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Parameter Estimation Techniques for Fiber Bragg Grating Sensors

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in fulfilment of the requirement for the degree of
Master of Engineering

2008

Statement of Originality

I hereby certify that the content of this thesis is the result of work done by me and has not been submitted for a higher degree to any other University or Institution.

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Date

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Tang Yu

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Summary

In recent decades, fiber Bragg grating sensors (FBGSs) have been widely implemented for measurements in various dynamic or static fields. The basic properties of the FBGSs are their wavelength-encoded measured information and their inherent self-referencing. When the light delivered from the light source reaches the FBGSs, signal waves that are close to the Bragg wavelength are reflected by the fiber Bragg grating (FBG) and the others waves pass through the FBG. Hence, the value of the Bragg wavelength can be obtained by measuring the reflected waves with an optical spectrum analyzer. Physical properties such as the fiber reflectance index and the Grating pitch will change with strain and temperature, thus causing a shift of the Bragg wavelength. Based on this principle, measuring the shift of the Bragg wavelength provides a means to monitor external strain and temperature variations. However, to produce an accurate result, the ability to detect small measured-induced wavelength shifts is critical.

This thesis looks into the development of algorithms to efficiently detect the wavelength shift of FBGSs under two kinds of noise. One is the interferometric signal, caused by the residual reflected light in the sensing system. Conventional filtering techniques could be quite effective in removing random Gaussian-white noise, but not so for the interferometric signal, which is very structured. Another kind of noise is the crosstalk caused by the reflections from multiple FBGs, of which the Bragg wavelengths are close to each other. Peak-detection techniques could effectively detect the peaks when the two Bragg wavelengths are far apart, but not so when the Bragg wavelengths are close to each other.

In this dissertation, a model of the reflected spectrum proposed in the literature is utilized. A Nonlinear Least-Squares (NLS) approach is firstly introduced to identify the unknown parameters in this spectrum model with interferometric signals. The interferometric signal's removal is then addressed. However, since the parameter estimation problem is nonlinear, the larger the number of parameters, the higher the chance that the algorithm will get trapped into a local minimum and fail to identify the correct parameters. It is also demonstrated in the experiment that the NLS approach is sensitive to the spectrum model, so any unknown perturbations will somehow affect the detection accuracy. To solve the problem, it is proposed to combine the nonlinear least squares method with a Kalman smoother. Hence, the number of parameters to be estimated by the nonlinear least squares algorithm will be greatly reduced and the robustness of the algorithm is improved at the same time. To test the feasibility of the proposal, two state-space formulations of the spectrum model are designed based on the NLS estimator. An interesting point to note is that these two models are both linear time-varying instead of nonlinear, thus not requiring an extended Kalman filter. It is demonstrated that NLS embedded with a Kalman filter can obtain excellent estimation results in both simulations and experiments. Similarly to the spectra with interferometric signals, an NLS approach is proposed to recover the FBGs reflected spectra distorted by crosstalk. After correctly constructing the spectrum model, the NLS approach provides more robust results than the conventional peak-detection techniques, which can be demonstrated from the simulations and experiments.

This thesis firstly gives a thorough literature review on the basic knowledge and relevant work of fiber Bragg grating sensors, followed by an introduction to the interferometric spectrum model. The NLS approach is employed to estimate the Bragg wavelength in both simulation and experimental studies. When we find that the performance of the NLS is not as good in the experiments as in the simulations, we improve it by further embedding a Kalman smoother within the NLS estimator. Having presented the algorithms for the removal of the interferometric signal, we

propose a similar NLS approach to deal with the crosstalk caused by multiple FBGs reflections. Simulation and experimental results are also provided.

Acronym

ADALINE	Adaptive Linear
AOTF	Acousto-Optic Tunable Filter
BGSs	Bragg Grating Sensors
CTE	Coefficient of Thermal Expansion
DMF	Digital Matched Filter
DSP	Digital Signal Processing
EMI	Electromagnetic Interference
FBG	Fiber Bragg Grating
FBGSs	Fiber Bragg Grating Sensors
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FPF	Fabry-Perot Filter
FS	Fixed Stage
FSR	Free Spectral Range
FWHM	Fullwidth at Half Maximum
GN	Gauss Newton
IIR	Infinite Impulse Response
KF	Kalman Filter
LLS	Linear Least Squares
MZIF	Mach-Zehnder Interferometer
NKMI	NLS with Kalman Filter Model I
NKMII	NLS with Kalman Filter Model II

Summary

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NLS	Nonlinear Least Square
PC	Personal Computer
RMS	Root Mean Square
SNR	Signal to Noise Ratio
SRS	Stimulated Raman Scattering
TS	Translation Stage

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Chapter 1

Introduction

In recent decades, fiber Bragg grating sensors (FBGSs) have been widely implemented in many areas, such as measurements of different dynamic or static fields, using time- and wavelength- division multiplexing [33]. A basic property of the FBGSs is their wavelength- encoded measured information so that these sensors are independent of fluctuating light levels and the system is immune to source power vibration. Moreover, the FBGSs can make reproducible measurements since they are immune to many environmental disturbances, such as electromagnetic interference(EMI), humidity etc. FBGSs are also becoming widely used in areas such as smart structure technology, process control and aerospace industries.

1.1 Motivation

1.1.1 Fiber Bragg Grating Sensors (FBGSs)

Fiber Bragg grating sensors are sensing systems with intrinsic Bragg grating as shown in Figure 1.1. The systems make use of the properties of Bragg grating and accurately process the light wave to satisfy different objectives. Since the light

Figure 1.1: Fiber Bragg Grating (Figure is obtained from internet open source)

wave, after passing through the Bragg grating, is wavelength encoded, the measurement of the light wave will not be deteriorated by most external environmental factors such as EMI, humidity. The measurements appear quite flexible and stable. The center wavelength of FBG will be changed with relative strains and temperatures, which will modify the physical properties of the FBG, thus causing the shift of the Bragg wavelength. The determination of the small measured-induced wavelength shifts become the key detection issue in FBG sensors. Detailed principles of FBGSs will be introduced in Chapter 2.

As we know, the Bragg wavelength, or resonance condition of a grating, is given by the expression[46]: $\lambda_B = 2n\Lambda$. Where Λ is the grating pitch and n is the effective index of the core. When the light from a broadband source is injected to the sensors, a small narrow band component of the spectral at the center wavelength is reflected by the Bragg grating, while in the transmitted light, the reflected light is missing. The wavelength shift of the sensors caused by the external strains and temperatures can be detected in either the reflected or transmitted spectrum. The strain variation will change the physical elongation of the sensor and the fiber index due to photoelastic effects, whereas the thermal variation will inherently

expand the fiber material and change the refractive index due to the temperature dependence. The relationship among the wavelength shift of Bragg grating, strain and temperature can be expressed as[33]:

$$\Delta\lambda_B = 2n\Lambda \left(\left\{ 1 - \left(\frac{n^2}{2}\right)[P_{12} - \nu(P_{11} + P_{12})] \right\} \varepsilon + \left[\alpha + \frac{\left(\frac{dn}{dT}\right)}{n} \right] \Delta T \right) \quad (1.1)$$

where ε is the applied strain, $P_{i,j}$ coefficients are the Pockel's(piezo) coefficients of the stress-optic tensor, ν is Poisson's ratio, and α is the coefficient of thermal expansion (CTE) of the fiber material, and ΔT is the temperature change. With relationship (1.1), FBGSs can be used to detect the strain and temperature variation by detecting the shift of the Bragg wavelength. In fact, Bragg gratings have already widely been adopted in smart structure technology. The FBGSs can be easily embedded into materials to provide internal strain field mapping with high localization, strain resolution, and measurement range. The sensors are also compatible with small diameter medical probe such as measurement of human body temperature.

In practice, to resolve a temperature change of 0.1°C or a strain change of $1\ \mu\text{strain}$, wavelength resolution of 1 pm is required. It is a challenge to resolve changes on this order in small packaged electro-optics unit. A wide variety of techniques have been demonstrated for monitoring Bragg wavelengths shift, only certain techniques have the potential for being reduced to practical system instrument used in real-world application. In recent years, tunable narrow band light sources are introduced in the FBGS systems. This kind of light source overcomes some shortages of the broadband light source such as low signal to noise ratio (SNR), but it also brings new technical problems to the researcher. Here we briefly review the progress in the detection of wavelength shifts and then present the topic of this thesis.

1.1.2 Background

In 1978, Hill and his coworkers [20, 25] firstly observed the photosensitivity in germanium-doped silica fiber in experiments at their Communication Research Center in Canada. In the experiment, they found that the increase in reflectivity was the result of the photoinduced refractive index grating over the 1-m fiber length. The grating found in the experiment was subsequently called Hill gratings. The grating was also called self-organized or self-induced since they formed spontaneously within the optical fiber. When the high-intensity light was launched into the fiber, it would alter the index of the refraction in the photosensitive fiber core permanently. If the refractive index perturbation had the same spatial periodicity as the interference pattern was formed, the refractive index grating would act as a distributed reflector that counter the forward-propagation light beam to backward. The backward-propagation light beam enhanced the back-reflective light and then increased the reflective index. The feedback was positive and would not stop until the reflectivity of the grating reached a saturation level. Yet, the fiber photosensitivity did not attract much attention of the international reacher after its discovery. People regarded the grating as a special phenomenon in this germanium-doped silica fiber. Almost a decade later, Stone [58] proved that another kind of optical fiber could have a similar photosensitivity. Nowadays, researcher overall the world are seeking and proving that different types of fiber could have the characteristic of photosensitivity.

In 1989 Meltz et al. [40] , following the work by Lam and Garside [35], irradiated a single-photon 5eV UV light to a germanium-doped fiber. Surprisingly, they found that a strong index of refraction change occurred. Later, they exposed the fiber under the intersection of two coherent 244-nm beams, the core index of the refraction was modulated by the interferometer. As they changed the angle between the intersecting beams, the spacing between the interference maxima altered, thus making possible reflectance at any wavelength. This discovery was a breakthrough.

Before that, the Bragg resonance wavelength was limited to 488 nm, same as the injected argon ion light wavelength, and the wavelength induced by straining of the fiber was small, so the devices such as induced gratings were not practical.

After discovering that the grating could be set to reflect any wavelength, the development of fiber Bragg grating became faster and faster. Later, the index change Δn could be modulated large enough to make a wide varieties of linear and nonlinear optical devices. Since the fiber Bragg Grating can couple the necessary light out of the waveguide to perform reflection diffraction and filtering in a highly efficient, low loss manner, fiber Bragg grating has a critical impact on telecommunication and optical fiber sensor areas.

In this thesis's work, the research will focus on FBGSs wavelength shift detection techniques, so we firstly make some introduction to the development of FBGSs here. Detailed literature review on wavelength shift detection techniques will be provided in Chapter 2.

FBGSs are fiber sensors with gratings as their simple intrinsic sensing elements. They have all the advantages normally attributed to fiber sensors and they are inherently self-referencing. In addition, the devices are excellent elements for multiplexing [46, 47, 45]. Various applications of FBGSs have emerged during the last decades. The primary area attracting most interest is the distributed embedded sensing in materials for creating "smart structures". FBGSs array are easily embedded in materials to provide damage detection and measurements of parameters such as load, temperature, strain and vibration. There are other applications for FBGSs: pressure sensors, accelerometers and chemical sensors. The sensors can also work well in hostile environment, such as high-pressure, high temperature, and borehole drilling applications.

To guarantee the accuracy of the sensor, reflected light beam from the FBG should be measured correctly. The physical properties such as the fiber reflectance index and the Grating pitch will change with strain and temperature, thus causing the

shift of the center wavelength. The small perturbation of strain and temperature will bring bias reading of the reflected wavelength, thus lower the performance of the sensor, so detection of the small measurand-induced wavelength shift is quite important.

Various schemes have been proposed to detect the wavelength drifts in the FBGSs. One kind of scheme uses a broadband source and various spectral analyzing techniques such as edge filters, scanning Fabry-Perot filters, acousto-optic tunable filters and Mach-Zehnder interferometers [27, 61]. However, the broadband source would provide a low power signal to the receiver, thus resulting in a low signal-to-noise ratio (SNR). Another kind of scheme makes use of a tunable laser source with a broadband detector [54]. This kind of method provides a high signal power to enhance the SNR. However, it would incur an unwanted noise, known as the interferometric signal. Previously, digital low-pass filters [11] were applied to enhance the accuracy of the measurements for the FBG sensors. However, the improvement was limited because the spectrum of the interferometric signal fell within the filter band. Later, neural network techniques [13] were proposed to deal with the interferometric signal of which the spectrum range was comparable to the full-width-at-half-maximum (FWHM) of the FBG. This method enhanced the accuracy slightly and partially succeeded in recovering the signal spectrum. The challenges of wavelength shift detection in FBGSs are not only in removing various noise, but also in recovering the spectrums from crosstalk where multiple FBGSs are interrogated in an array, and WDM or TDM techniques are used to multiplex large numbers of FBGSs [14].

1.2 Objective

The objective of this thesis is to design an iterative optimization algorithm to recover the Bragg wavelength from the interference structure noise and the crosstalk

distortion. On one hand, as we stated before, although the tunable laser source provides a high signal power to enhance the SNR, it will incur an unwanted interferometric signal. The noise components are close to the frequency of the reflected signal and can not be filtered out by normal filtering techniques. In 1998, J. Wei [23] gave a detailed exploration of the interferometric signal and derived the general formulas for wavelength detection. These formulas suggest that the interferometric signal could be regarded as structured noise so that we can apply optimization techniques to process the spectrum. On the other hand, crosstalk discussed in this thesis are caused by multiple FBGs reflection. Thus, the crosstalk spectrum could be represented by the sum of single FBGs formulas and then we can apply optimization techniques to recover the spectrums.

To design an optimization algorithm, we firstly consider the Nonlinear Least Squares (NLS) estimation techniques. Least squares fitting is a mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offsets (“the residuals”) of the points from the curve. NLS applies the Least squares fitting technique to nonlinear system model. Gauss-Newton method and Levenberg-Marquardt are the two popular NLS methods to find the minimum of the residuals. Both of them require at least a first order derivative of the system formulations. In [11, 12], system models of the FBG wavelength with interferometric signal and crosstalk are proposed. We need to transform the system model into parametric formulations and try to set proper ranges for the initialization of these parameters. After properly designing the parametric formulations, all the parameters including the Bragg wavelength can be recovered from the NLS estimation. Simulation and experimental results are provided to demonstrate the performance of the design.

Considering that the noise factors in reality are normally different from these in the theoretical design, if we want to guarantee the robustness of the algorithm, additional methods need to be incorporated with NLS. Kalman filter is a good choice to be combined with NLS since it can keep obtaining the best current state

of the system even with imprecise measurement data. State-space equations are required for the Kalman filter so that we need to reformulate the system model into state-space formulations, where the coefficient matrix of the formulations include all the parameters to be estimated. Simulation and experiment are further to be done to show the performance of the design.

1.3 Contributions of the Thesis

The following are the main contributions of this thesis:

- Representation of the system model in a parametric way
- NLS estimator for removing the interferometric signal
- NLS Embedded with Kalman Smoothing for removing the interferometric signal
- NLS estimator for restoring the system spectrums from crosstalk distortion

1.3.1 Representation of the System Model in a Parametric Way

Generally speaking, a good NLS estimation approach mainly consists of two problems. The first problem is a proper representation of the system. If the representation does not match the reality, the estimation will surely diverge from the true value. In [11], the system model had already been verified by previous work [23]. The second problem is how to choose the system parameters. Proper choose of the parameters not only increases the performance of the algorithm, but also improves the efficiency of the estimation. In the thesis, we assume sufficient parameters in

the system, fully covering the possible variables of the formulas. Details of the design will be demonstrated in Chapters 3 & 5.

1.3.2 NLS Estimator for Removing the Interferometric Signal

After constructing the NLS estimator, we soon found out that the estimation easily got trapped into a local minimum. We looked into the principle of NLS and realized that when the parameter estimation problem was nonlinear, the larger number of the parameters, the higher chance that the algorithm would get trapped into a local minimum and failed to identify the correct parameters. We studied carefully on the characteristics of the system model and found out that: when 2 parameters of the system were fixed, the other parameters could be represented as a linear relationship of these two fixed parameters, thus suggesting us to embed the NLS algorithm with a linear least squares estimator. Simulation results have demonstrated the efficiency of such modifications.

