Adaptive power control for multi-antenna and cooperative communications with imperfect channel knowledge

Zhang, Xiao Juan

2010

Zhang, X. J. (2010). Adaptive power control for multi-antenna and cooperative communications with imperfect channel knowledge. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/40924>

<https://doi.org/10.32657/10356/40924>

Adaptive Power Control for Multi-Antenna and Cooperative Communications with Imperfect Channel Knowledge

ZHANG Xiao Juan

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2010

Acknowledgment

I am sincerely grateful to my supervisor Prof. GONG Yi for his patient guidance, stimulating encouragement, and deep insights in research. His invaluable suggestions and advices have helped me identify the right research topic and always keep in the right track.

I would like to thank Prof. LI Kwok Hung, Prof. GUAN Yong Liang, Prof. Erry GUNAWAN and Prof. MA Maode for their enlightening courses. I would also like to thank Prof. Khaled Ben LETAIEF, Prof. ZHENG Lizhong and Prof. J. Nicholas LANEMAN for their stimulating discussions that helped me a lot in various aspects.

My sincere thanks also go to my friends in NTU and my former colleagues in Guangdong Nortel R&D Center, who cheered me up and encouraged me whenever necessary.

Finally, I wish to give thanks to my parents and sister for their continuous support and understanding. I would like to dedicate this work to my dearest parents and sister.

Abstract

In this thesis, we consider adaptive power control for multi-antenna and cooperative fading channels with imperfect channel state information (CSI). The thesis comprises two main parts.

In the first part, we study temporal power control in multi-antenna and cooperative relaying channels subject to long-term power constraints at the transmitters, with a focus on the asymptotically high signal-to-noise ratios (SNRs). Efficient power control schemes based on noisy CSI at the transmitters (CSIT) and/or receivers (CSIR) are proposed to maximize the achievable diversity-multiplexing tradeoff (DMT). We also provide a framework to systematically study the achievable DMT in multi-input multi-output (MIMO) and cooperative systems with noisy CSI. We firstly consider a MIMO system and develop novel two way training strategies to improve the DMT. To make more efficient use of the imperfect CSI, joint power and rate control schemes are proposed which drastically increase the achievable DMT, especially at high multiplexing gains. Next, we extend the DMT analysis to cooperative relaying channels. Both conventional amplify-and-forward (AF)/decode-and-forward (DF) and dynamic DF (DDF) relaying protocols are considered. We show that long-term power control based on imperfect CSIT significantly improves the DMT of the conventional AF and DF relaying protocols. Moreover, it is shown that the DDF relaying, which supports higher spectral efficiencies, also enjoys a further improved DMT over the conventional relaying protocols.

In the second part, we study spatial power control in cooperative relaying channels to minimize the system outage/error probability. In particular, we con-

sider adaptive power allocation (PA) among the source and relay(s) subject to a short-term sum power constraint at the source and relay(s). We first consider a dual-hop DF relay system with multiple antennas at the destination. With partial CSIT in the form of channel statistics, we take both PA and relay location into joint optimization. Secondly, we consider a multi-hop DF relay system with imperfect CSIT, where the imperfect CSIT is due to limited channel feedback from the destination to the source and relays.

Contents

Acknowledgment	ii
Abstract	iii
List of Abbreviations	xiii
List of Notations	xv
1 Introduction	1
1.1 Motivation	1
1.2 Problem Statement	7
1.3 Major Contributions of the Thesis	8
1.4 Organization of the Thesis	10
2 Preliminaries	11
2.1 MIMO Channels	11
2.1.1 Ergodic Capacity in Fast Fading Channels	13
2.1.2 Outage Capacity and Diversity-multiplexing Trade-off in Slow Fading Channels	15
2.2 Relay Channels	17
2.2.1 Three-Node Relay System	18

2.2.2	Dual-hop Multi-relay System	21
3	Power Control in MIMO Channels With Imperfect CSI: A DMT Perspective	23
3.1	System Model	24
3.2	Power Control with Imperfect CSIT and Perfect CSIR	24
3.3	Power Control with Imperfect CSIT and CSIR	37
3.3.1	DMT Analysis for Destination-Initiated Training	40
3.3.2	DMT Analysis for Source-Initiated Training	44
3.4	Joint Rate and Power Control with Imperfect CSIT and CSIR	47
4	Power Control in Cooperative Relaying Channels With Imperfect CSIT	57
4.1	Conventional AF/DF Relaying	58
4.1.1	System Model	58
4.1.2	Diversity-Multiplexing Tradeoff Analysis	61
4.1.3	Examples and Discussions	68
4.2	Dynamic Decode-and-Forward Relaying	71
4.2.1	System Model	71
4.2.2	Diversity-Multiplexing Tradeoff Analysis	72
4.2.3	Discussions	78
5	Adaptive Power Allocation in Dual-hop Relaying Channels	80
5.1	System Model	81
5.2	Outage Performance Analysis	83
5.3	Optimizing PA and Relay Location	86

5.3.1	Optimizing PA with Fixed Relay Location	86
5.3.2	Optimizing Relay Location with Fixed PA	88
5.3.3	Jointly Optimizing PA and Relay Location	88
5.4	Numerical Results	91
6	Adaptive Power Allocation in Multi-hop Relaying Channels	98
6.1	System Model	99
6.2	Optimal Power Allocation with Perfect CSI	100
6.3	Adaptive Power Allocation with Imperfect CSIT	103
6.4	Simulation Results and Discussions	107
7	Conclusion and Future Work	113
7.1	Conclusion	113
7.2	Future Work	115
	Appendices	117
A	Proof of Lemma 3.2.2	117
B	Proof of Lemma 3.2.3	119
C	Proof of Theorem 3.3.1	120
D	Proof for the DMT of Quantized-Power Controlled Training	124
E	Proof of $\sum_{n=1}^N (p - v_n)^+ < r_{min}$ Satisfying $\tau \sum_{n=1}^N w_n > \bar{r} - r_{min}$	126
F	Proof of the Convergence of Lower and Upper Bounds	127

G Proof of Equation (5.3.15)	129
H Proof of the Number of Roots to (5.3.15a)	130
Author's Publications	132
Bibliography	134

List of Figures

2.1	A MIMO system.	12
2.2	A three-node relay system.	19
2.3	A dual-hop multi-relay network.	21
3.1	Relationship of Ω_k and $\tilde{\Omega}_k$	34
3.2	DMT in a 3×3 MIMO channel. Note that $\mathbf{d}_o(\mathbf{r})$ in the legend denotes $d_{CSIT}^*(r)$	35
3.3	DMT in a 4×2 MIMO channel with $\alpha = 0.1$. Note that $\mathbf{d}_o(\mathbf{r})$ in the legend denotes $d_{CSIT}^*(r)$	35
3.4	Diversity gain versus channel quality α in a SIMO/MISO channel. The dashed line refers to the rate control scheme in [20].	37
3.5	Diversity gain at $r = N$ versus channel quality α in a 5×3 MIMO channel.	38
3.6	Illustration of two-way training. Note that $L \gg I \cdot T$	40
3.7	Diversity-multiplexing tradeoff in SIMO/MISO channels with two-way training. $\alpha = \beta = 1$	46
3.8	Diversity-multiplexing tradeoff in 2×2 MIMO channels with two-way training. $\beta = 1$	47

3.9	DMT in a 3×3 MIMO channel with $r_{min} = 0.5$ and $\alpha = 1/2$. The multiplexing gain r on x-axis refers to $\bar{r}$ in the case of rate control or joint power and rate control.	55
4.1	A dual-hop relay system with multiple destination antennas and direct link.	58
4.2	DMT in relaying channels with direct link.	69
4.3	DMT in relaying channels without direct link.	70
4.4	A three-node relay system with cut-sets.	78
4.5	DMT in relaying channels with imperfect CSIT. Note that for the conventional relaying, $0 \leq r \leq 1/2$	79
5.1	A dual-hop relay system with multiple destination antennas and no direct link.	82
5.2	Outage probability in a DF relay system with direct link. $N = 2$, $d = 0.5$	91
5.3	Performance of adaptive/semi-adaptive/fixed allocation algorithms in a DF relay system with direct link.	92
5.4	Optimal source power ratio versus relay location in a DF relay system with direct link under various $P_{N,T}$. The circled points are the jointly optimized solution. $N = 2$	93
5.5	Optimal source power ratio versus relay location in a DF relay system with direct link under various N . The circled points are the jointly optimized solution. $P_{N,T}/\sigma^2 = 20$ dB.	94
5.6	Outage probability in a DF relay system without direct link. $N = 3$, $d = 0.5$	95
5.7	Performance of adaptive/semi-adaptive/fixed allocation algorithms in a DF relay system without direct link. $N = 3$	95

5.8	Optimal source power ratio versus relay location in a DF relay system without direct link. The circled points are the jointly optimized solution. $N = 3$	96
5.9	Impact of N on the semi-adaptive allocation algorithms in a DF relay system without direct link.	97
6.1	Ratio 1: the probability of $P_T \leq 10 \left(\ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)} \right)$. Ratio 2: the probability of $P_T \leq \left(\ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)} \right)$. $M_r = 4, \alpha = 4$	103
6.2	SER performance of different PA algorithms. $\alpha = 2.6$	108
6.3	SER performance of different PA algorithms. $\alpha = 4$	108
6.4	SER performance of different PA algorithms. $\alpha = 4$. Relays are equally spaced between the source and destination.	110
6.5	SER performance of Algm 2 with different number of feedback bits. $\alpha = 4$	110
6.6	Achievable end-to-end rates of different PA algorithms. $\alpha = 2.6$. .	111
6.7	Outage probabilities of different PA algorithms. $\alpha = 2.6$	112

List of Tables

5.1	Joint optimal solutions $(d^*, \frac{P_{N,s}^*}{P_{N,T}})$ in DF relay system with direct link	90
5.2	Joint optimal solutions $(d^* = \frac{P_{N,s}^*}{P_{N,T}})$ in DF relay system without direct link	91

List of Abbreviations

3GPP	Third Generation Partnership Project
AF	Amplify-and-Forward
ARQ	Automatic Retransmission Request
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BS	Base Station
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
CF	Compress-and-Forward
CSI	Channel State Information
CSIT	CSI at the Transmitter
CSIR	CSI at the Receiver
DF	Decode-and-Forward
DMT	Diversity-Multiplexing Tradeoff
DSL	Digital Subscriber Line
DSTC	Distributed Space-Time Coded
FDD	Frequency Division Duplexing
i.i.d.	Independent and Identically Distributed
WLAN	Wireless Local Area Network
MGF	Moment Generating Function
MIMO	Multi-Input Multi-Output
MISO	Multi-Input Single-Output
ML	Maximum-Likelihood
MRC	Maximum Ratio Combining
MS	Mobile Station

NNUB	Nearest neighbor Union Bound
OFDM	Orthogonal Frequency Division Multiplexing
PA	Power Allocation
pdf	Probability Density Function
QoS	Quality of Service
R-D	Relay-Destination
SIMO	Single-Input Multi-Output
SNR	Signal-to-Noise Ratio
S-D	Source-Destination
S-R	Source-Relay
SVD	Singular Value Decomposition
SER	Symbol Error Rate
TDD	Time Division Duplexing
TDM	Time-Division Multiplexing
WF	Water-filling

List of Notations

$\triangleq$	variable definition
$E\{\cdot\}$	expectation operator
$\doteq$	exponential equality, $f(x) \doteq x^n \Leftrightarrow \lim_{x \rightarrow \infty} \log(f(x))/\log(x) = n$
$\mathcal{R}^N$	the set of real N -tuples
$\mathcal{R}^{N+}$	the set of non-negative N -tuples
$\mathcal{C}^{N \times M}$	the set of complex $N \times M$ matrices
a	scalar
$\mathbf{a}$	column or row vector
$\mathbf{A}$	matrix
$\mathbf{A}_{m,n}$	element in m -th row and n -th column of matrix $\mathbf{A}$
$\mathbf{A}^T$	transpose
$\mathbf{A}^*$	conjugate
$\mathbf{A}^\dagger$	conjugate transpose(Hermitian)
$\mathbf{A} \succcurlyeq 0$	means that $\mathbf{A}$ is positive semidefinite
$\det(\mathbf{A})$	determinant of $\mathbf{A}$
$\ \mathbf{A}\ _F$	Frobenius norm of $\mathbf{A}$
$\sigma_k(\mathbf{A})$	k -th singular value of matrix $\mathbf{A}$
$\lambda_k(\mathbf{A})$	k -th eigenvalue of matrix $\mathbf{A}$
$\text{Tr}(\mathbf{A})$	trace of $\mathbf{A}$
$(x)^+$	$\max(x, 0)$
$\mathcal{O}^+$	$\mathcal{O} \cap \mathcal{R}^{N+}$
$\mathcal{O}^c$	the complementary set of $\mathcal{O}$
$\emptyset$	the empty set
$x \in (a, b]$	the scalar x belongs to the interval $a < x \leq b$
$\mathcal{CN}(0, \sigma^2)$	the complex Gaussian distribution with mean 0 and variance σ^2

$\delta(\cdot)$	Dirac delta (unit impulse) function
$\mathbf{I}_N$	$N \times N$ identity matrix
$\log(\cdot)$	the base-2 logarithm
$\succeq$	the componentwise inequality
$\mathcal{Q}(x)$	$\mathcal{Q}$ -function, $\mathcal{Q}(x) = 1/\sqrt{2\pi} \int_0^\infty e^{-t^2/2}$
$\exp(x)$	e^x
$\Gamma(x)$	gamma function, $\Gamma(x) = \Gamma(x, 0) = \int_0^\infty u^{x-1} e^{-u}$
$\mathcal{P}(\mathcal{A})$	the probability of event $\mathcal{A}$

Chapter 1

Introduction

1.1 Motivation

Transmit power is a key degree of freedom in the design of wireless networks. Increasing the transmit power leads to a higher signal-to-noise ratio (SNR) and a reduced bit error rate (BER) at the receiver, and it allows a system to transmit at a higher data rate resulting in greater spectral efficiency. Nevertheless, higher transmit power also has some major drawbacks. For example, when the overall power consumption at the transmitting device is increased, it will lead to a reduced battery life. Further more, the interference to other users will also increase. This problem is particularly acute in interference-limited systems, such as code division multiple access (CDMA) systems where perfect orthogonality among users is difficult to maintain. Therefore, power control is of great practical importance to achieve good system performance. Since 1970s, power control in wireless networks has been systematically studied with respect to various optimizing metrics such as link data rate, network capacity, geographic coverage and range, and life of the network and network devices and so on. Power control algorithms have been used in many contexts, including cellular networks, sensor networks, wireless local area networks (WLANs), and digital subscriber line (DSL) modems [1].

In wireless systems, there is an increasing demand for higher data rates, bet-

ter quality of service (QoS), and higher network capacity. In recent years, Multi-Input Multi-Output (MIMO) systems, which have multiple antennas at both the transmitting end as well as the receiving end, have emerged as a most promising technology in these measures. On the one hand, multiple antennas can be used to increase diversity to combat channel fading [2, 3]. Each pair of transmit and receive antennas provides a signal path from the transmitter to the receiver. By sending signals that carry the same information through different paths, multiple independently faded replicas of the data symbol can be obtained at the receiver end; hence, more reliable reception is achieved. On the other hand, multiple antennas can be used to transmit independent information streams in parallel through the spatial channels, which increases the data rate thus providing spatial multiplexing gain [4, 5]. In summary, a MIMO system can provide two types of gains: diversity gain and spatial multiplexing gain. It was shown in [6] that both gains can, in fact, be simultaneously obtained, but there is a fundamental tradeoff between how much of each type of gain any coding scheme can extract: higher spatial multiplexing gain comes at the price of sacrificing diversity. The diversity-multiplexing tradeoff (DMT) presented in [6] provides a theoretical framework to analyze many existing diversity-oriented and multiplexing-oriented MIMO schemes. Later, the DMT analysis was extended to multiple-access channels in [7]. In the meantime, considerable research has been devoted to designing codes that can achieve the desired tradeoff of diversity and multiplexing gain [8]-[11]. The DMT for a MIMO channel at finite SNRs was also studied in [12, 13], indicating that the achievable diversity gains at realistic SNRs are significantly lower than those at asymptotically high SNRs.

It is noted that most work on DMT has assumed no channel state information (CSI) at the transmitter (CSIT) and perfect CSI at the receiver (CSIR), e.g., [6, 7]. When the transmitter has channel knowledge, the DMT can be further enhanced through power control. If CSIT is perfectly known, the outage events might be completely avoided in interference-free slow fading channels since the transmitter is always able to adjust its power adaptively according to the instantaneous channel conditions while keeping a long-term power constraint. For example, it can transmit with a higher power when the channel is poor and a lower power

when the channel is good. This is called *temporal power control*. However, in practice the CSIT is almost always imperfect due to imperfect CSI feedback from the receiver or imperfect channel estimation at the transmitter through pilots. For frequency division duplex (FDD) systems, partial channel knowledge can be made available at the transmitter through a quantized channel feedback from the receiver. The automatic retransmission request (ARQ) scheme was shown to be able to significantly increase the diversity gain by allowing retransmissions with the aid of decision feedback and power control in block-fading channels [14]. The works in [15, 16] showed that finite-bit channel information with power control led to a substantial (though finite) diversity improvement and the diversity gain increased unboundedly as the number of feedback bits increased in MIMO channels. Note that the above works have assumed error free feedback. Within a noisy feedback channel framework, Aggarwal *et al.* showed in [17] that a single bit of imperfect feedback was sufficient to double the maximum diversity gain compared to the case when there was no feedback in multi-user MIMO systems, and that using more feedback bits did not necessarily increase the maximum diversity gain. In time division duplex (TDD) systems, on the other hand, the transmitters can estimate the CSIT of forward channels using the pilots received in backward channels, due to the channel reciprocity. It was reported in [18] that transmitter training through pilots significantly increased the achievable diversity gain in a single-input multi-output (SIMO) link and the work was later extended to a two-way training scheme in [19]. It is not unexpected that rate control can also help to improve the achievable DMT. Lim and Lau showed in [20] that rate control was able to achieve the same DMT in SIMO/multi-input single-output (MISO) channels as power control did in [18], if we ignore the multiplexing gain loss due to training symbols.

Despite the promise shown by multiple antennas in mitigating the effects of fading, increasing the number of transmit antennas on small mobile devices is often impractical as a result of size and hardware complexity constraints. For such scenarios, cooperative relaying transmission was proposed as an alternative to multi-antennas. Due to the broadcast nature of wireless communications, when a source node transmits, besides the intended destination, other nodes within

earshot may also receive the transmitted signal. Then, these nodes can work as relays to re-transmit the source message to destination. Since the destination node receives multiple independently faded replicas of the source signal, diversity is obtained, which is also called *cooperative diversity* [21]. The following lists some of the main advantages of cooperative relaying transmissions:

1. Higher spatial diversity: resistance to both small scale and shadow fading
2. Higher throughput/lower delay: higher achievable data rates, fewer retransmissions, and lower transmission delay
3. Reduced interference/lower transmitted power: better frequency reuse in a cellular/WLAN deployment
4. Adaptability to network conditions: opportunistic use and redistribution of network energy and bandwidth

The classical relay channel introduced by Van der Meulen models a three-node communication channel [22]. Cover and El Gamal developed lower and upper bounds on the channel capacity for specific non-faded relay channel models [23]. Later, several works have studied the capacity of the relay channels and developed coding strategies that can achieve the ergodic capacity of the channel under certain scenarios [24]. In [25, 26], Laneman *et al.* proposed different cooperative diversity protocols and analyzed their performance in terms of outage behavior. Some most popular relaying strategies were introduced in these two works. In decode-and-forward (DF) relaying, the relay decodes the source signal, re-encodes it and forwards it to the destination. In amplify-and-forward (AF) relaying, the relay forwards the received signal to the destination after scaling it to satisfy its power constraint. In compress-and-forward (CF) relaying, the relay retransmits a quantized or compressed version of the received signal, exploiting the statistical dependencies between the signal received at the relay and that received at the destination. User cooperation diversity has also been proposed recently [27], where the authors implemented a two-user CDMA cooperative system. Another technique to achieve diversity that incorporates error-control-coding into cooperation is *coded diversity*, which was introduced by Hunter *et al.* in [28].

A lot of research has been devoted to developing the optimal power allocation (PA) algorithms at the source and relays under a total power constraint to improve the performance of cooperative relay systems. The total power constraint corresponds to the maximum power that a packet is allowed to consume throughout its propagation from the source to the destination. Since a short-term power constraint is employed leading to the same total power for each channel condition but adaptive PA among different nodes, we call these PA methods *spatial power control* (Chapters 5 and 6) in this thesis in contrast to *temporal power control* (Chapters 3 and 4). For three-node wireless relay channels, the authors in [29, 30] discussed optimal PA to maximize the instantaneous achievable rate (end-to-end SNR) in AF/DF relaying channels. Host-Madsen and Zhang studied upper/lower bounds on the outage/ergodic capacity and proposed adaptive PA solutions to maximize the ergodic capacity for DF relaying in [31]. The optimal and suboptimal transmission strategies were developed through appropriate transmit signaling and spatial PA were proposed in [32] to minimize the outage probability. Hammerström *et al.* examined PA over frequency and space sub-channels at the source and relay in an orthogonal frequency division multiplexing (OFDM) AF relaying communication system [33, 34]. For multi-relay dual-hop wireless relay channels, Maric and Yates proposed the optimal PA algorithms to maximize the capacity for AF and DF relaying, respectively, in [35] and [36]. Adaptive PA schemes were proposed to minimize the system outage probability in AF relaying [37, 38]. For DF relaying, the work in [39] derived a simple near-optimal solution, where a fixed fraction of the total power is allocated to the source and the remaining power is split equally among a set of selected relay nodes if the selected relay set is not empty, or otherwise is allocated to the source node. The work in [40] derived the optimal PA to minimize the system outage probability with AF/DF/distributed space-time coded (DSTC) relaying protocols. Adaptive PA schemes that maximize the output SNR in AF/DF relay systems were discussed in [41]- [43]. The work in [44] studied the PA strategy that minimizes the total power consumption given a target symbol error rate (SER).

Although most work in literature has focused on dual-hop relaying channels, relay assisted multi-hop cooperative transmission also has a lot of advantages.

The primary advantage of multi-hop relaying comes from the reduction in the overall path loss between the BS and mobile stations (MSs). Another benefit of multi-hop relaying is that the path diversity gain can be achieved by selecting the most favorable multi-hop path in shadowed environments. In a multi-hop cellular network, the MSs can choose to utilize the multi-hop relaying instead of the single-hop direct transmission. Therefore, the multi-hop relaying technology can provide a significant flexibility in the design and operation of cellular networks [45]. In fact, standardization efforts have also been made to incorporate multi-hop relaying into the third-generation (3G) mobile communication systems [46]. In [47], we proposed an adaptive PA at the source and relays that maximizes the system capacity in a multi-hop OFDM AF/DF relay system. The optimal PA subject to a total power budget that minimizes the system outage probability in a multi-hop system with DF relaying was studied in [48]. In [49], an adaptive PA strategy of minimizing the average SER for multi-hop DF relaying with binary phase shift keying (BPSK) modulation was proposed.

Despite so much work on temporal/spatial power control in MIMO and cooperative relay systems, there are still lots of open issues in this area. Some of them are listed below:

1. What is the optimal temporal power control scheme and the corresponding DMT performance in TDD MIMO channels if CSIT or CSIR is imperfect?
2. What is the achievable DMT performance in TDD MIMO channels with imperfect CSITR under joint power and rate control?
3. What is the impact of imperfect CSIT on the achievable diversity gain in TDD relaying channels with temporal power control?
4. What is the optimal spatial power control scheme in practical cooperative relay systems with multiple antennas under imperfect CSI?
5. What is the impact of relay location on the system performance in cooperative relay systems?

6. What is the optimal design if both spatial power control and relay location are jointly considered in cooperative relay systems?

In this thesis, we will address the above issues and propose optimal temporal/spatial power control schemes in MIMO and cooperative relay systems to improve the system performance.

1.2 Problem Statement

In TDD systems, it has been shown in [18] and [19] that the temporal power control under the assumption of imperfect CSIT and perfect CSIR significantly improves the diversity gain in SIMO channels. It is interesting to extend the results to MIMO systems and investigate whether there exist more efficient temporal power control schemes to further improve the achievable diversity gain. It is not unexpected that imperfect CSIR also has a great impact on the achievable diversity gain. Thus, it is essential to take both imperfect CSIT and CSIR into joint consideration and investigate the impact of imperfect CSITR on the DMT performance in MIMO channels. Further more, it has been shown that rate control is also able to boost the diversity gain in SIMO/MISO systems [20]. Noting that most previous work on DMT in MIMO channels with imperfect CSI considers either power control or rate control, it is of great interest to study how power control together with rate control jointly affects the DMT in MIMO channels.

Wireless relay systems that employ cooperative diversity have sometimes been referred to as virtual MIMO systems, so it is natural to extend our results for MIMO systems to cooperative relay systems. There has also been some work on the DMT in relaying channels with partial CSIT. For example, the impact of quantized CSIT feedback on the achievable diversity gain in a half-duplex DF relaying channel is systematically studied in [50] for a three-node system consisting of a source, relay and destination with one antenna at each node. Since the feedback channel usually has a finite capacity, the imperfect CSIT considered in [50] is mainly due to the quantization errors. Noting that the DMT in relaying channels with imperfect CSIT in TDD systems remains an open question, it is

of interest to use the DMT tool to investigate the diversity gain in cooperative relaying channels with imperfect CSIT, where the imperfect CSIT comes from channel estimation at the transmitters.

For spatial power control in dual-hop relay systems, most previous work has focused on very simple networks of terminals, for example, the traditional three-terminal relay channel, or straightforward extensions such as multiple relays in parallel with diversity combination at the destination. There has been minimal consideration of wireless relay systems where some or all of the cooperating terminals employ multiple antennas. To improve the system performance while keeping the computational complexity of the source and relay unchanged, a different system model, which consists of single-antenna source and relay, and multi-antenna destination node, is very attractive in practice. This model is reasonable as the complexity issue at the destination node such as the Base Station (BS) in an uplink is not as critical as that of mobile terminals. While most work assumes that perfect CSI is available at the transmitters for optimum PA, it is demanding to study the optimal PA based on imperfect CSIT, e.g., the channel statistics, since perfect CSIT requires a high rate feedback link between the receiver and the transmitter to keep the latter continuously informed about the channel state. Further more, noting that the relay location also has great impact on the system performance, it is valuable to jointly optimize PA and relay location to find the optimal solution. Finally, we investigate the optimal PA in multi-hop relay systems, especially with imperfect CSIT.

1.3 Major Contributions of the Thesis

This thesis analyzes the performance of multi-antenna and cooperative systems under the assumption of imperfect CSI. In the first part, we provide a framework to systematically study the achievable DMT in MIMO and cooperative systems with noisy CSI. In the second part, we propose optimal spatial power control schemes to minimize the outage probability and symbol error rate in DF relaying channels under imperfect CSI. The details are as follows:

In the first part, we firstly consider a MIMO channel. For the case of imperfect CSIT and perfect CSIR, we propose an optimal temporal power control scheme to significantly improve the achievable diversity gain. For the case where both CSIT and CSIR are imperfect, we propose an optimal power control scheme as well as novel two-way training strategies and show that a higher diversity gain as well as a more efficient DMT can be achieved than the case with no CSIT and perfect CSIR. Specifically, we show that the achievable diversity gain after one round of two-way training can either be a straight line, a collection of discontinuous line segments or a piecewise-linear line depending on the training strategy, channel qualities, and the achievable diversity gain after two rounds of two-way training is a single straight line. To further exploit imperfect CSIT, we take both rate and power control into account. We show that if there is no minimum instantaneous multiplexing gain constraint r_{min} , the achievable diversity gain is infinity with rate control itself; otherwise it is finite and limited by the achievable diversity gain at r_{min} . With joint power and rate control, the achievable diversity gain can be further increased significantly.

Next, we extend our results for MIMO channels to relaying channels. After considering orthogonal AF and DF relaying with imperfect CSIT and multiple destination antennas, we study a more efficient DDF relaying protocol. The DMT as a function of the CSIT quality of the source-relay (S-R) channel, source-destination (S-D) channel and relay-destination (R-D) channel is derived. We show that with temporal power control, one can achieve a higher diversity gain than that without power control. Imperfect CSIT at the relay itself does not improve the DMT if the source has no CSIT at all.

In the second part of this thesis, we firstly consider a dual-hop DF relay system with multiple destination antennas. We propose optimal PA algorithms between the source and relay subject to a total power constraint to minimize the system outage probability. Simulation results show that the presented optimal PA solutions significantly outperform the uniform PA. To further improve the system performance, we take both PA and relay location into optimization. Numerical results show that the proposed adaptive schemes significantly outperform the

fixed allocation schemes. It is also found that by using more destination antennas and/or choosing an appropriate relay location, less power will be needed at the relay. One can see that this is of great importance, particularly in practice.

Next, we move to a multi-hop DF relay system. We firstly present the optimal PA among source and relays under a total power constraint to minimize the system error probability when the perfect CSI is known at the transmitters. Then we propose adaptive PA schemes with limited feedback when the transmitters have no perfect knowledge of CSI. Numerical results show that the proposed PA scheme with perfect CSI significantly outperforms the existing PA solutions. When no perfect CSI is available at the transmitters, the proposed PA schemes with a small number of feedback bits are able to achieve very close performance to the adaptive PA with perfect CSI.

1.4 Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter 2 provides background knowledge relevant to the problems studied in this thesis. Chapter 3 analyzes the DMT in MIMO channels with the proposed power and/or rate control schemes under imperfect CSIT(R). The extension to cooperative relaying channels is addressed in Chapter 4. Chapter 5 addresses the joint optimization of PA and relay location in a dual-hop DF relay system with multiple destination antennas. Chapter 6 presents the adaptive PA algorithms among source and relays to minimize the system error probability in a multi-hop DF relay system with limited feedback. Chapter 7 provides some concluding remarks and a number of suggestions for future research.

Chapter 2

Preliminaries

2.1 MIMO Channels

One form of diversity, spatial diversity, can be provided via physical antenna arrays at either transmitters or receivers. Figure 2.1 shows an example multiple-antenna system, also known as a MIMO system. Suppose there are M transmit antennas and N receive antennas, the system model can be written as

$$\mathbf{y} = \sqrt{P/M} \mathbf{H} \mathbf{x} + \mathbf{w} \quad (2.1.1)$$

where $\mathbf{H} = \{\mathbf{H}_{n,m}\} \in \mathcal{C}^{N \times M}$ with $\mathbf{H}_{n,m}$, $m = 1, \dots, M$, $n = 1, \dots, N$, being the channel gain from the m -th transmit antenna to the n -th receive antenna; $\mathbf{x} = \{\mathbf{x}_m\} \in \mathcal{C}^{M \times 1}$ with $\mathbf{x}_m$, $m = 1, \dots, M$, being the symbols transmitted from antenna m ; $\mathbf{y} = \{\mathbf{y}_n\} \in \mathcal{C}^{N \times 1}$ with $\mathbf{y}_n$, $n = 1, \dots, N$, being the received signal at antenna n ; the additive noise $\mathbf{w} \in \mathcal{C}^{N \times 1}$ has independent and identically distributed (i.i.d.) entries $\mathbf{w}_n \sim \mathcal{CN}(0, \sigma^2)$, $n = 1, \dots, N$; P denotes the instantaneous sum transmit power with a long term power constraint $E\{P\} = \bar{P}$. In this chapter, we do not consider temporal power control and therefore, $P = \bar{P}$. Throughout this thesis, we assume that $\mathbf{H}$ is perfectly known at the receiver (i.e., perfect CSIR), unless otherwise specified.

Without loss of optimality, the input distribution can be taken to be Gaussian with a covariance matrix $\mathbf{Q}$ with $\text{Tr}(\mathbf{Q}) = M$. Thus the mutual information of

Figure 2.1: A MIMO system.

the system described by (2.1.1) is given by,

$$\mathcal{I}(\mathbf{H}, \mathbf{Q}) = \log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{Q} \mathbf{H}^\dagger \right) \quad (2.1.2)$$

where $\rho \triangleq P/\sigma^2$.

The capacity is usually defined as the maximum error-free data rate that a channel is able to support. Without loss of generality, we consider block transmission with a block length of L . When the channel coherence time is much smaller than the block length L , the channel is usually called a fast fading channel. Since any transmission block experiences a sufficiently large number of different channel realizations in fast fading channels, the achievable rate can be well represented by the *ergodic capacity* $\bar{C}$, which is the ensemble average of the information rate over the statistics of the channel matrix.

By contrast, if the channel coherence time is much larger than the block length L , the channel is usually called a slow fading channel. In slow (quasi-static) fading channels, the channel remains constant within one transmission block but changes independently from one block to another. A useful notation in this case is the *outage capacity*, i.e., the information rate guaranteed for a given percentage of the channel realizations. In other words, there is a certain probability that a given rate is not supported by a random channel realization, resulting in the declaration of an outage [51]. In the following, We will discuss ergodic capacity and outage capacity in detail.

2.1.1 Ergodic Capacity in Fast Fading Channels

Capacity with Perfect CSIT

The channel capacity with perfect CSIT is given by

$$C_{CSIT}(\mathbf{H}) = \max_{\mathbf{Q} \succeq 0, \text{Tr}(\mathbf{Q})=M} \log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{Q} \mathbf{H}^\dagger \right). \quad (2.1.3)$$

Now we find the optimum input covariance matrix $\mathbf{Q}^*$ when the transmitter has perfect CSIT. The transmission is first decoupled along the individual channel eigenmodes, so that $K = \min(M, N)$ parallel data streams are formed in the directions of the singular vectors of the channel matrix $\mathbf{H}$ at both the transmitter and receiver. Suppose that the optimal power allocation strategy $\{p_1^*, p_2^*, \dots, p_K^*\}$ is adopted along these eigenmodes. Then, $\mathbf{Q}^*$ can be expressed as

$$\mathbf{Q}^* = \mathbf{V} \text{diag}\{p_1^*, p_2^*, \dots, p_K^*\} \mathbf{V}^\dagger \quad (2.1.4)$$

where $\mathbf{V}$ is unitary and given by the singular value decomposition (SVD) of $\mathbf{H}$, i.e., $\mathbf{H} = \mathbf{U} \mathbf{D} \mathbf{V}^\dagger$. Here, $\mathbf{U}$ is unitary, $\mathbf{D}$ is diagonal with entries specified by $\mathbf{D} = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_K\}$, and $\sigma_k^2 \triangleq \lambda_k \triangleq \lambda_k(\mathbf{W})$ with

$$\mathbf{W} = \begin{cases} \mathbf{H} \mathbf{H}^\dagger, & M \geq N \\ \mathbf{H}^\dagger \mathbf{H}, & M < N. \end{cases} \quad (2.1.5)$$

It follows that the capacity with perfect CSIT can be expressed as

$$C_{CSIT}(\mathbf{H}) = \sum_{k=1}^K \log \left(1 + \frac{\rho p_k^* \lambda_k}{M} \right). \quad (2.1.6)$$

The optimal solution to the above optimization problem is the well-known water-filling solution [52]

$$p_k^* = \left(\eta - \frac{M}{\rho \lambda_k} \right)^+, \quad k = 1, 2, \dots, K \quad (2.1.7)$$

where η is chosen to satisfy the power constraint $\sum_{k=1}^K p_k^* = M$.

Consequently, the ergodic capacity is given by

$$\bar{C} = E\{C_{CSIT}(\mathbf{H})\} = E\left\{\sum_{k=1}^K \left(\log \frac{\rho \lambda_k \eta}{M}\right)^+\right\}. \quad (2.1.8)$$

Ergodic Capacity with Partial CSIT

Perfect channel knowledge at the transmitter requires a high rate feedback link between the receiver and the transmitter to keep the later continuously informed about the channel state and is thus impractical. Yet, it is possible for the transmitter to have partial CSIT in the form of channel statistical distribution. With the channel statistics, the ergodic capacity is defined as

$$\bar{C} = \max_{\mathbf{Q} \succcurlyeq 0, \text{Tr}(\mathbf{Q})=M} E\left\{\log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{Q} \mathbf{H}^\dagger\right)\right\}. \quad (2.1.9)$$

In i.i.d. Rayleigh fading channels, the ergodic capacity is achieved with a uniform PA scheme $\mathbf{Q}^* = \mathbf{I}_M$ [52], i.e.,

$$\begin{aligned} \bar{C} &= E\left\{\log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger\right)\right\} \\ &= E\left\{\sum_{k=1}^K \log \left(1 + \frac{\rho}{M} \lambda_k\right)\right\}. \end{aligned} \quad (2.1.10)$$

There is no closed-form expression to $\bar{C}$. However, lower and upper bounds are provided in [53] as

$$E\left\{\sum_{k=1}^K \log \left(1 + \frac{\rho}{M} \mathcal{X}_{2(N-k+1)}^2\right)\right\} \leq \bar{C} \leq E\left\{\sum_{k=1}^M \log \left(1 + \frac{\rho}{M} \mathcal{X}_{2N}^2\right)\right\} \quad (2.1.11)$$

where $\mathcal{X}_{2k}^2$ is chi-square distributed with $2k$ degrees of freedom.

