Micro-deformation characterization of electronic packaging materials and assemblies

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**MICRO-DEFORMATION CHARACTERIZATION
OF ELECTRONIC PACKAGING MATERIALS
AND ASSEMBLIES**

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SCHOOL OF MECHANICAL & AEROSPACE ENGINEERING
2008**

Micro-Deformation Characterization of Electronic Packaging Materials and Assemblies

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ABSTRACT

Abstract

The continuing miniaturization of microelectronic assemblies and devices requires experimental micro-deformation analysis techniques for material characterization and structure reliability analysis. This work investigates both theoretical aspects and practical applications of digital image correlation (DIC) techniques for in-plane micro- and nano-scale deformation characterization of electronic packaging materials and assemblies.

The accuracy of displacement measurement by existing subimage-based DIC algorithms in spatial and spectral domains is first compared in a quantitative manner. The effects of the image correlation algorithm in spatial domain on DIC analysis are further studied. An experimental factor, subimage quality index, is first defined and its effect on DIC analysis is studied. The effects of other experimental factors, such as DIC system stability and out-of-plane displacement, are also investigated quantitatively and their solutions are proposed. For instance, by applying the image frame averaging scheme, the maximum system error from μ -Disc system decreases from 0.18 pixel to 0.05 pixel.

A novel element-based DIC method is developed by introducing finite element framework into the digital image correlation process. Both numerical and experimental evaluations have verified the accuracy and robustness of the finite element formulation for digital image correlation. The existing subimage-based DIC method shows 3~4 times as variation in measured uniform strains as the new element-based DIC method. The element-based method is then successfully applied to the characterization of near crack-tip deformation in lead-free SnCu solder alloy, and of thermo-mechanical fatigue deformation in a solder joint of a sectioned electronic

ABSTRACT

assembly. The K_I values from DIC analysis and ASTM formula at the initial elastic phase show a good match within a difference of 6.8%, which verifies the crack-tip displacement measurements by DIC. In the thermal cycling experiment for fatigue deformation measurement, the difference of maximum accumulated shear strain is – 9.5% per cycle.

The effect of AFM (Atomic Force Microscope) scanner drift on the distortion of AFM images is investigated. A novel correction method is developed to reconstruct the AFM scanned images for DIC analysis. The proposed method generates a corrected image from two AFM images scanned at the angle of 0 and 90 degrees used to calibrate and remove the image distortion. A calibration test shows that an AFM image distortion decreases from 4.3% to 0.16% after the image correction. A displacement measurement accuracy of about 2 nm is achieved when the corrected AFM images of size $10\mu m \times 10\mu m$ is used in AFM-DIC analysis. Furthermore, an in-situ AFM-DIC experiment on a micro thermoelectric cooler for nanoscale deformation measurement is demonstrated using the correction approach.

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Abbreviations

AFM	Atomic force microscope
ATC	Accelerated thermal cycling
BGA	Ball grid array
CCD	Charge coupled device
CT	Compact tension
CTE	Coefficient of thermal expansion
CTOD	Crack tip opening displacement
DC	Direct current
DFT	Discrete Fourier transform
DIC	Digital image correlation
FEA	Finite element analysis
FEM	Finite element modeling
FFT	Fast Fourier transform
GUI	Graphic user interface
IC	Integrated circuit
IFFT	Inverse fast Fourier transform
I/O	Input/output
LEFM	Linear elastic fracture mechanics
MEMS	Micro electro-mechanical systems
PBGA	Plastic ball grid array
PCB	Printed circuit board
SAC	Lead-free solder alloy SnAgCu
SEM	Scanning electronic microscope
SPM	Scanning probe microscope
TE	Thermo-electric
TEC	Thermo-electric cooler
TMA	Thermo-mechanical analyzer

Nomenclature

I	Light intensity, gray scale value
u, U	Displacement component in the horizontal/ x direction
v, V	Displacement component in the vertical/ y direction
Δ	Difference
∇	Gradient
∇^2	Laplacian
ε	Strain
σ	Standard deviation/Stress
E	Young's modulus
E_R	Reference modulus
C	Correlation coefficient
G	Shear modulus
ν	Poisson's ratio
θ	Angle
K_I	Stress intensity factor (mode I)
Γ	Contour path
J	J integral
J_v	Generalized J integral
δ	CTOD or Subimage quality index
δ^e	Pseudo-CTOD
t, τ	Time
T	Temperature

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Chapter 1. Introduction

The miniaturization of integrated circuits (IC) on silicon chips and the shift to environment-friendly lead-free solder interconnection materials have introduced new challenges on reliability issues in electronic packaging technology. Experimental micro-deformation measurements are needed to characterize material thermo-mechanical properties, validate numerical modeling results, and identify failure site, mode and mechanism.

In an effort to take advantage of this opportunity, this thesis will investigate digital image correlation (DIC) techniques for microscale deformation characterization of electronic packaging materials and assemblies, and will also extend the DIC techniques for nanoscale deformation measurements by means of an atomic force microscope.

1.1 Background

In electronic assemblies, thermally induced stresses and strains are produced during service operation. These occur due to nonuniform thermal expansions of the different materials bonded together in an electronic package and assembly. Mechanical deformations and strains are often transmitted to solder interconnections in electronic assemblies. Cyclic strain can cause fatigue failure in solder interconnection materials. Creep deformation is another concern as electronic solders are used at high homologous temperature from 0.5 to 0.8 of their melting point.

Structural reliability analysis of electronic packages and assemblies typically employ numerical modeling and experimental reliability test methods. Numerical modeling studies use finite element analysis (FEA) modeling and simulation of complex package and assembly geometries. Such numerical modeling applications

CHAPTER 1. INTRODUCTION

require many assumptions and simplifications that need to be validated. The constitutive behavior of electronic packaging materials, such as the solder material, is another important area of research. Experimental investigations on these reliability problems by micro-deformation measurements can be employed to characterize electronic packaging material properties, validate finite element analysis results for complex electronic assemblies.

Moiré interferometry techniques have been utilized for microscopic in-plane deformation measurements of electronic packages and assemblies. The interferometric method provides sensitive, high-resolution and full-field micro-deformation measurements. However, the moiré technique has some disadvantages, such as sample preparation time and stringent requirements on the laser interferometry system.

Digital image correlation (DIC) techniques are emerging as an alternative non-contact and full-field micro-deformation measurement tool. The DIC method was first developed by the research group of Peters et al [1-4]. DIC has since been commonly applied in various research and engineering fields [5-10]. The DIC technique has great capabilities in adapting to digital images obtained from different imaging tools. For example, combined with optical microscope, digital image correlation can be used for deformation measurements at micrometer scale. With the aid of scanning electron microscope (SEM)/scanning probe microscope (SPM), the submicron- and nano-scale resolution of deformation measurements by DIC can be applied if distortions in SEM [11-12] or scan drifts in SPM [13-15] could be calibrated and corrected.

Most applications [16-21] of DIC in electronic packaging have been made to measure the in-plane deformations and strains of exposed cross-sections taken from surface mount packages, such as flip chip assemblies, chip scale packages and ball

grid array (BGA) packages. The thermally-induced deformation of solder interconnects due to temperature loading were monitored under SEM or optical microscope.

Some technical issues in the application of DIC deserve careful investigation. The measurement accuracy of this method is heavily dependent on the quality of the imaging tool used and the experimental procedures. The algorithms behind DIC are more complicated than those for moiré methods. The development and use of a DIC program require a thorough understanding on the DIC techniques.

The DIC techniques are advancing both in algorithm development and in application exploration. There are few articles [22-24] on nanoscale applications with the aid of atomic force microscope (AFM). The AFM scanner drift effect is a major concern and a method to compensate for the AFM scanner drifts is needed. The deformation measurement at the submicron scale is therefore one of most challenging issues in experimental stress and strain analysis in electronic packaging assemblies.

1.2 Objective

The objective of this study is to develop micro-deformation characterization employing improved digital image correlation methods and to extend the study to AFM-based image correlation technique. These developments for in-plane micro/nano-scale deformation measurements are specially applied to deformation analysis for electronic packaging materials and assemblies.

The specific research objectives are:

- 1) To investigate DIC analysis methods in spectral and spatial domains, and further study the DIC technique in the spatial domain;
- 2) To develop a finite element formulation for an element-based DIC method to incorporate displacement continuity in the DIC results;

- 3) To apply the developed DIC method to the measurements of in-plane crack-tip deformation and thermo-mechanical deformation of an electronic assembly;
- 4) To develop a method to correct the AFM image distortion and implement the extension from microscale to nanoscale deformation measurements by DIC.

1.3 Scope of work

Regarding experimental deformation characterization by digital image correlation, the accuracy of DIC analysis is essential. The type of DIC algorithms used and the speckle quality of digital images for DIC analysis affect the accuracy of displacement measurements by DIC techniques. Hence, this thesis firstly examines two sets of most used DIC algorithms. One is DIC algorithm of spatial domain; the other is DIC algorithm of spectral domain. Computer programs are written to implement the two sets of DIC algorithms. Case studies of deformation measurements are conducted to clarify their advantages and disadvantages. The set of DIC algorithms with a higher accuracy of DIC analysis will be chosen for further investigations. A new DIC method is supposed to be developed with the purpose of overcoming the disadvantages of the well-investigated DIC algorithms but retaining their advantages. The novel method, element-based DIC algorithms, is then employed for challenging micro-deformation characterization of electronic packaging materials and assemblies. Meanwhile, the experimental factors contributed to speckle image quality are studied during the experiments of micro-deformation measurements. Solutions to counteract the influence of experimental factors are explored, too.

In view of the increasing demand for sub-micro to nanoscale deformation measurements, AFM-DIC technique is investigated for nano-deformation characterization. The issue of AFM image distortion has to be resolved for a reliable AFM-DIC method. A correction approach for AFM-DIC technique is developed and

verified. The corrected AFM-DIC method is then used in a well-designed AFM-DIC experiment to demonstrate its applicability for nanoscale deformation measurements.

1.4 Structure of thesis

This thesis is organized into 9 chapters. The first and last chapters are the introduction and the conclusions respectively. A brief account of other chapters is given below.

In Chapter 2 literature review on the issues on solder joint reliability in electronic packaging, and the introduction of the techniques for interconnect reliability studies are reported. Both numerical method (finite element modeling) and experimental methods (moiré interferometry and digital image correlation) are reviewed.

In Chapter 3 digital image correlation (DIC) is investigated in detail considering that a thorough understanding of DIC analysis methods is essential to further improvements and successful applications of the technique. A comparative investigation is conducted on DIC in spatial domain and in spectral domain. After the identification of the advantage of DIC in spatial domain, the DIC method in spatial domain is further investigated. The studies on 4 input parameters (subimage quality index, subimage size, displacement functions and interpolation schemes) are reported.

Chapter 4 presents the development of the element-based image correlation method. The new DIC method was developed by overcoming the drawback of the existing subimage-based image correlation method that has no requirements for ensuring displacement continuity among subimages. The element-based image correlation method discretizes the entire image region of interest into finite elements that are involved in a common image correlation process by using a finite element formulation. Thus, the results from element-based image correlation meet the requirements of displacement continuity and derivative continuity.

CHAPTER 1. INTRODUCTION

Chapter 5 demonstrates the application of the element-based image correlation to the characterization of near crack-tip deformation in lead-free SnCu solder alloy. Chapter 6 reports on further application of the element-based image correlation to thermo-mechanical deformation characterization. The issues that influence thermally-induced deformation measurement accuracy by DIC are also investigated.

In Chapter 7 the development of AFM digital image correlation (AFM-DIC) method is presented. The developed method is to reconstruct AFM images for the use of digital image correlation by removing the image distortion induced by the scanner drifts. A corrected image is reconstructed from correlating two AFM images scanned at the angle of 0 and 90 degrees respectively. The verification of the proposed method has been demonstrated by correlating corrected AFM images associated with zero deformation. A buckling test of a thin plate under AFM has also been demonstrated. The in-plane displacement field at the center point of the buckling plate has been characterized using the proposed AFM image correlation technique.

Chapter 8 presents the experiment details on a micro thermoelectric cooler for nanoscale deformation measurement by the developed AFM-DIC technique. An examination of the in-plane deformation fields gives an understanding of the potential failure mode when the device works between its cooling and heating stages. To provide a comparison with experimental results, the thermally induced deformation behavior of the micro-assembly device was modeled by finite element analysis (FEA). Both thermal-electric analysis and thermal-structural analysis were conducted on a 3D finite element model of the device.

Chapter 2. Literature review

Solder interconnections are needed for electrical connection in electronic packages and assemblies. The failure of solder joints is a major cause of failure in electronic devices. Solder material and structural reliability of solder joints in electronic assemblies is one of the most important areas of research in the development of reliable electronic components. The reliability design and assessment methodology for new electronic package development involves experimental and numerical strain/stress analysis. Finite element modeling for electronic packaging design and reliability assessment can be validated by experimental mechanics techniques, such as moiré interferometry or digital image correlation for micro-deformation analysis.

2.1 Electronic packaging

In Singapore, electronic manufacturing is one of the most important industries. Production of high-quality, low-cost consumer electronics relies on proficiency in electronic packaging technologies, which focus primarily on how IC (integrated circuit) chips may be packaged cost-efficiently and reliably. A package's primary function is to provide a means for electrical connectivity from the chip to a printed circuit board (PCB). It provides a path for power to be applied to the chip as well as a way for the data signals to be transmitted into and out of the chip. Its secondary function is to house and protect the chip from harsh environment conditions, such as moisture, light and dust, which might hinder its performance. Finally, the package provides a pathway for dissipating the heat generated by the chip. The increase in chip complexity and integration has imposed strict requirement on packaging technology for higher I/O (input/output) counts, finer lead pitch, enhanced heat dissipation, and

high-speed, reliable interconnect [25].

Ball grid array (BGA) and area-array solder bumped flip-chip packaging become popular choices in advanced packaging design. The BGAs have small conductive area-array balls (see Fig. 2.1) that are soldered directly to the surface of PCBs, and thus gain the advantages in I/O count, I/O density, and heat dissipation. Flip chip packaging, widely used in chip scale package, has a silicon die/chip and mounted face down on a substrate. Advantages of flip chip include efficient die access, high assembly yields using solder attachment, elimination of an interconnect layer, lower inductance, and the potential for low cost. Using solder attachment, instead of conductive adhesives, provides the possibility of high assembly yields due to the self-alignment properties of the solder [26].

Fig. 2. 1 Ball grid array (BGA) package

2.2 Solder joint reliability

With the shrinking size of the packages and increasing number of components integrated into the packages, the reliability of the packages becomes one of the biggest concerns in the packaging development process. Solder joints, widely used in electronic packages, function not only as mechanical support of component but also

as electronic conduction joints. Hence, mechanical failure of solder joints will lead to electrical and product failure. The causes that are usually related to the failure solder joints include (1) strain accumulation due to thermal cyclic load; (2) large shear force due to excessive mismatch of thermal expansion; (3) inadequate material strength and resistance to shock and vibration. Solder joint reliability research [27-39] has become one of the most important areas of research in the development of reliable electronic components.

Another challenge to solder joint reliability is due to the transition from a technology using SnPb for electronic interconnections (with more than 50 years of experience) to a new lead-free solder technology. The established understanding of eutectic Pb-Sn solders needs to be extended to lead-free solders and reliability studies of solder joints are actively pursued [40-48].

2.3 Methodology for reliability design and assessment

In electronic packaging, both finite element analysis (FEA) and experimental characterization investigations are needed for the studies of solder joint reliability. They work together for reliability design and assessment as shown in Fig. 2.2. Finite element analysis involves modeling of the geometry, material behavior, boundary conditions, and load history. Experimental characterization and testing results provide the material characterization and validation of finite element simulation results.

Experimental stress/strain analysis relies on deformation measurement techniques. A variety of deformation measurement techniques [49-51] are available, such as strain gages, brittle coatings, moiré interferometry, speckle photography, and digital image correlation. Considering the small scale of these objects to be measured and the whole field data to be required, only a few methods are available for that purpose. As reported in the literature, moiré interferometry has been applied to

measure in-plane deformations [52-60] in electronic packaging. Digital image correlation (DIC) technique is suited for micro-deformation characterization of electronic packages and assemblies [61-63] because of its non-contact feature, easy specimen preparation, and vibration insensitivity.

Fig. 2. 2 Methodology for reliability design and assessment

2.4 Finite element analysis of soldered assemblies

Finite element analysis (FEA) is a numerical analytical technique that is employed in computing stresses/strains in solder joints for any geometrical package structures. FEA modeling is a complex process involving the interaction of thermal and mechanical phenomena. The finite element method is one of the necessary tools for electronic packaging design or solder joint reliability evaluation.

In terms of the reliability of solder joints, low cycle fatigue of solder joints is the typical mode of failure in thermal cycling test. Studies have been conducted for fatigue life prediction of solder joints [29-32]. In these studies, the accelerated thermal cycling (ATC) tests of electronic packages were simulated. Nonlinear, temperature and rate dependent behavior of solder material is taken into account. Simplified two-dimensional or three-dimensional finite element models of the packages were generated due to geometric symmetry and other considerations. The models were

subjected to the temperature cycling profile as shown in Fig. 2.3. Three or four cycles were run in the temperature cycling simulation; stress and strain distributions were output for fatigue life prediction of solder joints based on various empirical fatigue life models [27, 36, 64-66]. The fatigue models, such as Coffin-Manson model [27] and its variants [65], were generally obtained through the correlation of the fatigue lifetime from controlled fatigue tests based on inelastic strain and inelastic energy parameters.

Fig. 2. 3 Temperature cycling profile used for reliability analysis of solder joints

Compared with the simulation of ATC, power cycling simulation [67-70] increases the difficulties in accurate life prediction of solder joints, because the power cycling test involves complex, transient, anisothermal load condition. The power cycling test was several times faster than the ATC test. With the purpose of shortening the reliability test time, other accelerated thermal/mechanical cycling tests [28, 33, 71] have been developed and simulated.

Besides finite element simulation for life prediction of solder joints in a cyclic thermal/mechanical environment, numerical analysis has also been used for reliability optimization [27, 34, 37] and failure analysis of solder joints [34, 42-43]. Typical examples of solder joint reliability optimization include the studies of effects of solder joint geometry, material selection, and package structure on solder joint reliability.

2.5 Moiré interferometry for deformation measurement

As a sensitive, full-field optical method for in-plane deformation measurement, moiré

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interferometry has been used for almost 50 years since introduced by Weller [72]. As shown in Fig. 2.4, a cross-line diffraction grating is replicated onto the cross-section of the specimen that is the area of interest in the deformation measurement [73]. The grating attached to the specimen surface is usually referred to the specimen grating which deformed together with the underlying specimen. The specimen grating diffracts the incident laser beams ($B_1 - B_4$), and the diffracted beams form the interferometry fringe patterns. The two horizontal laser beams measure the horizontal deformation of the specimen, called the U field and the two vertical beams measure the vertical deformation of the specimen, called the V field. Fringe analysis is then applied to the resultant U field and V field fringe patterns to extract the in-plane displacements and strains. The general relationship between the moiré fringes and in-plane displacements is given by:

Fig. 2. 4 Schematic diagram of moiré interferometry [73]

$$\begin{aligned} U &= \frac{1}{f} \cdot N_x \\ V &= \frac{1}{f} \cdot N_y \end{aligned} \quad (2.1)$$

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where f is the frequency of the virtual reference grating, which is twice of the spatial frequency of the specimen grating, and N_x , N_y are the fringe orders in the U field and V field respectively. If the specimen grating has a groove density of 1200 lines/mm, a deformation sensitivity of 0.417 $\mu\text{m}/\text{fringe}$ will be achieved. The in-plane strains can be derived from the measured displacements based on the small strain relationship:

$$\begin{aligned}\varepsilon_x &= \frac{1}{f} \cdot \frac{\partial N_x}{\partial x} \\ \varepsilon_y &= \frac{1}{f} \cdot \frac{\partial N_y}{\partial y} \\ \gamma_{xy} &= \frac{1}{f} \cdot \left(\frac{\partial N_x}{\partial y} + \frac{\partial N_y}{\partial x} \right)\end{aligned}\tag{2.2}$$

where ε_x , ε_y are the in-plane normal strains in horizontal and vertical directions respectively, and γ_{xy} is the shear strain.

Moiré interferometry has been used as a tool to investigate the reliability of electronic packages due to extensive efforts in this area by Bastawros and Voloshin [52, 74-76], and researchers from IBM [56-57, 77-79]. Several of these investigations were conducted to determine the thermally induced in-plane displacements and strains occurring to exposed cross-sections taken from BGA, column grid array, and flip chip packages which contain multiple solder joints. In addition, moiré interferometry has also used to monitor solder joint fatigue testing under cyclic thermal loading [58, 80], and to evaluate effective coefficients of thermal expansion (CTE) of various electronic packages [59, 78]. Furthermore, correlation of interferometric moiré data with finite element predictions has also been explored [79, 81].

Moiré interferometry is well developed but there are some limitations in the application to thermally induced deformation measurements. For example, complicated optical apparatus is needed to produce moiré patterns in heating

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environment. The difficulty of grating reproduction on the specimen surface, the misalignment of the optical axes of the illuminating laser beams and the diffractive wave fronts of the grating are some issues. The null field of the moiré fringe patterns is difficult to be obtained before the thermal loading is introduced so that the influence of those initial fringes must be subtracted from the final displacement field through data post processing in order to obtain the net thermally induced displacements [74]. Sometimes, the whole field fringe interpretation is a tough job through only one fringe interferogram due to the need of manual processing and professional knowledge or experience. The phase shifting technique [52] offers an automatic processing way to solve the whole-field displacement field through capturing several images of the interferogram, with the fringes in each image being shifted a fractional wavelength of the laser source. But it was found that any small heating disturbance during the artificial phase shifting might introduce big errors in the solutions of the displacement phase.

Besides the cumbersome specimen preparation of grating replication and the sensitivity to disturbance, the grating/adhesive system, acting as the medium of the sample deformation, introduces a perturbation in the sample material structures, particularly small sized structures. As to some types of experimental studies such as the detection of microscopic crack, the grating/adhesive system can be considered as an obstacle to the measurement.

Concerning the measurements of nanoscale deformation, researchers resort to SEM/AFM moiré [82-85] that is the mechanism of geometric moiré. Geometric moiré fringe produces when specimen gratings are superimposed with corresponding reference gratings formed by SEM/AFM scanning lines. The applications of nano-moiré require the very fine specimen gratings that cannot be fabricated economically.

In the reference [84], crystal lattices of the sample itself form the required fine specimen gratings.

2.6 Digital image correlation for deformation measurement

The digital image correlation (DIC) method resolves the displacement field on the surface of a planar specimen by correlating subimages on a pair of images, one before deformation, and the other after deformation. Fig 2.5 illustrates the principle of digital image correlation. The displacements of a point in the center of a subimage in the reference image are obtained by the find of the corresponding subimage in the deformed image. To some extent, DIC can be considered as an extension to digital images of the speckle photography technique [86]. Speckle photography obtains the displacement field through the analysis of photographic plates that are exposed twice, before and after the specimen deformation.

Fig. 2. 5 The principle of digital image correlation

The development and broad application of DIC technique can be attributed to the rapid evolution of computers, CCD camera or recorders, and frame grabbers during the last decades. These low-cost and easy-to-use hardwares combined with software make digital image acquisition and the solution to displacement field from acquired images easy and rapid. In contrast, the conventional speckle photography requires a time-consuming analysis procedure of recording the specklegram on photographic plates and filtering this pattern to give Fourier or Young's fringe that represents the displacement undergone by the object between the two exposures. Compared with moiré methods, digital image correlation method has the advantages of (1) the ease in specimen preparation, (2) the insensitivity to vibration, (3) low-cost hardware and simple apparatus, (4) the easy implementation of measurement automation.

Fig. 2. 6 Digital image correlation in spatial domain and spectral domains

Digital image correlation extracts displacement fields associated with a pair of digitized images by means of a correlation algorithm. A fast, accurate and robust correlation algorithm will be very helpful for its popularity in engineering applications. The processing of an image can be carried out in one of two domains: the spatial domain and the spectral domain. Fig. 2.6 shows the general configuration of digital image correlation in the spatial domain or in the spectral domain and the relationship between them. When image correlation is performed in the spatial

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domain, the processing of digitized images is carried out directly by pixel processing. But if image correlation is to be performed in the spectral domain, the digitized images will first be transformed by discrete Fourier transform (DFT) or fast Fourier transform (FFT). Discrete correlation will then be carried out in the spectral domain. After the correlation is performed, the inverse fast Fourier transform (IFFT) will be carried out on the results to transform them back to the spatial domain.

The digital image correlation in the spatial domain was first proposed by the researchers [1-4] at University of South Carolina at the beginning of 1980's. Initially, they used coarse-fine search to find the accurate displacements in integral pixel, and to achieve the subpixel accuracy by combining with bilinear, polynomial or bicubic spline reconstruction of the intensity pattern of an image. The slow performance of DIC at that time lay on the iterative procedure of coarse-fine search. In 1989 [4], the use of Newton-Raphson method of partial differential correction speeds up the detection of the solution significantly and integrates the search procedures for the accurate displacements on integral pixel and subpixel. Vendroux and Knauss [5] in 1998 proposed a least-square correlation coefficient in place of cross-correlation coefficient and an approximate Hessian matrix in the Newton-Raphson iteration algorithm to simplify the calculation and achieve more robust convergence characteristics.

