Spheroidal antennas with metamaterials coatings and phase mode processing for wireless communications

Huang, Ming Da

2008

Huang, M. D. (2008). Spheroidal antennas with metamaterials coatings and phase mode processing for wireless communications. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/41414>

<https://doi.org/10.32657/10356/41414>

Spheroidal Antennas with Metamaterials Coatings and Phase Mode Processing for Wireless Communications

Huang Mingda

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2008

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

04/01/2008

Date

Huang Mingda

Huang Mingda

Acknowledgements

I would like to express my sincere gratitude and appreciation to my supervisor, Assoc. Prof. Tan Soon Yim, for his valuable support and advice throughout the course of this study. Without his professional and dedicated guidance, this thesis would not have been possible.

I would also like to thank Prof. Robert H. MacPhie of University of Waterloo, Canada, and Dr. Francis Cooray of CSIRO ICT Centre, Australia, for their fascinating discussions and advices in this project. My appreciation also goes to Assoc. Prof. Max Tegmark of Massachusetts Institute of Technology, USA, for his generous assistance and sharing.

I am deeply indebted to my parents Huang Xiang and Hu Ruomei for their support and encouragement which have enabled me to take full advantage of the highest educational opportunities. I would like to thank my girlfriend Xie Yingrong who endured this long process with me, always offering support and love. Thanks are also dedicated to all of my friends, faculties, and technicians in Communication Lab IV who make my life in NTU meaningful and enjoyable.

Last but not least, scholarships granted by School of Electrical and Electronics Engineering, Nanyang Technological University are gratefully acknowledged.

Summary

This thesis focuses on the theoretical study of electromagnetic scattering and radiation by a conducting spheroid with metamaterials (MTMs) coatings (Part I) and phase mode processing for spherical and spheroidal antenna arrays (Part II).

Firstly, scattering problems of a conducting prolate spheroid coated with MTMs are studied in Part I. Our results show that a conducting spheroid with double-negative (DNG) MTMs coating has different scattering characteristics compared with the conventional dielectric coating. This feature can be used to increase, decrease, smooth the scattering cross section (SCS), and even cancel the nulls of the SCS. We have found that by choosing appropriate thickness of the coatings, it is possible to dramatically enhance the total SCS to achieve “compact resonant structures” with DNG MTMs coatings or to drastically reduce the total SCS to achieve “transparency” of the objects with low-permittivity MTMs coatings.

Secondly, the radiation characteristics of the spheroidal antenna coated with DNG MTMs are investigated. It is found that the half wavelength confocal DNG MTMs radome is almost electrically transparent to the spheroidal antenna, even when the DNG MTMs radome is placed very close to the antenna. These features can be used to reduce the size of an antenna system with better performance. It

is also found that spheroidal antennas with DNG MTMs shell can realize efficient electrically small antennas because electrically small antennas always act as capacitive element and DNG shell can act as an inductive element. With careful design, the radiation power of an antenna system can be increased significantly and the fractional bandwidth (FBW) of antenna is much larger than the Chu limit which is the maximum FBW limit for the normal electrically small antenna.

In Part II, an improved icosahedron-based spherical antenna array for phase mode processing is proposed firstly to dynamically reconfigure the radiation pattern of an antenna array. In this topology, the electrical environment of all elements is almost identical. This feature is useful for three-dimensional beam scanning and for minimizing and compensating the effects of mutual coupling. The use of directional elements on this array can overcome the limitations of rapid variations in amplitude of the far-field mode over a wide frequency band and enable such array to synthesize wideband patterns. The aliasing of spatial sampling is analyzed theoretically for wideband applications.

Finally, the theory of spherical phase mode is extended to both prolate and oblate spheroidal antenna arrays in terms of spheroidal wave functions. Our results show that a spheroidal phase mode of the excitation function produces a far field radiation pattern with the same spheroidal phase mode form, and the elevation angle of that pattern increases (decreases) with the ratio of the interfocal distance of the prolate (oblate) spheroid antenna array to the wavenumber. These properties have a number of attractive applications three-dimensionally, such as broadband pattern synthesis, null steering, direction finding, and superresolution.

List of Abbreviations and Symbols

Abbreviations

EM	electromagnetic
MM	Moment Method
GTD	geometrical theory of diffraction
DSP	digital signal processing
MTMs	metamaterials
LH	left-handed
DNG	double-negative
DPS	double-positive
ENG	epsilon-negative
FBW	fractional bandwidth
MIC	microwave integrated circuit
EMC	electromagnetic compatibility
EMI	electromagnetic interference
RCS	radar cross section
e.m.f	electromotive force

MSA	microstrip antenna
MOM	method of moments
FE-BI	finite element-boundary integral
BW	backward-wave
SNG	single-negative
NIMs	negative-index materials
MNG	mu-negative
EBG	electromagnetic bandgap
SRRs	split ring resonators
MRI	magnetic resonance imaging
RPR	radiated power ratio
DF	direction finding

Symbols

η	spheroidal angular coordinates
ξ	spheroidal radial coordinates
ϕ	spheroidal azimuthal coordinates
d	interfocal distance of spheroid
S_{mn}	spheroidal angular functions
$R_{mn}^{(i)}$	spheroidal radial functions of i th kind
$\mathbf{S}$	Poynting vector
$\mathbf{E}$	electric field

$\mathbf{H}$	magnetic field
ϵ	permittivity
μ	permeability
ϵ_r	relative permittivity
μ_r	relative permeability
k	wavenumber
n	index of refraction
Z	wave impedance
λ	wave length
O	origin
a	semi-major axis
b	semi-minor axis
h	parameter in spheroidal wave functions ($= kd/2$)
$\psi_{on}^{(i)}$	spheroidal scalar wave functions
$\bar{M}$ and $\bar{N}$	spheroidal vector wave functions
$\hat{q}$	arbitrary coordinates vector
j	square root of -1
T	matrix transpose
r	spherical radial coordinates
θ	spherical polar coordinates
$\sigma(\theta, \phi)$	radar cross section
A	arbitrary scalar wave function
P	power

V	voltage
I	current
X	reactance
ω	angular frequency
f	frequency
Q	radiation quality factor
Y_l^m	spherical harmonics
P_l^m	associated Legendre functions
β	wavenumber
j_l	spherical Bessel functions
Z_n^m	spheroidal phase mode

Contents

Acknowledgements	i
Summary	ii
List of Abbreviations and Symbols	iv
List of Figures	xii
List of Tables	xx
1 Introduction	1
1.1 Background	1
1.2 Motivation	3
1.3 Objectives	6
1.4 Contributions of the Thesis	8
1.5 Organization of the Thesis	11
2 Background Theories of Spheroidal Antennas with Metamaterials	
Coating and Phase Mode Processing	13
2.1 Introduction	13
2.2 A Brief Survey of Spheroidal Wave Functions	16
2.2.1 Introduction of Spheroidal Wave Functions	16
2.2.2 Electromagnetic Scattering of Spheroids	20

2.2.3	Spheroidal Antennas	23
2.3	Metamaterials and Negative Refraction Index	28
2.4	Phase Mode Processing for Circular Antenna Arrays	32
2.5	Conclusion	34
I	Metamaterials	35
3	Electromagnetic Scattering by a Conducting Spheroid Coated with Double-Negative Metamaterials	36
3.1	Introduction	36
3.2	Scattering with DNG MTMs Coating	38
3.2.1	Prolate Spheroidal Scalar and Vector Wave Functions	39
3.2.2	Incident, Scattered, and Transmitted Fields	41
3.2.3	Imposing the Boundary Conditions	44
3.2.4	Far-Field Expressions	46
3.3	Results and Discussion	49
3.4	Conclusion	69
4	Spheroidal Antenna Coated with Double-Negative Metamaterials	70
4.1	Introduction	70
4.2	EM Radiation of Prolate Spheroidal Antenna	73
4.2.1	Geometry of Spheroidal Antenna	73
4.2.2	Formulation of the Problem	74
4.3	EM Radiation with DNG MTMs Radome	77

4.3.1	Geometry of a Spheroidal Antenna with a Confocal Radome	77
4.3.2	Obtaining the Auxiliary Scalar Wave Functions	78
4.3.3	Applying the Boundary Conditions	81
4.4	Efficient Electrically Small Antennas with DNG MTMs Shell	85
4.5	Results and Discussion	87
4.5.1	DNG Radome	87
4.5.2	Efficient Electrically small antennas	97
4.6	Conclusion	106

II Phase Mode Processing 108

5 Icosahedron-Based Spherical Antenna Array for Wideband Phase

Mode Processing	109
5.1	Introduction 109
5.2	Icosahedron-Based Spherical Antenna Arrays 112
5.2.1	Topology 112
5.2.2	Spherical Phase Mode 116
5.2.3	Directional Element Pattern 118
5.2.4	Icosahedron-based Arrays 121
5.3	Results and Discussion 125
5.4	Conclusion 132

6 Spheroidal Phase Mode Processing for Antenna Arrays 133

6.1	Introduction 133
-----	----------------------------

6.2	Spheroidal Phase Mode Processing	135
6.3	Results and Discussion	140
6.4	Conclusion	147
7	Conclusions and Recommendations	148
7.1	Conclusions	148
7.2	Recommendations for Future Work	151
	Author's Publications	153
	Bibliography	155
A	Matrices in (3.35)	170
B	Orthogonal Properties of the Spheroidal Angular Functions	172
C	Intermediate Terms of (4.43)	180

List of Figures

2.1	Left-handed system.	14
2.2	Prolate and oblate spheroidal coordinate systems.	17
2.3	Modelling different antennas using prolate and oblate spheroids. . .	23
2.4	EM waves reflected/transmitted at air/DNG MTMs interface. . . .	30
3.1	Geometry of the problem.	38
3.2	Normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 =$ 1.0001) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.0$ and $\mu_{r1} = \mu_{r2} = 1.0$). . .	50
3.3	Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 27$ mm, $a_0 = 30$ mm) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 6.0$ and $\mu_{r1} = \mu_{r2} = 1.0$).	50
3.4	Normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 =$ 1.0001) with DNG MTMs coating ($\epsilon_{r1} = \epsilon_{r2} = -2.0$ and $\mu_{r1} = \mu_{r2} =$ -1.0).	52
3.5	Normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 =$ 1.0001) with coatings ($\epsilon_{r1} = -\epsilon_{r2} = -2.0$ and $\mu_{r1} = -\mu_{r2} = -1.0$). . .	52

3.6	Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) versus the relative permittivity ϵ_{r1} of dielectric- DNG MTMs coatings ($\epsilon_{r2} = 2.0$ and $\mu_{r1} = -\mu_{r2} = -1.0$).	53
3.7	Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_0 = 40$ mm) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.0$ and $\mu_{r1} = \mu_{r2} = 1.0$) and DNG MTMs coating ($\epsilon_{r1} = \epsilon_{r2} = -2.0$ and $\mu_{r1} = \mu_{r2} = -1.0$).	54
3.8	Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_0 = 40$ mm) with different coatings at $f = 2$ GHz.	56
3.9	Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_0 = 40$ mm) with different coatings at $f = 2$ GHz.	57
3.10	Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, , $a_2 = 30$ mm, $a_1 = 40$ mm, $a_0 = 50$ mm) with Dielectric-DNG MTMs coating ($\epsilon_{r1} = -\epsilon_{r2} = -2.0$, $\mu_{r1} =$ $-\mu_{r2} = -1.0$) and DNG MTMs-Dielectric coating ($\epsilon_{r1} = -\epsilon_{r2} = 2.0$, $\mu_{r1} = -\mu_{r2} = 1.0$).	58
3.11	Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.2$ and $\mu_{r1} =$ $\mu_{r2} = 1.0$).	58

3.12	Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) with DNG MTMs coating ($\epsilon_{r_1} = \epsilon_{r_2} = -2.2$ and $\mu_{r_1} = \mu_{r_2} = -1.0$).	59
3.13	Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) with coatings ($\epsilon_{r_1} = -\epsilon_{r_2} = -2.2$ and $\mu_{r_1} = -\mu_{r_2} =$ -1.0).	59
3.14	Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 5.0$) with dielectric coating ($\epsilon_{r_1} = \epsilon_{r_2} = 2.2$ and $\mu_{r_1} =$ $\mu_{r_2} = 1.0$).	60
3.15	Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 5.0$) with DNG MTMs coating ($\epsilon_{r_1} = \epsilon_{r_2} = -2.2$ and $\mu_{r_1} = \mu_{r_2} = -1.0$).	60
3.16	Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 5.0$) with coatings ($\epsilon_{r_1} = -\epsilon_{r_2} = -2.2$ and $\mu_{r_1} = -\mu_{r_2} =$ -1.0).	61
3.17	Normalized total scattering cross section of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) coated with DNG MTMs $\epsilon_{r_1} = \epsilon_{r_2} = -1.0$ and $\mu_{r_1} = \mu_{r_2} = -1.0$	62
3.18	Ratios of a_2/a_0 at the resonant peaks of the normalized total scat- tering cross sections (TSC) of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) as a function of the relative permittivity ($\epsilon_r = \epsilon_{r_1} =$ ϵ_{r_2}) of coated DNG MTMs ($\mu_{r_1} = \mu_{r_2} = -1.0$).	64

3.19	Normalized total scattering cross section of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) coated with low-permittivity MTMs $\epsilon_{r_1} = \epsilon_{r_2} = 0.5$ and $\mu_{r_1} = \mu_{r_2} = 1.0$	66
3.20	Ratios of a_2/a_0 at the “transparency” points of the normalized total scattering cross sections of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) as a function of the relative permittivity ($\epsilon_r = \epsilon_{r_1} = \epsilon_{r_2}$) of coated low-permittivity MTMs ($\mu_{r_1} = \mu_{r_2} = 1.0$).	67
4.1	Geometry of a prolate spheroidal antenna.	73
4.2	Geometry of a prolate spheroidal antenna with radome	77
4.3	Normalized radiation patterns of prolate spheroidal antennas having slots at $\eta_0 = 0$, half-wavelength interfocal distance $d/\lambda_0 = 0.5$ and different radial coordinates of $\xi_1 = 1.077, 2.0$, and 5.0	88
4.4	Normalized radiation patterns of prolate spheroidal antennas with slots at $\eta = 0$ and different interfocal distances $D = d/\lambda_0 = 0.25, 0.5$, and 1.0 and radial coordinates (a) $\xi_1 = 1.077$ and (b) $\xi_1 = 2.0$	89
4.5	Normalized radiation patterns of prolate spheroidal antennas with half-wavelength interfocal distance $d/\lambda_0 = 0.5$, radial coordinate $\xi_1 = 1.077$ and different excitation slots at $\eta_0 = 0, 0.5, 0.707$, and 0.94	90

- 4.6 Normalized radiation patterns of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t+s)/\lambda_1 = 0.5$, $\epsilon_{r1} = \epsilon_{r2} = -2.97$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 2.97$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating). 91
- 4.7 Normalized radiation patterns of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t+s)/\lambda_1 = 0.5$, $\epsilon_{r1} = \epsilon_{r2} = -5.25$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 5.25$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating). 91
- 4.8 Normalized radiation patterns of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t+s)/\lambda_1 = 0.1$, $\epsilon_{r1} = \epsilon_{r2} = -9.6$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 9.6$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating). 92
- 4.9 Normalized radiation pattern of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t+s)/\lambda_1 = 0.25$, $\epsilon_{r1} = \epsilon_{r2} = -1.24$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 2.97$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating). 93
- 4.10 Normalized radiation patterns of prolate spheroidal antennas coated by a radome ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, $\eta_0 = 0$, and $t/\lambda_2 = 0.5$) placed at different separations s using different DNG MTMs and conventional dielectric materials. 94

4.11	Normalized radiation patterns of prolate spheroidal antennas with different excitation slots locations coated by a radome ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, $t/\lambda_2 = 0.5$, $\epsilon_{r_2} = -9.6$ and $\mu_{r_2} = -1.0$ for DNG MTMs radome and $\epsilon_{r_2} = 9.6$ and $\mu_{r_2} = 1.0$ for dielectric radome) placed at different separations s	96
4.12	Normalized radiation patterns of prolate spheroidal antennas coated by a radome ($d/\lambda_0 = 0.5$, $\xi_1 = 2.0$, $\eta_0 = 0$, and $t/\lambda_2 = 0.5$) placed at different separations s using DNG MTMs ($\epsilon_{r_2} = -9.6$ and $\mu_{r_2} = -1.0$) and conventional dielectric materials ($\epsilon_{r_2} = 9.6$ and $\mu_{r_2} = 1.0$).	97
4.13	Radiated power of an infinitesimal electric dipole ($2a_1 = 10.0$ mm $= \lambda_0/100$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) for various thicknesses t (or a_3) at different separations s normalized to the power radiated by the same infinitesimal electric dipole in free space.	98
4.14	Radiated power of an infinitesimal electric dipole ($2a_1 = 10.0$ mm $= \lambda_0/100$, $\xi_1 = 1.005$) coated with DNG shells, (a) $\mu_{r_2} = -1.0$ and different ϵ_{r_2} ; (b) $\epsilon_{r_2} = -3.0$ and different μ_{r_2} , at separation $s = \lambda_0/1000$ normalized to the power radiated by the same infinitesimal electric dipole in free space.	100

- 4.15 Frequency dependence of radiated power of an infinitesimal electric dipole ($2a_1 = 10.0 \text{ mm} = \lambda_0/100$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ with $a_3 = 7.64 \text{ mm}$ normalized to the power radiated by the same infinitesimal electric dipole in free space. 101
- 4.16 Radiated power of an electrically small dipole ($2a_1 = 20.0 \text{ mm} = \lambda_0/50$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same electrically small dipole in free space. 102
- 4.17 Frequency dependence of radiated power of an electrically small dipole ($2a_1 = 20.0 \text{ mm} = \lambda_0/50$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ with $a_3 = 14.78 \text{ mm}$ normalized to the power radiated by the same infinitesimal electric dipole in free space. 103
- 4.18 Radiated power of an electrically small dipole ($2a_1 = 100.0 \text{ mm} = \lambda_0/10$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same electrically small dipole in free space. 104
- 4.19 Radiated power of a $\lambda_0/4$ dipole ($2a_1 = 250.0 \text{ mm} = \lambda_0/4$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same dipole in free space. 105

4.20	Radiated power of a $\lambda_0/2$ dipole ($2a_1 = 500.0$ mm = $\lambda_0/2$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same dipole in free space.	105
5.1	Icosahedron with a subdivision scheme ($v = 3$) on one face.	112
5.2	Area equalization mapping on $z = a$ plane.	114
5.3	Spherical array geometry.	117
5.4	Far-field radiation pattern of icosahedron-based antenna array excited by spherical phase mode Y_3^2 with $a/\lambda = 3.33$ and $N = 3612$. (phase is represented on gray scale)	125
5.5	Theoretical far-field radiation pattern by an excitation of spherical phase mode Y_3^2 with $a/\lambda = 3.33$ presented in [1].	126
5.6	Amplitudes of the far-field mode excited by spherical phase modes Y_1^0 and Y_3^2 versus the radius of array in wavelengths a/λ	127
5.7	Amplitude of the error function e_l^m excited by Y_3^2 with $a/\lambda = 3.33$ and $d/\lambda \approx 0.46$ ($N = 812$).	128
5.8	Amplitudes of the far-field mode versus βa with rotated raised cosine element pattern for $l = 1, 5, 10, 20$	128
5.9	Amplitudes of the far-field mode excited by Y_3^2 versus a/λ for $N = 812$ with rotated raised cosine element pattern.	129

5.10	Compensated amplitudes of the far-field mode excited by Y_1^0 with rotated raised cosine element pattern for $a/\lambda = 10.0$ and $d/\lambda \approx 0.3$ at 6.0 GHz.	130
6.1	Spheroidal array geometry (d is the interfocal distance between two foci P and Q).	135
6.2	Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda \rightarrow 0$ and the semi-major axis $a/\lambda = 3.33$	141
6.3	Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda = 0.5$ and the semi-major axis $a/\lambda = 3.33$	142
6.4	Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda = 4$ and the semi-major axis $a/\lambda = 3.33$	143
6.5	Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda = 6.66$ and the semi-major axis $a/\lambda = 3.33$	144
6.6	The elevation angle of the far-field beam as a function of d/λ	145

List of Tables

3.1	Convergence study of normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) with MTMs coating ($\epsilon_{r_1} = \epsilon_{r_2} = -2.0$ and $\mu_{r_1} = \mu_{r_2} = -1.0$) and dielectric coating ($\epsilon_{r_1} = \epsilon_{r_2} = 2.0$ and $\mu_{r_1} = \mu_{r_2} = 1.0$)	49
4.1	Convergence study of normalized radiation patterns of prolate spheroidal antennas coated by a radome (interfocal distance $d/\lambda_0 = 0.5$, radial coordinate $\xi_1 = 1.077$, $\eta_0 = 0$, the separation $s/\lambda_0 = 0.25$, and the thickness $t/\lambda_2 = 0.5$; $\epsilon_{r_2} = -9.6$ and $\mu_{r_2} = -1.0$ for DNG MTMs radome; $\epsilon_{r_2} = 9.6$ and $\mu_{r_2} = 1.0$ for dielectric radome)	87

Chapter 1

Introduction

1.1 Background

Antennas are one of the most essential components in wireless communications systems, which have been used for more than 100 years [2–4]. Wireless communications began with James Clerk Maxwell’s postulates of electromagnetic (EM) waves in the 1860’s, and Heinrich Rudolf Hertz proved the existence of EM waves using reflector antennas and loops in the 1880’s. The development of wireless communications can be said to have commenced with the invention of wireless telegraph by Guglielmo Marconi in 1890’s, in which the use of wire antennas was one of the most crucial requirements to the success of his experiments. Since then the growth of the technology of antennas and antenna arrays for wireless communications has steadily increased.

A new era in antennas was launched from World War II [5]. Especially with the development of computer architecture and technology from the 1960’s, it became possible to do numerical prediction and optimization of previously intractable complex antenna system configurations due to the widely used computers. Sev-

CHAPTER 1. INTRODUCTION

eral asymptotic methods were introduced, such as Moment Method (MM), finite-difference and finite-element methods for low frequencies and geometrical theory of diffraction (GTD) for high frequencies. As the speed of the computer increases, it was possible to directly solve Maxwell's equations to obtain the radiated fields or the scattered fields by small to medium size antennas. The performance of antenna systems can be predicted with remarkable accuracy which makes antenna technology a true engineering art. Many successful designs of overall systems rely on the analysis and design of the proper performance of the antenna. In the first half of the last century, antenna design may have been seen as almost a "cut and try" operation and had to be considered a secondary issue in the system design.

Particularly in the past two decades, the upsurge of personal mobile communications made the cellular phone a significant symbol of telecommunications. Many types of basic antenna elements had been used in personal mobile communications due to low cost and simplicity, and the most common one was a short monopole antenna. After that, microstrip or patch antennas with more sophisticated designs were used in mobile handsets to obtain more radiation gain and save more power on both receive and transmit links.

Antenna arrays are used to achieve specific radiation pattern requirements, e.g. large directivity, without increasing the size of single antenna elements. Furthermore, antenna arrays can be used to scan their main lobes by controlling the relative phase excitation between the elements. With the number of users and the demand for wireless services increasing at a formidable rate, adaptive antennas or smart antenna systems have been proposed and developed, which combine an-

tenna technology with the technology of digital signal processing (DSP), especially spatial processing.

1.2 Motivation

Although considerable efforts have been invested into antenna design and a certain level of maturity of antenna engineering has been attained, many challenging problems still need to be solved. For instance, integration of new materials, e.g. metamaterials (MTMs), into antenna technology is one of the hottest topics recently. MTMs have many novel properties, such as negative refraction index. In such media, the electric field, magnetic field and the phase vector form a left-handed (LH) system. Therefore, all EM phenomena have to be reconsidered in MTMs. In Part I of this thesis (Chapter 3 and 4), We aim to investigate some scattering and radiation properties of MTMs. We assume that the MTMs studied in this thesis are “perfect” (lossless and nondispersive).

EM scattering is one of the most important problems in EM theory. The EM scattering of objects coated with MTMs always shows different characteristics compared with conventional materials coating [6]. The scattering properties of sub-wavelength spherical and cylindrical resonant structures with MTMs coatings have been investigated by several researchers, such as Alù and Engheta [7]. Their results show that, with MTMs coatings, the total scattering cross section can be

CHAPTER 1. INTRODUCTION

enhanced significantly, which means it is possible to achieve effectively electrically small scatterers or “compact resonators”. On the other hand, the total scattering cross section of spherical and cylindrical objects also can be drastically reduced with MTMs coatings, which means that these objects are nearly “invisible” or “transparent” to an outside observer [8]. However, there are few investigations on the properties of spheroids with MTMs coatings which can be used to model a variety of other shapes. In this thesis, we will study these properties.

There are many other features of double-negative (DNG) MTMs, and one of them is phase compensation. As a wave propagates through a two-layer structure, i.e. double-positive (DPS) materials pairing with a DNG MTMs, the phase difference developed by traversing DPS materials will be decreased and even compensated by traversing DNG MTMs [9]. Based on this idea, the prolate spheroidal antenna coated with a DNG MTMs radome will be investigated, which has many potential applications, for instance, antenna systems on aircraft.

In addition, it is well known that efficient and electrically small antennas with wide bandwidth are desirable for wireless communications, but these requirements are always contradictory. MTMs offer a new way to solve this problem. Ziolkowski et al. presented efficient electrically small antennas by coating them with a DNG or epsilon-negative (ENG) MTMs shell [10, 11]. In this thesis, electrically small spheroidal antenna coated with a confocal DNG shell will be analyzed in order to obtain both higher efficiency and wider bandwidth.

Since single-element antennas always have low values of directivity, antenna arrays should be used to obtain more directive characteristics. With arrays, it is

CHAPTER 1. INTRODUCTION

practical to scan the radiation pattern as well. In Part II of this thesis (Chapter 5 and 6), we aim to study some antenna arrays problems related with phase mode processing.

Phase mode processing can be used to analyze and synthesize circular antenna arrays [12]. This concept has been extended to spherical antenna arrays [1]. However, the spherical array presented in [1] is based on equiangular sampling schemes, which can only scan the beam in the azimuth angle but not electrically in the elevation angle. Furthermore, the sampling points for such array are much denser near the poles than at the equator due to the equiangular topology, which make the array suffer from severe mutual coupling effects near the poles. In order to overcome these limitations, an icosahedron-based topology may be used to improve this spherical array. And with the knowledge of circular arrays, the use of directional elements should increase the frequency bandwidth of antenna arrays. Based on these ideas, an improved spherical antenna array will be investigated.

The spherical antenna array is a special case of the spheroidal antenna array which has more general and flexible properties. The spherical phase mode processing will be extended to spheroidal phase mode processing, and its properties will be studied.

1.3 Objectives

Specifically, we aim to

- (i) Analyze the EM scattering properties of MTMs coatings and investigate potential applications. A conducting prolate spheroid coated with DNG MTMs for an axial incidence will be studied. An analytic solution will be derived using the method of separation of variables in the spheroidal coordinates. The scattering characteristics of conducting prolate spheroid with DNG MTMs coatings will be compared with those of the same spheroid with conventional dielectric coatings. The possibility and the conditions of achieving effective and electrically small spheroidal scatterers and “transparency” of spheroidal objects will be investigated.
- (ii) Investigate the EM radiation properties of a prolate spheroidal antenna coated with confocal DNG MTMs coatings or radome. The analytic solution of EM radiation from a prolate spheroidal antenna, excited by a delta voltage across a circumferential infinitesimally narrow slot at any arbitrary position along the major axis of the antenna, enclosed in one and two layers of confocal DNG MTMs coatings will be obtained. The potential applications of DNG MTMs coatings will be investigated, such as radomes, electrically small antennas, etc.
- (iii) Propose an improved icosahedron-based topology of spherical antenna arrays for phase mode processing in order to overcome the limitations of equiangular

CHAPTER 1. INTRODUCTION

topology. We will investigate the properties of this antenna array with the use of directional elements in order to achieve wideband synthesis in terms of phase mode processing. The errors of this sampling scheme will be analyzed.

- (iv) Further extend the concept of spherical phase mode processing to more general prolate and oblate spheroidal phase mode processing. We will provide the general formulations of the spheroidal phase mode processing using spheroidal wave functions and study the properties of spheroidal antenna arrays with different parameters, from prolate spheroid geometries to oblate ones.

1.4 Contributions of the Thesis

The contributions of the thesis include the following:

- (i) The problem of EM scattering by a conducting prolate (or oblate) spheroid coated with MTMs for an axial incidence is studied. An analytic solution is derived using the method of separation of variables in spheroidal coordinates. The incident and scattered fields in different regions are expanded in terms of spheroidal vector wave functions and unknown expansion coefficients for the scattered wave are obtained by imposing the appropriate boundary conditions.

The DNG MTMs coatings of a conducting spheroid have different scattering characteristics compared with conventional dielectric coatings, which can be used for increasing, decreasing, and smoothing the scattering cross section, and even cancel the nulls of the scattering cross section. These features are useful in antenna design, radar engineering, etc.

It is also found that the total scattering cross section can be dramatically enhanced with certain DNG MTMs coatings to achieve effective and electrically small scatterers (“compact resonant structures”) or drastically reduced with certain low-permittivity MTMs coatings to achieve “invisibility” of objects.

- (ii) A prolate spheroidal antenna coated with a confocal DNG MTMs coating or radome is investigated. The analytic solution of EM radiation from a prolate spheroidal antenna, excited by a delta voltage across a circumferential

CHAPTER 1. INTRODUCTION

infinitesimally narrow slot at any arbitrary position along the major axis of the antenna, enclosed in one and two layers of confocal DNG MTMs coatings is obtained using the method of separation of the spheroidal scalar wave functions in prolate spheroidal coordinates.

One application of DNG MTMs coatings is an antenna radome. Different from conventional dielectric radomes, the radiation patterns of the prolate spheroidal antenna coated with a half wavelength confocal DNG MTMs radome are almost the same as the uncoated antenna patterns. That means the DNG MTMs radome can be electrically transparent to the antenna and has almost no effect on the antenna's radiation pattern. And this DNG MTMs radome can be placed much closer to the antenna than a conventional dielectric one without loss of performance, meaning that the size of the antenna system can be significantly reduced.

Another possible application is to achieve electrically small antennas. It is found that the electrically small spheroidal antenna-DNG shell system is very efficient and has much larger fractional bandwidth (FBW) than that given by the Chu limit which is the maximum FBW limit for the normal electrically small antenna.

- (iii) An improved topology (based on an icosahedron) of spherical antenna arrays for phase mode processing is proposed. In this icosahedron-based topology, the inter-element spacing of all antenna elements is almost identical. This attractive property can be used for three-dimensional beam scanning and for

CHAPTER 1. INTRODUCTION

reducing the effects of mutual coupling. To overcome the limitations of rapid variations in amplitude of the far-field mode, raised cosine pattern elements are used in this array, which enables us to synthesize a wideband pattern without moving nulls. The distortion due to the finite inter-element spacing is analyzed which is useful to design a spherical antenna array. A number of attractive properties for applications are discussed such as electric beam scanning in the entire three-dimensional space and reducing the effects of mutual coupling.

- (iv) The spherical phase mode concept is extended to both prolate spheroidal and oblate spheroidal geometries using spheroidal wave functions. Theoretical and numerical results show that a spheroidal phase mode of the excitation function produces a far field radiation pattern with the same spheroidal phase mode form, and the elevation angle of that pattern increases (decreases) with the ratio of the interfocal distance of the prolate (oblate) spheroid antenna array to the wavenumber. The generality of the spheroidal geometry and the flexibility of controlling the far-field patterns are attractive for a number of applications, such as broadband beamforming and direction finding, etc.

1.5 Organization of the Thesis

The thesis is organized as follows.

Chapter 2 provides a general literature review. The background theory of spheroidal wave functions is introduced firstly. Then the problems of wave scattering from spheroids and spheroidal antennas are reviewed. A brief review of the characteristics and applications of MTMs are given. After that, the phase mode processing for circular antenna arrays is introduced.

Chapter 3 presents EM scattering by a conducting prolate spheroid coated with MTMs for an axial incidence. Firstly, the formulation of the problem is derived in a matrix form in terms of spheroidal vector wave functions. Then the characteristics of DNG MTMs coatings are compared with those of conventional dielectric coatings in order to show the potential applications of DNG MTMs. After that, the properties of achieving both effective electrically small scatterers and “transparency” of objects are presented and discussed.