2.1.2 Outage Capacity and Diversity-multiplexing Trade-off in Slow Fading Channels

In slow fading channels, the achievable rate cannot be described by a single ergodic quantity, because it may not be possible to fully average out the randomness of the channel. We thus resort to the concept of outage probability $\mathcal{P}_{out}$, which is defined as the probability that the mutual information of a channel realization does not support a target data rate $R(\rho)$

$$\mathcal{P}_{out} = \inf_{\mathbf{Q} \succeq 0, \text{Tr}(\mathbf{Q})=M} \mathcal{P} \left[\log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{Q} \mathbf{H}^\dagger \right) < R(\rho) \right]. \quad (2.1.12)$$

If the transmitter has perfect CSIT, it can control transmit power such that the rate $R(\rho)$ can be achieved no matter what the fading state is and therefore the outage probability will be zero.

When the transmitter does not have perfect CSIT, we are interested to quantify how $\mathcal{P}_{out}$ behaves as a function of the SNR ρ , especially when ρ goes to infinity. To this end, we introduce the important concepts of spatial multiplexing gain and diversity gain defined as follows [6]:

Definition 2.1.1 A diversity gain $d^*(r)$ is achieved at spatial multiplexing gain r if

$$r \triangleq \lim_{\rho \rightarrow \infty} \frac{R(\rho)}{\log \rho} \quad (2.1.13)$$

$$d^*(r) \triangleq - \lim_{\rho \rightarrow \infty} \frac{\mathcal{P}_{out}}{\log \rho}. \quad (2.1.14)$$

Intuitively, the multiplexing gain r indicates how fast the transmission rate increases with the SNR, whereas the diversity gain $d^*(r)$ represents how fast the outage probability decays with the SNR. It is well known that for a MIMO channel with zero CSIT and perfect CSIR, the maximum diversity gain is MN , which is the total number of fading gains that one can average over; whereas the maximum multiplexing gain is K , which is also the number of degrees of freedom. The curve $d^*(r)$ as a function of r is known as the DMT of the channel. The following theorem characterizes the DMT of a MIMO channel:

Theorem 2.1.1 [6] Under the assumption of i.i.d. quasi-static flat Rayleigh fading channels where the CSI is known at the N -antenna receiver but not at the M -antenna transmitter, for any integer $r \leq \min(M, N)$, the optimal diversity gain $d^*(r)$ (the supremum of the diversity gain over all coding schemes) is given by

$$d^*(r) = (M - r)(N - r). \quad (2.1.15)$$

Sketch of the Proof:

By choosing $\mathbf{Q} = \mathbf{I}_M$ and $\mathbf{Q} = M\mathbf{I}_M$, respectively, we can get the following upper and lower bounds of the outage probability

$$\mathcal{P} \left[\log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right] \geq \mathcal{P}_{out} \geq \mathcal{P} \left[\log \det \left(\mathbf{I}_N + \rho \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right]. \quad (2.1.16)$$

At high SNRs

$$\begin{aligned} & \lim_{\rho \rightarrow \infty} \frac{\mathcal{P} \left[\log \det \left(\mathbf{I}_N + \rho \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right]}{\log \rho} \\ &= \lim_{\rho \rightarrow \infty} \frac{\mathcal{P} \left[\log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right]}{\log \frac{\rho}{M}} \\ &= \lim_{\rho \rightarrow \infty} \frac{\mathcal{P} \left[\log \det \left(\mathbf{I}_N + \frac{\rho}{M} \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right]}{\log \rho}. \end{aligned} \quad (2.1.17)$$

Therefore, we have [6]

$$\begin{aligned} \mathcal{P}_{out} &\doteq \mathcal{P} \left[\log \det \left(\mathbf{I}_N + \rho \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right] \\ &\doteq \mathcal{P} \left[\prod_{k=1}^K (1 + \rho \lambda_k) < \rho^r \right] \\ &\doteq \mathcal{P} \left[\sum_{k=1}^K (1 - \phi_k)^+ < r \right] \\ &\doteq \int_{\mathcal{O}} p(\phi) d\phi \end{aligned} \quad (2.1.18)$$

where $\mathcal{O}$ is the outage event defined as

$$\mathcal{O} = \left\{ \phi : \sum_{k=1}^K (1 - \phi_k)^+ < r \right\} \quad (2.1.19)$$

ϕ_k is the exponential order of $1/\lambda_k$, i.e., $\phi_k \triangleq -\lim_{\rho \rightarrow \infty} \frac{\log(\lambda_k)}{\log(\rho)}$, $p(\phi)$ is the joint pdf of the random vector $\phi = [\phi_1, \phi_2, \dots, \phi_K]$ given by

$$p(\phi) = K_{M,N}^{-1} (\log \rho)^K \prod_{k=1}^K \rho^{-(M-N+1)\phi_k} \prod_{k < j} (\rho^{-\phi_k} - \rho^{-\phi_j})^2 \exp \left[- \sum_{k=1}^K \rho^{-\phi_k} \right] \quad (2.1.20)$$

and $K_{M,N}$ is a normalizing constant.

Since we are only interested in the SNR exponent of $\mathcal{P}_{out}$, we can neglect some irrelevant terms in $p(\phi)$ and get

$$p(\phi) \doteq \rho^{-\sum_{k=1}^K 2k-1+|M-N|\phi_k}. \quad (2.1.21)$$

With the Laplace principle [6], we get

$$\mathcal{P}_{out} \doteq \rho^{-d^*(r)} \quad (2.1.22)$$

where

$$d^*(r) = \inf_{\mathcal{O}^+} \sum_{k=1}^K 2k - 1 + |M - N|\phi_k. \quad (2.1.23)$$

Solving this optimization problem directly yields Theorem 2.1.1.

2.2 Relay Channels

Relay assisted cooperative transmission has recently become a prominent candidate to combat channel fading, improve channel capacity and enlarge coverage without needing to deploy physical antenna arrays. The basic idea of cooperative diversity is to allow distributed users in the network to help relay information for each other so as to explore inherent spatial diversity. In this thesis, we will focus on the following two relaying strategies [25, 26].

Amplify and Forward

AF relaying protocol is conceptually the simplest cooperative signaling method. The relay node receives a noisy version of the signal transmitted by source node and then amplifies and retransmits this noisy signal. The destination will combine the information sent by the source and the relay node and make a final decision on the transmitted symbol. Obviously the main advantage of AF strategy is its simplicity. Nevertheless, the performance of AF relaying is limited since the noise at the relay is also amplified and forwarded to the destination.

Decode and Forward

If the relay node employs DF protocol, it will decode, regenerate a new message and forward it to the destination subsequently. When the regenerated message is encoded to provide additional error protection to the original message, it is also referred to as coded cooperation. At the destination, signals from both the source and the relay paths are then combined for detection. The performance of DF strategy is limited by the S-R link. If S-R channel performs well, DF strategy is optimal, i.e., it achieves the capacity of the relay channel.

When CSI is not known to the transmitter, the spatial diversity gain is achieved by allowing users to have a fair share of each other's resources. With full or partial knowledge of the CSI, significant improvements in terms of BER, outage probability or capacity can be attained by applying optimal PA among cooperating nodes. In the following, we will quickly review the PA methods under different network topologies, relaying protocols with full CSI assumptions to maximize the system capacity. We first study the three-node topology shown in Figure 2.2, followed by the dual-hop multi-relay topology shown in Figure 2.3.

2.2.1 Three-Node Relay System

Consider the three-node relay system shown in Figure 2.2. A typical cooperative transmission can be modeled with two orthogonal phases, either in TDMA or

Figure 2.2: A three-node relay system.

FDMA, to avoid interference between the two phases. In the first phase, source S transmits symbol X_s to both relay R and destination D. The received signals at the relay and the destination can be expressed as

$$Y_r = \sqrt{P_s} h_{sr} X_s + Z_r \quad (2.2.1)$$

$$Y_{d,1} = \sqrt{P_s} h_{sd} X_s + Z_{d,1} \quad (2.2.2)$$

respectively, where X_s is the source transmitted signal with $E[|X_s|^2] = 1$, P_s denotes the source transmit power; Y_r and Z_r denote the signal and noise received at the relay; $Y_{d,1}$ and $Z_{d,1}$ denote the signal and noise received at the destination, h_{sr} and h_{sd} denote the instantaneous channel gains of the S-R link and S-D link.

In the second phase, R transmits symbol X_r ($E[|X_r|^2] = 1$) as a function of the received signal Y_r with power P_r . Consequently, the signal received at D can be written as

$$Y_{d,2} = \sqrt{P_r} h_{rd} X_r + Z_{d,2} \quad (2.2.3)$$

where $Y_{d,2}$ and $Z_{d,2}$ denote the signal and noise received at the destination; h_{rd} denotes the instantaneous channel gain of the R-D link. In the following discussions, we assume that all the noise terms are additive zero-mean white circular complex Gaussian variables with variance $\sigma^2 = 1$.

The main objective is to determine the optimal allocation of P_s and P_r to maximize the channel capacity, subject to the total power constraint $P_s + P_r \leq P_T$. The solution depends on whether the direct S-D link is taken into account (i.e., $Y_{d,1}$ is combined with $Y_{d,2}$ in signal detection).

We first examine the DF relaying channel. If there is no direct link between S and D, the capacity of the relay path will be equal to the minimum of the S-R

and R-D link capacities. Thus, the optimal PA becomes a standard max-min problem [30], i.e.,

$$C_{DF,1} = \max_{P_s, P_r} \min \left\{ \frac{1}{2} \log(1 + P_s |h_{sr}|^2), \frac{1}{2} \log(1 + P_r |h_{rd}|^2) \right\}. \quad (2.2.4)$$

The solution must yield an equal capacity (or SNR) for both links. Hence, we have

$$P_s^* = \frac{|h_{rd}|^2}{|h_{sr}|^2 + |h_{rd}|^2} P_T \quad (2.2.5)$$

$$P_r^* = \frac{|h_{sr}|^2}{|h_{sr}|^2 + |h_{rd}|^2} P_T. \quad (2.2.6)$$

If there is a direct link between S and D, both $Y_{d,1}$ and $Y_{d,2}$ are combined for detection at the destination. In DF relaying, especially full-duplex DF relaying, if the relay transmits a data block that is incorrectly detected/decoded, it is likely that there will be an error at the destination. This problem, usually called error propagation, limits the diversity order of the DF relaying. To overcome this problem, adaptive DF relaying was proposed [25], where R retransmits only when it correctly decodes the message and D is said to receive a message successfully only if the message is correctly received from both the source and relay. Under such a scenario, the PA problem can be formulated as

$$C_{DF,2} = \max_{P_s, P_r} \min \left\{ \frac{1}{2} \log(1 + P_s |h_{sr}|^2), \frac{1}{2} \log(1 + P_s |h_{sd}|^2 + P_r |h_{rd}|^2) \right\} \quad (2.2.7)$$

and the capacity is maximized with

$$P_s^* = \frac{|h_{rd}|^2}{|h_{sr}|^2 + |h_{rd}|^2 - |h_{sd}|^2} P_T \quad (2.2.8)$$

$$P_r^* = \frac{|h_{sr}|^2 - |h_{sd}|^2}{|h_{sr}|^2 + |h_{rd}|^2 - |h_{sd}|^2} P_T. \quad (2.2.9)$$

The optimal PA of AF relaying with respect to the end-to-end capacity can be derived similarly. For the case without direct link, the ratio between P_s and P_r becomes [29]

Figure 2.3: A dual-hop multi-relay network.

$$\frac{P_s^*}{P_r^*} = \sqrt{\frac{|h_{rd}|^2 P_T + \sigma^2}{|h_{sr}|^2 + \sigma^2}}. \quad (2.2.10)$$

With direct link, the PA problem exists only when the S-R link and R-D link are sufficiently good compared with the S-D link. Otherwise, one should simply allocate all the power to S. Specifically, when $|h_{sr}| \approx |h_{rd}|$ and are both sufficiently larger than $|h_{sd}|$, the ratio between P_s and P_r can be approximated as [29]

$$\frac{P_s^*}{P_r^*} = \frac{|h_{sr}|^2 |h_{rd}|^2 P_T + |h_{rd}|^2 |h_{sd}|^2 P_T + |h_{sd}|^2 \sigma^2}{|h_{sr}|^2 |h_{rd}|^2 P_T - |h_{sr}|^2 |h_{sd}|^2 P_T - |h_{sd}|^2 \sigma^2}. \quad (2.2.11)$$

2.2.2 Dual-hop Multi-relay System

Consider a dual-hop multi-relay system shown in Figure 2.3, where there are N relay nodes, denoted by R_k , $k = 1, \dots, N$. Let h_{sk} and h_{kd} denote the complex channel coefficients from S to the relay R_k and from R_k to D, respectively. A two-stage cooperation is adopted. That is, S broadcasts its message in the first stage and the set of relays $\{R_k, k = 1, \dots, N\}$ transmits simultaneously at the second stage. The transmit powers of S and R_k are denoted by P_s and P_k , respectively. The total power constraint is imposed on the summation of relay powers, i.e., $\sum_{k=1}^N P_k \leq P_R$. Since PA among P_s and P_R can be determined using the techniques derived in the previous subsection, we focus on PA among relay nodes in this subsection.

When full CSI is known at the relays, a precoding technique similar to that in MIMO systems can be used to compensate for both the channel gain and phase rotation experienced by the relays to achieve better detection performance. For orthogonal relaying channels, D receives N copies of the source symbol from the relay nodes with no interference among each other. With knowledge of perfect CSI, the N symbols can be combined coherently at D to increase the received SNR. With AF relaying, the capacity of the parallel relay channel can be found as [35]

$$C_{AF} = \frac{1}{2} \log \left(1 + \sum_{k=1}^N \frac{|h_{sk}|^2 |h_{kd}|^2 P_s P_k}{P_s |h_{sk}|^2 + P_s |h_{kd}|^2 + 1} \right) \quad (2.2.12)$$

and the capacity-maximizing PA strategy results in the following water-filling solution [35]

$$P_k^* = \frac{|h_{sk}|^2}{\sqrt{\gamma_k}} \left(\frac{1}{\sqrt{\eta}} - \frac{1}{\sqrt{\gamma_k}} \right)^+ \quad (2.2.13)$$

where $\gamma_k = (|h_{sk}|^2 |h_{kd}|^2) / (P_s |h_{sk}|^2 + 1)$ and the Lagrange multiplier, η , is chosen to meet the total power constraint of the relay nodes.

The optimal PA for the DF scheme with orthogonal relay channels was derived in [26] to maximize the capacity. Consider a set of relay nodes, denoted by R_D , who are able to correctly decode the source messages. That is, for all $k \in R_D$, the desired transmission rate is smaller than the capacity of the source to R_k (S- R_k) link. These relays decode and forward the messages to D, acting as multiple antennas on a single terminal. In the wideband or the low SNR regime [36], the capacity can be approximated by

$$C_{DF} \approx \frac{1}{2} \sum_{k \in \mathcal{R}_D} P_k |h_{kd}|^2, \quad \text{if } P_k |h_{kd}|^2 \leq 1. \quad (2.2.14)$$

The solution to the above optimization problem is to choose the relay node among R_D with the best channel towards D and allocate all the power to that node. This means that the selective relaying scheme is optimal for the DF scheme with orthogonal relay channels.

Chapter 3

Power Control in MIMO Channels With Imperfect CSI: A DMT Perspective

In this and the next chapters, we consider temporal power control and the transmit power satisfies a long term power constraint. We propose optimal temporal power control schemes to maximize the achievable DMT over MIMO channels in this chapter. We firstly consider the case of imperfect CSIT and perfect CSIR. Then we move to a more general case where both CSIT and CSIR are imperfect. Our major results in this chapter are summarized as follows:

- For a MIMO channel with imperfect CSIT and perfect CSIR, we propose an optimal temporal power control scheme and show that the achievable diversity gain can be significantly improved.
- When both CSIT and CSIR are imperfect, we propose a power control scheme with novel two-way training strategies, which turns out to make more efficient use of the imperfect CSITR. We show that the optimal DMT in the case of a single training round is either a straight line, a collection of discontinuous line segments, or a piecewise-linear curve, depending on the underlying training strategy and channel qualities. In the case of multiple training rounds, we show that the optimal DMT is much higher and can

be described as a single straight line. In particular, we show that at high multiplexing gains, no training can help get any diversity and thus the achievable diversity is zero.

- To further improve the system performance, we take both rate and power control into joint consideration. It is interesting to find that if there is no minimum instantaneous spatial multiplexing gain constraint r_{min} , rate control itself is already able to boost the diversity gain unboundedly with imperfect CSITR. If r_{min} is imposed, the diversity gain is limited by the achievable diversity gain at r_{min} . When joint rate and power control is taken into account, the DMT can be further improved significantly.

3.1 System Model

We consider a point-to-point wireless link with M transmit and N receive antennas as described in Section 2.1. Moreover, the forward and reverse channels are considered to be reciprocal. We assume $M \geq N$ in this chapter. As shown in [6], this assumption does not affect the DMT result. We consider quasi-static i.i.d. Rayleigh fading, where the channel gain matrix $\mathbf{H}$ is constant within one transmission block of length L , but changes independently from one block to another. We assume that the channel gains are independently complex circular symmetric Gaussian with zero mean and unit variance. Without loss of generality, we assume that the input (Gaussian) covariance matrix $\mathbf{Q} = \mathbf{I}_M$ [6] and thus the average SNR at each receive antenna can be written as $\rho = \bar{P}/\sigma^2$.

3.2 Power Control with Imperfect CSIT and Perfect CSIR

We assume that the receiver has perfect CSIR $\mathbf{H} \in \mathcal{C}^{N \times M}$, but the transmitter has imperfect CSIT $\hat{\mathbf{H}} \in \mathcal{C}^{N \times M}$, which is estimated from the reverse channel pilots

using ML estimation. Thus, $\hat{\mathbf{H}}$ can be modeled as [54]- [56]

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{E} \quad (3.2.1)$$

where the channel estimation error $\mathbf{E} \in \mathcal{C}^{N \times M}$ has i.i.d. entries $\mathbf{E}_{n,m} \sim \mathcal{CN}(0, \sigma_e^2)$, $n = 1, 2, \dots, N$, $m = 1, 2, \dots, M$, and is independent of $\mathbf{H}$. It can be shown in the next section that a companion CSIT model arising from MMSE estimation in the form of $\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}$, where $\mathbf{E}$ is independent of $\mathbf{H}$, does not lead to a DMT result different from the one based on (3.2.1). The quality of $\hat{\mathbf{H}}$ is thus characterized by σ_e^2 . If $\sigma_e^2 = 0$, the transmitter has perfect channel knowledge; if σ_e^2 increases, the transmitter has less reliable channel knowledge. We follow [20] to quantify the channel quality at the transmitter. The transmitter is said to have a *CSIT quality* α , if $\sigma_e^2 \doteq \rho^{-\alpha}$. The definition of α builds up a connection between the imperfect channel knowledge at transmitters and the forward channel SNR, ρ . Since the variance of the channel estimation error is inversely proportional to the pilots' SNR, i.e., $\sigma_e^2 \propto (SNR_{pilot})^{-1}$ [54], any value of α can be achieved by scaling the reverse channel power such that $SNR_{pilot} \doteq \rho^\alpha$. One can see that the selection of α value actually determines the cost of obtaining CSIT in terms of the reverse channel power consumption.

Based on the imperfect $\hat{\mathbf{H}}$, the outage probability of a MIMO channel with sum transmit power $P(\hat{\mathbf{H}})$ is given by

$$\mathcal{P}_{out,1} = \mathcal{P} \left(\log \det \left(\mathbf{I}_N + \frac{P(\hat{\mathbf{H}})}{M\sigma^2} \mathbf{H}\mathbf{H}^\dagger \right) < R(\rho) \right). \quad (3.2.2)$$

Note that the sum transmit power has to satisfy the long term constraint $E\{P(\hat{\mathbf{H}})\} = \bar{P}$. In the following, we will show how transmit power control based on the imperfect CSIT helps to significantly increase the diversity gain.

Letting $\mathbf{a} = [a_1, \dots, a_N]$, $0 < a_1 \leq \dots \leq a_N$, $\mathbf{b} = [b_1, \dots, b_N]$, $0 < b_1 \leq \dots \leq b_N$, and $\mathbf{c} = [c_1, \dots, c_N]$, $0 < c_1 \leq \dots \leq c_N$, denote the eigenvalue vectors of $\mathbf{H}\mathbf{H}^\dagger$,

$\hat{\mathbf{H}}\hat{\mathbf{H}}^\dagger$ and $\mathbf{E}\mathbf{E}^\dagger$, respectively, the pdfs of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ can be shown to be [57]

$$p(\mathbf{a}) = \xi^{-1} \prod_{n=1}^N a_n^{M-N} \prod_{n < j}^N (a_n - a_j)^2 \exp \left(- \sum_{n=1}^N a_n \right) \quad (3.2.3a)$$

$$p(\mathbf{b}) = \hat{\xi}^{-1} \prod_{n=1}^N b_n^{M-N} \prod_{n < j}^N (b_n - b_j)^2 \exp \left(- \frac{1}{1 + \sigma_e^2} \sum_{n=1}^N b_n \right) \quad (3.2.3b)$$

$$p(\mathbf{c}) = \tilde{\xi}^{-1} \prod_{n=1}^N c_n^{M-N} \prod_{n < j}^N (c_n - c_j)^2 \exp \left(- \frac{1}{\sigma_e^2} \sum_{n=1}^N c_n \right) \quad (3.2.3c)$$

where $\hat{\xi}^{-1} = \xi^{-1}(1 + \sigma_e^2)^{-MN}$, $\tilde{\xi}^{-1} = \xi^{-1}(\sigma_e^2)^{-MN}$ and ξ is a normalizing constant.

Lemma 3.2.1 The eigenvalues of $\hat{\mathbf{H}}\hat{\mathbf{H}}^\dagger$, $\mathbf{H}\mathbf{H}^\dagger$ and $\mathbf{E}\mathbf{E}^\dagger$ have the following relationship

$$b_n \leq 2(a_n + c_N), \quad n = 1, 2, \dots, N. \quad (3.2.4)$$

Proof. We obviously have the following equality

$$(\mathbf{H} + \mathbf{E})(\mathbf{H} + \mathbf{E})^\dagger + (\mathbf{H} - \mathbf{E})(\mathbf{H} - \mathbf{E})^\dagger = 2(\mathbf{H}\mathbf{H}^\dagger + \mathbf{E}\mathbf{E}^\dagger) \quad (3.2.5)$$

where both $(\mathbf{H} + \mathbf{E})(\mathbf{H} + \mathbf{E})^\dagger$ and $(\mathbf{H} - \mathbf{E})(\mathbf{H} - \mathbf{E})^\dagger$ are positive semidefinite matrices. We denote the vector of eigenvalues of $(\mathbf{H}\mathbf{H}^\dagger + \mathbf{E}\mathbf{E}^\dagger)$ as $\mathbf{d} = [d_1, \dots, d_N]$ with $d_1 \leq \dots \leq d_N$. Since the eigenvalues of the sum of two positive semidefinite matrices are at least as large as the eigenvalues of any one of the positive semidefinite matrices [58], we have $b_n \leq 2d_n$, $n = 1, 2, \dots, N$. Further, using the relationship of the eigenvalues of the sum of Hermitian matrices, we get $a_n + c_1 \leq d_n \leq a_n + c_N$, $n = 1, 2, \dots, N$. It thus directly leads to (3.2.4). $\square$

We propose the following power control scheme $P(\hat{\mathbf{H}})$ to mitigate the channel uncertainty

$$P(\hat{\mathbf{H}}) = \frac{\kappa \bar{P}}{\left(\prod_{n=1}^N b_n^{2n-1+M-N} \right)^t} \quad (3.2.6)$$

where $\kappa = \hat{\xi} \prod_{n=1}^N [(2n-1+M-N)(1-t)]$ and t ($0 \leq t < 1$) can be chosen arbitrarily close to 1.

It is easy to prove that the above power control scheme satisfies the average

power constraint $E\{P(\hat{\mathbf{H}})\} = \bar{P}$ with (3.2.3b). The idea behind this power control scheme is to mimic the well-known channel inversion scheme, where κ is used to keep the average power constraint. Since $\kappa = 0$ when $t = 1$, the choice of $t < 1$ keeps κ as a constant and from scaling with ρ , such that it can be ignored in the DMT derivation. In fact, the value of t does not affect the DMT result in the asymptotic perspective.

Theorem 3.2.1 Consider a MIMO channel with M transmit and N receive antennas ($M \geq N$) and a CSIT quality of α . The optimal DMT using the power control scheme in (3.2.6) is characterized by

Case 1: If $N = 1$ or $\alpha \geq \frac{1}{M-1}$, then

$$d_{CSIT}^*(r) = MN(1 + MN\alpha) - (M + N - 1)r. \quad (3.2.7)$$

Case 2: Otherwise, the optimal DMT $d_{CSIT}^*(r)$ is a collection of discontinuous line segments, with the two end points of line segment $d_k(r)$ ($k \in \mathcal{B}$) given by

$$\begin{aligned} \text{Left: } d_k(r) &= k(M - N + k)\tau(k), \text{ for } r = (N - k)\tau(k) \\ \text{Right: } \left\{ \begin{array}{l} d_k(r) = k\alpha(M - N + k)((N - k)(k - N - 1) + MN) + (k - k^2) \\ \quad \text{for } r = N, \quad \text{if } k = \min_{\bar{k}} \mathcal{B} \\ d_k(r) = ((N - k)(k - N - 1) + MN)\tau(k) - (2k - 1 + M - N)(N - \mathcal{I}(k))\tau(\mathcal{I}(k)), \\ \quad \text{for } r = (N - \mathcal{I}(k))\tau(\mathcal{I}(k)) \end{array} \right. \end{aligned} \quad (3.2.8)$$

where

$$\mathcal{B} = \left\{ k \mid (M - N + k)(N - k) < 1/\alpha, (N - k)\tau(k) < (N - \bar{k})\tau(\bar{k}), \forall \bar{k} < k, k = 1, \dots, N \right\},$$

$$\tau(k) = 1 + k\alpha(M - N + k) \text{ and } \mathcal{I}(k) = \max_{\bar{k} \in \mathcal{B}, \bar{k} < k} \bar{k}.$$

Proof. Substituting (3.2.6) in (3.2.2), we have

$$\begin{aligned}
 \mathcal{P}_{out,1} &= \mathcal{P} \left(\log \det \left(\mathbf{I}_N + \frac{\rho \kappa}{M \prod_{n=1}^N b_n^{(2n-1+M-N)t}} \mathbf{H} \mathbf{H}^\dagger \right) < R(\rho) \right) \\
 &= \mathcal{P} \left(\log \prod_{n=1}^N \left(1 + \frac{\rho \kappa a_n}{M \prod_{n=1}^N b_n^{(2n-1+M-N)t}} \right) < R(\rho) \right) \\
 &\stackrel{(a)}{\leq} \mathcal{P} \left[\log \prod_{n=1}^N \left(1 + \frac{\rho \kappa a_n}{M \prod_{n=1}^N (2a_n + 2c_N)^{(2n-1+M-N)t}} \right) < R(\rho) \right]
 \end{aligned} \tag{3.2.9}$$

where (a) is due to Lemma 3.2.1.

Let v_n and u_n denote the exponential orders of $1/a_n$ and $1/c_n$, respectively, i.e., $v_n = -\lim_{\rho \rightarrow \infty} \frac{\log(a_n)}{\log(\rho)}$, $u_n = -\lim_{\rho \rightarrow \infty} \frac{\log(c_n)}{\log(\rho)}$. Using (3.2.3a) and (3.2.3c), the joint pdfs of the random vectors $\mathbf{v} = [v_1, \dots, v_N]$ and $\mathbf{u} = [u_1, \dots, u_N]$ can be shown to be [6], respectively,

$$p(\mathbf{v}) \doteq \begin{cases} 0, & \text{for any } v_n < 0 \\ \prod_{n=1}^N \rho^{-(2n-1+M-N)v_n}, & \text{for all } v_n \geq 0 \end{cases} \tag{3.2.10a}$$

$$p(\mathbf{u}) \doteq \begin{cases} 0, & \text{for any } u_n < \alpha \\ \prod_{n=1}^N \rho^{-(2n-1+M-N)(u_n-\alpha)}, & \text{for all } u_n \geq \alpha. \end{cases} \tag{3.2.10b}$$

At high SNRs, with (3.2.10a) and (3.2.10b), (3.2.9) becomes

$$\mathcal{P}_{out,1} \doteq \mathcal{P} \left[\sum_{i=1}^N \left(1 - v_i + \sum_{n=1}^N t(2n-1+M-N) \min(v_n, u_N) \right)^+ < r \right] \tag{3.2.11}$$

where $\doteq$ is due to that the outage probability is dominated by the term with the largest SNR exponent when ρ grows to infinity. So, the outage event $\mathcal{O}$ in (3.2.11) is the set of $\{v_1, \dots, v_N, u_1, \dots, u_N\}$ that satisfies

$$\sum_{i=1}^N \left(1 - v_i + \sum_{n=1}^N t(2n-1+M-N) \min(v_n, u_N) \right)^+ < r \tag{3.2.12}$$

where $v_n \geq 0, u_n \geq \alpha \geq 0, n = 1, 2, \dots, N$. With Laplace's principle, we have

$$\mathcal{P}_{out,1} \doteq \rho^{-d_{CSIT}^*(r)}, \quad \text{for } d_{CSIT}^*(r) = \inf_{(\mathbf{u}, \mathbf{v}) \in \mathcal{O}} \sum_{n=1}^N (2n-1+M-N) (v_n + u_n - \alpha). \quad (3.2.13)$$

Next, we work on the explicit expression of $d_{CSIT}^*(r)$. Since the left hand side (LHS) of (3.2.12) is a non-decreasing function of u_N , decreasing u_N will not violate the outage condition in (3.2.12) while enjoying a reduced SNR exponent $\sum_{n=1}^N (2n-1+M-N) (v_n + u_n - \alpha)$. Combining with the fact $u_n \geq \alpha, n = 1, 2, \dots, N$, the optimal solution of $\mathbf{u}$ is found to be $u_1^* = \dots = u_N^* = \alpha$. Therefore, the outage event can be re-expressed as

$$\mathcal{O} = \left\{ \mathbf{v} \left| \sum_{i=1}^N \left(1 - v_i + \sum_{n=1}^N t(2n-1+M-N) \min(v_n, \alpha) \right)^+ < r, v_1, \dots, v_N \geq 0 \right. \right\}. \quad (3.2.14)$$

To solve the optimization problem of (3.2.13), we firstly divide $\mathcal{O}$ into $N+1$ disjoint subsets, $\mathcal{O}_0, \dots, \mathcal{O}_N$, and solve the following subproblems under each subset

$$d_k(r) \triangleq \inf_{(v_1, \dots, v_N) \in \mathcal{O}_k} \sum_{n=1}^N (2n-1+M-N) v_n \quad (3.2.15)$$

where subset $\mathcal{O}_k$ ($0 \leq k \leq N$) is defined as

$$\mathcal{O}_k = \left\{ v_1, \dots, v_N \left| \sum_{i=1}^N \left(1 - v_i + \sum_{n=1}^k t(2n-1+M-N) \alpha + \sum_{n=k+1}^N t(2n-1+M-N) v_n \right)^+ < r, \right. \right. \\ \left. \left. v_1 \geq \dots \geq v_k \geq \alpha \geq v_{k+1} \geq \dots \geq v_N \geq 0 \right. \right\}.$$

After obtaining $d_k(r)$, we will go back to solve the main problem in (3.2.13), which can be re-expressed as

$$d_{CSIT}^*(r) = \min \left(d_0(r), d_1(r), \dots, d_N(r) \right). \quad (3.2.16)$$

It implies that, among all the DMT curves $d_0(r), \dots, d_N(r)$, corresponding to the outage subsets $\mathcal{O}_0, \dots, \mathcal{O}_N$, respectively, the lowest one will be the DMT curve for

the entire outage event. Since t can be made arbitrarily close to 1, it is without loss of accuracy to set $t = 1$ in the rest of this section.

Solving Subproblems to Find $d_k(r)$

Firstly, we derive $d_0(r)$. Note that

$$\sum_{i=1}^N \left(1 - v_i + \sum_{n=1}^N (2n-1+M-N)v_n \right)^+ \geq N - \sum_{n=1}^N v_n + N \sum_{n=1}^N (2n-1+M-N)v_n \geq N \quad (3.2.17)$$

which suggests that it is possible to operate at spatial multiplexing gain $r \in [0, N]$ reliably without any outage, i.e., $d_0(r) = \infty$. So we can exclude $d_0(r)$ from the main optimization problem in (3.2.16).

Secondly, we derive $d_k(r)$ ($1 \leq k \leq N$). Note that the function $\sum_{i=1}^N \left(1 - v_i + \sum_{n=1}^k (2n-1+M-N)\alpha + \sum_{n=k+1}^N (2n-1+M-N)v_n \right)^+$ is an increasing function of $v_{k+1}, v_{k+2}, \dots, v_N$. That is, decreasing $v_{k+1}, v_{k+2}, \dots, v_N$ does not violate the outage condition for $\mathcal{O}_k$, while reducing the SNR exponent $\sum_{n=1}^N (2n-1+M-N)v_n$. Therefore, the optimal solutions of $v_{k+1}, v_{k+2}, \dots, v_N$ are $v_{k+1}^* = v_{k+2}^* = \dots = v_N^* = 0$. Consequently, the optimization subproblem in (3.2.15) can be reformulated as

$$d_k(r) = \inf_{(v_1, \dots, v_k) \in \check{\mathcal{O}}_k} \sum_{n=1}^k (2n-1+M-N)v_n. \quad (3.2.18)$$

Here the modified outage subset $\check{\mathcal{O}}_k$ is defined as

$$\check{\mathcal{O}}_k = \left\{ v_1, \dots, v_k \left| N\tau(k) - \sum_{n=1}^k v_n < r, \alpha \leq v_k \leq \dots \leq v_1 \leq \tau(k) \right. \right\} \quad (3.2.19)$$

where $\tau(k) = 1 + k\alpha(M - N + k)$. Since $N\tau(k) - \sum_{n=1}^k v_n \geq (N - k)\tau(k)$, there will be no outage (i.e., $d_k(r) = \infty$), if $r \leq (N - k)\tau(k)$ or $(N - k)\tau(k) \geq N$. Note that $(N - k)\tau(k) \geq N \Rightarrow (M - N + k)(N - k) \geq 1/\alpha$. In other words, if $(M - N + k)(N - k) < 1/\alpha$, there will be nonzero outage (i.e., $d_k(r) < \infty$), for

$r \in \Omega_k$, where Ω_k is defined as

$$\Omega_k \triangleq \left((N - k)\tau(k), N \right]. \quad (3.2.20)$$

For any $r \in \Omega_k$, we are able to explicitly calculate the optimal solutions of $v_1, \dots, v_k$ that minimize the SNR exponent $\sum_{n=1}^k (2n - 1 + M - N)v_n$. The results are summarized in the following.

1. If $r = (N - k')\tau(k) - (k - k')\alpha$, $k' = 1, 2, \dots, k$, then the diversity for outage event $\check{\mathcal{O}}_k$ is

$$d_k(r) = k'(M - N + k')\tau(k) + (k - k')(k + k' + M - N)\alpha. \quad (3.2.21)$$

The corresponding optimal solution of $v_1, \dots, v_k$ is $v_1^* = \dots = v_{k'}^* = \tau(k)$, $v_{k'+1}^* = \dots = v_k^* = \alpha$. Specifically, $d_k(r) = k(M - N + k)\tau(k)$ for $r = (N - k)\tau(k)$.

2. If $(N - k')\tau(k) - (k - k')\alpha < r < (N - k' + 1)\tau(k) - (k - k' + 1)\alpha$, $k' = 1, 2, \dots, k$, the diversity for outage event $\check{\mathcal{O}}_k$ is

$$d_k(r) = ((N - k')(k' - N - 1) + MN)\tau(k) + (k - k' + 1)(k - k')\alpha - (2k' - 1 + M - N)r. \quad (3.2.22)$$

The corresponding optimal solution of $v_1, \dots, v_k$ is $v_1^* = \dots = v_{k'}^* = \tau(k)$, $v_{k'+1}^* = (N - k' + 1)\tau(k) - (k - k')\alpha - r$, $v_{k'+1}^* = \dots = v_k^* = \alpha$.

That is, for a particular k' , when spatial multiplexing gain r is between $(N - k')\tau(k) - (k - k')\alpha$ and $(N - k' + 1)\tau(k) - (k - k' + 1)\alpha$, only one singular value of $\mathbf{H}$, corresponding to the typical outage event, needs to be adjusted to be barely large enough to support the data rate. Therefore, the DMT curve $d_k(r)$ is piecewise-linear with $(r, d_k(r))$ specified in (3.2.21) being its corner points.

Solving Main Problem to Find $d_{CSIT}^*(r)$

After calculating $d_1(r), \dots, d_N(r)$, we remain to solve $d_{CSIT}^*(r) = \min_k d_k(r)$, $k = 1, \dots, N$. Since $d_k(r) = \infty$ if $(M - N + k)(N - k) \geq 1/\alpha$, we only need to consider $k \in \mathcal{A}$, where set $\mathcal{A}$ is defined as

$$\mathcal{A} = \left\{ k \mid (M - N + k)(N - k) < 1/\alpha, k = 1, \dots, N \right\}. \quad (3.2.23)$$

Note that we always have $N \in \mathcal{A}$. We consider the following two cases.