Since 1990, Chen and coworkers [87-88] began to perform digital image correlation in the spectral domain, called digital speckle displacement measurement. The subpixel accuracy was achieved by biparabolically fitting the correlation coefficients around the peak position. The similar DIC method in the spectral domain, called electronic speckle photography, was also developed by Sjö Dahl and Benckert [89-91] at Luleå University of Technology. They proposed a well-defined correlation

surface for accurate subpixel search. Later the shifting technique in FFT [89] was used to maximize the overlap area of two correlated subimages and hence to maximize the correlation between them.

2.6.1 DIC in the spatial domain

The digital image correlation in the spatial domain is to perform correlation algorithm in the spatial domain. The mapping of one subimage in the reference image is to be found in the deformed image by maximizing the cross correlation coefficient between intensity patterns of two subimages:

$$C(\mathbf{p}) = \frac{\sum_{y=y_0-N/2}^{y_0+N/2} \sum_{x=x_0-M/2}^{x_0+M/2} g(x, y) h(x+u(x, y, \mathbf{p}), y+v(x, y, \mathbf{p}))}{\sqrt{\sum_{y=y_0-N/2}^{y_0+N/2} \sum_{x=x_0-M/2}^{x_0+M/2} g^2(x, y) \sum_{y=y_0-N/2}^{y_0+N/2} \sum_{x=x_0-M/2}^{x_0+M/2} h^2(x+u(x, y, \mathbf{p}), y+v(x, y, \mathbf{p}))}} \quad (2.3a)$$

or the least squares correlation coefficient:

$$C(\mathbf{p}) = 1 - \frac{\sum_{y=y_0-N/2}^{y_0+N/2} \sum_{x=x_0-M/2}^{x_0+M/2} [g(x, y) - h(x+u(x, y, \mathbf{p}), y+v(x, y, \mathbf{p}))]^2}{\sum_{y=y_0-N/2}^{y_0+N/2} \sum_{x=x_0-M/2}^{x_0+M/2} g^2(x, y)} \quad (2.3b)$$

where the coordinate (x_0, y_0) represents the center point of a subimage with its size $M \times N$; $g(x, y)$ is the light intensity at the point (x, y) in the reference image; $h(x+u(x, y, \mathbf{p}), y+v(x, y, \mathbf{p}))$ is the light intensity in the deformed image at the mapping position of the point (x, y) . The intensity at a sub-pixel position is obtained by using bicubic spline interpolation function:

$$I(x, y) = \sum_{n=0}^3 \sum_{m=0}^3 \alpha_{mn} x^m y^n \quad (2.4)$$

The coefficients α_{mn} are determined from the intensity values of pixel locations.

$u(x, y, \mathbf{p})$ and $v(x, y, \mathbf{p})$ are displacement functions which define how each of the

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subimage points located in the reference image at (x, y) is mapped to the deformed image. First-order displacement functions are often used:

$$\begin{aligned} u(x, y, \mathbf{p}) &= p_0 + p_2(x - x_0) + p_4(y - y_0) \\ v(x, y, \mathbf{p}) &= p_1 + p_3(x - x_0) + p_5(y - y_0) \end{aligned} \quad (2.5)$$

The parameter vector $\mathbf{p}(p_0, p_1, p_2, p_3, p_4, p_5)$ can be explicitly denoted

$$\mathbf{p}(u(x_0, y_0), v(x_0, y_0), \frac{\partial u}{\partial x}|_{x_0, y_0}, \frac{\partial v}{\partial x}|_{x_0, y_0}, \frac{\partial u}{\partial y}|_{x_0, y_0}, \frac{\partial v}{\partial y}|_{x_0, y_0}) \quad (\text{where } u(x_0, y_0) \text{ and}$$

$v(x_0, y_0)$ are the displacements of the subimage center in x and y directions respectively). To find the resultant vector $\mathbf{p}$ that maximizes the correlation coefficient (Eq. (2.3a-b)), the Newton-Raphson method is used to iteratively find the solution with a given approximation $\mathbf{p}_{old}$ to the solution. The next improved approximation $\mathbf{p}_{new}$ can be obtained using the following Newton-Raphson iteration formula:

$$\nabla^2 C(\mathbf{p}_{old})(\mathbf{p}_{new} - \mathbf{p}_{old}) = -\nabla C(\mathbf{p}_{old}) \quad (2.6)$$

Newton-Raphson method is known for its fast convergence capability if the function peak is well defined. Fig. 2.7 shows a typical peak profile of cross correlation coefficients around the solution. Usually, two or three iterations are enough to determine the peak position and thus solve the parameter vector $\mathbf{p}$, which represents the displacements and displacement gradients at the subimage center. It is practical that the displacement gradients at the subimage center are used for the direct calculation of strain components in some cases where average strains are the interest, instead of additional operations of displacement field smoothing [92-94] and derivative calculation.

The strain components of the subimage center are therefore determined using the following expressions:

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 \right] \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \\
\varepsilon_{xy} &= \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{1}{2} \left(\frac{\partial u}{\partial x} \cdot \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \cdot \frac{\partial v}{\partial y} \right)
\end{aligned} \tag{2.7}$$

As to the small rigid body rotations less than 10 deg, the angle of rotation θ may be approximated by the following formula [3]

$$\theta \cong \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right). \tag{2.8}$$

Fig. 2. 7 A typical peak profile of cross correlation coefficients around the solution

2.6.2 DIC in the spectral domain

If g denotes the pattern of a subimage with the size of $M \times N$ in the reference image, and h denoted the pattern of another subimage with the same size of $M \times N$ in the deformed image, the digital image correlation in the spectral domain is to obtain the displacements at the correlation peak by correlating two subimages based on the discrete correlation definition for two real sequences

$$c_{k,l} = \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} g_{x,y} h_{x+k,y+l} \quad (2.9)$$

$$(k = 0, 1, \dots, m-1, \quad l = 0, 1, \dots, n-1)$$

where $m \geq 2M-1$, $n \geq 2N-1$, g and h must be padded with zeros by as much as the size $m \times n$ in order to avoid wraparound errors. Both m and n must be an integer power of two in order to use fast Fourier transform. Since g and h are two real sequences, the use of discrete correlation theorem leads to

$$c_{k,l} = F^{-1}(G_{k,l} H_{k,l}^*) \quad (2.10)$$

$$G_{k,l} = F(g_{x,y}) = \frac{1}{mn} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} g_{x,y} e^{-2\pi i(kx/m + ly/n)}$$

$$H_{k,l} = F(h_{x,y}) = \frac{1}{mn} \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} h_{x,y} e^{-2\pi i(kx/m + ly/n)} \quad (2.11)$$

$$(k = 0, 1, \dots, m-1, \quad l = 0, 1, \dots, n-1)$$

where F denotes fast Fourier transform, F^{-1} denotes inverse Fourier transform, $*$ denotes the complex conjugate.

Fig. 2.8 shows a typical result from a discrete correlation calculation according to Eq. (2.9-2.11). It is noted in Fig. 2.8 that the indices of matrix c were transformed to correspond with displacements, u in x direction and v in y direction. The correlation peak position represents the best estimation of the displacements between the two correlated subimages. The displacements above fall on integral pixels. It is highly advantageous to repeat the above procedure by updating a choice of either the reference subimage or the deformed subimage, that is, shifting either of two subimages by the last estimated displacements so as to maximize the match of two subimages. This iteration procedure will stop till the estimated displacements are within ± 1 pixel. To achieve subpixel accuracy, either biparabolic or cubic spline scheme can be used to fit the correlation peak surface. Likewise, Sjö Dahl and Benckert [90] implemented it by interpolating those samples around the peak position

(u_0, v_0) using the following continuous correlation function

$$c(u, v) = \frac{1}{(2p+1)(2q+1)} \sum_{k=u_0-p}^{u_0+p} \sum_{l=v_0-q}^{v_0+q} c_{k,l} \times \frac{\sin(\pi(u-k))\sin(\pi(v-l))}{\sin(\pi(u-k)/(2p+1))\sin(\pi(v-l)/(2q+1))} \quad (2.12)$$

As suggested in the reference [90], p and q can be as small as 5 without affecting the subpixel accuracy.

Fig. 2. 8 A typical profile from a discrete correlation calculation

Again, shifting one of the subimages by the nonintegral pixel displacements can maximize the fit of two subimages. The non-integer shifting is implemented in the spectral domain instead of the spatial domain

$$G_{k,l}^{shift} = G_{k,l} e^{-2\pi i(uk/m + vl/n)} \quad (k = 0, 1, \dots, m-1, \quad l = 0, 1, \dots, n-1) \quad (2.13)$$

where u and v are the shifted nonintegral displacement values. Iterating the described procedure with $G_{k,l}^{shift}$ in place of $G_{k,l}$ in Eq. (2.11) will obtain a new estimation of nonintegral displacements.

2.6.3 DIC applications in electronic packaging

Digital image correlation has been actively applied to measure thermo-

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mechanical in-plane deformation and strains of solder joints in a cross-sectioned electronic assembly [16-21]. In these investigations, the cross-sectioned assemblies were subjected to thermal load in a chamber, and the images of the solder joints before load and after load were captured to do the image correlation and thus to obtain the displacement and strain fields.

DIC can also be employed to characterize mechanical properties of materials, such as coefficients of thermal expansion (CTE), and Poisson ratios [9, 95, 96]. The fracture toughness of Loctite 3536 underfill material [16], and the interfacial fracture toughness between silicon and underfill [97] were determined as a function of temperature by measuring the crack tip opening displacement using digital image correlation.

Other applications of DIC have also been found for crack detection and stress localization evaluation in semiconductor packages [98], and damage fracture prediction [99]. The method has also been used for low cycle thermal fatigue test for solders [100] where a non-contact DIC measurement system was employed to measure the deformation of the specimen gage length instead of a contacting extensometer.

2.7 Issues in deformation measurement by DIC techniques

Fundamentally, DIC techniques compare intensities (grayscale values) on an array of pixels from two different images and try to match them. It relies on the assumption that, if the pixel intensities in one subimage of one image match those of a different image, it is likely that the two subimages represent the same subregion of an observed region. Experimentally, digital image correlation for in-plane deformation measurement is simple. All that is required is two successive images of a flat surface before and after deformation. However, imaging conditions need to remain mostly

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unchanged during the occurrence of the deformation and the observed surface must be amenable to DIC analysis, though these requirements are much less stringent than those of interferometry techniques.

For deformation measurement by DIC techniques, the image intensity variation (image speckles) is the information carrier of deformation, and image correlation algorithm is the information analyzer of encoded deformation. Therefore, the resolution and accuracy of the measured displacements with DIC techniques are determined by the type of algorithms used and the image quality. So far, the studies on the influence of image correlation algorithms and especially the image quality on displacement measurement accuracy are limited. For example, the above section 2.6 has described two types of DIC algorithms, nevertheless a report on the comparison of the two types of algorithms for the displacement measurement accuracy can not be found yet in literature. Hence, a comparative study of DIC methods in the spatial domain and the spectral domain will be presented in this thesis.

The study to examine the accuracy of an image correlation algorithm [7, 101-102] can be performed by employing the algorithm on numerically-generated images, which encode the deformation ideally. A numerically-generated image is considered to be of perfect image quality because no noise and other experimental factors deteriorate its image quality for deformation information transformation. Compared with the studies on image correlation algorithms, the studies on image quality for DIC analysis are much more challenging and are almost ignored. The effects of the image quality (experimental factors) deserve more attentions considering that experimental factors degrade the theoretical accuracy of displacement measurement and make the practical accuracy of displacement measurement undetermined. The accuracy of displacement measurement can reach 0.01 pixel [7] using numerically-generated

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images. In real experiments, the accuracy can range from 0.01 to 0.1 pixel [103]. Related with experimental factors, the unique work to be done in this thesis includes the definition of subimage quality index and its effect on displacement measurement, the calibration of DIC system errors and its reduction solution, the effect of out-of-plane displacement and its solution, and AFM image distortion and its correction method.

Chapter 3. Investigation on digital image correlation techniques

This chapter will first describe the experimental setup for digital image correlation techniques, specimen preparation, and a comparison of DIC analysis methods in the spectral domain and the spatial domain. The DIC method in the spatial domain is taken for further study, which involves the investigations into the effects of the subimage quality index, subimage size, subimage displacement functions, and intensity interpolation functions on DIC analysis.

3.1 Experimental setup

The applications of digital image correlation is fast emerging as an experimental tool for micro-deformation measurements. The experimental setup for DIC deformation measurement consists of an imaging tool and a mechanical and/or thermal loading stage as shown in Fig. 3.1. Fig. 3.1a shows the DIC experimental setup that combined a digital microscope and a mechanical micro tester. The digital

Fig. 3. 1 Experimental setups for DIC analysis: (a) digital microscope combined with Instron micro tester, (b) micro DIC system combined with thermal loading stage.

CHAPTER 3. INVESTIGATION ON DIGITAL IMAGE CORRELATION TECHNIQUES

microscope comprises a CCD camera (Charge-Coupled Device, SONY XC-55/55BB, image size 640×480 pixels), a long working-distance microscope with white light source (OPTTEM 100), image grabber card and image acquisition software installed in a personal computer. This setup has been used for the work in this chapter and Chapter 4. Fig. 3.1b shows another DIC experimental setup that incorporates a thermal cycling loading stage. This system with a thermal chamber was designed for thermally induced deformation measurements, and is described in detail in Chapter 6.

The resolution of the DIC technique theoretically depends on the imaging system. Consider the imaging system of the digital microscope shown in Fig. 3.1a. The spatial resolution is determined by the optical magnification of the microscope lens system and pixel size of the CCD camera. The calibration of imaging resolution is given in $\mu m / pixel$. The formula for calibrating imaging resolution = (field of view) / (the number of pixels in the CCD camera or the image) respectively in horizontal and vertical directions. For example, the imaging resolution is $1.0 \mu m / pixel$ in both directions for a given field of view $0.64 \times 0.48 \text{ mm}$ and a corresponding image size 640×480 pixels.

3.2 Specimen preparation

Specimen preparation for DIC measurement and analysis require considerations on specimen material and imaging system configuration used. The surface of polished specimen must be flat for in-plane deformation measurement. The surface must present random and dense speckle pattern under the imaging system because image speckle pattern is the deformation information carrier in the DIC analysis. Under optical setups, polished surface of metal specimen can present the natural speckle pattern required for DIC analysis. Fine polishing provides proper speckle pattern at high optical magnification but is not suitable if a low optical magnification is used. In

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case the specimen surface has low reflectivity, such as for a polymer specimen surface, a random speckle pattern can be obtained by spraying white and black paint on the specimen surface. Fig. 3.2 shows the surface painted specimen of aluminum, which is utilized for the following investigation on DIC techniques.

Fig. 3. 2 DIC specimen with painted surface

As the DIC information carrier, the specimen surface condition for image speckle pattern affects DIC analysis. In Section 3.4, a parameter will be defined to quantify the image speckle quality for DIC analysis. The validity of the parameter and the effects of the parameter on DIC analysis have been investigated.

3.3 DIC analysis methods

Both DIC analysis in the spatial domain and in the spectral domain are investigated in this section. A comparison of the two methods is made to examine their possible application for micro-deformation measurements.

DIC in the spectral domain is expected to be faster than DIC in the spatial domain because image processing can be facilitated in the spectral domain for some operations, such as convolution and correlation. It is true that the solving of the Eq. (2.9) in the spectral domain is much faster than in the spatial domain because multiplication of large numbers of n digits can be done in time $O(n \log(n))$ with the Fast Fourier Transform instead of $O(n^2)$ with the classic algorithm. However, digital image correlation in the spatial domain does not resolve the Eq. (2.9) in the spatial

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domain. Newton-Raphson iteration method was employed in DIC in the spatial domain to search the correlation peak position. The fast search is achieved because of the use of function derivative in Newton-Raphson method so that much calculation for correlation coefficients at the locations around the correlation peak is skipped. In contrast, correlation coefficients of $m \times n$ (see Eq. (2.10)) are calculated in DIC in the spectral domain and compared to find out the peak position in integer pixel. Although fast Fourier transform is used in DIC in the spectral domain, it can still be concluded that DIC in the spatial domain runs faster than that in spectral domain based on the algorithm analysis and the following case studies.

To compare the accuracy of displacement measurement in DIC in the spatial domain with that in the spectral domain, the two DIC codes have been prepared. The flowcharts of DIC codes in the spatial domain and the spectral domain are shown in Fig. 3.3 and Fig. 3.4 respectively. A GUI (graphic user interface) program of DIC in the spatial domain was developed using Microsoft Visual C++ language. A routine of DIC in the spectral domain has been written using MATLAB language.

The comparative study uses 3 deformed images generated numerically from a same reference image with 8 bits grayscale information per pixel. The reference image was taken on a painted area of the DIC specimen as shown in Fig. 3.2. The relationship between a deformed image intensity profile $h(x, y)$ and its reference image intensity profile $g(x, y)$ is: $h(x, y) = g(x - u_0, y)$ for a case of translation u_0 in x direction, and $h(x, y) = g(x/(1 + \epsilon_0), y)$ for a case of uniform stretch ϵ_0 in x direction. One translated ($u_0 = 1.5$ pixel) and two stretched ($\epsilon_0 = 0.005$, $\epsilon_0 = 0.05$) images were used to run digital image correlation both in the spatial domain and the spectral domain. 840 measured points distributed evenly in the reference image were

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Fig. 3. 3 Flowchart for DIC in the spatial domain

Fig. 3. 4 Flowchart for DIC in the spectral domain

selected for image correlation with subimage size of 32×32 pixels. The accuracy of measured displacements was evaluated by the average absolute deviation between measured displacements and their ideal values. For the translation case, the ideal displacements of all measured points are $(u_0, v_0) = (1.5 \text{ pixel}, 0 \text{ pixel})$. For the uniform stretch case, the ideal displacements of a measured point at the coordinate (x, y) are $(u_0, v_0) = (\varepsilon_0 x, 0)$, where $\varepsilon_0 = 0.005$ or 0.05 .

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Table 3.1 demonstrates a comparison of the accuracy of displacement measurements using the two DIC codes. The results indicate that (1) the DIC in the spectral domain has the accuracy as good as that in the spatial domain only in the case of a translation by integer pixel ($v_0=0$ pixel), and larger average absolute deviation in non-integer pixel translation ($u_0=1.5$ pixel) and uniform deformation cases; (2) The accuracy with DIC in the spectral domain will decrease when the uniform deformation becomes larger. Lower accuracy of image correlation in the spectral domain can be attributed to the use of the implicit zero-order displacement function for the calculation of correlation coefficients in DIC in the spectral domain. The mismatch of displacement functions with actual deformation results in the error in correlation coefficient calculation, and the accuracy of correlation coefficient calculation has a direct effect on the accuracy of displacements obtained from the modeling of correlation coefficient peak. With regard to the influence of displacement functions, the details will be described in Section 3.6.

Table 3. 1 Comparison of displacement measurements using DIC methods in the spatial domain and the spectral domain

Method Cases	In spatial domain		In spectral domain	
	$ \Delta u $ (pixels)	$ \Delta v $ (pixels)	$ \Delta u $ (pixels)	$ \Delta v $ (pixels)
Translation $u_0=1.5$ pixels	0.0063	0.0066	0.0209	0.0054
Uniform stretch $\varepsilon_0=0.005$	0.01	0.0099	0.102	0.023
Uniform stretch $\varepsilon_0=0.05$	0.0098	0.011	0.17	0.021

The comparison of DIC in the spatial domain and the spectral domain indicates DIC algorithm in the spatial domain has the advantages over that in the spectral domain. Hence, DIC in the spatial domain is preferred and adopted for the subsequent investigation in this chapter. The finite element-based image correlation method

discussed in the next chapter is developed based on the DIC in the spatial domain.

3.4 Subimage quality index

Since digital image correlation matches the intensity patterns in two subimages, the intensity variation or distribution in a subimage is one of the important factors in the accuracy of displacement measurements by digital image correlation methods. However, the problem is how to quantify the intensity variation or distribution in a subimage. When digital image correlation is applied on laser speckle images, the laser speckle size is an indicator of the image quality for digital image correlation and the average speckle size [102] can be estimated by the parameters of optical setup. On the other hand, for white-light images, the speckles that are present on the specimen surface are captured, thus average speckle size is not available a priori. In addition, average speckle size is only a global indicator of the whole image quality for DIC. It is clearly preferable to find a parameter to define the local image quality on a basis of a subimage. By doing so, it is practicable to evaluate the effect of image quality of each image subimage on its displacement measurement, and thus to assign a confidence of displacement measurement to each measured point. Later, the confidence can be used to link to the weighting factor of each measured point in the equations for experimental data smoothing [93-94]. To serve this purpose, a parameter called subimage quality index is proposed here

$$\delta = \left(\sum_{S_p \in S} \sum_{i=1}^8 |I_p - I_i| \right) / (2^\beta MN) . \quad (3.1)$$

where δ is called subimage quality index, S_p denotes any pixel point in a subimage S with its size $M \times N$, I_p is the intensity at point S_p , I_i is the intensity at one of 8 pixel points neighboring point S_p , β is the pixel depth.

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Eq. (3.1) shows that subimage quality index is the average of absolute intensity deviations at any pixel point in the subimage from its 8 neighboring points (see Fig. 3.5a), and is normalized by the pixel depth of the image. Consider the binary image Fig. 3.5b. The intensity of a black or white pixel is 0 or 1 respectively. The absolute intensity deviation at any pixel point in the sample subimage from its 8 neighboring points is 4. The calculated subimage quality index is thus 4 for the sample subimage using Eq. (3.1). It is noted that 4 is the upper limit of subimage quality index.

Fig. 3. 5 Schematic of a digital image: (a) one pixel point surrounded by 8 pixel points; (b) a sample subimage with the size of 2×2 pixels.

Fig. 3. 6 Effect of subimage size on subimage quality index

A speckle image of the DIC specimen was used to show the typical relationship between subimage size and subimage quality index. Fig. 3.6 shows that the smaller the subimage is, the more scattered the subimage quality indexes in one image are.

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Clearly, the images with scattered subimage quality indexes can be used to investigate the effect of the subimage quality index on displacement measurement accuracy of a measured point. A pair of images with scattered subimage quality indexes was obtained under the rigid body translation test. One image was initially captured from the DIC specimen that was fixed on a positioning stage; the other image of the DIC specimen was captured after the positioning stage moved a few micrometers in the x/u direction. Digital image correlation was performed on the pair of images with the subimage size of 33×33 pixels. Displacements at 875 points were obtained in an image correlation area of 510×360 pixels with the step of 15×15 pixels. The absolute deviation of displacement u at each measured point was calculated by subtracting the mean u -displacement value. The calculation of subimage quality indexes at the same 875 points was performed by a C++ routine based on Eq. (3.1). Absolute u -displacement deviations and subimage quality indexes of all 875 measured points were plotted in Fig. 3.7. The sample range of absolute u -displacement deviation decreases along with the larger value of subimage quality index. For example, absolute u -displacement deviations of the measured points with high subimage quality indexes from 1 to 1.1 are less than 0.04 pixel.

Fig. 3. 7 Effect of subimage quality index on uniform displacement field measurement

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The influence of subimage quality index was studied by examining individual subimage quality indexes of one image. Next the influence of subimage quality index will be illustrated by examining different mean subimage quality indexes in different images. Considering that digital images with different mean δ could be obtained by changing the optical magnification, twelve pairs of white light images were acquired when translation tests were performed under 12 configurations of different optical magnification. Each pair of images includes one reference image and one translated image. The mean δ of the 12 reference images were computed at 35×25 array points with the subimage size of 33×33 pixels. Fig. 3.8a and Fig. 3.8b show the two reference images with the maximum and the minimum mean δ respectively. The displacements at those points were obtained by running digital image correlation on all 12 pairs of images. The standard deviation of u -displacement in each pair of images was calculated. The plot of standard deviation of u -displacement (σ_u) versus mean δ is shown in Fig. 3.9 where a curve from measured data fitting was also drawn. Fig. 3.9 demonstrates the decreasing standard deviation of measured displacement when the mean subimage quality index increases.

Fig. 3. 8 Digital images with higher (a) and lower (b) subimage quality indexes

As shown in Fig. 3.7 and Fig. 3.9, the accuracy of displacement measurements by

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DIC increases when the subimage quality index increases. Considering that the use of same subimage size but different subimage quality indexes produces results of different accuracy, it is valuable to define an effective subimage size which is independently associated to the accuracy of displacement measurement. An initial exploration of the specific form of effective subimage size has been made in Fig. 3.9 where the relation between subimage quality index and standard deviation was simply fitted to a curve $\sigma_u(\delta) = 1/(88.44 \times \exp(\delta) - 110.56)$ and thus the effective subimage size can be expressed as $M \times N \times \sigma_u(0.5)/\sigma_u(\delta)$, where $\sigma_u(0.5)$ is arbitrarily chosen as the normalization level.

Fig. 3. 9 Effect of mean subimage quality index on uniform displacement field measurement

3.5 Subimage size

The selection of the subimage size is important during the image correlation process because the subimage size has significant effect on the sub-pixel accuracy of measured displacements. The larger subimage contains more signal information but meanwhile may bring along larger error due to the mismatch of adopted displacement function and actual deformation associated in the subimage. The reduction of such

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systematic error would require a smaller subimage size. But the use of smaller subimages reduces the computing time of digital image correlation, but often results in larger standard deviations of calculated results due to the lack of signal information to suppress noise influence. Therefore, there is a trade-off for the determination of optimal subimage size for specific signal and deformation.