Chapter 4 investigates the EM radiation from a prolate spheroidal antenna enclosed in one and two layers of confocal DNG MTMs coatings. The solution of EM radiation of an uncoated prolate spheroidal antenna is given firstly. Then the formulations of EM radiation by the prolate spheroidal antenna enclosed in one and two layers of confocal DNG MTMs coatings are derived. After that, the properties of DNG MTMs radomes and electrically small antennas using DNG MTMs coatings are analyzed and discussed.

Chapter 5 presents an icosahedron-based spherical antenna array for phase

CHAPTER 1. INTRODUCTION

mode processing. Firstly, the icosahedron-based topology is given. Then the formulations of this array with directional antenna elements excited by a spherical phase mode are derived. Finally, a number of attractive properties of this icosahedron-based array are discussed.

Chapter 6 extends the concept of spherical phase mode processing used in Chapter 5 to spheroidal phase mode processing. The theoretical solutions for the prolate and oblate spheroidal phase mode excitations and their corresponding far-field radiation patterns are derived firstly. After that, the numerical results and potential applications are presented.

Chapter 7 summarizes the main issues addressed in this thesis and comments on future work.

Chapter 2

Background Theories of Spheroidal Antennas with Metamaterials Coating and Phase Mode Processing

2.1 Introduction

Spheroidal wave functions are special functions in mathematical physics which occur in many scientific and engineering contexts, from atomic nuclei to the cosmos [13]. In the electromagnetic (EM) theory, there are many practical applications, such as antenna analysis and design, microwave integrated circuit (MIC) design, electromagnetic compatibility (EMC) and electromagnetic interference (EMI), and radar cross section (RCS) calculations. Since a spheroid can be used to model many scatterers/antennas, such as wires, disks, aircraft noses, human heads, raindrops, etc. [14, 15], it is very important to investigate the EM scattering and radiation problems in the spheroidal coordinates.

Recently, metamaterials (MTMs) became popular in electromagnetics because of their negative refraction indexes. In such media, the electric field, magnetic field

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

and the phase vector form a left-handed (LH) system [16] as shown in Figure 2.1. Therefore, all the EM phenomena have to be reconsidered in MTMs. However, there are few reports in the literature on full-wave analysis of EM scattering and radiation with MTMs in spheroidal coordinates, which will be investigated in Part I (Chapter 3 and 4) of this thesis.

Figure 2.1: Left-handed system.

For many applications in modern wireless communications, antenna arrays have to be used in order to obtain the required values of directivity or beam width. The phased antenna array is one of the most practical arrays due to its ability to dynamically reconfigure its radiation pattern, to achieve an optimum signal reception in a changing environment. The configuration is often planar. However, a very large number of separately controllable radiators is needed in order to scan a reasonably stable antenna pattern over wide angles, which results in very expensive systems. Circular antenna arrays, with the development of the technique of phase mode processing, can achieve beam steering in the full azimuthal plane due to the symmetry and the ability of 360° coverage. When the array pattern needs to scan three-dimensionally, the cylindrical, spherical, or spheroidal antenna arrays with phase mode processing have to be used, which will be studied in Part II (Chapter

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

5 and 6) of this thesis.

This chapter is organized as follows. In Section 2.2, the background theory of spheroidal wave functions is introduced, and the scattering problems of spheroid and spheroidal antennas are reviewed. A brief review of the characteristics and applications of MTMs are given in section 2.3. Section 2.4 introduces the phase mode processing for circular antenna arrays. Conclusions are given in Section 2.5.

2.2 A Brief Survey of Spheroidal Wave Functions

2.2.1 Introduction of Spheroidal Wave Functions

The prolate and oblate spheroidal coordinate systems (η, ξ, ϕ) are systems of orthogonal curvilinear coordinates, which are formed by rotating the two-dimensional elliptic coordinate system, consisting of confocal ellipses and hyperbolas, about the major and minor axes of the ellipses, respectively, as shown in Figure 2.2. (η, ξ, ϕ) are respectively the spheroidal angular, radial, and azimuthal coordinates, which form a right-handed system. The interfocal distance of the spheroid is denoted as d . The prolate spheroidal coordinates are related to the Cartesian coordinates (x, y, z) by the transformation

$$x = \frac{d}{2}[(1 - \eta^2)(\xi^2 - 1)]^{\frac{1}{2}} \cos \phi \quad (2.1)$$

$$y = \frac{d}{2}[(1 - \eta^2)(\xi^2 - 1)]^{\frac{1}{2}} \sin \phi \quad (2.2)$$

$$z = \frac{d}{2}\eta\xi \quad (2.3)$$

with $-1 \leq \eta \leq 1$, $1 \leq \xi < \infty$, and $0 \leq \phi \leq 2\pi$. And oblate spheroidal coordinate are related to the Cartesian coordinates (x, y, z) by the transformation

$$x = \frac{d}{2}[(1 - \eta^2)(\xi^2 + 1)]^{\frac{1}{2}} \cos \phi \quad (2.4)$$

$$y = \frac{d}{2}[(1 - \eta^2)(\xi^2 + 1)]^{\frac{1}{2}} \sin \phi \quad (2.5)$$

$$z = \frac{d}{2}\eta\xi \quad (2.6)$$

with $-1 \leq \eta \leq 1$, $0 \leq \xi < \infty$, and $0 \leq \phi \leq 2\pi$. The details of the geometry of spheroidal coordinates are documented by C. Flammer [14], M. Abramowitz and I. A. Stegun [17], and P. Moon and D. E. Spencer [18].

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

(a) Prolate

(b) Oblate

Figure 2.2: Prolate and oblate spheroidal coordinate systems.

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

To investigate the EM fields in prolate and oblate spheroidal coordinates, the spheroidal wave functions are frequently used, especially in the full-wave analysis when boundary value problems in spheroidal structures are encountered [15]. Scalar Helmholtz equations are separable in the prolate and oblate spheroidal coordinate systems. By applying the separation of variables in the prolate and oblate spheroidal coordinates to Maxwell's equations, the spheroidal harmonics of EM waves can be obtained. A time dependence of $e^{j\omega t}$ is assumed and suppressed throughout. The separation of scalar variables results in three independent functions:

- 1) the spheroidal radial functions $R_{mn}^{(i)}(h; \xi)$ or $R_{mn}^{(i)}(-jh; j\xi)$,
- 2) the spheroidal angular functions $S_{mn}(h; \eta)$ or $S_{mn}(-jh; \eta)$,
- 3) the sine and cosine functions (or exponential harmonic functions $e^{jm\phi}$).

This separability is analogous to that of solving the Laplace equation in spherical coordinates. Here the last pair of trigonometric functions (sine and cosine functions) is well known, but the first two are not so easily computed. In general, spheroidal radial and angular functions in spheroidal coordinates are, respectively, the generalization of Legendre functions and spherical Bessel functions in the spherical polar coordinates [13]. The theoretical formulation of these harmonics is well documented by C. Flammer [14], J. A. Stratton [19], and L. W. Li [15].

However, the spheroidal vector wave functions are not separable in the prolate and oblate spheroidal coordinate systems due to the nonorthogonality of spheroidal radial functions. Therefore, it is difficult to obtain analytic solutions to those

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

vector boundary value problems. In computing EM or physical quantities, the nonorthogonality is involved in the method of eigenfunctional expansions, in which linear equations in the form of nonorthogonal summation are converted to a matrix equation system. The dimension of the matrix system is normally infinite, and the convergence depends mostly on the magnitudes of the interfocal distance and the applied frequency range. The convergence also depends on the representation of different kinds of spheroidal wave functions. Several kinds of spheroidal vector wave functions have been introduced by researchers [20–22]. However, most of them show fast convergence only for plane-wave scattering or far-field approximations in free space. Furthermore, some kinds of spheroidal vector wave functions converge much more slowly in the high-frequency region, although it is possible to use these vector wave functions in the low-frequency region.

The calculation of the spheroidal angular and radial functions is very complicated, which is another difficulty in obtaining the analytic solutions in spheroidal structures [14, 23]. Computation of the spheroidal radial or angular functions requires eigenvalue computations and the forward and backward recursion formulations. There are only a few computer programs available to the public for computing the spheroidal wave functions and their eigenvalues [24–27]. With the help of these open programs, we have developed an efficient algorithm for numerically computing these functions and eigenvalues using the C language.

2.2.2 Electromagnetic Scattering of Spheroids

The problems of EM-wave scattering of a single spheroid or a system of spheroids have been well studied, and many analytic solutions have been obtained to date. Moffatt studied the echo area of a perfectly conducting prolate spheroid [28, 29]. The approximate solution to EM-wave backscattering from a prolate spheroid was obtained by means of modifying the time-dependent backscattering waveform from a perfectly conducting sphere in order to conform with spheroidal geometry.

Asano and Yamamoto provided a solution of EM scattering by a homogeneous prolate (or oblate) spheroidal particle with an arbitrary size and refractive index at any angle of incidence using a separation of the vector wave equations in the spheroidal coordinates and an expansion in them in terms of spheroidal wave functions [30]. In [31], Asano found that the prolate spheroids at parallel incidence have steep and high resonance maxima in the scattering efficiency factors and broad and low forward scattering peaks in the intensity functions; on the other hand, the oblate spheroids at parallel incidence have broad and low resonance maxima and sharp and high forward scattering peaks. It was shown that, for oblique incidence, the scattering properties of a long slender prolate spheroid resemble those of an infinitely long circular cylinder.

Sinha and MacPhie obtained the exact solutions for the EM scattering of plane waves with arbitrary polarization and angle of incidence by conducting prolate spheroids in terms of spheroidal vector wave functions [20, 21]. In [21], translational addition theorems [32] were used in order to translate the wave functions in

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

different local coordinates. In their formulation, the column vectors of the expansion coefficients of the scattered field are obtained from the column vectors of the expansion coefficients of the incident field by means of a transformation matrix. The matrix depends only on the scatterers; therefore, the scattered field for any direction of incidence can be obtained with the same transformation matrix. This method was used by some other researchers [22, 33–37] and had many applications in physics and engineering.

Dalmas and Deleuil investigated multiple scattering of EM waves by two infinitely conducting prolate spheroids, using spheroidal vector wave functions constructed along the radius vector [38]. To solve this problem, translational addition theorems for the wave functions of any translation were derived [39]. Merchant et al. studied the complex pole patterns of the scattering amplitude for conducting spheroids using the Waterman T-matrix method [40].

Cooray and Ciric solved the problems of EM-wave scattering by a system of two perfectly conducting or dielectric spheroids of arbitrary orientation [22, 33]. Their method was similar to that of Sinha and MacPhie [20, 21]. In order to impose the boundary conditions, the scattering field of one spheroid is expressed in terms of the local spheroidal coordinate attached to the other spheroid using the rotational-translational addition theorems for spheroidal vector wave functions [41]. Nag and Sinha provided the solution of EM scattering by a system of two lossy dielectric prolate spheroids [34].

Cooray and Ciric also solved the problems of EM scattering by a system of n dielectric spheroids of arbitrary orientation [35] and EM-wave scattering by a

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

coated dielectric spheroid [36] in terms of the same method as in [41]. Sebak and Sinha provided the solution of a plane EM-wave scattering by a conducting prolate spheroid with a confocal dielectric coating at axial incidence [37].

The translational addition theorems applied to incoming or outgoing waves possess a convergence sphere near which the addition series fail to converge rapidly. To overcome this deficiency, Do-Nhat and MacPhie developed the quasi-translational addition expressions for prolate wave functions by using a physical-geometrical transformation [42], which are valid everywhere and, in particular, exhibit fast convergence characteristics for thin spheroids which are commonly used in physical and engineering applications.

2.2.3 Spheroidal Antennas

The oscillations of spheroidal structures and spheroidal antennas were investigated a long time ago [43–46], since spheroidal antennas can be used to model a variety of antenna shapes, from wire/cylindrical antennas via spherical antennas to disk antennas (using oblate spheroid) as shown in Figure 2.3.

Figure 2.3: Modelling different antennas using prolate and oblate spheroids.

Chu and Stratton studied the forced EM oscillations of a conducting prolate spheroid [47]. An external electromotive force (e.m.f.) was applied across an infinitesimal gap at the central section of the spheroid and the resultant field was expressed as an infinite sum of wave modes in terms of spheroidal wave functions, in which only few modes were considered.

Schelkunoff analyzed the prolate and oblate spheroidal antenna problem in [48]. In his work, the voltage is applied symmetrically between two halves of the spheroid by means of a biconical transmission line. In this case, the far field is independent of the ϕ coordinate. The input admittances for such antennas are obtained for various major to minor axial ratios and the first few spheroidal modes

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

of propagation.

Flammer also analyzed the radiation characteristics of a prolate spheroidal monopole antenna with a finite gap using a variational approach [49]. Wells [50] studied the near-field of the prolate spheroidal antennas, the current distribution over the antenna was determined.

Weeks [51] investigated the problem of the prolate spheroidal antenna from a slightly different view. Instead of having the excitation gap in the center of the antenna, the gap can be located arbitrarily along the antenna without disturbing the symmetry in the ϕ direction. In addition, the solution of the boundary value problem of a dielectric-coated spheroidal antenna was also obtained. His results were presented in the form of radiation patterns.

Lytle and Schultz [52] considered the problems of a gap-excited finite sized prolate spheroidal antenna, with and without a confocal prolate spheroidal vacuum sheath, operated in a uniform cold and in a uniform warm lossy plasma medium. Sato and Naito [53] studied a prolate spheroidal antenna covered with a confocal prolate spheroidal ferrite sleeve. Kotulski [54] presented an equivalent circuit to calculate the input admittance of the prolate spheroidal antenna for the first two modes.

Jen and Hu [55] derived a method to calculate the spheroidal wave functions asymptotically for any value of h greater than ten using Wentzel-Kramer-Brillouin and Langer transformations and the asymptotic characteristics of Airy functions. The radiated fields and input admittance of a metallic prolate spheroid excited by a circumferential slot were given. Hu [56] solved asymptotically the problem of

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

the radiation fields from the metallic prolate spheroid of any length excited by any asymmetrical sources.

The study of coupled spheroidal antennas has also received interest from researchers. Sinha and MacPhie [57] obtained the mutual admittance of a system of two center-fed parallel prolate spheroidal antennas by means of applying the translational addition theorem for spheroidal wave functions. With the rotational translational addition theorem, Ciric and Cooray [58] investigated the admittance characteristics and far-field patterns for coupled center-fed spheroidal dipole antennas in arbitrary orientation. For practical applications, the reduction in the coupling between the two spheroidal antennas for various orientations was evaluated quantitatively.

Do-Nhat and MacPhie extended the method of the ultraspherical distribution of cylindrical dipole antennas to the spheroidal dipoles [59]. They studied the input admittance of thin prolate spheroidal dipole antennas with three types of gap field: Dirac's function, uniform, and ultraspherical gap field. For various gap widths, three kinds of gap fields produce virtually the same admittance value for very thin spheroids but different values of the admittance for thicker spheroidal antennas. It was found that for prolate spheroidal antennas, the first resonance occurs when their lengths are slightly greater than half wavelength, in contrast to circular cylindrical antennas which resonate when their lengths are slightly less than half a wavelength.

Vinogradova investigated the radiation characteristics of a nonclosed surface spheroidal antenna using the spheroidal scalar wave functions [60]. The radiation

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

pattern of the nonclosed spheroidal antenna varies with the frequency in a narrow band.

Zhang and Sebak [61] studied the radiation characteristics of an asymmetrical slot antenna on a conducting prolate spheroid. In their work, the radiated fields were expanded in terms of the vector prolate spheroidal wave functions. The unknown expansion coefficients were obtained by a system of equations derived from the boundary conditions. The effects of the slot length and shape of the spheroid on the radiated field were also presented.

With the development of mobile communication systems, the microstrip antenna (MSA) is widely used, as it is adaptive in size, thin and lightweight, and has larger gain than comparable wire antennas. In some practical applications, such as global positioning systems or iridium satellite communication systems, the wider beamwidths of MSA's are desired. Taguchi et al. [62] and Yagyu et al. [63] analyzed the radiation characteristics of a circular MSA inserted in the center of an oblate spheroidal conductor using an integral equation and the method of moments (MOM). They found that the beamwidth in the E-plane of the antenna became broader than that of the normal circular patch MSA, and the beamwidth becomes wider as the antenna height increases.

The effects of coating and radomes on antennas are interesting topics as well, since antennas often have to be insulated from the surrounding environments, and the presence of dielectric coating or radomes can affect the radiation pattern. For wire, cylindrical dipole, or spherical antennas, this issue has been addressed extensively in literature, while similar studies on spheroidal structures are limited.

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

The full-wave analysis of spheroidal antennas coated with a confocal radome is presented in [64]. The results show that a radome placed at a distance of λ_0 (wavelength in free space) and beyond, with a thickness of multiples of $0.5\lambda_2$ (λ_2 is the wavelength in coating), has little effect on the radiation pattern.

Macon et al. [65] investigated the effect of curvature variation on the resonant input impedance of a cavity-backed, conformal slot antenna and a conformal patch antenna recessed in a perfectly conducting, electrically large prolate spheroid surface using the finite element-boundary integral (FE-BI) method. It was found that the input impedance of the slot antenna, for normal probe excitation, is sensitive to surface curvature variation, and the performance of a conformal antenna is dependent on the local geometry of the mounting surface.

Capoglu and Smith [66] studied the input admittance of the prolate-spheroidal monopole antenna fed by a magnetic current frill through an image plane. The theoretical results are in very good agreement with the experimental results.

2.3 Metamaterials and Negative Refraction Index

Metamaterials (meta is an ancient Greek prefix, which means “beyond”) are artificially constructed materials which are engineered media having qualitatively new response functions that do not occur or may not be easily available in nature [6]. They have been one of the most interesting topics of electromagnetics in recent years [6, 67–81]. In the late 1960s, Veselago first theoretically studied monochromatic uniform plane wave propagation in a homogeneous isotropic medium with both negative permittivity ϵ and permeability μ [67]. His results showed that the direction of the group velocity (the Poynting vector $\mathbf{S}$) of a monochromatic uniform plane wave is opposite to that of the phase velocity (phase vector $\mathbf{k}$) in such a medium, and the index of refraction n of this medium can be considered negative. Therefore, from Maxwell’s equations it is found that the electric field $\mathbf{E}$, the magnetic field $\mathbf{H}$, and the phase vector $\mathbf{k}$ form a LH system in this medium, which is referred to as “left-handed” medium by Veselago. There are many well-known phenomena due to EM-wave propagation in materials. All of these phenomena must be reconsidered in MTMs. For example, the Doppler effect is reversed, with a wave approaching an observer being down-shifted in frequency.

However, such media were not available until recently. Shelby et al. [68] constructed a composite structure based on a periodic array of small metallic wires to produce negative permittivity [69] and split ring resonators (SRRs) to produce negative permeability [70] and experimentally demonstrated a negative index of

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS
COATING AND PHASE MODE PROCESSING

refraction n in the microwave regime. Since then, many researchers are trying to improve the performance of microwave, wireless communications, microelectronics, and optical devices using MTMs.

Today, the class of MTMs consists of many related types of artificial materials [6]. Common examples are double-negative (DNG) MTMs [6, 74, 75] (also named as LH medium [67, 68], backward-wave (BW) media [76], and negative-index materials (NIMs) [77], etc.), single-negative (SNG) materials [78, 79] which include epsilon-negative (ENG) media and mu-negative (MNG) media, electromagnetic bandgap (EBG) structured materials, etc.

For lossless DNG MTMs, $\epsilon < 0$ and $\mu < 0$, the following relations are given in [74],

$$\sqrt{\epsilon} = \sqrt{-|\epsilon|} = -j\sqrt{|\epsilon|} \quad (2.7)$$

$$\sqrt{\mu} = \sqrt{-|\mu|} = -j\sqrt{|\mu|} \quad (2.8)$$

The definitions of the index of refraction, the wavenumber, and the wave impedance are given as follows,

$$n = \frac{\sqrt{\epsilon}\sqrt{\mu}}{\sqrt{\epsilon_0}\sqrt{\mu_0}} = -\sqrt{|\epsilon_r|}\sqrt{|\mu_r|} \quad (2.9)$$

$$k = \omega\sqrt{\epsilon}\sqrt{\mu} = -\omega\sqrt{|\epsilon|}\sqrt{|\mu|} \quad (2.10)$$

$$Z = \frac{\sqrt{\mu}}{\sqrt{\epsilon}} = \frac{\sqrt{|\mu|}}{\sqrt{|\epsilon|}} \quad (2.11)$$

where $\epsilon_r = \epsilon/\epsilon_0$ and $\mu_r = \mu/\mu_0$ are the relative permittivity and relative permeability, respectively, and ϵ_0 and μ_0 are the permittivity and permeability in free space.

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

As shown in Figure 2.4, an EM wave bends the “wrong way” relative to the surface normal when the wave enters the DNG MTM from free space, which does not bend toward the surface normal as in conventional materials. Figure 2.4 also shows that the wave propagates in a direction opposite to that of the flow of energy, or the phase vector $\mathbf{k}$ and the Poynting vector $\mathbf{S}$ are in the opposite direction in MTMs.

Figure 2.4: EM waves reflected/transmitted at air/DNG MTMs interface.

MTMs have many potential applications. One of the possible applications is in imaging due to the negative refraction. It is an accepted convention that the resolution of an image is limited by the wavelength of light used. However, the results of Pendry et al. show that a lens made of MTMs could focus light for objects less than a wavelength in size to a geometric point [70]. This is because a negative index lens focuses not only the propagating waves, but also the finer details (the evanescent waves in the near field) of the object [71]. Therefore, the

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

lens' resolution is enhanced, and sub-wavelength imaging is achievable. Grbic and Eleftheriades investigated the sub-wavelength focusing in planar 2-D structures made of negative-index transmission lines [82], which can achieve $\lambda/2$ resolution. This characteristic may be used in sub-wavelength magnetic resonance imaging (MRI) to increase the resolution for cancer detection.

MTMs also have many applications in communications. For example, Ziolkowski et al. constructed efficient electrically small antennas by surrounding them with a DNG or ENG MTM shell [10, 11]. It is well-known that the radiated power is much smaller than the capacitive reactive power for electrically small radiators [83–91]. However, from a circuit point of view, an MTM shell has inductive reactance. Therefore, the electrically small antenna and the DNG MTMs shell are matched to form a resonant CL circuit, and the radiated power is greatly enhanced. The scattering properties of small particles with MTMs coatings have also been investigated [7, 8, 92]. The results show both the possibility of making these objects nearly “invisible” or “transparent” to an outside observer and the possibility of enhancing the overall scattering cross section dramatically. Another interesting feature of DNG MTMs is phase compensation. Alù and Engheta studied the characteristics of pairs of concentric MTMs slabs [9, 93, 94]. Their results show that it is possible to realize sub-wavelength electrically small cavity resonators and waveguides with lateral dimensions below diffraction limits. Combining conventional and MTMs leaky-wave antennas, it is also possible to realize leaky-wave antennas with backward to forward scanning capability [95]. MTMs should have a wealth of applications in microwaves, wireless communications, microelectronics, optics, etc.

2.4 Phase Mode Processing for Circular Antenna Arrays

Circular antenna arrays have the principal advantage of 360° coverage and the ability to steer the beams in full azimuthal angles with little change of either beamwidth or sidelobe level. The most significant developments of analysis and general understanding of the properties of circular arrays were made by several groups of researchers in the 1960s when the concept of phase mode excitation was developed [12].

The concept of phase mode processing can be described as follows. The excitation of planar circular antenna arrays can be conveniently analyzed in terms of a Fourier series, and each term of the series is called a phase mode [12]. When a circular array is excited by a phase mode, the far-field radiation pattern corresponding to that phase mode has the same characteristic variation of phase with azimuth angle, maintained over a potentially very broad instantaneous bandwidth. And the amplitude of the far-field radiation pattern is a constant value which is given by a Bessel function coefficient. From the point of view of pattern synthesis, any desired far-field radiation pattern can be broken down into its complex Fourier components. Therefore, it is possible to separately excite each of these Fourier components around the array with appropriate Bessel function terms to obtain the corresponding far-field radiation pattern. In addition, taking the mutual coupling effects into account, all values of the radiation impedances of the antenna elements will be changed by the same amount for a single mode. For this

CHAPTER 2. BACKGROUND THEORIES OF SPHEROIDAL ANTENNAS WITH METAMATERIALS COATING AND PHASE MODE PROCESSING

reason, if the excitation network of an antenna array has a separate feed port for each phase mode, the effects of mutual coupling can be compensated by adjusting the different impedance mismatches at the respective input ports.

In [96], Davies presented a kind of Butler matrix network to implement circular antenna arrays which can use phase mode processing conveniently. Rahim and Davies investigated the effect of directional elements on circular antenna arrays fed by phase mode excitations [97]. Their results show that the use of directional elements can overcome the limitation of rapid variation in the amplitude of the far-field pattern and can enable circular arrays to synthesize wide-bandwidth directional patterns. Jones and Griffiths studied the problem of broadband pattern synthesis from a circular array [98]. Their experiment results are in good agreement with theoretical predictions. In [99], Griffiths and Eiges presented a technique to confine circular array phase modes to angular sectors, which can improve considerably the performance of direction-finding (DF) and null-steering systems. Eiges and Griffiths studied the application of superresolution techniques to circular arrays in mode space [100]. Their results show that coherent signals can be decorrelated by a mode space version of the spatial smoothing technique.

2.5 Conclusion

In this chapter, a brief survey of spheroidal wave functions is presented. An introduction to the prolate and oblate spheroidal coordinates and spheroidal wave functions are given. A literature review of EM scattering of spheroids and EM radiation of spheroidal antennas is provided.

There has been an increasing interest in MTMs recently. The history of development of MTMs is reviewed. And the properties of DNG MTMs, such as the negative refraction index, are introduced. The potential applications of MTMs are also presented. The EM scattering and radiation problems with MTMs in spheroidal coordinates will be investigated in this thesis.

The concept of phase mode processing for circular antenna arrays is introduced briefly. A literature review of phase mode processing is also presented. In order to electrically scan the beam in three dimensions, as an extension of the circular antenna array, spherical and spheroidal antenna arrays with phase mode processing will be studied in this thesis.

Part I

Metamaterials

Chapter 3

Electromagnetic Scattering by a Conducting Spheroid Coated with Double-Negative Metamaterials

3.1 Introduction

As mentioned in Chapter 2, the prolate and oblate spheroids are important canonical objects which can be used to model a variety of other shapes [14, 15]. The problem of scattering of electromagnetic (EM) waves by spheroids with a confocal dielectric coating has been studied [36, 37]. In [37] the scattering problem by a conducting spheroid coated with a confocal homogeneous dielectric layer is studied. In [36] the problem of scattering by a coated dielectric spheroid is solved. Recently, there has been an increasing interest in the investigation of the problem of scattering of coating metamaterials (MTMs) [6, 101, 102]. Especially for small particles, it is possible to achieve both effective electrically small scatterers and “invisible” (or “transparent”) objects such as cylinder, sphere, etc., using different MTMs coatings [7, 8, 92].

In this chapter, the EM scattering by a conducting prolate (or oblate) spheroid

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

coated with lossless MTMs for an axial incidence is studied in order to understand the scattering properties of MTMs coating and investigate potential applications. An analytic solution is derived using the method of separation of variables in the spheroidal coordinates. The incident and scattered fields in different regions are expanded in terms of spheroidal vector wave functions, and the unknown expansion coefficients for the scattered wave are obtained by imposing the appropriate boundary conditions.

Our results show that the scattering characteristics by a conducting prolate spheroid coated with MTMs exhibits different characteristics compared with that coated with a conventional material. This can be used to increase, decrease, and smooth the scattering cross section, and even cancel the nulls of the scattering cross section. These features are useful in antenna design, radar engineering, etc.

For example, with different MTMs coatings, the total scattering cross sections can be dramatically enhanced to achieve “compact resonant structures” or drastically reduced to achieve “invisibility” of objects. The results with different parameters, such as relative permittivity, the spheroidal geometry, and the ratio of semi-major axes of two layers, are provided and discussed.

This chapter is organized as follows. Section 3.2 derives the formulation of the problem in a matrix form. The numerical results and discussion are given in Section 3.3. And Section 3.4 concludes the work of this chapter.

3.2 Scattering with DNG MTMs Coating

The geometry of the problem studied in this chapter, a conducting prolate spheroid coated with confocal layers, is shown in Figure 3.1. The prolate spheroidal coor-

Figure 3.1: Geometry of the problem.

dinates (η, ξ, ϕ) form a right-handed system, where η , ξ , and ϕ are the spheroidal angular, radial, and azimuthal coordinates [14]. The major axes of these spheroids are along the z -axis of the Cartesian system, where the origin O is at the center of the spheroids. The surfaces from the inner to the outer spheroid are defined by $\xi = \xi_2$, $\xi = \xi_1$, and $\xi = \xi_0$ respectively; the length of the corresponding semimajor and semiminor axes are a_2 , a_1 , a_0 , and b_2 , b_1 , b_0 respectively. The surface of a

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

spheroid can be calculated by

$$\xi = \frac{a}{\sqrt{a^2 - b^2}} = \frac{2a}{d} \quad (3.1)$$

where d is the length of the common interfocal distance. The media of the coating layers are both assumed to be linear, homogeneous, isotropic, and lossless with permittivities ϵ_2 and ϵ_1 , and permeabilities μ_2 and μ_1 respectively. The exterior medium is assumed to be free space with permittivity ϵ_0 and permeability μ_0 .

3.2.1 Prolate Spheroidal Scalar and Vector Wave Functions

Using the method of separation of variables, the solutions of the scalar wave equation

$$(\nabla^2 + k^2)\psi = 0 \quad (3.2)$$

in prolate spheroidal coordinates, or the prolate spheroidal scalar wave functions, can be obtained in the form of the Lamé products

$$\psi_{e_{mn}}^{(i)}(h; \eta, \xi, \phi) = S_{mn}(h, \eta) R_{mn}^{(i)}(h, \xi) \frac{\cos}{\sin} m\phi \quad (3.3)$$

where $S_{mn}(h, \eta)$ are the prolate spheroidal angular functions of the first kind, $R_{mn}^{(i)}(h, \xi)$ are the prolate spheroidal radial functions of the i th kind with $i = 1, 2, 3$, or 4, and $h = \frac{1}{2}kd$ with k being the wavenumber. The subscript e stands for an even function and the subscript o indicates odd function [14].

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

The solutions of the vector wave equation in prolate spheroidal coordinates are obtained as follows,

$$\bar{M}_{e_{mn}}^{q(i)}(h; \eta, \xi, \phi) = \nabla \psi_{e_{mn}}^{(i)}(h; \eta, \xi, \phi) \times \hat{q} \quad (3.4)$$

$$\bar{N}_{e_{mn}}^{q(i)}(h; \eta, \xi, \phi) = \frac{1}{k} \nabla \times \bar{M}_{e_{mn}}^{q(i)}(h; \eta, \xi, \phi) \quad (3.5)$$

where $\hat{q}$ is an arbitrary constant vector ($\hat{q} = \hat{x}, \hat{y}$, or $\hat{z}$), or the position vector $\hat{r}$.

The following additional vector wave functions are also often used in the expansion of EM fields,

$$\bar{M}_{e_{m+1,n}}^{+(i)}(h; \eta, \xi, \phi) = \frac{1}{2} \left[\bar{M}_{e_{mn}}^{x(i)}(h; \eta, \xi, \phi) \mp \bar{M}_{e_{mn}}^{y(i)}(h; \eta, \xi, \phi) \right] \quad (3.6)$$

$$\bar{M}_{e_{m-1,n}}^{-(i)}(h; \eta, \xi, \phi) = \frac{1}{2} \left[\bar{M}_{e_{mn}}^{x(i)}(h; \eta, \xi, \phi) \pm \bar{M}_{e_{mn}}^{y(i)}(h; \eta, \xi, \phi) \right] \quad (3.7)$$

$$\bar{N}_{e_{m+1,n}}^{+(i)}(h; \eta, \xi, \phi) = \frac{1}{k} \nabla \times \bar{M}_{e_{m+1,n}}^{+(i)}(h; \eta, \xi, \phi) \quad (3.8)$$

$$\bar{N}_{e_{m-1,n}}^{-(i)}(h; \eta, \xi, \phi) = \frac{1}{k} \nabla \times \bar{M}_{e_{m-1,n}}^{-(i)}(h; \eta, \xi, \phi) \quad (3.9)$$

The explicit expressions of the prolate spheroidal vector wave functions are well documented in [14].

