Decentralized control for multi-input multi-output integrating processes

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**DECENTRALIZED CONTROL FOR MULTI-INPUT
MULTI-OUTPUT INTEGRATING PROCESSES**

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**SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING**

2010

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Abstract

This thesis presents a decentralized PI/PID controller design scheme for Multi-Input Multi-Output (MIMO) integrating processes with time delays. A new loop pairing criterion is proposed for multi-loop control of MIMO integrating processes, based on a new interaction measurement method called relative effective gain array (REGA) / relative normalized gain array (RNGA). Compared with its counterparts such as RGA or DRGA, this novel loop pairing technique considers not only the steady-state information but also the transient information of the process. It also comprehensively reflects the dynamic interactions among individual loops without requiring specification of the controller type and with a relatively low computational load. Based on the loop pairing results of a multivariable process, its Equivalent Transfer Function (ETF) matrix is then derived for the design of decentralized control systems. The ETF of a process in a given loop represents the approximation of its transfer function, and reflects the dynamic changes of the process, when the other loops are closed. A decentralized controller is then designed based on the ETF matrix. This design scheme guarantees the stability, causality, properness and compatibility of the overall control configuration for MIMO integrating processes. The REGA/RNGA-based design method is simple, straightforward and easy to understand for field control engineers. Several industrial MIMO integrating processes are presented to demonstrate the advantages of the method in terms of simplicity in design and effectiveness in control system performance.

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Chapter 1

Introduction

1.1 Multi-Input Multi-Output Control

In multi-input multi-output (MIMO) control, the closed-loop system's performance is affected by interactions caused by the non-zero off-diagonal entries in transfer function matrix [1, 2]. For this reason, a very important step in MIMO control system design is to determine the pairing strategy of main loops, i.e., to pair the manipulated variables and controlled variables. For decentralized control schemes, the control system design establishes which control output is used for which process input, so that minimum interactions from and to other loops can be achieved. For decoupling control schemes, the purpose of control structure configuration is to seek a balance on the dominance of the main loops and the causality of the resultant decouplers such that the control system can be implemented.

There are basically two categories of approaches to the determination of control structures, i.e., steady state based and dynamic based. For steady

state based control approaches, the most popular one is the Relative Gain Array (RGA) method, which is often used together with the Niederlinski Index (NI) for loop pairings to guarantee the closed-loop stability of system [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. By employing structured singular value (SSV) techniques, the μ -interaction measure is also used for both selection of loop pairings and design of decentralized controllers [1, 14, 15]. The method not only predicts the stability of decentralized control systems, but also determines the performance loss caused by the decentralized control structure. In particular, it provides a sufficient condition for achieving offset-free performance for closed-loop control systems. For processes with one-way interactions with triangular transfer function matrix, Hovd and Skogestad proposed a performance relative gain array (PRGA) to overcome the limitation of RGA based methods [14, 17].

Steady state based approaches have the shortcoming that it is derived solely from the steady-state gain matrix of process and does not utilize any information of dynamics, which may result in incorrect interaction measures and loop pairing decisions. Dynamic based control approaches take the process dynamics into consideration. Witcher and Mc Avoy presented a dynamic relative gain array (DRGA) approach [18] by employing the transfer function model in place of the steady-state gain matrix to calculate the DRGA. In each entry of the DRGA, the denominator achieves perfect control at all frequencies, while the numerator is simply the open-loop transfer function. A few other DRGA approaches have also been proposed and proved to be more accurate in terms of interaction assessment and loop pairing decision than their RGA counterpart [19, 20, 21, 22]. However, the calculation of

the DRGA is often more complicated than that of the RGA, and in certain cases the DRGA is controller dependent [22], making it more difficult for application of practical control engineers.

Utilizing both steady state gain and critical frequency information of the process open loop transfer function matrix, Xiong et al. presented a new loop pairing criterion by defining loop energy transmission ratio to combine both steady state and dynamic information into an effective gain matrix [23]. Then, the loop pairing procedure of the popular RGA method is directly extended for the system to determine the loop interactions. This effective gain matrix reflects dynamic loop interactions under finite phase crossover frequency control. However, the effectiveness of the method has caused concerns due to the definition of the two alternative critical frequencies, i.e., phase cross over frequency and bandwidth, for the reason that the pairing decision may depend strongly on how the critical frequency is defined. Bandwidths ought to be used as critical frequencies for transfer function matrices without phase crossover frequencies. However, phase crossover frequency, i.e., ultimate frequency is recommended for the calculation of the effective gain matrix [23].

A new loop pairing criterion featuring the relative virtual gain array was proposed by He et al., based on a new loop interaction measurement of MIMO processes [24]. This new loop interaction measurement utilizes both the steady-state gain matrix and the transient function matrix of a certain MIMO system, hence the loop pairing criterion investigates the dynamic relative interactions among the loops of the MIMO system. This loop pairing method is as simple to understand and implement as those based on the

relative gain array (RGA), hence it can be a useful tool in the design of decentralized and decoupling control systems. Moreover, it also provides comprehensive controller-independent descriptions of dynamic interactions among individual loops as this method is based on the dynamic relative gain array.

Based on the loop pairing results, He et al. introduced a design method for decentralized PID controller [25]. On the basis of structure decomposition, the dynamic relative interaction is defined and represented by the process model and controller explicitly. An initial decentralized controller is designed by using the diagonal elements and then implemented to estimate the dynamic relative interaction to individual control loop from all others. By using the dynamic relative interaction, the multiplicate model factor is derived and simplified to a pure time delay function at the neighborhood of each control loop's critical frequency to obtain the equivalent transfer function. Consequently by applying the Internal Model Control tuning rules for PID controller design of the equivalent transfer function, appropriate controller parameters for individual control loop are determined. What is not reported in [24] and [25] is the performance of their proposed decentralized controller design especially for high dimensional processes (e.g., 4×4 MIMO systems) and the robustness analysis against parametric and structural model errors.

1.2 Multi-Input Multi-Output Control for Integrating Processes

Integrating processes are encountered in chemical plants and other industrial processes. A proportional controller would be adequate for normal integrating processes that do not require tight level control. However, it may be important to control some integrating industrial process at a tight level, e.g., to keep the precise flooding area in heat transfer process or to maintain precise retention time for material process. A PD or P controller is needed to improve the transient performance of those processes.

Controller design for MIMO integrating processes often encounters some difficulties, however, due to the significant phase lags in low frequencies that the integrator contributes. On the other hand, processes having large time constant were encountered in a composition control loop of a high purity distillation column [27]. Early work that treated a slow dynamic process as an integrating process for controller design was given by Chien and Fruehauf [28] and by Tyreus and Luyben [29]. They found the controller designed in this way may significantly improve the slow dynamics. For the purposes aforementioned, controller design as well as tuning methods for PI controller was thus proposed [30, 31, 32]. Later, tuning methods for the PID controller has also been presented by Luyben [33], Wang and Cluett [34], and Visioli [35]. From the performance point of view, however, the system with an integrator is not really satisfactory with the single loop PI or PID controllers aforementioned. Lately, the Smith-predictor type of controller [37, 38] and cascaded type controller [36, 37] have also been considered. Some late developments

include the PID-P controller of Wang and Cai [39], and the PI-PD controller of Kaya [40] have also been reported. The methods aforementioned are in general good for SISO systems, and it is difficult to apply them directly to the control of MIMO processes. The reason is that the design of such controllers does not take into consideration the loop interactions which exist in MIMO processes. Detuning the controllers using the BLT method does not always give good results, and it is by no means a systematic approach.

As discussed in last subsection, RGA is a useful tool for interaction measurement and loop pairing in MIMO control. When computing RGA for MIMO integrating processes, however, infinite steady state gains arise from the integrating elements of the transfer function matrix. Efforts have been made to modify RGA to cope with MIMO integrating processes. A derivative variable is used to redefine the static gain matrix before re-computing the RGA [41]. This method encounters difficulties, however, when the redefined static gain matrix is singular. Another approach [42] involves the integrator as part of the RGA. This method works for two-input two-output systems, but it is unable to handle MIMO systems with dimensions higher than 2x2. Moreover, the approaches mentioned above are by no means direct for RGA computation for engineering application.

1.3 Objective and Outline of the Thesis

The main objective of this thesis is to present a novel decentralized control system configuration for multi-input multi-output processes containing pure integrators. A decentralized controller design scheme is presented, utilizing

Relative Effective Gain Array / Relative Normalized Gain Array loop pairing methodology and gain/phase margin PID controller selection for processes' equivalent transfer functions.

The contributions of this thesis include (1) the derivation of the average residence time of processes with Second Order plus Time Delay models, (2) the extension of Relative Effective Gain Array / Relative Normalized Gain Array loop pairing methodology to MIMO integrating processes, and (3) the application of effective transfer function and decentralized MIMO controller design schemes for MIMO integrating processes.

The organization of the thesis is as follows.

Chapter 2 investigates the issue of loop pairing in multivariable processes, and proposes REGA/RNGA loop pairing methodologies.

Chapter 3 presents the underlying principle of how effective transfer function reflects the dynamic property of multivariable processes with closed-loop feedback, and discusses how to derive the effective transfer functions of MIMO integrating processes.

Chapter 4 proposes a scheme of decentralized controller design based on gain/phase margin for MIMO integrating processes, by utilizing the novel loop pairing techniques.

Chapter 5 demonstrates by case studies the effectiveness of the proposed decentralized controller design in the thesis.

Chapter 6 concludes the thesis with discussion on future work in the design of decoupler matrix in multivariable control systems.

Chapter 2

REGA/RNGA-based Loop Pairing Criteria

2.1 Introduction

A MIMO control system can be configured in several different ways, depending on which input variable is paired with which output variable. Each “pairing” of input m_i and output y_j constitutes a loop configuration. The primary task of MIMO control system design is to determine its loop configuration, i.e. pair the manipulated variables and controlled variables to achieve the minimum interactions among control loops.

The RGA loop pairing techniques introduced in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] has been widely applied in industries. However, the RGA approach and several other DRGA approaches have their own shortcomings for loop pairing. This chapter proposes Relative Effective Gain Array / Relative Normalized Gain Array loop pairing criteria for MIMO integrating

processes to combine the advantages of both RGA and DRGA.

2.2 Preliminaries

Control system configuration problem is concerned with defining the control system structure, i.e., which of the available process inputs is to be used to control each of the process outputs. In any practical problem, there are (to a greater or lesser extent) interactions in the process. This implies that even if the control system is decentralized, subsystems of the closed-loop system are not independent of each other. When the process interactions are significant, the choice of a control system structure is far from trivial. For an $n \times n$ plant there are $n!$ possible SISO loop pairings. For plants beyond even a modest number of inputs and outputs, a brute force approach, i.e., to design controllers for every possible pairing and then select the design which provides the best closed-loop performance is impractical.

This complexity drives the need for analysis methods to determine achievable closed-loop system characteristics as a function of control system structure independent of controller design. With these tools, pairings with unacceptable closed-loop performance can be discarded before any controllers are designed. An important characteristic of the results developed is that they only require steady-state knowledge of the plant. This information is easily obtained from simple identification experiments or steady-state design models. Often the pairing question can be answered without the need for detailed dynamic modeling which, in itself, can be expensive and time consuming for large-scale systems, as shown below in this section.

An $n \times n$ MIMO system is shown in Figure 2.1, with process inputs (manipulated variables) denoted as m_j , $j = 1, 2, \dots, n$ and process outputs (controlled variables) as y_i , $i = 1, 2, \dots, n$. The transfer function model of the system is given by

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & g_{n2}(s) & \dots & g_{nn}(s) \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix}$$

i.e.,

$$\mathbf{y} = \mathbf{G}(s)\mathbf{m} \quad (2.1)$$

with $\mathbf{G}(s)$ the transfer function matrix of the process, $\mathbf{m}$ and $\mathbf{y}$ the vector form of n input variables and n output variables respectively.

Figure 2.1: n -input, n -output system

Define $\lambda_{ij}(s)$, the dynamic relative gain between output variable y_i and input variable m_j , as the ratio of two dynamic gains:

$$\lambda_{ij}(s) = \frac{\left(\frac{\partial y_i}{\partial m_j} \right)_{all \text{ loop open}}}{\left(\frac{\partial y_i}{\partial m_j} \right)_{all \text{ loop closed except } m_j \text{ loop}}} \quad (2.2)$$

When the dynamic relative gain is calculated for all the input/output combinations of a multivariable system, it results in an array of the form:

$$\Lambda(s) = \begin{bmatrix} \lambda_{11}(s) & \lambda_{12}(s) & \dots & \lambda_{1n}(s) \\ \lambda_{21}(s) & \lambda_{22}(s) & \dots & \lambda_{2n}(s) \\ \dots & \dots & \dots & \dots \\ \lambda_{n1}(s) & \lambda_{n2}(s) & \dots & \lambda_{nn}(s) \end{bmatrix}$$

the result is the *dynamic relative gain array* (DRGA) and can be used to measure loop interactions.

Important properties of the DRGA:

- The elements of the DRGA across any row, or down any column, sum up to 1, for $s \in (0, \infty)$, i.e.:

$$\sum_{i=1}^n \lambda_{ij}(s) = \sum_{j=1}^n \lambda_{ij}(s) = 1$$

- $\lambda_{ij}(s)$ is dimensionless, neither the units, nor the absolute values taken by the variables y_i or m_j affect it.
- The value of $\lambda_{ij}(s)$ is a measure of the *dynamic* interaction expected in the i th loop if its output y_i is paired with m_j .
- Let $g_{ij}^*(s)$ represent the loop i dynamic gain when all the other loops are closed, whereas $g_{ij}(s)$ represents the normalized, open-loop gain. By the definition of $\lambda_{ij}(s)$,

$$g_{ij}^*(s) = \frac{1}{\lambda_{ij}(s)} g_{ij}(s)$$

and $1/\lambda_{ij}(s)$ indicates by what factor the open-loop gain between y_i and m_j will be altered when the other loops are closed.

- When $\lambda_{ij}(s)$ is negative, it is indicative of a situation in which loop i , with all loops open, will produce a change in y_i in response to a change in m_j totally opposite in direction to that when the other loops are closed. Such input/output pairings are potentially unstable and should be avoided.

The open loop gain from input m_j to output y_i is $g_{ij}(s)$ when all other outputs are uncontrolled. Assuming $\mathbf{G}^{-1}(s)$ exists, and by writing equation (2.1) as

$$\mathbf{m} = \mathbf{G}^{-1}(s)\mathbf{y} \quad (2.3)$$

it can be seen that the gain from m_j to y_i is $1/[\mathbf{G}^{-1}(s)]_{ji}$ when all other outputs are perfectly controlled. The relative gain is the ratio of these "open-loop" and "closed-loop" gains. Thus a matrix of relative gains, the DRGA matrix, can be computed using the formula

$$\mathbf{\Lambda}(s) = \mathbf{G}(s) \otimes \mathbf{G}^{-T}(s) \quad (2.4)$$

where the $\otimes$ symbol denotes element by element multiplication (Hadamard or Schur product).

Given that each element in $\mathbf{G}(s)$ is stable and rational, let n individual feedback controllers (with integral action) be designed for each loop so that each of the resulting n feedback control loops is stable when all the other $n - 1$ loops are open. Define the Niederlinski index (NI) as

$$NI \equiv \frac{|\mathbf{G}(0)|}{\prod_{i=1}^n g_{ii}(0)} \quad (2.5)$$

NI is a supplementary analysis tool of the investigated system's closed-loop stability [10]. Under closed-loop conditions in all n loops, the system

will be unstable for all possible values of controller parameters if NI is negative. In other words, based on steady state information, $NI < 0$ provides a sufficient condition for the loop configuration to be unstable. Variables should be paired in such a way that the resulting pairing corresponds to an NI closest to 1.0. The NI interaction rule is based on empirical observations of the definition of the NI and is justified largely from the relationship between the size of the DRGA and that of the NI . The NI interaction rule has been found to be capable of avoiding ambiguities in using the DRGA interaction rule.

The inverse $\mathbf{G}^{-T}(s)$ may be non-proper or non-causal, and a physical interpretation in terms of perfect control is not meaningful except at steady-state. This has restricted the applications of the DRGA to plants with no RHP-zeros. To relax its restrictions, we consider $s = 0$, i.e., the process is at steady state. In this case, the DRGA becomes the RGA and the calculations are as follows:

$$\lim_{s \rightarrow 0} \mathbf{G}(s) = \mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{bmatrix} \quad (2.6)$$

where $\mathbf{K}$ is the matrix of steady-state gains of $\mathbf{G}(s)$.

Define λ_{ij} , the steady state relative gain between output variable y_i and input variable m_j , as the ratio of two steady state gains:

$$\lambda_{ij} = \lim_{s \rightarrow 0} \lambda_{ij}(s) = \lim_{s \rightarrow 0} \frac{\left(\frac{\partial y_i}{\partial m_j} \right)_{all \text{ loop open}}}{\left(\frac{\partial y_i}{\partial m_j} \right)_{all \text{ loop closed except } m_j \text{ loop}}} \quad (2.7)$$

When the relative gain is calculated for all the input/output combinations of a multivariable system, it results in an array of the form:

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \dots & \lambda_{nn} \end{bmatrix} \quad (2.8)$$

The result is the *relative gain array* (RGA) and has become the most widely used measure of interaction.

