A polynomial approach to robust channel estimator design for DS-CDMA systems

Cao, Chengtao

2008

Cao, C. (2008). Polynomial approach to robust channel estimator design for DS-CDMA systems. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/4107>

<https://doi.org/10.32657/10356/4107>

Nanyang Technological University

Downloaded on 26 Apr 2023 20:27:27 SGT

A Polynomial Approach to Robust Channel Estimator Design for DS-CDMA Systems

Cao Chengtao

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2008

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

.....

Date

.....

Cao Chengtao

*To my wife Song Ying and my parents,
for their encouragement and love.*

Acknowledgments

First and foremost, I would like to express my heartfelt gratitude to my supervisor, Professor Xie Lihua for professional guidance, invaluable advices and constant support throughout my study at Nanyang Technological University. Without his help, this thesis would not be finished. Also, especial appreciation is due to him for his great help during my medical operation and healing.

I am deeply thankful to Professor Zhang Huanshui, Shandong University, China, for his insightful discussion. I would also express my gratitude to Dr. Lars Lindbom, Uppsala University, Sweden. As one of the inventors of the Wiener LMS algorithm, Dr. Lars Lindbom provided a useful feedback on my query regarding the WLMS filter. I would like to record my gratitude to Zhang Hua and other Tianjin University alumni at NTU for their help and friendship. My sincere thanks also go to all friends in Computer Control Lab.

Finally, I am especially grateful to my dearest wife, Song Ying, for her great love, understanding, full support and boosting my morale in hard times. My gratitude to my parents' dedicated life-long support and tremendous care are beyond description. I would not have accomplished this thesis without their constant support.

Contents

Acknowledgments	iv
List of Figures	ix
List of Tables	xiii
Summary	xiv
Notation and Symbols	xvi
1 Introduction	1
1.1 Background and Motivation	1
1.1.1 Overview of wireless communication channels	3
1.1.2 Channel estimation in DS-CDMA systems	6
1.2 Objective and Contribution of the Thesis	7
1.3 Organization of the Thesis	11
2 Issues in Channel Estimation	13
2.1 DS-CDMA System Model for Channel Estimation Problems	13
2.2 Channel Modelling	20

2.2.1	Autoregressive channel model	20
2.2.2	How to obtain the parameters?	21
2.2.3	How to select the order of AR channel model?	23
2.3	Review of Channel Estimation Algorithms	24
3	Polynomial WLMS Channel Estimator	28
3.1	Introduction	28
3.2	Problem Statement and Structure of Joint Channel Estimator and DFE Detector	31
3.3	Polynomial WLMS Channel Tracking and Prediction	32
3.4	Design of Adaptive DFE Symbol Detector	38
3.5	Simulation Results	41
3.5.1	Performance of BER	42
3.5.2	The setting of step-size in WLMS algorithm	44
3.5.3	Error in previous decision	45
3.6	Conclusion	46
4	Polynomial MMSE Channel Estimation of DS-CDMA Systems with Stochastic Parametric Uncertainty	48
4.1	Introduction	48
4.2	Problem Statement	51
4.3	Linear Time-invariant Regression	51
4.4	Treatment of Parametric Uncertainties in System Models	55
4.4.1	Statistics of channel model uncertainties	55
4.4.2	Statistics of observation model uncertainties	58
4.5	Optimal Polynomial Estimator Design	59
4.6	Simulation Results	62

4.7	Conclusion	67
5	Polynomial H_∞ Channel Estimation of DS-CDMA Systems	68
5.1	Introduction	68
5.2	Preliminaries of Krein Space Theory	71
5.3	Problem Statement	76
5.4	H_∞ Estimator Design	77
5.4.1	Polynomial solution via Krein space	77
5.4.2	Design of polynomial H_∞ channel estimator	81
5.4.3	A closed-form solution to J -spectral factorization	85
5.5	Simulation Results	89
5.5.1	Performance with various observation noises	89
5.5.2	Performance with modeling errors	95
5.6	Conclusion	95
6	Polynomial Robust H_∞ Estimation for Systems with Stochastic Parametric Uncertainty and Its Application in DS-CDMA Systems	98
6.1	Introduction	98
6.2	Problem Statement	100
6.3	Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty	101
6.3.1	Auxiliary system without parametric uncertainties	102
6.3.2	Design of polynomial H_∞ channel estimator	103
6.4	Extension to General Stochastic H_∞ Estimation Problem	110
6.5	Simulation Results	118
6.6	Conclusion	122

7 Robust Polynomial MMSE Channel Estimation of DS-CDMA Systems with Deterministic Parametric Uncertainty	123
7.1 Introduction	123
7.2 Problem Statement	125
7.3 Robust Polynomial Channel Estimator	128
7.4 Simulation Results	133
7.5 Conclusion	136
8 Conclusions and Future Work	138
8.1 Conclusions	138
8.2 Future Work	141
Author's Publications	143
Bibliography	144
A Proof of Theorem 4.4.1	157
B Proof of Theorem 4.5.1	160
C Proof of Theorem 5.4.3	162

List of Figures

1.1	Typical mobile radio scenario illustrating multipath propagation in a terrestrial mobile radio environment [61].	3
1.2	Doppler shift with angle of arrival α and velocity v	4
1.3	Doppler Power Spectrum.	5
2.1	The model of uplink transmission in the DS-CDMA system.	14
2.2	Possible receiver structure of the DS-CDMA system.	14
2.3	The impulse response of frequency selective fading channel.	15
2.4	The received signal of the k th user after undergoes multipath channel.	18
2.5	Channel autocorrelation function when $f_d = 50$ Hz, $f_s = 10^4$ Hz.	23
3.1	Joint channel estimation and symbol detection block diagram	32
3.2	Equivalent model for Decision Feedback Equalization	38
3.3	BER performance of the proposed detector based on WLMS and the detector based on Kalman filter in time-invariant channel. The single-user bound is shown for comparison.	43
3.4	BER performance versus SNR. Two Raleigh fading paths for each user with $f_d T = 0.005$	44

List of Figures**x**

3.5	The WLMS channel estimation of the desired user when SNR is equal to 4 dB. Dashed line: the real channel response; solid line: the proposed WLMS estimation algorithm.	45
3.6	BER performance versus step-size μ under the different SNR. . . .	46
3.7	BER performance versus ε when SNR is equal to 2 dB.	47
4.1	Mean Square Error of proposed robust estimator, Kalman estimator, standard RLS estimator and WLMS estimator with $f_d = 50$ Hz. . .	63
4.2	Mean square error of proposed estimator that does not consider parameter uncertainties, proposed estimator considering parameter uncertainties, Kalman estimator, standard RLS estimator and WLMS estimator for the first user with different Doppler frequencies. Note that the SNR of all users is 15 dB.	64
4.3	True signal, channel estimates based on proposed algorithm that takes parameter uncertainties into consideration and one without the consideration of uncertainties with respect to $f_d = 75$ Hz. . . .	64
4.4	BER performance of the DFE detector with the proposed estimator, Kalman estimator and WLMS estimator. The BER performance based on actual channel coefficients is also shown for comparison. .	65
4.5	Relative near-far resistance of DFE detector with the proposed estimator, Kalman estimator and WLMS estimator. Curves present the first user's BER corresponding to other five users' SNR that varies from 0 dB to 30 dB.	66
5.1	Mean Square Error with respect to different SNR under a white Gaussian noise of variance $\sigma^2 = 0.01$	90

List of Figures

xi

5.2	BER with respect to different SNR under a white Gaussian noise of variance $\sigma^2 = 0.01$	90
5.3	The MSE under a colored noise generated by filter $H(z) = (1 - 0.9z)^{-1}$ with $f_d = 50$ Hz.	92
5.4	BER with respect to different SNR under a colored noise generated by filter $H(z) = (1 - 0.9z)^{-1}$ with $f_d = 50$ Hz.	92
5.5	The PDF of a two-term Gaussian Mixture noise where $\epsilon = 0.1$ and $\alpha^2 = 100$	93
5.6	The MSE under a two-term Gaussian Mixture noise with (pdf) $f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \alpha^2\sigma_n^2)$, where $\epsilon = 0.1$ and $\alpha^2 = 100$	94
5.7	BER with respect to different SNR under a two-term Gaussian Mixture noise	94
5.8	The MSE with respect to SNR when there is a model mismatch in the 1th user. Real Doppler frequency of the 1th user is 125 Hz whereas others remain 50 Hz.	96
5.9	The BER with respect to SNR when there is a model mismatch in the 1th user. Real Doppler frequency of the 1th user is 125 Hz whereas others remain 50 Hz.	96
6.1	Mean Square Error with respect to different SNR under a white Gaussian noise of variance $\sigma^2 = 0.01$. Real Doppler frequency of all users is 50 Hz but the fading channel model is corrupted by the white noise process with the variance of 0.001.	119
6.2	The MSE with respect to different SNR under a colored noise generated by filter $H(z) = (1 - 0.9z)^{-1}$	120
6.3	The MSE under a two-term Gaussian Mixture noise with (pdf) $f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \alpha^2\sigma_n^2)$, where $\epsilon = 0.1$ and $\alpha^2 = 100$	121

List of Figures

xii

7.1	The MSE with respect to different SNR for $\delta = 0$	134
7.2	The MSE with respect to different SNR for $\delta = 0.06$	135
7.3	The MSE with respect to different SNR for $\delta = 0.1$	135

List of Tables

2.1	The state transition coefficients of AR(2) model for a 900 MHz wire- less system with symbol duration $T_s = 100\mu s$	22
3.1	Summary of the WLMS filter	30
3.2	Computational complexity comparison between Kalman algorithm and WLMS algorithm when tracking multipath channel response . .	37

Summary

This thesis presents a series of research results on channel estimation problem for a DS-CDMA system in multipath fading channel environments. The focus of the thesis is a polynomial approach to robust estimation problems for uncertain channels with deterministic or stochastic parameter uncertainties under the *minimum mean square error* (MMSE) or H_∞ sense. The polynomial approach has the distinct advantage of lower computational complexity.

We begin with the application of the *Wiener least-mean-square* (LMS) algorithm on the channel estimation. The proposed estimator not only preserves the simplicity of the standard LMS algorithm but also exhibits desirable estimation performance in practical fading channel environments. Parallel to the derivation of the channel estimator, we also derive an adaptive *decision feedback equalizer* (DFE) multiuser detector under the *mean square error* (MSE) criterion with the consideration of the feedback of detection errors. It is employed to evaluate the impact of various estimation algorithms on the *bit-error-rate* (BER) performance.

Next, we develop a robust polynomial channel estimator based on the MMSE criterion. The involved system is considered to be subject to stochastic parametric uncertainties. The statistics of these uncertainties are first obtained via a frequency domain approach and are then taken into account in the estimator design. The

optimal polynomial MMSE filter is derived by employing a variational approach and the result is given in terms of a polynomial spectral factorization and the solution to a bilateral Diophantine equation.

We also derive a polynomial version of standard H_∞ estimator for DS-CDMA systems. By virtue of the exploitation of the mathematical equivalence between the H_∞ filtering problem in Hilbert spaces and H_2 filtering problems in Krein spaces, a complete polynomial solution is obtained elegantly by projection in Krein spaces. Furthermore, we derive a closed-form solution to a polynomial matrix J -spectral factorization operating in the proposed estimator. This result greatly decreases the computational complexity of a high dimension J -spectral factorization thereby improves the computational efficiency of the proposed algorithm. Simulation results show that the proposed estimator has good robust performance in several practical wireless channel environments.

By applying the same strategy for H_∞ filtering, we develop a robust polynomial H_∞ channel estimator for systems with stochastic parametric uncertainties in the channel state matrix. For such stochastic H_∞ estimation problem, we first convert it into an indefinite quadratic form and then derive a solution by readily employing the approach of the polynomial H_∞ estimation developed above. Furthermore, we also investigate the application of such algorithm in general stochastic H_∞ estimation problems.

Finally, we turn to the study of the robust polynomial MMSE channel estimation problem for systems with deterministic parametric uncertainties. Based on the *sum quadratic constraint* (SQC) of norm-bounded uncertainties, we convert the robust MMSE filtering problem into the optimization of such SQC. Next, we employ the optimal linear estimation approach in a Krein space system and obtain a robust polynomial MMSE estimator.

Notation and Symbols

$\mathcal{R}$	field of real numbers
$\mathcal{C}$	field of complex numbers
$\in$	belong to
$\otimes$	convolution operator
$\oplus$	direct sum or Kronecker sum
$trace(A)$	trace of A
$col\{x_1, \dots, x_N\}$	the column vector consists of $x_1, \dots, x_N$
$diag\{b_1, b_1, \dots, b_K\}$	a $K \times K$ diagonal matrix with elements $\{b_1, b_1, \dots, b_K\}$ on the main diagonal
A^T	transpose of A
A^*	Hermitian transpose of A
A^{-1}	inverse of A
A^{-*}	shorthand for $(A^{-1})^*$
A^+	pseudo inverse of A
I_N	$N \times N$ identity matrix
q^{-1}	unity delay operator $q^{-1}y_n = y_{n-1}$
q	unity forward operator $qy_n = y_{n+1}$
$G(q^{-1})$	polynomial matrix

$G(q^{-1}) \in C^{n \times m}$	a set of $n \times m$ matrices having polynomial entries in q^{-1}
$G^*(q^{-1})$	Hermitian transpose of the polynomial matrix $G(q^{-1})$ where $G^*(q^{-1}) = G^T(q)$
$E\{\cdot\}$	the statistical expectation operator
$E_B E_h\{\cdot\}$	the statistical expectation with respect to B and h , respectively
$\ x\ $	Euclidian norm of vector x
$\ x\ _2$	ℓ_2 -norm of $x \in \ell_2[0, \infty)$
H_∞	Babach space of matrix-valued functions that are bounded on the imaginary axis and analytic in the open right-half plane
$\mathcal{R}H_\infty$	the real rational subspace of H_∞ which contains all proper and real rational stable transfer matrices

Chapter 1

Introduction

1.1 Background and Motivation

In recent years, mobile communication service has been growing at an explosive rate, and this trend will continue in the near future. The beginning of this rapid development may be traced back to more than 40 years ago. First-generation (1G) wireless networks were developed in the late 1970s. This system is entirely based on analog technique due to the use of *frequency division multiple access* (FDMA) as its multiple access architecture for voice transmission. There followed second-generation (2G) wireless networks in the early 1990s. Compared with single voice service and the limited capacity of subscribers and accessibility of 1G systems, 2G systems aimed at providing a better spectral efficiency, a more robust communication, voice and low-speed data services, voice privacy, and authentication capabilities by virtue of the digital transmission techniques [90]. As a main representative of the 2G systems, *Global System for Mobile communications* (GSM) standard developed in Europe by adopting *time division multiple access* (TDMA) techniques has become a worldwide mobile communication standard and been widely applied in voice transmission service until then. At the beginning of the 21st century, the

1.1 Background and Motivation

2

third-generation (3G) mobile systems emerged to meet the increasing demands for a variety of wide band services (e.g., high-speed Internet access and high-quality video and images transmission). The initial push to the 3G systems began with the proposal of the system IMT-2000 (International Mobile Telecommunications-2000). IMT-2000 standards and specifications are being designed to support multimedia services and other broadband services with the maximum data rate up to 2 Mbit/s at a frequency range of 2 GHz, with the same quality as fixed networks. One of the main developments in 3G standards is the adoption of the wideband *direct sequence code division multiple access* (DS-CDMA) technology, which is designed to offer greater capacity and security for supporting wideband services, such as wireless internet services with peak download rate of 384 kb/s and video transmissions with data rate up to 2 Mb/s. However, the evolution of wireless technology will never cease. The research of the fourth generation mobile radio systems (4G) has been in progress. An important design goal for 4G systems is to integrate all broadband mobile services and extend frequency ranges up to 100 GHz.

Regardless of the standards and specifications developed by different standardization organizations or industry associations, there exists a main consideration in the design of systems that is their ability to perform with adequate margin over a wireless fading channel suffering from a lot of impairments. In fact, complicated fading environments degrade the system performance, thereby limit system capacity. This problem drives more sophisticated wireless applications to be developed. Indeed, the combat with fading channel scenarios occurred well prior to the wireless revolution and will continue to exist in the future. Thus, channel estimation and equalization techniques over fading channels are still important and necessary in modern wireless communication systems, which is also the impetus to this research.

1.1.1 Overview of wireless communication channels

Figure 1.1: Typical mobile radio scenario illustrating multipath propagation in a terrestrial mobile radio environment [61].

In mobile radio communications, radio frequency (RF) signals reach the receiving antenna not only along the direct path but also along several indirect paths due to natural and artificial obstacles on their ways. In practice, a transmitted signal arrives at the receiver several times from all directions with different time delays due to reflection, diffraction, and scattering caused by buildings, trees, and other obstacles, as illustrated in Figure 1.1. These signals are superimposed together to form a distorted signal. This effect is referred to as *multipath propagation*. In fact, this superposition may be a double-edged weapon. On the one hand, the energy of the received signal is enforced in this case relative to the single-path propagation. On the other hand, this characteristic may distort the frequency response of the original signal, that is, cause fluctuations in the receive signal's amplitude, phase, and angle of arrival. Moreover, due to signal's dispersion in time domain, it

1.1 Background and Motivation

4

overlaps with signals transmitted at adjacent times. This phenomenon is known as *intersymbol interference* (ISI). The terms, *frequency selective fading* and *flat fading* are used to describe the impact of radio channels on different data transmission. A channel is referred to as frequency selective fading if the data transmission rate is high enough to exceed the coherence bandwidth of the channel. The phenomenon is that multipath components of a signal are resolvable, thereby yield severe ISI distortion.

Figure 1.2: Doppler shift with angle of arrival α and velocity v .

Besides the multipath propagation, the *Doppler effect* is another negative phenomenon for the transmission characteristics of the mobile radio channel. Because of the movement of the mobile unit and some of the reflecting objects, signal frequency at the receiver is shifted due to the Doppler effect [61]. In the case of single-path reception, we consider a mobile receiver moving with velocity v from left to right and receiving a wave from base station as shown in Figure 1.2. The

1.1 Background and Motivation

5

change in frequency (the *Doppler shift*) is given by

$$f_d = \frac{v}{c} f_c \cos \alpha$$

where

v : speed of vehicle

c : speed of light

f_c : carrier frequency

α : spatial angle between the wave and the direction of motion

Figure 1.3: Doppler Power Spectrum.

In the case of a multipath environment, the original signal undergoes individual paths with different Doppler shifts due to different angles α . It results in that the received spectrum is broadened. For example, for well-known Clarke model that assumes arrival angles having a uniform distribution throughout $(0, 2\pi)$, the power density spectrum of the received signal is [19]

$$P(f) = \frac{1}{\pi \sqrt{f_d^2 - f^2}}, \quad \text{for } |f| < |f_d|. \quad (1.1)$$

Figure 1.3 illustrates the shape of the Doppler spectrum formulated in (1.1). Like the effect of multipath on channel dispersion, the Doppler effect is concerned with fading rapidity mechanism of the channel. The terms, *fast fading* and *slow fading*, describe such time-variant nature of the channel. In general, higher Doppler rate causes faster variation of the channel, and makes channel estimation more difficult.

1.1.2 Channel estimation in DS-CDMA systems

We note that in a practical radio channel, “Fading, Multipath, Doppler Effect” causes the loss of *signal-noise-ratio* (SNR) and the distortion of the received signal in time-frequency domain. As a result, the system may exhibit severe performance degradation, such as **burst errors**.

The task of channel estimation algorithm is to approximate the impulse response of the radio channel which is to be used in symbol detection of a communication system. Thus, accurate channel estimation is important for reliable data transmission. As far as DS-CDMA systems are concerned, some applications of channel estimation are listed below

- Coherent demodulation relies on a model of the fading channel [83].
- Adaptive channel equalizer can apply the channel estimate to mitigate the ISI in frequency selective fading.
- Diversity scheme is a technique to improve the loss of transmitted signal’s SNR due to fading environments. It also requires the knowledge of the channel coefficients.

Channel estimation is used not only in DS-CDMA systems but also in other wireless communication environments. In fact, for different applications, channel estimation performs different tasks. For example, in the application of DS-CDMA

coherent detection, a channel estimation algorithm captures the impulse response of the fading channel and uses it in the symbol detector. The estimated coefficients are complex and so include the phase information. Another useful application of channel estimation is in the joint symbol and channel parameter detector, in which channel estimation algorithm is employed to estimate timing offset for asynchronous CDMA systems [89] [82].

What is a “good” channel estimation algorithm? A commonly used evaluation criterion is based on the *mean square error* (MSE), which is the variance of the difference between the true channel response and the corresponding estimate. Broadly speaking, the smaller the MSE, the better the channel estimation performance. From the viewpoint of whole DS-CDMA system, however, the objective of the channel estimation approaches is only to provide the channel information to multiuser detector. Thus, based on a given multiuser detection algorithm, the *bit error rate* (BER) of a system is also used as an alternative performance index for channel estimation algorithms.

1.2 Objective and Contribution of the Thesis

Our main objective in this thesis is channel estimation in the reverse link of DS-CDMA systems. From a practical perspective, although the task of designing a channel estimator can be a direct application of various estimation theories, the following factors should be considered:

- Computational complexity. It involves the number of operations such as addition and multiplication and the size of memory to store the data. The computational efficiency of an algorithm also determines the possibility of its real-time operation.

1.2 Objective and Contribution of the Thesis

8

- Tracking ability. Due to the time-variant property of practical wireless fading channels, a channel estimator is required to continuously capture the variations of channel coefficients in time. This ability becomes especially important for tracking fast fading channels.
- Performance robustness. In most practical situations, there exist various uncertainties in the system model of interest. For example, the disturbance or measurement noise in a state-space model may have inaccurate statistical information due to the interferences from other users or exhibit non-Gaussian statistical property. Also, there inevitably exist modeling errors, resulting from, for example, the use of a finite order FIR model to approximate a higher order IIR model. Another type of uncertainties is termed as parametric uncertainties that appear in the state and output matrices. These uncertainties account for the performance degradation of several estimation algorithms and hence the study of robust estimation is of practical importance.

Above requirements motivate us to develop a framework of polynomial design approaches to solve different estimation problems for practical time-variant fading channel environments. The compact polynomial expression of the proposed algorithms can achieve computational efficiency and make them implementable in digital signal processor (DSP). The main focus of the thesis is on the investigation of robust channel estimation problems. While the state-space approach to robust estimation has been extensively studied, few work has been reported for a polynomial robust estimation approach. This is due to the limited machineries available for dealing with stability and performance problems via the polynomial approach as compared to the state-space approach and the fact that finding a tight bound of the effect of uncertain system parameters on system performance in frequency

1.2 Objective and Contribution of the Thesis

9

domain is not easy. In this thesis, our objective is to develop a systematic polynomial approach to robust estimation under H_2 and H_∞ performance for systems with deterministic and random parameter uncertainties and demonstrate their applications in DS-CDMA systems. In the light of uncertainties, corresponding robust estimation problems are solved in individual chapters.

The main contributions of our work are summarized as follows

- Develop a polynomial channel estimator based on the *Wiener Least Mean Square* (WLMS) algorithm. For a second-order AR channel model, a one-step predictor is given in a simple form. In addition, we propose a novel adaptive *decision feedback equalizer* (DFE) to accomplish the multiuser detection in DS-CDMA systems. The proposed WLMS channel estimator has not only low computational complexity but also good performance in tracking fast fading channels.
- Develop a robust polynomial channel estimator in DS-CDMA systems in terms of the MMSE criterion. The system model of interest suffers from stochastic parametric uncertainties in both the state and observation equations. We introduce an innovative approach to calculate the statistics of the matrices which represent stochastic parametric uncertainties from the perspective of the frequency domain. Furthermore, a polynomial optimal MMSE channel estimator is derived by a simple variational approach. The proposed estimator offers robust tracking performance in fast fading applications.
- Derive a polynomial version of standard H_∞ estimator for DS-CDMA systems. By virtue of Krein space estimation theory, a polynomial solution to the H_∞ estimation problem is obtained by the projection in Krein space. This approach not only offers an attracting alternative to the standard H_∞

estimator but also lays a mathematical basis for the application of Krein space theory in the development of other robust estimation. Another noteworthy contribution is to derive a closed-form solution to the polynomial matrix J -spectral factorization, which is required in the proposed algorithm. This result leads to a great reduction of the computational complexity for a high dimension polynomial matrix and hence improves the computational efficiency of the proposed estimator.

- Develop a robust polynomial H_∞ estimation approach for systems that are subject to stochastic parameteric uncertainties from the Krein space viewpoint. Compared with the state-space solution given in [29], the proposed algorithm is based on a simpler derivation in a polynomial framework. The application in DS-CDMA channel estimation demonstrates good tracking performance and low computational complexity.
- Derive a robust polynomial MMSE channel estimator for systems with real time-varying norm-bounded parametric uncertainties in the channel state matrix. Based on the sum quadratic constraint of these uncertainties, the robust MMSE estimation problem is transformed into seeking the minimum of an indefinite quadratic form. The corresponding polynomial solution is obtained by employing optimal linear estimation for this indefinite quadratic cost function in the Krein space, which extends the family of robust Kalman filters to a polynomial representation.

1.3 Organization of the Thesis

The rest of this thesis is organized as follows.

In Chapter 2, we present a DS-CDMA system model used in the rest of this thesis. Firstly, an *autoregressive* (AR) channel model is established in terms of Jakes' model [42]. We then list several special cases of AR models and briefly introduce how to estimate AR model parameters in terms of covariance statistics of fading channels. Secondly, we present the received baseband signal in the reverse link of a binary DS-CDMA communication system, which suffers from the multipath propagation. The described channel model will be used as a system model for estimator design in the following chapters.

In Chapter 3, we employ a WLMS algorithm for channel estimation in DS-CDMA systems. An adaptive DFE detector is derived with the consideration of the feedback of decision errors. The proposed multiuser detector in conjunction of WLMS channel estimator shows robust detection performance.

In Chapter 4, we study the optimal robust polynomial channel estimation problem with respect to the MMSE criterion. Here, we consider a channel model subject to stochastic parameter uncertainties and an observation model subject to feedback decision errors from multiuser detection output in the tracking case. Based on the polynomial expression of systems, the statistics of parameter uncertainties of both the channel model and observation model are derived under rather weak assumptions. Next, by treating these uncertainties as fictitious noises, we convert the estimation of uncertain channels into the standard MSE estimation without uncertainties. In terms of the linear estimation theory, an optimal linear channel estimator is designed based on the MMSE criterion and is given in terms of a polynomial spectral factorization and the solution to a bilateral Diophantine equation.

In Chapter 5, we focus on the problem of standard H_∞ estimator design for systems without accurate knowledge of statistics of disturbances and measurement noises. These cases often occur in practical data transmission environment, which suffers from the fading and interferences from co-channel users and users outside cell. By virtue of Krein space theory, an innovative linear H_∞ estimator is given in a polynomial form via a J -spectral factorization. Further, under some conditions, we derive a closed-form solution to this J -spectral factorization, which greatly reduces the computational load of estimator design.

In Chapter 6, we extend the results of Chapter 5 to systems with stochastic parametric uncertainties. This class of problem is called as stochastic robust H_∞ estimation problem. Studies first focus on stochastic uncertainties in the channel state and a polynomial solution to this problem is developed. Furthermore, the proposed approach is extended to solve the stochastic robust H_∞ estimation problem for more general uncertain systems with stochastic parametric uncertainties in the state and output matrices. The application in DS-CDMA systems is demonstrated. Simulations show that the proposed estimator exhibits robust performance.

In Chapter 7, we return to the study of robust polynomial MMSE channel estimation problems. The study focuses on uncertain systems with norm-bounded parametric uncertainties in the channel state matrix. The uncertainties are defined by a sum quadratic constraint. For this class of uncertain systems, a robust Kalman filter is derived via a polynomial approach in the Krein space.

In Chapter 8, conclusions are drawn together with possible future research directions.

Chapter 3 to Chapter 7 consist of the main body of this thesis. They provide a systematic polynomial approach to robust H_2 and H_∞ estimation for systems with deterministic or stochastic parametric uncertainties.

Chapter 2

Issues in Channel Estimation

2.1 DS-CDMA System Model for Channel Estimation Problems

In recent years, DS-CDMA has emerged as one of the most popular technologies for cellular mobile communication. In a DS-CDMA system, individual users multiplexed by distinct codes as spreading codes can share frequency and time space. Thus, compared with other multiple access techniques such as FDMA and TDMA, DS-CDMA has many advantages. It is able to achieve higher system capacity and has higher capability to employ frequency reuse and support the transmission of multi-code and multi-rate symbols. By virtue of the approximate orthogonality among all users' spreading codes, the recovery of transmitted signals can be achieved by a detector, which separates individual signals transmitted over the same multi-access channel. The model of uplink DS-CDMA communication with K users is presented in Figures 2.1 and 2.2.

In this thesis, we consider a K -user binary DS-CDMA communication system which has been widely used; see [83] [103] [86]. The transmitted baseband signal

2.1 DS-CDMA System Model for Channel Estimation Problems

14

Figure 2.1: The model of uplink transmission in the DS-CDMA system.

Figure 2.2: Possible receiver structure of the DS-CDMA system.

of the k th user is given by

$$x_k(t) = \sqrt{A_k} \sum_{n=-\infty}^{\infty} b_k(n) s_k(t - nT_s) \quad (2.1)$$

where A_k is the transmitted bit energy, T_s is the symbol duration, $b_k(n)$ denotes the information symbol sequence of the k th user and is chosen randomly from the set $\{-1, +1\}$ and s_k denotes the energy-normalized transmitted waveform (i.e., $\int s_k^2(t) dt = 1$). We assume that $s_k(t)$ is supported only on the interval $[0, T_s]$ and is of the form

$$s_k(t) = \sum_{i=0}^{N-1} \tilde{c}_k(i) \psi(t - iT_c)$$

2.1 DS-CDMA System Model for Channel Estimation Problems

15

where the sequence $\{\tilde{c}_k(i)\}_{i=0}^{N-1}$ denotes the spreading code of the k th user and ψ is the chip waveform with chip duration $T_c = T_s/N$ and $\int \psi^2(t)dt = N^{-1}$, where N is the spreading gain.

Figure 2.3: The impulse response of frequency selective fading channel.

As mentioned in Chapter 1, the frequency selective fading is a phenomenon which occurs when the transmitted signal occupies a greater bandwidth than the *coherence bandwidth* during which spectral components are distorted in a similar way. Due to the spreading process of the DS-CDMA system, the bandwidth of the transmitted signal is much wider than the channel *coherence bandwidth*. Therefore, the signals in the DS-CDMA system undergo frequency selective fading channel.

2.1 DS-CDMA System Model for Channel Estimation Problems

16

This type of fading can be modeled as a linear filter characterized by the complex-valued impulse response in Figure 2.3, which is also termed as the *input delay-spread function* [4]. For example, for the k th user's signal multipath propagation, the impulse response of fading channel is denoted by $\mathcal{H}_k(t)$. We assume that this time-varying multipath channel consists of $L + 1$ resolvable propagation paths. Then the chip-rate sampled channel impulse response is described by

$$\mathcal{H}_k(t) = \sum_{l=0}^L h_k^l(nN + i)\delta(t - lT_c), \quad nT_s + iT_c \leq t < nT_s + (i + 1)T_c \quad (2.2)$$

where $\delta(\cdot)$ is the Dirac function, l is the channel index and lT_c is the time delay introduced by the complex-valued fading coefficient $h_k^l(nN + i)$. From (2.1) and (2.2), the received signal component from the k th user is

$$\begin{aligned} y_k(t) &= x_k(t) \otimes \mathcal{H}_k(t) \\ &= \sqrt{A_k} \sum_{m=-\infty}^{\infty} b_k(m) s_k(t - mT_s) \otimes \mathcal{H}_k(t) \\ &= \sqrt{A_k} \sum_{m=-\infty}^{\infty} \sum_{i=0}^{N-1} \sum_{l=0}^L h_k^l(mN + i) \psi(t - mT_s - iT_c) \\ &\quad \times \left(b_k(m) \tilde{c}_k(i - l) + b_k(m - 1) \tilde{c}_k(N + i - l) \right) \end{aligned} \quad (2.3)$$

The total received signal at base station is the superposition of the signals of the K users in the presence of *additive white Gaussian noises* (AWGN), given by

$$r(t) = \sum_{k=1}^K y_k(t) + v(t)$$

where $v(t)$ is a zero mean complex Gaussian white noise. If we assume $L \leq N$, by sampling the output of a chip-matched filter at the chip rate, a discrete-time

signal of the k th user at the j th chip interval of the n th symbol is given by

$$\begin{aligned}
 & y_k(nN + j) \\
 = & \int_{nT_s + jT_c}^{nT_s + (j+1)T_c} y_k(t) \psi(t - nT_s - jT_c) dt \\
 = & \int_{nT_s + jT_c}^{nT_s + (j+1)T_c} \left(\sqrt{A_k} \sum_{m=-\infty}^{\infty} \sum_{i=0}^{N-1} \sum_{l=0}^L h_k^l(mN + i) \psi(t - mT_s - iT_c) \right. \\
 & \times \left. \left(b_k(m) \tilde{c}_k(i - l) + b_k(m - 1) \tilde{c}_k(N + i - l) \right) \right) \psi(t - nT_s - jT_c) dt \\
 = & \sqrt{A_k} \sum_{m=-\infty}^{\infty} \sum_{i=0}^{N-1} \sum_{l=0}^L h_k^l(mN + i) (b_k(m) \tilde{c}_k(i - l) + b_k(m - 1) \tilde{c}_k(N + i - l)) \\
 & \times \int_0^{T_c} \psi\left(t + (n - m)T_s + (j - i)T_c\right) \psi(t) dt \\
 \stackrel{m=n, i=j}{=} & \frac{\sqrt{A_k}}{N} \sum_{l=0}^L h_k^l(nN + j) \left(b_k(n) \tilde{c}_k(j - l) + b_k(n - 1) \tilde{c}_k(N + j - l) \right) \\
 = & \sum_{l=0}^L h_k^l(nN + j) \left(b_k(n) c_k(j - l) + b_k(n - 1) c_k(N + j - l) \right) \\
 & 0 \leq j \leq N - 1 \tag{2.4}
 \end{aligned}$$

where we define

$$c_k(i) = \begin{cases} \frac{\sqrt{A_k}}{N} \tilde{c}_k(i) & , \quad 0 \leq i \leq N - 1 \\ 0 & , \quad otherwise. \end{cases}$$

To simplify (2.4), we can assume that the channel taps $h_k^l(nN + j)$ are constant within one symbol interval. Moreover, we wish that such invariant channel tap has same time index as $b_k(n)$ or $b_k(n - 1)$ as shown in Figure 2.4. Hence, we introduce the following assumption:

Assumption 2.1.1. *For each multipath channel, the channel taps $h_k^l(nN + j)$ are constant within one symbol interval and are termed as $h_k^l(n)$.*

2.1 DS-CDMA System Model for Channel Estimation Problems

18

Figure 2.4: The received signal of the k th user after undergoes multipath channel.