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

3.2.2 Incident, Scattered, and Transmitted Fields

Consider a linearly polarized monochromatic plane EM wave incident on this prolate spheroid structure along the negative z axis with unit amplitude electric field $\bar{E}^i$ polarized along the positive y axis and the magnetic field $\bar{H}^i$ polarized along the positive x axis. The solution for the oblate spheroids can be obtained by the transformations $\xi \rightarrow j\xi$ and $h \rightarrow -jh$.

The incident electric and magnetic fields $\bar{E}^i$ and $\bar{H}^i$ can be expanded in terms of prolate spheroidal vector wave functions [14, 15, 37] as

$$\bar{E}^i = \frac{1}{k_0} \sum_{n=0}^{\infty} a_n \bar{M}_{e_{0n}}^{x(1)}(h_0; \eta, \xi, \phi) \quad (3.10)$$

$$\bar{H}^i = \frac{j}{k_0 Z_0} \sum_{n=0}^{\infty} a_n \bar{N}_{e_{0n}}^{x(1)}(h_0; \eta, \xi, \phi) \quad (3.11)$$

where

$$a_n = \frac{2j^{n-1} S_{0n}(h_0, 1)}{N_{0n}} \quad (3.12)$$

in which N_{0n} are the normalization factors, $h_0 = \frac{1}{2}k_0 d$, and k_0 is the wavenumber in free space. $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the characteristic impedance of free space. (3.10) can be expressed in matrix form as

$$\bar{\bar{E}}^i = \bar{\bar{M}}_i^{(1)T} \bar{\bar{I}} \quad (3.13)$$

where the double bar denotes a column matrix, T denotes the transpose of a matrix, and

$$\bar{\bar{M}}_i^{(1)T} = \begin{bmatrix} \bar{M}_{e_{00}}^{x(1)}(h_0; \eta, \xi, \phi) & \bar{M}_{e_{01}}^{x(1)}(h_0; \eta, \xi, \phi) & \bar{M}_{e_{02}}^{x(1)}(h_0; \eta, \xi, \phi) & \dots \end{bmatrix} \quad (3.14)$$

$$\bar{\bar{I}}^T = \begin{bmatrix} a_0 & a_1 & a_2 & \dots \end{bmatrix} \quad (3.15)$$

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Similarly, (3.11) can be written in matrix form as

$$\bar{H}^i = \frac{j}{k_0 Z_0} \bar{N}_i^{(1)T} \bar{I} \quad (3.16)$$

The components of $\bar{N}_i^{(1)}$ are obtained from the corresponding components of $\bar{M}_i^{(1)}$ by replacing $\bar{M}$ by $\bar{N}$.

The scattered field for $\xi \geq \xi_0$ can be expanded in terms of vector spheroidal wave functions [20, 37] as

$$\bar{E}^s = \sum_{n=0}^{\infty} \left[\alpha_n^+ \bar{M}_{e_{0n}}^{+(4)}(h_0; \eta, \xi, \phi) + \alpha_{n+1}^z \bar{M}_{e_{1,n+1}}^{z(4)}(h_0; \eta, \xi, \phi) \right] \quad (3.17)$$

$$\bar{H}^s = j/Z_0 \sum_{n=0}^{\infty} \left[\alpha_n^+ \bar{N}_{e_{0n}}^{+(4)}(h_0; \eta, \xi, \phi) + \alpha_{n+1}^z \bar{N}_{e_{1,n+1}}^{z(4)}(h_0; \eta, \xi, \phi) \right] \quad (3.18)$$

where α_n^+ and α_{n+1}^z are unknown expansion coefficients which can be determined using boundary conditions. (3.17) can be written in matrix form as

$$\bar{E}^s = \bar{M}_s^{(4)T} \bar{\alpha} \quad (3.19)$$

where

$$\bar{M}_s^{(4)T} = \begin{bmatrix} \bar{M}_0^{+(4)T} & \bar{M}_1^{z(4)T} \end{bmatrix} \quad (3.20)$$

in which

$$\begin{aligned} \bar{M}_0^{+(4)T} &= \begin{bmatrix} \bar{M}_{e_{00}}^{+(4)}(h_0; \eta, \xi, \phi) & \bar{M}_{e_{01}}^{+(4)}(h_0; \eta, \xi, \phi) & \bar{M}_{e_{02}}^{+(4)}(h_0; \eta, \xi, \phi) & \dots \end{bmatrix} \\ \bar{M}_1^{z(4)T} &= \begin{bmatrix} \bar{M}_{e_{11}}^{z(4)}(h_0; \eta, \xi, \phi) & \bar{M}_{e_{12}}^{z(4)}(h_0; \eta, \xi, \phi) & \bar{M}_{e_{13}}^{z(4)}(h_0; \eta, \xi, \phi) & \dots \end{bmatrix} \end{aligned} \quad (3.21)$$

and

$$\bar{\alpha}^T = \begin{bmatrix} \bar{\alpha}^{+T} & \bar{\alpha}^{zT} \end{bmatrix} \quad (3.22)$$

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

in which

$$\begin{aligned}\bar{\alpha}^{+T} &= \begin{bmatrix} \alpha_0^+ & \alpha_1^+ & \alpha_2^+ & \dots \end{bmatrix} \\ \bar{\alpha}^{zT} &= \begin{bmatrix} \alpha_1^z & \alpha_2^+ & \alpha_3^+ & \dots \end{bmatrix}\end{aligned}\quad (3.23)$$

(3.18) can be written as

$$\bar{H}^s = \frac{j}{Z_0} \bar{N}_s^{(4)T} \bar{\alpha} \quad (3.24)$$

The components of $\bar{N}_s^{(4)}$ are obtained from those of $\bar{M}_s^{(4)}$ by replacing $\bar{M}$ by $\bar{N}$.

The transmitted field can be expressed as

$$\begin{aligned}\bar{E}^{t_i} &= \sum_{n=0}^{\infty} \left[{}^i\beta_n^{+(1)} \bar{M}_{e_{0n}}^{+(1)}(h_i; \eta, \xi, \phi) + {}^i\beta_n^{+(2)} \bar{M}_{e_{0n}}^{+(2)}(h_i; \eta, \xi, \phi) \right. \\ &\quad \left. + {}^i\beta_{n+1}^{z(1)} \bar{M}_{e_{1,n+1}}^{z(1)}(h_i; \eta, \xi, \phi) + {}^i\beta_{n+1}^{z(2)} \bar{M}_{e_{1,n+1}}^{z(2)}(h_i; \eta, \xi, \phi) \right]\end{aligned}\quad (3.25)$$

$$\begin{aligned}\bar{H}^{t_i} &= j/Z_i \sum_{n=0}^{\infty} \left[{}^i\beta_n^{+(1)} \bar{N}_{e_{0n}}^{+(1)}(h_i; \eta, \xi, \phi) + {}^i\beta_n^{+(2)} \bar{N}_{e_{0n}}^{+(2)}(h_i; \eta, \xi, \phi) \right. \\ &\quad \left. + {}^i\beta_{n+1}^{z(1)} \bar{N}_{e_{1,n+1}}^{z(1)}(h_i; \eta, \xi, \phi) + {}^i\beta_{n+1}^{z(2)} \bar{N}_{e_{1,n+1}}^{z(2)}(h_i; \eta, \xi, \phi) \right]\end{aligned}\quad (3.26)$$

where $h_i = \frac{1}{2}k_id$ with $k_i = \sqrt{\frac{\mu_i \epsilon_i}{\mu_0 \epsilon_0}}k_0$, $Z_i = \sqrt{\frac{\mu_i}{\epsilon_i}}$, ${}^i\beta$ are the unknown expansion coefficients, and $i = 1, 2$ denote the coating regions $\xi_1 \leq \xi \leq \xi_0$ and $\xi_2 \leq \xi \leq \xi_1$ respectively. (3.25) and (3.26) have respective matrix forms

$$\bar{E}^{t_i} = \bar{M}_{t_i}^{(1)T} {}^i\bar{\beta}^{(1)} + \bar{M}_{t_i}^{(2)T} {}^i\bar{\beta}^{(2)} \quad (3.27)$$

and

$$\bar{H}^{t_i} = \frac{j}{Z_i} \left[\bar{N}_{t_i}^{(1)T} {}^i\bar{\beta}^{(1)} + \bar{N}_{t_i}^{(2)T} {}^i\bar{\beta}^{(2)} \right] \quad (3.28)$$

where the components of $\bar{M}_{t_i}^{(1)}$, $\bar{M}_{t_i}^{(2)}$, $\bar{N}_{t_i}^{(1)}$, and $\bar{N}_{t_i}^{(2)}$ are obtained from the corresponding components of $\bar{M}_s^{(4)}$ and $\bar{N}_s^{(4)}$ by replacing the spheroidal vector wave

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

functions of the fourth kind by those of the first kind and second kind appropriately, and h_0 by h_i . ${}^i\bar{\beta}^{(1)}$ and ${}^i\bar{\beta}^{(2)}$ are obtained from $\bar{\alpha}$ by replacing α by ${}^i\beta^{(1)}$ and ${}^i\beta^{(2)}$, respectively.

3.2.3 Imposing the Boundary Conditions

To determine the unknown expansion coefficients, the boundary conditions at each spheroidal surface are applied,

$$\bar{E}^{t_2} \times \hat{\xi} = 0, \quad \xi = \xi_2 \quad (3.29a)$$

$$\begin{aligned} \bar{E}^{t_1} \times \hat{\xi} &= \bar{E}^{t_2} \times \hat{\xi}, \quad \xi = \xi_1 \\ \bar{H}^{t_1} \times \hat{\xi} &= \bar{H}^{t_2} \times \hat{\xi}, \quad \xi = \xi_1 \end{aligned} \quad (3.29b)$$

$$\begin{aligned} \bar{E}^{t_1} \times \hat{\xi} &= (\bar{E}^i + \bar{E}^s) \times \hat{\xi}, \quad \xi = \xi_0 \\ \bar{H}^{t_1} \times \hat{\xi} &= (\bar{H}^i + \bar{H}^s) \times \hat{\xi}, \quad \xi = \xi_0 \end{aligned} \quad (3.29c)$$

Using the orthogonality properties of the trigonometric functions and the spheroidal angular functions, and integrating correspondingly over each spheroidal surface, the unknown coefficient column matrix $\bar{\bar{S}}$ is obtained by matrix transformation

$$\bar{\bar{S}} = [G]\bar{\bar{I}} \quad (3.30)$$

where

$$\bar{\bar{S}}^T = k_0 \begin{bmatrix} {}^2\bar{\beta}^{(1)} & {}^2\bar{\beta}^{(2)} & {}^1\bar{\beta}^{(1)} & {}^1\bar{\beta}^{(2)} & \bar{\alpha} \end{bmatrix} \quad (3.31)$$

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

$$[G] = [R]^{-1}[P] \quad (3.32)$$

in which

$$[R] = \begin{bmatrix} A^{(1)}(\xi_2, h_2) & A^{(2)}(\xi_2, h_2) & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ A^{(1)}(\xi_1, h_2) & A^{(2)}(\xi_1, h_2) & -A^{(1)}(\xi_1, h_1) & -A^{(2)}(\xi_1, h_1) & \mathbf{0} \\ \frac{B^{(1)}(\xi_1, h_2)}{k_2 Z_2} & \frac{B^{(2)}(\xi_1, h_2)}{k_2 Z_2} & -\frac{B^{(1)}(\xi_1, h_1)}{k_1 Z_1} & -\frac{B^{(2)}(\xi_1, h_1)}{k_1 Z_1} & \mathbf{0} \\ A^{(1)}(\xi_0, h_1) & A^{(2)}(\xi_0, h_1) & \mathbf{0} & \mathbf{0} & -A^{(4)}(\xi_0, h_0) \\ \frac{B^{(1)}(\xi_0, h_1)}{k_1 Z_1} & \frac{B^{(2)}(\xi_0, h_1)}{k_1 Z_1} & \mathbf{0} & \mathbf{0} & -\frac{B^{(4)}(\xi_0, h_0)}{k_0 Z_0} \end{bmatrix} \quad (3.33)$$

and

$$[P]^T = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & C^{(1)}(\xi_0, h_0) & \frac{D^{(1)}(\xi_0, h_0)}{k_0 Z_0} \end{bmatrix} \quad (3.34)$$

where

$$A^{(i)} = \begin{bmatrix} U_{Nn}^{(i)} & V_{N+1, n+1}^{(i)} \\ X_{Nn}^{(i)} & Y_{N+1, n+1}^{(i)} \end{bmatrix} \quad (3.35a)$$

$$B^{(i)} = \begin{bmatrix} T_{Nn}^{(i)} & W_{N+1, n+1}^{(i)} \\ Z_{Nn}^{(i)} & Q_{N+1, n+1}^{(i)} \end{bmatrix} \quad (3.35b)$$

$$C^{(1)} = \begin{bmatrix} 2U_{Nn}^{(1)} & 2X_{Nn}^{(1)} \end{bmatrix} \quad (3.35c)$$

$$D^{(1)} = \begin{bmatrix} 2T_{Nn}^{(1)} & 2Z_{Nn}^{(1)} \end{bmatrix} \quad (3.35d)$$

in which $i = 1, 2, 4$, $N = 0, 1, 2, \dots$, $n = 0, 1, 2, \dots$, and Q, T, U, V, W, X, Y, Z

are, theoretically, infinite matrixes which are given in Appendix A.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

3.2.4 Far-Field Expressions

Once the coefficient column matrix $\bar{\bar{S}}$ is solved, the fields outside the spheroidal structure and inside the coatings are known. In this chapter, we are only interested in the behavior of the scattered wave in the far-field zone. For very large distances from the scatterer (i.e., in the limit as $h\xi \rightarrow \infty$), the polar angle θ and the spherical radial coordinate r are related, respectively, to the spheroidal angle coordinate η and radial coordinate ξ by

$$\eta = \cos \theta, \quad \hat{\eta} = -\hat{\theta} \quad (3.36)$$

$$\frac{1}{2}d\xi = r, \quad h\xi = \frac{1}{2}kd\xi = kr \quad (3.37)$$

and the radial functions of the fourth kind become

$$R_{mn}^{(4)} = \frac{1}{h\xi} j^{n+1} e^{-jc\xi} \quad (3.38)$$

$$\frac{d}{d\xi} R_{mn}^{(4)} = \frac{1}{\xi} j^n e^{-jc\xi} \quad (3.39)$$

In the far-field zone, the scattered magnetic and electric fields are related by

$$\bar{H}^s = \frac{1}{Z_0} \hat{r} \times \bar{E}^s \quad (3.40)$$

Therefore, it is sufficient to consider only $\bar{E}^s$. Substituting (3.36)–(3.39) into the spheroidal vector wave functions and neglecting terms containing ξ^{-2} and higher inverse powers of ξ , we obtain

$$\bar{M}_{e0n}^{+(4)} = \frac{j^n k e^{-jkr}}{2 kr} \left[-S_{0n}(h_0, \eta) \sin \phi \hat{\eta} + \eta S_{0n}(h_0, \eta) \cos \phi \hat{\phi} \right] \quad (3.41)$$

$$\bar{M}_{e1,n+1}^{z(4)} = -\frac{j^{n+1} k e^{-jkr}}{2 kr} S_{1,n+1}(h_0, \eta) \sqrt{1 - \eta^2} \cos \phi \hat{\phi} \quad (3.42)$$

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

And the scattered electric field $\bar{E}_s$ becomes

$$\bar{E}_s = \frac{e^{-jk_0 r}}{k_0 r} \left[F_\theta(\theta, \phi) \hat{\theta} + F_\phi(\theta, \phi) \hat{\phi} \right] \quad (3.43)$$

where

$$F_\theta(\theta, \phi) = \sin \phi \sum_{n=1}^{\infty} \frac{j^n}{2} S_{0n}(h_0, \cos \theta) (k_0 \alpha_n^+) \quad (3.44)$$

$$F_\phi(\theta, \phi) = \cos \phi \sum_{n=1}^{\infty} \frac{j^n}{2} \left[\cos \theta S_{0n}(h_0, \cos \theta) (k_0 \alpha_n^+) \right. \\ \left. - 2j \sin \theta S_{1,n+1}(h_0, \cos \theta) (k_0 \alpha_{n+1}^z) \right] \quad (3.45)$$

and (r, θ, ϕ) are the spherical coordinates of the observation point in the far-field.

The bistatic radar cross section is defined as 4π times the ratio of the scattered power delivered per unit solid angle in the direction of the receiver in the far-field to the power per unit area incident on the scatterer and is obviously independent on r . This is given mathematically by

$$\sigma(\theta, \phi) = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|\bar{E}^s \cdot \hat{\tau}|^2}{|\bar{E}^i|^2} \quad (3.46)$$

where $\hat{\tau}$ is the polarization of the receiver at the observation point (r, θ, ϕ) . The normalized bistatic cross section is given by

$$\frac{\pi \sigma(\theta, \phi)}{\lambda_0^2} = |F_\theta(\theta, \phi)|^2 + |F_\phi(\theta, \phi)|^2 \quad (3.47)$$

where $\lambda_0 = 2\pi/k_0$. When $\theta = 0$ and $\phi = 0$, we obtain the normalized backscattering cross section

$$\frac{\pi \sigma(0, 0)}{\lambda_0^2} = |F_\theta(0, 0)|^2 + |F_\phi(0, 0)|^2 \quad (3.48)$$

Using the forward scattering theorem [103], the total cross section σ_t is related to the imaginary part of the scattering amplitude in the forward direction as

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

follows,

$$\sigma_t = -\frac{4\pi}{k_0} \text{Im} \left\{ \frac{1}{k_0} \left[F_\theta(\pi, 0) \hat{\theta} + F_\phi(\pi, 0) \hat{\phi} \right] \right\} \cdot \hat{e}_i \quad (3.49)$$

where Im denotes the “imaginary part”, and $\hat{e}_i$ is the unit vector in the direction of polarization of the incident wave. Noting that

$$\hat{e}_i = \hat{y} = \hat{r} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi \quad (3.50)$$

we get

$$\sigma_t = \frac{4\pi}{k_0^2} \text{Im} \left[\sum_{n=0}^{\infty} \frac{j^n}{2} S_{0n}(h_0, -1) (k_0 \alpha_n^+) \right] \quad (3.51)$$

where σ_t is the sum of the total scattering and the total absorption cross sections,

$$\sigma_t = \sigma_s + \sigma_a \quad (3.52)$$

Since the MTMs coatings are assumed to be lossless, $\sigma_a = 0$ and $\sigma_s = \sigma_t$. Therefore, the normalized total scattering cross section is given by

$$\frac{\pi \sigma_s}{\lambda_0^2} = \text{Im} \left[\sum_{n=0}^{\infty} \frac{j^n}{2} S_{0n}(h_0, -1) (k_0 \alpha_n^+) \right] \quad (3.53)$$

3.3 Results and Discussion

To obtain numerical results of the unknown expansion coefficients, the infinite transformation matrix $[G]$ in (3.30) must be truncated. The truncated number N with a required accuracy depends on the coating materials, the frequency and the size of the spheroid structure. For the coating materials, the frequency and the size considered in this chapter, it is found that $N = [k_0 a_0] + 12$ is sufficient to obtain three significant digits in calculating the scattering cross section ($[\]$ denotes the integer part function). For instance, a convergence study of bistatic cross sections is shown in Table 3.1. It is found that in order to obtain the same accuracy the truncated number N for DNG MTMs coating is larger than that for conventional dielectric coating due to the negative value of the wavenumber k of DNG MTMs.

Table 3.1: Convergence study of normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) with MTMs coating ($\epsilon_{r_1} = \epsilon_{r_2} = -2.0$ and $\mu_{r_1} = \mu_{r_2} = -1.0$) and dielectric coating ($\epsilon_{r_1} = \epsilon_{r_2} = 2.0$ and $\mu_{r_1} = \mu_{r_2} = 1.0$)

	MTMs ($\pi\sigma(80^\circ, 0^\circ)/\lambda^2$)	Dielectric ($\pi\sigma(80^\circ, 0^\circ)/\lambda^2$)
N=5	5.19	3.98
N=6	8.58	4.07
N=7	7.91	4.04
N=8	8.04	4.04
N=9	7.96	
N=10	7.96	

Figure 3.2 shows the normalized bistatic scattering cross section for a conducting quasi-sphere with a layer of the conventional dielectric coating ($a_2/b_2 = 1.0001$) with $\epsilon_{r_1} = \epsilon_{r_2} = 2.0$ and $\mu_{r_1} = \mu_{r_2} = 1.0$ for $\phi = 0^\circ$ and $\phi = 90^\circ$ respectively.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.2: Normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.0$ and $\mu_{r1} = \mu_{r2} = 1.0$).

Figure 3.3: Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 27$ mm, $a_0 = 30$ mm) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 6.0$ and $\mu_{r1} = \mu_{r2} = 1.0$).

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.3 shows the normalized backscattering cross section of a conducting quasi-sphere with a layer of the conventional dielectric coating ($a_2/b_2 = 1.0001$) with $\epsilon_{r1} = \epsilon_{r2} = 6.0$ and $\mu_{r1} = \mu_{r2} = 1.0$. It can be seen from Figure 3.2 and Figure 3.3 that our results are in excellent agreement with the results in [37, 104] and [105] respectively, which verifies the formulation described and the computer code written are correct.

In Figure 3.4, a layer of the DNG MTMs coating replaces the dielectric coating of the conducting quasi-sphere of Figure 3.2, with the same thickness and having the same absolute value and opposite sign of permittivity and permeability ($\epsilon_{r1} = \epsilon_{r2} = -2.0$ and $\mu_{r1} = \mu_{r2} = -1.0$). It is shown that the scattering cross section with DNG MTMs coating is different comparing with the dielectric coating in Figure 3.2. It is also seen that in general, the scattering cross section is smoother than that with dielectric coating in Figure 3.2, especially for the case of $\phi = 0^\circ$, where the scattering cross section keeps approximately constant from 0° to around 120° .

Figure 3.5 shows the normalized bistatic scattering cross section for the conducting quasi-sphere with two layers of coatings having the same absolute value and opposite sign of permittivity and permeability ($\epsilon_{r1} = -\epsilon_{r2} = -2.0$ and $\mu_{r1} = -\mu_{r2} = -1.0$). It is seen that the scattering cross section with both dielectric coating and DNG MTMs coating are more different compared with Figure 3.2 and Figure 3.4. It is also found that the back scattering cross section ($\theta = 0^\circ$) is reduced by about 60% and the forward scattering cross section ($\theta = 180^\circ$) is enhanced by more than 2.5 times compared with Figure 3.2.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.4: Normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) with DNG MTMs coating ($\epsilon_{r1} = \epsilon_{r2} = -2.0$ and $\mu_{r1} = \mu_{r2} = -1.0$).

Figure 3.5: Normalized bistatic cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) with coatings ($\epsilon_{r1} = -\epsilon_{r2} = -2.0$ and $\mu_{r1} = -\mu_{r2} = -1.0$).

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.6 shows that the backscattering cross section of a coated conducting quasi-sphere varies with the relative permittivity ϵ_{r1} of dielectric-DNG MTMs coatings. For comparison, the backscattering cross section for the dielectric coating ($\epsilon_{r1} = 2.0$) is provided. It is shown that the backscattering cross section with dielectric-DNG MTMs coatings is lower than that with dielectric coating between $\epsilon_{r1} = -1.93$ and $\epsilon_{r1} = -2.06$, and the minimum value of backscattering cross section is at around $\epsilon_{r1} = -2.0$.

Figure 3.6: Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$) versus the relative permittivity ϵ_{r1} of dielectric-DNG MTMs coatings ($\epsilon_{r2} = 2.0$ and $\mu_{r1} = -\mu_{r2} = -1.0$).

Figure 3.7 shows the frequency variation of the normalized backscattering cross section of a conducting quasi-sphere with a layer of the conventional dielectric coating and with a layer of the DNG MTMs coating. The thicknesses of these two coatings are identical. The absolute value of the permittivity and permeability of these two coatings are equal in magnitude but opposite in sign. It is shown that

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

the backscattering cross section for DNG MTMs coating is different compared with the dielectric coating. In general, the backscattering cross sections for DNG MTMs coating are larger and smoother than those for dielectric coating up to 6 GHz, and smaller than those for dielectric coating from 6 GHz to 10 GHz.

Figure 3.7: Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_0 = 40$ mm) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.0$ and $\mu_{r1} = \mu_{r2} = 1.0$) and DNG MTMs coating ($\epsilon_{r1} = \epsilon_{r2} = -2.0$ and $\mu_{r1} = \mu_{r2} = -1.0$).

Figure 3.8 and Figure 3.9 show the backscattering cross section of a coated conducting quasi-sphere versus the coating material's relative permittivity and relative permeability respectively. It is seen that the backscattering cross section has resonant points between $\epsilon_r = -0.5$ and $\epsilon_r = -2$ in Figure 3.8. In Figure 3.9, the backscattering cross section increases sharply at $\mu_r \approx -2$ when the permittivity and permeability are both negative, but at $\mu_r \approx 5.5$ when the permittivity and permeability are both positive. It is also seen that the backscattering cross sections

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

in both figures are approximately linear between -8 and -4 .

Figure 3.10 shows the normalized backscattering cross section of a conducting quasi-sphere with two layers of coatings having the same absolute value and opposite sign of permittivity and permeability (Dielectric-DNG MTMs coating and DNG MTMs-Dielectric coating), respectively. It is shown that the backscattering cross section for Dielectric-DNG MTMs coating is larger than that for only DNG MTMs coating or only dielectric coating in Figure 3.7, especially between 2 GHz and 4 GHz. And the backscattering cross section for DNG MTMs-Dielectric coating is smoother than that for only DNG MTMs coating or only dielectric coating in some frequency bands, such as 2–3 GHz and 6–7 GHz.

Figures 3.11–3.13 show the normalized bistatic scattering cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) coated with different materials. It is seen in Figure 3.12 that the bistatic scattering cross section with DNG MTMs coating for the case of $\phi = 0^\circ$ is approximately constant from 0° to around 120° and the nulls of the scattering cross section with dielectric coating are cancelled in this region, which is similar to the quasi-sphere case. It is also found that the backward and forward scattering cross sections with dielectric-DNG MTMs coating in Figure 3.13 are both enhanced by about 9 and 3.7 times, respectively, compared with Figure 3.11.

Figures 3.14–3.16 show the normalized bistatic scattering cross section for a conducting prolate spheroid ($a_2/b_2 = 5$) coated with different materials. It is found that the backward and forward scattering cross sections with dielectric-DNG MTMs coating are larger than those with only DNG MTMs coating, but

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $\mu_r = -1$

(b) $\mu_r = 1$

Figure 3.8: Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_0 = 40$ mm) with different coatings at $f = 2$ GHz.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $\epsilon_r = -2$

(b) $\epsilon_r = 2$

Figure 3.9: Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_0 = 40$ mm) with different coatings at $f = 2$ GHz.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.10: Normalized backscattering cross section for a conducting quasi-sphere ($a_2/b_2 = 1.0001$, $a_2 = 30$ mm, $a_1 = 40$ mm, $a_0 = 50$ mm) with Dielectric-DNG MTMs coating ($\epsilon_{r1} = -\epsilon_{r2} = -2.0$, $\mu_{r1} = -\mu_{r2} = -1.0$) and DNG MTMs-Dielectric coating ($\epsilon_{r1} = -\epsilon_{r2} = 2.0$, $\mu_{r1} = -\mu_{r2} = 1.0$).

Figure 3.11: Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.2$ and $\mu_{r1} = \mu_{r2} = 1.0$).

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.12: Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) with DNG MTMs coating ($\epsilon_{r1} = \epsilon_{r2} = -2.2$ and $\mu_{r1} = \mu_{r2} = -1.0$).

Figure 3.13: Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 1.5$) with coatings ($\epsilon_{r1} = -\epsilon_{r2} = -2.2$ and $\mu_{r1} = -\mu_{r2} = -1.0$).

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

the scattering cross section with DNG MTMs coating for the case of $\phi = 0^\circ$ is smoother than that with dielectric-DNG MTMs coating from 0° to around 120° .

Figure 3.14: Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 5.0$) with dielectric coating ($\epsilon_{r1} = \epsilon_{r2} = 2.2$ and $\mu_{r1} = \mu_{r2} = 1.0$).

Figure 3.15: Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 5.0$) with DNG MTMs coating ($\epsilon_{r1} = \epsilon_{r2} = -2.2$ and $\mu_{r1} = \mu_{r2} = -1.0$).

Using (3.53), we have computed the normalized total scattering cross sections

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.16: Normalized bistatic cross section for a conducting prolate spheroid ($a_2/b_2 = 5.0$) with coatings ($\epsilon_{r1} = -\epsilon_{r2} = -2.2$ and $\mu_{r1} = -\mu_{r2} = -1.0$).

of electrically small spheroidal scatterers with different values of a_2/b_2 coated with a layer of confocal spheroidal DNG MTMs as shown in Figure 3.17. It is found that there always exists a ratio of a_2/a_0 which dramatically enhances the total scattering cross sections. This resonant phenomenon may be intuitively explained by a compact L-C resonance [7] because the inner conducting spheroid acts as a small positive capacitance and the outer double-negative metamaterials coating acts as another small negative capacitance (equivalent with a large inductance). Therefore, pairing these two opposite capacitance may lead to resonance. In other words, it is possible to achieve effective electrically small scatterers or sub-wavelength “compact resonant structures” by choosing an appropriate value of the thickness ($t = a_2 - a_0$) of the MTMs coating due to the interface resonance when the complementary materials are paired together. It is seen that, in Figure 3.17 (a), when the spheroidal scatterer is almost a sphere ($a_2/b_2 = 1.0001$), the resonant peak

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $a_2/b_2 = 1.0001$

(b) $a_2/b_2 = 2.0$

(c) $a_2/b_2 = 5.0$

Figure 3.17: Normalized total scattering cross section of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) coated with DNG MTMs $\epsilon_{r1} = \epsilon_{r2} = -1.0$ and $\mu_{r1} = \mu_{r2} = -1.0$.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

occurs at $a_2/a_0 = 0.63$ which is consistent with the theory of electrically small spherical DPS particles with a layer of concentric DNG MTMs shells presented in [7]. It was found that the significant enhancement of wave scattering or resonance can be achieved if γ_{TM}^r (the ratio of inner sphere radius to outer sphere radius for a TM-polarized scattered wave) satisfies the following condition,

$$\gamma_{TM}^r \simeq \sqrt[2n+1]{\frac{[(n+1)\epsilon_0 + n\epsilon_{DNG}][(n+1)\epsilon_{DNG} + n\epsilon_{DPS}]}{n(n+1)(\epsilon_{DNG} - \epsilon_0)(\epsilon_{DNG} - \epsilon_{DPS})}} \quad (3.54)$$

where $n \geq 1$. It is worth noting that in the case of a TE wave, γ_{TE}^r is only related to permeability μ [7], which is not suitable for our case. In this case, where the inner sphere (or spheroid) is assumed to be perfectly conducting ($\epsilon_{DPS} \rightarrow -j\infty$), we get

$$\gamma_{TM}^r = \sqrt[2n+1]{\frac{(n+1)\epsilon_0 + n\epsilon_{DNG}}{(n+1)(\epsilon_0 - \epsilon_{DNG})}} \quad (3.55)$$

Clearly, γ_{TM}^r has to satisfy $0 < \gamma_{TM}^r < 1$ in order to have physical meaning.

Therefore, the values of ϵ_{DNG} have to satisfy the following relation:

$$-\frac{n+1}{n}\epsilon_0 < \epsilon_{DNG} < 0 \quad (3.56)$$

For the dominant term $n = 1$ of electrically small particles, this relation becomes $-2\epsilon_0 < \epsilon_{DNG} < 0$. And in the case of $\epsilon_{DNG} = -\epsilon_0$ ($\epsilon_{r1} = \epsilon_{r2} = -1.0$), the predicted resonant peak should occur at $\gamma_{TM}^r = a_2/a_0 = 0.63$ which is the same as shown in Figure 3.17 (a). It is also seen that, with a_2/b_2 increasing (spheroidal scatterers becoming thinner), the ratio of a_2/a_0 at which the resonant peak occurred moves from 0.63 towards 1.0; i.e., the thickness t of the coating becomes thinner.

