, the FFT cannot exactly recover the correct spectral

coefficients. Cyclic padding solves this problem by turning the linear convolution of the channel impulse response with the signal into a cyclic convolution.

2.3.1 DMT Transmitter

A block diagram of a DMT (or OFDM) transmitter is shown in Figure. 2.11. In the transmitter, the input bit stream is buffered. These bits are then assigned to each of the $N/2$ subchannels using a bit-loading algorithm [16]. In DMT systems, bit loading algorithms assign the bits and available power (Power assignment per subcarrier is only available in ADSL2 but not ADSL) to each subchannel according to the SNR in each subchannel, such that high SNR subchannels receive more bits than low SNR subchannels. Extremely low SNR subchannels are shut down. In non-adaptive OFDM systems, the number of bits in each channel is equal and constant. Thus, there is no need for a bit-loading algorithm. Adaptive OFDM systems require bit loading.

Figure 2.11 Block diagram of DMT transmitter [17]

The second step is the mapping of the assigned bits to subsymbols using a modulation method, such as QAM in ADSL modems. These subsymbols are complex-valued in general and can be thought of being in the frequency domain. The efficiency of DMT and OFDM lies in the modulation of the subcarriers. Instead of having $N/2$ independent modulators, the modulators are implemented with a N - point inverse FFT (IFFT). In order to obtain real-valued samples after the IFFT, the

$N/2$ subsymbols are duplicated with their conjugate symmetric counterparts. The obtained time domain samples are called DMT symbols.

A guard period between DMT symbols is used to reduce ISI. It is implemented by prepending a symbol with its last ν samples, which is called cyclic prefix (CP). Thus, one block consists of $N + \nu$ samples instead of N samples, which reduces the channel throughput by a factor of $(N + \nu)/N$. ISI is completely eliminated for channels with impulse response of length less than or equal to $\nu + 1$ assuming perfect synchronization between receiver and transmitter. After the convolution of the channel impulse and the cyclic prefix, the information of channel impulse is saved into these $\nu + 1$ values. The cyclic prefix is selected as the last ν samples of the symbol in order to convert the linear convolution effect of the channel into circular convolution and help the receiver perform symbol synchronization. Circular convolution can be implemented in the DFT domain by using the FFT. After the FFT in the receiver, the subsymbols are the product of the N -point FFT of the channel impulse response and the N -point FFT of the transmitted subsymbols.

2.3.2 DMT Receiver

Figure 2.12 shows the block diagram of DMT receiver. For data transmission, the receiver is basically the dual of the transmitter with the exception of the addition of time-domain and frequency domain equalizers and adaptive system to track and compensate for symbol timing and phase offsets, carrier frequency and phase offset, etc. The time-domain equalizer ensures that the cascaded equalized channel impulse response is shortened to be less than the length of the cyclic prefix. If the TEQ is successful, then the received complex subsymbols after the FFT are the multiplication of the transmitted subsymbols with the FFT of the shortened (equalized) channel

impulse response. The frequency domain equalizer (also called a one-tap equalizer) divides the received subsymbols by the FFT coefficients of the shortened channel impulse response. After mapping the subsymbols back to the corresponding bits using the QAM constellation, they are converted to serial bits.

Figure 2.12 Block diagram of DMT receiver [17]

Chapter 3

Time Domain Equalization

Methodology

3.1 Minimum Mean Squared Error (MMSE)

Maximum Likelihood (ML) receivers that minimize probability of error occurred in data transmission over channels with intersymbol interference can be realized in multicarrier demodulators or sequence (Viterbi) detectors. Sequence (Viterbi) detectors are commonly used to implement maximum-likelihood detection for single-carrier or baseband data transmission. Falconer and Magee [18] propose the Minimum Mean Squared Error equalization structure for maximum likelihood reception of data transmitted over intersymbol interference channels. The technique for training the prefilter is actually borrowed from an ad hoc technique developed for use in single-channel, Viterbi Algorithm based, maximum likelihood sequence estimation (MLSE) systems [18]. Sequence detectors require an infinite number of states to decode transmitted signals at rates near capacity on a bandlimited (or intersymbol interference) channel. The solution to this problem often leads to approximation of ML performance with a finite-complexity receiver. Complexity constraints that are

appropriate for multicarrier demodulators are imposed in the derivation of the corresponding front-end equalization structures that maximize a measure of performance for ML receivers. The measure of complexity for an ML detector will be specified by the length of the cyclic prefix, which is also the order of a finite-length polynomial that is used in the design of the finite-complexity ML receiver. Chow and Cioffi [19] suggest that finding the best choice of polynomial target impulse response can minimize the mean square error between the equalizer output and the filtered channel-input sequence. They also propose to implement an unit-tap constraint (UTC) to prevent an all-zero trivial solution for the equalizer taps during minimization [20].

Figure 3.1 Conventional structure of the minimum mean-squared error (MMSE) equalizer. The lower portion of diagram indicated the delayed target impulse response, which is not physically implemented. $\mathbf{w}_k$ represents channel equalizer of k taps.

The block diagram of MMSE TEQ design method is shown in Figure 3.1. To truncate the channel from a length of L to a usable length of $\nu + 1$, an adaptive prefilter (channel equalizer) $\mathbf{w}_k$ is used as shown in Fig 3.1. The prefilter is trained to approximate a delayed target end-to-end impulse response $\mathbf{b}_{k-\Delta}$. Performance loss occurs because the channel is artificially shortened (information loss), and the noise may no longer be white. After minimizing the MSE, the Viterbi algorithm can be used with $\mathbf{b}_{k-\Delta}$ forming the traceback trellis. MMSE design is used to jointly design both $\mathbf{b}_{k-\Delta}$ and $\mathbf{w}_k$, where the lengths of $\mathbf{b}_{k-\Delta}$ and $\mathbf{w}_k$ are chosen by the user. After MMSE

adaptation the error is uncorrelated with the data by the orthogonality principle, but not necessarily white. If b is a one-tap FIR, the resulting design is simply the conventional MMSE linear equalizer.

Al-Dhahir and Cioffi [21] prove that imposing an unit-energy constraint results in a lower mean square error at a similar computational complexity as compared with unit-tap constraint. Furthermore, they point out that under assumed constraint of finite-length filters, the relative delay between the equalizer and the TIR plays an important role in optimizing performance.

The channel impulse response $\mathbf{H}$ is assumed to be linear and time-invariant and consists of m complex taps $\{h_0, h_1, \dots, h_m\}$. Each h_i ($0 \leq i \leq m$) is a l dimension column vector where l is the sampling factor. A block of N_w symbols y_k each consisting l dimension column vectors as shown in Fig 3.1 can be written as

$$\begin{bmatrix} y_{k+N_w-1} \\ y_{k+N_w-2} \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \dots & h_m & 0 & \dots & 0 \\ 0 & h_0 & h_1 & \dots & h_m & 0 & \dots \\ \vdots & & & \dots & & & \vdots \\ 0 & \dots & 0 & h_0 & h_1 & \dots & h_m \end{bmatrix} \times \begin{bmatrix} x_{k+N_w-1} \\ x_{k+N_w-2} \\ \vdots \\ x_{k-m} \end{bmatrix} + \begin{bmatrix} n_{k+N_w-1} \\ n_{k+N_w-2} \\ \vdots \\ n_k \end{bmatrix} \quad (3.1)$$

Equation (3.1) can be further simplified to

$$\mathbf{Y}_k = \mathbf{H}\mathbf{X}_k + \mathbf{N}_k \quad (3.2)$$

The solution for $\mathbf{w}$ is found in terms of $\mathbf{b}$ as Eq. (3.3).

$$\mathbf{w} = \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx} \mathbf{b} \quad (3.3)$$

$\mathbf{b}$ is the eigenvector corresponding to the minimum eigenvalue of Eq. (3.4).

$$\mathbf{R} = \mathbf{R}_{xx} - \mathbf{R}_{xy} \mathbf{R}_{yy}^{-1} \mathbf{R}_{yx} \quad (3.4)$$

$$MSE = \mathbf{b}^T \mathbf{R} \mathbf{b} \quad (3.5)$$

unit tap constraint

$$\mathbf{b}^T \mathbf{b} = 1 \quad (3.6)$$

where $\mathbf{R}_{ab} = E\{\mathbf{a}_k \mathbf{b}_k^T\}$ so that $\mathbf{a}$ and $\mathbf{b}$ could be input $\mathbf{x}$ or output $\mathbf{y}$ of (3.1)[21].

Al-Dhahir and Cioffi then define another matrix $\mathbf{R}'$

$$\mathbf{R}' = \begin{bmatrix} \mathbf{0}_{(v+1) \times \Delta} & \mathbf{I}_{(v+1)} & \mathbf{0}_{(v+1) \times (N_w + m - \Delta - v - 1)} \end{bmatrix}^T \mathbf{R} \begin{bmatrix} \mathbf{0}_{(v+1) \times \Delta} & \mathbf{I}_{(v+1)} & \mathbf{0}_{(v+1) \times (N_w + m - \Delta - v - 1)} \end{bmatrix} \quad (3.7)$$

This solution is found by performing a minimization of the MSE in (3.7) with the Lagrangian constraint of (3.6). The target impulse response is

$$\mathbf{b} = \frac{\mathbf{R}'^{-1} \mathbf{u}_i}{\mathbf{R}'^{-1}(i, i)} \quad (3.8)$$

where $0 \leq i \leq v$ and $\mathbf{R}'^{-1}(i, i)$ is the i^{th} diagonal element of the matrix $\mathbf{R}'^{-1}$ and $\mathbf{u}_i$ is the i th unit vector of (3.6). The MSE is

$$MSE = \frac{1}{\mathbf{R}'^{-1}(i, i)} \quad (3.9)$$

The minimum MSE can be obtained when

$$i_{op} = \arg \max \{\mathbf{R}'^{-1}(i, i)\} \quad (3.10)$$

and the optimal $\mathbf{b}$ is achieved at

$$\mathbf{b}_{op} = \frac{\mathbf{R}'^{-1} \mathbf{u}_{i_{op}}}{\mathbf{R}'^{-1}(i_{op}, i_{op})} \quad (3.11)$$

and $\mathbf{w}_{op}$ can be obtained from (3.3) by using $\mathbf{b} = \mathbf{b}_{op}$. $\mathbf{b}_{op}$ is the eigenvector of $\mathbf{R}'$ corresponding to the minimum eigenvalue. MSE under unit energy constraint can be derived with the similar steps as above by imposing the condition of a unit-energy target response $\mathbf{b}^* \mathbf{b} = 1$ instead. Interestingly to note, the minimum MSE is equal to minimum eigenvalue of $\mathbf{R}'$.