In term of Assumption 2.1.1, $h_k^l(nN+j)$ and $h_k^l(n)$ satisfy the following relation:

$$\begin{cases} h_k^l(n) \triangleq h_k^l(nN+j) = \dots = h_k^l(nN+j+N-1) & , \quad j \geq l \\ h_k^l(n-1) \triangleq h_k^l((n-1)N+j) = \dots = h_k^l(nN+j-1) & , \quad j < l \end{cases} \quad (2.5)$$

From (2.5), (2.4) is reformulated as:

$$y_k(nN+j) = b_k(n) \sum_{l=0}^L h_k^l(n) c_k(j-l) + b_k(n-1) \sum_{l=0}^L h_k^l(n-1) c_k(N+j-l) \quad (2.6)$$

For (2.4), we note that there is a similar derivation in [14]. Even above derivation is involved only in the case when $L \leq N$, it is easy to extend to the case when $L > N$. When $L > N$, the $y_k(nN+j)$ has a similar form to (2.4) except that item $h_k^l(n-2)$ may be included. Next, the vector $\mathbf{y}_k(n)$ is formed by collecting N

2.1 DS-CDMA System Model for Channel Estimation Problems

19

successive samples as shown in Figure 2.4.

$$\begin{aligned}\mathbf{y}_k(n) &= [y_k(nN) \ \cdots \ y_k(nN + N - 1)]^T \\ &= b_k(n)\mathbf{C}_k^0\mathbf{h}_k(n) + b_k(n-1)\mathbf{C}_k^1\mathbf{h}_k(n-1)\end{aligned}\quad (2.7)$$

where

For a synchronous CDMA forward channel, the discrete-time observation vector of the received signal can be written as

$$\begin{aligned}\mathbf{r}(n) &= [r(nN) \ r(nN + 1) \ \cdots \ r(nN + N - 1)]^T \\ &= \sum_{k=1}^K \mathbf{y}_k(n) + \mathbf{v}(n) \\ &= \mathbf{C}_0\mathbf{B}(n)\mathbf{h}(n) + \mathbf{C}_1\mathbf{B}(n-1)\mathbf{h}(n-1) + \mathbf{v}(n)\end{aligned}\quad (2.8)$$

where

$$\mathbf{C}_0 = [\mathbf{C}_1^0 \ \cdots \ \mathbf{C}_K^0], \quad (2.9)$$

$$\mathbf{C}_1 = [\mathbf{C}_1^1 \ \cdots \ \mathbf{C}_K^1], \quad (2.10)$$

$$\mathbf{B}(n) = \text{diag}\{b_1(n)\mathbf{I}_{L+1}, \ \cdots, b_K(n)\mathbf{I}_{L+1}\},$$

$$\mathbf{h}(n) = [\mathbf{h}_1^T(n) \ \mathbf{h}_2^T(n) \ \cdots \ \mathbf{h}_K^T(n)]^T, \quad (2.11)$$

$$\mathbf{v}(n) = [v(nN) \ \cdots \ v(nN + N - 1)]^T, \quad (2.12)$$

$$v(nN + j) = \int_{nT_s + jT_c}^{nT_s + (j+1)T_c} v(t)\psi(t - nT_s - jT_c)dt, \quad 0 \leq j \leq N - 1.$$

It should be noted that the same model is adopted in [16]. For the forward channel, since every user's spreading code is known by the base station, coefficient matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ in (2.8) are known. Symbol $b_k(n)$ is the actual value from training sequences or information data of the k th user. When a multiuser detector has

much high BER performance, symbol values detected can also be used as actual values in the proposed estimator.

2.2 Channel Modelling

In fact, it is extremely difficult to obtain the statistical characterization of practical mobile radio channels because they are non-stationary. However, considering slow channel variations over a small interval of time, we can assume that the fading statistics of radio channels are stationary over a short period of time. In 1963, *Bello* proposed the *wide-sense stationary uncorrelated scattering* (WSSUS) channel model to describe the statistical characteristics of wireless fading channels in [4]. This model expresses two meanings in terms of its name, *wide-sense stationary* (WS) channel and *uncorrelated scattering* (US) channel. Based on the assumption of weakly stationary fading channels, the channel autocorrelation functions are viewed time-invariant during transmission time in a WS channel. It means that the statistics of channel fading does not depend on the specific time but the time difference. On the other hand, the channel coefficients from different paths are treated to be uncorrelated with each other in the US channel. Even though a WSSUS channel model is less accurate, it is still a much effective tool to describe many practical radio channels. Bell's model has withstood the test of time!

2.2.1 Autoregressive channel model

The discrete-time fading channel can be modeled exactly by Autoregressive Moving-Average model (ARMA). However, Autoregressive model (AR) is most commonly used to approximate channel fading process. It is often chosen due to the following reasons. First, as the IIR filter by nature, AR model may use the limited

order to match accurately desired theoretical statistics. Secondly, the parameters of AR models are easily calculated. Thus, AR models are successfully applied not only in fading channel estimation [78] [10] [79] [47] but also in fading channel simulation [3] [99].

We assume that fading channel coefficients are invariant at a symbol duration. Then fading coefficient h_k may be modeled as a p -order AR model as

$$h_k^l(n) = \sum_{j=1}^p d_{k,j}^l h_k^l(n-j) + e_k^l(n), \quad l = 0, \dots, L \quad (2.13)$$

where $d_{k,j}^l$ denotes the state transition coefficients of the k th user in the l th path and $e_k^l(n)$ is a circular complex Gaussian process with zero mean.

2.2.2 How to obtain the parameters?

According to the Bello model, h_k^i and h_k^j in (2.2) are uncorrelated because different path's channel taps are independent, i.e. $E\{h_k^i h_k^j\} = 0$ when $i \neq j$. However, given the power density spectrum as described in (1.1), the channel tap of a path has autocorrelation function as in [42]

$$E\{h_k^i(n) (h_k^i(n-j))^*\} \sim J_0(2\pi f_d T_s j), \quad j = 0, \pm 1, \dots, \quad (2.14)$$

where $J_0(\cdot)$ denotes the zero order Bessel function of the first kind and f_d is the Doppler frequency. With the autocorrelation model of (2.14), the parameters of channel model can be obtained by solving a Yule-Walker equation. We rewrite the p -order AR channel model (2.13) as follows

$$d_1 h(n-1) + d_2 h(n-2) + \dots + d_p h(n-p) = h(n) - e(n). \quad (2.15)$$

2.2 Channel Modelling

22

Multiplying row of (2.15) by $h^*(n-i)$, $1 \leq i \leq p$ and taking the expectation, we get p equations which are called Yule-Walker equations

$$\begin{bmatrix} r(0) & r(-1) & \cdots & r(-p+1) \\ r(1) & r(0) & \cdots & r(-p+2) \\ \vdots & \vdots & \ddots & \vdots \\ r(p-1) & r(p-2) & \cdots & r(0) \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ r(p) \end{bmatrix}$$

where $r(l)$ denotes the *autocorrelation function* (ACF) of $h(n)$, which can be obtained by virtue of the autocorrelation property in (2.14). Hence, state transition coefficients d_j can be obtained directly by solving the above system of equations. In the case of a large p , the above high dimensional system of equations can be solved efficiently by using Levinson-Robinson algorithm [64]. It should be noted that coefficients ' d_j ' only rely on autocorrelation function $r(i)$, which is only influenced by the maximum Doppler frequency f_d .

Example 2.2.1. For a 900 MHz wireless system with symbol duration $T_s = 100\mu s$, the state transition coefficients of AR(2) model corresponding to different Doppler frequency are given in Table 2.1.

Table 2.1: The state transition coefficients of AR(2) model for a 900 MHz wireless system with symbol duration $T_s = 100\mu s$

Mobile velocity (km/hr)	30	60	90	120	150	180
Doppler frequency f_d (Hz)	25	50	75	100	125	150
Doppler rate $f_d T_s$	0.0025	0.005	0.0075	0.01	0.0125	0.015
d_1	1.9998	1.9994	1.9986	1.9975	1.9961	1.9945
d_2	-1.0000	-0.9999	-0.9997	-0.9995	-0.9992	-0.9989

2.2.3 How to select the order of AR channel model?

It should be noted that an accurate representation of dynamic statistics would be obtained with an AR model of infinite order. However, low-order AR models (even a simple Gauss-Markov model) that match the Bessel ACF well for small lags can capture most of the channel dynamics and perform well in channel estimation algorithms. For example, a second-order AR (AR(2)) model has been adopted in [47] [52] due to its satisfactory trade-off between computation complexity and good matching of ACF. The latter is demonstrated by Figure 2.5. It shows that an AR(2) model provides a good autocorrelation match to Bessel ACF for lags less than 100, in which ACF value is important for the design of the estimator. Certainly, there inevitably exists leakage outside the passband for a limited order AR model. But the majority of energy still remains in the frequency range $[-f_d, f_d]$.

Figure 2.5: Channel autocorrelation function when $f_d = 50$ Hz, $f_s = 10^4$ Hz.

In this thesis, a Rayleigh fading simulator based on Clark-Jakes model [19] is

applied to produce experiment data ; see also [3]. The simulation results also show that the selection of an AR(2) model is reasonable for approximating real channel dynamics. Therefore, in Chapters 3-5, we restrict our attention to an AR(2) model due to its simplicity and close match with real channels. In terms of AR(2) model, multipath fading channels that all users undergo can be formed as the vector

$$\mathbf{h}(n) = \mathbf{D}_1 \mathbf{h}(n-1) + \mathbf{D}_2 \mathbf{h}(n-2) + \mathbf{e}(n) \quad (2.16)$$

where $\mathbf{h}(n)$ is as in (2.11) and

$$\begin{aligned} \mathbf{D}_j &= \text{diag}\{d_{1,j}^0, \dots, d_{1,j}^L, \dots, d_{K,j}^0, \dots, d_{K,j}^L\} \quad j = 1, 2, \\ \mathbf{e}(n) &= [e_1^0(n) \dots e_1^L(n) \dots e_K^0(n) \dots e_K^L(n)]^T. \end{aligned}$$

2.3 Review of Channel Estimation Algorithms

Broadly speaking, the channel estimation is an application of estimation theory in wireless communication systems. Therefore, many classic algorithms in estimation theory (for example, *maximum-likelihood* (ML), *least-squares* (LS), Bayesian estimation, Kalman filter and maximum entropy filter [44]) have been widely employed to solve vast complicated problems of channel estimation by treating the time-variant channels as stochastic and deterministic processes. Next, we will offer some references of several widely used estimation algorithms.

Given the knowledge of the symbols from training sequences, multiuser ML channel estimation algorithms maximize the likelihood function of the desired channel parameters, such as amplitude, phase and time delay, to approximate the channel characteristics. ML estimators for single path parameters estimation have been proposed in [89] [93] [6]. Corresponding estimation algorithms in multipath

propagation environment are considered in [71] [25] [7]. Moreover, ML techniques have also been widely used to combine multiuser detector for joint channel parameter estimation and symbol detection; see [50]. Even though the ML algorithm has very good estimation performance, it requires a tree search over many parameters. Therefore, this type of methods suffers from high computational complexity and is not suitable for the fast fading channel case. On the other hand, the ML algorithm is based on the assumption of the Gaussian noise channel. Hence, we have to consider other estimation schemes without probabilistic assumption if the statistics of the distribution of channel noise is not Gaussian or unavailable.

Subspace-based method is an alternative to handle estimation problems with no prior probabilistic knowledge. This method was proposed by Bensley and Aazhang in [5]. Due to the advantage of no requirement of channel statistics, subspace algorithm has been widely employed for blind estimation. Early works in this field include [75] [77]. However, this algorithm involves a complicated optimization process based on the block fading assumption. Hence, it is not suitable for fast-fading channels where channel changes rapidly within a block.

Other schemes to exploit the channel estimation problem have also been proposed, such as *Expectation-Maximization* (EM) algorithm [43] [41], Wavelets applications [94] [91].

In fact, as far as the channel estimation problem is concerned, a wide variety of works concentrate on the applications of *least mean square* (LMS), LS and Kalman algorithms. As the family of adaptive filters, these approaches do not require a priori information about the statistics of the channels and can converge to the optimum Wiener solution in some statistical sense by a recursive updating process [37]. We will visit some applications of these algorithms on the channel estimation problems. First, LMS filters have been employed early in the channel

estimation due to its simplicity and model-independence [54]. However, the LMS filter exhibits a relatively slow convergence behavior thereby it is not suitable to track fast-varying channels. Even so, the LMS filter is of practical importance, and besides, is a benchmark for the evaluation of other channel estimation algorithms. Unlike the LMS filter, the least-square filter is a model-dependent procedure and operates a batch of input data. In particular, its recursive version, the RLS filter, has a higher rate of convergence and smaller mean square error than those of the LMS filter. Therefore, RLS-based channel estimators are well suitable for fast fading channels estimation. For example, in [10] authors propose a rectangular-windowed RLS filter which provides a significant improvement in the estimation performance for a time-varying channel over the conventional LMS and the exponentially weighted RLS algorithms. Furthermore, an application of the RLS algorithm on the estimation of fast fading channels is also presented in [49]. Finally, we must emphasize the practical importance of the celebrated Kalman filter on the wireless channel estimation problems. It is well known that the Kalman filter is the optimal estimator in the sense of mean square error for a linear dynamical system with Gaussian white noise inputs and has been employed in numerous applications. Also, a great deal of Kalman-based channel estimation algorithms have been proposed to operate in various communication systems, see for example, [16] [47] [87] [11] [78] [31] [76] [46] [51]. However, the Kalman filter has the high computational complexity, which restricts itself from on-line implementations. Moreover, its performance is sensitive to the model uncertainties, which often occur in practical wireless applications. These problems are just what we will investigate in the following chapters.

Finally, as an interesting field, the study of the blind channel estimation and semiblind channel estimation has gained considerable attention recently. Blind

2.3 Review of Channel Estimation Algorithms

27

channel estimation is motivated by the improvement of bandwidth utilization for time-varying channels. The corresponding estimator operates without the knowledge of the transmitted signals thereby can capture the variation of channels as fast as possible. Semibind channel estimation aims at the estimation problems when the observations corresponding to both the known symbols and unknown symbols are available. Such study is of practical importance since in general some known data are incorporated in the data package, which can be used by the semibind estimator. A detailed review on these two kinds of estimation problems is presented in [81].

Chapter 3

Polynomial WLMS Channel Estimator

3.1 Introduction

As mentioned in Chapter 2, a family of adaptive filters has been widely applied to solve the channel estimation problem in various radio communication systems. Among them, the well known Kalman-based estimator has received considerable interest because it is the optimal linear MMSE estimator whenever the dynamics of systems can be described as a stochastic state-space model [12]. However, it is not suitable for channel tracking due to the high computational complexity. Whereas the LMS filter operates in a simple manner but its estimation performance is unacceptable. Hence, the challenge facing us is to strike a preferable balance between the complexity and performance in time-varying wireless channels. In [52] [53] Lindbom *et al.* propose a LMS-like filter, termed *Wiener LMS* (WLMS) algorithm, which can provide an appropriate tradeoff between such two requirements. The key idea behind this algorithm is to transform the estimation problem of interest to a standard Wiener filtering problem then design corresponding MMSE estimator by means of a polynomial approach. Our primary interest in the WLMS-based estimator is motivated by the fact that the WLMS filter bears as nearly the

same form and low computational complexity as that of the standard LMS, yet offers a significant performance improvement. By virtue of this scheme, we can have a straightforward design on a fixed-lag predictor, estimator, and smoother. Next we will outline the WLMS filter and corresponding operation steps.

Consider the following linear system:

$$h(n) = \frac{e(n)}{D(q^{-1})} = \frac{e(n)}{1 + d_1 q^{-1} + \dots + d_{n_D} q^{-n_D}} \quad (3.1)$$

$$y(n) = \varphi(n)^* h(n) + v(n) \quad (3.2)$$

where $h(n) \in \mathcal{C}^{N_h}$ denotes the state vector, $e(n) \in \mathcal{C}^{N_h}$ denotes the process noise, $y_n \in \mathcal{C}^{N_y}$ denotes the measured output, $v(n) \in \mathcal{C}^{N_y}$ is the measurement noise, $D(q^{-1})$ is polynomial with scalar coefficients and $\varphi(n)^* \in \mathcal{C}^{N_y \times N_h}$ is the known coefficient matrix. WLMS algorithm is based on the following assumption:

Assumption 3.1.1. $\mathbf{R} \triangleq \{\varphi(n)\varphi(n)^*\}$ is nonsingular and time-invariant.

This assumption can be satisfied for the system model adopted in this thesis. In Table 3.1 we present a summary of the WLMS algorithm, which generalizes the prediction and smoothing in the unified representation. A point to note is that, although the computation of the current estimate $\hat{\mathbf{h}}(n|n)$ is similar to the standard LMS, the corresponding prediction and smoothing are entirely based on $\mathcal{P}_m(q^{-1})$, which is a causal stable rational matrix and time-invariant with respect to the step parameter m . The appropriate $\mathcal{P}_{\mathbf{m}}(\mathbf{q}^{-1})$ depends on the dynamic of $\mathbf{h}(n)$ and the SNR. The solution to the optimal $\mathcal{P}_m(q^{-1})$ is obtained via by minimizing $\text{tr} \mathbf{P}$, where $\text{tr} \mathbf{P} \triangleq E\{ \tilde{\mathbf{h}}(n+1|n)^* \tilde{\mathbf{h}}(n+1|n) \}$ and prediction error vector $\tilde{\mathbf{h}}(n+1|n) \triangleq \mathbf{h}(n+1) - \hat{\mathbf{h}}(n+1|n)$. Generally, the process of obtaining an optimal solution to $\mathcal{P}$ is complex and needs to compute a polynomial spectral factorization. Fortunately, for the second-order AR model, the optimal $\mathcal{P}_{\mathbf{m}}$ has

Table 3.1: Summary of the WLMS filter

Prediction ($m > 0$) and Smoothing ($m < 0$):

For each instant time, $n = 1, 2, \dots$, compute

$$\mathcal{E}(n) = y(n) - \varphi(n)^* \hat{\mathbf{h}}(n|n-1),$$

$$\hat{\mathbf{h}}(n|n) = \hat{\mathbf{h}}(n|n-1) + \mu \mathbf{R}^{-1} \varphi(n) \mathcal{E}(n),$$

$$\hat{\mathbf{h}}(n+m|n) = \mathcal{P}_m(q^{-1}) \hat{\mathbf{h}}(n|n).$$

An example (Simplified WLMS algorithms):

Hpermodel:

$$h(n) = \frac{1}{1+d_1q^{-1}+d_2q^{-2}} e(n).$$

One-step Prediction:

$$\hat{\mathbf{h}}(n+1|n) = -p_1 \hat{\mathbf{h}}(n|n-1) + p_2 \hat{\mathbf{h}}(n|n) + d_2 \hat{\mathbf{h}}(n-1|n-1)$$

where

$$p_1 = \frac{d_1 d_2 (1-\mu)}{1-d_2(1-\mu)}, \quad p_2 = \frac{d_1}{1-d_2(1-\mu)}.$$

been obtained in a simple form. This filter is called *Simplified WLMS* (WLMS) algorithms and corresponding one-step predictor is also presented in Table 3.1. A complete derivation of the LMS filter is presented in [53] and the corresponding analysis of stability and performance is shown in [1].

In this chapter, we consider the problem of designing a simple polynomial channel estimator in the application of multiuser detection. A WLMS algorithm is employed to estimate the coefficients of the fading channels. Due to the attributes of WLMS filters, the proposed estimator has a performance similar to Kalman

3.2 Problem Statement and Structure of Joint Channel Estimator and DFE Detector

31

filter but with a computational complexity similar to the standard LMS. Besides the estimator design, we derive a decision-feedback equalizer (DFE) to implement multiuser detection under the MSE criterion. The optimal design of feedforward filter and feedback filter in the DFE detector also takes the decision errors into consideration. Thus, this MMSE-DFE is robust in the symbol detection. Another point needed to be stressed is that throughout this thesis, such DFE multiuser detector not only provides the symbol for channel estimators in the tracking case, but also is employed to evaluate various channels estimation algorithms in terms of BER performance.

3.2 Problem Statement and Structure of Joint Channel Estimator and DFE Detector

We consider the K -user DS-CDMA system model with N -length spreading code previously shown in Chapter 2:

$$\mathbf{h}(n) = D_1 \mathbf{h}(n-1) + D_2 \mathbf{h}(n-2) + \mathbf{e}(n), \quad (3.3)$$

$$\mathbf{r}(n) = \mathbf{C}_0 \mathbf{B}(n) \mathbf{h}(n) + \mathbf{C}_1 \mathbf{B}(n-1) \mathbf{h}(n-1) + \mathbf{v}(n) \quad (3.4)$$

where $\mathbf{h}(n) \in \mathcal{C}^{K(L+1)}$ is the vector of channel coefficients, $\mathbf{r}(n) \in \mathcal{C}^N$ denotes the observation, D_1 and $D_2 \in \mathcal{R}^{K(L+1) \times K(L+1)}$, $\mathbf{C}_0$ and $\mathbf{C}_1 \in \mathcal{R}^{N \times K(L+1)}$ are defined in Chapter 2 and the second-order AR model is adopted to describe the dynamics of the fading channels.

In this chapter, our aim is to design a joint channel estimation and multiuser detection algorithm which can perform the adaptive symbol detection without priori channel information. This detector uses a WLMS filter to track the channel

Figure 3.1: Joint channel estimation and symbol detection block diagram

coefficients and employs a MMSE-DEF to equalize received signals. The WLMS filter assumes that the DFE hard decisions are correct and uses them to estimate the channel response. On the other hand, DFE employs the estimated channel values to update both the feedforward filter and the feedback filter, and in turn, performs the task of equalization and hard-decision. This process is illustrated in Figure 3.1 where $\mathcal{W}$ and $\mathcal{G}$ denote the feedforward filter and feedback filter, respectively. In the following two sections, we will describe such two parts of the detector in details, respectively.

3.3 Polynomial WLMS Channel Tracking and Prediction

In view of the results in Table 2.1 and for the sake of reducing the algorithm complexity, we use d_1 and d_2 , the averages of all coefficient estimates $\hat{d}_{k,1}^l$ and

3.3 Polynomial WLMS Channel Tracking and Prediction

33

$\hat{d}_{k,2}^l$, respectively, to simplify (3.3), while taking their potential differences with the average values as uncertainties. Here, the hats emphasize that they are the estimates of real state transition coefficients. We rewrite (3.3) to a polynomial form as

$$d(q^{-1})\mathbf{h}(n) = \mathbf{e}(n) + \mathbf{e}_0(n) \quad (3.5)$$

where

$$\begin{aligned} d(q^{-1}) &= 1 - d_1 q^{-1} - d_2 q^{-2}, \\ \mathbf{e}_0(n) &= \Delta \mathbf{D}(q^{-1})\mathbf{h}(n), \\ \Delta \mathbf{D}(q^{-1}) &= \text{diag}\{\Delta d_{1,1}^0 q^{-1} + \Delta d_{1,2}^0 q^{-2}, \dots, \Delta d_{K,1}^L q^{-1} + \Delta d_{K,2}^L q^{-2}\}, \\ d_j &= \frac{1}{K(L+1)} \sum_{k=1}^K \sum_{l=0}^L \hat{d}_{k,j}^l \quad j = 1, 2, \\ \Delta d_{i,j}^m &= d_{i,j}^m - d_j \quad m = 0 \dots L, \quad i = 1 \dots K. \end{aligned}$$

Note that the coefficient $\Delta d_{i,j}^m$ is given by

$$\begin{aligned} \Delta d_{i,j}^m &= d_{i,j}^m - \frac{1}{K(L+1)} \sum_{k=1}^K \sum_{l=0}^L \hat{d}_{k,j}^l \\ &= d_{i,j}^m - \frac{1}{K(L+1)} \sum_{k=1}^K \sum_{l=0}^L (d_{k,j}^l + \Delta \hat{d}_{k,j}^l) \\ &= \frac{1}{K(L+1)} \sum_{k=1}^K \sum_{l=0}^L (d_{i,j}^m - d_{k,j}^l) - \frac{1}{K(L+1)} \sum_{k=1}^K \sum_{l=0}^L \Delta \hat{d}_{k,j}^l \quad (3.6) \end{aligned}$$

where $\Delta \hat{d}_{k,j}^l$ denotes the estimation error of coefficients. Since Table 2.1 shows that the state transition coefficients corresponding to different Doppler frequencies are very close, the first term in (3.6) is very small. Thus $\Delta \hat{d}_{k,j}^l$ mainly depends on the second item which is the coefficient estimation error and can be considered to be independent of $\mathbf{h}(n)$. Hence, the term $\mathbf{e}_0(n)$ is termed as *fictitious noise*. Rewrite

3.3 Polynomial WLMS Channel Tracking and Prediction

34

(3.4) as

$$\mathbf{r}(n) = \mathbf{F}(n) \mathbf{h}(n) + \tilde{\mathbf{F}}(n) \mathbf{h}(n-1) + \mathbf{v}(n)$$

where $\mathbf{F}(n) = \mathbf{C}_0 \mathbf{B}(n)$ and $\tilde{\mathbf{F}}(n) = \mathbf{C}_1 \mathbf{B}(n-1)$. Then,

$$\begin{aligned} & \mathbf{r}(n) - \tilde{\mathbf{F}}(n) \hat{\mathbf{h}}(n-1|n-1) \\ &= \mathbf{F}(n) \mathbf{h}(n) + \tilde{\mathbf{F}}(n) \mathbf{h}(n-1) - \tilde{\mathbf{F}}(n) \hat{\mathbf{h}}(n-1|n-1) + \mathbf{v}(n) \\ &= \mathbf{F}(n) \mathbf{h}(n) + \tilde{\mathbf{F}}(n) (\mathbf{h}(n-1) - \hat{\mathbf{h}}(n-1|n-1)) + \mathbf{v}(n). \end{aligned} \quad (3.7)$$

Define the estimation error vector

$$\tilde{\mathbf{h}}(n|n) \triangleq \mathbf{h}(n) - \hat{\mathbf{h}}(n|n)$$

where $\hat{\mathbf{h}}(n|n)$ is an estimate of $\mathbf{h}(n)$ at time T_n . Then (3.7) is simplified as follows:

$$\mathbf{z}(n) = \mathbf{F}(n) \mathbf{h}(n) + \tilde{\mathbf{v}}(n), \quad (3.8)$$

where

$$\mathbf{z}(n) = \mathbf{r}(n) - \tilde{\mathbf{F}}(n) \hat{\mathbf{h}}(n-1|n-1), \quad (3.9)$$

$$\tilde{\mathbf{v}}(n) = \tilde{\mathbf{F}}(n) \tilde{\mathbf{h}}(n-1|n-1) + \mathbf{v}(n). \quad (3.10)$$

The new vector $\mathbf{z}(n)$ is referred to as new measurement vector with $\tilde{\mathbf{v}}$ in (3.8) being the noise, which consists of estimation error and actual measurement noise. For $\mathbf{v}(n)$, we introduce the following assumption.

Assumption 3.3.1. $\mathbf{v}(n)$ is a white noise with zero mean and the covariance σ_v^2 .

Remark 3.3.1. *From the simulation results, the proposed algorithm has good channel estimation performance ($\tilde{\mathbf{h}}(n-1|n-1)$ comes close to 10^{-2}). Also, the variance of the measurement noises $\mathbf{v}(\mathbf{n})$ is relatively high. Hence, the impact of the estimate error $\tilde{\mathbf{h}}(n-1|n-1)$ on $\tilde{\mathbf{v}}(n)$ can be neglected. In view of Assumption 3.3.1, $\tilde{\mathbf{v}}(n)$ is approximately a white noise.*

The channel predictor process includes two phases: training phase and tracking phase. In the training phase, training sequences are sent and actual channel response can be obtained. It should be noted that in this thesis we apply uniformly distributed training sequences in the simulations. Following the training phase, channel predictor is switched to the tracking phase. In this case, the WLMS algorithm regards the decision $\hat{\mathbf{b}}(n)$ from DFE as symbols information and assumes it to be equal to the true $\mathbf{b}(n)$. According to the WLMS algorithm as demonstrated in Table 3.1, the channel tracking procedure based on the system (3.5)(3.8) is described as follows:

- 1) Calculate the matrices $\mathbf{F}(n)$ and $\tilde{\mathbf{F}}(n)$, where matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ of (3.4) are known previously from the fact that the base station masters every user's spread code.
- 2) Calculate the new observation vector $\mathbf{z}(n)$ via (3.9). $\hat{\mathbf{h}}(n-1|n-1)$ is acquired from the last iterative result.
- 3) $\mathcal{E}(\mathbf{n}) = \mathbf{z}(\mathbf{n}) - \mathbf{F}(\mathbf{n})^* \hat{\mathbf{h}}(\mathbf{n}|\mathbf{n}-1)$.
- 4) $\hat{\mathbf{h}}(n|n) = \hat{\mathbf{h}}(n|n-1) + \mu \mathbf{R}^{-1} \mathbf{F}(n)^* \mathcal{E}(\mathbf{n})$.
- 5) $\hat{\mathbf{h}}(n+1|n) = -\frac{d_1 d_2 (1-\mu)}{1-d_2(1-\mu)} \hat{\mathbf{h}}(n|n-1) + \frac{d_1}{1-d_2(1-\mu)} \hat{\mathbf{h}}(n|n) + d_2 \hat{\mathbf{h}}(n-1|n-1)$.

where μ is the step-size and $\mathbf{R}$ is the nonsingular covariance matrix of $\mathbf{F}(n)$ given

by

$$\begin{aligned}
\mathbf{R} &\triangleq E\{\mathbf{F}^*(n)\mathbf{F}(n)\} \\
&= E\{(\mathbf{C}_0 \mathbf{B}(n))^* \mathbf{C}_0 \mathbf{B}(n)\} \\
&= E\left\{ \begin{bmatrix} b_1(n)\mathbf{C}_1^{0*} \\ \vdots \\ b_K(n)\mathbf{C}_K^{0*} \end{bmatrix} \begin{bmatrix} b_1(n)\mathbf{C}_1^0 & \cdots & b_K(n)\mathbf{C}_K^0 \end{bmatrix} \right\}.
\end{aligned}$$

Since a user's symbols are independent and uncorrelated with those of other users, $E_{i \neq j}\{b_i b_j\} = 0$ and $b_i^2(n) = 1$ [38] [55]. We get

$$\mathbf{R} = \text{diag}\{\mathbf{C}_0^* \mathbf{C}_0, \dots, \mathbf{C}_{K-1}^* \mathbf{C}_{K-1}\}.$$

Hence $\mathbf{R}$ is an invariant block diagonal matrix and $\mathbf{R}^{-1}$ can be calculated.

Compared with the step-size in the standard LMS filter, μ in WLMS filter cannot be arbitrarily selected but is derived from the model parameter of $\mathbf{h}(n)$ and parameter-drift-to-noise ratio being denoted by γ , which is defined by

$$\gamma \triangleq \text{tr} \mathbf{R}_e / \text{tr}(\mathbf{R}^{-1} \mathbf{R}_\eta \mathbf{R}^{-1})$$

where

$$\begin{aligned}
\mathbf{R}_\eta &= E\{\eta(n) \eta^*(n)\}, \\
\mathbf{R}_e &= E\{(\mathbf{e}(n) + \mathbf{e}_0) (\mathbf{e}(n) + \mathbf{e}_0(n))^*\}, \\
\eta(n) &= (\mathbf{F}^*(n)\mathbf{F}(n) - \mathbf{R})\tilde{\mathbf{h}}(n|n-1) + \mathbf{F}^*(n)\tilde{\mathbf{v}}(n).
\end{aligned}$$

In practice, however, $\mathbf{R}_\eta$ may be hard to know in advance. Hence, the setting of

3.3 Polynomial WLMS Channel Tracking and Prediction

37

the optimal γ has to be based on the MMSE performance of channel estimation via simulations. This would also be the case for adaptive filters like RLS filters and LMS filters, which operate with some kind of adaptive “forgetting factor” or adaptive “step-size” schemes. As for the practical case, the tuning of γ is replaced by selecting the step-size μ , which can be acquired from either off-line simulations or on-line adaptive adjustment.

Table 3.2: Computational complexity comparison between Kalman algorithm and WLMS algorithm when tracking multipath channel response

Algorithm	Number of real multiplication	$N = 31, K = 4, L = 2$
Kalman	$16K^2L^2(3KL + N + 1) + 4KL(N^2 + 2N)$	90080
WLMS	$2K^2L^2 + 2KL(3N + 4)$	1680

Finally, we will compare the computational complexity of Kalman filter and WLMS filter. In comparison with the Kalman filter, the main advantage of WLSM algorithm is its low computational complexity. The channel vector $\mathbf{h}$ and the received signal $\mathbf{r}$ are assumed to be complex-valued. The coefficients of AR model, step-size μ and matrix $\mathbf{R}$ are real-valued. The comparison is based on the number of real multiplications per step. Multiplication between two complex numbers is counted as two real multiplications and two real additions. Both the algorithms are based on a second-order AR model. The result is presented in Table 3.2. Note that the number of operations on inverting matrix in Riccati update of the Kalman algorithm is not shown on this table. In addition, we show an example for $N = 31, K = 4, L = 2$. As the Table 3.2 is shown, the computational complexity of Kalman filter is proportional to $(KL)^3$, whereas that of the WLMS filter is the function of $(KL)^2$. Therefore, as the number of users or multipaths increases, the advantage of WLMS computational complexity is more evident. Hence, the WLMS channel tracking algorithm is well suitable for implementation in a practical wireless system.