1.3.3 NLS Embedded with Kalman Smoothing for Removing the Interferometric Signal

In the experiment, we found that the noise in reality might be a bit different from the expectation since the spectrum model was not perfectly Gaussian, which lead the bias to the estimation. All these factors would possibly result in a wrong estimate of the system state, thus lowering the wavelength shift detection accuracy. In order to improve the robustness of the algorithm, we embedded a Kalman filter within the NLS estimator. The design of the Kalman smoother in this thesis not only decreased the number of the system parameters, but also guaranteed the robust estimation against additional noise in the experiment. Details of the combined algorithm will be discussed in Chapter 4.

1.3.4 Estimator for Restoring the System Spectrums from Crosstalk Distortion

In a serial or parallel topology of FBGs array, crosstalk will bring a great difficulty in accurate wavelength shift detection. Normal techniques such as peak-detection, match edges filter cannot effectively recover the crosstalk spectrum since the reflected spectrums from two FBG will overlap with each other. The NLS algorithm proposed in this thesis implements multiple hypothesis to initialize the estimation, successfully separates two spectrums with crosstalk noise. Simulation and experiment results demonstrate the effectiveness of the design. Details of the algorithm for removing the crosstalk noise will be discussed in Chapter 5.

1.4 Organization of the Thesis

The thesis is organized as following:

Chapter 2 gives a detailed literature review on the necessary knowledge of FBGs and Bragg wavelength shift detection techniques.

Chapter 3 firstly introduces some basic knowledge of Nonlinear Least Squares estimation, following by the representation of the spectrum model in parametric mode. Two types of NLS estimation models are presented in this chapter. Detailed comparison and analysis are provided in later simulation and experiment.

Chapter 4 firstly introduces the basic knowledge about the Kalman filter and Kalman smoother. Detailed reformulation of the spectrum model into innovational state-space model is thereafter presented. Two types of Kalman filters are introduced in this chapter. Finally, simulation and experimental result are given.

Chapter 5 firstly introduces the spectrum models used in serial & parallel topology FBGs array, followed by the representation of the spectrum model in parametric

1.4 Organization of the Thesis

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mode. Simulation and experiment results are then provided to demonstrate the effectiveness of the algorithm.

Chapter 6 gives the conclusions of this thesis and provides recommendations of future research.

Chapter 2

Fiber Bragg Grating Sensors

This chapter will cover the necessary knowledge about the Fiber Bragg Grating Sensors (FBGSs), including photosensitivity, Fiber Bragg Grating and wavelength shift detection techniques. Photosensitivity is the basic of the latter two. It is the photosensitivity of fiber optics that enables the in-fiber index change, thus forming the grating. After discovery of the grating, the telecommunication and the fiber optical sensing field were greatly impacted. It is possible to make compact, lightweight and small accurate sensors. The FBGSs nowadays develop very fast and many of them have been commercialized. Yet, small wavelength drift detection is still a challenge to improve the accuracy of FBGSs, especially when many fiber Bragg grating components are multiplexed together.

2.1 Photosensitivity

Photosensitivity refers to a light-induced refraction index change in the fiber core when exposed to light with characteristic wavelength and intensity dependent on the material used in the core.

In 1978, Hill and his coworkers tried to observe the stimulated Raman scatter-

ing (SRS) phenomena in the germanium-doped-core high silica optical fiber [20]. When they tried to launch 514.5-nm light from their argon-ion laser into the core of a test strand of one of the fibers. They found that the transmitted light decayed after a few minutes. This phenomena suggested that a form of reflection hologram might be being written. Later, they tried to verify this suggestion by launching a 514.5-nm light into a test strand while monitoring light back-reflected from the fiber. Within a few seconds, an extremely strong buildup with time in the back-reflected light from the fiber amazingly occurred. After several trails, the researchers confirmed that they were dealing with a refractive index grating, or periodic modulation of the refractive index of the core of the fiber. The period of the modulation was equal to approximately half the wavelength of the writing source in the fiber medium. These in-fiber-core holograms are referred as “Hill gratings” or internally written Bragg gratings. The term “photosensitivity” has been widely accepted by the scientific community today and used to denote a light-induced index change that is permanent.

After the photosensitivity was observed, the researchers thought that the phenomena was a special case associated only with optical fiber having large germanium in the core and photoexcited with particular ultraviolet light. Yet, following years of research, people found that photosensitivity could be obtained by photoexcited at different UV light intensity and wavelength in many different optical fibers. Not all these fibers use germanium as the only dopant and some of them do not contain germanium at all.

2.2 Fiber Bragg Grating

Fiber Bragg Grating (FBG) is a promising in-fiber device emerging in the late 1980s. The device is simple but powerful. It sets revolutions to telecommunication and greatly impacts the optical fiber sensing area. Normally speaking, the

Figure 2.1: A schematic representation of a fiber Bragg grating (After[50])

periodicity of the index modulation of a fiber Bragg grating has a physical spacing that is one half of the wavelength of light propagating in the waveguide and the phase matching between the grating and incident light thus results in coherent back reflection. Figure 2.1 shows a simple fiber Bragg grating device. The device consists of a periodic modulation of refractive index along the core. For a simple Bragg gratings, the condition for the gratings to reflect the signal at a particular wavelength is as following:

$$\lambda_B = 2n_{eff}\Lambda \quad (2.1)$$

where λ_B is the center Bragg grating wavelength, n_{eff} is the effective refractive index of the fiber core and Λ is the grating spacing.

In this section, we will firstly introduce the types of FBGs and then present the properties of fiber Bragg gratings.

Figure 2.2: Illustration of a uniform Bragg grating with constant index of modulation amplitude and period (After[50])

2.2.1 Types of Fiber Bragg Gratings

Normally speaking, one can classify the FBGs according to their structures and properties. Corresponding to different grating pitch (spacing between grating planes) or tilt, FBGs can be classified as common fiber Bragg gratings, blazed Bragg gratings and chirped Bragg gratings. Corresponding to different photosensitivities, FBGs can be classified as Type I Bragg gratings, Type IIA Bragg gratings and Type II Bragg gratings. There are also other Bragg grating structures, such as superimposed multiple Bragg gratings [51], superstructure Bragg gratings[16], and phase-shifted Bragg gratings[44]. Interested readers can refer to related references.

- **Different Structures of Fiber Bragg Gratings**

- **Common Fiber Bragg Gratings**

The most common FBGs is Bragg reflector. The Bragg reflector has a constant pitch and serves as the simplest and most used fiber Bragg gratings. Figure 2.2 shows the structure of this common fiber Bragg gratings. Based on the pitch length and the induced index change, the Bragg reflector can be used as a broadband mirror, narrowband transmission and reflection filter.

Since the measurements from the Bragg reflector are wavelength encoded, the reflectors are considered as excellent component in measuring the strain and temperature. The characteristic of wavelength encoded can prevent the fiber sensor system from suffering the problems such as magnitude or intensity fluctuation. Moreover, based on this characteristic, each Bragg reflector has its own wavelength-encoded signature and it is thus possible to place a series of gratings in the same fiber, each having a distinct Bragg reflected signal. This design enables the grating to be used for wavelength division multiplexing and quasi-distributed sensing [33]. The gratings have also been implemented in tunable filter and semiconductor lasers [7, 21]. By varying the center Bragg wavelength, resonance feedback signal to the gratings can tune the laser wavelength. In [5], Ball and Morey designed an tunable laser using the above approach. Beside the applications mentioned above, by combining a series of Bragg reflectors, it is possible to design reflectors serving the dual purpose of tuning element and sensor, multi-point sensing [49, 6, 1].

Figure 2.3: (a) A schematic diagram of a blazed grating. (b) A vector diagram for the Bragg condition of a blazed grating (After[50])

– Blazed Bragg Gratings

The planes of blazed Bragg gratings tilt with respect to the fiber axis, which means that the grating planes are not perpendicular with the fiber axis. Figure 2.3 (a) demonstrates the structure of this types of gratings. As shown in the figure, tilting the grating planes with an angle to the fiber axis will result in light coupling into guided-cladding or radiation modes. The angle between the phase plane and the axis together with the index modulation will determine the coupling efficiency and the bandwidth of the light to be tapped out. The condition for a blazed Bragg gratings to reflect a resonance wavelength is similar to that of the Bragg reflector, except that an additional angle between the phase plane and the axis need to be considered. Figure 2.3 (b) shows the relationship between the angle and scattered wavevector[50]:

$$|v_i|^2 + |v_s|^2 - 2|v_i||v_s|\cos(\pi - 2\theta_b) = |K|^2 \quad (2.2)$$

The blazed Bragg gratings can be used to flatten the gain spectrum of erbium-doped fiber amplifiers [24]. This implementation is important in communication since it can uniform the signal to noise ratio at the output receiver when there are different wavelength signals propagating in the fibers. Another important implementation for blazed Bragg

Figure 2.4: A schematic diagram of a chirped grating with an aperiodic pitch.(After[50])

gratings is the mode conversion. In [19], Hill et al. demonstrated a successful mode conversion between forward propagation LP_{01} and LP_{11} modes.

– Chirped Bragg Gratings

The chirped Bragg gratings offers some advantages for specific applications in modern telecommunications. The chirped Bragg gratings has an aperiodic pitch, displaying monotonically varying period. Figure 2.4 illustrates the structure of this gratings. The monotonically varying period can be realized by axially modifying the period of the grating Λ or the refractive index or both. The relationship among the Bragg wavelength of this types of gratings, the Λ and the refractive index n_{eff} is as follows:

$$\lambda_B(z) = 2n_{eff}(z)\Lambda(z) \quad (2.3)$$

The gratings are applicable for dispersion compensation and the stable synthesis of multiple wavelength sources [52, 9]. Since the period of the gratings are varying, it is possible for the gratings to reflect different

frequencies present in the signal caused by chromatic dispersion, thus compress temporally broadened pulses. Furthermore, the grating could efficiently reflect the pump light from the optical fiber amplifier. In [17], Farries et al. demonstrated such use.

- **Photosensitivity Types of Fiber Bragg Gratings**

Based on the initial writing conditions (laser power and wavelength, CW or pulsed energy delivery) and fiber properties, the Bragg gratings can be classified into 3 distinct photosensitivity types.

- **Type I Bragg Gratings**

Type I Bragg gratings refers to gratings that are formed in normal photosensitive fibers under moderate intensities. For example, [53] presents a method to form the gratings by exposing the fiber to $47W/cm^2$ for 15 minutes. The spectrum response of Type I gratings is a typical uniform period. One fundamental characteristic of this gratings is that the reflective spectra is complementary to the transmission signal, suggesting that there is little reflection loss. Another characteristic of this gratings is that the gratings could be erased at relatively low temperature (approximately $200^\circ C$). Regardless of the characteristic of temperature erasure, the gratings work efficiently in daily telecommunication and sensor systems.

- **Type IIA Bragg Gratings**

Type IIA Bragg gratings is developed from Type I gratings [59]. This types of gratings are present most often in high Ge-content, small core fibers and are associated with the high internal fiber stress [48, 57]. The

spectral characteristic of Type IIA Bragg gratings are the same as Type I gratings. However, Type IIA Bragg gratings requires a long process to inscribe and it is not practical to be fabricated. Yet, compared with Type I gratings, Type IIA gratings demonstrates a much higher temperature erasure (high as 500°C).

– Type II Bragg Gratings

Type II Bragg gratings can be formed by injecting a single light pulse into small localized regions at the core/cladding boundary [3]. This type of gratings can produce very large refractive index modulations resulting from the physical damage caused by the localized fusion at the fiber core. One characteristic of this gratings is that there exists a threshold. When the incident pulse energy is above the threshold, a photoinduced modulation index will increase by two orders of magnitude. Below the threshold, the index grows linearly. Another characteristic of the gratings is their wavelength selective taps. The gratings pass wavelength that are longer than the Bragg wavelength, while the shorter wavelength are coupled into the cladding. Moreover, Type II Bragg gratings have superior temperature stability. Their temperature erasure is as high as approximately 1000°C . Last but not least, the most attractive features of Type II gratings is that they can be formed in a few nanoseconds.

2.2.2 Properties of Fiber Bragg Gratings

The properties of the FBGs enable the gratings to be practical implemented in telecommunication and sensing systems. The basic property of the gratings is their wavelength encoded characteristic, which prevents the measurement of the reflected or transmitted signal from deterioration by the light source and the other

perturbation. One of the most important properties is their temperature and strain sensitivity. Combining the above two characteristics, the FBGs can be used as strain and temperature sensors which can provide accurate measurement of strain and temperature variations. There are also other properties of FBGs, such as response to the exposure to intense UV irradiation, the impact of loss effects etc. In this section, we will mainly present the expression of the wavelength dependant reflection and the function of the strain and temperature perturbation.

- **Bragg Gratings Reflectivity**

The simplest form of fiber Bragg gratings are uniform fiber gratings. These types of gratings consist of a constant periodic modulation of the refractive index in the core of a single-mode optical fiber. The phase fronts of these gratings are perpendicular to the fiber's longitudinal axis. These simplest form of fiber Bragg gratings are the fundamental building block for most of the Bragg gratings structure. The light to be reflected by the Bragg gratings need to satisfy the Bragg condition (include the energy and momentum conservation), stated as follows:

(a) **Energy Conservation:**

$$\hbar\omega_f = \hbar\omega_i \quad (2.4)$$

where the frequency of the incident light is the same as the reflected light

(b) **Momentum Conservation:**

$$k_i + K = k_f \quad (2.5)$$

where k_i is the incident wavevector, K is the grating wavevector and k_f is the scattered light wavevector. The momentum conservation can also be expressed as:

$$2 \left[\frac{2\pi n_{eff}}{\lambda_B} \right] = \frac{2\pi}{\Lambda} \quad (2.6)$$

where λ_B is the Bragg grating wavelength, n_{eff} is the effective refractive index and Λ is the grating spacing shown in (2.2). The first-order simplification form of equation (2.6) can be written as equation (2.3).

For a uniform Bragg Grating, the index of refractive profile can be expressed as:

$$n(z) = n_0 + \Delta n \cos \left(\frac{2\pi z}{\Lambda} \right) \quad (2.7)$$

where n_0 is the average refractive index at the core of an optical fiber, Δn is the amplitude of the induced refractive index perturbation, and z is the distance along the fiber longitudinal axis. According to the coupled mode theory proposed by Lam and Garside [35], the reflectivity can be express as:

$$R(l, \lambda) = \frac{\Omega^2 \sinh^2(sl)}{\Delta k^2 \sinh^2(sl) + s^2 \cosh^2(sl)} \quad (2.8)$$

where $R(l, \lambda)$ is the reflectivity function of the grating length l and the wavelength λ . Ω is the coupling coefficient, $\Delta k = k - \pi/\lambda$ is the detuning wavevector, $k = 2\pi n_0/\lambda$ is the propagation constant, and $s^2 = \Omega^2 - \Delta k^2$.

• Strain and Temperature Sensitivity of Bragg Gratings

Although the fiber Bragg gratings can offer advantages such as low loss transmission, immunity to electromagnetic interference, humidity, and electrical

isolation, there are some external perturbations that can affect the Bragg gratings. It is shown by many experiments that any change in the center Bragg wavelength, associated with the external perturbation, is the sum of strain and temperature terms. When there is external strain on the gratings, the grating spacing and the refractive index will change according to the magnitude of the strain. If the external temperature changes, there will be a shift in the Bragg wavelength due to thermal expansion changes the grating spacing and the index of refraction. The relationship between the shift and these external perturbations is expressed as follows[50]:

$$\Delta\lambda_B = 2 \left(\Lambda \frac{\partial n_{eff}}{\partial l} + n_{eff} \frac{\partial \Lambda}{\partial l} \right) \Delta l + 2 \left(\Lambda \frac{\partial n_{eff}}{\partial T} + n_{eff} \frac{\partial \Lambda}{\partial T} \right) \Delta T \quad (2.9)$$

The first term of equation (2.9) represents the strain effect on an optical fiber. The second term in the equation represents the temperature effect on an optical fiber. These two terms can be expressed in another form according to [39].

