A new method for identifying mechanical impedance of soft viscoelastic materials

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A New Method for Identifying Mechanical Impedance of Soft Viscoelastic Materials

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ABSTRACT

Following the wider use of viscoelastic materials in industries over the past few decades, being able to characterize mechanical properties of such materials experimentally has become increasingly important. However, existing dynamic testing methods such as free decay method, forced vibration method, resonant method and wave propagation method share a common drawback. Each of these methods requires the use of sensors and actuators in exciting and measuring the load and deformation which renders the loading effect of sensors for small samples and the mass effect of sensors at high frequency testing inevitable. Furthermore, installation of sensors and actuators is difficult when the space is small. Samples preparation is also a major problem to overcome in applying the existing methods. Moreover, they are not suitable for in situ testing.

In this study, a new method, transduction matrix method, for testing complex modulus and loss factor of soft viscoelastic materials is designed by using Simultaneous Sensing and Actuating (SSA) technique. In transduction matrix method the actuator is described by a 2×2 transduction matrix which allows the mechanical impedance of the output port of the actuator to be detected from the electrical impedance of the input port while the actuator works as a sensor simultaneously. The detected mechanical impedance can then be analyzed to provide mechanical properties of viscoelastic materials.

Transduction matrix method was first applied on electrodynamic actuators. The theoretical model of the shaker was then derived and proved to be the closed form of the four transduction functions based on some simplification on the shaker. Transduction matrix

was also obtained and the accuracy was confirmed by the comparison of calibrated and measured masses while sensing capability of the shaker system was investigated. It is concluded that when using added mass to calibrate the shaker, the mass should be comparable to the equivalent mass of the shaker to get a higher accuracy and that the effect of the stiffness is much larger than that of the mass of the samples since the sensitivity is load-related.

The transduction matrix method was also applied on material characterization. The complex modulus and loss factor derived from the detected mechanical impedance based on an impedance model were found to match reasonably well with that obtained by DMA for some rubbers with the deviation of 8%. It is proved to be a simple method for characterizing materials quantitatively.

More importantly, we investigated a new technique for dynamic testing mechanical properties of human skin in vivo and a portable device called Dynamic Indenting Actuator for Skin (DIAS) was designed based on transduction matrix method. Its accuracy was obtained by testing on three viscoelastic samples (silicone gel). The complex moduli and damping ratio of three samples were with good coincidence with the results of DMA method. The deviation of complex modulus is around 11% and that of loss factor is around 18% at 100Hz. Tests on human skin of five subjects using the DIAS were conducted and the complex modulus and damping ratio of the skins were measured. The quantitative results (limited by small sample space of subjects) are believed to be good references for human skin researchers since there are no such kind of data been measured because of lack of technique and equipment.

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TABLE OF CONTENTS

Abstract..... i

Acknowledgement..... iii

Table of Contents iv

List of Figures viii

List of Tables ixv

1. Introduction

Backgrounds 1

Objectives4

Outline of the report.....5

2. Literature review

2.1 Introduction of viscoelastic materials7

2.1.1 Young’s modulus of viscoelastic materials8

2.1.2 Modeling of viscoelastic materials9

2.1.3 Creep and relaxation tests 11

2.2 Dynamic testing method of viscoelastic materials..... 12

2.2.1 Free decay method 13

2.2.2 Force vibration method 15

2.2.3 Resonance method 18

2.2.4 Wave propagation method 19

2.2.5 Comparison of the methods 22

2.3 Mechanical properties of human tissues 24

2.3.1 In vitro mechanical properties of human tissues..... 24

2.3.2 In vivo mechanical properties of human skin 25

2.3.3 Equipment for testing mechanical properties of human skin.... 28

2.4 Summary 31

3. Transduction matrix method and its application on shaker

3.1 Introduction 33

3.2 Transduction matrix method 35

3.2.1 Four-pole model..... 35

3.2.2 Transduction matrix model 37

3.3 Derivation of theoretical model of shaker..... 41

3.4 Characterization on transduction matrix of shaker 45

3.4.1 Experimental equipment 46

3.4.2 Direct method..... 47

3.4.3 Correction of common method 51

3.4.4 Adding mass method..... 54

3.5 Actuating and sensing capability of shaker 64

3.5.1 Actuating capability of shaker 64

3.5.2 Sensing capability of shaker 65

3.6 Decoupling effect of the transduction matrix method 73

3.7	Summary	78
 4. Soft viscoelastic materials characterization using shaker		
4.1	Introduction	80
4.2	Theoretical model on viscoelastic materials characterization.....	82
4.2.1	General equation of motion	82
4.2.2	Radiation impedance of the sphere	84
4.2.3	Effects of the parameters.....	88
4.3	Experiments on soft viscoelastic materials characterization	97
4.3.1	Experimental set up.....	97
4.3.2	Four pole model of shaker-testing-device.....	100
4.3.3	Mechanical properties of samples.....	104
4.3.4	Experimental results	105
4.4	Analysis transduction matrix method on soft viscoelastic materials characterization	111
4.4.1	Decoupling effect of transduction matrix method on soft viscoelastic materials characterization.....	111
4.4.2	Discussion on experimental results.....	113
4.5	Summary	115
 5. Characterization human skin in vivo by portable Dynamic Indenting Actuator for Skin		
5.1	Introduction	117

5.2	Design of DIAS.....	118
5.2.1	Requirements of DIAS.....	118
5.2.2	Linear actuator options	119
5.2.3	Flexure design of DIAS	122
5.3	Calibration of DIAS.....	126
5.3.1	Repeatability of DIAS.....	127
5.3.2	Mass calibration method	128
5.3.3	Experimental set up of mass test.....	131
5.3.4	Testing results and discussion.....	132
5.4	Testing on silicone gel samples	134
5.4.1	Preparation of silicone gel samples.....	135
5.4.2	Experimental set up of silicone gel test	137
5.4.3	Testing results and discussion.....	138
5.5	Testing on human skin in vivo.....	145
5.5.1	Details of the subjects	145
5.5.2	Testing results and discussion.....	147
5.6	Summary.....	154
 6. Conclusions and recommendations		
6.1	Conclusions.....	156
6.2	Recommendations.....	159
 References.....		161
 Appendix.....		174

LIST OF FIGURES

Fig. 2.1 Stress- t and strain- t of viscoelastic materials 8

Fig. 2.2 Common models of viscoelastic materials 10

Fig. 2.3 Discrete spectra model of viscoelastic materials 10

Fig. 2.4 Creep and relaxation functions of three models..... 12

Fig. 2.5 Different testing methods with frequencies 13

Fig. 2.6 Schematic diagram of apparatus for torsion pendulum method 14

Fig. 2.7 Amplitude X versus time t for damped vibrations..... 14

Fig. 2.8 Moving parts of GABO qualimeter DMA 16

Fig. 2.9 Impedance method configuration..... 18

Fig. 2.10 Resonance apparatus for measuring dynamic Young’s modulus 19

Fig. 2.11 Wave propagation method 21

Fig. 2.12 Tactile sensor setting up..... 29

Fig. 3.1 Schematic four-pole model 35

Fig. 3.2 Cascaded 2-port model 37

Fig. 3.3 Four-pole diagram of shaker 38

Fig. 3.4 Electromechanical model of the shaker 42

Fig. 3.5 Shaker as an SSA device characterization ---- free condition 48

Fig. 3.6 Shaker as an SSA device characterization ---- clamp condition..... 48

Fig. 3.7 Transduction element T_{11} 49

Fig. 3.8 Transduction element T_{12} 50

Fig. 3.9	Transduction element T_{21}	50
Fig. 3.10	Transduction element T_{22}	51
Fig. 3.11	Determinant of transduction matrix [T]	51
Fig. 3.12	Velocity in “clamped” boundary condition	53
Fig. 3.13	Corrected transduction element T_{11}	54
Fig. 3.14	Corrected transduction element T_{21}	54
Fig. 3.15	Schematic model of the adding mass system.....	56
Fig. 3.16	Picture of adding masses.....	59
Fig. 3.17	Experiments setting up (Adding mass)	59
Fig. 3.18	Electrical impedance of shaker system (added on masses).....	60
Fig. 3.19	Transduction element T_{11} (adding group 3 mass).....	61
Fig. 3.20	Transduction element T_{12} (adding group 3 mass).....	61
Fig. 3.21	Transduction element T_{21} (adding group 3 mass).....	62
Fig. 3.22	Transduction element T_{22} (adding group 3 mass)	62
Fig. 3.23	Determinant of the transduction matrix [T]	63
Fig. 3.24	Torque/speed characteristic of step motor	64
Fig. 3.25	Relationship between force and velocity at a series of frequencies.....	65
Fig. 3.26	Normalized magnitude errors vs. mean value of calibrated masses distribution	69
Fig. 3.27	Phase errors vs. mean value of calibrated masses distribution	69
Fig. 3.28	Normalized magnitude errors vs. standard deviation of calibrated masses distribution	70
Fig. 3.29	Phase errors vs. standard deviation calibrated masses distribution	70

Fig. 3.30 Resonant frequency vs. mass and spring ratio 73

Fig. 3.31 Comparison of T_{11} of two systems..... 74

Fig. 3.32 Comparison of T_{12} of two systems 75

Fig. 3.33 Comparison of T_{21} of two systems 75

Fig. 3.34 Comparison of T_{22} of two systems 76

Fig. 3.35 Comparison of the determinant of the two systems..... 76

Fig. 3.36 Comparison of electrical impedance testing on mass 10 via two
systems..... 77

Fig. 3.37 Comparison of calculated mass 10 via two systems 77

Fig. 4.1 Sphere coordinate..... 84

Fig. 4.2 Comparison of radiation impedance (tissue and incompressible
medium) 89

Fig. 4.3 Effect of radius of sphere on mechanical impedance 91

Fig. 4.4 Effect of density of medium on mechanical impedance 92

Fig. 4.5 Effect of shear modulus of medium on mechanical impedance 93

Fig. 4.6 Effect of Lamé parameter of medium on mechanical impedance 94

Fig. 4.7 Ratio of radius effect..... 95

Fig. 4.8 Ratio of density effect 95

Fig. 4.9 Ratio of shear modulus effect 96

Fig. 4.10 Ratio of Lamé parameter effect 96

Fig. 4.11 Experiment setting up for soft material characterization 98

Fig. 4.12 Schematic frame of characterization materials 100

Fig. 4.13	Transduction element T_{11} (total testing system)	101
Fig. 4.14	Transduction element T_{12} (total testing system)	102
Fig. 4.15	Transduction element T_{21} (total testing system)	102
Fig. 4.16	Transduction element T_{22} (total testing system).....	103
Fig. 4.17	Determinant of [T] of shaker testing system.....	103
Fig. 4.18	Electrical impedance when testing on commercial erasers.....	106
Fig. 4.19	Electrical impedance when testing on industrial rubbers.....	106
Fig. 4.20	Complex modulus of r5 (DMA and proposed method)	107
Fig. 4.21	Complex modulus of r2 (DMA and proposed method)	108
Fig. 4.22	Complex modulus of r1 (DMA and proposed method)	108
Fig. 4.22	Complex modulus of r3 (DMA and proposed method)	109
Fig. 4.24	Complex modulus of r4 (DMA and proposed method)	109
Fig. 4.25	EPDM 280C complex modulus and loss factor (DMA and proposed method).....	110
Fig. 4.26	NE 264C complex modulus and loss factor (DMA and proposed method)	110
Fig. 4.27	Na 263C complex modulus and loss factor (DMA and proposed method)	111
Fig. 4.28	Comparison of electrical impedance testing on r5 via two systems	112
Fig. 4.29	Comparison of complex modulus of r5 via two systems	113
Fig. 5.1	Voice coil actuator	120
Fig. 5.2	Simulated stress distribution of flexure	125

Fig. 5.3	Linear relationship of force and displacement of flexure	125
Fig. 5.4	Pictures of DIAS (schematic figure, main parts and main view)	127
Fig. 5.5	Schematic diagram of repeatability experiment.....	128
Fig. 5.6	Repeatability result of new device (electrical impedance of free situation)	128
Fig. 5.7	Photo of DIAS and different indenters	131
Fig. 5.8	Experimental set up of mass calibration	131
Fig. 5.9	Electrical impedance of DIAS with different indenters.....	132
Fig. 5.10	Characteristic Z_{ec} of DIAS.....	133
Fig. 5.11	Characteristic Z_{ec} of DIAS.....	133
Fig. 5.12	Mass comparison (true and calculated).....	134
Fig. 5.13	DMA results of three silicone gels.....	137
Fig. 5.14	Photo of experimental setting up for testing Silgard samples.....	138
Fig. 5.15	Zoom in photo on indenter and sample.....	138
Fig. 5.16	Electrical impedance when testing on Silgard samples (Indenter 1)	139
Fig. 5.17	Electrical impedance when testing on Silgard samples (Indenter 3)	139
Fig. 5.18	Electrical impedance when testing on Silgard samples (Scw5mm)	140
Fig. 5.19	Electrical impedance when testing on Silgard samples (Scw8mm)	140
Fig. 5.20	Electrical impedance when testing on Silgard samples (Scr10mm).....	141
Fig. 5.21	Complex modulus of Sample 11(measured by DIAS and DMA)	142
Fig. 5.22	Complex modulus of Sample 23(measured by DIAS and DMA)	142
Fig. 5.23	Complex modulus of Sample 12(measured by DIAS and DMA)	143

Fig. 5.24	Electrical impedance when testing with different indenters (Sample 11)	143
Fig. 5.25	Comparison of complex modulus using different indenter (Sample 11).....	144
Fig. 5.26	Comparison of complex modulus using different indenter (Sample 23).....	144
Fig. 5.27	Comparison of complex modulus using different indenter (Sample 12).....	145
Fig. 5.28	Photo of tests on subject 4, 5 and 1	146
Fig. 5.29	Electrical impedance of DIAS when testing on ventral forearm of subject 1 (on different days).....	147
Fig. 5.30	Electrical impedance of DIAS when testing on dorsum forearm of subject 1 (on different days).....	148
Fig. 5.31	Electrical impedance when testing on different parts of subject 1	149
Fig. 5.32	Complex modulus of skins of different part of subject 1	149
Fig. 5.33	Electrical impedance when testing on ventral and dorsum of 4subject 2's forearm.....	150
Fig. 5.34	Complex modulus of foreman skin of subject 2 (ventral and dorsum)	150
Fig. 5.35	Electrical impedance when testing on forearm of four subjects	151
Fig. 5.36	Complex modulus of human skin of four subjects	151
Fig. 5.37	Complex modulus of ventral foreman skin (subject 1 and subject 2).....	153
Fig. 5.38	Complex modulus of dorsum foreman skin (subject 1 and subject 2).....	153
Fig. 5.39	Complex modulus of soft tissues comparison (review and DIAS).....	154

LIST OF TABLES

Table 2.1 Mechanical properties of some common soft tissues and biomaterials25

Table 2.2 Young’s modulus of skin in vivo.....27

Table 2.2 Commercial equipment for skin testing30

Table 3.1 Common application of four-pole model.....36

Table 3.2 Equipment list for shaker calibration experiment46

Table 3.3 List of adding masses58

Table 3.4 Parameters of shaker63

Table 3.5 List of normalized mass and resonant frequency.....68

Table 3.6 Calculated mass errors (all masses)71

Table 3.7 Calculated mass errors (in group)72

Table 4.1 Equipment list for shaker calibration experiment.....98

Table 4.2 Materials properties of samples105

Table 5.1 Winding constant of LA0505000A.....121

Table 5.2 Actuator parameters of LA0505000A.....121

Table 5.3 Materials for flexure.....124

Table 5.4 Masses of indenters132

Table 5.5 Recommended curing time for Silgard 527136

Table 5.6 Details of measured subjects.....146

Chapter 1

Introduction

1.1 Background

Viscoelastic materials possess a combination of viscous and elastic characteristics. Amorphous polymers such as plastics and synthetic rubbers, fibrous materials (e.g., silk, rayon and cellulose), biomaterials (e.g., skin and muscle), and nonmetals including the modern composite materials are viscoelastic materials [1]. The concept of viscoelastic behavior of materials, though old in origin, has only recently come into the prominence of widespread attention and application [2]. Viscoelastic polymers have become widely used in many areas involving sound and vibration control, water-borne transmission and attenuation of acoustic waves [3]. At the same time newly developed biomaterials need to be qualified with accurate mechanical properties [4]. The frequency and temperature dependent mechanical properties of materials (modulus and loss) are of great interest throughout the plastic, polymer, and rubber world from initial design to routine production [5]. Viscoelastic materials are useful for isolation and damping of noise, shock and vibration. For an efficient design of components or systems with such viscoelastic materials, first of all it is necessary to have an accurate knowledge of the complex modulus of elasticity, that is, the dynamic Young's modulus and loss factor [6].

The complex modulus is typically used to represent viscoelastic behavior. It must be obtained experimentally for individual materials. The methods for testing mechanical properties of viscoelastic materials are divided into two kinds, static and dynamic. Many different mechanical testing techniques have been developed over years, such as creep and relaxation test under tension, indentation method [7-9], free decay method [10, 11], forced vibrated method (dynamic mechanical analysis) [3, 6, 12-17], resonance method [18-21], and wave propagation method [22-24]. There are also a lot of standards for testing the mechanical properties of viscoelastic materials. The methods above are selected by different intentions, such as the frequency domain, the deformation model and the force model. All these methods need actuators to vibrate the samples and then force cells and displacement sensors to measure the input force and the output displacement or vice versa. After that different models are used to extract mechanical properties of the materials.

There are some problems on the existed methods when testing mechanical properties of viscoelastic materials. First of all, the samples must be manufactured into special shapes to cancel the size effect in material characterization. For force vibration method, the length of the cylindrical specimen should not be so short that corrections must be applied, nor so long that creep or toppling occurs. ASEM 756 uses samples made in cantilever shape to measure the complex modulus and damping of the materials. The shape of the samples is critical for resonant method and wave propagation method too. Some times the sample preparation is the most difficult task for material characterization. Secondly, the loading effect of the sensors is inevitable especially when samples are small. In dynamic testing especially at high frequencies, the mass effect of the sensors makes major difficulties in achieving accurate

information. Thirdly, the space is too small to install proper sensors in some cases. Finally when in situ measurement are needed neither of the methods mentioned above are suitable for obtaining the mechanical properties of viscoelastic materials. So a new method is needed to overcome these problems.

Biomaterials, which possess viscoelastic characteristic, have been widely studied as biomedicine, being a research hotspot in the past decades. With the development of medical technique, more and more biomaterials are designed to substitute human tissue. In order to function properly, mechanical properties of the biomaterials should be the same as that of human tissues. For computer simulation, knowledge of mechanical properties of tissues is required to predict forces and deformation through a set of constitutive equations [25-28]. When there are pathological changes in tissues, the mechanical properties of the tissues are always different from healthy ones. So the changes of mechanical properties of tissues are usually used as an indicator for checking. In vivo tests are more useful compared with in vitro measurement for medical checking and materials characterization.

Skin as the largest tissue in human body is the superficial one which protects the body from diseases. Knowledge about the mechanical behaviour of the skin in vivo is of importance for cosmetic and clinical applications. It can help to quantify the effectiveness of cosmetic products and to evaluate both skin diseases and pressure sores [29]. Many researchers have studied the mechanical properties of human skin which are related to age, sunshine and so on, but they only focused on the elasticity while viscoelasticity of the skin are less investigated. The dynamic testing on skin was only done at very lower frequency (3~5Hz) or at ultrasound frequency via existed

equipment such as Cutometer, DermaLab, Linear Skin Rheometer and Venustron. And most of the results are qualitative not quantitative. A new technique for dynamic measuring the mechanical properties of human skin in vivo quantitatively is needed.

1.2 Objectives

The aim of this research work is to develop a new method for testing mechanical properties of viscoelastic materials which will overcome the difficulties mentioned above. In this method, the shape of the samples is less important for the tests and the samples are easy to manufacture, the loading effect of the sensors can be cancelled or no sensors are used to measure the mechanical variables, and it is more simply to implement to fit for in situ measurement. We will use a new method called transduction matrix method and a new technique, simultaneous sensing and actuating (SSA) that means the actuator as a sensor simultaneously then no sensors in the testing system. The main objectives are:

- To investigate transduction matrix method on electrodynamic shaker
- To measure the complex modulus and loss factor of soft incompressible viscoelastic materials using shake SSA device
- To investigate a new technique for dynamic testing complex modulus and loss factor of human skin in vivo quantitatively

1.3 Organization of the thesis

An overview of relevant knowledge and research work in the field of dynamic testing of viscoelastic materials and mechanical properties of human skin is presented in Chapter 2. The complex modulus and the different models of viscoelastic materials are introduced. Several testing methods, such as free delay method, forced vibration method, resonant method and wave propagation method are reviewed and compared. Mechanical properties of human tissues especially human skin in vitro and in vivo are reviewed in detail.