Let $\mathbf{R}$ be the transpose of the inverse of $\mathbf{K}$, i.e.:

$$\mathbf{R} = \mathbf{K}^{-T} \quad (2.9)$$

with elements r_{ij} . Then, the elements of the RGA can be obtained from the elements of these two matrices according to:

$$\mathbf{\Lambda} = \mathbf{K} \otimes \mathbf{R} \quad (2.10)$$

where $\otimes$ denotes element-by-element multiplication (Hadamard or Schur product).

Important properties of the RGA:

- The elements of the RGA across any row, or down any column, sum up to 1, i.e.:

$$\sum_{i=1}^n \lambda_{ij} = \sum_{j=1}^n \lambda_{ij} = 1$$

- λ_{ij} is dimensionless, neither the units, nor the absolute values taken by the variables y_i or m_j affect it.

- The value of λ_{ij} is a measure of the *dynamic* interaction expected in the i th loop if its output y_i is paired with m_j .
- Let $\hat{k}_{ij}$ represent the loop i dynamic gain when all the other loops are closed, whereas k_{ij} represents the normalized, open-loop gain. By the definition of λ_{ij} , we have:

$$\hat{k}_{ij} = \frac{1}{\lambda_{ij}} k_{ij}$$

and $1/\lambda_{ij}$ tells by what factor the open-loop gain between y_i and m_j will be altered when the other loops are closed.

- When λ_{ij} is negative, it indicates that loop i will produce a change in y_i in response to a change in m_j totally opposite in direction to that when the other loops are closed. Such input/output pairings are potentially unstable and should be avoided.

2.3 Relative Effective Gain Array

Assume each element in the transfer function matrix $\mathbf{G}(s)$ in (2.1) is linear, which can be expressed by

$$g_{ij}(j\omega) = k_{ij} g_{ij}^0(j\omega) e^{-jL_{ij}\omega} \quad (2.11)$$

where k_{ij} , $g_{ij}^0(j\omega)$ and $e^{-jL_{ij}\omega}$ are steady state gain, normalized transfer function and time delay of $g_{ij}(j\omega)$, respectively. The interaction among individual loop is described by the Relative Effective Gain Array (REGA).

To effectively reflect both steady-state and transient information of the open loop transfer function, we divide the solution into two parts which should be considered for the loop pairing decision:

- Steady-state gain k_{ij} . A process often operates at steady state. The steady gain is the ratio between controller input power and process output power. Given that the process output power is the same, the larger the steady state gain, the smaller the controller input power is required, and the lower the power consumption for the controller's operation is;
- The transient response $f_{ij}(t) = \mathcal{L} [g_{ij}(s)m_j]$. Since it gives an indication on how fast and in what kind of manner the output y_i can reach the steady state, it provides the dynamic information of the process.

Define e_{ij} of a particular transfer function as

$$e_{ij} = k_{ij} \int_0^{\omega_{c,ij}} |g_{ij}^0(j\omega) e^{-jL_{ij}\omega}| d\omega \quad (2.12)$$

where $\omega_{c,ij}$ for $i, j = 1, 2, \dots, n$ are the critical frequencies of the transfer function $g_{ij}(j\omega)$ and $|\bullet|$ is the absolute value of $\bullet$. In order to calculate e_{ij} , the critical frequency can be defined in two ways:

- $\omega_{c,ij} = \omega_{b,ij}$, where $\omega_{b,ij}$ for $i, j = 1, 2, \dots, n$ is the bandwidth of the transfer function $g_{ij}^0(j\omega)$ and determined by the frequency where the magnitude of frequency response is reduced to 0.707 time, i.e.,

$$|g_{ij}^0(j\omega_{b,ij})| = 0.707 |g_{ij}^0(0)| \quad (2.13)$$

- $\omega_{c,ij} = \omega_{u,ij}$, where $\omega_{u,ij}$ for $i, j = 1, 2, \dots, n$ are the ultimate frequencies of the transfer function $g_{ij}^0(j\omega)$ and determined by the frequency where the phase plot of frequency response across $-\pi$, i.e.,

$$\arg [g_{ij}(j\omega_{u,ij})] = -\pi \quad (2.14)$$

For the frequency response of $g_{ij}(j\omega)$, e_{ij} is the “area” covered by $g_{ij}(j\omega)$ up to $\omega_{u,ij}$. Since $|g_{ij}^0(j\omega)|$ represents the magnitude of the transfer function at various frequencies, e_{ij} is considered to be the energy transmission ratio from the manipulated variable u_j to the controlled variable y_i .

Express the energy transmission ratio array as

$$\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & \dots & e_{1n} \\ e_{21} & e_{22} & \dots & e_{2n} \\ \dots & \dots & \dots & \dots \\ e_{n1} & e_{n2} & \dots & e_{nn} \end{bmatrix} \quad (2.15)$$

To simplify the calculation, on energy basis we approximate the integration of e_{ij} by a rectangle area, i.e.,

$$e_{ij} \approx k_{ij}\omega_{c,ij} \quad i, j = 1, 2, \dots, n \quad (2.16)$$

Then, the energy transmission ratio array is given as:

$$\mathbf{E} = \mathbf{K} \otimes \mathbf{\Omega} \quad (2.17)$$

where

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \dots & \dots & \dots & \dots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{bmatrix} \quad (2.18)$$

and

$$\mathbf{\Omega} = \begin{bmatrix} \omega_{c,11} & \omega_{c,12} & \dots & \omega_{c,1n} \\ \omega_{c,21} & \omega_{c,22} & \dots & \omega_{c,2n} \\ \dots & \dots & \dots & \dots \\ \omega_{c,n1} & \omega_{c,n2} & \dots & \omega_{c,nn} \end{bmatrix} \quad (2.19)$$

are the steady state gain array and the critical frequency array, respectively. Since e_{ij} is an indication of energy transmission ratio for loop $y_i - u_j$, the bigger the e_{ij} value is, the more dominant of the loop will be.

2.4 REGA Based Loop Pairing Criterion

Similar to the definition of relative gain [3, 17], the relative effective gain, ϕ_{ij} , between output variable y_i and input variable m_j is defined as the ratio of two effective energy transmission ratios:

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \quad (2.20)$$

where $\hat{e}_{ij}$ is the effective energy transmission ratio between output variable y_i and input variable m_j when all other loops are closed. When the relative effective gains are calculated for all the input/output combinations of a multivariable process, it results in an array of the form, REGA, which can be calculated by

$$\Phi = \mathbf{E} \otimes \mathbf{E}^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \dots & \dots & \dots & \dots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} \end{bmatrix} \quad (2.21)$$

Remark 1. The introduction of energy transmission ratio is to mathematically represent the effectiveness of a control loop which is affected by two key factors, i.e., the steady state gain of the transfer function reflecting the effect of the manipulated variable to the controlled variable k_{ij} , as well as the response speed reflecting the sensitivity of the controlled variable to

the manipulated variable $\omega_{c,ij}$ and consequently the ability to reject the interactions from other loops. Since REGA is a relative measure, using the multiplication of the two parameters to approximate the energy transmission ratio can simplify the calculation while captures the key elements in a multivariable control system.

The shortcoming of the REGA method is that we can either choose bandwidth or phase cross over frequency as the critical frequency. It may lead to undesirable results if the critical frequency is not chosen properly. Empirical studies show that for a transfer function $g_{ij}(s) = \frac{k_{ij}}{T_{ij}s+1}e^{-L_{ij}s}$, when $T > 8L$, bandwidth should be chosen as the critical frequency; On the other hand, when $T \leq 8L$, phase cross over frequency should be chosen as the critical frequency.

For a second-order or higher order transfer function, we may firstly approximate it by a first-order-plus-time-delay transfer function, before we apply the rules stated above. The effectiveness of the above rule of thumb will be demonstrated by the following example.

Example 1. Consider a TITO process [21] with the transfer function matrix given as

$$\mathbf{G}(s) = \begin{pmatrix} \frac{5e^{-40s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-4s}}{10s+1} & \frac{5e^{-40s}}{100s+1} \end{pmatrix}$$

The RGA is

$$\mathbf{\Lambda}(\mathbf{G}) = \begin{pmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{pmatrix}$$

Using phase cross over frequency, $\omega_{c,ij} = \omega_{u,ij}$ as critical frequency, where

$\omega_{u,ij}$ is the frequency $\angle g_{ij}(\omega) = -\pi$, we have

$$\mathbf{\Omega}(\mathbf{G}) = \begin{pmatrix} \omega_{c,11} & \omega_{c,12} \\ \omega_{c,21} & \omega_{c,22} \end{pmatrix} = \begin{pmatrix} 1.57 & 0.448 \\ 0.448 & 1.57 \end{pmatrix}$$

and the REGA is

$$\mathbf{\Phi}_1(\mathbf{G}) = \begin{pmatrix} 0.9840 & 0.0160 \\ 0.0160 & 0.9840 \end{pmatrix}$$

However, select $\omega_{c,ij} = \sqrt{2}g_{ij}(j0)/2$ as the critical frequency (bandwidth), we obtain

$$\mathbf{\Omega}(\mathbf{G}) = \begin{pmatrix} \omega_{c,11} & \omega_{c,12} \\ \omega_{c,21} & \omega_{c,22} \end{pmatrix} = \begin{pmatrix} 0.01 & 0.1 \\ 0.1 & 0.01 \end{pmatrix}$$

and the REGA is

$$\mathbf{\Phi}_2(\mathbf{G}) = \begin{pmatrix} 0.0476 & 0.9524 \\ 0.9524 & 0.0476 \end{pmatrix}$$

By observing the rule of thumb mentioned above, we can tell that $T \leq 8L$ holds for all the four elements of the process matrix. Therefore we should adopt phase cross over frequency as the critical frequency for this process. The loop pairing result shows both the RGA and REGA with phase cross over frequency for $\mathbf{\Omega}$ indicate the same loop pairing, i.e. diagonal pairing, while using bandwidth for $\mathbf{\Omega}$ gives a different result, i.e. the off-diagonal pairing.

To demonstrate the control system performance, the IMC-PID design method is used to design decentralized PI controllers for each pairing selection [25]. The decentralized PI controller for diagonal pairing and for off-diagonal pairing are respectively given as

$$\mathbf{G}_{c1}(s) = \text{diag} \left\{ 0.5 \left(1 + \frac{1}{100s} \right), 0.5 \left(1 + \frac{1}{100s} \right) \right\}$$

Figure 2.2: MATLAB simulation diagrams of the TITO process in Example 1

$$\mathbf{G}_{c2}(s) = \text{diag} \left\{ 1.25 \left(1 + \frac{1}{10s} \right) \quad -0.25 \left(1 + \frac{1}{10s} \right) \right\}$$

The MATLAB simulation diagrams and the simulation results are shown as in Figures 2.2, 2.3 and 2.4. It is obvious that the off-diagonal pairing gives better overall control system performance. It demonstrates the effectiveness of the off-diagonal pairing result by using both REGA and the rule of thumb for critical frequency selection.

Example 2. Consider a 2×2 process [1] with transfer function matrix

$$\mathbf{G}(s) = \begin{bmatrix} \frac{5}{4s+1} & \frac{2.5e^{-5s}}{(2s+1)(15s+1)} \\ \frac{-4e^{-6s}}{20s+1} & \frac{1}{3s+1} \end{bmatrix}$$

$$\text{RGA} : \mathbf{\Lambda} = \begin{bmatrix} 0.3333 & 0.6667 \\ 0.6667 & 0.3333 \end{bmatrix}$$

Figure 2.3: MATLAB simulation diagrams of control system performance comparison

Figure 2.4: Control system performance comparison using REGA (dotted lines: diagonal pairing, solid lines: off-diagonal pairing)

The RGA-based loop pairing criterion indicates off-diagonal 1-2/2-1 pairing ($NI = 3 > 0$). However, by studying the transfer function matrix in frequency domain and analyzing magnitude and phase characteristics of the process' transfer functions, Grosdidier and Morari investigated the closed-loop stability condition of the process under decentralized control, and pointed out it appears undesirable to implement integral controller on off-diagonal 1-2/2-1 pairing due to the time delays. Due to slow dynamics of $g_{12}(s)$ and $g_{21}(s)$, the performance of off-diagonal 1-2/2-1 pairing process with decentralized PI controller implemented appears undesirable and is outperformed by that of diagonal 1-1/2-2 pairing one [1].

Applying the REGA approach on the process, firstly we need to select either the phase cross over frequency or the bandwidth as our critical frequency. Note that $g_{11}(s)$ and $g_{22}(s)$ do not have time delay, and more importantly, we cannot define its phase cross over frequency. Therefore we use the bandwidth as our critical frequency, although for $g_{12}(s)$ and $g_{21}(s)$, $T \leq 8L$ holds.

$$\mathbf{E} = \mathbf{K} \otimes \mathbf{\Omega} = \begin{bmatrix} 1.4330 & 0.3792 \\ -0.5996 & 0.3612 \end{bmatrix}$$

$$\mathbf{\Phi} = \mathbf{E} \otimes \mathbf{E}^{-T} = \begin{bmatrix} 0.6948 & 0.3052 \\ 0.3052 & 0.6948 \end{bmatrix}$$

which indicates diagonal 1-1/2-2 pairing ($NI = 1.5 > 0$). Compared with the approach in [1], the REGA-based technique leads to the same off-diagonal pairing and performs better than the RGA-indicated diagonal pairing. In the meantime the REGA approach is less computationally complicated than the one in Grosdidier's paper, hence it rates higher from a benefit-cost point

of view.

Example 3. Consider a 3×3 process [20] with transfer function matrix

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{s+1} & \frac{e^{-s}}{s+1} \\ \frac{1.5e^{-s}}{s+1} & \frac{e^{-s}}{s+1} & \frac{-2e^{-s}}{10s+1} \\ \frac{e^{-s}}{s+1} & \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{s+1} \end{bmatrix}$$

$$\text{RGA : } \mathbf{\Lambda} = \begin{bmatrix} -0.9302 & 1.1860 & 0.7442 \\ 1.1860 & 0.7442 & -0.9302 \\ 0.7442 & -0.9302 & 1.1860 \end{bmatrix}$$

From the resultant RGA, the desirability of 1-2/2-1/3-3 pairing is close to that of 1-3/2-2/3-1 pairing, and it cannot be determined whether 1-2/2-1/3-3 pairing outperforms 1-3/2-2/3-1 pairing. Without calculating the corresponding NI , closed-loop instability of the latter 1-3/2-2/3-1 pairing is not reflected from the RGA.

Applying the REGA approach on the process, firstly we need to determine whether we choose the phase cross over frequency or the bandwidth as our critical frequency. For $g_{12}(s)$ and $g_{13}(s)$, $T \leq 8L$ holds. On the other hand, $T > 8L$ holds for $g_{11}(s)$. We thus choose the bandwidth for $g_{11}(s)$ and the phase cross over frequency for $g_{12}(s)$ and $g_{13}(s)$. Any of the rest six transfer functions is the same with either $g_{11}(s)$ or $g_{12}(s)$ or $g_{13}(s)$.

$$\mathbf{E} = \mathbf{K} \otimes \mathbf{\Omega} = \begin{bmatrix} -0.1996 & 1.4964 & 0.9976 \\ 1.4964 & 0.9976 & -0.1996 \\ 0.9976 & -0.1996 & 1.4964 \end{bmatrix}$$

$$\Phi = \mathbf{E} \otimes \mathbf{E}^{-T} = \begin{bmatrix} 0.0555 & 0.6977 & 0.2468 \\ 0.6977 & 0.2468 & 0.0555 \\ 0.2468 & 0.0555 & 0.6977 \end{bmatrix}$$

which indicates 1-2/2-1/3-3 pairing ($NI = 0.2593 > 0$). Huang et al. used generalized dynamic relative gain array and also obtained 1-2/2-1/3-3 pairing [47]. The REGA-based methodology is much simpler, however, compared with the generalized dynamic relative gain array approach.

2.5 Relative Normalized Gain Array

For each element in the transfer function matrix $\mathbf{G}(s)$ in Equation (2.1), in time domain, its output response to a unit step input is shown as in Figure 2.5 for non-oscillatory processes and in Figure 2.6 for oscillatory processes, respectively.

Figure 2.5: Step response of a non-oscillatory open transfer function

The time domain solutions for both cases can be uniquely represented as

$$y_i(t) = \begin{cases} 0; & t < L_{ij} \\ k_{ij} f_{ij}(t); & t \geq L_{ij} \end{cases} \quad (2.22)$$

Figure 2.6: Step response of an oscillatory open transfer function

where k_{ij} and $f_{ij}(t)$ are the steady state gain and transient function, respectively.