The strain effect

$$\Delta\lambda_B = \lambda_B(1 - p_e)\varepsilon_z \quad (2.10)$$

where p_e is an strain-optical constant defined as[50]

$$p_e = \frac{n_{eff}^2}{2} [p_{12} - \nu(p_{11} + p_{12})] \quad (2.11)$$

The temperature effect

$$\Delta\lambda_B = \lambda_B(\alpha_\Lambda + \alpha_n)\Delta T \quad (2.12)$$

where $\alpha_\Lambda = (1/\Lambda)(\partial\Lambda/\partial T)$ is the thermal expansion coefficient for the fiber, $\alpha_n = (1/n_{eff})(\partial n_{eff}/\partial T)$ represents the thermo-optic coefficient.

- **Other Properties of Fiber Bragg Gratings**

When the grating is formed by exposing to UV light under the conditions for which the modulated index changes is saturated, the effective length will be reduced as the transmitted signal is depleted by reflection. As a result, two interesting features appear. Firstly, the spectrum will broaden a bit and is not a symmetric sine or Gaussian shape spectrum. Another feature is the small-shape spectral resonance on the short wavelength side of the grating centerline.

There are also other properties such as lifetime and reliability of the fiber Bragg gratings. Interested readers could refer to [50] for more information.

2.3 Wavelength Shift Detection Techniques

As we have discussed previously, fiber Bragg gratings are considered as excellent sensing elements, suitable for measuring external temperature and strain. Currently and in the future, the key detection issue for Bragg grating sensors (BGSs) is the determination of the often small measurand-induced wavelength shifts. Years of experiments show that, for an initial Bragg wavelength of $1.3\mu\text{m}$, to respectively resolve 0.1°C temperature and $1\mu\epsilon$ strain change, a wavelength resolution of 1pm is required. Such a requirement provokes the need for precise Bragg grating wavelength shift detection to achieve good sensor performance. In previous literature, various approaches have been proposed to demodulate the gratings. These various approaches can be classified into several schemes. These schemes use either a broadband source and various spectral analysis techniques such as edge filters, scanning Fabry-Perot filters, acousto-optic tunable filters and Mach-Zehnder interferometers [27], or a tunable laser source with a broadband detector [54]. We will cover most of them in this section and relate them to our work.

Figure 2.5: Edge filter for processing fiber Bragg grating return signals (After[50])

2.3.1 Spectral Analysis Techniques for Broadband Source

Edge Filters

There are two types of edge filters: Long Wave Pass filter and Short Wave Pass filter. In the long wave pass filter, the goal is to minimize transmission below a given wavelength and maximize transmission above it. In the short wave pass filter, the goal is to minimize transmission above a given wavelength and maximize transmission below it. Figure 2.5 shows a typical wavelength detection by implementing the edge filter.

When the cutoff is close to the signal wavelength, edge filter will provide a wavelength-dependent loss, offering a linear relationship between the wavelength shifts and the output intensity changes of the filter. In [37], Melle¹ constructed a system of such edge filter by using hi-birefringent fiber, each of which has its own reflected wavelength due to the slightly different index of refraction for each axis. Comparing light transmitted through the filter with light passed along a reference path recovers the wavelength shift of the sensor. They have demonstrated that the edge filter could provide a static strain resolution of $375\mu\epsilon$ and a dynamic strain range of $35,000\mu\epsilon$. This approach provides a very limited range and strictly requires

¹This document was published in 1991. The data and experimental results may seem to be out of date now. But at that time, the results were not bad.

the filter stability. In 1992, Melle et al. [38] further expanded the system of edge filters by removing the influences of light source fluctuations and losses in the optical fiber links via a ratiometric detection approach. The relationship between signal intensity I_s and the reference intensity I_R could be expressed as[50]

$$\frac{I_s}{I_R} = A \left(\lambda_B - \lambda_0 + \frac{\Delta \lambda}{\sqrt{\pi}} \right) \quad (2.13)$$

The system is low in cost, easy to use and has a high speed in measurements. It has been commercialized by Electrophotonics Co. of Canada.

Scanning Optical Filter

Scanning optical filter is one of the most successful demodulation approaches to track the Bragg grating sensor wavelength changes. It includes several well implemented filters such as tunable Fabry-Perot filter (FPF) [32], acousto-optic tunable filter (AOTF) [60], and Bragg grating-based filters [22]. In all cases, the tunable filter spectrum will convolute with light reflected from the gratings to form the output, and the optimal output is obtained when the spectrum of the tunable filter matches the reflected light from the grating. Tracking the tunable filter spectrum that optimizes the output will lead to the wavelength shift of the FBGS. Yet, at a given time, only a small slice of the optical spectrum is sampled. As a result, the measured resolution is dependent on the SNR of the return signal and the linewidths of the tunable filter and Bragg grating sensor (BGS). Implementation of the scanning optical filter demands a strong gratings and source be used for good resolution. For example, in [33], Kersey et al., demonstrated when the filter bandwidth is $x\%$ of the scanned wavelength range, the detectable energy per scan for close to 100% reflectivity is still only $x\%$ of the energy reflected by each grating per sampling period. Here $x\%$ refer to a proportion with any value.

Figure 2.6: Single Bragg grating strain sensor demodulation by employing a tunable Fabry-Perot filter. (After [33])

A tunable FPF is formed by mirroring the ends of two aligned fibers. The filters are characterized by band-pass resonances of Lorentzian lineshapes and a typical bandwidths of 0.3nm . The filter tuning is achieved by displacing the mirror separation to change the cavity spacing. The operating modes for FPF include a tracking (closed-loop) and a scanning mode. The former one is limited to addressing one BGS and the latter one is applicable to more than one BGS.

Figure 2.6 shows how a tunable FPF is used to demodulate the wavelength shift from a single Bragg grating. Light (tracking mode) from the broadband source transmits through the system and a narrow-band component is reflected from the BGS and directed to the FPF through the coupler. Here, the filter bandwidth is close to the grating bandwidth, where the free spectral range (FSR) between resonances is larger than the operating range of the grating. The sensor locks the FPF passband to the grating reflected signal, with a simple closed-loop design. After dithering the FPF resonance wavelength by a fraction of its passband at a frequency f_m , the filter produces a modulation in the optical output that contains components at f_m and its harmonics. When the wavelength of the BGS reflected signal are aligned with the wavelength of the FPF transmission peak, the funda-

mental component is nulled. Here, the amplitude of the modulation component at f_m serves as an error signal that can be used to lock the FPF passband to the Bragg wavelength, and the control voltage is used to give a measurement of the mechanical or thermal perturbation acting on the grating. In [33], Kersey et al. have reported a resolution of 1pm over the a working range in excess of 40nm for a single BGS. They also indicated that the strain detectability is better than $0.3\mu\text{m}$. The principle of the scanning mode for addressing more than one sensor is quite different from the tracking mode, where the nominal Bragg wavelengths and operational wavelength domains do not overlap and yet fall within the spectral envelope of the source and the FSR of the FPF. Interested readers could refer to [26, 15] for more information about the scanning mode.

Interferometric Wavelength-Shift Detection

The interferometers convert the wavelength shift from the grating into a phase changes at the interferometer output. Michelson and Mach-Zehnder are two common schemes of such interferometers. The output of the interferometer is a cosine output with a phase bias dependent on the strain of the grating. The main advantage of this approach is that the filtering open to many signal-processing schemes that offer very high phase resolution measurements and the filter could be used to sense the dynamic strain. Yet, the interferometer itself will cause the temperature-induced phase drift, where the drift is excluded from the detection of quasi-static strains. When the phase change in the interferometer reaches 2π , it will cause an limited unambiguous range, thus losing the absolute wavelength measurement.

Figure 2.7 shows the principle of employing a fiber Mach-Zehnder interferometer (MZIF). The light from the broadband source is coupled to the BGS and reflected to an unbalanced optical fiber Mach-Zehnder interferometer. The reflected light could be viewed as a tunable light source into the interferometer, where the wavelength shifts induced by the perturbation of the BGS resembling a wavelength modulated sources. In [29], Kersey et al. described an active heterodyne method,

Figure 2.7: Grating sensor system using Mach-Zehnder interferometer for dynamic strain measurements (After [30])

where a minimum detectable strain perturbation of $0.6\mu\epsilon/\sqrt{Hz}$ for frequencies greater than $100Hz$ was shown. This approach is clearly sensitive to dynamic strain changes. However, it is not suitable for quasi-static or static strain measurements due to drifts in the interferometer bias phase. More information and advanced work about the MZIF could be found on [31, 28, 55].

2.3.2 Wavelength-Shift Detection with Tunable Laser Source

For systems that use broadband sources, the received signal is usually low, resulting in a low signal to noise ratio (SNR), especially when time-division-multiplexing is used to multiplex a large number of sensors [27]. In 1994, Ball et al.[6] used a calibrated, narrow-linewidth, single-frequency, continuously wavelength-tunable erbium fiber to interrogate an array of three Bragg grating temperature sensors at $1550nm$. The approach immediately improved the signal-to-noise ratio as the measurement determined a maximum in grating reflected power and it dispensed with

the need for optical filtering. However, although the signal power was relatively high, residual reflections in the system might cause an unwanted interferometric signal [8, 11].

Jin [23] studied the interferometric signals in fiber-optic Bragg grating sensors that used tunable laser sources. The paper also proposed detailed formulations of the interferometric signals under various situations. Based on the interferometric signals model proposed by Jin, several filtering techniques [11, 10, 13] were proposed to remove the interferometric signals.

In [11], digital low-pass filters were employed to improve accuracy of the wavelength shift detection. Comparing with analogue devices, digital filters are cheap and convenient for their easy implementation and upgrade. Two types of digital filters were discussed in the paper: finite impulse response (FIR) and infinite impulse response (IIR) filters. As stated in the paper, the IIR filters will distort the signal due to the nonlinear phase response of the filter itself. The peak position of the filtered signal (IIR filter) may change with input signal and therefore introduce an error for peak wavelength detection from the filtered signal. The simulation and experiment results in the paper showed that the IIR filter performed worse than the FIR filter (A 0.78 pm RMS value for IIR vs 0.05 pm RMS value for FIR). The FIR filter in the paper obtained a good shift detection accuracy as high as 0.05 pm for simulation and a potential 0.625 pm for experiment. The good performance came from the fact that the wavelength shift depended only on the order of the FIR filter and was independent of the detailed input signal, and therefore did not affect the accuracy of wavelength shift detection. The frequency spectrum of the combined signal was firstly obtained by using FFT analysis, then the FIR filters could be properly designed based on this spectrum by using standard FIR design procedure. The effectiveness of the FIR technique was further tested for a range of noise amplitude and a range of noise frequencies by setting different intensity reflectivity and the optical difference path. However, the paper only showed the results when the noise frequency was relatively high. In the later part of the

experiment section, the authors mentioned that the measurement accuracy of FIR filter became worse when the noise was large in amplitude and low in frequency. As shown in Chapter 3 and Chapter 4 of this thesis, simple FIR filter is not so effective when large amplitude, low frequency noise exist.

In [10], a digital matched filter (DMF) was proposed to improve the wavelength detection accuracy. The DMF technique is being used widely in image processing and pattern recognition for the detection of weak signals in a noisy environment. For a DMF filter, a reference waveform is required in the system. The DMF has a frequency response exactly matched to the signal and allows for the passage of all the signal frequencies. By matching the filtered signal with the referenced signal, the noise power is reduced because the noise components that are not matched to the signals frequency are minimized. In the paper, Chan et.al. implemented the DMF to process the FBG spectrum with noise, using a computer program. The sampling range of the FBG spectrum was from 1556 to 1557 nm and the sampling points were from 128 to 8192. With the same simulation setup, the standard deviations of the wavelength measurement errors for 10 sets of measurement were found to be 1.74 and 5.65 pm for the DMF and the LPF, respectively. The experiment results also demonstrated an improvement of the DMF over LPF. Yet, there are two constraints regarding the DMF filter. Firstly, the DMF filters need a reference FBG to match the filtered signal, which will increase the complexity of the system and is not suitable for FBG sensor array. Secondly, the DMF filters have not provided robust results to show how the filters remove low frequency noise efficiently.

Later, an adaptive linear (ADALINE) network and a least mean squares (LMS) method [13] were applied to remove the signals with unwanted interferometric noise. The ADALINE network implemented a learning process with sufficient training. After proper training, the ADALINE network could obtain the optimal weight value from the LMS estimation. By multiplying the measurements with the optimal weight, the network could efficiently recover the spectrum from the

interferometric noise. Simulation results from the paper showed a 4pm wavelength detection error. Under the same simulation condition, the network provided better results than FIR filter did when the frequency of the noise was low. The experiment results again proof that the ADALINE network performed better than the FIR filter when the frequency of the noise was low. However, LMS estimation requires a strict modelling of the system. Any unexpected noise will affect the accuracy of the wavelength shift measurement. This thesis will propose an NLS estimator with embedded Kalman filter to solve this problem. The reader is referred to Chapter 4 of the thesis for more details.

2.3.3 Intrinsic Crosstalk Induced Wavelength-Shift Detection

The term “crosstalk” in FBGs refers to the unexpected effect of one reflected signal over another reflected signal in the same channel. Similar effects exist in other industry domains. For example, in telecommunication or telephony, crosstalk is often distinguishable as pieces of speech or signaling tones leaking from other people’s connections. While in wireless communication, crosstalk often denotes co-channel interference, and is related to adjacent-channel interference.

Recently, some interests have been focused on the interrogation of FBG sensors in serial topology [14]. Although the interrogation can improve signal to noise ratio and can be used for multiplexing a large number of sensors using TDM or a combination of WDM and TDM techniques in a single optical fiber, there are some drawbacks in using the serial topology. For example, the reflected signal from FBG separated in time but overlapping in wavelength can cause crosstalk, termed “spectral shadowing” and “multiple reflection” [33]. These kinds of intrinsic crosstalk will greatly affect the wavelength detection accuracy and the number of sensors in the array [12].

Multiple-reflection crosstalk arises due to light reflected from one grating which arrives in time window allotted to the detection of the downstream grating, because multiple reflection paths have delayed some of the first grating's signal [33]. In [34], Kersey et al. gave a detailed analysis of the multiple-reflection problem. Although the analysis was proposed to deal with the problem in Fabry-Perot array with ring reflectors, the definition of the multiple-reflection crosstalk is nearly identical to the case in FBGs, where we can find out that the effect is strongly dependent on the reflectivity of the gratings, and can be minimized by the use of the low reflectivity gratings. In [12], Chan et al., considered the first-order crosstalk pulses, which are the strongest multiple reflections. Consequently, a wavelength detection error formulation due to the multiple reflection crosstalk was derived and a 12 pm wavelength detection error in a 50-FBG sensor array was reported.

Spectral-shadowing crosstalk is defined as the distortion of a downstream FBG's spectrum that results from light having to pass through twice an upstream FBG. If the two gratings' center wavelengths are slightly offset, it appears that the downstream FBG is shifted further in the direction of the actual offset [33]. In [12], a 18pm wavelength detection error was reported for a 50-FBG array. When two FBG's width and reflectivity are equal, the worst case distortion occurs approximately when they are spectrally offset by their FWHM. Errors of $> 1 \mu\text{strain}$ are projected for a pair of interfering FBG's with a reflectivity $> 5\%$ [4].

The results in the later chapters of this thesis show that the algorithms proposed by us are not restricted to low-reflectivity gratings. They can achieve a detection accuracy of around 0.5pm in the simulations for parallel Bragg gratings with reflectivity $> 95\%$, which suggests that the proposed algorithm has a great potential to produce superior results even with a high-reflectivity array.

2.3.4 Merits of the Proposed Methods in This Thesis

Most of the techniques mentioned in Section 2.3.1 were designed to detect the wavelength shift in a general case by a proper hardware design (Scanning Optical Filters etc.) or simple DSP techniques (Edge Filters). On the other hand, this thesis proposes sophisticated pure DSP techniques not only to estimate the wavelength shift but also to remove some specific noise (interferometric signal etc.).

Those DSP techniques reviewed in Section 2.3.2 use a computer algorithm to remove the interferometric noise. Although the FIR filter, DMF and ADALINE could provide ways to detect the wavelength shift, their performances are limited due to the facts that the interferometric noise is quite structured and that the FSR is close to the FWHM of the FBG. To improve the wavelength shift detection ability, this thesis introduces an NLS estimator embedded with different filters (LLS filter, Kalman filter etc.). The proposed algorithm cannot only detect the wavelength shift accurately while the frequency of the interferometric noise is low, but also tolerate unexpected noise arising from the environment.