In Chapter 3, the transduction matrix method is presented and discussed in detail. Firstly the transduction matrix method is introduced and its application is reviewed. Then an electrodynamic shaker is investigated as a SSA transducer for the first time. Theoretical model of the shaker is derived. The shaker is calibrated via some methods and the correct characteristics are obtained. Decoupling of the transduction matrix method is confirmed from the experiments. And the sensing and actuating ability of the shaker system is developed which illustrate some rules on the feasibility of the transduction matrix method.

In Chapter 4 a theoretical model is introduced first. This model relates the radiation impedance of a sphere with the material properties of the media in which the sphere vibrates. The effect of the parameters is analyzed when the model is used in material characterization. Shaker is then used to measure the complex modulus and loss factor of soft incompressible viscoelastic materials. Eight samples are tested and the results

are compared with DMA results with deviation of 8% for softer samples. It showed that shaker can characterize soft viscoelastic materials.

In Chapter 5, a new technique for dynamic testing mechanical properties of human skin in vivo quantitatively is investigated. A portable device called Dynamic Indenting Actuator for Skin (DIAS) is developed for testing human skin in vivo. After calibration DIAS is used to measure the complex modulus and loss factor of three viscoelastic samples (silicone gel). The complex modulus and loss factor of the three samples confirms well with the DAM results. The deviation of complex modulus is 11% and that of loss factor is 18% at 100Hz. Testing on human skin in vivo of five subjects is done and the complex modulus and loss factor of human skin are obtained quantitatively. The results are limited because of the small sample space of the subjects.

Chapter 6 summarizes the research work and concluding remarks so far, as well as some recommendations of the project are given.

Chapter 2

Literature review

The concept of viscoelastic behaviour of materials has recently come into the prominence of widespread attention and application through it is old in origin. Viscoelastic materials are of importance for vibration isolation, biomaterials and cosmetic industry. The need of knowing the mechanical properties of viscoelastic materials has been put forward urgently. For the purpose of design and in-service prediction of the transmission of vibration and structure-borne sound through viscoelastic materials such as flexible pipes or vibration isolators, an accurate knowledge of the complex modulus of the materials is necessary [30]. Viscoelastic materials are firstly introduced in this chapter. Then dynamic testing methods for viscoelastic materials are reviewed. Mechanical properties of soft tissues such as human skin are summarized based on reviews of the literatures and commercial equipment is listed. Based on this a few problems of the testing methods of viscoelastic materials are promoted for further investigation.

2.1 Introduction of viscoelastic materials

The theory of elasticity may account for materials which have a capacity to store mechanical energy with no dissipation of the energy. On the other hand, a Newtonian viscous fluid in a nonhydrostatic stress state implies a capacity for dissipation energy, but none for storing it. Materials outside the scope of these two are those for which

some of the work done to deform them can be recovered. Such materials possess a capacity to both store and dissipate mechanical energy and are called viscoelastic materials. Amorphous polymers such as plastics and synthetic rubbers, fibrous materials (e.g., silk, rayon and cellulose), biomaterials (e.g., skin and muscle), and nonmetals including the modern composite materials are all viscoelastic materials.

2.1.1 Young's modulus of viscoelastic materials

An equation that describes a property of a material is called a constitutive equation of that material. The stress-strain relationship of viscoelastic materials is shown in Fig. 2.1.

Fig. 2.1 Stress-t and strain-t of viscoelastic materials

From Fig. 2.1 we can see, when a sinusoidal force (stress) is applied,

$$\sigma = \sigma_0 \sin \omega t \quad (2.1)$$

The mechanical response (strain) will also be a sinusoidal one with the same frequency, and a phase angle lag behind the force.

$$\varepsilon = \varepsilon_0 \sin(\omega t - \delta) \quad (2.2)$$

The constitutive equation of viscoelastic materials is as following:

$$\sigma = E^* \varepsilon = (E' + iE'') \varepsilon \quad (2.3)$$

Where E^* is called the complex Young's modulus, which is divided into two parts, that is, storage modulus E' and loss modulus E'' . The tangent of the angle called damping can be defined as:

$$\tan \delta = \frac{E''}{E'} \quad (2.4)$$

The complex modulus is typically used to represent viscoelastic behaviour. The determination of the components of the complex modulus must be done experimentally for individual materials by applying harmonic excitation over a range of frequency and temperature.

2.1.2 Modelling of viscoelastic materials

Traditional models for viscoelastic materials are one-dimensional lumped-element models made up of basic elastic and viscous elements, namely linear springs and dashpots (or dampers), respectively [1, 31]. Different combinations of springs and dashpots have been used to model different time-dependent and strain history-dependent viscoelastic behaviours, e.g., hysteresis, stress relaxation and strain creep. The simplest combinations are the Maxwell model, the Voigt model, and the standard linear model. Many other variations have also been developed by adding more springs and dashpots in series and in parallel. The commonly used models are shown in Fig. 2.2.

Fig. 2.2 Common models of viscoelastic materials

All of these models of discrete springs and dashpots are associated with a discrete spectrum of the relaxation function. The relaxation function of a model describes the gradual reduction of stress as a function of time, during the process of stress relaxation when a material (sample) is subject to a constant strain. Corresponding to a combination of an infinite numbers of Voigt and Maxwell elements, the relaxation function can be expressed as a sum of N exponential terms with N characteristic time constants, each associated with a characteristic coefficient [31]. The discrete model is shown in Fig. 2.3.

Fig. 2.3 Discrete spectra of viscoelastic materials

The equation of discrete model is

$$k(t) = \sum_{n=0}^N \alpha_n e^{-tV_n} \quad (2.5)$$

There are also some other constitutive models used in biomaterials such as quasilinear viscoelastic model and molecular model. Chan and Titze [28] used them to predict the viscoelastic shear properties of human vocal fold cover.

2.1.3 Creep and relaxation test

Viscoelastic materials show a time-dependent and history-dependent behavior. Relaxation and creep tests provide the simplest and most direct means of obtaining the mechanical properties involved in the linear theory of viscoelasticity. Relaxation function, creep function and some other properties such as relaxation and creep spectra can be procured by these tests.

When a material is subjected to a stress and then the stress is maintained constant afterward, the material continues to deform, and the phenomenon is called creep. The constant force is often applied by a dead weight, and the displacement is measured by a differential transformer, traveling microscope, or deflection of a light beam. When a body is suddenly strained and then the strain is maintained constant afterward, the corresponding stress induced in the body decreases with time, this phenomenon is called stress relaxation. The corresponding creep and relaxation functions for Maxwell, Voigt and Standard linear solid models are shown in Fig. 2.4.

Fig. 2.4 Creep and relaxation functions of three models

(a) Maxwell (b) Voigt (c) Standard solid

The uniaxial deformation test is the one normally used in creep and relaxation test, since its execution involves standard procedures and testing machines. We can also get the creep and relaxation functions via other deformation models such as shear and compress deformation.

2.2 Dynamic testing methods of viscoelastic materials

Dynamic mechanical analysis is to determine the dynamic mechanical properties of viscoelastic materials, that is, the storage modulus E' , the loss modulus E'' , and the

damping or internal friction, $\tan \delta = E''/E'$. As a result of this analysis, the relationship between the dynamic properties and the external variables (temperatures, pressure, time, frequency, type of deformation, etc.) can be explained. Several methods and instruments have been devised for the measurement of dynamic mechanical properties of viscoelastic materials in the past decades. Based on the frequency where the tests are given, it is divided into free vibration, forced vibration, resonant vibration and wave propagation. The suitable frequency domain for different methods is shown in Fig. 2.5.

Fig. 2.5 Different testing methods with frequencies

2.2.1 Free decay method

The most commonly used free decay method is known as torsion pendulum measurement which is an ISO standard [10]. A test specimen of uniform cross-section is gripped by two clamps, one of them fixed and the other connected to a disc, which acts as an inertial member, by a rod. The end of the specimen connected to the disc is excited, together with the disc, to execute freely decaying torsional oscillations. The inertial member is suspended either from the specimen or from a wire. Optical, electrical or other recording systems may be used provided they have no significant

influence on the oscillating system. The schematic diagram of apparatus is shown in Fig. 2.6.

Fig 2.6 Schematic diagram of apparatus for torsion pendulum method [10]

When a torsional impulse is added, the whole system vibrates freely decayed. The amplitude X versus time t for damped vibration is shown as Fig. 2.7.

Fig. 2.7 Amplitude X versus time t for damped vibrations

The logarithmic decrement for damped oscillations of pendulum plus specimen Λ may be calculated using the following equation:

$$\Lambda = \ln(X_p / X_{p+1}) \quad (2.6)$$

where X_p and X_{p+1} are the amplitudes of two successive oscillations in the same direction. The logarithmic decrement for damped oscillations of pendulum without the specimen is Λ_0 . Suppose the moment of inertia of the inertial member is I , frequency of the damped oscillating system is f_d , natural frequency of the pendulum is f_0 . Dimensional factor of the specimen is F_g . The damping correction factor is given by the equation:

$$F_d = 1 - (\Lambda / 2\pi)^2 \quad (2.7)$$

$$\text{then} \quad G' = 4\pi^2 I (f_d^2 F_d - f_0^2) F_g \quad (2.8)$$

$$G'' = 4\pi I f_d^2 (\Lambda - \Lambda_0) F_g \quad (2.9)$$

This method is usually used to measure complex modulus in the lower frequency. Hong and Lu measured viscosity based on an improved pendulum method [11].

2.2.2 Forced vibration method

When a system is subjected to harmonic excitation, it is forced to vibrate at the same frequency as that of the excitation. Dynamic Mechanical Analysis (DMA) is the most commonly used technique. The working principle is as shown in Fig. 2.1. It has been a commercial device widely used in lab and company for testing mechanical properties of viscoelastic materials. The commercial companies include TA Instruments, Rheometric Scientific, Gabo Qualimeter and so on. The deformation models include compression, tension, shear and bending. The testing models include single point measurement, temperature sweep, frequency sweep, temperature/frequency sweep, static/dynamic strain-, force-, or stress sweep [17].

Fig. 2.8 displays construction and operational characteristics of the GABO EPLEXOR. The pre-load is applied by the servomotor and introduced into the sample by the force transducer and clamping device. The transmission distance of the traverse is measured by the static elongation transducer and restricted by a limit switch. The electro-dynamic shaker generates the oscillating dynamic load of the sample. The dynamic oscillating deformation is being registered with the dynamic elongation transducer. The static and dynamic loading devices each have a cylindrical force line with flange, which are fastened with a clipping ring onto the desired clamping devices. The analog signals taken up by the force transducer (force) and elongation measuring system are being digitalised by A-D converters and then processed and evaluated by the processing computer. The advantages of DMA are that the size of the sample is small, there are a lot of testing models and it is feasible for a lot of testing standards. The disadvantage is the high price of the instrument.

The use of indentation tests as a tool for material characterization has increased rapidly in recent years. It can be done in a rather simple procedure by the use of a specimen with very limited size [7]. A lot of microindentors have the DMA loading model to test viscoelastic materials. Larsson and Carlsson [8] got data interpretation on microindentation of viscoelastic polymers. Shimizu, Yanagimoto et al. [9] investigated the time-dependent surface deformation of viscoelastic materials by the indentation load P versus depth h curves with the pyramidal indenter.

Dynamic stiffness technique is a kind of forced vibration method. It is divided into transmissibility and impedance method in the case of rod-type specimens. Pritz [12-14] excited a cylindrical or prismatic specimen into longitudinal vibration at one end, the other end being loaded by a mass, then he used a transfer function method to investigate the complex modulus of acoustic materials. Sim and Kim [3] developed a technique to estimate the properties of viscoelastic materials for finite element method application, on the basis of both theoretical considerations and data obtained from transmissibility measurements. Ödeen and Lundberg [15] determined the complex modulus of a linearly viscoelastic material by measuring the end-point accelerations of an impact-loaded rod specimen, while applying an iterative numerical scheme. Ahn and Kim [6] analyzed the sensitivity for estimation of complex modulus by dynamic stiffness method. If the specimen is cylindrical, of length L and cross-section area A , then the modulus E is obtained, provided that the specimen is not so short that corrections must be applied, nor so long that creep or toppling occurs. Fig. 2.9 shows the configuration of impedance method.

Fig. 2.9 Impedance method configuration [16]

2.2.3 Resonance method

When an oscillating force whose amplitude is fixed, but whose frequency may be varied, is applied to a mechanical system, the amplitude of the resulting vibration passes through a maximum at a frequency which is known as the resonant frequency of the system. The values of the resonant frequency depend on the elastic properties of the system, while the breadth of the resonance gives a measure of the dissipative forces. This is the principle of resonance method widely used in dynamic testing of viscoelastic materials.

In the resonant apparatus of Madigosky and Lee [18], a longitudinal wave was transmitted down a rod of material with accelerometers attached at both ends. The apparatus was shown in Fig. 2.10. An electromagnetic shaker (Bruel & Kjaer type 4810) was used to drive a test specimen at one end while the other end was allowed to move freely. Miniature accelerometers (Bruel & Kjaer type 8307) were mounted on each end to measure the driving point acceleration and acceleration at the free end. A random noise source was used to drive the test specimen. The complex Young's modulus was calculated at specific frequencies from the phase interference of the

extensional waves in the rod. The technique works well throughout the audio-range, but it is limited to one-dimensional sample geometry and a discrete set of frequencies.

Fig. 2.10 Resonance apparatus for measuring dynamic Young's modulus [18]

ASTM E756 [19] uses the cantilever beam technique to measure damping and complex modulus of materials. Results at different resonant frequencies are analyzed to get the mechanical properties. Grief and Hebert [20] used the resonant dwell technique and off-resonant test to characterize composite materials. Oyadiji and Tomlinson [21] determined the complex moduli of viscoelastic structural elements by resonance and non-resonance methods. Pritz [32, 33] measured damping ratio and complex moduli of some solid viscoelastic materials using resonant method. Lee, Lakers et al. [34] compared the resonant ultrasound spectroscopy and other resonant methods for the determination of viscoelastic properties.

2.2.4 Wave Propagation method

Several types of elastic (or anelastic) waves can propagate in solids. For example, flexural, extensional and torsional waves can travel along solids rods. In plates, only extensional and flexural waves need be considered. The velocity of propagation of all these waves depends on the elastic constants and the density of the material. Hence, dynamic elastic constants can be calculated from the velocity of the wave propagation in the given medium. When materials display viscous behaviour in addition to an elastic response, the parameters describing the dissipative forces can be determined from measurements of the amount of energy dissipated, phase relation between stresses and strain, and amplitude ratio of the measured stresses (or strains) at two different locations along the sample.

The propagation of disturbance along a filament is considered and accordingly the uniaxial complex modulus $E^*(i\omega)$ is the involved mechanical property. If the filament is very long and one end is subjected to a steady state harmonic oscillation condition, then harmonic waves are propagated along the filament with attenuation in the direction of propagation. The equation of motion is

$$\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\rho \omega^2 \bar{u}}{E^*(i\omega)} = 0 \quad (2.10)$$

where $\bar{u}(x, \omega)$ is the Fourier transformed displacement, has a solution in the form

$$u(x, t) = B e^{-\alpha x} e^{i\omega[t - (x/c)]} \quad (2.11)$$

where

$$c = |E^*| / \rho^{1/2} E_3, \quad \alpha = \rho^{1/2} \omega E_4 / |E^*| \quad (2.12)$$

$$[E^*(i\omega)]^{1/2} = E_3(\omega) + iE_4(\omega) \quad (2.13)$$

The solution (Eq.2.11) represents the propagation of a harmonic wave moving in the positive x direction, while c and α are the phase velocity and attenuation, respectively, and the complex constant B is to be determined from boundary conditions. Experimental measurements of the phase velocity and attenuation then allow the determination of the real and imaginary parts of the complex modulus $E^*(i\omega)$, at the frequency ω of the test. This method has been widely used to determine high frequency properties [2].

Pereira, Mansour et al. [22, 23] used this technique to measure the dynamic viscoelastic properties of excised skin when subjected to a low incremental strain. A strip of material was hung vertically from a small telephone speaker as shown in Fig. 2.11. A pulse generator connected to the speaker introduced a step change in voltage, which resulted in a displacement step propagating down the strip. The disturbance was measured by a phonographic pickup. The pickup was mounted on an arm instrumented with strain gages to measure the force between the pickup and sample.

Fig. 2.11 Wave propagation method

The complex extensional modulus was calculated by solving Eq. 2.12 and 2.13 for E_3 and E_4 , and substituting in the measured values of attenuation and velocity.

$$E_3 = \frac{\omega^2 c \sqrt{\rho}}{\omega^2 + c^2 \alpha^2}, \quad E_4 = \frac{\omega \sqrt{\rho} c^2 \alpha}{\omega^2 + c^2 \alpha^2} \quad (2.14)$$

where

$$E^* = E_1(\omega) + iE_2(\omega) = (E_3^2 - E_4^2) + 2iE_3E_4 \quad (2.15)$$

The storage modulus E_1 , and the loss modulus E_2 , were then found in terms of frequency, attenuation and velocity as:

$$E_1 = \frac{\rho c^2 \omega^2 (\omega^2 - \alpha^2 c^2)}{\omega^2 + \alpha^2 c^2}, \quad E_2 = \frac{2\rho \alpha (\omega c)^3}{\omega^2 + \alpha^2 c^2} \quad (2.16)$$

Mossberg, Hillstrom et al. [24] used the wave propagation experiment to identify viscoelastic materials. When the testing frequency is over 20kHz, the transferred wave is an ultrasonic wave. Ultrasonic measurement is widely used in medical checking which is introduced in the following section [35].

2.2.5 Comparison of the methods

The methods above are selected by different intentions, such as the frequency domain, the deformation model and the force model. Pendulum method can only get the mechanical properties at much lower frequencies. Madigosky and Lee's technique works well throughout the audio-range, but it is limited to one-dimensional sample geometry and a discrete set of frequencies. A disadvantage of resonance method is at once evident, namely that the complex modulus is obtained at only one frequency for each test. For DMA apparatus, the samples are very small (less than a few square centimeters), two-dimensional, clamped elements excited at very low frequencies

(usually at a few hertz) inside a small temperature-controlled chamber. This indirect method relies on the time–temperature superposition principle and the subsequent shift factors require constructing a master curve over a broad frequency range from measurements taken at multiple temperatures. Wave propagation method is only used in high frequency domain.

Obviously, besides various actuators excite the specimens, a great deal of load cells and displacement sensors are generally needed to measure the applied force and the corresponding deformation of viscoelastic materials. In tests it is generally noted that when a failure occurs in a sensor-actuator system, the fault is far more frequently due to sensor failure (or failure of related interfacing or wiring) than actuator failure. In dynamic testing especially at high frequencies, the mass effect of the sensors makes major difficulties in achieving accurate data. The installation of sensors is not feasible due to their size and mass at some conditions. When in situ measurement are needed neither of the methods can be used to obtain the mechanical properties of viscoelastic materials.

The samples must be manufactured into special shapes to cancel the shape effect in these methods. For force vibration method, the samples must be long and slim to make sure that the Young's modulus is measured. At the same time the length of the cylindrical specimen should not be so short that corrections must be applied, nor so long that creep or toppling occurs. ASEM 756 uses samples made in cantilever shape to measure the complex modulus and damping of the materials. The shape of the test samples are critical for resonant method and wave propagation method too. The

sample preparation is the most difficult thing for material characterization at some times.

A new method is needed to overcome these problems. In this new method, there are no specifications on the shape of the samples, no load effect of the sensor or no sensor for proper mechanical testing, and more simply to fit for in situ measurements.

2.3 Mechanical properties of human tissues

With the development of medical technique, more and more biomaterials are designed to substitute human tissues. In order to function properly, mechanical properties of the biomaterials should be as same as that of human tissues. For computer simulation, knowledge of mechanical properties of tissues is required to predict tissue forces and deformation through a set of constitutive equations [25-28]. When there are pathological changes in tissues, the mechanical properties of the tissues are always different from healthy ones. So the changes of mechanical properties of tissues are usually used as an indicator for checking.