Fig. 3. 10 Effect of subimage size on uniform displacement field measurement

It is preferable to use the images with almost uniform subimage quality index for the study of subimage size influence on displacement measurement by DIC so that the influence of subimage quality index can be minimized. Thus, considering the reference image Fig. 3.8a has the lowest and most uniform standard deviations in subimage quality index along with subimage size, the image and its translated image were selected to investigate the subimage size effect on the accuracy of displacement measurements. Digital image correlation was then performed on the pair of images in an area of 510×360 pixels with the step of 15×15 pixels, and the subimage size ranged from 21×21 to 81×81 pixels with the increment 6×6 pixels. The image correlation results were shown in Fig. 3.10. The results show that standard deviation and sample range in the translation u decrease along with the larger subimage size while the

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mean u keeps almost constant. Both the standard deviation and the sample range of u -displacement decrease faster at the initial stage. For example, the standard deviation of u -displacement (σ_u) decreased from 0.03328 to 0.01178 pixel when the subimage size increased from 21×21 to 51×51 pixels, yet from 0.01178 to 0.00816 pixel when the subimage size increased from 51×51 to 81×81 pixels. The sample range was further checked, and the sample distribution when using the subimage size of 51×51 pixels was demonstrated in the histogram of Fig. 3.11 with superimposed normal density. Fig. 3.11 indicates the sample distribution is approximate to normal distribution.

Fig. 3. 11 u -displacement histogram with superimposed normal density

A tensile experiment was also conducted on Instron micro tester (Fig. 3.1a) to study the subimage size effect on the accuracy of measured displacement gradients or strains that are directly from the resolving of image correlation algorithms, not from the displacement smoothing. For verification and comparison, a strain gauge was stuck to the other side of the DIC specimen shown earlier in Fig. 3.2. The stretched image was taken when the strain gauge indicated 1000 $\mu\epsilon$. Then, digital image correlation was implemented in the tensile test as the procedure for the pair of images in the translation test. Fig. 3.12 shows the results from image correlation in the tensile

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test. Fig. 3.12 shows sample range and standard deviation of the strain component ε_x decrease faster when the subimage size initially increases than when the subimage size further increases. When small subimage size of 21×21 pixels was adopted, a large error more than 50% was observed in the mean strain ε_x . When the subimage size is larger than 33×33 pixels, the mean strain difference from the actual value measured by the strain gauge is within $\pm 10 \mu\epsilon$ which represents an error less than 1%.

Fig. 3.12 Effect of subimage size on uniform strain field measurement

A comparison of Fig. 3.10 and Fig. 3.12 indicates a similar trend in sample range and standard deviation of measured uniform displacements and uniform strains versus subimage size. It is noted that the larger the subimage size, the smaller the standard deviations in the measurements of displacement and deformation parameters. The standard deviations decrease sharply at the initial stage, but very slowly later. Such trends can be attributed to that when the larger subimage is used, (1) more mutual pixels are involved in the correlation for neighboring measured points because the intervals among neighboring measured points are 15×15 pixels; (2) the influence from random errors becomes stable, which can be explained through theoretical analysis.

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Provided that ε_p is the intensity error at the mapping position of Point S_p in a subimage S , the least-square correlation coefficient can be given:

$$C = 1 - \frac{\sum_{S_p \in S} [I_p^r - (I_p^d + \varepsilon_p)]^2}{\sum_{S_p \in S} (I_p^r)^2} = 1 - \frac{\sum_{S_p \in S} (I_p^r - I_p^d - \varepsilon_p)^2}{\sum_{S_p \in S} (I_p^r)^2} \quad (3.2)$$

When the subimages match, we have $I_p^r = I_p^d$. The resultant correlation coefficient is:

$$\begin{aligned} C &= 1 - \frac{\sum_{S_p \in S} \varepsilon_p^2}{\sum_{S_p \in S} (I_p^r)^2} = 1 - \frac{\sum_{S_p \in S} (\varepsilon_p^2 - 2\bar{\varepsilon}\varepsilon_p + \bar{\varepsilon}^2 + \bar{\varepsilon}^2)}{\sum_{S_p \in S} (I_p^r)^2} \\ &= 1 - \frac{\sum_{S_p \in S} (\varepsilon_p - \bar{\varepsilon})^2 + \sum_{S_p \in S} \bar{\varepsilon}^2}{\sum_{S_p \in S} (I_p^r)^2} = 1 - \frac{(MN-1)\sigma_\varepsilon^2 + MN\bar{\varepsilon}^2}{\sum_{S_p \in S} (I_p^r)^2} \end{aligned} \quad (3.3)$$

where $\bar{\varepsilon}$ and σ_ε are the mean value and standard deviation of ε_p respectively, $M \times N$ is the subimage size in pixel.

One of important characteristics of random error is that its standard deviation σ_ε will keep constant and its mean value $\bar{\varepsilon}$ will tend to be zero as the sample number increases. Therefore, the resultant correlation coefficient of Eq. (3.3) is less disturbed by random errors when the subimage size increases.

Besides the random errors, the systematic errors degrade the accuracy on displacement measurement by DIC. One source of systematic errors is due to the mismatch of subimage displacement function (see Eq. (2.5)) used in the image correlation algorithm. This systematic error does not occur in the above tests because the first-order displacement function used is able to represent the uniform translation and uniform tension strain in a subimage. However, this is not generally the case in practical applications where actual displacement gradients are not uniform and may not be represented perfectly by any predictive displacement function.

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Numerical studies were employed to investigate the case when the mismatch of displacement function occurs. Nonlinear deformed images can be numerically generated by imposing the quadratic deformation field on a reference image:

$$\begin{aligned} x' &= (1 + \varepsilon)(x - W/2) + W/2, \quad y' = y \\ \varepsilon &= \varepsilon_0 \left[4(y/H - 0.5)^2 - 1 \right] \end{aligned} \quad (3.4)$$

where the intensity at coordinate (x', y') in generated images equals to that at coordinate (x, y) in the reference image, W and H are the width and height in pixel of the reference image, ε_0 is a constant, e.g. $\varepsilon_0 = 0.1$. A C++ program was written to implement Eq. (3.4) on a digital image. Using the program, a deformed grid (Fig. 3.13b, $\varepsilon_0 = 0.1$) was obtained by contracting a reference grid (Fig. 3.13a) more and more in the horizontal direction from top and bottom edges to the centerline.

Fig. 3. 13 Quadratic deformation: (a) reference grid, (b) deformed grid

Fig. 3. 14 Numerically generated image with quadratic deformation

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Fig. 3.15 Effect of subimage size on nonlinear displacement field measurement

Fig. 3.16 Effect of subimage size on non-uniform strain field measurement

The verified program was then used on the reference image (Fig. 3.8a) and two deformed images ($\epsilon_0 = 0.01, 0.05$) were numerically generated. One of the two deformed images was shown in Fig. 3.14 ($\epsilon_0 = 0.05$). Image correlation algorithm with first-order displacement functions was used to correlate the two pairs of images with the measured point layout same as above tests in the previous section. Fig. 3.15 and Fig. 3.16 show the standard deviations of displacement error Δu and strain error $\Delta \epsilon_x$

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respectively. In Fig. 3.15, the standard deviation $\sigma_{\Delta u}$ decreases initially with the larger subimages, but increases after the standard deviation reaches its minimum and when the subimage size continues to increase. The subimage size corresponding to the minimum standard deviation is 31×31 pixels for the larger deformed image ($\varepsilon_0=0.05$), and 51×51 pixels for the case $\varepsilon_0=0.01$. In Fig. 3.16, the standard deviation of $\Delta \varepsilon_x$ also decreases initially with the larger subimages, and the tiny increase of the standard deviation was observed until the subimage size is larger than 63×63 pixel for the case $\varepsilon_0=0.05$. The minimum standard deviation $\sigma_{\Delta \varepsilon}$ for the case $\varepsilon_0=0.01$ did not arise before the subimage size of 75×75 pixels. Obviously, the subimage sizes corresponding to the minimum standard deviations $\sigma_{\Delta u}$ and $\sigma_{\Delta \varepsilon}$ are different, and the standard deviation $\sigma_{\Delta u}$ increases faster than $\sigma_{\Delta \varepsilon}$ after their minimums. Fig. 3.17 shows the measured strain ε_x for the case $\varepsilon_0=0.05$ using the subimage size 63×63 pixels. The sample ranges of ε_x are also given in Fig. 3.17 because here ε_x is the average value at 35 measured points in a row.

Fig. 3.17 Strain ε_x versus y in the quadratic deformation field

A check of sample range of measured strains shown in Fig. 3.12 and Fig. 3.17

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shows the strain error in a single measured point falls within $\pm 0.1\%$ at the best of cases. In fact, a strain range insensitive to the image correlation algorithm can be observed for certain subimage size because a strain change in that range will not change the correlation coefficient. The strain error $\pm 0.1\%$ may be accepted in the measurement of large strain, for example, larger than 1%, but in the case of local small strain measurement where average smoothing cannot be used, the strains directly from the resolving of image correlation algorithm lacks accuracy. Thus, strain results of DIC analysis are often derived from the resolved and then smoothed displacement fields.

3.6 Displacement function

It is demonstrated in the previous section that systematic errors happen when adopting first-order displacement function to actual quadratic deformation measurement. H. Lu and P.D. Cary [105] first implemented a second-order displacement function in digital image correlation. They have shown second-order displacements encoded in numerically generated images could be measured more accurately by second-order displacement function than by first-order displacement function. Later, H.W. Schreier and M.A. Sutton [101] made further studies on systematic errors in digital image correlation due to undermatched subimage displacement functions. In their studies, quadratic displacement along x direction was introduced in numerically generated images:

$$\begin{aligned} x' &= x + ax^2, \quad y' = y \\ (a &= 2.5e-5, 5.0e-5, 7.5e-5, 10.0e-5) \end{aligned} \tag{3.5}$$

where the intensity at coordinate (x', y') in generated images equals to that at coordinate (x, y) in the reference image.

The second-order displacement functions embedded in DIC are as follows:

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$$\begin{aligned}
 u(x, y, \mathbf{p}) &= p_0 + p_2(x - x_0) + p_4(y - y_0) \\
 &+ p_6(x - x_0)^2/2 + p_8(y - y_0)^2/2 + p_{10}(x - x_0)(y - y_0) \\
 v(x, y, \mathbf{p}) &= p_1 + p_3(x - x_0) + p_5(y - y_0) \\
 &+ p_7(x - x_0)^2/2 + p_9(y - y_0)^2/2 + p_{11}(x - x_0)(y - y_0)
 \end{aligned} \tag{3.6}$$

The explicit form of the parameter vector $\mathbf{p}(p_0, p_1, p_2, \dots, p_{11})$ is

$$\mathbf{p} \left(u(x_0, y_0), v(x_0, y_0), \frac{\partial u}{\partial x} \Big|_{x_0, y_0}, \frac{\partial v}{\partial x} \Big|_{x_0, y_0}, \frac{\partial u}{\partial y} \Big|_{x_0, y_0}, \frac{\partial v}{\partial y} \Big|_{x_0, y_0}, \right. \\
 \left. \frac{\partial^2 u}{\partial x \partial x} \Big|_{x_0, y_0}, \frac{\partial^2 v}{\partial x \partial x} \Big|_{x_0, y_0}, \frac{\partial^2 u}{\partial y \partial y} \Big|_{x_0, y_0}, \frac{\partial^2 v}{\partial y \partial y} \Big|_{x_0, y_0}, \frac{\partial^2 u}{\partial x \partial y} \Big|_{x_0, y_0}, \frac{\partial^2 v}{\partial x \partial y} \Big|_{x_0, y_0} \right), \text{ and } u(x_0, y_0)$$

and $v(x_0, y_0)$ are the displacements of the subimage center at (x_0, y_0) in x and y directions respectively.

Fig. 3. 18 Displacement error Δu as a function of subimage size $M \times M$ and curvature of the displacement a for first- (a) and second-order (b) displacement functions [101]

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Fig. 3.19 Strain error $\Delta\epsilon$ as a function of subimage size $M \times M$ and curvature of the displacement a for first- (a) and second-order (b) displacement functions [101]

Fig. 3.18 shows the average displacement errors of 4 subimage centers from the correlation of the generated images with first- and second-order displacement functions. For the second-order displacement functions which match the actual quadratic displacement field, the displacement errors are zero. The displacement errors occur for the first-order displacement functions and increase along with larger subimage and larger curvature of displacement a . The average strain errors of the same 4 subimage centers for first- and second-order displacement functions are shown in Fig. 3.19. Likewise, the small strain errors are observed for the first-order displacement functions and increase slowly towards larger subimage and a . The second-order displacement functions also exhibit a constant strain error of $5 \mu\epsilon$ due

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to residual interpolation bias [102].

As expected, the quadratic displacement field can be measured more accurately by second-order displacement functions. However, for other deformation cases, even uniaxial tensile deformation, second-order displacement functions cannot generally be expected to measure them more accurately, and require more than double computing time of image correlation compared with using first-order displacement functions [101].

Fig. 3. 20 Intensities of 8×8 pixels in a subimage: (a) before interpolation, (b) using bilinear interpolation, (c) using bicubic spline interpolation

3.7 Interpolation function

Subpixel displacement measurements by digital image correlation are attributed to the interpolation function implemented in the image correlation algorithm, because subimage intensity patterns are represented by continuous data after interpolation and thus the match of two subimages is not restricted to be implemented at the position of integer pixel. Different interpolation methods can be used to reconstruct the image intensity pattern. Fig. 3.20a-c shows the intensity patterns before interpolation, after

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bilinear interpolation, and after bicubic spline interpolation respectively.

Since different interpolation methods can be used in the image correlation algorithm, their influences on the accuracy of subpixel displacement measurements are required to be evaluated. H.W. Schreier et al. [102] have made numerical studies with two kinds of speckle patterns (see Fig. 3.21) and three interpolators including cubic polynomial, cubic spline and quintic spline. Fig. 3.21a represents the kind of digital images with normal contrast often from white light source, while Fig. 3.21b with high contrast often from laser source. From the two reference images, translated and stretched images were generated numerically. Digital image correlations were then performed on those images.

Fig. 3. 21 Two kinds of intensity patterns used in [102]: (a) normal contrast, (b) high contrast

They found the use of interpolation functions causes the position-dependent bias on subpixel displacement measurements, and the results from simulated translation test and simulated stretch tests are consistent. As an example, the errors on subpixel displacement measurements in simulated translation tests caused only by interpolation were shown in Fig. 3.22. The results show that higher order interpolation is better for normal contrast images, while lower order interpolation is better for high contrast

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images, which agrees well with the results in literature [3] that bilinear interpolation caused less errors than higher order polynomial fit when image correlation were performed on high contrast speckle patterns. High order interpolation tends to over smooth the intensity data of high contrast images.

Fig. 3.22 Position-dependent errors [102] in simulated translation tests with different interpolation functions and with: (a) normal contrast images, (b) high contrast images.

3.8 Summary

A comparison of DIC in the spatial domain and the spectral domain shows that the former has advantages over the latter from accuracy and efficiency view point. The DIC in the spatial domain achieved subpixel accuracy and fast search for the maximum of correlation coefficient profile through the use of displacement function,

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interpolation function and Newton-Raphson iteration method. Hence, if not specified, the DIC technique used in this thesis denotes a DIC method in the spatial domain.

The investigation on subimage quality index (δ) illustrated the importance of speckle pattern quality or specimen preparation for DIC analysis. The quantity (δ) of speckle pattern quality, as well as the discussion of specimen preparation, will facilitate the application of DIC analysis.

An examination of both studies on subimage size and displacement functions indicates that (1) the subimage size ranging from 33×33 pixels to 57×57 pixels is the best choice which is a balance of random errors, potential systematic errors and algorithm performance; (2) larger subimages can be used if adopted displacement functions can closely approximate or exactly represent the actual displacements to be measured.

Although systematic errors occur and increase along with larger subimage size when there is a mismatch between displacement functions and underlying deformation, first-order displacement functions are usually utilized in practical applications because (1) the implementation of first-order displacement functions requires less computing time than that of higher-order displacement functions; (2) it is difficult to determine the order of actual deformation; (3) higher-order displacement functions will not measure actual lower-order deformation more accurately; (4) subimages larger than 57×57 pixels are seldom used.

Concerning the interpolation function, the selection of high or low order interpolation depends on the characteristic of image speckle pattern. Low order interpolation (linear interpolation) is good for images of high contrast, and high order interpolation (such as cubic spline) is good for images of normal contrast.

CHAPTER 3. INVESTIGATION ON DIGITAL IMAGE CORRELATION TECHNIQUES

The investigations in Section 3.5 show smoothing techniques are needed for DIC analysis results. One approach to smooth the DIC deformation analysis in spatial domain is to introduce a finite element-based DIC framework for incorporating displacement continuity in an element-based DIC formulation, which is described in the next chapter.

Chapter 4. Finite element-based image correlation

This chapter presents the finite element formulation for a new digital image correlation method that will determine directly the complete, two-dimensional displacement field during the image correlation process on digital images. This method discretizes the entire interested image area into finite elements that are involved in a common image correlation process and so is referred to element-based image correlation.

4.1 Motivation and concept

A drawback of the existing subimage-based digital image correlation method is that the obtained displacement field has no requirement of inter-subimage continuity during the correlation process. For example, errors in DIC results as noted by Vogel [96] (see Fig. 4.1) can be observed in digital image correlation. In addition, the discrete displacement field data that is obtained directly from existing image correlation algorithms usually need to be improved through post-processing using smoothing techniques [92-94].

The above issue is addressed in this chapter by introducing a finite element framework into the digital image correlation process. The interested image area is discretized into finite elements, and the displacements of element nodes can be solved in a common image correlation process with the developed algorithms, the inter-elements continuity requirement can be satisfied and the underlying deformation field can be improved by implicitly adopting finite element smoothing technique. The details of such a finite element formulation for a DIC method are presented in this chapter. Results from a series of numerical studies are also presented to verify the developed software code and provide basic data regarding the accuracy achieved by

the element-based DIC method.

Fig. 4. 1 Deformation data error in displacement vector field [96]

4.2 Development of element-based image correlation

The finite element formulation for digital image correlation is discussed in this section. The method is referred to as an element-based digital image correlation method, or element-based DIC method in short. The element-based DIC method discretizes the interested image area into segments/elements linked with nodes (see Fig. 4.2). Nodal displacements are obtained by maximizing the least squares correlation coefficient between all elements in the reference image and corresponding elements in the deformed image.

Fig. 4. 2 Schematic of finite element-based image correlation: (a) reference image, (b) deformed image

4.2.1 Correlation coefficient and displacement functions

Because element-based digital image correlation is to involve all elements to be correlated at one time, Eq. (2.3b) could be modified as:

$$C = 1 - \frac{\sum_{e \in S} \sum_{sp \in e} [g(x, y) - h(x + u(x, y, \mathbf{p}^e), y + v(x, y, \mathbf{p}^e))]^2}{\sum_{sp \in S} g^2(x, y)} \quad (4.1)$$

where S is a set of all elements on the reference image; e represents any one of elements; S_p represents any single point in S at position (x, y) ; $\mathbf{p}^e$ is the element displacement vector.

As shown in Eq. (4.1), the relationship between a point in the reference image at (x, y) and its mapping in the deformed image at (x', y') could be described as follows:

$$\begin{aligned} x' &= x + u(x, y, \mathbf{p}^e) \\ y' &= y + v(x, y, \mathbf{p}^e) \end{aligned} \quad (4.2)$$

In finite element method [105-106], the displacements of a point in an element are functions (known as displacement functions of the element) of the displacements of the element nodes.

$$\begin{aligned} u(x, y, \mathbf{p}^e) &= \sum_{i=1}^m N_i(x, y) u_i \\ v(x, y, \mathbf{p}^e) &= \sum_{i=1}^m N_i(x, y) v_i \end{aligned} \quad (4.3)$$

where m is the number of nodes in the element where the point is located; $N_i(x, y)$ are shape functions of the element; u_i and v_i are the components of the displacement at the i th node. Thus, the element displacement vector $\mathbf{p}^e = [u_1 \quad v_1 \quad \cdots \quad u_i \quad v_i \quad \cdots \quad u_m \quad v_m]^T$.

4.2.2 Determination of maximal correlation coefficient

Eq. (4.1) is used to find the optimum values for the displacement parameters. For tidiness, let K represent $1/\sum_{Sp \in S} g^2(x, y)$ because it can be considered as a constant.

Then, C can be written as:

$$C = 1 - K \sum_{e \in S} \sum_{Sp \in e} [g(Sp) - h(Sp, \mathbf{p}^e)]^2 \quad (4.4)$$

The maximum of C can be obtained by solving the following nonlinear equations,

$$\nabla C = \left(\frac{\partial C}{\partial \mathbf{p}_i^s} \right)_{i=1,2n} = 2K \sum_{e \in S} \mathbf{G}^T \left\{ \sum_{Sp \in e} (g(Sp) - h(Sp, \mathbf{p}^e)) \frac{\partial h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_j^e} \right\}_{j=1,2m} = \mathbf{0}. \quad (4.5)$$

where the global displacement vector $\mathbf{p}^s = [u_1 \ v_1 \ u_2 \ v_2 \ \cdots \ u_i \ v_i \ \cdots \ u_n \ v_n]^T$; n is the total number of nodes; $\mathbf{G}$ is known as nodal assembly matrix. The relationship between the element displacement vector $\mathbf{p}^e$ and the global displacement vector $\mathbf{p}^s$ is

$$\mathbf{p}^e = \mathbf{G} \mathbf{p}^s. \quad (4.6)$$

The size of matrix $\mathbf{G}$ is $2m \times 2n$. The determination of $\mathbf{G}$ can be illustrated by considering a finite element mesh consisting of two quadrilateral elements (see Fig. 4.3). The numbers inside the mesh are the local indices of element nodes; those outside are the global node indices. For instance, the matrix $\mathbf{G}$ for element 1 in Fig. 4.3 is

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4.7)$$

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Fig. 4. 3 Determination of nodal assembly matrices using the correspondence between global and element nodes

Newton-Raphson iteration is used to find the roots of Eq. (4.5). This method uses the derivative of a function to improve an approximation to the actual root. The Newton-Raphson iteration formula is given by:

$$\nabla^2 C(\mathbf{p}_{old}^s)(\mathbf{p}_{new}^s - \mathbf{p}_{old}^s) = -\nabla C(\mathbf{p}_{old}^s) \quad (4.8)$$

where $\mathbf{p}_{old}^s$ denotes an initial guess or previous approximation of the global displacement vector $\mathbf{p}^s$ to the root; $\mathbf{p}_{new}^s$ the next approximation to the root. The Hessian matrix $\nabla \nabla C(\mathbf{p}_{old}^s)$ can be determined by

$$\nabla^2 C = \left(\frac{\partial^2 C}{\partial \mathbf{p}_i^s \partial \mathbf{p}_j^s} \right)_{\substack{i=1,2n \\ j=1,2n}} = \sum_{e \in S} \mathbf{G}^T \left\{ \begin{array}{l} 2K \sum_{Sp \in e} \left(g(Sp) - h(Sp, \mathbf{p}^e) \right) \frac{\partial^2 h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_i^e \partial \mathbf{p}_k^e} \\ -2K \sum_{Sp \in e} \frac{\partial h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_i^e} \frac{\partial h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_k^e} \end{array} \right\}_{\substack{l=1,2m \\ k=1,2m}} \mathbf{G}. \quad (4.9)$$

Following Vendroux and Knauss [5], when $\mathbf{p}^s$ is close to the solution, consider

$g(S_p) \approx h(S_p, \mathbf{G}\mathbf{p}^s)$ and this leads to an approximation to the Hessian matrix

$$\frac{\partial^2 C}{\partial \mathbf{p}_i^s \partial \mathbf{p}_j^s} \approx 2K \sum_{e \in S} \mathbf{G}^T \sum_{Sp \in e} \frac{\partial h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_i^e} \frac{\partial h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_k^e} \mathbf{G}. \quad (4.10)$$

Given the displacement functions and the bicubic intensity interpolation from the deformed image, the first-order partial derivatives of the intensity with respect to the displacement parameters are readily derived

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$$\frac{\partial h(Sp, \mathbf{p}^e)}{\partial \mathbf{p}_k^e} = \frac{\partial h(x', y', \mathbf{p}^e)}{\partial x'} \frac{\partial x'}{\partial \mathbf{p}_k^e} + \frac{\partial h(x', y', \mathbf{p}^e)}{\partial y'} \frac{\partial y'}{\partial \mathbf{p}_k^e} . \quad (4.11)$$

The $\frac{\partial h(x', y', \mathbf{p}^e)}{\partial x'}$ and $\frac{\partial h(x', y', \mathbf{p}^e)}{\partial y'}$ terms can be derived from the bicubic interpolation polynomial Eq. (2.4) of grayscale values in the deformed image and the $\frac{\partial x'}{\partial \mathbf{p}_k^e}$ and $\frac{\partial y'}{\partial \mathbf{p}_k^e}$ terms from Eq. (4.2-4.3).

When the solution to $\mathbf{p}^s$ is obtained, that is, the displacements of all element nodes are determined, the strain components and rotation angle can be calculated from the known displacement field [106].

Initial guesses for the nodal displacements of the set S are required for the correlation process. The Newton-Raphson iteration formula with the approximate Hessian matrix [5] has the convergence radius of 7 pixels, which can be confirmed in our studies. Hence, an initial guess for the nodal displacements of any node is within ± 5 pixels of the actual displacements to ensure convergence. The initial guess of the global displacement vector can be initially zero or a constant for the small deformation case without rigid body translation or with large rigid body translation. In the case of large rotation or the deformation between correlated images larger than the convergence range, the initial $\mathbf{p}_{old}^s$ can be estimated by performing interpolation with user-defined displacement guesses for four or more arbitrary nodes distributed in the set S .