On the other hand, the crosstalk in the FBGSs array mentioned in Section 2.3.3 makes it difficult to measure the Bragg wavelength peak, thus decreasing the wavelength detection performance. The NLS algorithm proposed in this thesis implements multiple hypothesis to initialize the estimation, successfully separates the two spectrums with crosstalk noise. The experiment results suggest that the proposed algorithm have a great potential to produce superior results even with a high-reflectivity array, which is important to improve the efficiency of the FBGSs.

Chapter 3

Wavelength Shift Detection Using Nonlinear Least Squares (NLS)

The method of least squares is the cornerstone for most estimation theory, both classical and modern. Around 1795, Karl Gauss invented the least squares method to predict the motion of planets and comets using telescopic measurements. The motion of these bodies could be represented by 6 parameters. Gauss considered the problem of inferring the values of these 6 parameters from the measurement data.

The method of least squares firstly started with a linear model:

$$Z(k) = H(k)\theta + V(k) \quad (3.1)$$

where $Z(k) = \text{col}(z(k), z(k-1), \dots, z(k-N+1))$, $z(k) = h'(k)\theta + v(k)$, $h'(k)$ is the transpose of vector $h(k)$, $H(k)$ is a $N \times M$ observation matrix, and the estimation model for $Z(k)$ is

$$Z(k) = H(k)\hat{\theta}(k) \quad (3.2)$$

The least squares estimate of θ is denoted as $\hat{\theta}$.

Suppose that θ contains M parameters and $Z(k)$ contains N measurements. If $N > M$, we have more measurements than parameters, then the extra measurements can be used to offset the effects of measurement noise. The simplest approach for obtaining $\hat{\theta}(k)$ is to choose it so as to minimize the sum of the squared errors between its components and the respective components of θ , i.e., to minimize

$$\sum_{i=1}^M [\theta_i - \hat{\theta}_i(k)]^2 \quad (3.3)$$

In this chapter, some basic terms of least squares method will be introduced, followed by the formulation of the wavelength shift detection using nonlinear least squares estimation. To verify the validation of the estimation, simulation results and experiment results will also be provided.

3.1 Estimation

Estimation theory plays an important role in modern science and engineering. Most branches of science and engineering need estimation methods to complete the modelling of some particular systems. For example, Gauss firstly applied least squares estimation method to assist the prediction of planet motions. Based on Gauss's legacy, R. A. Fisher, Norbert Wiener and many others proposed plenties of estimation methods. As the development of digital computer, estimations from discrete-time viewpoint are greatly emphasized in the last century. It is considered that estimation theory is a natural adjunct to classical digital signal processing [42], "It produces time-varying digital filter designs that operate on random data in an optimal manner."

Estimation is part of modeling problems. The modeling normally consists of 4

Figure 3.1: Modeling problem (After [41]).

problems: representation, measurement, estimation and validation. Mendel [41] firstly forwarded the discussion on these problems. Representation problem deals with how something should be modelled. The measurement problem deals with which physical quantities should be measured and how they should be measured. The estimation problem deals with the determination of those physical quantities that cannot be measured from those that can be measured. The validation problem deals with demonstrating confidence in the model. Figure 3.1 describes the modeling problem.

Least squares estimation is a very basic and important technique and, under certain often-occurring conditions, other techniques reduce to it. In observational and experimental sciences, measurements errors are inevitable. Generally, averaging are applied to smooth out the errors in many cases, which requires more measurements than strictly necessary to determine parameters of system. This will result in the system overdetermined, leading no exact solution to the system. People usually project higher dimensional data into lower dimensional space to

suppress irrelevant detail. Least squares are those methods to accomplish such projection.

3.1.1 Linear Least Squares (LLS) Estimation

Linear least squares estimation applies least squares methods to linear model. The methods can be stated as follows:

For linear problems, an overdetermined linear system can be represented as

$$H\Theta \cong Z \quad (3.4)$$

with $m \times n$ matrix H , $m > n$. Least squares method is to find out the solution Θ to minimize the squared Euclidean norm of the residual vector $r = Z - H\Theta$,

$$\min_x \|r\|_2^2 = \min_x \|Z - H\Theta\|_2^2 \quad (3.5)$$

To minimize

$$\|r\|_2^2 = r^T r = (Z - H\Theta)^T (Z - H\Theta) = Z^T Z - 2\Theta^T A^T Z + \Theta^T A^T A \Theta \quad (3.6)$$

we take the first derivative with respect to Θ and set to 0, from which we can obtain the normal equations of the $n \times n$ linear system:

$$H^T H \Theta = H^T Z \quad (3.7)$$

3.1.2 Nonlinear Least Squares (NLS) Estimation

For real world data, linear specification are not “universal” in characterizing all economic relationship. An extension of linear specification is the nonlinear specification. Let’s consider a nonlinear specification

$$y = f(x; \theta) + e(\theta), \quad (3.8)$$

where f is a given function: $\mathbb{R}^\ell \times \Theta_1 \mapsto \mathbb{R}$, Θ_1 denotes the parameter space, a subspace of $\mathbb{R}^k$, x denotes an $\ell \times 1$ vector of explanatory variables, and θ denotes a $k \times 1$ vector of parameters, and $e(\theta)$ denotes the error of the specification. Given T observations of y and x , let

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix} \quad f(x_1, \dots, x_T; \theta) = \begin{bmatrix} f(x_1; \theta) \\ f(x_2; \theta) \\ \vdots \\ f(x_T; \theta) \end{bmatrix} \quad (3.9)$$

Equation (3.8) can be expressed as:

$$y = f(x_1, \dots, x_T; \theta) + e(\theta), \quad (3.10)$$

The nonlinear least squares method is to find a solution of θ to minimize the objective function

$$F(\theta) = \sum_{i=1}^T (y_i - f(x_i; \theta))^2, \quad (3.11)$$

A solution to this NLS minimization problem is some $\bar{\theta} \in \Theta$ that solve the first order function $\nabla_{\theta} F(\bar{\theta}) = 0$ and satisfy the second order condition: $\nabla_{\theta}^2 F(\bar{\theta})$ is positive definite.

When a solution to the first order condition of the NLS minimization problem cannot be obtained analytically, the NLS estimates must be computed using numerical methods. To optimize a nonlinear function, an iterative algorithm starts from some initial value of the argument in that function and then repeatedly calculates next available value according to a particular rule until an optimum is reached approximately. Gauss-Newton(GN) and Levenberg-Marquardt methods are two of such algorithms that are widely used. These two methods expand $f(x, \theta)$ in Taylor series around θ , making use of the first two orders of Taylor series and the first order condition of minimization to find out the improvement of $\bar{\theta}$ after each iteration. For more details regarding these two method, readers can refer to [18] for more information.

3.2 Problem Formulation

3.2.1 Spectrum Model

Figure 3.2 shows the system setup for our simulation and experiment. The narrow-band laser light delivered from the tunable light source passes through the coupler and reaches the FBG. Signal waves that are close to the Bragg wavelength are reflected by the FBG and the other waves pass through the FBG. Part of the transmissive waves are reflected by the reflection point and will interfere with the previous reflected waves from the FBG, thus forming the interferometric noise. Therefore, some methods are needed to recover the Bragg wavelength from these interferometric noise when the signals are detected by the photo-detector. In this section, we firstly construct the system model.

Figure 3.2: System Setup

A reflective type of FBG sensor using tunable laser sources is implemented in this thesis, as shown in Figure 3.3. The reflected spectrum of this reflective type can be written as[11]:

$$I_s(\lambda) = \frac{I_0}{4} R(\lambda - \lambda_B) \quad (3.12)$$

I_0 is the tunable source intensity and can be regarded as a constant within a small range around the Bragg wavelength. The divisor 4 is an empirical constant obtained from the derivation. $R(\lambda)$ represents the reflective spectrum of the FBG and is a rather complicated function of both operating wavelength and grating parameters. In [43], it is proposed that in many fiber sensor applications, the reflective spectrum could be reasonably approximated by a Gaussian function. The reflective spectrum is modeled as a Gaussian distribution with a spectral full-width-at-half-maximum (FWHM) of $\Delta\lambda_B$ and a center wavelength of λ_B :

$$R(\lambda) = \bar{R} \exp \left(-4 \ln 2 \left(\frac{\lambda - \lambda_B}{\Delta\lambda_B} \right)^2 \right) \quad (3.13)$$

where $\bar{R}$ is the peak reflectivity and λ is the wavelength in vacuum.

Figure 3.3: Signal and reflected waves in a reflective-type sensor: (a) signal wave, (b) first-order reflected waves from a point before the grating, (c) first-order re-reflected wave from a point behind the grating, (d) example of higher-order reflected wave.

The measured spectrum $I_{meas}(\lambda)$ can be represented by

$$I_{meas}(\lambda) = I_s(\lambda) + I_n(\lambda) \quad (3.14)$$

where $I_n(\lambda)$ is the noise. There are various types of noise in the system. The one that interests us is the interferometric signal caused by the interference between the signal wave and the residual reflected wave. We write the noise $I_n(\lambda)$ into $I_{sn}(\lambda) + I_{wn}(\lambda)$, where $I_{sn}(\lambda)$ is the interferometric signal and $I_{wn}(\lambda)$ represents the other noise, mainly random noise generated by the environment. The interferometric signal can be modelled as [23]:

$$I_{sn}(\lambda) = \frac{I_0}{2} \sqrt{R(\lambda - \lambda_B)\alpha^2} \cos\left(\frac{2\pi}{\lambda}n\Delta L + \phi\right) + \frac{\alpha^2 I_0}{4} \quad (3.15)$$

where α^2 is the intensity reflectivity of the reflection point, $n\Delta L$ is the optical path length difference between the signal wave and the reflected wave from the reflection point, and ϕ is a random phase factor caused by environmental disturbance. Under proper assumptions, the random noise $I_{wn}(\lambda)$ may be modelled as a Gaussian-white

Figure 3.4: An example of spectrums I_{meas} , I_s , I_{sn} in the computer simulation.

noise with mean σ^2 and variance $2\sigma^4/N$, where $N > 50$. (If the noise level were higher, a different noise model could be used to ensure that $I_{meas}(\lambda)$ was non-negative.)

Figure 3.4 shows a typical example of I_{meas} in the computer simulation, where the Gaussian model (3.13) is assumed with $\lambda_B = 1549.8\text{nm}$. (Details of other settings can be found in Section 3.3.) Note that due to the interferometric signal I_{sn} , the detection of the Bragg wavelength λ_B becomes a challenging task. When $n\Delta L$ is relatively large, the variation of I_{sn} is fast compared with I_s , and conventional filtering techniques are effective in removing I_{sn} . However, they fail if $n\Delta L$ is small. Since a mathematical model of the spectrum is available, this chapter treats the detection of λ_B as a parameter estimation problem instead of a filtering problem.

3.2.2 Nonlinear Least Square Estimation Model I

There have been many attempts to remove the interferometric signal I_{sn} from the measured spectrum $I_{meas} = I_s + I_{sn} + I_{wn}$ using conventional filtering techniques, so as to recover the original spectrum I_s [11, 13]. However, since I_{sn} has a mathematical model given by (3.15), the problem may also be treated as a parameter estimation problem rather than a filtering problem. More precisely, I_{meas} can be regarded as a signal with 7 unknown parameters:

$$I_{meas}(\lambda) = I_s(\lambda) + (I_{sn}(\lambda) + \sigma^2) + v(\lambda) \quad (3.16)$$

$$I_s(\lambda) = \rho_1 \exp \left(-4 \ln 2 \left(\frac{\lambda - \rho_2}{\rho_3} \right)^2 \right) \quad (3.17)$$

$$I_{sn}(\lambda) + \sigma^2 = \rho_4 \sqrt{I_s} \cos \left(\frac{2\pi}{\lambda} \rho_6 \right) - \rho_5 \sqrt{I_s} \sin \left(\frac{2\pi}{\lambda} \rho_6 \right) + \rho_7 \quad (3.18)$$

where $\rho_1, \rho_2, \dots, \rho_7$ are the unknown parameters given by $\rho_1 = \frac{I_0}{4} \bar{R}$, $\rho_2 = \lambda_B$, $\rho_3 = \Delta \lambda_B$, $\rho_4 = \sqrt{\alpha^2 I_0} \cos \phi$, $\rho_5 = \sqrt{\alpha^2 I_0} \sin \phi$, $\rho_6 = n \Delta L$, $\rho_7 = \frac{\alpha^2 I_0}{4} + \sigma^2$ and $v(\lambda)$ is a zero-mean Gaussian white noise. To distinguish it from the model to be introduced in next section, we name the system formulation (3.17) and (3.18) as Model I. Nonlinear least square (NLS) methods can be used to estimate these 7 parameters given the measurement I_{meas} . Once these 7 parameters are known, the interferometric noise I_{sn} can be easily reduced and the original spectrum I_s can be easily recovered. In fact, our main interest is only the value of ρ_2 , i.e., the Bragg wavelength λ_B .

3.2.3 Nonlinear Least Square Estimation Model II

Although the above model can provide quite effective estimation, as we will show in the simulation and experiment, we still need to make some modifications on the original estimation model to further improve the performance. Normally speaking,

the larger number of the parameters in the system, the easier for the estimation to be trapped into a local minimum, so we want to reduce the number of the parameters in the model. After reducing the number, we can not only increase the speed of the estimation but also reduce the risk of being trapped into a local minimum. On the other hand, the computer simulation may adopt a Gaussian model of $R(\lambda)$, but it might be inaccurate for experimental data and therefore will be dropped in the theoretical development below. Instead, $R(\lambda)/\bar{R}$ is assumed general but available, i.e., the shape of $R(\lambda - \lambda_B)$ is provided but not its magnitude $\bar{R}$ or its location λ_B . It will be obvious from the following development that the proposed method works with an arbitrary scaling of $R(\lambda)/\bar{R}$. Hence, for the simplicity of notation, $\bar{R}$ is assumed 1 and $R(\lambda)$ is assumed given.

We rewrite the spectrum model in Section 3.2.1 here and make some modifications. The spectrum model also uses the system setup as shown in Figure 3.2.

The spectrum at the photo-detector is:

$$I_{meas}(\lambda) = I_s(\lambda) + I_{sn}(\lambda) + I_{wn}(\lambda) \quad (3.19)$$

$$I_s(\lambda) = I_0/4 \cdot R(\lambda - \lambda_B) \quad (3.20)$$

$$I_{sn}(\lambda) = \sqrt{I_s(\lambda)} \cdot \alpha \sqrt{I_0} \cos(2\pi n \Delta L / \lambda + \phi) + \alpha^2 I_0 / 4 \quad (3.21)$$

$R(\lambda - \lambda_B)$ represents the reflective spectrum of the FBG, centered at the Bragg wavelength λ_B . As mentioned above, $R(\lambda - \lambda_B)$ is typically modelled by a Gaussian function (3.13), but it is not a required assumption here. $I_{wn}(\lambda)$ in (3.19) represents a random noise.

The spectrum model introduced above has six parameters: I_0 , λ_B , α , ϕ , $n\Delta L$ and σ^2 , which may not be known but can be estimated from the measured data. In the current case, NLS can be applied directly to estimate the six unknown parameters

from the measurement data $I_{meas}(\lambda)$ as what we have done in Model I. However, this is not the most effective way as the larger the number of parameters, the more time the NLS algorithm consumes and also the higher the chance that it gets trapped into a local minimum. Instead, this section formulates the problem at hand as an NLS problem of two variables, within which a linear least-square problem of 4 variables will be solved. To this end, (3.19) is rewritten as

$$\begin{aligned}
 I_{meas}(\lambda) = & q_1 R(\lambda - \lambda_B) \\
 & + q_2 \sqrt{R(\lambda - \lambda_B)} \cos(2\pi n \Delta L / \lambda) \\
 & - q_3 \sqrt{R(\lambda - \lambda_B)} \sin(2\pi n \Delta L / \lambda) \\
 & + q_4 + v(\lambda)
 \end{aligned} \tag{3.22}$$

where $q_1 = I_0/4$, $q_2 = \alpha I_0/2 \cdot \cos \phi$, $q_3 = \alpha I_0/2 \cdot \sin \phi$, $q_4 = \alpha^2 I_0/4 + \sigma^2$ and $v(\lambda)$ is a zero-mean Gaussian-white noise. Hence, when λ_B and $n\Delta L$ are fixed, q_i can be estimated from the measurement data I_{meas} as a linear least-square (LLS) problem. We name the system formulation (3.22) introduced above as Model II. This model reduces the original NLS problem into a problem of two variables only, namely λ_B and $n\Delta L$.