In MMSE design, the end-to-end impulse response does not precisely equal to the target impulse response within the target span. This distortion results because $\mathbf{w}$ is a projection of $\mathbf{b}$ in (3.3) which orthogonalizes the data in both $\mathbf{w}$ and $\mathbf{b}$ to the

error. $\mathbf{b}$ is projected into $\mathbf{w}$'s subspace. In an attempt to minimize total MSE, end-to-end distortion (MSE) within the target zone is balanced with the MSE due to both channel noise and ISI outside the target zone. This distortion is similar to the biasing phenomenon which occurs with the DFE [20]. MMSE design says nothing about the spectral location of error.

In terms of minimizing the mean squared error, UEC outperforms UTC with similar computation effort required [21]. But UEC requires eigenvector manipulation which cost significant amount of calculation. Falconer and Magee [18] propose a least mean squared (LMS) based algorithm to mitigate the computational complexity involved in obtaining the optimal TIR and TEQ. By applying unit-energy constraint after every iteration, the TIR and the TEQ are adapted with standard LMS.

Shortening the channel impulse response by means of an MMSE criterion has no direct relation to the optimal bit rate, which results in unsatisfactory performance. The non-triviality constraint (UEC and UTC) also has no relation to the bit rate, and hence it may be difficult to select the appropriate constraint. Besides the non-optimal capacity, another aspect of the unsatisfactory behavior of the MMSE equalization algorithm is the non-smoothness of the resulting bit rate as a function of the decision delay. Hence, an optimal delay is not easily selected beforehand, and exhaustive search over a large range of delays is needed during modem initialization.

K. Van Acker and G. Leus *et al.* [22] modify the MMSE method by including weight matrices in the cost function as well as in the applied constraint. The unused tones are explicitly disregarded. Optimizing the weighted MSE can be treated as minimizing a sort of $1/\text{SNR}$ over the subband of used tones. Their proposed method shows a significant performance increase in terms of bit rate.

Farhang-Boroujeny and Ding [23] [24] discover that deep notches are presented in the frequency response of MMSE based TEQ design. Figure 3.2 illustrates these notches from simulation using MMSE method. The transmission rate will be degraded around the notched region. They propose general guidelines to design a near-optimum TEQ. An eigen-approach is proposed to the design of TEQ and through numerical examples they are able to demonstrate that the results in TEQ designs are comparable to [21] with a much lower computational cost.

Figure 3.2 Notches in Equalized channel frequency response of MMSE method

Yap and McCanny [25] apply a variant of the Least-Mean Fourth (LMF) algorithm to the TEQ, with a Least-Mean Square variant applied to the Target Impulse Response (TIF) filter. The iterative-based TEQ algorithm results in a useful speed-up in the convergence rate. Their proposed method outperforms the classical MMSE design for a range of TEQ filter lengths and the improved performance outweighs the increases in computational overhead [12].

The minimum eigenvalue of a matrix and its associated eigenvector are the inverse of the maximum eigenvalue of the inverse of that matrix, and its associated eigenvector [26]. An eigenvector approach using modified power method is proposed by Nafie and Gatherer [27]. In order to make the iteration in their inverse power method to converge faster, LU decomposition is suggested [12].

Another iterative algorithm to solve the MMSE design is proposed by Lashkarian and Kiaei [28] propose a more computationally efficient and highly parallelizable algorithm. Research shows that the Hessian matrix provides an efficient method to obtain the global minimum of constrained optimization problem when combined with the Rayleigh minimization algorithm [12].

3.2 Maximum Shortening SNR (MSSNR)

MMSE tries to minimize the mean square error between the target impulse response and equalized response. MSSNR approaches the problem of equalization from the perspective of impulse response shortening by setting the length of TIR and minimizing the squared error difference between the equalized channel and TIR [29]. An optimum shortening SNR is defined as the energy ratio between the energy inside the TIR and energy outside. By maintaining constant energy inside the TIR window, the maximum shortening SNR (SSNR) is achieved at the minimum energy of the shortened impulse response (SIR) outside the window. ADSL channel impulse response, which is estimated by the channel FFT coefficients, is obtained by applying bit loading algorithm [16]. We follow the derivation of [29] to briefly illustrate the MSSNR equalization design here. The impulse responses of the SIR inside the target window can be written as the following matrix

$$\mathbf{h}_{\text{win}} = \begin{bmatrix} h(\Delta+1) & h(\Delta) & \cdots & h(\Delta - N_w + 2) \\ h(\Delta+1) & h(\Delta+1) & \cdots & h(\Delta - N_w + 3) \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta+\nu+1) & h(\Delta+\nu) & \cdots & h(\Delta - N_w + \nu + 2) \end{bmatrix} \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(N_w - 1) \end{bmatrix} \\ = \mathbf{H}_{\text{win}} \mathbf{w}$$

and the rest of impulse response as

$$\mathbf{h}_{\text{wall}} = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ h(1) & h(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta) & h(\Delta-1) & \cdots & h(\Delta - N_w + 1) \\ h(\Delta+\nu+2) & h(\Delta+\nu+1) & \cdots & h(\Delta - N_w + \nu + 3) \\ \vdots & \vdots & \ddots & \vdots \\ h(m-1) & h(m-2) & \cdots & h(m - N_w + 1) \\ 0 & h(m-1) & \cdots & h(m - N_w + 2) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{L-1}(m-1) \end{bmatrix} \begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(N_w - 1) \end{bmatrix} \\ = \mathbf{H}_{\text{wall}} \mathbf{w}$$

where N_w is the number of TEQ taps. The matrix $\mathbf{A}$ and $\mathbf{B}$ are defined in the following equations which show the energy inside and outside the target windows.

$$\begin{aligned} \mathbf{h}_{\text{win}}^T \mathbf{h}_{\text{win}} &= \mathbf{w}^T \underbrace{\mathbf{H}_{\text{win}}^T \mathbf{H}_{\text{win}}}_{\mathbf{B}} \mathbf{w} \\ \mathbf{h}_{\text{wall}}^T \mathbf{h}_{\text{wall}} &= \mathbf{w}^T \underbrace{\mathbf{H}_{\text{wall}}^T \mathbf{H}_{\text{wall}}}_{\mathbf{A}} \mathbf{w} \end{aligned} \quad (3.12)$$

The final optimum solution is obtained at

$$\min_{\mathbf{w}} \mathbf{w}^T \mathbf{H}_{\text{wall}}^T \mathbf{H}_{\text{wall}} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^T \mathbf{H}_{\text{win}}^T \mathbf{H}_{\text{win}} \mathbf{w} = 1 \quad (3.13)$$

when $\mathbf{B}$ has a non-empty null space, which means $\mathbf{H}_{\text{win}}$ has more columns than rows ($N_w > \nu$), solving (3.13) becomes difficult [29]. The optimal shortening SNR is expressed

$$SSNR_{op} = 10 \log \left(\frac{\mathbf{w}_{op}^T \mathbf{B} \mathbf{w}_{op}}{\mathbf{w}_{op}^T \mathbf{A} \mathbf{w}_{op}} \right) = -10 \log(\mathbf{w}_{op}^T \mathbf{A} \mathbf{w}_{op}) \quad (3.14)$$

The optimum equalizer is

$$\mathbf{w}_{\text{op}} = (\sqrt{\mathbf{B}})^{-1} \mathbf{l}_{\text{min}} \quad (3.15)$$

where $\sqrt{\mathbf{B}}$ denotes the Cholesky decomposition of $\mathbf{B}$, assuming matrix $\mathbf{B}$ is positive definite. $\mathbf{l}_{\text{min}}$ indicates the eigenvector corresponding to the minimum eigenvalue of the matrix $(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}^T})^{-1}$ [29]. Since $\mathbf{B}$ is of full rank, the matrix $(\sqrt{\mathbf{B}})^{-1}$ exists.

Yin and Yue [30] propose the solution for the case where $N_w > \nu$. They suggest achieving the optimum SSNR by maximizing $\mathbf{w}^T \mathbf{B} \mathbf{w}$ while maintaining $\mathbf{w}^T \mathbf{A} \mathbf{w}$ constant. So the optimum equalizer is $\mathbf{w}_{\text{op}} = (\sqrt{\mathbf{A}^T})^{-1} \mathbf{l}_{\text{max}}$. For most physical channels, $\mathbf{A}$ is positive definite and invertible. Our later discussion in efficient matrix computation of MSSNR method is based on Yin and Yue's method. Figure 3.3 shows the original and shortened channel of MSSNR method using CSA loop 1.

Figure 3.3 MSSNR Original & Shortened Channel

The MSSNR method attempts to minimize the energy outside the target impulse response window, which effectively triggers the ISI. This portion of energy cannot be eliminated thoroughly in general. There still will be some residual energy

left outside and this prevents us from achieving the maximum the channel capacity. MSSNR does not consider aligning these residual ISI power in high noise frequency domain [12].

Wang *et al.* [31] generalize the MSSNR and MMSE equalization method by introducing a frequency-weighted energy to define composite squared cost functions. Their algorithm can jointly shorten the channel impulse response and suppress the stopband noise and passband interference. To leverage the computational complexity due to eigenvalue and Cholesky decompositions incurred in MSSNR design, Chiu, Tsai, Liao, and Tzoulis [32] propose an inverse power method. This method reformulates the complicated matrix decompositions and inversions to directly iterate on two matrices to obtain the optimal TEQ in MSSNR design [12].

The infinite length MSSNR TEQ is symmetric or skew-symmetric always [33][34]. In practice, SSNR TEQs are nearly symmetric and not skew-symmetric. Symmetric TEQs have greatly reduced the design and implementation complexity without performance degradation with respect to the MSSNR design in [29].

3.3 Maximum Geometric SNR (MGSNR)

MGSNR is the first TEQ design method that explicitly targets maximizing the transmission data rate in equalizer design. Al-Dhahir and Cioffi's [35] proposed approach is based on optimizing the geometric SNR.