4.3 Verification and comparison

To verify the correctness and accuracy of element-based DIC method and compare it with the existing subimage-based DIC method (Section 2.6.1), both DIC software codes were written around the above-mentioned scheme. Both codes

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implement the two-dimensional correlation method with bicubic spline interpolation and with the approximate Hessian matrix formulation for the least squares correlation coefficient. Element-based DIC code uses rectangle elements with four nodes in order to approach to the displacement functions and the shape of subimages used in subimage-based DIC method. Although element-based DIC method has potential to adopt arbitrary shape and higher order elements, this work is to explore the advantage of element-based DIC method over subimage-based DIC method by meeting the displacement continuity among elements.

All images used in this work have the size of 640×480 pixels and are of 8-bit intensity data. 32×32 pixels were used as the subimage size and the step size during subimage-based image correlations to output results over fixed 16×11 array points above-mentioned. Correspondingly, the fixed 16×11 array points were selected as element nodes with the rectangular element size of 32×32 pixels during element-based image correlations. Therefore, the comparisons between subimage-based method and element-based method are based on the same data points.

With regard to the element-based image correlation process, it is found that the correlation errors of computed nodal displacements at the boundary are somewhat larger than those in the interior. That is because of the lack of adjoining elements to impose continuity constraints at the boundary. There are two approaches to eliminate this effect. One is to discard the results at the boundary; the other is to perform the second correlation using boundary elements with nodal displacements at the boundary as the unknowns and those inside the boundary as the knowns. The former was used in this work. Thus, above-mentioned 16×11 array points are interior nodes in the element-based correlation process.

4.3.1 Numerical studies

Simulated deformed images were employed to validate and evaluate two DIC codes since simulated images have the advantage to eliminate assessing instrumentation errors. Based on a reference image (Fig.4.4a), deformed images (e.g. Fig. 4.4b), with imposed translation, stretch, shear or rotation, were numerically generated by assigning intensity data from the reference image to the deformed image in corresponding positions, that is $h(x', y') = g(x, y)$. Taking the deformed image with imposed translation $u_0 = 0.5$ pixel only in $+x$ direction for example, a utility was written to new an image with its intensity data $h(x', y') = g(x' - 0.5, y')$.

Fig. 4. 4 Images used in numerical studies: (a) reference image, (b) image encoded with shear deformation

Table 4.1~4.3 show the results of element-based and subimage-based DIC codes using numerically translated images, stretched and skewed images, and the rotated image, respectively. A comparison of element-based DIC method and subimage-based DIC method results indicates that the element-based DIC method has much smaller standard deviations of strain components (Table 4.1-4.3) and rotation angle (Table 4.3) whereas the subimage-based DIC method has smaller variations in displacement data (Table 4.1).

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Table 4. 1 Results of simulated rigid body translations

Cases	Method	$\bar{u} - u_0$ (pixels)	$\bar{v} - v_0$ (pixels)	σ_u (pixels)	σ_v (pixels)	$\sigma_{\varepsilon_{xx}}$	$\sigma_{\varepsilon_{yy}}$	$\sigma_{\varepsilon_{xy}}$
Case 1	Element	8.39e-4	9.68e-3	10.73e-3	12.02e-3	2.23e-4	2.22e-4	2.97e-4
	Subimage	1.13e-4	8.37e-3	7.75e-3	7.99e-3	7.35e-4	7.36e-4	9.57e-4
Case 2	Element	8.94e-4	9.10e-4	8.88e-3	10.37e-3	1.90e-4	1.89e-4	3.22e-4
	Subimage	5.93e-4	-1.1e-4	7.09e-3	6.83e-3	7.80e-4	7.02e-4	9.82e-4

Case 1: $u_0 = 0.5 \text{ pixel}$ $v_0 = 0 \text{ pixel}$ Case 2: $u_0 = 0.5 \text{ pixel}$ $v_0 = 0.5 \text{ pixel}$ **Table 4. 2** Results of simulated uniform deformations

Cases	Method	$\bar{\varepsilon}_{xx} - \varepsilon_{xx}^0$	$\bar{\varepsilon}_{yy} - \varepsilon_{yy}^0$	$\bar{\varepsilon}_{xy} - \varepsilon_{xy}^0$	$\sigma_{\varepsilon_{xx}}$	$\sigma_{\varepsilon_{yy}}$	$\sigma_{\varepsilon_{xy}}$
Case 1	Element	-2.80e-5	-1.04e-6	-2.71e-6	3.57e-4	1.39e-4	2.72e-4
	Subimage	-1.3e-4	-8.37e-5	5.49e-6	1.96e-3	7.21e-4	8.22e-4
Case 2	Element	-2.43e-5	-2.05e-5	-8.67e-6	3.70e-4	3.88e-4	3.43e-4
	Subimage	-1.10e-4	-5.84e-5	-5.15e-5	1.93e-3	2.23e-3	1.08e-3
Case 3	Element	5.00e-7	3.98e-6	2.60e-7	2.38e-4	1.99e-4	2.89e-4
	Subimage	1.96e-4	3.37e-4	-2.1e-4	7.29e-4	7.55e-4	1.09e-3

Case 1: $\varepsilon_{xx}^0 = 0.0158730$ $\varepsilon_{yy}^0 = 0$ $\varepsilon_{xy}^0 = 0$ Case 2: $\varepsilon_{xx}^0 = 0.0158730$ $\varepsilon_{yy}^0 = 0.0169492$ $\varepsilon_{xy}^0 = 0$ Case 3: $\varepsilon_{xx}^0 = 0$ $\varepsilon_{yy}^0 = 0$ $\varepsilon_{xy}^0 = 0.0174533$ **Table 4. 3** Results of simulated rigid body rotation

Method	$\bar{\theta} - \theta_0$ (radians)	σ_{θ} (radians)	$\sigma_{\varepsilon_{xx}}$	$\sigma_{\varepsilon_{yy}}$	$\sigma_{\varepsilon_{xy}}$
Element	2.14e-4	2.50e-4	3.42e-4	3.19e-4	4.56e-4
Subimage	4.57e-4	6.80e-4	8.96e-4	9.13e-4	1.33e-3

 $\theta_0 = 0.0174533$

Fig. 4.5 and 4.6 graphically demonstrate the uniform strain varies of element-based method and subimage-based method respectively from Case 1 of Table 4.2. The large variations in strain shown in Fig. 4.6 accounts for that additional smoothing process of full-field displacements is often needed to obtain the displacement gradients when using subimage-based correlation method. In addition, corresponding standard deviations of Case 1 and Case 2 in Table 4.1 are at the same level even though Case 1 and Case 2 have different imposed translation. It is also true for a comparison of Case 1, 2 and 3 in Table 4.2.

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Fig. 4. 5 Variation in strain ϵ_{xx} from element-based method Fig. 4. 6 Variation in strain ϵ_{xx} from subimage-based method

4.3.2 Rotation experiment

A rotation experiment under controlled conditions was also performed to evaluate the two DIC codes. The rotation experiment was selected because it is not necessary to do CCD camera calibration, to evaluate the calibration error, or to consider the rigid body translation resulted from vibration.

The specimen was mounted on a rotation stage (see Fig. 4.7) with an angular resolution of 0.01 deg ($1.745\text{e-}4$ radians). Two images were acquired before and after

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the rotation angle change of 1 deg. Table 4.4 presents the element-based and subimage-based correlation results. The results show that the rotation error is 0.011 deg for the element-based correlation, and 0.027 deg for the subimage-based correlation. Comparison of the results in Table 4.3 and Table 4.4 shows that intensity noise slightly increases standard deviations in rotation angle and strains when using subimage-based correlation, but not when using element-based correlation.

Fig. 4. 7 DIC specimen mounted on a high-resolution rotation stage

Table 4. 4 Results of rigid body rotation experiment

Method	$\bar{\theta} - \theta_0$ (radians)	σ_{θ} (radians)	$\sigma_{\epsilon_{xx}}$	$\sigma_{\epsilon_{yy}}$	$\sigma_{\epsilon_{xy}}$
Element	1.95e-4	2.27e-4	3.04e-4	3.20e-4	4.54e-4
Subimage	4.72e-4	7.07e-4	1.04e-3	1.05e-3	1.62e-3

$$\theta_0 = 0.0174533$$

4.3.3 Summary

In summary, both the numerical studies and experimental work have preliminarily shown that (a) the proposed method is accurate and (b) the existing subimage-based image correlation method has 3~4 times as variation in the strains and rotation angle as the element-based method although the latter has slightly larger variation of displacement data in translation measurements. Such outcome can only be

attributed to the fact that the element-based image correlation method enforces displacement and/or displacement gradient continuity requirements among image elements.

4.4 Software development

In the work above, the image element meshes were implicitly generated. To make the method useful in practice would require the coding of a mesh generator, as well as post-processing of displacement and strain data. Hence, an application with graphic user interface was developed with Microsoft Visual C++ for the element-based image correlation analysis. The application was named Microdisc.

Fig. 4. 8 Architecture diagram of Microdisc

Fig. 4.8 shows the architectural overview of the software Microdisc. A user is to load images into Microdisc, mesh the image region of interest, and then specify the settings for running digital image correlation. After image correlation the user can view the displacement and strain data in multiple visualization styles and save the visualization figures in Bitmap format. In addition, displacement and strain data can be exported from Microdisc in ASCII text and Microsoft Excel formats. Microdisc project file will record the path of loaded image files and Microdisc data files, image

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meshes, and latest settings for running image correlation. The project file can be opened in Microdisc later. The core part of Microdisc is the module of “Image Correlation”. This module is to perform the image correlation and the output is stored in binary data format of Microdisc.

The graphic user interface of Microdisc has many of the familiar control elements found in other Windows applications. Fig. 4.9 shows the graphic user interface of Microdisc, where the meshes were generated and shown on the reference image. The most commonly used functions can be accessed by clicking buttons on the Toolbar. The panel of ‘Filelist View’ on the left of the main window provides a quick overview of input and output files. The ‘View Tab’ control at the bottom of window is a simple means to show which views/documents are currently open and to switch easily between them. The view tabs are also labeled by the view icons which represent different types of document views. The tab control is smart enough to reflect all operations to the views, such as view open and close.

Fig. 4.9 Graphic user interface of Microdisc with a mesh generator

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Fig. 4.10 shows the visualization of resultant displacement and strain data in Microdisc. The resultant displacement field can be rendered as vector and as deformed mesh. The two displacement components and three strain components can be visualized by banded contour, as well as isoline contour. A three dimensional surface plot of displacement and strain data will be ready in future.

Fig. 4. 10 Displacement and strain data visualization in Microdisc

Microdisc stores its data in binary file format, thus only Microdisc can read its data files properly. Another reasons using binary file format are including performance and efficiency consideration on data reading and writing, and the prevention of incautious changes to the resultant data. Fig. 4.11 shows a list view of resultant data at image element nodes in Microdisc. The displacement and strain data are listed in the order of node number. The Cartesian coordinates of nodes on the reference image are also recorded and associated with node displacement and strain data. Microdisc resultant data can be exported to ASCII text and Microsoft Excel files, which are popular recognized data files for third-party applications of data processing, such as Matlab and Microcal Origin.

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Fig. 4. 11 List view of exportable resultant data in Microdisc

Chapter 5. Characterization of near crack-tip deformation in solder alloy

Using the element-based image correlation technique, near crack-tip surface deformation was investigated for a stationary crack in SnCu solder alloy. The crack-tip region of about $2\text{ mm} \times 3\text{ mm}$ on a compact tension (CT) specimen was observed in-situ by a long working-distance microscope and a CCD camera under monotonic increasing loading. A sequence of digital images was acquired for deformation analysis by digital image correlation. Finite element modeling for fracture mechanics analysis of the crack-tip deformation was carried out using the actual loading conditions applied on the CT specimen. The rate-dependent plasticity of lead-free SnCu solder was modeled. The displacement and strain fields were calculated and compared to the DIC measurement results.

The displacement data at crack-tip region was used to determine the stress intensity factor (SIF) K_I at low loading conditions. The SIFs measured by DIC analysis were compared to the measurements from remote loads and the geometry and the predictions from finite element analysis.

As the solder alloy has a low melting point and exhibits time-dependent (creep) deformation at room temperature, Time-dependent fracture mechanics is applied to characterize the crack-tip behavior. The concepts of pseudo crack tip opening displacement (CTOD) and generalized J integral are introduced. Their values at different loads were calculated. The relationship between pseudo CTOD and generalized J integral was discussed in this study.

5.1 Fundamentals of crack-tip deformation field

Fracture mechanics concepts are well developed from linear elastic fracture mechanics (LEFM), to elastic-plastic fracture mechanics, and time-dependent fracture mechanics [107]. Stress intensity factor, J -integral, and crack tip opening displacement are important crack-tip deformation parameters investigated in this chapter.

5.1.1 K -controlled fracture

In linear elastic fracture mechanics (LEFM), the stresses near the crack tip in a linear elastic material vary as $1/\sqrt{r}$. In the cases of nonlinear materials, LEFM is still valid as long as the size of the crack-tip plastic zone becomes small relative to key dimensions such as the crack size, or a singularity dominated zone is present outside of the small plastic zone. The stress intensity factor K defines the amplitude of the singularity. In other words, one can completely define the stresses, strains and displacements in the singularity-dominated zone if the stress intensity factor is known. The detailed expressions [107] for crack-tip displacement fields for the opening mode (Mode I) of loading are as follows:

$$\begin{aligned} u(r, \theta) &= \frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right) \left[\kappa - 1 + 2 \sin^2\left(\frac{\theta}{2}\right) \right] \\ v(r, \theta) &= \frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \sin\left(\frac{\theta}{2}\right) \left[\kappa + 1 - 2 \cos^2\left(\frac{\theta}{2}\right) \right] \end{aligned} \quad (5.1)$$

where K_I is the stress intensity factor for Mode I loading; G is the shear modulus; $\kappa = 3 - 4\nu$ (plane strain, ν is Poisson's ratio) and $\kappa = (3 - \nu)/(1 + \nu)$ (plane stress); r , θ , and displacement components u , v are as illustrated in Fig. 5.1.

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Fig. 5. 1 Definition of the coordinate axis ahead of a crack tip

5.1.2 J -controlled fracture

The term J -controlled fracture corresponds to situations where J completely characterizes crack-tip conditions. Elastic-plastic fracture mechanics applies to materials that exhibit time-independent, nonlinear behavior; and the J contour integral describes crack-tip conditions in elastic-plastic materials. For these materials which exhibit rate-dependent deformation, linear elastic and elastic-plastic fracture mechanics, which assume quasistatic, rate-independent deformation, are inadequate. Recently, fracture mechanics has been extended to include time-dependent material behavior such as viscoplasticity and viscoelasticity based on the J contour integral.

From a continuum mechanics viewpoint, viscoplastic creep in metals is actually a special case of viscoelastic material behavior. While creep in metal is generally considered permanent deformation, the strains can recover with time in viscoelastic materials. Schapery [108-109] developed a generalized J integral applicable to a wide range of viscoelastic materials by forming an analogy with the nonlinear elastic case:

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$$J_v = \int_V \left(w^e dy - \sigma_{ij} n_j \frac{\partial u_i^e}{\partial x} ds \right) \quad (5.2)$$

where w^e is the pseudo-strain energy density:

$$w^e = \int \sigma_{ij} d\epsilon_{ij}^e \quad (5.3)$$

The stresses in Equation (5.2) are the actual values in the body, but the strains and displacements are pseudo elastic values. The pseudo strains and displacements are related to the actual strains ϵ_{ij} and displacements u_i by

$$\epsilon_{ij}^e = E_R^{-1} \int_0^t E(t-\tau, t) \frac{\partial \epsilon_{ij}}{\partial \tau} d\tau \quad \text{and} \quad u_i^e = E_R^{-1} \int_0^t E(t-\tau, t) \frac{\partial u_i}{\partial \tau} d\tau \quad (5.4)$$

where $0 < \tau < t$; E_R is a reference modulus; $E(t)$ is the relaxation modulus.

Fig. 5. 2 Failure zone at the crack tip in a viscoelastic material [108]

In fracture mechanics, the crack tip opening displacement (CTOD) is an alternative parameter to characterize crack-tip conditions. The two parameters are related. Schapery [108] derived a simplified relationship between J_v and CTOD by assuming a strip-yield-type failure zone ahead of the crack tip, where a closure stress σ_m acts over ρ , as illustrated in Fig. 5.2. While the material in the failure zone may be severely damaged and contain voids and other discontinuities, it is assumed that

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the surrounding material can be treated as a continuum. If σ_m does not vary with x , the J_v – CTOD relationship is given by

$$J_v = \sigma_m \delta^e \quad (5.5)$$

where δ^e is the pseudo crack tip opening displacement, which is related to the actual CTOD through a hereditary integral of the following form

$$\delta^e = E_R^{-1} \int_0^t E(t-\tau, t) \frac{\partial \delta}{\partial \tau} d\tau \quad (5.6)$$

5.2 Fracture experiment by DIC technique

The fracture experiment was conducted on a compact tension specimen and the crack-tip region was in-situ observed by the imaging tool of digital image correlation setup. The images of crack-tip region were then used for deformation analysis by the element-based DIC method.

Fig. 5. 3 Dimensions of the compact tension specimen

5.2.1 Specimen and material properties

The compact tension specimen for the fracture test was machined from solder alloy block as shown in Fig. 5.3. The notched fracture specimen was machined perpendicular to the rolling direction. The fatigue precracking was done at room

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temperature. Initial total crack length to specimen width ratio a/W is 0.45. The specimen width W is 20 mm, and the specimen thickness B is 5 mm.

The specimen material is 99.3Sn-0.7Cu solder, one of lead free solder alloys which are recommended as the substitutes for traditional tin-lead solders by the National Electronic Manufacturing Initiative (NEMI) [110].

The SnCu solder exhibits rate-dependent plasticity behavior. The elastic modulus, yield stress, and ultimate tensile strength (UTS) vary with strain rate, as well as temperature. Table 5.1 shows the strain rate dependent properties [111] for the bulk Sn-0.7Cu solder at the temperature of 25 °C. The Poisson's ratio ν of the material is 0.35.

Table 5. 1 Rate-dependent mechanical properties for SnCu solder

Strain rate	E (GPa)	σ_y (MPa)	σ_{UT} (MPa)
5.6×10^{-4}	20.2	31.1	35.8
5.6×10^{-3}	25.6	36.5	45.0
5.6×10^{-2}	32	47.8	52.0

The material constitutive law of SnCu solder can be described by Anand viscoplastic model [112, 113]. The Anand model consists of a set of equations which account for the physical phenomena of strain-rate and temperature sensitivity, strain rate history effects, strain hardening and the restoration process of dynamic recovery.

The internal variable s represents an averaged isotropic resistance to macroscopic plastic flow offered by the underlying isotropic strengthening mechanisms such as dislocation density, solid solution strengthening, subgrain and grain size effects, etc. The deformation resistance s is proportional to the equivalent

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stress σ .

$$\sigma = c \cdot s; \quad c < 1 \quad (5.7)$$

where c is a material parameter and constant in the constant strain rate test, which is defined as

$$c = \frac{1}{\xi} \sinh^{-1} \left[\left(\frac{\dot{\epsilon}_p}{A} e^{Q/RT} \right)^m \right] \quad (5.8)$$

where ξ is the multiplier of stress, $\dot{\epsilon}_p$ is the inelastic strain rate, A is the pre-exponential factor, Q is the activation energy, R is the gas constant, T is the absolute temperature, and m is the strain rate sensitivity, respectively.

The following functional form of the flow equation was obtained by combining Eq. (5.7) and Eq. (5.8)

$$\dot{\epsilon}_p = A \exp \left(-\frac{Q}{RT} \right) \left[\sinh \left(\xi \frac{\sigma}{s} \right) \right]^{1/m}. \quad (5.9)$$

The evolution equation for the internal variable s is assumed to be of the form as

$$\dot{s} = g(\sigma, s, T) \dot{\epsilon}_p. \quad (5.10)$$

where the function $g(\sigma, s, T)$ is associated with strain hardening and dynamic recovery. A simple form of evolution equation of Eq. (5.10) was given by Anand [112] as follows:

$$\dot{s} = \left\{ h_0 \left| 1 - \frac{s}{s^*} \right|^a \cdot \text{sign} \left(1 - \frac{s}{s^*} \right) \right\} \cdot \dot{\epsilon}_p; \quad a > 1 \quad (5.11)$$

$$\text{with } s^* = \hat{s} \left[\frac{\dot{\epsilon}_p}{A} \exp \left(\frac{Q}{RT} \right) \right]^n \quad (5.12)$$

where h_0 is the hardening/softening constant, a is the strain rate sensitivity of hardening/softening. s^* represents a saturation value of s associated with a set of

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given temperature and strain rate as shown in Eq. (5.12). $\hat{s}$ is a coefficient, and n is the strain rate sensitivity for the saturation value of deformation resistance.

From the above viscoplastic Anand model, there are nine material parameters: A , Q , ξ , m , h_0 , $\hat{s}$, n , a , and s_0 , the last one is the initial value of the deformation resistance, which is needed to determine the evolution of deformation resistance in Eq. (5.11). The nine parameters were determined from tensile and creep tests and are given in Table 5.2.

Table 5. 2 Parameters in Anand model for Sn-0.7Cu [111]

Parameters	A (s ⁻¹)	Q/R (K)	ξ	m	$\hat{s}$ (MPa)	n	h_0 (MPa)	a	s_0 (MPa)
Values	9875	5276	2	0.176	65.2	/	/	1.03	33.8

5.2.2 Experimental setup and procedures

White paint and black particles were used to create a speckle pattern on the surface of the specimen (Fig. 5.4) so that digital images of good quality can be obtained for deformation measurements by digital image correlation technique. The specimen was then mounted on an INSTRON micro tensile tester with digitally imaging system as shown in Fig. 5.5. The region of interest around the specimen crack tip was observed by a long working distance microscope, which was connected to a CCD camera and a computer with a frame grabbing card and an image acquisition application. A positioning stage with 3 degrees of freedom was used to adjust the position of the microscope. The magnification and depth of field of the microscope can be controlled by the computer through the control unit and hardware drive software.

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Fig. 5. 4 Compact tension specimen with painted surface

Fig. 5. 5 Experimental setup (left) and mounted specimen (right) for fracture test

The fracture experiment was performed at room temperature in an air environment. The experimental procedures were as follows:

- 1) Mount the compact tension specimen on the micro tensile tester.
- 2) Use the positioning stage to adjust the microscope position and locate the crack tip of the specimen; Adjust the magnification and depth of field of the microscope and thus bring the region of interest around the crack tip into focus. The size of the region was about $2\text{mm} \times 3\text{mm}$, and centered around the crack tip in this test.

- 3) Use load-line displacement control to load the fracture specimen. The applied loading rate was 0.37 mm/min .

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- 4) Start to load the specimen and acquire the images. The imaging of the crack tip region was captured at a frequency of 1 hertz.
- 5) Steadily rotate the knob of the positioning stage to move the microscope in the direction of specimen load-line displacement, which is to approximately maintain the position of the crack tip in the image during the experiment. The fracture test was stopped arbitrarily after 35 seconds. No crack growth was observed before the test was terminated. The force and load-line displacement curve was shown in Fig. 5.6. A total of 36 images, 480×640 pixels in size, were taken and stored.
- 6) Calibrate the magnification factor of the imaging system used in the experiment, which was 214 pixel/mm .

Fig. 5. 6 Loading curve from the fracture experiment

5.3 Finite element analysis of SnCu CT specimen

A finite element fracture analysis of the CT specimen shown in Fig. 5.3 was performed using actual load conditions to estimate displacement and strain fields. The

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analyses were performed using the ANSYS 10.0 finite element program. In order to compare with the experimental measurement, the specimen was full modeled and analyzed. The origin of the coordinate system was placed at the crack tip ($a/W = 0.45$). As shown in Fig. 5.7, the mesh contained a total of 2038 eight-node viscoplastic plane strain elements, with 6354 nodes and 12708 degrees of freedom. Near the crack tip region (Fig. 5.7b) which is corresponding to the region of interest in the fracture experiment, the mesh has 792 elements. A circular ring of quadrilateral elements at the crack tip are degenerated into triangles with the mid-side nodes placed at the quarter points. Such elements are called singular elements [114] which result in a $1/\sqrt{r}$ strain singularity for elastic problems. The element size at the crack tip is 0.02 mm .

Fig. 5. 7 Finite element meshes in the full model (a) and near crack-tip region (b)

The zero displacements in y direction were imposed on the symmetrical surface of the uncracked ligament. Rigid body translations were eliminated by fixing the crack tip, that is, $(u, v) = (0, 0)$ at $(x, y) = (0, 0)$. The force is distributed and

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applied to three nodes of two elements, which were marked in black in Fig. 5.7a. The elements on which the load acts were specified elastic instead of viscoplastic. By doing so, the load can be transferred to the body without wasting computer time solving a local indentation problem. The simulation was run under a total of 35 time-varying loads, which were extracted from the experimental load history.

Fig. 5. 8 Image mesh around a crack used in DIC analysis

Fig. 5. 9 Crack-tip v -displacement field with contributions of rigid-body motions

5.4 Results and discussions

Interpretation of the experimental results requires the analysis of displacement and strain at the crack tip. Digital images obtained from the fracture experiment were analyzed using the element-based image correlation program (see Chapter 4). Surface deformations were determined by correlating images of the deformed specimen to the reference image. The size of the analyzed region was 376×520 pixels as shown in Fig. 5.8 to ensure that the analyzed region was contained within the deformed images. The region of interest was meshed for element-based image correlation analysis. The

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meshes nearer to the crack tip become a bit denser to characterize higher gradients of displacements ahead of the crack tip.