To evaluate the dynamic properties of each transfer function element, we adopt the integrated error (IE) concept by defining the Normalized IE (NIE), σ_{ij} , as the difference between the unit step input m_j and the normalized open-loop process output $f_{ij}(t) = y_i(t)/k_{ij}$. For system performance assessments, IE provides simple linear calculation, which suits the requirement of subsequent controller design better than other integrated tracking errors (e.g. IAE or ISE). On the other hand, for robust design of either non-oscillatory or oscillatory processes, it is usually desired to achieve the highest possible attenuation of low-frequency disturbance [44]. For such a purpose, the low-frequency controller gain K_I should be maximized. For both PI- and PID-control, maximizing K_I is equivalent to minimizing the IE for load disturbances, which for robust designs with no overshoot is the same as minimizing the IAE [45].

Mathematically, the Normalized IE (NIE) σ_{ij} can be expressed by

$$\sigma_{ij} = \int_0^{\infty} [u_j(t) - f_{ij}(t)] dt \quad (2.23)$$

where $u(t)$ represents the unit-step function.

Obviously, the smaller the value of σ_{ij} , the better the dynamic response of the process is.

For interaction analysis and control system design, each transfer function element can be simplified by either analytical or empirical method [46] to a first-order plus time delay (FOPTD) model

$$g_{ij}(s) = \frac{k_{ij}}{T_{ij}s + 1} e^{-L_{ij}s} \quad (2.24)$$

for non-oscillatory unit step response, or a Second Order plus Time Delay (SOPTD) model

$$g_{ij}(s) = k_{ij} \frac{\omega_{n,ij}^2}{s^2 + 2\zeta\omega_{n,ij}s + \omega_{n,ij}^2} e^{-L_{ij}s} \quad (2.25)$$

for oscillatory unit step response.

In either case, the analytical solution for NIE, σ_{ij} , can be derived.

For the FOPTD model given in Equation (2.24), the time domain solution of output $f_{ij}(t)$ for a unit step change in m_j is shown as in Figure 2.5 and is mathematically expressed as in Equation (2.22) with

$$f_{ij}(t) = 1 - e^{-(t-L_{ij})/T_{ij}} \quad (2.26)$$

Substituting (2.26) into (2.23), we obtain

$$\begin{aligned}
 \sigma_{ij} &= \int_0^\infty [u_j(t) - \bar{y}_i(t)] dt \\
 &= \frac{\int_0^\infty [y_i(\infty) - y_i(t)] dt}{k_{ij}} \\
 &= \frac{\int_0^{L_{ij}} y_i(\infty) dt + \int_{L_{ij}}^\infty [y_i(\infty) - k_{ij} f_{ij}(t)] dt}{k_{ij}} \\
 &= \frac{\int_0^{L_{ij}} k_{ij} dt + k_{ij} \int_{L_{ij}}^\infty [1 - (1 - e^{-(t-L_{ij})/T_{ij}})] dt}{k_{ij}} \\
 &= T_{ij} + L_{ij}
 \end{aligned} \tag{2.27}$$

which is the average residence time of loop $y_i - u_j$.

For the SOPTD model given in Equation (2.25) with $0 < \zeta_{ij} < 1$, the time domain solution to unit step input is in the form of Equation (2.22) with its response and the response envelop curves are shown as in Figure 2.6 by red color curve and black color curve, respectively.

$$f_{ij}(t) = 1 - \frac{e^{-\zeta_{ij}\omega_{n,ij}(t-L_{ij})}}{\sqrt{1-\zeta_{ij}^2}} \sin \left[\omega_{n,ij} \sqrt{1-\zeta_{ij}^2} (t-L_{ij}) + \tan^{-1} \frac{\sqrt{1-\zeta_{ij}^2}}{\zeta_{ij}} \right], 0 < \zeta_{ij} < 1 \tag{2.28}$$

Substituting Equation (2.28) into Equation (2.23), we obtain

$$\begin{aligned}
 \sigma_{ij} &= \int_0^\infty [u_{ij}(t) - \bar{y}_{ij}(t)] dt \\
 &= \frac{\int_0^\infty [y_{ij}(\infty) - y_{ij}(t)] dt}{k_{ij}} \\
 &= \frac{\int_0^{L_{ij}} y_{ij}(\infty) dt + \int_{L_{ij}}^\infty [y_{ij}(\infty) - k_{ij} f_{ij}(t)] dt}{k_{ij}} \\
 &= \frac{\int_0^{L_{ij}} k_{ij} dt + k_{ij} \int_{L_{ij}}^\infty [1 - f_{ij}(t)] dt}{k_{ij}} \\
 &= L_{ij} + \int_{L_{ij}}^\infty [1 - f_{ij}(t)] dt \\
 &= L_{ij} + \int_{L_{ij}}^\infty \left[\frac{e^{-\zeta_{ij} \omega_{n,ij}(t-L_{ij})}}{\sqrt{1-\zeta^2}} \sin \left(\omega_{n,ij} \sqrt{1-\zeta^2} (t-L_{ij}) + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] dt \\
 &= L_{ij} + \int_0^\infty \left[\frac{e^{-\zeta_{ij} \omega_{n,ij} \tau}}{\sqrt{1-\zeta^2}} \sin \left(\omega_{n,ij} \sqrt{1-\zeta^2} \tau + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] d\tau
 \end{aligned} \tag{2.29}$$

Since

$$\begin{aligned}
& \int_0^\infty \left[\frac{e^{-\zeta_{ij}\omega_{n,ij}\tau}}{\sqrt{1-\zeta^2}} \sin \left(\omega_{n,ij} \sqrt{1-\zeta^2}\tau + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] d\tau \\
&= \frac{1}{\sqrt{1-\zeta^2}} \int_0^\infty \left[e^{-\zeta_{ij}\omega_{n,ij}\tau} \sin \left(\omega_{n,ij} \sqrt{1-\zeta^2}\tau + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] d\tau \\
&= \frac{1}{\sqrt{1-\zeta^2}} \int_0^\infty \left[e^{-\zeta_{ij}\omega_{n,ij}\tau} \sin \left(\omega_{n,ij} \sqrt{1-\zeta^2}\tau \right) \cos \left(\tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] d\tau \\
&\quad + \frac{1}{\sqrt{1-\zeta^2}} \int_0^\infty \left[e^{-\zeta_{ij}\omega_{n,ij}\tau} \cos \left(\omega_{n,ij} \sqrt{1-\zeta^2}\tau \right) \sin \left(\tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right] d\tau \\
&= \frac{\zeta}{\sqrt{1-\zeta^2}} \int_0^\infty \left[e^{-\zeta_{ij}\omega_{n,ij}\tau} \sin \left(\omega_{n,ij} \sqrt{1-\zeta^2}\tau \right) \right] d\tau \\
&\quad + \int_0^\infty \left[e^{-\zeta_{ij}\omega_{n,ij}\tau} \cos \left(\omega_{n,ij} \sqrt{1-\zeta^2}\tau \right) \right] d\tau \\
&= \frac{\zeta_{ij}}{\sqrt{1-\zeta^2}} \frac{1}{\omega_{n,ij}^2} \left[-\zeta_{ij}\omega_{n,ij} e^{-\zeta_{ij}\omega_{n,ij}\tau} \sin \left(\omega_{n,ij} \sqrt{1-\zeta_{ij}^2}\tau \right) \right] \Big|_0^\infty \\
&\quad - \frac{\zeta_{ij}}{\sqrt{1-\zeta^2}} \frac{1}{\omega_{n,ij}^2} \left[\omega_{n,ij} \sqrt{1-\zeta_{ij}^2} e^{-\zeta_{ij}\omega_{n,ij}\tau} \cos \left(\omega_{n,ij} \sqrt{1-\zeta_{ij}^2}\tau \right) \right] \Big|_0^\infty \\
&\quad + \frac{1}{\omega_{n,ij}^2} \left[\omega_{n,ij} \sqrt{1-\zeta_{ij}^2} e^{-\zeta_{ij}\omega_{n,ij}\tau} \sin \left(\omega_{n,ij} \sqrt{1-\zeta_{ij}^2}\tau \right) \right] \Big|_0^\infty \\
&\quad - \frac{1}{\omega_{n,ij}^2} \left[\zeta_{ij}\omega_{n,ij} e^{-\zeta_{ij}\omega_{n,ij}\tau} \cos \left(\omega_{n,ij} \sqrt{1-\zeta_{ij}^2}\tau \right) \right] \Big|_0^\infty \\
&= \frac{e^{-\zeta_{ij}\omega_{n,ij}\tau}}{\omega_{n,ij} \sqrt{1-\zeta_{ij}^2}} \left[(1-2\zeta_{ij}^2) \sin \left(\omega_{n,ij}\tau \sqrt{1-\zeta_{ij}^2} \right) - 2\zeta_{ij} \sqrt{1-\zeta_{ij}^2} \cos \left(\omega_{n,ij}\tau \sqrt{1-\zeta_{ij}^2} \right) \right] \Big|_0^\infty \\
&= 0 - \left(-\frac{2\zeta_{ij}}{\omega_{n,ij}} \right) \\
&= \frac{2\zeta_{ij}}{\omega_{n,ij}}
\end{aligned} \tag{2.30}$$

we have,

$$\sigma_{ij} = \frac{2\zeta_{ij}}{\omega_{n,ij}} + L_{ij} \triangleq T_{ij} + L_{ij} \tag{2.31}$$

where the time constant is $T_{ij} = \frac{2\zeta_{ij}}{\omega_{n,ij}}$.

Now, define the normalized gain of a transfer function $g_{ij}(s)$ by combining both steady state and transient information, as

$$k_{N,ij} = \frac{k_{ij}}{\sigma_{ij}} \quad (2.32)$$

The physical explanation of Equation (2.32) is that the larger $k_{N,ij}$ is, the more effective of input u_j to the output y_i will be.

Substituting Equation (2.27) or (2.31) into Equation (2.32), we can find a very simple expression for loop's effective gain:

$$k_{N,ij} = \frac{k_{ij}}{T_{ij} + L_{ij}} \quad i, j = 1, 2, \dots, n \quad (2.33)$$

Based on effective gain, we define effective gain matrix for the overall process as:

$$\mathbf{K}_N = \begin{bmatrix} k_{N,11} & k_{N,12} & \dots & k_{N,1n} \\ k_{N,21} & k_{N,22} & \dots & k_{N,2n} \\ \dots & \dots & \dots & \dots \\ k_{N,n1} & k_{N,n2} & \dots & k_{N,nn} \end{bmatrix} = \begin{bmatrix} \frac{k_{11}}{T_{11}+L_{11}} & \frac{k_{12}}{T_{12}+L_{12}} & \dots & \frac{k_{1n}}{T_{1n}+L_{1n}} \\ \frac{k_{21}}{T_{21}+L_{21}} & \frac{k_{22}}{T_{22}+L_{22}} & \dots & \frac{k_{2n}}{T_{2n}+L_{2n}} \\ \dots & \dots & \dots & \dots \\ \frac{k_{n1}}{T_{n1}+L_{n1}} & \frac{k_{n2}}{T_{n2}+L_{n2}} & \dots & \frac{k_{nn}}{T_{nn}+L_{nn}} \end{bmatrix}$$

or in matrix form,

$$\mathbf{K}_N = \mathbf{K} \odot (\mathbf{L} + \mathbf{T}) \quad (2.34)$$

where the $\odot$ symbol denotes element by element division.

2.6 RNGA Based Loop Pairing Criterion

Define a complementary system

$$\mathbf{y}_N = \mathbf{K}_N \mathbf{m}_N \quad (2.35)$$

where $\mathbf{K}_N$ is defined by Equation (2.34), $\mathbf{m}_N = [m_{N,1}, \dots, m_{N,n}]$ and $\mathbf{y}_N = [y_{N,1}, \dots, y_{N,n}]$ are the complementary system input and output vectors, respectively. Since both steady state and dynamic information are contained in a constant matrix $\mathbf{K}_N$, the definition and calculation procedures of RGA can be directly extended to the complementary system (2.35).

Define the relative normalized gain, $\lambda_{N,ij}$, as the ratio of two normalized gains between output variable $y_{N,i}$ and input variable $m_{N,j}$:

$$\lambda_{N,ij} = \frac{k_{N,ij}}{\hat{k}_{N,ij}} \quad (2.36)$$

where $\hat{k}_{N,ij}$ is the normalized gain between output variable and input variable when all other loops are closed. When the effective relative gains are calculated for all the input/output combinations of a multivariable process, it results in an array of the form similar to RGA, namely the Relative Normalized Gain Array (RNGA):

$$\mathbf{\Lambda}_N = \begin{bmatrix} \lambda_{N,11} & \lambda_{N,12} & \dots & \lambda_{N,1n} \\ \lambda_{N,21} & \lambda_{N,22} & \dots & \lambda_{N,2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{N,n1} & \lambda_{N,n2} & \dots & \lambda_{N,nn} \end{bmatrix} \quad (2.37)$$

which can be calculated by

$$\mathbf{\Lambda}_N = \mathbf{K}_N \otimes \mathbf{K}_N^{-T} \quad (2.38)$$

Consequently, the recommended controller pairing is determined in the same manner, and the properties of RGA can be directly extended to RNGA:

- The value of $\lambda_{N,ij}$ is a measure of the effective interaction expected in the i^{th} loop if its output $y_{N,i}$ is paired with input $m_{N,j}$.

- The elements of the RNGA across any row, or down any column, sum up to 1, i.e.

$$\sum_{i=1}^n \lambda_{N,ij} = \sum_{j=1}^n \lambda_{N,ij} = 1 \quad (2.39)$$

- Let $\hat{k}_{N,ij}$ represent the normalized gain of Loop i when all the other loops are closed, whereas $k_{N,ij}$ represents the normalized, open-loop virtual gain. By the definition of $\lambda_{N,ij}$:

$$\hat{k}_{N,ij} = \frac{1}{\lambda_{ij}} k_{N,ij} \quad (2.40)$$

- When $\lambda_{N,ij}$ is negative, it is indicative of a situation in which loop i , with other loops open, will produce a change in y_i in response to a change in m_j totally opposite in direction to that when the other loops are closed.
- Process with large RNGA elements are always ill-conditioned, it is sensitive to the modeling errors.

Define the Niederlinski index (NI) as previously,

$$NI \equiv \frac{|\mathbf{G}(0)|}{\prod_{i=1}^n g_{ii}(0)}$$

Similar to the RGA and NI based pairing rules, the RNGA and NI based loop pairing rules require that manipulated and controlled variables in a decentralized control system be paired:

- Corresponding RNGA elements are closest to 1.0.
- Corresponding RNGA elements are positive.

- The NI is positive.
- All paired RNGA elements are positive.
- Large RNGA elements should be avoided.

Here, both RNGA and NI play important roles for control structure selection. RNGA is used to measure interactions, while NI and RGA are used as a sufficient condition to rule out the closed loop unstable pairings. Since we are using effective gains instead of simply steady state gains, transient response can be effectively reflected. Therefore, comparing the RNGA based method with RGA/DRGA based one, we may expect that:

- In addition to steady state gains, only time constant and time delay are required in the RNGA method, it is far easier to calculate than those DRGA methods.
- RNGA combines both steady state gains and equivalent error in measuring the loop interactions, it should provide better pairing results than that of RGA based pairing and comparable with DRGA ones.
- Since RNGA only uses information of open loop process transfer functions, it is controller type independent.

More examples are provided as follows to demonstrate the effectiveness of the proposed RNGA-based loop pairing criterion.

Example 1 (Continued). A TITO process [21] with the transfer function matrix is given as

$$\mathbf{G}(s) = \begin{pmatrix} \frac{5e^{-40s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-4s}}{10s+1} & \frac{5e^{-40s}}{100s+1} \end{pmatrix}$$

The RGA is

$$\mathbf{\Lambda}(\mathbf{G}) = \begin{pmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{pmatrix}$$

Applying the RGA approach on the same process,

$$\mathbf{K}_N = \mathbf{K} \odot (\mathbf{L} + \mathbf{T}) = \begin{bmatrix} 0.0357 & 0.0714 \\ -0.3571 & 0.0357 \end{bmatrix}$$

$$\mathbf{\Lambda}_N = \mathbf{K}_N \otimes \mathbf{K}_N^{-T} = \begin{bmatrix} 0.0476 & 0.9524 \\ 0.9524 & 0.0476 \end{bmatrix}$$

which indicates off-diagonal 1-2/2-1 pairing ($NI = 6 > 0$). The conclusion was confirmed by Mc Avoy et al that off-diagonal 1-2/2-1 pairing outperforms diagonal 1-1/2-2 pairing [21]. The RGA approach, however, is simpler to apply and easier to understand, and it involves both time delay and time constant of the process dynamics in the loop pairing criterion.