Case 1: $\mathcal{A}$ has only one element, i.e., $\mathcal{A} = \{N\}$. In this case, we have $d_{CSIT}^*(r) = d_N(r)$. If $N = 1$, this condition is naturally satisfied, since there is only one element in $\mathcal{A}$ that is $k = 1$. If $N > 1$, we must require $(M - N + k)(N - k) \geq 1/\alpha$ for $k = 1, \dots, N - 1$, which leads to

$$\alpha \geq \frac{1}{M - 1}. \quad (3.2.24)$$

To obtain the explicit expression for $d_N(r)$, we now examine the corner points of $d_N(r)$. For corner point k' ($k' = 1, 2, \dots, N - 1$), from (3.2.21) we have $r = (N - k')\tau(N) - (N - k')\alpha = (N - k')(1 + \alpha MN - \alpha) \geq 1 + MN\alpha - \alpha \geq 1 + \frac{MN}{M-1} - \frac{1}{M-1} > N$. Therefore, there is only one corner point N , i.e., $(0, d_N(0))$, on curve $d_N(r)$ making $r \in \Omega_N$. As a result, $d_{CSIT}^*(r) = d_N(r)$ is a straight line between the two points $(0, d_N(0))$ and $(N, d_N(N))$. From (3.2.22), we have $d_N(N) = MN(1 + MN\alpha) - (M + N - 1)N$, so $d_{CSIT}^*(r)$ can be described as

$$d_{CSIT}^*(r) = MN(1 + MN\alpha) - (M + N - 1)r \quad \text{for } 0 \leq r \leq N. \quad (3.2.25)$$

Case 2: $\mathcal{A}$ has more than one element. Since $N \in \mathcal{A}$ and $\Omega_N = [0, N]$, Ω_k ($k \neq N, k \in \mathcal{A}$) overlaps with Ω_N . That is, there are some regions of spatial multiplexing gain r , leading to finite diversity gains on different DMT curves. A straightforward method to find $d_{CSIT}^*(r)$ is to numerically calculate $d_k(r)$ for all $k \in \mathcal{A}$, and choose the minimum value among them. However, this makes $d_{CSIT}^*(r)$ implicit and hardly insightful. To find the closed-form solution of

$d_{CSIT}^*(r)$, we wish to find out if there is any relationship among $d_1(r), \dots, d_N(r)$. This motivates the birth of the following Lemma, the proof of which is given in Appendix A.

Lemma 3.2.2 For any spatial multiplexing gain $r \in \Omega_{k_1} \cap \Omega_{k_2}$ ($1 \leq k_1, k_2 \leq N$), if $k_1 < k_2$, we have $d_{k_1}(r) < d_{k_2}(r)$.

This Lemma tells us if a spatial multiplexing gain r leads to finite diversity gains on two DMT curves, we only need to select the curve with lower index. For example, if $r \in \Omega_1 \cap \Omega_2 \cap \dots \cap \Omega_N$, then $d_{CSIT}^*(r) = d_1(r)$ since $d_1(r) < d_2(r) < \dots < d_N(r) < \infty$. Therefore, we can further expurgate bad k (i.e. $\Omega_k \subseteq \Omega_{\bar{k}}$, $\exists \bar{k} < k \in \mathcal{A}$) from $\mathcal{A}$ and only take into account $k \in \mathcal{B}$ for the optimization problem, where

$$\mathcal{B} = \left\{ k \mid (N-k)\tau(k) < (N-\bar{k})\tau(\bar{k}), \forall \bar{k} < k, \bar{k}, k \in \mathcal{A} \right\}. \quad (3.2.26)$$

Letting $|\mathcal{B}|$ denote the cardinality of $\mathcal{B}$, we further divide $r \in [0, N]$ into $|\mathcal{B}|$ non-overlapping regions with region $\tilde{\Omega}_k$ ($k \in \mathcal{B}$) defined as

$$\tilde{\Omega}_k = \begin{cases} \left[(N-k)\tau(k), N \right], & \text{if } k = \min_{\bar{k} \in \mathcal{B}} \bar{k} \\ \Omega_k \cap \tilde{\Omega}_{\bar{k}}^c, \forall \bar{k} < k \text{ \& } \bar{k} \in \mathcal{B} = \left[(N-k)\tau(k), (N-\mathcal{I}(k))\tau(\mathcal{I}(k)) \right), & \text{otherwise} \end{cases} \quad (3.2.27)$$

where $\mathcal{I}(k)$ indicates the immediately preceding element of k in $\mathcal{B}$, i.e., $\mathcal{I}(k) = \max_{\bar{k} < k, \bar{k} \in \mathcal{B}} \bar{k}$. Figure 3.1 is an example to illustrate the relationship between Ω_k and $\tilde{\Omega}_k$. Since for region $\tilde{\Omega}_k$, $d_k(r)$ has the lowest index among all the DMT curves leading to finite diversity, with the help of Lemma 3.2.2 we get $d_{CSIT}^*(r) = d_k(r)$ for any $r \in \tilde{\Omega}_k$.

To have an explicit picture what the DMT curve looks like over the entire outage event, we examine the corner points on curve $d_k(r)$ over $r \in \tilde{\Omega}_k$ and give the following Lemma, the proof of which is given in Appendix B.

Lemma 3.2.3 For $k \in \mathcal{B}$, there is only one corner point, i.e., $((N-k)\tau(k), k(M-N+k)\tau(k))$, on curve $d_k(r)$ making $r \in \tilde{\Omega}_k$.

Figure 3.1: Relationship of Ω_k and $\tilde{\Omega}_k$.

As a result of Lemma 3.2.3, $d_k(r)$ over $r \in \tilde{\Omega}_k$ is just a single line segment connecting the two corner points as described in (3.2.8). Finally, since $d_{CSIT}^*(r)$ is the union of $d_k(r)$ over $r \in \tilde{\Omega}_k$ for all $k \in \mathcal{B}$, the DMT curve over the entire spatial multiplexing gain region is the collection of all the involved line segments. This completes the proof of Theorem 3.2.1. $\square$

When $N > 1$, for example, when $M = N = 2$ and $\alpha < 1$, the DMT curve consists of two discontinuous line segments which are $(0, 16\alpha + 4) - (1 + \alpha, 13\alpha + 1)$ and $(1 + \alpha, 1 + \alpha) - (2, 2\alpha)$. When $r = 1 + \alpha$, the optimal diversity gain is $d_{CSIT}^*(r) = 1 + \alpha$ instead of $13\alpha + 1$.

Corollary 3.2.1 From Theorem 3.2.1, we can get $d_{CSIT}^*(0) = MN(1 + MN\alpha)$ and $d_{CSIT}^*(N) = \varphi\alpha(M - N + \varphi)(MN + (\varphi - N)(N - \varphi + 1)) - \varphi^2 + \varphi$, where $\varphi = \min_{k \in \mathcal{B}} k$. If $\alpha < \frac{1}{(N-1)(M-N+1)}$, which indicates $1 \in \mathcal{B}$, we will have $d_{CSIT}^*(N) = \alpha N(M - N + 1)^2$.

Discussions We now discuss the additional diversity gain $\Delta_d(r)$ brought by the imperfect CSIT through power control.

Case A) $N = 1$ (MISO/SIMO): According to (3.2.7), the imperfect CSIT provides an additional diversity gain of $\Delta_d(r) = M^2\alpha$ at any spatial multiplexing gain in the considered MISO/SIMO channel. Most remarkably, when $\alpha = 1/M$,

Figure 3.2: DMT in a 3×3 MIMO channel. Note that $d_o(r)$ in the legend denotes $d_{CSIT}^*(r)$.

Figure 3.3: DMT in a 4×2 MIMO channel with $\alpha = 0.1$. Note that $d_o(r)$ in the legend denotes $d_{CSIT}^*(r)$.

one can achieve both full diversity gain (i.e., M) and full spatial multiplexing gain (i.e., 1) at the same time, while α has to be equal to or greater than 1 to achieve the same performance in [20].

Case B) $\alpha \geq \frac{1}{M-1}$, $N > 1$: For such MIMO channel, according to (3.2.7),

the additional diversity gain is $\Delta_d(r) = (M^2N^2\alpha + r - r^2)$, for $r = 0, 1, \dots, N$. Specifically, $\Delta_d(0) = M^2N^2\alpha$ and $\Delta_d(N) = \alpha M^2N^2 + N - N^2 > MN^2 + N$. If $0 < r < N$, the additional diversity gain is between the two extreme values $\Delta_d(0)$ and $\Delta_d(N)$.

Case C) $\alpha < \frac{1}{M-1}$, $N > 1$: When $r = N$, $\Delta_d(N) = d_{CSIT}^*(N) \geq d_1(N) = \alpha N(M - N + 1)^2$. When $r < N$, for the convenience of comparison with [6], we consider integer spatial multiplexing gains, i.e., $r = N - k$, $k = 1, 2, \dots, N$. Since $r = N - k \leq (N - k)\tau(k)$, from Theorem 3.2.1, the optimal diversity gain is $d_{CSIT}^*(r) \geq k(M - N + k)\tau(k) = (M - r)(N - r)(1 + \alpha(M - r)(N - r))$. Recall that the optimal diversity gain without CSIT is $d^*(r) = (M - r)(N - r)$. Therefore, the additional achieved diversity gain with our scheme is $\Delta_d(r) \geq \alpha(M - r)^2(N - r)^2 = \alpha(d^*(r))^2$. It indicates that even a very small α leads to a significant diversity gain improvement.

We use numerical results to show the additional diversity gain achieved with imperfect CSIT. We compare the following two scenarios: 1) No CSIT [6]; and 2) Imperfect CSIT with power control. Figure 3.2 and Figure 3.3 plot the DMT curves for 3×3 and 4×2 MIMO fading channels, respectively. It is obvious that imperfect CSIT provides significant additional diversity gain improvement and offers non-zero diversity gain at any possible spatial multiplexing gain. Figure 3.2 also shows the impact of α value. When $\alpha \geq \frac{1}{M-1} = \frac{1}{2}$, we only have $d_N(r) < \infty$ and thus $\mathcal{B} = \{3\}$. Therefore, the DMT curve is a single line segment. However, when $\alpha = \frac{1}{3} < \frac{1}{M-1}$, $\mathcal{B} = \{1, 2, 3\}$. Therefore, the DMT curve is made up of three discontinuous line segments. Figure 3.3 shows how $d_{CSIT}^*(r)$ depends on $d_1(r)$ and $d_2(r)$ in a 4×2 MIMO channel with $\alpha = 0.1$. We observe that $d_2(r) \geq d_1(r)$ and there is only one corner point on $d_1(r)$ (or $d_2(r)$) over spatial multiplexing gain region $r \in \tilde{\Omega}_1$ (or $r \in \tilde{\Omega}_2$).

Next we illustrate the impact of α on DMT. Figure 3.4 plots the relationship between the optimal diversity gain and the channel quality α in a MISO/SIMO channel. It clearly shows that power control makes better use of the imperfect CSIT than the rate control in [20]. In other words, to achieve the same performance our scheme saves a great amount of pilot power and thus is more applica-

Figure 3.4: Diversity gain versus channel quality α in a SIMO/MISO channel. The dashed line refers to the rate control scheme in [20].

ble. Specifically, the diversity gain improvements over [6] and [20] are $M^2\alpha$ and $(M-1)M\alpha$, respectively, at any spatial multiplexing gain. It is no doubt that the achievable DMT increases with CSIT quality α . Figure 3.5 plots how the achievable diversity gain with power control improves with the channel quality α in a 5×3 MIMO channel at the full multiplexing gain. We observe that there are fast increases of diversity gain at $\alpha = 0.25$ and $\alpha = 0.1667$. These two values of α are actually the thresholds for $d_k(r) < \infty, k = 1, 2, 3$. When $\alpha \geq \frac{1}{M-1} = 0.25$, $\mathcal{B} = \{3\}$. Therefore, we have $d_{CSIT}^*(N) = d_3(N)$. When $0.1667 \leq \alpha < 0.25$, we have $\mathcal{B} = \{2, 3\}$ and $d_{CSIT}^*(N) = d_2(N)$. When $\alpha < 0.1667$, we have $\mathcal{B} = \{1, 2, 3\}$ and $d_{CSIT}^*(N) = d_1(N)$. Combining with the fact that $d_1(r) < d_2(r) < d_3(r)$ for any fixed α , it is not difficult to understand the cliffs on this curve.

3.3 Power Control with Imperfect CSIT and CSIR

In this section, we assume that neither the destination nor the source has any CSI initially. After two-way training, both of them will have imperfect CSI due to channel estimation errors. We consider the following two training strategies: 1) the destination initiates the training [19]; and 2) the source initiates the training.

Figure 3.5: Diversity gain at $r = N$ versus channel quality α in a 5×3 MIMO channel.

For illustration purpose, we will firstly mainly describe the case of a single training round. The multiple-round training will be mainly discussed in Subsections 3.3.1 and 3.3.2 in detail together with the related DMT results. Since ML channel estimation has been addressed in the previous section, we will discuss MMSE channel estimation in this section in order to show that both channel estimation methods can lead to the same DMT result.

Destination Initiates Training

If the destination initiates the training process, in the first phase the destination sends T ($T \ll L$) training symbols with fixed power $P_b = \bar{P}^\alpha$, $\alpha \geq 0$, to the source through the reverse channel. The source obtains imperfect CSIT $\hat{\mathbf{H}}_b \in \mathcal{C}^{N \times M}$ from reverse channel pilots using MMSE channel estimation. Thus, $\hat{\mathbf{H}}_b$ can be modeled as [60]

$$\mathbf{H} = \hat{\mathbf{H}}_b + \mathbf{E}_b \quad (3.3.1)$$

where the channel estimation error $\mathbf{E}_b \in \mathcal{C}^{N \times M}$ has i.i.d. entries $E_{1,n,m,b} \sim \mathcal{CN}(0, \sigma_{1,e,b}^2)$, $n = 1, \dots, N$, $m = 1, \dots, M$, and is independent of $\hat{\mathbf{H}}_b$. The quality of $\hat{\mathbf{H}}_b$ is thus characterized by $\sigma_{1,e,b}^2 \doteq \rho^{-\alpha}$.

In the second phase, the source sends T training symbols with an instantaneous power P_f subject to an average power constraint $\bar{P}^\beta$, i.e., $E\{P_f\} = \bar{P}^\beta$, $\beta \geq 0$, in the forward channel. We refer to these two training phases as a complete *two-way training round*. The destination then estimates the channel and gets $\hat{\mathbf{H}}_f$ with MMSE estimation. Thus, we have

$$\mathbf{H} = \hat{\mathbf{H}}_f + \mathbf{E}_f \quad (3.3.2)$$

where the channel estimation error $\mathbf{E}_f \in \mathcal{C}^{N \times M}$ has i.i.d. entries $E_{1,n,m,f} \sim \mathcal{CN}(0, \sigma_{1,e,f}^2)$, $n = 1, \dots, N$, $m = 1, \dots, M$, and is independent of $\hat{\mathbf{H}}_f$. Similarly, we have $\sigma_{1,e,f}^2 \propto P_f^{-1}$. For the forward training channel, β describes the average CSIR quality at the destination. After the two-phase training, the source will send data symbols with average power $\bar{P}$ to the destination.

In the above, we have described a single two-way training round. If there are multiple training rounds, in the next round, the destination will send power-controlled pilots to the source (Phase 1) and the source will then send power-controlled pilots to the destination (Phase 2). In the case of the final round, the source will send power-controlled data symbols after Phase 2 of training.

Source Initiates Training

As the source initially has no idea of the channel, it sends training symbols with fixed power $\bar{P}^\beta$, i.e., $P_f = \bar{P}^\beta$, in the forward channel. The destination estimates the channel and gets $\hat{\mathbf{H}}_f$ with MMSE channel estimation. Note that this is actually one-way training if there is only a single training round. After the training, the source will send data symbols with fixed power $\bar{P}$.

If there are two training rounds, the source will send pilots with fixed power $\bar{P}^\beta$ to the destination in the first round. Note that there is only one training phase in the first round in this case. In the second round, the destination will send power-controlled pilots to the source in the reverse channel (Phase 1) and then the source will send power-controlled pilots in the forward channel (Phase 2). After that, the source will send power-controlled data symbols to the destination.

Figure 3.6: Illustration of two-way training. Note that $L \gg I \cdot T$.

3.3.1 DMT Analysis for Destination-Initiated Training

Now we extend the single training round to multiple training rounds. Suppose there are totally I ($I \geq 1$) training rounds, after which the source sends power-controlled data symbols with average power $\bar{P}$ to the destination as shown in Figure 3.6. It is noted that although more pilots lead to a reduced achievable data rate, the spatial multiplexing gain might not decrease since the number of pilots does not increase with ρ .

The Genie-Aided Model

We assume that before performing channel estimation, the source and destination know via a genie exactly the power levels that have been used to send training symbols by the destination and source in the previous training phase, respectively¹.

We examine the i -th ($1 \leq i \leq I$) two-way training round. In Phase 1, since the destination knows² $\mathbf{v}_{i-1,f} = [v_{i-1,f,1}, \dots, v_{i-1,f,N}]$, we let the destination send T

¹There is no such issue for the case of a single training round, since α and β are predetermined and assumed to be known to the source and destination initially.

² $\mathbf{v}_{i-1,f}$ will be defined soon after. For $i = 1$, we set $\mathbf{v}_{0,f} = \mathbf{0}$.

training symbols to the source with power

$$P_b(i) \doteq \rho^{A_{i-1}} \quad (3.3.3)$$

where $A_{i-1} \triangleq \alpha + t \sum_{n=1}^N (2n-1+M-N)v_{i-1,f,n}$. It can be verified immediately that $A_0 = \alpha$ and $P_b(1) \doteq \rho^\alpha$. Knowing $P_b(i)$ via a genie, the source estimates the channel and gets $\hat{\mathbf{H}}_b(i)$, which is modeled as $\mathbf{H} = \hat{\mathbf{H}}_b(i) + \mathbf{E}_b(i)$, where the channel estimation error $\mathbf{E}_b(i) \in \mathcal{C}^{N \times M}$ has i.i.d. entries $E_{i,n,m,b} \sim \mathcal{CN}(0, \sigma_{i,e,b}^2)$, $n = 1, \dots, N$, $m = 1, \dots, M$, and $\sigma_{i,e,b}^2 \propto P_b(i)^{-1}$. We let $\hat{\lambda}_{b,n}(i)$, $n = 1, \dots, N$, denote the n -th smallest eigenvalue of $\hat{\mathbf{H}}_b(i)\hat{\mathbf{H}}_b(i)^\dagger$ and let $v_{i,b,n}$ denote the exponential order of $1/\hat{\lambda}_{b,n}(i)$, i.e., $v_{i,b,n} \triangleq -\lim_{\rho \rightarrow \infty} \frac{\log(\hat{\lambda}_{b,n}(i))}{\log(\rho)}$. We define $\mathbf{v}_{i,b} \triangleq [v_{i,b,1}, \dots, v_{i,b,N}]$.

In Phase 2, being aware of $\mathbf{v}_{i,b}$, the source transmits T training symbols with power

$$P_f(i) \doteq \rho^{B_i} \quad (3.3.4)$$

where $B_i \triangleq \beta + t \sum_{n=1}^N (2n-1+M-N)v_{i,b,n}$. With the knowledge of $P_f(i)$ via a genie, the destination estimates the channel and gets $\hat{\mathbf{H}}_f(i)$, which is modeled as $\mathbf{H} = \hat{\mathbf{H}}_f(i) + \mathbf{E}_f(i)$, where the channel estimation error $\mathbf{E}_f(i) \in \mathcal{C}^{N \times M}$ has i.i.d. entries $E_{i,n,m,f} \sim \mathcal{CN}(0, \sigma_{i,e,f}^2)$, $n = 1, \dots, N$, $m = 1, \dots, M$, and $\sigma_{i,e,f}^2 \propto P_f(i)^{-1}$. We let $\hat{\lambda}_{f,n}(i)$, $n = 1, \dots, N$, denote the n -th smallest eigenvalue of $\hat{\mathbf{H}}_f(i)\hat{\mathbf{H}}_f(i)^\dagger$ and let $v_{i,f,n}$ denote the exponential order of $1/\hat{\lambda}_{f,n}(i)$, i.e., $v_{i,f,n} \triangleq -\lim_{\rho \rightarrow \infty} \frac{\log(\hat{\lambda}_{f,n}(i))}{\log(\rho)}$. We define $\mathbf{v}_{i,f} \triangleq [v_{i,f,1}, \dots, v_{i,f,N}]$. After I training rounds ($I \cdot T \ll L$), the source will send data symbols with power

$$P \doteq \rho^{1+t \sum_{n=1}^N (2n-1+M-N)v_{I,b,n}}. \quad (3.3.5)$$

It is easy to prove that the above power control scheme satisfies the average power constraints $E\{P\} \doteq \bar{P}$, $E\{P_b(i)\} \doteq \bar{P}^\alpha$, and $E\{P_f(i)\} \doteq \bar{P}^\beta$.

Next we analyze the DMT performance under the above power control scheme. The system model in (2.1.1) can be rewritten as

$$\mathbf{y} = \sqrt{P/M}(\hat{\mathbf{H}}_f(I) + \mathbf{E}_f(I))\mathbf{x} + \mathbf{w}. \quad (3.3.6)$$

The mutual information of this channel is difficult to compute since the equivalent noise $\tilde{\mathbf{w}} = \sqrt{P/M}\mathbf{E}_f(I)\mathbf{x} + \mathbf{w}$ is not Gaussian distributed. However, since $\tilde{\mathbf{w}}$ has uncorrelated entries and is uncorrelated to the signals $\sqrt{P/M}\hat{\mathbf{H}}_f(I)\mathbf{x}$, it has the same first- and second-order moments as an AWGN with variance $\tilde{\sigma}^2 = \sigma^2 + P\|\mathbf{E}_f(I)\|_F^2/M$. Therefore, we may replace $\tilde{\mathbf{w}}$ by an AWGN with the same variance and the resulting mutual information would be a lower bound of the exact one [77]. Therefore, the outage probability of the considered MIMO channel with transmit power P can be upper bounded by

$$\mathcal{P}_{out,2} = \mathcal{P} \left(\log \det \left(\mathbf{I}_N + \frac{P\hat{\mathbf{H}}_f(I)\hat{\mathbf{H}}_f(I)^\dagger}{M\sigma^2 + P\|\mathbf{E}_f(I)\|_F^2} \right) < R(\rho) \right). \quad (3.3.7)$$

Throughout this section, the overhead due to two-way training is ignored for brevity. The DMT for the considered MIMO channel is characterized by the following theorem, the proof of which is given in Appendix C.

Theorem 3.3.1 Consider an $M \times N$ MIMO channel with CSIT quality α ($\alpha \geq 0$) and CSIR quality β ($\beta \geq 0$). After I ($I \geq 1$) rounds of two-way training, the achievable diversity gain is zero if $r > N \min(\beta, 1)$. For $r \leq N \min(\beta, 1)$, the achievable DMT is characterized by

Case 1: If $I = 1$ and $N > 1$ and $\alpha < \min(\beta, 1)/(M - 1)$, the achievable DMT $d_{CSITR}^*(r)$ is a collection of discontinuous line segments, with the two end points of line segment $d_k(r)$ ($k \in \mathcal{B}$) given by

$$\begin{aligned} \text{Left: } d_k(r) &= k(M - N + k)\hat{\tau}(k), \text{ for } r = (N - k)\hat{\tau}(k) \\ \text{Right: } \begin{cases} d_k(r) = k\alpha(M - N + k)((N - k)(k - N - 1) + MN) + (k - k^2)\min(\beta, 1) \\ \quad \text{for } r = N\min(\beta, 1), \quad \text{if } k = \min_{\bar{k}} \hat{\mathcal{B}} \\ d_k(r) = ((N - k)(k - N - 1) + MN)\hat{\tau}(k) - (2k - 1 + M - N)(N - \hat{\mathcal{I}}(k))\hat{\tau}(\hat{\mathcal{I}}(k)), \\ \quad \text{for } r = (N - \hat{\mathcal{I}}(k))\hat{\tau}(\hat{\mathcal{I}}(k)), \quad \text{otherwise} \end{cases} \end{aligned} \quad (3.3.8)$$

where

$$\hat{\mathcal{B}} = \left\{ k \mid (M - N + k)(N - k)\alpha < \min(\beta, 1), (N - k)\hat{\tau}(k) < (N - \bar{k})\hat{\tau}(\bar{k}), \forall \bar{k} < k, k = 1, \dots, N \right\},$$

$$\hat{\tau}(k) = \min(\beta, 1) + k\alpha(M - N + k), \text{ and } \hat{\mathcal{I}}(k) = \max_{\bar{k} \in \hat{\mathcal{B}}, \bar{k} < k} \bar{k}.$$

Case 2: Otherwise, the DMT is given by

$$d_{CSITR}^*(r) = MN(\min(\beta, 1) + MN\alpha(I, N)) - (M + N - 1)r \quad (3.3.9)$$

where

$$\alpha(i, N) = \begin{cases} \alpha + MN(MN\alpha(i - 1, N) + \beta), & i > 1 \\ \alpha, & i = 1. \end{cases} \quad (3.3.10)$$

Quantized-Power Controlled Training

In this subsection, we present a feasible training scheme to perform channel estimation, which avoids the use of a genie. We show that this scheme is able to achieve the same DMT result as the genie-aided model.

- In the first phase of the i -th ($1 \leq i \leq I$) round training, the destination trains the source with power $\tilde{P}_b(i) = \rho^{\tilde{A}_{i-1}}$, where $\tilde{A}_{i-1} = \alpha + \sum_{n=1}^N (2n - 1 + M - N)\tilde{v}_{i-1,f,n}$. We define $\tilde{v}_{i-1,f,n} \triangleq 0$, $n = 1, \dots, N$. With the knowledge of $\tilde{P}_b(i)$, the source performs MMSE channel estimation and obtains $\hat{\mathbf{H}}_b(i)$. Next, the source quantizes $v_{i,b,n}$ with codebook $\left\{0, \beta, 2\beta, \dots, \lfloor \frac{\tilde{A}_{i-1}}{\beta} \rfloor \beta, \tilde{A}_{i-1}\right\}$ and gets $\tilde{v}_{i,b,n} = \tilde{A}_{i-1}$ if $v_{i,b,n} \geq \tilde{A}_{i-1}$ and $\tilde{v}_{i,b,n} = \lfloor \frac{v_{i,b,n}}{\beta} \rfloor \beta$ otherwise.
- In the second phase of the i -th ($1 \leq i < I$) round training, the source trains the destination with power $\tilde{P}_f(i) = \rho^{\tilde{B}_i}$, where $\tilde{B}_i = \beta + \sum_{n=1}^N (2n - 1 + M - N)\tilde{v}_{i,b,n}$. With the knowledge of $\tilde{P}_f(i)$, the destination performs MMSE channel estimation and obtains $\hat{\mathbf{H}}_f(i)$. Next, the destination quantizes $v_{i,f,n}$, $n = 1, \dots, N$, with codebook $\left\{0, \alpha, 2\alpha, \dots, \lfloor \frac{\tilde{B}_i}{\alpha} \rfloor \alpha, \tilde{B}_i\right\}$ and gets $\tilde{v}_{i,f,n} = \tilde{B}_i$ if $v_{i,f,n} \geq \tilde{B}_i$ and $\tilde{v}_{i,f,n} = \lfloor \frac{v_{i,f,n}}{\alpha} \rfloor \alpha$ otherwise.
- In the second phase of the I -th round training, the source sends training symbols with power $\tilde{P}_f(I) = P_f(I) = \rho^{\beta + \sum_{n=1}^N (2n - 1 + M - N)v_{I,b,n}}$, after which the source sends data symbols with power $P = \rho^{1 + \sum_{n=1}^N (2n - 1 + M - N)v_{I,b,n}}$. In this last round, knowing $\sqrt{\tilde{P}_f(I)}\hat{\mathbf{H}}_f(I)$ is sufficient for the destination to

decode data. So, there is no need for the destination to know $\tilde{P}_f(I)$ and thus there is no need for the source to quantize $v_{I,b,n}$.

Since $\tilde{v}_{i,b,n} \leq v_{i,b,n}$ and $\tilde{v}_{i,f,n} \leq v_{i,f,n}$, we have $E\{\tilde{P}_f(i)\} \leq E\{P_f(i)\} \doteq \rho^\beta$ and $E\{\tilde{P}_b(i)\} \leq E\{P_b(i)\} \doteq \rho^\alpha$. The question left is: how to inform the source and destination of $\tilde{P}_b(i)$ and $\tilde{P}_f(i)$, respectively? We take the source for example. In the i -th training round, since the source knows $\tilde{B}_{i-1}$, we only need to let the destination send $\lfloor \frac{v_{i-1,f,n}}{\alpha} \rfloor$ if $v_{i-1,f,n} < \tilde{B}_{i-1}$ or $\lfloor \frac{\tilde{B}_{i-1}}{\alpha} \rfloor + 1$ if $v_{i-1,f,n} \geq \tilde{B}_{i-1}$ to the source. There are only $\log_2 \left\{ N \left(\lfloor \frac{\tilde{B}_{i-1}}{\alpha} \rfloor + 2 \right) \right\}$ bits to be conveyed. There are two possible ways to convey these finite bits: 1) we let the destination send these bits in a separate error free channel. This kind of channel has been widely used in the literature that addresses limited feedback. 2) We let the destination send these bits as a part of training symbols. Since the number of these bits is small, it is very likely for the source to decode these bits successfully. Recall that at high SNRs, there is no outage or infinite diversity gain at zero multiplexing gain. This quantized-power controlled training scheme is able to achieve the same DMT as the genie-aided model, the proof is given in Appendix D.

3.3.2 DMT Analysis for Source-Initiated Training

In this subsection, we consider the achievable DMT performance when the source initiates the training process and there are I ($I \geq 1$) training rounds. We follow Section 3.3.1 to carry out two-way training for the first round and the i -th ($1 < i \leq I$) round, respectively. If $i = I$, after the second phase, the source will send data symbols with power P as given in (3.3.5). We present the following DMT result, whose proof is similar to the proof of Theorem 3.3.1 and thus is omitted here.

Theorem 3.3.2 (Source initiates training) Consider an $M \times N$ MIMO channel with CSIT quality α and CSIR quality β . After I ($I \geq 1$) rounds of two-way training, the achievable diversity gain is zero if $r > N \min(\beta, 1)$. For $r \leq N \min(\beta, 1)$, the achievable DMT is characterized by

Case 1: If $I = 1$, the achievable DMT is given by a piecewise-linear function

connecting the points $(r, d_{CSITR}^*(r))$, for $r = 0, \min(\beta, 1), 2 \min(\beta, 1), \dots, N \min(\beta, 1)$, and

$$d_{CSITR}^*(r) = \left(M - \frac{r}{\min(\beta, 1)} \right) (N \min(\beta, 1) - r). \quad (3.3.11)$$

Case 2: If $I \geq 2$, the DMT is described by the following single straight line

$$d_{CSITR}^*(r) = MN(\min(\beta, 1) + MN\alpha(I, N)) - (M + N - 1)r \quad (3.3.12)$$

where

$$\alpha(i, N) = \begin{cases} \alpha + MN(MN\alpha(i-1, N) + \beta), & i > 1 \\ 0, & i = 1. \end{cases} \quad (3.3.13)$$

Both Theorems 3.3.1 and 3.3.2 show that in the case of a single training round ($I = 1$), there is no point to make $\beta > 1$. If $\beta = 1$, (3.3.11) actually reduces to the DMT result of no CSIT and perfect CSIR [6]. One can find that (3.3.11) is the same as (3.3.8) if α is 0. This is not unexpected since a fixed or limited training power ($\Leftrightarrow \alpha = 0$) does not contribute to the diversity gain and is equivalent to no training.

The Case 2 of Theorem 3.3.1 is similar to that of Theorem 3.3.2, except that $\alpha(1, N) = \alpha$ in Theorem 3.3.1 while $\alpha(1, N) = 0$ in Theorem 3.3.2. This is expected from the difference of the two training strategies.

Discussions We discuss the impact of power control on the DMT of the considered MIMO channel with noisy CSITR. Theorems 3.3.1 and 3.3.2 show that imperfect CSITR based on two-way training provides significant diversity gain improvement and offers non-zero diversity gain at any possible spatial multiplexing gain if $r \leq N \min(\beta, 1)$. On the other hand, if $r > N \min(\beta, 1)$, there will be no diversity gain for whatever α value, since the destination can never decode the source data correctly.

We first consider the destination-initiated training strategy. For a fair comparison with [19], we set $\alpha = \beta = I = N = 1$. The optimal diversity is therefore given as $d_{CSITR}^*(r) = M(1 + M - r)$ with the proposed power control scheme. The

Figure 3.7: Diversity-multiplexing tradeoff in SIMO/MISO channels with two-way training. $\alpha = \beta = 1$.

diversity gain improvements over [6] (perfect CSIR and no CSIT) and [19, Eqn. (46)] (imperfect CSITR) are M^2 and $M(M - 1)$, respectively, as illustrated in Figure 3.7. We should notice that when $I = 1$ and CSIR quality $\beta > 1$, there is no further diversity gain improvement over the case with $\beta = 1$. This is because the system has already achieved the best possible diversity with the perfect CSIR as shown in Section 3.2. So for $I = 1$, it is reasonable to let the source use the same power to transmit training symbols and data symbols, i.e., $\beta = 1$.

We next consider the source-initiated training strategy. If $\beta = I = 1$, which corresponds to the case when the source has no CSIT and trains the destination with fixed power $\bar{P}$, the achievable diversity is given by $d_{CSITR}^*(r) = (M - r)(N - r)$. This can be observed in Figures 3.7 and 3.8 and is found to be exactly the same as the optimal diversity given in [6] for the case of perfect CSIR and no CSIT. This implies that the DMT under the ideal condition of perfect CSIR can be equivalently achieved by spending reasonable training power.

If there are multiple training rounds ($I > 1$), much higher diversity can be achieved as shown in Figure 3.7 and Figure 3.8. However, it should be noted

Figure 3.8: Diversity-multiplexing tradeoff in 2×2 MIMO channels with two-way training. $\beta = 1$.

that more training rounds will lead to a lower achievable data rate due to more overheads involved.

3.4 Joint Rate and Power Control with Imperfect CSIT and CSIR

In this section, we not only consider the case where both CSIT and CSIR are imperfect in MIMO channels but also take both power and rate control into joint consideration. For simplicity, we assume only one round of the destination initiated two-way training is performed for the transmitter and receiver to obtain some partial channel knowledge before data transmission, which is described in the previous section. The extension to the source initiated training or multi-round training is straightforward. It is reasonable to let the transmitter spend the same amount of power sending data symbols and training symbols in the forward channel ($\beta = 1$), i.e., $P_f(1) = P$, where P is given by (3.3.5). Thus, we have $\sigma_{1,e,f}^2 \propto P^{-1}$. For notational convenience, we define $w_n \triangleq v_{1,b,n}$, $\mathbf{w} = [w_1, \dots, w_N]$,

$\psi_n \triangleq v_{1,f,n}$ and $\Psi = [\psi_1, \dots, \psi_N]$.

The system model in (2.1.1) can be rewritten as

$$\mathbf{y} = \sqrt{P/M}(\hat{\mathbf{H}}_{\mathbf{f}} + \mathbf{E}_{\mathbf{f}})\mathbf{x} + \mathbf{w}. \quad (3.4.1)$$

Based on the noisy $\hat{\mathbf{H}}_{\mathbf{b}}$, the transmitter performs power control P and rate control $R(\hat{\mathbf{H}}_{\mathbf{b}})$ (i.e., $r(\hat{\mathbf{H}}_{\mathbf{b}}) \triangleq \lim_{\rho \rightarrow \infty} R(\hat{\mathbf{H}}_{\mathbf{b}})/\log \rho$) for data transmission. Therefore, the outage probability of the considered MIMO channel is upper bounded by

$$\mathcal{P}_{out,3} = \mathcal{P} \left(\log \det \left(\mathbf{I}_N + \frac{P\hat{\mathbf{H}}_{\mathbf{f}}\hat{\mathbf{H}}_{\mathbf{f}}^{\dagger}}{M\sigma^2 + P\|\mathbf{E}_{\mathbf{f}}\|_F^2} \right) < R(\hat{\mathbf{H}}_{\mathbf{b}}) \right). \quad (3.4.2)$$

Note that the transmit rate has to satisfy the average spatial multiplexing gain constraint $E \left\{ r(\hat{\mathbf{H}}_{\mathbf{b}}) \right\} = \bar{r}$.

Rate Control Scheme without Minimum Multiplexing Gain Requirement: Given $\hat{\mathbf{H}}_{\mathbf{b}}$, the transmitter adaptively adjusts its data rate $R(\hat{\mathbf{H}}_{\mathbf{b}})$ in a way that

$$r(\hat{\mathbf{H}}_{\mathbf{b}}) = \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log \det(\hat{\mathbf{H}}_{\mathbf{b}}\hat{\mathbf{H}}_{\mathbf{b}}^{\dagger})}{\log(\rho)} \right)^+ \quad (3.4.3)$$

where τ is a finite constant satisfying $\tau \geq \max(1, 1/\alpha)$.

Now we prove the above rate control scheme satisfies the average multiplexing gain constraint $E\{r(\hat{\mathbf{H}}_{\mathbf{b}})\} = \bar{r}$.