2.3.1 In vitro mechanical properties of human tissues

Measurements have been made on the material properties of different kinds of tissues for many years. Tissue properties have been determined in the past by extracting samples from the organ of interest, and measuring their responses similar to those used to get the properties of engineering materials [4, 27, 28, 36-40]. The mechanical

properties of some common tissues and biomaterials are listed in Table 2.1. The data are based on the review of some literatures [23, 31, 41].

Table 2.1 Mechanical properties of some common soft tissues and biomaterials

Material	Young’s modulus (MPa)	Tensile strength (MPa)
Resilin	1.8	3 [31]
Abduction	1-4	[31]
Elastin	0.6	[31]
Skin and muscle	0.05-0.2	[41]
Brain	0.01-0.3	2100 (bulk modulus) [41]
Rabbit skin	0.173+i*0.01 (1500kPa)	[23]
Collagen (along fiber)	1×10 ³	50-100[31]
Bone (along osteones)	1×10 ⁴	100[31]
Biomedical elastomers	1.15-105	[41]
Lightly Vulcanized rubber	1.4	[31]
Oak	1×10 ⁴	100[31]
Mild steel	2×10 ⁵	500[31]

2.3.2 In vivo mechanical properties of human skin

Tissue samples removed from the living state can radically change their responses, as the blood supply to the cells is cut off, as temperature and humidity change, and as the boundary conditions of the sample in the organ are different from those in the testing device. Recent work in measuring tissue properties has begun looking at tissues in

vivo and in situ, that is in the living state and in the environment of the body, without removal [42, 43].

Elasticity imaging or elastography is widely used to measure and image the mechanical properties of tissues in vivo qualitatively. The general approach is to measure tissue motion caused by a force or displacement and use it to reconstruct the elastic parameters of the tissue. The excitation stress can be either static or dynamic (vibration). Dynamic excitation is of particular interest because it provides more comprehensive information about tissue properties in a spectrum of frequencies[44-47]. One kind of these methods is that an external source of forced vibration is imposed and the Doppler technique is used to determine the internal vibration of soft tissue and to generate the relevant image. Magnetic resonance elastography is a recently developed method that employs a mechanical actuator to vibrate the body surface and then measures the strain wave with phase-sensitive magnetic resonance imaging (MRI). Another method is vibro-acoustography where a localized oscillating radiation force is applied to the tissue and the acoustic response of the tissue is detected by a hydrophone.

Knowledge about the mechanical behaviour of the skin in vivo is of importance for cosmetic and clinical applications. It can help to quantify the effectiveness of cosmetic products and to evaluate both skin diseases and pressure sores [29]. A lot of researchers have studied the mechanical properties of skin[48, 49]. Several studies have been performed to determine the mechanical properties of the skin in vivo. The most frequently used techniques are tensile[50-53], indentation[54, 55], torsion[56] and suction[29, 54, 57-60]. These tests mostly measure the stress-strain curve to get

the Young’s modulus of skin under static loads. The testing sites were on the forearm skin without explanation. The results are listed in Table 2.2.

Table 2.2 Young’s modulus of skin in vivo

Method	Young’s modulus (MPa)	Indenter or aperture
Indentation	0.0011-0.002	20mm indenter[54]
Torsion	0.42-0.85	Disk of25mm diameter and guard ring of 35mm diameter[29]
	0.02-0.1	[59]
	1.1-1.32	[61]
Tensile	4.6-20	Load pads of 10×10mm with a distance of 5mm[29]
Suction	0.13-0.26	2mm aperture[57]
	0.153	6mm aperture with 100mbar suction[59]
	0.056	[29]
	18-57	[59]
	0.13-0.17	[59]
	0.20-0.32	(forehead)
	0.25	[59]
		270-350N/m upper back
		270-800N/m anterior part[59]

Only a few researches work on dynamic characterization of human skin [22, 25, 26, 29, 46, 57, 62-64]. Ultrasonic technique is used for dynamic testing in ultrasonic

frequencies and the results are qualitative. Shear wave propagation is a most promising approach for the evaluation of mechanical properties of human skin [57, 62]. This parameter was shown to be proportional to the viscoelasticity of the materials tested when acoustic waves in the frequency of 0.5-30kHz were used. Vexler, Polyansky et al [64] evaluated skin viscoelasticity and anisotropy by measurement of speed of shear wave propagation with Viscoelasticity Skin Analyzer (VESA).

2.3.3 Equipment for testing mechanical properties of skin

There are kinds of equipment for testing mechanical properties of tissues. These equipments are based on different techniques, such as mechanical vibration, ultrasound, shear resonant method and so on. The commercial equipment for testing mechanical properties of skin is reviewed and listed in Table 2.3.

Omata and Terunuma [65] and Lindahl, Omata et al. [66] set up a tactile sensor for detection of mechanical properties of human skin in vivo. The experimental set up is shown in Fig. 2.12. It consists of a spring loaded piezoelectric transducer, a vibration pickup and a displacement sensor. When an alternating voltage is applied across the electrode, the ceramic transducer is able to vibrate freely in the direction of its length. The vibration pickup detects the generated vibration and the feedback circuit system oscillates at the resonance frequency (61kHz) as the amplification is increased in the system. If the free end of the transducer is pressed against a surface, the resonance frequency changes depending upon the acoustic impedance of the object. The stiffness of the materials can be obtained by comparing to the results of the reference samples.

The same equipment was used in testing skin and other soft tissue by their group [35, 67, 68]. A set of commercial equipment (Venustron) is made based on this technique.

Table 2.3 Commercial equipment for skin testing

Equipment	Picture	Specification
Cutometer Courage-Khazaka Company		Suction [69]
DermaLab Elasticity module Cortex Technology, Hadsund, Denmark		Suction [70]
LSR		Tensile [71]
Venustron Axiom Company		Indentation 61kHz[72]

Fig. 2.12 Tactile sensor setting up

Linear Skin Rheometer (LSR) [73, 74] is a precision mechatronic instrument that was designed to measure the viscoelastic properties of the stratum corneum of the skin at 3~5 Hz. To make a measurement a probe is attached to the surface of the skin or larynx, and a sinusoidal force is then applied along its axis and thereby onto the tissue. Typically the peak force applied will be in the region of 3g for skin and 1g for the larynx. Inside the measuring head, the probe is attached to a load cell, which is moved along the probe axis by a motor-driven lead screw. An LVDT is used to monitor the position of the load cell and probe. The load cell used in the LSR is supplied by Maywood and has a full scale reading of 10g, with an overall accuracy of better than 0.02g. The LVDT is supplied by Solartron, type DF2.5, which has an accuracy of better than 4 Microns.

In order to ensure un-biased readings, the DermaLab (Cortex Technology, Hadsund, Denmark) features a light weight probe which, when glued to the skin using a double adhesive sticker, eliminates movement artifacts from holding the probe. With the probe in place, negative pressure will elevate the skin, and the differential negative

pressure needed to lift the skin a predetermined distance is used as input to calculate Young's modulus. The unit allows for adjusting the airflow according to the measurement site and actual skin condition. Cutometer (Courage & Khazaka, Cologne, Germany) is also a commercial instrument for testing skin. The vertical displacement of the skin surface is measured using an optical system in Cutometer and by variation of capacitance in the DermaLab.

Most often, a single stiffness or strength parameter is reported for the case of static or quasi-static test in Cutometer and DermaLab. LSR measures the viscoelasticity of skin at 3~5Hz but it is not portable. Vennustron tests skin at its resonant frequency (61kHz). Beyond the circumstances of testing, insufficient data is available regarding a complete description of the skin in question. The properties of the skin in the frequency of some hundred Hertz are not obtained for portable measurement. A new technique for dynamic testing mechanical properties of human skin in vivo quantitatively is needed.

2.4 Summary

Viscoelastic materials are introduced in this chapter firstly. Dynamic testing methods are reviewed in detail. These existing methods require actuators and sensors in exciting and then measuring the load and deformation which renders the loading effect of sensors, for example, the inevitable mass effect of sensors at high frequency testing. Furthermore, installation of sensors and actuators is difficult when the space is small. Samples preparation is also a major problem to overcome in applying the existing

methods. Moreover, they are not suitable for in situ testing. A new method is needed to measure the mechanical properties of viscoelastic materials.

Mechanical properties of soft tissues in vitro and in vivo are reviewed. The mechanical properties of skin are reviewed in detail since its mechanical properties are important for cosmetic and clinical applications. Even there are some kinds of commercial equipment for testing skin, there is still less knowledge of the skin at some hundreds Hertz. A new technique for dynamic testing mechanical properties of human skin in vivo quantitatively is needed.

Chapter 3

Transduction matrix method and its application on shaker

3.1 Introduction

From energy concept, transducers may generally divide into two classes: sensors and actuators. Sensors are devices which monitor a parameter of a system, hopefully without disturbing that parameter. In real sensors it is not possible to achieve the ideal of a transducer which monitors a parameter while having absolutely no effect on it, because the act of the energy exchange. For this reason, most sensors are low-power device and are small in order to minimize the load they impose on the system that they are monitoring. Actuators are devices which impose a state on a system, hopefully independent of the load applied to them. The ideal of a load-independent actuator can never be achieved for a transducer with finite energy, because there will always be a load exceeds its energy handling ability. In general, we minimize the effect of the load seen by the actuator by having high-power actuator, so that typical loads cause only a little perturbation. The distinctions between sensors and actuators explain the difficulties to build a transducer which function well, as both a sensor and an actuator.

While with the development of smart materials and structures, the coupling effects between electrical and mechanical domain have been widely studied. Hagood and Anderson [75] investigated the possibility of using a single piezoelectric element simultaneously as both a structural actuator and collocated sensor. A general

formulation for coupled electromechanical modeling was specialized to the analysis of transducers used for simultaneous sensing and actuating. Its application to controlled structures was also studied [76]. Studies on the coupling of the electrical impedance of piezoceramic materials and the mechanical impedance of structures were reported by the group of C. A. Rogers in Center for intelligent material systems and structures of Virginia Polytechnic Institute [77-80]. In their work, piezoceramic materials patches were bonded on or emerged into the structure under testing for structural health monitoring. Variation of the mechanical impedance of the structure was detected by measuring the electrical impedance of PZT because the changes make the electrical impedance of the PZT patches different from that of unchanged structure. Some researchers focused their work on a self-sensing technique which uses a transducer to both actuate and sense concurrently to control medium-scale electric motors [81, 82].

Ling et al. set up a new technique called transduction matrix which used a four-pole model to model the electromechanical system [83-87]. The mechanical impedance of the output port of the transducer can be deduced from the electrical impedance of the input port using this technique. It cancelled the sensors and had no load effect to the testing systems. This technique is firstly introduced in section 2. Then the theoretical formulae of transduction matrix model of an electrodynamic shaker are briefly derived. The characteristics of shaker are obtained by experimental method. The sensing capability of the shaker is analyzed in detail which shows the optimizing requirements of a shaker when used as a simultaneous sensing and actuating (SSA) device.

3.2 Transduction matrix method

3.2.1 Four-pole theory

From the viewpoint of system dynamics, a complex system can be separated as many subsystems. Each subsystem is a “black box” which can be viewed as a multi-port device. The relationship of variables at different ports can be described with a transfer matrix. The theory is first used for circuit network analysis. In 1960’s, the multi-port theory was applied on other subsystems including mechanical element, hydraulic element, etc. Thereafter, the theory is commonly used for system modeling and element coupling [88]. Among various multi-port theories, two-port or four-pole theory that has only one input port and one output port is more useful for its simple form and common usage while the schematic is shown in Fig. 3.1.

Fig. 3.1 Schematic four-pole model

The four-pole device exchanges energy with others at only two ports. At each port we define two variables, one of which is selected as being of primary practical interest to use. The product of the two variables gives the instantaneous power flowing through the port. We use the names effort variable and flow variable. The most common applications are shown in Table 3.1.

Table 3.1 Common application of four-pole model

Class of system	Effort variable (<i>e</i>)	Flow variable (<i>f</i>)
Electrical circuits	Voltage	Current
Mechanical systems	Force (Torque)	Velocity (Angle Velocity)
Hydraulic systems	Pressure	Volume flow rate

The conventional four-pole theory is severely restricted in application. It is generally regarded as a purely frequency domain approach (applicable to steady-state performance analysis only) that applies to four-pole whose elements are entirely linear and non-energetic, that means energy is neither stored nor dissipated [89]. Linear system which accords with superposition theorem and reciprocity theorem is mostly used because of its simplification on analysis.

The goal of four-pole theory is to develop the links between the effect and flow variables. There are six possible sets of choices for the independent variables used to represent a transducer. For each of these we may generate the functional representation linking the dependent and independent variables. If the system is linear, only one kind of variables is needed for the analysis because other kinds can be deduced.

A passive transducer is the one energy can flow in either direction, i.e., which can function as either a sensor or an actuator. For such transducer, it is possible to derive relation which must hold true and which reduce some of the problems associated with measuring the elements of the various 2-port matrices. Then it is immaterial which

port we label the input port and which is labeled the output port. Among these six possible sets of choices for the independent variables used to represent system, the transfer matrix is usually used:

$$\begin{Bmatrix} e_{in} \\ f_{in} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} e_{out} \\ f_{out} \end{Bmatrix} \quad (3.1)$$

If we couple a chain of n four-pole devices such that the output of one is the input of the next, seen in Fig. 3.2, we can easily generate the equations relating the effort and flow variables of the first and last devices as

$$\begin{Bmatrix} e_{li} \\ f_{li} \end{Bmatrix} = [M_1][M_2] \cdots [M_n] \begin{Bmatrix} e_{on} \\ f_{on} \end{Bmatrix} = [M_s] \begin{Bmatrix} e_{on} \\ f_{on} \end{Bmatrix} \quad (3.2)$$

where the M_j ($j = 1, 2, \dots, n$) is the 2-by-2 subsystem transfer matrix of each four-pole device, and M_s is a 2-by-2 system transfer matrix that is their left-to-right product. This process can be continued to combine any number of elements in a serried string.

Fig. 3.2 Cascaded 2-port model

3.2.2 Transduction matrix model

Making use of the theory introduced above, we can establish the relationship between the input and output variables of an electromechanical transducer. In the four-pole theory mentioned above, the input port and the output port variables are always the same, that is, both are electrical variables or mechanical variables. While in electromechanical systems, there are always two different variables in two ports, one

port electrical variables and the other mechanical variables. We get the four-pole model of it shown in Fig. 3.3. In the model, voltage and current are defined to be the variables at the input port, and force and velocity at the output port.

Fig. 3.3 Four-pole diagram of electromechanical transducer

The relationship between the electrical and mechanical parameters is given by

$$\begin{Bmatrix} E \\ I \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} \quad (3.3)$$

The four elements of the transduction matrix could be determined by clamped or free boundary condition.

$$T_{11} = \frac{E}{F} \Big|_{v=0} \quad \text{transfer function of voltage to force if the output point is clamped}$$

$$T_{21} = \frac{I}{F} \Big|_{v=0} \quad \text{transfer function of current to force if the output point is clamped}$$

$$T_{12} = \frac{E}{v} \Big|_{F=0} \quad \text{transfer function of voltage to velocity if the output point is free}$$

$$T_{22} = \frac{I}{v} \Big|_{F=0} \quad \text{transfer function of current to velocity if the output point is free}$$

From the definitions of the four transfer functions, we can get the transduction matrix which can describe the system in the round. The determinant of the transduction matrix should be 1 according to the reciprocity theorem of the linear system.

When we deduce Eq. 3.3, we can get

$$Z_m = \frac{F}{v} = \frac{T_{22}Z_e - T_{12}}{T_{11} - T_{21}Z_e} \quad (3.4)$$

where $Z_e = \frac{E}{I}$ is the electrical impedance of the input port

$Z_m = \frac{F}{v}$ is the mechanical impedance of the output port

From Eq. 3.4 we can see that the electrical impedance of the input port and the mechanical impedance of the output port are connected using the transduction matrix model of the electromechanical transducer. The four transduction elements can be obtained via theoretical or experimental methods. After getting the transduction matrix, the mechanical impedance of the output port can be deduced through the electrical impedance of the input port. On the measurement concept the electrical variables are much easier to obtain compared with the mechanical variables.

As we known, actuator is a transducer which changes the electrical power into the mechanical power, that is, from voltage and current to force and velocity. Also from the Eq. 3.4, we can get the mechanical impedance from the electrical impedance when we know the transduction matrix. From this point we can say the actuator can also work as the sensor simultaneously. So we called the device as the SSA (simultaneous sensing and actuating) device.

Compared to the traditional approach of using separate actuators and sensors, this method can offer several advantages. A reduction in the number of sensing and actuation devices, and associated power, wiring and interfacing, immediately reduces cost and complexity. The robustness is increased if an actuator can be used without

sensors; it is generally noted that when a failure occurs in a sensor–actuator system, the fault is far more frequently due to sensor failure (or failure of related interfacing or wiring) than actuator failure. It is also particularly suitable for devices where the addition of sensors is not feasible due to their size and mass. It is predicted that transduction matrix method will become an important part of micro-miniature devices and be very useful at in situ measurements.

Transduction matrix method has been applied on structural integrity monitoring, acoustic material characterization, wire bond monitoring, drilling monitoring and so on [83-87, 90-94]. A piezoceramic inertial actuator was used to monitor structural integrity and measure mechanical impedance of a structure. Because of elimination of sensors and compactness of piezoceramic inertial actuators, this method provides a neat and convenient way for mechanical impedance detection of structures on site. The specific acoustic impedance or absorption coefficient of acoustic materials was measured using a standing wave tube without a microphone via transduction matrix method. It is suitable for in situ study of acoustical or vibrational conditions of the material or structural specimen at the end of a duct with geometry making microphone installation difficult or impossible. Research on monitoring wire-bonding, ultrasonic welding, EDM (electrical discharge machining), drilling mechanism by transduction matrix method has been doing in recently years. The difficulties of measuring force and velocity at high frequency, drilling torque and drilling speed in situ and the complex working conditions were overcome since only the electrical impedance are needed for measuring.

Shaker is widely used as an actuator in dynamic testing. In the following the transduction matrix method applied on the shaker is then investigated for the first time.

3.3 Derivation of theoretical model of shaker

The electromagnetic shaker is widely used in mechanical testing working as an exciter. When current passes through a coil placed in a magnetic field, a force F proportional to the current I and the magnetic flux intensity D , is produced which accelerate the component placed on the shaker table:

$$F = DIl \quad (3.5)$$

where l is the length of the coil. The magnetic field is produced by permanent magnets in small shakers while electromagnets in large shakers. The magnitude of acceleration of the table or component depends on the maximum current and the masses of the component and the moving element of the shaker. If the current flowing through the coil varies harmonically with time (a.c. current), the force produced also varies harmonically. For an electrical conductor, in the form of a coil, moves in a magnetic field, a voltage E is generated in the conductor. The value of E in volts is given by

$$E = Dlv \quad (3.6)$$

where v is the velocity of the conductor relative to the magnetic field (meters/second), the magnetic field may be produced by either a permanent magnet or an electromagnet. Since the voltage output of an electromagnet transducer is proportional to the relative velocity of the coil, they are frequently used in “velocity pickups”.

From Eq. 3.5 and 3.6 we can get the coefficients

$$Dl = \frac{E}{\nu} = \frac{F}{I} \tag{3.7}$$

where F denotes the force acting on the coil while carrying a current I . This equation shows that the performance of an electromagnetic transducer can be reversed. When the coil is of circular configuration, of n turns of radius r and in a radial magnetic field of magnetic flux density D , one can estimate the coefficients from:

$$K_{f/i}^{\Delta} = \text{N/amp} = K_{e/\dot{x}}^{\Delta} = \text{V/(m/sec)} = 2\pi nrD \tag{3.8}$$

Fig. 3.4 Electromechanical model of the electromagnetic shaker (taken from [95])

Figure 3.4 shows the schematic model of an electromagnetic shaker which is taken from Doebelin [95]. The flexure K_f is an intentional soft spring (stiff, however, in the radial direction) that serves to guide the axial motion of the coil and table without the need for conventional sliding or rolling bearings whose friction, wear, and lost

motion are unacceptable in this type of application. Flexure damping B_f is usually intentional, fairly strong, and obtained not by mechanical dashpots but by laminated construction of the flexure spring, using layers of metal, elastomer, plastic, and so on. The coupling of the coil to the shaker “table” would ideally be rigid so that magnetic force is transmitted undistorted to the mechanical load. Thus K_{tc} and B_{tc} represent “parasitic” effects rather than intentional spring and damper elements. Spring stiffness K_{tc} is generally large while B_{tc} , due to internal losses in stressed metal, coil potting compounds, and so on is quite small.