Assuming that the subchannels of the discrete multitone are equally spaced (the number of subchannels $N/2+1$ is large). The achievable bit rate of a multicarrier channel can be written as a summation of the capacities of subchannels.

$$b_{DMT} = \sum_{i=1}^{\bar{N}} b_i = \sum_{i=1}^{\bar{N}} \log_2 \left(1 + \frac{SNR_i}{\Gamma} \right) \quad (3.16)$$

$$\Gamma \approx \frac{\gamma_m}{3\gamma_{eff}} (Q^{-1}(\frac{P_e}{2}))^2 \quad (3.17)$$

where i is the subchannel number, $\bar{N}$ is the used subchannels numbers which is bounded by $N/2 + 1$ subchannels, SNR_i is the i^{th} subchannel signal-to-noise ratio. Assumption is made that is SNR gap Γ remains constant throughout all used subchannels $\bar{N}$ [36]. P_e is the transmitted symbol error rate, γ_{eff} and γ_m are effective coding gain and system margin, respectively.

The matched filter bound (MFB) SNR ignores the effect of ISI and channel equalizers and it assumes that the input signal and noise are wide sense stationary. The gain of the channel spectrum $\mathbf{H}_i$ is assumed to be constant since the number of the subchannels $\bar{N}$ tends to be large. It is written as

$$SNR_i^{MFB} = \frac{S_i^x |\mathbf{H}_i|^2}{S_i^n} = \frac{S_i^x |\mathbf{H}_i|^2 |\mathbf{W}_i|^2}{S_i^n |\mathbf{W}_i|^2} \approx \frac{S_i^x |\mathbf{B}_i|^2}{S_i^n |\mathbf{W}_i|^2} \quad (3.18)$$

where $\mathbf{B}_i$ and $\mathbf{W}_i$ are the i^{th} FFT coefficients of the TIR and TEQ. The geometric SNR is defined as

$$SNR_{geo} = S_i^x \left[\prod_{i=1}^{\bar{N}} \left(\frac{|\mathbf{B}_i|^2}{S_i^n |\mathbf{W}_i|^2} \right) \right]^{1/\bar{N}} \quad (3.19)$$

which is related to channel capacity. By substituting (3.19) into (3.16). We have

$$b_{DMT} = \bar{N} \log_2 \left(1 + \frac{SNR_{geo}}{\Gamma} \right) \quad (3.20)$$

Geometric SNR can be thought of a geometric mean value of the all subchannel SNR. From (3.20), by obtaining the optimum geometric SNR value, overall channel data rate will be optimized concurrently. Maximizing (3.19) is equivalent of maximizing the following cost function

$$D(b) = \frac{1}{N} \sum_{i=1}^{\bar{N}} \ln |B_i|^2 \quad (3.21)$$

Since B_i is the i^{th} FFT coefficient of TIR,

$$D(b) = \frac{1}{N} \sum_{i=1}^{\bar{N}} \ln \left| \sum_{j=0}^{N-1} b_j e^{-j \frac{2\pi}{N} i j} \right|^2 \quad (3.22)$$

Additional unit-energy constraint is imposed on $\mathbf{b}$ to prevent trivial solution; it follows that $|\mathbf{B}_i|^2 = 1$ for each i to prevent an infinite gain in the TEQ. This leads to a zero forcing equalization of the channel. Zero forcing is not necessary as in DMT since it uses a guard band. Other constraint is imposed to keep the MSE below certain $MSE_{\max}$ so that zero forcing solution is avoided. Al-Dhahir and Cioffi show that the optimum geometric SNR problem is solved at

$$\max \sum_{i=1}^{\bar{N}} \ln |\mathbf{B}_i|^2 \text{ s.t. } \mathbf{b} * \mathbf{b} = 1 \text{ and } \mathbf{b}^T \mathbf{R} \mathbf{b} \leq MSE_{\max} \quad (3.23)$$

where $\mathbf{R}$ can be derived from (3.4). Currently this non-linear optimization problem can only be solved by numerical methods [37].

Al-Dhahir and Cioffi [38] proposed to include effects of ISI into (3.18) to modify the MGSNR method. A less computationally complex and more geometrically intuitive interactive MGSNR method is proposed in [39].

3.4 Minimum Intersymbol Interference (min-ISI)

The bit rate of a DMT system is optimized by maximizing the number of bits per DMT block, transmitted at a target error probability, by appropriate choice of subchannel constellation. A good approximation to the number of bits, which can be transmitted at a fixed error probability for a given modulation, is given by a

modification of the Shannon capacity in (3.16). The SNR gap to capacity Γ can be given in dB by

$$\Gamma = \gamma_m - \gamma_c + \gamma_d \quad (3.24)$$

where γ_m is the modulation gap, γ_c is the coding gain and γ_d is a design margin [36].

Arslan, Evans and Kiaei [12], [40] proposed the Maximum Bit Rate (MBR) before introducing Minimum ISI (min-ISI) TEQ design algorithm. The MBR TEQ maximizes the total bit rate, by redefining the subchannel signal to noise ratios SNR_i to incorporate an ISI term, and secondly by expressing the total bit rate as a function of the TEQ co-efficient $\mathbf{w}$ and performing a nonlinear optimization of total bit rate with respect to $\mathbf{w}$. Supposing the i^{th} subchannel signal to noise (including ISI) ratio is SNR_i^{ISI} , it is necessary to express the SNR_i^{ISI} as a function of $\mathbf{w}$. Knowing the cyclic prefix (or some other redundancy) can eliminate all ISI if the channel is sufficiently shortened, it seems reasonable that the ISI measure should reflect the shortcomings of the TEQ in achieving this goal. A signal path, a noise path and an ISI path should be defined in the system. This can be done by isolating the actual channel $h(n)$ by multiplication by a rectangular window $\Phi(n)$ as the following

$$h^{sig}(n) = \Phi(n)h(n) \quad (3.25)$$

where

$$\Phi(n) = \begin{cases} 1, & \Delta \leq n \leq \Delta + \nu \\ 0, & \text{otherwise} \end{cases} \quad (3.26)$$

By extension, the ISI path must be formed by the tails left after this windowing. This can be specified by multiplication of $h(n)$ by $\Psi(n) = 1 - \Phi(n)$

$$h^{ISI}(n) = \Psi(n)h(n) \quad (3.27)$$

The noise path $h^{noise}(n)$ is just the TEQ $w(n)$. In order to convert to matrix notation, define the column vectors

$$\Phi' = [\Phi(0) \quad \Phi(1) \quad \dots \quad \Phi(L-1)]^T \quad (3.28)$$

$$\Psi' = [\Psi(0) \quad \Psi(1) \quad \dots \quad \Psi(L-1)]^T \quad (3.29)$$

We then can define three L -length column vectors to describe the paths of interest.

$$\mathbf{h}^{sig} = \Phi \mathbf{H} \mathbf{w} \quad (3.30)$$

$$\mathbf{h}^{ISI} = \Psi \mathbf{H} \mathbf{w} \quad (3.31)$$

$$\mathbf{h}^{noise} = \Theta \mathbf{w} \quad (3.32)$$

with $\Phi = \text{diag}(\Phi')$, $\Psi = \text{diag}(\Psi')$ and

$$\Theta = \begin{bmatrix} \mathbf{1}_{N_w \times N_w} \\ \mathbf{0}_{(L-N_w) \times N_w} \end{bmatrix} \quad (3.33)$$

The function $\text{diag}()$ forms a diagonal matrix from its vector input. Since we are interested in the SNR_i^{ISI} on each subchannel, we take Fourier transforms of the above paths, in order to find their spectral responses. If the DFT vector is

$$\mathbf{q}_i = \begin{bmatrix} 1 & e^{j\frac{2\pi}{L}} & e^{j\frac{2\pi 2i}{L}} & \dots & e^{j\frac{2\pi(L-1)i}{L}} \end{bmatrix}^H \quad (3.34)$$

Then the i^{th} spectral co-efficient of the paths can be written as

$$\mathbf{H}_i^{sig} = \mathbf{q}_i^H \Phi \mathbf{H} \mathbf{w} \quad (3.35)$$

$$\mathbf{H}_i^{ISI} = \mathbf{q}_i^H \Psi \mathbf{H} \mathbf{w} \quad (3.36)$$

$$\mathbf{H}_i^{noise} = \mathbf{q}_i^H \Theta \mathbf{w} \quad (3.37)$$

The SNR_i^{ISI} can be written as

$$SNR_i^{ISI} = \frac{S_i^x |\mathbf{H}_i^{sig}|^2}{S_i^n |\mathbf{H}_i^{noise}|^2 + S_i^x |\mathbf{H}_i^{ISI}|^2}$$

$$= \frac{\mathbf{w}^T \mathbf{H}^T \Phi^T \mathbf{q}_i S_i^x \mathbf{q}_i^H \Phi \mathbf{H} \mathbf{w}}{\mathbf{w}^T [\mathbf{H}^T \Psi^T \mathbf{q}_i S_i^x \mathbf{q}_i^H \Psi \mathbf{H} + \Theta^T \mathbf{q}_i S_i^n \mathbf{q}_i^H \Theta] \mathbf{w}} \quad (3.38)$$

The optimum TEQ is achieved at

$$\mathbf{w}_{\text{opt}} = \arg \min_{\mathbf{w}} \sum_{i=0}^{N-1} \log_2 \left(1 + \frac{1}{\Gamma} \text{SNR}_i^{\text{ISI}} \right) \quad (3.39)$$

The theory was developed by Arslan *et al.* [40][12], who propose to minimize this nonlinear function of $\mathbf{w}$ by using the Broyden-Fletcher-Goldfarb-Shannon quasi-Newton algorithm in the Matlab optimization toolbox. They then propose a near-optimum solution, which is the min-ISI method.

To avoid the nonlinear optimization, it is possible to minimize the distortion power on each subchannel, which is the denominator of (3.38), by minimizing the sum of the distortion powers over all the subchannels. This is possible because power terms are always non-negative. It is proposed to reformulate the optimization as

$$\mathbf{w}_{\text{opt}} = \arg \min_{\mathbf{w}^T \mathbf{H}^T \Phi^T \Phi \mathbf{H} \mathbf{w} = 1} \left[\mathbf{w}^T \mathbf{H}^T \Psi^T \sum_{i=0}^{L-1} \mathbf{q}_i S_i^x \mathbf{q}_i^H \Psi \mathbf{H} \mathbf{w} + \mathbf{w}^T \Theta^T \sum_{i=0}^{L-1} \mathbf{q}_i S_i^n \mathbf{q}_i^H \Theta \mathbf{w} \right] \quad (3.40)$$

The UEC on the signal path is only to prevent the null solution, since now we have a generalized eigen-problem, which could otherwise have its minimum at the origin.