Figure 3.18 shows the ratios of a_2/a_0 at the resonant peaks of the normalized total scattering cross sections as they vary with the relative permittivity ϵ_r of DNG

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 3.18: Ratios of a_2/a_0 at the resonant peaks of the normalized total scattering cross sections (TSC) of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) as a function of the relative permittivity ($\epsilon_r = \epsilon_{r_1} = \epsilon_{r_2}$) of coated DNG MTMs ($\mu_{r_1} = \mu_{r_2} = -1.0$).

MTMs coatings. It is seen that, in the range of $-2.0 < \epsilon_r < 0$, there always exist resonant peaks for different spheroidal scatterers, and we also find through simulations that there is no resonant peak when ϵ_r is out of this range, i.e. $\epsilon_r < -2.0$ or $\epsilon_r > 0$. Our results show that the ratio of a_2/a_0 at the resonant peak increases as $|\epsilon_r|$ decreases. This may be explained by noting that the positive capacitance provided by the inner spheroid decreases with its size's increasing (or a_2/a_0 increasing) and therefore, the negative capacitance provided by the double-negative metamaterials coating (or $|\epsilon_r|$) need to decrease as well. It is also found that, with a given ϵ_r , this ratio of a_2/a_0 keeps increasing with a_2/b_2 . (Or with a given a_2/a_0 , $|\epsilon_r|$ decreases with a_2/b_2 decreasing.) This is also because with a fixed a_2/a_0 , the positive capacitance provided by the inner spheroid decreases with its size's in-

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

creasing (or a_2/b_2 decreasing) and therefore, the negative capacitance provided by the double-negative metamaterials coating (or $|\epsilon_r|$) also need to decrease.

Figure 3.19 shows the normalized total scattering cross sections of electrically small spheroidal scatterers with different values of a_2/b_2 coated with a layer of confocal spheroidal low-permittivity MTMs. It is found that there always exists a ratio of a_2/a_0 which drastically reduces the total scattering cross section. The total scattering cross sections of electrically small objects are generally dominated by the dipolar term in the multipole expansion, and this dipolar scattering may vanish or the “transparency” condition can be achieved by choosing appropriate values of the thickness t of the coating. It is seen that in Figure 3.19 (a), when the spheroidal scatterer is almost a sphere ($a_2/b_2 = 1.0001$), the total scattering cross section approaches zero at $a_2/a_0 = 0.63$. This is consistent with the theory of drastically reducing the total SCS from a small sphere using plasmonic and MTMs coatings with negative or low permittivity [8]. The “transparency” or “invisible” condition is given as,

$$\gamma_{TM}^t = \sqrt[2n+1]{\frac{(\epsilon_{MTMs} - \epsilon_0)[(n+1)\epsilon_{MTMs} + n\epsilon_{DPS}]}{(\epsilon_{MTMs} - \epsilon_{DPS})[(n+1)\epsilon_{MTMs} + n\epsilon_0]}} \quad (3.57)$$

Similarly, in the case of electrically small particles with a perfect conducting inner sphere, this condition becomes

$$\gamma_{TM}^t = \sqrt[3]{\frac{\epsilon_0 - \epsilon_{MTMs}}{2\epsilon_{MTMs} + \epsilon_0}} \quad (3.58)$$

which can be satisfied when $0 < \epsilon_{MTMs} < \epsilon_0$ (low permittivity DPS MTMs or plasmonic materials). Therefore, in the case of $\epsilon_{MTMs} = 0.5\epsilon_0$ ($\epsilon_{r1} = \epsilon_{r2} = 0.5$), the predicted “transparency” point should occur at $\gamma_{TM}^t = a_2/a_0 = 0.63$ which

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $a_2/b_2 = 1.0001$

(b) $a_2/b_2 = 2.0$

(c) $a_2/b_2 = 5.0$

Figure 3.19: Normalized total scattering cross section of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) coated with low-permittivity MTMs $\epsilon_{r1} = \epsilon_{r2} = 0.5$ and $\mu_{r1} = \mu_{r2} = 1.0$.

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

is the same as presented in figure 3.19 (a). It can also be found that, with a_2/b_2 increasing (spheroidal scatterers becoming thinner), the ratio of a_2/a_0 at which the “transparency” occurs moves from 0.63 towards 1.0; i.e., the thickness t of the coating becomes thinner.

Figure 3.20: Ratios of a_2/a_0 at the “transparency” points of the normalized total scattering cross sections of electrically small spheroidal scatterers ($a_0 = \lambda_0/100$) as a function of the relative permittivity ($\epsilon_r = \epsilon_{r1} = \epsilon_{r2}$) of coated low-permittivity MTMs ($\mu_{r1} = \mu_{r2} = 1.0$).

Figure 3.20 shows that the ratios of a_2/a_0 at the “transparency” points of the normalized total scattering cross sections vary with the relative permittivity ϵ_r of low-permittivity MTMs coatings. It is seen that, in the range $0 < \epsilon_r < 1.0$, the “transparency” condition can always be satisfied for different spheroidal scatterers. But when ϵ_r is out of this range, i.e. $\epsilon_r < 0$ or $\epsilon_r > 1.0$, the “transparency” condition can not be achieved. Our results show that the ratio of a_2/a_0 at the “transparency” point decreases as the magnitude of ϵ_r increases. As mentioned

CHAPTER 3. ELECTROMAGNETIC SCATTERING BY A CONDUCTING SPHEROID COATED WITH DOUBLE-NEGATIVE METAMATERIALS

before, the “transparency” phenomenon may be viewed as simply reducing the dipolar term which is the integral sum of the scatterer’s volume polarization. The polarization vector can be expressed as $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$ where ϵ is the material permittivity [8]. Therefore, when the permittivity ϵ of the outer layer coating is less than the background materials ϵ_0 , its dipolar term may cancel the dipolar term provided by inner spheroid to achieve “transparency”. In this Figure, with the size of inner spheroid increasing, its dipolar term increases and in order to cancel this increasing dipolar term, the magnitude of ϵ_r need to decrease (or $\mathbf{P} = (\epsilon - \epsilon_0)\mathbf{E}$ increasing) to increase the dipolar term provided by outer layer coating. And for a fixed ϵ_r , it is found that the ratio of a_2/a_0 increases with a_2/b_2 . (Or with a given a_2/a_0 , the magnitude of ϵ_r increases with a_2/b_2 increasing.) This is also because with a given a_2/a_0 , the dipolar term provided by inner spheroid decreases with it’s size decreasing, i.e. a_2/b_2 increasing. Therefore, the dipolar term provided by the outer layer coating should also decrease, i.e. ϵ_r increasing.

3.4 Conclusion

In this chapter, the analytic solution and numerical results for the problem of EM scattering by a conducting prolate spheroid coated with DNG MTMs at axial incidence is presented using the method of separation of variables in terms of spheroidal vector wave functions. The scattering cross sections of a conducting prolate spheroid coated with different materials, such as DNG MTMs, conventional dielectric materials coated with DNG MTMs, are shown. It is found that a conducting spheroid with DNG MTMs coatings has substantially different scattering cross section performance compared with that coated with conventional dielectric materials. The possibility of increasing, decreasing and smoothing the scattering cross section using DNG MTMs coating is demonstrated. These features are useful for antenna and radar designs. Our results also show that the total scattering cross sections can be dramatically enhanced with certain DNG MTMs coatings to achieve “compact resonant structures” or drastically reduced with certain low-permittivity MTMs coatings to achieve “invisibility” of objects. The results with different parameters, such as relative permittivity, the spheroidal geometry, and the ratio of semi-major axes of two layers, are provided and discussed, which are useful for the design of sub-wavelength resonant structures and transparent objects.

Chapter 4

Spheroidal Antenna Coated with Double-Negative Metamaterials

4.1 Introduction

In Chapter 3, the electromagnetic (EM) scattering problem by a conducting spheroid coated with DNG MTMs with a linearly polarized monochromatic plane wave incidence was studied. It was found that DNG MTMs coatings bring different scattering performance. Coatings of antennas, or antenna radomes, are often used to shield the antenna systems against environmental effects. For example, in general, the antenna used in aircraft applications is mounted on the nose of the aircraft, and a slot antenna is often used. On aircraft, in addition to giving protection, the radome streamlines the antenna, thereby reducing drag. This antenna configuration can be modelled by a slot antenna mounted on a spheroid coated with a confocal spheroidal radome. However, the presence of conventional dielectric coatings or radomes can distort the performance of the antenna, such as radiation patterns and antenna impedance [64].

In addition, with the development of wireless technologies for communications

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

and sensor networks, efficient electrically small antennas with wide bandwidth are desirable. However, these requirements are usually contradictory [11]. As mentioned in Chapter 2, efficient electrically small antennas can be achieved with DNG MTMs shells. It is well-known that electrically small dipole antennas in free space are inefficient radiators because of their small resistance and large capacitive reactance. However, the DNG MTMs shell has an inductive reactance effect, which may cancel the capacitive reactance of electrically small dipole antennas.

In this chapter, a prolate spheroidal antenna coated with confocal DNG MTMs coatings or radomes is investigated in order to improve the performance of the antenna system. The solution of EM radiation from a prolate spheroidal antenna, excited by a delta voltage across a circumferential infinitesimally narrow slot at any arbitrary position along the major axis of the antenna, enclosed in one and two layers of confocal DNG MTMs coatings, is obtained using the method of separation of the spheroidal scalar wave functions in prolate spheroidal coordinates. The unknown expansion coefficients are solved by applying the continuity of tangential EM fields on the boundaries of spheroidal surfaces.

Our results show that, unlike a conventional dielectric radome, the radiation pattern of the prolate spheroidal antenna coated with a half wavelength confocal DNG MTMs radome are almost the same as that of the uncoated antenna. That means the DNG MTMs radome can be electrically transparent to the antenna and has almost no effect on the antenna's radiation pattern. And this DNG MTMs radome can be placed much closer to the antenna than a conventional dielectric one without loss of performance, which means the size of the antenna system can

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

be significantly reduced.

We also analyze the electrically small spheroidal antenna coated with a confocal DNG shell. Our results show that this electrically small dipole-DNG shell system has very high radiation efficiency compared with the normal electrically small antenna due to the inductive effect of the MTMs shell that cancels the capacitive effect of the electrically small antenna. It is found that the spheroidal shell can achieve more a compact structure and a higher radiated power ratio than the corresponding spherical shell. And it is also shown that the fractional bandwidth (FBW) of this system is much larger than that predicted by the Chu limit which is the maximum FBW limit for the normal electrically small antenna. Dipole-DNG shell systems with different sizes are analyzed and discussed.

This chapter is organized as follows. In Section 4.2, the solution of EM radiation of an uncoated prolate spheroidal antenna is given. Section 4.3 derives the solution of EM radiation when the prolate spheroidal antenna is enclosed in one and two layers of confocal DNG MTMs coatings. In Section 4.4, the radiated power of efficient electrically small spheroidal antennas is analyzed. Section 4.5 presents the numerical results and discussions about DNG MTMs radomes and efficient electrically small antennas. The conclusion of this chapter is given in Section 4.6.

4.2 EM Radiation of Prolate Spheroidal Antenna

4.2.1 Geometry of Spheroidal Antenna

In this section, we consider a perfectly conducting prolate spheroidal antenna, which is assumed to be excited by a source field E^a over a gap on its surface as shown in Figure 4.1. It is assumed that the environment is a homogeneous, isotropic medium, and that the applied electric field intensity is circularly symmetry about the major axis.

Figure 4.1: Geometry of a prolate spheroidal antenna.

In prolate spheroidal coordinates (η, ξ, ϕ) , the semi-major and semi-minor axes of the spheroidal antenna are designated a and b , respectively, and the interfocal

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

distance is d . The excitation gap can be located anywhere on the antenna surface, i.e., at $\eta = \eta_0$. The surface of the spheroidal antenna is $\xi = \xi_1$, which is given as

$$\xi_1 = \frac{2a}{d} = \frac{a}{\sqrt{a^2 - b^2}} \quad (4.1)$$

4.2.2 Formulation of the Problem

Due to symmetry (i.e., $\partial/\partial\phi=0$), Maxwell's equations in free space are as follows,

$$jw\epsilon_0 E_\xi = \frac{1}{h_\eta h_\phi} \frac{\partial(h_\phi H_\phi)}{\partial\eta} \quad (4.2a)$$

$$jw\epsilon_0 E_\eta = -\frac{1}{h_\phi h_\xi} \frac{\partial(h_\phi H_\phi)}{\partial\xi} \quad (4.2b)$$

$$jw\mu_0 H_\phi = \frac{1}{h_\xi h_\eta} \left[\frac{\partial(h_\xi E_\xi)}{\partial\eta} - \frac{\partial(h_\eta E_\eta)}{\partial\xi} \right] \quad (4.2c)$$

where ϵ_0 and μ_0 are respectively the permittivity and the permeability of free space, and the metrical coefficients h_η , h_ξ , and h_ϕ in prolate spheroidal coordinates are defined by

$$h_\eta = \frac{d}{2} \left(\frac{\xi^2 - \eta^2}{1 - \eta^2} \right)^{\frac{1}{2}} \quad (4.3a)$$

$$h_\xi = \frac{d}{2} \left(\frac{\xi^2 - \eta^2}{\xi^2 - 1} \right)^{\frac{1}{2}} \quad (4.3b)$$

$$h_\phi = \frac{d}{2} [(\xi^2 - 1)(1 - \eta^2)]^{\frac{1}{2}} \quad (4.3c)$$

It is seen from (4.2) that if the source field on the gap has only an E_η component, the excited magnetic field will have only an H_ϕ component and $E_\phi = 0$. Thus following the method of Schelkunoff [48], it can be shown that the EM fields can

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

be determined in terms of a potential A , where A satisfies

$$(\xi^2 - 1) \frac{\partial^2 A}{\partial \xi^2} + (1 - \eta^2) \frac{\partial^2 A}{\partial \eta^2} + h^2(\xi^2 - \eta^2)A = 0 \quad (4.4)$$

and

$$h = \frac{k_0 d}{2} \quad (4.5)$$

k_0 is the wavenumber of free space. In general, in the exterior region ($\xi > \xi_1$), the potential A , or the auxiliary scalar wave function, is given by

$$A = \sum_{n=1,2}^{\infty} a_n U_n(h, \xi) V_n(h, \eta) \quad (4.6)$$

where the unknown coefficients a_n can be determined by boundary conditions.

$U_n(h, \xi)$ and $V_n(h, \eta)$ are defined as

$$U_n(h, \xi) = \sqrt{\xi^2 - 1} R_{1,n}^{(4)}(h, \xi) \quad (4.7)$$

$$V_n(h, \eta) = \sqrt{1 - \eta^2} S_{1,n}^{(1)}(h, \eta) \quad (4.8)$$

where $R_{1,n}^{(4)}$ and $S_{1,n}^{(1)}$ are the prolate spheroidal radial and angular functions, respectively. To satisfy the radiation condition, the radial function of the fourth kind has to be used. This ensures that at large distances from the center of the spheroid, the radiating wave must behave as a spherical diverging wave.

The potential A is related to the EM fields by

$$H_\phi = \frac{A}{h_\phi} \quad (4.9)$$

$$E_\xi = \frac{4}{jw\epsilon_0 d^2} \frac{1}{\sqrt{(\xi^2 - 1)(\xi^2 - \eta^2)}} \frac{\partial A}{\partial \eta} \quad (4.10)$$

$$E_\eta = -\frac{4}{jw\epsilon_0 d^2} \frac{1}{\sqrt{(1 - \eta^2)(\xi^2 - \eta^2)}} \frac{\partial A}{\partial \xi} \quad (4.11)$$

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Using the boundary conditions, the electric field on the surface of the antenna should equal the applied electric field. Therefore, we have the following equation:

$$E_\eta(\xi_1, \eta) = E_\eta^a(\xi_1, \eta) \quad (4.12)$$

Substituting (4.6) and (4.11) into (4.12), we get

$$E_\eta^a(\xi_1, \eta) = -\frac{4}{jw\epsilon_0 d^2} \frac{1}{\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \sum_{n=1,2}^{\infty} a_n U'_n(h, \xi_1) V_n(h, \eta) \quad (4.13)$$

Multiplying both sides of (4.13) by $(\xi^2 - \eta^2)$ and then integrating with respect to η from -1 to 1, finally using the orthogonality property of functions $V_n(c, \eta)$,

$$\int_{-1}^1 (1-\eta^2)^{-1} V_n(h, \eta) V_{n'}(h, \eta) d\eta = \begin{cases} N_{1,n}(h) & \text{if } n = n', \\ 0 & \text{if } n \neq n'. \end{cases} \quad (4.14)$$

we can obtain the unknown coefficients a_n

$$a_n = -\frac{jw\epsilon_0 d^2}{4N_{1,n}(h)U'_n(h, \xi_1)} \int_{-1}^1 E_\eta^a(\xi_1, \eta) \sqrt{\frac{\xi_1^2 - \eta^2}{1 - \eta^2}} V_n(h, \eta) d\eta \quad (4.15)$$

For the case of a delta gap, (4.15) can be expressed as

$$a_n = \frac{jw\epsilon_0 d}{2N_{1,n}(h)U'_n(h, \xi_1)} \bar{V} V_n(h, \eta_0) \quad (4.16)$$

where $\bar{V}$ is voltage across the gap.

As the expansion coefficients a_n are solved, the magnetic and electric fields in the far-field zone ($\xi \rightarrow \infty$) can be obtained by substituting (4.6) into (4.9).

$$H_\phi \approx \frac{2e^{-jk_0 r}}{k_0 r d \sin \theta} \sum_{n=1,2}^{\infty} e^{j(\frac{n+1}{2})\pi} a_n V_n(h, \cos \theta) \quad (4.17)$$

$$E_\theta \approx \sqrt{\frac{\mu_0}{\epsilon_0}} H_\phi \quad (4.18)$$

4.3 EM Radiation with DNG MTMs Radome

4.3.1 Geometry of a Spheroidal Antenna with a Confocal Radome

The geometry of a perfectly conducting prolate spheroidal antenna coated with a confocal spheroidal radome is shown in Figure 4.2. The surfaces from inner

Figure 4.2: Geometry of a prolate spheroidal antenna with radome

to outer spheroid are defined by $\xi = \xi_1$, $\xi = \xi_2$, and $\xi = \xi_3$, respectively, the lengths of the corresponding semimajor and semiminor axes are a_1, a_2, a_3 , and b_1, b_2, b_3 , respectively, and the length of the common interfocal distance is d . The thickness of the radome layer along the semiminor axis is denoted as $t = b_3 - b_2$. The separation between the antenna and the radome is denoted as $s = b_2 - b_1$. The media of the coating layers in region I, II and III are all assumed to be linear, homogeneous, and isotropic with relative permittivities $\epsilon_{r_1}, \epsilon_{r_2}$, and ϵ_{r_3} and relative

permeabilities μ_{r_1} , μ_{r_2} , and μ_{r_3} , respectively. In this section, regions I and III are assumed to be free space with permittivity ϵ_0 and permeability μ_0 .

4.3.2 Obtaining the Auxiliary Scalar Wave Functions

In region I ($\xi_1 \leq \xi \leq \xi_2$), both outgoing and incoming waves are considered due to the discontinuity at $\xi = \xi_2$. Therefore, the auxiliary scalar wave function can be expressed by linear combinations of $U_n(h_1, \xi)$ and $T_n(h_1, \xi)$

$$A_1 = \sum_{n=1,2}^{\infty} [M_n^1 U_n(h_1, \xi) + N_n^1 T_n(h_1, \xi)] V_n(h_1, \eta) \quad (4.19)$$

where M_n^1 and N_n^1 are the unknown expansion coefficients to be solved by boundary conditions, $h_1 = wd\sqrt{\epsilon_{r_1}\mu_{r_1}\epsilon_0\mu_0}/2$, and

$$T_n(h_1, \xi) = \sqrt{\xi^2 - 1} R_{1,n}^{(3)}(h_1, \xi) \quad (4.20)$$

in which $R_{1,n}^{(3)}(h_1, \xi)$ is the prolate spheroidal radial function of the third kind for $m = 1$.

The tangential electric and magnetic fields are given as

$$E_\eta^1 = -\frac{4}{jw\epsilon_1 d^2} \frac{1}{\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \frac{\partial A_1}{\partial \xi} \quad (4.21)$$

where $\epsilon_1 = \epsilon_{r_1}\epsilon_0$, and

$$H_\phi^1 = \frac{2A_1}{d\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \quad (4.22)$$

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

To obtain the relation between M_n^1 and N_n^1 , the tangential applied electric field E_η^a on the antenna surface is defined as

$$E_\eta^a(\xi_1, \eta) = -\frac{4}{jw\epsilon_1 d^2} \frac{1}{\sqrt{(1-\eta^2)(\xi_1^2-\eta^2)}} \sum_{n=1,2}^{\infty} p_n V_n(h_1, \eta) \quad (4.23)$$

Multiplying both sides of (4.23) by $(\xi^2 - \eta^2)$, integrating with respect to η from -1 to 1 and using the orthogonality property of functions $V_n(h, \eta)$ as in (4.14), we can obtain the unknown coefficients p_n as

$$p_n = -\frac{jw\epsilon_1 d^2}{4N_{1,n}(h_1)} \int_{-1}^1 E_\eta^a(\xi_1, \eta) \sqrt{\frac{\xi_1^2 - \eta^2}{1 - \eta^2}} V_n(h_1, \eta) d\eta \quad (4.24)$$

In the case of an infinitesimally thin slot, (4.24) can be expressed as

$$p_n \approx \frac{jw\epsilon_1 d}{2N_{1,n}(h_1)} \bar{V} V_n(h_1, \eta_0) \quad (4.25)$$

where $\bar{V}$ is the applied voltage across the slot.

On the antenna surface, it is obviously required that

$$E_\eta^1(\xi_1, \eta) = E_\eta^a(\xi_1, \eta) \quad (4.26)$$

Substituting (4.19) into (4.21), we obtain

$$\begin{aligned} E_\eta^1(\xi_1, \eta) = & -\frac{4}{jw\epsilon_1 d^2} \frac{1}{\sqrt{(1-\eta^2)(\xi_1^2-\eta^2)}} \\ & \cdot \sum_{n=1,2}^{\infty} [M_n^1 U'_n(h_1, \xi_1) + N_n^1 T'_n(h_1, \xi_1)] V_n(h_1, \eta) \end{aligned} \quad (4.27)$$

Comparing (4.27) with (4.23), it can be found that

$$p_n = M_n^1 U'_n(h_1, \xi_1) + N_n^1 T'_n(h_1, \xi_1) \quad (4.28)$$

Therefore,

$$N_n^1 = \frac{p_n - M_n^1 U'_n(h_1, \xi_1)}{T'_n(h_1, \xi_1)} \quad (4.29)$$

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Substituting (4.29) into (4.19), we obtain,

$$A_1 = \sum_{n=1,2}^{\infty} V_n(h_1, \eta) \left[M_n^1 \left(U_n(h_1, \xi) - \frac{U'_n(h_1, \xi_1) T_n(h_1, \xi)}{T'_n(h_1, \xi_1)} \right) + \frac{p_n T_n(h_1, \xi)}{T'_n(h_1, \xi_1)} \right] \quad (4.30)$$

Similarly, in region II ($\xi_2 \leq \xi \leq \xi_3$), the auxiliary scalar wave function A_2 is given by

$$A_2 = \sum_{n=1,2}^{\infty} [M_n^2 U_n(h_2, \xi) + N_n^2 T_n(h_2, \xi)] V_n(h_2, \eta) \quad (4.31)$$

where M_n^2 and N_n^2 are the unknown expansion coefficients to be solved by boundary conditions, and $h_2 = wd\sqrt{\epsilon_{r_2}\mu_{r_2}\epsilon_0\mu_0}/2$. The tangential electric and magnetic fields are given as

$$E_\eta^2 = -\frac{4}{jw\epsilon_2 d^2} \frac{1}{\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \frac{\partial A_2}{\partial \xi} \quad (4.32)$$

where $\epsilon_2 = \epsilon_{r_2}\epsilon_0$, and

$$H_\phi^2 = \frac{2A_2}{d\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \quad (4.33)$$

In region III ($\xi \geq \xi_3$), the auxiliary scalar wave function A_3 is expressed by

$$A_3 = \sum_{n=1,2}^{\infty} M_n^3 U_n(h_3, \xi) V_n(h_3, \eta) \quad (4.34)$$

where M_n^3 are the unknown expansion coefficients to be solved by boundary conditions, and $h_3 = wd\sqrt{\epsilon_{r_3}\mu_{r_3}\epsilon_0\mu_0}/2$. The only use of radial functions $U_n(h_3, \xi)$ is to satisfy the radiation condition, i.e., there is only an outgoing wave as $\xi \rightarrow \infty$.

The tangential electric and magnetic fields are given as

$$E_\eta^3 = -\frac{4}{jw\epsilon_3 d^2} \frac{1}{\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \frac{\partial A_3}{\partial \xi} \quad (4.35)$$

where $\epsilon_3 = \epsilon_{r_3}\epsilon_0$, and

$$H_\phi^3 = \frac{2A_3}{d\sqrt{(1-\eta^2)(\xi^2-\eta^2)}} \quad (4.36)$$

4.3.3 Applying the Boundary Conditions

To determine the unknown expansion coefficients, the boundary conditions at spheroidal surfaces $\xi = \xi_2$ and $\xi = \xi_3$ are imposed. The continuity of the tangential fields requires

$$H_\phi^1 = H_\phi^2 \Big|_{\xi=\xi_2} \quad (4.37a)$$

$$E_\eta^1 = E_\eta^2 \Big|_{\xi=\xi_2} \quad (4.37b)$$

$$H_\phi^2 = H_\phi^3 \Big|_{\xi=\xi_3} \quad (4.37c)$$

$$E_\eta^2 = E_\eta^3 \Big|_{\xi=\xi_3} \quad (4.37d)$$

Substituting (4.21), (4.22), (4.32), (4.33), (4.35), and (4.36) into (4.37), the boundary conditions are rewritten as

$$A_1 = A_2 \Big|_{\xi=\xi_2} \quad (4.38a)$$

$$\frac{1}{\epsilon_1} \frac{\partial A_1}{\partial \xi} = \frac{1}{\epsilon_2} \frac{\partial A_2}{\partial \xi} \Big|_{\xi=\xi_2} \quad (4.38b)$$

$$A_2 = A_3 \Big|_{\xi=\xi_3} \quad (4.38c)$$

$$\frac{1}{\epsilon_2} \frac{\partial A_2}{\partial \xi} = \frac{1}{\epsilon_3} \frac{\partial A_3}{\partial \xi} \Big|_{\xi=\xi_3} \quad (4.38d)$$

To solve these equations, the expansion form of V_n is used [14],

$$V_n(h, \eta) = \sum_{r=0,1}^{\infty} \sqrt{1 - \eta^2} d_r^{1,n}(h) P_{1+r}^1(\eta) \quad (4.39)$$

where $d_r^{1,n}(h)$ are the expansion coefficients which are nonzero when r is even (odd) and n is odd (even). $P_{1+r}^1(\eta)$ is the associated Legendre function. Therefore, the

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

auxiliary scalar wave functions become,

$$A_1 = \sqrt{1 - \eta^2} \sum_{r=0,1}^{\infty} P_{1+r}^1(\eta) \sum_{n=1,2}^{\infty} d_r^{1,n}(h_1) \cdot \left[M_n^1 \left(U_n(h_1, \xi) - \frac{U'_n(h_1, \xi_1) T_n(h_1, \xi)}{T'_n(h_1, \xi_1)} \right) + \frac{p_n T_n(h_1, \xi)}{T'_n(h_1, \xi_1)} \right] \quad (4.40)$$

$$A_2 = \sqrt{1 - \eta^2} \sum_{r=0,1}^{\infty} P_{1+r}^1(\eta) \sum_{n=1,2}^{\infty} d_r^{1,n}(h_2) [M_n^2 U_n(h_2, \xi) + N_n^2 T_n(h_2, \xi)] \quad (4.41)$$

$$A_3 = \sqrt{1 - \eta^2} \sum_{r=0,1}^{\infty} P_{1+r}^1(\eta) \sum_{n=1,2}^{\infty} d_r^{1,n}(h_3) [M_n^3 U_n(h_3, \xi)] \quad (4.42)$$

Substituting (4.40), (4.41), and (4.42) into the boundary conditions (4.38), and comparing the coefficients of $P_{1+r}^1(\eta)$ on both sides of the equations, we obtain an infinite number of linear equations

$$\sum_{n=1,2}^{\infty} A(r, n) M_n^1 - \sum_{n=1,2}^{\infty} C(r, n) M_n^2 - \sum_{n=1,2}^{\infty} D(r, n) N_n^2 = - \sum_{n=1,2}^{\infty} B(r, n) \quad (4.43a)$$

$$\sum_{n=1,2}^{\infty} E(r, n) M_n^1 - \sum_{n=1,2}^{\infty} G(r, n) M_n^2 - \sum_{n=1,2}^{\infty} H(r, n) N_n^2 = - \sum_{n=1,2}^{\infty} F(r, n) \quad (4.43b)$$

$$\sum_{n=1,2}^{\infty} J(r, n) M_n^2 + \sum_{n=1,2}^{\infty} K(r, n) N_n^2 - \sum_{n=1,2}^{\infty} L(r, n) M_n^3 = 0 \quad (4.43c)$$

$$\sum_{n=1,2}^{\infty} O(r, n) M_n^2 + \sum_{n=1,2}^{\infty} P(r, n) N_n^2 - \sum_{n=1,2}^{\infty} Q(r, n) M_n^3 = 0 \quad (4.43d)$$

where the intermediate terms $A - Q$ are given in Appendix C.

However, the series expansions of the fields have convergent representations. Therefore, we can truncate the series expansions, obtain a finite set of equations, and solve the unknown expansion coefficients M_n^1 , M_n^2 , N_n^2 , and M_n^3 . Since coefficients $d_r^{1,n}$ have sharp peaks near $r = n$, the difference between a finite set of equations and the infinite set is very small. In addition, since $d_r^{1,n}$ is nonzero

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

only when n is odd (even) and r is even (odd), the equations in the case of odd n are completely decoupled from those in the case of even n . Therefore, these linear equations can be solved separately for n odd and n even. And (4.43) can be rewritten in matrix form

$$\begin{bmatrix} [A] & -[C] & -[D] & \mathbf{0} \\ [E] & -[G] & -[H] & \mathbf{0} \\ \mathbf{0} & [J] & [K] & -[L] \\ \mathbf{0} & [O] & [P] & -[Q] \end{bmatrix} \begin{bmatrix} [M_n^1] \\ [M_n^2] \\ [N_n^2] \\ [M_n^3] \end{bmatrix} = \begin{bmatrix} -[B] \\ -[F] \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (4.44)$$

where $\mathbf{0}$ is the zero matrix.

$$[X] = \begin{cases} \begin{bmatrix} X_{01} & X_{03} & X_{05} & \dots \\ X_{21} & X_{23} & X_{25} & \dots \\ X_{41} & X_{43} & X_{45} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, & n \text{ odd} \\ \begin{bmatrix} X_{12} & X_{14} & X_{16} & \dots \\ X_{32} & X_{34} & X_{36} & \dots \\ X_{52} & X_{54} & X_{56} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}, & n \text{ even} \end{cases} \quad (4.45)$$

where X represents either $A, C, D, E, G, H, J, K, L, O, P$, or Q ;

$$[Y] = \begin{cases} [\sum Y_{0n} & \sum Y_{2n} & \sum Y_{4n} & \dots]^T, & n \text{ odd} \\ [\sum Y_{1n} & \sum Y_{3n} & \sum Y_{5n} & \dots]^T, & n \text{ even} \end{cases} \quad (4.46)$$

where Y represents either B or F ; and

$$[Z] = \begin{cases} [Z_1 & Z_3 & Z_5 & \dots]^T, & n \text{ odd} \\ [Z_2 & Z_4 & Z_6 & \dots]^T, & n \text{ even} \end{cases} \quad (4.47)$$

where Z represents either M^1, M^2, N^2 , or M^3 . Therefore, the matrix equation (4.44) can be solved for odd n and even n separately.