As mentioned above, new variables q_1, q_2, q_3, q_4 are introduced to redefine the set of 6 unknown parameters as $\lambda_B, n\Delta L, q_1, q_2, q_3$ and q_4 . When the first two parameters $\lambda_B, n\Delta L$ are fixed, the estimation problem becomes a linear least squares (LLS) problem in q_1, q_2, q_3 and q_4 . However, it does not mean that the values of λ_B and $n\Delta L$ are assumed a priori. In fact, all 6 parameters will be estimated using a nested algorithm. The nonlinear least squares (NLS) is used to estimate λ_B and $n\Delta L$. Within each iteration of this NLS estimation, λ_B and $n\Delta L$ can be regarded fixed and hence the optimal values of q_1, q_2, q_3, q_4 can be estimated using a standard linear least squares method. This is what we meant by the problem being reduced to a NLS problem with only 2 unknowns embedded with a 4-parameter

Table 3.1: Ranges of parameter values in the computer simulation for both models.

	minimum	maximum
$\ln(I_0/4 \cdot \bar{R})$	-0.1	0.1
λ_B	1550-0.1	1550+0.1
$\log(\Delta\lambda_B)$	-1	-0.3979
$\ln(2\alpha/\sqrt{\bar{R}})$	-2	-1
ϕ	$-\pi$	π
$\log(n\Delta L)$	$\log(2 \times 1550^2)$	$\log(12.5 \times 1550^2)$
$\ln(1 + 4\sigma^2/(\alpha^2 I_0))$	0.8	1.2

LLS engine in the NLS estimation Model II.

3.3 Simulation

To verify the effectiveness of the NLS approach, proper simulation and experiment have been designed to employ the estimation model. In this section, we will introduce the details of the simulation, including the principles to choose initialization, the performance after reducing the number of parameters, and a comparison between NLS and low-pass FIR filter etc.

The actual parameter values for Model I and Model II are randomly generated and distributed in the ranges shown in Table 3.1. The logarithm in Table 3.1 is to guarantee that the value of the parameters will not be negative.

3.3.1 Initialization of NLS

Initialization is quite important in estimation theory. Good initialization of the parameters can not only increase the efficiency of the program, it will also protect the estimation from being trapped into a local minimum.

In our simulation design, the initialization of the parameters come from the real experimental setup. λ_B is the parameter we want to detect, ranging around the

center wavelength of the FBG; I_0 , α , $n\Delta L$ and σ^2 could be known from the setup, but the value of these variables are provided with some uncertainties; ϕ is a random phase factor, the initialization of which need to be considered carefully.

The initializations of the parameters are generated from the simulation data. $I_0/4 \cdot \bar{R}$ is the peak value of reflected intensity. Since the measurements are generated with random Gaussian noise, moving average is firstly implemented in the algorithm to smooth the measurements, followed by finding the maximum of the data. We will use this maximum value as $I_0/4 \cdot \bar{R}$. λ_B is initialized as the wavelength corresponding to the peak intensity. $\Delta\lambda_B$ is the FWHM of the spectrum, which we can initialize by searching for the indexes corresponding to half of the maximum magnitude. $n\Delta L$ is the optical path length difference between the FBG reflected spectrum and the spectrum reflected by the reflect point. This parameter corresponds to the frequency of the interference signal and it should be within the coherence length of the incident light. In our experiment, the frequency of the interference signal is close to the FBG reflected spectrum so we initialize the frequency of the interference signal to be randomly within the range from 2Hz to 12.5Hz.

For Model I, the initialization of ϕ takes 4 different values 0, $\pi/2$, π and $3 \cdot \pi/2$, which means that we need to repeat the NLS estimation 4 times and then choose the one providing the best estimate. $2\alpha/\sqrt{\bar{R}}$ and ϕ are related with the intensity and the phase of the interferometric signal. $1 + 4\sigma^2/(\alpha^2 I_0)$ is related with the background noise level, which can be estimated from the measured spectrum.

3.3.2 Simulation for Model I and Model II

After proper initialization, we can now investigate the performance of these two models. Figure 3.5 and Figure 3.6 are the simulation result after implementing model I and model II to Figure 3.4.

Figure 3.5: Simulation result obtained by applying the NLS Model I to Figure 3.4. A refers to the measured spectrum I_{meas} with interferometric signal and white noise; B refers to the optimal estimate of I_{meas} given by Equation (3.23), which should be close to A for a successful estimation; C refers to the actual FBG signal spectrum I_s in the simulation; D refers to the FBG signal spectrum estimated by the NLS algorithm; E refers to the interferometric signal I_{sn} in the simulation; F refers to the interferometric signal estimated by the NLS algorithm.

Figure 3.5 shows the simulation result when Model I is applied to Figure 3.4 to estimate the 7 unknown parameters $\rho_1, \dots, \rho_7$. After the parameters are identified, the optimal estimates $\hat{I}_s$ of I_s and $\hat{I}_{sn}$ of I_{sn} can be constructed from Equations (3.17)–(3.18), and hence the optimal estimate $\hat{I}_{meas}$ of I_{meas} , given by

$$\hat{I}_{meas} = \hat{I}_s + \hat{I}_{sn} + \sigma^2 \quad (3.23)$$

Similar to Figure 3.5, Figure 3.6 shows the simulation result when Model II is applied to Figure 3.4 to estimate the parameters.

In the estimation, our main interest is the value of center Bragg wavelength λ_B and we use $er = \hat{\lambda}_B - \lambda_B$ as a basic indicator to evaluate the performance of the

Figure 3.6: Simulation result obtained by applying the NLS Model II to Figure 3.4. The definitions in the figure are the same as in Figure 3.5.

simulation result, where $\hat{\lambda}_B$ is the estimate of λ_B . In Figure 3.5, the value of er is $-13.8pm$; In Figure 3.6, the value of er is $-9.6pm$. Obviously, Model II provides a better estimate of λ_B than Model I.

Although the above results have shown an improvement of Model II, it is too hasty to conclude that Model II performs better than Model I. To verify the improvement of Model II over Model I in general, a total of 4,000 random spectrums similar to the one in Figure 3.4 are used in the computer simulation, with the actual values randomly generated and uniformly distributed in the ranges shown in Table 3.1. The measured spectrum I_{meas} is sampled at an 1pm interval. The results are summarized in Figure 3.7, labelled as “NLS-MI” for Model I and “NLS-MII” for Model II.

For comparison, a conventional low-pass FIR filter followed by peak-detection is also used, and the results are plotted on the same graph, labelled as “FIR”. Figure 3.7 shows the cumulative distribution function of $er = \hat{\lambda}_B - \lambda_B$ over the 4,000 random spectrums, where $\hat{\lambda}_B$ denotes the estimate of λ_B . The improvement of Model II over Model I and FIR is obvious. FIR has a 90% confidence interval of

Figure 3.7: Computer simulation results for λ_B

$[-33.5, 34.4]$ pm, Model I has a 90% confidence interval of $[-15.33, 14.6]$ pm, while Model II has a 90% confidence interval of $[-0.8112, 0.9007]$ pm. The detection accuracy has been improved by more than 30 times.

3.4 Experiment

The experimental setup is shown in Figure 3.8, the setting of which is the same as Figure 3.2. The light from a CL15-16ASE fiber source passed through a 3dB coupler and divided equally into two arms with one of them connected to an FBG sensor. The FBG was mounted on a fixed stage (FS) and a translation stage (TS) for the purpose of applying strain to change the Bragg wavelength λ_B . The reflected light from the FBG returned through the same coupler and was detected by Ms9710C optical spectrum analyzer. The tuning step and range of the wavelength from the tunable laser were set as 1pm and 1nm, respectively. The signal from the optical spectrum analyzer was synchronously recorded by a

Figure 3.8: Experiment Setup

personal computer (PC). Fig. 3.9 shows a typical measured spectrum I_{meas} in the experiments. Fig. 3.10 shows the estimation results after applying the NLS Model II to Fig. 3.9. For comparison, Fig. 3.11 shows the filtering results after applying the FIR filter to Fig. 3.9. It is surprising to observe that the FIR filtering provides a better estimate than the NLS algorithm.

To verify the effectiveness of the approaches in general, we need to collect more data from the experiment to do some statistical analysis. In the experiment, we made use of the translation stage to apply strain to change the center Bragg wavelength. For each value of center Bragg wavelength λ_B , a “good” spectrum without interferometric signal was first obtained to provide a reference spectrum $R(\bullet)$. Then 10 sets of I_{meas} , with different unwanted interferometric signals in different sets, were measured. The root-mean-square (RMS) error in the estimation of λ_B over these 10 sets was recorded. This procedure was repeated for 10 different values of λ_B and the 10 RMS values are summarized in Table 3.2. It was seen that FIR performed reasonably well because $n\Delta L$ was relatively large in the experimental data ($> 1.5\text{cm}$). NLS performed worse than FIR in the experiment. It is supposed that the system model we are using does not fully match the real spectrum.

Figure 3.9: Typical measured spectrum I_{meas} in the experiments.

Figure 3.10: Typical estimated spectrum in the experiments. A refers to the measured spectrum I_{meas} ; B refers to the estimation of I_{meas} ; C refers to the estimation of I_s ; D refers to the estimation of the interferometric signal I_{sn} .

Figure 3.11: Typical spectrum after applying a FIR low-pass filter.

In principles, FIR filtering explores the properties of the signal itself. On the other hand, the NLS estimation assumes that the signal is generated from a system model and tries to explore the properties of that system. It estimates the system parameters by projecting higher dimensional data into a lower dimensional space to suppress irrelevant details. Hence, the least squares method smoothes the data and is also a form of low-pass filtering. Since the NLS estimation attempts to fit a system model to the data, its success or not relies on whether the assumed structure of the model is correct. A classical example is to try to fit a straight-line model to a set of experimental data. The estimation cannot be good if the data do not really lie on a straight line. Although NLS is also a form of low-pass filtering, whether it will perform better or worse than the FIR filtering therefore depends strongly on the accuracy of the system model (such as the Gaussian model of the FBG spectrum). In the computer simulations, the model is accurate and therefore the results show that the NLS estimation out-performs the FIR filtering. On the other hand, the model is less accurate in real-life experiments and therefore the FIR filtering out-performs the NLS estimation there.

Table 3.2: Root-mean-square error in the estimation of λ_B (pm).

	1	2	3	4	5
FIR	5.5408	4.7854	5.4222	6.2530	4.2426
NLS-LLS	8.4669	5.4100	6.4496	7.7398	8.6305
	6	7	8	9	10
FIR	5.3944	6.8484	3.8210	2.7386	5.1672
NLS-LLS	6.9901	6.9841	3.9841	4.8732	7.8067

3.5 Conclusions

An NLS algorithm has been presented in this chapter for the removal of the unwanted interferometric signal from the contaminated FBG spectrums. Simulations have confirmed a good accuracy of Bragg wavelength detection. Improvements over conventional filtering approaches have also been observed in the simulations. Yet, with real experimental data, the algorithm provided worse estimation than the low-pass FIR filtering, which suggests that the NLS approach is sensitive to the model used and a more robust algorithm need to be developed to remove the unwanted interferometric signal. This leads us to the Kalman filtering algorithm in the next chapter.

Chapter 4

Wavelength Shift Detection Using an NLS Embedded with Kalman Smoothing

As we have discussed in Chapter 3, despite its success in the computer simulations, NLS estimation does not work equally well for experimental data. It is because NLS estimation is relatively model sensitive. The spectrum model in Section 3.2.1 is good but by no means perfect. To overcome this sensitivity to the spectrum model, a Kalman filter (KF) is proposed to replace the LLS engine inside the NLS estimation.

In this chapter, we will firstly introduce some basic principles of the Kalman filter and Kalman smoother, followed by embedding the Kalman filter into the NLS estimation. Simulation and experiment result will be provided to demonstrate the effectiveness of the embedded Kalman smoother.

4.1 Kalman Filter

Around 1959, Kalman developed a recursive mean-squared state filter with the name of Kalman filter. Originally, Kalman filter was designed to solve problems within control theory, but it turned out lately to be adopted in many other areas. Kalman filter is now widely accepted as a signal processing tool.

Consider the discrete linear time-invariant system given by:

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (4.1)$$

$$y_k = Hx_k + v_k \quad (4.2)$$

where x is the state vector, k is the time index, u is the known control input, y is the measurement, and w_k and v_k are uncorrelated zero-mean white noise input sequences. We use Q to denote the positive semidefinite covariance of w_k and R to denote the positive semidefinite covariance of v_k , and $\bar{x}$ to denote the expected value of x . The problem is to find an estimate $\hat{x}_{k+1}$ of x_{k+1} given the measurements $y_0, y_1, \dots, y_k$. The Kalman filter can be used to estimate the state vector as follows:

$$K_k = AP_k H^T (HP_k H^T + R)^{-1} \quad (4.3)$$

$$\hat{x}_{k+1} = Ax_k + Bu_k + K_k(y_k - H\hat{x}_k) \quad (4.4)$$

$$P_{k+1} = (AP_k - K_k HP_k)A^T + Q \quad (4.5)$$

The above equations are the prediction form of Kalman filter, so x_k is estimated based on the measurements up to and including time $k - 1$. The filter is initialized with $\hat{x}_0 = \bar{x}_0$, and $P_0 = E[(x_0 - \bar{x}_0)(x_0 - \bar{x}_0)^T]$. It can be shown [2] that the Kalman filter estimate has several attractive properties. It is unbiased, and of all affine estimators, it is the one that minimizes the variance of the estimation error. In addition, if x_0 , w_k and e_k are jointly Gaussian, $\hat{x}_{k+1}$ is the conditional mean of x_{k+1} given the previous measurements up to time k , i.e., $\hat{x}_{k+1} = E[x_{k+1}|y_0, y_1, \dots, y_k]$. Also the Kalman filter estimate is the one that maximizes the conditional probability density function of the state given the measurement history.

As we have discussed in Section 3.2.1, the FBG reflected spectrum is continuous in wavelength. In a digital implementation the state of the reflected spectrum is required at discrete instants, so we need discrete, or sampled, data to perform the estimation.

Suppose the system model is described by

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \quad (4.6)$$

with the measurements

$$y(t) = Hx(t) + v(t) \quad (4.7)$$

Let the initialization of x and P be $\bar{x}_0$ and P_0 . $w(t)$ and $v(t)$ are zero-mean white noise processes uncorrelated with each other and with $\bar{x}_0$, and have covariance matrices Q and R respectively. If the input $u(t)$ and measurement $y(t)$ are sampled at every T sec, we can apply the discrete Kalman filter to the continuous system

by discretizing it. Firstly, we need to find out the solution to (4.6)

$$x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^t e^{A(t-\tau)}Bu(\tau)d\tau + \int_{t_0}^t e^{A(t-\tau)}Gw(\tau)d\tau. \quad (4.8)$$

To discretize the solution, let $t_0 = kT$, $t = (k+1)T$ for an integer k . The sampled state function could be written as $x_k \triangleq x(kT)$. Then equation (4.8) becomes

$$x_{k+1} = e^{AT}x_k + \int_{kT}^{(k+1)T} e^{A[(k+1)T-\tau]}Bu(\tau)d\tau + \int_{kT}^{(k+1)T} e^{A[(k+1)T-\tau]}Gw(\tau)d\tau. \quad (4.9)$$

where $u(\tau)$ is constant over the integration interval since that input $u(t)$ is reconstructed from the discrete input sequence u_k by a zero-order hold. The third term of equation (4.9) describes a discrete white noise sequence. We define it as

$$w_k = \int_{kT}^{(k+1)T} e^{A[(k+1)T-\tau]}Gw(\tau)d\tau. \quad (4.10)$$

Then the sampled version of equation (4.6) could be written as

$$x_{k+1} = A^s x_k + B^s u_k + w_k \quad (4.11)$$

where

$$A^s = e^{AT} \quad (4.12)$$

$$B^s = \int_0^T e^{A\tau} B d\tau \quad (4.13)$$

$$Q^s = \overline{w_k w_k^T} = \int_0^T e^{A\tau} G Q G^T e^{AT\tau} d\tau. \quad (4.14)$$

The measurement equation is easy to be discretized since it has no dynamics:

$$y_k = H x_k + v_k \quad (4.15)$$

With proper calculation, the covariance of v_k become

$$R^s = \frac{R}{T}. \quad (4.16)$$

After discretization, Equation (4.3) – (4.5) could be applied to perform the estimation.