5.4.1 Near crack-tip deformation fields

As shown in Fig. 5.9, the experimental displacement data from DIC analysis contained the contributions of rigid-body motions. Rigid body motions can be eliminated from experimental crack-tip displacement fields by (a) defining a coordinate system at the crack tip in order to eliminate rigid body translation and (b) assuming symmetry for the u -field and anti-symmetry for the v -field and calculating a rotation angle about the crack tip to eliminate rigid body rotation. This correction procedure was implemented by fitting the experimental displacements in the following forms (see Appendix A):

$$u(r, \theta) = C_u r^{1/2} \cos\left(\frac{\theta}{2}\right) \left[1 - 2\nu + \sin^2\left(\frac{\theta}{2}\right) \right] - \phi_u r \sin(\theta) + u_0 \quad (5.13a)$$

$$v(r, \theta) = C_v r^{1/2} \sin\left(\frac{\theta}{2}\right) \left[2 - 2\nu - \cos^2\left(\frac{\theta}{2}\right) \right] + \phi_v r \cos(\theta) + v_0 \quad (5.13b)$$

In the two equations, the first right-hand terms represent respectively crack-tip u - and v -displacement fields for Mode I loading and plane strain condition (refer to Eq. (5.1) and Fig. (5.1) for the definition of r , θ , and ν , where C_u and C_v are constants); the second right-hand terms represent displacement contributions of rigid-body rotation, where ϕ_u and ϕ_v are rotation angles, and equal theoretically; the last right-hand terms, u_0 and v_0 , represent displacement contributions of rigid-body translation. All experimental displacement data were exported to the MATLAB program and least-squares fitted using the Eq. (5.13) to solve for the six

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constants. The experimental displacements, $\tilde{u}(r, \theta)$, and $\tilde{v}(r, \theta)$, were then corrected by subtracting the contributions of rigid-body motions.

$$u'(r, \theta) = \tilde{u}(r, \theta) + \phi_u r \sin(\theta) - u_0 \quad (5.14a)$$

$$v'(r, \theta) = \tilde{v}(r, \theta) - \phi_v r \cos(\theta) - v_0 \quad (5.14b)$$

where $u'(r, \theta)$ and $v'(r, \theta)$ represent the corrected displacement components at the position (r, θ) in x and y directions respectively.

The MATLAB program was used to present the corrected displacement fields in full-field contour plots, as well as all in-plane strain fields, which nevertheless were not influenced by rigid-body motions. Fig. 5.10 to 5.14 (pages 81-85) present the full-field distribution for the corrected displacement components u' and v' , and the strain components ϵ_{xx} , ϵ_{yy} , and ϵ_{xy} at four different loads ($P = 54.01N$, $P = 205.87N$, $P = 248.89N$, and $P = 287.59N$) during the test. For a stationary crack under the opening mode of loading, load-dependent increases in deformation are clearly shown in both the displacement fields and the strain fields; the maximum strain value for ϵ_{yy} increases from 0.0023 to 0.0425, when the load increases from 54.01N to 287.59N, which corresponds to a time change from 5 to 35 seconds.

In order to compare the experimental results with finite element solutions, finite elements at near crack-tip region were extracted from the full model to match the region of interest in the DIC analysis. It is noted that the first ring of singularity elements at the crack tip was taken away because the theoretical $1/\sqrt{r}$ singularity does not apply to the very near crack tip. Comparisons of the experimental and

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numerical results for all two displacement components and three strain components at two different loads are shown in Figs. 5.15-5.19 (pages 86-90).

Fig. 5. 10 U -displacement contour plots for CT specimen with a stationary crack

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Fig. 5. 11 V -displacement contour plots for CT specimen with a stationary crack

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Fig. 5. 12 Strain ϵ_{xx} contour plots for CT specimen with a stationary crack

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Fig. 5. 13 Strain ϵ_{yy} contour plots for CT specimen with a stationary crack

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Fig. 5. 14 Strain ε_{xy} contour plots for CT specimen with a stationary crack

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Fig. 5. 15 Comparison of near crack-tip u -displacement fields between experimental (left) and numerical (right) results

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Fig. 5. 16 Comparison of near crack-tip v -displacement fields between experimental (left) and numerical (right) results

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$P = 54.01N$

$P = 248.89N$

Fig. 5. 17 Comparison of near crack-tip strain ϵ_{xx} between experimental (left) and numerical (right) results

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$P = 54.01N$

$P = 248.89N$

Fig. 5. 18 Comparison of near crack-tip strain ϵ_{yy} between experimental (left) and numerical (right) results

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Fig. 5. 19 Comparison of near crack-tip strain ϵ_{xy} between experimental (left) and numerical (right) results

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The comparisons shown in Figs.5.15-5.19 are remarkably good, both for the displacement distribution patterns and the displacement quantities. Consider a constant absolute error in one dimensional displacement measurement at two points apart by DIC technique, the error for the strain (displacement gradient) decreases when the difference of the displacements at the two points increases. As the ranges of both u - and v -displacements increase with increasing load, a comparison of experimental and numerical results at the lower load is better than that at the higher load. It is the same reason that the experimental results of strain ϵ_{yy} are better than those of strain ϵ_{xx} since the range of v -displacements is larger than that of u -displacements. Large errors occurred to measured strains ϵ_{xx} and ϵ_{xy} , especially at the lower load, can be recognized by a detail comparison of experimental and numerical results shown in Figs. 5.17 and 5.19. For example, high strains appear behind the crack tip. It can be concluded that larger displacement errors happened at the location with high strain errors than elsewhere. The fact that the crack is to open under load makes the location around the crack susceptible to higher imaging errors and hence higher errors of displacement measurement by DIC technique. In addition, very near the crack tip, the strains predicted by the FEM model have higher gradients than those characterized by the DIC analysis, and the deviations between experimental results and finite element solutions for the extremes of three strain components, especially ϵ_{xy} , increase as the load increases with time. The observed deviations were partially caused by the limit of DIC analysis on mapping strains of very high gradients because the image element size in DIC analysis can not be too

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small in order to suppress the influence of random errors (see Chapter 3). The actual strain gradients should be overestimated by finite element solutions because the actual initial crack tip is not as sharp as that modeled in finite element analysis. Further studies are needed to understand this discrepancy comprehensively. For example, the higher magnification factor can be used to image the crack tip region and perform digital image correlation in order to examine the observed discrepancy.

5.4.2 Determination of stress intensity factor

At the initial stage of the fracture test, the load was low and the size of the plastic zone developed near crack tip at short times was limited. Therefore it is feasible to consider solder alloy as linear elastic behavior, in other words, to use stress intensity factor to characterize crack-tip conditions for the stationary crack at the beginning of the fracture test.

One method [115] to determine stress intensity factor is to use the theoretical equation (5.13b) to fit experimental v -displacement field. Compared with Eq. (5.1), the constant C_v in Eq. (5.13b) is equal to $K_I / (G\sqrt{2\pi})$. Therefore, the v displacement function of Eq. (5.13b) becomes:

$$v(r, \theta) = \frac{K_I}{G\sqrt{2\pi}} r^{1/2} \sin\left(\frac{\theta}{2}\right) \left[2 - 2\nu - \cos^2\left(\frac{\theta}{2}\right) \right] + \phi_v r \cos(\theta) + v_0 \quad (5.15)$$

where the shear modulus G is related to the elastic modulus E by $E/2(1+\nu)$. In this experiment, E and ν are taken as 20.2 GPa and 0.35 respectively considering that the load-line displacement rate of 0.37 mm/min in this fracture test is equivalent to the loading strain rate of 5.6×10^{-4} /s in the uniaxial tensile test. The coefficients K_I , ϕ_v , and v_0 were estimated by least squares fitting between theoretical and

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experimental displacements at image element nodes shown in Fig. 5.8. The least square fitting method determines the unknown coefficients by minimizing the total error between the predicted and measured values at all data points of interest. The minimization procedure in the least squares method is implemented by taking the partial derivatives of the total error with respect to the coefficients and setting them equal to zero. The total error for the whole field of v -displacement data are defined as:

$$ERR = \sum_{i=1}^n [v(r_i, \theta_i) - \tilde{v}(r_i, \theta_i)]^2 \quad (5.16)$$

where (r_i, θ_i) are polar coordinates of the i th data point at the crack-tip region.

It is noted that r and θ in equations above are measured from the crack tip; therefore, it becomes imperative to locate the crack tip precisely. In the analysis of the data, the fitting procedure was run repeatedly using different crack-tip positions. For each position, the total error summed from all data points was calculated. The optimum crack-tip position was determined to be the position which gave the minimum total error. An example is shown in Table 5.3 where K_I is for a load of 22.06 N. Using the above procedure, the K_I values at different loads can be computed and are shown in Table 5.4.

Table 5. 3 Determination of optimum crack-tip position at a load of 22.06 N

Crack tip position (pixels)		K_I (MPa $\sqrt{m}$)	Error ($10^{-12} m^2$)
Row	Column		
244	327	0.2552	7.89
245	327	0.2541	7.87
246	327	0.2528	7.90
245	326	0.2540	7.90
245	328	0.2540	7.89

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Table 5. 4 ASTM and fitted K_I values at different loads

Load (N)	ASTM K_I (MPa $\sqrt{m}$)	Fitted K_I (MPa $\sqrt{m}$)	% errors
11.06	0.1304	0.1216	-6.74
22.06	0.2602	0.2541	-2.34
32.88	0.3878	0.4084	5.32
54.01	0.6370	0.6777	6.39
74.46	0.8782	1.0243	16.63
94.23	1.1113	1.3534	21.78

In order to validate the estimation of K_I by DIC analysis, the quantities of K_I were also calculated using an expression in ASTM E 399-05 standard [116]. The formula from the standard is:

$$K_I = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right) \quad (5.17)$$

where

$$f\left(\frac{a}{W}\right) = \frac{\left(2 + \frac{a}{W}\right)}{\left(1 - \frac{a}{W}\right)^{3/2}} \left[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4 \right]$$

(See Fig.5.3 for definition of above variables).

Eq. (5.17) uses loading and specimen configuration for the experiment to calculate K_I whose values are also shown in Table 5.4. A comparison of fitted K_I vs ASTM K_I at different loads shows that (a) the K_I values from DIC analysis and ASTM agree well at the small loads, and (b) the fitted values increase faster along with the loads than the K_I values from ASTM. This discrepancy was caused by the increasing contribution of material plasticity and creep to the near crack tip deformation with the time and higher load. The discrepancy can also be confirmed by finite element analysis which calculated K_I values using a fit of the nodal

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displacements in the vicinity of the crack [114]. The comparison of K_I values from ASTM, DIC analysis, and finite element solutions is shown in Fig. 5.20.

Fig. 5. 20 Comparison of K_I values from ASTM, least squares fitting, and FEA

5.4.3 Relationship between CTOD and J integral

Eq. (5.5) describes the relationship between the generalized integral J_v and the pseudo crack tip opening displacement for viscoelastic materials. In the equation, the CTOD is defined as the displacement at the original crack tip. There are a number of alternative definitions of CTOD. The definition of CTOD presented here is the displacement at 0.5 mm behind the original crack tip. From the plastic hinge model [117], the CTOD used here is proportional to the CTOD in Eq. (5.5). Therefore, Eq. (5.5) can be modified as:

$$J_v = m\delta^e \quad (5.18)$$

where m is a constant with units of stress.

In the equation above, J_v can be obtained by using Eq. (5.2-5.4), while δ^e is related to the actual measured CTOD through Eq. (5.7). The determination of J_v and

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δ^e requires the determination of the bulk relaxation modulus $E(t)$. A tensile relaxation experiment was conducted to measure the relaxation modulus, which was obtained by measuring time-dependent stress at a fixed strain. The experimental data for the relaxation modulus at the constant strain of 0.5% are shown in Fig. 5.21 and fitted to the generalized Maxwell model in the form of $E(t) = E_\infty + \sum_i E_i e^{-t/\rho_i}$:

$$E(t) = 2.62483 + 0.8914e^{-t/5.39987} + 2.41993e^{-t/46.50308} \text{ GPa}. \quad (5.19)$$

Fig. 5. 21 Tensile relaxation modulus for solder alloy SnCu

In this work, the reference modulus E_R was arbitrarily set to equal the tensile relaxation modulus at $t=0$. The finite element solutions to the strains and displacements were used to compute their pseudo-elastic values through Eq. (5.4). The calculations were performed by numerical integration with a time step of 1 s. The predicted stresses, together with the pseudo strains and displacements are inserted into Eq. (5.2-5.3) to determine the generalized J_v . The J -integral contour path was the outline of the region of interest shown in Fig. 5.7b. A Macro was created for the calculation of the generalized J_v using ANSYS Parametric Design Language

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(APDL). Executing the Macro on finite element solutions of each load step gives J_v values at the end of each load/time step.

The pseudo crack tip opening displacement δ^e was calculated using the actual CTOD from DIC analysis. The integral equation (5.6) for calculating the pseudo CTODs can be expressed as:

$$\delta^e(t) = \sum_{\tau=0}^t \left[E(t-\tau) \times (\delta(\tau+1) - \delta(\tau)) \right] \quad t=1,2,3...35 \quad (5.20)$$

where $\delta(t)$ is the actual measured CTOD at time $t=1s, 2s, 3s...35s$ and $\delta(0)=0\mu m$. Once the generalized J_v and pseudo CTOD δ^e at the end of all time steps were obtained, their corresponding values were plotted in Fig. 5.22 where the data were linearly fitted to a line. As shown in Fig. 5.22, a good linear relationship is found between the pseudo CTOD and the generalized J integral if ignoring their nonlinearity at the initial stage. Small stresses at the initial stage accounted for this nonlinearity considering that the linear relationship was developed from the strip yield model (refer to Fig. 5.2).

Fig. 5. 22 Linear fitting between pseudo CTOD and generalized J integral

5.5 Summary

The experimental and data post-processing procedure for crack-tip deformation measurements by the DIC analysis was set up. Rate-dependent plasticity of SnCu solder alloy is modeled in fracture mechanics analysis. Comparisons of experimental results with numerical solutions were illustrated by means of contour plots of displacement and strain fields at the observed crack-tip region. Results from these comparisons indicate that the experimental and numerical displacement and strain data are in good agreement in the whole load history.

Stress intensity factors at low loading conditions were determined by DIC deformation analysis, ASTM formula, and finite element analysis. The results show the SIFs computed using the three approaches are in good agreement. The difference of SIFs calculated by DIC analysis and ASTM formula at any load less than 54.01 N is within $\pm 7\%$. Hence, it validated the expectation that viscoplastic solder alloy can be considered as linear elastic behavior at low stresses and short times. The agreement also verified the element-based DIC deformation analysis and the finite element analysis of crack-tip deformation field.

The pseudo crack tip opening displacement (CTOD) was calculated from the actual CTOD which was extracted from displacement fields measured by DIC analysis. The generalized J contour integral was calculated from finite element solutions to the crack-tip deformation fields. The linear relationship between pseudo CTOD and generalized J integral values was confirmed in this study.

Chapter 6. Thermo-mechanical deformation characterization

The measurement of in-plane full-field deformation is valuable to material characterization and reliability assessment. It can be used to calibrate and validate finite element models of microelectronic assemblies subject to thermal loading conditions. The application of digital image correlation technique to thermally induced strain measurements is an important contribution to electronic assemblies and solder joint reliability assessment. When a specimen undergoes thermal loading in a chamber the influences of critical factors, such as acquired image speckle quality, out-of-plane displacement, and rigid body translation and rotation due to thermal expansion must be considered. This chapter will present the calibration of the experimental setup used for thermally induced deformation measurement. The corresponding measures for error prevention and correction are developed. Case studies for the measurements of coefficient of thermal expansion (CTE) of a solder material and solder joint deformation pattern in a cross-sectioned PBGA assembly are reported in this chapter.

6.1 Calibration of experimental setup

Digital image correlation technique uses image speckle as the information carrier. The encoding of deformation information in digital image speckle is affected by experimental setup and test environment conditions. Hence, besides the influence of the DIC algorithm parameters to the deformation measurement accuracy, it is also important to examine the experimental setup and environmental effects. The displacement error from DIC deformation analysis can be within 0.01 pixel as shown earlier in Chapter 3 for simulated images (refer to Fig. 3.18 and Fig. 3.22). The

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displacement error for actual DIC experiments is several times of this ideal error due to the effects of signal and lighting variations. In this work, the effects of key factors in the DIC system on thermo-mechanical deformation measurements are evaluated. These key factors discussed here are DIC system errors, out-of-plane displacement, and rigid body motions.

6.1.1 DIC system errors

Digital image correlation measurement system is given by the loading stage under a computer vision system with digital image correlation software. For thermal-induced deformation measurements, a chamber or a heating stage is generally added instead of a mechanical loading stage. Fig. 6.1a shows the schematic illustration of the micro digital image correlation system with computer control functions. Fig. 6.1b shows the picture of the actual Micro-DIC system. The system consists of six main parts: a working table, a miniature heating and cooling chamber with a glass window, a three-degree-of-freedom motorized translation stage, an illuminator and a long working-distance microscope with controllable zoom and focus functions, a CCD (Charge-Coupled Device) camera for image acquisition, and a computer with system control software and image correlation software installed.

In this micro digital image correlation system, the system errors are a sum of optical errors from the chamber glass window, the long distance microscope, the CCD camera and environmental vibration, electronic errors from CCD camera and image acquisition card, and systematic errors [103-104] from digital image correlation algorithm. Although the system errors are complicated, a simple experiment can be designed to evaluate the system errors. Two images of a DIC specimen can be acquired at a static state without external loading or translation, then the correlation process can be applied to the two images, one as the reference image, the other as the

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Fig. 6. 1 Micro digital image correlation system: (a) schematic diagram, (b) photo of actual system

deformed image. The results of such experiments using the DIC system are shown in Fig. 6.2 and Fig. 6.3. Sample ranges of 0.11 and 0.19 pixel were observed respectively in the measurement system without and with the glass window in place. The comparison of Fig. 6.2 and Fig. 6.3 illustrates the optical performance of the chamber glass window, and the calibrated contributions to system errors. In order to examine the reproducibility and stability of the system errors, ten images were

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acquired in series and image correlation was performed using one of them as the reference image, the others as deformed images. The measured system errors by correlating the 9 images with the common reference image are shown in Table 6.1. The sample ranges and standard deviations of measured displacements are from 609 measured points in a region of interest 560×400 square pixels with the step of 20 pixels in x and y directions respectively. The results show the system errors are reproducible and stable among multiple measurements.

Fig. 6. 2 System errors of micro DIC system without chamber glass window

Fig. 6. 3 System errors of micro DIC system with chamber glass window

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Table 6. 1 System errors represented by sample range and standard deviation of displacements in multiple measurements (Unit: pixel)

No.	R_u	σ_u	R_v	σ_v
1	0.202	0.0287	0.155	0.0211
2	0.214	0.0322	0.158	0.0222
3	0.158	0.0268	0.151	0.0220
4	0.154	0.0252	0.166	0.0237
5	0.184	0.0324	0.221	0.0261
6	0.186	0.0288	0.169	0.0236
7	0.192	0.0317	0.145	0.0222
8	0.134	0.0211	0.118	0.0184
9	0.205	0.0317	0.156	0.0236

The fundamental cause of system errors is that if an arbitrary point of one image pixel size is captured multiple times at the same conditions by a camera, its gray value undulates. It is expected that the gray value undulates around its average value because the point is acquired at the same conditions. The average gray value can be obtained by sampling multiple times on the points. With this understanding, an attempt was made to reduce the system errors by applying an averaging scheme. The averaging scheme is to generate a reference image or a deformed image by averaging a series of image frames acquired at the same conditions. The image correlation procedure was then performed on the pair of generated images. The system errors in the images without averaging, with averaging 5 times and 10 times are presented in Table 6.2. A comparison between no averaging and averaged 5 or 10 times shows the averaging scheme is effective to reduce the system errors of the DIC system.

Table 6. 2 Comparison of system errors (Unit: pixel)

No.	R_u	σ_u	R_v	σ_v
1*	0.181	0.0287	0.160	0.0225
2*	0.069	0.0105	0.056	0.0093
3*	0.052	0.0077	0.050	0.0070

1* Mean values in Table 6.1

2* Images averaged 5 times

3* Images averaged 10 times

Fig. 6. 4 Artificial in-plane deformation due to out-of-place displacement

6.1.2 Influence of out-of-plane displacement

When the object is subjected to thermal loading in a chamber, the out-of-plane displacement will result from the out-of-plane thermal expansion of the specimen and the displacement that is transferred to the specimen. For a typical optical system, the out-of-plane displacement will lead to the artificial in-plane uniform deformation (see Fig. 6.4) observed by correlating two images captured before and after the out-of-plane movement. A typical lens has optical cone effect which are not perpendicular and thus produce parallax errors [118]. Consider Fig. 6.5, where all three locations (A , B and C) measures the same area, but in reality, are all of different sizes. The location B is the position of best focus. If Δz is denoted as the small distance between location B and another location, e.g. A , and the length of line segment A, B is L_A , L_B respectively, the artificial deformation ϵ^a can be expressed as

$$\epsilon^a = \frac{(L_B - L_A)}{L_B} = \frac{\Delta z}{(0.5L_B/\tan \alpha')} \quad (6.1)$$

where α' is the parallax angle (Fig. 6.5). The angle α' is related with optical magnification M and numerical aperture NA by $\sin \alpha' = NA/M$ [119]. The image

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size of either L_A or L_B is a constant L , and $L_B = L/M$. Since α' is a small angle $\tan \alpha' \approx \sin \alpha'$, Eq. (6.1) becomes

$$\varepsilon^a = \frac{2 \cdot \Delta z \cdot NA}{L}. \quad (6.2)$$

Eq. (6.2) shows the artificial strain is proportional to the distance Δz and the numerical aperture NA of the optical system used. It is noted that the NA of the OPTTEM optical system used increases along with the higher magnification. For example, the NA increases from 0.029 to 0.15 as the magnification increases from 1.6 to 19.6. Hence, the artificial strain due to out-of-plane thermal expansion will be notable at a high magnification. To show the artificial strain and verify Eq. (6.2), an experiment was done. 9 images of solder ball cross-section at a magnification of 0.78 $\text{pixel}/\mu\text{m}$ were acquired from 9 sequential focus steps. 5 among the 9 images were shown in Fig. 6.6a-e. Each situation represented the change from a defocused image through a focused image and then continuing to a defocused image.

Fig. 6. 5 Representation of a typical optical system [118]

To compare the focus of the image sequence, the focus function, called squared gradient [120-121]

$$F = \iint_{\text{image}} \left| \frac{\partial g(x, y)}{\partial x} \right|^2 dx dy \quad (6.3)$$

where $g(x, y)$ is the grey value at (x, y) , was used. The function for the image

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Fig. 6. 6 An image sequence (a –e) with different focus position. Image (c) was taken at the position of best focus.

sequence versus z position is given in Fig. 6.7. The image in the middle of the sequence was of best focus and its z position was denoted as zero in Fig. 6.7. The image of best focus was taken as the reference image, the others as deformed image. The deformation between the reference image and a deformed image was obtained by

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performing digital image correlation. The results are shown in Fig. 6.8. The artificial strain is almost linear with Δz , which agrees with the description of equation (6.2). The out-of-plane movement of $25\ \mu\text{m}$ shown in Fig. 6.8 has resulted in the artificial strain of about 0.1%.

Fig. 6. 7 Focus function normalized by the maximum function value for the image sequence

Fig. 6. 8 Artificial in-plane strain due to out-of-plane movement

To solve this issue, there are three methods: using telecentric optical system, maintaining the image-to-object distance by moving the microscope in imaging axis

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direction, and resultant strain correction combined with artificial strain calibration. A telecentric cone of rays emanating from an object point remains perfectly perpendicular to the object plane across the entire field-of-view. *A*, *B* and *C* in Fig. 6.9 will all appear the same size to the camera. The drawback of this method is that a telecentric microscope is much more expensive than a typical one. The key to the second method is the realtime maintainance of the image-to-object distance. The out-of-focus detection technique can be used to detect the occurrence of out-of-displacement and feed the information back to stage positioning controller, and the controller will give orders to move the microscope in *z* direction till the resumption of the best focus. This method requires a high-resolution stage for *z* positioning with a typical step size of no more than $1\ \mu\text{m}$. The requirement is beyond the capability of the existing motorized stage for *z* positioning with a step size of $5\ \mu\text{m}$. The system resorts to the internal focus modules for fine focusing. The third method of software correction is to subtract the artificial deformation from measured deformation. The calibration procedure is to build up the mapping of artificial deformation with image content change. For example, combining the Fig. 6.7 and Fig. 6.8, the focus function can be related to artificial strains. The disadvantage of this method is to require an extra calibration procedure during the experiment.

Fig. 6. 9 Representation of a telecentric optical system

6.1.3 Influence of rigid-body translation and rotation

Generally, the rigid body translation is ignorable in thermal deformation measurements at a low magnification, but it becomes important for the use of a high magnification and thus it is necessary to move the specimen or the microscope during thermal loading. Obviously, it is preferable to do it automatically. To automatically keep the inspected specimen surface in view, the center point of the initial image can be selected as an anchor point; its displacements can be monitored by performing realtime image correlation, and system positioning controller will give movement orders to the stage so that the anchor point can move back to the center position of the vision.