Once the expansion coefficients are solved, the EM fields of this antenna system are known. In the far-field zone ($\xi \rightarrow \infty$), the magnetic and electric fields are obtained as

$$H_\phi \approx \frac{2e^{-jk_3r}}{k_3rd \sin \theta} \sum_{n=1,2}^{\infty} e^{j(\frac{n+1}{2})\pi} M_n^3 V_n(h_3, \cos \theta) \quad (4.48)$$

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

where $k_3 = \sqrt{\epsilon_{r_3}\mu_{r_3}}k_0$, and

$$E_\theta \approx \sqrt{\frac{\mu_{r_3}\mu_0}{\epsilon_{r_3}\epsilon_0}} H_\phi \tag{4.49}$$

4.4 Efficient Electrically Small Antennas with DNG MTMs Shell

The EM fields of prolate spheroidal antennas with two layers of coatings are given in the last section. They can be used to evaluate electrically small antennas with DNG MTMs shell. The power radiated by an antenna-DNG shell system can be obtained in terms of the integral of the Poynting vector over a closed surface containing the spheroidal antenna

$$\begin{aligned} P_{\text{rad}} &= \frac{1}{2} \text{Re} \left[\oint_S (\mathbf{E} \times \mathbf{H}^*) \cdot \hat{\xi} dS \right] \\ &= \frac{1}{2} \text{Re} \left[\int_0^{2\pi} d\phi \int_{-1}^{+1} h_\phi h_\eta ((-\hat{\eta}) E_\eta \times \hat{\phi} H_\phi^*) \cdot \hat{\xi} d\eta \right] \\ &= \pi \text{Re} \left[\int_{-1}^{+1} h_\phi h_\eta E_\eta H_\phi^* d\eta \right] \end{aligned} \quad (4.50)$$

where S is the surface of the confocal spheroid $\xi = \xi_3$. Substituting (4.35), (4.36) and (4.34) into (4.50), we obtain

$$\begin{aligned} P_{\text{rad}} &= \text{Re} \left[\frac{j2\pi}{w\epsilon_3 d} \int_{-1}^{+1} (1 - \eta^2)^{-1} \frac{\partial A_3}{\partial \xi} A_3^* d\eta \right] \\ &= \text{Re} \left[\frac{j2\pi}{w\epsilon_3 d} \int_{-1}^{+1} (1 - \eta^2)^{-1} \left[\sum_{n=1,2}^{\infty} M_n^3 U_n'(h_3, \xi_3) V_n(h_3, \eta) \right] \right. \\ &\quad \left. \times \left[\sum_{n'=1,2}^{\infty} M_{n'}^3 U_{n'}'(h_3, \xi_3) V_{n'}(h_3, \eta) \right]^* \right] \end{aligned} \quad (4.51)$$

Using the orthogonality property of functions $V_n(h, \eta)$ (4.14), we get

$$P_{\text{rad}} = \text{Re} \left[\frac{j2\pi}{w\epsilon_3 d} \sum_{n=1,2}^{\infty} |M_n^3|^2 U_n'(h_3, \xi_3) U_n^*(h_3, \xi_3) N_{1,n}(h_3) \right] \quad (4.52)$$

The current distribution on the spheroidal surface in the η direction can be expressed as

$$I(\eta) = \int_0^{2\pi} J_\eta h_\phi d\phi = \int_0^{2\pi} H_\phi|_{\xi=\xi_1} h_\phi d\phi = 2\pi A_1|_{\xi=\xi_1} \quad (4.53)$$

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

where J_η is the surface current, which is equal to $H_\phi|_{\xi=\xi_1}$ in magnitude. The “average current” I_{av} over the length along the angular direction on the surface of the spheroidal dipole is defined as

$$I_{\text{av}} = \frac{\int_{-1}^{+1} I(\eta) h_\eta d\eta}{\int_{-1}^{+1} h_\eta d\eta} \quad (4.54)$$

Therefore, the radiated power by an antenna system with 1 A average current can be obtained as

$$P_{\text{norm}} = \frac{P_{\text{rad}}}{|I_{\text{av}}|^2} \quad (4.55)$$

Finally, the radiated power ratio (RPR) can be expressed as

$$RPR = \frac{P_{\text{norm, DNG shell}}}{P_{\text{norm, uncoated}}} \quad (4.56)$$

4.5 Results and Discussion

4.5.1 DNG Radome

To obtain numerical results of the unknown expansion coefficients, the series expansions are truncated at 30 terms in order to ensure the convergence of the solution with different dimensions and frequencies assumed in this chapter. It is found that in most cases, convergence can be achieved with fewer than 30 truncation terms. For instance, a convergence study of radiation patterns is shown in Table 4.1. It is shown that three significant digits of radiation patterns are achieved with the truncation number $N = 6$. It is also found that $N = [k_3 a_3] + 20$ is sufficient to obtain three significant digits in calculating the radiation pattern presented in this chapter ($[]$ denotes the integer part function).

Table 4.1: Convergence study of normalized radiation patterns of prolate spheroidal antennas coated by a radome (interfocal distance $d/\lambda_0 = 0.5$, radial coordinate $\xi_1 = 1.077$, $\eta_0 = 0$, the separation $s/\lambda_0 = 0.25$, and the thickness $t/\lambda_2 = 0.5$; $\epsilon_{r_2} = -9.6$ and $\mu_{r_2} = -1.0$ for DNG MTMs radome; $\epsilon_{r_2} = 9.6$ and $\mu_{r_2} = 1.0$ for dielectric radome)

	MTMs ($ H_\phi(40^\circ) / H_\phi _{\max}$)	Dielectric ($ H_\phi(40^\circ) / H_\phi _{\max}$)
N=2	0.559	0.559
N=4	0.574	0.342
N=6	0.573	0.334
N=8	0.573	0.334

Figure 4.3 shows the normalized radiation patterns of prolate spheroidal antennas with different radial coordinates. The length of the interfocal distance is a half-wavelength, i.e., $d/\lambda_0 = 0.5$. And the excitation slots are all located at

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.3: Normalized radiation patterns of prolate spheroidal antennas having slots at $\eta_0 = 0$, half-wavelength interfocal distance $d/\lambda_0 = 0.5$ and different radial coordinates of $\xi_1 = 1.077, 2.0$, and 5.0 .

the center of the antennas, i.e., $\eta_0 = 0$. It is seen that the radiation patterns vary greatly with the increase of the radial coordinate ξ_1 of the antenna. This is because when $\xi_1 \rightarrow 1$, the spheroidal antenna becomes a thin wire antenna, and when $\xi_1 \rightarrow \infty$, the spheroidal antenna becomes a spherical antenna.

Figure 4.4 shows the normalized radiation patterns of prolate spheroidal antennas with different interfocal distances. It is shown that only when the antenna is very thin, such as $\xi_1 = 1.077$, the spheroidal antennas with $d/\lambda_0 = 0.25$ and $d/\lambda_0 = 0.5$ can be used to approximately model quarter- and half-wavelength wire antennas. When the antenna becomes wider, such as $\xi_1 = 2.0$, the number of lobes increases significantly with increasing interfocal distance.

Figure 4.5 shows the normalized radiation patterns of prolate spheroidal antennas with different excitation slots. It is seen that the main lobes of the radiation

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $\xi_1 = 1.077$

(b) $\xi_1 = 2.0$

Figure 4.4: Normalized radiation patterns of prolate spheroidal antennas with slots at $\eta = 0$ and different interfocal distances $D = d/\lambda_0 = 0.25, 0.5$, and 1.0 and radial coordinates (a) $\xi_1 = 1.077$ and (b) $\xi_1 = 2.0$.

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.5: Normalized radiation patterns of prolate spheroidal antennas with half-wavelength interfocal distance $d/\lambda_0 = 0.5$, radial coordinate $\xi_1 = 1.077$ and different excitation slots at $\eta_0 = 0, 0.5, 0.707$, and 0.94 .

patterns are shifted away from the center to the opposite side of the excitation slot.

It is also seen that from Figures 4.3 – 4.5 that the results obtained are in excellent agreement with the results in [15], which verifies the developed computer codes written are correct.

In some cases, antennas are coated directly by a protective layer in order to prevent environmental effects. This problem can be solved by using the same materials in regions I and II in Figure 4.2.

Figures 4.6 and 4.7 show the effects when DNG MTMs are used for antenna coatings. For comparison, the uncoated antenna patterns are also given. It is seen that the antenna with DNG MTMs coatings divert more energy from the central lobe to the side lobes than those with dielectric coatings when having the

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.6: Normalized radiation patterns of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t + s)/\lambda_1 = 0.5$, $\epsilon_{r1} = \epsilon_{r2} = -2.97$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 2.97$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating).

Figure 4.7: Normalized radiation patterns of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t + s)/\lambda_1 = 0.5$, $\epsilon_{r1} = \epsilon_{r2} = -5.25$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 5.25$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating).

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

same thickness and the same absolute value but opposite sign of permittivity and permeability.

Figure 4.8: Normalized radiation patterns of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t + s)/\lambda_1 = 0.1$, $\epsilon_{r1} = \epsilon_{r2} = -9.6$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 9.6$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating).

Figure 4.8 shows that when the thickness is small compared with the wavelength in the coating, such as $T = 0.1$, the radiation patterns are both similar to that of the uncoated antenna. It is seen that the radiation pattern of the DNG MTMs coated antenna is slightly wider than that of the uncoated antenna, and in contrast, the radiation pattern of the dielectric coated antenna is slightly narrower than that of the uncoated antenna.

Figure 4.9 shows that the radiation pattern of the DNG MTMs coated antenna is the same as that of the conventional dielectric coated antenna. However, the thickness of the DNG MTMs coated antenna is smaller (23% in this case) than that of the dielectric coated antenna. Therefore, it is possible to reduce the size of

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.9: Normalized radiation pattern of prolate spheroidal antennas ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, and $\eta_0 = 0$) with different coatings ($T = (t + s)/\lambda_1 = 0.25$, $\epsilon_{r1} = \epsilon_{r2} = -1.24$, $\mu_{r1} = \mu_{r2} = -1.0$ for DNG MTMs coating, and $\epsilon_{r1} = \epsilon_{r2} = 2.97$, $\mu_{r1} = \mu_{r2} = 1.0$ for dielectric coating).

antenna systems using DNG MTMs.

When the antenna is coated by a radome, regions I and III are simply air in most practical cases, i.e., $\epsilon_{r1} = \epsilon_{r3} = 1.0$ and $\mu_{r1} = \mu_{r3} = 1.0$. Figure 4.10 shows the normalized radiation patterns of prolate spheroidal antennas coated with DNG MTMs radomes compared with those coated with conventional dielectric radomes. The thickness of the radome is $0.5\lambda_2$ because the half-wavelength wall is widely used. It can cancel the reflections at the two interfaces by making the reflections out-of-phase. This is desirable because the use of radomes is to protect the antenna from environmental effects but not to interfere with its operation [106]. It can be observed that DNG MTMs radomes have better performance than conventional dielectric radomes, even when placed very close to the antenna. This phenomenon is more significant when a material with large absolute value of the

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $\epsilon_{r_2} = -2.97$ and $\mu_{r_2} = -1.0$ for DNG MTMs radome; $\epsilon_{r_2} = 2.97$ and $\mu_{r_2} = 1.0$ for dielectric radome

(b) $\epsilon_{r_2} = -9.6$ and $\mu_{r_2} = -1.0$ for DNG MTMs radome; $\epsilon_{r_2} = 9.6$ and $\mu_{r_2} = 1.0$ for dielectric radome

Figure 4.10: Normalized radiation patterns of prolate spheroidal antennas coated by a radome ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, $\eta_0 = 0$, and $t/\lambda_2 = 0.5$) placed at different separations s using different DNG MTMs and conventional dielectric materials.

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

relative permittivity is used. As shown in Figure 4.10 (b), when the DNG MTMs radome is placed at $s/\lambda_0 = 0.25$, the radiation pattern is still the same as that of an uncoated antenna, whereas the radiation pattern of an antenna with conventional dielectric radome is narrower obviously. This is because the use of DNG MTMs radomes can decrease and even compensate the phase difference developed in region I. Therefore, it is possible to use an appropriate DNG MTMs radome to provide the same radiation pattern as that of an uncoated antenna, and reduce the size of the antenna system significantly at the same time.

Figure 4.11 shows the normalized radiation patterns of prolate spheroidal antennas coated with DNG MTMs radomes with different locations of the excitation slots. It is found that when the radomes are placed at a separation of λ_0 from the antenna, the DNG MTMs radomes still have better performance than conventional dielectric radomes in different excitation slots locations. When the radome is placed closer, e.g. $s/\lambda_0 = 0.25$, the effects of both kind of radomes become significant, but the DNG MTMs radome does not create more sidelobes in the radiation pattern compared to the conventional dielectric radome.

Figure 4.12 shows that when the prolate spheroidal antenna becomes wider, the radiation patterns of antennas with DNG MTMs radomes are still closer to that of the uncoated antenna than those of antennas with conventional dielectric radomes, especially when the separation between the antenna and the radome is small, e.g. $s/\lambda_0 = 0.25$.

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $\eta_0 = 0.5$

(b) $\eta_0 = 0.94$

Figure 4.11: Normalized radiation patterns of prolate spheroidal antennas with different excitation slots locations coated by a radome ($d/\lambda_0 = 0.5$, $\xi_1 = 1.077$, $t/\lambda_2 = 0.5$, $\epsilon_{r_2} = -9.6$ and $\mu_{r_2} = -1.0$ for DNG MTMs radome and $\epsilon_{r_2} = 9.6$ and $\mu_{r_2} = 1.0$ for dielectric radome) placed at different separations s .

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.12: Normalized radiation patterns of prolate spheroidal antennas coated by a radome ($d/\lambda_0 = 0.5$, $\xi_1 = 2.0$, $\eta_0 = 0$, and $t/\lambda_2 = 0.5$) placed at different separations s using DNG MTMs ($\epsilon_{r2} = -9.6$ and $\mu_{r2} = -1.0$) and conventional dielectric materials ($\epsilon_{r2} = 9.6$ and $\mu_{r2} = 1.0$).

4.5.2 Efficient Electrically small antennas

An electrically small antenna in free space is defined as $k_0 r_e \leq 0.5$ in [91]. For the frequency of interest here, $f_0 = 300$ MHz, the free-space wavelength is $\lambda_0 = 1.0$ m, and the electrically small antenna should be inside the sphere with radius $r_e = 79.58$ mm. For electrically small antennas, we only need to consider the dominant lowest order mode ($n = 1$) to calculate the RPRs using (4.56). Regions I and III as shown in Figure 4.2 are assumed to be free space, i.e. $\epsilon_{r1} = \epsilon_{r3} = \epsilon_0$ and $\mu_{r1} = \mu_{r3} = \mu_0$.

Figure 4.13 shows the RPRs of the infinitesimal electric dipole-DNG shell system versus the outer shell semi-major axis a_3 with different separations s between dipole and shell. The dipole antenna is modelled by a thin spheroid ($\xi_1 = 1.005$,

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.13: Radiated power of an infinitesimal electric dipole ($2a_1 = 10.0$ mm $= \lambda_0/100$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) for various thicknesses t (or a_3) at different separations s normalized to the power radiated by the same infinitesimal electric dipole in free space.

i.e. $a_1/b_1 = 10.0$). The total length of the antenna is $2a_1 = 10.0$ mm $= \lambda_0/100$.

It is shown that with a DNG shell ($\epsilon_{r_2} = -3.0$, $\mu_{r_2} = -1.0$), the infinitesimal electric dipole-DNG shell system has a natural resonance due to the cancellation of the inductive reactance introduced by the DNG MTMs shell and the capacitive reactance of the electrically small antenna. It is seen that the maximum RPR is 85.78 dB at $a_{3,\max} = 7.64$ mm with $s = \lambda_0/1000$. It is found that the maximum RPR is enhanced by more than 20 dB compared with the maximum RPR coated with the corresponding spherical ENG shell presented in [11], and the outer shell semi-major axis $a_{3,\max}$ is about 60% smaller than the corresponding radius of the outer ENG spherical shell ($r_{2,\max} = 18.79$ mm). It is also seen that the peak of the RPR occurs at an increasing outer shell semi-major axis a_3 as the separation s

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

increases.

Figures 4.14 (a) and (b) show the RPRs of the infinitesimal electric dipole-DNG shell system with different relative permittivity ϵ_{r_2} and different relative permeability μ_{r_2} of DNG shells, respectively. It is found that parameter a_3 , for which the peak of the RPR occurs, is approximately inversely proportional to the relative permittivity ϵ_{r_2} . However, the change of μ_{r_2} has no effect on the peak of the RPR. This is because only ϵ_{r_2} changes the effective reactance. This relation can be expressed as [11]

$$X_{\text{shell}} \propto jw_0 \frac{1}{w_0^2 |\epsilon_{r_2}| \Delta r} \quad (4.57)$$

where Δr is the thickness of the shell, and in our case, $\Delta r = t = b_3 - b_2$.

Figure 4.15 shows the frequency dependence of the RPRs of the infinitesimal electric dipole-DNG shell system which is the same as the one considered in Figure 4.13 with $a_3 = a_{3,\text{max}} = 7.64$ mm. Therefore, the FBW can be obtained by the relation [5, 11]

$$\text{FBW} = \frac{\Delta f_{3 \text{ dB}}}{f_{0 \text{ dB}}} = \frac{1}{Q_{\text{BW}}} \quad (4.58)$$

where $\Delta f_{3 \text{ dB}} = f_{+,3 \text{ dB}} - f_{-,3 \text{ dB}}$, and Q_{BW} is the radiation quality factor. $f_{+,3 \text{ dB}}$ and $f_{-,3 \text{ dB}}$ denote the frequencies above and below $f_{0 \text{ dB}}$ where the RPR is 3 dB lower than its maximum value. It is found in Figure 4.15 that the maximum value of RPR is 86.45 dB at $f_{0 \text{ dB}} = 297.19$ MHz, $f_{-,3 \text{ dB}} = 290.21$ MHz and $f_{+,3 \text{ dB}} = 304.01$ MHz. Therefore, $\text{FBW}_{\text{DNG}} = 4.64\%$ and the quality factor $Q_{\text{BW, DNG}} = 21.55$. The most widely cited Chu limit of the lowest possible radiation Q of a

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

(a) $\mu_{r2} = -1.0$

(b) $\epsilon_{r2} = -3.0$

Figure 4.14: Radiated power of an infinitesimal electric dipole ($2a_1 = 10.0$ mm $= \lambda_0/100$, $\xi_1 = 1.005$) coated with DNG shells, (a) $\mu_{r2} = -1.0$ and different ϵ_{r2} ; (b) $\epsilon_{r2} = -3.0$ and different μ_{r2} , at separation $s = \lambda_0/1000$ normalized to the power radiated by the same infinitesimal electric dipole in free space.

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.15: Frequency dependence of radiated power of an infinitesimal electric dipole ($2a_1 = 10.0$ mm $= \lambda_0/100$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ with $a_3 = 7.64$ mm normalized to the power radiated by the same infinitesimal electric dipole in free space.

given size antenna is given in [5, 11, 83, 90] as

$$Q_{\text{Chu limit}} = \frac{1 + 2(k_0 r_e)^2}{(k_0 r_e)^3 [1 + (k_0 r_e)^2]} \quad (4.59)$$

Therefore, with $f_{0 \text{ dB}} = 297.19$ MHz and $r_e = a_{3, \text{max}} = 7.64$ mm, the electric size of the antenna is $k_{0 \text{ dB}} a_{3, \text{max}} = 0.048$ and the minimum quality factor obtained by (4.59) is $Q_{\text{Chu limit}} = 9063.03$. However, it is found that the quality factor of the dipole-DNG shell antenna system presented in Figure 4.15 is significantly smaller than the value of the Chu limit, i.e. $Q_{\text{BW, DNG}} = 0.0024 Q_{\text{Chu limit}}$. This is because that the radiated power is increased dramatically due to the resonance of this dipole-DNG system corresponded to a decrease of the radiation Q. In other words, this infinitesimal electric dipole-DNG shell antenna system has much larger FBW than that predicted by the Chu limit for conventional antennas, i.e. $\text{FBW}_{\text{DNG}} =$

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

420.56 $\text{FBW}_{\text{Chu limit}}$.

Figure 4.16: Radiated power of an electrically small dipole ($2a_1 = 20.0$ mm = $\lambda_0/50$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same electrically small dipole in free space.

Figure 4.16 shows the RPR of an electrically small dipole ($2a_1 = 20.0$ mm = $\lambda_0/50$) coated with a DNG shell. It is found that at $a_{3,\text{max}} = 14.78$ mm, the maximum RPR is 68.85 dB which is 16.93 dB smaller than the one with $2a_1 = \lambda_0/100$ presented in Figure 4.13. This is because the size of this dipole-DNG antenna system is 43% larger than that considered in Figure 4.13.

In Figure 4.17, it is found that the maximum value of RPR is 68.94 dB at $f_{0\text{ dB}} = 298.07$ MHz, $f_{-,3\text{ dB}} = 284.39$ MHz and $f_{+,3\text{ dB}} = 311.15$ MHz. Therefore, $\text{FBW}_{\text{DNG}} = 8.98\%$ and the quality factor $Q_{\text{BW, DNG}} = 11.14$. However, in this condition, the electric size of the antenna is $k_{0\text{ dB}} r_e = k_{0\text{ dB}} a_{3,\text{max}} = 0.092$, and the minimum quality factor obtained by (4.59) is $Q_{\text{Chu limit}} = 1294.99$. Thus the radi-

Figure 4.17: Frequency dependence of radiated power of an electrically small dipole ($2a_1 = 20.0$ mm $= \lambda_0/50$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ with $a_3 = 14.78$ mm normalized to the power radiated by the same infinitesimal electric dipole in free space.

ation quality factor of this electrically small dipole-DNG antenna system is much smaller than that predicted by the Chu limit, i.e. $Q_{\text{BW, DNG}} = 0.0086 Q_{\text{Chu limit}}$, and the FBW is much larger than that predicted by the Chu limit, i.e. $\text{FBW}_{\text{DNG}} = 116.25 \text{ FBW}_{\text{Chu limit}}$. From above figures, it is also found that with increasing size of the antenna system, the maximum value of RPR and the quality factor Q decrease, while the FBW increases.

Figure 4.18 shows the RPR of an electrically small dipole ($2a_1 = 100.0$ mm $= \lambda_0/10$) coated with a DNG shell. It is found that the maximum RPR is 22.98 dB at $a_{3,\text{max}} = 79.39$ mm which is just a little bit smaller than the maximum radius of the electrically small antenna, 79.58 mm. It can be seen that the peak value of the RPR of this electrically small dipole-DNG system is reduced by 62.80 dB

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.18: Radiated power of an electrically small dipole ($2a_1 = 100.0$ mm $= \lambda_0/10$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same electrically small dipole in free space.

compared with the one with $2a_1 = \lambda_0/100$ presented in Figure 4.13 due to the larger size of antenna.

Figure 4.19 shows the RPRs of a $\lambda_0/4$ dipole-DNG shell antenna system. It is found that the maximum RPR is 6.83 dB at $a_{3,\max} = 161.51$ mm. It is seen that since the dipole is thin ($a_1/b_1 = 10.0$), the result given by only the lowest dominant mode ($n = 1$) is almost the same as the result given by the first 2 modes ($n = 1$ and 3).

Figure 4.20 shows the RPRs of a $\lambda_0/2$ dipole-DNG shell antenna system. It is seen that, due to the larger size of the antenna, we need to consider more modes ($n = 1, 3, 5, \dots$). From the first two modes, the maximum RPR is only 0.78 dB at $a_{3,\max} = 279.16$ mm.

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

Figure 4.19: Radiated power of a $\lambda_0/4$ dipole ($2a_1 = 250.0$ mm = $\lambda_0/4$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same dipole in free space.

Figure 4.20: Radiated power of a $\lambda_0/2$ dipole ($2a_1 = 500.0$ mm = $\lambda_0/2$, $\xi_1 = 1.005$) coated with a DNG shell ($\epsilon_{r_2} = -3.0$ and $\mu_{r_2} = -1.0$) at separation $s = \lambda_0/1000$ normalized to the power radiated by the same dipole in free space.

4.6 Conclusion

The problem of EM radiation from a prolate spheroidal antenna, excited by a delta voltage across a circumferential infinitesimally narrow slot at any arbitrary position along the major axis of the antenna, coated with a confocal DNG MTMs coating or radome is studied in this chapter. The method of separation of the spheroidal scalar wave functions in prolate spheroidal coordinate is used. The unknown expansion coefficients are solved by applying the continuity of tangential EM fields on the boundaries of the spheroidal surfaces. It is found that the use of DNG MTMs coating can reduce the size of the antenna system with the same radiation pattern compared with the antenna with conventional dielectric coating. Our results also show that, different from a conventional dielectric radome, the half-wavelength confocal DNG MTMs radome is almost electrically transparent to the spheroidal antenna and has almost no effect on the antenna's radiation pattern, even when the DNG MTMs radome is placed very close to the antenna which means that the size of the antenna system can be significantly reduced without loss of performance.

For the application of efficient electrically small antenna, it is also found that the electrically small dipole-DNG shell system has very high radiation efficiency compared with the normal electrically small antenna. It is found that the spheroidal shell can achieve a more compact structure and higher radiated power ratio than the corresponding spherical shell. And it is also shown that the fractional bandwidth (FBW) of this system is much larger than that predicted by the Chu

CHAPTER 4. SPHEROIDAL ANTENNA COATED WITH DOUBLE-NEGATIVE METAMATERIALS

limit which is the maximum FBW limit for the normal electrically small antenna. Dipole-DNG shell systems with different sizes (up to $\lambda/10$) are analyzed and discussed. It is found that the maximum value of RPR and the quality factor Q decrease due to the increasing size of the antenna system, while the FBW of this electrically small dipole-DNG system increases with antenna size.

Part II

Phase Mode Processing

Chapter 5

Icosahedron-Based Spherical Antenna Array for Wideband Phase Mode Processing

5.1 Introduction

In Chapter 4, the radiation characteristics of single-element antennas were analyzed and discussed. As the number of users and the demand for wireless services increases at an exponential rate, antenna arrays should be used to meet these requirements, such as higher gains, wider coverage, higher transmission quality, etc.

As mentioned in Chapter 2, the analysis and synthesis of circular antenna arrays has been studied using the concept of phase mode excitation over the past four decades [12, 96–100, 107]. The excitation of planar circular antenna arrays can be conveniently analyzed in terms of a Fourier series [12], and each term of the series is called a phase mode. When a circular array is excited by a phase mode, the far-field radiation pattern has the same phase variation with azimuth angle and a constant amplitude given by a Bessel function coefficient. These properties are mainly due to their symmetry and 360° coverage. This kind of circular antenna

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

array can be implemented conveniently by using a Butler matrix network [96].

The use of directional elements in circular arrays can overcome the limitation of rapid variation in the amplitude of the far-field and allow such arrays to be used in wideband applications [97].

Recently, this concept was extended to the spherical phase mode [1] and the spheroidal phase mode [108] based on spherical and spheroidal array geometries, respectively. These results show that similar characteristics to a circular phase mode have been obtained in three-dimensions. The interest of such spherical or spheroidal antenna arrays is based on the possibility to scan a single or multiple beams through the whole three-dimensional space with low grating lobe levels [1, 108–117]. However, spherical or spheroidal arrays based on equiangular sampling schemes can only scan the beam in the azimuth angle but not the elevation angle [1, 108]. This is because the distribution in an equiangular sampling scheme is not uniform in the elevation angle. Furthermore, the sampling points for such an array are much denser near the poles than at the equator due to the equiangular topology, which make the array suffer severe mutual coupling effects near the poles. In addition, the effects of the mutual coupling vary with the polar angle, which makes compensation of these effects inconvenient.

In this chapter, an improved topology of spherical antenna arrays for phase mode processing is presented in order to overcome the above-mentioned limitations. In this icosahedron-based topology, the inter-element spacing of all antenna elements is almost identical. This attractive property can be used for three-dimensional beam scanning and for reducing the effects of mutual coupling. The

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

use of directional elements in this array is discussed which shows that the amplitude of the far-field mode does not go to zero for any frequency because the zero points of spherical Bessel function coefficients with certain arguments are compensated so that this proposed array can be used for wideband synthesis. A theoretical analysis of aliasing is also developed, which gives useful design information for wideband applications.

This chapter is organized as follows. Section 5.2 presents spherical antenna arrays, based on the icosahedron topology excited by spherical phase modes, with directional antenna element pattern. In Section 5.3, the numerical results and discussion are given. And Section 5.4 concludes the work of this chapter.

5.2 Icosahedron-Based Spherical Antenna Arrays

5.2.1 Topology

A spherical antenna array with equal separation between neighboring elements is desirable for three-dimensional beam scanning and for minimizing and compensating the effects of mutual coupling. Designing such an array is equivalent to the problem of symmetrically dividing a spherical surface about a center point into areas of congruent polygons. The geometry so formed is a regular polyhedron [118]. However, there are only five regular polyhedra, i.e. tetrahedron, cube, octahedron, dodecahedron and icosahedron [119]. Each polyhedron has an inscribed sphere and a circumscribing sphere, the spheres being concentric. The best angular resolution among the five regular polyhedra is given by the icosahedron [118], which has 12 vertices, 20 equilateral triangular faces and 30 edges as shown in Figure 5.1.

Figure 5.1: Icosahedron with a subdivision scheme ($v = 3$) on one face.

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE
MODE PROCESSING

To increase the number of array elements, each equilateral triangle can be subdivided into a number of smaller equilateral triangles. The subdivision schemes are named ‘alternative breakdown scheme with frequency v ’ [120] as shown in Figure 5.1, where v is the number of subdivisions of the edge of the equilateral triangle. The number of the vertices of all the subdivided triangles N can be calculated by

$$N = 10v^2 + 2 \quad (5.1)$$

However, this is an approximate method in the sense that when these vertices are projected onto the inscribed or circumscribing spheres, the separation between neighboring elements is no longer exactly the same. This is because the vertices are now located at various points in between the inscribed and circumscribing spheres, instead of all lying on the circumscribing sphere for the original icosahedron.

In order to obtain equal distance between elements on the surface of the inscribed or circumscribing spheres, the vertices on the triangular surface need to be rearranged before mapping to the spherical surface. Unfortunately, it is a well-known group-theoretical result that there is no completely uniform distribution on the sphere for $N > 20$ [121]. In this chapter, we make use of the method of area equalization [121] to obtain approximately equal distance. Without loss of generality, we assume an inscribed sphere with radius a . Since each face of the icosahedron can be projected onto a region on the inscribed sphere bounded by three great circles, any face of this circumscribing icosahedron can be rotated to lie in the $z = a$ plane by multiplying with an appropriate rotation matrix. Therefore,

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

we only need to consider this triangular face. The area equalization is to find a mapping, $(x, y, a) \rightarrow (x', y', a)$, on this triangular face whose Jacobian is proportional to the inverse of the Jacobian of the mapping from the inscribed unit sphere to the triangular plane as shown in Figure 5.2, where (x, y, a) and (x', y', a) denote the coordinates of the vertices before and after mapping, respectively. Therefore,

Figure 5.2: Area equalization mapping on $z = a$ plane.

the Jacobian of these two mappings is a constant, and this combined mapping is a equal-area mapping. Because the Jacobian of the mapping from the sphere to the plane is $(\frac{a^2+x^2+y^2}{a^2})^{\frac{3}{2}}$, the mapping $(x, y, a) \rightarrow (x', y', a)$ should satisfy the nonlinear partial differential equation

$$\det \begin{pmatrix} \partial x' / \partial x & \partial x' / \partial y \\ \partial y' / \partial x & \partial y' / \partial y \end{pmatrix} = C \left(\frac{a^2 + x^2 + y^2}{a^2} \right)^{-\frac{3}{2}} \quad (5.2)$$

where C is a proportionality constant of mapping which is the ratio of the area of

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

the triangle to the area of the triangular region mapping on the inscribed sphere and is given by

$$C = \frac{\frac{\sqrt{3}}{4}[9 \tan^2(\frac{\pi}{5}) - 3]a^2}{\frac{4\pi a^2}{20}} \approx 1.21$$

Solving the partial differential equation (5.2), the coordinates of the vertices after mapping, (x', y', a) , can be calculated by the following equations

$$\begin{cases} y' = a \sqrt{\frac{2C}{\sqrt{3}}} \tan^{-1} \left[\sqrt{3} \frac{\sqrt{a^2 + 4y^2} - a}{\sqrt{a^2 + 4y^2 + 3a}} \right] \\ x' = \left(\frac{xy'}{y} \right) \sqrt{\frac{a^2 + 4y^2}{a^2 + x^2 + y^2}} \end{cases} \quad (5.3)$$

After projecting these adjusted vertices (x', y', a) onto the inscribed sphere's surface, and by placing an antenna element on each vertex, an icosahedron-based spherical antenna array is obtained which can provide approximately equal distance between the neighboring elements. The average distance between the neighboring elements d is approximately given by

$$d \approx \frac{3.81a}{\sqrt{N}} \quad (5.4)$$

5.2.2 Spherical Phase Mode

The spherical phase modes or spherical harmonics are the angular portion of the solution to the Helmholtz or the space-dependence of the electromagnetic wave equation in spherical coordinates, which is defined in [1] as

$$Y_l^m(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos \theta) e^{jm\phi} \quad (5.5)$$

where θ and ϕ are, respectively, the polar and azimuthal coordinates in the spherical coordinate system, l is the degree of the spherical phase mode, $j = \sqrt{-1}$. P_l^m are the associated Legendre functions which represent the standing spherical waves in θ , and the exponential terms $e^{jm\phi}$ denote the travelling spherical waves in ϕ [122].