3.4 Design of Adaptive DFE Symbol Detector

Figure 3.2: Equivalent model for Decision Feedback Equalization

To obtain the optimal filter coefficients of DFE under the MMSE criterion, we remodel the decision feedback equalizer in Figure 3.1, as shown in Figure 3.2. Due to the hard decision, the system becomes nonlinear and is hard to be analyzed. Therefore, many researchers convert this nonlinear problem into a simple linear problem by assuming that the decision output is correct. In this chapter, however, we take the decision errors into consideration in the design of adaptive DFE. In Figure 3.2, vector $\hat{\mathbf{b}}(n-1)$ represents the decision at $n-1$ and

$$\mathbf{e}_{\hat{\mathbf{b}}}(n-1) \triangleq \hat{\mathbf{b}}(n-1) - \mathbf{b}(n-1)$$

is defined as the corresponding error in the decision. For $\mathbf{e}_{\hat{\mathbf{b}}}(n-1)$, we have the following the weak assumption:

Assumption 3.4.1. *The decision error $\mathbf{e}_{\hat{\mathbf{b}}}(n-1)$ is uncorrelated with $\mathbf{b}(n)$ and $\mathbf{b}(n-1)$.*

3.4 Design of Adaptive DFE Symbol Detector

39

Remark 3.4.1. *Assumption 3.4.1 is based on two factors. First, in terms of the simulation results, the proposed DFE detector has lower BER, that is $\mathbf{e}_{\hat{\mathbf{b}}} = 0$ in most situations. Secondly, due to the multi-access environment, other users also affect the detection of the desired user. It leads to the weak impact of symbol $\mathbf{b}(n)$ on decision output $\hat{\mathbf{b}}(n)$.*

The task of designing MMSE-DFE is equivalent to seeking the optimal solutions to feedforward filter ($\mathcal{W}$) and feedback filter ($\mathcal{G}$) under the MMSE criterion. In terms of (3.4) and Figure 3.2, the MMSE-DFE error vector is given by

$$\begin{aligned}
 \mathbf{e}_b(n) &= \mathbf{b}(n) - \underline{\mathbf{b}}(n) \\
 &= \mathbf{b}(n) - \left\{ \mathcal{W}[\mathbf{C}\mathbf{H}(n)\mathbf{b}(n) + \tilde{\mathbf{C}}\mathbf{H}(n-1)\mathbf{b}(n-1) + \mathbf{v}(n)] \right. \\
 &\quad \left. - \mathcal{G}[\mathbf{b}(n-1) + \mathbf{e}_{\hat{\mathbf{b}}}(n-1)] \right\} \\
 &= \mathbf{b}(n) - \mathcal{W}[\mathbf{C}\mathbf{H}(n)\mathbf{b}(n) + \tilde{\mathbf{C}}\mathbf{H}(n-1)\mathbf{b}(n-1) + \mathbf{v}(n)] \\
 &\quad + \underbrace{\mathcal{G}[\mathbf{b}(n-1) + \mathbf{e}_{\hat{\mathbf{b}}}(n-1)]}_{\mathbf{x}_1}. \tag{3.11}
 \end{aligned}$$

Note that $\mathbf{b}(n)$ and $\mathbf{b}(n-1)$ are uncorrelated, and $E\{\mathbf{b}(n-1)\mathbf{b}(n-1)^*\} = \mathbf{I}$. Applying the *orthogonality principle* which states that $E\{\mathbf{e}_b(n)\mathbf{x}_1^*\} = 0$ and considering Assumption 3.4.1, the optimal feedforward filter $\mathcal{W}_{\text{opt}}$ and feedback filter $\mathcal{G}_{\text{opt}}$ have the following relation:

$$\begin{aligned}
 E\{\mathbf{e}_b(n)\mathbf{x}_1^*\} &= -\mathcal{W}_{\text{opt}}\tilde{\mathbf{C}}\mathbf{H}(n-1) + \mathcal{G}_{\text{opt}}(\mathbf{I} + \mathbf{R}_{\hat{\mathbf{b}}}) = \mathbf{0} \\
 \mathcal{G}_{\text{opt}} &= \mathcal{W}_{\text{opt}}\tilde{\mathbf{C}}\mathbf{H}(n-1)(\mathbf{I} + \mathbf{R}_{\hat{\mathbf{b}}})^{-1} \tag{3.12}
 \end{aligned}$$

where the auto-correlation matrix of decision errors is defined by

$$\mathbf{R}_{\hat{\mathbf{b}}} \triangleq E\{\mathbf{e}_{\hat{\mathbf{b}}}(n-1)\mathbf{e}_{\hat{\mathbf{b}}}(n-1)^*\}. \tag{3.13}$$

3.4 Design of Adaptive DFE Symbol Detector

40

Since a user's decision error is independent from others, $\mathbf{R}_{\mathbf{b}}$ is a diagonal matrix as follows:

$$\mathbf{R}_{\mathbf{b}} = \begin{bmatrix} \varepsilon_1 & & \\ & \ddots & \\ & & \varepsilon_K \end{bmatrix} \quad (3.14)$$

Next, we will derive the optimal feedforward filter $\mathcal{W}_{\text{opt}}$. By (3.11), the auto-correlation matrix of decision errors is equal to

$$\begin{aligned} & E\{\mathbf{e}_b(n)\mathbf{e}_b(n)^*\} \\ = & [\mathbf{I} - \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n})][\mathbf{I} - \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n})]^* \\ & + [\mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1) - \mathcal{G}] [\mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1) - \mathcal{G}]^* + \mathcal{W}\mathbf{R}_{\mathbf{v}}\mathcal{W}^* + \mathcal{G}\mathbf{R}_{\mathbf{b}}\mathcal{G}^* \\ = & \mathbf{I} - \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n}) - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\mathcal{W}^* + \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n})\mathbf{H}(\mathbf{n})^*\mathbf{C}^*\mathcal{W}^* \\ & + \mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1)\mathbf{H}(\mathbf{n}-1)^*\tilde{\mathbf{C}}^*\mathcal{W}^* - \mathcal{G}\mathbf{H}(\mathbf{n}-1)^*\tilde{\mathbf{C}}^*\mathcal{W}^* \\ & - \mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1)\mathcal{G}^* + \mathcal{G}\mathcal{G}^* + \mathcal{W}\mathbf{R}_{\mathbf{v}}\mathcal{W}^* + \mathcal{G}\mathbf{R}_{\mathbf{b}}\mathcal{G}^* \\ = & \mathbf{I} - \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n}) - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\mathcal{W}^* + \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n})\mathbf{H}(\mathbf{n})^*\mathbf{C}^*\mathcal{W}^* + \mathcal{W}\mathbf{R}_{\mathbf{b}}\mathcal{W}^* \\ & + \mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1)\bar{\mathbf{R}}\mathbf{H}(\mathbf{n}-1)^*\tilde{\mathbf{C}}^*\mathcal{W}^* \\ & + \underbrace{\left[\mathcal{G} - \mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1)(\mathbf{I} + \mathbf{R}_{\mathbf{b}})^{-1}\right]}_{\mathbf{G}}(\mathbf{I} + \mathbf{R}_{\mathbf{b}}) \underbrace{\left[\mathcal{G} - \mathcal{W}\tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1)(\mathbf{I} + \mathbf{R}_{\mathbf{b}})^{-1}\right]^*}_{\mathbf{G}^*} \\ = & \mathcal{W} \underbrace{\left[\mathbf{C}\mathbf{H}(\mathbf{n})\mathbf{H}(\mathbf{n})^*\mathbf{C}^* + \tilde{\mathbf{C}}\mathbf{H}(\mathbf{n}-1)\bar{\mathbf{R}}\mathbf{H}(\mathbf{n}-1)^*\tilde{\mathbf{C}}^* + \mathbf{R}_{\mathbf{v}}\right]}_{\phi} \mathcal{W}^* \\ & - \mathcal{W}\mathbf{C}\mathbf{H}(\mathbf{n}) - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\mathcal{W}^* + \mathbf{G}(\mathbf{I} + \mathbf{R}_{\mathbf{b}})\mathbf{G}^* + \mathbf{I} \\ = & \mathbf{I} - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\phi^{-1}\mathbf{C}\mathbf{H}(\mathbf{n}) \\ & + \underbrace{\left[\mathcal{W} - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\phi^{-1}\right]}_{\mathbf{W}} \phi \underbrace{\left[\mathcal{W} - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\phi^{-1}\right]^*}_{\mathbf{W}^*} + \mathbf{G}(\mathbf{I} + \mathbf{R}_{\mathbf{b}})\mathbf{G}^* \\ = & [\mathbf{I} - \mathbf{H}(\mathbf{n})^*\mathbf{C}^*\phi^{-1}\mathbf{C}\mathbf{H}(\mathbf{n})] + \mathbf{W} \phi \mathbf{W}^* + \mathbf{G}(\mathbf{I} + \mathbf{R}_{\mathbf{b}})\mathbf{G}^* \end{aligned} \quad (3.15)$$

where we introduce the matrices $\{\mathbf{G}, \phi, \mathbf{W}\}$ and $\bar{\mathbf{R}} = \text{diag}\{\frac{\varepsilon_1}{1+\varepsilon_1}, \dots, \frac{\varepsilon_K}{1+\varepsilon_K}\}$. It

is obvious that ϕ is invertible because $\mathbf{R}_v$ is nonsingular. In terms of (3.15), we immediately obtain the optimal $\mathcal{W}_{\text{opt}}$ as follows

$$\begin{aligned}\mathcal{W}_{\text{opt}} &= \mathbf{H}(n)^* \mathbf{C}^* \phi^{-1} \\ &= \mathbf{H}(n)^* \mathbf{C}^* [\mathbf{C} \mathbf{H}(n) \mathbf{H}(n)^* \mathbf{C}^* + \tilde{\mathbf{C}} \mathbf{H}(n-1) \bar{\mathbf{R}} \mathbf{H}(n-1)^* \tilde{\mathbf{C}}^* + \mathbf{R}_v]^{-1}.\end{aligned}\tag{3.16}$$

In a practical application of adaptive DFE, we substitute prediction and estimate of channel, $\hat{\mathbf{H}}(n|n-1)$ and $\hat{\mathbf{H}}(n-1|n-1)$ for $\mathbf{H}(n)$ and $\hat{\mathbf{H}}(n-1)$ in (3.12) and (3.16). Then the optimal adaptive feedforward filter and feedback filter are given by

$$\begin{aligned}\mathcal{W} &= \hat{\mathbf{H}}(n|n-1)^* \mathbf{C}^* \left[\mathbf{C} \hat{\mathbf{H}}(n|n-1) \hat{\mathbf{H}}(n|n-1)^* \mathbf{C}^* \right. \\ &\quad \left. + \tilde{\mathbf{C}} \hat{\mathbf{H}}(n-1|n-1) \bar{\mathbf{R}} \hat{\mathbf{H}}(n-1|n-1)^* \tilde{\mathbf{C}}^* + \mathbf{R}_v \right]^{-1}, \\ \mathcal{G} &= \mathcal{W} \tilde{\mathbf{C}} \hat{\mathbf{H}}(n-1|n-1) (\mathbf{I} + \mathbf{R}_{\hat{\mathbf{b}}})^{-1}.\end{aligned}$$

3.5 Simulation Results

In this chapter, several simulations are carried out to illustrate the performance of the proposed WLMS estimator and DFE detector. First, we will compare the performance of the detector based on WLMS with that of the Kalman-based detector in the case of time-variant channel and fast-fading Rayleigh channel. Then we demonstrate the relation between step-size μ and BER performance. Finally, the impact of previous decision error on BER is examined.

In the following simulations, signal-to-noise ratio (SNR) is defined as

$$SNR \triangleq 10 \log \frac{E_k}{N_0} = 10 \log \frac{E_k}{2\sigma^2}$$

where E_k denotes the sum of the k -th user's signal energy from all paths and σ^2 is the variance of the white Gaussian noise $\mathbf{v}(n)$ in (3.4). The system includes four users with equal power and $N = 31$ length Gold codes. Assume that previous decision is correct, that is $\varepsilon_k = 0$. In each trial, 40 symbols are sent as training sequences and followed by 1000 symbols as the information data. Training sequences are generated by making hard-decision of the the random data based on white Gaussian distribution. The BER performance is the average of 200 Monte Carlo trials in the decision-directed mode.

3.5.1 Performance of BER

The time-invariant channel

The time-invariant channels are used to compare the BER performance of the proposed detector based on WLMS with the detector based on Kalman filter. Figure 3.3 shows the BER versus SNR performance. The single user's lower bound is also shown on the same figure for comparison. It is quite evident that the proposed detector has nearly the same BER performance as the Kalman-based detector. In addition, both algorithms' performances are close to the single-user's lower bound at low SNR.

Fast fading channel

In this example, we demonstrate the performance of the proposed detector in fast fading channels. Each user's signal undergoes two Rayleigh channels, that is $L = 1$.

3.5 Simulation Results

43

Figure 3.3: BER performance of the proposed detector based on WLMS and the detector based on Kalman filter in time-invariant channel. The single-user bound is shown for comparison.

The mobile speed is 60 km/hr and carrier frequency is $f_c = 900\text{MHz}$, and signaling is 10^4 Hz. Then, the Doppler rate is $f_d T = 0.005$. From Figure 3.4 we observe that the proposed detector has nearly the same BER performance as the Kalman-based detector in the fast fading case. The performance gap between the BER curve of the WLMS-aided detector and the BER curve with accurate channel responses is caused by the fast fading channels. However, this gap is very small. The main reasons for the good performance are twofold. First, the WLMS channel estimator can keep accurate tracking of the fading channels. The estimated channel responses of the 1st and 2nd paths of the desired user when SNR is equal to 4 dB are shown in Figure 3.5. We observe that the WLMS channel estimation almost matches the real channel responses. The second reason is credited to the employment of the second-order AR channel model in WLMS channel estimation algorithm. Since a

Figure 3.4: BER performance versus SNR. Two Raleigh fading paths for each user with $f_d T = 0.005$.

second-order AR model is sufficient to approximate the real channel responses, the performance degradation due to the channel model mismatch will be reduced.

3.5.2 The setting of step-size in WLMS algorithm

In Section 3.3, we introduce the step-size μ , which is crucial to the optimization of the tracking performance and the stability of the algorithm. The tuning of the μ has impact on the BER performance of the detector. This parameter can be obtained from off-line Monte Carlo simulations. Figure 3.6 shows that the relationship between the BER and step-size with respect to different SNR. It is evident that the optimum step-size is from $[0.06, 0.1]$. Hence, we can employ a certain fixed step-size in the WLMS algorithm.

Figure 3.5: The WLMS channel estimation of the desired user when SNR is equal to 4 dB. Dashed line: the real channel response; solid line: the proposed WLMS estimation algorithm.

3.5.3 Error in previous decision

So far, we have assumed that previous DFE decision output is correct in the above simulations. In reality, this assumption does not always hold. In the design of DFE, we have considered the previous decision error which is presented by the power of the decision error ε_k . However, this parameter is not known beforehand. For a binary signal with level +1 and -1, the only possible values of the error are 2 and -2. Hence, the error power should be 4 times the BER. Since the BER is also dependent on the ε_k , it is difficult to obtain an explicit expression for the optimal value of ε . Figure 3.7 shows how the value of ε affects the BER performance when

Figure 3.6: BER performance versus step-size μ under the different SNR.

SNR is equal to 2dB. The optimum value of ε should be 0.05, which corresponds to nearly 4 times the BER.

3.6 Conclusion

In this chapter, a joint channel estimation and adaptive DFE multiuser detection algorithm has been presented. The optimal feedforward filter and feedback filter of DFE are obtained by minimizing the mean square error between symbols and equalized results. With regard to this algorithm, we consider that multiuser symbols undergo the multipath Raleigh fading channels and channel responses are modeled as a second order AR model. The WLMS algorithm is employed to estimate and predict the channel coefficients. Since the WLMS filter has very low computation complexity and good tracking behavior as compared to Kalman filter, it is well suitable for practical applications. In addition, the previous decision

Figure 3.7: BER performance versus ε when SNR is equal to 2 dB.

error is also taken into consideration to improve the performance of the proposed detector. Based on the assumption of such decision error, we have obtained an explicit formula for the feedforward and feedback filters. The simulation results for the performance of the proposed detector were given. Through the simulation in time-invariant and fast fading channels, the proposed detector shows a similar BER performance as the Kalman-based detector. In addition, the selection of step-size μ affects the tracking behavior of the WLMS filter. The simulation also shows the relationship between μ and the BER performance. Finally, even though it is hard to obtain the explicit statistical property of the decision errors, we may obtain an appropriate ε from the simulation results for practical applications.

Chapter 4

Polynomial MMSE Channel Estimation of DS-CDMA Systems with Stochastic Parametric Uncertainty

4.1 Introduction

From this chapter, we begin to study a polynomial approach to robust channel estimation problems with applications in the DS-CDMA system. Our interest is motivated by the fact that many uncertain elements existing in system models can result in severe performance degradation of many classic estimation algorithms. For example, the performance of the celebrated Kalman filter can be very sensitive to model uncertainties. The uncertainties can be in the form of parametric uncertainties or disturbances with unknown statistics. Hence, the study of robust estimation problem is of practical importance and has been attracting the interest of many researchers in the past decades.

In general, there are two categories of parametric uncertainties to be considered in robust estimator design, *deterministic uncertainties* and *stochastic uncertainties*.

In the first scenario, system model is assumed to be subject to certain time-varying norm bounded parameter uncertainties. The objective of optimal estimator design is to minimize the upper bound of the estimation error variance over all admissible system uncertainties. For example, Xie *et al* [88] proposed a solution to a robust Kalman filtering problem for systems subject to time-varying norm-bounded parameter uncertainties via a Riccati equation approach. Further studies on robust MMSE filter appeared in [67] [74] [84]. The corresponding polynomial approach for the H_2 optimal estimation has been proposed in [58]. As another type of parametric uncertainties, stochastic uncertainties are modeled as multiplicative white noise processes. For this scenario, the robust filtering aims at minimizing the expectation of the performance index, the variance of the estimation error (H_2) or the worst-case energy gain (H_∞), with respect to random parameters. An earlier work on the robust estimations for systems with stochastic uncertainties may be traced back to the late 1970s, during which the average MMSE approaches were proposed in [70] [18]. The corresponding robust deconvolution problem was investigated in [17]. A robust Kalman filter with stochastic uncertainties was proposed in [84], where the algorithm minimizes an upper bound of the estimation error variance at each step based on *linear matrix inequalities* (LMI). On the other hand, the study on robust estimation problems for systems with disturbance or noise with unknown statistics has been mainly concentrated on the H_∞ estimation. The objective of the H_∞ estimation is to minimize the worst-case effect for all possible disturbances. Hence, a H_∞ estimator is of robustness in nature.

In this chapter, we mainly study the MMSE channel estimation problem for systems with stochastic parametric uncertainties. The proposed algorithm is based on the minimum mean square error (MMSE) criterion and the second-order AR

channel model. Our main aim is to improve the robustness of this linear channel estimator for fast fading channels and accordingly improve the detection performance of practical DS-CDMA systems. In fact, with regard to a practical DS-CDMA system, there inevitably exist uncertainties in mobile radio channels. In particular, the fast moving between the base station and the user causes the fast fading of channels, which are difficult to be estimated accurately. Therefore, we will take the uncertainties of channel models into consideration when developing our estimation algorithm. By treating uncertainties as fictitious noises, we convert the estimation of uncertain channels into the standard MSE estimation of channels without uncertainty. A simple explicit calculation of the statistics of the fictitious noises is given. On the other hand, the knowledge of transmitted symbols is generally obtained from either training sequences or decision outputs. In the tracking mode where decisions are employed to estimate channels, decision errors will cause the degradation of the estimation performance. In this chapter, we also employ the characteristics of the decision errors in the design of tracking algorithm. Compared with the Kalman estimator, the proposed algorithm is of much lower computational complexity. With the aid of the DFE detector, simulations are carried out to compare the proposed algorithm with the Kalman-based estimator and the WLMS-based estimator proposed in Chapter 3. The results show that the proposed algorithm substantially improves the estimation performance in the presence of channel uncertainties and decision errors in the tracking mode.

4.2 Problem Statement

Consider a K -user DS-CDMA system model with N -length spreading code as shown in Chapter 3:

$$\mathbf{h}(n) = \mathbf{D}_1 \mathbf{h}(n-1) + \mathbf{D}_2 \mathbf{h}(n-2) + \mathbf{e}(n) \quad (4.1)$$

$$\mathbf{r}(n) = \mathbf{C}_0 \mathbf{B}(n) \mathbf{h}(n) + \mathbf{C}_1 \mathbf{B}(n-1) \mathbf{h}(n-1) + \mathbf{v}(n) \quad (4.2)$$

where $\mathbf{h}(n) \in \mathcal{C}^{K(L+1)}$ is the vector of channel coefficients, $\mathbf{r}(n) \in \mathcal{C}^N$ denotes the observation, $\mathbf{D}_1$ and $\mathbf{D}_2 \in \mathcal{R}^{K(L+1) \times K(L+1)}$, and $\mathbf{C}_0, \mathbf{C}_1 \in \mathcal{R}^{N \times K(L+1)}$ are defined in (2.9)-(2.10).

Our objective is to seek $\hat{\mathbf{h}}(n+m|n)$ based on the AR(2) model in (4.1) and measurement data $\mathbf{r}(n)$ in (4.2). $\hat{\mathbf{h}}(n+m|n)$ is an estimate of $\mathbf{h}(n+m)$ obtained at time n by filtering($m=0$), prediction($m>0$) or smoothing($m<0$). The criterion for designing the estimator is to minimize the estimation error variance matrix

$$\mathbf{P} = E\{\tilde{\mathbf{h}}(n+m|n)\tilde{\mathbf{h}}^*(n+m|n)\} \quad (4.3)$$

where $\tilde{\mathbf{h}}(n+m|n) = \mathbf{h}(n+m) - \hat{\mathbf{h}}(n+m|n)$. Furthermore, $\mathbf{D}_1$ and $\mathbf{D}_2$ in (4.1) include the stochastic uncertainties due to channel estimation error and channel variations, which should be taken into account when designing a channel estimator.

4.3 Linear Time-invariant Regression

It should be noted that the coefficient matrix $\mathbf{B}(n)$ in (4.2) is time-varying with its entries $b_k(n)$ of either 1 or -1 , $k = 1, \dots, K$. Therefore, we are required to constitute a linear time-invariant regression since the proposed algorithm is based

4.3 Linear Time-invariant Regression

52

on a polynomial approach, which is a steady-state design method.

There are two distinct cases to be considered, respectively. For the case when $N \geq K(L+1)$, we assume that $\mathbf{C}_0$ is of full column rank¹ and define the pseudo-inverse of $\mathbf{C}_0$

$$\mathbf{C}^+ = (\mathbf{C}_0^* \mathbf{C}_0)^{-1} \mathbf{C}_0^*.$$

By subtracting $\mathbf{C}_1 \mathbf{B}(n-1) \hat{\mathbf{h}}(n-1|n-1)$ and multiplying $\mathbf{B}(n) \mathbf{C}^+$ from the left, the observation vector (4.2) is rewritten as

$$\mathbf{f}(n) = \mathbf{B}(n) \mathbf{C}^+ \left(\mathbf{r}(n) - \mathbf{C}_1 \mathbf{B}(n-1) \hat{\mathbf{h}}(n-1|n-1) \right) \quad (4.4)$$

$$\mathbf{f}(n) = \mathbf{h}(n) + \mathbf{v}_0(n) \quad (4.5)$$

$$\begin{aligned} \mathbf{v}_0(n) = & \mathbf{B}(n) \mathbf{C}^+ \mathbf{C}_1 \mathbf{B}(n-1) \underbrace{\left(\mathbf{h}(n-1) - \hat{\mathbf{h}}(n-1|n-1) \right)}_{\tilde{\mathbf{h}}(n-1|n-1)} \\ & + \mathbf{B}(n) \mathbf{C}^+ \mathbf{v}(n) \end{aligned} \quad (4.6)$$

where the vector $\mathbf{v}(n)$ is a white noise with zero mean and covariance matrix $\mathbf{R}_v = \sigma_v^2 \mathbf{I}$ in terms of (2.12). Here, $\mathbf{f}(n)$ and $\mathbf{v}_0(n)$ are regarded as fictitious observation and noise. In the following estimator design, $\mathbf{v}_0(n)$ is assumed to be equal to the second term of (4.6). This assumption holds approximately since the first item of (4.6) is negligible. It is based on the fact that $\tilde{\mathbf{h}}(n-1|n-1)$ is small due to good estimation performance from simulation result and $\mathbf{C}_1$ is a sparse matrix, majority of whose components are 0. Therefore, $\mathbf{v}_0(n)$ can be considered to be a white noise with zero mean and covariance given by

$$\begin{aligned} \mathbf{R}_{v_0} &= E\{\mathbf{v}_0 \mathbf{v}_0^*\} \\ &\approx \sigma_v^2 E_{\mathbf{B}}\{\mathbf{B}(n) (\mathbf{C}_0^* \mathbf{C}_0)^{-*} \mathbf{B}(n)\} \end{aligned}$$

¹Since all users' spreading code $\{c_k(i)\}_{i=0}^{N-1}$ and their shifted versions are linearly independent, $\mathbf{C}_0$ generally has full column rank.

4.3 Linear Time-invariant Regression

53

$$\begin{aligned}
&= \sigma_v^2 E_{\mathbf{B}} \{ (\mathbf{B}(n) \mathbf{C}_0^* \mathbf{C}_0 \mathbf{B}(n))^{-*} \} \\
&= \sigma_v^2 E_{\mathbf{B}} \left\{ ([b_1(n) \mathbf{C}_1^0, \dots, b_K(n) \mathbf{C}_K^0]^* [b_1(n) \mathbf{C}_1^0, \dots, b_K(n) \mathbf{C}_K^0])^{-1} \right\} \\
&\approx \sigma_v^2 (\text{diag}\{\mathbf{C}_1^{0*} \mathbf{C}_1^0, \dots, \mathbf{C}_K^{0*} \mathbf{C}_K^0\})^{-1} \quad (4.7)
\end{aligned}$$

where the property that the inversion of $\mathbf{B}(n)$ is itself is employed in the second equality and the last equality follows from the fact that different user's symbols are uncorrelated and there are strong uncorrelation among $\mathbf{C}_k^0$, $k = 1 \dots K$.

For the case when $N < K(L + 1)$, (4.2) is an underdetermined system so that new observations should be included. Let the number of added observations be $\lambda = K(L + 1) + 1 - N$. By adding λ previous samples to the observation vector (4.2), we obtain

$$\begin{aligned}
\underbrace{\begin{bmatrix} r(nN - \lambda) \\ r(nN - \lambda + 1) \\ \vdots \\ r(nN - 1) \\ \hline \mathbf{r}(n) \end{bmatrix}}_{\tilde{\mathbf{r}}(n)} &= \underbrace{\begin{bmatrix} c_1(N - \lambda - 1) & | & \\ c_1(N - \lambda) & | & \mathbf{0} \\ \vdots & & \\ c_1(N - 2) & & \\ \hline c_1(N - 1) & & \mathbf{C}_0 \\ \mathbf{0} & & \end{bmatrix}}_{\tilde{\mathbf{C}}_0} \begin{bmatrix} b_1(n - 1) & 0 \\ \mathbf{0} & \mathbf{B}(n) \end{bmatrix} \begin{bmatrix} h_1^1(n - 1) \\ \mathbf{h}(n) \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{C}_2 \\ \tilde{\mathbf{C}}_1 \end{bmatrix} \tilde{\mathbf{B}}(n - 1) \tilde{\mathbf{h}}(n - 1) + \begin{bmatrix} v(nN - \lambda) \\ v(nN - \lambda + 1) \\ \vdots \\ v(nN - 1) \\ \hline \mathbf{v}(n) \end{bmatrix} \quad (4.8)
\end{aligned}$$

4.3 Linear Time-invariant Regression

54

where

$$\begin{aligned}
 \mathbf{C}_2 &= \begin{bmatrix} \mathbf{C}_1^2 & \mathbf{C}_2^2 & \dots & \mathbf{C}_K^2 \end{bmatrix}, \\
 \mathbf{C}_1^2 &= \begin{bmatrix} c_1(N-\lambda) & c_1(N-\lambda-2) & \dots & c_1(N-\lambda-L) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N-1) & c_1(N-3) & \dots & c_1(N-L-1) \end{bmatrix}_{\lambda \times L}, \\
 \mathbf{C}_k^2 &= \begin{bmatrix} c_k(N-\lambda) & c_k(N-\lambda-1) & \dots & c_k(N-\lambda-L) \\ \vdots & \vdots & \ddots & \vdots \\ c_k(N-1) & c_k(N-2) & \dots & c_k(N-L-1) \end{bmatrix}_{\lambda \times (L+1)} \quad k = 2, \dots, K, \\
 \tilde{\mathbf{C}}_1 &= \begin{bmatrix} \tilde{\mathbf{C}}_1^0 & \mathbf{C}_2^0 & \dots & \mathbf{C}_K^0 \end{bmatrix}, \\
 \tilde{\mathbf{C}}_1^0 &= \begin{bmatrix} c_1(0) & 0 & \dots & 0 \\ c_1(1) & 0 & \dots & 0 \\ c_1(2) & c_1(0) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_1(L) & c_1(L-1) & \dots & c_1(0) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N-1) & c_1(N-3) & \dots & c_1(N-L-1) \end{bmatrix}_{N \times L},
 \end{aligned}$$

$$\tilde{\mathbf{h}}(n-1) = [h_1^0(n-1) \ \dots \ h_1^L(n-1) \ \mathbf{h}_2^T(n-1) \ \dots \ \mathbf{h}_K^T(n-1)]^T,$$

$$\tilde{\mathbf{B}}(n-1) = \text{diag}\{b_1(n-1)\mathbf{I}_L, b_2(n-1)\mathbf{I}_{L+1}, \dots, b_K(n-1)\mathbf{I}_{L+1}\}.$$

From $\tilde{\mathbf{C}}_0$ of (4.8), it is possible to make $\tilde{\mathbf{C}}_0^* \tilde{\mathbf{C}}_0$ invertible by adding the spreading codes of the first user. Obviously, this transformation is not unique. In fact, $h_1^1(n-1)$ can be replaced by the channel coefficients of another path as long as the

corresponding $\tilde{\mathbf{C}}_0^* \tilde{\mathbf{C}}_0$ is nonsingular. Since (4.8) is an overdetermined system, we can employ a similar procedure as stated earlier to obtain a linear time-invariant regression as in (4.5).

4.4 Treatment of Parametric Uncertainties in System Models

4.4.1 Statistics of channel model uncertainties

In a practical CDMA system, the coefficients of the matrices $\mathbf{D}_1$ and $\mathbf{D}_2$ in the dynamic model (4.1) are obtained from independent estimation, which is performed during the training period. However, there inevitably exist uncertainties in the model coefficients due to channel estimation errors and channel variations. The key idea of our proposed robust estimator is to treat these coefficient uncertainties as noises and apply their statistics in the design of channel estimator.

Rewrite (4.1) in the form:

$$d(q^{-1})\mathbf{h}(n) = \mathbf{e}(n) + \mathbf{e}_0(n) \quad (4.9)$$

where

$$\begin{aligned} d(q^{-1}) &= 1 - d_1 q^{-1} - d_2 q^{-2}, \\ \mathbf{e}_0(n) &= \Delta \mathbf{D}(q^{-1})\mathbf{h}(n), \\ \Delta \mathbf{D}(q^{-1}) &= \text{diag}\{\Delta d_{1,1}^0 q^{-1} + \Delta d_{1,2}^0 q^{-2}, \dots, \Delta d_{K,1}^L q^{-1} + \Delta d_{K,2}^L q^{-2}\} \\ d_j &= \frac{1}{K(L+1)} \sum_{k=1}^K \sum_{l=0}^L \hat{d}_{k,j}^l, \quad j = 1, 2, \end{aligned} \quad (4.10)$$

where $\Delta \hat{d}_{k,j}^l$ denotes the estimation error of the coefficient. With regard to the definition of $\Delta d_{i,j}^m$ in (3.6), we introduce the following assumptions throughout this chapter.

Assumption 4.4.1. *Matrices $\Delta \mathbf{D}_1$ and $\Delta \mathbf{D}_2$ are independent of $\mathbf{h}(n)$ and $\mathbf{e}(n)$ where $\Delta \mathbf{D}_j$ are defined as*

$$\Delta \mathbf{D}_j = \text{diag}\{\Delta d_{1,j}^0 \cdots \Delta d_{K,j}^L\}, \quad j = 1, 2. \quad (4.11)$$

Assumption 4.4.2. *Matrices $\Delta \mathbf{D}_1$ and $\Delta \mathbf{D}_2$ have zero means, that is, $E\{\Delta \mathbf{D}_j\} = \mathbf{0}$ ($j = 1, 2$).*

Assumption 4.4.3. *$\mathbf{e}(n)$ is a white noise with zero mean and known covariance*

$$\mathbf{R}_e = E\{\mathbf{e}(n)\mathbf{e}^*(n)\} = \sigma_e^2 \mathbf{I}.$$

Remark 4.4.1. *Since $\hat{d}_{k,j}^l$ is an unbiased estimate of the actual coefficient $d_{k,j}^l$ and for different channels, it has been shown in Table 2.1 of Chapter 2 that their coefficients are close to each other, hence, their average can be approximately considered as an unbiased estimate of the actual coefficient $d_{i,j}^l$. Therefore, Assumption 4.4.2 follows.*

Remark 4.4.2. *In Assumption 4.4.3, the diagonal property of the covariance matrix results from the Bello model [4] where input signals to different channels are uncorrelated.*

From (4.9), it can be seen that the effect of these uncertainties is considered as a noise $\mathbf{e}_0$ whose statistics is obtained in the following theorem.

4.4 Treatment of Parametric Uncertainties in System Models

57

Theorem 4.4.1. $\mathbf{e}(n)$ and $\mathbf{e}_0(n)$ in (4.9) are uncorrelated. $\mathbf{e}_0(n)$ is a white noise with zero mean and diagonal covariance matrix $\mathbf{R}_{\mathbf{e}_0}$, which is given by

$$\mathbf{R}_{\mathbf{e}_0} = \sigma_e^2 \text{diag}\left\{\frac{\rho_1}{1 - \rho_1}, \dots, \frac{\rho_i}{1 - \rho_i}, \dots, \frac{\rho_{K(L+1)}}{1 - \rho_{K(L+1)}}\right\}$$

where

$$\begin{aligned} \rho_i &= \gamma_0 E\{\Delta D_{1,ii} \Delta D_{1,ii}^* + \Delta D_{2,ii} \Delta D_{2,ii}^*\} \\ &\quad + \gamma_1 E\{\Delta D_{1,ii}^* \Delta D_{2,ii}\} \\ &\quad + \gamma_2 E\{\Delta D_{1,ii} \Delta D_{2,ii}^*\} \end{aligned} \quad (4.12)$$

$$\gamma_0 = \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} \frac{dz}{z} \quad (4.13)$$

$$\gamma_1 = \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} \frac{dz}{z^2} \quad (4.14)$$

$$\gamma_2 = \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} dz \quad (4.15)$$

$\Delta D_{j,ii}$ denotes the i th element of the main diagonal of the matrix $\Delta \mathbf{D}_j$ ($j = 1, 2$).

Proof: See Appendix A.

Theorem 4.4.1 states that the $E\{\Delta D_{j,ii} \Delta D_{k,ii}^*\}$, ($j, k = 1, 2$) affect ρ_i , which further affects $\mathbf{R}_{\mathbf{e}_0}$ and $\mathbf{R}_{\Delta \mathbf{v}}$ defined in subsequent subsection. For time-varying channel estimation, their accurate values are hard to obtain. However, we may approximate the bounds of $E\{\Delta D_{j,ii} \Delta D_{k,ii}^*\}$, ($j, k = 1, 2$) by using the channel estimation error variances obtained from the last training sequence. Due to the nonlinearity between $E\{\Delta D_{j,ii} \Delta D_{k,ii}^*\}$ and $\mathbf{R}_{\mathbf{e}_0}$, it is difficult to analyze the influence of this approximation on the final estimation performance. However, we can observe the influence directly from the estimation performance with respect to $\mathbf{e}_0(n)$ and $\Delta \mathbf{v}$.