Proof. To calculate $E\{r(\hat{\mathbf{H}}_{\mathbf{b}})\}$, we only need to integrate over the range where $\det(\hat{\mathbf{H}}_{\mathbf{b}}\hat{\mathbf{H}}_{\mathbf{b}}^{\dagger}) = \prod_{n=1}^N \lambda_n(\hat{\mathbf{H}}_{\mathbf{b}}) \leq 1$, since for any $\lambda_n(\hat{\mathbf{H}}_{\mathbf{b}}) > 1$, the joint pdf of $\lambda_1(\hat{\mathbf{H}}_{\mathbf{b}}), \dots, \lambda_N(\hat{\mathbf{H}}_{\mathbf{b}})$ decays with SNR exponentially [6]. Therefore, we have

$$E \left\{ \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log 1}{\log(\rho)} \right) \right\} \geq E\{r(\hat{\mathbf{H}}_{\mathbf{b}})\} \geq E \left\{ \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log \det(\hat{\mathbf{H}}_{\mathbf{b}}\hat{\mathbf{H}}_{\mathbf{b}}^{\dagger})}{\log(\rho)} \right) \right\}. \quad (3.4.4)$$

According to [77], it follows that $\lim_{\rho \rightarrow \infty} \frac{E\{\log \det(\hat{\mathbf{H}}_{\mathbf{b}}\hat{\mathbf{H}}_{\mathbf{b}}^{\dagger})\}}{\log(\rho)} = \lim_{\rho \rightarrow \infty} \frac{\sum_{i=M-N+1}^N \mathcal{X}_{2i}^2}{\log(\rho)} = 0$, where $\mathcal{X}_{2i}^2$ is a chi-square random variable of dimension $2i$. Combined with the

fact that τ is a finite constant we have

$$E \left\{ \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log 1}{\log(\rho)} \right) \right\} = E \left\{ \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log \det(\hat{\mathbf{H}}_b \hat{\mathbf{H}}_b^\dagger)}{\log(\rho)} \right) \right\} = \bar{r} \quad (3.4.5)$$

which leads to $E\{r(\hat{\mathbf{H}}_b)\} = \bar{r}$. $\square$

Remarks: For τ to be a finite constant, i.e., not to scale with $\log(\rho)$, we must require $\alpha \neq 0$. In fact, it is shown later that the value of τ does not have any impact on the DMT, as long as τ is a constant satisfying $\tau \geq \max(1, 1/\alpha)$.

For some applications, a minimum rate $R_{min} > 0$ is required to meet an acceptable quality of service. Without R_{min} , it is also not very meaningful to discuss outage, because the transmitter may switch off transmission completely if the channel is too bad. To express R_{min} in the limit of high SNRs, the authors in [16] introduced the concept of minimum multiplexing gain, which is defined as $r_{min} = \lim_{\rho \rightarrow \infty} \frac{R_{min}}{\log(\rho)}$. In the next, we propose a new rate control scheme $R(\hat{\mathbf{H}}_b)$ in the presence of r_{min} . It can be proved that $E\{r(\hat{\mathbf{H}}_b)\} = \bar{r}$. The proof is similar to that for rate control scheme in (3.4.3) and is thus omitted.

Rate Control Scheme with r_{min} Requirement: Given $\hat{\mathbf{H}}_b$ and r_{min} , the transmitter adaptively adjusts its data rate $R(\hat{\mathbf{H}}_b)$ in a way that

$$r(\hat{\mathbf{H}}_b) = \max \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log \det(\hat{\mathbf{H}}_b \hat{\mathbf{H}}_b^\dagger)}{\log(\rho)}, r_{min} \right). \quad (3.4.6)$$

With the power control scheme in (3.3.5) and rate control schemes in (3.4.3) and (3.4.6), we can derive the DMT of the considered MIMO channel with noisy CSITR, which is described by the following theorem.

Theorem 3.4.1 Consider an $M \times N$ ($M \geq N$) MIMO channel with noisy CSITR obtained from the two-way training as described above. The achievable DMT with average spatial multiplexing gain $\bar{r}$ ($\bar{r} \leq N$) is characterized by:

Case 1: If there is no minimum instantaneous multiplexing gain constraint, with the rate control scheme in (3.4.3), the achievable diversity gain is infinity.

Case 2: If a minimum instantaneous multiplexing gain r_{min} ($r_{min} > 0$) is imposed, with only the rate control scheme in (3.4.6), the achievable DMT is given by $d^*(r_{min})$.

Case 3: In the presence of r_{min} requirement, with power control in (3.3.5) and rate control in (3.4.6), the achievable DMT is given by $d_{CSIT}^*(r_{min})$.

Remarks: It can be verified immediately that if $\bar{r} = r_{min}$, we will have $r(\hat{\mathbf{H}}_{\mathbf{b}}) = r_{min}$, which corresponds to the case with no rate control, and thus the results of Theorem 3.4.1 are consistent with Theorem 3.2.1 and Theorem 2.1.1.

Proof. We let p denote the exponential order of P , i.e., $p = \lim_{\rho \rightarrow \infty} \frac{\log(P)}{\log(\rho)}$. If there is no power control, we have $p = 1$. If the power control scheme in (3.3.5) is performed, we have $p = 1 + t \sum_{n=1}^N (2n - 1 + M - N)w_n \geq 1$. The joint pdf of $\mathbf{v}$, Ψ and $\mathbf{w}$ is given by

$$\begin{aligned} p(\mathbf{v}, \Psi, \mathbf{w}) &= p(\Psi|\mathbf{w}, \mathbf{v})p(\mathbf{w}|\mathbf{v})p(\mathbf{v}) \\ &\doteq \rho^{-\sum_{i=1}^{t_2} (2i-1+M-N)(\psi_i-p) - \sum_{j=1}^{t_1} (2j-1+M-N)(w_j-\alpha) - \sum_{n=1}^N (2n-1+M-N)v_n} \end{aligned} \quad (3.4.7)$$

if $\min(v_i, \psi_i) \geq p, \forall i = 1, \dots, t_2; 0 \leq v_i = \psi_i < p, \forall N \geq i > t_2; \min(v_j, w_j) \geq \alpha, \forall j = 1, \dots, t_1; 0 \leq v_j = w_j < \alpha, \forall N \geq j > t_1$; and $0 \leq t_1, t_2 \leq N$; otherwise $p(\mathbf{v}, \Psi, \mathbf{w}) = 0$.

Rate Control without Minimum Instantaneous Multiplexing Gain

At high SNRs, the outage probability with the rate control scheme in (3.4.3) is given by

$$\mathcal{P}_{out,3} \doteq \mathcal{P} \left(\sum_{n=1}^N (p - \psi_n)^+ + \tau \sum_{n=1}^N w_n < \bar{r} \right) \doteq \rho^{-d_{joint}^*(\bar{r})} \quad (3.4.8)$$

where

$$d_{joint}^*(\bar{r}) = \min_{w_n, \psi_n, v_n \in \mathcal{O}_{joint}^+} \sum_{i=1}^{t_2} (2i-1+M-N)(\psi_i-p) + \sum_{j=1}^{t_1} (2j-1+M-N)(w_j-\alpha) + \sum_{n=1}^N (2n-1+M-N)v_n \quad (3.4.9)$$

and

$$\mathcal{O}_{joint} = \left\{ w_n, \psi_n, v_n \mid \sum_{n=1}^N (p - \psi_n)^+ + \tau \sum_{n=1}^N w_n < \bar{r}; 0 \leq t_1, t_2 \leq N; \right. \\ \min(v_i, \psi_i) \geq p, \forall i = 1, \dots, t_2; 0 \leq v_i = \psi_i < p, \forall N \geq i > t_2; \\ \left. \min(v_j, w_j) \geq \alpha, \forall j = 1, \dots, t_1; 0 \leq v_j = w_j < \alpha, \forall N \geq j > t_1 \right\}.$$

We observe that under $\mathcal{O}_{joint}$, if $\psi_n < p$, we have $\psi_n = v_n$, which leads to $(p - \psi_n)^+ = (p - v_n)^+$. On the other hand, if $\psi_n \geq p$, we have $v_n \geq p$, which leads to $(p - \psi_n)^+ = 0 = (p - v_n)^+$. Therefore, we may simply replace ψ_n with v_n in (3.4.8) and reexpress the outage event as

$$\mathcal{O}_{joint} = \left\{ w_n, \psi_n, v_n \mid \sum_{n=1}^N (p - v_n)^+ + \tau \sum_{n=1}^N w_n < \bar{r}; 0 \leq t_1, t_2 \leq N; \right. \\ \min(v_i, \psi_i) \geq p, \forall i = 1, \dots, t_2; 0 \leq v_i = \psi_i < p, \forall N \geq i > t_2; \\ \left. \min(v_j, w_j) \geq \alpha, \forall j = 1, \dots, t_1; 0 \leq v_j = w_j < \alpha, \forall N \geq j > t_1 \right\}.$$

We examine the outage event and consider the following two cases:

Case 1: If $v_n \geq \alpha$, then $w_n \geq \alpha$ and therefore we have $(p - v_n)^+ + \tau w_n \geq \tau \alpha \geq 1$.

Case 2: If $v_n < \alpha$, then $v_n = w_n$ and therefore we have $(p - v_n)^+ + \tau w_n = (p - v_n)^+ + \tau v_n \geq p \geq 1$.

As a result, we have $\sum_{n=1}^N (p - v_n)^+ + \tau \sum_{n=1}^N w_n \geq N$, implying that $\mathcal{O}_{joint} = \emptyset$. It is thus possible to operate at average spatial multiplexing gain $\bar{r} \in [0, N]$ reliably without any outage by simply performing the rate control scheme in

(3.4.3). One possible scenario is that if the channel is too bad, the transmitter may choose not to send anything, i.e., $R(\hat{\mathbf{H}}_{\mathbf{b}}) \rightarrow 0$. In this way, no outage will occur at all.

Rate Control With Minimum Instantaneous Multiplexing Gain

Now we move to the case where the minimum instantaneous multiplexing gain r_{min} is imposed. At high SNRs, the outage probability with the rate control scheme in (3.4.6) is given by

$$\begin{aligned}
 \mathcal{P}_{out,3} &\doteq \mathcal{P} \left(\sum_{n=1}^N (p - \psi_n)^+ < \max \left(\bar{r} + \lim_{\rho \rightarrow \infty} \frac{\tau \log \det(\hat{\mathbf{H}}_{\mathbf{b}} \hat{\mathbf{H}}_{\mathbf{b}}^\dagger)}{\log(\rho)}, r_{min} \right) \right) \\
 &= \mathcal{P} \left(\sum_{n=1}^N (p - \psi_n)^+ < \bar{r} - \tau \sum_{n=1}^N w_n \cap \bar{r} - \tau \sum_{n=1}^N w_n \geq r_{min} \right) \\
 &+ \mathcal{P} \left(\sum_{n=1}^N (p - \psi_n)^+ < r_{min} \cap \bar{r} - \tau \sum_{n=1}^N w_n < r_{min} \right) \\
 &\stackrel{(a)}{=} \mathcal{P} \left(\sum_{n=1}^N (p - v_n)^+ < r_{min} \cap \bar{r} - \tau \sum_{n=1}^N w_n < r_{min} \right) \\
 &\stackrel{(b)}{=} \mathcal{P} \left(\sum_{n=1}^N (p - v_n)^+ < r_{min} \right) \\
 &\doteq \rho^{-d_{joint}^*(\bar{r})}
 \end{aligned} \tag{3.4.10}$$

where

$$\begin{aligned}
 d_{joint}^*(\bar{r}) &= \min_{w_n, \psi_n, v_n \in \mathcal{O}_{joint}^+} \sum_{i=1}^{t_2} (2i - 1 + M - N)(\psi_i - p) + \sum_{j=1}^{t_1} (2j - 1 + M - N)(w_j - \alpha) \\
 &\quad + \sum_{n=1}^N (2n - 1 + M - N)v_n
 \end{aligned} \tag{3.4.11}$$

and

$$\begin{aligned} \mathcal{O}_{joint} = & \left\{ w_n, \psi_n, v_n \mid \sum_{n=1}^N (p - v_n)^+ < r_{min}; 0 \leq t_1, t_2 \leq N; \min(v_i, \psi_i) \geq p, \right. \\ & \forall i = 1, \dots, t_2; 0 \leq v_i = \psi_i < p, \forall N \geq i > t_2; \min(v_j, w_j) \geq \alpha, \forall j = 1, \dots, t_1; \\ & \left. 0 \leq v_j = w_j < \alpha, \forall N \geq j > t_1 \right\}. \end{aligned} \quad (3.4.12)$$

In (3.4.10), (a) is due to the fact that $\mathcal{P} \left(\sum_{n=1}^N (p - \psi_n)^+ + \tau \sum_{n=1}^N w_n < \bar{r} \right) = 0$ and $(p - \psi_n)^+ = (p - v_n)^+$, as proved previously. The proof of (b) can be found in Appendix E.

Next we need to find the optimal solution of v_n, w_n, ψ_n that minimizes the SNR exponent $\mathcal{G} = \sum_{i=1}^{t_2} (2i - 1 + M - N)(\psi_i - p) + \sum_{j=1}^{t_1} (2j - 1 + M - N)(w_j - \alpha) + \sum_{n=1}^N (2n - 1 + M - N)v_n$ under $\mathcal{O}_{joint}$. Noting that $\mathcal{G}$ is an increasing function of $\psi_1, \dots, \psi_{t_2}$, the minimizing solution of $\psi_1, \dots, \psi_{t_2}$ should be

$$\psi_1^* = \dots = \psi_{t_2}^* = p. \quad (3.4.13)$$

Substituting (3.4.13) into $\mathcal{G}$, we get $\mathcal{G} = \sum_{j=1}^{t_1} (2j - 1 + M - N)(w_j - \alpha) + \sum_{n=1}^N (2n - 1 + M - N)v_n$, which is now an increasing function of $w_1, \dots, w_{t_1}$. Combined with the fact that decreasing $w_1, \dots, w_{t_1}$ does not violate the outage condition in $\mathcal{O}_{joint}$, i.e., $\sum_{n=1}^N (p - v_n)^+ < r_{min}$, the minimizing solution of $w_1, \dots, w_{t_1}$ should be

$$w_1^* = \dots = w_{t_1}^* = \alpha. \quad (3.4.14)$$

Substituting (3.4.13) and (3.4.14) into (3.4.11) and (3.4.12), we are able to simplify the optimization problem to

$$d_{joint}^*(\bar{r}) = \min_{v_n \in \tilde{\mathcal{O}}_{joint}^+} \sum_{n=1}^N (2n - 1 + M - N)v_n \quad (3.4.15)$$

and

$$\tilde{\mathcal{O}}_{joint} = \left\{ v_n \mid \sum_{n=1}^N (p - v_n)^+ < r_{min} \right\}. \quad (3.4.16)$$

where $p = 1$ is for the case without power control and $p = 1 + t \sum_{i=1}^N (2i - 1 + M - N) \min(\alpha, v_i)$ is for the case with power control.

It turns out that the original optimization problem (3.4.11) has been transformed into an optimization problem solved by literature. If the transmitter does not perform power control, i.e., $p = 1$, the optimization problem in (3.4.15) is equivalent to the problem solved in [6] for the MIMO channel with no CSIT, perfect CSIR and spatial multiplexing gain $r = r_{min}$. That is, $d_{joint}^*(\bar{r}) = d^*(r_{min})$. On the other hand, if power control is performed at the transmitter, i.e., $p = 1 + t \sum_{i=1}^N (2i - 1 + M - N) \min(\alpha, v_i)$, the optimization problem in (3.4.15) is equivalent to the problem solved Section 3.2 for the MIMO channel with imperfect CSIT of channel quality α , perfect CSIR and spatial multiplexing gain $r = r_{min}$. That is, $d_{joint}^*(\bar{r}) = d_{CSIT}^*(r_{min})$. Therefore, we complete the proof of Theorem 3.4.1. $\square$

Discussions As shown in Theorem 3.4.1, if there is no minimum instantaneous multiplexing gain constraint r_{min} , rate control itself is able to boost the diversity gain unboundedly, since the transmitter may choose sufficiently low data rate to accommodate very bad channels. It is in sharp contrast to the power control scheme which only yields a finite diversity improvement as shown in Theorem 3.2.1. It might imply that rate control is more effective than power control in this aspect.

However, if there exists minimum instantaneous multiplexing gain constraint r_{min} , outage events cannot be avoided since there is finite probability that the channel cannot support r_{min} . Therefore, rate control leads to a finite diversity increase and the diversity gain is limited by $d^*(r_{min})$. It is interesting to find that for these two cases (with and without r_{min}), the actual value of α (> 0) does not have any impact on the DMT, if rate control itself is performed. This is quite different from power control, where the achievable diversity gain improvement

Figure 3.9: DMT in a 3×3 MIMO channel with $r_{min} = 0.5$ and $\alpha = 1/2$. The multiplexing gain r on x-axis refers to $\bar{r}$ in the case of rate control or joint power and rate control.

greatly depends on the value of α .

We use numerical results to illustrate the impact of rate and power control on the DMT of MIMO channels. From Figure 3.9, we can see that rate control based on noisy CSITR leads to a significant diversity gain improvement over the diversity gain with no CSIT and perfect CSIR [6]. Together with power control, the achievable diversity gain (with noisy CSITR) is further increased significantly and is higher than the one with power control only (with noisy CSIT and perfect CSIR) as described in Section 3.2. While the diversity gain without rate control decreases with the spatial multiplexing gain r , rate control keeps the achievable diversity gain from decreasing with $\bar{r}$ for the entire range of average spatial multiplexing gains, i.e., $N \geq \bar{r} \geq r_{min}$.

We also observe that Theorem 3.4.1 shares some similarities with the result for coherent block-fading MIMO ARQ channels. In ARQ channels, the receiver feeds back to the transmitter a one-bit success/failure indicator. In the success case, the transmitter moves on to the next information message whereas in the failure case, the transmitter retransmits the same message [14]. By this means,

the transmission rate is adaptively adjusted according to the decoding status and the minimum instantaneous multiplexing gain is actually $\bar{r}/L$, where L is the maximum number of transmission rounds. If there is no delay constraint in ARQ channels, i.e., $L \rightarrow \infty$, the outage can be completely avoided, since the transmitter can always keep transmitting the same message until the accumulated mutual information exceeds the target data rate. This is coincident with Case 1 of Theorem 3.4.1, where there is no minimum instantaneous multiplexing gain constraint. On the other hand, if the ARQ protocol is allowed to use a given maximum number of rounds, i.e., $L < \infty$, the optimal diversity gain without power control is given by $d_{ARQ}^* = d^*(\bar{r}/L)$ [14]. This coincides with Case 2 in Theorem 3.4.1 with $r_{min} = \bar{r}/L$. In spite of these similarities, we should be aware of the major differences between our considered channels and ARQ channels. For instance, ARQ channels assume error-free backward links, while we consider noisy backward links. From the DMT point of view, our proposed rate control scheme with noisy feedback is able to achieve the same performance as the ARQ protocol with error-free feedback.

Chapter 4

Power Control in Cooperative Relaying Channels With Imperfect CSIT

There has been some work investigating the DMT in cooperative relaying channels. The achievable DMT for half-duplex relaying channels was firstly studied in [25] and [61]. Laneman *et al.* [25] studied fixed and adaptive relaying, where the relay listens to the source during the first half of the transmission block, and may or may not transmit during the second half, as well as incremental relaying, where there is a 1-bit feedback from the destination to both the source and relay and the relay is only used if the destination cannot decode the source signal during the first half of the transmission block. Specifically, the achievable DMT in this case is given by $d(r) = 2(1 - 2r)$ [25], where r is the spatial multiplexing gain defined in [6]. Note that this protocol can at most support a multiplexing gain of $1/2$ since two time slots are used to transmit one symbol. We refer to the fixed and adaptive relaying [25] as *conventional relaying* in this work. To make more efficient use of the resources, nonorthogonal amplify-and-forward (NAF) and dynamic DDF protocols, where the source is allowed to transmit simultaneously with the relay, were considered in [61]. With these protocols, the full multiplexing gain of 1 can be achieved. A slotted-AF scheme was later proposed in [62], which was shown to have a better DMT than the NAF scheme in [61].

Figure 4.1: A dual-hop relay system with multiple destination antennas and direct link.

The work in [63] considered a general multiple-antenna network with multiple sources, multiple destinations, and multiple relays. We notice that most of the above relaying schemes have assumed no CSIT.

In this chapter, we use the DMT tool to analyze the diversity gain in half-duplex cooperative relaying channels with CSIT estimated at the source and relay. We firstly consider conventional AF/DF relaying and then move to more efficient DDF relaying protocol. We propose temporal power control strategies that exploit imperfect CSIT. We show that under a long-term power constraint, temporal power control based on the imperfect CSIT significantly improves the achievable diversity gain. We also observe that imperfect CSIT at the relay itself does not improve the DMT if the source has no CSIT at all.

4.1 Conventional AF/DF Relaying

4.1.1 System Model

Consider a dual-hop relay system with a single-antenna source node, a single-antenna relay node and an N -antenna ($N \geq 1$) destination node, as shown in Figure 4.1. The relay node works in half-duplex mode. We consider quasi-static Rayleigh fading channels, where the mutually independent channel gains are con-

stant within one transmission block of L symbols, but change independently from one block to another. Furthermore, we assume that the transmission codeword spans within a single fading block. Throughout this section, we consider the following orthogonal relaying protocol.

Without loss of generality, we only take one transmission block into account. During the first half of the block, the source sends data while the relay and destination listen. The received signal at time j ($1 \leq j \leq L/2$) at the relay and the n -th ($n = 1, 2, \dots, N$) destination antenna can be written, respectively, as

$$y_r[j] = \sqrt{P_s} h_1 x_s[j] + z_r \quad (4.1.1)$$

$$y_{n,d}[j] = \sqrt{P_s} h_{2,n} x_s[j] + z_{n,d} \quad (4.1.2)$$

where $x_s[j]$ and P_s denote the source transmitted signal with unity mean power and the instantaneous source transmit power, respectively; $z_r \sim \mathcal{CN}(0, 1)$ and $z_{n,d} \sim \mathcal{CN}(0, 1)$ denote the additive noise at the relay node and the n -th destination antenna, respectively; $h_1 \sim \mathcal{CN}(0, 1)$ and $h_{2,n} \sim \mathcal{CN}(0, 1)$ denote the instantaneous channel gains of the S-R link and the source-to-the n -th destination antenna (S-D(n)) link, respectively. Let ρ_1 denote the transmit power at the source averaged over a large number of fading blocks (with respect to half of the block due to the considered orthogonal transmission), with a notation of $E\{P_s\} = \rho_1$.

During the second half of the block, the source remains silent and the relay transmits to the destination. The received signal at the n -th destination antenna can be written as

$$y_{n,d}[j] = \sqrt{P_r} h_{3,n} x_r[j] + z_{n,d}, \text{ for } L/2 < j \leq L \quad (4.1.3)$$

where $x_r[j]$ denotes the relay transmitted signal with unity mean power; P_r denotes the instantaneous relay transmit power; $h_{3,n} \sim \mathcal{CN}(0, 1)$ denotes the instantaneous channel gain of the relay-to-the n -th destination antenna (R-D(n)) link. Similarly, we let ρ_2 denote the transmit power at the relay averaged over a large number of fading blocks, i.e., $E\{P_r\} = \rho_2$. In view of the DMT analysis,

we assume $\rho_1 = \rho_2 = \rho$ in the rest of this section. Finally, the destination will decode its received signals by appropriately combining the signals from the two half blocks using maximum-ratio combining based on P_s , P_r , and the channel gains. The SNR at each destination antenna averaged over a large number of fading blocks can then be written as ρ .

We consider TDD transmission and therefore the transmitters can estimate the CSIT of forward channels using the pilots received in backward channels, due to the channel reciprocity. We assume that the receivers have perfect knowledge of h_1 and $h_{i,n}$, $i = 2, 3$. On the other hand, the transmitters only have estimated CSIT, $\hat{h}_1$ and $\hat{h}_{i,n}$, which are imperfect and can be modeled as noise-corrupted versions of the perfect CSIT. For instance, with MMSE estimation [54], $\hat{h}_1$ and $\hat{h}_{i,n}$ can be modeled as

$$h_1 = \hat{h}_1 + e_1 \quad (4.1.4a)$$

$$h_{i,n} = \hat{h}_{i,n} + e_{i,n}, \quad i = 2, 3, n = 1, \dots, N \quad (4.1.4b)$$

where the mutually independent estimation errors $e_1 \sim \mathcal{CN}(0, \sigma_{1,e}^2)$ and $e_{i,n} \sim \mathcal{CN}(0, \sigma_{i,e}^2)$ are independent of $\hat{h}_1$ and $\hat{h}_{i,n}$, respectively. According to (4.1.4), the quality of CSIT is thus characterized by $\sigma_{i,e}^2 = \rho^{-\alpha_i}$, $i = 1, 2, 3$. We assume that the channels between the source, relay and destination nodes may have different CSIT quality, i.e., $\alpha_1 \neq \alpha_2 \neq \alpha_3$.

For the sake of notational simplicity, we define $\gamma_1 \triangleq |h_1|^2$, $\hat{\gamma}_1 \triangleq |\hat{h}_1|^2$, $\epsilon_1 \triangleq |e_1|^2$, $\gamma_{i,n} \triangleq |h_{i,n}|^2$, $\hat{\gamma}_{i,n} \triangleq |\hat{h}_{i,n}|^2$, $\epsilon_{i,n} \triangleq |e_{i,n}|^2$, $\hat{\gamma}_i \triangleq \prod_{n=1}^N \hat{\gamma}_{i,n}$, and $\gamma_i \triangleq \sum_{n=1}^N \gamma_{i,n}$, $i = 2, 3$. It follows immediately that γ_1 , $\hat{\gamma}_1$, ϵ_1 , $\gamma_{i,n}$, $\hat{\gamma}_{i,n}$ and $\epsilon_{i,n}$ are exponentially distributed with rate parameters 1, $1/(1 - \sigma_{1,e}^2)$, $1/\sigma_{1,e}^2$, 1, $1/(1 - \sigma_{i,e}^2)$ and $1/\sigma_{i,e}^2$, respectively. Moreover, it can be shown that

$$(\sqrt{\gamma_1} - \sqrt{\epsilon_1})^2 \leq \hat{\gamma}_1 \leq (\sqrt{\gamma_1} + \sqrt{\epsilon_1})^2 \quad (4.1.5a)$$

$$(\sqrt{\gamma_{i,n}} - \sqrt{\epsilon_{i,n}})^2 \leq \hat{\gamma}_{i,n} \leq (\sqrt{\gamma_{i,n}} + \sqrt{\epsilon_{i,n}})^2. \quad (4.1.5b)$$

4.1.2 Diversity-Multiplexing Tradeoff Analysis

If the transmitters have perfect CSIT, they may adopt the optimal power control according to the instantaneous channel gains such that the outage might never happen. With only the imperfect CSIT, we extend our power control scheme in Chapter 3 and propose the following power control strategy

$$P_s = \frac{\kappa_1 \rho}{(\hat{\gamma}_1 \hat{\gamma}_2)^t} \quad (4.1.6a)$$

$$P_r = \frac{\kappa_2 \rho}{(\hat{\gamma}_3)^t} \quad (4.1.6b)$$

where $\kappa_1 = \frac{(1-\sigma_{1,e}^2)^t (1-\sigma_{2,e}^2)^{Nt}}{(\Gamma(1-t))^{1+N}}$ and $\kappa_2 = \frac{(1-\sigma_{3,e}^2)^{Nt}}{(\Gamma(1-t))^N}$ are normalizing factors; t ($0 \leq t < 1$) can be chosen to be arbitrarily close to 1.

It can be shown that the above power control scheme satisfies the long term power constraints $E\{P_s\} = \rho$ and $E\{P_r\} = \rho$. The proof is rather straightforward and hence, is omitted here for brevity. We believe that given the CSIT quality α_i , $i = 1, 2, 3$, the above power control scheme is one of the optimal schemes that maximizes the achievable diversity gain.

DF Relaying

We first consider adaptive DF relaying that avoids error propagation [25]. If the relay is able to decode the source signal, it processes $y_r[j]$ by decoding an estimate $\hat{x}_s[j]$ of the source transmitted signal and transmits $x_r[j] = \hat{x}_s[j - L/2]$, $j = L/2 + 1, \dots, L$, under a repetition-coded scheme at the relay. Otherwise, it keeps silent.

For adaptive DF relaying, the maximum system mutual information with repetition coding at the relay is given by

$$I_{DF} = \begin{cases} \frac{1}{2} \log(1 + P_s \gamma_2), & \text{if } P_s \gamma_1 < R \\ \frac{1}{2} \log(1 + P_s \gamma_2 + P_r \gamma_3), & \text{otherwise} \end{cases} \quad (4.1.7)$$

where $R \triangleq 2^{2R(\rho)} - 1$. Because of the adaptive relaying, we actually have $E\{P_r\} \leq$

ρ rather than $E\{P_r\} = \rho$. Since DMT is obtained at asymptotically high SNRs, this will not make any difference from the DMT point of view.

An outage occurs if the transmission cannot support the target data rate. From (4.1.7), the outage probability of the relaying channel is given by

$$\mathcal{P}_{out}^{DF} = \mathcal{P}(P_s\gamma_1 \geq R, P_s\gamma_2 + P_r\gamma_3 < R) + \mathcal{P}(P_s\gamma_1 < R, P_s\gamma_2 < R). \quad (4.1.8)$$

With (4.1.5) and (4.1.6), the lower bound and upper bound of the outage probability can be written, respectively, as

$$\mathcal{P}_l = \mathcal{P}\left(\frac{\kappa_1\rho\gamma_1}{A_1^t} \geq R, \frac{\kappa_1\rho\gamma_2}{A_2^t} + \frac{\kappa_2\rho\gamma_3}{B_2^t} < R\right) + \mathcal{P}\left(\frac{\kappa_1\rho\gamma_1}{A_2^t} < R, \frac{\kappa_1\rho\gamma_2}{A_2^t} < R\right) \quad (4.1.9)$$

$$\mathcal{P}_u = \mathcal{P}\left(\frac{\kappa_1\rho\gamma_1}{A_2^t} \geq R, \frac{\kappa_1\rho\gamma_2}{A_1^t} + \frac{\kappa_2\rho\gamma_3}{B_1^t} < R\right) + \mathcal{P}\left(\frac{\kappa_1\rho\gamma_1}{A_1^t} < R, \frac{\kappa_1\rho\gamma_2}{A_1^t} < R\right) \quad (4.1.10)$$

where $A_1 = (\sqrt{\gamma_1} + \sqrt{\epsilon_1})^2 \prod_{n=1}^N (\sqrt{\gamma_{2,n}} + \sqrt{\epsilon_{2,n}})^2$, $B_1 = \prod_{n=1}^N (\sqrt{\gamma_{3,n}} + \sqrt{\epsilon_{3,n}})^2$, $A_2 = (\sqrt{\gamma_1} - \sqrt{\epsilon_1})^2 \prod_{n=1}^N (\sqrt{\gamma_{2,n}} - \sqrt{\epsilon_{2,n}})^2$ and $B_2 = \prod_{n=1}^N (\sqrt{\gamma_{3,n}} - \sqrt{\epsilon_{3,n}})^2$.

We let $\gamma_1 \doteq \rho^{-v_1}$, $\gamma_{i,n} \doteq \rho^{-v_{i,n}}$, $\epsilon_1 \doteq \rho^{-u_1}$ and $\epsilon_{i,n} \doteq \rho^{-u_{i,n}}$. At high SNRs, the pdfs of v_1 , u_1 , $v_{i,n}$ and $u_{i,n}$ are given by [61]

$$p_{v_1}(v_1) \doteq \begin{cases} 0, & \text{for } v_1 < 0 \\ \rho^{-v_1}, & \text{for } v_1 \geq 0 \end{cases} \quad (4.1.11)$$

$$p_{u_1}(u_1) \doteq \begin{cases} 0, & \text{for } u_1 < \alpha_1 \\ \rho^{-(u_1 - \alpha_1)}, & \text{for } u_1 \geq \alpha_1 \end{cases} \quad (4.1.12)$$

$$p_{v_{i,n}}(v_{i,n}) \doteq \begin{cases} 0, & \text{for } v_{i,n} < 0 \\ \rho^{-v_{i,n}}, & \text{for } v_{i,n} \geq 0 \end{cases} \quad (4.1.13)$$

$$p_{u_{i,n}}(u_{i,n}) \doteq \begin{cases} 0, & \text{for } u_{i,n} < \alpha_i \\ \rho^{-(u_{i,n} - \alpha_i)}, & \text{for } u_{i,n} \geq \alpha_i. \end{cases} \quad (4.1.14)$$

The conditional pdf of $\frac{2\gamma_1}{(1-\sigma_{1,e}^2)}$ given ϵ_1 follows the noncentral chi-square distri-

bution with two degrees of freedom and non-centrality parameter $\lambda = \frac{2\epsilon_1}{1-\sigma_1^2}$ [64]:

$$p\left(\frac{2\gamma_1}{(1-\sigma_{1,e}^2)} \mid \epsilon_1\right) = \frac{1}{2} \exp\left(-\frac{\gamma_1 + \epsilon_1}{1-\sigma_{1,e}^2}\right) I_0\left(\sqrt{\frac{4\gamma_1\epsilon_1}{(1-\sigma_{1,e}^2)^2}}\right). \quad (4.1.15)$$

By changing variables, we get

$$\begin{aligned} p(v_1 \mid u_1) &= \frac{\rho^{-v_1} \ln \rho}{1-\sigma_{1,e}^2} \exp\left(-\frac{\rho^{-v_1} + \rho^{-u_1}}{1-\sigma_{1,e}^2}\right) I_0\left(\sqrt{\frac{4\rho^{-v_1-u_1}}{(1-\sigma_{1,e}^2)^2}}\right) \\ &\doteq \begin{cases} 0, & \text{for } v_1 < 0 \\ \rho^{-v_1}, & \text{for } v_1 \geq 0 \end{cases} \end{aligned} \quad (4.1.16)$$

which implies that v_1 is independent of u_1 . Following the same line, we can show that $v_{i,n}$ and $u_{i,n}$ are also independent. Thus, by letting $\mathbf{v} = (v_1, v_{2,1}, \dots, v_{3,N})$ and $\mathbf{u} = (u_1, u_{2,1}, \dots, u_{3,N})$, the joint pdf of $\mathbf{v}$ and $\mathbf{u}$, $p(\mathbf{v}, \mathbf{u})$, can be easily obtained using (4.1.11)-(4.1.14).

We also let $v_{i,\min} = \min\{v_{i,1}, \dots, v_{i,N}\}$, $i = 2, 3$. When ρ grows to infinity, it is shown in Appendix F that the two bounds in (4.1.9) and (4.1.10) converge. That is,

$$\begin{aligned} \lim_{\rho \rightarrow \infty} \mathcal{P}_l &= \lim_{\rho \rightarrow \infty} \mathcal{P}_u \\ &= \lim_{\rho \rightarrow \infty} \mathcal{P} \left(\frac{\kappa_1 \rho^{1-v_1}}{C_1^t} \geq \rho^{2r}, \frac{\kappa_1 \rho \sum_{n=1}^N \rho^{-v_{2,n}}}{C_1^t} + \frac{\kappa_2 \rho \sum_{n=1}^N \rho^{-v_{3,n}}}{C_2^t} < \rho^{2r} \right) \\ &\quad + \mathcal{P} \left(\frac{\kappa_1 \rho^{1-v_1}}{C_1^t} < \rho^{2r}, \frac{\kappa_1 \rho \sum_{n=1}^N \rho^{-v_{2,n}}}{C_1^t} < \rho^{2r} \right) \\ &= \int_{\mathcal{O}_{DF,1}} p(\mathbf{v}, \mathbf{u}) d\mathbf{v} d\mathbf{u} + \int_{\mathcal{O}_{DF,2}} p(\mathbf{v}, \mathbf{u}) d\mathbf{v} d\mathbf{u} \end{aligned} \quad (4.1.17)$$

where $C_1 = \rho^{-\min\{v_1, u_1\}} \prod_{n=1}^N \rho^{-\min\{v_{2,n}, u_{2,n}\}}$ and $C_2 = \prod_{n=1}^N \rho^{-\min\{v_{3,n}, u_{3,n}\}}$. The two exclusive sets $\mathcal{O}_{DF,1}$ and $\mathcal{O}_{DF,2}$ are defined, respectively, as

$$\mathcal{O}_{DF,1} \triangleq \{(\mathbf{v}, \mathbf{u}) \mid \text{P1, P2, P3, P4}\} \quad (4.1.18)$$

$$\mathcal{O}_{DF,2} \triangleq \{(\mathbf{v}, \mathbf{u}) \mid \overline{\text{P1}}, \text{P2, P4}\} \quad (4.1.19)$$

where

$$\begin{aligned}
 \text{P1} &\Leftrightarrow (1 - v_1 + t \min\{v_1, u_1\} + t \sum_{n=1}^N \min\{v_{2,n}, u_{2,n}\})^+ \geq 2r \\
 \text{P2} &\Leftrightarrow (1 - v_{2,\min} + t \min\{v_1, u_1\} + t \sum_{n=1}^N \min\{v_{2,n}, u_{2,n}\})^+ < 2r \\
 \text{P3} &\Leftrightarrow (1 - v_{3,\min} + t \sum_{n=1}^N \min\{v_{3,n}, u_{3,n}\})^+ < 2r \\
 \text{P4} &\Leftrightarrow v_1 \geq 0, v_{i,n} \geq 0, u_1 \geq \alpha_1, u_{i,n} \geq \alpha_i
 \end{aligned}$$

and $\overline{\text{P1}}$ denotes the complementary event of P1. For notational brevity, we denote $\mathcal{P}^{\mathcal{O}_{DF,m}} = \int_{\mathcal{O}_{DF,m}} p(\mathbf{v}, \mathbf{u}) d\mathbf{v} d\mathbf{u}$, $m = 1, 2$.