Due to the completeness property of spherical phase modes, any square integrable function $f(\theta, \phi)$ over the surface of the sphere can be expanded in a double series of spherical phase modes, which is given by [123]

$$f(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l f_{ml} Y_l^m(\theta, \phi) \quad (5.6)$$

The coefficients f_{ml} can be obtained using spherical Fourier transform

$$f_{ml} = \iint_s f(\theta, \phi) Y_l^{m*}(\theta, \phi) ds \quad (5.7)$$

where ds is the element of the solid angle. Therefore, an excitation function on the surface of a sphere can be expressed as a linear combination of spherical phase modes.

For illustration, consider one spherical phase mode $Y_l^{m'}$ as the excitation func-

CHAPTER 5. ICOSAHDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

tion, and the corresponding far-field radiation pattern can be represented by

$$D(\theta, \phi) = \frac{1}{4\pi} \iint_{s'} Y_{l'}^{m'}(\theta', \phi') e^{j\vec{\beta} \cdot \vec{a}} ds' \quad (5.8)$$

where a is the radius of the sphere, the direction of $\vec{a}$ is (θ', ϕ') , the wavenumber $\beta = 2\pi/\lambda$, the direction of $\vec{\beta}$ is (θ, ϕ) as shown in Figure 5.3. And (5.8) can be finally expressed by [1]

$$D(\theta, \phi) = j^{l'} j_{l'}(\beta a) Y_{l'}^{m'}(\theta, \phi) \quad (5.9)$$

where $j_{l'}(\beta a)$ is a spherical Bessel function of the first kind. (5.9) shows that the far-field radiation pattern has the same spherical phase mode form as the excitation function.

Figure 5.3: Spherical array geometry.

5.2.3 Directional Element Pattern

Although it is possible to obtain any desired far-field radiation pattern in three-dimension by means of breaking the pattern down into a series of spherical harmonics and exciting each of them around the spherical array with appropriate coefficients separately, (5.9) shows that the amplitude of the far-field mode $j_l(\beta a)$ varies rapidly with a/λ , which means the array is narrowband. [1] anticipates that the raised cosine pattern would be the optimum directional element pattern with spherical phase mode excitations, and [12, 97] show that the use of directional elements can overcome the similar problem in circular antenna arrays. Our theoretical calculations show that the use of directional elements can overcome these limitations and give almost the same amplitudes of the far-field mode if excited by different spherical phase modes. The simulation results will be given in Section 5.3.

Let us assume that the individual element pattern is a raised cosine pattern $(1 + \cos \theta'')$ rotated around the normal axis z'' of the elements on the sphere (a pencil-beam radiation pattern), where θ'' is the elevation angle in the local coordinate (x'', y'', z'') of the antenna element as shown in Figure 5.3. And each element has θ (or ϕ) polarization. The far-field radiation pattern excited by spherical phase mode $Y_l^m(\theta, \phi)$ can be represented by

$$D(\theta, \phi) = \hat{\theta} \frac{1}{4\pi} \iint_{s'} (1 + \cos \psi) Y_l^{m'}(\theta', \phi') e^{j\vec{\beta} \cdot \vec{a}} ds' \quad (5.10)$$

where ψ is the angle between $\vec{a}$ and $\vec{\beta}$. The plane wave in the far-field can be expressed by

$$e^{j\vec{\beta} \cdot \vec{a}} = \sum_{l=0}^{\infty} (2l+1) j^l j_l(\beta a) P_l(\cos \psi) \quad (5.11)$$

CHAPTER 5. ICOSAHDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

Differentiating both sides of (5.11) with respect to βa ,

$$j \cos \psi e^{j\vec{\beta} \cdot \vec{a}} = \sum_{l=0}^{\infty} (2l+1) j^l \frac{d}{d(\beta a)} j_l(\beta a) P_l(\cos \psi) \quad (5.12)$$

and using the spherical harmonic addition theorem [124],

$$P_l(\cos \psi) = \frac{4\pi}{2l+1} \sum_{m=-l}^l Y_l^{m*}(\theta', \phi') Y_l^m(\theta, \phi) \quad (5.13)$$

we can get,

$$\begin{aligned} (1 + \cos \psi) e^{j\vec{\beta} \cdot \vec{a}} &= 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^l j^l \left(j_l(\beta a) - j \frac{d}{d(\beta a)} j_l(\beta a) \right) \\ &\quad \cdot Y_l^{m*}(\theta', \phi') Y_l^m(\theta, \phi) \end{aligned} \quad (5.14)$$

Substituting (5.14) into (5.10), and using the orthogonality property of spherical harmonics over the sphere [124],

$$\iint_s Y_{l_1}^{m_1*}(\theta, \phi) Y_{l_2}^{m_2}(\theta, \phi) ds = \delta_{l_1, l_2} \delta_{m_1, m_2} \quad (5.15)$$

The far-field radiation is obtained as,

$$D(\theta, \phi) = \hat{\theta} j^{l'} \left(j_{l'}(\beta a) - j \frac{d}{d(\beta a)} j_{l'}(\beta a) \right) Y_{l'}^{m'}(\theta, \phi) \quad (5.16)$$

Comparing with (5.9), (5.16) shows that a spherical array with directional element pattern excited by a spherical phase mode also leads to a far-field radiation pattern with the same phase mode. But the amplitude of the far-field mode is no longer a single spherical Bessel function but a sum of such functions, which prevents the amplitude going to zero for any value of βa . Furthermore, for electrically large spherical arrays ($\beta a \gg l(l+1)/2$), the asymptotic expansions of spherical Bessel functions can be used [124],

$$j_l(x) \approx \frac{1}{x} \sin\left(x - \frac{l\pi}{2}\right) \quad (5.17)$$

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

and

$$\frac{d}{dx}j_l(x) \approx \frac{1}{x} \cos(x - \frac{l\pi}{2}) \quad (5.18)$$

Therefore the far-field radiation pattern becomes

$$D(\theta, \phi) = \hat{\theta} \frac{1}{(\beta a)} e^{j(\beta a - \frac{\pi}{2})} Y_l^{m'}(\theta, \phi) \quad (5.19)$$

(5.19) shows that for electrically large arrays, the amplitude of the far-field mode is inversely proportional to frequency and does not have any nulls. Furthermore, the amplitude does not depend on the degree of the exciting spherical phase mode. This means that for electrically large arrays the use of such directional element patterns can give almost the same amplitudes of the far-field mode when excited by different spherical phase modes. In addition, the phase of the far-field mode only varies with frequency linearly. Therefore, a linear coefficient with frequency in both amplitude and phase, such as $(\beta a e^{-j\beta a})$, can be added to the excitation functions in order to compensate this decay and keep the same radiation pattern in a wide bandwidth.

5.2.4 Icosahedron-based Arrays

For spherical arrays, N elements will be set in terms of the method presented in Section 5.2.1. The excitation at each element will be the value of the spherical harmonic Y_l^m at the coordinates of the element $(a_i = a, \theta_i, \phi_i)$, $i = 1, \dots, N$, which can be considered a product of Y_l^m with a spatial sampling function $S(\theta, \phi)$. The sampling function for this distribution is given as

$$S(\theta, \phi) = \sum_{i=0}^{N-1} w_i \delta_{(\theta_i, \phi_i)} \quad (5.20)$$

where the weights w_i of the array elements are all the same due to the symmetry of the topology, and are given by

$$w_i = \frac{4\pi}{N}, \quad i = 1, \dots, N \quad (5.21)$$

and the far-field radiation pattern (5.8) can be rewritten as

$$\begin{aligned} D(\theta, \phi) &= \frac{1}{4\pi} \sum_{i=0}^{N-1} w_i Y_l^m(\theta_i, \phi_i) e^{j\vec{\beta} \cdot \vec{a}_i} \\ &= \frac{1}{N} \sum_{i=0}^{N-1} Y_l^m(\theta_i, \phi_i) e^{j\vec{\beta} \cdot \vec{a}_i} \end{aligned} \quad (5.22)$$

When a spherical harmonic is excited in this array distribution, there will be other harmonics radiating in the far-field, which distort the far-field pattern. The sampling function (5.20) can be expressed by Fourier expansion on the sphere using (5.6) and (5.7),

$$\begin{aligned} S(\theta, \phi) &= \sum_{l=0}^{\infty} \sum_{m=-l}^l \left(\frac{4\pi}{N} \sum_{i=0}^{N-1} Y_l^{m*}(\theta_i, \phi_i) \right) Y_l^m(\theta, \phi) \\ &= 1 + \sum_{l=1}^{\infty} \sum_{m=-l}^l \left(\frac{4\pi}{N} \sum_{i=0}^{N-1} Y_l^{m*}(\theta_i, \phi_i) \right) Y_l^m(\theta, \phi) \end{aligned} \quad (5.23)$$

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

Therefore, in this array distribution, the far-field radiation pattern with the excitation of a spherical phase mode is given by

$$\begin{aligned}
 D(\theta, \phi) = & \frac{1}{4\pi} \iint_{s'} Y_{l_1}^{m_1}(\theta', \phi') \\
 & \cdot \left(1 + \sum_{l_2=1}^{\infty} \sum_{m_2=-l_2}^{l_2} \frac{4\pi}{N} \left(\sum_{i=0}^{N-1} Y_{l_2}^{m_2*}(\theta_i, \phi_i) \right) \right. \\
 & \cdot Y_{l_2}^{m_2}(\theta', \phi') \Big) e^{j\vec{\beta} \cdot \vec{a}} ds' \quad (5.24)
 \end{aligned}$$

Using the orthogonality property of spherical harmonics over the sphere,

$$\begin{aligned}
 D(\theta, \phi) = & j^{l_1} j_{l_1}(\beta a) Y_{l_1}^{m_1}(\theta, \phi) \\
 & + \sum_{l=0}^{\infty} \sum_{m=-l}^l j^l j_l(\beta a) Y_l^m(\theta, \phi) \\
 & \cdot \sum_{l_2=1}^{\infty} \sum_{m_2=-l_2}^{l_2} \frac{4\pi}{N} \left(\sum_{i=0}^{N-1} Y_{l_2}^{m_2*}(\theta_i, \phi_i) \right) \\
 & \cdot \iint_{s'} Y_{l_1}^{m_1}(\theta', \phi') Y_{l_2}^{m_2}(\theta', \phi') Y_l^{m*}(\theta', \phi') ds' \quad (5.25)
 \end{aligned}$$

we can find that the far-field pattern consists of two terms. The first term of (5.25) is the same as (5.9), and the second term of (5.25) is aliasing, which reflects the distortion. Using “group-theoretical methods” as in the quantum theory of angular momentum [125], the second term of (5.25) can be rewritten with Wigner $3j$ -symbols $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$, and (5.25) becomes

$$D(\theta, \phi) = j^{l_1} j_{l_1}(\beta a) Y_{l_1}^{m_1}(\theta, \phi) + \sum_{l=0}^{\infty} \sum_{m=-l}^l j^l j_l(\beta a) Y_l^m(\theta, \phi) e_l^m \quad (5.26)$$

CHAPTER 5. ICOSAHDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

where the error function e_l^m is expressed by,

$$e_l^m = \sum_{l_2=1}^{\infty} \sum_{m_2=-l_2}^{l_2} \frac{4\pi}{N} \left(\sum_{i=0}^{N-1} Y_{l_2}^{m_2*}(\theta_i, \phi_i) \right) \cdot \sqrt{\frac{(2l_1+1)(2l_2+1)(2l+1)}{4\pi}} \cdot \begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & -m \end{pmatrix} \quad (5.27)$$

and three restrictions

$$\begin{aligned} \text{(i)} \quad & |l_1 - l_2| \leq l \leq (l_1 + l_2) \\ \text{(ii)} \quad & m_1 + m_2 = m \\ \text{(iii)} \quad & l_1 + l_2 + l \equiv 0 \pmod{2} \end{aligned} \quad (5.28)$$

have to be satisfied or the product of two Wigner $3j$ -symbols in (5.27) vanishes.

From (5.26), we can see that the distortion terms consist of infinite spherical harmonics Y_l^m , but the spherical array only can excite the harmonics with degree $l \lesssim \beta a$ due to the attenuation of the spherical Bessel coefficients. This means that the upper limit of l in (5.26) is approximately given by $[\beta a]$, where $[\]$ denotes the integer part function. Therefore, there are $([\beta a] + 1)^2$ distortion terms. And it is apparent to see from (5.27) that the error function distortion terms are inversely proportional to the number of array elements N . Therefore, the amplitude of the error function e_l^m can be controlled by appropriately choosing the number of array elements N .

Furthermore, if directional elements, such as rotated raised cosine pattern elements mentioned in Section 5.2.3, are used in this array, spherical Bessel functions in (5.25) will become a sum of such functions,

$$j_{l_1}(\beta a) \rightarrow \left(j_{l_1}(\beta a) - j \frac{d}{d(\beta a)} j_{l_1}(\beta a) \right) \quad (5.29)$$

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

$$j_l(\beta a) \rightarrow \left(j_l(\beta a) - j \frac{d}{d(\beta a)} j_l(\beta a) \right) \quad (5.30)$$

which give practically the same value for different l when $\beta a > l$. For electrically large arrays, the amplitude of (5.29) and (5.30) will not depend on the degree l , and become the same value $1/\beta a$ as shown in Section 5.2.3. In this case, (5.25) becomes

$$D(\theta, \phi) \approx \hat{\theta} \frac{1}{(\beta a)} e^{j(\beta a - \frac{\pi}{2})} \left(Y_{l_1}^{m_1}(\theta, \phi) + \sum_{l=0}^{[\beta a]} \sum_{m=-l}^l Y_l^m(\theta, \phi) e_l^m \right) \quad (5.31)$$

Our calculations show that when the inter-element spacing d given in (5.4) is small, e.g., $d < \lambda/2$, or equivalently the number of array elements $N > 58(a/\lambda)^2$, the sum of the distortion terms of this icosahedron-based topology is negligible because of the attenuation of the error function e_l^m . The amplitude and the phase variation with frequency also can be compensated by multiplying by an appropriate coefficient, such as $(\beta a e^{-j\beta a})$, which is presented in Section 5.2.3. Therefore, these icosahedron-based arrays can be used in wideband applications such as UWB systems with the concept of spherical phase mode processing.

5.3 Results and Discussion

Figure 5.4 shows the far-field radiation pattern when the array is excited by a spherical phase mode Y_3^2 , with $a/\lambda = 3.33$ and the number of the array elements $N = 3612$ ($v = 19$ in (5.1)) for the comparison with the results in [1] with $N = 4096$. It shows that the pattern for this icosahedron-based antenna array is practically the same as the theoretical pattern in [1], as shown in Figure 5.5, for both amplitude and phase.

Figure 5.4: Far-field radiation pattern of icosahedron-based antenna array excited by spherical phase mode Y_3^2 with $a/\lambda = 3.33$ and $N = 3612$. (phase is represented on gray scale)

Figures 5.6 (a) and (b) show the amplitudes of the far-field mode excited by spherical phase mode Y_1^0 and Y_3^2 versus a/λ for both isotropic element pattern and the rotated raised cosine pattern, respectively, in the case of zero inter-element spacing. It can be noticed that the nulls are cancelled due to the effect of the directional pattern. This is because $j_l'(\beta a)$ is approximately a maximum when

CHAPTER 5. ICOSAHDREDON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

Figure 5.5: Theoretical far-field radiation pattern by an excitation of spherical phase mode Y_3^2 with $a/\lambda = 3.33$ presented in [1].

$j_l(\beta a) \rightarrow 0$, and vice versa (except for $\beta a \rightarrow 0$ when $l > 1$). Therefore, the concept of spherical phase modes can be used for wideband synthesis.

Figure 5.7 shows the amplitude of the error function e_l^m excited by Y_3^2 with $a/\lambda = 3.33$ and $d/\lambda \approx 0.46$ ($N = 812$). The degree l of the error function is limited up to $\beta a \approx 21$. It is found that the amplitudes of the error function for all distortion terms are all lower than 0.0035 (−49 dB), thus are negligible.

Figure 5.8 shows the amplitudes of the far-field modes versus βa with rotated raised cosinc pattern elements for degree $l = 1, 5, 10, 20$. Note that this array can not excite the harmonics when $l > \beta a$ due to the attenuation of the spherical Bessel functions. It is worth noting that when $\beta a > l$, the amplitudes of the far-field modes are practically the same for different l .

Figure. 5.9 shows the amplitudes of the far-field mode excited by spherical

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

(a) Y_1^0

(b) Y_3^2

Figure 5.6: Amplitudes of the far-field mode excited by spherical phase modes Y_1^0 and Y_3^2 versus the radius of array in wavelengths a/λ .

CHAPTER 5. ICOSAHDREDON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE
MODE PROCESSING

Figure 5.7: Amplitude of the error function e_l^m excited by Y_3^2 with $a/\lambda = 3.33$ and $d/\lambda \approx 0.46$ ($N = 812$).

Figure 5.8: Amplitudes of the far-field mode versus βa with rotated raised cosine element pattern for $l = 1, 5, 10, 20$.

CHAPTER 5. ICOSAHDREDON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

phase mode Y_3^2 versus a/λ for both zero inter-element spacing and the icosahedron-based topology with a rotated raised cosine element pattern. The number of array elements is $N = 812$ ($d/a \approx 0.134$). It shows that the use of directional elements prevents the amplitude of the far-field mode to go to zero. And it is also seen that the far-field patterns of the icosahedron-based array are approximately the same as those of the zero spacing array up to $a/\lambda \approx 4$, i.e. $d/\lambda \approx 0.5$. This is because when $d/\lambda < 0.5$, the distortion terms are negligible as presented in the previous section.

Figure 5.9: Amplitudes of the far-field mode excited by Y_3^2 versus a/λ for $N = 812$ with rotated raised cosine element pattern.

Figure 5.10 shows the amplitudes of the far-field mode excited by Y_1^0 with rotated raised cosine element pattern for $a/\lambda = 10.0$ and $d/\lambda \approx 0.3$ at 6.0 GHz with a compensation coefficient ($\beta a e^{-j\beta a}$). It shows that with the compensation, the amplitudes of the far-field mode are practically the same in a wide bandwidth

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

(2–11 GHz). This is because when the frequency is large, e.g., 2 GHz, the antenna array can be seen as an electrically large array. Therefore, the icosahedron-based array with directional element can be used in wideband synthesis with spherical phase mode processing including UWB (3.1–10.6 GHz) applications.

Figure 5.10: Compensated amplitudes of the far-field mode excited by Y_1^0 with rotated raised cosine element pattern for $a/\lambda = 10.0$ and $d/\lambda \approx 0.3$ at 6.0 GHz.

Due to the symmetry of the icosahedron-based antenna array, it is possible to scan the beam electronically in the entire 4π steradians of the space by rotating the excitation distribution through appropriate polar and azimuth angles. Therefore, a number of applications, such as broadband pattern synthesis, null steering, direction finding, and superresolution presented in [98–100, 107], can be used in both azimuth and elevation angles.

Furthermore, many antenna arrays suffer from the mutual coupling effects, and these detrimental effects intensify as the space between elements is decreased. As

CHAPTER 5. ICOSAHEDRON-BASED SPHERICAL ANTENNA ARRAY FOR WIDEBAND PHASE MODE PROCESSING

mentioned before, the effects of mutual coupling are so severe near the poles of the array using the equiangular grid that the radiation pattern would be strongly affected. But for the icosahedron-based topology, the elements distribution on the sphere are almost uniform which can reduce the effects of mutual coupling. To take into account the effect of mutual coupling, the embedded element pattern should be used, which can be calculated by the method presented in [116].

5.4 Conclusion

An improved spherical antenna array has been presented in this chapter. Our results show that this icosahedron-based topology can be used for phase mode processing. To overcome the limitations of rapid variations in the amplitude of the far-field mode, the raised cosine pattern elements are used in this array, which enables us to synthesize a wideband pattern without moving nulls. The distortion due to the finite inter-element spacing has been analyzed as it is useful for the design of spherical antenna arrays. And a number of attractive properties for applications are discussed such as electric beam scanning in the entire three-dimensional space and reducing the effects of mutual coupling.

Chapter 6

Spheroidal Phase Mode Processing for Antenna Arrays

6.1 Introduction

In Chapter 5, an improved topology, based on an icosahedron, of spherical antenna arrays was presented, in which the concept of spherical phase modes was used. When a spherical phase mode excites a spherical antenna array, the far-field radiation pattern would have the same spherical phase mode form [1]. This feature has a number of attractive applications, such as broadband pattern synthesis, null steering, direction finding, and superresolution for spherical antenna arrays.

As we know, the spherical antenna array is a special case of the spheroidal antenna array which has more general and flexible properties. And the spheroidal wave functions are special functions in mathematical physics which have many important properties and practical applications in science and engineering where the prolate or the oblate spheroidal coordinate system is used [15]. In this chapter, the spherical phase mode concept is extended to both prolate spheroidal and oblate spheroidal geometries using spheroidal wave functions. Theoretical and numerical

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

results show that a spheroidal phase mode of the excitation function produces a far field radiation pattern with the same spheroidal phase mode form, and the elevation angle of that pattern increases (decreases) with the ratio of the interfocal distance of the prolate (oblate) spheroid antenna array to the wavenumber.

This chapter is organized as follows. In Section 6.2, the theoretical formulations are presented. The numerical results and discussion are shown in Section 6.3. Finally, the conclusion of the work of this chapter is given in Section 6.4.

6.2 Spheroidal Phase Mode Processing

The prolate and oblate spheroidal coordinate systems are formed by rotating the two-dimensional elliptic coordinate system, consisting of confocal ellipses and hyperbolas, about the major and minor axes of the ellipses, respectively [14]. Figure 6.1 shows a prolate spheroidal coordinate system, described by (η, ξ, ϕ) with the interfocal distance d , where η , ξ , ϕ are respectively the prolate spheroidal angular, radial, and azimuthal coordinates. The surface of the prolate spheroid is given by

$$\xi = \xi_o = 2a/d \quad (6.1)$$

where a is the semi-major axis of the prolate spheroid.

Figure 6.1: Spheroidal array geometry (d is the interfocal distance between two foci P and Q).

In the prolate spheroidal coordinates, the scalar wave equation (3.2) can be

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

solved using the separation of scalar variables, which leads to three independent functions as shown in (3.3): (1) the prolate spheroidal radial function of the i th kind $R_{mn}^{(i)}(h; \xi)$, (2) the prolate spheroidal angular function of the first kind $S_{mn}^{(1)}(h; \eta)$, (3) sine and cosine functions or exponential harmonic functions $e^{jm\phi}$. In this chapter, the exponential harmonic functions $e^{jm\phi}$ are employed for convenience.

Consider an excitation function $E(\xi = \xi_o, \eta, \phi)$ on the surface of a prolate spheroid as shown in Figure 6.1 and define the prolate spheroidal phase mode $Z_n^m(h; \eta, \phi)$ which is analogous to the spherical counterparts as

$$Z_n^m(h; \eta, \phi) = \frac{1}{\sqrt{2\pi N_{mn}(h)}} S_{mn}^{(1)}(h; \eta) e^{jm\phi} \quad (6.2)$$

In (6.2), $S_{mn}^{(1)}(h; \eta)$ represents the amplitude of the phase mode, and $N_{mn}(h)$ is the normalization factor so that $S_{mn}^{(1)}(h; \eta)$ can be reduced exactly to the corresponding associated Legendre function when h approaches zero [14].

The far-field radiation pattern arising from one exciting prolate spheroidal phase mode can be expressed by

$$D(\theta_i, \phi_i) = \frac{1}{4\pi} \iint_S Z_{n'}^{m'}(h; \eta, \phi) e^{jkr \cos \psi} ds \quad (6.3)$$

where r is the distance between the origin to each excitation point on the spheroid surface, and ψ is the angle between the direction of propagation of the plane wave and the direction of $\mathbf{r}$ as shown in Figure 6.1. The solid angle is given by

$$ds = \frac{\hat{\mathbf{n}} \cdot d\mathbf{a}}{r^2} = \frac{\xi(\xi^2 - 1)}{(\xi^2 + \eta^2 - 1)^{\frac{3}{2}}} d\eta d\phi = d \cos \theta d\phi \quad (6.4)$$

where $\hat{\mathbf{n}}$ is a unit vector from the origin, $d\mathbf{a}$ is the differential area of a spheroidal surface patch, and θ is the elevation angle of r as shown in Figure 6.1. Therefore,

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

(6.3) can be rewritten as

$$D(\theta_i, \phi_i) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi Z_{n'}^{m'}(h; \cos \theta, \phi) e^{jkr \cos \psi} \sin \theta d\theta d\phi \quad (6.5)$$

A plane wave propagating along the direction (θ_i, ϕ_i) can be expanded in terms of spheroidal wave functions [32] as

$$\begin{aligned} e^{jkr \cos \psi} = & 2 \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{j^n}{N_{mn}(h)} S_{mn}^{(1)}(h; \cos \theta_i) \\ & \cdot S_{mn}^{(1)}(h; \cos \theta) R_{mn}^{(1)}(h; \xi_o) e^{jm(\phi_i - \phi)} \end{aligned} \quad (6.6)$$

Substituting (6.6) into (6.5) gives

$$\begin{aligned} D(\theta_i, \phi_i) = & \frac{1}{2\pi \sqrt{2\pi} N_{m'n'}(h)} \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{j^n}{N_{mn}(h)} S_{mn}^{(1)}(h; \cos \theta_i) R_{mn}^{(1)}(h; \xi_o) e^{jm\phi_i} \\ & \cdot \int_0^{2\pi} \int_0^\pi S_{mn}^{(1)}(h; \cos \theta) S_{m'n'}^{(1)}(h; \cos \theta) e^{j(m'-m)\phi} \sin \theta d\theta d\phi \end{aligned} \quad (6.7)$$

Notice that the prolate spheroidal angular functions $S_{mn}^{(1)}(h; \eta)$ form an orthogonal set on the interval $(-1, 1)$ [14],

$$\int_{-1}^1 S_{mn}^{(1)}(h; \eta) S_{m'n'}^{(1)}(h; \eta) d\eta = \delta_{nn'} N_{mn} \quad (6.8)$$

and

$$\int_0^{2\pi} e^{jm\phi} d\phi = 2\pi \delta_{0m} \quad (6.9)$$

Therefore, using the property of orthogonality of both $S_{mn}^{(1)}(h; \eta)$ and exponential harmonic functions over the prolate spheroid surface, the far-field radiation pattern is obtained as

$$D(\theta_i, \phi_i) = j^{n'} Z_{n'}^{m'}(h; \cos \theta_i, \phi_i) R_{m'n'}^{(1)}(h; \xi_o) \quad (6.10)$$

(6.10) shows that when a spheroidal antenna array is excited by a spheroidal phase mode $Z_{n'}^{m'}(h; \eta, \phi)$, the far-field radiation pattern has the same spheroidal phase

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

mode form $Z_{n'}^{m'}(h; \cos \theta_i, \phi_i)$ with a constant amplitude given by a spheroidal radial function coefficient, which shows the same characteristics as the spherical phase mode $Y_n^m(\theta, \phi)$ given in [1]. Additionally, it is worth noting that the far-field radiation pattern also depends on $h = \frac{1}{2}kd$ or d/λ , which can control the elevation angle of the pattern. The details will be discussed in the next section.

For the oblate spheroidal geometry, the oblate spheroidal phase mode can be deduced from the prolate one by using the relations $\xi \rightarrow j\xi$ and $h \rightarrow -jh$. Then the corresponding far-field radiation pattern and similar characteristics as those of the prolate spheroidal geometry can be obtained.

In the limiting case when the interfocal distance vanishes ($d \rightarrow 0$), both the prolate and oblate spheroidal coordinates (ξ, η, ϕ) reduce to the spherical coordinates (r, θ, ϕ) , and the spheroidal wave functions reduce to the corresponding counterparts in the spherical coordinate system as shown below.

$$S_{mn}^{(1)}(h; \eta) \rightarrow P_n^m(\cos \theta) \quad (6.11)$$

$$N_{mn}(h) \rightarrow \frac{2}{(2n+1)} \frac{(n+m)!}{(n-m)!} \quad (6.12)$$

$$R_{mn}^{(1)}(h; \xi) \rightarrow j_n(kr) \quad (6.13)$$

P_n^m are the associated Legendre functions, j_n is the spherical Bessel function of the first kind. The prolate spheroidal phase mode, (6.2), becomes

$$Y_n^m(\theta, \phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} P_n^m(\cos \theta) e^{jm\phi} \quad (6.14)$$

and the far-field radiation pattern reduces to

$$D(\theta_i, \phi_i) = j^{n'} j_{n'}(kr) Y_{n'}^{m'}(\theta_i, \phi_i) \quad (6.15)$$

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

(6.14) and (6.15) are respectively the spherical phase mode and its relative far-field radiation pattern given in [1].

6.3 Results and Discussion

Figure 6.2 (a) and (b), which are calculated by (6.10), show the far-field radiation patterns excited by prolate and oblate spheroidal phase modes Z_3^2 , respectively. The phase is represented on a gray scale in the figures of this section. For comparison with the spherical counterpart in [1], the semi-major axis is chosen to be the same as the radius of the sphere, $a/\lambda = 3.33$, the interfocal distance $d/\lambda \rightarrow 0$. It should be noted that a white-to-black transition is continuous since we use white to present 0° and black to present 360° to compare with the result presented in [1]. As expected, the far-field radiation patterns approach the spherical counterpart in both amplitude and phase.

Figures 6.3–6.5 show the far-field radiation patterns for prolate and oblate cases with $d/\lambda = 0.5, 4$ and $2a/\lambda$ or 6.66 while other variables are kept constant.

Compared the far-field radiation patterns in Figure 6.3 ($d/\lambda = 0.5$) with the pattern corresponding to the spherical harmonic Y_3^2 in [1], as shown in Figure 5.5, it is seen that the radiation pattern has the same variation in phase as its spherical counterpart. But the elevation angle of the far-field beam for the prolate (oblate) spheroidal phase mode is larger (smaller) than that for the spherical phase mode. Our results show that in general, the difference in elevation angle of the far-field beam is less than 10% for $d/\lambda < 1$.

As $d/\lambda > 1$, the difference of the elevation angle becomes much larger. For example, if $d/\lambda = 4$, the elevation angle of the far-field beam has 34% difference for the prolate case and 56% difference for the oblate case as shown in Figure 6.4 (a)

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

(a) Prolate

(b) Oblate

Figure 6.2: Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda \rightarrow 0$ and the semi-major axis $a/\lambda = 3.33$.

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

(a) Prolate

(b) Oblate

Figure 6.3: Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda = 0.5$ and the semi-major axis $a/\lambda = 3.33$.

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

(a) Prolate

(b) Oblate

Figure 6.4: Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda = 4$ and the semi-major axis $a/\lambda = 3.33$.

(a) Prolate

(b) Oblate

Figure 6.5: Far-field radiation pattern of an array excited by spheroidal phase mode Z_3^2 , with the interfocal distance $d/\lambda = 6.66$ and the semi-major axis $a/\lambda = 3.33$.

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

and Figure 6.4 (b), respectively. And it is also found that there is a 90° phase difference for the prolate case as shown in Figure 6.4 (a). This is because d/λ is one of variables of the spheroidal radial functions which control the amplitude of the radiation pattern. When d/λ increases, it leads to a sign change of the prolate spheroidal radial functions.

In the limiting case, when d/λ approaches $2a/\lambda$, the prolate spheroid becomes a straight line between the two foci, and the oblate spheroid becomes a circular dish. The radiation patterns are shown in Figure 6.5. It is found that the differences of the elevation angle are about 41% and 67% for prolate and oblate cases, respectively.

Figure 6.6: The elevation angle of the far-field beam as a function of d/λ .