4.4.2 Statistics of observation model uncertainties

In general, the training sequences are inserted periodically into the transmitted symbol sequences, which help the detector to estimate the variation of wireless channels. However, for fast fading channels, this method does not keep a good track of the fast variation of channels. Therefore, in the tracking mode, we apply the output of the detector to the channel estimator instead of actual symbols. However, the decision errors will degrade the estimation performance. Here, we regard the decision errors as uncertainties and try to obtain their characteristics. Define actual symbol matrix

$$\mathbf{B}(n) = \hat{\mathbf{B}}(n) + \Delta\mathbf{B}(n) \quad (4.16)$$

where $\hat{\mathbf{B}}(n)$ is the decision output of the detector and $\Delta\mathbf{B}(n)$ is the decision errors matrix, both of which are diagonal matrices. With (4.4) and (4.5), we have

$$\mathbf{f}(n) = \hat{\mathbf{B}}(n)\mathbf{C}^+ \left(\mathbf{r}(n) - \mathbf{C}_1\hat{\mathbf{B}}(n-1)\hat{\mathbf{h}}(n-1|n-1) \right), \quad (4.17)$$

$$= \mathbf{h}(n) + \underbrace{\hat{\mathbf{B}}(n)\Delta\mathbf{B}(n)\mathbf{h}(n)}_{\Delta\mathbf{v}} + \mathbf{v}_0(n), \quad (4.18)$$

$$\mathbf{v}_0(n) = \hat{\mathbf{B}}(n)\mathbf{C}^+\mathbf{C}_1\hat{\mathbf{B}}(n-1)\tilde{\mathbf{h}}(n-1|n-1) + \hat{\mathbf{B}}(n)\mathbf{C}^+\mathbf{v}(n). \quad (4.19)$$

In view of the properties of $\tilde{\mathbf{h}}(n-1|n-1)$ and $\mathbf{v}(n)$ in (4.19), we can consider that the uncertainty $\Delta\mathbf{v}$ is uncorrelated with $\mathbf{v}_0$ and is of zero mean. Its covariance matrix is given by

$$\begin{aligned} E_{\mathbf{B}}E_h\{\Delta\mathbf{v}\Delta\mathbf{v}^*\} &= E_{\mathbf{B}}\left\{ \hat{\mathbf{B}}(n)\Delta\mathbf{B}(n)E_h\{\mathbf{h}(n)\mathbf{h}^*(n)\}\Delta\mathbf{B}(n)\hat{\mathbf{B}}(n) \right\} \\ &= E_{\mathbf{B}}\left\{ \hat{\mathbf{B}}(n)\Delta\mathbf{B}(n) \right\} \end{aligned}$$

$$\begin{aligned}
 & \times \left(\frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} (\mathbf{R}_{e_0} + \mathbf{R}_e) \frac{dz}{z} \right) \Delta \mathbf{B}(n) \hat{\mathbf{B}}(n) \Big\} \\
 = & E_{\mathbf{B}} \left\{ \hat{\mathbf{B}}(n) \Delta \mathbf{B}(n) \right. \\
 & \times \gamma_0 \sigma_e^2 \text{diag} \left\{ \frac{1}{1 - \rho_1}, \dots, \frac{1}{1 - \rho_{K(L+1)}} \right\} \Delta \mathbf{B}(n) \hat{\mathbf{B}}(n) \Big\} \\
 = & \gamma_0 \sigma_e^2 \text{diag} \left\{ \frac{E_{\mathbf{B}} \{ \Delta b_1^2(n) \}}{1 - \rho_1}, \dots, \frac{E_{\mathbf{B}} \{ \Delta b_K^2(n) \}}{1 - \rho_{K(L+1)}} \right\}
 \end{aligned}$$

where we use Parseval's formula [2] in the second equality and Theorem 4.4.1 in the third equality. For a hard decision, $\Delta b_k \in \{0, -2, 2\}, k = 1, \dots, K$. Then $E_{\mathbf{B}} \{ \Delta b_k^2 \} = 4P_e$, where P_e denotes BER. Therefore, the covariance matrix of the uncertainties $\Delta \mathbf{v}$ can be bounded in terms of BER.

4.5 Optimal Polynomial Estimator Design

With (4.9) and (4.18), the system model (4.1)-(4.2) is rewritten as follows:

$$d(q^{-1})\mathbf{h}(n) = \mathbf{e}(n) + \mathbf{e}_0(n) \quad (4.20)$$

$$\mathbf{f}(n) = \mathbf{h}(n) + \mathbf{v}_1(n) \quad (4.21)$$

where $v_1 = v_0 + \Delta v$ with covariance $R_{v_1} = R_{v_0} + E_{\mathbf{B}} E_h \{ \Delta \mathbf{v} \Delta \mathbf{v}^* \}$.

Substituting (4.20) in (4.21) we obtain:

$$\mathbf{f}(n) = d^{-1} (\mathbf{e}(n) + \mathbf{e}_0(n) + d\mathbf{v}_1(n)). \quad (4.22)$$

In terms of the linear estimation theory, we design a stable causal matrix $\mathcal{R}$ that provides a linear m-step predictor of $\mathbf{h}(n+m)$:

$$\hat{\mathbf{h}}(n+m|n) = \mathcal{R} \mathbf{f}(n). \quad (4.23)$$

4.5 Optimal Polynomial Estimator Design

60

We now show the following main result.

Theorem 4.5.1. *Robust channel estimate $\hat{\mathbf{h}}(n + m|n)$ minimizing (4.3) is given by*

$$\hat{\mathbf{h}}(n + m|n) = \mathbf{L}_m \boldsymbol{\beta}^{-1} \mathbf{f}(n) \quad (4.24)$$

where the polynomial square matrix

$$\boldsymbol{\beta}(q^{-1}) = \mathbf{I} + \boldsymbol{\beta}_1 q^{-1} + \cdots + \boldsymbol{\beta}_{n_\beta} q^{-n_\beta} \quad (4.25)$$

is of dimension $K(L + 1) \times K(L + 1)$ and degree $n_\beta = 2$ and is the stable spectral factor obtained from

$$\boldsymbol{\beta} \mathbf{R}_\varepsilon \boldsymbol{\beta}^* = \mathbf{R}_e + \mathbf{R}_{e_0} + d d^* \mathbf{R}_{v_1} \quad (4.26)$$

and $\mathbf{L}_m$ is the unique solution to the bilateral Diophantine equation

$$\mathbf{L}_m \mathbf{R}_\varepsilon \boldsymbol{\beta}^* + q d \mathbf{Q}_m^* = q^m (\mathbf{R}_e + \mathbf{R}_{e_0}) \quad (4.27)$$

where polynomial matrices

$$\begin{aligned} \mathbf{L}_m(q^{-1}) &= \mathbf{L}_0^m + \mathbf{L}_1^m q^{-1}, \\ \mathbf{Q}_m^*(q) &= \mathbf{Q}_0^{m*} + \mathbf{Q}_1^{m*} q + \cdots + \mathbf{Q}_{n_Q}^{m*} q^{n_Q} \end{aligned}$$

are of dimension $K(L + 1) \times K(L + 1)$. The minimal estimation error variance $\mathbf{P}$ is given by

$$\mathbf{P}_{min} = \frac{1}{2\pi i} \oint_{|z|=1} (d d^*)^{-1} (\mathbf{I} - z^{-m} \mathbf{L}_m \boldsymbol{\beta}^{-1}) (\mathbf{R}_e + \mathbf{R}_{e_0}) \frac{dz}{z}.$$

Proof: See Appendix B.

4.5 Optimal Polynomial Estimator Design

61

For a given integer m , the corresponding $\mathbf{L}_m(q^{-1})$ and $\mathbf{Q}_m(q)$ can be solved by the Theorem 4.5.1, then the m -step predictor is obtained. On the other hand, if the m -step predictor is known, we can apply the following forward recursions to obtain the higher m -step predictor.

Corollary 4.5.1. *If the solution to the Diophantine equation (4.27) for an m -step predictor, $\mathbf{L}_m(q^{-1})$ and $\mathbf{Q}_m^*(q)$ are known, the solution for a higher m can be obtained through the following recursion process*

$$\begin{aligned}\mathbf{L}_{m+1}(q^{-1}) &= q(\mathbf{L}_m - d\mathbf{L}_0^m), \\ \mathbf{Q}_{m+1}^*(q) &= q\mathbf{Q}_m^* + \mathbf{L}_0^m \mathbf{R}_\varepsilon \boldsymbol{\beta}^*.\end{aligned}$$

This corollary can be verified by directly substituting $\mathbf{L}_{m+1}(q^{-1})$ and $\mathbf{Q}_{m+1}^*(q)$ into (4.27).

The robust MMSE estimation algorithm is summarized as:

1. Compute the spectral factor matrix $\boldsymbol{\beta}$ in terms of (4.26).
2. Obtain $\mathbf{L}_0$ and $\mathbf{Q}_0$ by solving the Diophantine equation (4.27) when $m = 0$.
Then apply Corollary 4.5.1 to obtain $\mathbf{L}_1$ and $\mathbf{Q}_1$.
3. Compute the modified observation vector $\mathbf{f}(n)$ via (4.4).
4. Finally, compute $\hat{\mathbf{h}}(n|n)$ and one-step prediction $\hat{\mathbf{h}}(n+1|n)$ via (4.24).

The complexity of the proposed algorithm comes mainly from the calculation of the Diophantine equation and the matrix spectral factorization, which can be resolved via the method proposed in [96]. Even though the first two steps of the proposed algorithm have a high complexity, they are just required to compute one time at a long transmission period. Therefore, we only focus on the computational

complexity of the 3rd and the 4th steps of the design procedure. For K users with channel length $L + 1$, the computational complexity of the proposed algorithm is approximately $4K^2L^2 + KL(2N + 1)$, which is of the order of $(KL)^2$. On the other hand, the Kalman-based estimator requires $16K^2L^2(3KL + N + 1) + 4KL(N^2 + 2N)$ computations, which is proportional to $(KL)^3$. Therefore, the proposed algorithm has a significant computational advantage.

Remark 4.5.1. *For the WLMS filter introduced in Chapter 3, the robust design is based on the average spectra of the coefficients of the AR model [52] [53]. In our design, the statistics of the uncertainties of the AR model can be approximately those from the previous training sequence and are directly incorporated into the design of the estimator.*

Remark 4.5.2. *Recall that C_1 defined in (2.10) is a sparse matrix. Then the impact of estimation errors $\tilde{\mathbf{h}}(n-1|n-1)$ on the noise $\mathbf{v}_0(n)$ is small even when the power of measurement noise $\mathbf{v}(n)$ is low. Whereas in the WLMS filter, estimation errors enter the measurement noise thereby indirectly affect future estimation. In particular when the system has high signal-to-noise ratio (SNR), measurement noise mainly depends on the estimation errors. Therefore, compared with the WLMS filter, the proposed algorithm is more suitable for the situation of fast-fading channels and high SNR. This will be demonstrated in the following simulation.*

4.6 Simulation Results

In this section, simulations are carried out to illustrate the performance of the proposed channel estimation algorithm. The performance is evaluated under fast fading Rayleigh channels in terms of the mean square error (MSE) criterion. For comparison, the Kalman-based estimator, the WLMS-based estimator and the

Figure 4.1: Mean Square Error of proposed robust estimator, Kalman estimator, standard RLS estimator and WLMS estimator with $f_d = 50$ Hz.

standard RLS-based estimator [37] are also included. We consider a DS-CDMA system with $K = 6$ users. The carrier frequency is $f_c = 900$ MHz, and symbol duration is $T_s = 100\mu s$. Golden codes with length $N = 31$ are used as spreading codes. Each user is assumed to undergo three Rayleigh channels ($L = 2$) with equal power.

The MSE of the channel estimation algorithms under the different SNR is shown in Figure 4.1. In this case, the mobile speed of each user is 60 km/hr, or equivalently, the Doppler frequency $f_d = 50$ Hz, which is assumed to be known. As seen in Figure 4.1, the proposed algorithm has performance close to that of the Kalman-based estimator, especially when the SNR is high. However, the standard RLS estimator has poor performance because the channel model characteristics are not taken into consideration. For the WLMS-based estimator, an obvious ‘floor’ appears when the SNR is high. The reason for this effect is that the MSE

4.6 Simulation Results

64

Figure 4.2: Mean square error of proposed estimator that does not consider parameter uncertainties, proposed estimator considering parameter uncertainties, Kalman estimator, standard RLS estimator and WLMS estimator for the first user with different Doppler frequencies. Note that the SNR of all users is 15 dB.

Figure 4.3: True signal, channel estimates based on proposed algorithm that takes parameter uncertainties into consideration and one without the consideration of uncertainties with respect to $f_d = 75$ Hz.

Figure 4.4: BER performance of the DFE detector with the proposed estimator, Kalman estimator and WLMS estimator. The BER performance based on actual channel coefficients is also shown for comparison.

of the WLMS-based estimator will not vanish but converge to a constant when $SNR \rightarrow \infty$ [52].

Figure 4.2 presents the robust performance of the proposed channel estimator under $SNR = 15$ dB. The Doppler frequency of user1 varies from 25 Hz to 125 Hz, which is equivalent to the mobile speed ranging from 30 km/hr to 150 km/hr. The Doppler frequency of other users remains to be 50 Hz. The channel model parameters of User1 are still based on those with $f_d = 50$ Hz. Figure 4.2 shows that the proposed robust algorithm can maintain a good performance under different Doppler frequencies. However, the RLS-based estimator performs poorly when the channel model can not be obtained accurately. With the increase of Doppler frequency, this problem becomes more serious. We can conclude that the proposed estimator has good performance and is insensitive to channel model uncertainties.

Figure 4.5: Relative near-far resistance of DFE detector with the proposed estimator, Kalman estimator and WLMS estimator. Curves present the first user's BER corresponding to other five users' SNR that varies from 0 dB to 30 dB.

Figure 4.3 shows the estimated channel coefficients of the proposed estimator and true signal with respect to $f_d = 75$ Hz.

Next, we investigate the average BER performance for all users from an adaptive decision feedback equalizer (DFE) detector with the proposed channel estimator, the Kalman estimator and the WLMS estimator in the tracking mode, as shown in Figure 4.4. All users have equal power. For comparison, a lower bound of BER is also shown which is obtained from the DFE detector applying actual channel coefficients. The BER performance is obtained by averaging 200 Monte Carlo trials. In each trial, 20 symbols are sent as the training sequence and then 1000 symbols follow as information data. It is noted that the DFE detector based on the proposed algorithm has a similar BER performance as the Kalman-based DFE detector. Moreover, the proposed algorithm also shows better performance

than the WLMS-based estimator.

Figure 4.5 examines the near-far resistance of DFE detector with the proposed channel estimator, the Kalman estimator and the WLMS estimator in the tracking mode. The power of user1 remains 5 dB throughout the experiment while the power of other five users varies from 5 dB to 30 dB. It is seen that the DFE detector with the proposed algorithm suffers much less from increasing *multi-access interference* (MAI) than that with the WLMS-based estimator. It shows that the proposed estimator is insensitive to the near-far effect.

4.7 Conclusion

In this chapter, a robust polynomial channel estimator has been derived for estimating time-varying fading channel in DS-CDMA systems via a polynomial approach. The proposed algorithm can provide the optimal robust channel estimation in terms of MMSE criterion. Moreover, channel model uncertainties are also considered in the design of the estimator. The proposed estimator can perform well not only in the training mode but in the tracking mode. The estimator applies the detected symbols in the tracking mode. We have also investigated the problem of decision errors and taken it into consideration in the design of the estimation algorithm. Simulation results showed that the proposed estimator has good robust performance, especially for high Doppler frequency cases. Thus, it is very suitable for fast-fading applications.

Chapter 5

Polynomial H_∞ Channel Estimation of DS-CDMA Systems

5.1 Introduction

In the preceding chapter, we have investigated the robust polynomial MMSE channel estimation for systems with parametric uncertainties due to inaccurate parameter estimation of model for fast speed data transmission. Broadly speaking, from the view of design criterion, an estimator based on the MMSE measure can be categorized as the H_2 algorithm which is based on the minimization of the average power of the estimation error. In this chapter, we will study an alternative polynomial solution under the H_∞ performance criterion. The interest of H_∞ estimation is motivated by practical multi-access channel environments. In addition to parametric uncertainties, as addressed in Chapter 4, there exist uncertainties in the process disturbances and observation noises in practical wireless systems. The first type of uncertainties results from the use of the limited-order model to describe practical time-varying fading channels. In general, this mismatch between

the actual channel and its model is intractable and it is hard to obtain any statistical information. The second kind of uncertainties is ascribed to the lack of statistics knowledge of noises. By far, our discussion of channel estimation problems has focused on the assumption of additive white Gaussian observation noises. Unfortunately, in many physical channels such as urban and indoor radio channels [56] [8], lots of experimental measurements show that the observation noise is definitely non-Gaussian because of the impulsive nature of human-made electromagnetic interferences and a lot of natural noises as well. Besides, the nature of multi-access techniques implies the existence of additive interferences from other users inside cell and outside cell, referred to as *multiple access interference* (MAI). The H_2 -based estimator may perform poorly because it may be sensitive to or not robust against this type of system uncertainties.

The above problem motivates our study of H_∞ estimator. The basic idea behind H_∞ filtering is to construct a filter which guarantees that the H_∞ -norm of the filtered dynamic error is less than a prescribed level. As an alternative to the classic H_2 estimator, it can achieve acceptable estimation performance in the absence of accurate knowledge of model dynamics and statistics of system noises. Therefore, the H_∞ algorithm is by nature of much interest when a robust estimate is desired in the absence of the precise knowledge of signals and systems. A vast of background materials and papers on the issue of H_∞ estimation have been published. Some of papers and textbooks are listed as [21] [68] and [32] [100].

With regard to the application of H_∞ filtering, more light is cast on the control field than communication study. The major reason is that most of important performance indices for wireless system applications, such as symbol detection and channel equalization, are based on BER that is the average value in the statistical sense, which naturally corresponds to the concept of H_2 filtering. The H_∞ filter

aims at the upper bound of the energy of the estimation error and thus sacrifices the average estimation performance. Recently, however, H_∞ filtering has gained many attentions due to its inherent robustness against uncertainties in channel model and observation model. For applications of H_∞ filtering in communication systems, recent works have been mainly devoted to the H_∞ equalization for wireless communication channels; see e.g. [102] [73]. Both the papers discuss adaptive H_∞ equalization and show its robustness for time-varying fading channels except that the former adopts a decision feedback equalizer scheme whereas the latter applies an FIR equalizer. In [23] [24], the authors apply a systematic method to obtain a closed-form solution to the linear H_∞ equalization problem in MIMO communication channels. The results show that for minimum phase channels, the proposed causal H_∞ equalizer can achieve the same performance as the best noncausal equalizer.

In this chapter, we will be interested in the multivariate channel estimation for a DS-CDMA system, which suffers from multipath time-variant fading channels. For a *single-input and single-output* (SISO) case, a similar solution has been proposed in [95]. To apply the polynomial approach, we first convert the time-varying regression model due to the time-varying nature of users' information symbols into a time-invariant one. Then, in virtue of Krein space theory [34] [35], an innovative linear H_∞ estimator is obtained in the polynomial form. Compared with the classic H_∞ estimator which requires updating at each step, the proposed estimator is of lower computational complexity. The estimator is obtained via a J -spectral factorization which has high computational complexity for high dimension polynomial matrix. In this chapter, under some conditions, we derive a closed-form solution to this J -spectral factorization, which only requires the calculation of a low dimension spectral factorization. Finally, simulations are given which demonstrate that the

proposed H_∞ estimator is of better estimation performance than other algorithms such as the RLS estimator and zero-forcing estimator. Especially, the multiuser detector based on the proposed H_∞ estimator shows a distinct improvement in the BER performance under channel model uncertainties.

5.2 Preliminaries of Krein Space Theory

We shall first briefly review linear estimation in Krein space which will be used in our development of H_∞ and robust H_∞ estimators.

Linear estimation in Krein space was initiated by Hassibi, Sayed and Kailath in a series of publications [36] [34] [35]. Detailed concepts of this theory can be found in books [9] [40] [39]. Being linear spaces, Krein spaces share many common properties with Hilbert spaces in some important ways. Two key differences between these two spaces are the existence of neutral and isotropic vectors in Krein spaces [34]. Neutral vector is referred to as a nonzero vector with zero length. An isotropic vector is a nonzero vector, which is orthogonal to every element in the same linear subspace. Obviously, there exist no such vectors in Euclidean and Hilbert spaces. These characteristics of Krein spaces result in the fact that unlike Hilbert spaces, projection in Krein spaces does not always exist and even if exists it may not be unique. Hence, the transformation of linear estimation problems from Hilbert spaces to Krein spaces must satisfy some specific equivalence conditions, which is indeed manifestation of such fact.

We first introduce an important theorem, which describes the equivalent relationship between deterministic problems in Hilbert spaces and stochastic problems in Krein spaces. This theorem was firstly proposed by Hassibi *et al.* in [34]. Whenever the Krein space elements and the Euclidean space elements satisfy the same

5.2 Preliminaries of Krein Space Theory

72

set of constraints, we will denote them by the same letters with the former being bold and the latter being normal in the following presentation.

Theorem 5.2.1. *Consider a scalar quadratic form*

$$J(z, y) = [z^* \ y^*] \begin{bmatrix} R_z & R_{zy} \\ R_{yz} & R_y \end{bmatrix}^{-1} \begin{bmatrix} z \\ y \end{bmatrix} \quad (5.1)$$

where R_z , R_{yz} , R_{zy} , and R_y are given matrices and R_y is nonsingular. Then the following holds:

1. The stationary point z_0 of $J(z, y)$ for a given y is equivalent to the projection of $\mathbf{z}$ onto the linear span $\mathcal{L}\{\mathbf{y}\}$ in Krein spaces.
2. The value of $J(z, y)$ at z_0 is

$$J(z_0, y) = y^* R_y^{-1} y. \quad (5.2)$$

3. $J(z_0, y)$ is minimum iff

$$R_z - R_{zy} R_y^{-1} R_{yz} > 0. \quad (5.3)$$

Proof. (5.2) and (5.3) readily follow from the use of triangular factorization on (5.1):

$$\begin{aligned} & [z^* \ y^*] \begin{bmatrix} R_z & R_{zy} \\ R_{yz} & R_y \end{bmatrix}^{-1} \begin{bmatrix} z \\ y \end{bmatrix} \\ = & [z^* \ y^*] \begin{bmatrix} I & 0 \\ -R_y^{-1} R_{yz} & I \end{bmatrix} \begin{bmatrix} R_z - R_{zy} R_y^{-1} R_{yz} & 0 \\ 0 & R_y \end{bmatrix}^{-1} \begin{bmatrix} I & -R_{zy} R_y^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} z \\ y \end{bmatrix} \end{aligned}$$

5.2 Preliminaries of Krein Space Theory

73

$$= \begin{bmatrix} z^* - y^* R_y^{-1} R_{yz} & y^* \end{bmatrix} \begin{bmatrix} R_z - R_{zy} R_y^{-1} R_{yz} & 0 \\ 0 & R_y \end{bmatrix}^{-1} \begin{bmatrix} z - R_{zy} R_y^{-1} y \\ y \end{bmatrix} \quad (5.4)$$

For statement 1, the proof can be found in Theorem 1 and Theorem 2 in [34]. $\square$

In Theorem 5.2.1, the deterministic variables z, y are viewed as stochastic variables $\mathbf{z}, \mathbf{y}$ in Krein space. For arbitrary coefficient matrices R_z, R_y, R_{yz} and R_{zy} in (5.1), they are treated as correlation matrices with regard to $\{\mathbf{z}, \mathbf{y}\}$ in Krein spaces. Therefore, we can transform the task of seeking the minimum of a quadratic form to a stochastic estimation problem in Krein spaces. It will lead to some complicated optimization problems to be simplified greatly in Krein spaces because many tools and results on stochastic filtering problems can be employed directly. For example, Hassibi *et. al.* have reconsidered LQG, H_2 and H_∞ theories from Krein Space perspective in [36]. The H_∞ fixed-lag smoothing problem has been solved by virtue of Krein Space theory in [97].

Equivalent to the condition for minimum in (5.3), an easier checking approach is presented in Lemma below and the detailed proof is in [34].

Lemma 5.2.1. *When $R_z > 0$, then there exists a minimum solution to (5.1) iff $R_y > 0$ and $R_y - R_{yz} R_z^{-1} R_{zy}$ have the same inertia¹.*

Next, we consider the following linear time-varying system

$$x_{n+1} = D_n x_n + G_n w_n \quad (5.5)$$

$$y_n = H_n x_n + v_n \quad (5.6)$$

where $D_n \in \mathcal{C}^{n \times n}$, $G_n \in \mathcal{C}^{n \times m}$ and $H_n \in \mathcal{C}^{p \times n}$ are parametric matrices, $x_0 \in \mathcal{C}^n$ is the initial state, and $w_n \in \mathcal{C}^m, v_n \in \mathcal{C}^p$ denotes the driving disturbance and the

¹Inertia denotes the number of positive, negative and zero eigenvalues of a matrix.

5.2 Preliminaries of Krein Space Theory

74

observation noise, respectively. y_n is regarded as known observation.

Next, we consider following the minimization problem:

$$\begin{aligned} \inf_{\{x_0, w_n, v_n\}} \min J &= x_0^* \Pi_0^{-1} x_0 + \sum_{n=0}^N [w_n^* \ v_n^*] \begin{bmatrix} Q_n & S_n \\ S_n^* & R_n \end{bmatrix}^{-1} \begin{bmatrix} w_n \\ v_n \end{bmatrix} \\ &\text{subject to (5.5) (5.6)} \end{aligned} \quad (5.7)$$

where $Q_n \in \mathcal{C}^{m \times m}$, $S_n \in \mathcal{C}^{m \times p}$, $R_n \in \mathcal{C}^{p \times p}$ and $\Pi_0 \in \mathcal{C}^{n \times n}$ are given Hermitian matrices. The optimization problem of the above deterministic quadratic form often arises in filtering problems. For example, we can note in the following section that H_∞ problem can take on an indefinite quadratic form where $Q_n = I$ and $R_n = \text{diag}\{I, -\gamma^2 I\}$.

According to system (5.5)-(5.6) and the quadratic form (5.7), we establish the following Krein space system:

$$\mathbf{x}_{n+1} = D_n \mathbf{x}_n + G_n \mathbf{w}_n, \quad (5.8)$$

$$\mathbf{y}_n = H_n \mathbf{x}_n + \mathbf{v}_n, \quad (5.9)$$

where $\{\mathbf{x}_0, \mathbf{w}_n, \mathbf{v}_n\}$ are white uncorrelated sequences in Krein space with

$$\left\langle \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{w}_n \\ \mathbf{v}_n \end{bmatrix}, \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{w}_n \\ \mathbf{v}_n \end{bmatrix} \right\rangle = \begin{bmatrix} \Pi_0 & 0 \\ 0 & \begin{bmatrix} Q_n & S_n \\ S_n^* & R_n \end{bmatrix} \end{bmatrix} \quad (5.10)$$

where $\langle \cdot, \cdot \rangle$ is the inner product in Krein space.

To show the equivalence between such two systems, J in (5.7) can be rewritten

5.2 Preliminaries of Krein Space Theory

75

as [34, Eq. (21)]:

$$J = \begin{bmatrix} x_0 \\ w \\ y \end{bmatrix}^* P^{-1} \begin{bmatrix} x_0 \\ w \\ y \end{bmatrix} \quad (5.11)$$

where w , y and P are described as:

$$\begin{aligned} w &= \text{col}\{w_0, \dots, w_N\}, \\ y &= \text{col}\{y_0, \dots, y_N\}, \\ P &= \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ \Phi & \Gamma & I \end{bmatrix} \begin{bmatrix} \Pi_0 & 0 \\ 0 & \begin{bmatrix} Q & S \\ S^* & R \end{bmatrix} \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ \Phi & \Gamma & I \end{bmatrix}^* \end{aligned}$$

where matrices Φ and Γ correspond to the map from $[x_0^*, w^*, v^*]^*$ to y and we have defined:

$$\begin{aligned} Q &\triangleq Q_0 \oplus \dots \oplus Q_N \\ R &\triangleq R_0 \oplus \dots \oplus R_N \\ S &\triangleq S_0 \oplus \dots \oplus S_N. \end{aligned}$$

Similarly, we substitute $\mathbf{y}_n$ into (5.9) to $\mathbf{v}_n$ in (5.10) and can obtain new correlation matrix as follows:

$$\left\langle \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix}, \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{w} \\ \mathbf{y} \end{bmatrix} \right\rangle = P. \quad (5.12)$$

According to Theorem 5.2.1 and the correspondence with (5.11) and (5.12), the stationary point of J for a given sequence y is equivalent to the projection of $\mathbf{x}_0$ and $\mathbf{w}$ onto the linear span $\mathbf{y}$ in Krein space. Hence, the minimization problem (5.7) can be solved through the linear estimation in the corresponding Krein space system (5.8)-(5.9).

In what follows, we will investigate the polynomial H_∞ channel estimation problem. It should be noted that our derivation comes from linear estimation in Krein spaces thereby is quite different from the H_∞ filter proposed in [35]

5.3 Problem Statement

Consider a K -user DS-CDMA system model with N -length spreading code below

$$h(n) = D_1 h(n-1) + D_2 h(n-2) + e(n), \quad (5.13)$$

$$r(n) = C_0 B(n) h(n) + C_1 B(n-1) h(n-1) + v(n), \quad (5.14)$$

where $h(n) \in \mathcal{C}^{K(L+1)}$ is the vector of channel coefficients, $r(n) \in \mathcal{C}^N$ denotes the observation, D_1 and $D_2 \in \mathcal{R}^{K(L+1) \times K(L+1)}$, and $C_0, C_1 \in \mathcal{R}^{N \times K(L+1)}$ are defined in (2.9) and (2.10), the driving disturbance $e(n) \in \mathcal{C}^{K(L+1)}$ and the measurement disturbance $v(n) \in \mathcal{C}^N$ are unknown complex vectors.

In this section, we will seek a H_∞ channel estimator for the channel model (5.13) and the observation model (5.14). Compared with the system models in the previous chapters, a difference is that here $e(n)$ and $v(n)$ are any signals with finite energy. As noted in the first section, $e(n)$ includes channel model mismatch due to the model truncation even though a AR(2) model matches the actual channel closely. Moreover, the transition coefficients D_1 and D_2 often estimated via the use of training sequences also contribute to the uncertainties of (5.13). These two

factors lead to the fact that $e(n)$ is generally not a white noise process. Similarly, observation noise $v(n)$ may be non-Gaussian.

In view of the concept of H_∞ filtering, the design objective of a H_∞ estimator is to guarantee the smallest estimation error energy over the worst-case effect from all possible bounded disturbances and noises. It can be formulated as follows:

Given a scalar $\gamma > 0$ and any positive integer $\mathcal{N}$, find a causal estimator $\mathcal{K}$ to satisfy

$$\inf_{\text{causal } \mathcal{K}} \sup_{e(n), v(n)} \frac{\sum_{n=1}^{\mathcal{N}} \|\hat{h}(n) - h(n)\|_Q^2}{\sum_{n=1}^{\mathcal{N}} \|e(n)\|_2^2 + \sum_{n=1}^{\mathcal{N}} \|v(n)\|_2^2} < \gamma^2 \quad (5.15)$$

where $\hat{h}(n)$ denotes the estimate of $h(n)$,

$$\|\hat{h}(n) - h(n)\|_Q^2 = \left(\hat{h}(n) - h(n) \right)^* Q \left(\hat{h}(n) - h(n) \right)$$

with $Q > 0$ being a weighting matrix. It should be noted that the estimator $\mathcal{K}$ is generally a suboptimal solution to the H_∞ problem because γ is not necessarily minimum.

5.4 H_∞ Estimator Design

5.4.1 Polynomial solution via Krein space

To apply the Krein space approach, we first reformulate (5.15) into the indefinite quadratic form:

$$J_{\mathcal{N}} = \sum_{n=1}^{\mathcal{N}} \|e(n)\|_2^2 + \sum_{n=1}^{\mathcal{N}} \|v(n)\|_2^2 - \gamma^{-2} \sum_{n=1}^{\mathcal{N}} \|\hat{h}(n) - h(n)\|_Q^2$$

$$\begin{aligned}
&= \sum_{n=1}^N e^*(n)e(n) + \left(\begin{bmatrix} r(n) \\ \hat{h}(n) \end{bmatrix} - \begin{bmatrix} C_0 B(n) \\ I_{K(L+1)} \end{bmatrix} h(n) \right. \\
&\quad \left. - \begin{bmatrix} C_1 B(n-1) \\ 0 \end{bmatrix} h(n-1) \right)^* \begin{bmatrix} I_N & 0 \\ 0 & -\gamma^{-2}Q \end{bmatrix} \\
&\quad \times \left(\begin{bmatrix} r(n) \\ \hat{h}(n) \end{bmatrix} - \begin{bmatrix} C_0 B(n) \\ I_{K(L+1)} \end{bmatrix} h(n) - \begin{bmatrix} C_1 B(n-1) \\ 0 \end{bmatrix} h(n-1) \right) \\
&> 0.
\end{aligned} \tag{5.16}$$

This inequality problem is equivalent to $\min J_N > 0$. Referring to (5.7), we introduce the following Krein space system

$$\begin{aligned}
\mathbf{h}(n) &= D_1 \mathbf{h}(n-1) + D_2 \mathbf{h}(n-2) + \mathbf{e}(n), \\
\begin{bmatrix} \mathbf{r}(n) \\ \hat{\mathbf{h}}(n) \end{bmatrix} &= \begin{bmatrix} C_0 B(n) \\ I_{K(L+1)} \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} C_1 B(n-1) \\ 0 \end{bmatrix} \mathbf{h}(n-1) + \begin{bmatrix} \mathbf{v}(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix},
\end{aligned} \tag{5.17}$$

(5.18)

where $\tilde{\mathbf{h}}(n) = \hat{\mathbf{h}}(n) - \mathbf{h}(n)$ and $\{\mathbf{e}(n), \mathbf{v}(n), \tilde{\mathbf{h}}(n)\}$ are white uncorrelated sequences in Krein space with

$$\left\langle \begin{bmatrix} \mathbf{e}(n) \\ \mathbf{v}(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix}, \begin{bmatrix} \mathbf{e}(n) \\ \mathbf{v}(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix} \right\rangle = \begin{bmatrix} I_{K(L+1)} & 0 & 0 \\ 0 & I_N & 0 \\ 0 & 0 & -\gamma^2 Q^{-1} \end{bmatrix}. \tag{5.19}$$

Since $B(n) \in \{-1, 1\}$, the above Krein space system model is time-varying for which a polynomial approach cannot be directly applied. Hence, we need to convert it to a time-invariant form, which can then be addressed by a polynomial

approach. (5.18) is rewritten as

$$\begin{bmatrix} \mathbf{r}(n) \\ \hat{\mathbf{h}}(n) \end{bmatrix} - \begin{bmatrix} C_1 B(n-1) \\ 0 \end{bmatrix} \mathbf{h}(n-1) = \begin{bmatrix} C_0 B(n) \\ I_{K(L+1)} \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{v}(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix},$$

$$\begin{bmatrix} \mathbf{f}(n) \\ \hat{\mathbf{h}}(n) \end{bmatrix} = \begin{bmatrix} I_N \\ I_N \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} B(n)C^+ \mathbf{v}(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix},$$

where

$$\begin{aligned} \mathbf{f}(n) &= B(n)C^+ (\mathbf{r}(n) - C_1 B(n-1)\mathbf{h}(n-1)), \\ C^+ &= (C_0^* C_0)^{-1} C_0^*. \end{aligned} \quad (5.20)$$

For simplicity, we use symbol I instead of I_N in the remainder of this section.

Then the polynomial Krein system model is as follows

$$D(q^{-1})\mathbf{h}(n) = e(n),$$

$$\begin{bmatrix} D(q^{-1}) & 0 \\ 0 & D(q^{-1}) \end{bmatrix} \begin{bmatrix} \mathbf{f}(n) \\ \hat{\mathbf{h}}(n) \end{bmatrix} = \begin{bmatrix} I & D(q^{-1})B(n)C^+ & 0 \\ I & 0 & D(q^{-1}) \end{bmatrix} \begin{bmatrix} \mathbf{e}(n) \\ \mathbf{v}(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix}, \quad (5.21)$$

where

$$D(q^{-1}) = I - D_1 q^{-1} - D_2 q^{-2}. \quad (5.22)$$

In view of (5.19), we have the following innovation model

$$\begin{bmatrix} D(q^{-1}) & 0 \\ 0 & D(q^{-1}) \end{bmatrix} \begin{bmatrix} \mathbf{f}(n) \\ \hat{\mathbf{h}}(n) \end{bmatrix} = \beta(q^{-1})\mathbf{w}(n) \quad (5.23)$$

where the square polynomial matrix

$$\beta(q^{-1}) = I + \beta_1 q^{-1} + \cdots + \beta_{n_\beta} q^{-n_\beta}$$

is of degree $n_\beta = 2$ and is the causal and causally invertible spectral factor obtained from

$$\begin{aligned} & \langle \beta(q^{-1})\mathbf{w}(n), \beta(q^{-1})\mathbf{w}(n) \rangle \\ &= \beta(q^{-1})R_{\mathbf{w}}\beta^*(q^{-1}) \end{aligned} \quad (5.24)$$

$$\approx \begin{bmatrix} I + D(q^{-1})R_c^{-1}D^*(q^{-1}) & I \\ I & I - \gamma^2 D(q^{-1})Q^{-1}D^*(q^{-1}) \end{bmatrix}. \quad (5.25)$$

In the above, we have applied the following simplification:

$$\begin{aligned} B(n)C^+C^{+*}B(n) &= B(n)(C_0^*C_0)^{-*}B(n) \\ &= (B(n)C_0^*C_0B(n))^{-*} \\ &= ([b_1(n)C_1^0 \cdots b_k(n)C_K^0]^*[b_1(n)C_1^0 \cdots b_k(n)C_K^0])^{-*} \\ &\approx \{\text{diag}\{C_1^{0*}C_1^0, \dots, C_K^{0*}C_K^0\}\}^{-1} \\ &\triangleq R_c^{-1} \end{aligned} \quad (5.26)$$

and $R_{\mathbf{w}}$ is the covariance of the innovations vector $\mathbf{w}(n)$. Note that the second equality of (5.26) is due to the fact that $B(n)$ is a diagonal matrix with diagonal elements of either 1 or -1 whereas the approximation by R_c^{-1} of (5.26) is based on the fact that different users' spreading codes $\{c_k(i)\}_{i=1}^N$ are nearly orthogonal.