Min-ISI method was proposed after they realized that MBR needs significant computation effort and it's not suitable for real-time application on a DSP processor [40]. In Min-ISI the residual ISI noise is shaped in the frequency domain in such a way that high ISI noise is put in the high SNR subchannels. It can be considered as a generalization of MSSNR method. Both methods would be equivalent if the SNR are constant over all subchannels and if all subchannels are in use. In simulation performed in [40], 95% of the bit rate of the MBR method can be achieved by min-ISI. Real time implementation of the Min-ISI on DSP chips is available now [40]. The dual-path TEQ [41] makes use of the Min-ISI design by placing a Min-ISI TEQ in

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parallel with a main TEQ. The newly added one uses the Min-ISI method for a small subset of subchannels. The subset contains the low frequency tones with expected high bit rate, and can be targeted to tones near the division between downstream and upstream bands to increase the overall bit rate.

Chapter 4

TEQ Modeling and Performance

Evaluation

4.1 Carrier Serving Area Loops

Digital Subscriber Line (DSL) technology is a modem technology that uses existing twisted-pair telephone lines to transport high-bandwidth data, such as multimedia and video, to service subscriber. The term xDSL covers a number of similar yet competing forms of DSL. xDSL is drawing significant attention from implementers and service providers because it promises to deliver high-bandwidth data rates to dispersed locations with relatively small changes to the existing Telco infrastructure. xDSL services are dedicated, point-to-point, public network access over twisted-pair copper wire on the local loop (“last mile”) between a network service provider (NSP’s) central office and the customer site, or on local loops created either intra-building or intra-campus.

The digital subscriber line’s physical characteristics will directly affect the overall performance of the xDSL system performance. Basically, the outside local

loop distribution plant consists of all the facilities necessary to distribute telephone service from the central office out to the subscribers. A subscriber loop may include several sections of copper twisted pairs with different gauge cables. The distribution cables from central office to the subscriber are either aerial (hung on poles) or underground. The modern trend is to use buried distribution cables all the way to the subscriber premises, but there are a large number of existing loop plants using aerial distribution cables. These distribution cables are then split up into residence aerial drop wires (one per phone line) at a pole closure (in aerial plant), or at a cable pair to service wire cross box in buried plant. Cables may be grouped together to form bundles. Grouping of cables affects the crosstalk noise in xDSL system.

Bridged tap refers to any portion of a loop that is not in the direct talking path between the central office and the service user's terminating equipment. A bridged tap may be an unused cable pair connected at an intermediate point or an extension of the circuit beyond the service user's location. A bridged tap creates an impedance mismatch within the transmission line, which creates signal reflections. These reflections are generally not noticed in standard (POTS) voicegrade service, but become significant with high frequency (xDSL) and digital transmission (DDS and DS1) services. Normally the further away from the customer's location, the less it will affect DSL. DSL signal reflects back through the cable pair from the end of a bridged tap. The deflected signal is out of phase and mixed up with the original signal. If the bridged tap is long, by the time the signal bounces back, the original signal is far ahead and more powerful. Therefore the modem will ignore the weaker deflected signal.

The loop plant was originally designed to provision customers with POTS. To ensure proper QoS, design rules were defined for subscriber loop provisioning. One

set of rules that govern distribution of twisted wire pairs for voice service from CO to the CP is the resistance design rules [42]. If a subscriber loop is provisioned with a length greater than the maximum defined by the resistance design rules, the loading coils must be inserted in the loop to achieve proper voice quality. However, that subscriber loops provisioned with loading coils are not suitable for support of wideband DSL services, because loading coils introduce too much attenuation of the frequencies outside the voice channel, which are required by the DSL system. In short, loading coils must be removed from any subscriber loop that is to support a DSL based service.

Another set of rules, called carrier serving area (CSA) rules, define the distribution of twisted pairs from digital loop carrier (DLC) systems. The radius covered by CSA rules is up to 2.7km of 26-gauge wire and up to 3.7 km on 24-gauge wire. The concept of CSA rules was originally developed for provisioning loops from digital loop carrier (DLC) systems in support of 56-kbps digital data service (DDS) and later slightly modified for the support of POTS from a DLC system [43]. The CSA rules are defined as follows:

- There should be only nonleaded cable.
- Multigauge cable is restricted to two gauges.
- Total bridged-tap length may not exceed 0.8 km, and no single bridged tap may exceed 0.6 km
- The amount of 26 AWG cable may not exceed a total length of 2.7 km, including bridged taps.
- For single gauge or multigauge cables containing only 19, 22, or 24 AWG cable, the total cable length may not exceed 3.6 km., including bridged taps.

- The total cable length including bridged taps of a multigauge cable that contains 26-gauge wire may not exceed $12 - (3 \times L_{26}) / (9 - L_{BTAP})$ kft where L_{26} is the total length of 26 gauge wire in the cable (excluding any 26 gauge bridged tap) and L_{BTAP} is the total length of bridged tap in the cable.

The above CSA guidelines do not include any wiring in the CO nor any drop wiring and any wiring in the CP. Figure 4.1 shows the configuration of the eight standard CSA loops. Numbers represent length/thickness in feet per gauge. The vertical lines represent bridge taps.

Figure 4.1 Configuration of the eight standard CSA loops [2]

The American Wire Gauge (AWG) is a U. S. standard set of non-ferrous wire conductor sizes. The “gauge” means the diameter. Non-ferrous includes copper and also aluminum and other materials, but is most frequently applied to copper

household electrical wiring and telephone wiring. The higher the gauge number, the smaller the diameter and the thinner the wire. Since thicker wire carries more current because it has less electrical resistance over a given length, thicker wire distance is better for longer distance. For this reason, where extended distance is critical, installing a network might prefer telephone wire with the lower-gauge, thicker wire of AWG 26 to 24.

As the transport medium for wideband signals, the twisted wire pairs introduce linear impairments into the signal. Specifically, the linear impairments are propagation loss, amplitude distortion, and delay distortion. The propagation loss in decibels is directly proportional to the distance of the loop. Amplitude distortion results from the fact that signal components at higher frequencies experience more amplitude loss than the components at low frequencies. To the first order, the amplitude response of the twisted-wire-pair channel rolls off at roughly the square root of frequency. Finally, the delay is such that at low frequencies (less than approximately 10 kHz) there is very sharp variation in the group delay, and at higher frequencies the delay, response is relatively constant [44].

4.2 Sources of Noise

Digital subscriber loop noise consists of unwanted interfering signal sources whether it comes from crosstalk, radio frequency interference, distortion, or random signals created by thermal energy. Noise impairs the detection of the smallest analog levels, which may be resolved within the demodulator. The noise level along with the maximum clip level of an analog signal path set the available amplitude dynamic range. The maximum data rate of a modem is limited by the available frequency range (bandwidth) and signal to noise ratio (SNR), which is amplitude dynamic range [43].

Crosstalk noise refers to the interference that enters a communication channel, such as twisted wire pairs, through some coupling path. It can be further divided into two groups called near-end crosstalk (NEXT) and far-end crosstalk (FEXT). NEXT noise occurs when the transmitter sends a signal and a nearby transceiver at the same end of link, through capacitive and inductive coupling, interfere with the signal. The noise is generated at the near end of the channel. FEXT occurs when the transmitter sends a signal and a transceiver at the far end of the link, through capacitive and inductive coupling, interferes the signal. FEXT will be of more concern in an asymmetrical system such as ADSL than symmetrical systems like HDSL. Since the power level of the signal attenuates due to the channel, FEXT noise is generally less powerful than NEXT noise.

In the loop plant, NEXT is generally far more damaging than FEXT because, unlike FEXT, NEXT directly disturbs the received signal transmitted from the far-end after it has experienced the propagation loss from traversing the distance from the far-end down the disturbed wire pair [43]. In a multipair cable, relative to one desired receive signal on one twisted wire pair, all the other wire pairs are sources of crosstalk. For DSL systems, the reference cable size for evaluating performance in the presence of crosstalk is a 50-pair cable [45]. Relative to the received signal on a particular wire pair, the other 49 wire pairs are sources of crosstalk (both near-end and far-end). Crosstalk noise is usually modeled as a coupling filter fed by a random signal [2]. The random signal has the similar bandwidth and statistical properties of the signal modulated with the modulation method being used by the adjacent loops. For the case of adjacent loops adopting ADSL, a multicarrier modulated random signal is used as the random signal. In modeling the electromagnetic coupling between two copper

wires, the coupling filter is generally a highpass filter with increasing gain as frequency increases.

Background noise levels in the telephone lines have been studied, and the results [43] show that these noise levels vary widely and frequency band dependent. The number typically given as the average ADSL background noise power density is -140 dBm/Hz, although some lines are quieter, at -153 dBm/Hz. Thermal noise is caused by the semiconductor device's resistance. White Gaussian noise is usually used to simulate the background noise in general.

4.3 TEQ Performance Analysis

Simulation results are presented in this section in order to analyze and compare the performance of the four TEQ design schemes described in Chapter 3. We use the eight standard carrier-serving-area (CSA) loops as our test channels. Linemod software [46] is adopted in generating the CSA loops' channel data. "Matlab DMT TEQ design toolbox" [57] developed by The University of Texas at Austin is used in our simulation. The simulation results are tally with the previous published paper [40] and research work [12] done by B. L. Evans' ADSL equalizer research group.

Sampling at 2.208 MHz, the CSA loops' channel impulse responses consist of 512 samples. In order to filter out pilot and synchronization channels (channel 0, 1-5, and the Nyquist channel) [43], a Chebyshev highpass filter is incorporated with the receiver. For a fair TEQ performance comparison of the four TEQ schemes, the simulation environment is set to be the same. A -140 dBm AWGN distributed over the entire frequency band, besides NEXT noise is incorporated as part of the channel noise. The NEXT noise is modeled with the transfer function

$$|H_{NEXT}(f)|^2 = x_{49} \times \left(\frac{N}{49}\right)^{0.6} f^{\frac{3}{2}}, \text{ where } x_{49} = \frac{1}{1.13 \times 10^{13}} \text{ is the coupling coefficient of}$$

49 NEXT disturbers. N is set to 10, which corresponds to 8 ADSL disturbers [2]. The eight ADSL disturbers in NEXT noise are described in the ANSI T1.413 1995 standard [9]. The FEXT noise is ignored for simplicity and it's relatively insignificant in the high frequency region. The current ADSL standards (ITU Recommendations G.992.1 and G.992.2) are adopted. It specifies a bit error rate of 10^{-7} with a margin of 6 dB. For uncoded PAM and QAM constellations, Γ (SNR gap) is set to 9.8 dB in order to achieve the same desired probability of bit error rate. We take coding gain as 4.2 dB, which is same as in [12]. Based on (3.24), the SNR gap is $9.8+6-4.2=11.6\text{dB}$. All bit rate results are calculated with the effect of TEQ based on the SNR distribution in each subchannel. Assumption is made that power allocation is constant over all used subchannels. The input signal power is used as 23 dBm and equally applied over entire CSA channel, and the FFT size is set to $N = 512$. Delay optimization has been performed over 1 to 50 samples, except for MGSNR scheme. As stated in chapter 3, MGSNR TEQ optimization depends on the optimal MMSE solution and its optimal delay. Another 2dB more MSE is added on to $MSE_{\max}$, which is obtained by MMSE method, in order to achieve the best MGSNR performance [47]. UEC is applied as the constraint for MMSE method in the simulation.