Fig. 6. 10 Overlapped u -displacement field due to thermal expansion and rigid body rotation

Rigid body rotation of the specimen do not bring along additional strains but displacements which will overlap with the displacements due to actual deformation. When a specimen is subjected to thermal loading in a chamber, an invisible rigid body rotation may come from the interaction between the specimen and its support. For example, an asymmetric support during thermal expansion will rotate the specimen unnoticeably. Fig. 6.10 shows a typical displacement field resulted from uniform

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thermal expansion and tiny rotation of the specimen. The contour bands lean because of the appearance of specimen rotation. In this case, it is not difficult to correct the measured displacement fields since the actual deformation mode is known. The similar case was presented in the previous chapter which has illustrated the removal of rotational displacements from the resultant crack-tip displacement fields. Rigid body rotation can also be detected by monitoring some special spots such as the specimen symmetry line. The rotational angle can then be figured out.

Since rigid body motions may degrade the accuracy of displacement measurements by DIC, it is very important to devise the sample fixture to minimize the rigid body motions.

6.2 Measurement of coefficient of thermal expansion (CTE)

Coefficient of thermal expansion (CTE) is one of important material properties for the thermal mechanical stress analysis of electronic packaging assemblies. Han and Guo [78] showed good reason for the importance of accurate measurements of CTE. The reason is that a minor error in CTE values of critical materials in electronic packages can make the predicted fatigue life of the solder joint much different. In this study, DIC method was applied to measure CTE of solder material. The measurement accuracy of CTE by DIC will be discussed and the result will be compared with that measured by a thermo-mechanical analyzer (TMA).

The following equation illustrates how CTE values are evaluated from deformations determined over a temperature interval ΔT .

$$\begin{aligned} CTE(ppm/^{\circ}C) &= \frac{\Delta L}{L\Delta T} \times 10^6 \\ &= \frac{\epsilon_{xx}}{\Delta T} \times 10^6 \quad \text{or} \quad \frac{\epsilon_{yy}}{\Delta T} \times 10^6 \end{aligned} \tag{6.4}$$

where ΔL is the dimension change of the gauge length L , ϵ_{xx} and ϵ_{yy} are strains in

horizontal/ x and vertical/ y directions respectively. CTEs in the two directions are the same for isotropic materials in theory, but not for anisotropic materials.

6.2.1 Measurement of CTE by DIC technique

Measurement was performed on a block specimen of lead-free solder material (SnAgCu) using micro DIC system. The specimen dimension is $10 \times 10 \times 4 \pm 0.1 \text{ mm}$, and polished with grind paper 1000. The temperature interval used in the measurement was 60°C . The out-of-plane displacement was calibrated by moving the microscope in the out-of-plane direction and resuming the position of best focus after the temperature ramp-up from 25°C to 85°C . Although there was probably an error $5 \mu\text{m}$ in the calibration of out-of-plane displacement due to the resolution of the stage, its influence on deformation measurements can be ignorable considering that the minimum optical magnification of the system, $0.093 \text{ pixel} / \mu\text{m}$, was used. After the calibration, 10 images at 85°C were captured. Then ramp down the temperature to 25°C , and move the microscope at the calibrated out-of-plane displacement. Another 10 images were captured. Both ten images were averaged to generate the reference image (Fig. 6.11a) at 25°C and the deformed image at 85°C (Fig. 6.11b).

Fig. 6. 11 Images used in CTE measurement: (a) reference image, (b) deformed image

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The displacement fields shown in Fig. 6.12a-b were obtained by correlating the two images. Linear fitting to the displacement fields generated constant strains in either direction. As such, ϵ_{xx} is 1.56×10^{-3} ; ϵ_{yy} 1.33×10^{-3} . CTEs are calculated by strains divided by ΔT . They are $26.0 \text{ ppm}/^\circ\text{C}$ and $22.1 \text{ ppm}/^\circ\text{C}$. The error of CTE measurement by DIC can be calculated by

$$e_{CTE} (\text{ppm}/^\circ\text{C}) = \frac{2\sigma_{\Delta L}}{L\Delta T} \times 10^6. \quad (6.5)$$

(a)

(b)

Fig. 6. 12 u -displacement (a) and v -displacement (b) fields in CTE measurement

Take CTE in x direction for example. The $\sigma_{\Delta L}$ in x direction, a correspondence of σ_u in Table 6.2, is 0.0077 pixel. The L in x direction, the width in pixel of the

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rectangular correlated image area, is 560 pixels. Thus, the measurement error of CTE in x direction is within $0.46 \text{ ppm}/^{\circ}\text{C}$. Likewise, the calculated error of CTE in y direction is within $0.58 \text{ ppm}/^{\circ}\text{C}$. In this case, the difference of measured CTEs in x and y directions did not caused by measurement errors, but because of the fabrication process of the specimen that was cut from a compact tension specimen.

Fig. 6.13 Curves of dimension change vs. temperature from TMA measurements: (a) in x direction, (b) in y direction

6.2.2 Measurement of CTE by thermo-mechanical analyzer

For a comparison, CTE measurements were also performed on the same specimen with TMA. It is important for the use of TMA that the specimen was ground to make two of its surfaces normal to the gauge length. Load the specimen with the gauge length along x direction; calibrate its gauge length and select temperature ramp-up rate $5^{\circ}\text{C}/\text{min}$ and temperature range from room temperature to 100°C . The curve of dimension change versus temperature is shown in Fig. 6.13a. Repeat the procedure above but load the specimen with the gauge length along y direction. Another curve of dimension change in y direction versus temperature was recorded as shown in Fig. 6.13b.

A linear curve fitting in Fig. 6.13 to data from 40°C to 100°C produced the CTE measured by TMA, which is $26.1\text{ ppm}/^{\circ}\text{C}$ in x direction, and $22.3\text{ ppm}/^{\circ}\text{C}$ in y direction. A comparison of CTEs measured by DIC and TMA has indicated that there are very little differences between them. Accordingly, DIC can be a tool for the accurate CTE measurement of materials.

6.3 Investigation of solder joint fatigue deformation

Micro-deformation and strain measurements by DIC techniques have been applied to electronic packaging applications with some success [16-21]. However, these studies focus on solder joint displacement fields induced by monotonic temperature change, instead of repeated thermal cycling loading. It is desirable to measure the accumulated deformations in solder joints during thermal cycling tests considering that low cycle thermal fatigue failure is one of the primary failure mode for solder joints. The moiré interferometry method was used to determine the average strains in solder joints of a BGA package subject to thermal cycling loading [58]. It

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was noted that the displacement accuracy of $0.417 \mu\text{m}/\text{fringe}$ was not adequate enough to obtain the detailed displacement fields in a solder joint subjected to thermal cycling. A higher displacement resolution can be achieved by using the Micro-DIC system, equipped with a long working-distance microscope. For example, if a solder joint with a physic width of $500 \mu\text{m}$ is filled in a digital image of 500 pixels wide, the displacement resolution by DIC deformation analysis is about $0.05 \text{ pixel} \times (500 \mu\text{m} / 500 \text{ pixels}) = 0.05 \mu\text{m}$. Hence, the accelerated temperature cycling test was conducted on an electronics assembly in which one of solder joints was in-situ observed under the Micro-DIC system.

6.3.1 Specimen and experimental procedure

The specimen used in the accelerated temperature cycling test is a BGA (ball grid array) assembly. The assembly was sectioned along the plane as shown in Fig. 6.14. The cross section of the BGA assembly is shown in Fig. 6.15. The sectioned surface was polished and etched so that a random pattern of sectioned solder joint can be observed under the imaging tool of DIC system. The solder joint just below the right edge of silicon chip (shown in Fig. 6.15) was chosen for this study. The specimen was put in the chamber with symmetrical support. The microscope magnification and focus was adjusted to bring the region of interest in clearest view by operating the system control software. The reference image of the solder joint captured at 25°C before the start of thermal cycling test is shown in Fig. 6.16, which is actually formed by averaging ten reference images. The specimen is subjected to thermal cycling between 25°C and 100°C with a cycle period of 1 hour. The cycle has three phases: a ramp-up time of 20 minutes, a hold time of 20 minutes at 100°C , and a ramp-down time of 20 minutes. As shown in Fig. 6.17, this temperature profile was set in the system control software which sent commands to the temperature controller to control

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the temperature inside the mini chamber. Deformed images for DIC analysis were acquired at the end of each cycle, that is, at 25°C . Since the reference image and deformed images were acquired at the same temperature, the effect of out-of-plane displacement on displacement measurement accuracy was eliminated. A total of 14 thermal cycles were monitored in this experiment. The calibration of used imaging resolution to the optical system was conducted at the end of the experiment. The imaging resolution used in the experiment is $1.295\ \mu\text{m}/\text{pixel}$, and thus the spatial resolution of the DIC system is $1.295\ \mu\text{m}/\text{pixel} \times 0.052\ \text{pixel} = 0.067\ \mu\text{m}$, where $0.052\ \text{pixel}$ is the displacement resolution of the DIC system.

Fig. 6. 14 Schematic of BGA and the position of cross-section plane

Fig. 6. 15 Half of the cross-section of BGA assembly

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Fig. 6. 16 Reference image of the solder joint in thermal cycling test

Fig. 6. 17 Temperature profile settings in the system control software

6.3.2 Experimental results

The reference image and deformed images were then correlated to obtain displacement fields using in-house DIC software, which implements element-based image correlation algorithms. The region of interest and image mesh for the DIC analysis is shown in Fig. 6.18. The resultant displacement fields at the end of the 7th and 14th temperature cycle are shown in Fig. 6.19 and Fig. 6.20 respectively. Both u

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fields in Fig. 6.19a and Fig. 6.20a show similar displacement contour patterns, and that is true for both v fields in Fig. 6.19b and Fig. 6.20b. Additionally, the ranges of displacement u and v in Fig. 6.20 are larger than those in Fig. 6.19, which indicates the effect of strain accumulation during thermal cycling. Globally, the u – displacement contours tend to be horizontal and v – displacement contours tend to be vertical, which led to significant displacement gradients $\partial u/\partial y$ and $\partial v/\partial x$ respectively. It is known that both shear deformation and rigid body rotation contribute displacement gradients $\partial u/\partial y$ and $\partial v/\partial x$. Because the components $\partial u/\partial y$ derived from the u – displacement field were generally positive while the components $\partial v/\partial x$ derived from v – displacement field negative, the opposite sign of the cross derivatives $\partial u/\partial y$ and $\partial v/\partial x$ indicates primary rotational contributions to the resultant displacement field.

Fig. 6. 18 Region of interest and image mesh on the solder joint for DIC analysis

The displacement gradients can be calculated from displacement data to obtain strains. A high density of displacement contour bands represents high strain. Consider the two positions at the interface of copper pads and solder bump marked by two ellipses in Fig. 6.19b, where dense and vertical v – displacement contours were found

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(a)

(b)

Fig. 6. 19 The displacement fields at the end of the 7th temperature cycle: (a) u field with axial strains shown at the four corners; (b) v field with peeling strains shown at the four corners and shear strains shown at the ellipse marks.

and so the large negative displacement gradient $\partial v / \partial x$ occurred there. At the same locations, the small positive displacement gradient $\partial u / \partial y$ was found in Fig. 6.19a. As a result, the negative shear strain occurred at the two locations. The shear strains were calculated and shown in Fig. 6.19b. The shear strains were found increasing slowly

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during thermal cycling when compared to the corresponding values shown in Fig. 6.20b.

Fig. 6. 20 The displacement fields at the end of the 14th temperature cycle: (a) u field; (b) v field with shear strains shown at the ellipse marks.

Near the four corners of the solder joint, both u – and v –displacement contours bent and presented the axial strain $\partial u/\partial x$ and the peeling strain $\partial v/\partial y$. As an example, the axial and peeling strains near the four corners of the solder joint

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occurred at the end of 7 thermal cycles were calculated and shown in Fig. 6.19a and Fig. 6.19b respectively.

6.3.3 Finite element analysis of sectioned BGA assembly

The finite element analysis of the sectioned assembly was conducted to obtain numerical results. The array layout of solder joints in the assembly and the section position are shown in Fig. 6.14. A finite element model (Fig. 6.21) of one half of the sectioned assembly was built due to symmetry. The detailed dimensions of solder joint, copper pad, FR-4 PCB, BT substrate, silicon chip and mold compound are listed in Table 6.3.

Fig. 6. 21 3D finite element model of BGA assembly: (a) global view, (b) details of package structure

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Table 6. 3 Structure dimensions in finite element model of BGA assembly

Structure	Length (mm)	Width (mm)	Height (mm)
FR-4 PCB	17.2	16.45	1.60
Copper pad	$\phi = 0.46$ (top) $\phi = 0.52$ (bottom)		0.04 (top) 0.07 (bottom)
Solder ball	$\phi = 0.46$ (top), $\phi = 0.52$ (bottom)		0.55
BT substrate	13.5	12.75	0.40
Silicon chip	4.0	3.25	0.40
Mold compound	12.0	11.25	1.2
Pitch of solder balls	1.5	1.5	

Table 6. 4 Mechanical properties of materials

Material	Young's modulus (GPa)	Poisson's ratio	CTE (<i>ppm/°C</i>)
FR-4	19.68 (x, y) 8.55 (z)	0.39 (xz, yz) 0.11 (xy)	17.6 (x, y) 64.1 (z)
Copper	129.93	0.34	16.8
Solder	See Table 5.5	0.4	21.0
BT substrate	15.11 (x, y) 6.59 (z)	0.39 (xz, yz) 0.11 (xy)	12.5 (x, y) 57.0 (z)
Silicon	130.36	0.28	2.5
Mold compound	16.52	0.25	14.8

Table 6. 5 Temperature-dependent elasticity of solder (63Sn/37Pb)

Temperature (K)	298	348	398
Young's modulus (GPa)	22.39	17.31	12.23

Except solder joint, the rest materials were simulated as elastic materials. The FR-4 PCB and BT substrate were modeled as linear orthotropic materials whose in-plane and out-of-plane mechanical properties were different. The mechanical properties of materials used are listed in Table 6.4, which are cited from literature [25]. Temperature-dependent Young's modulus of solder material is taken from literature [71] and listed in Table 6.5. Creep behavior was included for the solder material. Solder's creep law [122-123] based on standard sine hyperbolic relation was shown below

$$\xi^{cr} = A[\sinh(\beta\sigma)]^n \exp\left(-\frac{Q}{RT}\right) \quad (6.6)$$

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where ξ^{cr} is the uniaxial equivalent creep strain rate, σ is the equivalent stress, Q , the activation energy, equals 52890 J/kg, R , the universal gas constant, equals 8.314 J/kgK, T is the temperature in Kelvin, and A , β and n are material constants, they are 1128 K^{-1} , 0.064 mm^2/N , and 3.3 respectively.

Fig. 6. 22 The displacement fields at the end of the 7th temperature cycle from finite element analysis: (a) u field; (b) v field.

In the analysis, the symmetry boundary condition was applied along the centerline of the sectioned assembly. Other necessary fixed boundary conditions were imposed at an arbitrary point to prevent rigid body displacement and rotation. Thermal cycling

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with the same temperature profile as used in the experiment was applied in the simulation.

The simulation was conducted with the software of ANSYS version 10.0. Seven thermal fatigue cycles were run in the simulation. The numerical results of interest are on the section of the solder joint corresponding to that observed in DIC analysis. The displacement fields in the solder joint of interest at the end of the 7th thermal cycle are shown in Fig. 6.22.

6.3.4 Comparison between experimental and numerical results

Displacement fields (Fig. 6.19) from the DIC experiment superpose object displacements induced by strains, rigid body translation and rotation. Consequently, simple comparison of measurement displacements with the FEA data is seldom possible. Strain data is free of rigid body movements, but strain maps derived from measurement displacement maps are often messy because of degraded accuracy, and thus not good for visual comparison with FEA data. In fact, rigid body translation does not change the displacement pattern, but rigid body rotation does. If the displacements induced by rigid body rotation can be estimated, it is possible to make a good visual comparison between experimental and numerical displacement patterns through extracting the rotation displacements from experimental displacement data or superposing the rotation displacements on numerical displacement data. The latter approach was used in this work. The angle of rigid body rotation was estimated based on the difference of largest relative experimental and numerical u -displacement data. The range of experimental u -displacements is $23.99-23.70=0.29\ \mu\text{m}$. The range of numerical u -displacements shown in Fig. 6.22 is $0.466+0.388=0.85\ \mu\text{m}$. The rotation angle was calculated as $(0.854-0.29)\ \mu\text{m}/440\ \mu\text{m}=0.00128$ radian, where $440\ \mu\text{m}$ is the height of solder joint. The estimated rigid body rotation was then applied to the

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finite element model and so new displacement fields superposed with rotation displacements on previous FEA data (Fig. 6.22) were obtained and shown in Fig. 6.23. Now the comparison of experimental and numerical displacement patterns can be performed between Fig. 6.19 and Fig. 6.23, instead between Fig. 6.19 and Fig. 6.22. By comparison of Fig. 6.19 with Fig. 6.23, it can be shown that both the experimental and numerical u -displacement contours tend to be horizontal and indicate the obvious displacement gradient $\partial u/\partial y$; and both the experimental and numerical v -displacement contours tend to be vertical and indicate the obvious displacement gradient $\partial v/\partial x$.

Fig. 6. 23 The displacement fields obtained by combining rigid body rotation displacements with the displacements shown in Fig. 22: (a) u field; (b) v field.

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Fig. 6. 24 The strain distribution at the end of the 7th temperature cycle from finite element analysis: (a) axial strain; (b) peeling strain; (c) shear strain.

Fig. 6.24(a-c) shows the axial, peeling and shear strains on the sectioned solder joint from the finite element analysis. The normal strain concentration developed at the four corners of the solder joint shown in Fig. 6.24(a-b). At the interface between

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copper pads and solder ball there is peak negative shear strain which is important for fatigue life evaluation of solder joints. The simulation also shows the peak shear strain started at the interface and was growing into the solder joint body. A comparative analysis on the measured and predicted strains shows that 1) the positions where extreme normal and shear strains occur agree well between the measurement and the predication; 2) the larger shear strain values are found on the top/chip side of the solder joint, compared to those on the bottom/PCB side in both the experiment and the simulation; 3) however, the extreme values of predicted strains are generally greater than those of measured strains; for example the minimum value of predicted shear strains is -3.07×10^{-3} , while that of measured ones is -1.8×10^{-3} ; 4) after 7 thermal cycles, the large shear strains predicted by finite element analysis distribute more widely in the body of the solder joint than those measured in DIC experiment.

6.4 Summary

The issues related with thermal deformation measurement by DIC were investigated. The Micro-DIC system error can be significantly reduced by running image correlation on a pair of reference and deformed images generated from averaging multiple reference and deformed images. The influence of out-of-plane displacement on in-plane DIC deformation measurement was illustrated analytically and verified by DIC experiments. Artificial in-plane deformation caused by out-of-plane displacement is determined by the out-of-displacement and the configuration of an optical system. The solutions to correct this artificial in-plane deformation were discussed. The rigid body translation and rotation could degrade the measurement accuracy and should be minimized. The study on these issues was prepared for CTE characterization of a solder material and thermo-mechanical deformation measurement on a sectioned BGA assembly.

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The difference of CTEs measured by the DIC system and the thermo-mechanical analyzer was $(26.1-26.0)/26.1 = 0.38\%$ in x direction and $(22.3-22.1)/22.3 = 0.89\%$ in y direction. The in-plane deformation of exposed solder joint cross-section was monitored during the temperature cycling test. The displacement fields obtained from DIC analysis have the similar patterns with predicted by FEA. The measured extreme strains are lower than the corresponding predicted values. For example, the measured highest shear strain after 7 thermal cycles is -0.0018 and the predicted is -0.003, which represents an average difference $(-0.0018+0.003)/-0.0018/7 = -9.5\%$ per cycle. The deformation accumulated in a few temperature cycles can be obtained by in-situ DIC measurements, which is significant for structure design and reliability study of electronic packaging.

Chapter 7. Reconstruction of AFM images for DIC

This chapter investigates the correction of an AFM (atomic force microscope) scanner drift to extend the DIC method for nano-deformation measurements. A method is presented to reconstruct AFM images aiming to establish AFM image correlation for reliable and accurate measurements of displacement and strain. Case studies for the verification of the proposed method have also been illustrated.

7.1 Motivation

Digital image correlation treating optical images has been found a popular approach as a full-field displacement measurement technique because of its ease in specimen preparation, experiment operation, and experimental data processing. There is a potential to employ digital image correlation technique to handle AFM (a typical scanning probe microscope) images in expectation of the nano-scale deformation measurement with the aid of high spatial resolution of AFM. In addition, with the rapid development of micro/nano fabrication technologies, the characterizations of fabricated micro/nano structures are in great demand. Although AFM image correlation technique has increasing attention, there are few reports [22, 24] on the application of AFM image correlation technique.

The spurious distortion of AFM images results not only from the nonlinear motions of the AFM piezoelectric scanner when responding to the applied voltage, but also from its hysteresis and creep effects, even from the complex cross coupling between x , y and z motions. The nonlinearity corrections of AFMs have been investigated in some studies [13-15] and proved to be effective for the compensation of scanner's nonlinearity. The nonlinearity [124] of AFMs can be reduced within 10 percent by software solutions and to less than 1 percent by hardware solutions with

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external sensors [125-127] to positioning the scanner. 1 percent deviation may be unnoticed or can be ignored by a common user who examines the topographs of micro/nano- structures under AFM. However, the deviation must be taken into account for the application of digital image correlation to the accurate deformation measurement. In fact, the scanner drifts can easily be observed by digital image correlation of AFM images acquired at the same load condition. Fig. 7.1 shows the displacement field obtained from the correlation of two AFM images of size 512×512 $pixel^2$ or $2 \times 2 \mu m^2$ from two continuous scans without any load. Although the drift effect or the poor reproducibility has also been mentioned in literature [22, 24], the method to compensate the AFM nonlinearity and thus derive the accurate displacement field has not been given out.

Fig. 7. 1 Artificial displacement fields between two AFM images scanned at the same condition: (a) u field, (b) v field.

7.2 Atomic force microscope (AFM)

The AFM is a typical member in the scanning probe microscope family, the powerful modern research tools to investigate the morphology and the local properties of the sample surface from the atomic to the micron level. The experimental system

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(see Fig. 7.2) studied in this work is a Digital Instrument-3000 AFM. A sharp tip, typically less than $5\ \mu\text{m}$ and often less than $10\ \text{nm}$ in diameter at the apex, is located at the free end of a cantilever that is $100\text{--}500\ \mu\text{m}$ long. The interactive forces between the tip and the sample surface cause the cantilever to deflect. A position-sensitive photo-detector (see Fig. 7.3) measures the cantilever deflections as the sample is scanned under the tip. The measured cantilever deflections allow a computer to generate a map of surface topography. The interactive force primarily contributed to the deflection of an AFM cantilever is the van der Waals force. The relationship between the van der Waals force and the tip-to-sample separation are shown in Fig. 7.4. Depended on the distance from the sample surface that the probe tip is held, an AFM can be operated at contact, non-contact and intermittent contact modes. The selection of AFM modes is dependent on specific applications. Detailed information is available on the operation manual [128] from AFM vendors.

Fig. 7. 2 Schematic of the experimental AFM system [128]

Fig. 7. 3 Position-sensitive photo-detector system in AFM

Fig. 7. 4 Curve of van del Waals force vs. distance and AFM modes [124]

Scanning a sample surface in an AFM is performed by the piezoelectric scanner (Fig. 7.5). The scanner acts as a fine positioning stage to move the sample under the probe tip and thus makes a high spatial resolution of AFM possible. Like moving an electronic beam on the screen in the cathode ray tube of a television, the AFM control unit drives the scanner in a type of raster pattern as shown in Fig. 7.6. The tip goes along the first line in forward (fast-scan direction), and then back, then moves across the next line (slow-scan direction perpendicular to the fast scan direction), and so forth. During the scanning, data can be collected separately as the probe moves from left to right (the “trace”) and from right to left (the “retrace”). The data are sampled

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digitally at equally spaced intervals scheduled in the fast-scan direction and the slow-scan direction. The distance between the sample points is called the step size. The step size is calculated as the full scan size divided by the number of sample points per line. As a rule, the number of scan lines equals the number of sample points per line. Thus, the data set from a square grid of measurements, called the AFM frame, consists of a square matrix of data elements, whose size is typically 256×256 or 512×512 . The data elements are the cantilever deflection that reflects the surface topography of the

Fig. 7. 5 Tube-shaped piezoscanner in AFM. Voltage applied to the x and y axes produces the bend of the tube. Voltage applied to the z axis produces the extension of the tube.

Fig. 7. 6 Schematic description of scanner motion and point sampling

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scanned area. The AFM frame can be visualized on a computer as a three-dimensional topography or two-dimensional color/grayscale image (Fig. 7.7a-b), where the frame data elements fill image pixels.

Fig. 7. 7 AFM frame visualized as (a) a 3D topography or (b) 2D color/grayscale image

7.3 Scanner imperfections

While an ideally square grid of measurements is scheduled for the scanner, virtually there are a lot of difficulties in achieving perfect tip-over-sample motions in the plane of the scanned sample surface, as well as in the out-of-plane direction. Besides external influences, such as electronic noises, vibrations, acoustic noises and instability of tip-sample contact, the imperfections of a piezoelectric scanner itself cause the distortions in the AFM images. Here scanner imperfections, including intrinsic nonlinearity, hysteresis, creep and cross coupling, are discussed in the following paragraphs.