4.2 Kalman Smoother

The FBG reflected spectrum model is wavelength-based, so we can implement a smoothing method to increase the robustness of the estimation. A smoothing problem refers to using all the data, past and future, to determine the state estimation $\hat{x}_k$ for $k \in (0, N)$, where the data are available over the interval $(0, N]$. Consequently, smoothing cannot be carried out in real time, because we need all the data

before we compute a smoothed estimate. There are several types of smoothing, such as fixed-interval smoother, fixed-point smoother and fixed-lag smoother [42]. Since the sampling number of the system in this thesis is fixed, we consider fixed interval smoothing method in our algorithm.

A typical Kalman smoother combines a forward Kalman filtering with a backward Kalman filtering. For example, to find the optimal smoothing state estimate $\hat{x}_k$ at time k in the time interval $(0, N]$, we not only need to consider the data y_j for $0 < j \leq k$, but also for $k < j \leq N$. The forward Kalman filter is straight ahead. What we need to do is to denote the estimate at time k based on the data y_j for $0 < j \leq k$ by $\hat{x}_k^f$, then we perform the recursive estimate through equation (4.3) – (4.5).

A backward Kalman filter is a filter that runs backward in time beginning at N to provide the best estimation of x_k incorporating the data z_j for $k < j \leq N$. To combine the forward filter and backward filter, we need to derive the proper formation of the backward filter. Firstly, we write the system as

$$x_k = A^{-1}x_{k+1} - A^{-1}Bu_k - A^{-1}Gw_k. \quad (4.17)$$

This is a recursion for decreasing k beginning at $k = N$. The error covariance measurement update is

$$P_k^{-1} = (P_k^-)^{-1} + H^T R^{-1} H. \quad (4.18)$$

The time update for stochastic system (4.17) is

$$P_k^- = A^{-1}(P_{k+1} + GQG^T)A^{-T}, \quad (4.19)$$

where $A^{-T} \triangleq (A^{-1})^T$. Now, we define the information matrices

$$S_k \triangleq P_k^{-1} \quad (4.20)$$

$$S_k^- \triangleq (P_k^-)^{-1} \quad (4.21)$$

After substituting (4.20) into (4.19), we can rewrite the updates as:

$$S_k = S_k^- + H^T R^{-1} H S_k^- = A^T [S_{k+1} - S_{k+1} G (G^T S_{k+1} G + Q^{-1})^{-1} G^T S_{k+1}] A. \quad (4.22)$$

Let us define the gain matrix for the backward filter as

$$K_k = S_{k+1} G (G^T S_{k+1} G + Q^{-1})^{-1} \quad (4.23)$$

and also define intermediate variables as

$$\hat{z}_k^b = P_k^{-1} \hat{x}_k^b = S_k \hat{x}_k^b \quad (4.24)$$

$$\hat{z}_k^{b-} = (P_k^-)^{-1} \hat{x}_k^{b-} = S_k^- \hat{x}_k^{b-} \quad (4.25)$$

Then the measurement update for the backward filter can be written as

$$\hat{z}_k^b = \hat{z}_k^{b-} + H^T R^{-1} y_k. \quad (4.26)$$

The backward time update for the estimate for stochastic system is

$$\hat{z}_k^{b-} = A^T (I - K_k G^T) (\hat{z}_{k+1}^b - S_{k+1} B u_k). \quad (4.27)$$

The forward and backward filter update diagram is as depicted in Fig 4.1. We

Figure 4.1: Forward and backward filter update diagram.

should note that: for forward filter, the priori estimate $\hat{x}_k^-$ occurs to the left of k and the posteriori estimate $\hat{x}_k$ occurs to its right; while for backward filter, the priori estimate occurs to the right of k and the posteriori estimate occurs to its left.

The entire smoothing scheme, including a forward filter, backward filter and a smoother, is shown in Table 4.2.

Table 4.2 Optimal Smoothing Scheme

System and measurement model:

$$x_{k+1} = Ax_k + Bu_k + Gw_k$$

$$y_k = Hx_k + v_k,$$

$$x \sim (\bar{x}_0, P_0), w_k \sim (0, Q), v_k \sim (0, R).$$

$$x_0, w_k, v_k \text{ uncorrelated.}$$

Forward filter:

initialization:

$$\hat{x}_0^f = \bar{x}_0, P_0^f = P_0$$

time update:

$$P_{k+1}^{f-} = AP_k^f A^T + GQG^T$$

$$\hat{x}_{k+1}^{f-} = A\hat{x}_k^f + Bu_k$$

measurement update:

$$K_k^f = P_k^{f-} H^T (HP_k^{f-} H^T + R)^{-1}$$

$$\hat{x}_k^f = \hat{x}_k^{f-} + K_k^f (y_k - H\hat{x}_k^{f-})$$

$$P_{k+1}^f = (I - K_k^f H) P_k^{f-}$$

Backward filter:

initialization:

$$\hat{z}_N^{b-} = 0, S_N^- = 0$$

measurement update:

$$S_k = S_k^- + H^T R^{-1} H$$

$$\hat{z}_k^b = \hat{z}_k^{b-} + H^T R^{-1} y_k$$

time update:

$$K_k^b = S_{k+1} G (G^T S_{k+1} G + Q^{-1})^{-1}$$

$$S_k^- = A^T (I - K_k^b G^T) S_{k+1}$$

$$\hat{z}^{b-} = A^T (I - K_k^b G^T) (\hat{z}_{k+1}^b - S_{k+1} B u_k)$$

Smoother:

$$K_k = P_k^f S_k^- (I + P_k^f S_k^-)^{-1}$$

$$P_k = (I - K_k) P_k^f$$

$$\hat{x}_k = (I - K_k) \hat{x}_k^f + P_k \hat{z}_k^{b-}$$

4.3 Kalman Filter Embedded in an NLS Estimator

In Section 3.2.1, we have already presented the system spectrum model. We will use the same model in this section. Developing from the NLS estimation, we will here embed a Kalman filter in NLS estimator to obtain the optimal estimation of the system parameters.

The embedded Kalman filter methods involve reformulation of the problem into a state-space framework to efficiently optimize the cost function in a recursive manner. The optimal estimation is achieved by implementing the Kalman smoother listed in Table 4.2. Here, we will introduce two models of embedded Kalman filter based on NLS-MI and NLS-MII respectively. Finally, simulation and experimental result will be provided to show the performance of these two designs.

4.3.1 NLS with Kalman Filter Model I (NKMI)

Although the NLS Model I in Section 3.2.2 is effective, it has two limitations. Firstly, NLS is a nonlinear optimization problem, and methods such as the well-known Gauss-Newton method can only find a local optimum. A proper initialization is needed to get good results. The simulation shows that the initial guess of the unknown phase angle ϕ is critical. To guarantee good results, four different initial guesses of ϕ have been used and the best one selected. Also, for a general NLS problem, the chance of being trapped into a local optimum instead of a global one, will increase with the number of parameters ρ_i to be estimated. After we combine the NLS optimization with a Kalman smoother, the algorithm will reduce the number of parameters from 7 to 3 in NLS Model I, thus reducing the chance of being trapped in a local optimum. Furthermore, the unknown phase angle ϕ is one of those 4 parameters that will be eliminated.

The second limitation of the NLS method is the assumption of the residue I_{wn} being a white noise. If this assumption is not satisfied, the estimation can be very poor sometimes. Unfortunately, from the experimental data we observe in Section 3.4, this white-noise assumption is not very valid. There is usually some variations in the experimental spectrum, in addition to the random white noise, which exhibits a much lower-frequency variation than the latter. However, by combining the NLS method with Kalman smoothing, this unknown variations can be taken into account to a certain extent and hence good results can be obtained. We denote this model combining NLS and Kalman filter as Kalman filter Model I. The details of this model are given below.

The key element of Kalman filtering or Kalman smoothing is a state-space model. Let 4 state-variables x_1, x_2, x_3, x_4 be defined by

$$\begin{aligned} x_1(\lambda) &= \rho_1 \exp \left(-4 \ln 2 \left(\frac{\lambda - \rho_2}{\rho_3} \right)^2 \right) \\ x_2(\lambda) &= \sqrt{(\rho_4^2 + \rho_5^2) I_s} \cos \left(\frac{2\pi}{\lambda} \rho_6 + \phi \right) \\ x_3(\lambda) &= \sqrt{(\rho_4^2 + \rho_5^2) I_s} \sin \left(\frac{2\pi}{\lambda} \rho_6 + \phi \right) \\ x_4(\lambda) &= \rho_7 \end{aligned}$$

Then obviously $I_s(\lambda) = x_1(\lambda)$ and $I_{sn}(\lambda) + \sigma^2 = x_2(\lambda) + x_4(\lambda)$.

A state-space model for this state-vector $x = (x_1, x_2, x_3, x_4)^T$ is given by

$$\frac{d}{d\lambda} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -8 \ln 2 \left(\frac{\lambda - \rho_2}{\rho_3} \right) & 0 & 0 & 0 \\ 0 & -4 \ln 2 \left(\frac{\lambda - \rho_2}{\rho_3} \right) & \frac{2\pi}{\lambda^2} \rho_6 & 0 \\ 0 & -\frac{2\pi}{\lambda^2} \rho_6 & -4 \ln 2 \left(\frac{\lambda - \rho_2}{\rho_3} \right) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + u \quad (4.28)$$

$$I_{meas} = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + v \quad (4.29)$$

where u is called the system noise and v is called the measurement noise in the Kalman filter literature, both of which are Gaussian-white. Note that this model is a continuous-time linear time-varying model, which will be converted into a discrete-time linear time-varying model before Kalman filtering or smoothing is applied. It is also important to note that this state-space model is linear. Therefore, a standard Kalman filter is sufficient and no extended Kalman filter is required, and hence nice convergence properties remain.

Since the independent variable in the state-space model (4.28)–(4.29) is the wavelength λ instead of time, causality is not an issue and Kalman smoothing instead of Kalman filtering can be used to give much better results[36]. The Kalman smoother in this work adopts a formulation similar to that of Rauch-Tung-Striebel[56].

The state-space model (4.28)–(4.29) consists only of 3 parameters: ρ_2 , ρ_3 and ρ_6 . The other 4 parameters ρ_1 , ρ_4 , ρ_5 and ρ_7 are now hiding in the state-vector x . Hence, an NLS algorithm can be used to estimate ρ_2 , ρ_3 and ρ_6 . In each iteration where ρ_2 , ρ_3 and ρ_6 are fixed, Kalman smoothing can be applied to estimate the state x , which in turns will give an estimate of ρ_1 , ρ_4 , ρ_5 and ρ_7 . In other words, this is an NLS problem of 3 variables only, namely ρ_2 , ρ_3 and ρ_6 , and Kalman smoothing is a subroutine inside each NLS iteration. In this way, the dimension of the NLS problem is reduced from 7 to 3.

4.3.2 NLS with Kalman Filter Model II (NKMII)

Compared with NLS Model I, Model II with a LLS estimator embedded within NLS reduces the number of parameters, thus decreasing the risk of being trapped

into local minimum. Yet, the second limitation of NLS Model I still exists in Model II. It is that the NLS estimation is relatively model sensitive. Although we have reduced the number of parameters, we are still using the spectrum model in Section 3.2. The model is good but by no means perfect. As we demonstrated in previous experiment, NLS-LLS could not work equally well in the experiment. The fact suggests us that the system model is not as perfect as we hoped and there are some unknown perturbations in the system. To improve the robustness of the algorithm and overcome this problem, we embed a Kalman smoother instead of LLS in the NLS estimator. The underlying idea is to relax the assumption of the parameters q_i being a constant to a random process, allowing q_i to be slowly varying over λ . This relaxation can effectively reduce the sensitivity of the algorithm to the spectrum model. We will name this model as Kalman filter Model II. Note that the principles of the Kalman filter Model II and the Kalman filter Model I are somewhat different.

In Section 3.2.3, we have given the expression of the NLS Model II. Here, let h denote the sampling step of $I_{meas}(\lambda)$. A state-space model for $q = (q_1, q_2, q_3, q_4)$ can be constructed:

$$q(\lambda + h) = q(\lambda) + u(\lambda) \quad (4.30)$$

$$I_{meas}(\lambda) = C(\lambda)q(\lambda) + v(\lambda) \quad (4.31)$$

$$C(\lambda) = \begin{bmatrix} R(\lambda - \lambda_B) \\ \sqrt{R(\lambda - \lambda_B)} \cos(2\pi n \Delta L / \lambda) \\ -\sqrt{R(\lambda - \lambda_B)} \sin(2\pi n \Delta L / \lambda) \\ 1 \end{bmatrix}^T$$

where the “system noise” u and the “measurement noise” v are Gaussian-white noise. If the variance of u is zero, q is constant over λ as in Section 3.2.3. In general, by controlling the covariance matrices of u and v , various degrees of mismatch

between the four terms in Eq. (3.22) and the experimental data I_{meas} can be considered, making this method very flexible when a Kalman filter is applied to estimate the unknown state-vector q .

It is actually possible to apply the idea of Kalman filtering to estimate all seven unknown parameters of I_{meas} . However, this will lead to an extended Kalman filter. In this section we propose to replace only the LLS engine within the NLS estimator by a Kalman filter, which will result in a linear time(λ)-varying system with guaranteed properties of convergence. In the end, the estimation algorithm will have an outer NLS estimation with an inner Kalman filtering routine. This hybrid NLS-KF approach seems to show a big improvement over a pure Kalman filtering approach, analogous to the advantage observed in Section 3.2.3 when a combined NLS-LLS estimation was used instead of a pure NLS estimation.

Since the independent variable in the state-space model Eqs. (4.31)–(4.32) is the wavelength λ instead of time, causality is not an issue and a Kalman smoother instead of a Kalman filter can be employed to give better results[36] as for NKMI.

To show the whole estimation procedure clearly, we draw the flowchart 4.3 of NKMI here. For comparison, we also draw here the flowchart 4.2 of NLS-LLS estimation.

4.4 Simulation

In this section, we will provide the simulation results for NKMI and NKMI. Thoughtful analysis will be given to fully discover the excellent performance of the algorithm. Since these two models are embedding a Kalman filter in the NLS estimator, initializations of the parameters are also required. In the following simulations, the range of the parameters of NKMI and NKMI follows Table 3.1.

Figure 4.4 demonstrates the simulation result when the NKMI is applied to the

Figure 4.2: Flowchart of NLS estimation

Figure 4.3: Flowchart of NLS with Kalman filter Model II

Figure 4.4: Simulation results obtained by applying NKMI to Figure 3.4. A refers to the measured spectrum I_{meas} with interferometric signal and white noise; B refers to the optimal estimate of I_{meas} given by Equation (4.32) using NLS with Kalman smoothing; C refers to the actual FBG signal spectrum I_s in the simulation; D refers to the FBG signal spectrum estimated by the NKMI algorithm; E refers to the interferometric signal I_{sn} in the simulation; F refers to the interferometric signal estimated by NKMI.

example in Figure 3.4. The optimal estimate $\hat{I}_{meas}$ of I_{meas} can now be constructed from

$$\hat{I}_{meas} = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix} \quad (4.32)$$

where $\hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4)^T$ is the optimal estimate of the state x provided by the Kalman smoother.

Figure 4.5 demonstrates the simulation result when the NKMI is applied to the example in Figure 3.4.

To verify the general performance of the algorithm, the same 4,000 random examples as in Section 3.3 have been tested in the simulation for both Kalman models. Figure 4.6 shows the cumulative distribution function of $er = \hat{\lambda}_B - \lambda_B$ over the

Figure 4.5: Simulation results obtained by applying NKMII to Figure 3.4. A refers to the measured spectrum I_{meas} with interferometric signal and white noise; B refers to the optimal estimate of I_{meas} ; C refers to the actual FBG signal spectrum I_s in the simulation; D refers to the FBG signal spectrum estimated by the NKMII algorithm; E refers to the interferometric signal I_{sn} in the simulation; F refers to the interferometric signal estimated by NKMII.