Performance of the previously mentioned TEQ methods in terms of output bit rate is analyzed varying the length of equalizer N_w and cyclic prefix length ν . This is similar to what has been done in [40]. ν is fixed at 32 as required by the ADSL standards for downstream transmission. The simulation results with CSA loop 5 channel data are shown in Figure 4.2.

The min-ISI method achieves bit rates above 95% of the upper bound with only three TEQ taps. This matched with Arslan's findings [40]. For TEQ length larger than 3, the output data rate of the min-ISI method attempts to be more closely approaching the matched filter bound in a steady manner. The upper bound refers to the matched filter bound (MFB) which is the simplified SNR scenario excluding the effect of ISI. The SNR in the i th subchannels is defined as

$$SNR_i^{MFB} = \frac{S_{x,i} |\mathbf{H}_i|^2}{S_{n,i}} \quad (4.1)$$

The achievable bit rate equation is defined in (3.29). The subchannels are assumed to be narrow enough so that the channel frequency response and transmitted signal power spectrum can be considered flat in MFB. It is the maximum achievable output data rate.

The performance of MSSNR, MMSE and MGSNR TEQ methods in terms of output data rate are similar. Their output data rates climb to a peak and then oscillate in a declining trend as the length of TEQ N_w increases fixing the cyclic prefix to be 32. If the method is guaranteed to find the optimum bit rate, it should be monotonically increasing as N_w increases [40]. For the case of MSSNR, MMSE and MGSNR methods, the dependence of bit rate on N_w is not monotonic. MSSNR climbs roughly the same height as Min-ISI method for short TEQs ($N_w < 6$). MMSE achieves its global maximum output bit rate at $N_w = 6$. MSSNR outperforms MMSE roughly over all N_w values. MGSNR outperforms MMSE and MSSNR when $N_w > 9$. For $N_w = 32$, the bit rate for MSSNR, MMSE and MGSNR drops significantly comparing to their small N_w values. This oscillation may seem to contradict with the common understanding that larger number of equalizer taps produces better

performance. MSSNR, MMSE and MGSNR may not be good candidates in terms of maximizing the achievable bit rate. The min-ISI method can give higher bit rates than the other three methods for any TEQ size.

Figure 4.2 Achievable bit rate vs. the number of equalizer taps N_w for CSA loop 5

As cyclic prefix corresponds to the redundant information prepended in the transmitted symbols, the effective bit rate is reduced $N/(N+\upsilon)$. The two crucial parameters N_w and υ will affect the performance of TEQ. Then we will fix N_w to 32 and vary the length of cyclic prefix υ from 2 to 32. The simulation results are shown in Fig 4.3.

Differentiating form Figure 4.2, the performance of MSSNR, MMSE and MGSNR TEQ methods oscillate with an increasing trend first and then decrease potentially in Figure 4.3. The min-ISI method achieves the maximum bit rate for a cyclic prefix of 12 samples when a 17-tap TEQ is used. The MGSNR method

outperforms the MSSNR and MMSE method. MSSNR outperforms MMSE when $\nu > 13$. The MGSNR method is only competitive with min-ISI for short cyclic prefix length ($\nu = 5$). The MMSE achieve the same output data rate as min-ISI when $\nu = 3$. As the number of cyclic prefix increase, the matched filter bound is decreased as indicated in Figure 4.3. This is governed by bit rate reduction factor $N/(N + \nu)$.

Arslan, Evans, and Kiaei's [40] min-ISI TEQ scheme needs only a three-tap equalizer to achieve high output data rate. Figure 4.4 simulates the output performance of various TEQ schemes with the length of equalizer tap set to 3. Simulation results show that MSSNR methods catch up to min-ISI for $\nu = 24$ and achieve the upper bound bit rate for $\nu = 26$. Arslan, Evans, and Kiaei concluded that a three-tap equalizer and a cyclic prefix length of 26 adopting their min-ISI method outperform all other methods with up to 32 TEQ taps and a cyclic prefix length smaller than 32. The MMSE and the MGSNR TEQ methods are not competitive at all for small N_w . And this point has already been revealed in Figure 4.2.

Figure 4.3 Achievable bit rate versus the cyclic prefix length for CSA loop 5 $N_w = 17$

Figure 4.4 Achievable bit rate versus the cyclic prefix length for CSA loop 5 $N_w = 3$

Interestingly in Figure 4.3 and 4.4, we observed similarity between MMSE and MGSNR. There are sections of ‘ineffective regions’ or “unchanged in achievable bit rate regions” as the length of the cyclic prefix increases. We know that the optimum delay of MGSNR is obtained from the optimum condition of MMSE. MGSNR uses this information of MMSE to calculate its optimum TIR and TEQ taps. This is the reason why the trends of two curves sometimes look so similar. The cyclic prefix is redundant information we insert after the transmitting symbols. Overall MFB is decreasing as we increase the CP, which injects more redundant information into the system; hence, the effective data rate drops. The simulation shows the achievable data rate is not monotonically increasing as CP increases. As CP increased to the length of equalized channel impulse response, it no longer contributes to the increase of the data rate in terms of eliminating the intersymbol interference. So a portion of “saturated region” can be observed in Fig 4.4 with all four TEQ methods. With small CP in Fig 4.3 and 4.4, the performance of MGSNR and min-ISI is somehow similar. This can be explained that MGSNR and min-ISI both work on maximizing the subchannel SNR hence maximizing the achievable data rate. MGSNR makes the assumption that each channel has an SNR equal to GSNR. Min-ISI treats subchannel individually and set aside the ISI as subchannel noise. MGSNR still relies on CP to kill ISI. With small CP, for MGSNR the dominating factor is channel data rate increase, which is similar with min-ISI. As CP increases, the effect of increasing achievable bit rate with indiscriminate SNR of each subchannel deteriorates and redundancy CP brought in to reduce the bit rate takes the dominance. So MGSNR method does not perform well as compared to min-ISI in large CP region.

The bit rate results are listed in Figure 4.5 and Figure 4.6 for all eight channels. with $N_w = 17$, $\nu = 32$ and $N_w = 3$, $\nu = 32$ combination respectively. The bit rate

results take average of 25 simulations. Local MFB refers to the corresponding to the matched filter bound each CSA loop can achieve.

Figure 4.5 Achievable bit rates for the eight CSA loops equalized with MMSE, MGSNR, MSSNR and min-ISI in percentage with local MFB $N_w = 17$

Figure 4.6 Achievable bit rates for the eight CSA loops equalized with MMSE, MGSNR, MSSNR and min-ISI in percentage with local MFB $N_w = 3$

Figure 4.7 Local MFB for the eight CSA loops $N_w = 3$ and $N_w = 17$

Figure 4.5 and Figure 4.6 tells the range of achievable bit rates in percentage with respective of local MFB of each CSA loops with given equalizer length of 17 and 3 taps. Figure 4.7 shows that local MFB for each CSA loop remains almost the same regardless of the size of equalizer taps. With the above simulation results, we conclude that MSSNR have better bit rate performance with smaller TEQ length. In Chapter 5 and 6, we will further investigate the implementation of MSSNR.

Figure 4.8-4.11 plotted out the upper-bound SNR vs. subchannel SNR in dB simulated on CSA loop 5 with $N_w = 17$ and $\nu = 32$ cyclic prefix length, by using the same parameters as in Section 4.3 with MMSE-UEC, MSSNR, MGSNR and min-ISI TEQ respectively. By observation, we can tell that the subchannel SNR of min-ISI is the closest to the matched filter bound; MGSNR has its SNR, which corresponds to achievable bit rate, the second closest to the MFB. With (3.29), higher subchannels' SNR provide higher achievable bit rate. Simulation results of Figure 4.8-4.11 can indicate the output bit rate utilizing different TEQ schemes. Based on the closeness of SNR with MFB, the output bit rate relationship is $I_{\text{min-ISI}} > I_{\text{MGSNR}} > I_{\text{MSSNR}} > I_{\text{MMSE}}$.

The argument above is validated in Figure 4.3 with the rightmost points which indicating bit rates of various TEQ methods with $\nu = 32$.