If a package to be shaken is rigidly bolted to the table and behaves mainly as a mass element, we can analyze the entire system just as it appears in Fig. 3.4 since the package mass just becomes part of M_t . Similarly, if a specimen is essentially a single spring and/or damper and is fastened between the table and the foundation, it experiences the same relative motions as K_f and B_f and may thus be included with them in the analysis. We could also in this case lump the specimen’s mass at the table attachment point and include it in M_t . Our analysis will be useful for the bare shaker as shown in Fig. 3.4 or for the two kinds of specimen just describes. Application of Newton’s law to the table and coil masses gives:

$$-K_f x_t - B_f \dot{x}_t - K_{tc}(x_t - x_c) - B_{tc}(\dot{x}_t - \dot{x}_c) = M_t \ddot{x}_t \quad (3.9)$$

$$K_{tc}(x_t - x_c) + B_{tc}(\dot{x}_t - \dot{x}_c) + K_{f/i} i = M_c \ddot{x}_c \quad (3.10)$$

In the electrical circuit, e_i is the output voltage of the electronic power amplifier that drives the shaker, while R and L are the total circuit resistance and inductance, including contributions from both the shaker coil and the amplifier output circuit. Krichoff’s voltage loop law gives:

$$-e_i + iR + L \frac{di}{dt} + K_{e/\dot{x}} \dot{x}_c = 0 \quad (3.11)$$

As we have mentioned above, the table and the coil can be seen as a rigid body so the difference between x_t and x_c is ignored. That means:

$$x_t = x_c \quad \text{and} \quad \dot{x}_t = \dot{x}_c \quad (3.12)$$

Then Eq. 3.9 to 3.11 can be simplified as:

$$K_{f/i} i - K_f x_t - B_f \dot{x}_t = (M_t + M_c) \ddot{x}_t \quad (3.13)$$

$$iR + L \frac{di}{dt} + K_{e/\dot{x}} \dot{x}_t = e_i \quad (3.14)$$

Supposing we add a sinusoidal voltage signal to the shaker,

$$e_i = E e^{i\omega t} \quad (3.15)$$

Corresponding the current and displacement are also signals with the same frequency.

Then Eq. 3.13 and 3.14 can be deduced as:

$$K_{f/i} I - \left(\frac{K_f}{j\omega} + B_f + Mj\omega \right) v = F \quad (3.16)$$

$$(R + j\omega L) I + K_{e/\dot{x}} v = E \quad (3.17)$$

where $M = M_t + M_c$. We can also change the two equations in the form:

$$\frac{R + j\omega L}{K_{f/i}} F + \left[\frac{(R + j\omega L) \left(\frac{K_f}{j\omega} + B_f + Mj\omega \right)}{K_{f/i}} + K_{e/\dot{x}} \right] v = E \quad (3.18)$$

$$\frac{1}{K_{f/i}} F + \frac{\frac{K_f}{j\omega} + B_f + Mj\omega}{K_{f/i}} v = I \quad (3.19)$$

Eq. 3.18 and 3.19 are also the four-pole model of the shaker. We write it in the forms of matrix as mentioned in section 3.2.

$$\begin{Bmatrix} E \\ I \end{Bmatrix} = \begin{bmatrix} \frac{R + j\omega L}{K_{f/i}} & \frac{(R + j\omega L) \left(\frac{K_f}{j\omega} + B_f + Mj\omega \right)}{K_{f/i}} + K_{e/\dot{x}} \\ \frac{1}{K_{f/i}} & \frac{\frac{K_f}{j\omega} + B_f + Mj\omega}{K_{f/i}} \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} \quad (3.20)$$

The determinant of the transduction matrix is

$$|T| = \frac{R + j\omega L}{K_{f/i}} \cdot \frac{\frac{K_f}{j\omega} + B_f + Mj\omega}{K_{f/i}} - \frac{1}{K_{f/i}} \cdot \left[\frac{(R + j\omega L) \left(\frac{K_f}{j\omega} + B_f + Mj\omega \right)}{K_{f/i}} + K_{e/\dot{x}} \right] = -1 \quad (3.21)$$

which is the characteristic of the transduction matrix method. So the shaker can work as a SSA device from the theoretical analysis.

The four-pole model of the shaker is derived from the theoretical analysis. The elements of the transduction matrix are expressed by the mechanical and electrical parameters of the shaker in Eq. 3.20. Apparently they are the inherent characteristics of the shaker. We can measure all the parameters in the matrix and then calculate the four transduction functions via the theoretical model. But it is a little tedious to measure so many parameters, moreover, the damping is too difficult to measure with accuracy. We can also measure the transduction matrix directly. In next section the experimental method to get the four-pole model of the shaker system is introduced.

3.4 Characterization on transduction matrix of shaker

From section 3.2 we know that the electromechanical device can work as a SSA device using the transduction matrix method. We also analyse the theoretical four-pole model of the electromagnetic shaker at section 3.3. Next we will use a few methods to characterize the electromagnetic shaker. Later we will use the transduction matrix method to measure mechanical properties of viscoelastic materials, which will be introduced in Chapter 4.

3.4.1 Experimental equipment

A mini shaker investigated is driven by a power amplifier. An impedance head is screwed at the output port of the shaker to measure the force and velocity. Corresponding charge amplifier and conditioning amplifier are connected to the output of the impedance head to amplify the signals. A dynamic signal analyzer is used to gather the output signals of the charge amplifiers and voltage and current signal just before the input port of the shaker. It also works as a signal generator to send driven signal to the power amplifier. The equipment is listed in Table 3.2.

Table 3.2 Equipment list for experiments of shaker calibration

Equipment	Brand	Type
Mini Shaker	Brüel & Kjær	4810
Power Amplifier	Brüel & Kjær	2712
Impedance Head	Brüel & Kjær	8001
Condition Amplifier	Brüel & Kjær	2626
Charge Amplifier	Brüel & Kjær	2635
Dynamic Signal Analyzer	HP	35670A
Three-D Stage	Newport	ESP300

3.4.2 Direct method

Equation 3.3 suggests a number of possibilities for the implementation of characterization shaker using experimental measurements. We could arrange our apparatus $F \equiv 0$, apply a sinusoidal sweep, pulse, or random drive signal at E and compute

$$T_{12}(i\omega) = \frac{E(i\omega)}{v(i\omega)} \quad (3.22)$$

$$T_{22}(i\omega) = \frac{I(i\omega)}{v(i\omega)} \quad (3.23)$$

Returning to Eq. 3.3, repeating the above testing with $v \equiv 0$ would give

$$T_{11}(i\omega) = \frac{E(i\omega)}{F(i\omega)} \quad (3.24)$$

$$T_{21}(i\omega) = \frac{I(i\omega)}{F(i\omega)} \quad (3.25)$$

$F \equiv 0$ means the output force is zero which is the free condition in the mechanical port, while $v \equiv 0$ means the output velocity is zero which is the clamp condition in the mechanical port. Then we can get the four transduction elements using the free and clamped boundary condition. The four-pole parameters are in general sinusoidal transfer functions and when measured experimentally we have only graphs of amplitude ratio and phase versus frequency, not analytical functions. However, these numerical values are sufficient to carry out the operations of the four-pole model of the system. The experimental set up is shown in Fig. 3.5 and Fig. 3.6.

Fig. 3.5 Characterization of shaker as an SSA device ---- free condition

Fig. 3.6 Characterization of shaker as an SSA device ---- clamp condition

In Fig 3.5, which is the free boundary condition, the force of the output point is equal to zero. We can get the transduction elements of T_{12} and T_{22} . While in Fig. 3.6, we do not use the data of the velocity channel. The reason is that in clamped boundary condition the velocity is equal to zero. We can get the transduction elements of T_{11} and

T_{21} . In experiments the dynamic signal analyzer HP 35670 sends signals to the power amplifier to actuate the shaker. The voltage we set on HP 35670 is 200mV, the sweeping frequency domain is from 1Hz to 1600Hz. The integration time and the setting time are set at 20 cycles. The power amplifier is set at proper gain for the shaker to get good working condition. The results are shown in Fig. 3.7 - Fig. 3.10. Also we can get the determinant of the transduction matrix shown in Fig. 3.11.

From the figures we see that T_{11} and T_{21} are similar while T_{12} and T_{22} are similar. From Fig. 3.11 we can see the determinant of the transduction matrix is 1 or so and the phase is near 0 rad especially below 750Hz.

Fig 3.7 Transduction element T_{11}

Fig 3.8 Transduction element T_{12}

Fig 3.9 Transduction element T_{21}

Fig 3.10 Transduction element T_{22}

Fig. 3.11 Determinant of transduction matrix [T]

3.4.3 Correction of the direct method

The measurements described above are not always easy to make with necessary accuracy. The clamped boundary condition is extremely easy to apply in a theoretical analysis, simply by adding restrictions on the appropriate coordinates, while it is much more difficult to implement in the practical case. Making the velocity v exactly zero when measuring T_{11} and T_{21} is impossible since no “perfectly rigid wall” is available in a real-world laboratory. It is very difficult to provide a base or foundation which is sufficiently rigid to furnish the necessary grounding to the tested system. All structures have finite impedance (or a non-zero mobility) and thus cannot be regarded as truly rigid. It is less easy to approximate the grounded condition without taking extraordinary precautions when designing the support structure.

As shown in Fig. 3.6, the clamped condition is gained by connecting the indenter with a clamped 3-D stage. The velocity at the “clamped” boundary condition is shown in Fig. 3.12.

Fig. 3.12 Velocity in “clamped” boundary condition

We rewrite Eq. 3.3 as:

$$\begin{aligned} E &= T_{11}F + T_{12}v \\ I &= T_{21}F + T_{22}v \end{aligned} \quad \rightarrow \quad \begin{aligned} T_{11} &= \frac{E - T_{12}v}{F} \\ T_{21} &= \frac{I - T_{22}v}{F} \end{aligned} \quad (3.26)$$

The free situation is easily acquired in experimental set up shown in Fig. 3.5. So the two transduction elements in free situation are with enough accuracy. Then we can get the corrected elements T_{11} and T_{21} by Eq. 3.26 considering the velocity at the “clamped” experimental set up. Obviously the determinant of the transduction matrix does not change via this derivation. The corrected T_{11} and T_{21} are shown in Fig. 3.13 and 3.14. The results obtained by the direct method are also shown in these figures. It is clear that the corrected results are much smoother except at some points.

Fig. 3.13 Corrected transduction element T_{11}

Fig 3.14 Corrected transduction element T_{21}

The direct method and corrected direct method are based on the mechanical free and clamp boundary conditions which are the extreme situations in mechanical port. Even we consider the velocity of the “clamped” situation, there did be a lot of difficulties to realize the clamped boundary condition. Of course the motion need not be exactly zero; however, at high frequency all the motions become small and it becomes increasingly difficult to keep the motion of the “fixed point” small relative to that of intended moving point. So we should find other methods to calibrate the system, for example, adding other boundary conditions other than the extreme boundary conditions. One method is to add different masses at the output point.

3.4.4 Adding mass method

Instead of connecting the output point with a “rigid wall” which is impossible in real world, a compromise procedure can be applied in some cases in which the testing

system is connected at certain coordinates to another simple component of known mobility, such as a specific mass. This modified or “loaded” test-piece is then studied experimentally and the effects of the added component “removed” analytically. We will use different masses added on the impedance head to characterize the shaker system.

Four-pole model of the shaker and added mass system

As we know, from the four-pole theory, we can get the four-pole model of a mass [96]. Since the mass can be considered as a rigid body, the velocities of its connections are equal. The force required to give the mass a velocity v_1 is $j\omega m v_1$ (or $j\omega m v_2$). If a force F_2 is transmitted, this must be added to $j\omega m v_1$ to determine F_1 .

$$\begin{aligned} F_1 &= F_2 + j\omega m v_2 \\ v_1 &= v_2 \end{aligned} \quad (3.27)$$

We can write it in matrix form:

$$\begin{Bmatrix} F_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} 1 & j\omega m \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix} \quad (3.28)$$

Note that the determinant of transduction matrix is equal to unity. This is a characteristic of all systems made up of linear bilateral elements and is another indication of reciprocity.

From the introduction above and the theory in section 3.2, our testing system can be seen as two subsystems in series. The shaker-impedance head can be seen as the first electromechanical system which can be modeled as the first four-pole model, while

the added mass can be seen as the second mechanical system modeled as the second four-pole model. Its schematic model is shown in Fig. 3.15.

Fig. 3.15 Schematic model of the SSA and added mass system

The two subsystems' four-pole model equations are

$$\begin{Bmatrix} E_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} F_{o1} \\ v_{o1} \end{Bmatrix} \quad (3.29)$$

$$\begin{Bmatrix} F_{i1} \\ v_{i1} \end{Bmatrix} = \begin{bmatrix} 1 & j\omega m \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} F_2 \\ v_2 \end{Bmatrix} \quad (3.30)$$

From Eq. 3.30 when the mass is not connected to the third subsystem, the force at its output is zero. Then the force in its input port is:

$$F_{i1} = j\omega m v_2 = j\omega m v_{i1} \quad (3.31)$$

Since the two subsystems are connected, it is evident that

$$\begin{aligned} F_{o1} &= F_{i1} \\ v_{o1} &= v_{i1} \end{aligned} \quad (3.32)$$

then we can get the four-pole model of the first subsystem:

$$\begin{Bmatrix} E_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} j\omega m v_{i1} \\ v_{i1} \end{Bmatrix} \quad (3.33)$$

Since the added mass is already known, we can measure the velocity of the mass, the voltage and the current to calculate the transduction matrix of the first subsystem, that is, the shaker & impedance head system.

Experiments on added masses

A group of simultaneous equations are built as following based on a set of $j\omega m v$ and v obtained by adding various masses to the mechanical output port of shaker & impedance head testing system.

$$\begin{bmatrix} E_1 & I_1 \\ E_2 & I_2 \\ \vdots & \vdots \\ E_n & I_n \end{bmatrix} = \begin{bmatrix} j\omega m_1 v_1 & v_1 \\ j\omega m_2 v_2 & v_2 \\ \vdots & \vdots \\ j\omega m_n v_n & v_n \end{bmatrix} \begin{bmatrix} T_{11} & T_{21} \\ T_{12} & T_{22} \end{bmatrix} \quad (3.34)$$

When $n = 2$, the transduction functions can be calculated by the equations. While more masses are used the more accurate results of the transduction functions can be obtained. Theoretically there are no exact solutions for the over-determined simultaneous equations while only the least square solutions can be calculated for practical use.

There are 20 masses to be used for characterize the four pole model of the shaker. They are divided into 4 groups according to the diameter of the aluminum bar we manufactured and there are 5 masses in each group. The masses of the aluminum bars measured by a balance are list in Table 3.3. The picture of the added masses and the experimental set up is shown in Fig. 3.16 and 3.17.

Table 3.3 List of adding masses

No.	Mass (gram)	Group
1	1.57	I
2	2.26	
3	3.18	
4	4.10	
5	5.02	
6	4.90	II
7	8.09	
8	11.92	
9	15.87	
10	19.66	
11	18.87	III
12	30.70	
13	46.66	
14	62.17	
15	77.81	
16	80.08	IV
17	107.16	
18	133.15	
19	160.45	
20	187.01

Fig. 3.16 Picture of adding masses

Fig. 3.17 Experimental set up (Adding mass)

The electrical impedances when adding different masses at the output port of shaker & impedance head system are shown in Fig. 3.18. Only results of six masses are drawn in this figure. It is clear there is a peak on each curve which stands for the resonant frequency of the coupled mass and shaker system. The larger mass added, the lower the resonant frequency is.

Fig. 3.18 Electrical impedance of shaker system (with added masses)

Transduction matrix of electromagnetic shaker

The transduction matrix is calibrated via Eq. 3.34. As we know we can choose different n to calibrate the transduction matrix. Here we choose $n = 4$. The results are shown in Fig. 3.19 to 3.23 as the solid lines.

Since the simplified theoretical four pole model of the shaker is derived in section 3.3, nonlinear curve fitting method is used to get the inherent parameters of the shaker based on the results we just calibrated. The parameters are listed in Table 3.4 and the comparable results are also shown in Fig. 3.19 to 3.23 as the dashed lines. From the figures we notice that there are large differences between the calibrated and curve-fitted T_{11} and T_{21} . From the analysis of the theoretical model of the shaker we know

that $T_{21} = \frac{1}{K_{f/i}}$, it should be a real number. But the calibrated result shows us that

there is a phase angle which is also relevant to the frequency. The simplified model has not considered this kind of factors which caused the larger difference between the calibrated and curve-fit results.

Fig. 3.19 Transduction element T_{11} (adding group 3 mass)

Fig. 3.20 Transduction element T_{12} (adding group 3 mass)

Fig. 3.21 Transduction element T_{21} (adding group 3 mass)

Fig. 3.22 Transduction element T_{22} (adding group 3 mass)

Fig. 3.23 Four-pole determinate

Table 3.4 Parameters of shaker

<i>Parameter</i>	<i>Result</i>
R	3.3619Ω
L	$0.19092mH$
B_f	$8.8996Ns/m$
M_t	$0.04456Kg$
K_f	$5366N/m$
$K_{f/i}$	$3.3145N/A$

3.5 Actuating and sensing capabilities of shaker

3.5.1 Actuating capability of shaker

When an actuator is working under proper loading, the relationship between the force it exports and the velocity at the output are linear. The actual characteristic of a step motor is shown in Fig. 3.24.

Fig. 3.24 Torque/speed characteristic of step motor (taken from [97])

When a system is linear in time domain, it should be linear in frequency domain, which can be verified using the Laplace transform. We can verify the linearity of the force and velocity in the frequency domain since sweep sine signals is used in our experiments. At each frequency, we get the data of the force and velocity of each added mass. The relationship between the force and velocity at a series of frequencies is shown in Fig. 3.25. It is clear that the relationship between the force and velocity is linear in frequency domain.

Fig. 3.25 Relationship between force and velocity at a series of frequencies

3.5.2 Sensing capability of the electromagnetic shaker

Accuracy analysis using mass loads

When the shaker is used as a simultaneous sensing actuator, it works as an impedance sensor. The sensitivity is defined as

$$S_p = \frac{dZ_e}{dZ_m} = \frac{T_{11}T_{22} - T_{12}T_{21}}{(T_{21}Z_m + T_{22})^2} = \frac{1}{T_{21}^2} \frac{1}{(Z_m + Z_{mo})^2} \quad (3.35)$$

where $Z_{mo} = \frac{T_{22}}{T_{21}} = \frac{K_f}{j\omega} + B_f + j\omega M$ defined as the mechanical impedance of the shaker when the circuit is open.

It means that the sensitivity of the shaker when using as a simultaneous sensing actuator is related to the load. When we use a set of masses to calibrate the four pole model of the shaker, the sensitivity of the shaker is related to the set of mass we used. Ge found that better results can be obtained from an optimized set of known mass loads when study impedance characteristics of transducers and reciprocity calibration [98, 99]. Since we have used 20 masses to calibrate the four transduction functions of the shaker, we can analyze the sensitivity of the shaker using the results of the experiments with added masses.

When we get the four transduction functions of the shaker, we can calculate the mechanical impedance of the masses using Eq. 3.4. Then the calculated mass is:

$$m_{cal} = \frac{Z_m}{j\omega} \quad (3.36)$$

The calculated error for the mass is:

$$e_{mass} = m_{cal} - m_{true} \quad (3.37)$$

Normalized by the true value of the mass to get the normalized error:

$$Ne_{mass} = \frac{e_{mass}}{m_{true}} \times 100\% \quad (3.38)$$

The sensitivity of the four pole model is expressed by the statistics of the normalized error of the mass. Since the four pole model of the shaker is complex function, there

are two kinds of calculated errors, one is the magnitude and the other is the phase, or the real part and the imaginary part. We choose the former expressions, that is, magnitude and phase. The normalized mass error is written as:

$$Ne_{mass} = |Ne_{mass}| \angle Ne_{mass} \quad (3.39)$$

Firstly the masses are normalized by the mass of the table of the shaker (M_t) and listed in Table 3.5. Since we choose $n = 4$ to calibrate the shaker system, there are C_{20}^4 choices and C_{20}^4 sets of the four transduction functions when choose the four masses arbitrarily. The mean value and the standard deviation of the four masses are calculated first and written down as $\bar{m}$ and Δm . The mean values of the normalized mass errors are also calculated and the magnitude and phase are written down as $|\bar{Ne}_{mass}|$ and $\angle \bar{Ne}_{mass}$.