Remark 5.4.1. By (5.26), the spectrum matrix (5.24) is approximated by a time-invariant one even though the spreading codes in $B(n)$ of (5.21) are time-varying.

5.4.2 Design of polynomial H_∞ channel estimator

In view of Lemma 2 of [35], the scalar indefinite quadratic form J_N satisfies the condition (5.16) if, and only if, J_N has a minimum.

Theorem 5.4.1. *Given a scalar $\gamma > 0$, there exists an estimator such that J_N has a minimum iff*

$$R_{\mathbf{v}, \tilde{\mathbf{h}}} = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 Q^{-1} \end{bmatrix} \quad (5.27)$$

and $R_{\mathbf{w}}$ have the same inertia where $R_{\mathbf{w}}$ is given in (5.24). In this situation, this minimum of J_N is given by

$$J_N^0 = \sum_{n=1}^N \mathbf{w}^*(n) R_{\mathbf{w}}^{-1} \mathbf{w}(n). \quad (5.28)$$

Proof. Comparing the matrices $\{Q_n, S_n, R_n\}$ in (5.10) with the corresponding matrices in (5.19), we reformulate J_N according to (5.7) as

$$J_N = \sum_{n=1}^N \begin{bmatrix} e^*(n) & v^*(n) & \tilde{h}^*(n) \end{bmatrix} \begin{bmatrix} I_{k(L+1)} & 0 \\ 0 & \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 Q^{-1} \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} e(n) \\ v(n) \\ \tilde{h}(n) \end{bmatrix}. \quad (5.29)$$

On the other hand, it can be easily verified from (5.23) that $\mathbf{w}(n)$ of (5.23) is the innovation of

$$\mathbf{y}_u(n) = \begin{bmatrix} \mathbf{f}(n) \\ \hat{\mathbf{h}}(n) \end{bmatrix}. \quad (5.30)$$

5.4 H_∞ Estimator Design

82

Thus, there exists a matrix $L_{\mathbf{w}}$ such that

$$Y = L_{\mathbf{w}}W \quad (5.31)$$

where $W = \text{col}\{\mathbf{w}(0), \dots, \mathbf{w}(N)\}$ and $Y = \text{col}\{\mathbf{y}_u(0), \dots, \mathbf{y}_u(N)\}$. Therefore,

$$R_Y \triangleq \langle Y, Y \rangle = L_{\mathbf{w}}^{-1} \mathbf{R}_1 (L_{\mathbf{w}}^T)^{-1} \quad (5.32)$$

where $\mathbf{R}_1 = R_{\mathbf{w}} \oplus \dots \oplus R_{\mathbf{w}}$.

Defining

$$R_2 = R_{\mathbf{v}, \tilde{\mathbf{h}}} \oplus \dots \oplus R_{\mathbf{v}, \tilde{\mathbf{h}}} \quad (5.33)$$

where $R_{\mathbf{v}, \tilde{\mathbf{h}}}$ is as in (5.27). Applying Lemma 9 in [34] to (5.29), the minimum of $J_{\mathcal{N}}$ exists iff R_Y defined in (5.32) and R_2 have the same inertia. According to the definition of R_Y , R_1 and R_2 , the condition for the existence of a minimum follows immediately.

From Theorem 5.2.1, this minimum is

$$\begin{aligned} J_{\mathcal{N}}^0 &= Y^* R_Y^{-1} Y = Y^* L_{\mathbf{w}}^* R_1^{-1} L_{\mathbf{w}} Y \\ &= W^* R_1^{-1} W \\ &= \sum_{n=1}^{\mathcal{N}} \mathbf{w}^*(n) R_{\mathbf{w}}^{-1} \mathbf{w}(n). \end{aligned}$$

The proof is completed. □

Since $Q^{-1} > 0$ and $\gamma^2 > 0$, condition of Theorem 5.4.1 implies that $R_{\mathbf{w}}$ has a J -spectral factorization

$$R_{\mathbf{w}} = \Omega J \Omega^* \quad (5.34)$$

where $J = \text{diag}\{I_N, -I_N\}$ and Ω is a nonsingular matrix of dimension $2N \times 2N$.

By Theorem 5.4.1, the minimum of J_N is

$$\begin{aligned}
 J_N &= \sum_{n=1}^N \mathbf{w}^*(n) R_{\mathbf{w}}^{-1} \mathbf{w}(n) \\
 &= \sum_{n=1}^N \mathbf{w}^*(n) \Omega^{-*} J \Omega^{-1} \mathbf{w}(n) \\
 &= \sum_{n=1}^N \begin{bmatrix} w_{\mathbf{f}} & w_{\hat{\mathbf{h}}} \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} w_{\mathbf{f}} \\ w_{\hat{\mathbf{h}}} \end{bmatrix} \\
 &= \sum_{n=1}^N (\|w_{\mathbf{f}}(n)\|_2^2 - \|w_{\hat{\mathbf{h}}}(n)\|_2^2)
 \end{aligned} \tag{5.35}$$

where

$$w = \begin{bmatrix} w_{\mathbf{f}}(n) \\ w_{\hat{\mathbf{h}}}(n) \end{bmatrix} = \Omega^{-1} \mathbf{w}(n). \tag{5.36}$$

For the following theorem, we define

$$\bar{\beta}(q^{-1}) = \beta(q^{-1}) \Omega = \begin{bmatrix} \bar{\beta}_{11}(q^{-1}) & \bar{\beta}_{12}(q^{-1}) \\ \bar{\beta}_{21}(q^{-1}) & \bar{\beta}_{22}(q^{-1}) \end{bmatrix} \tag{5.37}$$

where $\bar{\beta}_{11}(q^{-1}), \bar{\beta}_{12}(q^{-1}), \bar{\beta}_{21}(q^{-1})$ and $\bar{\beta}_{22}(q^{-1}) \in \mathcal{C}^{Kl \times Kl}$. Then, we have the following result

Theorem 5.4.2. *Consider the system (5.14) and (5.13). If the J -spectral factorization (5.24) exists with $\bar{\beta}(\infty)$ being nonsingular and $\bar{\beta}_{11}^{-1}(q^{-1})$ stable, all H_∞ linear estimators that achieve a prescribed positive scalar γ are given by*

$$\hat{h}(n) = D^{-1}(q^{-1}) \bar{\beta}_2(q^{-1}) \bar{\beta}_1^{-1}(q^{-1}) D(q^{-1}) f(n) \tag{5.38}$$

where

$$\bar{\beta}_1(q^{-1}) = \bar{\beta}_{11}(q^{-1}) + \bar{\beta}_{12}(q^{-1})\theta(q^{-1}), \quad (5.39)$$

$$\bar{\beta}_2(q^{-1}) = \bar{\beta}_{21}(q^{-1}) + \bar{\beta}_{22}(q^{-1})\theta(q^{-1}), \quad (5.40)$$

$\theta(q^{-1})$ is any linear contractive transfer function matrix satisfying $\|\theta(q^{-1})\|_\infty < 1$, and $f(n)$ is given in (5.20).

Proof. Since all H_∞ estimators achieving the H_∞ performance γ must guarantee $J_N > 0$, it holds that $\|w_f(n)\|_2^2 > \|w_{\hat{\mathbf{h}}}(n)\|_2^2$ for all $0 \leq n \leq N$. Considering that $w_f(n)$ and $w_{\hat{\mathbf{h}}}(n)$ are causal functions, we can introduce a linear contractive transfer function matrix $\theta(q^{-1})$ satisfying $\|\theta(q^{-1})\|_\infty < 1$, then

$$w_{\hat{\mathbf{h}}}(n) = \theta(q^{-1})w_f(n). \quad (5.41)$$

We substitute (5.23) and (5.41) into (5.36) and obtain

$$\begin{bmatrix} w_f(n) \\ \theta(q^{-1})w_f(n) \end{bmatrix} = \Omega^{-1}\beta^{-1}(q^{-1}) \begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ D(q^{-1})\hat{\mathbf{h}}(n) \end{bmatrix}. \quad (5.42)$$

Applying the definition (5.37) and canceling the item $w_f(n)$ in (5.42) yield the final expression of all H_∞ linear estimators as in (5.38).

Next, we will verify that the estimator (5.38) is stable. It follows from (5.25) that

$$\begin{aligned} & \bar{\beta}_{11}(q^{-1})\bar{\beta}_{11}^*(q^{-1}) - \bar{\beta}_{12}(q^{-1})\bar{\beta}_{12}^*(q^{-1}) \\ &= I + D(q^{-1})R_c^{-1}D^*(q^{-1}) > 0 \end{aligned}$$

for $q = \exp(jw)$, which yields $\|\bar{\beta}_{11}^{-1}(q^{-1})\bar{\beta}_{12}(q^{-1})\|_\infty < 1$. Since $\|\theta(q^{-1})\|_\infty < 1$, it holds that

$$(\bar{\beta}_{11}^{-1}(q^{-1})\bar{\beta}_{12}(q^{-1})\theta(q^{-1}) + I)^{-1} \in \mathcal{RH}_\infty.$$

Hence,

$$\begin{aligned} & (\bar{\beta}_{11}^{-1}(q^{-1})\bar{\beta}_{12}(q^{-1})\theta(q^{-1}) + I)^{-1} \bar{\beta}_{11}^{-1}(q^{-1}) \\ &= (\bar{\beta}_{11}(q^{-1}) + \bar{\beta}_{12}(q^{-1})\theta(q^{-1}))^{-1} \\ &= \bar{\beta}_1^{-1}(q^{-1}) \in \mathcal{RH}_\infty. \end{aligned}$$

Since $D(q^{-1}), \bar{\beta}_2(q^{-1}) \in \mathcal{RH}_\infty$, (5.38) is a stable estimator. $\square$

Remark 5.4.2. With $\theta(q^{-1})=0$, the central solution to the H_∞ linear estimator is given by

$$\hat{h}(n) = D^{-1}(q^{-1})\bar{\beta}_{21}(q^{-1})\bar{\beta}_{11}^{-1}(q^{-1})D(q^{-1})f(n). \quad (5.43)$$

5.4.3 A closed-form solution to J -spectral factorization

From the result of Theorem 5.4.2, the computational complexity mainly comes from the polynomial matrix J -spectral factorization (5.24). Even though there exist many approaches for J -spectral factorization, the computational burden is still very heavy when the dimension of desired matrix is high. Here, we try to seek an explicit solution to (5.24) so as to avoid the complex calculation.

5.4 H_∞ Estimator Design

86

Theorem 5.4.3. *If $Q_c^{-1} - Q_h^{-1} > 0$, one canonical solution to the J -spectral factorization*

$$\begin{bmatrix} I + D(q^{-1})Q_c D^*(q^{-1}) & I \\ I & I - D(q^{-1})Q_h D^*(q^{-1}) \end{bmatrix} = M(q^{-1})JM^*(q^{-1}) \quad (5.44)$$

is given by

$$M(q^{-1}) = \begin{bmatrix} N(q^{-1})Q_c^{-\frac{1}{2}} + D(q^{-1})Q_c^{\frac{1}{2}} & N(q^{-1})Q_h^{-\frac{1}{2}} \\ N(q^{-1})Q_c^{-\frac{1}{2}} & N(q^{-1})Q_h^{-\frac{1}{2}} - D(q^{-1})Q_h^{\frac{1}{2}} \end{bmatrix} \quad (5.45)$$

where

$$N(q^{-1}) = G(q^{-1})P^{\frac{1}{2}} - D(q^{-1})P, \quad (5.46)$$

$$P^{-1} = Q_c^{-1} - Q_h^{-1}, \quad (5.47)$$

and $G(q^{-1})$ is obtained from the canonical factorization of

$$I + D(q^{-1})PD^*(q^{-1}) = G(q^{-1})G^*(q^{-1}). \quad (5.48)$$

Proof. See Appendix C. □

Remark 5.4.3. *It should be noted that the J -spectral factor of (5.44), if exists, is not unique. In fact, all other J -spectral factors have the form of $M(q^{-1})U$, where U is any J -unitary unimodular matrix satisfying*

$$UJU^* = J.$$

For example, the matrix U can be in the form as follows

$$U = \begin{bmatrix} I \cosh(\Theta) & I \sinh(\Theta) \\ I \sinh(\Theta) & I \cosh(\Theta) \end{bmatrix}$$

where $\Theta \in \mathcal{R}$.

It is easy to see that (5.25) is equal to (5.44) with $Q_c = R_c^{-1}$ and $Q_h = \gamma^2 Q^{-1}$. In view of Theorem 5.4.3, if $R_c - Q/\gamma^2 > 0$, the J -spectral factor of (5.44) $\beta(q^{-1})\Omega = M(q^{-1})U$. In fact, the condition $R_c - Q/\gamma^2 > 0$ can be satisfied by tuning the scalar γ . Of course, increasing γ also implies a worse H_∞ performance of the estimator.

Finally, we will show that the estimator (5.38) applying the solution (5.44) is stable.

Lemma 5.4.1. *If $R_c - Q/\gamma^2 > 0$, H_∞ linear estimators (5.38) exist with $\bar{\beta}(q^{-1})$ obtained from Theorem 5.4.3.*

Proof. In terms of Theorem 5.4.2, we need to verify that $\bar{\beta}(\infty)$ is nonsingular and $\bar{\beta}_{11}^{-1}(q^{-1})$ is stable when the solution (5.44) is applied.

First, it follows that

$$\bar{\beta}(\infty) = \begin{bmatrix} I + Q_c & I \\ I & I - Q_h \end{bmatrix}$$

Since $I + Q_c$ is nonsingular, $\bar{\beta}(\infty)$ has the following decomposition

$$\bar{\beta}(\infty) = \begin{bmatrix} I & 0 \\ (I + Q_c)^{-1} & I \end{bmatrix} \begin{bmatrix} I + Q_c & 0 \\ 0 & \Delta \end{bmatrix} \begin{bmatrix} I & (I + Q_c)^{-1} \\ 0 & I \end{bmatrix}$$

where

$$\begin{aligned}
 \Delta &= I - Q_h - (I + Q_c)^{-1} \\
 &= ((I - Q_h)(I + Q_c) - I)(I + Q_c)^{-1} \\
 &= -Q_h(I + Q_c^{-1} - Q_h^{-1})Q_c(I + Q_c)^{-1} \\
 &= -Q_h(I + P^{-1})Q_c(I + Q_c)^{-1}.
 \end{aligned}$$

Since Δ is nonsingular, $\bar{\beta}(\infty)$ is nonsingular.

Next, we will show the stability of $\bar{\beta}_{11}^{-1}(q^{-1})$. From (5.45), it follows that

$$\begin{aligned}
 \bar{\beta}_{11}^{-1}(q^{-1}) &= \left(\left(G(q^{-1})P^{\frac{1}{2}} - D(q^{-1})P \right) Q_c^{-\frac{1}{2}} + D(q^{-1})Q_c^{\frac{1}{2}} \right)^{-1} \\
 &= Q_c^{\frac{1}{2}} \left(GP^{\frac{1}{2}} + D(Q_c - P) \right)^{-1} \\
 &= Q_c^{\frac{1}{2}} P^{-\frac{1}{2}} \left(G + D(Q_c - P)P^{-\frac{1}{2}} \right)^{-1}.
 \end{aligned}$$

Indeed,

$$\begin{aligned}
 &GG^* - \left(D(Q_c - P)P^{-\frac{1}{2}} \right) \left(D(Q_c - P)P^{-\frac{1}{2}} \right)^* \\
 &= I + DPD^* - D(Q_c P^{-1} Q_c + P - 2Q_c) D^* \\
 &= I - DQ_c (P^{-1} - 2Q_c^{-1}) Q_c D^* \\
 &= I + DQ_c (Q_c^{-1} + Q_h^{-1}) Q_c D^* \\
 &> 0
 \end{aligned}$$

for $q = \exp(jw)$, which yields $\|G^{-1}(q^{-1})D(q^{-1})(Q_c - P)P^{-\frac{1}{2}}\|_\infty < 1$. It implies that

$$\left(G^{-1}(q^{-1})D(q^{-1})(Q_c - P)P^{-\frac{1}{2}} + I \right)^{-1} \in \mathcal{RH}_\infty.$$

Therefore,

$$\begin{aligned}
& Q_c^{\frac{1}{2}} P^{-\frac{1}{2}} \left(G^{-1}(q^{-1}) D(q^{-1}) (Q_c - P) P^{-\frac{1}{2}} + I \right)^{-1} G^{-1}(q^{-1}) \\
&= Q_c^{\frac{1}{2}} P^{-\frac{1}{2}} \left(G + D(Q_c - P) P^{-\frac{1}{2}} \right)^{-1} \\
&= \bar{\beta}_{11}^{-1}(q^{-1}) \in \mathcal{RH}_{\infty}.
\end{aligned}$$

□

5.5 Simulation Results

In this section, we conduct several simulations that demonstrate the performance of the proposed H_{∞} channel estimation algorithm. The performance is evaluated under frequency-selective Rayleigh channels in terms of the MSE criterion. For comparison, RLS estimator and zero forcing estimator are also included. We consider a DS-CDMA system with $K = 6$ users. The carrier frequency is $f_c = 900$ MHz, and symbol duration is $T_s = 100\mu s$. Golden codes with length $N = 31$ are used as spreading codes. Each user is assumed to undergo two Rayleigh channels ($L = 1$) with equal power. Moreover, we also investigate the different estimation algorithm's influence on the BER performance of the adaptive decision feedback equalizer (DFE) detector in the following trials. The BER performance is obtained by averaging 100 Monte Carlo trials.

5.5.1 Performance with various observation noises

Figure 5.1-Figure 5.7 show the MSE and BER performance of the system suffering from three kinds of observation noises. The mobile speed of six users remains at 60 km/hr and hence the Doppler frequency is $f_d = 50$ Hz, which is assumed

5.5 Simulation Results

90

Figure 5.1: Mean Square Error with respect to different SNR under a white Gaussian noise of variance $\sigma^2 = 0.01$.

Figure 5.2: BER with respect to different SNR under a white Gaussian noise of variance $\sigma^2 = 0.01$.

to be known. It means that the coefficient matrices D_1 and D_2 can be obtained accurately in advance.

In Figure 5.1 and Figure 5.2, the observation noise is an additive white Gaussian noise (AWGN) with variance $\sigma^2 = 0.01$. Figure 5.1 shows that the MSE performance of the proposed H_∞ algorithm is better than that of the other algorithms, especially at low SNR. However, all of them come closer with the increase of SNR. Figure 5.2 demonstrates the BER performance of the system applying a DFE detector under AWGN. The result shows that the DFE using the proposed H_∞ estimation algorithm has a much better BER performance than that based on other algorithms. Such superior BER performance of the proposed estimator results from the fact that there is no decision error when the upper bound of the H_∞ filter drops in the tolerance of the DFE detector. The corresponding phenomenon shown in Figure 5.2 is that when the SNR is higher than 12 dB, the BER of the proposed H_∞ estimator is equal to zero though there still exist estimation errors, as demonstrated in Figure 5.1.

In Figure 5.3-Figure 5.4, we check the performance of every estimator under an additive colored Gaussian noise, which is generated by a transfer function $F(z) = (1 - 0.9z)^{-1}$ and a Gaussian white noise input. It is easy to see that MSE performance in Figure 5.3 is similar to that in Figure 5.1. However, the BER performances based on all estimation algorithms shown in Figure 5.3 are worse than those in Figure 5.2. It is because the DFE detector is sensitive to the colored noise. Despite the degradation of BER performance, the detector based on the proposed H_∞ estimator still has better detection performance than other algorithms like the case in Figure 5.2.

Next, we investigate the estimation performance under the non-Gaussian observation noise. In general, there exist a variety of statistical models to describe

5.5 Simulation Results

92

Figure 5.3: The MSE under a colored noise generated by filter $H(z) = (1 - 0.9z)^{-1}$ with $f_d = 50$ Hz.

Figure 5.4: BER with respect to different SNR under a colored noise generated by filter $H(z) = (1 - 0.9z)^{-1}$ with $f_d = 50$ Hz.

Figure 5.5: The PDF of a two-term Gaussian Mixture noise where $\epsilon = 0.1$ and $\alpha^2 = 100$.

the non-Gaussian phenomena in practice. Here, we employ the widely adopted two-term Gaussian mixture model, the PDF of which is given by

$$f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \alpha^2\sigma_n^2) \quad (5.49)$$

where $\mathcal{N}(0, \sigma_n^2)$ represents a Gaussian noise and $\mathcal{N}(0, \alpha^2\sigma_n^2)$ represents the impulsive component with ϵ representing the probability for which impulses occur. This mixture noise density model is introduced in [63] and gives a good approximation to the more fundamental Middleton class A noise model [57] [92], which has been used extensively to model a variety of physical noises. If we let $\epsilon = 0.1$ and $\alpha^2 = 100$ as in [63], the PDF of two-term Gaussian mixture noises is demonstrated in Figure 5.5. Figure 5.6 and Figure 5.7 show that both the MSE and BER performance of the estimators suffering from non-Gaussian noise are similar to those under AWGN shown in Figure 5.1-Figure 5.2.

5.5 Simulation Results

94

Figure 5.6: The MSE under a two-term Gaussian Mixture noise with (pdf) $f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \alpha^2\sigma_n^2)$, where $\epsilon = 0.1$ and $\alpha^2 = 100$.

Figure 5.7: BER with respect to different SNR under a two-term Gaussian Mixture noise

In view of the above results, it should be noted that the DFE detector based on the H_∞ channel estimator has better BER performance improvement at high SNR in comparison with other algorithms. The reason behind this surprising result is that the inherent nature of the H_∞ algorithm guarantees that channel estimation error is less than a prescribed bound, which is usually inversely proportional to SNR. Therefore, when SNR is large, the detection error caused by the channel estimation error will be negligible because the hard decision component of the multiuser detector is nonlinear. Conversely, a sudden big estimation error of a MMSE-based estimator may be translated to a symbol error even though corresponding MSE, from the statistical perspective, is not much worse than that of the H_∞ estimator.

5.5.2 Performance with modeling errors

In this simulation, we present the robust performance of the proposed channel estimator against modeling errors in the fading channel model. The Doppler frequency of user1 changes from 50 Hz to 125 Hz. The Doppler frequency of other users remains to be 50 Hz. However, the channel model parameters of user1 are still based on those with $f_d = 50$ Hz. It means that there exists a channel mismatch for the first user. Figures 5.8-5.9 show that the proposed H_∞ algorithm still maintains a good performance and is insensitive to the channel modeling error.

5.6 Conclusion

In this chapter, a polynomial H_∞ channel estimator has been derived to estimate time-varying fading channels in DS-CDMA systems. By virtue of Krein space theory, the proposed algorithm gives a polynomial estimator in terms of the H_∞

5.6 Conclusion

96

Figure 5.8: The MSE with respect to SNR when there is a model mismatch in the 1th user. Real Doppler frequency of the 1th user is 125 Hz whereas others remain 50 Hz.

Figure 5.9: The BER with respect to SNR when there is a model mismatch in the 1th user. Real Doppler frequency of the 1th user is 125 Hz whereas others remain 50 Hz.

performance criterion. The polynomial form of the proposed estimator has lower computational complexity than the state-space H_∞ filter due to the latter's requirement of computing an H_∞ Riccati equation. Moreover, under certain condition we obtain a closed-form solution to the J -spectral factorization of (5.24) by which the estimator is calculated. This result leads to a great reduction of the computational complexity of a high dimension J -spectral factorization. Simulation results show that the proposed estimator has good robust performance against several kinds of noises and modeling errors of the fading channels. Especially, the H_∞ estimator-based DFE detector exhibits good BER performance at the high SNR.

Chapter 6

Polynomial Robust H_∞ Estimation for Systems with Stochastic Parametric Uncertainty and Its Application in DS-CDMA Systems

6.1 Introduction

In Chapter 4 and Chapter 5, we have investigated the problem of MMSE estimation for systems with stochastic parametric uncertainties and the problem of H_∞ estimation for systems without parametric uncertainties, respectively. This chapter is devoted to the polynomial H_∞ estimator design for the system subject to stochastic parametric uncertainties, which is also classified as a robust H_∞ estimation problem. In general, the term *robust* implies the design of an estimator/controller that has some guaranteed stability/performance for systems which contain either deterministic or stochastic uncertainties in the state and/or control matrices. Note

that stochastic parametric uncertainties in systems can be modeled as multiplicative white noises in many applications [59] [20]. A polynomial H_∞ discrete robust filter for stochastic systems is described in [33] which minimizes the peak of the estimation error in the frequency domain. By virtue of the stochastic *bounded real lemma* (BRL), the solutions to the discrete-time H_∞ control and estimation problems have been proposed in [22] [29] [60]. The finite-horizon output-feedback control and filtering problems are addressed in [29]. In [29], an optimal strategy is developed first by deriving a BRL based on the adjoint system and by applying this result to obtain a solution in terms of a Riccati-type recursion. Moreover, necessary and sufficient conditions for the existence of this solution are obtained. This algorithm has been applied to solve the problem of finite-horizon H_∞ tracking for linear time varying systems with stochastic parametric uncertainties in [28]. The infinite-horizon output-feedback control problem is solved in [22] where the solution involves a number of LMIs. The corresponding stochastic H_∞ filter in the stationary case has been derived in [30].

In this chapter, we are concerned with the polynomial robust H_∞ channel estimation problem for DS-CDMA systems. The systems under consideration here are subject to stochastic parametric uncertainties which appear in the channel state matrix. A solution to this estimation problem is obtained first by treating stochastic uncertainties as noises and deriving a sufficient solution to the corresponding H_∞ estimation for the new system. Different from the approach of applying discrete-time stochastic BRL in [29], the proposed algorithm is developed directly from Krein space perspective with derivation similar to that of the polynomial H_∞ estimator in Chapter 5. Furthermore, we also investigate the polynomial version of general robust H_∞ estimation problems, which contains the channel estimation as a special case.

6.2 Problem Statement

Consider a K -user DS-CDMA system model with stochastic uncertainties in the channel state matrix:

$$\mathbf{h}(n) = (\mathbf{D} + \Delta\mathbf{D})\mathbf{h}(n-1) + \mathbf{e}(n), \quad (6.1)$$

$$\mathbf{r}(n) = \mathbf{C}_0\mathbf{B}(n)\mathbf{h}(n) + \mathbf{C}_1\mathbf{B}(n-1)\mathbf{h}(n-1) + \mathbf{v}(n) \quad (6.2)$$

where $\mathbf{h}(n) \in \mathcal{R}^{K(L+1)}$ is the channel coefficient, $\mathbf{r}(n) \in \mathcal{R}^N$ denotes the observation, $\mathbf{D} \in \mathcal{R}^{K(L+1) \times K(L+1)}$ and $\{\mathbf{C}_0\}, \{\mathbf{C}_1\} \in \mathcal{R}^{N \times K(L+1)}$ are defined in Chapter 2, and $\Delta\mathbf{D}$ is an uncertain stochastic matrix satisfying

$$E\{\Delta\mathbf{D}^* \Delta\mathbf{D}\} = Q_D \quad \text{and} \quad E\{\Delta\mathbf{D}\} = 0. \quad (6.3)$$

It should be noted that in a practical DS-CDMA system, the coefficient matrix in channel model (6.1) is generally stochastic due to the motion of mobile terminals. Generally, a first-order autoregressive model like (6.1) can be adopted with

$$D(q^{-1}) = 1 - \cos \Omega q^{-1} \quad (6.4)$$

where $\Omega = 2\pi f_d T_s / \sqrt{2}$ and f_d is Doppler frequency. When Doppler frequency is varying, uncertainty in f_d can be treated as a stochastic item $\Delta\Omega$:

$$\Omega = 2\pi(f_d + \Delta f_d)T_s / \sqrt{2} = \Omega^o + \Delta\Omega.$$

Therefore, $\cos \Omega$ can be written as

$$\cos \Omega = \cos(\Omega^o + \Delta\Omega)$$

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

101

$$\begin{aligned}
&= \cos \Omega^o \cos \Delta \Omega - \sin \Omega^o \sin \Delta \Omega \\
&\approx \cos \Omega^o - \sin \Omega^o \Delta \Omega
\end{aligned}$$

where $\cos \Omega^o$ and $\sin \Omega^o \Delta \Omega$ correspond to $\mathbf{D}$ and $\Delta \mathbf{D}$ in (6.1), respectively.

On the other hand, there exists inevitably channel model mismatch because the limited order AR model is employed to approximate the dynamics of real fading channel. It leads to the fact that the $e(n)$ in (6.1) may not be a white noise process but a noise with unknown statistics. Likewise, the statistics of $v(n)$ in (6.2) is also hard to obtain because in wireless channels, the observation vector $r(n)$ is corrupted not only by non-Gaussian ambient noises but also by additive interferences from other users outside cell. Therefore, the robust H_∞ filtering technique becomes an effective tool to solve real wireless channel estimation problems.

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

The robust H_∞ estimation algorithm will realize two aims:

1. Minimize the worst-case energy gain from the disturbance signals to the estimation error without the knowledge of noise statistics. This is a distinct advantage of H_∞ -optimization.
2. Guarantee probabilistic average performance over all admissible parameter uncertainties by virtue of the priori knowledge of their statistics.

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty 102

Therefore, a robust H_∞ suboptimal estimation strategy can be stated as, for a given scalar $\gamma > 0$, finding an estimate of $\mathbf{h}(n)$ to satisfy

$$\begin{aligned} J_{\mathcal{N}} &= \sum_{n=1}^{\mathcal{N}} \|\mathbf{e}(n)\|_2^2 + \sum_{n=1}^{\mathcal{N}} \|\mathbf{v}(n)\|_2^2 - \gamma^{-2} E_{\{\Delta D\}} \left\{ \sum_{n=1}^{\mathcal{N}} \|\hat{\mathbf{h}}(n) - \mathbf{h}(n)\|_2^2 \right\} \\ &> 0. \end{aligned} \quad (6.5)$$

In [29], the problem (6.5) is solved by applying the approach of completion of squares. The resulting solution is termed as the discrete-time stochastic BRL, which relates to the stochastic H_∞ performance to a matrix Riccati equality or inequality and leads to a necessary and sufficient solution to the stochastic H_∞ estimation problem. For our proposed algorithm, the key idea is to build up an auxiliary system without parametric uncertainties by treating uncertain item $\Delta D\mathbf{h}(n-1)$ as a disturbance and then design a polynomial H_∞ estimator based on such system. The corresponding solution of the filter is obtained in virtue of Krein space theory.

6.3.1 Auxiliary system without parametric uncertainties

We begin by reformulating (6.1) in the following form

$$\mathbf{h}(n) = D\mathbf{h}(n-1) + \mathbf{e}_h(n) \quad (6.6)$$

where

$$\begin{aligned} \mathbf{e}_h(n) &= \Delta D\mathbf{h}(n-1) + \mathbf{e}(n), \\ E_{\{\Delta D\}} \{\mathbf{e}_h^*(n) \mathbf{e}_h(n)\} &= \mathbf{e}(n)^* \mathbf{e}(n) + E_{\{\Delta D\}} \{\mathbf{h}^*(n-1) E_{\Delta D} \{\Delta D^* \Delta D\} \mathbf{h}(n-1)\} \\ &= \mathbf{e}(n)^* \mathbf{e}(n) + E_{\{\Delta D\}} \{\mathbf{h}^*(n-1) Q_D \mathbf{h}(n-1)\}. \end{aligned} \quad (6.7)$$

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

103

From (6.7), the cost function (6.5) is rewritten as

$$\begin{aligned}
 J_{\mathcal{N}} &= \sum_{n=1}^{\mathcal{N}} E_{\{\Delta D\}} \{ \mathbf{e}_h(n)^* \mathbf{e}_h(n) \} - \sum_{n=1}^{\mathcal{N}} E_{\{\Delta D\}} \{ \mathbf{h}(n-1)^* Q_D \mathbf{h}(n-1) \} \\
 &\quad + \sum_{n=1}^{\mathcal{N}} \{ \mathbf{v}(n)^* \mathbf{v}(n) \} - \gamma^{-2} E_{\{\Delta D\}} \left\{ \sum_{n=1}^{\mathcal{N}} \|\hat{\mathbf{h}}(n) - \mathbf{h}(n)\|_2^2 \right\} \\
 &= E_{\{\Delta D\}} \left\{ \sum_{n=1}^{\mathcal{N}} \mathbf{e}_h(n)^* \mathbf{e}_h(n) + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* \mathbf{v}(n) - \sum_{n=1}^{\mathcal{N}} \mathbf{h}(n-1)^* Q_D \mathbf{h}(n-1) \right. \\
 &\quad \left. - \gamma^{-2} \sum_{n=1}^{\mathcal{N}} \|\hat{\mathbf{h}}(n) - \mathbf{h}(n)\|_2^2 \right\}.
 \end{aligned}$$

We define

$$\begin{aligned}
 \tilde{J}_{\mathcal{N}} &= \sum_{n=1}^{\mathcal{N}} \mathbf{e}_h(n)^* \mathbf{e}_h(n) + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* \mathbf{v}(n) - \sum_{n=1}^{\mathcal{N}} \mathbf{h}(n-1)^* Q_D \mathbf{h}(n-1) \\
 &\quad - \gamma^{-2} \sum_{n=1}^{\mathcal{N}} \|\hat{\mathbf{h}}(n) - \mathbf{h}(n)\|_2^2.
 \end{aligned}$$

Clearly $J_{\mathcal{N}} > 0$ holds for any $\{\mathbf{e}(n)\}, \{\mathbf{v}(n)\} \in l_2$ if $\tilde{J}_{\mathcal{N}} > 0$ for any $\{\mathbf{e}_h(n)\}, \{\mathbf{v}(n)\} \in l_2$. Next, we will seek a polynomial H_∞ estimator that satisfies $\tilde{J}_{\mathcal{N}} > 0$ for a given γ . Hence, the corresponding result gives a sufficient solution to $J_{\mathcal{N}} > 0$.