To check the control system performance, the IMC-PID design method is used to design decentralized PI controllers for each pairing selection [25]. The decentralized PI controller for diagonal pairing and for off-diagonal pairing are given, respectively, as

$$\mathbf{G}_{c1}(s) = \text{diag} \left\{ 0.5 \left(1 + \frac{1}{100s} \right) \quad 0.5 \left(1 + \frac{1}{100s} \right) \right\}$$

$$\mathbf{G}_{c2}(s) = \text{diag} \left\{ 1.25 \left(1 + \frac{1}{10s} \right) \quad -0.25 \left(1 + \frac{1}{10s} \right) \right\}$$

Figure 2.7: Control system performance using RGA (dotted lines: diagonal pairing, solid lines: off-diagonal pairing)

The simulation results are shown as in Figure 2.7, it is obvious that the off-diagonal pairing gives better overall control system performance.

Example 2 (Continued). Consider a 2×2 process [1] with transfer function matrix

$$\mathbf{G}(s) = \begin{bmatrix} \frac{5}{4s+1} & \frac{2.5e^{-5s}}{(2s+1)(15s+1)} \\ \frac{-4e^{-6s}}{20s+1} & \frac{1}{3s+1} \end{bmatrix}$$

$$\text{RGA : } \mathbf{\Lambda} = \begin{bmatrix} 0.3333 & 0.6667 \\ 0.6667 & 0.3333 \end{bmatrix}$$

The RGA-based loop pairing criterion indicates off-diagonal 1-2/2-1 pairing ($NI = 3 > 0$). However, by studying the transfer function matrix in frequency domain and analyzing magnitude and phase characteristics of the process' transfer functions, Grosdidier and Morari investigated the closed-loop stability condition of the process under decentralized control, and pointed

out it appears undesirable to implement integral controller on off-diagonal 1-2/2-1 pairing due to the time delays. Due to slow dynamics of $g_{12}(s)$ and $g_{21}(s)$, the performance of off-diagonal 1-2/2-1 pairing process with decentralized PI controller implemented appears undesirable and is outperformed by that of diagonal 1-1/2-2 pairing one [1].

Applying the RGA approach on the process,

$$\mathbf{K}_N = \mathbf{K} \odot (\mathbf{L} + \mathbf{T}) = \begin{bmatrix} 1.2500 & 0.1126 \\ -0.1539 & 0.3333 \end{bmatrix}$$

$$\mathbf{\Lambda}_N = \mathbf{K}_N \otimes \mathbf{K}_N^{-T} = \begin{bmatrix} 0.9597 & 0.0403 \\ 0.0403 & 0.9597 \end{bmatrix}$$

which indicates diagonal 1-1/2-2 pairing ($NI = 1.5 > 0$). By comparison with the approach in [1], the RGA-based technique is less computationally complicated, and hence rates higher from a benefit-cost point of view.

Example 3 (Continued). Consider a 3×3 process [20] with transfer function matrix

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{s+1} & \frac{e^{-s}}{s+1} \\ \frac{1.5e^{-s}}{s+1} & \frac{e^{-s}}{s+1} & \frac{-2e^{-s}}{10s+1} \\ \frac{e^{-s}}{s+1} & \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{s+1} \end{bmatrix}$$

$$\text{RGA : } \mathbf{\Lambda} = \begin{bmatrix} -0.9302 & 1.1860 & 0.7442 \\ 1.1860 & 0.7442 & -0.9302 \\ 0.7442 & -0.9302 & 1.1860 \end{bmatrix}$$

From the resultant RGA, the desirability of 1-2/2-1/3-3 pairing is close to that of 1-3/2-2/3-1 pairing, and it cannot be determined whether 1-2/2-1/3-3 pairing outperforms 1-3/2-2/3-1 pairing. Without calculating the cor-

responding NI , closed-loop instability of the latter 1-3/2-2/3-1 pairing is not reflected from the RGA.

Applying the RGA approach on the process,

$$\mathbf{K}_N = \mathbf{K} \odot (\mathbf{L} + \mathbf{T}) = \begin{bmatrix} -0.1818 & 0.7500 & 0.5000 \\ 0.7500 & 0.5000 & -0.1818 \\ 0.5000 & -0.1818 & 0.7500 \end{bmatrix}$$

$$\mathbf{\Lambda}_N = \mathbf{K}_N \otimes \mathbf{K}_N^{-T} = \begin{bmatrix} 0.0834 & 0.6574 & 0.2592 \\ 0.6574 & 0.2592 & 0.0834 \\ 0.2592 & 0.0834 & 0.6574 \end{bmatrix}$$

which indicates 1-2/2-1/3-3 pairing ($NI = 0.2593 > 0$). The same loop pairing decision was obtained by Huang et al. by the methodology of generalized dynamic relative gain array [47]. Comparatively, however, the RGA-based methodology is simpler.

We use an example that will be studied in detail in Chapter 5 to demonstrate how the modified RGA works for MIMO integrating systems. A TITO process has the transfer function matrix

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-1.1053}{s(19.4361s+1)}e^{-4.6744s} & \frac{1.4772}{s(32.3660s+1)}e^{-9.169s} \\ \frac{0.3029}{15.5059s+1}e^{-4.2528s} & \frac{-0.6682}{23.4279s+1}e^{-5.7224s} \end{bmatrix}$$

By letting $\varepsilon = 10^{-6}$, the steady state gain of the approximated transfer function matrix $\mathbf{G}(s)$ is

$$\mathbf{K} = \begin{bmatrix} \frac{-1.1053}{\varepsilon} & \frac{1.4772}{\varepsilon} \\ 0.3029 & -0.6682 \end{bmatrix} \quad (\varepsilon = 10^{-6})$$

Therefore, the RGA of the approximated transfer function matrix $\mathbf{G}(s)$

is

$$\begin{aligned}\Lambda &= \mathbf{K} \otimes (\mathbf{K}^{-1})^T \\ &\approx \begin{bmatrix} 2.5370 & -1.5370 \\ -1.5370 & 2.5370 \end{bmatrix}\end{aligned}$$

Also, $NI = 0.39 > 0$. Thus the RGA-based loop pairing criterion indicates diagonal 1-1/2-2 pairing.

2.7 Modified RNGA and REGA for MIMO Processes Containing Integrators

For the elements in the transfer function matrix $\mathbf{G}(s)$ in Equation (2.1), some of them might contain integrators. Huang et al. discussed [26] multiple loop PID controller design for not only the control of level loops but also that of large time constant processes. In a multivariable process $\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s) + \mathbf{D}(s)$, $\mathbf{Y}(s)$, $\mathbf{U}(s)$ and $\mathbf{D}(s)$ represent the output vector, the input vector and the disturbance vector respectively. A generalized representation of an integrating element in the process' transfer matrix $\mathbf{G}(s)$ is as follows:

$$g_{ij}(s) = \tilde{k}_{ij} \times \bar{g}_{ij}(s)$$

where $\tilde{k}_{ij}$ and $\bar{g}_{ij}(s)$ with $\bar{g}_{ij}(j0) = 1$ are the steady state gain and the normalized transfer function of $g_{ij}(s)$ respectively, and

$$\tilde{k}_{ij} = \begin{cases} k_{ij}, & \text{if } g_{ij}(s) \text{ does not contain pure integrator} \\ k_{ij}/\varepsilon, & \text{if } g_{ij}(s) \text{ contains a pure integrator} \end{cases}$$

Huang et al. discussed [26] multiple loop PID controller design for not only the control of level loops but also that of large time constant processes.

In the computation for RGA, the parameter, ε , is assigned to have a positive and very small number (e.g., an order of 10^{-6}) [26]. Therefore, Equation (2.10) is modified accordingly as

$$\mathbf{\Lambda} = \lim_{\varepsilon \rightarrow 0} \tilde{\mathbf{K}} \otimes \tilde{\mathbf{K}}^{-T}$$

In light of this, Equation (2.38) is modified accordingly as

$$\mathbf{\Lambda}_N = \lim_{\varepsilon \rightarrow 0} \mathbf{K}_N \otimes \mathbf{K}_N^{-T}$$

With this modification, $k_{N,ij}$ for typical processes are listed in Table 2.1.

Process	g_{ij}	σ_{ij}	$k_{N,ij}$
Pure time delay	$k_{ij}e^{-\theta_{ij}s}$	θ_{ij}	k_{ij}/θ_{ij}
First order	$\frac{k_{ij}}{\tau_{ij}s+1}$	τ_{ij}	k_{ij}/τ_{ij}
Integrating	$k_{ij}\frac{e^{-\theta_{ij}s}}{s}$	θ_{ij}	$k_{ij}/\varepsilon\theta_{ij}$
FOPDT	$\frac{k_{ij}}{\tau_{ij}s+1}e^{-\theta_{ij}s}$	$\theta_{ij} + \tau_{ij}$	$\frac{k_{ij}}{\tau_{ij}+\theta_{ij}}$
First order Integrating	$\frac{k_{ij}}{s(\tau_{ij}s+1)}e^{-\theta_{ij}s}$	$\theta_{ij} + \tau_{ij}$	$\frac{k_{ij}}{\varepsilon(\tau_{ij}+\theta_{ij})}$
SOPDT	$k_{ij}\frac{\omega_{n,ij}^2}{s^2+2\zeta_{ij}\omega_{n,ij}s+\omega_{n,ij}^2}e^{-\theta_{ij}s}$	$\frac{2\zeta_{ij}}{\omega_{n,ij}} + \theta_{ij}$	$\frac{k_{ij}}{2\zeta_{ij}/\omega_{n,ij}+\theta_{ij}}$

Table 2.1: $k_{N,ij}$ for typical processes

Similarly, for REGA calculation, Equation (3.7) is modified as

$$\mathbf{\Phi} = \lim_{\varepsilon \rightarrow 0} \mathbf{E} \otimes \mathbf{E}^{-T}$$

We use the same example in last section to demonstrate how the modified REGA/RNGA works for MIMO integrating systems. To apply REGA, by

letting $\varepsilon = 10^{-6}$,

$$\begin{aligned}\mathbf{E} &= \mathbf{K} \otimes \mathbf{\Omega} \\ &= \begin{bmatrix} \frac{-0.1241}{\varepsilon} & \frac{0.1678}{\varepsilon} \\ 0.0364 & -0.0795 \end{bmatrix}\end{aligned}$$

The REGA matrix is calculated as follows.

$$\begin{aligned}\Phi &= \mathbf{E} \otimes (\mathbf{E}^{-1})^T \\ &\approx \begin{bmatrix} 2.6278 & -1.6278 \\ -1.6278 & 2.6278 \end{bmatrix}\end{aligned}$$

which also indicates diagonal 1-1/2-2 pairing ($NI = 0.48 > 0$).

We may apply the RNGA approach on the same process in a similar way.

By letting $\varepsilon = 10^{-6}$,

$$\mathbf{K}_N = \begin{bmatrix} \frac{-0.0458}{\varepsilon} & \frac{0.0356}{\varepsilon} \\ 0.0153 & -0.0229 \end{bmatrix}$$

The RNGA matrix is calculated as follows.

$$\Lambda_N = \mathbf{K}_N \otimes (\mathbf{K}_N^{-1})^T \approx \begin{bmatrix} 2.0783 & -1.0783 \\ -1.0783 & 2.0783 \end{bmatrix}$$

which also indicates diagonal 1-1/2-2 pairing ($NI = 0.48 > 0$).

2.8 Conclusion

In this chapter, REGA/RNGA-based loop pairing techniques for MIMO integrating system were presented. Based on interaction measurements, the REGA and RNGA loop pairing methods combine the advantages of both the RGA and the DRGA approaches. They utilize both the steady state and

transient information of the process transfer function. A new loop pairing criterion based on the REGA/RNGA was then proposed for control structure configuration. The main advantages of both the REGA and RNGA methods are their simplicity together with their comprehensive description of dynamic interaction among individual loops without requiring the specification of the controller type. Application of the REGA and RNGA techniques to MIMO integrating systems were also discussed. Several examples, for which the RGA based loop pairing criterion gives inaccurate interaction assessments, were employed to demonstrate the effectiveness of the proposed interaction measurements and loop pairing criteria.

Chapter 3

Effective Transfer Function Based Decentralized Controller Design

3.1 Introduction

One key issue in decentralized control is to derive effective transfer functions of a MIMO process based on its loop pairing result. In a MIMO process, the effective transfer function (ETF) of an individual control loop explicitly reflects its dynamic relative interaction with all the other loops when they are closed. With the effective transfer functions, the dynamic relative interaction is explicitly represented by the process model. After we derive the ETF matrix of a MIMO process, the subsequent controller tuning is largely facilitated by SISO controller design techniques. This chapter presents the derivation of effective transfer function and extends the discussion to MIMO

integrating processes.

3.2 Relative Average Residence Time Array

In control system design, two parameters are most important in describing the dynamic properties of a transfer function, i.e.:

- Steady state gain $g_{ij}(j0)$: the steady state gain reflects the effect of the manipulated variable u_j to the controlled variable y_i ;
- Average residence time σ_{ij} : the average residence time is accountable for the response speed of the controlled variable y_i to manipulated variable u_j .

Based on the discussion in Section 2.5, we can use normalized integrated error to denote the average residence time, and we can consequently define the normalized gain $k_{N,ij}$ and the relative normalized gain $\lambda_{N,ij}$. To reveal the transfer function changes when all other loops are closed, we further define the relative average residence time, γ_{ij} , as the ratio of the average residence time of loop $y_i - u_j$ when other loops are closed and the average residence time of loop $y_i - u_j$ when other loops are open, i.e.

$$\gamma_{ij} \triangleq \frac{\hat{\sigma}_{ij}}{\sigma_{ij}} \quad i, j = 1, 2, \dots, n \quad (3.1)$$

where $\hat{\sigma}_{ij}$ is the average residence time of loop $i - j$ when other loops are closed. Using the definition of RNGA, we rewrite Equation (3.6) as

$$\hat{k}_{ij} \times \frac{1}{\hat{\sigma}_{ij}} = \frac{k_{ij} \times \frac{1}{\sigma_{ij}}}{\lambda_{N,ij}} \quad i, j = 1, 2, \dots, n \quad (3.2)$$

Equation (3.7) provides both gain and average residence time change information when all other loops are closed.

To separate these two changes, we first use the definition of RGA [48]

$$\hat{k}_{ij} = \frac{k_{ij}}{\lambda_{ij}} \quad i, j = 1, 2, \dots, n \quad (3.3)$$

where λ_{ij} can be calculated by letting $s = 0$ in Equation (2.2).

Then substitute Equation (3.3) into Equation (3.7) and rearrange, we obtain a formula for calculating γ_{ij} :

$$\gamma_{ij} = \frac{\hat{\sigma}_{ij}}{\sigma_{ij}} = \frac{\lambda_{N,ij}}{\lambda_{ij}} \quad i, j = 1, 2, \dots, n \quad (3.4)$$

When the relative average residence times are calculated for all the input/output combinations of a multivariable process, it results in an array of the form, i.e., Relative Average Residence Time Array (RARTA) defined as

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \dots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \dots & \gamma_{2n} \\ \dots & \dots & \dots & \dots \\ \gamma_{n1} & \gamma_{n2} & \dots & \gamma_{nn} \end{bmatrix}$$

and its elements can be calculated by

$$\mathbf{\Gamma} \triangleq \mathbf{\Lambda}_N \odot \mathbf{\Lambda} = \begin{bmatrix} \lambda_{N,11} & \lambda_{N,12} & \dots & \lambda_{N,1n} \\ \lambda_{N,21} & \lambda_{N,22} & \dots & \lambda_{N,2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{N,n1} & \lambda_{N,n2} & \dots & \lambda_{N,nn} \end{bmatrix} \odot \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \dots & \lambda_{nn} \end{bmatrix} \quad (3.5)$$

where the operator $\odot$ is the hadamard division.

Similarly, based on the discussion in Section 2.3, we can adopt the concept of energy transmission ratio to evaluate the dynamic properties of each transfer function element. By defining relative effective gain between output variable y_i and input variable u_j , ϕ_{ij} , as the ratio of two normalized gains, we have:

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \quad i, j = 1, 2, \dots, n \quad (3.6)$$

where $\hat{e}_{ij}$ is the effective gain between y_i and input variable u_j when all other loops are closed. The Relative Effective Gain Array (REGA) is hence

$$\Phi = \mathbf{E} \otimes \mathbf{E}^{-T} = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \dots & \dots & \dots & \dots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} \end{bmatrix} \quad (3.7)$$

where the operator $\otimes$ is the hadamard product.

Consequently, we define the Relative Average Residence Time Array based on REGA as

$$\Gamma \triangleq \Phi \odot \Lambda = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \dots & \dots & \dots & \dots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} \end{bmatrix} \odot \begin{bmatrix} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1n} \\ \lambda_{21} & \lambda_{22} & \dots & \lambda_{2n} \\ \dots & \dots & \dots & \dots \\ \lambda_{n1} & \lambda_{n2} & \dots & \lambda_{nn} \end{bmatrix} \quad (3.8)$$

where the operator $\odot$ is the hadamard division.

Since the relative average residence time is the ratio of the average residence times between when other loops are closed and when other loops are open, it represents the dynamic changes of the transfer function when other loops closed. By the definition of RARTA, we can write

$$\hat{\sigma}_{ij} = \gamma_{ij} \times \sigma_{ij} = \gamma_{ij} \times \tau_{ij} + \gamma_{ij} \times \theta_{ij} \quad i, j = 1, 2, \dots, n \quad (3.9)$$

The average resident time of loop $i - j$ when other loops are closed is the open loop average resident time scaled by a factor γ_{ij} .