Figure 3.26 shows the distribution of $\bar{m}$ and $|\bar{Ne}_{mass}|$ of all masses. Figure 3.27 shows the distribution of $\bar{m}$ and $\angle \bar{Ne}_{mass}$ of all masses. While Fig. 3.28 shows the distribution of Δm and $|\bar{Ne}_{mass}|$ of all masses and Fig. 3.29 Δm and $\angle \bar{Ne}_{mass}$ of all masses.

Table 3.5 List of Normalized mass and resonant frequency

No.	Normalized mass	Normalized f_{res}	Group
1	0.0352	0.1876	I
2	0.0507	0.2251	
3	0.0713	0.2670	
4	0.0920	0.3033	
5	0.1126	0.3355	
6	0.1099	0.3315	II
7	0.1815	0.4260	
8	0.2675	0.5172	
9	0.3561	0.5967	
10	0.4412	0.4412	
11	0.4234	0.6507	III
12	0.6889	0.83	
13	1.0471	1.0232	
14	1.3952	1.1812	
15	1.7462	1.3214	
16	1.7971	1.3405	IV
17	2.4048	1.5507	
18	2.9881	1.7286	
19	3.6008	1.8976	
20	4.1968	2.0486

Fig. 3.26 Normalized magnitude errors vs. mean value of calibrated masses distribution

Fig. 3.27 Phase errors vs. mean value of calibrated masses distribution

Fig. 3.28 Normalized magnitude errors vs. standard deviation of calibrated masses distribution

Fig. 3.29 Phase errors vs. standard deviation calibrated masses distribution

In Fig. 3.26 if we draw a horizontal line at 70 and a vertical line at 1.15 we notice that the normalized magnitude errors are smaller at the lower left part of the figure. Similarly if we draw a horizontal line at 25 and two vertical lines at 0.5 and 2.5, the phase errors are smaller in the lower area enclosed by the three lines in Fig. 3.27. In Fig. 3.28 a horizontal line at 70 and a vertical line at 0.35 are drawn and the normalized magnitude errors are smaller in the lower left area. In Fig. 3.29 a horizontal line at 25 and two vertical lines at 0.25 and 0.7 are drawn the phase errors are smaller at the lower area enclosed by the three lines. Considering the four figures we conclude that both the normalized magnitude errors and the phase errors are smaller when $\overline{m}$ is between 0.5 and 1.15 and Δm is between 0.25 and 0.35.

In order to confirm the conclusion we have got, four groups of the masses listed in Table 3.5 are picked up from the C_{20}^4 results. The statistics of the normalized magnitude errors and the phase errors are listed in Table 3.6. Especially when calculated the errors intra-group, the normalize magnitude errors and the phase errors of group 3 is the smallest between the four groups as shown in Table 3.7.

Table 3.6 Calculated mass errors (all masses)

	<i>Group I</i>	<i>Group II</i>	<i>Group III</i>	<i>Group IV</i>
Normalized magnitude error (%)	35.866	49.148	42.175	111.73
Phase (degree)	30.787	16.342	21.969	20.072

Table 3.7 Calculated mass errors (intra-group)

	<i>Group I</i>	<i>Group II</i>	<i>Group III</i>	<i>Group IV</i>
Normalized magnitude error (%)	17.653	9.6959	5.3005	7.5243
Phase (degree)	10.741	5.8132	3.0666	4.4634

Resonant frequency method

When add a set of masses on the output of the shaker, the electrical impedance of the shaker changes according to the electromechanical effect of the shaker, just as Fig. 3.18 shown. The maximum point of the magnitude of the electrical impedance is the resonant frequency of the whole system. Analysis on the changes of the resonant frequency of the system is another method to analyze the sensitivity of the electrodynamic shaker. Adding springs is a little more difficult when doing experiments compared with adding masses. And viscoelastic samples are always looked as combinations of mass and spring. Since we have obtained the inherent parameters via nonlinear curve fitting method, we can use these parameters to numerically simulate many cases which are difficult in experiments.

The masses and the spring are normalized by the corresponding parameters of the shaker, that is, the mass of the table (M_t) and the relevant spring (K_f) of the shaker. The maximum points of the magnitude of the electrical impedances when added different masses and springs are picked. The resonant frequency is normalized by the natural frequency of the shaker testing system when no load is added. The normalized resonant frequency for the added masses are calculated and listed in Table 3.5. Then

the differences away from the free condition (no load is added on the output port of the shaker) are calculated. The result is shown in Fig. 3.30 and the vertical coordinate is the changes of normalized frequency.

Fig. 3.30 Resonant frequency vs. mass and spring ratio

It is clear that the larger the masses added on the shaker, the larger changes on the resonant frequency of the whole system. It is same for the spring added on it. When mass and spring combines together, the effect is disparate corresponding to the different ratio of the mass and spring. In actual practice, considering the power capacity of the shaker, the larger masses and the larger spring cannot get the results as the simulations.

3.6 Decoupling effect of the transduction matrix method

When the shaker testing system is changed, the transduction matrix is changed accordingly. The impedance head is used in the experiments to test force and velocity

of the output of the shaker system. From the system concept, it can also be looked as a subsystem for the whole system. As shown in Fig. 3.15, the shaker and impedance head system is the whole system. It includes two subsystems, one is the shaker, the other is the impedance head. The transduction matrix of the shaker is also characterized using the adding mass method. The velocity of the output of the shaker is measured by an accelerometer (B&K 4374) and other experimental set up is same as section 3.4.4. The shaker is remarked as system 1 and the shaker & impedance head is remarked as system 2. The comparison on the four transduction functions of the two systems is shown in Fig. 3.31 to 3.34. It is very clear that the effect of the impedance head is like that of a rigid mass to the system. There is no change on T_{11} and T_{12} which is concerned on the electrical parameters of the shaker. The determinant is shown in Fig. 3.35 and they are identical.

Fig. 3.31 Comparison of T_{11} of two systems

Fig. 3.32 Comparison of T_{21} of two systems

Fig. 3.33 Comparison of T_{12} of two systems

Fig. 3.34 Comparison of T_{22} of two systems

Fig. 3.35 Comparison of the determinant of the two systems

When tested on masses, the electrical impedances are different due to the different testing systems. The electrical impedances with mass 10 via two systems are shown in Fig. 3.36.

Fig. 3.36 Comparison of electrical impedance with mass 10 via two systems

Fig. 3.37 Comparison of calculated mass 10 via two systems

The mechanical impedance is inherent characteristic of structure or samples. It should not change no matter what kind of method we used. The masses calculated using Eq. 3.36 is compared in Fig. 3.37. When using different testing system the exclusive mechanical impedance is deduced from the electrical impedance of the system and the decoupling effect of the four transduction functions is obtained.

3.7 Summary

Transduction matrix method is introduced first in this chapter. It is based on the four pole model of linear system which is utilized in electromechanical system especially. Since its application on piezoelectric actuators was studied, electromagnetic actuator is investigated as a SSA device for the first time in this project.

Transduction matrix method applied on electromagnetic shaker is investigated in detail. The theoretical model of the shaker is derived which is the close form of the four pole equation based on some simplifications on the shaker.

Three different calibrating methods are introduced also. Direct method can get two transduction functions accurately. Corrected method has improved the results, but some more accurate results are still needed for the transduction matrix. Adding mass method is used to cancel the difficulty of clamp boundary condition. Transduction matrix is obtained and the accuracy is confirmed by the comparison of calibrated and measured masses. Decoupling effect of the transduction matrix is also analyzed.

Sensing capability of the shaker system is investigated. Accuracy is analyzed via the errors of adding mass experiments. It is concluded that when using added mass to

calibrate the shaker SSA device, the mass should be comparable to the equivalent mass of the shaker. The mean value of calibrated masses is between 0.5 and 1.15 and the standard deviation of calibrated masses is between 0.25 and 0.35. The sensitivity is analyzed by the resonant frequency method based on the theoretical model of the transduction matrix. It shows us that the effect of the stiffness is much larger than that of the mass.

Chapter 4

Soft viscoelastic materials characterization using electrodynamic shaker

4.1 Introduction

Mechanical impedance has been utilized as means for characterizing viscoelastic materials. Madigosky, Swanson et al. [5] presented a dynamic hardness tester from which contact impedance between the indenter/waveguide and test sample was obtained for deducing the mechanical properties of samples. Zhang, Royston et al. [100] applied the same method for medical diagnosis. Since mechanical impedance is measured by an impedance head in these applications, the installation of the hardware devices is problematic when samples are small or in-situ characterization is necessary. Both works employed the impedance model put forward by Oestreicher [101].

Daugela, Fujii et al. [102] investigated method called audiosonic contact impedance evaluation to test rubberlike materials. It is capable of differentiating material samples of different properties, but possesses limitation in precisely quantifying material properties.

Xie and Ling [103] developed a method using the electrical impedance to characterize different materials. The mechanical contact impedance is evaluated from the measured input electrical impedance of an elaborately designed actuating device driven by a patch of piezoceramic material when the electromechanical interaction of this device is made known. Experiments have qualitatively demonstrated that the higher the

elastic modulus of sample, the greater the mechanical loading to the sensor cum actuator, thus the greater the shift of its resonance. Also, the bigger the indentation, the greater the shift of its resonance, the greater the mechanical loading to the sensor cum actuator.

Transduction matrix method has been introduced and its application on electromagnetic shaker has been expatiated on in last chapter. Mechanical impedance of the output port of a shaker can be deduced from the electrical impedance of the input port of the shaker. Once the mechanical impedance is connected with the mechanical properties of the materials we measured, transduction matrix method can be used to characterize viscoelastic materials. It thus facilitates sensor-less and in-situ dynamic measurement of mechanical systems and avoids difficulties of sensor installations, whether due to limited available space or troublesome procedures.

In this chapter the connection between the mechanical properties of the viscoelastic materials and mechanical impedance is firstly introduced by a theoretically simplified model. The effects of the parameters of the model are analyzed and the feasibility of the model is also investigated. Then characterization of the viscoelastic materials using transduction matrix method is explained. Eight sample tests are then presented and the results are compared with the complex modulus and loss factor measured via conventional DMA to validate the proposed method. And the decoupling effect of the transduction matrix method is explained further by characterizing viscoelastic materials.

4.2 Theoretical model on viscoelastic materials characterization

4.2.1 General equation of motion

The generalized Hooke's Law is:

$$\left\{ \begin{array}{l} \tau_{xx} = (\lambda + 2\mu)\epsilon_{xx} + \lambda(\epsilon_{yy} + \epsilon_{zz}) \\ \tau_{yy} = (\lambda + 2\mu)\epsilon_{yy} + \lambda(\epsilon_{xx} + \epsilon_{zz}) \\ \tau_{zz} = (\lambda + 2\mu)\epsilon_{zz} + \lambda(\epsilon_{xx} + \epsilon_{yy}) \\ \tau_{xy} = \mu\gamma_{xy} \\ \tau_{yz} = \mu\gamma_{yz} \\ \tau_{zx} = \mu\gamma_{zx} \end{array} \right. \quad (4.1a)$$

when written in more generalized format:

$$\tau_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} \quad (4.1b)$$

while continuous condition is:

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (4.2)$$

Then Hook's Law becomes:

$$\tau_{ij} = \lambda u_{k,k}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) \quad (4.3)$$

Consider Newton's second law on an infinitesimal volume element, equation of motion in terms of tensor is [104]:

$$\tau_{ij,j} + B_j = \rho\dot{V}_j = \rho u_{j,t} \quad (4.4)$$

The motion equation becomes:

$$\lambda u_{j,ji}\delta_{ij} + \mu(u_{i,jj} + u_{j,ij}) + B_j = \rho u_{j,t} \quad (4.5a)$$

$$(\lambda + \mu)u_{j,ji} + \mu u_{i,jj} + B_j = \rho u_{j,t} \quad (4.5b)$$

using the vector formatting, $\text{div}\vec{u} = \nabla \cdot \vec{u} = \bar{u}_{i,i}$ $\nabla^2 \vec{u} = \bar{u}_{,ii}$

the generalized motion equation is:

$$(\lambda + \mu) \text{grad} \text{div} \bar{u} + \mu \nabla^2 \bar{u} + \bar{B} = \rho \bar{u}_{,tt} \quad (4.6)$$

if there is no body forces, it can be written as:

$$(\lambda + \mu) \nabla \nabla \cdot \bar{u} + \mu \nabla^2 \bar{u} = \rho \bar{u}_{,tt} \quad (4.7)$$

In the frequency domain, with $u = u(r)e^{i\omega t}$ this is called the Navier-Cauchy equation

$$(\lambda + \mu) \nabla \nabla \cdot \bar{u} + \mu \nabla^2 \bar{u} + \rho \omega^2 \bar{u} = 0 \quad (4.8)$$

since $\bar{u}$, a displacement vector field which in dynamic elasticity, may be decomposed into an irrotational field, say $\bar{u}_{IR}$, associated with a scalar potential ϕ and a rotational (incompressible) field, $\bar{u}_R$, associated with a vector potential $\bar{\psi}$.

$$\bar{u} = \nabla \phi + \nabla \times \bar{\psi} \quad (4.9)$$

with

$$\nabla \cdot \bar{\psi} = 0 \quad (4.10)$$

Defining

$$k^2 = \frac{\rho \omega^2}{2\mu + \lambda} \quad (4.11)$$

$$h^2 = \frac{\rho \omega^2}{\mu} \quad (4.12)$$

Then the equation of motion is:

$$\nabla(\nabla^2 + k^2)\phi + \nabla \times (\nabla^2 + h^2)\bar{\psi} = 0 \quad (4.13)$$

For the irrotational compression wave,

$$(\nabla^2 + k^2)\phi = 0 \quad (4.14)$$

its solution is:

$$\phi = -\frac{h_n(kr)}{(kr)^n} \chi_n = -\frac{h_n(kr)}{(kr)^n} r^n S_n \quad (4.15)$$

where $h_n(kr)$ is second spherical Hankel's function, S_n is Laplace spherical surface harmonic function, also Legendre function, $\chi_n = r^n S_n$ is spherical solid harmonics. And the Hankel's function satisfies recurrence formula of the types

$$h'_n(\eta) = -\eta h_{n+1}(\eta) \quad (4.16)$$

$$\eta h'_n(\eta) + (2n+1)h_n(\eta) = h_{n-1}(\eta) \quad (4.17)$$

which makes it very convenient for derivation.

And for the incompressible shear wave,

$$(\nabla^2 + h^2)\tilde{\psi} = 0 \quad (4.18)$$

whose solution is:

$$\nabla \times \psi = (n+1) \frac{h_{n-1}(hr)}{(hr)^{n-1}} \text{grad} \chi_n - n \frac{h_{n+1}(hr)}{(hr)^{n-1}} r^{2n+1} \text{grad} \frac{\chi_n}{r^{2n+1}} \quad (4.19)$$

4.2.2 Radiation impedance of the sphere

When a rigid sphere oscillates back and forth at frequency ω with a displacement s_0 in an unlimited medium, the stress field around the sphere can be calculated. Spherical coordinate (r, ϑ, φ) shown in Fig. 4.1 is simpler than rectangular coordinate for our analysis.

Fig. 4.1 Sphere coordinate

Suppose the center of the sphere with the radius a , situated at the origin and vibrating along the x -axis with an amplitude s_0 . When a rigid sphere vibrates in a media, it means that $n = 1$, so just put $n = 1$ in Eq. 4.15 and 4.19:

$$\phi = -\frac{h_1(kr)}{kr} \chi_1 = -\frac{h_1(kr)}{kr} r S_1 = -\frac{h_1(kr)}{kr} x \quad (4.15a)$$

$$\nabla \times \bar{\psi} = 2h_0(hr) \text{grad} x - h_2(hr) r^3 \text{grad} \frac{x}{r^3} \quad (4.19a)$$

the displacement vector is:

$$\begin{aligned} \bar{u} &= (\xi_1, \xi_2, \xi_3) \\ &= \nabla \phi + \nabla \times \bar{\psi} \\ &= -A_1 \text{grad} \left(\frac{h_1(kr)}{kr} x \right) + B_1 \left[2h_0(hr) \text{grad} x - h_2(hr) r^3 \text{grad} \frac{x}{r^3} \right] \\ &= -A_1 \left[\left(\frac{h_1(kr)}{kr} \right) \text{grad} x + x \text{grad} \left(\frac{h_1(kr)}{kr} \right) \right] \\ &\quad + B_1 \left[2h_0(hr) \text{grad} x - h_2(hr) \text{grad} x - x h_2(hr) r^3 \text{grad} \frac{1}{r^3} \right] \end{aligned} \quad (4.20)$$

where A_1 and B_1 are constants which are determined by the boundary conditions.

It is clear that

$$\text{grad} x = l \bar{i} \quad \text{and} \quad \text{grad} r = \frac{x}{r} \bar{i} + \frac{y}{r} \bar{j} + \frac{z}{r} \bar{k} \quad (4.21)$$

then the components of the displacement vector are:

$$\xi_1 = -A_1 \left(\frac{h_1(kr)}{kr} \right) - A_1 k \left(\frac{h_1(kr)}{kr} \right) x \frac{x}{r} + 2B_1 h_0(hr) - B_1 h_2(hr) + 3B_1 h_2(hr) x \frac{x}{r^2} \quad (4.22a)$$

$$\xi_2 = -A_1 k \left(\frac{h_1(kr)}{kr} \right) x \frac{y}{r} + 3B_1 h_2(hr) x \frac{y}{r^2} \quad (4.22b)$$

$$\xi_3 = -A_1 k \left(\frac{h_1(kr)}{kr} \right) x \frac{z}{r} + 3B_1 h_2(hr) x \frac{z}{r^2} \quad (4.22c)$$

These formulae make

$$x\xi_1 + y\xi_2 + z\xi_3 = \left\{ -A_1 \left[\frac{h_1(kr)}{kr} + kr \left(\frac{h_1(kr)}{kr} \right)' \right] + 6B_1 \left(\frac{h_1(hr)}{hr} \right) \right\} x \quad (4.23)$$

$$z\xi_1 - x\xi_3 = \left\{ -A_1 \left(\frac{h_1(kr)}{kr} \right) + B_1 [2h_0(hr) - h_2(hr)] \right\} z \quad (4.24)$$

In our model the boundary condition is:

$$r = a: \quad \xi_1 = s_0 \quad \xi_2 = \xi_3 = 0 \quad (4.25)$$

we can get the relation between A_1 and B_1 from Eq. 4.23 and 4.24:

$$-A_1 \left[\frac{h_1(ka)}{ka} + ka \left(\frac{h_1(ka)}{ka} \right)' \right] + 6B_1 \left(\frac{h_1(ha)}{ha} \right) = 1 \quad (4.26)$$

$$-A_1 \left(\frac{h_1(ka)}{ka} \right) + B_1 [2h_0(ha) - h_2(ha)] = 1 \quad (4.27)$$

then get the expressions:

$$A_1 = - \frac{(3 + 3iha - h^2 a^2) k^3 a^3 e(ika)}{k^2 a^2 (1 + iha) + (2 + 2ika - k^2 a^2) h^2 a^2} \quad (4.28)$$

$$B_1 = \frac{(3 + 3ika - k^2 a^2) h^3 a^3 e(iha)}{3[k^2 a^2 (1 + iha) + (2 + 2ika - k^2 a^2) h^2 a^2]} \quad (4.29)$$

The components of the stress across the y, z -plane are p_{11} , p_{12} and p_{13} , analogous for p_{21} , p_{22} and p_{23} across the x, z -plane, p_{31} , p_{32} and p_{33} across the x, y -plane.