Figure 4.8 Subchannel SNR versus upper-bound SNR with MMSE-UEC TEQ

Figure 4.9 Subchannel SNR versus upper-bound SNR with MSSNR TEQ

Figure 4.10 Subchannel SNR versus upper-bound SNR with MGSNR TEQ

Figure 4.11 Subchannel SNR versus upper-bound SNR with min-ISI TEQ

Chapter 5

Efficient Matrix Computation Design in MSSNR

5.1 Optimization of shortening SNR

As discussed in 4.3, maximum shortening SNR method outperforms MMSE in terms of data rate with shorter TEQ length. MSSNR attempts to maximize the energy ratio between the interested window and the rest of the channel impulse response. The effective channel impulse response is defined as

$$\mathbf{h}_{eff(N_w+N_h-1 \times 1)} = \mathbf{H}_{(N_w+N_h-1 \times N_w)} * \mathbf{w}_{(N_w \times 1)}$$

$$\mathbf{H} = \begin{bmatrix} h(1) & 0 & \cdots & 0 \\ h(2) & h(1) & \ddots & \vdots \\ \vdots & \vdots & & \\ h(N_h) & h(N_h-1) & \cdots & h(N_h-N_w+1) \\ 0 & h(N_h) & \cdots & h(N_h-N_w+2) \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & & h(N_h) \end{bmatrix} \quad (5.1)$$

Let $\mathbf{h}_{win}$ represent a window of $\nu + 1$ consecutive samples of $\mathbf{h}_{eff}$. $\mathbf{h}_{win}$ covers the range from Δ to $\Delta + \nu$. Let $\mathbf{h}_{wall}$ be the remainder of the samples, which includes row 1 to $\Delta - 1$ and $\Delta + \nu + 1$ to $N_h + N_w - 1$ of $\mathbf{h}_{eff}$.

$$\mathbf{H}_{win} = \begin{bmatrix} h(\Delta) & h(\Delta - 1) & \cdots & h(\Delta - N_w + 1) \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta + \nu) & h(\Delta + \nu - 1) & \cdots & h(\Delta + \nu - N_w + 1) \end{bmatrix}, \mathbf{h}_{win} = \mathbf{H}_{win} * \mathbf{w} \quad (5.2)$$

So we have

$$\mathbf{h}_{eff} = \begin{bmatrix} \mathbf{h}_{out1} \\ \mathbf{h}_{win} \\ \mathbf{h}_{out2} \end{bmatrix} \quad \mathbf{h}_{wall} = \begin{bmatrix} \mathbf{h}_{out1} \\ \mathbf{h}_{out2} \end{bmatrix} \quad (5.3)$$

$\mathbf{h}_{win}$ and $\mathbf{h}_{wall}$ are Δ functions. The SSNR objective function [29] is

$$SSNR = 10 \log \left(\frac{\mathbf{h}_{win}^T \mathbf{h}_{win}}{\mathbf{h}_{wall}^T \mathbf{h}_{wall}} \right) \quad (5.4)$$

The energy inside $\mathbf{h}_{win}$ and $\mathbf{h}_{wall}$ is discussed in (3.19).

The optimal shortening problem is solved by assuming unit energy inside the shortened window and minimizing the energy outside it [29]. That is minimizing $\mathbf{w}^T \mathbf{A} \mathbf{w}$ while maintaining $\mathbf{w}^T \mathbf{B} \mathbf{w} = 1$. Matrix $\mathbf{B}$ is assumed to be positive definite and has Cholesky decomposition as $\mathbf{B} = \sqrt{\mathbf{B}} \sqrt{\mathbf{B}}^T$. The optimum solution for the TEQ is achieved together with the smallest unit-length eigenvector $\mathbf{l}_{min}$ of matrix $(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}}^T)^{-1}$. The optimal equalizer setting is achieved at $(\sqrt{\mathbf{B}}^T)^{-1} \mathbf{l}_{min}$.

In order to avoid matrix $\mathbf{B}$ to be singular and not invertible, Yin and Yue [30] proposed an alternative way of optimization by maximizing the numerator $\mathbf{w}^T \mathbf{B} \mathbf{w}$ and constraining the denominator $\mathbf{w}^T \mathbf{A} \mathbf{w} = 1$ of (3.20). Both methods require Cholesky decomposition and an eigen-decomposition of a $N_w \times N_w$ matrix for every

tested delay Δ in order to optimize the shortening SNR ratio. Yin and Yue's method makes the arbitrary MSSNR TEQ length design possible and it's adopted for our design in the following sections.

5.2 Dependency of Matrices $\mathbf{A}$ and $\mathbf{B}$

SSNR TEQ method searches for a range of Δ values to obtain the optimum Δ and optimum $\mathbf{w}$ based on the objective function in (5.4). From (5.2), (5.3), (5.4) and (3.19), we know that the matrices $\mathbf{A}$ and $\mathbf{B}$ are functions of Δ . New $\mathbf{A}$ and $\mathbf{B}$ are calculated for every new Δ . This adds tremendous computation complexity in the implementation of time domain equalizer. Previous studies in [48] raised the issue of the possible efficient matrix computation. Martin, Ding, Evans and Johnson, derived a method that focused on the MSSNR optimum problem [49].

In this section, we show that the matrices $\mathbf{A}$ and $\mathbf{B}$ are mathematically dependent on each other. This means once we know $\mathbf{A}$, we will know $\mathbf{B}$ with very little calculation. $\mathbf{A}$ and $\mathbf{B}$ are defined in (3.19)

$$\begin{aligned}\mathbf{A} &= \mathbf{H}_{wall}^T \mathbf{H}_{wall} \\ \mathbf{B} &= \mathbf{H}_{win}^T \mathbf{H}_{win}\end{aligned}\tag{5.5}$$

From (5.2), (5.3), we have

$$\begin{aligned}\mathbf{C} &= \mathbf{H}_{eff}^T \mathbf{H}_{eff} = (\mathbf{H}_{out1}^T \mathbf{H}_{out1} + \mathbf{H}_{out2}^T \mathbf{H}_{out2}) + \mathbf{H}_{win}^T \mathbf{H}_{win} \\ &= \mathbf{H}_{wall}^T \mathbf{H}_{wall} + \mathbf{H}_{win}^T \mathbf{H}_{win} = \mathbf{A} + \mathbf{B}\end{aligned}\tag{5.6}$$

From (5.6) we know that matrix $\mathbf{C}$ is independent of delay Δ . Once we know $\mathbf{A}$ or $\mathbf{B}$ for a specific delay Δ , the other can be determined by doing a subtraction.

5.3 Computation proposal based on matrix $\mathbf{B}$

Rather than a brute force calculation approach that has been addressed in [48], Martin [49] proposed to emphasize on the computation of $\mathbf{B}(\Delta+1)$ from $\mathbf{B}(\Delta)$ by anatomizing matrix $\mathbf{B}$ columnwise. They concluded that overall it takes $2N_w - 1 + \nu$ multiplication and addition in order to get $\mathbf{B}(\Delta+1)$ from $\mathbf{B}(\Delta)$. Actually $2N_w$ is enough.

From (5.5), we know that

$$\mathbf{H}_{win}(\Delta) = [\mathbf{S} \quad \mathbf{D}] \quad \mathbf{H}_{win}(\Delta+1) = [\mathbf{E} \quad \mathbf{S}] \quad (5.7)$$

where $\nu+1$ column vectors $\mathbf{D}$ and $\mathbf{E}$ correspond to the last column vector of $\mathbf{H}_{win}(\Delta)$ and the first column vector of $\mathbf{H}_{win}(\Delta+1)$ respectively, $\mathbf{S}$ is $(\nu+1) \times (N_w - 1)$ matrix. $\mathbf{B}(\Delta)$ and $\mathbf{B}(\Delta+1)$ are:

$$\mathbf{B}(\Delta) = \begin{bmatrix} \mathbf{S}^T \\ \mathbf{D}^T \end{bmatrix} [\mathbf{S} \quad \mathbf{D}] = \begin{bmatrix} \mathbf{S}^T \mathbf{S} & \mathbf{S}^T \mathbf{D} \\ \mathbf{D}^T \mathbf{S} & \mathbf{D}^T \mathbf{D} \end{bmatrix} \quad (5.8)$$

$$\mathbf{B}(\Delta+1) = \begin{bmatrix} \mathbf{E}^T \\ \mathbf{S}^T \end{bmatrix} [\mathbf{E} \quad \mathbf{S}] = \begin{bmatrix} \mathbf{E}^T \mathbf{E} & \mathbf{E}^T \mathbf{S} \\ \mathbf{S}^T \mathbf{E} & \mathbf{S}^T \mathbf{S} \end{bmatrix} \quad (5.9)$$

Observing $\mathbf{B}(\Delta+1)$ from (5.13), $\mathbf{E}^T \mathbf{E}$ and $\mathbf{S}^T \mathbf{E}$ are terms that are unknown to $\mathbf{B}(\Delta)$. $\mathbf{E}^T \mathbf{S}$ can be obtained by a transpose operation if we know $\mathbf{S}^T \mathbf{E}$. The problem is now changing to solve the first column of $\mathbf{B}(\Delta+1)$. Since $\mathbf{B}(\Delta+1)$ is almost Toeplitz, $\mathbf{E}^T \mathbf{E}$ and $\mathbf{S}^T \mathbf{E}$ can be determined from the first column $\mathbf{B}(\Delta)$. As we know

$$\mathbf{H}_{win}(\Delta) = \begin{bmatrix} h(\Delta) & h(\Delta-1) & \cdots & h(\Delta-N_w+1) \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta+\nu) & h(\Delta+\nu-1) & \cdots & h(\Delta+\nu-N_w+1) \end{bmatrix} \quad (5.10)$$

$$\mathbf{H}_{win}(\Delta+1) = \begin{bmatrix} h(\Delta+1) & h(\Delta) & \cdots & h(\Delta - N_w + 2) \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta + \nu + 1) & h(\Delta + \nu) & \cdots & h(\Delta + \nu - N_w + 2) \end{bmatrix} \quad (5.11)$$

Substituting (5.10) and (5.11) into (5.5), we derive the following

$$\underbrace{\mathbf{B}(\Delta)_{(i,1)}}_{i \in [1, N_w]} = \sum_{j=0}^{\nu} h^T(\Delta+1-i+j)h(\Delta+j) \quad (5.12)$$

We then derive

$$\mathbf{B}(\Delta+1)_{(i,1)} = \mathbf{B}(\Delta)_{(i,1)} - h^T(\Delta+1-i)h(\Delta) + h^T(\Delta+2-i+\nu)h(\Delta+1+\nu) \quad (5.13)$$

For every entry of the first column $\mathbf{B}(\Delta+1)$, we only need 2 MAC operations. And for N_w entries we need $2N_w$ MAC operations. In Martin *et al.*'s method, they solve the last entry of the column by $\nu+1$ MACs, which may be simplified to two MACs as well. As compared with their overall requirement of $2N_w - 1 + \nu$ MACs, we save $\nu - 1$ MACs for every delay evaluated.

Figure 5.1 Computation from $\mathbf{B}(\Delta)$ to $\mathbf{B}(\Delta+1)$ with (5.13) by using the first column of $\mathbf{B}(\Delta)$. The first row of $\mathbf{B}(\Delta+1)$ is a transpose of the first column. The green matrix remains the same.

5.4 Efficient computation of matrix $\mathbf{C}$

Besides matrix $\mathbf{B}$, the calculation of matrix $\mathbf{C}$ can also be simplified further. From (5.6), we know the matrix $\mathbf{C}$ is independent of the delay Δ . Since $\mathbf{C}$ is a Toeplitz matrix, any row or any column is enough to determine the whole matrix. We focus on the first row of $\mathbf{C}$, which is calculated using the first row of $\mathbf{H}^T$ and every column of $\mathbf{H}$.