3.3 Effective Transfer Function

In process control, steady state gain, time constant and time delay are the three model parameters which are uppermost for control system design. By using RGA and RARTA information, gain and phase changes of a transfer function element when other loops closed can be uniquely determined. That is, a transfer function element of a MIMO process when other loops are closed can be approximated by a transfer function element having the same form as the open-loop transfer function element, but the steady state gain, time constant and time delay are scaled RGA and RARTA, respectively. Using Equation 3.3 and 3.9, we can derive the effective transfer function of a certain process as

$$\hat{g}_{ij}(s) = \hat{k}_{ij} \times \frac{1}{\hat{\tau}_{ij}s + 1} e^{-\hat{\theta}_{ij}s} = \frac{k_{ij}}{\lambda_{ij}} \times \frac{1}{\gamma_{ij}\tau_{ij}s + 1} e^{-\gamma_{ij}\theta_{ij}s} \quad i, j = 1, 2, \dots, n \quad (3.10)$$

where $\hat{g}_{ij}$ in Equation (3.10) is optimal ETF of loop $i - j$ when other loops are closed under IE criterion. $\hat{g}_{ij}$ should resemble the dynamic response of corresponding true transfer function element when other loops are closed. ETFs for most commonly used transfer functions are listed in Table 3.1.

Process	g_{ij}	$\hat{g}_{ij}$
Pure time delay	$k_{ij}e^{-\theta_{ij}s}$	$\hat{k}_{ij}e^{-\hat{\theta}_{ij}s}$
First Order	$\frac{k_{ij}}{\tau_{ij}s+1}$	$\frac{\hat{k}_{ij}}{\hat{\tau}_{ij}s+1}$
FOPDT	$\frac{k_{ij}}{\tau_{ij}s+1}e^{-\theta_{ij}s}$	$\frac{\hat{k}_{ij}}{\hat{\tau}_{ij}s+1}e^{-\hat{\theta}_{ij}s}$
SOPDT	$k_{ij}\frac{\omega_{n,ij}^2}{s^2+2\zeta_{ij}\omega_{n,ij}s+\omega_{n,ij}^2}e^{-\theta_{ij}s}$	$\hat{k}_{ij}\frac{\omega_{n,ij}^2}{s^2+2\hat{\zeta}_{ij}\omega_{n,ij}s+\omega_{n,ij}^2}e^{-\hat{\theta}_{ij}s}$

Table 3.1: ETFs for most commonly used transfer functions

3.4 Effective Transfer Function for Integrating Processes

As discussed in Section 2.7, we define $\hat{g}_{ij}(s)$ as the effective open-loop transfer function, i.e., the open-loop transfer function from the selected input u_j to the output y_i . Before we start designing decentralized controller for a MIMO integrating process, we need to identify the which entries in the transfer matrix contains $1/s$, i.e., which elements of the process has integrating properties when the other loops are closed. Using this modified RGA loop pairing technique [26], we can conclude the following properties:

Property 1.

- $\lambda_{ij} \approx 1$ if there is one and only one integrating element, i.e., only one $g_{ij}(s)$ in the transfer matrix equals which contains an integrator.

Proof.

$$\lambda_{i,j} = \lim_{\varepsilon \rightarrow 0} \left\{ \frac{\frac{k_{i,j}}{\varepsilon}}{\frac{k_{i,j}}{\varepsilon} - k_i [K_{ix,jx}]^{-1} k_j} \right\}$$

where k_i denotes the i -th row of the K with $k_{i,j}$ being excluded; k_j denotes the j -th column of K with $k_{i,j}$ being excluded, and $K_{ix,jx}$ denotes the submatrix

of K with i -th row and j -th column being removed.

Since k_i , k_j and $K_{ix,jx}$ are all finite, we can conclude that

$$\lambda_{i,j} = \lim_{\varepsilon \rightarrow 0} \left\{ \frac{\frac{k_{i,j}}{\varepsilon}}{\frac{k_{i,j}}{\varepsilon}} \right\} = 1$$

In other words, whenever there is only one integrator contained in $\mathbf{G}(s)$, the input variable associated with the integrator will be decoupled from other loops at steady-state.

Property 2.

- When $g_{ij}(s)$ contains a pure integrator and λ_{ij} is a non-zero finite number, the resulting effective open-loop transfer function $\hat{g}_{ij}(s)$ is also an integrating element.

Proof. In a multi-loop system, when all other loops have perfect control at steady state, the effective open-loop process of the $y_i - u_j$ pair is as follows.

Since $\lambda_{i,j} = \lim_{\varepsilon \rightarrow 0} \left\{ \frac{\frac{k_{i,j}}{\varepsilon}}{\frac{k_{i,j}}{\varepsilon} - k_i [K_{ix,jx}]^{-1} k_j} \right\}$, when $\lambda_{i,j}$ is non-zero and finite, it means $k_i [K_{ix,jx}]^{-1} k_j = \frac{\varepsilon}{\beta}$ and $\beta \neq k_{i,j}$.

Therefore,

$$\lambda_{i,j} = \lim_{\varepsilon \rightarrow 0} \left\{ \frac{\frac{k_{i,j}}{\varepsilon}}{\frac{k_{i,j}}{\varepsilon} - k_i [K_{ix,jx}]^{-1} k_j} \right\} = \frac{\frac{k_{i,j}}{\varepsilon}}{\frac{k_{i,j}}{\varepsilon} - \beta} \neq 1$$

Notice that

$$g_i(s) = g_{i,j}(s) - g_i(s) [G_{ix,jx}(s)]^{-1} g_j(s)$$

and

$$\lim_{s \rightarrow 0} g_i(s) = \lim_{\varepsilon \rightarrow 0} \left\{ \frac{k_{i,j}}{\varepsilon} - k_i [K_{ix,jx}]^{-1} k_j \right\} = \lim_{s \rightarrow 0} \left\{ \frac{1}{\varepsilon} (k_{i,j} - \beta) \right\}$$

Thus, $\hat{g}_{ij}(s)$ has a pure integrator.

- When $g_{ij}(s)$ contains a pure integrator and λ_{ij} is zero, the resulting effective open-loop transfer function $\hat{g}_{ij}(s)$ has more integrators than $g_{ij}(s)$.

Proof. Similarly, when $\lambda_{i,j}$ approaches to 0, it means

$$\frac{k_{i,j}}{\varepsilon} \ll k_i [K_{ix,jx}]^{-1} k_j \approx \frac{\beta}{\varepsilon^n}$$

where $n > 1$. In other words, $\hat{g}_{ij}(s)$ has more than one pure integrators.

Property 3.

- When $g_{ij}(s)$ does not contain any pure integrator and λ_{ij} a non-zero finite number, the resulting effective open-loop transfer function $\hat{g}_{ij}(s)$ also has no integrator.

Proof. The fact that $\lambda_{i,j}$ is finite implies that $k_i [K_{ix,jx}]^{-1} k_j$ is finite, together with the fact k_{ij} is finite and that $g_{ij}(s)$ is finite. Therefore, $\hat{g}_{ij}(s)$ has no pure integrator.

Remark 2. It is important to identify whether a multivariable system contains integrating elements. Properties 2 and 3 listed above are useful for investigating whether a MIMO process has integrator(s).

The following case studies of 3×3 and 4×4 processes will illustrate how to use the proposed properties for the modified RGA calculation of integrating processes. The original process transfer function matrices are omitted and discussions are based on their corresponding steady state gain matrices $\tilde{\mathbf{K}} = \{\tilde{\mathbf{k}}_{ij}; i, j = 1, 2, \dots, n\}$.

Case 1. The 3×3 steady state gain matrix $\tilde{\mathbf{K}}$ for computing the modified RGA is given by

$$\tilde{\mathbf{K}} = \begin{bmatrix} \frac{-1.986}{\varepsilon} & 5.24 & 5.984 \\ 0.0204 & -0.33 & 2.38 \\ 0.374 & -11.3 & -9.811 \end{bmatrix}$$

Substituting $\varepsilon = 10^{-6}$, we have

$$\begin{aligned} \Lambda &= \lim_{\varepsilon \rightarrow 0} \tilde{\mathbf{K}} \otimes \tilde{\mathbf{K}}^{-T} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.107 & 0.893 \\ 0 & 0.893 & 0.107 \end{bmatrix} \end{aligned}$$

The third property holds that if $\tilde{\mathbf{K}}$ contains one and only one integrating element g_{11} , $\lambda_{ij} \approx 1$. The modified RGA indicates a loop pairing of 1-1/2-3/3-2. Based on **Property 1** and **3**, we can conclude that $\hat{g}_{11}(s)$ is an integrating element, but $\hat{g}_{23}(s)$ and $\hat{g}_{32}(s)$ are not.

Case 2. The 3×3 steady state gain matrix $\tilde{\mathbf{K}}$ for computing the modified RGA is given by

$$\tilde{\mathbf{K}} = \begin{bmatrix} \frac{145.5}{\varepsilon} & \frac{-4.12}{\varepsilon} & 53 \\ 98.6 & 101.5 & 19.9 \\ -3.3 & -3.025 & 1.435 \end{bmatrix}$$

Substituting $\varepsilon = 10^{-6}$, we have

$$\begin{aligned}\Lambda &= \lim_{\varepsilon \rightarrow 0} \tilde{\mathbf{K}} \otimes \tilde{\mathbf{K}}^{-T} \\ &= \begin{bmatrix} 0.972 & 0.028 & 0 \\ 0.019 & 0.688 & 0.293 \\ 0.009 & 0.284 & 0.707 \end{bmatrix}\end{aligned}$$

The modified RGA indicates a diagonal loop pairing. Based on the **Property 2** and **3**, we can conclude that $\hat{g}_{11}(s)$ is an integrating element, but $\hat{g}_{22}(s)$ and $\hat{g}_{33}(s)$ are not.

Case 3. The 3×3 steady state gain matrix $\tilde{\mathbf{K}}$ for computing the modified RGA is given by

$$\tilde{\mathbf{K}} = \begin{bmatrix} \frac{0.66}{\varepsilon} & 0.61 & 0.49 \\ \frac{0.75}{\varepsilon} & 0.45 & 0.55 \\ 0.72 & 0.8 & 0.9 \end{bmatrix}$$

Substituting $\varepsilon = 10^{-6}$, we have

$$\begin{aligned}\Lambda &= \lim_{\varepsilon \rightarrow 0} \tilde{\mathbf{K}} \otimes \tilde{\mathbf{K}}^{-T} \\ &= \begin{bmatrix} 0.1640 & 2.923 & -2.0873 \\ 0.8360 & -1.8978 & 2.0618 \\ 0 & -0.0256 & 1.0256 \end{bmatrix}\end{aligned}$$

The modified RGA indicates a loop pairing of 1-2/2-1/3-3. Based on **Property 2** and **3**, we can conclude that $\hat{g}_{21}(s)$ is an integrating element, but $\hat{g}_{12}(s)$ and $\hat{g}_{33}(s)$ are not.

Case 4. The 4×4 steady state gain matrix $\tilde{\mathbf{K}}$ for computing the modified RGA is given by

$$\tilde{\mathbf{K}} = \begin{bmatrix} \frac{2.22}{\varepsilon} & \frac{2.94}{\varepsilon} & 2.33 & 2.60 \\ \frac{3.33}{\varepsilon} & 2.9 & \frac{2.7}{\varepsilon} & 3.1 \\ 4.1 & 6 & 5.9 & 6 \\ 3.5 & 2.2 & 4.2 & 3.8 \end{bmatrix}$$

Substituting $\varepsilon = 10^{-6}$, we have

$$\begin{aligned} \Lambda &= \lim_{\varepsilon \rightarrow 0} \tilde{\mathbf{K}} \otimes \tilde{\mathbf{K}}^{-\text{T}} \\ &= \begin{bmatrix} 0.785 & 0.216 & 0 & 0 \\ -0.371 & 0 & 1.371 & 0 \\ -1.686 & 1.863 & 2.993 & -2.17 \\ 2.273 & -1.079 & -3.364 & 3.17 \end{bmatrix} \end{aligned}$$

The modified RGA indicates a loop pairing of 1-1/2-3/3-2/4-4. Based on **Property 2** and **3**, we can conclude that $\hat{g}_{11}(s)$ and $\hat{g}_{23}(s)$ are integrating elements, but $\hat{g}_{32}(s)$ and $\hat{g}_{44}(s)$ are not.

In summary, the proposed modified RGA and its properties are useful in identifying integrating elements and analyzing their desired loop pairing for multivariable control.

With the discussions on integrating process' loop pairing and RGA calculation, we can determine the effective transfer functions of integrating processes, summarized below in Table 3.2.

Process	g_{ij}	$\hat{g}_{ij}$
Integrating	$k_{ij} \frac{e^{-\theta_{ij}s}}{s}$	$\hat{k}_{ij} \frac{e^{-\hat{\theta}_{ij}s}}{s}$
First Order Integrating	$\frac{k_{ij}}{s(\tau_{ij}s+1)} e^{-\theta_{ij}s}$	$\frac{\hat{k}_{ij}}{s(\hat{\tau}_{ij}s+1)} e^{-\hat{\theta}_{ij}s}$

Table 3.2: ETFs for most commonly used transfer functions, including integrating transfer functions

3.5 Conclusion

In this chapter, interaction analysis was utilized to reflect the dynamic relative interactions to an individual control loop from all the other loops in a MIMO process. The ETF of an individual control loop was derived from the parameters of the process model. Furthermore, modified RGA loop pairing technique was used for deriving ETFs of MIMO integrating processes. ETF optimally approximates an individual loop's single-input single-output counterparts when all the other loops are closed. ETF will be used for decentralized controller design in Chapter 4.

Chapter 4

Decentralized Controller Design for MIMO Integrating Processes

4.1 Introduction

There are two basic MIMO control approaches. The first category is coordinated or centralized control, in which a single control algorithm uses all measurements to calculate all manipulated variables simultaneously. The second category is decentralized control, which is a straightforward extension of single loop control to many controlled variables in a single process. Compared with sophisticated centralized control system design methods for MIMO systems, decentralized control remains dominant in process control applications, due to its advantages of simplicity and loop failure tolerance.

Despite its wide application in industrial processes, decentralized PID

control has not been commonly used for integrating processes. This chapter focuses on the tuning rules of the decentralized PID controller for MIMO systems, especially for MIMO integrating processes.

4.2 Decentralized Controller Design

The design of controllers for multi-loop system usually uses, directly or indirectly, methods for single loop system with modifications. For example, Luyben [49] designed controllers for diagonal elements and detuned for multi-loop systems. Huang et al. [50] designed controllers for an effective open-loop process. Due to loop interactions, single loop controllers to be used in a multi-loop system must have better stability robustness or controllability than the one standing alone in a single loop. It is thus desirable, especially in the case of the integrating process, to have single loop controllers that have better controllability (e.g., larger ultimate gain).

Without loss of generality, we assume that each main loop, i.e., diagonal element in the transfer function matrix is represented by a second order plus dead time (SOPDT) model, which can be used to describe most of the industrial processes:

$$g_{ii}(s) = \frac{b_{0,i}}{a_{2,i}s^2 + a_{1,i}s + 1} e^{-\theta_{ii}s} \quad (4.1)$$

Similarly, ETF is represented as

$$\hat{g}_{ii}(s) = \frac{\hat{k}_{ii}}{\hat{a}_{2,i}s^2 + \hat{a}_{1,i}s + 1} e^{-\hat{\theta}_{ii}s} \quad (4.2)$$

The decentralized controllers can then be independently designed by single loop approaches based on the corresponding ETFs. Here we employ the

gain and phase margins approach. This is primary because the frequency response method provides good performance in the face of uncertainty in both plant model and disturbances.

The PID controller of each loop is assumed to be of the following standard form:

$$g_{c,i}(s) = \frac{k_{d,i}s^2 + k_{p,i}s + k_{i,i}}{s} = k_i \frac{A_i s^2 + B_i s + C_i}{s} \quad (4.3)$$

where $A_i = k_{d,i}/k_i$, $B_i = k_{p,i}/k_i$ and $C_i = k_{i,i}/k_i$.