With Laplace principle [6], we have

$$\begin{aligned}
 \mathcal{P}^{\mathcal{O}_{DF,m}} &\doteq \rho^{-d_{o,m}(r)}, \text{ for } d_{o,m}(r) = \inf_{(\mathbf{v}, \mathbf{u}) \in \mathcal{O}_{DF,m}} v_1 + u_1 - \alpha_1 \\
 &\quad + \sum_{i=2}^3 \sum_{n=1}^N (v_{i,n} + u_{i,n} - \alpha_i), \quad m = 1, 2.
 \end{aligned} \tag{4.1.20}$$

Therefore, the overall outage probability is given by

$$\mathcal{P}_{out}^{DF} \doteq \mathcal{P}^{\mathcal{O}_{DF,1}} + \mathcal{P}^{\mathcal{O}_{DF,2}} \doteq \rho^{-d_{DF}^*(r)} \tag{4.1.21}$$

where $d_{DF}^*(r) = \min\{d_{o,1}(r), d_{o,2}(r)\}$ is the achievable diversity gain at multiplexing gain r . Since t can be made arbitrarily close to 1, it is without loss of accuracy to set $t = 1$ in the rest of this section.

First we find $d_{o,1}(r)$. We temporarily relax outage condition P1 in (4.1.18) and get a bigger set $\tilde{\mathcal{O}}_{DF,1} \supseteq \mathcal{O}_{DF,1}$, i.e.,

$$\tilde{\mathcal{O}}_{DF,1} = \{(\mathbf{v}, \mathbf{u}) \mid \text{P2, P3, P4}\}. \tag{4.1.22}$$

Since decreasing the values of v_1, u_1 and $u_{i,n}$ does not violate the outage conditions in $\tilde{\mathcal{O}}_{DF,1}$, the optimal solution of these variables that minimizes the SNR exponent $v_1 + u_1 - \alpha_1 + \sum_{i=2}^3 \sum_{n=1}^N (v_{i,n} + u_{i,n} - \alpha_i)$ should be $v_1^* = 0$, $u_1^* = \alpha_1$,

$u_{i,n}^* = \alpha_i$. It is obvious that v_1^* , u_1^* , and $u_{i,n}^*$ satisfy P1, which implies that the optimal solution in set $\tilde{\mathcal{O}}_{DF,1}$ is also optimal in set $\mathcal{O}_{DF,1}$. Moreover, examination of outage condition P2 yields

$$v_{2,\min} > 1 - 2r + N \min\{v_{2,\min}, \alpha_2\}. \quad (4.1.23)$$

One has to choose $v_{2,\min} \geq \alpha_2$. Otherwise there will be a contradiction that $v_{2,\min} > 1 - 2r + Nv_{2,\min}$. Hence, the optimal solution that minimizes the SNR exponent should be $v_{2,1}^* = \dots = v_{2,N}^* = 1 - 2r + N\alpha_2$. Similarly, we can find the optimal solution of $v_{3,n}$ as $v_{3,1}^* = \dots = v_{3,N}^* = 1 - 2r + N\alpha_3$. As a result, we have

$$d_{o,1}(r) = N(2 - 4r + N\alpha_2 + N\alpha_3). \quad (4.1.24)$$

In a similar way, we can get

$$d_{o,2}(r) = (N + 1)(1 - 2r + \alpha_1 + N\alpha_2) \quad (4.1.25)$$

with the optimal solution being $v_1^* = v_{2,1}^* = \dots = v_{2,N}^* = 1 - 2r + \alpha_1 + N\alpha_2$, $u_1^* = \alpha_1$, $u_{2,1}^* = \dots = u_{2,N}^* = \alpha_2$, $v_{3,1}^* = \dots = v_{3,N}^* = 0$, and $u_{3,1}^* = \dots = u_{3,N}^* = \alpha_3$.

From (4.1.24) and (4.1.25), we reach the following theorem.

Theorem 4.1.1 Given the CSIT quality α_1 of the S-R link, α_2 of the S-D(n) link, and α_3 of the R-D(n) link, the DMT achieved by the adaptive DF scheme with N destination antennas is given by

$$d_{DF}^*(r) = \min\{(N+1)(1-2r+\alpha_1+N\alpha_2), N(2-4r+N\alpha_2+N\alpha_3)\}, \quad 0 \leq r \leq 1/2. \quad (4.1.26)$$

For fixed DF relaying, the maximum mutual information with repetition coding at the relay can be shown to be [25]

$$I_{FDF} = \frac{1}{2} \min\{\log(1 + P_s\gamma_1), \log(1 + P_s\gamma_2 + P_r\gamma_3)\}. \quad (4.1.27)$$

The outage probability in this case is given by

$$\mathcal{P}_{out}^{FDF} = \mathcal{P}(\min\{P_s\gamma_1, P_s\gamma_2 + P_r\gamma_3\} < R). \quad (4.1.28)$$

Similarly to the adaptive DF case, we obtain the following.

Theorem 4.1.2 The fixed DF scheme can achieve a DMT of

$$d_{FDF}^*(r) = \min\{1 - 2r + \alpha_1 + N\alpha_2, N(2 - 4r + N\alpha_2 + N\alpha_3)\}. \quad (4.1.29)$$

It is clear that fixed DF relaying limits the achievable diversity gain as the relay always transmits even if it decodes the source signal erroneously. It is not unexpected, however, that if the CSIT quality of the S-R channel is sufficiently high, the fixed DF scheme will be equivalent to the adaptive DF schemes from the DMT perspective.

AF Relaying

For AF relaying, the relay processes $y_r[j]$, and relays the information by transmitting $x_r(j) = G \cdot y_r[j - L/2]$ for $j = L/2 + 1, \dots, L$, where G denotes the relay's amplification gain and is chosen as $G = \sqrt{1/(P_s|h_1|^2 + 1)}$ to satisfy the power constraint at the relay. Note that for AF relaying, G usually needs to be known at the destination.

With AF relaying, the maximum mutual information between the source and destination can be shown to be [25]

$$I_{AF} = \frac{1}{2} \log \left(1 + P_s\gamma_2 + \sum_{n=1}^N \frac{P_s\gamma_1 P_r\gamma_{3,n}}{P_s\gamma_1 + P_r\gamma_{3,n} + 1} \right). \quad (4.1.30)$$

The outage probability of the AF relaying is given by

$$\mathcal{P}_{out}^{AF} = \mathcal{P} \left(P_s\gamma_2 + \sum_{n=1}^N \frac{P_s\gamma_1 P_r\gamma_{3,n}}{P_s\gamma_1 + P_r\gamma_{3,n} + 1} < R \right). \quad (4.1.31)$$

Substituting (4.1.6) into (4.1.31) and following similar steps as in the adaptive DF scheme, we get the lower and upper bounds of $\mathcal{P}_{out}^{AF}$, which can be shown to both converge to

$$\mathcal{P}_{out}^{AF} \doteq \mathcal{P} \left(\left(2 - v_1 - v_{3,\min} - \max \left\{ 1 - v_1 - t \sum_{n=1}^N \min \{ v_{3,n}, u_{3,n} \}, 1 - v_{3,\min} - t \min \{ v_1, u_1 \} - t \sum_{n=1}^N \min \{ v_{2,n}, u_{2,n} \} \right\} \right)^+ < 2r, (1 - v_{2,\min} + t \min \{ v_1, u_1 \} + t \sum_{n=1}^N \min \{ v_{2,n}, u_{2,n} \})^+ < 2r \right). \quad (4.1.32)$$

Close examination on (4.1.32) results in

$$\mathcal{P}_{out}^{AF} \doteq \mathcal{P}^{\tilde{\mathcal{O}}_{DF,1}} + \mathcal{P}^{\mathcal{O}_{DF,2}} \quad (4.1.33)$$

Therefore, we have the following theorem.

Theorem 4.1.3 The AF scheme achieves the same DMT as the adaptive DF scheme, which is given in (4.1.26).

DMT Upper Bound

We present an upper bound on the DMT of an orthogonal relaying channel. To obtain an upper bound, we assume that the source and relay have made full cooperation in the first half of the block such that the relay knows the source message perfectly. The maximum mutual information is then given by

$$I_{UP} = \frac{1}{2} \log(1 + P_s \gamma_2 + P_r \gamma_3). \quad (4.1.34)$$

The outage probability can therefore be written as

$$\mathcal{P}_{out}^{UP} = \mathcal{P}(P_s \gamma_2 + P_r \gamma_3 < R) \doteq \mathcal{P}^{\tilde{\mathcal{O}}_{DF,1}} \quad (4.1.35)$$

which immediately leads to the following theorem.

Theorem 4.1.4 The DMT of an orthogonal relaying channel is upper bounded

by

$$d_{UP}^*(r) = N(2 - 4r + N\alpha_2 + N\alpha_3), \quad 0 \leq r \leq 1/2. \quad (4.1.36)$$

It is clear from the above theorems that fixed/adaptive DF and AF schemes can achieve the upper bound in (4.1.36), if α_1 is sufficiently high. The required α_1 value for adaptive DF and AF schemes can be easily calculated from (4.1.26); while for fixed DF scheme, we require a much higher α_1 , which can be calculated from (4.1.29).

Next we relax the transmission orthogonality requirement and allow the source to transmit simultaneously with the relay. We consider a genie-aided strategy, where the relay is assumed to know the source message *a priori*. The relaying channel will therefore be equivalent to a $2 \times N$ channel except that the source and relay perform power control based on their individual CSIT, $\hat{\gamma}_2$ and $\hat{\gamma}_3$, respectively. A detailed analysis of such channel is beyond the scope of this section. However, for the special case of $N = 1$, the genie-aided DMT can be readily shown to be $d_{GA}^*(r) = 2(1 - r) + \alpha_2 + \alpha_3$, $0 \leq r \leq 1$.

So far, we have considered power control in forward channels. It can be shown that if there is also power control in backward channels subject to the average SNR of the backward channels, ρ^{α_i} , a higher diversity gain can be achieved.

4.1.3 Examples and Discussions

In this section, we give examples to show the impact of imperfect CSIT on the DMT. We assume that there is forward channel power control but no backward channel power control. Due to the inefficiency of fixed DF relaying, we only discuss the DMT of AF and adaptive DF schemes. We consider the following cases, which are also shown in Figure 4.2.

Case 1: $\alpha_i = 0$, $i = 1, 2, 3$. This corresponds to the case when there is no CSIT at all. In this case, we have $d_{DF}^*(r) = (N + 1)(1 - 2r)$, so the full diversity gain $N + 1$ is achieved at the spatial multiplexing gain $r = 0$ and zero diversity gain is achieved at the full spatial multiplexing gain $r = 1/2$.

Figure 4.2: DMT in relaying channels with direct link.

Case 2: $\alpha_1 = \alpha_2 = 0, \alpha_3 \neq 0$. This corresponds to the case when the source is not willing to do channel estimation or power control. We also have $d_{DF}^*(r) = (N+1)(1-2r)$, which implies that there is no additional diversity gain if no CSIT exists at the source, even when the relay has perfect CSIT.

Case 3: $\alpha_1 \neq 0, \alpha_2 = \alpha_3 = 0$. This is the case when the destination does not send pilots through backward channels to the source and relay nodes. Hence, there is no power control at the relay and the power control at the source is solely based on $\hat{\gamma}_1$. As a result, we have $d_{DF}^*(r) = \min\{(N+1)(1-2r+\alpha_1), 2N(1-2r)\}$. In particular, $d(1/2) = 0$ for any α_1 .

Case 4: $\alpha_1 = \alpha_3 = 0, \alpha_2 \neq 0$. In this case, there is no power control at the relay and the power control at the source is solely based on $\hat{\gamma}_2$. The optimal DMT is given by $d_{DF}^*(r) = \min\{(N+1)(1-2r+N\alpha_2), 2N(1-2r)+N^2\alpha_2\}$. In particular, $d(1/2) = N^2\alpha_2$.

Case 5: $\alpha_3 = 0, \alpha_1 \neq 0, \alpha_2 \neq 0$. This is the case when the relay does not do forward channel estimation or power control, and the optimal DMT is given by $d_{DF}^*(r) = \min\{(N+1)(1-2r+N\alpha_2+\alpha_1), 2N(1-2r)+N^2\alpha_2\}$. In particular,

Figure 4.3: DMT in relaying channels without direct link.

for any α_1 , we have $d(1/2) = N^2\alpha_2$.

Case 6: $\alpha_1 = 0, \alpha_2 \neq 0, \alpha_3 \neq 0$. This corresponds to the case when the source does not know the S-R link. So $\hat{\gamma}_1$ should be removed from (4.1.6a) for the power control at the source. We will then have $d_{DF}^*(r) = \min\{(N+1)(1-2r+N\alpha_2), N(2-4r+N\alpha_2+N\alpha_3)\}$.

Case 7: $\alpha_1 \neq 0, \alpha_2 \neq 0, \alpha_3 \neq 0$. This is the general case when both the source and relay have imperfect CSIT, and we have the general DMT as given in (4.1.26).

We end this section by considering a special scenario where there is no direct link between the source and destination. With forward channel power control similar to that in (4.1.6), the achievable DMT of AF relaying and adaptive/fixed DF relaying can be shown to be $d_{DF}^*(r) = \min\{1-2r+\alpha_1, N(1-2r+N\alpha_3)\}$. The DMT in this case is illustrated in Figure 4.3. Similarly to the case with direct link, imperfect CSIT provides an additional diversity gain only if the source has CSIT quality $\alpha_1 \neq 0$.

4.2 Dynamic Decode-and-Forward Relaying

In the previous section, we showed that with temporal power control, imperfect CSIT significantly improves the diversity gain in conventional AF/DF relaying channels, where the imperfect CSIT comes from imperfect channel estimation at the transmitters. In this section, we extend to the more efficient DDF relaying protocol.

4.2.1 System Model

We consider the system model used in the previous section, except that the destination node has only one antenna, i.e., $N = 1$. We consider the DDF protocol, where the source transmits data during each symbol interval in the codeword at a rate of $R(\rho)$ bits per channel use. The relay listens to the source until the accumulated mutual information between the received signal and the source signal reaches $LR(\rho)$. It then decodes and re-encodes the message using an independent Gaussian codebook and transmits it during the rest of the codeword. Letting L' denote the number of symbol intervals the relay waits before starting transmission, the received signal at the relay and destination nodes can be written as [61]

$$y_r[j] = \sqrt{P_s}h_1x_s[j] + z_r[j], \text{ for } j = 1, \dots, L' \quad (4.2.1)$$

$$y_d[j] = \sqrt{P_s}h_2x_s[j] + z_d[j], \text{ for } j = 1, \dots, L' \quad (4.2.2)$$

$$y_d[j] = \sqrt{P_s}h_2x_s[j] + \sqrt{P_r}h_3x_r[j] + z_d[j], \text{ for } j = L' + 1, \dots, L \quad (4.2.3)$$

where $z_d[j] \sim \mathcal{CN}(0, 1)$ denote the complex Gaussian noise received at the destination node; $h_2 \sim \mathcal{CN}(0, 1)$ and $h_3 \sim \mathcal{CN}(0, 1)$ denote the instantaneous channel gains of the S-D link and R-D link, respectively. We also consider the following temporal power constraint

$$E\{P_s\} = E\{P_r\} = \rho \quad (4.2.4)$$

where ρ also denotes the average SNR of the individual S-R link, S-D link and R-D link.

As in the previous section, the relay knows h_1 and the destination knows h_2 and h_3 , both perfectly. On the other hand, the transmitters only have imperfect CSIT of $\hat{h}_i$, $i = 1, 2, 3$, which are estimated from training pilots using MMSE estimation. Therefore, $\hat{h}_i$ can be modeled as

$$h_i = \hat{h}_i + e_i \quad (4.2.5)$$

where the channel estimation errors $e_i \sim \mathcal{CN}(0, \sigma_{i,e}^2)$ are i.i.d. random variables independent of $\hat{h}_i$. For notation convenience, we define $\gamma_i \triangleq |h_i|^2$, $\hat{\gamma}_i \triangleq |\hat{h}_i|^2$, $\epsilon_i \triangleq |e_i|^2$, $i = 1, 2, 3$. The quality of CSIT is characterized by $\sigma_{i,e}^2 \doteq \rho^{-\alpha_i}$, $i = 1, 2, 3$. Note that these definitions are consistent with the previous section if we set $N = 1$.

4.2.2 Diversity-Multiplexing Tradeoff Analysis

The mutual information of the considered relaying channel is given by [61]

$$I = f \log(1 + P_s \gamma_2) + (1 - f) \log(1 + P_s \gamma_2 + P_r \gamma_3) \quad (4.2.6)$$

where $f \triangleq L'/L \leq 1$. The outage probability is then given by

$$\mathcal{P}_{out}^{DDF} = \mathcal{P}(I < R(\rho)) \quad (4.2.7)$$

With only the imperfect CSIT, we present the following power control scheme

$$P_s = \frac{\kappa_3 \rho}{(\hat{\gamma}_2 \hat{\gamma}_1)^t} \quad (4.2.8a)$$

$$P_r = \frac{\kappa_4 \rho}{(\hat{\gamma}_3)^t} \quad (4.2.8b)$$

where $\kappa_3 = \frac{(1-\sigma_{1,e}^2)^t (1-\sigma_{2,e}^2)^t}{\Gamma^2(1-t)}$, $\kappa_4 = \frac{(1-\sigma_{3,e}^2)^t}{\Gamma(1-t)}$, and t ($0 \leq t < 1$) can be chosen to

be arbitrarily close to 1. It can be readily shown that the above power control scheme satisfies the temporal transmit power constraint $E\{P_s\} = E\{P_r\} = \rho$.

Theorem 4.2.1 For the considered DDF relaying channel, given the CSIT quality of α_1 of the S-R link, α_2 of the S-D link, and α_3 of the R-D link, the optimal diversity gain with the power control scheme in (4.1.6) is characterized by

$$d_{DDF}^*(r) = \min \left(\frac{1 + \alpha_2 - r}{r}, 2 - 2r + \alpha_2 + \alpha_3, 2 - 2r + 2\alpha_2 + 2\alpha_1 \right). \quad (4.2.9)$$

Proof. Substituting (4.2.6) and (4.2.8) into (4.2.7), the outage probability can be rewritten as

$$\mathcal{P}_{out}^{DDF} = \mathcal{P} \left(f \log \left(1 + \frac{\kappa_3 \rho}{(\hat{\gamma}_1 \hat{\gamma}_2)^t} \gamma_2 \right) + (1-f) \log \left(1 + \frac{\kappa_3 \rho}{(\hat{\gamma}_1 \hat{\gamma}_2)^t} \gamma_2 + \frac{\kappa_4 \rho}{(\hat{\gamma}_3)^t} \gamma_3 \right) < R(\rho) \right). \quad (4.2.10)$$

Letting v_i and $\hat{v}_i$ denote the exponential orders of $1/\gamma_i$ and $1/\hat{\gamma}_i$, respectively, i.e., $v_i = -\lim_{\rho \rightarrow \infty} \log(\gamma_i)/\log(\rho)$, $\hat{v}_i = -\lim_{\rho \rightarrow \infty} \log(\hat{\gamma}_i)/\log(\rho)$, the outage probability at high SNRs can be expressed as

$$P_{out}^{DDF} \doteq \mathcal{P} \left(f(1 + t\hat{v}_1 + t\hat{v}_2 - v_2)^+ + (1-f)(\max(1 + t\hat{v}_1 + t\hat{v}_2 - v_2, 1 + t\hat{v}_3 - v_3))^+ < r \right) \quad (4.2.11)$$

where $r \triangleq \lim_{\rho \rightarrow \infty} R(\rho)/\log(\rho)$. Since $R(\rho) = f \log(1 + P_s \gamma_2)$, it follows from (4.2.8a) that $r = f(1 + t\hat{v}_1 + t\hat{v}_2 - v_1)^+$.

To calculate the outage probability, we need to find the joint pdf of v_i and $\hat{v}_i$, $i = 1, 2, 3$. With (4.2.5), the pdf of $\hat{\gamma}_i$ is given by

$$p_{\hat{\gamma}_i}(\hat{\gamma}_i) = \frac{1}{1 - \sigma_{i,e}^2} \exp \left(-\frac{\hat{\gamma}_i}{1 - \sigma_{i,e}^2} \right). \quad (4.2.12)$$

From (4.2.12), the pdf of $\hat{v}_i$ at high SNRs can be shown to be

$$p(\hat{v}_i) \doteq \begin{cases} 0, & \text{for } \hat{v}_i < 0 \\ \rho^{-\hat{v}_i}, & \text{for } \hat{v}_i \geq 0. \end{cases} \quad (4.2.13)$$

The pdf of $2\gamma_i/\sigma_{i,e}^2$ conditioned on $\hat{\gamma}_i$ follows the noncentral chi-square distribution with two degrees of freedom and non-centrality parameter $\lambda = 2\hat{\gamma}_i/\sigma_{i,e}^2$ [64]. That is,

$$p\left(\frac{2\gamma_i}{\sigma_{i,e}^2} \mid \hat{\gamma}_i\right) = \frac{1}{2} \exp\left(-\frac{\gamma_i + \hat{\gamma}_i}{\sigma_{i,e}^2}\right) I_0\left(\sqrt{\frac{4\gamma_i \hat{\gamma}_i}{\sigma_{i,e}^4}}\right)$$

where $I_0(\cdot)$ is the modified zero-th order Bessel function of the first kind. By changing variables, we get

$$p(v_i \mid \hat{v}_i) = \rho^{-v_i+\alpha_i} \ln \rho \exp\left(-\rho^{-v_i+\alpha_i} - \rho^{-\hat{v}_i+\alpha_i}\right) I_0\left(\sqrt{4\rho^{-v_i-\hat{v}_i+2\alpha_i}}\right). \quad (4.2.14)$$

To simplify $p(v_i \mid \hat{v}_i)$, we consider the following two cases:

1) If $v_i + \hat{v}_i \geq 2\alpha_i$, then $I_0\left(\sqrt{4\rho^{-v_i-\hat{v}_i+2\alpha_i}}\right) \approx 1$. We have

$$\begin{aligned} p(v_i \mid \hat{v}_i) &\approx \rho^{-v_i+\alpha_i} \ln \rho \exp\left(-\rho^{-v_i+\alpha_i} - \rho^{-\hat{v}_i+\alpha_i}\right) \\ &\approx \rho^{-v_i+\alpha_i} \ln \rho \exp\left(-\rho^{-\min(v_i, \hat{v}_i)+\alpha_i}\right) \\ &\doteq \begin{cases} 0, & \text{if } \min(v_i, \hat{v}_i) < \alpha_i \\ \rho^{-v_i+\alpha_i}, & \text{if } \min(v_i, \hat{v}_i) \geq \alpha_i \end{cases} \end{aligned} \quad (4.2.15)$$

where we have used the results that $\rho^a + \rho^b \approx \rho^{\max(a,b)}$ if ρ approaches infinity, and the term $\exp(-\rho^a)$ decays with ρ exponentially if $a > 0$ and approaches 1 for $a < 0$ and e for $a = 0$.

2) If $v_i + \hat{v}_i < 2\alpha_i$, then $I_0\left(\sqrt{4\rho^{-v_i-\hat{v}_i+2\alpha_i}}\right) \approx \frac{\exp\left(\sqrt{4\rho^{-v_i-\hat{v}_i+2\alpha_i}}\right)}{\sqrt{2\pi}\sqrt{4\rho^{-v_i-\hat{v}_i+2\alpha_i}}}$ [65]. It follows that if $v_i \neq \hat{v}_i$, we have

$$\begin{aligned} p(v_i \mid \hat{v}_i) &\approx \frac{\rho^{-v_i+\alpha_i} \ln \rho \exp\left(-\left(\rho^{\frac{-v_i+\alpha_i}{2}} - \rho^{\frac{-\hat{v}_i+\alpha_i}{2}}\right)^2\right)}{\sqrt{2\pi}\sqrt{4\rho^{-v_i-\hat{v}_i+2\alpha_i}}} \\ &= \frac{\rho^{\frac{\hat{v}_i}{4} - \frac{3v_i}{4} + \frac{\alpha_i}{2}} \ln \rho \exp\left(-\rho^{-\min(v_i, \hat{v}_i)+\alpha_i}\right)}{\sqrt{4\pi}} \\ &\leq \frac{\rho^{-\min(v_i, \hat{v}_i)+\alpha_i} \ln \rho \exp\left(-\rho^{-\min(v_i, \hat{v}_i)+\alpha_i}\right)}{\sqrt{4\pi}} \doteq 0. \end{aligned} \quad (4.2.16)$$

If $v_i = \hat{v}_i$, we have

$$p(v_i | \hat{v}_i) = \delta(v_i - \hat{v}_i) \quad (4.2.17)$$

where $\delta(\cdot)$ is the Dirac delta function.

Together with (4.2.13), the joint pdf of v_i and $\hat{v}_i$, $i = 1, 2, 3$, is given by

$$p(v_i, \hat{v}_i) \doteq \begin{cases} \rho^{-v_i} \delta(v_i - \hat{v}_i), & \text{if } 0 \leq v_i = \hat{v}_i < \alpha_i \\ \rho^{-v_i - \hat{v}_i + \alpha_i}, & \text{if } v_i, \hat{v}_i \geq \alpha_i \\ 0, & \text{otherwise.} \end{cases} \quad (4.2.18)$$

Since t can be made arbitrarily close to 1, it is without loss of accuracy to set $t = 1$ in the rest of this section. With (4.2.18) and Laplace principle [6], we can rewrite (4.2.11) as

$$\mathcal{P}_{out}^{DDF} \doteq \rho^{-d_{DDF}^*(r)} \quad (4.2.19)$$

where

$$d_{DDF}^*(r) = \inf_{v_i, \hat{v}_i \in \mathcal{O}_{DDF}} \sum_{i=1}^3 (v_i + (\hat{v}_i - \alpha_i)^+) \quad (4.2.20)$$

and

$$\begin{aligned} \mathcal{O}_{DDF} = & \left\{ v_i, \hat{v}_i \mid (1 + \hat{v}_1 + \hat{v}_2 - v_2)^+ < r; f(1 + \hat{v}_1 + \hat{v}_2 - v_2)^+ + (1 - f)(1 + \hat{v}_3 - v_3)^+ < r; \right. \\ & \left. f(1 + \hat{v}_1 + \hat{v}_2 - v_1)^+ = r; \text{ if } v_i < \alpha_i \text{ then } v_i = \hat{v}_i \geq 0, \text{ otherwise } v_i, \hat{v}_i \geq \alpha_i, i = 1, 2, 3 \right\}. \end{aligned} \quad (4.2.21)$$

It can be observed that decreasing $\hat{v}_3$ does not violate the outage condition in $\mathcal{O}_{DDF}$ while reducing the SNR exponent $\sum_{i=1}^3 (v_i + (\hat{v}_i - \alpha_i)^+)$. Thus, the solution of $\hat{v}_3$ should be $\hat{v}_3^* = \min(v_3, \alpha_3)$. Substituting $\hat{v}_3^*$ into (4.2.21), we can equivalently reexpress the outage condition as

$$\begin{cases} v_1 = 1 + \hat{v}_1 + \hat{v}_2 - \frac{r}{f} \\ v_2 > 1 + \hat{v}_1 + \hat{v}_2 - r \\ v_2 > v_1 + \frac{1-f}{f} (1 + \min(v_3, \alpha_3) - v_3)^+ \end{cases} \quad (4.2.22)$$

where $v_2, \hat{v}_2 \geq \alpha_2$; if $\hat{v}_1 < \alpha_1$ then $v_1 = \hat{v}_1 \geq 0$, otherwise we have $v_1, \hat{v}_1 \geq \alpha_1$.

With (4.2.22), we can transform the optimization problem in (4.2.20) to

$$\begin{aligned} d_{DDF}^*(r) &= \min_{\hat{v}_1, \hat{v}_2, v_3, f} 2v_1 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ + g(v_3) \\ \text{s. t. } \hat{v}_2 &\geq \alpha_2; v_1 = 1 + \hat{v}_1 + \hat{v}_2 - \frac{r}{f}; \text{ if } v_1 < \alpha_1, \text{ then } v_1 = \hat{v}_1, \text{ otherwise } v_1, \hat{v}_1 \geq \alpha_1 \end{aligned} \quad (4.2.23)$$

where $g(v_3) \triangleq \frac{1-f}{f} \max(r, (1 + \min(v_3, \alpha_3) - v_3)^+) + v_3$.

To solve the above problem, we first find the solution of v_3 . We consider the following four cases to find the minimum of $g(v_3)$:

1) If $v_3 \leq \alpha_3$, then $g(v_3) = \frac{1-f}{f} + v_3$ is an increasing function of v_3 . The solution of v_3 is $v_3^* = 0$ and $g(v_3^*) = \frac{1-f}{f}$.

2) If $v_3 \geq 1 - r + \alpha_3$, then $g(v_3) = \frac{1-f}{f}r + v_3$ is an increasing function of v_3 . The solution of v_3 is $v_3^* = 1 + \alpha_3 - r$ and $g(v_3^*) = \frac{1-f}{f}r + 1 + \alpha_3 - r$.

3) If $\alpha_3 \leq v_3 < 1 + \alpha_3 - r$ and $f \geq 1/2$, then $g(v_3) = \frac{1-f}{f}(1 + \alpha_3 - v_3)^+ + v_3$ is an increasing function of v_3 . The solution of v_3 is $v_3^* = \alpha_3$ and $g(v_3^*) = \frac{1-f}{f} + \alpha_3$.

4) If $\alpha_3 \leq v_3 \leq 1 + \alpha_3 - r$ and $f < 1/2$, then $g(v_3) = \frac{1-f}{f}(1 + \alpha_3 - v_3)^+ + v_3$ is a decreasing function of v_3 . The solution of v_3 is $v_3^* = 1 + \alpha_3 - r$ and $g(v_3^*) = \frac{1-f}{f}r + 1 + \alpha_3 - r$.

From the above, we know that $\min_{v_3} g(v_3) = \min\left(\frac{1-f}{f}, \frac{1-f}{f}r + 1 + \alpha_3 - r\right)$. Therefore, the optimization problem in (4.2.23) can be rewritten as

$$d_{DDF}^*(r) = \min_{\hat{v}_1, \hat{v}_2, f} \min(\mathcal{G}_1, \mathcal{G}_2) \quad (4.2.24)$$

where $\mathcal{G}_1 \triangleq 2v_1 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ + \frac{1-f}{f}r + 1 + \alpha_3 - r$ and $\mathcal{G}_2 \triangleq 2v_1 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ + \frac{1-f}{f}$. Next, we consider the following two complementary and exclusive events:

1) If $v_1 = \hat{v}_1 < \alpha_1$, then $f = r/(1 + \hat{v}_2)$. The optimal diversity gain in this

case can be obtained as

$$\begin{aligned}
 d_1(r) &= \min_{\hat{v}_1, \hat{v}_2, f} \min(\mathcal{G}_1, \mathcal{G}_2) \\
 &= \min_{\hat{v}_1, \hat{v}_2} \min \left(2\hat{v}_1 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ + \hat{v}_2 + \alpha_3 + 2 - 2r, \right. \\
 &\quad \left. 2\hat{v}_1 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ + \frac{1 + \hat{v}_2 - r}{r} \right) \\
 &= \min \left(2 - 2r + \alpha_2 + \alpha_3, \frac{1 + \alpha_2 - r}{r} \right)
 \end{aligned}$$

with $\hat{v}_2^* = \alpha_2$ and $\hat{v}_1^* = 0$.

2) If $\hat{v}_1, v_1 \geq \alpha_1$, then $f \geq r/(1 + \hat{v}_2 + \hat{v}_1 - \alpha_1)$. The minimum of $\mathcal{G}_1$ can be found to be

$$\begin{aligned}
 d_2(r) &= \min_{\hat{v}_1, \hat{v}_2, f} \mathcal{G}_1 \\
 &= \min_{\hat{v}_1, \hat{v}_2, f} 2(1 + \hat{v}_1 + \hat{v}_2) - \frac{r}{f} - 2r + 1 + \alpha_3 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ \\
 &= \min_{\hat{v}_1, \hat{v}_2} 2 - 2r + \hat{v}_1 + \hat{v}_2 + \alpha_1 + \alpha_3 + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ \\
 &= 2 - 2r + \alpha_2 + 2\alpha_1 + \alpha_3
 \end{aligned}$$

with $f^* = r/(1 + \hat{v}_1^* + \hat{v}_2^* - \alpha_1)$ and $\hat{v}_1^* = \alpha_1$, $\hat{v}_2^* = \alpha_2$.

The minimum of $\mathcal{G}_2$ can be found to be

$$d_3(r) = \min_{\hat{v}_1, \hat{v}_2, f} \mathcal{G}_2 = \min_{\hat{v}_1, \hat{v}_2, f} 1 + 2(\hat{v}_1 + \hat{v}_2) + (\hat{v}_2 - \alpha_2)^+ + (\hat{v}_1 - \alpha_1)^+ + \frac{1 - 2r}{f}.$$

If $r \leq 1/2$, the optimal solution of f should be $f^* = 1$ and thus $d_3(r) = 2 - 2r + 2\alpha_1 + 2\alpha_2$ with $\hat{v}_1^* = \alpha_1$ and $\hat{v}_2^* = \alpha_2$. If $r > 1/2$, the optimal solution of f should be $f^* = r/(1 + \hat{v}_1 + \hat{v}_2 - \alpha_1)$ and thus $d_3(r) = 2\alpha_1 + (1 + \alpha_2 - r)/r$ with $\hat{v}_1^* = \alpha_1$ and $\hat{v}_2^* = \alpha_2$.

Finally, the optimal diversity gain $d_{DDF}^*(r)$ should be the minimum of $d_1(r)$, $d_2(r)$ and $d_3(r)$, which is given in (4.2.9). This completes the proof. $\square$

Figure 4.4: A three-node relay system with cut-sets.

4.2.3 Discussions

It can be verified immediately that if $\alpha_i = 0$, $i = 1, 2, 3$, the diversity gain $d_{DDF}^*(r)$ degrades to the case with perfect CSI at the receiver and no CSIT at all transmitters [61], i.e., $d_{DDF}^*(r) = \min(2(1-r), (1-r)/r)$. This is not unexpected since fixed or limited training power ($\Leftrightarrow \alpha_i = 0$) does not contribute to the diversity gain and is equivalent to no training.

In general, $d_{DDF}^*(r)$ can be intuitively interpreted as follows. For the considered DDF channel, we have two cut-sets $C_{S,RD}$ and $C_{SR,D}$, as shown in Figure 4.4. With imperfect CSIT, the value $(2 - 2r + 2\alpha_1 + 2\alpha_2)$ actually corresponds to the diversity gain of cut-set $C_{S,RD}$ and $(2 - 2r + \alpha_2 + \alpha_3)$ corresponds to the diversity gain of cut-set $C_{SR,D}$. It is not surprising that when the source node does not have any CSIT, i.e., $\alpha_1 = \alpha_2 = 0$, or both the S-D and R-D links do not have CSIT, i.e., $\alpha_2 = \alpha_3 = 0$, no diversity gain improvement can be achieved and thus in this case $d_{DDF}^*(r) = \min(2(1-r), (1-r)/r)$. To have more insights of $d_{DDF}^*(r)$, we rewrite $d_{DDF}^*(r)$ as

$$d_{DDF}^*(r) = \begin{cases} \frac{1+\alpha_2-r}{r}, & \text{for } \frac{1+\alpha_2}{2} < r \leq 1 \\ \min\left(\frac{1+\alpha_2-r}{r}, 2 - 2r + \alpha_2 + \alpha_3\right), & \text{for } \frac{1}{2} < r \leq \frac{1+\alpha_2}{2} \\ \min\left(\frac{1+\alpha_2-r}{r}, 2 - 2r + \alpha_2 + \alpha_3, 2 - 2r + 2\alpha_1 + 2\alpha_2\right) & \text{for } 0 \leq r \leq \frac{1}{2}. \end{cases} \quad (4.2.25)$$

We can see that at low multiplexing gains, α_1 , α_2 and α_3 all have impacts

Figure 4.5: DMT in relaying channels with imperfect CSIT. Note that for the conventional relaying, $0 \leq r \leq 1/2$.

on the overall diversity gain. At moderate multiplexing gains, both α_2 and α_3 contribute to the diversity gain. At high multiplexing gains, only α_2 contributes. This is because that at high multiplexing gains, the relay needs to listen to the source for a long duration and thus does not have much time left to transmit the high-rate source information. Therefore, the system looks like only having the S-D link and the DMT at multiplexing gain r only depends on α_2 . (4.2.25) also shows that if $\alpha_2 \geq \alpha_3$, the S-R link CSIT will not contribute to the overall diversity gain. Consequently, there is no need for the relay to send training pilots to the source for the S-R link channel estimation.