These figures demonstrate that the elevation angle varies with d/λ . This is because d/λ is a variable of the spheroidal angular functions which determine the pattern in the angular coordinate. Figure 6.6 shows how the elevation angle of the far-field beam varies with d/λ ranging from 0 to $2a/\lambda$. In this figure, it is seen

CHAPTER 6. SPHEROIDAL PHASE MODE PROCESSING FOR ANTENNA ARRAYS

that the elevation angle increases with d/λ from about 55° to 77° for the prolate case and decreases from about 55° to 18° for the oblate case.

6.4 Conclusion

In this chapter, the theoretical solutions and consequent numerical results for the prolate and oblate spheroidal phase mode excitations and their corresponding far-field radiation patterns have been presented in terms of spheroidal wave functions. It is shown that a spheroidal phase mode as excitation function leads to the same spheroidal phase mode in the far-field radiation pattern, just as the spherical phase mode. And it is found that the elevation angle of the far-field radiation pattern increases (decreases) with d/λ for prolate (oblate) spheroidal antenna arrays. The generality of the spheroidal geometry and the flexibility of controlling the far-field patterns are attractive for a number of applications, such as broadband beamforming and direction finding, etc.

Chapter 7

Conclusions and Recommendations

7.1 Conclusions

Antennas and antenna arrays play very important roles in wireless communications. And the objective of this thesis is to create some challenging opportunities and solve problems in antenna engineering. Our research activities have been focused on integration of metamaterials (MTMs) into spheroidal scatterers and antennas, and phase mode processing for spherical and spheroidal antenna arrays.

It is found that the scattering characteristics by a conducting prolate spheroid coated with MTMs exhibits different characteristics compared with that coated with a conventional material, and these features can be used for increasing, decreasing, and smoothing the scattering cross section, and even cancel the nulls of the scattering cross section. For instance, we found that with DNG MTMs coatings, it is possible to dramatically enhance the total SCS to achieve “compact resonant structures” by choosing appropriate thicknesses of the coatings due to the interface resonance when the complementary materials are paired together. It

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

is also possible to drastically reduce the total SCS to achieve “transparency” of objects by using low-permittivity MTMs coatings. This is because the total SCSs of electrically small objects are generally dominated by the dipolar term in the multipole expansion. This dipolar scattering may vanish, or the “transparency” condition can be achieved by choosing appropriate values of the thicknesses of the coatings.

The radiation problems of spheroidal antennas coated with DNG MTMs have been investigated after studying the scattering properties of MTMs coatings. Our results show that the radiation pattern of the prolate spheroidal antenna coated with a half-wavelength confocal DNG MTMs radome is almost the same as that of the uncoated antenna, which is different from the properties of a conventional dielectric radome. In other words, the DNG MTMs radome can be electrically transparent to the antenna and has almost no effect on the antenna’s radiation pattern. Furthermore, this DNG MTMs radome can be placed much closer to the antenna than a conventional dielectric one without loss of performance, which means the size of the antenna system can be significantly reduced. It is also shown that the electrically small spheroidal antenna coated with a confocal DNG shell has higher efficiency compared with the normal electrically small antenna and has much larger fractional bandwidth (FBW) than that given by the Chu limit which is the maximum FBW limit for normal electrically small antennas.

In Chapter 5, an improved topology of spherical antenna arrays for phase mode processing is proposed. In this icosahedron-based topology, the inter-element spacing of all antenna elements is almost identical. This attractive property can be

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

used for three-dimensional beam scanning and for reducing the effects of mutual coupling. To overcome the limitations of rapid variations in the amplitude of the far-field mode, raised cosine pattern elements are used in this array, which enables us to synthesize a wideband pattern without moving nulls. The distortion due to the finite inter-element spacing has been analyzed as it is useful to the design of a spherical antenna array. A number of attractive properties for applications are discussed such as electric beam scanning in the entire three-dimensional space and reducing the effects of mutual coupling.

Finally, spherical phase mode processing is extended to both prolate spheroidal and oblate spheroidal phase mode processing. It is shown that a spheroidal phase mode of the excitation function produces a far-field radiation pattern with the same spheroidal phase mode form, and the elevation angle of that pattern increases (decreases) with the ratio of the interfocal distance of the prolate (oblate) spheroid antenna array to the wavenumber. The generality of the spheroidal geometry and the flexibility of controlling the far-field patterns are attractive for a number of applications, such as broadband beamforming and direction finding, etc.

7.2 Recommendations for Future Work

Although an intensive study on spheroidal scatterers and spheroidal antennas coated with MTMs, and antenna arrays using phase mode processing has been carried out in this thesis, there still exists some issues that would be interesting to investigate. In this section, we present a number of possible extensions of current work.

- (i) Although the scattering properties of conducting spheroids coated with MTMs for an axial incidence have been studied, the EM scattering characteristics of dielectric spheroids coated with MTMs with arbitrary polarization and angle of incidence can provide more general information. It would be interesting in more detail to investigate the conditions of achieving “compact resonant” spheroids and “transparency” of spheroids using MTMs coatings, for instance, taking the dispersive and lossy effects of MTMs into account.
- (ii) The spheroidal antennas considered in this thesis are excited by a delta voltage across an infinitesimally narrow gap. Therefore, it will be an improvement if a finite gap width is taken into account. And the impedance matching of the antenna with the MTMs coatings can be further investigated. Since the dispersion and losses of MTMs may reduce the efficiency of the antenna system, it is necessary to study these effects in future work. Since mutual coupling affects the performance of antenna arrays, it will be a good opportunity to investigate the mutual coupling between the spheroidal antennas with MTMs coatings in order to compensate for these effects.

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

- (iii) In order to further develop the icosahedron-based spherical antenna arrays presented in this thesis, the polarization, mutual coupling effects, and the feed systems can be studied more detailed in future work.
- (iv) Spheroidal phase mode processing has been investigated theoretically in this thesis. How to sample the spheroidal geometry and to implement this kind of array is a challenging opportunity and a difficult problem to be solved.

Author's Publications

Journal Paper

1. M. D. Huang and S. Y. Tan, "Spheroidal phase mode processing for antenna arrays," *J. Electromag. Waves and Appl.*, USA, vol. 19, pp. 1431–1442, 2005.
2. M. D. Huang and S. Y. Tan, "An improved spherical antenna arrays for wideband phase mode processing," *Progress in Electromagnetics Research*, USA, PIER 66, pp. 27–40, 2006.
3. M. D. Huang and S. Y. Tan, "Achieving sub-wavelength resonant structures and transparency of a conducting spheroid with metamaterials coatings," *Waves in Random and Complex Media*, vol. 18, no. 1, pp. 161–171, 2008.
4. M. D. Huang and S. Y. Tan, "Efficient electrically small prolate spheroidal antennas with a shell of double-negative metamaterials," submitted to *IEEE Trans. Antennas Propagat.*, 2006.
5. M. D. Huang and S. Y. Tan, "Radiation of a prolate spheroidal antenna coated with a double-negative metamaterials radome," submitted to *Waves in Random and Complex Media*, 2007.

AUTHOR'S PUBLICATIONS

Conference Paper

1. M. D. Huang and S. Y. Tan, "Spheroidal phase mode processing for antenna arrays," in *IEEE International Workshop on Antenna Technology: Small antennas and Novel Metamaterials*, Singapore, pp. 359-362, Mar. 2005.
2. M. D. Huang and S. Y. Tan, "Spherical antenna arrays for phase mode processing," in *IEEE International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications*, Beijing, China, pp. 387-390, Aug. 2005.

Bibliography

- [1] E. De Witte, H. D. Griffiths, and P. V. Brennan, “Phase mode processing for spherical antenna arrays,” *Electron. Lett.*, vol. 39, no. 20, pp. 1430–1431, Oct. 2003.
- [2] A. D. Olver, “Trends in antenna design over 100 years,” in *IEE Int. Conf. on 100 Years of Radio*, Sep. 1995, pp. 83–88.
- [3] R. Prasad, “Overview of wireless personal communications: Microwave perspectives,” *IEEE Communications Magazine*, vol. 35, no. 4, pp. 104–108, Apr. 1997.
- [4] G. C. Corazza, “Marconi’s history,” in *Proc. IEEE*, July 1998, vol. 86, pp. 1307–1311.
- [5] C. A. Balanis, *Antenna Theory: Analysis and Design*, John Wiley & Sons, New York, third edition, 2005.
- [6] N. Engheta and R. W. Ziolkowski, “A positive future for double-negative metamaterials,” *IEEE Trans. Antennas Propag.*, vol. 53, no. 4, pp. 1535–1556, Apr. 2005.
- [7] A. Alù and N. Engheta, “Polarizabilities and effective parameters for collections of spherical nanoparticles formed by pairs of concentric double-negative, single-negative, and/or double-positive metamaterial layers,” *J. Appl. Phys.*, vol. 97, pp. 094310, Apr. 2005.

BIBLIOGRAPHY

- [8] A. Alù and N. Engheta, “Achieving transparency with plasmonic and meta-material coatings,” *Phys. Rev. E*, vol. 72, pp. 016623, July 2005.
- [9] N. Engheta, “An idea for thin subwavelength cavity resonators using meta-materials with negative permittivity and permeability,” *IEEE Antennas Wireless Propag. Lett.*, vol. 1, no. 1, pp. 10–13, 2002.
- [10] R. W. Ziolkowski and A. D. Kipple, “Application of double negative materials to increase the power radiated by electrically small antennas,” *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2626–2640, Oct. 2003.
- [11] R. W. Ziolkowski and A. Erentok, “Metamaterial-based efficient electrically small antennas,” *IEEE Trans. Antennas Propag.*, vol. 54, no. 7, pp. 2113–2130, July 2006.
- [12] D. E. N. Davies, *The Handbook of Antenna Design*, chapter 12, Peter Peregrinus, 1983.
- [13] W. J. Thompson, “Spheroidal wave functions,” *Comput. in Sci. Eng.*, vol. 13, pp. 84–87, May 1999.
- [14] C. Flammer, *Spheroidal Wave Functions*, Stanford University Press, Stanford, 1957.
- [15] L. W. Li, X. K. Kang, and M. S. Leong, *Spheroidal Wave Functions in Electromagnetic Theory*, John Wiley & Sons, New York, 2001.
- [16] D. R. Smith and N. Kroll, “Negative refractive index in left-handed materials,” *Phys. Rev. Lett.*, vol. 85, no. 14, pp. 2933–2936, Oct. 2000.
- [17] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Washington, D.C., 1972.

BIBLIOGRAPHY

- [18] P. Moon and D. E. Spencer, *Field Theory Handbook*, Springer-Verlag, Berlin, third edition, 1988.
- [19] J. A. Stratton, P. M. Morse, L. J. Chu, J. D. C. Little, and F. J. Corbato, *Spheroidal Wave Functions*, John Wiley & Sons, New York, 1956.
- [20] B. P. Sinha and R. H. MacPhie, “Electromagnetic scattering by prolate spheroids for plane waves with arbitrary polarization and angle of incidence,” *Radio Sci.*, vol. 12, no. 2, pp. 171–184, 1977.
- [21] B. P. Sinha and R. H. MacPhie, “Electromagnetic plane wave scattering by a system of two parallel conducting spheroids,” *IEEE Trans. Antennas Propag.*, vol. 31, no. 2, pp. 294–304, Mar. 1983.
- [22] M. F. R. Cooray and I. R. Ciric, “Electromagnetic wave scattering by a system of two spheroids of arbitrary orientation,” *IEEE Trans. Antennas Propag.*, vol. 37, no. 5, pp. 608–618, May 1989.
- [23] T. Do-Nhat and R. H. MacPhie, “Accurate values of prolate spheroidal radial functions of the second kind,” *Can. J. Phys.*, vol. 75, pp. 671–675, 1997.
- [24] S. J. Zhang and J. M. Jin, *Computation of Special Functions*, John Wiley & Sons, New York, 1996.
- [25] L. W. Li, T. S. Yeo, P. S. Kooi, M. S. Leong, and K. Y. Tan, “Exact computations of spheroidal harmonics with complex argument: A comparative study with an algorithm,” *Phys. Rev. E*, vol. 58, no. 5, pp. 6792–6806, 1998.
- [26] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press, New York, second edition, 1992.

BIBLIOGRAPHY

- [27] L. Baker, *C Mathematical Function Handbook*, McGraw-Hill, New York, 1992.
- [28] D. L. Moffatt and E. M. Kennaugh, "The axial echo area of a perfectly conducting prolate spheroid," *IEEE Trans. Antennas Propag.*, vol. 13, no. 3, pp. 401–409, May 1965.
- [29] D. L. Moffatt, "The echo area of a perfectly conducting prolate spheroid," *IEEE Trans. Antennas Propag.*, vol. 17, no. 3, pp. 299–307, May 1969.
- [30] S. Asano and G. Yamamoto, "Light scattering by a spheroidal particle," *Appl. Opt.*, vol. 14, no. 1, pp. 29–49, Jan. 1975.
- [31] S. Asano, "Light scattering properties of spheroidal particles," *Appl. Opt.*, vol. 18, no. 5, pp. 712–723, Mar. 1979.
- [32] B. P. Sinha and R. H. MacPhie, "Translational addition theorems for spheroidal scalar and vector wave functions," *Quart. Appl. Math.*, vol. 38, no. 2, pp. 143–158, July 1980.
- [33] M. F. R. Cooray and I. R. Ciric, "Scattering of electromagnetic waves by a system of two dielectric spheroids of arbitrary orientation," *IEEE Trans. Antennas Propag.*, vol. 39, no. 5, pp. 680–684, May 1991.
- [34] S. Nag and B. P. Sinha, "Electromagnetic plane wave scattering by a system of two uniformly lossy dielectric prolate spheroids in arbitrary orientation," *IEEE Trans. Antennas Propag.*, vol. 43, no. 3, pp. 322–327, Mar. 1995.
- [35] M. F. R. Cooray and I. R. Ciric, "Scattering by systems of spheroids in arbitrary configurations," *Comp. Phys. Commun.*, vol. 68, pp. 279–305, 1991.

BIBLIOGRAPHY

- [36] M. F. R. Cooray and I. R. Ciric, "Scattering of electromagnetic waves by a coated dielectric spheroid," *J. Electromagn. Waves Appl.*, vol. 6, pp. 1491–1507, 1992.
- [37] A. B. Sebak and B. P. Sinha, "Scattering by a conducting spheroidal object with dielectric coating at axial incidence," *IEEE Trans. Antennas Propag.*, vol. 40, no. 3, pp. 268–274, Mar. 1992.
- [38] J. Dalmas and R. Deleuil, "Multiple scattering of electromagnetic waves from two infinitely conducting prolate spheroids which are centered in a plane perpendicular to their axes of revolution," *Radio Sci.*, vol. 20, no. 3, pp. 575–581, May 1985.
- [39] J. Dalmas and R. Deleuil, "Translational addition theorems for prolate spheroidal vector wavefunctions m' and n' ," *Q. Appl. Math.*, vol. 44, pp. 213–222, 1986.
- [40] B. L. Merchant, P. J. Moser, A. Nagl, and H. Uberall, "Complex pole patterns of the scattering amplitude for conducting spheroids and finite-length cylinders," *IEEE Trans. Antennas Propag.*, vol. 36, no. 12, pp. 1769–1778, 1988.
- [41] R. H. MacPhie, J. Dalmas, and R. Deleuil, "Rotational-translational addition theorems for scalar spheroidal wave functions," *Q. Appl. Math.*, vol. 44, no. 4, pp. 737–749, Jan. 1987.
- [42] T. Do-Nhat and R. H. MacPhie, "Quasi-translational addition expressions of prolate scalar wave functions for thin prolate spheroids," *IEEE Trans. Antennas Propag.*, vol. 44, no. 7, pp. 960–963, July 1996.
- [43] L. Page and N. I. Adams, "The electrical oscillations of a prolate spheroid. paper I," *Phys. Rev.*, vol. 53, pp. 819–831, May 1938.

BIBLIOGRAPHY

- [44] L. Page, "The electrical oscillations of a prolate spheroid. paper II-III," *Phys. Rev.*, vol. 65, pp. 98–117, Feb. 1944.
- [45] R. M. Ryder, "The electrical oscillations of a perfectly conducting prolate spheroid," *J. Appl. Phys.*, vol. 13, no. 5, pp. 327–343, May 1942.
- [46] J. R. Wait, "Theories of prolate spheroidal antennas," *Radio Sci.*, vol. 1, no. 4, pp. 475–511, Apr. 1966.
- [47] L. J. Chu and J. A. Stratton, "Steady-state solutions of electromagnetic field problems: III. forced oscillations of a prolate spheroid," *J. Appl. Phys.*, vol. 12, pp. 241–248, Mar. 1941.
- [48] S. A. Schelkunoff, *Advanced Antenna Theory*, Wiley, New York, 1952.
- [49] C. Flammer, "The prolate spheroidal monopole antenna," *Tech. Rep. 22*, Contract AF 19(604)-1296, 1954.
- [50] C. P. Wells, "The prolate spheroidal antenna: Current and impedance," *IRE Trans. Antennas Propag.*, vol. 6, no. 1, pp. 125–128, Jan. 1958.
- [51] W. L. Weeks, *Electromagnetic Theory for Engineering Applications*, McGraw-Hill, 1964.
- [52] R. J. Lytle and F. V. Schultz, "Prolate spheroidal antennas in isotropic plasma media," *IEEE Trans. Antennas Propag.*, vol. 17, no. 4, pp. 496–506, July 1969.
- [53] S. Sato and Y. Naito, "Prolate spheroidal antenna covered with ferrite sleeve," in *Antennas and Propagation Society International Symposium Digest*, June 1974, vol. 12, pp. 64–67.

BIBLIOGRAPHY

- [54] J. D. Kotulski, *Transient Radiation from Antenna: Early Time Response of the Spherical Antenna and the Late Time Response of the Prolate Spheroidal Impedance Antenna*, Ph.D. dissertation, University of Illinois, 1983.
- [55] L. Jen and C. S. Hu, "Spheroidal wave functions of large frequency parameters $c = kf$ and the radiation fields of a metallic prolate spheroid excited by any circumferential slot," *IEEE Trans. Antennas Propag.*, vol. 31, no. 2, pp. 382–389, Mar. 1983.
- [56] C. S. Hu, "Prolate spheroidal wave functions of large frequency parameters $c = kf$ and their applications in electromagnetic theory," *IEEE Trans. Antennas Propag.*, vol. 34, no. 1, pp. 114–119, Jan. 1986.
- [57] B. P. Sinha and R. H. MacPhie, "Mutual admittance characteristics for two-element parallel prolate spheroidal antenna systems," *IEEE Trans. Antennas Propag.*, vol. 33, no. 11, pp. 1255–1263, Nov. 1985.
- [58] I. R. Ciric and M. F. R. Cooray, "Admittance characteristics and far-field patterns for coupled spheroidal dipole antennas in arbitrary configuration," *IEE Proc., Pt. H: Microwaves Antennas Propag.*, vol. 137, no. 6, pp. 337–342, Dec. 1990.
- [59] T. Do-Nhat and R. H. MacPhie, "The input admittance of thin prolate spheroidal dipole antennas with finite gap widths," *IEEE Trans. Antennas Propag.*, vol. 43, no. 11, pp. 1243–1252, Nov. 1995.
- [60] E. D. Vinogradova, "Analysis of radiation of a surface spheroidal antenna," in *Antennas and Propagation Society International Symposium Digest*, 1996, vol. 3, pp. 1844–1847.
- [61] M. Zhang and A. A. Sebak, "Radiation characteristics of a slot antenna on a conducting prolate spheroid," *Can. J. Phys.*, vol. 73, pp. 376–385, 1995.

BIBLIOGRAPHY

- [62] M. Taguchi, S. Nonaka, and K. Tanaka, "Analysis of oblate spheroidal antenna," in *Antennas and Propagation Society International Symposium Digest*, 1993, vol. 2, pp. 968–971.
- [63] D. Yagyu, K. Tanaka, T. Fujimoto, and M. Taguchi, "Analysis of rotationally symmetrical antenna with wider beamwidth," in *Antennas and Propagation Society International Symposium Digest*, 1998, vol. 1, pp. 324–327.
- [64] L. W. Li, M. S. Leong, T. S. Yeo, and Y. B. Gan, "Electromagnetic radiation from a prolate spheroidal antenna enclosed in a confocal spheroidal radome," *IEEE Trans. Antennas Propag.*, vol. 50, no. 11, pp. 1525–1533, Nov. 2002.
- [65] C. A. Macon, L. C. Kempel, S. W. Schneider, and K. D. Trott, "Modeling conformal antennas on metallic prolate spheroid surfaces using a hybrid finite element method," *IEEE Trans. Antennas Propag.*, vol. 52, no. 3, pp. 750–758, Mar. 2004.
- [66] I. R. Capoglu and G. S. Smith, "The input admittance of a prolate-spheroidal monopole antenna fed by a magnetic frill," *IEEE Trans. Antennas Propag.*, vol. 54, no. 2, pp. 572–585, Feb. 2006.
- [67] V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of ϵ and μ ," *Sov. Phys.—Usp.*, vol. 47, no. 4, pp. 509–514, 1968.
- [68] R. A. Shelby, D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, vol. 292, no. 5514, pp. 77–79, Apr. 2001.
- [69] J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, "Extremely low frequency plasmons in metallic mesostructure," *Phys. Rev. Lett.*, vol. 76, no. 25, pp. 4773–4776, June 1996.

BIBLIOGRAPHY

- [70] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 11, pp. 2075–2081, Nov. 1999.
- [71] J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett.*, vol. 85, no. 18, pp. 3966–3969, Oct. 2000.
- [72] N. Garcia and M. Nieto-Vesperinas, "Left-handed materials do not make a perfect lens," *Phys. Rev. Lett.*, vol. 88, no. 20, pp. 207403, May 2002.
- [73] J. B. Pendry, "Comment on 'left-handed materials do not make a perfect lens'," *Phys. Rev. Lett.*, vol. 91, no. 9, pp. 099701, Aug. 2003.
- [74] R. W. Ziolkowski and E. Heyman, "Wave propagation in media having negative permittivity and permeability," *Phys. Rev. E.*, vol. 64, no. 5, pp. 056625, Oct. 2001.
- [75] R. W. Ziolkowski, "Design, fabrication, and testing of double negative metamaterials," *IEEE Trans. Antennas Propag.*, vol. 51, no. 7, pp. 1516–1529, July 2003.
- [76] I. V. Lindell, S. A. Tretyakov, K. I. Nikoskinen, and S. Ilvonen, "BW media: media with negative parameters, capable of supporting backward waves," *Microwave Opt. Technol. Lett.*, vol. 31, no. 2, pp. 129–133, Oct. 2001.
- [77] P. M. Valanju, R. M. Walter, and A. P. Valanju, "Wave refraction in negative-index media: Always positive and very inhomogeneous," *Phys. Rev. Lett.*, vol. 88, no. 18, pp. 187401, May 2002.
- [78] D. R. Fredkin and A. Ron, "Effective left-handed (negative index) composite material," *Appl. Phys. Lett.*, vol. 81, no. 10, pp. 1753–1755, Sept. 2002.

BIBLIOGRAPHY

- [79] A. Alù and N. Engheta, “Pairing an epsilon-negative slab with a mu-negative slab: Resonance, tunneling and transparency,” *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, pp. 2558–2571, Oct. 2003.
- [80] S. Longhi, “Gap solitons in metamaterials,” *Waves Random Complex Media*, vol. 15, no. 1, pp. 119–126, Feb. 2006.
- [81] “Special issue on metamaterials,” *IEEE Trans. Antennas Propag.*, vol. 51, no. 10, Oct. 2003.
- [82] A. Grbic and G. V. Eleftheriades, “Overcoming the diffraction limit with a planar left-handed transmission line lens,” *Phys. Rev. Lett.*, vol. 92, no. 11, pp. 117403, Mar. 2004.
- [83] L. J. Chu, “Physical limitations of omnidirectional antennas,” *J. Appl. Phys.*, vol. 19, no. 12, pp. 1163–1175, Dec. 1948.
- [84] H. A. Wheeler, “Fundamental limitations of small antennas,” *IRE Proc.*, vol. 35, no. 12, pp. 1479–1484, Dec. 1947.
- [85] H. A. Wheeler, “Small antennas,” *IEEE Trans. Antennas Propag.*, vol. 23, no. 4, pp. 462–469, July 1975.
- [86] R. P. Harrington, *Time Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961.
- [87] R. E. Collin and S. Rothschild, “Evaluation of antenna Q,” *IEEE Trans. Antennas Propag.*, vol. 12, no. 1, pp. 23–27, Jan. 1964.
- [88] R. L. Fante, “Quality factor of general ideal antennas,” *IEEE Trans. Antennas Propag.*, vol. 17, no. 3, pp. 151–155, Mar. 1969.
- [89] R. C. Hansen, “Fundamental limitations in antennas,” *Proc. IEEE*, vol. 69, no. 2, pp. 170–181, Feb. 1981.

BIBLIOGRAPHY

- [90] J. S. McLean, "A re-examination of the fundamental limits on the radiation Q of electrically small antennas," *IEEE Trans. Antennas Propag.*, vol. 44, no. 5, pp. 672–676, May 1996.
- [91] S. R. Best, "A discussion on the properties of electrically small self-resonant wire antennas," *IEEE Antennas Propag. Mag.*, vol. 46, no. 6, pp. 9–22, Dec. 2004.
- [92] X. Zhou and G. Hu, "Design for electromagnetic wave transparency with metamaterials," *Phys. Rev. E.*, vol. 74, pp. 026607, Aug. 2006.
- [93] A. Alù and N. Engheta, "Guided modes in a waveguide filled with a pair of single-negative (SNG), double-negative (DNG), and/or double-positive (DPS) layers," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 1, pp. 199–210, Jan. 2004.
- [94] A. Alù and N. Engheta, *Negative Refraction Metamaterials: Fundamental Properties and Applications*, chapter 9, pp. 339–380, John Wiley & Sons, New York, 2005.
- [95] R. S. Kshetrimayum, "A brief intro to metamaterials," *IEEE Potentials*, vol. 23, no. 5, pp. 44–46, Dec. 2004 – Jan. 2005.
- [96] D. E. N. Davies, "A transformation between the phasing techniques required for linear and circular aerial arrays," *Proc. IEE*, vol. 112, no. 11, pp. 2041–2045, 1965.
- [97] T. Rahim and D. E. N. Davies, "Effect of directional elements on the directional response of circular antenna arrays," *Proc. IEE*, vol. 129, (Pt. H.), no. 1, pp. 18–22, 1982.

BIBLIOGRAPHY

- [98] M. R. Jones and H. D. Griffiths, "Broadband pattern synthesis from a circular array," in *Proc. 6th IEE Int. Conf. Antennas Propag.*, Coventry, UK, Apr. 1989, pp. 55–59.
- [99] H. D. Griffiths and R. Eiges, "Sectoral phase modes from circular antenna arrays," *Electron. Lett.*, vol. 28, no. 17, pp. 1581–1582, 1992.
- [100] R. Eiges and H. D. Griffiths, "Mode-space spatial spectral estimation for circular arrays," *IEE Proc. Radar, Sonar Navig.*, vol. 141, no. 6, pp. 300–306, 1994.
- [101] C. Li and Z. Shen, "Electromagnetic scattering by a conducting cylinder coated with metamaterials," *Progress in Electromagnetic Research*, vol. 42, pp. 91–105, 2003.
- [102] M. Pastorino, M. Raffetto, and A. Randazzo, "Interactions between electromagnetic waves and elliptically shaped metamaterials," *IEEE Antennas Wireless Propag. Lett.*, vol. 4, pp. 165–168, 2005.
- [103] A. Ishimaru, *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice Hall, Englewood Cliffs, 1991.
- [104] G. T. Ruck, D. E. Barrick, W.D. Stuart, and C. K. Krichbaum, *Radar Cross Section Handbook*, vol. I, Plenum, New York, 1970.
- [105] J. Rheinstein, "Scattering of electromagnetic waves from dielectric coated conducting spheres," *IEEE Trans. Antennas Propag.*, vol. 12, no. 3, pp. 334–340, May 1964.
- [106] JR. J. D. Walton, Ed., *Radome Engineering Handbook: Design and Principles*, Marcel Dekker, New York, 1970.

BIBLIOGRAPHY

- [107] H. D. Griffiths, N. Karavassilis, M. R. Jones, and D. E. N. Davies, "Broad-band nulls from a circular array," in *Proc. 4th IEE Int. Conf. Antennas Propag.*, Coventry, UK, Apr. 1985, pp. 304-308.
- [108] M. D. Huang and S. Y. Tan, "Spheroidal phase mode processing for antenna arrays," *J. Electromagn. Waves Appl.*, vol. 19, no. 11, pp. 1431-1442, 2005.
- [109] D. L. Sengupta, T. M. Smith, and R. W. Larson, "Radiation characteristics of a spherical array of circularly polarized elements," *IEEE Trans. Antennas Propag.*, vol. 16, no. 1, pp. 2-7, Jan. 1968.
- [110] B. Tomasic, J. Turtle, and S. Liu, "A geodesic sphere phased array for satellite control and communication," in *Proc. URSI XXVIIIth General Assembly*, Maastricht, The Netherlands, Aug. 2002, pp. 1274-1277.
- [111] Z. Sipus, N. Burum, and J. Bartolic, "Analysis of rectangular microstrip patch antennas on spherical structures," *Microwave Opt. Technol. Lett.*, vol. 36, no. 4, pp. 276-280, Feb. 2003.
- [112] N. Burum, S. Rupcic, and Z. Sipus, "Theoretical and experimental study of spherical arrays," in *Proc. 12th IEEE Mediterranean Electrotechnical Conf.*, Dubrovnik, Croatia, May 2004, vol. 2, pp. 503-506.
- [113] N. Burum, Z. Sipus, and J. Bartolic, "Mutual coupling between spherical-rectangular microstrip antennas," *Microwave Opt. Technol. Lett.*, vol. 40, no. 5, pp. 387-391, Mar. 2004.
- [114] J. Verhaevert, E. Van Lil, and A. Van de Capelle, "Uniform spherical distributions for adaptive array applications," in *IEEE Vehic. Technol. Conf.*, (*VTC'01 Spring*), Rhodes, Greece, May 2001, vol. 1, pp. 98-102.

BIBLIOGRAPHY

- [115] J. Verhaevert, E. Van Lil, and A. Van de Capelle, "Three-dimensional monopole antenna for direction of arrival determination," in *IEE Proc. Microw. Antennas Propag.*, Apr. 2004, vol. 151, pp. 121–126.
- [116] E. De Witte, P. V. Brennan, and H. D. Griffiths, "Mutual coupling analysis of a spherical array antenna," in *IASTED Int. Conf. on Antennas, Radar and Wave Propag.*, Banff, Canada, July 2005, pp. 324–329.
- [117] A. Hessel, Y. L. Liu, and J. Shmoys, "Mutual admittance between two circular apertures on large conducting sphere," *Radio Sci.*, vol. 14, no. 1, pp. 35–41, Feb. 1979.
- [118] S. Y. Tan and H. S. Tan, "Modelling and measurements of channel impulse response for an indoor wireless communication system," *IEE Proc. Microw. Antennas Propag.*, vol. 142, no. 5, pp. 405–410, Oct. 1995.
- [119] A. Gheorghiu and V. Dragomir, *Geometry of Structural Forms*, Applied Science, London, 1978.
- [120] P. K. Bondyopadhyay, "Geodesic sphere phased arrays for LEO satellite communications," in *Proc. IEEE AP-S Int. Symp.*, Salt Lake City, UT, July 2000, pp. 206–209.
- [121] M. Tegmark, "An icosahedron-based method for pixelizing the celestial sphere," *ApJ. Lett.*, vol. 470, pp. 81–84, 1996.
- [122] Boaz Rafaely, "Analysis and design of spherical microphone arrays," *IEEE Trans. Speech Audio Process.*, vol. 13, no. 1, pp. 135–143, Jan. 2005.
- [123] J. R. Driscoll and D. M. Dennis, JR., "Computing Fourier transforms and convolutions on the 2-sphere," *Adv. Appl. Math.*, vol. 15, pp. 202–250, 1994.

BIBLIOGRAPHY

- [124] George B. Arfken and Hans J. Weber, *Mathematical Methods for Physicists*, Academic, San Diego, CA, fifth edition, 2001.
- [125] L. C. Biedenharn and J. D. Louck, *Angular Momentum in Quantum Physics: Theory and Application*, Addison-Wesley, Reading, MA, 1981.