6.3.2 Design of polynomial H_∞ channel estimator

According to the above transformation, the design objective of a robust H_∞ estimator is changed to that for a given scalar γ , seek an estimate $\hat{\mathbf{h}}(n)$ to guarantee that

$$\begin{aligned}
 \tilde{J}_{\mathcal{N}} &= \sum_{n=1}^{\mathcal{N}} \mathbf{e}_h(n)^* \mathbf{e}_h(n) + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* \mathbf{v}(n) - \sum_{n=1}^{\mathcal{N}} \mathbf{h}(n-1)^* Q_D \mathbf{h}(n-1) \\
 &\quad - \gamma^{-2} \sum_{n=1}^{\mathcal{N}} \|\hat{\mathbf{h}}(n) - \mathbf{h}(n)\|_2^2
 \end{aligned}$$

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

104

$$\begin{aligned}
&= \sum_{n=1}^{\mathcal{N}} \mathbf{e}_h(n)^* \mathbf{e}_h(n) + \sum_{n=1}^{\mathcal{N}} \left(\begin{bmatrix} \mathbf{r}(n) \\ 0 \\ \hat{\mathbf{h}}(n) \end{bmatrix} - \begin{bmatrix} \mathbf{C}_0 \mathbf{B}(n) \\ 0 \\ \mathbf{I} \end{bmatrix} \mathbf{h}(n) \right. \\
&\quad + \begin{bmatrix} \mathbf{C}_1 \mathbf{B}(n-1) \\ \mathbf{I} \\ 0 \end{bmatrix} \mathbf{h}(n-1) \left. \right)^* \begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & -Q_D^{-1} & 0 \\ 0 & 0 & -\gamma^2 \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{r}(n) \\ 0 \\ \hat{\mathbf{h}}(n) \end{bmatrix} \\
&\quad - \begin{bmatrix} \mathbf{C}_0 \mathbf{B}(n) \\ 0 \\ \mathbf{I} \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{C}_1 \mathbf{B}(n-1) \\ \mathbf{I} \\ 0 \end{bmatrix} \mathbf{h}(n-1) \Bigg) \\
&> 0
\end{aligned} \tag{6.8}$$

where $Q_D > 0$ is assumed.

From (6.8), we introduce the following Krein space system:

$$\begin{aligned}
\mathbf{h}(n) &= \mathbf{D} \mathbf{h}(n-1) + \mathbf{e}_h(n), \\
\begin{bmatrix} \mathbf{r}(n) \\ 0 \\ \hat{\mathbf{h}}(n) \end{bmatrix} &= \begin{bmatrix} \mathbf{C}_0 \mathbf{B}(n) \\ 0 \\ \mathbf{I} \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{C}_1 \mathbf{B}(n-1) \\ \mathbf{I} \\ 0 \end{bmatrix} \mathbf{h}(n-1) + \begin{bmatrix} \mathbf{v}(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix}
\end{aligned} \tag{6.9}$$

(6.10)

where $\tilde{\mathbf{h}}(n) = \hat{\mathbf{h}}(n) - \mathbf{h}(n)$ and $\{\mathbf{e}_h(n), \mathbf{v}(n), \mathbf{v}_2(n), \tilde{\mathbf{h}}(n)\}$ are uncorrelated white sequences in Krein space with

$$\left\langle \begin{bmatrix} \mathbf{e}_h(n) \\ \mathbf{v}(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix}, \begin{bmatrix} \mathbf{e}_h(n) \\ \mathbf{v}(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix} \right\rangle = \begin{bmatrix} \mathbf{I}_{K(L+1)} & 0 & 0 & 0 \\ 0 & \mathbf{I}_N & 0 & 0 \\ 0 & 0 & -Q_D^{-1} & 0 \\ 0 & 0 & 0 & -\gamma^2 \end{bmatrix}. \tag{6.11}$$

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

105

Since $\mathbf{B}(n) \in \{-1, 1\}$, the above Krein space system model is time-varying for which a polynomial approach cannot be directly applied. Hence, we need to convert it to a time-invariant form which can then be addressed using a polynomial approach. To this end, (6.10) is rewritten as

$$\begin{bmatrix} \mathbf{f}(n) \\ 0 \\ \hat{\mathbf{h}}(n) \end{bmatrix} = \begin{bmatrix} \mathbf{I}_N \\ q^{-1}\mathbf{I}_N \\ \mathbf{I}_N \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{B}(n)\mathbf{C}^+\mathbf{v}(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix} \quad (6.12)$$

where

$$\begin{aligned} \mathbf{f}(n) &= \mathbf{B}(n)\mathbf{C}^+ (\mathbf{r}(n) - \mathbf{C}_1\mathbf{B}(n-1)\mathbf{h}(n-1)), \\ \mathbf{C}^+ &= (\mathbf{C}_0^*\mathbf{C}_0)^{-1}\mathbf{C}_0^*. \end{aligned} \quad (6.13)$$

It should be noted that there exist no parametric uncertainties in the Krein space system (6.9) and (6.10). Hence, the robust H_∞ estimation problem in Hilbert spaces is reduced to the design of a standard H_2 estimator for a Krein space system.

Then the polynomial Krein space system is represented as follows

$$\mathbf{D}(q^{-1})\mathbf{h}(n) = \mathbf{e}_h(n), \quad (6.14)$$

$$\begin{bmatrix} \mathbf{D}(q^{-1})\mathbf{f}(n) \\ 0 \\ \mathbf{D}(q^{-1})\hat{\mathbf{h}}(n) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{D}(q^{-1})\mathbf{B}(n)\mathbf{C}^+ & 0 & 0 \\ q^{-1}\mathbf{I} & 0 & \mathbf{D}(q^{-1}) & 0 \\ \mathbf{I} & 0 & 0 & \mathbf{D}(q^{-1}) \end{bmatrix} \begin{bmatrix} \mathbf{e}_h(n) \\ \mathbf{v}(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{h}}(n) \end{bmatrix} \quad (6.15)$$

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

106

where (6.15) follows by substituting (6.14) in (6.12) and

$$\mathbf{D}(q^{-1}) = \mathbf{I} - \mathbf{D}q^{-1}.$$

According to (6.11) and (6.15), we have the following innovation model

$$\begin{bmatrix} \mathbf{D}(q^{-1})\mathbf{f}(n) \\ 0 \\ \mathbf{D}(q^{-1})\hat{\mathbf{h}}(n) \end{bmatrix} = \boldsymbol{\beta}(q^{-1})\mathbf{w}(n) \quad (6.16)$$

where the square polynomial matrix

$$\boldsymbol{\beta}(q^{-1}) = \mathbf{I} + \boldsymbol{\beta}_1 q^{-1}$$

is a causal and causally invertible spectral factor obtained from

$$\begin{aligned} & \langle \boldsymbol{\beta}(q^{-1})\mathbf{w}(n), \boldsymbol{\beta}(q^{-1})\mathbf{w}(n) \rangle \\ &= \boldsymbol{\beta}(q^{-1})\mathbf{R}_w\boldsymbol{\beta}^*(q^{-1}) \end{aligned} \quad (6.17)$$

$$\approx \begin{bmatrix} \mathbf{I} + \mathbf{D}(q^{-1})\mathbf{R}_c^{-1}\mathbf{D}^*(q^{-1}) & q\mathbf{I} & \mathbf{I} \\ q^{-1}\mathbf{I} & \mathbf{I} - \mathbf{D}(q^{-1})Q_D^{-1}\mathbf{D}^*(q^{-1}) & \mathbf{I} \\ \mathbf{I} & \mathbf{I} & \mathbf{I} - \gamma^2\mathbf{D}(q^{-1})\mathbf{D}^*(q^{-1}) \end{bmatrix} \quad (6.18)$$

where $\mathbf{R}_w$ is the covariance of the innovation vector $\mathbf{w}(n)$ and the simplification process and the definition of $\mathbf{R}_c^{-1}$ are stated in (5.26).

In view of the Lemma 2 of [35], the scalar indefinite quadratic form $\tilde{J}_N$ satisfies the conditions (6.38) if, and only if, $\tilde{J}_N$ has a minimum. We apply Theorem 5.4.1 in Chapter 5 and have the following existence condition for an estimator of the

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

107

polynomial Krein space system.

Theorem 6.3.1. *Given a scalar $\gamma > 0$, there exists an estimator satisfying (6.8) if, and only if*

$$\mathbf{R} = \begin{bmatrix} I & 0 & 0 \\ 0 & -Q_D^{-1} & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix}$$

and $\mathbf{R}_w$ has the same inertia where $\mathbf{R}_w$ is given in (6.17). In this situation, a minimum of $\tilde{J}_N$ is given by

$$\tilde{J}_N^0 = \sum_{n=1}^N \mathbf{w}^*(n) \mathbf{R}_w^{-1} \mathbf{w}(n).$$

Since $Q_D^{-1} > 0$ and $\gamma^2 > 0$, the satisfaction of the condition in Theorem 6.3.1 implies that $\mathbf{R}_w$ has a J -spectral factorization

$$\mathbf{R}_w = \mathbf{\Omega} \mathbf{J} \mathbf{\Omega}^* \quad (6.19)$$

where $\mathbf{J} = \text{diag}\{I, , -I, -I\}$ and $\mathbf{\Omega}$ is a nonsingular matrix. By Theorem 5.4.1, the minimum of $\tilde{J}_N$ is

$$\begin{aligned} \tilde{J}_N &= \sum_{n=1}^N \mathbf{w}^*(n) \mathbf{R}_w^{-1} \mathbf{w}(n) \\ &= \sum_{n=1}^N \mathbf{w}^*(n) \mathbf{\Omega}^{-*} \mathbf{J} \mathbf{\Omega}^{-1} \mathbf{w}(n) \\ &= \sum_{n=1}^N \begin{bmatrix} \bar{\mathbf{w}}_f(n)^* & \bar{\mathbf{w}}_h(n)^* & \bar{\mathbf{w}}_{\hat{h}}(n)^* \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix} \begin{bmatrix} \bar{\mathbf{w}}_f(n) \\ \bar{\mathbf{w}}_h(n) \\ \bar{\mathbf{w}}_{\hat{h}}(n) \end{bmatrix} \end{aligned}$$

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

108

$$= \sum_{n=1}^{\mathcal{N}} (\|\bar{\mathbf{w}}_{\mathbf{f}}(n)\|_2^2 - \|\bar{\mathbf{w}}_{\mathbf{h}}(n)\|_2^2 - \|\bar{\mathbf{w}}_{\hat{\mathbf{h}}}(n)\|_2^2)$$

where we define

$$\bar{\mathbf{w}} = \begin{bmatrix} \bar{\mathbf{w}}_{\mathbf{f}}(n) \\ \bar{\mathbf{w}}_{\mathbf{h}}(n) \\ \bar{\mathbf{w}}_{\hat{\mathbf{h}}}(n) \end{bmatrix} = \mathbf{\Omega}^{-1} \mathbf{w}(n). \quad (6.20)$$

The main result is then stated in the following theorem.

Theorem 6.3.2. *Considering a DS-CDMA system (6.1)-(6.2) subject to stochastic parametric uncertainties which satisfy (6.3). Given a scalar $\gamma > 0$, all robust polynomial H_∞ estimators satisfying (6.8) can be characterized by*

$$\hat{\mathbf{h}}(n) = \mathbf{D}^{-1}(q^{-1}) \bar{\boldsymbol{\beta}}_2(q^{-1}) \bar{\boldsymbol{\beta}}_1^{-1}(q^{-1}) \mathbf{D}(q^{-1}) \mathbf{f}(n) \quad (6.21)$$

where $\bar{\boldsymbol{\beta}}_1$ and $\bar{\boldsymbol{\beta}}_2$ are defined in (6.28) and (6.29), respectively, and the central robust H_∞ linear estimator is given by

$$\hat{\mathbf{h}}(n) = \mathbf{D}^{-1}(q^{-1}) (\bar{\boldsymbol{\beta}}_{31} - \bar{\boldsymbol{\beta}}_{32} \bar{\boldsymbol{\beta}}_{22}^{-1} \bar{\boldsymbol{\beta}}_{21}) (\bar{\boldsymbol{\beta}}_{11} - \bar{\boldsymbol{\beta}}_{12} \bar{\boldsymbol{\beta}}_{22}^{-1} \bar{\boldsymbol{\beta}}_{21})^{-1} \mathbf{D}(q^{-1}) \mathbf{f}(n) \quad (6.22)$$

or

$$\hat{\mathbf{h}}(n) = \mathbf{D}^{-1}(q^{-1}) \begin{bmatrix} \bar{\boldsymbol{\beta}}_{31} & \bar{\boldsymbol{\beta}}_{32} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{\beta}}_{11} & \bar{\boldsymbol{\beta}}_{12} \\ \bar{\boldsymbol{\beta}}_{21} & \bar{\boldsymbol{\beta}}_{22} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{D}(q^{-1}) \mathbf{f}(n) \\ 0 \end{bmatrix} \quad (6.23)$$

where $\{\bar{\boldsymbol{\beta}}_{ij}(q^{-1})\}$ and $\mathbf{f}(n)$ are defined in (6.27) and (6.13), respectively.

Proof. Since all H_∞ estimators with the H_∞ performance γ must guarantee $\tilde{J}_{\mathcal{N}} > 0$, it holds that $\|\bar{\mathbf{w}}_{\mathbf{f}}(n)\|_2^2 > \|\bar{\mathbf{w}}_{\mathbf{h}}(n)\|_2^2 + \|\bar{\mathbf{w}}_{\hat{\mathbf{h}}}(n)\|_2^2$ for all $0 \leq n \leq \mathcal{N}$. Considering that $\bar{\mathbf{w}}_{\mathbf{f}}(n)$, $\bar{\mathbf{w}}_{\mathbf{h}}(n)$ and $\bar{\mathbf{w}}_{\hat{\mathbf{h}}}(n)$ are causal functions, we can introduce two linear

6.3 Robust Polynomial H_∞ Channel Estimator Design for Systems with Stochastic Uncertainty

109

contractive transfer function matrix $\theta_1(q^{-1})$ and $\theta_2(q^{-1})$ satisfying $\|\theta_2(q^{-1})\|_2^2 + \|\theta_1(q^{-1})\|_2^2 < 1$ such that

$$\bar{\mathbf{w}}_{\mathbf{h}}(n) = \theta_1(q^{-1})\bar{\mathbf{w}}_{\mathbf{f}}(n), \quad (6.24)$$

$$\bar{\mathbf{w}}_{\hat{\mathbf{h}}}(n) = \theta_2(q^{-1})\bar{\mathbf{w}}_{\mathbf{f}}(n). \quad (6.25)$$

We substitute (6.16), (6.24) and (6.25) to (6.20) and obtain

$$\begin{bmatrix} \bar{\mathbf{w}}_{\mathbf{f}}(n) \\ \theta_1(q^{-1})\bar{\mathbf{w}}_{\mathbf{f}}(n) \\ \theta_2(q^{-1})\bar{\mathbf{w}}_{\mathbf{f}}(n) \end{bmatrix} = \boldsymbol{\Omega}^{-1}\boldsymbol{\beta}^{-1}(q^{-1}) \begin{bmatrix} \mathbf{D}(q^{-1})\mathbf{f}(n) \\ 0 \\ \mathbf{D}(q^{-1})\hat{\mathbf{h}}(n) \end{bmatrix}$$

$$\begin{bmatrix} \bar{\boldsymbol{\beta}}_{11}(q^{-1}) & \bar{\boldsymbol{\beta}}_{12}(q^{-1}) & \bar{\boldsymbol{\beta}}_{13}(q^{-1}) \\ \bar{\boldsymbol{\beta}}_{21}(q^{-1}) & \bar{\boldsymbol{\beta}}_{22}(q^{-1}) & \bar{\boldsymbol{\beta}}_{23}(q^{-1}) \\ \bar{\boldsymbol{\beta}}_{31}(q^{-1}) & \bar{\boldsymbol{\beta}}_{32}(q^{-1}) & \bar{\boldsymbol{\beta}}_{33}(q^{-1}) \end{bmatrix} \begin{bmatrix} \bar{\mathbf{w}}_{\mathbf{f}}(n) \\ \theta_1(q^{-1})\bar{\mathbf{w}}_{\mathbf{f}}(n) \\ \theta_2(q^{-1})\bar{\mathbf{w}}_{\mathbf{f}}(n) \end{bmatrix} = \begin{bmatrix} \mathbf{D}(q^{-1})\mathbf{f}(n) \\ 0 \\ \mathbf{D}(q^{-1})\hat{\mathbf{h}}(n) \end{bmatrix} \quad (6.26)$$

where we define

$$\boldsymbol{\beta}(q^{-1})\boldsymbol{\Omega} = \begin{bmatrix} \bar{\boldsymbol{\beta}}_{11}(q^{-1}) & \bar{\boldsymbol{\beta}}_{12}(q^{-1}) & \bar{\boldsymbol{\beta}}_{13}(q^{-1}) \\ \bar{\boldsymbol{\beta}}_{21}(q^{-1}) & \bar{\boldsymbol{\beta}}_{22}(q^{-1}) & \bar{\boldsymbol{\beta}}_{23}(q^{-1}) \\ \bar{\boldsymbol{\beta}}_{31}(q^{-1}) & \bar{\boldsymbol{\beta}}_{32}(q^{-1}) & \bar{\boldsymbol{\beta}}_{33}(q^{-1}) \end{bmatrix} \quad (6.27)$$

and $\{\bar{\boldsymbol{\beta}}_{ij}(q^{-1})\} \in \mathcal{C}^{K(L+1) \times K(L+1)}$. Cancelling the items $\bar{\mathbf{w}}_{\mathbf{f}}(n)$ and $\theta_1(q^{-1})$ in (6.26), yields the final expression of all robust H_∞ linear estimators as

$$\hat{\mathbf{h}}(n) = \mathbf{D}^{-1}(q^{-1})\bar{\boldsymbol{\beta}}_2(q^{-1})\bar{\boldsymbol{\beta}}_1^{-1}(q^{-1})\mathbf{D}(q^{-1})\mathbf{f}(n)$$

where

$$\bar{\beta}_1(q^{-1}) = \bar{\beta}_{11} - \bar{\beta}_{12}\bar{\beta}_{22}^{-1}(\bar{\beta}_{21} + \bar{\beta}_{23}\theta_2) + \bar{\beta}_{13}\theta_2, \quad (6.28)$$

$$\bar{\beta}_2(q^{-1}) = \bar{\beta}_{31} - \bar{\beta}_{32}\bar{\beta}_{22}^{-1}(\bar{\beta}_{21} + \bar{\beta}_{23}\theta_2) + \bar{\beta}_{33}\theta_2. \quad (6.29)$$

With $\theta_2(q^{-1}) = 0$, the robust polynomial H_∞ linear estimators (6.21) is reduced to the central solution shown in (6.22). Another expression (6.23) readily follows from (6.26) by cancelling $\bar{\mathbf{w}}_{\mathbf{f}}(n)$ and $\theta_1(q^{-1})$ and setting $\theta_2(q^{-1}) = 0$. $\square$

6.4 Extension to General Stochastic H_∞ Estimation Problem

In the preceding section, we have derived the robust polynomial H_∞ channel estimators for DS-CDMA systems. A similar scheme can be applied to solve a general stochastic H_∞ estimation problem. We consider the following class of discrete-time uncertain systems:

$$x_{n+1} = (D + \Delta D_n)x_n + e_n, \quad (6.30)$$

$$y_n = (H + \Delta H_n)x_n + v_n, \quad (6.31)$$

$$z_n = Lx_n, \quad (6.32)$$

where

$x_n \in \mathcal{C}^{N_x}$ state vector

$y_n \in \mathcal{C}^{N_y}$ measured output

$z_n \in \mathcal{C}^{N_z}$ the state combination to be estimated

e_n and v_n disturbance signals

ΔD_n and ΔH_n stochastic uncertainty matrices with zero means and satisfying the following statistics

$$E\{\Delta D_i^* \Delta D_j\} = Q_{\Delta D} \delta_{ij}, \quad E\{\Delta H_i^* \Delta H_j\} = Q_{\Delta H} \delta_{ij}.$$

The matrices D, H , and L are all real and appropriately dimensioned. Without loss of generality, it is convenient to assume the initial condition $x_0 = 0$.

Therefore, a stochastic robust H_∞ suboptimal estimation strategy can be stated as, for a given scalar $\gamma > 0$, finding an estimate of $z_n, \hat{z}_n$, to satisfy

$$\begin{aligned} J_N &= \sum_{n=1}^N \|e_n\|_2^2 + \sum_{n=1}^N \|v_n\|_2^2 - \gamma^{-2} E_{\{\Delta D, \Delta H\}} \left\{ \sum_{n=1}^N \|\hat{z}_n - z_n\|_2^2 \right\} \\ &> 0. \end{aligned} \quad (6.33)$$

Similar to Section 6.3.1, we firstly reformulate (6.30) and (6.31) in the following form

$$x_{n+1} = Dx_n + e_x(n), \quad (6.34)$$

$$y_n = Hx_n + v_x(n) \quad (6.35)$$

where

$$e_x(n) = \Delta D_n x_n + e_n,$$

$$v_x(n) = \Delta H_n x_n + v_n.$$

Since ΔD_n and ΔH_n are independent zero-mean white noise processes, the

variances of e_x , v_x for uncertainties $\{\Delta D_n\}$, $\{\Delta H_n\}$ satisfy

$$\begin{aligned} E_{\{\Delta D_n\}}\{e_x^*(n)e_x(n)\} &= e_n^*e_n + E_{\{\Delta D_{n-1}\}}\{x_n^*E_{\Delta D_n}\{\Delta D_n^*\Delta D_n\}x_n\} \\ &= e_n^*e_n + E_{\{\Delta D_{n-1}\}}\{x_n^*Q_{\Delta D}x_n\}, \end{aligned} \quad (6.36)$$

$$\begin{aligned} E_{\{\Delta D_n, \Delta H_n\}}\{v_x^*(n)v_x(n)\} &= v_n^*v_n + E_{\{\Delta D_n, \Delta H_{n-1}\}}\{x_n^*E_{\Delta H_n}\{\Delta H_n^*\Delta H_n\}x_n\} \\ &= v_n^*v_n + E_{\{\Delta D_n, \Delta H_{n-1}\}}\{x_n^*Q_{\Delta H}x_n\}. \end{aligned} \quad (6.37)$$

Substituting (6.36) and (6.37) into (6.33), we rewrite the cost function as

$$\begin{aligned} J_{\mathcal{N}} &= \sum_{n=1}^{\mathcal{N}} E_{\{\Delta D_n\}}\{e_x^*(n)e_x(n)\} - \sum_{n=1}^{\mathcal{N}} E_{\{\Delta D_{n-1}\}}\{x_n^*Q_{\Delta D}x_n\} \\ &\quad + \sum_{n=1}^{\mathcal{N}} E_{\{\Delta D_n, \Delta H_n\}}\{v_x^*(n)v_x(n)\} - \sum_{n=1}^{\mathcal{N}} E_{\{\Delta D_n, \Delta H_{n-1}\}}\{x_n^*Q_{\Delta H}x_n\} \\ &\quad - \gamma^{-2} E_{\{\Delta D_n, \Delta H_n\}} \left\{ \sum_{n=1}^{\mathcal{N}} \|\hat{z}_n - z_n\|_2^2 \right\} \\ &= E_{\{\Delta D_n, \Delta H_n\}} \left\{ \sum_{n=1}^{\mathcal{N}} e_x^*(n)e_x(n) + \sum_{n=1}^{\mathcal{N}} v_x^*(n)v_x(n) - \sum_{n=1}^{\mathcal{N}} x_n^*(Q_{\Delta D} + Q_{\Delta H})x_n \right. \\ &\quad \left. - \gamma^{-2} \sum_{n=1}^{\mathcal{N}} \|\hat{z}_n - z_n\|_2^2 \right\}. \end{aligned}$$

We define

$$\begin{aligned} \tilde{J}_{\mathcal{N}} &= \sum_{n=1}^{\mathcal{N}} e_x^*(n)e_x(n) + \sum_{n=1}^{\mathcal{N}} v_x^*(n)v_x(n) - \sum_{n=1}^{\mathcal{N}} x_n^*(Q_{\Delta D} + Q_{\Delta H})x_n \\ &\quad - \gamma^{-2} \sum_{n=1}^{\mathcal{N}} \|\hat{z}_n - z_n\|_2^2. \end{aligned}$$

If $\tilde{J}_{\mathcal{N}} > 0$ for all $\{e_x(n)\}, \{v_x(n)\} \in l_2$, $J_{\mathcal{N}} > 0$ holds for all $\{e_n\}, \{v_n\} \in l_2$. That is, the former is only a sufficient solution for the latter. Therefore, the design objective of the stochastic robust H_∞ estimation problem is transformed to that

of seeking $\hat{z}_n$ to satisfy the following indefinite quadratic form with respect to the system (6.34) and (6.35):

$$\begin{aligned}
 & \tilde{J}_{\mathcal{N}} \\
 = & \sum_{n=1}^{\mathcal{N}} e_x^*(n) e_x(n) \\
 & + \sum_{n=1}^{\mathcal{N}} \left(\begin{bmatrix} y_n \\ 0 \\ \hat{z} \end{bmatrix} - \begin{bmatrix} H \\ I \\ L \end{bmatrix} x_n \right)^* \begin{bmatrix} I & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -\gamma^{-2}I \end{bmatrix} \left(\begin{bmatrix} y_n \\ 0 \\ \hat{z} \end{bmatrix} - \begin{bmatrix} H \\ I \\ L \end{bmatrix} x_n \right) \\
 > & 0 \tag{6.38}
 \end{aligned}$$

where

$$Q \triangleq Q_{\Delta D} + Q_{\Delta H} > 0.$$

From (6.38), we introduce the following Krein space system:

$$\mathbf{x}_{n+1} = D\mathbf{x}_n + \mathbf{e}_x(n), \tag{6.39}$$

$$\begin{bmatrix} \mathbf{y}(n) \\ 0 \\ \hat{z}(n) \end{bmatrix} = \begin{bmatrix} H \\ I \\ L \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} \mathbf{v}_x(n) \\ \mathbf{v}_2(n) \\ \tilde{z}_n \end{bmatrix} \tag{6.40}$$

with

$$\left\langle \begin{bmatrix} \mathbf{e}_x(n) \\ \mathbf{v}_x(n) \\ \mathbf{v}_2(n) \\ \tilde{z}_n \end{bmatrix}, \begin{bmatrix} \mathbf{e}_x(n) \\ \mathbf{v}_x(n) \\ \mathbf{v}_2(n) \\ \tilde{z}_n \end{bmatrix} \right\rangle = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -Q^{-1} & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}. \tag{6.41}$$

Note that there exist no parameter uncertainties in the auxiliary system (6.39)

and (6.40). Hence, the stochastic robust H_∞ estimation problem in Hilbert space is reduced to the design of a standard H_2 estimator for a Krein-space system.

For a time-invariant stochastic system, we can represent the state equation (6.39) in the polynomial form

$$\mathbf{D}(q^{-1})\mathbf{x}(n) = q^{-1}\mathbf{e}_x(n) \quad (6.42)$$

where

$$\mathbf{D}(q^{-1}) = I - q^{-1}D.$$

Substituting (6.42) into (6.40), we obtain

$$\begin{bmatrix} \mathbf{y}(n) \\ 0 \\ \hat{\mathbf{z}}(n) \end{bmatrix} = \begin{bmatrix} H\mathbf{D}(q^{-1})^{-1} \\ \mathbf{D}(q^{-1})^{-1} \\ L\mathbf{D}(q^{-1})^{-1} \end{bmatrix} q^{-1}\mathbf{e}_x(n) + \begin{bmatrix} \mathbf{v}_x(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{z}}_n \end{bmatrix} \quad (6.43)$$

To establish the same innovation model as (6.16), we need to calculate a J -spectral factor similar to the polynomial matrix $\beta(q^{-1})$ in (6.16). However, due to the existence of $\mathbf{D}(q^{-1})^{-1}$ on the right side of (6.43), it is difficult to obtain this J -spectral factor. Hence, in what follows we will introduce three schemes to solve this problem according to different cases:

Case 1: A general scheme is to seek the left polynomial fractions for the polynomial matrices $H\mathbf{D}(q^{-1})^{-1}$ and $L\mathbf{D}(q^{-1})^{-1}$, i.e.:

$$P(q^{-1}) \triangleq H\mathbf{D}(q^{-1})^{-1} = \tilde{\mathbf{D}}_1(q^{-1})^{-1}\tilde{H}, \quad (6.44)$$

$$G(q^{-1}) \triangleq L\mathbf{D}(q^{-1})^{-1} = \tilde{\mathbf{D}}_2(q^{-1})^{-1}\tilde{L} \quad (6.45)$$

where $\tilde{H}$ is a $N_y \times N_x$ matrix, $\tilde{L}$ is a $N_z \times N_x$ matrix, $\tilde{\mathbf{D}}_1(q^{-1})$ is a $N_y \times N_y$

polynomial matrix, and $\tilde{\mathbf{D}}_2(q^{-1})$ is a $N_z \times N_z$ polynomial matrix. In (6.44), $\mathbf{H}\mathbf{D}(q^{-1})^{-1}$ can be viewed as a right *matrix fraction description* (MFD) of a polynomial matrix $\mathbf{P}(q^{-1})$. Therefore the task of seeking matrices $\tilde{\mathbf{D}}_1(q^{-1})$ and $\tilde{\mathbf{H}}$ is equivalent to the calculation of left MFD of $\mathbf{P}(q^{-1})$. Same statements are also given for $\mathbf{G}(q^{-1})$ in (6.45). Refer to [65] for the detailed calculation of MFD.

Substitute (6.44) and (6.45) into (6.43) and premultiply $\mathbf{y}(n)$ and $\mathbf{z}(n)$ by $\tilde{\mathbf{D}}_1(q^{-1})$ and $\tilde{\mathbf{D}}_2(q^{-1})$, respectively, such that

$$\begin{bmatrix} \tilde{\mathbf{D}}_1(q^{-1})\mathbf{y}(n) \\ 0 \\ \tilde{\mathbf{D}}_2(q^{-1})\tilde{\mathbf{z}}(n) \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{H}} & \tilde{\mathbf{D}}_1(q^{-1}) & 0 & 0 \\ \mathbf{I} & 0 & \mathbf{D}(q^{-1}) & 0 \\ \tilde{\mathbf{L}} & 0 & 0 & \tilde{\mathbf{D}}_2(q^{-1}) \end{bmatrix} \begin{bmatrix} q^{-1}\mathbf{e}_x(n) \\ \mathbf{v}_x(n) \\ \mathbf{v}_2(n) \\ \tilde{\mathbf{z}}_n \end{bmatrix}. \quad (6.46)$$

Since (6.46) is similar to (6.15), we can readily apply the proposed approach and Theorem 6.3.2 in Section 6.3.2 to obtain a solution to the H_∞ estimation associated with the performance index (6.38).

Case 2: The first scheme requires the calculation of the left MFDs of two polynomial matrices. When the dealt polynomial matrix has high dimension, this operation is of high computational complexity. However, if the pseudo inverses of $\mathbf{H}$ and $\mathbf{L}$ exist, a straightforward method is to perform the following transformation

$$\begin{aligned} \mathbf{H}^+\mathbf{y}(n) &= \mathbf{x}_n + \mathbf{H}^+\mathbf{v}_x(n), \\ \mathbf{L}^+\mathbf{z}(n) &= \mathbf{x}_n + \mathbf{L}^+\tilde{\mathbf{z}}_n \end{aligned}$$

where

$$\begin{aligned} H^+ &= (H^*H)^{-1}H^*, \\ H^- &= (L^*L)^{-1}L^*. \end{aligned}$$

Then the corresponding polynomial system model is as follows

$$\begin{aligned} D(q^{-1})\mathbf{x}(n) &= q^{-1}\mathbf{e}_x(n) \\ \begin{bmatrix} D(q^{-1})H^+\mathbf{y}(n) \\ 0 \\ D(q^{-1})L^+\hat{\mathbf{z}}(n) \end{bmatrix} &= \begin{bmatrix} \mathbf{I} & D(q^{-1}) & 0 & 0 \\ \mathbf{I} & 0 & D(q^{-1}) & 0 \\ \mathbf{I} & 0 & 0 & D(q^{-1}) \end{bmatrix} \begin{bmatrix} q^{-1}\mathbf{e}_x(n) \\ H^+\mathbf{v}_x(n) \\ \mathbf{v}_2(n) \\ L^+\tilde{\mathbf{z}}_n \end{bmatrix}. \end{aligned} \quad (6.47)$$

The same algorithm as in Section 6.3.2 can be employed in the rest of derivation process. It should be noted that the DS-CDMA system (6.1)-(6.2) investigated before is indeed an application of this case where $L = \mathbf{I}$.

Case 3: For the above scheme, the assumption of the existence of the pseudo-inverse of L may be too strong in general. In fact, L is usually a flat matrix where $N_x > N_z$, therefore L^*L is usually not invertible. For this problem, we can formulate the equations corresponding to $-Q^{-1}$ and $-\gamma^2 I$ in (6.41) such that

$$\begin{bmatrix} 0 \\ \hat{\mathbf{z}}(n) \end{bmatrix} = \begin{bmatrix} I \\ L \end{bmatrix} \mathbf{x}_n + \begin{bmatrix} \mathbf{v}_2(n) \\ \tilde{\mathbf{z}}_n \end{bmatrix}$$

$$\begin{aligned} \Phi^{-1} \begin{bmatrix} I & L^* \end{bmatrix} \begin{bmatrix} 0 \\ \hat{\mathbf{z}}(n) \end{bmatrix} &= \mathbf{x}_n + \Phi^{-1} \begin{bmatrix} I & L^* \end{bmatrix} \begin{bmatrix} \mathbf{v}_2(n) \\ \tilde{\mathbf{z}}_n \end{bmatrix} \\ \Phi^{-1} L^* \hat{\mathbf{z}}(n) &= \mathbf{x}_n + \Phi^{-1} (\mathbf{v}_2(n) + L^* \tilde{\mathbf{z}}_n) \end{aligned} \quad (6.48)$$

where $\Phi = (I + L^*L)$ is invertible.

The corresponding polynomial system model is then as follows

$$\begin{bmatrix} \mathbf{D}(q^{-1})H^+\mathbf{y}(n) \\ \mathbf{D}(q^{-1})\Phi^{-1}L^*\hat{\mathbf{z}}(n) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{D}(q^{-1}) & 0 \\ \mathbf{I} & 0 & \mathbf{D}(q^{-1}) \end{bmatrix} \begin{bmatrix} q^{-1}\mathbf{e}_x(n) \\ H^+\mathbf{v}_x(n) \\ \Phi^{-1}(\mathbf{v}_2(n) + L^*\tilde{\mathbf{z}}_n) \end{bmatrix}$$

According to (6.41) and (6.49), we have the following expression

$$\begin{bmatrix} \mathbf{D}(q^{-1})H^+\mathbf{y}(n) \\ \mathbf{D}(q^{-1})\Phi^{-1}L^*\hat{\mathbf{z}}(n) \end{bmatrix} = \boldsymbol{\beta}(q^{-1})\mathbf{w}(n) \quad (6.49)$$

where the polynomial square matrix $\boldsymbol{\beta}(q^{-1})$ is the causal and causally invertible spectral factor obtained from

$$\begin{aligned} \langle \boldsymbol{\beta}(q^{-1})\mathbf{w}(n), \boldsymbol{\beta}(q^{-1})\mathbf{w}(n) \rangle &= \boldsymbol{\beta}(q^{-1})\mathbf{R}_w\boldsymbol{\beta}^*(q^{-1}) \\ &= \begin{bmatrix} \beta_{11}(q^{-1}) & \beta_{12}(q^{-1}) \\ \beta_{21}(q^{-1}) & \beta_{22}(q^{-1}) \end{bmatrix} \end{aligned}$$

where $\mathbf{R}_w$ is the covariance of the innovation vector $\mathbf{w}(n)$ and

$$\begin{aligned} \beta_{11}(q^{-1}) &= \mathbf{I} + \mathbf{D}(q^{-1})(H^*H)^{-1}\mathbf{D}^*(q^{-1}), \\ \beta_{22}(q^{-1}) &= \mathbf{I} - \mathbf{D}(q^{-1})\Phi^{-1}(Q^{-1} + \gamma^2 L^*L)^{-1}\Phi^{-1}\mathbf{D}^*(q^{-1}), \\ \beta_{12}(q^{-1}) &= \beta_{21}(q^{-1}) = \mathbf{I}. \end{aligned}$$

Clearly, (6.49) is similar to the model (5.23) in Chapter 5. Hence, we can apply the corresponding approach in Section 5.4.2 to obtain the robust polynomial H_∞ estimator. Since the dimension of the vector involved in (6.49) is lower than them in (6.46) and (6.47), the corresponding $\beta(q^{-1})$ in (6.49) requires the spectral factorization of a lower dimension polynomial matrix. It leads to that the computational complexity is reduced greatly. However, the process in (6.48) compresses the measurement data in nature which may result in that such robust H_∞ estimator has worse estimation performance than those of solutions obtained in the first and second cases.