By selecting $A_i = \hat{a}_{2,i}$, $B_i = \hat{a}_{1,i}$ and $C_i = 1$, the open loop transfer function becomes

$$g_{c,i}(s)\hat{g}_{ii}(s) = k_i \frac{\hat{k}_{ii}}{s} e^{-\theta_{ii}s} \quad (4.4)$$

Denoting the gain and phase margin specifications as $A_{m,i}$ and $\Psi_{m,i}$, and their crossover frequencies as $\omega_{g,i}$ and $\omega_{u,i}$, respectively, we have

$$\arg [g_{c,i}(j\omega_{g,i})\hat{g}_{ii}(j\omega_{g,i})] = -\pi$$

$$A_{m,i} |g_{c,i}(j\omega_{g,i})\hat{g}_{ii}(j\omega_{g,i})| = 1$$

$$|g_{c,i}(j\omega_{u,i})\hat{g}_{ii}(j\omega_{u,i})| = 1$$

$$\Psi_{m,i} = \pi + \arg [g_{c,i}(j\omega_{u,i})\hat{g}_{ii}(j\omega_{u,i})]$$

By substitution and simplification, we obtain from the above equations that

$$\omega_{g,i}\hat{\theta}_{ii} = \frac{\pi}{2}$$

$$A_{m,i} = \frac{\omega_{g,i}}{k_i \hat{k}_{ii}}$$

$$k_i \hat{k}_{ii} = \omega_{u,i}$$

$$\Psi_{m,i} = \frac{\pi}{2} - \omega_{u,i}\hat{\theta}_{ii}$$

which results in

$$\Psi_{m,i} = \frac{\pi}{2} \left(1 - \frac{1}{A_{m,i}}\right)$$

$$k_i = \frac{\pi}{2A_{m,i}\hat{\theta}_{ii}\hat{k}_{ii}}$$

By this formulation, the gain and phase margins are interrelated to each other, some possible gain and phase margin selections are given in Table 4.1.

$\Psi_{m,i}$	$\pi/4$	$\pi/3$	$3\pi/8$	$2\pi/5$
$A_{m,i}$	2	3	4	5

Table 4.1: Typical gain and phase margin values

The PID parameters are given by [39]

$$\begin{bmatrix} k_{p,i} \\ k_{i,i} \\ k_{d,i} \end{bmatrix} = \frac{\pi}{2A_{m,i}\hat{\theta}_{ii}\hat{k}_{ii}} \begin{bmatrix} \hat{a}_{1,i} \\ 1 \\ \hat{a}_{2,i} \end{bmatrix} \quad (4.5)$$

Since it is necessary that the controlled system possesses integrity property, i.e., the overall control system remains to be stable regardless whether other control loops are put in and/or taken out, $\hat{k}_{ii}$ and $\hat{\lambda}_{ii}$ in ETF must take different values for different combination of k_{ii} and λ_{ii} , which is shown in Figures 4.1-4.4 and are discussed below:

Case 1: $\lambda_{ii} \leq 1$, $\gamma_{ii} \leq 1$. In this case, according to Equations (3.3) and (3.9), $\hat{k}_{ii} \geq k_{ii}$, and $\hat{\theta}_{ii} \leq \theta_{ii}$.

- $\hat{k}_{ii} \geq k_{ii}$. This means that the magnitude of the frequency response when the other loops are closed is not less than that of when the other

Figure 4.1: Comparison of $g_{ii}(s)$ and $\hat{g}_{ii}(s)$, $\lambda_{ii} \leq 1$, $\gamma_{ii} \leq 1$

loops are open. Since the retaliatory effect from the other loops magnifies the main effect of u_j on y_i , we need to reduce the gain to assure system stability according to Equation (3.3), i.e., $\hat{k}_{ii} = k_{ii}/\lambda_{ii}$

- $\hat{\theta}_{ii} \leq \theta_{ii}$. This means that the time delay when the other loops are closed is no bigger than that of when the other loops are open. The reduced time delay will increase the phase margin. However, by considering control system integrity, the time delay needs to be kept as before, i.e., $\hat{\theta}_{ii} = \theta_{ii}$.

Case 2: $\lambda_{ii} \leq 1$, $\gamma_{ii} > 1$. In this case, according to Equations (3.3) and (3.9), $\hat{k}_{ii} \geq k_{ii}$, and $\hat{\theta}_{ii} > \theta_{ii}$.

- $\hat{k}_{ii} \geq k_{ii}$. Same as in **Case 1**, $\hat{k}_{ii} = k_{ii}/\lambda_{ii}$.
- $\hat{\theta}_{ii} > \theta_{ii}$. This means that the time delay when the other loops are

Figure 4.2: Comparison of $g_{ii}(s)$ and $\hat{g}_{ii}(s)$, $\lambda_{ii} \leq 1$, $\gamma_{ii} > 1$

closed is bigger than that of the other loops are open. The enlarged time delay will reduce the phase margin. In this case, the time delay is determined by Equation (3.9) i.e., $\hat{\theta}_{ii} = \gamma_{ii}\theta_{ii}$.

Case 3: $\lambda_{ii} > 1$, $\gamma_{ii} \leq 1$. In this case, according to Equations (3.3) and (3.9), $\hat{k}_{ii} < k_{ii}$, and $\hat{\theta}_{ii} \leq \theta_{ii}$.

- $\hat{k}_{ii} \geq k_{ii}$. This means that the magnitude of the frequency response when the other loops are closed is smaller than that of when the other loops are open. Even if the retaliatory effect from other loops acts in opposition to the main effect of u_i on y_i , we cannot enlarge the controller gain for better performance due to system integrity consideration. Hence, the gain should be unchanged, i.e., $\hat{k}_{ii} = k_{ii}$
- $\hat{\theta}_{ii} \leq \theta_{ii}$. Same as in **Case 1**, $\hat{\theta}_{ii} = \theta_{ii}$.

Figure 4.3: Comparison of $g_{ii}(s)$ and $\hat{g}_{ii}(s)$, $\lambda_{ii} > 1$, $\gamma_{ii} \leq 1$

Case 4: $\lambda_{ii} > 1$, $\gamma_{ii} > 1$. In this case, according to Equations (3.3) and (3.9), $\hat{k}_{ii} < k_{ii}$, and $\hat{\theta}_{ii} > \theta_{ii}$.

- $\hat{k}_{ii} < k_{ii}$. Same as in **Case 3**, $\hat{k}_{ii} = k_{ii}$
- $\hat{\theta}_{ii} > \theta_{ii}$. Same as in **Case 2**, $\hat{\theta}_{ii} = \gamma_{ii}\theta_{ii}$.

Remark 3. A unique problem for decentralized control of MIMO processes is the zero crossing [48]: stable or unstable zeros might be introduced into a particular control loop when other loops are closed. If an unstable zero is introduced, it will result in phase shift to the left in the frequency domain. In order to guarantee the entire system stability, the controllers are normally conservatively designed by conventional detuning approaches [48]. By introducing the relative critical frequency, γ_{ii} , to indicate phase changes

Figure 4.4: Comparison of $g_{ii}(s)$ and $\hat{g}_{ii}(s)$, $\lambda_{ii} > 1$, $\gamma_{ii} > 1$

after the other loops closed, the effects of unstable zeros can be accurately estimated in each control loop. Consequently, the resultant control systems will be much less conservative.

Applying Equation (4.5) for each case, we can obtain both the ETFs and the PID parameters which are summarized in Table 4.2.

To illustrate the decentralized controller design scheme, we have a example which is a continuation of **Example 1**. As discussed in Chapter 2, the decentralized PI controller are given as follows.

$$\mathbf{G}_{c1}(s) = \text{diag} \left\{ 0.5 \left(1 + \frac{1}{100s} \right), \quad 0.5 \left(1 + \frac{1}{100s} \right) \right\}$$

$$\mathbf{G}_{c2}(s) = \text{diag} \left\{ 1.25 \left(1 + \frac{1}{10s} \right) \quad -0.25 \left(1 + \frac{1}{10s} \right) \right\}$$

As shown in Figure 2.4, compared with the approach in [21], it is obvious

Mode	$\hat{g}_{ij}$	$k_{p,ii}$	$k_{i,ii}$	$k_{d,ii}$
$\lambda_{ii} \leq 1,$ $\gamma_{ii} \leq 1$	$\frac{k_{ii}/\lambda_{ii}}{a_{2,i}s^2+a_{1,i}s+1} e^{-\theta_{ii}s}$	$\frac{\pi\lambda_{ii}a_{1,i}}{2A_{m,i}\theta_{ii}k_{ii}}$	$\frac{\pi\lambda_{ii}}{2A_{m,i}\theta_{ii}k_{ii}}$	$\frac{\pi\lambda_{ii}a_{2,i}}{2A_{m,i}\theta_{ii}k_{ii}}$
$\lambda_{ii} \leq 1,$ $\gamma_{ii} > 1$	$\frac{k_{ii}/\lambda_{ii}}{\hat{a}_{2,i}s^2+\hat{a}_{1,i}s+1} e^{-\gamma_{ii}\theta_{ii}s}$	$\frac{\pi\lambda_{ii}\hat{a}_{1,i}}{2A_{m,i}\gamma_{ii}\theta_{ii}k_{ii}}$	$\frac{\pi\lambda_{ii}}{2A_{m,i}\gamma_{ii}\theta_{ii}k_{ii}}$	$\frac{\pi\lambda_{ii}\hat{a}_{2,i}}{2A_{m,i}\gamma_{ii}\theta_{ii}k_{ii}}$
$\lambda_{ii} > 1,$ $\gamma_{ii} \leq 1$	$\frac{k_{ii}}{a_{2,i}s^2+a_{1,i}s+1} e^{-\theta_{ii}s}$	$\frac{\pi a_{1,i}}{2A_{m,i}\theta_{ii}k_{ii}}$	$\frac{\pi}{2A_{m,i}\theta_{ii}k_{ii}}$	$\frac{\pi a_{2,i}}{2A_{m,i}\theta_{ii}k_{ii}}$
$\lambda_{ii} > 1,$ $\gamma_{ii} > 1$	$\frac{k_{ii}}{\hat{a}_{2,i}s^2+\hat{a}_{1,i}s+1} e^{-\gamma_{ii}\theta_{ii}s}$	$\frac{\pi \hat{a}_{1,i}}{2A_{m,i}\gamma_{ii}\theta_{ii}k_{ii}}$	$\frac{\pi}{2A_{m,i}\gamma_{ii}\theta_{ii}k_{ii}}$	$\frac{\pi \hat{a}_{2,i}}{2A_{m,i}\gamma_{ii}\theta_{ii}k_{ii}}$

Table 4.2: Decentralized PID controller design

that the proposed approach (with off-diagonal pairing) gives better overall control system performance.

4.3 Decentralized Controller Design for Integrating Processes

The application of PID control to integrating processes is less common. Here we also propose a decentralized controller design for integrating processes, featuring a P controller in an inner feedback loop for the integrating process. The following simple integrating process is studied here as an example, and its transfer function $g_{ii}(s)$ is as follows:

$$g_{ii}(s) = \frac{k_{ii}}{s(\tau_{ii}s + 1)} e^{-\theta_{ii}s} \quad (4.6)$$

Similarly, its ETF is represented as

$$\hat{g}_{ii}(s) = \frac{\hat{k}_{ii}}{s(\hat{\tau}_{ii}s + 1)} e^{-\hat{\theta}_{ii}s} \quad (4.7)$$

The block diagram of the proposed PID controller design for integrating processes with an inner feedback loop is shown in Figure 4.5. The P controller $k_{l,i}$ in the inner feedback loop plays an important role in changing the integrating process to a stable process [39].

Figure 4.5: PID controller design for integrating processes with an inner feedback loop

The internal closed-loop transfer function $g_{l,i}(s)$ can be obtained as

$$g_{l,i}(s) = \frac{\hat{g}_{ii}(s)}{1 + k_{l,i}\hat{g}_{ii}(s)} = \frac{\hat{k}_{ii}e^{-\hat{\theta}_{ii}s}}{\hat{\tau}_{ii}s^2 + s + \hat{k}_{ii}k_{l,i}e^{-\hat{\theta}_{ii}s}} \quad (4.8)$$

By using a Taylor series expansion, the time delay term in the denominator of Equation (4.8) can be approximated by

$$e^{-\hat{\theta}_{ii}s} \cong 1 - \hat{\theta}_{ii}s + 0.5\hat{\theta}_{ii}^2s^2 \quad (4.9)$$

Substituting Equation (4.9) into Equation (4.8), $g_{l,i}(s)$ can be expressed

as:

$$g_{l,i}(s) \cong g'_{ii}(s) = \frac{\hat{k}_{ii}e^{-\hat{\theta}_{ii}s}}{(\hat{\tau}_{ii} + 0.5\hat{k}_{ii}k_l\hat{\theta}_{ii}^2)s^2 + (1 - \hat{k}_{ii}k_l\hat{\theta}_{ii})s + \hat{k}_{ii}k_l} \quad (4.10)$$

Here, $g'_{ii}(s)$ denotes the second-order plus time-delay model obtained from the Taylor series expansion method. Since the characteristic equation of $g'_{ii}(s)$ should have negative poles to be stable, the following condition must be satisfied from the Routh-Hurwitz stability criterion

$$k_{l,i} < \frac{1}{\hat{k}_{ii}\hat{\theta}_{ii}}$$

For optimal disturbance rejection, it was proposed by [39] that:

$$k_{l,i} = \frac{1}{5\hat{k}_{ii}\hat{\theta}_{ii}} \quad (4.11)$$

If we choose Equation (4.11) as a design value of the P controller gain in the inner feedback loop, then Equation (4.10) is obtained as:

$$g'_{ii}(s) = \frac{5\hat{k}_{ii}\hat{\theta}_{ii}e^{-\hat{\theta}_{ii}s}}{5\hat{\theta}_{ii}(\hat{\tau}_{ii} + 0.1\hat{\theta}_{ii})s^2 + 4\hat{\theta}_{ii}s + 1} \quad (4.12)$$

The gain and phase margin design method can be directly used to design controller.

$$\begin{aligned} b_{0,i} &= 5\hat{k}_{ii}\hat{\theta}_{ii} \\ a_{2,i} &= 5\hat{\theta}_{ii}(\hat{\tau}_{ii} + 0.1\hat{\theta}_{ii}) \\ a_{1,i} &= 4\hat{\theta}_{ii} \end{aligned} \quad (4.13)$$

$$\begin{bmatrix} k_{p,i} \\ k_{i,i} \\ k_{d,i} \end{bmatrix} = \frac{\pi}{10A_{m,i}\hat{\theta}_{ii}\hat{k}_{ii}} \begin{bmatrix} 4 \\ 1/\hat{\theta}_{ii} \\ 5(\hat{\tau}_{ii} + 0.1\hat{\theta}_{ii}) \end{bmatrix} \quad (4.14)$$

The 3 examples which are to be studied in detail in Chapter 5 illustrate that the MIMO decentralized controller design scheme work effectively for integrating processes.

4.4 Conclusion

In this chapter, a simple yet effective scheme for decentralized controller design was presented. Appropriate parameters of each individual controller are determined by introducing an inner feedback loop and applying the IMC-PID tuning rules for the effective transfer function. With this proposed PID tuning method for integrating processes, we can obtain a control system which guarantees both robustness and performance. Simulation examples will be given in Chapter 5 to show the proposed decentralized controller design method outperforms other methods.

Chapter 5

Simulation Studies

5.1 Introduction

In this chapter, the REGA-based and RNGA-based decentralized controller design methods will be illustrated with simulation examples of MIMO integrating processes.

5.2 MIMO Decentralized Controller Design for a 2×2 Submarine Process

Example 4. Consider a 2×2 process of submarine [51] in Figure 5.1. The transfer function matrix of the process is as follows.

$$\mathbf{G}_0(s) = \begin{bmatrix} \frac{-0.0076(s+0.5461)(s+0.0494)}{s(s+0.0629)(s+0.0336+0.0472i)(s+0.0336-0.0472i)} & \frac{-0.0229(s+0.0604)(s-0.1815)}{s(s+0.0629)(s+0.0336+0.0472i)(s+0.0336-0.0472i)} \\ \frac{0.0017s(s+0.0306)}{s(s+0.0629)(s+0.0336+0.0472i)(s+0.0336-0.0472i)} & \frac{-0.0022s(s+0.0556)}{s(s+0.0629)(s+0.0336+0.0472i)(s+0.0336-0.0472i)} \end{bmatrix}$$

Using 1st order least-square method, the processes can be approximated to their first-order plus time-delay equivalents as follows:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{-1.1053}{s(19.4361s+1)}e^{-4.6744s} & \frac{1.4772}{s(32.3660s+1)}e^{-9.169s} \\ \frac{0.3029}{15.5059s+1}e^{-4.2528s} & \frac{-0.6682}{23.4279s+1}e^{-5.7224s} \end{bmatrix}$$

The step responses of original processes and those of the approximated FOPTD processes are compared in Figure 5.2, with the original processes' responses in dashed line and the approximated processes' in solid line.

By letting $\varepsilon = 10^{-6}$, the REGA result has been derived in Chapter 2, which indicates diagonal 1-1/2-2 pairing.

Figure 5.1: 2×2 submarine process

Figure 5.2: Comparison of original processes and approximated FOPTD processes

The Relative Average Residence Time Array (RARTA) of the process is

$$\begin{aligned}\mathbf{\Gamma} &= \mathbf{\Phi} \odot \mathbf{\Lambda} \\ &\approx \begin{bmatrix} 1.0360 & 1.0595 \\ 1.0595 & 1.0360 \end{bmatrix}\end{aligned}$$

Note that $\lambda_{11}, \lambda_{22} > 1$ and $\gamma_{11}, \gamma_{22} > 1$. Therefore,

$$\begin{aligned}\hat{k}_{ii} &= k_{ii} & i &= 1, 2 \\ \hat{\sigma}_{ii} &= \sigma_{ii} \times \gamma_{ii} & i &= 1, 2\end{aligned}$$

Based on the obtained effective transfer functions, we take two alternative controller design approaches: either without or with inner feedback loop for integrators.