Recall that the DMT under the conventional DF relaying protocol with partial CSIT is given by $d_{DF}^*(r) = \min(2(1 - 2r) + \alpha_2 + \alpha_3, 2(1 - 2r + \alpha_1 + \alpha_2))$ for $0 \leq r \leq 1/2$ according to the previous section. Comparison with (4.2.9) clearly indicates that the DDF protocol provides a better DMT than the conventional DF protocol, as shown in Figure 4.5. We can see that by simply performing power control at the source (e.g., $\alpha_2 = 1, \alpha_3 = 0$), a higher diversity gain can be achieved as compared to the case without CSIT. If the relay also performs power control (e.g., $\alpha_2 = \alpha_3 = 1$), a more considerable diversity improvement can be achieved. Note that since $\alpha_2 \geq \alpha_3$, the value of α_2 does not have any impact in this case.

Chapter 5

Adaptive Power Allocation in Dual-hop Relaying Channels

In Chapters 3 and 4, we focused on the asymptotically high SNR regimes and studied temporal power control in MIMO and cooperative relaying channels subject to long-term power constraints at the transmitters. From this chapter onwards, we will consider spatial power control schemes subject to short-term power constraints in cooperative relay systems in a broad regime of SNR. In this chapter, we consider a dual-hop DF relaying channel with multiple destination antennas. We wish to improve the system performance by employing multiple antennas at the destination while keeping the computational complexity of the source and relay unchanged. This is reasonable as the complexity issue at the destination node such as the BS in an uplink is not as critical as that of mobile terminals. Instead of focusing on PA only, like most of the previous works do, this chapter considers a more general case where we take both PA and relay location into optimization and study both systems with and without direct link. Firstly, with the fixed relay location, we derive the optimal PA at the source and relay under a joint power constraint to minimize the system outage probability. Secondly, conditioned on the fixed PA, we derive the optimum relay placement. Finally, we jointly optimize PA and relay location. Numerical results show that the proposed adaptive schemes significantly outperform the fixed allocation schemes. For a system without direct link, it is found that optimal relay positioning can fully exploit

the advantage of employing multiple destination antennas. For both systems with and without direct link, it is shown that employing multiple destination antennas and/or choosing an appropriate relay location can greatly save the relay power. One can see that this is of great importance, particularly in practice.

5.1 System Model

We consider a dual-hop DF relay system consisting of one single-antenna source node, one single-antenna relay node and one N -antenna destination node ($N \geq 1$). The system model with direct link is given in Figure 4.1. The system without direct link is shown in Figure 5.1. Without loss of generality, we assume that all the three nodes are located in a line. If the relay is originally not located in the line connecting the source and destination, after relay location optimization, the optimal relay location will turn out to lie in this line in order to minimize the effect of path loss. For simplicity, we normalize the distance between the source and destination. So, letting d denote the S-R distance, the R-D distance is given as $1 - d$. The relay node is regenerative and works in half-duplex mode. In a DF relay system, if the relay transmits a data block that is incorrectly detected/decoded, it is likely that there will be an error at the destination. This problem, usually called error propagation, limits the diversity order of digital relaying. Therefore, we consider adaptive protocols, which allow the relay to transmit only when it can detect/decode the source signal reliably. In this way, error propagation can be significantly reduced and the digital relaying can provide full diversity order [25].

We consider time-division multiplexing (TDM) transmissions. In the first time slot, the source node transmits data with power $P_{N,s}$. The received signal at the relay node can be written as

$$Y_r = \sqrt{P_{N,s}} h_{sr} X_s + Z_r \quad (5.1.1)$$

where X_s is the transmitted signal with $E[|X_s|^2] = 1$; h_{sr} denotes the instantaneous channel gain of S-R link; and Z_r denotes the additive noise received at the

Figure 5.1: A dual-hop relay system with multiple destination antennas and no direct link.

relay node.

If the direct link transmission exists as shown in Figure 4.1, the received signal at the n -th destination antenna can be written as

$$Y_{d,n,1} = \sqrt{P_{N,s}} h_{sd,n} X_s + Z_{d,n,1} \quad (5.1.2)$$

where $h_{sd,n}$ denotes the instantaneous channel gain of the link between source and the n -th destination antenna, and $Z_{d,n,1}$ denotes the additive noise.

In the second time slot, the relay node decodes the received signal and forwards the re-encoded data to the destination upon reliably detecting/decoding the source data. The received signal at the n -th destination antenna can be written as

$$Y_{d,n,2} = \sqrt{P_{N,r}} h_{rd,n} X_r + Z_{d,n,2} \quad (5.1.3)$$

where X_r denotes the transmitted signal by the relay node with $E[|X_r|^2] = 1$; $P_{N,r}$ denotes the relay transmit power; $h_{rd,n}$ denotes the instantaneous channel gain of the link between the relay and the n -th destination antenna; and $Z_{d,n,2}$ denotes the additive noise.

We assume that all channels are subject to independent Rayleigh fading and uniform path-loss with a path loss exponent α . Therefore, the variances of the channel gains of S-R link, R-D link, and the S-D link are given by $\sigma_{sr}^2 = \frac{1}{d^\alpha}$,

$\sigma_{rd}^2 = \frac{1}{(1-d)^\alpha}$, and $\sigma_{sd}^2 = 1$, respectively. The extension to the case of non-uniform path-loss is straightforward and thus is not considered in this chapter. It can be shown that our results can apply to non-uniform path-loss with slight modification. We further assume that all the noise terms are additive zero-mean white circular complex Gaussian variables with unit variance (i.e., $\sigma^2 = 1$).

5.2 Outage Performance Analysis

Firstly, we consider the system with direct link, where the destination is also receiving the original signal transmitted by the source as well as the relayed signal. The SNR at the relay node is given by $\gamma_{sr} = P_{N,s}|h_{sr}|^2$ and the corresponding pdf is given by $p_{\gamma_{sr}}(x) = \frac{d^\alpha}{P_{N,s}} \exp\left(-\frac{d^\alpha}{P_{N,s}}x\right)$.

If the relay node cannot reliably detect source data ($\gamma_{sr} < \gamma_{th}$), it will keep silent in the second time slot to avoid error propagation. In this case, the SNR at destination is given by

$$\gamma_{sd} = \sum_{n=1}^N P_{N,s}|h_{sd,n}|^2. \quad (5.2.1)$$

Since $P_{N,s}|h_{sd,n}|^2, n = 1, 2, \dots, N$, are i.i.d. exponential random variables with rate parameter $a = \frac{1}{P_{N,s}}$, we can express the pdf of γ_{sd} as [66]

$$p_{\gamma_{sd}}(x) = \frac{a^N x^{N-1}}{(N-1)!} \exp(-ax). \quad (5.2.2)$$

On the other hand, if the relay node can reliably detect the source data ($\gamma_{sr} \geq \gamma_{th}$), it will forward the re-encoded data to destination with repetition coding. Then the destination will jointly combine the signal received from the source node in the first time slot and the one received from the relay node in the second time slot using MRC with perfect knowledge of the instantaneous channel gains [67]. Therefore, the SNR at the destination can be expressed as

$$\gamma_{MRC} = \gamma_{sd} + \gamma_{rd} \quad (5.2.3)$$

where $\gamma_{rd} = \sum_{n=1}^N P_{N,r} |h_{rd,n}|^2$. Defining $b = (1-d)^\alpha / P_{N,r}$, the pdf of γ_{rd} is given by [66]

$$p_{\gamma_{rd}}(x) = \frac{b^N x^{N-1}}{(N-1)!} \exp(-bx). \quad (5.2.4)$$

Now we derive the pdf of γ_{MRC} . Since γ_{MRC} is the sum of N independent exponential random variables with rate a and N independent exponential random variables with rate b , the moment generating function (MGF) is given by

$$M_{\gamma_{MRC}}(s) = \left(\frac{a}{s+a} \right)^N \left(\frac{b}{s+b} \right)^N = \sum_{n=1}^N \left(\frac{A_n}{(s+a)^n} + \frac{B_n}{(s+b)^n} \right) \quad (5.2.5)$$

where $A_n = \frac{\Gamma(2N-n)(-1)^{N-n} b^N a^N}{(b-a)^{2N-n} \Gamma(N) \Gamma(N-n+1)}$ and $B_n = \frac{\Gamma(2N-n)(-1)^{N-n} b^N a^N}{(a-b)^{2N-n} \Gamma(N) \Gamma(N-n+1)}$. Applying inverse Laplace transform, the distribution of γ_{MRC} is given by

$$p_{\gamma_{MRC}}(x) = \sum_{n=1}^N \left(\frac{A_n x^{n-1}}{\Gamma(n)} \exp(-ax) + \frac{B_n x^{n-1}}{\Gamma(n)} \exp(-bx) \right). \quad (5.2.6)$$

Defining $c = d^\alpha / P_{N,s}$, the overall system outage probability with a predetermined threshold γ_{th} is given by

$$\begin{aligned} \mathcal{P}_{out,div} &= \mathcal{P}(\gamma_{sr} < \gamma_{th}) \mathcal{P}(\gamma_{sd} < \gamma_{th}) + \mathcal{P}(\gamma_{sr} \geq \gamma_{th}) \mathcal{P}(\gamma_{MRC} < \gamma_{th}) \\ &= \int_0^{\gamma_{th}} p_{\gamma_{sr}}(x) dx \int_0^{\gamma_{th}} p_{\gamma_{sd}}(y) dy + \int_{\gamma_{th}}^{\infty} p_{\gamma_{sr}}(x) dx \int_0^{\gamma_{th}} p_{\gamma_{MRC}}(z) dz \\ &= (1 - \exp(-c\gamma_{th})) \left(1 - \exp(-a\gamma_{th}) \sum_{i=0}^{N-1} \frac{(a\gamma_{th})^i}{\Gamma(i+1)} \right) + \exp(-c\gamma_{th}) \\ &\quad \sum_{n=1}^N \left(\frac{A_n}{a^n} \left(1 - \exp(-a\gamma_{th}) \sum_{i=0}^{n-1} \frac{(a\gamma_{th})^i}{\Gamma(i+1)} \right) + \frac{B_n}{b^n} \left(1 - \exp(-b\gamma_{th}) \sum_{i=0}^{n-1} \frac{(b\gamma_{th})^i}{\Gamma(i+1)} \right) \right). \end{aligned} \quad (5.2.7)$$

We aim to optimize the PA and relay location to minimize the system outage probability. However, it is difficult, if not impossible, to optimize over (5.2.7)

directly. Instead, we use its outage upper bound

$$\begin{aligned}
 \mathcal{P}_{out,div} &= \mathcal{P}(\gamma_{sr} < \gamma_{th})\mathcal{P}(\gamma_{sd} < \gamma_{th}) + \mathcal{P}(\gamma_{sr} \geq \gamma_{th})\mathcal{P}(\gamma_{MRC} < \gamma_{th}) \\
 &\leq \mathcal{P}(\gamma_{sr} < \gamma_{th})\mathcal{P}(\gamma_{sd} < \gamma_{th}) + \mathcal{P}(\gamma_{sr} \geq \gamma_{th})\mathcal{P}(\gamma_{sd} < \gamma_{th})\mathcal{P}(\gamma_{rd} < \gamma_{th}) \\
 &= \mathcal{P}(\gamma_{sd} < \gamma_{th}) (1 - \mathcal{P}(\gamma_{sr} \geq \gamma_{th})\mathcal{P}(\gamma_{rd} \geq \gamma_{th})) \\
 &= \left(1 - \exp(-a\gamma_{th}) \sum_{i=0}^{N-1} \frac{(a\gamma_{th})^i}{\Gamma(n+1)}\right) \left(1 - \exp(-c\gamma_{th}) \exp(-b\gamma_{th}) \sum_{i=0}^{N-1} \frac{(b\gamma_{th})^i}{\Gamma(n+1)}\right) \\
 &= \left(1 - \exp(-a\gamma_{th}) \left(\exp(a\gamma_{th}) - \sum_{i=N}^{\infty} \frac{(a\gamma_{th})^i}{\Gamma(n+1)}\right)\right) \\
 &\quad \cdot \left(1 - \exp(-c\gamma_{th}) \exp(-b\gamma_{th}) \left(\exp(b\gamma_{th}) - \sum_{i=N}^{\infty} \frac{(b\gamma_{th})^i}{\Gamma(n+1)}\right)\right) \\
 &= \exp(-a\gamma_{th}) \sum_{i=N}^{\infty} \frac{(a\gamma_{th})^i}{\Gamma(n+1)} \left(1 - \exp(-c\gamma_{th}) \left(1 - \exp(-b\gamma_{th}) \sum_{i=N}^{\infty} \frac{(b\gamma_{th})^i}{\Gamma(n+1)}\right)\right) \\
 &\approx \frac{(a\gamma_{th})^N}{\Gamma(N+1)} \left(1 - (1 - c\gamma_{th}) \left(1 - \frac{(b\gamma_{th})^N}{\Gamma(N+1)}\right)\right) \\
 &\approx B \left(\frac{Ad^\alpha}{P_{N,s}^{N+1}} + \frac{B(1-d)^{\alpha N}}{P_{N,s}^N P_{N,r}^N}\right) \tag{5.2.8}
 \end{aligned}$$

where $A = \gamma_{th}$, $B = \frac{(\gamma_{th})^N}{\Gamma(N+1)}$. Although the above approximation is obtained with the high SNR assumption and is due to the fact that for small x , $\exp(-x) \approx 1 - x$, we observe that the approximation is valid in a broad SNR regime as shown in Section 5.4.

The upper bound in (5.2.8) also corresponds to the outage probability of a system with selective diversity combining, where the destination keeps monitoring both the relayed and the direct transmitted signals, and at any instant of time only the signal with the highest instantaneous SNR is processed and detected. This type of selection diversity bypasses the need of synchronizing and coherently combining the signals received from the direct and relayed paths [48]. It should be noted that although the adaptive allocation algorithms to be proposed in the following are to minimize the outage upper bound, these algorithms do have significant performance gain over uniform allocation algorithms with respect to the actual system outage probabilities as shown by the numerical results in Section 5.4.

Now we consider a special case, where the destination is not able to receive signals from the source directly. This may result from long distance separation or deep shadowing between the two nodes. For the considered system, an outage in any of the two hops leads to an overall system outage. Equivalently, it is the complementary event of having both links operating above the threshold. Therefore, the system outage probability can be expressed as

$$\begin{aligned}\mathcal{P}_{out,nodiv} &= \mathcal{P}(\min(\gamma_{sr}, \gamma_{rd}) < \gamma_{th}) \\ &= 1 - \mathcal{P}(\gamma_{sr} \geq \gamma_{th})\mathcal{P}(\gamma_{rd} \geq \gamma_{th}) \approx \frac{Ad^\alpha}{P_{N,s}} + \frac{B(1-d)^{N\alpha}}{P_{N,r}^N}.\end{aligned}\quad (5.2.9)$$

5.3 Optimizing PA and Relay Location

5.3.1 Optimizing PA with Fixed Relay Location

With the fixed relay placement, d , we derive the optimal PA at the source and relay to minimize the system outage probability. The optimization problem for the system with direct link can be formulated as

$$P_{N,s}^*, P_{N,r}^* = \arg \min_{P_{N,s}, P_{N,r}} \mathcal{P}_{out,div} \quad (5.3.1)$$

$$\text{subject to: } P_{N,s} + P_{N,r} = P_{N,T} \quad (5.3.2a)$$

$$P_{N,s}, P_{N,r} > 0. \quad (5.3.2b)$$

Substituting $P_{N,r} = P_{N,T} - P_{N,s}$ into the objective function $\mathcal{P}_{out,div}$ and taking the second derivative of it with respect to $P_{N,s}$, we can get

$$\begin{aligned}\frac{\partial^2 \mathcal{P}_{out,div}}{\partial P_{N,s}^2} &= \frac{ABd^\alpha (N+1)(N+2)}{P_{N,s}^{N+3}} + \frac{B^2(1-d)^{N\alpha}(N^2+N)}{P_{N,s}^{N+2}(P_{N,T}-P_{N,s})^N} \\ &+ \frac{B^2(1-d)^{N\alpha}(N^2+N)}{P_{N,s}^N(P_{N,T}-P_{N,s})^{N+2}} - \frac{2B^2(1-d)^{N\alpha}N^2}{P_{N,s}^{N+1}(P_{N,T}-P_{N,s})^{N+1}}.\end{aligned}\quad (5.3.3)$$

It is easy to prove that (5.3.3) is positive in the interval $P_{N,s} \in (0, P_{N,T})$

using $X + Y \geq 2\sqrt{XY}, \forall X, Y \geq 0$. This implies that $\mathcal{P}_{out,div}$ is a strictly convex function of $P_{N,s}$ in the interval. Therefore, by taking derivative of it with respect to $P_{N,s}$ and equating the derivative to zero, we can find the optimal PA. The optimal source power $P_{N,s}^*$ is the root of the following equation:

$$\frac{(2P_{N,s} - P_{N,T})NB(1-d)^{\alpha N}}{A(P_{N,T} - P_{N,s})^{N+1}} - \frac{d^{\alpha}(N+1)}{P_{N,s}} = 0. \quad (5.3.4)$$

If $N \leq 3$, analytical solutions to (5.3.4) can be found. The solution with $N = 1$, $d \neq 0.5$ is given by

$$P_{1,s}^* = \frac{-(1-d)^{\alpha} + 4d^{\alpha} + \sqrt{(1-d)^{2\alpha} + 8(1-d)^{\alpha}d^{\alpha}}}{-4(1-d)^{\alpha} + 4d^{\alpha}} P_{1,T}. \quad (5.3.5)$$

If $d = 0.5$, the solution is $P_{1,s}^* = \frac{2}{3}P_{1,T}$. The solutions for $N = 2$ and $N = 3$ are omitted here for brevity. If the destination node is equipped with more than three antennas, we cannot find analytical PA solutions and the numerical solutions can be found using standard iterative root-finding algorithms with great efficiency, such as Bisection method and Newton's Method [68]. The optimal relay power is thus given as $P_{N,r}^* = P_{N,T} - P_{N,s}^*$.

With the similar method, we can derive the optimal PA solution for the system without direct link. The optimal relay power $P_{N,r}^*$ is the root of the following equation:

$$\frac{Ad^{\alpha}}{(P_{N,T} - P_{N,r})^2} - \frac{BN(1-d)^{\alpha N}}{(P_{N,r})^{N+1}} = 0. \quad (5.3.6)$$

Analytical solutions can be derived if $N \leq 3$ and are omitted here. Numerical solutions can be derived with root-finding algorithms if $N > 3$. The optimal source power is given by $P_{N,s}^* = P_{N,T} - P_{N,r}^*$.

In practice, the destination node such as a BS can work as a central controller to calculate the optimal PA solutions upon measuring the channel statistics and inform the source and relay through feedback channels. Since this needs only to be done once for channels with static second-order channel statistics, the implementation complexity as well as the signaling overhead is mild.

5.3.2 Optimizing Relay Location with Fixed PA

Since $\mathcal{P}_{out,div} = \frac{B}{P_{N,s}^N} \mathcal{P}_{out,nodiv}$, the optimal relay location is the same for both systems with/without direct link. Therefore, under a predetermined PA $(P_{N,s}, P_{N,r})$, the problem of finding the optimal relay location for both systems can be formulated as

$$d^* = \arg \min_d B \left(\frac{Ad^\alpha}{P_{N,s}^{N+1}} + \frac{B(1-d)^{\alpha N}}{P_{N,s}^N P_{N,r}^N} \right) \quad (5.3.7)$$

subject to

$$0 \leq d \leq 1. \quad (5.3.8)$$

Taking the second derivative of $\mathcal{P}_{out,div}$ with respect to d , we can get $\frac{\partial^2 \mathcal{P}_{out,div}}{\partial d^2} = \frac{B}{P_{N,s}^N} \left(\frac{A\alpha(\alpha-1)d^{\alpha-2}}{P_{N,s}} + \frac{B\alpha N(\alpha N-1)(1-d)^{\alpha N-2}}{P_{N,r}^N} \right) > 0$, which implies that the objective function is a strictly convex function of d in the interval $d \in (0, 1)$. Therefore, we can find the optimal solution by taking the first derivative of $\mathcal{P}_{out,div}$ with respect to d and equating it to zero. The optimal S-R distance d^* is the root of the following equation:

$$\frac{A\alpha d^{\alpha-1}}{P_{N,s}} - \frac{B\alpha N(1-d)^{\alpha N-1}}{P_{N,r}^N} = 0. \quad (5.3.9)$$

Similarly, standard iterative root-finding algorithms can be adopted to find the numerical solutions to d^* with great efficiency. If $N = 1$, the optimal solution is

$$d^* = \frac{P_{N,s}^{\frac{1}{\alpha-1}}}{P_{N,s}^{\frac{1}{\alpha-1}} + P_{N,r}^{\frac{1}{\alpha-1}}}. \quad (5.3.10)$$

From (5.3.10), we know that the optimal relay location is the middle point of the source and destination if uniform PA is adopted.

5.3.3 Jointly Optimizing PA and Relay Location

The system performance can be further improved if we jointly optimize PA and relay location at the same time. The optimization problem for the system

with direct link can be formulated as

$$d^*, P_{N,s}^* = \arg \min_{d, P_{N,s}} \left[B \left(\frac{Ad^\alpha}{P_{N,s}^{N+1}} + \frac{B(1-d)^{\alpha N}}{P_{N,s}^N (P_{N,T} - P_{N,s})^N} \right) \right] \quad (5.3.11)$$

subject to

$$0 < P_{N,s} < P_{N,T} \quad (5.3.12)$$

$$0 < d < 1. \quad (5.3.13)$$

It is obvious that the optimal solution must lie in the boundary points and thus satisfy

$$\frac{\partial \mathcal{P}_{out,div}}{\partial d} = \frac{AB\alpha d^{\alpha-1}}{P_{N,s}^{N+1}} - \frac{B^2\alpha N(1-d)^{\alpha N-1}}{P_{N,s}^N (P_{N,T} - P_{N,s})^N} = 0 \quad (5.3.14a)$$

$$\frac{\partial \mathcal{P}_{out,div}}{\partial P_{N,s}} = \frac{AB(N+1)d^\alpha}{P_{N,s}^{N+2}} - \frac{B^2N(1-d)^{\alpha N}(2P_{N,s} - P_{N,T})}{P_{N,s}^{N+1} (P_{N,T} - P_{N,s})^{N+1}} = 0. \quad (5.3.14b)$$

It is very hard to solve this non-linear equation set. However, after some mathematical manipulations as shown in Appendix G, (5.3.14) can be simplified as

$$AP_{N,T}^{N-1}d^{\alpha-1} - BN(2 + dN - d)^{N-1}(1 + dN)(1-d)^{\alpha N - N - 1} = 0 \quad (5.3.15a)$$

$$\frac{P_{N,s}^*}{P_{N,T}} = \frac{1 + dN}{2 + dN - d}. \quad (5.3.15b)$$

It is noted that (5.3.15a) is an equation with only one variable d . We can show that there is only one root d^* to (5.3.15a) if any of the following three conditions is satisfied: 1) $N \leq 6$ and $\alpha \geq 2$; 2) $N \leq 9$ and $\alpha \geq 2.6$; 3) $N \leq 18$ and $\alpha \geq 4$. The proof is given in Appendix H. Note that these three conditions are sufficient not necessary conditions. Therefore, the optimal d^* can be found by standard root finding algorithms and the optimal PA can be calculated according to (5.3.15b). Based on (5.3.15), the optimal solutions $(\frac{P_{N,s}^*}{P_{N,T}}, d^*)$ for various N and $P_{N,T}$ are listed in Table 5.1.

Similarly, the optimal solution for the system without direct link can be found by solving:

$$d^{\alpha-2} - \frac{(1-d)^{\alpha N-1-N}BN}{A(P_{N,T})^{N-1}} = 0 \quad (5.3.16a)$$

$$\frac{P_{N,s}^*}{P_{N,T}} = d. \quad (5.3.16b)$$

It is interesting to find that the optimal source power ratio, $\frac{P_{N,s}^*}{P_{N,T}}$, is equal to d^* . If $N = 1$, we have $d^* = \frac{1}{2}$, $P_{1,s}^* = P_{1,r}^* = \frac{1}{2}P_{1,T}$, which indicates that the optimal relay location is the middle point of the source and destination and the corresponding optimal PA is the uniform PA. The optimal solutions ($\frac{P_{N,s}^*}{P_{N,T}} = d^*$) for various N and $P_{N,T}$ are listed in Table 5.2. Tables 5.1 and 5.2 show that, for both systems with and without direct link, the optimal source power ratio and S-R distance do not change with the total power level when $N = 1$ but decrease monotonously with the total power level when $N > 1$. Furthermore, when N increases, the optimal S-R distance, d^* , monotonously decreases when $\frac{P_{N,T}}{\sigma^2} \geq 5$ dB. We also observe that the optimal source power ratio is always greater than 0.5 for the system with direct link, which indicates that the system will always allocate more power to the source than the relay in order to have a lower system outage probability. It should be noted that although numerical calculation is needed for the optimal relay location, the complexity of the proposed algorithm is very low, since we only need to calculate once for each system configuration.

Table 5.1: Joint optimal solutions (d^* , $\frac{P_{N,s}^*}{P_{N,T}}$) in DF relay system with direct link

$d^*, \frac{P_{N,s}^*}{P_{N,T}}$	$\frac{P_{N,T}}{\sigma^2} = 0$ dB	$\frac{P_{N,T}}{\sigma^2} = 5$ dB	$\frac{P_{N,T}}{\sigma^2} = 10$ dB	$\frac{P_{N,T}}{\sigma^2} = 15$ dB	$\frac{P_{N,T}}{\sigma^2} = 20$ dB
$N = 1$	0.6180, 0.8090	0.6180, 0.8090	0.6180, 0.8090	0.6180, 0.8090	0.6180, 0.8090
$N = 2$	0.6586, 0.8716	0.5904, 0.8419	0.5152, 0.8073	0.4355, 0.7682	0.3553, 0.7263
$N = 3$	0.6608, 0.8979	0.5620, 0.8598	0.4448, 0.8079	0.3173, 0.7409	0.1984, 0.6656
$N = 4$	0.6580, 0.9139	0.5392, 0.8726	0.3899, 0.8075	0.2259, 0.7109	0.0968, 0.6056
$N = 5$	0.6544, 0.9252	0.5206, 0.8826	0.3435, 0.8054	0.1505, 0.6735	0.0395, 0.5549

Table 5.2: Joint optimal solutions ($d^* = \frac{P_{N,s}^*}{P_{N,T}}$) in DF relay system without direct link

$d^* = \frac{P_{N,s}^*}{P_{N,T}}$	$\frac{P_{N,T}}{\sigma^2} = 0 \text{ dB}$	$\frac{P_{N,T}}{\sigma^2} = 5 \text{ dB}$	$\frac{P_{N,T}}{\sigma^2} = 10 \text{ dB}$	$\frac{P_{N,T}}{\sigma^2} = 15 \text{ dB}$	$\frac{P_{N,T}}{\sigma^2} = 20 \text{ dB}$
$N = 1$	0.5	0.5	0.5	0.5	0.5
$N = 2$	0.5158	0.4321	0.3460	0.2626	0.1879
$N = 3$	0.4876	0.3646	0.2382	0.1288	0.0561
$N = 4$	0.4556	0.3066	0.1583	0.0536	0.0121
$N = 5$	0.4249	0.2565	0.0987	0.0180	0.0020

Figure 5.2: Outage probability in a DF relay system with direct link. $N = 2$, $d = 0.5$.

5.4 Numerical Results

To verify our proposed adaptive algorithms, we show some numerical results in this section. We compare the outage performance of the following algorithms: 1) Uniform allocation algorithm ($P_{N,s} = P_{N,r}$ and $d = 0.5$); 2) Semi-adaptive allocation algorithm, where we adopt adaptive PA under $d = 0.5$ or we choose the optimal relay location under $P_{N,s} = P_{N,r}$; 3) Fully-adaptive allocation algorithm, where we jointly optimize PA and relay location at the same time. We set $\alpha = 4$ and $\gamma_{th} = 10 \text{ dB}$ throughout this section. It should be noted that all the results below are the exact system outage probabilities without any approximation unless

Figure 5.3: Performance of adaptive/semi-adaptive/fixed allocation algorithms in a DF relay system with direct link.

otherwise stated.

Firstly, we consider a DF relay system with direct link in Figures 5.2-5.5. Figure 5.2 compares the exact outage probability in (5.2.7) with the approximation in (5.2.8). We can see that the approximation well matches the actual values in a broad SNR range.

Figure 5.3 compares the outage performance of different allocation algorithms with $N = 1$ and $N = 2$. We can see that when there is only one destination antenna, the uniform allocation algorithm has very close performance to the semi-adaptive allocation algorithms. Actually, the optimal relay location under uniform PA is exactly at the middle point of the source and destination nodes. Therefore, the semi-adaptive allocation algorithm ($d = 0.5$) is equivalent to the uniform allocation algorithm. However, fully-adaptive allocation algorithm significantly outperforms both the uniform and semi-adaptive allocation algorithms. When there are more than one destination antennas, the adaptive PA under the fixed relay location outperforms the uniform allocation algorithm. It is interesting to find that jointly optimizing PA and relay location does not bring much per-

Figure 5.4: Optimal source power ratio versus relay location in a DF relay system with direct link under various $P_{N,T}$. The circled points are the jointly optimized solution. $N = 2$.

formance gain over the algorithm of optimizing relay location under the uniform PA.

Figures 5.4 and 5.5 plot the optimal source power ratio ($P_{N,s}^*/P_{N,T}$) versus the relay location under different total power constraints ($P_{N,T}$) and under different number of destination antennas, respectively. We can see that the closer the relay node moves to the destination, the less power will be needed at the relay node. This is because that with shorter R-D link (better second hop performance), the overall system performance is more dependent on the S-R link and S-D link. In this case, allocating more power to the source node will improve the overall system performance. On the other way around, we can observe that the more power allocated to source, the shorter the optimum R-D distance will be. This is because that with less relay power, the R-D link will have more impact on the overall system performance. Therefore, reducing R-D distance will improve the overall system performance. Further observations of Figure 5.4 show that with the same relay location, more total power leads to a higher source power ratio. It has been shown in [69] that employing more destination antennas also leads

Figure 5.5: Optimal source power ratio versus relay location in a DF relay system with direct link under various N . The circled points are the jointly optimized solution. $P_{N,T}/\sigma^2 = 20$ dB.

to a higher source power ratio. These observations are very beneficial since in practice we expect the relay to consume less power in re-transmitting the source data while keeping a good system performance. The circled points in Figures 5.4 and 5.5 are the optimal S-R distance and source power ratio pairs which correspond to the third row and the sixth column in Table 5.1, respectively.

In Figures 5.6-5.9, we consider a DF relay system without direct link. Figure 5.6 compares the exact outage probability with the approximation in (5.2.9). We can see that the approximation also well matches the actual values in a broad SNR regime.

Figure 5.7 compares the system outage performance under different allocation schemes with $N = 3$. We can see that with the fixed relay location (i.e., $d = 0.5$), the proposed adaptive PA significantly outperforms the fixed PA (i.e., $P_{N,s} = P_{N,r}$). On the other hand, under the fixed PA, choosing the optimal relay location significantly reduces system outage probability. When the PA and relay location are jointly optimized, the minimum outage probability is achieved.

Figure 5.6: Outage probability in a DF relay system without direct link. $N = 3$, $d = 0.5$.

Figure 5.7: Performance of adaptive/semi-adaptive/fixed allocation algorithms in a DF relay system without direct link. $N = 3$.

Figure 5.8 plots the optimal source power ratio ($P_{N,s}^*/P_{N,T}$) versus the relay location with different total power constraints ($P_{N,T}$). Similar to Figure 5.4, we

Figure 5.8: Optimal source power ratio versus relay location in a DF relay system without direct link. The circled points are the jointly optimized solution. $N = 3$.

can see that the closer the relay node moves to the destination, the less power will be needed at the relay node. With the same relay location, more total power leads to a higher source power ratio. The circled points in Figure 5.8 (corresponding to the fourth row in Table 5.2) are the optimal S-R distance and source power ratio pairs under different total power constraints. We can see that the optimal source power ratio is equal to the optimal S-R distance, which exactly matches the analytical solution.

Figure 5.9 illustrates the impact of N on the system error performance when semi-adaptive allocation algorithms are adopted. If we optimize PA under the fixed relay location of $d = 0.5$, deploying two destination antennas significantly outperforms deploying one destination antenna. However, deploying more than two destination antennas does not bring further performance improvement as shown by the dashed curves. It shows that by simply adaptively allocating power, we might not be able to fully exploit the advantage of using multiple destination antennas. However, if we fix the PA (i.e., $P_{N,s} = P_{N,r}$) and locate the relay node at the optimal position, the full diversity order can be achieved as shown by the solid curves. The similar result can be observed if we jointly optimize PA

Figure 5.9: Impact of N on the semi-adaptive allocation algorithms in a DF relay system without direct link.

and relay location at the same time. This indicates that choosing the optimal relay location can fully exploit the advantage of employing multiple destination antennas. These remarks can also be observed in Figure 5.7 for $N = 3$.

Figures 5.4 and 5.8 also present a simple way of finding the joint optimal solution (the optimal PA and relay location pair) under various total power constraints. With the semi-adaptive allocation algorithms, we can easily plot the curve of the optimal source power ratio versus the relay location or vice versa under a given total power level. Then the intersection of this curve with the straight line represented by the equation of $\frac{P_{N,s}^*}{P_{N,T}} = d^*$ for a system without direct link and with the line represented by $\frac{P_{N,s}^*}{P_{N,T}} = \frac{N}{N-1} - \frac{N+1}{d^*(N-1)^2+2(N-1)}$ for a system with direct link will be the joint optimal solution.

Chapter 6

Adaptive Power Allocation in Multi-hop Relaying Channels

In Chapter 5, we have investigated the optimal PA in dual-hop relay systems. In this chapter, we explore adaptive PA algorithms to minimize the system SER for multi-hop DF relaying channels with imperfect CSI due to limited feedback. Our main results are:

- We propose an analytical solution to the PA optimization problem of minimizing the instantaneous SER upper bound of a multi-hop DF relay system when the perfect CSI is known at all nodes. Since the analytical solution is provided, the computational complexity at each node is low. It also works as benchmark to compare the performance of other PA algorithms to be considered later on.
- In the situation when the channels are fast varying, we propose PA algorithms with limited feedback when no perfect CSI is available at the transmitters. Here, the transmitters include the source and all relay nodes. We discuss two feedback schemes: (1) the optimal PA is computed at the destination node and then is quantized and fed back to the transmitters; (2) a quantized version of CSI is fed back to the transmitters and the optimal PA is calculated at the transmitters [70]. We develop a codebook construction method based on the generalized Lloyd algorithm [71]. It is noted that the

quantizer for the first feedback scheme needs to be re-optimized for each different SNR and needs more than one codebook if the system has multiple operating SNR points. So, we also propose a design method that does not depend on the system operating SNR in the high SNR regime.

6.1 System Model

We consider a multi-hop system consisting of one source node, one destination node and $(M_r - 1)$ intermediate regenerative relay nodes. We assume that there is no direct path from the source to the destination, which may result from high shadowing caused by obstacles between the two nodes. We further assume that each relay node only forwards the data received from its immediate preceding node. That is, no diversity combining will be implemented at the receivers. Here, the receivers include the destination and all relay nodes. In this chapter, TDM transmissions are considered. In the first time slot, the source node sends data to the first relay node. In the m -th ($1 < m \leq M_r$) time slot, the $(m - 1)$ -th relay node decodes the signal received from the previous node, re-encodes it and forwards the re-encoded data to the next relay node (the destination node if $m = M_r$). The received signal over the m -th hop can be modeled as

$$Y(m) = \sqrt{p(m)}h(m)X(m) + Z(m) \quad (6.1.1)$$

where $X(m)$ denotes the transmitted signal with $E[|X(m)|^2] = 1$ using any modulation, $p(m)$ denotes the transmit power, $h(m)$ denotes the instantaneous channel gain, $Y(m)$ denotes the received signal, and $Z(m)$ denotes the additive noise at the receiver over the m -th hop. Without loss of generality, we assume the channels are subject to independent Rayleigh fading and uniform path-loss, with a path loss exponent α . Therefore, the channel coefficient between two nodes with distance d is distributed as $h \sim CN(0, d^{-\alpha})$. It should be noted that the PA algorithms to be proposed in the following sections can work with other fading distributions. The case when the channels in different hops encounter different path loss exponents can be equivalently evaluated by varying inter-node distances

with uniform path loss exponent. We consider quasi-static channels where the channels remain constant during the time for the training sequences to reach the destination and additional M_r time slots for multi-hop data transmission. We assume that all the noise terms are additive zero-mean white circular complex Gaussian variables with variance σ^2 .

6.2 Optimal Power Allocation with Perfect CSI

Let $\mathcal{P}_e$ denote the symbol error probability for a narrow-band system with instantaneous SNR, γ , over an AWGN channel. The nearest neighbor union bound (NNUB) of $\mathcal{P}_e$ is given by [72]

$$\mathcal{P}_e(\gamma) \leq N_e Q \left(\sqrt{\frac{1}{2} d_{min}^2 \gamma} \right) \quad (6.2.1)$$

where N_e is the average number of the nearest neighbors to points in S , $Q(\cdot)$ is the Q-function defined as $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{x^2}{2}} dx$, d_{min} is the minimum distance of S , and S is the constellation used in the transmission. If BPSK modulation is adopted in the transmission, then $d_{min} = 2$.