Figure 4.6: Computer simulation results for λ_B

4,000 random spectrums, where $\hat{\lambda}_B$ denotes the estimate of λ_B . For comparison, a conventional low-pass FIR filter followed by peak-detection is also used, and the results are plotted on the same graph, labelled as “FIR”. The results of NLS-MI and NLS-MII are also plotted on the graph.

The 90% confidence interval turned out to be $[-11.1, 11.3]$ pm for NKMI, note that the estimation is slightly better than NLS-MI and worse than NLS-MII in Section 3.3. However, when all the methods are applied to real experimental data, the advantage of using Kalman smoothing will be evident.

The 90% confidence interval turned out to be $[-0.7120, 0.7357]$ pm for NKMII, which was about 15% narrower than that of NLS-MII. It was a small surprise, since the spectrum model in the computer simulations was exact, unlike the experiments. Closer examinations revealed that NLS-MII had a higher chance of being trapped at a local minimum, while NKMII could avoid it by providing a better estimate of $n\Delta L$, especially when $n\Delta L$ was relatively large. This might indirectly improve the estimate of λ_B as a result.

One possible explanation can be obtained from the cumulative probability distribution of $n\Delta L$ in Figure 4.7. In theory, NLS might produce an estimate that is only locally optimal. Figure 4.7, as well as observations of the spectrums, suggest that the estimate of $n\Delta L$ can get trapped at a local minimum, which will indirectly result in a poorer estimate of λ_B . Figure 4.7 shows that NLS-KF actually give a better estimate of $n\Delta L$ than NLS-LLS. Especially when $n\Delta L$ is relatively large, NLS-LLS might fail to identify the actual value and thus underestimating it, while NLS-KF would succeed. Interestingly this difference is not observed when $n\Delta L$ is relatively small.

Figure 4.7: Cumulative probability distribution for $n\Delta L$

4.5 Experiment

The experimental setup is the same as in Figure 3.8. FIR, NLS-MII and NKMI are applied to estimate the Bragg wavelength λ_B .

Similar to last chapter, for each value of λ_B , a “good” spectrum with no interferometric noise is firstly obtained to provide a reference spectrum $R(\lambda)$. Then 10 sets of I_{meas} , with different interferometric noise in different sets, are measured. The root-mean-square (RMS) value of $\hat{\lambda}_B - \lambda_B$ over these 10 sets is recorded. This procedure is repeated for 10 different values of λ_B and the 10 RMS values are summarized in Table 4.1. Obviously, NKMI shows an improvement of about a factor of 2 vs NLS-MII.

Kalman filters assume that the signals or data are generated from a system model (more precisely, a state-space model here) and try to explore the model structure. Therefore, although Kalman smoother is effectively a low-pass filter, it can out-

Table 4.1: Root-mean-square error in the estimation of λ_B (pm).

	1	2	3	4	5
FIR	5.5408	4.7854	5.4222	6.2530	4.2426
NLS-MII	8.4669	5.4100	6.4496	7.7398	8.6305
NKMII	1.5681	1.0424	2.1458	1.8878	1.3971
	6	7	8	9	10
FIR	5.3944	6.8484	3.8210	2.7386	5.1672
NLS-MII	6.9901	6.9841	3.9841	4.8732	7.8067
NKMII	2.7301	2.0494	2.1921	2.2648	4.6950

perform the FIR filter if the model is accurate and its structure is fully utilized. As mentioned before, the interferometric signal is low in frequency compared with the system spectrum. As a result, the FIR low-pass filter cannot effectively remove the interferometric signal since it only explores the properties of the signal itself but not how it is generated (i.e., the system model).

4.6 Conclusions

An NLS algorithm with an embedded Kalman filter has been presented in this chapter for FBG spectrums with interferometric signal. Simulation and experimental results have confirmed a good accuracy of Bragg wavelength detection. Improvements over conventional filtering approaches have also been observed.

Chapter 5

Crosstalk Induced Wavelength Shift Detection

In this chapter, we bring forward another FBG wavelength shift detection problem where crosstalk, due to the reflection from multiple FBGs, exists. We firstly derive the system parametric model and then try to estimate the interested parameters with NLS algorithm. Simulation and experiment results are provided to verify the efficiency of the algorithm.

5.1 Principle of Operation

The schematic diagram of the FBG sensing system is shown in Fig. 5.1. The light is delivered from a broadband source and passes to a FBG sensing system via a 3dB 2×2 coupler. Half of the light intensity passes through one arm of the coupler and reaches the FBG1. Another half of the light intensity passes to the other arm of the same coupler and reaches the FBG2. The lights of the Bragg wavelengths are reflected by these two FBGs via the same coupler again and are detected by a broadband photodetector. If the center wavelengths of FBG1 and FBG2 are close

Figure 5.1: System Setup

to each other, the reflected signals from the FBGs will overlap, leading to crosstalk of the spectrums and difficulties in wavelength detection.

The spectrum at the photodetector can be written as:

$$I_{meas}(\lambda) = I_{s1}(\lambda) + I_{s2}(\lambda) + I_{wn}(\lambda) \quad (5.1)$$

where λ is the wavelength. $I_{s1}(\lambda)$ is the spectrum of FBG1 in ideal conditions and $I_{s2}(\lambda)$ is the spectrum of FBG2 in ideal conditions,

$$I_{s1}(\lambda) = I_0/4 \cdot R_1(\lambda - \lambda_{B1}) \quad (5.2)$$

$$I_{s2}(\lambda) = I_0/4 \cdot R_2(\lambda - \lambda_{B2}) \quad (5.3)$$

where $R_1(\lambda - \lambda_{B1})$ represents the reflective spectrum of the FBG1 centered at the Bragg wavelength λ_{B1} and $R_2(\lambda - \lambda_{B2})$ the reflective spectrum of the FBG2 centered at the Bragg wavelength λ_{B2} , respectively. I_0 is the intensity of the broadband source, which may be regarded as constant. $R_i(\bullet)$ is typically modeled as a Gaussian function with a peak reflectivity of $\bar{R}_i$ and a spectral full-width at

half-maximum (FWHM) of $\Delta\lambda_{Bi}$:

$$R_i(\lambda - \lambda_{Bi}) = \bar{R}_i \exp \left(-4 \ln 2 (\lambda - \lambda_{Bi})^2 / (\Delta\lambda_{Bi})^2 \right). \quad (5.4)$$

However, this Gaussian assumption is not needed in our proposed method. Similar to the discussion in Section 3.4, NLS assumes a system model that generates the signal or data, and this system model contains several unknown parameters that require to be estimated. The Gaussian model is commonly adopted as this system model in the literature, but is not a necessary assumption in our method. It is because the system model here need not take an analytical form (like the Gaussian model). It can be in a numerical form like pairs of wavelength-intensity values stored in a table. Such a numerical form can be obtained from the experimental data directly. How the system parameters affects the signal or data generated can then be obtained by looking up the numerical data stored in the table followed by interpolating these values. As long as the effects of the system parameters on the data generated can be computed, the NLS method we proposed can be used.

$I_{wn}(\lambda)$ in Eq. (5.1) represents a random noise from the broadband source. Under proper assumptions, $I_{wn}(\lambda)$ may be modelled as a Gaussian-white noise with mean σ^2 and variance $2\sigma^4/N$, with $N > 50$. In the NLS estimation proposed below, it suffices to assume that $I_{wn}(\lambda)$ is a nonzero-mean Gaussian-white noise.

Fig. 5.2 shows a typical example of the measured spectrum I_{meas} in the experiment to be presented in Section 5.3, where the shapes of the two ideal spectrums are not exactly Gaussian but they are similar because the two FBGs come from the same fabrication. When the center wavelengths of the two FBGs are far away from each other (case (a)), peak detection techniques can easily distinguish I_{s1} and I_{s2} . However, when the values of λ_{B1} and λ_{B2} are close (case (b)), peak detection cannot detect the Bragg wavelengths accurately. As shown in the figure, the peak of FBG1 would shift towards FBG2 while the peak of FBG2 would shift towards FBG1 due to their mutual interaction. Consequently peak detection would overestimate λ_{B1}

Figure 5.2: Examples of measured spectrum in the experiment: (a) when λ_{B1} and λ_{B2} are well-separated; (b) when λ_{B1} and λ_{B2} are close.

but underestimate λ_{B2} for this particular example. The method proposed in this paper aims mainly at the case where the two FBG spectra overlap and the peak detection technique will not work successfully.

The spectrum model of I_{meas} in Eq. (5.1) consists of 4 parameters, namely λ_{B1} , λ_{B2} , $I_0/4$ and σ^2 . The first two are the objects of interest and the last two are also subject to a small amount of uncertainties in the experiment. To estimate these unknown parameters from the measured spectrum, the method of nonlinear least-square can be used. However, NLS is basically a nonlinear optimization problem and therefore the effectiveness of these algorithms depend on a good initial guess. Sometimes even multiple initial guesses are preferred. In the proposed method, conventional peak detection are employed in attempt to detect two peaks, which will be used to provide two initializations of λ_{B1} and λ_{B2} (the first peak to λ_{B1} and the second peak to λ_{B2} , and then the other way round). Of course such initial guesses are not accurate estimates of λ_{B1} and λ_{B2} when they are close, due to the failure of the simple peak detection, but the NLS estimation will successfully recover the actual λ_{B1} and λ_{B2} from these false initial guesses, as shown in the simulation and the experimental results in Sections 5.2 and 5.3 below.

5.2 Simulation Results

In the computer simulation, the shapes of the two ideal spectrums $R_1(\bullet)$ and $R_2(\bullet)$ actually come from the experiment in Section 5.3, as shown in Fig. 5.2. The value of λ_{B1} is kept at 1545.9110nm while the value of λ_{B2} is varied decreasingly from 1546.4995nm to 1545.7005nm. Note that the experimental spectrums are in fact discrete: they are sampled at an 1pm step and therefore interpolation algorithms should be used when shifting the spectrums. The values of $I_0/4$ and σ^2 are randomly generated in the ranges of 90–110% and 0–200% of the values observed from the experimental data, respectively. The statistics reported in this section is the result after 50 runs of the computer simulation.

Fig. 5.3 shows the 2-norm of the residue:

$$\sqrt{\sum_k r(\lambda_k)^2}$$

averaged over the 50 runs of simulation. The residue $r(\lambda)$ is defined as the difference between the real $I_{meas}(\lambda)$ and the estimated $I_{meas}(\lambda)$ by the algorithm, and is sampled from 1545.6nm to 1546.6nm in an 1pm step. Despite that the norm of the residue is what the NLS algorithm is trying to minimize, our primary interests are the estimation errors in λ_{B1} and λ_{B2} . Fig. 5.4 shows the absolute estimation errors in λ_{B1} :

$$|\text{estimated } \lambda_{B1} - \text{actual } \lambda_{B1}|$$

and λ_{B2} :

$$|\text{estimated } \lambda_{B2} - \text{actual } \lambda_{B2}|$$

averaged over the 50 runs of simulation. Although λ_{B1} is not varied during the computer simulation, this information is not known by the NLS algorithm. The only reason of not varying λ_{B1} is to reproduce a scenario exactly equal the experimental setup in Section 5.3 for the ease of comparison.

Figure 5.3: Norm of the residue when λ_{B2} is varied in the simulation.

Figs. 5.3–5.4 show not only the performance of the proposed NLS algorithm, but also the performance of the conventional peak detection technique. Since the NLS method uses the peak detection to provide the initial guesses, it should certainly have a lower residue. Fig. 5.3 confirms that the norm of the residue has been reduced by 1–2 order of magnitude. As for the estimation errors in λ_{B1} and λ_{B2} , the performance limitation of the peak detection method would be around 0.5pm if there were no measurement noise, since the resolution of the measured spectrum is 1pm. Fig. 5.4 indicates that when the two Bragg wavelengths are well-separated, the estimation error using the peak detection is about 1pm, while that of the NLS method is about 3 order of magnitude lower, providing a much higher resolution effectively. When the two Bragg wavelengths move closer to each other, cross-talk occurs and the resolution will be reduced by about 1 order of magnitude in both cases. The worst estimation errors of the NLS method are 0.5859pm in λ_{B1} and 0.5365pm in λ_{B2} , which are still below 1pm. Of course all the above statistics are dependent on the level of the measurement noise, which is pretty small in this computer simulation ($\sigma^2 < 0.63 \times 10^{-6}$ pW). Still, it can be easily concluded that the improvement of the NLS method over the conventional peak detection is rather obvious, and the former remains very effective even when the two FBG spectrums have a significant overlap.

Figure 5.4: Estimation errors in λ_{B1} and λ_{B2} when λ_{B2} is varied in the simulation.

5.3 Experimental Results

Experiments are conducted using the setup shown in Fig. 5.1. The light from a broadband source passes through a 3dB coupler and divides equally into two arms with each of them connected to an FBG sensor. The FBG2 is mounted on a fixed stage (FS) and a translation stage (TS) for the purpose of applying strain to change the Bragg wavelength λ_{B2} . The Bragg wavelength λ_{B1} may be assumed fixed during the experiment. The reflected lights from FBG1 and FBG2 return through the same coupler and are detected by a photodetector. The resolution and range of the wavelength are set as 1pm and 1nm, respectively. The signal from the photodetector is synchronously recorded by a personal computer (PC). Altogether, 145 different values of the strain has been used. Fig. 5.2 shows two typical spectrums I_{meas} measured in the experiment.

Since the real λ_{B2} is unknown in the experiment, only the norm of the residue is used to assess the performance of the NLS algorithm, which is shown in Fig. 5.5. The improvement over the peak detection is not as significant as in the computer simulation, but the NLS method performs consistently even when the two FBG spectrums overlap. On the other hand, the peak detection method fails miserably when the overlap occurs, with two high peaks in the norm of the residue as in

Figure 5.5: Performance of the NLS method in the experiment.

Figure 5.6: Magnified plot of the estimated λ_{B1} and λ_{B2} during the overlap of the two FBG spectra.

the computer simulation. The corresponding NLS estimates of λ_{B1} and λ_{B2} are shown in the same figure, from which it appears that the transition of the estimates remains smooth during the cross-over of the spectra. A magnified plot of the estimates is given in Fig. 5.6, together with those estimates provided by the peak detection for comparison. From the magnified plot it is observed that the transition of the NLS method is indeed much smoother than that of the simple peak detection technique, confirming that the former is much more effective in handling the problem of spectrum shadowing.

5.4 Conclusions

An NLS algorithm has been presented in this chapter to distinguish two cross-talk FBG spectrums. Simulations and experimental results have confirmed a good accuracy of Bragg wavelength detection. Improvements over conventional peak-detection technique have also been observed.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

Detection of Fiber Bragg center wavelength shift is a challenging topic in optical domain. By accurate detection of the shift, valuable information, such as external strain change, temperature change etc, can be obtained. Yet, various noise greatly affects the result of detection. In this thesis, two kinds of noise are considered.

One of them is the interferometric signal, which normal spectrum analysis techniques could not deal with effectively. Nonlinear least squares (NLS) is a general approach in estimation theory. It involves a proper representation of the system, accurate measurements of the system quantities, correct estimation of the unknown parameters and efficient validation. After proper design, NLS can not only filter out the random white noise, but also recover the system parameters, which means that we can filter out the interferometric signal and obtain the accurate Bragg wavelength after estimation. To fast and efficiently recover the system parameters, good initializations of the parameters are required. Some of the initializations come from the experimental data and the others come from the experiment setup. Theoretically speaking, decreasing the number of the parameters will lower

the risk of being trapped into a local minimum. In order to reach such aims, we proposed an NLS estimation approach embedded with an LLS engine. Within one recursion of the NLS estimation, parts of the parameters are fixed and a linear least squares is applied to estimate the other parameters. As a result, only parts of the parameters are estimated by the nonlinear optimization, thus reducing the risk of being trapped into a local minimum. As a fact, the simulation result actually demonstrated such improvement after applying the NLS embedded with LLS.

However, NLS performs much worse than expectation in the experiment. After careful examination, we found that the system model we are using is good but not perfect. As NLS is an estimation method that is sensitive to system model, unexpected disturbance will greatly deteriorate the estimation accuracy. In order to increase the robustness of the algorithm, we incorporate the Kalman filter with NLS. Two NLS with Kalman filter models are included in our work. The main difference between these two models is their state space formulation. For the first model, the coefficient matrix A in equation (4.1) is changed with part of the parameters and H in equation (4.1) is constant. For the second model, the coefficient matrix A in equation (4.1) is constant and H is changed with part of the parameters. The second model relaxes the assumption of q_i being a constant to a random process, allowing q_i to be slowly varying over λ . This relaxation can effectively reduce the sensitivity of the algorithm to the spectrum model. Simulation results show that the second model of NLS with Kalman filter perform much better than the first model and exhibit a slight improvement compared with NLS-LLS method. The power of NLS with Kalman filter are shown in the experiment. Experiment results show a improvement of the NLS-KF over NLS-LLS by about a factor of 2.