Using the gain-phase-margin controller design methodology introduced in 4.2, the PID controllers are designed as follows (take $A_{m,i} = 5$).

$$\mathbf{G}_c(s) = \begin{bmatrix} -73.3636 - 787.8705s & 0 \\ 0 & -96.2445 - \frac{0.2644}{s} \end{bmatrix}$$

The simulation program is shown in Figures 5.3 and 5.4. Figure 5.4 is the part of “Multivariable Process” in Figure 5.3. The two PID controllers in Figure 5.3 are given by $\mathbf{G}_c$.

The two control inputs of the submarine process are z and θ , i.e., the depth and the pitch angle of the submarine, respectively. With $z_{ref} = 1$ and $\theta_{ref} = 0$, the responses of the submarine process without inner feedback loop are shown in Figure 5.5.

Alternatively, using the inner feedback loop controller design methodology introduced in Section 4.3, the PID controllers are designed as follows

Figure 5.3: Control system configuration of the submarine process, without inner feedback loop

Figure 5.4: Multivariable process in the submarine process' control system configuration, without inner feedback loop

Figure 5.5: Submarine process' responses to $z_{ref} = 1$ and $\theta_{ref} = 0$, without inner feedback loop

Figure 5.6: Multivariable process in the submarine process' control system configuration, with inner feedback loop

(take $A_{m,i} = 5$).

$$\mathbf{G}_c(s) = \begin{bmatrix} -12.5207 - \frac{0.0017}{s} - 201.7047s & 0 \\ 0 & -64.1630 - \frac{1.3218}{s} \end{bmatrix}$$

The simulation program is shown in Figures 5.3 and 5.6. The two PID controllers in Figure 5.3 are given by $\mathbf{G}_c$, and Figure 5.6 is the part of “Multivariable Process” in Figure 5.3.

Similarly, with $z_{ref} = 1$ and $\theta_{ref} = 0$, the responses of the submarine

Figure 5.7: Submarine process' responses to $z_{ref} = 1$ and $\theta_{ref} = 0$, with inner feedback loop

process with inner feedback loop are shown in Figure 5.7.

Liceaga-Castro et al. provided some non-PID control configurations in their paper [51]. We compare the step responses of the submarine process using our REGA-based control and that of [51], with the control system configuration shown in Figure 5.8.

The responses using the proposed REGA-based control in comparison with those in [51] are shown in Figures 5.9 and 5.11. It takes less time for submarine depth and approximately the same time for submarine pitch angle to converge to the desired reference signal with the presented PID controllers with inner feedback loop, compared with the control configuration in [51]. Therefore the proposed REGA-based method with inner feedback

Figure 5.8: Submarine's control configuration with REGA-based controller and with the controller of [51]

loop outperforms the method in [51]. A step disturbance of 0.1 to both the processes is also added to test the control system's stability in the presence of disturbance, and the comparison results are shown in the following figure 5.10. It is observed that the proposed REGA-based method is much more stable in the presence of disturbance.

On the other hand, compared with the control configuration in [51], it also takes less time for submarine depth and approximately the same time for submarine pitch angle to converge to the desired reference signal with the presented PID controllers with inner feedback loop. Therefore the proposed REGA-based method without inner feedback loop also outperforms the method in [51].

Moreover, if we set the two control inputs $z_{ref} = 1$ and $\theta_{ref} = 1$ in the presence of a step disturbance of 0.1, the unit step response of the proposed REGA-based control is much better than those of [51], as shown in Figure 5.12. In summary, the advantage of the presented REGA-based PID controller design, either with or without inner feedback loop, is the proposed control system performance is more stable and that the PID controllers is much easier to design and implement.

Similarly, we may apply the RNGA approach on the same process. By letting $\varepsilon = 10^{-6}$, the RNGA result has been derived in Chapter 2, which indicates diagonal 1-1/2-2 pairing.

The Relative Average Residence Time Array (RARTA) of the process is

$$\mathbf{\Gamma} = \mathbf{\Lambda}_N \odot \mathbf{\Lambda} \approx \begin{bmatrix} 0.8192 & 0.7016 \\ 0.7016 & 0.8192 \end{bmatrix}$$

Figure 5.9: Submarine process' responses to $z_{ref} = 1$ and $\theta_{ref} = 0$ using REGA-based control (without inner feedback loop) in comparison with Liceaga-Castro et al. [51]

Figure 5.10: Submarine process' responses to $z_{ref} = 1$ and $\theta_{ref} = 0$ using REGA-based control with disturbance in comparison with Liceaga-Castro et al. [51]

Figure 5.11: Submarine process' responses to $z_{ref} = 1$ and $\theta_{ref} = 0$ using REGA-based control (with inner feedback loop) in comparison with Liceaga-Castro et al. [51]

Figure 5.12: Submarine process' unit step responses to $z_{ref} = 1$ and $\theta_{ref} = 1$ with disturbance using REGA-based control in comparison with Liceaga-Castro et al. [51]

Note that $\lambda_{11}, \lambda_{22} > 1$ and $\gamma_{11}, \gamma_{22} < 1$. Therefore,

$$\hat{k}_{ii} = k_{ii} \quad i = 1, 2, \hat{\sigma}_{ii} = \sigma_{ii} \quad i = 1, 2$$

Based on the obtained effective transfer functions, we also take two alternative controller design approaches: either without or with inner feedback loop for integrators.

Using the gain-phase-margin controller design methodology introduced in 4.2, the PID controllers are designed as follows (take $A_{m,i} = 5$).

$$\mathbf{G}_c(s) = \begin{bmatrix} -60.8047 - 689.3867s & 0 \\ 0 & -256.6520 - \frac{1.3694}{s} \end{bmatrix}$$

With the same simulation program shown in Figures 5.3 and 5.4, and with the two control inputs $z_{ref} = 1$ and $\theta_{ref} = 0$, the responses of the submarine process without inner feedback loop are shown in Figure 5.13.

Alternatively, using the inner feedback loop controller design methodology introduced in Section 4.3, the PID controllers are designed as follows (take $A_{m,i} = 5$).

$$\mathbf{G}_c(s) = \begin{bmatrix} -12.1609 - \frac{0.0019}{s} - 201.7047s & 0 \\ 0 & -3.2081 - \frac{0.1369}{s} \end{bmatrix}$$

With the same simulation program shown in Figures 5.3 and 5.6, and with the two control inputs $z_{ref} = 1$ and $\theta_{ref} = 0$, the responses of the submarine process with inner feedback loop are shown in Figure 5.14.

Liceaga-Castro et al. provided some non-PID control configurations in their paper [51]. With the same simulation program shown in Figure 5.8, the responses using the proposed RNGA-based control in comparison with those

Figure 5.13: Submarine process' step responses without inner feedback loop

Figure 5.14: Submarine process' step responses with inner feedback loop

in [51] are shown in Figures 5.15 and 5.16. It takes less time for submarine depth and approximately the same time for submarine pitch angle to converge to the desired reference signal with the presented PID controllers with inner feedback loop, compared with the control configuration in [51]. Therefore the proposed RNGA-based method with inner feedback loop outperforms the method in [51].

On the other hand, compared with the control configuration in [51], it takes less time for both the submarine depth and the submarine pitch angle to converge to the desired reference signal with the presented PID controllers with inner feedback loop. Therefore the proposed RNGA-based method without inner feedback loop also outperforms the method in [51]. A step disturbance of 0.1 to both the processes is also added to test the control system's stability in the presence of disturbance, and the comparison results are shown in the following figure 5.17. It is observed that the proposed RNGA-based method is much more stable in the presence of disturbance.

Moreover, if we set the two control inputs $z_{ref} = 1$ and $\theta_{ref} = 1$ in the presence of a step disturbance of 0.1, the unit step responses of the proposed RNGA-based control is much better than those of [51], as shown in Figure 5.18. In summary, the advantage of the presented RNGA-based PID controller design, either with or without inner feedback loop, is the proposed control system performance is more stable and that the PID controllers is much easier to design and implement.

Figure 5.15: Submarine process' step responses using RGA-based control (without inner feedback loop) in comparison with Liceaga-Castro et al. [51]

Figure 5.16: Submarine process' step responses using RGA-based control (with inner feedback loop) in comparison with Liceaga-Castro et al. [51]

Figure 5.17: Submarine process' responses to $z_{ref} = 1$ and $\theta_{ref} = 0$ using RGA-based control with disturbance in comparison with Liceaga-Castro et al. [51]

Figure 5.18: Submarine process' unit step responses to $z_{ref} = 1$ and $\theta_{ref} = 1$ with disturbance using RNGA-based control in comparison with Liceaga-Castro et al. [51]

5.3 MIMO Decentralized Controller Design for a 3×3 Boiler Process

Consider a 3×3 process of boiler [52] in Figure 5.19. The transfer function matrix of the process is as follows.

$$\mathbf{G}_0(s) = \begin{bmatrix} \frac{(-0.16s^2+0.052s+0.0014) \times 10^{-3}}{s^2+0.0168s} & \frac{(3.1s-0.032) \times 10^{-3}}{s^2+0.0215s} & 0 \\ \frac{-0.0395 \times 10^{-3}}{s+0.018} & \frac{2.51 \times 10^{-3}}{s+0.0157} & \frac{(0.588s^2+0.2015s+0.0009) \times 10^{-3}}{s^2+0.0352s+0.000142} \\ \frac{-0.00118s+0.000139}{s^2+0.01852s+0.000091} & \frac{0.448s+0.0011}{s^2+0.0127s+0.000095} & \frac{0.582s-0.0243}{s^2+0.1076s+0.00104} \end{bmatrix}$$

Using 1st order least-square method, the processes can be approximated to their first-order plus time-delay equivalents as follows:

$$\mathbf{G}(s) = \begin{bmatrix} \frac{8.3 \times 10^{-5}}{s} e^{-2.8571s} & \frac{-3.2 \times 10^{-5}}{s(s+0.0215)} e^{-96.875s} & 0 \\ \frac{-0.0395 \times 10^{-3}}{s+0.018} & \frac{2.51 \times 10^{-3}}{s+0.0157} & \frac{1.9 \times 10^{-4}}{s+0.0320} e^{-0.2321s} \\ \frac{6.7 \times 10^{-3}}{s+0.0036} e^{-34.694s} & \frac{0.4819}{s+0.0145} e^{-0.5574s} & \frac{-0.2414}{s+0.0103} e^{-31.847s} \end{bmatrix}$$

By letting $\varepsilon = 10^{-6}$, the steady state gain of the approximated transfer function matrix $\mathbf{G}(s)$ is

$$\mathbf{K} = \begin{bmatrix} \frac{0.000083}{\varepsilon} & \frac{-0.0015}{\varepsilon} & 0 \\ -0.0022 & 0.1599 & 0.0063 \\ 1.8599 & 33.2267 & -23.4594 \end{bmatrix} (\varepsilon = 10^{-6})$$

Therefore, the RGA of the approximated transfer function matrix $\mathbf{G}(s)$ is

$$\begin{aligned} \mathbf{\Lambda} &= \mathbf{K} \otimes (\mathbf{K}^{-1})^T \\ &\approx \begin{bmatrix} 1.2191 & -0.2191 & 0 \\ -0.2841 & 1.1542 & 0.1299 \\ 0.0651 & 0.0648 & 0.8701 \end{bmatrix} \end{aligned}$$

Figure 5.19: 3×3 boiler process

Also, $NI = 0.8671 > 0$. Thus the RGA-based loop paring criterion indicates diagonal 1-1/2-2/3-3 pairing.

Similarly, we may apply the REGA approach on the same process. By letting $\varepsilon = 10^{-6}$,

$$\begin{aligned} \mathbf{E} &= \mathbf{K} \otimes \mathbf{\Omega} \\ &= \begin{bmatrix} \frac{1.22 \times 10^{-5}}{\varepsilon} & \frac{-2.13 \times 10^{-4}}{\varepsilon} & 0 \\ -0.0003 & 0.00226 & 0.0010 \\ 0.2343 & 4.7016 & -3.7699 \end{bmatrix} \end{aligned}$$

The REGA matrix is calculated as follows.

$$\begin{aligned} \mathbf{\Phi} &= \mathbf{E} \otimes (\mathbf{E}^{-1})^T \\ &\approx \begin{bmatrix} 1.2288 & -0.2288 & 0 \\ -0.2826 & 1.1670 & 0.1156 \\ 0.0538 & 0.0619 & 0.8844 \end{bmatrix} \end{aligned}$$

which also indicates diagonal 1-1/2-2/3-3 pairing ($NI = 0.48 > 0$).

The Relative Average Residence Time Array (RARTA) of the process is

$$\begin{aligned} \mathbf{\Gamma} &= \mathbf{\Phi} \odot \mathbf{\Lambda} \\ &\approx \begin{bmatrix} 1.0089 & 1.0497 & 0 \\ 0.9996 & 1.0119 & 0.8931 \\ 0.8309 & 0.9552 & 1.0159 \end{bmatrix} \end{aligned}$$

Note that $\lambda_{11}, \lambda_{22} > 1, \lambda_{33} < 1$ and $\gamma_{11}, \gamma_{22}, \gamma_{33} > 1$. Therefore,

$$\begin{aligned} \hat{k}_{ii} &= k_{ii} \quad i = 1, 2 \\ \hat{k}_{33} &= \frac{k_{33}}{\lambda_{33}} \\ \hat{\sigma}_{ii} &= \sigma_{ii} \times \gamma_{ii} \quad i = 1, 2, 3 \end{aligned}$$

Based on the obtained equivalent transfer functions, we take the controller design approach without inner feedback loop for integrators.

Using the gain-phase-margin controller design methodology introduced in 4.2, the PID controllers are designed as follows (take $A_{m,i} = 3$).

$$\mathbf{G}_c(s) = \begin{bmatrix} 2179.7 & 0 & 0 \\ 0 & 1266.5 + \frac{19.6506}{s} & 0 \\ 0 & 0 & -0.0593 - \frac{0.0006}{s} \end{bmatrix}$$

The simulation program is shown in Figures 5.20 and 5.21. Figure 5.21 is the part of “Multivariable Process” in Figure 5.20. The 3 PID controllers in Figure 5.20 are given by $\mathbf{G}_c$.

The three control inputs of the boiler process are the feedwater flow, the fuel flow and the attemperator spray flow of the boiler, respectively. Tan et al. introduced a 3×3 matrix of PI controller as their decentralized PID control design [52]. We compare the step responses of the boiler process using our REGA-based control and that of [52], with the control system configuration shown in Figure 5.22.

We show the comparison result in Figure 5.23. According to [52], the input attemperator spray flow changes from 0 to set-point 680 at time 0, fuel flow from 6.4 to set-point 6.8 at time 500, feedwater flow from 0 to set-point 1.3 at time 1000.

We also add a unit-step disturbance to both the processes, and compare their stability at the presence of disturbance as shown in the following figure 5.24.

It is observed that the process takes much less time to converge to the set-points with the PID controllers we propose, either with or without dis-

Figure 5.20: Control system configuration of the boiler process, without inner feedback loop

Figure 5.21: Multivariable process in the boiler process' control system configuration, without inner feedback loop

Figure 5.22: Boiler's control configuration with REGA-based controller and with the controller of [52]

Figure 5.23: Boiler process' step responses using REGA-based control in comparison with Tan et al. [52]

Figure 5.24: Boiler process' step responses with disturbance using REGA-based control in comparison with Tan et al. [52]

turbance. Moreover, the REGA-based PID controller is easier to design and implement.

Similarly, we may apply the RGA approach on the same process. By letting $\varepsilon = 10^{-6}$,

$$\mathbf{K}_N = \begin{bmatrix} \frac{2.91 \times 10^{-5}}{\varepsilon} & \frac{-1.04 \times 10^{-5}}{\varepsilon} & 0 \\ -3.95 \times 10^{-5} & 0.00251 & 0.00022 \\ 0.00598 & 0.47803 & -0.18183 \end{bmatrix}$$

The RGA matrix is calculated as follows.

$$\begin{aligned} \Lambda_N &= \mathbf{K}_N \otimes (\mathbf{K}_N^{-1})^T \\ &\approx \begin{bmatrix} 1.0037 & -0.0037 & 0 \\ -0.0046 & 0.8155 & 0.1891 \\ 0.0008 & 0.1882 & 0.8109 \end{bmatrix} \end{aligned}$$

which also indicates diagonal 1-1/2-2/3-3 pairing ($NI = 0.8671 > 0$).