6.5 Simulation Results

In this section, simulations will be given to illustrate the performance of the proposed robust H_∞ channel estimation algorithm against stochastic parametric uncertainties in the fading channel model. No priori knowledge of the statistics of disturbance noises is assumed. The performance is evaluated under frequency-selective Rayleigh channels in terms of the mean square error (MSE) criterion. For comparison, standard Kalman estimator, RLS estimator and zero forcing estimator are also included. We consider a DS-CDMA system with $K = 6$ users. The carrier frequency is $f_c = 900$ MHz, and symbol duration is $T_s = 100\mu s$. Golden codes with length $N = 31$ are used as spreading codes. Each user is assumed to undergo two Rayleigh channels with equal power. The mobile speeds of six users are 60 km/hr and hence their Doppler frequencies are $f_d = 50$ Hz. D in the channel model (6.1) is corrupted by the additive noise matrix $\Delta \mathbf{D}$ with the variance of $0.001I$. The comparison of MSE performance will be carried out in three kinds of observation noises.

6.5 Simulation Results

119

Figure 6.1: Mean Square Error with respect to different SNR under a white Gaussian noise of variance $\sigma^2 = 0.01$. Real Doppler frequency of all users is 50 Hz but the fading channel model is corrupted by the white noise process with the variance of 0.001.

In Figure 6.1, the simulation is carried out based on the additive white Gaussian observation noise with the variance $\sigma^2 = 0.01$. Figure 6.1 shows that MSE performance of the proposed robust H_∞ algorithm is better than that of RLS and Zero-forcing estimator, especially at low SNR. However, all of them come closer with the increase of SNR. We also notice that the robust H_∞ estimator exhibits better performance than the standard Kalman filter when SNR is higher than 20 dB. It is because the proposed algorithm takes into account parametric uncertainties in the fading channel model where the standard Kalman filter does not.

In Figure 6.2, we check the MSE performance of every estimator under an additive colored Gaussian noise, which is generated by a transfer function $F(z) = (1 - 0.9z)^{-1}$ and a Gaussian white noise input. Compared with the the case of additive white Gaussian observation noise in Figure 6.1, all estimators perform

Figure 6.2: The MSE with respect to different SNR under a colored noise generated by filter $H(z) = (1 - 0.9z)^{-1}$.

poorer in the case of additive colored Gaussian noise. The proposed robust H_∞ estimator still shows a superior performance. Moreover, it can achieve the same performance as the Kalman-based estimator at 17 dB of SNR in contrast to 20 dB in Figure 6.1. The reason of performance gain is that color observation noise environment deteriorates the estimation performance of the Kalman filter whereas the H_∞ algorithm does not assume the statistics of observation noises. In comparison with standard Kalman filter with higher computational cost, the proposed channel estimator not only operates in an efficient manner but also exhibits the robust estimation performance.

Finally, we check the MSE performance of the proposed estimator under the non-Gaussian channels. We adopt a two-term Gaussian mixture model introduced

Figure 6.3: The MSE under a two-term Gaussian Mixture noise with (pdf) $f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \alpha^2\sigma_n^2)$, where $\epsilon = 0.1$ and $\alpha^2 = 100$.

in Chapter 5, the probability density function (pdf) of which is given by

$$f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \alpha^2\sigma_n^2)$$

where $\mathcal{N}(0, \sigma_n^2)$ represents Gaussian noise and $\mathcal{N}(0, \alpha^2\sigma_n^2)$ represents the impulsive component with ϵ representing the probability for which impulses occur. If we let $\epsilon = 0.1$ and $\alpha^2 = 100$ as in Chapter 5, the results in Figure 6.3 show that the MSE performances of the estimators suffering from non-Gaussian noise are similar to those under AWGN shown in Figure 6.1 except some little performance degradation.

6.6 Conclusion

In this chapter, a polynomial robust H_∞ channel estimator has been derived for a real DS-CDMA system where the fading channel is modeled as linear systems with stochastic uncertainties due to modelling mismatch. An extension to robust H_∞ estimation problem for general stochastic uncertain systems has been made. Compared with the approach in [29], the proposed polynomial estimator is computationally much more efficient due to the involvement of a H_∞ Riccati recursion in [29]. Simulation results have demonstrated good robust performance of the proposed channel estimator in counteracting various kinds of noises and parametric uncertainties of the fading channels.

Chapter 7

Robust Polynomial MMSE Channel Estimation of DS-CDMA Systems with Deterministic Parametric Uncertainty

7.1 Introduction

In this chapter we return to the issue of robust MMSE channel estimation of DS-CDMA systems introduced in Chapter 4, in which the system model is supposed to be subject to stochastic parametric uncertainties. Here, our study will involve systems with deterministic parametric uncertainties and the corresponding robust polynomial MMSE channel estimator will be developed with the aid of Krein space approach.

Rather than employing the statistic of stochastic uncertainties in the robust estimator design as in Chapter 4, we generally describe deterministic uncertainties by norm-bounded blocks and then seek a filter that minimizes an upper bound on the estimation error variance over all possible values of parametric uncertainties. This type of problems is also termed as robust Kalman filtering problem, and

has motivated extensive studies over the past two decades. Early work on robust Kalman filter design for linear systems with norm-bounded uncertainties can be found in [88], where the stationary solution is obtained by applying a pair of Riccati difference equations. Further development of robust infinite horizon filtering appears in [62] [74] [101]. The corresponding finite horizon version is proposed in [26] by employing a semidefinite programming approach. Another class of robust estimation studies focuses on systems with polytopic uncertainty, where the uncertain state space matrices belong to a polytope. A stationary filter for the polytopic uncertain systems is given in [27] via an LMI approach. The treatment of [27] is improved in [69] where the conservatism of the filter is reduced by relaxing the quadratic stability requirements. Also the continuous time counterpart of [69] is given in [80] by employing a parameter-dependent Lyapunov function.

In this chapter we consider a robust polynomial Kalman filter design for a DS-CDMA system with deterministic parametric uncertainties in the channel state matrix. Note that a polynomial robust filtering has not been addressed in existing literature. The key idea is to combine parametric uncertainties with the driving disturbance and then seek all possible estimates of channels to satisfy *sum quadratic constraint* (SQC) for such uncertain discrete-time system. This approach has been employed to solve the finite horizon robust Kalman filtering problem via a state space approach in [48]. Following this application of SQC in robust Kalman filtering, we develop a robust polynomial MMSE channel estimator by seeking the optimal state estimate of a Krein space system.

7.2 Problem Statement

We consider a K -user DS-CDMA system model with N -length spreading code below

$$\mathbf{h}(n+1) = (\mathbf{D} + \Delta\mathbf{D})\mathbf{h}(n) + \mathbf{e}(n) \quad (7.1)$$

$$\mathbf{r}(n) = \mathbf{C}_0\mathbf{B}(n)\mathbf{h}(n) + \mathbf{C}_1\mathbf{B}(n-1)\mathbf{h}(n-1) + \mathbf{v}(n) \quad (7.2)$$

$\mathbf{h}(n) \in \mathcal{C}^{K(L+1)}$ is the channel coefficient, $\mathbf{r}(n) \in \mathcal{C}^N$ denotes the observation, $\mathbf{D} \in \mathcal{R}^{K(L+1) \times K(L+1)}$, $\mathbf{C}_0, \mathbf{C}_1 \in \mathcal{R}^{N \times K(L+1)}$ are defined in Chapter 2, $\mathbf{e}(n) \in \mathcal{C}^{K(L+1)}$, $\mathbf{v}(n) \in \mathcal{C}^N$ denote the driving disturbance and the observation noise, respectively. $\Delta\mathbf{D}$ is an unknown diagonal matrix which describes time-varying parametric uncertainty and is assumed to be of the form

$$\Delta\mathbf{D} = G\Delta_n E$$

where G and E are known constant matrices of appropriate dimension and $\Delta_n \in \mathcal{R}^{K(L+1) \times K(L+1)}$ is an unknown matrix satisfying

$$\Delta_n^T \Delta_n \leq I, \quad n = 1, 2, \dots \quad (7.3)$$

Moreover, we assume that the covariance matrices associated with $\mathbf{e}(n)$ and $\mathbf{v}(n)$ are known as follows

$$E\{\mathbf{e}(n)\mathbf{e}(m)^*\} = \begin{cases} R_e, & n = m \\ 0, & n \neq m \end{cases}, \quad E\{\mathbf{v}(n)\mathbf{v}(m)^*\} = \begin{cases} R_v, & n = m \\ 0, & n \neq m \end{cases}.$$

Our work is based on the assumption that given a constant α , the uncertain

7.2 Problem Statement

126

system (7.1)-(7.2) satisfies the following energy constraint:

$$\sum_{n=1}^{\mathcal{N}} \mathbf{e}(n)^* R_e^{-1} \mathbf{e}(n) + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* R_v^{-1} \mathbf{v}(n) \leq \alpha^2. \quad (7.4)$$

Letting $\mathbf{w}(n) = \triangle_n E \mathbf{h}(n)$, it follows from (7.3) that

$$\| \mathbf{w}(n) \| \leq \| E \mathbf{h}(n) \|.$$

Hence,

$$\sum_{n=1}^{\mathcal{N}} \| \mathbf{w}(n) \|^2 \leq \sum_{n=1}^{\mathcal{N}} \| E \mathbf{h}(n) \|^2. \quad (7.5)$$

Adding (7.5) to (7.4), we get

$$\sum_{n=1}^{\mathcal{N}} \mathbf{e}(n)^* R_e^{-1} \mathbf{e}(n) + \sum_{n=1}^{\mathcal{N}} \| \mathbf{w}(n) \|^2 + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* R_v^{-1} \mathbf{v}(n) \leq \alpha^2 + \sum_{n=1}^{\mathcal{N}} \| E \mathbf{h}(n) \|^2,$$

i.e.

$$\sum_{n=1}^{\mathcal{N}} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & R_e \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix} + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* R_v^{-1} \mathbf{v}(n) \leq \alpha^2 + \sum_{n=1}^{\mathcal{N}} \| E \mathbf{h}(n) \|^2.$$

To proceed with the application of SQC, we introduce the following auxiliary system:

$$\mathbf{h}(n+1) = \mathbf{D} \mathbf{h}(n) + \begin{bmatrix} G & I \end{bmatrix} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix} \quad (7.6)$$

$$\mathbf{r}(n) = \mathbf{C}_0 \mathbf{B}(n) \mathbf{h}(n) + \mathbf{C}_1 \mathbf{B}(n-1) \mathbf{h}(n-1) + \mathbf{v}(n) \quad (7.7)$$

$$\mathbf{z}(n) = E \mathbf{h}(n) \quad (7.8)$$

7.2 Problem Statement

127

where $[\mathbf{w}(n)^* \ \mathbf{e}(n)^*]^*$ and $\mathbf{v}(n)$ are viewed as the deterministic uncertainty inputs. For this auxiliary system, a SQC refers to that given a constant α , the positive definite symmetric matrices Q and R , the uncertain inputs $[\mathbf{w}(n)^* \ \mathbf{e}(n)^*]^*$ and $\mathbf{v}(n)$ satisfy the inequality:

$$\sum_{n=1}^{\mathcal{N}} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix}^* Q \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix} + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* R \mathbf{v}(n) \leq \alpha^2 + \sum_{n=1}^{\mathcal{N}} \|\mathbf{z}(n)\|^2. \quad (7.9)$$

If let $Q = \text{diag}\{I, R_e\}^{-1}$ and $R = R_v^{-1}$, we find that the SQC (7.9) of the auxiliary system implies the constraint (7.4) of the system (7.1)-(7.2). Hence, the objective of robust estimator design becomes that for the uncertain system (7.6)-(7.8), seek the corresponding state estimates of $\{\mathbf{h}(n)\}_1^{\mathcal{N}}$ that satisfy the constraint (7.9).

With the definition of Q and R , we rewrite (7.9) in the following indefinite form:

$$\begin{aligned} & J_{\mathcal{N}}(\mathbf{e}, \mathbf{v}, \mathbf{h}) \\ &= \sum_{n=1}^{\mathcal{N}} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & R_e \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix} + \sum_{n=1}^{\mathcal{N}} \mathbf{v}(n)^* R_v^{-1} \mathbf{v}(n) - \sum_{n=1}^{\mathcal{N}} \|E\mathbf{h}(n)\|^2 \\ &= \sum_{n=1}^{\mathcal{N}} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix}^* \begin{bmatrix} I & 0 \\ 0 & R_e \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix} + \left(\begin{bmatrix} \mathbf{r}(n) \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{C}_0 \mathbf{B}(n) \\ E \end{bmatrix} \mathbf{h}(n) \right. \\ &\quad \left. - \begin{bmatrix} \mathbf{C}_1 \mathbf{B}(n-1) \\ 0 \end{bmatrix} \mathbf{h}(n-1) \right)^* \begin{bmatrix} R_v & 0 \\ 0 & -I \end{bmatrix}^{-1} \\ &\quad \times \left(\begin{bmatrix} \mathbf{r}(n) \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{C}_0 \mathbf{B}(n) \\ E \end{bmatrix} \mathbf{h}(n) - \begin{bmatrix} \mathbf{C}_1 \mathbf{B}(n-1) \\ 0 \end{bmatrix} \mathbf{h}(n-1) \right). \quad (7.10) \end{aligned}$$

It should be noted that there exists a set of channel estimates satisfying SQC (7.9). For this estimation problem, the set of finite horizon state estimates has been obtained in the form of Kalman-like recursive update in [66]. Following this result, [48] develops a robust Kalman filter from the perspective of the SQC for parametric uncertainties. The main idea in [48] is to choose a point-valued from the set of all possible channel estimates satisfying SQC (7.9) so as to approach the minimum of the indefinite quadratic form (7.10). Such estimate is just the solution to robust Kalman filtering. Hence, the robust MMSE estimation problem is restated below:

Problem 7.2.1. [48] *Given $\{r(n)\}_1^N$ and uncertain system (7.6)-(7.8) subject to the SQC (7.9), find state estimate that corresponds to the minimum of the indefinite quadratic form (7.10).*

Rather than directly employing the finite horizon solution to the minimum of (7.10) via Krein space in [48], we derive the corresponding polynomial solution by applying variational approach for a Krein space system to get the optimal channel estimate.

7.3 Robust Polynomial Channel Estimator

With regards to (7.10) and robust MMSE estimation problem discussed in preceding section, we immediately introduce the following Krein space system:

$$\begin{aligned} \mathbf{h}(n+1) &= \mathbf{D}\mathbf{h}(n) + \begin{bmatrix} \mathbf{G} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \end{bmatrix}, \\ \begin{bmatrix} \mathbf{r}(n) \\ 0 \end{bmatrix} &= \begin{bmatrix} \mathbf{C}_0\mathbf{B}(n) \\ E \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{C}_1\mathbf{B}(n-1) \\ 0 \end{bmatrix} \mathbf{h}(n-1) + \begin{bmatrix} \mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix} \end{aligned} \quad (7.11)$$

7.3 Robust Polynomial Channel Estimator

129

(7.12)

where

$$\left\langle \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \\ \mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix}, \begin{bmatrix} \mathbf{w}(n) \\ \mathbf{e}(n) \\ \mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix} \right\rangle = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 \\ 0 & R_e & 0 & 0 \\ 0 & 0 & R_v & 0 \\ 0 & 0 & 0 & -\mathbf{I} \end{bmatrix}. \quad (7.13)$$

Similar to previous treatments of this time-varying system, we first convert (7.12) to a time-invariant form. To this end, we rewrite (7.12) as

$$\begin{bmatrix} \mathbf{r}(n) \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{C}_1 \mathbf{B}(n-1) \\ 0 \end{bmatrix} \mathbf{h}(n-1) = \begin{bmatrix} \mathbf{C}_0 \mathbf{B}(n) \\ E \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix},$$

which can be put into the form:

$$\begin{bmatrix} \mathbf{f}(n) \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ E \end{bmatrix} \mathbf{h}(n) + \begin{bmatrix} \mathbf{B}(n) \mathbf{C}^+ \mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix} \quad (7.14)$$

where

$$\begin{aligned} \mathbf{f}(n) &= \mathbf{B}(n) \mathbf{C}^+ (\mathbf{r}(n) - \mathbf{C}_1 \mathbf{B}(n-1) \mathbf{h}(n-1)), \\ \mathbf{C}^+ &= (\mathbf{C}_0^* \mathbf{C}_0)^{-1} \mathbf{C}_0^*. \end{aligned} \quad (7.15)$$

In terms of Krein space estimation theory introduced in Section 5.2, the channel estimate that achieves the minimum of the indefinite quadratic form (7.10) is the projection of $\{\mathbf{h}(n)\}$ onto the linear span $\{[\mathbf{f}(n)^* \ 0]^*\}$ in the Krein space system (7.11)-(7.12). Therefore, we can apply the approach in Chapter 4 to design a robust polynomial MMSE estimator, which performs the optimal MMSE estimation in the

7.3 Robust Polynomial Channel Estimator

130

Krein space system.

We represent the state equation (7.11) in the polynomial form:

$$\mathbf{D}(q^{-1})\mathbf{h}(n) = q^{-1}(G\mathbf{w}(n) + \mathbf{e}(n)) \quad (7.16)$$

where

$$\mathbf{D}(q^{-1}) = \mathbf{I} - \mathbf{D}q^{-1}.$$

Substituting (7.16) into (7.14), we obtain

$$\begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix} = \begin{bmatrix} D(q^{-1})^{-1} \\ ED(q^{-1})^{-1} \end{bmatrix} q^{-1}(G\mathbf{w}(n) + \mathbf{e}(n)) + \begin{bmatrix} \mathbf{B}(n)\mathbf{C}^+\mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix} \quad (7.17)$$

We seek the left polynomial fractions for $ED(q^{-1})^{-1}$, i.e:

$$ED(q^{-1})^{-1} = \tilde{D}(q^{-1})^{-1}\tilde{E} \quad (7.18)$$

Then substitute (7.18) into (7.17) such that

$$\begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & D(q^{-1})\mathbf{B}(n)\mathbf{C}^+ & 0 \\ \tilde{E} & 0 & \tilde{D}(q^{-1}) \end{bmatrix} \begin{bmatrix} q^{-1}(G\mathbf{w}(n) + \mathbf{e}(n)) \\ \mathbf{v}(n) \\ \mathbf{v}_h(n) \end{bmatrix} \quad (7.19)$$

7.3 Robust Polynomial Channel Estimator

131

According to (7.13) and (7.19), we have the following expression

$$\begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix} = \boldsymbol{\beta}(q^{-1})\mathbf{w}(n) \quad (7.20)$$

where the square polynomial matrix

$$\boldsymbol{\beta}(q^{-1}) = \mathbf{I} + \boldsymbol{\beta}_1 q^{-1}$$

is the causal and causally invertible spectral factor obtained from

$$\begin{aligned} & \langle \boldsymbol{\beta}(q^{-1})\mathbf{w}(n), \boldsymbol{\beta}(q^{-1})\mathbf{w}(n) \rangle \\ &= \boldsymbol{\beta}(q^{-1})\mathbf{R}_w\boldsymbol{\beta}^*(q^{-1}) \\ &\approx \begin{bmatrix} GG^* + R_e + \mathbf{D}(q^{-1})\mathbf{R}_c^{-1}\mathbf{D}^*(q^{-1}) & q(GG^* + R_e)\tilde{E}^* \\ q^{-1}\tilde{E}(GG^* + R_e) & \tilde{E}(GG^* + R_e)\tilde{E}^* - \mathbf{D}(q^{-1})\mathbf{D}^*(q^{-1}) \end{bmatrix} \end{aligned} \quad (7.21)$$

where the definition of $\mathbf{R}_c^{-1}$ is stated in (5.26).

Applying linear estimation theory in the Krein space system, we design a stable causal matrix $\mathcal{R}$ that provides a linear m-step predictor of $\mathbf{h}(n+m)$:

$$\hat{\mathbf{h}}(n+m|n) = D^{-1}(q^{-1})\mathcal{R} \begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix}. \quad (7.22)$$

We now give the following main result.

Theorem 7.3.1. *A robust MMSE channel estimate $\hat{\mathbf{h}}(n+m|n)$ minimizing (7.10)*

7.3 Robust Polynomial Channel Estimator

132

is given by

$$\hat{\mathbf{h}}(n+m|n) = \mathbf{L}_m \boldsymbol{\beta}^{-1} \begin{bmatrix} D(q^{-1}) \mathbf{f}(n) \\ 0 \end{bmatrix} \quad (7.23)$$

where $\boldsymbol{\beta}(q^{-1})$ is obtained from matrix spectral factorization (7.21) and $\mathbf{L}_m$ is the unique solution to the bilateral Diophantine equation

$$\mathbf{L}_m \mathbf{R}_w \boldsymbol{\beta}^* + z D \mathbf{Q}_m^* = z^m (G G^* + R_e) \begin{bmatrix} I_{K(L+1)} & I_{K(L+1)} \end{bmatrix} \quad (7.24)$$

where matrices $\mathbf{L}_m$ and $\mathbf{Q}_m$ are of dimension $K(L+1) \times 2K(L+1)$.

Proof. According to the variational approach in [2], all admissible variations of the estimate can be represented by

$$\boldsymbol{\xi} = \boldsymbol{\eta}(q^{-1}) \begin{bmatrix} D(q^{-1}) \mathbf{f}(n) \\ 0 \end{bmatrix} \quad (7.25)$$

where $\boldsymbol{\eta}(q^{-1})$ is an arbitrary, but stable and causal rational matrix. Then $\hat{\mathbf{h}}(n+m|n)$ is the optimal estimation based on the MMSE iff

$$E\{\tilde{\mathbf{h}}(n+m|n) \boldsymbol{\xi}^*\} = 0. \quad (7.26)$$

By using the Parseval's formula, the orthogonality relationship between $\tilde{\mathbf{h}}(n+m|n)$ and $\boldsymbol{\xi}^*$ can be converted into a frequency domain condition:

$$\begin{aligned} & E\{\tilde{\mathbf{h}}(n+m|n) \boldsymbol{\xi}^*\} \\ &= \frac{1}{2\pi i} \oint_{|z|=1} E \left\{ \left(z^m \mathbf{h} - D^{-1} \mathbf{R} \begin{bmatrix} D(q^{-1}) \mathbf{f}(n) \\ 0 \end{bmatrix} \right) \begin{bmatrix} D(q^{-1}) \mathbf{f}(n) \\ 0 \end{bmatrix}^* \boldsymbol{\xi}^* \right\} \frac{dz}{z} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2\pi i} \oint_{|z|=1} E \left\{ z^m D^{-1}(G\mathbf{w} + \mathbf{e}) \begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix}^* \right. \\
 &\quad \left. - D^{-1}\mathcal{R} \begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix} \begin{bmatrix} D(q^{-1})\mathbf{f}(n) \\ 0 \end{bmatrix}^* \right\} \boldsymbol{\xi}^* \frac{dz}{z} \\
 &= \frac{1}{2\pi i} \oint_{|z|=1} (z^m D^{-1}(GG^* + R_e) \begin{bmatrix} I_{K(L+1)} & I_{K(L+1)} \end{bmatrix} - D^{-1}\mathcal{R}\boldsymbol{\beta}\mathbf{R}_w\boldsymbol{\beta}^*) \boldsymbol{\xi}^* \frac{dz}{z} \\
 &= \frac{1}{2\pi i} \oint_{|z|=1} D^{-1} (z^m (GG^* + R_e) \begin{bmatrix} I_{K(L+1)} & I_{K(L+1)} \end{bmatrix} - \mathcal{R}\boldsymbol{\beta}\mathbf{R}_w\boldsymbol{\beta}^*) \boldsymbol{\xi}^* \frac{dz}{z} \quad (7.27)
 \end{aligned}$$

where the spectral factorization (7.21) is inserted in the last two equalities. Since $\boldsymbol{\xi}$ is causal and stable, thereby has no pole in $|z| < 1$. The orthogonality requirement of (7.26) holds iff there are no poles inside the integration path $|z| = 1$ in (7.27). Thus, we require that

$$z^m (GG^* + R_e) \begin{bmatrix} I_{K(L+1)} & I_{K(L+1)} \end{bmatrix} - \underbrace{\mathcal{R}\boldsymbol{\beta}}_{\mathbf{L}_m} \mathbf{R}_w \boldsymbol{\beta}^* = z D \mathbf{Q}_m^* \quad (7.28)$$

where $\mathbf{L}_m$ and $\mathbf{Q}_m$ are causal matrices to be solved. Hence, the filter $\mathcal{R}$ can be obtained by solving this Diophantine equation. $\square$

7.4 Simulation Results

In this section, we present the simulation results to validate the proposed robust polynomial MMSE channel estimator. The mean square error (MSE) is used as the performance criterion. We consider a DS-CDMA system with $K = 6$ users. The carrier frequency is $f_c = 900$ MHz, and symbol duration is $T_s = 100\mu s$. Golden codes with length $N = 31$ are used as spreading codes. Each user is assumed to undergo two Rayleigh channels with equal power.

Figure 7.1: The MSE with respect to different SNR for $\delta = 0$.

An example of an uncertain DS-CDMA system is presented as follows:

$$\mathbf{h}(n) = (\mathbf{I} + \delta \mathbf{I})\mathbf{h}(n-1) + \mathbf{e}(n) \quad (7.29)$$

$$\mathbf{r}(n) = \mathbf{C}_0 \mathbf{B}(n)\mathbf{h}(n) + \mathbf{C}_1 \mathbf{B}(n-1)\mathbf{h}(n-1) + \mathbf{v}(n) \quad (7.30)$$

where $\mathbf{e}(n)$ and $\mathbf{v}(n)$ are zero-mean white complex random vectors with the variance of 0.01, $\delta \mathbf{I}$ denotes the norm-bounded parametric uncertainties and the scalar δ satisfies $\|\delta\| \leq 0.1$. The mobile speeds of six users are all equal to 60 km/hr, or equivalently, the Doppler frequency $f_d = 50$ Hz which determines the parametric matrix of (7.29). Corresponding to the system description in (7.1), the uncertain term is given by:

$$\mathbf{G} = \mathbf{E} = \mathbf{I}.$$

Figure 7.1-Figure 7.3 correspond to the the parameter $\delta = 0$ ($\mathbf{D} = 0.998\mathbf{I}$), 0.06

7.4 Simulation Results

135

Figure 7.2: The MSE with respect to different SNR for $\delta = 0.06$. Figure 7.3: The MSE with respect to different SNR for $\delta = 0.1$.

($\mathbf{D} = 0.9398\mathbf{I}$), and 0.1 ($\mathbf{D} = 0.8198\mathbf{I}$), respectively. Each figure shows the MSE curves for the proposed robust MMSE, standard Kalman, RLS, and Zero-forcing algorithms. Based on these three figures, we make the following observations. First, Figure 7.1 shows the estimation performance of each algorithm when there are no parametric uncertainties in the system model, namely, the dynamic model of the practical system is entirely known. We see that the standard Kalman estimator exhibits the superior MSE performance over the RLS and ZF algorithms. On the contrary, the proposed robust estimator performs poorly when the SNR is not high. This result is not surprising because the proposed robust estimator is based on the conservative design due to the consideration of parametric uncertainties. Secondly, from Figure 7.2 and Figure 7.2 we observe that the MSE performance of the standard Kalman estimator deteriorates as the increase of the parameter δ , whereas the RLS and ZF algorithms are relatively insensitive to the variation in δ . It results in that the advantage of the standard Kalman estimator in Figure 7.1 is lost when $\delta = 0.1$, as demonstrated in Figure 7.3. Furthermore, the most noteworthy finding in Figure 7.2 and Figure 7.3 is that the proposed MMSE estimator performs significantly better than other estimators especially when the SNR is low. This result clearly shows that the proposed polynomial MMSE channel estimator is robust with respect to the parametric uncertainties.

7.5 Conclusion

In this chapter, we have presented a robust polynomial MMSE channel estimator design for systems subject to deterministic parametric uncertainties. The derivation is based on the description of deterministic uncertainties in [66] known as the SQC and its further application on robust Kalman filtering in [48]. In terms of

the equivalence relationship between the robust Kalman filtering problem and the optimization of the SQC for the corresponding uncertain system, the polynomial version of robust MMSE estimator is obtained by simply employing linear estimation in a Krein space system. The resulting estimator has a similar expression as the MMSE estimator derived in Chapter 4.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

In this thesis we have addressed several problems of polynomial channel estimation for DS-CDMA systems. The proposed channel estimators mainly operate in the multipath environment and provide the estimates of time-varying channel coefficients for multiuser symbol detection. For various physical environments, different filtering algorithms and optimization criteria have been adopted in the exploitation of polynomial channel estimators. In particular, the application of Krein space estimation theory provides a powerful technique for solving standard H_∞ , robust H_∞ and robust MMSE estimation problems via a polynomial approach. The polynomial solutions to these problems have been obtained elegantly from the perspective of Krein space. To summarize the main contributions of this thesis, we point out the following aspects:

We developed a WLMS channel estimator by directly applying the WLMS filter proposed in [53] to the state estimation of a multipath channel model. This algorithm exploits the computational advantage offered by a simple update process like standard LMS filter. Also a sample of the WLMS estimator based on the

second-order AR channel model was provided in detail. Furthermore, we showed the interaction between channel estimation and multiuser symbol detection and derived a robust DFE symbol detector that can perform in the tracking case.

We studied the problem of designing polynomial MMSE channel estimator which exhibits robust performance in the face of stochastic parametric uncertainties in both the state and observation equations. The statistics of these uncertainties are first obtained through the mathematical equivalence in the frequency domain and then are taken into account in the estimator design. The development of a polynomial MMSE estimator is based on the variational approach that guarantees optimality by means of evaluation of orthogonality in the frequency domain. The practical importance of the proposed polynomial estimator is due to its simplicity of implementation and robust performance in the real physical wireless environment.

We derived a polynomial version of H_∞ estimator for DS-CDMA systems. The algorithm is based on the exploitation of the mathematical equivalence between the H_∞ filtering problem and H_2 filtering problem in Krein space. Based on the establishment of an innovation model in Krein space, a complete solution in the polynomial form was obtained directly by projection in Krein space. This result extends existing H_∞ filtering approaches and performs an efficient filtering operation. It is also the basis for solving robust polynomial H_∞ estimation problems in Chapter 6 and robust polynomial MMSE estimation problems in Chapter 7. Another important contribution is that we derived a closed-form solution to the polynomial matrix J -spectral factorization required in the estimation algorithm, which leads to a great reduction of the computational complexity, especially for a high dimension J -spectral factorization. Finally, the simulation results also showed the robust performance of the proposed estimator in several practical wireless channel environments.

The polynomial H_∞ channel estimation problem for systems subject to stochastic uncertainties in the channel state matrix as in DS-CDMA systems was also analyzed. It has been shown in Chapter 6 that the stochastic H_∞ estimation problem can be converted into an indefinite quadratic form and a solution was obtained through a natural extension of the method of polynomial H_∞ estimation in Chapter 5. Furthermore, the same scheme was extended to solve general stochastic H_∞ estimation problems for uncertain systems with stochastic parametric uncertainties in the state and output matrices. Applications in DS-CDMA systems demonstrated that the proposed channel estimator exhibits robust estimation performance in the face of parametric uncertainties and various kinds of noises.

Benefiting from Krein space estimation theory, we developed a polynomial version of robust MMSE channel estimator. The systems of interest suffer from real time-varying norm-bounded parametric uncertainties in the channel state matrix. Our main idea originated from the description of these uncertainties, as Sum Quadratic Constraint. The solution to the robust polynomial MMSE estimation problem was obtained by directly employing optimal linear estimation approach in a Krein space system, which bears resemblance to the approach in Chapter 4. This result extends the state-space robust Kalman filtering approach to a polynomial approach which is of significant in both theoretical research and practical applications.

8.2 Future Work

We note that most of the existing robust estimation approaches is targeted at either stochastic uncertainties or deterministic uncertainties separately. In the same way, the robust polynomial estimation problems considered throughout this thesis are also based on the stochastic and deterministic descriptions for parametric uncertainties, respectively. The corresponding stochastic H_2 and H_∞ estimation problems have been investigated in Chapters 4 and 6 while for the latter, a robust polynomial MMSE estimator is derived in Chapter 7. However, for a practical wireless communication channel, parametric uncertainties in a system model generally exhibit both stochastic and deterministic properties. Hence, the study of robust estimation for systems with a mixed stochastic and deterministic uncertainties is of practical importance. This problem has been considered in [85] where two robust steady state-space filters based on minimum mean energy gain and minimum asymptotic mean-square error respectively, are developed by means of LMI-based convex optimization. It would be interesting to investigate the robust estimation problem via a polynomial approach.

It is noteworthy that the H_2 filter and the H_∞ filter are based on different optimization criteria and intended for different applications. A natural question arises, which is preferable? In theory, optimality in one case does not imply optimality in the other. This problem has motivated a great deal of interest in mixed H_2/H_∞ estimation [45] [72] [104] [15]. The corresponding optimization strategy is to maintain a good average performance in the H_2 sense while guarantee certain performance in the worst-case scenario. It is challenging to study the design of a polynomial version of mixed H_2/H_∞ estimator and its application in wireless channel estimation.

On the other hand, for a practical DS-CDMA system, the channel estimator

or equalizer plays an indirect role in the operation of symbol detection. The BER performance of whole multiuser detector including channel estimation is what we are concerned with and is also the important index for an estimator. Due to the non-linearity of hard detector, it is difficult to perform the theoretical analysis of the impact of channel estimator on the multiuser detection performance. The literature on this issue is not extensive, see [13] [102]. Our works are only limited in the demonstration of BER performance of the DFE detector based on simulations and lack further theoretical analysis. Hence, it would be interesting, though challenging, to analyze and compare different channel estimation approaches from a viewpoint of whole multiuser detection system. Furthermore, it opens up the possibility to reconsider channel estimation strategy directly based on the BER performance.

Finally, our treatment of channel estimation problems, especially robust channel estimation problems in DS-CDMA systems, based on polynomial approaches can be extended to the applications of other wireless communication systems such as MIMO or OFDM systems. With the establishment of the state-space model for a corresponding system, some results in this thesis can be employed readily. It will be a promising follow-up to our works.