Another noise considered in this thesis is the crosstalk in a topological array. To simplify the model, we design a system with two parallel FBGs whose wavelengths are slightly offset to each other. After we construct the correct formulation of the system, accurate separation of the two spectrums is realized by using NLS estimation. A challenge of the estimation is that when the two center wavelengths are

close to each other, the algorithm could be confused about which peak belongs to which FBG, even though the center wavelengths of both peaks are accurately detected. We solve this problem by multiple initializations of the peak. Simulation and experiment results show a good estimation of the spectrum by the NLS algorithm.

6.2 Future Work

Although the NLS model has shown effectiveness in recovering the FBG center wavelength from different noise, several possibilities exist to further enhance the work. Some important directions of future research are stated as follows:

- Compared with normal NLS algorithm, NLS-KF algorithm can provide much better estimation, but the processing time increases due to the more complex structure of the combined algorithm. Further investigations to cut down the computational burden will surely be beneficial.
- Initialization is important to NLS and other estimation methods. Poor initialization often leads to local minima. In our work, we proposed an approach to reduce the number of parameters so as to relax the dependence on the initialization. Still, other techniques could be explored to further improve the initialization process.
- In future, multi-center wavelength shift should be considered, which means that there are more than 2 FBG sensors in a topological array and the multiple crosstalk will bring difficulties in wavelength shift detection. With a larger number of FBG sensors, the computational burden of multiple initializations would also increase. A more efficient algorithm should be developed in this case.

Author's Publications

- (1) Yu Tang, Yun Chung Chu, Chi Chiu Chan, Shuang Liu, “Bragg wavelength detection in fiber Bragg grating sensor by combining nonlinear least squares with Kalman smoothing”, *Proceedings of SPIE, Optics East 2006*, Boston, Massachusetts, USA, 6379: 41–49, Oct 2006.
- (2) Yu Tang, Yun Chung Chu, Chi Chiu Chan, Jian Sun, “Nonlinear least squares algorithm with embedded Kalman filter for Bragg wavelength detection in fiber Bragg grating sensors”, *Optoelectronics and Advanced Materials — Rapid Communications*, Vol. 1, No. 4, p. 145–148, April 2007.
- (3) Yu Tang, Yun Chung Chu, Chi Chiu Chan, Jian Sun, “Crosstalk analysis of FBG using NLS estimation”, under preparation.

Bibliography

- [1] A. T. Alavie, S. E. Karr, A. Othonos, and R. M. Measures. A multiplexed Bragg grating fiber laser sensor system. *IEEE Photonics Technology Letters*, 5:1112–1114, 1993.
- [2] B. Anderson and J. Moore. *Optimal Filtering*. Prentice Hall, Eaglewood Cliffs, New Jersey, 1979.
- [3] J. L. Archambault, L. Reekie, and P. St. J. Russell. 100% reflectivity Bragg reflectors produced in optical fibres by single excimer laser pulses. *Electronics Letters*, 29:453–455, 1993.
- [4] C. G. Askins, M. A. Putnam, and E. J. Friebele. Instrumentation for interrogating many-clement fiber Bragg grating arrays embedded in fiber/resin composites. In *SPIE Smart Sensing, Processing and Instrumentation*, volume 2444, page 257, 1995.
- [5] B. A. Ball and W. W. Morey. Continuously tunable single-mode erbium fiber laser. *Optics Letters*, 17:420–422, 1992.
- [6] G. A. Ball, W. W. Morey, and P. K. Cheo. Fiber laser source/analyzer for Bragg grating sensor array interrogation. *IEEE Journal of Lightwave Technology*, 12:700–703, 1994.
- [7] G. A. Ball, W. W. Morey, and W. H. Glenn. Standing-wave monomode erbium fiber laser. *IEEE Photonics Technology Letters*, 3:613–615, 1991.

-
- [8] L. Bjerkan, D. R. Hjelle, and K. Johannessen. Bragg grating sensor demodulation scheme using semiconductor laser for measuring slamming forces of marine vehicle models. In *11th Int. Conf. Optical Fiber Sensors*, pages 236–239, 1996.
 - [9] G. P. Brady, K. Kalli, D. J. Webb, D. A. Jackson, L. Zhang, and I. Bennion. Extended range, coherence tuned, dual wavelength interferometry using a superfluorescent fibre source and chirped fibre Bragg gratings. *Optics Communications*, 134:341–346, 1997.
 - [10] C. C. Chan, J. M. Gong, C. Z. Shi, W. Jin, M. Zhang, L. M. Zhou, and M. S. Demokan. Improving measurement accuracy of fiber Bragg grating sensor using digital matched filter. *Sensors and Actuators A*, 104:19–24, 2003.
 - [11] C. C. Chan, W. Jin, and M. S. Demokan. Enhancement of measurement accuracy in fiber Bragg grating sensors by using digital signal processing. *Optics and Laser Technology*, 31:3199–3307, 1999.
 - [12] C. C. Chan, W. Jin, D. N. Wang, and M. S. Demokan. Intrinsic crosstalk analysis of a serial TDM FBG sensor array by using a tunable laser. *IEEE Lasers and Electro-Optics Society*, 2:533–534, Nov 2000.
 - [13] C. C. Chan, C. Z. Shi, W. Jin, and D. N. Wang. Improving the wavelength detection accuracy of FBG sensors using an adaline network. *IEEE Photonics Technology Letters*, 15(8):1126–1128, 2003.
 - [14] T. Coroy, L. M. Chappell, N. J. Guillermo, S. Y. Huang, R. M. Measures, and K. D. Chik. Peak detection demodulation of a Bragg fiber optic sensor using a gain-coupled distributed feedback tunable laser. In *Proc. of Optical Fiber Sensors Conf.*, volume 12, pages 210–212, 1997.
 - [15] M. A. Davis, D. G. Bellemore, M. A. Putnam, and A. D. Kersey. Interrogation of 60 fibre Bragg grating sensors with microstrain resolution capability. *Electronics Letters*, 32:1393–1394, 1996.
-

- [16] B. J. Eggleton, P.A. Krug, L. Poladian, and F. Ouellette. Long periodic superstructure Bragg gratings in optical fibres. *Electronic Letters*, 30:1620–1622, 1994.
- [17] M. C. Farries, C. M. Ragdale, and D. C. J. Reid. Broadband chirped fibre Bragg filters for pump rejection and recycling in erbium doped fibre amplifiers. *Electronics Letters*, 28:487–489, 1992.
- [18] M. T. Heath. *Scientific Computing: An Introductory Survey*. McGraw-Hill, New York, 2nd edition, 2005.
- [19] K. O. Hill, F. Bilodeau, B. Malo, and D. C. Johnson. Birefringent photosensitivity in monomode optical fiber: Application to external writing of rocking filters. *Electronics Letters*, 27:1548–1550, 1991.
- [20] K. O. Hill, Y. Fujii, D. C. Johnson, and B. S. Kawasaki. Photosensitivity in optical fiber waveguides: Application to reflection filter fabrication. *Applied Physics Letters*, 32:647–649, 1978.
- [21] H. Hillmer, H. L. Zhu, A. Grabmaier, S. Hansmann, H. Burkhard, and K. Magari. Novel tunable semiconductor lasers using continuously chirped distributed feedback gratings with ultrahigh spatial precision. *Applied Physics Letters*, 65:2130–2132, 1994.
- [22] D. A. Jackson, A. B. Lobo Ribero, L. Reekie, and J. L. Archambault. Simple multiplexing scheme for fiber-optic grating sensor network. *Optics Letters*, 18:1192–1194, 1993.
- [23] W. Jin. Investigation of interferometric noise in fiber-optic Bragg grating sensors by use of tunable laser sources. *Appl Opt*, 37:2517–25, 1998.
- [24] R. Kashyap, R. Wyatt, and P. F. McKee. Wavelength flattened saturated erbium amplifier using multiple side-tap Bragg gratings. *Electronics Letters*, 29:1025–1026, 1993.

- [25] B. S. Kawasaki, K. O. Hill, D. C. Johnson, and Y. Fujii. Narrow-band Bragg reflectors in optical fibers. *Optics Letters*, 3:66–68, 1978.
- [26] A. D. Kersey. Interrogation and multiplexing techniques for fiber Bragg grating strain-sensors. In *Distributed and Multiplexed Fiber Optic Sensors III*, volume 2071, pages 30–48, Boston, Sept 1993. Society of Photo-Optical Instrumentation Engineers.
- [27] A. D. Kersey. Multiplexing techniques for fiber-optic sensors. In J. Dakin and B. Culshaw, editors, *Optical Fiber Sensors: Applications, Analysis, and Future Trends*, pages 373–376. Artech House, Norwood, 1997.
- [28] A. D. Kersey and T. A. Berkoff. Fiber-optic Bragg-grating differential-temperature sensor. *IEEE Photonics Technology Letters*, 4:1183–1185, 1992.
- [29] A. D. Kersey, T. A. Berkoff, and W. W. Morey. Fiber-grating based strain sensor with phase sensitive detection. In *1st European Conference on Smart Structures and Materials*, Session(2), pages 61–67, Glasgow, Scotland, 1992.
- [30] A. D. Kersey, T. A. Berkoff, and W. W. Morey. High-resolution fibre-grating based strain sensor with interferometric wavelength-shift detection. *Electronics Letters*, 28:236–238, 1992.
- [31] A. D. Kersey, T. A. Berkoff, and W. W. Morey. Fiber-optic Bragg grating strain sensor with drift compensated high-resolution interferometric wavelength-shift detection. *Optics Letters*, 18:72–74, 1993.
- [32] A. D. Kersey, T. A. Berkoff, and W. W. Morey. Multiplexed fiber bragg grating strain-sensor system with a fiber fabry-perot wavelength filter. *Optics Letters*, 18:1370–1372, 1993.
- [33] A. D. Kersey, M. A. Davis, H. J. Patrick, M. LeBlanc, K.P. Koo, C.G. Askins, M.A. Putnam, and E.J. Friebele. Fiber grating sensors. *IEEE Journal of Lightwave Technology*, 15:1442–1463, 1997.

- [34] A. D. Kersey, K. L. Dorsey, and A. Dandridge. Cross talk in a fiber-optic Fabry-Perot sensor array with ring reflectors. *Optics Letters*, 14(1):93–95, January 1989.
- [35] D. K. Lam and B. K. Garside. Characterization of single-mode optical fiber filters. *Applied Optics*, 20:440–445, 1981.
- [36] F. Lewis. *Optimal Estimation*. John Wiley & Sons, Inc., New York, 1986.
- [37] S. M. Melle, K. Liu, and R. M. Measures. Strain sensing using a fiber Bragg grating. In *Fiber Optic Smart Structures and Skins IV, SPIE*, volume 1588, pages 255–263, Boston, Massachusetts, Sep 1991.
- [38] S. M. Melle, K. Liu, and R. M. Measures. A passive wavelength demodulation system for guided-wave bragg grating sensors. *IEEE Photonics Technology Letters*, 4:516–518, 1992.
- [39] G. Meltz and W. W. Morey. Bragg grating formation and germanosilicate fiber photosensitivity. In *International Workshop on Photoinduced Self-Organization Effects in Optical Fiber*, volume 1516, pages 185–199, Quebec City, Quebec, May 10–11, 1991. Proceedings SPIE.
- [40] G. Meltz, W. W. Morey, and W. H. Glenn. Formation of Bragg gratings in optical fibers by a transverse holographic method. *Optics Letters*, 14:823–825, 1989.
- [41] J. M. Mendel. *Discrete Techniques of Parameter Estimation: The Equation Error Formulation*. New York: Marcel Dekker, 1973.
- [42] J. M. Mendel. *Lessons in estimation theory for signal processing, communications, and control*. Prentice-Hall International, Inc, 1995.
- [43] V. Mizrahi and J. E. Sipe. Optical properties of photosensitive fiber phase gratings. *Journal of Lightwave Technology*, 11(10):1513–1517, Oct 1993.

-
- [44] W. W. Morey, G. A. Ball, and G. Meltz. Photo-induced Bragg gratings in optical fibers. In *Optics and Photonics News*, page 814. Optical Society of America, February 1994.
 - [45] W. W. Morey, J. R. Dunphy, and G. Meltz. Multiplexing fiber Bragg grating sensors. In *SPIE Distributed and Multiplexed Fiber Optic Sensors*, volume 1586, page 216, Boston, MA, Sept 1991.
 - [46] W. W. Morey, G. Meltz, and W. H. Glenn. Fiber Bragg grating sensors. In *SPIE Fiber Optic & Laser Sensors VII*, volume 1169, page 98, 1989.
 - [47] W. W. Morey, G. Meltz, and W.H. Glenn. Bragg-grating temperature and strain sensors. In *Optical Fiber Sensors Conf. '89*, page 526, Paris, France, 1989.
 - [48] P. Niay, P. Bernage, M. Douay, T. Taunay, W. X. Xie, G. Martenelli, J. F. Bayon, H. Poignant, and E. Delevaque. Bragg grating photoinscription within various types of fibers and glasses. In *Topical Meeting on Photosensitivity and Quadratic Nonlinearity in Glass Waveguides: Fundamentals and Applications*, volume 22 of *Technical Digest Series*, pages 66–69, Portland, OR, Paper SuA1, 1995. Optical Society of America, Washington, DC.
 - [49] A. Othonos, A. T. Alavie, S. M. Melle, S. E. Karr, and R. M. Measures. Fiber Bragg grating laser sensor. *Optical Engineering*, 32:2841–2846, 1993.
 - [50] A. Othonos and K. Kalli. *Fiber Bragg Gratings: Fundamentals and Applications in Telecommunications and Sensing*. MA: Artech House, Norwood, 1999.
 - [51] A. Othonos, X. Lee, and R. M. Measures. Superimposed multiple Bragg gratings. *Electronic Letters*, 30:1972–1973, 1994.
 - [52] F. Ouellette. Dispersion cancellation using linearly chirped Bragg grating filters in optical waveguides. *Optics Letters*, 12:847–849, 1987.
-

-
- [53] H. Patrick and S. L. Gilbert. Growth of Bragg gratings produced by continuous-wave ultraviolet light in optical fiber. *Optics Letters*, 18:1484–1486, 1993.
 - [54] Y. J. Rao. In-fiber Bragg grating sensor. *Measurement Science and Technology*, 8:355–375, 1997.
 - [55] Y. J. Rao, D. A. Jackson, L. Zhang, and I. Bennion. Dual-cavity interferometric wavelength-shift detection for in-fiber Bragg grating sensors. *Optics Letters*, 21:1556–1558, 1996.
 - [56] H. E. Rauch, F. Tung, and C. T. Striebel. Maximum likelihood estimates of linear dynamic systems. *AIAA Journal*, 3(8):1445–1450, August 1965.
 - [57] I. Riant and F. Haller. Study of the photosensitivity at 193nm and comparison with photosensitivity at 240nm influence of fiber tension: Type IIA aging. *IEEE Journal of Lightwave Technology*, 15:1464–1469, 1997.
 - [58] J. Stone. Photorefractivity in GeO_2 -doped silica fibers. *Journal of Applied Physics*, 62:4371–4374, 1987.
 - [59] W. X. Xie, P. Niay, P. Bernage, M. Douay, J. F. Bayon, T. Georges, M. Monerie, and B. Pournellec. Experimental evidence of two types of photorefractive effects occurring during photoinscriptions of Bragg gratings written within germanosilicate fibres. *Optics Communications*, 104:185–195, 1993.
 - [60] M. G. Xu, H. Geiger, J. L. Archambault, L. Reekie, and J. P. Dakin. Novel interrogation system for fibre Bragg grating sensors using an acousto-optic tunable filter. *Electronics Letters*, 29:1510–1511, 1993.
 - [61] M. G. Xu, H. Geiger, and J. P. Dakin. Modeling and performance analysis of a fiber Bragg grating interrogation system using an acousto-optic tunable filter. *Journal of Lightwave Technology*, 14:391–396, 1996.
-