The Relative Average Residence Time Array (RARTA) of the process is

$$\begin{aligned} \Lambda_N &= \mathbf{K}_N \otimes (\mathbf{K}_N^{-1})^T \\ &\approx \begin{bmatrix} 0.8234 & -0.0037 & 0 \\ -0.0046 & 0.8155 & 0.1891 \\ 0.0008 & 0.1882 & 0.8109 \end{bmatrix} \end{aligned}$$

Note that $\lambda_{11}, \lambda_{22} > 1, \lambda_{33} < 1$ and $\gamma_{11}, \gamma_{22}, \gamma_{33} < 1$. Therefore,

$$\begin{aligned} \hat{k}_{ii} &= k_{ii} & i &= 1, 2 \\ \hat{k}_{33} &= \frac{k_{33}}{\lambda_{33}} \\ \hat{\sigma}_{ii} &= \sigma_{ii} & i &= 1, 2, 3 \end{aligned}$$

Using the gain-phase-margin controller design methodology introduced in 4.2, the PID controllers are designed as follows (take $A_{m,i} = 3$).

$$\mathbf{G}_c(s) = \begin{bmatrix} 2199.1 & 0 & 0 \\ 0 & 1251.6 + \frac{19.6506}{s} & 0 \\ 0 & 0 & -0.0593 - \frac{0.0006}{s} \end{bmatrix}$$

Tan et al. introduced a 3×3 matrix of PI controller as their decentralized PID control design [52]. We compare the step responses of the boiler process using our RNGA-based control with the same simulation program shown in Figures 5.20 and 5.21 and that of [52], and show the comparison result in Figure 5.25. According to [52], the input attemperator spray flow changes from 0 to set-point 680 at Time 0, fuel flow from 6.4 to set-point 6.8 at Time 500, feedwater flow from 0 to set-point 1.3 at Time 1000.

We also add a unit-step disturbance to both the processes, and compare their stability at the presence of disturbance as shown in the following figure 5.26.

Obviously, our RNGA-based PID controller design outperforms that of [52], either with or without disturbance, by taking much less time to converge to the set-points. Meanwhile, our design method is also easier to design and implement.

Figure 5.25: Boiler process' step responses using RNGA-based control in comparison with Tan et al. [52]

Figure 5.26: Boiler process' step responses with disturbance using RNGA-based control in comparison with Tan et al. [52]

5.4 MIMO Decentralized Controller Design for a 4×4 Reactor Process

Consider a 4×4 process of reactor [53] in Figure 5.27. The transfer function matrix of the process is as follows.

$$\mathbf{G}_0(s) = \begin{bmatrix} 0 & \frac{3.96(197s+1)}{49.4s^2+14.1s+1} & \frac{0.536(258s+1)}{83.3s^2+18.3s+1} & \frac{9.7}{24.3s+1} \\ \frac{0.00111}{s} & \frac{0.044}{46.9s^2+s} & \frac{-0.0152e^{-32s}}{s} & \frac{0.039e^{-20s}}{s} \\ 0 & \frac{-232.2}{32.2s+1} & \frac{-15.96(529s+1)}{10417s^2+204s+1} & \frac{139.2}{7.27s+1} \\ \frac{-0.582}{s} & \frac{-2.54(8.11s+1)}{6.25s^3+5s^2+s} & \frac{0.0462(45.6s+1)e^{-35s}}{306s^3+35s^2+s} & \frac{-0.0358e^{-30s}}{s} \end{bmatrix}$$

Figure 5.27: 4×4 reactor process

Using 2nd order least-square method, the processes can be approximated

to their first-order plus time-delay equivalents as follows:

$$\mathbf{G}(s) = \begin{bmatrix} 0 & \frac{0.0802e^{-0.0098s}}{s^2+0.2854s+0.0202} & \frac{0.0064e^{-0.0117s}}{s^2+0.2197s+0.0120} & \frac{9.7}{24.3s+1} \\ \frac{0.00111}{s} & \frac{0.044}{46.9s^2+s} & \frac{-0.0152e^{-32s}}{s} & \frac{0.039e^{-20s}}{s} \\ 0 & \frac{-232.2}{32.2s+1} & \frac{-0.0015e^{-0.0005s}}{s^2+0.0196s+0.0001} & \frac{139.2}{7.27s+1} \\ \frac{-0.582}{s} & \frac{-2.54 \times 8.11/2.5}{2.5s^2+s} & \frac{0.0462 \times 45.6/35}{18s^2+s} & \frac{-0.0358e^{-30s}}{s} \end{bmatrix}$$

By letting $\varepsilon = 10^{-6}$, the steady state gain of the approximated transfer function matrix $\mathbf{G}(s)$ is

$$\mathbf{K} = \begin{bmatrix} 0 & 0.0802/0.0202 & 0.0064/0.0120 & 9.7 \\ \frac{0.00111}{\varepsilon} & \frac{0.044}{\varepsilon} & \frac{-0.0152}{\varepsilon} & \frac{0.039}{\varepsilon} \\ 0 & -232.2 & -0.0015/0.0001 & 139.2 \\ \frac{-0.582}{\varepsilon} & \frac{-2.54 \times 8.11/2.5}{\varepsilon} & \frac{0.0462 \times 45.6/35}{\varepsilon} & \frac{-0.0358}{\varepsilon} \end{bmatrix}$$

Therefore, the RGA of the approximated transfer function matrix $\mathbf{G}(s)$ is

$$\begin{aligned} \mathbf{\Lambda} &= \mathbf{K} \otimes (\mathbf{K}^{-1})^T \\ &\approx \begin{bmatrix} 0 & 0.1145 & 0.1359 & 0.7497 \\ -0.0763 & 0.1972 & 0.8324 & 0.0467 \\ 0 & 0.7588 & 0.0375 & 0.2038 \\ 1.0763 & -0.0704 & -0.0058 & -0.0001 \end{bmatrix} \end{aligned}$$

Also, $NI = 1.4953 > 0$. Thus the RGA-based loop pairing criterion indicates diagonal 1-4/2-3/3-2/4-1 pairing.

Similarly, we may apply the REGA approach on the same process. By letting $\varepsilon = 10^{-6}$,

$$\mathbf{E} = \mathbf{K} \otimes \mathbf{\Omega}$$

$$= \begin{bmatrix} 0 & 1.4810 & 0.1433 & 1.421 \\ \frac{0.0002}{\varepsilon} & \frac{0.0052}{\varepsilon} & \frac{-0.0021}{\varepsilon} & \frac{0.0055}{\varepsilon} \\ 0 & -33.623 & -2.2583 & 27.548 \\ \frac{-0.0824}{\varepsilon} & \frac{-1.3711}{\varepsilon} & \frac{0.0068}{\varepsilon} & \frac{-0.0051}{\varepsilon} \end{bmatrix}$$

The REGA matrix is calculated as follows.

$$\mathbf{\Phi} = \mathbf{E} \otimes (\mathbf{E}^{-1})^T$$

$$\approx \begin{bmatrix} 0 & 0.3181 & 0.1706 & 0.5112 \\ -0.0920 & 0.1733 & 0.8810 & 0.0376 \\ 0 & 0.5951 & -0.0463 & 0.4512 \\ 1.0920 & -0.0866 & -0.0053 & -0.0001 \end{bmatrix}$$

which also indicates diagonal 1-4/2-3/3-2/4-1 pairing ($NI = 1.4953 > 0$).

The Relative Average Residence Time Array (RARTA) of the process is

$$\mathbf{\Gamma} = \mathbf{\Phi} \odot \mathbf{\Lambda}$$

$$\approx \begin{bmatrix} 0 & 2.7792 & 1.2557 & 0.6820 \\ 1.2052 & 0.8789 & 1.0584 & 0.8062 \\ 0 & 0.7843 & -1.2358 & 2.2144 \\ 1.0145 & 1.2290 & 0.9215 & 0.8062 \end{bmatrix}$$

Note that $\lambda_{14}, \lambda_{23}, \lambda_{32} < 1, \lambda_{41} > 1$ and $\gamma_{14}, \gamma_{32} < 1, \gamma_{23} > 1$. Therefore,

$$\hat{k}_{14} = \frac{k_{14}}{\lambda_{14}}$$

$$\hat{k}_{23} = \frac{k_{23}}{\lambda_{23}}$$

$$\hat{k}_{32} = \frac{k_{32}}{\lambda_{32}}$$

$$\hat{k}_{41} = k_{41}$$

$$\hat{\sigma}_{14} = \sigma_{14}$$

$$\hat{\sigma}_{23} = \sigma_{23} \times \gamma_{23}$$

$$\hat{\sigma}_{32} = \sigma_{32}$$

Here γ_{41} is not discussed, as $g_{41} = \frac{-0.582}{s}$ is a pure integrating process and the concept of energy transmission ratio does not apply..

Based on the obtained effective transfer functions, we take the controller design approach without inner feedback loop for integrators.

Using the gain-phase-margin controller design methodology introduced in 4.2, the PID controllers are designed as follows (take $A_{m,i} = 3$).

$$\mathbf{G}_c(s) = \begin{bmatrix} 0 & 0 & 0 & 5.8999 + \frac{0.2428}{s} \\ 0 & 0 & -0.5031 - \frac{0.0012}{s} & 0 \\ 0 & -0.3306 - \frac{0.0103}{s} & 0 & 0 \\ -1.7182 - \frac{0.4296}{s} & 0 & 0 & 0 \end{bmatrix}$$

The simulation program is shown in Figures 5.28 and 5.29. Figure 5.29 is the part of “Multivariable Process” in Figure 5.28. The 4 PID controllers in Figure 5.28 are given by $\mathbf{G}_c$.

The four control inputs of the reactor process are A flow, B flow, the recycle valve and the purge valve of the reactor, respectively. There are also two disturbances, namely D split ratio and Mol. %D feed occurring at time 100

Figure 5.28: Control system configuration of the reactor process, without inner feedback loop

Figure 5.29: Multivariable process in the reactor process' control system configuration, without inner feedback loop

and 1700. Robinson et al. introduced a decentralized control configuration by combining feedback PID and feedforward PID control [53]. We compare the unit step responses of the reactor process using our REGA-based control and that of [53], with the control configurations of both methods shown in Figure 5.30.

The comparison result is shown in Figure 5.31. The input A flow changes from 0 to set-point 1 at time 0, B flow at time 1000, recycle valve at time 2000 and purge valve at time 3000.

It is observed that the 4×4 reactor process takes much less time to converge to the set-points with the PID controllers we propose. Moreover, the REGA-based PID controller is easier to design and implement.

Similarly, we may apply the RGA approach on the same process. By letting $\varepsilon = 10^{-6}$,

$$\mathbf{K}_N = \begin{bmatrix} 0 & 0.2807 & 0.0293 & 0.3992 \\ \frac{0.0011}{\varepsilon} & \frac{0.0009}{\varepsilon} & \frac{-0.0005}{\varepsilon} & \frac{0.0020}{\varepsilon} \\ 0 & -7.2112 & -0.0782 & 19.147 \\ \frac{-0.5820}{\varepsilon} & \frac{-3.2959}{\varepsilon} & \frac{0.0010}{\varepsilon} & \frac{-0.0012}{\varepsilon} \end{bmatrix}$$

The RGA matrix is calculated as follows.

$$\begin{aligned} \Lambda_N &= \mathbf{K}_N \otimes (\mathbf{K}_N^{-1})^T \\ &\approx \begin{bmatrix} 0 & 2.2565 & -2.3351 & 1.0786 \\ -3.3736 & 0.5012 & 3.5394 & 0.3330 \\ 0 & 1.6006 & -0.1894 & -0.4112 \\ 4.3736 & -3.3583 & -0.0149 & -0.0004 \end{bmatrix} \end{aligned}$$

which also indicates diagonal 1-4/2-3/3-2/4-1 pairing ($NI = 1.4953 > 0$).

Figure 5.30: Reactor's control configuration with REGA-based controller and with the controller of [53]

Figure 5.31: Reactor process' unit step responses using REGA-based control in comparison with Robinson et al. [53]

The Relative Average Residence Time Array (RARTA) of the process is

$$\Lambda_N = \mathbf{K}_N \otimes (\mathbf{K}_N^{-1})^T$$

$$\approx \begin{bmatrix} 0 & 19.7128 & -17.1853 & 1.4388 \\ 44.2104 & 2.5418 & 4.2518 & 7.1352 \\ 0 & 2.1095 & -5.0547 & -2.0181 \\ 4.0635 & 47.6839 & 2.5671 & 4.7568 \end{bmatrix}$$

Note that $\lambda_{14}, \lambda_{23}, \lambda_{32} < 1, \lambda_{41} > 1$ and $\gamma_{14}, \gamma_{23}, \gamma_{32} > 1$. Therefore,

$$\hat{k}_{14} = \frac{k_{14}}{\lambda_{14}}$$

$$\hat{k}_{23} = \frac{k_{23}}{\lambda_{23}}$$

$$\hat{k}_{32} = \frac{k_{32}}{\lambda_{32}}$$

$$\hat{k}_{41} = k_{41}$$

$$\hat{\sigma}_{14} = \sigma_{14}$$

$$\hat{\sigma}_{23} = \sigma_{23} \times \gamma_{23}$$

$$\hat{\sigma}_{32} = \sigma_{32}$$

Here γ_{41} is not discussed, as $g_{41} = \frac{-0.582}{s}$ is a pure integrating process and the concept of average residence time does not apply.

Based on the obtained effective transfer functions, we take the controller design approach without inner feedback loop for integrators.

Using the gain-phase-margin controller design methodology introduced in 4.2, the PID controllers are designed as follows (take $A_{m,i} = 3$).

$$\mathbf{G}_c(s) = \begin{bmatrix} 0 & 0 & 0 & 8.4888 + \frac{0.2428}{s} \\ 0 & 0 & -0.8970 - \frac{0.0037}{s} & 0 \\ 0 & -0.6973 - \frac{0.0103}{s} & 0 & 0 \\ -1.7182 - \frac{0.4296}{s} & 0 & 0 & 0 \end{bmatrix}$$

With the same simulation program shown in Figure 5.30, we compare the unit step responses of the reactor process using our RNGA-based control and that [53], and show the comparison result in Figure 5.32. The input A flow changes from 0 to set-point 1 at Time 0, B flow at Time 1000, recycle valve at Time 2000 and purge valve at Time 3000.

Figure 5.32: Reactor process' unit step responses using RNGA-based control in comparison with Robinson et al. [53]

It is observed that the 4×4 reactor process takes much less time to converge to the set-points with the PID controllers we propose. Moreover, the RNGA-based PID controller is easier to design and implement.

5.5 Conclusion

In this chapter, simulation studies were used to verify the performance of our proposed decentralized controller design for MIMO integration systems. From the simulation results of the 3 examples, it was observed that the overall control system performance using the proposed method is better than that using any of other tuning methods. On the other hand, the REGA/RNGA-based decentralized controller design is less computationally complicated, and is easier to understand and implement for field control engineers.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, a decentralized PI/PID controller design scheme for MIMO processes containing pure integrators and time delays was presented. Firstly, for multi-loop control of integrating MIMO processes, a new loop pairing criterion based on a new method for interaction measurement was proposed, especially for MIMO process containing pure integrators. Through investigating both the steady state and transient information of the process transfer function, the effective and normalized gains were defined to provide a more comprehensive description of each process input to output channel. The REGA and RNGA methods were then introduced for loop interaction measurements. Consequently, REGA/RNGA-based loop pairing techniques have been used for control structure configuration. The main advantages of this method are: (1) compared with RGA method, it considers not only the process' steady- state information but also its transient information; (2)

compared with DRGA method, it also provides a comprehensive description of dynamic interaction among individual loops without requiring the specification of the controller type and with much less computation; (3) it is very simple for field engineers to understand and work out pairing decisions in practical applications.

Based on the REGA/RNGA loop pairing result, the design of decoupling control systems is followed by the calculation of ETF, which reflects the dynamic relative interactions to an individual control loop from all the other loops in a MIMO process. For decentralized controller design, proper parameters are selected based on the ETF matrix of MIMO process. This design scheme guarantees the MIMO process' stability, causality and properness. The performance of the overall control system is compatible to those of other decoupling control schemes for MIMO processes with integrators. For field control engineers, the proposed REGA/RNGA based controller design method is simple, straightforward and easy to apply. An industrial multi-variable process is employed to demonstrate the advantages of the method in terms of simplicity in design and effectiveness in control system performance.

6.2 Future Work

For future work, it is desirable to design a stable, proper and causal ideal diagonal decoupler matrix of high dimensional MIMO processes, based on equivalent transfer function matrix. In process control industry, decoupling control for multivariable processes is a popular technique which uses a decoupler to eliminate the effect of the undesirable cross-couplings such that

the process can be treated as multi-single loops and less conservative single loop control design methods can be directly applied. In the ideal case, the decoupler causes the control loops to act as if they are totally independent of one another, thereby reducing the controller tuning task to that of tuning several, non-interacting controllers.

The method should be simple, straight forward and easy to understand for field engineers and it can be embedded into the computer control systems. Research topics on design of decoupling control system using equivalent transfer function matrix method may include the necessary and sufficient conditions for the existence of ideal diagonal decoupler in terms of equivalent transfer function matrix and optimal structure selection for partial decoupling, etc. These topics deserve for further investigation.

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