Letting $\mathcal{P}_{e,m}$ denote the instantaneous SER over the m -th hop, the instantaneous SER at the destination node in a dual-hop DF relay system with BPSK modulation is given by

$$\mathcal{P}_e = 1 - (1 - \mathcal{P}_{e,1}) \cdot (1 - \mathcal{P}_{e,2}) - \mathcal{P}_{e,1} \cdot \mathcal{P}_{e,2} \leq 1 - (1 - \mathcal{P}_{e,1}) \cdot (1 - \mathcal{P}_{e,2}) \leq \mathcal{P}_{e,1} + \mathcal{P}_{e,2}. \quad (6.2.2)$$

The upper bounds in (6.2.2) are based on the fact of relatively low SERs ($\ll 1$). In a similar way, the instantaneous SER in an M_r -hop DF relay system

can be upper bounded by

$$\begin{aligned} \mathcal{P}_e &\leq 1 - (1 - \mathcal{P}_{e,1}) \cdot (1 - \mathcal{P}_{e,2}) \dots (1 - \mathcal{P}_{e,M_r-1}) \cdot (1 - \mathcal{P}_{e,M_r}) \\ &\leq \sum_{m=1}^{M_r} N_e Q \left(\sqrt{\frac{1}{2} d_{min}^2 \frac{p(m)|h(m)|^2}{\sigma^2}} \right). \end{aligned} \quad (6.2.3)$$

Since the Q-function can be upper bounded by an exponential function [73], the instantaneous SER upper bound in (6.2.3) can be approximated as

$$\mathcal{P}_e \leq \sum_{m=1}^{M_r} N_e \frac{\exp \left(\frac{-d_{min}^2 p(m)|h(m)|^2}{4\sigma^2} \right)}{2}. \quad (6.2.4)$$

Defining $g(m) = -d_{min}^2 |h(m)|^2 / (4\sigma^2)$, the problem of adaptively allocating power among the source and relay nodes under a total power constraint P_T to minimize the instantaneous SER upper bound can be formulated mathematically as

$$[p^*(1)p^*(2)\dots p^*(M_r)]^T = \arg \min_{[p(1)p(2)\dots p(M_r)]^T} \sum_{m=1}^{M_r} N_e \frac{\exp(p(m)g(m))}{2} \quad (6.2.5)$$

$$\text{subject to: } \sum_{m=1}^{M_r} p(m) \leq P_T \quad (6.2.6a)$$

$$p(m) > 0. \quad (6.2.6b)$$

The SER upper bounds in (6.2.3) and (6.2.4) are sums of Q-functions and exponential functions, respectively. Since functions $Q(\sqrt{x})$ and $\exp(-x)$ are convex and nonincreasing functions, the SER upper bounds are also convex from the compositions preserving convexity [68]. The constraints stated in (6.2.6) are linear inequalities. Therefore, the PA optimization problem is convex.

To make the optimization problem tractable, we firstly loosen the constraint (6.2.6b) and solve a modified optimization problem. With the Lagrange multiplier

method, the modified objective function can be written as

$$J = \sum_{m=1}^{M_r} N_e \frac{\exp(p(m)g(m))}{2} - \mu \left(\sum_{m=1}^{M_r} p(m) - P_T \right). \quad (6.2.7)$$

Taking its derivative and equating it to zero, we can find the optimal solution as

$$p(m)^* = \frac{\ln\left(\frac{-2\mu}{N_e}\right) - \ln(-g(m))}{g(m)} \quad (6.2.8)$$

where μ has to be chosen in a way that the total power constraint (6.2.6a) is fulfilled.

Now we take the constraint (6.2.6b) into account and discuss the solution (6.2.8) in two cases:

Case 1: If $P_T > \ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)}$, the optimal PA solution and the corresponding μ are found as

$$p(m)^* = \frac{\frac{P_T + \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)}}{\sum_{n=1}^{M_r} \frac{1}{g(n)}} - \ln(-g(m))}{g(m)} \quad (6.2.9)$$

$$\ln\left(\frac{-2\mu}{N_e}\right) = \frac{P_T + \sum_{m=1}^{M_r} \frac{\ln(-g(m))}{g(m)}}{\sum_{m=1}^{M_r} \frac{1}{g(m)}}. \quad (6.2.10)$$

Case 2: If $P_T \leq \ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)}$, we cannot find an appropriate μ to satisfy $p(m)^* > 0$. That is, the solution to the modified optimization problem is not the solution to the original optimization problem. In this case, we alternatively adopt the PA algorithm of minimizing average SER in [49] by jointly considering the tradeoff between complexity and performance. Letting σ_m^2 denote the variance of channel gain over hop m , the PA solution is expressed as

$$p(m)^* = \frac{P_T}{\sigma_m \sum_{n=1}^{M_r} \frac{1}{\sigma_n}}. \quad (6.2.11)$$

Although (6.2.11) is suboptimal, simulation results show that the chance for the channels to fall into Case 2 is rare, especially in the high SNR regime as shown in Figure 6.1. Therefore, it does not degrade the average performance

Figure 6.1: Ratio 1: the probability of $P_T \leq 10 \left(\ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)} \right)$. Ratio 2: the probability of $P_T \leq \left(\ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)} \right)$. $M_r = 4$, $\alpha = 4$.

of the proposed PA algorithm. The combination of Case 1 and Case 2 leads to the final and complete solution to the optimization problem stated in (6.2.5). We refer to this algorithm as Alg1. Simulation results in Section 6.4 will show that this PA algorithm significantly outperforms existing PA algorithms in literature. It should be noted that since this PA algorithm requires instantaneous CSI, it will be more beneficial when the channels vary slowly.

6.3 Adaptive Power Allocation with Imperfect CSIT

The optimal PA algorithm presented in Section 6.2 is based on the assumption that all the transmitters and receivers have perfect CSI. However, it is not practical since it requires an infinite rate of feedback link. In this section, we will present PA algorithms with imperfect CSIT due to limited feedback.

We assume that the receivers have perfect knowledge of CSI. We further assume that the feedback link is reliable and error free. This is possible because the rate of feedback link is very low. The destination can discover the CSI in the entire system by training symbols. In practical systems, different nodes may transmit their training symbols simultaneously and use orthogonal codes. Therefore, the destination can distinguish between different nodes and determine the CSI over different hops accordingly. Considering the specific case where the destination cannot receive signals from some non-neighboring nodes (at least from the source node due to high shadowing between the source and destination nodes), the following procedure can be implemented for the destination to discover the entire system CSI. For simplicity, we take a two-hop system for example. The relay and the destination can discover S-R link channel gain $h(1)$ and R-D link channel gain $h(2)$ using training symbols T_1 and T_2 , respectively. We let the relay append S-R received training symbols T_1 into R-D transmit packet. The S-R gain is found by noting that when T_1 is received at the destination it will contain the product $h(1) \cdot h(2)$. Since $h(2)$ is known at the destination, it can be compensated for, and then $h(1)$ can be found [74]. For multi-hop relaying channels, the similar procedure can be done iteratively until the destination gets the CSI of all hops.

There are usually two feedback schemes: (1) the optimal PA is computed at the destination with known CSI and then is quantized and fed back to the transmitters; and (2) a quantized version of CSI is fed back to the transmitters and the optimal PA is calculated at the transmitters.

We mainly discuss how the first feedback scheme works in the rest of this section. The similar procedure can be implemented for the second scheme. If we have L_B feedback bits, the destination can inform the transmitters of 2^{L_B} transmit power levels depending on the observed CSI. This requires a mapping from the channel state space to a discrete set of power levels. The 2^{L_B} power levels for all hops form a codebook denoted by $\mathbf{A} = \{\hat{p}_j(m)\}_{2^{L_B} \times M_r}^T$, $j = 1, 2, \dots, 2^{L_B}$, $m = 1, 2, \dots, M_r$, where $\hat{p}_j(m)$ is the j -th power level for the m -th hop. The problem of minimizing the average SER upper bound with a finite number of transmit power levels can be formulated as the following vector quantization

problem

$$\mathbf{A}^* = \arg \min_{\mathbf{A}} \sum_{j=1}^{2^{L_B}} \int_{\gamma_j(h)} \sum_{m=1}^{M_r} \frac{N_e}{2} \exp(\hat{p}_j(m)g(m)) f(h)dh \quad (6.3.1)$$

$$\text{subject to: } \sum_{j=1}^{2^{L_B}} \int_{\gamma_j(h)} \sum_{m=1}^{M_r} \hat{p}_j(m) f(h)dh = P_T. \quad (6.3.2)$$

Herein, the channel state space is partitioned into 2^{L_B} subsets $\gamma_1(h), \gamma_2(h), \dots, \gamma_{2^{L_B}}(h)$, each of which is mapped onto a codeword in the codebook $\mathbf{A}$ (e.g., $\gamma_j(h)$ is mapped to $[\hat{p}_j(1)\hat{p}_j(2)\dots\hat{p}_j(M_r)]^T$), and $f(h)$ is the pdf of h . In (6.3.1), $\frac{N_e}{2} \exp(\hat{p}_j(m)g(m))$ is the SER upper bound of hop m with respect to a single channel realization $\gamma_j(h)$ and $\sum_{j=1}^{2^{L_B}} \int_{\gamma_j(h)} \sum_{m=1}^{M_r} \frac{N_e}{2} \exp(\hat{p}_j(m)g(m)) f(h)dh$ is the average SER upper bound over all hops and over all channel realizations. Constraint (6.3.2) guarantees the total power constraint over all channel realizations.

It is intractable to find an analytical solution to the optimization problem (6.3.1). Instead, the generalized Lloyd algorithm for vector quantization with a modified distortion metric (i.e., representing the optimal PA over each hop as closely as possible) can be adopted to find the optimal codebook iteratively. Similarly, the codebook for the second feedback scheme can be constructed with a distortion metric of representing the exact channel gain over each hop as closely as possible. For convenience, we rewrite $\mathbf{A}$ as $[\mathbf{a}^1 \mathbf{a}^2 \dots \mathbf{a}^{2^{L_B}}]$. The codebook $\mathbf{A}$ for the first feedback scheme is designed to minimize the following distortion function

$$D(\mathbf{A}) = E \left\{ \min_{1 \leq j \leq 2^{L_B}} \|\mathbf{a}^j - \mathbf{G}\|_2^2 \right\} \quad (6.3.3)$$

where $\mathbf{G} = [p^*(1)p^*(2)\dots p^*(M_r)]^T$ is the optimal power loading vector (see Section 6.2) with respect to a single channel realization $\gamma_j(h)$.

The Lloyd algorithm can be easily implemented by generating Q test channels according to the assumed channel distribution (e.g., complex Gaussian distribution for Rayleigh fading channels). The optimal power loading vectors with respect to each test channel form a set $\mathcal{B}_p \subseteq \Re^{M_r+}$. Firstly, we randomly generate a codebook $\mathbf{A}_0 = [\mathbf{a}_0^1 \mathbf{a}_0^2 \dots \mathbf{a}_0^{2^{L_B}}]$ and set $i = 1$. Then, the following two steps are

repeatedly performed until convergence.

1) (Nearest Neighbor Rule) Given a transmission codebook $\mathbf{A}_{i-1}$, divide the set of optimal power loading vectors into 2^{L_B} quantization regions with the k -th region defined as

$$\mathcal{C}_k = \left\{ \mathbf{G} \in \mathcal{B}_p \mid \|\mathbf{a}_{i-1}^k - \mathbf{G}\|_2^2 \leq \|\mathbf{a}_{i-1}^l - \mathbf{G}\|_2^2, \forall l \neq k \right\}. \quad (6.3.4)$$

2) (Centroid Condition) Given a certain partition $\{\mathcal{C}_1, \mathcal{C}_2, \dots, \mathcal{C}_{2^{L_B}}\}$, construct a new codebook $\mathbf{A}_i$ with the k -th column $\mathbf{a}_i^{k*}$ given by

$$\mathbf{a}_i^{k*} = \arg \min_{\mathbf{a}_i^k} E \left\{ \|\mathbf{a}_i^k - \mathbf{G}\|_2^2 \mid \mathbf{G} \in \mathcal{C}_k \right\}. \quad (6.3.5)$$

The set of optimal power loading vectors in the k -th quantization cell can be written as $\mathcal{C}_k = [\mathbf{G}_1 \ \mathbf{G}_2 \ \dots \ \mathbf{G}_{Q_k}]^T$, where Q_k is the cardinality of the k -th region. Since the optimization problem (6.3.5) is very tedious, one approximate method is to set $\mathbf{A}_i^k$ equal to the dominant right singular vector of matrix $\mathcal{C}_k$. A detailed proof can be found in [75].

The codebook can be designed offline (i.e., not as a function of instantaneous channel conditions) and stored at all nodes. Upon measuring CSI, the destination will select a codeword $\mathbf{a}^q$ from the codebook $\mathbf{A}$ with $q = \arg \min_{1 \leq j \leq 2^{L_B}} \|\mathbf{a}^j - \mathbf{G}\|_2^2$, where $\mathbf{G}$ is the optimal power loading vector with respect to this specific channel state. The index q to the selected codeword will be conveyed back to the source and all relay nodes through a feedback channel. Upon reception of q , the transmitters will know the power they need to use in the next transmission (e.g., the transmit power over hop m will be $\mathbf{A}_{m,q}$). We refer to the PA with this quantizer as Algm 2. As a comparison, for the second feedback scheme, the selected codeword represents the quantized CSI and the transmitters still need to calculate the optimal PA by themselves. This PA is referred to as Algm 4, which will compare with other algorithms in Section 6.4.

A drawback of the above quantizer for the first feedback scheme is that it is optimized for a particular SNR (or P_T). As a result, we need more than one codebook if the system has multiple operating SNR points. It is thus important

to find another design method that does not directly rely on P_T . The optimal power loading vector in (6.2.9) can be rewritten as

$$p(m)^* = \frac{P_T - \left[\ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)} \right]}{g(m) \sum_{n=1}^{M_r} \frac{1}{g(n)}}. \quad (6.3.6)$$

In the high SNR regime (i.e., $P_T \gg \left[\ln(-g(m)) \sum_{n=1}^{M_r} \frac{1}{g(n)} - \sum_{n=1}^{M_r} \frac{\ln(-g(n))}{g(n)} \right]$), we neglect the bracketed terms in the numerator of (6.3.6). So, the optimal PA fraction on each hop can be expressed as

$$p'(m) = \frac{p(m)^*}{P_T} \approx \frac{1}{g(m) \sum_{n=1}^{M_r} \frac{1}{g(n)}}. \quad (6.3.7)$$

Based on (6.3.7), we can design a codebook with its codewords representing the quantized power fractions instead of the exact PA. The codebook can be designed with the similar procedures described previously with a modified distortion function given by $D(\mathbf{A}) = E\{\min_{\mathbf{a} \in \mathbf{A}} \|\mathbf{a} - \mathbf{G}'\|_2^2\}$, where $\mathbf{G}' = [p'(1) p'(2) \dots p'(M_r - 1) p'(M_r)]^T$. To justify this approximation, we have shown in Figure 6.1 that the chance for the neglected terms to be comparable with P_T is very rare when $P_T/\sigma^2 > 15$ dB. The PA based on this new quantizer is referred to as Algm 3.

6.4 Simulation Results and Discussions

In this section, we simulate a four-hop DF relay system and assume all the nodes are located in a line and the distance between the source and destination is normalized to 1. Including Figure 6.1, the distances between the neighboring nodes from the source to the destination are proportional to 1:2:3:4 unless otherwise specified. Without loss of generality, BPSK modulation is assumed.

Figure 6.2: SER performance of different PA algorithms. $\alpha = 2.6$.

Figure 6.3: SER performance of different PA algorithms. $\alpha = 4$.

We compare the performance of the following PA algorithms: 1) Algm 1 - the proposed algorithm of minimizing the instantaneous SER upper bound under perfect CSI; 2) Algm 2 - the proposed algorithm using feedback of quantized exact PA; 3) Algm 3 - the proposed algorithm using feedback of quantized PA fraction based on (6.3.7); 4) Algm 4 - the proposed algorithm with feedback of quantized channel gain (the second feedback scheme); 5) the algorithm of minimizing average SER proposed in [49], namely Boyer et al.'s adaptive PA; 6) uniform PA; 7) direct transmission.

Figures 6.2 and 6.3 show the system SER performance for $\alpha = 2.6$ and $\alpha = 4$, respectively. Since Algm 1 minimizes the instantaneous SER with respect to any channel realization, it outperforms all other PA algorithms. Although it requires the system to re-calculate the optimal PA with each channel realization, the complexity is modest with the analytical solution. It is especially beneficial when the channels vary slowly. Further more, it can also work as a benchmark to compare the performance of the proposed PA algorithms with limited feedback. Among the three proposed PA algorithms with limited feedback, Algm 4 performs only slightly better than the PA algorithm in [49] with $\alpha = 2.6$ and even performs worse with $\alpha = 4$. This implies that more feedback bits are needed for Algm 4 to achieve good performance. However, Algm 2 significantly outperforms the PA in [49] and has very close performance to Algm 1, which is the ideal case. This shows that PA Algm 2 performs well even with a small number of feedback bits. It is worth noticing that Algm 3 works only slightly worse than Algm 2 at high SNRs. However, a great amount of storage can be saved at each node if Algm 3 is adopted, since the system needs only one codebook for all operating SNR points.

Figure 6.4: SER performance of different PA algorithms. $\alpha = 4$. Relays are equally spaced between the source and destination.

Figure 6.5: SER performance of Algm 2 with different number of feedback bits. $\alpha = 4$.

Figure 6.4 plots the SER performance for $\alpha = 4$, where all the relays are equally spaced between the source and destination nodes. In this case, the PA in [49] is equivalent to the uniform PA. We can see that all the proposed PA algorithms outperform the PA in [49] and the direct transmission. Among all the considered PA algorithms, Algm 1 achieves the minimum SER. However, it requires a huge amount of feedback bits to make perfect CSI available at all nodes. Algm 2 and Algm 3 have very close performance to Algm 1. This again shows that both Algm 2 and Algm 3 are able to achieve good performance with a small number of feedback bits.

Figure 6.5 shows the performance of Algm 2 with different number of feedback bits. It can be observed that when there is only one bit of feedback, the performance of Algm 2 is poor. However, when there are more than one feedback bits, a significant performance gain is achieved by Algm 2 over the PA in [49]. Although more feedback bits yield better performance, from the figure we can see that with two feedback bits, the performance of Algm 2 is already very close to the proposed PA algorithm with perfect CSI.

Figure 6.6: Achievable end-to-end rates of different PA algorithms. $\alpha = 2.6$.

Figure 6.7: Outage probabilities of different PA algorithms. $\alpha = 2.6$.

The achievable end-to-end rates and outage probabilities are shown in Figures 6.6 and 6.7, respectively. The link quality threshold for an outage is set to $\gamma_{th}/\sigma^2 = 10$ dB. One can see that although our PA algorithms are not originally designed to optimize these two performance metrics, they still have very good performance. Among all the considered PA algorithms, Algm 1 achieves the maximum average end-to-end rate and the minimum outage probability, since it exploits full CSI. Further more, both Algm 2 and Algm 3, which exploit limited feedback, have better performance than the PA of minimizing average SER in [49] and the uniform PA.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

In this thesis, we proposed efficient temporal and spatial power control schemes to improve the performance of non-cooperative and cooperative multi-antenna systems under the assumption of imperfect CSI.

In the first part of this thesis, we provided a framework to systematically study the achievable DMT in MIMO and cooperative systems with noisy CSI. For a TDD MIMO system with imperfect CSIT and perfect CSIR, we proposed an optimal temporal power control scheme to significantly improve the achievable diversity gain. For the case where both CSIT and CSIR are imperfect, we firstly proposed the optimal temporal power control schemes and novel two-way training strategies and showed that two-way training together with the proposed power control significantly increased the optimal diversity gain and offered non-zero diversity gain at any achievable multiplexing gains. We considered two training strategies: 1) the destination initiates the training and 2) the source initiates the training. It is found that the optimal DMT after more than two rounds of two-way training is a single straight line. However, if there is only one round of two-way training, the achievable DMT is much lower and can either be a straight line, a collection of discontinuous line segments or piecewise-linear line depending on the training strategy, system configuration and channel qualities. At sufficient

high multiplexing gains, no training can help bring any diversity gain. Secondly, we proposed rate control schemes under imperfect CSITR. We show that if there is no minimum instantaneous multiplexing gain constraint r_{min} , the diversity gain is infinity with rate control itself; otherwise it is finite and limited by the diversity gain at r_{min} . With joint power and rate control, the diversity gain can be further increased significantly.

Next, we extended our results in MIMO channels to cooperative relaying channels. We firstly studied DMT performance in orthogonal relaying channels with imperfect CSIT and multiple destination antennas. Through power control, we derived the achievable DMT of AF and adaptive/fixed DF schemes as well as a DMT upper bound of the relaying channel. The obtained DMT results show that exploiting imperfect CSIT significantly increases the achievable diversity gain. It is also shown that simple DF and AF schemes can achieve the DMT upper bound if the CSIT quality of the S-R channel is sufficiently high. Secondly, we derived the DMT of relaying schemes with higher spectral efficiency, namely DDF relaying protocol. It is shown that power control based on the imperfect CSIT significantly increases the optimal diversity gain. In particular, at high spatial multiplexing gains, the achievable diversity is dominated by the S-D link CSIT quality while the S-R link and R-D link CSIT quality does not help to improve the overall diversity. It is also shown that CSIT at the relay does not contribute to the overall diversity if the source node has no CSIT at all.

In the second part of this thesis, we investigated optimal spatial power control schemes in relaying channels to minimize the system outage probability or SER. First, we studied PA problems in a dual-hop DF relay system with multiple destination antennas. We studied both systems with and without direct link, respectively. We proposed adaptive PA at the source and relay under a joint power constraint to minimize the system outage probability with a fixed relay location. We also derived the optimal relay location conditioned on the fixed PA. Further, we jointly optimized PA and relay location at the same time. Numerical results show that the proposed adaptive allocation algorithms significantly outperform the fixed allocation schemes. In the system without direct link, it is shown that

choosing the optimal relay location can fully exploit the advantage of employing multiple destination antennas. In both systems with and without direct link, deploying multiple destination antennas and/or choosing the appropriate relay location can greatly save the needed power at the relay node.

Next, we investigated adaptive PA for a multi-hop DF relay system. We considered two scenarios. When the perfect CSI is known, we proposed an adaptive PA algorithm under a total power constraint to minimize the instantaneous system SER upper bound. When the transmitters have no perfect knowledge of CSI, we proposed PA algorithms with limited feedback. Simulation results show that the proposed PA algorithm with perfect CSI significantly outperforms the existing PA solutions. Further more, with a small number of feedback bits, the proposed PA algorithms based on limited feedback are able to achieve very close performance to the proposed adaptive PA with perfect CSI.

7.2 Future Work

The work presented in this thesis can be extended in many ways. Here are some examples for potential future research.

- **Distributed spatial power control for wireless relay networks**

In literature, most papers on the optimal power allocation algorithms have assumed that there is a centralized base station or access point that allocates power resources to each link. These centralized schemes require the feedback of CSI of all communication channels to the central controller for each channel realization, which may be infeasible in practice. As such, distributed strategies are needed, where the power is allocated and optimized in a distributed manner at each node.

- **Depicting the tradeoff between the diversity gain, multiplexing gain and multi-access gain under imperfect CSITR**

Despite many works on the impacts of imperfect CSITR on MIMO systems, the performance limits of MIMO multiple access channels are considerably

less understood. In Chapters 6 and 7, we derived the DMT in point-to-point MIMO channels and three-node relaying channels under imperfect CSITR. The techniques developed for MIMO/relaying channels cannot be readily extended to multi-access channels with imperfect CSITR, because different users may have different level of channel knowledge and thus should use different power/rate control schemes, which would greatly complicate the DMT analysis. Therefore, new power and rate control schemes need to be investigated. We should not restrict ourselves to the symmetric scenario where each user attains the same multiplexing gain and the scenario where different users have different rate and diversity requirements should be considered as well. The DMT performance of various suboptimal decoders such as successive interference cancelation based decoder can also be studied.

- **Extension to general multi-antenna cooperative networks over general fading channels**

In Chapter 4, we considered a simple three-node multi-antenna cooperative system over Rayleigh fading channel, while the source and relay have single antenna each and the destination has multiple antennas. It is interesting to extend our results to a general multiple-antenna network with multiple sources, multiple destinations, and multiple relays over other compound fading channels, e.g., Rician fading, general-K fading, where each node may have multiple antennas. In addition to AF/DF relaying protocols discussed in this thesis, it is also a very interesting topic to look into CF relaying with imperfect CSITR. Two-way relaying is also a good extension.

- **Secure transmission with imperfect CSITR**

The broadcast nature of wireless medium makes wireless networks susceptible to eavesdropping, and hence secure transmission is a fundamental issue in wireless networks. With partial CSI, efficient power and/or rate control schemes at the transmitters can be proposed to limit knowledge of the existence of the information at the adversary. Thus, it is interesting to use the tool developed in this thesis to characterize the achievable diversity versus secure multiplexing gain tradeoff through secrecy outage analysis.

Appendix A

Proof of Lemma 3.2.2

Let $v_{1,k_1}, \dots, v_{k_1,k_1}$ denote the solutions of $v_1, \dots, v_{k_1}$ that minimize $d_{k_1}(r)$, and $v_{1,k_2}, \dots, v_{k_2,k_2}$ denote the solutions of $v_1, \dots, v_{k_2}$ that minimize $d_{k_2}(r)$. According to (3.2.21) and (3.2.22), we may assume $v_{1,k_1} = \dots = v_{i-1,k_1} = \tau(k_1)$, $v_{i,k_1} = (N - i + 1)\tau(k_1) - (k_1 - i)\alpha - r$, $v_{i+1,k_1} = \dots = v_{k_1,k_1} = \alpha$, $v_{1,k_2} = \dots = v_{j-1,k_2} = \tau(k_2)$, $v_{j,k_2} = (N - j + 1)\tau(k_2) - (k_2 - j)\alpha - r$ and $v_{j+1,k_2} = \dots = v_{k_2,k_2} = \alpha$, where $\tau(k_1) > v_{i,k_1} \geq \alpha$, $\tau(k_2) > v_{j,k_2} \geq \alpha$. It follows that

$$B = \{(N - j + 1)\tau(k_2) - (k_2 - j)\alpha - v_{j,k_2}\} - \{(N - i + 1)\tau(k_1) - (k_1 - i)\alpha - v_{i,k_1}\} = 0. \quad (\text{A.1})$$

We consider the following two cases. Case 1) $j < i$: Since $v_{i,k_1} \geq \alpha$ and $v_{j,k_2} < \tau(k_2)$, we have

$$\begin{aligned} B &> \{(N - j)\tau(k_2) - (k_2 - j)\alpha\} - \{(N - i + 1)\tau(k_1) - (k_1 - i + 1)\alpha\} \\ &\geq \{(N - i + 1)\tau(k_2) - (k_2 - i + 1)\alpha\} - \{(N - i + 1)\tau(k_1) - (k_1 - i + 1)\alpha\} \\ &\geq \alpha(k_2 - k_1)((N - i + 1)(M - N + k_2 + k_1) - 1) > 0. \end{aligned} \quad (\text{A.2})$$

This contradicts with $B = 0$. Therefore, $j < i$ is not possible.

Case 2) $j \geq i$: It is observed that

$$v_{1,k_2} - v_{1,k_1} = \dots = v_{i-1,k_2} - v_{i-1,k_1} = k_2\alpha(M - N + k_2) - k_1\alpha(M - N + k_1) > 0$$

$$v_{i+1,k_2}, \dots, v_{k_2,k_2} \geq \alpha = v_{i+1,k_1} = \dots = v_{k_1,k_1}.$$

If $j > i$, we have $v_{i,k_2} - v_{i,k_1} > k_2\alpha(M - N + k_2) - k_1\alpha(M - N + k_1) > 0$; otherwise, we have $v_{i,k_2} - v_{i,k_1} = \alpha(k_2 - k_1)((N - i + 1)(M - N + k_2 + k_1) - 1) > 0$ from (A.1). Therefore,

$$d_{k_2}(r) - d_{k_1}(r) = \sum_{n=1}^{k_2} (2n - 1 + M - N)v_{n,k_2} - \sum_{n=1}^{k_1} (2n - 1 + M - N)v_{n,k_1} > 0. \quad (\text{A.3})$$

Combining the two cases, the proof of Lemma 3.2.2 is complete.

Appendix B

Proof of Lemma 3.2.3

According to (3.2.21), the spatial multiplexing gain of corner point k' ($k' = 1, \dots, k$) on the DMT curve $d_k(r)$ is given by $r_{k'} = (N - k')\tau(k) - (k - k')\alpha$. It is obvious that $r_k \in \tilde{\Omega}_k$. We now compare $r_{k'}$ ($k' = 1, \dots, k - 1$) with the lower boundary of Ω_{k-1} , i.e., $(N - k + 1)\tau(k - 1)$, and get $r_{k'} - (N - k + 1)\tau(k - 1) \geq ((N - k + 1)(M - N + 2k - 1) - 1)\alpha > 0$. If $r_{k'} < N$, we have $r_{k'} \in \Omega_{k-1}$. Otherwise, we get $r_{k'} \notin \Omega_k$. Since $\Omega_{k-1} \cap \tilde{\Omega}_k = \emptyset$ and $\tilde{\Omega}_k \subseteq \Omega_k$, both cases lead to $r_{k'} \notin \tilde{\Omega}_k$ for $k' = 1, \dots, k - 1$. This completes the proof of Lemma 3.2.3.

Appendix C

Proof of Theorem 3.3.1

We let z_n denote the exponential order of $1/\lambda_n(\mathbf{E}_f(I))$, i.e., $z_n = -\lim_{\rho \rightarrow \infty} \frac{\log(\lambda_n(\mathbf{E}_f(I)))}{\log(\rho)}$. The joint pdf of the random vector $\mathbf{z} = [z_1, \dots, z_N]$ conditioned on B_I can be shown to be [6],

$$p(\mathbf{z}|B_I) \doteq \begin{cases} 0, & \text{for any } z_n < B_I \\ \prod_{n=1}^N \rho^{-(2n-1+M-N)(z_n-B_I)}, & \text{for all } z_n \geq B_I. \end{cases} \quad (\text{C.1})$$

To calculate the outage probability, we only need to consider the case $z_n \geq B_I$. Therefore, the outage probability in (3.3.7) at high SNRs can be expressed as

$$\begin{aligned} \mathcal{P}_{out,2} &\doteq \mathcal{P} \left(\sum_{j=1}^N \left(1+t \sum_{n=1}^N (2n-1+M-N)v_{I,b,n} - v_{I,f,j} \right. \right. \\ &\quad \left. \left. - \left(1+t \sum_{n=1}^N (2n-1+M-N)v_{I,b,n} - z_N \right)^+ \right)^+ < r \right) \\ &\doteq \mathcal{P} \left(\sum_{j=1}^N \left(\min(\beta, 1) + t \sum_{n=1}^N (2n-1+M-N)v_{I,b,n} - v_{I,f,j} \right)^+ < r \right). \end{aligned} \quad (\text{C.2})$$

Given $\mathbf{v}$ and $\mathbf{v}_{i-1,f}$, the conditional joint pdf of $\mathbf{v}_{i,b}$ is given by [16] [76]

$$p(\mathbf{v}_{i,b}|\mathbf{v}, \mathbf{v}_{i-1,f}) \doteq \prod_{j=1}^{t_{i,b}} \rho^{-(2j-1+M-N)(v_{i,b,j}-A_{i-1})} \quad (\text{C.3})$$

if $\min(v_j, v_{i,b,j}) \geq A_{i-1}$, $\forall j = 1, \dots, t_{i,b}$ and $0 \leq v_j = v_{i,b,j} < A_{i-1}$, $\forall N \geq j > t_{i,b}$, $0 \leq t_{i,b} \leq N$; otherwise, $p(\mathbf{v}_{i,b}|\mathbf{v}, \mathbf{v}_{i-1,f}) = 0$. Given $\mathbf{v}$ and $\mathbf{v}_{i,b}$, the conditional joint pdf of $\mathbf{v}_{i,f}$ is given by

$$p(\mathbf{v}_{i,f}|\mathbf{v}, \mathbf{v}_{i,b}) \doteq \prod_{j=1}^{t_{i,f}} \rho^{-(2j-1+M-N)(v_{i,f,j}-B_i)} \quad (\text{C.4})$$

if $\min(v_j, v_{i,f,j}) \geq B_i$, $\forall j = 1, \dots, t_{i,f}$ and $0 \leq v_j = v_{i,f,j} < B_i$, $\forall N \geq j > t_{i,f}$, $0 \leq t_{i,f} \leq N$; otherwise, $p(\mathbf{v}_{i,f}|\mathbf{v}, \mathbf{v}_{i,b}) = 0$.

Combining (3.2.10a), (C.3) and (C.4), the joint pdf of $\mathbf{v}, \mathbf{v}_{1,f}, \dots, \mathbf{v}_{I,f}, \mathbf{v}_{1,b}, \dots, \mathbf{v}_{I,b}$ is given by

$$\begin{aligned} p(\mathbf{v}_{1,b}, \dots, \mathbf{v}_{I,b}, \mathbf{v}_{1,f}, \dots, \mathbf{v}_{I,f}, \mathbf{v}) &= \prod_{i=1}^I \left(p(\mathbf{v}_{i,f}|\mathbf{v}, \mathbf{v}_{i,b}) p(\mathbf{v}_{i,b}|\mathbf{v}, \mathbf{v}_{i-1,f}) \right) p(\mathbf{v}) \\ &\doteq \rho^{-\sum_{i=1}^I \left(\sum_{j=1}^{t_{i,b}} (2j-1+M-N)(v_{i,b,j}-A_{i-1}) + \sum_{j=1}^{t_{i,f}} (2j-1+M-N)(v_{i,f,j}-B_i) \right) - \sum_{n=1}^N (2n-1+M-N)v_n} \end{aligned} \quad (\text{C.5})$$

if $\min(v_j, v_{i,b,j}) \geq A_{i-1}$, $\forall j = 1, \dots, t_{i,b}$; $0 \leq v_j = v_{i,b,j} < A_{i-1}$, $\forall N \geq j > t_{i,b}$; $\min(v_j, v_{i,f,j}) \geq B_i$, $\forall j = 1, \dots, t_{i,f}$; $0 \leq v_j = v_{i,f,j} < B_i$, $\forall N \geq j > t_{i,f}$; and $0 \leq t_{i,b}, t_{i,f} \leq N$. Otherwise, we have $p(\mathbf{v}_{1,b}, \dots, \mathbf{v}_{I,b}, \mathbf{v}_{1,f}, \dots, \mathbf{v}_{I,f}, \mathbf{v}) = 0$.

In order to have $p(\mathbf{v}_{1,b}, \dots, \mathbf{v}_{I,b}, \mathbf{v}_{1,f}, \dots, \mathbf{v}_{I,f}, \mathbf{v}) \neq 0$, we either have $v_j = v_{I,f,j} < B_I$ or $v_j, v_{I,f,j} \geq B_I$. Both cases lead to $\left(\min(\beta, 1) + t \sum_{n=1}^N (2n-1+M-N)v_{I,b,n} - v_{I,f,j} \right)^+ = \left(\min(\beta, 1) + t \sum_{n=1}^N (2n-1+M-N)v_{I,b,n} - v_j \right)^+$. Therefore, we may simply replace $v_{I,f,j}$ with v_j in (C.2). Careful observation of (C.5) indicates that $v_{i,b,n} \geq \min(v_n, A_{i-1})$ and $v_{i,f,n} \geq \min(v_n, B_i)$. Since at high SNRs, the outage probability is dominated by the term with the largest SNR exponent, the dominant outage event will occur when $v_{i,b,n} = \min(v_n, A_{i-1})$ and $v_{i,f,n} = \min(v_n, B_i)$. Therefore, we can reexpress the outage probability as

$$\begin{aligned} \mathcal{P}_{out,2} &\doteq \mathcal{P} \left(\sum_{j=1}^N \left(\min(\beta, 1) + t \sum_{n=1}^N (2n-1+M-N) \min(v_n, A_{I-1}) - v_j \right)^+ < r \right) \\ &\doteq \rho^{-d_{CSITR}^*(r)} \end{aligned}$$

where

$$d_{CSITR}^*(r) = \min_{v_n \in \hat{\mathcal{O}}^+} \sum_{n=1}^N (2n-1+M-N)v_n \quad (\text{C.6})$$

and

$$\begin{aligned} \hat{\mathcal{O}} = & \left\{ v_n \mid \sum_{j=1}^N \left(\min(\beta, 1) + t \sum_{n=1}^N (2n-1+M-N) \min(A_{I-1}, v_n) - v_j \right)^+ < r, \right. \\ & A_i = \alpha + t \sum_{n=1}^N (2n-1+M-N) \min(B_i, v_n), A_0 = \alpha, \\ & \left. B_i = \beta + t \sum_{n=1}^N (2n-1+M-N) \min(A_{i-1}, v_n), i = 1, \dots, I \right\}. \end{aligned}$$

To solve the optimization problem in (C.6), we divide $\hat{\mathcal{O}}$ into $N+1$ disjoint subsets, $\hat{\mathcal{O}}_0, \dots, \hat{\mathcal{O}}_N$, and solve the following subproblem under each subset

$$\hat{d}_k(r) \triangleq \inf_{(v_1, \dots, v_N) \in \hat{\mathcal{O}}_k^+} \sum_{n=1}^N (2n-1+M-N)v_n \quad (\text{C.7})$$

where subset $\hat{\mathcal{O}}_k$ ($0 \leq k \leq N$) is defined as $\hat{\mathcal{O}}_k \triangleq \hat{\mathcal{O}} \cap \{v_1 \geq \dots \geq v_k \geq A_{I-1} \geq v_{k+1} \geq \dots \geq v_N\}$. After obtaining $\hat{d}_k(r)$, we will go back to solve the main problem in (C.6), which can be reexpressed as

$$d_{CSITR}^*(r) = \min(\hat{d}_0(r), \hat{d}_1(r), \dots, \hat{d}_N(r)) \quad (\text{C.8})$$

implying that among all the DMT curves $\hat{d}_0(r), \dots, \hat{d}_N(r)$, corresponding to the outage subsets $\hat{\mathcal{O}}_0, \dots, \hat{\mathcal{O}}_N$, respectively, the lowest one will be the DMT curve for the entire outage event. Since t can be made arbitrarily close to 1, it is without loss of accuracy to set $t = 1$.