Author's Publications

1. C. Cao, L. Xie, S. Xie and H. Zhang, "A robust channel estimator for DS-CDMA systems under multipath fading channels," in *Proc. IEEE Global Telecommunications Conf. (Globecom'04)*, Dallas, Texas USA, 2004, vol. 4, pp. 2525-2529.
2. C. Cao, L. Xie, H. Zhang and S. Xie, "A robust channel estimator for DS-CDMA systems under multipath fading channels," *IEEE Trans. Signal Processing*, vol. 54, no. 1, pp. 13-22, Jan. 2006.
3. C. Cao, L. Xie and H. Zhang, "A polynomial H_∞ channel estimator for DS-CDMA systems under multipath fading channels," in *Proc. the 45th IEEE Conf. on Decision and Control*, San Diego, Dec. 2006.
4. C. Cao, L. Xie and H. Zhang, " H_∞ channel estimator design for DS-CDMA systems: A polynomial approach in Krein space," *IEEE Trans. Veh. Technol.*, vol. 57, no. 2, pp. 819-827, Mar. 2008.
5. C. Cao and L. Xie, " H_∞ filter design for discrete-time systems with stochastic parameter uncertainties with application in DS-CDMA systems," *submitted to IEEE Trans. Signal Processing*.

Bibliography

- [1] A. Ahlén, L. Lindbom, and M. Sternad. Analysis of stability and performance of adaptation algorithms with time-invariant gains. *IEEE Trans. on Signal Processing*, 52:103–116, Jan. 2004.
- [2] A. Ahlén and M. Sternad. Wiener filter design using polynomial equations. *IEEE Trans. on Signal Processing*, 39:2550–2558, Nov. 1991.
- [3] K. E. Baddour and N. C. Beaulieu. Autoregressive models for fading channel simulation. In *IEEE Global Telecommun. Conf.(Globecom'01)*, volume 2, pages 1187–1192, 2001.
- [4] P. A. Bello. Characterization of randomly time-variant linear channels. *IEEE Trans. Commun. Syst.*, CS-11:360–393, Dec. 1963.
- [5] S. E. Bensley and B. Aazhang. Subspace-based channel estimation for code division multiple access communications systems. *IEEE Trans. on Commun.*, 44:1009–1020, Aug. 1996.
- [6] S. E. Bensley and B. Aazhang. Maximum likelihood synchronization of a single user for CDMA communication systems. *IEEE Trans. on Commun.*, 46(1):392–399, Mar. 1998.

- [7] S. Bhashyam and B. Aazhang. Multiuser channel estimation and tracking for long-code CDMA systems. *IEEE Trans. on Commun.*, 50:1081–1090, July 2002.
- [8] K. L. Blackard, T. S. Rappaport, and C. W. Bostian. Measurements and models of radio frequency impulsive noise for indoor wireless communications. *IEEE J. Select. Areas Commun.*, 40:991–1001, Sept. 1993.
- [9] J. Bogнар. *Indefinite Inner Product Spaces*. Springer-Verlag, New York, 1974.
- [10] D. K. Borah and B. D. Hart. Frequency-selective fading channel estimation with a polynomial time-varying channel model. *IEEE Trans. on Commun.*, 47:862–873, June 1999.
- [11] R. Bosisio, M. Nicoli, and U. Spagnolini. Kalman filter of channel modes in time-varying wireless systems. In *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP'05)*, volume 3, pages 785–788, 2005.
- [12] R. G. Brown and P. Y. C. Hwang. *Introduction to random signals and applied Kalman filtering*. John Wiley & Sons, Inc., 1992.
- [13] M. Budsabathon and S. Hara. When does H_∞ equalizer outperform H_2 equalizer in multipath fading environments? In *IEEE 51st Vehicular Technology Conference Proceedings*, volume 1, pages 608–611, Tokyo, May 2000. Spring.
- [14] S. Buzzi and H. V. Poor. Channel estimation and multiuser detection in long code DS/CDMA systems. *IEEE J. Select. Areas Commun.*, 19:1476–1487, Aug. 2001.

-
- [15] B. S. Chen, C. L. Tsai, and Y. F. Chen. Mixed H_2/H_∞ filtering design in multirate transmultiplexer systems: LMI approach. *IEEE Trans. on Signal Processing*, 49:2693–2701, Nov. 2001.
 - [16] L. M. Chen and B. S. Chen. A robust adaptive DFE receiver for DS-CDMA systems under multipath fading channels. *IEEE Trans. on Signal Processing*, 49:1523–1532, July 2001.
 - [17] Y. Chen and B. Chen. Minimax robust deconvolution filters under stochastic parametric and noise uncertainties. *IEEE Trans. on Signal Processing*, 42:32–45, Jan. 1994.
 - [18] C. Chung and P. R. Belanger. Minimum-sensitivity filter for linear time-invariant stochastic systems with uncertain parameters. *IEEE Trans. on Automatic Control*, 21:98–100, 1976.
 - [19] R. H. Clark. A statistical theory of mobile radio reception. *Bell Syst. Tech. J.*, 47(6):957–1000, Jul.-Aug. 1968.
 - [20] O. L. V. Costa and C. S. Kubrusly. State-feedback H-control for discrete-time infinite-dimensional stochastic bilinear systems. *J. Math. Syst. Estim. Control*, 6:1–32, 1996.
 - [21] J. C. Doyle, K. Glover, and B. A. Francis. State-space solution to standard H_2 and H_∞ control problems. *IEEE Trans. on Automatic Control*, 34(8):831–846, 1989.
 - [22] V. Dragan and A. Stoica. A γ attenuation problem for discrete-time time-varying stochastic systems with multiplicative noise. *Reprint series of the Institute of Mathematics of the Romanian Academy*, 10, 1998.

-
- [23] A. T. Erdogan, B. Hassibi, and T. Kailath. On linear H_∞ equalization of communication channels. *IEEE Trans. on Signal Processing*, 48:3227–3231, Nov. 2000.
 - [24] A. T. Erdogan, B. Hassibi, and T. Kailath. MIMO decision feedback equalization from an H_∞ perspective. *IEEE Trans. on Signal Processing*, 52:734–745, 2004.
 - [25] E. Ertin, U. Mitra, and S. Siwamogsatham. Maximum-likelihood-based multipath channel estimation for code-division multiple-access systems. *IEEE Trans. on Commun.*, 49:290–302, Feb. 2001.
 - [26] M. Fu, C. E. de Souza, and Z. Luo. Finite-horizon robust Kalman filter design. *IEEE Trans. on Signal Processing*, 49:2103–2112, Sep 2001.
 - [27] J. C. Geromel, M. C. de Oliveira, and J. Bernussou. Robust filtering of discrete-time linear systems with parameter dependent Lyapunov functions. In *Proc. IEEE Conference on Decision and Control*, Dec 1999.
 - [28] E. Gershon, D. J. N. Limebeer, U. Shaked, and I. Yaesh. Stochastic H_∞ tracking with preview for state-multiplicative systems. *IEEE Trans. on Automatic Control*, 49:2061–2068, Nov. 2004.
 - [29] E. Gershon, U. Shaked, and I. Yaesh. H_∞ control and filtering of discrete-time stochastic systems with multiplicative noise. *Automatica*, 37(3):409–417, 2001.
 - [30] E. Gershon, U. Shaked, and I. Yaesh. Robust H_∞ filtering of stationary discrete-time linear systems with stochastic parameter uncertainties. *System and Control Letters*, 45:257–269, 2002.

-
- [31] S. J. Grant and J. K. Cavers. Multiuser channel estimation for detection of cochannel signals. *IEEE Trans. on Commun.*, 49:1845–1855, Oct. 2001.
 - [32] M. Green and D. J. N. Limebeer. *Linear Robust Control*. Prentice-Hall, Eaglewood Cliffs, N.J., 1995.
 - [33] M. J. Grimble. Robust filter design for uncertain syetems defined by both hard and soft bounds. *IEEE Trans. on Signal Processing*, 44:1063–1071, May 1996.
 - [34] B. Hassibi, A. H. Sayed, and T. Kailath. Linear estimaton in Krein space- Part I: Theory. *IEEE Trans. on Automatic Control*, 41:18–33, Jan. 1996.
 - [35] B. Hassibi, A. H. Sayed, and T. Kailath. Linear estimaton in Krein space- Part II: Applications. *IEEE Trans. on Automatic Control*, 41:34–49, Jan. 1996.
 - [36] B. Hassibi, A. H. Sayed, and T. Kailath. *Indefinite-quadratic estimation and control: a unified approach to H_2 and H_∞ theories*. SIAM, Philadelphia, PA, 1999.
 - [37] S. Haykin. *Adaptive Filter Theory*. Prentice Hall, 4 edition, 2002.
 - [38] M. Hongig, U. Madhow, and S. Verdú. Blind adaptive multiuser detection. *IEEE Trans. Inform. Theory*, 41(4):944–960, July 1995.
 - [39] I. S. Iohvidov, M. G. Krein, and H. Langer. *Introduction to the Spectral Theory of Operatorod in Spaces with Indefinite Metric*. Mathematical Research. Akademie-Verlag, Berlin, Germany, 1982.
 - [40] V. I. Istratescu. *Inner Product Structures, Theory and Applications, Mathematics and Its Applications*. Reidel, Dordrecht, Holland, 1987.

-
- [41] H. Z. Jafarian and S. Pasupathy. EM-based recursive estimation of channel parameters. *IEEE Trans. on Commun.*, 47:1297–1302, Sep. 1999.
 - [42] W. C. Jakes. *Microwave Mobile Communications*. McGraw-Hill, New York, 1994.
 - [43] G. K. Kaleh and R. Vallet. Joint parameter estimation and symbol detection for linear or nonlinear unknown channels. *IEEE Trans. on Commun.*, 42:2406–2413, July 1994.
 - [44] S. M. Kay. *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, 1993.
 - [45] P. P. Khargonekar and M. A. Rotea. Mixed H_2/H_∞ filtering. In *Proc. IEEE Conference on Decision and Control*, pages 2299–2304. Tuscon, AZ, 1992.
 - [46] K. J. Kim and R. A. Iltis. Joint detection and channel estimation algorithms for QS-CDMA signals over time-varying channels. *IEEE Trans. on Commun.*, 50:845–855, May 2002.
 - [47] C. Komninakis, C. Fragouli, A. H. Sayed, and R. D. Wesel. Multi-input multi-output fading channel tracking and equalization using Kalman estimation. *IEEE Trans. on Signal Processing*, 50:1065–1075, May 2002.
 - [48] T. H. Lee, W. S. Ra, T. S. Yoon, and J. B. Park. Robust Kalman filtering via krein space estimation. *IEE Proc. Control Theory Appl.*, 151(1):59–63, Jan. 2004.
 - [49] S. Y. Leong, J. Wu, C. Xiao, and J. C. Olivier. Fast time-varying dispersive channel estimation and equalization for an 8-PSK cellular system. *IEEE Trans. on Veh. Technol.*, 55:1493–1502, Sep. 2006.

-
- [50] H. Li, J. Li, and S. L. Miller. Decoupled multiuser code-timing estimation for code-division multiple-access communication systems. *IEEE Trans. on Commun.*, 49:1425–1436, Aug. 2001.
- [51] T. J. Lim and L. K. Rasmussen. Adaptive symbol and parameter estimation in asynchronous multiuser CDMA detectors. *IEEE Trans. Commun.*, 45(2):213–220, Feb. 1997.
- [52] L. Lindbom, A. Ahlén, M. Sternad, and M. Falkenström. Tracking of time-varying mobile radio channels – part II: A case study. *IEEE Trans. on Commun.*, 50(12):156–166, Jan. 2002.
- [53] L. Lindbom, M. Sternad, and A. Ahlén. Tracking of time-varying mobile radio channels – part I: The Wiener LMS algorithm. *IEEE Trans. on Commun.*, 49(12):2207–2217, Dec. 2001.
- [54] G. Liu and C. Wei. Analysis of adaptive LMS estimator with cyclic sequences in complex frequency domain. *IEEE Trans. on Circuit and Systems II*, 42:829–832, Dec. 1995.
- [55] T. Luo, E. S. Sousa, and S. Pasupathy. Small delay multipath diversity in spread spectrum communication systems. *IEEE Trans. on Commun.*, 50(7):1160–1171, July 2002.
- [56] D. Middleton. Man-made noise in urban environments and transportation systems: Models and measurement. 21:1232–1241, Nov. 1973.
- [57] D. Middleton. Channel modeling and threshold signal processing in underwater acoustics: An analytical overview. *IEEE J. Ocean. Eng.*, 12:4–28, 1987.

- [58] R. H. Milocco and C. H. Muravchik. H_2 optimal linear robust sampled-data filtering design using polynomial approach. 51:1816–1824, July 2003.
- [59] M. M. Mohler and W. J. Kolodziej. An overview of stochastic bilinear control processes. *IEEE Trans. on Systems, Man and Cybernetics*, 10:913–919, 1980.
- [60] T. Morozan. Parameterized Riccati equations associated input-output operators for discrete-time systems with state-dependent noise. *Reprint series of the Institute of Mathematics of the Romanian Academy*, 1999.
- [61] M. Pätzold. *Mobile Fading Channels*. John Wiley & Sons, England, 2002.
- [62] I. R. Petersen and D. C. McFarlane. Optimal guaranteed cost filtering for uncertain discrete-time linear systems. *Int. J. Robust Nonlinear Contr.*, 6:267–280, 1996.
- [63] H. V. Poor and M. Tanda. Multiuser detection in flat fading non-gaussian channels. *IEEE Trans. on Commun.*, 50:1769–1777, Nov. 2002.
- [64] J. G. Proakis and D. G. Manolakis. *Digital Signal Processing: Principles, Algorithms and Applications*. Prentice-Hall, 3 edition, 1996.
- [65] W. J. Rugh. *Linear System Theory*. Prentice Hall, New Jersey, 2 edition, 1996.
- [66] A. V. Savkin and I. R. Petersen. Robust state estimation and model validation for discrete-time uncertain systems with a deterministic description of noise and uncertainty. *Automatica*, 34(2):271–274, 1998.
- [67] U. Shaked and C. E. de Souza. Robust minimum variance filtering. *IEEE Trans. on Signal Processing*, 43:2474–2483, Nov. 1995.

- [68] U. Shaked and Y. Theodor. H_∞ -optimal estimation: A tutorial. In *Proc. IEEE Conference on Decision and Control*, pages 2278–2286, Tucson, AZ, Dec. 1992.
- [69] U. Shaked, L. Xie, and Y. C. Soh. New approaches to robust minimum variance filter design. *IEEE Trans. on Signal Processing*, 49:2620–2629, Nov. 2001.
- [70] J. L. Speyer and D. E. Gustafson. An approximation method for estimation linear systems with parameter uncertainty. *IEEE Trans. on Automatic Control*, 20:354–359, 1975.
- [71] E. G. Ström and F. Malmsten. A maximum likelihood approach for estimating DS-CDMA multipath fading channels. *IEEE J. Select. Areas Commun.*, 18(1):132–140, Jan. 2000.
- [72] M. Sznajder. A exact solution to general SISO mixed H_2/H_∞ problems via convex optimization. *IEEE Trans. on Automatic Control*, 39(12):2511–2517, 1994.
- [73] A. M. Tehrani, B. Sayyarrodsari, B. Hassibi, J. P. How, and J. M. Cioffi. Estimation-based synthesis of H_∞ -optimal adaptive equalizers over wireless channels. In *IEEE Global Telecommun. Conf (GLOBECOM '99)*, 1999.
- [74] Y. Theodor and U. Shaked. Robust discrete-time minimum-variance filtering. *IEEE Trans. on Signal Processing*, 44:181–189, Feb. 1996.
- [75] M. Torlak and G. Xu. Blind multiuser channel estimation in asynchronous cdma systems. *IEEE Trans. on Signal Processing*, 45:137–147, Jan. 1997.

- [76] S. Tsai, T. F. Wong, and J. S. Lehnert. DS-CDMA system with joint channel estimation and MAP detection in time-selective fading channels. 19:121–131, Jan. 2001.
- [77] M. K. Tsatsanis and G. B. Giannakis. Subspace methods for blind estimation of time-varying FIR channels. *IEEE Trans. on Signal Processing*, 45:3084–3093, Dec. 1997.
- [78] M. K. Tsatsanis, G. B. Giannakis, and G. Zhou. Estimation and equalization of fading channels with random coefficients. In *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP-96)*, volume 2, pages 1093–1096, 1996.
- [79] M. K. Tsatsanis and Z. Xu. Pilot symbol assisted modulation in frequency selective fading wireless channels. *IEEE Trans. on Signal Processing*, 48:2353–2365, Aug. 2000.
- [80] H. D. Tuan, P. Apkarian, and T. Q. Nguyen. Robust and reduced-order filtering: New LMI-based characterizations and methods. *IEEE Trans. on Signal Processing*, 49:2975–2984, Dec. 2001.
- [81] J. K. Tugnait, L. Tong, and Z. Ding. Single-user channel estimation and equalization. *Signal Processing Magazine*, 17:16–28, May 2000.
- [82] M. K. Varanasi and S. Vasudevan. Multiuser detectors for synchronous cdma communication over non-selective rician fading channels. *IEEE Trans. on Commun.*, 42:711–722, Feb. 1994.
- [83] S. Verdú. *Multiuser Detection*. Cambridge University Press, Cambridge, UK, 1998.

-
- [84] F. Wang and V. Balakrishnan. Robust Kalman filters for linear time varying systems with stochastic parametric uncertainties. *IEEE Trans. on Signal Processing*, 50:803–813, Apr. 2002.
 - [85] F. Wang and V. Balakrishnan. Robust steady-state filtering for systems with deterministic and stochastic uncertainties. *IEEE Trans. on Signal Processing*, 51:2550–2558, Oct. 2003.
 - [86] X. Wang and H. V. Poor. *Wireless Communication Systems: Advanced Techniques for Signal Reception*. Prentice Hall, 2003.
 - [87] P. H. Y. Wu and A. Duel-Hallen. Multiuser detectors with disjoint Kalman channel estimators for synchronous CDMA mobile radio channels. *IEEE Trans. on Commun.*, 48:752–756, May 2000.
 - [88] L. Xie, Y. C. Soh, and C. E. de Souza. Robust Kalman filtering for uncertain discrete-time systems. *IEEE Trans. on Automatic Control*, 39:1310–1314, June 1994.
 - [89] Z. Xie, C. K. Rushforth, R. T. Short, and T. K. Moon. Joint signal detection and parameter estimation in multiuser communications. *IEEE Trans. on Commun.*, 41(7):1208–1216, 1993.
 - [90] M. D. Yacoub. *Wireless Technology: Protocols, Standards, and Techniques*. CRC Press, 2001.
 - [91] G. Yue, X. Zhou, and X. Wang. Performance comparisons of channel estimation techniques in multipath fading CDMA. *IEEE Trans. on Wireless Commun.*, 3:716–724, May 2004.

-
- [92] S. M. Zabin and H. V. Poor. Efficient estimation of the class A parameters via the EM algorithm. *IEEE Trans. Inform. Theory*, 37(1):60–72, Jan. 1991.
 - [93] D. Zhang, J. Li, and E. G. Ström. An efficient code-timing estimator for DS-CDMA signals. *IEEE Trans. on Signal Processing*, 46:82–89, Jan. 1997.
 - [94] H. Zhang, H. H. Fan, and A. Linsey. A wavelet packet based model for time-varying wireless communication channels. In *Proc. of Third IEEE Signal Processing Workshop on Signal Processing Advances in Wireless Communications*, pages 50–53, Taiwan, China, March 2001.
 - [95] H. Zhang, L. Xie, and Y. C. Soh. H_∞ deconvolution filtering, prediction, and smoothing : A Krein space polynomials approach. *IEEE Trans. on Signal Processing*, 48:888–892, Mar. 2000.
 - [96] H. Zhang, L. Xie, and Y. C. Soh. Discrete J-spectral factorization. *Systems Control Letters*, 43:275–286, 2001.
 - [97] H. Zhang, L. Xie, W. Wang, and X. Lu. An innovation approach to H_∞ fixed-lag smoothing for continuous time-varying systems. *IEEE Trans. on Automatic Control*, 49:2240–2244, Dec. 2004.
 - [98] H. Zhang, D. Zhang, L. Xie, and J. Lin. Robust filtering under stochastic parametric uncertainties. *Automatic*, 40:1583–1589, Sept.. 2004.
 - [99] Q. T. Zhang. A decomposition technique for efficient generation of correlated Nakagami fading channels. *IEEE J. Select. Areas Commun.*, 18:2385–2392, Nov. 2000.
 - [100] K. Zhou, J. C. Doyle, and K. Glover. *Robust and optimal control*. Prentice Hall, 1996.

-
- [101] X. Zhu., Y. C. Soh, and L. Xie. Design and analysis of discrete-time robust Kalman filters. *Automatica*, 38(6):1069–1077, 2002.
 - [102] W. Zhuang. Adaptive H_∞ channel equalization for wireless personal communications. *IEEE Trans. on Veh. Technol.*, 48:126–136, Jan. 1999.
 - [103] K. S. Zigangirov. *Theory of Code Division Multiple Access Communication*. John Wiley & Sons Inc., 2004.
 - [104] K. Zou, K. Glover, B. Bodenheimer, and J. Doyle. Mixed H_2 and H_∞ performance objectives I: Robust performnace analysis. *IEEE Trans. on Automatic Control*, 39(8):1564–1574, 1994.

Appendix A

Proof of Theorem 4.4.1

From (4.10), $\mathbf{e}_0(n)$ can be expressed as

$$\mathbf{e}_0(n) = \Delta \mathbf{D}_1 \mathbf{h}(n-1) + \Delta \mathbf{D}_2 \mathbf{h}(n-2) \quad (\text{A-1})$$

where $\Delta \mathbf{D}_1$ and $\Delta \mathbf{D}_2$ are respectively defined in (4.11). Under Assumptions 4.4.1 and 4.4.2, we have

$$\begin{aligned} E\{\mathbf{e}_0(n)\} &= E\{\Delta \mathbf{D}_1\}E\{\mathbf{h}(n-1)\} + E\{\Delta \mathbf{D}_2\}E\{\mathbf{h}(n-2)\} \\ &= 0, \end{aligned} \quad (\text{A-2})$$

$$\begin{aligned} E\{\mathbf{e}_0(n)\mathbf{e}_0^*(m)\} &= E\{\Delta \mathbf{D}_1\}E\{\mathbf{h}(n-1)\mathbf{e}_0^*(m)\} + E\{\Delta \mathbf{D}_2\}E\{\mathbf{h}(n-2)\mathbf{e}_0^*(m)\} \\ &= 0. \end{aligned} \quad (\text{A-3})$$

Thus, $\mathbf{e}(n)$ and $\mathbf{e}_0(m)$ are uncorrelated.

We rewrite (4.10) as [98]

$$\mathbf{e}_0(n) = \mathbf{D}_c(q^{-1})\mathbf{h}(n)q^{-1} \quad (\text{A-4})$$

where $\mathbf{D}_c(q^{-1}) = \Delta \mathbf{D}_1 + q^{-1} \Delta \mathbf{D}_2$. On the other hand, from (4.9), it follows that

$$\mathbf{h}(n) = d^{-1}(q^{-1})(\mathbf{e}(n) + \mathbf{e}_0(n)). \quad (\text{A-5})$$

Substituting (A-5) into (A-4), we have

$$\mathbf{e}_0(n) = d^{-1}(q^{-1}) \mathbf{D}_c(q^{-1}) (\mathbf{e}(n) + \mathbf{e}_0(n)) q^{-1}.$$

By using Parseval's formula [2], the correlation matrix of $\mathbf{e}_0(n)$ is given by

$$\begin{aligned} \mathbf{R}_{e_0} &= E\{\mathbf{e}_0(n) \mathbf{e}_0^*(n)\} \\ &= E \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} \mathbf{D}_c(\mathbf{R}_e + \mathbf{R}_{e_0}) \mathbf{D}_c^* \frac{dz}{z} \\ &= E \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} (\Delta \mathbf{D}_1 \mathbf{R} \Delta \mathbf{D}_1^* + z^{-1} \Delta \mathbf{D}_2 \mathbf{R} \Delta \mathbf{D}_1^* + z \Delta \mathbf{D}_1 \mathbf{R} \Delta \mathbf{D}_2^*) \frac{dz}{z} \\ &= \gamma_0 E\{\Delta \mathbf{D}_1 \mathbf{R} \Delta \mathbf{D}_1^* + \Delta \mathbf{D}_2 \mathbf{R} \Delta \mathbf{D}_2^*\} + \gamma_1 E\{\Delta \mathbf{D}_2 \mathbf{R} \Delta \mathbf{D}_1^*\} \\ &\quad + \gamma_2 E\{\Delta \mathbf{D}_1 \mathbf{R} \Delta \mathbf{D}_2^*\} \end{aligned}$$

where $\mathbf{R} = \mathbf{R}_e + \mathbf{R}_{e_0}$, γ_0 , γ_1 and γ_2 are defined in (4.13)-(4.15). Because $\Delta \mathbf{D}_1$ and $\Delta \mathbf{D}_2$ are diagonal matrices, then

$$\begin{aligned} R_{e_0,ii} &= (\gamma_0 E\{\Delta D_{1,ii} \Delta D_{1,ii}^* + \Delta D_{2,ii} \Delta D_{2,ii}^*\} + \gamma_1 E\{\Delta D_{1,ii}^* \Delta D_{2,ii}\} \\ &\quad + \gamma_2 E\{\Delta D_{1,ii} \Delta D_{2,ii}^*\})(R_{e_0,ii} + R_{e,ii}) \\ &= \rho_i (R_{e_0,ii} + R_{e,ii}). \end{aligned}$$

Then, $R_{e_0,ii} = \frac{\rho_i}{1-\rho_i} R_{e,ii}$ where ρ_i is defined in (4.12). In view of Assumption 3.3, the off-diagonal element of $\mathbf{R}_{e_0}$ is given by

$$R_{e_0,ij} = (\gamma_0 E\{\Delta D_{1,ii} \Delta D_{1,jj}^* + \Delta D_{2,ii} \Delta D_{2,jj}^*\} + \gamma_1 E\{\Delta D_{1,ii}^* \Delta D_{2,jj}\})$$

$$+\gamma_2 E\{\Delta D_{1,ii}\Delta D_{2,jj}^*\})R_{e_0,ij}, \quad i \neq j.$$

Since the scalar coefficient of $R_{e_0,ij}$ on the right hand side of the above equation is not equal to zero, all off-diagonal elements of the matrix $\mathbf{R}_{e_0}$ are zero and $\mathbf{R}_{e_0}$ is a diagonal matrix described in Theorem 4.4.1. The proof is completed.

Appendix B

Proof of Theorem 4.5.1

We use the variational approach to find the linear filter $\mathcal{R}$. All admissible variations which are generated by $\mathbf{f}(n)$ can be represented by

$$\boldsymbol{\xi} = \boldsymbol{\eta}(q^{-1})\mathbf{f}(n) \quad (\text{B-1})$$

where $\boldsymbol{\eta}(q^{-1})$ is a stable and causal rational matrix. From [53], $\hat{\mathbf{h}}(n+m|n)$ is the optimal estimation based on the MMSE iff

$$E\{\tilde{\mathbf{h}}(n+m|n)\boldsymbol{\xi}^*\} = 0. \quad (\text{B-2})$$

By using the Parseval's formula, the orthogonality relationship between $\tilde{\mathbf{h}}(n+m|n)$ and $\boldsymbol{\xi}^*$ can be converted into a frequency domain condition:

$$\begin{aligned} & E\{\tilde{\mathbf{h}}(n+m|n)\boldsymbol{\xi}^*\} \\ &= \frac{1}{2\pi i} \oint_{|z|=1} d^{-1}((z^m \mathbf{I} - \mathcal{R})(\mathbf{R}_e + \mathbf{R}_{e_0}) - \mathcal{R}d d^* \mathbf{R}_{v_1}) \boldsymbol{\eta}^* d^{-*} \frac{dz}{z} \\ &= \frac{1}{2\pi i} \oint_{|z|=1} d^{-1}(z^m(\mathbf{R}_e + \mathbf{R}_{e_0}) - \mathcal{R}\boldsymbol{\beta}\mathbf{R}_{\varepsilon}\boldsymbol{\beta}^*) \boldsymbol{\eta}^* d^{-*} \frac{dz}{z}. \end{aligned} \quad (\text{B-3})$$

In the last equality, the spectral factorization (4.26) was inserted. Since d and ξ are causal and stable, $\eta^* d^{-*}$ have no pole in $|z| < 1$. The orthogonality requirement of (B-2) holds iff there are no poles inside the integration path $|z| = 1$ in (B-3). Thus, we require that

$$z^m(\mathbf{R}_e + \mathbf{R}_{e_0}) - \underbrace{\mathcal{R}\beta}_{\mathbf{L}_m} \mathbf{R}_\varepsilon \beta^* = z d \mathbf{Q}_m^*. \quad (\text{B-4})$$

Hence, the filter $\mathcal{R}$ can be obtained by solving this Diophantine equation. For this bilateral Diophantine equation, the general solution can be presented by $(\mathbf{L}_m, \mathbf{Q}_m^*) = (\mathcal{L}_m + q d \mathbf{X}, \mathcal{Q}_m^* - \mathbf{X} \mathbf{R}_\varepsilon \beta^*)$ where $(\mathcal{L}_m, \mathcal{Q}_m^*)$ is one solution pair and $\mathbf{X}$ is an arbitrary polynomial matrix. Since $\mathbf{L}_m$ and $\mathbf{Q}_m^*$, respectively, are required to be polynomial matrices in q^{-1} and in q only, $\mathbf{X} = 0$ is the only choice. Thus, the solution to (B-4) is unique.

Applying the result in (4.26) and (B-4), we write the minimal estimation error variance $\mathbf{P}$ as

$$\begin{aligned} \mathbf{P}_{min} &= E\{\tilde{\mathbf{h}}(n+m|n)\tilde{\mathbf{h}}^*(n+m|n)\} \\ &= \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} [(z^m \mathbf{I} - \mathbf{L}\beta^{-1})(\mathbf{R}_e + \mathbf{R}_{e_0}) \\ &\quad \times (z^{-m} \mathbf{I} - \beta^{-*} \mathbf{L}_m^*) + \mathbf{L}_m \beta^{-1} d \mathbf{R}_{v_0} d^* \beta^{-*} \mathbf{L}_m^*] \frac{dz}{z} \\ &= \frac{1}{2\pi i} \oint_{|z|=1} (dd^*)^{-1} (\mathbf{I} - z^{-m} \mathbf{L}_m \beta^{-1})(\mathbf{R}_e + \mathbf{R}_{e_0}) \frac{dz}{z} \end{aligned}$$

where we use the fact that $d^{-*} \mathbf{Q}_m^* \beta^{-*} \mathbf{L}_m^*$ has no pole inside $|z| = 1$. The proof is completed.

Appendix C

Proof of Theorem 5.4.3

Assume that the spectral factor $M(q^{-1})$ is given by

$$M(q^{-1}) = \begin{bmatrix} M_1(q^{-1}) & M_2(q^{-1}) \\ M_3(q^{-1}) & M_4(q^{-1}) \end{bmatrix}. \quad (\text{C-1})$$

Then

$$M(q^{-1})JM^*(q^{-1}) = \begin{bmatrix} M_1M_1^* - M_2M_2^* & M_1M_3^* - M_2M_4^* \\ M_3M_1^* - M_4M_2^* & M_3M_3^* - M_4M_4^* \end{bmatrix}. \quad (\text{C-2})$$

Comparing both sides of (5.44), we have the following equations

$$M_1M_1^* - M_2M_2^* = I + D(q^{-1})Q_cD^*(q^{-1}), \quad (\text{C-3})$$

$$M_3M_3^* - M_4M_4^* = I - D(q^{-1})Q_hD^*(q^{-1}), \quad (\text{C-4})$$

$$M_1M_3^* - M_2M_4^* = I. \quad (\text{C-5})$$

By taking (C-3)+(C-4)-2×(C-5) and completing the squares, we obtain

$$(M_1 - M_3)(M_1 - M_3)^* - (M_2 - M_4)(M_2 - M_4)^* = D(q^{-1})(Q_c - Q_h)D^*(q^{-1}).$$

It implies that the above equality can be satisfied if the following expressions hold

$$M_1 - M_3 = D(q^{-1})Q_c^{\frac{1}{2}}, \quad (\text{C-6})$$

$$M_2 - M_4 = D(q^{-1})Q_h^{\frac{1}{2}}. \quad (\text{C-7})$$

From (C-5)-(C-7) and (C-3), it follows that

$$\begin{aligned} & M_1 M_3^* - M_2 M_4^* \\ &= M_1 \left(M_1 - D(q^{-1})Q_c^{\frac{1}{2}} \right)^* - M_2 \left(M_2 - D(q^{-1})Q_h^{\frac{1}{2}} \right)^* \\ &= M_1 M_1^* - M_2 M_2^* - (M_1 Q_c^{\frac{1}{2}} - M_2 Q_h^{\frac{1}{2}}) D^*(q^{-1}) \\ &= I + D(q^{-1})Q_c D^*(q^{-1}) - (M_1 Q_c^{\frac{1}{2}} - M_2 Q_h^{\frac{1}{2}}) D^*(q^{-1}) \\ &= I + \left(D(q^{-1})Q_c - (M_1 Q_c^{\frac{1}{2}} - M_2 Q_h^{\frac{1}{2}}) \right) D^*(q^{-1}) = I. \end{aligned}$$

Hence

$$M_1 Q_c^{\frac{1}{2}} = D(q^{-1})Q_c + M_2 Q_h^{\frac{1}{2}}. \quad (\text{C-8})$$

Substituting (C-8) into (C-3) leads to

$$\begin{aligned} & M_1 M_1^* - M_2 M_2^* - D(q^{-1})Q_c D^*(q^{-1}) \\ &= \left(D(q^{-1})Q_c + M_2 Q_h^{\frac{1}{2}} \right) Q_c^{-1} \left(D(q^{-1})Q_c + M_2 Q_h^{\frac{1}{2}} \right)^* - M_2 M_2^* - D(q^{-1})Q_c D^*(q^{-1}) \\ &= M_2 Q_h^{\frac{1}{2}} D^*(q^{-1}) + D(q^{-1})Q_h^{\frac{1}{2}} M_2^* + M_2 Q_h^{\frac{1}{2}} (Q_c^{-1} - Q_h^{-1}) Q_h^{\frac{1}{2}} M_2^* \\ &= I. \end{aligned}$$

Since $Q_c^{-1} - Q_h^{-1} > 0$, let $P^{-1} = Q_c^{-1} - Q_h^{-1}$. Then,

$$\begin{aligned} & \left(M_2 Q_h^{\frac{1}{2}} + D(q^{-1})P \right) P^{-1} \left(M_2 Q_h^{\frac{1}{2}} + D(q^{-1})P \right)^* \\ &= I + D(q^{-1})PD^*(q^{-1}) \\ &= G(q^{-1})G^*(q^{-1}). \end{aligned}$$

It is easy to see that

$$G(q^{-1}) = \left(M_2 Q_h^{\frac{1}{2}} + D(q^{-1})P \right) P^{-\frac{1}{2}}.$$

Hence

$$\begin{aligned} M_2 &= \left(G(q^{-1})P^{\frac{1}{2}} - D(q^{-1})P \right) Q_h^{-\frac{1}{2}} \\ &= N(q^{-1})Q_h^{-\frac{1}{2}}, \end{aligned}$$

where $N(q^{-1})$ is defined in (5.46). Finally, M_1 , M_3 and M_4 are given by

$$\begin{aligned} M_1 &= N(q^{-1})Q_c^{-\frac{1}{2}} + D(q^{-1})Q_c^{\frac{1}{2}}, \\ M_3 &= N(q^{-1})Q_c^{-\frac{1}{2}}, \\ M_4 &= N(q^{-1})Q_h^{-\frac{1}{2}} - D(q^{-1})Q_h^{\frac{1}{2}}, \end{aligned}$$

where M_1 , M_2 , M_3 , and M_4 are causal polynomial matrices. The proof is completed.