According to Eq. 4.3, the stress components are:

$$\begin{aligned} p_{11} &= 2\mu \frac{\partial \xi_1}{\partial x} + \lambda \text{divs} \\ p_{21} &= \mu \left(\frac{\partial \xi_1}{\partial y} + \frac{\partial \xi_2}{\partial x} \right) \\ p_{31} &= \mu \left(\frac{\partial \xi_1}{\partial z} + \frac{\partial \xi_3}{\partial x} \right) \end{aligned} \quad (4.30)$$

The components of stress across the surface of a sphere of radius r are p_{rx} , p_{ry} and p_{rz} , the one along the x -direction is:

$$p_{r1} = \frac{xp_{11} + yp_{21} + zp_{31}}{r} \quad (4.31)$$

then

$$rp_{r1} = x\lambda \text{divs} + \mu \left(r \frac{\partial}{\partial r} - 1 \right) \xi_1 + \mu \frac{\partial(x\xi_1 + y\xi_2 + z\xi_3)}{\partial x} \quad (4.32)$$

The net force on the sphere can be found by integrating the stress at the surface of the sphere:

$$P = \int_{\text{sphere}} p_{r1} dS = 2\pi a^2 \int_0^\pi p_{r1} \sin \vartheta d\vartheta \quad (4.33)$$

we obtain

$$P = \frac{4}{3} \pi A_1 \mu a \left[\frac{\lambda a k h_1(ka)}{\mu} + 2 \left(i + \frac{1}{ak} \right) \right] + 8 \pi \mu h B_1 a^2 h_0'(ha) \quad (4.34)$$

The radiation impedance of the sphere is equal to this force divided by the vibrating speed of the sphere, and represents the resistance the sphere will “feel” when pushing its surrounding medium back and forth. The velocity of the sphere is $v = \dot{s} = i\omega s_0$, so the radiation impedance of the sphere is:

$$Z_r = -\frac{4}{3} \pi \rho a^3 i \omega \times \frac{\left(1 - \frac{3i}{ah} - \frac{3}{a^2 h^2} \right) - 2 \left(\frac{i}{ah} + \frac{1}{a^2 h^2} \right) \left(3 - \frac{a^2 k^2}{aki+1} \right)}{\left(\frac{i}{ah} + \frac{1}{a^2 h^2} \right) \left(\frac{a^2 k^2}{aki+1} \right) + \left(2 - \frac{a^2 k^2}{aki+1} \right)} \quad (4.35)$$

When the media is viscoelastic, according to the linear viscoelastic-linear elastic correspondence principle (also called linear viscoelastic-linear elastic analogy), the

constitutive equation is replaced by the viscoelastic relations. We assume the media is linear viscoelastic, that means:

$$\mu = \mu_1 + i\omega\mu_2, \lambda = \lambda_1 + i\omega\lambda_2 \quad (4.36)$$

Then h and k are complex numbers which will not change the expression of Eq. 4.35.

If the medium is incompressible, that means $\lambda \rightarrow \infty$, $k \rightarrow 0$, the radiation impedance of the sphere is then simplified as:

$$Z_m = -\frac{2}{3}\pi a^3 \rho \omega i \left(1 - \frac{9i}{ah} - \frac{9}{a^2 h^2} \right) \quad (4.37)$$

From Eq. 4.12 and 4.37 we obtain

$$\mu = -\frac{7}{18}a^2 \rho \omega^2 - \frac{1}{6\pi a} i \omega Z_m \pm \sqrt{\frac{a \rho \omega^2}{6\pi} i \omega Z_m + \frac{5}{36}a^4 \rho^2 \omega^4} \quad (4.38)$$

Because the media is incompressible, Poisson's ratio $\nu \approx 0.5$ [104], the complex Young's modulus and the complex shear modulus are then related as

$$E = 2(1+\nu)\mu \cong 3\mu \quad (4.39)$$

4.2.3 Effects of the parameters

Oestrecher [101] applied typical tissue properties on the model and found it confirmed the experimental results very well. The typical properties of the human tissues used by Oestrecher are listed as following: $\mu_1 = 2500Pa$, $\mu_2 = 15Pa$, $\lambda_1 = 2.6 \times 10^9 Pa$, $\lambda_2 = 0$, $\rho = 1100 kg/m^3$ and $a = 0.01m$. We will use these parameters as the reference to analyze the impedance model.

Firstly Eq. 4.35 and 4.37 are compared. The magnitude and phase, resistance and reactance of the radiation impedance are shown in Fig. 4.2. It is clear that when frequency is not higher than 10kHz, the tissue can be looked as an incompressible medium.

Fig. 4.2 Comparison of radiation impedance (tissue and incompressible medium)

When using sphere with different radius, the mechanical impedance is different. It is same for other parameters, such as the density of the medium and material properties of the medium. The effects of these parameters on the impedance model are then analyzed. The ratios of different parameters are same, which are 0.1, 1, 5 and 10. The results are shown in Fig. 4.3 to 4.6. The magnitude and phase of the mechanical impedance as well as the resistance and the reactance are shown in order in these figures.

Fig. 4.3 Effect of radius of sphere on mechanical impedance

Fig. 4.4 Effect of density of medium on mechanical impedance

Fig. 4.5 Effect of shear modulus of medium on mechanical impedance

Fig. 4.6 Effect of Lamé property of medium on mechanical impedance

It is clear that the larger the radius of the sphere, the larger the mechanical impedance. It also changes the resonant frequency of the medium-sphere system. For density of the media, it is same that the larger the density, the larger the mechanical impedance. For shear modulus the trend is same. But for Lamé parameter λ , there is no obvious effect on the mechanical impedance. One of the reason is that it is too large compared with other parameters such as the shear modulus. So when it is changed the effect it caused to the mechanical impedance is small. It also stands that the Lamé parameter of the medium is not sensible for the mechanical impedance when using it to characterize the materials.

The effects of the parameters are further analyzed quantitatively. The ratio of 1 is used as referenced impedance accordingly and the results of the scaled mechanical impedance divided by the referenced impedance are calculated in different cases. The results are shown in Fig. 4.7 to 4.10. The Lamé property λ has no effect on the

mechanical impedance until the frequency is higher then 3kHz. For radius of the sphere, the ratio is the largest among the four parameters.

Fig. 4.7 Ratio of radius effect

Fig. 4.8 Ratio of density effect

Fig. 4.9 Ratio of shear modulus effect

Fig. 4.10 Ratio of Lamé property effect

The corresponding effects of the parameters (radius, density and shear modulus) are also analyzed on Eq. 4.37 and same results are obtained. Since when a media is chosen, the density and modulus of it can not be changed. So the most important parameter is the radius of the diameter. In Eq. 4.37 it is assumed that $\lambda \rightarrow \infty$, actually it is not realized. But we notice that when $ak \ll 1$, Eq. 4.37 can also derived from Eq. 4.35. As we know that for most materials, λ is at the level of $10^9 Pa$, when the frequency is not very high (lower than 10kHz), a smaller radius a can be chosen for material characterization by the simplified theoretical model.

4.3 Experiments on soft viscoelastic materials characterization

4.3.1 Experimental set up

Since the advantage of transduction matrix method is to avoid the difficulties of measuring mechanical impedance by measuring the electrical impedance of a SSA device, one of the applications is to characterize incompressible materials use the theoretical model introduced in last section.

The proposed material characterization method is investigated using the mini shaker with impedance head which we just characterized in last chapter. An indenter is fixed at its force output port which is used to vibrate the samples we tested. The indenter is a sphere of diameter 1mm. The impedance head is necessary for comparison and validation purpose and will not be required in applications. The indenter and the impedance head are screwed firmly onto the shaker table to ensure that three of them together form an integrated rigid body. For calibration and testing, the material characterization system in addition includes a 3-D stage, an optical sensor, a power

amplifier, a charge amplifier, a conditional amplifier and a dynamic signal analyzer. The schematic figure of the set up is shown in Fig. 4.11 and the equipment we used is listed in table 4.1.

Fig. 4.11 Experiment set up for soft material characterization

Table 4.1 Equipment list for shaker calibration experiment

Equipment	Brand	Type
Mini Shaker	Brüel & Kjær	4810
Power Amplifier	Brüel & Kjær	2712
Impedance Head	Brüel & Kjær	8001
Condition Amplifier	Brüel & Kjær	2626
Charge Amplifier	Brüel & Kjær	2635
Dynamic Signal Analyzer	HP	35670A
Three-D Stage	Newport	ESP300
MTI Fotonic Sensor	MTI	2000

In testing, swept sinusoidal signals from dynamic signal analyser are sent through the power amplifier to activate the mini shaker. The voltage we set on HP 35670 is 200mV, the sweeping frequency domain is from 1Hz to 300Hz. The integration time and the setting time are both set at 20 cycles. The power amplifier is set at proper amplify gain for the shaker to get good working condition. The measured voltage and current signals from the power amplifier to the shaker and the signals from the impedance head conditioned by charge amplifier are fed into the dynamic signal analyser for processing and analysis.

As the materials under testing are soft, gauging the position when the indenter starts to contact the specimen requires high-precision measurement. This is achieved using an optical sensor (MTI2000) which measures the surface displacement of the core of the shaker. In experiments, a specimen fixed at the moving part of the 3-D stage is moved downwards until the optical sensor indicates a displacement of the core surface due to the contact of the sphere indenter with the surface of specimen. Once reaching this starting-to-contact position, the 3-D stage is fixed in shape and the gauge of the 3-D stage records down the displacement of the specimen. From experiments, we learned that the error encountered in displacement measurements is less than $2\mu\text{m}$ which is quite acceptable for the experiment.

The actual indentation depth is the difference of the two displacement readings; one static from the optical displacement sensor and the other static from the gauge of the 3-D stage as mentioned above. The indentation depth used in the reported experiments is 0.5 mm, which means that half of the indenter is immersed into the specimen under testing. From this point, shaker starts exerting sinusoidal force to the specimen.

The theoretical model introduced earlier assumes the domain of the medium is semi-infinite. Von Gierke [105] made corrections for cases of finite domain. In our experimental studies, the correction was implemented and found necessary. After the correction was made, the effect of the preload and samples size became insignificant when the indenter is half-immersed into the specimen.

4.3.2 Four pole model of shaker-testing-device

We have obtained the four-pole model of shaker & impedance head in last chapter. As we mentioned in last section, an indenter is used as a probe to vibrate the specimens. The testing frame is shown in Fig. 4.12 and the testing device is in the dashed square.

Fig. 4.12 Schematic frame of characterization materials

The shaker & impedance head system's four-pole model is:

$$\begin{Bmatrix} E \\ I \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} F_{o1} \\ v_{o1} \end{Bmatrix} \quad (4.40)$$

while the indenter's four-pole model is:

$$\begin{Bmatrix} F_{i2} \\ v_{i2} \end{Bmatrix} = \begin{bmatrix} 1 & j\omega m \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} \quad (4.41)$$

m is the mass of the indenter (with the screw). So the total four-pole model of the testing system is:

$$\begin{Bmatrix} E \\ I \end{Bmatrix} = \begin{bmatrix} T'_{11} & T'_{12} \\ T'_{21} & T'_{22} \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 1 & j\omega m \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} = \begin{bmatrix} T_{11} & j\omega m T_{11} + T_{12} \\ T_{21} & j\omega m T_{21} + T_{22} \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} \quad (4.42)$$

The results of the four-pole model of the total system are shown in Fig. 4.13 to 4.16.

Fig. 4.13 Transduction element T_{11} (total testing system)

Fig. 4.14 Transduction element T_{12} (total testing system)

Fig. 4.15 Transduction element T_{21} (total testing system)

Fig. 4.16 Transduction element T_{22} (total testing system)

The determinant of it is same and is shown in Fig. 4.17.

Fig. 4.17 Determinant of $[T]$ of shaker testing system

From Fig. 4.17, it is observed that the determinant is slightly smaller than unity in lower frequency domain (lower than 50Hz) and is slightly greater than unity in higher

frequencies domain. For frequencies lower than 20Hz the obtained information should be discarded as the valid bandwidth of the employed power amplifier for the shaker is from 20Hz to 4kHz. This factor is reflected by the large deviation from unity in low frequency range in Fig. 4.19. Experiences from research works in a number of applications show that the proposed method provided acceptable accuracy in the frequency range where the determinant of the transduction matrix is between 0.9 and 1.1 [83-86]. Therefore, the valid frequency range of the employed SSA transducer in this study is taken as from 30Hz to 300Hz.

4.3.3 Mechanical properties of samples

With this experimental set up, a set of experiments were performed on five commercial erasers and three industrial rubbers. The three industrial rubbers are Ethylene Propylene (EPDM) rubber 280C, Neoprene rubber 264C, and Natural rubber 263C. They are produced by James Walker & Co Ltd, England, and distributed by RS Components Pte Ltd, Singapore. The properties of these materials provided by suppliers are summarized in Table 4.2. Note that the complex modulus of NA263C and NE 264C is one order higher than that of EPDM 280C and the commercial eraser. Samples are cut from a large bulk material into specimens of 30×30×20mm in size. For commercial erasers, no modifications are made on the samples.

The complex modulus and damping of the specimen are also tested using the GABO EPLEXOR® 500N DMA using compressed load mode at room temperature. The dynamic strain amplitude of GABO DMA is +/-1.5mm and its dynamic force amplitude is +/-500N. It is a modern material testing solution for the tire, rubber and

polymer industry[17]. The corresponding force and deformation are comparable to the shaker added on the specimen. The force is 0.7N and the strain is 0.3%. The swept-sinusoidal input is set from 2 Hz to 100Hz, the available frequency range for the DMA under use. For the five erasers, the complex modulus of eraser 4 is the largest, followed by eraser 3, eraser 2, eraser 1, and eraser 5 is the smallest. For the damping ratio, eraser 2 is the largest, followed by eraser 1, eraser 3, eraser 4, and eraser 5 is the smallest. These results will be the reference data for comparison.

Table 4.2 Material properties of samples

Material properties	EPDM 280C	NE 264C	NA 263C	Erasers
Hardness (IRHD)	60 to 75	55 to 70	55 to 70	Commercial
Density (Mg/m ³)	1.1±0.2	1.4±0.2	1.4±0.2	product.
Tensile strength (MPa)	5	5	5	Properties not
Elongation at break (%)	300	200	200	provided by
Compression set (%) (24h@100°C)	40	30	30	manufacturer.
Size Length(mm) x Width(mm) x Height(mm)	30x30x20	30x30x20	30x30x20	30x20x5 50x15x10

4.3.4 Experimental results

Figure 4.18 and 4.19 shows the electrical impedances obtained at the input port of the shaker-material-characterization system with the indenter half immersed and vibrating in the specimens. From these figures, we can find a peak on each curve which stands the resonant frequency of the whole testing system. The higher the frequency the

larger the complex modulus of the samples. For the damping ratio the half power bandwidth can give us a primary result. So qualitative characterization can be obtained from the electrical impedances we measured.

Fig. 4.18 Electrical impedance when testing on commercial erasers

Fig. 4.19 Electrical impedance when testing on industrial rubbers

The transduction matrix method shows that the relationship between the input electrical impedance and the output mechanical impedance is:

$$Z_m = \frac{T_{22}Z_e - T_{12}}{T_{11} - T_{21}Z_e} \quad (4.43)$$

Then the mechanical impedances are calculated using the electrical impedances shown in Fig. 4.18 and 4.19 and the transduction matrix obtained in section 4.3.2. Since the mechanical impedance of the material specimen is equal to the mechanical impedance loaded to the vibrating indenter, this calculated mechanical impedance can then be used to derive the complex modulus of material specimens via Eq. 4.38 and 4.39. To explore the applicability, the frequency range covered by the proposed method was from 2Hz to 300Hz, the designed frequency range of the employed shaker and its power amplifier. Fig. 4.20 to 4.27 provides the results obtained from the proposed methods and DMA method for comparison.

Fig. 4.20 Complex modulus of r5 (DMA and proposed method)

Fig. 4.21 Complex modulus of r2 (DMA and proposed method)

Fig. 4.22 Complex modulus of r1 (DMA and proposed method)

Fig. 4.23 Complex modulus of r3 (DMA and proposed method)

Fig. 4.24 Complex modulus of r4 (DMA and proposed method)

Fig. 4.25 EPDM 280C complex modulus and loss factor (DMA and proposed method)

Fig. 4.26 NE264C complex modulus and loss factor (DMA and proposed method)

Fig. 4.27 NA263C complex modulus and loss factor (DMA and proposed method)

Both results from DMA and the proposed method confirm that the complex modulus of NE 264C and NA263C is one order higher than that of EPDM 280C and the commercial erasers. In Fig. 4.20 to 4.24, both complex modulus and loss factor of the commercial erasers coincide well between the two methods in the frequency between 30 to 100Hz. In Fig. 4.25, the same happens to EPDM 280C in a smaller frequency range, between 50 to 100Hz. While in Fig. 4.26 and 4.27, the difference between results from the proposed method and DMA method is more significant in the same frequency range. For frequencies higher than 100Hz, the value of the complex modulus and loss factor continue their trend in lower frequency range. Discussions addressing these features are given in the following.

4.4 Analysis transduction matrix method on soft viscoelastic materials characterization

4.4.1 Decoupling of transduction matrix method on materials characterization

As section 3.6 mentioned, the decoupling effect of the transduction matrix method is also analyzed on material characterization. When testing on structures and materials, the electrical impedances are different due to the different testing systems. The mechanical impedance is inherent characteristic of structure or samples. It should not change whenever what kind of method we used. When using different testing system the exclusive mechanical properties of the samples is deduced from the electrical impedance of the system and the decoupling effect of the four transduction functions is obtained. The results of rubber 5 are shown in Fig. 4.28 and 4.29. System 1 is shaker system, while system 2 is shaker & impedance head system.

Fig. 4.28 Comparison of electrical impedance testing on r5 via two systems

Fig. 4.29 Comparison of complex modulus of r5 via two systems

4.4.2 Discussion on experimental results

It is well known [106] that, when characterizing viscoelastic materials, apparent discrepancies commonly arise in results obtained under different experimental conditions. Taking this factor into consideration, the discrepancies between results of DMA and the proposed method pointed out earlier in section 4.3.4 seems comfortably acceptable for commercial eraser and EPDM 280C, those softer incompressible viscoelastic materials. The deviation of transduction matrix method for softer viscoelastic materials is within 8% compared with the DMA results from 2 to 100Hz. The result for the harder materials, NE 264C and NA263C, shows more outstanding discrepancies.

Looking more closely into Fig. 4.18 and 4.19 it reveals that in measured electrical impedance there exists a relatively small kink at 54Hz for EPDM 280C and 52Hz for NE 264C and NA 263C. For commercial erasers, the kink still exists but is quite small. These kinks are due to the 50Hz resonance of the electrical power. In theory, since this inherent property of the shaker has already been included in the transduction functions (see Fig. 4.14 to 4.17), the mechanical impedance obtained by Eq. 4.43 should not contain the spikes as this resonant behavior of the shaker will be “decoupled” from the measured electrical impedance. This ideal “decoupling” does not work perfectly in practice and therefore the spikes around 50Hz remain in Fig. 4.20 to 4.27. Note that the error is more apparent in results of harder materials. These errors are from two sources. Firstly, errors in signal processing in the proposed method are inherently more serious than normal as the magnitude of transduction function T_{12} and T_{22} are about at least one order higher than that of T_{11} and T_{21} . Secondly, the shaker is assumed linear in the proposed method and this assumption is quite realistic if the material under testing is soft and becomes less valid when harder materials are tested. This nonlinearity is introduced into the shaker due to the excessive deformation of the suspension mechanism for the vibrating core of the shaker. In experiments, this deformation was 0.7mm when NE 264C and NA263C were tested and was only 0.2mm when the commercial erasers and EPDM 280C were tested.

There are two more kinks on electrical impedance of NE 264C and NA263C at 142Hz and 158Hz. The nonlinearity mentioned above seems responsible for these further discrepancies as well. In future studies, attention should be paid to the applicable range of a given shaker in terms of the “softness” of the testing materials. The authors reckon that a shaker with higher first resonance is needed for harder materials such as

NE 264C and NA 263C as the suspension mechanism of the core mass would be harder and less-deformed, and thus the required linearity would be better reserved in measurement.

4.5 Summary

A theoretical model is firstly derived which relates the radiation impedance of a sphere with the material properties of the media when the sphere is vibrating. Whatever the media is the model is maintained. When the media is incompressible the model can be simplified so the radiation impedance is only related to the shear modulus of the media which can be used as a good model for material characterization. The effects of the parameters are also analyzed and it is concluded that the radius of the sphere is more important to the model, followed by the shear modulus and the density while the effect of the Lamé parameter is negligible.