Fig. 7.8 shows the intrinsic nonlinearity and hysteretic behavior of a piezoelectric scanner. The intrinsic nonlinearity describes a nonlinear extension responding to the applied voltage, and the hysteretic behavior of a piezos scanner displays differences in

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forward and reverse motions. To make things worse the nonlinearity and hysteresis is also dependent on the range and even the rate of change of applied voltage. The intrinsic nonlinearity causes the distortion of the measurement grid shown in Fig. 7.5. The sample points are not equally spaced when the scanner does not move linearly with applied voltage. The hysteretic behavior causes the forward and reverse scan directions to behave differently. A surface with periodic structures, such as gratings, is generally used to calibrate an AFM. AFM images of such a structure are shown in Fig. 7.9. Both scans are in the down direction. Notice the differences in the spacing, size, and shape of the pits between the bottom and the top of each image. The effect of the hysteresis loop on each scan direction in xy plane is demonstrated.

Fig. 7. 8 Intrinsic nonlinearity and hysteresis curves of a piezoscanner for various peak voltages or scan sizes. The hysteresis loop is related to the scan size.

As for this nonlinear relationship, the correction solution in the AFM is the use of a calibration routine by applying a nonlinear voltage (see Fig. 7.10) in real-time to produce a linear scan along x - and y - axes in both tracing and retracing scan directions. The feature distortion in Fig. 7.9 can be reduced or eliminated by the application of such a solution, as shown in Fig. 7.11a. A quantitative investigation of the effect of hysteresis correction can be done by correlating two AFM images scanned in the tracing and retracing directions. Fig. 7.11b shows the displacement

Fig. 7.9 $100\ \mu\text{m} \times 100\ \mu\text{m}$ scans in the forward (trace) and reverse (retrace) directions of a two-dimensional $10\ \mu\text{m}$ pitch calibration grating without linearity correction [128]

Fig. 7.10 Nonlinear voltage waveform applied to the piezo electrodes to produce linear scanner movement [128]

field in the tracing/retracing direction from the correlation of such a pair of images. The result indicates good linearity of scanner motions along fast-scan direction. However, in fact the procedure does not work well for the linearity correction in slow-scan direction, which can be proved by the results (Fig. 7.1) from correlating two 'trace' images of the same area. The almost horizontal contour bands shown in Fig. 7.1(b) show large artificial deformation in slow-scan/vertical direction, but the contour bands of the same type in Fig. 7.1(a) denote tiny artificial deformation within

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a fast-scan/horizontal line. The effectiveness disparity of nonlinearity correction between in fast-scan and slow-scan directions ascribed to the disparity of scanner motions in both directions, which is shown in Fig. 7.6.

Fig. 7. 11 Effect of hysteresis correction: (a) $100\ \mu\text{m} \times 100\ \mu\text{m}$ scans of the same two-dimensional $10\ \mu\text{m}$ pitch calibration grating with linearity correction [128], (b) correlation between the images in the tracing and retracing directions.

Fig. 7. 12 Schematic time diagram of applied voltage change (blue line) vs. the scanner motion along x axis (red line)

Another drawback of a piezoscanner is the so-called creep or drift, a delay in the response to sudden change of the applied voltage. The schematic diagram of applied voltage change versus the scanner motions in xy plane is shown in Fig. 7.12. Strong influence of the creep occurs on the initial stage of the scanning process (see Fig.

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7.13), when making a change in x and y offsets (see Fig. 7.14), or when using the Frame Up and Frame Down commands to bring along abrupt voltage change applied to the scanner.

Fig. 7. 13 Artificial displacement fields in AFM images due to creep: (a) u field in the fast-scan direction, (b) v field in the slow-scan direction

Fig. 7. 14 Effect of creep from 300 nm offsets in x and y axes

Last but not the least, cross coupling exerts influence during AFM scanning. The cross coupling refers to the interaction between x , y and z axis scanner movements. The basis of geometric cross coupling is that the scanner is constructed by combining operated piezo electrodes for x , y and z into a single tube, and the movements of

the scanner in xy plane are achieved by bending the tube at the free end. The influences of cross coupling increase with scan size and surface roughness. Higher surface smoothness can reduce the effect of the cross coupling on in-plane deformation measurements by AFM image correlation.

7.4 Reconstruction of AFM images

Considering that Figs. 7.1 and 7.13 show the accuracy of the scan along fast-scan/horizontal direction is much better than that in slow-scan/vertical direction, it occurred to the author that the AFM image would have better quality if the image data in two directions were both from the accurate scan along fast-scan direction, and such an image can be produced by combining two AFM images of the same area scanned at the angle of 0 and 90 degrees respectively. Exactly speaking, new image can be generated from the reconstruction of the image scanned at the angle of 0 degree whose distortions are determined by digital image correlation with the image scanned at the angle of 90 degrees. The detailed procedure to for AFM image reconstruction is introduced as follows:

- 1) Acquire one AFM image at the scan angle of zero degree at some load condition.
- 2) Acquire the other AFM image at the scan angle of 90 degrees at the same load condition, and rotate the image 90 degrees clockwise.
- 3) With the image from step 1 as the reference image, and the image from step 2 as the deformed image, Correlate the two AFM images by digital image correlation technique and obtain the displacement field.
- 4) Determine the spurious displacements in the reference image based on the displacement field obtained from step 3.
- 5) Reconstruct the reference AFM image according to the spurious displacements

calculated from step 4.

In the procedure, steps 4 and 5 are the core of the method and are illustrated in the following paragraphs.

Fig. 7. 15 Schematic diagram for AFM image reconstruction: (a) normal grid to be scanned by the AFM, the grid with row spacing s and column spacing t , j as row index, i as column index, (b) distorted grid scanned at the scan angle of 0 degree, and used as reference image in DIC, (c) distorted grid scanned at the scan angle of 90 degree, used as deformed image in DIC, (d) Overlapping (a) and (b) shows displacements between corresponding rows (e) Overlapping (a) and (c) shows displacements between corresponding columns (f) correlation between the two distorted grids (b) and (c).

Considering that the u - and v -displacement contour bands are both in almost horizontal and parallel pattern in Fig. 7.1 and 7.13, it is reasonable to assign common u - and v -displacements to all points on a same horizontal/scan line. With this understanding, it can be imagined that the normal grid (Fig. 7.15a) scanned horizontally will become distorted as schematically shown in Fig. 7.15b; likewise, the normal grid scanned vertically will become distorted as schematically shown in Fig.

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7.15c. When Fig. 7.15a and 7.15b overlap, the offsets of horizontal scan lines will be exposed as shown in Fig. 7.15d, where u_j^r and v_j^r denote the horizontal and vertical offsets of horizontal scan line (row) j scanned in the reference image from its ideal position. Likewise, the overlapping of Fig. 7.15a and 7.15c will show up the offsets of vertical scan lines, as shown in Fig. 7.15e, where u_i^d and v_i^d denote the horizontal and vertical offsets of vertical scan line (column) i scanned in the deformed image from its ideal position. Now if a point is located both on the horizontal line j and the vertical line i in the normal grid, the point will have the position $(i + u_j^r, j + v_j^r)$ in the reference image, and have the position $(i + u_i^d, j + v_i^d)$ in the deformed image. If $U_{x,y}$ and $V_{x,y}$ denote the horizontal and vertical displacements between a point at the location (x, y) in the reference image and its match point in the deformed image, the displacements of the point at $(i + u_j^r, j + v_j^r)$ in the reference image can be denoted as $U_{i+u_j^r, j+v_j^r}$ in horizontal direction and $V_{i+u_j^r, j+v_j^r}$ in vertical direction, and expressed as:

$$\begin{aligned} U_{i+u_j^r, j+v_j^r} &= (i + u_i^d) - (i + u_j^r) = u_i^d - u_j^r \\ V_{i+u_j^r, j+v_j^r} &= (j + v_i^d) - (j + v_j^r) = v_i^d - v_j^r \end{aligned} \quad (7.1)$$

If the number of horizontal lines and of vertical lines are n ($j=1,2,\dots,n$) and m ($i=1,2,\dots,m$) respectively, the number of the unknowns (terms at the right side of Eq. (7.1)) is $2n + 2m$. Compared with the number of the knowns (terms at the left side of Eq. (7.1)) mn , we know that there is redundancy for the solution of Eq. (7.1). The redundancy can be used to reduce the deviation effect of measured displacements by image correlation. Consider random errors can usually be minimized with the use of the average approach. Sums of the displacements of all of m points on scan line j

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lead to:

$$\begin{aligned}\sum_{i=1}^m U_{i+u_j^r, j+v_j^r} &= \sum_{i=1}^m u_i^d - m u_j^r \\ \sum_{i=1}^m V_{i+u_j^r, j+v_j^r} &= \sum_{i=1}^m v_i^d - m v_j^r\end{aligned}\quad (7.2)$$

Eq. (7.2) can be rewritten as:

$$\begin{aligned}u_j^r &= -\frac{1}{m} \sum_{i=1}^m U_{i+u_j^r, j+v_j^r} + \frac{1}{m} \sum_{i=1}^m u_i^d \\ v_j^r &= -\frac{1}{m} \sum_{i=1}^m V_{i+u_j^r, j+v_j^r} + \frac{1}{m} \sum_{i=1}^m v_i^d\end{aligned}\quad (7.3)$$

The subtraction of displacements between two horizontal lines $j+k$ (k is an integer)

and j will cancel out the unknowns u_i^d and v_i^d , and is shown as follows:

$$\begin{aligned}u_{j+k}^r - u_j^r &= -\frac{1}{m} \sum_{i=1}^m \left(U_{i+u_{j+k}^r, j+k+v_{j+k}^r} - U_{i+u_j^r, j+v_j^r} \right) \\ v_{j+k}^r - v_j^r &= -\frac{1}{m} \sum_{i=1}^m \left(V_{i+u_{j+k}^r, j+k+v_{j+k}^r} - V_{i+u_j^r, j+v_j^r} \right)\end{aligned}\quad (7.4)$$

To reconstruct the reference image, a reference/anchor line is needed. Provided that the central horizontal line j_c is selected as the reference line, then $u_{j_c}^r = 0$ and $v_{j_c}^r = 0$. The displacements of horizontal line j_c+1 in the reference image can be rewritten based on Eq. (7.4):

$$\begin{aligned}u_{j_c+1}^r &= u_{j_c+1}^r - u_{j_c}^r = -\frac{1}{m} \sum_{i=1}^m \left(U_{i+u_{j_c+1}^r, j_c+1+v_{j_c+1}^r} - U_{i, j_c} \right) \\ v_{j_c+1}^r &= v_{j_c+1}^r - v_{j_c}^r = -\frac{1}{m} \sum_{i=1}^m \left(V_{i+u_{j_c+1}^r, j_c+1+v_{j_c+1}^r} - V_{i, j_c} \right)\end{aligned}\quad (7.5)$$

$u_{j_c+1}^r$ and $v_{j_c+1}^r$ in Eq. (7.5) can be solved by iteration technique by taking the initial guesses of them as zeros. The solutions are found when the iteration converges. The

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convergence is reached when the u_{jc+1}^r and v_{jc+1}^r differences between the current and the last estimates of them respectively are both lower than their convergence limits, 0.01 pixel. Likewise, we can obtain u_{jc-1}^r and v_{jc-1}^r , u_{jc+2}^r and v_{jc+2}^r , and so on. When the displacements of all horizontal lines scanned in the reference image are determined, the reference image can be reconstructed to remove its distortion. The reconstruction of the reference image is done based on the following expression:

$$\begin{aligned} \bar{f}(i, j) &= f(i + u_j^r, j + v_j^r) \\ (i &= 1, 2, \dots, m; j = 1, 2, \dots, n.) \end{aligned} \quad (7.6)$$

where $\bar{f}$ denotes the reconstructed image profile, and f denotes the reference image profile.

Fig. 7. 16 Software implementation for AFM image correction

In the implementation of the above algorithms for reconstruction of AFM images, the spurious displacements at a grid of control points located on the image are obtained from DIC. The values of the rest points are calculated using interpolation schemes, such as bilinear interpolation, which is to obtain the values of a point by linear interpolation between its four nearest neighbors. As shown in Fig. 7.16, this correction function has been added in the original DIC software.

7.5 Verification and buckling test

Fig. 7.17 shows the displacement field between two AFM images scanned at the angles of 0 and 90 degrees respectively but under the same other conditions. Either the displacement u field or v field displays the same pattern along a scan line, which agrees well with the supposed (see Fig. 7.15d).

Fig. 7. 17 Displacement ($3.906 \text{ nm} / \text{pixel}$) fields between two AFM images scanned at 0 and 90 degrees: (a) u field, (b) v field.

The second verification case is zero deformation measurement. Two images reconstructed from two pairs of AFM images scanned at the angle of 0 and 90 degrees are correlated using digital image correlation technique. The displacement field between one pair of images scanned at the angle of 0 and 90 degrees is shown in Fig. 7.17. The displacement field between both images scanned at the angle of 0 degree is shown in Fig. 7.18a-b, and the displacement field between two reconstructed images in Fig. 7.18c-d. The sample ranges of displacements u and v are 65.7 nm and 68.8 nm before image correction, 2.1 nm and 2.7 nm after image correction. In this instance, the maximum image distortion was decreased from 4.3% to 0.16% (correlated image area is $1.6 \mu\text{m} \times 1.6 \mu\text{m}$). Therefore, the correction method to AFM

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images is effective to cancel out the distortion due to the AFM scanner drifts. In addition, a comparison of measured strains before and after image correction has been listed in Table 7.1. After image correction, the average remaining strains, ϵ_x , ϵ_y and ϵ_{xy} , are $76.6 \mu\epsilon$, $-124 \mu\epsilon$ and $-2.83 \mu\epsilon$, and their standard deviations 0.00148, 0.00180 and 0.00124 respectively, which shows the promise of accurate deformation measurements by AFM image correlation.

Fig. 7.18 u and v displacement (3.906 nm/pixel) fields in zero deformation tests: (a-b) u , v fields before image correction, (c-d) u , v fields after image correction.

Table 7. 1 Remaining strains in zero deformation test

Strains	Before correction			After correction		
	ϵ_x	ϵ_y	ϵ_{xy}	ϵ_x	ϵ_y	ϵ_{xy}
mean(ϵ)	/	/	/	7.66e-5	-1.24e-4	-2.83e-6
min(ϵ)	-3.78e-3	3.01e-2	-3.18e-2	-4.92e-3	-7.69e-3	-4.25e-3
max(ϵ)	5.33e-3	5.48e-2	-1.16e-2	5.50e-3	6.78e-3	3.97e-3
$\sigma(\epsilon)$	/	/	/	1.48e-3	1.80e-3	1.24e-3

Fig. 7. 19 Thin plate (printed circuit board) for the buckling test **Fig. 7. 20** Schematic diagram of the fixture for the buckling test

A buckling test was conducted as an example of deformation measurements by AFM image correlation. The buckling specimen (see Fig. 7.19) is a thin PCB (printed circuit board) with cross copper lines located in the centerlines of the plate. The size of the specimen is $56 \times 48 \times 0.42 \text{ mm}$. The center area of the plate to be scanned was sputtered with nano particle to obtain AFM images with high contrast pattern. Nano indentation marks of $1 \mu\text{m}$ size were shaped for the assistance in the positioning of scan areas between two loading states. A simple fixture was designed and the schematic diagram of loading fixture was shown in Fig. 7.20. The fixture was required to be aligned with the AFM scan direction and be tightly fixed on the AFM

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stage. The specimen was first scanned at the lengthwise buckling state because of the ease to unload the specimen buckling during the test. An area of $10 \times 10 \mu\text{m}^2$ in the center of the plate surface was selected to be scanned considering that the smaller area will increase the difficulty in the search for the same area to be scanned at two separate scans. It is noted that the operation of probe offset was not used in this test because false variational displacements along fast scan axis have been found between AFM images scanned with probe offset and without probe offset. This nonlinearity of AFM scan has been calibrated and shown in Fig. 7.14 earlier. This behavior has also been reported by Helena [129].

Fig. 7.21 u (a) and v (b) displacement ($19.53 \text{ nm} / \text{pixel}$) fields in the buckling test

The interested area was scanned by AFM images at the scan angle of 0 and 90 degrees when the specimen was pushed 1.5 mm at one end to buckling deformation and was then unloaded. Acquired images were reconstructed by following the procedure in Section 7.4 and digital image correlation was conducted on the reconstructed images. The displacements u and v in the direction of specimen length and width have been shown in Fig. 7.21a and Fig. 7.21b. The tensile deformation shown in Fig. 7.21a is almost uniform. Finite element analysis (FEA) of the buckling

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test using ABAQUS shows that the strains in the center area of the buckling plate are uniform. Measured strains by digital image correlation and calculated strains by FEA are listed in Table 7.2. The results show the average strain in x direction measured by DIC agrees well with that calculated by FEA. The standard deviations of measured strains in Table 7.2 are less than those in Table 7.1. The difference can be attributed to the different scan sizes in two tests. Fig. 7.21b shows a displacement deviation of 0.11 *pixels* or 2.2 *nm* which is close to that in the zero deformation test. The minimum strain that can be identified in this instance is estimated at hundreds of micro strains.

Table 7. 2 Strains measured by DIC and from FEA

Strains		ϵ_x	ϵ_y	ϵ_{xy}
Measured	mean(ϵ)	3.96e-3	1.36e-5	2.35e-5
by DIC	$\sigma(\epsilon)$	6.96e-4	3.91e-4	3.58e-4
FEA solution		3.88e-3	6.59e-8	2.20e-9

7.6 Summary

a method was developed to reconstruct AFM images for nano-deformation measurements by digital image correlation. The developed method is to generate a corrected digital image from two AFM images scanned at the angle of 0 and 90 degrees respectively. The proposed method was shown valid for the correction of distorted AFM images by the calibration test which shows the displacement deviation in a reconstructed image is less than 0.16%. A buckling test of a thin plate was conducted and the uniform strains at the center of the plate were characterized by AFM image correlation. The measured value of the horizontal strain component agreed well with the numerical predication by an error of $(0.00396-0.00388)/0.00396 = 2\%$. The AFM-based DIC method is used for actual nano-deformation measurements discussed in next chapter.

Chapter 8. Nanoscale deformation measurement by AFM-DIC

The miniaturization of microelectronic interconnections places more demands on experimental deformation analysis at micro- and nano-scale for material characterization and structure reliability analysis. An application of digital image correlation (DIC) technique with the aid of an atomic force microscope (AFM) to nanoscale deformation measurement is described in this chapter. The AFM was used to scan and digitally image micro-interconnection solder assemblies in a micro thermoelectric cooler. AFM images of the scan region of interest were obtained separately when the microelectronic device was operating at its cooling and heating stages. The AFM images were then used to obtain the in-plane deformation fields in the observed region of the micro-assembly. AFM image correlation is performed for nanoscale deformation analysis using the in-house AFM-DIC program. The thermally induced deformation behavior of the micro-assembly device was modeled by finite element analysis (FEA). Both thermal-electric analysis and thermal stress analysis were conducted on a 3D finite element model of the device. The experimental DIC results were compared with the FEA results.

8.1 AFM-DIC Experiments

It is a challenge to conduct thermo-mechanical tests under AFM. Thermal loading can be exerted on a specimen readily by conducting a thermo-mechanical test. The issues of thermal drift of AFM scanner must be resolved and this led to the design of the thermo-mechanical experiments on a micro thermoelectric cooler under AFM.

8.1.1 Specimen preparation

A micro thermoelectric cooler (TEC) developed by Tan et al [130-131] was used

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in this work. A micro TEC is a tiny solid-state heat pump which is a critical component in many applications of integrated electronic packaging assemblies as it provides precise cooling and temperature stabilization. A typical TEC module comprises of two ceramic substrates that serve as a foundation and electrical insulation for p-type and n-type bismuth telluride (Bi_2Te_3) pellets that are connected electrically in series and thermally in parallel between the substrates (Fig. 8.1). Metallised pads are attached to the ceramic substrates to maintain the electrical connections inside the module. Solder is typically used at the connection joints to enhance the electrical connections and hold the module together. The prototype of the micro TECs with outer dimension of $L3.8 \times W3.0 \times H1.0 \text{ mm}$ is shown in Fig. 8.2. The dimension details of the micro TEC are listed in Table 8.1.

Fig. 8. 1 Structure of solid-state thermoelectric module

Fig. 8. 2 Prototype of micro thermoelectric cooler

Table 8. 1 Dimensions of the micro TEC module

Components	Dimensions
Top ceramic substrate	$3.0\text{mm} \times 3.0\text{mm} \times 0.28\text{mm}$
Bottom ceramic substrate	$3.8\text{mm} \times 3.0\text{mm} \times 0.28\text{mm}$
TE elements	$0.3\text{mm} \times 0.3\text{mm} \times 0.4\text{mm}$
TE element pitch	$0.45\text{mm} \times 0.45\text{mm}$
Solder interconnections	$0.3\text{mm} \times 0.3\text{mm} \times 0.02\text{mm}$

In order to observe the solder interconnections of the micro TEC under AFM, a cross-section (refer to Fig. 8.3) was created through the outmost row of thermal-

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electric elements and solder interconnections. Fine grinding and polishing was done through grit paper and alumina slurry respectively. The prepared sample was then checked under optical microscope and scanning electron microscope to confirm no precracks between interfaces occur due to sample preparation, and so the micro TEC module remained functional after the cross-sectioning.

Fig. 8. 3 Sample preparation and experimental test assembly

Fig. 8. 4 AFM used for nanoscale deformation measurement

8.1.2 Experimental preparation

After the specimen check, the hot junction side of the good TEC module was attached to a copper heat sink and thermal grease were filled between them. Two electric wires were soldered to the each terminal of the micro TEC, and then connected to a direct-current power supply. Before the deformation measurement by digital image correlation with the aid of AFM, a calibration check of the assembly

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performance was performed to obtain a configuration of input current, the surface temperature or temperature difference for the use in AFM-DIC experiments. For the calibration, a fine thermal couple was attached to the top surface of the TEC module. An input current of 0.8 A and a temperature difference of 50°C under the ambient temperature 25°C were determined for the following AFM-DIC experiments. In other words, in our experiments, the device working at its cooling stage was subjected to a temperature gradient of 25°C to -25°C from the bottom substrate to the top substrate, while working at its heating stage was subjected to a reverse temperature gradient of 25°C to 75°C from the bottom substrate to the top substrate. The multimode AFM (Fig. 8.4) of Dimension 3000 produced by Digital Instrument was used in this work. The assembly was then fixed on the stage of AFM with the region of interest facing the AFM scanner tip. The electric wires of the TEC module was fastened on the stage by the tape, squeezed tightly by the AFM shelter, and connected to a DC power supply.

8.1.3 Experimental procedure

The scanning area (shown in Fig. 8.3) was located at the right bottom corner of the solder interconnection which is connected to the bottom substrate. During the experiments, the bottom substrate maintained at the temperature of 25°C , and so the scanning area remained at a temperature of near 25°C , which allowed neglecting the thermal effect on the AFM scanner. The tapping mode of AFM was used to scan the specimen. All acquired AFM images were $5\mu\text{m}\times 5\mu\text{m}$ in size with 512 pixel resolution along each scan line. In the present test method, the topography of the area of interest was recorded via AFM at both the horizontal scan and vertical scan directions. The two AFM images were used to reconstruct the topography of the scanned area in order to reduce the drift of AFM scanning. The detail of AFM image

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correction for deformation measurements [132] was presented in the previous chapter. Based on the correct procedure, the resolution of AFM-DIC technique was found to be as good as 0.1 pixels.

Fig. 8. 5 AFM image of $5\mu m \times 5\mu m$ size captured for image correlation analysis

Once the scan parameters of AFM scan were determined, the region of interest in the solder interconnection of the sample was positioned and focused under the AFM. Without the current input, the $5\mu m \times 5\mu m$ region of interest was initially scanned in horizontal and vertical directions with the tapping mode, and thus a pair of AFM images before deformation was captured and later correlated to obtain a corrected AFM image as the reference image for DIC analysis. Next, a constant current of 0.8 A was applied to the micro TEC which was thus operating at its cooling stage. When the steady state condition was reached after approximately 2 minutes, the region of interest was scanned again with the same AFM settings and a pair of AFM images after deformation was captured. The pair of AFM images was later correlated to obtain a corrected AFM image which carried the deformation information due to the TEC's operating at its cooling stage. After that, the input current polarity was changed and so the micro TEC was operating at its heating stage. In other words, the hot side

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and the cool side of the TEC changed. Under this condition, the region of interest was scanned twice in the horizontal and vertical directions to obtain another pair of AFM images, which carried the deformation information due to the TEC's operating at its heating stage. A typical AFM image of the scanned region captured in this test is shown in Fig. 8.5.

8.2 Finite element coupled-field analysis

The finite element analysis was conducted for two reasons. First, it is used to provide a comparison with the experimental measurements. In addition, finite element simulation can show both the in-plane and out-of-plane whole field of displacements and strains in the whole device. In the above test, the observed region may not reveal the most severe strain in the micro TEC. Actually, higher strain is expected to be found at the corners of the solder interconnections joining the top ceramic substrate (refer to Fig. 8.3) because the solder interconnections further away the heat sink were subjected to the change of temperature in the testing. The region expected with more severe strain was not selected for experimental study considering that the temperature change can introduce thermal drift and distortion to the AFM images and thus introduce errors to AFM-DIC analysis.