Appendix A

Matrices in (3.35)

$$Q_{N+1,n+1}^{(i)}(\xi, h, g) = -2 \left[\frac{\xi}{(\xi^2 - 1)} R_{1,n+1}^{(i)}(h, \xi) I_{29,0Nn}(h, g) + \frac{d}{d\xi} R_{1,n+1}^{(i)}(h, \xi) I_{2,0Nn}(h, g) \right] \quad (\text{A.1a})$$

$$\begin{aligned} T_{Nn}^{(i)}(\xi, h, g) = & (\xi^2 - 1)^{\frac{3}{2}} \left[(\xi^2 - 1) \frac{d^2}{d\xi^2} R_{0n}^{(i)}(h, \xi) + \xi \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) \right] I_{3,0Nn}(h, g) \\ & + (\xi^2 - 1)^{\frac{3}{2}} \left[\frac{d^2}{d\xi^2} R_{0n}^{(i)}(h, \xi) + \frac{3\xi}{(\xi^2 - 1)} \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) \right] I_{14,0Nn}(h, g) \\ & + (\xi^2 - 1)^{\frac{3}{2}} \left[\xi \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) - \frac{1}{(\xi^2 - 1)} R_{0n}^{(i)}(h, \xi) \right] I_{4,0Nn}(h, g) \\ & + (\xi^2 - 1)^{\frac{1}{2}} \left[\xi \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) + \frac{(2\xi^2 - 1)}{(\xi^2 - 1)} R_{0n}^{(i)}(h, \xi) \right] I_{15,0Nn}(h, g) \\ & - \frac{R_{0n}^{(i)}(h, \xi)}{(\xi^2 - 1)^{\frac{1}{2}}} \left[2(\xi^2 - 1) I_{4,0Nn}(h, g) + I_{15,0Nn}(h, g) \right. \\ & \left. + (\xi^2 - 1)^2 I_{18,0Nn}(h, g) \right] \end{aligned} \quad (\text{A.1b})$$

$$U_{Nn}^{(i)}(\xi, h, g) = -(\xi^2 - 1)^{\frac{1}{2}} \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) I_{1,0Nn}(h, g) \quad (\text{A.1c})$$

$$V_{N+1,n+1}^{(i)}(\xi, h, g) = 2R_{1,n+1}^{(i)}(h, \xi) I_{2,0Nn}(h, g) \quad (\text{A.1d})$$

APPENDIX A. MATRICES IN (3.35)

$$\begin{aligned}
 W_{N+1,n+1}^{(i)}(\xi, h, g) = & 2 \left\{ (\xi^2 - 1)^2 \frac{d}{d\xi} R_{1,n+1}^{(i)}(h, \xi) I_{6,0Nn}(h, g) \right. \\
 & + \left[(\xi^2 - 1) \frac{d}{d\xi} R_{1,n+1}^{(i)}(h, \xi) + 2\xi R_{1,n+1}^{(i)}(h, \xi) \right] I_{24,0Nn}(h, g) \\
 & - (\xi^2 - 1)^2 \left[\xi \frac{d^2}{d\xi^2} R_{1,n+1}^{(i)}(h, \xi) + \frac{d}{d\xi} R_{1,n+1}^{(i)}(h, \xi) \right] I_{5,0Nn}(h, g) \\
 & - \left[\xi(\xi^2 - 1) \frac{d^2}{d\xi^2} R_{1,n+1}^{(i)}(h, \xi) \right. \\
 & \quad \left. + (3\xi^2 - 1) \frac{d}{d\xi} R_{1,n+1}^{(i)}(h, \xi) \right] I_{25,0Nn}(h, g) \\
 & + \frac{\xi}{(\xi^2 - 1)} R_{1,n+1}^{(i)}(h, \xi) \left[2(\xi^2 - 1) I_{5,0Nn}(h, g) \right. \\
 & \quad \left. + I_{25,0Nn}(h, g) + (\xi^2 - 1)^2 I_{26,0Nn}(h, g) \right] \left. \right\} \quad (\text{A.1e})
 \end{aligned}$$

$$\begin{aligned}
 X_{Nn}^{(i)}(\xi, h, g) = & \xi R_{0n}^{(i)}(h, \xi) I_{4,0Nn}(h, g) \\
 & + (\xi^2 - 1) \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) I_{3,0Nn}(h, g) \quad (\text{A.1f})
 \end{aligned}$$

$$\begin{aligned}
 Y_{N+1,n+1}^{(i)}(\xi, h, g) = & 2(\xi^2 - 1)^{\frac{1}{2}} \left[R_{1,n+1}^{(i)}(h, \xi) I_{6,0Nn}(h, g) \right. \\
 & \left. - \xi \frac{d}{d\xi} R_{1,n+1}^{(i)}(h, \xi) I_{5,0Nn}(h, g) \right] \quad (\text{A.1g})
 \end{aligned}$$

$$\begin{aligned}
 Z_{Nn}^{(i)}(\xi, h, g) = & (\xi^2 - 1)^{-\frac{1}{2}} R_{0n}^{(i)}(h, \xi) I_{27,0Nn}(h, g) \\
 & + \left[(\xi^2 - 1)^{\frac{1}{2}} \frac{d^2}{d\xi^2} R_{0n}^{(i)}(h, \xi) \right. \\
 & \left. + \xi(\xi^2 - 1)^{-\frac{1}{2}} \frac{d}{d\xi} R_{0n}^{(i)}(h, \xi) \right] I_{1,0Nn}(h, g) \quad (\text{A.1h})
 \end{aligned}$$

where $i = 1, 2$, or 4 . The integrals $I_{p,0Nn}(h, g)$, $p = 1, 2, \dots, 29$ (the orthogonal properties of the spheroidal angular functions) are given in Appendix B.

Appendix B

Orthogonal Properties of the Spheroidal Angular Functions

We use Ferrer's definition of the associated Legendre functions when $-1 \leq x \leq 1$.

$$P_n^m(x) = (1-x^2)^{\frac{m}{2}} \frac{d^m P_n(x)}{dx^m}, \quad -1 \leq x \leq 1 \quad (\text{B.1})$$

$P_n^m(x)$ and $P_n^{-m}(x)$ are related by

$$P_n^{-m}(x) = (-1)^m \frac{(n-m)!}{(n+m)!} P_n^m(x) \quad (\text{B.2})$$

Therefore, using the recurrence relations of the associated Legendre functions

$$P_n^m(x) = x P_{n-1}^m(x) + (n+m-1) \sqrt{1-x^2} P_{n-1}^{m-1}(x) \quad (\text{B.3a})$$

$$P_n^m(x) = \frac{2(m-1)}{\sqrt{1-x^2}} x P_n^{m-1}(x) - (n+m-1)(n-m+2) P_n^{m-2}(x) \quad (\text{B.3b})$$

$$P_n^m(x) = \frac{1}{n-m} [(2n-1)x P_{n-1}^m(x) - (n+m-1) P_{n-2}^m(x)] \quad (\text{B.3c})$$

$$P_n^m(x) = \frac{\sqrt{1-x^2}}{2m} [P_{n-1}^{m+1}(x) + (n+m)(n+m-1) P_{n-1}^{m-1}(x)] \quad (\text{B.3d})$$

$$\sqrt{1-x^2} P_{n-1}^{m-1}(x) = \frac{1}{(2n-1)} (P_n^m(x) - P_{n-2}^m(x)) \quad (\text{B.3e})$$

$$\frac{dP_n^m(x)}{dx} = \frac{mx}{1-x^2} P_n^m(x) - \frac{(n+m)(n-m+1)}{\sqrt{1-x^2}} P_n^{m-1}(x) \quad (\text{B.3f})$$

$$\frac{dP_n^m(x)}{dx} = \frac{1}{1-x^2} [(n+m) P_{n-1}^m(x) - nx P_n^m(x)] \quad (\text{B.3g})$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

$$\frac{dP_n^m(x)}{dx} = \frac{1}{2\sqrt{1-x^2}}[P_n^{m+1}(x) - (n+m)(n-m+1)P_n^{m-1}(x)] \quad (\text{B.3h})$$

and the integrals

$$\int_{-1}^1 P_\mu^m P_\nu^m d\eta = \frac{2}{2\mu+1} \frac{(\mu+m)!}{(\mu-m)!} \delta_{\mu\nu} \quad (\text{B.4a})$$

$$\begin{aligned} \int_{-1}^1 P_\mu^{m+2} P_\nu^m d\eta &= 0, \quad \nu > \mu \\ &= -\frac{2}{(2\nu+1)} \frac{(\nu+m)!}{(\nu-m-2)!}, \quad \nu = \mu \\ &= 2(m+1) \frac{(\nu+m)!}{(\nu-m)!} [1 + (-1)^{\mu+\nu}], \quad \nu < \mu \end{aligned} \quad (\text{B.4b})$$

we can get the orthogonality properties of the spheroidal angular functions S_{mn} , i.e. the integrals $I_{p,0Nn}(h, g)$, $p = 1, 2, \dots, 29$. The resulting expressions of these integrals are eventually given in terms of the expansion coefficients d_r^{mn} of the spheroidal angular functions; d_r^{mn} are nonzero for even integers r when $(n-m)$ is even, and for odd integers r when $(n-m)$ is odd.

The first integral is immediately written as

$$\begin{aligned} I_{1,mNn} &= \int_{-1}^1 S_{m,m+n} S_{m,m+N} d\eta = \int_{-1}^1 \sum_{q=0,1}^{\infty'} \sum_{r=0,1}^{\infty'} d_q^{m,m+n} d_r^{m,m+N} P_{m+q}^m P_{m+r}^m d\eta \\ &= 2\delta_{nN} \sum_{q=0,1}^{\infty'} \frac{(2m+q)!}{(2m+2q+1)q!} (d_q^{m,m+n})^2 \end{aligned} \quad (\text{B.5})$$

where δ_{nN} is the Kronecher delta. The prime over the summation sign indicates that the summation is over only even values of q (starting at 0) when n is even, and over only odd values of q (starting at 1) when n is odd. The same convention will be observed for other integrals which follow.

The second integral is

$$\begin{aligned} I_{2,mNn} &= \int_{-1}^1 \frac{\eta}{(1-\eta^2)^{\frac{1}{2}}} S_{m+1,m+n+1} S_{m,m+N} d\eta \\ &= \int_{-1}^1 \sum_{q=0,1}^{\infty'} \sum_{r=0,1}^{\infty'} d_q^{m+1,m+n+1} d_r^{m,m+N} \frac{\eta}{(1-\eta^2)^{\frac{1}{2}}} P_{m+q+1}^{m+1} P_{m+r}^m d\eta \end{aligned} \quad (\text{B.6a})$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

By applying the recurrence relations of the associated Legendre functions (B.3) and the integrals (B.4), we get

$$\frac{\eta}{(1-\eta^2)^{\frac{1}{2}}} P_{m+q+1}^{m+1} = \frac{[P_{m+q+1}^{m+2} + (2m+q+2)(q+1)P_{m+q+1}^m]}{2(m+1)} \quad (\text{B.6b})$$

$$\begin{aligned} \int_{-1}^1 \frac{1}{2(m+1)} P_{m+q+1}^{m+2} P_{m+r}^m d\eta &= -\frac{1}{2(m+1)} \frac{2}{(2m+2q+3)} \frac{(2m+q+1)!}{(q-1)!} \delta_{q+1,r} \\ &= \frac{(2m+r)!}{r!} [1 + (-1)^{2m+q+r+1}], \quad r < q+1 \end{aligned} \quad (\text{B.6c})$$

$$\begin{aligned} \int_{-1}^1 \frac{(2m+q+2)(q+1)}{2(m+1)} P_{m+q+1}^m P_{m+r}^m d\eta \\ = \frac{(2m+q+2)(q+1)}{2(m+1)} \frac{2}{(2m+2q+3)} \frac{(2m+q+1)!}{(q+1)!} \delta_{q+1,r} \end{aligned} \quad (\text{B.6d})$$

Substituting (B.6b), (B.6c) and (B.6d) into (B.6a), results in

$$\begin{aligned} I_{2,mNn} &= 2 \sum_{q=0,1}' \frac{(q+1)}{(2m+2q+3)} \frac{(2m+q+1)!}{(q+1)!} d_q^{m+1,m+n+1} d_{q+1}^{m,m+N} \\ &\quad + 2 \sum_{r=1,0}' \sum_{v=r+1}' \frac{(2m+r)!}{r!} d_v^{m+1,m+n+1} d_r^{m,m+N}, \quad (n+N) \text{ odd} \\ &= 0, \quad (n+N) \text{ even} \end{aligned} \quad (\text{B.6e})$$

These integrals are evaluated in a similar fashion by using recurrence relations of the associated Legendre functions (B.3) and the integrals (B.4). The end results which are used in this thesis are summarized below.

$$\begin{aligned} I_{3,mNn} &= \int_{-1}^1 \eta S_{m,m+n} S_{m,m+N} d\eta \\ &= 2 \left[\sum_{q=0,1}' \frac{(q+1)}{(2m+2q+1)(2m+2q+3)} \frac{(2m+q+1)!}{(q+1)!} d_q^{m,m+n} d_{q+1}^{m,m+N} \right. \\ &\quad \left. + \sum_{r=1,0}' \frac{(2m+r+1)}{(2m+2r+1)(2m+2r+3)} \frac{(2m+r)!}{r!} d_{r+1}^{m,m+n} d_r^{m,m+N} \right], \quad (n+N) \text{ odd} \\ &= 0, \quad (n+N) \text{ even} \end{aligned} \quad (\text{B.7})$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

$$\begin{aligned}
 I_{4,mNn} &= \int_{-1}^1 (1 - \eta^2) \frac{d}{d\eta} S_{m,m+n} S_{m,m+N} d\eta \\
 &= 2 \left[- \sum_{q=0,1}^{\infty} \frac{(m+q)(q+1)}{(2m+2q+1)(2m+2q+3)} \frac{(2m+q+1)!}{(q+1)!} d_q^{m,m+n} d_{q+1}^{m,m+N} \right. \\
 &\quad \left. + \sum_{r=1,0}^{\infty} \frac{(2m+r+1)(m+r+2)}{(2m+2r+1)(2m+2r+3)} \frac{(2m+r)!}{r!} d_{r+1}^{m,m+n} d_r^{m,m+N} \right], \quad (n+N) \text{ odd} \\
 &= 0, \quad (n+N) \text{ even}
 \end{aligned} \tag{B.8}$$

$$\begin{aligned}
 I_{5,mNn} &= \int_{-1}^1 (1 - \eta^2)^{\frac{1}{2}} S_{m+1,m+n+1} S_{m,m+N} d\eta \\
 &= 2 \sum_{q=0,1}^{\infty} \frac{1}{(2m+2q+3)} \left[\frac{(2m+q+1)(2m+q+2)}{(2m+2q+1)} \frac{(2m+q)!}{q!} d_q^{m,m+N} \right. \\
 &\quad \left. - \frac{(q+1)(q+2)}{(2m+2q+5)} \frac{(2m+q+2)!}{(q+2)!} d_{q+2}^{m,m+N} \right] d_q^{m+1,m+n+1}, \quad (n+N) \text{ even} \\
 &= 0, \quad (n+N) \text{ odd}
 \end{aligned} \tag{B.9}$$

$$\begin{aligned}
 I_{6,mNn} &= \int_{-1}^1 \eta (1 - \eta^2)^{\frac{1}{2}} \frac{d}{d\eta} S_{m+1,m+n+1} S_{m,m+N} d\eta \\
 &= -2 \sum_{q=0,1}^{\infty} \frac{(m+q+1)(q+1)}{(2m+2q+3)} \left[\frac{(2m+q+1)}{(2m+2q+1)} \frac{(2m+q)!}{q!} d_q^{m,m+N} \right. \\
 &\quad \left. + \frac{(q+2)}{(2m+2q+5)} \frac{(2m+q+2)!}{(q+2)!} d_{q+2}^{m,m+N} \right] d_q^{m+1,m+n+1} \\
 &\quad + 2 \sum_{q=0,1}^{\infty} \frac{q(m+1)}{(2m+2q+1)} \frac{(2m+q)!}{q!} d_q^{m+1,m+n+1} d_q^{m,m+N} \\
 &\quad + 2(m+1) \sum_{q=0,1}^{\infty} \sum_{v=q+2}^{\infty} \frac{(2m+q)!}{q!} d_v^{m+1,m+n+1} d_q^{m,m+N}, \quad (n+N) \text{ even} \\
 &= 0, \quad (n+N) \text{ odd}
 \end{aligned} \tag{B.10}$$

$$\begin{aligned}
 I_{7,mNn} &= \int_{-1}^1 S_{m+2,m+n+2} S_{m,m+N} d\eta \\
 &= -2 \sum_{q=0,1}^{\infty} \frac{(2m+q+2)!}{(2m+2q+5)q!} d_q^{m+2,m+n+2} d_{q+2}^{m,m+N} \\
 &\quad + 4(m+1) \sum_{q=0,1}^{\infty} \sum_{v=q}^{\infty} \frac{(2m+q)!}{q!} d_v^{m+2,m+n+2} d_q^{m,m+N} \quad (n+N) \text{ even} \\
 &= 0, \quad (n+N) \text{ odd}
 \end{aligned} \tag{B.11}$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

$$\begin{aligned}
I_{8,mNn} &= \int_{-1}^1 \eta S_{m+2,m+n+2} S_{m,m+N} d\eta \\
&= 2 \sum_{q=0,1}' \frac{(q+1)}{(2m+2q+5)} \left[\frac{2(m+1)(2m+2q+3) - q(2m+q+4)}{(2m+2q+3)} \right. \\
&\quad \times \frac{(2m+q+1)!}{(q+1)!} d_{q+1}^{m,m+N} - \frac{(2m+q+3)!}{(2m+2q+7)(q+1)!} d_{q+3}^{m,m+N} \left. \right] d_q^{m+2,m+n+2} \\
&\quad + 4(m+1) \sum_{r=1,0}' \sum_{v=r+1}' \frac{(2m+r)!}{r!} d_v^{m+2,m+n+2} d_r^{m,m+N}, \quad (n+N) \text{ odd} \\
&= 0, \quad (n+N) \text{ even}
\end{aligned} \tag{B.12}$$

$$\begin{aligned}
I_{9,mNn} &= \int_{-1}^1 (1-\eta^2) \frac{d}{d\eta} S_{m+2,m+n+2} S_{m,m+N} d\eta \\
&= 2 \sum_{q=0,1}' \frac{(q+1)}{(2m+2q+5)} \left[\frac{(m+q+2)}{(2m+2q+7)} \frac{(2m+q+3)!}{(q+1)!} d_{q+3}^{m,m+N} \right. \\
&\quad - \frac{(2m+q+4)(m+q+3)q + 2(m+1)(m+q+2)(2m+2q+3)}{(2m+2q+3)} \\
&\quad \times \frac{(2m+q+1)!}{(q+1)!} d_{q+1}^{m,m+N} \left. \right] d_q^{m+2,m+n+2} \\
&\quad + 4(m+1) \sum_{r=1,0}' \sum_{v=r+1}' \frac{(2m+v+4)(m+v+3) - (v+1)(m+v+2)}{(2m+2v+5)} \\
&\quad \times \frac{(2m+r)!}{r!} d_r^{m,m+N} d_v^{m+2,m+n+2}, \quad (n+N) \text{ odd} \\
&= 0, \quad (n+N) \text{ even}
\end{aligned} \tag{B.13}$$

$$\begin{aligned}
I_{10,mNn} &= \int_{-1}^1 (1-\eta^2)^{\frac{1}{2}} S_{0,n} S_{1,N+1} d\eta \\
&= 2 \sum_{q=0,1}' \frac{(q+1)(q+2)}{(2q+3)} \left[\frac{d_q^{0,n}}{(2q+1)} - \frac{d_{q+2}^{0,n}}{(2q+5)} \right] d_q^{1,N+1}, \quad (n+N) \text{ even} \\
&= 0, \quad (n+N) \text{ odd}
\end{aligned} \tag{B.14}$$

$$\begin{aligned}
I_{11,mNn} &= \int_{-1}^1 \eta (1-\eta^2)^{\frac{1}{2}} \frac{d}{d\eta} S_{0,n} S_{1,N+1} d\eta \\
&= 2 \sum_{q=0,1}' \frac{(q+1)(q+2)}{(2q+3)} \left[\frac{q d_q^{0,n}}{(2q+1)} + \frac{(q+3) d_{q+2}^{0,n}}{(2q+5)} \right] d_q^{1,N+1}, \quad (n+N) \text{ even} \\
&= 0, \quad (n+N) \text{ odd}
\end{aligned} \tag{B.15}$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

$$\begin{aligned}
 I_{14,mNn} &= \int_{-1}^1 (1 - \eta^2) \eta S_{m,m+n} S_{m,m+N} d\eta \\
 &= 2 \sum_{q=0,1}^{\infty} \left[-\frac{(q-2)(q-1)q}{(2m+2q-3)(2m+2q-1)} \left(\frac{d_{q-3}^{m,m+N}}{(2m+2q-5)} - \frac{d_{q-1}^{m,m+N}}{(2m+2q-1)} \right) \right. \\
 &\quad + \frac{(2m+1)(2m+q+1)q}{(2m+2q-1)(2m+2q+3)} \left(\frac{d_{q-1}^{m,m+N}}{(2m+2q-1)} - \frac{d_{q+1}^{m,m+N}}{(2m+2q+3)} \right) \\
 &\quad + \frac{(2m+q+1)(2m+q+2)(2m+q+3)}{(2m+2q+3)(2m+2q+5)} \left(\frac{d_{q+1}^{m,m+N}}{(2m+2q+3)} \right. \\
 &\quad \left. \left. - \frac{d_{q+3}^{m,m+N}}{(2m+2q+7)} \right) \right] \frac{(2m+q)!}{(2m+2q+1)q!} d_q^{m,m+n}, \quad (n+N) \text{ odd}, \\
 &= 0, \quad (n+N) \text{ even}
 \end{aligned} \tag{B.16}$$

$$\begin{aligned}
 I_{15,mNn} &= \int_{-1}^1 (1 - \eta^2)^2 \frac{d}{d\eta} S_{m,m+n} S_{m,m+N} d\eta \\
 &= 2 \sum_{q=0,1}^{\infty} \frac{(2m+q)! d_q^{m,m+n}}{(2m+2q+1)q!} \left[-\frac{(m+q+1)(q-2)(q-1)q}{(2m+2q-3)(2m+2q-1)} \left(\frac{d_{q-3}^{m,m+N}}{(2m+2q-5)} \right. \right. \\
 &\quad \left. \left. - \frac{d_{q-1}^{m,m+N}}{(2m+2q-1)} \right) + \frac{[2(m+q)^2 + 5m + 2q](2m+q+1)q}{(2m+2q-1)(2m+2q+3)} \right. \\
 &\quad \times \left(\frac{d_{q-1}^{m,m+N}}{(2m+2q-1)} - \frac{d_{q+1}^{m,m+N}}{(2m+2q+3)} \right) \\
 &\quad \left. - \frac{(m+q)(2m+q+1)(2m+q+2)(2m+q+3)}{(2m+2q+3)(2m+2q+5)} \left(\frac{d_{q+1}^{m,m+N}}{(2m+2q+3)} \right. \right. \\
 &\quad \left. \left. - \frac{d_{q+3}^{m,m+N}}{(2m+2q+7)} \right) \right], \quad (n+N) \text{ odd}, \\
 &= 0, \quad (n+N) \text{ even}
 \end{aligned} \tag{B.17}$$

$$\begin{aligned}
 I_{18,mNn} &= \int_{-1}^1 \frac{d}{d\eta} S_{m,m+n} S_{m,m+N} d\eta \\
 &= 2 \sum_{r=1,0}^{\infty} d_r^{0,N} \sum_{q=r+1}^{\infty} d_q^{0,n}, \quad (n+N) \text{ odd}, \quad m=0 \\
 &= \sum_{r=1,0}^{\infty} \frac{(2m+r)!}{r!} d_r^{m,m+N} \sum_{q=r+1}^{\infty} d_q^{m,m+n} \\
 &\quad - \sum_{q=0,1}^{\infty} \frac{(2m+q)!}{q!} d_q^{m,m+n} \sum_{r=q+1}^{\infty} d_r^{m,m+N}, \quad (n+N) \text{ odd}, \quad m \neq 0, \\
 &= 0, \quad (n+N) \text{ even}
 \end{aligned} \tag{B.18}$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

$$\begin{aligned}
 I_{24,mNn} &= \int_{-1}^1 \eta(1-\eta^2)^{\frac{3}{2}} \frac{d}{d\eta} S_{m+1,m+n+1} S_{m,m+N} d\eta \\
 &= 2 \sum_{q=0,1}^{\infty} \frac{(2m+q+2)! d_q^{m+1,m+n+1}}{(2m+2q+3)q!} \left[\frac{(m+q+2)q}{(2m+2q+1)} \right. \\
 &\quad \times \left(\frac{(q-1)d_{q-2}^{m,m+N}}{(2m+2q-3)(2m+2q-1)} + \frac{(2m+1)d_q^{m,m+N}}{(2m+2q-1)(2m+2q+3)} \right. \\
 &\quad \left. - \frac{(2m+q+2)d_{q+2}^{m,m+N}}{(2m+2q+3)(2m+2q+5)} \right) \\
 &\quad - \frac{(m+q+1)(2m+q+3)}{(2m+2q+5)} \left(\frac{(q+1)d_q^{m,m+N}}{(2m+2q+1)(2m+2q+3)} \right. \\
 &\quad \left. + \frac{(2m+1)d_{q+2}^{m,m+N}}{(2m+2q+3)(2m+2q+7)} \right. \\
 &\quad \left. - \frac{(2m+q+4)d_{q+4}^{m,m+N}}{(2m+2q+7)(2m+2q+9)} \right) \Big], \quad (n+N) \text{ even}, \\
 &= 0, \quad (n+N) \text{ odd}
 \end{aligned} \tag{B.19}$$

$$\begin{aligned}
 I_{25,mNn} &= \int_{-1}^1 (1-\eta^2)^{\frac{3}{2}} S_{m+1,m+n+1} S_{m,m+N} d\eta \\
 &= 2 \sum_{q=0,1}^{\infty} \frac{(2m+q+2)! d_q^{m+1,m+n+1}}{(2m+2q+3)q!} \left[\frac{(2m+q+3)(2m+q+4)}{(2m+2q+5)} \right. \\
 &\quad \times \left(\frac{d_q^{m,m+N}}{(2m+2q+1)(2m+2q+3)} - \frac{2d_{q+2}^{m,m+N}}{(2m+2q+3)(2m+2q+7)} \right. \\
 &\quad \left. + \frac{d_{q+4}^{m,m+N}}{(2m+2q+7)(2m+2q+9)} \right) - \frac{(q-1)q}{(2m+2q+1)} \\
 &\quad \times \left(\frac{d_{q-2}^{m,m+N}}{(2m+2q-3)(2m+2q-1)} - \frac{2d_q^{m,m+N}}{(2m+2q-1)(2m+2q+3)} \right. \\
 &\quad \left. \left. + \frac{d_{q+2}^{m,m+N}}{(2m+2q+3)(2m+2q+5)} \right) \right], \quad (n+N) \text{ even}, \\
 &= 0, \quad (n+N) \text{ odd}
 \end{aligned} \tag{B.20}$$

$$\begin{aligned}
 I_{26,mNn} &= \int_{-1}^1 \frac{1}{(1-\eta^2)^{\frac{1}{2}}} S_{m+1,m+n+1} S_{m,m+N} d\eta \\
 &= 2 \sum_{r=0,1}^{\infty} \frac{(2m+r)!}{r!} d_r^{m,m+N} \sum_{q=r}^{\infty} d_q^{m+1,m+n+1}, \quad (n+N) \text{ even} \\
 &= 0, \quad (n+N) \text{ odd}
 \end{aligned} \tag{B.21}$$

APPENDIX B. ORTHOGONAL PROPERTIES OF THE SPHEROIDAL ANGULAR FUNCTIONS

$$\begin{aligned}
 I_{27,mNn} &= \int_{-1}^1 (1-\eta^2)^{\frac{1}{2}} \frac{d}{d\eta} \left((1-\eta^2)^{\frac{1}{2}} \frac{d}{d\eta} S_{m,m+n} \right) S_{m,m+N} d\eta \\
 &= -2 \sum_{q=1,0}^{\infty} \frac{(q+1)^2}{(2q+3)} d_{q+1}^{0n} d_{q+1}^{0N} + 2 \sum_{r=0,1}^{\infty} d_r^{0N} \sum_{s=r+2}^{\infty} d_s^{0n}, \quad (n+N) \text{ even}, \quad m=0 \\
 &= - \sum_{q=0,1}^{\infty} \frac{(2m+q)!}{q!} \left(\frac{q(m+q) + (q+1)(m+q+1)}{(2m+2q+1)} d_q^{m,m+n} d_q^{m,m+N} \right. \\
 &\quad \left. - (m+1) d_q^{m,m+N} \sum_{r=q+2}^{\infty} d_r^{m,m+n} - (m-1) d_q^{m,m+n} \sum_{r=q+2}^{\infty} d_r^{m,m+N} \right), \\
 &\quad (n+N) \text{ even}, \quad m \neq 0 \\
 &= 0, \quad (n+N) \text{ odd} \tag{B.22}
 \end{aligned}$$

$$\begin{aligned}
 I_{29,mNn} &= \int_{-1}^1 (1-\eta^2)^{\frac{1}{2}} \frac{d}{d\eta} S_{m+1,m+n+1} S_{m,m+N} d\eta \\
 &= -2 \sum_{q=0,1}^{\infty} \frac{(m+q+1)(2m+q+1)!}{(2m+2q+3)q!} d_q^{m+1,m+n+1} d_{q+1}^{m,m+N} \\
 &\quad + 2(m+1) \sum_{r=1,0}^{\infty} \frac{(2m+r)!}{r!} d_r^{m,m+N} \sum_{v=r+1}^{\infty} d_v^{m+1,m+n+1}, \quad (n+N) \text{ odd} \\
 &= 0, \quad (n+N) \text{ even} \tag{B.23}
 \end{aligned}$$

Appendix C

Intermediate Terms of (4.43)

$$A(r, n) = d_r^{1,n}(h_1) \left(U_n(h_1, \xi_2) - \frac{U'_n(h_1, \xi_1) T_n(h_1, \xi_2)}{T'_n(h_1, \xi_1)} \right) \quad (\text{C.1a})$$

$$B(r, n) = d_r^{1,n}(h_1) \frac{p_n T_n(h_1, \xi_2)}{T'_n(h_1, \xi_1)} \quad (\text{C.1b})$$

$$C(r, n) = d_r^{1,n}(h_2) U_n(h_2, \xi_2) \quad (\text{C.1c})$$

$$D(r, n) = d_r^{1,n}(h_2) T_n(h_2, \xi_2) \quad (\text{C.1d})$$

$$E(r, n) = d_r^{1,n}(h_1) \left(U'_n(h_1, \xi_2) - \frac{U'_n(h_1, \xi_1) T'_n(h_1, \xi_2)}{T'_n(h_1, \xi_1)} \right) \quad (\text{C.1e})$$

$$F(r, n) = d_r^{1,n}(h_1) \frac{p_n T'_n(h_1, \xi_2)}{T'_n(h_1, \xi_1)} \quad (\text{C.1f})$$

$$G(r, n) = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} d_r^{1,n}(h_2) U'_n(h_2, \xi_2) \quad (\text{C.1g})$$

$$H(r, n) = \frac{\epsilon_{r_1}}{\epsilon_{r_2}} d_r^{1,n}(h_2) T'_n(h_2, \xi_2) \quad (\text{C.1h})$$

$$J(r, n) = d_r^{1,n}(h_2) U_n(h_2, \xi_3) \quad (\text{C.1i})$$

$$K(r, n) = d_r^{1,n}(h_2) T_n(h_2, \xi_3) \quad (\text{C.1j})$$

$$L(r, n) = d_r^{1,n}(h_3) U_n(h_3, \xi_3) \quad (\text{C.1k})$$

$$O(r, n) = d_r^{1,n}(h_2) U'_n(h_2, \xi_3) \quad (\text{C.1l})$$

$$P(r, n) = d_r^{1,n}(h_2) T'_n(h_2, \xi_3) \quad (\text{C.1m})$$

$$Q(r, n) = \frac{\epsilon_{r_2}}{\epsilon_{r_3}} d_r^{1,n}(h_3) U'_n(h_3, \xi_3) \quad (\text{C.1n})$$