The application of transduction matrix method on material characterization is investigated in the following. We propose and validate a method for characterizing soft incompressible viscoelastic materials. In the method, a shaker is used to dynamically indent a material specimen and the input electrical impedance of the shaker is measured to derive the mechanical impedance of the material under testing. The complex modulus and loss factor derived from the detected impedance based on the impedance model matches reasonably well with that obtained by DMA for six softer rubbers, with the deviation of 8%. For two harder material samples, the discrepancies are significantly bigger most likely due to the nonlinearity introduced to the shaker via excessive deformation of the suspension mechanism. Since the proposed method requires no sensor to measure mechanical impedance, the method is

particularly suitable for in-situ characterization of soft incompressible materials or situations where installing sensors is difficult due to limited available space such as in micro system. Part of this chapter has been published [92].

Chapter 5

Characterization human skin in vivo by portable Dynamic Indenting Actuator for Skin

5.1 Introduction

As mentioned in Chapter 2, mechanical properties of soft tissues are of importance for biomaterials, medical and cosmetic industry. A number of instruments have been made for measuring mechanical properties of soft tissues which use two kinds of loading mechanisms, one is ultrasonic technique, and the other is mechanical loading. When ultrasonic technique is used, the mechanical properties are measured at much higher frequency, for say, some MHz. The safety of subjects must be taken into account. While mechanical loading is taken, the dynamic properties can be measured at up to 10Hz or so. The dynamic response of the sensors and actuators will be a hindrance for the portable instrument at higher frequencies. For the commercial portable instrument, for example, Cutometer, DermaLab, Linear Skin Rheometer and Axiom Venustron, the properties measured are mainly static but not dynamic.

Since mechanical properties of soft viscoelastic materials have been measured with shaker SSA device, we may investigate a new method for testing mechanical properties of human skin in vivo. First of all a new portable device is developed based on transduction matrix method which we named it Dynamic Indenting Actuator for Skin (DIAS).

5.2 Design of DIAS

5.2.1 Requirements of DIAS

It is not convenient for three dimensions deformation of skin because skin is more like superficial tissue in human body. So small scale, one-axis deformations, with the capability to examine within a wide range of frequencies is the objective to be achieved by the DIAS describe herein.

A number of simple deformation modes can be imposed on skin under which extraction of material properties is fairly simple. Linear indentation, rotational shear and tangential shear are all possible deformation modes. Dynamic indentation is chosen for its convenience in testing.

Since linear properties are desired, this implies that only small deformation need to be (or alternatively, can be) applied, giving some guidance towards actuator selection. The magnitude of “small deformation” depends on the material and geometry of the structure being examined. On the one hand from the literature review, the deformation of the tissue by suction and indentation methods is in the level of millimetre. On the other hand the diameter of the sphere in the simplified theoretical model in Chapter is 1mm. A small dynamic deformation is enough for us to get the mechanical properties of the skin. The deformation the DIAS adds to the skin is not required too large, for example, smaller than 0.5mm is enough for us to get the proper deformation of the skin.

Considering linear indentation, the existing stiffness data and the use of semi-infinite body expressions provide guidelines for the force that an actuator should be able to provide to create deformation on the scale described. From literature review we know that typical density of biomaterials was assumed to be $\sim 1000 \pm 200 \text{ kg/m}^3$ and stiffness to be $\sim 5 \text{ MPa}$ [107]. For soft tissue, the volume elasticity λ_1 is at the order of 10^9 Pa . The shear elasticity μ_1 is at the order of 10^4 Pa [23, 31, 41, 42, 116].

When the maximum deformation was decided, the other concern is the force we need to add on the soft tissues. The force is concerned by the dynamic response of the designed device. Assume the mass of the tissue vibrated with the actuator is 0.5gram, the deformation is 0.5mm, then the dynamic force is: $F = ma = -m\omega^2 s$. When frequency is 500Hz, which is the dynamic working range of designed actuator, the force is 0.5N. So the block force for the actuator should be 0.5N.

These values represent minimum design goals. Because the device is intended for portable use, the basic requirement for it is small and light.

5.2.2 Linear actuators options

Based on the requirements just discussed, a number of candidate classes of actuators that support small deformations and forces are immediately apparent. Basic actuator usually uses electromagnetic, piezoelectric, electrostatic, shape memory, magnetostrictive and electrorheological technologies [108]. Typical examples that use electromagnetic principle are electromagnetic motors, solenoids and voice coil

actuators. Several designs are used in piezoelectric actuators: stacked design, extender design, tube design, bimorph design and hybrid design. These kinds of actuators are examined to determine whether they could be used to generator motions on the order of those considered here. Shape Memory Alloy (SMA) wire actuators are limited by the thermal time constant of the wire, which was defined as $\varphi = \rho c V / h A_s$, and provide only tensile loading, however strains of up to approximately 5% can be achieved. Piezoelectric and magnetostrictive materials respond up to very high frequencies, but can only generate strains on the order of parts per million; they would be difficult to use to create an actuator with millimetre-scale displacements. Electrostatic actuators require high operating voltage. Voice coils and solenoids can be designed to have large displacements, reasonable bandwidth, and apply both tensile and compressive loading.

It is better to choose voice coil as the linear actuator. The schematic drawing of voice coil actuator is shown in Fig. 5.1. Considering the requirement of section 5.2.1, the proper product, BEI Kimco Magnetics Division LA0505000A, was chosen after comparison on kinds of actuators. The specification of the actuator is listed in Table 5.1 and 5.2.

Fig. 5.1 Voice coil actuator [109]

Table 5.1 Winding constant of LA0505000A

Winding constants	Units	Tol	Symbol	WSG A
DC Resistance	Ohms	$\pm 12.5\%$	R	4.0
Voltage @ F_p	Volts	normal	V_p	4.35
Current @ F_p	Amperes	normal	I_p	1.09
Force sensitivity	N/Amp	$\pm 10\%$	K_F	0.575
Back EMF constant	V/m/s	$\pm 10\%$	K_B	0.574
Inductance	Milli-Henry	$\pm 10\%$	L	28.0

Table 5.2 Actuator parameters of LA0505000A

Actuator Parameters	Unit	Symbol	Value
Peak Force*	N	F_p	0.7
Constant Stall Force**	N	F_{st}	0.35
Actuator constant	$N / \sqrt{\text{Watt}}$	K_A	0.32
Electrical time constant	Milli-Second	τ_E	7.0
Mechanical time constant	Milli-Second	τ_M	8.9
Power $I^2 R$ @ F_p	Watt	P_p	5.83 Max.
Stroke	$\pm \text{mm}$		0.508
Clearance on each side of coil	mm		0.381
Thermal resistance of coil	$^{\circ}\text{C}/\text{Watt}$	θ_{TH}	74
Max. allowable coil temp.	$^{\circ}\text{C}$	TEMP	155
Weight of coil assembly	g	WT_C	0.907
Weight of field assembly	g	WT_F	8.50

* 10 second at 25°C ambient, 155°C coil temp.

** 25°C ambient and 155°C winding temperature

5.2.3 Flexure design of DIAS

After actuator is chosen, mechanical components are needed to transfer output force to samples and guideways are needed for linear displacement. For the guideways, sliding bearings introduce friction between the moving surfaces, and ball or roller bearings are not generally suited for very small, repetitive motions. Hydrostatic bearings (oil, water or air) were considered, but have additional difficulties such as sealing the system to prevent contamination of the skin (in addition to the precision manufacture of more parts). To provide fiction-free actuation, a flexural bearing system is needed for the new device.

The advantages of flexure includes that it is wear free, which means that, provided they are not abusively distorted, their line of action will remain constant throughout their life. Because there are no sliding pairs, the only likely wear mechanisms will be fretting and corrosion at fabricated interfaces where changing stresses are applied. Also displacements are smooth and continuous at all levels. Displacement can be accurately predicted from the application of known forces and, conversely, predictable forces can be generated by controlled displacements. If designed correctly, the springs will be closely linear in their force/displacement characteristics.

Corresponding to the advantages there are also some disadvantages of flexure. The force for a given deflection is dependent upon the elastic modulus of the flexure. It is surprisingly difficult to obtain values that are accurate to better than 1% for this parameter and it is often necessary to calibrate a flexure after it has been produced. Flexures are restricted to small displacements for a given size and stiffness. Out of plane stiffness tends to be relatively low and drive direction stiffness quite high compared to other bearing systems. Great care must be taken to ensure that the drive axis is collinear with the desired motion. They cannot tolerate large loads.

Considering the advantages and disadvantages of flexure, it is necessary to design it with caution to get the proper function. Firstly what kind of materials is chosen for the flexure. Elastic mechanisms exploit linear strain and so a material of low E might seem preferable. However, it is normally a key feature of these mechanisms that the flexures behave extremely repeatable. Hence the deflecting sections must remain well within the linear elastic region, with no hint of local yielding being acceptable. The maximum strain is limited by the yield strength $Y = E\varepsilon$, and, for this application, selection for high Y/E is desirable. The resonant frequency of a beam of fixed geometry is proportional to $(E/\rho)^{1/2}$. In most situations, decoupling unwanted vibration from a system is made easier if its resonances are at high frequencies, so for both static and quasi-static reasons E/ρ is chosen as a representative grouping. The usually used elastic materials are listed in Table 5.3. CuBe 2 is chosen based on the above considerations.

Table 5.3 Materials for flexure

Materials	$\rho(kg / m^3 \times 10^3)$	E (Gpa)	σ_y (Gpa)	$\sigma_y / E (\times 10^{-3})$
Steel	7.85	210	1	4.7
Al 7075	2.8	72	0.48	6.6
CuBe 2	8.9	126	0.75	6.0
Ti Alloys	4.5	109	0.9	8.3
Nylon	1.1	2.8	0.055	2.0

The design chosen for the new device is a fixed-free cantilever design, clamped to the moving armature and fixed housing at the inner and outer bosses respectively, with the cantilever wrapped around the central axis of the armature. To provide flexibility in design, a series of flexures were manufactured, each version with a different cantilever width, and thus a different stiffness. The flexures were manufactured by wire cutting 0.1mm thick beryllium bronze.

Finite element analysis was performed using ANSYS 7.0 to estimate the minimum stiffness of the slimmest cantilever design, and verify that normal range of motion would not exceed the fatigue strength of the alloy. A typical result is shown in Fig. 5.3, as are expected force-displacement data, showing that the flexures behave like linear springs within the normal range of motion. The stiffness of the flexure is 378N/m.

Fig. 5.2 Simulated stress distribution of flexure

Fig. 5.3 Linear relationship of force and displacement of flexure

Then other fixtures are designed and manufactured to assemble the new portable device DIAS. The schematic figure, the picture of main part and main view of DIAS

are shown in Fig. 5.4. The outer side of flexure was fixed with the outer-base of DIAS, and the inner side is fixed with the voice coil of the actuator and the indenter. The dimension is $\phi 37\text{mm} \times 38\text{mm}$ and weight is 30gram.

Fig. 5.4 Pictures of DIAS (schematic figure, main parts and main view)

5.3 Calibration of DIAS

Calibration is the process by which a sensor or actuator is provided with a factual statement about the errors this instrument will have. DIAS is firstly calibrated via mass experiments to obtain the characteristic. Responses when testing on inertial are used to verify that it performs as expected, and could be relied upon to accurately measure their properties prior to testing on human skin.

5.3.1 Repeatability of DIAS

Repeatability is a measure of transducer stability over time. Firstly the repeatability of DIAS is tested on the free condition, that is, no indenters are added on the connector of the moving table of DIAS. An Impedance Analyzer (Solartron 1260A) is used to send swept sine signal to DIAS and then measure the electrical impedance at the input

port with voltage setting at 0.4V and frequency setting from 100 to 400Hz. A schematic diagram is shown in Fig. 5.5 and the result is shown in Fig. 5.6 where twice experimental results are compared. It is obvious that the repeatability of DIAS is very good. Free condition is just used as an example and other experimental results show same as the one of free condition.

Fig. 5.5 Schematic diagram of repeatability experiment

Fig. 5.6 Repeatability result of new device (electrical impedance of free condition)

5.3.2 Calibration using mass method

From chapter 3 we know that the relationship between the input port and the output port of SSA device is:

$$Z_m = \frac{T_{22}Z_e - T_{12}}{T_{11} - T_{21}Z_e} \quad (5.1)$$

and the matrix form of the relationship is:

$$\begin{Bmatrix} E \\ I \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} F \\ v \end{Bmatrix} \quad (5.2)$$

Furthermore, if we let $F = 0$, $I = 0$, and $v = 0$ respectively in Eq. 5.2, the following three relationships are valid.

$$T_{12} = Z_{ef}T_{22} \quad (5.3)$$

$$T_{22} = -Z_{mo}T_{21} \quad (5.4)$$

$$T_{11} = Z_{ec}T_{21} \quad (5.5)$$

Substituting Eq. 5.3-5.5 into Eq. 5.1, the mechanical impedance at the output port Z_m can be represented by

$$Z_m = Z_{mo} \frac{Z_e - Z_{ef}}{Z_e - Z_{ec}} \quad (5.6)$$

where the quantities of Z_{ef} , Z_{ec} and Z_{mo} are defined as

$$Z_{ef} = \frac{E}{I} \Big|_{F=0} \text{ the input electrical impedance with the output free}$$

$$Z_{ec} = \frac{E}{I} \Big|_{v=0} \text{ the input electrical impedance with the output static}$$

$$Z_{mo} = \frac{F}{v} \Big|_{I=0} \text{ the output mechanical impedance with the input electrically open-circuited}$$

Among the above three impedances, Z_{ef} is easily calibrated by letting the output port free mechanically while Z_{mo} and Z_{ec} are much more difficult to calibrate. This is

because mechanical clamp conditions and electrical open-circuit conditions are difficult to be realized in the laboratory.

To overcome these problems, a method similar to section 3.4.4 is put forward to indirectly identify Z_{mo} and Z_{ec} from a set of the measured Z_{ef} and Z_e of the SSA device when various known masses are applied to its output port.

We can rewrite Eq. 5.6 as following:

$$\frac{Z_e - Z_{ef}}{Z_m} Z_{mo} + Z_{ec} = Z_e \quad (5.7)$$

or:

$$AZ_{mo} + BZ_{ec} = C \quad (5.8)$$

with $A = \frac{Z_e - Z_{ef}}{Z_m}$, $B = 1$, and $C = Z_e$.

For a given mass, A , C can be measured and calculated. We can build a group of simultaneous equations as following based on a set of Z_e and Z_m obtained by adding various masses to the mechanical port of the actuator.

$$\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \\ \cdots & \cdots \\ A_n & B_n \end{bmatrix} \cdot \begin{bmatrix} Z_{mo} \\ Z_{ec} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \cdots \\ C_n \end{bmatrix} \quad (5.9)$$

where $A_k = \frac{Z_{ek} - Z_{ef}}{Z_{mk}}$, $B_k = 1$, $C_k = Z_{ek}$, and $k = 1, 2, \dots, n$. n is the number of

experiment times of adding masses to the mechanical port of the actuator. Z_{mo} and Z_{ec} can be identified by solving these over-determined simultaneous equations.

5.3.3 Experimental set up of mass test

DIAS is clamped on a 3-D stage (Newport). Different screws whose masses are listed in Table 5.4 (shown in Fig. 5.7) are fastened on the connector of the moving table of DIAS. The experimental set up is shown in Fig. 5.8.

Fig. 5.7 Photo of DIAS and different indenters

Fig. 5.8 Experimental set up of mass calibration

Table 5.4 Masses of indenters

Indenter	Mass (gram)
Indenter 1mm	0.271
Indenter 3mm	0.376
Screw 5mm	0.6987
Screw 8mm	2.1781
Screw 10mm	3.4946

5.3.4 Testing results and discussion

Electrical impedances of DIAS with different screws fastened are shown in Fig. 5.9.

Z_{mo} and Z_{ec} are calculated via Eq. 5.9 and shown in Fig. 5.10 and 5.11.

Fig. 5.9 Electrical impedance of DIAS with different indenters

Fig. 5.10 Characteristic Z_{ec} of DIAS

Fig. 5.11 Characteristic Z_{mo} of DIAS

The calculated mass are used again to verify the calibrated parameters Z_{mo} and Z_{ec} . It is the reverse procedure of the calibration. The results are shown in Fig. 5. 12.

Fig. 5.12 Mass comparison (true and calculated)

The calibrated characteristics of DIAS are shown in Fig. 5.6, 5.10 and 5.11. We can clearly get the resonant frequency of DIAS from the figure of Z_{ef} , which is 220Hz. There is also a little peak at 300Hz. It is assumed another resonant frequency of DIAS. Resonant frequency is at 220Hz and anti-resonant frequency at 280Hz when the figure of Z_{MO} is observed. There is a little peak at 150Hz and another resonant frequency at 300Hz. For the figure of Z_{ec} , there is large change at 150Hz while a peak at 280Hz. As we know if we analysis the theoretical parameters of the DIAS, there should not be a peak for Z_{ec} . They are assumed to be caused by tolerance of the parts and process of assembly.

5.4 Testing on silicone gel samples

The second series of validation tests deal with a set of silicone gel samples, prepared

from mixtures of different proportions of two component liquids. By varying the proportions, gel with different stiffness could be created. The silicone gel is a material with viscoelastic properties, so measurements of variation in complex modulus and damping could be made, and compared between the DMA results.

5.4.1 Preparation of silicone gel samples

A spring-dashpot, such as a shock absorber would have a two parameter ideal model with both compliant and viscous elements. The behavior of such a system is the same as that of a Voigt body, described in section 2.2. As an example of a material more closely approximating tissue, a silicone gel (Silgard 527) is used to make phantoms with different stiffness and damping properties. It has been used as the model materials of soft tissue, such as brain [110-114] and breast [115], by other researchers. Silgard 527 is made of a mixture of two liquid components, part A and part B, which are normally mixed in equal amounts (weight or volume). Mixing them in different proportions yields gels with a range of elastic modulus. Three kinds of samples are prepared for the mimic of the soft tissue for our project. The ratios are 1:1, 2:3 and 1:2 for part A and part B of silgard 527. According to the manual of the silicone gel, when different ratio is applied for the mixture of the parts, different stiffness and viscosity are obtained.

The liquid of two parts are firstly deposited in two measuring cups then a precision balance is used to measure the weight of the liquids. After that the two liquids are poured into a vessel and mixed by a glass rod. According to the manual of Silgard 527, special care has to be taken to avoid trapping air bubbles in the solution after

application. So the speed of the mixture should be as slow as possible. The mixed time is 10 minutes. Curing is normally done for 24h at room temperature, and it can be accelerated at elevated temperatures. The usually used procedure of the curing time is shown in the Table 5.3.

Table 5.5 Recommended curing time for Silgard 527

Temperature (°C)	Time (hour)
65	4
100	1
150	0.25

The cure time for 1 hour in 100°C environment is chosen for the trade-off consideration. The vessel is put in the bench top type temperature and humidity chamber (ESPEC SH-240) with the temperature of the furnace setting at 100°C and the time at 1 hour. After these procedures the gel is cured and three samples are prepared.

The three samples are tested on GABO DMA to measure the complex modulus. The load mode is compression and the load added on is 0.3N. The results are shown in Fig. 5.13.

Fig. 5.13 DMA results of three silicone gels

5.4.2 Experimental set up of silicone gel test

DIAS with indenter fastened on its table is inverted and clamped on the 3-D stage (Newport) which can move up and down to control the depth of indentation. Silicone gel samples are put on a steel bar enclosed by aluminum alloy sheet. It is more convenient to put and remove the silicone gels from the steel bar with the aluminum alloy sheet enclosed because the silicone gel has self-healing properties. The experimental set up is shown in Fig. 5.14 and a zoom in picture is shown in Fig. 5.15. Other settings are same as explained in section 5.3.1.

Fig. 5.14 Photo of experimental setting up for testing Silgard samples

Fig. 5.15 Zoom in photo on indenter and sample

5.4.3 Testing results and discussion

Electrical impedances of DIAS with different indenters testing on different samples are measured. The results are shown in Fig. 5.16 to 5.20.

Fig. 5.16 Electrical impedance when testing on Silgard samples (Indenter 1)

Fig. 5.17 Electrical impedance when testing on Silgard samples (Indenter 3)

Fig. 5.18 Electrical impedance when testing on Silgard samples (Scw5mm)

Fig. 5.19 Electrical impedance when testing on Silgard samples (Scw8mm)

Fig. 5.20 Electrical impedance when testing on Silgard samples (Scw10mm)

From these figures, it is clearly that there are peaks at magnitude figures. The harder the materials, the higher frequency the peak is at the horizontal axis. Also the damping ratio can be simply assumed by half power bandwidth method since the materials are linear viscoelastic.