A 3D finite element model was created for thermal-electric analysis and thermal stress/strain analysis. Due to the lateral symmetry, half of the micro TEC module was simulated. The meshed model is shown in Fig. 8.6. The dimensions of the micro TEC module are listed in Table 8.1. Both the temperature-independent thermal-electric and linear elastic properties of materials used in present work can be found in Table 8.2, while Table 8.3 shows the temperature-dependent elastic-plastic properties used for the eutectic tin-lead solder. Thermal-electric analysis was to obtain the temperature distribution by simulating the thermal-electric behavior of Bi_2Te_3 elements. The nodal

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temperatures were then input to perform thermal stress/strain analysis.

Fig. 8. 6 3D finite element model of one half of the micro TEC

Table 8. 2 Material properties for thermal-electric and structural models

Material properties	TE element (n-type/p-type)	Cu strap	Solder	Ceramic
Thermal conductivity (<i>watt / m °C</i>)	1.3/1.2	385	57	25
Resistivity (<i>ohm*m</i>)	$1.05 \times 10^{-5} / 9.8 \times 10^{-6}$	1.7×10^{-8}	2.2×10^{-8}	10^{12}
Seebeck coefficient (<i>µvolts / °C</i>)	-165/210	/	/	/
Young's modulus (<i>GPa</i>)	31.7	120	Table 3	340
Poisson ratio	0.33	0.3	0.35	0.22
Coefficient of thermal expansion	1.3×10^{-5}	1.7×10^{-5}	2.24×10^{-5}	6.8×10^{-6}

Table 8. 3 Temperature-dependent material properties of eutectic tin-lead solder

Temperature (°C)	-40	0	30	60	90
Young's modulus (<i>GPa</i>)	40.554	34.474	29.914	25.354	20.794
Yield stress (<i>MPa</i>)	38	31	25.6	20.3	14.9
Stress (<i>MPa</i>) at strain 0.2%	46.4	39.1	32.2	25.5	17.9
Stress (<i>MPa</i>) at strain 0.5%	50.2	42.3	34.8	27.4	19.3

In thermal-electric analysis of the micro TEC operating at its cooling stage, a temperature of 25 °C was set on the bottom side of the TEC module, and a temperature of -25 °C on the top side of the TEC module. The input current of 0.8 A

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was applied and convection with the film coefficient of $10 \text{ W/m}^2\text{ }^\circ\text{C}$ was applied to the exposed lateral surface. The steady state analysis was performed. The temperature distribution was obtained from the thermal-electric simulation. The resultant nodal temperatures were taken as the thermal loading in the following thermal-structural analysis. The symmetry boundary condition was applied to the symmetrical plane. The out-of-plane displacement of a node was fixed to restraint the out-of-plane rigid body motion. This simulation procedure (see Fig. 8.7) was also used for the finite element simulation of the micro TEC operating at its heating stage, in which a temperature of 25°C was still used on the bottom side of the TEC module, but a temperature of 75°C was set on the top side of the TEC module, and a reverse current of 0.8 A was applied to the TEC circuit for thermal-electric analysis. In the structural analysis of the TEC operating at its heating stage, except that the thermal loading was taken from its own thermal-electric simulation, the same procedure and boundary conditions as that of the TEC operating at its cooling stage were employed.

Fig. 8. 7 Data flow for the sequential coupled thermal-electric-structural analysis

8.3 Experimental DIC results

The AFM images captured before and after deformation were correlated to obtain the deformation using the developed AFM-DIC software (Chapter 7). The resultant displacement fields due to the TEC's operating at its cooling stage and at its heating stage are shown in Fig. 8.8 and Fig. 8.9, respectively, where u and v represent the displacement fields in x /horizontal and y /vertical direction of images respectively (see Fig. 8.3). Fig. 8.8 shows the range of u -displacements, which is 3.29 nm , is less

Fig. 8. 8 Displacement fields of the observed region in the solder interconnection when the TEC is working at its cooling stage: (a) u field; (b) v field.

Fig. 8. 9 Displacement fields of the observed region in the solder interconnection when the TEC is working at its heating stage: (a) u field; (b) v field.

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than that of v -displacements which is 5.83 nm , and the u -displacement contour bands lean to the right while the v -displacement contour bands are almost horizontal. Thus, it can be estimated that the average strain in vertical direction $\bar{\epsilon}_y$ is several times larger than the average horizontal strain component $\bar{\epsilon}_x$ and the shear strain component $\bar{\epsilon}_{xy}$. Similarly, Fig. 8.9 shows the range of u -displacements, which is 4.07 nm , is less than that of v -displacements which is 7.12 nm , and the u -displacement contour bands incline from the vertical more than the v -displacement contour bands incline from the horizontal.

The heterogeneity of the displacement fields shown in Fig. 8.8 and Fig. 8.9 did not completely result from the actual deformation. The displacement errors from AFM-DIC analysis can contribute much to the heterogeneous displacement fields when the actual deformation is small. The gradients of strains on the small scanned area of $5\mu\text{m}\times 5\mu\text{m}$ are expected to be small. The investigation of DIC techniques discussed in Chapter 3 has shown that the average strains derived from linear-fitting the displacement fields can be very accurate. The calculation of average strains shows that $\bar{\epsilon}_x$, $\bar{\epsilon}_y$, and $\bar{\epsilon}_{xy}$ are 0.042%, 0.126%, and 0.005% respectively when the TEC was operating at its cooling stage. The calculated average strain $\bar{\epsilon}_x$, $\bar{\epsilon}_y$ and $\bar{\epsilon}_{xy}$ are 0.035%, -0.155%, and -0.07%, respectively, when the TEC was operating at its heating stage. Clearly, compared with $\bar{\epsilon}_x$ and $\bar{\epsilon}_{xy}$, the vertical strain component $\bar{\epsilon}_y$ is dominant in the observed region. When the operation of the TEC module was switched from the cooling mode to the heating mode, $\bar{\epsilon}_y$ changed from positive to negative, which means that the observed region of solder interconnection was subjected to cyclic stresses between tension and compression.

8.4 FEA modeling results

Fig. 8.10 and Fig. 8.11 show the respective warpage, or displacement in y direction when the TEC was operating at its cooling and heating stages. Working at the cooling stage, the TEC shows a concave top surface, while working at the heating stage, the TEC shows a convex top surface. The numerical results confirm that the switching of TEC's operation mode between cooling and heating causes cyclic stresses in its components. As expected, finite element analysis shows the changes of strains in the scanned area is negligible compared with the average strains of the area.

Fig. 8. 10 The TEC exhibited concave warpage (exaggerated view) when it worked at its cooling stage

Fig. 8. 11 The TEC exhibited convex warpage (exaggerated view) when it worked at its heating stage

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To compare the strains in the region of interest marked in Fig. 8.6 from experimental DIC analysis and FEA, the predicted strains at approximately a corresponding position of the scanning area in AFM-DIC experiments were averaged. A comparison of the average strains listed in Table 8.4 shows the numerical results from finite element analysis are in good agreement with the measured results from DIC. In other words, both the numerical and experimental results show that in the region of interest the axial strain ε_y is at a strain level with much higher amount than the peeling and shear strains ε_x and ε_{xy} .

Table 8. 4 Comparison between experimental and numerical strains

Operation mode	Strains	Experimental	Numerical	Difference (%)
Cooling stage	ε_y	0.126%	0.135%	-7.1%
	ε_{xy}	0.005%	0.004%	20%
Heating stage	ε_y	-0.155%	-0.143%	7.8%
	ε_{xy}	-0.07%	-0.061%	12.9%

On the other hand, it should be noted that the axial and shear strains ε_y and ε_{xy} are found at the similar level after a check of numerical strains at other positions of solder interconnections. For example, the maximum axial and shear strains ε_y and ε_{xy} in solder interconnections at the heating stage of TEC are 0.175% and 0.228% respectively. Since the amplitudes of cyclic strains in solder interconnections at the top side of TE elements are in general larger than those at the bottom side of TE elements, the most serious stresses occur to the solder interconnection at the top side of TE element, instead at the bottom side of TE element. The power cycling test [131] of the TEC module shows that cracks were predominately found in the solder interconnections at the top side of TE element, which agreed fairly well with the finite element analysis.

8.5 Summary

The AFM-DIC experiment for nanoscale deformation measurement on a micro thermoelectric cooler was presented. The functional specimen was operated in-situ under an AFM and AFM images of the scan region of interest were obtained. The AFM images were then used to obtain the in-plane deformation fields in the observed region of the micro-assembly. The results show that the observed region was subjected to cyclic strains when the device worked between its cooling and heating stages, and cyclic strain in the vertical direction was found to be the significant deformation mode. The measured average strain ε_y is 0.126% at the cooling stage and -0.155% at the heating stage. The thermally induced deformation behavior of the micro-assembly device was modeled by finite element analysis (FEA). Both thermal-electric analysis and thermal-structural coupled-field analysis were conducted on a 3D finite element model of the device. A comparison of measured and predicted strain ε_y shows that the experimental results were able to validate the finite element analysis results. FEA complemented the DIC deformation analysis. Cyclic shear strain, as well as cyclic strain in the vertical direction was found to be the significant deformation mode by FEA.

Chapter 9. Conclusions

This thesis has examined micro-deformation characterization of electronic interconnection material and assemblies by digital image correlation (DIC) technique. The DIC method was developed for in-plane micro-deformation measurements and was extended to nano-scale deformation using AFM-DIC for electronic packaging applications. Studies on the theoretical framework of digital image correlation, the problems encountered in the application of DIC to handling AFM images, and issues in thermally-induced deformation measurements were investigated for specific applications of DIC to deformation measurements at the micro- and nano-scale.

9.1 Investigation on DIC analysis methods

Two program codes were written for a comparative study on DIC analysis methods in the spatial domain and the spectral domain. Digital deformed images were generated and used for DIC analysis. The DIC results show that the spatial domain method has advantage over the spectral domain method. The advantage of DIC in the spatial domain was attributed to the combined use of the interpolation scheme of image intensity/grayscale and the fast search method of Newton-Raphson.

The parameters of the DIC analysis method in the spatial domain were investigated for a deeper understanding. These parameters are subimage quality index, subimage size, subimage displacement function, and subimage interpolation scheme. Experimental images from translation and tensile experiments, and numerically generated images encoded with nonlinear deformation were used in the study. The factorial investigation came to the following conclusions:

- 1) The subimage quality index the author defined is a valid parameter for quantifying image speckle pattern quality affecting DIC deformation analysis.

Digital images with higher subimage quality indexes are better for DIC analysis. The concept of subimage quality index can be used for (a) guidance for sample preparation in DIC experiments, (b) estimation of displacement measurement confidence, and (c) normalization of the chosen subimage size in digital image correlation.

- 2) The optimal subimage size is a tradeoff between random errors, potential systematic errors and algorithm performance. The subimage size ranging from 33×33 pixels to 57×57 pixels is recommended.
- 3) The advantage of higher-order displacement functions in DIC analysis is not clear. The first-order displacement functions are usually used.
- 4) High order intensity interpolation is good for digital image of normal contrast used in DIC analysis, and low order intensity interpolation is good for digital images of high contrast.

9.2 Development of finite element-based DIC method

With the purpose of overcoming the shortcoming of the existing digital image correlation method that does not ensure displacement continuity, a finite element formulation for a digital image correlation method was developed by introducing a finite element framework into the image correlation process. In the finite element-based DIC method, the image region of interest is discretized into the subimages interconnected by nodes (measured points). All the subimages in a reference image are correlated those in a deformed image simultaneously, instead sequentially in the former subimage-based DIC method. Two software codes were written to implement the subimage-based DIC method and the finite element-based DIC method respectively. Numerically-generated images were used to examine the performance of the two DIC codes. The DIC results have demonstrated that the proposed element-

based method is accurate and robust, and results in more continuous displacement gradients than the subimage-based image correlation. The subimage-based DIC method shows 3~4 times as variation in measured strains and rotation angle as the element-based DIC method. The rotation experiment was also performed and actual digital images were captured for digital image correlation. DIC results using experimental images agreed well with those using numerically-generated images.

9.3 Micro-deformation characterization by DIC

The applications of DIC technique to micro-mechanical and thermo-mechanical deformation measurements were carried out.

9.3.1 Crack-tip deformation measurement

The element-based image correlation method was applied to the characterization of near crack-tip deformation in the lead-free SnCu solder alloy. The fracture experiment of a CT (compact tension) specimen was conducted using an INSTRON micro tester combined with a DIC setup. Digital images of the crack-tip region on the CT specimen were acquired in situ during the loading history. Finite element modeling for time-dependent fracture mechanics analysis was carried out using the actual loading conditions applied on the CT specimen. The rate-dependent plasticity of SnCu solder alloy was modeled. The near crack-tip displacement fields characterized by DIC deformation analysis were compared with those predicted by finite element analysis. The SIF (stress intensity factor) and CTOD (crack tip opening displacement) were calculated from the experimental displacement fields. The generalized J integral was computed from the numerical displacement fields. The following conclusions/achievements can be made:

- 1) The near crack-tip displacement fields from experimental DIC analysis are in good agreement with those from FEA. The element-based DIC method is able

to characterize the high gradient displacement fields such as near crack-tip displacement fields.

- 2) The procedure to estimate SIFs using near crack-tip displacement data from DIC analysis is practicable. The SIFs measured by DIC analysis agree well with those calculated using ASTM formula and those predicted by FEA.
- 3) The linear relationship between pseudo CTOD and generalized J integral has been found in the fracture experiment. The ANSYS Macro that the author wrote to compute generalized J integral is useful.
- 4) The experimental and data processing procedures for fracture experiments combined with DIC deformation analysis were implemented for later use.

9.3.2 CTE measurement and solder joint fatigue strain measurement

Micro thermo-mechanical deformation characterization was demonstrated by CTE measurement of a solder material and solder joint fatigue deformation measurement using a sectioned BGA assembly. Meanwhile, the issues related with thermo-mechanical deformation measurements by DIC were investigated. The investigations led to the following conclusions:

- 1) System errors can be significantly reduced by running image correlation on a pair of reference and deformed images generated from averaging multiple reference and deformed images.
- 2) The influence of out-of-plane displacement on in-plane deformation measurement was determined analytically and experimentally. The resultant artificial in-plane deformation is proportional to the out-of-plane displacement and the numerical aperture of the optical system of a DIC setup.
- 3) The accuracy of CTE measurements by DIC analysis is comparable with that by a thermo-mechanical analyzer. The maximum difference of CTEs

measured by the two methods is 0.89%.

- 4) The displacement fields on the solder joint section obtained from DIC analysis have the similar patterns with predicted by FEA. The difference of highest measured and predicted shear strain accumulated in thermal cycles is -9.5% per cycle.

9.4 Development of AFM-DIC method

A new capability for nanoscale deformation measurement with AFM images has been developed. A method has been implemented to reconstruct AFM images and correct for the distortion of AFM images. The AFM-DIC method can generate a corrected image from two AFM images scanned at the angle of 0 and 90 degrees respectively. The proposed AFM-DIC method was calibrated for the correction of distorted AFM images and the calibration test shows the displacement deviation in a reconstructed image to be less than 0.16%. A buckling test on a thin plate was conducted and the uniform strains at the center of the plate were characterized by AFM image correlation. The measured value of the horizontal strain component agreed well with the numerical predication by an error of 2%. Therefore, the AFM images reconstructed with the correction method can be used for nanoscale deformation measurements by this AFM-DIC method.

9.5 Nano-deformation measurement by AFM-DIC

The experiment for nanoscale deformation measurement by the developed AFM-DIC technique was performed on a micro thermoelectric cooler. The specimen was operated in-situ under an AFM and AFM images of the scan region of interest were obtained. The AFM images were then used to obtain the in-plane deformation fields in the observed region of the micro-assembly. The results show that the observed

region was subjected to cyclic strains when the device worked between its cooling and heating stages, and cyclic strain in the vertical direction (ε_y) was found to be the significant deformation mode. The thermally induced deformation behavior of the micro-assembly device was modeled by finite element analysis (FEA). Both thermal-electric analysis and thermal-structural coupled-field analysis were conducted on a 3D finite element model of the device. It is shown that the experimental results were able to validate the finite element analysis results. The difference of cyclic strain ε_y between AFM-DIC measurement and FEA prediction is within $\pm 10\%$. FEA has complemented the AFM-DIC deformation analysis. Not only cyclic strain in the vertical direction but also the cyclic shear strain was found to be the significant deformation mode by FEA.

9.6 Summary of achievements and contributions

A summary of the major achievements and contributions arising from this PhD study are as follows:

- 1) A new finite element-based DIC method has been developed and implemented.
- 2) A new correction approach to reconstruct distorted AFM images has been developed and implemented in the AFM-DIC method for nanoscale deformation measurements.
- 3) A comparative study of DIC methods in the spatial domain and the spectral domain has been evaluated.
- 4) The factors affecting DIC deformation analysis have been investigated. A new notion of subimage quality index has been proposed and its effect on the accuracy of displacement has been studied. The DIC system has been

calibrated and the averaging scheme has been proposed to reduce system errors. The effect of out-of-plane displacement on in-plane displacement measurement has been first investigated and its solutions have been proposed.

- 5) The applications of DIC technique for micro- and nano-scale deformation measurements have been demonstrated. The unique case studies include the near crack-tip deformation measurement on a CT specimen of the SnCu solder alloy, solder joint fatigue deformation measurement using a sectioned BGA assembly, and cyclic strain deformation measurement on a functional specimen of TEC.

9.7 Recommendations for future work

Digital image correlation is an emerging technique for deformation measurement at a wide range of spatial scales by using optical, SEM and AFM images. The displacement resolution of DIC techniques is influenced both by image correlation algorithm and image quality (experimental factors). There may be different experimental factors affecting DIC analysis in different DIC experiments, which is an obstacle to the successful applications of DIC techniques.

Although the comparative study in this thesis shows that the DIC method in the spectral domain has less accuracy than the one in the spatial domain, the former can be improved if the algorithm can incorporate the interpolation scheme for intensity at pixels, and local deformation scheme (displacement function) for the search subimage. Regarding the new element-based DIC method, future work needs to be done on the influence of the element size and the element order on the displacement accuracy. To capture the high gradient strain, to what extent the element size can be reduced in the new method is unknown yet.

The displacement accuracy from DIC analysis is also dependent on the resolution

CHAPTER 9. CONCLUSIONS

of CCD camera used. The CCD camera used in this work has a resolution of 640×480 pixels. If a latest camera with a resolution e.g. 2048×1536 pixels is used for the example studies in this thesis, the displacement and strain results presented in Chapters 5 and 6 could be better. Consider that the whole cross-section of an electronic assembly has usually been investigated when using laser moiré interferometry technique. DIC technique shows the limitation that there is often a trade-off between field of view and spatial resolution. The limitation is directly linked to the imaging system.

The images from different AFMs can be distorted in different ways. The correction method developed for the distorted AFM images in this thesis might not be applicable to any AFM images. The procedure of the development of this correction method can be borrowed for later use. Similar approaches can be developed and implemented to handle the distortion of images from any other AFMs or imaging tools, such as scanning electron microscope. The applications of AFM-DIC to nanoscale deformation measurements are restricted by the difficulty to implement a mechanical or thermal stage to an AFM system without affecting the measurement stability. Nano-deformation characterization of small scale specimen can be further investigated by developing a mechanical or thermal stage integrated to an AFM system. As topographical AFM images of a sample are used, the effects of the roughness of the sample on DIC analysis need to be further investigated.

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Appendix A: Compensation for rigid body motion in crack-tip displacement measurement

The crack-tip displacement fields, $u(x, y)$ and $v(x, y)$, from DIC analysis involve the displacement contributions due to rigid body translation and rotation. Let u_0 and v_0 represent the measured displacements at the crack tip in x and y directions, respectively (see Fig. A.1). When the crack tip is selected as the reference point, its corrected displacements become zero, and the corrected displacements of any one point at (x, y) become $u(x, y) - u_0$ and $v(x, y) - v_0$. When the object rotates by a small rotational angle ϕ_r (Fig. A.2), the rotation-induced displacements u_r and v_r equal:

$$u_r = x' - x = r \cos(\theta + \phi_r) - r \cos \theta = r \cos \theta \cos \phi_r - r \sin \theta \sin \phi_r - r \cos \theta \quad (\text{A.1a})$$

$$v_r = y' - y = r \sin(\theta + \phi_r) - r \sin \theta = r \sin \theta \cos \phi_r + r \cos \theta \sin \phi_r - r \sin \theta \quad (\text{A.1b})$$

As the rotational angle ϕ_r is very small, $\cos \phi_r \approx 1$ and $\sin \phi_r \approx \phi_r$. Hence equation

(A.1) becomes:

$$u_r = -\phi_r r \sin \theta = -\phi_r (y - y_0) \quad (\text{A.2a})$$

$$v_r = \phi_r r \cos \theta = \phi_r (x - x_0) \quad (\text{A.2b})$$

The corrected displacements, which are compensated for both translation-induced and rotation-induced displacements, are:

$$u'(x, y) \approx u(x, y) + \phi_r (y - y_0) - u_0 \quad (\text{A.3a})$$

$$v'(x, y) \approx v(x, y) - \phi_r (x - x_0) - v_0 \quad (\text{A.3b})$$

Based on the theoretical crack-tip displacement fields for Mode I loading and plane strain condition, the corrected displacement fields, $u'(x, y)$ and $v'(x, y)$, can

Fig. A.1 Coordinate systems used in compensation for rigid body motion in crack-tip displacement measurement

Fig. A.2 Schematic of rotation-induced displacements

be expressed as follows:

$$u'(x, y) \approx C_u r^{1/2} \cos\left(\frac{\theta}{2}\right) \left[1 - 2\nu + \sin^2\left(\frac{\theta}{2}\right)\right] \quad (\text{A.4a})$$

$$v'(x, y) \approx C_v r^{1/2} \sin\left(\frac{\theta}{2}\right) \left[2 - 2\nu - \cos^2\left(\frac{\theta}{2}\right)\right] \quad (\text{A.4b})$$

where C_u and C_v are unknown constants, and ν is Poisson's ratio. Thus, the measured crack-tip displacement fields can be modeled as follows:

$$u(x, y) \approx C_u r^{1/2} \cos\left(\frac{\theta}{2}\right) \left[1 - 2\nu + \sin^2\left(\frac{\theta}{2}\right)\right] - \phi_r (y - y_0) + u_0 \quad (\text{A.5a})$$

$$v(x, y) \approx C_v r^{1/2} \sin\left(\frac{\theta}{2}\right) \left[2 - 2\nu - \cos^2\left(\frac{\theta}{2}\right)\right] + \phi_r (x - x_0) + v_0 \quad (\text{A.5b})$$

APPENDIX A

The unknown factors (C_u , C_v , ϕ_r , u_0 and v_0) can be solved by the least-square fitting based on equation (A.5). Once the factors are solved, the corrected displacement fields can be obtained based on equation (A.3).

A MATLAB program was written to perform the least-square fitting and solve the factors for the correction of measured crack-tip displacement fields. This program requires a two-dimensional input matrix, *Data*, which stored the coordinates (in pixels) and displacements (in pixels) of all data points in DIC analysis. Its format is shown below:

<i>Data</i>	<i>x coordinate</i>	<i>y coordinate</i>	<i>u</i> <i>displacement</i>	<i>v</i> <i>displacement</i>
Point 1	360	72	-7.7355	3.1915
Point 2	400	96	-7.6921	3.0813
...	...	...	...	...

Below is the MATLAB program:

```

nu = 0.35      ! Poisson's Ratio

x0 = 246      ! Coordinates of crack tip (in pixels)

y0 = 328

nn=size(Data,1) ! The number of data points

FOR i = 1:nn

    ! Compute the radius of a point on the crack-tip polar coordinates
    r(i) = SQRT((Data(i,1)-x0)^2+(Data(i,2)-y0)^2)

    ! Compute the angle of a point on the crack-tip polar coordinates
    theta(i) = ATAN2((Data(i,2)-y0),(Data(i,1)-x0))/2

    ! Matrix for the first right-hand terms in equation (A.5a)
    fu(i) = SQRT(r(i))*COS(theta(i))*(1-2*nu+SIN(theta(i))*SIN(theta(i)))

```

APPENDIX A

```

! Matrix for the first right-hand terms in equation (A.5b)
fv(i) = SQRT(r(i))*SIN(theta(i))*(2-2*nu-COS(theta(i))*COS(theta(i)))

END

! Matrix for the first right-hand terms in equation (A.5a)
xu(:,1) = fu(:)

! Matrix for the second right-hand terms in equation (A.5a)
xu(:,2) = y0-Data(:,2)

! Matrix for the third right-hand terms in equation (A.5a)
xu(:,3) = 1

! Compute unknown factors in equation (A.5a)
bu = inv(xu'*xu)*xu'*Data(:,3)

! Matrix for the first right-hand terms in equation (A.5b)
xv(:,1) = fv(:)

! Matrix for the second right-hand terms in equation (A.5b)
xv(:,2) = Data(:,1)-x0

! Matrix for the third right-hand terms in equation (A.5b)
xv(:,3) = 1

! Compute unknown factors in equation (A.5b)
bv = inv(xv'*xv)*xv'*Data(:,4)

FOR i = 1:nn

    ! Perform equation (A.3a) to compute corrected displacement  $u'$ 
    ut(i) = Data(i,3)-bu(2)*(y0-Data(i,2))-bu(3)

    ! Perform equation (A.3b) to compute corrected displacement  $v'$ 
    vt(i) = Data(i,4)-bv(2)*(Data(i,1)-x0)-bv(3)

END

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Appendix B: List of publications

- [1] Y. F. Sun, J. H. L. Pang, Experimental and numerical investigation of near crack-tip deformation in a solder alloy, *Acta Materialia* (2008), **56**, 537-548.
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