From (5.1), we know the first row of $\mathbf{H}^T$ is

$$[h(1) \quad h(2) \quad \cdots \quad h(N_h) \quad 0_{1 \times N_w-1}] \quad (5.14)$$

From the observation of row matrix in (5.14), only nonzero intersected terms in the first N_h terms of every column in $\mathbf{H}$ take part in column-row multiplication. And we count the number of multiplication and addition operations (MAC) needed for obtaining the first row of $\mathbf{C}$.

$\mathbf{C}_{1 \times 1}$ N_h Multiply and Adds

$\mathbf{C}_{1 \times 2}$ $N_h - 1$ Multiply and Adds

$\vdots$

$\mathbf{C}_{1 \times N_w}$ $N_h - N_w + 1$ Multiply and Adds

Altogether we need $\frac{(N_h + N_h - N_w + 1)N_w}{2}$ multiplication and addition

operations (MACs). Martin's algorithm [48], requires $N_h \times N_w$ MACs to obtain $\mathbf{C}$.

Ours only needs $(N_h - \frac{N_w - 1}{2}) \times N_w$ MAC. Obviously, we come to the conclusion that

our method saves $\frac{N_w - 1}{2} \times N_w$ MACs. If $N_w = 17$, we save 136 MACs in matrix **C** calculation.

5.5 Comparisons and recommendations

The complexity of Martin's method [49] as compared to our proposed method is presented in Table 5.1. Wu's method [48] is adopted in calculating the initial matrixes of **A** and **B**. Martin's method proceeds with matrix **B**(Δ) and our method saves $\upsilon - 1$ MACs for every delay evaluated. Initial matrices **A**_{ini} and **B**_{ini} do not incur additional complexity since both methods start from **B**_{ini}. Our method also saves MACs in the initial computation of matrix **C** as compared with Martin's method. For ADSL system downstream communication with $N_w = 16$ and $\upsilon = 32$, we save 120 MACs in matrix **C** and 31 MACs in every delay searched for the optimum equalizer. If the search is with 1 to 32, this could be 992 MACs which is significant. Our method could be proposed for use in VDSL DMT equalization scheme that has a υ ranging up to 512 and a N_w of tens or hundreds in DVB [49][50]. Table 5.2 shows the selection of cyclic extension as the number of subcarriers increases to achieve 4 kHz symbol rate [51]. Cyclic extension includes cyclic prefix and cyclic suffix as defined in [52]. Once the size of cyclic extension becomes significantly large, calculating Martin's extra $\upsilon - 1$ MACs in every delay as compared with ours is costly. In overall usage of arithmetic operations comparison in ADSL system, our proposed method beats Martin's since our method beats theirs in every item comparison in Table 5.1.

Our method is proposed to be a candidate in the design of DMT VDSL systems, where υ has a larger value than in ADSL system. Our method dominates the

algorithm in [49] in term of saving of MACs used in the amount of calculation required.

Table 5.1 Complexity Comparison of Martin et al [49] method and Proposed Method in MSSNR TEQ.

	Martin <i>et al</i> [49]	Proposed method
	MAC adds	MAC adds
C	$N_h N_w 0$	$(N_h - \frac{N_w - 1}{2}) N_w 0$
A_{ini}	$0 N_w^2$	$0 N_w^2$
B_{ini}	$N_w (N_w + \nu - 1) 0$	$N_w (N_w + \nu - 1) 0$
A(Δ)	$0 N_w^2$	$0 N_w^2$
B(Δ)	$2N_w + \nu - 1 0$	$2N_w 0$

Table 5.2 Selection of cyclic extension with respect to maximum subcarriers for VDSL

Cyclic extension expressed in samples	Maximum number of subcarriers
40	256
80	512
160	1024
320	2048
640	4096

The complexity analysis accounts for the floating-point addition and multiplication (MAC) only. Some other dimensions of the efficiency issues such as matrix subscripting, memory traffic, and other overheads associated with physical implementation have not been addressed. It somehow limits the accuracy in concluding which algorithm is superior. But it provides a general TEQ design algorithm guideline.

Chapter 6

Power Method in Eigenproblem

Solving

6.1 Algorithm of power method

The use of Power method [53] is to find the largest eigenvalue of a square matrix in absolute value and the corresponding eigenvector. The basic idea is to multiply the matrix repeatedly by a well-chosen starting vector, so that the component of that vector in the direction of the eigenvector with largest eigenvalue is magnified relative to other components. The speed of convergence of the power iteration depends on the ratio of the second largest eigenvalue (in absolute value) to the largest eigenvalue (in absolute value).

Iterations start from an initial eigenvector $\mathbf{x}_0$ in power method. The detailed algorithm is as follows

Given $\mathbf{x}_0$, we iterate

$$i = 0$$

repeat

$$\mathbf{y}_{i+1} = \mathbf{C}\mathbf{x}_i$$

$$\mathbf{x}_{i+1} = \mathbf{y}_{i+1} / \|\mathbf{y}_{i+1}\|_2 \quad (\text{Approximate eigenvector})$$

$$\tilde{\lambda}_{i+1} = \mathbf{x}_{i+1}^T \mathbf{C}\mathbf{x}_{i+1} \quad (\text{Approximate eigenvalue})$$

$$i = i + 1$$

until convergence

The power iteration method can be utilized in solving MSSNR problem. After obtaining the optimum matrix $\mathbf{A}$ and $\mathbf{B}$, the rest of the job is to determine a minimum unit-length eigenvector $\mathbf{l}_{\min}$ of matrix $(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}^T})^{-1}$ [29] or a maximum unit-length eigenvector $\mathbf{l}_{\max}$ of matrix $(\sqrt{\mathbf{A}})^{-1} \mathbf{B} (\sqrt{\mathbf{A}^T})^{-1}$ [30], from which the time domain equalizer is derived. Nafie and Gatherer first applied inverse power method in optimizing the MMSE equalizer in [27]. Chiu and Tsai introduced inverse power method in solving the optimum MSSNR equalizer taps [32]. Inverse power method has been proposed to solve the minimum eigenvector problem associated with an eigenvalue in [54]. Previous works attempted to determine the unit-length eigenvector $\mathbf{l}_{\min}$ corresponding to the minimum eigenvalue of $\lambda_{\min}$ of matrix $(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}^T})^{-1}$ [29]. Our approach of implementing power method is focused on the MSSNR algorithm proposed by Yin and Yue [30], which optimizes the shortening signal to noise ratio with arbitrary length time domain equalizer. When $N_w > \nu$, $\mathbf{B}$ is singular and $(\sqrt{\mathbf{B}})^{-1}$ does not exist. Regardless of the number of TEQ taps N_w , matrix $\mathbf{A}$ is

always positive definite when the physical channels are twisted copper wires. So there is always meaningful $(\sqrt{\mathbf{A}})^{-1}$. Utilizing power method to determine the eigenvector of matrix $(\sqrt{\mathbf{A}})^{-1} \mathbf{B} (\sqrt{\mathbf{A}}^T)^{-1}$ ensures the arbitrary length of TEQ. Simulation in the later part of this chapter shows that three iterations are enough for the maximum eigenvalue to converge.

6.2 New implementation of power method

Previous researchers have demonstrated the implementation of inverse power method to solve the eigenproblem. The algorithm is almost the same as the power method except that it works with the matrix $\mathbf{C}^{-1}$ instead of the original matrix $\mathbf{C}$. The vector will converge to the eigenvector associated with the dominant eigenvalue of $\mathbf{C}^{-1}$. Since the eigenvalue of $\mathbf{C}^{-1}$ and $\mathbf{C}$ are inverse of each other and their eigenvectors are identical, the iteration will converge to the eigenvector of $\mathbf{C}$ associated with the smallest absolute eigenvalue. Inverse power method fits well into the MSSNR method described in [29]. Comparing with power method, inverse power method requires more calculation when inverting an $N_w \times N_w$ square matrix. Our new power method implementation is based on the MSSNR algorithm [30]. We attempt to find the maximum unit-length eigenvector of the original matrix $\mathbf{C}$, which is $(\sqrt{\mathbf{A}})^{-1} \mathbf{B} (\sqrt{\mathbf{A}}^T)^{-1}$, by directly adopting the power method.

Several channels shortening TEQ using inverse power method, power method and Melsa's methods [29] are simulated. The current ADSL standards (ITU Recommendations G.992.1 and G.992.2) are adopted. It specifies a bit error rate of 10^{-7} and a margin of 6 dB [55]. For uncoded PAM and QAM constellations, Γ (SNR

gap) is equal to 9.8 dB for the same desired probability of bit error rate. We take coding gain as 4.2 dB.

The length of the target impulse response is chosen to be 32 and it is compatible with ADSL downstream 32 cyclic prefix. The sampling frequency is 2.208MHz. Test channel data for the CSA loop and AWGN plus NEXT noise was obtained directly from the Matlab DMTTEQ Toolbox developed by G. Arslan, B. Lu, and B. L. Evans [57]. CSA loop 4 is used in the simulation. The size of FFT is set to 512. The input signal power is flat with 23dBm throughout all used subchannels.

The initial eigenvectors in both power method and inverse power method are set to have unit energy with equal vector entries. Simulation results show that only three iterations are enough for eigenvalues to converge. The convergence margin is set at 10^{-20} . If the absolute values of two consecutive eigenvalues differ within this convergence margin, the optimum eigenvalue is achieved. Figs 6.1-6.4 show the original and shortened channel impulse response by implementing power (PM), inverse (InvPM) power, MSSNR and MMSE with unit energy constraint method (MMSE-UEC). The thicker line drawn on x-axis indicates the target window. Full range of optimum delay search is carried out for obtaining the maximum SSNR.