Since we have calibrated the characteristic of DIAS, it is more convenient for us to get the exact mechanical properties of the samples, just like explanation of Chapter 4. The mechanical impedances of the three samples are calculated via Eq. 5.6. Then the complex moduli of the samples are calculated using Eq. 4.38 and 4.39. If the indenter is not sphere, an amendatory parameter must be used for the more accurate results. According to Von Gierke [105] when using a piston 1.18 is used to correlate the results. The results tested via screw 10mm are used as an example to show the calculated complex modulus and damping ratio of silicone gel samples. The DMA results are compared also (see Fig. 5.21 to 5.23). Because of the different testing

frequency range, there are direct comparisons for these two methods. The deviation of complex modulus is around 11% and that of loss factor is around 18% at 100Hz.

Fig. 5.21 Complex modulus and loss factor of Sample 11 (measured by DIAS and DMA)

Fig. 5.22 Complex modulus and loss factor of Sample 23 (measured by DIAS and DMA)

Fig. 5.23 Complex modulus and loss factor of Sample 12 (measured by DIAS and DMA)

The electrical impedances of DIAS when testing on sample 11 with different indenters are redrawn in one figure shown as Fig. 5.24.

Fig. 5.24 Electrical impedance when testing with different indenters (Sample 11)

The characteristics of the different testing systems made up of DIAS and different indenters, are different definitely. While the calculated complex modulus and damping ratio are almost same when we compared the results (shown in Fig. 5.25 to Fig. 5.27). It shows that DIAS can be decoupled by the transduction matrix method.

Fig. 5.25 Comparison of complex modulus using different indenter (Sample 11)

Fig. 5.26 Comparison of complex modulus using different indenter (Sample 23)

Fig. 5.27 Comparison of complex modulus using different indenter (Sample 12)

5.5 Testing on human skin in vivo

The target of DIAS is to measure mechanical properties of human skin in vivo. A few tests are performed on five subjects on skins of different part of the body. The skin of the subjects had not been treated in any way prior to the examination. All measurements are taken at ambient conditions at a temperature of ~20°C and a relative humidity of ~30-45%.

5.5.1 Details of the subjects

The tests are made on subject 1 on different parts of the skin, which includes ventral of forearm, dorsum of forearm, cheek of face and forehead and also on the same part but at different times. For subject 2, measurements are carried out on ventral of

forearm and dorsum of forearm. For subject 3 to 5, tests are done only on ventral of forearm. The details of the subjects are listed in Table 5.6.

Table 5.6 Details of measured subjects

Subject	Gender	Age	Profession	Testing position
1	F	30	Researcher	Ventral and dorsum of forearm, cheer, forehead
2	M	26	Researcher	Ventral and dorsum of forearm
3	F	30	Researcher	Ventral of forearm
4	F	60	Accountant	Ventral of forearm
5	M	61	Treasurer	Ventral of forearm

It is no matter what kind of indenters used for skin testing since the decoupling effect of DIAS is perfect. Screw 10mm indenter is chosen because of the comfortable feeling of subjects. The experimental parameters are set as: frequency swept from 140 to 180Hz, others are same as the ones set as calibrations. When doing measurements on subjects, the device is just hold by hand, that is, connect the indenter to the skin of the subject, as shown in Fig. 5.28.

Fig. 5.28 Photo of tests on subject 4, 5 and 1

5.5.2 Testing results and discussion

Firstly the repeatability of the DIAS on human skin is investigated. Tests on ventral and dorsum of forearm of subject 1 were performed on different days. The results are shown in Fig. 5.29 and 5.30. It is clear that the repeatability is quite good.

Fig. 5.29 Electrical impedance of DIAS when testing on ventral forearm of subject 1
(on different days)

Fig. 5.30 Electrical impedance of DIAS when testing on dorsum forearm of subject 1
(on different days)

Then measurements were carried out on different part of the skin of subject 1. These different parts are ventral of forearm, dorsum of forearm, forehead and cheek face. The complex modulus of the skin is calculated based on the impedance model. While we just use the piston instead of sphere as the indenter, there should be a correlation coefficient for the results. According to [105] when using a piston 1.18 is used to correlate the results. The electrical impedance and complex modulus are shown in Fig. 5.31 and 5.32.

Fig. 5.31 Electrical impedance when testing on different parts of subject 1

Fig. 5.32 Complex modulus and loss factor of skins of different part of subject 1

Tests were also made on subject 2 on ventral and dorsum forearm. The results are shown in Fig. 5.33 and 5.34.

Fig. 5.33 Electrical impedance when testing on ventral and dorsum of subject 2's forearm

Fig. 5.34 Complex modulus of foreman skin of subject 2 (ventral and dorsum)

At last tests were performed on the ventral forearm of four different subjects. The four subjects are subject 1, subject 3, subject 4 and subject 5. The intension is to find the relation between the complex modulus of the skin and the age.

Fig. 5.35 Electrical impedance when testing on forearm of four subjects

Fig. 5.36 Complex modulus of human skin of four subjects

From Fig. 5.31 and 5.32 we can see that the complex moduli of the skin at different parts of the body are distinguished. The complex modulus of the forehead is the

largest, followed by ventral forearm, dorsum forearm and cheek face. For the damping ratio, the figure needs to be zoomed in to find the difference. The damping ratio of cheek is the largest, followed by dorsum, forehead and then ventral. Same results are found in Fig. 5.33 and 5.34 for the skin of subject 2. The complex modulus of ventral is larger than that of dorsum while the damping ratio of ventral is smaller than that of dorsum.

The age effect on skin is studied by testing on ventral of forearm on four subjects. There are two subjects over 60's and two at 30's. From Fig. 5.35 and 5.36 we can see that both the complex modulus and the damping ratio of young subjects are smaller than those of older subjects.

Comparing results of subject 4 and subject 5, sex has no effect on the mechanical properties of the skins. The comparisons of subject 1 and subject 2 on ventral and dorsum forearm are shown in Fig. 5.37 and 5.38. It is found that for ventral the complex modulus of subject 1 is larger than that of subject 2, while for dorsum the complex modulus of subject 2 is larger than that of subject 1. The relation between sex and mechanical properties of skin cannot be found clearly from the results of the two experiments.

Fig. 5.37 Complex modulus of ventral forearm skin (subject 1 and subject 2)

Fig. 5.38 Complex modulus of dorsum forearm skin (subject 1 and subject 2)

Since there are only 5 subjects for our experiments, we cannot get the conclusion on the relationship between age, sex, site of the skin and so on with the mechanical

properties of human skin. We can only get the phenomenon from these experiments. Refer to the reviewed results on human skin (Table 2.2) it is more reasonable that the ventral is stiffer than dorsum. Also the older skin is much stiffer than younger skin. For sex effect, many literatures concluded that there is no difference on the mechanical properties of human skin. But there is no review results on the damping ratio of the skins, our results should be a good reference for other researchers. Comparison was shown as Fig. 5.39 on the reviewed complex modulus of soft tissues and that of measured by DIAS.

Fig. 5.39 Complex modulus of soft tissues comparison (review and DIAS)

5.6 Summary

A new technique is investigated for dynamic testing mechanical properties of human skin in vivo quantitatively. A portable device called DIAS is developed for testing human skin. After the actuator is chosen found on the requirement of DIAS, design of

flexure part is the most important thing. The stiffness, weight and the dynamic characteristics of the device should be considered at the same time.

DIAS is calibrated by using a series of masses. The characteristic Z_{ef} , Z_{ec} and Z_{mo} are calibrated and confirmed by the reverse validation of the calculated masses. Then experiments are performed on three viscoelastic samples which are combined by two sets of silicone gels. Complex moduli and damping ratio of three samples are obtained which are with good coincidence with the results of DMA method. The deviation of complex modulus is about 11% and that of loss factor is around 18% at 100Hz. Decoupling effect of the DIAS also obtained since same mechanical properties of silicone gels are obtained via using different indenters to test.

Measurements on human skin of five subjects are carried out by DIAS. Since the number of subjects is too small only phenomena can be stated. For subject 1, the sequence of complex modulus of the skin on different site of the body is, from large to small, forehead, ventral forearm, dorsum forearm and cheek face. Similarly is the order of the damping ratio, from large to small, face cheek, dorsum, forehead and then ventral. Same phenomenon is found on the results of ventral and dorsum forearm of subject 2. Both the complex modulus and damping ratio of subject 4 and subject 5 (older) are larger than those of subject 1 and subject 3 (younger). The sex effect on subject 4 & subject 5 and subject 1 & subject 2 are different. These results are limited because of the small sample space of subjects.

Chapter 6

Conclusions and recommendations

6.1 Conclusions

Following the wider use of viscoelastic materials in the industries over the past few decades, being able to characterize mechanical properties of such materials experimentally has become increasingly important. However, existing dynamic testing methods such as free decay method, forced vibration method, resonant method and wave propagation method share a common drawback. Each of these methods requires the use of actuators and sensors in exciting and measuring the load and deformation which renders the loading effect of sensors for small samples and the mass effect of sensors at high frequency testing inevitable. Furthermore, installation of sensors and actuators is difficult when the space is small. Samples preparation is also a major problem to overcome in applying the existing methods. Moreover, they are not suitable for in situ testing.

In this study, a new method, transduction matrix method, for testing mechanical properties of soft viscoelastic materials is designed by using Simultaneous Sensing and Actuating (SSA) technique. The actuator is described by a 2×2 transduction matrix which allows the mechanical impedance of the output port of the actuator to be detected from the electrical impedance of the input port while the actuator works as a

sensor simultaneously. The detected mechanical impedance can then be analyzed to provide mechanical properties of viscoelastic materials based on a theoretical model.

It was the first time that transduction matrix method was applied on electrodynamic actuators. The theoretical model of the shaker was then derived and the closed forms of the four-pole equations based on some simplification on the shaker were obtained. Different experimental methods, they are, direct method, corrected method and adding mass method were investigated and implemented in this study. The transduction matrix of the shaker was obtained and the accuracy was confirmed by the comparison of calibrated and measured masses while sensing capability of the shaker system was also investigated.

It was concluded that when using added mass to calibrate the shaker, the mass should be comparable to the equivalent mass of the shaker to get a higher accuracy and that the effect of the stiffness is much larger than that of the mass of the samples since the sensitivity is load-related. For the shaker investigated in this project, the mean value of calibrated masses between 0.5 and 1.15 and the standard deviation of calibrated masses between 0.25 and 0.35 are the best parameters for the calibrated masses.

A theoretical model was derived which relates the radiation impedance of a sphere with the material properties of the media when the sphere is vibrating. When the media is incompressible the model can be simplified so the radiation impedance is only related to the shear modulus of the media which can be used as a good model for material characterization. The effects of the parameters were also analyzed and it was concluded that the radius of the sphere is more important to the model.

The application of transduction matrix method on material characterization was investigated in the following. The calibrated shaker was used to dynamically indent a material specimen and the input electrical impedance of the shaker was measured to derive the mechanical impedance of the material under testing. The complex modulus and loss factor derived from the detected impedance based on the impedance model matches reasonably well with that obtained by DMA for six softer rubbers, where the deviation is 8%. For two harder material samples, the discrepancies are significantly bigger most likely due to the nonlinearity introduced to the shaker via excessive deformation of the suspension mechanism.

The mechanical properties of skin were reviewed in detail since its mechanical properties are important for cosmetic and clinical applications. Even there are some kinds of commercial equipment for testing skin, there is still less knowledge of the skin at some hundreds Hertz. Based on former studies, a portable device called Dynamic Indenting Actuator for Skin (DIAS) was designed to dynamically test mechanical properties of human skin in vivo. The stiffness, weight and the dynamic characteristics of the device should be considered at the same time. DIAS was calibrated by using a series of masses. The characteristic Z_{ef} , Z_{ec} and Z_{mo} are calibrated and confirmed by the reverse validation of the calculated masses. Then experiments were performed on three viscoelastic samples which were combined by two sets of silicone gels. Complex moduli and damping ratio of three samples were obtained which were with good coincidence with the results of DMA method. The deviation of complex modulus is around 11% and that of loss factor is around 18% at

100Hz. Decoupling effect of the DIAS also obtained since same mechanical properties of silicone gels were obtained via using different indenters to test.

Tests on human skin of five subjects using the DIAS were conducted and the complex modulus and damping ratio of the skins were measured. Since the number of subjects was too small only phenomena could be stated. For subject 1, the sequence of complex modulus of the skin on different site of the body was, from large to small, forehead, ventral forearm, dorsum forearm and cheek face. Similarly was the order of the damping ratio, from large to small, face cheek, dorsum, forehead and then ventral. Same phenomenon was found on the results of ventral and dorsum forearm of subject 2. Both the complex modulus and damping ratio of subject 4 and subject 5 (older) were larger than those of subject 1 and subject 3 (younger). The sex effect on subject 4 & subject 5 and subject 1 & subject 2 were different. The quantitative results (limited by small sample space of subjects) are believed to be good references for human skin researchers since there are no such kinds of data been measured because of lack of technique and equipment.

6.2 Recommendations

Shaker has been investigated as a SSA device in this project. However, the presented SSA shaker is proved to work properly in the low frequency range ($<300\text{Hz}$ under our testing conditions). To broaden the working frequency range and application fields of SSA shaker, a new concept shaker should be developed in the future studies. More detailed specifications such as accuracy, linearity as well as range and resolution for the new concept shaker should be investigated.

Since the force (F) and velocity (v) of the output port can be detected from the voltage and current of the input port of shaker, the deformation x is easily obtained by $v = j\omega x$. Other mechanical properties of the materials, for example, hardness, stiffness and so on could be detected from the force and displacement curves. It is not limited to viscoelastic materials, which means that elastic and viscous materials can also be measured by the shaker using SSA technique. For elastic materials it is believed that only very lower oscillate signal is needed since the properties is not relevant to frequency. For viscous materials it should be same as tests on viscoelastic materials.

DIAS is developed to investigate the new technique for testing mechanical properties of human skin in vivo. The designed working frequency range for DIAS is from 100 to 200Hz. The thickness of the human skin has not been considered in testing. In later studies, measurement of the thickness of human skin should be combined with DIAS and more wide frequency range should be covered. Moreover, with the development of micro systems, miniature actuators could be further studied for testing the viscoelastic properties of cell and so on.

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Appendix

1. ASTM E756-98 Standard Test Method for Measuring Vibration-Damping Properties of Materials

1.1 Scope: This test method measures the vibration-damping properties of materials: the loss factor η , and Young's modulus E , or the shear modulus G . Accurate over a frequency range of 50 to 5000Hz and over the useful temperature range of materials. This method is useful in testing materials that have application in structural vibration, building acoustics, and the control of audible noise. Such materials include metals, enamels, ceramics, rubbers, plastics, reinforced epoxy materials, and woods that can be formed to cantilever beam test configurations.

1.2 Terminology: Figure A.1 shows the variation of modulus and materials loss factor with temperature. There are glassy region, rubber region and transition region of a damping material.

Fig. A.1 Variation of modulus and material loss factor with temperature
(frequency held constant)

1.3 Configuration: The configuration of the cantilever beam test specimen is selected based on the type of damping material to be tested. Figure A.2 shows four different test specimens used to investigate extensional and shear damping properties of materials over a broad range of modulus values.

Fig. A.2 Test Specimens

1.4 Testing: Once the test beam configuration has been selected and the test specimen has been prepared, the test specimen is clamped in a fixture and placed in an environmental chamber. Two transducers are used in the measurement, one to apply an excitation force to cause the test beam to vibrate, and one to measure the response of the test beam to the applied force. By measuring several resonance of the vibration beam, the effect of frequency on the material's damping properties can be established. By operating the test fixture inside an environmental chamber, the effects of temperature on the material properties are investigated. The test method determines the properties of a damping material by indirect measurement using damped cantilever beam theory. By applying beam theory, the resultant damping material properties are made independent of the geometry of the test specimen used to obtain them.

1.5 Assumptions: All damping measurements are made in the linear range, that is, the damping materials behave in accordance with linear viscoelastic theory. For linear beam behavior, the peak displacement from rest for a composite beam should be less than the thickness of the base beam. The amplitude of the force signal applied to the excitation transducer is maintained constant with frequency. If the force amplitude cannot be kept constant, then the response of the beam must be divided by the force amplitude. The ratio of response to force (referred to as the compliance or receptance) presented as a function of frequency must then be used for evaluating the damping.

1.6 Test specimen preparation: as shown in FigA.2 2d, sandwich specimen, is used for determining the damping properties of soft materials that will be subjected to shear deformation in their application. A metal spacer of the same thickness as the damping materials must be added in the root section between the two base beams of the test specimen. The spacer must be bonded in place with a stiff, structural adhesive system.

1.7 Procedure: Mount the beam in a heavy, rigid fixture providing clamping force around the root of the beam to simulate a fixed end, cantilever boundary condition. Place the test fixture, including the beam specimen, inside an environmental chamber. Position the transducers on or around the specimen as appropriate for the type of transducer. Set the environmental chamber to the desired temperature. Vibration response measurements must be performed at

intervals over a wide range of temperature. Measurement several resonant modes of the beam for each data acquisition temperature. Four or more modes area commonly measured starting with mode2. Mode 1 is usually not measured. Use the half-power bandwidth method to measurement the damping of the composite beam.

1.8 Calculation: Sandwich specimen- calculate the shear modulus and loss factor of the damping materials from the expression:

$$G_1 = [A - B - 2(A - B)^2 - 2(A\eta_s)^2] \left[\frac{\left(\frac{2\pi C_n E H H_1}{l^2} \right)}{\{(1 - 2A + 2B)^2 + 4(A\eta_s)^2\}} \right]$$

and

$$\eta_1 = \frac{(A\eta_s)}{[A - B - 2(A - B)^2 - 2(A\eta_s)^2]}$$

where

$$A = \left(f_s / f_n \right)^2 (2 + DT)(B/2)$$

$$B = 1/[6(1 + T)^2]$$

C_n = coefficient for model n, of clamped-free (uniform) beam

$D = \rho_1 / \rho$, density ratio

E = Young's modulus of base beam, Pa,

f_n = resonance frequency for mode n of Base beam, Hz

f_s = resonance frequency for mode s of composite beam, Hz

Δf_s = half-power bandwidth of mode s of composite beam, Hz

G_1 = shear modulus of damping material, Pa

H = thickness of base beam, m

H_1 = thickness of damping material, m

l = length of beam, m

s = index number: 1, 2, 3, ... ($s = n$)

$T = H_1 / H$, thickness ratio

η_1 = shear loss factor of damping material, dimensionless

$\eta_s = \Delta f_s / f_n$, loss factor of sandwiched specimen, dimensionless

$\rho_1 =$ density of damping material, kg/m³

$\rho =$ density of base beam, kg/m³

1.9 The reduced-Frequency Nomogram (RFN) is a data presentation method that allows the lengthy table of loss factor and modulus data obtained at various frequencies and temperatures to be represented as two curves. This permits the extrapolation of data to frequency, or temperature ranges where data are not available.

Fig A.3 Illustration of the Reduced Frequency Nomogram

2. ASTM D5023-01, D5024-01, D5026-01, D5279-01, D5418-01 Standard
Test Method for Plastics: Dynamic Mechanical Properties

2.1 Different deformation mode as:

- D5023-01 in flexure (three point bending)
- D5024-01 in compression

D5026-01 in tension

D5279-01 in torsion

D5418-01 in flexure (dual cantilever beam)

2.2 Scope of D5024-01: These test methods cover the use of dynamic mechanical instrumentation for gathering and reporting the viscoelastic properties of thermoplastic and thermosetting resins and composites systems in form of cylindrical specimens molded directly or cut from sheets, plates, or molded shapes. The compression data generated may be used to identify the thermomechanical properties of a plastic material or composition using a variety of dynamic mechanical instruments. It is intended to provide means for determining the modulus as a function of temperature of a wide variety of plastics materials using nonresonant forced-vibration techniques. Plots of the elastic (storage), loss (viscous) and complex moduli and tan delta as a function of frequency, time, or temperature are indicative of significant transitions in the thermomechanical performance of the polymeric material system. This test method is valid for a wide range of frequencies typically from 0.01 to 100Hz.

2.3 Test specimens may be cut from sheets, plates, or molded shapes, or may be molded to the desired finished dimensions. Typically, the cylindrical test specimen is up to 25mm, in diameter by 5mm in height (thickness). Cylindrical specimens of other dimensions can be used but should be clearly identified in the report.

2.4 Typical testing result is shown as Fig. A.4.

Fig. A.4. Dynamic mechanical properties in compression