Now we solve the subproblem in (C.7) to find $\hat{d}_k(r)$. Firstly, we derive $\hat{d}_0(r)$. Note that

$$\sum_{j=1}^N \left(\min(\beta, 1) - v_j + \sum_{n=1}^N (2n-1+M-N)v_n \right)^+ \geq N \min(\beta, 1) \quad (\text{C.9})$$

which suggests that if $r \leq N \min(\beta, 1)$, there is no outage, i.e., $\hat{d}_0(r) = \infty$.

Then we can exclude $\hat{d}_0(r)$ from the main optimization problem in (C.8). If $r > N \min(\beta, 1)$, the minimizing solution of $v_1, \dots, v_N$ can be found to be $v_1^* = \dots = v_N^* = 0$, yielding that $d_{CSITR}^*(r) = \hat{d}_0(r) = 0$.

Secondly, we derive $\hat{d}_k(r)$, $1 \leq k \leq N$, for $r \leq N \min(\beta, 1)$. Since $v_k \geq A_{I-1} = \alpha + \sum_{n=1}^N (2n-1+M-N) \min(B_{I-1}, v_n)$, we must have $v_k \geq B_{I-1}$; otherwise there will be a contradiction that $v_k \geq \alpha + (2k-1+M-N)v_k$. With $v_k \geq B_{I-1} = \beta + \sum_{n=1}^N (2n-1+M-N) \min(A_{I-2}, v_n)$, we can similarly get $v_k \geq A_{I-2}$. Following the same manner, we can iteratively get $v_1 \geq \dots \geq v_k \geq A_1, \dots, A_{I-1}, B_1, \dots, B_{I-1}$. Thus, A_i and B_i can be reexpressed as $A_i = \alpha + k(M-N+k)B_i + \sum_{n=k+1}^N (2n-1+M-N) \min(B_i, v_n)$ and $B_i = \beta + k(M-N+k)A_{i-1} + \sum_{n=k+1}^N (2n-1+M-N) \min(A_{i-1}, v_n)$. It now can be observed that decreasing $v_{k+1}, \dots, v_N$ does not violate the outage condition $\sum_{j=1}^N \left(\min(1, \beta) - v_j + \sum_{n=1}^k (2n-1+M-N)A_{I-1} + \sum_{n=k+1}^N (2n-1+M-N)v_n \right)^+ < r$, while reducing the SNR exponent $\sum_{n=1}^N (2n-1+M-N)v_n$. Therefore, the optimal solution of $v_{k+1}, \dots, v_N$ should be $v_{k+1}^* = \dots = v_N^* = 0$. Consequently, the optimization subproblem in (C.7) can be simplified to

$$\hat{d}_k(r) = \inf_{(v_1, \dots, v_k) \in \check{\mathcal{O}}_k} \sum_{n=1}^k (2n-1+M-N)v_n. \quad (\text{C.10})$$

Here the modified outage subset $\check{\mathcal{O}}_k$ is defined as

$$\check{\mathcal{O}}_k = \left\{ v_1, \dots, v_k \left| N\hat{\tau}(k) - \sum_{n=1}^k v_n < r, \alpha(I, k) \leq v_k \leq \dots \leq v_1 \leq \hat{\tau}(k) \right. \right\} \quad (\text{C.11})$$

where $\hat{\tau}(k) = \min(\beta, 1) + k(M-N+k)\alpha(I, k)$ and

$$\alpha(i, k) = \begin{cases} \alpha + k(M-N+k)(k(M-N+k)\alpha(i-1, k) + \beta), & i > 1 \\ \alpha, & i = 1. \end{cases} \quad (\text{C.12})$$

Following the same line of the proof of Theorem 3.2.1, we are able to derive the explicit solution of $\hat{d}_k(r)$ and $d_{CSITR}^*(r)$, which directly yields Theorem 3.3.1.

Appendix D

Proof for the DMT of Quantized-Power Controlled Training

As has been shown in the proof of Theorem 3.3.1, the dominant outage event will occur when $v_{i,b,n} = \min(v_n, \tilde{A}_{i-1})$ and $v_{i,f,n} = \min(v_n, \tilde{B}_i)$. Therefore, the outage probability of the considered system under quantized power control is given by $\tilde{P}_{out}^{CSITR} \doteq \rho^{-\tilde{d}^*(r)}$, where

$$\tilde{d}^*(r) = \min_{v_n \in \tilde{\mathcal{O}}^+} \sum_{n=1}^N (2n-1+M-N)v_n. \quad (\text{D.1})$$

Here, $\tilde{\mathcal{O}}$ is given by

$$\tilde{\mathcal{O}} = \left\{ v_n \mid \sum_{j=1}^N \left(\min(\beta, 1) + \sum_{n=1}^N (2n-1+M-N) \min(\tilde{A}_{I-1}, v_n) - v_j \right)^+ < r, \ i=1, \dots, I, \right. \\ \left. \tilde{A}_0 = \alpha, \ \tilde{A}_i = \alpha + \sum_{n=1}^N (2n-1+M-N) \tilde{v}_{i,f,n}, \ \tilde{B}_i = \beta + \sum_{n=1}^N (2n-1+M-N) \tilde{v}_{i,b,n} \right\}$$

where if $v_n \geq \tilde{B}_i$, $\tilde{v}_{i,f,n} = \tilde{B}_i$; if $v_n < \tilde{B}_i$, $\tilde{v}_{i,f,n} = \lfloor \frac{v_n}{\alpha} \rfloor \alpha$; if $v_n \geq \tilde{A}_{i-1}$, $\tilde{v}_{i,b,n} = \tilde{A}_{i-1}$; if $v_n < \tilde{A}_{i-1}$, $\tilde{v}_{i,b,n} = \lfloor \frac{v_n}{\beta} \rfloor \beta$.

We divide $\tilde{\mathcal{O}}$ into $N+1$ disjoint subsets, $\tilde{\mathcal{O}}_0, \dots, \tilde{\mathcal{O}}_N$, and solve the following

subproblem

$$\tilde{d}_k(r) = \min_{v_n \in \tilde{\mathcal{O}}_k^+} \sum_{n=1}^N (2n - 1 + M - N)v_n \quad (\text{D.2})$$

where $\tilde{\mathcal{O}}_k = \tilde{\mathcal{O}} \cap \{v_1 \geq \dots \geq v_k \geq \tilde{A}_{I-1} \geq v_{k+1} \geq \dots \geq v_N\}$. The solution of the main optimization problem in (D.1) should be $\tilde{d}^*(r) = \min(\tilde{d}_0(r), \dots, \tilde{d}_N(r))$.

It can be verified immediately that $\tilde{d}_0(r) = \hat{d}_0(r)$. We examine $\tilde{d}_k(r)$ for $k = 1, \dots, N$. Given $v_k \geq \tilde{A}_{I-1}$, we must require $v_k \geq \tilde{B}_{I-1}$; otherwise, there will be a contradiction that $\tilde{A}_{I-1} \geq \alpha + \tilde{v}_{I-1,f,k} = \alpha(1 + \lfloor \frac{v_k}{\alpha} \rfloor) > v_k$. Similarly, given $v_k \geq \tilde{B}_{I-1}$, we must require $v_k \geq \tilde{A}_{I-2}$ to avoid the contradiction that $\tilde{B}_{I-1} \geq \beta + \tilde{v}_{I-1,b,n} = \beta(1 + \lfloor \frac{v_k}{\beta} \rfloor) > v_k$. Following the same manner, we can iteratively get $v_1 \geq \dots \geq v_k \geq \tilde{B}_{I-1}, \dots, \tilde{B}_1, \tilde{A}_{I-1}, \dots, \tilde{A}_1$, which leads to $\tilde{v}_{i,f,1} = \dots = \tilde{v}_{i,f,k} = \tilde{B}_i$ and $\tilde{v}_{i,b,1} = \dots = \tilde{v}_{i,b,k} = \tilde{A}_{i-1}$. Substituting them into $\tilde{A}_i$ and $\tilde{B}_i$, we get $\tilde{A}_i = \alpha + k(M - N + k)\tilde{B}_i + \sum_{n=k+1}^N (2n - 1 + M - N)\tilde{v}_{i,f,n}$ and $\tilde{B}_i = \beta + k(M - N + k)\tilde{A}_{i-1} + \sum_{n=k+1}^N (2n - 1 + M - N)\tilde{v}_{i,b,n}$. Note that $\tilde{A}_{I-1}$ is an increasing function of $v_{k+1}, \dots, v_N$. Therefore, decreasing $v_{k+1}, \dots, v_N$ does not violate the outage condition $\sum_{j=1}^N \left(\min(\beta, 1) + k(M - N + k)\tilde{A}_{I-1} + \sum_{n=k+1}^N (2n - 1 + M - N)v_n - v_j \right)^+ < r$, while reducing the SNR exponent $\sum_{n=1}^N (2n - 1 + M - N)v_n$. As a result, the optimal solution of $v_{k+1}, \dots, v_N$ should be $v_{k+1}^* = \dots = v_N^* = 0$. Substituting this solution back into (D.2), we get the same simplified optimization problem as in (C.10) and thus $\tilde{d}_k(r) = \hat{d}_k(r)$. Subsequently, we have $\tilde{d}^*(r) = d_{CSITR}^*(r)$ and this completes the proof.

Appendix E

Proof of $\sum_{n=1}^N (p - v_n)^+ < r_{min}$

Satisfying $\tau \sum_{n=1}^N w_n > \bar{r} - r_{min}$

It suffices to prove that the set of $v_1, \dots, v_N$ satisfying $\sum_{n=1}^N (p - v_n)^+ < r_{min}$ (CON1) also satisfies $\tau \sum_{n=1}^N w_n > \bar{r} - r_{min}$ (CON2). Careful observation of $\mathcal{O}_{joint}$ indicates that we either have $w_n = v_n < \alpha$ or $\min(w_n, v_n) \geq \alpha$. That is, $w_n \geq \min(v_n, \alpha)$. As a result, we have $\tau \sum_{n=1}^N w_n \geq \tau \sum_{n=1}^N \min(v_n, \alpha) \geq \sum_{n=1}^N \min(\tau v_n, \tau \alpha)$. Since $\tau \geq \max(1, 1/\alpha)$, we get $\tau \sum_{n=1}^N w_n \geq \sum_{n=1}^N \min(v_n, 1) = \sum_{n=1}^N (1 - (1 - v_n)^+) = N - \sum_{n=1}^N (1 - v_n)^+$. With CON1, we have $\sum_{n=1}^N (1 - v_n)^+ \leq \sum_{n=1}^N (p - v_n)^+ < r_{min}$, which leads to $N - \sum_{n=1}^N (1 - v_n)^+ > N - r_{min}$. Therefore, we have $\tau \sum_{n=1}^N w_n > N - r_{min} \geq \bar{r} - r_{min}$ and this completes the proof.

Appendix F

Proof of the Convergence of Lower and Upper Bounds

We examine the asymptotic behaviors of $(\sqrt{\gamma_1} \pm \sqrt{\epsilon_1})^2$ and $(\sqrt{\gamma_{i,n}} \pm \sqrt{\epsilon_{i,n}})^2$, $i = 2, 3$, $n = 1, \dots, N$. When ρ grows to infinity, it can be shown that

$$(\sqrt{\gamma_1} + \sqrt{\epsilon_1})^2 \doteq \rho^{-\min\{v_1, u_1\}} \quad (\text{F.1a})$$

$$(\sqrt{\gamma_{i,n}} + \sqrt{\epsilon_{i,n}})^2 \doteq \rho^{-\min\{v_{i,n}, u_{i,n}\}}. \quad (\text{F.1b})$$

We now show that $(\sqrt{\gamma_1} - \sqrt{\epsilon_1})^2$ and $(\sqrt{\gamma_{i,n}} - \sqrt{\epsilon_{i,n}})^2$ have the same behaviors as in (F.1a) and (F.1b), respectively.

1) Case 1: γ_1 and ϵ_1 as well as all $\gamma_{i,n}$ and $\epsilon_{i,n}$ are of different orders, i.e., $v_1 \neq u_1$ and $v_{i,n} \neq u_{i,n}$, $\forall i \in \{2, 3\}$, $\forall n \in \{1, \dots, N\}$. In this case, we easily have

$$(\sqrt{\gamma_1} - \sqrt{\epsilon_1})^2 \doteq \rho^{-\min\{v_1, u_1\}} \quad (\text{F.2a})$$

$$(\sqrt{\gamma_{i,n}} - \sqrt{\epsilon_{i,n}})^2 \doteq \rho^{-\min\{v_{i,n}, u_{i,n}\}}. \quad (\text{F.2b})$$

2) Case 2: Any pair(s) of γ_1 and ϵ_1 or $\gamma_{i,n}$ and $\epsilon_{i,n}$ are of the same orders. Without loss of generality, we assume there are one pair $\gamma_{3,k}$ and $\epsilon_{3,k}$, $k \in \{1, \dots, N\}$,

with $v_{3,k} = u_{3,k}$ and $v_{3,n} \neq u_{3,n}$, $\forall n \neq k$. It then follows that

$$\left(\sqrt{\gamma_{3,k}} - \sqrt{\epsilon_{3,k}}\right)^2 \doteq \rho^{-(v_{3,k} + \delta_{3,k})}, \text{ for } \delta_{3,k} \geq 0. \quad (\text{F.3})$$

We examine (4.1.9) and (4.1.10) and find that the outage probability is dominated by the term with the largest SNR exponent as ρ goes to infinity. Among all the realizations of $\gamma_{3,k}$ and $\epsilon_{3,k}$ causing an outage, the set of $\gamma_{3,k}$ and $\epsilon_{3,k}$ with $\delta_{3,k} = 0$, dominates the outage probability for both $\mathcal{P}_l$ and $\mathcal{P}_u$. Therefore, $\left(\sqrt{\gamma_{3,k}} - \sqrt{\epsilon_{3,k}}\right)^2$ still has the same behavior as in (F.2b).

From (F.1)-(F.2), we get $A_1 \doteq A_2$ and $B_1 \doteq B_2$. Therefore, the two bounds in (4.1.9) and (4.1.10) converge as ρ goes to infinity.

Appendix G

Proof of Equation (5.3.15)

From (5.3.14), we can get

$$\frac{AB\alpha d^{\alpha-1}}{P_{N,s}^{N+1}} = \frac{B^2\alpha N(1-d)^{\alpha N-1}}{P_{N,s}^N(P_{N,T} - P_{N,s})^N} \quad (\text{G.1a})$$

$$\frac{AB(N+1)d^\alpha}{P_{N,s}^{N+2}} = \frac{B^2N(1-d)^{\alpha N}(2P_{N,s} - P_{N,T})}{P_{N,s}^{N+1}(P_{N,T} - P_{N,s})^{N+1}}. \quad (\text{G.1b})$$

We divide (G.1a) by (G.1b) and get

$$\frac{AB\alpha d^{\alpha-1}}{P_{N,s}^{N+1}} \cdot \frac{P_{N,s}^{N+2}}{AB(N+1)d^\alpha} = \frac{B^2\alpha N(1-d)^{\alpha N-1}}{P_{N,s}^N(P_{N,T} - P_{N,s})^N} \cdot \frac{P_{N,s}^{N+1}(P_{N,T} - P_{N,s})^{N+1}}{B^2N(1-d)^{\alpha N}(2P_{N,s} - P_{N,T})}. \quad (\text{G.2})$$

Simplifying (G.2) leads to

$$\frac{P_{N,s}}{P_{N,T}} = \frac{1 + dN}{2 + dN - d}. \quad (\text{G.3})$$

Substituting (G.3) into (G.1a), we get

$$\frac{AB\alpha d^{\alpha-1}}{\left(\frac{1+dN}{2+dN-d}P_{N,T}\right)^{N+1}} = \frac{B^2\alpha N(1-d)^{\alpha N-1}}{\left(\frac{1+dN}{2+dN-d}P_{N,T}\right)^N \left(P_{N,T} - \left(\frac{1+dN}{2+dN-d}P_{N,T}\right)\right)^N}. \quad (\text{G.4})$$

Simplifying (G.4) leads to

$$AP_{N,T}^{N-1}d^{\alpha-1} - BN(2 + dN - d)^{N-1}(1 + dN)(1 - d)^{\alpha N - N - 1} = 0. \quad (\text{G.5})$$

Appendix H

Proof of the Number of Roots to (5.3.15a)

For notation simplicity, we define a function $f(d) = Cd^{\alpha-1} - (2 + dN - d)^{N-1}(1 + dN)(1 - d)^{\alpha N - N - 1}$, where $C = \frac{AP_{N,T}^{N-1}}{BN}$. Thus, (5.3.15a) is equivalent to $f(d) = 0$. Now we check how many roots $f(d) = 0$ has. The first derivative of $f(d)$ is given by

$$\begin{aligned} \frac{\partial f(d)}{\partial d} &= C(\alpha-1)d^{\alpha-2} - (N-1)^2(2+dN-d)^{N-2}(1+dN)(1-d)^{\alpha N - N - 1} \\ &\quad - N(2+dN-d)^{N-1}(1-d)^{\alpha N - N - 1} + (\alpha N - N - 1)(2+dN-d)^{N-1}(1+dN)(1-d)^{\alpha N - N - 2} \\ &= C(\alpha-1)d^{\alpha-2} - (2+dN-d)^{N-1}(1+dN)(1-d)^{\alpha N - N - 1} \left(\frac{(N-1)^2}{2+dN-d} + \frac{N}{1+dN} - \frac{(\alpha N - N - 1)}{1-d} \right). \end{aligned} \quad (\text{H.1})$$

It is very difficult to determine the sign of $\frac{\partial f(d)}{\partial d}$ at arbitrary d . However, we can determine the sign of $\frac{\partial f(d)}{\partial d}$ at the root of $f(d) = 0$. Substituting $Cd^{\alpha-1} = (2 + dN - d)^{N-1}(1 + dN)(1 - d)^{\alpha N - N - 1}$ into (H.1), we will get

$$\frac{\partial f(d)}{\partial d} \Big|_{f(d)=0} = Cd^{\alpha-1} \left(\frac{(\alpha-1)}{d} - \frac{(N-1)^2}{2+dN-d} - \frac{N}{1+dN} + \frac{(\alpha N - N - 1)}{1-d} \right). \quad (\text{H.2})$$

If $\alpha \geq 2$ and $N = 1$, we will have $\frac{\partial f(d)}{\partial d} \Big|_{f(d)=0} = Cd^{\alpha-1} \left(\frac{\alpha-1}{d} - \frac{1}{1+d} + \frac{\alpha-2}{1-d} \right) > 0$ in the interval $d \in (0, 1)$. If $\alpha \geq 2$ and $N = 2$, we will have $\frac{\partial f(d)}{\partial d} \Big|_{f(d)=0} =$

$Cd^{\alpha-1} \left(\frac{\alpha-1}{d} - \frac{1}{2+d} - \frac{2}{1+2d} + \frac{2\alpha-3}{1-d} \right) > 0$ in the interval $d \in (0, 1)$. Similarly, we can prove that $\frac{\partial f(d)}{\partial d} |_{f(d)=0} > 0$ if any of the following three conditions is satisfied: 1) $N \leq 6$ and $\alpha \geq 2$; 2) $N \leq 9$ and $\alpha \geq 2.6$; 3) $N \leq 18$ and $\alpha \geq 4$. In fact, with (H.2) it is easy to find the maximal value of N making $\frac{\partial f(d)}{\partial d} |_{f(d)=0} > 0$ with arbitrary α . If $f(d) = 0$ has multiple roots, $\frac{\partial f(d)}{\partial d} |_{f(d)=0}$ should at least have one positive and one negative values in the interval $d \in (0, 1)$. It contradicts with $\frac{\partial f(d)}{\partial d} |_{f(d)=0} > 0$. Therefore, we conclude that there is only one root to $f(d) = 0$ under any of the aforementioned three conditions.

Author's Publications

- Journal Papers (published)

1. X. J. Zhang and Yi Gong, "Adaptive power allocation for regenerative relaying with multiple antennas at the destination," *IEEE Transactions on Wireless Communications*, vol. 8, pp. 2789-2794, June 2009.
2. X. J. Zhang and Yi Gong, "Adaptive power allocation for multihop regenerative relaying with limited feedback," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 7, pp. 3862-3867, Sep. 2009.
3. X. J. Zhang and Yi Gong, "Joint power allocation and relay positioning in multi-relay cooperative systems," *IET Communications*, vol. 3, no. 10, pp. 1683-1692, Oct. 2009.
4. X. J. Zhang, Yi Gong, and K. B. Letaief, "On the diversity gain in cooperative relaying channels with imperfect CSIT," *IEEE Transactions on Communications*, vol. 58, no. 4, pp. 1273-1279, Apr. 2010.
5. X. J. Zhang and Yi Gong, "A simple amplify-and-forward relaying scheme based on clipping and forwarding for dual-hop transmissions," *Journal of Communications - Special Issue on Coding and Modulation for Cooperative Communications*, vol. 5, no. 4, pp. 348-353, Apr. 2010.

- Journal Papers (under review)

1. X. J. Zhang and Yi Gong, "On the D-M tradeoff in dynamic decode and forward relaying channels with power control," in review in *IEEE Transactions on Communications*.

2. X. J. Zhang, Yi Gong, and K. B. Letaief, "Diversity and multiplexing tradeoff in MIMO channels: two-way training and power control," in review in *IEEE Transactions on Wireless Communications*.
3. X. J. Zhang and Yi Gong, "On the D-M tradeoff in MIMO channels with noisy CSITR: joint rate and power control," in review in *IEEE Transactions on Communications*.
4. X. J. Zhang and Yi Gong, "Joint optimization of power allocation and relay location for regenerative relaying with multi-antenna reception at destination," in review in *Wireless Communications and Mobile Computing*.

- Conference Papers

1. X. J. Zhang and Yi Gong, "Diversity and multiplexing tradeoff in SIMO/MISO channels: two-way training and power control," in *Proc. IEEE ICC'10*, May 2010.
2. X. J. Zhang and Yi Gong, "Impact of CSIT on the tradeoff of diversity and spatial multiplexing in MIMO channels," in *Proc. IEEE ISIT'09*, June 2009, pp. 769-773.
3. X. J. Zhang and Yi Gong, "Adaptive power allocation in two-way amplify-and-forward relay networks," in *Proc. IEEE ICC'09*, June 2009, pp. 1-5.
4. X. J. Zhang, Yi Gong, and G. X. Xiao "Multiuser detection for decode-and-forward cooperative relaying in DS-CDMA systems," in *Proc. IEEE VTC-Spring'09*, Apr. 2009, pp. 1-5.
5. X. J. Zhang and Yi Gong, "Power allocation for regenerative cooperative systems with multi-antenna destination," in *Proc. IEEE ICC'08*, May 2008, pp. 3755-3759.
6. X. J. Zhang and Yi Gong, "Adaptive power allocation for multi-hop OFDM relaying systems", in *Proc. IEEE ICICS'07*, Dec. 2007, pp. 1-5.

Bibliography

- [1] M. Chiang, P. Hande, T. Lan and C. W. Tan, *Power Control in Wireless Cellular Networks (Foundations and Trends in Networking)*. Hanover, MA, USA: Now Publishers Inc., 2008.
- [2] S. M. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.
- [3] Yi Gong and K. B. Letaief, “Concatenated space-time block coding with trellis coded modulation in fading channels,” *IEEE Trans. Wireless Commun.*, vol. 1, no. 4, pp. 580-590, Oct. 2002.
- [4] G. J. Foschini, “Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas,” *Bell Labs Tech. J.*, pp. 41-59, 1996.
- [5] S. Sfar, L. Dai, and K. B. Letaief, “Optimal diversity-multiplexing tradeoff with group detection for MIMO systems,” *IEEE Trans. Commun.*, vol. 53, pp. 1178-1190, July 2005.
- [6] L. Zheng and D. N. C. Tse, “Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels,” *IEEE Trans. Inform. Theory*, vol. 49, pp. 1073-1096, May 2003.
- [7] D. N. C. Tse, P. Viswanath, and L. Zheng, “Diversity-multiplexing tradeoff in multiple-access channels,” *IEEE Trans. Inform. Theory*, vol. 50, pp. 1859-1874, Sep. 2004.

- [8] S. Tavildar and P. Viswanath, "Approximately universal codes over slow-fading channels," *IEEE Trans. Inform. Theory*, vol. 52, pp. 3233-3258, July 2006.
- [9] H. El Gamal and A. R. Hammons, "On the design of algebraic space-time codes for MIMO block-fading channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 1, pp. 151-163, Jan. 2003.
- [10] H. El Gamal, G. Caire, and M. O. Damen, "Lattice coding and decoding achieve the optimal diversity-multiplexing tradeoff of MIMO channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 6, pp. 968-985, June 2004.
- [11] H. F. Lu and P. V. Kumar, "Rate-diversity tradeoff of space-time codes with fixed alphabet and optimal constructions for PSK modulation," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2747-2751, Oct. 2003.
- [12] R. Narasimhan, "Finite-SNR diversity performance of rate-adaptive MIMO systems," in *Proc. IEEE GLOBECOM'05*, Nov. 2005, pp. 1461-1465.
- [13] R. Narasimhan, "Finite-SNR diversity-multiplexing tradeoff for correlated Rayleigh and Rician MIMO channels," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3965-3979, Sep. 2006.
- [14] H. El Gamal, G. Caire, and M. O. Damen, "The MIMO ARQ channel: diversity-multiplexing-delay tradeoff," *IEEE Trans. Inform. Theory*, vol. 52, no. 8, pp. 3601-3621, Aug. 2006.
- [15] A. Khoshnevis and A. Sabharwal, "On the asymptotic performance of multiple antenna channels with quantized feedback ", *IEEE Trans. Wireless Commun.*, vol. 7, no. 10, pp. 3869-3877, Oct. 2008.
- [16] T. T. Kim and M. Skoglund, "Diversity-multiplexing tradeoff in MIMO channels with partial CSIT," *IEEE Trans. Inform. Theory*, vol. 53, pp. 2743-2759, Aug. 2007.
- [17] V. Aggarwal and A. Sabharwal, "Diversity order gain with noisy feedback in multiple access channels," in *Proc. IEEE ISIT'08*, July 2008, pp. 404-408.

- [18] C. Steger, A. Khoshnevis, A. Sabharwal, and B. Aazhang, "The case for transmitter training," in *Proc. IEEE International Symposium on Information Theory (ISIT'06)*, Seattle, WA, July 9-14, 2006, pp. 35-39.
- [19] C. Steger and A. Sabharwal, "Single-input two-way SIMO channel: diversity-multiplexing tradeoff with two-way training," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 4877-4885, Dec. 2008.
- [20] A. Lim and V. K. N. Lau, "On the fundamental tradeoff of spatial diversity and spatial multiplexing of MISO/SIMO links with imperfect CSIT," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 110-117, Jan. 2008.
- [21] K. J. R. Liu, A. K. Sadek, and W. Su, *Cooperative Communications and Networking*. Cambridge, UK: Cambridge University Press, 2004.
- [22] E. C. van der Meulen, "Transmission of information in a T-terminal discrete memoryless channel," Ph.D. dissertation, Univ. California, Berkeley, CA, June 1968.
- [23] T. M. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, pp. 572-584, Sep. 1979.
- [24] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, pp. 3037-3063, Sep. 2005.
- [25] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062-3080, Dec. 2004.
- [26] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2415-2425, Oct. 2003.
- [27] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity—Part I and Part II," *IEEE Trans. Commun.*, vol. 51, pp. 1927-1948, Nov. 2003.

- [28] T. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of coded co-operation," *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 375-391, Feb. 2006.
- [29] J. Zhang, Q. Zhang, C. Shao, Y. Wang, P. Zhang, and Z. Zhang, "Adaptive optimal transmit power allocation for two-hop non-regenerative wireless relay system," in *Proc. IEEE VTC'04 Spring*, May 2004, pp. 1213-1217.
- [30] Q. Zhang, J. Zhang, C. Shao, Y. Wang, P. Zhang, and R. Hu, "Power allocation for regenerative relay channel with Rayleigh fading," in *Proc. IEEE VTC'04 Spring*, May 2004, pp. 1167-1171.
- [31] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channel," *IEEE Trans. Inform. Theory*, vol. 51, pp. 2020-2040, June 2005.
- [32] Y. Zhu, Y. Xin and P. Kam, "Optimal transmission strategies for rayleigh fading relay channels," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 618-628, Feb. 2008.
- [33] I. Hammerström and A. Wittneben, "On the optimal power allocation for nonregenerative OFDM relay links," in *Proc. IEEE ICC'06*, Istanbul, Turkey, June 2006, pp. 4463-4468.
- [34] I. Hammerström and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 2798-2802, Aug. 2007.
- [35] I. Maric and R. D. Yates, "Bandwidth and power allocation for cooperative strategies in Gaussian relay networks," in *Proc. 38th Asilomar Conf. Signal, System Computers*, Nov. 2004, pp. 1907-1911.
- [36] I. Maric and R.D. Yates, "Forwarding strategies for Gaussian parallel-relay networks," in *Proc. IEEE ISIT'04*, July 2004, pp. 269.
- [37] Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 3114-3123, Aug. 2007.

- [38] K. G. Seddik, A. K. Sadek, W. Su and K. J. R. Liu, "Outage Analysis and Optimal Power Allocation for Multinode Relay Networks," *Signal Processing Letters*, vol. 14, pp. 377-380, June 2007.
- [39] J. Luo, R.S. Blum, L.J. Cimini, L.J. Greenstein and A.M. Haimovich, "Decode-and-forward cooperative diversity with power allocation in wireless networks," *IEEE Trans. Wireless Comm.*, vol. 6, pp. 793-799, Mar. 2007.
- [40] R. Annavaajjala, P. C. Cosman, and L. B. Milstein, "Statistical channel knowledge-based optimum power allocation for relaying protocols in the high SNR regime," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 292-305, Feb. 2007.
- [41] Y. Li, B. Vucetic, Z. Zhou, and M. Dohler, "Distributed adaptive power allocation for wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 948-958, Mar. 2007.
- [42] T. Q. S. Quek, M. Z. Win, H. Shin and M. Chiani, "Robust power allocation for amplify-and-forward relay networks", in *Proc. IEEE ICC'07*, June 2007, pp. 957-962.
- [43] T. Q. S. Quek, M. Z. Win, H. Shin and M. Chiani, "Optimal power allocation for amplify-and-forward relay networks via conic programming", in *Proc. IEEE ICC'07*, June 2007, pp. 5058-5063.
- [44] M. Chen, S. Serbetli and A. Yener, "Distributed power allocation strategies for parallel relay networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 552-561, Feb. 2008.
- [45] J. Cho and Z. J. Haas, "On the throughput enhancement of the downstream channel in cellular radio networks through multihop relaying," *IEEE J. Select. Areas Commun.*, vol. 22, pp. 1206-1219, Sep. 2004.
- [46] 3GPP TR25.924, "Opportunity driven multiple access," 3GPP, ver. 1.0.0, Dec. 1999.

- [47] X. J. Zhang and Yi Gong, "Adaptive power allocation for multi-hop OFDM relaying systems", in *Proc. IEEE ICICS'07*, Dec. 2007, pp. 1-5.
- [48] M. O. Hasna and M. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 1999-2004, Nov. 2004.
- [49] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, pp. 1820-1830, Oct. 2004.
- [50] T. T. Kim, G. Caire, and M. Skoglund, "Decode-and-forward relaying with quantized channel state feedback: an outage exponent analysis," *IEEE Trans. Inform. Theory*, vol. 54, pp. 4548-4564, Oct. 2008.
- [51] C. Oestges and B. Clerckx, *MIMO Wireless Communications: From Real-World Propagation to Space-Time Code Design*. Academic Press, June 2007.
- [52] I. E. Telatar, "Capacity of multiantenna Gaussian channels," *Eur. Trans. Commun.*, vol. 10, no. 6, pp. 585-595, 1999.
- [53] G. J. Foschini¹ and M.J. Gans, "On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, Mar. 1998.
- [54] G. Taricco and E. Biglieri, "Space-time decoding with imperfect channel estimation," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 1874-1888, July 2005.
- [55] R. Narasimhan, "Error propagation analysis of V-BLAST with channel-estimation errors," *IEEE Trans. Commun.*, vol. 53, pp. 27-31, Jan. 2005.
- [56] K. Lee and J. Chun, "Symbol detection in V-BLAST architectures under channel estimation errors" *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 593-597, Feb. 2007.
- [57] T. Ratnarajah, R. Vaillancourt and M. Alvo, "Eigenvalues and condition numbers of complex random matrices", *SIAM Journal on Matrix Analysis and Applications*, vol. 26, no. 2, pp. 441-456, 2005.

- [58] W. Fulton, “Eigenvalues, invariant factors, highest weights, and Schubert calculus”, *Bull. American Math. Soc.*, vol. 37, no. 3, pp. 209-249, 2000.
- [59] T. T. Kim and G. Caire, “Diversity gains of power control with noisy CSIT in MIMO channels,” *IEEE Trans. Inform. Theory*, vol. 55, no. 4, pp. 1618-1626, Apr. 2009.
- [60] E. Visotsky and U. Madhow, “Space-time transmit precoding with imperfect feedback,” *IEEE Trans. Inform. Theory*, vol. 47, pp. 2632-2639, Sep. 2001.
- [61] K. Azarian, H. El Gamal, and P. Schniter, “On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels,” *IEEE Trans. Inform. Theory*, vol. 51, pp. 4152-4172, Dec. 2005.
- [62] S. Yang and J.-C. Belfiore, “Towards the optimal amplify-and-forward cooperative diversity scheme,” *IEEE Trans. Inform. Theory*, vol. 53, pp. 3114-3126, Sep. 2007.
- [63] M. Yuksel and E. Erkip, “Multiple-antenna cooperative wireless systems: a diversity-multiplexing tradeoff perspective,” *IEEE Trans. Inform. Theory*, vol. 53, pp. 3371-3393, Oct. 2007.
- [64] M. K. Simon, *Probability Distributions Involving Gaussian Random Variables: A Handbook for Engineers and Scientists*. Springer Press, 2006.
- [65] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*. New York: Dover Publications, Inc., 1972.
- [66] K. Knight, *Mathematical Statistics (Texts in Statistical Science)*. Chapman & Hall/CRC, 1999.
- [67] D. G. Brennan, “Linear diversity combining techniques,” in *Proc. IRE*, vol. 47, pp. 1075-1102, June 1959.
- [68] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

- [69] X. J. Zhang and Yi Gong, "Adaptive power allocation for regenerative relaying with multiple antennas at the destination," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2789-2794, June 2009.
- [70] O. Kaya and S. Ulukus, "Achieving the capacity region boundary of fading CDMA channels via generalized iterative waterfilling," *IEEE Trans. Commun.*, vol. 54, pp. 3215-3223, Nov. 2006.
- [71] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Boston: Kluwer, 1992.
- [72] J. Cioffi, *Digital Transmission: Volume I*: Stanford University EE 379A. [Online] Available: www.stanford.edu/class/ee379a/
- [73] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. New York: John Wiley & Sons, 1965.
- [74] N. Ahmed, M. Khojastepour, A. Sabharwal, and B. Aazhang, "Outage minimization with limited feedback for the fading relay channel," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 659-669, April 2006.
- [75] D. J. Love and R. W. Heath, Jr., "OFDM power loading using limited feedback," *IEEE Trans. Veh. Technol.*, vol. 54, pp. 1773-1780, Sep. 2005.
- [76] V. Aggarwal and A. Sabharwal, "Power-controlled feedback and training for two-way MIMO channels," submitted to *IEEE Trans. Inform. Theory*, Jan. 2009.
- [77] L. Zheng and D. N. C. Tse, "Communication on the Grassmann manifold: a geometric approach to the noncoherent multiple-antenna channel," *IEEE Trans. Inform. Theory*, vol. 48, pp. 359-383, Feb. 2002.