Figure 6.1 Original Channel (dashed line, CSA loop4) and shortened implementing InvPM

Figure 6.2 Original channel (dashed line, CSA loop4) and shortened channel implementing PM

Figure 6.3 Original channel (dashed line, CSA loop4) and shortened channel implementing MSSNR

Figure 6.4 Original channel (dashed line, CSA loop4) and shortened channel implementing MMSE-UEC

6.3 Stability Issues of Numerical Algorithm for Eigen-Problem

The optimum delay, maximum SSNR value and achievable bit rate are tabulated in Table 6.1. MSSNR obtains the optimum eigenvalue by directly calling the function of `eig()` in Matlab. The optimum delays for inverse power method and MSSNR are the same. So are the values of the objective function. The aforementioned different values can be explained as the inverse power method and power method adopt different algorithms in solving the eigenproblem. Inverse power method and MSSNR attempt the eigenproblem by determining a minimum unit-length eigenvector $\mathbf{l}_{\min}$ of matrix $(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}}^T)^{-1}$ [29]. Power method approaches the problem by solving the maximum eigenvalue of matrix $(\sqrt{\mathbf{A}})^{-1} \mathbf{B} (\sqrt{\mathbf{A}}^T)^{-1}$ [30]. Inverse power method needs a square matrix inversion of $(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}}^T)^{-1}$. This will cause eigenvalue stability issues which leads to some variation in optimum delay. Furthermore, more matrix manipulation is needed as compared to power method. In Table 6.1, power method achieves the best SSNR value. The calculated TEQ based on the iteration with inverse power method and power methods both outperform the MSSNR method. This proves that the objective function in MSSNR method doesn't guarantee the optimization of the achievable bit rate. The MMSE with unit energy constraint produce the least data rate at the TEQ output. In terms of achievable bit rate, power method and inverse power method are almost the same. Our proposed power method needs fewer matrix computations since it doesn't require a matrix inversion as compared with inverse power method.

Table 6.1 Optimum Delay, Maximum SSNR and Achievable bit rate for different algorithm

Algorithm	Optimum Delay	Optimum SSNR	Bite Rate (Mbps)
PM	54	47.12	7.62
InvPM	50	46.87	7.63
MSSNR	50	46.85	7.59
MMSE-UEC	39	25.4	6.14

We then further investigate the bit rate vs. the length of time domain equalizer N_w . We use all the same settings as those selected in the previous simulation except that a more realistic optimum delay search is conducted from 15 to 35. By varying N_w from 2 to 32, we plot Fig 6.5. The simulation results show that the power method has more stable performance and higher achievable data rate than the inverse power method. Further simulations have been performed with small TEQ taps with different CSA loops in Fig 6.7-6.11, and similar performances of PM an InvPM are observed. Inverse power method uses the same algorithm but it has different achievable bit rate performance at small TEQ taps region as indicated by the dotted line in Fig 6.5. The inverse power method exhibits unstable characteristics in the $15 < N_w < 20$ region. Similar performance is reproducible with different CSA loops Fig 6.7-6.11. The output date rate has a sharp drop and then recovers to normal at the region. Another graph is drawn to focus on this stability issue in inverse power iteration method in Fig 6.6. Observed from Fig 6.5, the power method obtains the smoothest output data rate curve with the variation of N_w . There is an abrupt jump of bit rate from N_w value of 26 to 27. Similar abrupt changes can be observed in Fig 4.2 with MMSE and MSSNR. Simulations (Fig 6.7-6.12) have also been done using CSA loop 1-6; similar conclusions can be drawn as in Fig. 6.5 with acceptable tolerance. In CSA loop 1-4,

simulation results (Fig 6-10) show that power method presents better output data rate than inverse power method at most of TEQ taps from 2-32. Inverse power method has a region of instability from TEQ taps 10-20. The power method performs consistently in terms of output data rate as N_w increases. In CSA loop 5-6, the power method and inverse power perform almost similar in terms of output data rate and instability.

Figure 6.5 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM and MSSNR with CSA loop 4

In the power method and inverse power methods, the convergence rate depends on $|\lambda_2 / \lambda_1|$, where λ_2 and λ_1 is the second largest eigenvalue and the largest eigenvalue of the matrix in which we are interested. This ratio is generally smaller than 1, which allows adequate convergence. But there are cases where this ratio can be very close to 1, which causes very slow convergence [53]. Figure 6.13 shows the convergence ratio with different TEQ taps using power method and inverse power method with CSA loop 4. With a shorter TEQ length, power method may be more

effective since it has convergence ratio closer to 0. The convergence ratio is calculated after the optimum delay is obtained. As shown in Table 6.1, power method and inverse power method get different optimum delay with the same set of settings. The maximum and second maximum eigenvalues are determined by Matlab function `eigs()`. The convergence ratio graph with respect to N_w could be a good reference guide in determining whether power method can provide satisfied convergence rate in implementation. The simulation in Figure 6.13 shows that power method has a faster convergence ratio than inverse power method in most N_w .

Figure 6.6 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM and MSSNR method. Inverse power method has a sharp drop and then recover to normal in data rate.

Figure 6.7 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM with CSA loop 1

Figure 6.8 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM with CSA loop 2

Figure 6.9 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM with CSA loop 3

Figure 6.10 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM with CSA loop 4

Figure 6.11 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM with CSA loop 5

Figure 6.12 Achievable bit rate vs. the length of TEQ N_w for InvPM, PM with CSA loop 6

Figure 6.13 Convergence ratio of PM and InvPM vs. the length of TEQ N_w

Comparing the implementation complexity, the power method iteration does not require an inversion in square matrix which inverse power method needs. TEQ utilizing power method has a higher and more stable achievable bit rate curve than that of the inverse power method in most of CSA loops. Furthermore, power method has a faster convergence rate than inverse power method.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

A new efficient matrix calculation algorithm for the maximum shortening SNR method for time domain equalizer is proposed for ADSL transceivers. Its feasibility and effectiveness has been evaluated by theoretical analysis. When compared to the Maximum Shortening Signal-to-Noise Ratio (MSSNR) method for ADSL time domain equalizer (TEQ) design, our proposed power design not only yields improved bit rate at the TEQ output, but also has lower computational complexity than the fastest previously reported implementation of the MSSNR method. Our proposed method would be a better TEQ design option for DMT ADSL transceivers than the MSSNR method. An implementation of a new power method to solve eigenvector is proposed to obtain arbitrary length of time domain equalizer. Simulation also shows that our power method design performs well by providing high output data rate.

Another TEQ design method, the min-ISI TEQ method, outperforms all other previous design methods except filter bank TEQ designs (per tone equalization) in bit rate yet require fewer taps. The trade-off is the implementation complexity. For the

same TEQ length, the bit rate difference between min-ISI and the MMSE and MGSNR methods is up to 4 times more. Min-ISI's achievable bit rate is 4 times higher. The performance of the min-ISI method is better than that of the MSSNR method; although the performance difference is not significant. The min-ISI scheme generalizes the MSSNR design method. The basic difference is that the MSSNR method minimizes the energy of ISI while the min-ISI minimizes the output power of the ISI path impulse response.

Theoretical analysis shows that we can take advantage of the structure in the argument matrices, especially the Toeplitz structure of $\mathbf{H}$, in developing an efficient algorithm to compute the optimum matrix $\mathbf{A}$ and $\mathbf{B}$, which are the intermediate results for determining the minimum unit-length eigenvector $\mathbf{I}_{\min}$ of matrix

$(\sqrt{\mathbf{B}})^{-1} \mathbf{A} (\sqrt{\mathbf{B}}^T)^{-1}$ or maximum unit-length eigenvector $\mathbf{I}_{\max}$ of matrix $(\sqrt{\mathbf{A}})^{-1} \mathbf{B} (\sqrt{\mathbf{A}}^T)^{-1}$. One approach is to focus on matrix $\mathbf{B}$ in optimum delay searching.

We propose an algorithm using matrix $\mathbf{B}$ with less computational effort. The complexity of real time implementation depends on the arithmetic operations required for calculating the next delay matrix from the previous delay. The mathematical derivation shows our proposed method saves $0 - 1$ MACs for every delay computation and $\frac{N_w - 1}{2} \times N_w$ MACs for the initial matrix $\mathbf{C}$. Our proposed implementation scheme beats all the published MSSNR method in complexity in ADSL equalizer design. In DMT VDSL systems, the cyclic extension adopts a large value (CP is 80 in VDSL); our proposed method is a better choice in MSSNR TEQ design to save real time calculations.

We have proposed a new power method in solving the eigenvector problem after optimizing the objective function of SSNR. The original form of power method is utilized to determine the unit-length eigenvector for the optimum SSNR design scheme. The new design method offers not only an arbitrary length of time domain equalizer but also a better bit rate performance than in the inverse power method. Further simulation analysis suggests that the new iterative power method outperforms iterative inverse power method in terms of achievable bit rate and stability in most of the equalizer lengths in the range less than 32 tested in ADSL system.

7.2 Recommendations

7.2.1 Recommendations for implementation

- Computational complexity is still an important issue for matrix eigenvalue computation. The introduction of parallel computation in real time application might be able to lower the computation cost of the matrix eigenvalue calculation.
- Our proposed implementation method effectively reduces the computational complexity and it is proven to include the least number of multiplication and addition operations compared with published results in MSSNR TEQ design methods.

7.2.2 Recommendations for future work

- The TEQ design methods proposed in this thesis are based on the assumption that the physical channel impulse response is not time varying. When applying TEQ design method to time varying dynamic fading channels, one may encounter complicated problems in channel estimation. It would be desirable to modify current TEQ design schemes for time-varying channels.
- To further maximize the channel capacity, joint optimization of TEQ with other parts in DMT transceivers (e.g. the FEQ) could increase the data transmission rate for the same CSA loop. Pertone equalization is derived by shifting TEQ components into FEQ in the transceiver and it leads to better achievable bit rate over a TEQ. Joint TEQ-FEQ design can be a possible research direction in DMT systems, although the trade off is the implementation complexity.
- The simulation did not include noise sources such as far-end crosstalk (FEXT) or near-end echo to the model of the subchannel SNR. Near-end echo from the local transmitter is a powerful source of noise and it would be worthwhile to include it in future SNR models.

Author's Publications

1. Zhu Pengye, K. R. Subramanian, "Time-Domain Equalizers for OFDM/DMT Transceivers", In Proceedings of the International Symposium on Telecommunication (IST 2003), Isfahan Iran, August 2003.
2. Zhu Pengye, K. R. Subramanian, "Efficient Computation approach for Maximum Shortening SNR Time domain equalizer design", In proceedings of Network Technology Research Center Research forum 2004, Nanyang Technology University of Singapore, Singapore, Feb 2004.
3. Zhu Pengye, K. R. Subramanian, "Computationally Efficient Iterative Algorithm for Channel Shortening Equalizer Design", revised and submitted to the Journal of Communications and Networks 2004.
4. Zhu Pengye, K. R. Subramanian, "Fast and Efficient Computation Design in xDSL modem", accepted by IEEE Region 10 Infocom Colloquium on Broadband Access – Technology and Market, 21-22 October 2004, Singapore.

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